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UNSTABLE VELOCITY, VOLATILE EXCHANGE RATES, AND CURRENCY SUBSTITUTION: THE DEMAND FOR MONEY IN A MULTICURRENCY WORLD

by

Stephen Shawn Poloz

Department of Economics

Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy

Faculty of Graduate Studies
The University of Western Ontario
London, Ontario
October, 1981

ABSTRACT

The Thesis examines the problem of recent instability in the demand for money functions of the major OECD economies. Several clues in the literature lead to consideration of currency substitution, the presumption being that recent increases in the latter have resulted in unexpected exchange rate volatility as well as shifts in the world demand for money equations. The Thesis investigates the microeconomic foundations of the demand for money in a world where transactions occur in more than one currency, and where portfolios include foreign currency deposits. Three distinct but complementary theories are developed, each based on one of the three Keynesian motives for holding money: the transactions, precautionary and speculative motives. A variety of testable, discriminating hypotheses are derived.

The Thesis tests the predictions for both narrow and broad definitions of the money stock, for two alternative adjustment specifications of the demand for money, using data for Canada, the United States, and Germany. While the results for the popular 'partial-adjustment' model are largely negative, those for a 'polynomial distributed lag' model are supportive of the hypotheses derived in the theoretical chapters. In particular, expected rates of depreciation, foreign interest rates and foreign income are jointly statistically significant at the 0.95 level, indicating that a significant proportion of the variation in the demand for real cash
balances may be explained by 'currency substitution'.

Subsequently, the reliability of the demand for money specifications which take account of foreign influences is compared with that of traditional 'one-money' equations, by conducting ex-post forecasting experiments. It is found that the traditional specification performs best for Canada, while the expanded specification, which incorporates the hypotheses of this Thesis, achieves a lower root-mean-squared forecast error for both Germany and the United States.
ACKNOWLEDGEMENT

I should like to thank my Thesis Supervisor, Michael Parkin, and the other members of my Thesis Committee, Peter Howitt and Robin Carter, for their generous assistance in this undertaking. Helpful comments and criticisms, for which I am very grateful, were proffered by Bill Alexander, Jon Cockerline, Jim Frazer, Joel Fried, Tim Lane, John Murray, Ron Wirick, and Bing-Sun Wong, as well as the participants of the Money Workshop at The University of Western Ontario. I also benefitted from several fruitful discussions with David Laidler. Thanks also go out to Trien Nguyen, Gary Tompkins and Randy Wigle, all of whom provided lessons in mathematics from time to time.

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I bear full responsibility for that which follows.
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INTRODUCTION

A considerable amount of empirical work on the demand for money has been undertaken and published since the appearance of the late Harry G. Johnson's well-known survey article on monetary theory and policy. All of this work may be viewed as attempting to answer one or more of the questions which Johnson outlined (Johnson, 1962, 344-5), namely:

(a) What specific collection of assets corresponds most closely to the theoretical concept of money?

(b) What are the variables on which the demand for money, so defined, depends?

(c) Is the demand for money sufficiently stable to provide a better explanation of observable movements of money income than is provided by the income-expenditure models?

As pointed out by Khan (1974), these three issues are not mutually independent, so that the answers to any one of them depend upon what is presumed with regard to the other two, rendering empirical investigation difficult. However, barely a decade after the appearance of Johnson's article, an exhaustive study by Goldfeld (1973) drew together and built upon previous work to conclude that the above three issues had been resolved, at least for the U.S. data. Remarkably, the demand for real M1 (currency plus non-government demand deposits) was found to be a stable function of comparatively few variables: current real national income, the rate of interest on time deposits, the commercial paper rate, and the lagged dependent variable, the latter presumed to be reflecting partial stock adjustment.
Since 1973, however, instability of the demand for money function (in the sense that the regression coefficients have been found to change significantly over time) has been observed in the data for Canada (Cameron, 1979, for example), the United States (Goldfeld, 1976), the United Kingdom (Hacche, 1974), Australia (Lewis, 1978) and France and Italy (Boughton, 1979b). The situation with the U.S. data is of particular importance, since the relative size of that economy implies that disturbances which originate there inevitably spill over into all of the major economies, especially Canada. Furthermore, the observed periods of velocity instability approximately coincide with the commencement of money supply targeting in many of the major OECD economies. Since this phenomenon quite naturally has given rise to doubts about the efficacy of money supply targeting, it is important to see whether the instances of velocity instability may be explained so as to render the policy more effective.

It is with these problems that this Thesis is concerned. The Thesis has three parts: the first puts the study into perspective, the second develops the theory of the demand for money in a multicurrency world, and the third estimates the resulting demand for money functions and tests the hypotheses embodied in them. Chapter One documents the various instances of observed instability in the demand for money functions and shows that the numerous attempts to account for these instabilities have had only limited success. A new clue to the possible source of the instabilities is found in a recent paper by Brittain (1980), who reports that movements in the income velocity of money demand are correlated, sometimes strongly so, across the major
OECD economies. These results, combined with the observed volatility of exchange rates during the same period, prompt the suggestion that the instability in the demand for money function may be the consequence of currency substitution. A perusal of the currency substitution literature reveals a set of predictions which conform, in large measure, with the very phenomena which the demand for money literature seeks to explain. It is argued that, although the empirical evidence so far has been inconclusive in respect of the currency substitution hypothesis, there seems to be sufficient empirical support to warrant further investigation.

The following three chapters investigate the theoretical properties of the demand for money in a multicurrency setting. Chapters Two, Three and Four analyze, respectively, the transactions, precautionary and speculative motives for holding money in an environment in which the economic agent finds it necessary to use two currencies. The purpose in exploiting the Keynesian distinction between the three motives for holding cash, rather than simply considering two types of real balances as arguments of a utility function, is to derive the currency substitution hypothesis from first principles in a more restricted, more carefully defined setting, thereby analyzing its implications at a more fundamental level than that already explored in the existing literature. These theories are shown to be capable of explaining, qualitatively at least, the observed instability in the demand for money function. These results are brought together in Chapter Five; at the same time the relative merits of two broad approaches to the modelling of the demand for money are
discussed, that of this Thesis, namely the 'Keynesian motives' approach, and the 'marginal utility' approach, which places money in the utility function. Particular attention is paid to the relative precision of the predictions which may be derived from the two approaches.

The final three chapters (Part III) deal with empirical matters. Chapter Six analyzes in some detail the implications of the preceding theoretical analyses. It is shown that the data are broadly supportive of the hypotheses derived in Part II. In Chapter Seven an important distinction is drawn—that between statistical significance and policy significance. The results of a number of forecasting experiments are reported which indicate that the currency substitution hypothesis presently bears no significance for monetary policy in Canada, but that both the United States and Germany would probably find it worthwhile to consider foreign influences on the demand for real balances in formulating policy.

Finally, Chapter Eight offers some concluding remarks. Special attention is paid to the relationship between theories of the demand for money, the tradeoff between parsimony and relative stability of demand for money specifications, and a number of suggestions for further research are offered.
PART I:

PERSPECTIVE
CHAPTER ONE

UNSTABLE VELOCITY, VOLATILE EXCHANGE RATES
AND CURRENCY SUBSTITUTION

1.1 A Survey

(a) Introduction

Three distinct bodies of literature are relevant to this Thesis and are surveyed here. The first (Section (b)) contains the empirical evidence relating to the stability (or lack thereof) of the demand for money function. This evidence is mixed, a reflection of the interdependence between the issues outlined in Johnson (1962) and of the fact that the various researchers have made different presumptions with regard to issues (a) and (b) while attempting to resolve issue (c). However, the general conclusion which emerges is that the standard specification of the demand for money equation (as described above on page 1) does not explain the data for the 1970's satisfactorily, particularly in the U.S. but in several other countries as well. In the second body of literature (Section (c)) are found various attempts to achieve stable demand for money specifications. There has been some progress but not success. It is concluded that the evidence rather strongly indicates that international forces may be responsible, in part at least, for shifts in the demand for money functions around the world. This leads to consideration of the third body of literature (Section (d)), that on currency substitution, most
of which is relatively recent. Although the theoretical literature on this topic has provided strong predictions, the associated empirical work has met with only limited success. Section (e) provides a summary.

(b) The Stability of the Demand for Money

The reliability of investigations into the stability of regression relationships has improved substantially in the past decade. Prior to 1975 the only available econometric test for stability was the analysis of covariance, or Chow test, which is found in most econometrics textbooks (see Johnston, 1972, 207, for example). This procedure tests the null hypothesis that an additional set of observations belongs to the same regime as the pre-existing set. The difficulty with the test is that it requires that the researcher choose, a priori, a point at which a structural shift may have occurred. Moreover, if there is more than one such point care must be taken to nest the hypotheses properly. For these reasons some of the early research on the stability of the demand for money may have been misleading; for the purposes of this Thesis, then, it would seem preferable to focus on the more recent studies.

Three distinct approaches to stability testing have become popular in the past decade. The first, proposed by Brown, Durbin and Evans (1975) (BDE), is comprehensive, consisting of several tests (some of which were developed previously by other authors). The innovative ingredient of the approach is the recursive regression, where regressions are computed stepwise using increasing numbers of observations, and the 'recursive residuals' which are computed in the
process. A shift in the estimated regression coefficients is detected when one of the recursive residuals exceeds a critical level. The second is by Cooley and Prescott (1976) (CP) and entails the estimation of an equation under the presumption that the parameters are random variables, and asking whether at any point in time the parameters have deviated significantly from their means. The relative advantages and disadvantages of these two approaches have been thoroughly analysed both theoretically and by Monte Carlo experiment by Garbade (1977) and Cockerline (1978). The third approach is not really a formal stability test, but consists of performing out-of-sample forecasts to check the reliability of the equation. This approach has been used quite extensively for it simulates the sort of procedure which the policymaker must follow in practice.

For the Canadian demand for money function there is considerable evidence of instability. Using the BDE tests, Cockerline (1978) has found a shift in M1 in 1976, coinciding roughly with the adoption of M1 growth targets by the Bank of Canada. Shifts in the M2 (M1 plus savings deposits) function were detected in 1967 (the new Bank Act) and 1972 (the Winnipeg Agreement). Poloz (1979) also detected shifts in the M1 function in 1958 (the Conversion Loan) and in 1962 (the Diefenbaker exchange rate crisis) using the BDE tests. The conclusions of these studies have been amplified in some respects by Cameron (1979), who has reported instability for both M1 and M2 early in 1971, and also in 1978 for M2; and by Boughton (1979b), who has isolated an M1 shift in 1976 and an M2 shift in 1967. Finally, Rausser and Laumas (1976) have used the CP approach to find M1
unstable, M2 stable, while White (1976) has used ex-post forecasting to find a shift in the M1 demand equation in early to mid-1971.

For the U.K., Hacche (1974) has reported instability in the demand for M1 and M3 (M1 plus all time deposits) roughly coincident with the implementation of the Competition and Credit Control measures by the Bank of England in 1971. The results for M3 have been confirmed by the BDE tests used by Boughton (1979b). Lewis (1978) has found instability in the Australian money demand equation during the period 1969–73, whereas Pagan (1977) evidently has isolated a stable Australian demand function based on the BDE tests.

Results of BDE tests on the demand for money in the major OECD economies have been reported by Boughton (1979b). For France, there are several points in time where shifts in M1 and M2 demands have occurred. For the most part these may be explained by institutional changes, such as the prohibition of interest payments on demand deposits in 1967, the devaluation of the Franc in 1969 and exchange controls during the early 1970's. The demand for money in Germany appears to have been universally stable; at the same time, the demand for M1 in Japan has drifted since 1970 and the demand for M2 shifted in 1977, the latter shift apparently caused by a change in policy by commercial banks away from compensating balance requirements.

Early stability tests on the demand for money in the U.S. failed to find instability. Khan (1974) used the BDE techniques on annual data from 1901–65 and found M1 and M2 to be stable, with perhaps a slight aberration in 1948. Laumas and Mehra (1976) used the CP procedure on annual data from 1900–74 and obtained the same results.
Goldfeld (1973) used within-sample simulation to find that his simple formulation (described above in the Introduction) was stable for quarterly data on M1 and M2.

However, late in 1973 the Goldfeld equation began to overpredict the demand for M1 rather significantly, until Goldfeld (1976) reported a 9.8% ($22 billion) overprediction in 1976I. Boughton (1979b) confirmed these results, but found that M2 was much better behaved. Moreover, Boughton's tests indicated that the instability in M1 probably began in 1970 or 1971, not 1973 or 1974, perhaps suggesting that the dominant cause was the switch to money stock targeting rather than institutional changes which occurred later.² (c) Explaining the Observed Shifts

For the most part, the periodic shifts in demand for money functions which have been isolated coincide with some institutional change in the respective country's financial system. Some shifts, however, did not so coincide. Because of this, subsequent attention has been focused on the shifts which are less easy to explain: the 1976 shift in the Canadian M1 equation, the 1971 shift in the U.K. M3 equation, and that in the U.S. M1 equation.

Some indirect evidence on the 1976 shift in the Canadian M1 equation has been reported by Poloz (1980). There it was shown that the extent of simultaneous equations bias in the ordinary least squares (OLS) estimate of the equation had shifted quite dramatically in 1976II, precisely when the announced policy of money supply targeting began to take effect. Since BDE tests on the two-stage least squares (2SLS) estimate of the equation (Poloz, 1979) indicated that the equation was
stable through the period in question, it would appear that the demand for money function was misspecified. A complete model of the demand for money, including other equations which contain the exogenous variables which Poloz has used as instruments, evidently would be stable through the period.

The instability in the demand for M3 in the U.K. has been explained by Artis and Lewis (1976). Their hypothesis is that the money stock, rather than being demand-determined in the short run, is exogenous, and it is the rate of interest which is endogenous over the relevant horizon. Thus, they renormalize the demand for money equation on the rate of interest, and add an interest rate partial-adjustment mechanism. The resulting equation works well even through the 'unstable' period of the other formulation.

Attempts to explain the shift in the U.S. M1 function have been less successful. When Goldfeld (1976) first documented the seriousness of the shift he made several attempts to account for the 'missing money'. In particular, he noted that the demand for currency had remained stable, so that the source of instability was in the demand deposit portion of M1. Goldfeld was able to reduce the average prediction error over the ten quarters 1973IV to 1976II from $13 billion to about $5 billion by formulating the partial-adjustment process in nominal rather than real terms, using bank debits rather than GNE as a measure of transactions, and introducing 'previous peak' variables for income and interest rates. The remaining error has been attributed principally to the growth of NOW accounts, and in chequing accounts at mutual savings banks. Enzler, Johnson and Paulus (1976)
considered aggregation problems, difficulties in measuring the own rate of return on money, and changes in brokerage fees. However, in the end there was still a 10% ($22 billion) error in their equation's prediction for 1976I. Additional institutional explanations have been offered by Garcia and Pak (1978), who have stressed the apparent shift in business demand for money, due in part to the supply shocks of the early 1970's, which resulted in a redistribution of income in favour of agricultural and oil producers. Moreover, it seems that overnight investment of business demand deposits by banks has caused the measured money stock to be somewhat less than its effective daytime level. Thus, measured velocity would be higher than actual velocity.

Hamburger (1977b) has argued that undue emphasis on the transactions theory of the demand for money has led to use of the incorrect rate of interest in the money demand specification. Regarding money as a substitute for other durable goods such as physical capital rather than as a substitute for short-term bonds has led Hamburger to use a long-term rate of interest in his money demand equation. This change reduced the 1976I prediction error to $9 billion, a substantial improvement. Like Hamburger, Heller and Khan (1979) and Khan (1979) have attempted to shift the focus of research away from mere institutional explanations for the shift. In the former paper it is argued that whether one uses a short rate or a long rate of interest in the demand for money equation, using only one rate presumes that the entire term structure of interest rates is stable. Knowing this to be the case only rarely, Heller and Khan have devised a method whereby the entire term structure is included in
the regression while the usual problem of multicollinearity is avoided. According to the BDE stability tests, this specification is stable through 1972-74 at the 95% level of confidence. In the second paper, Klein (1979) argued in favour of including the level of the interest rate, rather than the logarithm, so allowing the interest elasticity of the demand for money to rise with the rate of interest. Adding this variation evidently produced a demand equation which was stable relative to the Goldfeld (1976) and Hamburger (1977b) models, for it simulated better after 1975 than these others.

This issue of logarithms versus levels of the rate of interest was also taken up by Laidler (1980). His results indicated that changing from logarithms to levels did improve the prediction errors after 1972, but on every other statistical criterion the semilogarithmic formulation was found to be inferior. Laidler went on to reconsider the work of Klein (1977) who hypothesized that uncertainty regarding the rate of inflation would reduce the quality of monetary services yielded by a given level of cash balances. Using a five-observation moving standard deviation from a ten-observation moving average of the rate of inflation as a proxy, Klein has established a positive relationship between this variable and the demand for money. Using Klein's data, Laidler has found that the predictions of the new equation after 1972 were extremely poor, and that the addition of four more years of data has changed the sign of the coefficient on Klein's variable. Thus, it would seem that the instability after 1972 was not caused by variations in the quality of money as measured by Klein.

Other key results presented by Laidler (1980) are:
(i) M2 demand equations perform much better after 1972 and, when a long rate of interest is included, the instability is almost completely accounted for.

(ii) the approach of Artis and Lewis (1976), which worked so well for the U.K., works not at all for the U.S., perhaps casting some doubt on its validity in general.

(iii) the various experiments performed indicate that, while institutional changes may have caused a shift in the demand for money function, specification problems are at work as well. Moreover, these latter problems relate to more than just an incorrect choice of interest rate, for Laidler's alternative stock adjustment specifications indicated that when real balances, rather than velocity, adjust towards a long-run equilibrium, the function performs better.

Further evidence that the U.S. equation, along with those of other countries, may be misspecified, is presented by Brittain (1980). The paper is based on the data in Table 1.1.1, (reproduced from Brittain, 1980, Table 2), and a possible explanation thereof. The large negative correlations are of particular interest, especially that between the U.S. and Germany, for they suggest that substitution between national monies may have been taking place on the demand side.

<table>
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<th>Italy</th>
<th>Switzerland</th>
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<td>-0.351</td>
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</tbody>
</table>

TABLE 1.1.1

CORRELATION COEFFICIENTS BETWEEN DEVIATIONS OF MONEY'S INCOME VELOCITY OF CIRCULATION FOR FIVE COUNTRIES
It is conceivable, therefore, that the currency substitution hypothesis will be capable of explaining the observed instabilities in the world demand for money equations which have been documented above.

(d) Currency Substitution

Although currency substitution has received considerable attention recently, the fundamental ideas evidently are not new, for they are contained in Boyer (1972). The essential ingredients of the argument are as follows. Since monies are assets, the decision to hold them depends not on their current relative prices (exchange rates) but on their current relative expected rates of return (expected rate of change in their exchange rates). In the limiting case where currencies are perfect substitutes any expectation that one may depreciate in value relative to the others will cause its demand to fall to zero. Hence, high currency substitution can lead to instability of the world financial system—a convincing argument for fixed exchange rates.

There seems to be fairly broad agreement on how one should model currency substitution. However, it is important to distinguish between 'effective' and 'direct' currency substitution, a distinction brought out by Boyer and Kingston (1980). Effective currency substitution may be derived from two conventional, currency-specific demand for money functions and the assumption of purchasing power parity. It is well-known that this will lead the relative demands for the two monies to depend upon the rate of change in the exchange rate. The implications of effective currency substitution, modelled in this way, have been explored in Calvo and Rodríguez (1977). On the other hand, direct currency substitution is modelled by the
inclusion of the expected rate of change of the exchange rate explicitly in each demand for money equation. It is this extension of the modelling of the demand for money which is of interest here. Thus, in a two-money model, the currency substitution hypothesis is that an increase in the expected rate of depreciation of one currency will cause a decline in its own demand, and an increase in the demand for the other currency.

The theoretical implications of direct currency substitution have been explored by Boyer (1978); King, Putnam and Wilford (1978); Girtón and Roper (1980); Kareken and Wallace (1980); and Boyer and Kingston (1980). A detailed description of this work would serve little purpose here, but it will be useful to consider the principal results.

(i) The degree of world currency substitution is limited by the degree of integration of world goods and capital markets. Thus, casual empiricism would lead one to expect the degree to which currencies are substitutable to have increased in the past decade.

(ii) If currency substitution were perfect, although the real quantity of money in the world would be well-defined, the nominal supplies of all currencies would be indeterminate, as would each price level and exchange rate. Indeed, as currency substitutability increases, fluctuations in exchange rates due to interest rate differentials become unbounded.

(iii) Currency substitution can seriously undermine or even eliminate the 'monetary independence' usually attributed to flexible exchange rate regimes. Official attempts to control liquidity may be frustrated by substitution on the demand side, while monetary shocks will be transformed quickly into exchange rate movements.

The implications of high currency substitutability, therefore, are nontrivial; thus, it is not surprising that there has been considerable effort made to measure the degree of currency substitution. On this
issue, however, there is considerable disagreement in the literature.

Evans and Laffer (1977) may have been the first to test the currency substitution hypothesis. They postulated that the demand for one currency depends on domestic income and a vector of the expected rates of depreciation of all other currencies. Utilizing the assumptions of purchasing power parity, exogenous money supplies, and symmetry of the interest rate responses, they derived an estimable exchange rate equation. The latter was estimated for Canada, France, Italy, Japan, and the U.K., with the U.S. as the reference country, with poor results. The point estimates indicate a high degree of currency substitution, but these estimates are statistically unreliable. Further, it is not clear why the authors did not simply estimate the system of demand for money equations to test for the degree of currency substitution directly.

More convincing, perhaps, is the evidence which has been brought to light by Hamburger (1977a). Without explicitly intending to test for currency substitution, Hamburger has found that foreign rates of interest are significant determinants of the demand for money in open economies, especially in Germany and the U.K. In addition, Brittain (1980) (who had every intention of testing for currency substitution) included domestic/foreign uncovered interest rate differentials in demand for money equations for Germany, the U.S. and the U.K., and found statistical significance in each case. These results, combined with his observed correlations in velocity shifts between the U.S. and Germany (see Section (c) above) represent strong evidence of currency substitution.

Miles (1978a, b) has presented results which seem to
indicate that currency substitution between Canada and the U.S. is high. He postulated a CES money services production function; maximizing the function subject to a wealth constraint yields the relative demand for money equation:

\[(1.1.1) \quad \log(M/SM^*) = \alpha + \beta \log\left[\frac{1+r^*}{1+r}\right]\]

where the asterisks denote foreign variables and \(S\) is the spot exchange rate, \(r\) is the interest rate, and \(M\) represents the money stock. Estimation of this equation for Canada and the U.S. yields high and statistically significant (during periods of exchange rate floating) estimates of \(\beta\). Bordo and Choudhri (1980), however, have argued that Equation (1.1.1) is misspecified, since it omits a scale variable (income). Correcting for this misspecification, they find \(\beta\) to be statistically not significant. Moreover, they have estimated a standard demand for money equation, with the addition of the expected rate of depreciation as an explanatory variable (proxied by the proportional difference between the forward and spot exchange rates) and have found the latter to be statistically not significant for the quarterly Canadian data. This result has been confirmed for Canada and the U.S. by Giddy and Schadler (1980). Two reasons are given in the latter paper for the apparent absence of currency substitution. First, it could be that inertia and uncertainty render currency substitution unimportant at the relatively low rates of inflation normally found in Canada and the U.S., and second, the well-developed Canadian and American capital and commodity markets provide other substitutes for depreciating currency.
Stronger evidence of currency substitution in the Canadian demand for money function has been presented by Alexander (1980). In this paper Alexander tests the statistical significance of two additional variables in the monthly demand for M1 equation. Both the proportional forward premium and the variance of the Canadian-U.S. exchange rate were found to be statistically significant. However, the importance of currency substitution for policy is measured not by statistical significance but by the size of the semi-elasticity of the demand for money with respect to the expected rate of depreciation. This number is found to be very small, implying that currency substitutability does not pose a policy problem of any significant magnitude for Canada.

A full-scale currency substitution model of exchange rate determination has been derived and tested by Brillembourg and Schadler (1979). These authors deployed a Brainard-Tobin portfolio model, assuming the demand for each financial asset to be a function of all rates of return in the portfolio and wealth. They went on to assume that the decision on what currencies to hold in the portfolio may be broken down into two stages. First, the agent decides on total cash holdings, second on how to allocate these holdings between the various currencies available. The latter stage may be viewed as determining the excess demand for each currency as a proportion of world currency holdings; these equations are rearranged so that they may be treated as exchange rate determination equations rather than demand for money equations. Full information maximum likelihood estimation of eight such equations simultaneously, with symmetry
imposed on the cross-elasticities of rates of return, yields two principal results:

(i) a restricted version of the model with the cross-elasticities constrained to zero is rejected by the data in favour of the more general model, lending strong support to the currency substitution hypothesis.

(ii) the cross-elasticities reveal that the continental European currencies behave as complements, rather than substitutes, while this same group of currencies generally behave as substitutes for the North American currencies.

These results are telling us that currency substitution has been occurring to a statistically significant degree, but at the same time the imprecision of the estimated coefficients makes it difficult to determine the extent of the phenomenon, and therefore its relevance for policy in any particular country. It should be clear that in order to improve upon this situation it will be necessary to do at least one of two things. First, it should be possible to sharpen our inferences from the data by strengthening our priors; that is, by introducing theoretical refinements so as to derive testable hypotheses which are more precise. Second, one could accept the present level of theoretical sophistication and attempt to improve upon the data and/or the statistical techniques which are used. This Thesis may be seen as an attempt to do both.

(e) Summary

The above survey of the literature may be summarized very simply by the following four points.

(i) The demand for money functions of the major economies have exhibited temporal instability in the last decade, only part of which has been explained by institutional shocks.

(ii) A point not covered above but which is often mentioned in the literature is that exchange rates, which have been floating
for approximately the same period, have exhibited a high degree of volatility, as may be seen in the graphs of Appendix A1.

(iii) The theory of currency substitution indicates that increased demand substitutability of currencies will result in a loss of monetary independence through shifts in demand from one money to another, and will also result in wider swings in exchange rates.

(iv) Although there is a significant amount of empirical evidence of the existence of currency substitution, its extent and therefore its relevance for policy have yet to be fully considered.

Speculation that demand substitutability between currencies may have increased during the past decade because of evasion of monetary targets and the frequent weakness of the U.S. dollar has appeared elsewhere in the literature (McClam, 1980; McKinnon, 1980; Vaubel, 1980). It seems to be a reasonable suggestion, given the evidence presented above, that this increase in currency substitution has been responsible both for the volatility of exchange rates and the periodic shifts in velocity which have been documented. Since the principle of money supply targeting, as it is currently implemented, rests on a stable demand for money function, the monetary policy regime which presently dominates the major economies may be in danger of being relaxed, or even abandoned.

Avoidance of this turn of events will require a greater understanding of the demand for money and of its role in determining fluctuations in exchange rates. It is the purpose of this thesis to contribute towards the realization of this goal.

1.2 Scope of the Thesis

Virtually all the existing currency substitution literature has taken as a point of departure an extension of a
conventional demand for money function, namely, the addition of the expected rate of depreciation as an explanatory variable. Only Boyer and Kingston (1980) and Bordo and Choudhri (1980) have attempted to derive such an extended formulation; however, in both cases the problem has been handled by placing real money balances (of each currency) in the utility function. Although the latter 'marginal utility' approach does provide a reasonable theoretic underpinning for the extended formulation, so far it has not generated testable hypotheses which are strong enough to yield a precise estimate of the degree of currency substitution. In an attempt to sharpen our knowledge in this respect, a potentially promising line of enquiry is that of deriving the demand for money function from a solid microeconomic foundation in a multicurrency setting. The most satisfactory theories of the demand for money in a two-asset (money and bonds) economy have focused on the Hicksian imperfections—transactions costs and uncertainty—as the fundamental reasons for holding real balances. Correspondingly, there is good reason to believe that this approach will result in a more tightly-specified and productive theory of the demand for money in a multicurrency world. It would seem likely to be the case that a theory which focuses on transactions costs and uncertainty will yield predictions which are both more precise and more amenable to empirical testing than those of the marginal utility approach. One of the byproducts of this Thesis will be a test of the latter hypothesis. Further discussion of this issue will be reserved for Chapter Five.

Transactions costs and uncertainty are viewed as market imperfections which give rise to non-zero holdings of money. It is
widely held that these holdings of money may be subdivided into three parts, each of which is identifiable with a distinct motive for holding cash. The distinction between these three motives—the transactions motive, the precautionary motive, and the speculative—was originally due to Keynes (1936, 195-6), but perhaps has been most clearly drawn by Arrow (1958). Briefly, analyses of the transactions motive generally focus upon the lack of synchronization between receipts and expenditures, both of which are known with certainty, and then minimize the costs of managing the inventory of cash. These costs are treated as arising from two separate sources—adjustment (transactions) and holding (opportunity) costs—which are offset one against the other in order to choose an optimal inventory of cash balances. The precautionary motive for holding cash arises from uncertainty with regard to the net-outlay stream. The latter is viewed as stochastic so that the agent can only minimize expected costs of meeting his cash requirements and, as a result, an optimal cushion of cash balances against an unforeseen outlay is held. The speculative motive recognizes uncertainty with regard to prices—in this context, asset prices or holding period yields—as a cause of portfolio diversification, where one of the assets in the portfolio is cash. In the coming chapters the three motives for holding money will be analyzed separately. However, the complementarity between the three analyses is very strong, in the sense that each may be viewed as treating explicitly a deficiency in one or both of the other two.

Thus, the first task of this Thesis will be to provide three theories of the demand for money, each pertaining to one of the
Keynesian motives, with currency substitutability incorporated throughout. These analyses will occupy Chapters Two, Three and Four, respectively, and it will be shown that the currency substitution hypothesis does have strong microeconomic foundations. Many new insights will also be gained in the process.

The Thesis will then go on to test the implications of the theory. The very nature of the problem implies that this endeavour must be global in perspective. However, the empirical analysis will place the greatest emphasis on Canada and the United States—the former for obvious reasons, and the latter because a minimum test for any theory in this field is that it be applicable to the U.S. data.

As mentioned earlier in this Chapter, the policy implications of the currency substitution hypothesis are nontrivial. However, the acid test for the importance of this analysis will be the empirical investigation of Chapters Six and Seven. Even if currency substitution is found to be statistically significant in the data, the size of the effects on the respective domestic economies may be so small as to render them irrelevant. An analysis of these policy implications, therefore, should serve to put the remainder of the Thesis into perspective relative to the literature described in this Chapter.
FOOTNOTES TO CHAPTER ONE

1 For details of this change in policy see The Bank of England Quarterly Bulletin, 1971, Number 2 (June), 189-93.

2 One possible interpretation is that the controls on monetary growth impose a constraint on the growth in profits to financial intermediation, leading to innovations in the industry which provide the market with a close, uncontrolled, substitute. As a consequence, the demand for the controlled asset may shift downward.

3 These instruments included the level of exports, government expenditures, the lagged money stock and, in particular, the U.S. rate of interest.
APPENDIX A1

GRAPHIC SUMMARY OF MOVEMENTS IN EXCHANGE RATES
FIGURE A1.1

RATe OF EXCHANGE OF THE CANADIAN DOLLAR
FIGURE A1.2

RATE OF EXCHANGE OF THE DEUTSCHE MARK
FIGURE A1.3

RATE OF EXCHANGE OF THE FRENCH FRANC
FIGURE A1.4

RATE OF EXCHANGE OF THE BRITISH POUND
PART II:

THEORY
CHAPTER TWO

THE TRANSACTIONS DEMAND FOR MONEY
IN A MULTICURRENCY WORLD

2.1 Introduction

Of the three Keynesian motives for holding money it is undoubtedly the transactions motive which gives rise to the largest component of total holdings of cash, or narrow money. Narrow money, which usually is viewed as consisting of currency and demand deposits, is the medium of exchange, and therefore is essential for the ultimate settlement of transactions. It is not surprising, therefore, to find a considerable literature on the transactions demand for cash. Since the pioneering papers by Baumol (1952) and Tobin (1956) the analysis of transactions balances has become rather sophisticated. For a review of these more recent contributions, the reader is referred to Barro and Fischer (1976). For the purposes of this Chapter, however, the level of sophistication found in the two pioneering articles will be quite adequate.

It is well-known that in a certain world where expenditures and receipts may be perfectly synchronized, transactions balances would be zero. As in Baumol (1952), the analysis to follow will assume that expenditures are continuous and are made at a constant rate through time, while receipts are regular but 'lumpy'. This lack of synchronization will give rise to the transactions demand for cash.
The presence of an interest-bearing asset will imply an opportunity, cost of holding cash and so will encourage the agent to economize on cash holdings. Transactions costs of exchanging cash for the interest-bearing asset and vice-versa, on the other hand, will make it infeasible for the agent to have a zero cash balance on average. Baumol has assumed that these transactions costs are a fixed amount per transaction. Tobin (1956), on the other hand, has assumed that these costs consist of two components—a fixed component and a proportional component, the latter being a fixed proportion of the size of each transaction—although his subsequent analysis has been simplified by assuming that the proportional component is zero. It is the view of this author that the 'fixed plus proportional', or mixed, brokerage fees lend a decidedly more realistic flavour to the analysis. Moreover, the Baumol fixed brokerage fees may be treated as a special case of the more general mixed specification.

In this Chapter the two-asset model of Baumol (1952), combined with the transactions costs of Tobin (1956), will be extended to a three-asset world, where the third asset is foreign money. The latter will not be treated simply as an available asset, but instead as a medium of exchange, a quantity of which is essential if the agent is to carry out his consumption plans. As is the case with the Baumol analysis, the model rests on modest assumptions. Yet the results contain a number of new insights into the phenomenon known as currency substitution, the implications of which seem even more significant when viewed from this fundamental level.
2.2 Two-Asset Model (One Currency)

In order to fix ideas and to provide a point of reference the Baumol (1952) model will first be set out using three different transactions cost specifications. This will also enable a useful comparison later in the Chapter.

(a) Fixed Brokerage Fees

This analysis is precisely that which was presented in Baumol (1952) and is well-known. The agent receives an income $y$ per period, all of which will be spent, $y - k$ of which is initially invested in bonds which pay $r$ per period, and $k$ of which is kept as cash. The latter is spent continuously until cash holdings are exhausted, at which time another $k$-worth of bonds is cashed in. Each trip to the broker involves a fixed brokerage fee of $b_0$. Average bond holdings, $B$, are given by the total of wealth and income net of average cash holdings, $M$:

\[(2.2.1) \quad B = W + y - M\]

Average cash holdings are clearly $k/2$ so that the agent seeks to maximize the following profit function.

\[(2.2.2) \quad \pi = rB - b_0 n\]

where:

\[B = W + y - k/2\]

\[n = 1 + (y - k)/k = y/k, \text{ the number of trips to the broker.}\]

Maximization of $\pi$ with respect to $k$ yields:
(2.2.3) \[ k = \left(2b_0y/r\right)^{1/2} \]

and,

(2.2.4) \[ M = k/2 \]

so that

(2.2.5) \[ M = \left(b_0y/2r\right)^{1/2} \]

For the purposes of this chapter nothing more needs to be said about this derivation.

(b) Proportional Brokerage Fees

In this case consider the same situation as above, except that at each transaction the broker charges the agent a proportion \( b_1 \) of the value of the transaction. If the agent invests \( y-k \) in bonds initially, then his total costs will be \( 2b_1(y-k) \) regardless of his strategy. Hence, once the retained amount of cash \( k \) is exhausted, the agent will begin to make continuous trips to the broker, holding a zero cash balance on average. The average cash holding will be \( k/2 \) for a proportion \( k/y \) of the period and zero for the balance of the period. Then:

(2.2.6) \[ M = k^2/2y + 0(y-k)/y \]

\[ = k^2/2y \]

The agent's objective is to maximize profits by choosing the optimal \( k \):

(2.2.7) \[ \pi = rB - 2b_1x \]

where:

\[ B = W + y - k^2/2y \]

\[ x = y-k \]
Maximizing over choice of \( k \) yields:

\[
(2.2.8) \quad k = 2b_1 y/r,
\]

so that, from (2.2.6),

\[
(2.2.9) \quad M = 2b_1^2 y/r^2.
\]

Since \( k \) is not zero it is clearly not optimal for the agent to invest in bonds and to make continuous trips to the broker beginning on payday. However, the amount of retained cash \( k \) will decline as the rate of interest rises.

(c) **Fixed Plus Proportional Brokerage Fees**

Now, suppose that the agent faces the same problem but must pay the broker a fixed fee, \( b_0 \), for each transaction, as well as a proportion, \( b_1 \), of the value of the transaction. Unlike Case (a) where only the fixed brokerage fee was paid it is clear here that the agent will not withhold the same amount of cash initially as he will cash in in subsequent trips to the broker. Suppose that the agent keeps \( k_1 \) in cash on payday, investing \( y-k_1 \) in bonds. When cash holdings are exhausted, he will cash in \( k_2 \)-worth of bonds, and subsequent trips to the broker will also involve cashing in \( k_2 \)-worth of bonds. Then average cash holdings will be \( k_1/2 \) for a proportion \( k_1/y \) of the period, and \( k_2/2 \) for the remainder of the period.

\[
(2.2.10) \quad M = k_1^2/2y + k_2(y-k_1)/2y
\]

\[
= (k_1^2 + k_2 y - k_1 k_2)/2y
\]

As before, the agent's objective is to maximize profits from his portfolio net of transactions costs:
\[(2.2.11) \quad \pi = rB - \{(b_0 + b_1x_1) - (b_0 + b_1x_2)n \} \]

where:

\[B = \bar{W} + y - \frac{k_1^2 + k_2y - k_1k_2}{2y} \]

\[x_1 = y - k_1 \], the size of the initial bond purchase,

\[x_2 = k_2 \], the value of subsequent bond transactions,

\[n = \frac{(y-k_1)}{k_2} \], the number of trips to the broker after the initial one.

