The Role of Transaction Volume in Producing Information About Asset Prices

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ABSTRACT

A dynamic equilibrium model is constructed in which agents with access to different information sets participate in the capital market. Agents must then use the equilibrium price of capital to make forecasts of the return to holding capital, and these forecasts must be consistent with Bayes' rule. The information structure is important because agent's consumption-investment decisions depend upon the expected return to holding capital. In equilibrium the price of capital reflects all the information available to the least informed agents in the economy. The differential information structure motivates some agents to buy or sell capital. It is shown by way of example that the volume of trade, as well as the change in the price of capital can be highly correlated with a measure of the information content of prices. This measure of information is the difference between the unconditional entropy of the dividend and the entropy of the dividend conditional on observing the price of capital. This helps give content to the view that changes in the price of capital and the volume of capital traded convey information about market fundamentals.
I. INTRODUCTION

In this paper a dynamic model is constructed in which optimizing agents who possess differential information choose portfolios based in part on expected future returns. Agents who receive less information must make inferences about the information observed by other agents and therefore also of the beliefs of these agents, after observing the equilibrium level of financial market variables. The presence of differential information gives rise to different portfolios or consumption choices by distinct agents and consequently to an endogenous volume of trade in capital. Volume of trade is greater when the price of capital is unusually high or low. It can be shown that in some instances the level of transaction volume, and the level of the price of capital are highly correlated with a measure of the "information" content of observed prices.

It has long been recognized that to gain an understanding of the dynamic behavior of asset prices and rates of return, dynamic models must be constructed and analyzed in order to see what features of an agents environment affect the equilibrium prices and rates of return to holding various capital assets. But while much progress has been made in analyzing asset prices in dynamic models, comparatively little progress has been made in analyzing the determinants of transaction volume in asset markets. This may seem strange to many, that so much attention has been devoted to the study of prices, and comparatively little research has been directed at analyzing the determinants of trading volume in the same markets.\(^1\) This is especially puzzling when one considers that it is likely to be the case that the same fundamental forces are likely to impinge on both of these phenomena, and that forces that impact on one of these factors may also likely affect the other as well.

The fact that a wide body of financial economics has not been able to construct many convincing equilibrium theories explaining the determinants of transaction volume is puzzling to say the least. Interestingly, LeRoy (1989) appears to take the view that

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\(^1\) To some extent this is exactly the opposite of what is done in much of the recent work on business cycles, where the primary focus of attention is on the behavior of various quantities, rather than on prices. One might be tempted to wonder how successful current business cycle theory would be considered if it were able to largely explain many of the fluctuations in many relative prices such as the real interest rate, but could not explain the behavior of the quantities, such as the procyclical nature of consumption and employment.
the observations on the "high" volume of trade on organized securities markets are rather
damming to some existing theories that imply market efficiency since, presumably, models
that imply market efficiency have no implications for the level of transaction volume.\(^2\)
That is, if all security prices follow a martingale then there appears to be no reason for
trade in these assets.

Karpoff (1987) presents a summary of the existing empirically-oriented literature
that characterized the relationship between price changes and trading volume. Two rela-
tions that seem to appear in the data are that volume is positively related to the magnitude
of the price change, and the level of the price change in equity markets. As Karpoff points
out, many of the existing studies use rather ad-hoc models as a motivation for the em-
pirical work.\(^3\) That is, many ad-hoc behavioral characteristics are attributed to agents a
priori, without any consideration for how these features could ever characterize optimal
behavior in any environment, much less in the economy in which they are analyzing. The
construction of explicitly dynamic general equilibrium models in which agents maximize
utility subject to budget constraints that reflect optimal participation in existing markets,
does not appear to be a goal of this literature.

There have been a few papers that have sought to characterize the nontrivial joint
behavior of asset prices and transaction volume in explicitly dynamic general equilibrium
environments where agents behave "optimally" (see Huffman (1987), (1989)). However, in
an attempt to characterize this relationship, certain assumptions on preferences are made
in order to obtain manageable decision rules. These decision rules have the unfortunate
property that the asset holdings, and therefore contemporaneous consumption, are inde-
pendent of expected future returns to capital. This may be viewed as problematic since

\(^2\) Presumably it is possible to argue that the opposite view is correct. That is, since
agents will attempt to insure themselves against any risk inherent in their income or
consumption stream, they should be buying or selling financial securities in order to attain
this goal. To the extent then that actual economic agents are not able to obtain "perfect
insurance," it may be viewed as puzzling that the volume of trade is not higher than is
currently observed. In any case, existing theories appear to have little to say concerning
whether the observed volume of trade is either "too high" or "not high enough."

\(^3\) Karpoff also makes reference to the pervasive view that it "takes volume to make
prices move." The models that is analyzed in this paper will give some theoretical content
to that view.
possibly an important reason why agents hold capital assets is for their expected future returns. However, it has not been easy to date to construct models in which agents decision rules depend non-trivially on the expected future returns of assets, and in which agents trade assets in equilibrium. Milgrom and Stokey (1982) have attempted to analyze such a model, but the result they obtain is that for a restricted set of economies, there will be no trade in equilibrium. One important reason why trade may not arise is related to ideas expressed by Grossman (1978), and Grossman and Stiglitz (1980). Consider an economy in which agents are initially endowed with diverse information sets, and this fact is common knowledge. Agents' portfolio decisions will be affected by their information sets, and consequently the equilibrium price (and possibly volume of trade) will reflect all this information. But if all information is reflected in financial market variables, then agents will not have diverse information in equilibrium, and there may then be no reason for agents to trade.

