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Tax Coordination in a Cross-Hauling Model: Conflict or Harmony of Interest?

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Abstract

The paper discusses strategic commodity taxation in a model with trade in a single private good, which is simultaneously imported by consumers of a high-tax region and exported by its producers. Sufficient conditions for the existence of a Nash equilibrium are given and an asymmetry is introduced through different preferences for public goods. It is shown that tax coordination generally benefits the high-tax country while the low-tax region will gain only if the intensity of tax competition is high in the initial equilibrium, or if governments are price-sensitive towards the effective marginal costs of public good supply.

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1 Introduction

It is a well-known problem of regional integration that tax or tariff differentials can give rise to trade flows which are exclusively based on tax considerations rather than comparative advantage. In the European Union (EU) the abolition of internal border controls has led to different tax treatments for imports of the same good, depending on whether the purchaser is a registered trader (destination principle) or a final consumer (origin principle), and has opened up the possibility that identical products are simultaneously exported and imported by a single country. As an example, French producers may have a comparative advantage in wine production and export wine to Germany at tax-exclusive prices. At the same time, French consumers have an incentive to cross the border and purchase their own wine in Germany, where commodity taxes are lower.

Similar arbitrage possibilities have long existed for consumers in the United States, where the enforcement of use taxes levied on out-of-state purchases is relatively difficult and mail ordering has become a growing source of concern [cf. Trandel (1992)]. Furthermore, even when the destination principle is technically in place for consumer purchases, it may be severely undermined in practice. A recent example is the forced reversal of Canada’s high-tax policy for cigarettes which proved unsustainable in view of the large tax differential to the United States. Canadian tobacco manufacturers exported their products net of tax to the United States from where they illegally re-entered the Canadian market, circumventing existing rules of destination\(^1\).

Cross-border shopping simultaneously redistributes tax revenues from the high-tax to the low-tax region and affects overall efficiency. An obvious source of welfare losses lies in the real resources used up in shipping goods abroad and re-importing them via consumer purchases but further (domestic) inefficiencies arise when both countries engage in strategic tax setting as a result of the fiscal externality. Possible tax coordination measures to reduce these welfare losses include minimum tax requirements imposed on the low-tax region and a tightening of rules of destination, for example in the field of mail-ordering. Both of these measures have been taken in the European Union and the last policy is also under active consideration in the United States [cf. ACIR (1986)]. Since the viability of such

\(^1\)According to estimates by the government of Quebec, two thirds of all cigarettes consumed in this province were bought illegally before tobacco tax reductions took effect in February 1994 (The Globe and Mail, January 29, 1994).
coordination measures depends, in large part, on the consent of the low-tax region the important question from a policy perspective is whether they can benefit both countries simultaneously.

To discuss these issues, the literature on strategic commodity taxation has taken two rather different approaches. Mintz and Tulkens (1986) use a model with a general government objective and balanced trade in two goods. In this framework, however, reaction functions are discontinuous and a Nash equilibrium cannot in general be proven to exist. Furthermore, while it can be shown under some additional restrictions that a tax rise in either country benefits the trading partner [de Crombrugghe and Tulkens (1990)], a more detailed discussion of tax coordination measures is precluded by the complexity of the model. Alternatively, Kanbur and Keen (1993) use a one good partial equilibrium model and assume that governments behave as revenue maximizers. In this setting, a rather optimistic result for the prospects of tax coordination is obtained: both a minimum tax requirement and an increase in the costs of cross-border shopping raise welfare in both countries since the mutual gains from reduced tax competition dominate purely redistributive effects. It is not clear, however, whether these results continue to hold under more general government objectives.

The approach taken in this paper is to combine elements of both models and we will argue that the assumption of cross-hauling – suggested by the examples given above – plays a crucial role in this task. In particular, it allows to incorporate a simple "partial equilibrium" specification of cross-border shopping in a (rudimentary) general equilibrium framework with balanced trade in a single good, which is simultaneously imported by consumers of the high-tax country and exported by its producers. An interior equilibrium is ensured by rising marginal transaction costs for consumer purchases abroad. With this specification sufficient conditions for the continuity of reaction functions can be specified and a Nash equilibrium can be shown to exist. As a side effect, this demonstrates the close links that exist to the parallel literature on capital tax competition [e.g. Bucovetsky (1991), Wilson (1991), Bucovetsky and Wilson (1991)].

Much of the recent work on both commodity and capital tax competition has focused on the role of country size as a source of tax differentials. While many tax havens actually tend to be small countries the reverse is not true in general, i.e., not all small countries have low tax rates. In the EU context Denmark may serve as a counterexample, pointing
to the relevance of economic "fundamentals" in the shaping of tax policy. The present
analysis will thus focus on differences in preferences for public versus private consumption
but the analysis of tax coordination generalizes to other sources of tax asymmetries.

The paper is organized as follows: section 2 presents the tax competition model with
trade in a single private good and derives the Nash equilibrium when preferences for public
goods differ across countries. Section 3 analyzes the effects of alternative coordination
measures on optimal tax rates and welfare in both countries. Section 4 summarizes the
argument and suggests possible extensions.

2 Model Description and Nash Equilibrium

There is a single representative consumer in each of two regions A and B. The consumer
in each country is endowed with one unit of output, valued at the producer price of unity
(i.e., countries are of equal size). Output can either be consumed as a private or as a local
public good. Preferences are described by the strictly quasi-concave utility function

\[ u^k(c^k, g^k) \quad \forall \quad k \in \{A, B\}, \]

where \( c^k \) and \( g^k \) denote country \( k \)'s overall consumption of the private and the public
good, respectively. To provide the public good, the government of region \( k \) purchases some
fraction of total output, which is financed by a commodity tax on all consumer purchases
that occur within its territory\(^2\). The marginal rate of transformation between the private
and the public use of output is thus equal to (-1).

