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by

Raveendra Batra
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and the Gains from Trade*

I. Introduction
Recent development in the pure theory of international trade has proceeded along two distinct lines. On the one hand, the usual two traded-commodity, two-factor model, which Bhagwati in his survey (1964) calls the Heckscher-Ohlin-Samuelson (H.O.S.), model has been generalized by introducing a third non-traded good in recognition of the phenomenon that the non-traded goods are a certain fact of life even in countries possessing a very large foreign trade sector: Komiya (1967), Kemp (1969, Ch. 6), and Findlay (1971) among others.¹ On the other hand, the assumption of perfect factor markets in the H.O.S. model has been recently relaxed by a number of economists in recognition of the observed presence of inter-industry factor-price differentials in the underdeveloped as well as developed countries: Hagen (1958), Bhagwati and Ramaswami (1963), Bhagwati and Srinivasan (1971), Batra and Pattanaik (1970) (1971a) (1971b), Herberg, Kemp and Magee (1970), Magee (1971) and Jones (1970) among others.² Both these developments, which constitute a lasting advance in the theory of international trade, have given us greater comprehension of how international trade affects allocation of resources and welfare in models which portray the more realistic situations by recognizing the existence of non-traded goods as well as factor market imperfections. However, the existing literature in trade theory still suffers from the lacunae in so far as it explores the implications of non-traded goods and factor market distortions separately. This procedure would be justified only if non-traded goods did not exist in a world

¹I am grateful to V. S. Rao for useful discussions. He, however, is not responsible for any error.
afflicted by factor market imperfections, and conversely. The purpose of this paper is to fill this gap by exploring the implications of inter-
industry wage-differentials for gains from trade in a three-good, two-factor model, where one of the commodities does not enter foreign trade. The main result is to show that free trade alone may be the second-best optimum policy in spite of the presence of the factor-price differential, provided i) it is paid by either the importable commodity, or ii) the differential exists among all three industries.

In order to facilitate comparison of our results with the existing results, it is necessary to present a brief statement of the "standard" theorems and those derived by Bhagwati and Ramaswami (1963) and Batra and Pattanaik (1970). By the standard or traditional theorems in the theory of gains from trade we mean the results that have been derived under the "ideal," first-best assumptions of Pareto-Optimality. If these conditions are satisfied, then it can be shown that i) free trade is superior to no trade and for a small country, it is an optimum policy (Samuelson, 1939), ii) a lower tariff is superior to the higher tariff (Kemp, 1962), and iii) an improvement in the terms of trade results in an improvement in welfare, and conversely (Krueger and Sonnenschein 1967).

By introducing the inter-industry wage-differential in the two-good model, these standard theorems have been modified in the following manner:

1. an improvement in the terms of trade may lead to a loss of welfare (Batra and Pattanaik, 1970),
2. a higher tariff may be superior to a lower tariff (Batra and Pattanaik, 1970),
3. free trade may be inferior to no trade (Bhagwati and Ramaswami, 1963),
4. and free trade alone is neither a first-best, nor a second-best optimum policy (Bhagwati and Ramaswami, 1963)

Of these four results, the first two will be referred to as the Batra-Pattanaik (B-P) theorem 1 and theorem 2, whereas the last two results will be called the Bhagwati-Ramaswami (B-R) theorem 1 and theorem 2. It may be pointed out here that the necessary condition for the validity of the B-P theorems and B-R theorem 1 is that the factor-price differential is paid by the importable commodity. Stated differently, all the relevant standard theorems continue to be valid in spite of the presence of the wage-differential, if the latter is paid by the exportable good. It turns out that the B-P and B-R theorems are special cases of the more general results derived in the present paper which, by allowing for the co-existence of non-traded goods and factor market imperfections, constitutes one more step closer towards the economic reality prevailing in both the developed and the underdeveloped world.

Section II lays down the general framework of our analysis; the implications of factor market imperfections for welfare are explored concerning an improvement in the terms of trade in Section III, a rise in the tariff rate in Section IV, and the optimality of free trade policy in Section V, which is also concerned with the policy implications that follow from our results.

II. Assumptions and the Model

Unless otherwise specified, the following assumptions will be maintained throughout the paper:
1. There is a small country which produces three goods, $X_1$, $X_2$ and $X_3$, with the help of given supplies of two factors of production, capital ($K$) and labor ($L$). In its trade relations with the rest of the world it takes the international price ratio as given, exports $X_2$ and imports $X_1$. The third good, $X_3$, is not traded.

2. There is perfect competition in product markets, and production functions are homogeneous of the first degree.

3. Factor markets are characterized by a) perfect internal mobility of factors, and perfect factor-price flexibility, and b) the reward of capital is the same in all three sectors, but the wage-rate differs among all three industries.

4. There is full employment of factors, which is ensured by the presence of factor-price flexibility.

5. Foreign trade is stable and inferior goods are absent. In addition all goods are gross substitutes.

6. The welfare of an economy can be represented by a well-behaved social utility function which is assumed to have the same properties as those of the individual indifference curves.

7. All three goods are produced and consumed under all conditions, so that there is no satiation in consumption and no specialization in production.

Let $U$ denote the total utility which a community derives from the consumption of the three goods, with their demand denoted by $D_i$ ($i=1,2,3$).

Then

(1) \[ U = U(D_1, D_2, D_3) \]

and
(2) \[ D_1 = X_1 + E_1, \]
(3) \[ D_2 = X_2 - E_2, \]
(4) \[ D_3 = X_3. \]

where \( E_1 \) equals the import of the first commodity and \( E_2 \) equals the export of the second commodity. Both \( E_1 \) and \( E_2 \) are defined to be non-negative.

