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Steel Compression Members with Partial-length Reinforcement

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ABSTRACT

Steel wide-flange compression members in existing truss bridges may be built up with flange cover plate reinforcement to increase compressive capacity at reduced cost compared to equivalent full-length cover plates. This thesis investigates the use of partial-length steel cover plates at the column mid-height to improve weak-axis buckling resistance.

This research first presents a critical review of literature concerning Euler buckling of members with partial-length reinforcement. Then inelastic buckling is simulated by a 3-D finite element analysis model that accounts for cover plate length, cover plate area, bolt hole perforations, yield strength of the column material, residual and locked-in dead load stresses, and initial out-of-straightness. The model is validated by a load test of a full-scale column with bolted reinforcement plates and then used to conduct a parametric analysis. It is shown that the capacity of the reinforced member is characterized by the transition between two failure modes: 1) failure initiating in the original column at the unreinforced end segments, and 2) failure initiating at column mid-height in the reinforced segment. Column steel grade and the presence of perforations have a significant effect on the capacity of the reinforced column. A simplified procedure for preliminary design is presented based on an equation developed through multiple linear regression.

**Keywords:** steel; compression; built-up sections; hybrid members; buckling; rehabilitation; design; finite element analysis; column tests; bolt holes
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NOMENCLATURE

$A$ design equation coefficient
$A_g$ gross cross-sectional area
$A_n$ net cross-sectional area
$A_o$ gross cross-sectional area of original column
$A_r$ gross cross-sectional area of reinforcing plates
$A_t$ combined gross cross-sectional area of original column and reinforcing plates
$b$ flange width
$B$ design equation coefficient
$b_p$ reinforcing plate width
$C$ design equation coefficient
$C_e$ Euler buckling capacity
$C_{eo}$ Euler buckling capacity of original column
$C_{e(D)}$ Euler buckling capacity of built-up section (Dinnik, 1932)
$C_{e(T)}$ Euler buckling capacity of built-up section (Timoshenko and Gere, 1961)
$C_o$ compressive resistance of original column
$C_r$ factored compressive resistance
$C_{r,\text{max}}$ maximum factored compressive resistance based on capacity of unreinforced ends of a column
$C_t$ total compressive resistance of hybrid reinforced column
$C_y$ compressive resistance of a column when $\lambda = 0$
$c_b$ labour cost to install reinforcing plates using bolts
$c_o$ overall cost of strengthening with reinforcing plates
$c_s$ material cost of steel reinforcing plates
$D$ design equation coefficient
$d$ depth of W-shape
$d_b$ bolt diameter
$E$ modulus of elasticity (200 000 MPa for structural steel)
$E_{sh}$ Strain-hardening modulus of elasticity
$F_e$ Euler buckling stress
$F_y$ specified yield strength
$F_{yo}$ yield strength of original column steel
$F_{yn}$ yield strength of new reinforcing plate steel
$F_u$ specified ultimate strength
$I_g$ gross second moment of area
$I_n$ net second moment of area
$I_o$ gross second moment of area of original column segments
gross second moment of area of built-up column segment
second moment of inertia at optimum cost of built-up section
effective length factor
column length
reinforcing plate length
length dimension used in column pre-bending
length dimension used in column pre-bending
applied moment
maximum moment sustained by column
design equation coefficient
maximum number of bolt rows (for minimum allowable spacing, \( s_{\text{min}} \))
minimum number of bolt rows (for maximum allowable spacing, \( s_{\text{max}} \))
parameter for compressive resistance (\( n = 1.34 \) for hot-rolled W-shapes)
applied load
failure load predicted by Finite Element Analysis
maximum load sustained by column
failure load from experimental testing
yield capacity of column
reinforcing plate thickness
plate thickness at optimum cost of built-up section
adjusted reinforcing plate thickness for a perforated column
cost scaling factor
radius of curvature
radius of gyration
radius of gyration for weak-axis buckling
elastic section modulus for the direction of weak-axis buckling
specified bolt pitch
maximum allowable bolt pitch
minimum allowable bolt pitch
temperature (°C)
flange thickness
out-of-straightness
web thickness
horizontal offset used to measure sweep and camber
horizontal offset used to measure sweep and camber
reinforced length fraction
reinforced length fraction at optimum cost of built-up section
coefficient of thermal expansion (\( 12 \times 10^{-6} \text{/°C} \) for structural steel)
\( \alpha' \) equivalent reinforced length fraction for column with bolt-hole perforations
\( \beta \) ratio of the second moment of area of the built-up section to the original section \((I_t/I_o)\)
\( \Delta \) deflection
\( \Delta \rho \) theoretical additional plate thickness for a perforated column
\( \Delta T \) thermal gradient \(^\circ\text{C}\)
\( \Delta \delta \) difference between the desired and initial sweep
\( \delta \) measured deflection
\( \delta_c \) camber
\( \delta_k \) skew
\( \delta_{max} \) maximum instantaneous mid-point deflection achieved during pre-bending procedure
\( \delta_s \) sweep
\( \delta_{so} \) initial measured column sweep before pre-bending
\( \delta_y \) deflection at yield
\( \varepsilon \) strain
\( \varepsilon_{max} \) maximum cross-sectional strain
\( \theta \) rotation
\( \lambda \) slenderness parameter
\( \lambda_u \) slenderness parameter for the unreinforced ends of column
\( \rho \) steel density
\( \sigma \) stress
\( \sigma_{LID} \) magnitude of locked-in dead load stress
\( \sigma_r \) maximum magnitude of residual stress
\( \varphi \) resistance factor for structural steel \((\varphi = 1.0\) for experimental work\)
CHAPTER 1: INTRODUCTION

1.1 INTRODUCTION

Increased traffic load demands may cause compression members of existing steel truss bridges to become structurally deficient. In particular, intermediate and slender compression members in bridges designed using the Working Stress Design provisions of the S6-52 Specification for Steel Highway Bridges (CSA, 1952) may be deficient (Shek, 2006) because these provisions are unconservative with respect to current provisions. Slenderness is quantified by the slenderness ratio, $kL/r$, where $k$ is an effective length factor, $L$ is the length of member, and $r$ is the radius of gyration. Compression members designed using S6-52 are particularly likely to be deficient for a critical range of slenderness ratios between 70 and 150 for main compression members or between 70 and 200 for secondary compression members (Shek, 2006).

One solution for increasing the capacity of a W- or I-shaped compression member is to create a hybrid member by adding cover plates on the flanges. As shown in Figure 1-1, partial-length reinforcement can be installed at mid-height over a length $L_r = \alpha L$, where $L$ is the original column length, $L_r$ is the reinforcing plate length, and $0<\alpha<1$. The total area of the reinforced section, $A_r$, is calculated as a sum of the gross area of the original W-shape, $A_o$, and the gross area of the reinforcing plates, $A_r$. Shek and Bartlett (2008) present the analysis of a hybrid member with full-length reinforcing plates, i.e., $\alpha = 1.0$, accounting for the variation in steel yield strengths between the new cover plates and existing member. They also present a simplified procedure for designing full-length reinforcement to attain a desired capacity. However, when the governing failure mode is Euler buckling, or to a lesser extent, inelastic buckling, it may be sufficient to reinforce only the middle region of the member, and so realize material and, particularly, labour cost savings.
1.2 INSTABILITY OF COLUMNS

The Canadian Highway Bridge Design Code (CHBDC) CAN/CSA-S6-14 (CSA 2014) presents the following general equation for the factored compressive resistance, $C_r$, of a simple column:

$$\ [1.1 \] \quad C_r = \phi A_o F_y (1 + \lambda^{2n})^{-1/n}$$

where $\phi$ is the resistance factor, $A_o$ is the cross-sectional area, $F_y$ is the specified minimum yield stress, and $n$ is an empirical constant equal to 1.34 for hot-rolled W-shapes. The dimensionless slenderness parameter, $\lambda$, equals the square root of the ratio of the yield stress to the Euler buckling stress, $F_e$:

$$\ [1.2 \] \quad \lambda = \frac{F_y}{\sqrt{F_e}} = \frac{kL}{r} \sqrt{\frac{F_y}{r^2 E}}$$
where $E$, the elastic modulus of steel, is 200,000 MPa. The compressive resistance becomes governed by cross-section strength as $\lambda$ approaches zero, by Euler (i.e. elastic) buckling as $\lambda$ approaches infinity, and by inelastic buckling for moderate values of $\lambda$, i.e., $0.5 \leq \lambda \leq 2.0$, that typically occur in practice.

Equations [1.1] and [1.2] may be used to calculate separately the resistances of the original section alone, using its radius of gyration, and the cover plates alone, using their radius of gyration, and the sum of these two capacities approximates the capacity of the hybrid reinforced column (Shek and Bartlett 2008). For stocky columns where the capacity is governed by yielding of the cross section (i.e. $\lambda = 0$) $A_r$ may be determined using:

$$[1.3] \quad A_r = A_o \left[ \frac{F_{yo}}{F_{yn}} \right] \left[ \frac{C_t}{C_o} - 1 \right]$$

where $F_{yn}$ and $F_{yo}$ are the yield strength of the new and the original steels, respectively, $C_t$ is the required capacity of the hybrid column, and $C_o$ is the capacity of the original unreinforced column as calculated using Equation [1.1]. Equation [1.3] is valid only if the column is reinforced for its entire length as the capacity is governed by yielding of the new and original material.

1.2.1 Factors Affecting Inelastic Buckling of Intermediate Slenderness Columns

Columns in the intermediate slenderness range, i.e. $50 \leq kL/r \leq 150$, fail by inelastic buckling, an instability that occurs when part of the cross-section has yielded. Shek (2006) identified the following factors that contribute to the capacity of columns that fail by inelastic buckling, and should thus also be considered in the present investigation:

**Yield Strength ($F_y$):** The minimum specified yield strength, $F_y$, of modern G40.21-350 steel is 350 MPa. However, yield strengths at the time when Working Stress Design was prevalent vary, depending on the date of the original construction.

**Residual Stress ($\sigma_r$):** The cooling of W-shape columns during the rolling process creates a distribution of self-equilibrated stresses including compressive stresses at the flange tips and tensile stresses at the flange-web intersection (Galambos 1998). The magnitude of the compressive stress has an effect on column capacity as the extreme fibres of the flange yield prematurely, reducing the rigidity of the section about both principal axes.

**Locked-in Dead Load Stress ($\sigma_{LD}$):** In practice, it may not be possible to remove the entire dead load at the time of connecting the reinforcement, so any dead load stresses present in the original member become “locked-in” when the reinforcement is installed. Additional axial loads cause stresses in both the reinforcement and the original member that add to the locked-in stresses in the original member.

**Out-of-Straightness ($\nu$):** Out-of-straightness is an initial imperfection in a column that initiates buckling instability (Galambos 1998). It is idealized as an initial displacement at column mid-point in the direction of buckling.

### 1.2.2 Feasibility of Partial-Length Reinforcing Plates

Figure 1-2 presents the feasibility region for partial-length reinforcement in terms of the required capacity increase, $C_t/C_o$, and the slenderness ratio of the original member with $F_y$ of 228MPa. The limit on capacity increase, $\varphi C_t/C_o$, is computed for varying $kL/r$, with the factored compressive resistance, $C_o$, computed using Equation [1.1], and $\varphi C_y$, the factored compressive resistance of the column with $\lambda$ of 0, computed as:

$$[1.4] \quad \varphi C_y = \varphi A_o F_{yo}$$
For the region above the curve, the yield strength of the original column limits the column capacity: reinforcing the entire length of the member is necessary to prevent failure of the cross section, so reinforcing plates can be designed using Equations [1.1] and [1.2] following the procedure proposed by Shek and Bartlett (2008). For the region below the curve, where the desired increase in capacity is less than the limit of $\phi C_y / C_o$ for a particular slenderness ratio, it is possible to use partial-length reinforcing plates. This alternative is likely to be particularly cost-effective if the required capacity increase is relatively small, or if the member is very slender. However, the simplified equations and methods currently available to design such reinforcement, Dinnik (1932) and Timoshenko and Gere (1961), address only the case of failure by Euler buckling, and are therefore insufficient to compute the capacity of columns that fail by inelastic buckling. It is therefore necessary to derive a simplified method calculating the compressive capacity of intermediate columns with partial-length reinforcement.

1.3 OBJECTIVES

The objective of the research presented in this thesis is to develop guidelines to determine the capacity of W- or I-shaped steel compression members with partial-
length flange reinforcing plates that fail by inelastic buckling. Particular attention will be paid to the weak-axis capacity as this is often critical.

Specific research objectives are:

1) Conduct a thorough literature review, including a critical analysis of the Euler-buckling-based design solution methodologies.
2) Develop a 3-D Finite Element Analysis (FEA) model that can accurately predict the capacity of a built-up hybrid column that fails by Euler buckling, inelastic buckling, or yielding and accounts for the various factors listed in Section 1.2.1.
3) Conduct full-scale laboratory testing to validate the structural response predicted using the FEA model.
4) Identify the variables that are most critical to the inelastic buckling capacity of built-up hybrid columns for various slenderness ratios, reinforced lengths, and reinforcing plate thicknesses.
5) Develop simplified equations that can be used to design reinforcing plates for weak-axis buckling.

1.4 THESIS ORGANIZATION

Chapter 2 critically reviews existing literature for steel columns with partial-length reinforcement that fail by Euler buckling. Two methods of calculating capacity for the Euler buckling case are compared to the inelastic buckling case to identify general trends, and a preliminary cost-benefit analysis is conducted.

Chapter 3 presents the 3-D finite element analysis (FEA) model developed to analyze built-up hybrid steel columns using ANSYS Mechanical Simulation (ANSYS, 2012). Typical initial conditions for a W-shape column, including residual stresses, initial out-of-straightness, and locked-in dead load stresses, are readily idealized using thermal loads and load-stepping controls to simulate a realistic response. This chapter also discusses the design simplifications used to model the
locations of bolt-holes in the flanges of the W-shape. Descriptions of the finite element mesh optimization and an overview of the sub-steps included in the column loading process are presented. The model of an unreinforced column is validated by comparing the output axial capacities with the column compressive resistances calculated according to CAN CSA-S6-14 for a range of slenderness ratios.

Chapter 4 summarizes a full-scale load test conducted to validate the FEA for a built-up hybrid steel column with partial-length reinforcement. All tests were undertaken at the University of Western Ontario Structures Laboratory in 2014/2015. The test specimen, apparatus, and procedure are described, including an overview of the auxiliary tests that were conducted to verify material properties and to achieve the desired sweep prior to load-testing. The experimental results and observations are presented, followed by a comparison of the observed and FEA-predicted capacities and, finally, conclusions regarding the accuracy of the FEA analysis.

Chapter 5 presents parametric sensitivity analysis for both a plain unperforated column and a column with flanges perforated by rows of bolt-holes. The basis for the sensitivity analysis is discussed with respect to realistic ranges of reinforcing plate dimensions, potential bolt-hole configurations, and scalability with respect to cross-section size. The parametric study investigates the capacity increases achieved in unperforated and perforated columns for various reinforcing plate length and area ratios across a range of slenderness ratios. Additional sensitivity analysis parameters, including steel grade, residual stresses, locked-in dead load stress, and initial out-ofstraightness are also investigated to determine their impact on column capacity.

Chapter 6 presents a simplified design method that can be used to quickly assess the weak-axis capacity of built-up hybrid columns with partial-length reinforcement. A design equation is used to determine the length and thickness of reinforcing plate required to satisfy a given increase in compressive capacity for an unperforated
column. If the column has perforated flanges, the reinforcing plate thickness is increased as a function of the net section area removed for the bolt-holes and the slenderness of the original column.

Chapter 7 presents a summary of and conclusions from this research project. Recommendations for future work are provided.
CHAPTER 2: CRITICAL REVIEW OF EULER BUCKLING CASE

2.1 INTRODUCTION

This chapter presents a critical review of the literature about compression members with partial-length reinforcement that fail by linear-elastic (i.e. Euler) buckling. Both linear-elastic and inelastic buckling represent bifurcation, or extreme deflection, when a column reaches a critical load. Linear-elastic buckling does not involve yielding and so is a function the modulus of elasticity, while inelastic buckling implies at least some yielding of the cross-section, and so is influenced by residual stresses, and initial out-of-straightness. Shek and Bartlett (2008) present the design of full-length reinforcing plates that fail by inelastic or linear-elastic buckling. Dinnik (1932) and Timoshenko and Gere (1961) present methodologies for the analysis of Euler buckling capacity, $C_e$, in compression members with variable stiffness along their length. Although none of these methods capture the inelastic-buckling failure for a member reinforced with partial-length flange reinforcing plates, they bound the design domain considered in the present investigation. Critical review of the Euler buckling case provides a basis for quantifying the behavior of slender columns that approach Euler buckling failure loads because their slenderness corresponds to the upper limit of the inelastic buckling range.

The objective of this chapter is to review the existing methodologies that apply Euler buckling theory to columns of variable cross section in the context of developing a new methodology to design partial-length reinforcement.

Section 2.2 presents the case of failure by Euler buckling for columns with partial-length reinforcement and compares these results to the available solutions for full-length reinforcement. Section 2.3 presents a preliminary design method based on cost optimization for the case of failure by Euler buckling.
2.2 EULER BUCKLING WITH PARTIAL-LENGTH REINFORCEMENT

For a hybrid member with partial-length reinforcement located symmetrically about its mid-height, the Euler buckling capacity of the built-up section, $C_{e(T)}$, may be approximated as (Timoshenko and Gere 1961):

$$[2.1] \quad C_{e(T)} = C_{eo} \times \beta \times \frac{1}{\alpha + (1-\alpha)\beta - \frac{(\beta-1)}{\pi} \sin(\pi\alpha)}$$

where $C_{eo} = \pi^2 E I_o / L^2$ is the Euler buckling capacity determined using the second moment of area of the original unreinforced column segments, $I_o$, and $\beta$ is the ratio of the total second moment of area of the built-up section to that of the original section, i.e., $I_t / I_o$. The normalized reinforcement plate length, $\alpha$, is simply the ratio $L_r / L$ as shown in Figure [1-1]. This approximate solution is derived using strain energy to determine the deflected shape, and is relatively conservative for the case of a single set of reinforcing plates of uniform thickness.

An exact Euler-buckling solution for partial-length reinforcing plates located at mid-height of the column is presented by Dinnik (1932). This solution is derived by considering the compatibility of the deflected shapes for the reinforced and unreinforced segments. Timoshenko and Gere (1961) obtain by substitution a transcendental equation that provides simpler calculation of the column capacity as previously proposed by Dinnik. Further rearranging to present the solution in non-dimensional terms, the relative capacity of the reinforced column, $C_{e(D)}/C_{eo}$, is calculated by trial-and-error using the transcendental equation:

$$[2.2] \quad \tan \left[ \sqrt{\frac{\pi^2 C_{e(D)}}{C_{eo}}} \left[ \frac{1-\alpha}{2} \right] \right] \tan \left[ \sqrt{ \frac{\pi^2 C_{e(D)}}{\beta C_{eo}} \left[ \alpha \right] } \right] = \sqrt{\beta}$$

This equation may alternatively be used to determine the required length of reinforcing plate, $\alpha L$, to achieve a desired value of $C_{e(D)}/C_{eo}$ for a particular reinforcing scheme with $\beta = L_r / L_o$. 
The weak-axis compressive capacities for the three methods are compared for an example case, shown in Figure 2-1, of a W310x158 with length \( L \) of 8000\( mm \) and reinforcing plate cross section of 310\( mm \times 10mm \) (i.e. \( A_r = 6200 \text{mm}^2 \)). For weak-axis buckling, the \( y \)-axis radius of gyration, \( r_y \), is used to calculate the slenderness ratio, \( kL/r_y \) of 101, and the second moments of area for the original and built-up section is also calculated in the \( y \)-axis for \( \beta \) of 1.4. Using the method proposed by Shek and Bartlett (2008), the original and hybrid section capacities for full-length reinforcement (\( \alpha = 1.0 \)) are calculated using Equations [1.1] to [1.3] for inelastic buckling, with \( F_{yn} \) of 350\( MPa \) and for \( F_{yo} \) of either 350\( MPa \) or 228\( MPa \) to account for the different yield strengths of the original unreinforced member. The increased capacities relative to Euler buckling capacity, \( C_{eo} \), are calculated for a range of \( \alpha \) using the methods proposed by Timoshenko and Gere (1961) and Dinnik (1932), i.e., Equations [2.1] and [2.2], respectively. Figure 2-1 a) presents the variation of the weak-axis compressive capacities of the reinforced column with the reinforced length ratio. The Euler-buckling-based methods proposed by Timoshenko and Gere and Dinnik predict higher compressive capacities than the inelastic buckling solutions for full-length reinforcement given by Shek and Bartlett (2008), because the initial inelastic-buckling capacity is less than the initial Euler-buckling capacity at this slenderness ratio. For the full range of reinforcement plate lengths shown, the method by Timoshenko and Gere (1961) is more conservative for \( 0.2 < \alpha < 0.8 \). Figure 2-1 b) presents the variation of the normalized increase in compressive capacity with the normalized reinforcement plate length. As \( \alpha \) approaches 1 both Euler-based methodologies approach the same result of \( C_{e(T)}/C_{eo} = C_{e(D)}/C_{eo} = \beta \), and also appear to approach the result obtained using the Shek and Bartlett (2008) method. The Euler-buckling-based solutions at \( \alpha \) of 1 appear slightly conservative for the inelastic case where \( F_{yo} \) of 228\( MPa \) (i.e. \( F_{yo} < F_{yn} \)) and slightly unconservative for \( F_{yo} \) of 350\( MPa \) (i.e. \( F_{yo} = F_{yn} \)). Ideally, a solution for columns with partial-length reinforcement that fail by inelastic buckling should be found that approaches the solution for full-length reinforcement.
Figure 2-1: Comparison of the existing methodologies for reinforcing plate design for a W310x158 shape with $kL/r_y$ of 101, $\beta$ of 1.4, and $F_{yn}$ of 350MPa in terms of:

a) Compressive capacity of the reinforced column

b) Increase in compressive capacity relative to the capacity of the original section
Figure 2-2 compares the Euler-based exact solution presented by Dinnik (1932) with the approximate solution of Timoshenko (1961) by showing the necessary values of $\alpha$ on each axis to obtain a specific $C_e/C_{eo}$ for various values of $\beta$. To achieve the same increase in capacity for a given increased stiffness of the reinforced region, the method of Timoshenko (1961) requires a reinforcing plate length that is 22% to 86% longer than that required using the method of Dinnik (1932). For example, at $\beta = 1.7$ and $C_e/C_{eo} = 1.5$, the Dinnik (1932) and Timoshenko (1961) methods require a reinforcing plate length ratio, $\alpha$, of 0.5 and 0.8, respectively.

Wang and Wang (2004) provide exact solutions of Euler Buckling Loads of compression members. The solution based on Dinnik (1932) is shown to calculate buckling loads exactly, and the solution proposed by Timoshenko and Gere (1961) is shown to be an approximation obtained by incorporating shear deformation theory. Wang and Wang do not directly compare the two approaches, or quantify the error associated with the use of Timoshenko approximation.

Figure 2-3 shows schematically the increase in weak-axis compressive capacity calculated using Dinnik’s (1932) method with respect to the reinforced length for varying reinforcement thicknesses. The Euler buckling capacity of the column with
partial-length reinforcement will lie in the shaded area, as bounded by the curve on the left, where the reinforced segment is infinitely stiff, and the lower curve, where the stiffness of the reinforced segment approaches that of the original column. The increase in compressive capacity is limited by yield of the original cross-section at the unreinforced ends of the member, depicted by the horizontal line at \( C_{e(D)}/C_{eo} = C_y/C_o \). This upper limit increases for slender columns and decreases for stocky columns because the inelastic-buckling capacity of the original column, \( C_o \), is calculated using Equations [1.1] and [1.2] as a function of slenderness, while the cross-section strength, \( C_y \), remains constant as calculated using Equation [1.4]. The heavy-dotted line represents a typical relationship based on Euler buckling failure for a given ratio of stiffness of the reinforced region relative to that of the original column, \( \beta \). As seen in Figure 2-1 b), as \( \alpha \) approaches 1, this line approaches the corresponding inelastic buckling case calculated for full-length reinforcement using the method proposed by Shek and Bartlett (2008).

![Figure 2-3](image-url)

**Figure 2-3:** General schematic of the Euler-buckling-based solutions for designing partial-length reinforcing plates
Figure 2-3 shows the limiting case where $\beta$, the ratio of the second moment of area of the reinforced section to that of the original section, becomes infinitely large. In this instance, the capacity is simply the Euler buckling capacity of the unreinforced column segment:

$$C_e = \frac{\pi^2 E I_o}{((1-\alpha)L)^2}$$

By rearranging Equation [2.3] to isolate $\alpha$ and setting $C_e = C_{eo} = \pi^2 E I_o/L^2$, the ratio of reinforcing plate length to length of the original member will reach a theoretical minimum critical length ratio, $\alpha_{crit}$, for a given value of $C_e/C_{eo}$, given by:

$$\alpha_{crit} = 1 - \sqrt{\frac{C_{eo}}{C_e}}$$

Figure 2-4 illustrates the effect of a column reinforced with a segment of infinite stiffness over the critical length. The Euler buckling capacity of the partially reinforced column is simply the Euler buckling capacity of an equivalent column formed from the two end segments with the stiffness of the original column. If $\alpha < \alpha_{crit}$ for a given value of $C_e/C_{eo}$, the capacity of the reinforced column will be insufficient irrespective of $I/I_o$. 
Figure 2-4: Critical length ratio for a column with reinforced segment of infinite stiffness

Figure 2-5 shows the exact solution for increased capacity, $C_{e(D)}/C_{eo}$, for the weak-axis Euler buckling of a W310x158 column with a length of 8 m ($kL/r_y = 101$) and various degrees of increased stiffness in the reinforced region, across a range of reinforced length ratios. The solution is given for 310 mm wide reinforcing plates with thicknesses of 10 mm, 20 mm, 40 mm, 80 mm and 225 mm, and infinity. The yield strengths of the both the original section and reinforcing plates are assumed to be 350 MPa, for calculating the increase in capacity of the full-length reinforced section according to Shek and Bartlett (2008). For the case of $p = 40$ mm, shown by the open square markers, it is clear that the compressive capacity increase due to full-length reinforcing plates must exceed that due to partial-length reinforcing plates, regardless of thickness, for $\alpha > 0.7$, where the capacity of the reinforced member is limited by the yield strength of the original section. As $\alpha$ approaches 1, the Euler-buckling-based solutions for $\beta = 1.8$ and $\beta = 1.4$ slightly overestimate the capacity increase because, as computed using the method of Shek and Bartlett (2008), the capacity is limited by inelastic-buckling.
Figure 2-5: Increased compressive resistance for built-section reinforced with varying lengths and thickness of reinforcing plate.

