Endogenous Price Fluctuations on Incomplete Markets

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1. **INTRODUCTION**

This paper will present an equilibrium model for a pure exchange
economy, in which the optimizing behavior of a finite number of agents will
determine current prices of a finite number of commodities and also current
prices of a finite number of financial assets with state-contingent returns
next period. The system of markets need not be complete in the sense of the
extended Arrow-Debreu-model (see Debreu (1959); or Arrow (1964)), and there is
no restriction on short sales.

However, the optimizing behavior of the agents depends upon their
individual expectations with respect to the future state of the world.
Information on the future state of the environment is provided by signals,
which agents receive individually each period, but also from past prices, or
more exactly, from past price functions. One can imagine that last period's
prices have an influence on the informational content of the signal, an agent
receives currently. Many observations of last periods' prices and current
signals probably allow some conclusion on the common distribution of past
price functions, current individual signals, and the current signals of the
other agents. Together with an idea on the common distribution of current
signals and future states of the world, current signals and last periods' 
price functions provide information on the relevant future states of the
world. Observe that this kind of rationalization is formally similar to that
usually applied to motivate rational expectation models (see Grossman
(1981)). These similarities, but also the differences, induced by the dynamic
structure of our model, will be discussed later in the paper. An equilibrium
here consists of a finite, closed sequence of price functions where each price function of the sequence refers to a certain period of time, and relates vectors of information signals to Radner equilibrium prices. The link between the price functions for the various periods is provided by the information structure: As outlined above, agents base their expectations with respect to the future state of the world on current signals and last periods' price functions.

It is an important feature of the model that the only link between the various periods of time is given by these assumptions on the information structure of the economy. Especially, there are no exogenous disturbances by e.g. changing initial endowments, or by utility functions, which are also dependent on the current state of the world. In addition to this, there is no ad hoc mechanism, governing the adjustment of expectations. Agents are assumed to make use of all the information, generated endogenously by the development of the economy.

These certainly extreme assumptions allow the consideration of a model, where price fluctuations, especially on asset markets, are only due to endogenously generated changes in individual information. And that is the main concern of the paper: To show that excessive price variability on asset markets can be caused by endogenous changes in expectations alone. The equilibrium concept, applied in this model, is suitable for an investigation of this idea.

In reality, of course, other factors can and will be responsible for fluctuations in market prices. There have been several attempted explanations of the comparably high degree of variability of financial asset prices,
among them a high degree of risk aversion exhibited by agents (see Grossman and Shiller (1981)), fluctuations in non-capital income or changes in the cost of producing new capital (see Huffman (1986)), or dependence of asset prices upon present and expected future realizations of factors in the environment (see Huberman (1984), Huffman (1985)). Observe that in contrast to the approach chosen in this paper, all these models explain price fluctuations by exogenous factors.

In addition to that, models dealing with the problem of arbitrage pricing of contingent claims, are usually either based on a perfect foresight approach, or, for applications, assume an exogenously given distribution of asset prices (compare e.g. the approach of Black and Scholes (1973), where stock prices are distributed according to a geometric Brownian motion; see also Harrison and Kreps (1979) for the general problem of arbitrage pricing). Again, there is no endogenized generation of price fluctuations.

There is another, quite interesting point with respect to short-term fluctuations of asset prices and also prices of other commodities: It often happens that after a period of excessive variability, prices suddenly settle down to a certain level. The present model provides the basis for one possible explanation of this phenomenon: The central idea is that agents can learn from price fluctuations. Assume e.g. that the economy is in a state of equilibrium, given by a closed sequence of equilibrium price functions, as explained above. Sophisticated agents will take into account this cyclical behavior of price functions behind the fluctuating actual prices, and, under certain circumstances, will be able to derive additional information on the future development of the economy. Of course, a change in the equilibrium
structure and even a breakdown of the price fluctuations may result from these behavioral changes. This issue will be discussed in more detail in a separate paper.

The remainder of this paper is as follows: Section 2 presents a detailed description of the model, including the information structure of the economy. The central concept of an 'equilibrium allowing for endogenously generated price fluctuations' is introduced in Section 3. This section also explains the relationship between this type of equilibrium and a rational expectations equilibrium. The problem of existence of such an equilibrium is then investigated in Section 4, including an analysis of the existence results. Special attention is also paid to the existence problem for Radner equilibria. Observe that the agents usually have asymmetric information with respect to the future development of the economy. Together with no restrictions on short sales of financial assets, this can cause problems for the existence of an equilibrium (see Radner (1972) or Werner (1986)). An extensive example in Section 5 illustrates the relevant parts of the preceding sections, and is followed by some concluding remarks.

2. THE MODEL

Our pure-exchange economy is characterized by a sequence of markets, each extending over two periods: At each date there are spot markets for a finite number \( I \) of commodities; furthermore, there are markets for a finite number \( N \) of financial assets with state contingent returns next period. The set of states of the world for each period of time, \( \Omega = \{ w_1, \ldots, w_n \} \), is
assumed to be finite. \( r(w) = (r_1(w), \ldots, r_N(w)) \in \mathbb{R}_+^N \) denotes the vector of returns of the financial assets, if \( w \in \Omega \) is the relevant state of the environment.

