1980

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WORKING PAPER NO. 8006

THE DISTRIBUTION OF GAINS FROM BILATERAL TARIFF REDUCTION

James R. Markusen

This paper contains preliminary findings from research still in progress and should not be quoted without prior approval of the author.
THE DISTRIBUTION OF GAINS FROM
BILATERAL TARIFF REDUCTION*

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Abstract

A two-good, two-country model is used to analyze the distribution of gains from proportionate, bilateral tariff reductions. It is shown that the distribution of gains is given by the solution to a rather complicated quadratic equation which exhibits multiple real roots. The country with the more elastic offer surface will be the relative gainer, for example, if it has either a very large or a very small tariff relative to the other country's tariff and relative to its own "optimal" (Nash-equilibrium) tariff.

January, 1980

*The author would like to thank John Whalley and Arthur Robson for helpful comments and suggestions.
I. Introduction

The across-the-board tariff-cutting proposals put forward by various countries at the recently concluded Tokyo Round of Multilateral Trade Negotiations represent something of a puzzle to economists. First, there is the question of precisely what each country thought it had to gain from its own proposal. Brown and Whalley (1980) have estimated that the U.S. would gain more from the Japanese proposal than from its own proposal. The Japanese and the EEC would gain less from their own proposals than from any of the other proposals considered. Each country has of course a few industries for which it wants special protection (e.g., textiles, footwear, agricultural products). But we are speaking here of broad, across-the-board formulae. Countries have always retained the right to exempt certain industries from these formulae, and have continued to do so in the Tokyo Round.

To the best of my knowledge, only a small number of theoretical papers have addressed these or closely related issues. Johnson (1954) analyzed the properties of Nash equilibria resulting from bilateral tariff imposition. He demonstrated that in such an equilibrium, one country may be better off relative to free trade. Turned on its head, this result implies that one of two countries may have no incentive to engage in large, bilateral tariff reductions.1 Small bilateral reductions, however, can always be found such that the new equilibrium is Pareto superior to the Nash equilibrium.

Gorman (1958) also dealt with the relationship between a tariff-ridden Nash equilibrium and a free trade equilibrium. He added to Johnson's
analysis by considering the effects of a tariff war on the volume of trade and showed how income as well as price elasticities of import demand influence the welfare effects of such a tariff war. Panchamukhi (1961) generalized the Gorman-Johnson problem to include more sophisticated types of behavior.

Cooper (1964) conducted a partial-equilibrium analysis of bilateral tariff reduction in the context of the Kennedy round. The major question posed by his paper concerned how the degree of dispersion of a country's tariff rates around its mean tariff value affects its relative gains from bilateral tariff reduction. One difficulty with his analysis is that bilateral tariff reduction is said to be biased against country i if country i's imports increase by more than country j's imports following the tariff reduction. There unfortunately exists no simple relationship between this criteria and the neoclassical welfare criteria used by Johnson and Gorman.

More recently, Hatta and Fukushima (1979) have analyzed the effects of multilateral tariff reductions on the real income of the world as a whole. They are able to show that proportionate reductions in all tariffs must raise world real income, thus ruling out perverse possibilities suggested by the theory of the second best.

None of these papers attempted to derive a theoretical relationship between the distribution of gains from bilateral tariff reduction and a specific tariff-cutting formula. The papers by Johnson and Gorman do, of course, shed some light on the issue but only if initial rates happen to be equal to the Nash equilibrium rates. The purpose of this paper, therefore, is to derive such a relationship for arbitrary initial tariff rates using
tariff-cutting formulae which conform closely to those actually proposed at the Tokyo round. In order to accomplish this purpose, the paper will develop a simple two-good, two-country trade model. While this might seem to limit the analysis, it will be shown that the results are rather complicated even in this simple case. I might also emphasize again that we are concerned with across-the-board tariff reductions, and thus I see no reason why we cannot adequately represent the import sector of each economy as consisting of a single good.

This model is first used to analyze the effects of bilateral tariff reduction (BTR) on the real income of each country. These effects are shown to break down into a volume-of-trade (VOT) effect, which always improves a country's welfare, and a terms-of-trade (TOT) effect. The TOT effect is rather complicated and can, in particular, be defined in several different ways. Under the definition adopted below, an improvement in the TOT resulting from BTR improves welfare if and only if the country's initial tariff is less than its "optimal" tariff. By "optimal" tariff we will mean the Nash equilibrium tariff rate analyzed by Johnson and earlier by Bickerdike (1906) and Graaff (1949). This tariff rate is equal to the inverse of the price elasticity of the foreign excess supply and, by convention, is referred to as an optimal tariff. It is not optimal in the Pareto sense. In any case, the sum of the VOT and TOT effects determine a country's overall welfare gain from BTR.

Using initially the U.S. formula proposed at the Tokyo round (post-cut tariffs proportional to pre-cut tariffs), the paper then considers the conditions under which a country will lose or gain from BTR. The results indicate that a country can lose only if it has the relatively large tariff and both tariffs are small compared to optimal tariff rates. In such a
situation, the country with the large tariff experiences a large, negative TOT effect and only a small, positive VOT effect. This finding may be of considerable importance insofar as empirical studies have consistently found that actual tariff rates are considerably less than the optimal rates defined above.

