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ABSTRACT

A three-good-two-primary-factor (3 × 2) general equilibrium model, along with parallel numerical illustrations, is developed to analyze the incidence and welfare cost of several taxes. The approach, blending theory and computed examples, enriches some well-known tax models and provides more insights than either the text-book two-by-two treatments or purely numerical models in areas such as environmental taxation and value-added tax (vat). It is ideal for considering factor taxes in intermediate-good industries (e.g., profit- and payroll taxes in mining industries) which are widely used but not much discussed in the literature. Their incidence, generally, turns out to be very different from similar taxes in final-good industries. A stylized application incorporating zero-rating and exemptions, two key features of the vat system in many countries, further illustrates the usefulness of this framework.

Key words: Intermediate-good taxation, general equilibrium, three-by-two.

JEL classification: H22

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1. INTRODUCTION

Most well-known analytical propositions about the incidence of various taxes have been derived in general equilibrium models with two final goods and one or two mobile factors of production, broadly following the seminal contribution of Harberger (1962). While three-input specifications featuring at least one factor that is sector-specific or immobile have led to rich and varied results (McLure (1971), Shome (1981)), there does not appear to be any theoretical work in the Harberger literature where the number of final goods has been increased beyond two, although quite a few existing tax models have complex production structures (Bhatia (1982)), three-sector specifications (manufacturing-wholesale-retail, for example, in discussions of a value-added tax (vat)), and more than two primary factors (some internationally mobile — Bovenberg and Goulder (1997)). Even in a two-sector model, say music CDs and computer games, one or both final goods may require the use of a third good in production — a produced input (software and/or hardware) — and if that itself is subject to an output or an input tax, its incidence, generally, cannot be inferred from conventional models with two goods and two or three primary factors.

With such applications in view, this paper is an attempt to develop a $3 \times 2$ framework. Its first contribution is to extend existing theoretical models which, in spite of similar (small) dimensions and other common elements, turn out to be rather different from what is being set up here. Such attempts immediately draw attention to computable general equilibrium (cge) models which typically have “large” dimensions, but the cge approach is better suited to answering specific questions than developing general insights into the working of various taxes. Numerical alternatives do not undermine the importance of theoretical work anyhow. The Harberger-type analytical models provide the foundations of modern taxation theory, which underlie the cge approach as well; therefore, enriching the theory and broadening its scope to include new applications are useful exercises per se, and they may also enhance the quality of numerical work in this area.

The second contribution of this analysis is to consider some tax settings that
are frequently observed but seldom analyzed. The optimum-tax literature admonishes against taxes on intermediate goods in a competitive economy\(^1\), but they continue to be levied, both on outputs and inputs. The former have been featured in Bhatia (1982), a competitive model; in Myles (1989), an imperfectly competitive specification; and in recent applications to environmental taxation (Poterba and Rotemberg (1995), Bovenberg and Goulder (1997), among others). There is, however, no corresponding treatment of factor taxes — a corporate profits tax (cpt) or payroll taxes — that apply to mining companies, heavy industry and other intermediate-good producers in several countries. In the analytical literature, such taxes are invariably placed in final-good industries, although they have been included in cge computations here and there, often in combination with complex vat regimes\(^2\). To anticipate one of the main conclusions of this analysis, final-good situations are not very helpful, and can be quite misleading, in predicting incidence outcomes when these taxes are levied in intermediate industries. How the burden of a cpt on steel is distributed cannot be inferred very well from an analysis of cpt on a final good that uses steel, say, household automobiles, even in the same model.

Such models, especially when more than one tax is involved, tend to be complex, even to the point of being analytically intractable sometimes. Numerical methods, then, are a researcher's main recourse. We shall therefore rely on a combination of algebraic and cge techniques, using each to its comparative advantage, and that is the third contribution of this paper. Tax studies more and more seem to be falling into one or the other camp, without much interface between them. Theoretical models are often too restrictive, and multi-sector numerical specific-

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\(^1\) According to Kay and King (1971), as a first principle, there should be no commodity taxes on intermediate goods at all. This is based on the argument made by Diamond and Mirrlees (1971) that, for optimal taxation, it is better to impose taxes only on final goods.

\(^2\) In Canada, to cite an overly simplified example, fertilizer companies, which produce a pure intermediate good (FIG) for the most part, are subject to a vat (called a goods-and-services tax or gst) and a cpt. The gst, but not the cpt, is refunded to food producers who purchase fertilizer. Similar provisions exist in the vat systems of U.K., France, and Germany (Bergwick and van den Broek (1987)). A 3 × 2 set-up is probably the smallest model in which such tax packages can be adequately formulated. Some aspects of these types of taxes will be taken up later in the paper, especially in Section 4.
cations so detailed and so complex that one loses track of the essential forces at work. When the two are combined, as in Bovenberg and Goulder (1997), a simple theoretical framework is used mostly to motivate the discussion and set down a few analytical results, really a curtain raiser for the numerical approach, but that means that effects of sectoral disaggregation, intermediate goods and other features that play an important part only in the numerical model cannot be readily isolated. Moreover, theoretical constructs are shut out, too quickly sometimes, from developing better priors for some of the expected results and analyzing them, and that is not conducive to good methodology. Several examples will emerge later in the paper where certain numerical outcomes cannot be explained except by referring to corresponding theoretical models. In this context, the main focus here is on deriving general propositions along with closely related cge computations for a series of specification within a consistent framework. In fact, the numbers are carefully chosen to illustrate some of the key points highlighted by the theoretical derivations, and when only numerical solutions can be reached, we shall try to relate them to existing analytical results as much as possible.

The final-goods-only (fgo) model and a corresponding numerical example are set out in the next section. Benchmark results for two taxes, a production tax and a partial factor tax, in an economy with three final goods are also presented there. One of the three goods is treated as a pure intermediate good (PIG) in Section 3 where exact analytical solutions for several taxes are derived and illustrated with the help of cge computations. Detailed results for taxes on intermediate goods and their relevance for some environmental-taxation debates are also taken up in that section. A vat/cpt type application is the subject of Section 4, and the paper concludes with a summary in Section 5.

2. THE fgo MODEL AND A NUMERICAL EXAMPLE

The theoretical framework, technically, merely adds one more commodity to the standard 2 × 2 Harberger model, and since it has been well documented and widely explored in the literature, a brief description will suffice.
Let there be three goods $X_1$, $X_2$, and $X_3$, the production technology of each being characterized by a linear homogeneous production function using two primary factors, labor ($L$) and capital ($K$). Labor and capital are assumed to be in fixed aggregate supply and fully employed. Denoting $a_{ij}$ as the quantity of input $i$ needed to produce one unit of good $j$, the full employment condition is:

$$
\begin{bmatrix}
  a_{L1} & a_{L2} & a_{L3} \\
  a_{K1} & a_{K2} & a_{K3}
\end{bmatrix}
\begin{bmatrix}
  X_1 \\
  X_2 \\
  X_3
\end{bmatrix}
= 
\begin{bmatrix}
  L \\
  K
\end{bmatrix}
$$

(1)

Perfect competition in all markets and mobility of capital and labor will ensure equality of wages ($w$) and capital rental ($r$) across industries, and each output price ($p_i$) will equal its corresponding average cost in equilibrium (the zero-profit condition):

$$
\begin{bmatrix}
  a_{L1} & a_{K1} \\
  a_{L2} & a_{K2} \\
  a_{L3} & a_{K3}
\end{bmatrix}
\begin{bmatrix}
  w \\
  r
\end{bmatrix}
= 
\begin{bmatrix}
  p_1 \\
  p_2 \\
  p_3
\end{bmatrix}
$$

(2)

In equilibrium, firms minimize unit cost, and all production coefficients depend on relative factor prices, i.e.

