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TURBULENT WAKE INTERACTIONS OF MULTIPLE STRUCTURES
IN AN ATMOSPHERIC BOUNDARY LAYER

by

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Submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
August 1996

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ISBN 0-612-15072-0
ABSTRACT

The complex turbulent flow in the wake of single isolated structures presents a formidable challenge to current modelling techniques. Models ranging from similarity based analytical models to fully numerical solutions of the flow have evolved to tackle this intriguing flow. Models also exist to predict the effect of the wake on the dispersion of contaminants entrapped by it. Such predictions are essential to public health and safety. In the industrial world, however, structures affecting the dispersion of contaminants are rarely single and isolated. More commonly, such buildings are located in small groups or clusters. To this end, this study examines the effectiveness of using single structure theory to predict the turbulent flow and the dispersion of contaminants in the wake of multiple structures located in a group.

A comprehensive study of turbulent flows and contaminant dispersion was conducted in an ‘open country’ type atmospheric boundary layer flow, using cross wire anemometers and flame ionization detectors. The primary structure groups studied consisted of a single row of three cubes of various clear separating distance. The separation distances examined range from closely packed to nearly isolated. Limited tests on non-cubic structures and double row arrangements were also conducted to round out the study.

The mean velocity deficits in the far wake of multiple structures are shown to coalesce into a single uni-modal wake. The combined wake is then demonstrated to decay in a universal manner when scaled by the overall width of the structure group. The single structure similarity theory of J.C. Hunt is subsequently modified and successfully applied to multiple structure wakes in this region of universal decay.

A Modified Area Source model is developed to predict mean contaminant concentrations in the wake of multiple structures. The model uses the universal decay feature of the mean velocity deficit in the wake to modify the advection velocity of the contaminant. Universal functions are also developed to describe measured turbulent
velocities. These functions are used to modify the turbulent spread parameters of the model. Predictions made by these modifications represent significant improvement over previous models, most of which incorporate arbitrary constants to fit the data.

keywords: wake flows, wake dispersion, cube wakes, building wakes
ACKNOWLEDGEMENTS

I would like to thank Dr. A.G. Davenport for his support, insightful comments, and helpful discussions throughout the duration of the research project.

The advice and support of the members of the advisory committee, Dr. S.R. Ramsay and Dr. N. Isyumov is also graciously acknowledged.

Thank you to all of my fellow graduate students for providing an amicable work space and inciting provocative dialogue. Also, thanks to the technical staff and support people of the Boundary Layer Wind Tunnel Laboratory for their assistance and companionship. The kind assistance of A. Zummaraga with some follow-up experimentation is appreciated.

I gratefully acknowledge the financial assistance of the Natural Sciences and Engineering Research Council through the Post Graduate Scholarship awarded to me, and through grants awarded to my supervisor, Dr. A.G. Davenport.
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NOMENCLATURE

\[ a \] exponent of power law fit equation
\[ A \] characteristic frontal area of structure
\[ b \] isolated structure width
\[ d \] isolated structure depth in streamwise direction
\[ D \] effective eddy diffusivity
\[ h \] nominal structure height
\[ H \] equivalent flat plate height
\[ i \] turbulence intensity (\textit{rms} over mean velocity)
\[ k \] von Karman’s constant \( \approx 0.4 \)
\[ K \] non-dimensional concentration coefficient (Eq. 2.21)
\[ L \] characteristic length scale
\[ n \] exponent of power law fit equation for velocity profile
\[ p \] growth rate exponent for turbulent eddy viscosity (Eq. 2.7)
\[ P \] leakage parameter (Eq. 2.16a)
\[ Q \] contaminant flow rate [\text{m}^3/\text{s}]
\[ R \] radius of homogenization
\[ S \] nominal clear separation distance between structures
\[ U \] constant reference mean velocity in streamwise (\textit{x}) direction
\[ u \] local mean velocity in streamwise (\textit{x}) direction
\[ \tilde{u} \] turbulent \textit{rms} velocity in (\textit{x}) direction
\[ \tilde{v} \] turbulent \textit{rms} velocity in lateral (\textit{y}) direction
\[ \tilde{w} \] turbulent \textit{rms} velocity in vertical (\textit{z}) direction
\[ W \] multiple structure scaling parameter
\[ x \] longitudinal streamwise component direction
\[ y \] lateral component direction
\[ z \] vertical component direction

* additional variables may not be included in this list but are defined locally in the thesis
Greek Symbols

$\beta$  porosity ratio for contaminant source
$\delta$  boundary layer depth
$\kappa$  thermal diffusivity
$\xi$   characteristic dimension of multiple structure group
$\sigma$ spread parameter, standard deviation of mean concentration profile
$\chi$  concentration of contaminant [g/m$^3$]
$\nu$   kinematic viscosity

Subscripts

$b$   background flow, no upstream structures
$\text{eff}$ effective stack height
$h$   value at building height $z = h$
$\text{pert}$ perturbation of background value
$r$   reference value
$w$   wake flow, downstream of structures

Acronyms

CFD    computational fluid dynamics
DNS    direct numerical simulation
FID    flame ionization detector
HS     Huber and Snyder (model)
HWA    hot wire anemometer
LES    large eddy simulation
MAS    Modified Area Source (model)
MPS    Modified Point Source (model)
MS     multiple structures
MSE    mean square error
MVD    mean velocity deficit
PSD    power spectral density
SIS    single isolated structure
CHAPTER 1

INTRODUCTION

A building located in the natural environment will disrupt the passing wind so as to cause a reduction in the mean velocity and an enhancement of the turbulent velocity fluctuations in the wake. This altering of the ambient wind flow can have significant implications on human activities downstream of wake generating structures. For example, hangars located beside runways may affect a crosswind in a manner dangerous to passing aircraft. Local ground level gustiness of the wind caused by gross architectural features of a building may prove detrimental to pedestrian comfort and safety. Perhaps most importantly, effluent of local pollution sources may be entrapped by a building wake thus affecting local ground level concentrations and causing potentially serious health risks.

An extensive body of literature exists on the nature of the flow around single isolated structures in an atmospheric boundary layer. A review of this literature is included in this introductory chapter to clarify the complex nature of general wake flows and the modelling difficulties that they present. In practice, buildings of concern from a pollution standpoint are rarely truly isolated in their surroundings. With the difficulties of predicting the flow in the wake of simple isolated structures, how can we expect to model the complex interactions of multiple structure wakes? Costly site specific scale model studies in wind tunnels are one possibility. The research suggests that another possibility may be to consider several buildings spaced sufficiently close together as a single wake generating unit (Huber 1984). While this approach has been suggested and, in some cases, attempted in the literature, clear definitions of the key terms ‘isolated’ and ‘sufficiently close’ have not emerged. It is one objective of this study to systematically examine the effect of building spacing on turbulent wake interactions to provide guidance to engineers wishing to pursue this modelling approach in the future.
1.1 SINGLE ISOLATED STRUCTURE WAKES

Flow in the vicinity of a sharp edged isolated structure in a turbulent shear flow is highly complex and three dimensional in nature. Flow separation at the leading edges results in regions of recirculating flow on the sides and top of the structure, as well as a large recirculation cavity in the lee. A frequently cited schematic based on the work of Hunt et. al. (1978) and showing the complexities of the flow is reproduced here from Hosker (1980), as Figure 1.1.

![Schematic of three-dimensional flow past a cubic structure in a turbulent shear flow](image)

**Figure 1.1:** Schematic of three-dimensional flow past a cubic structure in a turbulent shear flow (from Hosker, 1980)

Larousse et. al. (1992) suggest that the flow around the obstacle will look somewhat different from Figure 1.1 for a fully turbulent approach flow, such as that found in the present study. In particular, they find no evidence supporting the two secondary vortices upstream of the horseshoe vortex, and they demonstrate that reverse flow regions exist along the base of the sides of the structure. The authors reaffirm the complexities of the flow about such a seemingly simple geometry.
On the windward side of the structure is a frontal separation zone resulting from pressure gradients on the face of the cube. The extent of this region depends on the gradients of mean velocity in the approach flow boundary layer and fluctuates as leading edge vortices are alternately shed from the structure. This region is important from a structural perspective but less so from a pollution dispersion perspective as contaminants from rooftop sources are unlikely to be transported into this region.

A roof cavity region is formed when the flow separates from the leading edge of the cube. Similar regions exist on the sides of the cube and the extent of the recirculating regions are dependent on the direction of the approach flow, and the relative depth of the approaching boundary layer (h/δ). Wilson (1979) has shown that this region extends almost entirely across the roof of a cube for a flow normal to the front face.

A near wake cavity region is created in the lee of the structure when the flow separates from the trailing edge of the cube. Also dependent on h/δ, this region of highly turbulent recirculating flow is extremely unstable and varies greatly in longitudinal extent as reported in the literature. Meroney and Yang (1970) report lengths of 2h - 3h for this region while Counihan (1971) reports lengths as low as 1h-1.5h. Jones and Griffiths (1984) report that this region is highly unstable in full scale experiments and very difficult to observe, or to consistently measure.

A far wake region exists downstream, generally considered to begin beyond x/h=3. In this region a mean velocity deficit exists as a result of the momentum extraction from the mean flow by the presence of the obstacle. In addition, a significant excess of turbulence intensity exists above ambient levels due to the disruption of the flow by the structure. Both the mean velocity deficit and the excess turbulence intensity decay downstream with the mean velocity deficit recovering more rapidly than the excess turbulence intensity (Hunt, 1971). It is in this far wake region that the majority of this study is concerned.
1.2 RELEVANT STUDIES

There is a wealth of literature specifically directed at the nature of the flow and of the dispersion of contaminants in the wake of single isolated structures. Many comprehensive reviews already exist including Halitsky (1961), Hosker (1984), and more recently Musselman (1992).

Full scale experiments are limited due to the high costs involved and the inherent lack of experimental control in the natural environment. Those experiments that are successful are generally reported with very high uncertainty and are difficult to reproduce. For comparison of model experiments and analytic studies, however, full scale data is as valuable as it is rare.

Early full scale studies of turbulence and dispersion in building wakes were reported by Islitzer (1965), Culkowski (1967), and Hinds (1968). These works showed the increased turbulence and added mixing of the wake region and verified the need to consider wake effects on contaminant dispersion. Some experiments studied the extent of the recirculation zone, Region II of Figure 1.2, (Drivas and Shair, 1974, Jones and Griffiths, 1984). Others studied the far wake effects on dilution (Cagnetti, 1975). Ogawa et. al. (1982) specifically designed a full scale experiment to emulate the wind tunnel work of Castro and Robins (1977). As is the case with many full scale experiments, however, their conclusions were severely limited due to the unsteady behaviour of the natural environment.

Wind tunnel or water channel studies offer considerably more control than full scale studies and, if done carefully, provide a wealth of information. Data from controlled experiments are used to formulate empirical relations used to estimate concentration levels near structures, and to validate numerical models of the flow.

Higson et. al. (1994) compares results of building surface concentration measurements made in a wind tunnel with similar measurements made in a scaled field study. A unique aspect of the field study is the ability to realign the building according to the prevailing wind direction. Even with this rather novel approach, difficulties with large scale wind meandering in the natural environment make direct comparison with the wind tunnel data difficult. The study
suggests that wind tunnel studies are of limited use because they do not adequately model the large scale lateral turbulence found in the natural environment. In fact, the study demonstrates how the opposite is true. Large lateral turbulence scales result in a low frequency meandering of the mean flow direction that Higson et. al. (1994) tries to eliminate by realigning the building. The inability to control the direction of the mean flow is a serious limitation of field studies, as demonstrated in the recent work of Macdonald (1996). Lateral scales of concern from a dispersion standpoint are of the characteristic dimension of the plume. In a structure wake the contaminant plume scales with the characteristic dimension (h for a cubic structure). Such scales are typically an order of magnitude less than the scales which cause the meander of the mean flow. As a result, the effect of the large scale turbulence contributes to unrealistic statistics in the far wake, not representative of the effect of the building on the relative dispersion of the plume. Careful control of the smaller scales of lateral turbulence, of the order of the characteristic plume dimension, will provide meaningful, controlled, experimental results.

Lee et. al. (1991) demonstrate how flow visualization techniques, when calibrated by gas concentration measurements, give an informative description of turbulent dispersion in the wake of single isolated structures. The experiments graphically illustrate the effect on wake dispersion of geometric variations from the basic cube shape of the primary single isolated structure. Changing the height of the structure from $h/b = 1$ to $h/b = 3$ results in a slightly wider plume downstream. The authors attribute the widening to greater meander in the wake of the taller structure. As the structure becomes more two dimensional, flow over the roof will become less important and the alternating vortices of the wake will strengthen. The result is a more coherent 2-D type wake, and a wider concentration footprint.

The second geometric variation examined in Lee et. al. (1991) is that of a long building, $h/d = 1/3$ as opposed to $h/d = 1$ for a cube. The flow around a long building reattaches to the sides and top. The result is a narrower wake in the lee of the longer structure. The narrow wake effectively confines the dispersion of a contaminant released into it, resulting in a narrower concentration footprint, and higher ground level concentrations near the structure.
The last geometric variation of Lee et. al. (1991) is that of a wide building, \( h/b = 1/3 \) as opposed to \( h/b = 1 \) for a cube. The wide building produces a proportionally wider concentration footprint than the cube. The shape of the footprint remains consistent with that of the cube but the greater flow blockage results in greater concentrations in the near wake (due to reduced mean velocities), and lower concentrations in the far wake (due to stronger fluctuating velocities).

The experiments of Lee et. al. (1991) qualitatively demonstrate the effects of varying geometry on the dispersion in the wake of a single isolated structure and will be considered further in the context of the present study.

Huber (1991) also investigates the effect of varying the dimensions of a single isolated structure on the dispersion in the wake. Huber notes that the transition in the far wake from building scale turbulent dispersion to background scale turbulent dispersion is a function of both building dimension and background turbulence levels. He does not, however, quantify this dependence.

Hunt (1971) describes the effect of changing the aspect ratio of a single isolated structure from a cube shape to a wide, low, nearly two dimensional building. The theory of Hunt (1970) predicts that the rate of decay of the mean velocity deficit in the wake will change from a value of \(-3/2\) for a cube, to \(-1\) for a nearly two dimensional structure. The theory of Hunt (1970) further predicts the effects of adding shear to the approach flow on the far wake decay rate of the mean velocity deficit. For a cube the \(-3/2\) decay rate found in a shear flow is reduced to \(-1\) for a uniform flow. This prediction was confirmed experimentally by the recent LDA measurements reported by Musselman and Slawson (1995).

Lemberg (1973) compares the far wake of various single isolated structures in a boundary layer approach flow. In addition to examining a single cube aligned both normal and diagonal to the flow, Lemberg examines the wake of a tall rectangular prism, \( h = 1.5b = 1.5d \), at the same two inclinations to the flow. Lemberg also studied the far wake of circular cylinders of two different heights, \( h = d, 1.5d \). Despite the variety of shapes and aspect ratios examined, the far wakes of the structures were shown to be well described by the similarity
theory described by the author (Section 2.1). In particular, the decay rate of the mean velocity
deficit was similar for all structures and was not dependent on the relative height to boundary
layer thickness, \( h/\delta \), for the range studied. This observation has significance with respect to the
present study which focuses on only one of the various shapes explored by Lemberg, and uses
that shape to examine the effect of multiple structure wakes. Examining one aspect of the flow
at a time allows this highly complex problem to be broken into manageable building blocks
from which a more complete picture of the flow will emerge.

Comprehensive studies of far wake concentration measurements are presented by
Meroney and Yang (1970), and Robins and Castro (1977). In Castro and Robins (1977) and
Robins and Castro (1977) details are presented of experiments to study the flow around a cube
located in both a uniform freestream (also see Castro, 1973), and in a thick turbulent boundary
layer (also see Castro and Robins, 1975). The study shows that mean shear in the approach
flow significantly affects both the near field and the far wake flows. Studies intended for
application in the natural environment must carefully model the approach flow boundary layer
for practicality of results. Measurements of surface pressure coefficients and extensive
turbulent velocity data in the wake of a cube are also presented. While surface pressure
measurements are critical from an architectural and structural perspective, they are less so for
pollution dispersion considerations. All information that helps to describe the flow is welcome
and this important data is cited frequently in the literature, with portions of the work verified by

Additional works by Huber and Snyder (1982) and Huber (1989) describe several wind
tunnel simulations while studying the dispersion of contaminants near structures. These
experiments led to the development of an empirical model for concentrations discussed in
Section 2.2.4.

An important aerodynamic effect of a single isolated structure is the blockage resulting
in a mean velocity deficit in the wake. Numerous studies and theoretical discussions exist
concerning this important characteristic of the wake. Experimental studies by Lemberg (1973)
and Eskridge and Thompson (1982) complement theoretical discussions by Hunt (1970) and
Eskridge and Hunt (1979). The important works of Hunt (1970) and Lemberg (1973) are discussed further in Chapter 2.

1.3 MODELLING THE FLOW

Mathematically modelling the recirculating shear flows very near a structure is a formidable task. To solve the problem numerically requires, at best, solution to the fully three dimensional instantaneous Navier-Stokes equations with simultaneous solution of equations for continuity, energy, and the mass transport of a contaminant. Attempts to solve the problem leads to the traditional Reynolds decomposition of turbulence parameters into mean and fluctuating components, $u(t) = u + u'(t)$. This process, however, leads to additional terms in the equations of motion which must be modelled in order to close the system of equations. This famous closure problem of turbulence is described well by Rodi (1980). The $k$-$\varepsilon$ model of Launder and Spalding (1974) is the most common 2nd order closure model used in the numerical solution of turbulent flows. Musselman (1992) describes details of the $k$-$\varepsilon$ model not repeated here.

Many researchers have attempted to use the $k$-$\varepsilon$ turbulence model to predict the flow around cubic structures with little success (Crawford. 1977, Paterson and Apelt, 1990, for example). These models fail in the near wake region partly because they assume that a balance exists between the production and dissipation of turbulent kinetic energy in the flow. Martinuzzi et. al. (1993a) demonstrate that this equilibrium does not exist in the shear layer enveloping the obstacle or in the horseshoe vortex system. Equilibrium based models cannot hope to predict the concentration of contaminants in these regions.

An advanced and relatively recent modelling technique called Large Eddy Simulations (LES), holds promise to better model these complicated flows in the future.

Large Eddy Simulations

LES is a relatively new concept in turbulence modelling falling somewhere between Direct Numerical Simulation (DNS) and full equilibrium models such as $k$-$\varepsilon$. The
concept of LES is to solve the equations of motion for the smallest grid size available given the limitations of the computer. This artificially truncates the solution at eddy sizes no smaller than the grid dimensions. Most energy in the turbulent flow is contained in the large eddies so gross features of the flow are modelled well. Eddy scales smaller than the grid size, called Sub-Grid Scales (SGS) must be modelled using an equilibrium based model as previously discussed.

Murakami et. al. (1987) demonstrates that Large Eddy Simulations show promise in their ability to model the complicated flow near buildings and, as shown in Murakami et. al. (1990), are easily superior to full equilibrium models such as $k$-$\varepsilon$. Rodi (1996) uses LES techniques to simulate the flow very near a surface mounted square prism. Comparison with the comprehensive experimental results of Martinuzzi and Tropea (1993b) are encouraging. Additional studies by Baetke et. al. (1990) and discussions by Ferziger (1990) reiterate the advantages, however, to the knowledge of the writer, no researchers have yet expanded their simulations to include the dispersion of mass concentrations near the building. Until this advancement is made, applying the method to the interacting wakes of multiple structures seems premature. The nature of LES demands extensive computer resources and, as a result, we have only begun to realize the power of this modelling tool.
CHAPTER 2
THEORETICAL DEVELOPMENT

In this study the theoretical development, like the experimental work, is divided into two main parts: a) the turbulent flow field of the interacting wakes, and b) the concentration field downstream of the obstacle groupings. In each of these two main sections several important modelling methods are reviewed and the best methods are modified and applied to the present study.

2.1 THE TURBULENT FLOW FIELD

2.1.1 Relevant Studies

The majority of current research studies into advanced solutions of the flow equations in the wake emphasize CFD techniques. Little work has been done recently to advance the analytical solutions of the past. This trend in solution techniques, while understandable, forfeits the physical understanding of the flow problem that may be obtained through careful consideration of the governing equations. For this reason the theoretical works of Hunt (1970), and Lemberg (1973) will be discussed further.

Classical wake theory evolved from the early boundary layer work of Blasius (1908). Thin shear layer assumptions and self similarity concepts allow for analytic solutions to the equations of motion. Schlichting (1930) recognized that unbounded turbulent ‘free wakes’ also exhibited self similar behaviour far downstream of the originating body. The concept of self preservation in the far wake region is powerful and effective. The balance between the drag force on the object and the integrated momentum deficit is very well established. This pioneering work, however, deals primarily with the wakes of two dimensional cylinders and three dimensional axi-symmetric bodies in uniform approach flows. As a result, it does not adequately deal with the surface shear effects found in the flow behind a ground based obstacle in the natural environment. Improvements must be made to handle this complex flow.
The Theory of Hunt

The theoretical analysis of Hunt as described in Hunt and Smith (1969), and extended in Hunt (1970) treats the equations of motion in the wake of a single isolated structure through several simplifying assumptions. The most critical assumptions used in the analysis are:

1. The mean component of velocity in the wake, $u_w$, is equal to the mean component of velocity in the undisturbed boundary layer $u_b(x,z)$ plus a perturbation velocity $u_{pert}(x,y,z)$ (see Figure 2.1), so that $u_w = u_b(x,z) + u_{pert}(x,y,z)$. The theory requires that $u_{pert}(x,y,z) \ll u_b(x,z)$. This implies that the magnitude of the velocity deficit is small, and limits the theory of the far wake region where the mean velocity deficit has decayed substantially.

![Figure 2.1: Plan view of flow regions in the fully developed wake of a cube](image)

This assumption is consistent with classical wake analysis and allows higher order terms involving $u_{pert}$ to be neglected in the governing equation. It also limits the theory to the fully developed region, where local pressure gradients no longer control the flow.
The turbulent eddy viscosity term used to estimate the turbulent shear stresses is assumed to be constant and is calculated from the properties of the incident boundary layer. This assumption greatly simplifies the mathematics of the solution but puts limitations on the accuracy of the expected results. As an estimate of the mean eddy viscosity the authors use the expression (2.1), based on approach flow parameters,

$$\bar{\nu} = \frac{2k^2 n U_h h}{(2+n)} \quad (2.1)$$

where \( k = 0.4 \) is von Karman's constant

- \( n \) is the exponent of the power law representation of the turbulent boundary layer profile (0.1 < \( n \) < 0.2 typically)
- \( U_h \) is the value of \( u_h \) at \( y = h \)
- \( h \) is the characteristic building height

Using (2.1), lateral and vertical wake values of turbulent eddy viscosity are estimated as fractions of the mean value. These fractions are left as the free parameters, \( \gamma \) and \( \lambda \) (Appendix A).

Other key parameters in what is essentially a similarity solution to the simplified governing equations include an expression for the force couple exerted by the flow on the body. This expression forms an integral boundary condition for the equations of motion, given by (2.2),

$$C = -\rho \int \int \left[ z u_h (u_{ren}) \right] dz dy \quad (2.2)$$

Equation (2.2) replaces the conventional balance of drag force and momentum deficit commonly used by aerodynamicists. This replacement is justified on the assumption that (2.2) represents the effect of a surface mounted obstacle on the momentum deficit, without requiring knowledge \textit{a priori} of the friction drag on the wall.

The perturbation velocity, which represents the mean velocity deficit in the wake, is assumed to develop laterally as a point source diffusion problem. This result follows the classical axi-symmetric wake theory (Schlichting, 1955) which simplifies the governing
equation for mean velocity deficit into a simple diffusion equation. Such an equation is given as (2.3)

\[ u_b \frac{\partial u_{pert}}{\partial x} = \nu \frac{\partial^2 u_{pert}}{\partial y^2}. \]  

(2.3)

When subject to the boundary conditions

\[ u_{pert} \to 0 \text{ as } y \to \infty, \text{ and } \frac{\partial u_{pert}}{\partial y} = 0 \text{ at } y = 0 \]

(2.3) has an exponential solution of the form

\[ u_{pert} = \frac{u_b}{x} C \exp \left( -\left( \frac{y}{2} \right)^2 \frac{u_b}{\nu_x} \right) \]

\[ \sqrt{l} \]  

(2.4)

The constant of proportionality in (2.4) is dictated by an integral condition for drag, similar to (2.2), and the constant turbulent eddy viscosity of (2.1). \( l \) is a characteristic dimension of the body. While (2.4) was developed for an axi-symmetric wake, it is expected to apply, in principal, to the lateral profile of a surface mounted obstacle. Thus, the theory requires a Gaussian form in the lateral profiles.

These expressions, in combination with other mathematical relations and simplifications lead to the following similarity equation for the mean velocity in the wake

\[ \frac{u_{pert}}{U_h} = \frac{K_2 F_1(\tilde{z}, \tilde{y})}{[\{(x-a)/h\}]^{(\nu_n)/(2+\nu)}} \]  

(2.5)

where,

- \( u_{pert} \) is the perturbation velocity = \( u_w - u_b \),
- \( U_h \) is a reference background mean velocity at \( z = h \),
- \( K_2 \) is a constant which includes the force couple \( \overline{C} \), the constant eddy viscosity \( \overline{\nu} \), and an experimentally determined constant \( \lambda \).
$F_2$ is the similarity shape function; a function of height and lateral position which assumes a Gaussian profile in the lateral direction.

$z^*, \bar{y}$ are non-dimensional space co-ordinates which incorporate the longitudinal position $(x-a)/h$.

$a$ is an origin displacement factor determined from experiment.

Additional details of the theory necessary to obtain numerical solutions are included in Appendix A. The theory is applied by solving (2.5), utilizing the subsequent expressions (A2) to (A10), and values estimated for the constants $C_D, \gamma, \lambda,$ and $a$. Clearly analytical solutions to the governing equations are complex, even when simplified by such drastic assumptions as constant eddy viscosity.

Additional relationships are given for the $rms$ velocity components in the wake. However, based on the results presented in the original report, their agreement with experimental values appears poor at best. The theory is clearly much more effective in the prediction of the mean velocity deficit.

**The Theory of Lemberg**

The theoretical work of Lemberg (1973) extends that of Hunt (1970), with contributions from the theory of Sforza and Mons (1970). The primary contribution of the new theory is the replacement of the constant eddy viscosity with one that varies as a function of the height above the wall. While this change represents an improvement over the constant eddy viscosity approach, it still requires that the turbulent shear stresses be generated by local gradients of mean velocity and as such, is fundamentally incorrect in the far wake. Lemberg (1973), limits the application of the theory to the far wake region $(x/h>5)$ where the velocity deficits are small. This reduces the impact of this error. Using experimentally determined coefficients to describe the variation of eddy viscosity in the flow enables the model to reasonably predict the mean velocity deficit in the wake.

