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James R. Markusen
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TRADE, FACTOR PRICES, AND THE GAINS FROM TRADE
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James R. Markusen
and
James R. Melvin
Department of Economics
University of Western Ontario
London Canada

Abstract

A simple model is developed in order to show how increasing returns can form a basis for trade and to show the consequences of trade under this assumption. After analyzing the local and global properties of the production frontier it is shown that the existence of certain stable trading equilibria depends on relative country sizes, but not on the shapes of their production frontiers. Relative factor prices, which are not equalized by trade, are similarly determined. The possibility of negative gains from trade, however, depends very much on the shapes of the production frontiers as well as on the relative sizes of the countries in question. Under certain fairly general sets of circumstances, it is demonstrated that only the smaller country can lose.

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I. Introduction

Economists have long been interested in identifying the factors which lead to international trade and which determine the particular direction of trade. Two sets of factors are highlighted in our textbooks while other sets of candidates are almost totally ignored. Perhaps the most popular explanation is the Heckscher-Ohlin model, which posits that national differences in endowments of productive factors form the basis for trade. Running a poor second is the Ricardian model which assumes that differences in production technology form the basis for trade. In each case, the basis for trade combined with a number of other economic and technological assumptions has allowed researchers to build rather complete models of international trade. Variables explained by these models include (A) the volume and direction of trade, (B) the gains from trade, and (C) international factor price differentials. Closely related to and derivative from these last two categories are analyses of tariffs, factor migration, and certain comparative static properties of equilibrium such as the Stolper-Samuelson and Rybczynski effects.

Other explanations for trade include differences in demand patterns between countries, domestic distortions such as taxes and imperfect competition, and dynamic factors such as those modelled in the 'product-cycle' approach. A final explanation for trade is increasing returns to scale. None of these bases for trade has, however, led researchers to produce a model which is anywhere near as "complete" as the Heckscher-Ohlin and Ricardian models.

The lack of attention given to increasing returns to scale as a determinant of trade is actually quite remarkable given that increasing returns may be one of the most important bases for trade. It is interesting to note in this connection that location theory typically begins with the assumption that there are increasing returns in production and that industrial
organization economists have long devoted considerable energy to estimating the degree of returns to scale in various industries. The purpose of this paper, therefore, is to construct a simple model to show how increasing returns can form a basis for trade and to show the consequences of trade under this assumption.

In order to accomplish this purpose, the paper develops a familiar-looking two-good, two-factor, two-country model in which one good is produced with increasing returns to scale while the other good is produced with constant returns. Before explicitly dealing with trade questions, however, it is first necessary to analyze the shape of the production frontier. This is of importance insofar as it helps determine the differences in equilibrium factor prices, the likelihood that countries will be specialized in production, and the possibility that one country may experience negative gains from trade. Our contribution in this area is to extend earlier analyses, particularly that of Herberg and Kemp (1969), to include certain global properties of the production frontier and to show how it depends on elasticities of substitution in production as well as on the better-known roles of factor intensities and the degree of returns to scale.

Second, the model is used to show how increasing returns can influence the direction of trade. The natural assumption to make in this regard is that countries differ in size. Given this assumption, we are able to demonstrate that there will exist at least one stable equilibrium in which the large country exports the good with increasing returns. While this may seem rather obvious, we might emphasize again that this possibly very empirically important basis for trade is widely ignored in the literature. Perhaps more importantly the usual convexity problems associated with increasing returns to scale suggest that it is not at all obvious that such a stable equilibrium will exist independently of the shape of the production frontier.
Third, we examine the question of the gains from trade, building upon the earlier findings of Kemp (1969), Melvin (1969), and Henderson (1972). All of these authors have shown that increasing returns may imply that one country (but not both) is worse off at a free trade equilibrium relative to autarky. The contribution of the present paper is to show, given the direction of trade just mentioned, the circumstances under which each country may be a loser. In particular, it is shown that there exist fairly general sets of circumstances under which only the small country can lose.

Fourth, we examine the question of factor-price equalization and show that trade will not equalize factor prices. The general result is that the price of the factor used intensively in the good produced with increasing returns to scale will be relatively high in the large country regardless of whether countries are specialized or diversified in production at the trading equilibrium. Thus the price of the factor used intensively in the production of the export good will be relatively high in each country. In the Heckscher-Ohlin model, exactly the opposite occurs if one or both countries are specialized.

Finally, the paper will conclude with a short discussion of several further implications of the above results. Implicit in the work of Kemp, for example, is the fact that the Stolper-Samuelson and Rybczynski effects continue to hold in the present model provided that the production frontier is locally concave. A second interesting result concerns factor mobility. It is noted that if factors are perfectly mobile, then an equilibrium will find each country relatively well endowed with the factor used intensively in the production of its export good. In the Heckscher-Ohlin model this is, of course, the basis for trade whereas in the present model it is the result of trade.
II. General Equilibrium with Increasing Returns to Scale

Perhaps one reason for the neglect of increasing returns as a determinant of trade lies in the fact that increasing returns are difficult to model in general equilibrium. Two approaches may be taken. First, it can be assumed that economies of scale are external to firms and internal to the industry. This is the approach taken by Herberg and Kemp (1969), Melvin (1969), Kemp (1969), and Chipman (1970). The second approach is to assume that the increasing-returns-to-scale industry is monopolized. While this has yet to be explored, a paper by Melvin and Warne (1974) does provide a basis by analyzing the effects of a monopolized sector on the general-equilibrium solution under constant-returns-to-scale technology.

In the present paper, we shall adopt the first approach and hopefully explore the second approach in a later paper. Each requires a rather different analytical formulation and thus it is best not to attempt both in one paper.

Perhaps the easiest way to introduce the model is to set it out in point form. Seven principal assumptions complete the specification.

(1) (A) Two goods (X,Y) are produced, each using two factors of production (K,L). Production functions are identical between countries.
(B) Production functions of individual firms in both industries are homogeneous of degree 1.
(C) Factor intensities differ between industries and are non-reversing. X is assumed to be relatively labour intensive.
(D) There are economies of scale internal to the X industry such that the industry production function is homogeneous of a degree greater than one.
(E) There are two countries (L and S) which have identical capital/labour endowment ratios. Country L, however, has more of both factors.
(F) Demand in each country can be represented by a set of community indifference curves which are identical and homothetic in each country.
(G) There are no domestic distortions except the externality in (D).

