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1. INTRODUCTION

The recent survey by Baltensperger (1978) shows that the question of why bankers undertake non-price rationing of credit is still very much unanswered. Previous attempts to address the question have ended up merely assuming the answer. For example, the attempts prior to the 1960's all involved the assumption that interest rates could not adjust so as always to clear the market for credit, the attempt by Hodgman (1960) did not really address the issue of interest rate determination, and the later attempt by Modigliani and Jaffée (1969) involved the assumption that banks cannot charge different rates to all of their customers. The only analysis in the literature that is free from this criticism is that of Jaffée and Russell (1976), which focusses on moral hazard concerning default by dishonest customers.

Meanwhile, recent developments in the theory of labour contracts (for example, Azariadis, 1975, 1976; Baily, 1974, 1977; Gordon, 1973) have made progress in answering the closely analogous question of why firms lay workers off rather than adjusting wages. The purpose of the present paper is to show how these developments can be used and extended so as to provide a tentative answer to the question of why bankers ration credit.
In brief, our answer is that credit rationing exists as part of an equilibrium risk-sharing arrangement between a bank and its customers. A borrower and lender can benefit not only from trading loan contracts now, but also from an "understanding" or "implicit contract" concerning the amounts they will be willing to trade, and at what prices, under various conditions in the future. By means of such arrangements banks and their customers can share the risks associated with an uncertain future. Thus their arrangements may be similar to insurance contracts in which the less risk-averse party agrees for a fee to bear some of the risks to which the other party would otherwise be exposed. The example that we shall use is the risk of changing costs of funds to financial intermediaries. If loans were always negotiated in spot auction markets then customers would be exposed to the risk of fluctuating interest rates on their loans. A bank may be willing to insure the customer against part of such risks by a policy of keeping interest rates less variable than they would be on spot auction markets, in return for which the customers may be willing to compensate the bank in the form of a higher average interest rate. But by damping the movements in interest rates these arrangements open up the possibility of non-price rationing.

One merit of the present answer is that it directs attention back toward the issue often raised by bankers themselves when asked to explain why they ration credit; namely, the issue of customer relations. As in the market for labour, it is typical for the relationship between trading partners in the market for bank loans to be involved and highly personal. The object being traded is heterogeneous (in this case it involves the trustworthiness of the borrower) and on either side of the market
there are non-trivial costs involved in switching one's trading partner. Thus the normal arrangement between trading partners, whether in an explicit contract or an implicit understanding, will take into account the advantages to both sides of maintaining a continuous relationship, as well as the more immediate advantages from mutually beneficial trade. The present answer depends crucially upon the existence of such customer relations. In particular, the analysis below implies that if there were no costs to switching trading partners then credit rationing could never occur, because there would be no incentive for anyone to enter into such risk-sharing arrangements at all. Furthermore, the analysis offers an explanation of why there is a tendency for banks to ration least heavily those customers with the longest standing relationship.

The answer to why non-price credit rationing exists also directs attention away from the issue most heavily stressed by recent authors, namely that of default risk. The empirical evidence shows that the default risk on the typical bank loan is for all practical purposes negligible. This is not to suggest that banks do not spend a great deal in attempts to screen our potential defaulters. But it does suggest that any analysis that implies, as does that of Modigliani and Jaffée (1969), that customers whose probability of defaulting is negligible will be subjected to a negligible degree of non-price credit rationing, is inconsistent with such evidence. The present answer, as we shall see, is consistent with the assumption that all customers' risks of default are non-trivial. Furthermore, the cost of screening out potential defaulters is an important part of the answer. But it is also consistent with the evidence suggesting that even those customers most heavily rationed may have a negligible probability of defaulting.
The rest of the paper is organized as follows. Section 2 lays out a model of implicit contracts, formally quite similar to that of Azariadis (1976), and demonstrates some basic results, including the results that credit rationing may exist under some circumstances, and that long-established customers are less likely than others to be rationed. Section 3 discusses the nature of such rationing, especially with respect to the issues of whether or not it is consistent with the universal pursuit of self-interest, and in what sense the results depend upon the existence of "customer relations". Section 4 digresses briefly to show how the argument could be amended to include explicitly the risk of default. Section 5 characterizes the nature of equilibrium "arrangements", with particular attention to the influence of various exogenous variables upon the interest rates charged by banks and upon the incidence of credit rationing. Section 6 adds some concluding comments.