Aggregate supply of these financial assets is either positive or zero, and there are no restrictions on short sales of all or some of the assets. These assumptions especially allow the consideration of contingent claims, assets with returns depending on the returns of other assets. Clearly, this system of financial markets need not be complete in the sense of the extended Arrow-Debreu-model (compare Debreu (1959); Arrow (1964)). Inefficiency of equilibria can be a consequence of this incompleteness (compare e.g. Hart (1975)). In addition, no restrictions on short sales of financial assets may give rise to problems with the existence of equilibria, as the consumption set of the agents is no longer bounded below. See Radner (1972) and Werner (1986) for a consideration of this problem. We have to come back to this problem, too.

Our exchange economy further contains a finite number of agents, \( h=1, \ldots, H \). Trader \( h \) is characterized by the utility function \( u^h(x^1, I^2, w^2) \), depending on consumption \( x^1 \in \mathbb{R}_+^1 \) in the respective current period. Utility furthermore depends on income \( I^2 \in \mathbb{R}_+ \) and the state of the world \( w^2 \in \Omega \) in the next period of time. Observe that trader \( h \) is uncertain about future states of the environment! \( u^h(\cdot; \cdot, w^2): \mathbb{R}_+^1 \times \mathbb{R}_+ \to \mathbb{R} \) is assumed to be continuous, strictly monotonous and strictly quasi-concave for each \( w^2 \in \Omega \). Thus, the induced preference orderings on the consumption set \( \mathbb{R}_+^1 \times \mathbb{R}_+ \) are continuous, monotonous, and strictly convex. \( (e, \theta) \in \mathbb{R}_+^1 \times \mathbb{R}_+^N \) denotes the endowment of trader \( h \) with respect to commodities and financial
assets in the current period of time. We assume that \( \sum_{h=1}^{H} e^{-h} \gg 0 \).

Observe that there is no assumption on \( \theta_h \), \( h=1,\ldots,H \); especially, \( \theta_0 \) can be the zero vector in \( \mathbb{R}^W \).

For a complete description of the agents' behavior we have to introduce the information structure of the economy. As already mentioned, agents are uncertain about the state of the world in the respective next period of time. But agents receive certain signals \( y^h \in Y \subseteq \mathbb{R}^k \), \( h = 1,\ldots,H \), in each period of time, giving them some information about the future state of the world. The set \( Y \subseteq \mathbb{R}^k \) is assumed to be compact, there is no assumption on the dimension \( k \in \mathbb{N} \).

The central point however, is that agents are assumed to extract additional information on the development of the economy from last periods' price functions. Observe that prices on the various markets will certainly depend on the information signals \( (y^1,\ldots,y^H) \in Y^H \), which the agents receive in the period in question. Thus, a price function is in general a mapping

\[
p = (p,\pi): Y \rightarrow \mathbb{R}^+ \times \mathbb{R}^{1+H-1},
\]

associating commodity prices \( p(y) \) and asset prices \( \pi(y) \) with any signal vector \( y = (y^1,\ldots,y^H) \in Y^H \). Commodity 1 will be treated as a numeraire in this context.

The notion that current prices can convey information has been extensively used in the theory of rational expectations equilibria (see e.g. Jordan and Radner (1982) for a survey of the relevant literature). The basic idea is that in a market for commodities whose future utility is uncertain, the equilibrium prices will reflect the information and beliefs that the
traders bring to the market, as well as their tastes and endowments. If the traders have different non-price information, this situation presents an opportunity for each trader to make inferences from the market prices about other traders' information. The term rational expectations equilibrium is then usually applied to a model of market equilibrium that takes account of this potential informational feedback from market prices (compare) Jordan and Radner (1982), or Radner (1979).

The dynamic approach chosen here requires a different formulation of the information structure. The basic assumption is that there is a subjective or objective relationship between last periods' price functions and current information signals that the agents receive. If traders have then observed many realizations of prices and signals they should be able to recognize determinants of a common distribution of last periods' price functions, current own signals and current signals received by the other agents in the following way: By observing price functions and signals for each period of time, they can derive additional information on the other agents' signals by means of partially or fully revealing price functions (see Grossman (1981)). Given their own signal and the price function of the last period, they finally get information on the signals of all other agents. Thus, agents learn about the future state of the world, if we assume in addition the usual relationship between signals and states of the world.

One can also think of a situation, where current signals of the agents are modified in view of last periods' prices. In this case last periods' prices have a direct influence on how the agents interpret their signals, e.g. in a more optimistic or more pessimistic way. We then obtain an immediate
relationship between last periods' price function, current individual signals, and future states of the world.

The following formal presentation allows for both interpretations. Of course, it is possible to consider some kind of rational expectations equilibria in this framework, in view of the above assumptions on the information structure. This issue will be discussed in the next section, after the formal definition of an equilibrium price sequence.

In order to formalize the approach outlined above, assume that agent \( h, h = 1, \ldots, H \), acts according to a subjective or objective probability distribution \( \psi^h \) of \((y^h, p, w, \cdot) \in Y \times F_0(Y^H, R^{l+N-1}_+) \times \Omega \), hence of current signals, past price functions and future states of the world; price functions \( p \in F(Y^N, R^N_+) \) are assumed to be bounded, \( \psi^h: Y \times F_0(Y^H, R^{l+N-1}_+) \rightarrow \mathcal{M}(\Omega) \) determines the conditional distribution of \( w \in \Omega \) for agent \( h \), given \((y^h, p) \in Y \times F_0(Y^H, R^{l+N-1}_+) \); thereby \( \mathcal{M}(\Omega) \) denotes the set of all probability measures on \( \Omega \). Taking all agents into account, one obtains a mapping \( \psi: Y^H \times F_0(Y^H, R^{l+N-1}_+) \rightarrow \mathcal{M}(\Omega) \), completely describing the information structure of the economy.