Most of these assumptions deserve little comment except perhaps (E). The assumption that K/L endowment ratios are identical is necessary in order to "neutralize" the Heckscher-Ohlin basis for trade in the same way that (A) neutralizes the Ricardian basis for trade.
Following Herberg and Kemp and the others referenced above, we can specify the production functions for individual firms as follows.

\[(2) \quad X_i = (X^T)F(L_{ix}, K_{ix}) = (X^T)L_{ix}f(1, \frac{K_{ix}}{L_{ix}}) = (X^T)L_{ix}f(k_{ix}) \quad 0 < T < 1\]

\[Y_i = G(L_{iy}, K_{iy}) = L_{iy}G(1, \frac{K_{iy}}{L_{iy}}) = L_{iy}g(k_{iy}) \quad k = K/L\]

\(X_i\) and \(Y_i\) are outputs of identical individual firms while \(X\) and \(Y\) are industry outputs. \(F\) and \(G\) are homogeneous of degree one, allowing us to multiply through each function by \(L/L\) to obtain the well-known expressions on the right. \((X^T)\) is the external-economy effect in the \(X\) industry. (The reason that \(0 < T < 1\) is explained below.) Even though an increase in \(X_i\) affects \(X\) slightly, we will make the usual assumption that firm \(i\) regards industry output as parametric (see Kemp (1969)). This is equivalent to the assumption that competitive firms take prices as parametric whereas mathematically they must in fact have some small degree of market influence.

Summing over the individual firms, we obtain total industry outputs.

\[(3) \quad X = \sum_i X_i = (X^T)\sum_i L_{ix}f(k_{ix}) = (X^T)L_xf(k_x)\]

\[Y = \sum_i Y_i = \sum_i L_{iy}g(k_{iy}) = L_yg(k_y)\]

where \(L_x\) and \(L_y\) are total industry inputs of labour. Manipulating the equation for \(X\), we have

\[(4) \quad X = (L_xf(k_x))^{T_x} \quad \text{where } T_x = \frac{1}{1-T} > 1.\]

The industry production function for \(X\) is thus homogeneous of degree \(T_x\). The assumption that \(T\) lies between zero and one is equivalent to the assumption that \(T_x\) is greater than one.

Assuming price-taking behavior by firms in both industries and assuming that firms in industry \(X\) take total output as parametric, the first-order conditions
for profit maximization in each industry give us familiar value-of-marginal-product conditions.

\[ w = p(f-k_x f')(x^T) = (g-k_y g') \text{ where } p = p_x/p_y \]

\[ r = p f'(x^T) = g' \]

\( w \) and \( r \) are, respectively, the wage and rental rates as usually defined and \( p \) is the price of \( X \) in terms of \( Y \). \( f-k_x f'(x^T) \) and \( f'(x^T) \) are the marginal products of \( L \) and \( K \) for individual firms in the \( X \) industry and are derived by differentiation of (2) above. Dividing the two equations in (5) by each other we have standard marginal-rate-of-substitution conditions.

\[ \omega = \frac{k_x}{f_x} - k_x = \frac{k_y}{g_y} - k_y \quad \omega = w/r \]

Equation (5) also gives an output market condition

\[ p = g'/(f'(x^T)) = (g-k_y g')/((f-k_x f')x^T). \]

In the usual constant-returns-to-scale model, \( p \) would be equal to the marginal rate of transformation in production (the slope of the production-possibility curve). This is not true in the present model, however, due to the production externality in the \( X \) sector. The slope of the efficient production frontier can be solved for by maximizing \( X \) for various levels of \( Y \).

\[ \operatorname{Max}_{x} \left( L_x f(k_x) \right)^{T*} + \lambda \left( \min_{x} L_x g(k_y) \right), \]

where \( \lambda \) is a Lagrangian multiplier. The first-order conditions for this problem gives us the following:

\[ \text{MRS} = \frac{f}{k_x} - k_x = \frac{g}{g_y} - k_y = \omega \]

\[ \text{MRT} = 1/\lambda = g'/(f'(x^T)T*) = (g-k_y g')/((f-k_x f')x^T). \]

where MRS and MRT are, respectively, the marginal rate of substitution in
efficient production and the marginal rate of transformation along the efficient production frontier. Comparing (9) to (6) we see that competition will result in production on the efficient production frontier \((u=MRS)\). A comparison of (7) and (10), however, shows that production will not take place at a point of tangency between the production frontier and a price line or a community indifference curve. From (7) and (10), we have
\[(11) \quad T^* \cdot MRT = p > MRT.\]

This result, which is essentially due to Herberg and Kemp (1969) is shown in Figures IA, IB, and IC. Each diagram consists of a hypothetical production possibility curve and a representative community indifference curve. As we shall show subsequently, the production frontier with increasing returns may take on a variety of shapes. In each case, the competitive equilibrium will be at a point like \(E\) with the size of the distortion from a point of tangency dependent on \(T^*\) as shown in (11). If the production frontier is more convex than the indifference curves in Figure IA, an equilibrium will tend to be at a corner.

It should be noted that the production function for \(X\) given in (2) above avoids one difficulty pointed out by Herberg and Kemp (1969). Using a somewhat more general functional form (we replace their \(G(X)F\) with \(X^TF\)), they demonstrate that the "wedge" between the MRT and the price ratio is not constant, and thus \(p\) and the MRT may move in opposite directions as we move along the production frontier. This is not true in the present formulation. As shown in (11), a "perverse" price-output effect \((dX/dp < 0)\) can occur here if and only if the production frontier is locally convex.

We are now in a position to explicitly examine the shape of the production frontier. Since the MRT and \(p\) are related by \(T^*\) as just noted, we can infer exactly how the MRT changes as we change outputs by examining how \(p\) changes. Competitive behavior in \(X\) and \(Y\) implies that the price ratio will equal the
FIGURE IA

$T^* \text{ Large}$

$|1-k_x/k_y| \text{ Small}$

$E = \text{ Competitive Equilibrium}$

FIGURE IB

$T^* \text{ Small}$

$|1-k_x/k_y| \text{ Large}$

$\sigma_x \text{ Small}$

$\sigma_y \text{ Large}$

FIGURE IC

$T^* \text{ Small}$

$|1-k_x/k_y| \text{ Large}$

$\sigma_x \text{ Large}$

$\sigma_y \text{ Small}$
average cost of producing $X$ over the average cost of producing $Y$. This is written as follows:

$$p = \frac{\frac{wL_{ix} + rK_{ix}}{(X^T)I_{ix}f(k_{ix})}}{\frac{wL_{iy} + rK_{iy}}{I_{iy}g(k_{iy})}}.$$  

(12)

We can then multiply both sides by $(X^T)$ and multiply the right-hand side by $(I_{ix}/I_{ix})/(I_{iy}/I_{iy})$ and $r/r$ to obtain

$$p(X^T) = \frac{w + k_x(w)}{\bar{f}(k_x(w))} \left/ \frac{w + k_y(w)}{g(k_y(w))} \right.$$

(13)

The fact that $k_x$ and $k_y$ are single-value functions of $w$ follows from the homogeneity assumptions in (1B). Differentiation of (6) above will give us

$$\frac{dk_x}{dw} = \frac{-F''}{f} \quad \frac{dk_y}{dw} = \frac{-G''}{g}.$$  

(14)

Both derivatives are greater than zero by the assumption that $F$ and $G$ are concave ($F'' < 0, g'' < 0$).

