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by

John McMillan

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John McMillan

Department of Economics
University of Western Ontario
London N6A 5C2
Canada

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ABSTRACT

The free rider hypothesis states that, in the decision on public goods and private goods provision, individual incentives are such that public goods will tend to be under-supplied. This paper examines the free rider argument as it applies to public intermediate goods. The problem is set up in a game theoretic model. It is then shown that in a dynamic world there may exist incentives for firms to act cooperatively in determining the supply of public intermediate goods. In a dynamic context there is a cost to free riding: what one firm does now affects what others will do in the future. Provided future profits are not discounted too heavily, the free rider problem disappears when a time dimension is added to the theory.
THE EFFICIENT SUPPLY OF PUBLIC INPUTS

John McMillan

1. Introduction

The free rider hypothesis has gained wide acceptance among economists. The way this problem is usually stated is that, for a Pareto optimal solution to be reached, consumers must reveal their preferences for the public good. But since each individual consumes the total quantity of public good supplied, it is in any individual's interest to understate the satisfaction he gains from consuming the public good, thereby only slightly reducing the quantity of public good supplied but significantly reducing his own tax burden. Everyone reasons in this way and the public good will be under-supplied.

Not all public goods are consumption goods; some public goods serve as inputs to production processes. With a public intermediate good, the whole amount supplied enters the production functions of several firms; examples are scientific research, public information services, pollution control and uncongested roads. The concept was introduced into the literature by Meade with his discussion of "creation of atmosphere" (1952). Conditions for the efficient supply of public intermediate goods similar to the Samuelson (1954) rule for public consumption good supply have been derived by Kaizuka (1965), Sandmo (1972) and Negishi (1973); efficiency requires that the sum of the marginal revenue products of the public intermediate good be equated to its marginal cost. Laffont (1976) and Pestieau (1976) extend the efficiency conditions to encompass respectively technological uncertainty and distortionary taxation. McMillan (1977a) describes the optimal accumulation of a stock of public intermediate good. The free rider hypothesis says that these conditions for efficient public good supply will not be satisfied.
With public consumption goods, the free rider problem has two separate aspects. The first is that there is a need for individuals to reveal their preferences for the public good. The Samuelson condition, describing Pareto optimal supply of public consumption goods, involves individuals' marginal rates of substitution. Unless individuals honestly give this subjective information, the set of Pareto optimal outcomes cannot be known. The second aspect is that, for a Pareto optimum to be reached, individuals have to act cooperatively (compare with the "prisoners' dilemma" game). The free rider argument says that individuals are actually rewarded for free riding: any individual who tries to act cooperatively becomes worse off than if he had been a free rider.

With public intermediate goods it is easier to overcome the free rider problem because only the second of these aspects is present. The terms which occur in the Samuelson formula for efficient public input supply are marginal productivities: these, unlike marginal utilities, are objectively measurable. Naturally, firms may understate the marginal productivity of public intermediate goods, but it is clear that, in principle at least, external confirmation of their reports is available. Thus, if a source of cooperative behaviour can be found, a public input will be supplied efficiently.

There is a further reason why the free rider problem is less severe with public intermediate goods than with public consumption goods. Typically, a smaller number of agents will be affected by public intermediate goods than is the case with public consumption goods. The chances of some sort of cooperative behaviour developing are better if each agent can, acting alone, have a significant effect on the supply of the public good.

If the free rider mechanism were operating in real-world situations, we would expect to be able to observe a dramatic under-supply of public goods. There is little empirical evidence of such a misallocation of resources (Johansen, 1977); on the contrary, the evidence suggests that some public
intermediate goods are actually supplied in something like efficient quantities. An example of a public intermediate good being supplied in a decentralized fashion is offered by Coase (1974). Coase describes the historical development of the British lighthouse system. He concludes (p. 376) that "economists should not use the lighthouse as an example of a service which could only be provided by the government"; the British history shows that a lighthouse service can be provided by private enterprise. Another example of successful collective action by firms, contrary to the free rider argument, is the existence of trade associations providing public goods for their members (Stigler, 1974; Marsh 1976). A further example of coordinated behaviour of firms, confirmed by everyday experience, is the standardization of certain features of an industry's output (standard record speeds, standard sizes of nuts and bolts, etc.). Such standards are public intermediate goods. Bolnick (1976) describes the provision of education and health facilities in less developed countries through voluntary contributions to community development programmes. Bolnick makes the point that, although economists' theoretical understanding of the working of voluntary public goods interaction is weak, voluntary cooperation in the provision of public goods does occur in reality. These counter-examples to the free rider hypothesis suggest there may be incentives for collective action which the free rider argument ignores.