Substitution of this information into (2.2.11) yields the following:

\[(2.2.12) \quad \pi = r(\bar{W} + y - \frac{k_1^2 + k_2y - k_1k_2}{2y}) - \{(b_0 + b_1(y-k_1)) \}

\[- (b_0 + b_1k_2)(y-k_1)/k_2 \]

The first-order conditions are given by:

\[(2.2.13) \quad \frac{\partial \pi}{\partial k_1} = -r\frac{k_1}{y} + r\frac{k_2}{2y} + 2b_1 + b_0/k_2 = 0 \]

\[(2.2.14) \quad \frac{\partial \pi}{\partial k_2} = -r(y-k_1)/2y + b_0(y-k_1)/k_2 - b_0 = 0 \]

Equation (2.2.14) may be used to solve for \(k_2\); this may then be substituted into (2.2.13) to yield:

\[(2.2.15) \quad k_1 = (2b_0y/r)^{1/2} + 2b_1y/r \]

\[(2.2.16) \quad k_2 = (2b_0y/r)^{1/2} \]
First of all, it is clear that $k_1 \neq k_2$, so that it would be incorrect to set up the problem as if they were equal. Secondly, $k_2$ is the same as the $k$ derived in Case (a); that is, the optimal size of the transactions has not changed. However, $k_1 < k_2$, so that the agent retains more cash on payday than he would if he faced the fixed brokerage fees only. As a consequence, not surprisingly, the average cash holding will be higher than in the pure fixed brokerage fee case.

\[(2.2.17) \quad M = \left( b_0 y / 2r \right)^{1/2} + \left( b_1 / r \right) \left( 2b_0 y / r \right)^{1/2} + 2b_1 y / r^2 \]

Clearly, if $b_1 = 0$ and $b_0 \neq 0$ average cash holdings would be the same as in Case (a), and if $b_0 = 0$ and $b_1 \neq 0$, average cash holdings would be the same as in Case (b).

(d) Implications

The implications of this analysis are very simple. Since both Cases (a) and (b) are special cases of Case (c), it will suffice to conduct any further analysis using the 'fixed plus proportional' brokerage fee specification. Case (b) itself will be ignored in future, because the agent's time in visiting the broker must be recognized as a fixed cost per transaction. Hence, allowing the agent to make an infinite number of trips to the broker is simply too unrealistic. Thus, the analysis to follow will utilize the more general specification of brokerage fees, while the pure fixed brokerage fee model will be considered as a special case at the end of the analysis.

2.3 Three-Asset Model (Two Currencies)

A. The General Specification

The agent is now asked to solve this inventory problem in
a world of three assets—domestic bonds, paying $r$ per period, domestic currency, and foreign currency. The choices which the agent faces may be summarized in the following figure.

**Figure 2.3.1.**

**A Three-Asset Economy With Transactions Costs**

![Diagram](image)

The brokerage fees are denoted $b$ and $c$, and will be given by:

\begin{align}
(2.3.1) \quad (i) \quad b &= b_0 + b_1 x \\
(ii) \quad c &= c_0 + c_1 x
\end{align}

where $x$ is the value of the transaction in terms of the domestic unit of account. The agent makes his bond transactions with a broker, and his foreign exchange transactions with a foreign exchange dealer.

As before, the agent receives an income $y$ per period and will spend the entire amount, continuously. A fraction, $\alpha$, of his income will be spent using domestic currency while the remaining fraction, $1 - \alpha$, will be spent using foreign currency. We might think of $\alpha$ and $1 - \alpha$ as the exponents of a Cobb-Douglas utility function defined over domestic and foreign consumption, which has been maximized subject to an income constraint while ignoring transactions.
costs and uncertainty. In particular, these proportions will be independent of monetary phenomena, such as movements in the nominal exchange rate. The latter, the price of foreign currency in terms of the domestic unit of account, will be denoted $S$ (the spot rate), while $S^e$ will be used to represent the expected rate of change of $S$. Thus, $S^e$ is the expected rate of depreciation of the domestic currency. The formation of this expectation is assumed to be exogenous to this analysis. However, the agent has an estimate of $S^e$ and so he expects to either profit or lose from holding foreign currency. All variables will be expressed in units of domestic currency.

It was found in the preceding section that the brokerage fees as specified in (2.3.1) will cause the agent to retain a different amount of cash on payday than that amount which he will obtain from liquidating bonds at each subsequent transaction. In the present context, similar behaviour in respect of foreign exchange transactions is allowed for, although we shall see below that in many cases this generalization will be unnecessary. The general specification of the agent's profit function is given by:

\[(2.3.2)\]  
\[\pi = rB + s^eSM^* - (b_0 + b_1x_1) - (b_0 + b_1x_2)n_{11} - (c_0 + c_1x_3) - (c_0 + c_1x_4)n_{12}\]

where:

- $B = W + y = M - SM^*$, average bond holdings for the period,
- $M =$ average domestic currency holdings,
- $SM^*$ = average foreign currency holdings expressed in terms of the domestic unit of account,
\( x_1 = \) the value of the initial bond purchase,
\( x_2 = \) the value of all subsequent bond liquidations,
\( x_3 = \) the value of the initial purchase of foreign currency,
\( x_4 = \) the value of all subsequent foreign exchange transactions,

\( n_{11} = \) the number of bond sales transactions, and
\( n_{12} = \) the number of subsequent foreign exchange transactions.

The precise definitions of the \( x \)'s and \( n \)'s will change from case to case, and are described more fully below.

There are, in general, eight solutions to this problem. Three of these are 'interior' in the sense that the agent holds a non-zero quantity of bonds and conducts more than the single requisite foreign exchange transaction. Which of three solutions obtains will depend on whether \( s^e \) is positive, negative or zero. Four additional solutions occur when transactions costs lead the agent to choose either not to hold bonds or to make only the single foreign exchange transaction. These are referred to as 'semi-corner' solutions; when both phenomena occur simultaneously we observe the agent at the 'pure corner' solution, where no bonds are held and only one foreign exchange transaction occurs. The relationships between the various solutions should become clear as each is considered in turn.

(a) Non-Zero Bond Holdings, Multiple Foreign-Exchange Transactions

As mentioned above, these are the interior solutions, and are three in number, with the outcome depending upon the sign of \( s^e \). In each case it will be possible to derive conditions involving the decision variables which will ensure the interior solution. Failure of
these conditions to hold will cause the agent to move to one of the semi-corner solutions, or perhaps to the pure corner solution.

(i) \( s^e = 0 \)

With \( s^e = 0 \) the agent will expect neither to lose nor to profit by holding foreign exchange. Hence, there is no incentive to hold more or less foreign exchange and less or more domestic currency so as to reduce losses or increase profits. The agent will purchase \( x_1 = y-k_1 \) worth of bonds on payday, retaining \( k_1 \) in the form of cash. Then \( \alpha k_1 \) is kept in domestic currency while \( (1-\alpha)k_1 \) is held as foreign currency. Following Equation (2.2.10) the average cash holdings are given by:

\[
(2.3.3) \quad M + SM^* = k_1^2/2y + (y-k_1)k_2/2y
\]
\[
= (k_1^2 + k_2y - k_1k_2)/2y
\]

\[
(2.3.4) \quad M = \alpha(M+SM^*)
\]

\[
(2.3.5) \quad SM^* = (1-\alpha)(M+SM^*)
\]

The number of subsequent bond and foreign exchange transactions will be the same, given by:

\[
(2.3.6) \quad n_{11} = n_{12} = (y-k_1)/k_2
\]

Then the profit function (2.3.2) is given by the following expression.

\[
(2.3.7) \quad \pi = rW + ry - r(k_1^2 + k_2y - k_1k_2)/2y - \{b_0 + b_1(y-k_1)\}
\]
\[
- \{c_0 + c_1(1-\alpha)k_1\} - (b_0+b_1k_2)(y-k_1)/k_2
\]
\[- \{c_0 + c_1 (1-\alpha)k_2\} (y-k_1)/k_2 \]

The first-order conditions for maximization of this expression are:

\[
\frac{\partial \pi}{\partial k_1} = -rk_1/y + rk_2/2y + b_1 - c_1 (1-\alpha) + (b_0 + b_1 k_2)/k_2 \]

\[+ \{c_0 + c_1 (1-\alpha)k_2\}/k_2 = 0 \]

\[
\frac{\partial \pi}{\partial k_2} = -r(y-k_1)/2y + (c_0 + b_0)(y-k_1)/k_2^2 = 0
\]

Equation (2.3.9) may be solved for \(k_2\); substitution of this into (2.3.8) yields the solution for \(k_1\).

\[
k_1 = \{2(b_0+c_0)y/r\}^{1/2} + b_1 y/r
\]

\[
k_2 = \{2(b_0+c_0)y/r\}^{1/2}
\]

Hence,

\[
M+SM^* = (b_0+c_0)y/2r^{1/2} + (b_1/r)\{2(b_0+c_0)y/r\}^{1/2} + 2b_1^2 y/r^2
\]

and \(M\) and \(SM^*\) are given by Equations (2.3.4) and (2.3.5), respectively.

It should be clear that both \(n_{11}\) and \(n_{12}\) must be non-negative integers, and that the above procedure will not guarantee such an outcome. It will be assumed that the agent will round both \(n_{11}\) and \(n_{12}\) to the nearest integer and act accordingly. Because the profit function is quadratic and since the variable \(n_{11}\) is the number of bond
sales transactions, it is clear that if \( n_{11} < 0.5 \) the agent will not hold bonds at all. Similarly, \( n_{12} \) is the number of foreign exchange transactions in addition to the first one; the latter transaction is essential for the agent to carry out his consumption plans. Thus, if \( n_{12} < 0.5 \) the agent will conduct only one foreign exchange transaction, of value \((1-\alpha)y\).

This suggests that for the above interior solution to hold, the condition \( n_{11} = n_{12} > 0.5 \) must hold. Thus, the interior solution requires that:

\[
(2.3.13) \quad n_{11} = n_{12} = \left\{ \frac{ry}{2(b_0 + c_0)} \right\}^\frac{1}{b_1} - b_1 \left\{ \frac{2y}{r(b_0 + c_0)} \right\}^\frac{1}{b_1} - 1 > 0.5
\]

(ii) \( s_e < 0 \)

In this case the agent expects the foreign currency to depreciate during the period. Hence, he will be wary of holding more foreign currency, on average, than is necessary to carry out his plans. One way to reduce average foreign currency holdings while still spending \((1-\alpha)y\) in foreign currency is to reduce the size but increase the number of foreign exchange transactions. Assume, then, that the agent purchases \( y - k_1 \) worth of bonds on payday, and that \( \alpha k_1 \) is kept in domestic currency. The remainder, \((1-\alpha)k_1\), must eventually find its way into foreign currency. This is done first in an installment \( q_1 \), and thereafter in equal installments of \( q_2 \) each. It should be clear, however, that in this case \( q_1 = q_2 = q \) because, unlike the bond transactions, the foreign exchange transactions are all in the same direction. Regardless of other factors, \((1-\alpha)y\) must eventually find its way into
foreign currency. Below we shall encounter a case where foreign
exchange transactions will occur in both directions, and in that case
the above assumption need not hold. Thus, the supply of domestic
currency is subsequently replenished by liquidation of bonds in equal
lots of \( k_2 \) each, while the foreign currency requirement continues to be
satisfied in equal lots of \( q \) each. Average cash holdings during the
period are given by:

\[
(2.3.14) \quad M + SM^* = \frac{k_1^2}{2y} + \frac{k_2(y-k_1)}{2y} \\
= \frac{(k_1^2 + k_2y - k_1k_2)}{2y}
\]

\[
(2.3.15) \quad SM^* = \frac{q}{2}
\]

and \( M \) is determined residually. The number of trips (in addition to
the initial ones) to the broker and foreign exchange dealer are,
respectively,

\[
(2.3.16) \quad n_{21} = (y-k_1)k_2
\]

\[
(2.3.17) \quad n_{22} = \{(1-\alpha)y - q\}/q
\]

Then the profit function which the agent will maximize is given by the
following expression.

\[
(2.3.18) \quad \pi = rW + ry - r \left( \frac{k_1^2 + k_2y - k_1k_2}{2y} \right) + \frac{s^e}{2}q - \left\{ b_0 + b_1(y-k_1) \right\} \\
- (b_0 + b_1k_2)(y-k_1)/k_2 - (c_0 + c_1q)(1-\alpha)y/q
\]
The first-order conditions for maximization of this expression are given by:

\[ \frac{\partial \pi}{\partial k_1} = -r k_1 / y + r k_2 / 2y + b_1 + (b_0 + b_1 k_2) / k_2 = 0 \]  
(2.3.19)

\[ \frac{\partial \pi}{\partial k_2} = -r(y-k_1) / 2y + b_0(y-k_1) / k_2 = 0 \]  
(2.3.20)

\[ \frac{\partial \pi}{\partial q} = s / 2 + c_0 (1-\alpha)y / q^2 = \rho \]  
(2.3.21)

Solving these conditions for \( k_1, k_2, \) and \( q \) yields:

\[ k_1 = (2b_0 y / r)^{1/2} + 2b_1 y / r \]  
(2.3.22)

\[ k_2 = (2b_0 y / r) \]  
(2.3.23)

\[ q = \{2c_0(1-\alpha)y / s^{1/2} \}^{1/2} \]  
(2.3.24)

Notice that the latter solution is real-valued only when \( s^e < 0 \); this is an indication that the problem will be altered to some extent when \( s^e > 0 \).

The average cash holdings over the period are now given by:

\[ M + SM^* = (b_0 y / 2r)^{1/2} + (b_1 / r)(2b_0 y / r)^{1/2} + 2b_1^2 y / r \]  
(2.3.25)

\[ M = (b_0 y / 2r)^{1/2} + (b_1 / r)(2b_0 y / r)^{1/2} + 2b_1^2 y / r \]  
- \( \{c_0(1-\alpha)y / -2s^e \}^{1/2} \)
(2.3.27) \[ SM^* = \left(c_0(1-\alpha)\gamma - 2s^e\right)^{1/2} \]

Comparison of Equation (2.3.25) with Equation (2.2.17) reveals that total cash holdings are the same in this case as they were in the one-money model. However, total cash holdings are distributed between two currencies now, and that distribution depends on \( s^e \). A detailed examination of these results is reserved for Section 2.4 to follow.

The conditions under which the above interior solution holds are that \( n_{21} > 0.5 \) and \( n_{22} > 0.5 \). These conditions are found by substituting Equations (2.3.22) and (2.3.23) into Equations (2.3.16) and (2.3.17). Thus, for an interior solution,

(2.3.28) \[ n_{21} = \left(\frac{\gamma y}{2b_0}\right)^{1/2} - b_1\left(\frac{2y}{b_0 \cdot r}\right)^{1/2} - 1 > 0.5 \]

(2.3.29) \[ n_{22} = \left\{\left(-\left(1-\alpha\right)\gamma s^e/2c_0\right)^{1/2} - 1 \right\} > 0.5 \]

If either of these conditions fails to hold, we encounter a 'semi-corner' solution, as illustrated below.

(iii) \( s^e > 0 \)

In this case the agent stands to profit from holding foreign currency; thus, the interior solution will see him holding bonds but each time they are cashed in he will put his funds into foreign currency, converting gradually into domestic currency as the latter is needed. As before, suppose the agent purchases \( y-k_1 \) worth of bonds and subsequently cashes them in in lots of \( k_2 \) each: At each bond sale the funds are put into foreign currency, but a proportion \( \alpha \) must eventually find its way back into domestic currency. The first
such installment will be in the amount \( u_1 \), while subsequent installments will be in equal lots of \( u_2 \) each. Notice that in this case the foreign exchange transactions occur in both directions, implying that \( u_1 \) need not be the same as \( u_2 \). Notice also that each time that bonds are liquidated the agent will have to move his funds into domestic currency and then into foreign currency. Hence, in the intermediate stage of the transaction he may retain \( u_1 \) or \( u_2 \) as domestic currency, rather than converting the entire amount into foreign currency and then immediately reconverting \( u_1 \) or \( u_2 \) back into domestic currency. Thus, the first lot of domestic currency after each bond sale will come from the bond sale itself, not from his holdings of foreign currency.

The profit function is altered somewhat by the above considerations, and is given by:

\[
\pi = rB + s^e \text{SM}^* - (b_0 + b_1 x_1) - (b_0 + b_1 x_2) n_{31} - (c_0 + c_1 (y - x_1 - x_3)) - (c_0 + c_1 (x_2 - x_4)) n_{31} - (c_0 + c_1 x_4) n_{32}
\]

where:

\( x_1 = y - k_1 \), the initial bond purchase,
\( x_2 = k_2 \), the value of subsequent bond sales,
\( x_3 = u_1 \), the initial holding of domestic currency,
\( x_4 = u_2 \), subsequent purchases of domestic currency,
\( n_{31} = (y - k_1) / k_2 \), the number of bond sales transactions,
\[ n_{32} = \{y - u_1 - u_2(y-k_1)/k_2\}/u_2, \] the number of foreign exchange transactions after the required one,

\[ B = W + y - (M+SM^*), \]

\[ M+SM^* = (k_1^2+k_2y-k_1k_2)/2y, \]

\[ M = (u_1^2+u_2ay-u_1u_2)/2ay. \]

Substituting this information into (2.3.30) enables us to write the profit function as follows.

(2.3.31) \[ \pi = rW + ry + (s^e-r)(k_1^2+k_2y-k_1k_2)/2y - s^e(u_1^2+u_2ay-u_1u_2)/2ay \]

\[ - (b_0+b_1(y-k_1)) - (b_0+b_1k_2)(y-k_1)/k_2. \]

\[ - (c_0+c_1(k_1-u_1)) - (c_0+c_1(k_2-u_2))(y-k_1)/k_2. \]

\[ - (c_0+c_1u_2)(y - u_1 - u_2(y-k_1)/k_2)/u_2. \]

The first-order conditions for maximization of (2.3.31) are:

(2.3.32) \[ \frac{\partial \pi}{\partial k_1} = (s^e-r)(2k_1-k_2)/y + (b_1-c_1) + (b_0+b_1k_2+2c_0+c_1k_2)/k_2 = 0. \]

(2.3.33) \[ \frac{\partial \pi}{\partial k_2} = (s^e-r)(y-k_1)/2y + (y-k_1)(b_0+2c_0)/k_2 = 0. \]

(2.3.34) \[ \frac{\partial \pi}{\partial u_1} = -s^e(2u_1-u_2)/2ay + c_1(c_0+c_1u_2)/u_2 = 0. \]

(2.3.35) \[ \frac{\partial \pi}{\partial u_2} = -s^e(ay-u_1)/2ay + 2c_1(y-k_1)/k_2 + (c_0ay-c_0u_1)/u_2^2 = 0. \]

The first two conditions may be solved for \( k_1 \) and \( k_2 \); these solutions
are given by:

\[(2.3.36) \quad k_1 = \left\{2(b_0+2c_0)y/(r-s^e)\right\}^{1/2} + 2b_1 y/(r-s^e)\]

\[(2.3.37) \quad k_2 = \left\{2(b_0+2c_0)y/(r-s^e)\right\}^{1/2} \]

However, the other two conditions, once \(k_1\) and \(k_2\) have been substituted out, yield two complex quartic equations which seem insoluble. In order to solve the model, therefore, it has been assumed that \(u_1 = u_2\).

This assumption leaves the solutions for \(k_1\) and \(k_2\) unaffected, and yields a simple solution for \(u_1\) and \(u_2\):

\[(2.3.38) \quad u_1 = u_2 = \left\{2c_0 ay/s^e\right\}^{1/2}\]

Hence, average cash holdings for this agent are:

\[(2.3.39) \quad M+SM^* = \left\{(b_0+2c_0) y/(2(r-s^e))\right\}^{1/2} + \left\{2(b_0+2c_0)y/(r-s^e)\right\}^{1/2} b_1/(r-s^e)\]

\[+ \quad 2b_1^2 y/(r-s^e)^2\]

\[(2.3.40) \quad M = \left\{(c_0 ay/2s^e)\right\}^{1/2}\]

\[(2.3.41) \quad SM^* = \left\{(b_0+2c_0) y/(2(r-s^e))\right\}^{1/2} + \left\{2(b_0+2c_0)y/(r-s^e)\right\}^{1/2} b_1/(r-s^e)\]

\[+ \quad 2b_1^2 y/(r-s^e)^2 - \left\{(c_0 ay/2s^e)\right\}^{1/2}\]

As before, this interior solution will hold only when conditions are such that the agent will make more than one foreign exchange transaction and will purchase bonds. These conditions are:
(2.3.42) \[ n_{31} = \left( \frac{r-s^e}{2(b_0+2c_0)} \right)^{\frac{1}{2}} - b_1 \left[ \frac{2y}{(r-s^e)(b_0+2c_0)} \right]^{\frac{1}{2}} - 1 \geq 0.5 \]

(2.3.43) \[ n_{32} = \left( \frac{s^e y}{2c_0} \right)^{\frac{1}{2}} - \left( \frac{(r-s^e)}{2(b_0+2c_0)} \right)^{\frac{1}{2}} + b_1 \left[ \frac{2y}{(r-s^e)(b_0+2c_0)} \right]^{\frac{1}{2}} \geq 0.5 \]

Provided that \( n_{31} > 0.5 \), the agent will choose to purchase bonds on payday, rather than holding only cash; if \( n_{32} > 0.5 \) then the agent will choose to make more than the single required foreign exchange transaction. If either of these conditions fails, we will encounter a semi-corner solution, as illustrated below.

**Discussion**

With regard to the interior solutions when \( s^e \) is non-zero, it is important to notice that there is no a priori reason to expect \( n_{11} \) to be greater or less than \( n_{12} \). It is best to think of there being three pools of funds, according to Figure 2.3.1, where bond transactions feed the pool of domestic currency while deleting the pool of bonds, and foreign exchange transactions feeding the foreign currency pool while deleting the pool of domestic currency. Thus, the timing of bond and foreign exchange transactions are independent, making solutions where \( n_{11} > n_{12} \) or vice-versa equally feasible within the general solutions given above.

(b) **Non-Zero Bond Holdings, One Foreign Exchange Transaction**

When the agent chooses to hold bonds but at the same time to undertake only the single foreign exchange transaction which is necessary for him to complete his transactions, then he is said to be
at a semi-corner solution. This will occur when $n_{12} < 0.5$, where $i = 2, 3$. Notice that when $s^e = 0$, $n_{11} = n_{12}$ so that both conditions must pass or fail simultaneously. In this case the agent moves to the pure corner solution, which is described below. For that reason the semi-corner solutions will occur only when the expected rate of depreciation is non-zero.

(i) $s^e < 0$

Suppose that $n_{21} > 0.5$ but that $n_{22} < 0.5$; then the agent will choose an interior solution with respect to bond versus total cash holdings, but will make only one foreign exchange transaction. Then, from the interior solution (2.3.25) above it is clear that the agent's average cash holdings will be given by:

$$M + SM^* = (b_0 ay/2r)^{1/2} + b_1 (2b_0 ay/r)^{1/2} + 2b_1^2 ay/r + (1-\alpha)y/2$$  \hspace{1cm} (2.3.44)

$$M = (b_0 ay/2r)^{1/2} + b_1 (2b_0 ay/r)^{1/2} + 2b_1^2 ay/r$$  \hspace{1cm} (2.3.45)

$$SM^* = (1-\alpha)y/2$$  \hspace{1cm} (2.3.46)

where the latter solution follows from the fact that the single foreign exchange transaction must be of value $(1-\alpha)y$.

(ii) $s^e > 0$

When $s^e$ is a positive number and $n_{31} > 0.5$ but $n_{32} < 0.5$, then once again the agent will hold bonds but will choose to make only one foreign exchange transaction. Then the average cash holdings are given by:

$$M + SM^* = (b_0 + 2c_0) y/2(r-s^e)^{1/2} + b_1 (2(b_0 + 2c_0) y/(r-s^e))^{1/2} / (r-s^e)$$  \hspace{1cm} (2.3.47)
(2.3.47) (continued)
\[ + 2b_1^2y/(r-s^e)^2 \]

(2.3.48) \[ M = \left( b_0 + 2c_0 \right) y/2(r-s^e)^2 + b_1 \left( 2(b_0 + 2c_0) y/(r-s^e)^2 \right) + 2b_1^2y/(r-s^e)^2 - (1-\alpha)y/2 \]

(2.3.49) \[ SM^* = (1-\alpha)y/2 \]

(c) **Zero Bond Holdings, Multiple Foreign Exchange Transactions**

This is the other possible set of semi-corner solutions, where the agent elects to keep all of his income in the form of currency, and shuffle between the two currencies in some optimal way. As before, these semi-corner solutions apply only when \( s^e \) is non-zero.

(i) \( s^e < 0 \)

Suppose that \( n_{22} > 0.5 \) but that \( n_{21} < 0.5 \); then the agent will choose an interior solution with respect to his foreign exchange inventory, but will hold no bonds. Then his average cash holdings are given by:

(2.3.50) \[ M + SM^* = y/2 \]

(2.3.51) \[ M = y/2 - \left( c_0(l-\alpha)y/-2s^e \right)^{1/2} \]

(2.3.52) \[ SM^* = \left( c_0(l-\alpha)y/-2s^e \right)^{1/2} \]

(ii) \( s^e > 0 \)

Suppose that, with the expected rate of depreciation a positive number, \( n_{32} > 0.5 \) but \( n_{31} < 0.5 \). Then, as above, all income
will be held in the form of cash, with the total split according to the following solutions.

\[(2.3.53) \quad M + SM^* = y/2\]

\[(2.3.54) \quad M = (c_0 a y / 2s^e)^{1/2}\]

\[(2.3.55) \quad SM^* = y/2 - (c_0 a y / 2s^e)^{1/2}\]

(d) Zero Bond Holdings, One Foreign Exchange Transaction

This is the pure corner solution, where the agent splits his income into the two currencies on payday, and does not enter into any further financial transactions. There are three sets of conditions under which this solution will obtain. If either

(i) \( s^e = 0 \) and \( n_{11} = n_{12} < 0.5 \), or

(ii) \( s^e < 0 \) and \( n_{21} < 0.5 \) and \( n_{22} < 0.5 \), or

(iii) \( s^e > 0 \) and \( n_{31} < 0.5 \) and \( n_{32} < 0.5 \),

then the pure corner solution will occur. In each of these cases, average cash holdings are given by:

\[(2.3.56) \quad M + SM^* = y/2\]

\[(2.3.57) \quad M = ay/2\]

\[(2.3.58) \quad SM^* = (1-\alpha)y/2\]

This completes the taxonomy of the transactions model. We shall now move on to consider a special case of this model so as to strengthen the intuition behind the above results.
B. Special Case: Fixed Brokerage Fees

The properties of the above general solutions will now be considered for the case where \( b_1 = c_1 = 0 \). That is, the agent pays a fixed brokerage fee per transaction. This is the basis for the well-known Baumol 'square-root rule' found in Section 2.2 (a). It is of interest here because the solutions, which were rather complex in the general treatment above, take on a very simple intuitive form, analogous to the Baumol (1952) result. Moreover, the conditions under which the interior solutions will arise become much more self-evident.

Since the analysis follows Part A above exactly, it will suffice here to summarize the solutions as follows:

(a) Non-Zero Bond Holdings, Multiple Foreign Exchange Transactions

\[ s^e = 0 \]

\[ (2.3.59) \quad M + SM^* = \left( \frac{(b_0 + c_0)y}{2r} \right)^{1/2} \]

\[ (2.3.60) \quad M = \alpha \left( \frac{(b_0 + c_0)y}{2r} \right)^{1/2} \]

\[ (2.3.61) \quad SM^* = (1-\alpha) \left( \frac{(b_0 + c_0)y}{2r} \right)^{1/2} \]

This interior solution holds if

\[ (2.3.62) \quad n_{11}^* = n_{12}^* = \left( \frac{ry}{2(b_0 + c_0)} \right)^{1/2} - 1 \geq 0.5 \]

In other words, the interior solution holds if

\[ (2.3.63) \quad ry \geq 4.5(b_0 + c_0). \]

(ii) \( s^e < 0 \)

\[ (2.3.64) \quad M + SM^* = \left( \frac{b_0 y}{2r} \right)^{1/2} \]
\[(2.3.65)\quad M = \left(b_0 \frac{y}{2r}\right)^{1/2} - \left(c_0(1-\alpha)y/2s^e\right)^{1/2}\]

\[(2.3.66)\quad SM^* = \left(c_0(1-\alpha)y/2s^e\right)^{1/2}\]

The solution for total cash balances here is the well-known Baumol 'Square-root rule' from the two-asset model. The reader is referred to Equation (2.2.5). The distribution of cash holdings between foreign and domestic currency depends on \(s^e\); notice that the solution for \(SM^*\) is identical with Equation (2.3.27), the solution for the general brokerage fee specification.

The interior solution will hold provided that the following two conditions are satisfied.

\[(2.3.67)\quad ry > 4.5b_0\]

\[(2.3.68)\quad -(1-\alpha)y s^e > 4.5c_0\]

These conditions illustrate quite clearly that low-income agents will generally choose the corner solutions while high-income agents will be more likely to be found at the interior optimum.

\[(iii)\quad s^e > 0\]

\[(2.3.69)\quad M+SM^* = \left[(b_0+2c_0)y/2(r-s^e)\right]^{1/2}\]

\[(2.3.70)\quad M = \left(c_0\alpha y/2s^e\right)^{1/2}\]

\[(2.3.71)\quad SM^* = \left[(b_0+2c_0)y/2(r-s^e)\right]^{1/2} - \left(c_0\alpha y/2s^e\right)^{1/2}\]

Comparison of these solutions with those of the general model reveals that the demand for domestic currency is the same in both cases.
Recall that this was also true of foreign currency in the case of the interior solution when \( s^e < 0 \); this is a reflection of the symmetry of the model. The conditions under which this interior solution holds are given by:

\[
\begin{align*}
(2.3.72) \quad (r-s^e)y & \geq 4.5(b_0+2c_0) \\
(2.3.73) \quad (s^e ay/2c_0)^{1/2} & - \{(r-s^e)y/(b_0+2c_0)^{1/2}\} \geq 0.5
\end{align*}
\]

It is worth noting that a comparison of condition (2.3.72) with (2.3.67) reveals that the rate of interest and/or the agent’s income must be higher in order for him to hold bonds if \( s^e > 0 \) than if \( s^e < 0 \). This is a sensible result because when \( s^e > 0 \) foreign currency competes with domestic bonds as an income-earning asset.

(b) Non-Zero Bond Holdings, One Foreign Exchange Transaction

(i) \( s^e < 0 \).

If condition (2.3.68) fails to hold while (2.3.67) is satisfied, then the agent will hold bonds but make only one foreign exchange transaction. Then the average cash holdings are given by:

\[
\begin{align*}
(2.3.74) \quad M + SM^* & = (b_0 ay/2r)^{1/2} + (1-\alpha)y/2 \\
(2.3.75) \quad M & = (b_0 ay/2r)^{1/2} \\
(2.3.76) \quad SM^* & = (1-\alpha)y/2 \\
(11) \quad s^e & > 0^e
\end{align*}
\]

If condition (2.3.73) fails to hold while (2.3.72) is satisfied, then, as above, the agent will choose to hold bonds but will
make only the single required foreign exchange transaction. Then the average cash holdings are given by:

\[(2.3.77) \quad M + SM^* = \left\{ (b_0 + 2c_0) y / 2 (r - s^e) \right\}^{1/2} \]

\[(2.3.78) \quad M = \left\{ (b_0 + 2c_0) y / 2 (r - s^e) \right\}^{1/2} - (1-\alpha) y / 2 \]

\[(2.3.79) \quad SM^* = (1-\alpha) y / 2 \]

That is, total cash holdings are the same as when both conditions for the interior solution are satisfied, but average holdings of foreign currency are higher. This implies that average holdings of domestic currency must be lower than at the interior solution.

(c) Zero Bond Holdings, Multiple Foreign Exchange Transactions

(i) \( s^e < 0 \)

If condition (2.3.67) fails to hold while (2.3.68) is satisfied, then the average cash holdings are given by:

\[(2.3.80) \quad M + SM^* = y / 2 \]

\[(2.3.81) \quad M = y / 2 + \left\{ c_0 (1-\alpha) y / -2s^e \right\}^{1/2} \]

\[(2.3.82) \quad SM^* = \left\{ c_0 (1-\alpha) y / -2s^e \right\}^{1/2} \]

These solutions are identical with those of the general model; the reader is referred to Equations (2.3.50-52)

(ii) \( s^e > 0 \).

If condition (2.3.72) fails to hold while (2.3.73) is satisfied, then the agent will not hold bonds. Then:
(2.3.83) \[ M + SM^* = y/2 \]

(2.3.84) \[ M = \left(\frac{c_0 ay}{2s^e}\right)^{\frac{1}{2}} \]

(2.3.85) \[ SM^* = y/2 - \left(\frac{c_0 ay}{2s^e}\right)^{\frac{1}{2}} \]

This solution is identical with that obtained in the general model; the reader is referred to Equations (2.3.53-55).

(d) Zero Bond Holdings, One Foreign Exchange Transaction

If either

(i) condition (2.3.63) fails, or

(ii) conditions (2.3.67) and (2.3.68) fail; or

(iii) conditions (2.3.72) and (2.3.73) fail,

then the pure corner solution, as given above for the general model (Equations (2.3.56-58)), will obtain.

2.4 Implications

The large number of solutions will make it difficult to generalize the implications of the above analysis. It is important to note that the process of aggregation across agents may find a number of agents at each of the eight solutions. If information about \(s^e\) is fairly reliable, however, one would expect to observe 'bunching' at one group of solutions or another. To see the general characteristics of the above model, then, consider the summary of comparative statics given in Table 2.4.1:

With two exceptions, the signs are column-consistent, enabling us to make the more general statements about the comparative statics given in the last row. The two results in parentheses are,
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**AGGREGATE**

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<td>0 &gt; e</td>
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</tbody>
</table>

**COMPARATIVE STATICS**

**SOLUTION**

TABLE 2.4.1
strictly speaking, ambiguous in aggregate. With reference to this table and the other results then, the implications of the analysis are as follows.

(1) It is clear from the above table and from the solutions themselves that the currency substitution hypothesis has been confirmed, in the sense that a change in $s^e$ results in a shift in cash holdings towards the appreciating currency. In addition, the model is telling us that an increase in $s^e$ causes the total of cash holdings to rise, implying that bond holdings fall. This means that the increase in holdings of the appreciating currency exceeds the decrease in the depreciating currency. Taking this result one step further, we see that the total demand for money (in this model, currency) will be higher, ceterus paribus, under a floating exchange rate regime than under a fixed exchange rate. Notice that uncertainty about the exchange rate plays no role in this model, nor, therefore, in the generation of this result. As the variability of exchange rates increases, so does the demand for money, regardless of whether $s^e$ is perfectly predictable or not.

(2) The solutions reveal that, generally speaking, low-income earners will not hold bonds but will instead hold all cash; furthermore, they will typically make only one foreign/exchange transaction per period, where the size of this transaction is given by the total needs for foreign currency during that period. This prediction is not surprising, nor is it highly enlightening, given its obvious intuition and the fact that such behaviour frequently is observed in the real world.
(3) The model has something quite specific to say about the notion of interest rate parity. The latter can only be ensured by perfect international arbitrage via the foreign exchange market. However, conditions (2.3.68) and (2.3.73) indicate that there is a range for $s^e$ over which agents will act as if $s^e = 0$. For example, consider condition (2.3.68), which says that when $s^e < 0$ we must have $-s^e > 4.5c_0/(1-\alpha)y$ before the agent will react to any change in $s^e$. The form of condition (2.3.73) prohibits a similar calculation when $s^e > 0$, but it would seem likely that $s^e$ would need to be even larger (in absolute value) in this case, because of the higher brokerage costs involved. For the 'average' economic agent this band is potentially quite wide, as the insertion of some likely values into the above expression will readily demonstrate. For example, suppose that the time period in question is one month, in which the agent works 140 hours, and the cost of undertaking the additional transactions for the purposes of arbitrage is perceived as equivalent to one hour. Further, suppose that $\alpha = 0.8$.

Then:

\[(2.4.1) \quad 4.5c_0/(1-\alpha)y = (4.5)(1)/(0.2)(140)\]

\[> 16\%\]

which implies that $-s^e$ must exceed 16% per month to entice this agent into the international money market. Of course, this is a very unsophisticated calculation, but it does reinforce the belief that the main actors in the international money markets are specialists who have invested large fixed costs and operate at a
scale where marginal costs are virtually zero.\footnote{4}

(4) Theoretically, interest rate parity implies that $s^e = r - r^*$, where $r^*$ is the foreign rate of interest and, although the behaviour of private individuals in the face of transactions costs will not ensure this relationship in general, the domestic monetary authority may see fit to do so. Assuming that $r^*$ is independent of $r$ (the domestic economy is small relative to the foreign economy) and if the objective of the domestic monetary authority is to fix the rate of exchange of the currency, then movements in $r^*$ will be matched by movements in $r$ so as to keep $s^e$ constant, and movements in $s^e$ will be matched by movements in $r$ so as to keep $S$ constant. In the first case, where $\Delta r = \Delta r^*$ and $s^e$ is constant, no currency substitution will take place—holdings of real balances of all types will decline domestically while bond holdings rise. In the second case, on the other hand, where $\Delta r = \Delta s^e$, currency substitution will occur, with holdings being shuffled away from the depreciating currency. Thus, for example, a simultaneous increase in both $s^e$ and $r$ will cause bond holdings to rise, total domestic cash holdings to fall, and domestic holdings of foreign cash to rise. Hence, the measured stock of domestic money will overestimate the reduction in overall cash holdings domestically because of the currency substitution.

(5) The results of Table 2.4.1 also indicate that the effect on the demand for domestic balances of changes in $r$ will depend on $s^e$. In particular, as $s^e$ moves from a negative number towards zero and then becomes positive, the effect of changes in $r$ will
decline in absolute value. Thus, one would expect the predictive ability of a demand for money regression with $s^e$ omitted to decline during periods when $s^e$ is positive.

(6) It is important to note that to this point the discussion has focused on the domestic demand for domestic money. To complete the specification of the demand for domestic money function requires the addition of the foreign demand for domestic money. If we assume that a foreign agent faces a problem which is identical with that faced by the domestic agent, we would then have sixty-four possible solutions for the total demand for domestic money. To make the analysis manageable, suppose that the two agents agree on $s^e$, and that they both choose interior solutions. Furthermore, assume that they face only fixed brokerage fee schedules. Then there are three relevant solutions for the total demand for domestic money. First of all, if $s^e = 0$, we have:

\[ (2.4.2) \quad M_D = \left(\frac{b_0+c_0}{2r}\right)^{\frac{1}{2}} + S\left(1-a^*\right)^{\frac{1}{2}} \left(\frac{(b_0^*+c_0^*)y^*/2r^*}{2r}\right)^{\frac{1}{2}} \]

For $s^e < 0$, the total demand for domestic money is given by:

\[ (2.4.3) \quad M_D = \left(\frac{b_0 y/2r}{2}\right)^{\frac{1}{2}} - \left(c_0 (1-a) y/\left(-2s^e\right)\right)^{\frac{1}{2}} \]

\[ + S\left(\frac{(b_0^*+2c_0^*)y^*/2(r^*+s^e)}{2r}\right)^{\frac{1}{2}} \]

Finally, for $s^e > 0$, we have:

\[ (2.4.4) \quad M_D = \left(c_0 (1-a) y/2s^e\right)^{\frac{1}{2}} + S\left(c_0^* (1-a^*) y^*/2s^e\right)^{\frac{1}{2}} \]

These three solutions generally confirm the above discussion.
addition, it is now clear that a conventional demand for money specification which posits that money demand is a function of y and r only may be seriously misspecified.

