1976

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Citation of this paper:
RESEARCH REPORT 7606

EXPECTATIONS AND INCOME CLAIMS IN A
TWO-SECTOR MODEL OF INFLATION

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March 1976

*I am indebted to R. Boyer and C. Southey for useful comments on an earlier version of this paper.
EXPECTATIONS AND INCOME CLAIMS IN A
TWO-SECTOR MODEL OF INFLATION

Analysis of the effects of expectations on inflation has usually been undertaken in the context of assumed competitive markets. The present paper examines the consequences of incorporation of expectations into models of inflation for which some, but not all sectors, do not price according to the competitive norm. Such an investigation would seem necessary even if one takes the extreme view that non-competitive sectors are only a minor part of the economy, for recent experiments with price and wage control impose modes of behavior which are probably very unlike the competitive solutions.

The paper is in two parts, the first discussing a simple aggregative model and the second a simple disaggregated system, with the aggregated model being introductory to the later analysis. It is found that there are values of the parameters for which the results the system produces are virtually indistinguishable from those of a solely competitive model. In other cases the results resemble those of the competitive story, but the position of the Phillips curve, or the natural rate of unemployment is a function of parameters not present in the competitive case. Further, for some values of the parameters the present model produces results which in a wider context could not be sustainable, and hence must be thought of as holding temporarily until some structural or policy change occurs. Nevertheless, it seems possible that, if the authorities hold excess demand above some critical level, for the non-competitive elements to have a substantial impact on the nature of inflation, and for the impact probably to be sustainable for a long period.

I

In both models studied the lag structure is kept as simple as is reasonable, for even a moderately high order lag structure can lead to
substantial complexity.

Wages at time \( t \) \((W_t)\) are taken to be based on costs (represented by the general price level \( P_t \)) so that

\[
W_t = w P_t \\
w > 0, \text{ constant.}
\]

Such a method of wage determination could arise in several ways, for instance from cost-of-living clauses in wage contracts, or as part of an array of price and wage controls. Note that, because a zero lag is assumed, no expectational element need be incorporated in (1). The parameter \( w = W_t / P_t \) will be referred to as the real wage claim implicit in wage adjustment, and indeed the absence of a lag means that \( w \) is also the actual real wage.\(^2\)

The aggregate price level is supposed to be sensitive both to excess demand and to changes in variable production costs. One way in which this could occur (to be made explicit in part II) is that some markets are essentially competitive with prices being responsive to excess supply, and others are non-competitive. Alternatively, the price side of wage and price controls could impose a type of pricing based on costs. Consider first this cost element

in pricing. Expected variable costs of production are taken to be wage costs per unit of output \((W/\theta)\) where \( \theta \) is the level of output per head. Hence, assuming a one period lag in pricing, the desired price is

\[
D_t = (1 + \pi)(1 + \gamma_2 W_t^*) \frac{W_{t-1}}{\theta}, \quad \pi, \theta, \gamma_2 \text{ positive constants, } 0 \leq \gamma_2 \leq 1.
\]

or

\[
P_t^D = m(1 + \gamma_2 w_t^*) W_{t-1}
\]

where \( w_t^* = (W_t^* - W_{t-1}) / W_{t-1} \) is the expected rate of change of wages and \( \gamma_2 \) the extent to which wage costs are adjusted to incorporate this expectation. Demand determined prices move in accordance with overall excess demand for goods \((\xi)\) and with the expected rate of inflation, so that the general price
level is determined by

\[
\frac{p_t - p_{t-1}}{p_{t-1}} = p_t = k_1 (\xi + \gamma_1 p_t^*) + k_2 \left( \frac{p^D_t - p_{t-1}}{p_{t-1}} \right),
\]

\[k_1, k_2, \gamma_1 \text{ positive constants, } 0 < k_2 < 1.\]

Here \( \gamma_1 \) measures the extent of adjustment of demand determined prices to expected inflation. In (3) the rate of change of the general price level is a linear combination of the excess demand and cost elements. A possible interpretation of the \( k_1 \)'s is that they measure both the degree of response per unit period to disequilibria and the weight of each element in pricing. For future reference, note that this implies that \( k_1 + k_2 = 1 \), if the response to each element in pricing is complete. 3

The model can be solved when some assumption is made about the determination of excess demand (\( \xi \)). Several approaches may be taken to this issue, but here it is convenient to assume that \( \xi \) is a parameter controlled by the government through monetary and/or fiscal policy. This enables a relationship between the rate of change of prices and the level of excess demand to be derived which it is natural to call a Phillips curve.

