1970

Intermediate Products, the Transformation Curve and Optimal Trade Policy

Raveendra Batra

Follow this and additional works at: https://ir.lib.uwo.ca/economicsresrpt

Part of the Economics Commons

Citation of this paper:
RESEARCH REPORT 7036

INTERMEDIATE PRODUCTS, THE TRANSFORMATION CURVE AND OPTIMAL TRADE POLICY

by

Raveendra Batra

November, 1970
Intermediate Products, the Transformation Curve
and Optimal Trade Policy

The purpose of this paper is to explore the implications of such intermediate products as are produced to be utilized solely as inputs by the final products, for the shape of the transformation curve and some standard theorems in the theory of gains from trade which have been traditionally derived in the absence of intermediate goods. Most of the previous analyses concerned with intermediate products in trade theory assume that intermediate and final goods are identical,¹ even though, as Yates [18] has shown, the bulk of the world trade occurs in intermediate products which serve purely as material inputs. The effects of such intermediate goods have been analyzed by Khang [7] in the context of a dynamic trade model and by Ruffin [14] in the context of effective protection. In this paper, we utilize that part of Ruffin's model where intermediate goods are assumed to be produced domestically. We show, contrary to the assumption made by Ruffin,² that the transformation curve may become convex to the origin in the presence of, what Khang calls, "pure" intermediate goods. This result is derived in section I, which also deals with the conditions that ensure the usual concavity of the transformation curve to the origin. In section II, the implications of non-traded intermediate goods are briefly analyzed for gains from trade. Section III is concerned with optimal trade policy under the assumption that the transformation curve is still concave to the origin in spite of the presence of "pure" intermediate goods. The paper is concluded with some remarks in section IV.
I. A Closed Economy Model with Intermediate Products

It is assumed that an economy consists of three commodities, two final goods \((X_1\) and \(X_2\)) and one intermediate good \((X_3)\) which is produced only to serve as an input in the production of the final products. There are two primary factors of production, capital \((K)\) and labor \((L)\), which are utilized in the production of all three commodities. Full employment, perfect competition, constant returns to scale, diminishing returns to factor proportions, inelastic factor supplies, perfect factor mobility and non-reversibility of factor-intensities at all factor prices are also assumed. The three production functions are:

\[(1.1) \quad X_1 = F_1(K_1, L_1, \bar{X}_{31}) = L_1 f_1(k_1, \bar{x}_{31})\]

\[(1.2) \quad X_2 = F_2(K_2, L_2, \bar{X}_{32}) = L_2 f_2(k_2, \bar{x}_{32})\]

\[(1.3) \quad X_3 = F_3(K_3, L_3) = L_3 f_3(k_3)\]

where \(K_i\) and \(L_i\) are respectively the capital and labor inputs and \(k_i = K_i/L_i\) is the capital/labor ratio in the \(i^{th}\) sector \((i = 1, 2, 3)\) and \(\bar{X}_{3j}\) is the amount of \(X_3\) utilized as material input in the \(j^{th}\) final product \((j = 1, 2)\) and \(\bar{x}_{3j} = \bar{X}_{3j}/L_j\), where the bars indicate--what we assume hereafter--that the intermediate good is utilized by final products in fixed proportions. Let a\(\_j = (X_{3j}/X_j)\) denote the requirement of \(X_3\), the intermediate product, per unit of the \(j^{th}\) final product \((j = 1, 2)\). The material input coefficients, \(a_1\) and \(a_2\) are assumed to be constant. Equation \((1.3)\) can be written as:

\[(1.4) \quad X_3 = a_1X_1 + a_2X_2.\]

Let \(V_i\) stand for the marginal product of capital and \(U_i\) for the marginal product of labor in the \(i^{th}\) commodity. Then
\[ V_i = \frac{\partial f_i}{\partial k_i} = f'_i \]

and

\[ U_i = f_i - k_i f'_i. \]

\[(i = 1, 2, 3). \]

We assume that \( f'_i > 0 \) and \( f''_i < 0. \)

With perfect competition in both product and factor markets, the price of each factor equals its marginal value-added product and is equal in all three industries. Let \( w \) stand for the wage rate, \( r \) for the rental of capital, \( p_2 \) for the price of the second commodity, \( X_2 \), in terms of the first, \( X_1 \), and \( p_3 \) for the price of the intermediate product in terms of the first commodity. Factor prices expressed in terms of the first commodity are given by:

\[(1.5) \quad r = f'_1(1-a_1p_3) = f'_2(p_2-a_2p_3) = p_3f'_3 \]

\[(1.6) \quad w = (f'_1-k_1f'_1)(1-a_1p_3) = (f'_2-k_2f'_2)(p_2-a_2p_3) = p_3(f'_3-k_3f'_3). \]

With full employment,

\[(1.7) \quad L_1 + L_2 + L_3 = \bar{L} \]

\[(1.8) \quad L_1k_1 + L_2k_2 + L_3k_3 = \bar{k} \]

where the bars indicate that factor supplies are fixed. With this last equation, the supply side of our model is complete. To close the model, we need demand equations, but since these will not be needed until section III, we do not present them here.

A. The Slope of the Transformation Curve: It has been shown by Vanek [15] and Warne [17] that the slope of the transformation curve equals the negative of the commodity-price ratio even if the final commodities also serve as intermediate goods. We will now show that this result also holds in our model.
where the material input is not identical with the final goods. Totally differentiating (1.1)-(1.4) we have:

(1.9) \[ dX_1 = U_1 dL_1 + V_1 dK_1 \]

(1.10) \[ dX_2 = U_2 dL_2 + V_2 dK_2 \]

(1.11) \[ dX_3 = U_3 dL_3 + V_3 dK_3 \]

(1.12) \[ dX_3 = a_1 dX_1 + a_2 dX_2 \]

From (1.7) and (1.8),

(1.13) \[ dL_1 = -(dL_2 + dL_3) \]

(1.14) \[ dK_1 = -(dK_2 + dK_3) \]

Substituting (1.13), (1.14) and (1.11) in (1.9) and using (1.5) and (1.6), we obtain:

\[ dX_1 (1 - a_1 p_3) = -[dX_2 (p_2 - a_2 p_3) + p_3 dX_3] \]

Then substitution of (1.12) in this gives us,

(1.15) \[ \frac{dX_1}{dX_2} = -p_2 \]

which is nothing but the "standard" result that the slope of the transformation curve equals the negative of the commodity-price ratio.