$$
a_{ij} = a_{ij}(w, r), i = L, K; j = 1, 2, 3
$$

(3)

On the demand side, only one category of consumers is specified who, endowed with $K$ and $L$, are assumed to maximize a constant-elasticity-of-substitution (CES) utility function to generate final demands for the three goods. It is also assumed that the tax revenue is returned to them in a lump-sum fashion so that, to all intents and purposes, this is a single consumer economy. Table 1 provides a numerical depiction of such an economy in equilibrium. All initial prices

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3Simplicity and ease of comparison with the results in the literature are the main arguments for this patent unrealism assumption. Dealing with two types of consumers or more than one tax distortion in this three-good setting are daunting analytical tasks, although it can be easily handled by the ege approach. Witness, for example, the debate about “double dividend” from environmental taxes — improved environment plus possible reductions in other distortionary taxes through revenue recycling (Goulder (1995)). A lot is happening on the production side of this model, it will become clear presently, and that will be the focus of the rest of the analysis. Lump-sum tax refunds and a simple demand specification undoubtedly help in that task.
have been normalized at unity, so the numbers represent both physical and value units. Wage earnings and labor endowment, thus, are 195, $X_1$ employs 170 units of capital which earn exactly that amount in dollars, and so on.

**TABLE 1 HERE**

2.1. Comparing with Earlier Models

Since there are no intermediate inputs at this stage, and all goods are used for final consumption only, the obvious comparison is with other, small-dimension figo models, the most important among them being the Harberger two-by-two. For the most part, its logic, theoretical structure, and assumptions carry over to the present framework. The only difference, really, is that now there are more goods than factors of production, which will complicate some solution techniques. The common elements worth noting are that labor and capital do not differ intrinsically or in mobility assumptions, and each good is produced by the same technology and the same two inputs. Therefore, relabeling the goods, or interchanging $L$ and $K$, will have no substantive effect on the analysis. These models, in other words, have a symmetry property which ensures that partial factor- and output taxes operate analogously regardless of where and upon which factor they are levied. Moreover, ignoring the efficiency loss caused by distortionary taxes, one factor's income gain is matched by the other's income loss, there being only two factors of production in the model, both fully employed, and one input-price ratio, $r/w$. The interesting question is: How will the 2x2 results fare in the expanded framework?

Turning to models with an unequal number of goods and factors, there is not much in common beyond the uneven dimension, and even that does not lead to many similarities in solution procedures or outcomes. The theoretical section in Bovenberg and Goulder (1997), for example, has two final goods and three primary factors ($2 \times 3$), but two of the factor prices are exogenous and labor supply is endogenously determined. Closer to the set-up in this paper, a typical $2 \times 3$ specification has some restrictions on factor mobility, as in McLure (1971), where
each good is produced by a fixed and a mobile factor. Two of the three factors of production thus are immobile across activities, whereas Ratti and Shome (1977) specify only one such fixed input, land, in one of the two sectors which has a three-input production function. Production activities, therefore, are not interchangeable, so the same tax would operate differently in one industry than in another. The symmetry property actually breaks down before that because mobile and sector-specific inputs will lead to more than one ratio of input prices, and in spite of the common assumptions of full employment and perfect competition, many other complications set in (Shome 1981).

All in all, our model thus far is closer to the original Harberger specification than the $2 \times 3$ settings just described even though it has an unequal number of goods and factors. We shall return to this theme briefly in Section 3, after an intermediate good has been designated, because then also there will be more production inputs than final goods. For now we turn to a solution of the model and some benchmark results.

2.2. Taxes and Model Solutions

At most two of the three demand functions will be independent in this model, and different taxes can be readily incorporated into the relevant equations. A selective production tax, for instance, will affect one of the zero-profit conditions in (2) and the demand relation(s), whereas a partial factor tax will show up in the corresponding $a_{ij}$ equations also in (3). Starting with a no-tax equilibrium, any tax will eventually lead to an excess demand or supply for labor or capital, and full employment will be restored by adjustments in various output prices and factor rewards. The model is solved for a change in the rental-wage ratio, and that will reveal how the burden of a given tax has been borne.

This is also the logic of the text-book two-by-two and most higher-dimension models of this type. The eleven equations set out so far, along with two demand relations and three production functions will be sufficient, in principle, to determine the equilibrium values of the unknowns — two input prices, three out-
put prices, three output levels, three capital-labor ratios \((k_1, k_2, \text{ and } k_3)\) and six input-output coefficients — all except one (the model is underdetermined) which might be chosen as the numeraire. In practice, however, the procedure commonly used to solve such theoretical models (Jones (1965)) becomes quite tedious and unwieldy, and it is difficult to write down any analytical solutions\(^4\). A cge approach nonetheless enables us to identify some key results, and these can be examined further, numerically and analytically, in later sections.

2.2.1. Tax incidence: the computations

Beginning with the initial equilibrium displayed in Table 1 (no taxes), and deploying CES functional forms for the production relations and the utility functions (from which final demands are derived), a new equilibrium is computed in each tax situation. The wage rate, \(w\), is the numeraire, in keeping with much of the Harberger literature. Indicating a proportional change by an asterisk \((x^* = dx/x)\), movements in the rental-wage ratio \((\omega)\) can be computed from the factor rewards in each post-tax equilibrium, and these computed values of \(\omega^*\) are reported in Table 2. Negative signs indicate that this ratio falls, and in terms of the model outlined above, the tax in question hurts the owners of capital throughout the economy. An increase in the rental-wage ratio \((\omega^* > 0)\) of course will benefit capital. The numbers in parentheses are a utility index from which the welfare cost of different taxes can be readily computed (the pre-tax level of this index is unity).

\(^4\)A crucial step in the solution process is to totally differentiate the full employment equations, solve the resulting structural equations for proportional changes in output \((X_1^* \text{ or } X_2^*)\) and equate them to the corresponding demand equations. Here we shall have two structural equations in three unknowns \((X_1^*, X_2^*, \text{ and } X_5^*)\) which will not yield an unique solution for \(X_1^* \text{ or } X_2^*\). This difficulty does not arise in the immobile-factor specifications a la McLure (1971), because there are only two final goods, and solutions for \(r^*\) by and large can be derived as in the two-by-two mobile-factor model (Bhatia (1989)). Among the many ways of avoiding this complication in the present framework are aggregating two activities, as in Bhatia (1998), or defining a Hicksian composite good, and so on. For a comparison between the \(3 \times 3\)and \(2 \times 2\) specifications in a trade context, see Kemp (1969), chapter 5.

TABLE 2 HERE
2.2.2. Some benchmark results

Note, first, that according to the numbers in Table 1, $X_1$ is the most capital intensive of the three industries, followed by $X_2$ and $X_3$. Denoting $K_i/L_i$ by $k_i$, $k_1 > k_2 > k_3$. Among the results in Table 2, the only tax that always benefits owners of capital is the output tax on the most labor-intensive activity in this model, $X_3$. A tax on capital income in this industry ($t_{K3}$), too, will lead to this outcome, but only under highly restrictive conditions (row 2). The biggest drop in the rental-wage ratio as well as the highest welfare cost (the lowest utility index) are caused by $t_{K1}$, a partial tax on income from capital in the most capital-intensive industry.

The first result worth highlighting comes from the negative signs in Column 1 and the positive ones in Column 2: the factor used intensively in the taxed industry is being hurt in each instance regardless of the elasticities of substitution ($\sigma's$). The second key result pertains to a partial tax on capital income: $\omega^* < 0$ with $t_{K1}$ in Column 3, and also with $t_{K3}$ in Column 4 except in row 2 where the elasticity of substitution in the taxed industry ($\sigma_{KL}^3$) is small.