(7) To this point in the discussion, all variables have been nominal, so that it is important to ensure that the demand for real balances is homogeneous of degree zero in prices. For example, consider Equation (2.4.3); denoting real variables as x' where x is the nominal variable, we have:

\[
(2.4.5) \quad M_D' = (Pb_0'^{dy'/2r})^{1/2} - \{Pc_0'(1-\alpha)Py'-2s\}^{1/2}
\]

\[\quad - S(P^*c_0'^{y*/-2s})^{1/2} + S\{(P^*b_0'^{2P^*c_0'})P^*y*/2(r^*+s)\}^{1/2}\]

Then:

\[
(2.4.6) \quad \frac{M_D}{P} = (b_0'\alpha y'/2r)^{1/2} - \{c_0'(1-\alpha)\alpha y'-2s\}^{1/2}
\]

\[\quad - \frac{SP^*}{P}(c_0'^{\alpha y*/-2s})^{1/2} + \frac{SP^*}{P}\{(b_0'^{2P^*c_0'})\alpha y*'/2(r^*+s)\}^{1/2}\]

Clearly the latter expression is only homogeneous of degree zero in P if P and SP* move together; this is not to say that purchasing power parity must hold at all times, but that \(\Delta S=\Delta P\). This assumption cannot be presumed to hold in empirical application, so that the variable \(SP^*/P\) will have to be included as an argument of the
demand for real balances.

It is possible that $SP^*/P$ will have an additional, entirely separate effect on the demand for real balances. This is through the parameter $\alpha$, which was presumed to be independent of monetary phenomena but could well be a function of the real exchange rate. This possibility is empirically testable and will be discussed further below.

2.5 Summary

In this Chapter a deterministic inventory model has been used to derive the demand for money function in a three-asset world, where the other two assets are domestic bonds and foreign currency. The model embodies the currency substitution hypothesis, and also yields several new insights into the demand for money function.

The major shortcoming of the analysis is its ignorance of uncertainty. All variables are known to the agent with perfect certainty; this situation will be rectified in the next two chapters.
FOOTNOTES TO CHAPTER TWO

1 The validity of this assertion is easily checked by setting up the profit function (2.3.18) below using the assumption that \( q_1 \neq q_2 \). Maximization will yield the solution \( q_1 = q_2 \).

2 Notice that this assumes that \( s^e < r \); see Footnote 3 below.

3 One additional possibility deserves mention. Notice that if \( 0 < s^e > r \) then the solutions in Section (a)(iii) above become complex-valued. In fact, if \( s^e > r \) the agent would choose not to hold bonds (unless foreign exchange transactions are more costly than bond transactions) and would instead move to the semi-corner solution of Section (c)(ii) above. The occurrence of \( s^e > r \), however, seems rather unlikely.

4 It should be noted that this discussion could be modified by the introduction into the analysis of a fourth asset, foreign bonds, for this would provide an alternative channel through which arbitrage could be effected. On the other hand, it is not expected that the introduction of this fourth asset would alter the above results in any important way, for the only reasonable point of departure would be to assume that domestic and foreign bonds are perfect substitutes.
CHAPTER THREE

THE PRECAUTIONARY DEMAND FOR MONEY
IN A MULTICURRENCY WORLD

3.1 Introduction

The model of the transactions demand for money which was explored in the preceding chapter has provided a number of new insights into the demand for money in a two-money economy. As with every economic model, however, the precision of those results may be attributed to the degree to which the analysis abstracts from reality. The major abstraction in that case was the avoidance of uncertainty. In particular, the income and expenditure streams, as well as the holding period yield on bonds, are all presumed known. Furthermore, although it is the expected rate of appreciation of the domestic currency which plays a role, the formation of that expectation and its associated degree of certainty are not treated.

In the present chapter, although transactions and holding costs will still play a significant role, the focal point of the analysis will shift to uncertainty with regard to the expenditure stream. The latter will be modelled explicitly as a series of stochastic cash requirements, about which only the probability distribution is known a priori. The problem which the agent faces is to minimize expected costs of meeting the cash requirements. Perhaps the most comprehensive model of this variety is to be found in Gray
and Parkin (1973). In this Chapter the latter one-money framework will be extended to a two-currency world.

3.2 The Model

The analysis will be carried out for a three-asset world, which is identical with that analyzed in Chapter Two, where the three assets and the transactions costs involved in transferring funds between them are as illustrated in Figure 2.3.1. As in Chapter Two, the transactions costs are given by:

\[(3.2.1) \quad (i) \ c = c_0 + c_1x \]
\[(ii) \ b = b_0 + b_1x \]

where \(x\) is the value of the transaction in terms of the domestic unit of account. Following Gray and Parkin (1973), the agent's situation may be characterized as follows. The agent has a given stock of wealth, \(W\), which is to be allocated among the three assets. The allocations are reviewed at constant intervals; that is, the length of time between revisions is presumed fixed. Any adjustments in the portfolio which are made at the time of these reviews are assumed to be costless, on the grounds that such adjustments will be foreseen and the structure of the portfolio is such as to make these costs insignificant. At a single time point during the period two stochastic cash requirements will occur, one in domestic currency and the other in foreign currency. The agent is uncertain about the size of these cash requirements, but it is assumed that he knows their distributions. The latter are specified as follows.
(3.2.2) \[ p.d.f. \text{ of domestic cash requirements, } z \]
\[ = f(z), \quad 2 - \gamma < z < 2 + \gamma. \]
\[ = 0, \text{ otherwise.} \]

\[ p.d.f. \text{ of foreign cash requirements, } S_2 \]
\[ = g(S_2), \quad S_2 - \beta < S_2 < S_2 + \beta. \]
\[ = 0, \text{ otherwise.} \]

Although it will be assumed that these two probability density functions are independent, it will be convenient notationally to define \( F(z, S_2) \) as the joint p.d.f. of \( z \) and \( S_2 \). As before, \( S \) is the spot exchange rate, the price of one foreign unit of account in terms of the domestic currency. Thus, as in Chapter Two, all variables will be measured in terms of the domestic unit of account.

In meeting these cash requirements the agent will be forced to liquidate some of his assets. If neither cash requirement exceeds the respective levels of his holdings of cash, then he will obviously pay no liquidation costs. However, if \( z > M \) and/or \( S_2 > S_M \), he will be forced to move funds from one asset to another and incur a brokerage fee. It will be assumed that it is less costly to transfer funds between currencies than it is to liquidate bonds; this simply implies that cash holdings will be exhausted prior to any bond sales.

Based on his allocation of funds between \( M, S_M \) and \( B \), and on the specification of the distributions of \( z \) and \( S_2 \), the agent can calculate his total expected liquidation costs. These will be a function of the portfolio allocation, and will be denoted \( TLC(z, \gamma). \)

As before, the expected rate of depreciation of the
domestic currency will be denoted \( s_e \); formation of this expectation is assumed to be exogenous to this problem. The objective of the agent is to maximize the expected return from his portfolio, net of liquidation costs. The objective function is given by:

\[
\pi = rB + s^eSM^* - TLC^e(M, SM^*, B)
\]

This is maximized subject to the overall wealth constraint:

\[
W = M + SM^* + B
\]

It is convenient to substitute the constraint into the objective function to obtain a problem in two choice variables only, \( M \) and \( SM^* \).

\[
\Pi = r(W - M - SM^*) + s^eSM^* - TLC^e(M, SM^*)
\]

The first-order conditions are given by:

\[
r = -TLC^e_M(M, SM^*)
\]

\[
r - s^e = -TLC^e_{SM^*}(M, SM^*)
\]

Clearly, solution of the agent's problem requires the specification of the total expected liquidation cost function.

In deriving the total expected liquidation cost function, the possibility of bankruptcy will be ruled out; this results in a considerable simplification of the analysis to follow, without changing qualitatively any of the results. This assumption is imposed by restricting the values which the cash requirements can take on.

\[
(\zeta + \gamma) + (S2^* + R) = W
\]
That is, if the agent is faced with both the maximum domestic cash requirement and the maximum foreign cash requirement simultaneously, then his wealth will be just exhausted.

It is clear given that $c < b$ that the assets may be ordered according to liquidation cost. Then, a large domestic cash requirement will be met first out of domestic cash holdings, then out of excess foreign cash holdings, and finally out of bond holdings. Similarly, a large foreign cash requirement will be covered first with foreign currency, then out of excess domestic currency, and finally by liquidating bonds. This optimal liquidation strategy will be taken account of when the agent sets up his portfolio. To facilitate this, it is necessary to formalize the liquidation strategy; this may be achieved in the following way. Figure 3.2.1 divides the entire $z, S_z^*$ space into six regions each of which, were it to occur, would trigger a particular pattern of liquidations. The liquidations which occur, together with their associated costs, are set out in Table 3.2.1. Notice that the positions of $(\hat{z} - \gamma)$ and $(S_{2z}^* - \beta)$ are quite arbitrary; indeed, they might even be negative numbers, implying that the agent stands to add to his wealth in a given period. A more likely presumption might be that $(\hat{z} - \gamma) = (S_{2z}^* - \beta) = 0$. In any event, at this stage the specification of the distributions of $z$ and $S_z^*$ is kept perfectly general.

The total expected liquidation cost function is derived by the following three steps.

(a) First, by finding the probability of $z$ and $S_z^*$ falling in each of the six regions and then
FIGURE 3.2.1

CHARACTERIZATION OF $z$ AND $S_z^*$

TABLE 3.2.1

LIQUIDATION STRATEGIES

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<td>4</td>
<td>$B$</td>
<td>$S_z^* &lt; SM^<em>$ and $z + S_z^</em> &gt; M + SM^*$</td>
<td>$z - M + SM^* + S_z^*$</td>
<td>$b_0 + b_1 x$</td>
</tr>
<tr>
<td>5</td>
<td>$M$</td>
<td>$z &lt; M$ and $z + S_z^* &gt; M + SM^*$</td>
<td>$M - z$</td>
<td>$c_0 + c_1 x$</td>
</tr>
<tr>
<td>5</td>
<td>$B$</td>
<td>$z &lt; M$ and $z + S_z^* &gt; M + SM^*$</td>
<td>$S_z^* - SM^* + M - z$</td>
<td>$c_0 + b_0 + (c_1 + b_1) x$</td>
</tr>
<tr>
<td>6</td>
<td>$B$</td>
<td>$z &gt; M$ and $S_z^* &gt; SM^*$</td>
<td>$z - M$</td>
<td>$b_0 + b_1 x$</td>
</tr>
<tr>
<td>6</td>
<td>$B$</td>
<td>$z &gt; M$ and $S_z^* &gt; SM^*$</td>
<td>$S_z^* - SM^*$</td>
<td>$c_0 + b_0 + (c_1 + b_1) x$</td>
</tr>
</tbody>
</table>
multiplyng each probability by the associated fixed cost.

(b) Integrating over each region the product of the quantity, liquidated and its associated probability will then yield the expected liquidation quantities, each of which is then multiplied by the associated variable cost.

(c) Summing the components of (a) and (b) will give us the total expected liquidation cost function.

For example, consider a typical element of the total expected liquidation cost function $\text{TL}^e(M, SM^*)$, that element which arises from Region 2 of Figure 3.2.1:

\[
\begin{align*}
\text{LC}^e_z &= c_0 \int_{z=M}^{M+SM^*} \int_{s_2^*=s_2^*-\beta}^{M+SM^*-z} F(z, s_2^*) dS_2^* dz \\
&+ c_1 \int_{z=M}^{M+SM^*} \int_{s_2^*=s_2^*-\beta}^{(z-M)F(z, s_2^*)} dS_2^* dz
\end{align*}
\]

Complete details of the derivation have been relegated to the appendix to this Chapter, Appendix A3. The above three steps have been duplicated region by region so as to aid the reader in following the algebra. Differentiation of $\text{TL}^e(M, SM^*)$ with respect to $M$ and $SM^*$ is straightforward but tedious; since this step yields some thirty-seven terms for each first-order condition, the details are not presented.

Fortunately, considerable simplification is possible, so that the first-order conditions (3.2.6) and 3.2.7 may be written as equations (A3.2) and (A3.3) (see Appendix A3). Written, as they are, for a problem of
this generality, these conditions are not very informative. However, it is clear that the asset demand functions will have the following implicit form:

\[(3.2.10) \quad M = M_1(r, s^e, b_0, b_1, c_0, c_1, \bar{z}, Sz^*, \gamma, \beta)\]

\[(3.2.11) \quad SM^* = SM_1^*(r, s^e, b_0, b_1, c_0, c_1, \bar{z}, S\bar{z}^*, \gamma, \beta)\]

\[(3.2.12) \quad B = W = M - SM^*\]

If the agent may be viewed as implicitly choosing the level and range of possible expenditures, and increasing both as his income increases, and if it can be assumed that transactions costs are constant over the relevant horizon, then the demands for the two currencies may be rewritten:

\[(3.2.13) \quad M = M(r, s^e, y)\]

\[(3.2.14) \quad SM^* = SM^*(r, s^e, y)\]

where, as before, the demand for bonds is residually determined.

It will be necessary to parameterize the distribution \(F(z, Sz^*)\) before using comparative statics analysis to investigate the properties of the model. In the interests of analytical convenience it will be assumed that \(F(z, Sz^*)\) is uniform. Provided that \(F(z, Sz^*)\) is in fact unimodal the comparative statics of the uniform model should be representative. Then, assuming that the stochastic processes generating \(z\) and \(Sz^*\) are independent, the distributions may be specified as follows.
Substituting this information into the first-order conditions above enables the latter to be rewritten as (A3.4) and (A3.5) in the Appendix to this Chapter. Perhaps not surprisingly, it is not possible to solve these two equations for \( M \) and \( SM^* \). Instead, let us consider the comparative statics of the model. Strictly speaking, all of the comparative statics results are ambiguous in sign. However, in most cases reasonable assumptions about the variables may be imposed to yield fairly clear cut results. In Appendix A3, then, the following results are derived.

\[
\begin{align*}
(3.2.18) & \\
& \text{(a) } \frac{\partial M}{\partial r} < 0 \quad \text{(b) } \frac{\partial SM^*}{\partial r} < 0 \\
& \quad \text{(c) } \frac{\partial M}{\partial r} + \frac{\partial SM^*}{\partial r} < 0 \quad \text{(d) } \frac{\partial B}{\partial r} > 0 \\
& \quad \text{(e) } \frac{\partial M}{\partial a^e} < 0 \quad \text{(f) } \frac{\partial SM^*}{\partial a^e} > 0 \\
& \quad \text{(g) } \frac{\partial M}{\partial a^e} + \frac{\partial SM^*}{\partial a^e} > 0 \quad \text{(h) } \frac{\partial B}{\partial a^e} < 0
\end{align*}
\]

The conclusions about the signs of these effects assume in each case
that \( M \) is 'close to' \( \hat{\Sigma} \) and \( SM^* \) is 'close to' \( S^* \). Then the results are fully consistent with the results of Chapter Two. That is, all holdings of cash decline when the opportunity cost of holding them rises, so bond holdings rise at the same time. The comparative statics with respect to the expected rate of depreciation indicate that this model behaves in a manner consistent with the currency substitution hypothesis. That is, an increase in \( s^e \) causes a redistribution of cash holdings from domestic to foreign currency. Additionally, because the model contains a third asset, bonds, it is possible to predict that an increase in the expected rate of depreciation (or, equivalently, a decrease in the expected rate of appreciation) of the domestic currency will cause the agent to increase his cash holdings overall, while reducing his bond holdings.

As in Chapter Two, the explicit modelling of transactions costs makes it clear that interest rate parity cannot be assumed to hold. However, under a monetary policy the objective of which is to maintain the currency exchange rate, the domestic rate of interest will frequently move in sympathy with movements in \( s^e \). In this case, the following comparative statics are of interest.

\[
\frac{\partial M}{\partial r} + \frac{\partial M}{\partial s^e} < 0 \\
\frac{\partial SM^*}{\partial r} + \frac{\partial SM^*}{\partial s^e} > 0 \\
\frac{\partial M}{\partial r} + \frac{\partial M}{\partial s^e} + \frac{\partial SM^*}{\partial r} + \frac{\partial SM^*}{\partial s^e} \\
= -\frac{\partial B}{\partial r} - \frac{\partial B}{\partial s^e} < 0
\]

Additional assumptions are required for the above results. In particular, it is necessary that \( c_1 \beta \cdot b_1 (\gamma + \beta) \), which is ensured by our
earlier assumption that $c_1 < b_1$, for result (b) to hold. Also, for result (c) it is sufficient that $3\gamma > 52^*$, a reasonable assumption. The above results indicate that a simultaneous increase in both $r$ and $s^e$ will cause a decline in domestic currency holdings, an increase in foreign currency holdings, but an overall decline in cash balances. The latter implies an increase in bond holdings. Once again, the model not only confirms the currency substitution hypothesis but also tells us what happens to the total demand for money when $r$ or $s^e$ changes.

Unfortunately, it is not possible to sign the other comparative statics results of the above model. This is in spite of the fact that it is those other results which are most intuitive. For example, an increase in $z$ implies an increase in average domestic expenditures, and would require more domestic currency. Similarly, an increase in $\gamma$ would constitute an increase in the uncertainty surrounding the domestic cash requirement and presumably would lead to an increase in domestic precautionary balances. However, the effects of these changes on the above model are impossible to sort out.

3.3 Implications

The implications of the above analysis for the theory of the demand for money are as follows.

(1) The analysis has revealed that precautionary balances in an economy where expenditures may be required in more than one currency will react to movements in the expected rate of depreciation of one currency relative to the other. Furthermore, as pointed out in Chapter Two, it is important to notice that we
have considered only domestic precautionary balances, whereas there will be, in general, precautionary balances denominated in domestic currency which are held in the foreign country as well. These will be a function of the expected rate of depreciation, the foreign rate of interest and foreign wealth or income. Hence, the aggregate demand for precautionary balances denominated in domestic currency may be represented by:

\[(3.3.1) \quad M_p = M_p(r, s^e, y, r^*, y^*)\]

Therefore, to the extent that \(s^e\), \(r^*\) and \(y^*\) are important determinants of the demand for precautionary balances, the traditional specification (which contains only domestic variables) suffers from an omitted variables problem.

Two additional related points bear mentioning. First of all, it is not possible to carry out the customary check for homogeneity explicitly. However, it should be clear from the first-order conditions (A3.4) and (A3.5) that changing all of the decision variables and brokerage costs from nominal to real will leave the implicit solutions invariant. Secondly, the extension to a multicurrency environment, although extremely difficult in terms of the theoretical analysis, will be straightforward in empirical application, necessitating only the addition of further foreign variables (expected rates of depreciation, rates of interest, and income) to the above specification.

(2) The model presented above has generated predictions which are entirely consistent with the currency substitution hypothesis. An
increase in the expected rate of depreciation of the home currency will lead to a reshuffling of the currency holdings in the agent's portfolio in favour of the appreciating currency. At the same time, however, the analysis has revealed more. In particular, this same increase in $s^e$ will disturb the relationship between the rate of return to holding foreign cash and that on domestic bonds. The new portfolio will include fewer bonds, so that foreign cash holdings rise by more than the reduction in domestic cash holdings. 2 This prediction could be modified were the model to be expanded to include a fourth asset, foreign bonds, although since the only reasonable way to proceed would be to make foreign and domestic bonds perfect substitutes, it is doubtful whether any of the results would be altered significantly. 3

3) In addition, the model has revealed new insights into the operation of monetary policy in a small open economy. Suppose that the domestic economy is small relative to the foreign economy, so that the foreign rate of interest is exogenous to the domestic rate. Then if the policy of the domestic monetary authority is to attempt to maintain the current exchange rate, the domestic rate of interest will be set so as to keep $s^e$ equal to zero. Under such a policy, two contrasting scenarios are of interest. First, for a given $s^e$, movements in the foreign rate of interest will be matched by movements in the domestic rate. Hence, there will be no currency substitution, and the effects of the policy on real domestic balances will be highly predictable. Second, increases in $s^e$ will result in similar movements in the domestic interest rate
setting. In this case, there will be currency substitution. Total domestic cash holdings will fall, while the proportion of cash held in foreign currency, and bond holdings, will both rise. Now, consider the case where the objectives of the domestic monetary authority change to, say, a policy of reducing the rate of growth in the money stock by raising the rate of interest. The announced policy should lower inflationary expectations, and therefore cause a decline in $s^e$. Then the objective of the policy may be frustrated to a certain extent, as the increase in holdings of real domestic cash balances due to the fall in $s^e$ partially (or completely, or more than) offsets the decline in the demand for money due to the rise in the rate of interest. This prediction holds regardless of the sign of $s^e$, and therefore is distinct from the conclusion of Chapter Two that the effect of changes in the rate of interest on holdings of real balances will decline in absolute value when $s^e$ becomes a positive number.

(4) Finally, the above analysis has proceeded as if all solutions must be interior. This is because corner solutions do not so greatly affect the analysis as they did in Chapter Two. In this case, there are three relevant corner solutions—one where the agent does not hold foreign currency, one where he does not hold bonds, and a third where he holds neither. In the first case, it may be that the expected size of the agent's transaction and its variance are both very small, or zero. In this case he will respond to changes in $s^e$ only if $s^e$ becomes a large positive number. In the second case, the agent's wealth is too small to
warrant the purchase and resale of bonds, and so he will not respond to movements in the rate of interest unless the latter becomes very large. The third solution is the 'pure' corner solution, where the agent holds only domestic cash, and in general will not respond to movements in either $r$ or $s^e$ unless they become large. The important point to note here is that the behaviour of agents who adopt corner solutions will not offset the behaviour of those at the interior, but instead will dampen the response of the economy in aggregate to movements in the rate of interest or the expected rate of depreciation.

3.4 Summary

In this Chapter uncertainty with regard to expenditures has been modelled explicitly so as to investigate the properties of the demand function for precautionary cash balances in a two-currency world. Extensions to a multicurrency model, although desirable, would seem to add little to our intuition. In empirical work this extension of the analysis should be straightforward. The analysis has shown that, like transactions balances, precautionary balances behave in a manner consistent with the currency substitution hypothesis. Notice that in both models it has been explicitly assumed that the two currencies are non-substitutable in exchange. Thus, it is clear that such substitutability is not necessary for the implications of the currency substitution literature to matter.

Not surprisingly, the added bit of realism, uncertainty, caused the results of the model to be considerably less well-determined than those of the transactions model. However, the results were clear
enough to make firm inferences based on them.

To this point in the study, the rate of interest and the expected rate of depreciation of the home currency have been treated as if they were known with certainty. This shortcoming will be treated in the next chapter.
FOOTNOTES TO CHAPTER THREE

1. The case which allows the agent to be hit with cash requirements which exceed his resources has been worked through by the author. Some of the comparative statics become less unambiguous as a result, but all results are qualitatively the same. In spite of this, the assumption of no bankruptcy enables a considerable simplification of the algebra.

2. This result is similar to but not the same as that reported in Chapter Two. The transactions model of that chapter predicts non-symmetric responses on the part of the agent to changes in \( s^e \) depending upon the sign of the latter. In particular, when \( s^e < 0 \), the total money holding of the agent is unaffected by changes in \( s^e \), whereas if \( s^e > 0 \) an increase in \( s^e \) will lead to a reduction in bond holdings and therefore an increase in overall cash holdings. Because the foreign agent acts in exactly the opposite manner, this same experiment (an increase in \( s^e \) with \( s^e > 0 \)) will leave his total cash holdings unchanged, and therefore total world holdings of money have risen. In the precautionary model of the present chapter, however, behaviour of the agent is symmetric for values of \( s^e \) on either side of zero. Hence, an increase in total precautionary cash holdings on the part of the domestic agent will be partially, completely or more than offset by an opposite response on the part of the foreign agent.

3. Extending the above model to a four-asset world renders all the comparative statics uninterpretable.
APPENDIX A3

MATHEMATICAL DETAILS OF THE PRECAUTIONARY MODEL
APPENDIX A3

MATHEMATICAL DETAILS OF THE PRECAUTIONARY MODEL

This Appendix presents the full details of the model which was analyzed in the preceding chapter. Following Figure 3.2.1 and Table 3.2.1, the total expected liquidation cost function is given in its regional components as follows.

Region 2

\[
LC^e_2 = c_0 \int_{z=M}^{M+SM^*-z} F(z, Sz^*) dSzdz + c_1 \int_{z=M}^{M+SM^*-z} (z-M) F(z, Sz^*) dSzdz
\]

Region 3

\[
LC^e_3 = c_0 \int_{z=2-\gamma}^{M} F(z, Sz^*) dSzdz + c_1 \int_{z=2-\gamma}^{M} (Sz^*-SM^*) F(z, Sz^*) dSzdz
\]

Region 4a

\[
LC^e_{4a} = (b_0+c_0) \int_{z=M}^{M+SM^*-z} F(z, Sz^*) dSzdz + c_1 \int_{z=M}^{M+SM^*-z} (SM^*-Sz^*) F(z, Sz^*) dSzdz
\]

Region 4b

\[
LC^e_{4b} = (b_0+c_0) \int_{z=M+SM^*}^{2+\gamma} F(z, Sz^*) dSzdz
\]

\[
+ c_1 \int_{z=M+SM^*}^{2+\gamma} (SM^*-Sz^*) F(z, Sz^*) dSzdz
\]

\[
+ b_1 \int_{z=M+SM^*}^{2+\gamma} (z-M+SM^*+Sz^*) F(z, Sz^*) dSzdz
\]
Region 5a

\[ LC_{5a}^{e} = (b_{0} + 2c_{0}) \int_{z=2^{-\gamma}}^{M} F(z,Sz^{*}) dSz^{*} dz + c_{1} \int_{z=2^{-\gamma}}^{M} (M-z) F(z,Sz^{*}) dSz^{*} dz \]

\[ + (b_{0} + 2c_{0}) \int_{z=2^{-\gamma}}^{M} (z-M) F(z,Sz^{*}) dSz^{*} dz \]

Region 5b

\[ LC_{5b}^{e} = (b_{0} + 2c_{0}) \int_{z=2^{-\gamma}}^{M} F(z,Sz^{*}) dSz^{*} dz + c_{1} \int_{z=2^{-\gamma}}^{M} (M-z) F(z,Sz^{*}) dSz^{*} dz \]

\[ + (b_{0} + 2c_{0}) \int_{z=2^{-\gamma}}^{M} (z-M) F(z,Sz^{*}) dSz^{*} dz \]

Region 6

\[ LC_{6}^{e} = (2b_{0} + c_{0}) \int_{z=2^{+\gamma}}^{M} F(z,Sz^{*}) dSz^{*} dz + b_{1} \int_{z=2^{+\gamma}}^{M} (z-M) F(z,Sz^{*}) dSz^{*} dz \]

\[ + (b_{0} + c_{1}) \int_{z=2^{+\gamma}}^{M} (z-M) F(z,Sz^{*}) dSz^{*} dz \]

Then, the total expected liquidation cost function is given by:

\[ TLC^{e}(M,SM^{*}) = \sum_{i=1}^{6} LC_{i}^{e}, \quad i = 1 \ldots 6 \]

Differentiation of \( TLC^{e}(M,SM^{*}) \) with respect to both \( M \) and \( SM^{*} \) yields some thirty-seven terms for each. For obvious reasons these details will not be presented here. Fortunately, considerable simplification may be achieved through cancellation and aggregation of terms. Then the first-order conditions (3.2.6) and (3.2.7) may be rewritten as follows.
\[ (A.3.2) \quad r - r^e = c_0 \int_{\gamma} F(z, SM^*) dz + b_0 \int_{\gamma} F(M + SM^*, S\ast^*) dS\ast^* \]

\[ + b_1 \int_{\gamma} F(z, SM^*) dS\ast^* dz + (b_1 - c_1) \int_{\gamma} F(z, SM^*, S\ast^*) dS\ast^* dz \]

\[ + (b_1 - c_1) \int_{\gamma} F(z, SM^*) dS\ast^* dz \]

\[ + b_0 \int_{\gamma} F(z, SM^*) dS\ast^* dz \]

\[ + b_1 \int_{\gamma} F(z, SM^*) dS\ast^* dz + b_1 \int_{\gamma} F(z, SM^*, S\ast^*) dS\ast^* dz \]

\[ + (b_1 - c_1) \int_{\gamma} F(z, SM^*) dS\ast^* dz \]

\[ + (b_1 + c_1) \int_{\gamma} F(z, SM^*) dS\ast^* dz \]

\[ + (b_1 - c_1) \int_{\gamma} F(z, SM^*) dS\ast^* dz \]

\[ + (b_1 + c_1) \int_{\gamma} F(z, SM^*) dS\ast^* dz \]

\[ + (b_1 - c_1) \int_{\gamma} F(z, SM^*) dS\ast^* dz \]

\[ + (b_1 + c_1) \int_{\gamma} F(z, SM^*) dS\ast^* dz \]
Parameterization of these first-order conditions using the uniform distribution assumption for \( F(z, S_z^*) \) as described in the text yields:

\[
(A3.4) \quad r = \left( \frac{1}{4\gamma\beta} \right) \left( c_0(M+2SM^*+z-2S_z^*+\gamma) + b_0(M-\gamma+2\beta) \right) \\
+ c_1(\beta SM^*+\beta SM^*-2S_z^*+\gamma\beta \left( \frac{1}{2} S_z^* - \frac{3}{2} S_z^* + \gamma S_z^* \right) \\
+ b_1 \left( \frac{-1}{2} M^2 - \gamma M + 2M + S_z^* M - SM^* - \frac{1}{2} SM^* - \gamma SM^* - \beta SM^* \\
+ 2SM^* + S_z^* + \frac{1}{2} S_z^* + \beta S_z^* - \frac{3}{2} S_z^* + \gamma S_z^* \right) \left( \frac{3}{2} S_z^* - \frac{3}{2} S_z^* + \gamma S_z^* + \beta S_z^* \right) \right)
\]

\[
(A3.5) \quad r - s^e = \left( \frac{1}{4\gamma\beta} \right) \left( c_0(2M-2z+2\gamma) + b_0(SM^*-S_z^*+\beta+2\gamma) \right) \\
+ c_1(\beta SM^*+2SM^*+SM^*-3SM^*+\beta SM^*-\frac{1}{2} SM^*-S_z^* SM^* \\
- \beta^2 - S_z^* + \gamma S_z^* + \beta S_z^* + \gamma S_z^* - \frac{3}{2} S_z^* \right) \\
+ b_1 \left( \frac{-1}{2} M^2 - \gamma M + 2M + S_z^* M - SM^* - \frac{1}{2} SM^* - \gamma SM^* - \beta SM^* \\
+ 2SM^* + S_z^* + \frac{1}{2} S_z^* + \beta S_z^* - \frac{3}{2} S_z^* + \gamma S_z^* - \beta S_z^* \right) \left( \frac{3}{2} S_z^* - \frac{3}{2} S_z^* + \gamma S_z^* + \beta S_z^* \right) \right)
\]

Clearly, even the assumption of a uniform distribution for \( F(z, S_z^*) \) does not provide enough of a simplification to solve explicitly for the asset demands. Simultaneous solution of the two equations would require the solution of a complex quartic, only the most simple of which have a well-defined solution. Thus, it will be necessary to be
content with the comparative statics of the model instead. Then, total differentiation of these equations with respect to \( M^*, SM^* , r , s^e, \beta \) and \( S^2* \) yields the following system.

\[
(A3.6) \begin{bmatrix}
m_{11} & m_{12} \\
m_{21} & m_{22}
\end{bmatrix}
\begin{bmatrix}
dM \\
dSM^*
\end{bmatrix} =
\begin{bmatrix}
k_{11} & k_{12} & k_{13} & k_{14} \\
k_{21} & k_{22} & k_{23} & k_{24}
\end{bmatrix}
\begin{bmatrix}
dr \\
ds^e \\
d\beta \\
dS^2*
\end{bmatrix}
\]

where:

\[ m_{11} = (1/4\gamma\beta) \{c_0 + b_1 (-M-SM^*+2-\gamma S2*-\beta)\} \]

\[ m_{12} = (1/4\gamma\beta) \{2c_0 + c_1 (SM^*+\beta-S2*) + b_1 (-M-SM^*+2+\gamma+S2*-\beta)\} \]

\[ m_{21} = m_{12} \]

\[ m_{22} = (1/4\gamma\beta) \{b_0 + c_1 (M+SM^*+\beta-S2*-3\gamma-2) + b_1 (-M-SM^*+2-\gamma+\gamma+S2*-\beta)\} \]

\[ k_{11} = 1 \]

\[ k_{12} = 0 \]

\[ k_{13} = (1/4\gamma\beta) \{c_0 + b_0 + b_1 (-M-SM^*-2-\gamma+S2*-\beta)\} \]

\[ k_{14} = (1/4\gamma\beta) \{2c_0 + c_1 (SM^*+S2*-\beta) + b_1 (-M-SM^*+2-\gamma-S2*+\beta)\} \]

\[ k_{21} = 1 \]

\[ k_{22} = -1 \]

\[ k_{23} = (1/4\gamma\beta) \{2c_0 + c_1 (\gamma+SM^*) + b_1 (-M-SM^*+2-\gamma+S2*-\beta)\} \]
\[ k_{24} = \frac{1}{4\gamma \beta} \{ b_0 + c_1 (M + M^* + S^* - 3\gamma - \beta) + b_1 (-M - M^* + 2 - \gamma - S^* + \beta) \} \]

In considering the comparative statics of the model it will be assumed that the second-order conditions hold. That is,

(A3.7) (i) \( m_{11} < 0 \)

(ii) \( m_{22} < 0 \)

(iii) \( |m| = \begin{vmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{vmatrix} > 0 \)

The comparative statics of the above system, strictly speaking, are all ambiguous in sign. However, in most cases reasonable assumptions about the variables may be imposed to yield clear-cut results.

First of all, consider the effects of changes in the domestic rate of interest. Using Cramer's Rule, the results are:

(A3.8) \[ \frac{\partial M}{\partial \gamma} = \frac{1}{4\gamma \beta |m|} \{ b_0 - 2c_0 + c_1 (M - 3\gamma - 2) \} < 0 \]

(A3.9) \[ \frac{\partial M^*}{\partial \gamma} = \frac{1}{4\gamma \beta |m|} \{ -c_0 + b_0 + c_1 (-SM^* - \beta + S^*) \} < 0 \]

(A3.10) \[ \frac{\partial M}{\partial \gamma} + \frac{\partial M^*}{\partial \gamma} = \frac{1}{4\gamma \beta |m|} \{ 2b_0 - 3c_0 + c_1 (M - 3\gamma - 2 - SM^* - \beta + S^*) \} < 0 \]

(A3.11) \[ \frac{\partial B}{\partial \gamma} = -\frac{\partial M}{\partial \gamma} - \frac{\partial M^*}{\partial \gamma} > 0 \]

The conclusions about the signs of these effects assume in each case that \( M \) is 'close to' 2 and \( SM^* \) is 'close to' \( S^* \). These results are
discussed fully in the text. Now, consider the agent's reaction to a change in the expected rate of depreciation of the domestic currency.

\[ (A3.12) \quad \frac{\partial M}{\partial \varepsilon} = \left( \frac{1}{4} \gamma \beta |m| \right) \{ 2c_0 + c_1(M-SM^*-\gamma-S2^*) + b_1(-M-SM^*+\gamma+S2^*) - \beta \} \]

\[ < 0 \]

\[ (A3.13) \quad \frac{\partial SM*}{\partial \varepsilon} = \left( \frac{1}{4} \gamma \beta |m| \right) \{ -c_0 + b_0 + b_1(M+SM^*-2+\gamma-S2^*+\beta) \} > 0 \]

\[ (A3.14) \quad \frac{\partial M}{\partial \varepsilon} + \frac{\partial SM^*}{\partial \varepsilon} = \left( \frac{1}{4} \gamma \beta |m| \right) \{ c_0 - b_0 + c_1(SM^*+\beta-S2^*) \} > 0 \]

\[ (A3.15) \quad \frac{\partial B}{\partial \varepsilon} = -\frac{\partial M}{\partial \varepsilon} - \frac{\partial SM^*}{\partial \varepsilon} < 0 \]

Once again, these results assume that \( M \) is close to 2 and \( SM^* \) is close to \( S2^* \). The following results are also of interest and are discussed fully in the text.

\[ (A3.16) \quad \frac{\partial M}{\partial \varepsilon} + \frac{\partial M}{\partial \varepsilon} = \left( \frac{1}{4} \gamma \beta |m| \right) \{ b_0 + c_1(M+SM^*-3\gamma-2) \]

\[ + b_1(-M-SM^*+\gamma+S2^*-\beta) \} < 0 \]

\[ (A3.17) \quad \frac{\partial SM^*}{\partial \varepsilon} + \frac{\partial SM^*}{\partial \varepsilon} = \left( \frac{1}{4} \gamma \beta |m| \right) \{ -2c_0 + c_1(-SM^*-\beta+S2^*) \]

\[ + b_1(M+SM^*+\gamma-2+\beta-S2^*) \} > 0 \]

\[ (A3.18) \quad \frac{\partial M}{\partial \varepsilon} + \frac{\partial M}{\partial \varepsilon} + \frac{\partial SM^*}{\partial \varepsilon} + \frac{\partial SM^*}{\partial \varepsilon} = \]

\[ \left( \frac{1}{4} \gamma \beta |m| \right) \{ b_0 - 2c_0 + c_1(M-3\gamma-2-\beta+S2^*) \} < 0 \]

\[ (A3.19) \quad \frac{\partial B}{\partial \varepsilon} + \frac{\partial B}{\partial \varepsilon} = -\frac{\partial M}{\partial \varepsilon} - \frac{\partial M}{\partial \varepsilon} - \frac{\partial SM^*}{\partial \varepsilon} - \frac{\partial SM^*}{\partial \varepsilon} > 0 \]
Additional assumptions are required for the above results. In particular, it is necessary that $c_1 \beta < b_1 (\gamma + \beta)$, which is guaranteed by our earlier assumption that $c_1 \beta b_1$, to hold. Also, for (A3.18) it is sufficient that $3\gamma > S^2 \gamma$. 
CHAPTER FOUR

THE SPECULATIVE DEMAND FOR MONEY
IN A MULTICURRENCY WORLD

4.1 Introduction

The traditional Keynesian distinction between the transactions, precautionary and speculative motives for holding money has proved to be useful in developing theories of the demand for money in the past. Thus far, this has also been true of this Thesis. The transactions and precautionary frameworks explored in Chapters Two and Three have yielded a number of new insights into the problem at hand. However, neither framework has explicitly recognized the uncertainty surrounding the agent's perception of the various rates of return which are relevant to his problem. The theory of the speculative demand for money, as originally presented formally by Tobin (1958), explicitly deals with just this type of uncertainty. Thus, consideration of the speculative motive for holding cash will serve this Thesis in two ways: first, by completing a systematic inquiry into the three Keynesian motives for holding money and second, by seeking to correct deficiencies in earlier chapters by formally treating uncertainty.