The model is then used to analyze the distribution of gains when both countries are made better off. It is shown that the distribution of gains is given by the solution to a rather complicated quadratic equation which exhibits multiple real roots. The country with the more elastic offer surface will be the relative gainer, for example, if it has either a very large or a very small tariff relative to the other country's tariff and relative to its own optimal tariff. This outcome is actually fairly easily explained by earlier results. In the small tariff case, BTR improves the country's TOT, making it a relative gainer while in the large tariff case, BTR deteriorates the TOT. But with a tariff which is large relative to the optimal tariff, a deterioration in the TOT improves welfare as noted earlier. Converse statements apply to the country with the relatively inelastic offer surface.

The paper concludes with a short discussion of other tariff-cutting formulae proposed at the Tokyo Round.
II. Welfare Effects of Tariff Reduction

Two goods, $X_0$ and $X_1$, are produced and traded by countries $h$ and $f$. Factors are assumed to be in fixed and inelastic supply such that the aggregate transformation function for each country is given by

$$ X_0^i = T^i(X_1^i) \quad i = f, h. $$

Superscripts $h$ and $f$ will assign variables to countries throughout the paper. It is assumed that demand conditions in each country can be represented by a set of community indifference curves, so that social welfare in each country is given by

$$ U^i = U^i(C_0^i, C_1^i) \quad i = f, h. $$

where $C_0$ and $C_1$ are consumption quantities of $X_0$ and $X_1$ respectively.

The balance of payments constraint for each country is given by

$$ e_0^i + p^*e_1^i = 0; \quad e_j^i = C_j^i - X_j^i; \quad i = f, h; \quad j = 0, 1; $$

where $p^*$ is the world price of $X_1$ in terms of $X_0$ and $e_j^i$ is the excess demand of country $i$ for $X_j$. $X_0$ will be used as numeraire throughout the paper. Results that will follow are not entirely neutral with respect to the choice of numeraire, but the biases arising from any particular choice are easily analyzed as will be pointed out from time to time.

Given the symmetry between import and export tariffs which occurs in this type of model, it will prove analytically useful to regard one country as having an import tariff and the other country as having an export tax. Arbitrarily assuming that country $h$ imports $X_1$ and country $f$ imports $X_0$, we have
(4) \[ p^h = p^s(1 + \eta^h), \quad p^f = p^s(1 + \eta^f); \quad \eta^h > 0, \quad \eta^f < 0, \]

where \( p^i \) is the domestic price of \( X_1 \) in terms of \( X_0 \). In each country, the tariff has the effect of raising the domestic price of the import good above the world price, hence the tariff rates are of opposite sign. A certain symmetry occurs in this formulation insofar as equal tariff rates imply equal percentage deviations of the domestic prices from the world price ratio (i.e., \( \eta^h = -\eta^f \) implies \( (p^h - p^s)/p^s = -(p^f - p^s)/p^s \)).

Country \( i \)'s trading opportunities can be summarized by the offer surface of country \( j \). The excess demands of country \( i \) are equal to the excess supplies of country \( j \), which may in turn be specified as functions of \( p^s \) and \( \eta^j \). Since all relevant information can be summarized in terms of either good by virtue of the balance-of-payments constraint, the trading opportunities of country \( i \) are therefore given as follows:

\[
(5) \quad e^i_1 = -e^j_1 = E^j(p^s, \eta^j); \quad E^j_1 = \frac{\partial e^j_1}{\partial p^s}, \quad E^j_2 = \frac{\partial e^j_1}{\partial \eta^j}.
\]

Equation (5) is derived below. Except when operating on the "backward bending" section of \( j \)'s offer curve, \( E^j_1 > 0 \); i.e., an increase in \( p^s \) will increase \( j \)'s excess supplies of \( X_1 \). Unless otherwise explicitly stated, we will assume throughout the paper that \( E^j_1 > 0 \). \( E^j_2 \) will be positive under similar restrictions. For country \( f \), an increase in \( \eta^f \) (which is negative) represents a reduction in its export tax. This will, at constant world prices, lead \( f \) to increase its exports (excess supply) of \( X_1 \). For country \( h \), an increase in \( \eta^h \) leads to a reduction in its imports of \( X_1 \) at constant world prices. This represents a decrease in excess demand or in terms of (5), an increase in \( h \)'s excess supply of \( X_1 \). Thus all partial derivatives in (5) are assumed to be positive.
For each country, we can totally differentiate (2) and divide by \( U_0^i \).

\[
\frac{du^i}{U_0^i} = dc^i_0 + \frac{u^i_1}{U_0^i} dc^i_1 = dc^i_0 + p^i dc^i_1 = dy^i.
\]

\( U_1^i/U_0^i = p^i \) from the first-order conditions for utility maximization. Equation (6) gives the change in country i's real income in terms of \( X_0^i \), which will henceforth be denoted by \( dy^i \).

Similarly, we can totally differentiate (1) to get

\[
dx^i_0 - T^i_1 dx^i_1 = dx^i_0 + p^i dx^i_1 = 0.
\]

\( T^i_1 = -p^i \) from the first-order conditions for profit maximization in the absence of domestic distortions or non-convexities.