Using order of magnitude arguments the governing equations are reduced to the following differential equation
\[ \frac{\partial u}{\partial x} = az^{-1} \frac{\partial u}{\partial z} + z^m \left[ \frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial y^2} \right] \]  

(2.6)

where,

\[ u = \frac{u_b - u_m}{u_b} \] is the local normalized mean velocity deficit

\[ x, y, z = Kx/h, x/h, x/h, \] the longitudinal, lateral, and vertical dimensions, respectively.

The eddy viscosity is assumed to be

\[ \nu = Kz'' \]  

(2.7)

and \( m = p-n \), where \( n \) is the power law exponent for the mean approach flow.

The differential equation (2.6) is subject to the boundary conditions

\[ u = 0 \quad \text{at} \quad z = 0 \]

\[ u \rightarrow 0 \quad \text{as} \quad y, z \rightarrow \infty \]

and the integral condition

\[ M^* = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} z'''' u \, dz \, dy \]  

(2.8)

where \( M^* \) is the dimensionless overturning moment acting on the obstacle.

In order to solve the primary equation (2.6), Lemberg has had to impose some additional restrictions on the solution. Similarity profiles are assumed in the vertical direction and Gaussian profiles are assumed in the lateral direction, as in the theory of Hunt (1970). These assumptions combined with some mathematical simplifications allow for an analytical solution to the stated problem. The resulting expression for the mean velocity deficit is similar in form to that already given for the theory of Hunt (2.5).

Lemberg states that he is unable to distinguish between \( p = n \) and \( p = 2n \) in (2.7) due to insufficient accuracy in his carefully conducted experiments. For this reason, the \( p = 0 \) theory of Hunt (1970) (constant eddy viscosity) is likely to be just as useful and, as a result, will be considered further in the present study.
2.1.2 Application of the Theory to the Present Problem

All of the theoretical formulations discussed up to this point have dealt exclusively with predicting the flow in the wakes of single isolated structures (SIS). It is the premise of this study that under certain circumstances (i.e. close spacing, far downstream) the flow in the wake of obstacle groupings will behave sufficiently similar to a SIS wake to be modelled as such. Study of the theory leads us to examine certain aspects of the wake in order to identify if and when this modelling approach may be possible.

An essential assumption in the theory of Hunt (1970) is that the momentum deficit in the wake behaves downstream as if generated by a single point source in space. The profiles of mean velocity deficit are then considered to expand downstream while preserving their assumed Gaussian shape. If the wakes of multiple structures are to be described by such a model then it must be shown that the profiles of mean velocity deficit coalesce into a uni-modal Gaussian form. A form that is essentially indistinguishable from that generated by a single point source in space.

Another essential assumption of the theory is that the profiles of mean velocity deficit are self preserving in shape and may be described by a single shape function (in both lateral and vertical coordinate directions). Such a shape function incorporates a downstream scaling parameter characteristic of the wake generating object (l in (2.4) for example). A large grouping of closely spaced structures will create a greater mean velocity deficit than a smaller single isolated structure, due to the increased flow blockage. The more energy that is extracted from the flow by the larger wake generating structure will delay the recovery of the mean velocity deficit in the wake, creating a more persistent deficit downstream. A successful similarity solution must then apply the appropriate scaling parameters to collapse the profiles onto a self-similar curve. In the original single structure theory the appropriate parameter was the characteristic building dimension, \( h \) (see (2.5)). Preserving the approach flow parameters of the theory we hypothesize that the appropriate scaling parameter of the multiple structure cases, for this study, will be some function of the characteristic dimensions of the individual buildings, \( h \), and the characteristic building separation, \( S \). Such a scaling parameter, called \( \xi \),
should adequately describe the enhanced wake generating capabilities due to the increased frontal area of a multiple structure grouping.

The remaining consideration in applying the single structure theories to multiple structure wakes is the region of applicability. By assumption, the theory is limited to a region of small mean velocity deficit. Lemberg (1973) states that this condition may be satisfied as early as \( x/h = 5 \) for a single isolated structure, as previously discussed. This study shows that multiple structure wakes undergo a significant development region downstream, during which time they coalesce to a single uni-modal profile. Only after this coalescence is complete will the theory apply. To describe the point beyond which the wakes have adequately merged, the term \( \text{Radius of homogenization, } R \), is borrowed from the dispersion work of Plate and Baechlin (1988) (Section 2.2.2). Describing \( R \) as a function of the critical scaling parameter \( \xi \) is essential to the usefulness of the proposed extension to the theory.

In summary, before extending the single structure wake theory of Hunt (1970) to multiple structure wakes we must adequately establish that:

1. the individual wakes of the structures coalesce downstream into a uni-modal shape, similar in form to that found downstream of a single isolated structure,
2. the resulting uni-modal Gaussian type profiles of mean velocity deficit may be collapsed to a characteristic shape function described by the relevant scaling parameters,
3. the region of applicability of the theory may be quantified as a function of the relevant scaling parameters.

### 2.2 Contaminant Dispersion in the Wake

A building located in the path of a passing contaminant cloud will affect the concentrations of that contaminant by enhancing the turbulent mixing of the cloud and altering the magnitude and direction of the advective wind. Whether one exposed to the contaminant in the wake region sees higher or lower concentrations than one would in the absence of the building depends on the relative effects of the increased turbulent mixing and the blockage of the mean flow.
2.2.1 Concentration Studies in the Wake of Single Isolated Structures

Many studies conducted in recent years examine the extent of enhanced building dispersion in the wake of single isolated structures. The many references in the literature include studies on near wake and building surface concentrations such as Wilson and Britter (1982), as well as review articles on wake dispersion such as Meroney (1982). Of particular interest in the current research are those studies that deal primarily with measuring and predicting contaminant concentrations in the far wake region, outside of the initial wake cavity.

An early model of concentrations in the wake of buildings was presented by Gifford (1960) who suggested the centreline ground level values could be predicted by (2.9),

\[
\chi = \frac{Q}{u} \left( \pi \sigma_z + \lambda \Lambda \right)^{-1}
\]  

where \( \lambda \) is a constant of value between 0.5 and 2 (Meroney, 1982), depending on the flow.

A later study to gain widespread attention is described by Huber and Snyder (1982) (also see Huber and Snyder, 1976). In the paper the authors present an experimental study of mean contaminant concentrations measured in the wake. The rectangular model of width equal to twice its height and depth was located in the simulated atmospheric boundary layer of a wind tunnel. The paper also presents a dispersion model which accounts for the enhanced turbulent mixing effects of the building. The Huber and Snyder model (subsequently referred to in this report as the HS model) is based on the well established Gaussian plume model (Appendix B) which, for a continuous point source in a uniform cross wind (Appendix C), at effective stack height \( h_{\text{eff}} \), and with perfect ground reflection, may be written as (2.10) (see Sutton, 1953, Csanady, 1973, among others).
\[ \chi = \frac{Q}{2\pi \nu \sigma_y \sigma_e} \exp \left( \frac{y^2}{2\sigma_y^2} \right) \left[ \exp \left( \frac{z - h_{mf}}{2\sigma_e^2} \right) + \exp \left( \frac{z + h_{mf}}{2\sigma_e^2} \right) \right] \quad (2.10) \]

In (2.10) the turbulent mixing of the contaminant is accounted for by the values of \( \sigma_y \) and \( \sigma_e \), considered in the equation to be functions of \( x \) only. These spread parameters are generally estimated for various atmospheric conditions from published tables or curves (Turner, 1969). Limitations of equation (2.10) arise when applying it to non-homogeneous shear flows such as those of the natural environment. Restrictive assumptions used in its development are discussed in Appendix B, but the reality is that the model has had a great deal of success predicting mean concentrations. This success is largely attributed to the tremendous amount of empirical information incorporated in the chosen \( \sigma \) values.

In the wake of a building the turbulent mixing is enhanced above ambient levels and so the selection of \( \sigma \) values based on ambient flow conditions is inadequate. To account for the increased turbulent mixing Huber and Snyder (1982) present modified \( \sigma \) values (\( \sigma' \)) based on the results of their experiments. The building enhanced \( \sigma \) values suggested are of the form

\[ \frac{\sigma'}{h} = \left[ C_1 + C_2 \left( \frac{x}{h} \right)^{C_3} \right] \frac{\sigma}{h}, \quad \text{for} \ x/h \leq 10 \quad (2.11) \]

They chose the decay rate constant \( C_1 \) to match the observed decay rate of excess turbulence intensity in the wake (above background levels) and set \( C_1 = -1.8 \). The constants \( C_1 \) and \( C_2 \) were chosen to match the observed and predicted ground level concentrations after the initial plume spread in the wake cavity region \( x/h = 3 \). They found \( C_1 = 2 \) and \( C_2 = 35 \). \( \sigma \) in (2.11) represents the ambient values observed in the experiments in the absence of the building. The authors estimated the background values, \( \sigma_y \) and \( \sigma_e \), for their experiments, as (2.12),
\[
\frac{\sigma_y}{h} = \frac{\sigma_z}{h} = 0.115 \left( \frac{x}{h} \right)^{0.8}
\] (2.12)

A subsequent publication of Huber (1991) allows that the background values of \( \sigma_y \) and \( \sigma_z \) may not be equal beyond \( x/h = 10 \). Based on this observation and on the experimentally determined values of \( C_1, C_2, C_3, \) and \( \sigma_y, \sigma_z \) for \( x/h \leq 10 \), Huber (1991) presented the enhanced \( \sigma \) values as follows:

\[
\frac{\sigma_y}{h} = \frac{\sigma_z}{h} = 0.7 + 0.067 \left( \frac{x}{h} - 3 \right), \quad \frac{3}{h} \leq \frac{x}{h} \leq 10 \quad (2.13a)
\]

and

\[
\sigma_y' = \sigma_y \left( x + x_v \right), \quad \frac{x}{h} > 10 \quad (2.13b)
\]

where \( x_v \) and \( x_v \) are virtual source locations chosen so that (2.13a) and (2.13b) agree at \( x/h = 10 \). The authors indicate (2.13) may be used according to the flow regions of Figure 2.2.

**Figure 2.2:** Regions of applicability for HS model (after Huber and Snyder, 1982)

Huber (1984) presents model performance tests against ten field studies and shows reasonable agreement between model and measurements. However, there are limitations of (2.13) when applying it to general flow problems. In particular, the wake enhancement constants are based on the particular flow conditions and model dimensions of the originating experiments. In addition, the universal transition point for background dispersion levels of \( x/h = 10 \) shows no regard for obvious influential parameters such as...
building dimensions and background turbulence levels and scales. The model success is also sensitive to the particular source conditions of the experiment including stack height, stack location, and initial vertical velocity of the effluent. Several researchers including Ramsdell (1990), Musselman (1992), and Huber himself (1989, 1991) have identified these limitations and recommended various minor modifications. Nevertheless, the original HS model still finds its way into various government codes such as the Industrial Source Complex Model of the US EPA, and as such, is worthy of consideration with respect to the present study.

Robins and Castro (1977) describe a wind tunnel investigation of plume dispersion in the vicinity of a surface mounted cube. The study includes several source conditions including a low momentum source emitted through four porous walls of a cube (upstream face solid). The authors demonstrate that a simple point source Gaussian model such as that of (2.10) reasonably predicts centreline ground level concentrations beyond $x/h \approx 10$ if a virtual source of $x_v = -5h$ is incorporated into the model. The virtual source approach allows for the enhanced mixing of the building by increasing the effective $\sigma$ values near the actual source. This method does not, however, account for the reduction in the mean advection velocity in the near wake. Also, the authors do not specify their choice of $\sigma$ values used in the model which is paramount to any hope of generalizing the results.

Other researchers present time based models to account for the enhanced mixing of contaminants in the near wake region (Ramsdell, 1990, Weil, 1991). One such model currently under development is that of Weil (1991) which, although considerably more complex than the simple HS model, may prove effective in predicting concentrations in the building wake region. The added complexity of the model may not be justified by the results. Skepticism is due primarily to the presence of free parameters in the model which inhibit the generality of the results.
2.2.2 Dispersion Studies in Multiple Structure Wakes

It is clear that most pollution sources are not isolated cubic structures and some research has been conducted to examine the effect of building complexes on contaminant dispersion.

Halitsky (1977) presents full scale and wind tunnel data along with a model proposal for contaminants released in the wake of the EBR-II building complex. The EBR-II complex is a nuclear power generating facility located in Idaho. A variety of buildings make up the complex which is dominated by a dome capped cylindrical containment structure. The terrain surrounding the building cluster is relatively open, typical of such facilities.

The premise of the Halitsky model is to replace the entire building complex with a virtual rectangular plate of sufficient width and height to theoretically create similar wake generating and contaminant dispersing qualities as the actual complex.

Wake parameters for a flat plate (from Cooper and Lutzky, 1955) were used for the decay of the mean velocity deficit and the excess turbulence intensity, as well as the spatial growth of the wake. Fitting these parameters onto the data for the reactor complex, Halitsky estimates an effective height and width of the equivalent flat plate. The height, \( H \), is an average building height, but the width, \( W \), of the plate is difficult to estimate without wake velocity data for the actual building complex.

After removing the dimensions of the particular study, the mathematical dispersion model used by Halitsky (subsequently referred to as the HAL model) becomes

\[
\chi = \frac{3}{\sqrt{20\pi}} \frac{Q}{\sigma_y \sigma_z (u/u_o)} \left[ \left( 1 - \frac{y}{\sqrt{10} \sigma_y} \right)^2 \right] \exp \left( - \frac{z}{\sqrt{2} \sigma_z} \right) \]

\[
(u/u_o) = \left( 1 - 0.32 \left( \frac{x}{\sqrt{2} HW} \right)^{2/3} \left( \frac{W}{2H} \right)^{1/3} \right) \]

(2.14)

(2.15)
The spread parameters are defined as

\[ \sigma_y = P \frac{W}{2\sqrt{10}} + \frac{2.5}{\sqrt{10}} b_y \cdot x^* \]  \hfill (2.16a)

\[ \sigma_z = \frac{H}{2.5} + b_z \cdot x^* \]  \hfill (2.16b)

where \( b_y \) and \( a_z \) provide a fit for background \( \sigma \) values. \( P \) is a free parameter less than 1 describing the interruption in cavity mixing due to air seepage through the buildings. The concept is that as air passes through the complex the lateral spread will be less than that behind a solid plate of width \( W \).

The model assumes a Gaussian distribution of contaminant in the vertical direction and a parabolic distribution in the lateral. The author gives no guidance to estimate the key parameters of \( H, W, \) and \( P \) for general application of the model. Prior knowledge of the magnitude of \( P \) as a function of characteristic separation is necessary if we are attempting to model a complex wake generating cluster with a single wake generating structure such as a flat plate.

Another group of researchers examining contaminant dispersion in the wake of building clusters is Plate and Baechlin (1988), and Baechlin et. al. (1992). Although primarily concerned with concentrations within the building cluster, the researchers present some interesting concepts with respect to the far wake region. They define a Radius of homogenization, \( R \), beyond which the combined wakes of the various individual buildings are sufficiently intermixed to have effectively forgotten their individual origins. The standard Gaussian plume model used in the 1992 paper is not adequately modified to account for the effects of the building cluster, and as a result, the model does not acceptably predict the dispersion. The semi-empirical model described in the paper is not presented in a manner which would allow any generalization of the results. With criteria for prediction, however, the concept of a Radius of homogenization is appealing with respect to modelling in the wake of multiple structures.
2.2.3 Modified Point Source (MPS) Dispersion Model

Despite the violation of fundamental assumptions (homogeneous, stationary turbulence, constant eddy diffusivity, etc.) made in the derivation of the basic Gaussian plume model (2.10), the model remains reasonably effective at predicting contaminant concentrations in the natural environment. Careful choice of the turbulence parameters $\sigma_{x}$ and $\sigma_{z}$, and the mean velocity $u$, lead to reasonable agreement between measured and predicted values of the mean concentration field. It is because of this success and the relative simplicity of the model that it becomes the starting point for a model examined in this thesis.

Research shows that $\sigma$ values are enhanced in both $y$ and $z$ directions in the near wake of a building. Robins and Castro (1977) showed that using a virtual origin located upstream of the building, along with the basic model was one way to achieve the enhanced $\sigma$ values. This method is sufficient to match the far wake concentration field but the high concentrations in the near wake are underpredicted by such an enhanced model. This underprediction is primarily because of the substantial reduction in the mean advection velocity in the near wake, not accounted for by the model.

The development of the basic point source diffusion model as described in Appendix A assumed that gradients of $u$ in the $x$ direction were small. This assumption simplifies the governing equation allowing it to be solved analytically. For a plume spreading in a free atmosphere gradients of $u$ in $x$ may be negligible, but in the near wake of a building variations of $u(x)$ may be highly significant. Substituting $u = u(x)$ into the basic equation (2.10) accounts for the increased concentrations experienced in the low velocity near field. Provided the gradients represented by the expression $u = u(x)$ are small relative to concentration gradients in the same region, the governing equation of (B5) may still be expected to apply.

To extend the model to multiple structure wakes it is necessary to define a universal decay rate for the mean velocity deficit such as
\[
\frac{u_h - u_v}{u_h} \propto \left( \frac{x}{\xi} \right)^{\alpha_1}
\]  
(2.17)

where \( \xi \) is a characteristic length scale for the building group and \( \alpha_1 \) is a universal decay exponent.

In addition, modification of the spread parameters \( \sigma_y \) and \( \sigma_z \) in the wake, typically handled through a virtual origin, are hypothesized to relate directly to the excess turbulence intensity \( i = \bar{u} / U_r \), generated by the structure group as given by (2.18)

\[
\left( \frac{\sigma_y - \sigma_h}{\sigma_h} \right) \propto \left( \frac{i_y - i_h}{i_h} \right) \propto \left( \frac{x}{\xi} \right)^{\alpha_2}
\]  
(2.18)

(2.18) does not imply that the spread parameters are equal to the intensities, only that the buildings modify the background spread in the same manner that they modify the intensities. (2.18) is expected to apply to both lateral and vertical spread parameters with the exponent \( \alpha_2 \) determined from the velocity data, independently for each.

The wake flow modifications given by (2.17) and (2.18) allow the concentration model to be modified based on verifiable knowledge of the flow field in the far wake. This eliminates the need for virtual origins or other fit parameters which must be estimated from actual concentration measurements and seriously limit the predictive abilities of the model.

The research of Jerram et. al. (1993) suggests that the effect of the reduction in the mean velocity, (2.17), outweighs the effect of the added turbulent mixing, (2.18). The relative importance of the two effects is expected to vary depending on the particular grouping studied. A careful description of the mean velocity field and the turbulence intensity field is expected to be critical to the success of the model.

Even with the preceding modifications the model can only be reasonably expected to give moderate agreement with data. The assumption of constant eddy diffusivity and
constant mean velocity with height are expected to combine to cause an overprediction of \( \sigma_z \) in the model. Reasonable agreement at ground level, however, can be expected.

2.2.4 Modified Area Source (MAS) Dispersion Model

It is commonly assumed in the literature that a contaminant entrapped by the cavity region of a building will be fully mixed in that region before being transported downstream. As a consequence of this, the exact type of source that produced the contaminant is irrelevant once the gas becomes entrapped by the cavity region. This observation has led to the possibility of modelling the dispersion in the wake with a continuous area source model as opposed to the traditional point source model. While such a model is still limited by the shape of the area source used, this detail will be more quickly lost in the wake than the details of an infinite point source.

Consider an area source of initial strength \( q_a \) (g/m²s) in the \( yz \) plane with dimensions \( b \) by \( a \), and perpendicular to a uniform wind of velocity \( u \). We consider the source to be made of a number of distributed point sources such as that described by (C5). Each source is characterized by a source strength \( q_a dy'dz' \) such that the sum of all such sources may be given by

\[
\chi = \int_{-b/2}^{b/2} \int_{-a/2}^{a/2} \frac{q_a}{2\pi u\sigma^2} \exp\left(\frac{(z-z')^2 + (y-y')^2}{2\sigma^2}\right) dz'dy' \tag{2.19}
\]

We have assumed a single value for \( \sigma \), however \( \sigma \) need not equal \( \sigma_z \), provided they are functions of \( x \) only.

Now let

\[
t = \frac{(z-z')}{\sigma \sqrt{2}}, \quad s = \frac{(y-y')}{\sigma \sqrt{2}}
\]

such that

\[
dz' = -\sqrt{2}\sigma \; dt, \quad dy' = -\sqrt{2}\sigma \; ds
\]

New limits are given by

\[
z' = \frac{\pm a}{2} \Rightarrow t = \frac{1}{\sqrt{2}} \left( \frac{z + \sqrt{2}/2}{\sigma} \right), \quad y' = \frac{\pm b}{2} \Rightarrow s = \frac{1}{\sqrt{2}} \left( \frac{y + b/2}{\sigma} \right)
\]

such that (2.19) becomes
\[ \chi = \frac{q_u}{2\pi u\sigma^2} \int_{t_1}^{t_2} \int_{s_1}^{s_2} (-\sqrt{2}\sigma) \exp(-t^2)(-\sqrt{2}\sigma) \exp(-s^2) \, dt \, ds \]

from which we separate the exponentials and recognize the error function. The equation simplifies to become

\[ \chi_{\text{area}} = \frac{q_u}{4u} \left[ \text{erf} \left( \frac{a/2 + z}{\sqrt{2}\sigma} \right) + \text{erf} \left( \frac{a/2 - z}{\sqrt{2}\sigma} \right) \right] \left[ \text{erf} \left( \frac{b/2 + y}{\sqrt{2}\sigma} \right) + \text{erf} \left( \frac{b/2 - y}{\sqrt{2}\sigma} \right) \right] \]

(2.20)

To account for ground reflection we set \( a/2 = h \), where \( h \) is the height of the area source above the ground. Thus the vertical extent of the source is effectively doubled using the mirror source method for perfect ground reflection. The model is shown schematically in Figure 2.3.

![Figure 2.3: Schematic representation of Area Source model](image)

Such a model is based on the same assumptions as the point source model and, as such, will have similar limitations. The model will, however, more correctly predict the \( \sigma \) values in the near field because these values are forced by the physical dimensions of the building. The problem occurs beyond the initial cavity region where the model predicts the \( \sigma \) values to grow at background rates and does not account for the excess turbulence intensity in the wake. As a result, enhanced \( \sigma \) values are required in this model, as with the point source model, to correctly predict concentrations in the far wake. If a virtual
origin method is employed to accomplish this task then this origin must change from a value of zero at the building rear face, to a finite value downstream. Instead, the spread parameters are assumed to vary as the turbulence intensity as given by (2.18). As well, the mean velocity modification as given by (2.17) is again employed to correct for the substantial reduction of the mean advection velocity in the near field. As with the Modified Point Source model, this model is modified to account for the presence of the buildings only by knowledge of the flow field and not by fitting known concentration data.

It is expected that the MAS model will have an advantage over the MPS model in the near field where the MPS model is expected to overpredict the contaminant concentrations in most cases due to an unrealistic source condition. Comparable performance is expected between the MPS and MAS models at greater distances downstream where the details of the source are forgotten (Appendix C).

The modifications to the mean advection velocity and the turbulence spread parameters suggested in these models has the additional advantage of providing a natural transition to background spread conditions. Unlike the Huber and Snyder model, no transition to background dispersion is forced on the model without respect for the varying flow conditions occurring in the wake of various flow geometries.

2.2.5 Non-Dimensional Concentrations

In presenting the results of the experiments all concentration values are normalized as follows:

\[ K = \frac{U_h h^2 \chi}{Q} \]  \hspace{2cm} (2.21)

where, \( \chi \) is the measured concentration, \([\text{g/m}^3]\)

\( h^2 \) is the frontal area of the single isolated structure, \([\text{m}^2]\)

\( U_h \) is a reference velocity upstream at \( z = h \), \([\text{m/s}]\)

\( Q \) is the continuous source emission rate, \([\text{g/s}]\)

These are traditional normalizing variables, consistent with the literature.
2.3 CONCENTRATION FLUCTUATIONS

Without exception, all of the models of contaminant concentration considered in this thesis are concerned only with mean concentration values. It is well established that in many cases regulating the mean concentration alone is inadequate to ensure public safety. In particular, conditions where time dependent dosages, or odour problems are of concern, peak concentrations are of great interest. The models included in this thesis will be particularly inadequate in the near field where concentration fluctuations are the highest.

Considerable work on the turbulent fluctuations of concentration has been done (see Chatwin and Sullivan, 1979, for example). More research on the behaviour of fluctuating concentrations near single isolated structures must be done before the more complex flow regimes of the present study are examined. Such a dedicated study has been proposed by the writer and is beyond the scope of this thesis.
CHAPTER 3

PURPOSE AND METHODS OF THE EXPERIMENTS

3.1 PURPOSE OF THE EXPERIMENTS

The purpose of the experiments is to demonstrate how the wakes of multiple structures in an atmospheric boundary layer interact and coalesce downstream so that they might be modelled as if generated by a single, isolated, wake generating unit. This stated purpose will be achieved through three distinct experimental endeavors.

The first set of experiments utilize a pair of cross-wire type hot wire anemometers (HWA) to measure the development of mean and turbulent velocities in the far wake. It is the purpose of these experiments to demonstrate how the wakes of the individual structures combine and develop downstream. Measurements in the wake of a single isolated structure establish the credibility of the methods through comparison with published results from the literature.

The second set of experiments involve flow visualization in the wake using a smoke tracer illuminated by a horizontal laser light sheet. Overhead video photography and image post processing provide a visualization of the turbulent mixing properties of the combined wakes. It is the purpose of this experiment to demonstrate the relative importance of the reduced transport velocity in the wake (which results in higher contaminant concentrations), with the enhanced turbulent mixing (which results in lower contaminant concentrations). Knowledge of this balance is useful in designing models for contaminant dispersion in the wake, as well as for designing experiments to test these models.

The final set of experiments described involve fast response flame ionization detectors (FID) used to measure the propagation and dispersion of a continuous passive
ethane emission in the wake. The dispersion of contaminants in the wake of a single isolated structure is well documented in the literature and perhaps adequately modelled as well (see Section 2.2). Accepting the limitations of our ability to model dispersion in the wake of a single isolated structure, it is then the purpose of this experiment to demonstrate the relative dispersion and propagation of a similar emission in the wake of a structure group.

3.2 EQUIPMENT AND METHODS

3.2.1 The Wind Tunnel Facility

The experiments were conducted in the wind tunnel test facility, BLWT 1, at the Boundary Layer Wind Tunnel Laboratory, Faculty of Engineering Science, University of Western Ontario, in London, Ontario.

BLWT 1 is an open circuit type wind tunnel with an overall length of 35.5m. In the configuration of the present experiments the tunnel has a working test section area 2.44m wide and 1.96m nominal height. A fetch of 18.8m precedes a 3.2m long test section. The roof of the wind tunnel is adjustable to compensate for longitudinal pressure gradients in the tunnel. For the present experiments static pressure gradients were minimized along the extent of the tunnel by varying the roof height from 1.67m at the entrance to 1.99m at the end of the test section.

Tunnel flow is produced by a variable pitch fan, powered by a 26.9kW AC motor and located at the downstream end of the wind tunnel. Tunnel velocity is maintained within a tolerance of ±0.305 m/s through control software that adjusts the pitch of the fan based on feedback from two pitot-static probes located 0.46m from the ceiling of the tunnel. Barocel pressure transducers (Datametrics Inc., Waltham, Mass.) are used to convert the probe signals.

