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Spatially Dependent Interactions:
A Statistical Approach to Spatial Equilibria, Technological Standardization and Variety

Robin Cowan† and William Cowan ‡

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Abstract

This paper develops a technique for analyzing economies with a large but finite number of interacting agents. Heterogeneous agents are faced with a repeated choice problem, and are subject to externalities from the actions of other agents, but the nature of these externalities is dependent on the location of the agents. The concern is with the spatial distribution of economic activity in such an economy. We develop a technique that allows us to relate the probability of observing a particular state of the economy with the costs associated with that state. We use this technique to analyse a technology choice problem in which the concern is with the degree and spatial nature of technical standardization and the conditions under which variety is preserved.

Keywords: interacting agents; local and global interactions; spatial equilibria; technology choice; standardization

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In many contexts economic agents are affected by externalities from the actions of other agents. While general issues surrounding the existence of externalities have been studied in economics for many years, and are consequently reasonably well-understood, attention has been drawn recently to a particular type of externality, namely externalities that are spatially dependent. Because agents typically interact with, and generally affect agents who are spatially close to them, external effects, whether positive or negative, might be expected to be most important when the agents in question are near to each other. The most natural interpretation of 'space' in this context is geographic space—Krugman (1991 and elsewhere) and Arthur (1990) use the idea of spatially dependent externalities to address the spatial distribution of economic activity. Arthur looks at concentrations of particular industries; Krugman looks at the distribution of manufacturing. Schelling (1973) also considers spatial externalities in his work on the tipping phenomenon; here externalities exist because people care about what their residential neighbours are like, and will relocate in order to live in a neighbourhood to their taste. Space can be interpreted differently, however. Agents can be considered close together if they interact frequently. David and Foray (1993) address externalities arising from technological standards among firms and their major suppliers. Durlauf (1991) considers localized technological complementarities among similar industries and its effect on aggregate output. Prescott and Visscher (1977), and many others, have used the idea of spatial interactions to examine the location of products in product-characteristic space. Blume (1993), and Cowan and Miller (1994) return to geographic space, but in abstract, game-theoretic models of agents located on a lattice and interacting directly with only their nearest neighbours. Again, payoffs to any particular action are determined in part by the actions of an agent's neighbours.¹

All of this work shares a concern with local interaction effects, but finds them in many different contexts. The tools used to analyze the problems differ considerably, but there is a general concern with the relationship between the micro-economic structure of the economy, and the resulting patterns of economic activity at the macro level. This is expressed through attention to the connections between nature of local interactions and the global distribution of economic activity. This can be geographical distribution of industries or people; the distribution of market shares of competing technologies or standards; the temporal distribution of aggregate activity; or the spatial distribution of game-theoretic strategies. There is a second feature that appears in some of this work, namely agent

¹ For a general review of literature regarding economies with interacting agents, see Kirman (1994).
heterogeneity. This is expressed in the modelling context as agents displaying probabilistic behaviour. While agents may see a deterministic problem, from the point of view of the analyst, their behaviour is probabilistic, typically because there are factors that enter an agent’s decision that is is unreasonable to expect an analyst to be able to see.

The technique we develop to address the local externality problem is related to an issue surrounding representative agent models. Kirman (1993) has argued that representative agent models have a serious methodological problem, namely that it is unclear what the agent in those models represents.\(^2\) This creates problems for the analysis of any phenomenon that involves a reasonably large (though finite) number of agents; in a model that involves many agents, it does not take very much agent heterogeneity before we encounter serious tractability problems. This presents a dilemma—on the one horn representative agent models have a ‘representation problem’; on the other horn non-representative agent models have a tractability problem. One way to attack this dilemma, as Kirman has pointed out, is to embrace heterogeneity rather than to eschew it. If we begin by considering how agents interact with each other, we can divide agents into groups according to the types of interactions they have. Then we can use agent heterogeneity to permit us to concentrate on generic interactions, both generic within-group interactions, and generic between-group interactions. This approach cuts down dramatically on the tractability problem, without running into the ‘representation’ problem. One way of executing this approach is presented in this paper.

Technically, this paper presents a unified approach to problems involving local external effects. We consider situations in which agents are subject to repeated choice opportunities in which the payoffs from any choice are affected by two types of externalities, one local and one global. There are, however, factors outside the model, (or beneath the vision of the analyst) which affect agents’ behaviour. Because different agents are subject to different outside influences they appear heterogeneous to the analyst, and this is modelled statistically. The model itself is statistical in the sense that we are looking for results on population averages rather than on individual behaviour. We use the tool developed in the early sections of the paper to analyze a a technology choice or standardization problem, to address issues of the degree and spatial nature of technological standardization and the preservation of variety. We link individual behaviour and global outcomes in a way that

\(^2\) Kirman argues in fact that the agents do not represent anything of economic interest, and that therefore the models are inappropriate as tools for analysing economic phenomena.
sheds light on the spatial nature of economic activity, and on the conditions under which different types of behaviour (technologies in this example) co-exist in equilibrium.

