1971

A Two-Class Two-Sector Model of International Trade

V. Somasundara Rao

Follow this and additional works at: https://ir.lib.uwo.ca/economicsresrpt

Part of the Economics Commons

Citation of this paper:
RESEARCH REPORT 7105

A TWO-CLASS TWO-SECTOR MODEL OF
INTERNATIONAL TRADE

by ECONOMICS

V. Somasundara March 1971
A TWO-CLASS TWO-SECTOR MODEL OF
INTERNATIONAL TRADE

by

V. Somasundara Rao*

I. INTRODUCTION

In international trade models, aggregate national demand functions are
usually assumed as arising from each country maximizing a national utility
function. When factor ownership in each country is unchanging, such a repre-
sentation of aggregate demand assumes away the possible effect on demand of
changing income distribution between factor owners that takes place with changes
in factor prices. On the other hand, these trade models are used to carefully
analyze the effects of various trade policies on real factor rewards. As
Johnson [6] has pointed out "without an analysis of the connection between dis-
tribution and international demand, the general equilibrium model of interna-
tional trade is logically incomplete." In his fundamental paper, Johnson [6]
set out to fill this gap and demonstrated some of the analytical possibilities
that emerge as a result of considering disaggregated demand in the home country
in the two-country, two-commodity, two-factor Heckscher-Ohlin-Samuelson model.

In this article the idealization adopted by Johnson, that of treating
each factor as also a consuming class, is given a precise formulation and ex-
tended to include both the countries. Further, local as well as system

---

*This paper is a revised version of a chapter of a Ph.D. thesis submit-
ted to the University of Minnesota. The author is indebted to his
thesis adviser, Professor John S. Chipman, for valuable suggestions and com-
ments. He also wishes to express his gratitude to Professors Murray C. Kemp,
Anne O. Krueger, Takashi Negishi and Yoshihiko Otani for helpful comments on
an earlier draft.
stability of the model is studied for the Walrasian "tâtonnement" adjustment process for various possible patterns of world specialization. It is assumed that the utility functions of all individuals are homothetic (not necessarily identical), each capitalist supplies some fixed positive quantity of capital and each worker supplies some fixed positive amount of labor. In Section II, the basic two-class model is developed starting from assumptions about microeconomic units and deriving properties of aggregate excess demand functions. Utilizing these properties of excess demand functions stability of the model is studied in Section III, and some sufficient conditions for local stability and some necessary conditions for local instability are given in terms of factor intensities and consumption propensities. It is also found that system stability always obtains for the model considered.
II. THE TWO-CLASS MODEL

Initially the closed economy model is described. The following assumptions are made: two sectors, producing goods 1 and 2; two factors, in fixed supply and perfectly mobile; perfect competition and no externalities; neo classical production functions of constant returns to scale and diminishing returns to proportions; both factors are indispensable for positive production of either commodity; and no factor intensity reversals. There are two consuming groups: workers and capitalists, each worker supplies some fixed positive amount of capital; assumptions about individual preferences are: homothetic (not necessarily identical) and strictly quasi-concave utility functions, monotonicity of preferences and no consumption intensity reversals. The sense in which we could then consider two consuming groups is explained later.

The following notation is adopted: $K$ for capital, $L$ for labor, $\frac{K}{L} = k$ for overall capital labor ratio, $\frac{K_i}{L_i} = k_i$ for the $i$th sector capital labor ratio, wages as $w$, rental on capital as $r$, wage-rental ratio as $\omega$, $Y_i$ for the quantity produced of the $i$th good, the two classes: workers and capitalists are denoted by $\xi$, $c$; $X_{ij}$ denotes the demand for the $i$th good by the $j$th class, $E_i$ for aggregate excess demand for $i$th good, $p_i$ for the price of the $i$th good, $p$ for price of good 2 relative to good 1, aggregate income in terms of good 1 is denoted by $Z$, income of workers and capitalists (in terms of good 1) as $Z_{\xi}$, $Z_c$,

$\frac{Y_i}{L}$ as $y_i$ and $\frac{L_i}{L}$ as $\lambda_i$.

Production and Income Distribution

The production side of the model is written as:

\[\psi_i(L_i, 0) = \psi_i(0, K_i) = 0; \quad \psi_i(L_i, 0) = \psi_i(0, K_i) = 0; \quad \forall \lambda > 0; \quad \text{both factors are indispensable for positive production of either commodity}; \]

\[\psi_i(L_i, 0) = \psi_i(0, K_i) = 0; \quad \text{and marginal products are positive and diminishing}; \]

\[\frac{\partial \psi_i}{\partial L_i} > 0, \frac{\partial \psi_i}{\partial K_i} > 0, \frac{\partial^2 \psi_i}{\partial L_i^2} < 0, \frac{\partial^2 \psi_i}{\partial K_i^2} < 0 \text{ for all } L_i, K_i > 0. \quad K \text{ and } L \text{ of (4) and (5) will be taken as fixed throughout the remainder of the analysis.} \]

Further, in what follows it will be assumed there are no factor intensity reversals, i.e., either $k_1(\omega) > k_2(\omega)$ or $k_1(\omega) < k_2(\omega)$ for all $\omega$. 