At the entrance to the wind tunnel a series of honeycomb mesh screens are used to straighten the flow and minimize inlet turbulence. A vertical array of 7 horizontal bars
varying in thickness from $1.27(10^{-2}) \text{m}$ at $1.27(10^{-2}) \text{m}$ from the floor, to $0.64(10^{-2}) \text{m}$ at $0.254 \text{m}$ from the floor, are used to initialize the gradients of turbulence and trip the approach flow. A carpet covering the floor of the wind tunnel is sufficiently rough to generate an ‘open country’ terrain boundary layer. Details of the approach flow generated by this arrangement are provided in the experimental design description of Chapter 4. With the carpet in place the floor was carefully checked for level and any undulations were minimized.

3.2.2 Data Acquisition Equipment

For the majority of the experiments a Data Systems Design PDP-11 multitasking computer was used to control the wind tunnel and acquire the test data. Despite its age, the PDP-11 is effective in its task, providing consistent control of the wind tunnel parameters along with efficient data acquisition capabilities. Further experiments were controlled by an equally effective PC based system.

3.2.3 Probe Traversing

An airfoil shaped traversing mechanism spans the wind tunnel and provides three axes of motion for probes mounted to it. The mechanism was carefully checked to ensure that it was parallel to the tunnel floor at all times.

Vertical traversing is provided by a single stepper motor (Superior Electric Slo-Syn HS 50L) located on one end of the traversing wind. The motor provides 200 steps per revolution ($2.54(10^{-5}) \text{m/step}$) control and vertical location was verified to be within $0.5(10^{-3}) \text{m}$ at each measurement location. After each vertical traverse the home location was verified and the mechanism performed flawlessly throughout the experiments.

Lateral traversing is provided through a similar stepper motor as used in the vertical traverse. This motor is located at the opposite end of the traversing wing and drives a roller chain attached to the probe mounting fixture. Backlash and tension in the chain were carefully checked and lateral position of the probe was verified within $0.5(10^{-3})$
m. As with the vertical traversing mechanism, the lateral mechanism was continuously monitored and functioned flawlessly throughout the experiments.

Longitudinal traversing of the probes is provided by manually sliding the traversing wing along rails mounted on the tunnel walls. Locking nuts allow the longitudinal position of the mechanism to be fixed while lateral and vertical traverses are performed. Manual traversing is easily controlled within the measurable tolerance of 0.5(10⁻¹)m.

In order to be sufficiently rigid the traversing mechanism is, by necessity, rather large. With a maximum thickness of 0.089m it represents approximately 4% blockage of the wind tunnel. While the shape of the wing is designed to minimize its drag, flow variations near the wing are inevitable. In all experiments the measurement probes were mounted sufficiently far upstream from the traversing wing so that its effect on the local flow could be considered negligible. This was verified through comparative tests of various mounting arrangements. Figure 3.1 shows the traversing wing with two single wire probes attached.

![Figure 3.1: Traversing wing in the wind tunnel looking downstream](image-url)
3.2.4 Hot Wire Anemometers

Mean and turbulent \textit{rms} velocities were estimated from instantaneous velocity measurements using hot wire anemometers.

**Single Wire Probe**

Background flow parameters including vertical profiles of mean and turbulent velocities were measured with a single wire HWA (Thermo-Systems Inc. (TSI)). The single wire is mounted perpendicular to the flow, in a holder angled 45° to the flow in the vertical plane. In this arrangement the probe may be traversed within 0.005m of the wind tunnel floor.

**Cross-Wire Probes**

Some additional background measurements and most wake measurements of instantaneous velocity were made using two cross-wire (TSI) HWA probes. The two probes were aligned to give measurements of longitudinal/lateral and longitudinal/vertical velocity components. Positioned parallel to the floor at equal height ($\pm 0.5(10^{-3})$m) they were spaced laterally 0.05m $\pm 0.5(10^{-3})$m apart, a minimal distance dictated by mounting arrangements and blockage considerations. Because the probes must be aligned parallel to the flow, they are limited in their vertical range to no closer than 0.03m from the wind tunnel floor.

**HWA Signal Processing**

HWA voltages were controlled by matching TSI (model 1054B) anemometer modules with built in linearizing cards providing linear output signals within the velocity range 0-30 m/s.

The HWA probes and control modules were carefully calibrated before and after each experiment using Wind Tunnel Calibration Standard G-8 and a TSI (model 1125) calibration unit. In-tunnel calibration checks were conducted at the beginning and end of
each test session to ensure no significant deviations in calibration had occurred. Results of
the probe calibrations are included in Appendix E.

The data acquisition computer was set to sample instantaneous velocity
measurements at a rate of 500Hz. To eliminate aliasing the anemometer signals were low-
pass filtered at 250Hz using Krohn-Hite (model 3343R) and in-house built filters. Post
filtered signals were separated into alongwind and acrosswind components with the aid of
in-house constructed voltage adders designed to have a measured frequency response that
was flat from 0.5Hz to beyond 1kHz.

3.2.5 Flow Visualization Equipment

Flow visualization was accomplished through video and still photography of oil
smoke illuminated by a horizontal laser light sheet.

Laser Light Source

The laser used in the experiment is a 4W nominal output Argon-Ion laser (Spectra
Physics model 165). For the purpose of the experiments the laser was run at
approximately 2W output power, all-line TEM-00 n.

The laser source was located outside of the wind tunnel with the blue light beam
passing through the tunnel wall 5.0m upstream of the test section. Two 0.05m round
mirrors mounted inside the tunnel redirected the light to the test section where it was
passed through a 0.013m diameter circular prism to create a horizontal light sheet
approximately 0.01m thick. Light intensity was reasonably uniform ±45° from the beam
direction. A ‘hot spot’ region of high light intensity directly in line with the beam was
blocked by a black rod creating a shadow across the visualization field. This procedure
allowed for better image processing and analysis.

Flow Tracing Smoke

The flow was marked by an oil smoke generated by an Aerotech smoke generating
unit (model SGS-90). The hot oil smoke was gathered below the wind tunnel floor in a
large smoke containment device where it was allowed to come into thermo equilibrium with its surroundings. The large volume of the smoke containment device also ensured that a uniform supply of smoke was available to the flow. The smoke was drawn into the flow by the naturally occurring pressure gradient existing across the wind tunnel walls.

**Video Imaging Equipment**

An Elmo 1:1.6, 7.5mm black and white remote lens (CCD Camera model EM-102BW) was mounted overhead of the test section. The focal length of the lens allowed for a field of vision of approximately 1.0m across by 1.3m downwind, measured on the tunnel floor. These dimensions corresponded to a range of $1 < x/h < 26$ based on the characteristic model dimension described in the following chapter.

The overhead lens was connected by fibre-optic cable to a video camera/recorder located outside of the tunnel. A separate video monitor allowed for simultaneous observation of the recorded flow. The experiments were recorded on VHS format video cassette for further analysis using image processing software.

In addition to overhead video images, colour video images and colour still images were obtained to aid in the flow analysis.

For the purpose of the flow visualization experiments it was necessary to operate the tunnel at very low velocity ($<3$m/s). While this low velocity is necessary to ensure adequate tracer is present for image processing, it quite seriously affects the scaling parameters of the flow. These effects will be discussed further in the experimental design section, Chapter 4.

### 3.2.6 Contaminant Concentration Measurements

Instantaneous contaminant concentrations were made using two Cambustion (HFR 300) fast response flame ionization detector probes spaced laterally $0.05m \pm 0.5(10^{-1})m$ apart on the traversing mechanism. The advantage of using the Cambustion units is that the ionizing flame is located very near to the point of sampling and inside of the wind
tunnel. This arrangement allows for fast time response of the instruments. The compromise is that the sampling units are relatively large (0.02m by 0.06m perpendicular to the flow) and somewhat intrusive to the local flow. Longer sampling tubes move the units farther away from the point of measurement at the expense of response frequency. For the mean concentration measurements of the experiments, sampling tubes 0.125m in length were used resulting in an estimated maximum frequency response of 200Hz\(^1\). Instantaneous concentration measurements utilized a shorter 0.075m sampling tube capable of greater than 300Hz\(^2\) estimated response.

Ethane, C\(_2\)H\(_6\), was used as the tracer gas, combined to neutral buoyancy by a Linde (Union Carbide) (model FM4575) mass flowmeter/flow controller. The flow rate of the passive gas mixture is controlled by the unit to a tolerance\(^3\) of \(\pm 5.0 \times 10^{-7}\) m\(^3\)/s.

The fast FID units were carefully calibrated before and after each test run using analyzed gases of known hydrocarbon content. In addition, background concentrations were measured after each traverse to monitor ambient hydrocarbon levels in the wind tunnel. Calibration results are included in Appendix E.

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\(^1\) Cambustion Ltd. software SATFLAP1  
\(^2\) ibid 1  
\(^3\) Linde (Union Carbide) document L14-056
CHAPTER 4

EXPERIMENTAL DESIGN AND PROCEDURE

4.1 APPROACH FLOW CHARACTERISTICS

With the roughness carpet and trip fence in place as described in Section 3.2.1, vertical and lateral profiles of mean and turbulent velocity were taken throughout the test section to characterize the approach flow. In addition, instantaneous velocity measurements were made from which relevant turbulent length scales and turbulent energy spectra are estimated.

4.1.1 Velocity Profiles

Tunnel velocity was set at $U_\infty = 10.06 \text{ m/s}$, measured at the roof pitot-static probes. The boundary layer depth was estimated to be $\delta = 0.45\text{m}$ at the start of the test section. Taking reference values at the boundary layer height, $z_r = \delta$, the profiles of mean and turbulent velocities are presented in Figure 4.1 over the vertical extent relevant to the present experiments.

The mean velocity profile is fit by a power law equation of the form

$$\frac{u(z)}{U_r} = \left(\frac{z}{z_r}\right)^n$$  \hspace{1cm} (4.1)

where $n = 0.16$ is shown in Figure 4.1 to give good agreement with the measured data. The exponent 0.16 is consistent with the desired open country terrain (Davenport, 1960) in which an isolated wake generating structure is most likely to be located. The exponent is also consistent with that found most commonly in the literature cited in Sections 1.2 and 2.1 ($n = 0.15$, Lemberg, 1973, for example).

In the surface layer of the planetary boundary layer ($z < 50\text{-}100\text{m}$, Sutton, 1953) it is often reasonable to assume nearly constant shear stress as a function of height,
Figure 4.1: Profiles of mean velocity and turbulence intensity
\[ \tau(z) = \tau_n = \rho u_* k z \frac{\partial u(z)}{\partial z} \quad (4.2) \]

This assumption leads to a log-linear expression for the mean velocity profile in the surface layer. The so called 'log-law' is given by (4.3)

\[ \frac{u(z)}{u_*} = \frac{1}{k} \ln \left( \frac{z}{z_*} \right) \quad (4.3) \]

where, \( u_* \) is the surface friction velocity; a function of the surface shear stress, \( \tau_n \), and defined as \( u_*^2 \equiv \tau_n / \rho \)

\( k \) is von Karman's constant, \( k = 0.4 \)

\( z_* \) is the characteristic roughness length of the surface

The parameters \( u_* \) and \( z_* \) are not measured directly but are instead estimated by a linear regression of a \( u(z) \) vs. \( \ln(z) \) plot. Such a plot is given as Figure 4.2.

![Log-law velocity profile](image)

**Figure 4.2: Log-law velocity profile**

The linear regression analysis leads to the following estimates for the profile,

\[ u_* = 0.465 \text{ m/s} \]

\[ z_* = 1.12 \times 10^{-4} \text{ m} \]
These values of $u_*$ and $z_0$ are consistent with those estimated by Mounla (1995) for a similar velocity profile. Figure 4.2 shows that the measured profile is well represented by the log law (4.3) over the vertical extent of interest.

### 4.1.2 Turbulent Velocity Spectra

The profiles of longitudinal turbulence intensity of Figure 4.1 indicate that the magnitudes of local turbulence intensity vary from $>20\%$ near the surface, to $\sim 10\%$ at $z = 0.1 \delta$. These values are consistent with published reports of full scale atmospheric profiles (Pasquill, 1974).

Instantaneous velocity measurements at $z = 0.05$ m provided data from which the turbulent energy spectra of the approach flow could be estimated for $u$. Figure 4.3a shows the velocity spectra plotted so as to illustrate the inertial subrange decay rate consistent with the Kolmogorov -5/3 value expected. When the spectra is normalized as in Figure 4.3b, a -2/3 decay rate adequately describes the inertial subrange region, as it does in the natural environment.

![Figure 4.3a: Longitudinal PSD, $z = 0.05$m](image)
4.1.3 Length Scales of Turbulence

The length scales of turbulence in the approach flow are estimated from instantaneous velocity data using traditional analysis techniques. For details beyond those included here the reader is directed to Townsend (1956) and Hinze (1975).

To estimate the scales of turbulence we examine the general spatial correlation of arbitrary turbulent signals $s_i$ and $s_j$ separated in space by a distance $r$, defined by Townsend (1956) as

$$\overline{s_i' s_j'(r)} = \overline{s_i'(x_p, t)} \overline{s_j'(x_p + r, t)}$$  \hspace{1cm} (4.4)

A normalized correlation coefficient is then defined as

$$R_{ij}(r) = \frac{s_i'(x_p, t) s_j'(x_p + r, t)}{\sqrt{s_i'^2(x_p, t)} \sqrt{s_j'^2(x_p, t)}}$$ \hspace{1cm} (4.5)
The value of $R_{12}(r_i)$ will vary from a perfectly correlated value of 1 at $r_i = 0$, to an uncorrelated value of 0 as $r_i \to \infty$. The persistence of the correlation $R_{12}$ is held to be representative of the typical eddy scales in the turbulent signal.

Often, as in the present experiments, it is necessary to estimate turbulence scales from single probe measurements, variable in time and fixed in space. To do this we define a temporally varying correlation coefficient of the turbulent signal (here we use velocity $u'$ for example)

$$u' u' (\tau) = \overline{u' (x,t) u' (x, t+\tau)}$$

(4.6)

Analogous to (4.5) we define a normalized autocorrelation coefficient of the single turbulent velocity component $u'$ as

$$R_{uu} (\tau) = \frac{u' u' (\tau)}{u'^2}$$

(4.7)

Typical variation of the autocorrelation function $R_{uu}$ is shown in Figure 4.4, calculated for the present experiments at $z = 0.05m$.

![Figure 4.4: Decay of $R_{uu}$ at $z = 0.05m$](image)
Two essential scales of turbulence may be estimated from $R(\tau)$. The Taylor microscale is related to the initial radius of curvature of $R(\tau)$ and, for stationary, homogeneous turbulence, is given by Townsend (1956) as

$$t_E = \left| \frac{-2}{\frac{\partial^2 R(\tau)}{\partial \tau^2}} \right|_{0}^{1/2}$$

(4.8)

where the subscript $E$ is used to denote an Eularian time scale estimated from a fixed point in space.

The second Eularian time scale estimated from $R(\tau)$ is the integral time scale defined as

$$T_E = \int_{0}^{\tau} R(\tau) \, d\tau$$

(4.9)

$T_E$ is characteristic of the persistence of $R(\tau)$ by equivalent area and, as such, is considered representative of the typical size of eddies present in the flow.

Both (4.8) and (4.9) provide estimates of characteristic turbulent time scales and it is necessary to incorporate Taylor’s ‘frozen turbulence’ hypothesis (Hinze, 1975) in order to obtain characteristic turbulent length scales. The hypothesis suggests that the characteristic length scale is equal to the characteristic time scale when advected by the local mean velocity $u$. This functional form of the hypothesis is given by (4.10)

$$r_i = w_i$$

(4.10)

and is based on the assumption of stationary, homogeneous turbulence of less than 10% intensity. Cenedese et. al. (1991) points out that small turbulence structures may travel at much greater velocities than the mean flow causing serious errors in integral length scale estimates using Taylor’s hypothesis. For the general scaling estimates required for this study, however, these limitations are not severe, and length scale estimates using fixed point sampling techniques should be adequate.
Using the above techniques the following scales of turbulence were estimated at the beginning of the test section, $z = 0.05 \text{m}$. Subscript $E$ is implied hereon.

<table>
<thead>
<tr>
<th></th>
<th>micro-time $t$ (s)</th>
<th>micro-length $\lambda$ (m)</th>
<th>integral-time $T$ (s)</th>
<th>integral-length $L$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal, $u$</td>
<td>0.006</td>
<td>$4.03(10^{-2})$</td>
<td>0.044</td>
<td>$3.09(10^1)$</td>
</tr>
<tr>
<td>Lateral, $v$</td>
<td>0.003</td>
<td>$1.94(10^{-2})$</td>
<td>0.005</td>
<td>$4.26(10^2)$</td>
</tr>
</tbody>
</table>

4.2 MODEL DESIGN

4.2.1 Model Shape

Research in wake flows traditionally concentrates primarily on two basic three dimensional shapes of which many variations exist; the circular cylinder and the rectangular prism. The circular cylinder is attractive to theorists because its shape allows it to be modelled with variations of potential flow theory. Classical studies by von Karman (1921), and Schlichting (1930) were preceded by pioneers in the field as long ago as Galileo and da Vinci. Today, work in such areas as coherent structures (Hussain, 1986) continue to focus on the simple circular cylinder. Transitions in the surface boundary layer of a circular cylinder result in well known variations of drag with approach Reynolds number. Snyder (1972) notes that, generally speaking, the more streamlined an object is, the larger the critical Reynolds number for flow independence will be. Halitsky et. al. (1963) suggests that a Reynolds number greater than $7.9(10^4)$ is required for $Re$ independence in the flow about a cylinder with a hemispherical cap. As a result, experiments involving cylindrical shapes must take great care to control the Reynolds number regime, especially if applying the results to full scale prototypes.

The second fundamental wake generating shape found in the literature is the rectangular prism. Sharp edged structures of variable aspect ratio have been studied extensively in the literature. From the fundamental 2-D wake theories of Townsend
(1956), to current near wake interference studies in the interest of structural dynamics (Hangen, 1996), to surface mounted far wake studies like those considered in the present study (Hunt and Smith, 1969, Lemberg 1973, etc.), the rectangular prism is a well documented wake generating structure. Unlike its cylindrical counterpart, the sharp edges of the rectangular prism force clean separation of the flow that is independent of Re over a very large range (Re > 10^4, Golden, 1961). This lack of strong Re dependence allows for confident application of model scale data to full scale flows.

It may also be argued that the rectangular prism is representative of a greater percentage of the isolated structures found in the natural environment. Certainly when pollution dispersion is considered, the majority of the fundamental building influence studies (Huber and Snyder, 1982, Robins and Castro, 1977, etc.) deal exclusively with sharp edged wake generating structures.

In the present study we examine how multiple wake generating structures behave as a single wake generating unit. Possible Re effects associated with circular cylinders would seriously limit the generality of such a study. Clean, predictable wakes were desired and sharp edged structures were used in all cases. All models were carefully machined from acrylic or high density foam (ρ = 70 kg/m³) with clean, sharp edges.

4.2.2 Model Dimensions

Model dimensions were determined based on the desired scaling of the experiments, and to facilitate direct comparison to past research wherever possible.

From a mean velocity perspective typical structures of concern include aircraft hangars next to runways. Such buildings are typically 10m - 20m in height. From a pollution dispersion perspective structures of concern include power generating stations of 10m - 20m height, and nuclear containment vessels of 20m - 30m height. A useful study will then apply to structures 10m - 30m in height, reasonably isolated in their surroundings.
The atmospheric boundary layer approaching an isolated structure/structure group is typically of the 'open country' type as described in Section 4.1. In the real environment a natural open country boundary layer has characteristic parameters found in the literature. Using published parameters and the desired full scale building dimensions of 10m - 30m, a model height of $h = 0.05$m was determined to give the best agreement with key modelling parameters discussed in Section 4.2.3.

To facilitate direct comparison with the published work of Castro and Robins (1977), Hunt (1970), Lemberg (1973), and others, the primary single isolated structure considered in the study is cubic in shape, $b = d = h = 0.05$m.

### 4.2.3 Scaling Considerations

Scale modelling of atmospheric flows in a wind tunnel requires similarity of the geometric, kinematic, and dynamic properties of the flow, as well as similarity of important boundary conditions. Snyder (1972) describes how the governing equations of the flow may be non-dimensionalized to reveal several key parameters that must be matched between model and prototype scales. Conflicting requirements prohibit strict agreement of all parameters and physical arguments must be used to establish priority among scaling considerations.

From the dimensionless equations for the conservation of mass, momentum, energy, and for molecular diffusion come the following non-dimensional variables:

\[
Ro = \frac{U}{f(L\Omega,)} \quad \text{is the Rossby number}
\]
\[
Fr = \frac{U}{\sqrt{gL\delta T/T_n}} \quad \text{is the densimetric Froude number}
\]
\[
Re = \frac{U}{L\nu} \quad \text{is the Reynolds number}
\]
\[
Pr = \frac{\nu}{\kappa} \quad \text{is the Prandtl number}
\]
\[
Sc = \frac{\nu}{\alpha} \quad \text{is the Schmidt number}
\]

where $L$ is a characteristic length scale, $\Omega$ is angular velocity, $T$ is temperature and $\delta$ represents a small deviation, $\nu$ is the kinematic viscosity, $\kappa$ is the thermal diffusivity, and
\( \alpha \) is the molecular diffusivity. Subscript \( r \) denotes reference quantities and subscript \( o \) denotes neutral atmosphere quantities.

The Rossby number represents the ratio of local advective accelerations to Coriolis accelerations. In the wake of a building local accelerations dominate the flow and Coriolis accelerations may be neglected over horizontal scales of less than 5km (Csanady, 1973). This criteria is easily met in the present study which is limited to less than 1km downwind, full scale.

The Froude number is considered by Snyder (1972) to be the most important scaling parameter when modelling atmospheric flows. For the present study isothermal flow conditions are used to simulate neutral atmospheric conditions. This allows for similarity of the infinite Froude number, but limits the study to high wind, neutral conditions where mechanical turbulence dominates thermal turbulence. This is a reasonable assumption in the mechanically generated turbulent region of the building wake and some small deviations from neutral conditions will not affect the results greatly. Furthermore, contaminant releases in the study are isothermal, non-buoyant releases which also match the infinite Froude number of similar passive releases in the atmosphere.

The Reynolds number represents the ratio of inertial forces to viscous forces and similarity of this ratio between model and prototype cannot be neglected. Geometric scaling combined with velocity limitations in a wind tunnel generally give model \( Re \) orders of magnitude smaller than those found in the natural environment. Fortunately, as discussed in Section 4.2.1, rigorous similarity is not required provided the model \( Re \) exceeds a minimum value, beyond which the flow is effectively independent of \( Re \). For the sharp edged models of the present study the experimental value of \( Re_n = 2.3(10^4) \) exceeds the critical value of \( Re_n = 1.1(10^4) \) given by Golden (1961). To further reduce the possibility of viscous effects very near the building the surface roughness is exaggerated by the addition of stipletone paint. This method is demonstrated by Vet (1978) to be effective at creating an aerodynamically rough surface on the model.
Both the Prandtl number and the Schmidt number appear in product with the Reynolds number, representing respectively, the ratio of momentum diffusivity to thermal diffusivity, and the ratio of momentum diffusivity to mass diffusivity. Both numbers are fluid properties, not flow properties, and as such are modelled well in a wind tunnel. In product with the Reynolds number, however, they are underestimated in the model scale. In flows such as the present study, turbulence dominates the bulk transport of heat and mass and molecular diffusivities may be neglected provided Reynolds independence is obtained (Snyder, 1972).

Similarity of model and prototype also requires that the non-dimensional boundary conditions of the governing equations are conserved. This requires that important elements of the natural approach flow are duplicated in the experiments. Section 4.1 describes the open country type boundary layer approach flow used in the experiments. The approach flow is designed to be similar to full scale with respect to the variation with height of the first two moments of velocity, and the spectra of turbulent energy in the flow. Other parameters of the approach flow must be considered with respect to the characteristic model dimension for appropriate scaling considerations.

The vertical extent to which the model occupies the boundary layer is important because of the higher gradients of shear very near the ground, and the vertical scaling of the large turbulent eddies in the atmosphere. Castro and Robins (1975) demonstrated the effects of shear on the flow near a cube in a deep boundary layer. For the present study a ratio of model dimension to boundary layer height, \( h/\delta \approx 1/9 \) is consistent with other studies that are used for comparison (e.g. Hunt and Smith, 1969, \( h/\delta \approx 1/9 \), Lemberg, 1973, \( h/\delta \approx 1/8 \)).

Length scales of the turbulence in the approach flow must be considered, especially for studies of contaminant dispersion. Hunt (1971) suggests that typical longitudinal integral scales found in the atmosphere are of the order of 100m. The longitudinal integral length scales calculated in the present study (Section 4.1.3) estimate a ratio of \( h/L_u \approx 0.16 \); a value consistent with the 10m to 30m building scales of interest.
Lateral scales of turbulence are not large enough in the wind tunnel to model the low frequency meandering found in the natural environment. In the wake region the turbulence is generated at scales of the characteristic building dimension (Musselman and Slawson, 1995) and this turbulence will dominate dispersion until background flow parameters are recovered in the far wake. This recovery rate is maximized when scales of turbulence in the background are of the same order as the characteristic building dimension. In order to model wake dispersion we should obtain lateral scales of turbulence that are at least as large as the characteristic building dimension. For the present study a satisfactory ratio of $h/L_c \approx 1.17$ is obtained.

Vertical scales of turbulence are proportional to height above the ground (Csanady, 1973). As a result, similarity of vertical scales is automatically satisfied by the geometric scaling of the flow.

Jensen (1958) suggests that details in the model flow smaller than the estimated surface roughness, $z_o$, will have very little effect on the flow. As a minimum requirement we should obtain similarity of $z_o$ according to the geometric scaling of the test, i.e. $(h/z_o)_{model} = (h/z_o)_{prototype}$. Using a typical value of $z_o = 0.05$ for open country (Davenport, 1960) the building dimensions of interest give a Jensen number range of $200 < h/z_o < 600$. For the present study the values of $z_o$ estimated in Section 4.1.1 result in a ratio of $h/z_o \approx 450$, acceptably within the desired range.

Furthermore, Sutton (1949) suggests that aerodynamically rough flows, such as those occurring in the natural environment, will be correctly modelled if a minimum roughness Reynolds number, defined as

$$Re_{z_o} = \frac{u_z z_o}{v}$$

exceeds a value of 2.5. Based on the experimental values of $z_o$ and $u_z$ estimated in Section 4.1.1 the roughness Reynolds number of $Re_{z_o} = 3.47$ exceeds the minimum requirement.

Another boundary condition of the natural environment is zero longitudinal pressure gradient over the scales of interest. In the wind tunnel longitudinal pressure
gradients are minimized by varying the cross-sectional area (by raising the roof) downstream. Blockage due to the model was less than 0.05% and not considered significant. Monitoring roof static pressure taps ensured adequate control of this boundary condition.

