Population Policy and Optimal Economic Growth: A Proposed Methodology Applied to Ghana

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APPLIED TO GHANA

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POPULATION POLICY AND OPTIMAL ECONOMIC GROWTH:

A PROPOSED METHODOLOGY APPLIED TO GHANA

Åke G. Blomqvist
Gordon W. Davies*

I. INTRODUCTION

In recent years, an increasing number of LDCs have begun to implement policies aimed at making techniques of fertility control available to their populations. Partly, this can be seen simply as a response to a spontaneous demand for family planning services which will make it easier for couples to achieve their desired family size. In addition, however, governments in LDCs have become increasingly aware of the negative effects that high fertility and population growth rates will have on the growth rate of the economy and hence on future per capita income levels. Consequently, a reduction of actual family size and hence of population growth rates has also been considered a potential benefit of population policy. To this end, efforts have been made to provide incentives to fertility reduction in various ways: through propaganda, through the provision of family planning services free of charge or at highly subsidized fees, or even in some cases through direct government taxation and expenditure policies which make it profitable for couples to reduce fertility.

In this paper, we will consider population policy as one tool through which a society can achieve its objectives with respect to economic development, and the purpose of the paper is to analyze the interdependence between population policy, on the one hand, and the optimal rates of capital formation and growth of output, on the other. We will assume that, in general,
the objective of stimulating the rate of economic growth is derived from the desire to bring about a higher level of welfare in future years than would prevail in the absence of an explicit policy to accomplish this, and we will represent society's preferences between present and future welfare by an objective function similar to those used in the traditional analysis of optimal savings problems. We will then construct a simple model in which real output per worker depends only on the amount of capital per worker; the time path of this variable, and hence indirectly, the distribution of welfare over time, will depend on the share of output that is devoted to capital formation at any given time, and on the rate of growth of the labour force which depends on the level of fertility. Given a set of alternative policies with respect to fertility limitation, we will find the amounts of savings and capital accumulation which maximize the value of society's welfare function.

Explicit consideration will be given to the fact that parents derive welfare from raising children, so that any policy which induces couples to reduce their family size will have a welfare cost in addition to the direct resource costs of providing modern family planning services. For a particular population policy over time, the value of the social preference function which emerges as the solution to the corresponding optimal accumulation problem will thus reflect both the costs and benefits of that policy, so that the analysis will permit us to choose the best population policy among the alternative ones which we consider. The optimal combination of policies is therefore given by that time path of fertility rates which, in conjunction with the corresponding optimal time path of capital accumulation, results in the highest value of the maximand for all of the alternative
fertility policies considered (each of which has a unique optimal time
path of capital accumulation associated with it). The significance of our
analytical approach for the formulation of development policy is that choices
which are optimal when fertility limitation and capital accumulation policies
are formulated jointly may be very different from those that result when
either policy is considered in isolation.

In the following sections, we report on a preliminary attempt to
apply the proposed methodology to the choice between a number of alternative
policies with respect to fertility limitation in Ghana. While we have
used simplified approaches to the separate problems of the microeconomic
determinants of fertility, of the formulation of an optimal program for
capital accumulation, and of the specification of a demographic model,
our illustrative results nevertheless throw some interesting light on the
interdependence between the choice of population policy on the one hand and
the optimal rate of capital accumulation on the other. In addition, it
will be seen that the choice of the best population policy among the alterna-
tive ones considered is quite sensitive to the assumed parameter values in
the relationship between fertility and its economic determinants; this
points to the need for further empirical research on the values of those
parameters before one can answer the question whether a policy aimed at
rapid fertility reduction represents an efficient way of reaching a society's
development objectives.

The organization of the paper is as follows. First, in section II,
we briefly discuss the relationship of our approach to existing literature.
Section III contains a detailed description of the model we use, and in
section IV we discuss the choice of parameter values and describe some of
the results; a few brief qualifications and concluding comments are made in section V.

II. RELATIONSHIP TO EXISTING LITERATURE

In this section we briefly review the literature on the problem of evaluation of the benefits of population policies, the welfare cost of reducing family size, and the positive economic-demographic models to which the proposed one is similar.