---

1 The assumptions indicated above for the production functions (1) can be formulated as follows. They are homogeneous of degree one: $\psi_i(\lambda L_i, \lambda K_i) = \lambda \psi_i(L_i, K_i)$, $\lambda > 0$; both factors are indispensable for positive production of either commodity: $\psi_i(L_i, 0) = \psi_i(0, K_i) = 0$; and marginal products are positive and diminishing: $\frac{\partial \psi_i}{\partial L_i} > 0, \frac{\partial \psi_i}{\partial K_i} > 0, \frac{\partial^2 \psi_i}{\partial L_i^2} < 0, \frac{\partial^2 \psi_i}{\partial K_i^2} < 0$ for all $L_i, K_i > 0$. $K$ and $L$ of (4) and (5) will be taken as fixed throughout the remainder of the analysis. Further, in what follows it will be assumed there are no factor intensity reversals, i.e., either $k_1(\omega) > k_2(\omega)$ or $k_1(\omega) < k_2(\omega)$ for all $\omega$. 

---
(1) \( Y_i = \psi_i(L_i, K_i) \quad i = 1, 2 \)

(2) \( w \equiv p_i \psi_{i1} \quad i = 1, 2 \)

(3) \( r \equiv p_i \psi_{i2} \quad i = 1, 2 \)

(4) \( L = L_1 + L_2 \)

(5) \( K = K_1 + K_2 \)

with the equality sign holding in (2) and (3), which are derived from cost minimization, if the ith sector is using a positive quantity of the factor. These could be reduced to the following six equations, where \( \psi_i(k_i) \) are defined as:

\( \psi_i(k_i) = \psi_i(1, k_i), \)

(6) \( y_1 = \ell_1 \psi_1(k_1) = \frac{k_2 - k}{k_2 - k_1} \psi_1(k_1) \)

(7) \( y_2 = \ell_2 \psi_2(k_2) = \frac{k - k_1}{k_2 - k_1} \psi_2(k_2) \)

(8) \( w \equiv p_i [\psi_i(k_i) - k_i \psi_i'(k_i)] \quad i = 1, 2 \)

(9) \( r \equiv p_i \psi_i'(k_i) \quad i = 1, 2 \)

Assuming incomplete specialization,\(^2\) we get the relations between \( k_i \) and \( w \), and \( w \) and \( p \).\(^3\) From (8) and (9):

(10) \( w = \frac{w}{r} = \frac{\psi_i(k_i)}{\psi_i'(k_i)} - k_i = \bar{w}_i(k_i). \)

---

\(^2\)Later, in Section III, this assumption is relaxed and various patterns of specialization are considered.

\(^3\)Some of the well-known properties of the two-sector model are stated here without the actual derivations being shown. See Uzawa [12] or Kemp [7].
We have $\tilde{w}_i(k_i) > 0$, hence (10) can be inverted as

$$k_i = \tilde{k}_i(\omega).$$

From (9) and (11):

$$p = \frac{P_2}{P_1} = \frac{\psi'[\tilde{k}_1(\omega)]}{\psi[\tilde{k}_2(\omega)]} = B(\omega)$$

and we have:

$$B'(\omega) \begin{cases} > 0 \text{ if } k_1 > k_2 \\
< 0 \text{ if } k_1 < k_2 \end{cases}$$

Hence (12) can be inverted as:

$$w = b(p)$$

Substituting (14) in (11) we get:

$$k_i = \tilde{k}_i[b(p)].$$

Using (15), (6), and (7), (1) can be expressed as

$$Y_i = h_i(p) \quad i = 1, 2.$$  

Further, we have:

$$h_1'(p) < 0, \text{ and } h_2'(p) > 0.$$  

Aggregate income $Z$, in terms of good 1, can be written as:

$$Z = Y_1 + pY_2 = h_1(p) + ph_2(p) = \phi(p),$$

and we have:

$$\phi'(p) = h_1'(p) + ph_2'(p) + h_2(p) = h_2(p).$$

The relation between real factor rewards and commodity price ratio can also be readily obtained. Substituting (11) in (8) and (9) and assuming equalities hold in (8) and (9)):

$$\frac{w}{P_1} = \psi'[\tilde{k}_1(\omega)] - \tilde{k}_1(\omega)\psi'[\tilde{k}_1(\omega)] = \tilde{w}_i(\omega)$$

$$\frac{r}{P_1} = \psi'[\tilde{k}_1(\omega)] = \tilde{r}_i(\omega).$$
Differentiating (20) and (21) we get:

(22) \( \hat{w}_1'(w) > 0, \hat{r}_1'(w) < 0 \)

Income of workers and capitalists is expressed in terms of good 1, by substituting (14) in (20) and (21) as:

(23) \( Z_k = \hat{L} \hat{w}_1[b(p)] = \varphi_k(p) \)

(24) \( Z_c = K \hat{r}_1[b(p)] = \varphi_c(p) \)

Further, differentiating (23) and (24) and using (22) and (13) we get:

(25) \( \varphi_k(p) \begin{cases} > 0 & \text{if } k_1 > k_2 \\ < 0 & \text{if } k_1 < k_2 \end{cases} \)

(26) \( \varphi_c(p) \begin{cases} < 0 & \text{if } k_1 > k_2 \\ > 0 & \text{if } k_1 < k_2 \end{cases} \)

Given the over-all capital labor ratio the relative prices at which the economy is completely specialized is known. Suppose with \( k = \bar{k} \), the economy is completely specialized in good 1 at \( p_0 \) and in good 2 at \( p_1 \). The supply function of good 2 (16) can be written as:

(27) \( Y_2 = \begin{cases} 0 & \text{for } p = p_0 \\ h_2(p) & \text{for } p \in (p_0, p_1) \\ \bar{Y}_2 & \text{for } p = p_1 \end{cases} \)

where the open interval \( (p_0, p_1) \) is the price range compatible with incomplete specialization, and \( \bar{Y}_2 = L \psi_2(\bar{k}) \).

**Income Distribution and Demand Functions**

The demand side of the model will now be considered. A consequence of the homothety assumption is that the proportion in which commodities are demanded by any individual depends only on the relative commodity price. Consider, for instance, the utility function of the \( i \)th capitalist:

(28) \( U_c^i(x_{1c}^i, x_{2c}^i) = F_c^i[f_c^i(x_{1c}^i, x_{2c}^i)] \).
where $F^i_c$ is monotone increasing and $f^i_c$ is homogeneous of degree one. The budget constraint of the $i$th capitalist is: $\theta^i Z_c = X^i_1 + pX^i_2$; where $\theta^i$ is the fixed proportion of total capital supplied by him, with $\theta^i > 0$ and $\sum \theta^i = 1$. Maximizing $U^i_c$ subject to the budget constraint we get:

$$
\frac{U^i_{c2}}{U^i_{c1}} = \frac{F^i_c X^i_2}{F^i_c X^i_1} \frac{f^i_c (X^i_1, X^i_2)}{f^i_c (X^i_1, X^i_2)} = \frac{d^i_c X^i_2}{d^i_c X^i_1} = \frac{X^i_2}{X^i_1} = p,
$$

(29) where the property that $f^i_{c1}$ and $f^i_{c2}$ are homogeneous of degree zero has been made use of. Inverting (29), we get:

$$
\frac{x^i_{2c}}{x^i_{1c}} = d^i_c(p).
$$

(30) Substituting (30) in the budget constraint, we get the demand functions of the $i$th capitalist as:

$$
\frac{z^i c}{1 + p d^i_c(p)} = G^i_1(p, Z_c).
$$

(31) $X^i_1 = \frac{\theta^i Z_c}{1 + p d^i_c(p)} = G^i_1(p, Z_c).

(32) $X^i_2 = \frac{\theta^i Z_c d^i_c(p)}{1 + p d^i_c(p)} = G^i_2(p, Z_c).

Denoting by $\delta_c(p) = \sum_i \frac{\theta^i}{1 + p d^i_c(p)}$; the aggregate demand functions of the capitalist class are:

$$
\delta_c(p) = \sum_i \frac{\theta^i}{1 + p d^i_c(p)}.
$$

Invertibility of (29) follows from the assumption that $U^i_c$ is strictly quasi-concave.
(33) \( X_{1c} = \sum_{i} X_{1c}^{i} = Z \delta_{c}(p) = G_{1c}(p, Z_{c}) \)

(34) \( X_{2c} = \sum_{i} X_{2c}^{i} = \frac{Z_{c}}{p}[1 - \delta_{c}(p)] = G_{2c}(p, Z_{c}) \).

