Orthogonal Frequency Division Multiplexing System with Index Modulations for Wireless Communication

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A thesis submitted in partial fulfillment of the requirements for the Doctor of Philosophy degree in Electrical and Computer Engineering
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Abstract

Orthogonal Frequency Division Multiplexing with Index Modulation (OFDM-IM) system with arbitrary (ARB) and Look-Up Table (LUT) methods for selection of active subcarriers is presented. The performance of such a system is studied over fading and shadowing channels using the following metrics: i) Average Bit Error Probability (ABEP); ii) Outage probability ($P_{out}$); iii) Energy Efficiency (EE), and iv) Spectral Efficiency (SE). Closed-form expressions have been obtained for these metrics for an OFDM-IM system over both Nakagami-m and Generalized-K channels. The performance of OFDM-IM system with ARB method of selection of active subcarriers with Power Saving Policy (PSP) and Power Reallocation Policy (PRP) is also examined. It is observed that using PRP an SNR gain of nearly 3 dB and using PSP 3 dB enhancement in power efficiency can be achieved compared to an OFDM system without active subcarrier selection. Analysis of OFDM-IM system with LUT method has been investigated using a low complexity detection method for number of active subcarriers ($k$) in the system. It is shown that 3 dB improvement in system performance and EE improvement of 0.33 bits/J can be achieved in going from $k=1$ to $k=3$ in the system. Performance analysis of mobile OFDM-IM system over Generalized-K shadowing and fading channel in the presence of co-channel interference (CCI) has been investigated. Simplified expressions for system performance have been arrived at using Meijer G function and system performance is illustrated as a function of shadowing parameter ($c$). It is observed that the system performance improves by 5 dB when shadowing parameter changes from 1 to 2 for an OFDM-IM system compared to conventional OFDM system. When adaptive modulation is employed in an OFDM-IM system, SE can be enhanced. For example, SE improves by 1.5 bits/s/Hz in an adaptive OFDM-IM system with Quadrature Amplitude modulation (QAM).
EE of OFDM-IM system can be enhanced by up to 13% by increasing the number of active subcarriers in the system to $k=3$ from $k=2$. Finally, a Maximal Ratio Combing (MRC) diversity receiver for OFDM-IM system with QAM is presented and examined for its performance. It is observed that at low values of SNR, the system SE can be enhanced by 1.5 bits/s/Hz when the number of diversity channels is increased to 4 from 2. The EE of the system is a function of number of diversity channels and number of active subcarriers in the system. It is observed that system EE improves by 0.4 bits/J in going from $k=1$ to $k=3$ for diversity channels.

**Keywords:** Orthogonal Frequency Division Multiplex (OFDM), Index Modulation (IM), Average Bit Error Probability (ABEP), Spectral Efficiency (SE), Energy Efficiency (EE), Generalized-k , Nakagami-$m$
Orthogonal frequency division multiple access with index modulation (OFD-IM) is a novel multi-carrier technique. The design of OFDM-IM makes it more capable of increasing throughput and constructing energy efficiency transmit. The thesis aims to study the OFDM-IM system design and investigate the select method to implement the OFDM-IM transceiver. The evaluation of the system performance over various channel conditions and fading was considered in the thesis. As part of system evaluation, power policy was conducted to measure the system performance and the trade between spectral efficiency and energy efficiency. Closed-form expressions for OFD-IM power policy Pairwise Error Probability (PEP) and Average Bit Error Probability (ABEP) were derived from examining the system performance in severe channel conditions. The simulation results for different modulation levels over multipath fading channels showed that the energy efficiency decreases by increasing the OFDM-IM active carries. On the other hand, the system experiences enhancement in spectral efficiency. The adaptive modulation was considered to enhance the system performance, where the modulation levels change as per channel condition; the throughput increases when the channel is less severe. The signal diversity technique was considered along with the adaptive modulation in system performance evaluation. The results showed that performance raised almost twice when diversity is introduced to the OFD-IM system.
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<td>ABEP</td>
<td>Average Bit Error Probability</td>
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<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
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<td>ASI</td>
<td>Active Subcarriers Indices</td>
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<td>BER</td>
<td>Bit Error Rate</td>
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<td>Binary Phase Shift Keying</td>
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<td>ICI</td>
<td>Interchannel Interference</td>
</tr>
<tr>
<td>i.i.d</td>
<td>independent and identically distributed</td>
</tr>
<tr>
<td>IFFT</td>
<td>Inverse Fast Fourier Transform</td>
</tr>
<tr>
<td>IoT</td>
<td>Internet of Things</td>
</tr>
<tr>
<td>IM</td>
<td>Index Modulation</td>
</tr>
<tr>
<td>ISI</td>
<td>Intersymbol Interference</td>
</tr>
<tr>
<td>LED</td>
<td>Light Emitting Diode</td>
</tr>
<tr>
<td>LoS</td>
<td>Line of Sight</td>
</tr>
<tr>
<td>LCP</td>
<td>Linear Constellation Per-coding</td>
</tr>
<tr>
<td>LUT</td>
<td>Look-up Table</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multi Input Multi output</td>
</tr>
<tr>
<td>MIMO-OFDM</td>
<td>Multi Input Multi Output Orthogonal Frequency Division Multiplexing</td>
</tr>
<tr>
<td>ML</td>
<td>Maximum Likelihood</td>
</tr>
<tr>
<td>mmWave</td>
<td>millimeter Wave</td>
</tr>
<tr>
<td>MRC</td>
<td>Maximal Ratio Combining</td>
</tr>
<tr>
<td>NG</td>
<td>Nakagami-m Gamma</td>
</tr>
<tr>
<td>PAPR</td>
<td>Peak to Average Power Ratio</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>PEE</td>
<td>Pairwise Error Event</td>
</tr>
<tr>
<td>PEP</td>
<td>Pairwise Error Probability</td>
</tr>
<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
</tr>
<tr>
<td>PSP</td>
<td>Power Saving Policy</td>
</tr>
<tr>
<td>PRP</td>
<td>Power Reallocation Policy</td>
</tr>
<tr>
<td>PSK</td>
<td>Phase Shift Keying</td>
</tr>
<tr>
<td>QAM</td>
<td>Quadrature Amplitude Modulation</td>
</tr>
<tr>
<td>QoS</td>
<td>Quality of Service</td>
</tr>
<tr>
<td>QPSK</td>
<td>Quadrature Phase Shift Keying</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
</tr>
<tr>
<td>OFDM-IM</td>
<td>Orthogonal Frequency Division Multiplexing-Index Modulation</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>-----------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>OFDM-GIM1</td>
<td>Orthogonal Frequency Division Multiplexing-Generalized1</td>
</tr>
<tr>
<td>OFDM-GIM2</td>
<td>Orthogonal Frequency Division Multiplexing-Generalized2</td>
</tr>
<tr>
<td>SE</td>
<td>Spectral Efficiency</td>
</tr>
<tr>
<td>SER</td>
<td>Symbol Error Rate</td>
</tr>
<tr>
<td>SIR</td>
<td>Signal to Interference Ratio</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>SIM</td>
<td>Subcarrier Index Modulation</td>
</tr>
<tr>
<td>TDMA</td>
<td>Time Division Multiple Access</td>
</tr>
<tr>
<td>TWDP</td>
<td>Two Wave with Diffuse Power</td>
</tr>
<tr>
<td>VoIP</td>
<td>Voice over Internet Protocol</td>
</tr>
<tr>
<td>WDMA</td>
<td>Wavelength Division Multiple Access</td>
</tr>
<tr>
<td>5G</td>
<td>5 Generation</td>
</tr>
</tbody>
</table>
# List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>Expected value</td>
</tr>
<tr>
<td>$m$</td>
<td>Fading factor</td>
</tr>
<tr>
<td>$c$</td>
<td>Shadowing factor</td>
</tr>
<tr>
<td>$D_t$</td>
<td>Total number of data in bit</td>
</tr>
<tr>
<td>$K(.)$</td>
<td>Modified Bessel function</td>
</tr>
<tr>
<td>$k$</td>
<td>Number of active subcarriers</td>
</tr>
<tr>
<td>$g$</td>
<td>Number of sub-group</td>
</tr>
<tr>
<td>$H$</td>
<td>Channel fading coefficient</td>
</tr>
<tr>
<td>$h_{KG}$</td>
<td>Channel gain of composite shadowing and fading channel</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Average received power gain</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Gamma function</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Instantaneous SNR</td>
</tr>
<tr>
<td>$\bar{\gamma}$</td>
<td>Average SNR</td>
</tr>
<tr>
<td>$\gamma_R$</td>
<td>SNR boundaries</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance</td>
</tr>
<tr>
<td>$C(.)$</td>
<td>Binomial coefficient</td>
</tr>
<tr>
<td>$n$</td>
<td>Length of sub-block</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of subcarriers</td>
</tr>
</tbody>
</table>
\(N_R\) Number of adaptive regions
\(M\) Constellation size
\(P_o\) Outage probability
\(p_{1}\) Maximum number of possible active subcarriers
\(p_{2}\) Number of bit mapped to active subcarrier
\(p\) Total number of data carried by one sub-block
\(P_{eM}\) Bit error rate probability
\(P_c\) Circuit Power
\(P_T\) Total Power
\(R\) Adaptive region
\(s\) Transmitted signal
\(\delta\) Active index
\(\hat{\delta}\) Inactive index
\(E_s\) Symbol energy
\(T_s\) Symbol duration
\(f_c\) Carrier frequency
\(W(\cdot)\) Whittaker function
\(\eta_{EE}\) Energy efficiency
\(\bar{\eta}_{EE}\) Average energy efficiency
\(\eta_{SE}\) Spectral efficiency
\(\bar{\eta}_{SE}\) Average spectral efficiency
\(\alpha\) Channel gain
Chapter 1

Introduction to Thesis

1.1 Introduction

With the advancement in science and technology, wireless communication has emerged as an effective method for information exchange; highly motivated by the growing demands of modern society for higher data speed and efficiency at any time and place [6]. The demand for high-speed data transmission is growing exponentially due to huge demand for voice, audio, and video services [7]. One of the most economical methods for achieving this objective is by using wireless communication systems [8], [9]. These systems not only offer faster data transmission but also are easy to deploy [10]. In addition, technologies based on such systems are currently booming such as Voice over Internet Protocol (VoIP) [11], [12]. In this context, OFDM with index modulation (OFDM-IM) technique has emerged as an important technique because of its superior performance compared to conventional OFDM [13], [14], [15] and [11]. In an OFDM-IM system additional information can be transmitted over specifically selected subcarriers and by nulling the others [16]. This Chapter will review literature on OFDM-IM and provide justifications for the problems considered in the thesis.

The conventional method for wireless data transmission uses a single-carrier modulation technique [17], [18]. The biggest disadvantage of this modulation in a wireless communication system is its complexity due to channel equalization [19]. Unlike single-carrier modulation, the multi-carrier modulation scheme (such as OFDM) can significantly reduce system complexity due to channel equalization, which makes it highly attractive for wireless communications [20], [21].

OFDM is a modulation technique that consists of sending information using a set of orthogonal subcarriers [22], [14]. In OFDM, modulation and demodulation processes are
performed in discrete time using IFFT and the FFT, respectively [23], [13], making it spectrally efficient. Figure 1.1 shows spectral characteristics of such an OFDM system.

Figure 1.1: Spectral characteristics of OFDM system [1]

In an OFDM system at the transmitter side, the input bits are mapped using Quadrature Amplitude Modulation (QAM) or Phase Shift Keying (PSK) signal constellation resulting in complex symbols. The modulation of these complex symbols is carried out using the IFFT operation. The output of IFFT is then passed through parallel to serial (P/S) converter. A cyclic prefix (CP) is then added and transmitted over the channel. At the receiver side, inverse operations are carried out to obtain the estimate of transmitted data as shown in the Figure 1.2. Detailed description of OFDM system can be found in [24], [25], [26].

Figure 1.2: Block diagram of baseband OFDM system: (a) Transmitter and (b) Receiver.
1.2 Literature Survey and Motivation

1.2.1 Introduction

Active research is underway to improve existing wireless technology in the context of latency, data rate and bandwidth. It is predicted that all devices would be linked to the Internet resulting in what is recognized as the Internet of Things (IoT) [27]. Hence, it is essential to come up with energy-saving physical layer methods that are spectrally efficient as well as applicable to fifth generation (5G) wireless systems [28]. Certain methods like flexible waveform designs, millimeter wave (mmWave) communications and massive MIMO systems have already been established; nevertheless, experts are suggesting innovative and more effective physical layer methods for 5G wireless systems [29].

According to [30], the technique of Index Modulation (IM) can be used to send additional information in comparison to customary communication systems. Hence, IM is considered as a potential option for future wireless infrastructure. The prediction is based on the superior aspects of IM in the context of energy and spectral efficiencies. In addition, it simplifies the system hardware. The focus on IM method is receiving importance in recent years as it enhances spectrum and energy efficiencies of wireless systems. It is also simple and can be used with any digital modulation technique [31]. IM encompasses additional ways to send more data in contrast to traditional schemes of digital modulation that rely on frequency, phase or amplitude of a carrier wave for transmission of information [32]. In fact, IM technique maps additional bits of information by using the ON and OFF positions of an object with which it is combined, like time slots, modulation types, relays, radio frequency (RF) mirrors, subcarriers, transmit antennas, dispersion matrices, spreading codes, signal powers, precoder matrices, etc [16], [33]. That is, IM uses completely new dimensions for transmission of data [34]. In IM, a significant amount of the outward-bound bits are transferred inherently; consequently, their transmission energy is conserved and can be essentially used by transferred bits, which brings about an enhanced bit error rate (BER)\(^1\) in contrast to the traditional methods [35]. Thus, IM conveys data in an energy-efficient way [36]. Also, as Index

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1. The bit error rate (BER) is known as the number of the bits in errors divided by the total number of transmitted bits during an examined time-period in a communication system.
Modulation proposes novel dimensions for conveying digital data, the spectral efficiency of wireless systems can be efficiently boosted without increasing the complexity of the system hardware [37].

1.2.2 OFDM with Index Modulation

In recent years, IM is viewed as a novel area of research and is widely considered for designing 5G wireless systems [38], [39]. Originally, IM was used for spatial modulation in MIMO (multiple-input multiple-output) systems to carry data by antenna indices [40]. The idea of IM has been used, recently, in OFDM and is referred to as OFDM with Index Modulation (OFDM-IM). In an OFDM-IM system data is conveyed not only by data symbols but also by using active subcarrier indices (ASI) [41]. In contrast, in traditional OFDM system data bits are conveyed using complex symbols from modulation alphabets (for example, Phase Shift Keying (PSK) and Quadrature Amplitude Modulation (QAM)). In IM, extra data bits are sent via indexes of the transmission entities that take part in the communication process [42], [43]; subcarrier indexes in multicarrier systems and indexes of the transmit antennas in multi-antenna systems are instances of such transmission entities [44], [45].

In Figure 1.3, a simple example is illustrated for an OFDM-IM sub-block comprising of eight subcarriers, where 3 of them are triggered to transport data symbols. Additional bits are sent by the ASI to improve the usage of spectrum [2].

According to [46], a major benefit of IM is the use of subcarrier indices as information source so that the bit error rate (BER) is considerably reduced than that of a traditional OFDM system. Furthermore, the spectral efficiency (SE) of the OFDM-IM system can surpass that of a traditional OFDM system without increasing the signal constellation size, as ASI also conveys data [47], [48]. OFDM-IM has been examined in the recent years by several researchers. For example, subcarrier IM scheme has been studied in an OFDM system using soft-index modulation in [49]. Furthermore, an enhanced-subcarrier-IM has been suggested to circumvent the proliferation of error by using the idea of majority counting, and on/off status of subcarriers is employed to signify a particular bit [50]. The point that two subcarriers are employed to convey a particular bit increases the system bandwidth by two-fold in this technique.
A novel OFDM-IM scheme is proposed for frequency-selective and time-varying fading environments in [51]. The authors provided various low-complexity transceiver structures based on full likelihood detection, and conducted error performance examination under ideal and real-time channel conditions. The results showed that OFDM-IM system achieved considerably better error performance than traditional OFDM system. In [52] two manifestations of OFDM are proposed: OFDM with Generalized Index Modulation 1, (OFDM-GIM1) (non-fixed active subcarriers) and OFDM with Generalized Index Modulation 2, OFDM-GIM2 (IM conducted on the quadrature and in-phase component of every subcarrier) with the aim to enhance the SE and BER performance of the system. In [42], GSFIM (generalized space and frequency IM) is introduced where the indexes of active subcarriers and transmit antennas are used to transport data bits. The findings showed that GSFIM can attain better data rates in comparison to MIMO-OFDM. Furthermore, BER results showed prospects of GSFIM with enhanced performance as compared to MIMO-OFDM. In [53] and [54], the performance of OFDM-IM system is extended by considering channel state information (CSI) for M-ary signal constellation. The authors presented a grouping method which can get advantage from the diversity ef-

Figure 1.3: Illustration of: (a) 3 out of 8 subcarriers initiated in OFDM-IM system; (b) All 8 subcarriers employed in OFDM system [2].
fects over frequency-selective fading channels. The results showed that OFDM-IM system with interleaved grouping surpassed traditional OFDM system in terms of performance. Likewise, the authors in [20] suggested a simple transmit diversity scheme for OFDM-IM system to attain a diversity gain for index detection. The findings displayed that coded OFDM-IM along with the suggested transmit diversity scheme can offer an enhanced performance over a frequency selective fading channel.

In [55], an improved IM method is discussed that is simpler than the scheme presented in [56]. This system has lower power consumption profile in comparison to OFDM-IM system; with smaller complications it does not require data rate compensation, as it uses usage of the frequency hops in a wise and smart manner. Also, the system does not employ all the accessible subcarriers and hence is ICI-free and permits sending extra bits in the index domain. Consequently, this method is an ideal candidate for IoT applications and wireless communication systems.

In [57], a multiple-mode (MM-OFDM-IM) system is presented to convey data via several distinct signal constellations to enhance SE of the system. Nevertheless, systems with in-phase/quadrature IM cannot offer any diversity gain, which might be a vital requirement in ultra-trustworthy communication system. In order to enhance the diversity gain of MM-OFDM-IM system, linear constellation pre-coding (LCP) and coordinate interleaved (CI) methods are used. The joint use of both can attain a diversity gain in the order of two without the loss of SE.

Studies have shown that OFDM-IM suffers from complications due to a reduction in data rate when higher order modulations is used in its unused subcarriers ([41], [58] and [26]). Consequently, Dual Mode OFDM-IM (DM-OFDM-IM) system has been suggested to overcome the reduction in data rate. In DM-OFDM-IM, extra bits can be conveyed via indices of subcarriers modulated by similar signal constellation alphabet. The SE of the system can be enhanced at the expense of negligible loss in performance. For example, the authors in [47] have proposed a generalized DM-OFDM (GDM-OFDM-IM) system, which is capable of increasing SE of the system. But the GDM-OFDM is unable to use different kinds of modulations for various sub-blocks in an OFDM symbol. Therefore, an adaptive DM-OFDM-IM has been proposed to improve the error performance [59], for various channel conditions, and to get a considerable enhancement in system SE. To further enhance SE of DM-OFDM-IM system, a quadrature DM-OFDM-IM system is proposed.
which can independently perform dual-mode index modulation on the quadrature and in-phase dimensions. In [61], a novel constellation design and a bit mapping scheme for DM-OFDM-IM is proposed based on PEP (pairwise error probability) assessment. The simulation results have shown that the developed bit mapping scheme and constellation pair can enhance the error performance of DM-OFDM-IM system. OFDM-IM system for wireless communication is assessed using random transmissions among device users and BS (base station) in a cellular environment. The absence of coordination between BS and users results in loss of orthogonality among the subcarriers, and generates ICI (inter-carrier interference) [62], [2]. Consequently, offering services to asynchronous device users is a main challenge when OFDM-IM is used.

The theoretical examination of BER of OFDM-IM system shows that it outperforms BER of traditional OFDM system [63]. This benefit has resulted in the study of OFDM-IM systems with low Peak-to-average Power Ratio (PAPR) lately [55]. A comprehensive research report can be found in [64] and [65], where findings reveal: that (a) OFDM with IM attains the full data rate if and only if the subcarriers in every group face autonomous fading which can be confirmed in reality by implementing interleaved grouping. (b) The benefit of OFDM with IM can be utilized fully by selecting a specific number of inactive subcarriers, typically 1 or 2, for an assumed SNR, and is more susceptible under PSK input. The energy efficiency of OFDM with IM is examined in [66], where improved performance is highlighted. An upper bound on the error performance of OFDM with IM system is given in [67]. The BER of OFDM with IM in the presence of carrier frequency offset is examined in [68]. Some of the recent studies have maintained that OFDM-IM scheme can be a very stimulating solution and a high-profile modulation scheme for next generation 5G wireless systems, particularly for range-extended placement, where relaying is implemented.

For example, in [38], the error performance analysis and deployment of MIMO-OFDM-IM system for 5G networks are discussed. It has been shown by means of comprehensive computer simulations that the proposed scheme offers a remarkable tradeoff between SE and error performance. Likewise, in [9] error performance of dual-hop OFDM decode-and-forward (DF) relay system is examined, using SM-OFDM-IM as a function of number of active subcarriers in the system. The findings have shown that SM-OFDM-IM relay system is superior to conventional OFDM relay system for LoS communications.
Communication system architecture, created by the combination of IM and OFDM methods is presented in [69]. This is done by examining data bits and sending them on several orthogonal subcarriers and using indices of active secondary subcarriers. This OFDM-IM system can be combined with MIMO communications system for effective and reliable data communication. Such a system could be used in 5G and later mobile communications systems [16]. An innovative quadrature space-frequency IM method as an energy-efficient radio-access technology for 5G wireless systems is proposed in [70]. By using dual antenna constellation for quadrature and in-phase communication, the suggested method can improve data rate at no additional energy cost, causing additional enhancement in EE of the system. Distributed processing method for multi-relay assisted OFDM-IM is proposed in [71]. It is shown that optimum BER can be obtained for a two-hop DF OFDM-IM system.

1.3 Overview of Contributions of the Thesis

The primary objectives of this thesis based on literature survey and motivations presented in the previous section are mentioned below:

- Derivation of BER expressions for OFDM-IM system with arbitrary selection method over Nakagami-m fading channel:
  
  Closed-form expressions for BER of OFDM-IM system for arbitrary selection of active subcarriers’ indices over Nakagami-m fading channel, with two power polices, namely, power saving policy (PSP) and power reallocation policy (PRP), are derived. The PSP is employed to save the energy used in inactive subcarriers and the PRP is employed to redistribute the energy used in inactive subcarriers to active subcarriers. In the system M-ary QAM and PSK modulations are considered. It is shown that BER is a function of modulation level \(M\), fading parameter \(m\) and SNR. The results of OFDM-IM system are compared with that of conventional OFDM system. The theoretical results are validated by Monte Carlo simulation using MATLAB. Several examples are presented and illustrated as well.
• Investigation of BER metric of OFDM-IM with M-QAM system over Nakagami-m fading channel using Look-up Table (LUT):
The concept of using LUT to choose indices of active subcarriers is presented and examined for OFDM-IM system over Nakagami-m fading channel. Three methods are investigated for detection of OFDM-IM symbols. An analysis of pairwise error probability (PEP) and average bit error probability (ABEP) of the system is presented as a function of number of active subcarrier index ($k$) in LUT, modulation level ($M$) and fading parameter ($m$).

• Design and analysis of OFDM-IM system limited by co-channel interference using Meijer G function over fading and shadowing channel:
A comprehensive framework for analysis of OFDM-IM system, using Meijer G function, in the presence and absence of co-channel interference (CCI) is presented. It is noted that Meijer-G function simplifies analysis considerably. The propagation environment is assumed to be a composite of Nakagami-m and Gamma (NG) channels. The CCI is used to cover wide range of AWGN, fading and shadowing channel conditions. Closed-form expressions for PEP and ABEP are derived and illustrated as a function of interference ($N_I$), shadowing parameter ($c$), $m$, $M$ and SNR. In the system BPSK and M-QAM signal constellations are used. Also, a discussion on trade-offs among various system parameters is provided.

• Design and analysis of an adaptive OFDM-IM system with M-QAM technique using spectral efficiency (SE) and energy efficiency (EE) over fading and shadowing channel:
The SE and EE metrics of OFDM-IM system with M-QAM technique are investigated over composite fading and shadowing Nakagami-m Gamma (NG) channel. An adaptive OFDM-IM system is proposed to improve ABEP, SE and EE metrics of the system as a function of number of regions ($N_R$), $c$, $m$, $k$, and SNR. Closed-form expressions for PEP, ABEP, SE and EE are derived and evaluated over NG channel using Greedy Detection (GD) method.
• Design and analysis of an adaptive OFDM-IM system with M-QAM technique with diversity reception over Nakagami-m fading channel:

A low complexity maximum ratio combining (MRC) receiver, efficient at low values of SNR, is introduced for OFDM-IM system with M-QAM using GD method. Also, an adaptive system with MRC receiver is proposed to enhance SE, EE, PEP and ABEP\textsuperscript{2} metrics of the system. Closed-form expressions for SE, EE, PEP and ABEP are derived and illustrated as function of number of antennas ($N_L$), $N_R$, $m$, $k$, and SNR. It is shown that SE, EE and ABEP metrics of OFDM-IM with M-QAM system can be enhanced at low values of SNR by adjusting the values of $k$, $N_L$ and $N_R$.

1.4 Thesis Organization

The thesis is organized as follows:

In Chapter 2, fundamental concepts, mathematical descriptions and functional block diagrams of OFDM and OFDM-IM systems are presented. The idea of index modulation in an OFDM system is explained. Two types of actives subcarrier selections, arbitrary and look up-table (LUT) are described.

In Chapter 3, a brief overview of wireless channel models used in the thesis is given. The mathematical descriptions of Rayleigh, Nakagami-m and composite Nakagami-Gamma (NG) channel models are presented. These models can be used to describe multipath fading environment. The NG model is used to describe multipath fading with shadowing effects over practical wireless channels.

In Chapter 4, closed-form expressions for BER metric of OFDM-IM system with M-QAM and M-PSK signal mappers over Nakagami-m channel are derived for arbitrary index selection of active subcarriers. Two power policy methods are introduced for transmissions of data. Both, power reallocation policy (PRP) and power saving policy (PSP) are considered in the system.

Chapter 5 focuses on the performance of OFDM-IM system with M-QAM using

\footnote{2. The bit error probability (BEP) is the expectation value of the BER.}
look-up table (LUT) technique for active subcarriers selection. Three methods for symbol detection of active subcarriers are considered.

Chapter 6 is devoted to the study of OFDM-IM with M-QAM system performance over shadowing and fading channel, modeled as composite Nakagami-m and Gamma (NG) density function. The effect of shadowing (c) and fading (m) parameters on the system performance are investigated in the presence and absence of co-channel interference (CCI) using the Meijer G function. Closed-form expressions have been derived for pairwise error probability (PEP) and average bit error probability (ABEP).

In Chapter 7, closed-form expressions for PEP and ABEP metrics of OFDM-IM with M-QAM system over composite shadowing and fading channel are derived. An adaptive OFDM-IM with M-QAM system is proposed to improve spectral efficiency (SE) and energy efficiency (EE) of the system. The expressions for metrics of the system PEP, ABEP, SE, outage probability are derived and illustrated as a function of system and channel parameters.

In Chapter 8, the performance of OFDM-IM system with diversity reception over Nakagami-m fading channel is considered. The performance is illustrated as a function of number of active subcarriers \(k\), number of receiver antennas \(N_r\) and fading channel parameter \(m\). An adaptive technique is also proposed to improve OFDM-IM system performance. Closed-form expressions for PEP and ABEP for this adaptive system with MRC technique are derived using GD method over Nakagami-m fading channel.

In Chapter 9, contributions of this thesis and the conclusions from the results obtained are summarized. Based on the results of this thesis, suggestions are offered for design of OFDM system with index modulation. Also, areas for further research are outlined in the light of the work presented in the thesis. For a quick reference, an overview of the thesis outline is presented in Figure 1.4.

### 1.5 Chapter Summary

An introduction to the thesis is provided with emphasis on the literature survey and the motivations for the problems addressed in the thesis. The primary objectives of the thesis are outlined. Also, the organization of the thesis is given.
Figure 1.4: Thesis outline
Chapter 2
Overview of OFDM and OFDM-IM Systems

2.1 Introduction

The OFDM is the most popular multicarrier technique used in wireless communications due to its high spectral efficiency and ability to combat multipath interference. In the classical OFDM system, signalling constellations such as M-PSK and M-QAM are used to map a constant number of input data bits to a point in a two-dimensional signal space. OFDM with index modulation (IM) has recently been proposed as a novel technique which conveys data not only by using arbitrary a M-ary signal constellation, but also by using indices of the subcarriers in the system that are activated by incoming data bits. The primary objective of this Chapter is to present a brief overview of conventional OFDM and OFDM with IM systems.

2.2 OFDM System

The block diagram of a conventional OFDM system is shown in Figure 2.1. The three main subblocks, namely, transmitter, channel, and receiver are described next.

2.2.1 Transmitter

The data source is assumed to be a stream of binary digits \( \{a_0, a_1, a_2, \cdots \} \), where, \( a_i = 1 \) or \( 0 \) and \( i = 0, 1, 2, \cdots \) with \( P(a_i = 0) = P(a_i = 1) = 1/2 \). The data stream from the source is fed to serial to parallel (S/P) converter that divides the data stream into sub-blocks of \( b \) bits with bit duration equal to \( T_b \) sec. Each sub-block is mapped to one of \( M = 2^b \) complex numbers dictated by the signal constellation. For example, when
BPSK mapper is employed ($b = 1$), each sub-block represents one bit of data and for 4-QAM ($b = 2$) and 16-QAM ($b = 4$), each sub-block carries two and four bits of data, respectively. The constellation diagrams for BPSK and 4-QAM mappers are illustrated in Figure 2.2 [72]. The output signal from the mapper can be described as a discrete-time complex signal, which is shown in Figure 2.3 for a random OFDM symbol of $N$ complex number, where $N$ is the number of subcarriers in the system. For example, the input and output of BPSK and 4-QAM mappers are represented in Tables 2.1 and 2.2, for random binary input data sequences.

Table 2.1: Example of input and output of BPSK mapper ($b=1$ and $M=2$).

<table>
<thead>
<tr>
<th>Input</th>
<th>$a_0 = 1$</th>
<th>$a_1 = 1$</th>
<th>$a_2 = 0$</th>
<th>$\cdots$</th>
<th>$a_{N-1} = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>$C_0 = +1$</td>
<td>$C_1 = +1$</td>
<td>$C_2 = -1$</td>
<td>$\cdots$</td>
<td>$C_{N-1} = +1$</td>
</tr>
</tbody>
</table>
Figure 2.2: Signal constellation diagrams for (a) BPSK and (b) 4-QAM mappers.

Figure 2.3: Discrete-time complex signal at the output of signal mapper for a random binary input.

The S/P converter divides a block of input stream of data to $N$ parallel streams, depending on the size of mapper, and the output of mapper can be expressed as a vector of $N$
Table 2.2: Example of input and output of 4-QAM mapper (b=2 and M=4).

<table>
<thead>
<tr>
<th>Input</th>
<th>( a_0a_1 = 01 )</th>
<th>( a_2a_3 = 00 )</th>
<th>( a_4a_5 = 11 )</th>
<th>( \cdots )</th>
<th>( a_{2N-2}a_{2N-1} = 10 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>( C_0 = -\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} )</td>
<td>( C_1 = -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} )</td>
<td>( C_2 = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} )</td>
<td>( \cdots )</td>
<td>( C_{N-1} = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} )</td>
</tr>
</tbody>
</table>

complex data and is represented by the vector [72]:

\[
C = [C_0, C_1, \cdots, C_{N-1}] \tag{2.1}
\]

The complex discrete signal given by (2.1) is fed to an N-point Inverse Fast Fourier Transform (IFFT) block to obtain a transformed discrete signal in time-domain as [72]:

\[
D = [D_0, D_1, \cdots, D_{N-1}] \tag{2.2}
\]

The relationship between (2.1) and (2.2) is given by [72]:

\[
D_n = \sum_{k=0}^{N-1} C_k e^{j\frac{2\pi}{N}nk}, \quad n = 0, 1, \cdots, N - 1 \tag{2.3}
\]

The P/S block converts the parallel signal from the IFFT block to serial signal and is fed to D/A converter to obtain [72]:

\[
s(t) = \sum_{k=0}^{N-1} C_k e^{j2\pi f_k t}, \quad 0 \leq t \leq NT_b \tag{2.4}
\]

where \( f_k = \frac{k}{NT_s} \) and \( t = nT_b \). \( T_b \) is the time period of bit from the data source and \( f_k \) is the frequency of the \( k^{th} \) subcarrier. The OFDM symbol duration is given by \( T_s \).

For example, for BPSK mapper, \( T_s = NT_b \) and for 4-QAM mapper \( T_s = N(2T_b) \). The discrete signal \( D = [D_0, D_1, \cdots, D_{N-1}] \) can be obtained by sampling \( s(t) \) at time \( t = n(\log_2 M)T_b \), \( n = 0, 1, \ldots, N - 1 \) as [72]:

\[
x(n(\log_2 M)T_b) = D_n = \sum_{k=0}^{N-1} C_k e^{j2\pi f_k n(\log_2 M)T_b} \tag{2.5}
\]
The complex continuous time modulating signal, given by (2.4), can be expressed as [72]:

\[ s(t) = s_I(t) + js_Q(t) \]  \hspace{1cm} (2.6)

where

\[ s_I(t) = \text{Re}\{s(t)\} \]  \hspace{1cm} (2.7)

and

\[ s_Q(t) = \text{Im}\{s(t)\} \]  \hspace{1cm} (2.8)

The signals in (2.7) and (2.8) can be expressed, respectively as:

\[ s_I(t) = \sum_{k=0}^{N-1} \left( A_k \cos 2\pi f_k t - B_k \sin 2\pi f_k t \right) \]  \hspace{1cm} (2.9)

and

\[ s_Q(t) = \sum_{k=0}^{N-1} \left( A_k \sin 2\pi f_k t - B_k \cos 2\pi f_k t \right) \]  \hspace{1cm} (2.10)

where \( C_k = A_k + jB_k \). The modulated signal given by (2.6) is transmitted by antenna over wireless communication channel after it is modulation and amplified by up-converter. The modulated signal can be mathematically expressed as [72]:

\[ y(t) = s_I(t) \cos 2\pi f_c t - s_Q(t) \sin 2\pi f_c t \]  \hspace{1cm} (2.11)

where \( f_c \) is the carrier frequency of transmitted signal.

### 2.2.2 Channel

The OFDM signal is transmitted over a physical wireless channel, say air. The basic model of a wireless channel used in the thesis is shown in Figure 2.4.

Using this model, the received OFDM signal can be expressed as:

\[ y(t) = \alpha s(t) + n(t), \quad 0 \leq t \leq T_s \]  \hspace{1cm} (2.12)

where \( \alpha \) is the random attenuation factor introduced by the wireless channel, \( s(t) \) is
the transmitted OFDM signal and \( n(t) \) is the AWGN. The statistical models of \( \alpha \) are explained in the next Chapter.

### 2.2.3 Receiver

The received OFDM signal from the antenna is down converted and fed to A/D converter to obtain \([72]\)

\[
D'_n = y(t)|_{t=n(\log_2 M)T_b} = \sum_{k=0}^{N-1} C'_k e^{j2\pi f_k n(\log_2 M)T_b}, \quad n = 0, 1, \ldots, N - 1 \tag{2.13}
\]

where \([D'_0, D'_T, D'_{2T}, \ldots, D'_{(N-1)T_b}]\) carries information about transmitted data. The S/P block converts the serial vector to parallel form and is fed to FFT block to obtain:

\[
C' = \frac{1}{N} \sum_{n=0}^{N-1} D'_n e^{-j2\pi \frac{kn}{N}}, \quad k = 0, 1, \ldots, N - 1 \tag{2.14}
\]

The vector \([C'_0, C'_1, C'_2, \cdots, C'_{N-1}]\) is then fed to demapper to obtain estimates of transmitted data \(a_0, a_1, \ldots\). Using IFFT at the transmitter and FFT at receiver for modulation and demodulation, respectively, is much easier than using \(N\) orthogonal oscillators in continuous time domain. IFFT and FFT algorithms used in software makes OFDM system more suitable for Software Defined Radio (SDR) \([73]\).
2.3 OFDM-IM System

The OFDM-IM is referred to as Subcarrier Index Modulation (SIM), which utilizes the subcarrier index to convey data using On-Off keying (OOK) technique. The proposed transmission technique achieves better BER performance and hence better power efficiency over classical OFDM system; using efficient power distribution policies. Figure 2.5a illustrates an example of classical OFDM system employing all active subcarriers. The OFDM-IM technique divides the input data stream into two groups of the same length. The first group, \((B_{OOK})\), is located on the right side of the divided group as shown in Figure 2.5b, and the second group, \((B_{QAM})\), using QAM symbols, is on the left side.

\[
N_{maj} = \max \left\{ N_{ones}^{B_{OOK}}, \left( N_{FFT} - N_{ones}^{B_{OOK}} \right) \right\},
\]
where $N_{maj}$ is the majority bit value and $N_{ones}^{BOOK}$ is Hamming weight of $BOOK$. When $N_{ones}^{BOOK} \geq N_{FFT}/2$, the majority value is one otherwise majority value is zeros. When $B_{QAM}$ is converted using S/P converter, $B_{QAM}$ is multiplexed to modulate the active subcarriers. This methodology requires two functions, which are explained in the following section. The Table 2.3 illustrates a brief comparison between OFDM and OFDM-IM systems.

### Table 2.3: Comparison Between OFDM and OFDM-IM Systems

<table>
<thead>
<tr>
<th>OFDM</th>
<th>OFDM-IM</th>
</tr>
</thead>
<tbody>
<tr>
<td>All the subcarriers carry data.</td>
<td>Some of the subcarriers carry data while the others do not carry any.</td>
</tr>
<tr>
<td>The throughput depends on the modulation level.</td>
<td>The throughput depends on the modulation level and the number of active subcarriers, which leads to higher spectral efficiency compared to classical OFDM.</td>
</tr>
<tr>
<td>The energy efficiency is fixed.</td>
<td>The energy efficiency can be increased or decreased by changing the number of active subcarriers.</td>
</tr>
</tbody>
</table>

### 2.3.1 Majority Bit-Value Signalling

Since the size of $B_{QAM}$ equals to $N_{FFT}/2 < N_{maj}$ the number of excess subcarriers can be expressed as:

$$N_{ex} = N_{maj} - \frac{N_{FFT}}{2},$$

(2.16)

where the number of excess subcarriers ($N_{ex}$) is utilized as control subcarrier to determine the type of the majority bit value to de-map $BOOK$ at the receiver side. Figure 2.6 shows the flow-chart to find the majority bit value using Hamming weight. For example, when the majority bit values are equal to one, and $N_{ex}$ is equal to zero, no overhead signaling is required. On other hand, the signaling type can be canceled, and an excess subcarrier can be employed for conveying information that leads to better spectral efficiency. At the receiver side, a coherent OOK detector is utilized in order to estimate the status of every received subcarrier. However, the receiver can only discover two groups of a combination.
Figure 2.6: Flow-Chart to determine the majority bit-value.

of $N_{FFT} - N_{maj}$ inactive subcarriers and an $N_{maj}$ active subcarriers. Subsequently, the receiver employs two possibilities of the majority bit value, zero and one, to get two possible assumptions on $B_{OOK}$. Then, the two assumptions are linked separately to the predefined $B_{QAM}$ to compose two different type of the original stream $B$. Finally, the two streams are subjected to error control techniques to find the version of $B$ that is less prone to error [41].

2.4 OFDM-IM System with Arbitrary and Look-Up Table (LUT) Models

The block diagram of OFDM-IM transceiver with LUT is shown in Figure 2.7. A block of $D_t$ bits enter the transmitter to form one symbol. Every block of $D_t$ bits is divided into $g$ groups, each consisting of $p$ bits, i.e., $D_t = pg$. Each group of $p$ bits is assigned to one of $g$ OFDM sub-blocks. The length of each sub-block is $n$, where $n = N/g$ and $N$ represents the total number of subcarriers. In conventional OFDM system, all $N$ subcarriers are active and each subcarrier conveys one of M-QAM data, whereas in OFDM-IM, only $k$ out of $n$ subcarriers per group are active. The subcarriers, that are active, transmit M-QAM symbols, and the $(n - k)$ inactive subcarriers, are zero padded. Thus, no data symbols are conveyed by inactive subcarriers. The number of active subcarriers ($k$) should be smaller or equal to the length of a sub-block ($n$), i.e., $k \leq n$. Therefore, $p$ bits are split into two parts; the first part contains $p_1$ bits and
the second part contains \( p_2 \) bits, i.e., \( p = p_1 + p_2 \) as illustrated in Figure 2.7. The maximum number of possible active subcarrier combinations is found by the binomial coefficient of \( C(n, k) = n!/k!(n-k)! \), which modulates \( p_1 = \lfloor \log_2 C(n, k) \rfloor \), where \( \lfloor . \rfloor \) is floor function, i.e., the largest lower integer number. \( p_1 \) bits are employed to map the indices of the subcarriers that transmit QAM symbols, and \( p_2 = k \log_2 M \) bits are mapped to sub-block, i.e., \( s_\beta = [s_\beta(1), \cdots, s_\beta(N)] \), where \( \beta = 1, \cdots, g \) and \( i = 1, \cdots, k \). 

\[
D_1 = p_1 g = \lfloor \log_2 C(n, k) \rfloor g
\]

is the denoted total number of bits that are conveyed by the \( k \) active subcarriers in each OFDM-IM block. \( D_2 = p_2 g = (k \log_2 M) g \) indicates the total number of bits that are transmitted by \( k \) M-QAM symbols, where \( g = N/n \) is the number of sub-blocks in a single OFDM-IM block. The total number of bits that can be transmitted by one OFDM-IM block symbol is \( D_t = D_1 + D_2 \) [51]. The received OFDM-IM signal is characterized as \( y = xH + w \), where \( y = [y(1), \ldots, y(N)] \), \( x \) is an OFDM-IM signal block symbols, \( H = \text{diag}[h(1), \ldots, h(N)] \) denotes the channel fading coefficient, and \( w = [w(1), \ldots, w(N)] \) is additive white Gaussian noise (AWGN) [24].

