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ADVANTAGEOUS REALLOCATIONS AND MULTIPLE EQUILIBRIA:
RESULTS FOR THE THREE-AGENT TRANSFER PROBLEM

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MAY 19, 1983
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ADVANTAGEOUS REALLOCATIONS AND MULTIPLE EQUILIBRIA:
RESULTS FOR THE THREE-AGENT TRANSFER PROBLEM

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ABSTRACT

Advantageous reallocations occur in competitive economies when some agents rearrange their initial endowments and so alter prices that they are better off. From an example of this phenomenon (due to Léonard and Manning) it is shown that economies which permit advantageous reallocations imply the existence of economies with multiple equilibrium. The idea of advantageous reallocations is then applied to economies with multiple equilibria. By reallocating their endowments, a pair of agents may be able to guarantee the outcome they prefer. It is sufficient for this that the economy with multiple equilibria be constructable from an economy which permits advantageous reallocations.
INTRODUCTION

There has been considerable recent interest in the transfer problem, especially for the case with more than two agents. Three streams of literature have flowed together in this work, with contributions coming from development economists, international trade theorists, and mathematical economists. All have found aspects of the problem challenging. It is not the aim of this paper to present an assessment of all of this work, nor are all the papers cited here (Jones (1983b) may be consulted for more details). A few important references must be noted, however. The inspiration in development economics is the paper of Chichilnisky (1980). In international trade theory, Bhagwati, Brecher and Hatta (1982a,b), and Yano (1983) have been the forerunners of much other work, including that of Kemp and Kojima (1983). Mathematical aspects of the problem have been explored by Majumdar and Mitra (1983), and Safra (1983), each of which presents a "smooth" example of the transfer paradox, in which a giver is made better off by the price changes his action causes.

It was the note of Gale (1974) which generated much of this work. He gave an example of a three-agent, two-commodity exchange economy in which a pair of the agents improved their welfare by reallocating their own endowments. His example used "L-shaped" indifference curves, and Gale challenged economists to present an example with smooth preferences. Aumann and Peleg (1974) quickly responded. Their example has several objectionable features (for instance, one agent does not consume one commodity). These were noted by Léonard and Manning (1975), who had constructed a genuinely smooth example. They also noted that multiple equilibria were related to the Gale phenomenon. This claim was strenuously denied by some readers. Later, Guesnerie and Laffont (1978) extended Gale's example to show that there are economies in which an arbitrary collection
of different agents are able to reallocate their own endowments and become better off: They coined the term "advantageous reallocation" for this. In view of the recent developments, Léonard and Manning (1983) revised their earlier example. Its construction is similar in spirit to the proof of the theorem of Guesnerie and Laffont.

The present paper begins by explaining the example of Léonard and Manning. From this beginning it is shown that there are many economies with multiple equilibria for each economy which permits advantageous reallocations. Next, the work of Manning (1975) is developed. This section applies the idea of advantageous reallocations to overcome the indeterminacy of multiple equilibria. The essence of an advantageous reallocation is that a reallocation of their endowments will improve the outcome of the competitive economy for each member of a group. This can also be made precise when there are multiple equilibria, in which case the group can guarantee one equilibrium rather than the others by an equilibrium-preserving reallocation. Finally, the relationship between economies which permit advantageous reallocations, and economies which have multiple equilibria which will be avoided by equilibrium-preserving reallocations, is explored. It is found that a sufficient condition for a pair of agents to both want and be able to guarantee one outcome is that the economy with multiple equilibria can be constructed from an economy which permits an advantageous reallocation. This is not a necessary condition, however.

**CONSTRUCTION OF ECONOMIES WITH MULTIPLE EQUILIBRIA**

Léonard and Manning (1983) construct an example of an exchange economy in which a pair of agents can make a (finite) reallocation of their endowments and so alter the relative prices of the two commodities that they are made better off at the expense of the third agent. Before and after this advantageous
reallocating the equilibrium is unique, and is globally stable in the Walrasian sense. It is apparent from the construction that many examples of this phenomenon may be found. Indeed, in the specific example they ended up with, one parameter needed only to satisfy an inequality. The example is now explained, and is used to construct economies with multiple equilibria.

The utility functions of the agents involved in the reallocation are $x_1^{1/2} y_1^{1/2}$ and $x_2^{7/8} y_2^{1/8}$, while the third agent has the utility function $5.25 x_3 - 0.14 x_3^{3/3} + y_3$, where $x_i$ and $y_i$ are the $i$th agent's consumption of commodities 1 and 2, respectively.