From an analytical standpoint, the most notable aspect of these outcomes is their similarity with, rather than any striking differences from, the results associated with the standard $2 \times 2$ model. In that framework, it is well known that for a single-consumer specification, (i) factor intensities play a decisive role in determining the incidence of selective production taxes, and for partial factor taxes, elasticities of substitution also matter (Mieszkowski (1967)), (ii) for a partial tax on income from capital, the rental-wage ratio falls if the taxed industry is relatively capital intensive, and (iii) capital can benefit from this tax only if the taxed industry happens to be relatively labor intensive (a necessary condition) and its elasticity of substitution is small (Harberger (1962), proposition 1). These propositions are well reflected in the two benchmark computations highlighted above. The common thread running through all the cases is that the tax in question will lead to an excess supply of capital unless there is an offsetting influence.

The close correspondence between the analytical propositions and the com-
putations is reassuring, but one cannot be sure of the generality of the numerical outcomes without parallel algebraic solutions for \( \omega^* \), a task which can be easily accomplished in the next section, where \( X_3 \) is treated as an intermediate input.

3. THE MODEL WITH AN INTERMEDIATE GOOD

Many of the examples mentioned in the Introduction involve an intermediate input, so we designate \( X_3 \) for that role. Because of the symmetry property, it does not matter which good serves in that capacity. Now the model can be viewed as a \( 2 \times 3 \) framework — two final goods, \( X_1 \) and \( X_2 \), and three inputs, one of which, \( X_3 \), happens to be produced rather than primary in nature. There are more production inputs than final goods, and that enables us to surmount the difficulties encountered above in deriving analytical solutions for \( \omega^* \). This numerical feature is evident in the specifications of McLure (1971) and Shome (1981) as well, although their three inputs are all primary factors of production, at least one of them being immobile across industries. The theoretical section in Bovenberg and Goulder (1997) also has two final goods and three primary factors (intermediate inputs are introduced only in the numerical model), but as noted above, two factor prices are exogenously given (small open economy assumptions) and labor supply is endogenous.

In our set-up, \( X_3 \) becomes a channel for capital and labor to be used indirectly by the final-good industries. If there are no taxes in \( X_3 \), it provides untaxed capital and labor services for the taxed industries; otherwise, any taxes in \( X_3 \) will be transmitted in their original or a transmuted form to other segments of the economy. An output tax in this industry, thus, will become an input tax in \( X_1 \), if that is where it is used as an intermediate input, and a tax such as \( t_{K3} \) will be a tax on the income of capital indirectly engaged in the production of \( X_1 \). Its incidence nonetheless will be rather different from that of a tax directly imposed in \( X_1(t_{K1}) \), as the following derivations will show. We shall discuss factor taxes in some detail because earlier theoretical work with intermediate inputs (Bhatia (1982)) and applications — especially to environmental taxation (Goulder (1995),
Poterba and Rotemberg (1995) — have concentrated by and large on output (or excise) taxes\(^5\).

3.1. Solving the PIG Model

This section will mostly deal with the case in which the entire output of \(X_3\) is used as an intermediate input in \(X_1\). Other arrangements — \(X_3\) used in \(X_2\) or shared by the two final-good industries — do not pose any special problems and can be analyzed in an analogous manner. For computational purposes, the \(X_1\) Column in Table 1 now shows an additional input, \(X_3(100)\), and the final demand for \(X_3\) in the column headed by \((C + G)\) becomes zero.

The crucial difference between this and the earlier, final-goods-only specification is that \(X_1\) now uses capital and labor directly as well as indirectly through \(X_3\). If \(a_{ij}\) is the quantity of the \(i^{th}\) input per unit of the \(j^{th}\) output, the direct labor-output ratio in \(X_1\) is \(a_{L1} = L_1 / X_1\), whereas its total use of labor per unit of output is \(R_{L1} = a_{L1} + a_{L3}\), or \((L_1 + L_3)/X_1\). Definitions for \(R_{K1}, R_{L2}\), and \(R_{K2}\) (where necessary) will be analogous. Direct factor shares (\(\rho_{ij}\)'s) are based on \(a_{ij}\)'s, and total factor shares (\(\theta_{ij}\)'s) are computed from the \(R_{ij}\)'s or the \(\rho_{ij}\)'s (e.g. \(\rho_{L1} = w.a_{L1}\), and \(\theta_{L1} = \rho_{L1} + \rho_{31}\rho_{L3}\)). A hallmark of models of this type is that an industry may be relatively capital intensive in terms of the \(a_{ij}\)'s but labor intensive when the \(R_{ij}\)'s are compared, or vice versa — a sort of factor-intensity reversal — and that, indeed, is one feature of the data in this numerical illustration. In the initial equilibrium, \(\rho_{K1} > \rho_{K2}\), but \(X_3\) has the highest labor-capital ratio among the three activities, and when it is used as an intermediate input in \(X_1\), \(\theta_{K1} < \rho_{K2}\). The switch from \(a_{ij}\) to \(R_{ij}\) effectively merges the production relations for \(X_3\) and \(X_1\), and \(\rho_3\), likewise, is subsumed by \(p_1\) in the zero-profit conditions. Technically, then, the model reduces to a framework with two final goods and two primary factors, and it can be solved following the procedures

\(^5\)Carbon taxes, pollution charges, and environmental levies are examples of this type in the environment tax literature. In their numerical model, Bovenberg and Goulder (1997) mention an increase in the corporate income tax (cit) rate which, presumably, applies to intermediate-good industries also, but the incidence and other effects of such taxes per se are not discussed.
in Jones (1965), modified to include a pure intermediate input (Bhatia (1982)). Since this methodology has been widely used and documented, without recording every step in the derivations, we can write down some expressions for \( \omega^* \), the proportional change in the rental-wage ratio which can also be indicated by \( r^* \) because \( \omega^* = r^* - w^* \), and \( w \), being the numeraire, does not change.

Briefly, though, this is a comparative-statics exercise that compares the initial equilibrium with the new, post-tax, equilibrium in which all markets clear. Because capital and labor are fixed in aggregate supply, variations in input demands by the various industries must be equal and opposite. There are only two final goods in the model now, so only one of them, say \( X_1 \), will have an independent demand function. Any changes in the demand for \( X_1 \) must be matched exactly by an adjustment in its supply, and that becomes the focus of the solution process. Total differentiation of the full-employment condition in (1), after restating it in terms of \( R_{K1} \) and \( R_{L1} \), leads to an expression for \( X_1^*(S) \), the proportional change in the supply of \( X_1 \). The \( a_{ij} \) equations and the zero-profit conditions, appropriately modified for various taxes, similarly yield expressions for adjustments in input-output coefficients based on the rental-wage ratio and elasticities of substitution. The demand side is summarized by \( X_1^*(D) \), and finally \( X_1^*(S) \) is set equal to \( X_1^*(D) \) to solve for \( r^* \). As in Table 2, \( r^* \)-values are computed by a parallel cge algorithm which relies on numerical approximations of actual production and utility functions in an optimizing framework\(^7\).

\(^6\)To facilitate comparison with the existing results in the Harberger framework, a demand function commonly used in that literature \(-X_1^*(D) = \epsilon(p_1^* - p_2)\) is also adopted here. Given the assumption about the use of tax revenue, \( \epsilon \) — negative by definition — can be regarded as a compensated elasticity of demand. The main departure from the Harberger two-by-two model is that the \( X_1 \) production function will have three inputs, \( L_1 \), \( K_1 \), and \( X_3 \), which, in the most general case, will require the use of Allen-Uzawa partial elasticities of substitution rather than the simple elasticity of the two-input specification.