B. The Shape of the Transformation Curve: It is well known that the shape of the transformation curves in models where (1.15) holds can be derived from the response of the outputs to their prices. In particular, if the output of a commodity rises in response to a rise in its relative price, and conversely, the underlying transformation curve is concave to the origin. Otherwise, it is convex to the origin. Therefore, our task here is to find out the effect
of a change in $p_2$ on the output of $X_1$ and $X_2$.

Differentiating (1.5) and (1.6) with respect to $p_2$ we obtain:

$$\frac{dk_1}{dp_2} = \frac{f_2 f_3}{(1-a_1 p_3) f_1 B}$$

$$\frac{dk_2}{dp_2} = \frac{(1-a_1 p_3) f_1 f_3}{(p_2 - a_2 p_3)^2 f_2 B}$$

$$\frac{dk_3}{dp_2} = \frac{(1-a_1 p_3) f_1 f_2}{p_3 f_3 B}$$

$$\frac{dp_3}{dp_2} = \frac{(1-a_1 p_3) f_2 (k_3 - k_1)}{B}$$

where $B = f_3 (k_2 - k_1) + f_2 (k_3 - k_1) (a_2 - p_2 a_1)$.

Differentiating (1.1)-(1.4) and (1.7) and (1.8) with respect to $p_2$ and using (1.16)-(1.18), we obtain:

$$\frac{dX_1}{dp_2} = -\frac{p_2 (1-a_1 p_3)}{A B} \lambda$$

$$\frac{dX_2}{dp_2} = \frac{(1-a_1 p_3)}{A B} \lambda$$

$$\frac{dX_3}{dp_2} = \frac{(1-a_1 p_3) (a_2 - p_2 a_1)}{A B} \lambda$$

where

$$\lambda = \frac{L_1 f_1^2 f_3^2}{(1-a_1 p_3)^3 f_1} + \frac{L_2 f_1^2 f_3^2}{(p_2 - a_2 p_3)^3 f_2} + \frac{L_3 f_1^2 f_3^2}{p_3 f_3}$$

and
\[ A = a_1 f_1 (k_3 - k_2) + a_2 f_2 (k_1 - k_3) + f_3 (k_1 - k_2). \]

If the output response to commodity prices is to be "normal", then $dX_1/dp_2 < 0$ and $dX_2/dp_2 > 0$. From (1.20) and (1.21) it is clear that the signs of $dX_1/dp_2$ and $dX_2/dp_2$ depend on the signs of $\lambda$ and $A.B$. In view of $f''_1 < 0$, $\lambda < 0$. Therefore $dX_1/dp_2 < 0$ and $dX_2/dp_2 > 0$ only if $A.B < 0$.

If there are no intermediate products, $a_1 = a_2 = 0$ and $A.B = -f_3^2 (k_1 - k_2)^2 < 0$, so that the response of the outputs of final products to changes in their prices is normal. In the presence of intermediate products, an examination of $A$ and $B$ reveals that none of them may have a definite sign. However, in the following cases both $A$ and $B$ have definite signs:

i) \( k_1 > k_3 > k_2, \quad a_2 > p_2 a_1 \)

ii) \( k_1 > k_2 > k_3, \quad a_2 > p_2 a_1 \) and \( a_2 f_2 > a_1 f_1 \)

iii) \( k_2 > k_1 > k_3, \quad a_2 < p_2 a_1 \) and \( a_1 f_1 < a_2 f_2 \).

In case (i), suppose $k_1 > k_3 > k_2$. Then $A > 0$ and if $a_2 > p_2 a_1$, $B < 0$, and both $A$ and $B$ have definite signs. If $k_1 < k_3 < k_2$, $A < 0$, but with $a_2 > p_2 a_1$, $B > 0$. It may be observed, however, that when both $A$ and $B$ have definite signs, their signs are opposite so that $A.B < 0$. We have already shown that if $A.B < 0$, the output response to changes in prices is normal and the transformation curve is concave to the origin. The following theorem is then immediate.

**Theorem 1.1**: If the capital/labor ratio of the intermediate product lies between the capital/labor ratios of the final products, and the final commodity which is treated as numeraire (commodity 1) has a lower material input coefficient in value terms than the other final commodity, the transformation curve is concave to the origin.

The proof of this theorem follows from the discussion of case (i)
where \( k_3 \) lies between \( k_1 \) and \( k_2 \) and \( a_1 \), the material input coefficient of the first commodity which is our numeraire, is lower than \( \frac{a_2}{p_2} = \frac{x_{22}}{p_2 x_2} \), the ratio of the amount the intermediate good used in \( X_2 \) and the value of the output of \( X_2 \) in terms of the first commodity.

In case (ii), suppose \( k_1 > k_2 > k_3 \). Here \( (k_1 - k_2) > 0 \), and with \( a_2 f_2 > a_1 f_1 \), one can see that \( a_2 f_2 (k_1 - k_3) > a_1 f_1 (k_2 - k_3) \), because \( (k_1 - k_3) > (k_2 - k_3) \). Hence \( A > 0 \). Moreover, with \( a_2 > p_2 a_1 \), \( B < 0 \). Similarly, with \( k_1 < k_2 < k_3 \), \( a_2 > p_2 a_1 \) and \( a_2 f_2 < a_1 f_1 \), \( A < 0 \), \( B > 0 \) and \( A.B < 0 \). A similar pattern holds in case (iii). One can see that here again \( A.B < 0 \). We can now derive the following theorem:

**Theorem 1.2:** If the final commodity, whose capital/labor ratio lies between the capital/labor ratios of the other final commodity and the intermediate product, not only has a higher material input coefficient in value terms but is also more intensive in the use of the intermediate good, then the output response to changes in their prices is normal and the transformation curve is concave to the origin.