Differentiating (3) and (5) we have

\[
de^i_0 + p^i de^i_1 + e^i_1 dp^* = 0
\]

(8)

\[
de^i_1 = E^i_1 dp^* + E^i_2 d\theta^j.
\]

(9)

Substituting the identities \( dc^i_j = dx^i_j + de^i_j \) into (6)

\[
dy^i = dx^i_0 + de^i_0 + p^i dx^i_1 + p^i de^i_1.
\]

(10)

Substituting for \( dx^i_0 \) from (7)

\[
dy^i = -p^i dx^i_1 + de^i_0 + p^i dx^i_1 + p^i de^i_1 = de^i_0 + p^i de^i_1.
\]

(11)

Substituting for \( de^i_0 \) from (8)

\[
dy^i = -e^i_1 dp^* - p^i de^i_1 + p^i de^i_1 = p^* e^i_1 de^i_1 - e^i_1 dp^*
\]

(12) since \( p^i - p^* = p^* e^i_1 \). Equation (12) is a fairly well-known equation (see, for example, Jones (1967)) which states that the change in a country's real
income can be broken down into a VOT effect \( (p^i \theta^i e^i_1) \) and a TOT effect \( (e^i_1 dp^i) \). The VOT effect shows, for example, that since \( h \)'s relative price for \( x_1 \) exceeds the world price, \( h \)'s welfare is increased by any increase in imports at constant terms of trade. Country \( h \)'s welfare is, on the other hand, decreased by any increase in the world price of its import good. Similar comments apply to country \( f \).

Equation (9) can also be used to define the TOT effect in a different manner.

\[
\begin{align*}
\frac{\partial y^i}{\partial p^*} &= (p^i \theta^i e^i_1 - e^i_1 dp^* + p^i \theta^i e^i_2 d\theta^j.
\end{align*}
\]

Equations (13) and (14) note that an improvement in country \( h \)'s TOT \( (dp^* < 0) \) will improve \( h \)'s welfare \( (\partial y^h / \partial p^* < 0) \), other things equal, if and only if \( \theta^h < \frac{e^h_1}{p^* e^h_1} \). This latter expression is equal to the inverse of the price elasticity of country \( f \)'s excess supply of \( x_1 \), denoted by \( \eta^f \) in equation (14). This is precisely the formula for country \( h \)'s so-called "optimal tariff" as first developed by Bickerdike (1906). We will denote \( \frac{1}{\eta^f} \) as \( \theta^*_f \) throughout the paper and refer to it as \( h \)'s optimal tariff. Subsequent references to the TOT effect will refer to the effect as defined in (13) rather than as in (12).

Equations (13) and (14) also apply to country \( f \) \( (\theta^*_f < 0 \text{ since } e^f_1 < 0) \). In \( f \)'s case, \( \theta^f < \theta^*_f \) \( (|\theta^f| < |\theta^*_f|) \) constitutes a tariff smaller than its optimal value, and \( dp^* > 0 \) constitutes an improvement in the terms of trade. Finally, we should emphasize that \( \theta^*_h \) and \( \theta^*_f \) are variables whose values depend on where they are evaluated. Only in the case of constant-elasticity offer curves (assumed by Johnson and Gorman) are these tariff rates constants.

The intuition behind the results given in (13) and (14) become clear if we consider the case where a country unilaterally reduces its tariff, thereby deteriorating its terms of trade. According to (13) and (14),
this unilateral action will improve its welfare if and only if its current tariff exceeds its optimal value. This is, of course, precisely what is meant by "optimal tariff" or "Nash equilibrium tariff" as mentioned in the introduction.

III. Derivation of the Offer Surfaces (This section may be skipped without loss of continuity)

Differentiating (1) above, we have

\[ p^i = -\frac{dX^i_0}{dX^i_1} = -T^i \]

\[ dp^i = -T^i_{11} dx^i_1 \text{ or } dx^i_1 = (-T^i_{11})^{-1} dp^i. \]

Similarly, from Vandendorpe (1972), demand in each country can be specified as follows:

\[ c^i_0 = D^i(c^i_1, y^i); \quad p^i = -\frac{\partial c^i_0}{\partial c^i_1} = -D^i \]

\[ dp^i = -D^i_{11} dc^i_1 - D^i_{12} dy^i \text{ or } dc^i_1 = (-D^i_{11})^{-1} dp^i + (-D^i_{12}/D^i_{11})dy^i = (-D^i_{11})^{-1} dp^i + m^i_1 dy^i. \]

Note that \((-D^i_{12}/D^i_{11})\) is simply the marginal propensity to consume \(c^i_1\) and will be written as \(m^i_1\) in what follows. \((-T^i_{11}) > 0\) if the production set is strictly convex and \((-D^i_{11}) < 0\) under the usual neoclassical demand assumptions.

