A Model Of The Evolution Of Exchange Processes

George Chuchman

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LA THÈSE A ÉTÉ MICROFILMÉE TELLE QUE NOUS L'AVONS RÉCU
A MODEL OF THE
EVOLUTION OF EXCHANGE
PROCESS

by

George Chuchman

Department of Economics

Submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy

Faculty of Graduate Studies
The University of Western Ontario
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ABSTRACT

This dissertation develops a theoretical framework which describes how the evolution of exchange processes in a primitive decentralized economy, initially characterized by unorganized barter exchange, might proceed. The framework demonstrates how natural economic forces within this unorganized barter exchange economy can lead to the evolution of a system of organized markets where some traders have become middlemen and play a central role in coordinating exchange activity.

The approach taken is to develop a model of individual expected utility maximizing behavior in a two commodity unorganized barter exchange economy over time where time consists of successive fixed length trading periods. Actual exchange behavior during each trading period in the unorganized barter economy is characterized by a random bilateral contacting process involving the symmetrical exchange reservation bids between individuals. This has been modelled as a sequential decision-making process involving both price and quantity uncertainty.

The planned exchange behavior of an individual in successive trading periods is described by a stock-flow model which involves
enhancement of exchange opportunities through inventory accumulation. This provides the framework for describing the evolution of exchange processes through the distinction between short-run trading equilibria and long run intertemporal equilibrium. It is demonstrated how the exchange time path of an individual in successive trading periods, during the course of adjustment to intertemporal equilibrium, can be such that he will have evolved into a middleman.

The evolution of middlemen in an economy where, previously, only unorganized barter exchange among individuals existed would create changes in individual exchange opportunities and would alter the nature of the bilateral contacting process itself. Bilateral contacting with middlemen would be characterized by regular accessibility, by reduced price dispersion, and by the absence of quantity uncertainty. Since middlemen would be maintaining relatively large inventories, so that the evolution of one or more middlemen guarantees the emergence of an organized market system in the midst of the system of unorganized barter exchange. It is demonstrated how natural economic forces might lead to a situation where the organized market mediated by middlemen would emerge as the dominant exchange process in the economy leading to the virtual disappearance of unorganized barter exchange.

Also it is shown how the framework can be extended to describe how monetary exchange could evolve if middlemen and organized markets did not evolve evenly for some pairs of commodities. In this situation indirect exchanges in organized markets using a commonly traded commodity as a medium of exchange would be much more efficient than direct exchange in the unorganized barter sector.
To Irene and my parents,
my mother Maria and my late father John
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Also, I would like to thank my wife, Irene, for her patience, support and personal sacrifice during the years of my work and no less for her many hours of uncomplaining work in typing the various drafts of this thesis.
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Chapter 2

INTRODUCTION TO THE PROBLEM

The Walrasian general equilibrium model has proved unsatisfactory as a basis for developing a theory of the functioning of a monetary economy. Its shortcomings have been adequately discussed in the literature. Most of these shortcomings stem from the failure of the model to characterize in any meaningful way a very fundamental process of any economic system, the process of exchange. This deficiency restricts the usefulness of Walrasian theory to the study of long run equilibrium states. The relevance of Walrasian theory to the policy-oriented analysis of short-run disequilibrium phenomena in the economic system has become recognized to be very doubtful.

While there is widespread agreement about the shortcomings of the Walrasian general equilibrium theory, there has as yet emerged no unified body of economic theory to supplant or subsume it. However, a number of approaches have been pursued in the search for a more satisfying theory and considerable progress has been taking place in many problem areas.


2 These may be most simply defined as exchanges at non-market-clearing prices having the consequences of leading to co-ordination failures, multiplier effects and involuntary unemployment. See Leijonhufvud (1973) and Barro and Grossman (1976), Ch. 2.

3 See Clower (1975) for a synthesis of these issues.
One approach has been to modify the Walrasian framework through the introduction of explicit transaction costs.\textsuperscript{4} Studies along these lines have succeeded in their objectives of providing a rationale for the existence of and demand for a money commodity (i.e. a medium of exchange). As has been noted by J. Ostroy\textsuperscript{5}, however; the introduction of transaction costs has been accomplished while retaining, implicitly at least, the Walrasian assumption that exchange is centrally coordinated - by "the auctioneer." Hence disequilibrium trading processes have not been accommodated within these models.

Another approach has involved extensions of the Walrasian framework through the introduction of exchange constraints which reflect the existence in the economy of a unique money commodity - one which actually functions as a medium of exchange.\textsuperscript{6} This feature has operational significance in a model only when disequilibrium trading is permitted to occur, so the implication of this approach is the necessity of doing away with Walrasian auctioneer through explicit specification of non-tatonnement market adjustment processes. The results of these efforts have been models characterized by features

\textsuperscript{4} For example, see Foley (1970), Hahn (1971), Wlehans (1971), Hahn (1973), Karni (1974). Also, see Ulph and Ulph (1975) for a survey of this literature.

\textsuperscript{5} See Ostroy (1973); p. 598, f.n. 2.

\textsuperscript{6} See Clower (1965) and Clower (1967) for the original formulation of the dual decision hypothesis. Also, see Arrow and Hahn (1971) Ch. 13, 14, Grandmont and Younes (1972), Kurz and Wilson (1974), Benassy (1975) for various formulations of this approach, as well as Barro and Grossman (1971) and (1976) for a macroeconomic model based on non-market-clearing conditions.
variously classified as temporary equilibria, quasi-equilibria, non-market-clearing or disequilibrium phenomena of the kind now widely recognized as having concerned Keynes in the General Theory. Although these models allow for the occurrence of exchange out of equilibrium and assume the necessity of a medium of exchange, there are no decentralized exchange processes specified and hence no decentralized mechanism for price changes to occur in any microeconomically meaningful way. Also, in these models grave doubts arise about the stability and uniqueness of any general equilibrium that through various restrictions might be assumed to arise.

Another direction for departure from the Walrasian framework that has recently received a great deal of attention has been the introduction of uncertainty and, diverse and imperfect information into the general equilibrium framework. This has led to the notion of a rational expectations stochastic equilibrium (or to related notions of Bayesian equilibrium or informational equilibrium) and to implications that differ radically from those of the traditional perfect information Walrasian model.

Other approaches have focussed more directly on the

7 For example, see the discussion of the non-market clearing approach in Howitt (1979b).

8 As concluded by Arrow and Hahn (1971). Also, see Weintraub (1974).

9 The initial contribution in this literature was by Lucas (1972). For the most recent contributions, see Riley (1979), Radner (1979), Stiglitz (1979), Hellwig (1980), Grossman and Stiglitz (1980), Harris and Townsend (1981) and Grossman (1981).
microeconomic sources of dissatisfaction with the traditional theory. Ostroy has argued convincingly that Walrasian theory ignores the logistics of exchange since it defines equilibrium to exist when prices are such that the sum of individual excess demands is zero for each commodity and merely presumes, via the artifice of the "auctioneer", that all individual excess demands for each commodity will be zero as well. Therefore, the central feature of Ostroy's approach is that exchange is a "do-it-yourself affair". Thus, when the mode of exchange is decentralized barter in the form of a sequence of simultaneous bilateral trades, Ostroy has demonstrated that even if equilibrium prices are known to all individuals in the economy, the competitive equilibrium allocation (and hence Pareto efficiency) is unlikely to be attained in the economy.  

This is because the informational requirements for efficiency under decentralized exchange go beyond the knowledge of equilibrium prices and require the specification of how much is traded and with whom. Ostroy showed that this informational deficiency could be overcome by having a monetary system serving as a record-keeping device in the economy.

This line of inquiry has been pursued further by Ostroy and Starr. The obstacles (created by the informational deficiency of decentralized barter exchange) to efficient full execution of trading plans of individuals are shown to stem from the conflicting demands for

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commodities which are wanted both for final consumption and as a means of payment whenever traders' wants do not coincide precisely. Several ways of achieving full execution are modelled by Ostroy and Starr. One way is that the expenditure of sufficient time (i.e. multiple bilateral contacts between the same individuals) along with the occurrence of some indirect exchange can overcome the informational deficiency.

Another way is that some slackness in initial endowments could permit the separation of the two conflicting demands for commodities. This slackness of initial endowments could take the form of each individual having inventories of a certain commodity that are at least equal in value to the value of desired purchases of other commodities. Such a commodity could then assume a role as the medium of exchange (Ostroy's record-keeping device). Alternatively, the slackness of initial endowments could be in the form of one individual (or more) holding sufficient inventories of all commodities to permit him (or them) to assume the role of a middleman who satisfies all the excess demands and accepts all the excess supplies of the other individuals in the economy. The middleman's volume of trade may greatly exceed his own consumption requirements and therefore the inventories are required to ensure that all exchanges of every commodity can be expedited despite the unpredictable timing of sales and purchases of commodities by individuals.

Thus, the valuable contributions of the Ostroy and Starr analysis are the formal demonstration of the constraints to allocative
efficiency posed by decentralized barter exchange arrangements\textsuperscript{12} (even with the problem of determining equilibrium prices having been assumed away) and the identification of the exchange processes which circumvent these constraints.

Ostroy and Starr focussed on efficiency and the logistics of exchange and abstracted from the problem of price determination by retaining the Walrasian framework in the background as an efficiency benchmark and an exogeneous generator of "equilibrium prices". However, a long-standing source of dissatisfaction with the traditional Walrasian theory has always been its lack of a price adjustment mechanism that is founded on the kind of decentralized exchange processes that are known to characterize economies in the real world. This issue has been explored in a number of approaches that focus on microeconomic disequilibrum mechanisms and on the modelling of decentralized exchange processes that might provide the foundations for a more satisfactory alternative to the Walrasian theory.

One approach along these lines was formulated by P. Howitt\textsuperscript{13} who constructed a model of an economy with a framework that is sufficiently rich to accommodate decentralized disequilibrum trading and to contain middlemen and money. The middlemen in this model are a group of "shopkeepers" (equal in number to the number of non-money commodities). Each middleman serves as the intermediary to the

\textsuperscript{12} Also, see Feldman (1973) and Madden (1975).

\textsuperscript{13} See Howitt (1974).
exchanges of particular commodity by all the other transactors in the economy and also is responsible for setting the price of the commodity in each period. The money commodity serves as the medium of exchange and unit of account, and is held by transactors because it is required as a deposit on purchase orders placed with shopkeepers. Initially, the prices set by the shopkeepers are unlikely to be equilibrium prices so that disequilibrium trading takes place and shopkeepers must either ration transactors bids or else have on hand sufficient inventories of commodities and money to absorb all excess demands. In subsequent periods, shopkeepers adjust prices, raising (lowering) them when excess demands are positive (negative).

Howitt has shown that this system has long run equilibrium properties that are identical to those of the Walrasian model and that (subject to some restrictions) the equilibrium of the system, in which the quantity theory is valid, is a stable one but that in the short-run (in disequilibrium) the system can exhibit Keynesian multiplier effects even though prices are not "rigid".

In a series of papers, R. Clower has synthesized the key ideas in his earlier work with the contributions of Ostroy, Ostroy and Starr, and Howitt, to outline the microeconomic characteristics that might be considered essential in the specification of an adequate theory of a monetary economy. One of these characteristics, namely

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14 See Clower (1975) and Clower (1977). In Clower (1975), these ideas were outlined as the elements of "Neo-Marshallian theory". Also, see Clower & Leijonhufvud (1975).
that there should exist a money commodity (serving as the medium of exchange by being involved in virtually all transactions) has, as discussed earlier, already been extensively incorporated in many models. The other characteristics are that exchange is "an ongoing process in time", that inventories of frequently traded goods are held by all individuals, and that exchange processes are decentralized and characterized by bilateral trades involving inventory-holding middlemen (or "specialized traders") whose activities result in the emergence of organized markets for the exchange of all goods traded in the economy.

The implications of one or more of these characteristics have been investigated in a number of papers with the more recent contributions coming to attention only in the later stages of the work on this dissertation. A paper by J. Hirshleifer\textsuperscript{15} has explored the main elements of exchange processes under the assumption of perfect markets (i.e perfect information and single market price). Middleman exchange and inventory-holding behaviour by all traders as well as the institution of money as a medium of exchange and a temporary store of value are "explained" in terms of proportional and fixed transactions costs relationships. Some of the implications of alternative types of organized markets were investigated by C.H. Lee,\textsuperscript{16} who modelled a market where inventory-holding (capital-intensive) firms may act as a buffer to cushion customers from uncertainty by maintaining a known

\textsuperscript{15} See Hirshleifer (1973).
\textsuperscript{16} See Lee (1974).
constant price. He compared this market with a market where prices are variable so that customers must absorb uncertainty by searching (a labor-intensive activity) for the lowest price.

The modelling of exchange in organized markets as "an ongoing process in time" under conditions of imperfect information and search has been investigated in the literature on atomistic market structures and adjustment processes. One of the more intricate contributions to this literature was by J.D. Hey. Another contribution was by Y.M. Ioannides. Although some doubts concerning the main conclusions of this latter work have been raised, the framework itself represented an interesting attempt to integrate decentralized bilateral exchange behaviour of individuals into a model of market structure. Also, along these lines, a model of trading uncertainty and liquid asset holding behaviour was studied by D.K. Foley and M.F. Hellwig.

As a whole, this body of literature has yielded mixed

17 See Rothschild (1973) for a survey and critique of some of the earliest literature in this area.

18 See Hey (1974). This paper models a price adjustment process in an essentially perfectly competitive market when buyers search for the lowest price by contacting firms while firms maximize profits by price-setting according to their estimated demand curves.

19 See Ioannides (1974).

20 See Butters (1977) where the central assertion of the model, that market equillibrium would be characterized by price dispersion, is contradicted.

21 See Foley and Hellwig (1974).
results. Among the major shortcomings of these models are the simplistic assumptions about the attributes and motivations underlying the exchange behaviour of economic agents and about the implications of these for the assumed market price adjustment mechanisms. For example, the assumption that both "buyers" are identical and "sellers" (or "firms") are identical is commonly made. Thus, the uncertainty and price dispersion in such models are artificial constructs which degenerate so that market processes inevitably converge to very simple solutions. Further progress in the modelling of organized market structures and processes will likely be dependent on a richer modelling of the exchange behaviour of the economic agents who perform in the marketplace.

In this regard, studies of the behaviour of inventory-holding middlemen or "specialized traders" are, as already discussed, particularly important. Howitt has developed a model of speculation in which an inventory-holding speculator or a "large group of identical

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22 It is explicitly assumed in the Hey model. In the Ioannides model any apparent differences are made trivial by the combination of assumptions that all transactions are at the single unit level, that the buyers and sellers have only two options (zero utility or the utility from one unit transacted in the market) and that buyers and sellers know the distributions of the reservation and asking prices they face.

23 More recently, the literature on atomistic market structures and adjustment processes has been extended and subsumed by the informational equilibrium and signalling literature and considerable progress in this direction has been evident. See Riley (1979), Hellwig (1980), Harris and Townsend (1981), Grossman (1981). Also, see Braverman (1979).

24 See Howitt (1979a).
(inventory-holding) speculators*, striving to maximize their capital
and seeking the function of facilitating trade out of equilibrium
without the necessity of rationing. In this model, the "gopings" of
speculators, buffered by inventories, converge to the market
equilibrium price.

Another recent investigation of middleman behaviour has been
by F.O. Irvine, Jr. who formulated a model of a supermarket-type firm,
denoted a "Typical Inventory Carrying Store" or "TICS" firm, and was
able to demonstrate that the expected profit maximizing price
adjustment policy of such a firm is a "short-run inventory-based
pricing policy". This pricing policy involves setting a price above
the long-run expected profit maximizing price if inventories have
fallen below the desired optimum level and setting a price below the
long-run expected profit maximizing price if inventories have
accumulated above the desired optimum level. Also, Irvine shows how a
market composed of "TICS firms" following such a pricing policy would
exhibit a market price adjustment process that is consistent with the
"law of supply and demand".

The literature that has been reviewed here is representative
of the range of studies that have been undertaken in recent years in an
effort to overcome the inadequacies of the traditional economic theory
in the study of a monetary economy. One of the few unifying elements
in this literature has been the recognition that explicit

characterization of exchange processes should replace the myth of the Walrasian auctioneer. Nevertheless, a considerable conceptual leap is involved in moving from a theoretical framework based on the premise that exchange is a "do-it-yourself affair" (i.e. unorganized barter) to a theoretical framework where exchange mediated by inventory-holding middlemen within organized markets is the dominant exchange process in the economy. 26 Where a money commodity fulfills a role as a medium of exchange. One way to bridge this gap is to develop an analytical framework for describing how the evolution of exchange processes from a primitive system of decentralized barter to more complex exchange processes involving middlemen and/or a money commodity might proceed. The task of developing a framework of this type, focusing on the evolution of middleman exchange, will be undertaken within the pages that follow.

A model which demonstrated how media of exchange could emerge through "the uncoordinated market behaviour of individuals" was formulated by R.A. Jones. 27 This model assumed a decentralized barter economy where only bilateral exchanges of single units of commodities occur at exogeneously established equilibrium prices. Accordingly, individual exchange behaviour is planned so as to minimize the expected

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26 For example, Clower has asserted: "As of the present time, therefore, the proposition that a class of middlemen traders will emerge naturally out of a situation in which trade is initially carried out on a strictly individual basis must be regarded as simply a plausible conjecture". (See Clower (1975) p. 14.)

27 See Jones (1976).
time spent on exchange activity which is formulated as being inversely related to the probability of achieving a double coincidence of wants. Under some plausible assumptions about these probabilities, Jones has demonstrated how a very "common" good would emerge as a medium of exchange.

A framework within which the evolution of both middleman exchange and of a money commodity in a decentralized barter economy could be demonstrated was recently investigated by M. Ott. This model assumes a bilateral exchange process involving traders with identical preferences (but differing production possibilities) faced with costly search within a spatial grid and a costless bargaining process characterized by location-based asymmetry in trader information. The alternative to time-consuming exchange outside one's home base is time-consuming production activity and costless exchange at one's home base. Ott shows how under these conditions middlemen will evolve as some traders cease to be producers and become specialist traders, who earn income on the basis of buy-sell price spreads, who deal in many commodities and who trade with each other in order to clear excess demands between regions. The exchange process between producers and middlemen does not imply an immediate "quid pro quo" but is broken down into contractual stage and a completion stage at which actual quantities are traded at previously contracted prices. This creates backtracking and retrieval costs, which are reduced by the

movement of a system of collateral in the form of personalized tokens which also permit producers to vary the timing of transactions to optimize on inventory holding of consumed goods (purchases) and of produced goods (sales) independently of each other. Money evolves in this system as a balancing medium of exchange among middlemen and has the characteristics that it is a durable, widely traded, commodity whose buy-sell spread is least among all commodities.

The framework that is formulated here differs significantly from the models of Jones and Ott. The Jones model, of course, does not address the subject of middlemen. The most fundamental difference in the Ott model is that middleman activity is characterized by increasing returns and middlemen evolve from among the economic agents whose commodity productivities are relatively low. In contrast, the approach taken here is to model a pure exchange-two-commodity economy in which middlemen evolve because of differences in the inherent preferences of individuals even if there are no increasing returns to middleman exchange activity.

The framework for the evolution of exchange processes, developed in subsequent chapters, has been dictated by several fundamental considerations. First, both the modelling of an evolutionary process and the importance of inventories in middleman exchange activity (and hence of inventory accumulation in the modelling of an evolutionary process) require that the framework should incorporate intertemporal dimensions. These considerations either
imply the use of a model involving complex applications of dynamic programming or optimal control techniques\textsuperscript{29} or else can be managed more simply within the framework of a stock-flow model,\textsuperscript{30} with certain analytical limitations.\textsuperscript{31} Another consideration is that if one does away with the Walrasian auctioneer, exchange must be modelled as a "do-it-yourself affair" which, initially at least, takes the form of unorganized barter involving bilateral contacts between individuals. When no organized facilities for exchange exist, then the terms on which any two individuals agree to exchange with each other will depend on the probabilistic outcome of a bilateral contacting and bargaining process rather than upon conforming to any particular structure of prices (equilibrium or otherwise). The implication of this is that the characterization of exchange processes must be based on a probabilistic model which incorporates search or sequential decision-making under uncertainty.\textsuperscript{32}

In Chapter 2, a probabilistic model of individual utility maximizing behaviour in a two commodity unorganized barter exchange economy is formulated. The main element of the model is a symmetrical bilateral contacting process involving the exchange of reservation bids between individuals. A distinction is made between the planned

\textsuperscript{29} See Intriligator (1971).

\textsuperscript{30} See Clower (1977), p. 207. Also, see Bushaw and Clower (1954).

\textsuperscript{31} See Leviatan (1966).

\textsuperscript{32} See the literature on the economics of information: e.g. Stigler (1961), McCall (1965), Kohn and Shavell (1974), Weitzman (1979).
exchange behavior and the actual exchange behavior of individuals. Actual exchange behavior during a trading period has been modelled as a sequential decision-making process characterized by both price uncertainty and quantity uncertainty.

In Chapter 3, the specification of exchange opportunities functions provides the framework for formulating the planned exchange behaviour of a typical individual at the beginning of each trading period as an expected utility maximization problem subject to exchange opportunities constraints. The evolution of exchange processes is developed in terms of changes in the planned exchange behaviour of the typical individual in successive trading periods. In successive trading periods, the behaviour of the individual would be characterized by temporary "trading" equilibria which would involve inventory accumulation resulting in enhancement of exchange opportunities. An eventual intertemporal "stock-flow" equilibrium would be attained when desired inventory stocks have been accumulated.

The alternative exchange timepaths of the typical individual in successive trading periods are studied in Chapter 4. It is shown that, in some cases, the exchange timepath of an individual will be such that during the course of adjustment to an intertemporal equilibrium he will have evolved into a middleman. By accumulating inventories, he will be able to exploit exchange opportunities to become both a "buyer" and a "seller" of some commodities when initially his exchange behavior was unspecialized consumer behaviour (being a
"buyer" or a "seller" of a commodity, but not both, solely to modify his consumption bundle).

In Chapter 5, the framework is extended to describe a process by which the evolution of a class of middlemen providing an organized market system for exchange of commodities can lead to the virtual elimination of unorganized barter exchange. In Chapter 6, the framework is extended to describe how monetary exchange could evolve if middlemen and organized markets did not evolve evenly for some pairs of commodities because inventory accumulation and time spent exchanging these pairs was not sufficiently attractive compared to exchanging other commodity pairs.
Chapter 2

INDIVIDUAL EXCHANGE BEHAVIOUR IN AN UNORGANIZED BARTER ECONOMY

2.1 The General Framework

In the beginning, we have a decentralized exchange economy consisting of a large number of individuals (households) as the only economic agents. Every individual is faced with choosing a pattern of consumption that will maximize his utility over time, constrained by the resources and opportunities at his disposal. We can abstract from the activity of production by assuming that the resources of individuals consist of a fixed endowment flow of one or more consumption commodities. Time will be modelled in discrete terms so that the endowment is assumed to accrue at the beginning of each time period. The opportunities of individuals to improve their pattern of consumption are assumed to be opportunities to forego some leisure and engage in the activity of barter exchange with other individuals during each time period. In this primitive state of the world, however, we assume that only intratemporal exchange opportunities exist and every exchange involves the immediate transfer of the agreed upon quantities of the commodities involved. Hence no interest-bearing assets exist, although consumption may be redistributed intertemporally through the accumulation of inventories.

We will assume that individuals in this economy maximize their expected utility so that their behavior is consistent with
the expected utility hypothesis. The general form of the utility function of a typical individual may be expressed as:

\[(2.1) \quad U = U(x_{10}, x_{20}, \ldots, x_{k0}, x_{11}, \ldots, x_{1n}, \ldots, x_{kn}, L_0 - z_0, \ldots, L_n - z_n)\]

where \(x_{it}\) = the quantity of commodity \(i\) consumed during period \(t\), \(i = 1, \ldots, k; \ t = 0, 1, \ldots, n;\)

\(L_t\) = potential amount of leisure during period \(t;\)

\(z_t\) = amount of time spent engaging in exchange in period \(t;\)

and where \(U\) is quasi-concave and has the usual properties axiomatic in standard consumer theory. However, this form is somewhat unwieldy analytically. Therefore, since the holding of inventory stocks represents a way (the only way in this system) of intertemporally redistributing consumption endowments, this utility function can be replaced by the special case expressed as:

\[(2.2) \quad U = U(x_1, x_2, \ldots, x_k, \bar{x}_1, \bar{x}_2, \ldots, \bar{x}_k, \bar{L} - z)\]

where \(x_i\) = consumption of commodity \(i\) during the current period;

\(\bar{x}_i\) = stock of commodity \(i\) held at the end of the current period;

\(\bar{L}\) = fixed potential amount of leisure time during current period, assumed equal to the length of the period;

\(z\) = time spent engaging in exchange during current period.