Observe that this information structure, relating past prices and current information signals to future realizations of the state of the world, provides the only link between the various market dates. Especially, there is no assumption on an exogenous, stochastic development of the environment, in contrast to many papers dealing with fluctuations in asset prices (compare e.g. Huffman (1985) and the there given references). Furthermore, because of the fixed initial endowments, and because of the assumption that assets live only for two periods, fluctuations in asset prices cannot result from the
specification or a changing distribution of endowments. Fluctuations of asset prices can only result from unstable endowments. To show the existence of such endogenously generated price fluctuations is the main concern of this paper.

We can now complete the description of the behavior of the agents after this extensive discussion of the information structure of the economy. We assume that trader $h$ maximizes expected utility with respect to $w^2 \in \Omega$, given $(y^h, p^o) \in \mathcal{Y} \times F_b(Y^H, \mathbb{R}^{\ell+N-1}_+)$, i.e. current information signal $y^h$ and past price function $p^o$. Of course, trader $h$ has to observe additional constraints, so that the complete maximization problem reads as follows: Assume that the past price function $p^o \in F_b(Y^H, \mathbb{R}^{\ell+N-1}_+)$, current information signal $y^h \in \mathcal{Y}$, and current prices $p = (p, \pi) \in \mathbb{R}_+^\ell \times \mathbb{R}_+^N$ are given.

Then, agent $h$ maximizes $E[u(x; I, w) | \psi(y, p)]$ such that $p \cdot x^1 + 1 - \pi \cdot \theta \leq p \cdot e + \pi \cdot \theta$, and $I(w^2) = r(w^2) \cdot \theta^1 - \theta^2$; $(x^1, \theta)$ is the then resulting bundle of commodities and financial assets for trader $h$ in consideration. In order to rule out problems with bankruptcy, we assume that only such $\theta^1 \in \mathbb{R}^N$ are relevant, for which $r(w^2) \cdot \theta^1 \geq 0$ for all $w^2 \in \Omega$. Furthermore, we assume that the above maximization problem always has a solution. This can e.g. be guaranteed by choosing the utility function $u^h$ to be strictly concave in $x^1$ and $I^2$ for each $w^2 \in \Omega$. Compare Werner (1986), chapter 3, for a related approach.

It should be mentioned here that the above model can be modified with respect to some minor points. Instead of relating the information structure to past price functions, it should be possible, following the lines of
Jordan (1982), to base the information on actually observed past prices. Moreover, one can think of utility functions, depending explicitly on consumption bundles in the future period. This would, however, only unnecessarily complicate the model without providing additional insight at the moment (compare also the example given in Section 5).

The next section contains the discussion of equilibria, giving rise to endogenously generated price fluctuations.

3. AN EQUILIBRIUM CONCEPT FOR ENDOGENOUSLY GENERATED PRICE FLUCTUATIONS

The following equilibrium concept is based on the notion of a Radner equilibrium with asymmetric information. We recall this definition appropriately adjusted to our framework (see also Radner (1972), Werner (1986)):

**Definition 3.1:** For the model outlined in the preceding section assume that trader h's, $h = 1, \ldots, H$, expectations with respect to next period's state of the world are given by a probability distribution $\psi^h \in M(\Omega)$.

A Radner equilibrium with asymmetric information is then given by a price system $p = (p, \pi) \in \mathbb{R}_+^{1} \times \mathbb{R}_+^{N}$ and an allocation $(x^h, \theta^h), h = 1, \ldots, H$ such that:

(i) For all $h = 1, \ldots, H$, $(x^h, \theta^h)$ belongs to the budget set $B^h(p, \pi) = \left[ x^h \geq 0, \quad x^h \cdot \pi^h \leq p^h \cdot \pi^h \right. \left. + \theta^h \right]$

\[
\{ (x^h, \theta^h) \in \mathbb{R}_+^1 \times \mathbb{R}_+^N : p^h \cdot x^h + \pi^h \cdot \theta^h \leq p^h \cdot \pi^h + \theta^h \text{ and } r(w) \theta^h \}
\]
$\geq 0$ for all $w \in \Omega$, and there is no $(x, \theta) \in B(p, \pi)$ such that

$$E[u(x; I, w)|\psi] > E[u(x; I, w)|\psi].$$

(ii) $H \sim_{hl} H - h \sim_{hl} H - h$

$$= \sum_{h=1}^{H-1} e_h ; \sum_{h=1}^{H} \theta_h = \sum_{h=1}^{H} \theta_h.$$

It should be pointed out that the existence of a Radner equilibrium with asymmetric information is in general not guaranteed. The reason is that non-overlapping expectations with respect to future returns of the financial assets can give rise to arbitrage opportunities at any price $(p, \pi) \in \mathbb{R}_+^{l-1} \times \mathbb{R}_+^N$. Given unrestricted short sales of financial assets, unbounded demand and unbounded offers at any price would not allow the existence of a Radner equilibrium (compare Werner (1986) with respect to the problem of arbitrage possibilities; see also Hart (1975) for non-existence of a Radner equilibrium). Due to the possible incompleteness of the considered market system, the allocation of commodities resulting from a Radner-equilibrium, is not necessarily Pareto-efficient. This has implications for the equilibrium concept to be introduced below. See again (Hart 1975) and Werner (1986) for more details on the efficiency issue.

