1985

A General Equilibrium Model of 'Efficient' Unions

Peter Kuhn

Follow this and additional works at: https://ir.lib.uwo.ca/economicsresrpt
Part of the Economics Commons

Citation of this paper:
RESEARCH REPORT 8508

A GENERAL EQUILIBRIUM MODEL
OF 'EFFICIENT' UNIONS

by

Peter Kuhn

May, 1985

Abstract

Unions which are allowed to set both wages and employment are introduced into a general equilibrium model of firm formation. This allows us (a) to derive a number of relationships among equilibrium firm size, percentage of firms unionized, wages, profitability, and productivity, and (b) to examine the welfare effects of unions in a world in which unions do not have the traditional monopoly effects on employment. We find, among other things, that large firms are more likely to be unionized, unionized firms are more productive and "better managed" than union firms, and that unions reduce economic efficiency by distorting the "occupation choice" decision.

I would like to thank Glenn MacDonald and Chris Robinson, as well as participants in workshops at the NBER labor institute, Cornell University, Université de Québec à Montréal, McMaster University, and University of Western Ontario, for many helpful comments.
A GENERAL EQUILIBRIUM MODEL OF 'EFFICIENT' UNIONS

I. Introduction

Most existing models of trade unionism (for example, Dunlop (1944), Johnson and Mieszkowski (1970), Oswald (1982), Lazear (1983), Grossman (1983) and DeFina (1983)) treat unions as simple monopolists who optimize against employers' labor demand curves. Indeed, it appears that the nature, if not the magnitude, of the general-equilibrium effects of such unions on wages, employment, distribution, and efficiency are by now quite well understood.

Interestingly, although, it has long been recognized (see Leontief, 1946) that the above models implicitly assume that inefficient bargains are struck between often well-established pairs of bargainers, research based on the alternative hypothesis of "efficient" models of union behavior which allows unions to bargain over employment as well as wages has revived only recently (see for example, de Menil (1971), Hall and Lilien (1979), McDonald and Solow (1981), and McCurdy and Pencavel (1983)). To date, however, no model of the general-equilibrium effects of such unions has appeared. This paper attempts to fill that gap by introducing unions into a simple, one-sector general-equilibrium model of production and firm-formation like those presented in Tuck (1954), Lucas (1978), Kahlstrom and Laffont (1979), Rosen (1982), and Oi (1983).

The one-sector firm-formation model is chosen as the context in which to study "efficient" unions for two reasons. First, since "efficient" unions, by definition, transfer rents from firms to workers without (at the firm level) distorting the employment decisions of firms, any allocative effects they have at all must come from another source. A natural place to look is at the
supply, rather than the demand side of the labor market—i.e., at individual's choices of whether to be workers or not. This is the focal point of the firm-formation models. Indeed, when we endogenize the "supply" of firms and of workers in this way, we find that even "efficient" unions can have interesting allocative effects on employment, firm size, and firm structure that are generally not noted in the "monopoly unions" literature.

Second, including unions in a simple firm-formation model has a number of empirical implications for the incidence of unionism throughout the economy that also are not derived elsewhere. These include the prediction that large firms are more likely to be unionized than small firms—regardless of whether economies of scale in union operating costs exist—and that, comparing union and nonunion firms of the same size, the union firms will be less profitable, have better managers, and have higher measured productivity.

We proceed as follows. Section II of the paper sets up the basic model in an economy where there are no unions. Section III introduces unions into the model and characterizes the structure of equilibrium in that case, while Section IV compares allocative and efficiency aspects of the union- and nonunion equilibria in an interesting special case. Section V comments on possible extensions and alternative specifications of the model.

II. The Basic Model Without Unions

We assume an economy that lasts two periods, has no capital, and a fixed population of risk neutral individuals, which we normalize to equal one. Individuals possess endowments of two factors of production: simple labor, \( l \), and "entrepreneurial skill", \( \alpha \), which are distributed as follows. Each individual possesses one unit of simple labor, while \( \alpha \) is distributed according to the density function \( h(\alpha) \), with c.d.f. \( H(\alpha) \).
In the first, "setup" period, individuals are assumed to make an "occupation choice" decision that determines whether or not they will attempt to operate a firm (be an entrepreneur) or work for someone else in the second period, when production takes place and workers are matched with firms.