2. THE MODEL

We shall consider implicit contracts between a particular bank and its several customers. The random variable whose fluctuations introduce an element of risk is the bank's cost of funds, \( i \). This can be thought of as some economy-wide interest rate. The set \( I \subset \mathbb{R}_+ \) is the set of all possible values of this interest rate, and its random behavior is believed by all concerned to be governed by the probability distribution \( q(i) \), where \( q(i) > 0 \) for all \( i \in I \). Since the values of \( i \) define for our purposes the states of the world, we shall use the expression "in state \( i \)" to mean "when the cost of funds to banks--the interest rate--equals \( i \)."
The bank has two classes of customers, indexed by \( k = 0, 1 \). Class 0 customers are those with whom it has a long-established relationship; we call them the "old customers". Class 1 customers are "new". All customers within a particular class are identical. Formally, the only difference between the two classes is that, from the bank's viewpoint, lending to an old customer is less costly than lending to a new customer, for two reasons. First, old customers may be supposed already to have an account with this particular bank, which may produce some economies in administering loans. This factor is much stressed by the literature emphasizing the "joint-product" nature of the banking enterprise (for example, Hodgman, 1963). Second, part of this administrative cost takes the form of screening the customer to ensure that he is a "good risk". This cost may be incurred on every loan to a new customer but not to an old one, because the old one has already been screened in the past and because the bank also has, we may presume, some first-hand experience in dealing with the customer, which can be used to substitute for other resources in screening (see also Kane and Malkiel, 1965). To capture this difference between old and new customers we assume that the total administrative cost of lending to \( n^0 \) old customers and \( n^1 \) new customers is \( c(n^0 + n^1) \), where \( c \) is a cost function with positive and increasing marginal cost \( (c' > 0, c'' > 0) \), and \( 0 \leq \alpha < 1 \). In other words we assume that old and new customers are perfectly substitutable as far as administrative costs are concerned, but that lending to an old customer costs only the fraction \( \alpha \) as much as to a new one.

In keeping with the literature on contracts for labour we shall suppose that one side of the market is systematically more risk averse than
the other. In particular let us suppose that the customers are risk-averse but the bank is risk-neutral. In the former literature such an asymmetrical assumption is often justified by the natural selection that tends to make entrepreneurs systematically more willing to bear risk than are workers. Likewise, in the present case, the asymmetry can be justified by reference to the traditionally cited role of banks as financial intermediaries that specialize in bearing financial risks that neither its creditors nor its debtors could bear as efficiently in the absence of such intermediaries. In any event it should be noted that in what follows, while an extreme assumption of risk-neutral banks is crucial for the particular form of many of the results, such as the fixed loan-rate theorem, it is quite inessential to the primary purpose of rationalizing the phenomenon of credit rationing, which would still exist under a large variety of alternative assumptions. Thus the chief merit of our extreme assumption is the analytical simplicity that it permits.

Again in the interest of simplicity assume that each customer wishes to borrow one unit of money, and that the bank must grant this loan either in full or not at all. Let \( n^k(i) \) denote the number of class \( k \) customers (\( k = 0,1 \)) to whom the bank will lend in state \( i \). Let \( r^k(i) \) be the corresponding rate of interest charged on the loan. Then the bank will seek to maximize its expected profits:

\[
\frac{1}{I} \sum_{i \in I} \left[ \sum_{k=0}^{1} n^k(i) (r^k(i) - i) - c(\alpha n^0(i) + n^1(i)) \right] q(i).
\]

Customers of different classes differ from each other only with respect to administrative costs. Thus each customer is assumed to have a
utility function $u(\cdot)$, with $u' > 0$, $u'' < 0$, which depends upon the random return, $x$, to a project that he wishes to finance by means of a loan. If he gets the loan at a rate $r$ his expected utility is $w(r) \equiv E_x u(x-r)$. If he is rationed we assume that his utility is equal to $K$. Note that:

(1) \quad w', \quad w'' < 0.

A "contract" is a collection: $\delta = \{r^k(i), n^k(i), m^k\}_{k=0,1, i \in I}$, where $m^k$ denotes the number of class $k$ customers who enter into the contract, with $n^k(i) \leq m^k$ for all $i$, $k$. Whenever $n^k(i) < m^k$ we assume that the choice of which class $k$ customers to be rationed is made by purely randomly selection; in such a case the "understanding" of a class $k$ customer is that his probability of getting his loan equals $n^k(i)/m^k$. Thus the expected utility that a class $k$ customer derives from entering into a contract equals:

$$K + \sum_{i \in I} (w(r^k(i)) - K) \left(\frac{n^k(i)}{m^k}\right) q(i).$$

When choosing what contract to offer its customers the bank faces a competitive constraint. The contract must permit its class $k$ customers to attain at least the expected utility $\lambda^k$, $k=0,1$. We may assume that:

(2) \quad \lambda_k > K; \quad k=0,1.

A contract is said to be "optimal with respect to $(\lambda^0, \lambda^1)$" if it maximizes the bank's expected profit subject to the constraint that the expected utilities of each class $k$ customers is at least $\lambda^k$. In other words, $\lambda^k$ can be thought of as the price that the bank must pay in order to acquire a class $k$ customer, and an optimal contract can be likened to a point on his demand curve at those prices.

We shall also be interested in characterizing a situation of equilibrium, in which the "prices", $\lambda^k$, are determined endogenously. Suppose that this
bank is in an industry of identical banks in which there are a total of \( m^0 \) customers per bank who wish to establish a long-term relationship with some bank; and \( m^1 \) who, for whatever reason, prefer to remain unattached. Then an equilibrium is a situation in which the optimal contract of any bank involves \( m^0 \) old customers and \( m^1 \) new ones. Formally, we say that a contract is an "equilibrium with respect to \((m^0, m^1)\)" if it is optimal with respect to some \((\lambda^0, \lambda^1)\) and it is entered into by \( m^k \) customers of each class \( k \).