We are now in the position to introduce the concept of an equilibrium allowing for endogenously generated price fluctuations:

**Definition 3.2:** An equilibrium allowing for endogenously generated price fluctuations, or an 'equilibrium price sequence' is given by a finite sequence of price functions $p_1^*, \ldots, p_m^*$, all contained in $F_d(\gamma^H, \mathbb{R}_+^{l+N-1})$ such that:
(i) For all \( s = 1, \ldots, m \) and for all \( y = (y^1, \ldots, y^H)^T \in \mathcal{Y}^H \): \( p_{s+1}^*(y) \) is a Radner-equilibrium given the individual probability distributions of the next period's state of the world by: \( \psi^h(y^h, p_s^*) \in \mathcal{M}(\Omega), h=1, \ldots, H \).

(ii) The sequence \( p_1^*, \ldots, p_m^* \) is closed in the sense that \( p_{m+1}^* = p_1^* \).

Observe that the required finiteness of the length of the cycle of equilibrium price functions in the above definition indicates some kind of a stability property of this equilibrium concept: This equilibrium describes a situation which can, in principle, be repeated on the same level; a typical equilibrium property!

On the other side, however, if agents have observed many realizations of this equilibrium cycle, i.e. they have observed many realizations of individual signals combined with prices and realized states of the world, one can imagine that sophisticated traders 'learn' from this cyclical behavior of price functions. Assume that they can derive relevant additional information from this observation, and that they take into account this additional information in their utility maximization problem. Prices may then 'leave' the given cycle and settle down in a different equilibrium in a more or less stable way. This breaking down of price fluctuations is a frequent observation with respect to asset prices, but also with respect to prices of other commodities. One idea behind this observation, namely the above mentioned 'learning from price fluctuations', will be exploited in a further paper.

Of course, it can happen that the equilibrium price sequence consists of a single price function. From a purely formal point of view, this price function is then a rational expectations equilibrium, although there is a
difference with respect to the economic interpretation of both equilibrium concepts, mainly due to the dynamic framework of our model. Agents use past price functions to gather information on the future state of the world; but after current prices are known, they can also use the informational content of these current prices to check the optimality of their choices on the various markets. We speak of a rational expectations equilibrium in our model only, if both processes yield the same result in the sense of an identical past and current price function. As we don't postulate ex post optimality for equilibria allowing for endogenously generated price functions, these rational expectations equilibria are included in our framework as special cases.

A final remark refers to the origin of the price fluctuations in our equilibrium concept. This definition of an equilibrium price sequence is completely independent from the actual realizations of the stochastic process, governing the development of the states of the world. This is, of course, due to the specific form of the utility functions, which are independent of the current state of the environment, and to the independence of the initial endowments of the actual state of the world. Although extreme, this assumption nevertheless shows that price fluctuations can be generated alone by endogenous variations in expectations. There is no influence from outside at all! This result is in sharp contrast to other models, explaining fluctuations in asset prices: Many of these models derive fluctuations in asset prices from exogenously given fluctuations in other determinants, such as non-capital income, or endowment streams (see e.g. Huffman (1986) and the references given there). Other models, such as temporary equilibrium models are based on exogenously modelled ad hoc adjustments in expectations, whereas in the above model, agents make use of all the information available in
a certain period of time.

Of course, the conclusion is not that all the assumptions in other models are unrealistic; the conclusion is that in addition to exogenous factors, endogenous factors can contribute to price fluctuations. This can be seen e.g., if one assumes that utility functions and endowments are explicitly dependent on the respective current state of the world. In this case, the concept of an equilibrium would then depend on a given sequence of realizations of the stochastic process, governing the development of the states of the world. Both exogenous and endogenous factors would then contribute to price fluctuations on the various markets.

The following section considers the existence problem of equilibria allowing for endogenously generated price fluctuations.

4. The Existence Theorem

Although generic existence of rational expectations equilibria is guaranteed under fairly general assumptions, at least if the dimension of the signal space is not equal to the dimension of the price space (see Jordan and Radner (1982)), these existence theorems are not satisfying from our standpoint, as one cannot derive the existence of non-degenerate cycles of price functions from them. In addition, the basic assumptions in the various existence theorems are quite different, and do not allow a uniform approach.