1. Model

Consider an economy of $N$ heterogeneous agents. Each agent makes repeated discrete choices among a set of $P$ possible actions. The net cost to each action is determined by an idiosyncratic effect, a local externality effect and a global externality effect. These externalities may be either technical or pecuniary. With a fixed population of agents the net cost of action $p$ to agent $n$ can be written as $c_{n,p} = f(z_p) + g(z_{n,p}) + h(E_n, p)$, where $z_p$ denotes the proportion of the population that makes choice $p$, thus determining the strength of the global externality; and $z_{n,p}$ denotes the proportion of the population local to agent $n$ that makes choice $p$, which determines the strength of the local externality (‘local’ will be defined more carefully below); and where $h(E_n, p)$ denotes the idiosyncratic benefit to agent $n$ from choosing action $p$. The argument $E$ is explained below. At random times each agent is able to re-choose, and perhaps change his action. As a concrete example, if an action consists of the adoption of one of $P$ possible capital goods, consider capital goods that have a finite but random lifetime, each agent having inelastic demand for one unit. We assume that agents act myopically and non-strategically,3 so that at each moment of choice, an agent’s problem can be written as

$$\min_{p \in \{1..P\}} f(z_p) + g(z_{n,p}) + h(E_n, p).$$

The idiosyncratic effect, $h(\cdot)$, expresses agent heterogeneity. Factors external to the model affect the costs to an agent of pursuing a particular action. We assume that these factors change rapidly relative to the frequency with which agents are able to change their actions. Thus external factors $E$ interact with the current action $p$, of the agent $n$ to determine the current idiosyncratic effect on cost. Thus in the choice problem, agents will be choosing actions based external factors that change over time and space according to some probability distribution. If we consider the example of an agent choosing a technology with which to perform a task, the factor external to the model would be the task that the agent intends to perform. Tasks change rapidly, as they are finished and new ones assigned, and particular technologies are better or worse-suited for different tasks. This implies that the idiosyncratic effect has two aspects. First, agents differ from each other in

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3 It will be clear from the analysis that the assumptions of myopia and non-strategic behaviour are not restrictive, and can be relaxed without changing the qualitative results.
their evaluation of different actions—different agents will use the technology for different tasks. Second, an agent's evaluation of different actions may change over time, as would be the case, for example if he were performing different tasks at different points in time.

The central difficulty in models exhibiting spatially-dependent externalities is the issue of aggregation. The problem faced by any agent is relatively straight-forward, but problems of analysis arise because changes in behaviour of one agent in response to external effects can affect all other agents potentially in two ways—directly, through the global externality; and indirectly, through a series of local externality effects. This makes it difficult to establish equilibria except in relatively special cases. We deal with the problem statistically in order to establish properties of equilibrium in terms of behaviour of the aggregation of agents.

2. Aggregation

In this section we develop the technique for generating macro properties of equilibrium. The general approach is to analyse the properties of total cost in an economy with many agents, and extract from the results of that analysis information about agent behaviour. In essence, we divide the forces acting on an agent into two types, and treat the problem as one of interacting systems. The two systems are the idiosyncratic forces on the one hand, and the externalities on the other. Equilibrium between the systems is reached when the forces generated by each system on the other are balanced. One can think of the idiosyncratic effect, \( h(E_n, p) \), as generating randomness in the distribution of actions, as agents try to choose the action best suited to their current (or predicted) idiosyncratic needs. On the other hand though, the local and global effects, \( f(\cdot) \) and \( g(\cdot) \), act to remove randomness as agents try to co-ordinate with their neighbours and benefit from externalities. Both of these effects promote changes in the spatial distribution of actions. Clearly, equilibrium in the macro state is reached when the two forces promoting change are exactly off-setting.

To make this formal, we need to specify how the two systems interact. To do so, we treat the exogenous influences, \( E \), as being drawn from a vast reservoir of possible factors. This draw interacts with the current action, \( p \), of the agent, \( n \), to generate the agent specific effect, \( h(E_n, p) \). (There exists a large store of tasks that might be assigned to any agent when he is free, and the task assignment interacts with his current technology to determine his current value of \( h \)). As the external forces on an agent change this changes the total costs within the economy. We can model this by saying that these external factors
enter and leave the economy, thereby changing its total costs. The cost associated with
one of these factors when it is in the economy is \( f(\cdot) + g(\cdot) + h(E_n, p) \), so let us call this
value the cost of the factor when it resides in the reservoir. That is, the 'cost' of any factor
in the reservoir is the cost it has manifested or will manifest when it enters the economy.
This definition of 'cost' allows us to treat the combined system as one in which cost is
conserved.\(^4\)