In the simple model constructed in this paper, there will be two types of agents: those agents who are "informed" about the future rate of return on the single asset, and those agents who are not informed. Both types of agents maximize utility subject to their budget constraints and agents' decision rules will depend non-trivially on expected future returns of assets. The informed agents also receive another type of shock, which could be an endowment or preference shock. The equilibrium value of the capital asset will reflect all such shocks as well as expected future returns of the asset. The uninformed agents use the equilibrium price to make inferences about the underlying information set possessed by the informed agents. The uninformed agents thereby gain information from the equilibrium price, but in many instances they are not able to uncover exactly what the informed agents know. Consequently, there may still be trade in assets in equilibrium due to the fact that agents have differential information about future returns of the asset.

The model is consistent with what many may believe to be important ingredients of observed asset markets. In some periods the price of capital reveals more about the underlying fundamentals of the asset (or of some other feature of the economy) than in other periods. There are many different types of agents in the economy who have different information sets. Uninformed agents, or those agents with relatively little information about
future rates of return, attempt to infer the information possessed by the more informed agents through the implied actions of the latter agents. The differential information in the economy contributes to the purchase and sale of capital, and in equilibrium capital is actually traded. Furthermore, the volume of trade that is present is different from what would prevail if there were perfect information. Two examples are presented that are illustrative of the behavior of the economy in question. These examples show that when the price of capital is near its average level, the volume of trade is low, and little information is conveyed by financial market variables to the uninformed agents. On the other hand, when the price of capital deviates substantially from its average level, the volume of trade will be high and more information will be conveyed by financial market variables. These latter features can be shown to be consistent with the observed behavior of the corresponding financial market variables as described by Karpoff (1987). Furthermore, it is shown that the price of capital carries information about the underlying fundamental of the asset, which in this case is the dividend. A measure of the amount of this information is available and it is the difference between the unconditional entropy of the dividend, and the entropy of the dividend conditional on observing the price of capital. It is shown that this measure of information is highly correlated with the volume of capital traded, and that the information content of prices is high when the price of capital is unusually high or low.

The remainder of the paper is organized as follows. In Section II the economic environment is presented in detail with reference to a description of preferences, information sets, and the trading opportunities available to agents. The equilibrium of the economy is studied, and a proof of the existence of an equilibrium for the economy is presented in the Appendix. In Section III two examples are presented that illustrate the dynamic behavior of the economy. Section IV contains some comments of how these techniques could be employed to study environments with shocks of a different sort. Some conclusions are presented in Section V.
II. DESCRIPTION OF ECONOMY

Environment:

The environment under study is one in which time is discrete and is indexed by \( t = 1, 2, 3 \ldots \) At each date \( t \), where \( t \) is odd, there is a population of size \( N \) that enters the economy. The agents are present in the economy for 3 periods only. At dates \( t \), where \( t \) is even, there are no agents entering the economy. Each agent has identical preferences such that if they enter the economy at date \( t \), for \( t \) odd, they wish to consume at dates \( t + 1 \) and \( t + 2 \). An agent \((j)\) who enters the economy at date \((t - 1)\) has preferences described by the function

\[
\frac{(c_{t-1,t})^{1-\rho}}{1-\rho} + \frac{\alpha_{t+1}^j(c_{t-1,t+1})^{1-\rho}}{1-\rho} \quad \rho > 0, \rho \neq 1
\]

Here \( c_{t-1,s} \) denotes period \( s \) consumption by agent \((j)\) who entered the economy in period \((t - 1)\). The variable \( \alpha_{t+1}^j \) is a random preference shock for agent \( j \), that will be discussed in more detail below. There is productive capital in the economy that yields a random payment or dividend \((r_t)\) to its holder in every period \( t \), in units of the consumption good. The number of units of capital is normalized to \( N \), so that the per-capita quantity is unity. At date \( t = 1 \), there is a population of agents who widely hold the existing units of capital, and these agents exist in the economy for only period 1.

The behavior of the remainder of agents is as follows. An agent who enters the economy in period \( t \), where \( t \) is odd, arrives with an endowment of the consumption good. The size of each agent’s initial endowment is initialized to unity. Since this agent does not consume in this period, he will use the endowment to increase consumption in future periods, and in this case he will do so by purchasing some amount of the productive capital. In the next period the agent will want to consume, and the capital that he holds will also pay a dividend. The agent may then wish to sell off some more of his holdings of capital in order to increase consumption even further, or he may wish to use some of the dividend received in order to purchase more capital for the next period. Note that since for date \( t \), where \( t \) is even, the only agents present in the economy are those who entered it in period \( t - 1 \), it must be that if there are exchanges of capital, they must only take place between agents who entered the economy during the previous period. That is, in odd periods trade
is intergenerational, while in even periods it is intragenerational. In the last period of
an agents’ planning horizon, he receives a dividend proportional to his capital holdings,
and must then sell his capital (to the new generation entering the economy) in order to
maximize consumption in the last period of life.