Our methodology can be seen as an alternative to the one originally proposed by Enke [1960a, 1960b], which consisted in finding the present discounted value of the increase in per capita consumption consequent upon a fertility reduction and comparing this quantity with the resource costs (in the form of manpower and materials) of the family planning programs assumed to lead to the fertility reduction. Cost-benefit methodology, however, provides a framework designed to enable planners to test the profitability of allocating resources to different uses, given the total resources available to the economy at various times, but it provides no criteria for the evaluation of the impact of a project on the distribution of welfare. In our view, the essence of a policy aimed at reducing fertility is to increase the amount of resources per capita available to the population in the future, which essentially represents a change in the distribution of welfare as between present and future generations. On these grounds, we agree with Leibenstein's [1969] criticisms of the application of traditional cost-benefit techniques in this area, and we therefore seek to evaluate fertility limitation programs within a framework
which explicitly takes into account society's objectives with respect to the distribution of welfare over time.  

If one rejects the idea of applying partial cost-benefit techniques to the analysis of fertility limitation, it becomes necessary to make use of comprehensive and aggregative economic-demographic models. Such models have been extensively used to illustrate the economic consequences of changing fertility, beginning with the well-known study by Coale and Hoover [1958]. In some cases, the explicit purpose of these experiments has been to evaluate the benefits of reduced fertility in order to make possible an assessment of the profitability of family planning programs. This has been the case for the model developed by Enke and his associates [undated], by Demeny [1965], and by Blomqvist [1971]. Demeny measures the gains from fertility limitation in a given year as the amount of savings and investment actually made in the economy less the smaller amount of investment which would have been necessary in order to maintain per capita income at the same level as would have prevailed without fertility limitation.

In general, our fundamental approach is similar to that of Demeny in the sense that we also consider fertility limitation and capital accumulation as alternative methods of achieving increases in future per capita income. Our approach differs from his, however, in that we consider the problem of combining these two methods to obtain time paths for the economy which are optimal in the sense of maximizing economic welfare over time, rather than as alternative ways of reaching some arbitrary, preassigned time path for per capita income.
Our approach also differs from that of earlier work in our method of measuring the welfare cost of fertility limitation. This cost has typically been estimated in the context of analyzing family planning programs involving the supply of different types of contraceptive services (see, in particular, Enke [1966]). The cost per prevented birth has then been estimated by attempting to calculate the net reduction in the number of births that would result from a given program in relation to the costs of the program. Sometimes it has been argued (e.g., Enke [1960a, 1960b]) that in order to stimulate fertility limitation, the use of different types of contraception techniques should be subsidized (as is well known, such a policy has been tried in India where payments have been made to males undergoing vasectomies). In a cost-benefit calculation for a family planning program, it should be taken into account that a subsidy of this kind merely represents a transfer within the economy and should not be counted as a real cost to the program.

Leibenstein [1969] has noted that one serious difficulty with the type of cost calculation just outlined lies in the fact that many users of the modern types of contraceptive methods offered in family planning programs may simply be substituting that method for a different, "traditional" method, so that their use of modern family planning services may have little net impact on fertility. Blomqvist [1973] has also argued that if the objective of a population policy is to reduce fertility, the economically efficient way of accomplishing this objective is to design a tax policy which directly affects the opportunity cost to couples of raising
children, while leaving them free to choose whichever form of contraception they prefer at unsubsidized prices. As previously suggested, the real cost of such a policy corresponds to the welfare loss inherent in the fact that the tax/subsidy scheme raises the private opportunity cost to parents of raising children, over and above the opportunity cost of the economic resources used in this activity.

As is further discussed below, an attempt will be made in this study to assign a value to this welfare cost. In order to do so, we follow the lead in some of the recent theoretical and empirical work on fertility determination (see, e.g., the Proceedings of the N.B.E.R. Conference, "New Economic Approaches to Fertility" [1972]) and postulate that age-specific fertility rates are partially related to economic factors such as income levels and opportunity costs. Since little empirical work has been done with regard to fertility determination in Ghana, however, our methods for measuring the opportunity cost and assigning values to the parameters in this relationship are of necessity somewhat arbitrary.

We now briefly review the major economic-demographic models which have been used to date to describe the economic effects of population policies. These models all disaggregate the total population, usually by single years of age and sex, by specifying population cohorts: the result is a very high level of disaggregation without a corresponding degree of complexity because the cohorts are all predetermined. The models may be classified into two groups: those in which output is supply-determined and those in which it is demand-determined. The former have generally been applied to less developed countries and the latter to more developed countries.
The pioneering study of the first type is the one previously mentioned by Coale and Hoover [1958]. Others have worked with more detailed specifications, for example, Barlow [1969], Enke and Zind [1969], Denton and Spencer [1971], Enke et al. [undated], and Barlow and Davies [1974]. Two models of the second type are Davies [1972, 1975]. The model we use in this paper is a supply-constrained model which is somewhat less disaggregated than those mentioned above, in order to reduce the complexity of the computations.