\(^1\) For an overview of some of the literature see McCall (1970) and Rothschild (1973).
In this utility function the stock terms, $X_1, \ldots, X_k$, act as proxies for the future consumption terms ($x_{11}, \ldots, x_{k1}, \ldots, x_{1n}, \ldots, x_{kn}$ respectively). In circumstances where the individual wishes to redistribute consumption from the present to the future, he will be induced to accumulate inventory stocks of one or more consumption commodities (which ones, may depend on future exchange prospects and on storage costs).

There are several features of this type of formulation of the utility function that should be kept in mind. First of all, this formulation is a transformation of a multi-period utility function. It embodies, in the inventory stock terms, a subjective summation of hypothetical solution values of utility maximization over future periods discounted according to an endogenously determined structure of discount rates and based on certain expectations of endowments, inventories, exchange opportunities and leisure time allocations in future periods. Should these expectations become modified in a way that created intertemporal substitution effects, the form of the one-period utility function would generally change. Thus, the analytical uses of such a formulation are limited to studying "wealth effects" - the effects on current period decision variables of changes in initial wealth (the current period constraint).² In the model developed here, such a limitation will not pose any problems since the

² For a rigorous exposition of such transformations see Levia1an (1966).
aim is to provide a description of how "adaptive" decision making\(^3\) by individuals engaging in exchange might result in some of them evolving into middlemen and this involves conceptual experiments analogous to changes in initial wealth. However, one shortcoming of this formulation is that the endogenous subjective rate of time preference\(^4\) implicit in the inventory stock terms cannot be directly determined but must be deduced indirectly.

Without loss of generality but with great gains in expositional ease, the model will be developed for the case of two commodities, \(x\) and \(y\). Thus, the utility function of the typical individual will be expressed as:

\[
(2.3) \quad U = U(x_C, y_C, X, Y, l - z)
\]

where \(x_C\) and \(y_C\) are the quantities of the two commodities consumed during the current period and \(X\) and \(Y\) are the stocks of the two commodities held at the end of the period. As before, it is assumed that \(U\) is strictly quasi-concave and founded on the usual preference axioms of consumer theory. However, the individual's behaviour will be based on his expectations about \(U\) because of uncertain exchange opportunities.

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\(^3\) What Clower has called "dumb-beast" theories of decision-making. See Clower (1963), pp. 189-190.

\(^4\) Assuming for simplicity that a single positive subjective rate of time preference characterizes the individual's discount function, reflecting according to Strotz, "consistent" intertemporal behaviour (see Strotz (1955)).
The way in which exchange opportunities and the process of unorganized barter exchange are formulated will be very important. In a highly centralized and efficient market system with no uncertainty and no transactions costs, where a single relative price prevails between the two commodities, individual exchange opportunities would be represented by the familiar linear budget constraint of standard consumer theory. As illustrated in Figure 2.1, an individual with an endowment \((\bar{x}, \bar{y})\) could consume any bundle along AB by exchanging the appropriate quantities.

If matters are complicated by the existence of transactions costs that are proportional to the quantities exchanged, then there will exist two relative prices - a higher "buying" price and a lower "selling" price.\(^5\) Therefore, exchange opportunities will still be linear, as illustrated in Figure 2.2.

On the other hand, in our completely unorganized barter economy the concept of a single relative "price" for any pair of commodities is meaningless since there exists no compulsion for exchanges to be concluded on any particular terms and, in fact, no mechanism for ensuring the widespread transmission of information about any set of prices. The terms of exchange obtainable by the typical individual during a particular trading period will depend on the uncertain endowments, preferences, and expectations of one or more other individuals encountered randomly during the period. Thus, we

\(^5\) See, for example, Niehans (1971) or Hirschleifer (1973).
FIGURE 2.1
LINEAR BUDGET CONSTRAINT

FIGURE 2.2
BUDGET CONSTRAINT WITH TRANSACTIONS COSTS
have a situation where individuals face "price" uncertainty. Moreover, in a decentralized exchange economy without organized facilities for exchange, many individuals would find it difficult to obtain the desired quantity of a commodity from any single exchange with another individual irrespective of the terms of exchange. For example, a typical individual may find that he must expect to make several exchanges before a satisfactory quantity of the desired commodity can be obtained. This is because, initially at least, the probability of encountering other individuals, who are prepared to trade away sufficiently large quantities of the desired commodity at the particular moment of contact, may be low if the timing and the frequency with which individuals receive their endowments and set out to trade is not synchronized. Thus, in a decentralized exchange economy the typical individual who undertakes to engage in exchange activity will be faced not only with "price" uncertainty but also with "quantity" uncertainty.6

We will assume that an individual, upon receiving his endowment and deciding that prospects for increasing his utility through exchange are attractive, would set out to make bilateral contacts with other individuals for purposes of engaging in exchange.

6 A model incorporating both "quantity risk" and "price risk" has been developed by Hirschleifer (1975) to analyse speculation and hedging: however, the quantity uncertainty in that model arises out of uncertain endowments and therefore is not directly related to the present discussion of individual exchange opportunities. Also, a model with "trading" uncertainty taking the form of either price or quantity uncertainty was developed by Foley and Hellwig (1974) within the context of employment-unemployment contingencies.
The individual would determine according to some decision strategy whether or not to conclude an exchange with a particular contact. Such a decision strategy would be based on his expectations about the nature of his exchange opportunities. For simplicity, we also will assume that an individual who has engaged in exchange for several periods will have developed, through experience, some accurate expectations regarding the nature of his exchange opportunities and will be able to plan his exchange behaviour during the present and in the future periods accordingly.

It is important to differentiate and reconcile two aspects of individual exchange behaviour within the model. First of all, we will be interested in describing an individual's actual exchange behaviour during each trading period. By developing a suitably detailed model of the process of bilateral contacts, it will be possible to formulate this behaviour as a problem in sequential decision theory. This will then provide the foundation for the portrayal of a second aspect of individual behaviour, planned exchange behaviour in successive periods, which will be important in describing the evolution of exchange processes in later Chapters.

2.2 Exchange Opportunities and the Process of Bilateral Contacts

In an economy with both "price" and "quantity" uncertainty, exchange opportunities may be assumed to consist of a known (through experience) distribution of offers to exchange various "quantities" of
the commodities \( x \) or \( y \), at various "prices". Then, bilateral contacting could be assumed to occur as a random sampling process with each contact requiring a fixed interval of time irrespective of whether or not an exchange is concluded. The distribution of exchange opportunities and the process of making a bilateral contact must be modelled to reflect the symmetry inherent in unorganized barter exchange.

We may begin by observing that the expected terms on which an individual will be willing to engage in exchange activity will be determined by his preferences and his holdings of \( x \) and \( y \). This is illustrated in Figure 2.3 where the individual has holdings of \( x \) and \( y \) consisting of an endowment \((x,y)\) at point A on the indifference curve \( U \). In deciding to incur a loss of leisure time (say \( z' \)) to make a bilateral contact, the individual will expect to be compensated by attaining combinations of \( x \) and \( y \) no less preferable than those along the indifference curve \( U' \). It can be seen that, if the expected terms of exchange obtainable by making a bilateral contact represents a relatively low "price" of \( x \) (such as \( p \) or less of \( y \) per unit of \( x \)), then the individual would choose to obtain \( x \) by trading away \( y \). He would never be willing to trade away \( x \) to

---

7 This is not as unrealistic an assumption as might first appear since the timing and the frequency with which individuals receive their endowments and set out to make bilateral contacts vary from individual to individual so that the prospect of concluding exchanges at the particular moment of contact with individuals traded with in previous periods is uncertain. There is a problem not only of a double coincidence of wants but also of double coincidence of timing. For example, see Clower and Howitt (1978).
Figure 2.3
obtain $y$ at these terms of exchange. If, instead, the expected terms of exchange represented a relatively high "price" of $x$ (such as $p^H$ or more of $y$ per unit of $x$) then this individual would choose to trade away $x$ to obtain $y$. At these terms of exchange he would never be willing to obtain more $x$ by trading away $y$. Obviously, if the expected "price" for obtaining $x$ was appreciably greater than $p^L$ or for trading away $x$ was less than $p^H$ (of $y$ per unit of $x$), then this individual would not make any bilateral contacts since he would be unwilling to engage in exchange. This example shows that the convexity of the indifference curves which characterizes individual preferences and the leisure cost of exchange activity both contribute to the existence of a gap between the relatively low expected prices (i.e. around and below $p^L$) at which the individual would be willing to make a bilateral contact to obtain more $x$ and the relatively high expected prices (i.e. around and above $p^H$) at which the individual would be willing to make a bilateral contact to trade away $x$.

Thus, in Figure 2.3, $p^L$ and $p^H$ might be viewed respectively as approximations of the individual's maximum "demand price" and minimum "supply price" of $x$. The individual's actual maximum "demand price" and minimum "supply price", communicated when making a bilateral contact, will be denoted the reservation demand price, $p^d$, and the reservation supply price, $p^s$, respectively, and will be based on the expected utility maximizing behaviour of the individual within the given trading and bargaining environment. The
gap between \( p^d \) and \( p^s \) might be somewhat smaller if the sunk leisure time cost of making a bilateral contact can be overlooked during bargaining prior to exchange. On the other hand, the gap between \( p^d \) and \( p^s \) might be somewhat larger if the expected terms of exchange reflect exchange opportunities that permit the individual to expect either to obtain \( x \) at a price considerably lower than \( p^L \) or to trade away \( x \) at a price considerably higher than \( p^H \).

Also, it is obvious from the example in Figure 2.3 that the individual would choose to place some restrictions on the quantities of \( x \) and \( y \) that would be exchanged. For example, to be willing to make a bilateral contact to obtain more \( x \), he would have to expect to end up with combinations of \( x \) and \( y \) along \( BC \) (or better). These restrictions can be represented by reservation quantities, \( s^x \) and \( s^y \), denoting the maximum quantities of \( x \) and \( y \) that the individual would be willing to trade away at his stated reservation bid prices (\( p^s \) and \( p^d \)). There is obviously no need for setting minimum quantities, since after a bilateral contact has been made (involving a sunk leisure time cost), any positive quantities exchanged at equal to or better than reservation bid prices would increase utility.

The manner in which the individual would determine and vary (with each successive bilateral contact) these reservation prices (\( p^s \) and \( p^d \)) and the reservation quantities (\( s^x \) and \( s^y \)) so as to maximize his expected utility is a problem in sequential decision theory. The existence of a solution to this type of sequential
decision problem will be demonstrated later in this Chapter so that the intuitive ability of individuals to establish such expected utility maximizing reservation bids in a consistent manner may be assumed. For the present, it is sufficient to note that it may be assumed that the exchange information provided by every individual making a bilateral contact consists of a reservation bid set: \{(p^s,s^x),(p^d,s^y)\}. Here, to repeat, \( p^s \) is the relatively higher "minimum supply price" (quantity of \( y \) per unit \( x \)) at which the individual would be willing to trade away up to \( s^x \) of \( x \), and \( p^d \) is the relatively lower "maximum demand price" (quantity of \( y \) per unit \( x \)) that the individual would be willing to pay to acquire \( x \) for up to \( s^y \) of \( y \). 

It follows that when the typical individual makes a bilateral contact, the other individual encountered will be similarly providing a reservation bid such as \{(p^{s1},s^{x1}),(p^{d1},s^{y1})\}, which will also be characterized by a gap between a higher "supply price" for \( x \), \( p^{s1} \), and a lower "demand price" for \( x \), \( p^{d1} \). However, the reservation bids of the various types of individuals that might be encountered during bilateral contacting will differ according to their preference, their perceived exchange opportunities and their holdings of \( x \) and \( y \) at the time of the bilateral contact. The various possible bilateral contact situations can be illustrated within the framework of Edgeworth-Bowley Boxes as in Figure 2.4. The three possible outcomes after a bilateral contact are: (a) no exchange occurs; (b) the typical individual trades away ("sells") \( x \) to obtain \( y \); and (c) the typical individual obtains ("buys") \( x \) by trading
FIGURE 2.4

- REGIONS OF DOUBLE COINCIDENCE OF WANTB
In each of the three cases the reservation bid of the typical individual, \( \{(p^s, s^x), (p^d, s^y)\} \), is denoted by the segments BAC, as shown. The reservation bids of the other individuals having holdings of \( x^1 \) and \( y^1 \) have been denoted: \( \{(p^{s_1}, s^{x_1}), (p^{d_1}, s^{y_1})\} \), where \( i = 1,2,3 \) for the cases (a), (b), and (c) respectively. It can be seen that no exchange will occur if 
\( p^s > p^d_1 \) and \( p^s_1 > p^d^* \), as in case (a), while a double coincidence of wants exists in the other two cases. In case (b), where 
\( p^s < p^d_2 \), the typical individual will trade away \( x \) to obtain \( y \), while in case (c), where \( p^s < p^d^* \), the typical individual will be trading away \( y \) to obtain \( x \). In Figure 2.4, the regions of double coincidence of wants have been indicated by shading.

This may be contrasted with the Edgeworth-Uzawa framework of costless exchange (see Uzawa (1962)) with successive barterers between individuals, where all utilitatively improving trades are transacted. Within that framework the region of double coincidence of wants consists of all common \((x,y)\) combinations above the indifference curves passing through the endowment points of the two individuals.

The exchange opportunities of the typical individual will be determined by the distribution of the reservation bids of all the other individuals in the economy that could be encountered in the course of the random sampling process of making bilateral contacts. By superimposing a large number of Edgeworth-Bowley Boxes depicting all the different possible sets of reservation bids that the typical individual
might face, it is possible to generate (about point A representing  
his holdings \( \hat{x} \) and \( \hat{y} \)) the domains of the reservation bid joint  
probability functions, as illustrated in Figure 2.5. In particular, we  
have the box \( 0^00^1 \) representing a possible contact with an  
individual whose reservation bid is \( \{(p^{s1},s^{x1}),(p^{d1},s^{y1})\} \), the box  
\( 0^00^2 \) with reservation bid \( \{(p^{s2},s^{x2}),(p^{d2},s^{y2})\} \), and boxes such  
as \( 0^00^n \) and \( 0^00^{n+1} \) with respectively reservation bids  
\( \{(p^{sn},s^{xn}), (p^{dn},s^{yn})\} \) and \( \{(p^{sn+1},s^{xn+1}),(p^{dn+1},s^{yn+1})\} \).  
We may assume that the distribution of these reservation bids is  
described by a pair of continuous joint probability functions defined  
as follows.

Let \( p^s \), the reservation bid price at which \( x \) is offered  
for exchange in the economy, be a random variable which takes on values  
between a relatively low \( p^s_1 \) and a relatively high \( p^s_J \) and let  
\( S^x \), the reservation bid maximum quantity of \( x \) offered for exchange  
in the economy be a random variable which takes on values between \( 0 \)  
and \( S^x_K \). Then let \( h(p^s,s^x) \) be the continuous joint probability  
density function of reservation bid subsets indicating the willingness  
of individuals to trade away \( x \) to obtain \( y \). Hence, we have:

\[
\int_{p^s=p^s_1}^{p^s_J} \int_{s^x=0}^{S^x} h(p^s,s^x) ds^x dp^s = 1.
\]

Similarly, let \( p^d \), the reservation bid price at which \( x \)
\[ \text{Domain of } g(p^d, s^y) \]

\[ \text{Domain of } h(p^s, s^x) \]

**FIGURE 2.5**
is accepted in exchange for $y$, be a random variable which takes on values between $p^d_1$ and $p^d_G$ and let $S_Y^y$ be a random variable which takes on values between 0 and $s^y_H$. Then, let $g(p^d,s^y)$ be the continuous joint probability density function of reservation bid subsets indicating the willingness of individuals to obtain $x$ by trading away $y$. Hence, we have:

$$\int_{p^d_1}^{p^d_G} \int_{s^y=0}^{s^y_H} g(p^d,s^y) ds^y dp^d = 1$$

(2.5)

Thus, it will be assumed that when the typical individual makes a bilateral contact and exchanges reservation bids with another individual, he obtains an observation from each of the distributions represented by $h$ and $g$. Furthermore, it will be assumed that the typical individual has an accurate perception of these distributions (because of previous exchange experience) and is able to determine which of his own reservation bid subsets (i.e. $(p^s*,s^x*)$ or $(p^d*,s^y*)$) has a higher probability of resulting in successful exchanges or whether one or both of the subsets of his reservation bid may be trivial because the probability of its being accepted is (virtually) zero.

One obvious characteristic of the reservation bid joint probability distribution functions $g$ and $h$ that results from their being interrelated through the preferences $\alpha$ of the same population of individuals should be noted. Because the own reservation bid of every
individual in the economy has the property that $p_d x < p^*_s x$ (as previously demonstrated in Figure 2.3), the expected price offer obtainable for $x$ when "selling" $x$ to obtain $y$:

$E(p_d) = \int p_d \left\{ \int_{s_H}^{s_Y} g(p_d, s_y) ds_y \right\} dp_d$

will be lower than the expected price asked for $x$ when "buying" $x$ with $y$:

$E(p_s) = \int p_s \left\{ \int_{s_K}^{s_X} h(p_s, s_x) ds_x \right\} dp_s$

when the typical individual makes a bilateral contact. This characteristic, $E(p_d) < E(p_s)$ is illustrated in Figure 2.6.

Since the reservation bid of any single individual cannot influence appreciably the reservation bid probability distribution functions, it can be assumed that all individuals in the economy face the same reservation bid distribution and thus will have identical exchange opportunities. Furthermore, since the total quantity of $x$ or $y$ traded away during a period must be identical to the total quantity of $x$ or $y$ obtained in exchange during that period, the "average buying" price of each commodity must equal its "average selling" price for the economy as a whole. This, also, is illustrated in Figure 2.6 as line CAB. It can be readily shown that
\[ - \text{Domain of } g(p^d, s^Y) \]
\[ - \text{Domain of } h(p^s, s^x) \]

FIGURE 2.6
\[ E(P_d) < p^A < E(P_s) \] as illustrated.

This means that the price at which an individual can expect to trade away \( x \) to obtain \( y \) as an outcome of a single bilateral contact, \( E(P_d) \), will be less than the "average prevailing" price of \( x \) in the economy, \( p^A \). Also, this means that the price at which an individual can expect to obtain \( x \) for \( y \) as an outcome of a single bilateral contact, \( E(P_s) \), will be greater than \( p^A \). This indicates that an individual who expects to trade away \( x \) for \( y \) at an average price as high as \( p^A \), the "average prevailing" price of \( x \), or to obtain \( x \) for \( y \) at a price as low as \( p^A \) must also expect to make an "average" number of bilateral contacts (thereby devoting an "average" amount of time to exchange activity) and must expect to conclude exchanges with only a fraction of the individuals contacted. Moreover, it follows that an individual who expects both to trade away \( x \) for \( y \) at a price higher than \( p^A \) and to obtain \( x \) for \( y \) at a price lower than \( p^A \) must expect to make a greater number of bilateral contacts than the "above-average" number that an individual, who only expects to trade away \( x \) for \( y \) at a price higher than \( p^A \), must expect to make.

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8 Proof: The only other two possibilities, \( E(P_d) > p^A < E(P_s) \) or \( E(P_d) < p^A > E(P_s) \), can both be shown to be contradictions. Both \( E(P_d) > p^A \) and \( p^A > E(P_s) \) are only possible if a relatively large proportion of bilateral contacts made in the economy result in trade. The corresponding conditions, both \( p^A < E(P_s) \) and \( E(P_d) < p^A \) respectively, are only possible if only a small proportion of the bilateral contacts made in the economy result in trade. Since both situations cannot prevail in the economy simultaneously, the other two possibilities are contradictions.
One other aspect of bilateral contacting and exchange behaviour deserves further discussion here. The Edgeworth Box in Figure 2.7 illustrates a bilateral contact situation where a double coincidence of wants exists (similar as Figure 2.4 (c)) and an exchange will be concluded (since $p^d*p^s$). It is obvious that the precise price and quantities that will be agreed upon cannot be determined solely from the reservation bids. It is necessary to specify the exchange convention or to describe the bargaining process according to which indeterminate exchange situations become resolved in the economy in order to describe the final outcome of such exchange situations.

For example, it may be assumed that the exchange convention in the economy is that reservation bid quantities are binding constraints and that the exchange price, $p^e$, will be a weighted average of reservation bid prices (i.e. $p^e = ap^s + (1-a)p^d$) with the weighting parameter "a" predetermined by custom. Then the exchange outcome of the bilateral contact would be at $F$ (with $y_x^F$ of $y$ being exchanged for $x_y^F$ of $x$) in Figure 2.7.

A more realistic assumption might be that after reservation bids have been exchanged and a double coincidence of wants established, the bilateral contact would entail a bargaining process where the constraining reservation bid quantity (in this case $s^X$) might be varied if individual demands are somewhat elastic with respect to price. In the course of this bargaining process the individual desiring the exchange of larger quantities of $x$ and $y$ would have the opportunity to try to persuade the other individual to increase his
FIGURE 2.7
reservation bid quantity by offering to negotiate a more attractive exchange price. Then the exchange outcome of the bilateral contact situation in Figure 2.7 could be at a point such as G (with \( y_x^G \) of y being exchanged for \( x_y^G \) of x). Let us assume that these exchange outcomes can, in general, be represented by the continuous functions \( x_y(p^{d_x}, s^{y_x}, p^{s_y}, s^{x_y}) \) and \( y_x(p^{d_x}, s^{y_x}, p^{s_y}, s^{x_y}) \), representing respectively the actual quantities of x and y exchange as functions of the two reservation bids, where obviously, \( p^{s_y} < \frac{y_x}{x_y} < p^{d_x} \).

As discussed earlier, the typical individual engaging in exchange, making bilateral contacts, would be able to employ a sequential decision procedure to maximize his expected utility. He would reassess his situation after each bilateral contact and determine whether or not to continue bilateral contacting during that period and, if so, on what terms (whether to adjust reservation bids). This actual behaviour of the individual may be distinguished from his planned behaviour based on exchange plans formulated by the individual at the beginning of each period regarding his desired expected "consumption" of commodities and leisure, and holding of inventories at the end of the period. While the actual exchange behaviour (sequential decision procedure) would frequently involve departures from the original plan if the outcomes of bilateral contacts unexpectedly altered circumstances, over the long run, the actual behaviour can be assumed to correspond closely to planned behaviour if expectations about exchange opportunities are accurate.
In developing a manageable model of the evolution of exchange processes with middlemen, it is necessary to focus on this planned exchange behaviour of the individual from period to period rather than on the actual exchange behaviour during a period. This will be undertaken in the next Chapter, but first it is necessary to make a digression for the remainder of this Chapter to complete a description of actual exchange behaviour during a period by characterizing the sequential decision exchange behaviour of the individual in the unorganized barter economy.

2.3 Sequential Exchange Behaviour

The task at hand is to describe the optimal exchange behaviour of a typical individual who has imperfect knowledge of the price and quantity that will be obtainable during the course of exchange activity and who incurs costs (loss of leisure time) while engaging in exchange activity.

It may be recalled that time is divided into discrete trading periods of length \( \bar{t} \). At the beginning of each trading period the individual receives an endowment, \((x, y)\), of the two commodities. Adding this to inventories \((\bar{x}, \bar{y})\) held over from the previous period (if they exist), the individual begins each trading period with a particular commodity bundle, \((x + \bar{x}, y + \bar{y})\), which will be simply denoted as \((q^x_0, q^y_0)\). During each trading period the individual has
opportunities to engage in exchange, to consume some (or all) of his commodity bundle and to enjoy his leisure time. The analysis will be confined to describing the optimal exchange behaviour of the typical individual during a trading period so that we will abstract from the allocation decision between consumption and inventories in the following way.

It may be recalled that, earlier, the utility function of the typical individual was expressed as:

\[(2.3) \quad U = U(x_C, y_C, x, y, \bar{L} - z)\]

where \(x_C\) and \(y_C\) were the quantities of the two commodities consumed during the trading period, \(x\) and \(y\) were the inventories of the two commodities held at the end of the period, and \(\bar{L} - z\) was the amount of leisure time enjoyed during the period with \(z\) being the amount of time spent on exchange activity.