Given (14), the differentiation of (13) is straightforward but tedious. Since the result is implicit in Kemp, perhaps we can simply state it without going through the mechanics.

$$\frac{dp}{dw} p + T \frac{dX}{dw} \frac{w}{w + k_x} - \frac{w}{w + k_y} > 0 \text{ if } k_x < k_y.$$  

(15)

This result can be analyzed fairly easily using the factor box shown in Figure II. Given that $X$ is labour intensive and that both production functions are homogeneous, the contract curve $0_X, 0_Y$ must be smoothly bowed downward as shown. This in turn implies that $k_x, k_y$, and $w$ increase as we move up the contract curve from $0_X$ to $0_Y(dX > 0, dY < 0 \Rightarrow dw > 0)$. Referring back to
\[
\frac{dL}{L} = \frac{dK}{K} > 0, \quad d\left(\frac{Y}{X}\right) = 0 \Rightarrow dp < 0
\]
equation (15), we see that if $T = 0$ (constant returns in $X$) then $dp/dw$ would be positive, and hence the MRT would increase with $w$. This would imply that the production frontier has the "normal" concave shape locally.

For any given allocation of $K$ and $L$ between $X$ and $Y$, an increase in $T$ will not affect the value of the right-hand side of (15). This follows from the homogeneity assumption. $(dX/X)/(dw/w)$ will increase with $T$ for any given allocation, however. A movement up the contract curve has a larger effect on $X$ the larger the value of $T$. Thus as we increase $T$, the value of $(dp/p)/(dw/w)$ must fall and must eventually become negative. This is equivalent to stating that the production frontier will become convex as shown in Figure IA. Exactly when this will occur depends upon the difference in factor intensities between $X$ and $Y$. If there were no difference, $(k_x = k_y)$, then we see from (15) that the production frontier must be convex for all $T^* > 1$. In summary then, the production frontier is more likely to be concave the smaller the value of $T^*$ and the larger the value of $|1 - k_x/k_y|$.

In general, the production frontier is neither entirely convex nor concave. There may be an inflection point (Figure 1B) or even several inflection points (Figure 1C). Herberg and Kemp (1969) show that the production frontier must be convex in the neighborhood of $X=0$ as depicted in Figures IA, IB, and IC. This property must hold in the present model also since (2) is just a special case of the Herberg-Kemp formulation. Other global properties, not surprisingly, cannot be derived from a general functional specification. In order to better understand the possibilities, therefore, an appendix to the paper explores the problem for C.E.S. production functions. Denoting the elasticity of substitution in the $i^{th}$ industry as $\sigma_i$, the results show that if $\sigma_x \leq 1$ and $\sigma_y \geq 1$, then the production frontier has at most one inflection point. Thus for two Cobb-Douglas functions ($\sigma_x = \sigma_y = 1$), for example, Figures IA and IB give the only two possible shapes. For $\sigma_x > 1$ and $\sigma_y < 1$, the frontier may have a second inflection point (Figure 1C) and indeed must take on this shape as $T^* \to 1$ and $\sigma_x = \infty$. 
III. The Direction of Trade

The classic approach to determining the direction of trade is to ask what commodity prices would prevail in the absence of trade. In the present case, if we can show that p will be low in country L relative to p in country S in the absence of trade, we can then demonstrate that there will exist a stable trading equilibrium in which L exports X and imports Y.

This is fairly easy to do in the present case. The method is outlined in Figure III which consists of production frontiers for country L (obviously the outer one) and for country S. If we can show that the MRT for L is flatter than for S along the same ray from the origin then it must follow from the assumption that demand is identical and homothetic between countries that the autarky price ratio will be relatively low in L provided that these autarky equilibria are unique. We would expect this price relationship to occur since an increase in the supply of productive factors will lead to a percentage increase in the capacity to produce X which is greater than for Y by virtue of the fact that X exhibits increasing returns.

Along any ray from the origin, the ratio Y/X is constant. Since F and G are homogeneous, we can use Euler's equation to write this ratio as follows:

\[
\frac{Y}{X} = \frac{(g_k-g'_k)\overline{L} + g'_k\overline{K}}{(f_k-f'_k)\overline{L}_x + f'_k\overline{K}_x} - \frac{(g_k-g'_k)\overline{L}_x + g'_k\overline{K}_x}{(f_k-f'_k)\overline{L}_x + f'_k\overline{K}_x}
\]

The numerator of the right hand side of (16) makes use of the simple identities \(L_y = \overline{L} - \overline{L}_x\) and \(K_y = \overline{K} - \overline{K}_x\). Using (5) above, we can replace the expressions in g with expressions in f and p. This allows us to write (16) as

\[
\frac{Y}{X} = \frac{(f_k-f'_k)\overline{L} + f'_k\overline{K}}{(f_k-f'_k)\overline{L}_x + f'_k\overline{K}_x} - p = \frac{(\omega K)\overline{L}}{\overline{L}_x} - p = p\alpha - p; \quad \alpha > 1.
\]
\( \alpha \) is greater than one by virtue of the facts that \( \bar{k} > k_x \) (X is labour intensive) and \( \bar{L} > L_x \).

Now let us refer back to Figure II and assume that \( 0_x \) is the factor box for country S while \( 0'_x \) is the factor box for country L. Suppose also that A in Figure II corresponds to A or B in Figure III. If this is true, then point A' in Figure II cannot correspond to A' or B' in Figure III. The reason is that with increasing returns in X, the move from A to A' in Figure II increases X relatively more than Y, thus violating the condition that Y/X remains constant. Points A' and B' in Figure III must correspond to a point like B in Figure II.