III. THE MODEL

As noted in the introduction, the basic approach in this paper will be to consider a set of alternative population policies, and then, for each such policy, to formulate and solve the problem of finding the optimal time path of consumption and capital accumulation corresponding to that policy. In this section, we describe the character of the simple economic-demographic model used in these experiments. We first consider the demographic submodel.

A given population policy is taken to consist of hypothetical "target paths" for age-specific fertility rates over time. To simplify the analysis, we first postulate that the relative fertility rates of women in different age groups will stay unchanged over time. Given the total number of women in each age group, this assumption enables us to define a variable $F_t$ which is computed as a weighted average of women in different age brackets with the weights being proportional to relative fertility rates. We shall call $F_t$ the number of fertility-equivalent females. The only fertility variable which is required to compute the total number of births at a given
time, \( B_t \), is therefore \( h_t \), the number of births per equivalent female. We have

\[
B_t = h_t \cdot F_t, \quad \text{(1)}
\]

with a specified time path of \( h_t \) corresponding to each alternative assumed population policy. The relationship between \( h_t \) and the economic variables, and hence the magnitude of the incentive required to reach the fertility objective in each time period, will be discussed below.

The cohorts of males and females aged zero in the current period equal total live births in the previous period times the constant ratio of births of a given sex to total births, appropriately adjusted for mortality. For ages 1 through 59, the value of a cohort of a given age and sex in the current period equals the value of the cohort of the same sex but one year younger in the previous period, with an adjustment for mortality occurring in the interim. The value of the open-ended cohort of a given sex and of age sixty and above equals the value of the same cohort in the previous year, adjusted for mortality, plus the value of the cohort of the same sex but of age 59 in the previous period, again with an adjustment for mortality. Given initial values for the total population by age and sex, initial data on age-specific fertility rates, and taking age-specific mortality rates as exogenously given and constant over time, the relationships given above are sufficient to completely specify the time path of the total population by age and sex over the given planning period.

The labour force \((L_t)\) is determined by applying exogenous age- and sex-specific labour force participation rates to the corresponding population
subgroups, and the number of equivalent consumers \( N_t \) is defined as a weighted average of subgroups of the total population in which the weights are lower for the dependent population than for the rest of the population. We turn now to a description of the economic submodel.

Total output \( Y_t \) is determined by a two-factor Cobb-Douglas production function with disembodied or Hicks-neutral technical change and constant returns to scale:

\[
Y_t = \alpha_0 (1 + \alpha_1) L_t^\alpha_2 K_t^{1-\alpha_2}
\]

(2)

\[\alpha_0, \alpha_1, \alpha_2 > 0\]

where \( K \) is the stock of capital, \( \alpha_0 \) a scale parameter, \( \alpha_1 \) the rate of technical change, \( \alpha_2 \) the share of labour in the total product and \( 1-\alpha_2 \) the share of capital.

Total savings, \( S_t \), is regarded as a control variable, and its time path will be determined as the solution to the dynamic optimization problem for each hypothetical set of time paths for the demographic variables. Total savings can be taken to consist of voluntary private savings by business firms and households plus the surplus of government revenue over current expenditure; if private savings are regarded as endogenously determined, the government surplus will be the indirect control variable which has to be adjusted to reach the optimal level of total savings at each point in time. Limits may be assigned to the government surplus and/or total savings in order to reflect the administrative difficulties associated with high levels of taxation as well as limitations in the capacity of the economy to productively absorb capital formation at any given time, although this was not done in the
examples reported in section IV. Given $S_t$, the change in the capital stock will be given by

$$ (K_{t+1} - K_t) = S_t - \delta K_t $$  \hfill (3) 

where $\delta$ is the given and constant rate of depreciation of the capital stock.