The combined system, economy and reservoir, has a total cost \( C^t \), of which \( C \) is the
total cost in the economy, and so \( C^t - C \) is the cost in the reservoir. What is the probability
that we see a total cost of \( C \) in the economy, given a total combined-system cost of \( C^t \)?
In principle, a cost \( C \) can be produced in many ways, by arranging actions and external
factors differently among the agents. Let the number of ways be \( \Omega(C) \). Let the number of
ways that the remaining cost, \( C^t - C \), can be found in the reservoir be \( \Omega'(C^t - C) \). The
product, \( \Omega(C)\Omega'(C^t - C) \) represents a frequency distribution: the number of ways a given
combined-system cost can be split, leaving exactly \( C \) in the economy. Because \( \Omega'(\cdot)\Omega(\cdot) \) is
very large for any \( C \) and \( C^t \), the mean, mode and median of this distribution will be very
close to one another.\(^5\) The mode is the easiest to calculate, by differentiating the number
of states \( \Omega'(C^t - C)\Omega(C) \) with respect to \( C \) and setting the derivative to zero:

\[
\Omega'(C^t - C) \frac{d\Omega(C')}{dC'} \bigg|_{C' = C} = \Omega(C) \frac{d\Omega'(C')}{dC'} \bigg|_{C' = C^t - C}.
\]

Express this as

\[
\frac{d(\ln \Omega(C'))}{dC'} \bigg|_{C' = C} = \frac{d(\ln \Omega'(C'))}{dC'} \bigg|_{C' = C^t - C}.
\]  

If the external factors that reside in the reservoir are truly exogenous to activity in the
economy then the right hand side of equation (1) is independent of \( C \). Further, we can see
that to achieve equilibrium the economy adjusts to the reservoir rather than the reverse,
so the right hand side of the equation can usefully be expressed as a constant. Call it
\(-\beta \) where \( \beta \) is an (as yet) uninterpreted property of the reservoir of external factors.
Integrating (1), we now have

\[
\ln \Omega = -\beta(C - K),
\]

\(^4\) We appear to have departed from the usual practice of treating costs as incurred by agents,
and have instead described costs as 'attached to external factors that impinge on agents'. This
apparent departure is purely formal.

\(^5\) For a proof of this claim see Pathria, pp. 53-61, who describes the limit of this distribution
as the number of agents gets large: "In the limit the mean values, the most probable values—in
fact any values that occur with nonvanishing probability—become identical!"

5
where $K$ is an undefined normalization constant. Now we can state that

$$\Omega(C) = K' e^{-\beta C}$$

is the total number of states with cost $C$ in the economy when the reservoir of external factors has property $\beta$.\(^6\) Thus it is proportional to the probability of observing the economy in a state that has total cost $C$ when the source of agent-specific idiosyncracies has the property $\beta$. Assuming that every state that has the same total cost, $C$, is equally likely to occur implies that the probability of observing a state, $(\phi, \chi)$, is proportional to $e^{-\beta C((\phi, \chi))}$.\(^7\) Normalizing in the usual way, we have now derived the probability that an economy is in some state $(\phi, \chi)$, as a function of the total cost in that state, $C((\phi, \chi))$, given that the idiosyncratic effects on agents have property $\beta$:

$$\Pr(\phi, \chi) = \frac{e^{-\beta C((\phi, \chi))}}{\sum_{(\phi, \chi)' e^{-\beta C((\phi, \chi)')}}} . \quad (2)$$

In many cases external factors are changing relatively rapidly, so any observable feature of the economy will be associated with some average over them. In other words, the actual state, as defined by $(\phi, \chi)$, is not in general observable, since the vector $\phi$ changes rapidly. By contrast, though, the vector $\chi$ changes slowly, so it is in principle observable. By integrating equation (2) over the possible values of $\phi$ we get the probability of observing a vector of actions, $\chi$:

$$\Pr(\chi) = \frac{\int e^{-\beta C((\phi, \chi))} d\phi}{\sum_{(\phi, \chi)' e^{-\beta C((\phi, \chi)')}}} . \quad (3)$$

Noting that total cost is the sum of individual costs, $C = \sum_n c_n$,

$$\Pr(\chi) = \frac{\int e^{-\beta C((\phi_1, \chi))} \ldots e^{-\beta C((\phi_N, \chi))} d\phi}{\sum_{\chi'} \sum_{\phi'} e^{-\beta C((\phi', \chi)')}} .$$

\(^6\) A state is defined by a vector of ordered pairs: $(E_n, p_n)$; $n = 1 \ldots N$, describing the current external factor and the current action for each agent. Call the set of possible vectors $S$, and define $\phi$ and $\chi$ as vectors of external effects and actions respectively. We can then refer to a point in state space as $(\phi, \chi)$.