Differentiating (4), we also have

\[ dp^i = (1 + \theta^i)dp^* + p^i d\theta^i. \]

From (15), (16), and (17), country i's excess supply of \(X_1\) is given as follows

\[ de^i_1 = dx^i_1 - dc^i_1 = [(D^i_{11})^{-1} - (T^i_{11})^{-1}] dp^i - m^i_1 dy^i \]

\[ = [(D^i_{11})^{-1} - (T^i_{11})^{-1}][(1+\theta^i)dp^* + p^i d\theta^i] - m^i_1 dy^i. \]
Substituting from equation (12) we then have our result.

\[ -d_1^i = \left[ (D_{11}^i)^{-1} - (T_{11}^i)^{-1} \right] (1+\theta_i^i) \Delta p^*_1 + p_1^i \delta_i^i \]

\[ = -m_1^i p_1^i \theta_i^i \Delta e_1^i + m_1^i e_1^i \Delta p^*_1 \]

\[ -d_1^i = (1-m_1^i p_1^i \theta_i^i)^{-1} \left[ (D_{11}^i)^{-1} - (T_{11}^i)^{-1} \right] + m_1^i e_1^i (1+\theta_i^i)^{-1} (1+\theta_i^i) \Delta p^*_1 \]

\[ + (1-m_1^i p_1^i \theta_i^i)^{-1} \left[ (D_{11}^i)^{-1} - (T_{11}^i)^{-1} \right] p_1^i \delta_i^i. \]

Equation (20) provides the derivatives given in equation (9).

\[ E_1^i = (1-m_1^i p_1^i \theta_i^i)^{-1} \left[ (D_{11}^i)^{-1} - (T_{11}^i)^{-1} \right] + m_1^i e_1^i (1+\theta_i^i)^{-1} (1+\theta_i^i) \]

\[ E_2^i = (1-m_1^i p_1^i \theta_i^i)^{-1} \left[ (D_{11}^i)^{-1} - (T_{11}^i)^{-1} \right] p_1^i. \]

Both of these derivatives and hence the corresponding elasticities become larger as demand and supply become more responsive to price changes. For country \( h \), both derivatives may become negative as \( m_1^h \) and \( \theta^h \) become large. For country \( f \), \( E_1^i \) may similarly become negative as \( m_1^f \) and \( |\theta^f| \) become large, although for a different reason \( (e_1^f < 0) \). \( E_2^f \) is always positive.

One other factor ought to be mentioned here since it will become important later on in explaining the biases caused by the choice of numeraire. Given that \( e_1^h \) and \( \theta^h \) are greater than zero and that \( e_1^f \) and \( \theta^f \) are negative, it follows from (21) that

\[ E_1^h / E_2^h > E_1^f / E_2^f, \quad E_1^i / E_2^i = [\partial \theta_i^i / \partial p^*_1] d_1^i = 0. \]

\[ \frac{E_1^i}{E_2^i} = (1+\theta_i^i) / p_1^i + \frac{m_1^i e_1^i}{(D_{11}^i)^{-1} - (T_{11}^i)^{-1} p_1^i} \]
The first additive term in the second equation of (22) is positive for both countries, but larger for country h since $\theta^h > \theta^f$. The second additive term is positive for country h ($e^h_1 > 0$) but negative for country f ($e^f_1 < 0$).

Hence the relationship shown in the first equation of (22).

The ratios in (22) are, from (9), marginal rates of substitution between $\theta^i$ and $p^*$ holding excess supplies constant. For country h, $dp^* > 0$ deteriorates welfare at a constant volume of trade (equation 12), leading to reduced excess demand (increased excess supply) for $X_1$ via an income effect as well as a substitution effect. Thus a relatively large tariff change is required in order to keep imports constant. For country f, $dp^* > 0$ increases welfare at a constant volume of trade. The substitution effect tends to increase excess supplies while the income effect tends to reduce excess supplies. Thus only a relatively small tariff change is needed to hold exports constant.
IV. Bilateral Tariff Reduction

From an analytical point of view, there did not seem to be a great deal of difference among the proposals for across-the-board tariff reduction put forward at the Tokyo round. The U.S. proposed that tariffs be reduced in proportion to existing rates while the EEC, Japan, and Switzerland proposed more-than-proportional reductions (i.e., large tariffs reduced proportionately more than small tariffs). In this section we will therefore first consider the U.S. proposal and then analyze the effects of more-than proportionate reduction.

Both countries are assumed to engage in small tariff reductions such that \( d\theta^h/\theta^h = d\theta^f/\theta^f < 0 \) (note that for country \( f \), \( d\theta^f > 0 \) constitutes a reduction in \( \theta^f \)). From equation (9), the change in each country's excess demand for \( X^1 \) is given by

\[
(23) \quad d\theta^h = E^f_1 dp^* + E^f_2 \theta^f \frac{d\theta}{\theta}, \quad d\theta^f = E^h_1 dp^* + E^h_2 \theta^h \frac{d\theta}{\theta}
\]

\[
(24) \quad \frac{d\theta}{\theta} = \frac{d\theta^h}{\theta^h} = \frac{d\theta^f}{\theta^f} < 0
\]

Noting that the sum of \( d\theta^h \) and \( d\theta^f \) must equal zero, (23) gives us the terms of trade effect.

\[
(25) \quad \frac{dp^*}{d\theta/\theta} = \frac{-\theta^f E^f_2 - \theta^h E^h_2}{E^h_1 + E^f_1} = \frac{\theta^f E^f_2/e^f_2 - \theta^h E^h_2/e^h_2}{(p^* E^h_1/e^h_1 - p^* E^f_1/e^f_1)/p^*}
\]

\[
= \left( \frac{\theta^f - \theta^h}{(\eta^f - \eta^h)} \right)/p^* \quad \sigma^i = \frac{\theta^i}{e^i_1} \frac{de^i}{\theta^i} \quad \sigma^f, \sigma^h > 0; \eta^f > 0, \eta^h < 0.
\]