This notion of equilibrium is somewhat unorthodox. The bank sees \( \lambda^0 \) as the expected utility that must be offered in order to maintain any given stock of old customers in the long run. But in the short run we may presume that it could keep at least some of these customers by offering them less than \( \lambda^0 \), for if an old customer were to leave for any other bank he would now become a new customer. The analysis of Phelps and Winter (1970) of a similar problem suggests that the bank would attempt to exploit this temporary monopoly power to an extent depending upon its rate of time preference. We are implicitly assuming that the bank exhibits no positive time preference, and thus chooses to act perfectly competitively.

The Kuhn-Tucker conditions characterizing an optimal contract are:

1. \( q(i) n^k(i) + \phi^k q(i) n^k(i) w^r(k(i)) = 0; \forall i, k, \) (3)
2. \( q(i)[r^k(i) - i - \alpha^k c'(w^r(k(i)) - K)] - \mu^k(i) \leq 0, \) \( n^k(i) \leq 0; \forall i, k, \) (4)
3. \( \phi^k(K - \lambda^k) + \sum_i \mu^k(i) \leq 0, m^k \leq 0; \forall k, \) (5)
4. \( \sum_i q(i) n^k(i)[w(r^k(i)) - K] + m^k(K - \lambda^k) \leq 0; \phi^k \geq 0, \forall k, \) (6)
5. \( m^k - n^k(i) \geq 0, \mu^k(i) \geq 0; \forall i, k, \) (7)
with at least one equality in each complementary pair of conditions in (4) - (7), and where \( \alpha^0 = \alpha, \alpha^1 = 1, \) and the \( \phi^k \)'s and \( \mu^k(i) \)'s are Lagrangean multipliers.

Note that if \( \delta \) is an equilibrium with respect to \( (m^0, m^1) \), then:

\[
\phi^k > 0 \text{ if } m^k > 0; \quad k=0,1.
\]

For suppose that \( m^k > 0 \) and \( \phi^k \neq 0 \). Then, from (6), \( \phi^k = 0 \). Therefore, from (3), \( n^k(i) = 0 \) \( \forall i \in I \). Therefore, from (6), \( K - \lambda^k \geq 0 \), contradicting (2).

Our first result illustrates in extreme form the possibility that such contracts may involve the bank insuring its customers against risks of variations in the rate of interest:

Theorem 1: In any optimal contract, \( r^k(i) = r^k(i') \) for all \( i, i' \in I \) such that \( n^k(i) \neq 0 \neq n^k(i') \).

Proof: follows immediately from (1), (3), and (8).

Thus within each class whenever a loan is granted the same rate will be charged, independently of variations in the economy-wide interest rate, \( i \).

It follows from Theorem 1 that we may safely assume that \( r^k(i) \) is constant for all \( i \), \( (r^k(i) = r^k > 0 \forall i, k) \) since neither the bank nor the class \( k \) customer cares what the loan rate is if \( n^k(i) = 0 \). Next, note that, from (3), (4) and Theorem 1,

\[
q(i)[z(r^k) - i - \alpha^k c'(\cdot)] \geq \mu^k(i); \quad n^k(i) \geq 0; \quad \forall i, k
\]

with at least one equality in each complementary pair, where:

\[
z(r) \equiv r - [w'(r)]^{-1}[w(r) - K].
\]

The function \( z(\cdot) \) is important for the following analysis. It may be interpreted as the shadow-value to the bank of an additional loan at the
rate \( r \) in state \( i \); that is, the interest revenue plus the addition in expected profits that it realizes because the additional loan reduces the probability of rationing in state \( i \) by the amount \( \frac{1}{k} \) to each of its \( m^k \) customers, each of whom can be charged an interest rate that is higher than otherwise by the amount \( \frac{1}{m} \left[ w'(r) \right]^{-1} [w(r) - K] \) with no loss in utility.

Thus (9) asserts that if \( 0 < n^k(i) < m^k \) then the shadow-value of each loan must equal its marginal cost.

Next, note that, from the complementary equalities of (5) – (7),
\[
\sum_{i} q(i) n^k(i) (w(r^k) - K) \phi^k = \sum_{i} \mu(i)n^k(i); \ k = 0, 1.
\]
Therefore, from the sum of the complementary equalities in (4),
\[
(11) \quad \sum_{i} q(i) n^k(i) (r^k - i - \alpha c'(r)) = 0; \ k = 0, 1.
\]
In other words, the expected return on class \( k \) loans must equal their expected marginal cost.

Next, we see that there are indeed circumstances under which creditrationing occurs (i.e., \( n^k(i) < m^k \)):

**Theorem 2:** If \( \delta \) is an equilibrium with respect to \( (m^0, m^1) > 0 \) with \( n^k(i) = m^k \) \( \forall i, k \), then \( w(r^k) + w'(r^k)(i - E i) \geq K; \forall i, k \).