The definition of an equilibrium with endogenously generated price fluctuations depends on the concept of a Radner equilibrium with asymmetric information. Clearly, we first should formulate conditions, guaranteeing the existence of these equilibria.
The information structure in this model is governed by the mappings 
\[ \psi^h : Y \times F_b(Y^H, \mathbb{R}_+^{\mathbb{N}^* + N}) \to M(\Omega); \quad h = 1, \ldots, H, \] 
associating with any current information signal and with any past price function a probability distribution of the future states of the world. These probability distribution then determines the set of return vectors that agent \( h \) expects with positive probability, i.e. \( R^h(y^h, p) := \{ r(w) \in \mathbb{R}_+^N : \psi^h(y^h, p)(w) > 0 \} \). We then have to assume that these individual expectations with respect to the returns of the financial assets are overlapping in the following sense:

**Assumption 4.1:** To \( (y^h, p) \in Y \times F_b(Y^H, \mathbb{R}_+^{\mathbb{N}^* + N}) \) consider \( \psi^h(y^h, p) \in M(\Omega) \), and \( R^h(y^h, p) := \{ r(w) \in \mathbb{R}_+^N : \psi^h(y^h, p)(w) > 0 \text{ for } w \in \Omega \} \). The convex cone \( R^h(y^h, p), \text{ cco } R^h(y^h, p) \), is then defined as follows: \( \text{ cco } R^h(y^h, p) := \{ r \in \mathbb{R} : r = \sum_{i=1}^N \alpha_i \cdot r_i(w) \text{ such that } r_i(w) \in R^h(y^h, p) \text{ and } \alpha_i \in \mathbb{R} \text{ for all } i \} \).

Assume then that \( \cap_{h=1}^H \text{ r.i. cco } R^h(y^h, p) \neq \phi \) for all \( (y^h, p) \in Y \times F_b(Y^H, \mathbb{R}_+^{\mathbb{N}^* + N}) \), where 'r.i.' means 'relative interior'.

It is intuitively clear that this condition is necessary for the existence of a Radner equilibrium with asymmetric information given by \( \psi^h(y^h, p), \quad h = 1, \ldots, H \). Assume that \( \cap_{h=1}^H \text{ r.i. cco } R^h(y^h, p) = \phi \) for a specific \( (y^h, p) \in Y \times F_b(Y^H, \mathbb{R}_+^{\mathbb{N}^* + N}) \), \( h = 1, \ldots, H \). Take any price vector \( \pi \in \mathbb{R}_+^N \) for the financial assets, assume \( \pi \not\in \text{ r.i. cco } R^h(y^h, p) \). Then one can find a portfolio \( \theta \in \mathbb{R}^N \) such that \( \pi \cdot \theta^h < 0 \) and \( r(w) \cdot \theta^h \).
h \ h \\
> 0 \text{ for all } r(w) \in \mathbb{R}^0 \ (y, p). \text{ Thus, there are arbitrage opportunities}

for any price vector \( \pi \) of the financial assets, ruling out the existence of an equilibrium (compare also Werner (1986) for a more detailed consideration of this point).

However, the above condition is also sufficient for the existence of a Radner equilibrium with asymmetric information. The proof of this result is just a slight adaptation of a theorem stated in Werner (1986), and shall therefore not be repeated here.

Lemma 4.2: Assume that for each \( h = 1, \ldots, H \), and for each \( w^2 \in \Omega, \ u^1 \in \mathbb{R}^2 \times \mathbb{R}_+; \)

\[
\sum_{h=1}^{H} e_h > 0 \text{ and that Assumption 4.1 is valid for the information structure of the economy. Then there exists a Radner equilibrium } (p(y, p), \pi(y, p), (x(y, p), \theta(y, p)), h = 1, \ldots, H) \text{ to any }
\]

\[
(y, p) \in Y \times F(Y, \mathbb{R}_+^b), \text{ describing the information available in the economy.}
\]

For a proof compare Werner (1986), chapter 3. Observe that for a price function \( p \in F_b(Y, \mathbb{R}_+^b) \) the resulting equilibrium price functions

\[
y \rightarrow p(y, p), \text{ respectively } y \rightarrow \pi(y, p) \text{ are clearly bounded, so that }
\]

\[
y \rightarrow p(y, p) = (p(y, p), \pi(y, p)) \text{ again belongs to } F(Y, \mathbb{R}_+^b). \]
In order to continue with the consideration of the existence problem, we have to introduce some topological concepts:

The mapping \( \psi: Y^H \times F_b(Y, R^{l+N-1}) \to M^H(\Omega) \), describing the information structure of the economy, induces a mapping \( \psi: F(Y, R^+) \to F(Y, M(\Omega)) \) by: \( \psi(p)(y) = \psi(y, p) \). We endow these function spaces with the topology of uniform convergence, whereby \( M^H(\Omega) \) is endowed with the weak topology; observe that \( M^H(\Omega) \) is a compact metrizable topological space with respect to the weak topology.

Define sequences of sets \( (\wp^N)_{n \in N} \subseteq F(Y, M(\Omega)) \), and \( (p^R)_{n \in N} \subseteq F(Y, R^{l+N-1}) \) inductively in the following way:

\[ \wp^1 := \text{Im} \psi (= \text{image of } \psi) \subseteq F(Y, M(\Omega)). \]

\[ \wp^1 \subseteq \wp^2 \subseteq \cdots \subseteq \wp^N \subseteq \cdots \]

\[ P^1 := \{ p \in F(Y, R^{l+N-1}) : \text{There exists } \psi \in \wp^1 \text{ such that } p(y) \text{ is a Radner equilibrium for the economy for all } y^H \in Y^H \}, \text{ given the asymmetric information } \psi(y) \}. \]

\[ P^n := \text{Im}(\psi|_{P^{n-1}}) \subseteq F(Y, M(\Omega)), n \geq 2. \]

\[ P^n := \{ p \in F(Y, R^{l+N-1}) : \text{There exists } \psi \in \wp^n \text{ such that } p(y) \text{ is a Radner equilibrium for the economy for all } y^H \in Y^H \}, \text{ given the asymmetric information } \psi(y) \}. \]
information $\psi(y)$, $n \geq 2$.