The present Chapter comprises five additional sections. Section 4.2 provides an informal discussion of the formation of expectations in this context, while Section 4.3 develops the two-asset model of Tobin (1958) so as to aid the reader's intuition in
the more mathematical treatment in Section 4.4. The latter analysis utilizes the general mean-variance framework presented in Royama and Hamada (1987) for a three-asset world, and discusses extensions to worlds of four or more assets. The implications of the results of this inquiry are set down in Section 4.5, while some concluding remarks are given in Section 4.6.

4.2 Rates of Return

In the simple models explored in this Thesis there are three rates of return which must be known by the agent prior to solving the money holding problem. The first is the rate of interest on domestic bonds, and the second is the rate of depreciation of the domestic currency, which is the rate of return to holding foreign currency in terms of the domestic unit of account. The third rate, that on domestic money, has been implicitly zero until now. Furthermore, all three rates of return have been presumed to be known with certainty. This assumption has proved useful in isolating particular phenomena which give rise to motives for holding money--first, the mismatching of expenditures and receipts, which gives rise to the so-called transactions motive, and second the uncertainty with regard to expenditures, which gives rise to the precautionary motive. In this Chapter consideration of expenditures is dropped altogether, and the problem of allocating wealth across a portfolio of assets in the face of uncertainty with regard to the individual rates of return on those assets is considered. It is this particular type of uncertainty which gives rise to the third Keynesian motive for holding cash--the speculative motive.
The subject of this Chapter, then, is quite distinct from that of the previous two, in that the latter essentially dealt with the determination of transactions balances, whereas this Chapter is really about a broader notion of money. It would be relatively simple in the theory to follow to restrict ourselves to a world in which there is only one kind of riskless asset, money, and which pays a zero rate of return. However, the empirical evidence is abundantly clear on this point—there exist several riskless financial assets each of which pays a positive rate of return. An example of such an asset is the time deposit, or savings account. Since speculative balances are intended for wealth-holding, and not for either transactions or for unforeseen expenditures, then surely cash will be dominated as a candidate for speculative balances by the time deposit. This will also be true of the agent's holdings of foreign currency, which will be held not in cash but in a foreign currency-denominated time deposit.

Naturally, the distinction between the two 'types' of 'money' is clear neither in theory nor in practice. For rapid and continual portfolio adjustment may cause the agent to hold some of his speculative balances in the form of 'narrow' or transactions money, whereas the ease with which one can transfer funds between the two forms implies that 'broad' or speculative money is often held for transactions or, in particular, precautionary purposes in mind. In empirical testing this overlap will be explicitly recognized.

For the purposes of this Chapter, then, we will be concerned with three assets—domestic time deposits, domestic bonds,
and time deposits denominated in foreign currency, or simply 'foreign currency deposits'. There are, therefore, four rates of return involved. First, the rate of return on domestic bonds will be treated as uncertain, and it will be presumed that the agent may utilize public knowledge about financial market conditions in general and monetary policy in particular to derive an expectation of this rate of return for the holding period under consideration, and to attach a subjective measure of risk to that expectation. This situation will be approximated by presuming that the agent has a probability density function for this rate. The rate of return on the domestic time deposit will be treated identically. The rate of return on the foreign currency deposit will be decomposed into two parts—a certain, contracted interest rate on the deposit, and the expected rate of depreciation of the domestic currency. Furthermore, it will prove convenient analytically to suppose that the certain rate of return on foreign currency deposits is expressed as a proportion of the domestic currency equivalent of the deposit. Where $\rho^*$ is the contracted rate on foreign currency deposits, and $\delta^*W_0$ the number of domestic currency units deposited, then, this assumption will imply that at the end of the period the deposit will be worth $\delta^*W_0(1+\rho^*+\delta^*)$ in terms of the domestic unit of account. The expectation of the rate of depreciation of the domestic currency, $e^\delta$, will be derived from the current economic environment—in particular, the spread between the expected rates of inflation domestically and abroad. It is expected that the agent's subjective measure of risk concerning $e^\delta$ will, on average, be higher than those for the other
rates of return, given the often high degree of serial correlation in the latter series, and the large number of factors which may affect exchange rate expectations.

In summary, then, the three key rates of return—that on domestic bonds, on domestic time deposits, and that on foreign currency deposits—will be treated analytically as random variables which follow normal distributions. The agent's expectation will be taken to be the mean of each distribution, and his subjective measure of the risk associated with his expectation will be proxied by the standard deviation of each distribution. This will enable us to analyze the agent's portfolio choice in a risk-return framework.

4.3 Two-Asset Model

In this section the analysis of Tobin (1958) is adapted to consider the choice between two monies. Suppose that our domestic agent has an amount of wealth $W_0$ available for investment. The only assets available for investment are domestic money, $M$, paying a certain rate of return of $\rho$ per period, and foreign money, $M^*$, paying a certain rate of return of $\rho^*$ plus an uncertain rate of return $s$. It is assumed that $s$ may be described by a subjective probability distribution with mean $s^e$ and standard deviation $\sigma_s$. The latter is viewed as a measure of the risk associated with investment in $M^*$.

The objective of the agent is to maximize a utility function which is defined over expected wealth, $E(W)$ and risk; the marginal utility of expected wealth is positive but decreasing (implying risk-aversion) and that of risk is negative. Thus, in expected wealth-risk space with the former measured along the
ordinate and the latter along the abscissa, utility will be increasing in the northwesterly direction. Assuming that $p^* + s^e > p$ the agent's choice set and typical solution are as illustrated in Figure 4.3.1. The agent attains the highest possible indifference curve by choosing the tangency with the choice set, point A. At this point the agent has chosen a diversified portfolio. Where $\delta$ represents the fraction of the portfolio held in foreign currency, we have $0 < \delta^* < 1$ and the optimal allocation is given by:

\begin{align}
(4.3.1) \quad M &= (1-\delta^*)W_0 \\
(4.3.2) \quad SM^* &= \delta^*W_0
\end{align}

Strictly speaking, comparative statics results from this theory are all ambiguous in sign, in a way exactly analogous to those of the traditional consumer problem. Consider, for example, an increase in $s^e$. This causes the choice set in Figure 4.3.1 to become steeper, and to lie everywhere outside the original one. This implies that the agent can reach a higher indifference curve, at point C in Figure 4.3.2. The diagram illustrates the case where the fraction of the portfolio allocated to foreign currency holdings has risen, a reasonable outcome. However, it is quite conceivable that $\delta^*$ lie to the left of $\delta^*$, implying the opposite result. The difficulty is clarified when we decompose the movement from A to C into effects. The substitution effect, obviously positive, is the movement from A to B. The expected wealth effect moves the equilibrium in the opposite direction, partially or perhaps more than completely offsetting the substitution effect.
The outcome depends upon the character of the individual utility function.

In spite of this drawback, the model does tell us something about interior as opposed to corner solutions. It should be clear from the above diagrams that if \( \rho^* + s^e > \rho \) then the agent will choose the corner solution \( \delta = 0 \), or \( W_0 = M \). This is because it is pointless to bear risk when there is no expected profit from doing so. Indeed, unless the indifference curves become horizontal at the ordinate axis, there will have to be some positive \( \phi \) such that \( \rho^* + s^e - \rho > \phi \) before the agent will move away from the corner. Thus, domestic speculative holdings of domestic money will only be sensitive to movements in the expected rate of depreciation when we have \( \rho^* + s^e - \rho > \phi \). The variable \( \phi \) will undoubtedly depend on transactions costs which, of course, are not explicitly modelled in this analysis.

The above geometric analysis is obviously constrained to the consideration of two assets. Naturally, one would expect to learn much more from a three-asset model, where the third asset is domestic bonds. This complication, however, will require a more general, algebraic treatment of the problem, and is the subject of the following section.

4.4 Three-Asset Model

Royama and Hamada (1967) have presented a perfectly general analysis of portfolio choice under uncertainty. In this section their model is applied to a specific three-asset economy, where the three assets are domestic time deposits, often referred to simply
as 'money', foreign currency deposits, or foreign 'money', and domestic bonds. The following definitions of symbols will be used.

(a) domestic money, $M$, with expected return $\rho^e$, actual return $\rho$.

(b) domestic bonds, $B$, with expected return $r^e$, actual return $r$.

(c) foreign money, $M^*$, with expected return $\rho^* + s^e$, actual return $\rho^* + s$.

The rate of return $\rho^*$ is a contracted rate for the holding period under consideration and therefore is known with certainty.

The vector of assets will be denoted $A = (M, B, S M^*)$, where $S$ is the spot exchange rate, the price of one unit of foreign currency in terms of the domestic unit of account, while the vector of expected returns will be given by:

\[(4.4.1) \quad \mu = (\mu_1, \mu_2, \mu_3) = (1 + \rho^e, 1 + r^e, 1 + \rho^* + s^e)\]

and the actual returns by:

\[(4.4.2) \quad \bar{\mu} = (1 + \rho, 1 + r, 1 + \rho^* + s)\]

Now, define the variance-covariance matrix of returns as follows:

\[(4.4.3) \quad E(\mu - \bar{\mu})^2 = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 \end{bmatrix}\]

a symmetric, positive-definite matrix. Notice that the assets have been ordered $M=1$, $B=2$ and $SM^*=3$ so as to make the notation more
compact. At the beginning of the period the agent's wealth is given by:

$$W_0 = M + B + SM^*$$

and at the end of the period by:

$$W = M(1+\rho) + B(1+r) + SM^*(1+\rho^r+s)$$

It is assumed that the investor has a utility function of the von-Neumann-Morgenstern class. That is,

$$U(W) = W - \frac{1}{2}\theta W^2$$

where \(\theta\) is a positive fraction such that \(1-\theta W > 0\). This latter restriction ensures that the marginal utility of wealth is positive.

The agent is assumed to maximize expected utility subject to his wealth constraint. Then his problem is given algebraically as follows.

$$\text{MAX } E\{U(W)\} = E(W) - \frac{1}{2}\theta E(W^2)$$

subject to: \(W_0 = M + B + SM^*\)

After substituting for expected wealth the problem may rewritten as follows:

$$\text{MAX } E[U(W)] = \mu_1 M + \mu_2 B + \mu_3 SM^*$$

$$-\frac{1}{2}\theta((\sigma_1^2+\mu_1^2)M^2 + (\sigma_2^2+\mu_2^2)B^2 + (\sigma_3^2+\mu_3^2)SM^*^2 +$$

$$2(\sigma_1+\mu_1\mu_2)MB + 2(\sigma_1+\mu_1\mu_3)MSM^* + \ldots$$
(4.4.8) (continued)

\[ + 2(\sigma_2 + \mu_2 \mu_3)BSM^* \]  

subject to:  \( W_0 = M + B + SM^* \)

After defining the Lagrangian,

\[ \mathcal{L} = E[U(W)] - \lambda (M + B + SM^*) \]  

the first-order conditions for maximization will be given by:

\[ \begin{align*}
(4.4.10) & \\
(a) & \mu_1 - \sigma_1 \mu_1 M - \sigma_2 \mu_2 B - \sigma_3 \mu_3 SM^* - \lambda = 0 \\
(b) & \mu_2 - \sigma_1 \mu_1 M - \sigma_2 \mu_2 B - \sigma_3 \mu_3 SM^* - \lambda = 0 \\
(c) & \mu_3 - \sigma_1 \mu_1 M - \sigma_2 \mu_2 B - \sigma_3 \mu_3 SM^* - \lambda = 0 \\
(d) & (M + B + SM^* - W_0) = 0
\end{align*} \]

Total differentiation of these conditions assuming that \( \partial \sigma_1 = \partial \sigma_4 = \partial \sigma_2 \) = 0 yields the following system:

\[ \begin{bmatrix}
-\frac{\partial}{\partial M} \\
\frac{\partial}{\partial B} \\
\frac{\partial}{\partial SM^*} \\
\frac{\partial}{\partial \lambda}
\end{bmatrix}
\begin{bmatrix}
dM \\
dB \\
dSM^* \\
d\lambda
\end{bmatrix}
= \begin{bmatrix}
k \\
\frac{\partial}{\partial \mu_1} \\
\frac{\partial}{\partial \mu_2} \\
\frac{\partial}{\partial \mu_3} \\
\frac{\partial}{\partial \sigma_1} \\
\frac{\partial}{\partial \sigma_2} \\
\frac{\partial}{\partial \sigma_3} \\
\frac{\partial}{\partial W_0}
\end{bmatrix}
\begin{bmatrix}
d\mu_1 \\
d\mu_2 \\
d\mu_3 \\
d\sigma_1 \\
d\sigma_2 \\
d\sigma_3 \\
dW_0
\end{bmatrix} \]
the details of which are given in the appendix to this Chapter, Appendix A4. In addition, the details of the comparative statics of the model are given in full.

Consider, first of all, the effects of changes in the various rates of return. Using Cramer's Rule, the effect on the demand for asset $j$ of a change in the return $\mu_j$ is given in general by:

$$\frac{\partial A_j}{\partial \mu_j} = -(1 - \theta \sum_k A_{jk} |m_{kj}| / |\bar{m}|) + A_j \sum_k (E_{kj} |m_{kj}| / |\bar{m}|)$$

(4.4.12)

where $|m_{ij}|$ is the determinant of the $ij^{th}$ cofactor of the matrix $\bar{m}$. This expression is derived by expanding along the $\partial \mu_j$ column after it replaces the $\partial A_j$ column in $\bar{m}$. Royama and Hamada have shown that this equation is exactly analogous to the Slutsky equation of consumer theory. The total effect of a change in a rate of return is decomposed into two effects. The first term on the right is the substitution effect, while the second term is the expected wealth effect. These are the same two effects analyzed in Figure 4.3.2.

As was demonstrated in the diagrammatic analysis, the total effect will always be ambiguous because the two effects generally work in opposite directions. Preliminary inspection has revealed that this is indeed the case for this model. Furthermore, the expected wealth effect itself is ambiguous in sign for each of the assets in this model. Thus, for the purposes of this analysis, we will concentrate on the substitution effect of changes in rates of return only. This may be thought of as the change in asset demand once the agent has been compensated for any change in expected wealth due to the change in the rate of return. Then, we will be most concerned with the
matrix of partial derivatives given by:

\[ \frac{\partial \lambda_j}{\partial \mu_i} \bigg|_S = -(1 - \theta \sigma_{ij} m_j) \frac{m_i}{m}. \]

Of course, the other comparative statics experiments may be considered in full.

It will prove convenient to assume that the variance-covariance matrix of returns is diagonal; this assumption may be relaxed to diagonal dominance without affecting the results. Also, it should be clear that \( \mu_1 < \mu_2 \) and that \( \mu_3 \) since really \( \sigma_i^2 \) is either zero or extremely small. The relationship between \( \mu_3 \) and \( \mu_2 \) is open to question, and we will see that this will play a significant role in the following results.

The comparative statics are best viewed together, in a matrix. A summary of the results are given in Table 4.4.1. The ambiguous results are set down according to our priors and the conditions under which the results hold and are consistent with each other are also given. The notation \( \bigg|_S \) makes it clear that the result refers to a substitution effect only.

These results essentially reveal that the three assets behave as substitutes provided that one of the three conditions holds. Condition (2) is particularly easy to believe, as is condition (3). The latter is probably quite reasonable since the \( \mu \)'s are expectations, and we might expect interest rate parity to hold on average. Notice that Young's Theorem holds also—the substitution matrix is symmetric.

In the special case where the holding period yield on the
**Table 4.4.1**

**Summary of Comparative Statics**

If one of the following three conditions holds:

1. \( \mu_3 > \mu_2 \) and \( \sigma_2 \) is large, or
2. \( \mu_3 < \mu_2 \) and \( \sigma_3 \) is large, or
3. \( \mu_3 = \mu_2 \)

Then the following comparative statics results hold:

\[
\begin{align*}
\frac{\partial M}{\partial \mu_1} |_{S} &> 0 & \frac{\partial B}{\partial \mu_1} |_{S} &< 0 & \frac{\partial SM^*}{\partial \mu_1} |_{S} &< 0 \\
\frac{\partial M}{\partial \mu_2} |_{S} &< 0 & \frac{\partial B}{\partial \mu_2} |_{S} &> 0 & \frac{\partial SM^*}{\partial \mu_2} |_{S} &< 0 \\
\frac{\partial M}{\partial \mu_3} |_{S} &< 0 & \frac{\partial B}{\partial \mu_3} |_{S} &< 0 & \frac{\partial SM^*}{\partial \mu_3} |_{S} &> 0 \\
\frac{\partial M}{\partial \sigma_1^2} &< 0 & \frac{\partial B}{\partial \sigma_1^2} &> 0 & \frac{\partial SM^*}{\partial \sigma_1^2} &> 0 \\
\frac{\partial M}{\partial \sigma_2^2} &> 0 & \frac{\partial B}{\partial \sigma_2^2} &< 0 & \frac{\partial SM^*}{\partial \sigma_2^2} &> 0 \\
\frac{\partial M}{\partial \sigma_3^2} &> 0 & \frac{\partial B}{\partial \sigma_3^2} &> 0 & \frac{\partial SM^*}{\partial \sigma_3^2} &< 0
\end{align*}
\]
domestic time deposit is certain, \( \sigma^2 = \sigma_{12} = \sigma_{13} = \sigma_{21} = \sigma_{31} = 0 \). The comparative statics of Table 4.4.1 are unchanged by this consideration, except of course that changes in \( \sigma^2 \) are no longer relevant. However, the comparative statics results of changes in the level of wealth (not reported for the general model because two out of three are ambiguous; see the Appendix to this Chapter) are affected by this assumption. Specifically, the effect of an increase in wealth is to unambiguously increase the holdings of all three assets.

Extensions of the analysis to a higher number of assets is possible, in principle. However, investigation of the four-asset model (with foreign bonds added to the above specification) has shown that the three-asset model is the most general for which the cross effects may be signed. The own-effects of rate of return or risk changes are all signable by the second-order conditions, as demonstrated by Royama and Hamada (1967). The cross effects, as above, depend upon whether the assets are substitutes or complements, but the conditions under which one or the other holds soon become uninterpretable. An interesting empirical exercise might be to estimate the variance-covariance matrix of returns and then to use the above framework to find whether particular pairs of assets would ever behave as complements. This would serve not only as a test of the above model, but also serve as a check against econometric results, such as those reported by Brillembourg and Schadler (1979), or those reported in Chapter Six of this Thesis.

4.5 Implications

The implications of the above analysis for the theory of
the demand for money are as follows.

(1) The model of this Chapter is evidently the minimum requirement for demonstrating the full currency substitution hypothesis. In a two-asset model, the two instruments must behave as substitutes so as to satisfy the second-order conditions. Hence, when one rate of return rises relative to the other, the demand for the first asset rises while the demand for the substitute declines. The three-asset model set out above generally confirms the currency substitution hypothesis, but illustrates that there is a possibility that some currencies will behave as complements. Were this to occur in fact, the data might be interpreted as rejecting the currency substitution hypothesis. However, the above model has illustrated that such a result would be perfectly consistent with the theory. Furthermore, as the number of assets included in the model increases, the conditions under which two currencies will behave as complements become increasingly difficult to rule out. Hence, this theoretical possibility is one which should be remembered when empirical testing of the hypothesis is being contemplated.

(2) The changes in risks rarely receive much attention in the literature but this takes on some significance in the above analysis. Notice that the subjective estimates of the risk associated with the expected rate of depreciation will, in part, be determined by the relative noisiness of monetary policy in the two economies. Thus, for example, moving from a fixed interest rate policy to the adoption of money supply growth targets might be expected to
reduce the noise in the expected rate of inflation, and hence in $s^e$, while at the same time increasing the variance of the domestic rate of interest. According to the comparative statics results reported above, this change in the relative variances will cause the demand for domestic money to become less sensitive to changes in the rate of interest, and more sensitive to changes in the expected rate of depreciation. In a demand for money equation with $s^e$ omitted, the influence of an omitted variable will have increased while that of an included variable, the rate of interest, will have decreased. Obviously, the reliability of the regression equation will decline as a result. As noted in Chapter One, such was the case in the U.S. in 1973 and in Canada in 1976. Both instances occurred after the adoption of money supply growth targets. Naturally, this cannot be regarded as prima facie evidence of the above hypothesis, but is sufficiently encouraging to warrant further empirical investigation.

(3) As in previous chapters, it is important to note that we have considered only the domestic demand for domestic money. However, the discussion of Section 4.3 indicates that a foreign agent, when faced with an identical problem, will only include domestic money (foreign, to him) in his portfolio when $s^e < 0$, and sufficiently so to pull him away from his corner solution. Thus, the domestic agent will be at his corner solution when the foreign agent is at an interior solution, and vice versa. This will only be exactly true when the two agents agree on $s^e$ and require the same amount of steepness in the opportunity locus before adopting an interior
solution. In general, with uncertainty about \( S^e \), and different utility functions, there will be a considerable variety of interior and corner solutions adopted by agents in both economies. This consideration in no way affects the results of the analysis, however, provided that in aggregate at least some agents adopt interior solutions.

(4) In view of the discussion of Section 4.2, it would seem natural to expect the predictions of this analysis, insofar as they differ from those of the previous two chapters, to more emphatically be observed in demand functions for 'broad' measures of the money stock. A more detailed discussion of this issue and its empirical relevance is reserved for Chapter Five.

(5) Finally, the extension of this model to include additional assets, particularly currencies, has failed to provide any additional insights into the problem at hand. In fact, the four-asset model is less detailed in its predictions than the present model. As in previous chapters, however, the extension would seem straightforward in empirical application. The speculative demand for domestic money will depend upon the expected rates of depreciation of all major currencies, all foreign interest rates and all foreign wealth variables. In addition, changes in relative variances of the various rates of return may play a significant role.

One potentially important qualification should be repeated at this time. The comparative statics results reported above which related to changes in rates of return ignored the expected wealth effect. Although one would not expect the latter to reverse any important
results, it is a possibility which cannot be ignored.

4.6 Summary

In this Chapter uncertainty about rates of return and its effect on portfolio decisions has been explicitly modelled. As was the case in the analyses of the transactions and precautionary demands for money, the speculative demand for money evidently behaves in a manner consistent with the currency substitution hypothesis. However, the analysis has told us more; the theoretical possibility of currency complementarity and the effect of changes in the relative variances are cases in point. As in previous chapters, however, the empirical significance of these implications is an open question.
FOOTNOTES TO CHAPTER FOUR

This assumption is made to keep the analysis general; implicitly this means that the time deposit in question is a savings account, whose rate of return can vary over the holding period. The alternative, a fixed-rate time deposit whose holding period yield would be certain, will be considered as a special case below. In contrast, the foreign currency deposit is assumed to bear a certain, contracted rate of return, as well as an uncertain return in the form of $s^e$. This assumption simplifies the expression for the variance of the total rate of return on the foreign currency deposit and, indeed, is not at odds with instruments which are available in the Canadian money market.
APPENDIX A4

MATHEMATICAL DETAILS OF THE SPECULATIVE MODEL
APPENDIX A6

MATHEMATICAL DETAILS OF THE SPECULATIVE MODEL

The first-order conditions (4.4.10), after total differentiation, yield the system (4.4.11), which is given as follows.

\[
\begin{pmatrix}
  m_{11} & m_{12} & m_{13} & -1 \\
  m_{21} & m_{22} & m_{23} & -1 \\
  m_{31} & m_{32} & m_{33} & -1 \\
  -1 & -1 & -1 & 0 \\
\end{pmatrix}
\begin{pmatrix}
  dM \\
  dB \\
  dS\text{\(M\)}* \\
  d\lambda \\
\end{pmatrix}
= \begin{pmatrix}
  k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} & 0 \\
  k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} & 0 \\
  k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & -1 \\
\end{pmatrix}
\begin{pmatrix}
  d\mu_1 \\
  d\mu_2 \\
  d\mu_3 \\
  d\sigma_1 \hat{t} \\
  d\sigma_2 \hat{t} \\
  d\sigma_3 \hat{t} \\
  dW_0 \\
\end{pmatrix}
\]

where:

\[m_{11} = -\theta(\sigma_1^2 + \mu_1^2)\]
\[m_{12} = -\theta(\sigma_2^2 + \mu_1^2)\]
\[m_{13} = -\theta(\sigma_3^2 + \mu_1^2)\]
\[m_{21} = -\theta(\sigma_1^2 + \mu_2^2)\]
\[m_{22} = -\theta(\sigma_2^2 + \mu_2^2)\]
\[m_{23} = -\theta(\sigma_3^2 + \mu_2^2)\]
\[m_{31} = -\theta(\sigma_1^2 + \mu_3^2)\]
\[m_{32} = -\theta(\sigma_2^2 + \mu_3^2)\]
\[m_{33} = -\theta(\sigma_3^2 + \mu_3^2)\]
\[k_{11} = -(1 - \theta \mu_1 M - \theta \sigma_2 B - \theta \sigma_3 \text{\(M\)}*) + \theta \mu_1 M\]
\[k_{12} = \theta \mu_1 B\]
\[ k_{13} = \Theta \mu_1 \text{SM}^* \]
\[ k_{14} = \Theta \mu_1 \text{SM}^* \]
\[ k_{15} = 0 \]
\[ k_{16} = 0 \]
\[ k_{21} = \Theta \mu_2 \]
\[ k_{22} = -(1-\Theta \mu_1 \text{SM}^* - \Theta \mu_2 \text{SM}^* - \Theta \mu_3 \text{SM}^*) + \Theta \mu_2 \text{B} \]
\[ k_{23} = \Theta \mu_2 \text{SM}^* \]
\[ k_{24} = 0 \]
\[ k_{25} = \Theta \mu_2 \text{B} \]
\[ k_{26} = 0 \]
\[ k_{31} = \Theta \mu_3 \text{M} \]
\[ k_{32} = \Theta \mu_3 \text{B} \]
\[ k_{33} = -(1-\Theta \mu_1 \text{SM}^* - \Theta \mu_2 \text{SM}^* - \Theta \mu_3 \text{SM}^*) + \Theta \mu_3 \text{SM}^* \]
\[ k_{34} = 0 \]
\[ k_{35} = 0 \]
\[ k_{36} = \Theta \text{SM}^* \]

Notice that Equation (4.4.1) together with the assumption that \( d\rho = 0 \) implies that

\[(A\psi r2) \quad (i) \quad d\mu_1 = d^e \]
\[(ii) \quad d\mu_2 = d\rho^e \]
\[(iii) \quad d\mu_3 = d\sigma^e \]

The second-order conditions for an extremum are that the matrix \( m \) is negative definite. This implies that:
(A4.3) 

\begin{align*}
(1) \quad & |\bar{m}| < 0 \\
(2) \quad & |m_{11}|/|\bar{m}| < 0
\end{align*}

where \( |m_{11}| \) is the determinant of the \( i \)th cofactor of \( \bar{m} \). These second-order conditions will be assumed to hold when attempting to sign the comparative statics results.

It will prove useful to define

\begin{equation}
\Delta = -(1 - \theta \mu_1 M - \theta \mu_2 B - \theta \mu_3 SM) < 0.
\end{equation}

where the sign of \( \Delta \) is due to our earlier assumption about the parameter \( \theta \). Additionally, it will prove convenient to assume that the variance-covariance matrix of returns is diagonal; this assumption may be relaxed to diagonal dominance without affecting the results.

Finally, it should be clear that \( \mu_1 < \mu_2 \) and that \( \mu_1 < \mu_3 \) since really \( \sigma_1^2 \) is either zero or extremely small. The relationship between \( \mu_3 \) and \( \mu_2 \) is open to question, and we will see that this will affect the outcome of the results in a significant way.

Each of the comparative statics is now considered in turn.

**Changes in Rates of Return**

\[
\frac{\Delta M}{\Delta \mu_1} \Bigg|_S = \Delta |m_{11}|/|\bar{m}| > 0 \text{ by second-order conditions.}
\]

\[
\frac{\Delta M}{\Delta \mu_2} \Bigg|_S = \Delta |m_{21}|/|\bar{m}| = \left(\frac{\Delta}{|\bar{m}|}\right) \theta \{\mu_2 - \mu_3\}(\mu_3 - \mu_1) - \sigma_3^2
\]

\[
< 0 \text{ if } \mu_3 > \mu_2 \text{ or if } \mu_2 > \mu_3 \text{ and } \sigma_3^2 \text{ is large,}
\]

\[
> 0 \text{ if } \mu_2 > \mu_3 \text{ and } \sigma_3^2 \text{ is small.}
\]

\[
\frac{\Delta M}{\Delta \mu_3} \Bigg|_S = \Delta |m_{31}|/|\bar{m}| = \left(\frac{\Delta}{|\bar{m}|}\right) \theta \{\mu_3 - \mu_2\}(\mu_2 - \mu_1) - \sigma_3^2
\]

< 0 if \( \mu_3 < \mu_2 \) or if \( \mu_2 < \mu_3 \) and \( \sigma_3^2 \) is large,

\[ \frac{\partial B}{\partial \mu_1} \Big|_S = \frac{\Delta m_{12}}{|m|} = \Delta |m_{21}| / |m| = \frac{\partial B}{\partial \mu_2} \Big|_S \]

> 0 if \( \mu_2 < \mu_3 \) and \( \sigma_3^2 \) is small.

\[ \frac{\partial B}{\partial \mu_2} \Big|_S = \Delta |m_{22}| / |m| > 0 \text{ by second-order conditions.} \]

\[ \frac{\partial B}{\partial \mu_3} \Big|_S = \Delta |m_{32}| / |m| = (\Delta / |m|) \theta\{ (\mu_1 - \mu_2)(\mu_3 - \mu_1) - \sigma_1^2 \} \]

< 0 unambiguously.

\[ \frac{\partial B^*}{\partial \mu_1} \Big|_S = \frac{\Delta m_{13}}{|m|} = \frac{\Delta m_{31}}{|m|} = \frac{\partial B}{\partial \mu_3} \Big|_S \]

\[ \frac{\partial B^*}{\partial \mu_2} \Big|_S = \frac{\Delta m_{23}}{|m|} = \frac{\Delta m_{32}}{|m|} = \frac{\partial B}{\partial \mu_3} \Big|_S \]

\[ \frac{\partial B^*}{\partial \mu_3} \Big|_S = \Delta |m_{33}| / |m| > 0 \text{ by second-order conditions.} \]

These results are discussed fully in the text. Now consider the other comparative statics of the model.

**Changes in Risk and Wealth**

\[ \frac{\partial M}{\partial \sigma_1} = \frac{(\partial M / |m|)}{1} \{(\mu_3 - \mu_2)^2 + \sigma_2^2 - 2\sigma_{23} + \sigma_3^2\} < 0 \]

\[ \frac{\partial M}{\partial \sigma_2} = -\left(\frac{\partial B}{|m|}\right)\{(\mu_3 - \mu_2)(\mu_3 - \mu_1) + \sigma_3^2\} \]

> 0 if \( \mu_3 > \mu_2 \) or if \( \mu_2 > \mu_3 \) and \( \sigma_3^2 \) is large,

< 0 if \( \mu_2 > \mu_3 \) and \( \sigma_3^2 \) is small.

\[ \frac{\partial M}{\partial \sigma_3} = \frac{(\partial B^* / |m|)}{1} \{(\mu_2 - \mu_1)(\mu_3 - \mu_2) - \sigma_3^2\} \]
\( > 0 \) if \( \mu_2 > \mu_3 \) or if \( \mu_3 > \mu_2 \) and \( \sigma^2 \) is large,
\( < 0 \) if \( \mu_3 > \mu_2 \) and \( \sigma^2 \) is small.

\[
\frac{\partial M}{\partial \omega_0} = (\theta^2/|m|)\{\sigma_2^2(\mu_1\mu_3 - \mu_2\mu_3) + \sigma_3^2(-\mu_2 + \mu_1\mu_3)\} > 0
\]

\[
\frac{\partial B}{\partial \sigma_2^2} = \frac{\partial M}{\partial \sigma_2^2}
\]

\[
\frac{\partial B}{\partial \sigma_3^2} = (\theta^2/|m|)\{(\mu_3 - \mu_1)^2 + \sigma_3^2 - 2\sigma_{13} + \sigma_1^2\} < 0
\]

\[
\frac{\partial B}{\partial \sigma_3} = - (\theta^2/|m|)\{(\mu_2 - \mu_1)(\mu_3 - \mu_4) + \sigma_3^2\} > 0
\]

\[
\frac{\partial B}{\partial \omega_0} = -(\theta^2/|m|)\{\sigma_3^2(\mu_2^3 - \mu_2\mu_3) + \sigma_3^2(\mu_2^2 - \mu_1\mu_3 + \sigma_1^2)\} = \text{ambiguous}
\]

\[
\frac{\partial \sigma_3^2}{\partial \sigma_2^2} = \frac{\partial \omega_0}{\partial \sigma_2^2}
\]

\[
\frac{\partial \sigma_3^2}{\partial \sigma_3} = \frac{\partial \omega_0}{\partial \sigma_3^2}
\]

Notice that if \( \sigma_2^2 = \sigma_{12} = \sigma_{13} = \sigma_{21} = \sigma_{31} = 0 \) then we have:

\[
\frac{\partial B}{\partial \omega_0} = -(\theta^2/|m|)\{\sigma_3^2(\mu_2^3 - \mu_2\mu_3)\} > 0
\]

\[
\frac{\partial \sigma_3^2}{\partial \omega_0} = (\theta^2/|m|)\{\sigma_3^2(\mu_1\mu_3 - \mu_1)\} > 0
\]

These results are discussed in detail in the text.
CHAPTER FIVE

SUMMARY OF PART II

Part II of this Thesis has been largely theoretical in nature. In contrast, Part III will be concerned solely with empirical testing of the implications of the analysis of Part II. With this in mind, this summary will be presented in such a way as to demonstrate clearly the purpose behind each of the empirical tests to come. The ability of the data to discriminate between the three different theoretical approaches will be of great interest.

A logical starting point is to set out the empirical implications which are common to each of the transactions, precautionary and speculative theories presented above for they cannot be rejected by the data without rejecting all three theoretical frameworks.

(1) Since we live in a highly interactive, multicurrency world, it is clear that conventional 'one-currency' demand for money equations, which specify real balances as a function of domestic income or wealth and the interest rate, are misspecified. An investigation of the three Keynesian motives for holding money has made it clear that the expected rates of depreciation of all other relevant currencies will be determinants of the domestic demand for the domestic currency. Furthermore, to the extent that the latter is held by non-residents, the levels of real national income or wealth and interest rates from each of the other relevant
countries should also be determinants of the demand for real balances denominated in that currency. Thus, total demand for the domestic money in a two-money world is given by:

$$\frac{M_D}{P} = \frac{D}{P}(y, r, s^e, r^*, y^*, SP^*/P)$$

whereas, domestic demand for domestic money is given by:

$$\frac{M}{P} = \frac{D}{P}(y, r, s^e)$$

Thus, it is possible to test the extent to which domestic money is held by non-residents by testing the null hypothesis that $r^*$, $y^*$ and $SP^*/P$ do not belong in the demand equation.

(2) All three theories have predicted that there will be a range for $s^e$ over which agents will act as if $s^e = 0$. In the transactions and precautionary approaches this arises at a corner solution where transactions costs exceed the profit incentive to react. In the speculative approach, this is a theoretical possibility which arises when the agent's indifference curves are not horizontal at the ordinate axis. The size of this band will depend on the values of the parameters of the problem which pertain to a particular individual. It would seem that for the 'average' agent, the expected rate of depreciation will have to be very large to induce arbitraging behaviour. However, other agents face significantly lower transactions costs, and it is these agents who will ensure that the covered interest differentials will be minimized. This issue will not be explored further in this Thesis;
the reader is referred to Branson (1969) and Frenkel and Levich (1977). ¹

(3) Each of the three theories has been quite specific in its predictions of reactions to changes in $s^e$ at interior solutions, although the analysis of the speculative motive was restricted to substitution effects. The predictions are that an increase in $s^e$ will cause domestic cash holdings to fall, and foreign cash holdings to rise more than in proportion; hence, bond holdings decline. This presumes, of course, that the two currencies are substitutes, as they must be in a two-currency framework. The possibility of currency complementarity has been suggested during analysis of the speculative motive only, for to introduce additional currencies into the other two frameworks is to create undue complexity.

These, then are the three predictions which are common to all three approaches to the problem. Further predictions have been found specific to the transactions and speculative approaches, and these two sets are mutually independent. The precautionary approach has little more to offer and, in that sense, may be regarded as superfluous. However, it is reassuring that the precautionary framework conforms to the predictions of the other two.

The following three implications are specific to the transactions approach and therefore would be expected to show up more strongly in demand functions for narrow definitions of the money stock rather than broad definitions. To the extent that these effects are evident in broad definitions of the money supply, then, one would regard the latter as transactions balances.
The first implication is rather trivial and will not aid in the 'what is money' debate; however, it is worth mentioning. The analysis of the transactions motive made it clear that agents with 'small' incomes will not hold bonds and will make the minimum number of foreign exchange transactions necessary to complete their consumption plans.

The theory suggests that the interest elasticity of the demand for real balances will decline in absolute value when $s^e$ is positive. Hence, traditional specifications of the demand for money function would be expected to exhibit particular instability when $s^e$ is positive.

The theory of Chapter Two presumed that the parameter $\alpha$, the fraction of the agent's income which is spent using domestic currency, is fixed. Although it is reasonable to regard $\alpha$ as independent of monetary phenomena, such as the exchange rate, real phenomena (such as the real exchange rate) might be expected to affect $\alpha$ and hence the demand for real balances. This is distinct from the direct effect which changes in the terms of trade have on the demand for real balances. It would appear that the two effects are empirically inseparable; however, it is possible to test whether the changes in $SP^*/P$ affect real balances indirectly through their effect on real foreign income, or directly, as the transactions model would suggest. This requires testing the formulation (5.1) against the restricted alternative:

\[
\frac{M_{D}}{F} = \frac{M_{D}}{F}(y, r, s^e, r^*, SP^*y^*/P)
\]
One further set of implications remains to be considered, that which arises from the analysis of the speculative motive and not from the other theories. These predictions, therefore, would be expected to be of greater significance in demand functions for broadly-defined money. To the extent that transactions balances act as a buffer for speculative portfolio adjustments, then, these effects would also be observed in narrow demand for money specifications.