\( \sigma^i \) is thus defined as country \( i \)'s own tariff elasticity of excess demand,

\( \sigma^f \) and \( \sigma^h \) are both positive, while \( \eta^f > 0 \) and \( \eta^h < 0 \) as noted in (14) above.
Note that since \( \frac{d\theta}{\theta} < 0 \), a positive value for (25) indicates that \( dp^* < 0 \). Thus the terms of trade will deteriorate for country \( f \) if and only if \( f \)'s tariff elasticity exceeds the corresponding value for \( h \). Price elasticities help determine the magnitude but not the direction of the terms of trade change.

Substituting (24) and (25) into (13), we have

\[
\frac{dy^h}{d\theta} = (p^* \theta^h E_1^h - e_1^h)\left( -\frac{\theta^f E_2^f}{E_1^h + E_1^f} + p^* \theta^h E_2^h \right)
\]

\[
\frac{dy^f}{d\theta} = (p^* \theta^f E_1^f - e_1^f)\left( -\frac{\theta^h E_2^h}{E_1^f + E_1^h} + p^* \theta^f E_2^f \right)
\]

By factoring out \( (E_1^h + E_1^f)^{-1} \), these equations can be simplified (several terms cancel) to yield

\[
\frac{dy^h}{d\theta} = \left[(p^* \theta^h E_1^h + e_1^h)E_2^h \theta^h + (p^* \theta^h E_1^h + e_1^h)E_2^f \theta^f\right](E_1^h + E_1^f)^{-1}
\]

\[
\frac{dy^f}{d\theta} = \left[(p^* \theta^f E_1^f + e_1^f)E_2^f \theta^f + (p^* \theta^f E_1^f + e_1^f)E_2^h \theta^h\right](E_1^f + E_1^h)^{-1}
\]

Using the Nash equilibrium tariff rates given in (14), these equations can also be expressed as

\[
\frac{dy^h}{d\theta} = \left[(\theta^h - \theta^f)p^* E_1^h E_2^h \theta^h + (\theta^h - \theta^f)p^* E_1^f E_2^f \theta^f\right](E_1^h + E_1^f)^{-1}
\]

\[
\frac{dy^f}{d\theta} = \left[(\theta^f - \theta^h)p^* E_1^f E_2^f \theta^f + (\theta^f - \theta^h)p^* E_1^h E_2^h \theta^h\right](E_1^f + E_1^h)^{-1}
\]

Dividing the numerator and the denominator of each equation in (28) by \( e_1^h \), (28) can be expressed in elasticity form.
\[
\frac{\text{d}y}{\text{d}\theta} = \frac{[(\theta^* - \theta^i)\sigma^i - (\theta^* - \theta^i)\sigma^i\eta^i]}{\eta^i - \theta^i}
\]

Changes in welfare resulting from this simple tariff-cutting formula are thus rather complicated. Whether expressed in the form of (28) or (29), it is clear that welfare effects are a function of a number of interdependent variables even in this simple two good case. Equation (21), for example, allows us to solve for the interdependence between \( \eta^i \) and \( \sigma^i \).

\[
\eta^i = -\sigma^i\left[1 + \theta^i\right] - \frac{p^x_{11}}{1 - m^1_1 \theta^i}.\]

Given the restrictions we have imposed on signs, however, equations (28) and (29) do demonstrate that a necessary condition for a country to lose from this particular tariff-cutting formula is that its existing tariff is smaller than its optimal tariff (i.e., \( \text{d}y^i < 0 = |\theta^i| < |\theta^{i*}| \)). Recall here that ETR is defined as \( \text{d}\theta/\theta < 0 \), implying that negative values for (28) and (29) indicate a welfare improvement. We might also note from (28) that each country's welfare improvement is an everywhere increasing function of its partner's existing tariff and optimal tariff rates. A country's welfare improvement is, on the other hand, an everywhere decreasing function of its own optimal tariff rate. Given the optimal tariff formulas presented in (14) above, these statements can be rephrased to note that a country is more likely to gain from ETR, other things equal, if it has the relatively inelastic offer surface. This is probably quite consistent with our general notions about the problem.
More specific results can be obtained by analyzing the two equations in (28). The equations in (29) are unfortunately not very satisfactory due to the interdependencies noted in (30). Factoring out $p^SE_1E_2$ from each equation in (28), we have

$$\frac{dy^h}{d\theta} = [(\theta^h - \theta^h)\theta^h - C(\theta^f - \theta^f)\theta^f]p^SE_1E_2(E_1^h + E_1^f)^{-1}$$

$$\frac{dy^f}{d\theta} = [C(\theta^f - \theta^f)\theta^f - (\theta^h - \theta^h)\theta^h]p^SE_1E_2(E_1^h + E_1^f)^{-1}$$

$$C = \frac{E_1^f}{E_1^h} > 1.$$

Equation (33) follows from equation (22). $C$ is of course a variable. Nevertheless, (31) and (32) are in a form such that we can assess their qualitative properties without knowing precisely how $C$ is related to the other variables in the system. Given that $C$ and the $E_{ij}$ are everywhere positive, the qualitative properties of (31) and (32) can be assessed simply by examining the terms in the square brackets.