Groves and Loeb (1975) construct a mechanism for overcoming the free rider problem with public inputs. A central planning body taxes firms on the basis of the firms' reported revenue functions. The tax paid by any firm is computed as a special function of the reported revenue functions of all the other firms. Under the Groves-Loeb mechanism, each firm has an incentive to send truthful information so that an optimal quantity of the public input will be provided.
This paper proposes an alternative source of coordinated behaviour to that offered by Groves and Loeb. The free rider hypothesis (and the Groves-Loeb solution) are based on a static analysis. In a dynamic world there exist incentives for firms to act cooperatively in determining public intermediate good supply. In a dynamic context there is a cost to free riding: the effects of one firm's actions now can be observed by others and will influence the others' behaviour in the future. Provided future profits are not discounted too heavily, the free rider problem disappears when a time dimension is added to the theory.

2. **The Static Case**

The problem of the decision on the supply of a public intermediate good is formalized in the following model. Suppose there are \( n \) firms (producing the same or different outputs) using as inputs a single primary factor (labour) and a single public intermediate good. The \( i \)th firm's output, \( y_i \), is a function of \( L_i \), the quantity of labour it employs, and \( r \), the quantity of public input it uses (which is the same as the quantity of public input produced):

\[
y_i = f_i(L_i, r), \quad i=1, \ldots, n
\]  

(1)

The public intermediate good is itself produced from inputs of labour alone, under constant returns to scale:

\[
r = f_r(L_r).
\]  

(2)

Labour is in fixed total supply:

\[
\sum_{i=1}^{n} L_i + L_r = L_*
\]  

(3)

A necessary condition for efficient public input supply is the Samuelson summation condition (Kaizuka, 1965; Sandmo, 1972):
\[
\sum_{i=1}^{n} \frac{\partial f_i}{\partial x} = \frac{1}{d_f/dL_i} \tag{4}
\]

Consider now the decision problem of any firm \(i\). The firm faces a price for its output of \(p_i\) and pays labour a wage rate of \(w\). To abstract from the feedback effects of each firm's public good decision on prices, assume \(p_i, i=1,\ldots,n,\) and \(w\) are constant. (For instance, international trade may serve to fix the prices \(p_i\).) The firm chooses to pay a tax \(t_i \geq 0\) to the producer of the public intermediate good. Thus the firm will seek to maximize profit:

\[
\pi_i = p_i f_i (L_i, r) - wL_i - t_i \tag{5}
\]

Labour is the only input to the industry producing the public good. Assume this industry has zero profits (that is, if it is the government, its budget is balanced), so that the revenue raised for the public good is used only to pay the workers in that industry:

\[
\sum_{j=1}^{n} t_j = wL_r \tag{6}
\]

Then, from equations (2) and (6), the profit of the \(i\)th firm is a function of \(L_i, t_i\) and \(\sum_j t_j\):

\[
\pi_i (L_i, t_i, \sum_{j=1}^{n} t_j) = p_i f_i \left(L_i, f_r \left(\sum_{j=1}^{n} t_j \right) \right) - wL_i - t_i \tag{7}
\]

Any efficient level \(r^*\) of public input supply (any solution of condition (4)) can be supported by a vector of taxes \(t^* = (t^*_1, t^*_2, \ldots, t^*_n)\) on the \(n\) firms as follows. Set the \(i\)th firm's tax so that the tax paid per unit of public input supplied is equal to the marginal value product of the public inputs in firm \(i\) (this is a Lindahl pricing scheme):

\[
\frac{t_i^*}{r^*} = p_i \frac{\partial f_i}{\partial r}, \quad i=1,\ldots,n \tag{8}
\]
Summing equations (8) over all firms, and using the budget balance condition (6) and the fact that if the firm is using labour efficiently then \( w = p_i \frac{\partial f_i}{\partial L_i} \), gives:

\[
\sum_{i=1}^{n} \frac{\partial f_i}{\partial r} \frac{\partial r}{\partial L_i} = \frac{L_r}{r^*}
\]

which is the same as the efficiency condition (4) because, with constant returns to scale in the public good industry, average productivity of labour, \( r/L_r \), is the same as marginal productivity, \( df_r/dL_r \). Thus the cooperative tax payment \( t_{i}^* \), \( i=1,\ldots,n \), does lead to an optimal allocation.