2.3 DESIGN BASED ON MINIMUM COST

The overall cost of compression member strengthening, $c_o$, using steel flange plate reinforcement may be expressed as the sum of material, $c_s$, and labour, $c_b$, costs:

$$ c_o = Q(c_s) + \frac{1}{Q}(c_b) = Q\left[ A_r \alpha L \frac{\$ 3000}{\text{tonne}} \right] + 4 \frac{\alpha L}{s} \frac{\$ 100}{\text{bolt}} $$

where $\rho$ is the density of the steel, 7.850 $t/m^3$, $A_r$ is the area of reinforcing plates, and $Q$ is a generic cost scaling factor to facilitate sensitivity analysis. Material cost is the cost of steel plate, assumed to be $3000 per tonne. Labour cost is calculated as a function of the number of bolts installed, assuming that each bolt costs $100 based on the labour required to drill holes in the existing member and install the bolts. The number of bolts is calculated assuming two rows of bolts along each flange and so depends on the length of reinforcing plate and the bolt pitch, $s$. To satisfy CHBDC (CSA 2014) requirements for sealing, Clause 10.18.4.5, the maximum pitch, depends on the thickness of the smaller component:

$$ s = (100 + 4[\text{MIN}(t,p)]) \leq 180 \text{mm} $$
where \( t \) and \( p \) are the thicknesses of the flange and reinforcing plate, respectively. The requirements of Clause 10.18.4.6 for maximum stitch bolt spacing have been ignored herein in the interest of simplicity.

Figure 2-6 demonstrates the variation of cost with reinforcing plate length for different material and labour cost assumptions. The figure is derived based on increasing the weak-axis capacity of a column by 50%, i.e. \( C_{e(D)}/C_{eo} = 1.5 \), for a W310x158 section with \( L \) of 8m \((kL/r_y = 101)\). It can be shown that the enhancement of the strong axis capacity by the reinforcing plates causes the weak-axis capacity to govern. The standard material and labour costs are simulated using \( Q = 1.0 \). The cases of maximum material costs and maximum labour costs are simulated using \( Q = 2.0 \) and 0.5, respectively, in Equation [2.5]. The relationships converge to a vertical asymptote at \( \alpha_{crit} = 0.18 \) (i.e., the critical minimum value obtained from Equation [2.4]). For shorter reinforced lengths, the desired \( C_{e(D)}/C_{eo} \) cannot be achieved with even infinitely large reinforcement plate areas. Each cost relationship reaches an optimum value at a different reinforcement length, \( \alpha_{opt} \), and corresponds to a different plate thickness, \( p_{opt} \), as presented in Table 2-1. The thickness of the plate required to maintain a constant value of \( C_{e(D)}/C_{eo} \) increases as the length of reinforcing plate decreases, potentially resulting in impractical values as \( \alpha \) approaches \( \alpha_{opt} \), and plate area required to produce the desired \( C_{e(D)}/C_{eo} \) approaches infinity. Therefore, the figure also indicates the relative costs of rehabilitation assuming a practical maximum reinforcement plate thickness of 25\( mm \), which is approximately equal to the flange thickness of a W310x158 section. For this case, \( \alpha = 0.39 \) is necessary to achieve \( C_{e(D)} = 1.5C_{eo} \).

Table 2-1 also summarizes parameters and percentage deviations corresponding to the optima for the three cost relationships shown in Figure 2-6. There is little deviation in the value of \( \alpha_{opt} \), corresponding to the minimum cost, compared to the corresponding deviation of \( I_{opt}/I_o \), where \( I_{opt} \) is the second moment of inertia of the built-up section corresponding to \( p_{opt} \). This indicates that the optimum
reinforcement length is relatively insensitive to the assumed cost parameters: in all cases the number of bolts reduces as both the plate length decreases and, due to increased reinforcing plate thickness, the bolt spacing increases. The bolt spacing reaches the upper limit of 180\,mm in Equation [2.6] at plate thickness of 20\,mm and, typically, the optimum cost corresponds to a slightly greater thickness. For the case of the high labour cost, the optimum cost is therefore the most sharply defined.

Figure 2-6: Comparison of optimized costs to achieve $C_{e(D)}/C_{eo}$ of 1.5 for weak-axis buckling with various material and labour assumptions for a W310x158 section with length, $L$ of 8\,m ($kL/r_y$ of 101)

Table 2-1: Parameters corresponding to the optima under various cost assumptions for W310x158 section with $C_{e(D)}/C_{eo}$ of 1.5

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Deviations from $Q = 1.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{opt}$ (mm)</td>
<td>$\alpha_{opt}$</td>
</tr>
<tr>
<td>$Q = 1.0$</td>
<td>80</td>
</tr>
<tr>
<td>$Q = 2.0$</td>
<td>45</td>
</tr>
<tr>
<td>$Q = 0.5$</td>
<td>175</td>
</tr>
</tbody>
</table>
Figure 2-7 presents a comparison of the optimized costs for a W310x158 section with length $L$ of 8 m ($kL/r_y = 101$) for three different requirements of $C_{e(D)}/C_{eo}$. The cost variation is in all cases characterized by a sharp optimum as the plate thickness increases from that corresponding to the minimum cost, i.e., $\alpha = \alpha_{opt}$, and a soft optimum as the plate thickness decreases, i.e., $\alpha > \alpha_{opt}$. For $C_{e(D)}/C_{eo} = 1.5$, $\beta$ ranges from 25 to 4.8 for $\alpha$ ranging from $\alpha_{crit} = 0.18$ to $\alpha_{opt} = 0.24$, and from 4.2 to 1.6 for $\alpha$ ranging from 0.24 to 0.59. The cost to install the steel reinforcing plates is more sensitive to the length of the reinforcement than the thickness of the plates, particularly when the cost of labour increases. As previously noted for Figure 2-6, the practical limits of 25 mm for reinforcing plate thickness are marked as they correspond to $\alpha$ greater than $\alpha_{opt}$.

![Figure 2-7: Comparison of optimized cost to achieve varying required ratios of $C_{e(D)}/C_{eo}$ for weak-axis buckling of a W310x158 with length, $L$ of 8 m ($kL/r_y$ of 101)](image)

Table 2-2 summarizes the costs and cost savings of partial-length reinforcement as a function of the slenderness of the hybrid section with $F_{yo}$ of 228 MPa and $F_{yn}$ of 350 MPa, corresponding to $C_{e(D)}/C_{eo} = 1.5$. Equations [2.2] and [2.4] are independent of the slenderness ratio, therefore the calculated value of $\alpha$ will remain constant for a selected plate thickness at a given ratio of $C_{e(D)}/C_{eo}$, regardless of the variation in slenderness. The cost of full-length reinforcement is higher overall for
the high slenderness case, $kL/r_y = 130$, and, compared to the case of $kL/r_y = 80$, increases proportionally with the column length. Columns with lower slenderness ratios may be governed by the capacity of the cross-section, and require reinforcing plates over the entire length, as is the case with $kL/r_y = 70$ where the yield strength of the original cross section only permits partial reinforcement up to $C_e/C_{eo} = 1.33$. Thus the potential for cost savings is greatest for sections with a high slenderness ratio.

Table 2-2: Parameters and costs corresponding to the practical limits on reinforcing plate thickness for a W310x158 section with $F_{yo}$ of 228MPa and $F_{yn}$ of 350MPa

<table>
<thead>
<tr>
<th>$kL/r_y$</th>
<th>$C_{(D)}/(C_{eo} = 1.5$</th>
<th>$C_e/C_o = 1.5$</th>
<th>Potential Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha_{opt} = 0.24$</td>
<td>$\alpha = 0.39$</td>
<td>$\alpha = 1.0$</td>
</tr>
<tr>
<td></td>
<td>($p = 80mm$)</td>
<td>($p = 25mm$)</td>
<td>$p$ (mm)</td>
</tr>
<tr>
<td>70</td>
<td>N/A</td>
<td>N/A</td>
<td>17</td>
</tr>
<tr>
<td>101</td>
<td>10 900</td>
<td>14 900</td>
<td>12</td>
</tr>
<tr>
<td>130</td>
<td>13 000</td>
<td>19 000</td>
<td>9</td>
</tr>
</tbody>
</table>

2.4 SUMMARY AND CONCLUSIONS

This chapter has presented a critical review of two existing Euler-buckling-based methodologies of designing partial-length reinforcement for steel columns. The capacity increase of a reinforced column that fails by Euler buckling was examined in the context of limitations on capacity increases in the inelastic case for varying lengths and thicknesses of reinforcing plate. A preliminary cost-optimization analysis was conducted based on the Euler buckling capacities and a range of assumed material and labour costs.

The conclusions of the critical review of Euler-buckling-based solutions are:

1. Existing Euler-buckling-based analysis methods are inadequate and unsafe for sizing partial-length reinforcing plates if the reinforced member fails by inelastic buckling.
2. Three significant limits on compressive capacity have been identified: 1) Compressive capacity of a full-length reinforced section that fails by inelastic buckling, 2) Yield strength of the original cross section, and 3) Euler buckling capacity of the unreinforced column segments in the case where the reinforced segment has infinite stiffness.

3. The potential for cost savings is greatest for slender columns. The cost optimum for partial-length reinforcing plates is typically approached by minimizing the reinforcing plate length to minimize the cost of labour required for installation.
CHAPTER 3: FINITE ELEMENT ANALYSIS USING ANSYS SIMULATION

3.1 INTRODUCTION

This chapter presents the development and validation of a Finite Element Analysis (FEA) procedure used to model the non-linear buckling response of a W-section column reinforced with steel plates using bolted connections. A large sample of physical column tests would be required to determine a design procedure for the optimum sizing of reinforcement plates. This strategy would therefore be costly and time-consuming. Additionally, physical testing is constrained by capacity and column height restrictions of the actuator and by practical difficulties to control the material strength, residual stresses and column out-of-straightness, and fastener properties for the testing program. In contrast, FEA can be used to simulate rapidly a large number of tests and so investigate a large range of parameters. To ensure accuracy, the FEA results can be validated by a small number of physical tests.

Section 3.2 discusses the modeling capabilities of the Finite Element Analysis Software adopted, ANSYS Mechanical Simulation (ANSYS 2012), and describes preliminary trials conducted to validate its use for simple linear-elastic buckling. Section 3.3 summarizes the initial conditions for the non-linear analysis, including residual stresses, out-of-straightness, and modelling of the reinforcing plate fasteners. Section 3.4 describes the meshing parameters, and Section 3.5 presents the loading process used to simulate non-linear column buckling and validates the assumed initial conditions by comparison with the CSA S6-14 column curve. Samples of the plain text ANSYS input files are presented in Appendix A2.

3.2 ANSYS SIMULATION OF COLUMN BUCKLING

The finite element model was developed using ANSYS Mechanical 14.5 software, a commercial structural analysis tool capable of robust linear and non-linear analyses of 3-D meshes (ANSYS, 2012). A similar investigation, concerning the
capacity of corroded steel bridge compression members (Krisciunas, 2011) using Solidworks Simulation software, was referenced to define the necessary capabilities of the software as follows:

1. Applies temperature gradients to simulate residual stress and out-of-straightness.
2. Performs non-linear FEA with large displacements by gradually increasing the loading until instability occurs.
3. Simulates the effect of bolt holes on column capacity.

An additional advantage of ANSYS is its ability to recreate a model produced using the graphical Computer Aided Design (CAD) interface by writing scripts in ANSYS Parametric Design Language (APDL) format. These explicit text-based input files allow automation of the time-dependent loading scheme used in the non-linear analysis procedure, and changes in dimensional and material property parameters can be made more efficiently by adjusting the text inputs than through the CAD interface.

A trial model was constructed using SOLID187 elements and the eigenvalue buckling process as outlined in online tutorials (University of Alberta, 2002). As shown in Figure 3-1 a), the column is assumed to be symmetrical about mid-height, so only half of its length is modelled (i.e., a half-length column with an effective length factor, $k$, of 2.0 is assumed equivalent to a full-length column with pinned end connections). The top of the column is free to move in the x-direction to allow weak-axis buckling. The base of this half column is restrained from movement in the y-direction, but permitted conditional movement in the x- and z-directions. Figure 3-1 b) shows the bottom edge of a surface through the centre of the web that is restrained from movement in the z-direction for the entire length of the column to prevent out-of-plane motion corresponding to strong axis bending. Figure 3-1 b) also shows that the base is restrained at a single point at the flange-web interface to prevent the column from sliding but the flanges above the base are permitted to deform freely in the x- and z-directions to allow local buckling in
the horizontal plane. To spread the applied load to the full cross-section and prevent local distortion of the flanges and webs, the top of the column has a high-stiffness end cap as shown in Figure 3-1 c). Because the model is constrained to prevent strong-axis buckling, it is sufficient given symmetry considerations to model only one flange and half of the web as shown in Figure 3-1 b), to achieve a 50% reduction in computation time.

Preliminary ANSYS simulations were conducted to validate linear-elastic buckling results for a) a bare steel W-section column, and b) the same W-section column reinforced with mid-height steel flange plates located symmetrically about the column mid-height. The theoretical capacities for these cases were calculated as described in Chapter 2, using classic Euler buckling theory for the bare column, and the exact solution for the section with variable cross-section, formulated by Dinnik (1932). For a bare W100x19 column of Grade 350W steel with $I_y$ of $1.61 \times 10^6 \text{mm}^4$ about the weak axis and an effective length, $L$, of 2300 mm (i.e. $kL/r_y = 94$), the theoretical buckling capacity is 601kN. The capacity of predicted by ANSYS was 596.5kN, or within 0.8% of the theoretical buckling capacity. For the same column reinforced at the base with 12.7 mm x 103 mm steel plates (i.e. $I_t = 3.92 \times 10^6 \text{mm}^4$) for a reinforced length, $aL$, of 1100 mm, the theoretical buckling capacity computed using Equation [2.2] is 1110kN. The capacity predicted by ANSYS was 1099kN, or within 1% of the theoretical value. This indicates that the capacities determined using ANSYS for the idealized column are accurate for the linear-elastic buckling case.
Figure 3-1: Illustration of FEA model geometry for a) fixed connection at column midpoint, b) cross-section reduction due to symmetry, and, c) end cap stiffness
3.3 IDEALIZATION OF INITIAL CONDITIONS

The nominal column resistance calculated using Equation [2.1] defines the inelastic column buckling capacity as a function of geometric and material properties, including initial imperfections. Essentially Equation [2.1] is an empirical formula based on simulated column tests, that accounted for the magnitude and distribution of residual stresses created during the hot rolling process, and for the initial out-of-straightness (Galambos 1998). While ANSYS does have the capability to apply initial stresses and deflections, it is impractical to apply them accurately to the individual elements of a complex 3-D mesh, and so they were simulated using a procedure developed by Krisciunas (2011). As described in this section, realistic simulation of residual stresses and out-of-straightness can be obtained using thermal gradients.

3.3.1 Steel Stress-Strain Relationship

The linear-elastic behaviour of steel is characterized by stress-strain behaviour by which the material stress, $\sigma$, is accompanied by a corresponding material strain, $\varepsilon$, according to a linear relationship, $E = \sigma/\varepsilon$, up to the yield stress as specified by the steel grade. Post-yield, the steel undergoes plastic behaviour and the slope of the stress-strain relationship decreases to the strain-hardening modulus of elasticity, $E_{sh}$. Figure 3-2 shows the stress-strain relationships adopted for the 350MPa and 228MPa steel grades investigated in the present study. The 350MPa steel is assumed to exhibit a linear strain-hardening response between yield stress and the specified ultimate strength, $F_u$, of 450MPa (CSA 2014) at $\varepsilon = 0.03$, where the maximum strain was selected based on coupon tests described in section 4.2.1. The 228MPa steel is assumed to have a linear-elastic perfectly plastic relationship as shown.
3.3.2 Residual Stresses

Residual stresses are induced by uneven cooling across the cross section at the end of the hot-rolling procedure. Their presence reduces the buckling capacity for columns in the intermediate slenderness range, especially when combined with initial out-of-straightness (Galambos 1998). Flange tips develop compressive stresses because they cool first and prevent the regions at the flange-web intersections from contracting as they cool. Previous research has indicated that linear residual stress distributions with maximum magnitudes of $\pm 0.3 F_y$ are typical, where $F_y$ is the steel yield strength (Galambos 1998). Figure 3-3 shows the typical linear distribution of residual stresses across the width of the flange and depth of the web for a W-shape.

Residual stresses were created in the idealized column for ANSYS analysis using thermal convection loading and a coupled structural-thermal analysis. Figure 3-4 a) shows the thermal convection temperatures applied to the vertical edges of the flange tips and to the flange-web intersections. By trial and error, it was determined that the desired residual stress distributions would be achieved if the flange tips and centre of the web are heated to $+45.5 \, ^\circ C$ and $45 \, ^\circ C$, respectively, and the web-
Flange intersections are cooled to -44 °C. Figure 3-4 b) presents the resulting temperature gradient that develops across the cross section. The residual stresses are proportional to this temperature gradient, \( \sigma_r = T \alpha_s E \), where \( T \) is the temperature (°C), and \( \alpha_s \) is the coefficient of thermal expansion (12x10^{-6}/°C for structural steel).

**Figure 3-3:** Residual stress distribution due to thermal loading on flange and web

**Figure 3-4:** Illustrates a) use of thermal convection applied on vertical surfaces and b) resultant temperature gradient.
3.3.3 Out-of-straightness

Out-of-straightness of the column further reduces the inelastic buckling capacity (Galambos 1998). Initial out-of-straightness of up to $L/1000$ is permitted for a hot-rolled steel W-shape with a flange width greater than 150 mm, or $L/500$ with a flange width less than 150 mm (CISC 2010). Initial out-of-straightness may be simulated using a linear thermal gradient to produce a deformed shape that follows a circular arc. The necessary magnitude of the thermal gradient is (Krisciunas, 2011):

\[
\Delta T = \frac{\varepsilon_{\text{max}}}{\alpha_s} = \frac{(b/2)}{R\alpha_s},
\]

where $\Delta T$ is the required thermal gradient (°C) applied as a positive temperature on the convex-side flange and negative temperature on the concave-side flange, $\varepsilon_{\text{max}}$ is the maximum cross-sectional strain in the direction of weak-axis buckling, $\alpha_s$ is the coefficient of thermal expansion for structural steel ($12 \times 10^{-6}/°C$), and $b$ is the flange width (mm). The radius of curvature desired for the out-of-straightness, $R$, (mm), is:

\[
R = \frac{L}{2} \times \left[ \frac{1}{v} + \frac{v}{2} \right]
\]

where $v$ is the non-dimensional initial out-of-straightness (i.e. 0.001 mm/mm for $\Delta/L = 1 mm/1000 mm$). If the temperature gradient between the extreme fibres is linear, the out-of-straightness will be created without inducing any additional internal stresses. For the reinforced member, out-of-straightness was induced in both the original column and the reinforcing plates to simplify the meshing process. This is conservative, because any out-of-straightness of the reinforcing plates will likely be small compared to that of the original column.
3.3.4 Reinforcing Plate Attachment

Although the reinforcing plates may be bolted or welded to the original column, the focus of this investigation, as stated in Chapter 2, is on bolted connections. Such connections are inherently slip-critical and so rely on bolt pre-tensioning to generate sufficient friction between the flange and reinforcing plate to prevent slip. Small relative displacements, of the order of 1-2 mm as typically permitted for bolt-hole oversizing (CSA 2014), may occur when the shear applied to the elements exceeds these frictional forces and the bolts slip into bearing. Such slip is most likely to occur in the bolts at the extreme ends of the reinforcing plate where the compression forces in the original member are transferred. While ANSYS is capable of modelling pre-tensioned bolts, this was deemed to be beyond the scope of the present work. Investigation of the feasibility of bolted connections is limited to assessing the effect of the area removed by the bolt holes. Thus, in the present study, the original column and reinforcing plates are modelled as separate components joined perfectly across the surfaces in contact, to simulate a friction connection in which slip does not occur.

The effect of reducing the cross-section area due to regularly spaced bolt-holes will be investigated. The compression capacity of reinforced columns that include bolt-holes (i.e. perforated columns) will be compared with results from unperforated reinforced columns. Bolt-holes for the W100x19 section analyzed are assumed to be 18 mm diameter, as described in detail in Section 4.2. The minimum bolt pitch, $s_{\text{min}}$, requirements given in Clause 10.18.4.5.4 of the CHBDC (CSA 2014), are adopted: $s_{\text{min}}$ is equal to 3 bolt diameters, $d_b$. The end distance from the last bolt to the plate end is taken as 28 mm according to CHBDC Clause 10.18.4.9, and the edge distance is taken as 22 mm according to CHBDC Clause 10.18.4.8. The maximum number of bolt rows, $N_{\text{max}}$, in the model half-column is determined by trial-and-error such that:
\[ s_{\text{min}} = \frac{\left( \frac{aL}{2} \right) - 28}{\left( N_{\text{max}} - 0.5 \right)} > 3d_b \]

Similarly, the minimum number of bolt rows, \( N_{\text{min}} \), can be determined from the maximum allowable pitch, \( s_{\text{max}} \), for the sealing and stitch requirements for bolts (Clauses 10.18.4.5 and 10.18.4.6), which require that the bolt spacing cannot exceed 12 times the flange thickness, \( t \), or plate thickness, \( p \). Thus:

\[ s_{\text{max}} = \frac{\left( \frac{aL}{2} \right) - 28}{\left( N_{\text{min}} - 0.5 \right)} < 12[\text{MIN}(t,p)] \]

### 3.3.5 Locked-in Dead Load Stresses

Locked-in Dead Load Stresses for a compression member currently in service are modelled in a column that is subjected to axial dead load stresses before the reinforcement is installed. Their effect can be investigated as an initial condition for the column, applied using multiple load steps. The axial compressive load to be locked into the original column is applied to the column in its original state, after the residual stress and initial out-of-straightness have been created, resulting in development of internal stresses and deflections. The reinforcement plates are then attached and additional axial load is applied until failure occurs.

Compression members installed in existing bridges may also exhibit imperfections due to environmental effects, including corrosion and vehicular collisions, but an assessment of these effects is beyond the scope of the present investigation.

### 3.4 FINITE ELEMENT MESH

Typically SOLID187 10-node tetrahedron elements, shown in Figure 3-5, are used in all 3-D modelling (ANSYS 2012). A single element consists of 4 corner nodes and 6 mid-side nodes that deform quadratically under loading to capture non-linear displacements. These tetrahedron elements were developed to be used in non-
linear analyses, with stress stiffening automatically included when large deflection effects are simulated. The SOLID187 structural element is also compatible with the SOLID87 thermal element, used to apply the thermal convection loading that induces the initial residual stresses and out-of-straightness.

**Figure 3-5:** SOLID187, a higher order 3-D, 10-node element (ANSYS, 2012)

The model performance was evaluated for a range of mesh sizes to optimize solution accuracy and computation time. Figure 3-6 shows the variation of the compressive capacity with the number of elements for a sample column with the following geometric properties: W100x19 shape with $L$ of 2805mm ($kL/r_y = 110$); reinforcing plate length of 0.5$L$ with a thickness of 12mm; and twelve rows of bolt-holes through each flange and plate. The number of elements was controlled by adjusting the minimum and maximum element dimensions permitted for the modelled solid volumes. Figure 3-6 indicates that the computed compressive capacity approaches a lower limiting value of 421.5kN as the mesh size increases, with error up to 0.5% conservative due to incremental load stepping, described in Section 3.5. Using a fine and homogenous mesh with 140 000 elements yields a compressive capacity that is 1.5% lower than that obtained using a relatively coarse and gradated mesh with 11 000 elements. The 140 000 element mesh required a computation time of over three hours, which is excessively long given the scope of the parametric study proposed in Chapter 5. Using a fine mesh with interior elements six times larger than those on the edges produced 66 000 elements with a computation time of forty minutes, and yields a capacity that is
only 0.1% greater than that obtained using the fine mesh. A mesh size corresponding to approximately 40 000 elements was eventually adopted, which reduced the run time to approximately 20 minutes and consistently yielded capacities within 0.2% of the benchmark set by the fine mesh.

**Figure 3-6:** Variation of accuracy with the number of elements

Figure 3-7 shows the adopted mesh, for which parameters were selected to produce a minimum element size of $t/3$ at the edges of volumes and a maximum element size of $2t$ at the centres of volumes. Use of these mesh control parameters allowed a finite element model to be generated consisting of approximately 41 000 elements. As the number of elements changes depending on column and reinforcing plate lengths, and other factors, the time required to run each simulation ranged from between fifteen minutes to one hour.
3.5 LOADING PROCESS

Non-linear buckling in ANSYS requires the loading to be applied in user-defined load steps, with each step decreasing in magnitude to facilitate convergence on the critical load at which instability occurs. Several of these incremental sub-steps are saved as load step sets that can be reviewed to verify that convergence has been adequately completed. Figure 3-8 shows schematically the three-step approach used to create the initial state of the column and then apply axial loading until failure. The steps are as follows:

i. The original column is meshed at time \(-1\) sec. Thermal loadings necessary to obtain the desired residual stress state are applied to the section between
- 1 and 0 seconds using the predefined thermal physics environment and solved to generate the thermal gradients. The model reads in the physics environment using structural elements and the resulting stresses are saved to an “initial state” file, i.e. using the INISTATE command, to simulate residual stresses. The stresses and deformed shape are cleared.

ii. The reinforcing plates are meshed at time zero sec. The residual stresses from the initial state file are read back into the mesh without associated deformations, and column sweep required to cause weak-axis buckling is induced with an additional thermal gradient between zero and one second.

iii. An axial force is applied to the stiff end cap at the top of the column, and distributes evenly across the cross-section. Loading is applied between 1 and 5 seconds in incremental load steps rapidly up to 50% of the input compressive load. After 5 seconds, the load steps are reduced to 0.5% of the input value. Loading is increased until divergent behavior occurs when the deflected column cannot continue to resist the increased levels of compressive force being applied. This may occur from 50% to 100% of the input compressive force, depending on the accuracy of the initial estimate for the failure loading (i.e. input compressive force). Capacity readings are taken from the second last load step with a typical error of 0.5-1%.