Treating $\theta^h$ and $\theta^f$ as parametric, each country's welfare change is a quadratic function of its own tariff rate. Solving (31) and (32) for $dy^i = 0$, we have

$$\frac{dy^h}{d\theta} = 0 \Rightarrow \theta^f = \frac{\theta^h - \theta^h}{C(\theta^f - \theta^f)}$$

$$\frac{dy^f}{d\theta} = 0 \Rightarrow \theta^h = \frac{C(\theta^f - \theta^f)\theta^f}{(\theta^f - \theta^f)}$$

Both these equations are parabolas whose qualitative properties are independent of $C$ and the right-hand denominators (everywhere negative in (34) and (35)). For (34), $-\theta^f = 0$ at $\theta^h = 0$ and $\theta^h = \theta^h$. $-\theta^f > 0$ between these two values of $\theta^h$. Similar comments apply to (35).
These equations are graphed in Figures I, II, III, and IV. \( \theta^h \) is graphed on the horizontal axis in each case while \(-\theta^f \) is graphed on the vertical axis. In each case, \( dy^h = 0 \) is the parabola running between \((-\theta^f = 0, \theta^h = 0) \) and \((-\theta^f = 0, \theta^h = \theta^{h*}) \). Equation (31) shows that \( dy^h < 0 \) in the area below this parabola (vertical cross hatching) while \( dy^h > 0 \) (\( dy^h/(d\theta/\theta) < 0 \)) above this parabola. A similar parabola exists for \( dy^f = 0 \) in Figures I - IV where the horizontal cross hatching indicates the area in which \( dy^f < 0 \) (\( dy^f/(d\theta/\theta) > 0 \)).

These results thus demonstrate the finding noted in the introduction to the paper. A country may lose from bilateral tariff reduction if (A) it has the higher tariff relative to the optimal values, and (B) both tariffs are small relative to their optimal values. In such a situation, the losing country experiences a negative terms-of-trade effect. The size of this negative TOT effect is proportional to the volume of trade, \( e_1 \), as shown in (12). But \( e_1 \) is relatively large when tariffs are relatively small. Thus the negative TOT effect is relatively large when tariffs are relatively small. Equations (12) and (14) also show that the VOT effect, \( p^* \theta^d e_1 \) (which always helps a country) is proportional to the level of existing tariffs and disappears as a country's own tariff rate goes to zero. Thus the negative TOT effect dominates when tariffs are small. A country can never lose, however, if its tariff equals or exceeds its optimal value, regardless of the level of its partner's tariff.

Note finally that it is not true that a country will always gain if its partner's tariff exceeds its optimal value. An analysis of (31) and (32) will demonstrate that the situation shown in Figure IV can occur. The country with the relatively more elastic offer surface (country \( h \)) can lose despite the fact that \( |\theta^f| > |\theta^{f*}| \).
V. Total World Real Income and the Relative Distribution of Gains

As noted earlier, there is no necessary reason to believe that bilateral tariff reduction will always and everywhere increase total world real income, except when tariffs are reduced to zero. It is easy to show, however, that the tariff-cutting formula used in the previous section will always increase total real income. Adding equations (31) and (32), we have

$$\frac{dy^h}{d\theta} + \frac{dy^f}{d\theta} = [\theta^h - \theta^f + C(\theta^f - \theta^h)\theta^f]p^h p^f E^2(E^1 + E^f)^{-1} < 0.$$ \hspace{1cm} (36)

Equation (36) is everywhere negative given our sign restrictions, implying that total world real income must increase following this type of BTR. The present model, therefore, yields results which are consistent with those of Hatta and Fukushima.

The distribution of gains between country h and country f can be analyzed by subtracting equation (32) from equation (31).

$$\frac{dy^h}{d\theta} - \frac{dy^f}{d\theta} = [(2\theta^h - \theta^h - \theta^f + C(2\theta^f - \theta^f)\theta^f)] p^h p^f E^2(E^1 + E^f)^{-1}. \hspace{1cm} (37)$$

A negative value for (37) indicates that country h is the relative gainer while a positive value indicates relative gains for country f. It follows from (36) that the relative gainer must also gain in an absolute sense but that the relative loser may or may not gain in an absolute sense.

As in the case of equations (31), (32), and (36), the sign of (37) is equal to the sign of the term inside the square brackets. Focusing only on this term, it can be shown that the solution to this complicated quadratic for $dy^h - dy^f = 0$ is a "double parabola" with one parabola corresponding to each of the positive and negative branches of the quadratic in (37). It also turns out that the orientation of the parabolas depends on the relationship between $\theta^h$ and $C\theta^f$. 
The three possibilities are shown in Figures I, II, and III. In Figure I, it is assumed that $\theta^h > |C\theta^f|$. Two parabolas divide the diagram into regions in which country $h$ is the relative gainer (upward-sloping cross-hatching) and in which country $f$ is the relative gainer (negative-sloped cross-hatching). Figure II shows the corresponding situations for $\theta^h < |C\theta^f|$. The parabolas dividing the diagram run in the opposite direction. Figure III shows the situation for $\theta^h = |C\theta^f|$. In all three diagrams, the definitions of the hatching patterns are the same. Note finally that the region in which a country loses in absolute terms (as discussed above) is of course always a subregion of its region of relative losses.