Noting from Figure II that \( \alpha \) in (17) takes on the same value at A' as at A, we can differentiate (17) as follows:

\[
(18) \quad \frac{d(Y)}{X} = 0 = dp = -\frac{p}{(\alpha - 1)} \frac{dp}{d\alpha}, \quad \frac{dp}{d\alpha} < 0.
\]

\[
(19) \quad \frac{d\alpha}{dx} = \frac{\bar{L} \bar{L}_x [ (\omega + \bar{k}) \frac{d\omega}{\omega} - (\omega + \bar{k}) \frac{dk}{\omega} \bar{k} ] - (\omega + \bar{k}) \bar{L} \frac{d\bar{L}}{\bar{L}_x}}{(\omega + \bar{k})^2 \bar{L}_x^2}.
\]

\[
(20) \quad \frac{d\alpha}{dx} = \frac{\bar{L} [ (\omega + \bar{k}) \frac{d\omega}{\omega} - (\omega + \bar{k}) \frac{dk}{\omega} \bar{k} ] - \alpha \frac{d\bar{L}}{\bar{L}_x}}{(\omega + \bar{k})^2 \bar{L}_x^2} \quad dw, \; dk, \; d\bar{L}_x < 0 \Rightarrow d\alpha > 0.
\]

Comparing B to A' in Figure II, we see that \( \omega, k_x, \) and \( L_x \) are all smaller at B than at A'. From (17) and (20), it follows that \( \alpha \) is larger at B than at A' or A. Thus (18) in turn implies that p must be lower at B than at A if \( X/Y \) takes on the same value at each point. In terms of Figure III, the MRT must be lower at A' than at A (recall from (11) that \( T^* \cdot MRT = p \)). Similar
comments apply to B and B' in Figure III. Regardless of the shapes of the two production frontiers, the MRT in country L must be less than the MRT in country S for equal values of \(X/Y\).

This result can then be used to show that the equilibrium price ratio that would prevail in the absence of trade must be lower in country L than in country S. If A in Figure III is the autarky equilibrium for country S, then the autarky equilibrium for country L must be "downhill" from A' given that demand is identical and homothetic between the two countries. An equilibrium downhill from A' implies a higher consumption ratio \(X/Y\), implying in turn a lower equilibrium price ratio in country L. This will allow us to establish the following proposition.

**Proposition 1. (The Direction of Trade)**

Given the assumptions noted in 1 above, there will exist at least one stable equilibrium in which the large country exports the good with increasing returns to scale.

The existence of such an equilibrium can be established with the aid of Figure IVA. In that diagram, the excess demand of country L for X \(e_x\) is graphed on the horizontal axis while L's excess demand for Y is graphed on the vertical axis. \(e_x < 0\) and \(e_y > 0\) as shown if L exports X. OC\(_L\) and OC\(_S\) represent arbitrary offer curves for countries L and S respectively. These offer curves do, however, exhibit four properties which must characterize the offer curves for any production functions satisfying (1) above and any demand functions satisfying (1). First, we have just demonstrated that the slope of OC\(_L\) is less than the slope of OC\(_S\) at 0, the autarky point \(e_x = e_y = 0\). Second, the total exports of L cannot exceed its capacity to produce X, denoted \(\overline{X}_L\), and similarly the total exports of S cannot exceed its capacity to produce Y, denoted \(\overline{Y}_S\). Third, L's excess demand for Y must approach infinity as p
approaches infinity and S's excess demand for X must approach infinity as \( p \) approaches zero. Fourth, the offer curves for each country must be continuous (see Kemp (1969)).

Properties two, three, and four imply that \( OC_A \) must "exit the rectangle" \( OX \) through \( ZY \) and only through \( ZY \). Similarly, \( OC_B \) must exit through \( ZX \). This result, combined with the first property just mentioned, implies that \( OC_A \) and \( OC_B \) must cross at least once in Figure IVA, implying that there must exist at least one equilibrium in which country L exports X (equilibria may also exist such that L exports Y. (see Kemp)).

The assertion in Proposition 1 that a stable equilibrium exists can be established using Kemp's adjustment hypothesis. Figure IVA depicts the case discussed by Kemp in which both production frontiers are everywhere convex to the origin. The segment OR of \( OC_A \) corresponds to points of non-specialization while points above R on \( OC_A \) involve L specialized in X. OR's curvature is due to the convex production frontier. The segment corresponding to specialization may be either positively or negatively sloped provided that imports (\( e_y \)) increase with any increase in \( p \) (homotheticity in demand rules out Giffen goods).

Similar comments apply to \( OC_B \). Kemp assumes that at any point in Figure IVA, consumers but not necessarily producers are in equilibrium. Producers will reduce supplies if their supply price at the point in question is more than the demand price associated with that point.

Although Kemp deals with this model and its implications for stability quite thoroughly, it might be useful to briefly review the consequences of his formulation. Consider a point just below the equilibrium point A in Figure IVA. At that point, producers in country S will be producing more Y and less X than their desired amounts at that price ratio. With increasing returns in X, this
means that the supply price of X is higher than the demand price and hence production in S will shift out of X and into Y. S's exports of Y will increase. Producers in country L, however, will wish to expand production of X and contract their production of Y, resulting in an increased volume of exports of X. The arrows associated with the point just below A in Figure IVA summarize the resulting effects on the trade bundle. A similar analysis of other points in the neighborhood of A will demonstrate that A is a stable equilibrium. The equilibrium denoted by C in Figure IVA is also stable while B is unstable.

In establishing existence, we showed that (a) OC$_L$ is flatter than OC$_S$ at 0 in Figure IVA and (b) OC$_L$ and OC$_S$ must exit OX$_L$ through ZY$_S$ and ZX$_L$ respectively. Given the demand and production homotheticity assumptions in (1) above, this in turn implies that OC$_L$ and OC$_S$ must cross at least once in at least one of the ways depicted in Figure IVB (one or the other of OC$_L$ and OC$_S$ may be vertical or horizontal, but this is immaterial). All of the equilibria in Figure IVB are stable according to the Kemp hypothesis. This establishes the stability aspect of Proposition 1. Points A and C in Figure IVA, for example, correspond to quadrants IV and III in Figure IVB respectively. Point B in Figure IVA, which is unstable, cannot be the only direction in which OC$_L$ and OC$_S$ cross if (1) is satisfied, and thus no quadrant of Figure IVB corresponds to B. Another unstable equilibrium would occur if OC$_L$ and OC$_S$ crossed in the direction opposite to that shown in quadrant I of Figure IVB (i.e., interchange L and S). Since this equilibrium could not be the only equilibrium, it is similarly not shown in Figure IVB.
IV. The Gains from Trade

A second issue generally addressed by trade models is the question of gains from trade. The Heckscher-Ohlin model, for example, has as one output a well-known theorem which states that both of two trading countries will gain at a free trade equilibrium relative to autarky. While both countries may gain from trade in the present model, it follows from Melvin (1969) and Kemp (1969) that one may not. In the context of the present model, we can demonstrate the following fairly specific result.