Letting \(x_C + x = q^X\) and \(y_C + y = q^Y\), we may re-write \((2.3)\) as:

\[(2.8) \quad U = U(x_C, y_C, q^X - x_C, q^Y - y_C, \bar{L} - z)\]

Then the individual's utility function may be re-defined, in terms of holdings \(q^X\) and \(q^Y\) as being equal to the utility attainable with holdings \(q^X\) and \(q^Y\) when these are optimally allocated between consumption and inventories. Thus, we may define:

\[(2.9) \quad W = W(q^X, q^Y, \bar{L} - z) = \max_{\{x_C, y_C\}} U(x_C, y_C, q^X - x_C, q^Y - y_C, \bar{L} - z)\]
where by maximizing (2.9) with respect to \( x_c \) and \( y_c \), a unique level of utility can be determined for all \((q^x, q^y)\) since (quasi-concave) \( U \) has strictly convex indifference curves.

As previously discussed, exchange activity during a trading period consists of successive bilateral contacts with other individuals. It is assumed that each bilateral contact takes a fixed interval of time, \( z' \), to complete irrespective of whether or not an exchange has been concluded. It therefore follows that the individual can make at most \( N \) bilateral contacts during a trading period, where \( N \) is an integer such that \( \frac{L-z'}{z'} < N \leq \frac{L}{z'} \). Also, when this individual has terminated exchange activity after having made \( n \) bilateral contacts (where \( 0 \leq n \leq N \)) during the trading period, we may denote his leisure time during the trading period as \((L-nz')\). Therefore, using (2.9), we may denote his utility during the trading period as:

\[
W = W(q^x, q^y, L-nz')
\]

Furthermore, for simplicity, this utility function will be restricted to be of additive form so that,

\[
W = W(q^x, q^y, L-nz') = u(q^x, q^y) + W(L-nz')
\]

which means that the marginal utility of leisure is independent of the levels of holdings of \( x \) and \( y \) and that the marginal rate of substitution between \( x \) and \( y \) is independent of the level of
leisure. This will permit the leisure cost of exchange activity to be more simply identified. Also, since independent utilities and convex indifference curves will not necessarily imply diminishing marginal utility for each argument in the utility function, let us also assume that $w(\bar{c}-nz')$ is concave, (i.e. that $w' > 0$ and $w'' < 0$, implying positive but diminishing marginal utility of leisure) and that $u$ is quasi-concave. Then the cost of engaging in exchange activity may be measured in terms of the utility of leisure time with the cost of making the $i$th bilateral contact during the trading period being defined as:

\[(2.12) \quad c(i) = w(\bar{c}-(i-1)z') - w(\bar{c}-iz')\]

where, obviously, $c(0) = 0$, and where $c(i) > 0$ and $c(i) < c(i+1)$, $i = 1, 2, \ldots, N$.

Also let us make or recall the following assumptions:

1. The typical individual has already determined that he will be engaging in exchange only to obtain $x$ for $y$. Thus, he is "typical" in the sense that his situation is also perfectly

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9 These are well-known properties of independent utility functions. See for example Green (1971), pp. 89-94.


11 This will be true since $w' > 0$ and $(\bar{c}-(i+1)z') < (\bar{c}-iz') < (\bar{c}-(i-1)z')$, implies that $w(\bar{c}-(i+1)z') < w(\bar{c}-iz') < w(\bar{c}-(i-1)z')$ and therefore, $c(i+1) > 0$ and $c(i) > 0$. Also, since $w' < 0$ it is obvious that $c(i) < c(i+1)$. 
symmetrical to that of other individuals who have determined to engage in exchange only to trade away $x$ to obtain $y$. He will be interested only in the $(p^s, s^x)$ subset of any reservation bid, \{(p^s, s^x), (p^d, s^y)\}, obtained in the course of bilateral contacting and in how the \{(p^d*, s^y*)\} subset of his own reservation bid, \{(p^s*, s^x*), (p^d*, s^y*)\}, relates to this. Hence his concern will be with the distribution represented by $h(p^s, s^x)$ while the distribution $g(p^d, s^y)$ and the other subset of his own reservation bid will be irrelevant to the behaviour of this typical individual.

2. Again, it may be recalled that for simplicity, it has been assumed that the typical individual has already learned about the nature of the distributions $g$ and $h$ so that no search costs must be incurred in learning about the distributions of exchange opportunities that he faces.\(^{12}\)

3. When a bilateral contact is made with another individual whose reservation bid subset is $(p^s, s^x)$ and a double coincidence of wants becomes apparent (i.e. $p^s \leq p^d*$), it may be recalled that it has been assumed that the quantities exchanged will be $x_y(p^d*, s^y*, p^s, s^x)$ and $y_x(p^d*, s^y*, p^s, s^x)$, where

\(^{12}\) It has been shown that this simpler assumption in many instances does not differ from the more complex assumption that the probability distributions of the search process are unknown and must be learned through search and modification by Baye's rule. See Rothschild (1974) and Kohn and Shavell (1974).
\[ p^S \leq \frac{y}{x} \leq p^{d^*} \]. Therefore, by definition,
\[ x_y(p^{d^*}, y^*, x, p^S) = y_x(p^{d^*}, y^*, p^S, x) = 0 \text{ when } p^S > p^{d^*} \].

4. Finally, it is assumed that the typical individual has no opportunity to return and accept a previously rejected offer from a previously contacted individual, so that his behaviour will be described by a solution to a sequential decision problem without recall.

Within the framework of the exchange process described earlier, the typical individual will be faced, at the beginning of a trading period, with the decision whether to make a first bilateral contact or whether to forego any exchange activity and enjoy maximum leisure. It will be shown that this decision will be based on whether or not his initial holdings, \((q^x_0, q^y_0)\), permit him to choose an initial reservation bid, \(\{p_1^{d^*}, s_1^{y^*}\}\), that will maximize his expected utility and that has a sufficiently high probability of leading to a utility improving exchange. Then, supposing that he chooses to make the first bilateral contact, this reservation bid will also be the basis for determining whether or not to conclude an exchange after the contact has been made. After the first (and any subsequent) bilateral contact, irrespective of whether an exchange has been concluded, the typical individual will have to decide whether or not to make yet another bilateral contact on the basis of having determined what his new reservation bid will be.
Therefore, what distinguishes the sequential decision problem of the individual in an unorganized barter exchange economy from sequential decision problems found in the search theory and economics of information literature,\(^{13}\) is the presence of quantity uncertainty as well as price uncertainty. This means that the decision whether or not to exchange and whether to stop or continue making bilateral contacts are, in general, independent and cannot be based on a single optimal stopping decision rule.

In the search theory and economics of information literature, the single optimal decision rule has been expressed either as a reservation price\(^ {14}\) or as a switchpoint level of utility\(^ {15}\) (which may be formulated as an indirect utility expressed as a function of a reservation price). In the more general case of unorganized barter exchange where quantity uncertainty exists, a reservation price decision rule can be used only to determine whether or not to conclude an exchange. Thus, it may be called the exchange rule. However, another rule which may be called the stopping rule is required to determine whether to stop or to continue exchange activity.

The exchange rule is consistent with earlier discussion and may be expressed as:

---

\(^{13}\) See, for example, the survey by Rothschild (1973), and also Telser (1973), Kohn and Shavell (1974), Rothschild (1974), Ioannides (1975), Bosch-Domenech (1975), and Weitzman (1979).

\(^{14}\) For example, see Rothschild (1974), p. 694.

\(^{15}\) For example, see Kohn and Shavell (1974), p. 93.
\[ p^* > p^d* \]

where \( p^* \) is the reservation bid obtained from the possible values between \( p_1 \) and \( p_j^s \) after making the \( i^{th} \) bilateral contact, and \( p^d* \) is the decision-making individual's own expected utility maximizing reservation bid price chosen prior to making the \( i^{th} \) bilateral contact.

If \( p^* > p^d* \), then there would be no exchange of commodities since the minimum supply price for \( x \) during the contact would exceed the maximum price that the decision-making individual is willing to pay to obtain \( x \). If \( p^* \leq p^d* \), then the opposite situation exists and an exchange would be concluded, as per assumption 4. above.

In order to demonstrate the existence of expected utility maximizing reservation bids, \( \{(p^d*, s^y)\} \), and to derive the stopping rule pertinent to this problem, we may proceed as follows. Let the typical individual's expected utility of behaving according to the optimal rules after \( i \) bilateral contacts (when his holdings are \( q^x \) and \( q^y \)) be denoted respectively, as

\[
m_i^*(q^x, q^y) + w((L-i)z')
\]

when an \( i+1 \) bilateral contact has not yet been made and as

\[
m_i^*(q^x, q^y \mid p^*, s^x) + w((L-(i+1))z')
\]
when the $i+1$ contact has been made and a bid $\{(p^s,s^x)\}$ has been obtained. Then, recalling the notation of (2.12) and (2.13) and the assumptions made earlier, we may write:

(2.14) $m_1^*(q^x,q^y) = \max \left\{ u(q^x,q^y), \mathbb{E}[m_1^*(q^x,q^y | p^s,s^x) - c(i+1)] \right\}$

and

(2.15) $m_1^*(q^x,q^y | p^s,s^x)$

$$= \max \left\{ u(q^x+x_y(p^{d*},s^y*,p^s,s^x),q^y-y_x(p^{d*},s^y*,p^s,s^x)), \mathbb{E}[m_{i+1}^*(q^x+x_y(p^{d*},s^y*,p^s,s^x),q^y-y_x(p^{d*},s^y*,p^s,s^x) | p^s,s^x) - c(i+2)] \right\}$$

so that

(2.16) $m_1^*(q^x,q^y | p^s,s^x)$

$$= m_{i+1}^*(q^x+x_y(p^{d*},s^y*,p^s,s^x),q^y-y_x(p^{d*},s^y*,p^s,s^x))$$

where

$$x_y(p^{d*},s^y*,p^s,s^x) \begin{cases} > 0, & \text{if } p^s < p^{d*} \\ = 0, & \text{if } p^s > p^{d*} \end{cases}$$

and

$$y_x(p^{d*},s^y*,p^s,s^x) \begin{cases} > 0, & \text{if } p^s < p^{d*} \\ = 0, & \text{if } p^s > p^{d*} \end{cases}$$

are the quantities exchanged if an exchange is concluded or are zero otherwise.

This means that, in (2.14), $m_1^*(q^x,q^y)$ will be the
maximum of: (a) the (commodity) utility of stopping exchange activity and consuming on the basis of existing holdings \( q^X \) and \( q^Y \); or (b) the expected (commodity) utility of continuing exchange activity by making the \( i+1 \) bilateral contact and following the optimal rules thereafter. In (2.15), after setting \( \{(p^{d*}, s^{y*})\} \), making the \( i+1 \) bilateral contact and obtaining the reservation bid \( \{(p^{*}, s^{x})\} \), \( m^*_1(q^X, q^Y \mid p^X, s^X) \) will be the maximum of: (a) the (commodity) utility of stopping exchange activity irrespective of whether or not an exchange with the \( i+1 \) contact has occurred; or (b) the expected (commodity) of continuing exchange activity by making an \( i+2 \) contact and following the optimal rules thereafter irrespective of whether or not an exchange has occurred during the \( i+1 \) contact.

Obviously, if no exchange has occurred during the \( i+1 \) contact, then \( (q^{x+x}, q^{y-y}) = (q^X, q^Y) \).

Also, we may define

\[
(2.17) \quad \psi(1, q^X, q^Y) = E[m^*_1(q^X, q^Y \mid p^*, s^X) - c(i+1)]
\]

so that \( \psi(1, q^X, q^Y) \) denotes the expected net (of leisure cost) commodity utility after \( i \) bilateral contacts of continuing exchange by making an \( i+1 \) bilateral contact, after setting a reservation bid, and following the optimal rules thereafter.

Combining (2.14) and (2.17) we get

\[
(2.18) \quad m^*_1(q^X, q^Y) = \max \left\{ u(q^X, q^Y), \psi(1, q^X, q^Y) \right\}
\]

and by substituting \( i+1 \) for \( i \) in (2.17) and the result into (2.15);
then in (2.15) substituting \((p^s, s^x)\) for \((p^s, s^x)\) and substituting the result back into (2.17) we get:

\[
\begin{align*}
(2.19) \quad \psi(i, q^x, q^y) &= E[\max \{ u(q^x + x^y(p, s^y, p^s, s^x), q^y - y^x(p, s^y, p^s, s^x)), \\
&\quad \psi(i+1, q^x + x^y(p, s^y, p^s, s^x), q^y - y^x(p, s^y, p^s, s^x)) \} - c(i+1) \}.
\end{align*}
\]

Therefore, it can be seen that \(\psi(i, q^x, q^y)\), the expected net utility after \(i\) contacts of continuing exchange activity by setting the expected utility maximizing reservation bid, \(\{(p^d, s^y)\}\), making the \(i+1\) bilateral contact and following the optimal rules thereafter, will be equal to the expected value of the maximum of: (a) the net utility of consuming on the basis of the holdings after \(i+1\) contacts by stopping after \(i+1\) contacts less the leisure cost of making the \(i+1\) contact; or (b) the expected net utility of continuing after \(i+1\) contacts by making an \(i+2\) bilateral contact and following the optimal rules thereafter - less the leisure cost of making the \(i+1\) contact.

From (2.18), it can be seen that the decision whether or not to make an \(i+1\) contact after the \(i\) bilateral contact has been concluded depends on

\[
(2.20) \quad \psi(i, q^x, q^y) \geq u(q^x, q^y) ?
\]

If \(\psi(i, q^x, q^y) > u(q^x, q^y)\) then expected utility will be maximized by continuing exchange activity and making an \(i+1\) bilateral contact.
If, on the other hand, \( \Psi(i,q^X,q^Y) \leq u(q^X,q^Y) \) then expected utility will be maximized by stopping bilateral contacting after \( i \) contacts. If \( \Psi(i,q^X,q^Y) \) can be determined (as will be shown subsequently) for all \( i \) and all possible holdings \( (q^X,q^Y) \) by setting the appropriate expected utility maximizing reservation bids \( \{(p^*_d,s^*_y)\} \), then (2.20) will be the optimal stopping rule. The exchange rule, stated previously in (2.13) is similarly contingent on being able to set the appropriate expected utility maximizing reservation bids \( \{(p^*_d,s^*_y)\} \).

From (2.10), it follows that the determination of \( \Psi(i,q^X,q^Y) \) can be viewed as the solution of the maximizing problem:

\[
(2.21) \quad \Psi(i,q^X,q^Y) = \max_{\{p^d,s^y\}} E[A(i+1,q^X,q^Y,p^d,s^Y,p^s,S^X)] \\
= \max_{\{p^d,s^y\}} E[\max\{u(q^X+x_Y(p^d,s^Y,p^s,S^X),q^Y-y_X(p^d,s^Y,p^s,S^X)), \Psi(i+1,q^X+x_Y(p^d,s^Y,p^s,S^X),q^Y-y_X(p^d,s^Y,p^s,S^X))\} - c(i+1)]
\]

where, as defined earlier, \( p^s \) and \( S^X \) are random variables having the continuous joint probability density function \( h(p^s,S^X) \) where

\[
\int_{p^s=p_1}^{p^s=p_2} \int_{S^X=s_1}^{S^X=s_2} h(p^s,S^X)ds^Xdp^s = 1. \]

By definition, the expected utility \( \Psi(i,q^X,q^Y) \) will be equal to the sum of the utilities (or expected utilities) of all possible outcomes after making the \( i \) bilateral contact (and having set the optimal reservation bid), with each outcome being weighted by the probability of its occurrence.
Therefore, the maximization problem in (2.21) may be written as:

\[
\psi(i, q^x, q^y) = \max_{p^d, s^y} A(i+1, q^x, q^y, p^d, s^y, p^s, s^x) \\
= \max \left\{ \int_{p^d>0} \int_{K} h(p^s, s^x) ds^x dp^s \right\} \int_{p^d}^{s^x} \int_{p^s}^{s^x} \max \left\{ u(q^x, q^y), \psi(i+1, q^x, q^y) \right\} ds^x dp^s \\
+ \int_{p^d}^{s^x} \int_{p^s=0}^{s^x} \int_{0}^{1} \max \left\{ u(q^x+y, q^y(p^d, s^y, p^s, s^x), q^y-y(p^d, s^y, p^s, s^x)), u(q^x, q^y(p^d, s^y, p^s, s^x), q^y-y(p^d, s^y, p^s, s^x)) \right\} \right] \\
- c(i+1) \}
\]

Three distinct cases exist for the maximization problem (2.22) depending on the individual's preferences (utility function), his exchange opportunities (reflected by \( h(p^s, s^x) \)), his current holdings \((q^x, q^y)\) and given \( i \), the number of previous bilateral contacts. These cases are:

**Case 1:** Making the \( i+1 \) bilateral contact and stopping exchange activity thereafter irrespective of whether or not an exchange is concluded during the \( i+1 \) contact - (i.e. \( u(q^x, q^y) \geq \psi(i+1, q^x, q^y) \)).

**Case 2:** Making the \( i+1 \) bilateral contact and either continuing exchange activity if no exchange is concluded during the \( i+1 \) bilateral contact or, if an exchange is concluded during the \( i+1 \) bilateral contact, stopping or continuing exchange activity depending on the quantities.
exchanged during the \( i+1 \) bilateral contact -

\( (i.e. \quad u(q^X,q^Y) < \psi(i+1,q^X,q^Y) \) and

\[
\begin{align*}
&u(q^X+q^Y(p^d,q^Y,p^s,q^X),q^Y-y_X(p^d,q^Y,p^s,q^X)) \\
&\quad \geq \psi(i+1,q^X+q^Y(p^d,q^Y,p^s,q^X),q^Y-y_X(p^d,q^Y,p^s,q^X))
\end{align*}
\]

**Case 3:** Making the \( i+1 \) bilateral contact and continuing
exchange activity afterwards so that the \( i+2 \) bilateral
contact is made irrespective of what happens during the \( i+1 
\]

bilateral contact -

\( (i.e. \quad u(q^X+q^Y(p^d,q^Y,p^s,q^X),q^Y-y_X(p^d,q^Y,p^s,q^X)) \\
\quad < \psi(i+1,q^X+q^Y(p^d,q^Y,p^s,q^X),q^Y-y_X(p^d,q^Y,p^s,q^X)) \)

It is useful to distinguish between **Case 1**, **Case 2**, and **Case 3** because **Case 1** represents the set of all terminal exchange situations
for which optimal reservation bid values can be readily determined and
the resulting values for \( \psi(1, q^X, q^Y) \) can be used recursively to solve
for **Case 2** situations and subsequently to solve for **Case 3** situations.\(^{16}\) Therefore, for **Case 1** situations, determining the set of all
\( \psi(n-1,q^X,q^Y) \), the expected net utilities of continuing exchange
activity by making the \( n \)th bilateral contact and stopping irrespec-
tive of the exchange outcome (where \( n \leq N \) and \( \frac{L-z}{z} < N < \frac{L}{z} \))

---

\(^{16}\) The existence of solutions for sequential decision problems of this
\( \) type employing this "backward induction" technique has been
demonstrated and discussed by DeGroot (1970), Sec. 12.4, 12.5, p.
277 for cases where there is an upper bound on the number of
observations that can be taken (i.e. bilateral contacts cannot
97.
the maximization problem (2.22) may be written as:

\[
\psi(n-1, q_x, q_y) = \max \limits_{p^d, s^y} A(n, q_x, q_y, p^d, s^y, p^s, s^x)
\]

\[
= \max \left[ u(q_x, q_y) \cdot \int_{p^d}^{p_1} \int_{0}^{s^x} h(p^s, s^x) ds^x dp^s \right.
\]

\[
+ \int_{p^d}^{p_1} \int_{0}^{s^x} u(q^x + q_y(p^d, s^y, p^s, s^x), q^y - q_x(p^d, s^y, p^s, s^x)) h(p^s, s^x) ds^x dp^s
\]

\[
- c(n) \right].
\]

The necessary conditions for this maximization problem may be written as:

\[
(2.24) \frac{\partial A}{\partial p^d} = \int_{p^d}^{p_1} \int_{0}^{s^x} \left[ \frac{\partial u}{\partial q_x} \cdot \frac{\partial x_y}{\partial p^d} - \frac{\partial u}{\partial q_y} \cdot \frac{\partial y_x}{\partial p^d} \right] h(p^s, s^x) ds^x dp^s
\]

\[
+ \int_{p^d}^{p_1} \int_{0}^{s^x} u(q^x + q_y(p^d, s^y, p^d, s^x), q^y - q_x(p^d, s^y, p^d, s^x)) h(p^d, s^x) ds^x
\]

\[
- \int_{0}^{s^x} u(q^x, q_y) h(p^d, s^x) ds^x
\]

\[
= 0
\]

and
\[
\frac{\partial A}{\partial s_Y} = \int_p^d \int_{s_X}^{s_Y} \left[ \frac{\partial u}{\partial q_X} \frac{\partial x_Y}{\partial s_Y} - \frac{\partial u}{\partial q_Y} \frac{\partial y_X}{\partial s_Y} \right] h(p^s, s^x) ds^x dp^s \\
= 0.
\]

The solutions to (2.24) and (2.25) yield the optimal reservation bids for all situations where the individual makes his last bilateral contact prior to stopping exchange (i.e. for all \(\Psi(n,q^x,q^y)\) such that \(u(q^x,q^y) > \Psi(n,q^x,q^y) > \Psi(n,q^x,q^y)\) ) and may be written as:

\[
(2.26) \quad p^d = \begin{cases} 
p^d(n,q^x,q^y) \\
p^d(n,q^x,q^y) 
\end{cases} \\
s^Y = s^Y(n,q^x,q^y)
\]

For **Case 2**, the situation is considerably more complicated because whether bilateral contacting stops or continues after the \(i-1\) contact depends on the bid, \((p^s,s^x)\), received during the \(i-1\) contact. Thus, for **Case 2** situations, determining the set of all, \(\Psi(i,q^x,q^y)\) (for example, the set of \(\Psi(n-j,q^x,q^y)\) where some \(j=1\) or \(j\geq 2\) ), the maximization problem in (2.22) may be written as:

\[
(2.27) \quad \Psi(n-j,q^x,q^y) = \max_{p^d,s^x} \Lambda(n-j+1,q^x,q^y;p^d,s^y,p^s,s^x) \\
= \max_{p^d,s^x, p^s} \left[ \Psi(n-j+1,q^x,q^y) \int_{p^s}^{p^d} \int_{s_X}^{s_Y} h(p^s, s^x) ds^x dp^s \right. \\
+ \int_{p^s}^{p^d} \int_{s_X}^{s_Y} \max \left\{ \Psi(n-j+1,q^x+q^y(p^d,s^y,p^s,s^x),q^y+q^y(p^d,s^y,p^s,s^x)), \Psi(n-j+1,q^x+q^y(p^d,s^y,p^s,s^x),q^y+q^y(p^d,s^y,p^s,s^x)) \right\} \right] \\
- c(n-j+1).
\]
The first term on the right side of (2.27) is the probability density weighted expected utility of continuing exchange activity by making an \((n-j+1)\) bilateral contact without concluding an exchange and further continuing exchange activity (i.e. where \(j \geq 2\)) by making an \(n-j+2\) contact. The second term on the right side of (2.27) is the probability density weighted expected utility of making an \(n-j+1\) bilateral contact, concluding an exchange involving quantities \(x_y(p^d, s^y, p^s, s^x)\) and \(y_x(p^d, s^y, p^s, s^x)\), and either continuing exchange activity (i.e. a subset where \(j \geq 2\)) by making an \(n-j+2\) contact or stopping exchange activity (i.e. the subset where \(j=1\)) after exchanging during the \(n-j+1\) contact.

From (2.27), it can be seen that in order for \(\Psi(n-j,q^x,q^y)\) and the corresponding optimal reservation bids \(\{p^d(n-j+1,q^x,q^y), s^y(n-j+1,q^x,q^y)\}\) to be determined recursively for all Case 2 situations, the set of all \(\Psi(n-j+1,q^x,q^y)\), where \(j \geq 2\), must have been determined. The set of all Case 1 solutions obtained from (2.23) determine all \(\Psi(n-1,q^x,q^y)\) yielding values for all \(\Psi(n-j+1,q^x,q^y)\) where \(j=2\) permitting the subset of Case 2 situations consisting of all \(\Psi(n-2,q^x,q^y)\) to be determined from (2.27). These in turn permit an additional subset of Case 2 situations, \(\Psi(n-j,q^x,q^y)\) where \(j \geq 2\) to be solved using (2.27) and so on recursively until the set of all \(\Psi(n-j,q^x,q^y)\) and corresponding

---

17 It is possible to interpret \(j\) as the number of bilateral contacts that remain to be taken in a sequence of bilateral contacts during a trading period. Thus, for Case 1, \(j = 1\); for Case 2, \(j \geq 2\); and for Case 3, \(j \geq 2\).
\{p^{d*}(n-j+1, q^x, q^y), s^{y*}(n-j+1; q^x, q^y)\} has been determined for all remaining Case 2 situations. Similarly, the sets of all Case 1 and Case 2 solutions allow all Case 3 situations to be determined recursively. For Case 3 situations, \(\psi(i, q^x, q^y)\), the expected net utility of continuing exchange activity by making the \(i+1\) bilateral contact will be the sum of the probability density-weighted utilities of: (a) continuing exchange activity by making an \(i+2\) bilateral contact when no exchange during the \(i+1\) contact has occurred; and (b) continuing exchange activity by making an \(i+2\) bilateral contact after concluding an exchange during the \(i+1\) bilateral contact. Therefore, for Case 3 the maximization problem will be a special case\(^{18}\) of the problem in (2.22).