(7) The first is of minor importance, relating as it does to substitution effects only. Any empirical work will be capturing both substitution and expected wealth effects. However, it is worth mentioning that the speculative theory predicts that the substitution matrix of changes in \( s \) and the rates of interest will be symmetric. Notice that this holds only for the domestic demand for domestic money, not the aggregate demand for domestic money. To find that this hypothesis were true in the aggregate data would imply something very special about the expected wealth effects.

(8) The speculative framework has made it clear that currency complementarity is a possibility. A positive sign on a particular \( s_i^e \) in the \( j \)th country's estimated demand for money function would have been regarded as 'incorrect' without having considered the speculative framework. Traditional analysis has, like much of this Thesis, concentrated on only two currencies, in which case they must be substitutes for one another. It is interesting to note that the introduction of the third asset, bonds, into the speculative framework, makes complementarity
between any pair of assets a theoretical possibility, whereas the
three-asset model of Chapter Two made no such allowance. The
three-asset precautionary model in principle could have made
either prediction.

(9) Probably the most important prediction in this set is that a
reduction in the noise in \( s^e \) relative to the variance of the
domestic rate of interest will cause the demand for domestic money
to become less sensitive to changes in the rate of interest, and
more sensitive to changes in the expected rate of depreciation.
Thus, traditional demand for money specifications should exhibit
particular instability when the ratio of these two variances
changes substantially.

Thus, it is clear that we can distinguish theoretically,
and have at least an opportunity to distinguish empirically, between
the transactions and speculative theories of the demand for money.
This is the reward for having exploited the Keynesian motive
distinction from the beginning. For implication (1), and possibly
predictions (2), (7) and (8) as well, could have been derived
without resorting to the detailed analysis of the previous three
chapters. As will now be demonstrated, these implications may be
derived explicitly or at least be allowed for in the marginal utility
theory of the demand for money.

Following Friedman (1969) we may think of the agent as
maximizing a utility function defined over a vector of assets subject
to an overall wealth constraint. Total wealth is given by \( y/r \),
where \( y \) is the gross flow of real income and \( r \) is 'the' rate of
interest. Then the agent's problem may be represented as follows.

\[
(5.4) \quad \max U(M/P, SM^e/P, B/P) + r_M M/P + (r_{M^*} + s) S^e M^e/P + r_B B/P
\]

subject to: \( y = r_W W = r_M M/P + (r_{M^*} + s) S^e M^e/P + r_B B/P \)

where \( r_M \), \( r_{M^*} \), and \( r_B \) are the rates of return to holding domestic currency, foreign currency, and domestic bonds, respectively. The asset demands are derived in a straightforward manner; in particular, the domestic demand for domestic money is given by:

\[
(5.5) \quad M/P = M/P(y, r_M, r_{M^*} + s, r_B)
\]

Now, notice that \( r_M = r_{M^*} = 0 \). Then,

\[
(5.6) \quad M/P = M/P(y, r_B, s^e)
\]

Now, consider a foreign agent who faces an identical problem. His demand for domestic currency (foreign, to him) is derived analogously and is given by:

\[
(5.7) \quad M_{F}/P^* = M_{F}/P^*(y^*, r_B^*, s^e)
\]

where \( r_{B}^* \) is the rate of return on foreign bonds and \( y^* \) is real foreign income. Then, the aggregate demand for real domestic balances is given by:

\[
(5.8) \quad M_{D}/P = M/P + SM_{F} P^*/PP^*
\]

\[= M/P + (SP^*/P)(M_{F}/P^*)\]
Thus, in general,

\[(5.9) \quad M_D/P = M_D/P(y, \tau_B, s^e, \tau^e_B, y^*, SP^* / P)\]

It is clear from Equation (5.9) that implication (1), which is common to all three of the theories presented in earlier chapters is also implicit in this marginal utility theory. Although the above treatment has presumed an interior solution, it would be relatively easy to tell a corner solution story so as to draw out implication (2) as well. Finally, because (5.9) is the outcome of utility maximization, implications (7) and (8) apply as well. Prediction (7) actually will only apply to (5.9) if both the domestic and foreign utility functions and incomes are identical. However, as mentioned above, Young's Theorem implies result (7) for the domestic demand for domestic money, (5.6). This prediction is directly analogous to its counterpart in the fundamental theory of the consumer, and so applies here as well.

In spite of this admittedly fine performance from such a simple theory, the other five predictions which were obtained by analyzing a more complex, more restrictive series of frameworks, seem to be worth the effort, at least at a theoretical level. Furthermore, predictions (1) to (9) above are only the results which appear to be empirically falsifiable; there are several other theoretical results in Chapters Two - Four, which are certainly too detailed to come out of the marginal utility theory, but which do not appear to be testable. The marginal utility approach yields no testable prediction which can shed any light on the 'what is money?' debate, in particular.
However, the final word in this issue must be left to the data. For if the data cannot support at least one of the predictions which arise from one of the theories of the three Keynesian motives and not from the marginal utility theory of the demand for money, then it will become clear, theoretical insights aside, that the marginal utility theory is as specific as one can get. This issue will be reconsidered in light of the empirical results in the concluding chapter of this Thesis.
FOOTNOTES TO CHAPTER FIVE

1 Branson (1969) has calculated that the U.K.-U.S. and the U.S.-Canada covered interest rate differentials must exceed only 0.18 percent per annum to induce profitable arbitrage. Frenkel and Levich (1977), among other things, have shown that after allowing for transactions costs the data for the U.S., Germany and the U.K. do not display any unexploited profit opportunities due to arbitrage.

2 Assuming that the Marshall-Lerner conditions hold, then \( \alpha = \alpha(S/P) \) where \( \alpha' > 0 \).
PART III:

EMPIRICAL EVIDENCE
CHAPTER SIX

ESTIMATION AND TESTING

6.1 Introduction

In this Chapter a model of the demand for money which embodies the hypotheses derived above will be estimated, and in this and in the subsequent chapter the hypotheses outlined in Chapter Five will be tested. Throughout the empirical analysis the theory will be taken as the maintained hypothesis, and this theory will be presumed to apply regardless of which country or countries the data set may represent. Thus, the present investigation will be devoid of ad hoc specification adjustments due to statistical criteria. The thrust of the inquiry will be to determine to what extent the hypotheses derived above can improve upon the traditional specification of the demand for money function.

The present Chapter comprises four additional sections. A description of the data and the criteria employed in selecting the same will be presented in Section 6.2, with full details relegated to Appendix A6. Testing will first require the estimation of the demand for money function, and empirically relevant versions of the latter typically involve a convolution of many hypotheses. Thus, in order to focus on the tests which are central to this Thesis, a number of preliminary issues will be dealt with in Section 6.3. In Section 6.4, the estimation results will be presented, and it will prove convenient
to test some of the hypotheses outlined above at that time. Finally, the Chapter will conclude with Section 6.5, in which the test results will be summarized conveniently.

6.2 The Data

(a) Scope

The theory of earlier chapters has suggested that 'all relevant currencies' be represented in the demand for money equation of a particular country. Omitting any currency may be regarded as a linear equality restriction which will reduce the variance of the remaining coefficient estimators and at the same time introduce omitted variables bias. Thus, a statistical approach to deciding on the number of countries to include, assuming a quadratic loss function, would seek to minimize the mean-squared-error of the remaining coefficient estimators by excluding variables which give small increases in bias relative to the accompanying reduction in variance. A more economic approach, on the other hand, would be to select a ranking of countries which should be included based on the importance of their trade with one another, the availability and quality of data, and to cut off the ranking at some 'reasonable' point so that the estimating equations are manageable, if not parsimonious. The latter approach is the one which is adopted here, although the statistical criteria played a minor role.

With these considerations in mind, the 'world' will be viewed as consisting of five countries in the work to be reported on below—Canada, the United States, West Germany, France, and the United Kingdom—whereas a complete investigation of the hypotheses of
this Thesis will be undertaken only for the first three of these countries. The reasons for including these countries should be obvious—they constitute the largest of the OECD economies and each has close economic ties with the others. The decisions to exclude other countries, such as Japan, Italy and Switzerland, were highly judgemental, and involved a wide array of criteria. In particular, forward exchange rates (a variable which bears considerable importance for the ensuing analysis) are available for Japan and Italy only since the mid-1970's. The exclusion of Japan may seem particularly bothersome, given her importance in the international trade of Canada and the United States. However, as the reader may verify in Appendix A6, the Japanese spot exchange rate has historically moved very closely to the German Spot rate, implying that any omitted variables bias resulting from the exclusion of Japan should be very small.

(b) **Periodicity**

Evidence which has been summarized in Chapter One indicates that currency substitution effects are difficult to find in quarterly data, whereas Alexander (1980) and Brillembourg and Schadler (1979) have reported reasonable success with monthly data. Thus, for the purposes of this Thesis, monthly data are used. Although all relevant financial statistics are available on a monthly basis, however, national income is available on a quarterly basis only. In the literature two solutions to this problem have been suggested.

(1) White (1976) and Alexander (1980) use cubic Lagrange interpolation of quarterly data to obtain monthly observations of national income.
(2) Brillembourg and Schadler (1979) use a monthly industrial production index to interpolate the quarterly data. The first method is advantageous in that it is applicable to both real and nominal income, so that a monthly implicit price deflator may be derived also. The advantage of the second method is that it uses more information. However, the monthly production index is a real variable; hence, there is no equivalent way to interpolate nominal income, and so this approach will not provide an implicit price deflator. Of course, a monthly consumer price index is available and, although the bulk of prior empirical work in this area has not made use of this index, the approximation may not be a bad one. Ideally one should deflate money using the same variable which is used to deflate nominal income. However, the benefits of using monthly data are expected to outweigh the costs of using the consumer price index.

In any event, in the work to follow, national income series are generated from quarterly data by using the monthly industrial production index as an interpolator. The monthly price level series is proxied by the consumer price index. Although the approach is admittedly ad hoc, it is believed that the costs will be far outweighed by the richer variation in the financial variables and by the increased number of degrees of freedom, both of which are important benefits. The degrees of freedom consideration is particularly attractive because it will enable the use of a distributed lag model in addition to the now standard, but problem-ridden, partial-adjustment model of the demand for money. This problem of model selection will be developed further below.
(c) **Data Source**

The principal source for the data to be analyzed below is the OECD publication *Main Economic Indicators*. This source was chosen because the OECD is careful to document its sources (which are, for the most part, primary) and to publish complete historical series when a revision has been reported by the original reporting agency. Moreover, much of the data to be used here may be found conveniently in one historical statistics volume. An explicit, country-by-country description of the data is reserved for Appendix A6. Most of the data start in 1960; however, some series begin later, and for this reason the specifications which use narrow measures of the money stock will be estimated from 1963 onwards, and those which use broad measures will be estimated from 1967 onwards. The specific reasons are given in the Appendix.

(d) **Seasonal Adjustment**

Frequently the researcher is faced with the dilemma of choosing between using data which have been deseasonalized by some official agency or using the raw data in conjunction with binary variables which capture the seasonality. Occasionally the choice is made by availability; such is the case here, for the quarterly national income series in the OECD data and indeed, in most of the original central bank publications, are seasonally adjusted. Thus, in the work to be reported on below the data on the money stocks, price levels, and levels of national income will be seasonally adjusted prior to estimation. Before proceeding, then, the implications of this decision should be considered.
Work in this area has been carried out theoretically by Lovell (1963), Thomas and Wallis (1971) and Newbold (1975). These authors have pointed out that parameter estimates will be best linear unbiased only if the deseasonalization procedure is sum-preserving. Deseasonalization by ordinary least squares with binary variables is such an example; however, this procedure assumes that the seasonal pattern is constant throughout the sample. On the other hand, the ratio to moving average technique favoured by official agencies (as in the data to be used below) is not, in general, sum-preserving, but allows for a shifting seasonal pattern. However, it has been shown by spectral analysis (Nerlove, 1964) that this technique removes more than seasonal peaks from the data. Furthermore, deseasonalization of the data prior to regression analysis inevitably leads to overconfidence in the model; this is because the degrees of freedom used during the deseasonalization are not properly accounted for in the final regression. Clearly this is not a problem when the seasonal adjustment is carried out within the regression using binary variables. Finally, it is widely believed that the ratio to moving average technique will result in nonspherical regression residuals. This problem has been dealt with most effectively in Wallis (1972) where a test for fourth-order serial correlation is derived.

In the work to follow, then, the following additional precautions will be taken in dealing with seasonally adjusted data.

1. According to a suggestion made by Lovell (1963), the tests will be based on \( t-k-d \) degrees of freedom, where \( t \) is the number of observations, \( k \) is the number of estimated.
parameters, and \( d = 3m - 1 \), where \( m \) is the number of 'seasons'.

Then in this case, where monthly data is used, \( d = 35 \).

(2) Correspondingly, it is necessary to multiply the (mis)calculated standard errors by the factor \( q \), where:

\[
q = \left\{ \frac{(t-k)}{(t-k-d)} \right\}^{\frac{1}{2}}
\]

This raises the underestimated standard errors to the correct value. This adjustment applies also to the standard error of the regression (SER) as well as the calculation of \( R^2 \), \( R^2 \) corrected for degrees of freedom.

(3) Finally, a Box-Jenkins analysis of the residuals of the final estimated equations will be carried out to determine the degree of reliability of the results.

6.3 Preliminary Issues

Before proceeding to test the implications set out above, it is necessary to deal with a number of preliminary problems relating to estimation of demand for money equations. Although this will enable us to focus on the issues central to this Thesis, it will mean that each test will be of a joint hypothesis, with the conclusions qualified by the particular stands taken on the following matters.

(1) Definition of Money

It is not the express purpose of this Thesis to resolve the ongoing dispute between the proponents of broad versus narrow definitions of the money stock as targets for policy. Rather, it is hoped that as a byproduct the empirical work will shed light on this controversy and, with this in mind, two monetary aggregates will be analyzed for each country. These aggregates are commonly referred to
as M1 (narrow) and M2 or 'M1 plus quasi-money' (broad), although the precise definitions will vary somewhat between countries. As noted above, this distinction between narrow and broad money should also serve another purpose. This is because it would seem that the implications of the speculative theory presented in Chapter 4, should be more evident in a broad rather than a narrow monetary aggregate. Similarly, the implications of the transactions theory should more emphatically be demonstrated in the narrow money stock.

(2) **Scale Variable**

The choice between current national income, permanent income, or wealth as a scale variable in the demand for money function is a difficult one, but one which is often made by availability of data. The latter consideration rejects the use of wealth for most countries, with the exception of the United States. Permanent income is generally calculated from a distributed lag of current income in any event. Thus, from an econometric point of view it makes little difference which of the two variables is utilized, although of course the interpretation of the results will be affected. For example, use of a partial-adjustment model (described fully below) allows income to affect the demand for money with an infinite distributed lag, but this is not equivalent to saying that the demand for money depends on permanent income. In short, in estimating the short-run demand for money function current real national income will enter the specification in a distributed lag. As discussed below in the section on model selection, this lag will be interpreted as arising due to adjustment costs.
(3) Interest Rates

Judging from previous work on the demand for money function it would seem that two interest rates per country will be needed to complete the analysis. The first is a 'substitute' rate, in that it is a rate paid on an asset which is a close substitute for money. The second rate is an 'own' rate; that is, the rate which is paid on money. Setting aside arguments about the implicit rate of return on demand deposits, in a noninflationary environment the own rate on narrow money is equal to zero. Thus, narrow money demand equations typically contain only a substitute rate, such as the rate on commercial or finance company paper, the rate on call or day loans, or that on Treasury Bills. In the work reported here the same rate has not been used for each country; details are given in the Appendix.

The own rate on broad money, of course, is generally nonzero, so that it would seem that broad money demand equations should include both the substitute rate and the rate which is paid on time or savings deposits. Then, one would expect the substitute rate to have a negative effect on the demand for either narrow or broad money, and the own rate to have a positive effect on the demand for broad money.

An additional consideration here is whether to enter the interest rates as levels or as logarithms. In keeping with the vast majority of empirical work on the demand for money, the equations will be log-linear (that is, the log of real balances is expressed as a function of the log of real income). Entering the rate of interest also in logarithmic form has become extremely popular, mainly because of the convenience in obtaining constant elasticity estimates, but
also because preliminary inspection usually demonstrates that there is very little basis for choosing between this and the semilogarithmic form, where the rate of interest is entered in levels. Laidler (1980) has reported that the latter tends to perform better than the former during the 1970’s for the U.S. data. This is not too surprising given that the semilogarithmic form allows the interest elasticity to rise with the rate of interest, an attractive feature during periods of high interest rates, whereas the double-logarithmic form constrains the elasticity to be constant at all rates of interest. Because the choice seems to make no difference during the 1960’s and because we have observed very high rates of interest during the late 1970’s, the argument for using the semilogarithmic form would appear to be very compelling indeed. Therefore, the semilogarithmic form will be utilized throughout the ensuing empirical analysis.

Finally, it should be noted that, due to costs of portfolio adjustment, it will be necessary to allow lagged adjustment to changes in the rate of interest, just as with changes in national income. Alternative means of handling this will be considered below.

(4) The Expected Rate of Inflation

The theoretical analysis of earlier chapters implicitly assumed that the rate of return to holding cash was zero. In this case the opportunity cost of holding cash is simply the rate of interest on the alternative asset. This is also true when there is a positive but steady rate of inflation for, by the Fisher equation, the inflationary expectations become incorporated into the (nominal) rate of return on the alternative asset. However, during periods when the rate of
inflation is high and variable, inflationary expectations may not immediately and fully feed through to nominal rates of return. This has led some authors to suggest (see Boughton, 1979a, for example) that the expected rate of inflation be included in the demand for money equation. Although this seems a reasonable suggestion, incorporating it by using the actual inflation rate as a proxy for the rational expectation of the same is difficult to accept. Because the construction of a more reasonable proxy could be a considerable undertaking, and because rates of inflation in the countries under study have typically been relatively low, in the work to follow it will be presumed that inflationary expectations are fully incorporated into nominal rates of interest and/or expected rates of exchange rate depreciation.

(5) The Expected Rate of Exchange Depreciation

Clearly the expected rate of depreciation of the various currencies will play a significant role in testing the hypotheses of earlier chapters, and the question arises as to how one should measure this variable. Exchange rate theory and the concept of rational expectations are clear on this—the forward exchange rate must embody all currently available information and therefore is the best available indicator of expectations of the future spot rate. This position is supported by the stylized facts presented in Mussa (1979). However, recent work (see, for example, Hakkio, 1980; Freedman and Longworth, 1980) has cast some doubt on this hypothesis, arguing that the forward exchange market is inefficient. Freedman and Longworth have also suggested that the policy rule followed during their sample implies
that exchange market intervention was not the source of inefficiency. Hakkio (1980) has noted, quite correctly, that it is not possible to design a pure test of market efficiency—the test is also of the model employed, and most models assume (a) a constant risk premium (b) no distinction between permanent and transitory shocks and (c) no intervention. The only conclusion to be drawn from this literature is that this issue remains unresolved; even if the exchange market is inefficient, the burden of explaining why and suggesting a more appropriate measure of exchange rate expectations lies with the critics.

In the empirical work undertaken and reported on below, the forward exchange rate will be taken to be the rational expectation of the future spot rate. Then the expected rate of depreciation will be given by the difference between the logarithm of the forward rate and the logarithm of the spot rate, both in terms of the U.S. dollar.

In the Appendix this variable has been plotted in percentage terms for Canada, Germany, France and the United Kingdom for the period 1960.01–1980.09. Notice that the variable does indeed capture the major exchange rate crises, such as the Canadian dollar in 1976, and the U.S. dollar relative to the Deutschemark in 1972.

A related point concerns how one should model switches between fixed and floating exchange rate regimes. This is a difficult problem, and one which is of some importance since the data set to be used contains several such switches. However, the point is related to expectation formation only, and therefore is irrelevant here given the assumption that the forward rate embodies all such information.

(6) Model Selection
As pointed out by Laidler (1981), estimation of the short-run demand for money function has typically been achieved by adding a lagged dependent variable as an additional explanatory variable to the simple long-run specification. Various stories are available which can justify the addition; the most convincing is that of aggregation over agents who adjust their portfolios at different points during the period. For example, the partial-adjustment model is specified as:

\begin{equation}
(6.3.1) \quad m^*_t = \psi x_t
\end{equation}

\begin{equation}
(6.3.2) \quad m_t - m_{t-1} = \lambda (m^*_t - m_{t-1}) + \epsilon_t
\end{equation}

where $x$ is a vector of explanatory variables (usually only national income and an interest rate), $m$ is real money balances, the asterisk denotes a long-run desired level of real balances, $\psi$ is the transpose of a vector of coefficients, and $\epsilon$ is a white-noise error term.

Combining these yields the estimating equation:

\begin{equation}
(6.3.3) \quad m_t = \lambda \psi x_t + (1-\lambda) m_{t-1} + \epsilon_t
\end{equation}

This model has been used extensively in the literature. Notice that another interpretation is possible; where $L$ is the lag operator, we have:

\begin{equation}
(6.3.4) \quad m_t = \lambda \psi x_t / (1 - (1-\lambda)L) + \epsilon_t / (1 - (1-\lambda)L)
\end{equation}

\[= \sum_{i=0}^{\infty} (1-\lambda)^i x_{t-i} + u_t\]
so that the effect of changes of $x$ on $m$ is presumed to decline geometrically to infinity. This is the same for each element of $x$.

Setting aside the issues raised by Laidler concerning the interpretation of the specification (6.3.3), the purpose in introducing this modification was originally to allow lagged adjustment to changes in the independent variables without wasting precious degrees of freedom. Although this purpose is achieved by the specification, adjustments to changes in all of the independent variables contained in $x$ are implicitly assumed to follow the same geometrically declining lag pattern.

The second specification which is estimated here is the Almon polynomial distributed lag (PDL) model. This version allows different responses in money holdings to changes in different elements of $x$, perhaps because of aggregation across agents who face different adjustment costs:

$$m_t = \sum_{i} \sum_{j=0}^{T_i} \psi_{ij} x_{i,t-j} + \varepsilon_t$$

(6.3.5)

where $i$ indexes the elements of $x$ and $j$ indexes the lags. It is not necessarily true that the latter model is less restrictive than the partial-adjustment model. The lag structures of the PDL model are not constrained to decline geometrically, but are constrained to be of finite length and to lie along a low-order polynomial. However, both models have been used in the literature and considering both avoids the problem of choosing one a priori.

(7) Simultaneous Equations Bias

As is the case with any market, the observed money supply
and interest rate data are the outcome of the interaction between a
demand and a supply curve. Hence, estimation of the demand for money
equation in isolation may result in simultaneous equations bias in the
estimated elasticities.

The issue of simultaneity in respect of the demand for
money function has been analyzed by Feige (1967) and Goldfeld (1973)
for the U.S., and White (1976), Poloz (1980) and Gregory and McAleer
(1981) for Canada. In all studies the extent of bias has been found
to be statistically not significant, although it is argued in Poloz
(1980) (a) that statistical significance at the 95% level need not
correspond exactly to practical relevance and (b) that the bias has
exhibited an interesting pattern over time, corresponding closely to
changes in monetary policy regimes. Thus, the issue evidently
remains unresolved. However, a careful consideration of these
findings has led this author to believe the issue to be less
commanding in the present context. This is because the simultaneous
equations bias is detected (mainly in the interest elasticity, for
obvious reasons) by performing two-stage least squares with the foreign
(that is, the U.S., for the Canadian case) rate of interest treated
as exogenous. Thus, simultaneous equations bias in the Canadian demand
for money function arises only when r deviates significantly from r*.
Technically, two-stage least squares is quite like replacing r with r*
when r ≠ r*, and leaving r alone otherwise, so that the regression
equation behaves as if it contains r* rather than r. In the present
context, theory has dictated that each regression should contain both
r and r*. Thus, the 'simultaneous equations bias' which has been
measured previously might better be interpreted as a measure of the degree of misspecification caused by the omission of several potentially important variables. Viewed in this light, the consideration of simultaneous equations bias in the single-equation context no longer seems meaningful, unless one can think of a more appropriate set of instruments. A more sophisticated approach will be necessary to resolve the issue satisfactorily. In particular, one should replace the assumption that money demanded and supplied are equal, which is implicit in both Equations (6.3.3) and (6.3.5) with a money supply or reaction function, and estimate both simultaneously. Although this project would undoubtedly prove interesting, evidently it would double the proportions of the present undertaking, and therefore is considered beyond the scope of this Thesis. Thus, in the work to be presented below, the issue of simultaneous equations bias will not be considered. 6.4 Estimation and Testing (a) Equation Specification

As mentioned in the previous section, each hypothesis will be tested using two different models as the maintained hypothesis—the partial-adjustment (PA) formulation, and the Almon polynomial distributed lag (PDL) specification. It will prove convenient to classify the models as defined in Table 5.4.1. The reader will notice that the foreign variables in the PDL model (Model 4) have been constrained to affect the demand for real balances contemporaneously only, the reason being as follows. As mentioned above, the intention is to avoid ad hoc decisions (based on t-statistics, for example) about which variables to keep and which to discard. Thus, all variables
which are suggested by the theory should be given equal treatment.

Ideally, then, one would like to allow each of the variables to affect the demand for real balances according to some distributed lag, the form of which is determined by the data. This procedure has been followed with the traditional specification in order to construct a suitable maintained hypothesis for testing of the foreign variables. However, allowing a totally free specification exceeds the capabilities of the computing system readily available at the time of writing. In addition, some selective experimentation with lags on particular foreign variables which have displayed contemporaneous statistical significance has indicated that the assumption is not overly restrictive. Hence, the standard specification will be compared with the same model with a number of contemporaneous foreign variables added. Of course, this problem affects only the PDL model—the PA model forces each independent variable to take on a geometrically declining distributed lag, a feature which may be more restrictive than that of assuming contemporaneous interaction only.
TABLE 6.4.1

MODEL SPECIFICATIONS

(a) Narrow Money Stock

Model 1: PA without foreign variables (standard model)

\[ m_t = \alpha + \lambda y_t + \lambda y_t \gamma + (1-\lambda)m_{t-1} + \epsilon_t \]

Model 2: PA with foreign variables

\[ m_t = \alpha + \lambda y_t + \lambda y_t \gamma + (1-\lambda)m_{t-1} + \sum_{i=1}^{n-1} \mu_i s_{it} \]
\[ + \sum_{i=1}^{n-1} \delta_i s_t + \sum_{i=1}^{n-1} \eta_i y_{it} + \sum_{i=1}^{n-1} \phi_i S_{it} P_t / P_t + \epsilon_t \]

Model 3: PDL without foreign variables

\[ m_t = \alpha + \sum_{j=0}^{J} \beta_j y_{t-j} + \sum_{k=0}^{K} \gamma_k y_{t-k} + \epsilon_t \]

Model 4: PDL with foreign variables

\[ m_t = \alpha + \sum_{j=0}^{J} \beta_j y_{t-j} + \sum_{k=0}^{K} \gamma_k y_{t-k} + \sum_{i=1}^{n-1} \mu_i s_{it} \]
\[ + \sum_{i=1}^{n-1} \delta_i s_{it} + \sum_{i=1}^{n-1} \eta_i y_{it} + \sum_{i=1}^{n-1} \phi_i S_{it} P_t / P_t + \epsilon_t \]

(b) Broad Money Stock

Model 1: PA without foreign variables (standard model)

\[ m_t = \lambda + \lambda y_t + \lambda y_t \gamma + (1-\lambda)m_{t-1} + \epsilon_t \]
TABLE 6.4.1 (CONTINUED)

Model 2: PA with foreign variables

\[ m_t = \lambda \alpha + \lambda \beta y_t + \lambda \gamma_{1,R} s_t + \lambda \gamma_{2,R} o_t + (1-\lambda)m_{t-1} + \sum_{i=1}^{n-1} \delta_{1,i} s^e_{it} + \sum_{i=1}^{n-1} \delta_{2,i} o^e_{it} + \sum_{i=1}^{n-1} \eta_{i} y^e_{it} + \sum_{i=1}^{n-1} \mu_{i} s^e_{it} + \sum_{i=1}^{n-1} \phi_{i} S^e_{it} / P + \nu_t \]

Model 3: PDL without foreign variables

\[ m_t = \alpha + \sum_{j=0}^{J} \beta_{j} y_{t-j} + \sum_{k=0}^{K} \gamma_{k} s_{t-k} + \sum_{l=0}^{L} \gamma_{2,l} o_{t-l} + \nu_t \]

Model 4: PDL with foreign variables

\[ m_t = \alpha + \sum_{j=0}^{J} \beta_{j} y_{t-j} + \sum_{k=0}^{K} \gamma_{k} s_{t-k} + \sum_{l=0}^{L} \gamma_{2,l} o_{t-l} + \delta_{1,R} s^e_{it} + \delta_{2,R} o^e_{it} + \eta_{i} y^e_{it} + \mu_{i} s^e_{it} + \phi_{i} S^e_{it} / P + \nu_t \]

Definitions

\[ m = \text{natural logarithm of real money balances} \]
\[ y = \text{natural logarithm of real domestic national income} \]
\[ RS = \text{level of substitute rate of interest, domestically} \]
\[ RO = \text{level of own rate of interest, domestically} \]
\[ SP^e / P = \text{natural logarithm of the real exchange rate} \]
\[ * = \text{denotes a foreign equivalent of the domestic variable} \]
\[ \epsilon, \nu = \text{white-noise errors} \]
\[ n = \text{the number of countries under consideration; here } n=5 \]

Note: the indices J, K, and L need not be the same across different definitions of money.
(b) **Postal Strikes in the Canadian Equations**

The relatively frequent occurrence of postal strikes in Canada, and the fact that such occurrences cause a temporary surge in the demand for real balances, has made it necessary to include postal strike variables in the Canadian demand for money equation. Following Gregory and MacKinnon (1980), the following specifications will be estimated for Canada.

**PA Model**

\[
\begin{align*}
m_t &= \lambda \alpha + \lambda \beta y_t + \lambda \gamma R_t + (1-\lambda)m_{t-1} + \psi_1 y_t + (1-\lambda)\psi_1 m_{t-1} \\
&\quad + \psi_2 N_DAYS^h_t - (1-\lambda)\psi_2 N_DAYS^h_{t-1} + \omega_t
\end{align*}
\]

where all variables are as defined above, ST=1 for an observation affected by a strike, 0 otherwise, NDAY$S$ is the number of days which the strike lasts, and h=2 to make the effect of NDAY$S$ nonlinear. This equation contains two overidentifying nonlinear restrictions, which are evidently retained by the quarterly data used by Gregory and MacKinnon. These two restrictions are rejected by the monthly data used here; however, the two variables ST and NDAY$S^2$ are statistically significant and therefore will be retained in unrestricted form. The extension of the broad money demand specification is handled in exactly the same manner.

**PDL Model**

In the case of the PDL model it is necessary only to add the two variables, ST and NDAY$S^2$, to the specifications given in Table 6.4.1.
(c) Estimation of Model 1

The results of estimating Model 1 for Canada, the United States and Germany for both narrow and broad definitions of the money supply are given in Tables 6.4.2 and 6.4.3. Where necessary the equations have been estimated under the assumption that the error structure behaves according to a first-order autoregressive (AR1) process. The procedure used is that proposed by Beach and MacKinnon (1978), a procedure which retains the first observation.

Canada:

The two Canadian equations are quite robust, both with $R^2$ in excess of 0.99 and with all coefficients well-determined. Notice that for ease of comparison and expositional convenience the postal variables are not reported. The narrow money income elasticity of 0.8391 is a little high relative to those reported for quarterly data (for example, 0.730, Poloz, 1980). Similarly, the interest elasticity (evaluated at the mean) of 0.26 is high in absolute value, a reflection of the fact that the semilogarithmic form has been estimated, thus allowing the elasticity to rise with the rate of interest. As is typical with broader aggregates, the broad equation income elasticity is higher still, 1.1483, a number which is statistically greater than unity at the 0.95 level of confidence. The substitute and own rates of interest both achieve the expected signs, and their coefficients indicate that if all rates move together, a rise in interest rates generally will cause the broad aggregate to grow rather than to contract. Notice that the broad equation required AR1 estimation.
United States

The U.S. results are considerably less robust, with the long-run elasticities attaining rather unexpected values in the narrow equation due to the fact that the adjustment parameter is close to zero. This finding is consistent with those of Boughton (1979a) who found that \((1-\lambda) > 1\), thus completely preventing calculation of the long-run elasticities. This may be an indication that the U.S. money data are strongly trended. It should be clear that the narrow equation is highly dependent on the inclusion of the lagged money stock. The broad equation performs more satisfactorily, but the adjustment parameter is still very low, and the influence of real income is not statistically significant at the 0.95 level. As was the case with Canada, the two interest rates achieve the expected signs. Both equations required AR(1) estimation in order to bring the Durbin h statistic into the acceptable region (less than 1.96).

Germany

The two German equations are quite satisfactory, although also somewhat dominated by the lagged dependent variable. The narrow equation has properties quite like those of the Canadian version, although the income elasticity is higher than usual for a narrow definition of money. The broad equation is very similar, with all signs as expected, but with interest rates showing only a mild influence on the demand for real balances. Unlike the Canadian and American equations, the sum of the two interest rate coefficients is negative. Finally, the broad German equation required the AR(1) estimation procedure.
# Table 6.4.2

## Estimation Results of Model 1, Narrow Money Stock

<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>United States</th>
<th>Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0.0616 (3.71)</td>
<td>0.0064 (1.84)</td>
<td>0.0479 (2.67)</td>
</tr>
<tr>
<td>Rs</td>
<td>-0.0026 (5.59)</td>
<td>-0.0014 (5.36)</td>
<td>-0.0011 (4.00)</td>
</tr>
<tr>
<td>$m_{t-1}$</td>
<td>0.9266 (41.0)</td>
<td>0.9946 (120.4)</td>
<td>0.9572 (60.6)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.0652 (3.25)</td>
<td>-0.0067 (0.16)</td>
<td>-0.0921 (2.21)</td>
</tr>
</tbody>
</table>

## Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>United States</th>
<th>Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSR</td>
<td>0.0183</td>
<td>0.0043</td>
<td>0.0181</td>
</tr>
<tr>
<td>SER</td>
<td>0.0097</td>
<td>0.0046</td>
<td>0.0096</td>
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<tr>
<td>$R^2$</td>
<td>0.9971</td>
<td>0.9938</td>
<td>0.9983</td>
</tr>
<tr>
<td>log(L)</td>
<td>785.46</td>
<td>958.13</td>
<td>786.77</td>
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<tr>
<td>NOBS</td>
<td>237</td>
<td>237</td>
<td>237</td>
</tr>
<tr>
<td>DOF</td>
<td>194</td>
<td>198</td>
<td>198</td>
</tr>
<tr>
<td>DW</td>
<td>2.1332</td>
<td>1.9740</td>
<td>2.1163</td>
</tr>
<tr>
<td>$h^*$</td>
<td>1.0730</td>
<td>0.2013</td>
<td>0.9186</td>
</tr>
<tr>
<td>$\rho$</td>
<td>-</td>
<td>-0.1310 (2.00)</td>
<td>-</td>
</tr>
</tbody>
</table>

## Long-run Elasticities

<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>United States</th>
<th>Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0.8391 (13.4)</td>
<td>1.1663 (0.87)</td>
<td>1.1196 (15.1)</td>
</tr>
<tr>
<td>Rs</td>
<td>-0.0351 (3.82)</td>
<td>-0.2539 (0.79)</td>
<td>-0.0248 (2.95)</td>
</tr>
</tbody>
</table>

**Notes:**

1. Absolute values of t-statistics in parentheses
2. SSR = sum of squared residuals
3. SER = standard error of the regression
4. log(L) = log of likelihood function
5. NOBS = number of observations
6. DOF = degrees of freedom
7. DW = Durbin-Watson statistic
8. $h^*$ = Durbin's statistic for 1st-order serial correlation with a lagged dependent variable
9. $\rho$ = 1st-order serial correlation coefficient

Range: 1961:01 - 1980:09
## TABLE 6.4.3

**ESTIMATION RESULTS OF MODEL 1, BROAD MONEY STOCK**

<table>
<thead>
<tr>
<th></th>
<th>CANADA</th>
<th>UNITED STATES</th>
<th>GERMANY.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-0.0858 (3.05)</td>
<td>0.0373 (1.85)</td>
<td>0.0767 (2.63)</td>
</tr>
<tr>
<td>$RS$</td>
<td>-0.0011 (1.65)</td>
<td>-0.0019 (6.03)</td>
<td>-0.0007 (2.04)</td>
</tr>
<tr>
<td>$RO$</td>
<td>0.0022 (2.20)</td>
<td>0.0026 (0.96)</td>
<td>0.0005 (0.57)</td>
</tr>
<tr>
<td>$c_{t-1}$</td>
<td>0.9246 (40.1)</td>
<td>0.9701 (60.1)</td>
<td>0.9302 (43.6)</td>
</tr>
<tr>
<td>constant</td>
<td>-0.1155 (2.41)</td>
<td>-0.0540 (0.86)</td>
<td>-0.1653 (1.98)</td>
</tr>
</tbody>
</table>

**SUMMARY STATISTICS**

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SSR</td>
<td>0.0047</td>
<td>0.0018</td>
<td>0.0127</td>
</tr>
<tr>
<td>SER</td>
<td>0.0063</td>
<td>0.0038</td>
<td>0.0102</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.9987</td>
<td>0.9996</td>
<td>0.9957</td>
</tr>
<tr>
<td>log(L)</td>
<td>611.31</td>
<td>689.72</td>
<td>532.33</td>
</tr>
<tr>
<td>NOBS</td>
<td>161</td>
<td>161</td>
<td>161</td>
</tr>
<tr>
<td>DOF</td>
<td>117</td>
<td>121</td>
<td>121</td>
</tr>
<tr>
<td>DW</td>
<td>1.9689</td>
<td>2.0943</td>
<td>2.0180</td>
</tr>
<tr>
<td>$h$</td>
<td>0.2042</td>
<td>0.6083</td>
<td>0.0706</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.1715 (2.15)</td>
<td>0.4636 (6.58)</td>
<td>0.1461 (1.84)</td>
</tr>
</tbody>
</table>

**LONG-RUN ELASTICITIES**

<p>| | | | |</p>
<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>1.1483 (17.0)</td>
<td>1.2508 (3.40)</td>
<td>1.2384 (10.1)</td>
</tr>
<tr>
<td>$RS$</td>
<td>-0.0136 (1.41)</td>
<td>-0.0622 (1.99)</td>
<td>-0.0120 (1.56)</td>
</tr>
<tr>
<td>$RO$</td>
<td>0.0279 (2.30)</td>
<td>0.0888 (1.02)</td>
<td>0.0068 (0.53)</td>
</tr>
</tbody>
</table>

Range: 1967:05 - 1980:09
(d) Estimation of Model 3

In estimation of Model 3, the results of which are given in Tables 6.4.4 and 6.4.5, all equations utilize Almon lags on real income and interest rates. The Almon polynomials were all constrained to be of order two after some prior experimentation. In most cases, numerous rhs's were required in order to determine the optimal lag structures. In two cases, an AR1 specification of the error structure was assumed; in the others, an AR2 assumption was used. Because the PDL procedure in TSPL cannot at the same time assume non-white error structures, the estimation was done in stages. First the equation was estimated using ordinary least squares (OLS); then, the residuals were regressed on their lagged values to obtain an estimate of \( \rho_1 \). The entire model was then differenced using the coefficient \( \rho_1 \), and the resulting transformed model reestimated. When necessary (that is, the DW statistic was still unacceptable) the procedure was repeated on the transformed model. Thus, the parameters \( \rho_1 \) and \( \rho_2 \) do not have the usual interpretation of an AR2 model; rather, the errors are modeled as follows:

\[
\xi_t = \epsilon_t - (\rho_1 + \rho_2) \epsilon_{t-1} + (\rho_1)(\rho_2) \epsilon_{t-2}
\]

where \( \xi_t \) is the residual of the final equation, and \( \epsilon_t \) the residual of the original OLS equation. Thus, all reported \( \rho \)'s given below should be interpreted as above. It should also be noted that, because a few observations are available in front of the sample used, no observations are lost due to the AR2 procedure.
Canada

The narrow equation yields satisfactory estimates for the income and interest coefficients. The impact of income on real balances peaks in the previous and second-previous months, then dies out. The sum of the coefficients, 0.7360, is a reasonable value for the income elasticity, but is somewhat lower than that of the PA model. The interest rate effect peaks at lags three and four, being rather weak contemporaneously; the sum of the coefficients, evaluated at the mean interest rate, yields an interest elasticity of -0.10, a number which is low in absolute value relative to that of the PA model. The $R^2$ is considerably lower than in the PA model, giving some indication of the amount of variation which the lagged dependent variable captures in the PA specification.