The essential game-theoretic nature of the problem lies in the fact that any firm's profit depends on the tax payment (strategy choice) of every firm.

A non-zero sum game can be either cooperative or non-cooperative. In a cooperative game, where agents announce their decisions before acting and binding agreements among the agents are possible, the equilibrium can be optimal. If institutional arrangements are such that the public choice problem is a cooperative game then there is no free rider problem. The implicit assumption behind discussions of the free rider problem is that of a non-cooperative game; since we are concerned with examining the free rider argument, let us make this assumption.

The strategy vector \( t^* \) represents a cooperative solution to the decision on public input supply. The free rider problem says that, under a decentralized system, if each firm were to pay for the quantity of public input that maximizes its own profits, without regard to the actions of the other firms, then the outcome that would result would not be efficient. The equilibrium of this non-cooperative game would not be optimal; the public input would be under-supplied compared with the efficiency condition (4) (Kaizuka, 1965, p. 118; Groves and Loeb, 1975, pp. 212-213).

It is now shown that the free rider hypothesis does apply in the model
set up here. A non-cooperative equilibrium (Nash, 1951) is defined to be a vector of strategies for the n firms \( t^* = (t_1^*, t_2^*, \ldots, t_n^*) \) that satisfies, for all \( i=1, \ldots, n \):

\[
\pi_i^*(t_i^*, \sum_{j=1}^{n} t_j^*) \geq \pi_i^*(t_i^*, \sum_{j=1}^{n} t_j^* + t_i^*) \quad \text{for all } t_i^* \geq 0
\]  

(10)

where

\[
\pi_i^*(t_i, \sum_{j=1}^{n} t_j) = \max_{L_i, t_i} \pi_i(L_i, t_i, \sum_{j=1}^{n} t_j)
\]  

(11)

That is, the game is at a Nash equilibrium if no firm can alter its tax payment so as to increase its profits, assuming no other firm changes its tax payment. With \( f_i \) concave in all its arguments, the Nash equilibrium is unique.

**Proposition 1:** At the Nash equilibrium the supply of the public input is smaller than optimal.

**Proof:** The Nash equilibrium is found by differentiating the ith firm's profit function (6) with respect to the variables firm \( i \) can control, namely \( L_i \) and \( t_i \) (and taking the actions of the other firms \( t_j^*, j=1, \ldots, i-1, i+1, \ldots, n \), as fixed).

This gives the following necessary conditions for maximum profit:

\[
P_i \frac{\partial f_i}{\partial L_i} - w = 0
\]  

(12)

\[
P_i \frac{\partial f_i}{\partial r} \frac{df_r}{dL_r} \frac{1}{w} - 1 = 0
\]  

(13)

(Equation (12) makes use of the budget balance equation (6).) Substituting (12) in (13) and rearranging yields:

\[
\frac{\partial f_i}{\partial r} \frac{\partial f_i}{\partial L_i} = \frac{1}{df_r/dL_r}
\]  

(14)

The ith firm, to maximize profits, equates the marginal rate of substitution in its own production process to the marginal difference it, acting alone, can make
to public good supply. Summing equations (14) over all firms yields:

\[
\sum_{i=1}^{n} \frac{\partial f_i}{\partial x} = n \cdot \frac{1}{\frac{df}{dL}} \tag{15}
\]

Compare the competitive solution (equation (15)) with the efficient solution (equation (4)). Provided \( n > 1 \), the competitive outcome is not efficient; in the competitive solution the sum of the marginal rates of substitution between public input and labour is \( n \) times too large \(^6\) (remembering that, with constant returns to scale in the public input industry, \( \frac{df}{dL} \) is constant). From the convexity of the isoquants, the supply of the public input at the Nash equilibrium is smaller than optimal. \(^7\)

For later purposes denote by \( t^f_i \) the non-cooperative equilibrium contribution by firm \( i \) to public input production.