The welfare function normally used in finding the optimal time paths of capital accumulation has as arguments the levels of consumption in each time period, $C_t$, given simply by

$$ C_t = Y_t - S_t. $$  \hfill (4) 

As noted in the introduction, however, our optimization procedure will also take into account the reduction in individuals' welfare inherent in a policy of fertility reduction. To accomplish this, the welfare function in this paper will have as one of its arguments a variable $C^*_t$ which we term "adjusted consumption," given by

$$ C^*_t = C_t - E_t $$  \hfill (5) 

where $E_t$ is the welfare loss resulting from the population policy. We now turn to the problem of evaluating $E_t$.

Fundamentally, the line of approach will be to argue that, at least to some extent, couples are influenced by economic considerations in their decisions with respect to family size, and in particular, by the opportunity cost, in terms of foregone consumption, of raising children. As argued above, the economically efficient way of accomplishing the objective of fertility
reduction under those circumstances is to implement some form of a tax-subsidy scheme which directly affects this opportunity cost. Since such a tax-subsidy scheme will raise the cost to parents of child-raising over and above the social opportunity cost of the resources used for this activity, it will imply a welfare loss analogous to the "excess burden" that arises when an excise tax is placed on the consumption of an ordinary commodity. It is this welfare loss that has to be weighed against the benefits that fertility reduction will have in terms of raising the rate of economic growth and hence future levels of per capita income.

In order to obtain a tentative estimate of the welfare loss over time for each numerical experiment, we proceed as follows. We assume that at a given time, and given the various noneconomic factors that enter into decisions with respect to family size, couples will attempt to regulate their fertility behaviour so as to attain that family size which is optimal in the light of their lifetime income and the opportunity cost over time of raising children. We assume that the relevant income concept is income per equivalent consumer, and we further make the simplifying assumption that the relevant lifetime income at a point in time is proportional to actual income at that time. Furthermore, since child-rearing is a labour-intensive activity, the opportunity cost over time is taken to depend on the wage rate (which is assumed equal to the marginal product of labour and can be found from the production function), as well as any tax or subsidy scheme. Again, we simplify the analysis by letting the relevant discounted opportunity cost at a given time simply depend on the actual wage rate and subsidy level at that time.
We are thus postulating that desired family size at a given time depends on income and the wage rate at that time. Desired family size is then taken to influence $h_t$, the number of births per fertility-equivalent female as defined above.

The discussion can now be summarized by writing:

$$h_t = h\left(\frac{Y_t}{N_t}, p^*_t\right)$$

(6)

where $p^*_t$ represents the expected discounted opportunity cost of raising children. The variable $p^*$ is defined by

$$p^*_t = p(w^*_t) + \tau_t$$

(7)

where $p(w^*_t)$ is that opportunity cost which would prevail in the absence of a tax-subsidy scheme and which depends on the wage rate $w^*_t = \frac{\partial Y_t}{\partial L_t}$. The variable $\tau_t$ corresponds to the increase in the opportunity cost of raising children resulting from the imposition of a tax-subsidy scheme.

The welfare loss associated with a given degree of fertility reduction may now be evaluated as follows. For a given level of income, $h_t$ will be a function of $p^*_t$ (see Fig. 1). As discussed above, however, the value of $h_t$ is regarded as exogenously determined by the policy-makers. Suppose that at a given time, the target value of $h_t$ is $h^*$, and suppose that in the absence of any tax or subsidy (i.e., with $\tau_t = 0$) we would have $p(w^*_t) = p^*_o$. At $p^*_o$, it is seen that the number of births per fertility-equivalent female ($h^*_o$ in Fig. 1) exceeds the target level $h^*$, so that a tax or subsidy would have to be imposed, raising the opportunity cost to $p^*_1 = p^*_o + \tau_t$. The number of births prevented
per female during this time period will be \( h^* - h \); if the discounted opportunity cost in terms of consumption foregone is assumed to be constant, it is easy to show, using reasoning analogous to that used in the analysis of an excise tax on a commodity, that the excess burden of the population policy during this time interval can be approximated by the triangular area ACD.

![Figure 1](image)

Once the functions \( h(\frac{Y}{N_t}, p_t^*) \) and \( p(w_t) \) are specified, the value of the excess burden may be found in each time period for each numerical experiment. For purposes of the present paper, it was assumed for simplicity that \( p(w_t) = w_t \), and the function \( h \) was assumed to be of the constant-elasticity form, i.e.,

\[
h_t = B \left( \frac{Y}{N_t} \right)^{\beta_1} \left( p_t^* \right)^{\beta_2}
\]  

\[
(8)
\]
where $\beta_1$ and $\beta_2$ are the elasticities of $h$ with respect to income and the opportunity cost, respectively. The value of the excess burden computed in this manner is per fertility-equivalent female; to get the total value of the excess burden, $E_t$, we simply multiply by the number of fertility-equivalent females. The result was subtracted from total consumption of goods and services $C_t$ in order to get the "adjusted consumption" $C_t^*$ which was used in the welfare functional of the optimal savings problem.