Proposition 2 (the Gains from Trade)

Given the assumptions noted in (1) and the direction of trade noted in Proposition 1 above, one country may lose from trade. A necessary condition for the small country to gain from trade is that trade increases the price (relative to autarky) of its export good. This same condition, however, is a sufficient condition for the large country to gain from trade. If both production frontiers are concave over the relevant region, only the small country can lose from trade.

The underlying factor producing this result is that the production externality is essentially a domestic distortion. The general proposition of the theory of the second best states that removing one distortion from an economy (in this case, barriers to trade) may not move the system toward an optimum if other distortions exist in the system (in this case, the production externality).

Figures VA and VB illustrate the problem. In Figure VA, countries L and S are producing and consuming at points A' and A respectively in the absence of trade. In free trade, country S produces at P and consumes at C. P' and C' are the corresponding points for country L. The trade vectors PC and P'C'
have the equal slopes and lengths required for a trading equilibrium. Similar comments apply to Figure VB. In both diagrams country L is shown as gaining while S is shown as losing from trade.

Let $C_x$ and $C_y$ denote a country's consumption of $X$ and $Y$ respectively. Either economy can then be summarized by three equations.

(21) $U = U(C_x, C_y), F(X,Y) = 0, e_y + p^x e_x = 0$

where $e_y = C - Y$, $e_x = C - X$, and $p^*$ is the world price of $X$ in terms of $Y$.

These functions are, respectively, the community utility function, the aggregate transformation function, and the balance of payments constraint. Differentiating each function, we have

(22) $\frac{dU}{y} = \frac{U}{y} \frac{dC}{y} + \frac{X}{y} \frac{dC}{x} = \frac{dC}{y} + p^* dC_x$, since $\frac{U}{y} = p^*$

(23) $dY + \frac{F}{F} dX = dY + (p^*/T^*) dX = 0$, since $\frac{F}{F} = MRT = p^*/T^*$

(24) $e_y + p^* e_x + e_x dp^* = 0$.

Equation (22), which gives community real income in terms of $Y$, follows from the assumption of utility maximization. Equation (23) follows from (11) and from profit maximization. Substituting (24) into (22), we have

(25) $\frac{dU}{y} = dY + e_x + p^* dX + p^* e_x$.

Substituting for $e_x$ from (24) and for $dY$ from (23), gives us

(26) $\frac{dU}{y} = -(p^*/T^*) dX - p^* e_x - e_x dp^* + p^* dX + p^* e_x$,

$= p^*(1-1/T^*) dX - e_x dp^* + (p^*-p^*) e_x$.

Substituting from (4) above, we have a simple expression for the change in real income.
(27) \[ \frac{dU}{U_y} = p'TdX - e_x dp^* \] since \( T = (1-1/T^*) \)

The first term \( p'TdX \) relates to the existence of the externality and notes that since each country is underproducing \( X \) at a competitive equilibrium, an increase in \( X \) improves welfare at constant terms of trade. The second term in (27) is a terms-of-trade effect. Note from Figure IVA that it is not necessarily true that the free trade price ratio will lie between the two autarky price ratios. At points A (C) in that diagram, trade raises (lowers) the domestic price ratio of both countries relative to autarky. This possibility has previously been pointed out by Kemp.

Integration of (27) between the autarky and free trade equilibria would give us an expression for the total gains or losses from trade. Using superscripts \( f \) and \( a \) to denote free trade and autarky points respectively, we know that L's production of \( X \) is greater than autarky production \( (X > X^a) \) and that L's excess demand for \( X \) is negative \( (e_x < 0) \) over the relevant region. Opposite comments apply to S \( (X < X^a \text{ and } e_x > 0) \). It follows from (27) that a necessary condition for S to gain from trade is that trade raises the price of its export good \( (p^f < p^a) \). As noted in Proposition 2, however, this same requirement (in this case \( p^f > p^a \)) is a sufficient condition for L to gain from trade. Thus L may gain from trade even if trade deteriorates the price of its export good (e.g., point C in Figure IVA).

One interesting case arises when the production frontiers of both countries are strictly concave in the region between the free trade and the autarky equilibria (Figure VA). In this situation, we cannot construct a situation in which country L is worse off.
Given the concavity assumption, it follows that the free trade production bundle, evaluated at the price ratio tangent to the production frontier, is at least as valuable as the autarky production bundle. From (11) above, this price ratio is $p^f/T^*$ or, from (4), $p^f(1-T^*)$. The condition on the value of production bundles can then be written as

\[(28) \quad Y^f + p^f(1-T)X^f \geq Y^a + p^f(1-T)X^a \text{ where} \]

\[Y^a = C^a_y, \quad X^a = C^a_x, \quad \text{and} \quad Y^f + p^fX^f = C^f_y + p^fC^f_x.\]

The second line of (28) simply notes that (28) must satisfy the balance of payments constraint in (21) and that in autarky, supply must equal demand for each commodity. Substituting these constraints into the first equation of (28), we have

\[(29) \quad [C^f_y + p^fC^f_x] \geq [C^a_y + p^aC^a_x] + p^fT[X^f - X^a]\]

For the large country, $X^f > X^a$ and thus at free trade prices, the value of consumption at the free trade equilibrium exceeds the value at the autarky equilibrium. This represents an unambiguous gain from trade by the usual revealed-preference argument. The small country may lose as in Figure VA, however, since $X^f < X^a$. 
V. Factor-Price Equalization

Another issue which has long been of interest to trade economists is the degree to which trade will equalize factor prices. In the Heckscher-Ohlin model, the general result is that if trade equalizes commodity prices, it will equalize factor prices. Such a result generally does not occur in the Ricardian model.

The corresponding proposition for the present model is straightforward and easy to demonstrate.