The greedy detector is utilized into two processes: the location of the active subcarriers and modulated data symbol. Each process is performed independently. The first process detects the active subcarriers by finding the largest received power of signal on every subcarrier, i.e., \( r_\delta = \max |s_t(\delta)|^2 \), where \( r_\delta \) is the index of active subcarrier and \( t \) is the number of active subcarriers in a sub-block \( s \), i.e., \( t = 1, \cdots, N \). The subcarriers, which have higher received power, are estimated as ones. Whereas, the subcarriers, which have lower received power, are estimated as zero, i.e., \( r_\delta = \max |w(\hat{\delta})|^2 \). The second process, the Maximum-Likelihood (ML) is applied to estimate the data symbols in the active sub-blocks [74], [75]. The receiver side of the LUT-based transceiver block-diagram, shown in Figures 2.7, contains three sub-blocks: greedy detection, LUT and de-mapper blocks. The greedy detection sub-block is used to find the index of the subcarriers based on the power-profile of the subcarriers and the number of active subcarriers (\( k \)). The subcarriers with higher power are estimated to be one and the subcarriers with lower power are estimated to be zero. The resulting combinations of the estimated bit-pattern in then sent to the LUT sub-block. The LUT sub-block looks for the match from all the combinations used on the transmitter side to find \( p_1 \). The de-mapper sub-block aids the final detection process with the help of Maximum Likelihood (ML) sub-block, which is then fed with the subcarriers with higher power found by greedy detection sub-block. In
case of ARB selection-based transceiver, shown in Chapter 4 (Figure 4.1), the indices of the subcarriers are selected randomly depending on the data stream at the transmitter side. At the receiver side, ARB selection unit contains two sub-blocks: index finder and de-mapper. The index finder is used to form the sub-block of index-modulated signal by finding the active subcarrier, based on the highest power from the power profile of the subcarriers and the majority of the bits sent by the transmitter. The majority of the bits in the bit-stream is selected based on the Hamming weight. The de-mapper block applies the ML detection strategy on the active subcarriers and aids the index finding method.

**Figure 2.7: OFDM-IM transceiver [3].**

Table 2.4 below summarizes the main differences between look-up table (LUT) and arbitrary (ARB) selection method.

**Table 2.4: Comparison between look-up table (LUT) and arbitrary (ARB) selection**

<table>
<thead>
<tr>
<th>LUT</th>
<th>ARB</th>
</tr>
</thead>
<tbody>
<tr>
<td>LUT provides more flexibility in system design by selecting different indices.</td>
<td>The indices are selected randomly depending on the data stream.</td>
</tr>
<tr>
<td>Knowing the majority bits is not essential.</td>
<td>Hamming weight is used to know the majority bits.</td>
</tr>
<tr>
<td>The OFDM-IM symbol is divided into sub-block.</td>
<td>The concept of sub-block is not used.</td>
</tr>
<tr>
<td>The active subcarriers ( (k) ) have the important factor in enhancing the energy efficiency.</td>
<td>The energy efficiency depends on the power policy method (PRP or PSP).</td>
</tr>
</tbody>
</table>
2.5 Chapter Summary

In this Chapter, an overview of OFDM and OFDM-IM systems relevant to the thesis are presented. In particular, a system block diagram of a conventional OFDM consisting of transmitter, channel, and receiver is presented and explained. The concepts used in an OFDM-IM system for arbitrary selection and look-up methods for selection of active subcarriers are explained in detail. Statistical descriptions of channel models used in the thesis are given in the next Chapter in detail.
Chapter 3
Propagation Environment and Statistical Models

3.1 Introduction

The main purpose of a communications system is to transmit information from a source to a destination over a transmission channel. The signal transmitted over a wireless communications channel is received at the receiver as a direct component or as multiple echoes [76]. These signals arrive at the receiver with random amplitude, phase and arrival times. This behavior of the channel is referred to as multipath propagation and results in signal fading due to multiple copies transmitted signal received at the receiver [77]. The main objective of this Chapter is to briefly review statistical descriptions of wireless communication channel models used in the thesis, namely Rayleigh, Nakagami-m and Gamma-Nakagami-m models.

3.2 Wireless Channel Characteristics

Radio signals are influenced by three phenomena, namely, reflection, diffraction and scattering as shown in Figure 3.1. Reflection happens when signals hit objects that are larger in wavelength then the wavelength of signals causing them to bounce in another direction and scattering occurs when signals, with wavelength equal to or larger than object dimensions, hits objects when radio waves are blocked due to obstruction, the waves propagate through or bend introducing a phenomenon known as diffraction. This introduces additional attenuation, which is also known as shadowing. These phenomena causes the signal received at the receiver to be sum of multiple copies of the transmitted signal with random amplitude, phase, and delay. The received signal power is a function of transmitter-receiver distance and is shown in Figure 3.2. Multipath fading occurs when
a transmitted signal reaches the receiver from different directions with random amplitude, phase, and delay. These multiple paths are created by the channel between transmitter and receiver due to obstacles such as buildings, mountains, trees, etc. Present in the channel and results in random fluctuation of received signal power due to fading and shadowing. Pathloss is observed due to decreasing signal power decreases and depends on the distance the signal travels between transmitter and receiver. A model of a wireless channel is shown in Figure 3.3. The received signal over a wireless channel is given by:

\[ y(t) = \alpha S(t) + n(t), \]  

where \( \alpha \), the fading coefficient, is a random variable, \( S(t) \) is the transmitted signal and \( n(t) \) is the Additive White Gaussian Noise (AWGN), assumed zero-mean with two-sided of power spectral density (PSD) of \( N_0/2 \) Watts/Hz. The signal to noise ratio (SNR) over the channel is \( \gamma = \frac{\alpha^2 P}{BN_0} \) and its average is \( \bar{\gamma} = \frac{\Omega P}{BN_0} \), where \( \Omega = \mathbb{E}[\alpha^2] \), \( P = \int_0^{T_s} S^2(t) \, dt \) with \( T_s \) the symbol duration and \( B \) is channel bandwidth.

### 3.2.1 Rayleigh Fading Channel

The Rayleigh distribution is used when there exists no direct line of sight (LoS) path between transmitter and receiver [78]. The communication link is characterized by the sum of signals received over multipaths. A majority of mobile channels fall in this category and
Figure 3.2: Illustration of the characteristic of channel fading [4]

are referred to as Rayleigh distributed channel. Communication channels inside buildings lack direct LoS path between transmitter and receiver and are modelled as Rayleigh [79], [80]. For mobile radio channels, the Raleigh distribution has been extensively employed to describe variation of received signal strength. The Rayleigh probability density function is given by [81]:

$$P_\alpha (\alpha) = \frac{2\alpha}{\Omega} \exp \left( -\frac{2\alpha^2}{\Omega} \right), \quad \alpha \geq 0$$ (3.2)

where $\Omega = \mathbb{E} [\alpha^2]$. The probability density function (PDF) of SNR $\gamma$ is given by[82]:

Figure 3.3: Model of a wireless channel with AWGN
The PDF of Rayleigh Random variable $\alpha$ is plotted in Figure 3.4.

$$P_{\gamma}(\gamma) = \frac{1}{\gamma} \exp \left( -\frac{\gamma}{\bar{\gamma}} \right), \quad \gamma \geq 0$$  \hspace{1cm} (3.3)

3.2.2 Nakagami-m fading Channel

A more generalized model of fading over wireless channels is given by Nakagami distribution. The Nakagami-m distribution offers a good empirical fit for a wide variety of communication channels [83]. The Nakagami-m probability density function of random variable $\alpha$ is given by [82]:

$$P_{\alpha}(\alpha) = \frac{2m^{m} \alpha^{2m-1}}{\Omega^{m} \Gamma(m)} \exp \left( -\frac{m\alpha^{2}}{\Omega} \right), \quad m \geq \frac{1}{2}$$  \hspace{1cm} (3.4)

where $\Gamma(m)$ is the gamma function with $\Gamma = (m-1)!$, $\Omega = E[\alpha^{2}]$ is the average received power gain, and $m = \Omega / E \left( (\alpha^{2} - \Omega)^{2} \right)$ characterizes the type of fading channel. When $m$ is equal to 1, it is equivalent to a Rayleigh fading channel. When $m > 1$ the Nakagami-m distribution approaches that of the Ricean fading channel. The probability density function (PDF) of SNR $\gamma$ is given by [82]:

$$P_{\gamma}(\gamma) = \frac{m^{m} \gamma^{m-1}}{\bar{\gamma}^{m} \Gamma(m)} \exp \left( -\frac{m\gamma}{\bar{\gamma}} \right), \quad m \geq \frac{1}{2}$$  \hspace{1cm} (3.5)
where $\gamma$ is the instantaneous SNR and $\bar{\gamma}$ is the average SNR. The PDF of $\gamma$ is plotted as a function of $m$ for Nakagami distribution in Figure. 3.5 ($\bar{\gamma}=5\text{dB}$). The Nakagami-$m$ random variable can be obtained in MATLAB by employing transformation of Gamma distribution as $\text{gamrnd}(m, 1/m, 1)$ [84].

![Figure 3.5: Probability density function of $\gamma$ given by (3.5)](image)

### 3.2.3 Generalized-K Model

When Nakagami-$m$ distribution is combined with Gamma distribution a more generalized model of a channel that exhibits both shadowing and fading is obtained. When shadowing is present over the channel, the envelope of signal follows the Nakagami-$m$ distribution, and the power of the envelope follows Gamma distribution. This model is referred to as Generalized-K model. The PDF of Gamma distribution is given by

$$P_{\gamma_f}(\gamma_f) = \frac{1}{\Gamma(c)} \left( \frac{c}{\bar{\gamma}} \right)^c \gamma_f^{c-1} \exp \left( -\gamma_f \frac{c}{\bar{\gamma}} \right)$$  \hspace{1cm} (3.6)

where $\gamma_f$ is the instantaneous SNR, $\bar{\gamma}$ is the average SNR and $c$ is shadowing factor. The combined PDF of random variable that describes shadowing and fading can be obtained by averaging the conditional PDF of Nakagami-$m$ over the PDF of Gamma, and is given by:

$$P(\gamma) = \int_0^\infty P(\gamma | \gamma_f) P_{\gamma_f}(\gamma_f) d\gamma_f$$  \hspace{1cm} (3.7)
The PDF of Nakagami-m-Gamma (NG) model is also known as generalized \((K_G)\) model and its PDF can be obtained by substituting (3.5) and (3.6) into (3.7) \[85\] as:

\[
P(\gamma) = \frac{cm^m \gamma^{m-1}}{\Gamma(c)\Gamma(m)\gamma^{c}} \int_{0}^{\infty} \gamma^{c-m-1} e^{-\frac{m\gamma}{\gamma_f}} d\gamma_f
\]

(3.8)

By using Table of Integrals \[86, (3.471.9)\], a closed-form expression of NG model is obtained as \[87\]:

\[
P(\gamma) = \frac{2}{\Gamma(c)\Gamma(m)} \left( \frac{cm}{\gamma} \right)^{\frac{c+m}{2}} \gamma^{\frac{c+m-2}{2}} K_{c-m} \left( 2\sqrt{\frac{cm\gamma}{\gamma}} \right)
\]

(3.9)

where the \(c\) and \(m\) are shadowing and fading parameters of NG model, respectively. \(K_{c-m}(\cdot)\) is the modified Bessel function of order \((c - m)\). Here, it can be shown that \(\int_{0}^{\infty} P(\gamma) d\gamma = 1\). The NG model can be used to characterize various shadowing and fading situations over a wireless channel, by employing different combinations of \(c\) and \(m\). For instance, when \(c\) approaches infinity, the NG model approximates that of Nakagami-m model, and \(m=1\), describes the Rayleigh log-normal (R-L) model. As \(c\) and \(m\) increase in value, the effect of shadowing and fading decreases. As \(c\) and \(m\) approaches infinity, the NG model approaches Additive white Gaussian Noise (AWGN) channel. The PDF of \(\gamma\) is plotted as a function of \(c\) and \(m\) in Figure. 3.6, for \(\tilde{\gamma}\) dB.

![Figure 3.6: Probability density function of \(\gamma\) given by (3.9)](image)

The NG random variable \(\alpha_{NG}\) can be obtained in MATLAB using the transformation
given by [88]:

\[ \alpha_{NG} = 2\sqrt{\frac{G_m \times G_c}{|c-m| + 1}} \]  

(3.10)

where \( G_m \) and \( G_c \) denote Gamma densities distribution for fading (m) and shadowing (c) factors, respectively. For the AWGN channel, the channel parameters \( m \) and \( c \) are set to infinity with channel gain \( \alpha_{NG} = 1 \). In contrast, for the Rayleigh channel, the channel parameters are set to \( m = 1 \) and \( c = \infty \).

### 3.2.4 Path Loss Model

Path Loss model is used to compute the average signal energy deterioration as a function of distance from transmitter over a wireless channel. A simplified model is used to describe path loss in wireless communication systems and is given by [89]:

\[ P_r = P_t K \left[ \frac{d_0}{d} \right]^\vartheta \]  

(3.11)

where \( d_0 \) is the reference distance from transmitter, \( d \) is the distance between transmitter and receiver, and \( \vartheta \) is the path loss exponent, \( 2 \leq \vartheta \leq 6 \). The path loss typically is expressed in dB as [89]:

\[ P_r(dBm) = P_t(dBm) + K(dBm) - 10\vartheta \log_{10} \left[ \frac{d_0}{d} \right]^\vartheta \]  

(3.12)

The quantity \( K \) is a constant and depends on the wavelength, \( \Gamma \), of radio wave signal and reference distance, \( d_0 \) and is given by [89]:

\[ k(dBm) = 20\log_{10} \left( \frac{\lambda}{4\pi d_0} \right) \]  

(3.13)

### 3.3 Chapter Summary

In this Chapter, a brief overview of propagation environment and models used for wireless communication channels is presented. Three channel models (Rayleigh, Nakagami-m and combined Gamma-Nakagami-m distribution) are discussed and their statistical descriptions are given. A path loss model used to compute average signal energy as a function
of distance from transmitter antenna is also presented. These models are used in the analysis and design of OFDM-IM systems in Chapters 4 to 7.
Chapter 4
OFDM-IM System with Arbitrary Index Selection Method

4.1 Introduction

In this Chapter, OFDM-IM system with arbitrary index selection for carrying additional information about data bits is considered. The method of arbitrary selection is described. The structure of transceiver of the system is given. In the system M-QAM and M-PSK signal constellations are used. Expressions are derived for system metrics such as SE, EE, and BER for Nakagami-m fading channel. Simulation of these metrics of the system are also carried out and both theoretical and simulation results are presented. A comparison of behavior of system metrics with metrics of conventional OFDM system is also given as a function system and channel parameters.

4.2 Arbitrary Index Selection Method

The arbitrary index selection method is introduced in Chapter 2. The transceiver of OFDM-IM system with arbitrary selection method is shown in Figure 4.1. A subblock named index selector is added to the conventional OFDM system to form the OFDM-IM system. The idea of index selector is explained next. Two sub-streams $B_{OOK}$ and $B_{QAM}$ or $B_{PSK}$ be of equal length are formed. Let $l_1$ and $l_2$ denote estimates of $B_{OOK}$ and

Figure 4.1: Structure of transceiver of OFDM-IM system with arbitrary index selection method

\( B_{QAM} \) or \( B_{PSK} \), respectively, with \( P(l_1) = P(l_2) = 1/2 \). Denoting \( P_{b, OOK} \) as the error probability of \( l_1 \) (the activated subcarriers) and \( P_{b, M-ary} \) as the error probability of \( l_2 \), the probability of error of the OFDM-IM system, \( (P_e(E)) \), can be shown to be given by [90]:

\[
P_e(E) = P_e(E|l_1) P(l_1) + P_e(E|l_2) P(l_2)
= \frac{1}{2} P_{b, OOK}(E) + \frac{1}{2} P_{b, M-ary}(E)
\]  

(4.1)

where \( P_{b, M-ary} \) is either \( P_{b, MQAM} \) or \( P_{b, MPSK} \) the system error probability given by (4.1) is studied using two power policy methods [90].

### 4.3 Power Policies

Two ideas are presented for distribution of power of inactive subcarriers to active subcarriers: (i) power reallocation policy (PRP) and (ii) power saving policy (PSP). In PRP, after power redistribution, the powers in active subcarriers are increased. In case of traditional OFDM system all subcarriers are active and carry equal power. The BER performance of the OFDM-IM system is therefore better than that of OFDM system. For example, when \( N_{FFT} = 16 \) and \( N_{maj} = 12 \), the power per active subcarrier is increased by \( \frac{1}{12} - \frac{1}{16} = \frac{1}{48} W = 0.021 W \). The average signal to noise ratio (SNR) of active
subcarriers when PRP is used, $\tilde{\gamma}^{PRP}$, is given by [90]:

$$\tilde{\gamma}^{PRP} = \frac{P_t}{E[N_{maj}]/BN_0} \quad (4.2)$$

where $P_t$ is the total transmit power of each OFDM symbol, $E[N_{maj}]$ is the average number of majority bit-value zeros or ones, $N_0$ is the per-subcarrier average additive white Gaussian noise (AWGN) power and $B$ is the bandwidth. The number of the majority bits for a sequence of independent data stream is binomially distributed random variable, and hence $E[N_{maj}] \approx \frac{N_{FFT}}{2}$.

In the case of PSP, the powers assigned to inactive subcarriers are simply saved, this results in a better power efficiency and the average SNR of active subcarriers , $\tilde{\gamma}^{PSP}$, is given by:

$$\tilde{\gamma}^{PSP} = \frac{P_t}{N_{FFT}/BN_0} \quad (4.3)$$

where $N_{FFT}$ is the total number of subcarriers.

### 4.4 Spectral Efficiency $\eta_{SE}$

The spectral efficiency, $\eta_{SE}$, defines the number of bits that can be transmitted over unit Hz of channel bandwidth per second. For example, $\eta_{SE}$ of OFDM-IM system with $M=2$ is given by $\eta_{OFDM-IM}^{SE} = 1 + \frac{1}{2} \log_2 (M=2) = 1.5 \text{ bit/s}$, while for OFDM system this is given by $\eta_{OFDM-IM}^{SE} = \log_2 (M=2) = 1 \text{ bit/s}$. Assuming all active subcarriers can convey data, the $\eta_{SE}$ of OFDM-IM and conventional OFDM systems can be expressed as follows [90]:

$$\eta_{SE}^{OFDM-IM} = 1 + \frac{E[N_{maj}]}{N_{FFT}} \log_2 (M) \quad [\text{bits/s/Hz}] \quad (4.4)$$

$$\eta_{SE}^{OFDM} = \log_2 (M) \quad [\text{bits/s/Hz}]$$

### 4.5 Energy Efficiency $\eta_{EE}$

The energy efficiency, $\eta_{EE}$, defines the maximum data rate, $R$, in bits/sec, that can be transmitted by the system per unit amount of transmission power, $P_t$, in Watt. The $\eta_{EE}$
of OFDM-IM and OFDM systems can be shown to be given by:

\[
\eta_{EE}^{OFDM-IM} = \frac{1 + E \left[ N_{maj} \right]}{N_{FFT}} \frac{\log_2(M)}{\gamma} [b/TNEU]
\]

\[
\eta_{EE}^{OFDM} = \frac{\log_2(M)}{\gamma} [b/TNEU]
\]  

(4.5)

4.6 BER Metric of OFDM-IM System with OOK

In order to compute BER metric of OFDM-IM system, given by (4.2), first \( P_{b,OOK}(E) \) needs to be computed. The BER metric of OOK over AWGN channel is given by [91]:

\[
P_b(E | \gamma) = \frac{1}{2} \exp \left( -\frac{\gamma}{2} \right) \quad (4.6)
\]

where \( \gamma \) is the SNR over the channel. The BER of OFDM with OOK can be determined using:

\[
P_b(E) = \int_0^\infty P_b(E | \gamma) P_\gamma(\gamma) d\gamma \quad (4.7)
\]

where \( P_b(E | \gamma) \) given by (4.6) is the conditional BER given the value of \( \gamma \), and \( P_\gamma(\gamma) \) is probability density function (PDF) of Nakagami-m fading channel given by (3.5). Thus, (4.7) can be written as:

\[
P_{b,OOK} = \int_0^\infty 0.5 e^{\frac{\gamma}{2}} \times \frac{m^m \gamma^{m-1}}{\bar{\gamma}^m \Gamma(m)} e^{-\frac{\gamma m}{2\bar{\gamma}}} d\gamma \quad (4.8)
\]

\[
P_{b,OOK} = \frac{0.5 m^m}{\bar{\gamma}^m \Gamma(m)} \int_0^\infty \gamma^{m-1} e^{-\gamma \left( \frac{1}{2} + \frac{m}{\bar{\gamma}} \right)} d\gamma \quad (4.9)
\]

\[
P_{b,OOK} = \frac{0.5 m^m}{\bar{\gamma}^m \Gamma(m)} \int_0^\infty \gamma^{m-1} \exp \left( -\gamma \left( \frac{\bar{\gamma} + 2m}{2\bar{\gamma}} \right) \right) d\gamma \quad (4.10)
\]
Using Table of integrals [86, (3.381.4)], \( \int_{0}^{\infty} x^{v-1} e^{-\mu x} dx = \frac{1}{\mu v} \Gamma(v) \), the BER metric \( P_{b,OOK} \) is given by:

\[
\int_{0}^{\infty} x^{v-1} e^{-\mu x} dx = \frac{1}{\mu v} \Gamma(v)
\]

\[
P_{b,OOK} = \frac{0.5m^m}{\tilde{\gamma}^m \Gamma(m)} \times \left( \frac{2\tilde{\gamma}}{\tilde{\gamma} + 2m} \right)^m \Gamma(m)
\] (4.11)

\[
P_{b,OOK} = \frac{0.5m^m}{\tilde{\gamma}^m (\tilde{\gamma} + 2m)^m} = \frac{0.5m^m \times 2^m}{(\tilde{\gamma} + 2m)^m}
\] (4.12)

\[
P_{b,OOK} = 0.5 \left( \frac{2m}{\tilde{\gamma} + 2m} \right)^m
\] (4.13)

where \( \tilde{\gamma} \triangleq E(\alpha^2) \frac{E_b}{N_0} \) represents the average SNR per bit. \( \alpha \) and \( E_b \) are the random fading amplitude and bit energy, respectively. The quantity \( E_b \) is computed by dividing the total available transmitted power \( P_t \) assigned to an OFDM symbol by the number of subcarriers. Next, \( P_{b,OOK} \) when PRP and PSP power policies are used in the system are presented.

### 4.6.1 BER Metric with PRP

In this case, the transmitted power per symbol is the same, as the power of inactive subcarriers is redistributed among the active subcarriers i.e, \( \gamma^{PRP} = P_t \frac{N_{maj}}{N_maj} \), where \( N_{maj} \) is the number of active subcarriers and \( P_t \) is the power of one symbol. With this power policy, the BER metric is given by:

\[
P_{b,OOK}^{PRP} = 0.5 \left( \frac{2m}{\tilde{\gamma} + 2m} \right)^m
\] (4.14)

### 4.6.2 BER Metric with PSP

When the PSP power policy is applied, the transmitted power per symbol is decreased by an amount allocated to inactive subcarriers. Therefore, \( \gamma^{PSP} = \frac{P_t}{N_{FFT}} \), where \( N_{FFT} \) is the total number of subcarriers and the BER metric can be shown to be given by:
\[ P_{b_{OSK}}^{PSP} = 0.5 \left( \frac{2m}{0.5\gamma + 2m} \right)^m \] (4.15)

### 4.7 BER Metric of OFDM-IM System with M-QAM

The upper bound on BER metric of coherent M-QAM over AWGN channel is given by [92]:

\[ P_{QAM}(E|\gamma) \simeq 0.2 \exp \left( \frac{-3\gamma}{2(M-1)} \right), \quad \gamma \geq 0 \] (4.16)

where \( \gamma \) is the SNR and \( M \) denotes constellation size of M-QAM. The bound in (4.16) is valid for \( M \geq 4 \) and is tight for \( BER \leq 10^{-2} \). The BER metric of OFDM-IM system with M-QAM over Nakagami-m fading channel can be evaluated as:

\[ P_{b_{MQAM}}(E) = \int_0^\infty P_b(E|\gamma) P_\gamma(\gamma) \, d\gamma \] (4.17)

Substituting (3.5) and (4.16) gives:

\[ P_{b_{MQAM}} = \frac{0.2m^m}{\tilde{\gamma}^m \Gamma(m)} \int_0^\infty \gamma^{m-1} e^{-\frac{\gamma}{\tilde{\gamma}} - \frac{3\gamma}{2(M-1)}} \, d\gamma \] (4.18)

Using Table of integrals [86, (3.381.4)], the BER metric can be expressed as:

\[ P_{b_{MQAM}} = \frac{0.2m^m}{\tilde{\gamma}^m \Gamma(m)} \int_0^\infty \gamma^{m-1} e^{-\gamma \left( \frac{3}{2(M-1)} + \frac{m}{\tilde{\gamma}} \right)} \, d\gamma \] (4.19)
\[ P_{b,MQAM} = \left( \frac{0.2m^m}{\left( \frac{3\bar{\gamma}}{2(M-1)} + \frac{m}{\bar{\gamma}} \right)^m} \right) = \left( \frac{0.2m^m}{\left( \frac{3\bar{\gamma} + m2(M-1)}{2(M-1)} \right)^m} \right) = 0.2 \left( \frac{m}{\frac{3\bar{\gamma} + m2(M-1)}{2(M-1)}} \right)^m \]  

(4.21)

\[ P_{b,MQAM} = 0.2 \left( \frac{2m(M-1)}{3\bar{\gamma} + 2m(M-1)} \right)^m \]  

(4.22)

The effects of using PRP and PSP power policies on BER metric are considered next.

### 4.7.1 BER Metric with PRP

When PRP policy is used, the power is redistributed among the active subcarriers \( \approx \frac{N_{FEF}}{2} \), which makes the average SNR twice that of conventional OFDM system. The BER metric, thus, is given by:

\[ P_{b,MQAM}^{PRP} = 0.2 \left( \frac{2m(M-1)}{6\bar{\gamma} + 2m(M-1)} \right)^m \]  

(4.23)

Substituting (4.23) and (4.13) into (4.1), the overall BER metric of OFDM-IM system with M-QAM over Nakagami-m channel is given by:

\[ P_{e,MQAM/OOK}^{PRP}(E) = \frac{1}{4} \left( \frac{2m}{\bar{\gamma} + 2m} \right)^m + \left( \frac{1}{10} \right) \left( \frac{2m(M-1)}{6\bar{\gamma} + 2m(M-1)} \right)^m \]  

(4.24)

### 4.7.2 BER Metric with PSP

When PSP policy is used in the system, the power remains unchanged because it is saved in the inactive subcarriers. The BER metric is thus given by

\[ P_{b,MQAM}^{PSP} = 0.2 \left( \frac{2m(M-1)}{3\bar{\gamma} + 2m(M-1)} \right)^m \]  

(4.25)

Substituting (4.25) and (4.13) into (4.1), the total BER metric of OFDM-IM system with MQAM over Nakagami-m channel is given by:
\[
P_{e,MQAM/OOK}^{PSP}(E) = \frac{1}{4} \left( \frac{2m}{0.5\gamma + 2m} \right)^m + \left( \frac{1}{10} \right)^m \left( \frac{2m(M - 1)}{3\gamma + 2m(M - 1)} \right)^m \tag{4.26}
\]

### 4.8 BER Metric of OFDM-IM System with M-PSK

In this section steps similar to those used in previous section are used to derive BER metric of OFDM-IM system with MPSK over Nakagami-m channel. The symbol error rate (SER) metric of M-PSK over AWGN channel is given by [91]:

\[
P_E \approx 2Q\left( \sqrt{\frac{2\bar{\gamma}_s}{N_0}} \sin \left( \frac{\Pi}{M} \right) \right), \tag{4.27}
\]

where \( \bar{\gamma}_s \triangleq \bar{\gamma} \log_2(M) \). \( \bar{\gamma}_s \) and \( \bar{\gamma} \) represent average SNR per symbol and average SNR per bit, respectively. The relationship between BER \( (P_b) \) and SER \( (P_E) \) is given by \( P_b = \frac{M/2}{M-1}P_E \). By employing the approximation [93]:

\[
Q(x) \leq \frac{1}{2} \exp\left(-\frac{x^2}{2}\right), \tag{4.28}
\]

in (4.27), we get:

\[
P_b \approx \left( \frac{M/2}{M-1} \right)^{\gamma} \exp(-\gamma \log_2 M \sin^2\left( \frac{\pi}{M} \right)) \tag{4.29}
\]

Substituting (3.5) and (4.29) into (4.7), the BER metric of M-PSK can be given as:

\[
P_{b\text{-MPSK}} = \left( \frac{M/2}{M-1} \right)^{\gamma} \int_0^\infty \frac{m^m \gamma^{m-1}}{\Gamma(m)} e^{-\frac{m\gamma}{\gamma}} d\gamma \tag{4.30}
\]

\[
P_{b\text{-MPSK}} = \left( \frac{M/2}{M-1} \right)^{\gamma} \int_0^\infty \frac{m^m \gamma^{m-1}}{\Gamma(m)\gamma^m} e^{-\gamma \log_2 M \sin^2\left( \frac{\pi}{M} \right)} \left( \frac{\gamma}{\gamma} \right)^{m} d\gamma \tag{4.31}
\]

Using Table of integrals [86, (3.381.4)], a closed form expression for BER of OFDM-IM system with M-ary PSK over a Nakagami-m fading channel can be derived and is given
by:

\[ P_{b,MPSK} = \left( \frac{M/2}{M-1} \right)^m \frac{\Gamma(m) \gamma^m}{\Gamma(m) \gamma^m} \left( \frac{\log M \sin^2 \left( \frac{\pi}{M} \right) + m \gamma}{\frac{\gamma}{M}} \right) \times \Gamma(m) \]  \hspace{1cm} (4.32)

\[ P_{b,MPSK} = \left( \frac{M/2}{M-1} \right)^m \frac{m^m}{\left( \gamma \log M \sin^2 \left( \frac{\pi}{M} \right) + \frac{m \gamma}{\frac{\gamma}{M}} \right)^m} \]  \hspace{1cm} (4.33)

\[ P_{b,MPSK} = \left( \frac{M/2}{M-1} \right)^m \frac{m^m}{\left( m + \gamma \log_2 M \sin^2 \left( \frac{\pi}{M} \right) \right)^m} \]  \hspace{1cm} (4.34)

The BER metrics when PRP and PSP power policies are used are derived next.

### 4.8.1 BER Metric with PRP

Using PRP policy in the system, BER metric for the case of M-PSK can be shown to be given by:

\[ P_{PRP} b_{MPSK} = \left( \frac{M/2}{M-1} \right)^m \frac{m^m}{\left( \gamma \log M \sin^2 \left( \frac{\pi}{M} \right) + \frac{m \gamma}{\frac{\gamma}{M}} \right)^m} \]  \hspace{1cm} (4.35)

Substituting (4.35) and (4.13) into (4.1), the overall BER metric of OFDM-IM system with M-PSK is given by:

\[ P_{e,MPSK/OOK}^\text{PRP}(E) = \frac{1}{4} \left( \frac{2m}{\gamma + 2m} \right)^m + \frac{1}{2} \left( \frac{M/2}{M-1} \right)^m \frac{m}{\left( m + \gamma \log_2 M \sin^2 \left( \frac{\pi}{M} \right) \right)^m} \]  \hspace{1cm} (4.36)

### 4.8.2 BER Metric with PSP

Following steps similar to those used in Section (4.4.2), the BER metric for the case of M-PSK for PSP is given by:

\[ P_{b,MPSK}^{PSP} = \left( \frac{M/2}{M-1} \right)^m \frac{m^m}{\left( m + \gamma \log_2 M \sin^2 \left( \frac{\pi}{M} \right) \right)^m} \]  \hspace{1cm} (4.37)
Substituting (4.37) and (4.13) into (4.1), the total BER metric M-PSK scheme using PSP method with index modulation can be shown to be given by:

\[
P_{e,MPSK/OOK}^{PSP}(E) = \frac{1}{4} \left( \frac{2m}{0.5\bar{\gamma} + 2m} \right)^m + \frac{1}{2} \left( \frac{M/2}{M - 1} \right) \left( \frac{m}{m + \bar{\gamma} \log_2 M \sin^2(\frac{\pi}{M})} \right)^m
\]

(4.38)

### 4.9 Numerical Results and Discussion

To understand the influence of fading channel, the BER performance of OFDM-IM system is examined to improve the system design. The results of BER metric of OFDM-IM system for PSP and PRP with different constellation size is presented over the Nakagami-m fading channel. Figure 4.2 presents BER expressions in equations (4.24), (4.26) and (4.38) for \( m = 1 \), and compares it with the BER of Rayleigh fading channel. It is observed that PSP performs poorly in severe fading while PRP operates better than PSP at the same transmitted power. This is the reason of using PRP in designing OFDM-IM system under severe fading, and to enhance the system performance. Also, for designing efficient OFDM-IM system, the PSP can be used to get better power efficiency, once the channel is estimated in good condition. For example, PSP is given 3 dB less than PRP in Figure 4.2. Figure 4.3 and Figure 4.4 show the performance of OFDM-IM system when design system with 4-QAM symbol using PSP and PRP for different \( m \) fading environment. It is observed that the BER metric of the OFDM-IM system at 4-QAM symbol PRP outperforms PSP in the severe fading channel, for example, when \( m = 1 \) PRP requires an SNR of 24 dB and a PSP of 27.5 dB at a BER of \( 10^{-3} \). Considering PRP in system design, it will reserve 3.5 dB of transmitted power. Likewise, increasing the system throughput at 16-QAM PRP is still maintaining better performance than PSP when \( m = 2, 3, \) and 4 at a BER of \( 10^{-6} \) in Figure 4.5 and Figure 4.6. Overall introducing PRP to the OFDM-IM system will improve system performance. The simulation algorithm, and the associated parameters in Table 4.1 have been outlined below:

1) The generated data \( B \) are divided into two sub-streams \( B_{OOK} \) and \( B_{QAM} \), respectively, and the length of \( B_{OOK} \) must be equal to the number of subcarriers
2) The majority bits of \( B_{OOK} \), zeros or ones, are found using Hamming weight.

3) The majority bits type is employed to indicate the active subcarrier and the \( B_{QAM} \) is mapped to the active subcarrier to form the IM-Matrix

4) The IM-Matrix is applied to IFFT. After that cyclic prefix is added and the modulated data is sent over the Nakagami-m fading channel.

5) On the receiver side, two ordered estimation processes are utilized to estimate the signal. First, the location of active subcarrier is detected by finding the power of the signal. Second, the M-ary QAM or PSK symbol is estimated at the active subcarriers.

6) If the power of the active subcarrier is bigger than the threshold \((y)\), the subcarrier is active. The estimated matrix \((\tilde{B}_{OOK})\) is equal to one and the estimated M-ary modulated signal matrix \((\tilde{B}_{QAM})\) is equal to the received \((\tilde{B}_{IM})\) Matrix at the active subcarriers.

7) \((\tilde{B}_{OOK})\) and \((\tilde{B}_{QAM})\) are combined to form the original data stream matrix \(B\).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power Policy</td>
<td>PSP and PRP</td>
</tr>
<tr>
<td>Number of Subcarriers</td>
<td>64</td>
</tr>
<tr>
<td>SNR</td>
<td>0 dB to 30 dB</td>
</tr>
<tr>
<td>Fading Factor ((m))</td>
<td>1, 2, 3 and 4</td>
</tr>
<tr>
<td>Number of Symbols</td>
<td>10000</td>
</tr>
<tr>
<td>Constellation size of QAM</td>
<td>4 and 16</td>
</tr>
<tr>
<td>Iteration</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 4.1: Simulation parameters used for evaluation of BER metric of OFDM-IM system.

4.10 Chapter Summary

In this Chapter, OFDM-IM system has been analyzed and evaluated over Nakagamai-m fading channel and new general expressions closed-form has been derived for OFDM-IM
Table 4.2: Comparison of BER metrics of OFDM-IM system for 4-QAM and 16-QA, M as a function of SNR over Nakagami-m channels.

<table>
<thead>
<tr>
<th>BER</th>
<th>Nakagami-m factor</th>
<th>4-QAM</th>
<th>4-QAM</th>
<th>16-QAM</th>
<th>16-QAM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>PRP</td>
<td>PSP</td>
<td>PRP</td>
<td>PSP</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>$m=1$</td>
<td>24 dB</td>
<td>27.5 dB</td>
<td>24 dB</td>
<td>27 dB</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>$m=2$</td>
<td>30 dB</td>
<td>34 dB</td>
<td>30 dB</td>
<td>33 dB</td>
</tr>
<tr>
<td></td>
<td>$m=3$</td>
<td>22.5 dB</td>
<td>26 dB</td>
<td>23 dB</td>
<td>26 dB</td>
</tr>
<tr>
<td></td>
<td>$m=4$</td>
<td>19 dB</td>
<td>22.5 dB</td>
<td>22.5 dB</td>
<td>22.5 dB</td>
</tr>
</tbody>
</table>

Figure 4.2: BER performances OFDM and OFDM-IM systems for 4-QAM and 4-PSK, as a function of SNR over Nakagami-m ($m=1$) or Rayleigh channels.

with M-ary QAM and PSK modulation schemes for BER over different fading factors ($m$). The accuracy of the closed-form solution has been validated through simulations.
Figure 4.3: BER performance of OFDM-IM system (4-QAM and \( N = 64 \)) using PSP over Nakagami-m fading channel as a function of SNR and m

Figure 4.4: BER performances of OFDM-IM system (4-QAM and \( N = 64 \)) using PRP over Nakagami-m fading channel as a function of SNR and m
Figure 4.5: BER performance of OFDM-IM system (16-QAM and $N = 64$) using PSP over Nakagami-$m$ fading channel as a function of SNR and $m$.

Figure 4.6: BER performance of OFDM-IM system (16-QAM and $N = 64$) using PRP over Nakagami-$m$ fading channel as a function of SNR and $m$. 

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5.1 Introduction

In this Chapter, OFDM-IM system using LUT selection method is considered. The performance of the system using a low complexity detection technique known as greedy detection (GD) is examined over Nakagami-m channel. Three scenarios of detection method of received OFDM-IM symbol are presented. The first is the mis-detection of both the M-ary modulated symbol and the active subcarriers. The second is the correct detection of M-ary modulated symbol and the mis-detection of active subcarriers. Finally, third scenario with mis-detection of M-ary modulated symbol and correct detection of the active subcarriers. The analysis of the system is carried out and pairwise error probability (PEP) and average bit error probability (ABEP) metrics are derived and assessed as a function of channel parameters, constellation size and number of subcarriers in the system. The theoretical and simulation results are determined and compared.


5.2 Look-up Table (LUT) selection Method

The block diagram of OFDM-IM transceiver using the LUT selection method is shown in Figure 5.1 [16]. The total number of data $D_t$ bits enter the transmitter to form one block of OFDM symbol. Every $D_t$ bits is divided into $g$ groups, each consisting of $p$ bits, i.e., $D_t = pg$. Each group of $p$ bits is assigned to one of $g$ OFDM sub-blocks. The length of the sub-block is $n$, where $n = N/g$ and $N$ represents the total number of subcarriers in the system. In conventional OFDM system, all $N$ subcarriers are active and each subcarrier conveys $\log_2 M$ bits of information, whereas in OFDM-IM, only $k$ out of $n$ subcarriers per group are active. The subcarriers, which are active, convey one of $M$ symbols of M-QAM symbols, and the inactive subcarriers, $(n-k)$ are zero padded. Thus, no data are conveyed by inactive subcarriers. The number of active subcarriers ($k$) should be smaller or equal to the length of sub-block ($n$), i.e., $k \leq n$. Therefore, $p$ bits are split into two parts; the first part contains $p_1$ bits and the second part contains $p_2$ bits, i.e., $p = p_1 + p_2$ as illustrated in Figure 5.1. The maximum number of possible active subcarrier combinations can be found by using $C(n, k) = n!/k!(n-k)!$. Thus, modulates $p_1 = \lfloor \log_2 C(n, k) \rfloor$, where $\lfloor \cdot \rfloor$ is floor function, i.e., the largest lower integer number. $p_1$ bits are employed to map the indices of the subcarriers that transmit M-QAM symbols, and $p_2 = k \log_2 M$ bits are mapped to sub-block, i.e., $s_\beta = [s_\beta(1), \cdots, s_\beta(N)]$, where $\beta = 1, \cdots, g$ and $i = 1, \cdots, k$. $D_1 = p_1 g = \lfloor \log_2 C(n, k) \rfloor g$ is the total number of bits that are conveyed by the $k$ active subcarriers in each OFDM-IM block. $D_2 = p_2 g = (k \log_2 M)g$ indicates the total number of bits that are transmitted by $k$ M-ary QAM symbols, where $g = N/n$ is the number of sub-blocks in a single OFDM-IM block. The total number of bits that can be transmitted by one OFDM-IM symbol is $D_t = D_1 + D_2$ [16]. The received OFDM-IM signal is characterized as $y = xH + w$, where $y = [y(1), \ldots, y(N)]$, $x$ is the OFDM-IM signal, $H = diag[h(1), \ldots, h(N)]$ denotes the channel fading coefficient, and $w = [w(1), \ldots, w(N)]$ is the additive white Gaussian noise (AWGN)[94]. The greedy detector is utilized in two processes: the location of the active subcarriers and modulated data symbol. Each process is performed independently. The first process detects the active subcarriers by finding the greatest received power of signal on every subcarrier, i.e., $r_\delta = \max |s_t(\delta)|^2$, where $r_\delta$ is the index of active subcarrier and $t$ is the number of active subcarriers in sub-block $s$, i.e., $t = 1, \cdots, N$. The subcarriers, with have
higher received power, are estimated as ones. Whereas, the subcarriers, which have lower received power, are estimated as zero, i.e., $r_\delta = \max |w(\delta)|^2$. In the second process, the Maximum-Likelihood (ML) principle is applied to estimate the data symbols in the active sub-block $s$ [74].

$$r_\delta = \max |w(\delta)|^2.$$ 

5.3 Pairwise Error Probability (PEP) Metric of OFDM-IM System

In this section, the PEP metric of OFDM-IM system with M-QAM modulation is derived for Nakagami-m fading channel. It is assumed that the greedy detection method is used at the receiver in the system. The greedy detector first detects the indices of the active subcarriers (identified by index 1) and inactive subcarriers (identified by index 0). Also, the detector estimates data symbol of the active subcarriers by employing the Maximum-Likelihood (ML) principle. When the active subcarrier index ($\delta$) is incorrectly estimated at the receiver as inactive index ($\hat{\delta}$), this event is known as the pairwise error event (PEE), i.e., $(\delta \rightarrow \hat{\delta})$ and $\delta \neq \hat{\delta}$. The conditional pairwise error probability (PEP) of this event, $PEP(\delta \rightarrow \hat{\delta})$, is given by [95]:

$$PEP(\delta \rightarrow \hat{\delta})$$.
\[ PEP(\delta \to \tilde{\delta}) = 1 - \sum_{q=0}^{n-k} \left( \frac{n-k}{q} \right)^{-1} \frac{1}{q+1} e^{-\gamma \frac{-q}{q+1}}, \quad (5.1) \]

where \( \gamma \) is the instantaneous signal to noise ratio (SNR) of the active subcarrier. The expression in (5.1) can be approximated by [75]:

\[ PEP(\delta \to \tilde{\delta}) \simeq \frac{n-k}{2} e^{-\frac{\gamma \delta}{2}} \quad (5.2) \]

An upper bound on PEP of OFDM-IM system after averaging over \( \delta \) is given by [75]:

\[ PEP_t \leq \frac{k}{n} \sum_{\delta=1}^{n} PEP(\delta \to \hat{\delta}) \leq k PEP(\delta \to \hat{\delta}) \quad (5.3) \]

For simplicity of notations, \( PEP(\delta \to \hat{\delta}) \) is denoted by \( PEP \) and \( \gamma_{\delta} \) by \( \gamma \) from hereafter. Thus PEP of OFDM-IM system over the Nakagami-m fading channel can be written as:

\[ PEP_{\text{Naka}} = \int_{0}^{\infty} PEP \, p_{\gamma}(\gamma) d\gamma \quad (5.4) \]

where \( PEP \) is either the approximate or the exact pairwise error probability and \( p_{\gamma}(\gamma) d\gamma \) is density function of \( \gamma \). For the Nakagami-m channel, \( p_{\gamma}(\gamma) d\gamma \) is given by (3.5). Thus, (5.4) can be written as:

\[ PEP_{\text{Naka}} = \int_{0}^{\infty} \frac{m^{m \gamma m-1}}{\Gamma(m)} e^{-\frac{m \gamma}{m}} d\gamma - \int_{0}^{\infty} \frac{m^{m \gamma m-1}}{\gamma m \Gamma(m)} e^{-\frac{m \gamma}{m}} d\gamma \times \sum_{q=0}^{n-k} \left( \frac{n-k}{q} \right)(-1)^q \frac{1}{q+1} \frac{-\gamma q}{e^{q+1}} d\gamma \quad (5.5) \]

The the expression in (5.5) can be simplified and is given by:

\[ PEP_{\text{Naka}} = 1 - \sum_{q=0}^{n-k} \left( \frac{n-k}{q} \right)(-1)^q \frac{1}{q+1} \frac{m^{m \gamma m-1}}{\gamma m \Gamma(m)} \int_{0}^{\infty} \gamma^{m-1} e^{-\gamma(m \gamma + q + 1)} d\gamma \quad (5.6) \]
Using Table of Integrals [86, (3.381.4)], a closed-form expression for the upper bound on PEP over the Nakagami-m fading channel can be derived and is given by:

\[
PEP_{\text{Naka}} = 1 - \sum_{q=0}^{n-k} \binom{n-k}{q} (-1)^q \frac{1}{q+1} \times \frac{m^m}{\bar{\gamma}^m \Gamma(m)} \times \frac{1}{\left( \frac{m}{\bar{\gamma}} + \frac{q}{q+1} \right)^m} \quad (5.7)
\]

upon simplification, the above equation can be written as:

\[
PEP_{\text{Naka}} = 1 - \sum_{q=0}^{n-k} \binom{n-k}{q} (-1)^q \frac{1}{q+1} \left( \frac{m(q+1)}{\bar{\gamma}q +mq +m} \right)^m. \quad (5.8)
\]

The upper bound on total probability of PEP of OFDM-IM system can be obtained by substituting (5.8) into (5.3) and is given by:

\[
PEP_{t,\text{Naka}} = k \left[ 1 - \sum_{q=0}^{n-k} \binom{n-k}{q} (-1)^q \frac{1}{q+1} \left( \frac{m(q+1)}{\bar{\gamma}q +mq +m} \right)^m \right] \quad (5.9)
\]

It is noted that, for \( m = 1 \) in (5.9) becomes:

\[
PEP_{t,\text{Naka}} = k \left[ 1 - \sum_{q=0}^{n-k} \binom{n-k}{q} (-1)^q \frac{1}{q+1} \left( \frac{m(q+1)}{\bar{\gamma}q +q +1} \right)^m \right] \quad (5.10)
\]

The expression (5.10) is the upper bound on total probability of PEP of OFDM-IM system over well-known Rayleigh fading channel.

