Employment Insurance and Competitive Equilibrium Labor Contracts

Chin LIm

Citation of this paper:
RESEARCH REPORT 8004

EMPLOYMENT INSURANCE AND COMPETITIVE EQUILIBRIUM LABOR CONTRACTS

by

Chin Lim

February 1980
EMPLOYMENT INSURANCE AND COMPETITIVE EQUILIBRIUM LABOR CONTRACTS

by

Chin Lim*

Department of Economics
University of Western Ontario
London, Ontario N6A 5C2
Canada

January 1980
ABSTRACT

In a model where labor supply and demand are viewed as analogous to the demand and supply of employment insurance respectively, this paper demonstrates the properties of the competitive equilibrium labor contract structure. If risk neutral firms are insufficient in number to provide full contract-employment (i.e. every available worker receiving a contract), then competitive equilibrium must support both risk neutral and risk averse firms with the former offering either the layoff (fixed-wage/variable-employment) or the salary (fixed-wage/fixed-employment) contract; and the latter offering either the layoff or the variable-wage (variable-wage/fixed-employment) contract. The exact configuration of the equilibrium contract structure depends critically on the value of leisure and unemployment compensation. The Azariadis [2]-Baily [3] model which admits only the layoff and salary contracts into the equilibrium contract set is shown to be a limiting special case—an outcome of Frank Knight's market self-selection process which eliminates risk averse firms from participating in the market if there are a sufficiently large number of risk neutral firms capable of providing full contract-employment. Furthermore, increasing uncertainty of output demand is shown to have varying effects on the wages contained in different contracts.
I. INTRODUCTION

In an effort to rationalize the joint phenomena of wage rigidity and layoff unemployment—or, equivalently, the fixed-wage/variable-employment contract—the implicit contract theory of Azariadix [2] and Baily [3] (henceforth known as the A-B model) has appealed to a critical but rather restrictive assumption, i.e., the assumption of risk-asymmetry: Firms are risk neutral but workers are risk averse.\(^1\)

The purpose of this assumption was to establish the efficiency of the class of fixed-wage contracts (which admit both fixed-wage/fixed-employment and fixed-wage/variable-employment contracts) so that, subsequently, the efficiency of the fixed-wage/variable-employment contract could be demonstrated. As it turns out, however, even within the confines of implicit contract theory, the risk-asymmetry assumption, by itself, is neither sufficient (see Akerlof and Miyazaki [1] and Solow [14]) nor, as will be shown later, necessary for the efficiency of the fixed-wage/variable-employment contract.\(^3\) Thus, the assumption that all firms are risk neutral is not only restrictive but sterile.

In general, firms may differ, for a variety of reasons, in their degree of risk aversion. Taking this as an a priori assumption, the purpose of this paper is to consider the implications of this more general assumption on the properties of the competitive equilibrium labor contract structure within the risk-sharing framework of implicit contract theory. It is shown that competitive equilibrium in the labor market is characterized by a set of contracts which not only admits fixed-wage contracts (an outcome peculiar to the A-B model because of its risk-asymmetry assumption), but may also entertain variable-wage contracts which are by no means uncommon.
Furthermore, increasing uncertainty of output demand has varying effects on the wage rates of different contracts. In so doing, this paper provides a natural extension to the implicit contract theory which has so far been limited by the narrow confines of the A-B model.

The model begins by viewing labor supply and demand to be analogous to the demand and supply of employment insurance respectively. The fixed-wage/variable-employment contract is essentially one which contains no employment insurance since employment may vary across the states of the world. It also represents one polar case in which all the risk of fluctuating output demand is absorbed by workers. On the other hand, the fixed-employment contracts are those in which workers purchase an employment insurance to guarantee employment in all states of the world. However, depending on the amount of insurance purchased, the resultant contract could be either the fixed-wage/fixed-employment contract (which is the opposite polar case in which workers shift all the risk to firms by purchasing a full insurance coverage) or the variable-wage/fixed-employment contract (which is the intermediate case where both firms and workers share risk by way of workers purchasing only a partial insurance coverage).

The highlights of the results of the paper are as follows. First, the A-B result that the equilibrium contract set admits only the fixed-wage contracts, i.e., the fixed-wage/fixed-employment and fixed-wage/variable-employment contracts, is derived as a limiting special case which occurs when only risk neutral firms exist in competitive equilibrium. The sole existence of risk neutral firms, rather than being an a priori assumption as in the A-B model, occurs here as a competitive equilibrium phenomenon akin to Frank Knight's market self-selection process. Specifically, if there are sufficient numbers of risk neutral
firms capable of providing "full contract-employment" (i.e., all available workers being given a contract), then in competitive equilibrium, no risk averse firms may survive.

Second, if the number of risk neutral firms is insufficient to provide full contract-employment, then competitive equilibrium must support both risk neutral and risk averse firms with the former offering either the fixed-wage/variable-employment or the fixed-wage/fixed-employment contract and the latter offering either the fixed-wage/variable-employment or the variable-wage/fixed-employment contract. Also, in equilibrium, risk neutral firms may extract, but not always, an excess expected profit or a 'rent' for being able to absorb more risk than are risk averse firms. The exact configuration of the competitive equilibrium contract set depends on the value of z (defined as the value of leisure and government unemployment compensation). The larger is z, the less attractive to workers are those fixed-employment contracts which contain a guaranteed-employment insurance and the more likely is the fixed-wage/variable-employment contract which contains no employment insurance. On the other hand, the smaller is z, the less likely is the fixed-wage/variable-employment contract, and the more likely will equilibrium contracts be those guaranteed-employment insurance contracts like the fixed-wage/fixed-employment and the variable-wage/fixed-employment contracts.

Third, if only risk neutral firms are supported in competitive equilibrium, then the equilibrium contract set, which may admit only the fixed-wage/variable-employment and the fixed-wage/fixed-employment contracts, is affected by increasing uncertainty of output prices in the following manner: In the fixed-wage/variable-employment contract,
the wage increases; in the fixed-wage/fixed-employment contract, the wage remains constant.

Finally, if, on the other hand, both risk neutral and risk averse firms are supported in competitive equilibrium, then the equilibrium contract set, which may admit the fixed-wage/variable-employment, fixed-wage/fixed-employment and variable-wage/fixed-employment contracts, would be affected by increasing uncertainty as follows: In the fixed-wage/variable-employment contract, the wage tends to increase (decrease) the smaller (larger) is the degree of risk aversion among risk averse firms; in the fixed-wage/fixed-employment contract, the wage remains constant; and in the variable-wage/fixed-employment contract, the variation in wages tends to be reduced across the states of the world.

The rest of the paper is organized as follows. In Section II, a model is presented in which the supply and demand of labor are analogously viewed as the demand and supply of employment insurance, respectively. In this model, the efficient labor contracts are characterized. Section III then demonstrates the properties of the competitive equilibrium labor contract structure. In Section IV a comparative statics analysis is used to examine how the competitive equilibrium contracts are affected by increasing uncertainty regarding the output prices. Finally a conclusion is provided in Section V.
II. OPTIMAL EMPLOYMENT INSURANCE

In a world where uncertainty exists to cause unforeseen fluctuations in profits and labor income, it is useful to approach the labor market from an insurance viewpoint. In this paper, the supply and demand of labor services are viewed as analogous to the demand and supply of employment insurance, respectively. This analogy will become more apparent when we consider the model presented below.