We assume that the private good is tradeable while the public good is not. Producer
trade in the private good occurs at net-of-tax prices, as is true under both the European
value-added tax or a retail sales tax as applied in the United States. In the absence of
producer transportation costs producer prices of the private good must be equalized across
countries and differences in tax rates are reflected in different consumer prices. If cross-
border purchases by final consumers are taxed at the rate of the origin country residents

\(^2\)The assumption that no other tax instruments are available to the government is, of course, restrictive.
However, simple models with two tax instruments usually yield the result that only one of the tax rates
is positive in equilibrium [see, e.g., Bucovetsky and Wilson (1991)] and a model with enough structure to
ensure, for example, positive commodity and income tax rates is beyond the scope of the present paper.
Our analysis in section 3 will, however, implicitly discuss the availability of alternative tax instruments.
of the high-tax region have an incentive to purchase at least part of their demand for the private good abroad. Overall private consumption in the high-tax country then consists of domestic purchases, $c^k_k$, and cross-border purchases, $c^k_l$ (where $k, l \in [A, B]$ and $k \neq l$):

$$c^k \equiv c^k_k \quad \text{if} \quad t^k \leq t^l,$$
$$c^k = c^k_k + c^k_l \quad \text{if} \quad t^k > t^l. \quad (2)$$

Consumer purchases cause a trade deficit for the high-tax country, which is balanced by producer exports of the same good. Thus cross-hauling of the private commodity occurs as a result of different tax arbitrage conditions for producers on the one hand and consumers on the other. In the absence of transportation costs residents of the high-tax country would exclusively shop abroad and tax revenues in the high-tax country would be zero. Clearly, any tax equilibrium would then have to imply equal tax rates across countries. There are, however, obvious constraints on direct consumer purchases abroad since residents of the high-tax country must either physically cross the border or place mail orders in order to take advantage of the lower tax rate. These constraints are captured by a strictly convex transaction cost function for consumer purchases abroad. This function is assumed to be three times differentiable and the marginal cost of purchasing the first unit in the foreign country is zero so that any tax differential will lead to positive levels of cross-border shopping. Furthermore, we initially allow for country-specific differences in the transaction cost function. Thus

$$\tau^{k,l} > 0, \quad \tau^{k,n} > 0, \quad \tau^{k,n}(0) = \tau^{k,l}(0) = 0, \quad \tau^{k,n}(0) > 0 \quad \forall k. \quad (3)$$

Cross-hauling introduces a degree of freedom for specifying the decision to cross-border shop. In particular, it allows a "partial equilibrium" specification where cross-border shopping is an exclusive function of the international tax differential and transaction costs. It

3A similar argument has recently been made by Richardson (1992) in the context of free trade areas. It also underlies the analysis of trade deflection under the restricted origin principle [e.g. Georgakopoulos and Hitiris (1992)].

4In partial equilibrium models convex transportation costs for cross-border shopping are often rationalized by assuming heterogeneous consumers which live at varying distances from the border [Kanbur and Keen (1993), Christiansen (1993)]. Alternatively, one can think of a continuum of consumer goods, some of which are easily transported while others impose virtually prohibitive transaction costs. A still different interpretation focuses on illegal forms of tax arbitrage, arguing that higher levels of cross-border shopping increase the risk of being caught more than proportionally.
can then be derived from the consumer arbitrage condition
\[ r^k t^l(c^k_l) = t^k - t^l \quad \text{if} \quad t^k > t^l \] (4)

by inverting the marginal transaction cost function. Thus
\[ c^k_t = c^k_t(r^k t^l)^{-1}(t^k - t^l) \quad \text{if} \quad t^k > t^l. \] (5)

With a single private good, general equilibrium repercussions are limited to the income effects of taxes and transportation costs. Disregarding these in (5) is equivalent to assuming that all income changes are absorbed by domestic consumption, which automatically adjusts (via producer exports) to balance the trade account.

The high-tax and the low-tax country face asymmetric budget constraints. In regime I country \( k \) is the high-tax country whereas \( t^k < t^l \) holds in regime II. From the perspective of region \( k \) the regime-specific constraints for private consumption are given by

\[ R I : \quad (1 + t^k) c^k_t + (1 + t^l) c^k_l + r^k(c^k_l) = 1, \] (6)
\[ R II : \quad (1 + t^k) c^k_k = 1. \] (7)

In regime I, overall expenditure by residents of country \( k \) include the costs of cross-border shopping, which constitute a direct efficiency loss\(^5\). Domestic spending carries the domestic tax whereas cross-border shopping carries the foreign tax. Implicit in this budget constraint are aggregate producer exports to country \( l \). Tax revenues are redistributed between countries since residents of country \( k \) purchase their imports from country \( l \) on a gross-of-tax basis while balancing producer exports from \( k \) to \( l \) occur net of tax. This redistribution of tax revenues is, however, external from the perspective of cross-border shoppers. Instead, the private budget constraint is relaxed by cross-border shopping since, from the convexity of the transaction cost function, the sum of foreign taxes and transaction costs is always less in an arbitrage equilibrium than if the same amount of goods were purchased at home.

\(^5\)More generally, it is obvious that all trade must be inefficient in a one-good model. There are, however, no difficulties in principle to extend the present framework to the case of multiple private goods. Overall import demand is then determined by preferences and production while cross-border shopping is independently determined by (5); any residual gives simultaneous imports or exports by traders. In this framework, "efficient" producer trade and "inefficient" cross-hauling occur simultaneously. Similar efficiency losses arise when selective tariffs cause a substitution of higher (transportation) cost suppliers for lower cost suppliers [see, e.g., Melvin (1985)].
In regime II residents of country \( k \) purchase all goods at home. Since revenues from foreign cross-border shopping accrue to country \( k \)'s government they do not affect private consumption possibilities for a given domestic tax level.

The government budget constraints are easily derived: in regime I, the tax base of country \( k \) is given by the domestic purchases of its own residents. In regime II the tax base of country \( k \) consists of domestic purchases plus foreign cross-border shopping \( c^k_k \):

\[
RI : \quad g^k = t^k c^k_k = t^k (c^k - c^k_f),
\]

\[
R II : \quad g^k = t^k (c^k + c^k_k).
\]

It is useful to summarize the effects of domestic and foreign tax changes on each of the endogenous variables. By the above assumptions, cross-border shopping is influenced only by substitution effects. A rise in the domestic tax rate must always lower domestic purchases of the private good since income and substitution effects are both negative. In contrast, a tax change in the foreign country causes counteracting income and substitution effects in regime I, and its net effect on domestic purchases in country \( k \) – and thus tax revenues and public good supply – is ambiguous a priori. Total private consumption in country \( k \) must, however, fall in response to a rise in \( t^l \). Finally, optimizing governments will never choose tax rates such that an increase in the tax rate lowers tax revenues. Thus

\[
RI : \quad \frac{\partial c^l_k}{\partial t^l} > 0, \quad \frac{\partial c^k_k}{\partial t^k} < 0, \quad \frac{\partial c^k_k}{\partial t^l} < 0, \quad \frac{\partial g^k}{\partial t^l} > 0,
\]

\[
\frac{\partial c^l_k}{\partial t^l} < 0, \quad \frac{\partial c^k_k}{\partial t^l} <> 0, \quad \frac{\partial c^k_k}{\partial t^l} < 0, \quad \frac{\partial g^k}{\partial t^l} <> 0,
\]

\[
R II : \quad \frac{\partial c^l_k}{\partial t^l} < 0, \quad \frac{\partial c^k_k}{\partial t^l} \equiv \frac{\partial c^k_k}{\partial t^k} < 0, \quad \frac{\partial g^k}{\partial t^l} > 0,
\]

\[
\frac{\partial c^l_k}{\partial t^l} > 0, \quad \frac{\partial c^k_k}{\partial t^l} \equiv \frac{\partial c^k_k}{\partial t^k} = 0, \quad \frac{\partial g^k}{\partial t^l} > 0.
\]