\(^7\) This last assumption is known as the ergodic assumption, or equipartition assumption. It is the assumption that to use the tool being developed here, each state that incurs some cost, $C$, is equally likely to occur. This is in fact much weaker than it seems. If there is a phenomenon under analysis that would, on the face of it, violate this assumption because some states are more likely to occur than others for some reason, we can re-establish equipartition by re-defining states. In our re-definition of the state space, we would clone the more likely states, to create a group of similar states all of which have the same probability. This generates a world which does not violate equipartition and so is susceptible to the use of this tool, but which is observationally identical to a world which appears to violate the assumption.
or
\[
Pr(\chi) = \frac{\int e^{-\beta C((\phi_1, x))} \cdots e^{-\beta C((\phi_N, x))} d\phi}{\sum_{\chi'} \sum_{\phi'} e^{-\beta C((\phi, x'))}}.
\]

(4)

Because in this model external factors are independent across agents, we can see that
\[
\int \cdot d\phi = \int \cdots \int \cdot d\rho(E_1) d\rho(E_2) \cdots d\rho(E_n).
\]
This yields
\[
Pr(\chi) = \frac{\prod_{n=1}^N e^{-\beta \bar{c}_n(x)}}{\sum_{\chi'} \prod_{n=1}^N e^{-\beta \bar{c}_n(x')}},
\]

(5)

where \( \bar{c}_n = \int c_n(E_n, p_n; \chi) dE_n \), that is, the average cost to agent \( n \), given that the actions of all agents are given by vector \( \chi \), agent \( n \) is performing action \( p_n \), and where the average is taken over the distribution of external factors that might be assigned to agent \( n \).

2.1 The relationship between \( \beta \) and cost

A result that follows immediately from this argument and equation (2) (or equally, from equation (5)), is that expected costs in the economy are a monotonically decreasing function of \( \beta \). We can write the total expected cost as
\[
\langle C \rangle = \frac{\sum_C C e^{-\beta C}}{\sum_C e^{-\beta C}}.
\]

(6)

So
\[
\frac{\partial \langle C \rangle}{\partial \beta} = \frac{\left[ \sum_C C \exp(-\beta C) \right]^2 - \left[ \sum_C \exp(-\beta C) \right] \left[ \sum_C C^2 \exp(-\beta C) \right]}{\left[ \sum_C \exp(-\beta C) \right]^2},
\]

(7)

which can be written as
\[
\frac{\partial \langle C \rangle}{\partial \beta} = \langle C \rangle^2 - \langle C^2 \rangle,
\]
or
\[
\frac{\partial \langle C \rangle}{\partial \beta} = -\langle (C - \langle C \rangle)^2 \rangle,
\]

which gives that
\[
\frac{\partial \langle C \rangle}{\partial \beta} \leq 0,
\]

(8)

with the equality holding only if the cost, \( C \), is the same under every possible configuration of external effects across agents. This result indicates the importance of an interpretation for the parameter \( \beta \).
3. The Interpretation of $\beta$.

To facilitate the interpretation of $\beta$, consider a model in which there is no agent interaction, but in which all agents are subject to the same external effect, $E$, and are perfectly informed about its distribution. In such a world, all agents would perform the same action, namely the one that minimizes expected cost. Suppose now that agents are not perfectly informed about $E$ and so are making decisions based on past experiences—in the extreme case, based only on current experience. We can use the formalism just developed to derive the proportions of agents performing different actions. We know that in this model the total economy cost is the sum of individual costs: $\sum \bar{h}_n$. Thus by equation (5), if the reservoir of external factors has property $\beta$, the probability that the economy has total cost $\sum \bar{h}_n$ is proportional to $\exp\{-\beta \sum \bar{h}_n\}$, or equivalently, is proportional to $\exp\{-\beta \bar{h}_1\} \exp\{-\beta \bar{h}_2\} \ldots \exp\{-\beta \bar{h}_N\}$. Given that the economy is in this state, we can ask what is the probability that agent $n$ has cost $\bar{h}_n$. This is found by calculating marginal distributions, by integrating over other agents. The effects of the other agents cancel with the normalization term, and the probability that agent $n$ incurs cost $\bar{h}_n$ is proportional to $e^{-\beta \bar{h}_n}$.