The broad model fits the data very well by comparison. The income elasticity is higher than unity, as in the PA results. The seven-month lag which was used for RS in the narrow equation was shortened to four months here, the income lag was lengthened by one month, while the effects of changes in the own rate take seven months to die out. All signs are as expected.

United States

The U.S. narrow equation gives a poor performance. National income has an almost imperceptible effect on real balances; on the other hand, the interest rate coefficients are relatively well-behaved, with the only problem being a (very small) positive sign on the contemporaneous interest rate. A great deal of experimentation with this equation has failed to bring it into line with the others.
Interestingly, the model works well when estimated for the 1960's only, and fairly well when estimated for the 1970's only. This matter will be developed more fully as the results are assembled. The broad model, in contrast, behaves quite conventionally, with all signs as expected and all coefficients well-determined. The lag structures in both equations differ from those of the Canadian equations. In the narrow specification, three lags are allowed on income but eight lags are allowed on RS. In the broad model, these same two structures are supplemented by eight lags on the own rate of interest. Notice that, as with the Canadian broad equation, simultaneous increases in both RS and RO will cause the demand for broadly-defined money to rise.

Germany

Both equations utilize the same lag structure as the U.S. equations. The narrow equation is very impressive when compared with the remainder of those in this set. All of the desired properties are displayed, with the exception of the error structure, which required AR2 estimation to bring the DW statistic into line. This is in contrast to the other two narrow equations, which use an AR1 specification. The broad equation yields a curious result—the signs on the substitute and own rates of interest are exactly opposite to those which were expected. The contemporaneous effect in each case is of the expected sign, but all other lags give a positive sign on RS, and a negative sign on RO. Experimentation with the specification could not shake this result, which may be due to the fact that RO for Germany is not a proper own-rate but is instead the central bank discount rate, as the former was unavailable. On the other hand,
both elasticities are not significantly different from zero at the 0.95 level of confidence, so this finding should be weighed accordingly.
### TABLE 6.4.4

**ESTIMATION RESULTS OF MODEL 3, NARROW MONEY STOCK**

<table>
<thead>
<tr>
<th></th>
<th>CANADA</th>
<th>UNITED STATES</th>
<th>GERMANY</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>-0.0467 (2.73)</td>
<td>0.1371 (9.91)</td>
<td>-0.2278 (8.11)</td>
</tr>
<tr>
<td>y</td>
<td>0.1555 (1.89)</td>
<td>0.0286 (0.82)</td>
<td>0.2044 (5.31)</td>
</tr>
<tr>
<td>y(-1)</td>
<td>0.2208 (14.8)</td>
<td>0.0154 (0.67)</td>
<td>0.3297 (29.3)</td>
</tr>
<tr>
<td>y(-2)</td>
<td>0.2167 (4.99)</td>
<td>0.0618 (0.22)</td>
<td>0.3375 (15.1)</td>
</tr>
<tr>
<td>y(-3)</td>
<td>0.1431 (3.41)</td>
<td>0.0201 (0.05)</td>
<td>0.2276 (11.1)</td>
</tr>
<tr>
<td>y(-4)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Σ</td>
<td>0.7360 (14.8)</td>
<td>0.0512 (0.67)</td>
<td>1.0991 (26.7)</td>
</tr>
</tbody>
</table>

**SUMMARY STATISTICS**

<table>
<thead>
<tr>
<th></th>
<th>CANADA</th>
<th>UNITED STATES</th>
<th>GERMANY</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSR</td>
<td>0.0198</td>
<td>0.0042</td>
<td>0.0223</td>
</tr>
<tr>
<td>SER</td>
<td>0.0108</td>
<td>0.0050</td>
<td>0.0114</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.4979</td>
<td>0.1047</td>
<td>0.7656</td>
</tr>
<tr>
<td>log(L)</td>
<td>682.58</td>
<td>846.63</td>
<td>670.14</td>
</tr>
<tr>
<td>NOBS</td>
<td>212</td>
<td>212</td>
<td>212</td>
</tr>
<tr>
<td>DOF</td>
<td>170</td>
<td>172</td>
<td>172</td>
</tr>
<tr>
<td>DW</td>
<td>2.0195</td>
<td>1.0516</td>
<td>2.0017</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>0.9213 (33.8)</td>
<td>0.9745 (71.3)</td>
<td>0.8616 (24.3)</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td></td>
<td></td>
<td>0.2384 (3.55)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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<th>CANADA</th>
<th>UNITED STATES</th>
<th>GERMANY</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.1626 (7.70)</td>
<td>0.0512 (1.78)</td>
<td>-0.1632 (3.13)</td>
</tr>
<tr>
<td>y(1)</td>
<td>0.2563 (4.07)</td>
<td>0.1686 (4.94)</td>
<td>0.2091 (4.71)</td>
</tr>
<tr>
<td>y(-1)</td>
<td>0.2889 (16.7)</td>
<td>0.2237 (6.86)</td>
<td>0.3335 (12.7)</td>
</tr>
<tr>
<td>y(-2)</td>
<td>0.2796 (12.0)</td>
<td>0.2139 (5.77)</td>
<td>0.3002 (9.76)</td>
</tr>
<tr>
<td>y(-3)</td>
<td>0.2283 (6.79)</td>
<td>0.1394 (5.08)</td>
<td>0.2290 (8.06)</td>
</tr>
<tr>
<td>y(-4)</td>
<td>0.1351 (5.16)</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>y(-5)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Σ</td>
<td>1.1883 (22.5)</td>
<td>0.7456 (6.88)</td>
<td>1.1118 (12.7)</td>
</tr>
</tbody>
</table>

| RS       | -0.0010 (1.01) | -0.0012 (2.28) | -0.0003 (0.78)  |
| RS(-1)   | -0.0025 (2.57) | -0.0016 (3.72) | 0.0006 (1.02)   |
| RS(-2)   | -0.0031 (2.77) | -0.0019 (4.26) | 0.0013 (1.64)   |
| RS(-3)   | -0.0029 (2.71) | -0.0020 (4.18) | 0.0017 (1.89)   |
| RS(-4)   | -0.0019 (2.65) | -0.0020 (3.97) | 0.0020 (2.02)   |
| RS(-5)   | 0.0000        | -0.0019 (3.77) | 0.0020 (2.10)   |
| RS(-6)   | 0.0000        | -0.0016 (3.61) | 0.0018 (2.16)   |
| RS(-7)   | 0.0000        | -0.0012 (3.48) | 0.0014 (2.19)   |
| RS(-8)   | 0.0000        | -0.0007 (3.38) | 0.0008 (2.22)   |
| RS(-9)   | 0.0000        | 0.0000         | 0.0000         |
| Σ        | -0.0114 (2.71)| -0.0142 (4.23)| 0.0114 (1.83)   |

| RO       | 0.0045 (2.34)  | 0.0081 (1.29)  | 0.0003 (0.09)   |
| RO(-1)   | 0.0043 (3.24)  | 0.0123 (2.77)  | -0.0008 (0.41)  |
| RO(-2)   | 0.0040 (4.16)  | 0.0153 (3.92)  | -0.0015 (1.05)  |
| RO(-3)   | 0.0036 (4.33)  | 0.0170 (4.06)  | -0.0021 (1.27)  |
| RO(-4)   | 0.0030 (3.77)  | 0.0174 (3.80)  | -0.0024 (1.24)  |
| RO(-5)   | 0.0024 (3.14)  | 0.0165 (3.53)  | -0.0024 (1.17)  |
| RO(-6)   | 0.0017 (2.66)  | 0.0143 (3.32)  | -0.0022 (1.12)  |
| RO(-7)   | 0.0009 (2.31)  | 0.0108 (3.16)  | -0.0017 (1.08)  |
| RO(-8)   | 0.0000        | 0.0060 (3.04)  | -0.0010 (1.06)  |
| RO(-9)   | 0.0000        | 0.0000         | 0.0000         |
| Σ        | 0.0245 (4.33)  | 0.1176 (4.09)  | 0.0137 (1.25)   |

**SUMMARY STATISTICS**

| SSR      | 0.0055       | 0.0024       | 0.0167       |
| SFE      | 0.0071       | 0.0047       | 0.0123       |
| R²       | 0.8576       | 0.4468       | 0.4909       |
| Log(L)   | 565.21       | 627.69       | 480.60       |
| NQBE     | 153          | 153          | 153          |
| DF       | 109          | 111          | 111          |
| DW       | 2.0665       | 1.8595       | 2.0689       |
| ρ_1      | 0.8256 (18.4) | 0.8720 (21.6) | 0.8197 (17.8) |
| ρ_2      | 0.4835 (6.79) | 0.6741 (10.9) | 0.4998 (7.19) |

Range: 1968:01 - 1980:09
(e) Specification Testing

Prior to presenting estimation results for Models 2 and 4, the results of a variety of specification tests will be presented. This procedure serves two purposes. First, it follows the same route of inquiry pursued in the original work and, secondly, it will enable the presentation of only one set of results, rather than several. The results of the tests are summarized in Table 6.4.6; all tests were made at the 0.95 level of confidence.

The first test was of the null hypothesis that the term $SP^*y^*/P$ should not be split into two parts, namely $SP^*/P$ and $y^*$. The reader should note that when the results of this test were not unanimous across the three countries for a particular specification, then the majority ruled, and the same specification was used for each. This would introduce bias only if in one equation the variables were not split whereas the data said that they should have been. As it turns out, this did not occur in any of the equations to be reported on below.

For the narrow PA model, Model 2, the hypothesis is rejected for Canada and Germany, so subsequent results for Model 2 split the variable. The hypothesis is unanimously accepted by the broad PA model, so subsequent results for the Model 2 broad specifications will not split the variable. The hypothesis is unanimously rejected for the narrow PDL model, and for the U.S. and Germany in the broad PDL model, so Model 4 results will all split the variable.

According to the theory presented in earlier chapters, the effect of $SP^*/P$ could be direct (arising from the foreign demand for domestic
money) or could arise indirectly through the parameters α and α* in the transactions model. Although the effects are impossible to separate here, the broad PA results indicate that neither effect is statistically significant at the 0.95 level. This may be interpreted as only slight evidence that the broad money stock is less amenable to the transactions notion than is the narrow stock of money.

Subsequent tests ask if particular sets of variables should be included or excluded. Each test is discussed in turn.

**PA Model**

The narrow PA models have unanimously rejected the hypothesis that the foreign income and interest rate variables be excluded, when the total sample is used. Notice that this test leaves the s^e_s in the specification; thus, the test is whether or not non-residents are holding the domestic money. The two subsamples, roughly the two decades of the 1960's and 1970's, are less decisive. The hypothesis that all of the foreign variables, including the s^e_s, should be excluded, is rejected only for the U.S. Thus, the broad consensus is that the hypotheses of this Thesis are not aiding the PA model to a significant degree. The foreign income and interest rate variables evidently act by displacing some of the variation originally captured by domestic income and the domestic rate of interest.

Finally, all tests indicate that the s^e_s by themselves contribute nothing to the specification.

The broad PA results are perhaps a little more promising. For Canada the foreign variables make no difference whatsoever; however, for the U.S. and Germany the joint exclusion hypothesis is
rejected, and it is clear that the real foreign income and interest rate variables should not be excluded. For Germany, the expected rates of depreciation also make a significant difference to the results. Notice that the broad tests are not conducted on the split sample; this is because the broad sample only begins in 1967.

**PDL Model**

The narrow PDL model is very decisive and highly supportive of the hypotheses derived earlier in this Thesis. Virtually every null hypothesis is rejected by the data, and some very resoundingly. It is clear that the foreign variables make a substantial improvement on the specification, for the total sample and for the two subsamples. The foreign income and interest rate variables are significant in their own right, as are the expected rates of depreciation. Not surprisingly, the F-statistics are generally much larger for a test which spans the 1970's, than for a test which is restricted to the 1960's. However, the data of the 1960's are also supportive of the currency substitution hypothesis.

The same results hold for the broad specifications. For all three countries all three null hypotheses are soundly rejected. The foreign variables should be included, and the importance is shared by the foreign income and interest rate variables and the expected rates of depreciation.

**Discussion**

The results of these tests are somewhat mixed, the conclusions depending upon which of the two models is the more appropriate. Essentially, the currency substitution hypothesis is
very important for the PDL model, but only marginally so in the PA model. Presumably this is because the autoregressive nature of the PA model leaves a smaller residual variance for the foreign variables to explain. Thus, a choice between the models is not possible on these grounds, and is instead left until after the forecasting experiments of Chapter Seven.

The results do indicate with fair generality that a formulation which includes foreign income and interest rate variables as well as the expected rates of depreciation is superior to one which excludes the first two but retains the latter. According to our theory, this implies that real domestic balances are being held and adjusted by non-residents, for each of three countries considered. Thus, although the expected rate of depreciation is not playing an important role in the data, foreign holdings of domestic money are statistically significant.
**TABLE 6.4.6**

**SUMMARY OF F-TESTS: PDL MODEL, NARROW MONEY STOCK**

(critical values of $F_{0.05}$ given in parentheses)

<table>
<thead>
<tr>
<th>NULL HYPOTHESIS</th>
<th>CANADA</th>
<th>UNITED STATES</th>
<th>GERMANY</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SP^<em>y^</em>/P$ should not be split</td>
<td>2.63 (2.37)</td>
<td>1.34 (2.37)</td>
<td>3.32 (2.37)</td>
</tr>
<tr>
<td>$\mu_1 = \delta_1 = \eta_1 = \phi_1 = 0$ (full sample)</td>
<td>1.57 (1.65)</td>
<td>2.22 (1.65)</td>
<td>1.51 (1.65)</td>
</tr>
<tr>
<td>$\mu_1 = \delta_1 = \eta_1 = \phi_1 = 0$ (1960's only)</td>
<td>0.37 (2.92)</td>
<td>0.67 (2.70)</td>
<td>0.35 (2.70)</td>
</tr>
<tr>
<td>$\mu_1 = \eta_1 = \phi_1 = 0$ (1970's only)</td>
<td>0.95 (2.30)</td>
<td>2.27 (2.28)</td>
<td>1.33 (2.28)</td>
</tr>
<tr>
<td>$\eta_1 = \phi_1 = 0$ (full sample)</td>
<td>1.86 (1.75)</td>
<td>2.99 (1.75)</td>
<td>1.82 (1.75)</td>
</tr>
<tr>
<td>$\eta_1 = \phi_1 = 0$ (1960's only)</td>
<td>0.40 (3.03)</td>
<td>0.62 (2.84)</td>
<td>0.37 (2.84)</td>
</tr>
<tr>
<td>$\eta_1 = \phi_1 = 0$ (1970's only)</td>
<td>1.11 (2.50)</td>
<td>2.70 (2.46)</td>
<td>1.52 (2.46)</td>
</tr>
<tr>
<td>$\mu_1 = 0$ (full sample)</td>
<td>0.69 (2.37)</td>
<td>0.55 (2.37)</td>
<td>0.53 (2.37)</td>
</tr>
<tr>
<td>$\mu_1 = 0$ (1960's only)</td>
<td>0.37 (2.65)</td>
<td>0.93 (2.61)</td>
<td>0.38 (2.61)</td>
</tr>
<tr>
<td>$\mu_1 = 0$ (1970's only)</td>
<td>0.48 (2.50)</td>
<td>0.57 (2.50)</td>
<td>0.68 (2.50)</td>
</tr>
</tbody>
</table>

**SUMMARY OF F-TESTS: PDL MODEL, NARROW MONEY STOCK**

<p>| $SP^<em>y^</em>/P$ should not be split | 7.12 (2.37) | 19.34 (2.37) | 14.30 (2.37) |
| $\mu_1 = \delta_1 = \eta_1 = \phi_1 = 0$ (full sample) | 23.54 (1.69) | 42.07 (1.69) | 26.50 (1.69) |</p>
<table>
<thead>
<tr>
<th>NULL HYPOTHESIS</th>
<th>CANADA</th>
<th>UNITED STATES</th>
<th>GERMANY</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1 = \delta_1 = \eta_1 = \phi_1 = 0$</td>
<td>2.75 (2.07)</td>
<td>6.27 (2.05)</td>
<td>1.38 (2.05)</td>
</tr>
<tr>
<td>(1960's only)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_1 = \delta_1 = \eta_1 = \phi_1 = 0$</td>
<td>20.49 (1.80)</td>
<td>11.55 (1.80)</td>
<td>7.81 (1.80)</td>
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<td>(1970's only)</td>
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<tr>
<td>$\delta_1 = \eta_1 = \phi_1 = 0$</td>
<td>20.26 (1.75)</td>
<td>39.23 (1.75)</td>
<td>31.79 (1.75)</td>
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<td>(full sample)</td>
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</tr>
<tr>
<td>$\delta_1 = \eta_1 = \phi_1 = 0$</td>
<td>2.62 (2.16)</td>
<td>0.98 (2.13)</td>
<td>1.76 (2.13)</td>
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<tr>
<td>(1960's only)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$\delta_1 = \eta_1 = \phi_1 = 0$</td>
<td>14.70 (1.90)</td>
<td>8.28 (1.90)</td>
<td>9.12 (1.90)</td>
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<tr>
<td>(1970's only)</td>
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<td></td>
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</tr>
<tr>
<td>$\mu_1 = 0$</td>
<td>14.18 (2.37)</td>
<td>13.55 (2.37)</td>
<td>3.33 (2.37)</td>
</tr>
<tr>
<td>(full sample)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_1 = 0$</td>
<td>2.06 (2.64)</td>
<td>5.85 (2.62)</td>
<td>0.20 (2.62)</td>
</tr>
<tr>
<td>(1960's only)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_1 = 0$</td>
<td>12.71 (2.50)</td>
<td>10.54 (2.50)</td>
<td>1.82 (2.50)</td>
</tr>
<tr>
<td>Null Hypotheses</td>
<td>Canada</td>
<td>United States</td>
<td>Germany</td>
</tr>
<tr>
<td>-----------------</td>
<td>-----------</td>
<td>---------------</td>
<td>-----------</td>
</tr>
<tr>
<td>$SP^y/P$ should not be split</td>
<td>1.04 (2.50)</td>
<td>1.34 (2.45)</td>
<td>1.28 (2.45)</td>
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<tr>
<td>$\mu_1 = \delta_1 = \delta_2 = \eta_1 = \phi_1 = 0$ (full sample)</td>
<td>0.92 (1.86)</td>
<td>2.55 (1.79)</td>
<td>1.86 (1.76)</td>
</tr>
<tr>
<td>$\mu_1 = \delta_1 = \delta_2 = \eta_1 = \phi_1 = 0$ (full sample)</td>
<td>1.34 (1.86)</td>
<td>3.18 (1.85)</td>
<td>11.24 (1.85)</td>
</tr>
<tr>
<td>$\mu_1 = 0$ (full sample)</td>
<td>0.41 (2.46)</td>
<td>0.56 (2.45)</td>
<td>3.62 (2.45)</td>
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</table>

**Summary of F-Tests: PDL Model, Broad Money Stock**

<table>
<thead>
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<th>Null Hypotheses</th>
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<th>United States</th>
<th>Germany</th>
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</thead>
<tbody>
<tr>
<td>$SP^y/P$ should not be split</td>
<td>2.22 (2.49)</td>
<td>14.22 (2.49)</td>
<td>29.86 (2.49)</td>
</tr>
<tr>
<td>$\mu_1 = \delta_1 = \delta_2 = \eta_1 = \phi_1 = 0$ (full sample)</td>
<td>10.63 (1.79)</td>
<td>45.52 (1.70)</td>
<td>77.37 (1.70)</td>
</tr>
<tr>
<td>$\mu_1 = \delta_1 = \delta_2 = \eta_1 = \phi_1 = 0$ (full sample)</td>
<td>10.85 (1.78)</td>
<td>46.06 (1.78)</td>
<td>17.72 (1.78)</td>
</tr>
<tr>
<td>$\mu_1 = 0$ (full sample)</td>
<td>3.91 (2.47)</td>
<td>5.60 (2.47)</td>
<td>8.67 (2.87)</td>
</tr>
</tbody>
</table>
(f) Estimation of Models 2 and 4

The results of estimation of Models 2 and 4 are given in Tables 6.4.7 - 6.4.12. Where necessary, the equations of Model 2 have been estimated under an AR1 assumption. All of the equations of Model 4 required an AR2 transformation to bring the DW statistic into line. Because some readers may be troubled by this assumption, both the untransformed (OLS) and transformed (AR2) results for Model 4 are presented. Since the residual analysis which is presented in Subsection (g) casts some doubt on the AR2 assumption, the reader is advised that the untransformed results may be the most relevant estimates of Model 4, in spite of the nonspherical error structure. The AR1 results of Model 2, on the other hand, would seem to be quite reliable, for the coefficients are altered only slightly by the transformation; besides, four of the six Model 2 equations are acceptably estimated by OLS.

It should be noted that multicollinearity will be a factor throughout the results. The national income and interest rate variables are quite highly correlated across the countries under study; for specific correlation results the reader is referred to the Appendix to this Chapter. This may result in low t-statistics (accompanied by joint statistical significance) and, occasionally, unexpected signs. Furthermore, it can turn out that one variable in a correlated set of variables dominates a regression because it has the highest variance. Then the 'wrong' interest rate may be the most significant in a particular regression, for example.
depreciation are all expressed in terms of the U.S. dollar. Thus, a negative sign on an $s^e$ in either the Canadian or German equations indicates currency substitutability, whereas a negative sign in the U.S. equations indicates currency complementarity. In contrast, the terms of trade variables, $SP^*/P$, are all constructed so that the domestic economy is in whose money demand equation the variable is found. Thus, an increase in $SP^*/P$ is expected to reduce imports, increase exports, and thereby increase the demand for real domestic balances.

**Model 2**

It would seem sensible to restrict attention to the estimates of the long-run coefficients. The results indicate that the U.S. narrow money stock is a substitute for the Canadian, German and French currencies. The latter two behave as complements, as do the Canadian dollar and the Deutschemark, but as substitutes for the U.S. dollar.

Some unexpected signs are in evidence in the narrow equations. The sign problems on the interest rates and real income variables probably may be attributed to multicollinearity. However, the strong negative sign on the U.S. terms of trade in the Canadian equation deserves attention. One possible explanation is that the two economies are so strongly linked that an improvement in the U.S. terms of trade relative to the rest of the world, including Canada, causes Canada's trade balance to improve rather than worsen, as the surge in U.S. national income pulls Canada along. A similar linkage argument may also hold for the Germany-France-U.K. trio.

In the broad versions of Model 2 recall that the terms of
trade and real income variables are not split. Here it is found that
broad U.S. money is a substitute for the Deutschemark and the Franc,
but a complement for Canadian and British broad money. The relationship
between Germany and Canada is ambiguous—the Canadian equation
says complements, the German equation, substitutes, but both
coefficients could, statistically speaking, be of either sign.
Evidently broad Canadian money is a complement for French, and a
substitute for British, while the German, French and British broad
money stocks behave as complements.

For the most part, the coefficients on foreign income
variables are of the expected sign. This is true of the Canada–U.S.–
Germany trio with the exception of the Canadian variable in the
German equation. Finally, as expected, multicollinearity makes the
interest rate coefficients virtually uninterpretable.

Although the work, which is reported on here, is not
directly comparable to any previously reported work, it is most closely
related to that of Arango and Nadiri (1981). In that paper the
partial-adjustment model of the demand for money is fitted for Canada,
Germany, the U.K. and the U.S., with narrow real balances a function
of permanent income, domestic and an average of foreign interest rates,
the spot exchange rate and the expected rate of depreciation, and the
rate of inflation. The data are quarterly, so once again it must be
stressed that the results are not directly comparable. However, it is
interesting to note that \( \sigma^e \) is generally statistically significant at
the 0.95 level (except for Canada) and that the results indicate that
all four currencies are substitutes for one another. This is in
contrast with the results presented here, but this is undoubtedly a result of the differences between the two specifications and the data which were used.

Model 4

As mentioned above, there may be some doubt about the validity of the AR2 assumption, so discussion will focus on the untransformed results. Nevertheless, the effect of the transformation on the performance of the six equations carries with it some information. All of the results are altered to some degree by the transformation, but particularly the narrow U.S. results, indicating that a good deal of the statistical significance which is indicated in the untransformed equation is due to trend in the data. The lack of statistical significance of the foreign variables in the PA model tends to reinforce this view—in the PA model, virtually all trend will be captured by the lagged dependent variable. This discussion leads naturally to the suggestion that time series modelling or a combination of time series and econometric analysis might provide a clearer picture of the problem at hand. This idea is brought out more fully in the concluding chapter of the Thesis.

In the narrow equations, the predictions regarding currency substitutability/complementarity are broadly, although not perfectly, consistent with those of the PA model, Model 2. The results and a comparison with those of Model 4 are summarized in tabular form below. The German results are perfectly consistent with those of the PA model, signwise, whereas the results for Canada and the U.S. are somewhat ambiguous—the Canadian equation indicating that the two are
substitutes, while the U.S. equation indicates that they are complements. This is probably because the rest of the world treats the two major North American currencies as close substitutes for each other, so that major fluctuations in the U.S. currency relative to the European currencies pull the Canadian dollar along. The broad equations yield the same result. In the narrow equations the relationship between the Canadian and German currencies is ambiguous, whereas in the broad equations they are strong substitutes.

Detailed description of the other results would add little to the information contained in the tables. However, it is perhaps worth noting that both the narrow and broad U.S. equations find an inverse relationship between real Canadian income and the demand for U.S. money, whereas U.S. income affects the demand for real Canadian balances directly. Presumably, this is once again evidence of multicollinearity, although additional explanations may be possible. Notice that a similar problem is in evidence in the narrow German and U.S. equations.

Discussion

To facilitate a final summary, the substitutes/complements results are now brought together in tabular form, in Table 6.4.13. The results which are consistent with one another for the same model and consistent across models for the same definition of money are presumed to be the strongest. These are marked with an asterisk for the narrow equations, and a plus sign for the broad equations. It should be clear that the broad results are less definitive.
### TABLE 6.4.13

**Substitutability and Complementarity Between Monies**

<table>
<thead>
<tr>
<th></th>
<th>Narrow Money Stock</th>
<th></th>
<th></th>
<th>Broad Money Stock</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model 2</td>
<td>Model 4</td>
<td></td>
<td></td>
<td>Model 2</td>
<td>Model 4</td>
</tr>
<tr>
<td>CAN</td>
<td>-</td>
<td>S</td>
<td>C</td>
<td>CAN</td>
<td>-</td>
<td>C</td>
</tr>
<tr>
<td></td>
<td>US</td>
<td>S</td>
<td>-</td>
<td>US</td>
<td>C</td>
<td>S</td>
</tr>
<tr>
<td></td>
<td>GER</td>
<td>C*</td>
<td>-</td>
<td>GER</td>
<td>C</td>
<td>S</td>
</tr>
<tr>
<td></td>
<td>FR</td>
<td>S*</td>
<td>C*</td>
<td>FR</td>
<td>C*</td>
<td>S*</td>
</tr>
<tr>
<td></td>
<td>UK</td>
<td>S*</td>
<td>C*</td>
<td>UK</td>
<td>S*</td>
<td>C*</td>
</tr>
</tbody>
</table>

It can be said on the basis of this table with some certainty that narrow Canadian and German monies behave as complements, and that narrow German and U.S. monies behave as substitutes. All other pairs contain some element of ambiguity or are subsidiary, the latter in the sense that since our system is rectangular rather than square, some of the signs cannot be cross-checked for internal consistency.
<table>
<thead>
<tr>
<th></th>
<th>CANADA</th>
<th>UNITED STATES</th>
<th>GERMANY</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m(-1)$</td>
<td>0.8346</td>
<td>0.9679 (55.2)</td>
<td>0.8619 (24.1)</td>
</tr>
<tr>
<td>constant</td>
<td>-0.0663 (0.41)</td>
<td>-0.1600 (1.67)</td>
<td>0.3413 (1.90)</td>
</tr>
<tr>
<td>$y$(CAN)</td>
<td>0.0701 (1.64)</td>
<td>-0.0236 (1.06)</td>
<td>0.1314 (2.72)</td>
</tr>
<tr>
<td>$y$(US)</td>
<td>0.0435 (1.18)</td>
<td>0.0465 (2.03)</td>
<td>-0.0768 (1.87)</td>
</tr>
<tr>
<td>$y$(GER)</td>
<td>-0.0345 (1.14)</td>
<td>0.0139 (0.91)</td>
<td>0.0875 (2.34)</td>
</tr>
<tr>
<td>$y$(FR)</td>
<td>0.0267 (1.09)</td>
<td>-0.0140 (1.21)</td>
<td>0.0089 (0.36)</td>
</tr>
<tr>
<td>$y$(UK)</td>
<td>0.0047 (0.14)</td>
<td>0.0171 (1.90)</td>
<td>-0.0602 (1.65)</td>
</tr>
<tr>
<td>$R^2$(CAN)</td>
<td>-0.0043 (4.19)</td>
<td>-0.0018 (3.44)</td>
<td>0.0006 (0.50)</td>
</tr>
<tr>
<td>$R^2$(US)</td>
<td>0.0019 (1.63)</td>
<td>0.0609 (1.61)</td>
<td>-0.0016 (1.29)</td>
</tr>
<tr>
<td>$R^2$(GER)</td>
<td>-0.0008 (2.37)</td>
<td>-0.0003 (1.77)</td>
<td>-0.0003 (0.84)</td>
</tr>
<tr>
<td>$R^2$(FR)</td>
<td>0.0003 (0.43)</td>
<td>-0.0002 (0.54)</td>
<td>-0.0017 (2.04)</td>
</tr>
<tr>
<td>$R^2$(UK)</td>
<td>0.0010 (1.68)</td>
<td>-0.0001 (0.27)</td>
<td>-0.0003 (0.49)</td>
</tr>
<tr>
<td>$Sp^*/P$(CAN)</td>
<td>-0.1634 (3.52)</td>
<td>-0.0112 (1.57)</td>
<td>-0.1319 (2.84)</td>
</tr>
<tr>
<td>$Sp^*/P$(US)</td>
<td>0.0153 (1.04)</td>
<td>-0.0122 (1.57)</td>
<td>-0.0060 (0.26)</td>
</tr>
<tr>
<td>$Sp^*/P$(GER)</td>
<td>-0.0213 (1.05)</td>
<td>0.0068 (0.61)</td>
<td>-0.1911 (1.26)</td>
</tr>
<tr>
<td>$Sp^*/P$(FR)</td>
<td>0.0321 (2.52)</td>
<td>0.0036 (0.50)</td>
<td>0.0009 (0.21)</td>
</tr>
<tr>
<td>$s^0$(CAN)</td>
<td>-0.0031 (0.82)</td>
<td>0.0017 (0.83)</td>
<td>-0.0066 (3.10)</td>
</tr>
<tr>
<td>$s^0$(GER)</td>
<td>0.0016 (0.92)</td>
<td>0.0014 (1.50)</td>
<td>-0.0066 (3.10)</td>
</tr>
<tr>
<td>$s^0$(FR)</td>
<td>-0.0027 (1.15)</td>
<td>0.0008 (0.63)</td>
<td>0.0043 (1.69)</td>
</tr>
<tr>
<td>$s^0$(UK)</td>
<td>-0.0002 (0.19)</td>
<td>-0.0003 (0.38)</td>
<td>-0.0006 (0.44)</td>
</tr>
</tbody>
</table>

**SUMMARY STATISTICS**

- $SSR$   | 0.0137 | 0.0033 | 0.0148 |
- $SER$   | 0.0094 | 0.0046 | 0.0097 |
- $R^2$   | 0.9978 | 0.9897 | 0.9976 |
- $log(L)$| 722.06 | 872.28 | 713.50 |
- $NOBS$  | 212    | 212    | 212   |
- $DOF$   | 153    | 157    | 157   |
- $DW$    | 2.0390 | 1.9123 | -2.1871 |
- $h$     | -0.32  | 0.66   | -1.54 |
- $\rho_1$| -0.1861 (2.60) | - | - |

(continued on following page)
### Table 6.4.7 (Continued)

**Estimation Results of Model 2, Narrow Money Stock**

**Long-Run Coefficients**

<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>United States</th>
<th>Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y(CAN)$</td>
<td>0.4218</td>
<td>-0.7369</td>
<td>0.9513</td>
</tr>
<tr>
<td>$y(US)$</td>
<td>0.2642</td>
<td>1.4496</td>
<td>-0.5560</td>
</tr>
<tr>
<td>$y(GER)$</td>
<td>-0.2079</td>
<td>0.4336</td>
<td>0.6335</td>
</tr>
<tr>
<td>$y(FR)$</td>
<td>0.1617</td>
<td>-0.4366</td>
<td>0.0641</td>
</tr>
<tr>
<td>$y(UK)$</td>
<td>0.0297</td>
<td>0.5325</td>
<td>-0.4360</td>
</tr>
<tr>
<td>$RS(CAN)$</td>
<td>-0.0257</td>
<td>-0.0568</td>
<td>0.0040</td>
</tr>
<tr>
<td>$RS(US)$</td>
<td>0.0115</td>
<td>0.0268</td>
<td>-0.0119</td>
</tr>
<tr>
<td>$RS(GER)$</td>
<td>-0.0049</td>
<td>-0.0092</td>
<td>-0.0021</td>
</tr>
<tr>
<td>$RS(FR)$</td>
<td>0.0019</td>
<td>-0.0062</td>
<td>-0.0123</td>
</tr>
<tr>
<td>$RS(UK)$</td>
<td>0.0060</td>
<td>0.0027</td>
<td>-0.0023</td>
</tr>
<tr>
<td>$SP*/P(CAN)$</td>
<td>-</td>
<td>1.2217</td>
<td>-0.9208</td>
</tr>
<tr>
<td>$SP*/P(US)$</td>
<td>-0.9889</td>
<td></td>
<td>0.9552</td>
</tr>
<tr>
<td>$SP*/P(GER)$</td>
<td>0.0916</td>
<td>-0.3794</td>
<td>-</td>
</tr>
<tr>
<td>$SP*/P(FR)$</td>
<td>-0.1280</td>
<td>0.2115</td>
<td>-0.0436</td>
</tr>
<tr>
<td>$SP*/P(UK)$</td>
<td>0.1937</td>
<td>0.1114</td>
<td>-0.1383</td>
</tr>
<tr>
<td>$s^e(CAN)$</td>
<td>-0.0189</td>
<td>0.0528</td>
<td>0.0065</td>
</tr>
<tr>
<td>$s^e(GER)$</td>
<td>0.0098</td>
<td>0.0430</td>
<td>-0.0475</td>
</tr>
<tr>
<td>$s^e(FR)$</td>
<td>-0.0161</td>
<td>0.0239</td>
<td>0.0312</td>
</tr>
<tr>
<td>$s^e(UK)$</td>
<td>-0.0015</td>
<td>0.0078</td>
<td>-0.0046</td>
</tr>
</tbody>
</table>

