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The Alia musica and the Carolingian Conception of Mode

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A thesis submitted in partial fulfillment of the requirements for the Doctor of Philosophy degree in Music

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Abstract

The *Alia musica* is perhaps the most idiosyncratic of the early treatises on the ecclesiastical modes. It is a composite made up of at least three independent treatises and additional commentary, and the majority of the scholarly attention that it has thus far received has been devoted to questions of dating and authorship, as well as to the place of the *Alia musica* in the development of the octave species paradigm of modality. However, the majority of the treatise is dedicated to the explanation of a complex harmonic numerology that applies the fundamental relation 12:9:8:6 (which generate the intervals of an octave, a fifth, and a fourth) in a unique way to each of the eight ecclesiastical modes to define a set of intervals thought to be particularly characteristic of the chants in each mode.

This dissertation reviews the previous studies about the *Alia musica*, as well as the manuscript sources, examines the theoretical context within which the treatise was written, and analyzes the numerological system both from the evidence of the text of the *Alia* itself and from an analysis of the chants that the *Alia* cites as exemplifying the numerical relations proposed for each mode. The intervals represented by these numbers show only partial consistency and can generally be explained as being constructed from the simplest, or perhaps the most numerologically meaningful, multiples of the four base numbers from 12:9:8:6 that successfully filter out the intervals that are not considered to characterize a particular mode.

Keywords: *Alia musica*; medieval music theory; Carolingian music theory; ecclesiastical modes; octave species; harmonics.

Summary for Lay Audience

Every musical composition (or part of a composition) has a characteristic mood that results from its mode. For the last few centuries, most Western music has used only two modes: major and minor. These two modes may use the same notes (for instance all the white keys on a piano) but treat a different note as the point of origin of the musical scale (in this example, C for major and A for minor). It is also possible to start the scale from other notes, creating other modes that have been used in other historical contexts and are still used, in a somewhat different manner, in folk music, jazz, and some forms of popular music.

However, the concept of mode, which goes back to ancient Greece, has been defined in many different ways throughout history and was understood very differently in the Middle Ages. Around the beginning of the ninth century, musicians adapted mode to the classification of ecclesiastical chant according to a set of recognizable musical characteristics, to facilitate the joining together of chants with similar characteristics. From that time onward, although the modal system continued to develop, a degree of continuity in the modal concept was maintained right down to the present, making the ninth century essentially the point of origin of the current system of modes. The *Alia musica* is one the very few treatises from this period that provides technical details about them, describing them according to a set of mathematical ratios that correspond to favoured musical spans. These details are completely unlike any other known description, and this aspect of the treatise has not been well explained in previous studies. This dissertation analyzes the description of the modes in the *Alia musica* to clarify the meaning of its unusual definitions and increase our knowledge of the earliest stages of the current Western modal system.

In any attempt to construct a system out of human affairs, there will always be grey areas for things that do not quite fit – things that belong yet don't belong. And despite the traditional admonition that 'difficult cases make bad law', the exceptional cases will often figure disproportionately in defining the system's boundaries – occasionally even its central concerns. This is nowhere more true than of attempts to define tonal or modal systems for large bodies of music, such as what we know as Gregorian chant.

– Keith Falconer,
"The Modes Before the Modes"

In der Gesichtsforschung soll man nicht immer erst warten, bis es möglich ist, eine Aufgabe völlig zu lösen.

[In historical research, one should not always wait until it is possible to fully untangle a problem.]

– Aloys Schulte,
Der hohe Adel im Leben des mittelalterlichen Köln

If you think that you understand quantum mechanics, you don't understand quantum mechanics.

– unknown,
traditionally attributed to Niels Bohr

Dedication

This work is dedicated to my wife, Dawn, for her support, understanding, and patience during the long process of writing this dissertation.