Thus, in this single-period decision problem it is in each firm's interest to try to free ride on the supply of public input provided by the other firms. But each firm will try to do this, with the result that the non-cooperative equilibrium \( t^f \) is attained. Individual rationality leads to a collectively irrational outcome; a smaller than optimal quantity of the public intermediate good is supplied. All firms potentially could benefit from an increase in the supply of the public intermediate good.

3. **The Dynamic Equilibrium**

The public choice problem described in the previous section is now put in an explicitly dynamic setting. The vehicle used to make the model dynamic is the idea of a supergame (due to Luce and Raiffa, 1957, pp. 97-102): the game described in Section 2 is played repeatedly, once in each time period over an infinite horizon. To abstract from questions of distribution over time, it is assumed that the game is identical in each period; specifically \( f_i, f_r \) and \( L \)
do not change over time, and \( r \) is not accumulated as a stock. Assume each firm
knows exactly how much public input it is using, that is, how much is supplied.

The \( i \)th firm's objective now is to choose a sequence of strategies
\[ T_i = (t_{i1}, t_{i2}, \ldots, t_{i\tau}, \ldots) \]
(where the second subscript refers to the time period) to maximize discounted profit:

\[
\Pi_i = \sum_{\tau=0}^{\infty} d^\tau \prod_{i=1}^{n} \pi_i(t_{i\tau}, \sum_{j=1}^{n} t_{j\tau})
\]  
(16)

where \( d \) is the discount parameter, \( 0 < d < 1 \), and \( \pi_i \) is defined by (11).

It seems intuitively reasonable that in the supergame a steady state
would consist of repeated plays of a static game optimum, because over time
agents would tend to move away from a non-optimal point, since all potentially
could gain from such a shift. That is, the steady state of a non-cooperative
game would resemble a cooperative solution. This suggestion is given formal
content in this section.

Consider the supergame as a single large game, so that a strategy in
the supergame consists of a sequence of ordinary game strategies. The Nash
definition applies to the supergame considered in this way; one Nash equilibrium
of the supergame is for each firm repeatedly to play its static equilibrium
(non-cooperative) strategy
\[ T_i^f = (t_{i1}^f, t_{i2}^f, \ldots, t_{i\tau}^f, \ldots). \]
But in a dynamic setting,
no longer should firm \( i \) take the strategies of the other firms as given. The
effects of firm \( i \)'s actions in this period can be observed by the other firms,
and may affect the actions they take in succeeding periods. There are many
Nash equilibria, apart from \( T_i^f \), for the supergame. Consider the following
class of supergame strategies. Let \( S \) be the set of ordinary game strategy
vectors which give higher profits to each firm than the non-cooperative strategy
vector \( t_i^f \); that is:

\[
S \equiv \{ t' = (t_{1}', t_{2}', \ldots, t_{n}') : \pi_i(t_{i1}', \sum_{j=1}^{n} t_{j1}') > \pi_i(t_{i1}^f, \sum_{j=1}^{n} t_{j1}^f), \quad i=1, \ldots, n \}
\]
Define a supergame strategy for firm $i$, $T'_i = (t'_{i1}, t'_{i2}, \ldots, t'_{iT}, \ldots)$, by (for any $t' \in S$):

$$
\begin{align*}
  t'_{i1} &= t'_{i} \\
  t'_{iT} &= t'_{i} \quad \text{if } t'_{ju} = t'_{j}, \; j \neq i, \; u=1,\ldots,T-1, \; \tau=2,3,\ldots \\
  t'_{iT} &= \frac{t'_{i}}{t'_{i}} \quad \text{otherwise}
\end{align*}
$$

That is, firm $i$ plays strategy $t'_{i}$ repeatedly until some other firm $j$ chooses some strategy other than $t'_{j}$. (Even if firm $i$ cannot actually observe firm $j$'s strategy choice, the signal for this change in strategy will be a fall in the quantity of public good produced.) If in any period some firm plays a strategy different from $t'_{j}$, then firm $i$ will play non-cooperatively from the next period on. (It is assumed, for the reasons discussed in Section 1, that firm $i$ knows the other firms' production functions.)

Provided the agents do not discount the future too heavily, $T' = (T'_1, T'_2, \ldots, T'_n)$ is an equilibrium of the supergame. To make this statement precise, consider the possibilities open to firm $i$ at any point in time. (Call the present time period $\tau=0$). Assume the other firms are using supergame strategy $T'_j$ as defined by (17).

