CALIBRATION OF AN ANALYTICAL METHOD FOR VEHICLE-INDUCED VIBRATIONS ON FLEXIBLE BRIDGES

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ABSTRACT

The results of full scale trials of heavy vehicle-induced vibrations on the Pierre Laporte Bridge are presented. The Pierre Laporte Bridge spans approximately 1 km having the longest main span of any suspension bridge in Canada. The tests were performed during short periods of complete bridge closure on October 27 and 28, 2014. During the tests, the acceleration of the bridge was recorded at multiple locations in addition to measurements of the vertical and pitching accelerations of the testing vehicle itself. The results of these tests are used to calibrate an analytical tool developed to screen bridge designs for potential serviceability concerns due to vehicle-induced vibrations. Numerical simulations are performed using this analytical method and compared directly to the results of the full scale trials. Good agreement is found between the peak predictions and the measurements highlighting the effectiveness of the screening tool during the design of medium- to long-span bridges. No cases of pedestrian discomfort to any kind of dynamic excitations have been reported since the Pierre Laporte Bridge opened in 1970, and the results measured during the full scale trials and those simulated numerically reflect these observations.

Keywords: Vehicle-induced vibrations, Bridge dynamics, Pedestrian comfort, Bridge serviceability

1. INTRODUCTION

Medium- and long-span bridges are generally considered flexible structures susceptible to vibrations from various sources including wind-, pedestrian-, and vehicle-induced excitation. Should the response of the structure from one or more of these excitation sources exceed certain thresholds then it is possible that users of the bridge (i.e., pedestrians, cyclists, motorists) may feel discomfort. To mitigate these serviceability concerns for the bridge, it is recommended that pre-screening tools be used to assess the potential acceleration levels expected from each potential excitation source. The focus of the current work is on a pre-screening tool capable to assess the response of a structure to heavy vehicle-induced excitations.
A set of full scale trials and numerical simulations have been performed in order to calibrate the method previously described by Stoyanoff et al. (2006) to assess vehicle-induced vibrations. These full scale trials took place in the very early morning on October 28, 2014 on the Pierre Laporte Bridge in Quebec City, Quebec, Canada. The Pierre Laporte Bridge spans the Saint Lawrence Seaway connecting Quebec City with Lévis and currently has the longest main span of any suspension bridge in Canada with a main span of 668 m and a total suspended span of 1041 m. The bridge deck is 28.8 m wide with three lanes of traffic in each direction for a total of six lanes. There is also a sidewalk on each side of the bridge as shown in Figure 1. The full scale experiments were performed as part of a larger test program carried out by the Ministère des Transports, de la Mobilité durable et de l’Électrification des transports (MTMDET) including identification of the fundamental frequencies of the bridge, estimation of damping in the lowest modes of vibration, and measurements of deflections under patterns of heavy traffic. It should be noted that no instances of discomfort have been reported on the Pierre Laporte Bridge due to vehicle-induced vibrations. These experiments were performed to take advantage of given rare opportunity to calibrate both the finite element model of the bridge itself and, the analytical tools used to predict vehicle-induced vibrations.

In the assessment of human discomfort on bridges there is a wide variety of published codes and sources. A summary of the thresholds found in various codes and standards are summarized in Figure 2. As shown in Figure 2, the typical criteria for human comfort are expressed in terms of allowable accelerations. Since human comfort for each individual depends on a combination of physiological and psychological factors, there is a wide spread in the data and many factors which govern the perception of comfort or discomfort. For example, it is well established that the acceleration thresholds vary with both exposure time and the frequency of the response (ISO 2631, 1997). In general, any discomfort due to wind-induced vibration on a bridge occurs for a single mode of vibration with a period longer than 1 s. However, for vehicle-induced vibration the acceleration of the structure may occur in several modes of vibration with short time periods. Thus, the exposure time to vibrations must be considered in addition to the level of acceleration when assessing the comfort of pedestrians due to vehicle-induced vibrations. As recommended by ISO 10137 (2007), a 1-second window-averaged root-mean square (rms) appropriately represents the time required to perceive acceleration levels. Therefore, the comfort level of pedestrians can be assessed by the proposed mean square moving average to filter the time series of the response as shown in Eq. 1 where \(a(t)\) is the response of the structure, \(\tau\) is variable of integration over time, and \(t\) is time in seconds.

\[
[1] \quad a_{\text{comfort}}(t) = 1.414 \times \left[ \int_{-0.5}^{0.5} a(t + \tau)^2 d\tau \right]^{1/2}.
\]

The focus of the current work is to evaluate the performance of an analytical model to predict the acceleration of the structure. Few previous studies have been performed comparing full scale measurements to validate the analytical approaches for vehicle-induced vibrations (e.g., Green and Cebon, 1994) therefore the current dataset is especially
useful for calibration purposes. Typically measurements of vehicle-induced vibrations have only been performed on long-span suspension bridges that are open to traffic (e.g., Kim et al. 2003) making it difficult to control the parameters of the experiment. Numerical simulations are carried out and compared with the full scale measurements from the Pierre Laporte Bridge. These comparisons are performed to evaluate the adequacy of the methodology as a screening tool to predict the potential for pedestrian discomfort due to vehicle-induced vibrations at the design stage of medium- and long-span bridges.