The balance of trade equilibrium requires that

(5) \[ P_1E_1 = P_2E_2, \]

where \( P_1 \) and \( P_2 \) are the foreign prices of the first and the second commodity, respectively. Both \( P_1 \) and \( P_2 \) are exogenously determined for the small country under consideration. The three linearly homogeneous productions are given by

(6) \[ X_i = F_i(K_i, L_i) = L_i f_i(k_i) \quad (i=1,2,3), \]

where \( K_i \) and \( L_i \) are the capital and labor inputs and \( k_i = K_i / L_i \) is the capital/labor ratio in the \( i^{th} \) commodity. Let \( F_{Ki} \) and \( F_{Li} \) be the marginal productivity of capital and labor, respectively, in the \( i^{th} \) commodity. Then \( F_{Ki} = f_i' \) and \( F_{Li} = f_i - k_i f_i' \). The price of each factor of production equals the value of its marginal product. With the reward of capital (\( r \)) assumed to be the same in all industries,

(7) \[ r = P_1 f_1' = P_2 f_2' = P_3 f_3', \]

where \( P_3 \) is the price of the non-traded good. Let \( w_i \) stand for the wage-rate in the \( i^{th} \) industry. With the existence of the wage-differentials in all industries

\[
\frac{w_1}{\alpha_1} = \frac{w_2}{\alpha_2} = \frac{w_3}{\alpha_3}, \quad \text{or}
\]
\begin{equation}
\frac{P_1(f_1-k_1f_1')}{\alpha_1} = \frac{P_2(f_2-k_2f_2')}{\alpha_2} = \frac{P_3(f_3-k_3f_3')}{\alpha_3},
\end{equation}

where \( \alpha_i \geq 1 \) \((i = 1, 2, 3)\) is a constant. In the absence of wage-differentials, \( \alpha_1 = \alpha_2 = \alpha_3 = 1 \). In the presence of the wage-differential, however, the following possibilities may arise:

1. \( \alpha_1 = \alpha_2 < \alpha_3 \), and the wage-differential is paid by the non-traded good.
2. \( \alpha_1 = \alpha_3 < \alpha_2 \), and the differential is paid by the exportable good.
3. \( \alpha_2 = \alpha_3 < \alpha_1 \), and the differential is paid by the importable good.

In addition to these, there are six more possibilities if the wage-differential exists among all the industries. These possibilities are:

\begin{align*}
\alpha_1 & \geq \alpha_2 \geq \alpha_3, \text{ and } w_1 \geq w_2 \geq w_3; \\
\alpha_1 & \geq \alpha_3 \geq \alpha_2, \text{ and } w_1 \geq w_3 \geq w_2; \\
\alpha_2 & \geq \alpha_1 \geq \alpha_3, \text{ and } w_2 \geq w_1 \geq w_3.
\end{align*}

Under full employment,

\begin{equation}
L_1 + L_2 + L_3 = L
\end{equation}

and

\begin{equation}
K_1 + K_2 + K_3 = L_1k_1 + L_2k_2 + L_3k_3 = K.
\end{equation}

This completes the specification of our model which allows for the presence of the factor-price differential as well as a non-traded good. With the prices of the traded goods \((P_1 \text{ and } P_2)\) determined internationally, a subset of equations (7) and (8) consisting of two equations

\begin{align*}
P_{1}f_{1}' &= P_{2}f_{2}' \\
\frac{P_{1}(f_{1}-k_{1}f_{1}')}{\alpha_{1}} &= \frac{P_{2}(f_{2}-k_{2}f_{2}')}{\alpha_{2}}
\end{align*}
can be solved for two unknowns $k_1$ and $k_2$, which in turn determine factor prices in the traded commodities. If we assume that the factor-intensities are non-reversible, the solution to this subset will be unique. Once factor-intensities are determined in the traded goods, $P_3$ and $k_3$ can be determined from the two equations

$$P_j f_j' = P_3 f_3'$$

and

$$\frac{P_j (f_j' - k_j f_j')}{\alpha_j} = \frac{P_3 (f_3' - k_3 f_3')}{\alpha_3}.$$  

(j = 1, or 2).

Thus once the prices of traded goods are given, the price of the non-traded good, factor prices and factor proportions in the three industries are uniquely determined. With factor and commodity prices thus determined, the supply and demand for commodities will also be uniquely determined.

The Relationships Among $X_1$, $X_2$ and $X_3$: For a small country $P_1$, $P_2$ and $P_3$ are given, so that $k_i$ is constant. Differentiating $X_1$ and $X_2$ from (6) with respect to $X_3$, we obtain

$$\frac{dX_i}{dX_3} = \frac{f_i}{f_i} \frac{dL_i}{dX_3}.$$  

(i = 1,2,3).
Substituting this in the differentiation of the full employment equations (9) and (10) then yields

\[
\frac{1}{f_1} \frac{dx_1}{dx_3} + \frac{1}{f_2} \frac{dx_2}{dx_3} = - \frac{1}{f_3},
\]

\[
\frac{k_1}{f_1} \frac{dx_1}{dx_3} + \frac{k_2}{f_2} \frac{dx_2}{dx_3} = - \frac{k_3}{f_3},
\]

which in turn can be solved to obtain

\[\frac{dx_1}{dx_3} = \frac{f_1 (k_3 - k_2)}{f_3 (k_2 - k_1)} \quad (11)\]

\[\frac{dx_2}{dx_3} = \frac{f_2 (k_1 - k_3)}{f_3 (k_2 - k_1)} \quad (12)\]

Equations (11) and (12) reveal the importance of factor-proportions in the three industries in the determination of the relationships among \(X_1\), \(X_2\) and \(X_3\). These relationships can be obtained in yet another way which tends to remind us of the presence of the inter-industry wage-differential. Totally differentiating \(X_1\) and \(X_2\) in (6) and dividing through by \(dx_2\), we obtain

\[\frac{dx_1}{dx_2} = \frac{F_{K1} dK + F_{L1} dL}{F_{K2} dK + F_{L2} dL} \quad (13)\]

where \(F_{Ki}\) and \(F_{Li}\) are, respectively, the marginal productivities of capital and labor in the \(i\)th commodity. From (9) and (10),

\[dL_1 + dL_2 + dL_3 = 0, \quad \text{and}\]

\[dK_1 + dK_2 + dK_3 = 0.\]

Using these relations and (7) and (8), (13) can be written as,

\[\frac{dx_1}{dx_2} = - \frac{P_2}{P_1} \left( \frac{F_{K2} (dK_2 + dK_3) + (\alpha_1/\alpha_2) F_{L2} (dL_2 + dL_3)}{F_{K2} dK_2 + F_{L2} dL_2} \right), \quad \text{or}\]

\[\frac{dx_1}{dx_2} = - \frac{P_2}{P_1} \left( \beta + \frac{F_{K2} dK_3 + (\alpha_1/\alpha_2) F_{L2} dL_3}{F_{K2} dK_2 + F_{L2} dL_2} \right), \quad (14)\]
where \( \beta = \frac{F_{K2} dK_2 + (\alpha_1 / \alpha_2) F_{L2} dL_2}{F_{K2} dK_2 + F_{L2} dL_2} \geq 1 \) according as \( (\alpha_1 / \alpha_2) \geq 1 \). In the analogous manner,