Supposing the system is in a steady state with \( p_t = p_t^* = w_t = w_t^* \) 4 then two cases of interest arise, namely when there are no expectational adjustments \( (\gamma_1 = \gamma_2 = 0) \), and where expectational adjustments are made \( (\gamma_1 > 0, \gamma_2 > 0) \).

In the first case, substituting (1) and (2') in (3), the (steady state) rate of inflation at any time is given by

\[
p = k_1 \xi + k_2 (m_w - 1)
\]

This linear relation between \( p \) and \( \xi \) is plotted in figure 1, and it is seen to produce a conventionally sloped Phillips curve. The intercept of this curve \( (m_w - 1) \) is clearly of some importance, for if it is zero, price stability with zero excess demand is possible, whereas, if it is positive prices rise at a
constant rate when excess demand is zero. It has already been observed that $\omega$ can be interpreted as a claim for real wages. Examining (2) it can be seen that the type of pricing involved implies a real wage offer to labor such that (in a steady state) $W/P = 1/m$. Hence $\mu_0 = 1$ may be called a situation of consistent income claims, and $\mu > 1$ a situation of excessive income claims.

Thus whether the Phillips curve predicts price stability or inflation with zero excess demand depends on whether income claims are excessive or consistent.

Turning to the case in which expectations are influential, and solving for the steady state rate of inflation from (1) (2') and (3) gives

\[(4') \quad p[1 - (k_1 \gamma_1 + k_2 \mu_0 \gamma_2)] = k_1 \xi + k_2(\mu_0 - 1)\]

It is immediate that the Phillips curve has the usual positive slope if $k_1 \gamma_1 + k_2 \mu_0 \gamma_2 < 1$ and is vertical (thus defining a natural rate unemployment) if $k_1 \gamma_1 + k_2 \mu_0 \gamma_2 = 1$. It is also possible to suggest that $k_1 \gamma_1 + k_2 \mu_0 \gamma_2 > 1$ implies instability in the rate of inflation.
Given that \( k_1 + k_2 \leq 1 \) and \( \gamma_1 \leq 1 \), it follows that

(a) a vertical Phillips curve can occur without full adjustment to expectations provided income claims are sufficiently excessive (\( mw > 1 \)), but its occurrence is then coincidental, and could be classified as likely to occur with probability zero. Nevertheless, it is of interest to note that if \( mw > 1 \) the right hand side of \((4')\) implies a "natural rate" \( \frac{\gamma}{m} = k_2(1-mw)/k_1 \), which is negative.

(b) a more natural case for a vertical long-run Phillips curve would be produced by a combination of consistent income claims (\( mw = 1 \), full adjustment per unit period to disequilibrium (\( k_1 + k_2 = 1 \)) and full adjustment to expectations (\( \gamma_1 = \gamma_2 = 1 \)).

(c) The parameter values which will produce a positively sloped Phillips curve can easily be observed from \((4')\). The curve will have a positive intercept at \( \frac{\gamma}{m} = 0 \) if \( mw > 1 \) (provided \( k_2 > 0 \)).

In the next section the system will be disaggregated on the price side to separate out the competitive and non-competitive sectors of the economy.