The notion of intensity in the use of the intermediate good is implied in the comparison between \( a_2 f_2 \) and \( a_1 f_1 \), because \( a_j f_j = x_{3j} / L_j \) (\( j=1,2 \)).

In the three cases discussed above, both \( A \) and \( B \) have definite but opposite signs and \( A.B < 0 \). However, this need not necessarily be the case. For example, in case (i) if \( a_2 < p_2 a_1 \), then \( A \) still has a definite sign and \( B \) does not, and if \( B \) has the same sign as \( A \), \( A.B > 0 \). In this case the outputs of the final products will respond "perversely" to changes in their prices, so that the transformation curve will become convex to the origin. Another interesting possibility is that for some values of \( p_2 \), \( A.B < 0 \), but for other values, \( A.B > 0 \). Here the transformation curve will have local
convexity towards the origin. The following theorem may then be derived:

**Theorem 1.3:** If \( A, B > 0 \) for all levels of \( p_2 \), the transformation curve is globally convex to the origin; if \( A, B \) is positive for some values of \( p_2 \) and negative for other values, the transformation curve will be locally convex to the origin.

Thus the transformation curve, contrary to the traditional belief, may not be concave to the origin in our model with intermediate goods. It may have any of the shapes described in Figure 1. The output response of the final products with respect to changes in their prices and hence the shape of the transformation curve can also be determined diagrammatically by recourse to Melvin's geometry [12]. Consider Figure 2 where \( O_3, O_2 \) and \( O_1 \) represent, respectively, the origins for \( X_3, X_2 \) and \( X_1 \); \( O_2E_0_1 \) is the contract curve between the final products \( X_1 \) and \( X_2 \), \( O_2E \) is the equilibrium output of \( X_2 \) and \( O_1E \) the equilibrium output of \( X_1 \), and these together utilize \( O_3O_2 \) output of the intermediate good. The capital/labor ratios in \( X_2 \) and \( X_1 \) are respectively given by the slopes of the lines \( O_2E \) and \( O_1E \), whereas the capital/labor ratio in \( X_3 \) is given by the slope of \( O_3O_2 \). Now suppose there is a rise in the price of \( X_2 \), so that \( p_2 \) rises.

From equations (1.16)-(1.18) we know that \( \frac{dk_1}{dp_2}, \frac{dk_2}{dp_2} \) and \( \frac{dk_3}{dp_2} \) have the same signs, which means that a change in \( p_2 \) changes the capital/labor ratios in all three commodities in the same direction. Suppose that \( B \) has the same sign as \( (k_2-k_1) \). If \( k_2 > k_1 \), as is the case in Figure 2, then \( B > 0 \). From equations (1.16)-(1.18), it is clear that \( \frac{dk_1}{dp_2} < 0 \). In other words, a rise in \( p_2 \) will lower the capital/labor ratio in all three commodities. In terms of Figure 2, the production point will move along the contract curve towards the origin \( O_1 \). For the time being, suppose that the
Figure 1
capital/labor ratio and the output of the intermediate product remain unchanged, and the production point on the contract curve is given by $E'$ which shows that the capital/labor ratios in $X_2$ and $X_1$ have declined to the slopes of $O_2E'$ and $O_1E'$, respectively. Now suppose the capital/labor ratio in $X_3$ has declined to the slope of $O_3S$, and let $S$ be the point where the output of $X_3$ has remained unchanged (i.e., $O_3S = O_2O_2$). This is possible if $a_2 = p_2a_1$, as is evident from (1.22). If the output of $X_3$ remains unchanged at $S$, the origin for $X_2$ will now shift to point $S$, and there will be a new contract curve for the final products between the points $S$ and $O_1$. The new production point will lie on the new contract curve, but the capital/labor ratios in $X_2$ and $X_1$ will still equal the slopes of $O_2E'$ and $O_1E'$, respectively. Such a production point is given by $N$ which is obtained by drawing $SN$, which intersects $O_1E'$ at $N$, parallel to $O_2E'$. Thus if the output of $X_3$ remains unchanged, because $a_2 = p_2a_1$, the output of $X_2$ is given by $SN$ and that of $X_1$ by $O_1N$. Since $SN > O_2E'$ and $O_1N < O_1E'$, the output of $X_2$ has risen and that of $X_1$ declined as a result of the rise in the relative price of $X_2$, $P_2$. If $a_2 \neq p_2a_1$, the output of $X_3$ will not remain unchanged after $p_2$ has changed. If the output of $X_3$ rises in the new situation, the new origin for $X_2$ will move along $O_3S$ towards $G$ and lie between $S$ and $G$, because at point $G$, the output of $X_1$ will be zero and the economy will be completely specialized in $X_2$. On the other hand, if the output of $X_3$ declines as a result of the rise in $p_2$, the new $X_2$ origin will move along $O_3S$ towards $O_3$. Suppose the length $O_1M$ equals $O_1E'$, the original output of $X_1$. From $M$ draw $MR$ parallel to $O_2E'$. Then the new $X_2$ origin must lie between $S$ and $R$. This is because if the new $X_2$ origin lies at point $R$, the output of $X_1 (=O_1M = O_1E)$ remains unchanged, whereas the output of $X_2 (=MR > O_2E)$ rises. This result is clearly inconsistent with the reduced output of $X_3$. If the new $X_2$ origin lies
anywhere between $O_3$ and $R$, this inconsistency remains. It is only when the new origin of $X_2$ lies between $R$ and $S$ that this inconsistency disappears.

Whatever the level of output of $X_3$ in the new situation, the new $X_2$ origin will lie anywhere between $R$ and $G$, which means that the new output of $X_2$ will be higher and the new output of $X_1$ lower than before. In other words, the response of the output of the final products in terms of Figure 2 is shown to be normal, so that the transformation curve will be globally concave to the origin.