Firms in this economy are run by a single entrepreneur,\(^1\) with the production function:

\[ q = F(\alpha, n(\alpha)) \quad (1) \]

where \( n(\alpha) \) is the number of workers hired at random from the available pool by an entrepreneur of type \( \alpha \). We assume \( F \) is concave, with \( F_1 > 0, F_2 > 0, F_{12} \geq 0, F_{11} \leq 0 \) and, \( F_{22} < 0 \). The "profits", or income, of a nonunionized entrepreneur of type \( \alpha \) are given by:

\[ \pi(\alpha) = F(\alpha, n^n(\alpha)) - w^n n^n(\alpha) \quad (2) \]

where \( w^n \) is the (nonunion) wage paid to workers, and \( n^n(\alpha) \) is the number of workers hired in such a firm. Finally, to incorporate the notion that it takes time to set up a firm, but that the option of working is always available to all, we shall assume that the occupation choice decision is reversible in the second period for those who become entrepreneurs, but not for workers.\(^2\) This implies that we can define the rents, \( R(\alpha) \), earned by an entrepreneur as the difference between his income and what he could get by becoming a worker:

\[ R(\alpha) = \pi(\alpha) - w^n = F(\alpha, n^n(\alpha)) - w^n [n^n(\alpha) + 1] \quad (3) \]
which is just "profits" minus the opportunity cost of the entrepreneur's own time.

To characterize the equilibrium in the basic model without unions, we begin by noting that this involves simply finding a (nonunion) wage rate, \( w^n \), an allocation of individuals to occupations, and of workers to firms that satisfies three conditions: First, each entrepreneur chooses an \( n \), or firm size, that maximizes his profits. Second, \textit{ex ante}, all entrepreneurs must (weakly) prefer to be entrepreneurs, and similarly for workers. (This must also be true \textit{ex post} for entrepreneurs but not necessarily for workers.) Third, the number of workers hired by all the entrepreneurs must equal the total number of individuals who choose to be workers. These three conditions are presented formally in turn below; together they describe the equilibrium of the economy.

(i) Each entrepreneur's choice of optimal firm size results from maximizing profits with respect to \( n \). This yields the set of conditions:

\[
F_2[\alpha, n^*(\alpha)] = w^n \tag{4}
\]

Condition (4) yields a (nonunion) labor demand function \( N^n(\alpha, w^n) \) with the following properties:

\[
\frac{\partial n}{\partial \alpha} = - \frac{F_{21}}{F_{22}} > 0 \tag{5}
\]

\[
\frac{\partial n}{\partial w} = \frac{1}{F_{22}} < 0 \tag{6}
\]
FIGURE 1

Equilibrium (nonunion) Wages and "Occupation Choice" in the Model with and without Unions

---

model without unions

---

model with unions
and an indirect conditional profit function \( \pi(\alpha, w^n) \) with the properties:

\[
\frac{\partial \pi}{\partial \alpha} \frac{dw}{da} = F > 0
\]

(7)

\[
\frac{\partial \pi}{\partial n} < 0
\]

(8)

Thus, more able entrepreneurs hire more workers and earn greater profits; higher wages lower their labor demand and profits simultaneously.

(ii) Given the result immediately above, it is apparent that equilibrium selection into worker and entrepreneur status requires only that the marginal entrepreneur earn zero rents, that is:

\[
R(\alpha^*) = F[\alpha^*, n(\alpha^*)] - w[n(\alpha^*) + 1] = 0
\]

(9)

or

\[
\pi(\alpha^*, w^n) - w = 0.
\]

All entrepreneurs but the marginal ones (with talent \( \alpha^* \)) will earn strictly positive rents.

(iii) The "adding up" condition for our economy is simply:

\[
H(\alpha^*) = \int_{\alpha=\alpha^*}^{\infty} n(\alpha) h(\alpha) d\alpha
\]

(10)

The right-hand side adds up the labor demanded by all the entrepreneurs, which must be equal to the number of workers (the left-hand side). Condition (9) implies that an "across-the-board" fall in \( n(\alpha) \) (induced, say, by a rise in wages) would require a decrease in \( \alpha^* \), with more individuals deciding to be workers and fewer choosing entrepreneurship.
Equilibrium in this simple economy is completely characterized by equations (4), (9), and (10). We note four main aspects of this equilibrium in turn here.