II

Many ways of constructing disaggregated models of the general type being investigated are possible. A reasonable version corresponding to the model of section 1 is given by

\[
\begin{align*}
(5) & \quad p_t^1 = \alpha \left( R_t^1 - Q_1 \right) \frac{Q_1}{m} + \beta_1 p_t^{2*}, \quad \alpha, \beta_1 \text{ positive constants, } \alpha \leq 1, \beta_1 \leq 1. \\
(6) & \quad p_t^2 = m(1 + \beta_2 \omega_t^*) W_{t-1}, \quad \beta_2 \text{ positive constant, } \beta_2 \leq 1. \\
(7) & \quad W_t = \omega p_t \\
(8) & \quad p_t = a p_t^1 + (1 - a)p_t^2, \quad \text{a positive constant, } 0 < a < 1.
\end{align*}
\]

\( p^1 \) is the level of demand determined prices and \( p^1 \) its proportional rate of
change. The cost determined price level, and its rate of change are denoted by $P^2$ and $p^2$, respectively, and the aggregate price is taken to be a fixed weight index of $P^1$ and $P^2$. The expectation adjustment coefficients are now $\beta_1$ and $\beta_2$, and the degree of response per unit period of $P^1$ to excess demand is $\alpha$.

The supply assumptions are simply that the output of each good ($Q_1$ and $Q_2$) is fixed. It will be seen that this proposition simplifies the analysis, and also is justified in particular circumstances. One point of interest will be to identify the circumstances, and also to point out when the proposition of fixed supplies would break down in a reasonable extended version of the model.

Because the analysis will be confined to examination of steady states no theory of expectation formation need be given.

To complete the system it must be specified how real demand for goods 1 and 2 is determined. The conventional assumption will be made that in each market the quantity demanded depends upon the relative price $r$ ($= P^2/P^1$), and real expenditure $E$ (measured in terms of good 1). The level of $E$ is taken to be a parameter, which through monetary and fiscal policy, is supposed to be under the control of the authorities. Expressing these assumptions algebraically, the quantities demanded ($R^1_t$) are given by

$$
\begin{cases}
R^2_t = R^2(t, E), \quad R^2 \in C^1, \quad \frac{\partial R^2}{\partial t} < 0, \quad \frac{\partial R^2}{\partial E} > 0, \\
E = R^1_t + R^2_t.
\end{cases}
$$

These assumptions are implied by a situation in which it is assumed that the underlying demand curves $R^i = R^i(P^1, P^2, Z)$, where $Z$ is nominal expenditure, are homogeneous of degree zero in absolute prices, and possess the properties
\[ \frac{\partial R^1}{\partial z^1} < 0, \quad \frac{\partial R^1}{\partial p^2} > 0 \] ; and in addition commodity 1 is a gross substitute for commodity 2 \( \left( \frac{\partial R^1}{\partial p^2} > 0 \right) \).

It is useful to have a measure of the overall excess demand for labor. On the production side it will be assumed that there are fixed input coefficients with respect to labor, so that in each sector labor demand bears a fixed proportion to goods demand. Hence total excess demand for labor \( x \) is given by

\[ (10) \quad x = (R^1 - Q_1)\mu_1 + (R^2 - Q_2)\mu_2, \quad \mu_1, \mu_2, \text{ positive constants} \]

It will later be shown that in steady states \( E \) and \( x \) are positively related so that higher real expenditure raises excess labor demand. Note that the model abstracts from frictional and search unemployment.

From now on steady states only will be considered, unless otherwise stated. A key variable is the steady state relative price, \( r \), and it can be seen from \( (6) \), \( (7) \) and \( (8) \) that

\[ (11) \quad r = \left( \frac{P^2_t}{P^1_t} \right) = m \omega(1 + \beta_2w^*_t)[a(P^1_{t-1}/P^1_t) + (1 - a)(P^2_{t-1}/P^1_t)] \]

\[ = \frac{a m \omega(1 - \beta_2) + a m \omega \beta_2 \lambda}{\lambda - (1 - a)m \omega(1 - \beta_2) - (1 - a)m \omega \beta_2 \lambda} \]

where \( \lambda = 1 + p \).