\(^7\)The \( r^* \)-computations are based on MPS/GE software (Rutherford (1988)), and they involve the computing of pre- and post-tax equilibria for the economy. In the analytical literature (e.g. Harberger (1962)), empirical results are typically derived by plugging in the values of capital-labor ratios, factor shares, and elasticities of substitution and demand into the analytical expressions for \( r^* \), whereas factor shares and ratios will be computed endogenously by the cge algorithm for a given set of elasticities. Analytically, the two approaches are equivalent, so they should yield similar outcomes at least for small tax changes such as the ones considered here.
Some terms involving capital-labor ratios, factor shares, and elasticities of substitution will occur again and again in the expressions for \( r^* \), so it will be helpful to define and sign them in advance. Accordingly, \( A_1 = R_{K1}/a_{K2} - R_{L1}/a_{L2} \), an indicator of the relative factor intensity of \( X_1 \) based on its total usage of labor and capital, \( B = \theta_{K1}L_1/L_2 + \theta_{L1}K_1/K_2 \), and \( G \) is a weighted sum of all the elasticities of substitution in \( X_1 \) and \( X_3 \), using various factor shares as weights. \( A_1 \) can have any sign, depending on the numbers, although in the present example where \( X_3 \) is used entirely by \( X_1 \), \( A_1 < 0 \). Since all \( \sigma \)'s are defined to be positive, both \( B \) and \( G \) will be positive, or zero in the highly unlikely event that all input-output ratios are fixed.

3.1.1. Output taxes in \( X_1 \) and \( X_3 \)

Even though the entire output of \( X_3 \) is used up in the production of \( X_1 \), these two taxes operate rather differently. While the tax on \( X_1 \) is a straightforward output tax, the tax on \( X_3 \), to all intents and purposes, is a partial input tax in \( X_1 \). In terms of the automobile-steel example mentioned in the Introduction, although the automobile industry uses all the steel produced, a tax on the latter is different from a tax on the output of cars.

A selective output tax levied on \( X_1 \) at an ad valorem rate of 100\% percent, with the tax revenue returned to the consumers in a lump-sum fashion, leads to the following solution:

\[
r^* = A_1 \varepsilon \frac{T^*_1}{D_1}
\]

(4)

where \( \varepsilon \) (defined to be negative) is the compensated elasticity of demand for \( X_1 \), \( T^*_1 = dt/(1 + t) \) is the proportional change in the tax rate, and \( D_1 = A_1 \varepsilon (\rho_{K2} - \theta_{K1}) + \sigma^2_{KL} + BG \). Every term in \( D_1 \) is positive: in the first, \( A_1 \) and \( (\theta_{K1} - \rho_{K2}) \) have opposite signs and \( \varepsilon < 0 \); \( \sigma^2_{KL} > 0 \), by definition; and \( BG \) is a product of two positive entities noted above. It follows that the sign of expression (4), and also of the other \( r^* \)-solutions to be derived below, will depend on the numerator of \( r^* \).
Result 1: The direction of change in the rental-wage ratio in response to a selective output tax on a final good depends only on relative factor intensities even when a pure intermediate good is present.

This is an extension of the Mieszkowski result referred to in Section 2, and it is analogous to the first benchmark identified above in Table 2, the only difference being the inclusion of indirect factor usage also in determining the relative factor-intensity of \( X_1 \). Since \( \epsilon < 0 \), it follows that \( r^* \gtrless 0 \) as \( A_1 \lesssim 0 \). In particular, if \( X_1 \) is relatively capital intensive, as this industry contracts in response to the output tax, capital will be in a larger excess supply than labor, so the rental-wage ratio must fall. If the untaxed industry can substitute capital for labor very easily, however, the factor-price ratio will not change: As \( \sigma_{KL}^2 \to \infty \), \( r^* \to 0 \) in (4). As a corollary, it can be shown that if \( X_3 \) is used entirely by \( X_2 \), which is then subjected to an output tax, the rental-wage ratio will rise because the taxed sector (comprising of these two activities together) will be relatively labor intensive.

With the numbers in Table 1 rearranged to reflect intermediate usage, \( A_1 < 0 \); therefore, \( r^* > 0 \) in Column 1, Table 3. Recall that for the same tax, \( r^* < 0 \) in Table 2, Column 1. This switch in signs is strictly due to the factor-intensity reversal noted above. In Table 2, \( X_1 \) has the highest capital-labor ratio, whereas here, based on total factor use, this industry is more labor intensive than \( X_2 \). Whether a factor-intensity switch of this sort comes about or not is by and large an empirical matter (for example, it does not happen when \( X_3 \) is coupled with \( X_2 \) as an intermediate input in this data set). The key point worth underscoring is that a factor-intensity condition, defined by the sign of \( A_1 \), is sufficient to determine the sign of \( r^* \), as in a model with only final goods.

TABLE 3 HERE

A production tax on \( X_3 \).- This tax will be carried over to \( X_1 \) through the production linkage. Capital and labor services already subject to a tax and
embodied in $X_3$ are indirectly being used in the production of $X_1$. Therefore, like factor taxes in the final-good model, whether an excess demand or supply of capital comes about will depend on factor intensities as well as on substitution possibilities in $X_1$; more precisely, on whether $K_1$ or $L_1$ is a better substitute for the intermediate input. Defining $T_3^*$ analogously to $T_1^*$, the following solution emerges:

$$r^* = (A_1 \epsilon \rho_{31} + B \rho_{31} (\rho_{L3} \rho_{K1} \sigma_{K3}^1 - \rho_{K3} \rho_{L1} \sigma_{L3}^1) / \theta_{L1} \theta_{K1}) T_3^* / D_1 \quad (5)$$

Expression (5) is visibly different from (4), so it seems reasonable to conclude that the incidence of $t_3$ is not determined in a manner symmetrical to that of $t_1$, although there are some common elements ($A_1$, $\epsilon$, and $D_1$) between the two expressions. This is one example of the breakdown of the symmetry property noted in Section 2.

**Result 2.: Factor intensities alone cannot determine the direction of change in the rental-wage ratio when a pure intermediate good is subject to a selective output tax.**

Here is a selective production tax, yet it is clear from (5) that no condition on $A_1$ alone will be sufficient to sign $r^*$, contrary to Result 1. Some restrictions on $\sigma_{K3}^1$ and/or $\sigma_{L3}^1$ will also be required. Even if $X_1$ is relatively capital intensive, owners of capital can benefit from a tax on $X_3$ because it may increase the aggregate demand for capital if it is highly substitutable for the intermediate input (in (5), only a “large” value of $\sigma_{K3}^1$ can make $r^*$ positive). When labor and capital cannot be substituted for the intermediate good ($\sigma_{K3}^1 = \sigma_{L3}^1 = 0$), Result 1 will hold. In a similar vein, equation (5) also suggests a number of sufficient conditions for signing $r^*$: $r^* < 0$ if $A_1 \geq 0$ and $\sigma_{K3}^1 = 0$, $r^* > 0$ if $A_1 \leq 0$ and $\sigma_{L3}^1 = 0$, and as in the case of an output tax on $X_1$, as $\sigma_{KL}^2 \rightarrow \infty$, $r^* \rightarrow 0$.

Turning to numerical computations, in Table 3, $A_1 < 0$ and $r^* > 0$ everywhere in Column 2. The taxed industry (or the “taxed sector” which aggregates $X_1$ and $X_3$) is relatively labor intensive, the tax therefore leads to an excess supply of labor, and the elasticity of substitution between labor and the intermediate good
in $X_1$ is not large enough to offset that. The rental-wage ratio, accordingly, rises in the post-tax equilibrium. It is evident in (5), however, that somewhat different parameter values, especially a large $\sigma^1_{L3}$ relative to $\sigma^1_{K3}$ for instance, would lead to a negative $r^*$. In fact, setting $\sigma^1_{L3} = 0.5$ and $\sigma^1_{K3} = 0.1$ in row 1, column 2 of Table 3 yields an $r^* = -0.0007$. These $\sigma$-values, incidentally, are generally within the range indicated by econometric estimates of three-input production functions involving, for instance capital, labor, and energy (KLE, Berndt and Wood (1979)). $K - E$ complementarity, which also is emphasized in this literature ($\sigma^1_{K3} < 0$), would increase the odds for a negative $r^*$.