By following similar steps as above, an expression for approximate PEP of OFDM-IM system over Nakagami-m fading channel can be derived and is given by:

\[
PEP_{\text{approx}}^{\text{Naka}} \approx \int_0^{\infty} \frac{m^m \gamma^{m-1}}{\bar{\gamma}^m \Gamma(m)} e^{-m\gamma} \times \frac{n-k}{2} e^{-\frac{\gamma}{2}} d\gamma \approx \frac{n-k}{2} \frac{m^m}{\Gamma(m) \bar{\gamma}^m} \int_0^{\infty} \gamma^{m-1} e^{-\gamma \left( \frac{1}{2} + \frac{m}{\bar{\gamma}} \right)} d\gamma \quad (5.11)
\]
Using Table of Integrals [86, eq. (3.381.4)], (5.11) simplified to:

\[
PEP_{Naka}^{approx} \approx \frac{n - k}{2} \frac{m^m}{\Gamma(m) \bar{\gamma}^m} \times \frac{1}{\left(\frac{1}{2} + \frac{m}{\bar{\gamma}m}\right)^m \Gamma(m)} = \frac{n - k}{2} \frac{m^m}{\bar{\gamma}^m \left(\frac{1}{2} + \frac{m}{\bar{\gamma}m}\right)^m} (5.12)
\]

After simplification, (5.12) can be written as:

\[
PEP_{Naka}^{approx} \approx \frac{(n - k)}{2} \left(\frac{2m}{\bar{\gamma} + 2m}\right)^m (5.13)
\]

By using (5.3) in (5.13), the expression for approximate PEP of OFDM-IM system over Nakagami-\(m\) channel becomes:

\[
PEP_{t,Naka}^{approx} \approx k\left[\frac{(n - k)}{2} \left(\frac{2m}{\bar{\gamma} + 2m}\right)^m \right] (5.14)
\]

### 5.4 Metric of OFDM-IM System ABEP over Nakagami-\(m\) Fading Channel

The bit error event of OFDM-IM is a result of mis-detection of active subcarrier and of that M-ary symbol. This mis-detection appears in three errored situations [67]: 1) mis-detection of the index of active subcarrier and that of the M-ary symbol. 2) mis-detection of the index of the active subcarrier and correct detection of the M-ary symbol. 3) correct detection of the index of the active subcarrier and mis-detection of the M-ary symbol.

The first error situation can be represented as the probability of mis-detection of the index of the active subcarrier \((PEP)\) weighted by the probability of mis-detection of the M-ary QAM symbol \((P_{e_M})\) and is defined as [67]:

\[
e_{IM1} = PEP \ P_{e_M} (5.15)
\]

The second error situation can be described as the probability of mis-detection of the index of the active subcarrier \((PEP)\) weighted by the probability of correct detection of M-ary QAM symbol, \((1 - P_{e_M})\), and is given by [67]:

\[
e_{IM2} = PEP \ (1 - P_{e_M}) (5.16)
\]
The combined probability mis-detection of index of active subcarrier is given by adding (5.15) and (5.16) as:

\[ e_{IM} = e_{IM1} + e_{IM2} = PEP P_{e_M} + PEP (1 - P_{e_M}) = PEP \] (5.17)

The third error situation, the probability of correct detection of index of active subcarrier, \((1 - PEP)\), weighted by the probability of mis-detection M-ary QAM symbol \((P_{e_M})\) is the overall probability of this event and is given by [67]:

\[ e_{IM3} = (1 - PEP)P_{e_M} \] (5.18)

The total bit error probability (BEP) can be obtained by adding (5.17) and (5.18) [67] as:

\[ BEP = e_{IM} + e_{IM3} = PEP + (1 - PEP)P_{e_M} \] (5.19)

where \(BEP\) is the bit error probability over AWGN channel of the MQAM modulation. Substituting (5.19) into (5.3), the overall BEP expression can be written as:

\[
BEP_t \leq \frac{k}{n} \sum_{\delta=1}^{n} [PEP + (1 - PEP)P_{e_M}]
\leq [kP_{e_M} + (1 - k) PEP + P_{e_M} - \frac{1}{k} PEP \times P_{e_M}]
\] (5.20)

The BER of coherent M-ary QAM over the AWGN channel with gray coding is given by [91]:

\[ P_{e_{M_{-ary}}} \simeq 0.2 \exp \left( \frac{-3\gamma}{2(M-1)} \right) \] (5.21)

where \(\gamma\) is the instantaneous SNR and \(M\) denotes modulation level. The exact and approximate upper bounds ABEP of OFDM-IM system can be obtained by averaging (5.20) over PDF of \(\gamma\), (3.9), and is given by: as:

\[ ABEP_{Naka} = \int_{0}^{\infty} BEP_t \times P_\gamma (\gamma) d\gamma \] (5.22)
Inserting (5.21), (5.1) and (5.2), the ABEP can be expressed as, respectively:

\[
ABEP_{Naka} = k \int_0^\infty PEP \times P_\gamma(\gamma) \, d\gamma + 1/k \int_0^\infty P_{eM} \times P_\gamma(\gamma) \, d\gamma - k \int_0^\infty PEP \times P_{eM} \\
\times P_\gamma(\gamma) \, d\gamma
\]

\[
= k \int_0^\infty \left[ 1 - \sum_{q=0}^{n-k} \binom{n-k}{q} -1^q e^{-\gamma\left(\frac{-q}{q+1}\right)} \right] \times \frac{m^m \gamma^{m-1}}{\gamma^m \Gamma(m)} e^{-\gamma} \, d\gamma + 1/k \int_0^\infty 0.2e^{\left(\frac{-3\gamma}{2(M-1)}\right)} \\
\times \frac{m^m \gamma^{m-1}}{\gamma^m \Gamma(m)} e^{-\gamma} \, d\gamma
\]

And,

\[
ABEP_{Naka,approx} = k \int_0^\infty PEP \times P_\gamma(\gamma) \, d\gamma + 1/k \int_0^\infty P_{eM} \times P_\gamma(\gamma) \, d\gamma - k \int_0^\infty PEP \times P_{eM} \\
\times P_\gamma(\gamma) \, d\gamma
\]

\[
= k \int_0^\infty \frac{(n-k)}{2} e^{-\gamma} \times \frac{m^m \gamma^{m-1}}{\gamma^m \Gamma(m)} e^{-\gamma} \, d\gamma + 1/k \int_0^\infty 0.2e^{\left(\frac{-3\gamma}{2(M-1)}\right)} \times \frac{m^m \gamma^{m-1}}{\gamma^m \Gamma(m)} e^{-\gamma} \, d\gamma
\]

\[
- k \int_0^\infty \frac{(n-k)}{2} e^{-\gamma} \times 0.2e^{\left(\frac{-3\gamma}{2(M-1)}\right)} \times \frac{m^m \gamma^{m-1}}{\gamma^m \Gamma(m)} e^{-\gamma} \, d\gamma
\]

(5.23)

And, (5.24)

The derivation of (5.23) and (5.24) are given in Appendix (A). Similarly, the exact and approximate ABEP upper bounds in closed-form can be obtained when greedy detection is used at the receiver and are given by:
\[ ABEP_{\text{Naka}}^{\text{QAM}} = k \left[ 1 - \sum_{q=0}^{N-k} \binom{N-k}{q} \frac{-1^q}{q+1} \left( \frac{m}{L\bar{\gamma} + m} \right)^m + \frac{1}{k-1} \left( \frac{m}{\alpha\bar{\gamma} + m} \right)^m + \right. \\
\left. 0.2 \sum_{q=0}^{N-k} \binom{N-k}{q} \frac{-1^q}{q+1} \left( \frac{m}{(\alpha + L)\bar{\gamma} + m} \right)^m \right] \]

And

\[ ABEP_{\text{Naka,approx}}^{\text{QAM}} = k \left[ \left( \frac{n-k}{2} \right) \left( \frac{2m}{\bar{\gamma} + 2m} \right)^m + 0.2 \left( \frac{m}{\alpha_1\bar{\gamma} + m} \right)^m - \right. \\
\left. 0.1 (n-k) \left( \frac{m}{(\alpha_1 + 0.5)\bar{\gamma} + m} \right)^m \right] \]

where \( L = \frac{q}{q+1} \) and \( \alpha_1 = \frac{3}{2(M-1)} \). \( m \) and \( M \) are the fading factor and constellation size, respectively.

### 5.5 Numerical Results and Discussion

It is essential in the system design to evaluate the BER performance as per the BER matrix of OFDM-IM for various fading channel conditions. The results demonstrate that any improvement in channel condition will enhance system performance in terms of PEP. Figure 5.2 shows the comparison between the mathematical analysis and the simulated results. For example, when \( m=1 \), the theoretical result matches the simulation, likewise for another Nakagami-m channel parameters \( m=2, 3, \) and 4. Also, it is observed that there is no continuity in simulation result at \( m=3 \) and 4 beyond certain SNR. This is because with higher \( m \) values (better channel condition) combined with higher SNR, the received signal does not encounter any error. The PEP is considered in the system design to evaluate system performance for BER. The ABEP system performance improves when K active subcarriers increase. Also, the ABEP system performance was assessed in various channel severity fadings. It is noticed that ABEP performance improves when the Nakagami-m parameter increases from \( m=1 \) to 4 as illustrated in Figure 5.3. The performance is significantly enhanced when \( m=4 \) and \( k=3 \). 4-QAM and 16-QAM with OFDM-IM system are considered in ABEP of system performance as shown in Figure 5.4.
and Figure 5.5. Both figures provide comparison between the analytical and simulation result over Nakamagi-m fading parameters \( m=1, 2, 3 \) and 4. Table 5.1 compares the BEP of OFDM-IM system for 4-QAM and 16-QAM modulation schemes as a function of \( m \) fading parameters at \( ABEP = 10^{-3} \). The results shows the improvement of the system performance in terms of ABEP when \( m \) increases. Increment of modulation size requires higher SNR to achieve the same average BEP at different fading parameters \( m \). Therefore, ABEP system performance at \( 10^{-3} \) for 16-QAM requires more transmitted power almost 2 dB compared for 4-QAM at same fading condition.

The ABEP of OFDM-IM is simulated for greedy detection over the Nakagami-m fading channel using MATLAB. The channel gain \( (h_{Naka}) \), which characterizes Nakagami-m fading, is given by [5]:

\[
h_{Naka} = \sqrt{\text{gamrand}(m, 1/m, 1)}
\]

(5.27)

where \( h_{Naka} \) denotes the pdf of Nakagami-m for the fading \( (m) \) factor. For the AWGN channel, the channel parameter \( (m) \) is set to infinity to get channel gain \( (h_{Naka}=1) \), and for the Rayleigh channel, the channel factor is set to \( (m=1) \). The look-up table (LUT) and simulation parameters are shown in Table 5.1 and 5.2, respectively. LUT is used to map each sub-block \( (k) \) to the sub-block \( (n) \). As outlined in Table 5.2, simulations are carried out on the OFDM-IM with FFT size 128 subcarriers, guard-interval of 16 subcarriers and with SNR from 0 to 30 dB. The simulation scheme, and the associated parameters are formed as follows:

1) The generated serial data stream \( (D_t) \) is divided into \( g = \frac{N}{n} \) groups, where each have \( p \) bits, which is split into two sub-streams \( p_1 \) and \( p_2 \), depending on \( n \) and \( k \) as explained in section 5.3.

2) Every sub-block is mapped using a look-up table (LUT). Using \( p_1 = \lceil \log_2 C(n, k) \rceil \). For example, the LUT for \( n = 4, k = 2 \) and \( p_1 = 2 \) is shown in Table 5.1.

3) \( P_1 \) bits are employed to indicate the active subcarrier and the \( p_2 \) bits are mapped to the active subcarrier to form one BPSK or M-ary QAM sub-block.
4) The sub-blocks are applied to inverse fast Fourier transform (IFFT) process block, and then a cyclic prefix is added. Finally, the modulated data are transmitted over the Nakagami-m fading channel.

5) On the receiver side, two estimation steps are used to apply the greedy detection. In the first step, the received subcarriers in every sub-block that have the greatest power are estimated to be 1, i.e., active. The remaining bits, which have low received power, are estimated to be 0, i.e., inactive. \( \hat{p}_1 \) bits are retrieved by known LUT at the receiver that is employed at the transmitter.

6) At the second step, we estimate \( \hat{s} \) by applying ML decision to each active subcarrier.

7) \( \hat{p}_1 \) and \( \hat{p}_2 \) are combined to form \( \hat{p} \). Then, the original data stream \( \hat{D} \) is reshaped from parallel to serial.

<table>
<thead>
<tr>
<th>Table 5.1: LUT for ( n = 4 ) and ( k = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_1 ) bits</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>00</td>
</tr>
<tr>
<td>01</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>11</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 5.2: Simulation Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
</tr>
<tr>
<td>FFT Size</td>
</tr>
<tr>
<td>Guard interval</td>
</tr>
<tr>
<td>Length of sub-block (( n ))</td>
</tr>
<tr>
<td>Number of active subcarrier</td>
</tr>
<tr>
<td>Fading factors</td>
</tr>
<tr>
<td>Number of symbols</td>
</tr>
<tr>
<td>Constellation size</td>
</tr>
<tr>
<td>SNR</td>
</tr>
<tr>
<td>Iterations</td>
</tr>
</tbody>
</table>
Table 5.3: Comparison of BEP of OFDM-IM system for 4-QAM and 16-QAM as a function of SNR over Nakagami fading channel

<table>
<thead>
<tr>
<th>ABEP</th>
<th>Fading parameters</th>
<th>4-QAM</th>
<th>16-QAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-3}$</td>
<td>m=1</td>
<td>32</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>m=2</td>
<td>18</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>m=3</td>
<td>15</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>m=4</td>
<td>13</td>
<td>15</td>
</tr>
</tbody>
</table>

Figure 5.2: Average PEP of OFDM-IM with greedy detection over Nakagami-m fading channel as function of SNR and $m$

5.6 Chapter Summary

In this Chapter, the performance of OFDM-IM system is analyzed and assessed over Nakagami-m fading channel for different detection scenarios between index subcarriers ($k$) and QAM modulation level ($M$). The derived closed-form expressions of PEP and ABEP have been validated through simulations. The results show that increasing modulation levels and number of active subcarriers decreases energy efficiency while enhancing the spectral efficiency.
Figure 5.3: ABEP for OFDM-IM with 4-QAM as a function of SNR over different number of active subcarrier and fading parameters.

Figure 5.4: ABEP for OFDM-IM with 4-QAM as a function of SNR over various fading parameters and active subcarrier $k=2$. 
Figure 5.5: ABEP for OFDM-IM with 16-QAM as a function of SNR over various fading parameters.
Chapter 6
Performance analysis of OFDM-IM over Faded Shadowing Channel with co-channel interference

6.1 Introduction

A generalized model is presented to study most of the effect (shadowing and fading) in a OFDM-IM wireless system channel environment using the composite Nakagami-m and Gamma (NG) model. The NG channel model is studied because it is able to describe a large number of shadowing and fading channel conditions from moderate to severe scenarios in the system. The Meijer G function is used to simplify mathematical complexity in the encountered previous works. Also, the effect of proposed NG model in OFDM-IM system is investigated in presence and absence of co-channel interference (CCI). Moreover, the performance of OFDM-IM system is evaluated for BPSK and M-ary QAM in the absence of CCI over the shadowing and fading channels using the composite model. Then, the performance is evaluated in the presence of CCI for various number of channel interferences using this model. The numerical closed-form expressions have been derived for PEP, and the average bit error probability (ABEP) considering M-ary QAM in the absence and presence of CCI.

6.2 Pairwise Error Probability (PEP) over Fading and Shadowing Channel

In wireless communication, signal power usually varies from region to region within a given environment between the transmitter and receiver. This kind of behavior is determined in an urban propagation environment and therefore contributes shadowing due to buildings, terrain and other obstacles in propagation area. When a wireless systems are used in such an environment, it is necessary to take into account both fading and shadowing effects over wireless channel for computing average bit error rate of the system. A composite Nakagima-m and Gamma (NG) distribution is utilized to express both fading and shadowing effects over the propagation area. The parameters of NG model can be set to adapt a large value of fading and shadowing channels. An Alternate approach to assess PEP and ABEP using Meijer G function for OFDM-IM over the shadowed fading channel is given by:

$$PEP_{Exact} = \int_{0}^{\infty} PEP \, P(\gamma) \, d\gamma$$  \hspace{1cm} (6.1)

where \(PEP\) is given by (5.1) and (5.2) needs to be averaged employing \(P(\gamma)\) given by (3.5). Thus, PEP can be written as:

$$PEP_{Exact} = \int_{0}^{\infty} PEP \, f(\gamma) \, d\gamma = \int_{0}^{\infty} \left[ 1 - \sum_{q=0}^{n-k} \binom{n-k}{q} \frac{-1}{q+1} e^{\frac{-q}{q+1}} \right] \times \frac{2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right) \frac{c+m}{\gamma} \frac{c+m-2}{2} K_{c-m} \left( 2 \sqrt{\frac{cm}{\gamma}} \right) \, d\gamma$$ \hspace{1cm} (6.2)

Expressing the modified Bessel function in (6.2) using the Meijer G-function as \(K_{c-m} \left( 2 \sqrt{\frac{cm}{\gamma}} \right) = \frac{1}{2} G_{0,2}^{1,0} \left( \frac{cm}{\gamma} \left| \begin{array}{cc} 0 \\ \frac{c-m}{2}, \frac{m-c}{2} \end{array} \right. \right) \) \hspace{1cm} [96, (8.4.23.1)], the (6.2) can be expressed as:
\[
\begin{align*}
\text{PEP}_{\text{Exact}} &= \int_0^\infty \frac{2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\bar{\gamma}} \right)^{c+m} \gamma^{c+m-2} K_{c-m} \left( 2\sqrt{\frac{cm}{\bar{\gamma}}} \right) d\gamma - \frac{1}{2} \frac{2}{\Gamma(c) \Gamma(m)} \\
&\quad \times \sum_{q=0}^{n-k} \left( \frac{n-k}{q} \right) \frac{-1}{q+1} \int_0^\infty \gamma^{c+m-2} e^{\frac{-q\gamma}{q+1}} \times G_{2,0}^{2,0} \left( \frac{cm\gamma}{\bar{\gamma}} \bigg| \frac{0}{c-m, m-c} \right) d\gamma \\
&= 1 - \frac{1}{\beta} \sum_{q=0}^{n-k} \left( \frac{n-k}{q} \right) \frac{-1}{q+1} L^{-\nu} G_{1,2}^{2,1} \left( \frac{\beta}{L} \bigg| 1 - \frac{\nu}{c-m, m-c} \right) (6.3)
\end{align*}
\]

The first part of (6.3) is equal to one when composite fading and shadowing is integrated from zero to infinity and with the aid of the Table of Integrals [86, (7.813.1)], the second part of (6.3) can be integrated as:

\[
\begin{align*}
\text{PEP}_{\text{Exact}} &= 1 - \frac{1}{\beta} \sum_{q=0}^{n-k} \left( \frac{n-k}{q} \right) \frac{-1}{q+1} L^{-\nu} G_{1,2}^{2,1} \left( \frac{\beta}{L} \bigg| 1 - \frac{\nu}{c-m, m-c} \right) (6.4)
\end{align*}
\]

The detail of exact average closed-form for PEP is given in Appendix A.3, where \( \beta = \frac{cm}{\bar{\gamma}} \), \( \nu = \frac{c+m-1}{2} \), \( L = \frac{q}{q+1} \) and \( G(.) \) indicates the Meijer’s function.

Substituting (6.4) into (5.3), the total probability of overall PEP over the shadowed and fading channel is formulated as:

\[
\begin{align*}
\text{PEP}_{t,\text{Exact}} &\leq k \left[ 1 - \frac{1}{\beta} \sum_{q=0}^{n-k} \left( \frac{n-k}{q} \right) \frac{-1}{q+1} L^{-\nu} G_{1,2}^{2,1} \left( \frac{\beta}{L} \bigg| 1 - \frac{\nu}{c-m, m-c} \right) \right] (6.5)
\end{align*}
\]

By following this approach to express the modified Bessel function in terms of the Meijer G-function as \( K_{c-m} \left( 2\sqrt{\frac{cm\gamma}{\bar{\gamma}}} \right) = \frac{1}{2} G_{0,2}^{2,0} \left( \frac{cm\gamma}{\bar{\gamma}} \bigg| \frac{0}{c-m, m-c} \right) \) [96, (8.423/1)], the approximate PEP for OFDM-IM over the shadowed and fading channel is obtained by averaging (5.2) over (3.5) as:

\[
\begin{align*}
\text{PEP}_{\text{approx}} &\approx \frac{n-k}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\bar{\gamma}} \right)^{c+m} \gamma^{c+m-2} \int_0^\infty \gamma^{c+m-1} e^{-\frac{1}{2} \gamma} G_{0,2}^{2,0} \left( \frac{cm\gamma}{\bar{\gamma}} \bigg| \frac{0}{c-m, m-c} \right) d\gamma (6.6)
\end{align*}
\]
By applying the Table of Integrals [86, (7.813.1)] in (6.6), the approximate closed-form expression for PEP can be obtained as:

\[ PEP_{approx} \simeq \frac{n - k}{\Gamma(c) \Gamma(m)} (2\beta)^\nu G_{1,2}^{2,1} \left( \begin{array}{c} 2 \beta \\ 1 - \frac{(c+m)}{2} \end{array}, \frac{c-m}{2}, \frac{m-c}{2} \right) \]  

(6.7)

The approximation of overall PEP is determined by employing (6.7) into (5.3) as:

\[ PEP_{t,approx} \simeq k \left[ \frac{n - k}{2\Gamma(c) \Gamma(m)} (2\beta)^\nu G_{1,2}^{2,1} \left( \begin{array}{c} 2 \beta \\ 1 - \frac{(c+m)}{2} \end{array}, \frac{c-m}{2}, \frac{m-c}{2} \right) \right] \]  

(6.8)

The detail of approximate average closed-form for PEP is given in Appendix A.3

### 6.3 Average Bit Error Probability (ABEP) over Fading and Shadowing Channel

ABEP is a result of mis-detection in the active subcarrier and M-ary symbol. The BER of coherent M-ary QAM and BPSK over the AWGN channel are given by [82] and [91], respectively as:

\[ P_e_M \simeq 0.2 \exp \left( \frac{-3\gamma}{2(M-1)} \right) \]  

(6.9)

\[ P_{e,PBSK} = Q\left( \sqrt{2\gamma} \right) = \frac{1}{2} \text{erfc}(\sqrt{\gamma}) \]  

(6.10)

where \( \gamma \) indicates instantaneous SNR and \( M \) denotes modulation level. The average exact and approximate upper bound of BEP (ABEP) for OFDM-IM over the shadowed
fading channel can be obtained by averaging (5.20) over (3.9) as:

\[
ABEP = \int_0^\infty k \left[ PEP + P_{eM} - PEP \times P_{eM} \right] \times \frac{2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\bar{\gamma}} \right)^{\frac{c+m}{2}} \bar{\gamma}^{-\frac{c+m-2}{2}}
\]

\[
K_{c-m} \left( 2 \sqrt{\frac{cm}{\bar{\gamma}}} \right) d\gamma
\]

\[
ABEP = k \frac{2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\bar{\gamma}} \right)^{\frac{c+m}{2}} \int_0^\infty PEP \times \frac{\gamma^{\frac{c+m-2}{2}}}{K_{c-m} \left( 2 \sqrt{\frac{cm}{\bar{\gamma}}} \right)} d\gamma (6.11)
\]

\[
+ \int_0^\infty P_{eM} \times \gamma^{\frac{c+m-2}{2}} K_{c-m} \left( 2 \sqrt{\frac{cm}{\bar{\gamma}}} \right) d\gamma - \int_0^\infty PEP \times P_{eM} \times \gamma^{\frac{c+m-2}{2}}
\]

\[
K_{c-m} \left( 2 \sqrt{\frac{cm}{\bar{\gamma}}} \right) d\gamma
\]

inserting equations (6.9) and (5.1) or (5.2) into (6.11), and expressing the modified Bessel function in terms of the Meijer G-function as \( K_{c-m} \left( 2 \sqrt{\frac{cm}{\bar{\gamma}}} \right) = \frac{1}{2} G_{2,0}^{2,0} \left( \frac{cm \bar{\gamma}}{\gamma} \left| \begin{array}{c} 0 \\ c-m, m-c \end{array} \right. \right) \) [96, (8.423/1)], the (6.11) can be expressed as:

\[
ABEP = k \frac{2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\bar{\gamma}} \right)^{\frac{c+m}{2}} \left[ \frac{1}{2} \int_0^\infty \gamma^{\frac{c+m-2}{2}} \times PEP G_{2,0}^{2,0} \left( \frac{cm \bar{\gamma}}{\gamma} \left| \begin{array}{c} 0 \\ c-m, m-c \end{array} \right. \right) d\gamma
\]

\[
+ \frac{0.2}{2} \int_0^\infty \gamma^{\frac{c+m-2}{2}} \times \exp \left( \frac{-3\gamma}{2(M-1)} \right) G_{0,2}^{2,0} \left( \frac{cm \bar{\gamma}}{\gamma} \left| \begin{array}{c} 0 \\ c-m, m-c \end{array} \right. \right) d\gamma - \frac{0.2}{2} \int_0^\infty \gamma^{\frac{c+m-2}{2}}
\]

\[
	imes PEP \times \exp \left( \frac{-3\gamma}{2(M-1)} \right) \times G_{0,2}^{2,0} \left( \frac{cm \bar{\gamma}}{\gamma} \left| \begin{array}{c} 0 \\ c-m, m-c \end{array} \right. \right) d\gamma
\]

(6.12)

Applying the Table of Integrals [86, (7.813.1)] in (6.12), the exact and approximate upper bound of closed-form expressions of the M-ary QAM-OFDM-IM scheme for the
The approximate upper bound of BPSK-OFDM-IM scheme can be obtained by averaging (3.9) over (5.20), where \( P_e^M \) is replaced by \( P_{e,\text{PBSK}} \) using (6.10) for PBSK as:

\[
ABEP = \int_0^\infty k \left[ PEP + P_{e,\text{PBSK}} - PEP \times P_{e,\text{PBSK}} \right] \\
\times \frac{2}{\Gamma(c)\Gamma(m)} \left( \frac{cm}{\gamma} \right)^{\frac{c+m}{2}} \gamma^{\frac{c-m-2}{2}} K_{c-m} \left( 2\sqrt{\frac{cm}{\gamma}} \right) \, d\gamma
\]

(6.15)

Substituting (5.2) and applying an approximation of the Chernoff bound, \( Q(x) \leq \frac{1}{2} e^{-\frac{x^2}{2}} \) into (6.10) as \( P_e^M \leq \frac{1}{2} e^{-\gamma} \) on (6.15) [93]. The approximate ABEP of BPSK-OFDM-IM is expressed as:

\[
ABEP^QAM = k - \frac{k}{\Gamma(c)\Gamma(m)} \sum_{q=0}^{n-k} \binom{n-k}{q} \sum_{q=0}^{n-k} \binom{n-k}{q} q + 1 \binom{G_{1,2}^2}{1 - c, 1 - m} + 0.2 \binom{G_{1,2}^2}{1 - c, 1 - m}
\]

(6.13)

\[
ABEP^QAM_{\text{approx}} \approx \frac{n-k}{2\Gamma(c)\Gamma(m)} (\beta)^{c+m} \left[ 2^{\frac{c+m}{2}} - P_{e,\text{PBSK}} - PEP \times P_{e,\text{PBSK}} \right]
\]

(6.14)

where \( \beta = \frac{cm}{\gamma} \), \( L = \frac{q+1}{2(M-1)} \), and \( c, m \) and \( M \) are the shadowing factor, the fading factor and constellation size, respectively. The detail of derivation is explained in Appendix (A.3.1) and (A.3.2).
ABEP = \frac{k}{2} \int_0^\infty \frac{2^{c+m-2}}{\Gamma(c) \Gamma(m)} \left( \frac{c m}{\gamma} \right)^{\frac{c+m}{2}} e^{-\frac{\gamma}{2}} \times \gamma^{\frac{c+m-2}{2}} K_{c-m} \left( 2 \sqrt{\frac{c m}{\gamma}} \right) d\gamma

+ \int_0^\infty \frac{1}{2} e^{-\gamma} \times \gamma^{\frac{c+m-2}{2}} K_{c-m} \left( 2 \sqrt{\frac{c m}{\gamma}} \right) d\gamma - \int_0^\infty \frac{n-k}{2} e^{-\frac{\gamma}{2}} \times \frac{1}{2} e^{-\gamma} \times \gamma^{\frac{c+m-2}{2}} K_{c-m} \left( 2 \sqrt{\frac{c m}{\gamma}} \right) d\gamma

(6.16)

Applying the Table of Integrals [86, (7.813.1)] in (6.16) and compensating the modified Bessel function in terms of the Meijer G-function as $K_{c-m} \left( 2 \sqrt{\frac{c m}{\gamma}} \right) = \frac{1}{2}$

$G_{0,2}^2 \left( \frac{c m}{\gamma}, 0 \right) \left| \begin{array}{c} c-m, m-c \\ \frac{c m}{\gamma}, c \end{array} \right|$ [96, (8.4.23.1)], the approximate upper bound of closed-form expressions of the BPSK-OFDM-IM scheme for the greedy detection is expressed as:

$\begin{align*}
ABEP_{BPSK}^{approx} &= \frac{k}{2} \int_0^\infty \frac{2^{c+m-2}}{\Gamma(c) \Gamma(m)} \left( \frac{c m}{\gamma} \right)^{\frac{c+m}{2}} e^{-\frac{\gamma}{2}} \times \gamma^{\frac{c+m-2}{2}} G_{2,1}^{1,2} \left( \frac{1}{2\beta} \right| 1-c, 1-m, 0 \right) \\
&- \frac{(n-k)}{2} \int_0^\infty \frac{1}{2} e^{-\gamma} \times \gamma^{\frac{c+m-2}{2}} G_{2,1}^{1,2} \left( \frac{2}{3\beta} \right| 1-c, 1-m, 0 \right)
\end{align*}$

(6.17)

### 6.4 Probability Density Function of Signal-to-Interference Ratio (SIR) over Fading and Shadowing Channel

We study wireless mobile systems where the desired and co-channel interfered signals are subject to shadowing (gamma distribution) and fading (Nakagami-m distribution). We assume that the desired signal and the interfered co-channel signal are mutually and independently faded and shadowed with an equal average interfering power at the same distance from the receiver, which is true in the case of a multi-antenna or a cluster interfere. The signal to interference ratio (SIR) is described over the composite NG model as $\gamma \equiv \frac{\gamma_D}{\gamma_I}$, where $\gamma_D$ is the instantaneous SNR of the desired signal and $\gamma_I$ is the
instantaneous SNR of the interfering signal. From (3.9), the PDFs of the desired signal \( \gamma_D \) and the interfering signal are presented by:

\[
f_D(\gamma_D) = \frac{2}{\Gamma(c_D) \Gamma(m_D)} \left( \frac{c_D m_D}{\gamma_D} \right)^{c_D + m_D / 2} \gamma_D^{c_D + m_D - 2} \frac{K_{c_D - m_D}}{2} \left( 2 \sqrt{\frac{c_D m_D \gamma_D}{\gamma_D}} \right)
\]  

(6.18)

\[
f_I(\gamma_I) = \frac{2}{\Gamma(c_I) \Gamma(m_I)} \left( \frac{c_I m_I}{\gamma_I} \right)^{c_I + m_I / 2} \gamma_I^{c_I + m_I - 2} \frac{K_{c_I - m_I}}{2} \left( 2 \sqrt{\frac{c_I m_I \gamma_I}{\gamma_I}} \right)
\]  

(6.19)

where \( \bar{\gamma}_D \) is the average SNR of the desired signal, \( c_D \) and \( m_D \) are the shadowing and the fading parameters of the desired signal, respectively. And \( \bar{\gamma}_I \) is the average SNR of the interfered signal, \( c_I \), and \( m_I \) are the shadowing and the fading parameter of the interfered signal, respectively. The PDF of signal to interference ratio (SIR) in the compositied shadowing and fading model can be accomplished [97] as:

\[
f_{D/I}(\gamma) = \int_0^\infty f_I(\gamma_I) f_D(\gamma | \gamma_I) \gamma_I d\gamma_I
\]  

(6.20)

where \( f_I(\gamma_I) \) denotes the PDF of interfering signal, and \( f_D(\gamma | \gamma_I) \) indicates the PDF of desired signal, which is derived by substituting \( \gamma_D \) from \( \gamma = \bar{\gamma}_D \). Substituting (6.18) and (6.19) into (6.20), the PDF of SIR will be given as:

\[
f_{D/I}(\gamma) = A \gamma^{c_D + m_D - 2} \int_0^\infty \gamma_I^{c_I + m_I + c_D + m_D - 1} K_{c_D - m_D} \left( 2 \sqrt{\frac{c_I m_I \gamma_I}{\gamma_I}} \right) K_{c_I - m_I} \left( 2 \sqrt{\frac{c_D m_D \gamma_D}{\gamma_D}} \right) d\gamma_I
\]  

(6.21)

Where \( A = \frac{4(c_D m_D)^{c_D + m_D} (c_I m_I)^{c_I + m_I}}{\Gamma(c_D) \Gamma(m_D) \Gamma(c_I) \Gamma(m_I)} \). With the help of [98, (07.34.21.0011.01)] and after some simplification, the closed-form of PDF of SIR in (6.21) can be expressed as:
\[ f_{D/I}(\gamma) = \frac{c_D+m_D}{\beta_D^2} \frac{\beta_I}{\gamma^2} \frac{\Gamma(c_D)\Gamma(m_D)\Gamma(c_I)\Gamma(m_I)}{\Gamma(c_D+c_I+m_I)} \gamma^{c_D+m_D-2} \]

\[ G_{2,2}^{2,2} \left( \frac{\beta_D^2}{\beta_I} \left| \begin{array}{c} 1-c_D - \frac{c_D+m_D}{2}, 1-m_D - \frac{c_D+m_D}{2} \\ c_D^2, m_D^2 \end{array} \right. \right) \] (6.22)

where \( \beta_D = \frac{c_D m_D}{\gamma_D} \), \( \beta_I = \frac{c_I m_I}{\gamma_I} \). The detail of derivation is shown in Appendix (A.3.3).

### 6.5 Average Bit Error Probability Subject to Interference

The average bit error probability (ABEP) of M-ary QAM and PBSK modulations can be calculated by inserting (6.9), (6.10) and (5.2) into (5.20) and averaging them over (6.22) as:

\[ ABEP = \int_0^\infty BEP_t(\gamma) f_{D/I}(\gamma) d\gamma \] (6.23)

changing the exponential in (6.9) into the Meijer G-function using [96, (8.4.3.1)] and with the help of [98, eq. (07.34.21.0011.01)], the approximate upper bound of ABEP for BPSK and M-ary QAM modulations are given as, respectively:

\[ ABEP_{D/I, \text{approx}}^{BPSK} = \frac{k}{2\Gamma(c_D)\Gamma(m_D)\Gamma(c_I)\Gamma(m_I)} \left( \begin{array}{c} k \\ c_I \end{array} \right) \left( \begin{array}{c} c_D^2, m_D^2 \end{array} \right) \left( \begin{array}{c} c_I, m_I \end{array} \right) \]

\[ G_{2,3}^{3,2} \left( \frac{c_I m_I}{c_D^2 c_D^2} \rho \left| \begin{array}{c} 1-c_D, 1-m_D \\ 0, c_I, m_I \end{array} \right. \right) 
+ G_{2,3}^{3,2} \left( \frac{c_I m_I}{c_D^2 c_D^2} \rho \left| \begin{array}{c} 1-c_D, 1-m_D \\ 0, c_I, m_I \end{array} \right. \right) \]

\[ - \frac{(n-k)}{2} G_{2,3}^{3,2} \left( \frac{2c_I m_I}{c_D^2 c_D^2} \rho \left| \begin{array}{c} 1-c_D, 1-m_D \\ 0, c_I, m_I \end{array} \right. \right) \] (6.24)
\[ ABEP^{QAM}_{D/I, \text{approx}} = \frac{k}{\Gamma(c_D)\Gamma(m_D)\Gamma(c_I)\Gamma(m_I)} \left[ \frac{(n-k)}{2} \right] \]

\[ G_{2,3}^{3.2} \left( \frac{c_I m_I}{2c_D c_D \rho} \begin{vmatrix} 1 - c_D, 1 - m_D \\ 0, c_I, m_I \end{vmatrix} \right) \]

\[ + 0.2 G_{2,3}^{3.2} \left( \frac{\alpha c_I m_I}{c_D c_D \rho} \begin{vmatrix} 1 - c_D, 1 - m_D \\ 0, c_I, m_I \end{vmatrix} \right) \]

\[ - 0.1 (n - k) G_{2,3}^{3.2} \left( \frac{(\alpha + 0.5) c_I m_I}{c_D c_D \rho} \begin{vmatrix} 1 - c_D, 1 - m_D \\ 0, c_I, m_I \end{vmatrix} \right) \]

(6.25)

The detail of derivation is explained in Appendix (A.3.4).

### 6.6 Average Bit Error Probability Subject to Multiple Interferes

The desired signal are subject to multiple interfering signals when a multi-antenna interfere with closely spaced antennas with \( m_i = m_I, i = 1, 2, ..., N \) and \( f_I(\gamma_I) \) is given by [99] as:

\[ f_{N_I}(\gamma_{N_I}) = \frac{2}{\Gamma(c_I)\Gamma(N_I m_I)} \left( \frac{c_I m_I}{\bar{\gamma}_I} \right)^{c_I + N_I m_I - 2} \frac{\gamma_I}{\bar{\gamma}_I} K_{c_I - N_I m_I} \left( 2\sqrt{\frac{c_I m_I \gamma_I}{\bar{\gamma}_I}} \right) \]

(6.26)

where \( N_I \) is the sum of Gamma distributed of random variables (RVs) fading parameter \( m_I \). The multiple interference PDF ( \( f_{D/I}(\gamma) \)) in composited shadowing and fading model can be obtained by substituting (6.18) and (6.26) into (6.20) as:

\[ f_{D/N_I}(\gamma) = A \gamma^{c_D + m_D - 2} \int_0^{\infty} \gamma_D^{c_I + N_I m_I + c_D + m_D - 1} K_{c_D - m_D} \left( 2\sqrt{\frac{c_D m_D \gamma_I}{\bar{\gamma}_D}} \right) K_{c_I - N_I m_I} \left( 2\sqrt{\frac{c_I m_I \gamma_I}{\bar{\gamma}_I}} \right) d\gamma_I \]

(6.27)

Where \( A = \frac{4 (c_D m_D)^{c_D + m_D}}{\Gamma(c_D)\Gamma(m_D)\Gamma(c_I)\Gamma(N_I m_I)} \). The detail of integration is shown in Appendix (A.3.5) and with the help of [98, (07.34.21.0011.01)] and after some simplification,
the closed-form of PDF of multiple SIR in (6.21) can be expressed as:

\[
\begin{align*}
    f_{D/N_I}(\gamma) &= \frac{C_{D+mD}}{\beta_D} - \frac{(C_{D+mD})^{\gamma}}{\Gamma(c_D)\Gamma(m_D)\Gamma(c_I)\Gamma(N_I m_I)\gamma^{c_D+mD-2}} \\
    &= G_{2,2}^2 \left( \frac{\beta_D \gamma}{\beta_I} \right) \left( 1 - c_I - \frac{(c_D+mD)}{2}, 1 - N_I m_I - \frac{(c_D+mD)}{2} \right)
\end{align*}
\]

(6.28)

where \( \beta_D = \frac{c_D m_D}{\gamma_D} \quad \beta_I = \frac{c_I m_I}{\gamma_I} \).