As shown, the process of solving \(\psi(i, q^x, q^y)\) (which will include \(\psi(0, q^x_0, q^y_0)\) the expected utility, given initial holdings \((q^x_0, q^y_0)\), of making the first bilateral contact), for all \(i\), \(q^x\) and \(q^y\), involves working back recursively from the set of all possible states of known utility where exchange activity will have been stopped (i.e. \(\psi(n, q^x, q^y) < u(q^x, q^y)\)), along a subset of all possible sequences of optimized expected utility, \{\(\psi(n-1, q^x, q^y)\), \(\psi(n-2, q^x, q^y), \ldots, \psi(n-(j-1), q^x; q^y), \psi(n-j, q^x, q^y)\}\} where \(q^x \geq q^x_0\) and \(q^y \leq q^y_0\), to the set of optimized expected utilities \{\(\psi(n-j, q^x_0, q^y_0)\}\) which correspond to the individual's initial holdings: \((q^x_0, q^y_0)\). It may be observed that this method of determining \(\psi(i, q^x, q^y)\) and the corresponding optimal reservation bid \(\{(p^{d*}(1, q^x, q^y), s^{y*}(1, q^x, q^y))\}\) is

\(^{18}\) It will be converse of the formulation in (2.23).
analogous to the solution of a stochastic multi-stage optimization problem by dynamic programming since the Principle of Optimality\textsuperscript{19} is applicable. Based on (2.22), the backward induction process may be expressed by the generalized recurrence relation:

\begin{equation}
\psi(n-j, q^X, q^Y)
= \max \left\{ u(q^X,q^Y), \psi(n-(j-1), q^X,q^Y) \right\} + \int_{p^s>p^d} \int_{p^s}^{p^d} s^x \int_{p^s}^{p^d} s^y \int_{p^s}^{p^d} s^z \psi(n-(j-1), q^X+q^Y(p^d, s^y, s^z), q^Y-q^X(p^d, s^y, s^z)) \right.d.p^d \left.d.s^z \left.d.s^y \right.d.s^x \right. \\
- c(n-(j-1))
\end{equation}

where \( n-j \) corresponds to \( j \) in (2.22) and \( j \) denotes the number of bilateral contacts remaining to be made before stopping exchange activity.

\section*{2.4 Some Additional Results}

It may be useful to illustrate the nature of the bilateral exchange process and to distinguish between the three cases diagramatically. First it is essential to derive some results that

\textsuperscript{19} See Intrilligator (1971), Ch. 13, and DeGroot (1970), p. 278. In the terminology of dynamic programming \( \psi(1,q^X,q^Y) \) will be the optimal performance function and \( \{ p^d(i+1,q^X,q^Y), s^y(i+1,q^X,q^Y) \} \) the set of control variables.
will permit a clearer comprehension of the optimal stopping rule derived previously, in (2.20).

Using (2.20), let us consider the locus of points \((q^{x*}, q^y)\) where \(q^{x*}\) is determined given \(q^y\) and for a given \(i\) by

\[
(2.29) \quad \psi(i, q^{x*}, q^y) = u(q^{x*}, q^y).
\]

For every given \(i = 0, 1, 2, \ldots, N\), this locus will partition the commodity space \((q^x, q^y)\) into two sets, the continuation set and the stopping set. We may re-write (2.29) as

\[
(2.29a) \quad \psi(i, q^{x*}, q^y) - u(q^{x*}, q^y) = 0.
\]

By totally differentiating (2.29a), and letting \(d(\psi - u) = 0\) we obtain,

\[
(2.30) \quad d(\psi - u) = \frac{\partial \psi}{\partial q^{x*}} \cdot dq^x + \frac{\partial \psi}{\partial q^y} \cdot dq^y
\]

\[
- \frac{\partial u}{\partial q^{x*}} \cdot dq^x - \frac{\partial u}{\partial q^y} \cdot dq^y = 0
\]

so that, rearranging, we have

\[
(2.31) \quad \frac{dq^{x*}}{dq^y} = - \left[ \frac{\frac{\partial \psi}{\partial q^y} - \frac{\partial u}{\partial q^y}}{\frac{\partial \psi}{\partial q^{x*}} - \frac{\partial u}{\partial q^{x*}}} \right].
\]

Since this typical individual has preferences and exchange opportunities that predispose him towards engaging in exchange activity to obtain more \(x\) and trade away \(y\), it is reasonable to assume that
an increase in $q^Y$, his holdings of the commodity that he desires to trade away will increase his expected utility of continuing exchange activity, $\Psi$, by more than it increases $u^i$, his utility of consuming without further exchange activity.\(^{20}\) Hence \(\frac{\partial \psi}{\partial q^Y} > \frac{\partial u}{\partial q^Y} > 0\). On the other hand, it is reasonable to assume that an increase in $q^x$, his holdings of the commodity that he is seeking to obtain more of, will increase his expected utility of continuing exchange, $\Psi$, by less than the increase in $u^i$, his utility of consuming without further exchange activity.\(^{21}\) Hence \(0 < \frac{\partial \psi}{\partial q^x} < \frac{\partial u}{\partial q^x}\).

Therefore, in the ratio on the right side of (2.22), the numerator will be positive and the denominator negative so that \(\frac{dq^x}{dq^Y} > 0\).\(^{22}\)

In (2.29a), it is possible to solve implicitly for $q^x$ in terms of $q^Y$ and $i$. Let this solution be written as:

\[(2.32) \quad q^x = b^*(i, q^Y)\]

\(^{20}\) This would be true if $x$ is a normal good. If, however, $x$ is an inferior good then the income effect on the demand for $q^x$ with respect to the increases in $q^Y$ could account for the increase in the expected utility of continuing exchange, $\Psi$, being less than the increase in the utility, $u^i$, of consuming without further exchange activity.

\(^{21}\) Again this would be true if $y$ is a normal good. If, however, $y$ is an inferior good then the income effect of an increase in $q^x$ might mean that the increase in the expected utility of disposing of $y$ through continuing exchange would exceed the increase in the utility of consuming without further exchange.

\(^{22}\) Of course, if either $x$ or $y$ is an inferior good then $\frac{dq^x}{dq^Y} < 0$ is possible.
From the earlier discussion it follows that

\[(2.33) \frac{dq^*}{dq^y} = \frac{\partial b^*(i,q^y)}{\partial q^y} > 0 \]

It can be shown that, given holdings \( (q^xS,q^yS) = \{q^x,q^y\} | q^x > b^*(i,q^y) \) after the \( i \)th bilateral contact, the individual will prefer to stop exchange activity without making the \( i+1 \) contact, since \( u(q^xS,q^yS) > \Psi(i,q^xS,q^yS) \). Conversely, given holdings \( (q^xC,q^yC) = \{q^x,q^y\} | q^x < b^*(i,q^y) \) after the \( i \)th bilateral contact, the individual will prefer to continue exchange activity by making the \( i+1 \) contact, since \( u(q^xS,q^yS) < \Psi(i,q^xS,q^yC) \).

Thus, the stopping rule in (2.20) can alternatively be represented as

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23 Proof: Using a Taylor series expansion in the neighbourhood of \( q^x = q^x^* = b^*(i,q^y) \) we may write:

\[
\psi(i,q^x,q^y) - u(q^x,q^y) = [\psi(i,q^x^*,q^y) - u(q^x^*,q^y)]
\]

\[
+ \left[ \frac{\partial \psi(i,q^x+c,q^y)}{\partial q^x} - \frac{\partial u(q^x+c,q^y)}{\partial q^x} \right](q^x - q^x^*)
\]

where, by (2.29a), the first expression on the right side is zero; while for some \( c \) such that \( q^x > q^x^* + c > q^x^* \) (when \( q^x < q^x^* \)), the second expression on the right side is the remainder term. Since, in (2.31), \( \frac{\partial \psi}{\partial q^x^*} < \frac{\partial u}{\partial q^x^*} \), with \( \frac{\partial \psi}{\partial q^x^*} \approx 0 \) and \( \frac{\partial u}{\partial q^x^*} \approx 0 \) both assumed to be derivatives, and \( q^x \) can be chosen arbitrarily close to \( q^x^* \) then \( \frac{\partial \psi(i,q^x+c,q^y)}{\partial q^x} < \frac{u(q^x+c,q^y)}{\partial q^x} \) and the second expression on the right side will be negative. Therefore, given \( q^x = q^xS > q^x^* \) and \( q^y = q^yS \), we have \( \psi(i,q^xS,q^yS) < u(q^xS,q^yS) \).
(2.34): \( q^x \geq b^*(1,q^y) \)?

It is also of interest to determine if the optimal stopping holdings of \( x, q^x \), will be increasing or declining (for given \( q^y \)) as an additional bilateral contact is made. Since \( i \) is an integer and with \( \Delta i = 1 \), this question may be represented as

\[
\frac{\Delta b^*(i,q^y)}{\Delta i} < 0
\]

Using (2.29a), it can be shown that:

(2.35) \( \frac{\Delta b^*(i,q^y)}{\Delta i} \simeq \left[ \frac{T}{\Delta i} \right] \cdot \frac{\Delta \psi}{\Delta i} \cdot \frac{\Delta u}{\Delta q} - \frac{\Delta q^x}{\Delta q^x} \cdot \frac{\Delta q^x}{\Delta q^x} \)

From the discussion of (2.31), it follows that the first expression on the right side of (2.35) will be positive and since it can be shown

\[24\text{ Proof: Let } \Delta \left[ \psi(i,q^x,q^y) - u(q^x,q^y) \right] = 0 \text{ with } \Delta q^y = 0.
\]

Therefore, \( \Delta \psi \cdot \Delta i + \Delta q^x \cdot \Delta q^y - \Delta q^x \cdot \Delta q^y \simeq 0 \)

so that, rearranging, we have

\[
\frac{\Delta q^x}{\Delta i} \simeq \left[ \frac{\Delta u}{\Delta q^x} \cdot \frac{\Delta q^x}{\Delta q^y} \right] \cdot \frac{\Delta \psi}{\Delta i} \].
that \( \frac{\Delta \psi}{\Delta i} > 0 \) \( 25 \), therefore,

\[
(2.36) \quad \frac{\Delta b^*(i,q^y)}{\Delta i} = \Delta b^*(i+1,q^y) - \Delta b^*(i,q^y) < 0.
\]

This means that for given holdings \( q^y \), the optimal stopping

---

25 **Proof:** Using (2.19) we get \( \psi(i,q^x,q^y) = E[\max\{\ldots\}-c(i+1)] \)

and \( \psi(i+1,q^x,q^y) = E[\max\{\ldots\}-c(i+2)] \) so that

\[
\frac{\Delta \psi}{\Delta i} = \psi(i+1,q^x,q^y) - \psi(i,q^x,q^y)
= E[\max\{u(q^x,q^y), \psi(i+2,q^x,q^y)\}]
- E[\max\{u(q^x,q^y), \psi(i+1,q^x,q^y)\}]
+ c(i+1) - c(i+2).
\]

Several cases are possible depending on the \( E[\max\{\ldots\}] \) expressions:

**Case 1:** \( \frac{\Delta \psi}{\Delta i} = E(u(q^x,q^y)) - E(u(q^x,q^y)) + c(i+1) - c(i+2) \)

\( = c(i+1) - c(i+2) < 0 \)

**Case 2:** \( \frac{\Delta \psi}{\Delta i} = E(u(q^x,q^y)) - E(\psi(i+1,q^x,q^y)) + c(i+1) - c(i+2) \)

\( = u(q^x,q^y) - E(u(q^x,q^y)) \)

\( = c(i+1) - c(i+2) < 0 \)

**Case 3:** \( \frac{\Delta \psi}{\Delta i} = E(\psi(i+2,q^x,q^y)) - E(\psi(i+1,q^x,q^y)) + c(i+1) \)

\( - c(i+2) \)

\( = E(E(\psi(i+3,q^x,q^y) - c(i+3)) - E(\psi(i+2,q^x,q^y) - c(i+2)) + c(i+1) - c(i+2) \)

\( = E(E(\ldots E(\psi(i+j+1,q^x,q^y) - \sum_{j=2}^{n-(i+1)} c(i+j+1)) \ldots)) \)

\( - E(E(\ldots E(\psi(i+j,q^x,q^y) - \sum_{j=2}^{n-1} c(i+j)) \ldots)) \)

\( = c(i+1) - c(n) < 0 \)

since \( c(i) < c(i+1) \) for all \( i \), as defined in relation to

(2.12). Therefore, in all cases \( \frac{\Delta \psi}{\Delta i} < 0 \).
holdings, \( q^* = b^*(1,q^Y) \), will decline as additional bilateral contacts are made (provided, as discussed earlier, that \( x \) is a normal good).

Using these properties, the nature of the stopping rule implied by (2.20) or (2.34) can be illustrated diagrammatically. A set of curves describing \( q^* = b^*(1,q^Y) \) can be generated for all \( i = 0,1,2, \ldots, N \leq \frac{L}{Z} \) in \((q^x,q^y)\) commodity space as shown in Figure 2.8. Then, it can be seen that the decision of the typical individual whether to stop or make at least one more bilateral contact after \( i \) contacts will depend solely on his holdings \((q^x,q^y)\). For example, if initially (when \( i=0 \)) the individual has holdings \((q^x_0,q^y_0)\) represented by point \( A \), in Figure 2.8, several observations can be made about how he will decide to behave. Since it can be seen that \( b^*(j,q^y_0) > q^x_0 > b^*(j+1,q^y_0) \), then it follows that the maximum possible number of bilateral contacts that the individual will ever consider undertaking will be \( j+1 \) (where \( j > 4 \) is illustrated). Of course, such a possibility would only occur if he has had the unusual misfortune of not having concluded any exchanges after making \( j \) bilateral contacts so that his holdings have remained unchanged. In that case the individual will stop bilateral contacting after making the \( j+1 \) contact irrespective of whether an exchange has been concluded. It is much more probable, however, that the individual will succeed in making an exchange prior to making a \( j^{th} \) bilateral contact. Let us suppose that an exchange is concluded with the first contact resulting in new holdings, \((q^x_1,q^y_1)\).
$W = \bar{u}(q_x, q_y) + w(L)$

$A = b^* (j, q_y)$

$B = b^* (1, q_y)$

$C = b^* (2, q_y)$

$D = b^* (0, q_y)$

FIGURE 2.8

- NO BILATERAL CONTACTS MADE TO TRADE AWAY Y FOR X
represented by point B, in Figure 2.8. It can be seen that, since \( b^{*}(1, q^{Y}) > q^{x} > b^{*}(2, q^{Y}) \), the individual will decide to make a second bilateral contact but that he will stop thereafter without making a third bilateral contact irrespective of whether an exchange is concluded with the second contact.

Thus, it can be seen that, when the individual concludes an exchange after a bilateral contact so that the holdings of \( y \) are reduced and holdings of \( x \) are increased, the maximum possible number of bilateral contacts that the individual might choose to undertake will be reduced. Also, as each additional bilateral contact is made, the holdings of \( x \) required to make the individual decide to stop bilateral contacting (i.e. \( h^{*}(1, q^{Y}) \)) become lower for any possible holdings of \( y, q^{Y} \). Therefore, it is inevitable that the individual will converge towards a state of stopping bilateral contacting and thereby ceasing all exchange activity.

It is now possible to illustrate the reservation bid setting process and to distinguish between the three previously identified cases of distinctive exchange situations diagrammatically. This has been done in Figure 2.9, Figure 2.10 and Figure 2.11. In each Figure, point A gives the holdings, \( (q^{X}, q^{Y}) \), after 1 bilateral contacts have occurred, while the cross-hatched area bounded by ADE encloses the set of all potential holdings after the \( i+1 \) bilateral contact, \( (q^{X}, q^{Y}) \), as dictated by the distribution of reservation bids, \( h(p^{5}, s^{X}) \), from which the offer of the \( i+1 \) bilateral contact will be drawn. For example, in Figure 2.9, the point B represents the reserva-
\[ \bar{w} = \bar{w} - c(i+1) = \bar{u}(q, q^*) + w(L-(i+1)z^*) \]

\[ \bar{w} = \bar{u}(q, q^*) + w(L-iz^*) \]

CASE 1:

EXCHANGE ACTIVITY CEASES AFTER THE i+1 BILATERAL CONTACT

- DISTRIBUTION OF EXCHANGE OFFERS THAT WOULD NOT RESULT IN AN EXCHANGE DURING THE i+1 BILATERAL CONTACT;

- DISTRIBUTION OF EXCHANGE OFFERS THAT RESULT IN EXCHANGE DURING THE i+1 BILATERAL CONTACT AND AFTER WHICH EXCHANGE IS STOPPED.

FIGURE 2.9
\[ W = \bar{W} - c(i+1) = \bar{u}(q^x, q^y) + w(\bar{L} - (i+1)z') \]

\[ \bar{W} = \bar{u}(q^x, q^y) + w(\bar{L} - iz') \]

CASE 2:

EXCHANGE ACTIVITY CEASES OR CONTINUES DEPENDING ON THE NATURE OF THE OFFER DURING THE i+1 CONTACT.  

- DISTRIBUTION OF EXCHANGE OFFERS THAT WOULD NOT RESULT IN AN EXCHANGE DURING THE i+1 BILATERAL CONTACT AND i+2 CONTACT WOULD BE MADE;

- DISTRIBUTION OF EXCHANGE OFFERS THAT RESULT IN EXCHANGE DURING THE i+1 BILATERAL CONTACT AND CONTAINS ALL EXCHANGE OUTCOMES FOR WHICH EXCHANGE IS CONTINUED BY MAKING AN i+2 BILATERAL CONTACT;

- DISTRIBUTION OF EXCHANGE OFFERS THAT RESULT IN EXCHANGE DURING THE i+1 BILATERAL CONTACT AND CONTAINS ALL EXCHANGE OUTCOMES FOR WHICH EXCHANGE IS STOPPED AFTER THE i+1 CONTACT.

FIGURE 2.10
\[ \tilde{W} = \tilde{W} - c(i+1) = \tilde{u}(q^x, q^y) + w(l-(i+1)z') \]
\[ \tilde{W} = \tilde{u}(q^x, q^y) + w(l-iz') \]

CASE 3:
EXCHANGE ACTIVITY CONTINUES (i.e. THE i+2 CONTACT IS MADE) IRRESPECTIVE OF THE OFFER DURING THE i+1 CONTACT.

DISTRIBUTION OF EXCHANGE OFFERS THAT WOULD NOT RESULT IN AN EXCHANGE DURING THE i+1 BILATERAL CONTACT AND AN i+2 CONTACT WOULD BE MADE;

DISTRIBUTION OF EXCHANGE OFFERS THAT RESULT IN EXCHANGE DURING THE i+1 BILATERAL CONTACT AND AFTER WHICH EXCHANGE IS CONTINUED BY MAKING AN i+2 BILATERAL CONTACT.

FIGURE 2.11
tion bid, \((p^{s1}, s^{x1})\), as shown, which if received from the \(i+1\) contact would result in the new holdings,

\((q^{x}, y_{x}(p^{d*}, s^{y*}, p^{s1}, s^{x1}), q^{y}-y_{x}(p^{d*}, s^{y*}, p^{s1}, s^{x1}))\),

represented by point \(C\). In all of the figures the differently cross-hatched areas illustrate the various possible outcomes of an \(i+1\) bilateral contact as corresponding to a sampled reservation bid, \(\{p^{s}, s^{x}\}\), as determined by the individual's own illustrated optimal reservation bid, \(\{(p^{d*}(i+1, q, y), s^{y}(i+1, q, y))\}\) which is the basis for the exchange rule and by the illustrated locus, \(b^{*}(i+1, q, y)\), which is the basis for the stopping rule.
Chapter 3

A MODEL OF INDIVIDUAL PLANNED EXCHANGE BEHAVIOUR

3.1 Exchange Opportunities Functions

To characterize the planned exchange behaviour of the individual formally, let us introduce the concept of an exchange opportunities function. Let \( x_y \) be the expected quantity of \( x \) obtainable in exchange for \( y \) if the individual engages in exchange activity. It follows from earlier discussion of bilateral exchange opportunities that the magnitude of \( x_y \) will be determined directly by the amount of \( y \) the individual is willing to trade away and the amount of time the individual plans to devote to this exchange activity. It is worth noting that if the individual planned to make only a single bilateral contact, then \( x_y \) could be readily determined from \( E(PS) \) of the distribution \( h(p^s, s^x) \), defined earlier, provided that the assumed exchange convention or bargaining process was taken into account to reconcile exchange bids with exchange outcomes.

Also, it will be argued that \( x_y \) will be influenced by the size of the individual's holdings of both commodities (consisting of endowment and inventories), as well as by the nature of his exchange activity (if any) to obtain \( y \) by trading away \( x \). Therefore, we may write one of his exchange opportunities functions as:

\[
(3.1) \quad x_y = E_{xy}(y^s, x^s, x^s_x, y^s, z_{xy}, z_{yx})
\]

where \( y^s \) is the amount of \( y \) the individual is planning to trade away
to obtain \( x \), \( x^S \) is the amount of \( x \) he plans to trade away to obtain \( y \), \( z_{xy} \) is the amount of time spent trading away \( y \) to obtain \( x \), \( z_{yx} \) is the amount of time spent trading away \( x \) to obtain \( y \), while as defined earlier, \( X \) and \( Y \) are inventories of \( x \) and \( y \) respectively, and \( x \) and \( y \) are fixed endowments of \( x \) and \( y \). Also, let us assume that \( E_{xy} \) is a single-valued continuous function which is twice differentiable with respect to all of its arguments.

In this two-commodity economy, every individual will be faced with two exchange opportunities functions. The other one will be:

\[
y_x = E_{yx}(x^S_y, y^S_x, x + x', y + y', z_{yx}, z_{xy})
\]

and its properties will be similar to those of \( E_{xy} \). Since the individual's endowments are assumed to remain fixed and exogenous, they will subsequently be omitted from the exchange opportunities function expressions.

The properties of these exchange opportunities functions may be inferred from the earlier discussion of the bilateral contacting process and of reservation bid distributions. In particular, for the exchange opportunities function \( E_{xy} \), let us suppose that in

\[1\] In view of the way the process of bilateral contacting was modelled earlier, it may appear somewhat paradoxical that a distinction should be drawn between \( z_{xy} \) and \( z_{yx} \). However, it suffices to note that this specification provides a simple way of taking into account the intentions of the individual when denoting exchange opportunities.
addition to fixed endowments \( x \) and \( y \), both Inventories and time
spent in exchange activity are fixed at \( \overline{x}, \overline{y}, \) and \( z_{xy} \)
respectively, and without loss of generality that \( x^s_y = 0 \) and \( z_{yx} = 0 \)
(i.e. no \( x \) is being traded away for \( y \) during the period). Then, it
follows that the fixed amount of time devoted to exchange activity to
trade away \( y \) for \( x \) means that the number of random bilateral
contacts that will be made during the period will be fixed.
Consequently, the expected quantity of \( x \) obtainable in exchange for
\( y \) cannot exceed the expected sum of the endowments and inventory
holdings of \( x \) possessed by the fixed-size random sample of other
individuals contacted and will depend on how much \( y \) the individual
wants to trade away (i.e. on \( y^s_x \)). If he is unwilling to trade away
any \( y \) (i.e. \( y^s_x = 0 \)), we have that \( x_y = E_{xy}(0, 0, \overline{x}, \overline{y}, z_{xy}, 0) = 0 \).