\[
\frac{dX_3}{dX_2} = \frac{P_2}{P_3} \left[ \frac{F_{K2} dK_3 + (\alpha_3 / \alpha_2) F_{L2} dL_3}{F_{K2} dK_2 + F_{L2} dL_2} \right] = \frac{P_2}{P_3} \left[ \frac{F_{K2} dK_3 + (\alpha_1 / \alpha_2) F_{L2} dL_3}{(F_{K2} dK_2 + F_{L2} dL_2) \lambda} \right].
\]

where \( \frac{1}{\lambda} = \frac{F_{K2} dK_3 + (\alpha_3 / \alpha_2) F_{L2} dL_3}{F_{K2} dK_3 + (\alpha_1 / \alpha_2) F_{L2} dL_3} \geq 1 \), if \( (\alpha_3 / \alpha_2) \geq (\alpha_1 / \alpha_2) \), or \( \lambda \leq 1 \), according as \( (\alpha_1 / \alpha_3) \leq 1 \). It may now be seen that the second term within the brackets of (14) equals \( (\lambda P_3 / P_2) (dX_3 / dX_2) \). Hence

\[
(15^*) \quad \frac{dX_1}{dX_2} = - \frac{P_2}{P_1} \left[ \beta + \frac{\lambda P_3}{P_2} \cdot \frac{dX_3}{dX_2} \right], \quad \text{or}
\]

\[
(15) \quad P_1 dX_1 + \beta P_2 dX_2 + \lambda P_3 dX_3 = 0.
\]

Equation (15) presents an alternative form of the relationship among \( X_1, X_2 \) and \( X_3 \). In the absence of the wage differential, \( \beta = \lambda = 1 \), so that (15) reduces to \( P_1 dX_1 + P_2 dX_2 + P_3 dX_3 = 0 \). If, in addition, there is no non-traded good, we arrive at the traditional result that \( (dX_1 / dX_2) = - (P_2 / P_1) \), or that the marginal rate of transformation equals the negative of the price-ratio between traded commodities. Equation (15) will be extensively used in the derivation of general equations concerning welfare propositions in the presence of the wage-differential and the non-traded good.

III. Welfare and the Change in the Terms of Trade

In this section we examine the Batra-Pattnaik (1970) theorem derived in the absence of the non-traded goods that an improvement in the terms of trade may result in a loss of welfare, and vice-versa, if the wage-differential is paid by the importable good. In our model, \( X_2 \) is the exportable good and \( X_1 \) the importable good.
Therefore an exogenous improvement (deterioration) in the terms of trade can result from a rise (fall) in \( P_2 \) alone, or a fall (rise) in \( P_1 \) alone. Let us consider the implications for welfare of a slight change in \( P_2 \) only, keeping \( P_1 \) constant. Differentiating the social utility function given by (1) totally with respect to \( P_2 \), and dividing through by \( U_1 \), we get

\[
(16^*) \quad \frac{1}{U_1} \frac{dU}{dP_2} = \frac{dD_1}{dP_2} + \frac{U_2}{U_1} \cdot \frac{dD_2}{dP_2} + \frac{U_3}{U_1} \cdot \frac{dD_3}{dP_2},
\]

where \( U_i = \frac{\partial U}{\partial D_i} \) is the marginal utility of the \( i \)th commodity. Under conditions of consumer-equilibrium, the marginal rate of substitution equals the price ratio, or \( \frac{U_2}{U_1} = \frac{P_2}{P_1} \) and \( \frac{U_3}{U_1} = \frac{P_3}{P_1} \). Therefore \( (16^*) \) becomes

\[
(16) \quad \frac{1}{U_1} \frac{dU}{dP_2} = \frac{dD_1}{dP_2} + \frac{P_2}{P_1} \cdot \frac{dD_2}{dP_2} + \frac{P_3}{P_1} \cdot \frac{dD_3}{dP_2}.
\]

Differentiating \( (2)-(5) \) with respect to \( P_2 \), we obtain

\[
(2^*) \quad \frac{dD_1}{dP_2} = \frac{dX_1}{dP_2} + \frac{dE_1}{dP_2},
\]

\[
(3^*) \quad \frac{dD_2}{dP_2} = \frac{dX_2}{dP_2} - \frac{dE_2}{dP_2},
\]

\[
(4^*) \quad \frac{dD_3}{dP_2} = \frac{dX_3}{dP_2},
\]

\[
(5^*) \quad \frac{dE_1}{P_1 dP_2} = \frac{dE_2}{P_2 dP_2} + E_2.
\]

Substituting \( (2^*)-(5^*) \) in \( (16) \), we get

\[
(17^*) \quad \frac{1}{U_1} \frac{dU}{dP_2} = \frac{E_2}{P_1} + \frac{dX_1}{dP_2} + \frac{P_2}{P_1} \cdot \frac{dX_2}{dP_2} + \frac{P_3}{P_1} \cdot \frac{dD_3}{dP_2}.
\]

Dividing through \( (15) \) by \( P_1 dP_2 \) and substituting \( \frac{dX_1}{dP_2} \) in \( (17^*) \) and remembering that \( \frac{dD_3/dP_2}{dP_2} = (dX_3/dP_2) \), we obtain
\[
(17) \quad \frac{1}{U_1} \frac{dU}{dP_2} = \frac{E_2}{P_1} + \frac{P_2}{P_1} \cdot \frac{dX_2}{dP_2} (1-\beta) + \frac{P_3}{P_1} \frac{dD_3}{dP_2} (1-\lambda).
\]

This last equation furnishes the long sought-after relationship between the terms of trade and welfare measured in terms of the first commodity. The following results may now be derived.

1. If there is no wage-differential, then \( \beta = \lambda = 1 \), and (17) reduces to \( (1/U_1)(dU/dP_2) = E_2/P_1 > 0 \). In other words, an improvement in the terms of trade results in an improvement in welfare, and conversely, if there is no wage-differential. This is a straightforward generalization of the standard result to our model which permits the existence of a non-traded good.\(^5\)

2. Suppose the wage-differential exists, but the non-traded good does not. Following Batra and Pattanaik, it is assumed that \( dX_2/dP_2 > 0 \) in spite of the wage-differential.\(^6\) Then if \( \beta > 1 \), it can be easily seen that an improvement (deterioration) in the terms of trade may lead to a loss (rise) in welfare. Now \( \beta > 1 \) implies that \( \alpha_1 > \alpha_2 \), which from (8) means that the differential is paid by \( X_1 \), the importable commodity. This is a simple proof of the B-R theorem 1, and is obtained as a special case of the general equation (17).