An Agent’s Optimization Problem:

Let \( x_{t-1}^j \) denote the holdings of capital by an agent (\( j \)) who entered the economy
at date \( t - 1 \), where \( t - 1 \) is odd. If \( P_{t-1} \) is the price of capital during period \( t - 1 \) then it
must be that \( P_{t-1}x_{t-1}^j = 1 \), since the agent’s endowment is normalized to unity. During
the next two periods the agent must then choose \( x_t^j \) to maximize

\[
E_{t}^{j} \left( \frac{(c_{t-1,t}^j)^{1-\rho}}{1-\rho} + \alpha_{t+1}^j \frac{(c_{t-1,t+1}^j)^{1-\rho}}{1-\rho} \right)
\]

subject to the constraints

\[
\begin{align*}
    c_{t-1,t}^j &\leq (P_t + r_t)(x_{t-1}^j) - (P_t x_t^j) \\
c_{t-1,t+1}^j &\leq (P_{t+1} + r_{t+1})x_t^j \\
c_{t-1,t}^j, c_{t-1,t+1}^j, x_t^j &\geq 0.
\end{align*}
\]

The form of the expectation operator \( E_{t}^{j}(\cdot) \) is meant to reflect the fact that some
agents may have different information sets available to them at the same date. This feature
will be described in much detail in the next sections. The fact that the number of units of
capital has been normalized to \( N \) implies that the equilibrium condition for capital is the
following for all \( t \geq 1 \)

\[
\sum_{j=1}^{N} x_t^j = N.
\]

Remark:

The utility function that is employed is meant to be of a general nature, while
excluding certain uninteresting cases. For ease of exposition it has been assumed that the
“risk aversion” or intertemporal substitution parameter (\( \rho \)) is the same for period 1 and 2
consumption. For the utility of first period consumption, (\( \rho \)) must be greater than zero,
and otherwise no restrictions need apply.\footnote{In fact, Section (II) contains two examples in which the first period utility is of this form but for \((\rho) = 1\), while second period utility has \((\rho) = 0\). These parameterizations will easily fit into the present analysis.} For the utility of second period consumption, \((\rho)\) cannot equal unity for this is the case of logarithmic utility, and in this instance the agent's saving-consumption decision rules are independent of the expected return on capital. This would take away any interesting possible avenue for agents to infer information from the contemporaneous price of capital since agents would never act on this information.

**Information Structure:**

One of the important features of this model is that some agents will have more information about certain random variables than will other agents. In particular, for a group of agents who enter the economy at date \(t - 1\), where \(t\) is even, all agents can be considered identical. However, in period \(t\), a fraction \(\lambda\) of these agents will receive two signals. First, they will receive a signal about the value of their own preference shock for the next period \(\alpha_{t+1}^i\). As a benchmark, it will be assumed that this signal is perfectly informative and is common among agents that receive such signals, although these are certainly not necessary assumptions. Therefore, at date \(t\), for \(t\) even, a fraction \(\lambda\) of agents know with certainty the value of their own random preference shock in the next period. The remainder of the agents who do not receive such a signal have preference disturbances \((\alpha_{t+1}^i)\) that are constant, and known to be constant. Secondly, the same fraction \(\lambda\) of the agents who received the random preference shocks also receive another signal. At date \(t\) these agents observe a signal of the level of dividends in the next period \((r_{t+1})\). Again, as a benchmark it is assumed that this signal is perfectly informative, and hence to these agents \(r_{t+1}\) is predictable at date \(t\). The remainder of the agents receive no such signal, and consequently barring any other information their objective probability distribution over the level of dividends in the following next period is the unconditional probability distribution for this random variable. It is important that these "signals" observed by agents about their preference shocks or the future level of dividends, are private information, and the agent has no way to show other agents exactly what his signal actually was. Additionally, he cannot tell whether any other agent has or has not received any signals about these
random variables. Hence, these agents have no way in which to insure against adverse realizations of these disturbances. Henceforth, the agents \( (i) \) who receive the “signals” will be referred to as the informed agents and the other agents \( (u) \) will be referred to as the uninformed agents.

The assumption that the same agents receive the two information signals is for convenience only. In this setup there are only two groups of agents to deal with and thereby the model is somewhat tractable. If the signals were randomly given to agents in the economy, then there could be four groups of agents to deal with: those who observe only their preference shock, those who observe only the future value of the dividend, those who receive both pieces of information, and those who receive neither. Although this latter setup may perhaps be a more accurate pattern of how information is dispersed throughout actual economies, its implementation here would appear to only complicate the present analysis and would not produce greater insights. The structure assumed here still contains the main ingredients for the analysis of the interaction of the information or signals about payoffs of assets on the one hand, and the statistical relationship between the price and volume of capital traded on the other. Similarly, the assumption that these signals are perfectly informative could easily be modified, although this would not appear to lead to any further substantive results.

The informed agents know that they will be receiving their information signals, and that the uninformed agents will receive no such information. Similarly, the uninformed agents know that the informed agents will receive a signal about dividends and their own preference shock. Furthermore, these facts are common knowledge. That is, the informed agents know that the uninformed agents know that the informed agents will receive information about the future dividend and preference shock, and so on. In other words, both groups know exactly what they know, and what they do not know, and are similarly informed about the other agents.

There is an avenue, however, through which an uninformed agent may infer the information received by the informed agents. By observing the price of capital at which they can trade, the uninformed agent can infer the behavior of the informed agents. They can do this as follows. The decision rules of the informed agents will reflect the information
they have obtained on the dividend and their own preference disturbance, and therefore the price of capital being a function of the decision rules of all agents, will also reflect this information. Since the uninformed agents know the unconditional probability distributions of both the dividend and the preference shocks for the informed agents, and they observe the price of capital at which trade can take place, they can then compute a conditional probability distribution over the future dividend for capital which reflects the information embodied in the price of capital. Furthermore, as one would expect, it must be the case that the uninformed agent's conditional and unconditional probability distributions over the future level of dividends must be consistent with Bayes' rule. Hence, one group of agents observes financial market variables and must make inferences about the information contained in these variables.