IV. APPLICATION TO GHANA

In this section we describe some preliminary results from an application of the methodology just described to the case of Ghana. Because of limitations on data availability and lack of good information on the values of some of the parameters used in the analysis, the results should clearly be taken primarily as illustrations of the methodology rather than as concrete policy recommendations. In addition, they can be seen as providing useful indications of the types of data and parameter estimates that will be required before the methodology can be used for purposes of actual policy formulation.

In general, the type of problem which we considered was as follows. Given the initial population, capital stock and level of output, suppose that the policy-makers' goal were to bring about a growth rate of 2% per annum in per capita income over the planning horizon which was taken to be 80 years. The rise in per capita income over the planning period could be expected to lower fertility even in the absence of any specific population policy. Based on results from regressions using cross-section data from a set of less developed
and industrialized countries, it was assumed that the elasticity of age-specific fertility rates with respect to per capita income would be \(0.27\). Given the terminal per capita income level implied by the growth target of 2% per annum, we used this assumption to compute the proportional reduction in the fertility variable \(h_t\) that would occur between the initial and terminal time periods.\(^5\) Four alternative population policies were then considered. All of them involved the assumption that over the planning period, the value of \(h_t\) would fall from its initial level to the computed terminal level, but they differed in terms of the speed with which this decline would occur; in policy A, the decline was assumed to take place in equal linear decreases in each of the 80 years, whereas for policies B, C and D, it declined to its terminal level in the 40th, 20th, and 10th years respectively, and then remained constant at this level for the rest of the planning period. Since the reduction in \(h_t\) from the initial to the terminal level was as large as 35% of its original value, policy D which assumes that the decline is to take place in 10 years clearly would require an extremely active policy of fertility reduction, and policies C or B may in fact represent more realistic targets for actual population policy.

As discussed in the previous section, mortality rates are taken as exogenously given, so that the assumed alternative time paths of the fertility level permit the computation of a set of complete projections of the population by age and sex over the planning period. Together with given values of age-specific labour force participation rates and equivalent-consumer weights, the total labour force \(L_t\) and the number of equivalent consumers \(N_t\) can also be projected for each alternative fertility assumption. Selected summary statistics for the demographic projections are given in Table I.
-17-

Table I

Demographic Projections

<table>
<thead>
<tr>
<th>Policy</th>
<th>POP_40</th>
<th>L_40</th>
<th>N_40</th>
<th>POP_80</th>
<th>L_80</th>
<th>N_80</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(millions)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>14.4</td>
<td>6.2</td>
<td>11.7</td>
<td>25.0</td>
<td>11.8</td>
<td>20.7</td>
</tr>
<tr>
<td>B</td>
<td>13.0</td>
<td>6.0</td>
<td>10.7</td>
<td>19.0</td>
<td>9.2</td>
<td>15.8</td>
</tr>
<tr>
<td>C</td>
<td>11.7</td>
<td>5.6</td>
<td>9.7</td>
<td>16.2</td>
<td>7.8</td>
<td>13.5</td>
</tr>
<tr>
<td>D</td>
<td>11.0</td>
<td>5.3</td>
<td>9.1</td>
<td>15.0</td>
<td>7.3</td>
<td>12.5</td>
</tr>
</tbody>
</table>

The optimal savings problem corresponding to each of the four alternative demographic projections involved the maximization of a social welfare function \( W \) given by:

\[
W = \sum_{t=1}^{80} \left[ \frac{1}{1-\mu} \left( \frac{C^*}{N_t} \right)^{1-\mu} \cdot N_t \right] \frac{1}{(1+R)^{t-1}} (7)
\]

where \( C^*_t \) is "adjusted consumption" as previously defined, \( R \) is the social rate of time preference, set at 5% in all of the numerical experiments, and \( \mu \) is a parameter of the utility function. It can easily be verified that this form for the utility function has the property of constant elasticity of marginal utility of consumption and that the elasticity is in fact equal to \( -\mu \). In the initial example, the value of \( \mu \) was set equal to 2, but as reported below, alternative calculations were also made with \( \mu = 1.25 \).