Observe that $\Psi^{n+1}_R \subseteq \Psi^n_R$ and $P^{n+1}_R \subseteq P^n_R$ for all $n \in \mathbb{N}$. This follows immediately from the definition of these sets. Furthermore, under the assumptions of Lemma 4.2, $P = \phi$ for all $n \in \mathbb{N}$. Of course, we cannot rule out multiple equilibria.

In a natural way we then obtain a correspondence $H^o_{n+1} : P^o_R \rightarrow P^o_R$ for an arbitrary $n \in \mathbb{N}$, defined as follows: For $p \in P^o_R$ consider:

$$
H^o_{n+1}(p) := \{ p' \in P^o_R : p(y) \text{ is a Radner equilibrium, where } \psi(y,p) \}
$$

describes the asymmetric information of the traders for all $y^H \in y^H_R$. For $n, m \in \mathbb{N}$, $m > n$, we can similarly define a correspondence $H^o_{m+2} : P^o_R \rightarrow P^o_R$ by:

$$
H^o_{m+2}(p) := H^o_{m+1}(H^o_{m+1}(H^o_{m+1}(\ldots H^o_n(p))))
$$

We then have the following results for the existence of an equilibrium allowing for endogenously generated price fluctuations:
Theorem 4.3: The validity of Lemma 4.2 is assumed for the following statements:

(i) There exists an equilibrium allowing for endogenously generated price fluctuations if and only if there exist \( n^o, m^o \in \mathbb{N}, m^o > n^o \), such that the correspondence \( H : P^o \rightarrow P^o \) has a fixed point.

(ii) The following condition is sufficient for the existence of an equilibrium price sequence: There exist \( n^o, m^o \in \mathbb{N}, m^o > n^o \), there exists a convex and compact subset \( P^o \subseteq P^o \) such that a selection \( H^o \) of the correspondence \( H^o \) is continuous on \( P^o \).

and \( H^o (P^o) \subseteq P^o \cap P^o \).

(iii) The following condition is sufficient for the existence of an equilibrium price sequence: There exists \( m^o \in \mathbb{N} \) such that the correspondence \( H^o : F(Y, R^o) \rightarrow P^o \) has a continuous and compact selection \( H^o \).

(iv) Assume that there exists \( n^o \in \mathbb{N} \) and a compact subset \( P^o \subseteq P^o \),
such that a selection \( H^o \) of \( H : P \to P \) is continuous
\[
\Lambda_n^o \Lambda_n^{o+1} \quad \text{on } P ^o \quad \Lambda_n^{o+1} \quad \text{and such that }\ H_o \subseteq \bigcap P \quad \Lambda_n \quad \text{The following condition is then sufficient for the existence of an equilibrium allowing for endogenously generated price fluctuations: There exists}
\]
\[
\Lambda_n^{o+1} \quad \text{on } P ^o \quad \Lambda_n \quad \text{such that the sequence } p_1 = p, p_2 = (p_1)^o, \ldots
\]
\[
\Lambda_n^{o+1} \quad \text{in } P ^o \quad \text{has only finitely many accumulation points.}
\]

Proof of Theorem 4.3:

(i) "⇒": Assume that \( \{p_1, \ldots, p_m\} \subseteq \bigcap F \subseteq \bigcap Y^H \otimes \mathbb{R}^{l+n-l} \) is an equilibrium price sequence. Define \( n^o = 1, m^o = m+1 \); then, by the definition of this equilibrium, \( p_1^* \in H^o(p_1) \).

"⇐": Assume that \( p^* \in P^o \) is a fixed point of \( H : P \to P \), \( \Lambda_n^o \quad \text{on } P ^o \quad \Lambda_m^o \quad \text{i.e. } p^* \in H^o(p_1) \); then, because \( H(p^o) = H^o \quad \text{on } P ^o \quad \text{and } H(p^o) = H^o \quad \text{on } P ^o \quad \text{there exists a sequence } p^*, \ldots, p^* \quad \text{in } \bigcap F (Y, \mathbb{R}^+) \quad \text{and } p^* \quad \text{in } \bigcap F (Y, \mathbb{R}^+) \).
such that $p^*_s \in \mathbb{P}$ for $s=n, \ldots, m$, and such that $p^*_{s+1} \in H^*_s (p)$,

where $p^*_{m+1} = p^*_{-m}$. Obviously $\{p^*_0, \ldots, p^*_m\}$ is an equilibrium

allowing for endogenously generated price fluctuations.

(ii) Observe that $H^*_0$ is a continuous mapping defined on a convex and

compact subset of the Banach space $F_b^H (Y, \mathbb{R}^{l+N-1})$. According to

Schauder's Theorem, there exists a fixed point of the mapping

$$
H^*_0 : \mathbb{R}^m \rightarrow \mathbb{R}^m.
$$

(iii) This statement is an immediate consequence of Schauder's second theorem

(compare e.g. Smart (1974), Theorem 4.1.1). Just observe that $F_b^H (Y, \mathbb{R}^{l+N-1})$ is a convex subset of the Banach space $F_b^H (Y, \mathbb{R}^{l+N-1})$.