Suppose now that there are two possible actions having average costs (over the distribution of external factors) of $\bar{h}_1$ and $\bar{h}_2$. The probability that agent $n$ makes choice 1 is proportional to $e^{-\beta \bar{h}_1}$, and the corresponding probability for choice 2 is proportional to $e^{-\beta \bar{h}_2}$. Since the probabilities must sum to one, the probability of observing choice 1 is

$$\frac{e^{-\beta \bar{h}_1}}{e^{-\beta \bar{h}_1} + e^{-\beta \bar{h}_2}},$$

and the probability of observing action 2 is

$$\frac{e^{-\beta \bar{h}_2}}{e^{-\beta \bar{h}_1} + e^{-\beta \bar{h}_2}}.$$ 

Notice that as $\beta$ goes to infinity these results reduce to 0 and 1, depending on the relative sizes of $\bar{h}_1$ and $\bar{h}_2$. Thus the $\beta = \infty$ solution is the same as the solution when agents have identical, perfect knowledge about the distribution of external factors.\(^8\) Notice also that the solution for $\beta = 0$ is 1/2 for both actions.

\(^8\) We can interpret this either as agents having the same current experiences (where myopic decision-making is the rule so knowledge of $E$ amounts to one's current experience), or, if agents use their past experiences to calculate expected future costs, agents have had identical histories and use the same map from historical experiences to beliefs about $E$, over which expected values are taken.
With $\beta = \infty$ we recovered a simple case, which was characterized by every agent drawing from exactly the same distribution of exogenous influences. In fact, if we assume that agents' beliefs about the distribution of exogenous factors are determined by their experiences, this identity of distributions will obtain only if their experiences have been identical and their memories are infinite. Neither is the case in practice. Let us consider finite memory, with exogenous factors being drawn from the same distribution for every agent. Then the expected cost, $\bar{h}_n$, on which agents base their choices, is a random variable that varies from site to site. Its mean is $\int h(E, p)d\rho(E)$. If the agents' memory is $j^2$ events in length, and the standard deviation of the true distribution of external factors, $\rho(E)$, is $s$ then the standard deviation of the random variable $\bar{h}$ is $s/j$, and the distribution tends to a normal distribution as $j$ increases. Since the values of the random variable for each possible action vary from agent to agent some agents will prefer one action, some agents another, even though the true mean of the variable is the same for every agent, and the proportion of agents by whom a given action is preferred depends only on the differences between the mean cost for each action, measured in terms of the standard deviation. Thus, relating this to the equations of probabilities of agents choosing one or the other action, large values of $\beta$ describe systems in which the agents are subject to an external environment that varies very little from agent to agent; small values of $\beta$ describe systems in which agents are subject to external environments that vary greatly from agent to agent. We will use this interpretation of $\beta$.

We can now interpret the result derived at the end of section 2: the greater is the heterogeneity among agents, in the sense of the environment in which they operate, the higher will be the total costs in the economy. The more heterogeneity there is among the agents in the economy, the more difficult it becomes to coordinate so as to capture the benefits from externalities.

4. Technological Standardization and Variety

An area that has received considerable attention with regard to standardization and the existence of variety is technology choice. There is a relatively large literature now which examines the conditions under which standardization occurs or fails to occur, and under which standardization on the "right" technology takes place. In almost all of the theoretical literature, though, externalities from adoption are only positive, and are unbounded in the number of adopters. Furthermore, these externalities tend to be global in the sense that the adoption decision of any agent affects the payoffs of every other agent in the economy no
matter where located. Complete standardization is a common outcome in these models—in
a race for market share if one technology happens to gain a lead, this creates an incentive
for adopters to choose it, and by this mechanism a bandwagon can form.\footnote{There are some exceptions to the ‘global externalities’ assumption. See for example David and Foray, who consider localized externalities. In their model, though, all externalities from adoption are positive, and complete standardization always obtains.} We present here a model which includes both local and global externalities, and which generates a relation between the nature and degree of standardization and several parameters.

Consider an economy of $N$ agents who are located in two distinct spatial regions. These may be two firms, or two countries which speak different languages. Within a region agents interact costlessly, but there is no direct interaction between agents in different regions. Thus in what follows, “local” is defined as “within a single region”. Agents choose between two technologies with which they perform tasks.\footnote{This generalizes transparently to $R$ regions and $Q$ technologies.} There are positive local externalities to adoption, arising from local network externalities, but negative global externalities.\footnote{These may stem from decreasing returns in the production of the capital good which embodies the technology.} Tasks are relatively short-lived, whereas technology choices are long-lived, which means that the chosen technology will be used for several tasks before the opportunity to re-choose arises. Agents choose myopically and non-strategically, so at each moment of choice the problem for agent $n$ will be to minimize the net costs of his choice, and so can be written as