These outcomes also touch upon a methodological issue raised in the Introduction about an interface between the analytical and cge approaches: if our expectations are based on the results from a fgo model, whereas intermediate inputs in fact are a part of the actual computations, there can be some odd surprises. Comparing Tables 2 and 3 for $t_3$, for instance (Column 2), $r^* > 0$ everywhere, but its magnitude differs by a factor of two in rows 3 and 4. Since the industry directly subject to the tax is the most labor intensive in both cases, the production linkage appears to be magnifying the increase in the rental-wage ratio. Factor intensities alone mattered in Section 2. Here also the $A_1$-term is positive in (5), and although $r^*$ could be negative in principle, the $\sigma$-terms, fortuitously, do not outweigh the effect of relative factor intensities.

3.1.2. Output taxes: Some examples

Taxes such as $t_1$ and $t_3$ are being widely used. In Canada, for instance, tobacco is subsidized (directly or indirectly) whereas cigarettes are taxed, and production of crude oil is taxed along with further levies on gasoline used for final consumption. Output taxes, often described as "carbon taxes," "green levies," or environmental excises more broadly, are at the heart of many environment-tax debates. Results 1 and 2 and the models deployed thus far can shed light on some of the issues considered in this literature.

In an environmental context, if $X_3$ is coal and $X_1$ denotes electricity for house-
hold use, a carbon tax will apply only to $X_3$ in the fgo case, and its effects are illustrated in Table 2, Column 2. If electricity is generated by a coal-fired plant, the model in this section will apply, and that raises a question of policy interest: Should a carbon tax be levied on the coal-mining company or the power-generating facility, as an output tax on the intermediate good, as an input tax on the final good it helps produce, or something else? The analytical models here do not offer any propositions about efficiency cost as such, but the results set out above suggest that the two taxes can have rather different distributional effects even in this simple framework. The numbers in Table 3 show that there is nothing to choose between them on efficiency grounds, so only distributional considerations matter in this illustration.

It is also useful to note in this context that slight modifications of the model can lead to some specifications used in earlier work in this area. For example, when both $X_3$ and $X_2$ are imported, and the latter also uses some $X_3$ in production, we have the formulation in Poterba and Rotemberg (1995) (the simple case, without joint production). They focus on import neutrality, on how to counter the effect of a tax such as $t_3$, rather than its incidence per se, which is relatively straightforward in their set-up because labor is the only primary factor of production. Here, labor and capital are the two primary factors, and $X_3$ is domestically produced. Other changes along these lines can allow for both domestic production and imports of $X_3$, as is true of many energy-producing countries, a case considered by Burgess (1989) for instance. Different types of environmental externalities — strip mining of coal, pollution caused by using fossil fuels — can also be incorporated into this framework without much difficulty. The taxes considered above then can be regarded as Pigouvian taxes designed to deal with these externalities.

Instead of exploring these modified models and other applications in greater detail, we turn to some conventional factor taxes which are levied in intermediate activities in many countries but have not received much attention in the tax literature.
3.1.3. Partial factor taxes in $X_1$ or $X_3$

We shall discuss a partial factor tax, a tax on earnings of capital ($t_{K1}$ or $t_{K3}$) to highlight how differently these two taxes work in this model and how they contrast with the fgo case. Referring back to the steel-automobiles example, the main question is: How will the incidence of a profits tax in the automobile industry differ from that of a corresponding tax in steel production? Much of what we know about the incidence of factor taxes has been learnt from taxes such as $t_{K1}$, either analysed in fgo models or applied to final-good industries even when pure intermediate goods are present (Bhatia (1982)). The relevant analytical literature just has not dealt with taxes such as $t_{K3}$, the implicit assumption being that they are not much different from any other factor taxes.

A partial tax on capital income in $X_1(t_{K1})$. — The base of this tax is the income of capital directly employed in $X_1$, and its incidence will be affected by many things, including capital employed in $X_3$. This industry now serves as a conduit through which untaxed capital services flow into $X_1$, and it also absorbs some of the capital and labor released by $X_1$ as the economy moves to a post-tax equilibrium. In other respects, this tax operates in a manner similar to $t_{K1}$ in the Harberger two-by-two model, for it distorts cost-minimizing input choices by individual firms, and ultimately, through the zero-profit conditions, it affects output prices and demand decisions. The main additional complication arises because $X_1$ has three inputs so that a few more elasticities of substitution get involved. The solution for $r^*$ in the present case is:

$$r^* = (A_1e \rho_{K1} - B(\rho_{L1}\rho_{K1}\sigma_{KL}^1 + \rho_{K1}\rho_{L3}\rho_{31}\sigma_{K3}^1) / \theta_{L1}\theta_{K1})T_{K1}/D_1$$

(6)

where $T_{K1}$ is the proportional change in the tax rate, and the other terms have been defined above. Factor intensities, of course, will take into account both the direct and indirect usage of capital and labor, and $\sigma_{K3}^1$ is the additional elasticity of substitution that will affect the sign of $r^*$. Keeping in mind factor-intensity-
reversal possibilities, the incidence of this tax could be very different from what happens in the fgo case. Focusing on similarities, however, note that the key analytical propositions derived in Section 2 are reflected in (6): if $A_1 \geq 0$, $r^* < 0$ because $\epsilon < 0$ and $B > 0$, and the rental-wage ratio can rise only if the taxed industry is relatively labor intensive.

**Result 3.:** Even when a pure intermediate good is present, the incidence of a partial factor tax in a final-good industry by and large depends on the same considerations as in a final-goods-only framework.

The sufficient condition mentioned above is illustrated in Column 5, Table 3 where $X_3$ is used entirely by $X_2$, and the taxed industry ($X_1$) has a higher capital-labor ratio than the other two industries combined. The rental-wage ratio falls in every row, as (6) predicts, and that is also the outcome in Column 3 of Table 2. A necessary and sufficient factor-intensity condition follows if $\sigma_{KL}^1 = \sigma_{K3}^1 = 0$, for $r^* \geq 0$ as $A_1 \leq 0$ in expression (6).

A partial tax on capital income in $X_3(t_{K3})$. —As far as we can tell, this is the first treatment of such a factor tax in an analytical model. $X_3$ now supplies a taxed input to $X_1$; more precisely, it provides capital services already subject to a tax and embodied in $X_3$. Direct employment of capital in $X_1$, in contrast to the $t_{K1}$ case, is now a source of untaxed capital for this industry. Input choices in both $X_1$ and $X_3$ will be distorted on account of $t_{K3}$, and all elasticities of substitution involving the taxed input will affect the direction in which the rental-wage ratio moves. The ball will be set rolling, of course, by $\sigma_{KL}^3$ which will influence the amount of capital and labor released from $X_3$ in the first place. The solution for $r^*$ turns out to be:

$$
    r^* = \rho_{31}\rho_{K3}(A_1\epsilon - B \rho_{13}(\sigma_{KL}^3 - \rho_{K1}\sigma_{K3}^1) / \theta_{L1}\theta_{K1} - \rho_{L1}\sigma_{L3}^1(K_1/K_2\theta_{K1} \\
    + L_1\rho_{K3}/L_2\theta_{L1}))T_{K3}/D_1
$$

(7)

$\sigma$ in the two-factor final-good specification is always positive. This complication is not pursued further in order to focus on the differences between the two capital taxes in this framework.
All the terms have been defined above, and $T_{K3}^*$ is simply the proportional change in $t_{K3}$. Any industry that uses $X_3$ will be a part of the taxed sector now because $t_{K3}$ will be carried to it. The three $\sigma$'s appearing in the numerator of (7) identify the margins of substitution directly or indirectly affected by $t_{K3}$. This is undoubtedly the most complex expression for $r^*$ in the entire paper, but it brings out the contrast between the two factor taxes in sharp relief. There are similarities too, for a relatively capital-intensive taxed sector ($A_1 > 0$) and ease of substitution between the taxed and untaxed inputs there (large values of $\sigma_{KL}^3$ and $\sigma_{L3}^1$ in this case) will still be detrimental to the interests of capital owners.