The same is not true in Figure 3 where, as before, the origins for $X_3$, $X_2$ and $X_1$ before the change in $p_2$ are given by $O_3$, $O_2$ and $O_1$, respectively; $O_2E$ and $O_1E$ are the outputs of $X_2$ and $X_1$, and $O_3O_2$ is the output of $X_3$. In Figure 3, $k_2$ still exceeds $k_1$, but $k_2$, unlike the case in Figure 2, is now lower. This is because $O_3O_2$ in Figure 3 is flatter than $O_3O_2$ in Figure 2.

As $p_2$ increases, the new origin for $X_2$ is again given by $S$, if the output of $X_3$ remains unchanged. If the output of $X_3$ does not remain the same, the new $X_2$ origin will again lie between $S$ and $G$ if the output of $X_3$ rises. However, if the output of $X_3$ declines in the new situation, there are two possibilities. The new $X_2$ origin may lie between $R$ and $S$, or even at points on $O_3S$ closer to $O_3$. This is because, although at point $R$ and at some other points on $O_3R$ to the left of $R$, the lower output of $X_3$ is inconsistent with the same output of $X_1$ (as at $R$ where $O_1M = O_1E$) and a higher output of $X_2$, or with the higher outputs of both $X_1$ and $X_2$, yet there are some points on $O_3S$ to the left of $R$ where the output of $X_1$ is higher but that of $X_2$ is lower than the two outputs in the pre-price change situation. This means that there are some points on $O_3S$ to the left of $R$ which give consistent configurations for output levels for all three commodities. Point $H$ is one such point, and if the new $X_2$
origin lies at H, the new output of $X_2 (=HQ)^5$ is lower than its previous output ($=0_2E$), whereas the new output of $X_1 (=O_1Q)$ is higher than its previous output ($=0_1E$). In other words, the output response of final products to a rise in $p_2$ has been shown to be perverse, which means that the transformation curve may be locally or globally convex to the origin.

C. **Other Properties of the Model:** So far we have shown that in our model with intermediate products, the transformation curve may locally or globally become convex to the origin. Another interesting result concerns the relationship between $p_3$ and $p_2$, which is described by (1.19), where it is clear that $dp_3/dp_2$ may be positive, negative, or even zero. The relative price of the intermediate product $p_3$, can be treated in two ways: First, it can be treated as the price of a factor employed in the final commodities, and, therefore, should behave like prices of the primary factors in response to changes in prices of the final products; second, $p_3$ is the relative price of the intermediate product which is produced with the help of the primary factors, so that $p_3$ should also change as a result of changes in the prices of the primary factors. Due to this dual role of $p_3$, the effect of a change in $p_2$ on $p_3$ may have any sign, even if the implications for the change in primary factor prices are determinate. This can be seen as follows:

Differentiating (1.5) and (1.6) and utilizing any of $dk_i/dp_2$, we obtain:

\[
(1.23) \quad \frac{dr}{dp_2} = \frac{f_2(1-a_1p_3)(a_1f_1+f_3)}{B}
\]

\[
(1.24) \quad \frac{dw}{dp_2} = \frac{f_2(1-a_1p_3)(k_1f_3+a_1f_1k_3)}{B}
\]
Suppose that $B$ has a definite sign, which means that it has the same sign as $(k_2 - k_1)$. Then if $k_2 > k_1$, $B > 0$, $dr/dp_2 > 0$ and $dw/dp_2 < 0$. In other words, a rise in the relative price of the second commodity raises the real reward of capital, the primary factor utilized intensively by it, and lowers the real reward of labor, the other primary factor, and conversely. This is, of course, the famous Stolper-Samuelson theorem. However, $dp_3/dp_2$ from (1.19) is positive if $k_3 > k_1$ and negative if $k_3 < k_1$. In other words, the effect of a change in $p_2$ on $p_3$ may go in any direction independent of its effect on $r$ and $w$. The relationship between $p_3$ and $p_2$ will be utilized in the subsequent section on optimal trade policy.

II. Intermediate Goods Treated as Non-Traded Goods

Consider now the implications of the introduction of international trade in final products for gains from trade. If the transformation curve is globally concave to the origin, free trade is necessarily superior to no trade. This is depicted in Figure 1a, where $S$ is the point of self-sufficiency equilibrium on a transformation curve given by TT'. The introduction of trade at the foreign price ratio, FP, results in production at $P$, consumption at C and welfare at $U_2$. Since $U_2$ lies above $U_1$, the autarky level of welfare, free trade is necessarily superior to no trade.

If the transformation curve is either locally (Figure 1b) or globally (Figure 1c) concave to the origin, the analysis of gains from trade in our model with intermediate goods is similar to that in the traditional model with increasing returns to scale. Here free trade may or may not be superior to no trade. In Figure 1b, free trade at the
foreign-price ratio, \( FP \) (parallel to \( FP' \)) may lead to lower welfare, \( U_0 \), or to higher welfare, \( U_2 \). The same result holds in Figure 1c where the transformation curve is globally convex to the origin. Because of this convexity, the production point in free trade may be given by either \( T \) or \( T' \) (see Melvin [13], p. 393). In the former case, free trade is inferior and in the latter case superior to no trade.\(^7\)