In each case, the welfare function was maximized subject to the constraints on consumption and capital formation implied by the model of the economy in the previous section, and also subject to the condition that the terminal level of the capital stock be sufficiently high in order for the
economy to reach the level of per capita income implied by the target of a 2% per annum increase in per capita income. The values of the main parameters characterizing the economic model were as follows. The rate of technical change, \( \alpha_1 \), was set at 1% per annum. In the initial calculations, the share of labour in output, \( \alpha_2 \), was set at 70%, while \( \delta \), the rate of depreciation of capital was taken to be 10%, but results are also given below for an example where the value of \( \alpha_2 \) was assumed to be 75%.

Particular interest is attached to the choice of parameter values for the relationship (8) between the fertility variable \( h_t \) and its economic determinants. The excess burden associated with a hypothesized tax-subsidy scheme, and hence the social profitability of a fertility reducing policy, depends crucially on the elasticity of fertility with respect to the opportunity cost of raising children, or, in terms of (8), on the parameter \( \beta_2 \). Because of the importance of this parameter, a sensitivity analysis was carried out by performing each one of the optimizations with two alternative values for it, viz., \( \beta_2 = -.4 \) and \( \beta_2 = -.1 \). The value of the income elasticity of \( h_t \) was assigned through the following procedure. As noted above, a cross-section regression of a fertility variable with per capita income as the only explanatory variable yielded an estimated elasticity of -0.27.

It is likely that the opportunity cost of raising children is related to per capita income, however, so that this estimate can be seen as a composite of the partial elasticity of fertility with respect to per capita income and the partial elasticity of fertility with respect to the opportunity cost multiplied by the elasticity of the opportunity cost with respect to per capita income. In our case, where the opportunity cost is simply measured by the wage rate and where the production function is of the Cobb-Douglas form, it
is easy to show that the latter elasticity equals unity, so that an observed elasticity of fertility with respect to income would simply be the sum of the partial income and opportunity cost elasticities. In the absence of better information, we assumed that the estimate of this sum from the international cross-section results would be applicable over time in Ghana, so that the value of $\beta_1$, which represents the partial income elasticity, was determined by the condition $\beta_2 + \beta_1 = -0.27$; in other words, $\beta_1$ was assigned the values of $+0.13$ and $-0.17$ corresponding to the two assumed values of $\beta_2$ of $-0.4$ and $-0.1$, respectively.

We turn now to a brief description of the results of the numerical experiments. Appendix Table A1 contains selected values of $(C_t/N_t)$, i.e., consumption of goods and services per equivalent consumer, and $(E_t/N_t)$, the excess burden per equivalent consumer, for the optimal solutions with the original set of parameter values as described above. With $\beta_2 = -0.4$, the results for policies B, C and D are given in columns 1, 3 and 5, respectively. The highest value of the objective function in this case is attained for policy D, i.e., the most active population policy in which fertility declines to its terminal level in 10 years. Even though policy D has a somewhat higher excess burden resulting from this rapid decrease in fertility, especially around the 10th and 20th year, the fact that it is the best policy implies that this is more then offset by the impact on the welfare function of the higher levels of per capita consumption of goods and services made possible by lower fertility.

When the value of $\beta_2$ was set at $-0.1$, the time paths of $(C_t/N_t)$ for all policies remained essentially unchanged but, as is evident from columns 2, 4, and 6 in Table A1, the magnitude of the excess burden is now generally
much greater than in the previous case, especially for cases C and D. The alternative which yields the highest value of the objective function is now policy C; in other words, for this low value of the elasticity of fertility with respect to the opportunity cost, the welfare cost implied by the rapid fertility reduction in alternative D is now high enough to offset the increase in consumption per equivalent consumer as one moves from policy C to D.