Before the resource reallocation the first two agents had endowment vectors $(10,0)$ and $(4,0)$, respectively. After the reallocation they have endowment vectors $(6,0)$ and $(8,0)$. The third agent has the endowment vector $(0,c)$, $c > 12$. To be precise, let $c = 14$.

By construction, the (unique) equilibrium relative price before the reallocation was $p = 1$, while it is $p = 3$ afterwards, where commodity 2 is the numeraire.

Figure 1 represents this example (and provides a useful diagrammatic technique for the analysis of 3--or more--agent exchange economies). The sides of the box are each 14, the total endowment of the two commodities. From $O_1$ is measured the endowment and consumption of agent 1 (similarly for agent 2 from $O_2$). The endowments are initially at $w_1$ and $w_2$ respectively. The difference between these is $w_3$, the endowment of the third agent. If $p = 1$, the budget constraints of the first two agents are $w_1 c_1$ and $w_2 c_2$, and these agents consume at $c_1$ and $c_2$, respectively. By construction, the third agent willingly consumes the difference between $c_1$ and $c_2$ (that is, what the others do not consume). After the advantageous reallocation, the
endowments of the first two agents are at $r_1$ and $r_2$: The endowment of the third agent is unaffected. If $p = 3$, the budget constraints of the first two agents are $r_1 d_1$ and $r_2 d_2$, and they consume at $d_1$ and $d_2$. Again by construction the third agent consumes what the others do not, now the difference between $d_1$ and $d_2$. Figure 2 shows the endowment point $w_3$, and the budget constraints $w_3 c_3$ and $w_3 d_3$ which confront the third agent before and after the resource reallocation by the first two, and his consumptions $c_3$ and $d_3$ in these situations.

It is evident that relative prices must change when an advantageous reallocation occurs (or indeed when any of the three-agent transfer paradoxes arise). From this necessary condition it follows that the equilibrium budget constraints for each agent before and after the reallocation must intersect. In the example just explained, the constraints $w_1 c_1$ and $r_1 d_1$ intersect at points labelled $m_i$, $i = 1, 2, 3$. See Figures 1 and 2. Simple calculations reveal $m_1 = (4, 6)$ and $m_2 = (10, -6)$. Of course, $m_3 = w_3$: That is, the non-participant in the advantageous reallocation has budget constraints which intersect at his endowment.

In the example, the points $m_1$ and $m_3$ are non-negative, but $m_2$ has one negative component. Without loss of generality, suppose that the first agent makes the transfer to the second agent in the general case. It is then true that the intersection of the budget constraints which apply before and after the advantageous reallocation, at points labelled $m_1$, $m_2$ and $m_3$ for consistency with the example, have these properties: $m_1$ non-negative, $m_3 = w_3$ non-negative, but $m_2$ may have one negative component. Only the first of these claims needs any proof. Simple "revealed preference" arguments suffice.
Let \((w_{11}, w_{12})\) and \((r_{11}, r_{12})\) be the endowments of the first agent before and after the advantageous reallocation, while \((c_{11}, c_{12})\) and \((d_{11}, d_{12})\) are his consumption vectors in these two situations. These vectors are all non-negative. Let \(p_1\) and \(p_2\) be the (relative) prices of commodity 1 before and after the transfer. Suppose \(p_2 > p_1\). The budget constraints before and after the transfer intersect at \((m_{11}, m_{12})\). Of course,

\[
\begin{align*}
p_1 m_{11} + m_{12} &= p_1 c_{11} + c_{12} \\
p_2 m_{11} + m_{12} &= p_2 d_{11} + d_{12}
\end{align*}
\]

so that

\[
(1) \quad (p_1 - p_2) m_{11} = p_1 c_{11} + c_{12} - p_2 d_{11} - d_{12}
\]

But \((d_{11}, d_{12})\) is preferred to \((c_{11}, c_{12})\), by hypothesis. Therefore

\[
(2) \quad p_1 d_{11} + d_{12} > p_1 c_{11} + c_{12}
\]

It is immediate from (1) and (2) that \(m_{11} > d_{11} \geq 0\). On the other hand,

\[
\begin{align*}
p_1 m_{11} + m_{12} &= p_1 w_{11} + w_{12} \\
p_2 m_{11} + m_{12} &= p_2 r_{11} + r_{12}
\end{align*}
\]

so that

\[
(3) \quad \left(\frac{1}{p_2} - \frac{1}{p_1}\right) m_{12} = r_{11} + \frac{r_{12}}{p_2} - \frac{w_{11}}{p_1} - \frac{w_{12}}{p_1}
\]

But \((r_{11}, r_{12})\) is less valuable than \((w_{11}, w_{12})\) at the original prices. Therefore

\[
(4) \quad p_1 r_{11} + r_{12} < p_1 w_{11} + w_{12}
\]

It is immediate from (3) and (4) that \(m_{12} > r_{12} \geq 0\). So \(m_1 = (m_{11}, m_{12})\) is non-negative, as claimed.