$$\min_{q \in \{1, 2\}} g(z_{n,q}) + h_{n,q} + f(Z_q),$$

where $Z_q$ is the proportion of the global population using technology $q$, and $f(Z_q)$ is the global externality; $z_{n,q}$ is the proportion of agents local to agent $n$ using technology $q$, and $g(z_{n,q})$ is the local externality effect; and $h_{n,q}$ represents an idiosyncratic effect, determined by the interaction of the technology choice and the particular task that agent $n$ is performing. Assume that the expected value of the idiosyncratic effect, taken over the distribution of tasks, is zero. We shall assume that $f(\cdot)$ and $g(\cdot)$ are linear: $f(Z_q) = W Z_q$, and $g(z_{n,q}) = V z_{n,q}$, where $W > 0$ (a negative externality implies an increase in costs), and $V < 0$ (a positive externality implies a decrease in costs).
If we treat tasks as being assigned randomly, such that when an agent completes one task another is drawn (imagine a large reservoir of tasks waiting to be completed) we can use the formalism just developed to analyse the distribution of technologies.

The total costs incurred in the economy at any moment are the sum of individual costs:

\[ \sum_{n=1}^{N} g(z_{n,q}) + h_{n,q} + f(Z_q). \]

As in the general model, in this economy, any quantity observable by the analyst will be an average, either over time or space, so averaging by integrating over the distribution of tasks:

\[ \bar{C} = \sum_{n=1}^{N} g(z_{n,q}) + f(Z_q), \]

since the mean of \( h_{n,q} \) is zero.

From equation (5) we know the relationship between the probability that a state of the economy occurs and the total costs incurred in that state.\(^{12}\) We would like to know, however, about the proportions of agents using different technologies in each region. We can address this by asking about the probability that an arbitrary agent in region 1 uses technology one.

Consider equilibrium. An agent can be in one of two states, 0 or 1 (using technology 0 or technology 1). What is the probability that an arbitrary agent, \( a \), uses technology 1? If we fix the technologies of all other agents, we can ask about the probability of the joint event: \( \Pr(a = 1; X) \), where \( X \) represents the state of all agents apart from agent \( a \). But the event \( (a = 1; X) \) completely describes the technology allocation, so we know its probability:

\[ \Pr(a = 1; X) = \frac{e^{-\beta \bar{C}(a=1;X)}}{\int_X e^{-\beta \bar{C}(x)}} , \]

where the integral over \( \chi \) is over all possible technology allocations of the economy. Now to find the probability that \( a = 1 \) we integrate over all possible values of \( X \):

\[ \Pr(a = 1) = \frac{\int_{X'} e^{-\beta \bar{C}(a=1;X')}}{\int_X e^{-\beta \bar{C}(x)}} . \]

\(^{12}\) In the appendix we show that the equipartition assumption is acceptable and thus this problem can be analysed using the results summarized in equation (5).
Breaking $\chi$ into agent $a$ and all other agents we can rewrite the denominator to get

$$\Pr(a = 1) = \frac{\int_{X'} e^{-\beta \bar{C}(a=1;X')} \, dX}{\int_{a=\{0,1\}} \int_{X'} e^{-\beta \bar{C}(a,X')} \, dX}.$$  \hspace{1cm} (11)

How do we perform these integrals? Rewrite the costs in the following way. The cost incurred by any single agent is determined by his interactions with other agents, so $\bar{C}(a = 1;X')$ can be divided into two parts---$C_a$, those generated by interactions involving agent $a$, and $C_{-a}$, those generated without $a$. We can re-write equation (11) as

$$\Pr(a = 1) = \frac{\int_{X'} e^{-\beta C_a(a=1;X')} e^{-\beta C_{-a}(a=1;X')}}{\int_{a=\{0,1\}} \int_{X'} e^{-\beta C_a(a,X')} e^{-\beta C_{-a}(a,X')} \, dX}.$$  \hspace{1cm} (12)

To evaluate the integrals we make two observations. First, the solution will be in the form of two simultaneous equations, one for each region. Thus we can treat the agents in region two as fixed, which implies a constant value for the proportion of agents in region two using technology one, $z_{2,1}$. Second, agents in region one are indistinguishable to the analyst, so each agent has the same probability of using technology one. This must be the same as the proportion using technology one, $z_{1,1}$. Since the cost to an agents is determined by these two proportions, any costs generated by interactions not involving agent $a$ will be constant. Thus all of the $C_{-a}$ terms cancel from equation (12), leaving

$$\Pr(a = 1) = \frac{\int_{X'} e^{-\beta C_a(a=1;X')}}{\int_{X'} \int_{a=\{0,1\}} e^{-\beta C_a(a;X')} \, dX}.$$  \hspace{1cm} (13)

But when considering the costs to agent $a$ we observe that they are determined by the proportions using technology 1 in each of the two regions. Thus we can perform a change of variables, and integrate, not over $X$, but over $z_{1,1}$ (noting again that $z_{2,1}$ is fixed). But by the argument about the indistinguishability of agents within a region, $z_{1,1}$ is a constant, so the integral over $X$ cancels, and we are left with

$$\Pr(a = 1) = \frac{e^{-\beta C_a(1)}}{\int_{a=\{0,1\}} e^{-\beta C_a(a)} \, dX}.$$  \hspace{1cm} (14)