Supply of Employment Insurance

Consider a competitive firm which uses labor, \( n \), as its only variable factor of production and whose production function, \( f(n) \), is assumed to be a strictly increasing concave function of \( n \). Let \( F \) be some fixed capital cost, \( p \) be the output price, and \( w \) be the wage rate. Then the firm's profit, \( \pi = pf(n) - wn - F \). Furthermore, let \( u^f(\pi) \) be the firm's utility function with the following properties: \( u^f(0) = 0, u'^f(\cdot) > 0, \) and \( u''^f(\cdot) \leq 0 \), so that the firm can be either risk neutral or risk averse.

The risk that the firm faces is assumed to arise from the fluctuations in output prices. For simplicity, let there be only two states of the world so that in state 1, output price is \( p_1 \) which occurs with probability \( (1-q) \) and in state 2, output price is \( p_2 \) which occurs with probability \( q \). It is assumed that \( p_1 > p_2 \) so that state 1 can be considered the better state.

Now given the random fluctuations in output prices, two employment strategies are possible for any given competitive wage \( w \). One possible strategy is to choose employment \textit{ex post} to the realization of output prices. Denoting \( E[\cdot] \) as an expectations operator, this strategy is equivalent to
\[ E[\maximize u^F(p_f(n) - w_n - F)] \]
\[ \text{which yields the employment solution } n_i (i=1,2) \text{ which satisfies the necessary (and sufficient) condition for optimality:} \]
\[ p^f_i(n_i) - w = 0 \text{ , } i=1,2. \] (1)

Note that \( n_1 > n_2 \) since \( p_1 > p_2 \). In state 1, \( n_1 \) workers are employed and in state 2, \( (n_1 - n_2) \) workers are laid off. Such a strategy will henceforth be referred to as the **layoff** or the fixed-wage/variable-employment strategy. The other possible strategy is to commit to an employment level **ex ante** to the realization of output prices, i.e.,

\[ \maximize E[u^F(p_f(n) - w_n - F)] \]
\[ \text{In this strategy, the firm guarantees employment at wage } w \text{ no matter which state of the world occurs. Clearly, for the same fixed wage } w \text{, the layoff strategy dominates since} \]
\[ E[\max E[u^F(p_f(n) - w_n - F)] \geq E[u^F(p_f(n) - w_n - F)] \]
\[ \text{by virtue of the fact that the layoff strategy, whose employment solution } n_i (i=1,2) \text{ is chosen **ex post** to the revelation of output prices } p_i (i=1,2), \]
\[ \text{is **ex post** more flexible than the fixed-employment strategy whose employment } \]
\[ \text{n is chosen **ex ante** to the revelation of output prices.} \]

In adopting the layoff strategy, the firm's profits in states 1 and 2 are, respectively,
\[ \pi_{1L}(w) \equiv p_1 f(n_1) - w_n - F \] (2)
\[ \pi_{2L}(w) \equiv p_2 f(n_2) - w_n - F \] (3)
and its expected utility
\[ v^F_L(w) \equiv (1-q)u^F(\pi_{1L}(w)) + q u^F(\pi_{2L}(w)). \] (4)
It is important to emphasize that the above layoff strategy always dominates the guaranteed-employment strategy provided the fixed wage $w$ is common to both strategies. Clearly, a guaranteed-employment strategy which pays wages lower than $w$ may possibly dominate the layoff strategy which pays wage $w$. To motivate this, recall that a worker in the layoff program which pays wage $w$ does not have the guarantee or insurance of employment in state 2. Since he is generally risk averse, he might be better off purchasing from the firm an employment insurance which guarantees employment in all states of the world. Consider an insurance scheme in which a worker, if he participates, pays a premium $\alpha$ no matter which state occurs and who in return receives not only guaranteed employment but also a wage $w_2 \equiv \alpha - \alpha$ in state 2. Such an insurance scheme guarantees employment but not necessarily an identical wage income in both states of the world. The state 1 income, $(w-\alpha)$, may differ from state 2 wage income, $w_2$, depending on the amount of insurance the worker purchases. On the part of the firm, the offer of such employment insurance program yields the states 1 and 2 profits, respectively, as

$$\pi_{1I}(w,\alpha) \equiv p_{1I}(n^*) - (w-\alpha)n^* - F$$  \hspace{1cm} (5)

and

$$\pi_{2I}(w,\alpha) \equiv p_{2I}(n^*) - w_2n^* - F$$  \hspace{1cm} (6)

where the employment solution $n^*$ satisfies the necessary (and sufficient)\(^6\) condition for optimality:

$$(1-q)u^f(\pi_{1I}) + qu^f(\pi_{2I}) = 0$$  \hspace{1cm} (7)

The firm's expected utility under this employment insurance scheme is

$$V^f_{I}(w,\alpha, w_2) \equiv (1-q)u^f(\pi_{1I}(w,\alpha)) + qu^f(\pi_{2I}(w, w_2))$$  \hspace{1cm} (8)

Clearly, employment insurance will be offered if and only if

$$V^f_{I}(w,\alpha, w_2) \geq V^f_{L}(w)$$  \hspace{1cm} (9)
Demand for Employment Insurance

Let a worker's preferences over consumption, c, and leisure, L, be defined by a monotone bounded utility \( v^e(c, L) \) which is strictly concave in c. Leisure is assumed to be indivisible so that L takes on the value of 0 or 1. Suppose a worker receives some asset income, A, and, if he is employed (in which case \( L = 0 \)), a wage income, \( y > 0 \), then we define a strictly increasing concave utility function

\[
u^e(y) = v^e(A + y, 0), \quad y > 0.
\]

However, if he is laid off and hence unemployed (in which case \( L = 1 \)), let him receive some government unemployment compensation, g. His utility from being laid off is therefore \( v^e(A + g, 1) \). Since leisure is generally a normal good, and assuming nonsatisfaction, \( u^e(g) = v^e(A + g, 0) < v^e(A + g, 1) \). There must therefore exist a positive constant, \( k \), the money equivalence of unit leisure such that \( u^e(g + k) = v^e(A + g + k, 0) = v^e(A + g, 1) \). Thus, the utility of a worker who is unemployed is

\[
u^e(z) = v^e(A + z, 0)
\]

where \( z = g + k \), which consists of the government unemployment compensation and the money equivalence of leisure, is the total nonasset component of his income when he is unemployed.