Given the private decisions to cross-border shop, the government of each country decides simultaneously on public and (overall) private consumption by choosing the domestic tax rate. For each regime, the optimal tax problem can thus be described as maximizing (1) subject to the independent constraints (6) and (8), or (7) and (9), respectively. Substituting out for \( c^k_k \) using (2) leads to the regime-specific Lagrangians

\[
L^k_t = u^k(c^k, g^k) + \lambda_1 \left[ 1 - (1 + t^k)c^k + (t^k - t^l)c^k - \tau(c^k_f) \right] + \lambda_2 \left[ t^k(c^k - c^k_f) - g^k \right],
\]

\[\text{It will later be shown that this effect can be signed using the first-order condition of the high-tax country's optimal tax problem.} \]
\[ L_{II}^k = u^k(c^k, g^k) + \lambda_1 \left[ 1 - (1 + t^k)c^k \right] + \lambda_2 \left[ t^k(c^k + c^k_l) - g^k \right]. \]  

(13)

The problems (12)–(13) can conceptually be solved in two stages. The government first optimizes over \( g^k \) and \( c^k \), holding \( t^k \) constant. This yields shadow values for the two constraints, which are identical across regimes. Denoting marginal utilities by subscript letters gives
\[
\lambda_1 = \frac{u^k_c + t^k u^k_{g}}{(1 + t^k)}, \quad \lambda_2 = u^k_g, \quad \lambda_2 - \lambda_1 = \frac{u^k_g - u^k_c}{(1 + t^k)}. \]

(14)

In a second step, \( t^k \) is varied for constant values of \( g^k \) and \( c^k \) and the Lagrange parameters are substituted. This yields best response functions in implicit form, which consist of the two branches\(^7\)
\[
F_{II}^k(t^k, t^i) = -\frac{t^k}{r^k} + \frac{c^k}{(1 + t^k)} \left( 1 - \frac{u^k_c}{u^k_g} \right) = 0 \quad \text{if} \quad t^k > t^i, \]

(15)

if country \( k \) is in regime I and
\[
F_{II}^k(t^k, t^i) = -\frac{t^k}{r^k} + c^i + \frac{c^k}{(1 + t^k)} \left( 1 - \frac{u^k_c}{u^k_g} \right) = 0 \quad \text{if} \quad t^k \leq t^i \]

(16)

if it is in regime II. Following Mintz and Tulkens (1986, p. 148) we denote the first term in both (15) and (16) as a \textit{public consumption effect} (or tax base effect), which is negative for an increase in \( t^k \) in both regimes by the convexity of the transaction cost function. Note, however, that in regime I it is the response of domestic cross-border shoppers and thus the marginal transportation cost schedule in the home country \( k \) which determines the size of this effect while the foreign marginal cost schedule matters in regime II. In regime I the sign of the last term is thus unambiguously determined: in a non-cooperative equilibrium, the marginal rate of substitution (of \( g^k \) for \( c^k \)) must be less than one in absolute value, thus indicating an undersupply of public goods in country \( k \). A second fiscal externality, labelled a \textit{private consumption effect}, occurs in regime II only\(^8\): if residents of country \( l \)

\(^7\)The derivation uses the rule for differentiating the inverse function (5)
\[
\frac{\partial c^k}{\partial t^k} = \left[ (r^k)^{-1} \right]'(+1) = \frac{1}{r^k u(c^k)}. \]

\(^8\)Note, however, that the latter effect describes a redistribution from cross-border shoppers in country \( l \) to the \textit{government} of country \( k \). While there are obvious similarities to the more familiar terms of trade effects we avoid the latter term here because private consumption effects and terms of trade effects may simultaneously occur in the same model [e.g. Lockwood (1993)].
shop in region \( k \), there is an incentive for region \( k \)'s government to raise the domestic tax in order to extract more revenues from foreigners. Therefore, public goods may be under- or oversupplied in regime II depending on whether the public or the private consumption effect dominates in the Nash equilibrium.

The overall (regime-independent) reaction function for each country \( k \in [A, B] \) consists of the two regime-specific branches given in (15)–(16). Within each regime, all variables change continuously with a variation in \( t^k \) so that the critical point for the continuity of the overall best response functions occurs at the switch of regimes. Proposition 1 spells out the condition for the reaction functions to be continuous in this point.

**Proposition 1:** The reaction function for each country is continuous if and only if the slopes of the country-specific marginal transaction cost schedules are identical in \( t^k = t' \).

**Proof:** We determine the limits of the two constrained reaction functions (15)–(16) as \( t^k \) approaches \( t' \). This gives

\[
\frac{\lim_{t^k \to t'} F^k_I}{\tau^k n(0)} = -\frac{t^k}{(1 + t^k)} \left( 1 - \frac{u^k_c}{u^k_g} \right),
\]

\[
\frac{\lim_{t^k \to t'} F^k_{II}}{\tau^k n(0)} = -\frac{t^k}{(1 + t^k)} \left( 1 - \frac{u^k_c}{u^k_g} \right). \tag{17}
\]

As the tax differential approaches zero, cross-border shopping goes to zero and the private consumption effect is eliminated. Furthermore, tax bases in both regimes are then identical. The only source of discontinuity thus lies in the second derivatives of the country-specific transaction cost functions, evaluated at \( t^k - t' = 0 \).

The intuition for this result is straightforward: if conditions on both sides of the border are asymmetric, the government of country \( k \) will perceive a different change in the tax base for a marginal deviation from the foreign country's tax rate, depending on whether \( t^k \) approaches \( t' \) from above or below. To stay within a framework of homogeneous consumers, if all foreigners live close to the border while domestic residents live relatively far away, then the benefits of marginally undercutting the foreign tax rate are higher in absolute terms than the losses incurred by setting the tax rate slightly above the foreign. As a consequence, the trade-off between domestic efficiency and a favorable international distribution of tax revenues changes discretely at this point, leading to a "jump" in the reaction function\(^9\).

