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Information and Dynamic Adjustment in Life Insurance Markets

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Abstract

Genetic tests can be expected to produce a large amount of economically important information in the future. What are the effects on a life insurance market if more information becomes available over time, for individuals and possibly also for insurers? Should people buy insurance before or after becoming informed? How do earlier trades influence the market equilibrium in later periods? We also analyze the scope for a Pareto improving regulation of the insurance market.

Keywords: insurance, information, adverse selection.
JEL code: D8.

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1 Introduction

The information possessed by the insurer and by the insured has important implications on the resulting insurance market equilibrium. It is well known that under symmetric information, the risk allocation in the market equilibrium is optimal, while under asymmetric information, insurance opportunities tend to shrink, at least for some risk types, and equilibrium may cease to exist. However, the effects of the evolution of information; i.e., that individuals learn more about their risk type over time, have not been studied intensively. The fact that individuals have different information at different stages of their lives about their risk type as well as about other demand characteristics raises many interesting questions. When should an individual buy their insurance policy? Will consideration of future changes in risk type or demand characteristics affect the amount of insurance to buy now rather than later? How will earlier insurance purchases influence the insurance market at later times? And is there a possibility for a Pareto improving government intervention in the market? In this paper we analyze these questions in a life insurance framework.

A particularly important application for our results will be the new information which will be generated by expected widespread use of genetic testing in the medium term future. The impending completion of the Human Genome Project, which is a massive international effort to map and sequence the entire human genome, will accelerate recent successes in the discovery of disease genes and the development of associated genetic screening tests. This will lead to genetic screening tests becoming available at considerably lower costs, which will then be used more frequently and generate a large amount of economically important information.

The policy relevance of our analysis is clear from the fact that, at present, there are different regulatory approaches to the use of genetic information in insurance contracts. The insurance industries in France, the Netherlands and the UK have established a moratorium on using results from genetic screening tests. Several other countries, including Austria, Belgium and Norway, have regulations prohibiting the use of genetic information for ratemaking purposes, even if this information is provided voluntarily by an applicant. In Germany, on the other hand, there are no restrictions at all with respect to the use of genetic information by insurance companies. The primary responsibility for insurance regulation in the United States lies with individual states; therefore, a variety of regulations exists.\footnote{The information in this paragraph is based on Lemmens and Bahamin (1996).}

The aim of regulatory regimes which prohibit or at least inhibit the use of available
information for ratemaking purposes is usually to protect people who have, or who will likely have in the future, a very unpleasant illness, from also suffering financially from this illness. Prohibiting insurers from using such information, however, creates a situation of asymmetric information and related adverse selection costs. Thus, regulatory considerations face the competing goals of equity and efficiency. For the standard model of insurance markets,\(^2\) Rothschild and Stiglitz (1976) show that, in the case where a Nash equilibrium exists, allowing insurers to require customers to reveal information about their risk types, if this is possible, will lead to a Pareto improvement in welfare. In particular, the high risk types receive the same insurance coverage whether or not insurers are privy to the insureds information about risk type, while the low risk types are offered an efficient, full insurance, contract only in the case in which information about risk type is shared. If the proportion of high risk types in the insurance pool is below some critical level, a Nash equilibrium doesn't exist. However, for equilibrium concepts which allow for "nonmyopic" behaviour on the part of firms,\(^3\) Crocker and Snow (1986) show that allowing firms to use otherwise private information about insureds risk type always enhances efficiency, even in the case of imperfect information. Moreover, a tax-subsidy scheme can be set up to translate this efficiency gain into an actual (as opposed to hypothetical) Pareto improvement (see Crocker and Snow, 1985).\(^4\) These results may lead one to conclude that policies designed to restrict the use by firms of information about insureds risk type is ill-conceived. In this paper we show that appropriate regulation of the use of such information is substantially more complex when one takes into account the dynamic nature of information concerning both risk type and insurance needs.

An empirically observed feature of life insurance purchasing, which is difficult to understand in the traditional adverse selection model of insurance markets, is that most people do not purchase their life insurance at the first available opportunity in their lifetimes (i.e., at the time an individual can sign a legally binding contract).\(^5\) Waiting to purchase insurance only generates premium risk. If the information the individual receives is favorable, premiums may decrease below the average premium, but if the

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\(^2\) By the standard model we mean that introduced by Rothschild and Stiglitz (1976), which is characterized by: (i) a potential financial loss of fixed amount, (ii) a static or one period framework in which decisions are made, and (iii) no difference in tastes except as induced by differences in loss probabilities.

\(^3\) See Miyazaki (1977), Spence ( ), and Wilson (1977) for a description of these "nonmyopic" equilibria.

\(^4\) In the absence of such a tax-subsidy scheme, equity effects may be an issue (e.g., see Hoy, 1984).

\(^5\) Research based on a sample of 9,060 individual life insurance policies purchased in Canada during 1996 indicates that 48% of policies were purchased by individuals aged 35 years and over. (Canadian Life and Health Insurance Facts, 1998)
information is unfavorable, premiums will increase. A risk averse individual should wish to insure against this premium risk by buying the desired amount of insurance early in life, presumably as soon as one is legally able to contract for it, rather than wait until the information has arrived. This result is reinforced if the information remains private and there is adverse selection in the insurance market.

We analyze a model where people can buy life insurance at two different times in their lives. In the first period, all people are uninformed about two relevant characteristics: their risk type and their "demand type", where the latter type captures the idea that individuals may derive different utilities from the money they leave in case of their death. In the second period, individuals receive information about their risk and their demand type. There can be many reasons why individuals may not know at an early age their future insurance needs as this presumably depends on marital status, number of children, spouses earning capacity (where relevant), and other factors. We think that uncertainty about the demand type is an economically important reason why people postpone life insurance purchases. If people are not sure whether they actually need life insurance and if selling back insurance policies is problematic (in case one learns that one has no use for insurance), then it might make sense to wait with life insurance purchases nevertheless.

While we always assume that the demand type is unobservable for insurers, we analyze the cases of symmetric and asymmetric information concerning the risk type in the second period. Furthermore, we distinguish between the case that the individual can re-sell life insurance bought in period 1 for a fair price and the case where this is not possible.

Our main results are as follows: If individuals only receive information about their risk type in the second period, all insurance purchases are made in the first period in order to avoid the premium risk. If individuals only receive information about their demand type, it is useful to wait with buying insurance until the second period. If information about both risk types arrives in the second period, people buy some life insurance in the first period, and high demand types buy additional life insurance in the second period. If life insurance is a normal good and information about the risk type is symmetric, high demand–low risk types end up with the highest amount of life insurance.

Concerning the ex ante welfare of individuals, we derive a surprising result. It may be the case that all individuals prefer ex ante that there is asymmetric information in the second period rather than symmetric information. The reason is that asymmetric information effectively provides a kind of insurance against bad information outcomes,
since with asymmetric information, the average premium may be quite favorable for high risk individuals as compared to the risk type specific actuarially fair rate which they would have to pay under symmetric information. Asymmetric information is most likely to be better than symmetric information if the demand type risk is considerable and the second period information about the risk type is such that very few people receive very bad news while for most people the probability of death is barely changed in comparison with the ex ante state. At least until now, this scenario is quite realistic for the genetic information case; hence this model provides a possible rationale for regulations which limit the use of genetic information for ratemaking purposes.

Brugiavini (1993) develops a dynamic model of the annuity market. He shows that if only risk type information arrives in the second period, the full amount of annuities is bought in the first period, a result which is parallel to our Proposition 1. However, he does not analyze the interaction between risk type and demand type information and the welfare under different informational regimes, which are the key contributions of this paper. Other papers analyze the effects of changes in information among the actors of a life insurance market in a static setting. Doherty and Thistle (1996) analyze the value of information in a standard insurance model if insurers cannot use this information for ratemaking.6 The timing is such that individuals first decide whether to become informed or not and then the market for insurance is opened. In this setting, it is shown that individuals have a private incentive to acquire information about their risk type. However, if individuals had a choice when to buy their insurance, they would do this before becoming informed.

Hoy and Polborn (1999) analyze the welfare effects of additional information in a life insurance setting. They show that if the information is initially symmetrically distributed, then private information about the risk type has a negative social (and ex ante private) value. However, if there is initially asymmetric information, additional information for some individuals may be welfare improving or decreasing, depending on circumstances. Also this model is basically static; individuals have no choice whether to buy insurance before or after becoming informed. Although static models, such as those of Doherty and Thistle (1996) and Hoy and Polborn (1999), can explain the implications of changes in the availability and regulation of information in insurance markets from the perspective of individuals who face two possible information sets at some fixed point in time, this model considers explicitly the choice of individuals between buying their life insurance earlier or later in their life. In so doing, we do not

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6Among other papers which analyze the use of information about risk type in the standard, static insurance models are Crocker and Snow (1986), Ligon and Thistle (1996), Doherty and Posey (1998), Hoy (1984, 1989), and Tabarrock (1994).
artificially restrict when individuals can purchase insurance contracts in relation to the timing of the revelation of information.

In the next section, the model is presented. Section 3 contains the main results, and the last section concludes.

2 The model

There are infinitely many individuals in the economy who are ex ante homogeneous. Individuals live for at least two periods \((t = 1, \text{ also called } \textit{ex ante}, \text{ and } t = 2)\). At the beginning of the third period \((t = 3)\), individuals may die or may stay alive for this period; this is the only source of risk in the economy. All individuals living in \(t = 3\) will die at the end of the period. The reason of existence of the interim time period \(t = 2\) is that it allows us to model the effects of information which is revealed before the final state of the world in period 3 is known, but after an initial period in which "uninformed" insurance purchases can be made.