Table 4.2 summarizes key parameters of the experiments, along with target values and reference source.

Table 4.2: Scaling Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>0.16</td>
<td>$0.15^1,0.15^2,0.20^3$</td>
</tr>
<tr>
<td>$h/\delta$</td>
<td>$1/9$</td>
<td>$1/8^1,1/9^2,1/10^3,1/8^4$</td>
</tr>
<tr>
<td>$h/L_o$</td>
<td>450</td>
<td>200 - 600 $^5$</td>
</tr>
<tr>
<td>$h/L_a$</td>
<td>0.16</td>
<td>0.1 - 0.3 $^6$</td>
</tr>
<tr>
<td>$h/L_v$</td>
<td>1.17</td>
<td>~1.0 $^7$</td>
</tr>
<tr>
<td>$Re_z$</td>
<td>3.47</td>
<td>&gt; 2.5 $^8$</td>
</tr>
<tr>
<td>$Re_h$</td>
<td>$2.3(10^4)$</td>
<td>&gt; $1.1(10^4)^9$</td>
</tr>
</tbody>
</table>

$^1$ Hunt and Smith, 1969  
$^2$ Lemberg, 1973  
$^3$ Castro and Robins, 1975  
$^4$ Counihan, 1969  
$^5$ Davenport, 1960  
$^6$ Hunt, 1971  
$^7$ Musselman and Slawson, 1995  
$^8$ Sutton, 1949  
$^9$ Golden, 1961
4.2.4 Model Arrangements

The model arrangements studied in the experiments are summarized in Table 4.3.

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Arrangement, Plan View</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>▼</td>
<td>Background Flow, no structures</td>
</tr>
<tr>
<td>1</td>
<td>★</td>
<td>Single Isolated Structure (SIS), $S/h = \infty$</td>
</tr>
<tr>
<td>2</td>
<td>★★</td>
<td>Rectangular Prism, $b/h = 2.0$</td>
</tr>
<tr>
<td>3 - 7</td>
<td>★★ ★★</td>
<td>Three Cubic Structures, $S/h = 0.3, 0.6, 1.0, 1.5, 3.0$</td>
</tr>
<tr>
<td>8 - 9</td>
<td>★★ ★★</td>
<td>Five Cubic Structures, $S/h = 0.5, 1.0$</td>
</tr>
</tbody>
</table>

In all experiments background conditions with no structures present is considered Case 0.

The primary SIS considered is a single cube of characteristic dimension $h$. This is Case 1, used to validate experimental results through direct comparison with the literature.

Case 2 is a rectangular prism of width equal to twice the structure height, $b/h = 2$. Like the primary SIS of Case 1, Case 2 is also a single wake generating unit. Case 2 is designed to examine the effect of aspect ratio on the resulting wake without the complicating effects of the gaps that occur between multiple structures.

Cases 3 - 7 involve the primary SIS of Case 1 flanked by two identical cubic structures spaced equidistant, $S$, from the centre cube. The three cubes constitute a wake
generating unit that is symmetrical and normal to the oncoming flow. By considering symmetrical groupings it is possible to relate the effects of the outside wake generating structures back to the primary SIS, without complicating non-symmetrical effects. The field studies of Macdonald (1996) show that the effect of misalignment to the flow is to cause a more uniform, better mixed wake. We may speculate that flow misalignment will actually accelerate any wake coalescence that is observed for the normal flow arrangement. This hypothesis is not tested in the present experiments. The symmetrical arrangements and the normal alignment to the flow are limitations of the study deemed necessary to isolate the influences of the external wakes, and to quantify the rate of wake coalescence.

The separation distances between the three cube arrangements of Cases 3 - 7 varied from $S/h = 0.3$ to $S/h = 3.0$. Based on preliminary experiments these distances were selected to demonstrate two cases that would most likely merge ($S/h = 0.3$, $S/h = 0.6$), two cases that may or may not merge ($S/h = 1.0$, $S/h = 1.5$), and one case that most likely would not merge within the study region ($S/h = 3.0$).

Of some concern in the experiments was the effect of flow jetting through the gaps of the three cube arrangements, Cases 3 - 7. To examine the effect of jetting two additional test cases were designed using five identical cubic structures of characteristic dimension $h$. The additional cubes were placed directly in line with the gaps, a distance $S$ behind of the original three cubes. This five cube arrangement is shown as Cases 8 and 9 in Table 4.3. This arrangement permitted a single representative spacing parameter, $S$, to be used while not compromising the lateral scaling of the wakes, as dictated by the leading three structures. Two representative spacing values, $S/h = 0.5$ and $S/h = 1.0$, were studied in this arrangement. Figure 4.5 shows model cubes in such an arrangement.

For the purpose of examining the effect of structure spacing on the far wake flow the study is designed to emphasize Cases 1, 3 - 7.
4.2.5 Contaminant Source Geometry

An additional consideration of model design pertains specifically to the contaminant concentration measurements, Section 3.2.6, and involves the design of the contaminant source geometry.

Review of the literature reveals that many studies involving the effect of buildings on contaminant dispersion spend considerable time considering stack geometry and how the contaminant will be entrained by the building wake (see Huber and Snyder, 1982, Weil, 1991, for example). Whether or not the contaminant will be partially or fully entrained by the building wake depends on factors such as stack exit velocity and plume buoyancy, as well as stack height and location relative to the building. Many studies examine the effects of these various factors with respect to single isolated structures (Snyder and Lawson, 1976, Halitsky, 1965, among others).
The present study is concerned with examining the effect of multiple structure wakes on the downstream concentration of a fully entrained, passive (non-buoyant), contaminant in a neutral environment. To achieve this while avoiding the complications associated with stack geometry, the wake is flooded with a passive contaminant of minimal excess momentum using a method similar to the 'porous cube' of Robins and Castro (1977). Meroney and Yang (1970) indicate that the strong turbulent mixing in the near wake obscures the details of the source geometry. A uniform release in the near wake may be considered representative of many different source configurations.

To emit a uniform contaminant into the cavity region the rear face of the primary structure is perforated with \(4.0(10^{-3})\)m diameter holes. Using the low momentum criteria of Robins and Castro (1977) given by

\[
\frac{Q}{h^2 \beta U(h)} \ll 1.0
\]  

(4.10)

where \(\beta\) is a characteristic porosity ratio for the surface \((\beta = 0.18)\), the momentum ratio achieved for the present study was \(6.0(10^{-3})\). Such a low value suggests that the emission of the contaminant will not significantly alter the velocity field as it is studied in the absence of the contaminant. This assumption was verified by HWA measurements made with and without the contaminant source operating.

Figure 4.6 is a schematic representation of the source geometry with the spacial spread parameters also shown.

Figure 4.6: Schematic representation of source geometry
4.3 EXPERIMENTAL PROCEDURE

4.3.1 Origin of the Measurements

For all experiments of this study measurements were made relative to an origin defined as \((x/h, y/h, z/h) = (0, 0, 0)\) and located at the base of the windward face of the primary cube, at the lateral centreline of the wind tunnel. Figure 4.7 is a schematic showing the origin and characteristic dimensions.

![Flow diagram](image)

Figure 4.7: Schematic of origin and characteristic dimensions

4.3.2 Measurement Locations

Using two cross wire probes (Section 3.2.4) an extensive array of mean and turbulent velocity measurements were made in both the unobstructed background, and in the wakes of the various test cases (Table 4.3). For all cases lateral profiles of longitudinal mean velocity and intensity were taken at \(z/h = 1.0, -1.0 \leq y/h \leq 9.0\), for downstream locations \(x/h = 2.0, 3.0, 4.0, 5.0, 6.0, 8.0, 10.0, 12.5, 15.0, 20.0, 30.0\).

Past research indicates that the maximum turbulence intensity in the wake occurs along the lateral centreline, at or near the cube height, \(h\). Examining the wake at \(z = h\) is consistent with the literature (Lemberg, 1973, Robins and Castro, 1977) and is considered to be representative of the behaviour of the turbulence in the wake. Musselman (1992) demonstrates that observations made at the structure height, \(h\), are consistent with
observations made at the structure half height, \( h/2 \). While the magnitudes of the measurements differ, the trends do not. Lateral profiles at the structure height, \( h \), are considered most representative of the wake but are, admittedly, a limited two dimensional view of a three dimensional problem. To help alleviate the problem somewhat, vertical profiles were taken at the lateral centreline, for various downstream locations.

Of some concern is the reliability of HWA measurements in the near wake region, \( x/h = 2.0 \). Because the profiles are taken at \( z/h = 1.0 \), the probes are located above the recirculation zone as described in Section 1.1. With a positive mean velocity, bias errors due to the directional insensitivity of the probes were estimated to be less than 4% given local intensities of no more than 50%\(^1\). Further error estimates are included in Appendix E.

Flow visualization experiments (Section 3.2.5) were conducted in the wakes of all cube arrangements (Table 4.3). The horizontal laser light sheet was located parallel to the wind tunnel floor at \( z/h = 1.0 \), consistent with the flow velocity measurements.

Profiles of mean and turbulent contaminant concentrations were taken in the wakes of all test cases of Table 4.3, at measurement locations duplicating that of the HWA measurements. Background concentration measurements were taken downstream of a continuous point source located at \( z = 0.05 \text{m} \) and of finite diameter \( d = 0.01 \text{m} \). Configuration of the Cambustion FID probes (Section 3.2.6) permitted vertical profiles of concentration measurements to begin nearer to the wind tunnel floor than their HWA counterparts.

Additional HWA profiles necessary to verify modifications to the theory of Hunt (1970), as described in Section 5.1.4, were made using a single wire probe (Section 3.2.4).

\(^1\) based on an assumed normal probability density function
CHAPTER 5
THE FLOW FIELD IN THE WAKE

5.1 MEAN VELOCITY FLOW FIELD

The obstruction to the mean flow created by the presence of a wake generating body results in a local extraction of streamwise flow momentum and a mean velocity deficit in the wake. Using von Karman’s momentum integral for the flow past an obstacle, it is estimated that the extraction of momentum is balanced by a measurable drag on the object, given by

$$\text{Drag} = \rho \int_0^\infty \int_0^\infty u(U_w - u) dy dz$$

(5.1)

Many studies examine the drag created by bluff bodies of various geometry, in various approach flows (Roshko, 1954, among others). Hunt (1970) suggests that a more appropriate measure of momentum extraction for a surface mounted object is the force couple exerted on the body as given by (2.2). Regardless of how we model the flow mathematically, the net result of the obstruction to the flow is a reduction of mean velocity in the wake as shown schematically in Figure 5.1.

![Figure 5.1: Schematic representation of flow regions near a single structure](image-url)
The mean streamlines are diverted around the object, separating at the sharp edges and accelerating around the corners. The upstream mean velocity and pressure fields are recovered downstream by the entrainment of surrounding flow into the deficit region of the near wake.

The presence of the deficit region in the wake is of considerable engineering importance. The sheltering effects on downstream structures may allow for a reduction of design wind forces and a cost savings, as examined by Mounla (1995). The wind shear in the wake may have implications with respect to the safety of light aircraft or helicopter operations near hangars, as indicated by Hunt (1971). Furthermore, the mean velocity field represents the primary transport mechanism for environmental contaminants entrapped by the wake. This important consideration is essential to the present study and will be carefully considered with respect to the contaminant concentration measurements in the wake (Chapter 3).

### 5.1.1 Contours of Equal Mean Velocity Deficit

To illustrate the effect of multiple structures on the mean velocity deficit in the wake, the numerous lateral profiles are reduced to contours of equal mean velocity deficit. Before constructing the contour lines the background mean velocity is removed from the data and the resulting deficit is given as a fraction of the background value. The mean velocity deficit ($MVD$) is plotted as a positive value as given by (5.2),

$$ MVD = \frac{-(u_x - u_b)}{u_b} \tag{5.2} $$

Figure 5.2 shows data in the wake of the single isolated structure, Case 1, as it has been transformed by (5.2) for the purpose of contour plotting (data shown at $x/h = 2.0$, $z/h = 1.0$). Symmetry of the lateral profiles is assumed for $y/h < -1.0$. Full symmetry of the flow was confirmed at a single downstream point for all cases, and otherwise assumed.
Figure 5.2: Transformation of wake velocity data to positive mean velocity deficit

In all contour plots of mean velocity deficit the outermost contour is equal, and is given by $MVD = 0.1$. The innermost contour is dictated by the maximum $MVD$ measured in the near wake for each case studied. Contours are estimated by triangulating surrounding data points and linearly interpolating. No smoothing techniques have been employed.

Figures 5.3 to 5.7 show the effect of decreasing the spacing between three cubes from $S/h = 3.0$ to $0.3$. The overall width of the wake-creating structures is then further reduced in the rectangular prism case shown as Figure 5.8, and the isolated cube case shown as Figure 5.9.
Figure 5.4: Contours of longitudinal mean velocity deficit, Case 6, $S/h=1.5$
Figure 5.5: Contours of longitudinal mean velocity deficit, Case 5, S/h=1.0.
Figure 5.6: Contours of longitudinal mean velocity deficit, Case 4, $S/h=0.6$
Figure 5.7: Contours of longitudinal mean velocity deficit. Case 3, S/h=0.3
Figure 5.8: Contours of longitudinal mean velocity deficit, Case 2, b/h=2.0
Figure 5.9: Contours of longitudinal mean velocity deficit, Case 1, SIS, b/h=1.0
Large Spacing, $S/h = 3.0$

From Figure 5.3 it appears that the profiles of mean velocity deficit in the wake of three cubes separated by a distance of $3.0h$ decay without significant interaction to the 0.1 level shown in the figure. The momentum wakes have recovered to within 10% of the background mean velocity before $x/h = 7$. It is demonstrated in Section 5.1.2 that a very weak secondary flow is present between the structures even at this large spacing. These secondary flows strengthen as the separation distance is decreased and they will be discussed further with respect to the intermediately spaced structures.

Intermediate Spacing, $S/h = 1.5, 1.0$

The mean flow behaviour of three intermediately spaced structures is observed in Figures 5.4 and 5.5. A secondary flow field develops in the wake, centred between the structures. The streamlines around the individual objects no longer recover independently. The deceleration of the jets that form between the structures combines with the divergence of the recovering streamlines to form these secondary deficit regions. Such a flow is shown schematically in Figure 5.10.

Structure geometry dictates the kinematics of the initial flow regions. Low pressure wakes created behind each body (A) due to flow separation, result in a tri-modal form of the lateral velocity deficit profile.

Following the initial flow regions is a complex area of combining vortices and strong velocity fluctuations. The resulting mean flow patterns may be explained by the simplified description to follow. Streamlines diverted around the bodies are concentrated to form jets in the gaps (B). The streamlines are then drawn back towards the low pressure cavity regions. By mass continuity, these diverging streamlines cause a flow reduction at (C), much like the flow in a diverging nozzle. These secondary regions of mean velocity deficit are further strengthened by the transfer of momentum to the turbulent Reynolds stresses which are maximized along the shear layers of the individual
structure wakes. This phenomena is graphically illustrated in Section 5.3. The resulting lateral profiles of mean velocity deficit are bi-modal in shape.

![Diagram showing regions of significant mean velocity deficit](image)

**Figure 5.10: Schematic representation of flow regions around multiple structures**

An excess of fluid along the centreline due to converged streamlines results in a third region of reduced flow (D), uni-modal in form. Here the excess turbulent energy of the flow has redistributed itself across the wake and is no longer concentrated on the shear layers between the structures. This far downstream, convective accelerations due to turbulent stresses are maximum near the centreline, strengthening the mean velocity deficit by transferring momentum to turbulent Reynolds stresses. This uni-modal region becomes more visible as the spacing is reduced from $S/h = 1.5$ to $S/h = 1.0$.

**Closely Spaced, $S/h = 0.6, 0.3$**

In Figures 5.6 and 5.7 the contours of mean velocity deficit for the closely spaced three cube arrangements of Case 4 ($S/h = 0.6$) and Case 3 ($S/h = 0.3$) are observed. As the spacing is decreased in this range the structures increasingly act as a single wake generating unit. The developing region in which secondary flow fields are generated by
the flow passing between the structures is further reduced as more and more fluid bypasses the entire structure group. The result is a strengthening of the tertiary, uni-modal, flow region that is clearly similar to that found downstream of a single isolated structure. While the first two developing flow regions remain visible in the $S/h = 0.6$ case (Figure 5.6), they have almost completely disappeared in the $S/h = 0.3$ case (Figure 5.7).

**Single Structures, $b/h = 2.0$, $b/h = 1.0$**

From Figures 5.8 and 5.9 the result of further reducing the effective width of the wake generating unit on the contours of mean velocity deficit in the wake is observed. The similarities with the far wake form of the closely spaced three cube structures (Figures 5.6 and 5.7) are apparent. While the shape of the far wake deficits are similar, the magnitudes of the deficits are clearly not similar. As the equal height wake generating structures become narrower to the flow, the extent of the mean velocity deficit in the far wake is reduced. This is expected with the reduced blockage presented by the smaller structures, as predicted by (5.1). Similarity of the flow in the far wake is a function of the characteristic dimensions of the structure; a topic that is discussed at length in the subsequent sections.

**Summary of Single Row Groupings**

Figures 5.3 to 5.9 indicate that the flow in the wake of bluff bodies follow a reasonably organized cascade of structure as the wakes recover momentum and coalesce downstream. The creation of secondary and tertiary flow fields allows substantial mean velocity deficit to exist much farther downstream than that of the isolated structures.

In the near wake region the mean velocity deficit is uni-modal and centred behind each structure. If the bodies are close enough together to interact ($S/h \leq 1.5$ is indicated by the experiments) then substantial secondary flow deficit regions will develop, centred between the initial structures. This reduction by one of deficit peaks will continue in the far wake until a single uni-modal deficit region exists. This fully developed wake then relaxes to background levels. The rate at which this final uni-modal wake structure
develops and decays is of considerable importance from a modelling standpoint and will be examined further.

**Double Row Groupings, $S/h = 0.5, 1.0$, (Five Cubes)**

The five cube groupings of Case 8 ($S/h = 0.5$) and Case 9 ($S/h = 1.0$) present an interesting variation of the far wake structures previously examined. Figures 5.11 and 5.12 show the contours of mean velocity deficit obtained in these cases.

The more closely spaced structure of Case 8 (Figure 5.11) causes a large percentage of the flow to divert around the entire group. For a solid structure this would result in a wide, uniform contour of mean velocity deficit. Some flow does move through the group, however, generating the apparent ‘wings’ of mean velocity deficit. This is shown schematically in Figure 5.13.

Increasing the characteristic spacing to $S/h = 1.0$ as in Case 9 (Figure 5.12) permits more flow to move through the structure and the wings apparent in Figure 5.11 do not appear at the 0.1 level shown.

Blocking the jets of flow between the initial three bodies forces the flow into a bi-modal deficit in the near wake of the trailing bodies. The structure of the wake then cascades to a uni-modal shape as found in the three cube cases previously discussed. The leading cubes dictate the extent of the wake and complicate the edges of the wake, but the trailing bodies clearly accelerate the development of uni-modal behaviour. This is illustrated by direct comparison of Figures 5.5 and 5.12 which both have a characteristic separation of $S/h = 1.0$.

In summary, the partial blocking of the jets between the leading bodies promotes wake coalescence within the strongest regions of mean velocity deficit, along the wake centreline.
Figure 5.11: Contours of longitudinal mean velocity deficit. Case 8, $S/h=0.5$, 5 cubes.
Figure 5.12: Contours of longitudinal mean velocity deficit, Case 9, S/h=1.0, 5 cubes
Figure 5.13: Schematic representation of flow regions around five structures
5.1.2 Describing Wake Coalescence using Normal Functions

The similarity based analytical wake theories discussed in Section 2.1.1 assumed that the momentum deficit expanded from a single point origin as a Gaussian type function. Although this may be a reasonable assumption for the single isolated structures for which the theory was developed, the deficit profiles in the near wake of multiple structures clearly cannot have originated from a single point. As the multiple wakes develop and merge together they appear to become uni-modal Gaussian in shape and, beyond that point, may reasonably be considered to have originated from a single point in space.

In this section we quantify the rate at which the merging wakes combine into a single Gaussian type profile in the lateral direction. To accomplish this task the data is modelled by tri-modal, bi-modal, and uni-modal normal curves. The ‘goodness of fit’ of each model is estimated. As the residual error of the standard uni-modal normal curve decreases, coalescence of the wakes is occurring.

The data is fit with the following functions:

uni-modal:

\[ f_1(y) = \frac{1}{\sigma_1 \sqrt{2\pi}} \exp\left( -\frac{1}{2} \left( \frac{y - m_1}{\sigma_1} \right)^2 \right) \]  \hspace{1cm} (5.3)

bi-modal:

\[ f_2(y) = \frac{1}{\sigma_2 \sqrt{2\pi}} \left[ \exp\left( -\frac{1}{2} \left( \frac{y - m_2}{\sigma_2} \right)^2 \right) + \exp\left( -\frac{1}{2} \left( \frac{y + m_2}{\sigma_2} \right)^2 \right) \right] \]  \hspace{1cm} (5.4)

tri-modal:

\[ f_3(y) = \frac{1}{\sigma_3 \sqrt{2\pi}} \left[ \exp\left( -\frac{1}{2} \left( \frac{y - m_3}{\sigma_3} \right)^2 \right) + \exp\left( -\frac{1}{2} \left( \frac{y + m_1}{\sigma_3} \right)^2 \right) + \exp\left( -\frac{1}{2} \left( \frac{y + m_2}{\sigma_3} \right)^2 \right) \right] \]  \hspace{1cm} (5.5)

where, \( m_1 \) is the mean of the uni-modal equation located at \( y = 0 \)
$m_2$ is the mean of the bi-modal equation located at the mid-point between the structures at $y = (S + h)/2$

$m_1$ is the mean of the tri-modal equation centred behind each structure at $y = (S + h)$

The standard deviations $\sigma_1$, $\sigma_2$, and $\sigma_3$ are estimated for each model as the weighted deviation from the respective means. The methods are similar to those described in Appendix D.

Each function is weighted by a scaling factor estimated by (5.6)

$$a_i = \frac{\sum u_k - u_b}{\int f(x)dy}$$

A normalized mean square error of the resulting function is then estimated to represent the goodness of fit. The function that best represents the shape of the profile will result in the lowest error such that $(1 - \text{MSE}) \rightarrow 1$.

**Single Isolated Structure**

For the single isolated structure of Case 1 a uni-modal normal curve represents the profile well until it has completely decayed. Figure 5.14a shows the success downstream of the uni-modal curve fit. Figure 5.14b gives some chosen examples of the goodness of fit exhibited by the model.

For mathematical purposes in this section only, symmetry is assumed at the centreline, and negative data points are assumed zero. This assumption reduces the random error associated with scatter of the data in the tails, but gives unreasonably well behaved results outside of the central region. The modified data in the tails are included only to accentuate the primary features of the profile and not to suggest perfection in the experimental results.
Figure 5.14a: Error of uni-modal normal fit to single isolated structure wake

Figure 5.14b: Three representative profiles for the single isolated structure
Multiple Structures

For multiple structures of close and intermediate spacing the contour plots indicate a cascade from tri-modal to bi-modal to uni-modal shape. This cascade will be quantified by examining the goodness of fit of the three normal models. Figure 5.15 is a schematic representation of the expected regions on an error plot of the models. The transition from tri-modal to bi-modal, and from bi-modal to uni-modal will vary with structure separation. As the separation distance is decreased, the rate of coalescence increases and the persistence of the tri-modal and bi-modal regions is diminished. For increasing separation, the developing region is extended, and the dominance of the uni-modal fit is delayed to higher $x/h$.

![Schematic of distinct flow regions described by normal fit equations](image)

**Figure 5.15:** Schematic of distinct flow regions described by normal fit equations

Figures 5.16a to 5.20a show the transitions of the three distinct flow regions as the separation distance increases from $S/h = 0.3$ to $S/h = 3.0$. Also shown with the figures are representative curve fits for each flow region. These results are given as Figures 5.16b to 5.20b.
Figure 5.16a: Error results of curve fitting for Case 3, $S/h = 0.3$

Figure 5.16b: Representative curve fit results for Case 3, $S/h = 0.3$
Figure 5.17a: Error results of curve fitting for Case 4, $S/h = 0.6$

Figure 5.17b: Representative curve fit results for Case 4, $S/h = 0.6$
Figure 5.18a: Error results of curve fitting for Case 5, $S/h = 1.0$

Figure 5.18b: Representative curve fit results for Case 5, $S/h = 1.0$
Figure 5.19a: Error results of curve fitting for Case 6, $S/h = 1.5$

Figure 5.19b: Representative curve fit results for Case 6, $S/h = 1.5$
Figure 5.20a: Error results of curve fitting for Case 7, $S/h = 3.0$

Figure 5.20b: Representative curve fit results for Case 7, $S/h = 3.0$
The results for Case 3, $S/h = 0.3$, Figure 5.16, confirm that the individual wakes of the three structures are already obscured as early as $x/h = 2$. A very brief period of bi-modal dominance is quickly followed by a persistent and well defined uni-modal wake. The developing region is confined to $x/h < 7.5$.

When the separation is increased to $S/h = 0.6$, Figure 5.17, the tri-modal function is clearly superior in the very near field. The bi-modal region is shown to extend as far downstream as $x/h = 5.0$. A well formed uni-modal wake clearly dominates soon after and persists much farther downstream than in the single structure case of Figure 5.14.

For the intermediately spaced case of $S/h = 1.0$, Figure 5.18, the secondary flow region of bi-modal form persists in a dominant manner beyond $x/h = 10$. Clear coalescence to uni-modal form is delayed until $x/h > 15$ for this case.

The intermediately spaced case of $S/h = 1.5$, Figure 5.19, shows the most obvious resemblance to the schematic of Figure 5.15. The secondary bi-modal region is well illustrated by this method and is shown to persist as far as $x/h = 15$. Coalesced uni-modal form is limited to the very far wake of $x/h > 20$.

The final case examined is the largely spaced Case 7, $S/h = 3.0$, Figure 5.20. The contour plots of the previous section indicate that each structure generates a velocity deficit which decays independently. Even at this large spacing, however, a weak secondary flow is present in the far wake. This secondary flow region is slow to appear and a region of nearly zero mean velocity deficit is found between $6 < x/h < 8$. This explains the surprising success of the uni-modal model in this region. Farther downstream, $10 < x/h < 15$, the bi-modal profiles are apparent, but very weak. These secondary flows are real and are clearly not present downstream of the single isolated structure of Case 1. We recognize that some interaction of the mean wakes is present, even at this large separation distance. No significant uni-modal wake appears downstream of this case, as shown by the representative curve fits of Figure 5.20b.
The results of this section confirm the coalescence to uni-modal behaviour for the mean velocity deficit of multiple structures where $S/h \leq 1.5$.

5.1.3 Universal Decay of Mean Velocity Deficit

Consider the additional requirement of the theory that the profiles of mean velocity deficit behave in a self preserving manner. The theory of Hunt (1970) suggests a decay rate of -1.5 for the downstream variation of the maximum mean velocity deficit. Knowledge of this decay rate is essential to plotting self similar profiles in the wake.