**Proof:** Suppose that \( \delta \) is such a contract. Then from (11),
\[
(12) \quad r^k = E_i + \alpha c'(cm^0 + m^1); \ k = 0, 1
\]
Also, from (7) and (9),
\[
(13) \quad z(r^k) - i - \alpha c'(cm^0 + m^1) \geq 0 \ \forall i, k.
\]
The theorem follows from (10), (12), and (13).

These necessary conditions are illustrated in Figure 1, which is constructed as follows. If no rationing is to occur, then \( r^0 \) and \( r^1 \) can
Figure 1
be calculated from (12). Thus the lines \( R^0_0 \) and \( R^1_1 \) can be constructed by finding the tangent to \( w(i) \) at each \( r^k \) and shifting it in so as to pass through \( E_i \). Now according to Theorem 2 if there is to be no credit-rationing then there can be no value of \( i \) in \( I \) greater than \( i^1_{\text{max}} \) or \( i^0_{\text{max}} \). Of course this condition will not always hold on the basis of the assumptions made so far. Thus it is possible (but not necessary) for credit rationing to occur as part of an equilibrium contract.

It can be verified directly from Figure 1 that the likelihood of credit rationing occurring is larger the larger is: (1) the mean interest rate \( E_i \) (because an increase in \( E_i \) shifts the entire distribution to the right but it thereby makes \( R^0_0 \) and \( R^1_1 \) lower and steeper, thus reducing \( i^k_{\text{max}} - E_i \)), (2) the variance of interest rates, \( \sigma_i^2 \), (3) the marginal cost of loans, \( c'(\cdot) \), (4) the ratio \( m^1/m^0 \) for a given value of \( m^0 + m^1 \), (5) the expected utility derived by rationed customers, \( K \), or (6) the inverse of \( Ex \), the expected return from the typical customer's project (because reducing \( Ex \) shifts the \( w(\cdot) \) curve down and makes it steeper, thereby doing the same to \( R^0_0 \) and \( R^1_1 \)).

The last result of the present section shows that, as long as \( r^0 \leq r^1 \), customers with an established relationship will not be rationed as heavily as those without:

Theorem 3: If \( \delta \) is an equilibrium with respect to \( (m^0, m^1) > 0 \), with \( r^0 \leq r^1 \), then \( n^0(i) < m^0 \) only if \( n^1(i) = 0 \).

Proof: See Azariadis (1976, Theorem 3).
3. THE NATURE OF RATIONING

The most difficult problem involved in trying to explain the existence of credit rationing is how to reconcile it with the universal pursuit of self-interest. Why does the rationed customer not negotiate with a bank so as either to secure a loan at the same rate at which other, identical, customers are securing loans, or at least to bid up the rate of interest on such loans? For at a slightly higher rate the customer would still want a loan, and the bank would expect to earn more profits. Thus it seems as if credit rationing implies unexploited gains from trade that could be perceived by both parties.

The answer to this question given by the present approach is that, although the rationed customer would still gain from getting a loan at a slightly higher rate, the bank would not. For if in that particular state of the world the rate were raised on loans to all customers the bank would have to make other adjustments to its contract which, if the present contract were indeed optimal, would be inefficient. On the other hand if the customer undertook a separate negotiation with the bank to change the rate on his loan only then the only equilibrium would, as we shall show in the next paragraph, be one in which no loan is granted. In other words, even though the rationed customers would prefer to have a loan at the going rate rather than no loan, he would not prefer to have a loan at the best rate that could be negotiated separately with the bank. The bank is unwilling to grant loans to this particular customer at a slightly higher rate because it is making short-run losses on loans at such a rate in order to fulfill its contractual commitments. Thus, as in the example of labour markets, (Barro, 1977) there are no gains from trade that remain unexploited, despite the apparent presence of an excess demand for loans.
To prove this assertion, note that the minimum rate at which the bank would be willing to grant a loan under private negotiation would be the marginal cost: \( i + \alpha c'() \). But, from (2) and (6),

\[
(14) \quad w(r^k) > K; \quad k=0,1.
\]

So, from (4), (7), and (14), if class \( k \) customers are being rationed then \( r^k \) is no more than this minimum value. Thus, by (1), (10), (7), and (9)

\[
w(i + \alpha c'( )) - K < w(r^k) - K - w'(r^k)[r^k - i - \alpha c'( )] = - w'(r^k)(z(r^k) - i - \alpha c'( )) \leq 0
\]

if class \( k \) customers are rationed. In other words, the gain to the customer, \( w(i + \alpha c'( )) - K \) from securing a loan at this minimal rate is not positive.

As in the example of labour markets (Barro, 1977), the kind of price-stickiness implied by this explanation of rationing is real, not nominal. Suppose, for example, that the only source of underlying uncertainty was a variable rate of inflation over the course of any loan. Then the analysis could proceed as above with \( i \) and \( r^k \) interpreted as real rates of interest. In this case \( i \) would be constant, there would be only one "state of the world", Theorem 1 would imply a constant real rate of interest on loans, and Figure 1 would imply that no credit rationing would occur in this single state. It is important to recognize this distinction, because it shows that the present analysis does not rest upon or imply the stickiness of any nominal price when the price level is varying. It also shows that the present analysis is not directly refuted by the experience of fluctuations in the nominal rate charged on loans.