The first property to establish about the model is the meaning of excessive or consistent income claims in this wider context. The notion of an income claim does not apply to producers in sector 1, for this output is produced under conditions of fixed input coefficients, and price is demand determined. Inconsistency of claims may therefore only occur as between wage earners and producers in sector 2. Now wage earners always receive a real wage \( \omega \) (from \( (7) \)), so inconsistency arises if entrepreneurs in sector 1 achieve a wage-price ratio less than the claimed price ratio \( P^2/\bar{W} = m \). In
a steady state the actual wage-price ratio is, from (7) and (8)

\[ \frac{W}{P^2} = \omega \left( \frac{a}{r} + (1 - a) \right) \]

so claims are realized when

\[ \frac{W}{P^2} = \omega \left( \frac{a}{r} + (1 - a) \right) = \frac{1}{m} \]
or

\[ r = \frac{m \omega a}{1 - (1 - a)m \omega} \]

Thus consistency of claims requires that a particular relative price, \( r^c = r^c (m \omega) \), defined by (14), is achieved. The actual steady state relative price has already been given, so that equating (11) and (14) it is found that income claims are achieved in a steady state if

\[ p = 0 \text{ and/or } \beta_2 = 1. \]

The converse is also true as may be seen by substituting \( p = 0 (\lambda = 1) \) and/or \( \beta_2 = 1 \) in (11).

Figure 2 illustrates the relationship between \( m \omega \), relative price \( r \), and demand for commodities, as it affects salient features of the model. \( Q_1 \) and \( Q_2 \) are the fixed supplies of the two goods, and with a price line FE tangential to
the indifference curve i, the two goods market of the economy are in equilibrium. The marginal rate of substitution of \( R_1 \) for \( R_2 \) defines a unique relative price \( r_c^* \), which together with the unique real expenditure level \( E_o \), determine this equilibrium. Now note that \( r_c^* = r_c^* (mw) \) from (14) and that

\[
\frac{dr_c^*}{dmw} = \frac{a}{(1 - (1 - a)mw)^2} > 0.
\]

Thus, it is reasonable to expect that \( m^0_w^0 \) exists such that \( r_c^* = r_c^* (m^0_w^0) \), and it is natural to define the corresponding \( E_o \) as \( E_o = E(m^0_w^0) \). If \( mw > m^0_w^0 \) it follows that equilibrium in both goods markets is impossible.

Two cases giving different qualitative results will be examined, namely, fully anticipated inflation \( (\beta_1 = \beta_2 = 1) \), and partially anticipated inflation \( (\beta_1 < 1, \beta_2 \leq 1) \).

(a) \( \beta_1 = \beta_2 = 1 \).

From (15) it can be seen that income claims will be consistent in this case, and from (5) that the rate of inflation will be indeterminate in a steady state, and must involve zero excess demand in sector 1 \( (R^1 = Q_1) \). The question arises as to what determines the natural rate of excess demand for labor \( (x_n) \) associated with this state, and what magnitude \( x_n \) will have.

First, examine the problem geometrically. In figure 3 it is assumed that initially \( mw = m^0_w^0 \) and \( E = E_o \) so that to begin with the natural rate of unemployment is zero, and both markets are in equilibrium. Now suppose \( mw \) rises above \( m^0_w^0 \). Given that good 1 is a gross substitute for good 2, demand shifts to point B. But, for the economy to be at the natural rate demand for good 1 should be \( R^1 = Q_1 \), and hence expenditure must be adjusted to \( E_1 \) so that point C (on the income consumption curve through B) is attained. The market for good 2 is now in excess supply, and this defines a negative natural rate \( x_n \).

To prove this note that equilibrium in sector 1, and \( r = r_c^* \) imply

\[
R^1(r_c^*(mw), E) = Q_1,
\]
and that as \( r^c \) changes

\[
\frac{d r^c}{d E} \bigg|_{R^1=Q_1} = - \frac{\partial R^1}{\partial E} \frac{1}{\partial r} < 0.
\]

Demand for good 2 is given by \( R^2[r^c(mw), E(r^c(mw))] \), where the relationship between \( E \) and \( r^c \) must be such as to satisfy (17). Hence, raising \( mw \), while keeping the demand for good 1 zero, implies

\[
\frac{d R^2}{dmw} = \frac{\partial R^2}{\partial r} \frac{dr^c}{dmw} + \frac{\partial R^2}{\partial E} \frac{d E}{dr^c} \bigg|_{R^1=Q_1} \frac{dr^c}{dmw}.
\]

Inspection of the signs of the component terms of (19) verifies that it is negative, so that the higher \( mw \) (above \( m^0 w^0 \)) the lower \( R^2 \) and hence the more negative the natural rate.