Therefore, this discussion gives somewhat more meaning to the expectation operator given in equation (1). This expectation is conditional on all information available to the agent, which includes any observed signals plus any information inferred by observing the contemporaneous price of capital.

In the words of Grossman and Stiglitz (1980), behavior of the economy under study must be informationally efficient in the sense that in equilibrium, agents must be trying to obtain as much information from the observed pattern of financial variables as is possible, and must as well be using this information in maximizing their expected utility. In equilibrium, the financial market variables will reflect as much information as is available to the least informed agent who is participating in the market.

The computation of an equilibrium is now tricky. It must be the case that uninformed agents offer to buy or sell an amount of capital at the equilibrium price of capital, in such a manner that the price of capital does not reflect more information about future dividends than is present in the information set of the uninformed agents after the agents have observed the price of capital.

A Stationary Competitive Equilibrium

The competitive equilibrium under study will be stationary. To this end, note that
the economy behaves in a sort of trivial fashion in periods \( t \), where \( t \) is odd.\(^5\) In these periods, agents who are leaving the economy supply all capital inelasticity in return for the consumption good, while agents who are entering the economy supply all their endowment in return for the capital. Hence it follows that the price of capital will be \( P_t = 1 \), for \( t \) odd. Additionally, all units of capital are traded in each such period. Hence, the odd periods will not be of a great deal of interest. In the periods where \( t \) is even, however, the price of capital and the volume of trade can fluctuate, as will be shown.

In light of the description of the information structure, it is now possible to give a somewhat loose description of the stationary competitive equilibrium. Let \( \omega_t \) denote the state vector for this economy. In periods \( t \), for \( t \) even, let \( \omega_t = (r_t, \alpha_{t+1}^i) \) denote the state vector, since it will encompass all relevant variables for the agent's decision. For periods \( t \), for \( t \) odd, let \( \omega_t \) be constant since each of these periods are mirror images of each other.

**Definition:** A Stationary Competitive Equilibrium for this economy is characterized by a collection of functions \([P(w), x^j(w)]\) defined for all \( w = (r, \alpha^i), j = (i, u)\), such that for \( t = (2, 4, 6 \ldots)\)

i) Given the pricing functions \((P_t, P_{t+1}) = (P(w), 1)\), where \( w = (r_{t+1}, \alpha_{t+1}^i)\), the asset choices \((x_{t-1}^i, x_t^j) = (1, x^j(w))\) solve the optimization problem for agent \( j \) \((j = i, u)\)

given by equations (1) - (3) as described above.

ii) For all \( t \geq 1 \), and all \( \omega_t \), \( \sum_{j=i,u} x_t^j = N \).

**Remark:** It is implicit (if not explicit) from the above description that in condition i) of the definition, that the decision rules solve the optimization problem of maximizing the conditional expected utility of agent \( i \), where the respective "conditional" expectations reflect all the information to which agents have access, including that imbedded in the capital pricing function itself.

It is shown in the Appendix that a stationary equilibrium exists for a wide class of

\(^5\) It should be clear from the description of the economy that alternatively it could be viewed as a single 3 period model. The infinite horizon was employed in order to permit the characterization of the stationary stochastic properties of the economy and so as to not restrict the analysis to that of a single realization for the 3 period economy.
economies of the sort described above.

Consider now the optimization problem for an informed agent \((i)\) who enters the economy at date \((t - 1)\). The optimization condition for the agent is

\[
P_t[r_i x_{i-1}^i + P_t(x_{i-1}^i - x_i^i)]^{-\rho} = E_t^i \left( \alpha_{i+1}^i \left( [P_{t+1} + r_{t+1}] x_{i}^i \right) \right)^{-\rho} \cdot \frac{[P_{t+1} + r_{t+1}]}{[P_{t+1} + r_{t+1}]} \tag{5}
\]

However, the vector \((P_{t+1}, r_{t+1}, \alpha_{t+1}^i)\) is in the agent's information set at date \(t\). Hence this expression can be rewritten as

\[
[P_t x_i^i]^{\rho} [r_i x_{i-1}^i + P_t(x_{i-1}^i - x_i^i)]^{-\rho} = \alpha_{i+1}^i \left( \frac{P_{t+1} + r_{t+1}}{P_t} \right)^{1-\rho} \tag{6}
\]

This expression makes it clear that the value of the agent’s holdings of capital after period \(t\) will be an increasing function of the rate of return on capital for \(\rho \in (0, 1)\).

For an uninformed agent \((u)\) on the other hand, the optimization problem is identical to that for agent \((i)\) given by equation (5). However, \(r_{t+1}\) is not known with certainty to such an agent, and so this optimization condition can be rewritten as

\[
[P_t x_i^n]^{\rho} [r_i x_{i-1}^n + P_t(x_{i-1}^n - x_i^n)]^{-\rho} = E_t^n \left[ \alpha^n \left( \frac{P_{t+1} + r_{t+1}}{P_t} \right)^{1-\rho} \right] \tag{7}
\]