As mentioned in the introduction this explanation does rely heavily on the presence of costs to switching trading partners. For suppose there were no such costs and that the implicit contract between a bank and customer represented a commitment only on the part of the bank to meet the terms of the contract. In particular, suppose that customers could always costlessly
search out another bank in order to get a loan at the going market rate. Then, to begin with, the distinction between new and old customers would vanish (otherwise there would be a cost to switching; namely that an old switcher would lose his "old" status). Also, the utility of not getting a loan would now become \( \max(w(r'(i)), K) \), where \( r'(i) \) is the "market" rate on loans in state \( i \), rather than just \( K \). Thus a bank that wished to retain any customers at all in state \( i \) could charge no more than \( r'(i) \), and optimal contracts would be defined as solutions to the problem of maximizing

\[
\sum_{i \in I} q(i) (r(i) - i) - c(n(i)) \quad \text{s.t.} \quad r(i) \leq r'(i), \quad n(i) \geq 0 \quad \forall i.
\]

That is, whenever \( n(i) > 0 \), \( r'(i) = r(i) = i + c'(n(i)) \). But, given perfect mobility of customers \( r'(i) \) is an equilibrium rate only if \( n(i) \) equals the number of customers per bank actually desiring a loan at the rate \( r'(i) \). Thus, an equilibrium is a contract with:

\[
(r(i), n(i)) = \begin{cases} 
(i + c'(m), m) & \text{if } w(i + c'(m)) > K, \text{ or} \\
(w^{-1}(K), \max(0, s(w^{-1}(K) - i))) & \text{otherwise}
\end{cases}
\]

where \( m \) is the given number of customers per bank, and \( s(\ ) \) is the inverse of the function \( c'(\ ) \). It follows from (15) that Theorem 1 no longer holds; that is, the loan rate now fluctuates. Also, the possibility of credit rationing now disappears because, according to (15), any customer desiring a loan is always granted one. In fact, we would hardly say that this equilibrium is one with contracts at all. For the customers see themselves as always able to go to any bank at all for a loan at the market rate and banks see themselves as able to attract any number of customers at that rate, irrespective of any understandings. The price-quantity combinations (15) are nothing other than the competitive auction-market combinations.

What the above example represents is a breakdown of contracts because of moral hazard: If a bank commits itself to lend at a given rate over a range
of i, sometimes taking short-run profits or short-run losses, it will find
more firms borrowing over the loss range than when it would obtain short-
run profits. In the limit a bank maintaining a fixed loan rate would
have no customers when profits were positive and an arbitrarily large number
of customers when profits were negative. Now, since there are gains to
both customers and banks from insulating firms from interest rate fluctuations
it may well pay the borrower and lender to enter into explicit contracts
that impose switching costs to customers in exchange for insurance against
fluctuating loan rates. If monitoring costs were low we would expect these
contracts to arise in the absence of other costs. 7

4. DEFAULT RISK

To incorporate default risk, suppose, as do Modigliani and Jaffeé
(1969) and others, that the bank's expected revenue from a loan at any
rate r is: e(r) = \E_x \min(x, r), and that the customer's expected utility
from getting a loan at the rate r is: g(r) = \E_x u(x - \min(x, r)). It can
be shown that e(r) > 0, 0 < e'(r) < 1, e''(r) < 0, and g'(r) < 0. Assume
also that g''(r) < 0. Then the problem that must be solved by any optimal
contract is to maximize \[ \sum_{i \in I} \frac{1}{k} n^k(i)[g(r^k(i)) - r - c_n^k(i)] \]
subject to: \[ \sum_{i \in I} n^k(i)[g(r^k(i)) - K] + m^k[K - \lambda^k] \leq 0, k = 0, 1, \text{and} \]
m^k - n^k(i) \geq 0, m^k \geq 0, n^k(i) \geq 0, \forall k, i. The "fixed-rate" Theorem (1) and
the "preferred customer" Theorem (3) go through as before. Furthermore, the
analogue to Figure 1 can be drawn with e on the horizontal axis and g = r(e) =
g(r(e)) replacing the function w(r), where r(e) is the inverse of the function
e(r). Thus by analogous reasoning credit rationing will occur unless for all i \in I
\[ g(r^k) + (g'(r^k)/e'(r^k))(i - E_i) \leq K \]
where \[ r^k = r(E_i + c \alpha \epsilon(r^k + \mu + m^k)); k = 0, 1. \]
Thus the analysis can easily be extended to cover the case of explicit default
risk, although only at the cost of extraneous notational complexity.
5. CHARACTERIZING EQUILIBRIUM CONTRACTS

Let us return to the model analyzed in Section 2. We showed there the conditions necessary for no rationing to occur in equilibrium. The present section further characterizes the equilibrium when the possibility of rationing exists. First, Figure 2 shows how the number of successful customers, \( n^0(i) + n^1(i) \) varies in equilibrium as a function of \( i \), under the assumption that \( r^0 \leq r^1 \). Along this schedule the values of \( r^0 \) and \( r^1 \) as well as those of all exogenous variables are held constant. The meaning of the figure is self-explanatory, and its exact form follows directly from (7) and (9).