Consider now the economy with initial endowments, \(m_1\), \(m_2\) and \(m_3\) for the three agents, and with the same preferences as in the economy which permitted the advantageous reallocation. Two cases may be distinguished: Either \(m_2\)
is non-negative, or \( m_2 \) has one negative component. The second case is discussed later. In the former event, the economy has the features usually assumed, and it has multiple equilibria which include the prices found before and after the advantageous reallocation.\(^3\) In this case, an economy with multiple equilibria has been constructed from an economy which allowed an advantageous reallocation. Furthermore, the total endowments of these two economies are the same: Indeed, the sum of the endowments of the first two agents are the same. That is

\[(m_{11} + m_{21}, m_{12} + m_{22}) = (w_{11} + w_{21}, w_{12} + w_{22})\]

(This is also true when \( m_2 \) has a negative component: In the example shown in Figure 1, both of these vectors equal \((14,0)\).) This follows from the added budget constraints

\[p_1(m_{11} + m_{21}) + m_{12} + m_{22} = p_1(w_{11} + w_{21}) + w_{12} + w_{22}\]

\[p_2(m_{11} + m_{21}) + m_{12} + m_{22} = p_2(r_{11} + r_{21}) + r_{12} + r_{22}\]

and the requirement that the first two agents reallocate between themselves, so that

\[r_{11} + r_{21} = w_{11} + w_{21}\]

\[r_{12} + r_{22} = w_{12} + w_{22}\]

(Figure 1 shows the essential geometry of the result: \( w_{11}, m_1 \) and \( w_{21}, m_2 \) are congruent triangles.) This fact proves to be of use later.

Two approaches may be made to the case in which \( m_2 \) has a negative component. The first, direct, procedure is to allow agents to have negative endowments of some commodities. Although some would find this objectionable,\(^4\) it yields immediately the conclusion noted previously: Associated with an
economy which permits an advantageous reallocation is another economy which
has multiple equilibria. The alternative approach is to transform the economy
so that \( m_2 \) is non-negative. While this is less obvious, it does give more
insight.

Instead of the Léonard-Manning example, consider an economy in which
agents 1 and 3 are as they assumed, but in which agent 2 has a Stone-Geary LES
utility function \( x_2^{7/8}(y_2 - 6)^{1/8} \) for \( y_2 \geq 6 \), while for \( y_2 < 6 \), utility is \( y_2 - 6 \),
and an endowment \((4,6)\). Figure 1 illustrates this economy too, provided that
the endowment and consumption of agent 2 are measured from \( 0_2 \). If \( p = 1 \), the
consumption of this agent is at \( c_2 \), which is \((7/2,13/2)\) with respect to
the new origin. If agent 1 transfers 4 units of commodity 1 to agent 2, their
endowments become \( r_1 \) and \( r_2 \), where \( r_2 = (8,6) \). For this endowment, if \( p = 3 \),
agent 2 consumes at \( d_2 \) which is \((7,9)\) measured from \( 0_2 \). This reallocation is
advantageous, as in the Léonard-Manning example. In this case, however, \( m_2 \)
is non-negative. By a transformation of the preferences and the endowment
of the second agent alone, an economy with multiple equilibria and all
endowments non-negative has been constructed from the Léonard-Manning example
of advantageous reallocations. A moments reflection makes it clear that the
transformation used is not the only one which preserves the essential geometry
of the original example, but that it is the smallest such change which ensures
that \( m_2 \) is non-negative. For instance, an agent could be endowed with \( \beta_{22} \)
units of commodity 2 and his utility function be \( x_2^{7/8}(y_2 - \beta_{22})^{1/8} \), \( \beta_{22} > 6 \),
but then \( m_{22} \) will be positive.