Initially (in \(t = 1\)), all individuals are endowed with a present value of wealth of \(y\); each period, individuals have to decide how much to spend for consumption in that period, for saving and for life insurance purchases. Individuals' utility function is

\[
u(C^1) + u(C^2) + pv(C^{3D}, \theta) + (1 - p)w(C^{3L})\tag{1}
\]

Here, \(C^t\) is consumption in period \(t, t = 1, 2\); \(u(\cdot)\) is the first and second period utility of consumption.\(^7\) In the third period, the individual is either dead or alive; if he is dead (this happens with probability \(p\), the utility from leaving money to people about whom the individual cares is described by the function \(v(\cdot)\); note that this utility does not only depend on the consumption in the death state \(C^{3D}\), but also on the "demand type" \(\theta\) of the individual; we will discuss this in more detail below. If the individual is alive in the third period (this happens with probability \(1 - p\)), the individual consumes \(C^{3L}\) and derives a utility of \(w(C^{3L})\); the utility function \(w(\cdot)\) may be thought of as incorporating third period consumption as well as a possible bequest motive. For all three utility functions, we assume that they are strictly concave in consumption.

We assume that there are two demand types; a proportion \(r_1\) of the population are low demand types \((\theta_1)\) while the rest \((r_2 = 1 - r_1)\) are high demand types \((\theta_2 > \theta_1)\), with a standard single crossing condition \(
\frac{\partial^2 v}{\partial \theta \partial C^{3D}} > 0
\): The marginal utility of money

\(^7\)Superscripts in variables denote the time (and possibly also the state of the world in period 3). Subscripts will indicate the individual's type.

\(^8\)It is assumed that the first and second period utility functions are equal in order to save on notation, but no qualitative result would change if they were different.
in the death state is increasing in the demand parameter $\theta$. We assume that at least the $\theta_2$ consumers have a strictly positive demand for life insurance if it is offered for a risk type specific fair premium. Ex ante each individual only knows that it will be a low demander with probability $r_1$ and a high demander with probability $r_2$. We assume throughout the paper that an individual’s demand type is his private information and cannot be credibly revealed to insurance companies.

Also, there are two risk types $p_L$ and $p_H$, with $p_L < p_H$. Ex ante, all individuals are identical; in period $t = 2$ a proportion $q_L$ learns that they are low risks and a proportion $q_H = 1 - q_L$ learns that they are high risks. We assume that the demand type and the risk type are statistically independent (i.e., the probability of being a low risk and low demand type is $q_Lr_1$ and so on). We will consider both the case in which the information about the risk type is private (and cannot be credibly revealed to an insurance company) and the case where the insurer and the individual have symmetric information about the risk type.

A major difference between “standard” insurance markets and the life insurance market is that in life insurance, an individual can buy several contracts from different insurers, and consequently insurers can effectively only offer contracts in which they fix the premium, but leave the determination of the quantity purchased to the consumer. This is in contrast to the Rothschild and Stiglitz (1976) approach to “standard” insurance markets in which insurers propose price–quantity contracts in order to separate risk types. As Hoy and Polborn (1999), have shown, this change in the model framework is important: The life insurance market is probably the most important insurance market influenced by recent developments in genetics and the enormous amounts of new information which will become available through this research; and the welfare conclusion from life insurance models may be different from those drawn from an analysis of standard insurance models.

3 Results

It is useful to start with an analysis of the two polar cases in which there is only uncertainty about the risk type $p$ or only uncertainty about the demand type $\theta$. In the third subsection, we will analyze the most interesting and difficult case where there is uncertainty about both the risk type and the demand type. Our general procedure is as

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9 If this assumption were not satisfied, there would be no demand for life insurance in equilibrium. Note however that we impose no restriction on the low demand types' demand for life insurance; this may be positive or zero.
follows: In order to analyze the individuals’ optimal decisions concerning consumption, saving and life insurance purchases, it is useful first to consider each type’s optimal decisions in the second period. If we plug in the optimal second period decisions in the second period objective functions (there are different solutions and different objective functions for each type), we get the type specific value functions; these depend on the first period’s consumption and life insurance purchases. We then use these value functions in order to calculate the optimal decisions in the first period.

3.1 Uncertainty only about the risk type

In this subsection, we assume that individuals already know their demand type (so \( \theta_1 = \theta_2 = \theta \)) and only receive information concerning their risk type in the second period. In the second period, an individual of type \( i \) faces the following optimization problem:

\[
\max_{C^2, S^2, L} u(C^2) + p_i(v(S^2 + L, \theta) + (1 - p_i)w(S^2))
\]

(2)

s.t. \( C^2 + S^2 + \pi_i(L - L^1) \leq y - C^1 - \bar{\pi}L^1 \)

(3)

The right hand side of (3) is the money which is still left in the second period: initial income \( y \) minus first period consumption \( C^1 \) minus expenditures for first period insurance purchases, \( \pi L^1 \), where \( \bar{\pi} \) is the proportional premium to be paid for life insurance bought in the first period. Since there is symmetric information in the first period, \( \bar{\pi} \) is the pooled fair premium: \( \bar{\pi} = q_{LPL} + q_{PH} \).

On the left hand side of (3), the available money can be spent for second period consumption \( C^2 \), for savings\(^{10}\) \( S^2 \) and expenditures for additional life insurance purchases. We denote \( L \) the total amount of life insurance (measured by the payoff in the death state) an individual buys; so \( L - L^1 \) is the additional cover purchased in period \( t = 2 \). Here, we do not constrain \( L \) to be greater or equal to \( L^1 \), so \( L < L^1 \) has to be interpreted as the individual selling insurance which was purchased in period 1; as there will be no sales of insurance cover in the optimum, this assumption is unproblematic here. The individual’s decisions in period 2 will result in a consumption of \( C^2 \) in period 2, of \( S^2 \) in the life state in period 3, and of \( S^2 + L \) in the death state.

\(^{10}\)We call the unconditional transfer of money “saving”; that is, a dollar which is “saved” in \( t = 2 \) is available for consumption in period \( t = 3 \) in the life state as well as in the death state. In order to keep notation simple, we assume that the interest rate is 0; our results are robust against changes in this assumption.

7
For further analysis, we have to distinguish two cases concerning the nature of information in the second period:

**Symmetric information:** $\pi_L = p_L$, $\pi_H = p_H$. In the case of symmetric information in the second period, the premium will reflect the buyer's risk type. The first order conditions for the second period optimization problem are (where in a slight abuse of notation $v'$ denotes the derivative of $v$ with respect to the first argument, consumption)

\begin{align*}
v'(C^2) - \lambda_i &= 0 \\
p_i v'(S^2 + L, \theta) + (1 - p_i) w'(S^2) - \lambda_i &= 0, \quad (5) \\
p_i v'(S^2 + L, \theta) - \lambda_i p_i &= 0. \quad (6)
\end{align*}

From this, we can conclude that in the optimum, the marginal utilities in $t = 2$, in the life state and in the death state are equal: $u'(C^2) = v'(S^2 + L, \theta) = w'(S^2)$. This equality, together with the budget constraint, determines the optimal allocation of the available money to second period consumption, saving and life insurance purchases.

The interesting question is now whether high risks or low risks buy more insurance. This relation is not a priori clear: For high risks, the death state is more probable ($p_H > p_L$) and the premium is higher ($\pi_H > \pi_L$).

Let $Z_i(y - C^1 + (\pi_i - \bar{\pi})L^1)$ denote the value function of the optimization problem given in (2) and (3). From the envelope theorem, we have $Z'_i = \lambda_i$. In the first period, the individual maximizes

\[ \max_{C^1, L^1} u(C^1) + q_L Z_L(y - C^1 + (\pi_L - \bar{\pi})L^1) + q_H Z_H(y - C^1 + (\pi_H - \bar{\pi})L^1) \quad (7) \]

The first order conditions for this problem are:

\begin{align*}
v'(C^1) - q_L Z'_L - q_H Z'_H &= 0, \quad (8) \\
q_L (\pi_L - \bar{\pi}) Z'_L + q_H (\pi_H - \bar{\pi}) Z'_H &= 0. \quad (9)
\end{align*}

Using $\bar{\pi} = q_{LP} + q_{HP}$, it follows from (9) that $Z'_L = Z'_H$ and hence $\lambda_L = \lambda_H$. Going back to the second period problem, it follows then that low risks' and high risks' second period consumption is equal, and so are their consumptions in the life state and in the death state in the third period. The only way this can happen is that $L_L = L_H = L^1$: The whole amount of insurance cover is already bought in $t = 1$, and neither risk type wants to buy (or sell) additional cover in $t = 2$.

The intuition for this result is as follows. Suppose first that in the optimum both risk types were to buy (possibly different amounts of) insurance in period 2. But then the high risk type has to spend more for additional insurance than the low risk
type; effectively, the high risk type is poorer than the low risk type, and hence his marginal utility is higher than that of the low risk type. But in this case, buying additional insurance in period 1 (for the average fair premium) could transfer income from the low marginal utility state to the high marginal utility state without altering the expected value of wealth and hence would be preferable.