Some general results are worth noting before comparing these taxes in more detail.

Result 4.: Relative factor intensities are neither necessary nor sufficient to determine the direction in which the rental-wage ratio moves when a partial tax on capital income is levied in a pure-intermediate-good industry.

In expression (6), as well as in the 2 × 2 final goods model, unless the taxed industry (or sector) is relatively labor intensive, the rental-wage ratio will fall: $A_1 \geq 0$ is sufficient for $r^*$ to be negative, and it can be positive only if $A_1 < 0$. There are no necessary or sufficient factor-intensity conditions in (7) that will clinch the sign of $r^*$. It does not follow, however, that the direction of change in the rental-wage ratio cannot be determined without detailed numerical calculations. Some easily observed elasticity combinations can lead to definitive a priori results.

Result 5.: If the taxed sector is not relatively labor intensive, and it cannot substitute capital for the intermediate input, the rental-wage ratio will fall.

It is clear from (7) that if $A_1 \geq 0$ and $\sigma_{K3}^1 = 0$, $r^* < 0$. The tax will lead to an excess supply of capital unless capital directly employed in $X_1$, which is not subject to a tax now, can be easily substituted for the intermediate good. Actually, with $A_1 \geq 0$, equality of $\sigma_{K3}^1$ and $\sigma_{KL}^2$ will be sufficient to ensure a fall in the rental-wage ratio because these two elasticities work at cross purposes:
high values of $\sigma_{KL}^3$ contribute to a larger excess supply of capital, while a large $\sigma_{L3}^1$ will have the opposite effect.\footnote{This point is important in case of differential taxation when, for example, $X_1$ and $X_3$ are located in different (say, high and low) tax jurisdictions. However, if $t_{K1} = t_{K3}$ (both activities subject to the same cpt, for instance), the conflict between the roles of $\sigma_{KL}^3$ and $\sigma_{L3}^1$ disappears because all capital employed directly or indirectly by $X_1$ is subject to the same tax. In that situation, $A_1 \geq 0$ again will be sufficient to make $r^* < 0$, and the rental-wage ratio can rise only if the taxed sector ($X_1$ and $X_3$ together) is relatively labor intensive, as in the $2 \times 2$ fgo model. These propositions can be easily proved by adding together expressions (6) and (7), after setting $t_{K1} = t_{K3}$.} In (7), since $\rho_{K1} < 1$, the only potentially positive term in the numerator will be negative. A good illustration of Result 5 is in Column 4, Table 3. Since $X_3$ is being used exclusively by $X_2$, $A_1 > 0$, and in Rows 1 and 3, the elasticities of substitution in the taxed sector are equal, so $r^* < 0$.

3.1.4. $t_{K3}$ versus $t_{K1}$, further comparisons

The fact that $t_{K3}$ directly affects an intermediate-good producer whereas $t_{K1}$ is a tax on capital earnings in a final-good industry is at the heart of why these two taxes operate so differently. This angle is worth exploring further, apart from what is implied by Results 3 and 4, especially because of its relevance for the application in the next section. In this context, the most notable aspect of expression (7) is that its salient features cannot be inferred from (6), and that leads to a number of other implications:

1. Recall that since the denominator, $D_1$, is positive, the sign of $r^*$ is determined by the terms in the numerator. It is not affected by the two elasticities, $\sigma_{KL}^3$ and $\sigma_{L3}^1$ in (6), and by $\sigma_{KL}^4$ in (7). These $\sigma'$s flag the margins of substitution on which the two taxes operate. For $t_{K1}$, $\sigma_{KL}^4$ plays a crucial role in determining how much capital and labor will be released by the taxed industry in the first instance, and $\sigma_{KL}^3$ and $\sigma_{L3}^1$, appearing in the denominator of (6), merely affect the size of $r^*$. By contrast, for $t_{K3}$, $\sigma_{KL}^4$ takes over the role of $\sigma_{KL}^3$, relegating it to the denominator of (7), and since $X_3$ embodies taxed capital, substitution possibilities between labor and $X_3(\sigma_{L3}^1)$ also directly affect the excess supply of capital and hence the direction of change in the rental-wage ratio.
2. The elasticity of substitution between capital and $X_3$ in $X_1(\sigma_{K3}^1)$ matters for both taxes, but with opposite effects. For $t_{K1}$, a large value of this elasticity contributes a negative term to the numerator of (6) and will be detrimental to the interests of capital: Other things being equal, the larger is $\sigma_{K3}^1$, the greater will be the excess supply of capital, whereas for $t_{K3}$ the opposite effect will ensue because $K_1$ is untaxed capital in this case and $X_3$ embodies the taxed input. This point is reflected in the numerator of (7) where the term involving $\sigma_{K3}^1$ is positive, so it will tend to increase the rental-wage ratio.

3. Factor intensities have a similar effect on the incidence of the two taxes, and a capital-intensive taxed sector ($A_1 > 0$) will contribute to a lower rental-wage ratio. This, however, is not sufficient to give a negative sign to $r^*$ (Result 4); a restriction on $\sigma_{K3}^1$ may also be needed, as in Result 5, for this outcome.

Although expressions (6) and (7) pertain to the case in which $X_3$ is used exclusively in $X_1$, the underlying logic also applies to the case in which $X_2$ uses all of $X_3$. Therefore, instead of deriving more analytical expressions, we refer to the computations for $t_{K3}$ and an equal-yield $t_{K1}$ in Columns 4 and 5 of Table 3. The results differ dramatically: $r^*$ reverses signs twice, and three of the four $r^*$-values are not the same. The main reason for such striking changes in the computed $r^*$'s is that although the organization of production is the same in the two columns, in terms of the analytics of the model, when $t_{K3}$ alone is imposed, the taxed sector consists of $X_2$ and $X_3$, and its combined labor-capital ratio is higher than that of $X_1$. The taxed sector, in other words, is relatively labor intensive, but when $t_{K1}$ is the only tax in the system, the taxed industry is relatively capital intensive. With reference to the question posed at the beginning of this subsection, forecasting incidence results for one of these taxes from an analysis of the other can be quite misleading even in the same model with a given production structure. In particular, intuition trained by taxes such as $t_{K1}$ would miss much of the complexity caused by factor taxes levied in intermediate-good activities.

More generally, since $K$ and $L$ are interchangeable due to the symmetry postulate, the ultimate burden of a partial factor tax can be shared very differently,
depending on whether it is levied in a final-good \((X_1)\) or an intermediate-input industry \((X_3)\) even if the output of the latter is fully used up in producing \(X_1\). A similar conclusion is also reached when the models in Sections 2 and 3 are compared: the computed \(r^*\)'s in Columns 3 and 4 of Table 2 (the fgo case) differ sufficiently from those in Table 3 (the PIG version) to justify such an inference. Incidentally, \(t_{K1}\) and \(t_{K3}\) are now indistinguishable in terms of welfare cost, unlike the situation in Table 2.