First, although equation (4) actually represents a continuum of conditions, the model's equilibrium \( \alpha^* \) and \( w^n \) can in fact be expressed simply as the intersection of a "demand for workers" and "supply of workers" curve, as follows. First, imagine the continuum of solutions \( n^n(\alpha) \) obtained from (4) for all possible values of \( \alpha \) and substitute them into (10). Since each \( n^n(\alpha) \) falls with \( w^n \), this generates a downward-sloping relationship between \( w^n \) and \( \alpha^* \) which we call the "demand for workers" curve, or \( \alpha^* = D(w^n) \), \( D' < 0 \).

Second, consider the behavior of condition (9) when \( w^n \) changes; in other words, the behavior of the marginal entrepreneur's maximized rents when wages change. Profits fall with \( w^n \), so to restore the equality in (9), \( \alpha^* \) must rise when wages rise (the marginal entrepreneur will now be a more talented person, as some less able entrepreneurs have decided to become workers). Call this positive relationship between \( w^n \) and \( \alpha^* \) the "supply of workers" curve, or \( \alpha^* = S(w) \). The wage and "occupation choice", \( \alpha^* \), that equilibrates this economy must be at point a in Figure 1; because \( D' < 0 \) and \( S' > 0 \), we know this equilibrium is unique.

Second, the model has a number of implications for the distribution of wages, profits, and productivity by firm size, which are easily summarized. First, in equilibrium, all workers receive a wage, \( w^n \), which is independent of firm size. Second, because both profits and firm size increase with \( \alpha \), it is clear that larger firms earn higher profits. Although profits must increase with \( \alpha \) at a declining rate if \( F \) is concave, profits may increase with \( n^n(\alpha) \) at either an increasing or decreasing rate. Finally, imagine that we attempt to estimate production functions for the cross section of firms in this
equilibrium, but are unable to control for $\alpha$ in our estimation procedure. It is clear that such estimates could lead us to conclude, erroneously, that increasing returns to scale existed. Larger firms appear to be more "productive" than smaller firms because they have more of an unobserved factor of production, $\alpha$.

A third crucial feature of the model is the dependence of the equilibrium distribution of incomes and firm sizes on the shape of both production function and the distribution of talent. For example, a greater "complementarity" of $\alpha$ and $n$, as measured by the magnitude of $F_{12}$, implies that the distribution of firm sizes will be more unequal. Also, a change in the population's endowment of skills (say, through human capital formation) changes the equilibrium ratio of workers to entrepreneurs and, with it, the entire distribution of firm sizes.

Fourth, we note that the model generates a socially efficient outcome in the sense that the allocation of individuals to tasks and the distribution of firm sizes that results is the one that maximizes total output produced by this economy. To see this, simply maximize total output,

$$\int_{\alpha=\alpha^*}^{\infty} F(\alpha, n(\alpha)) h(\alpha) d\alpha,$$

subject to the factor supply constraint (10).

The resulting first-order conditions are the same as (4), (9) and (10) with the Lagrange multiplier on the output constraint substituting for $w^n$ as the shadow price of a worker. This outcome is also Pareto-efficient since, in this simple world, utility is derived only from the single consumption good produced.

III. Unions in the Basic Model

In this section we relax our assumption that unionization is impossible, and assume instead that workers at any given firm can form and operate a union
at a cost given by:

\[ C(n) + \theta; \theta \sim g(\theta), E(\theta) = 0, C' \geq 0. \quad (11) \]

\( C(n) \) is assumed to be the same for all firms; its structure will indicate whether or not there are economies of scale in operating unions. \( \theta \) is a firm-specific shock, and can be thought of as the outcome of a legal decision or lobbying process, which is revealed to workers once they are hired by a particular firm. The rights conferred upon workers who do unionize are the right to unilaterally set both wages and employment in the firm (this is the essence of the "efficient unions" argument), subject only to the constraint that the entrepreneur remain in business. This means that unions, when they exist, will maximize:

\[
\max_{n^U(\alpha)} R^u(\alpha) = R(\alpha) - C(n) \\
= \pi(\alpha) - \bar{w} - C(n) \\
= F(\alpha, n(\alpha)) - w n(\alpha) - \bar{w} - C(n) 
\]

where \( \bar{w} \), defined below, is the entrepreneur's opportunity wage and \( n^U(\alpha) \) is the labor demanded by a unionized firm. Workers will form unions whenever the maximized value of \( R^u(\alpha) \) exceeds the value of \( \theta \) in their firm, i.e., when the total net gains to unionization are positive.