(iv) Assume that $\tilde{p}_1, \ldots, \tilde{p}_m \in \mathbb{P}$ are the only accumulation points of

the sequence $(p^*_n)_{n \in \mathbb{Z}}$. Thus, there exists a subsequence $(p^*_n)_{n, k \in \mathbb{N}}$

of $(p^*_n)_{n \in \mathbb{Z}}$, converging to $\tilde{p}$. Assume w.l.o.g. that all

subsequences $(p^*_n)_{n + s, k \in \mathbb{N}}$, $s = 1, \ldots, m$, converge too. Obviously,
they then converge to one of the accumulation points in \( \{ p_1, \ldots, p_m \} \).

As there are \( m+1 \) converging subsequences, at least two of them must have a common limit. Assume that \( \lim_{k \to \infty} p_k = \lim_{k \to \infty} p_k \),

and assume furthermore w.l.o.g. that \( \lim_{k \to \infty} p_k \). \( \eta + s \) for \( s = 1, \ldots, m-1 \). By the continuity of \( H \) we obtain: \( \tilde{p}_1 = \lim_{n \to \infty} \tilde{p}_1 \).

\[
\lim_{k \to \infty} p_k \eta + m = \lim_{k \to \infty} (p_k \eta + m) = \lim_{k \to \infty} H(p_k \eta + m) = \tilde{p}_1 \eta + n
\]

(\( \lim_{k \to \infty} p_k \eta + m \)). Thus, there exists a fixed point of the correspondence \( H \) with \( m = n + m \).

The above proofs show that each equilibrium price sequence \( \{ p_1, \ldots, p_m \} \subseteq F(Y, R) \) is contained in each \( P_n \), \( n \in \mathbb{N} \). Thus, the existence of an equilibrium always implies that both \( \bigcap_{n=1}^{\infty} P_n \) and \( \bigcap_{n=1}^{\infty} \Psi_n \) are non-empty.

Especially statement (iv) shows that the critical assumption for the existence of an equilibrium is the finiteness of the set of accumulation points of a specific sequence \( (p_n)_{n \in \mathbb{N}} \subseteq F_X(Y, R^+) \). This finiteness assumption
corresponds in principle to the assumption of a finite \( m^0 \in \mathbb{N} \) in all the
other statements of the theorem. Without these assumptions a finite, closed
sequence of price functions \( p^*_1, \ldots, p^*_m \) need no longer exist, although
continuity and compactness assumptions would allow the existence of
'approximately closed' price sequences. We will not further discuss this
point.

Similarly to the problem of existence of rational expectations
equilibria, the problem of existence of equilibria allowing for endogenously
generated price fluctuations has something to do with discontinuities of the
function \( \psi: F(Y, \mathbb{R}^H) \to F(Y, H(\Omega)) \); it can well happen that the
limit \( p \) of a sequence of price functions \( (p_n) \) \( n \in \mathbb{N} \)
\( n \in \mathbb{N} \) reveals a qualitatively quite different information with respect to the future
state of the world then all members of the sequence \( (p_n) \). This means
that \( \psi(p)(y') \) is contained in a stratum of \( M(\Omega) \), which is itself not a
subset of the closure of the stratum containing almost all \( \psi(p_n)(y'), n \in \mathbb{N}, \)
for at least one \( y^H \in X^H \). Of course, these discontinuities can be ruled out
by postulating the continuity of the mapping \( \psi \). Together with an
unproblematic compactness assumption and the assumed existence of a continuous
selection of the Radner equilibrium correspondence, the existence of an
equilibrium price sequence would be an easy consequence (compare e.g.
statement (iii) of the theorem). So the restriction of the continuity
assumption in the above theorem to arbitrarily small compact and convex
subsets allows on the other side for a wide range of realistic
discontinuities of the mapping \( \psi: F(Y, R^{\mathbb{Z} + N-1}) \rightarrow F(Y, H^H(\Omega)) \).

So far the comments on the proof of the main theorem. It should be mentioned here that due to the large variety of fixed point theorems (compare e.g. the theorems stated in Smart (1974)) many other conditions could be formulated, which are sufficient for the existence of an equilibrium allowing for endogenously generated price fluctuations. We will not continue the discussion of this point here, as the substantial ideas are contained in the above given proofs and the corresponding remarks.

5. AN EXAMPLE

The following examples does not immediately fit into the framework so far outlined in this paper. It will, however, become clear that our model can be suitably adjusted, so that it includes this example. The main difference to our model is the assumption of a continuum of possible states of the world and the existence of commodity futures.

The example is an extended and appropriately reinterpreted version of an example given by Jordan and Radner (1982). They use it to show the possibility of non-existence of a rational expectations equilibrium for an open set of economics. Our modified version shows the existence of an equilibrium with endogenous price fluctuations, although again a rational expectations equilibrium does not exist.

In this economy there are two traders and two commodities. Trader 1 is fully informed before the market opens, i.e. trader 1's signal is
s ∈ [0,1] = : Ω, and trader 2 is only incompletely informed: Formally, 
trader 2 receives the same signal as trader 1, but trader 2 can only infer 
from signal s, that either state s or state (1−s) will occur next period. 
Traders' utility functions are defined by:

\[ u^1(x,y;s) = (1+s)\ln x + (2-s)\ln y, \]

\[ u^2(x,y;s) = (2-s^2)\ln x + (1+s^2)\ln y. \]

Each trader has the endowment (1,1) independently of s ∈ [0,1]. Let p, 0 < p 
< 1, denote the price of commodity x, and 1−p the price of commodity y; a 
price function is then given by p: [0,1] → ]0,1[, where s ∈ [0,1] represents 
both traders' signal, according to the above convention.