![Figure 3-8: Time-dependent application of loading](image-url)
The consideration of locked-in deal load stresses requires adjustments to the loading process. In this case, Step (iii) above begins with temporary deactivation of the reinforcing plate elements, i.e., using the EKILL command, for the duration of the initial period of rapid loading. A stepped load is applied up to the magnitude of the locked-in stress, typically 30% of the unreinforced column capacity, during which the original column is permitted deflection following the initial out-of-straightness. The reinforcing plate mesh is then reactivated, i.e., using the EALIVE command, with zeroed stresses and an initial out-of-straightness on par with the instantaneous weak-axis deformation of the original column. Subsequent load steps proceed at 0.5% of the input value.

3.5.1 Assumptions for Model Validation

The finite element model is validated based on the following assumptions:

1. The base of the column model is fixed against rotation while the top is free to deflect in the direction of weak axis buckling (i.e., height = L/2, k = 2.0)
2. Compressive loading is distributed evenly across the cross-section at the top of the column
3. Modulus of elasticity for steel is 200 000 MPa
4. The yield strength, $F_{yo}$, is 350 MPa and the stress-strain relationship is as shown in Figure 3-2
5. Residual stresses vary linearly with a maximum magnitude of $\pm 0.3F_y$ as shown in Figure 3-3
6. Columns have an initial out-of-straightness imperfection characterized by a mid-height sweep of $L/1000$

3.5.2 Validation of bare column with CAN/CSA S6-14 Criteria

The FEA model was validated by comparison of the predicted capacities with those computed using the column resistance equation specified in the CHBDC, i.e., Equation [1.1]. The column resistance equation was derived by Loov (1996) to replace equations proposed by Bjorhovde (1972) based on numerical simulation
of columns with varying out-of-straightness and residual stress distributions. Krisciunas (2011) and Shek (2006) also demonstrated the accuracy of their numerical solutions by comparison with Equation [1.1].

Figure 3-9 compares the nominal column compressive resistance calculated for a W100x19 section using the empirical relationship given in CAN/CSA S6-14 (CSA 2014) with the nominal axial capacity predicted using ANSYS FEA. The FEA results were found to agree closely with the code provisions, using a resistance factor $\varphi$ of 1.0. Results are almost identical for columns with a slenderness ratio of 110. The FEA-predicted capacity becomes up to 4.7% conservative as the slenderness ratio increases to 180, and becomes up to 2.5% unconservative as the slenderness ratio reduces to 20. These differences are acceptable given that the code equations are empirically derived.

Figure 3-9: ANSYS FEA-predicted capacities compared with CSA S6-14 (CSA 2014) for a W100x19 column
3.6 SUMMARY AND CONCLUSIONS

This chapter has described the idealizations and modelling necessary to use ANSYS Mechanical FEA to predict the weak-axis compressive capacity of a steel W-shape column reinforced with partial-length flange cover plates. Realistic initial conditions were simulated using a combination of temperature gradients and incremental load steps to induce appropriate residual stress distributions, out-of-straightness, and locked-in dead load stresses. The reinforcing plate attachment was idealized as a simple friction connection between the original column flange and reinforcing plate. A method of modelling bolt-hole geometry was presented to allow examination of perforations on the stiffness of the reinforced region.

The conclusions of the chapter are as follows:

1. The FEA model accurately predicts the Euler buckling capacity of a reinforced column when compared to results generated by the Euler-buckling-based methodology proposed by Dinnik (1932).
2. A 3-D non-linear FEA model that accounts for a linear residual stress pattern with a magnitude up to $0.3F_{yo}$ and an initial out-of-straightness of $L/1000$ can accurately reproduce the empirically-based column capacity curve given in CAN/CSA S6-14 for an unreinforced column.
CHAPTER 4: EXPERIMENTAL VALIDATION OF FEA MODEL

4.1 INTRODUCTION

This chapter summarizes a full-scale load test of a steel column with partial-length flange reinforcing plates. The purpose of the experiment is to validate the column capacities and failure modes for weak-axis buckling as predicted by the ANSYS FEA model presented in Chapter 3 for a reinforced column with properties that are representative of those investigated in the parametric study.

Section 4.2 describes the initial state of the test specimen, including additional tests to determine yield strength and residual stress magnitudes, and the procedures used to induce an initial out-of-straightness and pre-tension the bolted connections. Sections 4.3 and 4.4 describe the test apparatus and test procedure, respectively. Section 4.5 presents a summary of test results, and Section 4.6 compares these results to an ANSYS simulation with input parameters based on the initial state of the test specimen. Appendix B1 presents the descriptions and ANSYS FEA-predicted capacities for 3 additional columns that have been prepared for future testing.

4.2 SPECIMEN DESCRIPTION

A W100x19 shape was selected for the full-scale test because it is the lightest W-shape commercially available. Height restrictions on the available load frame in the UWO Structures Lab necessitate a lighter column section to bring the slenderness ratio \((kL/r_y)\) for weak-axis instability into the range representative of intermediate columns that fail by inelastic buckling. The maximum column length is 2300\(mm\), or 2400\(mm\) between pin bearings, which permits testing a W100x19 at a slenderness of \(kL/r_y = 94\) for weak-axis buckling. Additionally, the MTS 243.70 actuator in the University of Western Ontario Structures Laboratory has a maximum listed capacity of 1500\(kN\), but it was deemed prudent that the load should not exceed 1000\(kN\) for column instability testing, as described further in
Section 4.3.2. The W100x19 has a cross-sectional capacity of 868 kN for steel with a yield strength ($F_y$) of 350 MPa, which places an upper limit on the capacity reached using partial-length reinforcement regardless of the plate thickness. This ensures that the loading will remain within the actuator capabilities even when a column is reinforced with oversized plates that cause the column capacity to approach the capacity of the cross-section.

The specific objective of the experiment was to observe a case in which inelastic buckling occurs due to yielding in the unreinforced segment of original column at the ends of the reinforcing plate. This failure mode would be most easily obtained using thick, short reinforcing plates. It would be challenging, however, to achieve sufficient stiffness of the reinforced region to quantify the upper limit of the capacity increase with respect to plate thickness while keeping the test specimen at a practical weight and size for handling and bolt installation. It was therefore decided to observe an intermediate case where the reinforced segment will exhibit deformations, but inelastic buckling may still occur in an unreinforced segment. For comparison, the use of a thinner and longer reinforcing plate is predicted to fail by inelastic buckling due to partial yielding of both the reinforcing plates and original column near the column mid-height likely initiated by local yielding near a bolt-hole.

Figure 4-1 presents the specimen dimensions. The reinforcing plate dimensions were constrained by the plate thicknesses that were readily available: 0.25", 0.5", and 0.75"; or 6.35 mm, 12.7 mm, and 19.05 mm, respectively. The 12.7 mm plate was selected to approximate $A_t = 2A_o$, and a reinforced length of 1100 mm, or 0.46$\alpha$, was selected so that the specimen would be representative of the median plate dimensions of the parametric study. The bolt-hole diameters were constrained by the small flange width: space is needed to install the bolts while providing the minimum edge distance at the flange tip. ASTM A325 bolts with a diameter of 5/8" (16 mm) were selected, so the associated hole diameter is 11/16", or 18 mm. The bolt spacing was 80 mm, to satisfy CHBDC spacing requirements as
defined using Equations [3.3] and [3.4]. Appendix B3 presents the shop drawings for fabrication of Column 1 and the additional untested columns.

Figure 4-1: Test specimen geometry, reinforcement plate dimensions, and bolt installation

4.2.1 Material Properties and Residual Stresses

Table 4-1 summarizes the steel yield strengths, ultimate strengths, and residual stress magnitudes obtained from the tensile coupon tests and stub column test. Tensile coupons were taken from the W-shape web and flange, and from the 12.7 mm reinforcing plate. The coupons were loaded to failure at a constant load rate of approx. 175 MPa/minute, or strain rate of approx. 15 με/sec, in accordance with ASTM 370 (ASTM 2013). Although coupons were tested for both the flange and web of the W-shape, only the flange strengths were used in calculations as
the flanges must yield first during weak-axis buckling, and therefore govern the axial capacity.

Table 4-1: Material data from tensile coupon and stub column tests

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Tensile Coupons</th>
<th>Stub Column</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( F_{y,\text{flange}} ) (MPa)</td>
<td>( F_{u,\text{flange}} ) (MPa)</td>
</tr>
<tr>
<td>Column 1</td>
<td>353</td>
<td>487</td>
</tr>
</tbody>
</table>

Stub-column tests were conducted on short members to quantify the maximum magnitude of the residual stresses in the W-section. The stub column tests were performed according to the procedure recommended by Tall (1961), for which the residual stress is the difference between the initial deviation from linearity and section yield in the column stress-strain response. Non-linearity occurs at the initial yielding of the flange tips, with the assumption that an equal compressive stress occurs at the centre of the web, and equal tensile stresses occur at the junction of flange and web (Galambos 1998). Further details of the tensile coupon and stub column tests are presented in Appendix B2.

4.2.2 Initial Geometry

Table 4-2 presents average geometrical properties of the test specimen. Cross-section dimensions were measured at the top, mid-height, and bottom of the column using a Vernier caliper with an accuracy of one-tenth of a millimetre. These measurements confirmed that skew, \( \delta_k \), and camber, \( \delta_c \), were negligible and verified the gross areas of steel used as input parameters in the ANSYS simulation discussed in Section 4.6.

Table 4-2: Initial geometry

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Measurements (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( d )</td>
</tr>
<tr>
<td>Column 1</td>
<td>108.1</td>
</tr>
</tbody>
</table>
Camber and sweep were measured as shown in Figure 4-2 by using a theodolite to sight down the length of the column, as previously described in Krisciunas (2011). The theodolite is positioned in line with points (a) and (d), and used to sight the offsets at the column midpoint (b) and at the far corner point (c) with respect to the near corner point (a). It is assumed that the maximum out-of-straightness, $\delta$, would occur at the column mid-point, calculated as:

\[ \delta = x_m - \frac{x_e}{2} \]

where $x_m$ and $x_e$ are the horizontal offset from Line (a-d) to Points (b) and (c), respectively, as shown in Figure 4-2. Accuracy of the measured cambers and sweeps was considered to be equal to the width of the theodolite cross-hair at $\pm 0.5 \, \text{mm}$. The column sweep was initially measured as approximately $0.5 \, \text{mm}$, which would indicate an out-of-straightness approaching $L/5000$. This was deemed to be too straight to accurately predict the direction of weak-axis buckling or to delineate the progression of the instability, and so the column was loaded transversely to induce a permanent out-of-straightness of greater than $L/1000$.

Figure 4-2: Determination of initial camber and sweep

Figure 4-3 shows the 4-point loading system used to induce an inelastic deformation to achieve additional column out-of-straightness. The maximum mid-point deflection of the column to be achieved during the pre-bending procedure, $\delta_{\text{max}}$, was determined by:
where $\delta_y$ is the mid-point column deflection at yield, and $\Delta \delta$ is the difference between the desired sweep, $\delta_s$, and the measured initial sweep, $\delta_{so}$. The mid-point column deflection at yield corresponds to the load that induces yielding of the extreme fibres of the flanges:

$$P_y = \frac{2M_{max}}{(l_1 - l_2)/2} = \frac{4S_yF_y}{(l_1 - l_2)}$$

where $S_y$ is the elastic section modulus about the $y$-axis (i.e. for weak-axis buckling), and $l_1$ and $l_2$ are as indicated on Figure 4-3 a). Appendix B2 presents the complete calculation of $\delta_y$. The deflection at yield was verified during the bending procedure by noting the deflection at which the observed load-deflection curve began to depart from linearity. Loading was applied on the web of the beam to preclude local damage to the flanges. The steel blocks used to transfer load to the web were sized to ensure that the punching shear resistance of the column web at the load points would not be exceeded.

### 4.2.3 Installation of Bolted Connections

As shown in Figure 4-4 a), the reinforcing plate was first attached loosely to the column at two corners and the four middle bolt-holes. The middle bolts were then snug-tightened and pre-tensioned using the turn-of-nut method in accordance with Clause 10.24.6.6 of CHBDC (CSA 2014). Bolt installation and pre-tensioning was performed beginning at the column mid-point and extending equally along the reinforcing plate length in both directions.
**Figure 4-3:** Induced pre-bending of bare column using 4-point loading system

a) Key dimensions

b) Apparatus with spreader bar used to distribute load equally to two points

**Figure 4-4:** Installation of bolted connections
4.3 TEST APPARATUS

4.3.1 Column End Supports

Figure 4-5 a) shows the components of the column end support assembly. Custom column ends, shown in Figure 4-5 b), were re-fitted from a previous apparatus (Krisciunas, 2011). Additional FEA was not required to check for deformation and internal stress concentrations within the end plates, as the loads applied in the current study did not exceed those encountered previously (880 kN). Drawings of the original column end supports and modifications are presented in Appendix B3.

4.3.2 Load Frame and Actuator

Figure 4-6 shows the load frame and actuator as installed in the UWO Structures Lab. Sway-bracing designed by Krisciunas (2011) makes the frame deflections negligible. Loading eccentricity during the test may initiate an out-of-plane deflection at the horizontal cross-beam. An initial failure load was predicted using ANSYS FEA. It assumed a bilinear stress-strain relationship as shown in Figure 3-2 for $F_{yo}$ of 350 MPa and modelled both the original column and the reinforcing plates without bolt-holes. The predicted capacity, 617 kN, is therefore conservative and so places the proposed column test well within load frame limitations.

4.3.3 Instrumentation

Figure 4-7 shows the column specimen between the pin plates with displacement transducers and strain gauges attached. The column was coated with a hydrated lime whitewash to improve the visual detection of mill scale flaking at locations of local yielding, and so the onset of buckling. Instrumentation was installed according to guidelines suggested by Tall and Tebedge (1971) to monitor lateral displacement, twist and overall shortening of the column.
**Figure 4-5:** Column end support assembly

a) Apparatus Schematic (after Krisciunas 2011)

b) Pin Plate Detail
Figure 4-6: Overview of load frame and actuator

Figure 4-7: Column resting between pin plates
Figures 4-8 shows the locations of the nine displacement transducers, each with an accuracy of approx. ±0.1 mm. Details of each transducer are as follows:

- **Transducer 0**: Located at the actuator head. Aligned vertically on the top face of the top plate to measure vertical displacement of the specimen.
- **Transducer 6**: Aligned horizontally at the centre of the web at mid-height to monitor weak-axis lateral bending deflections.
- **Transducers 5&7**: Aligned horizontally at the centre of the web in line with the bolts at the top and bottom ends of the reinforcing plate, respectively, to monitor the curvature of the reinforced region relative to the unreinforced regions of the column.
- **Transducers 1&8**: Aligned horizontally and centered on the web near the faces of the pin plates (i.e. 35 mm), to monitor lateral deflections of the bearings.
- **Transducer 2**: Aligned horizontally at the vertical face of top plate, monitoring lateral movement of actuator head to capture any out-of-plane displacement of the load frame.
- **Transducers 3&4**: Aligned horizontally at column flange at mid-height (i.e. spaced 80 mm apart) to monitor strong axis lateral bending and torsion deformations.
Strain gauges were also installed in locations predicted to develop high stresses and associated deformation due to local yielding. Figure 4-9 shows the stresses and amplified out-of-plane displacements along a short length of the column flange at the end of the reinforcing plate, as predicted by FEA. Locations with the greatest magnitude of displacement were identified in the unreinforced column segment at $15\,\text{mm}$ from the edge of the reinforcing plate, and in the reinforced segment at $70\,\text{mm}$ from the edge of the reinforcing plate, or between the first two bolt-holes.
Both locations also coincide with the onset of local yielding with stresses exceeding 353 MPa.

Figure 4-9: Predicted locations of maximum flange tip stresses at column failure \( (P = 567 kN) \)

Figure 4-10 displays the locations of sixteen 5 mm foil strain gauges attached to the original column on the interior surface of the flange, the side adjacent to the web, and oriented parallel to the height of the column. Gauges 1, 2, 7, 8, 9, 10, 15, and 16 were placed on the flange tips of the original column about 15 mm above the interface of the flange plates, while Gauges 3, 4, 5, 6, 11, 12, 13, and 14 were placed on the flange tips centred between the first two bolts at the edge of the reinforcing plate. An additional four gauges were attached at 30 mm offsets from the top and bottom of the column, respectively, for a total of 24 strain gauges installed. Gauges 17-24 were used to confirm that the loading was applied concentrically.
4.4 TEST PROCEDURE

The procedure adopted is based on Krisciunas (2011) and is in accordance with Tall and Tebedge (1971), as follows:

1. Strain gauges were attached to the column flanges. The remainder of the column was then coated with whitewash.
2. Top and bottom end plates were attached with their free rotational axes parallel to the load frame, and aligned using a plumb bob to a horizontal tolerance of ±2 mm.
3. Pin plates were clamped to the test column ends using the set-screws.
4. The column was lifted by crane to position the lower pin plate on the bottom end plate, and the actuator head lowered to secure the top pin plate.

5. A minimal load (approx. 1kN) was applied to hold the column in place while the displacement transducers were installed.

6. Loading was applied at two rates in the following sequence suggested by Tall and Tebedge (1971)
   a. 28 MPa/minute up to 5% of the anticipated failure load, at which point all displacement and strain measuring devices were zeroed to minimize error due to initial mis-alignment at bearing seats.
   b. 28 MPa/minute while the column response is still linear elastic and before bolt slip is anticipated, up to 20% of the anticipated failure load
   c. 7 MPa/minute until maximum response of the column was reached
   d. 0 MPa/minute while the maximum static load was being determined
   e. 7 MPa/minute until full buckling of the column

Actual loading rates were applied in mm/min, as shown in Table 4-3. The actuator was operated in stroke-controlled mode to prevent the occurrence of a dangerous sudden instability failure.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Stroke controlled loading rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_{FEA,initial} (MPa)</td>
<td>&lt; 5%P_{FEA,initial} (mm/min)</td>
</tr>
<tr>
<td>Column 1</td>
<td>617</td>
</tr>
</tbody>
</table>
4.5 TEST RESULTS

The peak compressive capacity recorded in the full-scale test, \( P_r \), was 572\( kN \). Other test observations and results will be first discussed in general terms, and then compared with the FEA predicted response in Section 4.6.

4.5.1 Experimental Observations

The induced sweep was sufficient to predict the buckling direction correctly. Figure 4-11 shows the axial load versus weak-axis mid-height deflection for the duration of the column load test. The column displayed a ductile failure, with mid-height deflections increasing steadily over time while the applied load remained constant. At approximately 0.4mm and axial load of 200\( kN \), there is a slight increase in the rate of deflection with respect to load, from 0.002mm/kN to 0.005mm/kN, which maintains a linear relationship between axial load and mid-height deflection until rapid increases in the rate of deflection beyond 450\( kN \). For safety, the test was ended before local buckling occurred and the observed unloading response was near-linear.

![Figure 4-11: Comparison of weak axis deformations predicted by FEA to the Column 1 test data](image-url)

\[ P_{T,max} @ \delta = 9.2mm \]
\[ P_{FEA,max} @ \delta = 9.3mm \]
Weak axis sway steadily increased throughout the early part of the test, stabilizing at a magnitude of approximately 1 \( \text{mm} \) beyond an axial load of 200\( kN \), as shown in Figure 4-12. This is not a trivial amount considering that weak axis mid-height deformation at this loading was only 1.4\( \text{mm} \), however the average sway did not increase significantly as the applied load was increased. At maximum load the deflections measured by displacement transducers 3 and 4 differed by 0.4\( \text{mm} \), indicating that slight twisting was present at the time of failure.

![Figure 4-12: Out of plane deformations from Column 1 test data](image)

Figure 4-13 presents the deflected shape of the test column at the maximum applied load. Throughout the test, the displacement transducers indicated that there was a distinctly smaller curvature in the reinforced region compared to the unreinforced region. Additionally, the weak-axis deflections at the top and bottom of the flange plate were not symmetrical, with slightly larger deflections observed in the lower half of the column.
Figures 4-14 a) and b) show whitewash flaking adjacent to the reinforced section of the column on the inner and outer surfaces of the flange, respectively. Horizontal and diagonal Lueder lines on the compression flanges were not strongly apparent until after the maximum load had been reached. As shown in Figure 4-14 a), one of the Lueder lines was intercepted by Strain Gauge 1, at 15mm above the edge of the reinforcing plate. Flaked whitewash was also observed radiating at diagonals from the top bolt-holes on the compression side of the flanges, but only on the interior face of the flange. Whitewash flaking was not observed on any area of the reinforcing plate surfaces.

**Figure 4-13:** Web displacements in direction of weak-axis buckling over the column height
4.5.2 Strain Gauge Readings

The locations of the strain gauges are as shown previously in Figure 4-10. Figure 4-15 a) and b) present the strain variation with axial load for the portions of the back column flange subject to compression and tension forces, respectively. Strain Gauge 1, located on the front flange above the reinforcing plate, indicates a rapid increase in compressive strain immediately after the failure load was reached. The heightened compressive strain is consistent with the previous observation that Gauge 1 was intercepted by a Lueder line, and provides evidence of yield of the
original section flange directly adjacent to the ends of the reinforcing plate. The direction of global buckling is indicated in Figure 4-15 b) by the incremental strain reversal that initiates at a load of approximately $450\,kN$, resulting in tensile strain at Gauge 11 after the peak axial load has been reached. The strain reversal corresponds to the load at which the rate of mid-height deflection begins to increase as shown in Figure 4-11. Erratic strain readings were observed for axial loads up to $200\,kN$, at which point each gauge recorded a sudden increase in compressive strain. Additionally, Gauge 9 recorded compressive strain magnitudes greater than any other on the back tension flange. While it is expected that gauges on all the flanges should record similar compressive strains up until significant mid-height deflections occur, it is improbable that gauges on the extreme tension fibre should have consistently higher compressive strains than the extreme compression fibre. Malfunction of the data logger is suspected as a possible source of error in recording the strain magnitudes.

Figure 4-16 a) and b) present the strain variation with axial load for the front flange of the column. The observed strains were generally consistent with those recorded at the corresponding locations on the back flange, with the exception that Gauge 2 did not indicate the same large compressive strains recorded by Gauge 1. Comparing Figures 4-15 and 4-16, the strains on opposing flanges are similar for given applied loads, indicating that loading is reasonably concentric until failure.

### 4.5.3 Evidence of Bolt Slip

Displacement transducers and strain gauge readings both show subtle changes in behavior at an axial load of approximately $200\,kN$. Figures 4-15 b) and 4-16 b) record fluctuations in strain gauge readings around axial loads of $100\,kN$ and $200\,kN$, which may result from reinforcing plates coming into bearing with bolts. Any bolt slip has a negligible effect on global deformation over time.
Figure 4-15: Strain gauge data for back flange of Column 1

a) Compression flange
b) Tension flange
Figure 4-16: Strain gauge data for front flange of Column 1

a) Compression flange

b) Tension flange

4.6 COMPARISON TO RESPONSE PREDICTED USING ANSYS

The axial capacity predicted using the ANSYS FEA model presented in Chapter 3, accounting for the geometric and material properties of the specimen as described in Section 4.2, is $567\text{kN}$. As shown in Table 4-4, the axial capacity predicted by FEA underestimates the capacity observed in the full-scale test by less than 1%, indicating that the FEA accurately predicted the column failure load. The close agreement between observed and predicted axial capacity indicates that the assumption of a perfectly bonded reinforcement plate with initial curvature
matching that of the original column is an acceptable simplification of the actual connection.

### Table 4-4: Comparison of compressive capacities

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$P_{FEA}$ (kN)</th>
<th>$P_T$ (kN)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column 1</td>
<td>567</td>
<td>572</td>
<td>0.88</td>
</tr>
</tbody>
</table>

Figure 4-11 compares axial loading versus mid-height deflection observed for the full-scale column test with that predicted using the FEA simulation. The observed deflection is typically less than that predicted by FEA until the two curves converge at almost identical magnitudes of mid-height deflection at their respective maximum load.

Figure 4-13 indicates that the curvature of the column recorded at the last stable load step in the FEA simulation closely matches the deflection profile observed when the maximum test load was reached. Curvature of the test column was slightly less than the predicted by the FEA at the two points above the column mid-height. This may be due to asymmetry of the initial out-of-straightness, which becomes more exaggerated as column buckling progresses. In contrast, the FEA-predicted curvature is perfectly symmetrical about the mid-height section due to constraints applied to the model as described in Chapter 3.2. Overall, the ANSYS FEA model predicts the column deflection with excellent accuracy.

Figure 4-14 c) shows the stress distribution predicted by the FEA at the maximum applied load. The left image shows the distribution on the inside of the flange, i.e., the web side, and the right image shows the distribution on the outer flange face. There is good correlation between the extent of the whitewash flaking shown in Figures 4-14 a) and b), and the areas of the predicted stress distribution where the steel is above the yield stress of 353 MPa, shown in orange in Figure 4-14 c). In particular, the location of the maximum stress was predicted with such accuracy that Strain Gauge 1 was successfully positioned to capture the stress-strain
response at failure and during unloading, as shown in Figure 4-14 a). Additionally, the diagonals of flaked whitewash around the top bolt in Figure 4-14 a) match the extent of local yielding predicted around the matching bolt-hole in the web side view of the FEA stress distribution shown in Figure 4-14 c).