Now, if a worker opts for the layoff program which pays wage \( w \) or equivalently purchases no employment insurance from the firm, his expected utility from the firm's random layoff strategy is

\[
V^e_L(w) = (1-q)u^e(w) + rqu^e(w) + (1-r)qu^e(z)
\]

(10)

where \( r = n_2/n_1 \) is the probability of being retained given state 2 has occurred. However, if he purchases employment insurance at price \( (\alpha, \omega_2) \), which offers wage income \((w-\alpha)\) in state 1 and \( \omega_2 \) in state 2, his expected utility is
Clearly a worker would purchase employment insurance from the firm if and only if

\[ V_1^{e}(w-\alpha, w_2) \geq V_L^e(w) \]  

**Optimal Employment Insurance**

A typical worker is therefore faced with two decisions. First he must decide on whether or not to purchase employment insurance. Purchasing an insurance guarantees him employment in both states of the world whereas not purchasing an insurance is equivalent to opting for the layoff contract which pays wage \( w \) but which contains a probability of being laid off in state 2. Second, if employment insurance is deemed desirable, he must decide on the amount of insurance, \( \alpha \), to purchase which consequently determines his state 1 wage income, \( (w-\alpha) \), and state 2 wage income, \( w_2 \).

On the first decision, it is evident after using (10) and (11), that the tendency to not purchase employment insurance (the violation of inequality (12)) increases with the value of \( z \). In other words, the larger the value of leisure and government unemployment compensation, the more attractive is the layoff contract.

If, however, \( z \) is sufficiently small in that inequality (12) is satisfied, the optimal contract is one in which guaranteed-employment insurance is purchased. The optimal amount of insurance purchased can be easily derived by maximizing the worker's expected utility, \( V_1^{e}(w-\alpha, w_2) \), with respect to \( \alpha \) and \( w_2 \) subject to condition (9) which ensures that the firm is no worse off offering an insurance contract as opposed to the layoff contract. This yields the well-known optimality condition.
\[
\frac{u'(x)}{u'(w, x)} = \frac{u'^p(p_1(x) - (w-x)w - F)}{u'^p(p_2(x) - w^xw - F)}
\]

which states that optimal insurance is one which equalizes the worker's and the firm's marginal rate of substitution between their incomes in both states of the world.

Both conditions (12) and (13) imply

**Proposition 1:** The optimal labor contract structure admits

(a) the layoff (fixed-wage/variable-employment) and the salary (fixed-wage/fixed-employment) contracts if the firm is risk neutral, and

(b) the layoff (fixed-wage/variable-employment) and the variable-wage (variable-wage/fixed-employment) contracts if the firm is risk averse.

**Proof:** If \( z \) is large so that inequality (12) is violated, then no employment insurance is purchased which implies the optimal contract is the layoff contract paying wage \( w \). If, however, \( z \) is small so that inequality (12) is satisfied, then employment insurance is purchased but the optimal amount of insurance depends on whether the firm is risk neutral or risk averse.

(a) If the firm is risk neutral, then \( u'^p(x) \) is a constant and (13) reduces to \( u'^p(x) = u'^p(w, x) = (w-x) = w, \) a salary contract which guarantees employment and an identical wage income in both states of the world.

(b) If the firm is risk averse, \( u'^p(x) < 0 \). Noting also that \( u'^p(x) < 0 \) and \( p_1 > p_2 \), it is easily verified that the equality in (13) will be violated for contracts where \( w^x > (w-x) \). The only contracts that may sustain equality (13)
are those where \((w-\alpha) > w_2\) ⇒ a **variable-wage** contract which guarantees employment in both states of the world but in which the state 1 wage income exceeds the state 2 wage income.

It is worth noting that the layoff contract represents one polar case in which all the risk of fluctuating output demand is absorbed by workers whereas the salary contract is the opposite polar case where all the risk is shifted to the risk neutral firms by way of workers purchasing a full insurance coverage. The intermediate case is provided by the variable-wage contract in which risk is shared by both risk averse firms and workers by way of workers purchasing a partial insurance coverage.

In the A-B model which was aimed at explaining the efficiency of the layoff contract, much stress was placed at justifying the assumption that all firms should, \textit{a priori}, be treated as risk neutral. However, it is now clear that this assumption is unnecessarily restrictive for at least two reasons. First, Proposition 1 implies that the risk neutrality of firms is neither necessary nor sufficient in generating the efficiency of the layoff contract. The crucial factor in explaining the efficiency of the layoff contract rests, instead, on the sufficiently large value of \(z\) to make the purchase of employment insurance an unattractive option. Second, if all firms are assumed risk neutral, then the efficient contract set is confined to contain only fixed-wage contracts (Proposition 1(a)) and to the exclusion of variable-wage contracts. This feature of the A-B model is not totally appealing since variable-wage contracts are by no means uncommon. A more satisfactory theory should be one that is sufficiently general to explain the possible existence of both fixed and variable-wage contracts.
For the above reasons, the unnecessarily restrictive assumption that all firms are identically risk neutral will be abandoned. Instead, it is assumed that firms may generally differ, for a variety of reasons, in their attitude to risk. The implications of this more general assumption on the properties of competitive equilibrium contracts will be the focus of the following section.

III. COMPETITIVE EQUILIBRIUM CONTRACTS

For simplicity, it is assumed that there are two groups of firms—one risk neutral and the other identically risk averse. Workers, on the other hand, are assumed to be all identically risk averse. Under these circumstances, competitive equilibrium in the labor market is defined to be a situation which satisfies (i) the 'full contract-employment' condition—i.e., all available workers receiving a contract that yields an identical expected utility; and (ii) the null entry-exit condition—i.e., no incentive exists for the entry of firms into or the exit of firms from the industry. Condition (i) is self-evident but condition (ii), given the fact that there exists, a priori, both risk neutral and risk averse firms, is not so straightforward and requires further elaboration.

In the first instance, it might appear the zero expected profit condition would provide the most natural candidate for competitive equilibrium. The reason is contracts that yield zero expected profit also yield, given $u'(0) = 0$, a negative expected utility to risk averse firms; and hence zero expected profit contracts which satisfy the full contract-employment condition would also serve to deter the entry and exit of all firms, whether or not they are risk neutral or risk averse. However, because zero expected profit contracts can sustain only risk neutral firms, such an equilibrium is
feasible only if there exists, a priori, a sufficiently large number of risk neutral firms capable of providing full contract-employment.

What if the firms who are risk neutral are somehow limited in number so that, by themselves, they are unable to satisfy the full contract-employment condition or provide a contract to every available worker? In this case, the zero expected profit contracts, which can sustain only risk-neutral firms, cannot qualify as equilibrium contracts. The market adjustment process would take place in the form of the bidding down, by available workers without a contract, of the wage structure such that existing contracts offered by the limited number of risk neutral firms would now yield a sufficiently positive expected profit to attract the entry of risk averse firms and thereby increasing the level of contract-employment. In the final equilibrium, (i) all contracts must therefore yield positive expected profits so that they can sustain both risk neutral and risk averse firms to provide full contract-employment; and furthermore (ii) contracts offered by risk averse firms must only yield zero expected utility to satisfy the null entry-exit condition.

The rest of this section will formalize the above discussions and demonstrate the properties of the competitive equilibrium contract structure. Before proceeding, however, it is useful at the outset to consider some basic relationships between the zero expected profit and the zero expected utility contracts.