Proposition 3 (Factor-Price Equalization)

If trade equalizes commodity prices, the assumptions given in (1) above imply that the relative price of the factor used intensively in the good produced with increasing returns to scale will be higher in the large country. If both countries are diversified in production at the trading equilibrium, the absolute (real) price of the second factor will be lower in the large country, whereas if one or both countries are specialized, the absolute price of the second factor (and hence the real prices of both factors) may be higher in the large country.

The relationship between factor prices in country L and country S when each country is diversified can be seen from equation (13). If T=0 (constant returns in X), then relative factor prices (ω) depend only on relative commodity prices (p). Provided that factor intensities are non-reversing as assumed in (1) above, we then have the factor-price equalization result of the Heckscher-Ohlin model. In the present case (T > 0), however, the
equalization of commodity prices can only equalize factor prices between countries if their outputs of $X$ are equal. This cannot occur or more specifically it must be the case that $X_L$ exceeds $X_S$ at the trading equilibrium. Even in the absence of trade, our discussion of the shape of the production set in Figure III and in equations (16)-(18) implies that the relative and hence absolute amount of $X$ produced in country L must exceed the amount produced in country S. Since the introduction of trade increases the production of $X$ further in L and decreases it in S, it must follow that at a free trade equilibrium, $X_L$ exceeds $X_S$.

This result implies that the left-hand side of (13) is larger for country L than for country S when trade equalizes commodity prices. Holding $p$ constant, we can differentiate (13) in order to obtain a relationship between $X$ and $\omega$.

The result, which is implicit in (15), is given as follows:

$$T \frac{dX}{d\omega} \omega = \frac{\omega}{\omega + k_x} - \frac{\omega}{\omega + k_y} \quad \text{or} \quad (29)$$

$$\frac{dX}{d\omega} = \frac{X}{T} \frac{(k_y - k_x)}{(w + k_x)(w + k_y)} > 0 \text{ iff } k_x < k_y. \quad \text{(30)}$$

Equation (30) states that when a country is diversified in production, $\omega$ is positively related to the output of $X$ at any fixed terms of trade. Since $X_L$ exceeds $X_S$ at a free trade equilibrium (price ratios equalized in the two countries), it follows that $\omega_L > \omega_S$ when both countries are diversified. The point for country L which yields the same price ratio obtaining at point A for country S must be above point $A'$ at a point such as C in Figure II.

Finally, we can note from equation (5) that if both countries are diversified, then $r = g'(k_y)$. Since from Figure II we see that $k_y$ at C exceeds $k_y$ at A, it must also follow from the concavity of $g$ that real price of capital is lower in country L. Thus if countries are diversified at the trading
equilibrium, the price of both factors cannot be higher in L as noted in Proposition 3.

The parts of Proposition 3 relating to specialization at the trading equilibrium are also examined in Figure II. The endowments of country S (point $0_Y$) and country L (point $0_Y'$) lie along the same ray from the origin by assumption (1E). Along such a ray, the slope of X isoquants are always steeper than the slopes of Y isoquants by virtue of the facts that X is labour intensive and that both production functions are homogeneous (assumptions 1B, 1D, and 1C). If a country is specialized, then its wage/rental ratio is given by the slope of the relevant isoquant through its endowment point. If a country is diversified, then its wage/rental ratio is some convex combination of the slopes of the X and Y isoquants through its endowment point. In the present model, only three patterns of specialization can occur: (a) L is specialized in X and S is specialized in Y; (b) L is diversified but S is specialized in Y, and (c) L is specialized in X but S is diversified. In all three situations, it must be the case that $\omega_L > \omega_S$. Combined with our findings when both countries are diversified, we thus have the result of Proposition 3 that the relative price of labour is higher in country L regardless of whether or not countries specialize.

The possibility that, with specialization, the real prices of both factors can be higher in country L should not require a great deal of discussion. If L is specialized in X and S is specialized in Y, then it must be the case that the ratio of the value of L's output relative to S's output exceeds the ratio of their factor endowments due to the increasing returns in X. L is thus better off on a "per capita" basis. While it must be the case that $\omega_L > \omega_S$, this relative price difference may be outweighed by the "per capita" difference implying that $r_L > r_S$. 
VI. **Comparative Static Properties of Equilibrium**

The results of the preceding sections have a number of interesting implications regarding the effects of tariffs, factor mobility, and other economic changes on equilibrium. Several of these implications are summarized as follows:

**Proposition 4**

Given the assumptions noted in (1) above, the following results can be established.

(1) The Stolper-Samuelson and Rybczynski effects hold provided the production frontier is locally concave.

(2) A lowering of tariffs may drive factor prices in L and S further apart.

(3) Factor mobility between L and S may lead to an increase in the volume of trade.

(4) Perfect factor mobility will imply an equilibrium in which each country is relatively well endowed with the factor used intensively in the production of its export good.

Part (1) follows from the general formulas derived by Kemp. Let us simply point out that the Stolper-Samuelson effect can be easily established with the aid of equation (15) above. Note from (15) that \((dp/p)/(dω/ω) < 1\). Thus if \(dp/dω > 0\) (i.e., the production frontier is locally concave), it must follow that \((dω/ω)/(dp/p) > 1\). This is, of course, the Stolper-Samuelson or "magnification" effect.

Part (2), which is at variance with the predictions of the Heckscher-Ohlin model, can be established by considering the case where both countries are
diversified in production. In such a situation, the institution of tariffs will generally raise the price of the factor used intensively in the import industry in each country (this is not always the case, see Kemp). Given the results of Proposition 3, this implies that small tariffs tend to make factor prices more equal in the two countries. Part (2) of Proposition 4 turns this statement on its head.

Parts (3) and (4) also stand in contradiction to the usual Heckscher-Ohlin results. Consider again the case in which both countries are initially diversified. Given the factor-price results of Proposition 3, factor mobility would imply a movement of labour to the country exporting the labour intensive good and a movement of capital in the opposite direction. By the usual Heckscher-Ohlin analysis, this movement will result in increased specialization and in an increase in the volume of trade.

Part (4) can be established by noting from Proposition 3 that free trade will not equalize factor prices if both countries are diversified. Second, we can note from Figure II that with specialization, factor prices cannot be equalized if capital/labour endowment ratios are equal. If labour was the mobile factor, it would move from country S to country L which can be represented in Figure II as a horizontal leftward shift of point $Q^*_Y$ and a horizontal rightward shift of point $Q_Y$. This would eventually lead to an equalization in relative factor prices as the MRS falls in L and increases in S. Absolute differences in factor prices would also disappear since this process would tend to depress $p$ (the production of X increases and the production of Y decreases).
VII. **Summary and Conclusions**

Eight points summarize the principal findings of this paper.