Figure 4-17 shows the FEA-predicted strains at the compression flange tip as the axial load increases. The relationships between strain and axial load for the two distinct strain gauge locations are compared to the experimental observations shown in Figures 4-15 a) and 4-16 a). For the gauges located between the end bolt holes (i.e., strain gauges 3, 4, 5, and 6), the predicted strain at peak axial load is 900x10^{-6}. This is a magnitude 58% less than the peak strains observed in the experimental strain gauge data at gauges 5, previously shown in Figure 4-15 a). In both the FEA-predicted and experimental observations, the compressive strain increases at a near constant rate as axial load is increased. For the gauges located in the unreinforced region at the edge of the reinforcing plate (i.e., strain gauges 1, 2, 7, and 8), the peak strain of 2000x10^{-6} is 30-40% greater in magnitude than the peak strains observed at gauge 2, shown in Figure 4-16 a). Additionally, the rate of strain at this location is predicted to increase beginning at 80% of the peak axial load. A similar relationship was observed at strain gauges 2 and 4, located on the front flange, but was absent at strain gauges 1 and 3, located on the back flange, for which strain increased near-linearly as axial load increased. Overall, the FEA-generated strains are sufficient for predicting strains in the unreinforced column segment, but do not adequately predict the strains between the bolt holes. Further investigation into the effect of bolting the reinforcement is recommended for future study.

Figure 4-18 shows the FEA-predicted strains at the tension flange tip as the axial load increases. The predicted strains for the location between the end bolt holes (i.e., gauges 11, 12, 13, and 14) and for the location in the original column at the reinforcing plate edge (i.e., gauges 9, 10, 15, and 16) are overall lesser in magnitude than the experimental observations, shown on Figures 4-15 b) and 4-
16 b). The relationships are similar for predicted and observed strains up to 200\(kN\). For the applied loads between 200\(kN\) and failure the observed strains from the test specimen become erratic and indicate higher magnitudes than were generated by FEA at all strain gauge locations. It is uncertain whether the recorded strains are a result of bolt slip interactions in the column or instrumentation error, thus additional tests may be required to investigate the effects of bolted connections.

**Figure 4-17**: FEA-generated strain gauge data for compression flange; elastic and plastic strains in the y-direction

**Figure 4-18**: FEA-generated strain gauge data for tension flange; elastic strains in the y-direction
4.7 SUMMARY AND CONCLUSIONS

This chapter has presented the full-scale load test conducted to validate the FEA-predicted strengths of a steel column specimen selected as representative of those to be investigated in the planned sensitivity study. All tests were undertaken at the University of Western Ontario Structures Laboratory in 2014/2015. The test specimen, apparatus, and procedure are described, including an overview of the auxiliary tests that were conducted to verify material properties and to adjust initial geometry prior to load-testing. The experimental results and observations were presented, followed by a comparison of the test results with the FEA analysis and conclusions regarding the suitability of the model.

The conclusions drawn from the research presented in this chapter are as follows:

1. The predicted failure load using ANSYS was less than 1% lesser than that observed in the test.
2. The deflected shape at failure and stress distributions predicted by the ANSYS model were in agreement with those observed during the test.
3. The observed failure mode involved local yielding of the original column above the reinforced segment and was accurately predicted by the FEA model.
4. Additional stress concentrations were observed radiating at diagonals from the end bolt-holes on the compression flanges which are again consistent with the FEA predictions.
5. It is therefore concluded that the ANSYS model accuracy is validated by
CHAPTER 5: SENSITIVITY ANALYSIS

5.1 INTRODUCTION

The objective of this chapter is to determine the sensitivity of the compressive strength of a steel W-shape column with partial-length flange reinforcing plates to various geometric and loading parameters. This sensitivity analysis will be conducted using the ANSYS FEA model presented in Chapter 3 as validated by the experimental results presented in Chapter 4. All data are presented as dimensionless ratios of the compressive strength of the reinforced column, $C_r$, to that of the original unreinforced column, $C_o$. Compressive strength is defined as the maximum load that the column can sustain, limited either by large transverse deflections that trigger member instability, or by local buckling of the flange. This chapter will present the development of the sensitivity analysis and the effects of varying selected parameters.

Section 5.2 presents the range of various geometric properties investigated for the sensitivity analysis. These include the reinforcing plate dimensions, bolted reinforcement connections, and W-shape selected. Section 5.3 summarizes the sensitivity of the compressive capacity to variations of the reinforcing plate length, reinforcing plate area, steel grade, bolt-hole perforations, residual stresses, out-of-straightness, and locked-in dead load stresses.

5.2 BASIS FOR SENSITIVITY ANALYSIS

Previous studies of columns with full-length flange reinforcing plates (Shek, 2006) indicate that failure occurs either by instability initiated at the column mid-height, or by cross-section yielding at the unreinforced ends of the member. Similarly, columns with partial-length reinforcement can fail by two instability modes: 1) Failure initiating at mid-height of the reinforced column; or, 2) Failure initiating in the original column at its interface with the reinforcing plates. The first mode is more likely when thin reinforcing plates are used that approach full-length
reinforcement. Further investigation is required to quantify the strength of columns with thicker and shorter lengths of reinforcement that fail by the second instability mode.

5.2.1 Reinforcing Plate Dimensions

The reinforcing plate length, $L_r$, is again expressed as a dimensionless fraction of the original column length, $L$, that is: $L_r = \alpha L$, where $0<\alpha<1$. Reinforcing plate length fractions of $\alpha = 0.2$, 0.5, and 0.8 were chosen to represent the case of half-length reinforcement and the extreme cases previously investigated, using Equation [2.5].

The increase in cross-sectional area at the reinforced segment is expressed as a dimensionless area ratio, $A_t/A_o$. The total area of the reinforced section, $A_t$, shown in Figure [1-1], is computed as:

\[ A_t = A_r + A_o = ((2 \times p \times b_p) + A_o) \]

where $A_o$ is the gross area of the original unreinforced column, $A_r$ is the gross area of the reinforcing plates alone, and $p$ and $b_p$ are the thickness and width, respectively, of a single reinforcing plate. It was demonstrated previously by Shek (2006) that varying the width of reinforcing plate has a minimal effect as long as it is at least 2/3 the width of the W-shape flange. To reduce the number of simulations, it is assumed that the width of the reinforcing plate is equal to the width of the flange, $b$.

Reinforcing plate thicknesses were selected as a function of $A_t/A_o$ to be representative of a practical range of cases. The maximum area ratio of $A_t/A_o = 3$ corresponds to a case where each flange reinforcing plate has an area equal to that of the original unreinforced column. For a W100x19 shape, this corresponds to $p = 24mm$ for $b_p = b = 103mm$. This reinforcing plate thickness is approximately equal to $2.7t$, where $t$ is the flange thickness of the original column. The minimum area ratio selected is $A_t/A_o = 1.25$, which corresponds to $p = 3mm = 0.34t$ for the same W100x19 shape. Additional plate area ratios were chosen to provide $A_t/A_o =$
1.5, 1.75, 2, and 2.5, reflecting earlier observations from the linear-elastic buckling analyses that there are diminishing returns on the increase in capacity as the mid-height stiffness increases.

Figure 5-1 shows typical von Mises stress distributions through the cross-section of the original column at the onset of weak-axis buckling superimposed on the original residual stress distribution in the unloaded member. The initial parameters correspond to the W100x19 column described in Chapter 3: $kL/r_y = 94$; $F_y = 353$ MPa; and, $\alpha = 0.5$. The distance along the flange, indicated by the horizontal axis of each graph, is measured relative to the extreme compression fibre at the left side of the graph. As shown in Figure 5-1 a), the column with thin reinforcement, $p = 6\, mm \approx 0.7\, t$, yields at the compression flange tip over a length from the column midpoint to the end of the reinforced region. The column with thicker reinforcing plates, $p = 18\, mm \approx 2\, t$, shown in Figure 5-1 b), does not yield at the column mid-height when failure initiates by yielding of the original section at the ends of the reinforced region.

5.2.2 Bolted reinforcement connections

Bolts attach the reinforcing plates to the original column in the experimental study, so it is also necessary to assess the effect of bolt holes on column capacity. Columns with bolt holes on the flanges are referred to throughout this study as “perforated” columns, and columns without bolt holes are referred to as “unperforated”. Bolt holes reduce the net area and moment of inertia of the cross section relative to the unperforated column, and so reduce the strength. If the reinforcement plate thicknesses are small, the increase of column capacity they provide may not offset the strength loss due to the holes.
Figure 5-1: Key von Mises stresses for a W100x19 column with $kL/r_y$ of 94 and $\alpha$ of 0.5, for a) $A_t/A_o$ of 1.5, and b) $A_t/A_o$ of 2.5.
Figure 5-2 shows the distribution of total stresses around the bolt holes in an unreinforced column loaded with an axial compressive force. The wedge-shaped stress concentrations are consistent with documented elastic analyses indicating that an axial force applied to the ends of an infinite plate with circular holes in the middle results in regions of stress amplification radiating from the bolt-hole edge at 40-45 degree angles to the direction of the applied force (Wang, 1946). In these cases involving bolt holes through the flange, the applied compressive force causes stress concentrations around the bolt holes that intersect the flange tips, increasing the lateral deflections and so reducing the overall stability. The amplification of stresses around bolt holes in the column flange was confirmed experimentally, as described in Section 4.7.

**Figure 5-2:** von Mises stresses around bolt holes at failure (i.e. $P = 248kN$)

Unperforated and perforated stub columns were analyzed to determine the relative effect of the reduced cross sections on the axial and flexural stiffnesses. Figure 5-3 compares the relative deformations of the stub columns with $F_y$ of 350MPa, neglecting residual stresses. The perforations cause the axial or flexural
stiffnesses to be reduced, so the associated deformations at a given level of applied load increase. Superimposed on the graphs are pairs of images showing the stress distributions in the unperforated (upper image) and perforated (lower image) columns at a given level of applied load. For the net section taken through the centres of the bolt-holes, the ratios of net-to-gross area, $A_n/A_g$, and weak-axis moment of inertia, $I_n/I_g$, are 0.745 and 0.647, respectively.

Figure 5-3 a) shows the relative reduction of the axial stiffness caused by the perforations as the applied load, $P$, is increased to the maximum capacity of the perforated column, $P_{\text{max}}$. The relative stiffness of the perforated column is approximately constant, at 82%, up to $P/P_{\text{max}}$ of 0.6. At this load level, stresses between the two rows of bolt-holes are similar in magnitude to those in the unperforated column, but yielding is initiating in small areas at the outside edges of the bolt-holes. At greater load levels, the stiffness of the perforated column decreases linearly to 68% of that of the unperforated column at $P/P_{\text{max}}$ of 0.925. At this load level, yielding has occurred in wedge-shaped regions extending from the outer edge of the bolt-hole to the flange tip and the stress magnitudes between the two rows of bolt-holes are typically greater than those for the unperforated case. At greater load levels, the relative stiffness decreases more rapidly. Failure of the perforated column corresponds to the stress in the regions between the bolt-hole and flange tip reaching the ultimate strength of the steel. At this load, no region in the unperforated column has yielded.

Figure 5-3 b) shows the relative reduction of weak-axis flexural stiffness as the applied moment, $M$, is increased to the moment at failure of the perforated member, $M_{\text{max}}$. The relative stiffness of the perforated member remains constant at 83% up to $M/M_{\text{max}}$ of 0.65. At this load level, yielding initiates at the corner bolt-holes near the member mid-point on the tension side and near the member ends on the compression side. At greater load levels, the relative stiffness of the perforated member decreases markedly. Failure of the perforated member occurs when the stress at the flange tips approaches the ultimate strength of the steel.
Although the flange tips of the unperforated member begin yielding at $M/M_{\text{max}} = 0.90$, the unperforated member exhibits only slight stiffness loss at $M/M_{\text{max}} = 1.0$.

\[ \text{Figure 5-3: Relative deformations and von Mises stress distributions for an unperforated short column relative to the corresponding perforated column, versus the applied loading, for the cases of: a) vertical deflection under pure axial force, and b) angle of rotation under pure bending.} \]
The extent of the perforated length is limited to a segment about the column mid-point that varies depending on the length of plates. Figure 5-4 shows the relative capacities of a perforated column without reinforcement compared to an equivalent unperforated column for various normalized perforated lengths, $\alpha'L$. The column investigated has a length of 2805\,mm (i.e. $kL/r_y = 110$ about the weak axis) and 18\,mm diameter bolt-holes at a pitch of 80\,mm. The relative capacity of the perforated column decreases as the perforated length at mid-height increases. This value approaches a lower limit of approximately 73% of the unperforated column capacity when $\alpha'$ is greater than approximately 2/3. This is similar to the linear-elastic buckling analysis results shown in Figure 2-4, where the reinforcing plates increase the stiffness of the mid-height region, but increasing the reinforced length for a given reinforcing plate area has a diminishing effect on the increase in compressive capacity, particularly for $\alpha > 0.8$. Thus whether reducing the column stiffness by bolt-hole perforations or increasing the column stiffness with reinforcing plates, extending the weakening or strengthening to the column ends has relatively little effect on the capacity.

Figure 5-4 also considers the case where the lost bolt-hole areas are simulated by reducing the flange width in the mid-height region to obtain the same net second moment of area, $I_n$, as for the perforated column. In this case $I_n$ was assigned to be $1.07 \times 10^6$ mm$^4$, or 67% of $I_o$. As the length of the segment with the reduced flange width increases the relative capacity again approaches asymptotically a lower limit. The comparison here is not perfect: the use of an equivalent $I_n$ is slightly conservative for longer plate lengths and slightly unconservative for shorter plate lengths.
Columns with bolt-holes were investigated for both the minimum and maximum hole pitches defined by CHBDC requirements for sealing, stitch, and bolt diameter (CSA 2014). Typical results are shown in Figure 5-5. For the W100x19 shape the number of bolts was calculated based on a typical minimum bolt pitch of 55\textit{mm} and maximum bolt pitch of 105\textit{mm}. The bolt holes caused significant capacity reductions compared to the unperforated reinforced columns, however there were only slight differences for the minimum and maximum pitch cases. Minimum pitch will be assumed for the remainder of the present study to provide a conservative estimate of capacity.
Figure 5-5: Comparison of the unperforated case and the perforated case for the minimum and maximum allowable bolt pitch, at a slenderness of $kL/r_y$ of 94.

Figure 5-6 shows the von Mises stress distribution at failure in both flanges of a W100x19 column with $kL/r_y = 110$ and $\alpha = 0.8$, comparing reinforced columns for the unperforated and perforated cases. Each column is represented by a pair of images: the left shows the outer face of the flanges, while the right image reverses the x-axis and so shows the inner face of the flanges on the web side. Figure 5-6 a) shows the stresses corresponding to $A_t/A_o$ of 2.5, at a maximum compressive capacity increase, $C_t/C_o$, of 2.42 and 2.13 for the unperforated and perforated column, respectively. The unperforated column does not yield at mid-height, which corresponds to the bottom of the half-model, while the perforated column flanges yield adjacent to the bolt-holes. In both cases there are significant large stresses in the unreinforced regions at the ends of the original column. Figure 5-6 b) shows the stress distribution for $A_t/A_o$ of 1.75, for which $C_t/C_o$ is 2.02 and 1.44 for the unperforated and perforated column, respectively. The columns yield at both column mid-height and in the unreinforced region at the end of the original column, however the extent of yielding in the unreinforced end is more significant for the unperforated column, because the applied load is greater.
Figure 5-6: Comparison of von Mises stresses for W100x19 reinforced columns with $kL/r_y$ of 110, $\alpha$ of 0.8, and a) $A_i/A_o$ of 2.5 and b) $A_i/A_o$ of 1.75
Comparing the four columns in Figure 5-6, the relative magnitude of the capacity increase corresponds to the stiffness of the reinforced regions, which influences the extent of the stress distribution at the unreinforced ends of the column. For example, the stress distributions at failure are virtually identical for the perforated column with $A_t/A_o$ of 2.5 in Figure 5-6 a) and the unperforated column with $A_t/A_o$ of 1.75 in Figure 5-6 b). The slightly higher stresses at the unreinforced ends of the perforated column, Figure 5-6 a), correspond to its 5% greater compressive capacity. Calculating second moment of area based on the gross area, a column with $A_t/A_o$ of 2.5 has an $I_t/I_o$ of 3.03, and a column with $A_t/A_o$ of 1.75 has an $I_t/I_o$ of 2.02. As previously shown in Figure 5-4, the stiffness of the perforated column is reduced compared to an unperforated column, correlated to the loss of cross-section area caused by the bolt holes. A perforated column with $A_t/A_o$ of 2.5 has an $I_t/I_o$ of 1.96. For the capacity increase of the perforated column in Figure 5-6 a) to be greater than that of the unperforated column in Figure 5-6 b), the effective second moment of area for a perforated column must lie between $I_t$, calculated at the bolt-holes, and $I_n$, calculated between the bolt-holes. An objective of the parametric study should be to determine if this observation applies across a range of slenderness ratios and $A_t/A_o$ values.

5.2.3 Effect of Cross-section size

The present study is based on results obtained experimentally and theoretically for a W100x19 column, which is a very small shape for practical applications. It must therefore be determined whether these findings also apply for larger column cross-sections typically used in practice.

Using the parameters shown in Table 5-1, it was verified by ANSYS analysis that the capacity increase for a reinforced unperforated W310x158 column is identical to that of a W100x19 column if the reinforced length fraction ($\alpha$), section area ($A_t/A_o$), and slenderness ($kL/r_y$) are maintained. The same increase in capacity, $C_t/C_o = 1.45$, was achieved for both columns when $kL/r_y = 94$, $\alpha = 0.5$, and $A_t/A_o = 1.75$. 
Table 5-1: Length and area parameters for scaling between W100x19 and W310x158 where \( kL/r_y = 94 \), \( \alpha = 0.5 \), and \( A_t/A_o = 1.75 \)

<table>
<thead>
<tr>
<th></th>
<th>( r_y ) (mm)</th>
<th>L (mm)</th>
<th>( \alpha L ) (mm)</th>
<th>( A_o ) (mm(^2))</th>
<th>( A_t ) (mm(^2))</th>
<th>( p ) (mm)</th>
<th>( C_o ) (kN)</th>
<th>( C_t ) (kN)</th>
<th>( C_t/C_o )</th>
</tr>
</thead>
<tbody>
<tr>
<td>W100x19</td>
<td>25.5</td>
<td>2400</td>
<td>1200</td>
<td>2480</td>
<td>4340</td>
<td>9</td>
<td>402.5</td>
<td>583</td>
<td>1.45</td>
</tr>
<tr>
<td>W310x158</td>
<td>78.9</td>
<td>7426</td>
<td>3712</td>
<td>20100</td>
<td>35175</td>
<td>24.3</td>
<td>3300</td>
<td>4770</td>
<td>1.45</td>
</tr>
</tbody>
</table>

Table 5-2 compares the two W-shape columns for the perforated case, using the bolt-hole layout shown in Figure 5-7 and assuming minimum pitch. The net-to-gross area ratios, \( A_n/A_g \), are 0.71 and 0.75 for the W100x19 and W310x158 shapes, respectively, indicating that the net area of the lighter shape is relatively smaller. This implies that the larger section should have a relatively greater capacity for the stocky column case. The converse is true when comparing the ratio of the net-to-gross moments of inertia about the y-axis, \( I_n/I_g \), which is 0.65 for the W100x19 and 0.62 for the W310x158. This implies that the lighter shape should have a relatively larger capacity for a slender columns. For the same scaling assumptions as the unperforated column (i.e. \( kL/r_y = 94 \), \( \alpha = 0.5 \), and \( A_t/A_o = 1.75 \)) the values of \( C_t/C_o \) are 1.27 and 1.28 for the reinforced W100x19 and W320x158 members, respectively. Thus findings determined by investigation of W100x19 shapes are applicable to heavier shapes.

Table 5-2: Comparison of section area and second moment of area for perforated W100x19 and W310x158 sections

<table>
<thead>
<tr>
<th></th>
<th>( d_b ) (mm)</th>
<th>N (mm)</th>
<th>( A_n/A_g )</th>
<th>( I_n/I_g )</th>
<th>( C_o ) (kN)</th>
<th>( C_t ) (kN)</th>
<th>( C_t/C_o )</th>
</tr>
</thead>
<tbody>
<tr>
<td>W100x19</td>
<td>18</td>
<td>20</td>
<td>0.71</td>
<td>0.65</td>
<td>402.5</td>
<td>511</td>
<td>1.27</td>
</tr>
<tr>
<td>W310x158</td>
<td>22</td>
<td>56</td>
<td>0.75</td>
<td>0.62</td>
<td>3300</td>
<td>4224</td>
<td>1.28</td>
</tr>
</tbody>
</table>
Figure 5-7: Perforated W100x19 and W310x158 with $kL/r_y$ of 94, $\alpha$ of 0.5, and $A_f/A_o$ of 1.75

5.3 PARAMETRIC STUDY

Table 5-3 outlines the parameters and parameter ranges investigated. These include the reinforcement plate thicknesses, reinforcement plate lengths, and perforations as discussed in Section 5.2, and additional parameters that are known to typically affect the strength and behaviour of steel components. For example, Shek’s (2006) investigation of buckling indicated that the weak-axis capacities were more sensitive to locked-in dead load stresses, residual stresses, and yield strengths. The ranges of these parameters are the same as those investigated by Shek to facilitate comparison.
Table 5-3: Parameter matrix for W100x19 shape

<table>
<thead>
<tr>
<th>Variable</th>
<th>Constants</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha$ (MPa)</td>
</tr>
<tr>
<td>$\alpha$ 0.30F_{yo}</td>
<td>350</td>
</tr>
<tr>
<td>$A_t/A_o$ 0.30F_{yo} 1.25,1.5,1.75,2.0,2.5,3.0</td>
<td>0.2, 0.5, 0.8</td>
</tr>
<tr>
<td>$A_t/A_o$ 0.30F_{yo} 1.25,1.5,1.75,2.0,2.5,3.0</td>
<td>0.2, 0.5, 0.8</td>
</tr>
<tr>
<td>$\sigma_{R,max}$ 0.15F_{yo},0.30F_{yo},0.45F_{yo}</td>
<td>0, 0.2, 0.5, 0.8</td>
</tr>
<tr>
<td>$\sigma_{LiD}$ 0.30F_{yo}</td>
<td>0.2, 0.5, 0.8</td>
</tr>
<tr>
<td>$\nu$ 0.30F_{yo}</td>
<td>0.2, 0.5, 0.8</td>
</tr>
</tbody>
</table>

Sub-Total 122
Total 610

For the standard case of $F_{yo} = F_{yn} = 350\text{MPa}$, $\sigma_r = 30\% F_{yo}$, $\sigma_{LiD} = 0\% F_{yo}$, and $\nu = L/1000$, the compressive resistance of the unreinforced column ($C_o$) is calculated for a W100x19 section at slenderness ratios of 70, 90, 110, 130, and 150, as 556\text{kN}, 427\text{kN}, 321\text{kN}, 242\text{kN}, and 189\text{kN}, respectively. Appendix C1 presents graphs with the compressive capacity calculated for reinforced columns ($C_i$) and additional unreinforced capacities as required for the range of parameters outlined in Table 5-3.

Figure 5-8 a) shows the variation in compressive resistance for various slenderness ratios with respect to the variation in reinforcing plate thickness, for $\alpha$.
of 0.5. In general, use of thicker reinforcing plates does not increase the capacity of stockier columns because, in these cases, failure occurs by yielding or inelastic buckling in the unreinforced ends of the member. The stockier columns still reach the highest absolute values of \( C_t \). Figure 5-8 b) shows the same case using normalized axes \( C_t/C_o \) and \( A_t/A_o \). Clearly, the stockier columns benefit least from partial-length reinforcement and slender columns benefit most.

**Figure 5-8**: Set of slenderness ratios considered for the sensitivity analysis, presented for an unperforated reinforced segment with \( F_{yo} \) of 350MPa, \( \alpha \) of 0.5 in terms of:

a) Variation in compressive resistance at different reinforcing plate thicknesses

b) Increase in capacity achieved with respect to the increase in section area.
5.3.1 Effect of Reinforcing Plate Length

Figures 5-9 a) and b) show the compressive resistance and the relative capacity increases, respectively, for different lengths of reinforcing plates and different slenderness ratios, for reinforcement area $A_r/A_o$ equal to 2.5. As shown on Figure 5-9 a), the column with full-length reinforcement, $\alpha$ of 1.0, and the column with the longest partial-length reinforcing plates, $\alpha$ of 0.8, both reach limiting compressive resistances of $1017kN$ and $835kN$, respectively, at a slenderness ratio of 70. For a W100x19 shape with $F_{yo} = 350MPa$, the limit on compressive capacity is $868kN$ according to the capacity of the original column cross-section, calculated with Equation [1.4]. Failure of the column with full-length reinforcing plates initiates at mid-height of the column, and surpasses the yield capacity of the original shape because the failure involves combined inelastic buckling of the original column and reinforcing plates. As the slenderness ratio decreases, the compressive capacity of the reinforced column approaches that of the original unreinforced column because failure initiates in the unreinforced regions at the ends of the column. As the slenderness ratio increases, the compressive resistance of the columns with $\alpha$ of 0.8 and 1.0 tend to converge, indicating that failure of the reinforced column is changing to initiate at the column mid-height. Figure 5-8 b) shows that for each reinforcing plate length, $\alpha$, the capacity increase approaches a maximum value as slenderness increases. This maximum value is based on the relative stiffness of the reinforced region to the unreinforced region, as the column slenderness transitions from the intermediate to Euler buckling range, discussed in Chapter 2.
Figure 5-9: The effect of reinforcing plate length across slenderness ratios in the inelastic range for an unperforated member and $A_r/A_o$ of 2.5 in terms of a) Compressive resistance, and b) Capacity Increase
5.3.2 Effect of Reinforcing Plate Area

This section investigates the increase in weak-axis compressive capacity attainable with partial-length flange reinforcing plates as $A_t/A_o$ is varied for the standard initial state assumptions, i.e., $F_{yo} = F_{yn} = 350\text{MPa}$, $\sigma_r = 0.3F_{yo}$, $\sigma_{LID} = 0$, and $v = L/1000$.