To show that $T = (T'_1, \ldots, T'_n)$ is a Nash equilibrium of the supergame, we have to show that firm $i$ cannot change its supergame strategy so as to increase its profits. Firm $i$ can use the supergame strategy $T'_i$ (that is, continue to play the static game strategy $t'_{i}$). Firm $i$'s profit is then $\pi_i(t'_{i}, \sum_{j=1}^{n} t'_j)$ in each period. Or it can gain a short-term advantage by playing some strategy $t''_{i} < t'_{i}$ which gives it a higher profit when played against strategies $t'_j$, $j=1,\ldots,i-1,i+1,\ldots,n$. (That is, it can free ride on the public input paid for by the others.) Firm $i$'s profit in period zero is then $\pi_i(t''_{i}, \sum_{j=1}^{n} t'_j + t''_{i})$ (which exceeds $\pi_i(t'_{i}, \sum_{j=1}^{n} t'_j)$ - otherwise firm $i$ would not do this). The cost to firm $i$ of changing its strategy is that it will cause the other firms (if they are using supergame strategy (17))
to use their non-cooperative strategies $t^f_j$ from period $\tau=1$ on, thus reducing firm $i$'s profits in these periods. Firm $i$'s profit is then $\pi^x(t^f_i, \sum_{j=1}^n t^f_j) < \pi^x(t'_i, \sum_{j=1}^n t'_j)$ since by definition of $t^f$, once the other firms start playing their non-cooperative strategies $t^f_j$, firm $i$'s best reply is $t^f_i$.

**Proposition 2:** The supergame strategy vector $T' = (T'_1, \ldots, T'_n)$ defined by (17) is a Nash equilibrium for the supergame provided, for all firms $i=1, \ldots, n$,

$$\frac{1 - d}{d} \leq \frac{\pi^x(t^f_i, \sum_{j=1}^n t^f_j) - \pi^x(t'_i, \sum_{j=1}^n t'_j)}{\pi^x(t''_i, \sum_{j=1}^n t'_j + t''_j - \pi^x(t'_i, \sum_{j=1}^n t'_j)} \quad \text{for all } t'_i \geq 0$$  \hspace{1cm} (18)

**Proof:** Firm $i$'s profit if it uses supergame strategy $T'_i$ is

$$\pi'_i = \sum_{\tau=0}^{\infty} d^\tau \pi^x(t'_i, \sum_{j=1}^n t'_j). \hspace{1cm} (19)$$

If, in period zero, firm $i$ deviates from this strategy and contributes $t''_i < t'_i$, its discounted profit will be

$$\pi''_i = \pi^x(t''_i, \sum_{j=1}^n t'_j + t''_j) + \sum_{\tau=1}^{\infty} d^\tau \pi^x(t'_i, \sum_{j=1}^n t'_j) \quad \text{for all } t''_i \geq 0; \hspace{1cm} (20)$$

It is in firm $i$'s interest to use strategy $T'_i$ if $\pi'_i$ exceeds $\pi''_i$; that is if

$$\pi^x(t'_i, \sum_{j=1}^n t'_j) - \pi^x(t''_i, \sum_{j=1}^n t'_j) \leq \sum_{\tau=1}^{\infty} d^\tau [\pi^x(t'_i, \sum_{j=1}^n t'_j) - \pi^x(t''_i, \sum_{j=1}^n t''_j)] \quad \text{for all } t''_i \geq 0; \hspace{1cm} (21)$$

that is, the extra profit from free riding on the public good supplied by the other firms (in period zero) is less than the discounted sum of extra profits (from period one on) from being at $T' = (T'_1, \ldots, T'_n)$ rather than the non-cooperative equilibrium. Rearranging, and using the formula for a geometric series, inequality (21) becomes inequality (18). If (18) holds for all $i$, then no firm can increase
its (discounted) profits by changing from using supergame strategy $T'_i$ provided
the remaining firms, $j=1,...,i-1,i+1,...,n$, continue to use $T'_j$. $T' = (T'_1',...,T'_n')$
is a Nash equilibrium of the supergame.