The average bit error probability (ABEP) subject to multiple interferes of M-ary QAM and PBSK modulations can be calculated by inserting (6.9), (6.10) and (5.2) into (5.20) and averaging them over (6.29) as:

\[
ABEP_{N_I} = \int_0^\infty BEP_t(\gamma) f_{D/N_I}(\gamma) d\gamma
\]

(6.29)

changing the exponential in (6.9) into the Meijer G-function using [96, (8.4.3.1)] and with the help of [98, eq. (07.34.21.0011.01)], the approximate upper bound of \( ABEP_{N_I} \) for BPSK and M-ary QAM modulations are given as, respectively

\[
ABEP_{D/N_I,approx}^{BPSK} = \frac{k}{2 \Gamma(c_D)\Gamma(m_D)\Gamma(c_I)\Gamma(N_I m_I)} \left[ (n - k) \right]
\]

(6.30)
\[
\text{ABEP}^{QAM}_{D/N_{I,\text{appox}}(k)} = \frac{k}{\Gamma(c_D)\Gamma(m_D)\Gamma(c_I)\Gamma(N_{I,m_I})} \left[ \frac{(n-k)}{2} \right]
\]

\[
G_{2,3}^{3.2} \left( \frac{c_I m_I}{c_D c_D} \rho \left[ \begin{array}{c} 1 - c_D, 1 - m_D \\ 0, c_I, N_{I,m_I} \end{array} \right] \right) + 0.2 G_{2,3}^{3.2} \left( \frac{\alpha c_I m_I}{c_D c_D} \rho \left[ \begin{array}{c} 1 - c_D, 1 - m_D \\ 0, c_I, N_{I,m_I} \end{array} \right] \right)
\]

\[
-0.1 (n-k) G_{2,3}^{3.2} \left( \frac{(\alpha+0.5) c_I m_I}{c_D c_D} \rho \left[ \begin{array}{c} 1 - c_D, 1 - m_D \\ 0, c_I, N_{I,m_I} \end{array} \right] \right)
\]

The detail of derivation is explained in Appendix (A.3.6).

### 6.7 Numerical Results and Discussion

To attain a better understanding of how fading, shadowing parameters and co-channel interference impact the average bit error probability, some plots are given in Figures 6.1 to 6.8. Figure 6.1 of the pairwise error probability. This validates the results presented in [75] and [5]. Moreover, the composite fading and shadowing channel approximately matches Rayleigh and Nakagami-m \((m=1)\) fading channels at \(c=2.2\) and \(m=1\). Figure 6.2 and Figure 6.3 illustrate average bit error probability (ABEP) of BPSK and 4-QAM-OFDM modulation scheme over the compound shadowing and fading channel in the absence of co-channel interference (CCI), respectively. From Figure 6.2 and Figure 6.3, it is observed that as shadowing and fading parameters become less severe, the ABEP get better. Figure 6.4 and Figure 6.5 show the effect of shadowing and fading parameters in the presence of co-channel interference in ABEP BPSK and 4-QAM, respectively. From Figure 6.5, it is observed that for \(m = 2\), \(c= 1\), and for \(m = 1\), \(c= 2.5\), the ABEP performance are very similar to each other. This similar result comes from the combined effect of two interfering parameters where in the first case the fading parameter is higher than the shadowing parameter while in the second case the shadowing parameter is higher than the fading parameter. Figure 6.6 and Figure 6.7 depict ABEP for 4-QAM-OFDM-IM in the presence of CCI with various number of active subcarriers as a function of SNR, shadowing and fading parameters. We can see from the figures, that ABEP decreases when the number of active subcarriers \((k)\) decreases. Figure 6.8 compares the effect of various shadowing \((c=1, 1.5, 2, \text{and } 2.5)\), fading parameters \((m=1, \text{and } 2)\), and
Table 6.1: Comparison of ABEP of OFDM-IM system for 4-QAM as a function of SNR over shadowing and fading channel with co-channel interference

<table>
<thead>
<tr>
<th></th>
<th>$c_D = c_I = 1$</th>
<th>$c_D = c_I = 1.5$</th>
<th>$c_D = c_I = 2$</th>
<th>$c_D = c_I = 2.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-3}$</td>
<td>$m = 1$</td>
<td>$m = 1$</td>
<td>$m = 1$</td>
<td>$m = 1$</td>
</tr>
<tr>
<td>$k = 1$</td>
<td>37.5dB</td>
<td>32dB</td>
<td>26dB</td>
<td>23.75dB</td>
</tr>
<tr>
<td>$k = 2$</td>
<td>39.25dB</td>
<td>33.65dB</td>
<td>27.75dB</td>
<td>25.15dB</td>
</tr>
<tr>
<td>$k = 3$</td>
<td>39.5dB</td>
<td>33.75dB</td>
<td>27.9dB</td>
<td>23.9dB</td>
</tr>
</tbody>
</table>

active subcarriers ($k=1, 2, \text{ and } 3$). Figure 6.8 and Table (6.1) present the effect of $m, c$ parameters and active number of subcarriers at $ABEP = 10^{-3}$ in the presence of CCI for 4-QAM-OFDM-IM modulation schemes. We can see clearly that energy efficiency gets better when fading and shadowing parameters become less severe. For example, there is a decrease of 5dB in energy use when fading parameter gets from $m = 1$ to $m = 2$ at a shadowing parameter of $c = 1$. Also, about 2dB difference in energy consumption is observed when the number of active subcarriers ($k$) increases from $k = 1$ to $k = 2$. This difference is about 0.12dB as number of active subcarriers increases form $k = 1$ to $k = 2$.

Figure 6.1: Comparison of average PEP of OFDM-IM with greedy detection over the different fading channel as the function of SNR, fading factor ($m$) and shadowing factor ($c$).
Figure 6.2: ABEP for OFDM-IM-BPSK as a function of SNR over various shadowing and fading parameters.

Figure 6.3: ABEP for OFDM-IM with 4-QAM as a function of SNR over various shadowing and fading parameters

6.8 Chapter Summary

In this Chapter, three error probabilities of index subcarriers, BPSK and the M-ary QAM schemes for OFDM-IM were studied over compound shadowing and fading model. The
Figure 6.4: ABEP for OFDM-IM-BPSK with co-channel interference as a function of SNR over various shadowing and fading parameters.

Figure 6.5: ABEP for 4-QAM-OFDM-IM with co-channel interference as a function of SNR over various shadowing and fading parameters.

closed-form expressions of pairwise error probability (PEP) and average bit error probability (ABEP) have been derived in the absence and presence of co-channel interference (CCI). The results highlight significant effects on the fading severity and shadowing scat-
Figure 6.6: ABEP for 4-QAM-OFDM-IM with co-channel interference as a function of SNR over shadowing \(c = 1\) and fading \(m = 1\) and different number of active subcarrier.

Figure 6.7: ABEP for 4-QAM-OFDM-IM with co-channel interference as a function of SNR over shadowing \(c = 2\) and fading \(m = 1\) and different number of active subcarrier.

ter on the ABEP performance. The results have been validated by matching the previous works of Nakagami-m and Rayleigh fading at \(c = 2.2\) and \(m = 1\) fading parameters.
Figure 6.8: ABEP for 4-QAM-OFDM-IM with co-channel interference as a function of SNR over various shadowing and fading parameters and different number of active subcarrier.
Chapter 7
Adaptive OFDM-IM System over Faded shadowing channel\textsuperscript{1,2}

7.1 Introduction

Spectral efficiency (SE) and energy efficiency (EE) play major roles in evaluating the quality of service (QoS) of a wireless communication system. Designing an efficient wireless communication system requires trade-off between these two parameters. Orthogonal frequency division multiplexing technique with Index Modulation (OFDM-IM) has been introduced in the literature to increase the SE compared to traditional OFDM and several techniques have been introduced to improve the performance of an OFDM-IM based wireless communication system over shadowing and fading channel conditions. One of these techniques is adaptive modulation, which is employed to enhance spectral efficiency (SE) over wireless channels based on fading and shadowing. In this chapter, We are proposing adaptive modulation for an OFDM-IM based system to maximize the energy efficiency (EE) along with the EE by adapting different MQAM constellation sizes and active subcarriers of OFDM-IM system. It has been demonstrated that by adaptively varying the number of active subcarriers and modulation level in M-QAM scheme, maximum EE can


also be achieved for an acceptable ABEP. Here the impact of wireless channels is studied using the greedy detection method, and numerical closed-form expressions have been derived for average pairwise error probability (PEP), and average bit error probability (ABEP). Also, ABEP, SE, and EE of adaptive OFDM-IM with M-QAM modulation are investigated and evaluated over composite fading and shadowing conditions. In addition, the performance of an adaptive OFDM-IM system with M-QAM modulation is evaluated in terms of efficiency metrics, outage probability, and ABEP. The obtained results show that the adaptive scheme offers high potential for accomplishing significant improvement in SE and EE while maintaining acceptable ABEP even under severe channel impairment.

7.2 Block Diagram of OFDM-IM System Transceiver with Feedback Path

The block diagram of the adaptive OFDM-IM transceiver system is illustrated in Figure 7.1. The main concept of adaptive modulation is that the modulator and demodulator have to be configured at any time for the same constellation size, M, in the active subcarriers OFDM-IM and according to estimated , which requires achieving maximum throughput. The transmitter structure is explained in detail in section (2.4) of Chapter 2. The receiver structure of adaptive OFDM-IM system is similar to the receiver of OFDM-IM system in Chapter 2 except, we add \( \gamma \) estimator and rate control block with a feedback path. The \( \gamma \) estimator is used to estimate \( \gamma \) from the channel, while the rate control block is used to find the modulation size of transmitted symbol. The feedback path is employed to increase or decrease the modulation size depending on the channel condition at the receiver side.

7.3 Pairwise Error Probability (PEP) over Fading and Shadowing Channel

In this section, the PEP of OFDM-IM for M-ary QAM schemes is evaluated under shadowing and fading, which is modeled by Nakagami-m-Gamma in wireless channels using
the greedy detection method. The greedy detector involves two processes. The first process detects the indices of the active subcarriers (index 1) and inactive subcarriers (index 0). The second process modulates a data symbol at the active subcarriers by employing the Maximum-Likelihood (ML) detection method. When the active subcarrier index (δ) is incorrectly estimated at the receiver as an inactive index (δ̂), this event is known as the pairwise error event (PEE), i.e., (δ → δ̂) and δ ≠ δ̂. The conditional pairwise error probability (PEP) can be described as the probability having the PEE, PEP(δ → δ̂) and the PEP have been addressed in (5.1). The average PEP for OFDM-IM over the shadowed fading channel can be calculated by averaging (5.1) over (3.8) as:

\[
PEP_{NG} = \int_0^\infty \left[ 1 - \sum_{q=0}^{n-k} \left( \frac{n-k}{q} \right) \frac{-1^q}{q+1} \exp\left( \frac{-q\gamma}{q+1} \right) \right] \times \frac{2}{\Gamma(c)\Gamma(m)} \left( \frac{cm}{\gamma} \right)^{\frac{c+m}{2}} e^{\frac{c+m-2}{2}} K_{c-m} \left( 2\sqrt{\frac{cm}{\gamma}} \right) d\gamma
\]

Using the Table of Integrals [86, (6.643.3)] in (7.1), the average analytical expression of PEP can be expressed as:

\[
PEP_{NG} = 1 - (\beta)^\nu \sum_{q=0}^{n-k} \left( \frac{n-k}{q} \right) \frac{-1^q}{q+1} e^{\frac{\beta(q+1)}{2q}} \left( \frac{q}{q+1} \right)^{-\nu} W_{-\nu,\nu} \left( \frac{\beta(q + 1)}{2q} \right)
\]

Figure 7.1: Block Diagram of OFDM-IM Transceiver
where $\beta = \frac{cm + 1}{\gamma}$, $\nu = \frac{c + m - 1}{2}$, $\iota = \frac{c - m}{2}$ and $W(.)$ is the Whittaker function. The detail of integration of (7.2) is shown in Appendix (A.4.1).

Substituting (7.2) into (5.3), the overall PEP over the shadowed and fading channel is formulated as:

$$\text{PEP}_{NG} = k[1 - (\beta)^{\nu} \sum_{q=0}^{n-k} \binom{n-k}{q} \frac{1}{q + 1} e^{2q} e^{-2q} \frac{q + 1}{q + 1} W_{-\nu, \iota} \left( \frac{\beta(q + 1)}{2q} \right)]$$

(7.3)

By following this approach, the approximate PEP for OFDM-IM over the shadowed and fading channel is found by averaging (5.1) over (3.8) as:

$$\text{PEP}_{NG} \approx \int_0^{\infty} \frac{n - k}{2} e^{-\gamma} \frac{\Gamma(e) \Gamma(m)}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right)^{\frac{c + m}{2}} \gamma^{\frac{c + m - 2}{2}} K_{c-m} \left( 2 \sqrt{\frac{cm}{\gamma}} \right) d\gamma$$

(7.4)

The detail of integration of (7.4) is shown in Appendix (A.4.2). The approximate closed-form expression for PEP can be obtained by applying the Table of Integrals [86, (6.643.3)] in (7.4) as:

$$\text{PEP}_{NG}^{\text{approx}} \approx \frac{n - k}{2} (\beta)^{\nu} e^{\beta} \left( \frac{1}{2} \right)^{-\nu} W_{-\nu, \iota} (2\beta)$$

(7.5)

The approximation of overall PEP is determined by substituting (7.5) into (3.8) as:

$$\text{PEP}_{t,NG}^{\text{approx}} \approx k \left[ \frac{n - k}{2} (\beta)^{\nu} e^{\beta} \left( \frac{1}{2} \right)^{-\nu} W_{-\nu, \iota} (2\beta) \right]$$

(7.6)

### 7.3.1 Average Bit Error Probability (ABEP) over Fading and Shadowing Channel

The bit error rate ($P_{eM}$) of OFDM-IM is a result of mis-detection in the active subcarrier and M-ary symbol. This mis-detection appears in three errored situations. The first one results from the mis-detection in the index of the active subcarrier and M-ary symbol, and it is expressed as $e_{IM_1} = \text{PEP}_{t} P_{eM}$. The second one results from the mis-detection of the index of the active subcarrier, but correct detection of the M-ary symbol, and it
is expressed as $e_{IM2} = PEP(1 - P_{eM})$. The third one results from the correct detection of the index of the active subcarrier, but the mis-detection of the M-ary symbol, and it is expressed as $e_{IM3} = (1 - PEP)P_{eM}$. Therefore, the total bit error probability (BEP) can be obtained as $BEP = PEP + (1 - PEP)P_{eM}$, where BEP is the bit error probability over the (AWGN) channel of the M-ary modulation schemes. The total overall BEP for $(k)$ active subcarrier is $BEP_t \leq k[PEP + (1 - PEP)P_{eM}]$ and it is addressed in (5.20).

Averaging (5.20) over (3.8) and inserting (5.1), (5.2) and (5.21), the upper bound and approximate average BEP (ABEP) of OFDM-IM over the shadowing and fading channel can be found as

$$ABEP_{KG} = \int_{0}^{\infty} BEP_t P_\gamma(\gamma) \, d\gamma$$

(7.7)

The upper bound and approximate closed-form expressions of the M-ary QAM-OFDM-IM scheme for the greedy detection can be evaluated as shown in equation (7.8) and (7.9), respectively:

$$ABEP_{KG}^{QAM} \leq K \left[ 1 - (\beta)^\nu \sum_{q=0}^{n-k} \binom{n-q}{n-k} \frac{-1}{q+1} e^{\frac{\beta}{q+1}} (L)^{-\nu} W_{-\nu,\nu}(L) \right]$$

(7.8)

$$ABEP_{KG,approx}^{QAM} \leq k \left[ \frac{n-k}{2} (\beta)^\nu e^\beta \left( \frac{1}{2} \right)^{-\nu} W_{-\nu,\nu}(2\beta) + 0.2 (\beta)^\nu e^{\frac{\beta}{2\pi}} \left( \frac{1}{\alpha} \right)^{\nu} W_{-\nu,\nu}(\beta\alpha) - \frac{n-k}{10} (\beta)^\nu e^{\frac{\beta}{2\pi}} (D)^{-\nu} W_{-\nu,\nu}(\frac{\beta}{D}) \right]$$

(7.9)

where the parameters are: $\beta = \frac{cm}{\gamma}$, $L = \frac{q}{q+1}$, $\alpha = \frac{2(M-1)}{3}$, $\nu = \frac{c+m-1}{2}$, $\iota = \frac{c-m}{2}$ and $D = \frac{M+2}{M-1}$. The terms $c$, $m$ and $M$ are the shadowing factor, the fading factor and the constellation size, respectively. The detail of integration is shown in Appendix (A.10.3) and (A.10.4).
7.3.2 Simulation of ABEP

In this section, the ABEP of OFDM-IM is obtained by using Monte Carlo simulation, which is accomplished by using Matlab, for greedy detection over shadowing and fading channel. The channel gain \( h_{KG} \), which describes shadowing and fading, can be generated as [100], [88]:

\[
h_{KG} = 2 \sqrt{\frac{G_m \times G_c}{|c - m| + 1}}
\]  

(7.10)

where \( G_m \) and \( G_c \) denote the pdf of Gamma distribution for the fading \( m \) and shadowing \( c \) factors, respectively. For the AWGN channel, the channel parameters \( m \) and \( c \) are set to infinity to derive channel gain \( h_{KG} = 1 \). In contrast, for the Rayleigh channel, the channel parameters are set to \( m = 1 \) and \( c = \infty \). The simulation scheme is formed from two parts, the transmitter and the receiver as follows: at the transmitter end (see Figure 7.1 for clarity), serial random data stream \( D_t \) is generated and divided into \( g = \frac{N}{n} \) groups. In each group, \( p \) bits are split into two sub-groups \( p_1 \) and \( p_2 \), relying on \( n \) and \( k \) as demonstrated in Section (5.2) in Chapter 5. A look-up table (LUT) is used to map each sub-block. For instance, the LUT for \( n = 4 \), \( k = 2 \) and \( p_1 = 2 \) is shown in Table 7.1.

<table>
<thead>
<tr>
<th>( p_1 ) bits</th>
<th>sub-block</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>1010</td>
</tr>
<tr>
<td>01</td>
<td>0101</td>
</tr>
<tr>
<td>10</td>
<td>1001</td>
</tr>
<tr>
<td>11</td>
<td>0110</td>
</tr>
</tbody>
</table>

Here, \( p_1 \) bits are employed to indicate the active subcarrier and the \( p_2 \) bits are mapped to the active subcarrier to create one of the M-ary Modulation symbols in the sub-block. The OFDM-IM sub-block creator (see Figure 7.1 for clarity) combines \( g \) sub-blocks to create one OFDM-IM symbol. This symbol is passed through Inverse Fast Fourier Transform (IFFT), and then a cyclic prefix is applied. The fading and shadowing channel \( h_{KG} \) parameters considered in the simulation are shown in Table 7.2.
Table 7.2:
Simulation parameters used for evaluation BER of OFDM-IM system over faded and shadowing channel

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>FFT Size</td>
<td>64</td>
</tr>
<tr>
<td>Guard interval</td>
<td>16</td>
</tr>
<tr>
<td>Length of sub-block (n)</td>
<td>4</td>
</tr>
<tr>
<td>Number of active subcarrier</td>
<td>2</td>
</tr>
<tr>
<td>Fading and shadowing factors</td>
<td>1, 2, 3, and 4</td>
</tr>
<tr>
<td>Number of Symbols</td>
<td>10000</td>
</tr>
<tr>
<td>Constellation size</td>
<td>4 and 16 QAM</td>
</tr>
<tr>
<td>SNR</td>
<td>0 dB to 40 dB</td>
</tr>
<tr>
<td>Iterations</td>
<td>100</td>
</tr>
</tbody>
</table>

On the receiver side (see Figure. 7.1 for clarity), two processes are required to predict the index modulation and M-ary modulation symbol using the greedy detection. In the first step, the received subcarriers in every sub-block that have the largest power are estimated to be 1, i.e., active. The remaining bits, which have low received power, are estimated to be 0, i.e., inactive. \( \hat{p}_1 \) bits are retrieved by known LUT at the receiver that is employed at the transmitter. In the second step, ML decision is applied to estimate \( \hat{s} \) in every active subcarrier, i.e., \( \hat{p}_2 \). Finally, \( \hat{p}_1 \) and \( \hat{p}_2 \) are integrated to compose \( \hat{p} \). The original data stream is reshaped from parallel to serial stream as \( \hat{D} \). The average PEP for BPSK is plotted in Figure 7.2 over the composite Nakagami-m and Gamma (NG) model for various fading \( (m) \) and shadowing \( (c) \) parameters. The results validates the analytical result that is presented in [75] and [5]. From Figure 7.2, it is observed that the composite fading and shadowing channel approximately matches the Rayleigh and Nakagami-m \( (m = 1) \) fading channels at \( c = 2.15 \) and \( m = 1 \). Figure 7.3 and Figure 7.4 illustrate BEP for 4 and 16 QAM modulation schemes with various fading and shadowing factors, respectively. And these figures demonstrate the validity of the analytical results with the aid of the simulated results.
Figure 7.2: Comparison of average PEP of OFDM-IM with greedy detection over the different fading channels as the function of SNR.

Figure 7.3: BEP for OFDM-IM with 4-QAM as a function of SNR over various shadowing and fading parameters.
Figure 7.4: BEP for OFDM-IM with 16-QAM as a function of SNR over various shadowing and fading parameters.
7.4 Efficiency Metrics of Adaptive OFDM-IM System with M-QAM Technique

7.4.1 Adaptive OFDM-IM System with M-QAM Technique

The block diagram of the adaptive OFDM-IM transceiver system is illustrated in Figure 7.1. As mentioned before, the main concept of adaptive modulation is that the modulator and the demodulator have to be configured at any time for the same constellation size $M$ in the active OFDM-IM subcarriers and according to the estimated $\gamma$, which requires achieving maximum throughput as shown in Algorithm 1. Thus, depending on $\gamma$, the modulation size is chosen to satisfy a certain average bit error probability ($10^{-3}$ or $10^{-6}$).

The range of $\gamma$ is divided into $R$ regions to determine which M-ary QAM scheme is used when the estimated $\gamma$ falls in the $R^{th}$ region. Each $R$ region has assigned boundaries $(\gamma_R - \gamma_{R+1})$, as specified in Table 7.3, where $M_R$ represents the modulation levels. It is assumed that when $\gamma$ is below certain threshold $\gamma_1$, the BPSK modulation is used in the transmitter side and the channel state information is assumed to be known at the receiver side.

Table 7.3: Switching boundaries of $\gamma_R$ for target BER=$10^{-6}$ and BER=$10^{-3}$

<table>
<thead>
<tr>
<th>Region (R)</th>
<th>$M_R$</th>
<th>$\gamma_R$</th>
<th>$\gamma_{R+1}[dB]$</th>
<th>$\gamma_R$</th>
<th>$\gamma_{R+1}[dB]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>13.8 - 17.6</td>
<td>-</td>
<td>10.3 - 13.9</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>17.6 - 20.9</td>
<td>-</td>
<td>13.9 - 17.2</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
<td>20.9 - 24.0</td>
<td>-</td>
<td>17.2 - 20.4</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>32</td>
<td>24.0 - 27.1</td>
<td>-</td>
<td>20.4 - 23.5</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>64</td>
<td>27.1 - 30.1</td>
<td>-</td>
<td>23.5 - 26.5</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>128</td>
<td>30.1 - $\infty$</td>
<td>26.5 - $\infty$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Algorithm 1: Adaptive OFDM-IM modulation with greedy detection

\( N_R = 4 \), number of region (\( R \)).
\( n = 4 \), length of sub-block.
\( k = 2 \), number of active subcarriers.
\( M_R = 4 \), modulation level of (\( R \)).
\( \hat{\gamma} \) = SNR.
\( \gamma_R \) = SNR of \( R \).
\( y_i(\hat{\gamma}) \) = received signal of subcarrier (\( i \)).

for \( j = 1 \) to \( N_R \) do

At TX: Transmitter side

generate OFDM – IM using \( n, k \) and \( M_R \)
of Table (7.1) and (7.3)
generate channel effect using (7.10)

At RX: Receiver side

apply A/D, remove CP, and FFT
estimate Index Modulation (IM) of active subcarriers
set count = 0
set estimated received signal matrix to zeros
\( \hat{S} = [\hat{s}(i) \ldots \hat{s}(n)] \)

for \( i = 1 \) to \( n \) do

if count \( \leq k \) then

find subcarrier has greater received power
\( \hat{\gamma} = \arg\max |y_i(\hat{\gamma})|^2 \)
\( \hat{s}(i) = 1 \)

end if

count = count + 1
end for

find \( P_1 \) from matrix (\( \hat{S} \)) using Table (7.1).
at \( \hat{s}(i) = 1 \) Find \( P_2 \) using Maximum Likelihood
\( \hat{P} = [\hat{P}_1 \hat{P}_2] \).
if \( \hat{\gamma} \ll \gamma_R \) then

rate control feedback – 1 to TX
\( M_R = M_R - 1 \)
else

if \( \gamma_R \leq \hat{\gamma} \leq \gamma_R + 1 \) then

feedback 1 increase \( M_R \)
\( M_R = M_R + 1 \)
end if
end if
end if
7.4.2 System Outage Probability

The system outage probability, $P_{\text{out}}$, also known as outage probability is the probability that the received SNR $\gamma$, drops below the required threshold $\gamma_{th}$, which is computed to meet the target BER of $10^{-3}$ or $10^{-6}$ as outlined in Table 7.3. $P_{\text{out}}$ can be calculated using (3.8) as:

$$P_{\text{out}} = \int_{0}^{\gamma_{th}} P_\gamma(\gamma) d\gamma$$  \hspace{1cm} (7.11)

Substituting (3.7) into (7.11) and using the Table of Integrals [86, (3.381.1), (8.352.6), (3.471.9)], the $P_{\text{out}}$ is expressed as:

$$P_{\text{out}} = 1 - \frac{2}{\Gamma(c)} \sum_{i=0}^{m-1} \frac{1}{i!} (\beta \gamma_{th})^{c+i} K_{c-i}(2\sqrt{\beta \gamma_{th}})$$  \hspace{1cm} (7.12)

Where $\beta = \frac{cm}{\gamma}$ and $\gamma_{th}$ is the received signal threshold. Figure 7.5 illustrates the outage probability for various values of the composite shadowing and fading channel with target BERs of $10^{-3}$ and $10^{-6}$, respectively. It is observed that the outage probability $P_{\text{out}}$ gets better with an increase in $m$ and $c$ values. This clearly shows that an increase in $m$ and $c$ values decreases the fading and shadowing effects on system’s performance.

7.4.3 Spectral and Energy Efficiencies over Fading and Shadowing Channel

The main design parameters of a communication system are spectral efficiency (SE) and energy efficiency (EE). Both of these parameters should be sustained to satisfy certain quality of service (QoS) demands. Minimizing energy consumption in wireless communication systems has become a crucial factor; however, increasing data usage leads to more energy use. By employing OFDM-IM, the SE can be enhanced and, at the same time the EE can be reduced. In this section the upper bound of SE and EE for OFDM-IM are investigated.
Figure 7.5: Outage probability target BER $10^{-6}$ and $10^{-3}$ as a function of SNR over various shadowing and fading parameters.

### 7.4.3.1 Average System Spectral Efficiency (bit/s/Hz)

Spectral Efficiency, $\eta_{SE}$, is known as the maximum received bits per channel bandwidth (in Hz) that a wireless channel can convey to satisfy certain QoS requirements and it is given by [101]:

$$
\eta_{SE} = \frac{I(X;Y)}{BT_s}, \quad (b/s/Hz)
$$

(7.13)

where $BT_s$ in Hz/baud is the bandwidth used by the symbol duration. The term $I(X;Y)$ is the achievable average data rate of the modulation scheme and it is measured in bits/channel use. The average spectral efficiency, $\overline{\eta_{SE}}$, is the sum of the data rate ([$\log_2 C_n^k M_k$]) linked to R region each weighted by the probability of which that $\gamma$ falls in the $R^{th}$ region, $Pr(R)$, and is presented by [102]:

$$
\overline{\eta_{SE}} = \sum_{R=1}^{N_R} \left( \left[ \log_2 C_n^k M_k \right] \right) Pr(R)
$$

(7.14)
where $P_r(R) = \int_{\gamma_R}^{\gamma_{R+1}} P_\gamma(\gamma) d\gamma$ which can be written as:

$$P_r(R) = \int_0^{\gamma_{R+1}} P_\gamma(\gamma) d\gamma - \int_0^{\gamma_R} P_\gamma(\gamma) d\gamma$$

(7.15)

$P_r(R)$ can be obtained directly from (7.12) as:

$$P_r(R) = \frac{2}{\Gamma(c)} \sum_{i=0}^{m-1} \frac{1}{i!} \left[ (\beta\gamma_R)^{\frac{c+i}{2}} K_{c-i}(2\sqrt{\beta\gamma_R}) - (\beta\gamma_{R+1})^{\frac{c+i}{2}} K_{c-i}(2\sqrt{\beta\gamma_{R+1}}) \right]$$

(7.16)

Where $\gamma_R$ and $\gamma_{R+1}$ are the received signal thresholds of the region $R$ and $R+1$, respectively. The derivation detail is shown in Appendix (A.10.6).

Figure 7.6, Figure 7.7, Figure 7.8 and Figure 7.9 show the average SE ($\text{bits/s/Hz}$) of adaptive OFDM-IM with M-QAM modulation for four ($N_R = 4$) and five ($N_R = 5$) regions as function of SNR for various fading ($m$), shadowing channel parameters ($c$) and active subcarriers ($k$). In Figure 7.6 and Figure 7.7, the improvement in channel condition that reflected by $m=3$ and $c=4$ led to significant enhancement in SE. Figure 7.10 illustrates the average SE of the proposed system in three dimensions (3D) as a function of SNR for different values of $m$, $c$, $N_R$ and $k$. It is noticed that when the adaptive scheme employs a higher constellation level, the system SE is increased. For instance, system SE is enhanced by 0.5 $\text{bit/s/Hz}$ when the adaptive modulation increases modulation level from $M = 32$, i.e., $N_R = 4$, to $M = 64$, i.e., $N_R = 5$. From Figure 7.6 and Figure 7.7, it is observed that the total system SE is improved by almost 1 $\text{bit/s/Hz}$ as the active subcarriers ($k$) are increased. Similar results are observed in Figure 7.8 and Figure 7.9. Also, it is noted that the adaptive modulation offers better adaptation with any impaired channel conditions.

### 7.4.3.2 Average System Energy Efficiency (bits/TNEU)

Energy efficiency, $\eta_{EE}$, is defined as the maximum data rate ($R$) in (bit/s) that can be received by wireless communication systems per unit of transmitted power, $p_t$, and that is [103]:

$$\eta_{EE} = \frac{R}{p_t}$$
Figure 7.6: Spectral efficiency of adaptive M-QAM for OFDM-IM over shadowing and fading channels as a function of SNR for $N_R = 4$, $k = 2$, $n = 4$ and different values of $m$ and $c$.

Figure 7.7: Spectral efficiency of adaptive M-QAM for OFDM-IM over shadowing and fading channels as a function of SNR for $N_R = 4$, $k = 3$, $n = 4$ and different values of $m$ and $c$. 
Figure 7.8: Spectral efficiency of adaptive M-QAM for OFDM-IM shadowing and fading channels as a function of SNR for $N_R = 5$, $k = 2$, $n = 4$ and different values of $m$ and $c$.

Figure 7.9: Spectral efficiency of adaptive M-QAM for OFDM-IM over shadowing and fading channels as a function of SNR for $N_R = 5$, $k = 3$, $n = 4$ and different values of $m$ and $c$.

\[
\eta_{EE} = \frac{R}{P_t}, \quad (\text{bit/s/Watt})
\]  \hspace{1cm} (7.17)
Another definition that includes the effect of wireless channel on energy efficiency has been presented as the number of bits per thermal noise signal energy unit (TNEU). Therefore, energy efficiency is associated with SNR, $\gamma = \frac{P_t}{B N_0}$, and it can be expressed as [104]:

$$\eta_{EE} = \eta_s \frac{b}{TENU}$$  \hspace{1cm} (7.18)

When the SE is achieved, the EE of the adaptive transmission scheme is directly obtained from (7.18) as:

$$\eta_{EE} = \frac{\eta_{SE}}{\gamma} = \sum_{R=1}^{N_R} \left( \left\lfloor \log_2 \frac{C_n^k M^k}{n} \right\rfloor P_r(R) \right) \frac{b}{TENU}$$  \hspace{1cm} (7.19)

The EE is shown in Figure 7.11 and Fig. 7.12. Figure 7.13 illustrates the EE as a function of SNR for two regions ($N_R = 4$ and 5), with $c = 4$, $m = 4$ and various values of $k$. It is obvious that the EE gets better as the channel impairment decreases. Also, the EE is improved as the number of active subcarriers ($k$) increases. For example, in the case of $m = 1$ and $c = 1$, the EE is enhanced by 13% when the number of active subcarrier is reduced from $k = 3$ to $k = 2$. Also, it is clear from Figure 7.13 that the lower $N_R$ provides better EE by 15%.
Figure 7.11: Energy efficiency of adaptive M-QAM for OFDM-IM over shadowing and fading channels as a function of SNR for $N_R = 4$, $n = 4$ and different values of $c$, $m$ and $k$.

Figure 7.12: Energy efficiency of adaptive M-QAM for OFDM-IM over shadowing and fading channels as a function of SNR for $N_R = 5$, $n = 4$ and different values of $c$, $m$ and $k$. 
7.4.4 Average Bit Error Probability (ABEP)

The presented adaptive Modulation with OFDM-IM schemes is designed based on the separated SNR regions that aim to maximize EE and SE by changing the number of active subcarriers \( k \) in each subblock \( n \). However, it must maintain a certain QoS, which is measured in terms of ABEP. Therefore, ABEP needs to be determined by averaging the BEP for all \( \gamma \) regions, and this can be computed exactly as the ratio of the average number of bits in error over the average total number of transmitted bits as [102]:

\[
ABEP = \frac{\sum_{R=1}^{N_R} \left( \left\lfloor \log_2(M_R^k C_R^k n) \right\rfloor \right) P_R(R)}{\eta_{SE} = \sum_{R=1}^{N_R} \left( \left\lfloor \log_2(M_R^k C_R^k n) \right\rfloor \right) P_R(R)},
\]

(7.20)

where \( BEP_R \) is the average bit error probability over composite fading/shadowing channel for \( R^{th} \) region and can be obtained using:

\[
BEPR = \int_{\gamma_R}^{\gamma_{R+1}} BEP \cdot P_\gamma(\gamma)d\gamma,
\]

(7.21)

where \( P_\gamma(\gamma) \) is given by (3.9) and \( BEP \) is expressed by (5.1). Thus, \( BEP_R \) can be written as:

\[
BEPR = \frac{k}{\eta_{SE}} \sum_{\delta=1}^{n} \left[ \int_{\gamma_R}^{\gamma_{R+1}} PEP \cdot P_\gamma(\gamma)d\gamma \right]
\]

\[
- \int_{\gamma_R}^{\gamma_{R+1}} PEPR \cdot P_{\gamma M-ary}(\gamma)d\gamma
\]

\[
+ \int_{\gamma_R}^{\gamma_{R+1}} PEPR \cdot PEPR \cdot P_{\gamma M-ary}(\gamma)d\gamma
\]

(7.22)
By substituting (5.1), (5.2) and (3.9) into (7.22) and with the aid of the Table of Integrals [86, (3.381.1), (8.352.6), (3.471.9)], the overall upper bound $\overline{\text{BEP}}_R$ can be expressed as:

\[
\overline{\text{BEP}}_R = \frac{k}{n} \sum_{\delta=1}^{n} \left[\overline{\text{BEP}}^1_R + \overline{\text{BEP}}^2_R + \overline{\text{BEP}}^3_R\right] = k\left[\overline{\text{BEP}}^1_R + \overline{\text{BEP}}^2_R + \overline{\text{BEP}}^3_R\right]
\] (7.23)

The three different bit error probabilities ($\overline{\text{BEP}}^1_R, \overline{\text{BEP}}^2_R, \overline{\text{BEP}}^3_R$) of exact and approximate upper bounds in (7.23) for all $R^{th}$ regions are evaluated as:

\[
\overline{\text{BEP}}^1_{R,\text{approx}} = \frac{(n-k)\beta l}{\Gamma(c)} \left(\frac{1}{2}\right)^{-m} \left[\gamma_R e^{-\gamma_R} \sum_{i=0}^{m-1} \frac{1}{i!}\right]^{\frac{\gamma R}{2}} K_{c-m}(2\sqrt{\beta \gamma R})
\] (7.24)

\[
\overline{\text{BER}}^2_{R,\text{approx}} = \frac{0.4\beta l}{\Gamma(c)} \left[\gamma_R e^{-\gamma_R} \sum_{i=0}^{m-1} \frac{1}{i!}\right]^{\frac{\gamma R}{\alpha}} K_{c-m}(2\sqrt{\beta \gamma R})
\] (7.25)

\[
\overline{\text{BER}}^3_{R,\text{approx}} = \frac{0.4\beta l}{\Gamma(c)} \left(D \frac{1}{2}\right)^{-m} \left[\gamma_R e^{-\frac{D\gamma R}{2}} \sum_{i=0}^{m-1} \frac{1}{i!}\right]^{\frac{D\gamma R}{2}} K_{c-m}(2\sqrt{\beta \gamma R})
\] (7.26)
\[
\overline{\text{BEP}}_{R,\text{exact}}^1 = \frac{2}{\Gamma(c)} \sum_{i=0}^{m-1} \frac{1}{i!} \left( c \gamma_n \right)^{\frac{c+i}{2}} K_{c-i} \left( 2\sqrt{c \gamma_n} \right) - \left( c \gamma_n + 1 \right)^{\frac{c}{2}} K_{c} \left( 2\sqrt{c \gamma_n} \right)
\]

(7.27)

\[
\overline{\text{BEP}}_{R,\text{exact}}^2 = \frac{0.4}{\Gamma(c)} \frac{1}{k-1} \beta l \alpha m \sum_{i=0}^{m-1} \left( \gamma_n e^{\frac{\gamma_n}{\alpha}} \right)^i
\]

\[
- c \left( L \gamma_n + c \gamma_n \right) K_{c-m} \left( 2\sqrt{c \gamma_n} \right) - e^{C_1 L \gamma_n + c \gamma_n} K_{c-m} \left( 2\sqrt{c \gamma_n} \right)
\]

(7.28)

\[
\overline{\text{BEP}}_{R,\text{exact}}^3 = -\frac{0.4}{\Gamma(c)} \beta l \alpha m \sum_{i=0}^{m-1} \frac{1}{i!} \left( \gamma_n e^{\frac{\gamma_n}{\alpha}} \right)^i
\]

\[
\left[ \gamma_n e^{\frac{L+1}{\alpha}} \gamma_n \sum_{i=0}^{m-1} \frac{1}{i!} \left( L \gamma_n + c \gamma_n \right) K_{c-m} \left( 2\sqrt{c \gamma_n} \right) - \gamma_{n+1} e^{\frac{L+1}{\alpha}} \gamma_{n+1} \sum_{i=0}^{m-1} \frac{1}{i!} \left( L \gamma_{n+1} + c \gamma_{n+1} \right) K_{c-m} \left( 2\sqrt{c \gamma_{n+1}} \right) \right]
\]

(7.29)

where \( l_1 = \frac{c+1}{2}, \ v = \frac{c-m}{2}, \ D = \left( \frac{M+2}{2(M-1)} \right), \ \gamma_R \) is the SNR threshold for region \( R \) and \( \gamma_{R+1} \) is the SNR threshold for region \( R + 1 \).

\( ABEP \) of the proposed OFDM-IM with M-QAM modulation is obtained from (7.20) for the different \( N_R \) regions. Figure 7.14 and Fig. 7.15 illustrate \( ABEP \) of \( 10^{-3} \) and \( 10^{-6} \) as a function of SNR for \( N_R = 4, \ k = 2, \ n = 4 \) and various values of \( m \) and \( c \), respectively. The results in Figure 7.14 show an increase in \( ABEP \) above the target error-rate limit of \( 10^{-3} \) till about 10 dB, and then it starts to decrease with an increase in the SNR. The reason behind the initial increase is that, from 0 dB to 10 dB, the adaptive scheme does not set-in as outlined in Table 7.3. Similar conclusion can be drawn from Figure 7.15 for the target error-rate of \( 10^{-6} \). Figure 7.16 depicts the effects of different values of
the $k$ and channel parameters. It is evident that the adaptive OFDM-IM with M-QAM modulation preserves a certain QoS for all the values of $m$ and $c$ in the targeted $\gamma$ range. Also, Figure 7.16 shows the effect of increasing $k$ in ABEP for various values of channel parameters. Here, for high values of $\gamma$, the performance of the system is controlled by the M-QAM modulation level.

![Figure 7.13: Energy efficiency of adaptive M-QAM for OFDM-IM over shadowing and fading channels as a function of SNR for $N_R = 4$ and $5$, $n = 4$, $c = 4$, $m = 3$ and different values of $k$.](image)

**7.5 Chapter Summary**

In this Chapter, the performance of OFDM-IM system has been investigated in a multipath fading channel superimposed on a shadowing environment, by using a composite Nakagami-m-Gamma (NG) model. The closed-form expressions for PEP and ABEP have been derived over the proposed channel and validated using Monte Carlo simulation. Also, the closed-form boundaries of SE and EE for OFDM-IM with the M-QAM modulation scheme have been examined in regards to active subcarriers ($k$), $c$ and $m$ channel parameters, and SNR. We observe that SE and EE are highly dependent on the channel parameters. Thus, the adaptive modulation scheme was proposed to improve
Figure 7.14: ABEP of adaptive M-QAM for OFDM-IM over shadowing and fading channels as a function of SNR for $N_R = 4$, $k = 2$, $n = 4$ and different values of $m$ and $c$ targeting an ABEP $10^{-3}$.

Figure 7.15: ABEP of adaptive M-QAM for OFDM-IM over shadowing and fading channels as a function of SNR for $N_R = 4$, $k = 2$, $n = 4$ and different values of $m$ and $c$ targeting an ABEP $10^{-6}$. 