Figures 5-10 a), b) and c) show the capacity increase for reinforced length ratios of $\alpha = 0.8$, 0.5, and 0.2, respectively, for $A_t/A_o$ ranging from 1.25 to 3. Figure 5-10 a) demonstrates two distinct limits on the increase in capacity as a function of slenderness ratio for a column with a long reinforced length ratio, $\alpha$, of 0.8. The first limit corresponds to the capacity of the original column at the unreinforced ends, and affects stocky columns with thick reinforcing plates. For slenderness ratios between 70 and 90, increasing the area ratio, $A_t/A_o$, greater than 1.75 has a negligible effect on the capacity increase of the column. Similarly, area ratios greater than $A_t/A_o$ of 2.5 have a minimal effect on the capacity increase at a slenderness ratio of 110. In these cases, the capacity of the reinforced column is limited by the capacity of the unreinforced ends of the original column and increasing the reinforcing plate area has no impact. The second limit corresponds to Euler buckling, and affects slender columns with thin reinforcing plates. For each reinforcement area ratio, the increase in capacity approaches a limiting maximum value as the slenderness of the column increases. This occurs because failure of columns with high slenderness ratios initiates at the column mid-height, so the reinforcing plates reach their maximum effectiveness. In this case, the capacity increase is due to the increased area of the reinforced section compared to the original section, particularly for $A_t/A_o$ between 1.25 and 1.75 for slenderness ratios greater than 110. For $\alpha$ of 0.5, shown in Figure 5-10 b), the capacity increase for each $A_t/A_o$ value is less than that for the case of $\alpha$ of 0.8. The limits on the capacity increase as the slenderness ratio approaches 70 and 150 are also less noticeable. 5-10 c) shows the capacity increase for $\alpha$ of 0.2: increasing $A_t/A_o$ has relatively little effect.
Figure 5-10: Increase in compressive capacities of an unperforated column reinforced with a range of reinforcing plate area ratios (At/Ao), for $F_{yo}$ of 350 MPa and a) $\alpha$ of 0.8, b) $\alpha$ of 0.5, and c) $\alpha$ of 0.2.
5.3.3 Effect of Steel Grades of Original W-Section

In this section, the effect of the steel grade of the original W-shape is investigated for the case where the partial-length steel reinforcing plates have a higher steel grade than that of the original column. In older structures, the original steel may have a much lower yield strength than the reinforcing plate steel (Shek, 2008). For the present investigation, the steel in the original column is assumed to have a bilinear, elastic/perfectly plastic, stress-strain relationship, with a yield strength of 228 MPa. The reinforcing plate steel is assumed to have a bi-linear stress-strain relationship shown in Figure 3-2 with a yield strength of 350 MPa. All other parameters are as described in Section 3.5.1.

Figures 5-11 a) and b) show the capacity increases for reinforced lengths \( \alpha \) of 0.8 and 0.5, respectively, for reinforcement areas \( A_t/A_o \) ranging from 1.25 to 3. Figure 5-11 a) shows that the relationship between the capacity increase and the slenderness ratio is generally limited by a maximum capacity increase that applies to all reinforcement areas. As with the columns with \( F_{yo} \) of 350 MPa, Figure 5-10 a), the limit is due to the failure initiating with yield or instability in the unreinforced ends of the column. Unlike the previous case, the failure mode that defines capacity increase switches to Euler buckling initiating at column mid-height. There is little transition between these two failure modes. The magnitude of the capacity increase at \( A_t/A_o \) of 3 is also less than the corresponding values for the higher grade steel. Figure 5-11 b) shows that the relationship between capacity increase and slenderness ratio for \( \alpha \) of 0.5 and \( F_{yo} \) of 228 MPa are slightly lesser in magnitude compared to the corresponding relationships for \( F_{yo} \) of 350 MPa, but otherwise exhibit similar trends.
Figure 5-11: Increase in compressive capacities of an unperforated column reinforced with a range of reinforcing plate area ratios ($A_t/A_o$), for $F_{yo}$ of 228 MPa and a) $\alpha$ of 0.8, and b) $\alpha$ of 0.5

Figure 5-12 shows that the compressive capacity corresponding to $A_t/A_o$ of 2.5 and $\alpha$ of 0.8 reaches a maximum of 490 kN, limited by the capacity of the original cross section as previously described for the column with a yield strength of 350 MPa. However, for an unreinforced W100x19 shape with $F_{yo}$ of 228 MPa, the nominal yield capacity is 565 kN, as computed using Equation [1.4], or 10% higher than the FEA-predicted capacity. A more detailed investigation of the stress state at failure is required to determine the reason for the significantly lower capacity predicted by FEA.
Figure 5-12: Compressive resistance of an unperforated column reinforced with a range of reinforcing plate area lengths (α), for $F_{yo}$ of 228MPa

Figure 5-13 shows the von Mises flange stresses at failure of a W100x19 column with $kL/ry$ of 110 and α of 0.8, comparing cases of $F_{yo}$ of 350MPa and 228MPa. Each column is again shown as a pair of images depicting the outer and inner faces of the flange. Both columns yield in the unreinforced region at the end of the reinforcement. However, the column with $F_{yo}$ of 350MPa also has near-yield stresses of 0.99$F_{yo}$ at column mid-height, caused by weak-axis bending. There is little indication of weak-axis bending stresses at mid-height in the column with $F_{yo}$ of 228MPa, because the higher grade reinforcing plates, with $F_{yn}$ of 350MPa, redistribute stresses across the entire width of the flange at the unreinforced end segment of the original column. This high concentration of additional stresses may result in the unreinforced end segment of the original column reaching yield earlier than expected, resulting in the FEA-predicted capacity being significantly lower than the calculated capacity.

This analysis indicates that the effect of the steel grade of the original W-shape and the difference in yield strength between the original and reinforcing steels are significant. The design of partial-length reinforcing plates must account for the actual steel grades used.
Figure 5-13: Comparison of von Mises stresses for W100x19 reinforced columns with $kL/r_y$ of 110, $\alpha$ of 0.8, and $A_t/A_o$ of 2.5

5.3.4 Effect of Perforations

This section investigates the effect of perforations (i.e. bolt-holes) on the increase in compressive capacity attainable with partial-length flange reinforcing plates, using the same range of geometric parameters and initial state assumptions as stated in Section 5.3.2.

Figures 5-14 a) and b) compare the FEA-predicted strengths for the unperforated and perforated column at a $kL/r_y$ of 110, $F_{yo}$ of 350 MPa, and $\alpha$ of 0.8, 0.5, and 0.2. Figure 5-14 a) shows the variation of the capacity increases with respect to the normalized cross-section area of the reinforced segment. The perforated column capacity is consistently less than the unperforated column capacity at the same $A_t/A_o$. The capacity reduction due to the perforations is most severe for reinforcing plates with smaller areas and longer lengths. The effect of the perforations on the capacity of the unreinforced columns is indicated for $A_t/A_o$ of 1, where the capacity
of the perforated columns are calculated assuming $\alpha = \alpha'$. Similar to the results shown in Figure 5-4, the capacity of the unreinforced column decreases as the perforated length increases: $C_t/C_o$ approaches 1 for $\alpha$ of 0.2 and decreases as $\alpha$ increases. As the stiffness of the reinforced region increases due to increasing reinforcing plate area, the capacity increases for the three reinforcing plate lengths are roughly equal at a point between $A_t/A_o$ of 1.25 and 1.5 and at $C_t/C_o$ slightly less than 1. The FEA-predicted capacity increases for the perforated column approach those for the corresponding unperforated column as the area of reinforcement increases. This is most evident for $\alpha$ of 0.2, where the capacity increases have nearly converged at $A_t/A_o$ of 2.0 and are identical for $A_t/A_o$ of 3.0. Figure 5-14 b) shows the variation of the capacity increase with the stiffness of the reinforced region, expressed in terms of the gross and net second moment of area (i.e. $I_t/I_o$ and $I_n/I_o$) for the unperforated and perforated columns, respectively. The capacity of the perforated column is consistently greater than that of the unperforated column, because taking the net second moment of area does not account for the large areas between bolt-holes that mitigate the loss of cross-section area in bending, as also shown previously in Figure 5-6.

![Figure 5-14: Comparison of the unperforated and perforated column, $F_{yo}$ of 350MPa, $kL/r_y$ of 110, with respect to a) cross-section area of the reinforced segment, $A_t/A_o$, b) net second moment of area in the reinforced segment, $I_n/I_o$](image-url)
Figures 5-15 a), b), c), d) and e) compare the FEA-predicted strengths for the unperforated and perforated columns at $F_{yo}$ of 350 MPa, and $\alpha$ of 0.8, 0.5, and 0.2, for $kL/r_y$ of 150, 130, 110, 90, and 70, respectively. The range of the vertical axes are restricted to $C_t/C_o$ greater than 1 to focus on the comparison between the unperforated and perforated column with respect to $A_t/Ao$. Figure 5-15 c) shows the same data as Figure 5-13. The slender column with $kL/r_y$ of 150, Figure 5-15 a), shows a similar relationship between the FEA-generated curves for the perforated and unperforated columns. The areas of reinforcing plate required for the perforated column to equal the capacity of the original unreinforced column is slightly less than for $kL/r_y$ of 110, reflecting the reduced influence of cross-section capacity for inelastic buckling of slender columns. The stockier column with $kL/r_y$ of 70, Figure 5-15 e), shows the capacity increase for the unperforated and perforated columns converging, irrespective of the reinforced length, at $A_t/Ao$ of 2.5. This trend was previously noted for $\alpha$ of 0.2 with $kL/r_y$ of 110, but the convergence corresponds to a higher $A_t/Ao$ ratio because the cross-section capacity has a greater influence on the inelastic buckling strength of stockier columns. Figures 5-15 b) and d), corresponding to $kL/r_y$ of 130 and 90, respectively, display similar trends to those previously described.

Overall, perforations markedly lower the capacity of the reinforced column, due to the decreased axial and flexural stiffnesses of the reinforced segment. The effect of bolt-hole perforations must be considered when determining the compressive capacity of columns with partial-length reinforcing plates.
Figure 5-15: Comparison of the unperforated and perforated column, $F_{yo}$ of 350MPa, for a) $kL/r_y$ of 150, b) $kL/r_y$ of 130, c) $kL/r_y$ of 110, d) $kL/r_y$ of 90, and e) $kL/r_y$ of 70.
5.3.5 Effect of Residual Stresses

Residual stresses in the W-shape vary linearly across the width of the flange and the depth of the web as described in Section 3.3.1. For the purposes of this study the residual stresses in the steel cover plates are assumed to be negligible, though in practice there may be small magnitudes of residual stresses depending on the methods used to cut the plate edges.

Figures 5-16 a) and b) show the effect of the maximum residual stress magnitude on the capacity increase for area ratios $A_t/A_o$ of 1.5 and 2.5, respectively. The capacity increase is calculated relative to the capacity of the unreinforced column with the same residual stress distribution. The effect of varying the maximum residual stress from 15% to 45% of the yield strength (i.e. $0.15F_{yo}$ to $0.45F_{yo}$) has relatively little effect on the capacity increases of the unperforated reinforced W100x19 column over wide ranges of reinforcement lengths and thickness, particularly at higher slenderness ratios. This is consistent with extensive literature (e.g., Galambos 1998) highlighting the sensitivity of columns of intermediate slenderness ratios, that fail by inelastic buckling, to residual stress magnitudes.

Figures 5-17 a) and b) present the relative capacity increases for columns with maximum residual stresses magnitudes of $0.15F_{yo}$ and $0.3F_{yo}$, for $A_t/A_o$ of 2.5 for unperforated and perforated columns, respectively. For the unperforated case, Figure 5-17 a), the unreinforced column ($\alpha = 0$) with $\sigma_r$ of $0.15F_{yo}$ typically reaches greater capacities than the same column with $\sigma_r$ of $0.3F_{yo}$. The strength increase is 8% if the column has a slenderness ratio of 70. Thus, for stockier columns, the probable reduction of the residual compression stress at the flange tip caused by the heat from welding the reinforcing plates in place may enhance the capacity and so be beneficial. Columns with shorter reinforced lengths, $\alpha$ of 0.2 and 0.5, exhibit slightly greater relative capacities at higher slenderness ratios. The magnitude of the maximum residual stress has little effect on the relative capacity for $\alpha$ of 0.8. The increase in the magnitude of the maximum residual stress has little impact on the relative capacity for a perforated column, Figure 5-17 b). For $\alpha$
of 0.2, 0.5, or 0.8, the capacity increase approaches that for the unreinforced column, with $\alpha$ of 0, particularly for higher slenderness ratios. At lower slenderness ratios, the capacity increase is the same as for the corresponding unperforated case.

Figures 5-18 a) and b) present the relative capacity decreases when the maximum residual stress magnitude increases to $0.45F_{yo}$ from $0.3F_{yo}$, for $A_t/A_o$ of 2.5 for the unperforated and perforated columns, respectively. The unreinforced column, with $\alpha$ of 0, is again most sensitive to the magnitude of the maximum residual stress, with a capacity 8% less when $\sigma_r$ increases to $0.45F_{yo}$ from $0.3F_{yo}$ at a slenderness ratio of 70. Results for the full range of reinforced lengths and slenderness ratios otherwise mirror those shown in Figures 5-17 a) and b).

**Figure 5-16:** Increase in compressive capacity for an unperforated column with $\sigma_r$ of $0.15F_{yo}$, $\sigma_r$ of $0.3F_{yo}$, and $\sigma_r$ of $0.45F_{yo}$ for a) $A_t/A_o$ of 1.5, b) $A_t/A_o$ of 2.5
Overall, the weak-axis capacity of the column with partial-length reinforcement is sensitive to increasing residual stresses, but varying the maximum residual stress magnitude from 15% to 45% of $F_{yo}$ has only a slight impact on the compressive capacity of the reinforced column. Thus, design procedure for partial-length reinforcing plates can be based on $\sigma_r$ of $0.3F_{yo}$. Shek (2006) reached a similar conclusion in her study of columns with full-length reinforcing plates.
5.3.6 Effect of Locked-In Dead Load Stresses

As described in Section 3.3.4, it may not be possible to relieve the entire dead load in the original column at the time that the reinforcement is installed, so there is a potential for locked-in dead load stresses. Shek (2006) deemed that a magnitude of 30% locked-in stress (i.e. $\sigma_{LiD} = 0.3F_{yo}$) was representative, combined with varying residual stresses. The present study uses a maximum residual stress, $\sigma_r$, of $0.3F_{yo}$.

Figures 5-19 a) and b) show the compressive capacity increases for locked-in dead load stresses $\sigma_{LiD}$ of 0 and $0.3F_{yo}$, for $A_y/A_o$ of 1.5 and 2.5, respectively. In both cases the locked-in dead load stress has a minimal effect on the capacity increase for a reinforced length, $\alpha$, of 0.2, and a slightly greater effect for columns with $\alpha$ of 0.5 or 0.8. The effect of locked-in dead load stress becomes less prominent as reinforcing plate length decreases and the response of the unreinforced ends is the same regardless of the stiffness in the reinforced region.

Figures 5-20 a) and b) present the relative compression capacities of columns with $\sigma_{LiD}$ of zero and $0.3F_{yo}$, for unperforated and perforated columns, respectively. The unperforated column with $\sigma_{LiD}$ of $0.3F_{yo}$ is 7% weaker than the identical member with no locked-in dead load stresses, for $\alpha$ of 0.8 at higher slenderness ratios of 130 to 150. The impact is less at lower slenderness ratios. The relative magnitudes of compression capacity are consistent with data generated by Shek (2006) for columns with full-length reinforcing plates at the same locked-in dead load stress and maximum residual stress. The locked-in dead load stress magnitude has a more significant effect for the perforated column, with a capacity decrease of 10% to 12% for the slenderness ratios between 110 and 150. Capacity is lost during the pre-loading phase due to the perforated column segment having a lower stiffness than an unperforated column. As a result, the stresses are slightly higher than for an unperforated column that sustains the same magnitude of dead load at the time that flange reinforcing plates are added.
Figure 5-19: Increase in compressive capacity for an unperforated column with $\sigma_{LID}$ of 0.3$F_{yo}$ and $\sigma_{LID}$ of 0$F_{yo}$ for a) $A_t/A_o$ of 1.5, b) $A_t/A_o$ of 2.5.

Figure 5-20: Compressive capacity of a column with $\sigma_{LID}$ of 0.3$F_{yo}$ relative to $\sigma_{LID}$ of 0$F_{yo}$, with 2.5$A_o$ and for a) unperforated case, and b) perforated case.
The effect of locked-in dead load stresses is not very significant for the unperforated case, with a capacity decrease of less than 10% when they are increased from 0 to $0.3F_{yo}$. Shek (2008) reached a similar conclusion. However the magnitude of locked-in dead load stress should be considered if reinforcement is bolted, particularly for long reinforcing plate lengths.

5.3.7 Effect of Initial Out-of-Straightness

The magnitude of the initial out-of-straightness is investigated for two cases, $\nu$ of $L/500$ and $L/1000$. The latter corresponds to the out-of-straightness permitted in current design standards (e.g., CSA 2015) and the former is twice this limit. Figures 5-21 a) and b) show the effect of the initial out-of-straightness on the capacity increase, for $A_t/A_o$ of 1.5 and 2.5, respectively. The effect of the initial out-of-straightness is least for the shorter reinforced lengths, $\alpha$ of 0.2 and 0.5, and for higher slenderness ratios. As shown in Figure 5-21 b), the capacity increase for longer reinforcing plates with $\alpha$ of 0.8 and $A_t/A_o$ of 2.5 is significantly greater when $\nu$ is $L/500$ instead of $L/1000$, indicating that longer and thicker reinforcing plates are effective at mitigating the effect of the increased initial imperfection.

Figure 5-22 presents the relative compressive capacity increases for columns with $\nu$ of $L/500$ and $L/1000$, for $A_t/A_o$ of 2.5. The relative capacity is lowest for an unreinforced column or one with a short reinforced length, $\alpha$ of 0.2. The maximum difference, however, is only 11% in the slenderness ratio range from 90 to 110. As this relatively small difference corresponds to the effect of doubling the initial out-of-straightness it can be concluded that weak-axis capacity of a column with partial-length reinforcement is relatively insensitive to initial out-of-straightness.
Figure 5-21: Increase in compressive capacity for an unperforated column with \( v \) of \( L/500 \) and \( L/1000 \) for a) \( A_t/A_o \) of 1.5, b) \( A_t/A_o \) of 2.5

Figure 5-22: Compressive capacity of a column with \( v \) of \( L/500 \) relative to \( v \) of \( L/1000 \), for an unperforated case with 2.5\( A_o \)
5.4 SUMMARY AND CONCLUSIONS

This chapter has presented a sensitivity analysis of the weak-axis compressive resistance of columns reinforced with partial-length flange reinforcing plates. The rationale for the selection and ranges of the parameters was presented. These included the effect of the reinforcing plate length and area ratios, the effect of perforations due to attaching the reinforcement by bolting, and the effect of larger sized W-shapes. Parameters were investigated over a range of slenderness ratios from 70 to 150, which is typical for intermediate columns. The sensitivity study also investigated the effect of the yield strength of the W-shape, and the effects of residual stress magnitudes, locked-in dead load stress magnitudes, and out-of-straightness.

The conclusions of the sensitivity analysis are:

1. Two distinct failure modes limit the capacity increases for columns with partial-length flange reinforcing plates: 1) Compressive capacity of the original column, with failure initiating at the unreinforced end segments; and 2) Inelastic buckling capacity of the reinforced region, with failure initiating at column mid-height. The transition between the first failure mode, occurring at lower reinforced length and greater reinforcement areas, and the second failure mode, occurring at higher reinforced length and lower reinforcement areas, is most dramatic for columns of high slenderness.

2. The presence of perforations has a significant effect on the column capacity for the range of reinforcing plate lengths and areas investigated. The capacity decreases due to the reduction of axial and flexural stiffness caused by the holes in the reinforced region. A design method for calculating the weak-axis compressive capacity of a column reinforced with partial-length reinforcing plates must account for the presence of bolt-hole perforations.

3. The steel grade of the original column is significant because the capacity of columns with partial-length reinforcing plates can be limited by the capacity
of the original member at its unreinforced ends. A design method for calculating the weak-axis compressive capacity of a column reinforced with partial-length reinforcing plates must account for the steel grade of the original column.

4. Locked-in dead-load stresses have a relatively small effect on the compressive capacity of an unperforated column. For $\sigma_{LID} = 0.3F_{yo}$ the most severe decrease in capacity is 7% compared to the case of no locked-in dead-load stress. This is consistent with the findings of Shek (2006). However, locked-in dead load stresses have a more significant effect for the perforated case, with capacity decreases up to 12% for $\sigma_{LID} = 0.3F_{yo}$ for columns with high slenderness ratios. The need to account for locked-in-dead load stresses may therefore be evaluated on a case-by-case basis for perforated columns, particularly if they have high slenderness ratios.

5. Residual stresses have a relatively small effect on the capacity of a column reinforced with partial-length reinforcing plates. This is again consistent with the findings of Shek (2006).

6. Out-of-straightness has a relatively small effect on the capacity of a column reinforced with partial-length reinforcing plates.
CHAPTER 6: SIMPLIFIED DESIGN CRITERIA

6.1 INTRODUCTION

The accuracy of the Finite Element Analysis (FEA) model presented in Chapter 3 was validated in Chapter 4 based on tests of the capacity of columns reinforced with partial-length flange plates. The FEA model would be an appropriate tool for analyzing a proposed design, but is too time-consuming to use for the preliminary selection of the cover plate length and thickness. The objective of this chapter is therefore to present a simplified preliminary design method for partial-length flange reinforcing plates in hybrid steel compression members.

Sections 6.2 presents an empirical coefficient-based design equation for unperforated columns and verifies its accuracy using FEA-predicted results. Section 6.3 presents additional design procedures to account for the presence of bolt-holes in the column flanges. Section 6.4 presents the full simplified procedure accompanied by an example that includes cost optimization.

6.2 SIMPLIFIED DESIGN EQUATIONS FOR WEAK-AXIS BUCKING OF UNPERFORATED COLUMNS

It is proposed to consolidate the slenderness ratio \((kL/r_y)\), the reinforced length ratio \((\alpha)\), and reinforcement area ratio \((A_r/A_o)\) into a single design equation that can accurately predict the capacity increase for an unperforated column with partial-length reinforcement. The domain of the equation is confined to the realistic limits of plate thickness and length as previously justified in Section 5.2, specifically:

- slenderness of original column: \(70 < kL/r_y < 150\)
- reinforced length ratio: \(0.2 < \alpha < 0.8\)
- reinforcement area ratio (unperforated): \(1.25 < A_r/A_o < 3\)
- yield strength of original column: \(F_{y0} = 350\, MPa \& 228\, MPa\)
- yield strength of reinforcing plates: \(F_{yn} = 350\, MPa\)
The accuracy of the design equation will be quantified using FEA-generated results for: residual stresses in the original column that do not exceed 30% of $F_y$; initial out-of-straightness of $L/1000$; and, locked-in-dead load stresses neglected.

Previous results from Section 5.3 indicate that the ultimate column capacity is defined by two distinct failure mechanisms: i) Instability or yield of the original column initiating in the unreinforced region; or ii) Instability initiating in the reinforced region. A multi-step approach is necessary to determine the required plate length for a given area ratio, to achieve a desired increase of compressive capacity.

### 6.2.1 Design Equation for Buckling Failure of the Reinforced Member

The simplified design equation for calculating the capacity increase from use of partial-length reinforcement plates has the form:

$$\frac{C_t}{C_o} = \left\{ A \left[ \left( \frac{A_t}{A_o} \right)^{\frac{m}{2}} \right] + B[\alpha^m] + \frac{C}{10} \left[ \left( \frac{A_t}{A_o} \right)^m \right] + D \right\}^2$$

where $A$, $B$, $C$, $D$, and $m$ are coefficients obtained through multiple linear regression analysis for each slenderness ratio investigated. The form of the design equation was selected by trial and error to minimize the standard error, $\sqrt{\text{MSE}}$, using the stepwise procedure presented in Appendix D1. Similar to the Euler-buckling-based procedure proposed by Dinnik (1932), Equation [6.1] does not allow a closed-form solution for $A_t/A_o$ given $C_t/C_o$ and $\alpha$, or for $\alpha$ given $C_t/C_o$ and $A_t/A_o$.

The various coefficients necessary to use Equation [6.1] are summarized in Table 6-1 for an unperforated steel column with yield strength of 350 MPa at slenderness ratios of 70, 90, 110, 130, and 150. All but the $B$ estimate for $kL/r_y$ of 150 and the $D$ estimate for $kL/r_y$ of 70 are statistically significant (i.e. $B(kL/r_y = 150) = 0$, and $D(kL/r_y = 70) = 0$). Coefficients corresponding to intermediate slenderness ratios may be obtained by interpolation.
Table 6-1: Unadjusted coefficients and error for equation with $F_{yo}$ of 350MPa

<table>
<thead>
<tr>
<th>$kL/r_y$</th>
<th>Coefficients</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>70</td>
<td>0.97</td>
<td>0.21</td>
</tr>
<tr>
<td>90</td>
<td>1.69</td>
<td>0.20</td>
</tr>
<tr>
<td>110</td>
<td>2.28</td>
<td>0.14</td>
</tr>
<tr>
<td>130</td>
<td>2.18</td>
<td>0.05</td>
</tr>
<tr>
<td>150</td>
<td>2.00</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Figures 6-1 a) to e) compare the capacity increases computed using Equation [6.1] to the FEA-predicted strengths for reinforcement plate length ratios, $\alpha$, of 0.8, 0.5, and 0.2, for $A_t/A_o$ ranging from 1.25 to 3.0. Generally the equation accurately matches the capacities predicted by FEA, particularly for higher slenderness ratios.