(1) Section II above developed a two-good, two-factor, two-country general equilibrium model in which one sector had increasing returns to scale internal to the industry but external to firms. Since a number of important questions depend on the shape of the production frontier, the model was first used to analyze this problem. By assuming C.E.S. technology in the two sectors, we were able to extend earlier findings on the local properties of the frontier to include global properties. It was shown in particular how these global properties depend on elasticities of substitution in the two industries as well as on the degree of increasing returns and on the difference in factor intensities between the two industries.

(2) In order to analyze the equilibrium direction of trade, the relative factor endowments in the two countries were assumed to be equal although one country was assumed to have an absolutely larger endowment of both factors. This neutralized the factor which forms the basis for trade in the Heckscher-Ohlin model. Using a Kemp-type stability analysis, it was shown that there must exist at least stable equilibrium in which the large country exports the good with increasing returns to scale.

(3) Because of the production externality, it was demonstrated that a necessary condition for the small country to gain from trade is that trade increases the price (relative to autarky) of its export good. This same condition, however, is a sufficient condition for the large country to gain from trade. One implication of this asymmetry is that only the small country can lose if both production frontiers are concave over the relevant region.

(4) Contrary to the results of the Heckscher-Ohlin model, the equalization of domestic commodity prices will not lead to an equalization of factor prices in the present model. The general result is that the relative price
of the factor used intensively in the good produced with increasing returns will be higher in the large country. If at least one country is specialized at the trading equilibrium, it may also be true that the real prices of both factors are higher in the large country.

(5) One effect of tariffs in the model provides an interesting contrast to the results of the Heckscher-Ohlin model. Factor price differences between the countries under free trade exceed the factor price differences that would obtain if one or both countries had a small tariff.

(6) An analysis of factor mobility also produces several possible outcomes opposite to the Heckscher-Ohlin results. It was noted, for example, that factor mobility may lead to an increase in the volume of commodity trade. Thus trade in factors and trade in commodities may be "complements" rather than "substitutes" as in the Heckscher-Ohlin model.

(7) A second result of the analysis of factor mobility provides an interesting implication for empirical tests of the Heckscher-Ohlin model. It was shown that if both goods and factors are perfectly mobile, then a trading equilibrium must involve at least one country specializing and relative factor endowments characterized by the fact that the large country is relatively well endowed with the factor used intensively in the good produced with increasing returns (the large country's export good). A test of the Heckscher-Ohlin theorem would thus tend to be positive even though the underlying cause both of trade and of unequal factor endowments was increasing returns to scale.

(8) A final result of this analysis was that the Stolper-Samuelson and Rybczynski effects continue to hold in the present model provided that the production frontier is locally concave. This represents one of the few cases in which the present results are consistent with those of the Heckscher-Ohlin model.
Technical Appendix

In this section, we shall analyze the properties of the production set when both industries have C.E.S. technology. Production functions are specified as follows:

\( X = (X^T)^{\alpha} (aL^\beta_x + bK^\beta_x)^{-1/\beta} \quad Y = (cL^\gamma_y + dK^\gamma_y)^{-1/\gamma} \quad 1 < \beta, \gamma < \infty \)

where \((a,b,c,d) > 0\) and \(a+b = c+d = 1\). The marginal rates of substitution corresponding to (6) above are given by

\( w = \frac{a}{b} \left( \frac{K}{L_x} \right)^{\beta+1} = \frac{c}{d} \left( \frac{X}{Y} \right)^{\gamma+1} \).

The definition of the elasticity of substitution allows us to write

\( \frac{\sigma_x}{w} = \frac{\frac{dK_x}{K_x}}{\frac{dL_x}{L_x}} - \frac{\frac{dL_x}{L_x}}{\frac{dK_x}{K_x}} = \frac{1}{1+\beta} \)

where \(\sigma_x\) is the elasticity of substitution in the X industry.

Differentiating the production function for \(X\) and dividing through by \(X\), we have

\( \frac{dX}{X} = T \frac{dX}{X} + \frac{aL^{-\beta}}{aL^{-\beta} + bK^{-\beta}} \frac{dL_x}{L_x} + \frac{bK^{-\beta}}{aL^{-\beta} + bK^{-\beta}} \frac{dK_x}{K_x} \)

Replacing \(dK_x/K_x\) in (A4) with the same expression in (A3), gives us

\( \frac{dX}{X} = \left(1-T\right) \frac{dX}{w} \frac{dL_x}{L_x} \frac{dL_x}{w} + \frac{bK^{-\beta}}{aL^{-\beta} + bK^{-\beta}} \sigma_x \quad \text{where} \quad \sigma_x = \sigma_x \left[ \frac{a}{b} \left( \frac{K}{X} \right)^{-\beta} + 1 \right]^{-1} \)

Since \(L_x/K_x\) falls with an increase in \(X\), the second term on the right-hand side of (A5) increases with \(X\) if \(\beta < 0\) (\(\sigma_x > 1\)), falls with \(X\) if \(\beta > 0\) (\(\sigma_x < 1\)), and remains constant with \(X\) in the special case of unitary elasticity of substitution (the Cobb-Douglas case). Turning to the first term on the right-hand side of (A5), we note from (A3) that this can be written as
\[
\frac{dL_x}{L_x} / \frac{dw}{w} = \sigma \left[ \frac{dK_x}{K_x} / \frac{dL_x}{L_x} - 1 \right]^{-1}.
\]

If we now (A) totally differentiate both sides of (A2), (B) divide both sides through by \(dL_x\), and (C) collect terms, we will have

\[
\left[ (\beta+1) \left( \frac{a}{b} \right) \left( \frac{L_x}{L_x} \right) \left( \frac{1}{L_x} \right) + (\gamma+1) \left( \frac{c}{d} \right) \left( \frac{K-K_x}{L-L_x} \right) \left( \frac{1}{L-L_x} \right) \right] \frac{dK_x}{dL_x} =
\]

\[
\left[ (\beta+1) \left( \frac{a}{b} \right) \left( \frac{L_x}{L_x} \right) \left( \frac{1}{L_x} \right) + (\gamma+1) \left( \frac{c}{d} \right) \left( \frac{K-K_x}{L-L_x} \right) \left( \frac{1}{L-L_x} \right) \right].
\]