Essentially, in our "efficient unions" model, because unions appropriate all the entrepreneur's rents, each union chooses an employment level that maximizes those rents, net of union operating costs, and "pays" the
entrepreneur his opportunity wage. Note that the shadow price of labor, from
the "efficient" union's point of view, is workers' alternative (i.e.,
nonunion) wage, $w^n$, and that, given sufficient scope for side-payments, we
expect rent-maximization, as defined above, to characterize union behavior
regardless of who actually collects those rents. Despite this, in what
follows we shall occasionally define a "union wage" in a firm of type $\alpha$, for
"accounting" purposes, under the assumption that the rents appropriated by the
union are shared equally among its members.

Equilibrium in our economy with unions must satisfy the same three
conditions as the economy without unions, i.e., (i) each firm (or union in
this case) chooses the employment level it prefers, treating $w^n$ as a
parameter, (ii) entrepreneurs must (at least weakly) prefer to remain
entrepreneurs both ex post and ex ante while all workers prefer to work
ex ante, and (iii) total labor demand equals labor supply. We develop these
conditions below.

(i) There are now two types of firms, union and nonunion, coexisting in
our economy. The labor demand function of nonunion firms, $N^n(\alpha, w^n)$,
satisfies condition (4), as before. In union firms, labor demand will satisfy
the first-order conditions for a maximum of (12), i.e.,

$$F_2 = w^n + C'$$

which implies that, unless $C' = 0$, union firms are smaller than nonunion
firms with the same $\alpha$. As long as the second-order conditions for a maximum
of (12) are satisfied (i.e., $C'' - F_{22} > 0$), condition (13) yields a labor
demand function for unionized firms, $N^u(\alpha, w^n)$ with the following properties:
\[ \frac{\partial N}{\partial \alpha} = \frac{F}{C'' - F} > 0 \quad (14) \]

\[ \frac{\partial N}{\partial w} = \frac{-1}{C'' - F} < 0 \quad (15) \]

and allows us to characterize maximized net rents, \( R^u(\alpha, w^n) \), accruing to the union as:

\[ \frac{\partial R}{\partial \alpha} = \frac{F}{1} > 0 \quad (16) \]

\[ \frac{\partial R}{\partial w} < 0 \quad (17) \]

Thus, union firms' labor demand curves are also negatively inclined, while a higher opportunity cost of labor, \( w^n \), reduces the rents unions earn. Both employment and total maximized rents increase with the talent of the unionized entrepreneur.

(ii) Because the profits of nonunionized entrepreneurs increase with \( \alpha \) while the profits (i.e., income) of unionized entrepreneurs are set, by unions, equal to the expected wage of a worker (plus epsilon), equilibrium "occupational choice" in the model with unions again entails a critical \( \alpha^* \), above which everyone becomes an entrepreneur. Since marginal entrepreneurs in this world will never be unionized (they are indifferent between working and managing and so earn no rents unions can extract) the "occupation choice" condition is simply:
\[ \pi(a^k, w^R) - \tilde{w} = 0 \]  

(18)

where \( \tilde{w} \) is the expected wage of a worker, incorporating the probability of getting a union job and the rents that can be extracted from employers in that case.

To define \( \tilde{w} \), we note that the total (net) rents extracted by unions from all type-\( a \) firms is:

\[
\int_{\theta=0}^{u} \frac{R(\alpha)}{H(\alpha)} \frac{u}{(R(\alpha) - \theta) h(\alpha)} d\theta \]  

(19)

and, because jobs are allocated randomly and workers are risk neutral, the expected wage of a worker (before he knows which firm he will be matched with) is just \( w^n \), plus the total rents extracted by unions from all firms divided by the number of workers, \( H(a^*) \), i.e.:

\[
\tilde{w} = w^n + \frac{1}{H(a^*)} \int_{\alpha=a^*}^{u} \frac{R(\alpha)}{(R(\alpha) - \theta) h(\alpha)} d\theta d\alpha \]  

(20)

Thus (20) becomes:

\[
\pi(a^k, w) - w = \frac{1}{H(a^*)} \int_{\alpha=a^*}^{u} \frac{R(\alpha)}{(R(\alpha) - \theta) h(\alpha)} d\theta d\alpha \]  

(21)

(iii) A firm in our economy will be unionized whenever \( R^u(\alpha) > \theta \), which implies that the proportion of type-\( a \) firms which are unionized is just:

\[ p(\alpha) = G(R^u(\alpha)) \]  