3. Suppose that the non-traded good exists. We have already established that \( \beta > 1 \), if \( (\alpha_1/\alpha_2) > 1 \) and that \( \lambda > 1 \), if \( (\alpha_1/\alpha_3) > 1 \). Let us first consider the case where the differential is paid by the non-traded good. Here \( 1 = \alpha_1 = \alpha_2 < \alpha_3 \), so that \( \beta = 1 \) and \( \lambda < 1 \).

Under our assumption that all commodities are gross substitutes, \( (dD_3/dP_2) > 0 \). Hence from (17), we can see that \( (1/U_1)(dU/dP_2) > 0 \). Thus we conclude that if the wage-differential is paid by the non-traded good, and if the commodities are gross substitutes, then an improvement (deterioration) in the terms of trade results in an improvement (deterioration) in welfare.\(^7\)
Furthermore, the change in welfare is even greater than that would result in the absence of the wage-differential.

4. Next consider the case where the differential is paid by the exportable good \( (X_2) \) in the presence of the non-traded good. Here \( 1 = \alpha_1 = \alpha_3 < \alpha_2 \). Therefore \( \beta < 1 \) and \( \lambda = 1 \). The sign of \( \frac{dX_2}{dP_2} \) is now ambiguous. Even if the presence of the wage-differential causes no modification to the positive sign of \( \frac{dX_2}{dP_2} \), the presence of the non-traded good may. As demonstrated by Komiy (1967), the sign of \( \frac{dX_2}{dP_2} \) is ambiguous in the presence of the third non-traded good, even if gross substitutability among commodities is assumed. Thus if \( \frac{dX_2}{dP_2} > 0 \), \( \frac{1}{U_1}(dU/dP_2) > 0 \) and another theorem by Batra and Pattanaik (1970) that an improvement in the terms of trade leads to an improvement in welfare in spite of the wage-differential, provided the differential is paid by the exportable good, continues to hold when a non-traded good is introduced. However, if \( \frac{dX_2}{dP_2} < 0 \), this theorem may not hold.

5. If the differential is paid by the importable good, then \( 1 = \alpha_2 = \alpha_3 < \alpha_1 \). Here \( \beta > 1 \) and \( \lambda > 1 \). If \( \frac{dX_2}{dP_2} > 0 \), the B-P theorem contained in result (2) is strengthened; if \( \frac{dX_2}{dP_2} < 0 \), this theorem is weakened.

6. Of greater interest is the case where result (5) can be shown to be crucially dependent on factor-proportions in all three industries. For (17) can be written as

\[
\frac{1}{U_1} \frac{dU}{dP_2} = \frac{E_2}{P_1} + \frac{P_3 (1-\lambda)}{P_1} . \frac{dD_2}{dP_2} [-1 + \frac{P_2}{P_3} \frac{(1-\beta)}{(1-\lambda)} \cdot \frac{dX_2}{dX_3}].
\]

Substituting \( \frac{dX_2}{dX_3} \) from (12) in this, we get

\[
\frac{1}{U_1} \cdot \frac{dU}{dP_2} = \frac{E_2}{P_1} + \frac{P_3 (1-\lambda)}{P_1} \frac{dD_2}{dP_2} [-1 + \frac{P_2 (1-\beta)}{P_3 (1-\lambda)} \cdot \frac{f_2(k_1-k_3)}{f_3(k_2-k_1)}].
\]

With the wage-differential paid by the importable good, \( \lambda \) and \( \beta \) are both greater
than unity, so that \((1-\lambda) < 0\) and \((1-\beta)/(1-\lambda) > 0\). The effect of a change in \(P_2\) on welfare then depends on the sign of the terms within the brackets, and hence on the factor-proportions in the three industries. A sufficient condition for the terms of trade to have a positive relationship with social welfare is that the square bracketed term is zero or negative, that is

\[
\frac{P_3(1-\lambda)}{P_2(1-\beta)} \leq \frac{f_2(k_3-k_1)}{f_3(k_2-k_1)}. \tag{19}
\]

Since the left-hand side of (19) is positive, a necessary condition for (19) to be satisfied is that its right-hand side be positive, which in turn requires that either \(k_1 \geq k_3 \geq k_2\) or \(k_1 \geq k_2 \geq k_3\). However, condition (19) is violated if \(k_1\) lies between \(k_2\) and \(k_3\), so that with \(\lambda > 1\), an improvement in the terms of trade may then lead to a loss of welfare, and conversely. This is a variant of the B-P theorem 1, and it highlights the role of factor-proportions of the three industries in determining the implications of a change in the terms of trade for welfare, when the differential is paid by the importable good and all goods are gross substitutes.

One may also like to analyze the effects of the wage differentials prevailing in all the industries. In this case, the results derived above may be reinforced or accentuated. It is of some interest to note here that the effects of the wage-differentials among all three industries may cancel out each other and leave the standard theorem unscathed. The necessary condition for this "cancelling effect" is again given by condition (19) with equality.

IV. Higher Versus Lower Tariffs

This section is concerned with the implications of a rise in the rate of tariff on welfare. Assume that a small tariff already exists on the imports
of the first commodity, so that

\[ p_1^* = p_1 (1+t), \]

where \( p_1^* \) is the domestic, tariff-inclusive price for the importable commodity and \( t \) is the rate of non-prohibitive tariff. With \( p_1 \) constant,

\[ \frac{dp_1^*}{dt} = p_1 > 0. \tag{20} \]

Differentiating (1)-(5) with respect to \( t \) and remembering that here \( U_2/U_1 = p_2/p_1^* \) and \( U_3/U_1 = p_3/p_1^* \), we obtain

\[
\frac{1}{U_1} \frac{dU}{dt} = \frac{dE_1}{dt} + \frac{dX_1}{dt} + \frac{p_2}{p_1^*} \frac{dX_2}{dt} - \frac{p_1}{p_2} \frac{dE_2}{dt} + \frac{p_3}{p_1^*} \frac{dX_3}{dt} \tag{21}
\]

\[ = \left[ \frac{t}{(1+t)} \right] \frac{dE_1}{dt} + \frac{dX_1}{dt} + \frac{p_2}{p_1^*} \frac{dX_2}{dt} + \frac{p_3}{p_1^*} \frac{dX_3}{dt} \] (because \( \frac{p_1}{p_1^*} = \frac{1}{(1+t)} \)).