To summarize, the natural rate depends on the level of income claims, and will be lower (that is the natural rate of unemployment will be higher) the higher are income claims. Of course, such a natural rate could not be
sustained for a long period as excess supply of good 2 would eventually lead to a breakdown in some underlying constant of the system, for instance, a reduction in \( Q_2 \) or an induced fall in \( mw \) could occur. However, if real demand \( E \) were expanded sufficiently so that point \( F \) on the income consumption line were attained, excess supply of good 2 would be eliminated. At this point the economy is above the natural rate of excess demand, and accelerating inflation could be expected.

(b) \( \beta_1 < 1, \beta_2 \leq 1 \).

Suppose first that \( \beta_2 \) is strictly less than unity. It will later be seen that virtually the same results apply for the case in which \( \beta_2 = 1 \).

Now the real wage is a function of the rate of inflation (see (11)) and the rate of inflation can be shown to be an increasing function of real expenditure \( E \) and also of excess demand for labor \( x \). Proof of this requires several steps. First, from (5)

\[
p = \frac{\alpha}{(1 - \beta_1)Q_1}(R^1 - Q_1), \text{ or}
\]

\[
(20') \quad p = \alpha'[(R^1(r(p),E) - Q_1]
\]

which has the slope

\[
(21) \quad \frac{dp}{dE} = \frac{\alpha' \frac{\partial R^1}{\partial E}}{1 - \alpha' \frac{\partial R^1}{\partial r} \frac{dr}{dp}}
\]

and from (11)

\[
(22) \quad \frac{dr}{dp} = -\frac{1}{D^2}(1 - \beta_2) \quad \text{a m w} < 0,
\]

where \( D \) is the denominator of the right-hand side expression of (11). Substituting (22) into (21) it is found that the denominator of (21) is positive, so that higher real expenditure \( E \) raises the steady state rate of inflation.

Moreover, \( x \) increases with \( E \) for, differentiating \( x \) with respect to \( E \) gives
\begin{align*}
\frac{dx}{dE} &= \mu_1 \left( \frac{\partial R}{\partial r} \frac{dr}{dp} \frac{dp}{dE} + \frac{\partial R}{\partial E} \right) + \mu_2 \left( \frac{\partial R}{\partial r} \frac{dr}{dp} + \frac{\partial R}{\partial E} \right) \\
&= \mu_1 \left( \frac{\partial R}{\partial r} \frac{dr}{dp} + 1 \right) + \mu_2 \left( \frac{\partial R}{\partial r} \frac{dr}{dp} + \frac{\partial R}{\partial E} \right) \\
&= \mu_1 \left( \frac{\partial R}{\partial r} \frac{dr}{dp} \frac{1}{1 - \alpha} \right) + \mu_2 \left( \frac{\partial R}{\partial r} \frac{dr}{dp} + \frac{\partial R}{\partial E} \right) \\
&> 0.
\end{align*}

For future reference it is useful to note that from (23) it can be inferred that raising \( E \) raises excess demand for goods in both markets. (21) and (23) confirm that \( \frac{dp}{dx} > 0 \), and hence that the Phillips curve has the normal slope (which given that inflation is being related to excess demand, rather than unemployment is positive).