(22)
where \( G \) is the c.d.f. corresponding to \( g \). Our adding-up condition is then:

\[
H(\alpha^*) = \int_{\alpha=\alpha^*}^{\infty} \left\{ G(R(\alpha)) \ n(\alpha) + [1-G(R(\alpha))] \ n(\alpha) \right\} h(\alpha) \ d\alpha \quad (23)
\]

Equilibrium in the economy with unions is completely characterized by equations (4), (13), (21), and (23), and can be thought of in the same way as the nonunion equilibrium. Substitution of the firm-level labor demands (4) and (13) into the adding-up condition (23) yields a "demand-for-workers" curve relating \( \alpha^* \) and \( w^n \) (no longer necessarily monotonically decreasing). The occupation-choice condition (21) yields a "supply of workers" curve in \( \alpha^* \) and \( w^n \) (no longer necessarily monotonically increasing). Equilibrium is at the intersections of those curves. It is analyzed below in two stages: First, some brief comments are made about the way the equilibrium values of variables like \( \alpha^* \), \( w^n \), and \( \tilde{w} \) will differ from the nonunion equilibrium, although this question is examined in more detail, using a specific example, in Section IV below. Second, the structure of the equilibrium distribution of unionism, wage rates, profits, and productivity in the union equilibrium is described in detail.

1. **Comparison of Union and Nonunion Equilibria**

Table 1 lists the equilibrium conditions of the union and nonunion models in a form that allows for easy comparison. Consider first condition (1) which indicates that, for any given \( w^n \), \( n^u(\alpha) \) is below \( n^n(\alpha) \). When this is substituted into condition (3) to obtain the "demand for workers" curve, it is apparent that the integrand on the right of (3), for any given \( w^n \), is lower in the union than in the nonunion equation, thus necessitating a lower \( \alpha^* \).

In other words, regardless of the shape of \( D(w^n) \) in the union equilibrium, it must always lie to the left of \( D(w^n) \) in the nonunion equilibrium, as shown in Figure 1.
TABLE 1
A Comparison of the Model With and Without Unions

<table>
<thead>
<tr>
<th>Condition</th>
<th>Without Unions</th>
<th>With Unions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Demand for Labor in each firm</td>
<td>( n^n(\alpha) = N^n(\alpha, w^n) )</td>
<td>( n^n(\alpha) = N^n(\alpha, w^n) )</td>
</tr>
<tr>
<td></td>
<td>( n^u(\alpha) = N^u(\alpha, w^n) )</td>
<td>( n^u(\alpha) = N^u(\alpha, w^n) )</td>
</tr>
<tr>
<td>2. Zero rents of marginal entrepreneurs</td>
<td>( \pi(\alpha^*, w^n) - w^n = 0 )</td>
<td>( \pi(\alpha^*, w^n) - \tilde{w} = 0 )</td>
</tr>
<tr>
<td></td>
<td>( \pi(\alpha^<em>, w^n) - w^n = \frac{1}{H(\alpha^</em>)} \int_{\alpha=\alpha^*}^{\infty} \int_{\theta=0}^{R^u(\alpha, w^n)} (R^u(\alpha, w^n) - \theta) h(\alpha) , d\theta , d\alpha = 0 )</td>
<td>or ( \pi(\alpha^<em>, w^n) - w^n = \frac{1}{H(\alpha^</em>)} \int_{\alpha=\alpha^*}^{\infty} \int_{\theta=0}^{R^u(\alpha, w^n)} (R^u(\alpha, w^n) - \theta) h(\alpha) , d\theta , d\alpha = 0 )</td>
</tr>
<tr>
<td>3. &quot;Adding up&quot; condition</td>
<td>( H(\alpha^<em>) = \int_{\alpha=\alpha^</em>}^{\infty} n^n(\alpha) h(\alpha) , d\alpha )</td>
<td>( H(\alpha^<em>) = \int_{\alpha=\alpha^</em>}^{\infty} \left[ G(R^u(\alpha, w^n)) n^u(\alpha) + [1 - G(R^u(\alpha, w^n))] n^n(\alpha) \right] h(\alpha) , d\alpha )</td>
</tr>
</tbody>
</table>
Consider now condition (2). It is easily seen here that, since \( \bar{w} > w^n \), for any given \( w^n \), \( x^* \) along the supply of workers curve must be greater than before. Because workers now have a chance at earning union rents, equilibrium "occupation choice" at any \( w^n \) requires the marginal entrepreneur to be more able. Thus \( S(w^n) \) for the union equilibrium is everywhere to the right of its previous position, as shown in Fig. 1.