3.1.1 The Amount Traded Away

As the Individual increases the amount of \( y \) that he is
willing to trade away, the amount of \( x \) that he will expect to obtain
for it will increase initially. Moreover, it might be the case that
when the individual is willing to trade away only small amounts of \( y \),
the fixed sample of other individuals that he contacts will
predominantly be attempting to trade away relatively large quantities
of \( x \). Therefore, as discussed earlier in relation to Figure 2.7,
they may be willing to offer him \( x \) on better terms of exchange in
order to persuade him to engage in larger volume trades. In this case,
as \( y^s_x \) is increased, \( x_y \) will be increasing more rapidly so that the
expected amount of \( x \) obtainable per unit of \( y \) traded away will be
rising. However, as \( y^s_x \) is increased to the point where the individual wants to trade away relatively large quantities of \( y \), it will be necessary for him to conclude exchanges with more than one individual out of the fixed sample of other individuals that he makes bilateral contacts with. Therefore, although the amount of \( x \) obtainable will still be increasing as \( y^s_x \) is increased, the average expected amount of \( x \) obtainable per unit of \( y \) will be declining as \( y^s_x \) becomes larger requiring an increasing proportion of exchanges to be concluded with the fixed sample of individuals contacted. The other possibility is that this latter situation prevails from the outset with the amount of \( x \) obtainable increasing as \( y^s_x \) is increased but with the average expected amount of \( x \) obtainable per unit of \( y \) falling as \( y^s_x \) increased at all levels of \( y^s_x \).

At some point, as \( y^s_x \) is increased, the average expected terms of exchange will be such that within the fixed-size random sample of individuals contacted, every individual contacted will be willing to trade away a maximum quantity of \( x \). Then \( x_y = E_{xy}(y^s_x, 0, \bar{x}, \bar{y}, \bar{z}_{xy}, 0) \) will reach a maximum at this value of \( y^s_x \) (let us denote it as \( y^{s1} \)) and will be less than or equal to the expected sum of the holdings of \( x \) possessed by the fixed-size random sample of individuals.
contacted. Of course, the individual would normally find himself constrained by his holdings of \( y \) to trading away considerably less than \( y^{S1} \) of \( y \).

This describes the analytically relevant portion of the exchange opportunities function with respect to \( y^S_x \) since, even if he were able, the individual would never attempt to trade away more than \( y^{S1} \) of \( y \) while devoting only \( z_{xy} \) of his time to exchange activity.

These properties of an exchange opportunities function with respect to changes in \( y^S_x \) can be illustrated diagrammatically. The two possible forms of what may be called exchange opportunities curves are illustrated in Figure 3.1 and Figure 3.2 along with the shift resulting when \( z_{xy} \), the time spent trading away \( y \) to obtain \( x \), is increased, as will be discussed next.

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2 In exhibiting a willingness to trade away virtually all their holdings of \( x \), the individuals contacted might not merely be revealing a consumption demand for \( y \) but could largely be demonstrating what might be termed a speculative demand for \( y \). This would be based on the conviction, that the terms on which \( y \) is obtainable are so attractive in the light of their own exchange opportunities that gains from trade will be assured when \( y \) is re-exchanged back for \( x \).

3 If the individual did attempt to trade away ever larger quantities of \( y \) beyond \( y^{S1} \), he would never be able to obtain more \( x \) and the terms of exchange would begin to decline more rapidly than \( y \) increases, (an analogous situation to a monopolist attempting to sell a larger quantity when his demand curve has become inelastic) so that \( E_{xy}(y^S_x, x, y, z_{xy}) \) would be declining. In fact, in order to be in such a situation, the individual would have to be enjoying some sort of monopoly position as a supplier of \( y \).
3.1.2 The Amount of Time Spent on Exchange Activity

When the time spent trading away \( y \) to obtain \( x \) increases from \( z_{xy} \) to say, \( z_{xy}^* \), it will mean that the number of planned bilateral contacts to be made during the period will have increased. Therefore, for a given quantity of \( y \) being traded away, (i.e. for every level of \( y_x^s \)) the individual will have increased his options to reject less attractive exchange offers and to search for the most favourable ones. Thus, the expected quantity of \( x \) obtainable in exchange for \( y \) will increase for all values of \( y_x^s \) (i.e. \( E_{xy}(y_x^s,0,x,y,z_{xy},0) > E_{xy}(y_x^s,0,x,y,z_{xy},0) \), when \( z_{xy} < z_{xy}^* \)), as shown in Figure 3.1 and Figure 3.2.4

The properties of the exchange opportunities function, \( E_{xy}(y_x^s,0,x,y,z_{xy},0) \), with respect to \( z_{xy} \) can be more completely described by fixing \( y_x^s \), say \( y_x^s = y_x^s \) (with inventories fixed at \( \bar{x} \) and \( \bar{y} \) as before). As discussed previously, each bilateral contact is assumed to require a fixed interval of time to undertake so that the number of bilateral contacts will be directly proportional to \( z_{xy} \). Of course, if no bilateral contacts are made, (i.e. \( z_{xy} = 0 \), then

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4 Also, because the number of bilateral contacts is larger, the expected sum of the holdings of \( x \) possessed by the sample of individuals contacted will be larger. Therefore, the maximum expected quantity of \( x \) obtainable for \( y \) (when say, \( y_x^s = y_x^s^2 \)) will be greater.
\[ x_y = E_{xy}(y_x^s, 0, \bar{x}, \bar{y}, 0, 0) = 0. \] If \( z_{xy} \) is such that one bilateral contact can be made, then the expected quantity of \( x \) obtainable in exchange for \( y_x^s \) will be positive. As the number of bilateral contacts made is increased, the expected quantity of \( x \) obtainable for \( y_x^s \) will increase. In other words, as \( z_{xy} \) is increased, the expected average terms of exchange will improve. As stated previously, this will be because the additional bilateral contacts will provide the individual with more opportunities to reject unattractive exchange offers and, where \( y_x^s \) is relatively large, there will be more freedom to make exchanges involving smaller average quantities of \( y \).

However, each additional bilateral contact will be improving the expected terms of exchange by a smaller amount so that there will be diminishing returns to increasing \( z_{xy} \). This particular property is a standard result in search theory derived by Stigler\(^5\) and is portrayed graphically in Figure 3.3. It is also shown that if \( y_x^s \) were increased to \( \bar{y}_x^s \), the exchange opportunities curve would shift upwards, which is consistent with the earlier discussion pertaining to Figures 3.1 and 3.2.

3.1.3 Inventories

Now, let us discuss the properties of the exchange opportunities function, \( E_{xy} \), with respect to inventory stocks, \( X \) and \( Y \). First of all, the holding of inventory stocks of \( Y \) may attract potential traders so that the larger \( Y \) is, the greater will be the 

FIGURE 3.3
number of bilateral contacts encountered during a given amount of time devoted to exchange activity. Moreover, as $Y$ is increased the individual will be able to take more advantage of attractive exchange opportunities that are contingent upon larger volume transactions. This is because a potential trading partner who wishes to trade away a large quantity of $x$ can be persuaded to lower his terms of exchange if the entire quantity is exchanged for $y$ in one transaction since he will be gaining additional leisure time by not having to make any further bilateral contacts for the purpose of concluding several small volume exchanges.

Another way of looking at this is that an individual will discover that if he holds inventories of $y$, other individuals (those willing to trade away $x$) will seek him out as a trading partner because he will be equipped to undertake trades involving larger quantities and also because they will find that contacting individuals who have in previous periods held large inventories of $y$ will increase the likelihood of concluding exchanges by increasing the probability of a double coincidence of wants. For both of these reasons, search costs for other individuals will be lower and the inventory-holding individual will find himself able to negotiate

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6 Also, the individual may find that the frequency of bilateral contacts has increased to the point where, rather than actively seeking bilateral contacts, he finds it possible to fix his location and passively wait for other individuals to contact him, thereby reducing the amount of time required per contact. However, it is outside the present scope of this model to distinguish between various exchange technologies along these lines.
exchanges on more favourable terms.

These exchange-enhancing characteristics of inventory stocks, though important, are not in themselves sufficient to account for the entire relationship between exchange opportunities and inventories. It is obvious that an individual trading away $y$ to obtain $x$ and possessing a large inventory $Y$, would soon find this inventory exhausted if he tried to exploit fully the associated advantages of increased frequency of bilateral contacts and larger volume transactions—unless, he found some means by which to replenish these inventories. Invariably, such inventories can only be replenished by simultaneously engaging in exchange to obtain $y$ by trading away $x$ (and, of course, therefore holding also an inventory of $X$). Thus, the sustained maintenance of inventories, that are sufficiently large to influence exchange opportunities in the manner discussed earlier, is only possible if the individual finds it desirable to engage in middleman exchange. Therefore, the properties of the exchange opportunities function with respect to inventories are important primarily within the context of middleman exchange activity.

Within this context the main role of inventories in enhancing exchange opportunities can be postulated in the following way. Let us suppose that the individual is comparing exchange opportunities in two situations where the same amount of time, say $T_{xy}$, is spent trading away various possible amounts of $y, y_X$, which exceed his endowment, say $Y$ (i.e., $y_X > Y$). The first situation involves exchange while
not holding an inventory of $y$, while the second exchange situation involves holding an inventory of $y$, $\bar{y} = y^S_x - y$, and in both situations the inventory of $x$ held, say $\bar{x}$, is the same.

In the first situation, the individual would, during the trading period, find it necessary to trade away some $x$ (either out of the inventory $\bar{x}$, or out of the $x$ obtained for $y$ during the same period) in order to obtain the extra $y$, over and above $\bar{y}$, to make up $y^S_x$. Not having the entire quantity of $y$, $y^S_x$, on hand at the beginning of the period would impose a constraint on the individual's ability to trade away $y$ during the period. He would find that on some occasions during the period he could not take full advantage of attractive opportunities to trade away $y$, because he did not have enough (or any) $y$ on hand at the particular time (even though later in the period, he would be trading away more $y$ obtained after first trading away some $x$). This means that the individual would have to alternate his exchange activities between trading away $y$ and obtaining $y$ according to a pattern dictated by his immediate holdings of $y$ rather than his exchange opportunities. For example, at the beginning of the period the individual would not be able to take full advantage of exchange opportunities during bilateral contacts with other individuals who offer to trade away more than $\bar{y}$ of $y$ and, if he happened to trade away this endowment, he could expect to miss some subsequent opportunities to trade away more $y$ until he had traded away some $x$ and obtained more $y$.
This first situation may be compared with the second situation where the individual would hold a large enough inventory of \( y^s \), \( \bar{y} \), to ensure that he is able to trade away the \( y^s_x \) of \( y \) that he desires, while taking full advantage of the most attractive exchange opportunities, without concern about being temporarily constrained by a shortage of \( y \) during the period. Also, exchange activity to trade away \( x \) obtaining \( y \) would be undertaken to replenish the inventory, \( Y \), rather than to obtain \( y \) for immediate re-exchange and therefore, both types of exchange could proceed independently during the period.\(^7\)

This comparison may be restated in another way. With \( z_{xy} \) fixed at \( z_{xy} \), the number of random bilateral contacts that the individual can make during the period to trade away \( y \) will be fixed and, let us assume, (ignoring the advertisement effect of inventories) the same in both situations. If \( z_{xy} \) is sufficiently large then, within such a fixed size random sample of individuals contacted, there will be a subset consisting of the bilateral contacts with whom the exchange of a total of at least \( y^s_x \) of \( y \) could be successfully concluded and who offer more attractive terms of exchange than the remaining bilateral contacts not in the subset. From this definition it follows that the expected amount of \( x \) obtainable for \( y^s_x \) of \( y \) through exchange with this subset of bilateral contacts will be higher than for any other possible subset - let it be denoted \( x_{xy}(y^s_x) \). Also, it is clear that exchange with this expected subset of the

\(^7\) These ideas are clearly related to the Ostroy and Starr (1974) discussion of the role of inventories in facilitating full execution of trading plans although the framework is different.
Individuals contacted during the period can be expected to be concluded for the maximum amounts possible only if the individual expects to have on hand a sufficient quantity of y during the entire exchange period. This can be assured if the individual holds an adequate inventory of y, say \( Y \) as assumed earlier, which corresponds to the second situation discussed earlier.

On the other hand, if the individual holds no inventory of y so that a significant part of the amount of y, \( y_x^S \), traded away during the period must be acquired by simultaneously trading away x, then exchange with one or more bilateral contacts who belong to the earlier defined subset (offering the most favorable terms of exchange) can be expected to be constrained by temporary shortages of y. Therefore, the amount of y traded away during exchange with this subset of bilateral contacts can be expected to be less than the desired and possible \( y_x^S \). In order to trade away \( y_x^S \) of y, the individual would expect to have to conclude one or more exchanges with bilateral contacts not belonging to this subset at less favorable expected terms of exchange than those offered by any bilateral contact belonging to this subset. It follows that in this situation, which corresponds to the first situation discussed earlier, the expected amount of x obtainable for \( y_x^S \) will be less than \( \bar{x}_y(y_x^S) \), let it be denoted \( \bar{x}_y(y_x^S) \). Both \( \bar{x}_y(y_x^S) \) and \( \bar{y}_y(y_x^S) \) will vary as \( y_x^S \); the amount of y the individual desires to trade away, is varied and by definition will be equivalent to the exchange opportunities curves for the two situations (i.e., \( \bar{x}_y(y_x^S) = E_{xy}(y_x^S, \bar{x}_y, \bar{y}, z_{xy}, \bar{z}_y) \) and
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\( \bar{x}_y(y^s) = E_{xy}(y^s, x^s, \bar{x}, 0, \bar{z}_{xy}, \bar{z}_{yx}) \). These are illustrated in Figure 3.4. It can be seen that these exchange opportunities curves coincide as long as \( y^s_x \) is not very much greater than \( y \), since lack of inventory becomes a constraint only when a quantity of \( y \) larger than the endowment, \( y \), is being traded away.

On the basis of this discussion of the role of inventories in facilitating exchange, the properties of the exchange opportunities function, \( E_{xy} \), with respect to the size of inventory \( Y \), can be further described if \( y^s_x \) is fixed at \( \bar{y}^s_x \). It can be postulated that as the inventory \( Y \) is increased, the expected amount of \( x \) obtainable for \( \bar{y}^s_x \) (i.e., the expected terms of exchange) may increase very little until \( Y \) is of significant size (perhaps approaching the average amount of \( y \) traded in a typical exchange transaction). As the inventory \( Y \) is increased beyond that point, the expected amount of \( x \) obtainable for \( \bar{y}^s_x \) will increase at an increasing rate and then at a decreasing rate until at some point where further increases in \( Y \) will yield no further improvements in the expected terms of exchange. These properties of the exchange opportunities function with respect to inventory \( Y \) are illustrated in Figure 3.5 and imply that initially there are increasing returns to increases in inventory holding, then decreasing returns to a point where there are no further returns.

It may, however, be the case (if the endowment \( Y \) is relatively large or if \( Y \) is a commodity that is usually exchanged in very small quantities) that diminishing returns set in immediately as
\[ \bar{x}(y_{x}^{e}) = E_{xy}(y_{x}^{e}, \bar{x}_{y}, \bar{x}, \bar{y}, \bar{z}_{xy}, \bar{z}_{yx}) \]

\[ \bar{x}(y_{x}^{e}) = E_{xy}(y_{x}^{e}, \bar{x}_{y}, \bar{x}, 0, \bar{z}_{xy}, \bar{z}_{yx}) \]
inventory \( Y \) is increased. Then, the expected amount of \( x \), obtainable for \( y^S_x \), will increase at a decreasing rate up to the point where no further improvements in the expected terms of exchange occur, as is illustrated in Figure 3.6.

Similarly, it may be assumed that the holding of inventories of \( x \), the commodity being sought, will improve the bargaining power of the individual, thereby facilitating exchanges on more favorable terms. Therefore, the properties of the exchange opportunities function, \( E_{xy} \) with respect to \( X \) will be similar to those discussed with respect to \( Y \). Also, if the individual engages in middleman exchange activity, an inventory \( X \) may have a further indirect effect on the exchange opportunities function \( E_{xy} \). This could occur only if the inventory of \( y \), say \( Y \), is below the level where the desired exchange quantity of \( y \), \( y^S_x \), can be traded away during the period without being constrained by a temporary shortage of \( y \). Then, an inventory \( X \), by enhancing exchange opportunities to obtain \( y \), could reduce the constraints imposed by temporary shortages of \( y \) during the period and thereby indirectly enhance the exchange opportunities to obtain \( x \).

3.1.4 Simplified Exchange Opportunities Functions

If the individual engages in middleman exchange, both the amount of \( x \) being traded away, \( x^S_y \), and the amount of time spent trading away \( x \) to obtain \( y \), \( z_{yx} \), can be postulated to have an
indirect effect on $E_{XY}$, the exchange opportunities to obtain $x$ by trading away $y$. This indirect effect would be similar to the previously discussed indirect effect of inventory $\alpha$. If the inventory of $y$, $\overline{y}$ is inadequate, increases in $x_y$ and/or $z_{yx}$ would enhance opportunities to obtain $y$ and thereby reduce the constraint to trading away $\overline{y_x}$ resulting from temporary shortages of $y$ during the trading period.

Also, it is conceivable that other spillover effects exist so that time spent making bilateral contacts while seeking to trade away $y$ provides opportunities or information about opportunities to trade away $x$ and obtain $y$. Moreover, many other factors are likely to be determinants of exchange opportunities and hence could be included in the exchange opportunities function. Among these are the individual’s bartering skills, the information he possesses about his surroundings, his location relative to other individuals, as well as the preferences, endowments, inventory stocks, and trading habits of other individuals. However, these factors will be assumed to be exogenous to the exchange opportunities function formulated here, since they are not essential to the present task.

In order to avoid unnecessary complications in the model, it will be useful to simplify somewhat the formulations of the exchange opportunities functions discussed in the preceding sections by

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8 This most certainly would occur in view of the way the process of bilateral contacting was described earlier.
eliminating the variables which have been postulated to affect exchange opportunities only indirectly. These are \( x^s_y \) and \( z_{yx} \) in \( E_{xy}(x^s_y, x^s_y, x, y, z_{xy}, z_{yx}) \) as well as \( y^s_x \) and \( z_{xy} \) in \( E_{yx}(x^s_y, y^s_x, x, y, z_{yx}, z_{xy}) \). Therefore, equations (3.1) and (3.2) will be respectively simplified to:

\[
(3.1a) \quad x_y = E_{xy}(y^s_x, x, y, z_{xy})
\]

and

\[
(3.2a) \quad y_x = E_{yx}(x^s_y, x, y, z_{yx})
\]

Also, the previously postulated properties of these simplified exchange opportunities functions may be summarized for \( E_{xy} \) as follows:

\[
E_{xy}(0, x, y, z_{xy}) = 0; \quad E_{xy}(y^s_x, x, y, 0) = 0;
\]

\[
E_{xys} \geq 0, E_{xyss} < 0 \text{ as } y^s_x \frac{<}{>} k_s = k(x, y, z_{xy}) \text{ with } k_s \geq 0;
\]

\[
E_{xyz} \geq 0, E_{xyzz} \leq 0;
\]

\[
E_{xyy} \geq 0, E_{xyyy} \leq 0 \text{ as } y \frac{<}{>} y \text{ with } k_y = m(y^s_x, x, z_{xy}) \text{ with } k_y \geq 0;
\]

\[
E_{xyx} \geq 0, E_{xyxx} \leq 0 \text{ as } x \frac{<}{>} x \text{ with } k_x = j(y^s_x, y, z_{xy}) \text{ with } k_x \geq 0; \quad 9
\]

Finally, it is assumed that for \( (y^s_x, x, y, z_{xy}) \geq (k_x, k_y, k_x, 0) \):

\[
E_{xy}(h y^s_x, h x, h y, h z_{xy}) \leq h E_{xy}(y^s_x, x, y, z_{xy}) \text{ for all } h > 1.
\]

---

9 \( E_{xys} = \frac{\partial E_{xy}}{\partial y^s_x}, E_{xyz} = \frac{\partial E_{xy}}{\partial z_{xy}} \), etc.: and \( k_x \), \( k_y \), and \( k_x \) are inflection points which depend on the values of the other variables.
3.2 Utility Maximization Through Exchange

With the endowments received by the individual at the
beginning of each period fixed at \( x \) and \( y \), and assuming that his
inventory holdings at the beginning of the initial period are \( x \) and
\( y \), we may characterize this individual's planned behaviour by the
utility maximization problem:\(^{10}\)

\[
\text{(3.3) } \max U = U(x_c, y_c, x, y, \bar{z}) \quad ^{11}
\]

subject to constraints:

\[
\text{(3.4) } \bar{x} \geq x_c + (x - \bar{x}) + x^S - E_{x,y}(y^S, \bar{x}, \bar{y}, z_{xy}) ;
\]

\[
\text{(3.5) } \bar{y} \geq y_c + (y - \bar{y}) + y^S - E_{x,y}(x^S, \bar{x}, \bar{y}, z_{yx}) ;
\]

\[
\text{(3.6) } z = z_{xy} + z_{yx} ;
\]

and

\[
\text{(3.7) } x_c \geq 0, y_c \geq 0, x \geq 0, y \geq 0, z_{xy} \geq 0, z_{yx} \geq 0, y^S \geq 0, x^S \geq 0.
\]

The first two constraints state that the quantity of a commodity
consumed during the period plus the change in inventory stocks of the

\(^{10}\) It should be clarified here, that, since the exchange opportunities
functions have been defined in terms of expected quantities
obtainable during exchange activity, the maximization problem
entails uncertainty and hence is, strictly speaking, one of
maximizing expected utility. In order that the analysis does not
become unnecessarily complicated by the multivariate probability
distributions underlying the exchange opportunities functions, it is
necessary to assume that the individual is risk neutral, so that:
\( E(U(x_c, y_c, x, y, \bar{z})) = U(E(x_c, y_c, x, y, \bar{z})). \)

\(^{11}\) Recalling the formulation of equation (2.3).
commodity over the period plus the quantity of the commodity traded away during the period less the expected quantity of the commodity obtained as a result of exchange activity during the period, must not exceed the endowment of the commodity in that period. The third constraint expresses the identity that the total amount of time spent on exchange (leisure foregone) equals the sum of the amounts of time devoted to different exchange activities. The last constraints are non-negativity constraints on the solution.

As formulated above, this is a short-run problem because the initial inventory holdings, \( \overline{X} \) and \( \overline{Y} \), are not necessarily at intertemporal (stock-flow) equilibrium levels so that the solution to the problem constitutes a one period trading equilibrium in which inventory stock accumulation (or decumulation) takes place (but movement to the long-run intertemporal equilibrium within a single period is generally either impossible or undesirable).