In the interest of simplicity suppose that \( i \) always remains inside the interval \((0, i_B)\) in Figure 2; that is, that old customers are never rationed. Suppose also that the function \( c'(\cdot) \) is linear, so that \( s'(\cdot) \), the inverse of \( c'(\cdot) \), is a constant, \( \tilde{s} > 0 \). Furthermore, suppose that the initial set of parameter values for all the conceptual experiments below is such that the partition: \( \{I_A = \{i \in I: 0 \leq i \leq i_A\}, I_B = \{i \in I: i_A < i < i_B\}\} \) remains unaffected by small enough parameter changes. Let \( \tilde{z}(r, K) \) denote the value of \( z(r) \) with the dependence upon \( K \) made explicit, and note that \( \partial \tilde{z} / \partial K < 0 \).

We shall consider the effects on two endogenous variables: (a) \( r^1 \)--the rate of interest on loans to new customers, and (b) \( \xi = 1 - \sum_{i} q(i) n^1(i)/m^1 \)--the probability of any given new customer being rationed, of changes in the following four parameters; (i) \( K \) --the utility of not acquiring a loan, (ii) \( E_x \) --the expected return to a customer's project, (iii) \( E_i \) --the expected economy-wide rate of interest, and (iv) \( \sigma \) --the variance of \( i \).

(a) The effects on \( r^1 \) can be calculated using the following equilibrium condition, which follows directly from (11):

\[
\sum_{i \in I_A} q(i)m^1[r^1 - i - c'(\alpha m^0 + m^1)] + \sum_{i \in I_B} q(i)[s(z(r^1) - i) - \alpha m^0][r^1 - z(r^1)] = 0.
\]
Figure 2

\[ 0 = (\frac{1}{1}) \mu \]

\[ \mu / (\mu / (1 - \mu) \mu) / \nu = (\frac{1}{1}) \mu \]

\[ 0 = (\frac{1}{1}) \mu + (\frac{1}{1}) \mu \]

\[ 0 = (\frac{1}{1}) \mu + (\frac{1}{1}) \mu \]

\[ (\frac{1}{1}) \mu + (\frac{1}{1}) \mu \]
The derivative of this expression with respect to \( r^1 \) is:

\[
J_r = \sum_{\mathcal{A}} q(i)m^1 + \sum_{\mathcal{B}} q(i) \cdot s \cdot \frac{\partial z^1}{\partial r} \cdot [r^1 - z^1] \\
+ \sum_{\mathcal{B}} q(i)(s(z^1 - i) - \alpha m^0)(1 - \frac{\partial z^1}{\partial r}) > 0.
\]

Thus the first result is that:

\[
\frac{\partial r^1}{\partial \mathcal{A}} = -J^{-1} \frac{\partial z^1}{\partial \mathcal{A}} \sum_{\mathcal{B}} q(i)(s(z^1 - i) - n^1(i)) < 0.
\]

To calculate the effects of \( \mathcal{E} \) we consider the effect of a uniform rightward shift in the distribution of \( x \). Note that \( \frac{\partial w(r)}{\partial \mathcal{E} x} = -w'(r) > 0, \)
\( \frac{\partial w'(r)}{\partial \mathcal{E} x} > 0, \) and \( \frac{\partial z(r)}{\partial \mathcal{E} x} = 1 + \frac{(\partial w'(r)/\partial \mathcal{E} x)(w(r) - K)/(w'(r))^2 > 0.} \)

Thus,

\[
\frac{\partial r}{\partial \mathcal{E} x} = -J^{-1} \frac{\partial z}{\partial \mathcal{E} x} \sum_{\mathcal{B}} q(i)(s(z^1 - i) - n^1(i)) > 0.
\]

Similarly, we calculate the effects of \( \mathcal{E} i \) as:

\[
\frac{\partial r^1}{\partial \mathcal{E} i} = J^{-1} \sum_{\mathcal{A}} q(i)m^1 + (r^1 - z^1)s \sum_{\mathcal{B}} q(i) \\
+ \sum_{\mathcal{A}} q(i)(s(z^1 - i) - n^1(i)) > 0.
\]

the sign of which is indeterminate. But the effect of the variance, \( \sigma \), can be calculating by defining \( i = \mathcal{E} i + \sigma(i' - \mathcal{E} i) \) for a fixed set of \( i' \), and differentiating with respect to \( \sigma \) at \( i = i' \):

\[
\frac{\partial r^1}{\partial \sigma} = J^{-1} \sum_{\mathcal{A}} q(i)m^1 + (r^1 - z^1)s \sum_{\mathcal{B}} (i - \mathcal{E} i)q(i) < 0.
\]