Table 6-1 also displays the maximum errors associated with Equation [6.1] for the various slenderness ratios. Instances where Equation [6.1] predicts a capacity that is greater than the FEA-predicted capacity, and so is unconservative, are shown as negative error percentages. Most $\sqrt{MSE}$ values are less than 0.04 and the maximum error magnitudes are less than 6%. For slenderness ratios of 150 and 130, shown in Figures 6-1 a) and b), respectively, the FEA-predicted strengths typically correspond to instability of the reinforced region, and Equation [6.1] produces suitable results for the entire range of $\alpha$ and $A_t/A_o$, with unconservative errors less than 2.5% and conservative errors less than 3.9%. For a slenderness ratio of 110, Figure 6-1 c), the unconservative error for Equation [6.1] increases to 3.5%. This is partially an effect of the capacity increase computed using Equation [6.1] for $\alpha$ of 0.2 reaching a local maximum value at $A_t/A_o$ of 2.5, instead of $A_t/A_o$ of 3, due to its parabolic nature. The greater error magnitude results from the transition between column capacities limited by the instability of the reinforced region to those limited by the capacity of the unreinforced regions at the ends of the member. As described in Section 5.3.2, the capacities of the unreinforced ends of the original column typically govern the capacity of columns with higher area ratios and shorter reinforced lengths. For columns with slenderness ratios of 90
and 70, Figures 6-1 d) and e), respectively, the unconservative and conservative errors are less than 5.6% and 4.7%, respectively. For these slenderness ratios, the compressive capacity is typically limited by the unreinforced end sections for $\alpha$ of 0.8, for relatively large ranges of $A_t/A_o$ greater than 1.75. Additionally, the maximum capacity increases computed using Equation [6.1] are reached at $A_t/A_o$ of 2.5 for all $\alpha$ values, as previously noted at a slenderness ratio of 110. It is thus not recommended to use Equation [6-1] for $A_t/A_o$ greater than 2.5 for slenderness ratios of 70 to 110.

Table 6-2 presents the $p$-values associated with each parameter estimate, where $p$-values less than 0.05 correspond to parameter estimates that are statistically significant and those greater than 0.05 indicate that the parameter estimate is not significantly different from zero. Parameters A and C are significant for the entire range of slenderness ratios investigated. These parameters are both associated with different combinations of $A_t/A_o$ and $\alpha$, indicating that the relationship between these two variables has a significant effect on column capacity regardless of the column stiffness. Parameter B is significant for columns with slenderness ratios of 70 to 130 due to its partial correlation with Parameter C, which also contains the term $\alpha^m$. Increasing Parameter B reduces the maximum column capacity predicted using Equation [6.1], which particularly applies to stockier columns that fail by inelastic buckling, limited by the capacity of the original column at the unreinforced ends. Parameter B becomes not significant for columns in the slenderness range of 150 where the failure mode is elastic buckling initiating in the reinforced region. Parameter D is not statistically significant for columns with a slenderness ratio of 70.

Two alternate sets of equations were derived for slenderness ratios of 150, 110, and 70. The first set is based on the tangent function used in the Euler-buckling approach proposed by Dinnik (1932), that models the combined instability of the reinforced regions and capacity of the original column simultaneously. The second set adjusts the dimensionless slenderness parameter, $\lambda$, and so is similar to the
general equation for factored compressive resistance presented in the CHBDC (CSA 2014). However, neither approach was particularly robust with respect to small variations in \( \alpha \), \( A_t/A_o \), and \( kL/r_y \). A brief overview of the alternate equation sets is presented in Appendix D2.

<table>
<thead>
<tr>
<th>( kL/r_y )</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.48</td>
</tr>
<tr>
<td>90</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.01</td>
</tr>
<tr>
<td>110</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>130</td>
<td>0</td>
<td>0.01</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>150</td>
<td>0</td>
<td>0.20</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

It is a good practice to perform a simple check of the capacity of the unreinforced ends of the original column to confirm that partial-length reinforcement is possible, before using Equation [6-1]. As the stiffness of the reinforced region approaches infinity, the unreinforced ends of the original column can be idealized as a relatively stocky column with a short length, as shown in Figure 2-4. Thus the associated compressive capacity is:

\[
C_{r,\text{max}} = \varphi A_o F_y (1 + \lambda_u^{2n})^{-1/n}
\]

where \( \varphi \) is the resistance factor, and \( \lambda_u \) is the dimensionless slenderness parameter for the unreinforced ends, calculated as:

\[
\lambda_u = \frac{kL(1-\alpha)}{r_y} \sqrt{\frac{F_{yo}}{\pi^2 E}}
\]

where \( r_y \) is the weak-axis radius of gyration, \( k \) is equal to 1 for pinned ends, \( F_{yo} \) is the yield strength of the original column, and \( E \) is the elastic modulus of steel.
Figure 6-1: Partial-length flange reinforcing plate design equation for weak-axis buckling of an unperforated steel column with $F_{yo}$ of 350 MPa, using coefficients from Table 6-1, compared to FEA-predicted strengths for a) $kL/r_y$ of 150, b) $kL/r_y$ of 130, c) $kL/r_y$ of 110, d) $kL/r_y$ of 90, and e) $kL/r_y$ of 70
6.2.2 Revised Design Equation \((F_{yo} = 350\text{MPa})\)

Table 6-3 presents revised coefficients for use in Equation [6.1], when the column and reinforcing plate yield strengths are 350\text{MPa}, which ensure that the computed results are generally conservative. These coefficients have been derived by trial-and-error from those shown in Table 6-1. Figures 6-2 a) to e) compare the capacity increases obtained using the coefficients in Table 6-3 to the FEA results for slenderness ratios of 150, 130, 110, 90, and 70. Table 6-3 also summarizes the error range obtained using Equation [6.1] with the Table 6-3 coefficients. The maximum unconservative difference between the FEA-predicted strengths and the calculated capacities is less than 1%, which is acceptable.

Table 6-3: Conservative adjusted coefficients for simplified design with \(F_{yo}\) of 350\text{MPa}

<table>
<thead>
<tr>
<th>(kL/r_y)</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>m</th>
<th>low</th>
<th>high</th>
<th>(k_p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>0.83</td>
<td>0.19</td>
<td>-0.62</td>
<td>0.16</td>
<td>1.50</td>
<td>-0.5</td>
<td>6.8</td>
<td>0.8</td>
</tr>
<tr>
<td>90</td>
<td>1.64</td>
<td>0.16</td>
<td>-0.9</td>
<td>-0.66</td>
<td>1.25</td>
<td>-0.9</td>
<td>8.5</td>
<td>0.75</td>
</tr>
<tr>
<td>110</td>
<td>2.28</td>
<td>0.13</td>
<td>-1.13</td>
<td>-1.31</td>
<td>1.15</td>
<td>-0.1</td>
<td>6.9</td>
<td>0.7</td>
</tr>
<tr>
<td>130</td>
<td>2.18</td>
<td>0.04</td>
<td>-0.68</td>
<td>-1.18</td>
<td>0.90</td>
<td>-0.5</td>
<td>5.8</td>
<td>0.65</td>
</tr>
<tr>
<td>150</td>
<td>1.98</td>
<td>0</td>
<td>-0.4</td>
<td>-0.97</td>
<td>0.75</td>
<td>-0.6</td>
<td>6.6</td>
<td>0.6</td>
</tr>
</tbody>
</table>
Figure 6-2: Partial-length flange reinforcing plate design equation for weak-axis buckling of an unperforated steel column with $F_{yo}$ of 350 MPa, using revised coefficients from Table 6-3, compared to FEA-predicted strengths for a) $kL/r_y$ of 150, b) $kL/r_y$ of 130, c) $kL/r_y$ of 110, d) $kL/r_y$ of 90, and e) $kL/r_y$ of 70.
6.2.3 Adjusted Design Coefficients \((F_{yo} = 228\text{MPa})\)

Table 6-4 presents the revised coefficients to be used in Equation [6.1] to obtain generally conservative results when the yield strength of the original column is 228\text{MPa} and reinforcing plate yield strength is 350\text{MPa}. The original coefficients obtained through multiple linear regression analysis are presented in Appendix D1.

**Table 6-4: Conservative adjusted coefficients for simplified design with \(F_{yo}\) of 228\text{MPa}**

<table>
<thead>
<tr>
<th>(kL/r_y)</th>
<th>Coefficients</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(A)</td>
<td>(B)</td>
</tr>
<tr>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>110</td>
<td>1.30</td>
<td>0.17</td>
</tr>
<tr>
<td>130</td>
<td>2.20</td>
<td>0.15</td>
</tr>
<tr>
<td>150</td>
<td>2.59</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Figures 6-3 a), b) and c) compare the FEA-predicted strengths for a column with \(F_{yo}\) of 228\text{MPa} to values computed using Equation [6.1] with the Table 6-4 coefficients, for slenderness ratios of 150, 130, and 110, respectively. The maximum unconservative difference between FEA predicted strengths and the calculated capacity is less than 1%, and so considered acceptable. However, the conservative differences are relatively high, reaching 13.1% for a slenderness ratio of 110. The differences increase as the reinforcing plate length increases and thickness decreases; or within the narrow transition zone between failure at the unreinforced ends of the original column and failure due to inelastic buckling of the reinforced region, as previously described in Section 5.3.3. Coefficients are not shown for columns with slenderness ratios of 90 and 70 because the differences between Equation [6.1] and the FEA-predicted strengths are greater than 15% conservative if the largest unconservative difference is constrained to be less than 1%.
Figure 6-3: Partial-length flange reinforcing plate design equation for weak-axis buckling of an unperforated steel column with \( F_{yo} \) of 228 MPa, using revised coefficients from Table 6-4, compared to FEA-predicted strengths for a) \( kL/r_y \) of 150, b) \( kL/r_y \) of 130, and c) \( kL/r_y \) of 110.
6.3 DESIGN PROCEDURE TO ACCOUNT FOR PRESENCE OF BOLT HOLES

It has been shown in Chapter 5 that loss of section area and second moment of area due to bolt holes markedly reduces the capacity of the reinforced column, and thus must be considered by the reinforcement designer if the plates are to be attached by bolting. As shown in Section 5.3.4, the FEA-predicted strengths of perforated and unperforated columns converge at the same capacity increases, though the perforated column requires a higher $A_v/A_o$ ratio due to the decreased axial and flexural stiffnesses of the reinforced segment. In general, the perforated and unperforated columns exhibit similar compressive capacities and stress distributions at failure when their reinforced segments have similar flexural stiffnesses. A simple design strategy is therefore to replace the lost bolt-hole area with an equal additional reinforcing-plate area. Thus the theoretical additional plate thickness, $\Delta p$, to compensate for the bolt holes is:

$$\Delta p = \frac{1}{2} \times \frac{N \times (t+p) \times d_b}{b - \left( \frac{N}{2} \right) d_b}$$

where $N$ is the number of bolt-holes in the net cross-section, $p$ is the thickness of one reinforcing plate, $t$ and $b$ are the thickness and width of the original W-shape flange, respectively, and $d_b$ is the bolt-hole diameter.

The adjusted plate thickness required to maintain $C_v/C_o$ with the addition of bolt-holes, $p_2$, is therefore:

$$p_2 = p + k_p \Delta p$$

where $k_p$ is an empirically determined factor less than 1.0 as described below.

The capacity increase that a perforated column achieves with additional reinforcing plate thickness can be compared to the FEA predicted strengths for the
corresponding unperforated column by calculating an adjusted area ratio, \((A_t/A_o)p\), at a given value of \(C_t/C_o\):

\[ [6.6] \quad (A_t/A_o)p = ((2 \times p_2 \times b_p) + A_o)/A_o \]

where \(b_p\) is the thickness of a single reinforcing plate.

Figures 6-4 a) to e) compare the FEA-predicted strengths for a perforated column with a 350MPa steel grade to the column capacities calculated using Equation [6.1] with coefficients from Table 6-2, including the associated adjusted area ratios calculated from Equations [6.4], [6.5] and [6.6] for \(k_p\) of 1. The column capacities are extremely conservative, especially considering that the FEA simulations use the maximum possible number of bolt-hole perforations (i.e., by minimizing the pitch). Thus it is practical to introduce the adjustment factor, \(k_p\), for the thickness of added reinforcing plates.

### 6.3.1 Adjusted Plate Coefficients \((F_yo = 350MPa)\)

The right column in Table [6-3] presents values of the \(k_p\) factor for the range of slenderness ratios investigated. Values corresponding to each slenderness ratio were selected by trial-and-error to best fit the FEA-predicted strength data. The magnitude of \(k_p\) varies linearly with the slenderness ratio reducing from 0.8 for a slenderness ratio of 70 to 0.6 at a slenderness ratio of 150.

Figures 6-5 a) to e) compare the capacity increases computed using Equations 6.1 and 6.5 and \(k_p\) values from Table 6-3 to the FEA-predicted increases. The values computed using the simplified equation are almost universally conservative with respect to those predicted using FEA. The few instances of slight unconservativeness correspond to ranges of \(kL/r_y\), \(\alpha\), and \(A_t/A_o\) that are also slightly unconservative in Figure 6-2. As the area ratio, \(A_t/A_o\), is based on the gross area, the upper limit on area ratio is increased to 4 for the perforated column. Appendix D3 presents the FEA-predicted capacities generated for perforated columns with \(A_t/A_o\) of 4.
Figure 6-4: Partial-length flange reinforcing plate design for weak-axis buckling of a perforated steel column with $F_{yo}$ of 350MPa and adjusted $A/A_o$, using revised coefficients from Table 6-3 and $k_p$ of 1.0, compared to FEA-predicted strengths for:

a) $kL/r_y$ of 150, b) $kL/r_y$ of 130, c) $kL/r_y$ of 110, d) $kL/r_y$ of 90, and e) $kL/r_y$ of 70
Figure 6-5: Partial-length flange reinforcing plate design for weak-axis buckling of a perforated steel column with $F_{yo}$ of 350MPa and adjusted $A_t/A_o$, using revised coefficients and $k_p$ from Table 6-3, compared to FEA-predicted strengths for: a) $kL/r_y$ of 150, b) $kL/r_y$ of 130, c) $kL/r_y$ of 110, d) $kL/r_y$ of 90, and e) $kL/r_y$ of 70
6.3.2 Adjusted Plate Coefficients ($F_{yo} = 228\text{MPa}$)

Figures 6-6 a), b), and c) compare a limited number of capacity increases calculated using Equation [6.1] and [6.5], respectively, using coefficients and $k_p$ values from the right column of Table 6-3 to FEA-predicted strength increases. The range of values presented is sufficient to confirm that the relationships between the perforated and unperforated columns with $F_{yo}$ of $228\text{MPa}$ are comparable to the previously discussed columns with $F_{yo}$ of $350\text{MPa}$. Appendix D3 presents the FEA-predicted capacities for perforated columns with $A_t/A_o$ of 1.5 and 2.5.

Figure 6-6: Partial-length flange reinforcing plate design for weak-axis buckling of a perforated steel column with $F_{yo} = 228\text{MPa}$ and adjusted $A_t/A_o$, using revised coefficients and $k_p$ from Table 6-3, compared to FEA-predicted strengths for:

a) $kL/r_y$ of 150, b) $kL/r_y$ of 130, and c) $kL/r_y$ of 110.
6.4 SIMPLIFIED ASSESSMENT PROCEDURE

The following procedure may be used to design partial-length reinforcement for a column to achieve a desired increase of capacity, \( C_t/C_o \), with or without perforations:

1. Calculate the weak-axis slenderness ratio, \( kL/r_y \), and capacity of original column, \( C_{ro} \).
2. Determine whether partial-length reinforcing plates may be a possible solution by checking \( C_t/C_o < (C_{r,max}/C_{ro}) \), with \( C_{r,max} \) computed using Equations [6.2] and [6.3] with \( \alpha = 0.8 \). If \( C_t/C_o \geq (C_{r,max}/C_{ro}) \), use full-length reinforcing plate design proposed by Shek (2006).
3. Check minimum area of plate needed, \((A_t/A_o)_{min}\) using Equation [1.3]. If \((A_t/A_o)_{min} \geq 3\), use full-length reinforcing plate design proposed by Shek (2006).
4. Select coefficients A, B, C, D, and m, from Table [6-3] if the yield strengths of the column is 350MPa or from Table [6-4] if the yield strength of the column is 228MPa.
5. Optimize cost of partial-length reinforcement by selecting an initial value of \( A_t/A_o \geq (A_t/A_o)_{min} \), and use goal-seek or convergence techniques to solve for \( \alpha \) using Equation [6.1]. If \( \alpha > 0.8 \) or \( \alpha < 0.2 \), the solution is either outside of the specified range or unconvvergent and thus inadmissable. Increase \( A_t/A_o \) and solve for \( \alpha \) again.
6. Minimum thickness of reinforcing plates, \( p = A_o(A_t/A_o - 1)/(2b_p) \).

Minimum length of reinforcing plates, \( L_r = \alpha L \).
7. Select practical reinforcing plate thickness and length. Calculate actual value of \( C_t/C_o \).
Additional steps for the bolt-hole perforated case:

8. Calculate nominal additional plate thickness, $\Delta p$, using Equation [6.4].
9. Select adjustment factor, $k_p$, from Table [6-3] if the yield strengths of the column is 350MPa or from Table [6-4] if the yield strength of the column is 228MPa.
10. Calculate adjusted reinforcing plate thickness, $p_2$, using Equation [6.5].
11. Select practical reinforcing plate thickness and length. Calculate actual value of $C_t/C_o$.

6.4.1 Design Examples

Example 1: Unperforated W310x158 column for Grade 350W steel

Design steel flange reinforcing plates to provide a capacity increase of $C_t/C_o = 1.75$ for a W310x158 column rolled from Grade 350W steel, with an effective length of 10m. Using the simplified assessment procedure, the length and thickness of reinforcing plates required are computed as follows:

1. The column slenderness ratio is computed for weak-axis buckling of the original column, with $r_y = 78.9\,mm$:
   
   \[ kL/r_y = 1.0 \times 10000\,mm/78.9\,mm = 126.7. \]

   The non-dimensional slenderness parameter is computed:
   
   \[ \lambda = kL/r_y \times (F_{yd}/(\pi^2 \times 200000\,MPa))^{1/2} \]
   
   \[ = 126.7 \times (350\,MPa/(\pi^2 \times 200000\,MPa))^{1/2} = 1.69 \]

   The capacity of the original column is therefore:
   
   \[ C_{ro} = \phi A_o F_{yd}(1 + \lambda^{2n})^{-1/n} \]
   
   \[ = 0.9 \times 20100\,mm^2 \times 350\,MPa \times (1+1.69(2 \times 1.34))^{-1/1.34} \times 10^{-3}kN/N = 1882\,kN \]
2. Compute $C_{r, \text{max}}$ for $\lambda_u = 0.8$. The dimensionless slenderness parameter is calculated:

$$\lambda_u = 0.2 \times \lambda = 0.2 \times 1.69 = 0.338$$

The maximum strength is computed as:

$$C_{r, \text{max}} = \varphi A_o F_{yo}(1 + \lambda_u^{2n})^{-1/n}$$

$$= 0.9 \times 20100 \text{mm}^2 \times 350 \text{MPa} \times (1 + 0.338(2 \times 1.34))^{-1/1.34}$$

$$= 6085kN$$

Therefore, $(C_{r, \text{max}}/C_{ro}) = (6085/1882) = 3.23 > 1.75$. A partial-length reinforcement solution is clearly possible.

3. The minimum plate area for a full-length reinforcement is computed as:

$$A_t/A_o = (F_{yo}/F_{yn}) \times (C_t/C_o - 1) = (350/350) \times (1.75 - 1) = 0.75.$$ 

This implies $A_t/A_o = 0.75 + 1 = 1.75 < 3.0$

The reinforcing plate area is in an acceptable range for a solution with partial-length reinforcement.

4. Using linear interpolation between the listed values for $kL/r_y$ of 110 and 130 on Table 6-3, the coefficients are computed as follows:

$$A = 2.196, \ B = 0.055, \ C = -0.75, \ D = -1.2, \ m = 1.21.$$ 

5. An input area is selected as $A_t/A_o$ of 1.75. Using goal-seeking techniques, $\alpha$ is determined from equating the two sides of the following equation:

$$\frac{C_t}{C_o} = \left\{ A \left[ \left( \frac{A_t}{A_o} \right)^{\alpha/2} \right] + B [\alpha^m] + \frac{C}{10} \left[ \left( \frac{A_t}{A_o} \right)^2 \alpha^m \right] + D \right\}^2$$

$$[6.1a] \quad 1.75 = \left\{ 2.19 \left[ (1.75)^{\alpha/2} \right] + 0.055[\alpha^{1.21}] + \frac{-0.75}{10} \left[ (1.75)^2 \alpha^{1.21} \right] + -1.2 \right\}^2$$ 

The equation is balanced with $\alpha$ of 0.639.

6. The reinforcing plate thickness is calculated:

$$p = A_o(A_t/A_o - 1)/(2b_p) = 20100 \text{mm}^2(1.75 - 1)/(2 \times 310) = 24.3 \text{mm}$$

Length of reinforcing plates is calculated:

$$L_r = \alpha L = 0.64 \times 10000 \text{mm} = 6400 \text{mm}$$
7. Reinforcing plate dimensions are selected as $p = 25\text{mm}$ and $L_r = 6400\text{mm}$.
   Recalculating Equation [6.1] with the actual values of $A_t/A_o = 1.771$ and $\alpha = 0.64$, $C_t/C_o = 1.769 > 1.75 \text{ ok}$

**Example 2a: Perforated W310x158 column for Grade 350W steel**

For the same case as Example 1, assume the column has 8 bolt-hole perforations over its cross section with a spacing of $100\text{mm}$ and diameter of $22\text{mm}$ each. Adjust the plate thickness using the following steps:

8. The theoretical additional plate thickness is calculated as:
   $\Delta p = \frac{1}{2} \times (N \times (t+p) \times d_b)/(b-((N/2) \times d_b))$
   $= 0.5 \times (8 \times (25.3\text{mm}+24.3\text{mm}) \times 22\text{mm})/(310\text{mm} -((8/2) \times 22\text{mm}))$
   $= 19.7\text{mm}$

9. Using linear interpolation between the values listed in Table 6-3, $k_p = 0.708$

10. The adjusted plate thickness is: $p_2 = p + k_p \Delta p = 24.3\text{mm} + (0.708 \times 19.7\text{mm})$
    $= 38.2\text{mm}$

11. Reinforcing plate thickness is selected as $p_2 = 40\text{mm}$.
    Reinforcing plate length is selected as $L_r = 6360\text{mm}$ to satisfy 64 rows of bolt-holes with $100\text{mm}$ spacing and a $30\text{mm}$ end distance.
    Recalculating Equation [6.1] with the actual values of $A_t/A_o = 1.792$ and $\alpha = 0.636$, $C_t/C_o = 1.781 > 1.75 \text{ ok}$

Thus, plate length is unchanged and plate thickness increases by 60%.
Example 2b: Perforated W310x158 column for Grade 350W steel (fixed plate area)

For the same perforated column as Example 2a, the plate thickness is fixed at \( p = 60mm \) to meet design criteria and optimize costs. Thus the adjusted plate thickness is known and steps 8 to 10 are performed in reverse order to determine the plate thickness of an equivalent unperforated column:

1. The adjustment factor from Table 6-3 remains the same at \( k_p = 0.708 \)

2. Rearranging Equations [6.4] and [6.5], the theoretical additional plate thickness is:
   \[
   \Delta p = \frac{N_d b (t+p)}{(2*(b-(N_d b/2))+(k_p N_d b))}
   \]
   \[
   = 8 \times 22mm \times (25.3mm + 60mm)/(2*(310mm-(8 \times 22mm/2))) + (0.708 \times 8 \times 22mm))
   \]
   \[
   = 26.4mm
   \]

3. Rearranging Equation [6.5], the plate thickness for the equivalent unperforated column is:
   \[
   p = p_2 - k_p \Delta p = 60mm - (0.708 \times 26.4mm) = 41.3mm
   \]
   This plate thickness corresponds to \( A_t/A_o \) of 2.274

4. Using goal-seeking techniques, \( \alpha \) is determined from equating the two sides of the Equation [6.1]. The equation is balanced with \( \alpha \) of 0.403.
   The adjusted reinforcing plate thickness is \( p_2 = 60mm \)
   Reinforcing plate length is selected as \( L_r = 4160mm \) to satisfy 42 rows of bolt-holes with 100mm spacing and a 30mm end distance.
   Recalculating Equation [6.1] with the actual values of \( A_t/A_o = 2.274 \) and \( \alpha = 0.416 \),
   \[
   C_t/C_o = 1.778 > 1.75 \text{ ok}
   \]
6.4.2 Cost Optimization

Section 2.3 previously discussed design of partial-length reinforcing plates based on minimum cost. Using the material and labour cost assumptions included in Equation [2.5] with a standard Q of 1.0, the overall cost of compression member strengthening tends to be lower as reinforcing plate length decreases and the corresponding plate thickness increases to maintain a desired increase in capacity. The overall costs of strengthening the member to satisfy \( C_t/C_o = 1.75 \) are estimated for the reinforcing plates designed in Examples 2a and 2b as $32,170 and $22,250, respectively, using Equation [2.5]. Increasing the plate thickness in Example 2b allows for use of shorter reinforcing plates than Example 2a, and thus reduces the cost by 36%. Table 6-5 summarizes the costs and cost savings of Example 2b compared to 2a for cases when either material or labour costs are maximized. Using the plate dimensions calculated in Example 2b provide cost savings of 30% and 40% for maximized material costs and maximized labour costs, respectively, demonstrating that savings are relatively insensitive to the cost assumptions. Thus the cost can generally be minimized by selecting \( A_t/A_o \) for the greatest allowable reinforcing plate thickness, up to a maximum of \( A_t/A_o = 2.5 \).