It is strange that in spite of the prevalence of factor taxes in activities that produce intermediate goods and services, these taxes have been mostly considered in final-good settings in the analytical literature. One argument for focusing on final goods, although difficult to justify empirically, is that intermediate inputs might be used in fixed proportions, so that output in, say \(X_1\), can be defined as value-added, rather than in gross terms, and value-added is a fixed proportion of \(X_1\). That assumption will undoubtedly simplify all the expressions for \(r^*\), but the sharp differences between the incidence of taxes such as \(t_{K1}\) and \(t_{K3}\) highlighted above would remain. For instance, even if \(σ_{K3}^1\) and \(σ_{L3}^1\) are set equal to zero, expressions (6) and (7) will not become identical, although they will come closer and also have a common sufficient condition for \(r^*\) to be negative \((A_1 ≥ 0)\). Two different elasticities of substitution, \(σ_{KL}^1\) and \(σ_{KL}^2\), nonetheless will apply, and it is doubtful that computed \(r^*\)'s will be the same, except by chance.

4. TAXES ON INTERMEDIATE INPUTS: A STYLIZED APPLICATION

The PIG model and the results presented in the preceding section suggest a straightforward application to an analysis of some tax combinations mentioned in the Introduction, especially where differential factor taxes are combined with a vat regime. A standard feature of such regimes is that the value-added tax collected by intermediate-good producers will be credited to their customers even if they, in turn, do not charge any vat themselves, unless they are "exempt."\(^{10}\)

\(^{10}\)In vat usage, there is a distinction between these two types of activities: the former are "zero rated" while the latter are "exempt." For instance, in Canada, food is zero rated, so food
but there is no corresponding adjustment for factor taxes (cpi, payroll taxes) paid by them.

The \( r^* \)-expressions derived above were based on the stipulation that \( X_3 \) was an intermediate input in \( X_1 \). Now, for a somewhat different specification, let \( X_2 \) denote food which uses fertilizer \( (X_3) \) in production, and if \( X_1 \) represents a composite of other final goods, the logic of the analytical results presented in Section 3 will apply. The incidence and efficiency effects of a number of tax combinations then can be examined. For this purpose, three taxes, \( t_1 \), \( t_3 \), and \( t_{K3} \) in our notation, will be sufficient. They denote, respectively, a vat (or equivalently, an ad valorem output tax) on \( X_1 \) and \( X_3 \), and a tax on capital earnings in the intermediate activity.

Note first that since intermediate inputs are not needed for the production of \( X_1 \) and \( X_3 \), the output tax in these activities will be the same as a vat or an equal-rate tax on the incomes of the two primary factors, labor and capital. In \( X_2 \), a tax on value added can be specified as an equal rate tax on labor and capital directly employed there. Starting once again with the pre-tax equilibrium in Table 1 (no initial taxes), we shall consider several tax packages that yield the same total tax revenue.

The three taxes mentioned above are simultaneously introduced for the computations reported in Column 1, Table 4. There is no direct tax on \( X_2 \), although taxes levied in \( X_3 \) are being carried over to that industry. The data indicate that \( k_1 > k_2 \) in this situation, and in terms of the analytics of the PIG model, the taxed industry is comparatively capital intensive for \( t_1 \), but the taxed sector (combining \( X_2 \) and \( X_3 \)) is relatively labor intensive for the other two taxes. From the analytical results presented earlier, the rental-wage ratio should fall because of \( t_1 \), rise on account of \( t_3 \), and the effect of \( t_{K3} \) will be ambiguous in the most producers do not collect the gst from their customers, but they will get a credit for the gst paid on, say, their fertilizer purchases. Health service providers, however, are exempt. No gst is charged by them, nor can they claim a refund of the gst on their intermediate purchases. Taxes such as cpi or payroll taxes paid by intermediate-good producers nonetheless are not refunded. Selected real estate transactions in France and Germany, and residential rents in Sweden are additional examples of vat exemptions.
general case. The net incidence effect of this tax package, therefore, is difficult to predict beforehand. The CGE computations nonetheless show that the rental-wage ratio falls in two cases (when all elasticities of substitution are equal) and rises in the other two. The welfare cost is slightly larger than that of $t_{K3}$ alone (Column 4, Table 3).

**TABLE 4 HERE**

Turning to vat, for the computations in Column 2 of Table 4, $X_2$ is zero rated, which implies that food producers do not collect any tax on sales but get a refund of the vat paid on their purchases of $X_3$. In other words, when all is said and done, there are, in effect, only two taxes in the system — a five-percent levy on capital earnings in $X_3$, and a vat on $X_1$ — and the "equal-yield" rate for the latter turns out to be 0.014. On account of these taxes, the rental-wage ratio declines in every case, and in rows 1 and 3, where this was already happening in Column 1, the drop in $r/w$ is even bigger. Capital owners, therefore, bear the brunt of this move to a vat set-up. The efficiency cost of the vat regime, however, is lower than that of the taxes it replaces in three of the four cases considered.

Unlike the taxes in Column 1, the results stated earlier in the paper can explain some of these outcomes. For $t_{K3}$, a set of $r^*$-values has been presented already in Column 4 of Table 3. The value-added tax in $X_1$ is equivalent to an output tax here (no intermediate input in this industry), and since $k_1 > k_2$, there will be a downward pressure on the rental-wage ratio as Result 1 suggests. This is one reason why the computed $r^*$'s in Column 2, Table 4 are smaller than the $r^*$-values for $t_{K3}$ alone (Column 4, Table 3), even turning from positive to negative in two instances.

In this setting, if all three activities are treated alike for vat purposes, so that food producers also charge the same vat and do not get a tax refund, the vat rate can drop to 0.007 and still raise the same total revenue for the government. The results, summarized in Column 3, are better for owners of capital than the ones in Column 2. Scrapping $t_{K3}$ and subjecting every industry to a proportionately
higher vat (a rate of 0.009 will keep tax revenue constant) will lead to zero welfare cost and an unchanged rental-wage ratio. Labor and capital then will bear the tax burden in proportion to their initial contribution to national income.

4.1. Zero-rating Versus Vat Exemption

Vat exemptions are usually justified on administrative grounds (many small businesses or farms) or social welfare considerations (lower taxes on basic necessities, preferential treatment for low-income households), although pure political economy arguments also cannot be ruled out. Exemptions are sometimes accompanied by a series of deductions which have more or less the same effect as zero-rating (e.g. agriculture in France). Whatever the reason, exemption, when it is not zero-rating by another name, implies that exempt activities do not charge any vat on their sales, nor do they claim a refund of the vat paid on their purchases of intermediate inputs.

If $X_2$ is given an exempt status in lieu of its zero-rating in the present example, while retaining the other taxes, we are back in Column 1, Table 4. Capital owners will benefit from this switch because, compared to the computations in Column 2, the negative $r^*$'s are smaller in absolute value, and in two cases, the rental-wage ratio actually rises. The welfare cost also goes up in three of the four rows, so a vat-exemption for $X_2$ will not be a good idea from a social point of view.

It is worth noting, finally, that the production structure underlying all the computations in Table 4 is the same as in Column 4, Table 3 ($t_{K3} = 0.05$, $X_3$ used by $X_2$), but in every column of Table 4, at least three of the four $r^*$'s are different from those in the other table, and the level of the utility index is lower everywhere, indicating a greater welfare cost because of the additional taxes.