It follows that the proportion in which commodities are demanded by the "aggregate capitalist" also depends on the relative commodity price only and not on the income of the capitalist class, that is,

\[
\frac{X_{2c}}{X_{1c}} = \frac{1 - \delta_{c}(p)}{p \delta_{c}(p)} = d_{c}(p).
\]

Differentiating (35), we get

\[
d_{c}(p) = \frac{\delta_{c}(p)[\delta_{c}(p) - 1] - p \delta_{c}(p)}{[p \delta_{c}(p)]^2}.
\]

Differentiating \( \delta_{c}(p) \), we get

\[
\delta_{c}'(p) = \sum_{i} \frac{-\theta_{i}^{i} d_{c}^{i}(p) + \theta_{i}^{i}(p)}{[1 + p d_{c}^{i}(p)]^2} = \sum_{i} \frac{\theta_{i}^{i} d_{c}^{i}(p)(\sigma_{i} - 1)}{[1 + p d_{c}^{i}(p)]^2},
\]

where \( \sigma_{i} \) denotes the elasticity of substitution for the \( i \)th capitalist:

\[
\sigma_{i} = \frac{d_{c}^{i}[x_{1c}^{i}]}{dp} = \frac{px_{2c}^{i} - p d_{c}^{i}(p)}{x_{1c}^{i} d_{c}^{i}(p)}.
\]

Hence, substituting (37) in (36) and simplifying

\[
d_{c}(p) = \frac{1}{px_{1c}^{i}} \left[ \sum_{i} \frac{-(\theta_{i}^{i} x_{1c}^{i} - x_{1c}^{i})^2}{p \theta_{i}^{i}} - \sum_{i} \frac{x_{1c}^{i} x_{2c}^{i} \sigma_{i}}{\theta_{i}^{i}} \right].
\]

---

5 Defining consumption intensity of the \( j \)th class as \( d_{j} = \frac{x_{2j}}{x_{1j}} \); the assumption that there are no consumption intensity reversals implies either \( d_{c}(p) > d_{4}(p) \) or \( d_{c}(p) < d_{4}(p) \) for all \( p \).
Consider the expression in brackets on the right side of (38): the first term is negative and as \( \sigma_i > 0 \) for all individuals the second term is also negative. Therefore \( d_c'(p) < 0 \) and the function (35) is invertible.

\[
(39) \quad p = D_c \left[ \frac{X_{2c}}{X_{1c}} \right].
\]

Assuming that \( D_c \) is continuous and differentiable, the differential equation

\[
- \frac{dX_{1c}}{dX_{2c}} = D_c \left[ \frac{X_{2c}}{X_{1c}} \right],
\]

yields in this two-good case a unique set of community indifference curves.\(^6\)

It is in this sense we can say the capitalist class behaves as if it were a rational unit maximizing an aggregate utility function and further as a rational unit with a homothetic utility function. That is the "Engel" curves of the aggregate capitalist are all straight lines emanating from the origin, though different for different price sets.

It should, however, be pointed out that this aggregate utility function will not be invariant with respect to redistribution of total capital among the capitalist class. As Samuelson [11] has shown, it will be invariant for all redistributions if and only if all capitalists have identical homothetic utility functions.

Similarly, we have the demand functions for the worker class. Summarizing, the demand function for the ith good of the jth class can be expressed, using (23) and (24), (33) and (34) as:

\[
(40) \quad X_{ij} = G_{ij} [p, \phi_j(p)] = g_{ij}(p).
\]

Collecting information from (16) and (40), aggregate excess demand function for the ith good is defined as:

---

\(^6\)See Samuelson [10], and also Chipman [3] and Eisenberg [4].
(41) \[ E_i = X_{ic} + X_{i\ell} - Y_i = \{ G_{ic}[p, \varphi_c(p)] + G_{i\ell}[p, \varphi_\ell(p)] - h_i(p) \} = H_i(p). \]

**International Equilibrium**

The closed economy model is extended to open economies in the following way. The world is assumed to consist of two countries: the home country and the rest of the world; production functions are identical among countries; factors are perfectly mobile within countries and completely immobile among countries and that there is free world trade.

The world excess demand function for the ith good is defined as (an asterisk relates to the variable of the rest of the world):

(42) \[ E_i + E_i^* = H_i(p) + H_i^*(p) = J_i(p) \quad i = 1, 2. \]

Recall the assumptions about consumers, production functions and resource endowments: strictly quasi-concave utility functions, monotonicity of preferences, positive endowments of some factor for all individuals; production functions of constant returns to scale and diminishing returns to proportions. These assumptions are known to be sufficient for the continuity and single-valuedness of world excess demand functions and are sufficient for the existence of an equilibrium (see Chipman [3] and Nishida [9]): that is a \( \bar{p} \) such that

\[ J_i(\bar{p}) = 0 \quad i = 1, 2. \]
III. STABILITY OF INTERNATIONAL EQUILIBRIUM

We will turn now to a study of the stability of the static model described. First, we consider the stability of an equilibrium position in the sense of the convergence of some hypothesized dynamic adjustment process. Secondly, utilizing the properties of the aggregate excess demand functions we also study the stability properties of the total system under consideration rather than of a particular equilibrium position. The adjustment process that will be considered is the Walrasian "tâtonnement" adjustment process, in which no exchange or production takes place except at the final equilibrium. There are six markets in the model: two good markets; and for each country two factor markets. It is assumed that the factor markets always adjust instantaneously. By assuming that Walras' law holds for each country, i.e., each country is always on its offer curve, we need to study only one of the world goods markets. Assuming that the rate of change in p is a continuous and differentiable function of world excess demand for good 2, we have:

\[(43) \quad \frac{dp}{dt} = Q[H_2(p) + H_2^*(p)]; \quad Q' > 0, \quad Q(0) = 0.\]