2. FULL SCALE EXPERIMENTS

As described in the Introduction, full scale experiments were organized by the MTMDET with some assistance from RWDI in late October, 2014. There were four distinct 1 hour bridge closures to allow for the experiments to be completed. During one of those bridge closures, an instrumented heavy truck passed over the closed bridge and the bridge and vehicle responses were recorded. The following sections describe these experiments in more detail.

2.1 Bridge instrumentation

The bridge was instrumented with six (6) SENSR CX1 network tri-axial accelerometers. These accelerometers were networked over a local area network using Ethernet connections to a data acquisition system. Selected accelerometers have a range of ±1.5 g, where g is the acceleration due to gravity, and a resolution of $1 \times 10^{-5}$ g. The placement of the accelerometers has been marked on the typical section and the plan view of the bridge in Figure 1. The data were sampled at a rate of 400 Hz simultaneously between all six accelerometers and digitally low pass filtered at 10 Hz.

2.2 Truck

A 10-wheel loaded dump truck was used for the full scale experiments with a photo of this vehicle shown in Figure 3. Each axle of the truck was weighed with the parameters summarized in Table 1.

The truck was instrumented with tri-axial SENSR GP2xL USB accelerometers with one accelerometer located at the centre of the front bumper of the truck while the other was located at the centre of the rear bumper. These accelerometers have a measurement range of ±6 g while having a shock limit of up to 1000 g before any damage is sustained. USB cables were run from each accelerometer to a data acquisition computer in the cab of the truck where the data were sampled at a rate of 400 Hz from each accelerometer and digitally low pass filtered at 20 Hz.
In addition to the accelerometers used to measure the response of the truck, a GPS system was used to quantify the speed, position and time of each trial across the bridge. Two different trials were run with the truck in each direction over the bridge at a constant speed of 80 km/h in the same lane.

### Table 1: Parameters of the heavy truck used in the full scale trials

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total mass</td>
<td>25950</td>
<td>kg</td>
</tr>
<tr>
<td>Mass over rear axle 1</td>
<td>9580</td>
<td>kg</td>
</tr>
<tr>
<td>Mass over rear axle 2</td>
<td>9310</td>
<td>kg</td>
</tr>
<tr>
<td>Spacing from front axle to rear axle 1</td>
<td>4.00</td>
<td>m</td>
</tr>
<tr>
<td>Spacing from rear axle 1 to rear axle 2</td>
<td>1.53</td>
<td>m</td>
</tr>
<tr>
<td>Frequency of sprung mass</td>
<td>2.9</td>
<td>Hz</td>
</tr>
<tr>
<td>Damping of sprung mass</td>
<td>6</td>
<td>% critical</td>
</tr>
<tr>
<td>Frequency of unsprung mass</td>
<td>18</td>
<td>Hz</td>
</tr>
<tr>
<td>Damping of unsprung mass</td>
<td>0.1</td>
<td>% critical</td>
</tr>
</tbody>
</table>

### 3. NUMERICAL SIMULATIONS

#### 3.1 Vehicle

A two degree of freedom (2DOF) system representing the first vertical mode of vibration and the wheel-hop frequency for the heavy vehicle is assumed. It is recommended that during the screening process multiple fundamental frequencies are assumed for the heavy vehicle to cover the range of expected forcing frequencies among the wide variety of trucks on the road. The 2DOF system for the vehicle is written

\[
\begin{bmatrix}
    m_u & 0 \\
    0 & m_s
\end{bmatrix}
\begin{bmatrix}
    \ddot{z}_u \\
    \ddot{z}_s
\end{bmatrix}
+ \begin{bmatrix}
    c_u + c_s & -c_s \\
    -c_s & c_s
\end{bmatrix}
\begin{bmatrix}
    \dot{z}_u \\
    \dot{z}_s
\end{bmatrix}
+ \begin{bmatrix}
    k_u + k_s & -k_s \\
    -k_s & k_s
\end{bmatrix}
\begin{bmatrix}
    z_u \\
    z_s
\end{bmatrix}
= \begin{bmatrix}
    -c_u (\dot{z}_b + \frac{dr}{dx} U_t) - k_u (z_b + r) \\
    0
\end{bmatrix}
\]

where, \( z(t) \) describes the motion of the sprung (subscript, s) or unsprung (subscript, u) mass; \( z_b(t) \) describes the response of the bridge; \( r \) is the roughness profile of the road expressed as a function of the distance along the span, \( x \); \( m, c, k \) represent the mass, damping and stiffness of the sprung and unsprung masses; and \( U_t \) is the constant velocity of the truck.