Consider, first, all contracts that yield an identical expected profit. Then they must satisfy

\[ E[\pi_L(w)] = E[\pi_T(w; \alpha, w_2)] \]

which, after using (2), (3), (5), and (6), becomes

\[ w_2 = \phi(w) + \frac{(1-q)}{q} \alpha \]

(14)
where \( \varphi(w) = \frac{[E[p]f(n^*) - (1-q)w_n^* - F - E[\pi_L(w)]]/qn^*}{(n_1-n^*) + qn^*/q^*n^*} > 0 \) since \( n_2 < n^* < n_1 \). For the risk neutral firm to be indifferent between the lay-off contract (employing \( n_1 \) and \( n_2 \) workers in state 1 and 2 respectively) and the guaranteed-employment contract (employing \( n^* \) workers), the wage, \( w \), in the former contract must be related to the wage structure, \((w-\alpha), w_2\), in the latter contract according to (14). Condition (14) is graphed in Figure 1 as the discontinuous iso-expected-profit schedule \( \frac{-1}{\pi_i} N^i C^i_L (i=0,1,2,...) \). At the expected profit level \( \frac{-1}{\pi} \), the risk neutral firm is indifferent between offering guaranteed-employment insurance contracts along the continuous segment \( \frac{-1}{\pi} N^i \) and the layoff contract \( C^i_L \) at wage \( w^i \). Let \( \frac{-1}{\pi} N_0 C^0_L \) be the zero expected profit schedule, then all contracts to its left yields positive expected profits with expected profit level \( \frac{-1}{\pi} > \frac{-1}{\pi} \) (\( j > i \)).

**Figure 1**

[Diagram showing the relationship between state 2 wage rate and state 1 wage rate with various profit schedules and indifference curves labeled.]
In contrast to the zero expected profit contracts, consider now those contracts that satisfy zero expected utility of profit to risk averse firms. Let \( w^* \) satisfy \( E[u^f(\pi_L(w^*))] = 0 \), then all zero expected utility contracts satisfy
\[
E[u^f(\pi_L(w^*))] = E[u^f(\pi_L(w^*; \alpha, w_2))]
\]
or equivalently
\[
(1-q)u^f(\pi_{1L}(w^*)) + qu^f(\pi_{2L}(w^*)) = (1-q)u^f(\pi_{1L}(w^*; \alpha)) + qu^f(\pi_{2L}(w^*, w_2)).
\]
After using the strict concavity of \( u^f(\cdot) \), it is easily verified that
\[
(1-q)\pi_{1L}(w^*) + q\pi_{2L}(w^*) < (1-q)\pi_{1L}(w^*; \alpha) + q\pi_{2L}(w^*, w_2).
\]
Hence among all contracts to which a risk-averse firm is indifferent, the expected profit required from employment insurance contracts (right-hand side term in (15)) must be greater than that obtained from the no-insurance lay-off contract (left-hand side term in (15)). The reason is that a risk averse firm who offers an insurance contract must be compensated for some risk bearing whereas in the layoff contract, all risk is shifted to workers. The inequality in (15), after using (2), (3), (5), and (6), reduces to
\[
w_2 < \phi(w^*) + \frac{(1-q)}{q} \alpha
\]
Note that \( w^* \) satisfies \( E[u^f(\pi_L(w^*))] = 0 \) and \( w^0 \) (see Figure 1) satisfies
\( E[\pi_L(w^0)] = 0 \); so that because \( u^f(0) = 0, u^f(\cdot) > 0, \) and \( u^{f'}(\cdot) < 0 \) for risk-averse firms, \( E[\pi_L(w^*; \alpha)] > E[\pi_L(w^0)] = 0 \Rightarrow w^* < w^0 \). Condition (16) is graphed as the zero expected utility schedule \( V^{f*\pi^* \alpha \pi^* L} \) which contains a discontinuous segment \( A^{\pi^* \pi^* L} \). Although all the employment insurance contracts offered along the continuous segment \( V^{f* \pi^* L} \) and the no-insurance layoff contract \( C^{\pi^* L} \) (which pays wage \( w^* \)) yield zero expected utility, they yield different
levels of expected profit depending on the amount of insurance offered. For instance, the no-insurance lay-off contract \( C_L^* \) yields expected profit level \( \overline{\pi}^* \); the insurance contracts \( S \) (denoting small amount of insurance) and \( G \) (denoting greater amount of insurance) yield expected profits \( \overline{\pi}^1 \) and \( \overline{\pi}^2 \) respectively; and \( \overline{\pi}^2 > \overline{\pi}^1 > \overline{\pi}^* > \overline{\pi}^0 = 0 \).

A. Unlimited Number of Risk Neutral Firms

In Figure 1, it is clear that the zero expected utility contracts along schedule \( V^f A^* C_L^* \) are less attractive to workers compared to the zero expected profit contracts along schedule \( \overline{\pi}^0 N^0 C_L^0 \). It follows that if the risk neutral firms are sufficiently large in number such that by themselves they can ensure full contract-employment, then in competitive equilibrium, only the zero expected profit contracts along \( \overline{\pi}^0 N^0 C_L^0 \) will be supplied. In other words, risk averse firms, whose best contract offers to a worker are along the \( V^f A^* C_L^* \) schedule, cannot survive since all workers will be attracted toward the superior contracts along the \( \overline{\pi}^0 N^0 C_L^0 \) schedule. This is an example of Frank Knight's market self-selection process which eliminates the more risk averse firms from existing in competitive equilibrium.

Among the competitive supply of contracts along \( \overline{\pi}^0 N^0 C_L^0 \) schedule offered only by risk neutral firms, the equilibrium contracts are readily inferred from proposition 1(a) and are described in

**Proposition 2:** If risk neutral firms are unlimited in number such that they can provide full contract-employment, then competitive equilibrium supports only risk neutral firms; and the equilibrium contract set consists of the (a) layoff contract if \( z > \overline{z} \), (b) layoff or salary contract if \( z = \overline{z} \) and (c) the salary contract if \( z < \overline{z} \).
The above result is presented in Figure 2 and which becomes more apparent after the following considerations. First a worker is indifferent between purchasing and not purchasing insurance if and only if \( v^e_L(w) = v^e_I(w-\alpha, w_2) \), which, after using (10) and (11), reduces to

\[
(1-q)u^e(w) + ru^e(w) + (1-r)qu^e(z) = (1-q)u^e(w-\alpha) + qu^e(w_2). \tag{17}
\]

If no insurance is purchased, i.e., \( \alpha = 0 \), then the above becomes

\[
u^e(w_2) = ru^e(w) + (1-r)u^e(z)
\]

\[
\Rightarrow w_2 = u^{-1}_e\left( ru^e(w) + (1-r)u^e(z) \right) \equiv \gamma(w,z), \text{ with } \frac{\partial \gamma}{\partial w} > 0, \frac{\partial \gamma}{\partial z} > 0. \tag{18}
\]

Condition (18) says that if no insurance is purchased (i.e., \( \alpha = 0 \)), then a worker would be indifferent between a layoff contract and a fixed-employment contract provided the state 2 wage, \( \gamma(w,z) \), in the latter yields a utility equal to the expected utility from the former contract given state 2 has occurred. In other words, \( \gamma(w,z) \) is the compensation for the foregoing of expected utility consisting of \( (1-r)u^e(z) \) (arising from the receipt of \( z \) if a worker is not retained) and \( ru^e(w) \) (arising from receipt of wage \( w \) if he is retained) which a worker would otherwise obtain in state 2 in the layoff contract. Thus, combining (17) and (18), it is evident that a worker's indifference curve is discontinuous as exemplified by the discontinuous \( v^e_{HC_L} \) schedule in Figure 2 whenever \( z = \tilde{z} \).