The new \( D(w^n) \) and \( S(w^n) \) can only intersect in the shaded region of Fig. 1 at a point like (b); thus we know unambiguously that \( w^n \) falls as a result of unionization in the economy. Nonunion workers lose as a result of unionization, while firms that remain nonunion gain and expand in size as a result of unionization (from \( \frac{\partial n}{\partial w} > 0, \frac{\partial n}{\partial w} > 0 \)). The proportion of the population who are workers, \( x^* \), may, however, rise or fall, depending on whether the reduction in demand for workers outweighs the increase in supply, and it is also easily seen that \( \bar{w} \), the expected wage of a worker in the union equilibrium, may be above or below the nonunion wage that would prevail in the absence of unions. Conditions conducive to a drop in \( \bar{w} \) below the original \( w^n \) include a large drop in demand for workers, a small increase in supply, inelastic demand, and elastic supply.

The reason why the possibility of unionization can make workers worse off in an expected sense can be understood as follows. We are in a world in which individuals who are planning to become workers may be able, once a firm is set up, to expropriate the rents it produces. Furthermore, they cannot making binding commitments to potential entrepreneurs not to do so, should the occasion rise. This inability to bind oneself (in essence, to sign a "yellow
dog" contract\(^9\)) can make workers worse off by lowering the supply of entrepreneurs and the demand for labor than if they had no access to a unionization technology in the first place.

Finally, it is interesting to note that the equilibrium with unions will not in general be Pareto-efficient since it does not solve the output-maximization problem described at the end of Section II. There are three distinct reasons for this. First, equilibrium selection of individuals into "occupations", i.e., \(\alpha^*\), will not in general be efficient. This is the "new" source of allocative inefficiency noted in this paper. Second, because of variable operating costs of unions, otherwise identical union and nonunion firms will choose different employment levels--a demand-side effect very similar to that generated in "monopoly" models of unions which typically ignore union operating costs. Third, if \(C(n)\) and \(\theta\) do in fact involve resource costs rather than, for example, simple lump-sum bribes to other agents in the economy, those resources are directly wasted in the model; this could be thought of as a loss due to "rent-seeking".

2. Structure of the Union Equilibrium

We consider first how wages, profits, productivity and union incidence vary with firm size in the model, and then draw comparisons between union and nonunion firms of the same size.

Among nonunion firms, as before, all workers get the same wage, \(\bar{w}^n\), while profits, \(\alpha\), and firm size all increase together. Because \(n(\alpha)\) increases with \(\alpha\), larger firms will appear to be more "productive". Among union firms, it is also easy to see (from 14 and 16) that higher \(\alpha\) implies both higher maximized net union rents, \(R^u\), larger firms, and higher apparent productivity. Profits are invariant across union firms (all unionized entrepreneurs receive \(\bar{w}\)) while (net) union wages vary across firms according to:
\[
w(a, \theta) = w + \frac{u}{n(a)} \left[ R(a) - E(\theta | \theta < R(a)) \right]
\] (25)

which involves the mean of the upper-truncated distribution of \( \theta \),
\[E(\theta | \theta < R(\theta)).\]

Contrary to a number of arguments which invoke rent-sharing by firms
with workers as a possible reason for the observed positive firm-size wage
relationship (see for example Mellow, 1982), equation (26) indicates that, for
the case of rent-sharing due to unions, there is no strong reason to expect
union wages to increase with firm size except among the smallest firms in the
economy. This occurs despite the fact that there are more total rents to take
from large firms \( R(\theta) \) increases with \( a \) because \( \theta \) rents per worker,
\[
\frac{u}{R(a)} \text{ increases with } a \text{ only if the elasticity of rents with respect to } a,
\]
\[
\frac{\alpha dR}{u} \frac{d\theta}{R} \text{, exceeds the elasticity of labor demand with respect to } a,
\]
\[ u \ \frac{\alpha}{dn} - u \ \frac{\alpha}{da} \], and (b) as firm size increases, so does the mean cost of
unionization for those firms that are unionized \((E(\theta|\theta < R^U(\alpha))\) rises with \(\alpha\). Only when \(R^U(\alpha) = 0\) (in the marginal firm) can we be certain that the
effect of increasing \(R^U(\alpha)\) predominates, in which case the union wage (and
hence the union-nonunion differential) increase with firm size.