Again, this equation displays a similar relationship to that of the corresponding equation (6) for informed agents. The only difference is that equation (7) has the expected value of a function of the return on capital on the right side, rather than a function of the actual return on holding capital. The quantity of capital purchased by informed agents will reflect the actual rate of return on capital, whereas the quantity of capital purchased by uninformed agents will reflect expectations of (a function of) the rate of return on capital. The capital market clearing condition can then be used to show that the actual price must be a function of the actual return to capital. Consequently the equilibrium price of capital must then reflect the actual return on capital. Therefore, uninformed agents will then use their observation on the contemporaneous price of capital to make inferences about its future return and this will thereby affect the manner in which the expected value in equation (7) is calculated. Letting \(R_{t+1} = \left( \frac{P_{t+1} + r_{t+1}}{P_t} \right)\), equation (6) can be written as
\[
\left( \frac{P_t x_t^i}{x_{t-1}^i (P_t + r_t) - P_t x_t^i} \right) = \left[ \alpha^i_t (R_{t+1})^{1-\rho} \right]^{1/\rho} \equiv \Theta^{i+1}_t
\] (8)

or

\[
(P_t x_t^i) = \left( \frac{\Theta^{i+1}_t}{1 + \Theta^{i+1}_t} \right) \left[ x_{t-1}^i (P_t + r_t) \right].
\]

Similarly, for uninformed agents equation (7) can be written as

\[
(P_t x_t^u) = \left( \frac{\Theta^{u+1}_t}{1 + \Theta^{u+1}_t} \right) \left[ x_{t-1}^u (P_t + r_t) \right].
\]

where

\[
\Theta^{u+1}_t = [\alpha^u_t E^u_t (R_{t+1})^{1-\rho}]^{1/\rho}
\] (9)

These conditions together with equation (4), and with the fact that for \( t \) even, \( x_{t-1}^u = x_{t-1}^i = 1 \) imply

\[
P_t = (P_t + r_t) \left[ \left( \frac{(1 - \lambda) \Theta^{u+1}_t}{1 + \Theta^{u+1}_t} \right) + \left( \frac{\lambda \Theta^{i+1}_t}{1 + \Theta^{i+1}_t} \right) \right].
\] (10)

Obviously this is an implicit equation for the price of capital since \( \Theta^{i+1}_t \) and \( \Theta^{u+1}_t \) are functions of \( P_t \). From this expression it is clear that given an observation on \( P_t \) and \( (r_t) \), and given an uninformed agents expectation of \( (R_{t+1})^{1-\rho} \), this latter expectation must be consistent with the conditional probability distribution of \( r_{t+1} \) (or \( R_{t+1} \)) implied by equations (8) and (9). Rather than further analyzing this somewhat general case, it is more illuminating to analyze two specific examples. This is taken up in the next section.

**III. TWO EXAMPLES**

In this section two examples are presented to illustrate the behavior of such an economy, and to further investigate a measure of the "information" implied by prices and the volume of trade changes as a function of this information. In both cases, to ease the exposition it will be assumed that the utility function for all agents is of the form

\[
\ell_n(c_{t-1,t}^i) + \alpha^i_{t+1}(c_{t-1,t+1}^i).
\]

In the first example the probability distribution for the exogenous random variables is discrete, while in the second case it is continuous.
Example 1:

Let the probability distribution of the independent random variables be of the form

\[ r_t = \begin{cases} 
0.25 & \text{with probability } 1/2 \\
0.1111 & \text{with probability } 1/2
\end{cases} \]

\[ \alpha_t^i = \begin{cases} 
(0.9) & \text{with probability } 1/2 \\
(0.8) & \text{with probability } 1/2
\end{cases} \]

for \( t \) odd. Also \( r_t = 1 \) for \( t \) even, \( \alpha^u = 0.85 \), and \( \lambda = 0.2 \).

Now the optimization condition for an informed agent (equation (6)) can be written as

\[
\left[ \frac{P_t}{x_{t-1}^i(P_t + r_t) - x_t^iP_t} \right] = \alpha_{t+1}^i(P_{t+1} + r_{t+1}) = \alpha_{t+1}^i(1 + r_{t+1})
\]

The example is constructed so that the right side of this equation has the following distribution

\[
\alpha_{t+1}^i(1 + r_{t+1}) = \begin{cases} 
1.125 & \text{with probability } 1/4 \\
1.0 & \text{with probability } 1/2 \\
0.889 & \text{with probability } 1/4
\end{cases}.
\]

The decision rule for the informed agent will be of the form

\[
(P_t x_t^i) = x_{t-1}^i(P_t + r_t) - \left[ \frac{P_t}{\alpha_{t+1}^i(1 + r_{t+1})} \right]
\]

Similarly, for uninformed agents their optimization condition can be written as

\[
(P_t x_t^u) = x_{t-1}^u(P_t + r_t) - \left[ \frac{P_t}{\alpha^u E_t^u(1 + r_{t+1})} \right]
\]

Using these decision rules in equation (10) produces the following equilibrium capital pricing equation

\[
P_t = (r_t) \left[ \left( \frac{\lambda}{\alpha_{t+1}^i(1 + r_{t+1})} \right) + \left( \frac{(1 - \lambda)}{\alpha^u E_t^u(1 + r_{t+1})} \right) \right]^{-1}
\]
It is then straightforward to show that there exists an equilibrium with the properties listed in Table 1.