If the hypothesis is true that multiple structure wakes eventually forget their origin and behave as single structure wakes, then a common decay rate for mean velocity deficit should be expected beyond the radius of homogenization. Furthermore, this decay rate should equal the single structure decay rate of -1.5 as given in the literature.

In Section 5.1.1 it was shown that the momentum deficit created by the multiple structure groups is substantially greater than that of the single isolated structure. As a result, the combined deficit is much more tenacious downstream, persisting long after the SIS deficit has recovered to background levels. Clearly profiles of mean velocity deficit for all multiple structure cases will not be similar at identical absolute downstream locations $x$, but instead must be scaled by a characteristic scaling factor in order to display similar behaviour.

We suppose that the appropriate scaling factor must somehow characterize the important qualities of the multiple structure groupings and reduce them to a common descriptive parameter. We let $\xi$ represent such a parameter and initially suspect that it will be a function of the physical structure dimensions, $h$, $b$, and $d$, the typical spacing, $S$, and the number of structures, $N$. As well, the background flow parameters, $L$, $n$, $z_0$, $\delta$, and the shape of the individual structures, $\psi$. We write the functional dependence of $\xi$ as in (5.8).

$$\xi = f(h,b,d,S,N,L,n,z_0,\delta,\psi)$$ (5.8)
The design of the experiments permits the suspected functional form of $\xi$ to be substantially simplified from (5.8). By keeping a number of variables in (5.8) constant in the experiments it is possible to isolate the effects of a few key parameters. Generalizing the observed effects of these key parameters beyond the fixed values of the remaining variables is a bold but necessary step, to be performed with confidence, but not without caution.

For the present experiments the key parameters remaining in (5.8) are the number of structures, $N$, and the representative spacing of the structures, $S$. Mounla (1995) demonstrated that the first upwind row of structures in an array will cause the bulk of the drag on the wind. The first row may then be considered to dictate the lateral scaling of the mean velocity deficit created in the wake. For this reason we take $N$ to be the number of structures located on the front row, normal to the approach flow. We hypothesize the following relation for the characteristic scaling parameter $\xi$,

$$\xi = (N)b + P(N - 1)S$$ (5.9)

where $P$ is a ‘leakage factor’, after Halitsky (1975), representative of the momentum leakage between the structures. For all cases except Case 1 and Case 2, (5.9) may be written as

$$\xi = (3)h + P(2)S$$ (5.10)

Downstream distance $x$ is non-dimensionalized by $\xi$, and the mean velocity deficit at $y/h = 0.0$, $z/h = 1.0$ is plotted against $x/\xi$ for all cases. $P$ from (5.10) is determined to provide the best fit to a common decay curve. It was determined that the value $P = 1.0$ provided the most complete collapse of the data for the cases studied. Using the value of 1.0 for $P$ in (5.10), $\xi$ becomes simply the overall width, $W$, of the wake generating structure. Thus the characteristic scaling parameter becomes, simply

$$\xi = W$$ (5.11)

where, $W = h$, for Case 1, single isolated cube

$W = 2h$ for Case 2, single isolated rectangular prism
\[ W = 3h + 2S \quad \text{for Cases 3 - 9, multiple structure groupings (Table 4.3)} \]

The appropriateness of \( \xi \) as the overall width of the structure group is indicated in the horizontal contour plots of Section 5.1.1. All of the plots demonstrate significant interaction of the body wakes for \( S/h < 1.5 \). These combined wakes clearly scale laterally with the overall width of the structure group. We reasonably expect that the overall width of the wake generating group is an important scaling parameter with respect to the interacting wakes.

The mean velocity deficit is plotted against \( x/W \) for all cases except 6 and 7 in Figure 5.21 (the test section is not long enough to observe the universal decay for these cases). After an initial development region, \( x/W < 3 \), the decay of the mean velocity deficit, when scaled by the overall width of the wake generating structure, collapses to a single decay curve for the many varied cases shown in Figure 5.21. The solid line on Figure 5.21 shows the decay rate of -1.5 predicted by the single structure theory of Hunt (1970).

**Observations on the Universal Decay of Mean Velocity Deficit**

Figure 5.21 shows that the decay of mean velocity deficit is described well by a power law decay rate of -1.5 beyond \( x/W = 3 \), for the closely spaced structure cases studied. Prior to the downstream point \( x/W = 3 \), the multiple structure wakes are combining together and coalescing as described in Section 5.1.1. In this developing wake region the centreline mean velocity deficit is augmented by the adjacent flow deficits resulting in a substantial delay of the recovery of the background mean velocity. If the deficit profiles of the individual structures decay significantly before they have grown laterally enough to combine (as in the large \( S/h \) of Case 7) then no common decay region will be observed.

Examining the SIS case in Figure 5.21 \((b/h=1.0)\) shows that the background mean velocity is effectively recovered by \( x/h = 10 \). As \( S \) increases beyond \( 1.0h \) then \( x/W = 3 \) equates to \( x/h = 15 \) and it seems unlikely that the wakes will coalesce before substantially
Figure 5.21: Universal decay of mean velocity deficit for multiple structures
decaying. This is confirmed by the data. For the three cube cases of \( S/h < 1.0 \) (Cases 3 and 4, Table 4.3) good collapse to universal behaviour is shown to exist with respect to centreline mean velocity deficit.

For the case of \( S/h = 1.0 \) (Case 5, Table 4.3) the results indicate a very limited region of universal behaviour within the confines of the wind tunnel test section. \( S/h = 1.0 \) appears to be near the limit of applicability for universal behaviour in this configuration. Case 6, \( S/h = 1.5 \), shows coalescing qualities far downstream as demonstrated in the previous section. However, the developing region extents throughout the entire test section and no significant region of universal decay may be observed. For Case 7, \( S/h = 3.0 \), the individual mean velocity deficits completely decay without ever significantly interacting downstream.

The five cube groupings of Case 8 and Case 9 (Table 4.3) also collapse well on the universal decay curve of Figure 5.21. It appears that the added drag of the trailing cubes enhance the mean velocity deficit, accelerating the development region and permitting fully coalesced universal wake behaviour to be observed more rapidly than equally spaced three cube arrangements. Figure 5.21 demonstrates that despite the addition of the leeward cubes, the overall width of the structure group, \( W \), remains an appropriate scaling factor for this arrangement as well.

The rectangular prism of Case 2, as shown in Figure 5.21 (\( h/h = 2.0 \)), clearly demonstrates the appropriateness of \( W \) as the relevant scaling parameter for the decay of the mean velocity deficit for this case.

**Quantifying the Extent of the Development Region**

With the clear emergence of \( W \) as the relevant scaling parameter it is possible to quantify the development region for the in-line arrangements, as described in Section 5.1.2. Figure 5.21 indicates that the development region extends to approximately \( x/W = 3 \), however, the precise extent of the development region is related to the separation
distance. Using polynomials to describe the rate of coalescence to uni-modal normal behaviour in Figures 5.16a to 5.19a, we quantify this development as a function of separation. Table 5.1 shows the extent of the development region estimated at the 99% level.

**Table 5.1: Estimated extent of the development region, Cases 3-6**

<table>
<thead>
<tr>
<th>$S/h$</th>
<th>$x/h$</th>
<th>$x/W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>7.26</td>
<td>2.02</td>
</tr>
<tr>
<td>0.6</td>
<td>15.27</td>
<td>3.64</td>
</tr>
<tr>
<td>1.0</td>
<td>16.58</td>
<td>3.32</td>
</tr>
<tr>
<td>1.5</td>
<td>27.73</td>
<td>4.62</td>
</tr>
</tbody>
</table>

The results presented in Table 5.1 are based on a least squares fit of the uni-modal curves of Figures 5.16a to 5.19a. Inspection of these curves along with the quantitative estimates of Table 5.1 indicate that the flows are fully developed as early as $x/W = 2$. If the bodies are near enough to interact then they are almost certainly fully developed by $x/W = 5$. The functional relationship between separation distance and the extent of the development region is quantified in Section 5.2.

**5.1.4 Application of the Theory of Hunt**

The normal function fits of the lateral profiles of mean velocity deficit (Section 5.1.2) demonstrated how the multiple structure wakes coalesced into single, Gaussian type wake structures. From this observation we expect that a similarity solution based on a point source diffusion of momentum, such as that of Hunt (1970) (Section 2.1.1), may adequately describe the profiles of mean velocity in the far wake. In order for the single structure theory of Hunt to be generalized to multiple structures, however, it is necessary
to consider the relevant scaling parameters that will allow the various sized wakes to collapse onto similar curves.

Section 5.1.3 demonstrated that the maximum mean velocity deficit in the wake collapses for closely spaced structures according to the overall width, $W$, of the wake generating structure. Using the scaling parameter $W$, vertical and lateral profiles of mean velocity deficit were obtained downstream of each test case at $x/W = 5$. This value was chosen to be sufficiently far downstream for the similarity solution to apply (Lemberg, 1973), and not so far as to be beyond the test section for the wider test cases ($S/h \leq 1.0$).

In order to effectively apply the theory of Hunt the essential similarity equation (2.5) is modified to account for the universal scaling parameter $W$, as in (5.12)

$$\frac{u_{per}}{U_h} = \frac{K_z F_z \left( \frac{\bar{z}}{z} \right)}{\left[ \left( x - a \right)/W \right]^{(3+\eta)/(2+\eta)}}$$

(5.12)

This change permits the combined wakes of the multiple structures to be described by the same similarity functions as the single isolated structure.

The free parameters of Table 5.2 are necessary to evaluate equation (5.12), using the expressions given by (A2) through (A11).

**Table 5.2: Values used in theoretical estimate**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Equation</th>
<th>Value</th>
<th>Hunt (1970)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_D$</td>
<td>(A11)</td>
<td>1.6</td>
<td>1.6</td>
</tr>
<tr>
<td>$a$</td>
<td>(5.12) / (2.5)</td>
<td>-0.56W / -0.56h</td>
<td>-0.56h</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>(A6)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>(A6)</td>
<td>2.5</td>
<td>1.35</td>
</tr>
</tbody>
</table>
Figure 5.22 shows the vertical profiles of mean velocity deficit plotted on axes suggested by the theory. The many different wake generating structures studied show remarkably similar profiles when plotted in this way. The solid line through the data demonstrates that the theory of Hunt (1970) adequately predicts the vertical mean velocity deficit profile for multiple structures as well as for single isolated structures, when the appropriate scaling measures are employed.

Figure 5.23 shows the lateral profiles of mean velocity deficit for the various wake generating structures. Again, the data is plotted with axes suggested by the theory. Although not as tight as the collapse of the vertical profile data, the horizontal profiles demonstrate very similar behaviour when appropriately scaled. The theoretical prediction for the horizontal profile is shown to provide a good prediction of the profile shape.

As with the universal decay of mean velocity deficit discussed in 5.1.3, the collapse of the data is limited to the closely spaced structures of Cases 1 through 5, and Cases 8 and 9. The case of $S/h = 1.5$, Cases 6, does not develop sufficiently within the test section ($x/W = 5.0$ occurs outside of the test section for this case), and as a result, cannot be plotted in the same manner. The case of $S/h = 3.0$, Case 7, does not significantly interact before decaying to background levels and so cannot be described by the same scaling parameter $W$.

The constant value of $C_D$ required by the theory appears to be valid for all cases beyond the development region. The fully developed region will not form until all lateral pressure variations in the flow become negligible. As the spacing between structures increases, lateral variations are strengthened and persist farther downstream. Consider a second row of structures placed in the wake such as the 5 cube arrangements of Cases 8 and 9. These structures serve to homogenize the wake with respect to momentum deficit and lateral pressure variations. The extent of the development region is reduced for these cases. This homogenization of the wake is taken to the extreme for the solid body isolated structures of Cases 1 and 2. For these cases the longitudinal extent of the development region is the shortest of all.
Figure 5.22: Vertical profiles of non-dimensional mean velocity deficit
Figure 5.23: Lateral profiles of non-dimensional mean velocity deficit
Caveat on the Theory

In applying the theory of Hunt (1970) every attempt was made to use the values of constants from the original work. Engineering predictions based on theory are not entirely practical if the equations are full of 'best fit' parameters which require data in order to estimate.

Table 5.2 indicates that the value of \( \gamma \) used in (2.2) was modified from Hunt's value of 1.35 to a new value of 2.5. The parameter was selected in both cases to provide best agreement between theory and experiment. Hunt (1970) claims that the experimental work presented in Hunt and Smith (1969) is not reliable with respect to mean velocity measurements. As a result, differences in the value of the experimental fit parameter \( \gamma \) are expected.

5.2 EFFECT OF STRUCTURE DIMENSIONS ON THE MEAN FLOW

As a result of the intriguing universal wake behaviour of the multiple structure groups, as demonstrated in Section 5.1.3, subsequent experiments were performed to further examine this phenomena. It was desired to examine the effect of individual structure dimensions on the generality of the far wake by varying the models from the basic cubic shape of the primary experiments.

5.2.1 Additional Structure Dimensions Considered

The centreline decay of excess mean velocity deficit was examined at the model height, \( h \), as a function of clear separation for a single row of three identical structures. Three characteristic dimensions were examined: a) wide, low structures, \( h/b = 0.5 \), b) medium height structures, \( h/b = 1.5 \), and c) tall structures, \( h/b = 2.0 \). All of these cases allow for critical experimental parameters considered in Chapter 4 (\( h/\delta \), \( h/z_o \), etc.) to remain within limits necessary to model low rise structures in a deep, open country boundary layer.
5.2.2 Using Structure Width as a Scaling Parameter

Changing the characteristic dimensions of a single isolated structure will alter the drag characteristics and result in a change in magnitude of the mean velocity deficit in the wake at a given downstream distance. Lemberg (1973) demonstrate: that while the magnitude of the mean velocity deficit varies for the various shapes considered in his study, the decay rate of the resulting mean velocity deficit is reasonably constant.

The results presented in Section 5.1 suggest that it is the width of the isolated structure that dictates the magnitude of the mean velocity deficit at the structure height, \( h \). Thus, when downstream distance is normalized by the structure width, \( b \), the decay curves for the resulting mean velocity deficit collapse to a single curve. Data from the additional experiments described in this section allow for further examination of \( b \) as a normalizing scalar for the decay of the mean velocity deficit in the wake of single isolated structures.

Considering data for the five isolated structure shapes available, Figure 5.24 demonstrates the appropriateness of normalizing by \( b \) (\( W = b \) for a single isolated structure). Allowing for the large variation in structure dimensions considered, the collapse of the data as presented in Figure 5.24 is quite good. Significant scatter is limited to that region beyond which the mean velocity deficit has recovered to within 10% of the background velocities. In this region of weak mean velocity deficit the complete recovery of background velocity is delayed as the distance from the ground is increased (i.e. for the taller structures). This trend is apparent in the far wakes as indicated in Figure 5.24.

Along with the data in Figure 5.24 is presented the -1.5 decay rate predicted by the theory of Hunt (1970), as discussed in Section 5.1. This fit is shown to adequately describe the decay rate of all five geometries studied, when normalized by the body width. Also shown in the Figure 5.24 is the -1.0 decay rate predicted for two dimensional structures. The data suggests that this rate may become more appropriate as the object aspect ratio \( (b/h) \) continues to decrease beyond the 0.5 limit examined in the experiments.
Figure 5.24: Decay of mean velocity deficit for all single isolated structures
A final comment of Figure 5.24 is added with respect to the repeatability of results presented in this thesis. The decay curve shown in the figure was developed using the same constant of proportionality (1.5) as that presented in Figure 5.21, as well as subsequent Figures 5.25 to 5.27. The experiments described in this section were obtained using a single wire HWA (described in Chapter 3), in February, 1996 and provided data for three of the five cases plotted in Figure 5.24. The remaining two cases plotted in Figure 5.24 were tested using cross wire HWA probes in August, 1994. Careful experimental set up and measurement techniques has led to repeatable results that may be reviewed with confidence.

5.2.3 Single Row of Wide, Low Structures

A single row of three wide, low structures of width \( h/h = 1.5 \), depth \( d/h = 1.0 \), and height \( h = 0.05 \text{m} \) was examined for clear separations of \( S/h = 0.3, 0.6, 0.8, 1.0, 1.5, 3.0 \), and \( \infty \). The results of the excess mean velocity deficit measured at the structure height, \( h \), are given in Figure 5.25. The figure shows the same characteristic effect of separation exists for the wider structures, as was previously demonstrated for the cubic structures.

For intermediate separation distance (eg. \( S/h = 0.8 \), Figure 5.25) there is an initial period where the mean velocity deficit behaves as if the structure was isolated. In this region it is appropriate to consider the mean velocity deficit to decay as \((x/h)^{1.5}\). Beyond this there exists a region of coalescence in which the secondary flows caused by the presence of the peripheral structures stabilize the mean velocity deficit. In this region it appears that a nearly constant mean velocity deficit may be appropriate along the centreline. After wake coalescence is complete there exists a region of common decay. In this final region the data indicates that the mean velocity deficit once again decays at a -1.5 power rate. A universal decay is observed if we normalize downstream distance \( x \) by \( W \), where for the wide, low structures examined in this section we define \( W \) as

\[
W = 3h + 2S
\]

As with the cubic structures examined in Section 5.1, significant universal behaviour was not observed within the test region for normalized separation distances greater than unity.
Figure 5.25: Universal decay of mean velocity deficit for 3 structures, $b/h = 1.5$
5.2.4 Single Row of Tall, Narrow Structures

The effect of structure separation on the decay of mean velocity deficit in the wake of a single row of tall, narrow structures was examined for two geometries, $h/b = 1.5$, and $h/b = 2.0$. For both cases $b/d = 1.0$, and the width $b = 0.05m$. The clear separation distances examined were the same absolute distances as for the lower structures. As a result, $b$ is used to normalize the distance $S$ for the taller structures.

Figures 5.26 and 5.27 show the results of the mean velocity deficit measurements in the wake of the medium height, $h/b = 1.5$, and the tall structures, $h/b = 2.0$, respectively. The same characteristic regions of wake development, as a function of increasing separation distance, are observed as are for the lower structures.

Once again, the multiple structure wakes appear to combine and then decay as a single unit at an apparent -1.5 power rate. The difference here is that in order to observe the universal decay as a function of the single tall structure width, we must define the normalizing variable $W$ as

$$W = 3h + 2S$$

where $W$ no longer represents simply the overall width of the wake generating group.

As the structures become taller the development region of approximately constant mean velocity deficit appears to persist farther downstream than for the lower structures. This is most likely due to the increasingly two dimensional nature of the flow which encourages stronger secondary flow regions to develop in the wake. As a result, the momentum transfer required to coalesce the bi-modal secondary wakes to uni-modal form is not so well aided by flow over the top of the structures as it is for the lower structure cases. The structures are also extending up into higher velocity regions of the sheared approach flow. More persistent regions of pressure variation and a reduced effect of ground shear combine to delay the development of the constant drag region.
Figure 5.26: Universal decay of mean velocity deficit for 3 structures, $b/h = 0.67$
Figure 5.27: Universal decay of mean velocity deficit for 3 structures, $b/h = 0.5$
It is not surprising that as we change the height of the structures we change the nature of the three dimensional flow. The relevance of the overall width as the critical scaling factor is well established for the cases where the structures are no taller than they are wide. As the structure rises into the boundary layer, its height must be accounted for when choosing the scaling parameter. At the same time we recall that the measurements are taken at the structure height, \( h \). Measurements taken at the structure height will become less and less representative of the wake as the aspect ratio of the structure increases.

### 5.2.5 Describing the Decay of the Mean Velocity Deficit

Observations of the decay of the mean velocity deficit as presented in Sections 5.1 and 5.2 allows us to describe an empirical model in the wake of multiple structures of characteristic clear separation distance, \( S \). Such a model may be used to predict the downstream recovery of mean velocity along the lateral centreline and at the characteristic structure height, \( h \).

When plotted on log-log axes the mean velocity deficit demonstrates three distinct regions of development, as shown in Figure 5.28.

Region I is a near wake region where the recovery of the mean velocity deficit is unaffected by the peripheral structures. In this region the decay of the mean velocity deficit is dictated by the dimensions of the individual structures. The data suggests that the mean velocity deficit may be estimated by (5.14)

\[
- \left( \frac{u_w - u_b}{u_b} \right) = 1.5 \left( \frac{x}{b} + 0.5 \right)^{-1.5}
\]

(5.14)

where \( x \) is measured from the leading edge of the structure, accounting for the factor of 0.5 in (5.14). Region I will persist until the effects of the peripheral structures are experienced along the centreline. This occurs a constant distance downstream for a given value of the separation distance, \( S \), and individual structure width, \( b \), and is given as
Figure 5.28: Schematic representation of decay regions for mean velocity deficit
\[
\left( \frac{x}{b} \right) > f(S, b)
\]

where the data suggests a linear function given as

\[
f(S, b) = A + B \left( \frac{S}{b} \right)
\]

(5.15)

The constants \( A \) and \( B \) are determined by regression analysis for the cases considered. The results are summarized in Table 5.3.

Table 5.3: Constants for equation 5.15

<table>
<thead>
<tr>
<th>( h/h )</th>
<th>( A )</th>
<th>( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.46</td>
<td>3.88</td>
</tr>
<tr>
<td>1.0</td>
<td>0.85</td>
<td>2.28</td>
</tr>
<tr>
<td>0.67</td>
<td>1.20</td>
<td>2.55</td>
</tr>
<tr>
<td>0.5</td>
<td>1.27</td>
<td>2.18</td>
</tr>
</tbody>
</table>

Region I will only exist for single row structure groups and will not develop if downstream structures are present in the wake.

Region II is a developing wake region where the secondary flows described in Section 5.1 produce a bi-modal wake and the centreline mean velocity deficit remains nearly constant. For this region we estimate

\[
- \frac{(u_w - u_b)}{u_b} = 1.5(f(S, b) + 0.5)^{-1.5}
\]

(5.16)

Region II will persist until the wakes have fully coalesced to uni-modal form, marking the beginning of Region III.
Region III is a region of universal decay where the individual wakes of the structures have fully coalesced to uni-modal form. In this region we use (5.17) to estimate the mean velocity deficit with \( W \) defined as (5.18)

\[
-\frac{(u_w - u_h)}{u_h} = 1.5 \left( \frac{x}{W} + 0.5 \right)^{1.5}
\]

\[ W = (N) L + (N - 1) S \]

where, \( N \) is the number of structures in the windward row

\( L \) is the larger of the individual structure windward dimensions \( b \), or \( h \)

\( S \) is the characteristic clear separation distance between the structures

As with Region I, \( x \) is measured from the leading edge of the primary structure. The onset of Region III is also a function of \( S \) and \( b \), and appears to be given by

\[
\left( \frac{x}{W} \right) > f(S,b)
\]

where \( f(S,b) \) is again given by (5.15). Region III will exist until the mean velocity in the wake has fully recovered to background levels, generally beyond \( x/W = 15 \).

The model described in this section may be applied for clear separation distances no larger than the lesser of \( b \), or \( h \). The results of Section 5.1 indicate that this limit may be increased to 1.5 times \( b \) or \( h \). However, the regression analysis of this section does not include this separation due to the limited extent of the test section. For normalized separation distances greater than 1.5, Region III is not expected to develop within the longitudinal extent of the mean velocity wake.

An example of the fit provided by the model is presented for the case of the low wide structures, given as Figure 5.29. The linear relationship between \( S/h \) and the onset of Region II appears justified by the fit.
Figure 5.29: Model fit to centreline decay of mean velocity deficit for $b/h = 1.5$
5.3 TURBULENT VELOCITY FLOW FIELD

While description of the mean velocity flow field is necessary to predict the mean advection of contaminants in a wake, description of the fluctuating or turbulent flow field is necessary to predict the turbulent spread of contaminants about the centre of mass.

In the wake of a structure there exists a strong increase in turbulence intensity above background levels. The flow separation at the sharp edges of the body generates turbulent motions in the near wake that scale with the characteristic dimension of the wake generating structure (Musselman and Slawson, 1995). As the characteristic dimensions of the wake grow downstream the excess turbulent energy generated at the structure is moderated by entrained ambient fluid. With the source of the turbulent energy now far removed from the local flow the intensity decays, eventually returning to background levels downstream.

Castro (1973) showed that for a single isolated structure velocity fluctuations in the wake are greatest along the shear layers at the wake boundaries. Also, excess intensity in the wake persists in the far wake long after the mean velocity field has recovered. This tenacity of the turbulent energy in the far wake encourages many researchers to concentrate only on the turbulent flow field when considering the dispersion of contaminants in the wake.

Considering the decay rate of longitudinal rms velocity above background reference levels in the wake of the single cube, Figure 5.30 shows that the data agrees well with the power law decay exponent of -0.24 reported by Lemberg (1973).

5.3.1 Contours of Equal Longitudinal Turbulence Intensity

In a manner similar to that presented for mean velocity deficit (Section 5.1.1), contours of equal longitudinal turbulence intensity are constructed to demonstrate the effect of multiple structures on the turbulent flow field. The contours presented are equal
value excess turbulence intensity, as given by (5.13)

\[ i_v = \frac{\left( \frac{\bar{u}}{u} \right)_w - \left( \frac{\bar{u}}{u} \right)_h}{\left( \frac{\bar{u}}{u} \right)_h} \]  

(5.13)

Figures 5.31 to 5.35 show the effect of decreasing the spacing between three cubes from $S/h = 3.0$ to 0.3. The overall width of the wake generating structures is further reduced to $W = 2.0h$ (Case 2) in Figure 5.36, and to $W = h$ (Case 1) in Figure 5.37.
Figure 5.31: Contours of excess longitudinal turbulence intensity, Case 7, S/h=3.0
Figure 5.32: Contours of excess longitudinal turbulence intensity, Case 6, S/h=1.5
Figure 5.34: Contours of excess longitudinal turbulence intensity, Case 4, $S/h=0.6$. 
Figure 5.35: Contours of excess longitudinal turbulence intensity, Case 3, S/h=0.3
Figure 5.36: Contours of excess longitudinal turbulence intensity, Case 2, b/h=2.0
Figure 5.37: Contours of excess longitudinal turbulence intensity, Case 1, S1S, b/h=1.0
Plotting contours of local turbulence intensity, $\bar{u}/u$, instead of absolute turbulence intensity, $\bar{u}/U_{*}$, incorporates the previously discussed effects of reduced mean velocity deficit in the combined wakes. By contrasting these plots to the contours of equal mean velocity deficit we can see those regions where the turbulent velocity dominates the mean velocity effects. This balance is important when considering the wake effects on the dispersion of contaminants.

**Large Spacing, $S/h = 3.0$**

It is apparent that the contours of excess longitudinal turbulence intensity given as Figures 5.31 to 5.37 are very similar in appearance to those given for the mean velocity deficit, Figures 5.3 to 5.9. Perhaps the most graphical difference is demonstrated by the large space case, $S/h = 3.0$, given as Figure 5.31.