With the imposition of the tariff, \( p_1 \) is replaced by \( p_1^* \) in (15), so that

\[ p_1^* dX_1 + \beta p_2 dX_2 + \lambda p_3 dX_3 = 0. \tag{22} \]

Dividing through (22) by \( p_1^* dt \), substituting for \( dX_1/dt \) in (21) and with some manipulation with the help of (20), we obtain

\[
\frac{1}{U_1} \cdot \frac{dU}{dt} = \frac{1}{1+t} \left[ p_1 t \frac{dE_1}{dP_1^*} + p_2 \frac{dX_2}{dP_1^*} (1-\beta) + p_3 \frac{dX_3}{dP_1^*} (1-\lambda) \right]. \tag{23}
\]

Equation (23), which indicates the effects of a change in the tariff rate on welfare, may now be utilized to derive the following results.

1. To begin with, consider these effects in the absence of the wage-differential, so that with \( \beta = \lambda = 1 \), (23) reduces to

\[ \frac{1}{U_1} \frac{dU}{dt} = \frac{p_1 t}{1+t} \cdot \frac{dE_1}{dP_1^*}. \]
In the absence of non-traded goods, \( \frac{dE_1}{dP_1^*} < 0 \), that is, a rise in the domestic price of the importable good resulting from the rise in tariff lowers the demand for imports, so that \((1/U_1)(dU/dt) < 0\). In other words, a rise in the tariff rate leads to a decline in welfare. This is the standard theorem derived by Kemp (1962). However, when a non-traded good is introduced, the sign of \( \frac{dE_1}{dP_1^*} \) becomes ambiguous, because now the repercussions of a change in the domestic price of the importable good on the price of \( X_3 \) have also to be considered. Even if all goods are assumed to be gross substitutes, Komiya (1967) has shown that the sign of \( \frac{dE_1}{dP_1^*} \) may be uncertain, simply because of the ambiguity in the sign of \( \frac{dX_1}{dP_1^*} \), which is possible when a third non-traded good is incorporated into the model. Thus we conclude that the effects of a rise in the tariff rate on social welfare are uncertain in the presence of the non-traded good. What is of great interest is that a higher tariff may lead to higher welfare than a lower tariff in the presence of the non-traded good, in spite of the absence of the wage-differential, so that the standard theorem by Kemp may not hold.

2. Suppose the wage-differential exists, but the non-traded good does not. Following Batra and Pattanaik (1970), it is assumed that \( \frac{dX_2}{dP_1^*} < 0 \) in spite of the presence of the wage-differential which also ensures that \( \frac{dE_1}{dP_1^*} < 0 \). In the absence of the non-traded good, (23) reduces to

\[
\frac{1}{U_1} \cdot \frac{dU}{dt} = \frac{1}{(1+\epsilon)} \left[ (1-t) \frac{dE_1}{dP_1^*} + P_2 \frac{dX_2}{dP_1^*} (1-\beta) \right].
\]

If \( \beta < 1 \), that is \( \alpha_1 \leq \alpha_2 \), so that the wage-differential is paid by the exportable good, \((1/U_1)(dU/dt) < 0\), and the standard theorem by Kemp continues to hold. However, if \( \beta > 1 \), that is \( \alpha_1 > \alpha_2 \), so that the differential is paid by the importable good, it is possible that \((1/U_1)(dU/dt) > 0\), and a higher tariff
may lead to an improvement in welfare, and vice-versa. This is a simple proof of the B-P theorem 2.

3. Suppose that the non-traded good exists. Let us first consider the case where the differential is paid by the non-traded good, so that \(1 = \alpha_1 = \alpha_2 < \alpha_3\), and \(\beta = 1\), but \(\lambda < 1\). Here (23) reduces to
\[
\frac{1}{U_1} \cdot \frac{dU}{dt} = \frac{1}{1+t} \left[ P_1 \frac{dE_1}{dP_1} + P_3 \frac{dD_3}{dP_1} (1-\lambda) \right].
\]

As stated before, the sign of \((dE_1/dP_1^*)\) is uncertain in the presence of the non-traded good. Even if we assign a "normal" negative sign to \(dE_1/dP_1^*\), then, under the gross-substitutability assumption which renders \((dD_3/dP_1^*) > 0\), the standard theorem by Kemp may not hold when the wage-differential is paid by the non-traded good. Thus we conclude that in the presence of a wage-differential paid by the non-traded good, a rise in tariff may lead to a rise in welfare, and conversely.

4. Next consider the case where the differential is paid by the exportable good in the presence of the non-traded good. Here \(1 = \alpha_1 = \alpha_3 < \alpha_2\), so that \(\beta < 1\) and \(\lambda = 1\). Equation (23) now reduces to
\[
\frac{1}{U_1} \frac{dU}{dt} = \frac{1}{1+t} \left[ P_1 \frac{dE_1}{dP_1} + P_2 \frac{dX_2}{dP_1^*} (1-\beta) \right].
\]

If, following the two-good, Batra-Pattanaik model (1970), we assume that \((dE_1/dP_1^*) < 0\) and \((dX_2/dP_1^*) < 0\) regardless of the presence of the non-traded good and the wage-differential, one can see that \((1/U_1)(dU/dt)\) is unambiguously negative, so that the rise in tariff necessarily leads to the loss in welfare, and the standard theorem by Kemp holds. However, if both \((dX_2/dP_1^*)\) and \((dE_1/dP_1^*)\) are positive, the standard theorem may not hold.
5. If the differential is paid by the importable good, then \( \lambda = \alpha_2 = \alpha_3 < \alpha_1 \), and both \( \beta \) and \( \lambda \) exceed unity. Reverting to (23), it may be seen that if \( \frac{dX_2}{dP_1^*} < 0 \), then the B-P theorem 2 continues to hold and is reinforced if \( \frac{dD_3}{dP_1^*} > 0 \).

6. By following a procedure similar to the one used in obtaining result 6 in the previous section, one can also establish the significance of factor-proportions in the three industries for the change in welfare when the differential is paid by the importable good. This possibility will not be pursued further. The interested reader can derive his own conclusions with the help of condition (19).

If the wage-differentials exist among all three industries, the results derived above may be strengthened or weakened. Again it may be noted that the effects of the wage-differential may cancel out and the last two terms in the square brackets of (23) may add to zero. But the standard theorem by Kemp may still be invalid because of the possibility of the positive sign of \( \frac{dE_1}{dP_1^*} \).

V. Free Trade Versus No Trade and Optimum Policy

In the last section we have shown that there is no monotonic relationship between the tariff rate and the level of social welfare when the inter-industry wage-differential is present even if \( \frac{dE_1}{dP_1^*} \) has the "normal" negative sign. Since free trade represents the absence of the tariff and no trade may be considered to be a situation with prohibitive tariff, the results derived in the previous section are applicable to the questions under consideration in the present section. In the absence of the monotonic relationship between the rate of tariff and social welfare, we conclude that in the presence of the inter-industry wage-differential, free trade may or may not be superior to no trade.
In other words, the B-R theorem I continues to hold when a non-traded good is introduced in their model incorporating factor market imperfections, although the condition that the wage-differential should be paid by the importable good ceases to be the necessary condition.