An analogous argument excludes the case that both risk types sell insurance in the second period. So the only logically possible remaining case is that one risk type buys insurance in period 2, and the other one sells it. In this case, \( v' \), the marginal utility in the death state, would be higher for the selling type than for the buying type (by concavity); since optimality in the second period requires equality of marginal utility in all three states, \( u' = u' = u' \), we can conclude that the buying type would have to consume more than the selling type \textit{in all three states}. However, this is not possible: The money used by the one type to buy insurance will decrease either his second period consumption or his savings (possibly both), and the money earned from selling life insurance by the other type will either increase his second period consumption or his savings. Hence the case where both types neither buy nor sell in the second period must be the optimum.

From another point of view, the result is even more direct. If people were not to buy insurance in the first period, the information revealed in the second period would create a risk: If you are found to be a low risk, that's good (even financially, since it is now possible to buy life insurance cheaper), on the other hand, if you are found to be a high risk, that's bad. Buying insurance in the first period "insures" against this kind of good and bad news: A unit of insurance bought in the first period is worth more in the second period to an individual who turns out to be a high risk in the second period than to an individual who turns out to be a low risk. Buying all life insurance already in period 1 provides a full insurance against good or bad news in the second period.

\textbf{Asymmetric information:} \( \pi_L = \pi_H \). If information about the individual's risk type is private and cannot be shared with the insurer, or if the use of the risk type for tarification purposes is forbidden by law, then the premium the individual faces in the second period cannot depend on his risk type. In the market equilibrium, the premium will be equal to the "average clientele risk", which is a weighted average of the death probabilities of insurance buyers, where the weights take into account the amount of insurance bought by each type. The average clientele risk is generally higher than the average risk in the population, because high risks tend to buy more insurance under asymmetric information than low risks.\(^{11}\)

\(^{11}\)See Abel (1986), Hoy and Polborn (1999).
In any case, insurers will be able to distinguish whether an individual wants to buy or to sell insurance, and may charge or offer different prices for these two cases. Clearly, the equilibrium selling price (that is, the price for which an insurance company sells additional cover) as well as the the equilibrium buying price (the price consumers get in period 2 in case they sell cover back) must lie in the interval \([p_L; p_H]\). Otherwise insurers would either make a sure loss or a sure profit on second period transactions.

Suppose all individuals buy the same amount of insurance in period 1 as under symmetric information (the case analyzed above). Consider a high risk individual; we know from above that for a premium \(\pi_H = p_H\), this individual neither wants to buy nor to sell additional insurance in period 2. If the equilibrium selling price under asymmetric information is below \(p_H\), all high risk individuals would like to buy insurance. If the equilibrium selling price is between \(p_L\) and \(p_H\) (as it must be, as argued above), no low risk individual would want to buy additional insurance cover. However, if only high risks buy, then any selling price below \(p_H\) will lead to a loss for insurers; consequently, the only possible equilibrium selling price in period \(t = 2\) is \(p_H\). An analogous argument shows that the only possible equilibrium buying price in \(t = 2\) is \(p_L\). Given these prices, no individual in period 2 would like to buy or sell insurance cover. The allocation is the same as under symmetric information in period 2: All individuals buy their insurance cover already in period 1 in which there is symmetric information and do not recontract in period 2. In particular, high risks and low risks end up with the same amount of life insurance cover.

The intuition for this result is quite easy: Asymmetric information makes trading in the second period life insurance market less efficient than under symmetric information. Given that the second period market does not really exist even under symmetric information (in the sense that there are no trades in this market), it is no surprise that the second period market will also be non-existent under asymmetric information.

We gather our findings of this section in the following proposition:

**Proposition 1** If all individuals are identical in period \(t = 1\) and it is known that in period \(t = 2\) new information about the individuals’ risk type will arrive, then all individuals buy the complete amount of life insurance in period 1 (there is no recontracting in period 2); in particular, high risks and low risks end up with the same amount of life insurance cover.

These results are valid independent of whether insurance companies in period 2 can use the individual's risk type for ratemaking purposes (symmetric information) or not (asymmetric information).
Finally consider an individual who (for whatever reason) bought in period 1 more or less than the optimal amount of insurance. For this individual, the possibilities to recontract in period 2 are at least weakly worse than under symmetric information, and for a low risk individual who would like to buy additional insurance or for a high risk individual who would like to sell some of its insurance bought in the first period, the situation under asymmetric information is strictly worse than under symmetric information.

3.2 Uncertainty only about the demand type

In this subsection, all individuals have the same risk type (so \( p_L = p_H = p \)). In the second period, there is only information about the demand type \( \theta \). After receiving this information, an individual of demand type \( j \) faces the following optimization problem:

\[
\max_{C^2, S^2, L} u(C^2) + pv(S^2 + L, \theta_j) + (1 - p)w(S^2)
\]

s.t. \( C^2 + S^2 + \pi(L - L^1) \leq y - C^1 - \pi L^1 \)

(11)

Proposition 2

1. The final life insurance demand of type \( \theta_2 \) consumers is higher than that of type \( \theta_1 \) consumers.

2. For both types \( \theta_1 \) and \( \theta_2 \), the optimal final insurance demand is independent of the first period insurance purchases \( L^1 \).

Proof: We will prove the first claim in a more general setting in Proposition 3, so we omit it here. For the second claim, note that the first period and the second period price of insurance are equal, since the probability of death does not change. The first period life insurance purchases, \( L^1 \), cancel from the constraint (11), so the solution of problem (10), (11) does not depend on \( L^1 \) QED.

Intuitively, since the price of insurance does not change between period 1 and 2, buying life insurance in the first period is just another form of saving. Final life insurance demand only depends on the realization of the demand type (\( \theta_1 \) or \( \theta_2 \)) and the aggregate amount of resources available in the second period: endowment \( y \) minus first period consumption \( C^1 \).

In the last subsection, buying insurance in the first period was effectively an insurance against receiving unfavorable information in the second period. In the uncertain demand scenario, individuals would also like to buy "insurance" concerning the second period information. Here, they would like to have a contract which gives them more
money in the high demand state, since in this state they have the higher marginal utility. However, buying life insurance is no suitable instrument for transferring money between the different demand states, since the life risk is independent of the demand type risk.

The solution of problem (10), (11) just depends on overall savings $y - C^1$, but not on $L^1$. So there are many possibilities (different with respect to $L^1$) to implement the optimum. One possibility is not to buy insurance in the first period and then, in the second period, each demand type purchases his optimal life insurance demand. Another possibility is to buy in the first period the amount which is optimal for a high demand person and, if the person turns out to be a low demand type, he must sell the excess units of life insurance. Of course, there are also infinitely many other possibilities to implement the consumer's optimum.

However, if selling life insurance is impossible or at least the rate at which insurers buy back life insurance is worse than the rate for which they sell it, then it is clearly better not to buy in the first period (or at least not to buy more than the low demand types final insurance demand) and to wait for the information about the demand parameter to arrive in the second period. So we see that future information about their demand type will induce people to postpone their life insurance purchases until the information has arrived while future information about their risk type will induce people to buy before the information has arrived. We will study the trade-off between both effects in the presence of both kinds of uncertainty in the next subsection.

### 3.3 Uncertainty about $p$ and $\theta$

Before we proceed with the formal analysis, it is useful to discuss two institutional and informational features which have a big influence on the market outcome, namely whether the information about the risk type can be used in the second period for tarification (i.e. symmetric or asymmetric information) and whether life insurance which was bought in period 1 by the individual can be sold in period 2, and if yes, for which price.

**Symmetric vs. asymmetric information** In the case where only information about $p$ arrived in the second period, the effects of asymmetric information versus symmetric information in the second period were not too big, since all individuals bought the complete amount of insurance they needed in the first period, and there was no recontracting in period 2. However, if the individuals demand type is also uncertain, buying the complete amount of cover in the first period is impossible simply because it is unknown how large the demand is. So if both
types are uncertain, there could be transactions in the second period market, and the question whether information is symmetric or asymmetric there will become more important.

We would also like to stress that in the context of genetic information and its use for tariffation, there are two interpretations of "asymmetric information": The standard interpretation is that the individual knows his risk type, but it is not possible to communicate this risk type to the insurer, and therefore the insurance premium cannot depend on risk type. In the context of information generated by advances in genetic testing, this interpretation is not too convincing; after all, if an individual has a test result, it is technically quite easy to show this to the insurer.\textsuperscript{12}

Another interpretation of "asymmetric information" is that the transmission of information is technically possible, but it is prohibited by law to use this information for the purpose of determining the premium. As mentioned in the introduction, several countries have banned the use of genetic information for ratemaking purposes. The argument used for the introduction of this policy was a redistributive one: People who are terminally ill should not additionally suffer from much higher insurance premiums. In the classical Rothschild and Stiglitz (1976) model of adverse selection in insurance markets, an attempt to redistribute income from low risk to high risk people by prohibiting insurance companies from using available risk type information is bound to fail: Instead of the symmetric information outcome in which both risk types receive full cover for their risk type specific fair premiums, under asymmetric information high risk people still get a full cover contract for a high risk premium, while low risk people are constrained in the amount of cover they can buy. Hence, in the Rothschild and Stiglitz framework, a regulation banning the use of available information by insurance companies will not succeed in redistributing income from low risk to high risk individuals, but rather lead to a Pareto deterioration.