Acknowledgments

I would like to extend my sincere gratitude to my supervisor, Dr. James Grier, not only for his excellent advice on the more complicated issues underlying this dissertation, but also for his careful eye for typos and grammatical errors, and most especially for his advice on my translations from Latin. I am also grateful to him for the amount of time he was willing to invest as I neared my deadline and for putting up with my unceasing habit of overrunning the intended length of my work; in these respects, I am grateful also to Dr. Kevin Mooney, who served as a second reader for this dissertation.

Additionally, I wish to acknowledge the influence and assistance of Dr. Charles Atkinson on this work. With the exception of the various published editions of the *Alia musica* and its constituent parts, the chapter on the *Alia musica* in Atkinson's *Critical Nexus* (Oxford University Press, 2009) forms nearly the entirety of the secondary literature on the treatise and is certainly the highest quality discussion of the treatise up to the present. This would already be enough to make Atkinson highly influential on this dissertation. In addition, Atkinson served as an external examiner at the defence of this dissertation, and above and beyond the discussions at the defence, he also provided extensive written feedback. Much of the feedback that he provided is reflected in this revision of the original text presented at the defence.

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Terminology, Notation, and Related Matters

Throughout this dissertation, I principally use the pseudo-Odonian system of pitch nomenclature and octave differentiation. In this system, new octaves begin and end with the pitch-class A. The lowest complete octave of the traditional range, effectively covering the bass clef (though I shall normally notate in transposed treble clef in the manner common for tenor voices) is denoted in majuscule letters. Ordinarily, this system only contains one pitch below this range, a G that is denoted with the equivalent Greek letter, *gamma* (Γ); from time to time throughout this dissertation, it will be convenient to refer to hypothetical pitches below this range, and I shall follow the pattern implied by this usage and label this range with Greek equivalents. Specifically, I shall label the F in this range with the Greek letter *phi* (Φ). The notes of the octave above the majuscules are labelled with minuscule letters. The notes of the highest octave, of which only the lowest notes are required, are indicated by stacking two minuscule letters in a column.

Traditionally, only natural pitches arise in this gamut, with important exception of B-flat, which can appear in either of the upper two octaves; in this system, B-flat is indicated by a rounded letterform resembling the modern flat sign (\flat), while B-natural is indicated by a squared letterform (\natural). The B-flat in the majuscule octave is not traditionally considered to be a part of the system, but it does seem to appear in some of the repertoire and in the gamut used by many of the treatises coeval to the *Alia musica*; I shall use the symbol \flat for this pitch.¹ Other sharp and flat pitches also occasionally arise, and I shall occasionally find it convenient to place a sharp or flat symbol after the pseudo-Odonian symbol, notwithstanding the anachronism of this practice; in such cases, I shall employ only a single sharp or flat sign for the highest octave, not a pair stacked up into a column (as with the letter names).

¹ This symbol appears to have been invented by Hermannus Contractus; cf. Ellinwood, *The Musica of Hermannus Contractus*, 82–83, esp. fn. 61.

A note about Greek and Latin vocabulary, transliterations, and orthography:

The *Alia musica* is a Latin theory treatise that is widely known for the way that it incorporates Greek music theory into Latin ecclesiastical chant theory. It would hardly be possible to discuss this treatise without extensive use of Greek terminology, which brings with it the ever-vexing issue of how to transliterate Greek terminology. Where there is no suitable English translation, I have chosen to remain as faithful as possible to the original Greek orthography. In many cases, that means that I have retained the use of the Greek script, an ideal solution that avoids problems of transliteration and orthography.

However, I have also often found it necessary in some cases to transliterate the Greek terms into the Latin alphabet, and I have usually done so according to the same principles, rather than using Latinized versions. Specifically, I have generally maintained the Greek inflections, such as second-declension nominatives in -os (m.) and -on (n.) instead of the Latin -us and -um. I have also retained the traditional diphthongs -ai- and -oi- in place of the -ae- and -oe- that typically replace them in Latin. Both of these are amply demonstrated in the Greek tetrachord name *hyperbolaion*. Additionally, I have transliterated the Greek letters κ and χ as k and kh, respectively, as in the term *oktōēkhos* (commonly transcribed in other sources as *octoechos*). I have also transliterated long o and e with a macron because they are distinct letters in the Greek script, while I have ignored other long vowels that are not distinct in Greek script.