<table>
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<tr>
<th>((\alpha_{i+1}^t, r_{i+1}))</th>
<th>Prob</th>
<th>(P_t)</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
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<td>1/4</td>
<td>1.074438202</td>
<td>.004333</td>
</tr>
<tr>
<td>ii) (.9, .1111)</td>
<td>1/4</td>
<td>1.002775851</td>
<td>.000277</td>
</tr>
<tr>
<td>iii) (.8, .25)</td>
<td>1/4</td>
<td>1.002775851</td>
<td>.000277</td>
</tr>
<tr>
<td>iv) (.8, .1111)</td>
<td>1/4</td>
<td>.932784637</td>
<td>.005110</td>
</tr>
</tbody>
</table>

When the preference shock for informed agents is high and the future dividend is high, the uninformed agents can infer from the price of capital that the future dividend will be high with certainty. Similarly, when the preference shock for the informed agents is low and the future dividend is low, the uninformed agents can infer that the future dividend will low with certainty. In the other two cases, however the price of capital does not tell the uninformed agents “enough” information because the price of capital is identical in both states and consequently they are forced to compute a conditional expected value of the future dividend. It should also be noted that the volume of trade is highly correlated with the level of information conveyed by the price. Volume is highest when both agents are certain about the values of both stocks.

**Example 2:**

In this example continuous probability distributions for the exogenous random variables will be employed. In particular, it is assumed that the independent random variables \(\log(\alpha_{i+1}^t)\) and \(\log(1 + r_{i+1})\), for \(t\) odd, are both uniformly distributed on the interval \((0,1)\). Additionally, let \(\lambda = 0.5\), \(r_t = 0.5\) for \(t\) even, \(\alpha^u = E(\alpha_{i+1}^t) = (\epsilon - 1)\), and \(\beta = 0.8\). The utility function for both types of agents is as follows

\[
\ln(c_{i-1,t}^d) + \beta \alpha_{i+1}^t (c_{i-1,t+1}^d).
\]

The uninformed agents can infer the value of the random variable \((\alpha_{i+1}^t (1 + r_{i+1}))\) from an observation on the price of capital. Hence these agents must compute the conditional probability distribution of \((1 + r_{i+1})\) implied by this observation. It can be shown that
\[ \text{Prob}\{(1 + r_{t+1}) \leq z | \alpha_t^i(1 + r_{t+1}) = y \leq e\} = \begin{cases} 0 & \text{for } z < 1 \\ \log(z) / \log(y) & \text{for } 1 \leq z \leq y \\ 1 & \text{for } z \geq y \end{cases} \]

\[ \text{Prob}\{(1 + r_{t+1}) \leq z | \alpha_t^i(1 + r_{t+1}) = y \geq e\} = \begin{cases} 0 & \text{for } z \leq (y/e) \\ \frac{\log(z) + 1 - \log(y)}{2 - \log(y)} & \text{for } (y/e) \leq z \leq e \\ 1 & \text{for } z \geq e \end{cases} \]

It can then be shown that

\[ E(1 + r_{t+1} | \alpha_t^i(1 + r_{t+1}) = y \leq e) = \left[ \frac{y - 1}{\log(y)} \right], \]

and

\[ E(1 + r_{t+1} | \alpha_t^i(1 + r_{t+1}) = y \geq e) = \frac{[e - (y/e)]}{[2 - \log(y)]}. \]

The resulting pattern of observations in this economy for transaction volume and the price of capital are shown in Figure 1. Transaction volume is highest when the price of capital is unusually high or low. Note also that since the price of capital is constant with a value of unity in odd periods, Figure 1 then shows there is a positive correlation between the absolute value of the price change and the transaction volume. This was true in the previous example as well.

Considering the structure of the economy, it is not surprising that the price of capital conveys information about the future payoff of the asset. In particular, high (low) values for the price of capital imply high (low) values for the expected value of future dividends. Fortunately, there is a method available from the statistics literature for calculating the "information content" of prices in this economy (see Papoulis (1984)). Consider a continuously-distributed random variable \( x \) with probability density \( f(\bullet) \). The entropy of \( x \) is defined to be

\[ H(x) = E\{-\ln(f(x))\} = -\int_{-\infty}^{\infty} f(x) \ln(f(x)) \, dx. \]
Now consider random variables \( x \) and \( y \) which are jointly continuously distributed with joint density \( F(x, y) \), and with marginal probability densities \( f(\cdot) \) and \( g(\cdot) \), and the probability density of \( x \) conditional on observing \( y \) in denoted \( m(x|y) \). The entropy of the random variable \( x \) conditional on observing \( y \) is defined to be

\[
H(x|y) = E\{-\ln(m(x|y))|y\} = -\int_{-\infty}^{\infty} m(x|y)\ln(m(x|y))\,dx
\]

The mutual information, or the information about \( x \) contained in an observation of \( y \) is then defined as

\[
I(x, y) \equiv E\{\ln\left(\frac{F(x, y)}{f(x)g(y)}\right)\} = H(x) - H(x|y).
\]

It can be shown that \( I(x, y) \geq 0 \), and that \( I(x, y) = 0 \) if and only if \( x \) and \( y \) are independent.

Now for the economy under study, the uninformed agents know the unconditional distribution of the dividend, and can calculate the distribution of the dividend conditional on observing the price of capital. Hence the information content of the price of capital can be computed as the difference between the unconditional entropy of the dividend, and the entropy of the dividend conditional on observing the price of capital.