Local Stability

To show local stability of the adjustment process, it is sufficient to consider a linear system approximating the process in the neighbourhood of an equilibrium position. It is known that stability of the approximating linear system is sufficient for local stability of the original system. This yields from (43), the well-known condition that an equilibrium will be locally stable if and only if

7 If in addition \[J_2'(p) \neq 0]\ then stability of the approximating linear system is also necessary for local stability. Hereafter, it will be assumed that this condition holds.
(44) \( J'_2(p) = H'_2(p) + H'_2(p) < 0 \),

where the derivatives are evaluated at the equilibrium point \( \tilde{p} \). By using (41), \( J'_2(p) \) can be expressed as:

\[
(45) \quad J'_2(p) = S_{2c} - X_{2c} m_{2c} + m_{2c} \phi'_c(p) + S_{2l} - X_{2l} m_{2l} + m_{2l} \phi'_l(p) - h'_2(p) + S^*_{2c} - X^*_{2c} m^*_{2c} + m^*_{2c} \phi^*_c(p) + S^*_{2l} - X^*_{2l} m^*_{2l} + m^*_{2l} \phi^*_l(p) - h^*_2(p),
\]

where \( m_{2j} = \frac{\partial G_{2j}}{\partial z_j} \) and the substitution terms \( S_{2j} = \frac{\partial G_{2j}}{\partial p} + X_{2j} m_{2j} \). Further using the following notation:

\[
S_2 = S_{2c} + S_{2l} - h'_2(p); \quad R_j = X_{2j} - \phi'_j(p);
\]

\[
\chi(p) = -R_{2c} m_{2c} - R_{2l} m_{2l} - R^*_{2c} m^*_{2c} - R^*_{2l} m^*_{2l},
\]

(45) can be reduced to

\[
(46) \quad J'_2(p) = S_2 + S^*_2 + \chi(p).
\]

We note that \( S_2 + S^*_2 \) is always negative as it is the sum of the substitution terms in consumption and the production effect contained in (17). Therefore it will be sufficient for local stability if \( \chi(p) \) is non-positive. In order to obtain sufficient conditions for \( \chi(p) \) to be non-positive we need to consider various patterns of world specialization. We can distinguish: i) both countries are incompletely specialized; ii) one of the countries is incompletely specialized, the other completely specialized; iii) both countries are completely specialized. Consider (2,3) or (8,9): if a country is completely specialized in one of the industries factor rewards are equated to the value of marginal products in that industry. Consider, for instance, the home country.

(Similar results hold for the foreign country.) If the home country is incompletely
specialized, from the definition of $R_j = X_{2j} - \psi_j^i(p)$ and from (25) and (26) if follows that if $k_1 > k_2$, $\psi_c^i(p) < 0$ and $R_c = X_{2c} - \psi_c^i(p) > 0$. Income of workers in terms of good 2 is: $1/p[\psi_c^i(p)]$. If $k_1 > k_2$, income of workers in terms of any commodity price increases with $p$: that is,

$$\frac{d}{dp} \left[ \frac{\psi_c^i}{p} \right] = \frac{\psi_c^i}{p} - \frac{\psi_c^i}{p^2} > 0.$$ 

Substituting for $\psi_c^i = X_{2c} + pX_{3c}$, we get $\psi_c^i - X_{2c} > \frac{X_{1c}}{p}$. From this it follows that $R_c = X_{2c} - \psi_c^i(p) < 0$ if $k_1 > k_2$. That is: if $k_1 > k_2$, $R_c < 0$ and $R_c > 0$. Similarly if $k_1 < k_2$, $R_c > 0$ and $R_c < 0$. If the home country is specialized in the production of the second commodity, (23) and (24) will be:

$$\psi_c^i(p) = p[\bar{Y}_2 - K \psi_2^i(k)]$$

$$\psi_c^i(p) = p K \psi_2^i(k),$$

where $K$ is the capital endowment, $k$ the over-all capital-labor ratio and

$$\bar{Y}_i = L \psi_i^i(k) \quad (i=1,2).$$

Hence $\psi_c^i(p) = \psi_c^i/p$ and $\psi_c^i(p) = \psi_c^i/p$. It follows that $R_j = X_{2j} - \psi_j^i(p) = - X_{1j}/p < 0$. If the home country is specialized in the production of the first commodity, (23) and (24) will be:

$$\psi_c^i(p) = \bar{Y}_1 - K \psi_1^i(k)$$

$$\psi_c^i(p) = K \psi_1^i(k).$$

It follows that $\psi_j^i(p) = 0$ and $R_j = X_{2j} > 0$. Collecting information, we have the signs of $R_j$ for the home country for various patterns of specialization:

(47) (i) incompletely specialized

$$R_c \begin{cases} < 0 \text{ if } k_1 > k_2 \\ > 0 \text{ if } k_1 < k_2 \end{cases}$$

$$R_c \begin{cases} > 0 \text{ if } k_1 > k_2 \\ < 0 \text{ if } k_1 < k_2 \end{cases}$$
(ii) produces only the first commodity
\[ R_j = X_{2j} > 0 \quad j = \ell, c \]

(iii) produces only the second commodity
\[ R_j = \frac{-X_{1j}}{p} < 0 \quad j = \ell, c. \]

We are now in a position to derive sufficient conditions for local stability of an equilibrium, in terms of consumption propensities and factor intensities, for various possible patterns of world specialization.

1. Both countries are incompletely specialized:

\( \chi(p) \) of (46) can be expressed in one of the following forms:

\[
\chi(p) = R_c (m_{2c} - m_{2c}^*) + R_{\ell} (m_{2c}^* - m_{2c}^*) + E_2 (m_{2c}^* - m_{2c}) \\
= R_{\ell} (m_{2c} - m_{2c}^*) + R_c (m_{2c}^* - m_{2c}^*) + E_2 (m_{2c}^* - m_{2c})
\]

Consider the following conditions A:

A. (1) \( k_1 > k_2, \ E_2 > 0, \) and
\[ m_{2c} \equiv m_{2c}^*, \ m_{2c}^* \equiv m_{2c}^*, \ m_{2c} \equiv m_{2c}^* \]

(2) \( k_1 > k_2, \ E_2 < 0, \) and
\[ m_{2c} \equiv m_{2c}^*, \ m_{2c}^* \equiv m_{2c}^*, \ m_{2c} \equiv m_{2c} \]

(3) \( k_1 < k_2, \ E_2 > 0, \) and
\[ m_{2c} \equiv m_{2c}^*, \ m_{2c}^* \equiv m_{2c}^*, \ m_{2c} \equiv m_{2c} \]

(4) \( k_1 < k_2, \ E_2 < 0, \) and
\[ m_{2c} \equiv m_{2c}^*, \ m_{2c}^* \equiv m_{2c}^*, \ m_{2c} \equiv m_{2c}^* \]

It can be readily seen from (47) and (48) that any of the conditions A are sufficient for \( \chi(p) \leq 0. \) Hence they are sufficient for \( J_2(p) < 0 \) and are, therefore, sufficient for an equilibrium position to be locally stable.
Conditions A can be given the following interpretation. There are four "aggregate" citizens in the world, in each country an "aggregate worker" and an "aggregate capitalist" class. For each country, denote the marginal propensity to consume the labor-intensive good of capitalists and workers as: $\alpha_c$ and $\alpha'_k$; marginal propensity to consume the country's importables of capitalists and workers as $\beta_{ic}$, $\beta'_k$; marginal propensity to consume the country's exportables of capitalists and workers as $\beta_{ec}$, $\beta'_e$; and define $\beta_i = \max_j \beta_{ij}$, $\beta_e = \min_j \beta_{ej}$. Then any of conditions A is equivalent to the set of two conditions:

A'. (i) $\alpha_c \equiv \alpha'_k$, $\alpha^*_c \equiv \alpha^*_k$

(ii) $\beta_i \equiv \beta^*_e$

A' summarizes the relations between consumption propensities and factor intensities sufficient to guarantee local stability. Three special cases when A and A' are satisfied can be given:

(i) The four "aggregate citizens" have identical, homothetic utility functions.

(ii) In each country the two classes have identical homothetic utility functions; and each country has a higher marginal propensity to consume its importables than the other country its exportables ($\beta_i > \beta^*_e$).

(iii) The two "aggregate workers" have identical homothetic utility functions and similarly the two "aggregate capitalists". (This can be viewed as an analogue to the assumption of identical production functions among countries.) In each country, capitalists have a higher marginal propensity to consume the labor-intensive good than the workers ($\alpha_c > \alpha'_k$, $\alpha^*_c > \alpha^*_k$).