Table 6-5: Costs and cost savings with varying material and labour assumptions

<table>
<thead>
<tr>
<th></th>
<th>Example 2a ( (\alpha = 0.636) )</th>
<th>Example 2b ( (\alpha = 0.416) )</th>
<th>Potential Savings (Example 2a compared to 2b)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( (A_t/A_o = 1.792) )</td>
<td>( (A_t/A_o = 2.274) )</td>
<td></td>
</tr>
<tr>
<td>( Q )</td>
<td>Cost ($)</td>
<td>Cost ($)</td>
<td></td>
</tr>
<tr>
<td>Standard</td>
<td>1.0</td>
<td>32 170</td>
<td>22 250</td>
</tr>
<tr>
<td>Max. Material Cost</td>
<td>2.0</td>
<td>26 170</td>
<td>19 550</td>
</tr>
<tr>
<td>Max. Labour Cost</td>
<td>0.5</td>
<td>54 240</td>
<td>36 090</td>
</tr>
</tbody>
</table>
6.5 SUMMARY AND CONCLUSIONS

This chapter has presented a simplified equation for the preliminary design of partial-length flange reinforcing plates to increase weak-axis buckling capacity of W-shape columns. The equation uses sets of coefficients that depend on the slenderness ratio and on the steel grade of the original column. The effect of bolt holes is accounted for by calculating an increased plate thickness that is a function of the area removed by the bolt holes.

1. The simplified design procedure may be used with $F_{yo} = F_{yn} = 350\,\text{MPa}$ for slenderness ratios from 70 to 150, or with $F_{yo} = 228\,\text{MPa}$, and $F_{yn} = 350\,\text{MPa}$ for slenderness ratios ranging from 110 to 150.

2. The values calculated with Equation [6.1] are generally conservative with respect to FEA-predicted column capacities. Equation [6.1] is over-conservative at the discontinuity between failure mode by inelastic buckling of the reinforced section and failure in the unreinforced ends of the original column.

3. As noted for the linear-Elastic-buckling-based procedures in Section 2.3, the cost optimum is approached by decreasing the plate length and increasing the plate area to minimize the cost of labour without markedly increasing the cost of materials.
CHAPTER 7: SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS FOR FUTURE WORK

7.1 SUMMARY

Compression members in existing steel truss bridges designed using the Working Stress Design provisions of the S6-52 Specification for Steel Highway Bridges (CSA, 1952) may be deficient in capacity according to CAN/CSA S6-14 (CSA 2014). The capacity of a W- or I-shaped compression member can be increased by adding steel plates to the flanges to create a hybrid member with improved compressive capacity. Previous design solutions have been developed to design flange reinforcing plates along the entire length of the column, but full-length reinforcement may not be the most economical. The current thesis has investigated weak-axis buckling in columns with partial-length flange reinforcement about column mid-height and developed a simplified equation for preliminary design.

Existing Euler-buckling-based methodologies for columns with cross-sections of varying stiffness were reviewed in the context of designing partial-length reinforcement. The capacity increase of a reinforced column that fails by Euler buckling was examined for varying lengths and thicknesses of reinforcing plate, and compared to the inelastic buckling case to identify general trends. Additionally, a preliminary cost-optimization procedure based on Euler-buckling capacity of partial-length reinforced columns was presented for a range of assumed material and labour costs.

ANSYS Mechanical (ANSYS 2012) was used to develop a 3-D Finite Element Analysis (FEA) model to simulate inelastic buckling of steel compression members. A combination of temperature gradients and incremental load steps were used to simulate realistic initial conditions, including residual stress distributions, out-of-straightness, and locked-in dead load stresses. Bolt-hole geometry was modelled to allow examination of local effects around perforations,
though the attachment of original column and reinforcing plate was idealized as a simple friction connection.

After the FEA model was validated for unreinforced columns, an experimental test was conducted to validate its accuracy in predicting the compressive capacity of a column reinforced with partial-length flange plates. The full-scale column test was performed in the Structures Lab at The University of Western Ontario. The selected column specimen was selected to represent a median case where length of reinforcing plates approaches half of the column length and the plate thickness is slightly larger than the thickness of flange being reinforced. The reinforcing plates were attached to the column flanges by bolted connections. Ancillary tests were conducted to induce initial-out-of-straightness and to determine material properties.

A sensitivity analysis was conducted, using the validated FEA model, for both an unperforated column and for a column with flanges perforated by bolt holes in the reinforced region. Practical requirements and the underlying mechanics of buckling were considered in the selection of the ranges of reinforcing plate dimensions, the bolt-hole configurations, and the specific W-shape modelled. The parametric study investigated reinforcing plate lengths and area ratios over a range of slenderness ratios representing the intermediate range of columns, repeated for an unperforated and perforated case and for two different steel grades. Additional parameters investigated were: residual stresses, locked-in dead load stress, and initial out-of-straightness.

Using the FEA-predicted strengths from the sensitivity analysis, simplified equations were developed for the preliminary design of partial-length flange reinforcing plates to increase weak-axis buckling capacity of W-shape columns. The simplified design procedure is capable of accounting for the effect of perforations, involving additional steps to calculate an increased reinforcing plate thickness.
7.2 CONCLUSIONS

Limitations of the research presented in this thesis are as follows:

- Possible corrosion of the original column in-service is ignored.
- The range of steel yield strengths of the original column and reinforcing plates investigated is limited.
- Non-slip connection between the flange of the original column and the reinforcing plate is assumed at all applied load levels.
- Possible slip of the bolts into bearing is ignored.

Subject to these limitations, the following conclusions are made:

1. The empirically-based unreinforced column curve given in CAN/CSA S6-14 can be accurately reproduced using non-linear FEA that accounts for a linear residual stress pattern with a magnitude up to \(0.3F_{yo}\) and an initial out-of-straightness of \(L/1000\).
2. ANSYS Mechanical Simulation can effectively predict the weak-axis capacity increase for a column reinforced with partial-length flange reinforcing plates. The failure load observed for the experimental test column was within 1% of the FEA-predicted failure load.
3. The capacity increases for columns with partial-length flange reinforcing plates are limited by two distinct failure modes: 1) Compressive capacity of the original column, initiating at the unreinforced end segments; and 2) Inelastic buckling capacity of the reinforced region, initiating at column mid-height. For columns with longer reinforced lengths there is a dramatic transition between the first failure mode occurring at lower slenderness ratios and greater reinforcement areas, and the second failure mode occurring at higher slenderness ratios and lower reinforcement areas.
4. Bolt-hole perforations have a significant effect on the column capacity due to the reduction of axial and flexural stiffnesses in the reinforced region, for the range of reinforcing plate lengths and areas investigated. The capacity
decrease can be quantified as a function of the net area at a cross-section through the perforations.

5. The steel grade of the original column has a significant effect on the column capacity. The capacities of low-grade columns with partial-length reinforcing plates are often limited by the capacity of the original member at its unreinforced ends.

6. Locked-in dead load stresses have a minimal effect on the compressive capacity of an unperforated column. However, locked-in dead load stresses have a more significant effect for a perforated column, and may therefore be evaluated on a case-by-case basis.

7. Residual stresses have minimal effect on the capacity of a column reinforced with partial-length reinforcing plates.

8. Out-of-straightness has a minimal effect on the capacity of a column reinforced with partial-length reinforcing plates.

9. The proposed simplified equation provides a generally conservative means of estimating the size of partial-length flange reinforcing plates to achieve a desired column capacity increase. It is applicable for W-shape columns with $F_{yo} = F_{yn} = 350 \text{MPa}$ for slenderness ratios $70 < \frac{kL}{r_y} < 150$, and for W-shape columns with $F_{yo} = 228 \text{MPa}$ and $F_{yn} = 350 \text{MPa}$ for slenderness ratios $110 < \frac{kL}{r_y} < 150$.

10. The potential for cost savings is more significant for more slender columns. The cost optimum for partial-length reinforcing plates is typically approached by minimizing the reinforcing plate length to minimize the cost of labour required for installation.
7.3 RECOMMENDATIONS FOR FUTURE WORK

Recommendations for future work are as follows:

1. Investigate behavior and provide simplified design methods for strong-axis buckling.

2. Develop simplified design methods that account for locked-in dead load stresses.

3. Conduct in-depth investigation of bolted attachments, such as accounting for initial stresses at the edges of reinforcing plates, effects of bolt-slip, etc.

4. Investigate the impact of weld stresses in partial-length reinforcing plates on the capacity of the reinforced member.

5. Investigate tolerances on plate widths that are greater than or less than the width of the reinforced flange.

6. Expanded model validation and investigation of locked-in dead load stresses.
REFERENCES


APPENDIX A1

APPLIED CONVENTIONAL FORMULAE
A1.1 EQUATIONS FOR MOMENT OF INERTIA

- The weak-axis second moment of area, $I_w$, used throughout this thesis are generally based on the specified value for a given W-shape.
- The gross weak-axis second moment of area for a built-up section with flange reinforcing plates, $I_t$, is therefore:

$$[A1-1] \quad I_t = I_o + \frac{pb_p^3}{6}$$

where $p$ is the plate thickness and $b_p$ is the plate width.

- The net weak-axis second moment of area, $I_n$, is calculated depending on the number of bolt holes in the flange. For a column with four boltholes in the net section (i.e. two on each flange):

$$[A1-2] \quad I_n = I_t - 4d_b(t+p)\left(\frac{b_p}{2} - s_{edge}\right)^2 - \frac{N(t+p)d_b^3}{12}$$

where $d_b$ is the bolt diameter, $s_{edge}$ is the distance from centre of bolt to edge of flange tip, and $N$ is the number of bolts in the cross-section.
APPENDIX A2

ANSYS APLD INPUT FILES
A2.1 APLD FOR UNPERFORATED AND PERFORATED COLUMNS

FINISH
/CLEAR

/title, W-Section Column Compressive Resistance

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

! Input dimensional parameters
  \(d=106\) ! W-section depth (mm)
  \(b=103\) ! W-section width (mm)
  \(t=8.8\) ! Flange thickness (mm)
  \(w=7.1\) ! Web thickness (mm)
  \(r=6.7\) ! Radius of fillets (mm)
  \(L=2805\) ! Choose length of real-world column (mm)
  \(a=0.5\) ! Proportion of reinforced length. \(0 < a < 1\).

  \(L_r=a*L\) ! Calculate reinforcing plate length, or input (mm)
  \(p=12\) ! Reinforcing plate thickness (mm).

  *if,p,GT,0,then
  /title, W-section Reinforced Column Compressive Resistance
  reinforced=1 ! "1" for reinforced and "0" for unreinforced
  *endif

! Properties of fasteners
  ! Dimensions determined prior to program input
  \(Quantity=10\) ! Number of longitudinal fasteners in half-column

  \(Diameter=18\) ! Diameter of bolt hole (mm)
  \(Rows=1\) ! "1" for 4 transverse rows or "2" for 8 rows
  \(End=28\) ! Bolt hole end distance
  \(EdgeA=22\) ! Distance from outer bolt hole to flange tip
  \(EdgeB=95\) ! Distance from inner bolt hole to flange tip
  \(Spacing=((L_r/2)-End)/(Quantity-0.5)\) ! Bolt hole longitudinal spacing

! Input loading parameters
  \(Force=-700000\) ! Input Compressive Force (N) (negative sign)
  \(SC=-321000\) ! Compressive capacity of unreinforced column (N)
  \(LID=0\) ! proportion of Locked-in dead load.

  *if,LID,GT,0,then
  lockedstr=1 ! "1" for LID case and "0" for no LID
  FLID=LID*SC ! Force applied for Locked-in dead load stress
  *endif

! Input properties of Steel Column
  \(EWSection=200000\) ! Modulus of Elasticity of Column (MPa)
  \(Fy=350\) ! Yield stress in column steel (MPa)
Fu=450 ! Ultimate stress (MPa)
OOS=1000 ! Initial out-of-straightness (i.e. 1/OOS)

ThermPOS=45.5 ! Approximate residual stresses
ThermNEG=-44 ! for residual stress of 0.3Fy for 350MPa
ThermWEB=45
ThermExp=12-006 ! Coefficient of Thermal Expansion
ThermCOND=655 ! Coefficient of thermal conductivity

! Properties of reinforcing plate
ERFT=200000 ! Modulus of Elasticity
Fyr=350 ! Yield Stress
Fur=450 ! Ultimate Stress

! Calculate Thermal properties
IniRadDef=((L/2)*((1/OOS)+(OOS/4))) ! Calculate Radius of Curvature
ThermGRAD=((b/2)/(IniRadDef*ThermExp)) ! Calculate thermal gradient

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

! Create W-Section from solid elements on the X-Z plane (orient weak axis on the z-axis)

! Pre-Processor
/PREP7

! Dimensional Properties
K,1, -(w/2) , ,0, ! Define Keypoints for x-z cross-section
K,2, (w/2) , ,0,
K,3, -b/2 , ,d/2,
K,4, -b/2 , ,((d/2)-t),
K,5, -((w/2)+r), ,((d/2)-t),
K,6, -w/2 , ,((d/2)-(t+r)),
K,7, 0 , ,d/2,
K,8, 0 , ,d/2-
K,9, -((w/2)+r), ,((d/2)-(t+r)),

LARC,5,6,9,r, ! Create Arcs
KDELE, 9 ! Delete Keypoints to conserve space

LSTR,3,4 ! Create Lines
LSTR,4,5

FLST,3,3,4,ORDE,2 ! Reflect Lines (& keypoints) in z-axis
FITEM,3,1
FITEM,3,-3
LSYMM,X,P51X, , ,0,0

LSTR, 3, 8 ! Add Lines to complete shape
LSTR, 8, 11
LSTR, 6, 7
*if, reinforced, EQ, 1, then
   K, 13, -(b/2), (d/2)+p
   K, 14, b/2, (d/2)+p
*else
   K, 13, -(b/2), (d/2)+1
   K, 14, b/2, (d/2)+1
*endif
*do,i,1,Quantity
CYL4,-((b/2)-EdgeA),(Spacing/2)+((i-1)*Spacing),(Diameter/2), , , ,((d/2)+p+1)
*enddo

*do,i,1,Quantity
CYL4,((b/2)-EdgeA),(Spacing/2)+((i-1)*Spacing),(Diameter/2), , , ,((d/2)+p+1)
*enddo

*if,Rows,EQ,2,then ! Additional boltholes for two rows
*do,i,1,Quantity
CYL4,-((b/2)-Edgeb),(Spacing/2)+((i-1)*Spacing),(Diameter/2), , , ,((d/2)+p+1)
*enddo

*do,i,1,Quantity
CYL4,((b/2)-Edgeb),(Spacing/2)+((i-1)*Spacing),(Diameter/2), , , ,((d/2)+p+1)
*endif

vsbv,all,1
vsbv,all,2

*do,i,8,(Quantity*(Rows*2))+5
vsbv,all,i
*enddo

*endif

! Define Volume areas
FlangeR=6
FlangeL=5
PlateVol=4
WebVol=3

*if,Quantity,GT,0,then
  *if,Rows,EQ,2,then ! Two rows of boltholes (always even #)
    FlangeR=1
    FlangeL=4
    PlateVol=6
  *elseif,mod(Quantity,2),EQ,0,then ! Single row of bolts with even number
    FlangeR=1
    FlangeL=4
    PlateVol=6
  *else, ! Single row of boltholes with odd number
    FlangeR=1
    FlangeL=5
    PlateVol=4
*endif

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
! Mesh main column and define Structural properties

! Define Thermal Material properties and mesh volume of main column
ESIZE,t/3 ! Typical element size near section edges.
MOPT,EXPND,6 ! Elements at mid-section expand up to 6x larger
ET,1,SOLID87 ! Thermal element used for the column
MP,kXX,1,ThermCOND

VMESH,FlangeL ! Mesh main column
VMESH,FlangeR
VMESH,WebVol

! Define Thermal Material properties and mesh volume of end cap
ESIZE,t ! Element size for the end cap
ET,2,SOLID87 ! End cap set as Type 2 material
MP,kXX,2,ThermCOND
VATT,2,2,2 ! Mesh Type 2 material
VMESH,7

! Write Thermal Material Properties
PHYSICS,WRITE,THERMAL ! Write physics environment as thermal
PHYSICS,CLEAR ! Clear the environment

! Structural Material Properites
ET,1,SOLID187 ! Column Structural elements
MP,EX,1,EWSection
MP,PRXY,1,0.3
MP,CTEX,1,ThermExp

TB,MISO,1,1,2 ! Non-linear stress-strain profile
TBPT,DEFI,Fy/EWSection,Fy ! Slope equal to modulus of elasticity
TBPT,DEFI,.03,Fu ! Ultimate stress at strain of 0.03

ET,2,SOLID187 ! Structural elements
MP,EX,2,200000000000 ! End cap Modulus of Elasticity
MP,PRXY,2,0.3
MP,CTEX,2,ThermExp

!Write Structural Material Properties
PHYSICS,WRITE,STRUCT ! Write physics for structural properties
PHYSICS,CLEAR ! Clear all physics

FINISH

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

! Solution
/SOLU
! Pre-Load Step 1 => Apply Thermal Convection to induce residual stresses
NLGEOM,ON ! Large-deflection effects on (includes SSTIF)
LNSRCH,ON ! Line search on
ANTYPE,STATIC ! Static analysis
PHYSICS,READ, THERMAL ! Read in the thermal environment

SFA,37,1,CONV,ThermCOND,ThermPOS ! Apply temperatures on flange tips
SFA,41,1,CONV,ThermCOND,ThermPOS
SFA,20,1,CONV,ThermCOND,ThermWEB ! Apply temperature at web centre
SFA,38,1,CONV,ThermCOND,ThermNEG ! Apply temp at web-flange interface

SOLVE
FINISH

/SOLU

! Pre-Load Step 1 => Solve Stresses for Temperatures to induce residual stresses
PHYSICS,READ,STRUCT ! Read in Structural environment
LDREAD,TEMP,,, , , 'file', 'rth', ', ' ! Apply loads from thermal environment
TREF,0

! Constraints - DOF
DA,1,UY, ! Vertical deflection constraint at column base
DA,2,UY,
DA,3,UY,
DA,4,UY, ! vertical deflection at reinforcing plate base
DA,5,UY,
DA,12,UY,
DA,18,UY,

DK,7, , , , 0,UX, , , , , ! Symmetry constraint
DA,20,UZ,

SOLVE
INISTATE,WRITE,1,,,,0,s ! Stress state induced will be saved
SOLVE ! Solve and save stress state
INISTATE,WRITE,0,,,,0,s ! Overwrite stress state (but don't save)
SOLVE
UPCOORD,-1,ON ! Return coordinates to their original position
FINISH

PHYSICS,CLEAR

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
! Mesh Reinforcing Plates
*if,reinforced,EQ,1,then
/PREP7

ET,1,SOLID87
MP,kXX,1,ThermCOND
VATT,1,1,1

ET,2,SOLID87
MP,kXX,2,ThermCOND
VATT,2,2,2

ESIZE,t/3
MOPT,EXPND,6
ET,3,SOLID87
MP,kXX,3,ThermCOND
VATT,3,3,3
VMESH,PlateVol

PHYSICS,WRITE,THERMAL
PHYSICS,CLEAR

ET,1,SOLID187
MP,EX,1,EWSction
MP,PRXY,1,0.3
MP,CTEX,1,ThermExp
TB,MISO,1,1,2
TBPT,DEFI,Fy/EWSction,Fy
TBPT,DEFI,.03,Fu

ET,2,SOLID187
MP,EX,2,200000000000
MP,PRXY,2,0.3
MP,CTEX,2,ThermExp

ET,3,SOLID187
MP,EX,3,ERFT
MP,PRXY,3,0.3
MP,CTEX,3,ThermExp
TB,MISO,3,1,2
TBPT,DEFI,Fyr/ERFT,Fyr
TBPT,DEFI,.03,Fur

PHYSICS,WRITE,STRUCT
PHYSICS,CLEAR
*endif
! Pre-Load Step 2 => Apply Thermal Gradient to induce bending
   NLGEOM,ON ! Large-deflection effects on (includes SSTIF)
   LNSRCH,ON ! Line search on
   ANTYPE,STATIC ! Static analysis
   PHYSICS,READ,THERMAL ! Read in the thermal environment

   SFA,37,1,CONV,ThermCOND,ThermGRAD ! Apply thermal gradient on flange tips
   SFA,41,1,CONV,ThermCOND,-ThermGRAD
   SFA,20,1,CONV,ThermCOND,0
   SFA,38,1,CONV,ThermCOND,0
   *if,reinforced,EQ,1,then ! Apply thermal gradient on RFT plates
      SFA,27,1,CONV,ThermCOND,ThermGRAD
      SFA,30,1,CONV,ThermCOND,-ThermGRAD
   *endif

   SOLVE
   FINISH

! Pre-Load Step 2 => Solve Stresses for Temperatures to induce bending
   PHYSICS,READ,STRUCT ! Read in Structural environment
   INISTATE,READ,FILE,IST ! Read in saved stresses as an initial state
   INISTATE,LIST
   OUTRES,ALL,ALL

   LDREAD,TEMP,,,,'file','rth',' '
   TREF,0 ! Constraints - DOF

   DA,1,UY, ! Vertical deflection constraint at column base
   DA,2,UY,
   DA,3,UY,
   DA,4,UY,

   DK,7,,0,UX,, , , ! Use symmetry to model only half of column
   DA,20,UX, ! Fix base from sliding

! Load Step 1 => Set initial residual stress and out-of-straightness state
   TIME,1 ! Time = 1 after load step
   NSUBST,1,1,1,ON ! Only one substep in this load step
   NLGEOM,ON ! Large-deflection effects on (includes SSTIF)
   LNSRCH,ON ! Line search on
   ANTYPE,STATIC, ! Static analysis
   NROPT,FULL
   OUTRES,ALL,ALL ! Write all items from all substeps
LSWRITE,1

! Load Step 2 -> Apply initial Axial Loading
TIME,5
DELTIM,0.5,0.5,0.5
NLGEOM,ON
LNSRCH,ON
ANTYPE,STATIC,
NROPT,FULL
OUTRES,ALL,ALL

*if,lockedstr,EQ,1,then
VSEL,S,,PlateVol
ESLV,S
EKILL,ALL
ESEL,ALL
*endif

DA,1,UY,
DA,2,UY,
DA,3,UY,
DA,4,UY,

DK,7,,0,UX,,
DA,20,UZ,

! Initial loading applied relatively rapidly to reduce computation time

*if,lockedstr,EQ,1,then
FK,31,FY,(FLID/2)
*else
FK,31,FY,((Force/2)/2)
*endif

LSWRITE,2

! Load Step 3 -> Decrease size of load steps for greater accuracy
TIME,10
DELTIM,0.05,0.05,0.05
NLGEOM,ON
LNSRCH,ON
ANTYPE,STATIC,
NROPT,FULL
OUTRES,ALL,ALL

*if,lockedstr,EQ,1,then
VSEL,S, , ,PlateVol
ESLV,S
EALIVE,ALL
ESEL,ALL
*endif

DA,1,UY, ! Vertical deflection constraint at column base
DA,2,UY,
DA,3,UY,
DA,4,UY,
DK,7, , ,0,UX, , , , , ! Use symmetry to model only half of column
DA,20,UZ, ! Fix base from sliding

FK,31,FY,(Force/2) ! Half force for T-shape half column
LSWRITE,3 ! Write current data into Load Step File 3

! Solve
LSSOLVE, 1,3,1 ! Solves all Load steps together

FINISH

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

! Post-Processing
VSEL,S, , ,FlangeL
VSEL,A, , ,FlangeR
VSEL,A, , ,WebVol
ESLV,S

/POST1

! set commands to view appropriate load step
SET,LAST ! Severe distortion is typical on the last step
SET,PREVIOUS ! Readings taken from the 2nd last step.
PLDISP,0

A2.2 APLD FOR OUTPUT MACRO

! Initial State Measurements taken at first step
SET,FIRST
PLDISP,0

! Initial X-Deflection output on compression flange (column mid-height (base) - free end)
/output,ODEFLX,TXT
PATH,DeflCol,2,30,(L/2)/10,
PPATH,1,0,-(w/2),0,0,0,
PPATH,2,0,-(w/2),(L/2),0,0,
/PBC,PATH,1
/REPLOT
/PBC,PATH,0
AVPRIN,0,,
PDEF,,U,X,AVG
/PBC,PATH,,0
PRPATH,UX

! Initial Web Von Mises Stresses
/output,OWEBSTR,TXT
 PATH,Web,2,30,25,
 PPATH,1,0,0,(3/8)*L,(d-t)/2,0,
 PPATH,2,0,0,(3/8)*L,0,0,
 AVPRIN,0,,
PDEF,ResiStr,S,EQV,AVG
/PBC,PATH,,0
PRPATH,RESISTR

! Initial Flange Von Mises Stresses
/output,OFLSTR,TXT
 PATH,TopFL,2,30,50,
 PPATH,1,0,-(b/2),(3/8)*L,(d-t)/2,0,
 PPATH,2,0,(b/2),(3/8)*L,(d-t)/2,0,
 AVPRIN,0,,
PDEF,ResiStr,S,EQV,AVG
/PBC,PATH,,0
PRPATH,RESISTR

! Assume all measurements taken from 2nd last step
SET,LAST ! Severe distortion is typical on the last step
SET,PREVIOUS ! Readings taken from the 2nd last step.
PLDISP,0

! Force output for equivalent T-shape. Double for W-shape compressive capacity.
/output,TFORCECOL,TXT
PRRSOL,FY

! X-Deflection output on compression flange from column mid-height (base) to free end.
/output,TDEFLX,TXT
 PATH,DeflCol,2,30,(L/2)/10,
 PPATH,1,0,-(w/2),0,0,0,
 PPATH,2,0,-(w/2),(L/2),0,0,
! Z-Deflection output on compression flange from column mid-height (base) to free end.
/output,TDEFLZ.TXT
   PATH,DeflCol,2,30,(Length/2)/10,
   PPATH,1,0,(b/2),0,(d/2)-t,0,
   PPATH,2,0,(b/2),(Length/2),(d/2)-t,0,
   /PBC,PATH,1
   /REPL0T
   /PBC,PATH,0
   AVPRIN,0, ,
   PDEF, ,U,X,AVG
   /PBC,PATH, ,0
   PRPATH,UX

! Von Mises Stresses on compression flange edge (column mid-height (base) - free end)
/output,TSTRESS.TXT
   PATH,StressDist,2,30,(L/2)/10
   PPATH,1,0,(b/2),0,(d/2)-t,0,
   PPATH,2,0,(b/2),(L/2),(d/2)-t,0,
   /PBC,PATH,1
   /REPL0T
   /PBC,PATH,0
   AVPRIN,0, ,
   PDEF,ResiStr,S,EQV,AVG
   /PBC,PATH, ,0
   PRPATH,RESISTR

! Flange Von Mises Stresses in original column at mid-height (i.e. base of model)
/output,TMIDSTR.TXT
   PATH,Flng2,2,30,50,
   PPATH,1,0,b/2,0,(d-t)/2,0,
   PPATH,2,0,-b/2,0,(d-t)/2,0,
   AVPRIN,0, ,
   PDEF,ResiStr,S,EQV,AVG
   /PBC,PATH, ,0
   PRPATH,RESISTR
! Flange Von Mises Stresses in original column at edge of reinforcing plate
/output,TINTSTR,TXT
  PATH,Flng1,2,30,50,
  PPATH,1,0,b/2,Lr/2,(d-t)/2,0,
  PPATH,2,0,-b/2,Lr/2,(d-t)/2,0,
  AVPRIN,0,
  PDEF,ResiStr2,S,EQV,AVG
/PBC,PATH,0
PRPATH,RESISTR2
B1.1 INTRODUCTION

- 3 additional column specimens were planned for compressive capacity testing. Completed: pre-bending, bolt installation, and strain gauge installation.
- Tests could not be conducted due to a mechanical failure of the load actuator that resulted in the deflection controls being unusable.
- These tests are not critical to obtaining meaningful conclusions, thus they have been excluded from the present thesis and will be conducted at a later date.
- The column descriptions will be recorded here, including: initial geometry, FEA-predicted compressive capacity, stress distributions at failure, and strain gauge locations.