I have made a few exceptions to this practice, concerning Greek words that have been so thoroughly adopted into Latin that they have become part of the core Latin music-theoretical vocabulary. Though I shall occasionally use the Greek orthography when context recommends it, I make free use of the Latinized forms *tonus* and *tropus*, and generally do not use Greek script for such standard terms as the intervals *diapente*, *diatessaron*, and *diapason*. Furthermore, although the ordinal numbers *protus*, *deuterus*, and so on, are clearly derived from Greek (as opposed to the native Latin ordinals *primus*, *secundus*, etc.), they are, in fact, corrupted Greek (in particular, there is no such Greek word as “tetrardus”

by any conventional transliteration), and I have, therefore, generally treated these words as inherently Latin words, loosely synonymous to the native Latin ordinals but used exclusively to refer to the four pairs of authentic and plagal modes and their associated *finales* and *affinales*.

Additionally, except where I am directly citing a particular manuscript, I have taken the liberty to standardize orthography in both languages. This includes the names of chants; as a particularly prominent example, the manuscripts make frequent citations to *Rorate celi*, which I have standardized to *Rorate caeli*, and so on. I have not, however, made any attempt to rectify the titles of treatises, such as the *Alia musica*, *Musica enchiriadis*, or *Scolica enchiriadis*, all of which appear to have faulty Greek elements in their titles, but the interpretation of which has generally not reached complete scholarly consensus. Chailley, in his edition of the *Alia musica*, silently standardizes the orthography, including in ways that do not suit my preferences; while I shall leave Chailley's own prose as he wrote it, when citing his edition of the *Alia* text (and other Latin treatises) I shall, following his example, rectify the orthography to reflect my usage without further comment, particularly in removing the anachronistic use of j and v (though, for simplicity, I shall retain the v in treatise titles, such as *Nova expositio*).

Finally, there are also a handful of Latin terms that I have frequently chosen to leave in Latin, even though there are reasonable English translations for them. In these cases, it has been my judgment that the meaning of these terms in a musical sense differs somewhat from their conventional English cognates, or that the use of the English cognate in some way causes a degree of ambiguity that can hinder comprehension. A good example is the *finalis*; although the English translation of a "final pitch" adequately conveys the original meaning and derivation of the term, it does not truly convey the tonic-like qualities of a *finalis*, and can also cause some ambiguity with the use of the English word "final" as an adjective. Other words of this kind that I have retained in Latin include *medius*, *comma*, and *colon*. Obviously, I have also retained a handful of technical terms that do not translate well into English, such as a *locum* (which is not strictly the same thing as a *locus*).

A glossary of key Greek and Latin terminology is included in the appendices.

About the use of the terms 'Greek,' 'Roman' and 'Byzantine'

When discussing the older influences upon the theory of the *Alia musica*, there are two distinct “Greek” traditions. The first is that associated with most notably Pythagoras, Aristoxenus, Ptolemy, and other such writers from the ancient world, passed on to the later Middle Ages by Roman authors such as Martianus Capella, Boethius, Cassiodorus and Isidore. I shall often refer to this theoretical tradition as ancient Greek, notwithstanding the fact that several of the authors are neither Greek nor ancient (Isidore, at least, is unambiguously medieval) and that the Carolingian theorists probably never read the original Greek treatises and learned the theory principally from the Roman authors. I shall not often use the term Roman, but where I find it useful, it also refers to this same theoretical tradition, Romans not really possessing an independent theoretical tradition – unless one were to count the theoretical tradition of the Eastern Empire after the fall of the Western Empire. The people of the Eastern Empire continued to think of themselves as Roman, but the people of the former Western Empire called them Greek, a usage also commonly employed by modern musicologists. This second Greek theoretical tradition is barely, if at all, related to the ancient Greek tradition, but it also had important repercussions for the theory of the Carolingian Renaissance. To avoid ambiguity in the use of either the terms Greek or Roman, I shall refer to this later Greek tradition as Byzantine.²