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Figure 7.16: ABEP of adaptive M-QAM for OFDM-IM over shadowing and fading channels as a function of SNR for $N_R = 4$, $n = 4$ and different values of $k$, $m$ and $c$ targeting an ABEP of $10^{-6}$

the efficiency of OFDM-IM system with M-QAM modulation scheme. The performance of the adaptive modulation with an OFDM-IM system has been investigated in terms of outage probability, average SE, average EE, and average bit error probability. The results show that by varying the number of active subcarriers ($k$), the proposed adaptive OFDM-IM system not only enhances the system SE and EE but also adds flexibility in designing wireless communication system by trading-off between efficiency metrics and ABEP performance.
Chapter 8

OFDM-IM System with Low Complexity Detection and Diversity Reception Over Fading Channels

8.1 Introduction

This Chapter examines the performance of a low complexity detection technique and diversity reception for an adaptive OFDM-IM system with M-QAM modulation scheme, over Nakagami-m fading channel. The closed-form expressions of pairwise error probability (PEP) and average bit error probability (ABEP) for the adaptive OFDM-IM system with M-QAM modulation technique are derived using greedy detection (GD) and maximal ratio combining (MRC) over Nakagami-m fading channel. In section 8.2, the PEP and ABEP expressions for the system has been derived for the Nakagami-m Fading Channel using GD algorithm outlined in Chapter 7, while in section 8.3 the analysis has been extended by adding Diversity combining with Maximal Ration Combining (MRC) technique. The derived closed-form expressions of PEP and ABEP are presented in general terms by varying $m$ parameter, and the theoretical results are compared with the existing research studies. In addition, the theoretical results are validated by simulation results. The results show that the adaptive OFDM-IM with diversity, outperforms the conventional OFDM-IM system in terms of SE and EE.

8.2 Performance Analysis of Adaptive OFDM-IM System over Nakagami-m Fading Channel

8.2.1 Model of Adaptive OFDM-IM System with M-QAM Technique

The block diagram of the adaptive OFDM-IM transceiver system, which has been shown in chapter 7, is illustrated in Figure 8.1 with Nakagami-m fading channel and diversity combiner blocks. Diversity reception reduces the probability of occurrence of communication failures (outages) caused by fades by combining several copies of the same message received over different paths. Here, Maximal Ratio Combining (MRC), also known as Maximum Ratio Combining, is used as a diversity combining scheme. In MRC technique, each signal branch is multiplied by a weight factor that is proportional to the signal amplitude. That is, branches with strong signal are further amplified, while weak signals are attenuated. For detailed description of this block diagram we can refer to section 7.2 and 7.4.1.

![Figure 8.1: OFDM-IM transceiver [5]](image)

8.2.2 System Outage Probability

System outage probability has already been discussed in section 7.4.2. Here, we will present some results for the system in the presence of Nakagami-m channel model. Figure
8.2 illustrates the outage probability for various m-values of fading channel with target BER of $10^{-3}$. As before, it is observed that the outage probability $P_{out}$ gets better with an increase in $m$ values.

![Outage Probability Graph](image)

Figure 8.2: Outage probability with a target BER of $10^{-3}$ as a function of SNR over various fading parameters $m$.

### 8.2.3 Spectral and Energy Efficiencies over Nakagami-m Fading Channel

#### 8.2.3.1 The Analytical Results

Based on the background information provided in sections 7.4.3, the probability of $\gamma$ that falls in the $Rth$ (please see Table 7.3) region $P_r(R)$ can be obtained as follows:

$$P_r(R) = \frac{\Gamma \left( m, \frac{m}{\gamma} R \right) - \Gamma \left( m, \frac{m}{\gamma} R + 1 \right)}{\Gamma(m)}$$

or

$$= \frac{\gamma \left( m, \frac{m}{\gamma} R + 1 \right) - \gamma \left( m, \frac{m}{\gamma} R \right)}{\Gamma(m)} \quad (8.1)$$
where $\beta = \frac{m}{\gamma}$, $\Gamma(.,.)$ is the upper incomplete gamma function and $\gamma(.,.)$ is the lower incomplete gamma function.

By following the similar analytical approach outlined in section 7.3.4.2 the average spectral efficiency (SE) and average energy efficiency (EE) for the system in the presence of Nakagami-m channel can be obtained as:

$$\eta_{SE} = \eta_{EE} = \sum_{R=1}^{N_R} \left( \left\lfloor \log_2 C_{kn}^k M_k^k \right\rfloor \right) P_r(R) \gamma$$

(8.2)

where, $P_r(R)$ from equation 8.1 is substituted to get the desired theoretical expression.

![Figure 8.3: Spectral efficiency of adaptive M-QAM for OFDM-IM over Nakagami-m fading channel with a target ABEP of $10^{-3}$ as a function of SNR, $k$ and various fading parameters ($m$).](image)

### 8.2.3.2 The Numerical and Simulation Results

The numerical and simulation results of adaptive OFDM-IM system for SE and EE are presented in this section with various fading parameters ($m$) and active subcarriers ($k$). The adaptive OFDM-IM system with $N = 64$ subcarriers, have been grouped into $g$
Figure 8.4: Energy efficiency of adaptive M-QAM for OFDM-IM over Nakagami-m fading channel with a target ABEP of $10^{-3}$ as a function of SNR, $k$ and various fading parameters ($m$).

groups of $n$ subcarriers, where $n = 4$, $k \leq 3$, and $M \in \{2, 4, 8, 16, 32, 64\}$. As each group of $n$ subcarriers is detected individually, $N$ has no effect on the system performance, and the value of $n$ is selected to be much less than $N$ for a low complexity detection process. The following parameters in Table 8.1 and Table 8.2 have been used for the simulation results:

Table 8.1:
LUT for $n = 4$ and $k = 2$

<table>
<thead>
<tr>
<th>$p_1$ bits</th>
<th>sub-block</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>1010</td>
</tr>
<tr>
<td>01</td>
<td>0101</td>
</tr>
<tr>
<td>10</td>
<td>1001</td>
</tr>
<tr>
<td>11</td>
<td>0110</td>
</tr>
</tbody>
</table>

Figure 8.3 depicts the SE performance of GD method for adaptive OFDM-IM system over Nakagami-m fading channel with $n = 4$, $k = \{1, 2, 3\}$, $m = \{1, 2, 4\}$ and $R = 5$ tar-
Table 8.2:
Simulation Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>FFT Size</td>
<td>64</td>
</tr>
<tr>
<td>Length of sub-block (n)</td>
<td>4</td>
</tr>
<tr>
<td>Number of active subcarrier</td>
<td>1, 2, 3</td>
</tr>
<tr>
<td>Fading factors</td>
<td>1, 2, 3, 4</td>
</tr>
<tr>
<td>Number of symbols</td>
<td>10000</td>
</tr>
<tr>
<td>SNR</td>
<td>0 dB to 35 dB</td>
</tr>
<tr>
<td>Iterations</td>
<td>100</td>
</tr>
</tbody>
</table>

getting $ABEP = 10^{-3}$. It is observed that the Adaptive OFDM-IM system significantly enhances the SE as $k$ is increased. For example, by changing $k$ from $k = 1$ to $k = 2$, the SE is improved by 1 bit at 20 dB, while the SE is improved by 1.5 bits at 30 dB. Moreover, Figure 8.3 shows that the adaptive OFDM-IM system achieves better SE than conventional OFDM system from 10 dB and beyond. Figure 8.4 illustrates the EE of the proposed adaptive OFDM-IM scheme with target $ABEP = 10^{-3}$ over Nakagami-m fading channel. It is observed that with less severe channel impairments the system gets better in EE. For example, the EE is improved by about 3% from $m = 1$ to $m = 2$ while the EE is improved by about 6% from $m = 1$ to $m = 4$ at $k = 3$. Also, it is noted that the EE is enhanced as the number of active subcarriers ($k$) decreases. For instance, the EE gets better by 5% from $k = 3$ to $k = 1$ at $m = 1$.

8.2.4 ABEP over Nakagami-m Fading Channel

Based on the theoretical analysis presented in section 7.4.4, the average bit error probability (ABEP) over Nakagami-m fading channel for $R_{th}$ region can be written as:
\[
\overline{\text{BEP}}_R = \frac{k}{n} \sum_{\delta=1}^{n} \left[ \int_{\gamma_R}^{\gamma_{R+1}} \text{PEP} \cdot P_\gamma(\gamma) d\gamma \right] \\
\overline{\text{BEP}}^1_R - \int_{\gamma_R}^{\gamma_{R+1}} P_{\text{M-ary}} \cdot P_\gamma(\gamma) d\gamma \\
\overline{\text{BEP}}^2_R + \int_{\gamma_R}^{\gamma_{R+1}} \text{PEP} \cdot P_{\text{M-ary}} \cdot P_\gamma(\gamma) d\gamma \right]
\] (8.3)

By substituting (3.5), (4.22) and (5.2) into (8.3) and with the aid of the Table of Integrals [86, (3.381.1), (8.381.3)], the total upper bound \(\overline{\text{BEP}}_R\) can be expressed as:

\[
\overline{\text{BEP}}_R = \frac{k}{n} \sum_{\delta=1}^{n} [\overline{\text{BEP}}^1_R + \overline{\text{BEP}}^2_R + \overline{\text{BEP}}^3_R] = k[\overline{\text{BEP}}^1_R + \overline{\text{BEP}}^2_R + \overline{\text{BEP}}^3_R]
\] (8.4)

The three different bit error probabilities \(\overline{\text{BEP}}^1_R, \overline{\text{BEP}}^2_R, \overline{\text{BEP}}^3_R\) of approximate upper bounds in (8.4) for all the \(R^{th}\) regions are evaluated as:

\[
\overline{\text{BEP}}^1_{R,\text{approx}} = \overline{\text{BEP}}^1_R = \frac{(n - k)}{2\Gamma(m)} \left( \frac{m}{m + 0.5\bar{\gamma}} \right)^m \left[ \Gamma(m, (\beta + 0.5) \gamma_R) - \Gamma(m, (\beta + 0.5) \gamma_{R+1}) \right] \] (8.5)

\[
\overline{\text{BEP}}^2_{R,\text{approx}} = \frac{0.2}{\Gamma(m)} \left( \frac{m}{\alpha_1 \bar{\gamma} + m} \right)^m \left[ \Gamma(m, (\beta + \alpha_1) \gamma_R) - \Gamma(m, (\beta + \alpha_1) \gamma_{R+1}) \right] \] (8.6)

\[
\overline{\text{BEP}}^3_{R,\text{approx}} = \frac{-0.1(n - k)}{\Gamma(m)} \left( \frac{m}{\alpha_1 \bar{\gamma} + m + 0.5} \right)^m \left[ \Gamma(m, (\beta + \alpha_1 + 0.5) \gamma_R) - \Gamma(m, (\beta + \alpha_1 + 0.5) \gamma_{R+1}) \right] \] (8.7)
where $\bar{\gamma}$ is the average the SNR, $\beta = \frac{m}{\bar{\gamma}}$, $\alpha_1 = \frac{3}{2(M-1)}$, $\gamma_R$ is the SNR for region $R$, $\gamma_{R+1}$ is the SNR for region $R+1$, $m$ and $M$ are the fading factor and constellation size, respectively.

8.3 Performance Analysis of OFDM-IM System with Diversity Reception over Nakagami-m Fading Channel

8.3.1 PEP for GD with MRC Reception

The PDF of the effective SNR for MRC is given by [105]:

$$P_{N\gamma} (\gamma) = \frac{m^{N_L} \gamma^{N_L m - 1}}{\bar{\gamma}^{N_L m} \Gamma(N_L m)} e^{-\frac{m\gamma}{\bar{\gamma}}}, \tag{8.8}$$

where $N_L$ is the number of the receiver antenna, $m$ is the fading parameter and $\Gamma(.)$ is the Gamma function.

Using (8.8), (5.1) and (5.2), the exact average PEP for OFDM-IM system with GD-MRC technique in the Nakagami-m channel can be formulated as:

$$PEP_{MRC}^{Naka} = \int_0^\infty PEP_t f (\gamma) d\gamma = k \int_0^\infty \left[ 1 - \sum_{q=0}^{n-k} \binom{n-k}{q} \frac{q^q}{q+1} \exp\left(\frac{-p\gamma}{q+1}\right) \right]$$

$$\times \frac{m^{N_L m - 1}}{\bar{\gamma}^{N_L m} \Gamma(N_L m)} e^{-\frac{m\gamma}{\bar{\gamma}}} d\gamma, \tag{8.9}$$

where $k$ is number of the active subcarriers. With the aid of the Table of Integrals [86, eq. (3.381.4)] in (8.9), the closed-form expression of the total average PEP can be expressed as:

$$PEP_{MRC}^{Naka} = k - k \sum_{q=0}^{n-k} \binom{n-k}{q} \frac{q^q}{q+1} \left( \frac{qm+m}{q\gamma+qm+m} \right)^{N_L m}, \tag{8.10}$$

where $\beta = \frac{m}{\bar{\gamma}}$, $L = \frac{q}{q+1}$. The detail mathematical derivation of the exact PEP is shown in Appendix A [(A.4.8)].
Similarly, the approximate total average PEP for OFDM-IM with GD-MRC over Nakagami-
m fading channel is obtained by averaging (8.8) over (5.2) as:

\[
PEP_{MRC}^{N_{aka}} \simeq \frac{(N-k)}{2} \frac{N_{L}^m m^{N_{L}^m}}{\bar{\gamma}^{N_{L}^m} \Gamma(N_{L}^m)} \int_{0}^{\infty} \frac{\gamma^{N_{L}^m-1}}{e^{\frac{\gamma}{2}} e^{-m\gamma/\bar{\gamma}}} d\gamma
\]  

(8.11)

The approximate closed-form expression of total average PEP for OFDM-IM with GD-
MRC can be evaluated by applying the Table of Integrals [86, eq. (3.381.4)] in (8.11) as:

\[
PEP_{MRC}^{Approx-Naka} \simeq \frac{k(n-k)}{2} \left( \frac{2m}{\bar{\gamma}+2m} \right)^{N_{L}^m}
\]  

(8.12)

The detail mathematical derivation of the approximate PEP is shown in Appendix A

[(A.10.8)].

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure8.5.png}
\caption{comparison of exact and approximate PEP of OFDM-IM with GD as the
function of SNR and fading factor \(m\) for different values of \(N_{L}\).}
\end{figure}

### 8.3.2 ABEP for GD with MRC Reception

Based on the background information provided in sections 7.3.1, the total average ex-
act and approximate upper bound of BEP (ABEP) for OFDM-IM system with MRC
reception over the fading channel can be characterized as

\[ ABEP_{Naka} = \int _{0}^{\infty } P_{BEP}(\gamma ) f_\gamma (\gamma ) \, d\gamma \]  

(8.13)

The total exact and the approximate upper bound of the closed-form expressions for the M-ary QAM-OFDM-IM scheme for the greedy detection are given in (8.14) and (8.15), respectively:

\[ ABEP_{Naka}^{MRC} = k - k \sum _{q=0}^{n-k} \binom{n-k}{q} -1^q \frac{\beta}{q+1} \left( \frac{\beta}{L + \beta} \right) ^{N_L m} \]

\[ + 0.2(1-k) \left( \frac{\beta}{\alpha + \beta} \right) ^{N_L m} + 0.2 \sum _{q=0}^{n-k} \binom{n-k}{q} -1^q \frac{\beta}{q+1} \times \left( \frac{\beta}{\alpha + \beta + L} \right) ^{N_L m} \]  

(8.14)
\[ \text{ABEP}_{Naka,\text{Approx}}^\text{MRC} \approx \frac{k(n-k)}{2} \left( \frac{2m}{\bar{\gamma} + 2m} \right)^{NLm} + 0.2k \left( \frac{m}{\alpha\bar{\gamma} + m} \right)^{NLm} - 0.1k(n-k) \]

\[ \times \left( \frac{m}{0.5\bar{\gamma} + \beta + \alpha} \right)^{NLm} \]

where \( \beta = \frac{cm}{q} \), \( L = \frac{q}{q+1} \), \( \alpha = \frac{3}{2(M-1)} \), \( m \) and \( M \) are the fading factor and constellation size, respectively. The detail mathematical derivation is shown in Appendix A [(A.10.8) and (A.10.9)].

In Figure 8.5 a comparison between the exact and approximate upper bounded average PEP expressions for GD-MRC is shown for OFDM-IM system when \( n = 4 \), \( k = 2 \) and \( NL = \{1,2,3,4\} \). It is observed that the maximal difference between (8.10) and (8.12) remains at around less than 0.5 dB at low SNR while the difference is significantly small at higher SNR. Furthermore, the approximate expression gets closer to the exact one as the value of \( NL \) increases and this confirms the accuracy of the approximate PEP in Figure 8.5. Figure 8.6 shows the effect of different values of fading parameter \( m \) for PEP of OFDM-IM with GD-MRC. It is observed that the PEP is improved as the number of antennas increases. For example, for \( NL = 1 \) the PEP=0.119, for \( NL = 2 \) the PEP=0.007, for \( NL = 3 \) the PEP=0.0004, and for \( NL = 4 \) the PEP=2.5\(^{-5}\) at 15 dB.

Figure 8.7 compares the exact and approximate upper bounded ABEP of 4-QAM OFDM-IM scheme with \( n = 4 \), \( k = 2 \), \( NL = 1,2,4,6 \). The maximum difference of 1dB between (8.14) and (8.15) is noted at low SNRs while at higher SNR and \( NL \) values the difference gets even smaller (\( \leq 1 \) dB). This confirms the validity of the approximate expression. Figure 8.8 illustrates the ABEP of 4-QAM OFDM-IM with \( n = 4 \), \( k = 2 \), \( NL = \{1,2,4,6\} \) and \( m = \{1,2\} \). Similar to PEP, it is found that the average BEP improves significantly as \( NL \) increases. For example, the power gain of about 17 dB can be accomplished for ABEP of \( 10^{-3} \) between \( NL = 1 \) and \( NL = 2 \) at \( m = 1 \). Also, it is observed that with the higher value of \( m \), the power gain decreases. For example, the power gain of up to 10 dB can be achieved for ABEP of \( 10^{-3} \) between \( NL = 1 \) and \( NL = 2 \) at \( m = 2 \), which is smaller compared to the corresponding scenario for \( m = 1 \) where a gain of about 17 dB is observed.

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Figure 8.7: Comparison of exact and approximate ABEP of 4-QAM and OFDM-IM with GD as the function of SNR and fading factor ($m$)

### 8.4 Performance of Adaptive OFDM-IM System with Diversity over Nakagami-$m$ Fading Channel

#### 8.4.1 System Outage Probability

The required threshold $\gamma_{th}$ of outage probability ($P_{out}^{MRC}$) that is computed to meet a specific target of ABEP for MRC reception can be calculated following the same steps outlined in chapter 7, and evaluated as:

$$P_{out}^{MRC} = \frac{m^{N_L m}}{\gamma^{N_L m} \Gamma (N_L m)} \int_{0}^{\gamma_{th}} \gamma^{N_L m - 1} e^{-m\gamma} d\gamma$$

using the Table of Integrals [86, (3.381.1), (3.381.3)], $P_{out}^{MRC}$ can be evaluated as:

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where $N_L$ is the number of antenna and $\gamma_{th}$ is the threshold of SNR, which is calculated according to an APEB $10^{-3}$, and is indicated in Table 7.3. The detail mathematical derivation is shown in Appendix A (A.10.10). In figure 8.9, the outage probability of OFDM-IM with MRC is plotted for $m = \{1, 2\}$, and $N_L = \{1, 2, 4, 6\}$ targeting an $ABEP = 10^{-3}$. It can be noted that the $P_{out}$ is improved as the number of antenna increases. For example, $P_{out}$ of $m = 1$ and $N_L = 2$ is equal for $m = 2$ and $N_L = 1$. Also, $P_{out}$ for $m = 1$ and $N_L = 4$ is equal for $m = 2$ and $N_L = 2$.

### 8.4.2 System Spectral Efficiency (bit/s/Hz) of GD-MRC System

Using the same steps outlined in Chapter 7, the average SE for OFDM-IM for GD-MRC is obtained as:
Figure 8.9: Outage probability target BER $10^{-3}$ for MRC as a function of SNR over various fading parameters ($m$) and number of antennas $N_L$.

\[
\eta_{SE} = \sum_{R=1}^{N_R} \left( \left\lceil \log_2 C_n^k M^k \right\rceil \right) \left( P_r^{MRC}(R) = 1 - \frac{\Gamma \left( N_L m, \frac{m}{\gamma} \gamma_R \right)}{\Gamma (N_L m)} - \frac{\Gamma \left( N_L m, \frac{m}{\gamma} \gamma_{R+1} \right)}{\Gamma (N_L m)} \right)
\]

or

\[
\eta_{SE} = \sum_{R=1}^{N_R} \left( \left\lceil \log_2 C_n^k M^k \right\rceil \right) \left( \frac{\gamma \left( N_L m, \frac{m}{\gamma} \gamma_{R+1} \right)}{\Gamma (N_L m)} \right) - \frac{\gamma \left( N_L m, \frac{m}{\gamma} \gamma_R \right)}{\Gamma (N_L m)}
\]

Figure 8.10 depicts the SE for 4-QAM-OFDM-IM with the number of active subcarriers, $k = \{1, 2, 3\}$, the number of antenna, $N_L = \{1, 2, 4, 6\}$, and $m = 4$. From figure 8.10, it is observed that the overall system SE is enhanced as the adaptive technique increases the $k$ and $N_L$ values. For example, about 1.5 bit/s/Hz improvement is observed in SE as $N_L$ increases at low SNR. Also, the SE is improved by 1 bit/s/Hz when $k$ is increased.
from $k = 1$ to $k = 2$ for adaptive OFDM-IM.

Figure 8.10: Spectral efficiency of adaptive M-QAM for OFDM-IM over Nakagami-m fading channel with a target BER of $10^{-3}$ as a function of SNR, $k$ and number of antennas (NL) with ($m = 4$).

### 8.4.3 System Energy Efficiency (bits/TNEU) of GD-MRC System

Using the same steps outlined in Chapter 7, the average EE for OFDM-IM for GD-MRC is obtained as:
\[
\overline{\eta_{SE}} = \sum_{R=1}^{N_R} \left( \left\lfloor \log_2 C_n^k M^k \right\rfloor \right) \left( P_r^{MRC}(R) = 1 - \frac{\Gamma \left( N_{Lm}, \frac{m}{\gamma} \gamma_R \right)}{\Gamma \left( N_{Lm} \right)} \frac{1}{\gamma} \right) - \frac{\Gamma \left( N_{Lm}, \frac{m}{\gamma} \gamma_{R+1} \right)}{\Gamma \left( N_{Lm} \right)} \frac{1}{\gamma}
\]

or
\[
\sum_{R=1}^{N_R} \left( \left\lfloor \log_2 C_n^k M^k \right\rfloor \right) \left( \frac{\gamma \left( N_{Lm}, \frac{m}{\gamma} \gamma_R \right)}{\Gamma \left( N_{Lm} \right)} \frac{1}{\gamma} \right) - \frac{\gamma \left( N_{Lm}, \frac{m}{\gamma} \gamma_{R+1} \right)}{\Gamma \left( N_{Lm} \right)} \frac{1}{\gamma}
\]

Figure 8.11 illustrates the EE of adaptive OFDM-IM for 4-QAM modulation with \( n = 4 \), \( k = \{1, 2, 3\} \), \( N_L = \{1, 2, 4, 6\} \) and \( m = 1 \). It can be noted from this figure that the EE improves significantly as the number of \( k \) decreases. For example, the EE is improved by 5% as \( k \) increases from \( k = 1 \) to \( k = 3 \).

Figure 8.11: Energy efficiency of adaptive M-QAM for OFDM-IM over Nakagami-m fading channel with a target BER of \( 10^{-3} \) as a function of SNR, \( k \) and number of antennas \( (N_L) \) with \( m = 4 \).
8.4.4 ABEP of GD-MRC System

The total average approximate upper bound of BEP (ABEP) for OFDM-IM over the fading channel can be characterized using the same steps outlined in Chapter 7, as:

\[
\text{BEP}_{R}^{MRC} = \frac{\gamma_{R+1}}{\gamma_{R}} \int_{\gamma_{R}}^{\gamma_{R+1}} \text{BEP}(R)P_{\gamma} (\gamma) \, d\gamma
\]  \hspace{1cm} (8.20)

\[
\text{BEP}_{R}^{MRC,1} = \frac{k \gamma_{R}}{2\Gamma (N_{Lm}) \Gamma (\beta + 0.5)} N_{Lm}^{\beta + 0.5} \left[ \Gamma (N_{Lm}, (\beta + 0.5) \gamma_{R}) - \Gamma (N_{Lm}, (\beta + 0.5) \gamma_{R+1}) \right]
\]  \hspace{1cm} (8.21)

\[
\text{BEP}_{R}^{MRC,2} = \frac{0.2k \gamma_{R}}{\Gamma (N_{Lm}) \Gamma (\beta + \alpha)} N_{Lm}^{\beta + \alpha} \left[ \Gamma (N_{Lm}, (\beta + \alpha) \gamma_{R}) - \Gamma (N_{Lm}, (\beta + \alpha) \gamma_{R+1}) \right]
\]  \hspace{1cm} (8.22)

\[
\text{BEP}_{R}^{MRC,3} = \frac{0.1k (N - k)}{2\Gamma (N_{Lm}) \Gamma (\beta + \alpha + 0.5)} N_{Lm}^{\beta + \alpha + 0.5} \left[ \Gamma (N_{Lm}, (\beta + \alpha + 0.5) \gamma_{R}) - \Gamma (N_{Lm}, (\beta + \alpha + 0.5) \gamma_{R+1}) \right]
\]  \hspace{1cm} (8.23)

In Figure 8.12, the ABEP of adaptive OFDM-IM system as a function of average SNR, \(\bar{\gamma}\), for various values of \(N_{L}\) and \(m = 1\) is plotted to maintain a maximum ABEP of \(10^{-3}\). It is noted that ABEP gets better with an increase in the number of antennas \((N_{L})\). For instance, ABEP is improved from \(10^{-3}\) to \(10^{-4}\) when \(N_{L}\) is increased for 1 to 2 at 20 dB. Moreover, it is observed that the carves in Figure 8.12 at \(N_{L}=4\) and \(N_{L}=6\) drop down around SNR=10 dB because of the switching area between BPSK and 4-QAM. Figure 8.13 illustrates the behavior of the ABEP against the number of active subcarriers \((k = \{1, 2, 3\})\) and fading parameter \((m = \{1, 2\})\) to maintain an ABEP of \(10^{-3}\). It is observed that the ABEP is going down around 10 dB because MRC works better at low SNR and it improves more when the number of antennas increases.
Figure 8.12: Adaptive ABEP performance of OFDM-IM with GD-MRC for $m = 2$, $k = 2$, and $N_L = 1, 2, 4, 6$ (target ABEP of $10^{-3}$ for Adaptive operation).

8.5 Chapter Summary

In this Chapter, the performance of adaptive OFDM-IM system and MRC diversity receiver has been examined over Nakagami-m fading channel. The closed-form expressions for average PEP and ABEP of the OFDM-IM system are derived over the proposed channel by taking into account the detection approaches of GD and GD-MRC. Moreover, exact average PEP, ABEP, SE, and EE are compared with the approximate closed-form expressions derived for the adaptive OFDM-IM system over Nakagami-m fading channel. The derived expressions provided the insight into the effect of index of the active subcarriers, antennas number, and adaptive level of M-QAM modulation on ABEP, SE and EE. The results show that the adaptive OFDM-IM system performs better than the conventional OFDM-IM system in terms of SE and EE.
Figure 8.13: Adaptive ABEP performance of OFDM-IM with GD-MRC for $m = 1, 2$, $k = 1, 2, 3$ (target ABEP of $10^{-3}$ for Adaptive operation).
Chapter 9
Concluding Remarks and Suggestion for Further Research

9.1 Introduction

This Chapter provides a summary of contributions of the thesis and offers suggestion for future research in the light of the results obtained. In Section 9.2, a summary of the thesis contributions are given, and the new directions for further research are presented in Section 9.3.

9.2 Summary of contributions

Chapter 2 provided an overview of an OFDM system. The transmitter and the receiver of the system have been presented and described with the aid of the system block-diagram. Also, this Chapter explained the mathematical representations of the transmitted and the received signals of an OFDM system. Moreover, the concept of index modulation using ON-OFF keying modulation scheme is presented.

In Chapter 3, wireless channel models that have been used in this thesis are presented and explained. In particular, Nakagami-m model is utilized for modeling multipath fading channel, and shadowed multipath Gamma-Nakagmi (GN) model is utilized to model the fading channel. The analytical descriptions of these models are given and the simulation methods that are used to simulate those in MATLAB are also discussed in this Chapter. These models are suitable to design and analyze wireless communication systems as the parameters of these models can be modified to realize many real wireless channel environments.

In Chapter 4, expressions for BER metric of ON-OFF keying, M-QAM and M-PSK Modulation schemes over Nakagami-m are derived in closed form, and are examined using
power saving policy (PSP) and power reallocation policy (PRP) as a function of SNR and fading channel parameter $m$. The numerical results show that PSP saves about 50% of consumed power compared to PRP, while PRP provides better system performance with identical fading parameters.

Chapter 5 investigated the PEP and ABEP of OFDM-IM system over Nakagami-$m$ fading channel. Closed-form expressions for approximate and exact upper bounds of PEP and ABEP have been derived using greedy detection method. The PEP and ABEP are then examined as a function of the index of the subcarriers ($k$), QAM modulation level ($M$), fading factor ($m$) and SNR. Then, the results have been validated through Monte Carlo simulation using MATLAB. The numerical results show that the OFDM-IM system performance deteriorates as fading impairments over channel become severe. For example, ABEP of 4-QAM with OFDM-IM increases approximately by 50% for severe multipath fading ($m = 1$) compared to ABEP of the same system in AWGN channel. Increasing modulation levels and number of active subcarriers decreases the energy efficiency while enhances the spectral efficiency.

In Chapter 6, analysis and evaluation of PEP and ABEP for OFDM-IM system are studied over compound shadowing and fading model (Nakagami-$m$ and Gamma (NG)) employing Meijer $G$ function approach. In the analysis of OFDM-IM system, multipath fading and shadowing, and co-channel interference (CCI) effects over the channel are considered for evaluating the error probabilities of indexed subcarriers, BPSK and the M-ary QAM schemes. The closed-form expressions of pairwise error probability (PEP) and average bit error probability (ABEP) are derived in the absence and presence of co-channel interference (CCI), and are illustrated as a function of number of active subcarriers ($k$), fading parameter ($m$), shadowing parameter ($c$), number of interferer ($N_I$), constellation level ($M$) and SNR. The numerical results on the ABEP performance highlight the effects of the fading severity and shadowing scatterers. For example, ABEP improves by nearly 30% at 20 dB when shadowing factor decreases from $c = 1$ to $c = 2$. The theoretical results have been validated by matching the corresponding results for Nakagami-$m$ and Rayleigh fading with $c = 2.2$ and $m = 1$ fading parameters.

Chapter 7 provides a framework for enhancing the spectral efficiency (SE) and energy efficiency (EE) of an OFDM-IM system in a multi-path fading channel superimposed on a shadowing environment, where a composite Nakagami-m-Gamma (NG)
channel model has been used. The closed-form expressions for PEP and ABEP are derived over the proposed channel model, and validated using Monte Carlo simulation. Also, the closed-form boundaries of SE and EE for OFDM-IM system with the M-QAM modulation scheme are investigated in regards to active subcarriers \((k, c, m)\) channel parameters, and SNR. It is observed that the SE and the EE are highly dependent on the channel parameters. Thus, the adaptive modulation scheme has been proposed to improve the efficiency of OFDM-IM system with M-QAM modulation scheme. The performance of the adaptive modulation with the OFDM-IM system is examined in terms of outage probability, average SE, average EE, and average bit error probability. The results show that by varying the number of active subcarriers \((k)\), the proposed adaptive OFDM-IM system not only enhances the system SE and EE but also adds flexibility in designing wireless communication system by trading-off between the efficiency metrics and ABEP performance.

In Chapter 8, Adaptive OFDM-IM scheme and Maximal Ratio Combing (MRC) diversity receiver are introduced over Nakagami-m fading channel to improve SE and EE. Closed-form expressions of approximate and exact upper boundaries of average PEP and ABEP are derived for OFDM-IM system over the proposed channel by taking into account the GD and GD-MRC approaches. Also, closed-form expressions of PEP, ABEP, SE, and EE are derived for adaptive OFDM-IM system over Nakagami-m fading channel. Adaptive OFDM-IM system appears to be a suitable strategy to design an efficient wireless communication system better than a conventional OFDM-IM system by adapting different value of active subcarriers, number of antennas, and modulation level according to SNR estimation. The results also show that the proposed adaptive OFDM-IM system is better than a conventional OFDM-IM system in terms of SE and EE.

9.3 Suggestions for Further Research

This thesis investigated the OFDM system with index modulation for wireless communication system. Index modulation is a relatively new idea used to provide an added dimension to increase the data rate of a wireless communication system and to reduce the energy consumption of the OFDM subcarriers.
One of the major goals of this research was to design a wireless system by combining OFDM system with index modulation (OFDM-IM) that can provide high date-rate with low energy consumption at an acceptable bit error rate, so that it can be used for future wireless technology. Based on this goal, the future research directions can focus on some of the following ideas:

1. PAPR in OFDM-IM: It is important to investigate the Peak-to-Average Power Ratio (PAPR) for OFDM-IM with different reduction techniques to enhance wireless communication system performance.

2. Channel Coding: It is worthwhile considering OFDM-IM with channel coding over fading channel to improve system performance and reduce power penalty.

3. Wireless Sensors: It is important to examine the implementation of OFDM-IM system with wireless sensors to decrease the energy consumption for green communication.

4. Security in Physical-layer: It is worthwhile considering OFDM-IM to improve the physical layer security in mobile communication by changing the active subcarriers dependent on the channel quality. Also, it is useful to employ OFDM-IM to control the secure outage probability (SOP) and average secrecy capacity (ASC) to enhance the quality of service of mobile communication [106]. Moreover, the secured system performance at the physical-layer in terms of SOP and BER ensures better performance in the higher layers, in terms of data-packet loss-rate [107].

5. Internet of Things: Security is one of the biggest issues in Internet of Things (IoT), and by employing OFDM-IM system another level of security to IoT systems can be added.

6. Mobile Devices: It is possible to extend the lifetime of a mobile device by using Adaptive OFDM-IM and maintain certain data rate by changing the number of active subcarriers and modulation level.
Appendix A

A.1 Mathematical Functions

1. The Gamma function is defined in [86, eq. (8.310.1)] as:

\[ \Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt, \quad [Re \ z > 0] \tag{A.1} \]

where the symbol \( Re \) indicates the real part. For positive integer \( z \), we have \( \Gamma(z) = (z - 1)! \) [86, eq. (8.339.1)].

The incomplete gamma function is defined in [86, eq. (8.350.1, 2)] as:

\[ \gamma(\alpha, x) = \int_0^x e^{-t} t^{\alpha-1} dt \quad [Re \ \alpha > 1] \tag{A.2} \]

where \( \gamma(\alpha, x) \) is incomplete Gamma function of corresponding elements of \( \alpha \) and \( x \).

\[ \Gamma(\alpha, x) = \int_x^\infty e^{-t} t^{\alpha-1} dt \tag{A.3} \]

where \( \Gamma(\alpha, x) \) is the inverse of incomplete Gamma function of corresponding elements of \( \alpha \) and \( x \). The Special cases of in \( \gamma(n, x) \) is given [86, eq. (8.352.6)] as:

\[ \gamma(\alpha, x) = (n - 1)! \left[ 1 - e^{-x} \sum_{m=0}^{n-1} \frac{x^m}{m!} \right] \tag{A.4} \]

2. The combinations of exponential and arbitrary power function, which used in this thesis, is given in [86, eq. (3.381.4, 1, 2, 3)] and [86, eq. (3.471.9)] as:

\[ \int_0^\infty x^{v-1} e^{-\mu x} dx = \frac{1}{\mu^v} \Gamma(v), \quad [Re \ \mu > 0, Re \ v > 0] \tag{A.5} \]
\[ \int_0^u x^{v-1} e^{\mu x} dx = \mu^{-v} \gamma(v, \mu u), \quad [\text{Re } v > 0] \quad (A.6) \]

\[ \int_0^u x^{p-1} e^{-x} dx = \sum_{k=0}^{\infty} (-1)^k \frac{u^{p+k}}{k! (p+k)} \]

\[ = e^{-u} \sum_{k=0}^{\infty} \frac{u^{p+k}}{\rho (p+1) \cdots (p+k)} \quad (A.7) \]

\[ \int_u^{\infty} x^{v-1} e^{-\mu x} dx = \mu^{-v} \Gamma(v, \mu u), \quad [u > 0, \text{Re } > 0] \quad (A.8) \]

\[ \int_0^{\infty} x^{v-1} e^{-\beta x - \gamma x} dx = 2 \left( \frac{\beta}{\gamma} \right)^{\frac{v}{2}} K_v \left( 2 \sqrt{\beta \gamma} \right) \quad (A.9) \]

3. The combinations of Bessel function of more complicated argument, exponential and power

The integration of combination of Bessel function, exponential and power is given in [86, eq. (6.643.3)] as:

\[ \int_0^{\infty} x^{\mu - \frac{1}{2}} e^{-\alpha x} K_{2v} (2\beta \sqrt{x}) dx = \frac{\Gamma \left( \mu + v + \frac{1}{2} \right) \Gamma \left( \mu - v + \frac{1}{2} \right)}{2\beta} \beta^{-1} e^{2\alpha} e^{-\mu} W_{-\mu,v} \left( \frac{\beta^2}{\alpha} \right) \left[ \text{Re} \left( \mu + v + \frac{1}{2} \right) \right] \quad (A.10) \]

4. The transformation and integration of Meijer G-function

The traditional notation of Meijer G-function is given as:

\[ G_{p,q}^{m,n} \left( z x \left| \begin{array}{c} a_1, \cdots, a_p \\ b_1, \cdots, b_q \end{array} \right. \right), \quad [98, (07.34.02.0001.01)] \quad (A.11) \]
The mathematica standard form notation that is used in programs like Matlab is expressed as:

\[ MeijerG \left[ \left\{ \{a_1 \cdot \cdot \cdot , a_n \} , \{a_{n+1} \cdot \cdot \cdot , a_p \} \right\} , \left\{ \{b_1 \cdot \cdot \cdot , b_m \} , \{b_{m+1} \cdot \cdot \cdot , a_q \} \right\} , z \right], \]

\[ [98, (07.34.02.0001.01)] \]

\[ e^{-x} = G_{0,1}^{1,0} \left( x \left| \begin{array}{c} 0 \\ \cdot \end{array} \right. \right), \] \[ [96, eq.(8.4.3.1)] \] \[ (A.12) \]

\[ K_v \left( 2\sqrt{x} \right) = \frac{1}{2} G_{0,2}^{2,0} \left( x \left| \begin{array}{c} 0 \\ -\frac{v}{2}, \frac{v}{2} \end{array} \right. \right), \] \[ [96, eq.(7.4.23.1)] \] \[ (A.13) \]

\[ \int_0^\infty x^{\alpha-1} e^{-\beta x} G_{p,q}^{m,n} \left( \alpha x \left| \begin{array}{c} a_1, \cdot \cdot \cdot , a_p \\ b_1, \cdot \cdot \cdot , b_q \end{array} \right. \right) dx = \beta^{\rho-1} G_{p+1,q}^{m,n+1} \left( \alpha \left| \begin{array}{c} \rho, a_1, \cdot \cdot \cdot , a_p \\ b_1, \cdot \cdot \cdot , b_q \end{array} \right. \right), \] \[ [86, eq.(7.813.1)] \] \[ (A.14) \]

The classical Meijer G-function integral from two functions is given as:

\[ \int_0^\infty x^{\alpha-1} G_{a,t}^{b,u} \left( \tau w \left| \begin{array}{c} c_1, \cdot \cdot \cdot , c_t, c_{t+1}, \cdot \cdot \cdot , c_u \\ d_1, \cdot \cdot \cdot , d_{s+1}, d_1, \cdot \cdot \cdot , d_u \end{array} \right. \right) G_{p,q}^{m,n} \left( \tau z \left| \begin{array}{c} a_1, \cdot \cdot \cdot , a_n, a_{n+1}, \cdot \cdot \cdot , a_p \\ b_1, \cdot \cdot \cdot , b_m, b_{m+1}, \cdot \cdot \cdot , b_q \end{array} \right. \right) d\tau = w^{-\alpha} G_{v+p,u+q}^{m+t,n+s} \left( \frac{z}{w} \left| \begin{array}{c} a_1, \cdot \cdot \cdot , a_n, 1 - \alpha - d_1, \cdot \cdot \cdot , 1 - \alpha - d_s, \cdot \cdot \cdot , 1 - \alpha - d_{s+1}, \cdot \cdot \cdot , 1 - \alpha - d_v, a_{n+1}, \cdot \cdot \cdot , a_p \\ b_1, \cdot \cdot \cdot , b_m, 1 - \alpha - c_1, \cdot \cdot \cdot , 1 - \alpha - c_t, \cdot \cdot \cdot , 1 - \alpha - c_{t+1}, \cdot \cdot \cdot , 1 - \alpha - c_u, b_{m+1}, \cdot \cdot \cdot , b_q \end{array} \right. \right), \] \[ [98, (07.34.21.0011.01)] \] \[ (A.15) \]
A.2 Derivation of Equation (5.25) and (5.26) in Chapter 5

A.2.1 Derivation of the exact ABEP

The exact ABEP of OFDM-IM with M-QAM scheme is given in equation (5.23) as:

\[
ABEP_{Naka} = k \int_0^\infty \left[ PEP \times P_\gamma(\gamma) d\gamma + 1/k \int_0^\infty P_{\epsilon M} \times P_\gamma(\gamma) d\gamma - k \int_0^\infty PEP \times P_{\epsilon M} \times P_\gamma(\gamma) d\gamma \right] d\gamma
\]

\[
+ 1/k \int_0^\infty 0.2e^{\left(-3\gamma\right)\left(2(M-1)\right)} \times \frac{m^m \gamma^{m-1}}{\gamma^m \Gamma(m)} e^{-\gamma^m \gamma} d\gamma
\]

\[
- k \int_0^\infty \left[ 1 - \sum_{q=0}^{n-k} \left( \frac{n-k}{q} \right) \frac{-1^q}{q+1} e^{-\gamma\left(q+1\right)} \right] \times 0.2e^{\left(-3\gamma\right)\left(2(M-1)\right)} \times \frac{m^m \gamma^{m-1}}{\gamma^m \Gamma(m)} e^{-\gamma^m \gamma} d\gamma
\]

\[
(A.17)
\]

The equation (A.17) can be determined by dividing into the parts, \(ABEP_{Naka}^1, ABEP_{Naka}^2, ABEP_{Naka}^3\) and each part of ABEP is going to integrated sprightly:

\[
ABEP_{Naka}^1 = k \int_0^\infty \left[ 1 - \sum_{q=0}^{n-k} \left( \frac{n-k}{q} \right) \frac{-1^q}{q+1} e^{-\gamma\left(q+1\right)} \right] \times \frac{m^m \gamma^{m-1}}{\gamma^m \Gamma(m)} e^{-\gamma^m \gamma} d\gamma \quad (A.18)
\]
\[ ABEP_{Naka}^1 = k \int_0^\infty \frac{m^m \gamma^{m-1} e^{-\gamma/m} \gamma^m \Gamma(m)}{\Gamma(m)} \frac{1}{\gamma^m \Gamma(m)} e^{-\gamma/m} d\gamma - k \int_0^\infty \frac{m^m \gamma^{m-1} e^{-m\gamma}}{\gamma^m \Gamma(m)} e^{-\gamma/m} \sum_{q=0}^{n-k} \left( \begin{array}{c} n-k \\ q \end{array} \right) (-1)^q \frac{1}{q+1} \frac{e^{-\gamma/q+1}}{e^{-\gamma/q+1}} d\gamma \]  
(A.19)

The (5.5) can be simplify to:

\[ ABEP_{Naka}^1 = k - k \sum_{q=0}^{n-k} \left( \begin{array}{c} n-k \\ q \end{array} \right) (-1)^q \frac{1}{q+1} \frac{m^m}{\gamma^m \Gamma(m)} \int_0^\infty \gamma^{m-1} e^{-\gamma/m \left( \frac{m}{\gamma} + \frac{q}{q+1} \right)} d\gamma \]  
(A.20)

Using the Table of Integrals [86, (3.381.4)], the first part of ABEP over the Nakagami-m fading channel can be formulated as:

\[ ABEP_{Naka}^1 = k - k \sum_{q=0}^{n-k} \left( \begin{array}{c} n-k \\ q \end{array} \right) (-1)^q \frac{1}{q+1} \frac{m^m}{\gamma^m \Gamma(m)} \times \frac{1}{\left( \frac{m}{\gamma} + \frac{q}{q+1} \right)^m} \]  
(A.21)

The equation (A.21) can be simplify as:

\[ ABEP_{Naka}^1 = k - k \sum_{q=0}^{n-k} \left( \begin{array}{c} n-k \\ q \end{array} \right) (-1)^q \frac{1}{q+1} \frac{m^m}{\gamma^m \Gamma(m)} \times \left( \frac{m}{\gamma \left( \frac{m}{\gamma} + \frac{q}{q+1} \right)} \right)^m \]  
(A.22)

Where \( L = \frac{q}{q+1} \).