III. Optimal Trade Policy with Trade in Intermediate Products

If the transformation curve is not globally concave to the origin, the optimal policy is not clearly defined. To avoid this indeterminacy, we assume in this section that the transformation curve is strictly concave to the origin. Furthermore, we assume that intermediate goods are also traded. Let \( U \) be the level of welfare attained in the home country. This utility or social welfare function, viewed as a concave Scitovsky index of welfare, is a function of the amounts of the two final commodities consumed by the home country: \(^8\)

\[
(3.1) \quad U = U(D_1, D_2)
\]

\[
(3.2) \quad D_j = X_j + E_j
\]

\[
(3.3) \quad D_2 = X_2 + E_2
\]

where \( D_j \) is the home consumption and \( E_j \) the home excess demand for the \( j \)th final product \( (j=1,2) \). If there is trade in intermediate products also, then

\[
(3.4) \quad X_3 = D_3 - E_3
\]

where \( D_3 = a_1 X_1 + a_2 X_2 \) and \( E_3 \) is the home excess demand for the intermediate product. Let the asterisk denote the symbols for the foreign country.
Then international trade market equilibrium requires that

\[(3.5) \quad E_i + E_i^* = 0, \quad i=1,2,3\]

so that \((3.2), (3.3)\) and \((3.4)\) can be written as:

\[(3.6) \quad D_1 = X_1 - E_1^*\]
\[(3.7) \quad D_2 = X_2 - E_2^*\]
\[(3.8) \quad X_3 = D_3 + E_3^*\]

The foreign country is, furthermore, subject to the budget constraint:

\[(3.9) \quad E_1^* + p_2 E_2^* + p_3 E_3^* = 0\]

where \(p_2^*\) and \(p_3^*\) denote the world prices of the second and the third commodities in terms of the first. It is assumed that \(E_1^* < 0\) and \(E_2^* > 0\), so that the foreign country exports the first commodity and imports the second commodity; \(E_3^*\) may be positive or negative depending on whether the intermediate product is imported or exported by the foreign country.

When the intermediate good also enters trade, the marginal rate of transformation no longer equals the commodity-price ratio. This can be seen by differentiating \((1.1), (1.2)\) and \((3.8)\) and then following the same procedure as in section I. Here

\[(3.10) \quad \frac{dX_1}{dX_2} = - \left(p_2 dX_2 + p_3 dE_3^* \right)\]

or

\[(3.11) \quad \frac{dX_1}{dX_2} = - \left(p_2 + p_3 \frac{dE_3^*}{dX_2} \right)\]

It is clear from \((3.11)\) that the marginal rate of transformation is no longer equal to \(-p_2^*\), as was the case in section I, where \(E_3^*\) was assumed to be zero.

Let us now proceed to obtain the conditions under which the level of
welfare will be maximized. Assume that no satiation in consumption takes place. Then differentiating (3.1) we have

\[ \frac{dU}{U_1} = dD_1 + \frac{U_2}{U_1} dD_2 \]

where \( \frac{dU}{U_j} = \frac{\partial U}{\partial D_j} \), \( j = 1, 2 \).

With the domestic-price ratio equal to the marginal rate of substitution, \( U_2/U_1 = p_2 \); whereas \( dU/U_1 \) is an index of changes in real national income \((dy)\). Therefore,

\[ dy = dD_1 + p_2 dD_2. \]

Differentiating (3.6) and (3.7) and substituting in (3.13), we have:

\[ dy = (dX_1 + p_2 dX_2) - (dE_1^* + p_2 dE_2^*) \]

\[ = (dX_1 + p_2 dX_2) - (dE_1^* + p_2^* dE_2^*) + (p_2 - p_2^*) dE_2^*. \]

From (3.10) \( dX_1 + p_2 dX_2 = p_3 dE_3^* \) and from (3.9)

\[ - (dE_1^* + p_2^* dE_2^*) = E_2^* dP_2 + p_3^* dE_3^* + E_3^* dP_3. \]

Substituting these in (3.14), we obtain:

\[ dy = E_2^* dP_2 + E_3^* dP_3 + (p_2 - p_2^*) dE_2^* + (p_3 - p_3^*) dE_3^*. \]

From this last equation, we can derive optimal policies depending on whether the country under consideration is a small country or a large country enjoying monopoly power in international trade.

A. **The Small Country Case**: First, take the case where the home country is small and is a price taker. World prices in this case are fixed, so that \( dp_2^* = dp_3^* = 0 \). For an interior maximum we require that \( dy = 0 \). From (3.15)
it is clear that with $d_p^* = d_p^* = 0$, $dy = 0$ only if $p_2^* = p_2$ and $p_3^* = p_3$.

In other words, welfare maximization requires that the foreign prices of all traded goods be equalized to the domestic prices. Hence the optimal policy is one of laissez-faire for trade in all products. This is the standard result and the introduction of intermediate products necessitates no additional qualification provided, of course, the transformation curve is concave to the origin.

B. Monopoly Power in Trade: If the home country enjoys monopoly power in international trade, then $p_2^*$ and $p_3^*$ are no longer fixed, but dependent on $E_2^*$ and $E_3^*$. As before, the interior maximum requires that $dy = 0$, and it is evident from (3.15) that $dy = 0$ only if $p_2^* \neq p_2$ and $p_3^* \neq p_3$. Hence a tariff by the home country on the first commodity which introduces inequalities between $p_2^*$ and $p_2$ on the one hand and $p_3^*$ and $p_3$ on the other, becomes an optimal policy. If, in addition, the home country imports the intermediate product, a tariff must to be imposed on this product also. From (3.15), we can write:

\[
(3.16) \quad \frac{dy}{dp_2^*} = E_2^*(1 + \frac{p_2^* - p_2^*}{p_2^*} \eta_2^*) + E_3^*(1 + \frac{p_3^* - p_3^*}{p_3^*} \eta_3^*) \frac{dp_3^*}{dp_2^*}
\]

where $\eta_2^* = \frac{dE_2^*}{dp_2^*} \cdot \frac{p_2^*}{E_2^*}$ is the foreign elasticity of demand for imports of the second commodity, and $\eta_3^* = \frac{dE_3^*}{dp_3^*} \cdot \frac{p_3^*}{E_3^*}$ is the foreign elasticity of demand (supply) for imports (exports) of the intermediate product. It is assumed that $\eta_2^* < 0$, whereas $\eta_3^* \leq 0$ depending on whether $E_3^* \geq 0$. Let $t_1$ denote the tariff imposed
by the home country on the first commodity, so that
\[ p_2(1+t_1^*) = p_2^*, \text{ and } p_3(1+t_1^*) = p_3^*. \]