Since the proportion of firms unionized, \(p(\alpha) = G(R^U(\alpha))\), depends only
on total net rents our model predicts that this will increase with firm size,
as Freeman and Medoff (1983, p. 33) have in fact shown. It should be
emphasized that this is true in the present model, regardless of whether
economies of scale in union operating costs exist, as long as an internal
solution occurs in (13).

Finally, comparing union and nonunion firms of the same size (i.e.,
comparing union and union firms with identical measured characteristics in
this model) we note that, because union firms are smaller for the same \(\alpha\),
union firms will have more talented entrepreneurs (higher \(\alpha\)) than nonunion
entrepreneurs of the same size. Thus we expect lower profits but higher
productivity in union firms (see, for example, Brown and Medoff, 1979, and
Ruback and Zimmerman, 1984, for evidence consistent with this prediction).

IV. Union vs Nonunion Equilibria: A Simple Example

Most monopoly union models ignore the effects of variable union
operating costs. It is instructive to do the same in the present model, in
order to highlight the basic differences between this model and others. The
case of zero variable union operating costs \((C'(n) \equiv 0)\) is also a useful
"benchmark" case in which it is easy to characterize the effect of
unionization in the economy on equilibrium values of \(\alpha^*, w^*, \tilde{w}, \) etc.
When \( C'(n) \equiv 0 \), union and nonunion firms will have the same labor demand function, \( N^u(\alpha, w^n) \). This implies that Conditions (1) and (3) in Table 1 are the same in both the union and nonunion models, and hence that the "demand for workers" curve, \( D(w^n) \), is unaffected by the presence of unions in the economy. \( S(w^n) \) is still shifted right by the presence of unions, so the equilibrium will be at point (c). Thus, in the absence of union operating costs (and of other factors like union monopoly behavior that make union firms smaller than comparable nonunion firms) we know not only that the effect of unionism on the economy is to lower \( w^n \), but also, unambiguously, to increase the proportion of the population who choose to be workers rather than entrepreneurs. This is because the only effect of unionism is to make it more attractive to be a worker. The particular individuals who change "occupations" as a result of unionization can in fact be singled out—they are the people who ran the smallest firms in the economy before unionization was introduced.

Other consequences of unionization are, as before, that each surviving firm becomes larger (this must occur if there are more workers and fewer firms) and, if nonunion, increases its profits. Since \( \alpha^* \) rises and \( w^n \) falls we can now also say something unambiguous about \( \bar{w} \)--the expected wage of a worker: it rises. To see this, recall that \( \bar{w} \) equals the profits of the marginal entrepreneur (who we know is nonunion). This entrepreneur is a more talented individual than before and faces a lower \( w^n \), which implies that his profits (i.e., income) must be higher. Thus the possibility that the expected wage of a worker can fall as a result of unionization occurs only when variable union operating costs (or, incidentally, monopoly effects that make union firms smaller) are introduced into the model. Our example also highlights very clearly the additional allocative effect of unions that occurs
in firm-formation models: unions make it more attractive to work and less attractive to manage, and thus in the absence of countervailing monopoly effects on labor demand will lower entrepreneurship levels and increase the worker/firm ratio above the socially efficient level.

V. Alternative Formulations

This section briefly comments on some interesting extensions to the present model, as well as the effects of altering some key assumptions used in the modelling process.

Additional factors, such as capital, are easily introduced into the basic model and are indeed present in Lucas (1978) and Oi (1983). When capital is fully mobile ex post, the main changes that occur with unions are that (9) the price of capital may rise or fall as a result of unionization; it may even bear the entire “burden” of unionization so the nonunion wage rises when unionism is introduced, and (b) if capital and skill (i.e., entrepreneurship in this context) are complements, as has often been suggested, the model predicts higher capital intensity in larger firms than smaller ones, and in union versus nonunion firms of the same size. If capital is hostage to the union ex post, more serious consequences ensue (see, for example, Grout, 1984).

Alternative models of union behavior, such as the familiar monopoly model, are easily incorporated into the same firm-formation model. Monopoly effects are very similar to variable costs of union operation.