The second part, \( ABEP_{Naka}^2 \) of equation (A.17) is given by:
\[ ABEP^2_{\text{Naka}} = \frac{1}{k} \int_0^\infty 0.2 e^{-\frac{3\gamma}{2(M-1)}} \times \frac{m^m \gamma^{m-1}}{\gamma^m \Gamma(m)} e^{-\frac{m \gamma}{\gamma}} d\gamma \]

\[ = \frac{0.2 m^m}{k \gamma^m \Gamma(m)} \int_0^\infty \gamma^{m-1} e^{-\gamma \left(\frac{3}{2(M-1)} + \frac{m}{\gamma}\right)} d\gamma \]

(A.23)

Using the Table of integrals [86, (3.381.4)], the \( ABEP^2_{\text{Naka}} \) can be expressed as:

\[ ABEP^2_{\text{Naka}} = \frac{0.2 m^m}{k \gamma^m \Gamma(m) \alpha} \times \frac{1}{\left(\frac{3}{2(M-1)} + m \gamma\right)^m \Gamma(m)} = \frac{0.2 m^m}{k \gamma^m \left(\frac{3}{2(M-1)} + m \gamma\right)^m} = \frac{0.2 m^m}{k \left(\frac{3}{2(M-1)} + \frac{m \gamma}{\gamma}\right)^m} \]

(A.24)

The equation (A.24) can be expressed as:

\[ ABEP^2_{\text{Naka}} = \frac{0.2}{k} \left(\frac{m}{\frac{3}{2(M-1)} + m}\right)^m = \frac{0.2}{k} \left(\frac{m}{\alpha \gamma + m}\right)^m \]  

(A.25)

where \( \alpha = \frac{3}{2(M-1)} \).

The third part, \( ABEP^3_{\text{Naka}} \) of equation (A.17) is given by:

\[ ABEP^3_{\text{Naka}} = k \int_0^\infty \left[1 - \sum_{q=0}^{n-k} \binom{n-k}{q} \left(-\frac{1}{q+1} e^{-\gamma \left(\frac{-q}{q+1}\right)} \right) \right] \times 0.2 e^{-\frac{3\gamma}{2(M-1)}} \times \frac{m^m \gamma^{m-1}}{\gamma^m \Gamma(m)} e^{-\frac{m \gamma}{\gamma}} d\gamma \]

(A.26)
The equation (A.26) is simplified to

\[ ABEP_{\text{Naka}}^3 = 0.2 \, k \, \frac{m^m \gamma^{m-1}}{\gamma^m \Gamma(m)} \int_0^\infty e^{\left(\frac{-3\gamma}{2(M-1)}\right)} \times e^{\frac{-m\gamma}{\gamma}} \, d\gamma - 0.2k \sum_{q=0}^{n-k} \frac{(n-k)}{q} \frac{-1^q}{q+1} \]

\[ \times \frac{m^m \gamma^{m-1}}{\gamma^m \Gamma(m)} \int_0^\infty e^{-\gamma\left(\frac{-q}{q+1}\right)} \times e^{\left(\frac{-3\gamma}{2(M-1)}\right)} \times e^{\frac{-m\gamma}{\gamma}} \, d\gamma \]

(A.27)

The first part of equation (A.27) is equal to equation (A.25). Therefore, equation (A.27) can be written as:

\[ ABEP_{\text{Naka}}^3 = 0.2 \, k \, \left(\frac{m}{\alpha \gamma + m}\right)^m \frac{m^m}{\gamma^m \Gamma(m)} \int_0^\infty \gamma^{m-1} e^{-\gamma\left(\frac{q}{q+1} + \frac{3}{2(M-1)} + \frac{\gamma}{\gamma}\right)} \, d\gamma \]

(A.28)

Using the Table of Integrals, [86, (3.381.4)], the ABEP\(_{\text{Naka}}^3\) can be given as:

\[ ABEP_{\text{Naka}}^3 = 0.2 \, k \, \left(\frac{m}{\alpha \gamma + m}\right)^m - 0.2k \sum_{q=0}^{n-k} \frac{(n-k)}{q} \frac{-1^q}{q+1} \times \frac{m^m}{\gamma^m \Gamma(m)} \]

\[ \times \left(\frac{q}{q+1} + \frac{3}{2(M-1)} + \frac{\gamma}{\gamma}\right)^m \Gamma(m) \]

(A.29)

\[ = 0.2 \, k \, \left(\frac{m}{\alpha \gamma + m}\right)^m - 0.2k \sum_{q=0}^{n-k} \frac{(n-k)}{q} \frac{-1^q}{q+1} \left(\frac{m}{(L+\alpha) \gamma + m}\right)^m \]

(A.30)

The third part of ABEP can be obtained from equation (A.29) as:

\[ ABEP_{\text{Naka}}^3 = 0.2 \, k \, \left(\frac{m}{\alpha \gamma + m}\right)^m - 0.2k \sum_{q=0}^{n-k} \frac{(n-k)}{q} \frac{-1^q}{q+1} \left(\frac{m}{(L+\alpha) \gamma + m}\right)^m \]

(A.30)

The overall exact ABEP for OFDM-IM over Nakagami-m fading channel is obtained by
combining $ABEP_{Naka}^1$, $ABEP_{Naka}^2$ and $ABEP_{Naka}^3$ as:

$$ABEP_{Naka} = k - k \sum_{q=0}^{n-k} \binom{n-k}{q} (-1)^q \frac{1}{q + 1} \left( \frac{m}{L \hat{\gamma} + m} \right)^m + 0.2 k \left( \frac{m}{\alpha \hat{\gamma} + m} \right)^m$$

$$-0.2 k \left( \frac{m}{\alpha \hat{\gamma} + m} \right)^m + 0.2 k \sum_{q=0}^{n-k} \binom{n-k}{q} \frac{-1}{q + 1} \left( \frac{m}{(L + \alpha \hat{\gamma}) + m} \right)^m$$

$$= k \left[ 1 - \sum_{q=0}^{n-k} \binom{n-k}{q} (-1)^q \frac{1}{q + 1} \left( \frac{m}{L \hat{\gamma} + m} \right)^m + (1/k - 1) \left( \frac{m}{\alpha \hat{\gamma} + m} \right)^m \right]$$

(A.31)

### A.2.2 Derivation of the Approximate ABEP

The approximate ABEP of OFDM-IM with M-QAM scheme is given in equation (5.24) as:

$$ABEP_{Naka,approx} = k \int_0^\infty PEP \times P_\gamma(\gamma) d\gamma + 1/k \int_0^\infty P_eM \times P_\gamma(\gamma) d\gamma - k \int_0^\infty PEP \times P_eM$$

$$\times P_\gamma(\gamma) d\gamma$$

$$= k \int_0^\infty \frac{(n-k)}{2} e^{-\gamma} \frac{m\gamma^{m-1}}{\Gamma(m)} e^{-m \gamma} d\gamma + 1/k \int_0^\infty 0.2 e^{(-3\gamma)2(M-1)} \frac{m\gamma^{m-1}}{\Gamma(m)} e^{-m \gamma} d\gamma$$

$$- k \int_0^\infty \frac{(n-k)}{2} e^{-\gamma} \times 0.2 e^{(-3\gamma)2(M-1)} \frac{m\gamma^{m-1}}{\Gamma(m)} e^{-m \gamma} d\gamma$$

(A.32)

The second part of equation (A.32) is equal to $ABEP_{Naka}^2$ in equation (A.25). Using the Table of Integrals, [86, (3.381.4)], the $ABEP_{Naka,approx}$ can be expressed as:
After some manipulation in the equation (A.33), the approximate ABEP of OFDM-IM with M-QAM scheme is given as:

\[
ABEP_{Naka,\text{approx}} = \frac{k(n-k)}{2} \frac{m^m}{\bar{\gamma}m\Gamma(m)} \int_{0}^{\infty} \gamma^{m-1} e^{-\gamma\left(\frac{1}{2} + \frac{m}{\bar{\gamma}}\right)} d\gamma + \frac{0.2}{k} \left(\frac{m}{\alpha\bar{\gamma} + m}\right)^m
\]

After some manipulation in the equation (A.33), the approximate ABEP of OFDM-IM with M-QAM scheme is given as:

\[
ABEP_{Naka,\text{approx}} = \frac{0.2k(n-k)}{2} \left(\frac{m}{0.5\bar{\gamma} + m}\right)^m + \frac{0.2}{k} \left(\frac{m}{\alpha\bar{\gamma} + m}\right)^m - \frac{0.2k(n-k)}{2} \left(\frac{m}{(0.5 + \alpha)\bar{\gamma} + m}\right)^m
\]  

(A.34)

A.3 Derivation of Equation (6.5), (6.8), (6.13), (6.14), (6.22), (6.25), (6.28), and (6.31) in Chapter 6

The derivation detail of equation (6.3) is given as:

\[
PEP_{\text{Exact}} = \int_{0}^{\infty} \frac{2}{\Gamma(c)\Gamma(m)} \left(\frac{cm}{\bar{\gamma}}\right)^{\frac{c+m-2}{2}} K_{c-m} \left(2\sqrt{\frac{cm}{\bar{\gamma}}}\right) d\gamma - \frac{1}{2} \left(\frac{cm}{c-m\gamma}\right)^{0} \frac{2}{(c-m\gamma)^{m}}
\]

\[
PEP^{1}_{\text{Exact}} = \sum_{q=0}^{n-k} \left(\frac{n-k}{q}\right) \int_{0}^{\infty} \frac{c+m-2}{2} e^{\left(-q\frac{c}{q+1}\right)} \times G_{0,2}^{2,0} \left(\frac{cm\bar{\gamma}}{c-m\gamma}, \frac{0}{m-c\gamma} \right) d\gamma
\]

\[
PEP^{2}_{\text{Exact}}
\]

(A.35)
\[ PEP_{\text{Exact}}^{1} = \int_{0}^{\infty} \frac{2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\bar{\gamma}} \right)^{\frac{c+m}{2}} \gamma^{\frac{c+m-2}{2}} K_{c-m} \left( 2\sqrt{\frac{cm}{\bar{\gamma}}} \gamma \right) d\gamma \]  

(A.36)

The second part of equation (A.35) can be integrated using (A.15) as

\[ PEP_{\text{Exact}}^{2} = -\frac{1}{2} \int_{0}^{\infty} \gamma^{\frac{c+m-2}{2}} e^{\left( \frac{-q\gamma}{q+1} \right)} \frac{1}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\bar{\gamma}} \right)^{\frac{c+m}{2}} \sum_{q=0}^{\infty} \left( \frac{n-k}{q+1} \right)^{q+1} \left( \frac{q}{q+1} \right)^{-\left( \frac{c+m-1}{2} \right)} \right] \times G_{0,2}^{2,0} \left( \frac{cm\gamma}{\bar{\gamma}} \left| \frac{0}{c-m, m-c} \right. \right) d\gamma \]  

= -\frac{1}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\bar{\gamma}} \right)^{\frac{c+m}{2}} \sum_{q=0}^{\infty} \left( \frac{n-k}{q+1} \right)^{q+1} \left( \frac{q}{q+1} \right)^{-\left( \frac{c+m-1}{2} \right)} \right] \times G_{1,2}^{2,1} \left( \frac{cm\gamma}{\bar{\gamma}} \left| \frac{1 - \left( \frac{c+m}{2} \right)}{c-m, m-c} \right. \right) d\gamma \]  

(A.37)

After some manipulation, the overall exact PEP is expressed as:

\[ PEP_{\text{Exact}} = k \left[ 1 - \frac{1}{2\Gamma(c) \Gamma(m)} \beta^{n-k} \sum_{q=0}^{\infty} \left( \frac{n-k}{q+1} \right)^{q+1} \left( \frac{q}{q+1} \right)^{-\left( \frac{c+m-1}{2} \right)} \right] \times L^{-\nu} G_{1,2}^{2,1} \left( \frac{\beta}{L} \left| \frac{1 - \left( \frac{c+m}{2} \right)}{c-m, m-c} \right. \right) \]  

(A.38)

where \( \beta = \frac{cm}{\bar{\gamma}} \), \( \nu = \frac{c+m-1}{2} \), and \( L = \frac{q}{q+1} \).

The approximate of PEP in (6.6) can be integrated following same step of above equation using (A.15) as:

\[ PEP_{\text{approx}} \approx \frac{n-k}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\bar{\gamma}} \right)^{\frac{c+m}{2}} \int_{0}^{\infty} \gamma^{\frac{c+m-1}{2}} e^{-\frac{\gamma}{2}} \frac{1}{\Gamma(c) \Gamma(m)} \left( \frac{cm\gamma}{\bar{\gamma}} \left| \frac{0}{c-m, m-c} \right. \right) d\gamma \]  

\[ \approx \frac{n-k}{2\Gamma(c) \Gamma(m)} \left( \frac{2cm}{\bar{\gamma}} \right)^{\frac{c+m}{2}} \times \frac{1}{\Gamma} G_{1,2}^{2,1} \left( \frac{cm\gamma}{\bar{\gamma}} \left| \frac{1 - \left( \frac{c+m}{2} \right)}{c-m, m-c} \right. \right) \]  

(A.39)
After some changing $\beta = \frac{cm}{\gamma}$, $\nu = \frac{c+m-1}{2}$, the overall approximate PEP is expressed as:

$$PEP_{\text{approx}} \simeq k \left[ \frac{n-k}{2\Gamma(c)\Gamma(m)} (2\beta)^\nu G_1^{2,1} \left( 2\beta \left| \frac{1 - \frac{(c+m)}{2}}{\frac{c-m}{2}, \frac{m-c}{2}} \right. \right) \right]$$

(A.40)

A.3.1 Exact Average Bit Error Probability (ABEP) over Fading and Shadowing Channel

$$ABEP = 2 \Gamma(c) \Gamma(m) \left( \frac{cm}{\gamma} \right) \frac{c+m}{2} \frac{1}{\gamma} \int_0^\infty \frac{e^{\frac{c+m-2}{2}}}{\gamma} \times PEP \times G_0^{2,0} \left( \frac{cm}{\gamma} \left| \frac{0}{\frac{c-m}{2}, \frac{m-c}{2}} \right. \right) d\gamma$$

$$+ \frac{0.2}{2} \frac{2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right) \frac{c+m}{2} \int_0^\infty \frac{e^{\frac{-3\gamma}{2(M-1)}}}{\gamma} \times G_0^{2,0} \left( \frac{cm}{\gamma} \left| \frac{0}{\frac{c-m}{2}, \frac{m-c}{2}} \right. \right) d\gamma$$

$$- \frac{0.2}{2} \frac{2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right) \frac{c+m}{2} \int_0^\infty \frac{e^{\frac{-3\gamma}{2(M-1)}}}{\gamma} \times PEP \times G_0^{2,0} \left( \frac{cm}{\gamma} \left| \frac{0}{\frac{c-m}{2}, \frac{m-c}{2}} \right. \right) d\gamma$$

(A.41)

The $ABEP_{\text{Exact}}^{QAM,1}$ can be obtained by substituting (5.1) and (A.13) into (A.42) as:
After changing $\beta = \frac{cm}{\gamma}$ and $L = \frac{q}{q+1}$, the overall $ABEP^{1}_{QAM}$ is expressed as:

$$ABEP^{QAM,1}_{Exact} = 1 - \frac{1}{\Gamma(c) \Gamma(m)} \sum_{q=0}^{n-k} \left( \begin{array}{c} n-k \\ q \end{array} \right) \frac{-1^q}{q+1} \exp\left( -\frac{\gamma}{q+1} \right) G_{0,1}^{0,0} \left( \frac{cm}{\gamma}, \frac{c}{2}, \frac{m-c}{2}, 0 \right)$$

$$\times \left( \begin{array}{c} \frac{\gamma}{q+1} \\ \frac{m}{2} \end{array} \right)$$

The $ABEP^{QAM,2}_{Exact}$ is integrated using (A.16) and (A.13) to change the exponential to Meijer G-function, $e^{\frac{-\gamma}{2(M-1)}} = G_{0,1}^{1,0} \left( \begin{array}{c} -\frac{\gamma}{2(M-1)} \\ 0 \end{array} \right)$ as:

$$ABEP^{QAM,2}_{Exact} = 1 - \frac{1}{\Gamma(c) \Gamma(m)} \sum_{q=0}^{n-k} \left( \begin{array}{c} n-k \\ q \end{array} \right) \frac{-1^q}{q+1} G_{1,2}^{2,1} \left( \frac{L}{\beta}, \frac{1-c, 1-m}{0} \right)$$

(A.43)
After changing $\beta = \frac{cm}{\gamma}$ and $\alpha = \frac{3}{2(M-1)}$, the overall $ABEP_{Exact}^{QAM,2}$ is expressed as:

$$ABEP_{Exact}^{QAM,2} = 0.2 \frac{2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right)^{\frac{c+m}{2}} \Gamma(c) \Gamma(m) \int_{0}^{\infty} \frac{c+m-2}{2} \times G_{0,1}^{1,0} \left( \frac{-3}{2(M-1)} \bigg| 0 \right) \times G_{0,2}^{2,1} \left( \frac{0}{c-m, m-c} \right) d\gamma$$

$$= \frac{0.2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right)^{\frac{c+m}{2}} \left( \frac{cm}{\gamma} \right)^{-\frac{c+m}{2}} \left( \frac{c+m}{2} \right)$$

$$= \frac{0.2}{\Gamma(c) \Gamma(m)} G_{1+1,0+2}^{0+0+1} \left( \frac{3}{2(M-1)} \bigg| \begin{array}{c} 1 - \frac{c}{2} - \frac{m}{2}, \frac{c}{2} - \frac{m}{2}, 1 - \frac{c}{2} - \frac{m}{2}, 1 - \frac{c}{2} + \frac{m}{2} \end{array} \right)$$

$$= \frac{0.2}{\Gamma(c) \Gamma(m)} G_{1,2}^{2,1} \left( \frac{3}{2(M-1)} \bigg| \begin{array}{c} 1 - c, 1 - m \end{array} \right)$$

(A.44)

The $ABEP_{Exact}^{QAM,3}$ can be obtained by substituting (5.1) and (A.13) into (A.46) as:
$$ABEP_{Exact}^{QAM,3} = \frac{0.2}{2} \frac{2}{\Gamma(c) \Gamma(m)} \left(\frac{cm}{\bar{\gamma}}\right)^{\frac{c+m}{2}} \gamma^\frac{c+m-2}{2} \int_0^\infty \gamma^\frac{c+m-2}{2} \left[ 1 - \sum_{q=0}^{n-k} \frac{(n-k)}{q} \left( -1 \right)^q e^{-\gamma (\frac{q}{q+1})} \right]$$

$$\times \left( \frac{-3\gamma}{2(M-1)} \right) \times G_{0,2}^{2,0} \left( \frac{cm\gamma}{\bar{\gamma}} \left| \begin{array}{c} 0 \\ c-m, m-c \end{array} \right. \right) d\gamma$$

$$= \frac{0.2}{\Gamma(c) \Gamma(m)} \left(\frac{cm}{\bar{\gamma}}\right)^{\frac{c+m}{2}} \gamma^\frac{c+m-2}{2} \times e^{-\gamma (\frac{q}{q+1} + \frac{3}{2(M-1)})} \times G_{0,2}^{2,0} \left( \frac{cm\gamma}{\bar{\gamma}} \left| \begin{array}{c} 0 \\ c-m, m-c \end{array} \right. \right) d\gamma$$

$$- \sum_{q=0}^{n-k} \frac{(n-k)}{q+1} \times \left( \frac{cm}{\bar{\gamma}}\right)^{\frac{c+m}{2}} \gamma^\frac{c+m-2}{2} \times e^{-\gamma (\frac{q}{q+1} + \frac{3}{2(M-1)})} \times G_{0,2}^{2,0} \left( \frac{cm\gamma}{\bar{\gamma}} \left| \begin{array}{c} 0 \\ c-m, m-c \end{array} \right. \right) d\gamma$$

The first part of (A.46) is equal to $ABEP_{Exact}^{QAM,3}$ in (A.42) and after convert exponential to Mejeir G function, it is given by:

$$ABEP_{Exact}^{QAM,3} = \frac{0.2}{\Gamma(c) \Gamma(m)} G_{1,2}^{2,1} \left( \frac{\alpha}{\beta} \left| \begin{array}{c} 1-c, 1-m \\ 0 \end{array} \right. \right) - \frac{0.2}{\Gamma(c) \Gamma(m)} \left(\frac{cm}{\bar{\gamma}}\right)^{\frac{c+m}{2}}$$

$$\sum_{q=0}^{n-k} \frac{(n-k)}{q+1} \int_0^\infty \gamma^\frac{c+m-2}{2} \times e^{-\gamma (\frac{q}{q+1} + \frac{3}{2(M-1)})} \times G_{0,2}^{2,0} \left( \frac{cm\gamma}{\bar{\gamma}} \left| \begin{array}{c} 0 \\ c-m, m-c \end{array} \right. \right) d\gamma$$

$$= \frac{0.2}{\Gamma(c) \Gamma(m)} G_{1,2}^{2,1} \left( \frac{\alpha}{\beta} \left| \begin{array}{c} 1-c, 1-m \\ 0 \end{array} \right. \right) - \frac{0.2}{\Gamma(c) \Gamma(m)} \left(\frac{cm}{\bar{\gamma}}\right)^{\frac{c+m}{2}}$$

$$\sum_{q=0}^{n-k} \frac{(n-k)}{q+1} \int_0^\infty \gamma^\frac{c+m-2}{2} \times G_{0,1}^{1,0} \left( -\gamma \left( \frac{q}{q+1} + \frac{3}{2(M-1)} \right) \left| \begin{array}{c} 0 \\ \end{array} \right. \right) \times$$

$$G_{0,2}^{2,0} \left( \frac{cm\gamma}{\bar{\gamma}} \left| \begin{array}{c} 0 \\ c-m, m-c \end{array} \right. \right) d\gamma$$

The above equation can be integrated using (A.16) as:

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\[
\text{ABEP}_{\text{Exact}}^{QAM,3} = \frac{0.2}{\Gamma(c) \Gamma(m)} G_{1,2}^{2,1} \left( \alpha \left| \begin{array}{c} 1 - c, 1 - m \\ 0 \end{array} \right. \right) - \frac{0.2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\bar{\gamma}} \right)^{c+m \frac{L}{2}} \\
\sum_{q=0}^{n-k} \left( \begin{array}{c} n-k \\ q \end{array} \right) \frac{-1}{q+1} \int_{0}^{\infty} \frac{c+m-2}{\gamma} \times G_{0,1}^{1,0} \left( -\gamma \left( \frac{q}{q+1} + \frac{3}{2(M-1)} \right) \right) \bigg|_{0} \\
G_{0,2}^{2,0} \left( \frac{cm\gamma}{\bar{\gamma}} \left| \begin{array}{c} 0 \\ \frac{c-m}{2}, \frac{m-c}{2} \end{array} \right. \right) d\gamma
\]

\[
= \frac{0.2}{\Gamma(c) \Gamma(m)} G_{1,2}^{2,1} \left( \alpha \left| \begin{array}{c} 1 - c, 1 - m \\ 0 \end{array} \right. \right) - \frac{0.2}{\Gamma(c) \Gamma(m)} \frac{n-k}{q+1} \sum_{q=0}^{n-k} \left( \begin{array}{c} n-k \\ q \end{array} \right) \frac{-1}{q+1} \\
\times \left( \frac{cm}{\bar{\gamma}} \right)^{c+m \frac{L}{2}} G_{0,1}^{1,0} \left( \frac{q+1 + \frac{3}{2(M-1)}}{\frac{cm}{\bar{\gamma}}} \left| \begin{array}{c} 1 - c - m, -\frac{c}{2} + \frac{m}{2}, 1 - \frac{q}{2} - \frac{m}{2}, -\frac{q}{2} + \frac{m}{2} \\ 0 \end{array} \right. \right)
\]

\[
= \frac{0.2}{\Gamma(c) \Gamma(m)} G_{1,2}^{2,1} \left( \alpha \left| \begin{array}{c} 1 - c, 1 - m \\ 0 \end{array} \right. \right) - \frac{0.2}{\Gamma(c) \Gamma(m)} \frac{n-k}{q+1} \sum_{q=0}^{n-k} \left( \begin{array}{c} n-k \\ q \end{array} \right) \frac{-1}{q+1} \\
G_{0,1}^{1,0} \left( \frac{q+1 + \frac{3}{2(M-1)}}{\frac{cm}{\bar{\gamma}}} \left| \begin{array}{c} 1 - c - m, -\frac{c}{2} + \frac{m}{2}, 1 - \frac{q}{2} - \frac{m}{2}, -\frac{q}{2} + \frac{m}{2} \\ 0 \end{array} \right. \right)
\]

(A.48)

The \(\text{ABEP}_{\text{Exact}}^{QAM,3}\) is expressed After changing \(\beta = \frac{cm}{\bar{\gamma}}, L = \frac{q}{q+1}\) and \(\alpha = \frac{3}{2(M-1)}\) as:

\[
\text{ABEP}_{\text{Exact}}^{QAM,3} = \frac{0.2}{\Gamma(c) \Gamma(m)} G_{1,2}^{2,1} \left( \alpha \left| \begin{array}{c} 1 - c, 1 - m \\ 0 \end{array} \right. \right) - \frac{0.2}{\Gamma(c) \Gamma(m)} \frac{n-k}{q+1} \sum_{q=0}^{n-k} \left( \begin{array}{c} n-k \\ q \end{array} \right) \frac{-1}{q+1} \\
G_{0,1}^{1,0} \left( \frac{L + \alpha}{\beta} \left| \begin{array}{c} 1 - c, 1 - m \\ 0 \end{array} \right. \right)
\]

(A.49)

The exact overall \(\text{ABEP}\) can be obtained by combing \(\text{ABEP}_{\text{Exact}}^{QAM,1}\), \(\text{ABEP}_{\text{Exact}}^{QAM,2}\) and \(\text{ABEP}_{\text{Exact}}^{QAM,3}\).
\[
ABEP_{Exact}^{\text{QAM}} = k - \frac{k}{\Gamma(c) \Gamma(m)} \sum_{q=0}^{n-k} \binom{n-k}{q} \frac{(-1)^q}{q+1} \left[ G_{1,2}^{2,1} \left( \frac{L}{\beta} \middle| \frac{1-c, 1-m}{0} \right) + \right. \\
0.2 G_{1,2}^{2,1} \left( \frac{L + \alpha}{\beta} \middle| \frac{1-c, 1-m}{0} \right) \right] + \frac{0.2 (1-k)}{\Gamma(c) \Gamma(m)} G_{1,2}^{2,1} \left( \frac{\alpha}{\beta} \middle| \frac{1-c, 1-m}{0} \right) 
\] (A.50)

A.3.2 Approximate Average Bit Error Probability (ABEP) over Fading and Shadowing Channel

Substituting (5.2) into the first part of (A.41) and with aid of (A.15), the approximate ABEP\textsuperscript{QAM,1} can be obtained as:

\[
ABEP_{\text{Approx}}^{\text{QAM,1}} = \frac{2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\bar{\gamma}} \right)^{\frac{c+m}{2}} \left( \frac{1}{2} \right)^{\frac{1}{2}} \int_0^\infty \gamma^{\frac{c+m-2}{2}} \times \left[ PEP = \frac{n-k}{2} e^{-\frac{\gamma}{2}} \right] \\
\times G_{0,2}^{2,0} \left( \frac{cm}{\gamma} \middle| \frac{c-m, m-c}{0} \right) d\gamma 
\]

\[
= \frac{1}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\bar{\gamma}} \right)^{\frac{c+m}{2}} \left( \frac{1}{2} \right)^{\frac{1}{2}} \times G_{0+1,2}^{2,0+1} \left( \frac{cm}{\gamma} \middle| \frac{1-c + \frac{c+m}{2}}{c-m, m-c} \right) \\
= \frac{1}{\Gamma(c) \Gamma(m)} \left( \frac{2cm}{\bar{\gamma}} \right)^{\frac{c+m}{2}} \times G_{1,2}^{2,1} \left( \frac{2cm}{\gamma} \middle| \frac{1-c + \frac{c+m}{2}}{c-m, m-c} \right) 
\]

(A.51)

After changing \( \beta = \frac{cm}{\bar{\gamma}} \), the ABEP\textsuperscript{QAM,1}_{\text{Approx}} is expressed as:

\[
ABEP_{\text{Approx}}^{\text{QAM,1}} = \frac{1}{\Gamma(c) \Gamma(m)} \left( 2\beta \right)^{\frac{c+m}{2}} \times G_{1,2}^{2,1} \left( 2\beta \middle| \frac{1-c + \frac{c+m}{2}}{c-m, m-c} \right) 
\] (A.52)
The $ABEP_{\text{Approx}}^{QAM,2}$ can be obtained from (A.17) with help of (A.15) as:

$$ABEP_{\text{Approx}}^{QAM,2} = \frac{0.2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right)^{c+m/2} \int_{0}^{\infty} \gamma^{c+m-2/2} \times e\left(\frac{-3\gamma}{2(M-1)}\right) d\gamma$$

$$= \frac{0.2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right)^{c+m/2} \left( \frac{3}{2(M-1)} \right)^{1-(c+m/2)} \times G_{0,2}^{2,0} \left( \frac{cm/\gamma}{c-m/2, m-c/2} \right) \left( \frac{1-(c+m/2)}{2(M-1)} \right)$$

After changing $\beta = \frac{cm}{\gamma}$ and $\alpha = \frac{3}{2(M-1)}$, the $ABEP_{\text{Approx}}^{QAM,2}$ is expressed as:

$$ABEP_{\text{Approx}}^{QAM,2} = \frac{0.2}{\Gamma(c) \Gamma(m)} \left( \frac{\beta}{\alpha} \right)^{c+m/2} \times G_{1,2}^{2,1} \left( \frac{\beta}{\alpha} \left| \frac{1-(c+m/2)}{c-m/2, m-c/2} \right. \right)$$

(A.54)

The $ABEP_{\text{Approx}}^{QAM,3}$ can be obtained from (A.17) with help of (A.15) as:

$$ABEP_{\text{Approx}}^{QAM,3} = \frac{0.2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right)^{c+m/2} \int_{0}^{\infty} \gamma^{c+m-2/2} \times PEP = \frac{n-k}{2} e^{-\gamma/2}$$

$$\times e\left(\frac{-3\gamma}{2(M-1)}\right) \times G_{0,2}^{2,0} \left( \frac{cm/\gamma}{c-m/2, m-c/2} \right) \left( \frac{1-(c+m/2)}{2(M-1)} \right)$$

$$= \frac{0.2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right)^{c+m/2} \left( \frac{1}{2} + \frac{3}{2(M-1)} \right)^{1-(c+m/2)} \times G_{0,2}^{2,0} \left( \frac{cm/\gamma}{c-m/2, m-c/2} \right) \left( \frac{1-(c+m/2)}{2(M-1)} \right)$$

$$= \frac{0.2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right)^{c+m/2} \left( \frac{1}{2} + \frac{3}{2(M-1)} \right)^{1-(c+m/2)} \times G_{1,2}^{2,1} \left( \frac{cm/\gamma}{c-m/2, m-c/2} \right) \left( \frac{1-(c+m/2)}{2(M-1)} \right)$$

(A.55)
After changing $\beta = \frac{cm}{\gamma}$ and $\alpha = \frac{3}{2(M-1)}$, the $ABEP^{QAM,3}_{\text{Approx}}$ is expressed as:

$$ABEP^{QAM,2}_{\text{Approx}} = \frac{0.2}{\Gamma(c) \Gamma(m)} \left( \frac{\beta}{\alpha} \right)^{\frac{c+m}{2}} \times G_{1,2}^{2,1} \left( \frac{\beta}{\alpha} \left| \frac{1 - (\frac{c+m}{2})}{\frac{c-m}{2}, \frac{m-c}{2}} \right. \right)$$

(A.56)

The approximate overall $ABEP$ can be obtained by combing $ABEP^{QAM,1}_{\text{Approx}}$, $ABEP^{QAM,2}_{\text{Approx}}$ and $ABEP^{QAM,3}_{\text{Approx}}$:

$$ABEP_{\text{approx}} \simeq \frac{n-k}{2\Gamma(c) \Gamma(m)} (\beta)^{\frac{c+m}{2}} \left[ 2 \frac{c+m}{2} - 1 \right] G_{1,2}^{2,1} \left( 2 \beta \left| \frac{1 - (\frac{c+m}{2})}{\frac{c-m}{2}, \frac{m-c}{2}} \right. \right)$$

$$-0.1 \left( \frac{1}{2} + \alpha \right)^{-\frac{c+m}{2}} G_{1,2}^{2,1} \left( \frac{\beta}{1/2 + \alpha} \left| \frac{0, 0}{\frac{c-m}{2}, \frac{m-c}{2}} \right. \right) + \frac{0.2}{\Gamma(c) \Gamma(m)} \left( \frac{\beta}{\alpha} \frac{c+m}{2} \right)$$

$$G_{1,2}^{2,1} \left( \frac{\beta}{\alpha} \left| \frac{1 - (\frac{c+m}{2})}{\frac{c-m}{2}, \frac{m-c}{2}} \right. \right)$$

(A.57)

**A.3.3 Probability Density Function of Signal to interference ratio (SIR) over Fading and Shadowing Channel**

The PDF of signal to interference ratio (SIR) in the composited shadowing and fading model can be accomplished by compensating $\gamma_D = \gamma_I$ into (A.58) and averaging (6.18)
and (6.19) as:

\[
f_{D/I}(\gamma) = \int_{0}^{\infty} f_{I}(\gamma_{I}) f_{D}(\gamma_{I}) \gamma_{I} d\gamma_{I}
\]

\[
= \frac{2}{\Gamma(c_{I}) \Gamma(m_{I}) \Gamma(c_{D}) \Gamma(m_{D})} \left( \frac{c_{D}^{m_{D}}}{\bar{\gamma}_{D}} \right)^{\frac{c_{D}+m_{D}}{2}} \left( \frac{c_{I}^{m_{I}}}{\bar{\gamma}_{I}} \right)^{\frac{c_{I}+m_{I}}{2}} \int_{0}^{\infty} (\gamma_{I})^{\frac{c_{D}+m_{D}-2}{2}} \\
K_{c_{D}-m_{D}} \left( 2 \sqrt{\frac{c_{D}^{m_{D}} \gamma_{I} \bar{\gamma}_{D}}{\bar{\gamma}_{D}}} \right) \gamma_{I}^{\frac{c_{I}+m_{I}-2}{2}} K_{c_{I}-m_{I}} \left( 2 \sqrt{\frac{c_{I}^{m_{I}} \gamma_{I}}{\bar{\gamma}_{I}}} \right) d\gamma_{I}
\]

Applying A.14 into A.58 and with the help of A.16, the PDF of SIR can be expressed after some simplification as:

\[
f_{D/I}(\gamma) = \frac{4}{\Gamma(c_{I}) \Gamma(m_{I}) \Gamma(c_{D}) \Gamma(m_{D})} \left( \frac{c_{D}^{m_{D}}}{\bar{\gamma}_{D}} \right)^{\frac{c_{D}+m_{D}}{2}} \left( \frac{c_{I}^{m_{I}}}{\bar{\gamma}_{I}} \right)^{\frac{c_{I}+m_{I}}{2}} \gamma^{\frac{c_{D}+m_{D}-2}{2}} \\
\int_{0}^{\infty} \frac{(c_{I}+m_{I}+c_{D}+m_{D}-1) \sqrt{2} G_{0,2} \left( \frac{c_{D}^{m_{D}} \gamma_{I}}{\bar{\gamma}_{D}} \right) \gamma_{I}^{\frac{c_{I}+m_{I}+c_{D}+m_{D}-2}{2}}}{\Gamma(c_{I}) \Gamma(m_{I}) \Gamma(c_{D}) \Gamma(m_{D})} \left( \frac{c_{I}^{m_{I}} \gamma_{I}}{\bar{\gamma}_{I}} \right)^{\frac{c_{I}+m_{I}}{2}} \gamma^{\frac{c_{D}+m_{D}-2}{2}} d\gamma_{I}
\]

\[
\times G_{2+0,0+2}^{2+0,0+2} \left( \frac{c_{D}^{m_{D}} \gamma_{I}}{\bar{\gamma}_{I}} \right) \left( \frac{c_{I}^{m_{I}} \gamma_{I}}{\bar{\gamma}_{I}} \right)^{\frac{c_{I}+m_{I}}{2}} - \frac{c_{D}-m_{D}-c_{I}-m_{I}}{2} \left( \frac{c_{I}^{m_{I}} \gamma_{I}}{\bar{\gamma}_{I}} \right)^{\frac{c_{I}+m_{I}}{2}} \frac{c_{D}^{m_{D}} \gamma_{D}}{\bar{\gamma}_{D}}^{\frac{c_{D}+m_{D}-2}{2}}
\]

\[
= \frac{c_{D}^{m_{D}} \gamma_{I}}{\bar{\gamma}_{I}} \left( \frac{c_{I}^{m_{I}} \gamma_{I}}{\bar{\gamma}_{I}} \right)^{\frac{c_{I}+m_{I}}{2}} \gamma^{\frac{c_{D}+m_{D}-2}{2}} \\
\times G_{2,2}^{2,2} \left( \frac{c_{D}^{m_{D}} \gamma_{I}}{\bar{\gamma}_{I}} \right) \left( \frac{c_{I}^{m_{I}} \gamma_{I}}{\bar{\gamma}_{I}} \right)^{\frac{c_{I}+m_{I}}{2}} - \frac{c_{D}-m_{D}-c_{I}-m_{I}}{2} \left( \frac{c_{I}^{m_{I}} \gamma_{I}}{\bar{\gamma}_{I}} \right)^{\frac{c_{I}+m_{I}}{2}} \frac{c_{D}^{m_{D}} \gamma_{D}}{\bar{\gamma}_{D}}^{\frac{c_{D}+m_{D}-2}{2}}
\]

(A.59)
The PDF of SIR can be expressed after some simplification and changing $\beta_D = \frac{c D^{m_D}}{\gamma_D}$ and $\beta_I = \frac{c I^{m_I}}{\gamma_I}$ as:

$$f_{D/I}(\gamma) = \frac{\beta_D}{A} \frac{c D^{m_D}}{2} \beta_I \frac{c D^{m_D}}{2} \gamma \frac{c D^{m_D} - 2}{2}$$

$$\times G_{2,2}^{2,2} \left( \frac{\beta D \gamma}{\beta I} \left| 1 - c I - \frac{(c D^{m_D})}{2}, 1 - m I - \frac{(c D^{m_D})}{2} \right. \right)$$

(A.60)

### A.3.4 Average Bit Error Probability Subject to Interference

The ABEP of M-QAM modulation that is given in (6.25) can be calculated by inserting (A.11) and applying (A.16) as:

$$ABEP_{D/I,approx}^{QAM} = \int_0^\infty BEP_I(\gamma) f_{D/I}(\gamma) d\gamma$$

$$= k \int_0^\infty [PEP + P_e - PEP \times P_e] f_{D/I}(\gamma) d\gamma$$

$$= A \frac{n - k}{2} \int_0^\infty \gamma \frac{c D^{m_D - 2}}{2} e^{-\gamma} G_{2,2}^{2,2} \left( \frac{\beta D \gamma}{\beta I} \left| 1 - c I - \frac{(c D^{m_D})}{2}, 1 - m I - \frac{(c D^{m_D})}{2} \right. \right) d\gamma$$

$$+ 0.2 A \int_0^\infty \gamma \frac{c D^{m_D - 2}}{2} e^{-\gamma} G_{2,2}^{2,2} \left( \frac{\beta D \gamma}{\beta I} \left| 1 - c I - \frac{(c D^{m_D})}{2}, 1 - m I - \frac{(c D^{m_D})}{2} \right. \right) d\gamma$$

$$- 0.2 A \frac{n - k}{2} \int_0^\infty \gamma \frac{c D^{m_D - 2}}{2} e^{-\gamma} G_{2,2}^{2,2} \left( \frac{\beta D \gamma}{\beta I} \left| 1 - c I - \frac{(c D^{m_D})}{2}, 1 - m I - \frac{(c D^{m_D})}{2} \right. \right) d\gamma$$

(A.61)
The first part of (A.61) can be integrated as:

\[
ABEP^{D/I, 1}_{QAM} = A \frac{n-k}{2} \int_{0}^{\infty} \gamma^{\frac{c_D+m_D-2}{2}} e^{-\gamma} d\gamma
\]

\[
G_{2,2}^{2,2} \left( \frac{\beta_D \gamma}{\beta_I} \right) \left| \begin{array}{c}
1 - c_I - \frac{(c_D+m_D)}{2}, 1 - m_I - \frac{(c_D+m_D)}{2} \\
\end{array} \right| d\gamma
\]

\[
= A \frac{n-k}{2} \int_{0}^{\infty} \gamma^{\frac{c_D+m_D-2}{2}} e^{-\gamma} d\gamma
\]

\[
G_{1,0}^{1,0} \left( \frac{\gamma}{2} \right) G_{2,2}^{2,2} \left( \frac{c_D m_D \gamma}{\gamma_D \gamma_I} \right) \left| \begin{array}{c}
1 - c_I - \frac{(c_D+m_D)}{2}, 1 - m_I - \frac{(c_D+m_D)}{2} \\
\end{array} \right| d\gamma
\]

\[
= n-k \frac{c_D m_D \gamma}{\gamma_D \gamma_I} \left( \frac{c_D m_D \gamma}{\gamma_D \gamma_I} \right) ^{c_D m_D \gamma}
\]

\[
G_{2+0,2+1}^{1+2,0+2+2} \left( \frac{1}{2} \right) \left| \begin{array}{c}
1 - c_D \frac{m_D}{2}, 1 - m_D \frac{m_I}{2} \\
0, 1 - c_I \frac{m_D}{2}, 1 - m_I \frac{m_D}{2} \\
\end{array} \right| \left( \frac{1}{2} \right) \gamma_D \gamma_I
\]

The PDF of \( ABEP^{D/I, 1}_{QAM} \) can be obtained after some simplification as:

\[
ABEP^{D/I, 1}_{QAM} = \frac{(n-k)}{2 \Gamma(c_D) \Gamma(m_D) \Gamma(c_I) \Gamma(m_I)} G_{2,3}^{3,2} \left( \frac{c_D m_I}{2 \Gamma^2(c_D) \Gamma(m_D) \rho} \right) \left| \begin{array}{c}
1 - c_D, 1 - m_D \\
0, c_I, m_I \\
\end{array} \right|
\]

where \( \rho = \frac{\gamma_D}{\gamma_I} \).