The method proposed here seeks to maintain accuracy while allowing for certain simplifications to remain an efficient screening tool at the design stage. However, it should be noted that the method can be extended to include further
refinements to the vehicle dynamic model (e.g., incorporation of the pitching mode of vibration). For the numerical simulations representing the full scale experiments on the Pierre Laporte Bridge, a two degree of freedom truck was simulated to match the parameters described in Table 1. The dynamic characteristics of the sprung and unsprung masses have been estimated from measurements performed on the vehicle in a parking lot and from published sources (e.g., Papagiannakis and Gujarathi, 1995).

3.2 Road surface

The road surface is described using the displacement power spectral density based on the assumption of constant velocity in ISO 8608 (1995). In this case the power spectral density (PSD), $S_d(f)$, takes the following general form

$$S_d(f) = S_d(f_0) \left( \frac{f}{f_0} \right)^2$$

where, $f$ is the spatial frequency and $f_0$ is a reference spatial frequency ($f_0 = 0.1$ cycles/m). The road surface PSD is generated using the inverse Fourier transform with the phase angle assigned randomly for each frequency. The integration range for the inverse Fourier transform is taken as $f = [0.01, 10]$ cycles/m and the values for $S_d(f_0)$ are provided based on the road condition in Table 2.

Once the road surface profile is generated, a smoothing process in the analysis is applied to reflect the spatial averaging characteristics of the tire contact patch. Based on the distribution of pressure on a tire contact patch, a Gaussian smoothing kernel is convolved with the road surface profile to reflect the characteristics of how the road surface is transmitted to the vehicle suspension system.

Table 2: Values of $S_d(f_0)$ for different road conditions

<table>
<thead>
<tr>
<th>Description*</th>
<th>Road Class</th>
<th>$S_d(f_0) \times 10^{-6} \text{ m}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Good</td>
<td>A</td>
<td>16</td>
</tr>
<tr>
<td>Good</td>
<td>B</td>
<td>64</td>
</tr>
<tr>
<td>Good- Average</td>
<td>C</td>
<td>256</td>
</tr>
<tr>
<td>Average-Poor</td>
<td>D</td>
<td>1024</td>
</tr>
<tr>
<td>Poor</td>
<td>E</td>
<td>4096</td>
</tr>
</tbody>
</table>

* Description of classes is from Stoyanoff et al. (2006)

The primary excitation for the truck suspension system is the road surface roughness. Due to the recent resurfacing of the bridge deck, corroborated by observations during the test campaign, the road roughness was assumed to be in “Very Good” or “Class A” condition. The MTMDET separately measured the profile of the road and the comparison between these measurements and the simulated road roughness is shown in Figure 5.

In addition to road roughness, additional repeating or solitary features of a bridge may be included in the road profile such as groove patterns or expansion joints at their appropriate locations on the bridge. For the current numerical
simulations, the effect of the expansion joints on the bridge has been examined by modeling the roadway both with and without expansion joints. The expansion joints are included as rectangular impulse functions to represent their dimension along the bridge and the depth of the joint. The Gaussian kernel described above is applied after the inclusion of the expansion joints with the goal of mimicking the smoothing effect of the tire passing the expansion joints.

3.3 Structural response

The numerical model used in the current study is a simplified strip version of the full finite element model of the bridge with the discretization shown along with the coordinate system in Figure 6. The total number of strips included in the model is 719 including: 169 strips representing the deck; 30 representing each tower; 85 nodes for each main cable; and 320 representing the hangers. The mass and mass moment of inertia of the structure are lumped to each strip and the six DOF response is solved in the time domain. For more details on the bridge response solution methodology the reader is referred to Stoyanoff (2001).

The loading from the vehicle is transmitted to the structure via triangular influence functions under each wheel as shown schematically in Figure 4. The use of triangular influence functions is expected to retain some conservatism in the analysis.

The numerical simulations were carried out using the analytical method described in Stoyanoff et al. (2006) using the 2DOF model specified in Eq. 2. The solution is carried out in the time domain using a truncated set of modes to describe the mass, damping and stiffness of the structure. Due to the excitation of the vehicle being on the order of 1 Hz, it is recommended to truncate the modes at approximately 10 Hz, which in this case meant that 920 modes of vibration were retained in the analysis. The time step used in the simulations was 0.003125 s which corresponds to a frequency of 320 Hz.