Second, a worker's marginal rate of substitution between incomes in both states of the world is \( (1-q)u^e'(w-\alpha)/qu^e'(w_2) \) which is equal to \( (1-q)/q \) at the 45° line when \( (w-\alpha) = w_2 \).

Given the competitive supply of contracts along the discontinuous zero expected profit schedule \( \tilde{n}^N^O_{C_L} \), it is apparent from Figure 2 that when \( z = \tilde{z} \), the competitive equilibrium contract is either the layoff contract \( C^O_L \) (paying wage \( w^0 \)) or the salary contract \( C^O_s \) (paying wage \( s^0 \)). The reader
can easily verify that if the continuous segment, $\overline{V_{HN}}$, of the worker's indifference curve in Figure 2 is displaced vertically downward (in which case $z < \bar{z}$) or upward (in which case $z > \bar{z}$), then the competitive equilibrium contract is only the salary contract $C_s^o$ or the layoff contract $C_L^o$ respectively.

The exclusion of the variable-wage contracts from the competitive equilibrium contract set in this case is a direct consequence of the non-survival of risk averse firms. In competitive equilibrium these firms are totally crowded out by the plentiful number of risk neutral firms.

B. **Limited Number of Risk-Neutral Firms**

What if the number of firms who are risk neutral is insufficient to provide full contract-employment? In this case, we have
Proposition 3: If risk neutral firms are limited in number such that they cannot provide full contract-employment, then competitive equilibrium must support both risk neutral and risk averse firms, and the equilibrium contract set has the following properties:

<table>
<thead>
<tr>
<th>Value of $z$</th>
<th>Equilibrium Contract(s) Offered by Risk Neutral Firms</th>
<th>Equilibrium Contract(s) Offered by Risk Averse Firms</th>
<th>Relationship between Risk Neutral Firm's expected profit ($\pi^N$) and Risk Averse Firms Expected Profit ($\pi^A$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z &gt; \hat{z}$</td>
<td>layoff</td>
<td>layoff</td>
<td>$\frac{\pi^N}{\pi^A} &gt; 0$</td>
</tr>
<tr>
<td>$z = \hat{z}$</td>
<td>layoff or salary</td>
<td>layoff</td>
<td>$\frac{\pi^N}{\pi^A} &gt; 0$</td>
</tr>
<tr>
<td>$\hat{z} &gt; z &gt; \hat{z}$</td>
<td>salary</td>
<td>layoff</td>
<td>$\frac{\pi^N}{\pi^A} &gt; 0$</td>
</tr>
<tr>
<td>$z = \hat{z}$</td>
<td>salary</td>
<td>layoff or variable-wage</td>
<td>$\frac{\pi^N}{\pi^A} &gt; 0$</td>
</tr>
<tr>
<td>$z &lt; \hat{z}$</td>
<td>salary</td>
<td>variable-wage</td>
<td>$\frac{\pi^N}{\pi^A} &gt; 0$</td>
</tr>
</tbody>
</table>

The interesting array of equilibrium contract sets is demonstrated below by graphical illustrations. In every case, the graphical construction is based on the properties of competitive equilibrium discussed previously--i.e., if risk neutral firms are insufficient in number to provide full contract-employment, then the competitive equilibrium conditions are: (i) To achieve full contract-employment, both risk neutral and risk-averse firms must co-exist in equilibrium. This means identical expected utility among workers must be
supported by contracts offered by both types of firms. (ii) To satisfy the null entry-exit condition, contracts offered by risk-averse firms must earn only zero expected utility of profit.

Figure 3

Case where \( z > \tilde{z} \): In Figure 3, recall (from Figure 1) that the discontinuous \( V^* A^* C_L^* \) schedule depicts the zero expected utility contour of the risk-averse firm and specifically the layoff contract \( C_L^* \) yields a positive expected profit given by the discontinuous iso-expected profit schedule \( \tilde{\pi}^* N^* C_L^* \). (The zero expected profit schedule \( \tilde{\pi}^* N^* C_L^0 \) is drawn for the sake of reference). When \( z = \tilde{z} \), and \( w = w^* \), a worker's indifference curve is given by the discontinuous schedule \( \tilde{\pi}^* \tilde{w}^* C_L^* \). In this case, the competitive equilibrium
contracts are: the layoff contract $C_L^*$ or the salary contract $C_s^*$ (which yields a positive expected profit $\pi^*$) being offered by risk-neutral firms; and only the layoff contract $C_L^*$ (which yields the same expected profit $\pi^*$ but a zero expected utility) being offered by risk-averse firms. These contracts satisfy both the competitive equilibrium conditions (i) and (ii). The reader can verify that if $z > \tilde{z}$ in which case the continuous segment $\gamma_{\tilde{z}}$, of the worker's indifference curve is displaced vertically upward, then the competitive equilibrium contract set consists of only the layoff contract $C_L^*$ being offered by both risk neutral and risk averse firms.

**Figure 4**
Case where $z < \hat{z}$: When $z = \hat{z}$, and given $w = w^*$, the worker's indifference curve is given by the discontinuous schedule $\hat{V}e^{HC}_L$ in Figure 4. The competitive equilibrium contract set supporting this indifference curve are: The salary contract $C^2_s$ offered by risk-neutral firms (which yields expected profit $\pi^2$); the zero expected utility contracts offered by risk averse firms which are the variable-wage contract $C^1_v$ (yielding expected profit $\pi^1$) and the layoff contract $C^*_L$ (yielding expected profit $\pi^*$). Note that $\pi^2 > \pi^1 > \pi^*$. All these contracts satisfy the competitive equilibrium conditions (i) and (ii). Suppose $z < \hat{z}$, in which case the continuous segment, $\hat{V}e^r_H$, of the worker's indifference curve $\hat{V}e^{HC}_L$ is displaced downward, then it is readily verified that the no-insurance layoff contract $C^*_L$ will not be purchased since the insurance contracts $C^2_s$ and $C^1_v$ yield a higher worker's expected utility. In this case the competitive equilibrium contract set reduces to only $C^2_s$ being offered by risk-neutral firms and $C^1_v$ being offered by risk-averse firms.

Figure 5
Case where $\hat{z} < z < \bar{z}$: Let $\hat{z} < z^{**} < \bar{z}$ so that in Figure 5, the worker’s indifference curve is given by the discontinuous schedule $V^{e**} H^{C_L^*}$. In this case, the competitive equilibrium contracts are: the layoff contract $C_L^*$ offered only by risk averse firms (which yields zero expected utility but a positive expected profit $\pi^*$); the salary contract $C_s^{**}$ offered by risk neutral firms (which yields expected profit $\pi^{**}$). Since $\pi^{**} > \pi^*$, the layoff contract $C_L^*$ will not be offered by risk neutral firms. The above contract set satisfies the competitive equilibrium conditions (i) and (ii).