This brings us directly to the question of optimal policy in the presence of the wage-differential and the non-traded good. Here it is fruitful to draw a distinction between the optimal policy in the presence of a stable inter-industry factor-price differential, which we shall call the second-best policy, and the optimal policy that serves to eliminate the factor-price differential to the producers, so that the optimum optimorum is achieved. This latter policy will be referred to as the first-best policy.

In a recent article, Bhagwati, Ramaswami and Srinivasan (1969) have mathematically derived the first and second-best policies derived originally by Bhagwati and Ramaswami (1963) in the presence of the distortionary wage-differential. The method of analysis used in this section runs parallel to the one followed by Bhagwati, Ramaswami and Srinivasan. Specifically, we derive the expression for the change in social welfare resulting from a slight deviation from the initial situation of laissez faire. If this expression is non-zero, then an increase in welfare could be secured by introducing a suitable policy; otherwise free trade alone is the optimal policy. The expression for the change in welfare can be obtained by totally differentiating (1)-(5) and following essentially the same procedure as that used in
Section III. Thus we obtain

\[ \frac{dU}{U_1} = \frac{P_2}{P_1} \ dx_2 (1-\beta) + \frac{P_3}{P_1} \ dx_3 (1-\lambda). \]

The following results are then immediate:

1. The necessary condition for an interior maximum is that \((dU/U_1) = 0\).

If the wage-differential does not exist so that \(\beta = \lambda = 1\), this condition is automatically satisfied. In other words, free trade is the first-best policy in the absence of the distortionary wage-differential even when a non-traded good is introduced.

2. If the non-traded good does not exist, (24) reduces to

\[ \frac{dU}{U_1} = \frac{P_2}{P_1} \ dx_2 (1-\beta). \]

Here \(\beta\) reflects the divergence between the marginal rate of transformation and the international-price ratio, because from (15*) \((dx_1/dx_2) = -(\beta P_2/P_1)\) when the non-traded goods are absent. The optimal policy in the presence of the wage-differential requires a change in \(x_2\) such that the divergence between the marginal rate of transformation and the given international-price ratio is eliminated, so that the value of \(\beta\) to the producers comes to equal unity. The second-best policy in other words requires the imposition of production-tax-cum-subsidy on second commodity. This is the Bhagwati-Ramaswami theorem. The production tax-cum-subsidy required to attain the second-best situation is given by

\[ \beta = - \frac{(dx_1/dx_2)}{P_2/P_1} = \frac{f_1}{f_2} \frac{k_2}{k_2} = \frac{\phi_2}{\phi_1}, \]

where \(\phi_1\) and \(\phi_2\) are, respectively, the relative shares of capital in the first and the second commodity. Thus the amount of the required production
tax-cum-subsidy depends on the relative share of capital in the two commodities.

The first-best policy, of course, consists in the elimination of the effective wage-differential to the producers by means of the factor tax-cum-subsidy policy, such that \((\alpha_1/\alpha_2)\) and \(\beta\) are equated to one. This is the proof of the first-best policy prescribed by Bhagwati and Ramaswami.

3. Suppose the non-traded good exists. Then if the wage-differential is paid either by the non-traded or the exportable good, so that only \(\beta\) or \(\lambda\) differs from unity, the second-best policy requires the imposition of the production tax-cum-subsidy on the second commodity, or a consumption tax-cum-subsidy on the third commodity such that the divergence to the producers between \((dX_1/dX_2)\) and \((P_2/P_1)[1 + (P_3/P_2)(dX_3/dX_2)]\) is eliminated, that is to say, the effective value of \(\beta\) or \(\lambda\), as the case may be, is equalized to unity. 10

Here again the first-best policy should aim at removing the root cause of the problem, that is, at the elimination of the wage-differential by equating either \(\beta\) or \(\lambda\) to unity by means of the factor tax-cum-subsidy policy.

4. However, if the wage-differential is paid by the importable good, then, given that the non-traded good exists, both \(\beta\) and \(\lambda\) exceed unity. Here is is possible that \((dU/U_1) = 0\), provided

\[
P_2 dX_2(1-\beta) + P_3 dX_3(1-\lambda) = 0, \text{ or}
\]

\[
\frac{P_3(1-\lambda)}{P_2(1-\beta)} = \frac{dX_2}{dX_3}.
\]

In this case free trade alone is the second-best policy and the imposition
of the production tax-cum-subsidy is not needed. Substituting \( \frac{dX_2}{dX_3} \) from (12), we obtain our familiar condition (19) with equality, i.e.,

\[
\frac{P_3(1-\lambda)}{P_2(1-\beta)} = \frac{f_2(k_3-k_1)}{f_3(k_2-k_1)}. \tag{19*}
\]

It is clear that if either \( \lambda \), or \( \beta \), or both equal unity, condition (19*) cannot be satisfied if factor-proportions among industries differ, which has been usually assumed in the existing literature on non-traded goods. Now \( \beta \) or \( \lambda \) equals unity if the wage-differential is paid by the non-traded or the exportable good. Thus we conclude that if the differential is paid by either the exportable or the non-traded industry, the second-best optimum can be achieved only by following the Bhagwati-Ramaswami prescription. However, if the differential is paid by the importable good, so that both \( \beta \) and \( \lambda \) exceed unity, it is possible that condition (19*) is satisfied and free trade alone becomes the optimum policy in the presence of the wage-differential. It may be pointed out here that it is precisely this type of wage-differential (i.e., that paid by the importable good) that generates the possibility of free trade being inferior to no trade in the Bhagwati-Ramaswami model. But when a non-traded good is introduced, it is this type of wage differential which may obviate the necessity of imposing the production tax-cum-subsidy in order to attain the second-best welfare. Since \( (1-\lambda)/(1-\beta) > 0 \) in this case, a sufficient condition for (19*) to be satisfied is that \( k_1 \gtrsim k_3 \gtrsim k_2 \), which incidentally is similar to the condition derived recently by Findlay (1971) in order to ensure the stability of the long-run equilibrium in the presence of a non-traded, capital good.
5. The result derived above is open to the reasonable objection that the optimality of free trade in the presence of the wage-differential may occur only by chance. Moreover, if the differential is not paid by the importable good, free trade alone cannot be the second-best policy. If the wage-differentials exist among all three industries, there is a greater likelihood of the fulfilment of condition (19\*). Because now none of \( \beta \) and \( \lambda \) will equal unity. The essential point is that there exists a certain configuration of the wage-differentials which will satisfy condition (19\*).