The information content of prices, measured as described above as the information about the dividend contained in the current price of capital is shown in Figure 2. It is clear that more information is conveyed by unusually high or low prices. A comparison of Figures 1 and 2 reveals that the information content of the price of capital is high (low) when the volume of trade is high (low). This is further illustrated in Figure 3 where these variables alone are shown. In this instance the correlation between the volume of trade and the measure of information is \( .9005 \). Figure 4 shows the relationship between the information content of prices on the one hand, and the level of the preference shock and dividend on the other.\(^6\)

\(^6\) In each of the diagrams the space of preference shocks \( (\alpha_t) \) and dividends \( (r_t) \) was discretized so that there were 63 possible values for each random variables (90 for Figure 4). Clearly more information is conveyed by the price when the preference shock and dividend are either both high, or both low.
To see the impact that the information structure alone has on these patterns, it may be enlightening to compare this economy with one which is exactly the same except that the uninformed agents also receive the same information about future dividends that is received by informed agents (i.e. all agents are informed). Figure 5 shows the pattern of transaction volume and the price of capital that is now produced when there is perfect information. This is to be compared with Figure 1. The presence of uninformed agents appears to impose considerably more structure on this pattern then would otherwise appear if all agents were informed.⁷

IV. EXTENSIONS TO OTHER ENVIRONMENTS

It would appear that there is plenty of latitude for introducing such “signal extraction” issues into environments of this sort, as there appears to be room for a whole host of factors which are fundamental to the payoff of an asset that may be imbedded as information in the current price of an asset in a manner similar to the effect that other non-fundamental factors would have on the price. In many instances there would then not necessarily be any need to resort to the use of preference shocks so that the price of capital does not convey all information. For example, consider a similar environment in which agents do not receive preference shocks, and instead some random fraction of the population receives information about the future rate of return on capital. Again, the uninformed agents must make inferences about this rate of return based on their observation of the price of capital. In periods in which the price of capital is, say, slightly above average, the uninformed agents cannot distinguish whether this means that there are a very few informed agents who know that the future rate of return will be very high, or whether there are plenty of informed agents who know that the rate of return on capital will be slightly above average. This could lead them to under or overestimate the actual rate of return on the asset.

⁷ One might question whether the analysis would change if uninformed agents had access to the contemporaneous transaction volume as well as to the price of capital. Fortunately, giving agents this information would not alter the analysis. This can be seen by noticing that in Figure 1 there is a unique level of transaction volume for each price of capital, and consequently if the agent sees the price of capital he already knows the level of transaction volume that will prevail in the current period.
In the examples of Section III, it was not necessary to have the informed agents know exactly what the future dividend was going to be. This device was employed to ease the exposition of the model. Instead it could be that the informed agents receive a signal about the probability distribution of the future dividend, and there is less uncertainty associated with this distribution than with the probability distribution used by the uninformed agents. This would not change the general nature of the problem that confronts the uninformed agents. Similarly, it was not necessary that the environment be such that the price of capital be constant (unity) in odd periods. This was merely an artificial device designed to simplify the analysis.

There are also many other features of the environment about which the uninformed agents might not be aware. For example, they might not know with certainty the parameter governing intertemporal substitution of other agents in the economy. Alternatively, some agents could have a random endowment of the consumption good in either the first or second period of their life, and the uninformed agents may not be as well informed about this feature of the environment as are the other agents in the economy. It is not hard to see that this could affect the equilibrium price of capital. A higher equilibrium price of capital might reflect the fact that informed agents expect the return on capital to be high, or might instead reflect the fact that their first period endowments are higher than usual. It could also be that there are random variations in the supply of capital about which some agents are better informed than other agents. Therefore, the price of capital would then have to reflect the expectations of these events, as well as expectations of expected returns.

The random “fundamental” feature of the asset in question so far has been the realization of the dividend, but this could be altered in many ways. For example, the uncertainty about the payoff of capital could be the price for which it will sell in the subsequent period. This price could be random because of uncertainty about the number of agents entering the economy in the subsequent period, or because of uncertainty about the level of the endowments of the agents who will be making purchases of capital. Therefore, the uncertainty in the environment could be extended so that there are many random fundamental factors that influence the return on capital, about which some frac-
tion of the population are somewhat better informed, while there are also many random non-fundamental factors which also influence the equilibrium price of capital.

It is also conceivable that some agents could be better informed about the returns on alternative assets than other agents and that this could have an impact on the volume of trade. In this instance, a low price of a particular asset might reflect that informed agents expect low returns on this asset, or it might reflect that informed agents merely expect higher returns on alternative assets. Uninformed agents must then make inferences about such information from the equilibrium price of capital.

Lastly, the model is not without some policy implications. It is straightforward to see that if a government is introduced which can institute either an endowment tax or a capital income tax, then these policies are seen to influence the information content of asset prices and therefore influence the information conveyed to the least informed agents in the economy. For example, consider the economy described in Example 2. If the government in this economy were to levy a proportional tax on dividends in all periods and use the resulting revenue for government consumption that did not influence private consumption, this policy would affect the level of the price of capital. This in turn would influence the information content of prices. This is illustrated in Figure 6 where the information content of the price of capital is plotted under the regimes of a 25% dividend tax, and no tax at all. Similarly, if the tax were made stochastic this would also influence the information content of prices.

V. CONCLUSION

It is the purpose of this study to provide a somewhat specialized general equilibrium framework within which an analysis could be conducted into the relationship between a measure of the information content of asset prices on the one hand, and the statistical relationship between volume of trade and price of an asset on the other. Agent's optimizing actions lead to endogenous trade in assets. The agents with the least information face a "signal-extraction" problem that is reminiscent of that employed in Lucas (1972). The model that has been constructed and analyzed is consistent with many of the features that many people would view to be accurate characterizations of how observed asset markets
behave. Namely, in some periods the price of capital reveals more about the underlying fundamentals of the economy than in other periods. In all periods the price of capital reveals all the information available to the least informed agents. There are different types of agents, with some agents being better informed about future returns than are other agents. Uninformed agents use the price of capital to infer the information that is available to the informed agents, and then use this information in making a forecast of the future returns to capital. The differential information in the economy contributes to purchases and sales of transaction volume in capital. Examples illustrate that the pattern of transaction volume differs substantially from that which would exist if there were perfect information in the economy. In addition, the examples revealed that the relationship between the transaction volume and the price of capital closely conforms with that observed in actual asset markets. That is, when the transaction volume is highly correlated with the change in the price of capital. This general equilibrium model then gives content to a long-prevailing view that there is a relationship between the volume of trade in financial markets and the behavior of the change in the price of capital.
APPENDIX