The necessary and sufficient conditions for a maximum in the above problem can be derived by maximizing the Lagrangian:

\[
W = U(x_c, y_c, x, y, \overline{I} - z) \\
+ \lambda_x \left[ x + E_{xy}(y_x, \overline{X}, \overline{Y}, z_{xy}) - x_c - (x - \overline{X}) - x^S \right] \\
+ \lambda_y \left[ y + E_{yx}(y_y, \overline{X}, \overline{Y}, z_{yx}) - y_c - (y - \overline{Y}) - y^S \right] \\
+ \sigma (z - z_{xy} - z_{yx})
\]
The necessary Kuhn-Tucker conditions\textsuperscript{12} for a maximum are:

\begin{align}
(3.9) \quad & U_x - \lambda_x^* \leq 0, \quad (U_x - \lambda_x^*)x^*_c = 0, \quad x^*_c \geq 0 \quad 13; \\
& U_y - \lambda_y^* \leq 0, \quad (U_y - \lambda_y^*)y^*_c = 0, \quad y^*_c \geq 0; \\
& U_x - \lambda_x^* \leq 0, \quad (U_x - \lambda_x^*)x^* = 0, \quad x^* \geq 0; \\
& U_y - \lambda_y^* \leq 0, \quad (U_y - \lambda_y^*)y^* = 0, \quad y^* \geq 0; \\

(3.10) \quad & -\frac{U}{L-z} + \sigma^* \leq 0, \quad (-\frac{U}{L-z} + \sigma^*)z^* = 0, \quad z^* \geq 0; \\

(3.11) \quad & \lambda_x^*E_{xys} - \lambda_y^* \leq 0, \quad (\lambda_x^*E_{xys} - \lambda_y^*)y^*_{s} = 0, \quad y^*_{s} \geq 0; \\
& \lambda_y^*E_{yxs} - \lambda_x^* \leq 0, \quad (\lambda_y^*E_{yxs} - \lambda_x^*)x^*_{y} = 0, \quad x^*_{y} \geq 0; \\

(3.12) \quad & \lambda_x^*E_{xyz} - \sigma^* \leq 0, \quad (\lambda_x^*E_{xyz} - \sigma^*)z^*_{xy} = 0, \quad z^*_{xy} \geq 0; \\
& \lambda_y^*E_{yxz} - \sigma^* \leq 0, \quad (\lambda_y^*E_{yxz} - \sigma^*)z^*_{yx} = 0, \quad z^*_{yx} \geq 0; \\

(3.13) \quad & \bar{x} + E_{xy}^* - x^*_c - \frac{X^* - \bar{x}}{X^* - \bar{x}} = y^* x^* \geq 0, \\
& \bar{y} + E_{yx}^* - y^*_c - \frac{Y^* - \bar{y}}{Y^* - \bar{y}} = x^* y^* \geq 0; \\
& (\bar{x} + E_{xy}^* - x^*_c - \frac{X^* - \bar{x}}{X^* - \bar{x}} - x^* \lambda^*_x = 0, \quad \lambda^*_x \geq 0; \\
& (\bar{y} + E_{yx}^* - y^*_c - \frac{Y^* - \bar{y}}{Y^* - \bar{y}} - y^* \lambda^*_y = 0, \quad \lambda^*_y \geq 0; \\

\text{\textsuperscript{12} These will be sufficient for a twice-differentiable quasi-concave utility function having non-zero first derivatives and for exchange opportunities functions that are concave with respect to the maximizing variables in the neighbourhood of the solution (see Arrow and Enthoven (1961), p. 783). This can be ensured if it is assumed that there are no increasing returns with respect to the variables in the exchange opportunities functions.} \\
\text{\textsuperscript{13} * denotes solution values.}
\( (3.14) \quad z^* - z_{xy}^* - z_{yx}^* = 0 \)

These first order conditions can be converted into familiar marginal conditions to provide a clearer interpretation of the planned consumption, exchange and inventory-holding behaviour of the individual during the period. For positive solution values of all of the decision variables (i.e. \( y_x^* > 0, x_y^* > 0, x_c^* > 0, y_c^* > 0, x^* > 0, y^* > 0, z_{xy}^* > 0 \) and \( z_{yx}^* > 0 \)), we have the following conditions:

\( (3.15) \quad E_{yxs} = \frac{\lambda_x^*}{\lambda_y^*} = \frac{U_x}{U_y} = \frac{U_x}{U_y} = \frac{1}{E_{yxs}} \)

\( (3.16) \quad E_{xyz} = \frac{\sigma^*}{\lambda_x^*} = \frac{U_{L-Z}}{U_x} = \frac{E_{xyz}}{\lambda_y^*} = \frac{U_{L-Z}}{U_y} \)

and

\( (3.17) \quad \frac{U_x}{U_x} = \frac{U_y}{U_y} = 1 \)

Respectively, these have the interpretation that commodities should be bartered away to the point where their marginal rates of transformation through expected exchange equal their marginal rates of substitution in consumption, that time should be devoted to exchange to the point where the marginal productivity of time in improving the expected terms of exchange (measured by the expected additional amount of the commodity obtainable) equals the marginal rate of substitution of leisure for the
commodity in consumption, and that inventory holdings should be
adjusted to the point where the marginal rate of substitution between
the holdings as inventory and the consumption of each commodity should
equal the marginal rate of transformation between consuming the
commodity and adding it to inventories which is 1.

These marginal conditions will be modified if some of the
variables have zero solution values. Thus, if the individual does not
find it attractive to trade away \( x \) to obtain \( y \) (i.e. \( x_y^* = 0, \ z_{yx}^* = 0 \)), it must be, from (3.9) and (3.11), that

\[
(3.18) \quad \frac{\lambda_x^*}{\lambda_y^*} = \frac{U_x}{U_y} = \frac{U_x}{U_y} = \frac{1}{E_{yxys}}
\]

and from (3.12) that

\[
(3.19) \quad \frac{\sigma_x^*}{\sigma_y^*} = \frac{U_x^L - z}{U_x}; \quad \frac{\sigma_y^*}{\lambda_y^*} = \frac{U_x^L - z}{U_y}
\]

If, in addition, he disposes of his inventory of \( x \) and plans to hold
none at the end of the period (i.e. \( x^* = 0 \)), then we have that

\[
(3.20) \quad \frac{U_x}{U_y} = \frac{\lambda_x^*}{\lambda_y^*} = \frac{U_x}{U_y} = \frac{1}{E_{yxys}} \quad ; \quad \frac{1}{E_{yxys}} \leq \frac{1}{E_{yxys}}
\]

and that

\[
(3.21) \quad \frac{U_x}{U_x} \leq \frac{U_y}{U_y} = 1
\]
Similarly, other possibilities and combinations of corner solutions are readily conceivable.

The solution of the system (3.9)-(3.14) determines the planned short-run "trading" equilibrium of the individual by establishing the quantities of each commodity consumed, traded away and acquired in exchange during the period, the quantities held in inventory stocks at the end of the period and the amount of time spent on each type of exchange activity during the period (which gives the amount of leisure enjoyed during the period) as formulated in the plans of the individual.

In each subsequent period, the planned short-run "trading" equilibrium will be different as long as inventory accumulation (or decumulation) occurs. In the long run, provided certain conditions hold, an "intertemporal" (stock-flow) equilibrium will be reached where planned inventory changes will have ceased (i.e. \( X^e - \bar{X} = 0 \) and \( Y^e - \bar{Y} = 0 \)) and where the planned trading equilibria in all subsequent periods are identical. The intertemporal equilibrium solution values\(^{14}\) represent a steady state that, given the constant endowment flows assumed earlier, is contingent upon the exchange opportunities and the expectations of the individual remaining static over time. Convergence towards such an intertemporal equilibrium will be assured provided that the intertemporal preferences implicit in the individual's utility function reflect a positive subjective time.

\(^{14}\) Obtained by adding the conditions \( X^e - \bar{X} = 0 \), \( Y^e - \bar{Y} = 0 \) to the system (3.9) - (3.14).
preference as assumed earlier (see Ch.2., f.n. 4), and provided that the utility function is sufficiently non-convex.\textsuperscript{15} This condition will also be assumed - in effect, we assume that conditions for the convergence of successive trading equilibria to an intertemporal equilibrium exist.

3.3 The Subjective Rate of Time Preference

It is possible to deduce indirectly the subjective rate of time preference, $\rho$, that is implicit in the intertemporal equilibrium solution. This is not to say that the individual necessarily employs such a single discount rate in his decisions but merely that he acts as if he does.

In the steady-state the cost in utility terms of consuming a small increment, $dx$, less of commodity $x$, and adding it to inventory is $-\lambda x^* dx$ (approximately).\textsuperscript{16} Similarly, the benefit of this addition to inventory will be $\lambda x^* dX$ (approximately), where $dX = dx$. This benefit will equal the present discounted value in utility terms of the expected increase in returns resulting from this increase in inventory, so that (assuming for simplicity an infinite horizon) we may write:

\textsuperscript{15} The usual assumption in intertemporal optimization problems is that the utility function is concave. For example, see Intrilligator (1971), p. 406.

\textsuperscript{16} $*$ denotes intertemporal equilibrium solution values.
(3.22) \[ U^* dX = \left[ \sum_{t=1} \frac{1}{(1+\rho)^t} \left( \lambda_x^* E_{xyX}^* + \lambda_y^* E_{yxX}^* \right) \right] dX. \]

This benefit will be less than or equal to the cost in the steady state if no inventories are held or it will be equal to the cost in the steady state when inventories are held. Thus, (substituting \( dX \) for \( dX \)) we have

(3.23) \[ \lambda_x^* dx \geq \left[ \sum_{t=1} \frac{1}{(1+\rho)^t} \left( \lambda_x^* E_{xyX}^* + \lambda_y^* E_{yxX}^* \right) \right] dx, \]

which simplifies to

(3.24) \[ \lambda_x^* dx \geq \left[ \frac{\lambda_x^* E_{xyX}^* + \lambda_y^* E_{yxX}^*}{\rho} \right] dx. \]

Rearranging, we have

(3.25) \[ \rho \geq E_{xyX}^* + \frac{\lambda_y^* E_{yxX}^*}{\lambda_x^*}. \]

Similarly,

(3.26) \[ \rho \geq E_{yxY}^* + \frac{\lambda_x^* E_{xyY}^*}{\lambda_y^*}. \]

Both (3.25) and (3.26) hold with approximate equality if \( X^* > 0 \) and \( Y^* > 0 \), respectively. We may designate \( \rho \) as the implicit subjective rate of time preference.
Chapter 4

EXCHANGE TIME PATHS

4.1 Definitions and Assumptions

Within the framework of this model it is now possible to examine various possible time paths of successive short-run trading equilibria along which an individual might evolve from a particular initial endowment-exchange opportunity position. We will be interested in studying the paths evolving towards two broad categories of intertemporal equilibrium. The first intertemporal equilibrium would be one in which the individual continued to obtain his livelihood from his endowment, engaging in exchange only to improve his immediate consumption bundle by being either a "buyer" or a "seller" of a particular commodity (but not both) and holding only modest inventory stocks, if any, of one or both commodities. We will designate this category of intertemporal equilibrium as the consumer case. The second category of intertemporal equilibrium will be designated the middleman case because the individual will have found it advantageous to become both a "buyer" and a "seller" of each commodity, thereby profiting from middleman exchange activity in addition to his endowment flow. Such activity will be advantageous only if exchange opportunities are such that it is possible to obtain each commodity on better terms of exchange than it is necessary to offer when trading it away.

The consumer and middleman cases correspond to the nonspecialist and specialist traders described in Clower (1977), p. 209.
In demonstrating the evolution of middleman exchange processes from more primitive barter exchange, it will be of particular interest to study the time paths that might evolve from initial conditions under which middleman exchange is not an immediately advantageous proposition so that the initial short-run trading equilibrium of the individual conforms to the consumer case: This state of the world can be ensured by making some assumptions about the nature of exchange opportunities in the economy.²

Let us assume that all individuals in the economy are faced with identical exchange opportunities and that in the initial period no one holds any inventories. Thus, individuals are equally circumstanced except for their endowment flows and preferences. Also, let us assume an initial situation where every individual attempting to engage in middleman exchange (i.e. in both types of exchange activity) during a period finds that the terms of exchange at which he can expect to obtain \( y \) (by trading away \( x \)) are not sufficiently favorable compared to those that he expects to have to offer when trading away \( y \) for \( x \), since he cannot expect to be compensated adequately for the additional leisure lost no matter how he divides his time between these exchange activities. This situation can prevail even though in the economy as a whole the average terms of exchange at which \( x \) is traded away to obtain \( y \) during a period must be identical to the average

² Presumably, such a state of the economy would have emerged (as a result of exogenous technical, demographic, or social changes) from a more primitive barter state in which exchange opportunities were so dispersed and uncertain, and bilateral contacting so time consuming that no previous prospects for middleman exchange activity existed.
terms on which \( x \) is obtained by trading away \( y \).  

This is illustrated in Figure 4.1 and Figure 4.2 where point \( A, (x,y) \), is the endowment of a typical individual and where CAB reflects the various combinations of \( x \) and \( y \) obtainable if the average terms of exchange prevalent in the economy were attained. Although CAB has no precise analytical meaning in this model, it is useful as a benchmark indicating the average terms of exchange obtainable if, in some sense, "average" combinations of time and of quantity (of the commodity being traded away) were expended on exchange activity during the period.

Figure 4.1 shows that by concentrating all available leisure time, \( \overline{L} \), only on exchange activity to trade away \( y \) obtaining \( x \), the individual can expect to attain combinations of \( x \) and \( y \) along the exchange opportunities curve \( AF \) (which is a transformation of \( x_y = E_{x,y}(y^s, 0, 0, \overline{L}) \) where \( x_y = x-x \) and \( y^s = y-y \)). Spending a greater than average amount of time on this exchange activity permits him to obtain average terms of exchange that are considerably higher than the average terms prevalent in the economy. On the other hand, if

\[ \Sigma x_y = \frac{1}{\Sigma y^s} \]

\[ \Sigma y^s = \frac{\Sigma y_x}{\Sigma y^s} \]

A similar observation was made earlier in relation to \( p_A \) in Figure 2.6.

This is because in the economy as a whole the aggregate quantity of a commodity traded away during a period is identical to the aggregate quantity obtained in exchange. Thus, letting \( \Sigma \) denote aggregation over all individuals, we have that

\[ \Sigma x_y = \Sigma x^s_y \quad \text{and} \quad \Sigma y_x = \Sigma y^s_x \]

and therefore, the average terms of exchange are given by:

\[ \frac{\Sigma x_y}{\Sigma y^s} \quad \text{and} \quad \frac{\Sigma y_x}{\Sigma y^s} \]

earlier in relation to \( p_A \) in Figure 2.6.
\[ y - \tilde{y} = E_{yx}(\tilde{x} - x, 0, 0, L) \]
\[ x - \tilde{x} = E_{xy}(\tilde{y} - y, 0, 0, L) \]

**Figure 4.1**
only a very small amount of time, \( z_{xy} \), is spent on this exchange activity, then only lower average terms of exchange than prevalent in the economy can be expected to be obtainable as reflected in the exchange opportunities curve \( AG \) (a transformation of \( x_y = E_{xy}(y_x^S,0,0,z_{xy}) \)). In these instances, since \( x_y^S = 0 \), DAF and DAG will respectively depict the full set of constrained exchange opportunities.

Alternatively, higher than prevalent average terms of exchange also are attainable if all available leisure time, \( \bar{L} \), is expended to obtain \( y \) for \( x \) and these are reflected in the combinations of \( x \) and \( y \) along the exchange opportunities curve \( AH \) (which is a transformation of \( y_x = E_{yx}(x_y^S,0,0,\bar{L}) \) with \( y_x = y - \bar{y} \) and \( x_y^S = \bar{x} - x \)). Otherwise, if only a very small amount of time \( z_{yx} \), is expended to obtain \( y \) for \( x \) then only average terms of exchange lower than the prevailing average terms (indicated by \( AC \)) can be expected. These are reflected in the exchange opportunities curve \( AJ \) (a transformation of \( y_x = E_{yx}(x_y^S,0,0,z_{yx}) \)). Since \( y_x^S = 0 \), KAH and KAJ respectively depict the full set of constrained exchange opportunities.

If, however, the individual were to choose to devote all his leisure time, \( \bar{L} \), to middleman exchange activity, say, dividing it equally between both trading away \( y \) for \( x \) and \( x \) for \( y \), then, the situation might be as shown in Figure 4.2. Exchange opportunities could even be such that middleman exchange activity would be (up to a point) profitable in terms of being able to trade away \( y \) to obtain \( x \)
on better terms of exchange than obtaining \( y \) by trading away \( x \) as indicated by exchange opportunities curves AM (a transformation of \( x_y = E_{xy}(y_x, 0, 0, \bar{c}/2) \), and AN (a transformation of \( y_x = E_{yx}(x_y, 0, 0, \bar{c}/2) \)). These exchange opportunities curves make it possible to attain through middleman exchange activity combinations of \( x \) and \( y \) along the envelope curve RVS.

In these circumstances a sufficient condition for middleman exchange activity not to be taking place in the economy initially would be if all individuals in the economy had preferences such that the commodity gains from middleman exchange activity were not sufficient to compensate for the necessary loss of virtually all leisure time. An example of this situation is illustrated in Figure 4.2. The typical individual finds that the maximum expected utility attainable through middleman activity in these circumstances would be

\[
U(x_c, y_c, x_1-x_c, y_1-y_c, 0)
\]

involving the \((x_1, y_1)\) combination at \( V \), while a higher level of expected utility,

\[
U(x_c, y_c, x_2-x_c, y_2-y_c, \bar{c}-\bar{c}/2)
\]

is attainable at \( W \) where only \( y \) is traded away to obtain \( x \), while greater time (otherwise spent trading away \( x \) to obtain \( y \)) is expended on leisure.

With identical exchange opportunities and with initial conditions assumed to be such that exchange activity in the economy proceeds without middlemen, three distinct types of individuals will exist (depending on preferences and endowments): those who trade away \( y \) to obtain \( x \), those who trade away \( x \) to obtain \( y \), and those who prefer not to engage in any exchange activity.
FIGURE 4.2
4.2 Consumer Time Paths

Now, let us proceed to examine some of the time paths which a typical individual might follow towards attaining an intertemporal equilibrium. The first is a version of the consumer case where the initial short run trading equilibrium is also the intertemporal equilibrium because the individual finds it preferable not to accumulate inventories. Therefore, he begins each new period with only his endowment and undertakes exchange only to modify his current consumption bundle by engaging only in one type of exchange activity, say trading away \( y \) to obtain \( x \). This means that the intertemporal equilibrium solution values have the following characteristics:

\[ x^{**} > x > 0, \quad y^{**} = 0, \quad z^{**} = z > 0, \quad \frac{z_{xy}}{z_{yx}} < 0, \quad 0 < y_{x}, \quad y_{y}, \quad \text{and} \quad x_{y}^{**} = 0. \]

Then, the intertemporal equilibrium conforms with the following conditions. From equations (3.18) and (3.19) it follows that

\[ E^{**} \leq \frac{U^{**}_{x}}{U^{**}_{y}} ; \quad E^{**} = \frac{U^{**}_{x}}{U^{**}_{y}} ; \quad E^{**} \leq \frac{U^{**}_{x}}{U^{**}_{y}} ; \quad \text{and} \quad E^{**} = \frac{U^{**}_{x}}{U^{**}_{y}} \]

(with the inequalities holding with equality only by unusual concidence). Also, the absence of inventory accumulation means

\[ \frac{U^{**}_{x}}{U^{**}_{x}} \leq 1 \quad \text{and} \quad \frac{U^{**}_{y}}{U^{**}_{y}} \leq 1 \] (from equation (3.21)) and indicates

that the expected returns from the holding of inventories are insufficient to compensate for the subjective cost of postponing

\[ \text{Recall that } ** \text{ denotes intertemporal solution values.} \]
consumption reflected in the subjective rate of time preference, $\rho$.

While $\rho$ cannot be determined in this case (since we have a corner solution), it follows from equations (3.25) and (3.26) that

$$\rho \geq E^s_{xy}(y^s_x, 0, 0, z^s_{xy}) \quad \text{and} \quad \rho \geq \frac{\lambda^s_{xy}}{\lambda^s_{y}} E^s_{xy}(y^s_x, 0, 0, z^s_{xy})$$

(since $E^s_{yx} = 0$ and $E^s_{yy} = 0$, when $z^s_{yx} = 0$ and $z^s_{y} = 0$).

This case is illustrated in Figure 4.3 by means of a multi-quadrant diagram. In this diagram point $A$ depicts the initial endowment $(x, y)$. Point $B$ is the intersection of two curves:

$OPT_y y^s_x(x_{xy}, x_0, y_0, x, y)$, which is defined as the locus of utility-maximizing amounts of $y$ to be traded away when the amounts of time devoted to exchange activity are fixed at different levels such as $z^1_{xy}, z^2_{xy},$ or $z^3_{xy}$; and $OPT_z y^s_x(x_{xy}, x_0, y_0, x, y)$, which is defined as the locus of utility-maximizing amounts of time devoted to exchange when the amounts of $y$ to be traded are fixed at various levels such as $y^s_x$ or $y_x$. Both curves are contingent upon the given initial level of inventories: $x_0 = 0, y_0 = 0,$ and upon endowments. Therefore, the intersection of these two curves, determines the utility maximizing combination of $y^s_x$ and $z_{xy}$, which establishes AG (a transformation of $x_y = E_{xy}(y^s_x, 0, 0, z^s_{xy})$) as the appropriate exchange opportunities curve and the point $C$ (where $y^s_x = y^s_{xy}$) as the short-run trading equilibrium point. It follows then that $DD^1$ and $FF^1$ are consumption-inventory allocation constraints for $x$ and $y$ respectively. The highest indifference curves attainable along $DD^1$ and $FF^1$ occur at $D$ and $F$ respectively and these contact points
are shown to conform the conditions \( \frac{U_x}{U_X} < 1 \) and \( \frac{U_y}{U_Y} < 1 \) respectively. Also, the condition \( \rho > E_{xy}^{**} \) is shown to hold at \( D \).

Another version of the consumer case is very similar to the previous one except that some inventory accumulation is undertaken by the individual prior to the attainment of intertemporal equilibrium. Again, the individual finds it feasible to engage in only one type of exchange activity, trading away \( y \) to obtain \( x \), with the aim of modifying the composition of the bundle he is endowed with so as to maximize utility. He finds it attractive to forego some current-period consumption of one or both commodities in order to accumulate inventories during the period and this permits him to go from a situation

where \( \frac{U_x}{U_X} > 1 \) and/or \( \frac{U_y}{U_Y} > 1 \) to a point where \( \frac{U_x}{U_X} = 1 \)

and/or \( \frac{U_y}{U_Y} = 1 \), as in equation (3.17). This is because the present value of the expected improvement in exchange opportunities resulting from the accumulated inventories (which in our formulation is reflected in the marginal utility of inventory holding) exceeds the marginal utility of currently consuming the commodity. Hence, over the course of inventory accumulation in successive periods, it must be that

\[ \rho < E_{xy}^*(y^*_x, \bar{x}, \bar{y}, z^*_y) \] and/or \[ \rho < \frac{\lambda^*_x}{\lambda^*_y} E_{xy}^*(y^*_x, \bar{x}, \bar{y}, z^*_y), \]

the exchange productivity of inventories exceeds the implicit subjective rate of
time preference.\(^5\)

When inventory accumulation has ceased and intertemporal equilibrium has been reached, it must be that \( \rho = E_{x,y}(x^\ast, y, z_{xy}^\ast) \)

and/or \( \rho = \frac{\lambda x^\ast}{\lambda y} E_{x,y}(x^\ast, y, z_{xy}^\ast) \). In intertemporal equilibrium, the individual still engages in only one type of exchange activity the sole aim being to obtain a more preferred consumption bundle.

This case is illustrated in Figure 4.4. Again, A is the initial endowment point \((x, y)\), while B, is the intersection of \(OPT_x^S(x, y, x_0, y, x, y)\) and \(OPT_y^S(y, x, x_0, y, x, y)\) which establishes \(z_{xy}^\ast\), and \(y_{x_1}^\ast\), the trading equilibrium amount of time devoted to exchanging away y to obtain x during the first period. This establishes that \(C_1\), on the exchange opportunities curve \(E_{x,y}(x_{x_1}^S, y, x_{xy}^\ast)\), represents the expected post-exchange commodity bundle. Then it follows that \(KK^1\) and \(LL^1\) are the consumption-inventory allocation constraints for x and y respectively. The points \(C_1\) and \(F_1\), on \(KK^1\) and \(LL^1\) respectively, are points of tangency between the consumption-inventory allocation constraint and the indifference curves that correspond to the highest utility attainable during the period, reflecting the conditions \(\frac{U_x}{U_y} = 1\) and \(\frac{U_y}{U_x} = 1\). These establish how commodity holdings are to be

\[5\] Recall equations (3.25) and (3.26).
allocated between consumption and inventories giving the first period trading equilibrium solution values \( x^*_c, y^*_c, x^*_1 \) and \( y^*_1 \), depicted as points \( D_1 \) and \( J_1 \) respectively. Also, the proportions of \( x^*_1 \) to \( y^*_1 \) will be such that the marginal rate of transformation through exchange at \( C_1 \) and the marginal rate of substitution at \( D_1 \) are the same, conforming to the conditions of equation (23) that \( E_{xys} = \frac{U_Y}{U_X} \).

The individual, therefore, begins the second period with inventory holdings, \( (x^*_1, y^*_1) \), which enhance his exchange opportunities so that exchange activity in the second period can be expected to be carried on at somewhat more attractive terms of exchange. In Figure 4.4 this is depicted as a shift outward of the exchange opportunities curve towards \( D^{**} \).

The trading equilibrium in the second period and for numerous subsequent periods will be similar to the first trading equilibrium with inventories being accumulated and with only exchange opportunities to obtain \( x \) for \( y \) being pursued. These successive trading equilibria have not been explicitly depicted in Figure 4.4 but their implied time paths are illustrated by arrowed lines.

Eventually, when inventory holdings \( x^{**} \) and \( y^{**} \) have been accumulated (point \( J^{**} \) in Figure 4.4), the individual's behaviour will have converged to the intertemporal equilibrium. This is illustrated in Figure 4.4 where \( B^{**} \), the intersection of \( OPT_{z_{xy}}(y^*_x, x, y) \) and \( OPT_{x_{xy}}(z_{xy}, x, y) \) gives \( y^{**}_x \) and \( z^{**}_{xy} \), the equilibrium quantity of \( y \) to be traded away and the
equilibrium amount of time spent on exchange activity. This establishes the point \( D^{**} \) where \( x_c^{**} \) and \( y_c^{**} \) are the intertemporal equilibrium consumption quantities on the exchange opportunities curve \( E_{xy}(y_x^{**}, x^{**}, y^{**}, z_{xy}) \). At \( D^{**} \) the exchange opportunities curve will be tangent to the indifference curve that corresponds to the intertemporal utility maximum and conforms to the condition,

\[
E_{xys}^{**} = \frac{U^{**}}{U^{**}_x} \]

Also, it follows that \( RR^1 \) and \( SS^1 \) will be the consumption-inventory allocation constraints for \( x \) and \( y \) where \( G^{**} \) and \( F^{**} \) respectively will be the points of tangency confirming to the conditions \( \frac{U^{**}}{U^{**}_x} = 1 \) and \( \frac{U^{**}}{U^{**}_y} = 1 \).