Consider now what will be happening to excess demand in individual markets. It is easier to analyze this first for the case \( \beta_2 = 1 \) (\( \beta_1 < 1 \)). With \( \beta_2 \) unity, as has been shown \( r = r_c(mw) \), where \( r_c \) is defined by

\[(14') \quad r_c = \frac{mw a}{1 - (1 - a)mw}.
\]

If \( mw = m^0w^0 \), demand for the two goods will be determined by the points on the income consumption curve passing through point A in figure 2. If \( mw > m^0w^0 \) the income consumption curve through G in figure 4 gives points of demand for the two goods for each level of \( E \). At C there is excess supply of good 2 and zero excess demand for good 1, as was noted earlier. Raising real expenditure raises real demand for both goods, for both expenditure effects have been assumed to be positive and \( r \) is constant. At the expenditure level which is just sufficient to generate point F there is zero excess demand for good 2 and positive excess demand for good 1. The \( x \) defined in figure 4 I have elsewhere called the minimum rate of excess demand.\(^{10}\) Its importance in this
context is that at \( x \geq \bar{x} \) there will be no excess supply in any market in the economy. From (20') an associated minimum rate of inflation (\( p \)) may be calculated, and is easily seen to be positive. The "price" that must be paid for the "favorable" employment situation at \( x \geq \bar{x} \) is, therefore, a rate of inflation \( p \geq \bar{p} \). There seems no good reason why such a situation could not persist for a significant period of time.

Suppose now that \( \beta_2 < 1 \) so that \( r \) is not constant, but it is a function of the rate of inflation (from (11)). It was shown in (22) that as \( p \) increases \( r \) falls, so that raising \( E \) and thus \( p \), lowers \( r \). The locus of demands for goods as \( E \) rises would follow a path which would lie to the left of GF and cross \( Q_2 \) between A and F. The effect of \( \beta_2 \) being less than unity is, therefore, as might be expected, to create a lower value of the minimum rate of excess demand.
Both for $\beta_2 < 1$ and $\beta_2 = 1$, the aggregative results of part I would be valid so long as $x \geq x$, but would be misleading for $x < x$ as in this phase the assumption of constant $mW$ and $Q_L$ could not be reasonably sustained.

To complete the analysis it is required to examine the way in which the height of the Phillips curve (for any given $x$) is related to income claims. Both the case $\beta_2 < 1$ and $\beta_2 = 1$ can be handled simultaneously, noting that $\beta_2 = 1$, from (22) implies $\frac{dr}{dp} = 0$.

Suppose $x$ is held constant at some level $\bar{x}$, then from (20')

\begin{equation}
\frac{dp}{dmw} = \alpha' \left\{ \frac{\frac{dr}{dr}}{dmw} + \frac{\frac{dE}{dE}}{dmw} \right\}_{x=\bar{x}}
\end{equation}

\begin{equation}
= \alpha' \frac{dr}{dmw} \left\{ \frac{\frac{dr}{dr}}{x=x} + \frac{\frac{dE}{dE}}{dr} \right\}_{x=\bar{x}}
\end{equation}

and from (10)

\begin{equation}
\frac{dE}{dr}_{x=\bar{x}} = - \frac{\frac{dR}{dr} \mu_1 + \frac{dR}{dE} \mu_2}{\frac{dR}{dE} \mu_1 + \frac{dR}{dE} \mu_2}
\end{equation}

Substituting for (26) in (25) it is found that

\begin{equation}
\left(25'\right) \quad \frac{dp}{dmw} = \alpha' \frac{dr}{dmw} Z
\end{equation}

where

\[
Z = \left( \frac{\frac{dR}{dr} \frac{dR}{dE} - \frac{dR}{dE} \frac{dR}{dr}}{\frac{dR}{dE} \mu_1 + \frac{dR}{dE} \mu_2} \right) \mu_2 > 0.
\]

Examination of (11) shows that it can be written

\begin{equation}
\left(11'\right) \quad r = r(mw, p(mw))
\end{equation}

and hence,

\begin{equation}
\left(27\right) \quad \frac{dr}{dmw} = \frac{dr}{dmw} \left( \frac{dr}{dp} \right)_{p=p(mw)} \cdot \frac{dp}{dmw}, \quad \bar{p} \text{ and } \bar{mW} \text{ constants.}
\end{equation}
The derivative \( \frac{dr}{dp} \bigg|_{m_0 = m_0} \) has already been given in (22) and was found to be negative. From (11),

\[
(28) \quad \frac{dr}{dm_0} \bigg|_{p = p} = \frac{\lambda a (1 - \beta_2 + \beta_2 \lambda)}{D^2} > 0.
\]