Can insurance be sold in the second period? In a frictionless market with perfect information, an individual is able to sell insurance for the same price for which it could buy further insurance. An individual will probably wish to sell if

\textsuperscript{12}The reader might wonder why the individual should show the test result to the insurer in case the test result is unfavorable to the insured. The answer is that if it is known that all individuals have acquired the information about their risk type, then if an individual does not show a favorable test result, the insurers will assume that this individual had an unfavorable test result.
it finds that it is a low demand type. For a price of $p_i$ per unit life insurance, the selling life insurance company would be indifferent to buying back the policy. However, the insurance company might also try to appropriate a part of the rent generated by the buy-back transaction, so the price might also be less favorable for the individual. For example, suppose that life insurance is absolutely useless for low demand types; then any positive price below $p_i$ will constitute a mutually favorable trade, and if the bargaining power of the insurance company is great, the price might be quite low. In reality, there might also be substantial transaction costs which make it quite unattractive to sell life insurance.

We capture this continuum of possible cases by considering the two polar cases: In the first one, all bargaining power is with the consumer and we say that life insurance is “saleable”: Under symmetric information, life insurance can be sold at the same price at which it can be bought by this consumer (i.e. a high risk consumer can sell his life insurance for a higher price than a low risk consumer); under asymmetric information, the buying and the selling price cannot be equal, but at least the insurance companies offer a price for buying back life insurance which makes a zero profit, given the average risk of those who want to sell their life insurance policies. We say that in this first case “insurance can be sold” (meaning, for a fair price).

In the second scenario (“life insurance is unsaleable”), transaction costs are so high that it is unattractive for all types to sell their life insurance bought in the first period. Alternatively, this case might be interpreted as the insurance company having a much higher bargaining power in the second period than the consumer. The truth is certainly somewhere between the two cases considered, although it is unclear whether nearer to the first or to the second case.

We will now start analyzing the four cases which result from the different combinations of the information environment (symmetric or asymmetric) and the resaleability. In the second period, an individual of type $ij$ faces the following optimization problem:

$$\max_{c^2, s^2, L} u(c^2) + p_i v(s^2 + L, \theta_j) + (1 - p_i) w(s^2)$$

s.t. $c^2 + s^2 + \pi_i(L - L^1) \leq y - C_1 - \pi L^1$

$^{13}$There may also be an influence of the risk type, but here there are two balancing effects: First, life insurance is more important for high risk individuals, but second, it is also more expensive. We will discuss this issue further below.
and if life insurance cannot be re-sold in period 2, we have as an additional constraint

\[ L \geq L^1 \]  

(14)

3.3.1 Symmetric Information, and Insurance is Saleable

It is easiest to start with the case where insurance can be sold, and under symmetric information; in this case, (12) and (13), with \( \pi_i = p_i \), define the second period problem and there are no further constraints to consider.

**Proposition 3** The optimal solution of problem (12), (13) and the optimal first period insurance purchases can be characterized as follows:

1. High demand individuals have a higher life insurance demand than low demand individuals of the same risk type: \( L_{1L} < L_{2L} \) and \( L_{1H} < L_{2H} \).

2. If \( L^1 > 0 \), at least one type buys and at least one type sells life insurance in the second period. More specifically, either type 2L or type 2H (or both) will buy and either type 1L or type 1H (or both) will sell.

3. Consumption in all three states \( (C^2, S^2, S^2 + L) \) is a normal good: \( \frac{\partial C^2}{\partial y} > 0 \), \( \frac{\partial S^2}{\partial y} > 0 \), and \( \frac{\partial (S^2 + L)}{\partial y} > 0 \). However, final life insurance demand \( L \) may be normal or inferior: \( \frac{\partial L}{\partial y} \leq 0 \).

4. Let \( \lambda_{ij} \) denote the Lagrange multiplier of the budget constraint in the problem (12), (13) of an individual of type \( ij \). If at zero first period insurance purchases, the expected marginal utility of a low risk individual in the second period, \( r_1 \lambda_{1L} + r_2 \lambda_{2L} \), is greater than the expected marginal utility of a high risk individual in the second period, \( r_1 \lambda_{1H} + r_2 \lambda_{2H} \), \( L^1 = 0 \) is optimal. Otherwise, \( L^1 \) is chosen such that \( r_1 \lambda_{1L} + r_2 \lambda_{2L} = r_1 \lambda_{1H} + r_2 \lambda_{2H} \).

5. If \( L^1 > 0 \) and if life insurance is a normal good, then \( L_{1L} < L_{1H} < L^1 < L_{2H} < L_{2L} \): Low demand types sell insurance in the second period, and low demand low risk types sell more than low demand high risk types; high demand low risk types buy more insurance in the second period than high demand high risk types.

If \( L^1 = 0 \) and if life insurance is a normal good, then \( L_{1L} = L_{1H} = L^1 = 0 < L_{2H} < L_{2L} \).

If \( L^1 > 0 \) and if life insurance is an inferior good, then \( L_{1H} < L_{1L} < L^1 < L_{2L} < L_{2H} \): Low demand types sell insurance in the second period, and low demand high risk types sell more than low demand low risk types; high demand high risk types
buy more insurance in the second period than high demand low risk types.
If \( L^1 = 0 \) and if life insurance is an inferior good, then \( L_{1L} = L_{1H} = L^1 = 0 < \)
\( L_{2L} < L_{2H} \).

The proof of this and the following propositions can be found in the appendix. We
will concentrate on the intuition here. As for the first claim, a higher \( \theta \) means a higher
marginal utility in the death state. Consequently, an individual with a high \( \theta \) should
transfer more resources to this state (by buying life insurance) than a low \( \theta \) individual.

For the second claim, the same intuition applies as already given in the subsection
about uncertainty about \( p \) only: If all types were to buy insurance in the second period,
then individuals could decrease their risk (without changing the expected value of their
wealth) by buying more insurance in the first period. Since they are risk averse, they
will benefit from that. An analogous argument applies if all types were to sell insurance
in the second period, which consequently cannot be optimal, too. As final insurance
demand varies at least between low and high demanders, some have to sell and some
have to buy in the second period if \( L^1 > 0 \). If \( L^1 = 0,^{14} \) then there is of course no one
selling insurance in period 2.

In a separable utility function like (12), all three state consumptions must be normal
goods; higher income \( y \) means a higher consumption in all three states. Note however
that this does not imply that life insurance is a normal good, since life insurance is
only a part of death state consumption; if savings increase sufficiently with income, life
insurance purchases might actually decrease with income. A precise condition when
this happens can be found in the appendix.

The fourth claim is a generalization of the result from the case where only \( p \) was
uncertain: the marginal utility of low risk and of high risk individuals is equal.\(^{15} \) No
insurance is possible against the demand type risk; high demand types will have a
higher marginal utility than low demand types of the same risk type. Therefore, the
best thing which can be done in period 1 is to buy insurance in the first period such
that the expected marginal utilities of high risks and of low risks in the second period
are equal, where the expectation is taken with respect to the demand type risk.

Since individuals buy a medium amount of insurance in period 1 and high demand
individuals will increase their insurance holdings in period 2, an income effect arises:
High risks have to pay more for additional insurance than low risks; hence high demand

---

\(^{14}\)Our numerical example later on shows that this may indeed happen if low demanders have a zero
final insurance demand.

\(^{15}\)This is valid at least if the optimal solution for \( L^1 \) is an interior one. The condition stated in
the proposition when it is optimal not to buy insurance in the first period is the usual Kuhn Tucker
condition. We will discuss this case in more detail at the numerical example below.
low risk consumers are effectively more wealthy than high demand high risk consumers. If life insurance is a normal good, then high demand low risk consumers will end up with more insurance than high demand high risk consumers;\(^{16}\) if insurance is an inferior good, then it is the other way around.

Low demand types are selling insurance in the second period; high risk types can sell for a higher price than low risk types, and consequently they are effectively richer. Then, if life insurance is a normal good, low demand high risk consumers keep more insurance cover (i.e., they sell less) than low demand low risk individuals. If life insurance is an inferior good, then the relationship is the other way around.

3.3.2 Symmetric Information, but Insurance cannot be Sold

In the case that insurance cannot be sold in the second period, the second period optimization problem for an individual of type \(ij\) is given by (12), (13) and the additional constraint (14). Consider an individual who bought as much insurance in period 1 as in the case where insurance can be sold; if this individual turns out to be a low demand type, Proposition 3 implies that the constraint (14) is binding for this individual.

Since now there is no possibility to get rid of unwanted insurance in period 2, it seems very reasonable to decrease first period insurance purchases. Although this reduces the insurance against bad news in the second period, it increases flexibility in the second period and reduces the waste too much insurance means for the low demand type.

**Proposition 4** Let \(L^{1*}\) denote the optimal first period insurance purchases if insurance can be sold in period 2. If insurance cannot be sold in period 2, ex ante expected utility increases if first period insurance purchases are reduced below \(L^{1*}\).

Again, the proof is in the Appendix.

3.3.3 Asymmetric Information, Insurance cannot be Sold

In the case that information about the risk type cannot be used for tarification, in the second period the proportional premium is equal for high risks and low risks. If insurance cannot be sold in the second period, the second period optimization problem for an individual of type \(ij\) is given by (12), (13) and (14), where \(\pi_L = \pi_H = \pi^2\).