### TABLE 6.4.8

ESTIMATION RESULTS OF MODEL 2, BROAD MONEY STOCK

<table>
<thead>
<tr>
<th></th>
<th>CANADA</th>
<th>UNITED STATES</th>
<th>GERMANY</th>
</tr>
</thead>
<tbody>
<tr>
<td>m(-1)</td>
<td>0.9055 (30.8)</td>
<td>0.9164 (29.3)</td>
<td>0.9202 (28.8)</td>
</tr>
<tr>
<td>constant</td>
<td>-0.1704 (1.27)</td>
<td>0.0649 (0.46)</td>
<td>-0.0877 (0.53)</td>
</tr>
<tr>
<td>y(CAN)</td>
<td>0.0793 (2.12)</td>
<td>0.0254 (1.95)</td>
<td>-0.0108 (0.46)</td>
</tr>
<tr>
<td>y(US)</td>
<td>0.0111 (0.55)</td>
<td>0.0480 (2.20)</td>
<td>0.0192 (0.62)</td>
</tr>
<tr>
<td>y(GER)</td>
<td>0.0133 (0.87)</td>
<td>0.0113 (1.10)</td>
<td>0.1156 (2.96)</td>
</tr>
<tr>
<td>y(FR)</td>
<td>-0.0064 (0.56)</td>
<td>-0.0034 (0.46)</td>
<td>-0.0090 (0.53)</td>
</tr>
<tr>
<td>y(UK)</td>
<td>-0.0035 (0.27)</td>
<td>0.0025 (0.30)</td>
<td>-0.0469 (2.50)</td>
</tr>
<tr>
<td>s(CAN)</td>
<td>-0.0015 (1.48)</td>
<td>-0.0005 (0.77)</td>
<td>-0.0011 (0.75)</td>
</tr>
<tr>
<td>s(US)</td>
<td>-0.0001 (0.08)</td>
<td>-0.0007 (1.31)</td>
<td>-0.0001 (0.10)</td>
</tr>
<tr>
<td>s(GER)</td>
<td>-0.0003 (1.04)</td>
<td>-0.0008 (2.63)</td>
<td>-0.0005 (1.26)</td>
</tr>
<tr>
<td>s(FR)</td>
<td>0.0012 (1.64)</td>
<td>-0.0001 (0.18)</td>
<td>0.0003 (0.34)</td>
</tr>
<tr>
<td>s(UK)</td>
<td>0.0007 (1.05)</td>
<td>0.0000 (0.03)</td>
<td>-0.0009 (0.91)</td>
</tr>
<tr>
<td>s(CAN)</td>
<td>0.0016 (1.42)</td>
<td>-0.0012 (1.57)</td>
<td>0.0010 (0.64)</td>
</tr>
<tr>
<td>s(US)</td>
<td>0.0079 (1.21)</td>
<td>0.0031 (0.76)</td>
<td>-0.0004 (0.04)</td>
</tr>
<tr>
<td>s(GER)</td>
<td>0.0003 (0.41)</td>
<td>0.0001 (0.10)</td>
<td>0.0018 (1.53)</td>
</tr>
<tr>
<td>s(FR)</td>
<td>-0.0018 (2.08)</td>
<td>-0.0010 (1.76)</td>
<td>-0.0027 (2.06)</td>
</tr>
<tr>
<td>s(UK)</td>
<td>0.0003 (0.65)</td>
<td>0.0004 (1.23)</td>
<td>0.0010 (1.37)</td>
</tr>
</tbody>
</table>

**SUMMARY STATISTICS**

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SSR</td>
<td>0.0041</td>
<td>-0.0013</td>
<td>0.0098</td>
</tr>
<tr>
<td>SER</td>
<td>0.0064</td>
<td>0.0035</td>
<td>0.0097</td>
</tr>
<tr>
<td>R²</td>
<td>0.9991</td>
<td>0.9995</td>
<td>0.9973</td>
</tr>
<tr>
<td>log(L)</td>
<td>622.27</td>
<td>716.25</td>
<td>552.59</td>
</tr>
<tr>
<td>NOBS</td>
<td>161</td>
<td>161</td>
<td>161</td>
</tr>
<tr>
<td>DOF</td>
<td>101</td>
<td>105</td>
<td>105</td>
</tr>
<tr>
<td>DW</td>
<td>1.8847</td>
<td>2.0213</td>
<td>2.0141</td>
</tr>
<tr>
<td>h</td>
<td>0.79</td>
<td>-0.14</td>
<td>-0.10</td>
</tr>
<tr>
<td>ρ₁</td>
<td></td>
<td>0.3169 (3.97)</td>
<td></td>
</tr>
</tbody>
</table>

(continued on following page)
TABLE 6.4.8 (CONTINUED)

ESTIMATION RESULTS OF MODEL 2—BROAD MONEY STOCK

LONG-RUN COEFFICIENTS

<table>
<thead>
<tr>
<th>Canada</th>
<th>United States</th>
<th>Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y(\text{CAN})$</td>
<td>$0.8397 (2.85)$</td>
<td>$0.3065 (2.02)$</td>
</tr>
<tr>
<td>$y(\text{US})$</td>
<td>$0.1180 (0.57)$</td>
<td>$0.5918 (2.66)$</td>
</tr>
<tr>
<td>$y(\text{GER})$</td>
<td>$0.1406 (0.79)$</td>
<td>$0.1337 (1.16)$</td>
</tr>
<tr>
<td>$y(\text{FR})$</td>
<td>$-0.0675 (0.54)$</td>
<td>$-0.0368 (0.40)$</td>
</tr>
<tr>
<td>$y(\text{UK})$</td>
<td>$-0.0370 (0.26)$</td>
<td>$0.0200 (0.19)$</td>
</tr>
<tr>
<td>$\text{RS(\text{CAN})}$</td>
<td>$-0.0161 (1.49)$</td>
<td>$-0.0061 (0.74)$</td>
</tr>
<tr>
<td>$\text{RS(\text{US})}$</td>
<td>$-0.0008 (0.08)$</td>
<td>$-0.0078 (1.09)$</td>
</tr>
<tr>
<td>$\text{RS(\text{GER})}$</td>
<td>$-0.0003 (1.04)$</td>
<td>$-0.0041 (1.82)$</td>
</tr>
<tr>
<td>$\text{RS(\text{FR})}$</td>
<td>$0.0123 (1.59)$</td>
<td>$-0.0009 (0.16)$</td>
</tr>
<tr>
<td>$\text{RS(\text{UK})}$</td>
<td>$0.0073 (0.95)$</td>
<td>$0.0001 (0.02)$</td>
</tr>
<tr>
<td>$\text{RO(\text{CAN})}$</td>
<td>$0.0167 (1.47)$</td>
<td>$-0.0143 (1.79)$</td>
</tr>
<tr>
<td>$\text{RO(\text{US})}$</td>
<td>$0.0838 (1.30)$</td>
<td>$0.0347 (0.68)$</td>
</tr>
<tr>
<td>$\text{RO(\text{GER})}$</td>
<td>$-0.0036 (0.49)$</td>
<td>$0.0013 (0.17)$</td>
</tr>
<tr>
<td>$\text{RO(\text{FR})}$</td>
<td>$-0.0188 (1.66)$</td>
<td>$-0.0117 (1.38)$</td>
</tr>
<tr>
<td>$\text{RO(\text{UK})}$</td>
<td>$0.0030 (0.65)$</td>
<td>$0.0050 (1.38)$</td>
</tr>
<tr>
<td>$\text{se(\text{CAN})}$</td>
<td>$0.0186 (0.57)$</td>
<td>$-0.0133 (0.66)$</td>
</tr>
<tr>
<td>$\text{se(\text{GER})}$</td>
<td>$0.0018 (0.11)$</td>
<td>$0.0085 (0.80)$</td>
</tr>
<tr>
<td>$\text{se(\text{FR})}$</td>
<td>$0.0068 (0.38)$</td>
<td>$-0.0060 (0.49)$</td>
</tr>
<tr>
<td>$\text{se(\text{UK})}$</td>
<td>$-0.0113 (0.97)$</td>
<td>$-0.0038 (0.53)$</td>
</tr>
</tbody>
</table>

### TABLE 6.4-9

**ESTIMATION RESULTS OF MODEL 4, UNTRANSFORMED, NARROW MONEY STOCK**

<table>
<thead>
<tr>
<th></th>
<th>CANADA</th>
<th>UNITED STATES</th>
<th>GERMANY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.3925 (1.23)</td>
<td>2.2377 (5.83)</td>
<td>0.0834 (0.27)</td>
</tr>
<tr>
<td>y ((E_t))</td>
<td>0.6201 (6.65)</td>
<td>0.8000 (8.32)</td>
<td>0.8311 (12.8)</td>
</tr>
<tr>
<td>RS ((E_t))</td>
<td>-0.0127 (6.24)</td>
<td>0.0122 (3.23)</td>
<td>-0.0130 (10.4)</td>
</tr>
<tr>
<td>y(CAN)</td>
<td>-</td>
<td>-0.3852 (3.71)</td>
<td>0.4041 (4.85)</td>
</tr>
<tr>
<td>y(US)</td>
<td>0.0685 (0.85)</td>
<td>-</td>
<td>-0.2830 (4.18)</td>
</tr>
<tr>
<td>y(GER)</td>
<td>-0.2373 (4.20)</td>
<td>-0.0948 (1.39)</td>
<td>-</td>
</tr>
<tr>
<td>y(FR)</td>
<td>0.1366 (2.97)</td>
<td>0.0083 (0.16)</td>
<td>-0.0149 (0.39)</td>
</tr>
<tr>
<td>y(UK)</td>
<td>0.0586 (0.93)</td>
<td>0.0514 (0.40)</td>
<td>-0.0539 (0.93)</td>
</tr>
<tr>
<td>RS (CAN)</td>
<td>-</td>
<td>-0.0111 (4.39)</td>
<td>-0.0004 (0.22)</td>
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<tr>
<td>RS (US)</td>
<td>0.0004 (0.18)</td>
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<td>0.0007 (0.36)</td>
</tr>
<tr>
<td>RS (GER)</td>
<td>-0.0005 (0.86)</td>
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</tr>
<tr>
<td>RS (FR)</td>
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<td>-0.0056 (4.33)</td>
</tr>
<tr>
<td>RS (UK)</td>
<td>0.0038 (3.25)</td>
<td>0.0021 (1.51)</td>
<td>-0.0001 (0.05)</td>
</tr>
<tr>
<td>SP*/P(CAN)</td>
<td>-</td>
<td>0.8096 (10.6)</td>
<td>-0.4791 (8.59)</td>
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<tr>
<td>SP*/P(US)</td>
<td>-0.6544 (8.35)</td>
<td>-</td>
<td>0.3789 (5.48)</td>
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<tr>
<td>SP*/P(GER)</td>
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<td>-0.0205 (0.51)</td>
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</tr>
<tr>
<td>SP*/P(FR)</td>
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<td>-0.0278 (0.70)</td>
</tr>
<tr>
<td>SP*/P(UK)</td>
<td>0.0896 (3.35)</td>
<td>-0.0851 (2.54)</td>
<td>-0.0172 (0.65)</td>
</tr>
<tr>
<td>s* (CAN)</td>
<td>-0.0162 (2.50)</td>
<td>-0.0403 (4.93)</td>
<td>0.0119 (1.82)</td>
</tr>
<tr>
<td>s* (GER)</td>
<td>0.0003 (0.08)</td>
<td>0.0006 (0.13)</td>
<td>-0.0049 (1.40)</td>
</tr>
<tr>
<td>s* (FR)</td>
<td>-0.0007 (0.15)</td>
<td>-0.0095 (1.79)</td>
<td>0.0098 (2.43)</td>
</tr>
<tr>
<td>s* (UK)</td>
<td>-0.0016 (0.63)</td>
<td>-0.0018 (0.60)</td>
<td>-0.0039 (1.74)</td>
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</table>

**SUMMARY STATISTICS**

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<tr>
<td>SSR</td>
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<td>0.0637</td>
<td>0.0360</td>
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<tr>
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<td>0.0202</td>
<td>0.0152</td>
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<td>R²</td>
<td>0.9887</td>
<td>0.8004</td>
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<td>log(L)</td>
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<td>DOF</td>
<td>154</td>
<td>156</td>
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<tr>
<td>DW</td>
<td>0.9671</td>
<td>0.7256</td>
<td>0.8250</td>
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**Note:** Lag structures on y and RS (in own equations) are as given in Table 6.4.4.

**Range:** 1963:01 - 1980:09
### TABLE 6.4.10

**ESTIMATION RESULTS OF MODEL 4, TRANSFORMED, NARROW MONEY STOCK**

<table>
<thead>
<tr>
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<th>GERMANY</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>-0.0743 (0.81)</td>
<td>0.4206 (10.8)</td>
<td>-0.3327 (2.86)</td>
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<tr>
<td>y (Δ8)</td>
<td>0.7327 (9.33)</td>
<td>0.1659 (1.90)</td>
<td>0.9009 (10.8)</td>
</tr>
<tr>
<td>RS(Δy)</td>
<td>-0.0123 (4.04)</td>
<td>-0.0110 (3.33)</td>
<td>-0.0132 (8.15)</td>
</tr>
<tr>
<td>y(CAN)</td>
<td>-</td>
<td>-0.0272 (0.59)</td>
<td>0.2068 (2.71)</td>
</tr>
<tr>
<td>y(US)</td>
<td>-0.0664 (0.78)</td>
<td>-</td>
<td>-0.0683 (0.88)</td>
</tr>
<tr>
<td>y(GER)</td>
<td>0.0367 (0.87)</td>
<td>0.0089 (0.49)</td>
<td></td>
</tr>
<tr>
<td>y(FR)</td>
<td>0.0079 (0.33)</td>
<td>-0.0095 (0.96)</td>
<td>0.0002 (0.01)</td>
</tr>
<tr>
<td>y(UK)</td>
<td>0.0030 (0.08)</td>
<td>-0.0085 (0.55)</td>
<td>-0.0297 (0.82)</td>
</tr>
<tr>
<td>RS(CAN)</td>
<td>-</td>
<td>-0.0002 (0.18)</td>
<td>0.0002 (0.15)</td>
</tr>
<tr>
<td>RS(US)</td>
<td>-0.0012 (0.80)</td>
<td>-</td>
<td>-0.0007 (0.47)</td>
</tr>
<tr>
<td>RS(GER)</td>
<td>-0.0002 (0.53)</td>
<td>-0.0001 (0.61)</td>
<td></td>
</tr>
<tr>
<td>RS(FR)</td>
<td>0.0007 (0.47)</td>
<td>-0.0001 (0.12)</td>
<td>-0.0047 (3.53)</td>
</tr>
<tr>
<td>RS(UK)</td>
<td>0.0005 (0.41)</td>
<td>-0.0000 (0.01)</td>
<td>0.0004 (0.37)</td>
</tr>
<tr>
<td>S/P*P(CAN)</td>
<td>-</td>
<td>0.1119 (2.56)</td>
<td>-0.2976 (4.55)</td>
</tr>
<tr>
<td>S/P*P(US)</td>
<td>-0.1937 (2.15)</td>
<td>-</td>
<td>0.1835 (2.38)</td>
</tr>
<tr>
<td>S/P*P(GER)</td>
<td>0.0352 (0.64)</td>
<td>0.0189 (0.62)</td>
<td></td>
</tr>
<tr>
<td>S/P*P(FR)</td>
<td>-0.0179 (0.31)</td>
<td>-0.0284 (0.91)</td>
<td>0.0111 (0.22)</td>
</tr>
<tr>
<td>S/P*P(UK)</td>
<td>-0.0132 (0.32)</td>
<td>-0.0399 (1.70)</td>
<td>-0.0056 (0.16)</td>
</tr>
<tr>
<td>s(CAN)</td>
<td>-0.0022 (0.45)</td>
<td>-0.0016 (0.71)</td>
<td>0.0013 (0.26)</td>
</tr>
<tr>
<td>s(GER)</td>
<td>-0.0086 (0.26)</td>
<td>0.0004 (0.36)</td>
<td>-0.0023 (0.99)</td>
</tr>
<tr>
<td>s(FR)</td>
<td>0.0018 (0.59)</td>
<td>0.0009 (0.67)</td>
<td>0.0033 (1.11)</td>
</tr>
<tr>
<td>s(UK)</td>
<td>-0.0029 (1.59)</td>
<td>-0.0011 (1.29)</td>
<td>-0.0004 (0.24)</td>
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#### SUMMARY STATISTICS

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<tr>
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<th>CANADA</th>
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<tr>
<td>SSR</td>
<td>0.0223</td>
<td>0.0059</td>
<td>0.0194</td>
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<tr>
<td>SER</td>
<td>0.0120</td>
<td>0.0061</td>
<td>0.0112</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.8945</td>
<td>-0.0407</td>
<td>0.9725</td>
</tr>
<tr>
<td>log(L)</td>
<td>670.14</td>
<td>810.97</td>
<td>684.73</td>
</tr>
<tr>
<td>NOBS</td>
<td>212</td>
<td>212</td>
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</tr>
<tr>
<td>DOF</td>
<td>154</td>
<td>156</td>
<td>156</td>
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<tr>
<td>D</td>
<td>2.0424</td>
<td>1.7486</td>
<td>1.9253</td>
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<tr>
<td>$\rho_1$</td>
<td>0.6009 (11.0)</td>
<td>0.6341 (11.2)</td>
<td>0.4811 (7.94)</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>0.4219 (6.74)</td>
<td>0.7554 (16.8)</td>
<td>0.3776 (5.90)</td>
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Range: 1963:01 - 1980:09
## Table 6.4.11
ESTIMATION RESULTS OF MODEL 4, UNTRANSFORMED, BROAD MONEY STOCK

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<tr>
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<td>-3.0810</td>
<td>1.4505</td>
<td>-0.0541</td>
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<tr>
<td>$\hat{y}$ ($\Sigma_{1}$)</td>
<td>0.7464</td>
<td>0.8828</td>
<td>0.5937</td>
</tr>
<tr>
<td>$R_{(y_{14})}$</td>
<td>-0.0036</td>
<td>0.0012</td>
<td>0.0151</td>
</tr>
<tr>
<td>$RO(y_{24})$</td>
<td>0.0184</td>
<td>0.0254</td>
<td>0.1033</td>
</tr>
<tr>
<td>$y$(CAN)</td>
<td>-0.0500</td>
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<td>0.2273</td>
</tr>
<tr>
<td>$y$(US)</td>
<td>0.4741</td>
<td>-0.0560</td>
<td>0.3425</td>
</tr>
<tr>
<td>$y$(GER)</td>
<td>0.0273</td>
<td>-0.1687</td>
<td>0.0026</td>
</tr>
<tr>
<td>$y$(FR)</td>
<td>-0.0310</td>
<td>0.0099</td>
<td>0.0012</td>
</tr>
<tr>
<td>$y$(UK)</td>
<td>-0.0126</td>
<td>0.0161</td>
<td>0.0029</td>
</tr>
<tr>
<td>$R_{(S)}$</td>
<td>0.0025</td>
<td>-0.1035</td>
<td>0.0026</td>
</tr>
<tr>
<td>$R_{(G)}$</td>
<td>0.0001</td>
<td>0.0002</td>
<td>0.0005</td>
</tr>
<tr>
<td>$R_{(F)}$</td>
<td>0.0045</td>
<td>-0.0016</td>
<td>0.0005</td>
</tr>
<tr>
<td>$R_{(U)}$</td>
<td>0.0053</td>
<td>-0.0011</td>
<td>0.0005</td>
</tr>
<tr>
<td>$R_{(C)}$</td>
<td>-0.0126</td>
<td>-0.0126</td>
<td>0.0005</td>
</tr>
<tr>
<td>$SP/P$(CAN)</td>
<td>0.3905</td>
<td>-0.1035</td>
<td>0.0026</td>
</tr>
<tr>
<td>$SP/P$(US)</td>
<td>0.1140</td>
<td>0.0161</td>
<td>0.0029</td>
</tr>
<tr>
<td>$SP/P$(GER)</td>
<td>-0.0468</td>
<td>0.0161</td>
<td>0.0029</td>
</tr>
<tr>
<td>$SP/P$(FR)</td>
<td>-0.0441</td>
<td>0.0099</td>
<td>0.0005</td>
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<tr>
<td>$SP/P$(UK)</td>
<td>-0.0584</td>
<td>-0.0011</td>
<td>0.0005</td>
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<tr>
<td>$s_{(C)}$(CAN)</td>
<td>-0.0391</td>
<td>0.2515</td>
<td>0.0005</td>
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<td>$s_{(G)}$(GER)</td>
<td>-0.0242</td>
<td>-0.1035</td>
<td>0.0026</td>
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<tr>
<td>$s_{(F)}$(UK)</td>
<td>0.0351</td>
<td>-0.1035</td>
<td>0.0026</td>
</tr>
<tr>
<td>$s_{(UK)}$</td>
<td>-0.0024</td>
<td>-0.1035</td>
<td>0.0026</td>
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</table>

### SUMMARY STATISTICS

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<tbody>
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<td>SSR</td>
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<td>0.0283</td>
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<td>SER</td>
<td>0.0120</td>
<td>0.0090</td>
<td>0.0176</td>
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<tr>
<td>$R^{2}$</td>
<td>0.9965</td>
<td>0.9953</td>
<td>0.9986</td>
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<tr>
<td>log(L)</td>
<td>501.01</td>
<td>542.88</td>
<td>440.31</td>
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<td>153</td>
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<td>DOF</td>
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<td>91</td>
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<tr>
<td>DW</td>
<td>0.8769</td>
<td>1.5022</td>
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Note: Lag structures on year RS, and RO (in own equations) are as given in Table 6.4.5.
Range: 1968:01 - 1980:09
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<th>GERMANY</th>
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<td>0.7908 (3.92)</td>
<td>0.0001 (0.00)</td>
</tr>
<tr>
<td>$y$ ($\beta_{1}$)</td>
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<td>0.0173 (3.06)</td>
</tr>
<tr>
<td>$RO^{(C)}$</td>
<td>0.0175 (3.97)</td>
<td>0.0144 (1.00)</td>
<td>-0.0171 (1.82)</td>
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<tr>
<td>$y$ (US)</td>
<td>0.1842 (2.71)</td>
<td>-</td>
<td>0.2319 (2.13)</td>
</tr>
<tr>
<td>$y$ (GER)</td>
<td>0.0027 (0.08)</td>
<td>-0.0338 (0.86)</td>
<td>-</td>
</tr>
<tr>
<td>$y$ (FR)</td>
<td>0.0089 (0.46)</td>
<td>0.0260 (1.12)</td>
<td>0.0252 (0.80)</td>
</tr>
<tr>
<td>$y$ (UK)</td>
<td>0.0014 (0.52)</td>
<td>0.0024 (0.07)</td>
<td>-0.0516 (1.16)</td>
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<tr>
<td>$s_{e}^{(C)}$</td>
<td>-</td>
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<td>-0.0007 (0.31)</td>
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<tr>
<td>$RO^{(C)}$</td>
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<td>-</td>
<td>0.0011 (0.55)</td>
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<tr>
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<td>-0.0000 (0.04)</td>
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</tr>
<tr>
<td>$s_{e}^{(C)}$</td>
<td>0.0017 (1.40)</td>
<td>0.0010 (0.92)</td>
<td>-0.0025 (1.27)</td>
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<tr>
<td>$s_{e}^{(C)}$</td>
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<td>-0.0016 (1.38)</td>
<td>-0.0022 (1.11)</td>
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<tr>
<td>$s_{e}^{(C)}$</td>
<td>-</td>
<td>-0.0092 (4.70)</td>
<td>-0.0001 (0.03)</td>
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<td>$s_{e}^{(C)}$</td>
<td>0.0107 (0.97)</td>
<td>-</td>
<td>-0.0272 (1.41)</td>
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<td>$s_{e}^{(C)}$</td>
<td>-0.0034 (1.93)</td>
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<td>-</td>
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<td>$s_{e}^{(C)}$</td>
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<td>0.0012 (0.75)</td>
<td>0.0023 (0.81)</td>
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<td>$s_{e}^{(C)}$</td>
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<td>0.0043 (5.04)</td>
<td>0.0047 (3.14)</td>
</tr>
<tr>
<td>$s_{e}^{(C)}$</td>
<td>-</td>
<td>0.1887 (3.51)</td>
<td>0.0671 (0.78)</td>
</tr>
<tr>
<td>$s_{e}^{(C)}$</td>
<td>0.0986 (1.56)</td>
<td>-</td>
<td>-0.2044 (2.05)</td>
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<tr>
<td>$s_{e}^{(C)}$</td>
<td>-0.0376 (0.95)</td>
<td>0.1027 (3.12)</td>
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<tr>
<td>$s_{e}^{(C)}$</td>
<td>-0.0046 (0.12)</td>
<td>0.0537 (1.67)</td>
<td>-0.0784 (1.22)</td>
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<tr>
<td>$s_{e}^{(C)}$</td>
<td>0.0128 (0.38)</td>
<td>-0.0212 (0.59)</td>
<td>0.0874 (1.54)</td>
</tr>
<tr>
<td>$s_{e}^{(C)}$</td>
<td>-0.0023 (0.63)</td>
<td>-0.0067 (1.71)</td>
<td>-0.0051 (0.86)</td>
</tr>
<tr>
<td>$s_{e}^{(C)}$</td>
<td>0.0002 (0.11)</td>
<td>-0.0012 (0.62)</td>
<td>-0.0062 (2.32)</td>
</tr>
<tr>
<td>$s_{e}^{(C)}$</td>
<td>0.0034 (1.59)</td>
<td>0.0029 (1.20)</td>
<td>-0.0013 (0.37)</td>
</tr>
<tr>
<td>$s_{e}^{(C)}$</td>
<td>-0.0017 (1.33)</td>
<td>-0.0019 (1.25)</td>
<td>-0.0012 (0.56)</td>
</tr>
</tbody>
</table>

**SUMMARY STATISTICS:**

- $SSR$ 0.0052 0.0060 0.0150
- $SER$ 0.0077 0.0081 0.0128
- $R^2$ 0.9831 0.9861 0.9249
- $log(L)$ 569.91 559.03 488.99
- $NOBS$ 153 153 153
- $DOF$ 89 91 91
- $DW$ 1.5391 1.7241 1.8776
- $\rho_1$ 0.4128 (5.61) 0.2086 (2.61) 0.4634 (6.53)
- $\rho_2$ 0.5044 (7.22) 0.3370 (4.39) 0.4742 (6.74)

(g) Residual Analysis

The purpose of this Subsection is two-fold. First, the appropriateness of the AR2 assumption used in estimating Model 4 above will be considered, using Box-Jenkins time series analysis. Second, the relationship between these same residuals across countries will be investigated.

The first twenty-four autocorrelations and the first twelve partial autocorrelations of the six equations of Model 4, both before and after the AR2 transformation, are given in Tables 6.4.14-17. The last two tables indicate that, generally speaking, the AR2 assumption is a poor one, in spite of the fact that it is required to bring the Durbin-Watson statistic into the acceptable region. Only Germany, in its two equations, approaches anything resembling whiteness in the error structure. Tables 6.4.14 and 6.4.15 present the same analysis for the untransformed residuals. These indicate that the AR2 assumption has served mainly to muddy the already complex error structures. A reasonable starting point for the narrow Canadian equation might be to assume AR3, although there are other problems at longer lags. The narrow U.S. equation might be improved somewhat by assuming an AR5 structure, while the German results are a little mixed. The broad results indicate that AR1 should have been a fair approximation for the U.S. and AR2 for Germany, but the former assumption yields an unfavourable DW statistic. For the broad Canadian equation the lag pattern is quite complex and would require further testing.

Clearly, all six equations would benefit in terms of
efficiency from an estimation procedure which could properly model these high-order autoregressive error structures. On the other hand, perhaps a more fruitful approach would be to combine time series and econometric modelling from the beginning; that is, to construct a time series model for each variable first, and then to fit the residuals into an econometric model.

Ideally, the final estimation step of this Thesis would be to estimate the three demand for money equations simultaneously, in a seemingly unrelated regression, so as to capitalize on the (presumed) correlations between the residuals across equations. Unfortunately, the hardware at hand at the time of writing had insufficient working space to calculate this regression, even for Model 2. In order to obtain some estimate of the efficiency gain which could be achieved here, I have calculated the correlations between the residuals of the six equations of Models 3 and 4. These results are given in Table 6.4.18.

First of all, it is interesting to note the high correlation between Germany and the U.S. in both narrow and broad versions of Model 3, and the significantly lower correlations for those same pairs for Model 4. Furthermore, the narrow correlations are negative, indicating further support for the conclusion that these two monies are substitutes, whereas the broad correlations are positive, a further indication of complementarity. The other pairs are not changed a great deal by moving from Model 3 to Model 4, with the exception of the broad Canada-U.S. pairing, the coefficient of which actually changes sign.
### TABLE 6.4.18

**CORRELATION OF ESTIMATED RESIDUALS**

**NARROW MONEY STOCK**

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**BROAD MONEY STOCK**

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More importantly, however, it is clear that some gain in efficiency could be made by using a seemingly unrelated estimator, regardless of which model is used. Thus, we have isolated three sources of estimator imprecision in Model 4—multicollinearity, time-series behaviour in the residuals, and cross-correlation of the residuals between equations.
### Table 6.4.14

**Box-Jenkins Analysis of Estimation Residuals**

**Model 4, Untransformed, Narrow Money Stock**

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<th>Germany</th>
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</tr>
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<td>0.10 (0.07)</td>
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<td>-0.08 (0.07)</td>
</tr>
<tr>
<td>8</td>
<td>0.19* (0.07)</td>
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<td>9</td>
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<td>0.04 (0.10)</td>
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<tr>
<td>12</td>
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*Note: Standard errors in parentheses; * denotes statistical significance at the 0.95 level.*
## TABLE 6.4.15

**BOX–JENKINS ANALYSIS OF ESTIMATION RESIDUALS**

**MODEL 4, UNTRANSFORMED, BROAD MONEY STOCK**

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| 2   | -0.12 (0.08)             | -0.02 (0.08) | 0.00 (0.08)  |
| 3   | -0.04 (0.08)             | -0.12 (0.08) | 0.02 (0.08)  |
| 4   | -0.13 (0.08)             | 0.02 (0.08)  | -0.01 (0.08) |
| 5   | 0.01* (0.08)             | -0.03 (0.08) | -0.01 (0.08) |
| 6   | -0.15 (0.08)             | -0.04 (0.08) | 0.05 (0.08)  |
| 7   | 0.05 (0.08)              | 0.06 (0.08)  | -0.12 (0.08) |
| 8   | -0.20* (0.08)            | 0.10 (0.08)  | 0.10 (0.08)  |
| 9   | 0.04 (0.08)              | -0.04 (0.08) | -0.02 (0.08) |
| 10  | -0.00 (0.08)             | -0.04 (0.08) | -0.03 (0.08) |
| 11  | 0.02 (0.08)              | 0.01 (0.08)  | -0.11 (0.08) |
| 12  | 0.08 (0.08)              | 0.05 (0.08)  | -0.03 (0.08) |</p>
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<td>0.03 (0.12)</td>
<td>-0.02 (0.07)</td>
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</tbody>
</table>

**PARTIAL AUTOCORRELATIONS**

| .1  | -0.02 (0.07)| 0.12 (0.07)   | 0.03 (0.07) |
| 2   | 0.12 (0.07)| 0.27*(0.07)   | 0.07 (0.07) |
| 3   | 0.22*(0.07)| 0.44*(0.07)   | 0.06 (0.07) |
| 4   | 0.06 (0.07)| 0.15*(0.07)   | -0.03 (0.07)|
| 5   | 0.09 (0.07)| 0.24*(0.07)   | -0.00 (0.07)|
| 6   | 0.14*(0.07)| 0.23*(0.07)   | 0.04 (0.07) |
| 7   | 0.06 (0.07)| 0.04 (0.07)   | -0.04 (0.07)|
| 8   | 0.13 (0.07)| 0.01 (0.07)   | 0.06 (0.07) |
| 9   | 0.15*(0.07)| -0.02 (0.07)|  0.18*(0.07)|
| 10  | 0.11 (0.07)| -0.09 (0.07)  | 0.05 (0.07) |
| 11  | 0.13 (0.07)| 0.05 (0.07)   | -0.03 (0.07)|
| 12  | -0.12 (0.07)| 0.09 (0.07)   | -0.05 (0.07)|
### Table 6.4.17

**Box-Jenkins Analysis of Estimation Residuals**

**Model 4, Transformed, Broad Money Stock.**

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<th>Canada</th>
<th>United States</th>
<th>Germany</th>
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(h) Two Further Tests

Two hypotheses which were outlined in Chapter Five above have yet to be tested. The first arose from the transactions model of Chapter Two—that the impact of the effect of changes in interest rates on the demand for real balances would fall in absolute value as the expected rate of depreciation of the domestic currency became a positive number. In other words, the hypothesis is that the interest elasticity and \( s^e \) will be negatively correlated, where the elasticity is given in absolute value terms. The second hypothesis, although related to the first, came from the speculative model of Chapter Four—that the interest elasticity would be positively correlated with the ratio of the variance of \( s^e \) to the variance of the domestic rate of interest, \( \sigma_s^2/\sigma_{RS}^2 \).

These hypotheses were tested for Canada and Germany by calculating recursive estimates of the interest elasticity for both broad and narrow specifications of Model 1. By recursive is meant that the first estimate of the elasticity used data from 1963.01 to 1969.12, (1967.01 to 1969.12 for the broad equations), the second estimate used the sample 1963.01 to 1970.06, and so on, taking six-month increments in the sample at each step. A similar procedure was used to calculate \( \sigma_s^2/\sigma_{RS}^2 \), although the value of \( s^e \) used in the first hypothesis test was that which held during the last month of the particular recursive sample. This procedure provided twenty-two data points from four demand for money equations; all such data are given in the appendix to this Chapter. The results of correlation analysis are given in Table 6.4.19. As is evident from the table, the first hypothesis is
not well-supported by the data, although the result from the narrow
German equation is quite striking. The hypothesis definitely is rejected
by the broad specifications. However, it is clear that the data
strongly support the second hypothesis, and the support is most
evident in the broad specifications of the demand for money.

TABLE 6.4.19

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The data seem to be telling us that one can discriminate
between the various theories of the demand for money, and that there
is a definite tendency for narrow money specifications to favour the
transactions model, and for broad money specifications to favour the
speculative model. This is not surprising, but comforting. Further
discussion of this implication will be reserved for the concluding
chapter of the Thesis, where a number of such implications may be
brought together.
6.5 Summary

The purpose of this Chapter was to test six of the nine hypotheses outlined in Chapter 5 of the Thesis. At this time it seems natural to summarize the outcomes of the tests in the same way in which the hypotheses were originally presented.

First of all, the evidence strongly suggests that for the PDL model the additional variables which are suggested by the three theories of this Thesis should be included in the demand for money specification. For the PA model this is true for the U.S. (narrow and broad equations) and for the broad German specification. On the other hand, the hypothesis that non-residents hold domestic money enjoys broad support from both models. Unfortunately, multicollinearity has made it impossible to separate out the importance of the domestic and foreign components of the total. Comparison of the $R^2$ statistics of the narrow PDL model indicates that for Canada, for example, the explanatory power of the equation rises some 80%, from 50% of the variation in real balances to 89%. The difference in the PA model, of course, is by comparison extremely small.

Secondly, both substitutability and complementarity are exhibited between the currencies under study, although the low precision of the estimates gives reason for caution in interpreting the results. There seems to be fairly solid evidence of complementarity between the European currencies, of complementarity between narrow Canadian and German money, and of substitutability between narrow American and German money. All other pairs contain some element of ambiguity.
The hypothesis suggested specifically by the transactions theory of Chapter Two that the interest elasticity of the demand for real balances will decline in absolute value when $s^e$ is positive has not been supported by the data, except for the narrow German specification. The prediction that an independent effect of changes in the terms of trade will arise through $\alpha$ has been given only slight support, in the sense that the broad PA specifications have rejected the hypothesis. Because of the inseparably of this effect from the direct effect due to aggregation this outcome is not surprising.

The possibility of currency complementarity which was suggested by the speculative theory has been demonstrated in several instances in the data. More importantly, however, the prediction of the speculative framework that the interest elasticity of the demand for real balances will be positively correlated with the ratio of the variance of the expected rate of depreciation to the variance of the rate of interest, has been convincingly confirmed by the data. Because the results are strongest for the broad money specifications, this is seen as further evidence that the data do have the ability to discriminate between theories; that is, that broad money demand is more appropriately modelled in a framework such as the speculative one given in Chapter Four.

Thus, the data have lent support to five of the six hypotheses, at least two of which were not predictions of the marginal utility theory outlined in Chapter Five. It is clear, therefore, that there has been some gain from utilizing the more detailed, restrictive
Keynesian distinctions between the motives for holding money in analyzing the problem at hand.

One final prediction yet remains to be tested—the theory suggests that, since the traditional analysis results in a misspecified demand for money function, then the correct version presented above should be more stable, in some statistical sense, than the traditional formulations. This hypothesis is tested in Chapter Seven; this will lead naturally into a thorough discussion of the policy implications of the empirical evidence.
FOOTNOTES TO CHAPTER SIX

1 Preliminary work on the quarterly Canadian data has revealed that prior seasonal adjustment does not significantly affect the estimated parameters of the demand for money equation in any case.

2 A precise statement about the feasibility of zero restrictions would be inappropriate here. Essentially, it was found that when one of the foreign variables was statistically significant, adding only the lagged value of that particular variable did not improve the performance of the equation to a significant degree. A proper joint test of the statistical significance of lags in all of the foreign variables was not possible because of insufficient computer scratch space.

3 As a starting point the lag structures used by White (1976) and Alexander (1980) for Canada were investigated. Then these lags were lengthened and shortened and the final choice was determined by goodness of fit and the consistency of the signs of the coefficients.

4 It should also be noted that each of the Almon lags was estimated subject to a zero endpoint constraint. The constraint was added to the additional lag once the optimal lag length had been chosen.

5 This may not be strictly true. If the theory had been based on a four-asset model, presumably $r^*$ would have appeared as an argument of the domestic demand for domestic money. Thus, the conclusion should be qualified to the extent that foreign bonds are held in domestic portfolios.
APPENDIX A6

DATA

GRAPHICAL EXPOSITION OF $s^e$

CORRELATION MATRICES

RECURSIVE ESTIMATES
APPENDIX A6

A6.1 Data

Virtually all data are taken from the OECD publication Main Economic Indicators. Characteristics which are common to each country are as follows. The price level is given by the monthly consumer price index. The expected rate of depreciation is given by the difference between the natural logarithms of the forward (90 days) and spot exchange rates. With the exception of France, the scale variable is quarterly gross national product, interpolated to monthly using the total real domestic product index. Specifically, we assume that the quarterly observation applies to the middle month of the quarter. The ratio of the quarterly observation to the monthly observation of the index at that middle month is then multiplied by the two surrounding monthly observations to derive the two new income observations. We now move on to consider features which differ between countries.

Canada

Narrow money is the OECD's M1 and corresponds to the Bank of Canada definition M1, occasionally referred to as M1-A. For example, in December 1979 M1 was equal to $23.36 billion (CAN). This aggregate consists of currency outside banks plus non-government demand deposits net of float. Broad money is M1 plus savings deposits, and therefore falls about $2 billion (or 2%) short of the Bank of Canada definition of M2. However, a consistent M2 series is available only from 1968. For December 1979, M2 = $89.45 billion whereas 'broad
money' as used in this study is $87.56 billion. The money market (substitute) rate of interest used was that on 90-day finance company paper. The own rate on broad money used was that on non-chequable savings deposits, and was obtained from CANSIM, B14019; this series is available only from 1967 onwards.