5. SUMMARY AND CONCLUSIONS

This paper has developed a general equilibrium tax model with three goods and two primary factors of production to compare and contrast with the standard two-by-two model and related three-factor-two-good specifications which have been
the workhorse of tax-incidence theory during the last three decades or so. The main contributions fall into three broad categories: First, the theoretical models developed here turn out to be rather different from their precursors in the Harberger literature in spite of similarities in dimension and other features, although many well-known tax-incidence propositions based on the two-by-two case carry over to the three-by-two fgo model. The expanded framework, understandably more complex, enables us to consider some tax settings and production structures which were beyond the pale of earlier analytical models. When a pure intermediate good is introduced, the symmetry property of the final-goods-only model breaks down, additional elasticities of substitution come into play, and configurations of parameters which were sufficient for generating certain outcomes in the fgo case cease to be so. Second, relying on a close blend of algebraic and numerical techniques, a series of analytical results are derived and illustrated for several theoretical specifications in a consistent manner from the same set of numbers. Earlier work has tended to relegate one or the other approach to a relatively minor role. Third, we have the new results, particularly notable for factor taxes in intermediate-good industries (profit taxes on mining companies, payroll taxes) which have not been incorporated into an analytical model before. The numerical example is extended to a vat regime which has zero rating, exemptions, and factor taxes, and the analytical results help in explaining several aspects of the numerical outcomes. While the framework can be applied to other situations also (environmental taxation, for example), the paper adds a general note of caution to discussions of tax policy, that developing priors about the incidence of various taxes from an analysis of final-good settings can be quite misleading when intermediate goods are present.
References


# TABLE 1: A NUMERICAL EXAMPLE

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<td>135</td>
<td>100</td>
<td>465</td>
<td>465</td>
</tr>
</tbody>
</table>

* $X_i$ denotes the output of the $i^{th}$ industry, household consumption and government spending are given by $(C + G)$, and $K$ and $L$ respectively denote the earnings of labor and capital. All prices are set to unity, so these numbers denote dollar values as well as physical quantities.
### TABLE 2: PROPORTIONAL CHANGE IN r/w DUE TO TAXES

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<th>(2)</th>
<th>(3)</th>
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<td>.003</td>
<td>-.032</td>
<td>-.004</td>
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<tr>
<td></td>
<td>(.99999)</td>
<td>(.99999)</td>
<td>(.9998)</td>
<td>(.9999)</td>
</tr>
<tr>
<td><strong>2. Taxed-industry - σ = 0.01, other σ's = 1.0</strong></td>
<td>-.005</td>
<td>.004</td>
<td>-.020</td>
<td>.004</td>
</tr>
<tr>
<td></td>
<td>(.99999)</td>
<td>(.99999)</td>
<td>(.9998)</td>
<td>(.9999)</td>
</tr>
<tr>
<td><strong>3. All σ = 0.5</strong></td>
<td>-.005</td>
<td>.006</td>
<td>-.037</td>
<td>-.0002</td>
</tr>
<tr>
<td></td>
<td>(.99999)</td>
<td>(.99999)</td>
<td>(.9998)</td>
<td>(.9999)</td>
</tr>
<tr>
<td><strong>4. Taxed-industry σ = 0.5, other σ's = 2.0</strong></td>
<td>-.003</td>
<td>.002</td>
<td>-.018</td>
<td>-.0001</td>
</tr>
<tr>
<td></td>
<td>(.99999)</td>
<td>(.99999)</td>
<td>(.9998)</td>
<td>(.9999)</td>
</tr>
</tbody>
</table>

*The elasticity of substitution in demand is assumed to be unity. σ is the elasticity of substitution between capital and labor in the ith industry.

**Col. 1:** One percent ad valorem tax on X₁.

**Col. 2:** One percent ad valorem tax on X₃.

**Col. 3:** Five percent tax on capital earnings in X₁.

**Col. 4:** Five percent tax on capital earnings in X₃.
**TABLE 3: PROPORTIONAL CHANGE IN r/w DUE TO TAXES**

<table>
<thead>
<tr>
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<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. All σ's = 1.0</td>
<td>.0001</td>
<td>.003</td>
<td>-.004</td>
<td>-.004</td>
<td>-.004</td>
</tr>
<tr>
<td></td>
<td>(.99999)</td>
<td>(.99999)</td>
<td>(.99999)</td>
<td>(.99999)</td>
<td>(.99999)</td>
</tr>
<tr>
<td>2. Taxed-industry σ's = .01, other σ's = 1</td>
<td>.0002</td>
<td>.004</td>
<td>.004</td>
<td>.004</td>
<td>-.002</td>
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<tr>
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<td>(.99999)</td>
<td>(.99999)</td>
<td>(.99999)</td>
<td>(.99999)</td>
<td>(.99999)</td>
</tr>
<tr>
<td>3. All σ's = 0.5</td>
<td>.0002</td>
<td>.003</td>
<td>-.004</td>
<td>-.002</td>
<td>-.005</td>
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<tr>
<td></td>
<td>(.99999)</td>
<td>(.99999)</td>
<td>(.99999)</td>
<td>(.99999)</td>
<td>(.99999)</td>
</tr>
<tr>
<td>4. Taxed industry σ's = 0.5, other σ's = 2.0</td>
<td>.0001</td>
<td>.004</td>
<td>.002</td>
<td>.001</td>
<td>-.002</td>
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<tr>
<td></td>
<td>(.99999)</td>
<td>(.99999)</td>
<td>(.99999)</td>
<td>(.99999)</td>
<td>(.99999)</td>
</tr>
</tbody>
</table>

*The elasticity of substitution in demand is assumed to be unity. σ^i is the elasticity of substitution between capital and labor in the i-th industry.

Col. 1: One percent ad valorem output tax on X_1 \cdot X_3 is an intermediate input in X_1.

Col. 2: One percent ad valorem output tax on X_3, and X_3 is used as an intermediate input in X_1.

Col. 3: Five percent tax on capital earnings in X_3, and X_3 is used as an intermediate input in X_1.

Col. 4: Five percent tax on capital earnings in X_3, and X_3 is used as an intermediate input in X_2.

Col. 5: Tax on capital earnings in X_1 at the rate of 0.0059 to raise the same revenue as in col. 4 and X_3 is used as an intermediate input in X_2.
### TABLE 4: PROPORTIONAL CHANGES IN r/w DUE TO TAXES: X_3 USED AS AN INTERMEDIATE GOOD IN X_2

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. All σ's = 1.0</strong></td>
<td>-.004</td>
<td>-.008</td>
<td>-.004</td>
</tr>
<tr>
<td></td>
<td>(.999993)</td>
<td>(.999995)</td>
<td>(.999995)</td>
</tr>
<tr>
<td><strong>2. Taxed-industry - σ = 0.01,</strong></td>
<td>.008</td>
<td>-.002</td>
<td>.007</td>
</tr>
<tr>
<td><strong>other σ's = 1.0</strong></td>
<td>(.999998)</td>
<td>(.999998)</td>
<td>(.999999)</td>
</tr>
<tr>
<td><strong>3. All σ's = 0.5</strong></td>
<td>-.004</td>
<td>-.010</td>
<td>-.003</td>
</tr>
<tr>
<td></td>
<td>(.999996)</td>
<td>(.999997)</td>
<td>(.999997)</td>
</tr>
<tr>
<td><strong>4. Taxed-industry σ = 0.5,</strong></td>
<td>.004</td>
<td>-.002</td>
<td>.003</td>
</tr>
<tr>
<td><strong>other σ's = 2.0</strong></td>
<td>(.999993)</td>
<td>(.999995)</td>
<td>(.999994)</td>
</tr>
</tbody>
</table>

**Col. 1:** One percent ad valorem tax on X_1 and X_3, and a five percent tax on capital earnings in X_3.

**2:** X_2 zero-rated for vat, "equal yield vat on X_1 (rate = .014) and a five percent tax on capital earnings in X_3.

**3:** "Equal yield" value-added tax (vat, rate = 0.007) in all activities, and a five percent tax on capital earnings in X_3.