\(^{16}\) We should emphasize here that we measure life insurance by the amount which is paid out in the death state; even if this amount is higher for low risks than for high risks, it is well possible that the value of the life insurance holdings (measured by the amount one would have to spend in order to buy it in period 2) is higher for high risks.
In the equilibrium, \( \pi^2 \) is determined such that insurers earn zero profit on life insurance purchases made in period 2:

\[
\pi^2 = \frac{[r_1 q_L(L_{1L} - L^1) + r_2 q_L(L_{2L} - L^1)]p_L + [r_1 q_H(L_{1H} - L^1) + r_2 q_H(L_{2H} - L^1)]p_H}{[r_1 q_L(L_{1L} - L^1) + r_2 q_L(L_{2L} - L^1)] + [r_1 q_H(L_{1H} - L^1) + r_2 q_H(L_{2H} - L^1)]},
\]

where \( L_{ij} \), the final life insurance demand of a type \( ij \) individual, depends of course on the price of insurance, \( \pi^2 \). The right hand side is known in the literature as “average clientele risk”: It is a weighted average of the death probabilities \( p_L \) and \( p_H \), where the weights are the quantities of insurance bought by each type. It is easy to see that high risk individuals have a higher insurance demand than low risk individuals; consequently, the average clientele risk of life insurance buyers is higher than the average risk of the whole population (where the weights are the proportions of each type in the population). The existence of a solution to (15) is guaranteed since the right hand side of (15) is continuous in \( \pi^2 \) and for \( \pi^2 = p_L \), the right hand side is greater or equal to \( p_L \) and for \( \pi^2 = p_H \), the right hand side is lower or equal to \( p_H \); hence there must exist at least one \( \pi^2 \) between \( p_L \) and \( p_H \) such that the left hand side and the right hand side of (15) are equal. Uniqueness of the equilibrium is not guaranteed; however, it is argued in Hoy and Polborn (1999) that, if multiple solutions to (15) exist, the “reasonable” equilibrium premium is the smallest \( \pi^2 \) which satisfies (15): For all other “equilibrium” premiums, a single insurer could deviate and charge a premium which is slightly higher than the lowest possible equilibrium premium, attract all customers and make a profit; so only the smallest fixed point of (15) can be a Nash equilibrium in a full fledged version of the game between insurance companies.

**Proposition 5** If insurance cannot be re-sold by consumers in the second period and if there is asymmetric information in the second period insurance market, the equilibrium has the following properties:

1. \( L_{1L} \leq \min(L_{1H}, L_{2L}) \leq \max(L_{1H}, L_{2L}) \leq L_{2H} \): Type 1L has the smallest final life insurance demand, type 2H has the greatest final life insurance demand and the other two types cannot be ranked in general.

2. The optimal first period insurance purchases, \( L^1 \), are chosen such that type 1L does not buy insurance in the second period and type 2H buys additional insurance.

3. There exists \( \epsilon > 0 \) such that if \( |\partial L_{ij}/\partial y| < \epsilon \) and \( r_1 L_{1H} + r_2 L_{2H} > L_{2L} \), then an increase in \( L^1 \) leads to a higher equilibrium premium in the second period.
The proof can be found in the appendix.

Claim 1 of proposition 5 is very intuitive: As in proposition 3, high demand individuals have a higher life insurance demand than low demand individuals; high risk individuals have a higher insurance demand than low risk individuals (of the same demand type), since all consumers pay the same premium and the death state is more probable for high risk consumers. The following example shows that the final life insurance demand of type $1H$ and of type $2L$ cannot be ranked in general: Suppose $v(S_2 + L, \theta)$ can be written as $\theta v(S_2 + L)$; then type $1H$ has a higher expected marginal utility from a unit of money transferred to the death state than type $2L$ if and only if $\theta_1 p_H > \theta_2 p_L$, and since the budget constraint is equal for both types, type $1H$'s final life insurance demand is greater than type $2L$'s final life insurance demand if and only if this inequality holds. Since $\theta_2 > \theta_1$ and $p_L < p_H$, the inequality can, but need not hold.

For claim 2, suppose that all types were buying insurance in period 2. In proposition 3, we argued that then increased life insurance purchases could reduce the individual's risk of receiving bad news about his health status. Here, this risk is already reduced by the asymmetric information in period 2: High risks will not have to pay a higher premium for their additional life insurance purchases in period 2 than low risks. However, the second period life insurance market is subject to adverse selection, since high risk individuals have a higher life insurance demand than low risk individuals of the same demand type. Therefore, the premium which has to be paid in period 2 is higher than the average fair premium which has to be paid in period 1. If all individuals are certain that they would have to buy additional life insurance in period 2, buying it now dominates saving the money in period 1 and buying in period 2, a contradiction. On the other hand, it cannot be true that all individuals want to sell life insurance in period 2, since then it would be strictly better to buy less life insurance in the first period.

The conditions in claim 3 means that the conclusion can be derived if high risk individuals on average buy more insurance than high demand low risk individuals and income effects are not too big; it is only a sufficient condition which is probably stronger than needed. The intuition for the premium increase in the second period is as follows: If income effects are small, increasing life insurance purchases in period 1 reduce the life insurance demand in period 2 about 1:1 (i.e., final life insurance demand remains constant, and since more was bought in the first period, less needs to be bought in the second period). The proportional reduction is higher for individuals who turn out to be low risks than for those who turn out to be high risks: If final life insurance demand
were 10 for high risks and 5 for low risks, then buying 1 unit of life insurance in period 1 reduces the high risks’ second period demand to 9 (i.e. by 10%), but the low risks’ second period demand to 4 (i.e. by 20%). Hence the demand weighted second period “average clientele risk” is higher in this case than when no life insurance at all is bought in period 1.

We think that it is plausible that first period life insurance purchases lead to more adverse selection in the second period. If this is the case, it has interesting policy implications, since then taxing or otherwise discouraging first period life insurance purchases in order to decrease adverse selection in the insurance market in the second period would be welfare improving: Marginally, the welfare loss from consumers not buying the preferred amount of life insurance in the first period is a second order effect; however, the welfare gain from decreased adverse selection in the second period is a first order effect.

3.3.4 Asymmetric Information, Insurance can be Sold

In comparison to the last case, individuals here have the possibility to sell insurance in the second period. Information is asymmetric in the second period, but insurers can recognize whether an individual wants to buy further life insurance or wants to sell the life insurance; since the average risk of those individuals who wish to buy is possibly different from the average risk of those who wish to sell, the buying and the selling price of insurance may be different. More specifically, the price at which insurers sell further insurance is given by the solution of

\[
\pi^{2S} = \frac{[r_1 q_L (\tilde{L}_{1L} - L^1) + r_2 q_L (\tilde{L}_{2L} - L^1)] p_L + [r_1 q_H (\tilde{L}_{1H} - L^1) + r_2 q_H (\tilde{L}_{2H} - L^1)] p_H}{[r_1 q_L (\tilde{L}_{1L} - L^1) + r_2 q_L (\tilde{L}_{2L} - L^1)] + [r_1 q_H (\tilde{L}_{1H} - L^1) + r_2 q_H (\tilde{L}_{2H} - L^1)]},
\] (16)

where \( \tilde{L}_{ij} = \max(L^1, L_{ij}) \); so \( \tilde{L}_{ij} - L^1 \) is the effective demand of an individual on the second period market, given that additional insurance is sold at price \( \pi^{2S} \).

The price at which insurers buy back insurance from consumers is given by the solution of

\[
\pi^{2B} = \frac{[r_1 q_L (L^1 - \hat{L}_{1L}) + r_2 q_L (L^1 - \hat{L}_{2L})] p_L + [r_1 q_H (L^1 - \hat{L}_{1H}) + r_2 q_H (L^1 - \hat{L}_{2H})] p_H}{[r_1 q_L (L^1 - \hat{L}_{1L}) + r_2 q_L (L^1 - \hat{L}_{2L})] + [r_1 q_H (L^1 - \hat{L}_{1H}) + r_2 q_H (L^1 - \hat{L}_{2H})]},
\] (17)

where \( \hat{L}_{ij} = \min(L^1, L_{ij}) \); so \( L^1 - \hat{L}_{ij} \) is the effective amount of life insurance an individual of type \( ij \) would like to sell on the second period market, given that he gets a price for his policy of \( \pi^{2B} \).
It is shown in Villeneuve (1999) in a similar model\textsuperscript{17} that $\pi^{2B} < \bar{\pi} < \pi^{2S}$. The intuitive reason is that a disproportionate share of life insurance buyers will be high risks (therefore, the average risk of life insurance buyers will be higher than the population average), and among individuals who sell life insurance, a disproportionate share will be low risks, since high risks will be less inclined to sell their life insurance stocks. In the context of a continuum of possible types, Villeneuve (1999) shows that there will be in general some types who are inactive (neither buying a positive amount of insurance nor selling a positive amount of insurance). The reason is the wedge between the buying and the selling price; for some individuals, the price at which they could buy additional insurance may be too high to make this worthwhile, but at the same time, the price for re-selling insurance may be considerably lower, and thus it may also not be worthwhile for them to sell the life insurance they have.

The existing literature is static and so takes as given the insurance people already have (if any) before trading on the insurance market under adverse selection. However, the possibility to trade in the first period, before adverse selection can occur, has a surprising influence: As is shown in the numerical example in the next section, we can have an “overshooting” in the first period which means that first period insurance purchases are higher than the amount high demand individuals would buy if they faced a fair price; in the second period, type $2H$ holds on to his first period purchase while all other types sell at least some insurance back. Generally, we have:

\textbf{Proposition 6} If insurance can be re-sold by consumers in the second period and if there is asymmetric information in the second period insurance market, the equilibrium has the following properties:

1. $L_{1L} \leq \min(L_{1H}, L_{2L}) \leq \max(L_{1H}, L_{2L}) \leq L_{2H}$: Type $1L$ has the smallest final life insurance demand, type $2H$ has the greatest life insurance demand and the other two types cannot be ranked in general.