Now (A) multiply both sides by \(L_x^2/K_x\), and (B) divide both sides by \((K_x/L_x)^{-\beta}\), (A7) becomes

\[
\frac{dK_x}{K_x} / \frac{dL_x}{L_x} = \frac{s+r(k_x/k_y)}{s+r} > 1 \quad \text{where}
\]

\[
s = (\beta+1) \left( \frac{a}{b} \right) \quad r = (\gamma+1) \left( \frac{c}{d} \right) \left( L_x/L_y \right) \left( k_y^{-\gamma} / k_x^{-\beta} \right).
\]

Since both \(s\) and \(r\) are greater than zero, (A8) is greater than 1 by virtue of the fact that \((k_x/k_y) > 1\). Second, note from (A3) that \(k_x/k_y\) is non-decreasing in \(X\) if and only if \(\sigma_x \leq \sigma_y\). Third, we can similarly note that \((k_x^{-\gamma}/k_x^{-\beta})\) is non-decreasing in \(X\) if (but not only if) \(\sigma_x \geq 1\) and \(\sigma_y \leq 1\). Finally, note that if these elasticity restrictions are met, \(r\) and hence (A8) increase with \(X\) since \((L_x/L_y)\) increases with \(X\) and \((k_x/k_y) > 1\). Referring back to equations (A5) and (A6), we then have the following results for \(\sigma_x \leq 1\) and \(\sigma_y \geq 1\).

\[
\left[ \frac{dK_x}{K_x} / \frac{dL_x}{L_x} - 1 \right]^{-1} > 0 \quad \text{and falls monotonically with } X, \text{ implying}
\]

\[
\frac{dL_x}{L_x} / \frac{dw}{w} \quad \text{falls monotonically with } X \text{ (from (A6)), implying}
\]

\[
\frac{dX}{X} / \frac{dw}{w} \quad \text{falls monotonically with } X.
\]
Now let us rewrite equation (15) as follows:

$$\left(10\right) \frac{d\frac{p}{w}}{d\frac{w}{w}} + T \frac{dX}{X} = \frac{1}{1+k_{x}/w} - \frac{1}{1+k_{y}/w}$$

$k_{x}/w$ decreases with $X$ if and only if $\sigma_{x} < 1$. Similar comments apply to $k_{y}/w$. Thus sufficient conditions for the right-hand side of (A10) to be non-decreasing in $X$ are that $\sigma_{x} \leq 1$ and $\sigma_{y} \geq 1$. Note that the right-hand side of (A10) is constant when $\sigma_{x} = \sigma_{y} = 1$. Combined with (A9), this gives us

$$\left(11\right) \frac{d\frac{p}{w}}{d\frac{w}{w}} \text{ rises monotonically with } X \text{ if } \sigma_{x} \leq 1 \text{ and } \sigma_{y} \geq 1.$$  

Since Kemp has shown that (A11) must be negative in the neighborhood of $X=0$ (i.e., the production possibility curve must be convex in the neighborhood of $X=0$), this implies that the production possibility curve has at most one inflection point and that it cannot be everywhere concave.

As $\sigma_{x} = \infty$, the contract curve in Figure II will run from $O_{x}$ to a point like $Z$, and then along a ray from $Z$ to $O_{y}$. In moving from $O_{x}$ to $Z$, $X$ exhibits decreasing costs (there are no diminishing returns to $L_{x}$ as $\sigma_{x} = \infty$) while $Y$ exhibits increasing costs. For the function in (A1), the curvature of the production frontier along $O_{x}Z$ is given as follows:

$$\left(12\right) \frac{d^{2}Y}{dX^{2}} \geq 0 \text{ as } L_{x}/L_{y} \leq (T^{*} - 1)/d \quad T^{*} > 1.$$  

where $d$ is the parameter given in (A1). As shown by Herberg and Kemp, (A12) is positive in the neighborhood of $O_{x}$ ($L_{x} = 0$), but for a sufficiently small value of $T^{*}$, (A12) becomes negative as $Z$ is approached in Figure II. Moving along a ray from $Z$ to $O_{y}$, the production frontier again becomes convex (Figure IC) since $Y$ exhibits constant costs while $X$ exhibits decreasing costs.
Footnotes

1 For a discussion of commodity taxes as a determinant of trade, see Melvin (1970). Melvin and Warne (1973) discuss imperfect competition as a basis for trade.

2 The present paper does not represent the first attempt to deal with trade under increasing returns. Papers by Herberg and Kemp (1969), Melvin (1969), Kemp (1969), McMillan and Manning (1979), and Henderson (1979) have addressed various aspects of this question. Herberg and Kemp examine the shape of the production set and the relationship between the marginal rate of transformation in production and the equilibrium price ratio. The later work of Kemp adds an analysis of the Stolper-Samuelson and Rybczynski effects and a discussion of the stability of trading equilibria. Melvin shows that increasing returns can lead to trade even if two countries are absolutely identical in all respects. In a more recent paper, Ethier (1976) reformulates the theory by arguing that external economies depend on world rather than on national output. In another recent paper, however, McMillan and Manning (1979) note a very important reason why some external economies may be strictly national in scope: the existence of public intermediate goods. Henderson (1979) extends the analysis into urban economics by showing the relationship between a city's size and the returns to scale inherent in the production of its export good.

3 An interesting critique of trade theory assumptions in the context of regional economics is provided by Richardson (1969), Chapter 12.

4 Chipman (1965, 1970), Kemp and Herberg (1969), and Kemp (1969) derive some local properties of the production frontier under variable returns to scale. By using general functional forms they are, however, unable to show global properties. Ethier extends the analysis to the case of world external economies.

5 Isard (1977) presents an interesting discussion of agglomeration economies and their effects on trade and economic development.

6 Marginal products for labour and capital from the point of view of the X industry as a whole are \((f-k, f') X T^*\) and \(f' X T^*\) respectively as will be seen in equation (10). Thus marginal products from the point of view of the industry exceed marginal products from the point of view of the firm. By dividing the latter by X, one can also see that the latter equal average products. Thus although industry marginal products exceed average products, there is no "adding-up" problem in the present formulation.

7 Similar results are derived by Melvin (1969) in a model in which both goods have identical degrees of returns to scale, such that there is no distortion between \(p\) and \(MRT\).

8 For a discussion of the effects of a capital movement when ownership remains in the investing country, see Markusen and Melvin (1979).

9 Melvin (1970) notes a similar possibility in the presence of commodity taxes.

10 The methodology used here is related to that used by Herberg and Kemp (1971) in dealing with factor market distortions.
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