From Figure 5.31 it is observed that significant excess turbulence intensity is generated along the junction of the individual wake boundaries. Only when the secondary flow regions discussed in Section 5.1.1 join with the combining turbulent velocities between the structures, do such significant excesses of intensity exist. Figure 5.31 illustrates why care must be taken when using only centreline measurements to determine whether or not a structure is truly isolated from its neighbours.

**Intermediate Spacing, $S/h = 1.5, 1.0$**

From Figures 5.32 and 5.33 the downstream variation of excess longitudinal turbulence intensity is observed for structures of intermediate spacing. The combining turbulent energy along the wake junctions is again apparent by the development of bi-modal peaks in the intermediate wake, centred between the structures.

Farther downstream the turbulent energy redistributes itself more uniformly across the entire wake with the maximum intensity existing along the centreline of the wake generating group. The lateral profile of longitudinal turbulence intensity becomes unimodal in shape at this point and appears much like the contour of mean velocity deficit.
(Figure 5.4 and 5.5). For these intermediately spaced structures the effect of reduced mean velocity in the wake is dominating the contours of local turbulence intensity.

**Closely Spaced, \( S/h = 0.6, 0.3 \)**

As the spacing between the individual cubes is reduced to \( S/h = 0.6 \) (Figure 5.34) and further to \( S/h = 0.3 \) (Figure 5.35) the contours of longitudinal turbulence intensity show a more rapid coalescence to a uni-modal wake form. For \( S/h = 0.6 \) the effect of flow through the gaps is not apparent beyond \( x/h = 10 \). For \( S/h = 0.3 \) the wake is clearly uni-modal as early as \( x/h = 5 \).

For these cases the decreasing significance of the reduced mean velocity in the wake is noticeable as the outer contours become more ragged and less pointed downstream. When significant excess turbulent velocity exists beyond the region of substantial mean velocity deficit, the outer contours occupy the wake more uniformly. As more and more flow is diverted around the structure group and the overall width is reduced, the intensity contour begins to redistribute itself to the outer shear layers.

**Single Isolated Structures, \( b/h = 2.0, b/h = 1.0 \)**

The effect of further reducing the overall width of the wake generating structures to \( W = 2.0h \) and to \( W = h \) is shown in Figures 5.36 and 5.37 respectively. The persistence of the high intensity wake is reduced for the smaller wake generating structures. For these cases the effect of reduced mean velocity in the wake is less important. With the flow blockage through the structure complete, the characteristic high intensities along the outer shear layers become more prevalent. For the case of the cube, Figure 5.37, the lowest contour has been reduced to 10% of background to illustrate this feature more clearly.

**Summary of Single Row Groupings**

The contours of equal excess longitudinal turbulence intensity show a similar cascade from tri-modal to bi-modal to uni-modal form downstream of single row three structure groups as do the contours of equal mean velocity deficit. The excess of
turbulent velocity along the shear layers at the outer limits of the individual wakes allow for significant interaction between structures separated by as much as $3.0h$. As the structures move closer together the lateral profiles of turbulence intensity more quickly merge together to form a uni-modal profile, consistent with that downstream of a single isolated structure.

The persistence downstream of the excess longitudinal turbulence intensity increases as the overall width of the wake generating group increases. This trend continues so long as the turbulent fluctuations generated by the structures maintain substantial energy to the point where they merge downstream. For separations greater than $3.0h$, the excess turbulent fluctuations in the wake decay before energy from all three structures can combine into a single wake.

Comparisons with the contours of mean velocity deficit indicate that the effect of enhanced turbulence intensity in the wake dominates over the effect of reduced mean velocity for the single isolated structure wakes. For intermediately spaced structures, however, the effect of reduced mean velocity may be equally as important as enhanced turbulence intensity in the combined wake. This has important implications with respect to the dispersion of contaminants in the wake of multiple structures.

**Double Row Groupings, $S/h = 0.5, 1.0$, Five Structures**

Contours of equal excess longitudinal turbulence intensity are given for the five structure arrangements with $S/h = 0.5$ and 1.0, as Figures 5.38 and 5.39, respectively.

Partially blocking the flow through the gaps of the lead row results in stronger shear layers developing outside of the entire structure group. For the closely spaced grouping of Case 8, Figure 5.38, these outer shear layers are strong enough to form wings of high turbulence intensity, characteristic of an isolated structure. The high intensities generated along the centreline are very well mixed and demonstrate very little memory of the details of their origin.
Figure 5.38: Contours of excess longitudinal turbulence intensity, Case 8, S/h=0.5, 5 cubes
Figure 5.39: Contours of excess longitudinal turbulence intensity, Case 9, S/h=1.0, 5 cubes
Increasing the separation of the double row grouping to \( S/h = 1.0 \), Figure 5.39, weakens, but does not eliminate, the wings of high intensity present at the outer edges of the overall wake. The interior of the near wake shows attributes of its multi-structure origin but this memory quickly evaporates as the high intensities merge to form a single characteristic wake. In the far wake the contours look similar to those formed downstream of the single row grouping of comparable spacing (Figure 5.33). The most significant effect of the second row is to accelerate the coalescence to uni-modal wake behaviour and to strengthen the shear layers on the outer edge of the combined wake.

5.3.2 Decay of Turbulent Velocities

The decay of excess turbulent velocities is examined for all three component directions, downstream along the lateral centreline and at the structure height, \( z = h \). While the contour plots of the previous section demonstrated the cross sectional form of the wakes for the longitudinal component of turbulence intensity, centreline values are used to describe the differences between the three components of turbulent velocity. For all but the largest separation examined, centreline values far downstream are characteristic of the overall wake.

Longitudinal Component of Turbulent Velocity, \( \bar{u} \)

Figure 5.40a shows the downstream variation of excess turbulent \( rms \) velocity, above background levels, for the longitudinal component. Plotted in this manner the SIS data for the single cube of Case 1 has an estimated power law decay rate of -1.78 (based on a least squares fit), as shown in the Figure. Peterka and Cermak (1975) estimated a decay rate of -1.8 for their experiments, as reported by Huber and Snyder (1982). Martinuzzi (1996) demonstrates through dimensional considerations that a -1.75 power law decay rate may be expected when a -1.5 rate is assumed for the decay of the mean velocity deficit (see Appendix F). Knowing the rate at which the excess turbulent velocity decays in the wake of the single isolated structure, the effect of the peripheral structures
Figure 5.40a: Centreline decay of longitudinal turbulent velocity above background levels

Figure 5.40b: Centreline decay of longitudinal turbulent velocity above SIS levels
can most readily be seen by subtracting this rate out. The result of this calculation is presented as Figure 5.40b.

As the contour plots of Section 5.2.1 have shown, the longitudinal component of turbulent velocity is affected by the peripheral structures much as the mean velocity deficit is. Figure 5.40 shows that as the separation is increased from \( S/h = 0.3 \) to \( S/h = 3.0 \) the appearance of enhanced longitudinal fluctuations along the centreline is further delayed. Once present, there is a period of growth during which time the wakes are coalescing. This growth of excess turbulent velocities above single structure levels appears to be greater as the overall width (ie. the separation) is increased. In fact, the data suggests that this trend would continue even for the greatest separation case, \( S/h = 3.0 \), if the field of study was extended downstream.

**Lateral Component of Turbulent Velocity, \( \bar{v} \)**

The proximity of peripheral structures clearly affects the decay of the lateral component of turbulent velocity as illustrated by Figure 5.41.

Consider first the decay of excess lateral velocity fluctuations above background levels as shown in Figure 5.41a. Like the longitudinal component, the lateral component of turbulent velocity displays a well behaved decay in the wake of the single isolated structure of Case 1. A decay rate of -0.25 provides the best power law fit to the SIS data for this component. Figure 5.41b shows how the measured lateral turbulent velocities for the separations studied compare to this SIS decay rate.

The lateral component of turbulent velocity appear from the data to be most seriously affected by the relative blockage of the structure group. The most closely spaced group considered, \( S/h = 0.3 \), has a large percentage of the flow diverted around it. The onset of significant excess lateral turbulent velocities is substantially delayed beyond the much narrower SIS case; a consequence of the centreline measurement location. When the fluctuations for the closely spaced group do peak along the centreline, however, they are stronger than any other three structure group studied. This is important because
Figure 5.41a: Centreline decay of lateral turbulent velocity above background levels

Figure 5.41b: Centreline decay of lateral turbulent velocity above SIS levels
it suggests that increased flow leakage through larger and larger gaps results in less intense lateral fluctuations in the far wake. While the presence of peripheral structures does contribute to an increase above SIS levels, the maximum does not positively correlate to the overall width of the wake generating group as the mean velocities and longitudinal turbulent velocities appear to. In fact, the opposite is true and the enhancement of turbulent velocity is diminished with increasing structure separation.

**Vertical Component of Turbulent Velocity, \( \bar{w} \)**

Excess fluctuating turbulent velocities in the vertical component direction are shown in Figure 5.42. Once again, the decay of the single isolated structure is reasonably well described by a power law decay curve, with the best fit for the vertical component provided by a -1.21 exponent.

As with the lateral components, flow blockage of the wake generating group has the most significant effect on the amount of excess vertical turbulent velocity generated in the wake. As the spacing between structures is increased the appearance of significant excess turbulent velocity is delayed. The maximum excess turbulent velocity generated is reduced as the spacing increases, as more and more flow chooses to pass through the gaps, instead of over the structure group.

Figure 5.42b clearly indicates the significant excess vertical velocity fluctuations generated by the closely spaced wake generating group. As a function of SIS levels, both lateral and vertical components of turbulent velocity are greatly enhanced in the wake of the more closely spaced structure group.

**Summary of Observations on the Turbulent Flow Field**

In the following section the dispersion of contaminants in the wake of various wake generating groups is examined. From this perspective some important observations on the turbulent flow field are summarized.
Figure 5.42a: Centreline decay of vertical turbulent velocity above background levels

Figure 5.42b: Centreline decay of vertical turbulent velocity above SIS levels
The recovery of mean velocity deficit in the wake is shown to be related to the overall width of the wake generating group. Based on this observation the mean contaminant transport in the far wake will be reduced as separation distance is increased. By itself, this affect would lead to larger contaminant concentrations persisting farther downstream for increasing structure separation distance.

In the development of the contaminant dispersion equation (Appendix B) the longitudinal fluctuations are neglected relative to the mean transport in the \( x \)-direction. Based on this assumption it is only the lateral and vertical components of turbulent velocity which contribute significantly to the turbulent spread of a contaminant cloud. Both the lateral and vertical components of turbulent velocity are demonstrated to be seriously affected by the relative blockage presented by the wake generating group. The most substantial turbulent dispersion should then be expected in the wake of the most closely packed structures. Thus, considering only the effect of velocity fluctuations on dispersion, we expect an increase in measured concentration with increased structure separation.

The effect of the peripheral structures and gap spacing on the observed contaminant dispersion in the wake depends on the relative importance of the two conflicting mechanisms modified by their presence: the reduced mean transport velocity, and the enhanced turbulent mixing. The data suggests that the effect of reduced advection velocity may be more significant in the wake of multiple structures than it is in the wake of single structures. Determining the relative importance of the effect of the reduced velocity and of the enhanced turbulent mixing is a formidable problem, and the primary goal of the flow visualization experiments.
5.4 **UNCERTAINTY OF RESULTS**

Results of Chapter 5 are presented subject to the maximum uncertainties of Table 5.4. A discussion of uncertainties is included as Appendix E.

**Table 5.4: Maximum uncertainties for hot wire measurements**

<table>
<thead>
<tr>
<th></th>
<th>Bias Error</th>
<th>Precision Error</th>
<th>Total Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>0.051</td>
<td>0.0054</td>
<td>0.052</td>
</tr>
<tr>
<td>$\bar{v}$</td>
<td>6.036</td>
<td>0.008</td>
<td>0.039</td>
</tr>
<tr>
<td>$\frac{\bar{u}}{u}$</td>
<td>0.062</td>
<td>0.010</td>
<td>0.065</td>
</tr>
<tr>
<td>$\frac{u_w - u_b}{u_b}$</td>
<td>0.072</td>
<td>0.0076</td>
<td>0.074</td>
</tr>
<tr>
<td>$\frac{(\bar{u}_w - \bar{u}_h)}{u_h}$</td>
<td>0.087</td>
<td>0.014</td>
<td>0.091</td>
</tr>
</tbody>
</table>
CHAPTER 6

THE CONCENTRATION FIELD

6.1 FLOW VISUALIZATION

The flow visualization experiments were conducted as described in Section 3.2.5. Still photos and colour video was used to aid in understanding various aspects of the flow. In order to obtain reasonable flow visualization results it is necessary to run the wind tunnel at a relatively low free stream velocity of $U_\infty = 2.2 \text{ m/s}$. At this speed the Reynolds number based on cube dimension is $Re_x = 0.7(10^3)$. Viscous effects may be unreasonably dominant at this $Re$ and wake growth and boundary layer development may not yet be $Re$ independent. These factors limit these flow visualization results as dependable quantitative dispersion data. Relative flow characteristics between cases may, however, be illustrated with some clarity. It is with relative observations in mind that the results of overhead video photography experiments are presented for the various cases of three inline wake generating structures.

Presented results were obtained from overhead black and white video images using Image-Pro Plus image processing software by Media Cybernetics, Silver Spring, MD. Each figure is the result of averaging 64 individual video frames of laser illuminated oil smoke. A universal spectrum of pseudo-colouring is then applied to each figure.

The results are shown for Cases 1, 7 to 3 as Figure 6.1, parts a to f. The downstream extent of each figure is approximately $x/h = 20$. A shadow is cast across the smoke plume in line with the direct laser light source. This shadow is centred at a downstream distance of approximately $x/h = 15$. 
Figure 6.1: Flow visualization results for a) Case 1, SIS, b) Case 7, $S/h = 3.0$, and c) Case 6, $S/h = 1.5$
Figure 6.1: Flow visualization results for d) Case 5, $S/h = 1.0$, e) Case 4, $S/h = 0.6$, and f) Case 3, $S/h = 0.3$
6.1.1 Discussion of Flow Visualization Results

\( S/h = 3.0 \)

Consider the large separation case, \( S/h = 3.0 \), of Figure 6.1b in comparison to the infinite separation case of the single isolated structure, Figure 6.1a. The longitudinal extent of the mean concentration 'footprint' is slightly reduced, as are the mean concentrations in the centre of the wake. These observations indicate enhanced turbulent mixing downstream of the \( S/h = 3.0 \) case. Some wake interaction of turbulent mixing velocities is occurring along the shear layers, even at this relative large separation. This interaction may be accelerated by the exaggerated lateral growth of the wakes that occurs at such low Reynolds numbers. Figure 6.1b indicates no obvious interaction of mean advection velocities in the wake which would result in a longer concentration footprint.

\( S/h = 1.5 \)

Figure 6.1c shows the effect of reducing the clear separation of the structures to \( S/h = 1.5 \). The interaction of the turbulent mixing velocities is apparent when the figure is compared to the higher concentrations observed downstream of the single isolated structure of Figure 6.1a. The higher concentrations in the centre of the wake, when compared to the \( S/h = 3.0 \) case indicate reduced mean advection velocities in the wake resulting from the interaction of the mean velocity wakes. Despite the enhanced spreading, the longitudinal extent of the footprint is not reduced, a result of the interacting mean velocity wakes as described in Section 5.1.

\( S/h = 1.0 \)

Figure 6.1d shows the effect of reducing the clear separation of the structures to \( S/h = 1.0 \). At this spacing the reduction in mean advection velocity due to interacting wakes is clearly of considerable importance. While the turbulent spread velocities of the interacting wakes are more intense then the \( S/h = 1.5 \) case, the concentrations along the wake centre have actually increased slightly. In addition, the concentration footprint
downstream of the light source shadow is virtually unchanged. Clearly for this intermediate spacing the effect of reduced mean advection velocity is as important, if not more important, than the effect of enhanced spread caused by the presence of the peripheral structures.

$S/h = 0.6$

Figure 6.1e shows the effect of further reducing the clear separation distance to $S/h = 0.6$. At this spacing the enhanced turbulent spread velocities begin to dominate over the effect of the reduced mean advection velocity. When compared to the previous case of $S/h = 1.0$, Figure 6.1e illustrates a significantly wider and shorter footprint, with much lower concentrations along the wake centreline. This switch in dominance is expected when the flow results of Chapter 5 are considered. The interaction of the mean velocity wakes are shown in Section 5.1 to be a function of the overall width of the wake generating group. As a result, the longitudinal extent of the coalesced mean velocity deficit continues to reduce as the separation distance decreases. At the same time, Figures 5.31 and 5.32 of Section 5.3 illustrate that the enhancement of lateral and vertical turbulent velocities, which cause enhanced mixing in the wake, continue to increase as the separation decreases. Thus, the observed transition in the far field from the dominance of reduced mean advection velocities for the intermediately spaced cases, to the dominance of enhanced turbulent velocities for the closely spaced cases, is both expected and observed.

$S/h = 0.3$

Figure 6.1f shows the effect of further reducing the clear separation distance to $S/h = 0.3$. When compared to the previous case of $S/h = 0.6$, the enhancement in lateral spreading is clearly more immediate and significant. The further reduction of centreline mean concentrations and the longitudinal extent of the footprint indicate a continued trend towards enhanced spread velocities as the dominant factor affecting concentrations in the wake. Increased concentrations due to reduced mean velocities is limited to the very near field.
6.1.2 Summary of Flow Visualization

Results of the flow visualization experiments allow us to observe the relative importance of the reduced mean advection velocity and the enhanced turbulent spread velocities in the wakes of the various test cases.

For closely spaced groupings ($S/h = 0.3, 0.6$) the enhanced lateral and vertical spread caused by the interacting turbulent wakes dominate the dispersion of contaminants. This results in wider, shorter footprints of mean concentration when compared to the results for the single isolated structure.

As the clear separation distances increase to an intermediate range ($S/h = 1.0, 1.5$) the effect of reduced mean advection velocity begins to dominate in the far wake. This dominance over the enhanced spread velocities results in a concentration footprint with higher centreline values that persist farther downstream. This is a result of the interacting flow phenomena created in the wake of the structures at this spacing, as discussed in Section 5.1.

At a largely spaced clear separation distance of $S/h = 3.0$ the mean velocities in the wake no longer significantly interact. Interaction of the turbulent velocities along the shear layers between the structures results in slightly lower observed concentrations than for the single isolated structure case.

The results of the visualization experiments indicate that both the effect of reduced mean advection velocity, and the effect of enhanced turbulent spread, must be considered in an effective model of contaminant dispersion in the wake of multiple structures. The visualization experiments suggests that the flow results of Chapter 5 provide a good starting point for predicting the behaviour of the contaminants. The appropriateness of this hypothesis will be verified at the more proper high $Re_h$ flow, through the use of the quantitatively reliable concentration measurements presented in the following section.
6.2 CONTOURS OF EQUAL CONTAMINANT CONCENTRATION

The new models for mean contaminant concentration in the wake of multiple structures, as described in Chapter 2, suggest that the flow results of Chapter 5 may be used directly to predict the effect of peripheral structures on contaminant concentrations. The flow results in question pertain to the increased turbulence intensity and reduced mean velocity in the wake. Contour plots of equal contaminant concentration are used to demonstrate the direct relationship between the observed flow variations and measured contaminant concentrations.

Figure 6.2 shows the concentration contours downstream of the single isolated structure, Case 1, for the horizontal plane at the structure height, \( h \), and for the vertical plane at the lateral centreline of the structure. Figures 6.3 to 6.7 show similar plots for the three structure groupings, Cases 7 - 3, with clear separation distances decreasing from \( S/h = 3.0 \) to 0.3.

Considering first contours in the horizontal plane, we compare Figure 6.2a to Figure 6.3a and see that for the large separation case, \( S/h = 3.0 \), the weak secondary flows have contributed to a subtle increase in the length of the mean concentration footprint. This result was suggested by the observations of Chapter 5, but was not revealed by the less reliable flow visualization results of Section 6.1. Thus, even at this large separation distance there is an effect on the concentration field in the wake and this effect is attributed to the mean velocity field as opposed to the turbulent velocity field.

As the separation distance is reduced to \( S/h = 1.5 \), Figure 6.4a, we expect an even longer mean concentration footprint due to the significant reduction in the mean advection velocity downstream of this arrangement. Increases in the turbulent mixing velocities widen the plume in the mid wake region but this effect is still overshadowed by the mean velocity effect.

At a separation distance of \( S/h = 1.0 \) a strong well developed mean wake forms downstream of the structure group. The turbulent velocity plots show that the excess of
Figure 6.2a: Horizontal contours of mean concentration, $K$, at $y/h=0.0$, Case 1, SIS, b/h=1.0

Figure 6.2b: Vertical contours of mean concentration, $K$, at $y/h=0.0$, Case 1, SIS, b/h=1.0
Figure 6.3a: Horizontal contours of mean concentration, $K$, at $z/h=1.0$, Case 7, $S/h=3.0$

Figure 6.3b: Vertical contours of mean concentration, $K$, at $y/h=0.0$, Case 7, $S/h=3.0$
Figure 6.4a: Horizontal contours of mean concentration, $K$, at $z/h=1.0$, Case 6, $S/h=1.5$

Figure 6.4b: Vertical contours of mean concentration, $K$, at $y/h=0.0$, Case 6, $S/h=1.5$
Figure 6.5a: Horizontal contours of mean concentration, $K$, at $z/h=1.0$, Case 5, $S/h=1.0$

Figure 6.5b: Vertical contours of mean concentration, $K$, at $y/h=0.0$, Case 5, $S/h=1.0$
Figure 6.6a: Horizontal contours of mean concentration, $K$, at $z/h=1.0$, Case 4, $S/h=0.6$

Figure 6.6b: Vertical contours of mean concentration, $K$, at $y/h=0.0$, Case 4, $S/h=0.6$
Figure 6.7a: Horizontal contours of mean concentration, K, at $y/h = 1.0$, Case 3, S/h = 0.3

Figure 6.7b: Vertical contours of mean concentration, K, at $y/h = 0.0$, Case 3, S/h = 0.3
lateral and vertical turbulent velocities are only beginning to show significant enhancement above SIS levels. From these observations we expect the contaminant plume to be slightly wider than the SIS plume in the mid wake region, but more significantly, we expect the wake to be considerably longer than that of the SIS wake. These expectations are confirmed by the contour plot of contaminant concentration, Figure 6.5a.

As the clear separation distance is further reduced to $S/h = 0.6$, Figure 6.6a, the effect of enhanced turbulent mixing begins to dominate over the effect of reduced advection velocities. The overall width, and subsequently the overall length, of the momentum wake is reduced. At the same time, the turbulent mixing in the wake is becoming stronger as the separation decreases. Here we see this change in dominant mechanism demonstrated by the widening in the mid wake and a reduction in the downstream extent of the concentration footprint.

Further reducing the separation distance to $S/h = 0.3$, Figure 6.7a, shows that the enhanced turbulent mixing of the turbulent velocities clearly dominates in this case. The concentration footprint is clearly much wider than the SIS case. The increase in mean concentration due to the reduction in the mean flow is limited to the near field for this relatively narrow structure group. Thus, the concentration footprint is not extended downstream for this closely packed case.

The horizontal contours of mean contaminant concentration demonstrate that the flow observations of Chapter 5 correspond directly to the mean concentration wake. In summary, we expect the horizontal profiles to become narrower (due to the reduced excess lateral intensity), and longer (due to increased reduction in the mean advection velocity), as $S/h$ increases from 0.3 to 1.0. For $S/h > 1.0$, we expect the contour width to begin to return to SIS levels, as will the overall length of the contour, as the effect of the peripheral structures is reduced with large separations.

Figure 5.42 shows the effect of the multiple structure arrangements on the vertical component of turbulence intensity in the wake. With similar variation between cases as the lateral response, we expect similar effects on the vertical contours of contaminant
concentration in the wake. That is, substantial increases in vertical spread for the most closely spaced arrangements, with the effect of reduced advection velocity becoming more prevalent as separation distance is increased to $S/h = 1.5$. The vertical concentration contour plots of Figures 6.2b to 6.7b show that this expectation is confirmed by the experiments.

Contour plots are informative but difficult to view simultaneously for various cases. To further illustrate the direct relationship between the observed increase in vertical turbulence intensity and added vertical spread of contaminant, the vertical spread rate $\sigma_v$ has been estimated for each point downstream. The excess spread above SIS levels plotted in Figure 6.8. Comparison with Figure 5.42b for the excess vertical intensity in the wake reveals obvious similarities.

![Diagram](image)

**Figure 6.8:** Comparison of vertical spread parameter, $\sigma_v$, with SIS case
The observations of this section provide confidence to proceed with the modelling modifications suggested in Chapter 2, based entirely on flow observations from Chapter 5. A concentration model based only on flow variations has the considerable advantage of not requiring direct concentration measurements to estimate fit parameters or virtual origin locations.

6.3 MODELLING CONTAMINANT CONCENTRATIONS IN THE WAKE

6.3.1 Plume Spread in the Background Flow Field

The dispersion models considered in this thesis modify the behaviour of the well known Gaussian Plume model which works well for unobstructed flows. For full scale applications of the models the background dispersion rates, \( \sigma_y \) and \( \sigma_z \), would be determined in the customary manner from tables or curves (eg. Pasquill-Gifford curves as presented in Turner, 1969). The models then modify these dispersion rates to account for the presence of the structures. For the purpose of these experiments the rate at which a gas plume spreads downstream of a continuous point source was measured in the absence of any flow obstructions. From these measurements the background \( \sigma \) values are determined for use in the wake models. The simulated point source used is as described in Section 4.2. \( \sigma \) values are estimated from profiles of measured mean concentration as described in the Appendix D. The estimated values of \( \sigma_y \) and \( \sigma_z \) are plotted against \( x/h \) in Figure 6.8. \( h \) is used as the normalizing length for consistency. Figure 6.9 illustrates that a reasonable estimate of the data is provided by the equation

\[
\frac{\sigma_y}{h} = 0.131 \left( \frac{x}{h} \right)^{0.75}
\]  

(6.1)

This compares favourably to the expression found by Huber and Snyder (1982) (Eq. 2.12). The chosen power law representation is consistent with the literature.
Fewer estimates of $\sigma_z$ are available due to the elevated nature of the continuous point source. For consistency, a minimum sum of squares power law fit is used with the data available suggesting that (6.2) may adequately describe the variation of $\sigma_z$:

$$\frac{\sigma_z}{h} = 0.473 \left( \frac{x}{h} \right)^{0.25} \tag{6.2}$$

The lower observed growth rate of $\sigma_z$ is consistent with the observations of Halitsky (1977).

---

**Figure 6.9:** Downstream variation of $\sigma_y$ and $\sigma_z$ for background flow, Case 0
6.3.2 Concentration in the Wake of a Single Isolated Structure

It is the purpose of this section to establish the variation of contaminant concentrations in the wake of single isolated structures. Estimates of mean concentration using the models of Section 2.2 are compared with the data to demonstrate the strengths and weaknesses of the models. Satisfied with the ability and limitations of these models, the subsequent sections will illustrate the effect of multiple structure wakes on the contaminant concentrations, as they relate to this important case.