In the second set of calculations, the share of labour in total output was set at 75% (with the depreciation parameter remaining at 10%) rather than 70%. For each given population policy, this produces a significant change in per capita consumption (see Table A2), both in terms of its general level and its pattern of change over time. With $\beta_2 = -.4$, and $\beta_1 = +.13$, the objective function attains its highest value for policy D, as in the previous case. When $\beta_2 = -.10$ and $\beta_1 = -.17$, however, the best policy becomes alternative B, i.e., that tax-subsidy scheme which reduces fertility to its terminal level over 40 years rather than over 10 or 20 years. It is interesting to note that in the previous example (i.e., with the share of labour at 70%), when $\beta_2$ was set at -.10, the preferred policy was alternative C. An intuitive explanation of why the implied change in the productivity of capital leads to a change in the choice of population policy may be given along the following lines. For a given time path of the population and labour force, the lower productivity of capital implies that along the optimal path, the rate of capital accumulation and hence the rate of growth of the capital/labour ratio and the wage rate will be slower in the early stages of the programme. The slower rate of growth of the wage rate (and hence of the opportunity cost of raising children), however, means that
the level of taxation or subsidy required to bring about a given decline in the fertility level will have to be higher than it would be with a more rapid increase in the wage rate. For a given opportunity cost elasticity of fertility, the higher level of taxation or subsidy will imply a larger excess burden (compare, e.g., col. 6 in Table A1 with col. 6 in A2); this, however, makes it more probable that a policy of a somewhat slower decrease of fertility will be preferred to one involving a rapid decrease.

The final set of examples involved a change in the assumed elasticity of the marginal utility of consumption from -2.0 to -1.25 (i.e., a change in the parameter \( \mu \) in the utility function from +2.0 to +1.25). This modification leads to lower levels of consumption and a higher rate of capital accumulation than in the first example for the first five or so years, but thereafter, income and consumption are higher than in the first example. It is interesting to note that in this case, policy D yields the highest value of the objective function, both when \( \beta_2 = -.4 \) and when it is -.10. Again, the intuitive interpretation of this result would be that the early rapid growth in the capital stock and hence in the wage rate and the opportunity cost of raising children will lead to a rapid reduction in fertility even with a relatively low level of taxation or subsidy, and therefore to a relatively low excess burden (compare, e.g., col. 6, Table A1 with col. 6, Table A3). While one may doubt that the optimal time paths in this example represent realistic alternatives in an actual policy-making situation, a comparison of the results here with those in the first case nevertheless throw an interesting light on the interdependence between the choice of the "best" population policy and the time paths of consumption, capital accumulation, and the opportunity cost of raising children.
V. CONCLUSIONS AND QUALIFICATIONS

As has been noted in the introduction, the existing cost-benefit approach to the evaluation of the profitability of various types of population policy has been subject to considerable criticism, both on the grounds that it represents partial analysis and because it has neglected the welfare costs implicit in a policy of fertility reduction when it is recognized that parents derive utility from the activity of raising children. The work on economic-demographic projection models has served the purpose of illustrating the consequences of fertility reduction in a general equilibrium context, but has generally not incorporated a criterion for the consistent evaluation of different types of population policy in the context of development planning, and has also neglected the welfare costs of fertility reduction. The methodology developed in this paper, on the other hand, constitutes a first step toward the goal of integrating the evaluation of population policy into the general formulation of an optimal development plan, and provides a way in which a choice between different policies can be made on the basis of their impact on individual welfare over time, including the welfare that is derived from raising children. While the numerical results of the application to the Ghanaian case are still preliminary and incomplete in many respects, they clearly indicate the interdependence in the formulation of a population policy and of a general economic plan, and also illustrate how the optimal choice of a population policy is strongly dependent on certain parameters in the relationship between fertility and its economic determinants.

As we indicated above, the numerical results derived in the application to the case of Ghana should, at this stage, primarily be regarded as
illustrations of the methodology rather than as actual policy recommendations. The principal reason for that, of course, is that many of the parameters and data used in the experiments are more in the nature of arbitrary guesses rather than empirical estimates, especially with respect to the demographic part of the model. In future work, one might want to modify, for example, the assumption that age-specific mortality remains constant over time, or that the relative fertility of women in different age groups remains constant over time; changes in the spacing of births, and hence in the age-distribution of fertility, may in fact constitute one of the more important responses to changing income and opportunity cost of raising children. Furthermore, we assumed no time lags between changing economic incentives to child-bearing and the resulting change in fertility. If such lags are important, the appropriate timing of the introduction of incentives to reduce fertility will obviously be different from the one implied in our examples. Finally, we have not discussed the practical problems associated with the formulation of a tax-subsidy scheme which would affect the opportunity cost of raising children, and the possible conflicts that such a scheme would raise with the policy objective of achieving an equitable distribution of income at a given time; in a real world situation, attention would of course also have to be paid to the implications that different forms of a tax-subsidy scheme would have for the government budget.