Thus there are many Nash equilibria for the supergame. Aumann (1960) defines
a stronger equilibrium notion. Consider the possibility of several players making
themselves better off by simultaneously changing their strategies while the
remaining players retain their original strategies. (With a non-cooperative
game this simultaneous action cannot happen as a result of deliberate agreement;
however it may occur either by accident or through tacit cooperation developing
over time.) Allowing this possibility means that a Nash equilibrium may not be
stable. A strong equilibrium point is defined to be an $n$-tuple $T$ of supergame
strategies such that for no set of agents $B$ can all the members of $B$ increase
their payoffs by adopting strategies different from those at $T$ while the re-
main ing agents play as they did at $T$ (Aumann, 1960, p. 415; Yanovskaya, 1972,
p. 210). That is, $T'_i = (T'_1',...,T'_n')$ is a strong equilibrium point of the super-
game if, for all $i$,

$$\prod_{i} (T'_i, \sum_{j=1}^{n} T'_j) \geq \prod_{i} (T_i, \sum_{j \in B} T_j + \sum_{j \in N-B} T'_j), \text{ for all } T_j \geq 0, \quad (22)$$

and for all $B \subseteq N$ where $\prod_i$ represents the discounted profit stream of firm $i$
and $N$ is the set of all firms. The strong equilibrium point represents a long-run

It is easy to see now that a strong equilibrium point of the supergame
consists of repeated playing of an ordinary game socially optimal solution.

Define a supergame strategy $T^*_i = (t_{1i},...,t_{ir},...)$ by $^{10}$

$$t_{1i} = t^*_i$$

$$t_{ir} = t^*_i \text{ if } t_{ju} = t^*_j, \text{ if } u=1,2,...,r-1; r=1,2,...$$

$$t_{ir} = t^*_i \text{ otherwise} \quad (23)$$
(where \( t^*_i \) is the socially optimal static game strategy defined by (8)). That is, firm \( i \) plays its cooperative strategy until it observes some other firm attempting to free ride; then firm \( i \) plays non-cooperatively. Assume that, for all firms \( i = 1, \ldots, n \),

\[
\frac{1 - d_i}{d} \leq C_i
\]

\[
\pi_i^*(t_i^*, \sum_{j=1}^{n} t_j^*) - \pi_i^*(t_i^f, \sum_{j=1}^{n} t_j^f)
\]

where \( C_i = \min_{t_i'' \geq 0} \) \( \pi_i^*(t_i'', \sum_{j=1}^{n} t_j^* + t_i'') - \pi_i^*(t_i^*, \sum_{j=1}^{n} t_j^*) \) (25)

Then Proposition 2 implies that the supergame strategy vector \( T^* = (T_1^*, \ldots, T_n^*) \) defined by (23) is a strong equilibrium point for the supergame.

**Proposition 3:** Suppose inequality (24) holds. Then the cooperative supergame strategy (23) is the only strong equilibrium point of the supergame.

**Proof:** Suppose this is not the case: suppose there is a strong equilibrium point consisting of the playing at each period of a vector of non-efficient ordinary game strategies. The public input will not be produced in efficient quantities. There exists an alternative strategy vector which would increase some firms' profits and decrease none, namely \( T^* \). The mere existence of this alternative (better) strategy vector means that the system cannot be at a strong equilibrium point. Thus at a strong equilibrium point each firm is repeatedly paying an efficient level of tax \( t_i^* \).

The right hand side of (25) shows, on the denominator, the extra profit gained by the \( i \)th firm in period 0 by free riding (playing \( t_i'' \)); and, on the numerator, the extra profit it would get in each succeeding period if instead it continued to play \( t_i^* \). \( C_i \) is an "index of cooperation" for firm \( i \): the larger is the index of cooperation, the more likely it is that firm \( i \) will use the (cooperative) strategy \( t_i^* \). (Compare with the index of cooperation defined
by Rapoport and Chammah (1965) for the two-person two-strategy "prisoners' dilemma" game.)