Substituting (27) in (25') and rearranging

\[
(29) \quad \frac{dp}{dm_0} = \frac{\alpha' z \frac{dr}{dm_0} \bigg|_{p = p}}{1 - \alpha' z \frac{dr}{dp} \bigg|_{m_0 = m_0}}
\]

which is positive (from which it follows that \( \frac{dr}{dm_0} \) is also positive). Therefore, if \( m_0 \) rises and \( E \) is manipulated to keep \( x \) constant the steady state rate of inflation will rise.

III

It has therefore been shown that if income claims are such that \( m_0 = m_0^0 \) the present model gives results which would be practically indistinguishable from the competitive case. The natural rate of excess demand would be close to zero, or the Phillips curve would pass through the origin. If \( \beta_1 < 1 \), then as long as the level of excess demand is held above some minimum \( (x > x) \), \( m_0 > m_0^0 \) would produce inflation at a faster rate than in the competitive case, but this situation could be sustainable for a reasonable period. If \( x < x \), unemployment must develop in the non-competitive sector and output in that sector must fall. This process goes beyond the strict assumptions of the model, but it is not difficult to imagine that \( n \) and \( \omega \) would also fall.

Similarly, although a negative natural rate of excess demand results from \( m_0 < m_0^0 \), such a natural rate would require \( E \) such that \( x > x > x_n \) (and hence probably accelerating inflation) to support it. At \( x = x_n \), output of good 2 and income claims would probably eventually fall.
Finally, it should be noted that the simple aggregative model of section I, while it does not provide the insights on the relation between relative prices and income claims, and on the distribution of demand between the two sectors, nevertheless gives results broadly verified by the two sector case. Its most misleading aspect was the implication that the natural rate would be negative due to excessive only with a particular set of values of parameters. A negative natural rate is not a coincidence in the two sector model.
Footnotes

1 The present paper builds on work by Turnovsky and Pitchford [2] and Pitchford [1].

2 A lag in wage adjustment, and also demand determined wages are incorporated in Turnovsky and Pitchford [2]. This leads to a considerably more difficult problem than that presently examined.

3 By complete adjustment is meant a rise in price just sufficient to eliminate last periods nominal excess demand (for competitive prices), or just sufficient to wipe out the gap between last periods desired and actual price (for non-competitive prices).

4 Note that labor productivity has been assumed constant in the definition of $p^*$ in equation (2).

5 Consider the case in which expected rates of price and wage change are equal ($p^*_t = \omega^*_t$) but are not necessarily equal to the actual rate of inflation ($p_t$). From (1), (2) and (3),

$$p_t - p^*_t = - \left[ 1 - k_1 \gamma_1 - k_2 \gamma_2 \mu_{\omega} \right] p^*_t + k_1 \xi + k_2 (\mu_{\omega} - 1).$$

If $k_1 \gamma_1 + k_2 \gamma_2 \mu_{\omega} > 1$ and $p^*_t > 0$ then $p_t > p^*_t$ (assuming $\xi \geq 0$ and $\mu_{\omega} \geq 1$). Any adaptive expectations mechanism which raises $p^*_t$ in response to the gap $(p_t - p^*_t)$ must result in a continually rising $p_t$.

6 To see this write

$$R^1 = R^1 \left( 1, \frac{E_1}{P_1}, \frac{Z}{P} \right) = R^1 (x, E).$$

Then $\frac{1}{P} \frac{\partial R^1}{\partial x} = \frac{\partial R^1}{\partial P} > 0$ implies the gross substitution assumption which, in turn,

implies $\frac{\partial R^1}{\partial x} > 0$.

7 $\mu_i$ is the labor input-output coefficient for sector i.

8 Note that producers in sector 1 must cover variable production costs in order to continue operating. It will be assumed that $\mu_1$ do not reach levels which prevent this happening.
The intercept of the price line on the $R^1$ axis in Figure 2 defines $E(m^o \omega^o)$.

See Pitchford [1].
References