A Note About Translations

Throughout this dissertation, I cite many excerpts from theory treatises in both Greek and Latin, as well as excerpts from secondary literature in French, German, and Italian. In these citations, I provide the

² In fact, this term, too, is a bit of a misnomer; as will be described in Chapter 06, the theory (commonly referred to as the *oktōēkhos*) probably originated even further east, in territories that had once been part of the Eastern Roman (*i.e.*, Byzantine) Empire, but by the time of the Carolingian Renaissance in the West, had already been lost to the Muslim Conquests for some two centuries.

original language (in the case of treatises, drawn, where possible, from published critical editions) followed by a translation. I generally include in my citations references to published translations, except in the case of the *Alia musica* itself: the only available English translation for most of the treatise is that of Edmund Heard, and his translation is too problematic to be of value; however, Edward Nowacki provides a translation of the final portion of the *Alia musica*, while Charles Atkinson translates several important excerpts; their translations are of high quality and I cite them where relevant. It should be understood that, except where otherwise noted, the translations that I have provided in the text are not those that I have included in the citations accompanying the excerpts in the original language; I cite these previously published translations only for the purposes of comparison and verification. For the Latin translations, I am indebted to my dissertation advisor, Dr. James Grier, for his advice on some of the more complicated issues, and for the Greek treatises, I have leaned upon translations of other scholars (chiefly Andrew Barker); in all cases, however, in both Greek and Latin, the translations that I provide in the text (and any errors therein) are ultimately my own.

Likewise, with the secondary resources, all translations from French and German are my own. The Italian of Feretti's *Estetica Gregoriana* is a special case: I have cited from Feretti's original Italian edition, but I acknowledge having read the majority of the book solely in Agaësse's French translation; the only passages that I have read in the original Italian are those that I found to be valuable to cite.

About My Use of the Term "Numerology"

At the defence of this dissertation, it was suggested to me that my use of the term "numerology" in connection with the *Alia musica* is misleading. It was suggested that for many people, the word invokes elements of the occult or supernatural. I agree that this description does not correspond to anything found within the *Alia musica*, except perhaps in the brief section of the treatise that compares the recurrence of a pitch every octave to the recurrence of an unusually large wave every eight waves or an

unusually loud thunder clap every eight; eight notes corresponding to eight winds; five perfect consonances corresponding to five zones of the heavens; and so forth. While I acknowledge the concern, my use of the term is frequent enough in the dissertation that I felt that it was not desirable to revise every passage in which I used it. Instead, I have chosen to retain the term, but I wish to clarify here that I use it in a more general sense. I call the number system of the *Alia musica* “numerology” not because I suppose the numbers to carry any kind of occult or mystical significance, but more simply because in many cases they seem to lack practical significance. That is, the numbers appear in many instances to be selected so that the relationships between them are symbolically important for their own sake, without having any practical applicability to actual musical harmonics.

Section I: Background

1. Introduction (2)
2. Historiography (8)
3. Contents of the Treatise (52)
4. Manuscript Study (68)

Chapter 01: Introduction

The *Alia musica* is a composite treatise about the ecclesiastical modes, presumed to have been compiled by interleaving the work of three or four anonymous authors (or perhaps more), written during the late ninth or tenth centuries.³ This composite treatise is organized around an original source treatise of unknown origin (perhaps northern France),⁴ which contains a unique and elaborate but incompletely explained numerological system that attempts to relate the ecclesiastical modes to ancient Greek harmonic theory; this source treatise also provides enough examples of chants in each mode to constitute a rudimentary tonary (though it lacks the traditional discussion of *differentiae*). This text is appended to the end of the composite treatise, the bulk of which comprises a revised edition of the same text, as well as supplementary information drawn from the ancient Greek and Roman tradition (especially Boethius), and a commentary upon all of this material, all of which was once considered to be the work of a single editor, but is now presumed to be the work of two separate authors,⁵ and may also include brief passages incorporated from earlier sources. This relatively unified conglomerate is interleaved with another treatise from an entirely separate tradition (perhaps originating in Aquitaine)⁶ that functions as a more complete tonary, including lists of *differentiae*. Opinions vary as to which (if any) of the aforementioned contributors created the compilation.