The discount parameter $d$ lies between zero and one. If $d = 0$ we are back in the static situation and it is in the interest of firm $i$ to free ride; the only Nash equilibrium in the supergame will be non-cooperative. (The left hand side of (24) is infinite; (24) can never be satisfied.) If $d = 1$, that is the future is not discounted, inequality (24) is always satisfied and it will always be in firm $i$'s interest to play strategy $t^*_i$ rather than $t^f_i$. (The left hand side of inequality (24) is zero; (24) will always be satisfied.) Thus for each $i$, there will be some $d_i$, $0 \leq d_i \leq 1$, such that (24) holds for all $d \geq d_i$. If the discount parameter is close enough to unity that condition (24) is satisfied for all $i = 1, \ldots, n$ (that is, $d \geq \text{Maximum} (d_1, \ldots, d_n)$) then $T^* = (T^*_1, T^*_2, \ldots, T^*_n)$ is a Nash equilibrium and a strong equilibrium point for the supergame.

Thus, when a time dimension is added to the public good supply game, the optimum is an equilibrium (provided the future is not discounted too heavily). Individually rational behaviour leads to an efficient outcome. It is interesting to note that the free rider solution, which, it is normally argued, is the inevitable result of rational behaviour, is only reached in this dynamic model if some agent acts irrationally.

4. The Planning Horizon

The result derived above that an optimal quantity of public input can be produced in a dynamic model depends on the assumption that the time horizon is infinite. If there are a finite number of periods then the same considerations apply as in the static case: it is in each firm's interest to free ride. The supergame with a finite number of periods has a basically similar structure to the single-play game.
The reason why the solution offered in the previous section breaks down is that, with a finite time horizon, there is a final period: each firm knows when the game is going to end. Call this last period $P$. Then it is in firm $i$'s interest to play cooperatively for the first $P-1$ periods and then to free ride in the last period, because it would get an increase in profit in period $P$ without cost to itself; there being no future periods in which the other firms can retaliate. But every firm reasons in this way: every firm will try to free ride in period $P$. Therefore it is in firm $i$'s interests to try to free ride in period $P-1$. But, again, all firms will reason in this way. It is in the interest of each firm to be the first to play non-cooperatively instead of cooperatively. The Nash equilibrium strategy for each firm is to play non-cooperatively from the first period. The public input will not be supplied optimally in any period.

For the cooperative solution to be a Nash equilibrium of the supergame, the essential requirement is that there be no final period. This can be achieved in a finite horizon model if the players do not know when the game is going to end. Suppose the length of the time horizon, $P$, is a random variable. Suppose the probability distribution of $P$ is known to the firms, and suppose it has the property that it is independent of the number of periods that have already elapsed.

Each firm must weigh the expected gains and losses from free riding. If all firms are using supergame strategy (23) then the expected profit to firm $i$ from continuing to play $t_i^*$ is

$$E[\Pi^*_i] = E[\sum_{i=1}^{n} \sum_{j=1}^{t_i^*} \sum_{\tau=0}^{P} d_{i,j}^\tau (t_i^* - t_j^*)]$$

(26)

Firm $i$'s expected profit if it free rides in period zero is
\[
E[\Pi_i''] = E[\pi^x_i(t^x_i, \Sigma t^x_j + t^x_i') + \sum_{j=1}^{p} \delta^x_{j=1} \pi^x_i(t^f_i, \Sigma t^f_j)] 
\]

The profit from contributing optimally to public input supply exceeds the profit from free riding if \(E[\Pi_i^x] \geq E[\Pi_i'']\).

**Proposition 4:** Suppose \(d=1\). Then the cooperative solution (23) is a Nash equilibrium of the supergame if

\[
E[P] \leq C_i \quad \text{for all } i=1, \ldots, n
\]

where \(C_i\) is, as defined in (25), the "index of cooperation".

**Proof:** \(E[\Pi_i^x] \geq E[\Pi_i'']\) implies

\[
E[P(\pi^x_i(t^x_i, \Sigma t^x_j) - \pi^x_i(t^f_i, \Sigma t^f_j))] \geq \pi^x_i(t^x_i, \Sigma t^x_j + t^x_i') 
\]

\[
- \pi^x_i(t^f_i, \Sigma t^f_j),
\]

which becomes (28) upon rearranging, given that \(\pi^x_i(t^x_i, \Sigma t^x_j)\) and \(\pi^x_i(t^f_i, \Sigma t^f_j)\) do not change over time.

If the future is not discounted, then the larger is the expected length of the time horizon, the more likely it is that the public input will be supplied optimally.