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\(^9\)In Kanbur and Keen (1993) differences in population densities ("country size") have very similar
Proposition 1 is, however, in contrast to the result by Mintz and Tulkens (1986, Proposition 5) that reaction functions cannot be continuous as the country moves from one regime to another. This can be traced to differences in the modelling of cross-border shopping, which are in turn crucially linked to our cross-hauling assumption: setting up a specification as in (5), which depends only on tax differentials and transaction costs, requires that overall private consumption is independent of the level of cross-border shopping, except for income effects. This in turn implies counterbalancing exports of the same good. In contrast, if cross-border shopping is modelled as a regular demand function which "sets in" at the switch of regimes, the private consumption effect will not go to zero as the domestic tax rate approaches the foreign from below and there is a discrete gain for each region from moving slightly into regime II [cf. Mintz/Tulkens (1986, p. 152)].

In the following, we will assume identical transaction cost functions in both countries. This is sufficient to ensure continuity of the reaction functions and in the special case of quadratic costs (to be introduced in the next section) it is also necessary.

**Assumption 1:** Transaction cost functions are identical in both countries so that

\[ \tau^k(c^k_l) = \tau(c^l_l) \quad \forall \; k, l \in [A, B], \; k \neq l. \]

We further assume that the second-order conditions of the constrained maximization problems (12)–(13) are fulfilled, i.e., we assume the Lagrangians \( L^k_1, L^k_{II} \) (the payoff functions, in game-theoretic terms) to be quasiconcave in \( t^k \). This ensures that a unique optimal tax rate \( t^k \) exists for any given \( t^l \). Sufficient – but not necessary – conditions for this differ across regimes and can be inferred from the first-order conditions (15)–(16): the third derivative of the transaction cost function must be nonpositive (\( \tau''' \leq 0 \)) in regime I but nonnegative in regime II. Therefore, only a quadratic function fulfills both requirements simultaneously. Second, a domestic transfer of purchasing power from the private to the public sector must raise the marginal rate of substitution in absolute value. These restrictions will later be placed on transaction costs and preferences but introducing them here would unnecessarily reduce the generality of the following discussion.

Under these assumptions at least one Nash equilibrium must exist. Furthermore, it immediately follows from (17) that identical preferences lead to a symmetric Nash equilibrium, and are responsible for the discontinuity of the small region's reaction function (which disappears when countries are equally large).
rium with the public good being undersupplied in both countries\textsuperscript{10}. This symmetric Nash equilibrium can \textit{not} occur in the model of Mintz and Tulkens (1986, p. 156), demonstrating that the continuity of reaction functions is not merely a technical problem but has important implications for model results. On the other hand, the existence of a symmetric Nash equilibrium is well known from the analysis of source-based taxes on capital income\textsuperscript{11}. The specification of cross-border shopping (5) thus helps to point out the close links that exist between the analyses of capital and commodity tax competition.

We now introduce differences in preferences for public vs. private consumption in the trading countries. In particular, assume that preferences for public goods are higher in country A as compared to country B. Starting from a symmetric equilibrium this can be modelled as an outward shift in country A’s marginal utility schedule for the public good, \( \partial u_g^A / \partial \alpha > 0 \), while preferences in country B are unchanged.

**Proposition 2:** \textit{The country with a higher preference for the public good strictly has the higher tax rate in the Nash equilibrium.}

**Proof:** If the initial equilibrium is symmetric (i.e., if \( t^A = t^B \)) the direct effect of a change in \( \alpha \) is obtained from (17) by

\[
\frac{\partial F^A}{\partial \alpha} = \frac{c_A^A}{(1 + t^A)} \frac{u_g^A}{(u_g^A)^2} \frac{\partial u_g^A}{\partial \alpha} > 0. \tag{18}
\]

Overall tax changes are determined by this direct effect and the induced adjustment of optimal tax rates in the two countries (indirect effects). It is shown in the appendix that, by the second-order conditions of the optimal tax problems, the change in country A’s tax rate must have the same sign as the direct effect of the change in the shift parameter. Second, stability conditions ensure that country B’s tax rate cannot change by more than the tax rate in country A, even if the slope of country B’s reaction function cannot be determined

\textsuperscript{10}Note, however, that this symmetric equilibrium need not be the only one. A counterexample, which I owe to David de Meza, can be constructed by assuming a quadratic transaction cost function and fixed revenue requirements in both countries. This special case allows to derive reaction functions in closed form and yields two asymmetric Nash equilibria in addition to the symmetric one. However, only the symmetric equilibrium is usually considered under symmetric assumptions since there are no systematic reasons to assign a high tax rate (and a low tax base) to one of the countries and a low tax rate to the other.

\textsuperscript{11}Cf. Wilson (1991, pp. 429-430) for a graphical exposition and further references.
in general. Since tax rates are equal initially, this demonstrates the proposition\(^\text{12}\).

A final prerequisite for the analysis of tax coordination is to determine the effects of a change in one country's tax rate on the welfare of the other region. It is well known that a mutual tax increase benefits both countries in a symmetric Nash equilibrium since only tax base externalities are effective. In an asymmetric situation matters are more complicated for the region which is in regime I (country A) since it faces counteracting private and public consumption effects. It has been shown by de Crombrugghe and Tulkens (1990) that the net effect of a foreign tax increase must always be beneficial for the country in regime I, i.e., the positive tax base effect (public consumption effect) must dominate the negative private consumption effect. Their proof rests, however, on the additional assumption that the Lagrangian objective is concave within each regime (1990, pp. 340, 347–348). In the simpler present model, the net effect can be signed using the first-order condition of country A's optimal tax problem, with no further restrictions on \(L^A\).

**Proposition 3:** In any asymmetric Nash equilibrium, each country must benefit from a small tax increase in the other region.