The results in Table 1 have several interesting implications. First, equilibrium contracts offered by risk neutral firms are either the layoff or the salary contract (see column 2) and those offered by risk averse firms are either the layoff or variable-wage contract (see column 3). Thus risk neutrality of firms is neither necessary nor sufficient to generate equilibrium layoff contracts. Second, (going up the rows in columns 2 and 3), the tendency of occurrence of the equilibrium layoff contracts increases with the value of $z$, i.e., the value of leisure and government unemployment compensation. Third, the tendency of occurrence of guaranteed employment contracts (going down the rows of columns 2 and 3), like the salary and variable-wage contracts, increases with the decrease in value of $z$. Only in the extreme case when $z$ is very small (row 5) would full employment occur without any layoffs. Fourth, in equilibrium, the expected profits obtained by risk neutral firms are no smaller than those obtained by risk averse firms (see column 4). In other words, risk neutral firms may exact a 'rent' for being able to absorb more risk compared to risk averse firms. What is more interesting is that such a rent may be absent as in the case when $z \geq \bar{z}$ (see first two rows of column 4). Thus, contrary to intuition, it is not true that in equilibrium, risk neutral firms always exact a positive rent or an expected profit higher than that of risk averse firms.
IV. THE EFFECTS OF RISK ON THE COMPETITIVE EQUILIBRIUM CONTRACTS

In this section, the effects of increasing risk on the competitive equilibrium contracts are analyzed. Recall that the only source of uncertainty in this model is the output prices whose mean is \( E[p] = (1-q)p_1 + qp_2 \). The mean-preserving increase in risk, following Rothschild and Stiglitz [11] is defined to be the changing of the probability density by a mean-preserving spread which is adequately captured by allowing \( dp_1 > 0 \) and \( dp_2 < 0 \) such that

\[
dE[p] = (1-q)dp_1 + qdp_2 = 0
\]  
(19)

It turns out that increasing risk has varying effects on the competitive equilibrium contracts depending on whether equilibrium contracts are offered only by risk neutral firms (i.e. contracts characterized in proposition 2) or by both risk neutral and risk averse firms (i.e. contracts characterized in proposition 3).

**Proposition 4:** If only risk neutral firms exist in competitive equilibrium, then the equilibrium contracts which are (from proposition 2) the layoff and salary contracts react to increasing uncertainty of the output prices in the following manner: the layoff contract wage increases but the salary contract wage remains constant.

In Figure 2, observe that the equilibrium layoff contract wage, \( w^*_l \), satisfies the zero expected profit condition

\[
0 = (1-q)(p_1 f(n_1) - w^*_l n_1 - F) + q(p_2 f(n_2) - w^*_l n_2 - F),
\]

where \( n_i (i=1,2) \) satisfies (1). Totally differentiating the above with respect to \( w^*_l \), \( p_1 \) and \( p_2 \), and then using (19), it is easily verified that
\[ \frac{\partial \omega^0}{\partial p_1} \bigg|_{(19)} = \frac{(1-q)[f(n_1) - f(n_2)]}{(1-q)n_1 + qn_2} > 0 \]  

(20)

The above result arises from the fact that in the layoff contract, the 'indirect' profit function is an increasing convex function of output price \(^{10}\) (refer also Walter Oi [10]). Thus a mean-preserving increase in uncertainty of output prices, by Jensen's inequality, would increase the expected profit from its original equilibrium value of zero to some positive number. To re-establish equilibrium, i.e., zero expected profit, the layoff contract wage \(w^0\) must rise.

Regarding the equilibrium salary contract, note that the salary wage \(s^0\) in Figure 2 also satisfies the zero expected profit condition

\[ 0 = (1-q)(p_1 f(n^*) - s^0 n^* - F) + q(p_2 f(n^*) - s^0 n^* - F) \]

where \(n^*\) satisfies (7). Totally differentiating the above with respect to \(s^0\), \(p_1\) and \(p_2\), and then using (19), yields \(^{11}\)

\[ \frac{\partial s^0}{\partial p_1} \bigg|_{(19)} = 0. \]

(21)

This result is due to the fact that the 'indirect' profit function in the salary contract is a linear function of the output prices. \(^{12}\) Hence, a mean-preserving increase in uncertainty about the output prices, by Jensen's inequality, does not alter the expected profit level. Thus the equilibrium salary contract wage \(s^0\) remains constant.

**Proposition 5:** If both risk neutral and risk averse firms exist in competitive equilibrium, then the equilibrium contracts which are (from Proposition 3) the layoff contract, salary contract, and variable-wage contracts react to increasing uncertainty of the output prices in the following manner:
The layoff contract wage tends to increase (decrease) ceteris paribus, the smaller (larger) the degree of risk aversion on the part of the risk averse firms; the salary contract wage remains constant; and the variation in wages across the states of the world in the variable-wage contract tends to be reduced.

In Proposition 3, recall that if both risk neutral and risk averse firms exist in competitive equilibrium, then they both may offer the layoff contract. However, regardless of the source of offer, it is easily seen from Figures 2 to 4 that the layoff contract $C^*_L$ which pays wage $w^*$ must satisfy the zero expected utility of profit to risk-averse firms:

$$0 = (1-q)u^f(\pi_1L) + qu^f(\pi_2L)$$

$$= (1-q)u^f(p_1f(n_1) - w^*n_1 - F) + qu^f(p_2f(n_2) - w^*n_2 - F)$$

where $n_i (i=1,2)$ satisfies (1). Totally differentiating the above with respect to $w^*, p_1, p_2$, and then using (19) yields

$$\frac{\partial w^*}{\partial p_1} = \frac{(1-q)[u^f' (\pi_1L)f(n_1) - u^f' (\pi_2L)f(n_2)]}{(1-q)u^f (\pi_1L)n_1 + qu^f (\pi_2L)n_2}$$

(22)

$$\geq 0 \text{ as } f(n_1)[1 - R^f(\pi_2L)(\pi_1L - \pi_2L)] \geq f(n_2)$$

where $R^f(\cdot) = -\frac{u^f''(\cdot)}{u^f(\cdot)}$ is the Arrow-Pratt measure of the firm's absolute risk aversion. Thus, the wage $w^*$ tends to increase (decrease) ceteris paribus, smaller (larger) the degree of absolute risk aversion on the part of the risk averse firms. Note that (22) reduces to (20) when the degree of risk aversion tends to zero; in which case, $w^*$ will increase with risk. The reason for the ambiguity in the sign of (22) when $R^f(\cdot)$ is
positive is as follows. The utility function $u^f(\cdot)$ is concave in profit for the risk averse firms but the indirect profit function for the layoff contract is convex in output prices; hence, the utility function is likely to be convex (concave) in output prices the less (more) the firm is risk averse.\textsuperscript{13} To the extent that the firm's degree of risk aversion is small so that its utility function is convex in output prices, then a mean-preserving increase in uncertainty of the output prices would, by Jensen's inequality, increase the expected utility level. To return to the equilibrium expected utility level of zero, the wage $w^*$ must rise. The opposite reasoning would establish that $w^*$ must fall if the firm's degree of risk aversion is large and which results in its utility being a concave function of output prices.