Substituting these in (3.16) and equating \( dy^*/dp_2^* \) to zero, we have:

\[
(3.17) \quad (1 + \frac{t_1}{1+t_1^*} \eta_2^*) + \frac{E_3^*}{E_2^*} \cdot \frac{dp_3^*}{dp_2^*} (1 + \frac{t_1}{1+t_1^*} \eta_3^*) = 0,
\]

whence the optimum tariff, \( t_0^* \), can be shown to be

\[
(3.18) \quad t_0^* = \frac{-\alpha_3^* \beta_3^*}{(1+\eta_2^*) + \alpha_3^* \beta_3^* (1+\eta_3^*)}
\]

where \( \alpha_3^* = \frac{p_3^* E_3^*}{p_2^* E_2^*} \) and \( \beta_3^* = \frac{dp_3^*}{dp_2^* p_3^*} \) is the elasticity of the world relative price of the intermediate product with respect to the world relative price of the second commodity. If the foreign country imports the intermediate product, \( \alpha_3^* > 0 \); if it exports the intermediate product, \( \alpha_3^* < 0 \). \( \beta_3^* \) may be positive or negative depending on the sign of \( dp_3^*/dp_2^* \) which can be determined from (1.19) in section I.

Let us start with the case where the intermediate product is not traded. Here \( \alpha_3^* = 0 \), so that the optimum tariff reduces to

\[ t_0^* = \frac{-1}{1+\eta_2^*} \]

This is the traditional optimal tariff formula, and it may be observed that the optimum tariff is positive only if \( |\eta_2^*| > 1 \), i.e., only if the foreign import demand is elastic. This is the necessary condition for a positive optimum tariff to exist in the traditional model without intermediate goods, and the result...
remains unchanged even if the intermediate good is introduced in the model, provided it does not enter trade. However, if the intermediate product is traded, then \( \alpha_3^* \neq 0 \). First, consider the case where \( \alpha_3^* > 0 \), so that the foreign country also imports the intermediate product. Here \( \eta_3^* < 0 \). If \( \beta_3^* > 0 \), then the numerator of (3.18) is negative. Therefore if \( |\eta_3^*| \leq 1, |\eta_2^*| > 1 \) is still a necessary condition for the optimum tariff to be positive. Furthermore, even if \( |\eta_2^*| > 0 \), the optimum tariff may be negative if \( |\eta_3^*| < 1 \). Hence a sufficient condition for optimum tariff to be positive is that both foreign import demands are elastic.

In the absence of trade in the intermediate good, a tariff on the importables results in an improvement in the terms of trade (i.e., \( p_2^* \) rises), which tends to raise welfare. Welfare is maximized when the slope of the foreign offer curve equals the home country's internal commodity-price ratio. Since the latter is positive, the former has to be positive, and as is well known, a positive slope of the foreign offer curve implies an elastic foreign import demand (see Vanek [16], Ch. 16). However when the foreign country imports the intermediate good also, there are two terms of trade involved, and a tariff by the home country on its final commodity importables, improves them both (i.e., \( p_2^* \) rises and \( p_3^* \) rises when \( \beta_3^* > 0 \)). The result is symmetric. For optimum tariff to be positive, we require that both foreign import demands be elastic.

Next consider the case where \( \beta_3^* < 0 \). When \( \alpha_3^* > 0, \alpha_3^* \beta_3^* < 0 \). Here then arises the possibility that the optimum tariff may be zero. This possibility from (3.18) requires that \( \alpha_3^* \beta_3^* = -1 \). The economic explanation for this result is that when \( \beta_3^* < 0 \), a tariff by the home country improves its terms of trade with respect to the second commodity, but worsens them
with respect to the third. The former effect tends to raise welfare, whereas the latter effect tends to lower it. The final result will depend on the relative strength of these two forces working in the opposite direction. When $\alpha_3^{*}\beta_3^{*} = -1$, the relative strength of the two forces is equal, because

$$\alpha_3^{*}\beta_3^{*} = \frac{p_3^{*}}{p_2^{*}} \cdot \frac{E_3^{*}}{E_2^{*}} \cdot \frac{dp_3^{*}}{dp_2^{*}} = \frac{E_3^{*}}{E_2^{*}} \cdot \frac{dp_3^{*}}{dp_2^{*}}.$$

When $\beta_3^{*} = -1$, $p_3^{*}$ declines in the same proportion as the rise in $p_2^{*}$, and since $-E_3^{*}dp_3^{*} = E_2^{*}dp_2^{*}$, a rise in the value of home exports of the second commodity is exactly balanced by the decline in the value of home exports of the third commodity. Thus when $\alpha_3^{*}\beta_3^{*} = -1$, the home country neither benefits nor loses from the imposition of the tariffs, so that $t_o$ is zero.\(^9\)

However, when $|\alpha_3^{*}\beta_3^{*}| < 1$, the numerator of (3.18) is negative, a sufficient condition for $t_o > 0$ is that $|\eta_2^{*}| > 1$ and $|\eta_3^{*}| < 1$. In this case, it is no longer necessary that both foreign demands be elastic for $t_o > 0$. If, on the other hand, $|\alpha_3^{*}\beta_3^{*}| > 1$, the numerator of (3.18) is positive. A sufficient condition for $t_o$ to be positive in this case is that $|\eta_2^{*}| < 1$ and $|\eta_3^{*}| > 1$. Here, again both foreign import demands need not be elastic for the home country's optimum tariff to be positive. Thus we conclude that when $|\alpha_3^{*}\beta_3^{*}| \neq 1$, a sufficient condition for the optimum tariff to be positive is that at least one of the foreign import demands is elastic.