**Proof:** For country B, which is in regime II, the welfare change can be directly determined from (13). This gives

\[
\frac{\partial L^B}{\partial t^A} = \lambda_2 t^B \frac{\partial c^A_B}{\partial t^A} = u^B \frac{\partial c^A_B}{\partial t^A} > 0.
\]

(19)

The increase in \(t^A\) leaves private consumption in country B unaffected while tax revenues rise, thus strictly increasing country B's welfare. For country A the Lagrangian (12) is differentiated with respect to \(t^B\) to get

\[
\frac{\partial L^A}{\partial t^B} = \lambda_1 (-c^A_B) + \lambda_2 \left( -t^A \frac{\partial c^A_B}{\partial t^B} \right).
\]

(20)

The first effect is the negative private consumption effect whereas the positive second effect gives the increase in tax revenues due to reduced cross-border shopping. To evaluate the net effect, the induced change in the volume of *domestic* purchases in country A plays a

\(^{12}\)As a caveat, it should be noted here that this analysis applies strictly only to small parameter changes. If differences in preferences are large, and if multiple Nash equilibria exist, the system might adjust "globally" and reach a different equilibrium. Cf. Wilson (1991, p. 435–436) and footnote 10 above.
crucial role. Solving (6) for \( c_A^A \) and differentiating with respect to \( t^B \) gives

\[
\frac{\partial c_A^A}{\partial t^B} = \frac{1}{(1 + t^A)} \left( -c_B^B - t^B \frac{\partial c_A^A}{\partial t^B} - \tau' \frac{\partial c_B^B}{\partial t^B} \right) = \frac{1}{(1 + t^A)} \left( -c_B^B - t^B \frac{\partial c_A^A}{\partial t^B} \right),
\]

where the second equality follows from the consumer arbitrage condition (4). There is an obvious link between the requirement for a rise in \( t^B \) to increase country A’s domestic tax base in (21) and the condition for an overall welfare increase in (20). To see this (20) is rewritten as

\[
\frac{\partial L^A}{\partial t^B} = \lambda_1 \left( -c_B^B \frac{\partial c_A^A}{\partial t^B} \right) + (\lambda_2 - \lambda_1) \left( -t^A \frac{\partial c_B^B}{\partial t^B} \right).
\]

Assume first that \( -c_B^B - t^A (\partial c_B^B/\partial t^B) > 0 \) so that domestic consumption rises in country A. In the reformulation (22) the first effect gives the net income change, evaluated at a weighted average of marginal utilities for private and public consumption [eq. (14)]; this effect is positive by assumption. The second effect describes the welfare change from the implicit redistribution of purchasing power within country A. This effect must also be positive since country A undersupplies the public good in regime I. In other words, assuming that a rise in \( t^B \) increases \( c_A^A \), and thus country A’s tax base from (8), is sufficient to ensure that country A gains from the increase in \( t^B \).

Consider now the alternative case \( -c_B^B - t^A (\partial c_B^B/\partial t^B) < 0 \). From (21) this implies that the tax base falls in country A and both private and public consumption fall. Therefore, country A must unambiguously lose in this case. We can then reverse the above argument and use the information on the sign of the overall welfare change to infer the conditions that must prevail in the initial equilibrium. For \( \partial L^A/\partial t^B \) to be unambiguously negative it must be true that \( (\lambda_2 - \lambda_1) \) is negative initially, then indicating an oversupply of public goods in country A. This, however, is incompatible with the first-order condition of the optimal tax problem in regime I [eq. (15)]. Therefore, \( \partial c_A^A/\partial t^B < 0 \) cannot occur in this regime and country A must gain from a tax increase in the other region.

3 Tax Coordination

We are now ready to discuss the effects of alternative tax coordination measures. In order to sign comparative static effects it is necessary to impose some restrictions on preferences and consider the special case of quadratic transaction cost functions.
Assumption 2: The elasticity of substitution between the public and the private good is positive in both countries:

\[
\sigma^k \equiv \frac{\frac{d}{dMRS^k}}{MRS^k} \frac{MRS^k}{(g^k/c^k)} = \frac{\frac{d}{d\frac{u_c^k}{u^k}}}{\frac{d}{\frac{u_c^k}{u^k}}} (\frac{u_c^k}{u^k}) (\frac{g^k}{c^k}) > 0 \quad \forall \quad k \in [A, B].
\]

Assumption 3: The transaction cost function is quadratic in the volume of cross-border shopping and given by \(\tau(c^A_B) = (1/2) \beta (c^A_B)^2\).

Assumptions 2 and 3 guarantee that the second-order conditions of both optimal tax problems (12)–(13) are fulfilled. Assumption 2 corresponds to the ‘normal case’ of gross substitutability between two goods: an increase in private consumption possibilities increases demand for the public good and vice versa. Alternatively, \(\sigma^k\) can be interpreted as the elasticity of substitution between alternative sources of tax revenue: a high elasticity implies that other taxes (e.g. income taxes) can readily be adjusted to compensate for the revenue effects of changes in the commodity tax rate. In contrast, a low \(\sigma^k\) indicates that the revenue requirement from commodity taxation is inflexible.

Assumption 3 implies that cross-border shopping is a linear function of the tax differential. Differentiating \(\tau\), inverting, and using (4) gives

\[
c^A_B = \frac{(t^A - t^B)}{\beta}. \tag{23}
\]

A first measure, which has been implemented in the European Union, is a minimum tax requirement imposed on the low-tax country B. If this constraint is binding, country A must clearly gain by Proposition 3 whereas country B will gain if and only if its tax increase triggers a simultaneous tax rise in country A.\(^{13}\)

To determine the slope of the two regions’ best response functions, (15)–(16) are differentiated with respect to the tax rate of the union partner, using Assumptions 2 and 3. This gives for country A (which is in regime I)

\[
\frac{\partial F^A}{\partial t^B} = \frac{(1 - \frac{u_c^A}{u^A})}{(1 + t^A)} \frac{\partial c^A}{\partial t^B} + \frac{1}{\sigma^A} \frac{c^A}{(1 + t^A)} \frac{u_c^A}{u^A} \frac{\partial (g^A/c^A)}{\partial t^B}, \tag{24}
\]

and similarly for country B (regime II)

\[
\frac{\partial F^B}{\partial t^A} = \frac{\partial c^B}{\partial t^A} - \frac{1}{\sigma^B} \frac{c^B}{(1 + t^B)} \frac{u_c^B}{u^B} \frac{\partial (g^B/c^B)}{\partial t^A}. \tag{25}
\]

\(^{13}\)Of course, if the required increase in \(t^B\) is discrete rather than marginal, country B will lose from the change in its own tax rate which, on impact, forces it to move away unilaterally from the non-cooperative Nash equilibrium.
By Assumption 3, the second derivative of the transaction cost function is a constant so that, for each region $k$, the size of the negative public consumption effect (the "marginal costs" of a domestic tax increase) is independent of the other country's tax rate. This allows to focus on how a change in the foreign tax rate alters the "marginal benefits" of a domestic tax increase. For country A, the first effect is positive since it undersupplies the public good and $\frac{\partial c^A}{\partial t^B} > 0$ from (21). Intuitively a higher $t^B$ increases the amount of extra revenue that country A can collect from marginally raising its tax rate. The second effect is negative, however, since the increase in $t^B$ raises government revenue in country A while reducing private consumption [cf. (10)]. By Assumption 2 this will unambiguously raise the marginal rate of substitution and reduce the marginal benefits from redistributing purchasing power towards the public sector. For country B, the first effect is also positive since a rise in $t^A$ increases the role of the private consumption effect, thus giving an incentive to country B to raise its own tax rate at the (partial) expense of foreigners. However, an increase in $t^A$ simultaneously raises public good supply so that the second effect is negative.