**Proposition 5:** Suppose \(0 \leq d < 1\). Then the cooperative solution (23) is a Nash equilibrium of the supergame if

\[
\frac{1 - d}{d} \cdot \frac{1}{1 - E[P]} \leq C_i \quad \text{for all } i=1, \ldots, n
\]

where \(C_i\) is as defined in (25).

**Proof:** \(E[\Pi_i^x] \geq E[\Pi_i'']\) implies
\[ E[\eta_i^{*}(t_i^*, \Sigma t_i^*) - \eta_i^{*}(t_i^*, \Sigma t_i^*) + t_i^*'] + (\eta_i^{*}(t_i^*, \Sigma t_i^*) \left( \eta_i^{*}(t_i^*, \Sigma t_i^*) - \eta_i^{*}(t_i^*, \Sigma t_i^*) \right) \frac{d(1-d_i^P)}{1 - d_i^P} \geq 0 \] (31)

provided \( d \neq 1 \). Rearranging

\[ E[d_i^P] \leq 1 - \frac{1 - d_i^P}{d_i^P} \cdot \frac{1}{C_i} \] (32)

On rearranging, inequality (27) is obtained.

With \( d < 1 \), the right hand side of (32) is strictly less than one. If firm 1 is certain that the present period is the final period then \( E[d_i^P] = 1 \) (\( P=0 \)) and firm 1 should play non-cooperatively regardless of the sizes of the other parameters. This is the case considered at the start of this section.

For other probability distributions of \( P \), the greater the probability weighting of larger values of \( P \), the more likely it is that inequality (32) will be satisfied for all 1 and thus a cooperative solution will be an equilibrium.

Also, for similar reasoning to the previous section, inequality (32) is more likely to be satisfied the closer \( d \) is to one; the greater the dynamic gains from playing cooperatively; and the smaller the static gains from free riding.

5. **Conclusion**

Stephen Smale (1976) remarks that "sometimes static theories pose paradoxes whose resolution lies in a dynamic perspective." This paper examined the free rider paradox, in which firms, by seeking to maximize profits, cause a situation where each has lower profit than might otherwise be the case. The conclusion is that, provided the perspective is dynamic enough, in the sense that the future is not discounted too heavily, then there are incentives for firms to act cooperatively in determining the supply of a public intermediate good. When a time dimension is added, the public input may be supplied efficiently even in a decentralized system. It is an empirical question whether, in any particular case, the assumptions behind this result are satisfied so that there is no free rider problem.
References


Footnotes

*This paper is based on Chapter 4 of McMillan (1977). I wish to thank R. Manning and the members of the Economic Theory Workshop at the University of Western Ontario for helpful comments.

1See for example Samuelson (1954) and Musgrave (1959) Ch. 4; for a survey of writings on this question from Wicksell on see Green and Laffont (1976).

2A similar point is made by Arrow (1965, p. 6) and by Musgrave (1969, p. 800). Musgrave lists the following examples of evaluating the benefits of public intermediate goods: "the benefits from irrigation may be measured in terms of increased agricultural output; flood control results in cost-saving since measurable damage to capital assets or resources is avoided; better roads reduce automotive costs and save trucking time, which can be valued; public health measures reduce remedial care cost; investment in education raises earning power."

3Constant returns to scale in the public good industry is assumed to simplify the algebra, in particular the relationship between tax payments and public good supply (equation (6) below).

4It is assumed r* is unique, because unless this is the case there will be ambiguity in equation (8). r* will be unique, for example, if the economy is trading on world markets: compare with Manning and McMillan (1977) and McMillan (1977b, Ch. 2).

5t_i will be described interchangeably as the strategy and the tax payment of the ith firm.
This suggests another solution to the free rider problem: make $n=1$. If all the firms affected by the public intermediate good were to merge then equations (15) and (4) imply the resulting monopolist would supply the public intermediate good efficiently. It would be in the interests of profit-maximizing firms to amalgamate in this situation.

Malinvaud (1972, p. 214) derives a similar result for public consumption goods.

Notation is: capital $T$ is a supergame strategy, lower case $t$ is an ordinary game strategy; subscripts denote firms; and $a_t$ or $T$ without a superscript denotes a strategy vector for all $n$ firms.

This supergame strategy was put forward by J. W. Friedman (1971) in the context of oligopoly theory.

It is necessary to assume here that all firms earn higher profits at the Lindahl equilibrium $t^*$ than at the noncooperative equilibrium $t^f$. 