B1.2 SPECIMEN DESCRIPTIONS

- Table B1-1 summarizes the initial geometry of test specimens 2, 3 and 4.
- Table B1-2 shows the compressive capacity predicted for columns 2, 3 and 4.
- Table B1-3 shows the von Mises stress distributions at failure of the 3 columns.

Column 2

- Perforated Column. Selected to investigate the effects of the bolt hole perforations when there is no reinforcement present. Figure B1-1 a) shows the location of 10 strain gauges installed at the flange tips on the surface of the flange adjacent to the flange.

Column 3

- Unperforated Column. Selected to compare against the reinforced column(s) to determine whether predicted capacity increases are in a reasonable range. Compare to column 2 to determine the influence of pre-bending to induce out-of-straightness on compressive capacity. Figure B1-1 b) shows the location of 14 strain gauges installed at the flange tips on the interior surface of the flange.

Column 4

- Perforated column reinforced with short plate. Bolts pre-tensioned to 2/3 turn, which is beyond the typical recommendations. Selected to check consistency of results compared to Column 1, as reinforcing plate length is further decreased. Figure B1-1 c) shows the location of 14 strain gauges installed at the flange tips on the interior surface of the flange.
**Figure B1-1:** Strain Gauge Locations on a) Column 2, b) Column 3, and c) Column 4
Table B1-1: Summary of Initial Geometry

Measurements (mm)

<table>
<thead>
<tr>
<th>Specimen</th>
<th>d</th>
<th>b</th>
<th>t</th>
<th>w</th>
<th>d_b</th>
<th>g_1</th>
<th>s</th>
<th>L_r</th>
<th>p</th>
<th>δ_c</th>
<th>δ_k</th>
<th>δ_s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column 2</td>
<td>108.1</td>
<td>102.1</td>
<td>9</td>
<td>6.8</td>
<td>18</td>
<td>22</td>
<td>80</td>
<td>-</td>
<td>-</td>
<td>0.25</td>
<td>1</td>
<td>3.5</td>
</tr>
<tr>
<td>Column 3</td>
<td>108.1</td>
<td>102.1</td>
<td>9</td>
<td>6.8</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.5</td>
<td>1</td>
<td>3.2</td>
</tr>
<tr>
<td>Column 4</td>
<td>108.2</td>
<td>102.1</td>
<td>9</td>
<td>6.8</td>
<td>18</td>
<td>22</td>
<td>80</td>
<td>620</td>
<td>12.7</td>
<td>0.1</td>
<td>0</td>
<td>3.2</td>
</tr>
</tbody>
</table>

Table B1-2: Predicted Compressive Capacity

<table>
<thead>
<tr>
<th>Specimen</th>
<th>δ_{max} (mm)</th>
<th>P_{FEA} (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column 2</td>
<td>11.9</td>
<td>298</td>
</tr>
<tr>
<td>Column 3</td>
<td>15.7</td>
<td>410</td>
</tr>
<tr>
<td>Column 4</td>
<td>12.8</td>
<td>486</td>
</tr>
</tbody>
</table>

Table B1-3: Predicted Stress Distributions

von Mises Stresses, front view
APPENDIX B2

SUPPLEMENTARY TESTS
B2.1 INTRODUCTION
- This section presents the steel properties of the test column used in the full-scale load test and the additional columns planned to be tested.
- Table B2-1 provides an overview of the material data
- Intermediate calculations for estimating the required pre-bending deflection are also included

B2.2 TENSILE COUPON TESTS
- Conducted in accordance with ASTM 370 – 13 (ASTM 2013)
- Figure B2-1 shows the stress-strain profile for Test Group 1 (Columns 1&2).
- The full stress-strain profile is unavailable for Test Group 2 (Columns 3&4) due to errors with data logger.
  o Flange coupon: $F_y = 350\text{MPa}; F_u = 496\text{MPa}$
  o Web coupon: $F_y = 376\text{MPa}; F_u = 504\text{MPa}$
- Figure B2-2 shows the stress-strain profile for the 12.7mm reinforcing plate.

B2.3 STUB COLUMN TESTS
- Conducted in accordance with Tall (1961)
- Figures B2-3 and B2-4 show the stress-strain profiles obtained for Test Group 1 and Test Group 2, respectively. The maximum magnitude of residual stresses was obtained with good delineation.
- The tests were ended before reaching the yield plateaus due to the occurrence of excessive deflection and directional whitewash flaking as the flange yield stresses were being approached.

B2.4 CALCULATIONS FOR COLUMN PRE-BENDING PROCEDURE
- Equations B2-1 to B2-4 provide the intermediate steps for estimating $\delta_y$ used to calculate the magnitude of $\delta_{\text{max}}$ required for the pre-bending procedure:

\[
[B2-1] \quad \delta_y = \delta_{\text{ends}} + \delta_{\text{mid}} - \delta_{\text{DL}} - \delta_{\text{so}}
\]

\[
[B2-2] \quad \delta_{\text{ends}} = \frac{L - \ell_1}{2} \times \frac{P_y}{2} \times \left( \frac{\ell_1 - \ell_2}{2} \times \left( \frac{\ell_2 - \ell_1}{2} \right) \right)
\]
\[
\delta_{\text{mid}} = \frac{P_y}{2} \times \frac{\ell_1 - \ell_2}{2} \times \left[ \frac{3\ell_2 - 4 \left( \frac{\ell_1 - \ell_2}{2} \right)^2}{24EI_y} \right]
\]

\[
\delta_{DL} = \frac{5W\ell_2^4}{384EI_y} + \left[ \frac{(L - \ell_1) \times W\ell_2^3}{24EI_y} \right]
\]

where \( W \) is the column weight, \( I_y \) is the second moment of area in the weak axis, \( E \) is the modulus of elasticity, \( P_y \) is the predicted axial load at yielding, and \( L, \ell_1, \) and \( \ell_2 \) are dimensions defined in Figure 4-3 a).

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Tensile Coupons</th>
<th>Stub Column</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Flange ( F_y ) (MPa)</td>
<td>Web ( F_y ) (MPa)</td>
</tr>
<tr>
<td>Column 2</td>
<td>353</td>
<td>380</td>
</tr>
<tr>
<td>Column 3</td>
<td>350</td>
<td>376</td>
</tr>
<tr>
<td>Column 4</td>
<td>350</td>
<td>376</td>
</tr>
</tbody>
</table>
Figure B2-1: Tensile Coupon Test 1 for Columns 1 and 2

Figure B2-2: Tensile Coupon Test 2 for 12.7 mm Reinforcing Plate
Figure B2-3: Stub Column Test 1 for Columns 1 and 2

Figure B2-4: Stub Column Test 1 for Columns 3 and 4

$F_y = 353 \text{ MPa} \text{ (from coupon test)}$

$\sigma_r = 296 \text{ MPa}$

$F_y = 350 \text{ MPa}$

$\sigma_r = 310 \text{ MPa}$
APPENDIX B3

TEST APPARATUS DRAWINGS
B3.1 PILOT TEST DRAWINGS

Part 01: Assembly Base
- Dimensions: 300 x 300 mm
- Notes:
  1. Items outlined in red indicate the modifications to the existing apparatus.
  2. Part 02 is assumed to be bolted to the assembly base using 1/2" screws.
  3. It has been assumed that the screw-jacks from the apparatus will be reused (unless they are found to have been damaged, in which case new ones should be ordered).
  4. The portion of the screw-jack embedded into part 03 is to have the threads stripped, and the diameter of the holes are to matched the diameter of the stripped screw.

Part 02: Steel Spacers
- Dimensions: 83 x 50 mm

Part 03: Steel Clamp Detail
- Dimensions: 87 x 50 mm
- Notes:
  - screw-jack location

---

Assembly Base

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DWG 01
1. W100x19 Sections
2. 1/2" plate

3a. Typical tensile coupon
3b. Narrow tensile coupon for W100x19 flange

List of Materials:
- 1 x 40ft. length W100x19 section (delivered in 2-20 ft. length segments)
- 1 x 18ft. length steel plate (4x1/2", cold-rolled)
- 60 x 5/8"x2" A325 bolts

Machining for Phase 1:
- Re-fitting of end plates (see DWG 01)
- 2 x tensile coupons from W100x19 (see 3a and 3b)
- 1 x tensile coupon from steel plate (see 3a)
- 1 x 2300mm length cut from W100x19
- 1 x 480mm "stub column" length cut from W100x19
- 2 x 1100mm lengths cut from 1/2" plate
- 56 x Drill bolt holes, through column & steel plate (see DWG 03)

Notes:
1) Hatched areas indicate spare and/or cut-away material
2) Tolerances for Tensile coupons may be found in the notes for Figure 3 of ASTM A370-13
3) Tolerance on stub columns is +30mm –18mm (full 1911)
1. Exploded view. W100x19 steel section. 2 - 4"x1/2" steel plate.

1. Dimensions and Locations of Bolt Holes

Notes:
1) Holes are to be drilled for 5/8" diameter bolts. Oversize at 11/16" diameter (17.5 mm)
2) Reinforcing plates and bolt holes are to be centred on the beam at mid-height.

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Bolt Holes for Pilot Test

DWG 03
B3.2 PHASE TWO DRAWINGS

Notes:
1) Hatched areas indicate spare and/or previously used material

List of Materials
Column and Plate obtained previously
- 34 x 5/8" x 2" A325 bolts

Machining for Phase 2
- 2 x Tensile coupons from W100x19 (see DWG 2)
- 1 x 460 mm "stub column" length cut from W100x19 (see DWG 2)
- 3 x 2300 mm length cut from W100x19
- 2 x 620 mm lengths cut from 1/2" plate
- 56 x Drill bolt holes through column (see DWG 03)
- 32 x Drill bolt holes through column & steel plate (see DWG 04)

1. 2 - W100x19 Sections
2. 1/2" plate

Steel Sections (Phase 2)

SolidWorks Student Edition,
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1. web tensile coupon

2. flange tensile coupon

3. stub column

Notes:
1) Tolerances for Tensile coupons may be found in the notes for figure 3 of ASTM A370-13
2) Tolerance on stub columns is +30mm -18mm (fall 1961)
Notes:
1) Holes for column segment #2 are to be drilled for 5/8" diameter bolts. Oversize at 11/16" diameter (17.5 mm)
2) Bolt holes are to be centred on the beam at mid-height.

1. Column Segment #2
1 - W100x19 steel section

2. Column Segment #3
1 - W100x19 steel section
Notes:
1) Holes are to be drilled for 5/8" diameter bolts, Oversize at 11/16" diameter (17.5 mm)
2) Reinforcing plates and bolt holes are to be centred on the beam at mid-height.
3) 34 - 5/8" x 2" A325 bolts (Type 1.N)

1. Column Segment #4
   1 - W100x19 steel section

2. Reinforcing plate Segments
   2 - 1/2" steel plate

620 mm plate (Phase 2)
APPENDIX C1

FEA-GENERATED COLUMN CAPACITIES
a) **Figure C1-1**: Capacity at varying $\alpha$, unperforated, $F_{yo}$ of 350 MPa, for a) $A_t/A_o$ of 1.5, and b) $A_t/A_o$ of 2.5

b) **Figure C1-2**: Capacity at varying $\alpha$, perforated, $F_{yo}$ of 350 MPa, for a) $A_t/A_o$ of 1.5, and b) $A_t/A_o$ of 2.5
Figure C1-3: Capacity at varying $A_t/A_o$, unperforated, $F_{yo}$ of 350MPa, for a) $kL/r$ of 150, b) $kL/r$ of 130, c) $kL/r$ of 110, d) $kL/r$ of 90, e) $kL/r$ of 70
Figure C1-4: Capacity at varying $A_t/A_o$, perforated, $F_{yo}$ of 350 MPa, for a) $kL/r$ of 150, b) $kL/r$ of 130, c) $kL/r$ of 110, d) $kL/r$ of 90, e) $kL/r$ of 70
Figure C1-5: Capacity at varying $A_t/A_o$, unperforated, $F_{yo}$ of 228MPa, for a) $kL/r$ of 150, b) $kL/r$ of 130, c) $kL/r$ of 110, d) $kL/r$ of 90, e) $kL/r$ of 70
Figure C1-6: Capacity at $\sigma_r$ of 0.15$F_{yo}$, unperforated, $F_{yo}$ of 350MPa, for a) $A_t/A_o$ of 1.5, and b) $A_t/A_o$ of 2.5

Figure C1-7: Capacity at $\sigma_r$ of 0.45$F_{yo}$, unperforated, $F_{yo}$ of 350MPa, for a) $A_t/A_o$ of 1.5, and b) $A_t/A_o$ of 2.5
Figure C1-8: Capacity at $\sigma$ of 0.15$F_{yo}$, perforated, $F_{yo}$ of 350 MPa, for a) $A_t/A_o$ of 1.5, and b) $A_t/A_o$ of 2.5

Figure C1-9: Capacity at $\sigma$ of 0.45$F_{yo}$, perforated, $F_{yo}$ of 350 MPa, for a) $A_t/A_o$ of 1.5, and b) $A_t/A_o$ of 2.5
Figure C1-10: Capacity at $\sigma_{LID}$ of 0.30$F_{yo}$, unperforated, $F_{yo}$ of 350MPa, for a) $A_t/A_o$ of 1.5, and b) $A_t/A_o$ of 2.5

Figure C1-11: Capacity at $\sigma_{LID}$ of 0.30$F_{yo}$, perforated, $F_{yo}$ of 350MPa, for a) $A_t/A_o$ of 1.5, and b) $A_t/A_o$ of 2.5

Figure C1-12: Capacity at $v$ of $L/1000$, unperforated, $F_{yo}$ of 350MPa, for a) $A_t/A_o$ of 1.5, and b) $A_t/A_o$ of 2.5
APPENDIX D1

STEPWISE PROCEDURE TO DEVELOP DESIGN EQUATION
D1.1 INTRODUCTION

- Recognized that slenderness of the original column is a constant for the original column, but $A_t/A_o$ and $\alpha$ must be on a continuum to find the optimal $C_t/C_o$. Thus regression analysis was performed separately for each slenderness ratio to determine appropriate coefficients. The same general form of equation must be used in all cases so that interpolation between slenderness ratios is possible.

D1.2 REGRESSION ANALYSIS

- Figure D1-1 shows an example of the spreadsheet layout used for regression analysis by the method of least squares at $kL/r_y$ of 110. Each component is numbered as follows:

1. Input values for $A_t/A_o$, $\alpha$, and the FEA-generated values of $(C_t/C_o)_{observed}$.
2. Generate parameters from the input values; the known x’s.
3. Generate transformed parameters for the known y’s, i.e. $\sqrt{(C_t/C_o)_{observed}}$.
4. Calculate parameter estimates from the general relationship $y = Ax_1+Bx_2+Cx_3+D$, using the function LINEST(known_y’s, known_x’s, const, stats). The known y’s and known x’s are described in 2) and 3). If const is TRUE, the function calculates a value for D, and if FALSE, D is set to 0. If stats is TRUE, the function returns additional statistics.
5. Calculate untransformed $(C_t/C_o)_{predicted}$ using the parameter estimates, i.e.:
   $$(C_t/C_o)_{predicted} = (Ax_1+Bx_2+Cx_3+D)^2$$
6. Calculate an approximate quantification of mean square error, i.e.:
   $$\sqrt{MSE} = \frac{[(C_t/C_o)_{observed} - (C_t/C_o)_{predicted}]^2}{(N_d\text{-DOF})} < 0.05$$
   where
   - $N_d$ is the number of data entries (i.e. 18),
   - DOF is the parameter degrees of freedom (i.e. 4)
7. Check that each parameter is statistically significant by calculating the p-values. The p-value is based on a t-distribution of the parameter estimate divided by the parameter error, and the parameter is statistically significant for p-values < 0.05.
8. Manually adjust the coefficient $m$ as a function of slenderness ratio. This coefficient was added after it was noted that the exponential of $\alpha$ to produce the lowest error followed a consistent relationship with respect to $kL/r_y$. 
9. Calculate percent differences of the FEA-generated capacity increases compared to the calculated $C_i/C_o$.

- Tables D1-1, D1-2, and D1-3 show examples of the parameter combinations tried with linear regression for $kL/r_y$ of 150, 110, and 70 respectively.
- After completing the above steps, the parameter estimates are manually adjusted to minimize the unconservative error. Tables 6-1 and D1-4 show the initial parameter estimates generated for $F_y$ of 350MPa and 228MPa, respectively.
**Figure D1-1:** Example of Linear Regression by Method of Least Squares
### Table D1-1: Sample parameter trials with \(F_{yo}\) of 350MPa and \(kL/r_y\) of 150

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Max Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>(\alpha*(At/Ao))</td>
<td>(\alpha)</td>
</tr>
<tr>
<td>(\alpha*V(At/Ao))</td>
<td>(\alpha)</td>
</tr>
<tr>
<td>(\alpha*(At/Ao))</td>
<td>(\alpha)</td>
</tr>
<tr>
<td>(\alpha*V(At/Ao))</td>
<td>(\alpha)</td>
</tr>
<tr>
<td>(\alpha*V(At/Ao))</td>
<td>(\alpha)</td>
</tr>
<tr>
<td>(\alpha*V(At/Ao))</td>
<td>(\alpha)</td>
</tr>
<tr>
<td>(\alpha/V(At/Ao))</td>
<td>(\alpha)</td>
</tr>
<tr>
<td>(\alpha*V(At/Ao))</td>
<td>(\alpha)</td>
</tr>
<tr>
<td>((At/Ao)^{a^2})</td>
<td>(a^m)</td>
</tr>
<tr>
<td>((At/Ao)^{a^2})</td>
<td>(a^m)</td>
</tr>
<tr>
<td>((At/Ao)^{a^2})</td>
<td>(a^m)</td>
</tr>
<tr>
<td>((At/Ao)^{a^2})</td>
<td>(a^m)</td>
</tr>
<tr>
<td>((At/Ao)^{a^2})</td>
<td>(a^m)</td>
</tr>
</tbody>
</table>

### Table D1-2: Sample parameter trials with \(F_{yo}\) of 350MPa and \(kL/r_y\) of 110

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Max Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>(\alpha*(At/Ao))</td>
<td>(\alpha)</td>
</tr>
<tr>
<td>(\alpha/V(At/Ao))</td>
<td>(\alpha)</td>
</tr>
<tr>
<td>((At/Ao)^{a^2})</td>
<td>(a^m)</td>
</tr>
<tr>
<td>((At/Ao)^{a^2})</td>
<td>(a^m)</td>
</tr>
<tr>
<td>((At/Ao)^{a^2})</td>
<td>(a^m)</td>
</tr>
<tr>
<td>((At/Ao)^{a^2})</td>
<td>(a^m)</td>
</tr>
<tr>
<td>((At/Ao)^{a^2})</td>
<td>(a^m)</td>
</tr>
</tbody>
</table>

### Table D1-3: Sample parameter trials with \(F_{yo}\) of 350MPa and \(kL/r_y\) of 70

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Max Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>(\sqrt{(At/Ao)})</td>
<td>(\alpha)</td>
</tr>
<tr>
<td>((At/Ao)^{a^2})</td>
<td>(a^m)</td>
</tr>
<tr>
<td>((At/Ao)^{a^2})</td>
<td>(a^m)</td>
</tr>
<tr>
<td>((At/Ao)^{a^2})</td>
<td>(a^m)</td>
</tr>
</tbody>
</table>
Table D1-4: Unadjusted coefficients and error for equation with $F_{yo}$ of 228MPa

<table>
<thead>
<tr>
<th>$kL/r_y$</th>
<th>Coefficients</th>
<th>Error (%)</th>
<th>(\sqrt{\text{MSE}})</th>
<th>Error (%)</th>
<th>(\sqrt{\text{MSE}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>110</td>
<td>1.32</td>
<td>0.24</td>
<td>-0.90</td>
<td>-0.32</td>
<td>1.5</td>
</tr>
<tr>
<td>130</td>
<td>2.19</td>
<td>0.19</td>
<td>-1.25</td>
<td>-1.21</td>
<td>1.2</td>
</tr>
<tr>
<td>150</td>
<td>2.61</td>
<td>0.07</td>
<td>-1.18</td>
<td>-1.61</td>
<td>1.05</td>
</tr>
</tbody>
</table>
APPENDIX D2

ALTERNATE DESIGN EQUATIONS
D2.1 ALTERNATE SOLUTION A

- A tangent-based solution was developed $F_{yo}$ of 350MPa at a slenderness ratio of 110. Regression analysis was used to determine a general equation:

$$\sqrt{\frac{A_t}{A_o}} = A \left[ \tan \sqrt{\frac{\pi^2 C_t}{C_o}} \left( \frac{1-\alpha}{2} \right) \times \tan \sqrt{\frac{\pi^2 A_o C_t}{A_t C_o}} \left( \frac{\alpha}{2} \right) \right] + B \left[ \frac{A_o}{A_t} \right] + C$$

where A, B, and C are coefficients that vary for $\alpha$ of 0.2, 0.5, and 0.8.

- Figure D2-1 shows the FEA-predicted capacity increase compared to those calculated with Equation [D2.1]. The results are accurate within 2%, but the coefficients with respect to $\alpha$ have non-linear relationships that do not fit well to simple functions. In particular, behaviour at $\alpha$ of 0.2 appears discontinuous with the larger reinforced lengths.

- This design solution was discarded for being too overly complex for general use.

D2.2 ALTERNATE SOLUTION B

- A solution developed using regression analysis at slenderness ratio of 110 to calculate compressive resistance based on Equation [1-1]:

$$C_r = \phi A_o F_{y} (1 + \lambda_{eq})^{2n}$$

where all parameters are the same as for the original column, except for $\lambda_{eq}$ is an equivalent non-dimensional slenderness parameter for the reinforced column:

$$\lambda_{eq} = \frac{\lambda}{\lambda_n}$$

where $\lambda$ is the slenderness parameter of the original column, and $\lambda_n$ is the effective slenderness parameter in the unreinforced end segments of the built-up column:

$$\lambda_n = A - \frac{1}{\left[ \frac{A_t}{A_o} \right]} + B$$

where A and B are coefficients that vary for $\alpha$ of 0.2, 0.5, and 0.8.

- Figure D2-2 shows the FEA-predicted capacity increase at a slenderness ratio of 110 compared to those calculated with Equation [D2.2]. The near-quadratic
relationship of the coefficients leads to a much simpler continuous relationship between $A_t/A_o$ and $\alpha$ with the general forms $A = -A_1 \alpha^2 + A_2 \alpha - A_3$ and $B = B_1 \alpha^2 - B_2 \alpha + B_3$. However, there is a decrease in accuracy compared to Alternate Solution A, with calculated results for $\alpha$ of 0.8 being unconservative up to 7% between $A_t/A_o$ of 1.5 to 2.0, and overly-conservative up to 18% below $A_t/A_o$ of 1.5.

- As shown in Figure D2-3, at a slenderness ratio of 150 the error for $\alpha$ of 0.8 increases to being 10% unconservative and 20% conservative.

- This design solution was discarded due to unacceptable error, combined with complexities in the computation process.
**Figure D2-1:** Partial-length reinforcement with Alternate Solution A at $kL/r_y$ of 110
a) Calculated Capacity Increase, and b) Coefficients

**Figure D2-2:** Partial-length reinforcement with Alternate Solution B at $kL/r_y$ of 110
a) Calculated Capacity Increase, and b) Coefficients

**Figure D2-3:** Partial-length reinforcement with Alternate Solution B at $kL/r_y$ of 150
a) Calculated Capacity Increase, and b) Coefficients
APPENDIX D3

ADDITONAL FEA-GENERATED COLUMN CAPACITIES
Figure D3-1: Capacity at varying $\alpha$, perforated, for $F_{yo}$ of 350MPa and $A_t/A_o$ of 4

Figure D3-2: Capacity at varying $\alpha$, perforated, for $F_{yo}$ of 228MPa, and a) $A_t/A_o$ of 1.5, and b) $A_t/A_o$ of 2.5
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2011  Dean’s Honours List in the Faculty of Engineering Science
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      The University of Western Ontario

2008  The Western Scholarship of Excellence
      The University of Western Ontario

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Related Publications