The second part of (A.61) can be derived using the same step in the first part as:

\[
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\]
The PDF of $\text{ABEP}_{D/I,2}^{QAM}$ can be obtained after some simplification as:

$$\text{ABEP}_{D/I,2}^{QAM} = 0.2A \int_0^{\infty} \gamma \frac{e^{-\gamma}}{2^{(M-1)}} \left( \frac{\beta_D \gamma}{\beta_I} \right)^{\frac{c_D + m_D - 2}{2}} \left( \frac{c_D + m_D}{2} \right)^{1 - c_I - \frac{(c_D + m_D)}{2} - \frac{1 - m_I - \frac{(c_D + m_D)}{2}}{2}} d\gamma$$

$$G_{2,2} \left( \frac{\beta_D \gamma}{\beta_I} \right)^{\frac{c_D + m_D}{2}} \left( \frac{c_D + m_D}{2} \right)^{1 - c_I - \frac{(c_D + m_D)}{2} - \frac{1 - m_I - \frac{(c_D + m_D)}{2}}{2}} d\gamma$$

$$= 0.2 \int_0^{\infty} \gamma \frac{e^{-\gamma}}{2^{(M-1)}} \left( \frac{\beta_D \gamma}{\beta_I} \right)^{\frac{c_D + m_D}{2}} \left( \frac{c_D + m_D}{2} \right)^{1 - c_I - \frac{(c_D + m_D)}{2} - \frac{1 - m_I - \frac{(c_D + m_D)}{2}}{2}} d\gamma$$

$$G_{1+2,0+2} \left( \frac{-3\gamma}{2(M-1)} \right) \cdot G_{2,2} \left( \frac{c_D m_D \gamma}{\beta_D \beta_I} \right)^{\frac{1 - c_D}{2}} \left( \frac{1 - m_D}{2} \right)^{1 - c_I - \frac{(c_D + m_D)}{2} - \frac{1 - m_I - \frac{(c_D + m_D)}{2}}{2}} d\gamma$$

The PDF of $\text{ABEP}_{D/I,2}^{QAM}$ can be obtained after some simplification as:

$$\text{ABEP}_{D/I,2}^{QAM} = \frac{0.2}{\Gamma(c_D)\Gamma(m_D)\Gamma(c_I)\Gamma(m_I)} G_{2,3}^{3,2} \left( \frac{\alpha c_I m_I}{c_D m_D} \right) \left( \frac{1 - c_D, 1 - m_D}{0, c_I, m_I} \right)$$

(A.65)

where $\rho = \frac{\gamma_D}{\gamma_I}$ and $\alpha = \frac{3}{2(M-1)}$.

The third part of (A.61) can be derived using the same step in the first and second parts as:
\[ \text{ABEP}_{QAM}^{D/I,3} = \frac{0.2A(n-k)}{2} \int_{0}^{\infty} \gamma \frac{e^{-\gamma}}{\frac{3}{2}(M-1) + \frac{5}{2}} \gamma \] 

\[ G_{2,2}^{2,2} \left( \frac{\beta_{D}}{\beta_{I}} \right) 1 - c_{I} - \frac{(c_{D} + m_{D})}{2}, 1 - m_{I} - \frac{(c_{D} + m_{D})}{2} \right) d\gamma = \frac{0.2A(n-k)}{2} \int_{0}^{\infty} \gamma \frac{e^{-\gamma}}{\frac{3}{2}(M-1) + \frac{5}{2}} \gamma \] 

\[ G_{1,0}^{1,0} \left( \frac{3}{2(M-1)} + \frac{1}{2} \right) \gamma \right) G_{2,2}^{2,2} \left( \frac{c_{D} m_{D}}{\gamma_{D}} 1 - c_{I} - \frac{(c_{D} + m_{D})}{2}, 1 - m_{I} - \frac{(c_{D} + m_{D})}{2} \right) d\gamma \] 

\[ = \frac{0.2(n-k)}{2} \frac{c_{D} m_{D}}{\Gamma(c_{D}) \Gamma(m_{D})} \frac{\Gamma(c_{I}) \Gamma(m_{I})}{\Gamma(c_{I} + m_{I})} \frac{e^{\frac{1}{2}(c_{D} + m_{D})}}{e^{\frac{1}{2}(c_{D} + m_{D})}} \frac{1}{\gamma_{D}} \] 

The PDF of \( \text{ABEP}_{QAM}^{D/I,3} \) can be obtained after some simplification as:

\[ \text{ABEP}_{QAM}^{D/I,3} = \frac{0.1(n-k)}{\Gamma(c_{D}) \Gamma(m_{D}) \Gamma(c_{I}) \Gamma(m_{I})} G_{2,3}^{3,2} \left( \frac{(\alpha + 0.5)c_{D} m_{D}}{c_{D} c_{I}} \rho \left| \begin{array}{c} 1 - c_{D}, 1 - m_{D} \end{array} \right. \right) \] 

\[ \left( 0, c_{I}, m_{I} \right) \] 

The overall \( \text{ABEP}_{QAM}^{D/I} \) can be obtained by combing \( \text{ABEP}_{QAM}^{D/I,1} \), \( \text{ABEP}_{QAM}^{D/I,2} \) and \( \text{ABEP}_{QAM}^{D/I,3} \).

\[ \text{ABEP}_{QAM}^{D/I,\text{approx}} = \frac{\Gamma(c_{D}) \Gamma(m_{D}) \Gamma(c_{I}) \Gamma(m_{I})}{(n-k)} \] 

\[ +0.2G_{2,3}^{3,2} \left( \frac{\alpha c_{D} m_{D}}{c_{D} c_{I}} \rho \left| \begin{array}{c} 1 - c_{D}, 1 - m_{D} \end{array} \right. \right) \] 

\[ \left( 0, c_{I}, m_{I} \right) \] 

\[ -0.1(n-k)G_{2,3}^{3,2} \left( \frac{(\alpha + 0.5)c_{D} m_{D}}{c_{D} c_{I}} \rho \left| \begin{array}{c} 1 - c_{D}, 1 - m_{D} \end{array} \right. \right) \] 

\[ \left( 0, c_{I}, m_{I} \right) \]
A.3.5 Probability Density Function of Signal to Multiple Interference Ratio (SIR) over Fading and Shadowing Channel

The PDF of signal to multiple interference ratio (SIR) in the composited shadowing and fading model can be obtained by compensating $\gamma_D = \gamma \gamma_I$ into (A.69) and averaging (6.18) and (6.26) as:

$$f_{D/N_I}(\gamma) = \int_0^\infty f_{N_I}(\gamma N_I) f_D(\gamma I \gamma) \gamma_I d\gamma_I$$

$$= \frac{2}{\Gamma(c_I) \Gamma(N_I m_I) \Gamma(c_D) \Gamma(m_D)} \left( \frac{c_D m_D}{\gamma_D} \right)^{c_D+m_D/2} \left( \frac{c_I m_I}{\gamma_I} \right)^{N_I m_I + c_I} \int_0^\infty (\gamma I)^{c_D+m_D-2}$$

$$K_{c_D-m_D} \left( 2 \sqrt{\frac{c_D m_D \gamma I}{\gamma_D}} \right) \gamma_I^{c_I+N_I m_I-2} K_{c_I-N_I m_I} \left( 2 \sqrt{\frac{c_I N_I m_I \gamma I}{\gamma_I}} \right)$$

$$= \frac{4}{\Gamma(c_I) \Gamma(N_I m_I) \Gamma(c_D) \Gamma(m_D)} \left( \frac{c_D m_D}{\gamma_D} \right)^{c_D+m_D/2} \left( \frac{c_I m_I}{\gamma_I} \right)^{N_I m_I + c_I} \gamma^{c_D+m_D-2}$$

$$\int_0^{\infty} \gamma_I^{c_I+N_I m_I+c_D+m_D-1} K_{c_D-m_D} \left( 2 \sqrt{\frac{c_D m_D \gamma I}{\gamma_D}} \right) K_{c_I-N_I m_I} \left( 2 \sqrt{\frac{c_I N_I m_I \gamma I}{\gamma_I}} \right) d\gamma_I$$

(A.69)
Applying A.14 into A.69 and with the help of A.16, the PDF of multiple SIR can be expressed after some simplification as:

\[
f_{D/NI}(\gamma) = \frac{4}{\Gamma(c_I) \Gamma(N_I m_I) \Gamma(c_D) \Gamma(m_D)} \left( \frac{c_D m_D}{\gamma D} \right)^{c_D + m_D} \left( \frac{c_D m_I}{\gamma I} \right)^{\frac{c_D + m_I}{2}} \left( \frac{c_D + m_D - 2}{\gamma D} \right) \gamma^{c_D + m_D - 2} \times \int_{0}^{\infty} \frac{1}{\gamma I} \left( \frac{c_D m_D \gamma D}{\gamma I} \right)^{0,0} \left( \frac{c_D m_I \gamma I}{\gamma D} \right)^{0,0} \frac{1}{2} G_{0.2}^{2,0} \left( \frac{c_D m_I \gamma I}{\gamma I} \right) \left( \frac{c_D - m_D - c_D}{c_D} \right) \left( \frac{m_I \gamma I}{c_D \gamma I} \right) \left( \frac{\gamma I - N_I m_I - c_D}{2} \right) \frac{d\gamma I}{c_D m_D + c_D + N_I m_I} \right)
\]

The PDF of multiple SIR can be expressed after some simplification as:

\[
f_{D/NI}(\gamma) = \frac{\beta_D \frac{c_D + m_D}{2} \beta_I \frac{c_D + m_D}{2} \gamma^{c_D + m_D - 2}}{\Gamma(c_I) \Gamma(N_I m_I) \Gamma(c_D) \Gamma(m_D)} A_1 \times G_{2,2}^{2,2} \left( \frac{\beta_D \gamma}{\beta I} \left| 1 - c_I - \frac{(c_D + m_D)}{2} \right. \left. - \frac{(c_D - m_D)}{2} \right. \left. - \frac{m_D - c_D}{m_I \gamma I} \right) \right)
\]

where \( \beta_D = \frac{c_D m_D}{\gamma D} \) and \( \beta_I = \frac{c_D m_I}{\gamma I} \).
A.3.6 Average Bit Error Probability Subject to Multiple Interference

The ABEP subject to multiple interferes of M-QAM modulation that is given in (6.31) can be calculated by inserting (A.11) and applying (A.16) as:

\[
\text{ABEP}_{D/N_1,\text{approx}}^{\text{QAM}} = A_1 \frac{n-k}{2} \int_0^\infty \text{BEP}_t(\gamma) f_{D/N_1}(\gamma) d\gamma = k \int_0^\infty \left[ \text{PEP} + P_e - \text{PEP} \times P_e \right] f_{D/N_1}(\gamma) d\gamma
\]

\[
= A_1 \frac{n-k}{2} \int_0^\infty \gamma^{c_D+m_D-2} e^{-\gamma} G_{2,2}^{2,2} \left( \frac{\beta_D \gamma}{\beta_I} \right) \left| 1 - c_I - \frac{(c_D+m_D)}{2}, 1 - N_I m_I - \frac{(c_D+m_D)}{2} \right| \frac{e^{\frac{-3\gamma}{2(M-1)}}}{e^{\frac{-3\gamma}{2}}} d\gamma
\]

\[
+ 0.2A_1 \int_0^\infty \gamma^{c_D+m_D-2} e^{-\gamma} G_{2,2}^{2,2} \left( \frac{\beta_D \gamma}{\beta_I} \right) \left| 1 - c_I - \frac{(c_D+m_D)}{2}, 1 - N_I m_I - \frac{(c_D+m_D)}{2} \right| \frac{e^{\frac{-3\gamma}{2(M-1)}}}{e^{\frac{-3\gamma}{2}}} d\gamma
\]

\[
- 0.2A_1 \frac{n-k}{2} \int_0^\infty \gamma^{c_D+m_D-2} e^{-\gamma} \left( \frac{3}{2(M-1)} + \frac{1}{2} \right) G_{2,2}^{2,2} \left( \frac{\beta_D \gamma}{\beta_I} \right) \left| 1 - c_I - \frac{(c_D+m_D)}{2}, 1 - N_I m_I - \frac{(c_D+m_D)}{2} \right| \frac{e^{\frac{-3\gamma}{2(M-1)}}}{e^{\frac{-3\gamma}{2}}} d\gamma
\]

(A.72)
The first part of (A.72) can be integrated as:

\[
ABEP_{QAM}^{D/N_{I}, 1} = A_1 \frac{n-k}{2} \int_0^\infty \gamma \frac{e^{-\gamma}}{\frac{c_D + m_D}{2} \gamma} \left( 1 - c_I - \frac{(c_D + m_D)}{2}, 1 - N_{I}m_I - \frac{(c_D + m_D)}{2} \right) d\gamma
\]

\[
G_{2.2}^{2.2} \left( \frac{\beta_D D}{\beta_I I} \right) \left| 1 - c_I - \frac{(c_D + m_D)}{2}, 1 - N_{I}m_I - \frac{(c_D + m_D)}{2} \right| d\gamma
\]

\[
= A_1 \frac{n-k}{2} \int_0^\infty \gamma \frac{e^{-\gamma}}{\frac{c_D + m_D}{2} \gamma} \left( 1 - c_I - \frac{(c_D + m_D)}{2}, 1 - N_{I}m_I - \frac{(c_D + m_D)}{2} \right) d\gamma
\]

\[
G_{0,1}^{1,0} \left( \frac{\gamma}{2} \right) G_{2.2}^{2.2} \left( \frac{c_D m_D \gamma}{\gamma_D D}, \frac{c_I m_I I}{\gamma_I I} \right) \left| 1 - c_I - \frac{(c_D + m_D)}{2}, 1 - N_{I}m_I - \frac{(c_D + m_D)}{2} \right| d\gamma
\]

\[
= n-k + \frac{c_D m_D \gamma}{\gamma_D D}, \frac{c_D m_D \gamma}{\gamma_D D}, \frac{c_I m_I I}{\gamma_I I}, \frac{c_I m_I I}{\gamma_I I} \left| 1 - c_D, 1 - m_D \right| d\gamma
\]

(A.73)

The PDF of \(ABEP_{QAM}^{D/N_{I}, 1}\) can be obtained after some simplification as:

\[
ABEP_{QAM}^{D/N_{I}, 1} = \frac{(n-k)}{2(\gamma_D)\Gamma(m_D)\Gamma(c_I)\Gamma(N_{I}m_I)} G_{2.3}^{3.2} \left( \frac{c_I m_I I}{\gamma_D D}, \frac{c_I m_I I}{\gamma_I I} \right) \left| 1 - c_D, 1 - m_D \right| d\gamma
\]

(A.74)

where \(\rho = \frac{\gamma_D D}{\gamma_I I}\).

The second part of (A.61) can be derived using same step in first part as:

\[
G_{2+0,2+1}^{2,0+2} \left( \frac{c_D m_D \gamma}{\gamma_D D}, \frac{c_I m_I I}{\gamma_I I} \right) \left| 1 - c_D - \frac{m_D}{2}, 1 - m_D + \frac{c_D}{2}, 1 - c_D + \frac{m_D}{2}, 1 - m_D + \frac{c_D}{2} \right| d\gamma
\]

(A.75)
\[
ABEP_{QAM}^{D/N,2} = 0.2A \int_0^\infty \gamma \frac{e^{-\frac{\gamma(c_D+m_D)}{2}}}{\frac{c_D-m_D}{2}, \frac{m_D-c_D}{2}} d\gamma
\]

\[
G_{2,2}^{2,2} \left( \frac{\beta_D}{\beta_I} \right) \left| 1 - c_I - \frac{(c_D+m_D)}{2}, 1 - N_I m_I - \frac{(c_D+m_D)}{2} \right) d\gamma
= 0.2A \int_0^\infty \gamma \frac{e^{-\frac{\gamma(c_D+m_D)}{2}}}{\frac{c_D-m_D}{2}, \frac{m_D-c_D}{2}} d\gamma
\]

\[
G_{0,1}^{1,0} \left( \frac{-3\gamma}{2(M-1)} \right) \cdot G_{2,2}^{2,2} \left( \frac{c_D m_D}{\gamma_I} \right) \left| 1 - c_I - \frac{(c_D+m_D)}{2}, 1 - N_I m_I - \frac{(c_D+m_D)}{2} \right) d\gamma
= 0.2A \frac{\Gamma(c_I) \Gamma(N_I m_I) \Gamma(c_D) \Gamma(m_D)}{\gamma_I} e^{-\frac{\gamma(c_D+m_D)}{2}}
\]

The PDF of \(ABEP_{QAM}^{D/N,2}\) can be obtained after some simplification as:

\[
ABEP_{QAM}^{D/N,2} = \frac{0.2A \Gamma(c_D) \Gamma(m_D) \Gamma(c_I) \Gamma(N_I m_I)}{\Gamma(c_D) \Gamma(m_D) \Gamma(c_I) \Gamma(N_I m_I)} G_{2,3}^{3,2} \left( \frac{\alpha c_I m_I}{c_D c_D} \right) \left| 1 - c_D, 1 - m_D \right) \rho, c_I, N_I m_I \right)
\]

where \(\rho = \frac{\gamma_I}{\gamma_I} \) and \(\alpha = \frac{3}{2(M-1)}\).

The third part of (A.61) can be derived using the same step in the first and second parts as:

\[
G_{2+0,2+1}^{1+2,0+2} \left( \frac{-3\gamma}{2(M-1)} \right) \cdot \left| 1 - c_D \cdot c_I - m_D \cdot m_I \right) d\gamma
\]
\[
ABEP_{QAM}^{D/N_i,3} = 0.2A_1(n-k) \int_0^\infty \gamma^{c_D+m_D-2} e^{-\left(\frac{3}{2}+\frac{1}{2}\right)\gamma} d\gamma = 0.2A_1(n-k) \int_0^\infty \gamma^{c_D+m_D-2} d\gamma
\]

\[
G_{2,2}^2 \left( \frac{\beta_D^\gamma}{\beta_I^\gamma} \right) \left| 1 - c_I - \frac{(c_D+m_D)}{2}I - \frac{(c_D+m_D)}{2} \right| \Gamma \left( I - \frac{1}{2} \right) \Gamma \left( I + \frac{1}{2} \right) \]n_I \[m_N \]

\[
G_{1,0}^1 \left( \frac{3}{2(M-1)} + \frac{1}{2} \right) \Gamma \left( I - \frac{1}{2} \right) \Gamma \left( I + \frac{1}{2} \right) \]n_I \[m_N \]

\[
G_{2,0+2+1}^{1+2+2} \left( \frac{3}{2(M-1)} + \frac{1}{2} \right) \Gamma \left( I - \frac{1}{2} \right) \Gamma \left( I + \frac{1}{2} \right) \]n_I \[m_N \]

The PDF of \(ABEP_{QAM}^{D/N_i,3}\) can be obtained after some simplification as:

\[
ABEP_{QAM}^{D/N_i,3} = \frac{0.1(n-k)}{\Gamma(c_D)\Gamma(m_D)\Gamma(c_I)\Gamma(N_i m_I)} G_{2,3}^{3,2} \left( \frac{(\alpha+0.5)c_I m_I}{c_D c_D} \right) \left| 1 - c_D, 1 - m_D \right| \left( \frac{0, c_I, N_I m_I}{0, c_I, N_I m_I} \right)
\]

The overall \(ABEP_{QAM}^{D/N_i}\) subject to multiple interferes can be obtained by combining \(ABEP_{QAM}^{D/N_i,1}\), \(ABEP_{QAM}^{D/N_i,2}\) and \(ABEP_{QAM}^{D/N_i,3}\) as:

\[
ABEP_{QAM}^{D/N_i,\approx} = \frac{k}{\Gamma(c_D)\Gamma(m_D)\Gamma(c_I)\Gamma(N_i m_I)} \left( \frac{(n-k)}{2} \right)
\]

\[
G_{2,3}^{3,2} \left( \frac{c_I m_I}{c_D c_D} \right) \left| 1 - c_D, 1 - m_D \right| \left( \frac{0, c_I, N_I m_I}{0, c_I, N_I m_I} \right)
\]

\[
+0.2G_{2,3}^{3,2} \left( \frac{\alpha c_I m_I}{c_D c_D} \right) \left| 1 - c_D, 1 - m_D \right| \left( \frac{0, c_I, N_I m_I}{0, c_I, N_I m_I} \right)
\]

\[
-0.1(n-k)G_{2,3}^{3,2} \left( \frac{(\alpha+0.5)c_I m_I}{c_D c_D} \right) \left| 1 - c_D, 1 - m_D \right| \left( \frac{0, c_I, N_I m_I}{0, c_I, N_I m_I} \right)
\]

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A.4 Derivation of Equation (7.3), (7.6), (7.8), (7.9), (7.12) and (7.20) Adaptive OFDM-IM System over Faded Shadowing Channel

A.4.1 Exact Pairwise Error Probability (PEP) over Fading and Shadowing Channel

The average PEP for OFDM-IM over the shadowed channel fading, which is given in (7.1), can be calculated by averaging (5.1) over (3.8) and using (A.10) as:

\[
PEP_{NG} = \int_0^\infty \left[ 1 - \sum_{q=0}^{n-k} \binom{n-k}{q} \frac{-1}{q+1} \exp \left( -\frac{q\gamma}{q+1} \right) \right]
\]

\[
\times \frac{2}{\Gamma(c)\Gamma(m)} \left( \frac{cm}{\gamma} \right)^{\frac{c+m}{2}} \gamma^{\frac{c+m-2}{2}} K_{c-m} \left( 2 \sqrt{\frac{cm}{\gamma}} \right) d\gamma
\]

\[
= \frac{2}{\Gamma(c)\Gamma(m)} \left( \frac{cm}{\gamma} \right)^{\frac{c+m}{2}} \sum_{q=0}^{n-k} \binom{n-k}{q} \frac{-1}{q+1} \int_0^\infty \gamma^{\frac{c+m-2}{2}} e^{\left( \frac{-q\gamma}{q+1} \right)} K_{c-m} \left( 2 \sqrt{\frac{cm}{\gamma}} \right) d\gamma
\]

\[
= 1 - \frac{2}{\Gamma(c)\Gamma(m)} \left( \frac{cm}{\gamma} \right)^{\frac{c+m}{2}} \sum_{q=0}^{n-k} \binom{n-k}{q} \frac{-1}{q+1} \Gamma \left( \frac{c}{2} \right) \Gamma \left( \frac{m}{2} \right) \left( \frac{q + m + 1}{2} \right) \left( \frac{q + m}{2} \right) \Gamma \left( \frac{c}{2} + \frac{m}{2} \right) \Gamma \left( \frac{q}{2} + 1 \right) \Gamma \left( \frac{m}{2} + \frac{1}{2} \right) \gamma^{\frac{c}{2} + \frac{m}{2} - 1} W_{\left( \frac{c+m-1}{2}, \frac{c-m}{2} \right)} \left( \frac{2q}{q+1} \right)^{2} \left( \sqrt{\frac{cm}{\gamma}} \right)^{2}
\]

\]

(A.80)
The above equation can be simplified to:

\[
PEP_{NG} = 1 - (\beta)^{\nu} \sum_{q=0}^{n-k} \binom{n-k}{q} (-1)^q \frac{\beta(q+1)}{q+1} \left( \frac{q}{q+1} \right)^{-\nu} W_{-\nu, \ell} \left( \frac{\beta(q+1)}{2q} \right) \tag{A.81}
\]

where \( \beta = \frac{cm}{\gamma}, \nu = \frac{c+m-1}{2}, \ell = \frac{c-m}{2} \) and \( W(.) \) is the Whittaker function.

### A.4.2 Approximate Pairwise Error Probability (PEP) over Fading and Shadowing Channel

By following the same step in exact ABEP, the approximate PEP for OFDM-IM over the shadowed and fading channel in (7.4) is found by averaging (5.2) over (3.8) and using (A.10) as:

\[
PEP_{NG} \approx \frac{n-k}{2} \frac{2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right)^{c+m/2} \times \frac{\Gamma \left( \frac{c+ny+X}{2} + \frac{c - my + X}{2} \right)}{\left( \frac{c+m-1}{2} \right)} \frac{\Gamma \left( \frac{q + m - x + q + m + y + X}{2} \right)}{\left( \frac{c-m+1}{2} \right)}
\]

\[
\times K_{c-m} \left( 2 \sqrt{\frac{cm}{\gamma}} \right) \gamma^2 \frac{\Gamma \left( \frac{c+ny+X}{2} + \frac{c - my + X}{2} \right)}{\left( \frac{c+m-1}{2} \right)} \frac{\Gamma \left( \frac{q + m - x + q + m + y + X}{2} \right)}{\left( \frac{c-m+1}{2} \right)} \left( \frac{\sqrt{cm}}{\gamma} \right)^2
\]

\[
= \frac{n-k}{2} \frac{2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right)^{c+m/2} \times \frac{\Gamma \left( \frac{c+ny+X}{2} + \frac{c - my + X}{2} \right)}{\left( \frac{c+m-1}{2} \right)} \frac{\Gamma \left( \frac{q + m - x + q + m + y + X}{2} \right)}{\left( \frac{c-m+1}{2} \right)} \left( \frac{\sqrt{cm}}{\gamma} \right)^2 \tag{A.82}
\]

The approximate closed-form expression for PEP can be obtained by using (A.10) as:

\[
PEP_{NG}^{approx} \approx \frac{n-k}{2} (\beta)^{\nu} e^\beta \left( \frac{1}{2} \right)^{-\nu} W_{-\nu, \ell} \left( 2\beta \right) \tag{A.83}
\]

where \( \beta = \frac{cm}{\gamma}, \nu = \frac{c+m-1}{2}, \ell = \frac{c-m}{2} \) and \( W(.) \) is the Whittaker function.
A.4.3 Exact Average Bit Error Probability (ABEP) over Fading and Shadowing Channel

The exact ABEP of M-QAM OFDM-IM over the shadowing and fading channel in (7.7) can be calculated by averaging (5.20) over (3.8) and inserting (5.1) and (5.21), as

\[ ABEP_{QAM}^{K_G} = \int_0^\infty \text{BEP}_t P_\gamma(\gamma) \, d\gamma \]

\[ = \int_0^\infty \left[ P_{EP} + P_e - P_{EP} \times P_e \right] P_\gamma(\gamma) \, d\gamma \]

\[ = \int_0^\infty \left[ \sum_{q=0}^{n-k} \binom{n-k}{q} \frac{-1^q}{q+1} e^{-\gamma(\frac{q}{q+1})} \right] \times \frac{2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right)^{\frac{c+m-2}{2}} \gamma^{\frac{c+m-2}{2}} K_{c-m} \left( \frac{2 \sqrt{cm}}{\gamma} \right) \, d\gamma \]

\[ + \int_0^\infty 0.2 e^{\left(\frac{\gamma}{M-1}\right)} \times \frac{2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right)^{\frac{c+m-2}{2}} \gamma^{\frac{c+m-2}{2}} K_{c-m} \left( \frac{2 \sqrt{cm}}{\gamma} \right) \, d\gamma \]

\[ - \int_0^\infty \left[ \sum_{q=0}^{n-k} \binom{n-k}{q} \frac{-1^q}{q+1} e^{-\gamma(\frac{q}{q+1})} \right] \times 0.2 e^{\left(\frac{\gamma}{M-1}\right)} \frac{2 \gamma}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right)^{\frac{c+m}{2}} K_{c-m} \left( \frac{2 \sqrt{cm}}{\gamma} \right) \, d\gamma \]

(A.84)

The first part, \( ABEP_{QAM,1}^{K_G} \), of (A.84) is equal to (A.80) and it is given as:

\[ ABEP_{QAM,1}^{K_G} = 1 - (\beta)^\nu \sum_{q=0}^{n-k} \binom{n-k}{q} \frac{-1^q}{q+1} e^{-\gamma(\frac{q}{q+1})} \frac{\beta(q+1)}{2q} \left( \frac{q}{q+1} \right)^{-\nu} W_{-\nu,\nu} \left( \frac{\beta(q+1)}{2q} \right) \]

(A.85)

The second part, \( ABEP_{QAM,2}^{K_G} \), of (A.84) can be integrated using (A.10) as:
The first part of $\text{ABEP}_{K_G}^{QAM,2}$, of (A.84) can be integrated using (A.10) as:

$$ABEP_{K_G}^{QAM,2} = 0.2 \left( \frac{cm}{\gamma} \right)^{\frac{c+m}{2}} \int_0^\infty \frac{e^{\frac{c}{2}} \left( -\frac{3\gamma}{2} \right)^{\frac{c+m}{2}} e^{-\frac{3\gamma}{2}}}{\frac{\sqrt{\gamma}}{2(M-1)}} K_{c-m} \left( 2 \sqrt{\frac{cm}{\gamma}} \right) d\gamma$$

$$= 0.2 \frac{2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right)^{\frac{c+m}{2}} \times \frac{e^{-\frac{3\gamma}{2}}}{\frac{\sqrt{\gamma}}{2(M-1)}} W_{-\frac{c+m}{2}, \frac{c-m}{2}} \left( \frac{cm}{\gamma} \right)^{-\frac{3\gamma}{2}} W_{-\frac{c+m}{2}, \frac{c-m}{2}} \left( \frac{cm}{\gamma} \right)^{\frac{c+m}{2}}$$

The $ABEP_{K_G}^{QAM,2}$ can be obtain after some simplification as:

$$ABEP_{K_G}^{QAM,2} = 0.2 \left( \frac{cm}{\gamma} \right)^{\frac{c+m}{2}} e^{\frac{\beta}{2 \pi}} \left( L \right)^{-\nu} W_{-\nu,\beta} \left( L \right)$$

where the parameters are: $\beta = \frac{cm}{\gamma}$, $L = \frac{q}{q+1}$, $\alpha = \frac{2(M-1)}{3}$, $\nu = \frac{c+m-1}{2}$, and $\nu = \frac{c-m}{2}$.

The terms $c$, $m$ and $M$ are the shadowing factor, the fading factor and the constellation size, respectively. The third part, $ABEP_{K_G}^{QAM,3}$, of (A.84) can be integrated using (A.10) as:

$$ABEP_{K_G}^{QAM,3} = 0.2 \frac{2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right)^{\frac{c+m}{2}} \int_0^\infty \frac{e^{\frac{c}{2}} \left( -\frac{3\gamma}{2} \right)^{\frac{c+m}{2}} e^{-\frac{3\gamma}{2}}}{\frac{\sqrt{\gamma}}{2(M-1)}} K_{c-m} \left( 2 \sqrt{\frac{cm}{\gamma}} \right) d\gamma$$

$$-0.2 \frac{2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right)^{\frac{c+m}{2}} \sum_{q=0}^{n-k} \frac{n-k}{q} \left( \frac{cm}{\gamma} \right)^{\frac{c+m}{2}} \int_0^\infty \frac{e^{\frac{c}{2}} \left( -\frac{3\gamma}{2} \right)^{\frac{c+m}{2}} e^{-\frac{3\gamma}{2}}}{\frac{\sqrt{\gamma}}{2(M-1)}}$$

$$\times K_{c-m} \left( 2 \sqrt{\frac{cm}{\gamma}} \right) d\gamma$$

The first part of $ABEP_{K_G}^{QAM,3}$ in (A.88) is equal to $ABEP_{K_G}^{QAM,2}$ and it is simplified to:

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\[
ABEP_{K_G}^{QAM,3} = 0.2 (\beta)^v e^{\frac{\beta}{2\pi}} (L)^{-v} W_{v,\nu,\tau}\left(\frac{\beta}{\bar{L}}\right) - 0.2 \frac{2}{\Gamma(c) \Gamma(m)} \left(\frac{cm}{c+1}\right)^{e^m} \times \sum_{q=0}^{n-k} \binom{n-k}{q} \frac{(-1)^q}{q+1} \frac{\Gamma\left(\frac{c+2q}{2}\right)}{\Gamma\left(\frac{2q}{2}\right)} \frac{\Gamma\left(\frac{m+2q}{2}\right)}{\Gamma\left(\frac{2q}{2}\right)} \frac{\Gamma\left(\frac{c+2n}{2}\right)}{\Gamma\left(\frac{2n}{2}\right)}
\]

\[
\frac{\left(\sqrt{\frac{cm}{c+1}}\right)^2}{\left(\frac{3}{2(M-1)} + \frac{q}{q+1}\right)} \times e^{\frac{\beta}{2(L+\frac{1}{\alpha})}} \left(\frac{3}{2(M-1)} + \frac{q}{q+1}\right) W\left(\frac{c+m-1}{2},\frac{c-m}{2}\right) \left(\frac{3}{2(M-1)} + \frac{q}{q+1}\right)
\]

After some manipulation, the above equation can be expressed as:

\[
ABEP_{K_G}^{QAM,3} = 0.2 (\beta)^v e^{\frac{\beta}{2\pi}} (L)^{-v} W_{v,\nu,\tau}\left(\frac{\beta}{\bar{L}}\right) - 0.2 (\beta)^v \sum_{q=0}^{n-k} \binom{n-k}{q} \frac{(-1)^q}{q+1} \frac{\beta}{q+1} e^{\frac{\beta}{2(L+\frac{1}{\alpha})}} \left(\frac{L+\frac{1}{\alpha}}{L+\frac{1}{\alpha}}\right)^{-v} W_{v,\nu,\tau}\left(\frac{\beta}{L+\frac{1}{\alpha}}\right)
\]

The overall \(ABEP_{K_G}^{QAM}\) can be obtained over composited fading and shadowing channel by combing \(ABEP_{K_G}^{QAM,1}\), \(ABEP_{K_G}^{QAM,2}\) and \(ABEP_{K_G}^{QAM,3}\) as:

\[
ABEP_{K_G}^{QAM} \leq K \left[1 - (\beta)^v \sum_{q=0}^{n-k} \binom{n-k}{q} \frac{(-1)^q}{q+1} e^{\frac{\beta}{2\pi}} (L)^{-v} W_{v,\nu,\tau}\left(\frac{\beta}{L}\right)
\]
\[
+ 0.2 (\beta)^v \sum_{q=0}^{n-k} \binom{n-k}{q} \frac{(-1)^q}{q+1} e^{2(L+\frac{1}{\alpha})} (L+\frac{1}{\alpha})^{-v} W_{v,\nu,\tau}\left(\frac{\beta}{L+\frac{1}{\alpha}}\right)\right]
\]

(A.91)
A.4.4 Average Bit Error Probability (ABEP) over Fading and Shadowing Channel

The approximate ABEP of M-QAM OFDM-IM over the shadowing and fading channel in (7.9) can be calculated by averaging (5.20) over (3.8) and inserting (5.2) and (5.21), as

\[
ABEP_{K_G,\text{approx}}^{QAM} = \int_0^\infty \text{BEP}_t P_\gamma (\gamma) \, d\gamma
\]

\[
= \int_0^\infty [\text{PEP} + P_e - \text{PEP} \times P_e] P_\gamma (\gamma) \, d\gamma
\]

\[
= \int_0^\infty \frac{n - k}{2} e^{-\frac{\gamma}{2}} \frac{2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right) \frac{c+m}{2} \gamma^{-\frac{c+m-2}{2}} K_{c-m} \left( 2\sqrt{\frac{cm}{\gamma}} \right) \, d\gamma
\]

\begin{align*}
&= \int_0^\infty 0.2 e^{-\frac{3\gamma}{2} (M-1)} \times \frac{2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right) \frac{c+m}{2} \gamma^{-\frac{c+m-2}{2}} K_{c-m} \left( 2\sqrt{\frac{cm}{\gamma}} \right) \, d\gamma \\
&\quad + \int_0^\infty 0.2 e^{-\frac{3\gamma}{2} (M-1)} \times \frac{2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right) \frac{c+m}{2} \gamma^{-\frac{c+m-2}{2}} K_{c-m} \left( 2\sqrt{\frac{cm}{\gamma}} \right) \, d\gamma \\
&\quad - \int_0^\infty \frac{n - k}{2} e^{-\frac{\gamma}{2}} \times 0.2 e^{-\frac{3\gamma}{2} (M-1)} \frac{2\gamma^{\frac{c+m-2}{2}}}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right) \frac{c+m}{2} K_{c-m} \left( 2\sqrt{\frac{cm}{\gamma}} \right) \, d\gamma
\end{align*}

(A.92)

The first part, \(ABEP_{K_G,\text{approx}}^{QAM,1}\), of (A.92) is equal to (A.83) and it is given as:

\[
ABEP_{K_G}^{QAM,\text{approx},1} \approx \frac{n - k}{2} \beta^\nu e^\beta \left( \frac{1}{2} \right)^{-\nu} W_{-\nu,l} (2\beta)
\]

(A.93)

The second part, \(ABEP_{K_G,\text{approx}}^{QAM,2}\), of (A.92) is equal to (A.87) and it is given as:

\[
ABEP_{K_G,\text{approx}}^{QAM,2} = 0.2 \beta^\nu e^{\frac{\beta}{2\nu}} (L)^{-\nu} W_{-\nu,l} \left( \frac{\beta}{L} \right)
\]

(A.94)
The third part, $ABEP_{QAM, K_G, \text{approx}}$, of (A.92) can be integrated using (A.10) as:

$$ABEP_{QAM, K_G, \text{approx}} = 0.2 \frac{n - k}{2} \frac{2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right)^{\frac{c + m - 2}{2}} \int_0^\infty \gamma^{\frac{c + m - 2}{2} - \frac{1}{2}} e^{-\gamma} e^{\left( \frac{cm}{\gamma} \right)^{\frac{c + m - 2}{2}}} K_{c-m} \left( 2 \sqrt{\frac{cm}{\gamma}} \right) \frac{d\gamma}{\gamma}$$

$$= 0.2 \frac{n - k}{2} \frac{2}{\Gamma(c) \Gamma(m)} \left( \frac{cm}{\gamma} \right)^{\frac{c + m - 2}{2}} \times e^{2(\frac{\sqrt{cm}}{2(M-1)} + \frac{3}{2})} \left( \frac{3}{2(M-1)} + \frac{1}{2} \right)^{-(\frac{c + m - 2}{2} - \frac{1}{2})} W_{-(\frac{c + m - 2}{2} - \frac{1}{2}), \frac{cm}{\gamma}^{\frac{c + m - 2}{2}}} \left( \frac{3}{2(M-1)} + \frac{3}{2} \right)$$

$$= 0.2 \frac{n - k}{2} \left( \frac{cm}{\gamma} \right)^{\frac{c + m - 2}{2}} e^{2(\frac{\sqrt{cm}}{2(M-1)} + \frac{3}{2})} \left( \frac{3}{2(M-1)} + \frac{1}{2} \right)^{-(\frac{c + m - 2}{2} - \frac{1}{2})} W_{-(\frac{c + m - 2}{2} - \frac{1}{2}), \frac{cm}{\gamma}^{\frac{c + m - 2}{2}}} \left( \frac{3}{2(M-1)} + \frac{3}{2} \right)$$

(A.95)

The $ABEP_{QAM,3}^{\text{approx}}$ can be obtain after some simplification as:

$$ABEP_{QAM,3}^{\text{approx}, K_G} = 0.2 (\beta) v e^{\frac{3}{2D}} \left( \frac{\beta}{D} \right)^{-v} W_{-v,\ell} \left( \frac{\beta}{D} \right)$$

(A.96)

where $D = \frac{M+2}{M-1}$.