It is quite clear that the procedure which worked for the Léonard-Manning
example can be mimicked in general. From any three-agent economy which permits
an advantageous reallocation from agent 1 to agent 2 can be constructed other
economies with the same feature, indeed which have the same before and after transfer equilibrium prices and trades. If $u_2(x_2, y_2)$ is the utility function, and $(w_{12}, w_{22})$ is the endowment, of agent 2 in the economy which permits an advantageous reallocation, then the same reallocation is advantageous in the economy in which agent 2 has endowment $(w_{21} + \beta_{21}, w_{22} + \beta_{22})$ and utility function $u_2(x_2 - \beta_{21}, y_2 - \beta_{22})$. What is more, for $\beta_{21}, \beta_{22}$ sufficiently large, the before- and after-transfer budget constraints of agent 2 will intersect for non-negative values. Many economies with multiple equilibria may be associated in this way with any economy which permits an advantageous reallocation. Finally, note that the endowments and preferences of the remaining agents may also be manipulated to maintain the same net trading patterns as in any case which permits advantageous reallocations.

The conclusions reached so far are now summarized.

PROPOSITION I: Associated with any three-agent, two-commodity exchange economy which has a unique, Walrasian stable equilibrium, but which permits advantageous reallocations between two of the agents, are many exchange economies with multiple equilibria (which include the prices found before and after advantageous reallocations). The total resources of such an economy with multiple equilibria will in general differ from those of the economy which permits advantageous reallocations from which it was constructed. Where these totals are the same, however, it is only the endowments of the agents involved in the advantageous reallocation which are different.
MULTIPLE EQUILIBRIA RECONSIDERED

Multiple equilibria are an embarrassment to economic equilibrium theory. That theory seeks to explain the prices which prevail in markets. When there are many price vectors which equate supply and demand for all commodities, economic theory is silent on which will emerge: The theory cannot do what it set out to do. Strong results have been proved to show that the class of economies capable of giving a continuum of equilibria is negligibly small. Debreu (1970) is the classic paper on this topic. Unfortunately, the class of economies which give finite numbers of equilibria is not at all small. Indeed, this is suggested by the discussion in the previous section: There are many examples of advantageous reallocations, and from each example many economies which have multiple equilibria can be constructed. An alternative way of handling the problem of multiple equilibria is needed. The idea of advantageous reallocations suggests how to proceed. The essential point is that agents may be able to manipulate the outcome of the competitive economy by reallocating their initial endowments. There are certainly features of multiple equilibria which agents find unattractive, and they would like to prevent these if it is possible.

Figure 3 illustrates the problem of multiple equilibria, and its resolution, for a three-agent, two-commodity, exchange economy. The sides of the box measure the total endowments of the two commodities. From $O_1$ the endowment and consumption of agent $i$ are measured ($i=1,2$). $w_1$ and $w_2$ are these endowment points. $w_3$ is the difference between them. It is supposed that this initial endowment, and the preferences of the agents, are consistent with three equilibrium price ratios, $p_1$, $p_2$, $p_3$ (where commodity 2 is the numeraire). At these prices, agent $i$ consumes at $c_{i1}$, $d_{i1}$, and $e_{i1}$, respectively ($i=1,2$). The third agent consumes what is left.
Equilibrium theory cannot predict which equilibrium price ratio will emerge. Nor will the agents themselves be able to predict the outcome, since they do not know the value of their initial endowments. They can assess the value of their endowments conditional on each possible equilibrium price ratio, but there is no rational basis for combining these conditional valuations into a composite valuation over the outcome space. For instance, the expected utility calculus is not appropriate, since no probabilities can be attached to the outcomes.

However, all three agents have induced preferences on the set of possible equilibrium prices. These follow from their indirect utility functions $v_i(p, pw_i)$. In general, the ordering of the possible equilibrium prices will differ between agents, since each equilibrium implies a point on the utility possibility frontier. When there are only two agents the orderings are completely opposed: what one likes best, the other dislikes most. When there are more than two agents it may happen that a group of them agree, in whole or in part, on the ordering. Thus, in the example illustrated in Figure 3,

\[ v_1(p_1, p_1w_1) > v_1(p_2, p_2w_1) > v_1(p_3, p_3w_1) \]
\[ v_2(p_1, p_1w_2) > v_2(p_2, p_2w_2) > v_2(p_3, p_3w_2) \]

so that the first two agents are unanimous in preferring $p_1$ to the other possible equilibria.

It is now clear that all of a group of agents may prefer one outcome of multiple equilibria to the other possibilities. In the present case, the first two agents can make transfers between themselves which preserve the equilibrium price $p_1$ which they like best, while altering the other prices. Call these equilibrium-preserving reallocations. Geometrically, the reallocation
from $w_1$ to $r_1$, where $w_1 r_1 = w_2 r_2$, preserves the equilibrium price ratio $p_1$.