Regarding the equilibrium salary contract, which (from Proposition 3) is offered only by risk neutral firms, the equilibrium salary wage does not change with risk for reasons provided previously to justify (21).

Lastly, the effect of increasing risk on the equilibrium variable-wage contract can be shown as follows. From Proposition 3, the variable-wage contracts are offered only by risk averse firms which in equilibrium obtain zero expected utility of profit:

$$0 = (1-q)u^f(p_{11}f(n^*_1) - w_1n^*_1 - F) + qu^f(p_{21}f(n^*_2) - w_2n^*_2 - F)$$

(23)

where $w_1 = (w - \alpha)$. Condition (23) together with (7) describe the competitive supply of insurance contracts by each risk averse firm to its $n^*_i$ workers. The demand for employment insurance is given by (13), which has been shown to establish that
\[ v \equiv \frac{w_1}{w_2} > 1 \quad (24) \]

The competitive equilibrium system, made up of (23), (7), (13) and (24), determines \( n^*, w_1, w_2, \) and \( v \), given the values of \( p_1 \) and \( p_2 \). It turns out that increasing risk has an ambiguous effect on \( w_1 \) and \( w_2 \) but

\[
\frac{\partial v}{\partial p_1} \bigg|_{(19)} = \frac{v(1-q)\left[u^f(\pi_{1z}) - u^f(\pi_{2z})\right]}{n^*[(1-q)u^f(\pi_{1z})w_1 - qu^f(\pi_{2z})w_2]} < 0 \quad (25)
\]

because the numerator is negative since \( u^f(\cdot) \) is strictly concave for risk averse firms; and the denominator is positive if the stability of equilibrium system is to be satisfied.\(^{14}\) Thus increasing risk would tend to reduce the variation (across the states of the world) of wages in the variable-wage contract.
V. CONCLUSION

A model of the labor market is presented in which the supply and demand of labor services are viewed respectively as the demand and supply of employment insurance. The competitive equilibrium that ensues from such a model is a set of contracts that may include the no-insurance layoff (fixed-wage/variable-employment) contract, and the guaranteed-employment insurance contracts like the salary (fixed-wage/fixed-employment) and the variable-wage (variable-wage/fixed-employment) contracts. While the layoff contract represents one polar case in which all the risk of fluctuating output demand is absorbed by workers, the salary contract is the opposite polar case where all the risk is shifted to risk neutral firms by way of workers purchasing a full insurance coverage. In between, the variable-wage contract depicts the situation where the risk is shared by both risk averse firms and workers.

The A-B result that equilibrium may admit only fixed-wage contracts like the layoff and salary contracts is derived as a limiting special case which occurs when only all firms are risk neutral. However, whereas the sole existence of risk neutral firms in the A-B model is imposed as an a priori assumption, it arises in our model as an equilibrium phenomenon akin to Frank Knight's market self-selection process. That is, if firms who are risk neutral are sufficiently plentiful to provide full contract-employment, then in competitive equilibrium, risk averse firms cannot survive since they cannot offer at least equally attractive contracts relative to risk neutral firms.

If, on the other hand, risk neutral firms are limited in number so that by themselves they cannot provide full contract-employment, then competitive equilibrium must support both risk neutral and risk averse firms with the former offering either the layoff or the salary contract and
the latter offering either the layoff or the variable-wage contract. Also, in equilibrium, risk neutral firms may, but not always, extract an excess expected profit or 'rent' for being able to absorb more risk than are risk averse firms. The exact configuration of the competitive equilibrium contract set in this case depends on the value of $z$ (defined as the value of leisure and government unemployment compensation). The smaller is $z$ the less attractive to workers is the no-insurance layoff contract, and the more frequently would those employment insurance contracts, like the salary and variable-wage contracts, be observed in equilibrium. On the other hand, the larger is $z$, the less attractive to workers are the salary and variable-wage contracts, and the more frequently would the layoff contract occur in equilibrium. This suggests a testable hypothesis that if women have a higher value of leisure due to their higher productivity of home production, then they should be observed more frequently on layoff contracts than their male counterparts.

The impacts of increasing uncertainty or risk of the output prices on the competitive equilibrium contracts vary depending on whether or not equilibrium supports both risk neutral and risk averse firms. If only risk neutral firms are supported in competitive equilibrium, then increasing risk would affect the equilibrium contracts, which include only the layoff and salary contracts, in the following manner: the layoff contract wage increases; but the salary contract wage remains unaltered. If, on the other hand, both risk-neutral and risk averse firms are supported in competitive equilibrium, then the equilibrium contract set, which may include the layoff, salary and variable-wage contracts, react to increasing risk as follows: the layoff contract wage tends to increase (decrease) the smaller (larger) degree of risk aversion on the part of risk-averse firms; the salary contract wage remains unchanged; and the wages in the variable-wage contract tends to be less variable across the states of the world.
The analysis presented so far views labor supply as an individual decision problem. Since most individuals belong to families, a natural direction to proceed is to examine how the equilibrium contract set would emerge from the standpoint of a family decision problem. For instance, what predictions can be drawn regarding the choice, from among the various labor contracts, between a husband and a wife; and between men, with and without working wives?
Footnotes

*The author is grateful to R. Harris, J. Markusen, P. Howitt, and N. Tomes for helpful comments in the earlier draft of this paper. All errors are the responsibility of the author.

1 The risk-asymmetry assumption has been rationalized in the A-B model on various grounds. One is that firms are generally owned by wealthy shareholders who are able to minimize risk through their diversified asset portfolio holdings. Second, stockholders, because of their greater wealth are effectively less risk averse than workers. However, even these arguments do not justify that all firms should be treated as completely risk neutral.

2 If firms are risk neutral, then they would be indifferent between a variable-wage and fixed-wage contract provided the fixed wage in the latter is equal to the expected wage of the former. However a worker who is risk averse would be willing to pay a risk premium by accepting, in the fixed-wage contract, a fixed wage lower than the expected wage of the variable-wage contract. Hence, by the risk-asymmetry assumption, all fixed-wage contracts dominate variable-wage contracts.

3 Ever since the A-B model, alternate models based on transactional costs, turnover costs, and so on, have been presented to explain the occurrence of wage rigidity and layoff unemployment. See for example Varian [15], Salop [13], and Mayers and Thaler [9] amongst others.

4 A formalization of Knight's [5] market self-selection process in which firms, in equilibrium, are formed by less risk averse individuals has been presented by Kihlstrom and Laffont [4]. Their model, however, ignores the possibility of various types of contracts to exist in equilibrium.
The sufficiency condition is automatically satisfied because of the strict concavity of $f(\cdot)$.

The sufficiency condition is automatically satisfied because of the concavity of $u^f(\cdot)$ and the strict concavity of $f(\cdot)$.