(To get this result note that \( \sum (i - \mathcal{E} i)q(i) = (E(i | \mathcal{I}_A) - \mathcal{E} i) \sum q(i) < 0, \)
\( \mathcal{I}_A \)

and similarly \( \sum (i - \mathcal{E} i)q(i) > 0. \)
(b) To analyze the effects of parameter changes on $\xi$, the probability of rationing, note that for parameter changes (i) and (ii) the effects depend entirely upon whether the overall effect on $z(r^1)$ is positive or negative. If it is positive, then Figure 2 shows that the first two linear pieces of the schedule shift rightward, implying that $n^1(i)$ never decreases but may increase. Thus, from the definition of $\xi$,

$$\text{sgn} \left( \frac{\partial \xi}{\partial K} \right) = - \text{sgn} \left( \frac{dz^1}{dK} \right)$$

But, from inspection of (17) and (18.i),

$$\frac{dz^1}{dK} = \frac{\partial z^1}{\partial K} = \frac{\partial z^1}{\partial r} \cdot \frac{\partial r^1}{\partial K}$$

$$= \frac{\partial z^1}{\partial K} \left[ \sum_{i} q(i) m^1_i + \sum_{i} q(i) s^1_i \frac{\partial z^1}{\partial r} (r^1 - z^1_i) + \sum_{i} q(i) n^1_i (1 - \frac{\partial z^1}{\partial r}) \right]_{I_A}$$

$$- \frac{\partial z^1}{\partial r} \sum_{i} q(i) (s(r^1 - z^1_i) - n^1_i) \right]_{I_B} J^{-1}_r$$

$$= \frac{\partial z^1}{\partial K} \left[ \sum_{i} q(i) m^1_i + \sum_{i} q(i) n^1_i (1) \right]_{I_A} J^{-1}_r < 0 .$$

Therefore,

$$\frac{\partial \xi}{\partial K} > 0 .$$

Next, note that, by inspection of (18.i) and (18.ii):

$$\frac{\partial r^1}{\partial Ex} = \frac{\partial r^1}{\partial K} \cdot \frac{\partial z^1}{\partial Ex} / \frac{\partial z^1}{\partial K}$$

So, from (21) and (19);
\[
\begin{align*}
\frac{dz^1}{dEx} &= \frac{\partial z^1}{\partial r} \cdot \frac{\partial r}{\partial Ex} \\
&= \frac{\partial z^1}{\partial r} \cdot \frac{\partial r}{\partial \Delta k} \cdot \frac{\partial \Delta k}{\partial Ex} \\
&= \left( \frac{\partial z^1}{\partial Ex} \right) \left( \frac{\partial z^1}{\partial \Delta k} + \frac{\partial z^1}{\partial r} \cdot \frac{\partial r}{\partial \Delta k} \right) \\
&= \frac{\partial z^1}{\partial Ex} \cdot \frac{dz^1}{d\Delta k} / \frac{\partial z^1}{\partial \Delta k} > 0 \\
(+) & \quad (-) & \quad (-)
\end{align*}
\]

Therefore,

(20.ii) \[ \text{sgn} \frac{\partial r}{\partial Ex} = - \text{sgn} \frac{dz^1}{dEx} < 0. \]

As can be seen from Figure 2, the effect on \( \xi \) of an increase in \( E_i \) depends upon whether or not \( z^1 \) increases by more than \( E_i \). But, from (17) and (18.iii),

\[
\begin{align*}
\frac{dz^1}{dE_i} - 1 &= \frac{\partial z^1}{\partial r} \cdot \frac{\partial r}{\partial E_i} - 1 \\
&= \int_x \left[ \frac{\partial z^1}{\partial r} \sum q(1) m^1 + s \cdot \frac{\partial z^1}{\partial r} \right] (r^1 - z^1) \sum q(i) \\
&\quad \left( I^1_A \right) \\
&\quad - \left( \sum q(1) m^1 - s \frac{\partial z^1}{\partial r} \right) (r^1 - z^1) \sum q(i) \\
&\quad \left( I^1_B \right) \\
&\quad - \left( \sum q(i) n^1(i) - (1 - \frac{\partial z^1}{\partial r}) \right) \\
&\quad \left( I^1_B \right) \\
&= \int_x \left( \frac{\partial z^1}{\partial r} - 1 \right) \left[ \sum q(1) m^1 + \sum q(i) n^1(i) \right] < 0. \\
(+) & \quad (-) & \quad (+)
\end{align*}
\]

Therefore,

(20.iii) \[ \frac{\partial r}{\partial E_i} > 0. \]

Finally, we note, for the sake of completeness, that

(20.iv) \[ \text{sgn} \frac{\partial \xi}{\partial \Delta \xi} \] is indeterminate.
6. CONCLUDING REMARKS