The overall $ABEP_{QAM,1}^{\text{approx}, K_G}$ can be obtained over composited fading and shadowing channel by combining $ABEP_{QAM,1}^{\text{approx}, K_G}$, $ABEP_{QAM,2}^{\text{approx}, K_G}$ and $ABEP_{QAM,3}^{\text{approx}, K_G}$ as:

$$ABEP_{QAM, K_G, \text{approx}} \leq k \left[ \frac{n - k}{2} (\beta) v e^{\beta \left( \frac{1}{2} \right)} W_{-v,\ell} \left( 2\beta \right) + 0.2 (\beta) v e^{\beta \alpha \left( \frac{1}{\alpha} \right)} W_{-v,\ell} \left( \beta \alpha \right) - \frac{n - k}{10} (\beta) v e^{\beta D} \left( D \right)^{-v} W_{-v,\ell} \left( \frac{\beta}{D} \right) \right]$$

(A.97)
A.4.5 Outage Probability

\( P_{\text{out}} \) in (7.11) can be calculated by substituting (3.8) and using (A.6)

\[
P_{\text{out}} = \int_0^{\gamma_{th}} P_\gamma(\gamma) d\gamma
\]

(A.98)

The \( P_{\text{out}} \) can be obtained by changing the order of integration and using (A.6) as:

\[
P_{\text{out}} = \int_0^{\gamma_{th}} \frac{c^c m^m \gamma^{m-1}}{\Gamma(c) \Gamma(m)} \int_0^{\gamma_{th}} e^{-\gamma_f/\gamma} \frac{m \gamma_f - c \gamma_f}{\gamma_f} d\gamma_f
\]

(A.99)

The \( P_{\text{out}} \) can be achieved by applying (A.4) to convert incomplete Gamma then using (A.5) to solve the integration as:

\[
P_{\text{out}} = 1 - \frac{c^c c^m m^m}{\Gamma(c) \Gamma(m)} \sum_{i=0}^{m-1} \frac{(m \gamma_{th})^i}{i!} \int_0^{\gamma_{th}-i} e^{-\gamma_f/\gamma} \frac{m \gamma_f - c \gamma_f}{\gamma_f} d\gamma_f
\]

(A.100)

The integration in (A.100) can be solved by using (A.9) as:

\[
P_{\text{out}} = 1 - \frac{c^c c^m m^m}{\Gamma(c) \Gamma(m)} \sum_{i=0}^{m-1} \frac{(m \gamma_{th})^i}{i!} \int_0^{\gamma_{th}-i} e^{-\gamma_f/\gamma} \frac{m \gamma_f - c \gamma_f}{\gamma_f} d\gamma_f
\]

(A.101)
The above equation of $P_{\text{out}}$ can be simplified as:

$$1 - \frac{2}{\Gamma(c)} \sum_{i=0}^{m-1} \frac{1}{i!} (\beta \gamma_{th})^{c+i} K_{c-i} \left( 2\sqrt{\beta \gamma_{th}} \right)$$  \hspace{1cm} (A.102)

### A.4.6 Probability of $R^{th}$ region

The probability of $R^{th}$ region that is described in (7.15) can be obtained as:

$$P_r(R) = \frac{\gamma_{R+1}}{\gamma_R} \int_{\gamma_R}^{\gamma_{R+1}} P_\gamma(\gamma) d\gamma = \int_{0}^{\gamma_{R+1}} P_\gamma(\gamma) d\gamma - \int_{0}^{\gamma_R} P_\gamma(\gamma) d\gamma$$  \hspace{1cm} (A.103)

$P_r(R)$ can be gotten directly from $P_{\text{out}}$ by changing the threshold ($\gamma_{th}$) to region threshold ($\gamma_R$) as:

$$P_r(R) = 1 - \frac{2}{\Gamma(c)} \sum_{i=0}^{m-1} \frac{1}{i!} (\beta \gamma_{R+1})^{c+i} K_{c-i} \left( 2\sqrt{\beta \gamma_{R+1}} \right) - 1 + \frac{2}{\Gamma(c)} \sum_{i=0}^{m-1} \frac{1}{i!} (\beta \gamma_R)^{c+i} K_{c-i} \left( 2\sqrt{\beta \gamma_R} \right)$$

$$P_r(R) = \frac{2}{\Gamma(c)} \sum_{i=0}^{m-1} \frac{1}{i!} [(\beta \gamma_R)^{c+i} K_{c-i}(2\sqrt{\beta \gamma_R}) - (\beta \gamma_{R+1})^{c+i} K_{c-i}(2\sqrt{\beta \gamma_{R+1}})]$$  \hspace{1cm} (A.104)

### A.4.7 Average Bit Error Probability (ABEP)

The $BEP_R$ in (7.22) is given as:

$$BEP_R = \frac{k}{n} \sum_{\delta=1}^{n} \left[ \int_{\gamma_R}^{\gamma_{R+1}} P_{\text{EP}} \cdot P_\gamma(\gamma) d\gamma - \int_{\gamma_R}^{\gamma_{R+1}} P_{e_{M-ary}} \cdot P_\gamma(\gamma) d\gamma \right]$$

$$= \int_{\gamma_R}^{\gamma_{R+1}} P_{\text{EP}} \cdot P_{e_{M-ary}} \cdot P_\gamma(\gamma) d\gamma$$  \hspace{1cm} (A.105)
\( \text{BEP}_R \) in (A.105) can be expressed by substituting (5.2) and (3.8) as:

\[
\text{BEP}_R^1 = \int_{\gamma_R}^{\gamma_R+1} \text{PEP} \cdot P_{\gamma}(\gamma) d\gamma = \int_{\gamma_R}^{\gamma_R+1} \frac{(n-k)}{2} e^{-\gamma} \frac{c^m m^{m-1} \gamma^{-m} m\gamma_{\gamma} - c\gamma_f}{\Gamma(c) \Gamma(m) \gamma^c} \int_0^\infty \gamma e^{-\gamma} d\gamma_f
\]

(A.106)

The \( \text{BEP}_R^1 \) can be simplified by changing the order of integration as:

\[
\text{BEP}_R^1 = \int_{\gamma_R}^{\gamma_R+1} \text{PEP} \cdot P_{\gamma}(\gamma) d\gamma = \int_{\gamma_R}^{\gamma_R+1} \frac{(n-k)}{2} e^{-\gamma} \frac{c^m m^{m-1} \gamma^{-m} m\gamma_{\gamma} - c\gamma_f}{\Gamma(c) \Gamma(m) \gamma^c} \int_0^\infty \gamma e^{-\gamma} d\gamma_f
\]

\[
= \frac{(n-k)}{2} \frac{c^m m^m}{\Gamma(c) \Gamma(m) \gamma^c} \int_0^{\gamma_R+1} \gamma^{m-1} e^{-\gamma} d\gamma \int_0^\infty \gamma e^{-\gamma} d\gamma_f - \frac{(n-k)}{2} \frac{c^m m^m}{\Gamma(c) \Gamma(m) \gamma^c} \int_0^{\gamma_R+1} \gamma^{m-1} e^{-\gamma} d\gamma \int_0^\infty \gamma e^{-\gamma} d\gamma_f = \text{BEP}_R^{1,a} + \text{BEP}_R^{1,b}
\]

(A.107)

The \( \text{BEP}_R^1 \) is divided into two parts \( \text{BEP}_R^{1,a} \) and \( \text{BEP}_R^{1,b} \). The \( \text{BEP}_R^{1,a} \) can be integrated by and using (A.103) and (A.4) as:

\[
\text{BEP}_R^{1,a} = \frac{(n-k)}{2} \frac{c^m m^m}{\Gamma(c) \Gamma(m) \gamma^c} \int_0^{\gamma_R+1} \gamma^{m-1} e^{-\gamma} d\gamma \int_0^\infty \gamma e^{-\gamma} d\gamma_f = \frac{(n-k)}{2} \frac{c^m m^m}{\Gamma(c) \Gamma(m) \gamma^c} \int_0^{\gamma_R+1} \gamma^{m-1} e^{-\gamma} d\gamma \int_0^\infty \gamma e^{-\gamma} d\gamma_f
\]

(A.108)
using (A.6), the above equation can be simplified as:

\[
\overline{\text{BEP}}_{R}^{1,a} = \frac{(n - k)}{2} \frac{c^e m^m}{\Gamma(c) \Gamma(m) \gamma^c} \int_{0}^{\infty} \gamma_f^{c-m-1} e^{-\frac{c \gamma_f}{\gamma}} \left( \frac{1}{2} + \frac{m}{\gamma_f} \right)^{-m} (m - 1)! \\
\times \left[ 1 - e^{-\left( \frac{1}{2} + \frac{m}{\gamma_f} \right) \gamma_{R+1}} \sum_{i=0}^{m-1} \frac{1}{i!} \left( \frac{\gamma_{R+1}}{2} \right)^i \right]
\]

\[
= \frac{(n - k)}{2} \frac{c^e m^m \left( \frac{1}{2} \right)^{-m}}{\Gamma(c) \Gamma(m) \gamma^c} \int_{0}^{\infty} \gamma_f^{c-m-1} e^{-\frac{c \gamma_f}{\gamma}} (m - 1)! - \frac{(n - k)}{2} \frac{c^e m^m \left( \frac{1}{2} \right)^{-m}}{\Gamma(c) \Gamma(m) \gamma^c} \sum_{i=0}^{m-1} \frac{1}{i!} \left( \frac{\gamma_{R+1}}{2} \right)^i (m - 1)! \int_{0}^{\infty} \gamma_f^{c-m-1} e^{-\frac{m}{\gamma_f} \gamma_{R+1} + e^{-\frac{c \gamma_f}{\gamma}} \gamma_f}
\]

(A.109)

using (A.5), the above equation can be simplified as:

\[
\overline{\text{BEP}}_{R}^{1,a} = \frac{(n - k)}{2} \frac{c^e m^m \Gamma(c - m) \left( \frac{c}{\gamma} \right)^{-c+m}}{\Gamma(c) \gamma^c} - \frac{(n - k)}{2} \frac{c^e m^m \gamma_{R+1}}{\Gamma(c) \gamma^c} e^{-\frac{\gamma_{R+1}}{2}}
\]

\[
= \frac{(n - k)}{2} \frac{c^e m^m \Gamma(c - m) \left( \frac{c}{\gamma} \right)^{-c+m}}{\Gamma(c) \gamma^c} - \frac{(n - k)}{2} \frac{c^e m^m \gamma_{R+1}}{\Gamma(c) \gamma^c} e^{-\frac{\gamma_{R+1}}{2}}
\]

(A.110)

\[
\overline{\text{BEP}}_{R}^{1,b} \text{ can be obtained by following same step that used to get } \overline{\text{BEP}}_{R}^{1,a} \text{ as:}
\]

\[
\overline{\text{BEP}}_{R}^{1,b} = \frac{(n - k)}{2} \frac{c^e m^m \Gamma(c - m) \left( \frac{c}{\gamma} \right)^{-c+m}}{\Gamma(c) \gamma^c} - \frac{(n - k)}{2} \frac{c^e m^m \gamma_{R+1}}{\Gamma(c) \gamma^c} e^{-\frac{\gamma_{R+1}}{2}}
\]

\[
= \frac{(n - k)}{2} \frac{c^e m^m \Gamma(c - m) \left( \frac{c}{\gamma} \right)^{-c+m}}{\Gamma(c) \gamma^c} - \frac{(n - k)}{2} \frac{c^e m^m \gamma_{R+1}}{\Gamma(c) \gamma^c} e^{-\frac{\gamma_{R+1}}{2}}
\]

(A.111)

The \( \overline{\text{BEP}}_{R}^{1} \) can be found by subtracting \( \overline{\text{BEP}}_{R}^{1,a} \) from \( \overline{\text{BEP}}_{R}^{1,b} \) as:
After some simplification, above equation can be written as:

\[
\text{BEP}_{R,\text{approx}} = \frac{(n - k) \beta^{l_1}}{\Gamma(c)} \left( \frac{1}{2} \right)^{-m} \left\{ \gamma_R^{l_1} e^{-\frac{\gamma_R}{2}} \sum_{i=0}^{m-1} \frac{1}{i!} \left( \frac{\gamma_R}{2} \right)^i K_{c-m}(2\sqrt{\beta \gamma_R}) \right. \\
\left. - \gamma_{R+1}^{l_1} e^{-\frac{\gamma_{R+1}}{2}} \sum_{i=0}^{m-1} \frac{1}{i!} \left( \frac{\gamma_{R+1}}{2} \right)^i K_{c-m}(2\sqrt{\beta \gamma_{R+1}}) \right\}
\]

where \( l_1 = \frac{c+m}{2}, \) \( \iota = \frac{c-m}{2}, \) \( \gamma_R \) is SNR threshold for region \( R \) and \( \gamma_{R+1} \) is SNR threshold for region \( R + 1 \).
\[ \overline{\text{BEP}}_R^2 = \int_{\gamma_R}^{\gamma_{R+1}} P_{e_{M-arg}} P_{\gamma(\gamma)} d\gamma = \int_{\gamma_R}^{\gamma_{R+1}} 0.2e^{\left(\frac{-3\gamma}{2(M-1)}\right)} \frac{c^m m^{\gamma-1}}{\Gamma(c)\Gamma(m)} \int_0^\infty \gamma_f^{c-1} e^{\frac{m\gamma}{\gamma_f} - \frac{c\gamma}{\gamma_f}} d\gamma_f \]

\begin{align*}
\frac{0.2c^m m^m}{\Gamma(c)\Gamma(m)} & \int_0^{\gamma_{R+1}} \gamma^m e^{\left(\frac{-3\gamma}{2(M-1)}\right)} d\gamma \int_0^\infty \gamma_f^{c-1} e^{\frac{m\gamma}{\gamma_f} - \frac{c\gamma}{\gamma_f}} d\gamma_f - \frac{0.2c^m m^m}{\Gamma(c)\Gamma(m)} \int_0^{\gamma_{R+1}} \gamma^m e^{\left(\frac{-3\gamma}{2(M-1)}\right)} d\gamma \int_0^\infty \gamma_f^{c-1} e^{\frac{m\gamma}{\gamma_f} - \frac{c\gamma}{\gamma_f}} d\gamma_f \\
& = 0.2c^m m^m \int_0^\infty \gamma_f^{c-1} e^{\frac{-c\gamma_f}{\gamma_f}} d\gamma_f \int_0^{\gamma_{R+1}} \gamma^m e^{\left(\frac{-3\gamma}{2(M-1)}\right)} d\gamma
\end{align*}

\[ \overline{\text{BEP}}_R^{2a} \]

\[ \overline{\text{BEP}}_R^{2b} \]

The \( \overline{\text{BEP}}_R^2 \) is divided into two parts \( \overline{\text{BEP}}_R^{2a} \) and \( \overline{\text{BEP}}_R^{2b} \). The \( \overline{\text{BEP}}_R^{2a} \) can be integrated by and using (A.103) and (A.4) as:

\[ \overline{\text{BEP}}_R^{2a} = \frac{0.2c^m m^m}{\Gamma(c)\Gamma(m)} \int_0^\infty \gamma_f^{c-1} e^{\frac{-c\gamma_f}{\gamma_f}} d\gamma_f \int_0^{\gamma_{R+1}} \gamma^m e^{\left(\frac{-3\gamma}{2(M-1)}\right)} d\gamma = \frac{0.2c^m m^m}{\Gamma(c)\Gamma(m)} \int_0^\infty \gamma_f^{c-1} e^{\frac{-c\gamma_f}{\gamma_f}} d\gamma_f \left( \frac{3\gamma}{2(M-1)} + \frac{m}{\gamma_f} \right)^{-m} \gamma \left( m, \left( \frac{3\gamma}{2(M-1)} + \frac{m}{\gamma_f} \right) \right) \gamma_{R+1} \]

using (A.6), the above equation can be simplified as:
Using (A.5), the above equation can be simplified as:

\[
\begin{align*}
\text{BEP}_{2,R}^{2,a} &= \frac{0.2e^c m^m}{\Gamma(c)\Gamma(m)\gamma^c} \int_0^\infty \gamma^{c-m} e^{-\gamma} \gamma_f \left( \frac{3}{2(M-1)} + \frac{m}{\gamma_f} \right)^{-m} d\gamma_f \\
&\quad \left[ 1 - e^{-\left(\frac{3}{2(M-1)} + \frac{m}{\gamma_f}\right)\gamma_f} \sum_{i=0}^{m-1} \frac{1}{i!} \left( \frac{\gamma_f}{2} \right)^i \right] \\
&= \frac{0.2e^c m^m \left(\frac{3}{2(M-1)}\right)^{-m}}{\Gamma(c)\Gamma(m)\gamma^c} \int_0^\infty \gamma^{c-m} e^{-\gamma} \gamma_f d\gamma_f - \frac{0.2e^c m^m \left(\frac{3}{2(M-1)}\right)^{-m}}{\Gamma(c)\Gamma(m)\gamma^c} e^{-\frac{3\gamma R + 1}{2(M-1)}} \sum_{i=0}^{m-1} \frac{1}{i!} \left( \frac{\gamma_f}{2} \right)^i (m-1)! \int_0^\infty \gamma^{c-m} e^{-\gamma} \gamma_f d\gamma_f \\
&= 0.
\end{align*}
\]

(A.116)

The \(\text{BEP}_{2,R}^{2,b}\) can be obtained by following same step that used to get \(\text{BEP}_{2,R}^{2,a}\) as:

\[
\begin{align*}
\text{BEP}_{2,R}^{2,b} &= \frac{0.2e^c m^m \Gamma(c-m)\left(\frac{c}{\gamma}\right)^{-c+m} \left(\frac{3}{2(M-1)}\right)^{-m}}{\Gamma(c)\gamma^c} - \frac{0.2e^c m^m}{\Gamma(c)\gamma^c} e^{-\frac{3\gamma R + 1}{2(M-1)}} \left(\frac{3\gamma R + 1}{2(M-1)}\right)^{\frac{c-m}{2}} \gamma_f^{\frac{c-m}{2}} \gamma_f \\
&\quad \sum_{i=0}^{m-1} \frac{1}{i!} \left( \frac{3\gamma R + 1}{2(M-1)} \right)^i (m-1)! \int_0^\infty \gamma^{c-m} e^{-\gamma} \gamma_f d\gamma_f \\
&= 0.
\end{align*}
\]

(A.117)

The \(\text{BEP}_{2,R}^2\) can be found by subtracting \(\text{BEP}_{2,R}^{2,a}\) from \(\text{BEP}_{2,R}^{2,b}\) as:
\[ \text{BEP}_R = \frac{0.2e^{c_m m \Gamma (c - m)} \left( \frac{c}{\gamma} \right)^{c + m} \left( \frac{3}{2(M-1)} \right)^m}{\Gamma(c) \gamma^c} - \frac{0.2e^{c_m m}}{\Gamma(c) 2^m} \]

\[ e^{-\frac{3\gamma_{R+1}}{2(M-1)}} \left( \frac{3}{2(M-1)} \right)^{-m} \left( \frac{m \gamma_{R+1}}{c} \right)^\frac{c-m}{2} \sum_{i=0}^{m-1} \frac{1}{i!} \left( \frac{3\gamma_{R+1}}{2(M-1)} \right)^i \]

\[ K_{c-m} \left( 2\sqrt{\frac{cm \gamma_{R+1}}{\gamma}} \right) = \frac{0.2e^{c_m m}}{\Gamma(c) 2^m} e^{-\frac{3\gamma_R}{2(M-1)}} \left( \frac{3}{2(M-1)} \right)^{-m} \left( \frac{m \gamma_R \bar{\gamma}}{c} \right)^\frac{c-m}{2} \sum_{i=0}^{m-1} \frac{1}{i!} \left( \frac{3\gamma_R}{2(M-1)} \right)^i \]

After some simplification, above equation can be written as:

\[ \text{BER}_{R, \text{approx}}^2 = \frac{0.4\beta l_1 (\alpha)^{-m}}{\Gamma(c)} \left[ \gamma_R e^{-\frac{\gamma_R}{\alpha}} \sum_{i=0}^{m-1} \frac{1}{i!} \left( \frac{\gamma_R}{\alpha} \right)^i K_{c-m} \left( 2\sqrt{\beta \gamma_R} \right) \right. \]

\[ \left. -\gamma_{R+1} e^{-\frac{\gamma_{R+1}}{\alpha}} \sum_{i=0}^{m-1} \frac{1}{i!} \left( \frac{\gamma_{R+1}}{\alpha} \right)^i K_{c-m} \left( 2\sqrt{\beta \gamma_{R+1}} \right) \right] \]

where \( l_1 = \frac{c+m}{2}, \ l = \frac{c-m}{2}, \ \alpha = \frac{3}{2(M-1)}, \ \gamma_R \) is SNR threshold for region \( R \) and \( \gamma_{R+1} \) is SNR threshold for region \( R + 1 \). 

\( \text{BEP}_R^3 \) in (A.105) can be expressed by changing the order of integration and substituting
The $\text{BEP}_R^3$ is divided into two parts $\text{BEP}_R^{3,a}$ and $\text{BEP}_R^{3,b}$. The $\text{BEP}_R^{3,a}$ can be integrated by and using (A.103) and (A.4) as:

$$\text{BEP}_R^{3,a} = \frac{n-k}{2} \frac{0.2e^c m^m}{\Gamma(c)\Gamma(m)\gamma^c} \int_0^\infty \gamma_f e^{-\gamma_f} d\gamma_f \int_0^{\gamma_R+1} \gamma^m e^{-\gamma_f \gamma} d\gamma_f \int_0^{\gamma_R+1} e^{-\gamma_f \gamma} (\frac{M+2}{2(M-1)})^\gamma d\gamma$$

using (A.6), the above equation can be simplified as:
using (A.5), the above equation can be simplified as:

\[
\begin{align*}
\text{BEP}^{2,a}_R &= \frac{0.2c^c m^m}{\Gamma(c)\Gamma(m)\bar{\gamma}^c} \int_0^\infty \gamma^{c-m-1} e^{-\frac{\gamma}{\bar{\gamma}}} d\gamma \left( \frac{M + 2}{2(M - 1)} + \frac{m}{\gamma_f} \right)^{-m} (m - 1)!
\end{align*}
\]

\[
\begin{align*}
\left[ 1 - e^{-\left(\frac{M+2}{2(M-1)} \frac{m}{\gamma_f}\right)^{\gamma_R+1}} \sum_{i=0}^{m-1} \frac{1}{i!} \left( \frac{\gamma_R+1}{2} \right)^i \int_0^\infty \gamma^{c-m-1} e^{-\frac{\gamma}{\bar{\gamma}}} d\gamma \right] &= \frac{0.2(n-k)c^c m^m \left( \frac{M+2}{2(M-1)} \right)^{-m}}{2\Gamma(c)\Gamma(m)\bar{\gamma}^c} (m - 1)!
\end{align*}
\]

\[
\begin{align*}
\int_0^\infty \gamma^{c-m-1} e^{-\frac{\gamma}{\bar{\gamma}}} d\gamma &= \frac{0.2(n-k)c^c m^m \left( \frac{M+2}{2(M-1)} \right)^{-m}}{2\Gamma(c)\Gamma(m)\bar{\gamma}^c}
\end{align*}
\]

(A.123)

\[
\begin{align*}
e^{-\frac{3\gamma_R+1}{2(M-1)}} \sum_{i=0}^{m-1} \frac{1}{i!} \left( \frac{\gamma_R+1}{2} \right)^i (m - 1)! \int_0^\infty \gamma^{c-m-1} e^{-\frac{\gamma}{\bar{\gamma}}} d\gamma \left( \frac{M+2}{2(M-1)} \right)^{\gamma_R+1} i
\end{align*}
\]

\[
\begin{align*}
K_{c-m} \left( 2 \sqrt{\frac{cm\gamma_R+1}{\bar{\gamma}}} \right)
\end{align*}
\]

(A.124)

\(\text{BEP}^{3,a}_R\) can be obtained by following same step that used to get \(\text{BEP}^{2,a}_R\) as:

\[
\begin{align*}
\text{BEP}^{3,b}_R &= \frac{0.2(n-k)c^c m^m \Gamma(c-m)\left( \frac{c}{\gamma} \right)^{-c+m} \left( \frac{M+2}{2(M-1)} \right)^{-m}}{2\Gamma(c)\gamma^c} \sum_{i=0}^{m-1} \frac{1}{i!} \left( \frac{M+2}{2(M-1)} \gamma_i \right)^i \left( \frac{m\gamma_R+1}{c} \right) \left( \frac{M+2}{2(M-1)} \right)^{\gamma_R+1} i
\end{align*}
\]

\[
\begin{align*}
K_{c-m} \left( 2 \sqrt{\frac{cm\gamma_R+1}{\bar{\gamma}}} \right)
\end{align*}
\]

(A.125)

The \(\text{BEP}^3_R\) can be found by Subtracting \(\text{BEP}^{3,a}_R\) from \(\text{BEP}^{3,b}_R\) as:
After some simplification, above equation can be written as:

\[
\begin{align*}
\text{BEP}_R^{3,a} &= \frac{0.2 (n - k) c^e m^m \Gamma(c - m) \left(\frac{\xi}{\gamma}\right)^{-c+m} \left(\frac{M+2}{2(M-1)}\right)^{-m}}{2\Gamma(c)\gamma^c} - \frac{0.2 (n - k) c^e m^m}{2\Gamma(c)2^m} \\
e^{-\left(\frac{M+2}{2(M-1)}\right)\gamma R + 1} \left(\frac{M + 2}{2(M - 1)}\right)^{-m} \frac{(m\gamma R+1\gamma)}{c} \frac{c-m}{2} \sum_{i=0}^{m-1} \frac{1}{i!} \left(\left(\frac{M + 2}{2(M - 1)}\right)\gamma R + 1\right)^i K_{c-m} \left(2\sqrt{\frac{cm\gamma R+1}{\gamma}}\right)
\end{align*}
\]

\[
\begin{align*}
&= \frac{0.2 (n - k) c^e m^m}{2\Gamma(c)2^m} e^{-\left(\frac{M+2}{2(M-1)}\right)\gamma R} \left(\frac{M + 2}{2(M - 1)}\right)^{-m} \frac{(m\gamma R+1\gamma)}{c} \frac{c-m}{2} \sum_{i=0}^{m-1} \frac{1}{i!} \left(\left(\frac{M + 2}{2(M - 1)}\right)\gamma R + 1\right)^i K_{c-m} \left(2\sqrt{\frac{cm\gamma R}{\gamma}}\right) - \frac{0.2 (n - k) c^e m^m}{2\Gamma(c)2^m} \\
&\quad \sum_{i=0}^{m-1} \frac{1}{i!} \left(\left(\frac{M + 2}{2(M - 1)}\right)\gamma R + 1\right)^i K_{c-m} \left(2\sqrt{\frac{cm\gamma R+1}{\gamma}}\right)
\end{align*}
\]

(A.126)

\[
\text{BER}_R^{3,a,approx} = \frac{0.4\beta^l_1}{\Gamma(c)} \left(\frac{D}{2}\right)^{-m} \left[\gamma R^e - \frac{D\gamma^R}{2} \sum_{i=0}^{m-1} \frac{1}{i!} \left(\frac{D\gamma R}{2}\right)^i \right] K_{c-m} \left(2\sqrt{\beta \gamma R}\right) - \gamma R^e \frac{D\gamma R+1}{2} \sum_{i=0}^{m-1} \frac{1}{i!} \left(\frac{D\gamma R+1}{2}\right)^i K_{c-m} \left(2\sqrt{\beta \gamma R+1}\right)
\]

(A.127)

where \(l_1 = \frac{c+m}{2}, \ i = \frac{c-m}{2}, \ D = \left(\frac{M+2}{2(M-1)}\right), \ \gamma R \) is SNR threshold for region \( R \) and \( \gamma R+1 \) is SNR threshold for region \( R + 1 \).

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The exact ABEP can be obtained by following same approach in (A.105).

A.5 Derivation of Equation (8.10), (8.14), (8.17) and (8.20) Adaptive OFDM-IM System with Diversity over Nakagami-m Fading Channel

A.5.1 Average PEP for GD with MRC Reception (GD-MRC)

The exact APEP with GD-MRC in (8.9) can be obtained by substituting (8.8) and (5.1) and using Table of Integrals [86, eq. (3.381.4)] as:

$$
PEP_{MRC}^{Naka} = k \int_0^\infty PEP_t f(\gamma) \, d\gamma
$$

$$
= k \int_0^\infty \left[ 1 - \sum_{q=0}^{n-k} \frac{(n-k)}{q+1} \exp\left(-\frac{q\gamma}{q+1}\right) \right] \times \frac{m^{N_L} q^{N_L (m-1)} e^{-\gamma}}{\frac{q!}{m!} \Gamma(N_L)} \, d\gamma
$$

$$
= \frac{m^{N_L} q^{N_L}}{\Gamma(m N_L) \gamma^{m N_L}} \int_0^\infty \gamma^{m N_L - 1} e^{-\gamma} \left( k - \sum_{q=0}^{n-k} \frac{(n-k)}{q+1} \right) \, d\gamma
$$

$$
= 1 - \sum_{q=0}^{n-k} \frac{(n-k)}{q+1} \int_0^\infty \gamma^{m N_L - 1} \exp\left(-\frac{m\gamma}{\gamma + 1}\right) \, d\gamma
$$

(A.128)

The $PEP_{MRC}^{Naka}$ can be expressed after some simplification as

$$
PEP_{MRC}^{Naka} = k - k \sum_{q=0}^{n-k} \frac{(n-k)}{q+1} \left( \frac{q m + m}{q+1} \right)^{m N_L}
$$

(A.129)

where $\beta = \frac{m}{\gamma}$, $L = \frac{q}{q+1}$.

Similarly, the approximate APEP in (8.11) for OFDM-IM with GD-MRC over Nakagami-m fading channel is obtained by averaging (8.8) over (5.2) and using Table of Integrals [86, eq. (3.381.4)] as:

$$
PEP_{MRC, \text{Approx}, Naka} \approx \left( \frac{N-k}{2} \right) \frac{m^{N_L}}{\gamma^{m N_L} \Gamma(m N_L)} \int_0^\infty \gamma^{m N_L - 1} e^{-\gamma} \frac{m \gamma}{\frac{q!}{m!} \Gamma(N_L)} \, d\gamma
$$

$$
= \left( \frac{N-k}{\gamma^{m N_L} \Gamma(m N_L)} \frac{1}{\frac{m}{\gamma} - \frac{1}{2}} \right)^{m N_L} \Gamma(m N_L)
$$

(A.130)
The \( PEP_{MRC, \text{Approx}, Naka} \) can be expressed after some simplification as:

\[
PEP_{MRC, \text{Approx} - Naka} \simeq \frac{k(n-k)}{2} \left( \frac{2m}{\gamma+2m} \right)^{N_Lm}
\]  

(A.131)

### A.5.2 Average ABEP for GD with MRC Reception (GD-MRC)

The approximate (ABEP) for OFDM-IM in (8.13) over the fading channel can be characterized by averaging (5.20) over (8.8) and inserting (5.2) and (5.21) as:

\[
ABEP_{QAM, \text{Naka, Approx}}^\text{GD-MRC} = \int_0^\infty \BEP f_\gamma(\gamma) \, d\gamma = \int_0^\infty \left[ PEP + P_e - PEP \times P_e \right] f_\gamma(\gamma) \, d\gamma
\]

\[
= \frac{n-k}{2} \times \frac{m^{N_Lm}}{\gamma^{N_Lm} \Gamma (N_Lm)} \int_0^\infty \gamma^{N_Lm-1} e^{-\gamma} e^{-\frac{m}{\gamma}} d\gamma + \frac{0.2m^{N_Lm}}{\gamma^{N_Lm} \Gamma (N_Lm)} \int_0^\infty \gamma^{N_Lm-1} e^{-\gamma\left(\frac{3m}{2(M-1)}\right)} e^{-\frac{m}{\gamma}} d\gamma
\]

\[
= \frac{(n-k) m^{N_Lm}}{2\gamma^{N_Lm} \Gamma (N_Lm)} \left( \frac{m}{\gamma} + \frac{1}{2} \right)^{N_Lm} + \frac{0.2m^{N_Lm}}{\gamma^{N_Lm} \Gamma (N_Lm)} \left( \frac{m}{\gamma} + \frac{3}{2(M-1)} + \frac{1}{2} \right)^{N_Lm} \Gamma (N_Lm)
\]

(A.132)

The \( ABEP_{QAM, \text{Naka, Approx}}^\text{GD-MRC} \) can be expressed after some simplification as:

\[
ABEP_{\text{Naka, Approx}}^\text{MRC} \simeq \frac{k(n-k)}{2} \left( \frac{2m}{\gamma+2m} \right)^{N_Lm} + 0.2k \left( \frac{m}{\alpha \gamma + m} \right)^{N_Lm} - 0.1k(n-k)
\]  

(A.133)

where the parameters are \( \beta = \frac{cm}{\gamma} \), \( \alpha = \frac{3}{2(M-1)} \), \( m \) and \( M \) are the fading factor and constellation size, respectively.

The exact (ABEP) for OFDM-IM in (8.13) over the fading channel can be characterized by averaging (5.20) over (8.8) and inserting (5.1) and (5.21) as:
\[ ABEP^{\text{QAM}_{\text{Naka, Approx}}} = \int_0^\infty \text{BEP}_1 P_\gamma (\gamma) \, d\gamma = \int_0^\infty [\text{PEP} + P_e - \text{PEP} \times P_e] P_\gamma (\gamma) \, d\gamma \]

\[ = \frac{m_{NLM}}{\gamma_{NLM}\Gamma (N_{LM})} \int_0^\infty \gamma_{NLM}^{N_{LM}-1} e^{-\frac{\gamma}{\gamma}} \, d\gamma - \frac{m_{NLM}}{\gamma_{NLM}\Gamma (N_{LM})} \sum_{q=0}^{n-k} \frac{(-1)^q}{q+1} \left[ \frac{N_{LM} \Gamma (N_{LM})}{\gamma_{NLM}\Gamma (N_{LM})} \right] \gamma_{NLM}^{N_{LM}-1} e^{-\frac{\gamma}{\gamma}} \]

\[ + \frac{0.2m_{NLM}}{\gamma_{NLM}\Gamma (N_{LM})} \int_0^\infty \gamma_{NLM}^{N_{LM}-1} e^{-\frac{3\gamma}{2(M-1)}} e^{-\frac{\gamma}{\gamma}} \, d\gamma - \frac{0.2m_{NLM}}{\gamma_{NLM}\Gamma (N_{LM})} \sum_{q=0}^{n-k} \frac{(-1)^q}{q+1} \left[ \frac{N_{LM} \Gamma (N_{LM})}{\gamma_{NLM}\Gamma (N_{LM})} \right] \gamma_{NLM}^{N_{LM}-1} e^{-\frac{3\gamma}{2(M-1)}} e^{-\frac{\gamma}{\gamma}} \]

\[ = 1 - \frac{m_{NLM}}{\gamma_{NLM}\Gamma (N_{LM})} \sum_{q=0}^{n-k} \frac{(-1)^q}{q+1} \left[ \frac{N_{LM} \Gamma (N_{LM})}{\gamma_{NLM}\Gamma (N_{LM})} \right] \gamma_{NLM}^{N_{LM}-1} e^{-\frac{3\gamma}{2(M-1)}} e^{-\frac{\gamma}{\gamma}} \]

\[ \times \frac{1}{\left( \frac{m}{\gamma} + \frac{3}{2(M-1)} \right) N_{LM} \Gamma (N_{LM})} - \frac{0.2m_{NLM}}{\gamma_{NLM}\Gamma (N_{LM})} \sum_{q=0}^{n-k} \frac{(-1)^q}{q+1} \left[ \frac{N_{LM} \Gamma (N_{LM})}{\gamma_{NLM}\Gamma (N_{LM})} \right] \gamma_{NLM}^{N_{LM}-1} e^{-\frac{3\gamma}{2(M-1)}} e^{-\frac{\gamma}{\gamma}} \]

\[ + \frac{0.2m_{NLM}}{\gamma_{NLM}\Gamma (N_{LM})} \sum_{q=0}^{n-k} \frac{(-1)^q}{q+1} \left[ \frac{N_{LM} \Gamma (N_{LM})}{\gamma_{NLM}\Gamma (N_{LM})} \right] \gamma_{NLM}^{N_{LM}-1} e^{-\frac{3\gamma}{2(M-1)}} e^{-\frac{\gamma}{\gamma}} \]

\[ \times \frac{1}{\left( \frac{m}{\gamma} + \frac{3}{2(M-1)} \right) N_{LM} \Gamma (N_{LM})} \]

(A.134)
The exact $ABEP^{QAM}_{Naka}$ can be expressed after some simplification as:

$$ABEP^{MRC}_{Naka} = k - k \sum_{q=0}^{n-k} \left( \begin{array}{c} n-k \\ q \end{array} \right) \frac{-1^q}{q+1} \left( \frac{\beta}{L+\beta} \right)^{NLm}$$

$$+ 0.2(1-k) \left( \frac{\beta}{\alpha + \beta} \right)^{NLm} + 0.2 \sum_{q=0}^{n-k} \left( \begin{array}{c} n-k \\ q \end{array} \right) \frac{-1^q}{q+1} \left( \frac{\beta}{\alpha + \beta + L} \right)^{NLm}$$

(A.135)

where the parameters are $\beta = \frac{cm}{\gamma}$, $L = \frac{q}{q+1}$, $\alpha = \frac{3}{2(M-1)}$, $m$ and $M$ are the fading factor and constellation size, respectively.

### A.5.3 Outage Probability

The $P_{out}^{MRC}$ in (8.16) can be calculated by average (8.8) and Table of Integrals [86, (3.381.1), (3.381.3)] as:

$$P_{out}^{MRC} = \frac{m^{NLm}}{\gamma^{NLm}\Gamma(NLm)} \int_{0}^{\gamma_{th}} \gamma^{NLm-1}e^{-\frac{m\gamma}{\gamma}}d\gamma = 1 - \frac{m^{NLm}}{\gamma^{NLm}\Gamma(NLm)} \int_{\gamma_{th}}^{\infty} \gamma^{NLm-1}e^{-\frac{m\gamma}{\gamma}}d\gamma$$

$$= \frac{m^{NLm}}{\gamma^{NLm}\Gamma(NLm)} \left( \frac{m}{\gamma} \right)^{-NLm} \gamma(m, \frac{m}{\gamma}\gamma_{th}) = 1 - \frac{m^{NLm}}{\gamma^{NLm}\Gamma(NLm)} \left( \frac{m}{\gamma} \right)^{-NLm} \Gamma \left( m, \frac{m}{\gamma}\gamma_{th} \right)$$

(A.136)

The $P_{out}^{MRC}$ is expressed as:

$$P_{out}^{MRC} = 1 - \frac{\Gamma \left( NLm, \frac{m}{\gamma}\gamma_{th} \right)}{\Gamma (NLm)} \text{ or } \frac{\gamma \left( NLm, \frac{m}{\gamma}\gamma_{th} \right)}{\Gamma (NLm)}$$

(A.137)

where $NL$ is the number of antenna and $\gamma_{th}$ is the threshold of SNR.

### A.5.4 Adaptive ABEP for GD-MRC

$\overline{BEP}_{R}^{MRC}$ in (8.20) can be characterized as:
\[
\overline{\text{BEP}}_{R}^{MRC} = \int_{\gamma_{R}}^{\gamma_{R+1}} \text{BEP}(R) P_{\gamma}(\gamma) \, d\gamma = \int_{\gamma_{R}}^{\gamma_{R+1}} [\text{PEP} + P_{e} - \text{PEP} \times P_{e}] \, P_{\gamma}(\gamma) \, d\gamma \\
= \frac{n - k}{2} \times \frac{m_{N_{L}m}^{N_{L}m}}{\gamma_{R}^{N_{L}m} \Gamma(N_{L}m)} \int_{\gamma_{R}}^{\gamma_{R+1}} \gamma_{N_{L}m-1} e^{-\frac{\gamma}{2}} e^{-\frac{m_{\gamma}}{\gamma}} \, d\gamma \\
+ 0.2m_{N_{L}m}^{N_{L}m} \gamma_{R+1}^{N_{L}m-1} e^{-\frac{3\gamma}{2(M-1)}} e^{-\frac{m_{\gamma}}{\gamma}} \, d\gamma \\
- \frac{n - k}{2} \times \frac{m_{N_{L}m}^{N_{L}m}}{\gamma_{R}^{N_{L}m} \Gamma(N_{L}m)} \int_{\gamma_{R}}^{\gamma_{R+1}} \gamma_{N_{L}m-1} e^{-\frac{\gamma}{2}} e^{-\frac{m_{\gamma}}{\gamma}} \, d\gamma
\]

The first part of \(\overline{\text{BEP}}_{R}^{MRC,1}\) can be solved using Table of Integrals \([86, (3.381.1)]\) as:

\[
\overline{\text{BEP}}_{R}^{MRC,1} = \frac{n - k}{2} \times \frac{m_{N_{L}m}^{N_{L}m}}{\gamma_{R}^{N_{L}m} \Gamma(N_{L}m)} \int_{\gamma_{R}}^{\gamma_{R+1}} \gamma_{N_{L}m-1} e^{-\frac{\gamma}{2}} e^{-\frac{m_{\gamma}}{\gamma}} \, d\gamma \\
= \frac{n - k}{2} \times \frac{m_{N_{L}m}^{N_{L}m}}{\gamma_{R}^{N_{L}m} \Gamma(N_{L}m)} \left[ \int_{0}^{\gamma_{R+1}} \gamma_{N_{L}m-1} e^{-\frac{\gamma}{2}} e^{-\frac{m_{\gamma}}{\gamma}} \, d\gamma - \int_{0}^{\gamma_{R}} \gamma_{N_{L}m-1} e^{-\frac{\gamma}{2}} e^{-\frac{m_{\gamma}}{\gamma}} \, d\gamma \right]
\]

\[
= \frac{n - k}{2} \times \frac{m_{N_{L}m}^{N_{L}m}}{\gamma_{R}^{N_{L}m} \Gamma(N_{L}m)} \left[ \left( \frac{m_{\gamma}}{2} + \frac{1}{2} \right) \gamma_{N_{L}m} \left( \frac{m_{\gamma}}{2} + \frac{1}{2} \right) \gamma_{R+1} \right] \left( \frac{m_{\gamma}}{2} + \frac{1}{2} \right) \gamma_{N_{L}m} \left( \frac{m_{\gamma}}{2} + \frac{1}{2} \right) \gamma_{R+1}
\]

The above equation of \(\overline{\text{BEP}}_{R}^{MRC,1}\) can be simplified as:

\[
\overline{\text{BEP}}_{R}^{MRC,1} = \frac{k(n - k)}{2 \Gamma(N_{L}m)} \left( \frac{\beta}{\beta + 0.5} \right)^{N_{L}m} \left[ \Gamma \left( N_{L}m, (\beta + 0.5) \gamma_{R} \right) - \Gamma \left( N_{L}m, (\beta + 0.5) \gamma_{R+1} \right) \right]
\]

The second part of \(\overline{\text{BEP}}_{R}^{MRC}\) can be solved using Table of Integrals \([86, (3.381.1)]\) as:
The above equation of $\overline{BEP}_R^{MRC,2}$ can be simplified as:

$$\overline{BEP}_R^{MRC,2} = \frac{0.2k}{\Gamma(N_{Lm})} \left( \frac{\beta}{\beta + \alpha} \right)^{N_{Lm}} \left[ \Gamma(N_{Lm}, (\beta + \alpha) \gamma_R) - \Gamma(N_{Lm}, (\beta + \alpha) \gamma_{R+1}) \right]$$

(A.142)

where the parameters are $\beta = \frac{cm}{\gamma}$, and $\alpha = \frac{3}{2(M-1)}$.

The third part of $\overline{BEP}_R^{MRC}$ can be solved using Table of Integrals [86, (3.381.1)] as:

$$\overline{BEP}_R^{MRC,3} = \frac{0.2m^{N_{Lm}}}{\gamma^{N_{Lm}} \Gamma(N_{Lm})} \left[ \int_{\gamma_R}^{\gamma_{R+1}} \gamma^{N_{Lm}-1} e^{-\gamma} e^{\frac{-3\gamma}{2(M-1)}} e^{-\frac{m\gamma}{\gamma}} d\gamma \right]$$

$$= \frac{0.2m^{N_{Lm}}}{\gamma^{N_{Lm}} \Gamma(N_{Lm})} \left[ \int_{\gamma_R}^{\gamma_{R+1}} \gamma^{N_{Lm}-1} e^{-\gamma} e^{\frac{-3\gamma}{2(M-1)}} e^{-\frac{m\gamma}{\gamma}} d\gamma \right]$$

$$= \frac{0.2m^{N_{Lm}}}{\gamma^{N_{Lm}} \Gamma(N_{Lm})} \left[ \gamma^{N_{Lm}-1} e^{-\gamma} e^{\frac{-3\gamma}{2(M-1)}} e^{-\frac{m\gamma}{\gamma}} d\gamma \right]$$

(A.143)

The above equation of $\overline{BEP}_R^{MRC,3}$ can be simplified as:

$$\overline{BEP}_R^{MRC,3} = \frac{0.1k(N - k)}{2\Gamma(N_{Lm})} \left( \frac{\beta}{\beta + \alpha + 0.5} \right)^{N_{Lm}} \left[ \Gamma(N_{Lm}, (\beta + \alpha + 0.5) \gamma_R) - \Gamma(N_{Lm}, (\beta + \alpha + 0.5) \gamma_{R+1}) \right]$$

(A.144)

where the parameters are $\beta = \frac{cm}{\gamma}$, and $\alpha = \frac{3}{2(M-1)}$. 

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References


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