Note that if governments behave as revenue-maximizers, as in Kanbur and Keen (1993), the marginal utility of private consumption is zero and the negative second effects are eliminated in (24)–(25). Both reaction functions will then be upward sloping and a (small) mandated rise in country B's tax rate will benefit both regions. In the more general case, however, where governments care about the "right mix" between private and public consumption, the revenue inflow stemming from a foreign tax increase (and, in the case of country A, the simultaneous reduction in private purchasing power) will by itself tend to cause a downward adjustment of the domestic tax rate, other things being equal. A limiting case is when governments have to raise a fixed level of revenues from commodity taxation, i.e., $\sigma^k \to 0$. In this case, the negative second effect always dominates and the slope of best response functions is unambiguously negative.

A second coordination policy consists of legal or administrative measures which tighten the enforcement of the destination principle. Such measures may include stricter controls and penalties for illegal smuggling, or the closing of loopholes as in the mail order industry. There are two advantages of this coordination policy over minimum rate requirements: first, the central government may not have the legal powers to impose tax rate harmonization, as is the case for retail sales taxes set by U.S. states. Second, from a perspective of
global efficiency, a coordination policy aimed at (partially) restoring the destination principle for consumer purchases will generally be preferred to (partial) tax rate harmonization. The reason for this is seen from a comparison of the two benchmark cases: a general application of the destination principle is compatible with different national preferences for public goods whereas complete tax rate equalization is not.

A tighter enforcement of the destination principle can be represented by raising the transaction cost parameter $\beta$ for private consumer purchases abroad. Again, it has been shown by Kanbur and Keen (1993, Proposition 6) that this measure strictly benefits both countries when revenue maximization is the objective. In the present framework, its effects on private and public consumption are derived from (6)–(9), using Assumption 3:

\[
\frac{\partial c_A^x}{\partial \beta} = -\frac{c_B^A}{\beta} < 0, \quad \frac{\partial g_A^x}{\partial \beta} > 0, \quad \frac{\partial c_A^l}{\partial \beta} < 0, \quad \frac{\partial g_A^l}{\partial \beta} < 0, \quad \frac{\partial c_B^l}{\partial \beta} = 0, \quad \frac{\partial g_B^l}{\partial \beta} < 0. \tag{26}
\]

These effects are immediately intuitive: an increase in the costs of cross-border shopping redirects purchases by residents of the high-tax country towards the home market. This lowers overall private consumption by country A's residents and raises the tax base in country A while reducing that of country B.

The overall effect of a change in the transaction cost parameter $\beta$ on national welfare can be decomposed into a direct (impact) effect, and the induced change in the tax rate of the other country [cf. Dixit (1986)]. Algebraically,

\[
\frac{dL^k}{d\beta} = \frac{\partial L^k}{\partial \beta} + \frac{\partial L^k}{\partial t^l} \frac{dt^l}{d\beta} \quad \forall \ k, l \in [A, B], \ k \neq l. \tag{27}
\]

It is straightforward to sign the impact effect for each country:

**Proposition 4a:** At unchanged tax rates, an increase in marginal transaction costs increases welfare in country A and lowers welfare in country B.

**Proof:** Partially differentiate the payoff functions (12)–(13) with respect to $\beta$, using (26).

It follows that, for country B to gain from a coordinated rise in marginal transaction costs, country A's tax rate must rise by enough to overcompensate the negative impact effect. The changes in optimal tax rates in response to a variation in $\beta$ are derived in the appendix and are given by

\[
\frac{dt^k}{d\beta} = \frac{1}{\Delta} \left[ -\frac{\partial F^l}{\partial \beta} \frac{\partial F^k}{\partial \beta} + \frac{\partial F^k}{\partial t^l} \frac{\partial F^l}{\partial \beta} \right] \quad \forall \ k, l \in [A, B], \ k \neq l, \tag{28}
\]
where $\partial F^i/\partial t^i < 0$ follows from the second-order conditions and $\Delta > 0$ from stability [cf. (A.3) in the appendix]. The first effect in (28) gives the direct response of country $k$'s optimal tax rate to the parameter change whereas the second effect describes the response to the induced change in the optimal tax rate of country $l$. Differentiating the best response functions (15)–(16) with respect to $\beta$ gives for country A (regime I)

$$\frac{\partial F^A}{\partial \beta} = \frac{t^A}{\beta^2} + \frac{(1 - u^A_c/u^A_g)}{(1 + t^A)} \frac{\partial c^A}{\partial \beta} - \frac{1}{\sigma^A} \frac{c^A}{(1 + t^A)} \frac{u^A_c/u^A_g}{(g^A/c^A)} \frac{\partial (g^A/c^A)}{\partial \beta},$$

and for country B (regime II)

$$\frac{\partial F^B}{\partial \beta} = \frac{t^B}{\beta^2} + \frac{\partial c^B}{\partial \beta} - \frac{1}{\sigma^B} \frac{c^B}{(1 + t^B)} \frac{u^B_c/u^B_g}{(g^B/c^B)} \frac{\partial (g^B/c^B)}{\partial \beta}.$$ 

The first effect in both (29) and (30) is positive: a higher $\beta$ makes cross-border shopping less responsive to tax changes, thus lowering the marginal costs of a domestic tax increase in both countries. Note that this effect is large when the initial value of $\beta$ is low and tax competition through cross-border shopping is intense in the initial equilibrium. The benefits of a domestic tax rise are again ambiguous in both countries: for country A the reasoning is essentially the same as given in equation (24). Using (26) the second effect is positive but the rise in $\beta$ reallocates domestic purchasing power from the private to the public sector and the third effect tends to reduce the optimal tax rate. For country B the signs of the last two effects are now reversed in comparison to (25) because a rise in $\beta$ lowers cross-border shopping by country A’s residents. Thus the private consumption effect becomes less important and tends to reduce the optimal tax rate whereas the reduction in government revenues makes the third effect positive.