Variation of Maximum Ground Level Concentrations

Traditionally much emphasis is placed on maximum concentrations at ground level where most human and plant contact is expected. Figure 6.10 shows the decay of ground level mean concentrations, along the lateral centreline, downstream of the single isolated structure of Case 1. Along with the data are shown the mean concentration estimates of the four models described in Section 2.2. These models are the Modified Point Source (MPS) model, the Modified Area Source (MAS) model, the Halitsky (HAL) model, and the Huber and Snyder (HS) model.

For the MPS and MAS models the lateral and vertical spread parameters are modified by the equations describing the variation of lateral and vertical intensity in the wake as given in Section 5.3.2. The equations are as given in (6.4)

\[
\left( \frac{\sigma_x}{\sigma_y} \right) = 1 + 0.37 \left( \frac{x}{h} \right)^{-0.25} \quad (6.4a)
\]

\[
\left( \frac{\sigma_x}{\sigma_z} \right) = 1 + 3.32 \left( \frac{x}{h} \right)^{-1.21} \quad (6.4b)
\]

The reduced mean velocity deficit is modified as per the model described in Section 5.2.5, given here as (6.5)
\[
\left( \frac{u_x}{u_h} \right) = 1 - 1.5 \left( \frac{x}{h} + 0.5 \right)^{-1.5}
\]  
(6.5)

where the factor 0.5 corrects for the origin shift to the leading edge of the structure.

The MPS model is assumed to originate from the leading edge of the structure, while the MAS model is assumed to originate from the lee face of the structure.

Figure 6.10 shows that with these modifications both the MPS and MAS models adequately predict maximum ground level concentrations with no arbitrary origins or fit parameters involved.

To assist in the performance of both the HAL model and the HS model, background spread parameters determined as per (6.1) and (6.2)\(^1\) replace the values given in the respective originating papers.

The HS model was further modified by allowing for an effective stack height of \(h_{eff} = h/2\) and a 0.5 multiplier on \(\sigma_z\) to account for the narrower structure width than that examined by Huber and Snyder (1982). These modifications permitted reasonable, while slightly conservative estimates of maximum ground level concentrations downstream, as demonstrated in Figure 6.10.

The Halitsky model described in Section 2.2.2 was used with a reduction factor \(P = 1.0\). As with the HS model, the HAL model predictions of maximum ground level concentration, as shown in Figure 6.10, are reasonable, if slightly conservative.

A representative horizontal profile of ground level concentration is given at \(x/h = 10\), as Figure 6.11. Clearly all four models underpredict the lateral spread of contaminant at ground level. This result is caused by the use of background lateral spread parameters obtained at \(z = h\). Near ground level the additional turbulence in the background flow significantly enhances lateral spread. This is confirmed by the better prediction of lateral spread demonstrated by all models at \(z = h\), as illustrated in Figure 6.12. Full scale

\(^1\) (6.2) applies to HAL model only. HS model assumes \(\sigma_z = \sigma_y\) for \(x/h < 10\)
Figure 6.10: Decay of centreline ground level concentrations for single isolated structure, Case 1
Figure 6.11: Lateral profile of mean concentration at ground level, SIS, $x/h = 10$
Figure 6.12: Lateral profile of mean concentration at $z = h$, SIS, $x/h = 10$
estimates of $\sigma_y$ are generally obtained at ground level and model performance may be even better than that demonstrated in Figure 6.11.

Figure 6.13 shows the centreline vertical variation of non-dimensional concentration, $K$, at the same representative downstream point, $x/h = 10$. As discussed in Section 2.2.3, $\sigma_z$ is overpredicted by all of the models as expected.

Figures 6.10 to 6.13 show that all models considered in this thesis demonstrate comparable performance for the SIS case. The primary advantage of the MPS and MAS models thus far is that they do not depend on any experimentally determined fit parameters for concentration. Instead, estimated background spread parameters are modified by wake flow variations which are frequently verified in the literature. The real power in this method is demonstrated in the ability to adapt to more complex wake structures such as those generated by multiple in-line structures.

6.3.3 Concentrations in the Wake of Multiple Structures

For the three structure groupings of various separations, Cases 3 - 7, the performance of the Modified Point Source and the Modified Area Source models are examined. Of the Huber and Snyder model and the Halitsky model, only the Halitsky model attempts to extend to multiple structure wakes. Therefore, the Halitsky model performance is included for comparison.

Modifiers to the MPS and MAS models

The MPS and MAS models are as described in Section 2.2, with more details available as a result of the observations presented in Chapter 5. Non-dimensional concentrations are estimated for the MPS and MAS models as per (6.6) and (6.7) respectively,

$$K_{MPS} = \frac{h^2}{2\pi\sigma_y\sigma_z} \exp\left(\frac{y^2}{2\sigma_y^2}\right) \exp\left(-\frac{z^2}{\sigma_z^2}\right)$$  \hspace{1cm} (6.6)
Figure 6.13: Vertical profile of mean concentration, SIS, $x/h = 10$
\[ K_{\text{MAS}} = \frac{1}{4} \left( \frac{u_w}{u_h} \right)^{15} \left[ \text{erf} \left( \frac{h/2 + y}{\sqrt{2} \sigma_h} \right) + \text{erf} \left( \frac{h/2 - y}{\sqrt{2} \sigma_h} \right) \right] \left[ \text{erf} \left( \frac{h + z}{\sqrt{2} \sigma_z} \right) + \text{erf} \left( \frac{h - z}{\sqrt{2} \sigma_z} \right) \right] \] (6.7)

where the velocity modifier \( u_w/u_h \) is estimated as described by the mean velocity deficit model of Section 5.25, i.e.

\[ \frac{u_w}{u_h} = 1 - 15 \left( \frac{x}{h} + 0.5 \right)^{-15} \] for \( \frac{x}{h} < f(S,b) \) (6.8a)

\[ \frac{u_w}{u_h} = 1 - 15 \left( f(S,b) + 0.5 \right)^{-15} \] for \( \frac{x}{h} > f(S,b) \), and \( \frac{x}{h} < f(S,b) \) (6.8b)

\[ \frac{u_w}{u_h} = 1 - 15 \left( \frac{x}{W} + 0.5 \right)^{-15} \] for \( \frac{x}{W} > f(S,b) \) (6.8c)

where, from Table 5.1 \( f(S,b) = 0.85 + 2.28 \left( \frac{S}{h} \right) \) for \( b = h \) (6.9)

and \( W = 3(h) + 2(S) \)

where the origin is located at the leading edge of the structure.

The spread parameters, \( \sigma_y \) and \( \sigma_z \), are assumed to vary above background levels in the same manner as the lateral and vertical excess turbulence intensities, as described in Section 5.3.2. The centreline variations of \( \bar{v} \) and \( \bar{w} \), as given in Figures 5.41 and 5.42, suggest the following variation of \( \sigma_y \) and \( \sigma_z \) with \( x \), for characteristic width, \( W \)

\[ \left( \frac{\sigma_y}{\sigma_h} \right)_x = 1 + 25 \left( \frac{h}{W} \right) \left( \frac{x}{h} \right)^4 \] (6.10a)

\[ \left( \frac{\sigma_z}{\sigma_h} \right)_z = 1 + 40 \left( \frac{h}{W} \right) \left( \frac{x}{h} \right)^4 \] (6.11a)
The expressions (6.10a) and (6.11a) must be truncated in the near field and so the same relation which dictates the coalescence of the mean velocity wakes is used, ie.

\[
\frac{\sigma_x}{\sigma_b} = i + 25\left(\frac{h}{W}\right)\left(f(S,b)\frac{W}{h}\right)^{-1}
\]

(6.10b)

and

\[
\frac{\sigma_x}{\sigma_b} = 1 + 40\left(\frac{h}{W}\right)\left(f(S,b)\frac{W}{h}\right)^{-1} \quad \text{for} \quad \frac{x}{W} < f(S,b)
\]

(6.11b)

where \(f(S,b)\) is determined from (6.9). This step is justified by its simplicity and a lack of evidence to the contrary.

Using the modifications of (6.8) to (6.11), both the MPS and MAS models are expected to compensate for the various flow conditions observed in the wake of the multiple structure arrangements. Because it represents a more realistic source condition, the MAS model is expected to perform better in the near field, where the MPS model will overpredict \(K\) in most cases.

**Halitsky Model Parameters**

The HAL model as described in Section 2.2.2 requires estimates of \(H\) and \(W\) for the equivalent flat plate dimensions. In addition, a value for the flow leakage parameter, \(P\), is required. For the model predictions presented in this section the following estimates were used: \(H = h\), the nominal structure height, and \(W\) is the overall width of the structure group as defined by (5.18), ie. \(W = 3b + 2S\). The leakage parameter is required to be less than unity and is adjusted for each case to provide the best fit to the data. This free parameter limits the generality of the model, however, presentation of the appropriate \(P\) values may provide a guide for future estimates where concentration data is not available.

**6.3.4 Model Predictions of Ground Level Concentration for Multiple Structure Wakes**

Figures 6.14 to 6.23 show the decay of maximum ground level concentration for the various three structure cases with \(S/h\) increasing from 0.3 to 3.0. Also shown are
representative lateral profiles at ground level for the intermediate downstream location, \( x/h = 10 \). The decay curves are plotted on linear axes for the more familiar exponential type decay.

**Case 3, \( S/h = 0.3 \)**

Figure 6.14 shows the decay of maximum ground level concentrations in the wake of the most closely spaced arrangement of the study. For this case the enhancement of the spread parameters is high in the near field. As a result, the predictions are very sensitive to source location and the small change in origin between the MPS and the MAS models results in a significant difference in predicted concentrations in the very near field. Despite this sensitivity, both model predict the reduced concentrations (below SIS levels), beyond \( x/h = 3 \), very well.

The results of the HAL model predictions for this case are also shown in Figure 6.14. The model adequately accounts for the reduction of mean velocity in the near field and correctly predicts \( K \) in this region. In the far field \( K \) values are overpredicted due to an underestimate of the enhanced \( \sigma \) values. The maximum allowable value of \( P = 1.0 \) is used in this case.

Figure 6.15 shows how the models predict the lateral profiles of ground level concentration at the representative location of \( x/h = 10 \). Clearly, all of the models do a good job of correctly predicting the lateral spread at ground level for this case. The contour plots of the previous section reveal that for the multiple structure arrangements the contaminant dispersal is more limited to the interior wake regions, away from the wake edge shear layers. This is apparent in the lack of enhanced ground level spread noticed in the vertical contours. Thus, the turbulence field to which the contaminant is exposed is more homogeneous in nature, a result that is more consistent with the assumptions of the model. This has the direct and positive effect of permitting \( \sigma \) values that are based on flow parameters estimated at \( z = h \) to better represent ground level values. Thus, reduced three dimensional variations allow the simple models to work better for the multiple structures than for the single isolated structure.
Figure 6.14: Decay of centreline ground level concentrations for 3 in-line structures, Case 3, $S/h = 0.3$
Figure 6.15: Lateral profile of mean concentration at ground level, Case 3, $S/h = 0.3, x/h = 10$
Figure 6.16: Decay of centreline ground level concentrations for 3 in-line structures, Case 4, $S/h = 0.6$
Figure 6.17: Lateral profile of mean concentration at ground level, Case 4, $S/h = 0.6$, $x/h = 10$
Figure 6.18: Decay of centreline ground level concentrations for 3 in-line structures, Case 5, $S/h = 1.0$
Figure 6.19: Lateral profile of mean concentration at ground level, Case 5, $S/h = 1.0$, $x/h = 10$
Figure 6.20: Decay of centreline ground level concentrations for 3 in-line structures, Case 6, $S/h = 1.5$
Figure 6.21: Lateral profile of mean concentration at ground level, Case 6,
$S/h = 1.5, x/h = 10$
Figure 6.22: Decay of centreline ground level concentrations for 3 in-line structures, Case 7, $S/h = 3.0$
Figure 6.23: Lateral profile of mean concentration at ground level, Case 7, 
$S/h = 3.0, x/h = 10$
**Case 4, $S/h = 0.6$**

Case 4 is the most difficult case to predict concentrations in the near field. While significantly enhanced spreading reduces $K$ below SIS levels in the far field, the contaminant is confined in the near field by flow jets between the structures. This phenomena results in higher observed near field concentrations than the simple models will predict. Despite no attempt by the models to account for contaminant confinement, both the MPS and the MAS models predict reasonably high concentrations in the near field. These results are illustrated in Figure 6.16.

For cases where significant jetting is suspected conservative safety factors in the near field should be employed, or single structure modifiers could be employed to more correctly estimate the high near field concentrations. In contrast, the far field shows significantly lower concentrations than those found in the wake of the single structures and this quality is well predicted by both the MPS and MAS models. The balance between enhanced mixing and reduced advection velocity is well modelled in the far wake region.

For this case a leakage parameter of $P = 0.9$ was used in the HAL model results. With this free parameter set, the model does a good job of predicting $K$ in the far field, but underpredicts $K$ in the near field.

Figure 6.17 shows that all of the models correctly predict the lateral variation of ground level concentration downstream.

**Case 5, $S/h = 1.0$**

Figure 6.18 shows the measured maximum ground level concentrations in the wake of the intermediately spaced arrangement of Case 5. For this case the effects of the peripheral structures are well predicted by the model throughout most of the wake region. In the very near field the MPS model is beginning to overpredict as suspected, due to the reduced effect of the enhanced $\sigma$'s and the less realistic source condition. The MPS model does recover well however, and is comparable to the MAS model beyond $x/h = 3$. 
The MAS model predicts the variation of $K$ with $x$ well in the far field but underpredicts slightly in the near field.

The HAL model slightly underpredicts $K$ in the near field but matches the observed $K$ values in the far field if a free parameter of $P = 0.5$ is utilized, as shown.

The spread rate is predicted well by the models as illustrated in Figure 6.19.

**Case 6, $S/h = 1.5$**

The prediction of maximum ground level concentrations downstream of the large spaced, but still interactive arrangement of Case 6 is presented in Figure 6.20. This case represents the most impressive predictions for the MAS model. $K$ values are correctly predicted by the model throughout the range of the study, as is the lateral variation at ground level represented by Figure 6.21.

The MPS model also does a good job of predicting the $K$ values beyond the very near wake region, $x/h > 3$. In the very near field the MPS model is seriously overpredicting $K$ values.

With an arbitrary leakage parameter of $P = 0.4$, the HAL model predicts the $K$ values well throughout the full range of the study and is much improved in the near field, when compared to its performance on the previous cases.

**Case 7, $S/h = 3.0$**

Figure 6.22 shows the variation of $K$ in the wake of the large separation arrangement of Case 7. For this case the peripheral structures are effectively isolated from the centre structure. The effect of the multiple structure modifications to the concentration models are shown for comparison to the SIS predictions of the previous section.

Clearly the use of the modified advection velocity model leads to an overprediction of the $K$ values, especially by the MPS model. In presenting the mean velocity deficit
model in Section 5.2.5, it was applied only to separation distances less than $S/h = 1.5$. For this case of $S/h = 3.0$ the mean velocity modifier should properly be based on the isolated structure dimension $x/h$, instead of the multiple structure dimension $x/W$. This change would allow the model to perform comparably to the SIS model of the previous section.

Despite the inappropriate use of the velocity modifier, the MAS model is reasonable but consistently conservative in its estimates, throughout the range of the study. This suggests that the same multiple structure model may be used for isolated structures, provided a value of $S/h = 3.0$ is assumed.

The HAL model is displayed using a leakage parameter of $P = 0.4$ and shows good predictive qualities for this case.

Figure 6.23 shows that the lateral spread at ground level is once again underpredicted by the models, with the return of inhomogeneous variations in the wake.

**Summary of Model Performance**

In this section universal flow parameters describing multiple structure wakes are used to modify predictive models for mean contaminant concentration. The application of these modifiers is limited to the interacting wakes of closely spaced structures, $S/h < 3.0$. It is demonstrated that the effects of peripheral structures on the dispersion of contaminants in the wake can be correctly predicted by understanding the turbulent flow field. The use of measurable flow parameters to modify the predictive models offers a significant improvement in the simplicity and general application of the models, over the historic use of arbitrary constants and virtual origins. Universal behaviour of these flow properties, as demonstrated in Chapter 5, allows for universal modifiers to be proposed as a function of structure separation. It is expected that this approach to modelling contaminant dispersion will be equally successful for other structure arrangements that may be examined in the future.
Of the dispersion models included in this study the Modified Area Source model offers the best predictive abilities throughout the range of the study. The MAS model demonstrates a maximum prediction error of 37% in the very near field, and 9% in the far field. In the very near field we should be very cautious of applying a mean concentration model anyway. This model should work well far downstream of any source condition which results in full contaminant entrainment of the near wake cavity region.

6.4 UNCERTAINTY OF RESULTS

The results of Chapter 6 are presented subject to the maximum uncertainties of Table 6.1. A complete uncertainty analysis is included as Appendix E.

**Table 6.1:** Maximum Uncertainties for Concentration Measurements

<table>
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<th>Bias Error</th>
<th>Precision Error</th>
<th>Total Uncertainty</th>
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</thead>
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<td>0.011</td>
<td>0.101</td>
</tr>
<tr>
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</tr>
<tr>
<td>$K$</td>
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<td>0.154</td>
</tr>
</tbody>
</table>
CHAPTER 7
CONCLUSIONS AND RECOMMENDATIONS

7.1 CONCLUSIONS

7.1.1 Overview

A review of the literature shows that much work has been done to study the flow about single isolated structures. The dispersion of contaminants in the wake of such isolated structures is also well documented in the literature. Often building structures in the natural environment are not single and isolated but, instead, are found in groups. Knowledge gained from the study of single structure wakes may be extended to the combined wakes of multiple structure groups. This thesis aims to further our understanding of this complex flow by examining the effect of peripheral structures on the combined wake.

A systematic study of wake interactions for three in-line cubic structures of various separation is described. Flow regions in the combining wakes are identified from hot wire anemometer measurements. For clear separation distances of $S/h < 1.5$ the far wakes are shown to coalesce and form a single momentum wake. The development length of this final wake region is proportional to the separating distance of the structures. The critical scaling parameter for the multiple structure groups in this study is $W$, the overall width of the group including the clear separation distance $S$. All mean velocity wakes examined were fully developed beyond $x/W = 5$. Once fully developed the mean velocity deficit decays at a universal rate of $(x/W)^{1.5}$. This universal decay rate was demonstrated to exist for all interacting wake structures examined, including: 1) three in-line cubes of clear separation $S/h < 1.5$, 2) five cubes in two rows of clear separation $S/h < 1.0$, and 3) single isolated structures of width $h/h < 2.0$. 

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Recent advances in CFD techniques suggest that Large Eddy Simulations (LES) may provide satisfactory solutions to the flow about single isolated structures. To the knowledge of the writer, this technique has not yet been extended to include contaminant dispersion in the wake. LES techniques may provide reasonable predictions of turbulent velocities in the far wake. Such results would be interesting to compare to the results of this study on multiple structure wakes.

This study suggests that the wakes of multiple structures combine in such a way that they may be modelled by single structure theory. To demonstrate such behaviour, similarity theory is the best choice. Classical 2-D and axi-symmetric wake theories do not adequately account for the effect of ground shear in the wake of a surface mounted obstacle. The advanced similarity theory of Hunt (1970) reasonably predicts the mean velocity deficit in the far wake of surface mounted, isolated structures. This theory also predicts that the mean velocity deficit will decay as \( (x/h)^{1.5} \), the same rate observed for the fully developed multiple structure wakes. When the profile data is replotted using \( W \) as the critical scaling parameter, the single structure wake theory is demonstrated to predict the mean velocity deficit profiles of all the fully developed multiple structure wakes.

To examine the dispersion of contaminants in the wake, ethane was emitted continuously from the primary structure and concentration measurements were made using flame ionization detectors. While fluctuations of concentration are important and worthy of further study, predictive theories of wake concentration are currently limited to mean concentrations only. To study the effect of multiple structures and our ability to apply single structure dispersion models to the combined wakes, this study focuses on the prediction of mean contaminant concentrations.

Several wake dispersion models are examined in this study including the well known Huber and Snyder (1982) (HS) model. While giving reasonable results for the single isolated structure, the HS model, as presented, is not able to accommodate the
complicating effects of multiple structure wakes. Rather than modify the HS model for multiple structures, two improved models are developed.

The first model is a Modified Point Source model (MPS) using the conventional Gaussian plume model as its starting point. Flow visualization results indicate that the spread rate and mean transport of contaminant may be related to observed flow parameters in the multiple structure wakes. As a result, turbulent spread parameters are modified and based on observed variations of turbulent velocities in the wake. It is assumed that the lateral and vertical spread of contaminant is enhanced in wakes in the same way that the components of turbulent velocity are. Similarly, the mean advection velocity is modified and based on observed universal behaviour for multiple structure wakes. The modification of the spread parameters and the mean advection velocity is based on flow measurements and does not require concentration measurements to complete the model.

The second model is a Modified Area Source model (MAS) using an area source model to more accurately reflect the source conditions of a fully contaminated wake cavity. The turbulent spread parameters and mean advection velocity are modified in the same manner as the Modified Point Source model. The area source technique is superior to the point source model in the near field region.

A review of the literature reveals few concentration models that adequately deal with multiple structure wakes. One exception is the model of Halitsky (1977), selected in this study for comparison purposes. The model uses the flow characteristics in the wake of a virtual flat plate to modify the dispersion in the wake of a multiple structure group. A free parameter, \( P \), is retained and defined as a leakage parameter to account for flow between buildings. The parameter must be modified to provide a best fit to the data. A lack of guidance on the value of the free parameter \( P \), as well as the virtual plate dimensions, \( H \) and \( W \), represent a significant deterrent to the general application of the model. Values of these parameters giving best results for the present study are presented as a guide to improve the generality of the model.
7.1.2 The Flow Field

With respect to the mean velocity deficit in the wake of multiple structures, the following conclusions are made:

1. The mean velocity deficit of the wake persists much farther downstream for multiple structures with interacting wakes than for single isolated structures. The mean velocity deficit remains measurable for distances up to $x/W = 15$, where $W$ is the overall width of the wake generating group. This is equivalent to as much as $x/h = 90$ for structures separated by $S/h = 1.5$. The enhanced significance of the mean velocity deficit for multiple structures when compared to isolated structures is apparent.

2. For clear separation distances of less than $S/h = 1.5$ the lateral profiles of mean velocity deficit coalesce in the far wake. This universal wake development is complete by $x/W = 5$ for the interacting wakes examined. For clear separation distances $S/h > 3.0$, the mean velocity wakes decay independently to within 10% of background levels for each structure. Weak secondary flows are present at this large separation distance but full universal wake development is not observed within the measurable wake regime.

3. The result of wake coalescence is a wake profile which demonstrates similar shapes, in both the vertical and horizontal planes, as those profiles generated by single isolated structures. Single structure similarity theory of Hunt (1970) may be applied in this fully developed region, $x/W > 5$, provided the relevant scaling parameter used in the theory is modified from the single structure value.

4. Beyond the point of wake coalescence, the decay of the mean velocity deficit follows a universal decay rate of $(x/W)^{-1.5}$. This decay exponent matches that shown to exist for single isolated structures and derived in the theoretical work of Hunt (1970). Single structures of various aspect ratios were demonstrated to follow the same -1.5 decay rate when normalized by the body width, $b$. 
Study of turbulent velocities in the wake of multiple structures permits the following conclusions to be made:

1. Observed turbulence intensity in the wake of multiple structures significantly exceeds the turbulence intensity in the wake of a single structure in all three component directions. This excess of turbulence intensity becomes more substantial as the separation of the structures is decreased.

2. Profiles of turbulence intensity coalesce to form a uniform wake for interacting structure groups, \( S/h < 1.5 \). For separation distances as large as \( S/h = 3.0 \) there exists significant excess turbulence along the shear layers centred between the structures. At this large separation a uniform wake is not realized downstream.

7.1.3 The Concentration Field

With respect to contaminant dispersion in the wakes of multiple structures, the following conclusions are made:

1. As expected, the presence of peripheral structures significantly affects the dispersion of contaminants in the wake. As a consequence, dispersion models based on single isolated structure wakes are inadequate.

2. For closely spaced structures, ie. \( S/h \leq 0.6 \), the additional turbulence intensity in the near wake dominates dispersion. Blockage effects resulting in a reduction of the mean advection velocity is confined to the near field. The result is a much wider concentration footprint with higher concentrations in the near field than occur in the wake of a single isolated structure. Beyond the very near field, \( x/h > 5 \), concentration levels in the wake are lower than single structure levels due to the significantly enhanced turbulent mixing.

3. For larger separation distances, \( S/h \geq 1.0 \), the effect of increased mean velocity deficit in the wake takes on a stronger role and must be accounted for in any dispersion model. Enhanced lateral and vertical mixing results in a wider plume, but the dominance of the mean velocity effect results in a significantly longer concentration footprint than
occurs in the wake of an isolated structure. As the separation increases beyond $S/h = 1.5$, the structure wakes interact less and the plume dimensions returns to single structure levels. These clear separation distances of $S/h = 1$ result in critically high concentration levels in the wake. This has possible implications for urban planning as the downstream impact region may be significantly reduced if separation distances are maintained above $S/h = 1.5$.

4. The basic Gaussian Plume Model may be modified to account for the presence of the structures using only flow measurements of mean velocity and turbulence intensity in the wake. The use of a virtual origin alone to enhance the spread parameters is inadequate to predict the effect of multiple structure wakes on dispersion. The effect of the peripheral structures on the mean velocity deficit is too substantial to neglect.

5. The lateral and vertical spread parameters, $\sigma$, and $\sigma_z$, are shown to vary above background levels in a manner similar to the excess lateral and vertical turbulence intensity.

6. Modifications of the mean advection velocity to account for the momentum deficit in the wake is necessary for correct prediction of mean contaminant concentration. This may be done as a linear function of separation distance where both the developing and fully developed regions of the wake are accounted for. A model which neglects the effect of reduced mean velocity deficit will not be successful in predicting the effect of the multiple structure wakes on dispersion.

7. Of the models tested, the Modified Area Source model presented in the thesis provides the best estimates of mean concentration in the wake of multiple structures. The best predictive qualities of the model will be realized when source conditions result in the full entrapment of contaminant by the near wake cavity region. Using no arbitrary fit parameters the variations of ground level concentration observed in the various wakes are well predicted by the model. Maximum error of 37% in the very near field ($x/h = 2$) and 9% in the far field is realized by the model.
7.2 RECOMMENDATIONS

The following recommendations are suggested as a result of the experimental study and literature review:

1. The effect of peripheral structures on the reattachment of separated flow will have consequences on surface concentrations and far wake structures and should be examined in detail.

2. The relationship of peak to mean concentrations in the near wake of single or multiple structures is worthy of further investigation. Also, a detailed study of fluctuating concentrations and higher moments of concentration in the wake should be examined.

3. The present study may be expanded to include different approach flow conditions and possibly different wind angles.

4. More complex structure groupings should be systematically examined to further investigate the general validity of the theories presented in this study. In the limit, random city settings or industrial complexes of characteristic plan density may be examined.

5. Any full scale measurements of wake velocities or contaminant concentrations in similar flow conditions to those modelled in this study would be very informative.
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Snyder, W.H., Lawson, R.E. Jr., (1976), "Determination of a Necessary Height for a Stack Close to a Building-A Wind Tunnel Study", Atmos. Env. 10, 683-691.