An interesting policy device to attain the second-best welfare suggests itself from this discussion. Suppose the level of the inter-industry factor-price differential itself depends on the government policy. For example, Johnson (1966), referring to Harberger (1962), has suggested that the inter-industry factor-price differential will occur if the government imposes a corporation income tax (which is a tax on the use of capital in the corporate sector alone), that drives a wedge between the gross rewards of capital in the corporate and the non-corporate sector. If the government must impose the corporation income tax in order to raise revenue, etc., it could be imposed in such a way as to satisfy condition (19\*). For a small country, the commodity prices and hence factor-proportions, \( f_2 \) and \( f_3 \) are given. All we then need is to select that level of taxes on the use of capital, and hence the levels of \( \lambda \) and \( \beta \), that will equate \((1-\lambda)/(1-\beta)\) to \( P_2 f_2 (k_3-k_1)/P_3 f_3 (k_2-k_1) \). Although \( \lambda \) and \( \beta \) represent the wage-differentials among the three industries under the assumption of similar reward of capital in all industries, they may also represent the case where the rewards of capital differ among industries but the wage rates are similar.
Depending on the situation, the level of the tax on the use of capital could be determined in the following manner:

i) The entire revenue can be raised from taxing the use of capital alone in the importable good. If the other two industries consistute the non-corporate sector, this tax can be passed on as the corporation income tax.

ii) The use of capital can be taxed in the exportable and the non-traded good industries, but in order to attain the second-best optimum, the tax rate will have to be different in the two industries. This may require taxing the use of capital in the corporate as well as the non-corporate sector. The need for the different tax rates arises from the need to keep both $\beta$ and $\lambda$ from unity. This enables us to hazard a conjecture that the single corporation income tax rate on all incorporated industries in vogue in some countries (such as U.S.A., Canada, etc.) might have had distortionary effects on the second-best level of social welfare.

iii) Finally, the differential rate of tax on the use of capital can be imposed on all three industries. It may be emphasized again that the tax rate differential should be such as to select those levels of $\beta$ and $\lambda$ that satisfy condition (19*).
Thus there are several revenue generating measures at the disposal of the government which will satisfy the dual objective of raising the revenue without lowering the level of welfare below the second-best level.

Estimating the rates of tax on the use of capital is another matter. However, this problem is not as formidable as may superficially appear. The magnitudes of both $\beta$ and $\lambda$ depend on $dK_1$, $dL_1$ and the marginal productivities of factors which are determined once the factor-proportions are given. With factor-proportions and total factor supplies given, $dK_1$ and $dL_1$ can be determined from the following three equations:

$$dL_1 + dL_2 + dL_3 = 0$$

$$k_1 dL_1 + k_2 dL_2 + k_3 dL_3 = 0$$

$$dL_3 = \frac{dX_3}{f_3} (= \frac{dD_3}{f_3}), \text{ and } \frac{dK_1}{K_1} = \frac{dL_1}{L_1} \text{ (because } dk_1 = 0).$$

Hence with $F_{K1}$, $F_{Li}$, $dK_1$ and $dL_1$ so determined, such values of $\beta$ and $\lambda$ can be selected by the government as satisfy (19*), so that the decline in welfare is minimized, that is to say, welfare does not fall below the second-best optimum.

If the wage-differential already exists, then, as stated earlier, the optimum optimorum (or the first-best situation) can be attained only by the imposition of factor tax-cum-subsidy which eliminates the factor-price ratio differential to the producers. For this policy will also eliminate the production inefficiency that results from the presence of the factor-price ratio differential, an inefficiency which in the B-R and B-P models causes the transformation curves to shrink towards the origin.
VI. Conclusions

In the foregoing analysis, we have explored the implications for welfare in a small country of the simultaneous introduction of a non-traded good and the inter-industry factor-price differential. The major result is that if i) the factor-price differential is paid by the importable good, or ii) the differential exists among all industries, free trade alone may turn out to be the second-best policy. This implies that if the creation of the differential itself is subject to government policy (like the one resulting from the corporation income tax), it should be done in such a manner that welfare does not fall below the second-best level. If free trade is not the second-best policy, then there may be two ways to attain the second-best optimum: i) the imposition of a suitable production tax-cum-subsidy on the traded goods only, and/or ii) a consumption tax-cum-subsidy on the non-traded good along with free trade. The first-best policy, of course, requires the elimination of the factor-price ratio differentials to the producers by means of a suitable factor tax-cum-subsidy policy combined again with free trade.

Other interesting results offered by this paper are as follows:

In the absence of the factor-price differential, the traditional result that an improvement (deterioration) in the terms of trade results in a rise (decline) in welfare derived in the absence of non-traded goods continues to be valid when such goods are taken into account. However, another conventional result that a higher tariff is inferior to a lower tariff may not hold when non-traded goods are incorporated in the model. Nevertheless, free trade still turns out to be the first-best
policy even when non-traded goods exist.

The B-P theorems 1 and 2 derived in the presence of the wage-differential continue to be valid when a non-traded good is introduced. However, their condition that the differential be paid by the importable good ceases to be necessary.

In the two-good case, the factor-intensity rankings of commodities do not play any role in determining of the effects of changes in the parameters on welfare. In the presence of the third, non-traded good, however, the conclusions are significantly affected by factor-proportions in all three commodities.
Footnotes

1. There are several commodities in a country which do not enter international trade at all. As Kemp (1969, p. 134) puts it so nicely, "Every country, even Kuwait, produces commodities which are neither imported nor exported." Reasons for the existence of such goods have been provided in detail by Komiyama (1967, p. 132) and Kemp (1969).

2. Inter-industry factor-price differentials may exist for several reasons in spite of perfect mobility of factors, an assumption which has been made by all writers analyzing the implications of such factor market distortions for resource allocation and welfare. There is, of course, the possibility that the factor-price differentials may be more apparent than real so that in reality there is no distortion in the factor markets. This point has been emphasized by Bhagwati and Ramaswami (1963) who provide six reasons why a differential may exist between the import and the export sectors of an underdeveloped economy. In what follows we assume that the factor-price differential is distortionary. Such differentials in a developed economy may exist due to different policies followed by unions in different sectors, or due to discriminatory imposition of a tax on the use of capital on one sector (the corporate sector) but not on the other (Harberger, 1962 and Johnson 1966). For further details on the causes of the factor-price differentials, see Kemp (1969, p. 279).