Equation (24)–(25) and (29)–(30) provide all the information needed to evaluate (28). Since each of these effects is ambiguous, general results – which hold for all parameter values – can clearly not be expected. Individual effects can be signed, however, when substitution elasticities are below or above some critical value in each of the two countries. We introduce

**Lemma 1:** (a) There exists a $\bar{\sigma}^A$ ($\tilde{\sigma}^A$) such that for all $\sigma^A < \bar{\sigma}^A$ ($\sigma^A > \tilde{\sigma}^A$) it is true that $\partial F^A/\partial \beta < 0$ and $\partial F^A/\partial t^B < 0$ ($\partial F^A/\partial \beta > 0$ and $\partial F^A/\partial t^B > 0$).

(b) There exists a $\bar{\sigma}^B$ ($\tilde{\sigma}^B$) such that for all $\sigma^B < \bar{\sigma}^B$ ($\sigma^B > \tilde{\sigma}^B$) it is true that $\partial F^B/\partial \beta > 0$ and $\partial F^B/\partial t^A < 0$ ($\partial F^B/\partial \beta > 0$).
Proof: The RHS of equations (24), (25), and (29) are all monotonously falling in \(1/\sigma^k\). Therefore, if \(\sigma^k\) is low enough, the negative effects must dominate in each of these equations whereas the positive effects must dominate for sufficiently high \(\sigma^k\). In contrast, eq. (30) is monotonously increasing in \((1/\sigma^B)\) so that this expression must be positive for low enough values of \(\sigma^B\), irrespective of the net effect of the counteracting first two terms. (Note that Lemma 1 makes no statement on the sign of \(\partial F^B/\partial \beta\) if \(\sigma^B > \bar{\sigma}^B\).)

Using Lemma 1, two cases can be identified where best responses to the parameter change are unambiguous in both countries. This is given in

**Proposition 4b:** CASE 1: If \(\sigma^A < \bar{\sigma}^A\) and \(\sigma^B < \bar{\sigma}^B\) then an increase in the transaction cost parameter \(\beta\) lowers the optimal tax rate in country A and raises it in country B.

CASE 2: If \(\sigma^A > \bar{\sigma}^A\) and \(\sigma^B > \bar{\sigma}^B\) and country B undersupplies the public good initially then an increase in \(\beta\) raises the optimal tax rate in both countries.

Proof: Insert the results from Lemma 1 into (28). In case 2, \(2t^B - t^A > 0\) follows from (16) if country B is in undersupply initially and cross-border shopping is given by (23). Using this and (26) in (30) implies \(\partial F^B/\partial \beta > 0\).

Recall that a rise in \(\beta\) can be interpreted as a partial move towards a rigidly enforced destination principle (which is here equivalent to a closed economy). In case 1, low substitution elasticities imply rigid revenue requirements in both countries so that country A will respond to the loss in the tax base after the opening of borders by raising the domestic tax rate while country B will respond to the increase in its tax base by reducing the tax. As \(\beta\) increases through policy coordination the tax base in each country reverts towards its closed-economy level and the direct effect in (29)–(30) is to lower \(t^A\) and raise \(t^B\). Furthermore, since both reaction functions are downward sloping in this case, the indirect (second) effects in (28) reinforce the direct effects in both countries, further reducing \(t^A\) and increasing \(t^B\).

In case 2, high substitution elasticities imply that optimal tax rates in both countries are below their closed-economy levels in the initial Nash equilibrium. A rise in \(\beta\) effectively lowers the (opportunity) costs of public good supply and will thus raise optimal tax rates in (29)–(30). Since both reaction functions are upward sloping in this case indirect effects are also positive in both countries, reinforcing the direct effects. This case always applies
under the assumption of revenue maximizing governments\textsuperscript{14}.

The implications for the effects of an increase in $\beta$ on national welfare are obvious. In case 1 country A unambiguously gains from the coordination measure since both the impact effect and the induced change in country B's tax rate are positive. Vice versa, country B unambiguously loses from both the impact and the induced change in $t^A$. Therefore, there is a clear conflict of interests in this case once borders have been opened. In contrast, both countries will gain from tax coordination in case 2 if the induced rise in $t^A$ is strong enough to outweigh the negative impact on country B's welfare.

To illustrate, and to indicate the range of the different cases discussed, we present the results of some simulations using CES utility functions of the form

$$u^k = \left[ s_k^{1/\sigma} \left( g^k \right)^{(\sigma-1)/\sigma} + \left( 1 - s_k^{1/\sigma} \right) \left( c^k \right)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \quad \forall \ k \in [A, B],$$

where the weights of the public good are assumed to be $s_A = 1/4$ and $s_B = 1/7$. In a closed economy this implies optimal tax rates of $t^A = 0.333$ and $t^B = 0.167$. The elasticity of substitution $\sigma$ is constrained to be equal across countries. Figure 1 plots optimal tax rates as a function of the transaction cost parameter $\beta$ for alternative values of $\sigma$.

If the elasticity of substitution is low ($\sigma = 0.15$), country A's tax rate falls towards its closed-economy level as $\beta$ rises, as described in case 1. Note, however, that the adjustment is not monotonous: for very low initial values of $\beta$, which indicate intense fiscal competition\textsuperscript{15}, the "marginal cost" effect [the first effect in (29)] dominates and a rise in $\beta$ will initially raise the optimal $t^A$. In the intermediate case ($\sigma = 0.4$) country A's optimal tax rate is virtually unchanged for all $\beta \geq 3$ whereas it is monotonously rising in $\beta$ if the elasticity of substitution is relatively high ($\sigma = 1.0$). This last example thus corresponds to case 2 in Proposition 4b. It is also seen from Figure 1 that country B's optimal tax rate is affected very little by changing the elasticity of substitution, and is rising in $\beta$ in all the cases shown.

\textsuperscript{14}With $u^*_k = 0$, the partial effects $\partial F^A / \partial t^B$, $\partial F^B / \partial t^A$, and $\partial F^A / \partial \beta$ are all positive and country B always undersupplies the public good relative to its objective so that $\partial F^B / \partial \beta > 0$ (see the proof to Proposition 4b).