United States

Narrow money is the OECD's M1 and corresponds to the Federal Reserve Board's definition M1-B. As such, it includes currency outside banks plus demand deposits other than those due to domestic banks, the U.S. government, foreign banks and official institutions, less float (that is, M1-A) plus negotiable order of withdrawal (NOW) and automatic transfer service accounts at banks and thrift institutions, credit union share draft accounts, and demand deposits at mutual savings banks. In December 1979, M1-B was 386.4 billion (US) or some $17 billion (4-5%) greater than M1-A. As mentioned in Chapter One, there is reason to believe that the increase in M1-B relative to M1-A during the mid-1970's may have been partly responsible for the apparent rise in velocity (defined in terms of M1-A) in the U.S. Thus, it is probably wise to use M1-B so as to eliminate known exogenous shifts in the demand for money function a priori. Broad money is the OECD's 'M1 plus quasi-money' which corresponds to the current Federal Reserve Board definition M3. M3 includes M2 as well as 'large-denomination' time deposits; M2 is M1-B plus savings and small-denomination time deposits, overnight repurchase agreements, overnight Eurodollars held by U.S. residents, and money market mutual fund shares. In December 1979, M3 was $1773.6 billion. The money market
interest rate used was that on U.S. 3-month Treasury Bills; the own-rate on broad money was taken from various issues of the Federal Reserve Bulletin, and is given by the maximum interest rate payable on savings accounts at commercial banks.

Germany

Narrow money is the OECD's M1, and was equal to DM 237.2 billion in December 1979. Broad money is the OECD's 'M1 plus quasi money' and was DM 383.3 billion in December 1979. The substitute rate of interest used was that on Call Money (Frankfurt); a proper own-rate was not available so the central bank discount rate was used as a proxy.

France

A quarterly GNP series for France was not available; as a proxy I have used 'household consumption of industrial products', which is a monthly series. The rates of interest were that on Call Money and the central bank discount rate.

United Kingdom

The rates of interest used were that on British Treasury Bills and the Bank of England Discount rate.

All money stocks, price indices and scale variables are seasonally adjusted, while all other variables (interest rates, exchange rates) are seasonally unadjusted. The latter approach is quite conventional, as it is widely believed that these variables do not exhibit a discernible seasonal pattern. Applying the deseasonalization procedure from TSP to these series results in seasonality.
coefficients which are very close to unity, implying that there is no
significant seasonal pattern.
FIGURE A6.1

EXPECTED RATE OF DEPRECIATION (PERCENT)
CANADIAN DOLLAR
FIGURE A6.2

EXPECTED RATE OF DEPRECIATION (PERCENT)
GERMAN DEUTSCHEMARK

[Graph showing expected rate of depreciation for German Deutschemark with years and percentage values]
FIGURE A6.3

EXPECTED RATE OF DEPRECIATION (PERCENT)
FRENCH FRANC
FIGURE A6.4

EXPECTED RATE OF DEPRECIATION (PERCENT)
BRITISH POUND

# A6.3 Correlation Matrices


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## 5. Own Rates Of Interest, RO, 1967.01-1980.09

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### A6.4 Recursive Estimates

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<td>0.0351</td>
<td>0.0150</td>
<td>0.0095</td>
<td>0.0217</td>
</tr>
<tr>
<td>1973.06</td>
<td>0.0365</td>
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<td>0.0104</td>
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<td>1973.12</td>
<td>0.0408</td>
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<tr>
<td>1976.06</td>
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<td>0.0093</td>
<td>0.0095</td>
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<tr>
<td>1976.12</td>
<td>0.0361</td>
<td>0.0144</td>
<td>0.0118</td>
<td>0.0122</td>
</tr>
<tr>
<td>1977.06</td>
<td>0.0365</td>
<td>0.0153</td>
<td>0.0117</td>
<td>0.0127</td>
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<tr>
<td>1977.12</td>
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<td>0.0173</td>
<td>0.0118</td>
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<tr>
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<tr>
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<td>1979.12</td>
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<td>0.0116</td>
<td>0.0122</td>
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</table>

#### (b) \( \frac{\sigma_{x_e}^2}{\sigma_X^2} \) Germany

<table>
<thead>
<tr>
<th>Endpoint</th>
<th>Canada</th>
<th>Germany</th>
<th>Canada</th>
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<tbody>
<tr>
<td>1969.12</td>
<td>0.1134</td>
<td>0.3989</td>
<td>0.0322</td>
<td>-0.3683</td>
</tr>
<tr>
<td>1970.06</td>
<td>0.1080</td>
<td>0.2930</td>
<td>-0.3202</td>
<td>0.0363</td>
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<tr>
<td>1970.12</td>
<td>0.1079</td>
<td>0.2645</td>
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<tr>
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<td>0.0296</td>
<td>-0.9735</td>
</tr>
<tr>
<td>1972.12</td>
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<td>0.2504</td>
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<tr>
<td>1973.06</td>
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<td>0.2360</td>
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<td>-0.7007</td>
</tr>
<tr>
<td>1973.12</td>
<td>0.1334</td>
<td>0.1815</td>
<td>0.0800</td>
<td>0.2706</td>
</tr>
<tr>
<td>1974.06</td>
<td>0.1171</td>
<td>0.1749</td>
<td>-0.3978</td>
<td>-0.6621</td>
</tr>
<tr>
<td>1974.12</td>
<td>0.1062</td>
<td>0.1715</td>
<td>-0.0595</td>
<td>-0.5049</td>
</tr>
<tr>
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<td>0.1065</td>
<td>0.1708</td>
<td>0.1547</td>
<td>-0.4230</td>
</tr>
<tr>
<td>1975.12</td>
<td>0.1142</td>
<td>0.1723</td>
<td>0.8677</td>
<td>-0.4969</td>
</tr>
<tr>
<td>1976.06</td>
<td>0.1361</td>
<td>0.1721</td>
<td>0.9831</td>
<td>-0.3853</td>
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<tr>
<td>1976.12</td>
<td>0.1545</td>
<td>0.1717</td>
<td>0.7496</td>
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<tr>
<td>1977.06</td>
<td>0.1564</td>
<td>0.1722</td>
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<td>-0.4665</td>
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<tr>
<td>1977.12</td>
<td>0.1566</td>
<td>0.1739</td>
<td>-0.0547</td>
<td>-1.1926</td>
</tr>
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<td>1978.06</td>
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<tr>
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<td>0.2105</td>
<td>-1.4443</td>
</tr>
<tr>
<td>1980.06</td>
<td>0.1221</td>
<td>0.2078</td>
<td>0.4499</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
CHAPTER SEVEN

POLICY IMPLICATIONS

7.1 Introduction

The empirical evidence which has been presented thus far may be interpreted as either supporting or rejecting the currency substitution hypothesis, depending upon which model of the demand for money is preferred. Which model would best suit the needs of policy-makers, therefore, remains an open question, and it is the purpose of this Chapter to shed some light on this issue. The criterion which is used in this Chapter to discriminate between the models is ex-post forecasting reliability. That is, given that policy-makers would like to be able to reliably forecast the demand for money during the next few months, which model should be used? The fact that we will be using ex-post forecasts implies simply that the policy-maker’s forecasts of the exogenous variables are assumed to be correct.

The Chapter comprises three additional sections. Section 7.2 conducts several forecasting experiments, and compares the performances of the models by summarizing the results in two forms. First, the root-mean-squared-error (RMSE) of each forecast will be presented, and second, a plot of the forecast errors will be given. The focus of comparison will be first between the pairs of models, Models 1 and 2, and Models 3 and 4; and then on which model is best overall for a particular country. Section 7.3 will combine these
results and those of Chapter Six to discuss the implications for policy of the empirical evidence. In Section 7.4 the Chapter will be summarized briefly.

7.2 Forecasting

(a) **Experiment 1**

This first experiment involves only the twelve models of the narrow money stock. Each equation is estimated from 1963.01 to 1969.12 and then the rest of the sample, from 1970.01 to 1980.09, is forecast. The PA equations, Models 1 and 2, are forecast dynamically; that is, rather than using the actual money stock lagged, we use the forecast money stock from the previous month as the lagged dependent variable. This experiment is asking a great deal from the equations, given the sort of turbulence which has been observed during the 1970's. However, the forecast errors are of interest especially from an historical viewpoint.

The RMSE of each forecast is given in Table 7.2.1; the asterisk denotes the best performance for a given country. Notice that the results are not comparable across countries, because the units vary, but within the same country they are perfectly comparable.

**Table 7.2.1**

<table>
<thead>
<tr>
<th>Model</th>
<th>Canada</th>
<th>United States</th>
<th>Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>0.0424</td>
<td>0.1470</td>
<td>0.0883</td>
</tr>
<tr>
<td>Model 2</td>
<td>0.2466</td>
<td>0.2252</td>
<td>0.2166</td>
</tr>
<tr>
<td>Model 3</td>
<td>0.0400*</td>
<td>0.1162*</td>
<td>0.0949</td>
</tr>
<tr>
<td>Model 4</td>
<td>0.2071</td>
<td>0.1778</td>
<td>0.0557*</td>
</tr>
</tbody>
</table>
It is clear from the above table that empirically implementing the currency substitution hypothesis has improved the forecasting ability of the German equation only. For Canada and the U.S. it is the simple PDL specification, Model 3, which outperforms the other models. In both cases the performance of Model 4 is substantially worse than both Models 3 and 1. However, Model 2 gives the worst performance in all three countries. This is probably because the PA model forces all variables to take on an infinite, geometrically declining lag structure, whereas Model 3 takes the opposite extreme, allowing only a contemporaneous effect from the foreign variables. The results suggest that the latter specification is less restrictive than the former.

The plots of the forecast errors follow immediately after this subsection. Some interesting observations arise from an examination of these plots:

Canada

It should be noted before proceeding that the Canadian forecasts assume that the monetary authority cannot forecast a postal strike, so that the postal strike variables are set equal to zero for the ex-post forecast. It is clear from the results of both Model 1 and Model 3 that the downward shift in the demand for money equation late in 1976 was actually preceded by an upward shift in mid-1972 or 1973. This finding is a new one, probably because discussion of the 1976 shift occurred ex-post when demand for money equations were being estimated until 1975 or so and then forecast out; thus, the estimation period had already incorporated the upward shift. The two
major postal strikes in the spring of 1974 and the end of 1975 are readily discernable as two large negative forecast errors.

**United States**

The U.S. specifications are all in trouble through a significant proportion of the 1970's. The timing of the shift varies from as early as mid-1970 for Model 2 to mid-1973 for Model 3. Since the latter model gives the best overall performance, one would have to agree that the shift occurred in the third quarter of 1973. Since the money stock which has been used is M1-B, this shift cannot be explained by shifts from M1-A into M1-B.

**Germany**

Notice that the German forecast errors are, broadly speaking, opposite in sign to those in the U.S. equations. The U.S. equation experienced a downward shift in the autumn of 1973 (Model 3), while Models 1 and 3 for Germany indicate that the German equation shifted upwards late in 1974 or early 1975. There is considerable noise in the forecasts, especially Models 3 and 4, at precisely the time when the U.S. equation shifted down, which was just during a major depreciation of the U.S. dollar relative to the Deutschmark. Equation 4 evidently has done a superb job of tracking these phenomena, with the forecast errors consistently staying near the zero line. When viewed in terms of the length of time required of the forecast, the performance of Model 4 for Germany is quite remarkable.
FIGURE 7.2.1

EX-POST FORECAST ERRORS, MODEL 1
CANADA, NARROW MONEY STOCK

EX-POST FORECAST ERRORS, MODEL 2
CANADA, NARROW MONEY STOCK
FIGURE 7.2.2

EX-POST FORECAST ERRORS, MODEL 3
CANADA, NARROW MONEY STOCK

EX-POST FORECAST ERRORS, MODEL 4
CANADA, NARROW MONEY STOCK
FIGURE 7.2.3

EX-POST FORECAST ERRORS, MODEL 1
UNITED STATES, NARROW MONEY STOCK

EX-POST FORECAST ERRORS, MODEL 2
UNITED STATES, NARROW MONEY STOCK
FIGURE 7.2.4

EX-POST FORECAST ERRORS, MODEL 3
UNITED STATES, NARROW MONEY STOCK

EX-POST FORECAST ERRORS, MODEL 4
UNITED STATES, NARROW MONEY STOCK
FIGURE 7.2.5

EX-POST FORECAST ERRORS, MODEL 1
GERMANY, NARROW MONEY STOCK

EX-POST FORECAST ERRORS, MODEL 2
GERMANY, NARROW MONEY STOCK
FIGURE 7.2.6

EX-POST FORECAST ERRORS, MODEL 3
GERMANY, NARROW MONEY STOCK

EX-POST FORECAST ERRORS, MODEL 4
GERMANY, NARROW MONEY STOCK
(b) Experiment 2

In this case we perform an identical experiment on all twenty-four models (four models, two definitions of money each, and three countries). The estimation period is 1968.01 to 1973.12, and the forecast period is from 1974.01 to 1980.09. The RMSE's of the forecasts are given in Table 7.2.2.

<table>
<thead>
<tr>
<th></th>
<th>CANADA</th>
<th>UNITED STATES</th>
<th>GERMANY</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Narrow Money Stock</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 1</td>
<td>0.0794</td>
<td>0.0883*</td>
<td>0.0775</td>
</tr>
<tr>
<td>Model 2</td>
<td>0.1378</td>
<td>0.1292</td>
<td>0.1814</td>
</tr>
<tr>
<td>Model 3</td>
<td>0.0458*</td>
<td>0.1153</td>
<td>0.0500*</td>
</tr>
<tr>
<td>Model 4</td>
<td>0.0693</td>
<td>0.1277</td>
<td>0.1285</td>
</tr>
<tr>
<td>(b) Broad Money Stock</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 1</td>
<td>0.0396</td>
<td>0.0911</td>
<td>0.1227</td>
</tr>
<tr>
<td>Model 2</td>
<td>0.0391*</td>
<td>0.1884</td>
<td>0.1082</td>
</tr>
<tr>
<td>Model 3</td>
<td>0.0800</td>
<td>0.0497*</td>
<td>0.0911*</td>
</tr>
<tr>
<td>Model 4</td>
<td>0.0406*</td>
<td>0.0529</td>
<td>0.1136</td>
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</table>

These results present a slightly different impression. For the narrow money stock, the best model for Canada is still Model 3, although Model 4 has now taken second place from Model 1. As before, Model 2 performs worst for all three countries. For the U.S. it is Model 1 and, surprisingly, for Germany Model 3 which perform the best in terms of RMSE. In the German case Model 4 is no longer performing well and has been displaced by both Model 3 and Model 1.
For the broad money stock, there is no broad pattern to be discerned. For Germany and the U.S. Model 3 gives the best performance, while for Canada it is the lowly Model 2 which tops the list. However, notice that Models 1, 2 and 4 produce results which are all approximately the same for Canada.

The plots of the forecast errors follow immediately at the end of this subsection. While detailed comments would seem redundant, a few features are worth highlighting.

For Canada, the downward shift in the narrow specification in 1976 is readily apparent, and it is obvious why the earlier upward shift has received little attention. Models 3 and 4, however, appear to be getting back on track through 1979, whereas in 1980, Model 3 again begins to overpredict the demand for real balances, while Model 4 is performing extremely well. Thus, it is difficult to decide which model is really doing the best job.

For the U.S. the narrow results reveal nothing new, but it is immediately apparent that Model 3 is a relatively reliable forecaster of the broad money stock. Thus, it is clear that the Fed has a ready alternative demand for money equation to that for M1-A or M1-B in this broad specification, which uses M3.

For Germany, the results are rather mixed, with only the narrow Model 3 forecast exhibiting desirable properties. For the broad money stock it is Model 4 which gets back on track first but, of course, Model 3 is the best in terms of overall RMSE.
FIGURE 7.2.7

EX-POST FORECAST ERRORS, MODEL 1
CANADA, NARROW MONEY STOCK

EX-POST FORECAST ERRORS, MODEL 2
CANADA, NARROW MONEY STOCK
FIGURE 7.2.8

EX-POST FORECAST ERRORS, MODEL 3
CANADA, NARROW MONEY STOCK

EX-POST FORECAST ERRORS, MODEL 4
CANADA, NARROW MONEY STOCK
FIGURE 7.2.9

EX-POST FORECAST ERRORS, MODEL 1
CANADA, BROAD MONEY STOCK

EX-POST FORECAST ERRORS, MODEL 2
CANADA, BROAD MONEY STOCK
FIGURE 7.2.10

EX-POST FORECAST ERRORS, MODEL 3
CANADA, BROAD MONEY STOCK

EX-POST FORECAST ERRORS, MODEL 4
CANADA, BROAD MONEY STOCK

FIGURE 7.2.11

EX-POST FORECAST ERRORS, MODEL 1
UNITED STATES, NARROW MONEY STOCK

EX-POST FORECAST ERRORS, MODEL 2
UNITED STATES, NARROW MONEY STOCK
EX-POST FORECAST ERRORS, MODEL 3
UNITED STATES, NARROW MONEY STOCK

EX-POST FORECAST ERRORS, MODEL 4
UNITED STATES, NARROW MONEY STOCK

FIGURE 7.2.13

EX-POST FORECAST ERRORS, MODEL 1
UNITED STATES; BROAD MONEY STOCK

EX-POST FORECAST ERRORS, MODEL 2
UNITED STATES; BROAD MONEY STOCK
FIGURE 7.2.14

EX-POST FORECAST ERRORS, MODEL 3
UNITED STATES, BROAD MONEY STOCK

EX-POST FORECAST ERRORS, MODEL 4
UNITED STATES, BROAD MONEY STOCK
FIGURE 7.2.15

EX-POST FORECAST ERRORS, MODEL 1
GERMANY, NARROW MONEY STOCK

EX-POST FORECAST ERRORS, MODEL 2
GERMANY, NARROW MONEY STOCK
FIGURE 7.2.16

EX-POST FORECAST ERRORS, MODEL 3
GERMANY, NARROW MONEY STOCK

EX-POST FORECAST ERRORS, MODEL 4
GERMANY, NARROW MONEY STOCK

EX-POST FORECAST ERRORS, MODEL 1
GERMANY, BROAD MONEY STOCK.

EX-POST FORECAST ERRORS, MODEL 2
GERMANY, BROAD MONEY STOCK.
EX-POST FORECAST ERRORS, MODEL 3
GERMANY, BROAD MONEY STOCK

EX-POST FORECAST ERRORS, MODEL 4
GERMANY, BROAD MONEY STOCK
(c) **Experiment 3**

Experiment 3 reflects a worry on the part of the author that the earlier experiments have asked too much of Model 4. In particular, it may be that Model 4 achieves a major portion of the statistical significance for the foreign variables from the data of the same period which so far we have asked the model to forecast. Also, previous plots have all indicated that, with one or two exceptions, there is something about the 1970's which makes those years quite different from the 1960's. This may be due to long-run trends in velocity as a result of innovation in financial intermediation, for example. In any event, the present experiment is restricted to the 1970's only, and involves all twenty-four equations. The estimation period is from 1970.01 to 1978.12 and the forecast period is from 1979.01 to 1980.09, twenty-one observations. Because the forecast period is so short it would seem redundant to present twenty-four additional forecast plots. Instead, only the RMSE's will be presented; these are given in Table 7.2.3.

For the narrow models we see that only Germany benefits greatly from the inclusion of the foreign variables, although Model 2 does outperform Model 1 for the U.S. as well. From the beginning of this Thesis it was supposed that a great deal of currency substitution has gone on between the U.S. and Germany during the 1970's, and we see by the table that some improvement is made in forecasting reliability for those two countries by remodelling the demand for real balances to take this into account. For both definitions of the money stock the most reliable model for Canada is the simplest of all models of the
### Table 7.2.3

RMSE for Experiment 3

<table>
<thead>
<tr>
<th>CANADA</th>
<th>UNITED STATES</th>
<th>GERMANY</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Narrow Money Stock</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 1</td>
<td>0.0175*</td>
<td>0.0375</td>
</tr>
<tr>
<td>Model 2</td>
<td>0.0535</td>
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<tr>
<td>Model 3</td>
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<td>0.0403</td>
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<tr>
<td>Model 4</td>
<td>0.0594</td>
<td>0.0672</td>
</tr>
<tr>
<td>(b) Broad Money Stock</td>
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<td></td>
</tr>
<tr>
<td>Model 1</td>
<td>0.0245*</td>
<td>0.0198</td>
</tr>
<tr>
<td>Model 2</td>
<td>0.0277</td>
<td>0.0183</td>
</tr>
<tr>
<td>Model 3</td>
<td>0.0370</td>
<td>0.0372</td>
</tr>
<tr>
<td>Model 4</td>
<td>0.0408</td>
<td>0.0132*</td>
</tr>
</tbody>
</table>
demand for money, the closed-economy partial-adjustment model. For the broad money stock, the U.S. gains from the currency substitution hypothesis, but, as in the previous experiment, the German equation does not.

Discussion

Although the results are somewhat mixed, it is clear that Model 4 in particular has performed less well relative to Model 3 than expected. Because the added variables made rather a significant improvement in the regression results of Model 3, it seemed likely that Model 4 would outperform Model 3 by a substantial margin. (The relationship between Models 1 and 2 was less clear a priori, and so the results there are much less surprising.) The reason for the lacklustre performance of Model 4, however, is not difficult to ascertain, for it is the same problem which was noted several times in Chapter Six—imprecision of the coefficient estimates. There can be little doubt that the forecast performance of Model 4 could be improved by an increase in precision; indeed, the performance of any of the models could be so improved, but it is Model 4 which shows the most potential in this regard.

A country-by-country summary seems to be in order. The results for Canada indicate that there is nothing to be gained in terms of forecasting reliability (at the present time) in taking account of foreign influences in the demand for money equation. This is the same conclusion which was reached by Alexander (1980). For the United States the recent data indicate some gain to be realized from accounting for foreign influences in the demand for real balances.
This is in direct contrast with the longer-term experiments, which indicated that Model 1 or Model 3 were superior. In fact, none of the models performs exceptionally, indicating that efforts on the institutional side of this problem will prove to be valuable.

For Germany, the data fairly consistently (two experiments out of three) indicate that there is much to be gained in forecasting the narrow money stock by incorporating foreign influences in the demand for money function. However, the simpler models, Model 1 and 3, are the best in terms of forecasting reliability for the broad money stock.

Now that the results are all at hand, it is possible to evaluate the policy implications of the currency substitution hypothesis.

7.3 Policy Implications

In the last decade the notion that central banks should control the rate of growth in the money stock so as to control the rate of growth of nominal income has become widely accepted. A clearer statement of the objectives of such a policy has been given in White (1979, 591):

...the most important of these objectives is traditionally assumed to be minimizing the variance of nominal income around some desired long-run trend.... A subsidiary if important short-run objective of most policy-makers is that sharp fluctuations ('disorder') in financial and exchange rate markets should be avoided because of the welfare costs which may be associated with such movements.

Given the desire to control the money stock towards these ends, the policy may be formulated in two stages. First, one should choose the
monetary target (that is, the definition of money to be controlled) and second, one should choose the instrument of control. In this Section the second stage of the process will be abstracted from; although some input into the second decision may arise from the choice of aggregate, for the most part the choice of instrument issue seems to relate to institutional arrangements and will not be considered here.

According to White (1979), the choice of aggregate may be made according to three criteria, all of which relate to characteristics of estimated demand for money equations: money as an indicator of national income, implications of control of that aggregate for other financial variables, and controllability. However, getting to this stage presumes that one has available an array of 'stable' estimated demand for money equations; by 'stable' is meant that as a starting point each of the equations must have passed some minimum test of stability, such as the Brown, Durbin and Evans (1975) tests referred to in Chapter One. However, such tests are, by definition, statistical, and therefore can make only a probability statement at some arbitrary level of significance about the stability of the regression equation. That this arbitrary level of significance need not necessarily correspond to the level of precision required for policy purposes should be obvious.

Surely the concept of stability is a relative one, not absolute, as the statistical tests would have us believe. It is clear that Harry Johnson (1962) felt that a relative concept of stability should be used in resolving these issues. The tests conducted in this Chapter enable one to make statements about whether one specification
of the demand for money is more or less reliable (stable) than another, and that seems to be the relevant question to ask of the data.

The criteria for choice of a monetary aggregate are altered somewhat by these considerations. As pointed out by White (1979, 597) there is generally little to choose between narrow and broader aggregates in terms of their correlation with national income. This finding has been verified by this study, with the exception of the narrow U.S. specification, which is unusually poorly behaved. Furthermore, the effect on other financial variables of controlling a particular aggregate is fairly well-known, since it is widely believed that broad demand for money equations are less interest elastic than narrow equations. This, too, has been a feature of the equations considered here. Finally, controllability (a feature which is given less emphasis by White) relates rather essentially to the instrument which is chosen. Then the problem of choosing a monetary aggregate collapses to one of finding the most reliable demand for money equation. When this criterion fails to indicate a clear choice, some bias towards the narrow (more interest elastic) aggregate may be desirable in terms of minimizing 'disorder' in financial markets. Prior choice of the interest rate as an instrument of policy will serve to reinforce this bias, whereas choice of base control as a means of controlling the aggregate would be unlikely to affect the choice of monetary target.

One additional consideration will be likely to play a role, and it is this consideration about which it will be most difficult to make positive statements. In conducting monetary policy,
the policy-maker must be able to insert his best forecasts of the exogenous variables into the estimated demand for money equation and thereby forecast the implications of the various scenarios under consideration. It is immediately clear, therefore, that the various models of this Thesis will not be given equal weights in the objective function of the policy-maker. The latter will obviously favour the specification which contains fewer exogenous variables for him to forecast. This means that wherever Models 2 or 4 have outperformed Models 1 or 3 in ex-post forecasting, the policy-maker will have to decide whether the improvement in ex-post forecasting performance is likely to carry over to ex-ante performance. It is quite possible that the more complex model, while it has a lower RMSE for ex-post experiments, will have a higher RMSE for a real (ex-ante) situation, because of the added source of error, the estimates of the exogenous variables. The decision might be made easier by simulating both models in the real world for a trial period. In any event, this element of the policy formulation process cannot be treated adequately here.

In considering this issue, then, we will use only the results of Experiments 2 and 3, with more weight given to the latter, since it most closely resembles a policy situation. For Canada, the decision between the broad version of Model 1 and the narrow version of Model 1, is a difficult one. If 'disorder' in financial markets is of importance, then, the narrow aggregate should be chosen, and Model 1 has been outperforming Model 3 recently. It is clear that for policy purposes the more complex models which embody the currency substitution
hypothesis have not, to this point in time, produced an alternative which is superior to the simple, 'one-currency' specification of the demand for money.

For the United States, the results clearly indicate that choice of the broad aggregate would be preferred, and the results of Models 3 and 4 are quite similar in Experiment 2, whereas in Experiment 3 Model 4 is superior. Then, if the cost of forecasting the exogenous variables is not considered to be prohibitive, the U.S. should conduct policy using the broad specification of Model 4. If the cost is too high, then Model 1 would appear to be best, although with some additional innovations in the specification, Model 3 might prove to be superior to Model 1.

For Germany, the evidence is somewhat mixed, but would appear to favour the narrow money stock as choice of aggregate. However, choice of the narrow class of models seems to make consideration of the foreign influences necessary, whereas choice of the broader aggregate would allow the use of the simpler Model 1 or Model 3.

It should be emphasized that none of the equations which are recommended above are, strictly speaking, stable, but are only the best available from the set considered in this Thesis. The plots of forecast errors make it clear that there is some distance to go before the demand for real balances will have been perfectly specified.

6.4 Summary

The results presented in this Chapter indicate that, while the theory and, in large part, the data have told us to include the foreign variables in the demand for money function, the resulting
specification is not necessarily the best in terms of ex-post forecasting reliability, the latter depending crucially on the relative precision of the estimates of the equations which are being compared. Moreover, a brief analysis of the monetary policy problem has indicated that even if the fully specified currency substitution model performs best ex-post, it still may not be the most desirable model for policy purposes.  

With these qualifications in mind, it has been suggested that (a) Canada continue with a narrow aggregate and to use either Model 1 or Model 3, recent results favouring the former, (b) the United States conduct policy according to the broad version of Model 4 and (c) Germany has a choice between the narrow version of Model 2 or 4, or the broad version of Model 1 or 3. The latter choice would, of course, be the simplest, but other considerations may affect the decision.

At this point it is time to bring together all of the results of the Thesis in order to put it into perspective with the previous literature cited in Chapter One. That is the task of the eighth, and concluding, Chapter.
FOOTNOTES TO CHAPTER SEVEN

1 Of course, once a postal strike has occurred, the Bank of Canada can allow for it by providing the banking system with additional cash reserves.

2 This upward shift may have been due to the Winnipeg Agreement; in which the chartered banks agreed to ceilings on wholesale deposit rates; the ceilings could have caused agents to hold more transactions balances than they did before the ceilings were imposed.

3 By absolute I mean that a statistical test says that, at a pre-selected level of significance, either the regression is stable or it is not. Thus, this sort of technique is not helpful in comparing two equations which both either pass or fail the test. A statistical procedure is only useful in discriminating between good and bad, not between several good or several bad candidates.

4 One further possibility is that the foreign variables are needed in the regression to eliminate omitted variables bias, and therefore to obtain the correct income and interest elasticity estimates, but could subsequently be ignored in forecasting. This seems doubtful, since by definition a statistically insignificant variable does not significantly reduce omitted variables bias. Furthermore, this idea could only be tested after the solution of the multicollinearity problem.

5 It should be noted that the difficulty in explaining recent movements in the money stock may be caused by one of the maintained hypotheses—the dynamic specification. To test alternative specifications would be to exceed the bounds of this thesis. The issue of dynamic misspecification in the demand for money function is taken up by Hendry (1978).
CHAPTER EIGHT

CONCLUSION

This Thesis began with a quote from Harry G. Johnson (1962) which has been the guide for research into the properties of the demand for money function for nearly two decades. By the end of the first decade it appeared as though most of the issues had been resolved, but in the second decade that feeling has been replaced with one of considerable doubt. Several instances of instability in the world demand for money equations were noted, as well as numerous ad hoc (and generally unsuccessful) attempts to improve them. A number of clues led to a perusal of the currency substitution literature, which indicated that it might have been possible to predict the instability in the world demand for money functions. Since preliminary empirical work in the area had proved generally to be inconclusive, it was felt that some additional research into the problem might be useful. In particular, it was suggested that strengthening the priors which underlie empirical testing of the currency substitution hypothesis might render the latter more informative.

The stated purpose of the Thesis, then, was twofold: first, to derive the demand for money function from a firm microeconomic foundation in which it was explicitly recognized that we live in a multicurrency world, and second, to test the implications of this theory against the data. The theoretical part of the Thesis then
proceeded to adopt three distinct but complementary approaches to analysis of the demand for money, investigating in turn each of the three Keynesian motives for holding money. The three motives were treated in a slightly unconventional, but enlightening manner, in that each was presumed to give rise to a demand for a distinct quantity of money. Specifically, the transactions theory of Chapter Two was viewed as analyzing only the demand for money which arises from the deterministic portion of the agent's expenditures. Then, the precautionary theory may be interpreted as determining the additional amount of money demand for transactions purposes due to the stochastic portion of his expenditures, where the latter has a zero mean but a known distribution. In both frameworks it is simply assumed that some of the expenditures typically will require the use of foreign money. Thus, foreign and domestic currencies are perfectly non-substitutable in exchange, and yet all of the implications of the currency substitution hypothesis were derived. After the agent has allocated the income of a particular period among bonds (which will be cashed in by the end of the period), transactions and precautionary balances, the remainder is invested in his portfolio. The latter will be adjusted when variables relevant to the allocation change. It is supposed that, in most cases, any money holdings which arise from this portfolio allocation will be in the form of broad money balances. Hence, any predictions which arise from the speculative theory would be expected to be most strongly verified in the broader definitions of money. In contrast, predictions specific to the transactions model would be expected to be most evident in the narrower definitions of money.
In summarizing the implications of the three theories, it was found that some of the hypotheses were common to all three, but that there were three hypotheses specific to each of the transactions and speculative models. The precautionary model, although it was the most complex mathematically, generated no hypotheses which would enable empirical discrimination between it and the other two theories. However, it was noted that the transactions and speculative theories were capable of generating predictions which could not be derived from the simpler alternative, the marginal utility approach. By placing real balances of two different currencies in the utility function and maximizing subject to a wealth constraint, it is possible to derive several of the hypotheses which were generated from the more restrictive frameworks based on the Keynesian motives. However, two of the theories did generate discriminating hypotheses, so that it is possible to conclude that one can get more specific than the simple (yet powerful) marginal utility approach.

In the case of the PDL model the data were found to support virtually all of the hypotheses which were derived in Part II of the Thesis. In contrast, the simple PA model generally explains the data very well without the addition of the foreign variables suggested by the theory presented here. This fact suggests a potential benefit from using some combination of time series and econometric modelling on the demand for money function, for the PA model, in some sense, is just that. Two additional points related to the testing stage bear repeating. First, at least two of the hypotheses which were generated by the Keynesian theories of the demand for money but
not by the marginal utility approach were confirmed by the data. Thus, it is clearly beneficial to work with theories which are more restrictive, more detailed, and more specific than that which simply places real balances in the utility function. Second, the data seemed to lend stronger support for the narrow relative to the broad specifications in respect of an hypothesis which was derived specifically from the transactions theory; furthermore, a different hypothesis, one which was drawn specifically from the speculative model, was more strongly supported for the broad demand for money specifications. Although this evidence cannot be regarded as conclusive, the data did seem to be indicating that narrow money is a closer approximation to transactions balances than is broad money, and that the latter is more likely to be part of an investment portfolio than the former.

Each of the specifications was then tested for ex-post forecasting reliability, presumably a good measure of relative temporal stability, and RMSE's and plots of the forecast errors were compared. It was discovered that statistical significance of the foreign variables at the 0.95 level did not necessarily imply that the expanded model would perform better in a policy situation. Specifically, the policy implication of the present analysis for Canada is that the issue of currency substitutability effectively may be ignored. On the other hand, it was found that monetary policy in both the United States and Germany could be made more precise by incorporating the hypotheses of this Thesis into the current policy regimes. A qualification was added, however: ex-post forecasting performance is certainly no guarantee of ex-ante performance, the latter depending
crucially on the quality of the policy-maker's forecasts of the exogenous variables.

Several problems with the empirical work were noted, and these quite naturally give rise to suggestions for further research. Specifically, it was noted that Model 4 in particular was plagued with imprecision of parameter estimates, and the problem was traced to three possible sources. First, and probably most importantly, the error structures of the equations were found to be generated from high-order autoregressive or moving average (or perhaps mixed ARMA) processes, a complication which, given current software limitations, makes proper GLS estimation impossible. Second, these same residuals were found to be strongly correlated across equations, implying that SUR estimation would be a potential source of improved precision. However, the estimation proved to be too large for the computing facility at hand. Third, the correlation matrices given in Appendix A6 make it clear that each equation which contains the foreign variables suffers from multicollinearity. This problem could be approached with ridge estimation. However, all three of these suggestions for improving the precision of the estimates of Model 4 must be regarded as piecemeal, unless all were applied simultaneously. This would be a difficult task, indeed, and for the present the first problem, that of full GLS estimation, is both the most important and the most elusive of solution.

Perhaps a more fruitful avenue would be to adopt an entirely new approach, rather than attempting to 'patch up' a poorly-behaved econometric model. It would seem that the suggestion of
Zellner (1979), to combine the concepts of time series and econometric modelling, could be very helpful here. One suggestion might be to construct a time series model of each of the variables involved, and then to use the residuals of those models in an econometric model. Then there could be no question that statistical significance represents causality (running in one direction or the other) and an accurate measure of the reliability of the model could be obtained simply from the regression results.

In addition, data considerations should not be minimized. Many of the decisions which were made a priori about the data which were used could affect the results, although the priors of this author are that the effects would be minor. To erase any doubts some experiments with the alternatives would be in order.

On the theoretical front, the developments of Chapters Two, Three and Four make it clear that the demand for money equation in a multicurrency world should be specified as depending not only on domestic income, interest rates, and the expected rate of depreciation, but additionally on foreign income and interest rates as well. The implications of this addition on the body of literature which analyzes the theoretical implications of currency substitution should be considered.

One final difficulty suggests itself, its significance depending very much upon the views of the individual, so that the final word on the matter may never be written. To some, the costs of having achieved some improvement in the U.S. and German demand for money specifications may seem unduly high in terms of the complexity
of the new model which has been suggested. Surely Friedman would regard
the costs as being too high for, as he has said in his 'Restatement of
the Quantity Theory' (Friedman, 1969, 62-3):

...the quantity theorist must sharply limit,
and be prepared to specify explicitly, the
variables that it is empirically important
to include in the function. For to expand
the number of variables regarded as significant
is to empty the hypothesis of its empirical
content; there is indeed little if any difference
between asserting that the demand for money is
highly unstable and asserting that it is a
perfectly stable function of an indefinitely
large number of variables.

The purist might easily regard the complexity of Model 4 of this Thesis
as violating the principles laid down by Friedman. However, this view
would be a misconception which has been brought about by a slightly
misdirected quest for a 'stable' demand for money specification. The
latter is obviously desirable from the point of view of the policy-
maker. But it is clear that the concept of stability which both
Friedman (1969,62) and Johnson (1962,344-5) have proposed as a
requirement for an acceptable demand for money equation is that the
latter be relatively more stable than other functions, such as that
describing consumption expenditures, which are suggested as key
relationships by the income-expenditure models. Thus, an important
test to conduct would be to compare the relative stability of a simple
demand for money equation to that of an estimate of the consumption
function.

The quest for a demand for money equation which is a more
reliable tool of policy than the best which is currently available must
continue, but the need to do so must be seen as distinct from the need
to confirm one of the tenets of Monetarism.
FOOTNOTES TO CHAPTER EIGHT

1 In a theoretical analysis the assumption of purchasing power parity would probably be made, so that the terms of trade variable would disappear from the specification.
BIBLIOGRAPHY


Barro, R.J. and S. Fischer (1976), "Recent Developments in Monetary Theory", Journal of Monetary Economics, 2, 133-68.


Evans, P. and A. Laffer (1977), "Demand Substitutability Across Currencies", mimeo.


Kareken, J.H. and N. Wallace (1977), "Samuelson's Consumption-Loan Model with County-Specific Flat Moneys", Staff Report #24, Federal Reserve Bank of Minneapolis.


Newbold, P. (1975), "A Note on Relations Between Seasonally Adjusted Variables", mimeo.


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