Special cases (i) and (ii) are well-known. They are included here for the sake of completeness.
Finally, conditions A and A' are also sufficient for the commodities to be "gross substitutes" in the world market, when the definition of gross substitutes is modified to take into account the general equilibrium nature of the model. Changes in prices not only affect demand but also production and income distribution. Suppose p varies, gross substitutes can be defined as: \( J_1'(p) + H_1(p) + H_1^*(p) > 0 \). Walras' law holds identically in p for each country, summing them we get:

\[
J_1'(p) + pJ_2(p) = H_1(p) + H_1^*(p) + p[H_2(p) + H_2^*(p)] = 0.
\]

Differentiating (49) and evaluating at an equilibrium point gives

\[
J_1'(p) + pJ_2'(p) + J_2(p) = J_1'(p) + pJ_2(p) = 0.
\]

Hence \( J_2'(p) < 0 \) implies that \( J_1'(p) > 0 \), therefore A and A' are sufficient for the goods to be gross substitutes. It is known that this gross substitutes case is also sufficient for an equilibrium to be unique and globally stable. (Arrow, Block and Hurwicz [2].) It also follows that a necessary condition for an equilibrium to be locally unstable, is that conditions A are not satisfied. (In our model this is also necessary for "gross complementarity".) Even though both goods are non-inferior for both factor owners in both countries, the income distribution effect may cause the aggregate demand to increase with price and thus produce the equivalent of a "giffen paradox" in the aggregate for some price ranges. In international trade theory, Johnson [6] was the first to point out the importance of the income distribution effect for stability analysis.

2. **Home country incompletely specialized, foreign country specialized in production of the first commodity:**

This possibility can occur if: either i) \( k^* > k \) and \( k_1 > k_2 \), or

\[9\] Hicks [5] and Mosak [8] were also aware of the importance of income effects for stability analysis.
ii) $k^* < k$ and $k_1 < k_2$, where $k$ and $k^*$ are the over-all capital-labor ratios of the two countries. Corresponding to these possibilities $\chi(p)$ can be expressed from (47), at an equilibrium position, in one of the following forms:

$$
(51) \quad \begin{align*}
&i) \quad \chi(p) = R_c (m_{2c} - m_2^*) + X_{2c}^* (m_{2c} - m_2^*) + X_{2c}^* (m_{2c} - m_2^*) \\
&ii) \quad = R_{\ell} (m_{2c} - m_2^*) + X_{2c}^* (m_{2c} - m_2^*) + X_{2c}^* (m_{2c} - m_2^*)
\end{align*}
$$

It follows that $\chi(p) \leq 0$ if the following condition holds:

$$
(B.1) \quad \alpha_c \geq \alpha_{\ell}, \quad \beta_{ic}^* \geq \beta_e^*, \quad \beta_{i\ell}^* \geq \beta_e^*.
$$

The interpretation of (B.1) is that whichever home factor is used intensively in the production of the second commodity (home exportables) has the lowest marginal propensity to consume the second commodity of all four world classes.

3. Home country incompletely specialized, foreign country specialized in the production of the second commodity:

This possibility can arise if: either i) $k > k^*$ and $k_1 > k_2$, or ii) $k < k^*$ and $k_1 < k_2$; and corresponding to these $\chi(p)$ can be expressed from (47) in one of the following forms:

$$
(52) \quad \begin{align*}
&i) \quad \chi(p) = R_{\ell} (m_{2c} - m_{2c}) + \frac{X_{1c}^*}{p} (m_{2c}^* - m_{2c}) + \frac{X_{1\ell}^*}{p} (m_{2c}^* - m_{2c}) \\
&ii) \quad = R_c (m_{2c} - m_{2c}) + \frac{X_{1c}^*}{p} (m_{2c}^* - m_{2c}) + \frac{X_{1\ell}^*}{p} (m_{2c}^* - m_{2c})
\end{align*}
$$

It follows that the following condition is sufficient for $\chi(p) \leq 0$:

$$
(B.2) \quad \alpha_c \geq \alpha_{\ell}, \quad \beta_{i} \geq \beta_{ic}^*, \quad \beta_{i} \geq \beta_{e\ell}^*.
$$

The interpretation of (B.2) is that the home factor used intensively in the production of the first commodity (home exportables) has the highest marginal propensity to consume the second commodity of all four world classes.
4. **Home country specialized in the production of the first commodity, foreign country incompletely specialized:**

This possibility can occur if: either i) $k > k^*$ and $k_1 > k_2$, or ii) $k < k^*$ and $k_1 < k_2$; and $\chi(p)$ can be expressed for these possibilities from (47), at an equilibrium position, as:

\begin{align*}
(53) \quad i) \quad \chi(p) &= R^*(m^*_{2c} - m^*_{2l}) + X_{2c}(m^*_{2c} - m^*_{2c}) + X_{2l}(m^*_{2c} - m^*_{2l}) \\
&= R^*_c(m^*_{2c} - m^*_{2l}) + X_{2c}(m^*_{2c} - m^*_{2c}) + X_{2l}(m^*_{2c} - m^*_{2l})
\end{align*}

\(\chi(p) \leq 0\) if the following holds:

\begin{align}
(B.3) \quad \alpha_c^* &\geq \alpha_{2c}^* \quad \beta_{ic}^* \geq \beta_{e}^* \quad \beta_{il}^* \geq \beta_{e}^*;
\end{align}

and (B.3) can be interpreted as that the foreign factor used intensively in the production of the second commodity (foreign exportables) has the lowest marginal propensity to consume the second commodity of all four world classes.