Individuals may be endowed with varying amounts of simple labor, l. In that case, the opportunity wage varies across entrepreneurs, and if better entrepreneurs are, on average, better workers too, our result that there are always more rents to be extracted from larger firms can be reversed (except among firms close to the margin). The same applies if heterogeneity in the
value of leisure exists and individuals can drop out of the market \textit{ex post} in our model (if better entrepreneurs are more productive in the home, there may be less to take from them), or if the variable $\theta$ is reinterpreted as an entrepreneur's ability to "fight" unions and it is positively correlated with $\alpha$.

Exactly what managers do in the production process could be specified more explicitly, as in Rosen (1982) and Oi (1983). For example, Oi has shown that, if managers divide their fixed budget of time between "decision making" and monitoring of workers, then the higher opportunity costs of monitoring for more skilled entrepreneurs imply they will hire fewer workers, but more skilled workers. If union firms are run by more skilled entrepreneurs, the same prediction should apply to union/nonunion comparisons.

A final, more fundamental question concerns the nature of property rights to union jobs--or, more specifically, to rents accruing from unionization. By assumption, the present model gives each individual a chance at getting some of those rents if he or she elects to become a worker--indeed the model is \textit{designed} to look at the effects of workers' ability to extract these rents on the entrepreneur decision. As an alternative assumption, we can imagine allocating property rights to union rents to a third party--for example a union "boss" or a group of individuals who were all workers in the old, nonunion equilibrium. In this case, while the structural features of the union equilibrium (i.e., the cross-sectional relationships between firm size, profits, wages, productivity, etc.) remain the same, unions may have \textit{no} allocative effects at all. In fact, if $C' \equiv 0$, then because the opportunity wage of an entrepreneur is $w^N$ in this world, unionization is compatible with a fully Pareto-optimal allocation of resources.
1 This is a result of the model if we assume global concavity of the production function plus indivisibility of the entrepreneur's own $\alpha$ (he or she cannot run many small firms simultaneously). If the production function were not concave everywhere, due, for example to public-goods aspects of managerial decision making as Rosen (1982) has suggested, a more elaborate analysis, which is beyond the scope of the present paper, is required.

2 This is actually of no consequence in the basic model without unions (all will initially "decide" to be an entrepreneur and some will change their minds later) and only becomes relevant in the union model when individuals decide to become workers based on expected wages, but some are "unlucky" and drawn nonunion jobs which are less attractive than entrepreneurship for those workers.

3 To see both of these propositions, note that $$\frac{d^2 \pi}{d\alpha} = \frac{F}{11} + F \frac{dn}{12} \text{d}$$

$$\frac{\text{F}}{12}$$

$$\frac{\text{F}}{11}$$

$$\frac{\text{F}}{22}$$

which is negative if the production function is concave, and

$$\frac{d\pi}{dn} = - \frac{\frac{\text{F}}{12}}{\frac{\text{F}}{22}} > 0.$$  

4 The relationship between labor input and output across firms is given by:

$$\frac{dF}{dn} = F \frac{d\alpha}{1dn} + F = F - F \frac{22}{2} - F \frac{1}{12}$$

from which it is easy to see that $$\frac{d^2 F}{dn}$$
may be positive even with $F_{22} < 0$.

5 The assumption of uncertainty in $\theta$ is largely for cosmetic purposes— if $\theta$ were nonrandom, the relationship between firm size and percent unionized would be a step function, with all firms above a certain size unionized, rather than a smoothly increasing one as we develop here.

6 Nothing of substance in the model changes if unionized entrepreneurs, due to bargaining considerations, earn strictly positive rents, as long as the bargaining solution between firms and unions is on the "contract curve", as is maintained by the "efficient unions" hypothesis.

7 Since, by assumption, workers who do not receive union jobs have no alternative, ex post but to be a nonunion worker.

8 More precisely, because individuals care only about their expected wage as a worker (or entrepreneur) in our model, whether the rents accrue to a small minority of union members, to everyone in the union firms, or even are redistributed to workers in nonunion firms by an economy-wide union, is immaterial to the solution of the model. If, however, rents go to some third party, the results do change (see Section V).

9 "Yellow-dog contracts" are contracts signed by workers in which they promise not to form a union as a precondition of employment. These were declared unenforceable in a court of law in the Norris-La Guardia Act of 1932 and made illegal in the Wagner Act of 1935.

10 With the notable exception of MacDonald and Robinson (1985).
REFERENCES


Oi, W., "Heterogeneous Firms and the Organization of Production," *Economic Inquiry*, April 1983.