2. The optimal first period insurance purchases, $L^{1}$, are chosen such that type $1L$ does not buy additional insurance in the second period and type $2H$ does not sell insurance.

3. Suppose that $\frac{\partial w(S)}{\partial L_{2L}} < w'(S)$ for all $S$. In the second period, types $1L$, $1H$ and $2L$ sell insurance, and type $2H$ either buys additional insurance or is inactive.

\textsuperscript{17}Villeneuve's model is a one period model of the life insurance and annuity market, where individuals just differ in their risk type (so there is no demand type)
The proof for the first two claims of Proposition 6 is almost completely parallel to the proof of proposition 5 and therefore omitted. The intuition for the third claim is as follows: The condition given is a sufficient condition that in equilibrium both low demand types sell all of their insurance in the second period. Suppose that type 2L were not to sell a positive amount of insurance. Then the second period buying price equals the population average fair premium, $\pi^{2B} = \bar{\pi}$. Due to adverse selection among high demand types, $\pi^{2S} > \bar{\pi}$. But for these second period prices, it is always attractive to buy more insurance in the first period: If the individual turns out to be a low demand type, the additional insurance bought can be sold again, at no loss in comparison to saving; however, if the individual turns out to be a high demand type, it is strictly better to have the additional insurance. Therefore, this cannot be an equilibrium. Buying more insurance in period 1 just becomes unattractive if the second period buying price $\pi^{2B}$ is lower than $\bar{\pi}$, and consequently 2L types have to sell some of their first period purchases.

Note that if there are only very few high risk individuals (which is a correct assumption in the context of genetic information, for example), then this implies that we will see almost everyone in the economy selling life insurance in the second period.

### 3.4 Numerical Example

Our numerical example takes the following form:\footnote{Details of the computation are available from the author upon request.} Let the utility function be

$$\ln(C^1) + \ln(C^2) + p\theta \ln(S^2 + L - k) + (1 - p) \ln(S^2).$$

The following table provides some information on the optimal choices of individuals for the four different institutional settings for the following parameter constellation: $\theta_1 = 0, \theta_2 = 1, r_0 = 1/2, r_1 = 1/2, p_L = 0.0496, p_H = 0.4496, q_L = 0.999, q_H = 0.001, y = 1, k = 1/2$.\footnote{The parametrization $\theta_1 = 0$ simplifies calculations considerably since it is obvious that all low demand types will buy the lowest amount of insurance feasible, independent of their risk type. While this is of course an extreme case, it is probably not unrealistic to assume that there are some people who would not derive any positive (marginal) utility from a life insurance policy.} So in this example, only few individuals, the high risk types, have a large shift in their probability of death while for the low risks, the a priori probability is almost unchanged. Although we do not claim that these parameters depict a particular real life situation, the fact that there are only very few high risk individuals seems quite reasonable in the context of genetic testing, since only very few individuals have serious genetic defects influencing considerably their life expectancy.
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</tr>
</tbody>
</table>

If insurance is sellable and information is symmetric, it is optimal to buy some insurance in the first period ($L^1 = 0.1675$). In the second period, low demand types sell their insurance and high demand types buy additional insurance; note that, among high demand types, the final insurance demand of both risk types is equal; this is because there are no income effects for the utility function considered. Premiums in the second period are the risk type dependent fair premiums.

If insurance is not sellable and information is symmetric, the optimal first period insurance purchases are 0. Both high demand types have the same final life insurance demand, again due to the absence of income effects. In both cases with symmetric information, the high risk types consume considerably less than the low risk types in all three states (period 2, the life and the death state), since they have to spend more on insurance.

If insurance is sellable and information is asymmetric, the optimal first period insurance purchases are considerably larger than the amount high demand types buy under symmetric information ($L^1 = 0.8114$). High demand high risk types keep all their first period insurance purchases while all other types sell; low demand types sell everything, and high demand types keep a little less than 0.5.20

What is the reason for this apparently strange "overshooting" behavior (buying so much insurance in the first period that over 99.9% of the population sell again in period 2)? Suppose to the contrary that $L^1$ was such that only low demand types sold. Then the buying price would be 0.05. But in this case, it is attractive to buy more insurance in the first period: If one turns out to be a high demand type, each unit of insurance is worth more than 0.05 since the second period selling price will be higher that that, due to adverse selection among second period life insurance buyers; if one turns out to be a low demand type, nothing is lost in comparison to saving. Hence this cannot

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20The reason is that low demand types face a second period price which is slightly higher than their probability of death.
be an equilibrium. The second period buying price must be smaller than 0.05, and consequently first period life insurance purchases must be such that high demand low risk types sell some of their first period purchases in equilibrium.

If insurance is not sellable and information is asymmetric, the optimal first period insurance purchases are 0. Adverse selection in the second period drives up the premium to 0.052137. The reason is that high risks buy much more insurance than low risks (3.08 vs. 0.48).

We now turn to the question of comparing the welfare of consumers in the different informational and institutional regimes. Our welfare measure is the individuals’ ex ante expected utility under the different regimes; since all individuals are equal in $t = 1$, this is the reasonable welfare measure. Intuitively, the situation in which there is symmetric information in the second period and insurance can be sold by consumers, is closest to the case of a perfect market: Individuals can buy insurance in both periods, for a fair (risk type dependent) price; if individuals find in period 2 that they don’t need the insurance they bought in period 1, they can sell it, again for a fair (risk type specific) price. The only risk which cannot be insured is the demand type risk, but this is not possible in one of the other 3 scenario, either.

On the other hand, the asymmetric information case in which insurance cannot be sold in period 2 any more, seems to be least efficient, first due to adverse selection in the second period market, and second because possibly low demand types are stuck with insurance they do not really need. It is therefore quite surprising that it is possible that, from an ex ante point of view, all individuals prefer the situation in which information is asymmetric and insurance cannot be sold to those with symmetric information, whether insurance can or cannot be sold.

The following table provides information on the relative ranking of the four different cases for different levels of $k$ (that is, different utility functions). We take the symmetric information sellability case as benchmark and give the equivalent variation for the other 3 scenarios for an initial income of 1,000,000$. For example, the –2 in the first line means that taking 2 $ from an individual in the benchmark case hurts this individual as much as having asymmetric information instead of symmetric information in the second period (but keeping sellability of insurance). A positive number means that the respective scenario is better for individuals ex ante.
<table>
<thead>
<tr>
<th></th>
<th>symmetric info, saleable</th>
<th>symmetric info not saleable</th>
<th>asymmetric info, saleable</th>
<th>asymmetric info not saleable</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 1/2$</td>
<td>n.a.</td>
<td>-4</td>
<td>-2</td>
<td>-415</td>
</tr>
<tr>
<td>$k = 1$</td>
<td>n.a.</td>
<td>-310</td>
<td>+23</td>
<td>-409</td>
</tr>
<tr>
<td>$k = 4$</td>
<td>n.a.</td>
<td>-69000</td>
<td>+351</td>
<td>+380</td>
</tr>
</tbody>
</table>

It is clear that the utility level attainable under symmetric information and non-sellability is (weakly) lower than that under symmetric information and sellability because simply a further constraint is added. For $k = 1$ and $k = 4$, asymmetric information and sellability is better than the benchmark case, and for $k = 4$, asymmetric information and non-sellability is the best scenario for individuals ex ante.

This result is a typical second best result in the tradition of Hart (1975). Markets in the benchmark scenario are incomplete since the demand type risk cannot be insured. Adding a further market imperfection may then benefit all individuals. Also, for $k = 4$, the possibility to sell unwanted insurance in the second period decreases ex ante welfare although all individuals trade voluntarily.

Why could all individuals prefer asymmetric information in the second period? Let us first compare the symmetric information–sellability benchmark with the case of asymmetric information and non–sellability. Asymmetric information effectively provides a kind of insurance against being a high risk type, as high risks can buy insurance for a very low price (in comparison to their risk, and in comparison to the price they would have to pay under symmetric information. The disadvantage of asymmetric information is that there is adverse selection and hence the premium in the second period is higher than the average fair premium. However, if there are only few high risks and if the utility function is such that high risks insurance demand is not much higher than that of low risks, then there is not much adverse selection and premiums are not much higher in the second period than the average fair premium in the first period; in the limit case, they are equal, so there is no disadvantage of waiting with life insurance purchases.

On the other hand, if there is symmetric information, the individual could buy no (or not enough) insurance in the first period; then there is a considerable (premium) risk to be found a high risk individual. The individual could also buy the amount of insurance which would be optimal for high demand individuals at a fair price; this eliminates the premium risk, but has another disadvantage: Among low demand types, low risks have a higher marginal utility, since they have more possibilities to spend the

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21 This is certainly possible since the utility function could become very flat at values slightly larger than the optimum of the low risks.
money. Buying insurance in the first period under symmetric information amounts to transferring money from the low risk state to the high risk state, but for low demand types, this is the wrong direction. Both options under symmetric information are strictly worse than waiting and seeing whether one needs insurance under asymmetric information, if adverse selection in the second period market is sufficiently small.