3. This result is actually a special case of result 3, because free trade represents a situation of zero tariff and no trade describes a situation of prohibitive-tariff, a tariff high enough to prevent any good from being imported and hence being exported. However, in view of
the importance that has been accorded to this special case in the
literature on the gains from trade, result 3 will be treated as a
separate theorem.
4. The earlier trade literature on wage-differentials has tended
to make an illusory distinction between a wage-differential which
leaves the pattern of trade that would prevail in the absence of this
differential unchanged and the one that causes a change in this
pattern. With the former type of the wage-differential, the standard
theorems remain unscathed, whereas with the latter type, they may not
(Hagen (1958), Bhagwati and Ramaswami (1963), etc.). However, it
appears that this is an unsatisfactory way of presenting the conclusions,
for whether or not the differential causes a reversal in the earlier
pattern of trade, there are always two possibilities, namely, 1) the
differential is paid by the exportable good, and 2) it is paid by the
importable good. Suppose in the absence of the wage-differential,
one commodity is imported, and suppose now that the differential is in-
troduced which is unfavorable to this importable good. Obviously,
the earlier pattern of trade cannot be reversed in this case. The
conventional literature would then create the impression that the standard
theorems will now be valid because the earlier pattern of trade is not
reversed, which is not true. One does not have to bother about the
reversal of the pattern of trade. In a two-good model, as stated
above, there are only two possibilities: Under one the standard results
hold, under the other, they may not. In this paper, this is how the
conventional results and our own results will be presented.
5. It is worth noting that the income distribution effects of changes in the terms of trade on welfare have not been ignored. They are implicit in the choice of the social welfare function. This comment is of relevance to the entire discussion in this paper.

6. Recent advances in the theory of wage-differentials have made it clear that this may not be so. Exciting new results have been derived by Bhagwati and Srinivasan (1971), Kemp and Herberg (1971) and Jones (1970) that in the presence of the wage-differential a rise in the price of a commodity may actually lower its output, irrespective of whether the transformation curve is concave or convex to the origin. However, in order to facilitate comparison with earlier results, we assume "normal" price-output relations, so that \( \frac{dX_2}{dP_2} > 0 \).

7. That the wage-differential is paid by the non-traded good sector is not a mere theoretical possibility. There is a logical explanation for why the contrary may be true. Since the traded goods face the strong competition from foreign producers which the non-traded goods do not, the employers in the non-traded goods may be more prone to accepting union demands for higher wages than the employers in the traded goods. Recent restrictions introduced in the United States by President Nixon on the spiralling wage gains by unions in the construction industry, which is a very good example of a non-traded good industry, would lend credence to our hypothesis. Similarly, in Canada the wage increases secured by unions in the housing industry have far outpaced the increases in other sectors, some of which do enter international trade. Thus, if there was no wage-differential in the economy before, one will now be created in such a way that the higher wages will be paid by the non-traded good sector.
8. It is worth pointing out here that this procedure does not apply to the case where the wage-differential does not exist. As shown above, (23) now reduces to \((1/U_1)(dU/dt) = (P_1t/l+t)(dE_1/dP_1^*)\), and the ambiguity in the sign of \((dE_1/dP_1^*)\) in the presence of the non-traded good may lead one to the faulty conclusion that free trade may be inferior to no trade. In the subsequent sections free trade is shown to be the optimal policy in the absence of the wage-differential regardless of the presence of non-traded goods. A simple proof of this result actually follows from (23) if we assume that there is no tariff in the initial situation, so that \(t=0\) and with \(\beta = \lambda = 1\), \((1/U_1)(dU/dt) = 0\). In the presence of the wage-differential, however, it is easily seen that this is not true.

9. The derivation of this condition is very simple. From (11) and (12), \((dX_1/dX_2) = -(f_1k_2/f_2k_1)\) when \(k_3 = 0\) in the absence of the non-traded good. From (7), \((P_2/P_1) = (f_1/f_2)\). Therefore

\[
\beta = -\frac{(dX_1/dX_2)}{(P_2/P_1)} = \frac{f_1}{k_1f_1} \cdot \frac{k_2f_2'}{f_2}
\]

Now the relative share of capital in the \(j\)th commodity is given by

\[
\phi_j = \frac{k_jf_j'}{x_j} = \frac{k_jf_j'}{L_jf_j} = \frac{k_jf_j'}{f_j} \quad (j=1,2).
\]

Using this relationship, we can write

\[
\beta = \frac{\phi_2}{\phi_1}
\]

as shown in the text.
10. Note that this policy does not apply to the traded goods. Since the distortion is in the domestic production sector of the economy, the thrust of the cure should also lie in removing the inequality between \((dX_1/dX_2)\) and \((P_2/P_1)[1 + (P_3/P_2)(dX_3/dX_2)]\), just like in the two-good model, the second-best optimum is achieved by changing the output of \(X_1\) and \(X_2\) is such a way that the difference between \((dX_1/dX_2)\) and \((P_2/P_1)\) is eliminated. If the consumption tax (or subsidy) were to be imposed on one of the traded goods, there will not be any change in outputs, for \((P_2/P_1)\) which is exogenously determined will remain unchanged for the producers. However, in the case of the non-traded good, the domestic output of \(X_3\) equals its domestic consumption. Therefore, if the wage-differential is paid by \(X_3\) producers, the second-best optimum will be attained only by increasing its output to such an extent that to its producers \(\lambda\) comes to equal unity. However, in order to raise the output of \(X_3\) without creating a discrepancy in its domestic demand and supply, a consumption subsidy to the non-traded good, rather than a production subsidy, is required. The output of \(X_3\) will automatically rise when its demand increases, for in the three-good, two-factor model, as demonstrated by Melvin (1968) and Kemp (1969), any number of output configurations are compatible with a given set of commodity prices. The production possibility surface is completely described by straight lines and each line is a locus of possible output configurations for the three commodities corresponding to a given set of commodity prices. Hence when a consumption subsidy (or tax) is given to the non-traded good, a new production point can be selected for the
given set of prices in such a way that (15) is satisfied. It may also be noted from (24) that when \( \lambda < 1 \), so that the wage-differential is paid by the non-traded good, a positive sign of \( dD_3 = dX_3 \) will raise welfare.

On the other hand, if a production tax (or subsidy) was imposed on the non-traded good, its output would change, but not its consumption, so that a disequilibrium situation will arise which will not be self correcting because the price of the non-traded good to the consumers remains constant under the small country assumption.
References