\textsuperscript{15}From (23), $\beta = 1$ implies that a 15 percent tax differential leads to cross-border shopping of 0.15 in absolute value. For $t^A = 0.3$ this corresponds to roughly 20 percent of total private consumption in country A. In contrast, if $\beta = 10$ the ratio of cross-border shopping to overall private consumption induced by the same tax differential is only 2 percent.
Figure 1: Optimal Tax Rates

![Graph showing optimal tax rates as a function of the transaction cost parameter $\beta$. The graph compares two tax rates $t^A(\beta)$ and $t^B(\beta)$ for different values of $\sigma$. The legend indicates $\sigma = 0.15$, $\sigma = 0.4$, and $\sigma = 1.0$.](image-url)
Table 1: Welfare Change in the Low-Tax Country: sign of $dL^B/d\beta$

<table>
<thead>
<tr>
<th></th>
<th>$\beta = 0.5$</th>
<th>$\beta = 1.0$</th>
<th>$\beta = 2.0$</th>
<th>$\beta = 5.0$</th>
<th>$\beta = 10.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma = 0.15$</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma = 0.4$</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma = 1.0$</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma = 2.0$</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma = 5.0$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma = 10.0$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

A rise in country A's tax rate is only a necessary, but not a sufficient condition for the low-tax country B to gain from a coordinated increase in the costs of cross-border shopping. Table 1 summarizes the overall welfare change in country B for alternative initial values of $\beta$ and substitution elasticities $\sigma$. (The welfare change in country A is positive for all the cases reported and is thus not listed explicitly.) The interpretation of these results is straightforward from the above discussion: country B is likely to gain from coordination if the intensity of tax competition is high initially (i.e., if $\beta$ is low) since these conditions will induce a sufficiently strong tax increase in country A for a wide range of substitution elasticities. Symmetrically, the higher is $\sigma$ the wider is the range of initial $\beta$'s under which a small increase in transaction costs benefits the low-tax country.

Clearly, these are only illustrative examples. They indicate, however, under which conditions the mutual gains from reduced tax competition dominate redistributive effects: tax coordination is likely to benefit both countries if either fiscal competition is intense (low-$\beta$ case) or if governments respond elastically to the marginal costs of using commodity taxes in the presence of cross-border shopping (high-$\sigma$ case). Many observers of both EU and North American conditions have argued, however, that tax competition through cross-border shopping is moderate. If, in addition, governments are price-insensitive due to rigid revenue requirements and lack of flexibility to adjust other taxes the upper right area of Table 1 may be the relevant one, implying that the low-tax country stands to lose from coordination measures which are globally welfare-improving. Some (reverse) redistribution is then likely to be involved in efforts to protect the tax base of a region with high preferences for public goods. This may explain the insistence of the European Commission
to achieve an agreement on coordination measures before the opening of internal borders. In this case the model also predicts conflicting interests between low- and high-tax states in North America and elsewhere if rules of destination were more tightly enforced for mail ordering and other forms of cross-border shopping.

4 Concluding Remarks

This paper has introduced cross-hauling to discuss tax competition and tax coordination in a simple trade setting which captures the essential elements of cross-border shopping. Allowing for cross-hauling and introducing consumer transaction costs implies that the same good can be simultaneously traded under both the origin and the destination principle. This specification has the theoretical merit that all profit opportunities are exhausted when producers and consumers face different arbitrage conditions. It also turned out to be crucial for the continuity of reaction functions, and the existence of a Nash equilibrium, when a 'private consumption' externality exists in addition to the usual tax base effects.

The analysis has focused on the effects of tax coordination when optimal tax rates differ across countries due to different preferences for public goods. Two coordination measures were considered, a mandated tax increase in the low-tax region and a policy-induced increase in the (transaction) costs of cross-border shopping. While tax coordination generally benefits the high-tax country, the welfare change in the low-tax country depends crucially on two factors: (a) the intensity of tax competition, which can be directly related to the transaction costs of cross-border purchases and (b) the elasticity of substitution between public and private consumption or, alternatively, the elasticity of substitution between alternative sources of tax revenue. In principle at least, both of these parameters should be accessible to empirical investigation in order to determine whether low-tax countries have an incentive to agree to coordination measures in a given regional and institutional setting.

The limitations of our analysis must be emphasized, however. In particular, the model used here has assumed fixed endowments, thus neglecting any effects of taxes on the efficiency of production. It is known from previous contributions [Trandel (1992), Keen and Lahiri (1994)] that tax competition may improve overall efficiency, and tax coordination may lower it, when firms operate in an imperfectly competitive environment. Whether this
change in the basic framework also reverses the effects on *regional* welfare – on which the present work has focused – is a question that we leave to future research.

**Appendix**

**Proof of Proposition 2:** The first-order conditions (15)–(16) are rewritten in a compact form:

\[
\begin{align*}
F^A[t^A, t^B, \theta^A] &= 0, \\
F^B[t^A, t^B, \theta^B] &= 0,
\end{align*}
\]

where \(\theta^k\) are shift parameters. Totally differentiating the first-order conditions (A.1) yields

\[
\begin{pmatrix}
\frac{\partial F^A}{\partial t^A} & \frac{\partial F^A}{\partial t^B} \\
\frac{\partial F^B}{\partial t^A} & \frac{\partial F^B}{\partial t^B}
\end{pmatrix}
\begin{pmatrix}
dt^A \\
dt^B
\end{pmatrix}
= \begin{pmatrix}
-\frac{\partial F^A}{\partial \theta^A} & d\theta^A \\
-\frac{\partial F^B}{\partial \theta^B} & d\theta^B
\end{pmatrix}
\]

Inverting gives

\[
\begin{pmatrix}
dt^A \\
dt^B
\end{pmatrix}
= \frac{1}{\Delta} \begin{pmatrix}
-\frac{\partial F^B}{\partial t^B} & \frac{\partial F^A}{\partial t^B} \\
\frac{\partial F^B}{\partial t^A} & -\frac{\partial F^A}{\partial t^A}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial F^A}{\partial \theta^A} & d\theta^A \\
\frac{\partial F^B}{\partial \theta^B} & d\theta^B
\end{pmatrix}
\]

where stability of this system requires [cf. Dixit (1986, p. 110)]:

\[
\frac{\partial F^A}{\partial t^A} < 0, \quad \frac{\partial F^B}{\partial t^B} < 0, \quad \Delta \equiv \frac{\partial F^A}{\partial t^A} \frac{\partial F^B}{\partial t^B} - \frac{\partial F^A}{\partial t^B} \frac{\partial F^B}{\partial t^A} > 0.
\]

Setting \(d\theta^A = d\alpha\) and \(d\theta^B = 0\) in (A.2) and noting that a symmetric initial equilibrium implies \(\partial F^k / \partial t^l = \partial F^l / \partial t^k \forall k, l \in [A, B]\) shows that Proposition 2 follows directly from \(\Delta > 0\). In this symmetric case \(\Delta > 0\) is thus equivalent to the more familiar (but stronger) stability requirement that the slope of both reaction functions must be less than unity in absolute value.

**Derivation of Equation (28):** Equation set (A.2) simultaneously includes the solution for the more general case when best responses in both countries are altered by shift parameters: setting \(d\theta^A = d\theta^B = d\beta\) yields eq. (28) in the main text.
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