Since the equilibrium condition in the capital market as well as the individual optimization conditions are already imposed in the capital pricing equation (10), all that remains to be verified is that there exists a pricing function for capital such that when the uninformed agents observe the price of capital they use the information conveyed by the price. Given that an uninformed agent knows the value of $\Theta^w$, as well as $\lambda$ and $r_t$, an observation of the price $P_t$ for such an agent conveys the same information as an observation on $\alpha^i_{t+1}(1 + r_{t+1})^{1-\rho}$. Let $\Gamma(r_{t+1}|\alpha^i_{t+1}(1 + r_{t+1})^{1-\rho})$ denote the resulting conditional distribution of $r_{t+1}$, given this information. Then let

$$
\Theta^u_{t+1} = \left[ \alpha^u \int \left[ \frac{(1 + r_{t+1})^{1-\rho}}{P_t} \right]^{1-\rho} \Gamma(dr_{t+1}|\alpha^i_{t+1}(1 + r_{t+1})^{1-\rho}) \right]^{1/\rho} \tag{A1}
$$

as implied by equation (9).

Equation (10) can be rewritten as

$$
P_t \left[ 1 - \left[ \frac{(1 - \lambda)\Theta^u_{t+1}}{(1 + \Theta^u_{t+1})} + \frac{\lambda \Theta^i_{t+1}}{(1 + \Theta^i_{t+1})} \right] \right] = r_t \left[ \frac{(1 - \lambda)\Theta^u_{t+1}}{(1 + \Theta^u_{t+1})} + \frac{\lambda \Theta^i_{t+1}}{(1 + \Theta^i_{t+1})} \right] \tag{A2}
$$

where

$$
\Theta^i_{t+1} = \left[ \alpha^i_{t+1} \left( \frac{1 + r_{t+1}}{P_t} \right)^{1-\rho} \right]^{1/\rho}
$$

and $\Theta^u_{t+1}$ is given by equation A1 above. Now there are two different cases to consider.

Case (i): $0 < \rho < 1$.

Note that the right side of A2 is strictly increasing in $\Theta^u_{t+1}$ and $\Theta^i_{t+1}$, while both of these latter expressions are strictly decreasing in $P_t$. Hence, the right side of (A2), viewed as a function of $P_t$ on $(0, \infty)$ is a strictly decreasing function taking values from $+\infty$ to 0. Similarly the left side of (A2) is strictly decreasing in $\Theta^u_{t+1}$ and $\Theta^i_{t+1}$ and hence viewed as a function of $P_t$ on $(0, \infty)$, the left side of A2 is strictly increasing taking values from 0 to $+\infty$. Therefore, because of the continuity of A2 in $P_t$, by the intermediate value theorem there is a unique fixed point solution to equation A2 for $P_t$. 

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Case (ii): \( \rho > 1 \)

In this case both \( \Theta_{t+1}^{u} \) and \( \Theta_{t+1}^{d} \) are both increasing in \( P_t \) and so the left side of A2, viewed as a function of \( P_t \) on \((0, \infty)\) takes values from 0 to \(+\infty\). The right side of (A2) viewed similarly increases from 0 to \( r_t \). Hence to show there exists a solution to equation A3 in \( P_t \), it need only be shown that there exists a value of \( P_t \) such that when both sides of A2 are evaluated at this value, the right side is greater than the left side. To establish this note that for given \( \epsilon > 0 \), there exists a \( \delta > 0 \) such that \( |P_t| < \delta \) implies

\[
\left| P_t - P_t \left[ 1 - \left( \frac{(1 - \lambda) \Theta_{t+1}^{u}}{1 + \Theta_{t+1}^{u}} + \frac{\lambda \Theta_{t+1}^{d}}{1 + \Theta_{t+1}^{d}} \right) \right] \right| < \epsilon.
\]

Hence for \( |P_t| < \delta \), the left side of A2 is less than \( (P_t - \epsilon) \) and hence is strictly less than \( P_t \). Taking the derivative of the right side of A2 with respect to \( P_t \) reveals,

\[
\frac{d}{dP_t} \left[ \frac{(1 - \lambda) \Theta_{t+1}^{u}}{1 + \Theta_{t+1}^{u}} + \frac{\lambda \Theta_{t+1}^{d}}{1 + \Theta_{t+1}^{d}} \right] \to \infty \text{ as } P_t \to 0
\]

for \( \rho > 1 \). Hence the right side of A3, viewed as a function of \( P_t \) can be made larger than \( P_t - \epsilon \). Since A2 is continuous in \( P_t \), the intermediate value theorem again can be used to establish the existence of a fixed point to this equation. Employing equation (A1) in equation (10) produces an equilibrium pricing function for capital in which the price of capital conveys information to the uninformed agents, who use the same information in their decisions which are reflected in the price.
REFERENCES


Huffman, Gregory W. "An Analysis of Transaction Volume and Asset Pricing in a Representative Agent Economy", Manuscript, University of Western Ontario.


FIGURE 6

* o without tax

* o with tax

Information

Price of Capital