The term "full contract-employment" is coined to contrast with the "full employment" equilibrium condition of the certainty model. Since the equilibrium contract set may include layoff contracts, the "full contract-employment" condition, while it ensures every worker a contract, does not guarantee full or fixed employment to those on layoff contracts.

All zero expected utility contracts satisfy

$$
(1-q) [u^f(\pi_{1L}) - u^f(\pi_{1L})] + q[u^f(\pi_{2L}) - u^f(\pi_{2L})] = 0
$$

By the concavity of $u^f(\cdot)$, we have

$$
u^f(\pi_{1L}) - u^f(\pi_{1L}) \leq u^f(\pi_{1L})[\pi_{1L} - \pi_{1L}]
$$

and

$$
u^f(\pi_{2L}) - u^f(\pi_{2L}) \leq u^f(\pi_{2L})[\pi_{2L} - \pi_{2L}]
$$

Substituting (F2) and (F3) into (F1) yields

$$
u^f(\pi_{1L})(1-q)[\pi_{1L} - \pi_{1L}] + u^f(\pi_{2L})q[\pi_{2L} - \pi_{2L}] \geq 0
$$

$$
u^f(\pi_{2L})[(1-q)[\pi_{1L} - \pi_{1L}] + q[\pi_{2L} - \pi_{2L}]] > 0
$$

because $\pi_{2L} < \pi_{1L}$ and $u^f(\cdot)$ is strictly concave for risk averse firms. Hence condition (15) because $u^f(\cdot) > 0$.

The fixed cost term, $F$, in the zero expected profit condition,

$$
0 = (1-q)(p_1 f(n_1) - w^o n_1 - F) + q(p_2 f(n_2) - w^o n_2 - F)
$$

satisfies $p_i f_i(n_i) - w^o = 0$, serves the purpose of generating an interior solution $n_i > 0$. Without $F$, it is possible for the zero profit condition to degenerate into a solution where $n_i = 0$. 


10 In the layoff contract, the profit function \( \pi_L = pf(n) - wn - F \)
where \( n \) satisfies \( pf'(n) - w = 0 \). Thus \( n = h\left(\frac{w}{p}\right) \) where \( h \equiv f^{-1} \) and \( h' = 1/f'' < 0 \).
Now substitute \( h\left(\frac{w}{p}\right) \) for \( n \) to obtain the 'indirect profit' (as a function of \( p \))
\[ \pi_L^*(p) = p f(h\left(\frac{w}{p}\right)) - wh\left(\frac{w}{p}\right) - F. \]
This function is an increasing convex function of \( p \) because \( \pi_L'(p) = f(h\left(\frac{w}{p}\right)) > 0 \) and \( \pi_L''(p) = -\frac{f'}{f''} \frac{w}{p^2} > 0 \) since \( f'' < 0 \).

11 Consider the zero expected profit condition
\[ 0 = (1-q)(p_1 f(n^*) - s^n_0 n^* - F) + q(p_2 f(n^*) - s^n_0 n^* - F) \quad \text{(F4)} \]
where \( n^* \) satisfies
\[ E[p]f'(n^*) - s^0 = 0 \quad \text{(F5)} \]
Totally differentiating (F4), and then using (F5), yields
\[ n^* ds^0 = f(n^*)[(1-q)dp_1 + qdp_2] \]
\[ = 0 \quad dp_1 \]
because of (19). Hence we have (21).

12 In the salary contract, profit \( \pi_I = pf(n^*) - s^n_0 n^* - F \)
and \( n^* \) is invariant to \( p \). Thus the 'indirect profit' (as a function of \( p \))
\( \pi_I^*(p) \) is linear in price \( p \) because \( \pi_I'^*(p) = f(n^*) > 0 \) and \( \pi_I''(p) = 0 \).

13 In the layoff contract, we know (from footnote 10) that the indirect profit function \( \pi_L^*(p) \) is strictly convex with \( \pi_L'^*(p) > 0 \) and \( \pi_L''(p) > 0 \). Now the utility as a function of price \( p \) can be defined as \( u^{f^*}(p) \equiv u^f(\pi_L^*(p)) \) with
\[ u^{f^*'}(p) = u^{f'}(\pi_L^*(p)) > 0 \] and \( u^{f^*''}(p) = u^{f''}(\pi_L^*(p)) \geq 0 \) as
\[ R^f(\pi_L^*) \leq \pi_L''(p)/\pi_L'^*(p)^2 \]
where \( R^f(\cdot) \) is the Arrow-Pratt measure of absolute risk aversion. Thus \( u^{f^*}(\cdot) \) is convex (concave) the smaller (larger) is the degree of risk aversion on the part of firms.
Totally differentiating the equilibrium system (23), (7), (13) and (24) with respect to the endogeneous variables \( w_1, w_2, v, \) and \( n^* \), and the exogeneous variables \( p_1 \) and \( p_2 \), we obtain, after eliminating \( dw_1 \) and \( dw_2 \) by substitution,

\[
\begin{bmatrix}
  a_{11} & a_{12} \\
  a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
  \frac{\partial n^*}{\partial p_1}(19) \\
  \frac{\partial v}{\partial p_1}(19)
\end{bmatrix}
= \begin{bmatrix}
  -(1-q)\left( u^{f'}(\pi_{11}) - u^{f'}(\pi_{21}) \right) \\
  -[f(n^*)R^f(\pi_{11}) + \frac{(1-q)}{q}R^f(\pi_{21})] \\
  + \beta \left( \frac{\partial n^*}{\partial p_1} - \frac{(1-q)}{q} \frac{\partial n^*}{\partial p_2} \right)
\end{bmatrix}
\]

where \( a_{11} = 0; a_{21} = \beta \equiv R^f(\pi_{11})(p_1 f'(n^*) - w_1) - R^f(\pi_{21})(p_2 f'(n^*) - w_2) > 0 \)

because \( p_1 f'(n^*) > w_1 \) and \( p_2 f'(n^*) < w_2 \); \( a_{12} = -\frac{R}{v} \left[ (1-q)u^{f'}(\pi_{11})w_1 - qu^{f'}(\pi_{21})w_2 \right] \);

\( a_{22} = \frac{1}{v} \left[ \beta \left( \frac{\partial n^*}{\partial w_1} - \frac{\partial n^*}{\partial w_2} \right) - n^* (R^f(\pi_{11})w_1 + R^f(\pi_{21})w_2) - (R^e(w_1)w_1 + R^e(w_2)w_2) \right] \);

and \( R^f(\cdot) \) and \( R^e(\cdot) \) are respectively the firm's and worker's absolute risk aversion. Now, the stability of the above system requires (by the Routh theorem) (i) \( a_{11} + a_{22} < 0 \) and (ii) the determinant of the left-hand-side matrix \( \Delta = a_{11} a_{22} - a_{12} a_{21} = \beta \frac{R}{v} \left[ (1-q)f'(\pi_{11})w_1 - qu^{f'}(\pi_{21})w_2 \right] > 0 \)

\( = [(1-q)u^{f'}(\pi_{11})w_1 - qu^{f'}(\pi_{21})w_2] > 0 \) since \( \beta > 0 \). The denominator in (25) is therefore positive.
References