Finally, we need to establish $C_a(1)$. Agent $a$ is located in region 1, so $z_{1,a}$ can be interpreted as the proportion of agents in region 1 using technology $q$. If $a$ uses technology
1, costs incurred by him are $Vz_{1,1} + WZ_1$. This is generated entirely by interacting with agents who use the same technology. Each of those agents has an analogous interaction with $a$, so the costs associated with interactions in which $a$ is the "second party" will be identical to those in which $a$ is the "first party". Thus $C_a(1) = 2(Vz_{1,1} + WZ_1)$. So

$$z_{1,1} = \frac{e^{-2\beta(Vz_{1,1}+WZ_1)}}{e^{-2\beta(Vz_{1,0}+WZ_0)} + e^{-2\beta(Vz_{1,1}+WZ_1)}}.$$  \hfill (15)

The same procedure can be performed for region two:

$$z_{2,1} = \frac{e^{-2\beta(Vz_{2,1}+WZ_1)}}{e^{-2\beta(Vz_{2,0}+WZ_0)} + e^{-2\beta(Vz_{2,1}+WZ_1)}}.$$  \hfill (15a)

An observation that makes numerical solution particularly simple is that since there are only two regions equation (12) reduces to

$$z_{1,1} - z_{1,0} = \tanh(\beta(V(z_{1,1} - z_{1,0}) - W(z_{1,1} - z_{1,0} + z_{2,1} - z_{2,0})))$$

and similarly for region 2. These can be jointly solved to find the equilibria in terms of proportions of agents using each technology in each region.

4.1 Equilibria

In general it is not possible to solve equations (15) and (15a) analytically. We have solved them numerically for many parameter values, and we discuss the nature of those solutions here.\footnote{For more detail see Cowan and Cowan(1994).}

There are several types of equilibria, which depend on parameter values. If agent heterogeneity is high, and local externalities are weak, then there is a single equilibrium, in which there is no structure. In both regions, the proportion of agents using technology 1 is 1/2. This equilibrium obtains if $V < 2\beta$. Under these parameters agents' choices are determined largely by the fact that some technologies are well-, and some badly-suited to the task performed. The gains from coordinating with neighbours are outweighed by this consideration.

As the degree of heterogeneity falls relative to the strength of local increasing returns, the equilibrium takes on structure—each region standardizes on a single technology, as
the relative benefits from coordinating with nearby neighbours increases. This equilibrium emerges when \( V > 2\beta \), in which region the unstructured equilibrium ceases to exist. If the global effects are strong, each region will standardize on a different technology. Global variety is maintained, but local variety disappears. Which technology dominates in which region is not determined by the model of course, but is rather determined by the detailed history of the choices of particular adopters.

If the global effect is weak there are two types of equilibria. The one just discussed, with local standardization and global variety continues to exist, but another, in which both regions standardize on the same technology, emerges. The condition on parameters for the existence of this equilibrium is that \( V > 2\beta + 5W/2 \). Here even global variety disappears. An agent considering a choice opportunity sees gains from adopting a globally unusual technology, but those gains are out-weighed by the losses that would be suffered through inability to benefit from local externalities. The degree of local standardization is lower in this equilibrium than in the previous, and total costs incurred in the economy are higher.

In all of this, the role of the global externality is curious. Provided it is non-zero, it plays little role in determining the nature of the outcome. Whether or not any local standardization occurs is determined solely by the degree of heterogeneity and strength of local externalities—the strength of global externalities is unimportant. Furthermore, the degree of local standardization (measured as the proportion of users in a region using the dominant technology of that region) is determined, again, solely by heterogeneity and local effects.\(^1\) The only role played by the global externality is to prevent global standardization if it is strong, and to allow (but not force) it if it is weak.

There is one more interesting feature of this model. This is the existence of a first order phase change as the degree of heterogeneity relative to the strength of the local externality changes. It is not surprising that as heterogeneity falls the extent of local standardization increases. It is not a smooth relationship though. For high heterogeneity, the equilibrium is totally disordered, with the proportions of agents using technology 1 in any region being equal to 1/2. As heterogeneity falls, it passes through a critical value, below which local standardization begins to take place. As heterogeneity continues to fall,

\(^{14}\) This can be seen by observing that there will be symmetry in the solution, so that \( z_{1,1} - z_{1,0} = z_{1,0} - z_{2,1} \). Solving equations (15) and (15a) with this condition yields that \( z_{i,1} - z_{i,0} = \tanh \beta V (z_{i,1} - z_{i,0})/2 \) which does not include \( W \), the global externality effect.
the proportion using the dominant technology in any region moves very quickly to either 0 or 1, and almost complete regional standardization occurs. There is only a relatively small range of values in which intermediate degrees of standardization are possible. This makes it clear that in models like this the role of agent heterogeneity is crucial.