For a complete description of the information structure of this economy,
we have to define the mappings \( \psi^2: [0,1] \times F([0,1], [0,1]) \to M(\Omega) \); observe 
that \( \psi^1 \) is already defined by the above assumptions. Then, it is only 
necessary to introduce \( \psi^2(s,p) \) for \( p^*_1: s \to (1/6)(3 + s - s^2) \), and \( p^*_2: 
\)
\( s \to (1/12)(s + 4s - 2s^2) \); in these cases:

\[ \psi^2(s^0, p^*_1) = \begin{cases} 
1/2 , & s = s^0 \\
1/2 , & s = 1-s^0 
\end{cases} \]

\[ \psi^2(s^0, p^*_2) = \begin{cases} 
1 , & s = s^0 \\
0 , & s \neq s^0. 
\end{cases} \]

Observe that \( p^*_1(s) = p^*_1(1-s) \) for all s ∈ [0,1], but \( p^*_2 \) is one-to-one. To 
motivate these definitions assume that trader 2 does not learn directly about 
the current state of the world. Only if the last period's price function
is one-to-one, trader 2 learns the actual state of the world by comparing
prices and the information signal. Trader 2 then assumes this relationship
between signals and future state of the world also for the next period. If,
however, \( p^*(s) = p^*(1-s) \) only incomplete information about the current state
of the world is available, and no further conclusions with respect to the
future environment can be drawn.

We show that \( \{p_1^*, p_2^*\} \) is an equilibrium allowing for endogenously
generated price fluctuations. First, \( p_1^*(s) \) is a Radner-equilibrium given the
information structure \( \psi^1(s, p_2^*) \), resp. \( \psi^2(s, p_2^*) \), which implies full
information for both traders on next period's state of the world. This is
just an easy calculation, given the assumptions on the model. Then, also
\( p_2^*(s) \) is a Radner equilibrium for the information structure \( \psi^1(s, p_1^*) \), resp.
\( \psi^2(s, p_1^*) \); again an easy calculation. Thus, \( \{p_1^*, p_2^*\} \) is an equilibrium
price sequence!

On the other side, there does not exist a rational expectations
equilibrium for this economy. Any rational expectations equilibrium \( p^* \in F([0,1], [0,1]) \) either reveals the true next period's state of the world to
trader 2, or it does not. In the first case, however, \( p_1^*(s) \) is the Radner
equilibrium given \( \psi^1(s, p^*) \) and \( \psi^2(s, p^*) \), respectively; and in the second
case \( p_2^*(s) \) is the Radner equilibrium given \( \psi^1(s, p^*) \) and \( \psi^2(s, p^*) \),
respectively. Thus, a price function \( p^* \) such that \( p^*(s) \) is a Radner
equilibrium to \( \psi^1(s, p^*) \) and \( \psi^2(s, p^*) \), representing the information
structure, does not exist. It should be pointed out that this example
satisfies the assumptions of the existence theorem, at least after a
suitable adjustment. Take e.g. statement (iv) of Theorem 4.3 and define

\[
A^{*} = \{ p_1^*, p_2^* \}; \text{ then we obtain the sequence } p_1 = p_2 = p^* \implies H(p_1) = p_2.
\]
p : \( H(p) = p \) and so on; thus \( p_n \) alternates between \( p_1 \) and \( p_2 \) in \( n \in \mathbb{N} \), and clearly has only a finite number of accumulation points.

6. **Summary**

The preceding sections explain extensively the concept of an equilibrium allowing for endogenously generated price fluctuations. This concept is characterized by a finite and closed sequence of equilibrium price functions, linked together by an appropriate assumption on the information structure of the economy. A simple investigation shows that the possibility of a non-degenerate cycle of equilibrium price functions is due to the assumption that agents make efficient use of information contained in price functions only up to the respective last period's price function. Taking into account the current period's price function would lead to the concept of a rational expectations equilibrium, and non-degenerate cycles of equilibrium price functions would disappear. One should observe on the other side that the assumptions on the information gathering process are sufficiently general to allow for a wide range of more specific behavioral assumptions, so that 'ad hoc-mechanisms' are not responsible for the obtained results.

The treatment of the existence problem is still incomplete in an obvious sense. What one would like to have is an explicit relationship between a set of assumptions on the model and the existence of such a closed and non-degenerate sequence of equilibrium price functions. Observe that the existence theorems for rational expectations equilibria do not provide a solution to this problem, although rational expectations equilibria can be interpreted as special equilibria in the sense of this model after a slight
adjustment in the definition. Thus, the existence problem should be further investigated. Probably an interesting relationship between the number of commodities and/or the number of financial assets could result from such considerations.

Summing up, the present paper studies an equilibrium concept, suitable for a dynamic system of incomplete markets, where agents make efficient use of the endogenously generated information in equilibrium price functions and of exogenously received signals. Price fluctuations are thus endogenously generated in contrast to other models, explaining price variability. Another important, yet to be exploited feature of this model is that it allows an investigation of the problem of learning from price fluctuations. This will probably provide an explanation for the frequent observation of breaking down short term price fluctuations.
REFERENCES