As part of the full scale trials during the bridge closures in late October, 2014, tests were performed to determine the structural damping and natural frequencies of the lowest modes of vibration for the bridge. Further discussion of these

Figure 6: Strip model used for the numerical simulations of the vehicle-induced vibrations.
tests is outside the scope of the current work; however, it should be noted that the frequencies of the lowest modes of vibration from the finite element model of the bridge were calibrated based on these tests and used in the current analysis. Likewise, the structural damping is set to 0.3% of critical in each mode of vibration which was shown to be representative of the lower modes of vibration during the full scale tests; however, it should be noted that the damping may be lower at the small amplitudes observed during the trials with the heavy vehicle.

4. RESULTS

4.1 Results of the numerical simulations

The general output from the simulations is the 1 second \textit{rms} window-averaged peak acceleration estimated for each location along the walkways of the bridge. If these peak accelerations exceed the published standards regarding human comfort then it may indicate the need for remedial measures in the design of the bridge. An example of this output is shown in Figure 7 along with the comfort classes established by Setra (2006) for pedestrian comfort. As observed in Figure 7 all of the peak accelerations fall within the maximum comfort zone indicating that it is highly unlikely that any pedestrians would feel uncomfortable on Pierre Laporte Bridge due to the passage of heavy vehicles. It should be noted that these observations are in-line with the fact that no issues of comfort have been reported over the lifetime of the bridge which opened to the public in 1970.

4.2 Comparisons between numerical simulations and experiments

The comparisons performed here are based on the trials when the truck was passed over the bridge at a constant speed of 80 km/h (e.g., the speed limit on this bridge). The external load acting on the structure is the dead load of the truck plus the dynamic loads induced by the motion of its sprung and unsprung masses. Therefore, the first comparison is between the accelerations measured on the truck with the accelerations predicted in the simulation in Figure 8. Both simulations with and without the inclusion of expansion joints are included in the comparison. It is clear from measurements shown in Figure 8 that the truck responds to the expansion joints. The time series from the simulated 2DOF vehicle is shown to match the relative magnitude of the measurements, and the peaks are better captured with the inclusion of the expansion joints. In both cases good agreement is found between the power spectral densities of the simulation and the measurements. In the comparison of the power spectral densities it is observed that the pitching mode of the truck is not captured by the 2DOF model. It is expected that exclusion of this pitching mode of vibration is conservative in terms of the predicted bridge response. However, this remains a topic for future consideration.
A direct comparison of the acceleration time series measured at the site can be performed with the acceleration time series from the numerical simulations as shown in Figure 9 with the power spectral densities shown in Figure 10. As above for the response of the heavy vehicle, the results are shown for simulations both with and without the expansion joints. Figure 9 demonstrates that the case where the expansion joints have been modeled better replicates the accelerations measured on the bridge due to the passage of a heavy vehicle. In general, the peak amplitudes of acceleration measured on the bridge are well captured by the measurements (as shown in Figure 7 and Figure 9). Assessment of the power spectral densities in Figure 10 reveals a good match between measurements and simulations over the range of frequencies of the input force from the heavy vehicle. However, a significant amount of energy in the acceleration spectra is observed for frequencies below the primary forcing frequency of 2.9 Hz. As mentioned above, one possible explanation for this is the assumed damping of the structure. While 0.3% was shown to be a
reasonable estimate of the structural damping based on field measurements in primary torsional and sway modes, there were modes with significantly lower damping at low amplitudes of motion. Furthermore, it was not possible to estimate the damping in the primary vertical modes of vibration. Thus, it is possible that more energy was damped out of these modes during the simulations than what was observed during the full scale trials.

5. CONCLUSIONS

The analytical method developed in Stoyanoff et al. (2006) and extended to 2DOF in the current study has been shown to be a suitable design tool for medium- to long-span bridges to screen pedestrian comfort in terms of vehicle-induced vibrations. The results of numerical simulations of a heavy vehicle crossing the Pierre Laporte Bridge have been compared with a unique set of field measurements. The conditions of the field measurements were replicated as closely as possible considering the limitations inherent in the simplified analytical solution. The following represent the key findings of this study:

1. In general, good agreement was observed between the peak accelerations measured on the bridge and those predicted by the numerical simulations.
2. The road roughness as predicted using the spectral formulation was shown to match the measured road roughness reflecting a “very good” road condition.
3. The accelerations predicted for the sprung/unsprung mass 2DOF system were observed to reasonably match the results obtained from measurements on the vehicle during the full scale trials.
4. The predicted peak acceleration response of the bridge better matched the measurements when the expansion joints were included.

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