4.3 The Evolution of Middlemen

The two time paths described above are representative of individuals who engage in exchange only to alter their endowment bundles for consumption purposes but who do not derive any livelihood from exchange activity by trading away for profit commodities previously obtained through exchange activity. Next, we turn to describing time paths that might be representative of individuals who come to derive some livelihood from exchange activity by becoming middlemen.

The first such time path is in its initial phase very similar to one described earlier in Figure 4.4. The individual finds it most preferable to engage in only one type of exchange activity, say trading
away \ y \ to \ obtain \ x. \ Thus, \ in \ the \ early \ periods, \ say \ periods

\[ i = 1, 2, \ldots, k-1, \] we have that \[ E_{yxs}^{i*} < \frac{U_x}{i*}, \ E_{yxz}^{i*} < \frac{U_y}{i*}, \ E_{xyz}^{i*} < \frac{U_z}{i*}, \]

\[ E_{xys}^{i*} = \frac{U_y}{i*}, \] and \[ E_{xyz}^{i*} = \frac{U_z}{i*} \] as in equations (3.18) and (3.19)

so that \[ x_{y_1}^s = 0, \ z_{y_1}^s = 0, \ y_{x_1}^s > 0, \ z_{xy_1}^s > 0. \] Also, in each of these periods the individual prefers to accumulate inventories by not consuming all of his post-exchange bundle, \((x + x^*, y - y_{x_1}^s)\), to the point where \[ \frac{U_x}{i*} = 1 \] and \[ \frac{U_y}{i*} = 1 \] conforming with equation (3.17). Also, for periods \[ i = 1, 2, \ldots, k-1, \] it must be the case (from equations (3.25) and (3.26)) that

\[ \rho < E_{xyx}^{i*}(y_{x_1}^s, x_{i-1}' - x_{i-1}' - x_{xy_1}^s) \] and \[ \rho < \frac{X_{x_1}^*}{\lambda_{y_i}^*} E_{xyx}^{i*}(y_{x_1}^s, x_{i-1}' - y_{i-1}' - z_{xy_1}^s) \]

since otherwise inventories would not be accumulated.

The accumulating inventories enhance exchange opportunities to the extent that in period \[ k, \] a situation arises where the individual finds it profitable to engage in both types of exchange activities. He finds that middleman exchange activity in these circumstances allows for sufficient quantities of each commodity to be traded away on average terms of exchange, which are so much more
favorable (than the terms offered in obtaining it) that any additional loss of leisure time is more than compensated by the gains from trade. In this situation and in periods after period $k$, the trading equilibrium will conform to equations (3.15), (3.16) and (3.17) so that

$$E_{xyz} = \frac{k^*}{U_X}, E_{yxs} = \frac{k^*}{U_Y}, E_{xyz} = \frac{U_{x-z}}{U_X}, E_{yxz} = \frac{U_{y-z}}{U_Y},$$

while

$$\frac{U_{x^*}}{U_X} = \frac{U_{y^*}}{U_Y} = 1.$$ Since intertemporal equilibrium will not, except by coincidence, be attained during period $k$, it may be assumed that inventory accumulation continues during and after period $k$ until some later period, $k+m$, when inventory stocks $X^{**}$ and $Y^{**}$ have been accumulated.

This case is illustrated in Figure 4.5. The diagramatic framework is the same for the previous cases except that most of the secondary quadrants have been omitted to permit a larger and clearer presentation of the main part of the diagram. Point $A$ again designates the endowment bundle, $(x,y)$, accruing at the beginning of each period. Again the individual is assumed to have no inventory stocks in the initial period. By spending more than the "average" amount of time on exchange activity trading away a relatively small amount of $y$ to obtain $x$, he is able to obtain $x$ on better terms than the "average" terms of exchange that prevail in the economy (denoted by the slope of $HAF$). The initial trading equilibrium occurs at point $B_1$ on the exchange opportunities curve $E_{xy}(y^*_x,0,0,z^*_xy_1)$ and the
individual accumulates inventories \((X^*, Y^*)\) given by point \(D_1\) by choosing to consume, the bundle \((x^*_1, y^*_1)\), at point \(D_1\). The slope of the tangent to the exchange opportunities curve at \(B_1\) will be equal to the slope of the tangent to the indifference curve at \(D_1\), reflecting the first order condition in equation (3.18).

The trading equilibrium in the second period and in numerous subsequent periods prior to the \(k^{th}\) period will be similar to the first trading equilibrium with only exchange opportunities to obtain \(x\) for \(y\) being pursued and with inventories being accumulated. These trading equilibria are not explicitly shown in Figure 4.5 but their time path will lie along the arrowed line between \(B_1\) and \(B_{k-1}\) with the equilibrium in period \(k-1\) occurring at \(B_{k-1}\), on the exchange opportunities curve \(E_{xy}(y^*_x, \bar{X}_{k-2}, \bar{Y}_{k-2}, z^*_x y_{k-1})\). The corresponding equilibrium consumption bundles will lie along the arrowed line between \(D_1\) and \(D_{k-1}\).

In period \(k\) exchange opportunities have become so enhanced that it is profitable to spend time engaging in exchange to obtain \(y\) for \(x\) in addition to obtaining \(x\) for \(y\). It has become possible to obtain a certain range of quantities of \(y\) on better terms of exchange than have to be offered when trading away \(y\) provided that appropriate amounts of time are devoted to each exchange activity. In order that a sufficient amount of time, \(z_{yx_k}\), can be spent on the new exchange activity (i.e. to attain the exchange opportunities curve \(E_{yx}(x^*_y, \bar{X}_{k-1}, \bar{Y}_{k-1}, z^*_y x_k)\), the individual may find it necessary to reduce somewhat the amount of time spent on exchange opportunities.
to obtain $x$ for $y$, so that $z_{xy_k}^* < z_{xy_{k-1}}^*$. Such a substitution effect would mean that the exchange opportunities curve $E_{xy}(y_x^s, \bar{x}_{k-1}, \bar{y}_{k-1}, z_{xy_k}^*)$ has fallen below the exchange opportunities curve $E_{xy}(y_x^s, \bar{x}_{k-2}, \bar{y}_{k-2}, z_{xy_{k-1}}^*)$. Without loss of generality, this possibility has been illustrated in Figure 4.5. Thus we have that in period $k$ the individual engages in exchange to obtain $x$ for $y$ along the exchange opportunities curve $E_{xy}(y_x^s, \bar{x}_{k-2}, \bar{y}_{k-2}, z_{xy_k}^*)$ up to the point $G_k$ and to obtain $y$ for $x$ along the exchange opportunities curve $E_{yx}(x_y^s, \bar{x}_{k-1}, \bar{y}_{k-1}, z_{yx_k}^*)$ up to the point $C$ so that the net result is a trading equilibrium at point $B_k$. The individual consumes the bundle $(x_{C_k}^*, y_{C_k}^*)$ at point $D_k$ because he chooses to continue to accumulate inventories.

The trading equilibria in the period $k+1$ and in subsequent periods will be similar to that of period $k$. Inventory accumulation will continue and the scale of middleman activity will be increasing as indicated by the growth in the amounts of each commodity obtained and traded away during exchange activity. The exchange opportunities curves associated with these successive equilibria have not been illustrated but the time paths lie along the arrowed lines. The outcomes of exchange to obtain $x$ for $y$ and of exchange to obtain $y$ for $x$ occur along the arrowed lines between points $G_k$ and $G^{**}$.  

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6 Graphically this is depicted as a transposition of the curve $AC_k$ where $A$ is transposed onto $G_k$ so that $C_k$ determines $B_k$.  


and between points $C_k$ and $C^{**}$ respectively. The net result is that the successive trading equilibria will lie along the arrowed line between points $B_k$ and $D^{**}$ with consumption occurring along the arrowed line between points $D_k$ and $D^{**}$, while inventory accumulation increases inventory holdings along the arrowed line between points $J_{k-1}$ and $J^{**}$. In period $k+m$, inventory holdings $X^{**}$ and $Y^{**}/(point \ J^{**})$ will have been accumulated and convergence to intertemporal equilibrium behaviour will have occurred. This corresponds to the trading equilibrium and consumption at point $D^{**}$ based on exchange activity along the exchange opportunities curve, $E_{xy}(y^S_{xy},x^{**},y^{**},z^{xy})$ at $C^{**}$ and along the exchange opportunities curve, $E_{yx}(x^S_{xy},x^{**},y^{**},z^{yx})$ at $C^{**}$. The slope of the tangent to the indifference curve $U^{**}$ at $D^{**}$ is equal to the slopes of the exchange opportunities curves at $C^{**}$ and $G^{**}$ conforming to the conditions in equation (3.15).

This time path of the evolution of an individual into a middleman represents a situation where the length of each trading period, $T$, is sufficient to permit a relatively "large" number of bilateral contacts to be made so that it can be profitable to undertake both types of exchange activity simultaneously when large enough inventories are held. However, another kind of time path can be visualized. Middleman exchange could evolve even if both types of exchange activity cannot be carried out profitably simultaneously during a trading period. It may be possible to expect to obtain sufficiently better than average prevailing terms of exchange by spending a relatively large amount of time during the period either
obtaining \( x \) for \( y \) or obtaining \( y \) for \( x \), but not both simultaneously.

In this case the individual might accumulate an inventory of only one commodity, say \( X \), while engaging in only one type of exchange, say trading away \( y \) to obtain \( x \) during the first \( k \) trading periods. Then, in period \( k \), when a sufficiently large inventory of \( X \) has been accumulated he would commence a middleman exchange pattern characterized by oscillating trading equilibria in successive periods. In period \( k \), he would trade away most of his (now relatively large) holdings (inventory plus endowment) of \( x \), except for a quantity consumed during period \( k \), \( x_{ck} \), and would thereby obtain a (relatively large) quantity of \( y \) most of which would be carried over as inventory into period \( k+1 \), except for period \( k \) consumption, \( y_{ck} \). Then, in period \( k+1 \), the holdings of \( y \) (consisting of the endowment \( y \) and the large new inventory of \( y \)) would be traded away, except for consumption \( y_{ck+1} \), to obtain \( x \). Most of this (relatively large) quantity of \( x \) would be carried over as inventory into period \( k+2 \), except for period \( k+1 \) consumption, \( x_{ck+1} \). Thus, the individual would have evolved into a middleman but with a middleman exchange activity cycle spread over two trading periods.

Between period \( k \) and some period \( k+m \) (\( m \geq 0 \)) this type of middleman exchange would continue and consumption of \( x \) and \( y \) would be such that the alternating inventories \( X \) and \( Y \) would continue to increase. In period \( k+m \), intertemporal equilibrium
would be achieved, characterized, for all subsequent periods, by a constant level of consumption of \(x\) and \(y\), \((x_c^{**}, y_c^{**})\), and by oscillating exchange activity and end-of-period inventories, \((X,Y,\ldots)\), that fluctuate between \((X^{**},0)\) and \((0,Y^{**})\). In this case, the intertemporal equilibrium solution values, for any period \(i\) after period \(k+m\), would be as follows:

\[
x_c^{**} = x_c^i = \begin{cases} 
 x + E_{xy}(y_c^{**}, 0, y^{**}, X^{**}) - X^{**} & \text{when } i = 2t \\
 X + X^{**} - x_c^{**} & \text{when } i = 2t+1
\end{cases}
\]

\[
y_c^{**} = y_c^i = \begin{cases} 
 y + Y^{**} - y_c^{**} & \text{when } i = 2t \\
 y + E_{yx}(x_c^{**}, X^{**}, 0, Z^{**}) & \text{when } i = 2t+1
\end{cases}
\]

\[
z_1^{**} = z_1^i = \begin{cases} 
 z & \text{when } i = 2t \\
 z_{yx}^{**} & \text{when } i = 2t+1
\end{cases}
\]

\[
x_1 = \begin{cases} 
 x^{**} & \text{when } i = 2t \\
 0 & \text{when } i = 2t+1
\end{cases}, \quad Y_1 = \begin{cases} 
 0 & \text{when } i = 2t \\
 y^{**} & \text{when } i = 2t+1
\end{cases}
\]

\[
Y_x^{**} = \begin{cases} 
 y + Y^{**} - y_c^{**} & \text{when } i = 2t, 1 \\
 0 & \text{when } i = 2t+1
\end{cases}
\]

And

---

7 Of course, the individual may choose not to completely deplete these inventories in alternating periods, retaining relatively small buffer (or precautionary) stocks \(S_x^{**}\) and \(S_y^{**}\). Then inventories would be fluctuating between \((X^{**}, S_y^{**})\) and \((S_x^{**}, Y^{**})\).
\[
\begin{align*}
xy_1 &= x_{y1}^* - \left\{ \begin{array}{l}
0 \\
\frac{0}{x + x^{**} - x_{C_1}}
\end{array} \right. \\
&\text{when } \begin{cases}
i = 2t \\
i = 2t+1
\end{cases}
\end{align*}
\]

where \( t = 0,1,2, \ldots \).

4.4 A Brief Review

In this chapter, it has been demonstrated within the framework of an individual exchange model, that in an economy where utility enhancing opportunities to undertake exchange exist, it can be possible and, for some individuals, desirable to become middlemen. Middleman exchange activity can increase utility if commodities can be re-exchanged at higher terms of exchange than they were obtained for. It was shown that individuals who evolve into middlemen must be able to exploit opportunities to obtain commodities on terms of exchange that are higher than the "average" terms of exchange prevailing in the economy. If exchange opportunities are the same for all individuals, this necessitates that middlemen devote a greater than "average" amount of time during each period to exchange activity, concluding exchanges primarily with individuals whose preferences incline them to spend a less than "average" amount of time engaging in exchange and who are prepared to accept less than "average" terms of exchange.

Also, it has been shown that, even if sufficient time is devoted to exchange activity to create opportunities for middleman exchange, the endowment of the individual may be too small to permit middleman exchange activity to be undertaken at a high enough volume to compensate sufficiently (or to the extent possible) for the loss of
leisure. This constraint could be overcome by sacrificing some consumption for several periods in order to accumulate inventories of the traded commodities which would permit a higher volume of middlemen exchange to be carried out.

In this simple exchange economy, exchange between two middlemen will obviously be unlikely (since all middlemen will be seeking exchanges on better than "average" terms). Also, higher exchange volumes necessitate that a middleman undertake exchanges with several (or many) other individuals during each period. Therefore, in this economy, only a small proportion of individuals can be middlemen, although no barriers to middleman exchange exist (other than the necessity of having accumulated inventories). The vast majority of individuals will be induced by their preferences to remain consumers.

Two characteristics will be important determinants of whether or not an individual evolves into a middleman. First, because middleman exchange activity involves devoting a relatively large amount of time to exchange activity, thereby foregoing leisure, and since foregone leisure is the only transaction cost, an individual's preference for leisure relative to consumption of commodities x and y will be an important determinant of exchange behaviour.

Second, it has been shown that middleman exchange activity will be constrained (if not made impossible) unless relatively large inventories of the traded commodities are held. Therefore, only individuals whose preferences make them willing to accumulate and
maintain sufficient inventories will be able to become middlemen. The accumulation of inventories for the purpose of middlemen exchange involves a decision to redistribute consumption intertemporally. Initially, less consumption is tolerated, while inventories are accumulated, with the expectation that middleman exchange will allow a higher level of consumption to be enjoyed in future periods. An individual with a relatively low subjective rate of time preference would accumulate inventories and become a middleman if a relatively small increase in utility in future periods could be expected. In contrast, an individual with a relatively high subjective time preference rate would discount future consumption more heavily and would find a relatively low increase in expected future utility insufficient to compensate him for the loss of present utility resulting from consuming less to accumulate inventories.

The emergence of a few middlemen from within a large number of individual consumer traders is unlikely to have appreciable impact on the exchange opportunities of either the consumers or the middlemen. However, the evolution of a class of middlemen, sufficient in number, to provide organized facilities for exchange must invariably modify the exchange opportunities of all individuals. For the same reason the exchange opportunities of middlemen will be modified by competition among themselves. Since only a small proportion of individuals can be middlemen, competition will determine which individuals become and remain middlemen. In this respect, individuals with a low relative preference for leisure and a low subjective time preference rate will have a distinct competitive advantage, since they
will be willing to continue middleman exchange activity even if exchange opportunities become less lucrative. These and other aspects of the evolution of exchange processes will receive more systematic analysis and discussion in the pages that follow.
Chapter 5
MIDDLEMEN AND THE REPLACEMENT OF UNORGANIZED EXCHANGE
BY AN ORGANIZED MARKET

5.1 The Evolution of Middlemen and Organized Markets

The evolution of middlemen in an economy where previously only unorganized barter exchange among individuals existed would create changes in individual exchange opportunities and in the overall patterns of trade in the economy. It is possible to discuss these changes within the framework of the model of the evolution of exchange processes by returning to the previously discussed concepts that underlie the exchange opportunities function.

In Chapter 2, unorganized barter exchange was characterized as a random sampling process where an individual sets out to make bilateral contacts with other individuals. The bilateral contacts involve trading offers to exchange \( x \) for \( y \) or \( y \) for \( x \). It was assumed that individuals have accurate expectations about the distribution of exchange offers obtainable by bilateral contacting and hence about the nature of their exchange opportunities function, and that each bilateral contact requires a fixed interval of time to undertake irrespective of whether an exchange is agreed to or not.

The evolution of middlemen in the economy would not only alter the distribution of exchange offers obtainable by individuals making bilateral contacts and perhaps the length of time required to
make each bilateral contact, but also - more fundamentally - would alter the nature of the bilateral contacting process itself. Bilateral contacts with middlemen would be qualitatively different from bilateral contacts with other individuals. It can be conjectured that bilateral contacting with middlemen would have several distinguishing characteristics.

First of all, because middlemen would be devoting a large proportion of available time to exchange activity, they would be regularly accessible to individual seeking to trade. It can be visualized that either middlemen would make bilateral contacts with individuals regularly by acting as itinerant merchants or else they would make known their readiness to be contacted for exchange purposes by individuals at established locations.

Another characteristic of bilateral contacting with middlemen is that there would be no quantity uncertainty because it would be known that middlemen hold relatively large inventories of both x and y for exchange purposes. It is important to recall that under unorganized barter exchange both "price" and "quantity" uncertainty are aspects of bilateral contacting. An individual engaging in unorganized barter exchange can expect a significant proportion of the bilateral contacts to be with individuals with whom he will be able to trade away only part of the quantity that he is planning to trade away. Thus, he may find that he must continue bilateral contacting, if he wants to trade away the planned quantity of the commodity, or else (if the additional leisure cost is too great) he must settle for consuming a
combination of commodities that is less preferred than the planned (expected) combination. In unorganized barter exchange, such occurrences would be more common than might appear at first glance. This is because, besides contacting individuals who are prepared to exchange only small quantities of commodities initially, an individual would be contacting some individuals who have already made exchanges previously during their current trading period and have only small quantities remaining to be exchanged.

On the other hand, in making bilateral contacts with middlemen an individual could virtually eliminate such quantity uncertainty. Middlemen will be able and willing to accept the exchange of any quantity of a commodity if the terms of exchange are agreeable. Therefore, an individual can be virtually certain that a single successful exchange with a middleman will be sufficient to permit him to attain his most preferred combination of commodities feasible at the agreed terms of exchange.

The characteristics of regular accessibility for exchange and of no quantity uncertainty that distinguish bilateral contacting with middlemen mean that the evolution of one or more middlemen guarantees the emergence of an organized market system for exchange of \( x \) and \( y \) in the midst of a system of unorganized barter exchange. Thus, it is evident that processes in this economy will have evolved to where there are two groups of individuals: a relatively small group of middlemen providing an organized market and a relatively large group of consumers. Consumers will have the option of continuing unorganized
barter exchange activity through bilateral contacts with other consumers or of undertaking exchange in the organized market through bilateral contacts with middlemen. It remains to be demonstrated that under certain circumstances natural economic forces can lead to the virtual disappearance of unorganized barter exchange after the emergence of an organized market mediated by middlemen.

5.2 Middlemen and Price Dispersion in the Organized Market

In addition to the process of bilateral contacting in the organized market with middlemen being different, it can be demonstrated that the distribution of the terms of exchange obtainable from middlemen will also be different. It may be recalled that in Chapter 2 the distribution of exchange offers faced by an individual making a bilateral contact in the unorganized barter economy was derived and illustrated in Figure 2.6 in relation to the holdings, \((x, y)\), of the individual at point A. This is reproduced in Figure 5.1. It may be recalled that in this Figure, \(p^A\) (slope of line CAB), denotes the "average" terms of exchange prevailing in the economy, while \(E(p_d)\) and \(E(p_s)\), respectively, denote the expected price offer (i.e. in units of \(y\) per unit \(x\)) receivable when trading away \(x\) to obtain \(y\) and the expected asking price (in units of \(y\) per unit \(x\)) receivable for \(x\) when obtaining \(y\) by trading away \(x\) during a bilateral contact. It was shown that \(E(p_d) < p^A < E(p_s)\). It may be recalled that an individual must expect to make an "average" number of bilateral contacts and must reject less attractive offers if he expects either to trade away \(x\) for \(y\) at an average price as high as \(p^A\).
- Domain of $g(p^d, s^y)$
- Domain of $h(p^s, s^x)$

FIGURE 5.1
of \( y \) per unit \( x \) or to obtain \( x \) at an average price as low as \( p^A \) of \( y \) per unit \( x \).

Using these relationships, it is now possible to make some observations about the distribution of prices that individuals would face when making bilateral contacts with middlemen in the organized market. Let \( p^{sm} \) denote a price (in units of \( y \) per unit \( x \)) at which individuals will be able to obtain \( x \) by trading away \( y \) during a bilateral contact with a typical middleman. Also, let \( p^{dm} \) denote a price (in units of \( y \) per unit \( x \)) at which individuals will be able to trade away \( x \) to obtain \( y \) during a bilateral contact with the middleman. Since the middleman will be earning a return from his exchange activities, it is necessary that \( p^{dm} < p^{sm} \).

Furthermore, a typical middleman would not be offering a price as high as \( p^A \) (the "average" price of \( x \), in units of \( y \) per unit \( x \), prevailing in the unorganized barter economy) when obtaining \( x \) by trading away \( y \), nor would he be willing to accept terms of exchange as low as \( p^A \) when trading away \( x \) to obtain \( y \), so that \( p^{dm} < p^A < p^{sm} \). This is because neither of the other two exchange price possibilities, \( p^{dm} < p^{sm} < p^A \) or \( p^A < p^{dm} < p^{sm} \), could be maintained by a middleman without creating imbalances in his
inventories. 

Also, the typical middleman will be offering a price higher than \( E(p_d) \) when obtaining \( x \) for \( y \) (i.e. \( p^{dm} > E(p_d) \)) and will be willing to accept a price lower than \( E(p_s) \) when trading away \( x \) for \( y \) (i.e. \( p^{sm} < E(p_s) \)). This is because, otherwise, he would find himself able to conclude exchanges with only a small fraction of individuals contacted (only those unwilling to make further bilateral contacts in search of a more favorable price). If the price obtainable from a middleman was not more favorable than the expected price (\( E(p_d) \) or \( E(p_s) \)) obtainable through making a single bilateral contact, then most individuals prepared to make a number of bilateral contacts in search of a more favorable price (say in the neighborhood of \( p^A \)) would prefer to continue bilateral contacting in the unorganized barter economy rather than deal with the middleman in the organized market.