In spite of these qualifications, we nevertheless feel that the methodology proposed in this paper offers a good deal of promise as a method of integrating the problem of formulating a population policy into the more general problem of development planning. The results of the numerical experiments, moreover, suggest that substantial gains in economic welfare can be made by a rational choice of population policy, and we believe that further work on improving the precision with which this choice can be made would be of considerable interest.
APPENDIX

(i) Selected numerical results

Tables A1 to A3 contain results from some of the numerical experiments described in the text. The tables are arranged as follows. Table A1 corresponds to the initial experiment in which the parameter $\mu$ was set at 2.00 and the share of labour, $\alpha_2$, was set at .70. Table A2 corresponds to the case where $\mu$ remains equal to 2.00 but $\alpha_2$ is set at .75, and A3 to that with $\mu = 1.25$ and $\alpha_2 = .70$. Policies B, C, and D in the column headings imply fertility reductions to the terminal level over 40 years, 20 years, and 10 years, respectively. For each policy, the left-hand column corresponds to $(\beta_2 = -.4, \beta_1 = +.13)$ and the right-hand column to $(\beta_2 = -.1, \beta_1 = -.17)$, as described in the text. For each year shown, the larger number is the value of $(C_t/N_t)$, and the smaller number immediately below it is $(E_t/N_t)$. "Adjusted consumption" per equivalent consumer, $C^*_t/N_t$, is obtained by subtracting the latter from the former.
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TABLE A3
(ii) Sources of data for initial conditions

With respect to the initial data for the demographic projections, the main sources were the 1960 Population Census (data for the 1970 Census were not available at the time of writing) and the work of S. K. Gaisie (undated). Total population by sex and single years of age and sex came from the 1960 Census. Age-specific death rates were taken from Coale and Demeny (1966), North Level 9, p. 228.

Age-specific fertility rates were assumed proportional to those given in Gaisie, p. 10, Table 3, Col. 3 (graduated fertility). The initial value of h, i.e., the number of births per fertility-equivalent female, was computed as that value which, when multiplied by the number of fertility-equivalent females, yielded a crude fertility rate of approximately 51 per thousand, which is the midpoint in Gaisie's range for the true value of that variable in Ghana in 1960 (Gaisie, p. 2). The proportion of male births to total births was assumed to be .509. Age-specific labour force participation rates by sex were obtained from T. M. Brown (1972), Table A2. The weights needed to compute the number of equivalent consumers were the same as those employed by Coale and Hoover (p. 238, footnote 1), i.e., children up to 10 years of age are given the weight .5, males over 10 weighted by 1.0 and females over 10 by .9.

The only information required for the economic model, in addition to the parameters discussed in the text, were the initial levels of total income and capital stock in the starting year, which was taken to be 1960. These were obtained from T. M. Brown (1972), Tables C1 and D1. The figures used were those for "revised GNP" and "Business Stock," respectively.
*Department of Economics, University of Western Ontario. The authors wish to acknowledge their indebtedness to J. Barry Smith and Peter L. Jackson who developed the computer programs used in the numerical applications. Ellen Hall and S. Margaret Grant provided other forms of research assistance. The research is supported by the Economic Development Workshop at the University of Western Ontario which is financed by the Canadian International Development Agency under a twinning arrangement with the University of Ghana Department of Economics.

1 For further discussion of these issues, see Blomqvist [1973].

2 For a notable exception, see Caldwell [1967].

3 It may be noted that if population policy is taken to consist of a tax on the activity of child-rearing, then the methodology just discussed implies that the level of this tax is solved for endogenously. A more natural approach would be to take this tax as predetermined over time and then find the optimal capital accumulation policy. The difficulty with this approach is that if fertility is taken to depend on per capita income and on the opportunity cost of raising children, then fertility and hence the time paths of the number of people in different age brackets become endogenous, which leads to unmanageable computational difficulties.
4 Kenya and Ghana are two of the few countries in Africa south of the Sahara in which population policy is receiving serious attention and in which modest starts of actual family planning programs have been made. (See Omaboe [1969] and Miller in Brown and Sweezy [1972].)

5 The cross section regressions were performed using gross reproduction rates as the dependent variable. If the relative age-specific fertility rates remain constant over time, as assumed in this paper, age-specific fertility rates will be proportional to the gross reproduction rate.


7 The value of $\alpha_2$ was again set at .7 in this case.
REFERENCES