In this paper a theory of non-price credit rationing has been developed using the theory of contracts that has recently been applied to labour markets. We believe our model is superior to previous attempts at explaining non-price rationing because we show why the loan rate should be rigid whereas most other studies have simply supposed that it is. In previous work three elements, customer relations (cf. Hodgman (1963)), monopoly power (cf. Modigliani and Jaffé (1969)) and default risk (cf. Jaffé and Russell (1976)) have been stressed. The analysis incorporates all three although none is strictly necessary for the existence of credit rationing. Customer relations are included because we believe they are important aspects of the loan market, not because our analysis implies that old customers will never be rationed. It does not: there are circumstances where all customers obtaining loans are old customers while all new customers and some old customers are rationed. Short-run monopoly power of lenders exists in our model but is never used. It exists because there is a difference between the utility the borrower gets if he obtains a loan, \( w(r) \), and what he obtains if he is rationed by an individual bank, \( K \), a difference that could be exploited by the lender in the short run. We need this difference in a regime of implicit contracts to provide an incentive to borrowers to "keep them honest", i.e., not to borrow in a spot market when \( i \) is low and from their bank when \( i \) is high. An explicit contract including clauses requiring the customer to borrow only from a specific bank under a specific set of states would also resolve that problem and would not generate potential short-run monopoly profits. Default risk is also not necessary to explain non-price credit rationing but is included to lend plausibility to the assumption that the utility of a rationed customer is less than if he was not rationed under an implicit contract.
Two assumptions used that simplified our analysis were that banks were risk neutral and that borrowers either had their loan needs fulfilled completely or did not receive any loan at all from the bank. The first of these is necessary for the fixed loan rate theorem but is not necessarily crucial for the existence of rationing. Had we required that banks were relatively less risk averse than borrowers the loan rate would generally vary with \( i \) but gains would still be obtained from an implicit contract that had an inelastic response of \( r \) to \( i \) and permitted some rationing in some states. The second assumption meant that this form of the model cannot describe the partial rationing of borrowers that concerned Jaffé and Russell (1976). The model can be expanded to permit partial rationing but greater technical detail associated with \( K \) and the bank cost function would have to be specified.

While it is not necessary that a bank will sometimes ration customers it is shown that the likelihood of rationing contracts increases with increases in the mean and the variance of the cost of funds to banks, with increases in marginal cost of making loans, (including that caused by decreases in the ratio of old to new customers) and by decreases in the expected gain to borrowers from obtaining a loan relative to not getting one.

We also show if rationing contracts are optimal and \( r^0 \leq r^1 \), then the amount of rationing is lower and the loan rate is higher: the greater is the expected return to firms from obtaining a loan or the lower is the utility to borrowers in not obtaining one, the greater is the contracted loan rate and the smaller the expected level of rationing. Also, the greater is the expected cost of funds to banks (\( E_i \)), the greater the expected level of rationing but the loan rate may go up or down. The greater is the variance of \( i \), the lower will be the loan rate but the change in expected rationing is ambiguous. It should be stressed, however, that the contracts discussed
in this paper only deal with one dimension of the risks to borrowers and lenders in the real world, namely that associated with the cost of obtaining funds by banks. Nothing precludes the existence of additional implicit contracts that insure one participant or the other from risks arising from other sources. Before any predictions on the macroeconomic level can be confidently stated, these other possible contracts should be explored.
REFERENCES


Phelps, E. S. and Winter, S., "Optimal Price Policy Under Atomistic Competition,"
in Microeconomic Foundations of Employment and Inflation Theory, edited
by E. S. Phelps, New York, Norton, 1970.
FOOTNOTES

1 The analysis of Koskela (1976, ch. 6) is the only other attempt in the literature to address the question of credit rationing in these terms. Koskela's analysis, however, was directed at the much narrower issue of the conditions under which the loan rate in an optimal implicit contract would be independent of variations in the states of the world, which, as Theorem 2 below shows, is not sufficient to demonstrate the existence of credit rationing.

2 As pointed out below, these switching costs may be voluntarily imposed in an explicit contract. The use of customer relations is an attempt to lend realism to the analysis by generating switching costs without the necessity of explicit contracts.

3 Data compiled by Merrill Lynch Royal Securities Ltd. from the financial statements of the Canadian banks indicate that over the last decade the average value of defaults constituted less than .5% of total loans for the Canadian banking system. Further, no individual bank had loan losses greater than .7% of its loan portfolio. Data for the U.S. indicate similar qualitative results.

We would like to thank M. Jensen of the University of Western Ontario for pointing out these figures to us.

4 Another reason for administrative economies for customers with established accounts is that the effective cost of funds to the bank is lower in an intertemporal context. That possibility is not directly explored in this paper.
Large companies that have been "rated nationally" may entail no screening costs to the individual bank. While we do not make such gradations, an implication would be that *cet. par.*, nationally rated corporations will be treated more like old customers than like new customers in terms of non-price rationing.

For example, see the closely-related analysis on labour-market contracts by Markusen (1979), in which both sides are assumed to be risk averse. It is an obvious consequence of continuity that if the loan rate is constant and credit rationing may occur when the bank is risk neutral [Theorems 1 and 2 below], then the loan rate will be almost constant and credit rationing may still occur if the bank is almost risk neutral. This in itself shows that risk neutrality is inessential for our main objective.

One example of such voluntarily imposed switching costs on borrowers is interest rate penalties on Canadian mortgages. It should also be noted that, in principle, it is not necessary that banks must be the ones to provide insurance instead of a third party. Underwriters in the bond markets may, in fact, be providing just such a service to firms that choose to borrow in the securities market.