---

1 If the middleman were to offer to supply \( x \) at a price equal to or below the "average" prevailing in the economy (i.e. if \( p^{sm} \leq p^A \)), he might expect to succeed in concluding exchanges with almost all individuals who contact him wanting to obtain \( x \) for \( y \). This would imply relatively rapid reductions and increases in his inventories of \( x \) and \( y \) respectively. Simultaneously, the price at which the middleman would be accepting \( x \) by trading away \( y \) would have to be considerably lower than the "average prevailing in the economy" (i.e. \( p^{dm} \leq p^A \)) in order to maintain a profitable spread between \( p^{dm} \) and \( p^{sm} \). Therefore, the middleman could expect to conclude exchanges with only a fraction of the individuals who contact him wanting to trade away \( x \) for \( y \) (only those who planned to make relatively few bilateral contacts and expected to trade away \( x \) at relatively low prices). This would only imply relatively slow increases and reductions in his inventories of \( x \) and \( y \) respectively. The net effect would be that the middleman could expect his inventory of \( x \) to be falling and his inventory of \( y \) to be growing, rather than both remaining relatively stable (at intertemporal equilibrium levels, \((X^{**}, Y^{**})\)). Similarly, if \( p^A \geq p^{dm} < p^{sm} \), the middleman would expect to find his inventory of \( y \) falling and his inventory of \( x \) increasing, rather than both remaining stable.
The relationship of the typical middleman's prices to the distributions of the exchange offers obtainable by an individual when making a bilateral contact in the unorganized barter exchange economy can now be summarized as \( E(p_d) < p_{dm} < p_A < p_{sm} < E(p_s) \) as illustrated in Figure 5.1.

If a number of middlemen have emerged in this economy, then individuals would view making bilateral contacts with middlemen as a random sampling process (since middlemen would be under no compulsion to set identical prices). We may assume that the distribution of prices obtainable from bilateral contacts with middlemen in the organized market can be described by a pair of continuous probability distribution functions defined as follows.

Let \( p_{sm} \), the price at which a middleman will be supplying \( x \) for \( y \), be a random variable which takes on values between \( p_{sm}^1 \) and \( p_{sm}^B \). Then, let \( d(p_{sm}) \) be the continuous probability density function describing the distribution of middleman "selling" prices, where

\[
\int_{p_{sm}^1}^{p_{sm}^B} d(p_{sm}) dp_{sm} = 1
\]

Similarly, let \( p_{dm} \), the price at which a middleman will be accepting \( x \) in exchange for \( y \), be a random variable which takes on values between \( p_{dm}^1 \) and \( p_{dm}^K \). Then, let \( f(p_{dm}) \) be the continuous probability density function describing the distribution of middleman "buying" prices, where

\[
\int_{p_{dm}^1}^{p_{dm}^K} f(p_{dm}) dp_{dm} = 1
\]
The above discussion has demonstrated that middlemen will be constrained to setting prices less favorable than \( p^A \) by the necessity of earning a profit and maintaining balanced inventories and that middlemen will be forced by the competition from the unorganized barter sector to set prices more favorable than \( E(p^S) \) and \( E(p^d) \). In view of this, it is now possible to compare the distributions of prices faced by individuals making bilateral contacts with middlemen to the distributions of exchange offers faced by individuals making bilateral contacts in the unorganized barter economy. This is done in Figure 5.2 in relation to the holdings \((\mathbf{X,Y})\) of a typical individual. It is illustrated that \( E(p^d) < p^{dm} < p^{dm} < p^A < p^{sm} < p^{sm} < E(p^S) \).

Therefore, it may be noted that individuals will be faced with less price dispersion if they make bilateral contacts with middlemen in the organized market as compared to making bilateral contacts with other individuals in the unorganized barter exchange sector. Moreover, since it can be seen that \( E(p^{dm}) > E(p^d) \) and \( E(p^{sm}) < E(p^S)^2 \), it is obvious that an individual would obtain a more favorable expected price when making a single bilateral contact with a middleman in the organized market as compared to making a single bilateral contact with another individual in the unorganized barter sector. Therefore, an individual would have to plan to make a number of bilateral contacts

\[
E(p^{dm}) = \int_{p^c}^{p^{dm}} \int_{dm}^{K} p^{dm}(p^{dm}) dp^{dm}
\]

and \( E(p^{sm}) = \int_{p^c}^{p^{sm}} \int_{sm}^{B} p^{sm}(p^{sm}) dp^{sm} \).
FIGURE 5.2
with other individuals and to reject some offers before he could expect to obtain a price as favorable as the expected price obtainable when making a single bilateral contact with a middleman.

5.3 Exchange Opportunities Functions in the Organized Market

It has been argued that the main characteristics distinguishing an individual's exchange opportunities in the organized market with middlemen from his unorganized barter exchange opportunities with other individuals would be: (a) regular accessibility; (b) no quantity uncertainty; (c) reduced price dispersion; and (d) initially, a more favorable expected price. It follows that this distinction would influence the "planned" behaviour of individuals and may be represented analytically by a different set of exchange opportunities functions. Thus, after the evolution of middlemen, individuals would begin to perceive separate organized market exchange opportunities functions as alternatives to the unorganized barter exchange opportunities that prevailed traditionally.\(^3\)

It may be recalled that in Chapter 3 a simplified unorganized barter exchange opportunities functions involving trading away \( y \) to obtain \( x \) was specified as:

\[^3\text{For the time being, it is useful to assume \textit{ceteris paribus} - that the evolution of middlemen and the emergence of organized market exchange opportunities functions has occurred without having modified or shifted the existing unorganized barter exchange opportunities function.}\]
(3.1a) \[ x_y = E_{xy}(y_x^S, X, Y, z_{xy}) \]

where \( x_y \) was the expected quantity of \( x \) obtainable in exchange for \( y \), \( y_x^S \) was the amount of \( y \) being traded away for \( x \), \( z_{xy} \) was the amount of time spent trading away \( y \) to obtain \( x \), and \( X \) and \( Y \) were inventory holdings of \( x \) and \( y \) respectively. A corresponding organized market exchange opportunities function could be formulated as:

(5.1) \[ x_y = M_{xy}(y_x^S, z_{xy}) \]

having properties similar to \( E_{xy} \) except that inventories do not appear in the specification. This is because it seems more reasonable to make the assumption than an individual's inventory holdings (over and above the amount offered for exchange) have no influence on the terms of exchange obtainable when making bilateral contacts with middlemen.

This organized market exchange opportunities function can be readily compared with the corresponding unorganized barter exchange opportunities function by using diagrams. In Figure 5.3(a), the exchange opportunities curves corresponding to two levels of time devoted to exchange activity, \( z_{xy}^0 \) and \( z_{xy} \), have been depicted and labelled accordingly. The organized market exchange opportunities curves, \( M_{xy}(y_x^S, z_{xy}^0) \) and \( M_{xy}(y_x^S, z_{xy}) \), are shown to be linear because exchange with middlemen eliminates quantity uncertainty so that
FIGURE 5.3
an individual can expect to trade away all the \( y \) he wants in a single transaction if the terms of exchange offered by the middleman are acceptable. The corresponding non-linear unorganized barter exchange opportunities curves are \( E_{xy}(y_x^S,x,y,z_{xy}^0) \) and \( E_{yx}(y_x^S,x,y,z_{xy}) \).

It can be seen that if a single bilateral contact, requiring only the small amount of time \( z_{xy}^0 \), is made then the expected amount of \( x \) obtainable when trading away a quantity of \( y \), say \( y_x^S \), will be greater if the individual contacts a middleman than if he makes the bilateral contact with another individual in the unorganized barter sector (i.e. \( x_{xy}^1 x_{xy}^2 \)). This follows directly from the earlier discussion pertaining to Figure 5.2 (where \( E(p^{SM}) > E(p^S) \)). Also, if a relatively large amount of time, \( z_{xy} \), is spent on exchange, the expected amount of \( x \) obtainable for \( y_x^S \) will be greater for both organized and unorganized exchange. However, it is possible (and even likely) that, with \( z_{xy} \) of time spent on exchange, the expected amount of \( x \) obtainable for \( y_x^S \) will be greater if the individual makes bilateral contacts in the unorganized barter sector than if he searches for the most favorable terms of exchange among middlemen in the organized market, as illustrated in Figure 5.3(a) (where \( x_{xy}^2 > x_{xy}^3 \)). This again follows from the earlier discussion pertaining to Figure 5.2, where it was shown that there is less price dispersion in organized markets with middlemen than in unorganized barter exchange.

Consequently, each increment of additional time spent on exchange activity increases the expected amount of \( x \) obtainable for \( y_x^S \) from middlemen in organized markets by less than it increases the amount of \( x \) obtainable for \( y_x^S \) from other individuals in unorganized barter
exchange. This is illustrated in Figure 5.3(b). It shows how the expected amount of \( x \) obtainable for \( \bar{y}^S_x \) and for a somewhat larger quantity of \( y \), \( \bar{y}^S_x \), increases as the amount of time \( z_{xy} \) is increased, under unorganized barter exchange as compared to organized market exchange.

5.4 The Elimination of Unorganized Barter Exchange

It is now possible to demonstrate how natural economic forces in the economy might lead to the emergence of the organized market as the dominant exchange process in the economy and to the virtual disappearance of unorganized barter exchange.

The individual utility maximization problem formulated in Chapter 3 may be modified to accommodate the choice facing the typical individual consumer between engaging in exchange with middlemen in the newly emerging organized market or continuing unorganized barter exchange activity. The modified utility maximization problem would be:

\[
\max U = U(x_C, y_C, \bar{x}, \bar{y}, \bar{z}-z)
\]

subject to constraints:

\[
\bar{x} \geq x_C + \bar{x} - (x-x) + y^S \bar{y} - [E_{xy}(y^S_x, \bar{x}, \bar{y}, z_{xy}) + M_{xy}(y^S_x, z_{xy} - z_{xy}^U)]^5.
\]

4 This will be particularly true if \( \bar{z}_{xy} \) is a quantity of time sufficiently large to permit an individual to canvas virtually all known middlemen.

5 Here the caeteris paribus assumption that the emergence of the organized market has not disturbed the unorganized barter exchange opportunities is still retained.
\begin{align}
(3.5a) \quad \bar{y} \geq y_c + (y - \bar{y}) + y_x^s - [E_{yx}(x_y, \bar{x}, \bar{y}, z_{yx}) + M_{yx}(x_y, z_{yx}, z_{yx} - z_{yx})]; \\
(3.6) \quad z = z_{xy} + z_{yx};
\end{align}

and

\begin{align}
(3.7a) \quad x_c \geq 0, \quad y_c \geq 0, \quad x \geq 0, \quad y \geq 0, \quad z_{xy} \geq 0, \quad z_{yx} \geq 0, \quad y_x^s \geq 0, \\
\quad x_y^s \geq 0, \quad z_{xy} \geq z_{yx}^u \geq 0, \quad z_{yx} \geq z_{yx}^u \geq 0.
\end{align}

where $z_{xy}^u$ and $z_{yx}^u$ are the amounts of time spent trading away $y$ for $x$ and trading away $x$ for $y$ respectively in the unorganized barter sector and where the modified constraints (3.4a) and (3.5a)
state that the quantity of a commodity consumed during the period plus
the change in inventory stocks of the commodity over the period plus
the quantity of the commodity traded away during the period less the
maximum expected quantity of the commodity obtainable from either
unorganized barter exchange or exchange with a middleman, must not exceed the endowment of the commodity in that period.

It is unnecessary to develop the more complicated set of conditions for a solution to this modified problem since the similarities to the original problem are sufficiently close to permit the use of the earlier diagrammatic framework to analyze the impact on individual consumer exchange behaviour of the emergence of middlemen. It should be adequate to look at individual consumer cases similar to those previously described in Chapter 4 and represented in Figure 4.3 and Figure 4.4. This involves showing how various types of consumer individuals who previously engaged in unorganized barter exchange would
find that they are able to increase their utility by switching to exchange with middlemen in the organized market.

The most obvious such case is illustrated in Figure 5.4. It shows a typical individual consumer of the type having relatively high preference for leisure. The intertemporal equilibrium of this individual under unorganized barter exchange occurred at point C along the exchange opportunities curve AG. In each period, it allowed him to expect to consume the bundle \((x^*_C, y^*_C)\), after spending a relatively small (below "average") amount of time, \(z_{yx}^{**}\), on exchange activity (trading away \(y\) for \(x\)) while holding no inventories or relatively small inventories (i.e. say \(X^{**}\) and \(Y^{**}\), as illustrated). With the emergence of the organized market, this typical individual would realize that he is able to increase his expected utility and move to a new trading equilibrium at \(H\) (along the organized market exchange opportunities curve AF) by making a single bilateral contact with a middleman rather than continuing unorganized barter exchange. This would give him both a more preferred expected commodity consumption bundle, \((x^*_C, y^*_C)\), and more leisure, \(L^{-z_{xy}^{**}}\), rather than \(L^{-z_{xy}^{**}}\).

Another case is illustrated in Figure 5.5. It shows a typical individual consumer whose intertemporal equilibrium under unorganized barter exchange occurred at point E along the exchange opportunities curve AJ. In each period, he could expect to consume the bundle \((x^{**}_C, y^{**}_C)\), after spending around an "average" amount of
FIGURE 5.4
Figure 5.5
time, \(^6\) \(z_{xy}^{**}\), on exchange activity (trading away \(y\) for \(x\)) while holding only relatively small inventories, \((X^{**}, Y^{**})\), if any. With the emergence of the organized market this typical individual would realize that he is able to increase his expected utility by moving to a new trading equilibrium at point \(K\) along the organized market exchange opportunities curve \(AF\) after making a single bilateral contact with a middleman. Although he would then be expecting to consume a somewhat less preferred bundle \((x_{c}^{*}, y_{c}^{*})\), he would be enjoying considerably more leisure, \(\bar{z}_{xy}^{0}\), (rather than \(\bar{z}_{xy}^{**}\)) which would amply compensate him.

It is obvious that the foregoing two types of individual consumers would account for a large proportion of all the individuals engaging in exchange in an unorganized barter economy. They would represent virtually all individuals in the economy who, on average, traded away \(y\) for \(x\) (or \(x\) for \(y\)) at below the "average" prevailing terms of exchange as well as a significant proportion of all the individuals who, on average, traded away \(y\) for \(x\) (or \(x\) for \(y\)) at around the "average" terms of exchange. This indicates that the evolution of middlemen and the emergence of an organized market would induce a large proportion of individual consumers in the economy to switch from unorganized barter to organized market exchange. The

\(^6\) It may be recalled that in Chapter 4 (see f.n. 2) an "average" amount of time and "average" terms of exchange were benchmarks denoting the average prevailing exchange behaviour in the economy.
effect of this on unorganized barter exchange opportunities would be very significant.

First of all, it would no longer be possible to expect to make bilateral contacts with individuals willing to accept terms of exchange less favorable than those expected to be obtainable from middlemen. Secondly, it would become apparent to individuals remaining in the unorganized barter sector that more time would have to be spent on exchange activity in order to expect to attain the same average terms of exchange as before. Thus, individuals who previously had spent the "average" prevailing amount of time on unorganized barter exchange activity and could, on average, expect to obtain the "average" prevailing terms of exchange would now either have to spend significantly more time on exchange activity or be content with concluding exchanges at less than the "average" terms of exchange prevailing in the economy. Furthermore, since those individuals continuing unorganized barter exchange activity would have become reduced in number and hence more time-consuming to contact, the amount of time required to make each bilateral contact in the unorganized barter sector would increase.

All of these effects imply transformations of unorganized barter exchange opportunities functions in the direction of making unorganized barter exchanges less attractive. These shifts are illustrated in Figure 5.6. This would make exchange with middlemen relatively more attractive to a significant proportion of the individuals remaining in the unorganized market inducing a further deterioration of unorganized barter exchange opportunities. It is not
FIGURE 5.6
possible to show conclusively that the outcome of this process would always be the complete elimination of the unorganized barter sector but only that natural economic forces tending in that direction would exist. This is because it is conceivable that there may be some individuals in the economy who gain very little or no utility from leisure time and choose to devote a great deal of time to unorganized barter exchange activity to obtain better terms of exchange than those offered by middlemen but who have a high subjective rate of time preference that precludes them from choosing to accumulate inventories and becoming middlemen themselves.
Chapter 6

SOME EXTENSIONS, AND CONCLUDING COMMENTS

6.1 Monetary Exchange

The framework developed to discuss the evolution of middlemen (and emergence of organized markets from out of a system of unorganized barter exchange) can be extended to provide some additional insights on monetary exchange. Monetary exchange would be characterized by the existence of one or more "money" commodities serving as media of exchange. This would mean that a large proportion (if not all) of pairwise exchanges in the economy would involve a "money" commodity. For this to occur it would be necessary that most individuals find it preferable to accept a "money" commodity when trading away non-"money" commodities and to use a "money" commodity when engaging to obtain other commodities for consumption.

By extending the unorganized barter exchange model to assume the existence of a third commodity, $v$, in addition to $x$ and $y$, it is quite straightforward to show, using the concept of exchange opportunities functions, that one of the commodities would come to be employed as a medium of exchange. This is illustrated in Figure 6.1. In Figure 5.1(a) the exchange opportunities curve $E_{xy}(y_x, \bar{x}, \bar{y}, \bar{v}, z_{xy})$ is represented by OA showing the amounts of $x$ an individual can expect to obtain for various quantities of $y$, $y_x$, while spending $z_{xy}$ time on exchange (and holding fixed inventories $\bar{x}, \bar{y}, \bar{v}$). In an economy with a third commodity, there would exist opportunities to
FIGURE 6.1
obtain \( v \) by trading away \( y \) and to obtain \( x \) by trading away \( v \), as indicated by the exchange opportunities curves, \( E_{vy}(y^s_x, \overline{x}, \overline{y}, \overline{v}, \overline{z}_{vy}) \) and \( E_{xv}(y^s_x, \overline{x}, \overline{y}, \overline{v}, \overline{z}_{xv}) \), shown in Figure 6.1(b) and (d) respectively. Therefore, it would be possible to obtain \( x \) for \( y \) indirectly using \( v \) as a medium of exchange. It is conceivable that the distribution of endowments and preferences in the economy might create exchange opportunities such that, for a given total amount of time spent on exchange activity \( \overline{z}_{xy} = \overline{z}_{vy} + \overline{z}_{xv} \), an individual could expect to obtain more \( x \) by exchanging indirectly for \( y \) by first obtaining \( v \) for \( y \), than by trading away \( y \) to obtain \( x \) directly.\(^1\)

Such a situation is illustrated by the indirect exchange opportunities curve \( E_{xv}(E_{vy}(y^s_x, \overline{x}, \overline{y}, \overline{v}, \overline{z}_{vy}), \overline{z}_{xv}) \) represented by OB in Figure 6.1(a) which is generated by projections through 6.1(b), 6.1(c) and 6.1(d). It can be seen that indirect exchange using \( v \) as a medium of exchange will be more rewarding provided possibly that more than some minimum quantity of \( y \) (shown as \( y^s_x \)) is being traded away.

An even stronger potential for the emergence of monetary exchange can be demonstrated in an economy where organized markets with middlemen have evolved. In Chapter 4, the evolution of middleman exchange was demonstrated for an economy of two commodities \( x \) and \( y \). In Chapter 5, it was shown that the existence of an organized market with one or more middlemen, could lead to the elimination of

\(^1\) For example, this might occur if \( v \) was a commodity consumed regularly by all individuals while \( x \) and \( y \) were consumed irregularly and/or only by some individuals.
unorganized barter exchange involving \( x \) and \( y \). In generalizing these results to an economy with three or more commodities several conjectures appear reasonable. First of all, in an unorganized barter economy, there is no reason why middleman exchange would evolve simultaneously for all combinations of pairs of commodities. Moreover, because exchange opportunities functions for each pair of commodities would be determined by the distribution of endowments and by relative preferences, there is no reason why the evolution of middleman exchange and the emergence of distinctive organized markets for some pairs of commodities would necessarily lead to the evolution of middlemen and emergence of distinctive organized markets for all pair combinations of commodities. In fact, in an economy with very many commodities, it is conceivable that no organized markets would exist for a very large proportion of all commodity pair combinations because the exchange opportunities would not be sufficiently attractive to induce and sustain middleman exchange activity on a scale sufficient to distinguish it from unorganized barter exchange.

In such an economy, even if middleman exchange and organized markets evolved for all or most commodities, only some small subset (if not only one) of the most commonly exchanged and widely consumed commodities, \( S \), would have a large number of distinctive organized markets, and be paired for exchange with all or most other commodities. This means that for most other commodities there would exist only a small number of organized markets (perhaps only one) each involving the exchange of, or for, one of the commodities from the subset \( S \). For the remaining possible combinations of pairings of
commodities, unorganized barter exchange would be the only possible form of direct exchange.

In this situation, indirect exchange in organized markets, using one of the commodities from the subset \( S \) as a medium of exchange, would inevitably be more attractive than direct unorganized barter exchange. Earlier, in the discussion relating to Figure 5.3, it was shown how organized market exchange opportunities could be more attractive than unorganized barter exchange opportunities if only a small amount of time is devoted to exchange activity. This advantage of exchange in organized markets would obviously reinforce the attractiveness of engaging in indirect exchange in organized markets, using a medium of exchange (from \( S \)), when dealing with commodity pair combinations that can only be exchanged directly through unorganized barter. Thus, we would have a monetary exchange economy. Of course, all commodities from \( S \) would not necessarily be equally attractive (or efficient) for use as media of exchange in these indirect exchanges so that only one or a few might end up performing a monetary function.

This situation is illustrated in Figure 6.2, where no organized market for direct exchange of \( x \) and \( y \) exists but where separate organized markets for the exchange of \( x \) and \( v \) and the exchange of \( y \) and \( v \) exist. Thus, by making only two bilateral contacts with middlemen in these markets and spending \( 20 \) time on each contact, an individual seeking to obtain \( x \) for \( y \) could trade away \( y \) to obtain \( v \) and then trade away \( v \) to obtain \( x \) along the
FIGURE 6.2
organized market exchange opportunities curves \( M_{yy}(y^S, z^0) \) and \( M_{xy}(x^S, z^0) \), as shown in Figure 6.2(b), 6.2(c) and 6.2(d). The possible expected outcomes of this exchange activity are indicated in Figure 6.2(a) by the indirect exchange opportunities curve \( M_{yx}(M_{yy}(y^S, z^0), z^0) \) which can be seen to be more rewarding than spending \( 2z^0 \) of time on direct exchange along the unorganized barter exchange opportunities curve \( E_{xy}(y^S, \bar{x}, \bar{y}, 2z^0) \).

6.2 Concluding Remarks

The main objective of this dissertation has been to present a framework to describe the evolution of exchange processes from a system of unorganized barter exchange to a system of organized markets where some traders have become middlemen and play a central role in coordinating exchange activity. This was accomplished by developing an intertemporal model of individual expected utility maximization constrained, initially, by unorganized barter exchange opportunities which were based on a very simple formulation of the exchange environment. Time was modelled in discrete terms consisting of successive trading periods of the same fixed length for every individual but without necessarily any overlap between the trading periods of any two or more individuals.

Within a trading period, the actual exchange process for a typical individual was assumed to consist of making random bilateral contacts with other individuals. It was assumed that each bilateral contact involved the symmetrical exchange of information about
reservation prices and corresponding exchange quantities, and required a fixed interval of time to complete representing a transaction cost in the form of loss of leisure utility. This work has provided an extension of the search theory literature to the case of symmetrical unorganized barter exchange involving "quantity" uncertainty as well as "price" uncertainty.

Of course, there remain some unresolved problems. In particular, the specification of the bilateral contacting process has been quite restrictive. It included the assumption that information exchanged during a contact consisted of a price-quantity vector (with exchange terms determined by convention) rather than of some type of offer curve (involving a more open-ended bargaining process). This assumption may not be consistent with Pareto Optimality in the sense that after an exchange is concluded some utility-improving basis for further trade between the two individuals may have remained unexploited. In view of the emphasis on the efficiency of bilateral trading processes in the literature\(^2\), this might appear to be a serious deficiency. Nevertheless, it seems possible to define Pareto Optimality more broadly and assume that the unexploited utility gains from trade are not optimal, since they could be attained only at the cost of spending more time on exchange involving even greater losses of leisure utility. Alternatively, the assumption about the information exchanged during a bilateral contact may be viewed as a special case which can be generalized to cases where the information exchanged

\(^2\) See, in particular, Feldman (1973) and Madden (1975).
consists of a schedule of price-quantity vectors (i.e. an offer curve or an excess demand curve).

The outcome, at the end of a trading period, of the actual sequential exchange behaviour of an individual during a trading period was assumed to correspond closely, on average in the long run, to planned exchange behaviour at the beginning of a trading period. The implications of this assumption have not been explored in this dissertation and indicate a direction for further research.

The modelling of planned exchange behaviour, based on an accurate perception of expected exchange opportunities functions which explicitly formulate the possibility of accumulating and holding inventories of commodities intertemporally, has provided the framework for the evolution of exchange processes through the distinction between short-run trading equilibria and long-run intertemporal equilibrium. Although aggregate economic implications of this evolutionary process leading to the emergence and supremacy of organized markets and of monetary exchange have been explored, there remains considerable scope for a more systematic study of this subject.
BIBLIOGRAPHY