Under asymmetric information and sellability of insurance, the market equilibrium is completely different than under non-sellability: Individuals buy a large amount of insurance in the first period, and almost all of them are selling in the second period market. However, the basic advantage of this scenario as compared to symmetric information is the same as above: High risk high demand types are better off than under symmetric information, since they simply can keep the amount of insurance which they bought in the first period for a fair price. The basic problem in this scenario is that high demand low risk types spoil the second period buying price by selling some of their insurance. Individuals do not take this externality into account when deciding on how much insurance to buy in the first period; from a social point of view, they buy too much in the first period. So a tax on first period insurance purchases could help improve welfare in this scenario.

4 Conclusion

This paper analyzed a dynamic model of the life insurance market, where individuals can choose when to buy life insurance. If they do it early in their life, the advantage is that they know nothing about their risk type and hence can buy insurance for a fair average premium. If they wait and information in the second period is symmetric, the premium is risky (lower for those with good news about their death probability, but higher for those with bad news); if information in the second period is asymmetric, there is adverse selection in the market and hence the premium is higher than in the first period. The advantage of waiting until the second period is that people also learn whether they really need life insurance since they have to insure the well being of a spouse and/or children, or whether life insurance has no use for them.

Concerning the ex ante welfare of individuals, we have shown that it may be the case that all individuals prefer ex ante that there is asymmetric information in the second period rather than symmetric information. The reason is that asymmetric information effectively provides a kind of insurance against bad genetic information. However, if too

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22To see this, just substitute $\theta = 0$ and $p = 0$ or $p = 1$ in the utility function. Then "high risks" must spend all their money in the second period, which means that they have a low marginal utility as compared to "low risks" who can spread their consumption over two periods.
many high risks are in the market in the second period, then adverse selection becomes a severe problem and can drive premiums so high that no low risks buy. If this is the case, it is better to have symmetric information in the second period.

At present, only few people have taken a genetic test, so at present we may well be in a situation where a ban on using this kind of information for insurance purposes could be reasonable. However, as genetic testing will become more and more widespread, it becomes more probable that a ban on using genetic tests as a basis for pricing life insurance would only lead to an elimination of the insurance market.

A simplifying assumption in this paper was that ex ante all individuals were equally uninformed. It may be interesting to consider a model in which individuals are already somewhat heterogeneous in the first period. Some people may know about illnesses in their family before a genetic test can be performed whether they themselves have inherited the dangerous gene. Also, analyzing a model with more than just two time periods in which insurance can be bought may be worthwhile.

5 Appendix

5.1 Proof of Proposition 3

1. Substitution of the budget constraint for \( C^2 \) and differentiation of the objective function with respect to \( S^2 \) and \( L \) yield the following first order conditions:

\[
-u'(y - C_1 - \bar{\pi}L^1 - S^2 - p_i(L - L^1)) + p_iu'(S^2 + L, \theta) + (1 - p_i)w'(S^2) = 0
\]

\[
-u'(y - C_1 - \bar{\pi}L^1 - S^2 - p_i(L - L^1)) + v'(S^2 + L, \theta) = 0
\]

where in a slight abuse of notation \( u' \) denotes the derivative of \( u \) with respect to the first argument (consumption). Differentiating the system (19) and (20) with respect to \( \theta \) yields

\[
\begin{bmatrix}
    p_i(u'' + v'') & u'' + p_iu'' + (1 - p_i)w'' \\
    p_iu'' + v'' & u'' + v''
\end{bmatrix}
\begin{bmatrix}
    \frac{\partial L}{\partial \theta} \\
    \frac{\partial S^2}{\partial \theta}
\end{bmatrix} =
\begin{bmatrix}
    -p_i \frac{\partial^2 v}{\partial u \partial \theta} \\
    -\frac{\partial^2 v}{\partial y \partial \theta}
\end{bmatrix}
\]

(21)

From this we get

\[
\frac{\partial L}{\partial \theta} = \frac{\partial^2 v}{\partial C \partial \theta}(1 - p_i)(u'' + w'')/ \det > 0
\]

(22)

and

\[
\frac{\partial S^2}{\partial \theta} = -\frac{\partial^2 v}{\partial C \partial \theta} p_i(1 - p_i)u''/ \det < 0
\]

(23)
where det = \(-(1-p_i)^2u''v''-(1-p_i)w''(p_iu''+v'')\) < 0. Consequently, \(L_{2L} > L_{1L}\) and \(L_{2H} > L_{1H}\), as claimed.

2. The proof of this claim proceeds in two steps:
Step 1: Suppose \(L_{iL} > L^1\) and \(L_{iH} > L_1\); then \(\lambda_{iH} > \lambda_{iL}\); where \(\lambda_{ij}\) is the Lagrange multiplier of the budget constraint in problem (12), (13).
Proof of step 1: Let \(L\) denote the Lagrangian of problem (12), (13). From the envelope theorem, we have

\[
\frac{\partial L}{\partial p} = u(S^2 + L, \theta) - w(S^2) - \lambda(L - L^1) \tag{24}
\]

and consequently

\[
\frac{\partial^2 L}{\partial p \partial y} = u'(S^2 + L, \theta) \left( \frac{\partial S^2}{\partial y} + \frac{\partial L}{\partial y} \right) - w'(S^2) \frac{\partial S^2}{\partial y} - \frac{\partial \lambda}{\partial y} (L - L^1) - \frac{\partial L}{\partial y} \lambda \tag{25}
\]

\[
= -\frac{\partial \lambda}{\partial y} (L - L^1) \tag{26}
\]

since by the first order conditions \(u' = w' = \lambda\). Since for a concave objective function (12) obviously \(\partial \lambda/\partial y < 0\) (decreasing marginal utility of money), we have that the expression in (26) is positive, implying that \(\lambda_{iH} > \lambda_{iL}\) if \(L_{iL} > L^1\) and \(L_{iH} > L_1\).

Step 2: If \(\min(L_{1L}, L_{1H}, L_{2L}, L_{2H}) > L_1\), then increasing \(L^1\) increases ex ante expected utility.
Proof of step 2: Suppose that in the optimum, all types are buying in the second period, so we have \(\lambda_{1H} > \lambda_{1L}\) and \(\lambda_{2H} > \lambda_{2L}\). Now consider increasing first period life insurance purchases, \(L^1\), by one unit; effectively, this increases the income of a type \(H\) individual by \((pH - \pi_0)\) and decreases the income of a type \(L\) individual by \((\pi_0 - pL)\). Multiplying by the respective marginal utilities of income and the probabilities of being a \(ij\) type, we get that the change in utility brought about by the increase in \(L^1\) was

\[
r_1[qH\lambda_{1H}(pH - \pi_0) + qL\lambda_{1L}(pL - \pi_0)] + r_2[qH\lambda_{2H}(pH - \pi_0) + qL\lambda_{2L}(pL - \pi_0)] >
\]

\[
r_1[qH\lambda_{1L}(pH - \pi_0) + qL\lambda_{1L}(pL - \pi_0)] + r_2[qH\lambda_{2L}(pH - \pi_0) + qL\lambda_{2L}(pL - \pi_0)] = 0 \tag{27}
\]

Since the change of \(L^1\) increases ex ante expected utility, we were not in an optimum before the change.

An analogous argument can be used to exclude the case that all types are selling in period 2. Also, it is possible to exclude the case that some types (necessarily
high demanders) are inactive and the other types sell: For then, a decrease in $L^1$ would leave high demanders unaffected to the first order (since $\lambda_{2L} = \lambda_{2H}$), while low demanders would benefit from the reduction in $L^1$. Finally, it is also possible to exclude the case that some types (low demanders) are inactive and the other types buy if $L^1 > 0$! (by an analogous argument). However: If some types have zero final insurance demand, then marginal utility of the low demand low risk type is greater than the marginal utility of the low demand high risk type (reason: the low risk type has better opportunities to spend the money, since there is no need to hurry, he will probably be around also in period 3, other than the high risk type). In this case, buying insurance in period 1 need not be optimal since this is always transferring money from the low risk state to the high risk state, and for low demand individuals this is bad from the beginning on. Buying no insurance in period 1 under symmetric second period information is for example optimal in the numerical example.

3. Differentiation the first order conditions with respect to $y$ yields

$$
\begin{bmatrix}
p_i(u'' + v'') & u'' + p_i u'' + (1 - p_i) w'' \\
p_i u'' + v' & u'' + v''
\end{bmatrix}
\begin{bmatrix}
\partial L / \partial y \\
\partial S^2 / \partial y
\end{bmatrix}
= 
\begin{bmatrix}
u'' \\
w''
\end{bmatrix}.
$$

(28)

Solving these equations gives

$$
\frac{\partial L}{\partial y} = (1 - p_i)u''(v'' - w'') / \det
$$

(29)

which is positive if and only if $v'' < w''$ and

$$
\frac{\partial S^2}{\partial y} = -(1 - p_i)u''v'' / \det > 0
$$

(30)

Death state consumption is a normal good since

$$
\frac{\partial L}{\partial y} + \frac{\partial S^2}{\partial y} = -(1 - p_i)u''w'' / \det > 0
$$

(31)

Second period consumption is a normal good since

$$
\frac{\partial L}{\partial y} + \frac{\partial S^2}{\partial y} = \frac{(1 - p_i)u''v'' + p_i(1 - p_i)u''w''}{(1 - p_i)^2u''v'' + p_i(1 - p_i)u''w'' + (1 - p_i)w''v''} < 1
$$

(32)

4. Consider the first period optimization problem:

$$
\max_{C_1, L^1} u(C_1) + q_L [r_1 Z_{1L}(y - C_1 - (\bar{\pi} - p_L)L^1) + r_2 Z_{2L}(y - C_1 - (\bar{\pi} - p_L)L^1)]
+ q_H [r_1 Z_{1H}(y - C_1 - (\bar{\pi} - p_H)L^1) + r_2 Z_{2H}(y - C_1 - (\bar{\pi} - p_H)L^1)]
$$

(33)
where \( Z_{ij} \) is the value function of the second period optimization problem (12) and (13). Differentiating this with respect to \( C_1 \) and \( L^1 \) yields

\[
\begin{align*}
u'(C_1) + q_L [r_1 Z_{1L}^1 + r_2 Z_{2L}^1] + q_H [r_1 Z_{1H}^1 + r_2 Z_{2H}^1] &= 0 \quad (34) \\
q_L (p_L - \bar{\pi}) [r_1 Z_{1L}^1 + r_2 Z_{2L}^1] + q_H (p_H - \bar{\pi}) [r_1 Z_{1H}^1 + r_2 Z_{2H}^1] &= 0 \quad (35)
\end{align*}
\]

as first order conditions for an interior solution. Since \( q_L (p_L - \bar{\pi}) + q_H (p_H - \bar{\pi}) = 0 \), (34) implies that \( r_1 Z_{1L}^1 + r_2 Z_{2L}^1 = r_1 Z_{1H}^1 + r_2 Z_{2H}^1 \), as claimed.