A demonstration of these results via the application of the quadratic formula to (37) is straightforward but extremely tedious. We can easily provide a quick check on these diagrams, however, by considering the point $(\theta^h*, \theta^f*)$. Rearranging the terms in (37), we have

$$
\begin{align*}
\frac{dy^h}{d\theta^h} \frac{dy^f}{d\theta^f} &= [(2\theta^h - \theta^f)\theta^h - C(2\theta^f - \theta^h)\theta^f + (C-1)\theta^f \theta^h]A \\
A &= p^*_1 E^*_1 (E^*_1 + E^*_1)^{-1} > 0.
\end{align*}
$$

At $\theta^h = \theta^h*$, $\theta^f = \theta^f*$, equation (38) can be written as

$$
\begin{align*}
\frac{dy^h}{d\theta^h} \frac{dy^f}{d\theta^f} &= [(\theta^h*^2 - C\theta^f*^2 + (C-1)\theta^h* \theta^f*)A \\
A &= (\theta^h* + C\theta^f*)(\theta^h*^2 - \theta^f*)A.
\end{align*}
$$

Evaluating (39), we have

$$
\begin{align*}
\frac{dy^h}{d\theta^h} \frac{dy^f}{d\theta^f} > 0 \text{ at } \theta^h = \theta^h*, \theta^f = \theta^f* \iff \theta^h* > |C\theta^f*|.
\end{align*}
$$
FIGURE III

\[ \theta^h = |C \theta^f| \]

FIGURE IV
In Figure I, \( \theta^h < |C\theta^f| \) and thus the point \((\theta^h, \theta^f)\) lies in the region of relative gains for country \(h\) (i.e., equation (39) is negative). Similar comments apply to the situation in which \(\theta^h > |C\theta^f|\) (Figure II). Figure III shows the case in which \(\theta^h = |C\theta^f|\). As is clear from (39), both countries will experience equal gains at \((\theta^h, \theta^f)\). The curvature of the dividing curves as shown in Figure III can be demonstrated by differentiation of (38) holding \(\theta^h = C\theta^f\).

Before concluding, it would probably be useful to comment briefly on the asymmetry shown in Figures I-IV, which is caused by the fact that \(C \neq 1\). If \(C = 1\), the relevant parabolas in Figures I-IV would pass through the point \((\theta^h = 2\theta^h, \theta^f = 2\theta^f)\), and Figures I-III would be distinguished by the condition \(\theta^h \geq |\theta^f|\) rather than \(\theta^h \leq |C\theta^f|\). Both dividing curves in Figure III would degenerate into straight lines. With \(C > 1\) as noted in equation (33), an examination of (38) will show that the point \((\theta^h = 2\theta^h, \theta^f = 2\theta^f)\) always lies in the region of relative gains for country \(h\) (i.e., (38) is negative).

It is my guess that the explanation for this asymmetry lies in the choice of numeraire. Given any choice of numeraire, a type of index number problem arises. With income defined in terms of one good \(X_0\), the measured changes in real income in this model are biased against the country which imports the numeraire good (country \(f\)). The reason is that tariff reduction reduces the real price of each country's import good. Country \(f\)'s gains from tariff reduction are related to a fall in its domestic price of \(X_0\) while country \(h\)'s gains are related to a rise in its domestic price of \(X_0\). Evaluating each country's income in terms of \(X_0\) would tend to make it appear that country \(h\) is gaining more from BTR even if both countries are gaining the same in utility terms. Thus Figures I-IV show a type of artificial bias in favor
of country h. About the only alternative method of analysis open to us is to assume specific functional forms and compute the change in utility. But given that we are able to assess the bias imparted by the choice of numeraire (i.e., the consequences of C ≠ 1), it would seem that little additional information of a general nature can be obtained by resorting to specific functional forms.

As a final point, I would like to comment briefly on the other tariff-cutting formulae proposed at the Tokyo Round. In all cases other than the U.S. proposal analyzed above, the proposals called for large tariffs to be reduced slightly more than in proportion to small tariffs. A somewhat surprising result is that the welfare effects of these other formulae are not very different from the effects derived above. Denoting the post-cut tariff as \( \theta_n \) and the pre-cut tariff as \( \theta_0 \), the U.S. and Japanese formulae can, for example, be written as follows:

\[
\theta_n = \alpha \theta_0 \quad \text{(U.S. proposal)} \quad 0 < \alpha < 1
\]

\[
\theta_n = \beta \theta_0 + \gamma \quad \text{(Japanese proposal)} \quad 0 < \gamma < \beta < 1.
\]

An analysis of the Japanese proposal equivalent to that presented above would yield an equation equivalent to (38).