So far we have assumed that the foreign country imports the intermediate product. Let us now consider the case where the foreign country exports the intermediate product. Here $\eta_3^{*} > 0$. It is evident from (3.18) that if $\beta_3^{*} < 0$, the foreign import demand should be very elastic for $t_o$ to
be positive. Because $\alpha_3^* < 0$, $\beta_3^* < 0$, and $\alpha_3^* \beta_3^* > 0$, so that the numerator of (3.18) is negative. In the denominator, $\alpha_3^* \beta_3^*(1+\eta_3^*)$ is positive so that for the denominator to be negative we require that $|\eta_2^*| > [1 + \alpha_3^* \beta_3^*(1+\eta_3^*)] > 1$. Hence now the foreign import demand has to be even more elastic. On the other hand, if $\beta_3^* > 0$, $t_o$ may again be zero provided, of course, $\alpha_3^* \beta_3^* = -1$. If $|\alpha_3^* \beta_3^*| < 1$, so that the numerator of (3.18) is negative, $t_o > 0$ even if $|\eta_2^*| < 1$. In other words, the home country's optimum tariff on its final commodity imports may be positive even if the foreign import demand is inelastic or even zero. If $|\alpha_3^* \beta_3^*| > 1$, then $t_o$ can never be positive. Here the numerator is positive, but the denominator is necessarily negative, even if $\eta_2^* = 0$. Here then arises the need for a tariff on the imports of the intermediate product.

A. Home Country's Tariff on Both Importables

So far we have assumed that the home country imposes a tariff only on its imports of the final commodity. Suppose it also imposes a tariff on its imports of the intermediate good. Here

\[ p_2 = p_2^*(1+t_1), \quad \text{and} \]
\[ p_3(1+t_1) = p_3^*(1+t_3) \]

where $t_3$ is the home country's tariff on the intermediate product. Substituting these in (3.16) and denoting $t_o^*$ for $t_1$ at the optimum position, we obtain:

\[ t_o^* = t_o + \frac{\alpha_3^* \beta_3^*(1+t_3 \eta_3^*)}{(1+\eta_2^*) + \alpha_3^* \beta_3^*(1+\eta_3^*)}. \tag{3.19} \]

It may be observed from (3.19) that $t_o^*$, the optimum tariff obtained in the presence of a tariff on the imports of the intermediate good, may be greater or less than $t_o$, the optimum tariff obtained in the absence of a tariff on the intermediate product, depending on whether the second expression in the right hand side of (3.19) is positive or negative. If $\alpha_3^* \beta_3^* = -1$, we know
from our previous analysis that $t_0 = 0$. However, if $|\eta_2^*| > \eta_2^*$, $t_o^*$ is positive. If $\alpha_3^* \beta_3^*$ is positive, then $t_o^* > t_o$ if $|\eta_2^*| \leq 1$. Other interesting results may be similarly derived by placing restrictions on $\alpha_3^* \beta_3^*$ and $\eta_2^*$.

C. A Tariff only on the Intermediate Product: Suppose for some administrative or political reasons, the home country cannot impose a tariff on its imports of the final commodity. The country may still be able to reap benefits from its monopoly power in trade, by imposing a tariff on the imports of the intermediate product. In this case $p_2^* = p_2$ and $p_3 = p_3(1 + t_3^*)$. Substituting these in (3.16) and replacing $t_3$ by $t_3^*$ in the optimum position, we obtain:

$$t_3^* = \frac{1 + \alpha_3^* \beta_3^*}{\alpha_3^* \beta_3^* \eta_3^*}.$$  

Since $\eta_3^* > 0$, $t_3^* > 0$ if $\alpha_3^* \beta_3^* > 0$, which, with $\alpha_3^* < 0$, requires that $\beta_3^* < 0$. In other words, the optimum tariff on the intermediate product alone is positive if an improvement in the home country's terms of trade with respect to the intermediate good as a result of the tariff (so that $p_3^*$ declines) also results in an improvement in its terms of trade with respect to the second commodity (so that $p_2^*$ rises, which makes $\beta_3^* < 0$). Again if $\alpha_3^* \beta_3^* = -1$, $t_3^* = 0$. If $|\alpha_3^* \beta_3^*| > 1$, $t_3^*$ is still positive. Thus for a positive $t_3^*$, we require either a positive $\alpha_3^* \beta_3^*$ or a negative $\alpha_3^* \beta_3^*$ with absolute value greater than unity.

**Conclusions**

Introducing an intermediate product in the traditional two-country,
two-commodity, two-factor model we have derived the following conclusions:

1. The transformation curve may become locally or globally convex to the origin if, in addition to the primary factors, the final products also utilize the material input in the process of production.

2. Given the possibility of the convexity of the transformation curve towards the origin, free trade may or may not be superior to no trade. This result stands in sharp contrast to that derived in the traditional models of intermediate goods, where the latter are assumed to be identical with the traded final goods.

3. If we assume that the transformation curve is concave to the origin, then the traditional result concerning gains from free trade holds. Furthermore, if there is monopoly power in international trade, welfare maximization requires an optimum tariff. The formula for the optimum is the same as the traditional one, if intermediate products are not traded, so that an elastic foreign import demand is a necessary condition for optimum tariff to be positive. If trade is allowed in intermediate products, the optimum tariff may be positive even if the foreign import demand of the final product is inelastic.

4. If in addition to a final commodity, the intermediate good is imported, there may be two optimum tariffs: one on the final product imports and the other on the imports of the intermediate product. Both tariffs are, of course, interrelated.
Footnotes

1 See, for example, Vanek [15], Mckinnon [10], and Melvin [11] among others.

2 See Ruffin [14], p. 267. Ruffin's model has also been utilized by Batra and Singh [2] to analyze the stability properties of a two-sector growth model.

3 However, where \( \frac{dX_1}{dX_2} \neq -p \), the shape of the transformation curve may not be dependent on the response of the outputs to their prices. For example, in the presence of an inter-industry wage-differential where \( \frac{dX_1}{dX_2} \neq -p \), Kemp and Herberg [8] and Bhagwati and Srinavasan [6] have shown that even if the output response to price-change is "perverse," the transformation curve need not be convex to the origin; conversely, even if the output response to price change is "normal," the transformation curve may be convex to the origin. For other analyses of the wage-differential, see Bhagwati and Ramaswami [5], Batra and Pattanaik [1], and Batra and Scully [4].