Such reallocations are precisely defined by the requirements that

$$p_i r_i = p_i w_i, \quad i=1,2$$

$$r_1 + r_2 = w_1 + w_2$$

That is, the reallocation preserves the wealth of the agents involved, and makes no call on resources other than their own. It is trivial to check that the equilibrium is indeed the same before and after this transfer.

Under convexity of preferences, equilibrium prices are an upper-semi-continuous correspondence of the initial endowments, with the property that sufficiently small changes in endowments will preserve the number of equilibria. That is to say, in the case illustrated in Figure 3, that there is a neighborhood of the endowment $(w_1, w_2, w_3)$ in which there are always three equilibria. Suppose, however, that outside of this neighborhood, equilibrium is unique. In particular, suppose that $(r_1, r_2, w_3)$ has a unique equilibrium. The equilibrium-preserving reallocation between agents 1 and 2 shifts the economy from $(w_1, w_2, w_3)$ to $(r_1, r_2, w_3)$ and guarantees the first two agents the outcome they both like best among the multiple equilibria. The analogy with advantageous reallocations is clear: in each case, the agents have preferences over the outcomes of competition, and a group is able to reallocate the endowments of its members in order to achieve what they prefer. The implication of this behavior when there are multiple equilibria is striking: Multiple equilibria are not a problem, for individuals act to ensure uniqueness.

Two requirements must be satisfied for this resolution of the problem of multiple equilibria. There must be agents who agree on the equilibrium price ratio to be maintained, and there must be economies with unique equilibria.
that they can reach by reallocations between themselves. The second requirement is taken up in the next section. The first requirement is much less restrictive than it looks.

Suppose that there are $s$ equilibrium price ratios associated with some economy, and there are $n$ agents. Each agent has an induced preference ordering over the equilibrium prices. There are, at most, $s$ different ways of ranking one of these prices first, so that if $n > s$ at least two agents regard the same price ratio as best. When there are many agents, it becomes more likely that a group can be found which agrees on the best equilibrium.

Of course, some agents may wish to prevent the outcome that they all regard as worst among the set of multiple equilibria. Although the discussion has been in terms of a group agreeing to seek what they all regard as best, the analysis applies with equal force when there is not unanimity within the group on what is best. Recall that no probabilities attach to the various equilibria. Agents may wish to avoid the incalculable chance of the worst outcome, and seek something that they all think superior, even if each also judges something else to be better still. The adaptation of the idea of advantageous reallocations to resolve the problem of multiple equilibria requires only this.

The previous analysis in this paper has been expressed in terms of three agents. It is clear that nothing has in fact relied on this. The third agent is required to act as a complement to the pair singled out for attention. In view of this, the third agent may be interpreted as the "rest-of-the-economy", and the analysis then applies to $n$-agent exchange economies, with $n \geq 3$. In particular, in the case of multiple equilibria, agents 1 and 2 are chosen because they agree on an equilibrium price, and all remaining agents are aggregated to become "agent 3".10
The conclusions of this section are now summarized.

PROPOSITION II: If a three-agent, two commodity exchange economy has multiple equilibria, then a pair of agents may agree on an equilibrium price ratio, and be able to guarantee it by an equilibrium-preserving reallocation of their own endowments.

ADVANTAGEOUS REALLOCATIONS AND MULTIPLE EQUILIBRIA

It was first shown that some economies with multiple equilibria can be associated with economies which permit advantageous reallocations. By adapting the idea of advantageous reallocations, it was then shown that some economies with multiple equilibria permit reallocations between a pair of agents which remove the indeterminacy. Not all economies with multiple equilibria enjoy this property, but an interesting, non-trivial, class does. This class may be identified by reconsidering the two previous bits of analysis.

In Figure 3, the endowments $r_1$, $r_2$ and $w_3$ imply a unique equilibrium, by hypothesis. Suppose that a different endowment pattern $\theta_1$, $\theta_2$, $w_3$ also implied a unique equilibrium, and that $r_1 + r_2 = \theta_1 + \theta_2$. Then, beginning from the economy $\theta_1$, $\theta_2$, $w_3$ the first pair of agents can engage in an advantageous reallocation to form the economy $r_1$, $r_2$, $w_3$. In this case, the economy with multiple equilibria from which a pair of agents can engage in an equilibrium-preserving reallocation, is associated with another economy with a unique equilibrium which satisfies the conditions for an advantageous reallocation. Unfortunately this is not true in general, since the neighborhood of $r_1$, $r_2$, $w_3$ containing economies with unique equilibria need not include $\theta_1$, $\theta_2$, $w_3$. It cannot be shown that every economy with multiple equilibria which can be removed by an equilibrium-preserving reallocation implies the existence of
another economy (with a unique equilibrium) which permits advantageous
reallocations. The converse does follow, however.