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For the purpose of this appendix, the original notation of Hunt (1970) is preserved where possible. The exception is in the use of standard lateral and vertical coordinate designations, \( y \) and \( z \) respectively, consistent with the rest of the thesis.

The essential similarity equation (2.3) for the mean velocity in the wake is rewritten here as (A1)

\[
\frac{u_{\text{pert}}}{U_h} = \frac{K_2 F_2(\tilde{z}^*, \tilde{y})}{\left[(x-a)/h\right]^{(1+n)/(2+n)}} \tag{A1}
\]

where,

- \( u_{\text{pert}} \) is the perturbation velocity = \( u_x - u_h \)
- \( U_h \) is a reference background mean velocity at \( z = h \)
- \( K_2 \) is a constant which includes the force couple \( \bar{C} \), the constant eddy viscosity \( \nu^* \), and an experimentally determined constant \( \lambda \)

\[
K_2 = \frac{0.21\bar{C}\left[\frac{\nu^*}{U_h}\right]^{\frac{1}{(1+n)/(2+n)}}}{\frac{\nu^*}{\lambda h^{\frac{1}{2+n}}} U_h} \tag{A2}
\]

- \( F_2 \) is the similarity shape function as a function of height and lateral position

\[
F_2(\tilde{z}^*, \tilde{y}) = \frac{\left[\frac{1}{\tilde{z}^*} \exp\left(-\frac{\tilde{y}^2}{1.5 + \tilde{\eta}}\right)\right]}{\sqrt{\tilde{\eta} + 1.5}} \tag{A3}
\]

\( \tilde{z}^*, \tilde{y} \) are non-dimensional space coordinates which incorporate the longitudinal position \((x-a)/h\), given as
\( \tilde{z}^* = \frac{\begin{pmatrix} z \\ h \end{pmatrix}}{\begin{pmatrix} (x-a)^{\frac{1}{2+n}} \\ h \end{pmatrix}} \)  \hspace{1cm} (A4) \\

\( \tilde{y} = \frac{\begin{pmatrix} y \\ h \end{pmatrix}}{\begin{pmatrix} (x-a)^{\frac{1}{2+n}} \\ h \end{pmatrix}} \frac{hU_h}{\lambda \bar{v}_1} \)  \hspace{1cm} (A5) \\

\( a \) is the position of the virtual origin for the similarity solution to be determined experimentally.

\( \bar{v}_1, \bar{v}_2 \) are components of eddy viscosity, defined as factors of the mean eddy viscosity (2.1),

\[ \bar{v}_1 = \gamma \bar{v} \text{ and } \bar{v}_2 = \lambda \bar{v} \]  \hspace{1cm} (A6) \\

\( \lambda, \gamma \) are constants determined to give best fit to data.

In order to obtain numerical estimates of (A1) the following expressions are also required,

\[ \bar{\eta} = (\tilde{z}^*)^{(2+n)} \frac{hU_h}{(2+n)^2 \bar{v}_1} \]  \hspace{1cm} (A7) \\

\[ \bar{\eta} = (\tilde{z}^*)^{(2+n)} \frac{hU_h}{(2+n)^2 \bar{v}} \]  \hspace{1cm} (A8) \\

\[ \tilde{z}' = \frac{\begin{pmatrix} z \\ h \end{pmatrix}}{\begin{pmatrix} (x)^{\frac{1}{2+n}} \\ h \end{pmatrix}} \]  \hspace{1cm} (A9) \\

(2.1) is rewritten as,

\[ \bar{v} = \frac{2k^2_nU_h h}{(2+n)} \]  \hspace{1cm} (A10)
and $\overline{C}$ in (A2) is estimated as

$$\overline{C} = \frac{-C_D U_h^2 h^{(2-2n)b}}{4\delta \left[ \frac{\overline{V_1 \delta^n}}{U_h} \right]_{(1+n)}}$$  \hspace{1cm} (A11)

where $b$ is the width of the building, and $C_D$ is the non-dimensional drag coefficient.

The theory is applied by solving (A1), utilizing the subsequent expressions (A2) to (A.10), and values estimated for the constants $C_D$, $\gamma$, $\lambda$, and $a$. The values used for the example given in the thesis (plotted as solid lines in Figures 5.22 and 5.23) are given in Table 5.2.
APPENDIX B: DEVELOPMENT OF A GAUSSIAN PLUME TYPE DISPERSION MODEL

Consider an arbitrary control volume \( V \) containing a total mass of contaminant \( M = M(t) \) [g], where

\[
M = \int \chi \, dV
\]  
(B1)

and \( \chi = \chi(x,y,z,t) \) [g/m\(^3\)] is the contaminant concentration.

We represent the flux of contaminant across the surface of the control volume by a vector \( \mathbf{F} = \mathbf{F}(x,y,z,t) \) [g/(m\(^2\)s)]. In the absence of any sources or sinks within the control volume, the total change in mass per unit time must be equal to the flux across the surface of the control volume

\[
-\frac{\partial M}{\partial t} = \int_S \mathbf{F} \cdot \mathbf{n} \, dS = \int_V \nabla \cdot \mathbf{F} \, dV
\]  
(B2)

where \( \mathbf{n} \) is the outward normal vector and we have applied the divergence theorem to the surface integral.

Combining (B1) and (B2) we get

\[
\int_V \left( \frac{\partial \chi}{\partial t} + \nabla \cdot \mathbf{F} \right) dV = 0
\]  
(B3)

which must be true for any point within the volume, as well as the whole, requiring that

\[
\frac{\partial \chi}{\partial t} + \nabla \cdot \mathbf{F} = 0
\]  
(B4)

\( \mathbf{F} \) is the total flux of contaminant and (B4) applies regardless of whether the flux is caused by any or all of the following three components:

\[\mathbf{F} = \mathbf{F}_{hm} + \mathbf{F}_{mol} + \mathbf{F}_{turb}\]

where, \( \mathbf{F}_{hm} \) is the flux caused by the bulk motion of the fluid. For a fluid velocity \( \mathbf{u} \)

\[\mathbf{F}_{hm} = \mathbf{u} \chi\]

\( \mathbf{F}_{mol} \) is the molecular mass diffusion by Fick's law

\[\mathbf{F}_{mol} = -D_{mol} \nabla \chi\]
where $D_{mol}$ is the molecular diffusivity given as a product of a characteristic mixing length and velocity scale $l_{mol} u_{mol}$ and the last component of the flux vector is that of the turbulent mass diffusion $F_{turb}$. By analogy to Fick's law for molecular diffusion an effective *eddy diffusivity* is proposed as the product of a characteristic turbulent length scale and velocity scale, $D_{turb} = l_{turb} u_{turb}$ such that

$$F_{turb} = -D_{turb} \nabla \chi$$

Now we assume

- $D_{turb} \gg D_{mol}$ such that $\nabla \cdot F_{mol} \approx 0$
- $D_{turb} = D(t)$ only, such that $\nabla (-D_{turb} \nabla \chi) = -D \nabla^2 \chi$, where $\nabla^2$ is the Laplacian operator
- $u \gg v, w$ and $u$ gradients are small in the $x$ direction such that $\nabla (u \chi) \approx u \frac{\partial \chi}{\partial x}$

These assumptions result in (B4) being written as

$$\frac{\partial \chi}{\partial t} + u \frac{\partial \chi}{\partial x} = D \nabla^2 \chi \quad \text{(B5)}$$

(B5) is subject to the condition of mass continuity, which for a source $Q$ [g/s] may be written as

$$Q = \int_0^1 \int u \chi \, dy \, dz \quad \text{(B6)}$$

Csanady (1974) shows that the second moment of the distribution of $\chi$ resulting from a solution to (B5) gives one definition of a standard deviation $\sigma$. Representing the spread of the concentration distribution, $\sigma$ is related to the *eddy diffusivity* by the expression

$$\sigma^2 = 2Dt = 2D \frac{x}{u} \quad \text{(B7)}$$

(B5) is solved for a continuous point source in a uniform cross flow in Appendix C.
APPENDIX C: MODELLING A CONTINUOUS POINT SOURCE IN A UNIFORM CROSS FLOW

• assume a continuous point source emission rate of $q_p dt'$ (g/s·s) over all time periods $t \rightarrow t'+dt'$ so that for each puff $\sigma^2 = 2D(t-t')$

• assume a mean velocity $u$ in the $x$-direction so that $x'=x-ut$ represents the original position of each puff with no crossflow

The governing diffusion equation becomes

$$\frac{\partial \chi}{\partial t} + u \frac{\partial \chi}{\partial x} = D \nabla^2 \chi$$ (C1)

under the following assumptions

• $u \gg v, w$

• $D_x = D_y = D_z = D(t)$ only

• $\sigma_x = \sigma_y = \sigma_z = \sigma$

After satisfying continuity, (C1) has a solution of the form

$$\chi = \frac{q_p}{(\sqrt{2\pi \sigma})^3} \exp\left(-\frac{x'^2 + y'^2 + z'^2}{2\sigma^2}\right)$$ (C2)

Into (C2) we substitute the expression $x'=x-ut$ to account for the cross flow $u$, and we substitute $\sigma^2 = 2D(t-t')$ to account for the continuous flow.

Next we integrate over all time (successive puffs in the $x$-direction) to determine the concentration field.

This gives

$$\chi = \frac{q_p}{8(\pi D)^{3/2}} \int_0^{t'} \exp\left(-\frac{(x-u(t-t'))^2 + y'^2 + z'^2}{4D(t-t')}\right) \frac{dt'}{(t-t')^{3/2}}$$ (C3)

where $q_p = q_p(t')$

now expand $(x-u(t-t'))^2 = x^2 - 2xu(t-t') + u^2(t-t')^2$

and let $a = (4D(t-t'))^{-1/2}$
so that \[ da = \frac{2D dt'}{(4D)^{\frac{1}{2}} (t - t')^{\frac{1}{2}}} \]
giving, \[ \frac{dt'}{(t - t')^{\frac{1}{2}}} = 4D^{\frac{1}{2}} \, da \]
where the new limits become
\[ t' = 0 \implies a = (4D t)^{-\frac{1}{2}} = 0 \text{, for large } t \]
\[ t' \to t \implies a \to \infty \]
so (C3) becomes
\[ \chi = \frac{q_p}{8(\pi D)^{\frac{1}{2}}} 4D \int_0^\infty \exp \left( -\frac{r^2 a^2}{4D} - \frac{2xu}{4D} + \frac{u^2}{4D^2 a^2} \right) da \]
where \( r^2 = x^2 + y^2 + z^2 \).
If we break up the exponential and substitute \( b = r^2 \), and \( c = u^2/(4D)^2 \) we obtain a recognizable integral of the form
\[ \chi = \frac{q_p}{2\pi r^2 D} \exp \left( -\frac{xu}{2D} \right) \left( \int_0^\infty \exp \left( -ba^2 + \frac{c}{a^2} \right) da \right) \]
which solves and simplifies to give the following expression for a continuous point source in a uniform cross flow
\[ \chi = \frac{q_p}{4\pi rD} \exp \left( -\frac{u}{2D} (r - x) \right) \] \hspace{1cm} (C4)
Now the equation (C4) may be further simplified by a slender plume assumption that \( x >> y, z \), and \( u \frac{\partial}{\partial x} \frac{\partial^2 \chi}{\partial x^2} \) in the governing equation (C1).

If we express \( r = \chi \left( 1 + \frac{y^2 + z^2}{x^2} \right)^{\frac{1}{2}} \), and let \( b = \frac{y^2 + z^2}{x^2} \), where \( b^2 < 1 \) for a slender plume, this allows us to expand \( r \) as \( r \approx \chi (1 + b / 2 + 3b^2 / 8 + ...) \approx x + \frac{xb}{2} \),
from which \( (r - x) \approx \frac{x}{2} \left( \frac{y^2 + z^2}{x^2} \right) \).
We substitute this expression into (C4) along with \( t = x/u \), and \( \sigma^2 = 2Dt = 2Dx/u \) to get the following approximate expression for the concentration of a continuous point source in a uniform cross wind

\[
\chi = \frac{q_p}{2\pi u \sigma^2} \exp \left( \frac{y^2 + z^2}{\sigma^2} \right)
\]

Estimate of equal centreline concentration for Area Source and Point Source models

In order to determine at what point downstream we can expect to see similar centreline downstream concentrations predicted by both the Modified Point Source model (C5) and the Modified Area Source model (2.20) we examine the characteristics of the error function. In both equations the time variation of the diffusion is found in the \( \sigma \) values which vary in the \( x \) direction. In the limit as \( x \) becomes large the arguments of the error functions in (2.20) become small. For small arguments we observe that \( \text{erf} (\eta) \approx \eta \). If we allow that \( x \) is sufficiently large for this to occur then at ground level centreline, (2.20) becomes

\[
\chi_{\text{area}} = \frac{q_p}{4u} \left[ \frac{a}{\sqrt{2\sigma}} \sqrt{b} \right] = \frac{q}{8u\sigma^2} ab
\]

Similarly if we consider (C5) at sufficiently large \( x \) we may approximate it as

\[
\chi_{\text{point}} \approx \frac{q_p}{2\pi u \sigma^2}
\]

In order that \( \chi_{\text{area}} = \chi_{\text{point}} \) we must have sufficiently large \( x \) as well as a point source of approximate strength \( q_p = (q/\pi)ab \).

To determine what is sufficiently large \( x \) we notice that for

\[
\text{erf} (\eta) \approx \eta \quad \rightarrow \quad \eta \leq \frac{2}{\sqrt{3}}.
\]

For ground level centreline concentrations this condition is first satisfied in (2.20) when

\[
\frac{a/2}{\sqrt{2\sigma}} \leq \frac{2}{\sqrt{3}}
\]
or,

\[ \sigma \geq \frac{3a}{4\sqrt{2}} \]  \hspace{1cm} (C8)

from which we may estimate \( x \) given \( \sigma = \sigma(x) \) for the particular flow conditions. Beyond this point we may expect similar predictions from both models. Using \( a = 2h \) as in (2.20) then (C8) indicates

\[ \frac{\sigma}{h} > 1 \]

Figure 6.9 indicates that this growth is obtained in our flow beyond

\[ \frac{x}{h} > 15 \]

Observing the MPS and MAS model behaviour beyond this point (Figures 6.14 to 6.22) confirms this approximate analysis.
APPENDIX D: ESTIMATING SPREAD PARAMETERS, $\sigma$

The spread parameters, $\sigma$, represent the spatial variation of measured mean concentration for a given downstream location, $x_i$. The values of $\sigma$ must be estimated from the data as $\sigma(x_i) = f(y_i, \chi(y_i))$.

We assume the measured values of $\chi$ represents the number of observations of a random variable $Y$, for a given $y_i$. Thus, we generate a discrete probability mass function (PMF), $p(y_i)$ as

$$p(y_i) = \frac{\chi(y_i)}{\sum_{all \chi}}$$  \hspace{1cm} (D1)

Now we can estimate the Expected Value, $E(Y)$, or the mean value, $m_Y$, for each $x_i$, as

$$E(Y) = m_Y = \sum_{all \chi} y_i \cdot p(y_i)$$  \hspace{1cm} (D2)

If $y$ is the lateral spatial coordinate measured from the centreline of the contaminant source, then $E(Y) = m_Y = 0$.

Now we estimate the spread parameter, $\sigma$, as the standard deviation of the PMF, given by (D3)

$$\sigma_Y = \left[ \sum_{all \chi} (y_i - m_Y)^2 p(y_i) \right]^{1/2}$$  \hspace{1cm} (D3)

and this gives us an estimate of $\sigma$ at a given downstream location, $x_i$.

Values of $\chi(y_i)$ corresponding to $y$ locations outside of the range of measurements are estimated based on an assumed symmetry at the plume centreline.

The method described above is also used to estimate $\sigma_z$, with symmetry assumed at the ground plane where appropriate.
APPENDIX E:  UNCERTAINTY ANALYSIS

E.1 TYPES OF ERRORS

Results presented in Chapters 5 and 6 are subject to uncertainties created by systematic or bias errors \( (B_i) \), and precision errors \( (P_i) \).

Bias errors relate to the displacement of a measured value from its true value. Bias errors are considered to remain constant for a given experiment, and must be estimated from experience, for each set up.

In contrast, precision errors pertain to changes in the measured quantity within experimental runs. Precision errors may be estimated by statistical methods, for a given experiment.

Total bias and precision errors for a given variable may result from a combination of various uncorrelated error sources. Such individual error sources may be combined by a root-sum-square method as described by Coleman and Steele (1989), i.e.

\[
B^2 = (B_1^2 + B_2^2 + B_3^2 + \ldots) \quad (E1)
\]

\[
P^2 = (P_1^2 + P_2^2 + P_3^2 + \ldots) \quad (E2)
\]

The total uncertainty of a measured result is a combination of the bias and precision errors. The combined uncertainty is estimated after Albernethy et. al. (1985) as

\[
U = \left( B^2 + (tP)^2 \right)^{\frac{1}{2}} \quad (E3)
\]

where \( t \) is a Student's \( t \) multiplier taken as \( t = 2 \) for a 95% confidence interval in sample sizes greater than 30.
E.2 PROPAGATION OF ERRORS THROUGH A RESULTANT

When several variables are combined in an equation, the total uncertainty of each variable will propagate through to give a unique uncertainty to the resultant. For a resultant \( r \), given arbitrarily as

\[
r = r(U_1, U_2)
\]

we use the method of Coleman and Steele (1989) to estimate the uncertainty of \( r \)

\[
\left( \frac{B_r}{r} \right)^2 = \left( \frac{1}{r} \frac{\partial r}{\partial U_1} B_1 \right)^2 + \left( \frac{1}{r} \frac{\partial r}{\partial U_2} B_2 \right)^2 + 2 \frac{\partial r}{\partial U_1} \frac{\partial r}{\partial U_2} \rho_{12} B_1 B_2
\]

(E4)

where \( \rho_{12} \) is a correlation coefficient for the variables in \( r \).

A similar expression is used for the precision error, \( P_r \), and the total uncertainty of the resultant is then estimated by (E3).

E.3 HOT WIRE MEASUREMENTS

E.3.1 Bias Errors

Bias errors considered significant in the present study consisted of the following:

1. Calibration errors
2. Reverse flow truncation errors

Other sources of bias error were determined to be insignificant in the present study. Among those sources of error in this category were probe misalignment, changes in ambient conditions from calibration to test, and electronic filter offset drift. The reader is directed towards one of the many excellent references on the errors associated with hot wire measurements for a more complete description (e.g. Bruun (1995)).
Calibration Errors

Probe calibration was conducted very carefully according to the manufacturer's guidelines and using a TSI model 1125 probe calibrator. The results of the calibration are presented in Figure E1 for the X-array probe, and Figure E2 for the single wire probe.

Agreement of calibrations conducted before the experiments with those conducted after the experiments, more than 1 week later, is very good. In-tunnel calibrations were conducted several times a day to ensure the consistency of the equipment. Based on the calibration results and the manufacturer's documentation, systematic error in mean velocity measurements attributed to the calibration procedure is estimated as 5%, or

\[ B_{ucal} = 0.05 \]

Reverse Flow Truncation Error

Hot wire probes are insensitive to flow direction and will give erroneous results in reversing flow situations. For highly turbulent flows of greater than \(-35\%\) intensity, a small percentage of the flow may actually be in the opposite direction to the mean flow. Thus, errors may result even when the mean flow is not reversed. In these situations a bias will result in both the mean and turbulent (rms) velocities measured. By assuming a normal probability density function for the instantaneous velocity measurements, it is possible to estimate this bias as a function of the measured intensity. For the present experiments a maximum intensity of 48% was obtained. At this level the bias error due to truncation of reversing flows was estimated to be 3.6% in the rms, and 0.8% in the mean, i.e.

\[ B_{urff} = 0.036 \]

\[ B_{urff} = 0.008 \]
E.3.2 Precision Errors

Precision errors are estimated using the statistical methods of Castro (1989). For mean velocity the precision error is estimated by (E5)

\[ P_U = 100 \frac{z_{\alpha/2}}{\sqrt{N}} \frac{\bar{u}}{u} \]  

(E5)

where \( z_{\alpha/2} \) is the standard normal variate of 1.96 for a 95% confidence interval, \( N \) is the number of samples used to estimate \( u \) and \( \bar{u} \) \((N=30000)\).

In the present experiments the precision error for the mean velocity was estimated to be in the range of 0.11% to 0.54%, therefore,

\[ P_U = 0.0054 \]

Precision error for the rms velocity is estimated after Castro (1989), by (E6)

\[ P_{\bar{u}} = 100 \frac{z_{\alpha/2}}{\sqrt{2N}} \]  

(E6)

and is calculated for the present experiments to be 0.8%, or,

\[ P_{\bar{u}} = 0.008 \]

E.3.3 Total Uncertainties for Flow Velocity Measurements

Using the preceding uncertainties and the methods of Section E1, the results of Chapter 5 are presented subject to the maximum uncertainties outlined in Table E1.
Table E1: Maximum Uncertainties for Hot Wire Measurements

<table>
<thead>
<tr>
<th></th>
<th>Bias Error</th>
<th>Precision Error</th>
<th>Total Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>0.051</td>
<td>0.0054</td>
<td>0.052</td>
</tr>
<tr>
<td>$\bar{u}$</td>
<td>0.036</td>
<td>0.008</td>
<td>0.039</td>
</tr>
<tr>
<td>$\bar{u}/u$</td>
<td>0.062</td>
<td>0.010</td>
<td>0.065</td>
</tr>
<tr>
<td>$u_w - u_h$</td>
<td>0.072</td>
<td>0.0076</td>
<td>0.074</td>
</tr>
<tr>
<td>$(\bar{u}/u)_w - (\bar{u}/u)_h$</td>
<td>0.087</td>
<td>0.014</td>
<td>0.091</td>
</tr>
</tbody>
</table>

E.4 CONCENTRATION MEASUREMENTS

E.4.1 Bias Errors for FID Measurements

Results of FID measurements as presented in Chapter 6 are subject to significant bias errors as a result of the following:

1. Calibration errors
2. Ambient accumulation of hydrocarbons

Additional systematic errors resulting from electronic filter offset drift, and hydrocarbon analyser offset drift, were determined to be negligible for the experiments.

Calibration Errors

The Cambustion fast FID units were calibrated and operated according to the manufacturer's guidelines. Typical calibration results for electronic gain settings of 100V/µA and 500 V/µA are presented as Figure E4 and E5 respectively. Such calibrations were conducted several times per day to minimize systematic error. As a
result of the calibration results and the manufacturer's documentation, bias error in the mean concentration due to calibration error was determined to range from 2% to 10%, therefore,

\[ B_{x_{cal}} = 0.10 \]

**Ambient Accumulation of Hydrocarbons**

A systematic bias of the measured concentrations may result due to an accumulation of hydrocarbons in the ambient flow. To minimize this problem measurements of ambient concentration were made at regular intervals throughout the experiments. Mean concentrations are always reported relative to the background levels. Bias error in measurements of background levels are perfectly correlated to the bias errors in the wake measurements and no additional uncertainty is represented by this effect.

**E.4.2 Precision Errors for FID Measurements**

Precision errors for the mean contaminant concentrations are estimated in the same manner as for the mean velocity, however, fluctuations in concentration are much more significant than for velocity measurements. Concentration intensities as high as 100% are frequently measured resulting in an estimated precision error in the mean of 1.1%, or

\[ P_x = 0.011 \]

**E.4.3 Errors in Contaminant Flow Rate**

The contaminant flow rate \( Q \) was obtained from the Linde FM4575 mass flowmeter/flow controller as reported in Chapter 3. The manufacturer reports the following uncertainty levels pertaining to the equipment,

\[ B_Q = 0.030 \]
\[ P_Q = 0.0025 \]
E.4.4 Total Uncertainties for Concentration Measurements

Using the preceding uncertainties and the methods of Section E1, the results of Chapter 6 are presented subject to maximum uncertainties as outlined in Table E2.

<table>
<thead>
<tr>
<th></th>
<th>Bias Error</th>
<th>Precision Error</th>
<th>Total Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi$</td>
<td>0.100</td>
<td>0.011</td>
<td>0.101</td>
</tr>
<tr>
<td>$Q$</td>
<td>0.030</td>
<td>0.0025</td>
<td>0.030</td>
</tr>
<tr>
<td>$K$</td>
<td>0.153</td>
<td>0.013</td>
<td>0.154</td>
</tr>
</tbody>
</table>
Figure E1: Calibration curves for X-array hot wire probes

Probe a, \( \text{(m/s)} = 1.90757(V) + 0.293296 \)

Probe b, \( \text{(m/s)} = 2.00892(V) + 0.258628 \)
Figure E2: Calibration curves for single wire hot wire probes
Figure E3:  Typical calibration curves for FID probes, Gain = 100V/μA
Figure E4: Typical calibration curves for FID probes, Gain = 500V/μA
APPENDIX F: DIMENSIONAL CONSIDERATIONS

Martinuzzi (1996) suggests the following development for the decay of excess turbulence intensity in the far wake:

Consider the transport equation for turbulent kinetic energy \( k \), given as (F1)

\[
\frac{Dk}{Dt} = \frac{\partial}{\partial x_i} \left[ -u_i u_j \frac{\partial k}{\partial x_j} - p \frac{\partial u_i}{\partial x_j} + v \frac{\partial k}{\partial x_j} \right] + \left( -u_i \frac{\partial U_j}{\partial x_j} \right) - \varepsilon \tag{F1}
\]

Assume that the diffusion and production of turbulent kinetic energy (the first two terms on the right hand side of (F1)) are negligible in the far wake, leaving only the dissipation term \( \varepsilon \).

Further assume that the left hand side may be approximated as

\[
\frac{Dk}{Dt} \approx u \frac{dk}{dx}
\]

such that (F1) becomes

\[
u \frac{dk}{dx} = \varepsilon \tag{F2}
\]

Now observe that the dissipation term, \( \varepsilon \), has the units of velocity\(^3\)/length so that we may suppose that

\[
\varepsilon \propto \frac{u_{perr}^3}{h}
\]

from which it follows that (F2) suggests

\[
k \propto \frac{u_{perr}^3 x}{u h} = C \left( \frac{u_{perr}}{u} \right)^3 x \tag{F3}
\]

Substituting the observed single structure decay rate of -3/2 into (F3) we get

\[
k \propto \left( \frac{x}{h} \right)^{-3/2} x \quad \text{or} \quad k \propto x^{\frac{7}{2}}
\]
Defining turbulent kinetic energy as \( k = 0.5 \left( u'^2 + v'^2 + w'^2 \right) \) it follows that \( \bar{u} \propto k^{1/2} \) or

\[
\bar{u} \propto x^{-1.75}
\]  \hspace{1cm} (F4)

which agrees well with the -1.78 exponent obtained in the present experiments.

It is interesting to note that if we used a decay rate of -1.0 for the mean velocity deficit in (F3), we would expect a variation of \( \bar{u} \propto x^{-1.0} \). Such a decay rate may be expected for strongly two dimensional structures.