4 It can be seen that if B has the same sign as \( (k_2 - k_1) \), it always has a definite sign. This, however, does not mean that A also has a definite sign.

5 \( HQ \) is drawn parallel to \( O_2E' \) in Figure 3.

6 See Mathews [9] and Melvin [13].

7 The results in section III have been derived under the assumption of the absence of trade in intermediate products. This has been done for the sake of geometrical simplicity. For, when intermediate goods are traded, the marginal rate of transformation, as shown in section III, is no longer equal to the commodity-price ratio. Furthermore, the autarky and the trade transformation curves differ when intermediate goods are also traded. This is because trade in intermediate goods is equivalent to the exchange of the primary factors employed in the production of such goods. This effectively changes the available factor supplies and in turn makes the autarky transformation curve different from the one obtained in trade. No such complication is encountered when intermediate goods do not enter trade. For details on this point, see Batra and Casas [3].

8 It may be noted that the income distribution effects of the optimal policies have not been ignored, but are implicit in the selection of the social welfare function.

9 If \( \alpha_3^* \beta_3^* = -1 \), but \( \beta_3^* \neq 1 \) and \( \alpha_3^* \neq 1 \), the result and the reasoning behind it remain the same.
For example, we can derive the optimum tariff on the imports of the intermediate product from (3.19). A conclusion of some interest is that the optimum tariff on any of the two importables is not independent of the tariff on the other product.

Throughout our analysis, we have assumed that a tariff on any one of the importables improves the home country's relevant terms of trade. This, however, may not be always true. But a detailed analysis of this problem is not within the scope of this paper.
References


Mathematical Appendix

Some of the equations which appear in section I of the text without proof will be derived here. Totally differentiating equations (1.5) and (1.6) with respect to $p_2$, taking the members of each equation two by two, and writing in matrix form, we have

\[
\begin{vmatrix}
(1-a_1 p_3) f''_1 & -(p_2-a_2 p_3) f'_2 & 0 & a_2 f'_2 - a_1 f'_1 & dk_1 / dp_2 & f'_2 \\
(1-a_1 p_3) f''_1 & 0 & -p_3 f'_3 & -a_1 f'_1 & - f'_3 & dk_2 / dp_2 & 0 \\
-(1-a_1 p_3) k_1 f''_1 & (p_2-a_2 p_3) k_2 f''_2 & 0 & a_2 (f'_2-k_2 f'_2)-a_1 (f'_1-k_1 f'_1) & dk_3 / dp_2 & f'_2-k_2 f'_2 \\
-(1-a_1 p_3) k_1 f''_1 & 0 & p_3 k_3 f'_3 & -a_1 (f'_1-k_1 f'_1)-(f'_3-k_3 f'_3) & dp_3 / dp_2 & = 0
\end{vmatrix}
\]

(A.1)

Solving the system of equations (A.1) then gives equations (1.16) - (1.19).

Differentiating (1.3), (1.4), (1.7) and (1.8) with respect to $p_2$, we obtain:

\[
\begin{vmatrix}
1 & 1 & 1 & dL_1 / dp_2 \\
k_1 & k_2 & k_3 & dL_2 / dp_2 \\
a_1 f'_1 & a_2 f'_2 & - f'_3 & dL_3 / dp_2
\end{vmatrix} = \begin{vmatrix}
\frac{dk_1}{dp_2} + \frac{dk_2}{dp_2} + \frac{dk_3}{dp_2} & 0 \\
\frac{dL_1}{dp_2} + \frac{dL_2}{dp_2} + \frac{dL_3}{dp_2} & 0 \\
\frac{a_1 L_1 f'_1}{dp_2} + \frac{a_2 L_2 f'_2}{dp_2} - \frac{L_3 f'_3}{dp_2} & 0
\end{vmatrix}
\]

(A.2)

Substituting from (1.16) - (1.18), and solving, we have:
\[ (A.3) \quad \frac{dL_1}{dp_2} = -\frac{(1-a_1 p_3)}{A.B} \left[ \frac{L_1 f_2 f_3 \left\{ f_3 + a_2 f_2 + a_1 f_1 (k_3 - k_2) \right\}}{(1-a_1 p_3)^2 f_1} + \frac{p_2 L_2 f_1 f_3^2}{(p_2 - a_2 p_3)^3 f_2''} + \frac{p_2 L_3 f_1^2 f_2^2}{p_3 f_3} \right] \]

\[ (A.4) \quad \frac{dL_2}{dp_2} = -\frac{(1-a_1 p_3)}{A.B} \left[ \frac{L_1 f_2 f_3^2}{1 \ f_2} + \frac{L_2 f_1 f_3 \left\{ f_3 + a_2 f_1 + a_1 f_2 (k_3 - k_1) \right\}}{(1-a_1 p_3)^3 f_1} + \frac{p_2 L_2 f_1 f_2^2}{(p_2 - a_2 p_3)^2 f_2''} + \frac{3 f_2''}{p_3 f_3} \right] \]

Differentiating (1.1), (1.2) and (1.4) with respect to \( p_2 \), we have:

\[ (A.5) \quad \frac{dX_1}{dp_2} = L_1 f_1 \frac{dk_1}{dp_2} + f_1 \frac{dL_1}{dp_2} \]

\[ (A.6) \quad \frac{dX_2}{dp_2} = L_2 f_2 \frac{dk_2}{dp_2} + f_2 \frac{dL_2}{dp_2} \]

\[ (A.7) \quad \frac{dX_3}{dp_2} = a_1 \frac{dX_1}{dp_2} + a_2 \frac{dX_2}{dp_2} . \]

Substituting (1.16), (1.17), (A.3) and (A.4) in (A.5) and (A.6) we can derive equations (1.20) and (1.21). Then substituting (1.20) and (1.21) in (A.7) gives us (1.22).