Consider an economy with multiple equilibria which can be constructed
from an economy with a unique equilibrium which permits an advantageous
reallocation. Figure 1 provides an illustration, remember. Then two agents
certainly agree that one relative price is better than another. What is more,
by an equilibrium-preserving reallocation, this pair of agents can reach
initial endowments which have their preferred prices as a unique equilibrium.

This argument is now summarized.

PROPOSITION III: If a three-agent, two-commodity exchange economy with multiple
equilibria can be constructed from an economy which permits
advantageous reallocations, then there is a pair of agents
who want, and are able, to reallocate their own resources to
guarantee that one of the equilibrium price ratios prevails.

CONCLUDING COMMENTS

This paper has established some deep connections between economies with
multiple equilibria and economies which permit advantageous reallocations. These
connections are of two kinds. First, the latter imply the existence of the
former. Secondly, economies with multiple equilibria admit the possibility of
mutually beneficial rearrangements of endowments, so that this kind of behavior
is possible in a wider range of environments than previously realized. This has
the important implication that, in some circumstances, the unpredictability
associated with a multiplicity of equilibria is removed.

Although the arguments here have been for two commodities, nothing
depended on that. Just as the analysis can be extended to more than three agents
(as was pointed out in the discussion of multiple equilibria), so can many
commodities be included. Nothing new is involved.
Finally, an important open question must be posed. Under what conditions is there a pattern of resource ownership which no group in the economy has an incentive, and the ability to modify? All that advantageous reallocations, or equilibrium-preserving reallocations from an economy with multiple equilibria, show is that some pattern of ownership cannot be maintained. The economy reached by the reallocation may itself be prone to manipulation by some group.
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For their comments (at widely different dates) on the problems considered in this paper my gratitude is expressed to Geoff Fishburn, Roger Guesnerie, Ron Jones, Murray Kemp, Daniel Léonard and Jim Markusen. Many of the ideas here were presented in a talk to the Second Annual University of Western Ontario Conference on International Trade Theory, 26-7 March 1983, but they are developed from Manning (1975).

1 The construction by Léonard and Manning was of the preferences of the third agent, which is why this function has such non-obvious parameter values.

2 The example of Léonard and Manning (1975) was developed from considerations of just this sort of box diagram.

3 Remember that the number of equilibria must be odd (except in a negligible number of cases). Since only two prices are used in this construction, there must be at least one more price in the set of equilibrium prices.

4 One reason why negative endowments may be objectionable is that existence proofs use endowments which are within the consumption sets of agents. See Debreu (1959, Theorem 5.7.1). These are sufficient not necessary conditions, however. In the present case, the construction of economies with multiple equilibria starts from economies which do have equilibria, so there can be no problem of existence.

5 I am grateful to Jim Markusen who reminded me of the nice features of the LES system.

6 This section draws heavily on the arguments of Manning (1975).
It should be clearly understood that this does not prove that multiple equilibria are "more common" than unique equilibria, in some (or any) sense. In particular, it has not been proved that there is no overlap in the sets of economies with multiple equilibria associated with different economies which permit advantageous reallocations.

Figure 1 could also be used, but that diagram has worked hard enough.

For simplicity, p and \( w_1 \) are expressed here and in what follows as (column and row) vectors, so \( pw_1 \) is wealth.

The only loss in doing this is that "agent 3" has no utility function. But this is negligible, since no use is made of it anyway.

A weaker concept of "advantageous reallocations", to permit transfers from economies with multiple equilibria, would allow such a conclusion. This refinement is outside of the scope of the existing literature. Guesnerie and Laffout, for example, for simplicity avoided extending their concept to this case, while other writers have been concerned to avoid anything connected with multiple equilibria.
Figure 1: The Léonard-Manning Example of Advantageous Reallocations
Figure 2: The Choices of the Third Agent in The Léonard-Manning Example
Figure 3: Multiple Equilibria and Their Resolution in a Three-Agent, Two-Commodity Exchange Economy
1983

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