5. Conclusions

Economies that have many interacting agents can create problems of analysis for economists, and these problems are especially acute if the interactions are spatially dependent. The technique developed in this paper provides a way of addressing the difficulties created by the presence of many heterogeneous agents. Macro-economic results about economic activity, and in particular about the spatial distribution of it, can be derived from a micro-economic problem involving relatively complex, spatially dependent interactions among many agents. The technique is very general and can be generalized in many ways.

Models of technology choice have focussed on relatively simple forms of increasing returns to adoption and have tended to produce results in which all agents standardize on a single technology. The model in this paper, to which the technique we develop is applied, introduces several generalizations to these models. These generalizations change the results, and indicate different sorts of conditions under which variety in technology use can occur. The model presented is relatively stylized, but can readily be generalized, using the same analytical tools, to include for example, regional biases in technology choices, perhaps arising from nationalistic purchasing policies; or fuzzy borders between regions, perhaps caused by multi-national corporations that operate in many regions. Finally, the techniques we have developed here to address the issue of technology choice and standardization can be applied to other economic contexts in which identifiable, possibly complicated interactions among agents at the micro level determine the macro structure of the economy.
Appendix

Here we show how equipartition applies naturally to this model. There are several ways this can be done. One is to describe a transformation that would generate a uniform distribution from the actual distribution of states, and to show that this transformation does not invalidate statements made about the phenomenon being modelled. Another is to carry the multiplicities of the actual distribution through the derivation of equation (5), and to show that this has no effect on equation (5). Here we do the former. For a general derivation using the second technique, see Waldram (1985, pages 30-32).

The equipartition assumption was invoked in order to generate statements about states from statements about costs. We derived the frequency distribution of costs in the economy, \( \Omega(C) = K' e^{-\beta C} \). If the states are uniformly distributed, then the probability of observing state \((\phi, \chi)\) will be proportional to \( e^{-\beta C(\phi, \chi)} \). But it is unlikely that the states will be uniformly distributed. Suppose that \( \rho(\phi, \chi) \) is the actual distribution of states in the technology choice problem. Assume for the moment that \( \rho(\phi, \chi) \) is rational for all \((\phi, \chi)\). Then there is some integer \( M \) such that \( M \times \rho(\phi, \chi) \) is a whole number for all \((\phi, \chi)\). Define \( M_{\phi, \chi} = M \times \rho(\phi, \chi) \). Now define a new set of states using the following algorithm: \( \forall (\phi, \chi) \in S \) create states \((\phi, \chi, 1), (\phi, \chi, 2), \ldots, (\phi, \chi, M_{\phi, \chi}) \). These states define the new state space \( S' \). Assign to each state the probability, \( \rho'(\phi, \chi, m) = \rho(\phi, \chi)/M_{\phi, \chi} \forall m = 1 \ldots M_{\phi, \chi} \). This creates a new state space having a uniform distribution of states. On this state space the equipartition assumption holds by construction, so the derivation of equation (5) is intact.

Can this state space be used to make predictions about the state space on which the distribution \( \rho(\cdot) \) is the true distribution? Define \( S'_a = \{(\phi, \chi, m) \mid \text{agent } a \text{ uses technology 1}\} \); and \( S_a = \{(\phi, \chi) \mid \text{agent } a \text{ uses technology 1}\} \). Thus the probability of observing agent \( a \) using technology 1 under the new definition of the state space is given by \( \Pr(a = 1) = \sum_{(\phi, \chi, m) \in S'_a} \rho'(\phi, \chi, m) \). But since by construction \( \rho'(\phi, \chi, m) = \rho(\phi, \chi)/M_{\phi, \chi} \forall m \), we can write \( \Pr(a = 1) = \sum_{(\phi, \chi) \in S_a} M_{\phi, \chi} \times \rho'(\phi, \chi) = \sum_{(\phi, \chi) \in S_a} M_{\phi, \chi} \times \rho(\phi, \chi)/M_{\phi, \chi} = \sum_{(\phi, \chi) \in S_a} \rho(\phi, \chi) \), which is exactly the probability that agent \( a \) uses technology one in the original state space \( S \). Thus predictions made using the transformed state space can be transferred directly to the "actual" state space.

Clearly, if \( \rho(\phi, \chi) \) is irrational for some \((\phi, \chi)\), then predictions using the new state space \( S' \) and distribution \( \rho' \) can be made arbitrarily close to predictions made using \( \rho \) by making \( M \) large.
References