For a corner solution with respect to \( L^1 \), the Kuhn Tucker condition

\[
q_L (p_L - \bar{\pi}) [r_1 Z_{1L}^1 + r_2 Z_{2L}^1] + q_H (p_H - \bar{\pi}) [r_1 Z_{1H}^1 + r_2 Z_{2H}^1] \leq 0 \quad (36)
\]

implies \( r_1 Z_{1L}^1 + r_2 Z_{2L}^1 \geq r_1 Z_{1H}^1 + r_2 Z_{2H}^1 \). If the expected marginal utility of low risk types is greater than the expected marginal utility of high risk types, then transferring money from the low risk state to the high risk state (as done with first period insurance purchases) is counterproductive, and therefore zero first period insurance purchases are optimal in this case.

5. We prove the claim in the proposition for \( L^1 > 0 \); it should be obvious afterwards how to prove the claim for \( L^1 = 0 \).

Differentiating the first order conditions with respect to \( p \) and solving for \( \partial L / \partial p \) and \( \partial S^2 / \partial p \) yields

\[
\frac{\partial L}{\partial p} = -(L - L^1) \frac{\partial L}{\partial y} \quad (37)
\]

and

\[
\frac{\partial S^2}{\partial p} = -(L - L^1) \frac{\partial S^2}{\partial y} \quad (38)
\]

We know already from above that either type 2L or type 2H must be insurance buyers in the second period. Consider the case that insurance purchases are a normal good (\( \partial L / \partial y > 0 \)). If type 2H is a second period insurance buyer (\( L_{2H} - L^1 > 0 \)), then \( \partial L_{2H} / \partial p < 0 \) for \( p \in [p_L, p_H] \), so \( L_{2L} > L_{2H} \), as claimed. If type 2H were selling insurance in the second period (\( L_{2H} < L^1 \), then \( \partial L_{2H} / \partial p > 0 \) and consequently \( L_{2L} < L_{2H} < L^1 \). By the first claim of this proposition, this would mean that all individuals in the market sell insurance in the second period, which is a contradiction to the second claim of this proposition. Hence \( L^1 < L_{2H} < L_{2L} \) if insurance is a normal good.
Analogously, if type \( 1H \) is a second period insurance seller \( (L_{1H} - L^1 < 0) \), then \( \partial L_{1j} / \partial p > 0 \), so \( L_{1L} < L_{1H} \), as claimed. If type \( 1H \) were buying insurance in the second period \( (L_{1H} > L^1) \), then \( \partial L_{1j} / \partial p < 0 \) and consequently \( L_{1L} > L_{1H} > L^1 \). Again, by the first claim of this proposition, this would mean that all individuals in the market buy insurance in the second period, which is a contradiction to the second claim of this proposition. Hence \( L_{1L} < L_{1H} < L^1 \) if insurance is a normal good. The proof for the case that insurance is an inferior good is analogous and omitted here.

5.2 Proof of Proposition 4

Let \( \mu_{ij} \) denote the Lagrange multiplier associated with the constraint (13). Clearly, \( \partial Z_{ij} / \partial L^1 = (p_j - \bar{\pi}) \lambda_{ij} - \mu_{ij} \). The first period problem of the individual is again given by (33). Differentiating with respect to \( C^1 \) gives (34) and differentiating with respect to \( L^1 \) gives

\[
q_L (p_L - \bar{\pi}) \{ r_1 Z_{1L}^i + r_2 Z_{2L}^i \} + q_H (p_H - \bar{\pi}) \{ r_1 Z_{1H}^i + r_2 Z_{2H}^i \}
- r_1 q_L \mu_{1L} - r_1 q_H \mu_{1H} = 0
\]  

(39)

where it is assumed that (14) is not binding for high demand types\(^{23}\) and consequently \( \mu_{2L} = \mu_{2H} = 0 \). Clearly, (39) is satisfied for a smaller value of \( L^1 \) than (34).

5.3 Proof of Proposition 5

1. As in Proposition 3, high demand demand consumers must have an at least weakly higher final life insurance demand than low demand consumers of the same risk type. Since the premium is the same for both risk types, short inspection of the optimization problem (12), (13) shows that a higher \( p \) also increases \( p_i v'(S^2 + L_j \theta_j) \) and hence has the same effects as an increase in \( \theta \): High risk types have a higher final insurance demand than low risk types of the same demand type. An example that \( L_{1H} \) and \( L_{2L} \) cannot be ranked in general is given in the text.

2. Suppose first that in the optimum \( L^1 < L_{1L} \). A feasible plan then is to keep first period consumption and second period saving constant (in comparison to the assumed optimum), increase first period insurance purchases by \( L_{1L} - L_1 \) and decrease second period insurance purchases by the same amount. Since \( \pi^2 > \bar{\pi} \), the individual can consume \( (\pi^2 - \bar{\pi})(L_{1L} - L_1) > 0 \) more in the second period.

\(^{23}\)It is quite obvious that this must be the case
than in the assumed optimum, and in all other states, consumption is unchanged. This yields the desired contradiction.

Suppose $L^1 \geq L_{2H}$. Then the unconstrained final life insurance demand of the other 3 types $(1H, 2L, 1L)$ would all be lower than $L^1$. But then decreasing $L^1$ would mean either a gain or a second order loss for type $2H$ and a first order gain for the other 3 types. Hence the optimal $L^1$ satisfies $L^1 < L_{2H}$.

3. The right hand side of (15) will increase in $L^1$ if and only if

$$\frac{[r_1q_H(L_{1H} - L^1) + r_2q_H(L_{2H} - L^1)]}{[r_1q_L(L_{1L} - L^1) + r_2q_L(L_{2L} - L^1)] + [r_1q_H(L_{1H} - L^1) + r_2q_H(L_{2H} - L^1)]}$$

is increasing in $L^1$, since it is a convex combination of $p_L$ and $p_H$ and (40) is the weight on $p_H$. Note that $L_{1L} = L^1$, by the second part of this proposition.

Taking logarithms and differentiating with respect to $L^1$, (15) will increase in $L^1$ if and only if

$$\frac{r_2 \frac{\partial L_{2H}}{\partial L^1} + r_1 \frac{\partial L_{1H}}{\partial L^1} - 1}{r_2 L_{2H} + r_1 L_{1H} - L^1} - \frac{\partial L_{2L}}{\partial L_L} - 1 > 0$$

Since $\partial L_{ij}/\partial L^1 = (\pi^2 - \bar{\pi}) \partial L_{ij}/\partial y$ and $r_2 L_{2H} + r_1 L_{1H} > L_{2L}$ by assumption, a sufficient condition for (41) to hold is that $\partial L_{ij}/\partial y$ is close enough to 0.

### 5.4 Proof of Proposition 6

It follows immediately from the first order conditions and the fact that $\pi^{2B} \geq p_L$ that under the given condition, it is optimal for $1H$ types to sell all of their life insurance; the same is true for $1L$ types.

Suppose that type $2H$ buys additional insurance and that type $2L$ either buys or is inactive. Let $Z_{ij}$ be appropriately defined value functions as in the proof of Proposition 3 and note that $Z'_{2H} > Z'_{2L}$ if type $2H$ buys additional insurance in period 2 but type $2L$ does not, because then type $2H$ will have to decrease his consumption relative to that of type $2L$. Furthermore, let $\pi_{2L}$ be the second period premium for which type $2L$ would be willing to buy further insurance; if type $2L$ buys in equilibrium, then $\pi_{2L} = \pi^{2S}$, if $2L$ is inactive, we must have $p_L < \pi_{2L} < \pi^{2S}$.

Optimization over first period insurance purchases $L^1$ requires as in (35) that

$$r_1(\bar{\pi} - \bar{\pi})[Z'_{1L} + Z'_{1H}] + r_2q_L(\pi_{2L} - \bar{\pi})Z'_{2L} + r_2q_H(\pi_{2H} - \bar{\pi})Z'_{2H} = 0$$

which cannot hold, given $\pi_{2L} > p_L$, $Z'_{2H} > Z'_{2L}$ and $\bar{\pi} = q_{LPL} + q_{HPH}$; the left hand side of (42) is always positive. Consequently, type $2L$ must be selling in equilibrium.
References