\[
\frac{dy}{d\theta^h} - \frac{dy}{d\theta^f} = [(2\theta^h - \theta^h + \theta^f)(\theta^h - \delta) - C(2\theta^f - \theta^f - \theta^h)(\theta^f + \delta)]A
\]

where \( \delta = \gamma/(1-\beta) \) (this formula only applies to \( |\theta^i| > \delta \)). Figures I-IV can be used to display equation (42) by simply relabeling the origin as \( \theta^h = |\theta^f| = \delta \). If \( \delta \) was greater than \( \theta^{h*} \) and \( \theta^{f*} \), then neither country could lose. The other qualitative properties shown in Figures I-IV remain essentially unchanged.
VI. Summary and Conclusions

The above analysis demonstrated that the effects of bilateral tariff reduction (BTR) on a country's welfare can be broken down into a volume-of-trade (VOT) effect and a terms-of-trade (TOT) effect. The VOT effect, which is proportional to a country's existing tariff rate, always leads to an improvement in welfare. The TOT effect is somewhat more complicated and can, in particular, be defined in two different ways. Any change in the TOT, holding the VOT constant, improves a country's welfare if it leads to an increase in the relative price of the country's export good. The effect of a change in the TOT holding the foreign tariff rate constant, however, depends upon whether the country's existing tariff is greater than or less than its "optimal" (Nash equilibrium) tariff. An increase in the price of the country's export good under this definition improves welfare if and only if its existing tariff is less than its optimal tariff. This latter definition of the TOT effect proved to be relatively more useful in interpreting subsequent results.

It was not surprising in light of the work of Johnson, Gorman and Hatta and Fukushima that we were able to demonstrate that one country, but not both, can lose from BTR. The contribution here was to show, for a particular tariff-cutting formula (proportionate reductions), the precise conditions under which a country can lose. The results indicated that a country can lose if (A) it has the higher tariff of the two countries relative to the optimal values of these tariffs, and (B) both tariffs are small relative to their optimal values. In such a situation, it was explained why the losing country will have an adverse TOT effect which dominates a favorable VOT effect. A country cannot lose if its tariff equals or exceeds its optimal tariff.
These findings are of empirical interest for three reasons. First, they emphasize that no judgement can be made about the effects of BTR on the basis of knowing tariff rates alone. A detailed knowledge of import and export elasticities is a necessary prerequisite for even fairly crude welfare estimates. Second, empirical studies have consistently found that actual tariff rates are considerable less than the optimal or Nash equilibrium rates. Our findings might thus lead us to hypothesize that countries with relatively high tariff rates may indeed possibly lose as a result of the Tokyo Round. Third, the results note that the VOT effects are proportional to the level of existing tariffs. Thus both the level of expected gains and the probability that a country will gain at all become small as the level of existing tariffs becomes small. Both factors suggest that a country should have little interest in pursuing BTR when existing tariffs are already small.

The analysis also showed that the distribution of gains when both countries gain is given by the solution to a rather complicated quadratic. The country with the more elastic offer surface will be the relative gainer if it has either a very large or a very small tariff relative to the other country's tariff and relative to its own optimal tariff. In the small tariff case, BTR improves the country's TOT, making it a relative gainer while in the large tariff case, BTR deteriorates the TOT. But with a tariff which is large relative to the optimal tariff, a deterioration in the TOT improves welfare as noted earlier.

This finding is of empirical interest insofar as import and export elasticities are generally thought to be closely related to a country's size. Large countries are thought to have large elasticities and hence relatively
large optimal tariffs (i.e., the inverse elasticity of the small country (the large country's optimal tariff) is larger than the inverse elasticity of the large country). The theoretical findings thus suggest, for example, that the large country will be the relative loser if both countries have tariffs equal to their optimal values (the Johnson-Gorman problem). In per capita terms, the large country would presumably be a much larger relative loser. Thus if the large country is to be an equal gainer in absolute much less in per capita terms, it must have an initial tariff which is significantly in excess of the small country's tariff.
Footnotes

1 Markusen (1975) makes a similar point regarding transnational externalities: Relative to certain suboptimal equilibria, countries may only be able to achieve a Pareto optimum if they engage in transfer payments.

2 Gorman also corrected an error made by Johnson regarding constant-elasticity offer curves, and provided a more general treatment of these offer curves. Johnson incorporated this correction into the 1961 reprint of his paper.

3 Markusen (1977) and Markusen and Melvin (1979) show, for example, that in the presence of tariffs, the movement of a factor from where its reward is low to where it is high may decrease world real income.

4 The Johnson-Gorman papers are also restricted to pure-exchange economies.

5 An analysis of terms-of-trade effects resulting from the formation of the EEC forms a major part of a paper by Petith (1977).

6 The precise elasticity formula for the optimal tariff depends upon what elasticity one is talking about. The one used in the present paper goes back to Graaff (1949). Other formulae are discussed by Johnson (1954) and Kemp (1969).

7 The Gorman-Johnson analysis is restricted to the properties of the Nash equilibrium. In terms of Figures I-IV, they are concerned only with the gains or losses associated with the point ($\theta^h = \theta^h^*$, $\theta^f = \theta^f^*$).

8 In actual fact, $\delta$ equalled .05 in the Japanese proposal. This value is quite low relative to empirical estimates of the relevant elasticities.

9 For the proportionate-reductions case considered here, Hatta and Fukushima have shown that world real income must increase following BTR, implying that both countries cannot lose.
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