5. **Home country specialized in the production of the second commodity, foreign country incompletely specialized:**

This possibility can occur if: either i) $k > k^*$, $k_2 > k_1$, or ii) $k < k^*$, $k_2 < k_1$. $\chi(p)$ corresponding to these possibilities can be expressed from (47) as:

\begin{align*}
(54) \quad i) \quad \chi(p) &= R^*_c(m^*_{2c} - m^*_{2l}) + \frac{X_{1c}(m^*_{2c} - m^*_{2c})}{p} + \frac{X_{1l}(m^*_{2c} - m^*_{2l})}{p} \\
&= R^*_c(m^*_{2c} - m^*_{2l}) + \frac{X_{1c}(m^*_{2c} - m^*_{2c})}{p} + \frac{X_{1l}(m^*_{2c} - m^*_{2l})}{p}.
\end{align*}

It follows that the following condition is sufficient for $\chi(p) \leq 0$:

\begin{align}
(B.4) \quad \alpha_c^* &\geq \alpha_{2c}^* \quad \beta_i^* \geq \beta_{ec}^* \quad \beta_i^* \geq \beta_{e}^*.
\end{align}

(B.4) can be given the interpretation that whichever foreign factor is used intensively in the production of the first commodity (foreign exportables) has
the highest marginal propensity to consume the second commodity of all four world classes.

6. **Home country specialized in the production of the first commodity, foreign country specialized in the production of the second commodity:**

This possibility can arise if: either i) \( k > k^* \), \( k_1 > k_2 \), or

ii) \( k < k^* \), \( k_1 < k_2 \), and \( \chi(p) \) can be expressed for these possibilities from (47) as:

\[
\chi(p) = X^*_{2c}(m_{2c} - m_{2c}) + \frac{X^*_{1c}}{p}(m^*_{2c} - m_{2c}) + \frac{X^*_{lc}}{p}(m^*_{2c} - m_{2c})
\]

(i) \( X^*_{2c}(m_{2c} - m_{2c}) + \frac{X^*_{1c}}{p}(m^*_{2c} - m_{2c}) + \frac{X^*_{lc}}{p}(m^*_{2c} - m_{2c}) \)

It can be readily seen that \( \chi(p) \leq 0 \) if both home classes have a higher marginal propensity to consume the second commodity (home importables) than both foreign classes.

7. **Home country specialized in the production of the second commodity, foreign country specialized in the production of the first commodity:**

Without going through the analysis we note that for this pattern of specialization, \( \chi(p) \leq 0 \) if both home classes have a lower marginal propensity to consume the second commodity (home exportables) than both foreign classes.

**System Stability:**

More important is the possibility of multiple equilibria and whether system stability obtains or not when the sufficient conditions for \( \chi_2(p) \leq 0 \) do not hold. That system stability always obtains can be established by using Walras' law and the continuity and single-valuedness property of the
world excess demand functions. Without loss of generality, assume good 2 is labor intensive, i.e., \( k_1 > k_2 \), and \( k^* > k \). Given the over-all capital-labor ratios, the relative prices at which the countries are completely specialized is known. Let these be \( p_1, p_0 \) for the home country and \( p_1^*, p_0^* \) for foreign country with \( p_1 > p_0 \) and \( p_1^* > p_0^* \). Then the world is completely specialized in good 2 at \( p_1^* \), and in good 1 at \( p_0^* \). Walras' law holds for each country, hence (49) holds, i.e., \( J_1(p) + p J_2(p) = 0 \). As good 1 is not produced at \( p_1^* \): \( J_1(p_1^*) > 0 \), therefore, from (49) \( J_2(p_1^*) < 0 \). At \( p_0^* \), good 2 is not produced, hence \( J_2(p_0^*) > 0 \). Consider the Walrasian adjustment process:

\[
\frac{dp}{dt} = J_2(p) .
\]

Consider any initial price \( p_2 \) such that \( p_2 \in (p_1^*, p_0^*) \). If \( J_2(p_2) > 0 \), as \( J_2(p_1^*) < 0 \) and \( J_2 \) is continuous there exists an equilibrium \( p_3 \) such that \( p_3 > p_2 \) and \( J_2(p_3) = 0 \); therefore, (56) converges to some equilibrium \( \tilde{p} \in (p_2, p_3) \).

Similarly, if \( J_2(p_2) < 0 \), as \( J_2(p_0^*) > 0 \), there exists some \( \tilde{p} \in (p_0^*, p_2) \) such that (56) converges to \( \tilde{p} \) and \( J_2(\tilde{p}) = 0 \). This establishes system stability in the sense that the world economy converges to some equilibrium from any initial price, under the "tâtonnement" adjustment process.

\[10\] Without using Walras' law, Arrow and Hurwicz [1] have demonstrated system stability for the two-good exchange model.
References