Although the *Alia musica* is routinely mentioned in modern musicology in connection with early ecclesiastical modes, there have been surprisingly few large-scale examinations of this treatise. There have been, to the best of my knowledge, no articles about the *Alia musica* itself published in any journals

³ Huglo, *Tonaires*, 252. Huglo refers to this period as the “Second Carolingian Renaissance,” though the term is not as well known as the original Carolingian Renaissance. The Second Carolingian Renaissance took place under the reigns of Charlemagne’s immediate successors; the Ottonian Renaissance is also sometimes characterized as the “Third Carolingian Renaissance.”

⁴ Chartier, “Hucbald”, ¶1; Huglo, *Tonaires*, 252.

⁵ Atkinson, *Critical Nexus*, 175 and fn. 12.

⁶ Huglo, *Tonaires*, 58, 129.

or compilations, though the tonary with which the principal treatise is interleaved also appears elsewhere in the manuscript tradition, and a few articles have been dedicated to it by Karl-Werner Gumpel (1977 and 2007) and Michael Bernhard (1987). I have been able to find only a single conference presentation about the *Alia musica* (by Cynthia Cyrus at AMS 1992, for which only the abstract is now readily available). There have been two doctoral dissertations on the *Alia musica*, by Wilhelm Mühlmann (1914) and Edmund Heard (1966), and a critical edition with commentary by Jacques Chailley (1965), superseding Gerbert's 1784 edition. Additionally, the *Alia musica* is the subject of a substantial chapter in Charles Atkinson's *The Critical Nexus* (2009), and it receives considerable attention in parts of Michel Huglo's *Les Tonaires* (1971). Most recently, translation and commentary of a small portion of the treatise has been published by Edward Nowacki in his *Greek and Latin Music Theory: Principles and Challenges* (2020). This appears to be the extent of the literature dedicated to this treatise (this historiography will be considered in greater detail in the next chapter).

A substantial portion of the scholarship dedicated to the *Alia musica* has been devoted to the attempt to sort out the difficult question of authorship, generally labelling the authors with descriptive names (no specific attributions to known theorists have been made, except for the attribution to Hucbald found in one manuscript and reproduced in Gerbert's edition, which is no longer given much credence); much attention has also been given to determining the range of dates over which the treatises must have been written – but interestingly, rather less to identifying the geographic source of the treatise. Most of the remaining scholarship has focused on a few brief but critical passages that seem to represent the first comprehensive attempt to merge Carolingian ecclesiastical modal theory (which is, itself, most closely related to the Byzantine *oktōēkhos* system) with the prestigious harmonic and modal theory of ancient Greece (as transmitted to the medieval West by Boethius); these passages seem to be responsible for the use of Greek nomenclature for decidedly non-Greek modes, a practice that endures to the present day (seriously misrepresenting Greek theory in the process). In addition to these topics, a certain amount of

attention has been given to the tonary aspects of the *Alia musica* (especially by Huglo) and to its use of intonation formulae (especially by Terence Bailey).⁷

Because of the overwhelming attention paid to authorship and to the application of Greek ideas to ecclesiastical modes, very little attention has been given to the numerological elements of the treatise. This omission is odd, since the description of the numerology occupies more of the author's attention than any other single topic in the treatise; it is directly presented at least four times (once in the source treatise, once in the revised edition of the source treatise, then again in a table near the end of the treatise, and finally in a prose summary following the table), is elaborated upon twice (in the source treatise and the revised edition), and is the subject of much of the commentary.

The narrow focus of previous studies has left a hole in the scholarship about the *Alia musica*. This treatise does not seem to fit nicely with any of the other Carolingian treatises (most of which seem to work nicely together, notwithstanding a division regarding the structure of the gamut),⁸ which may explain why it has been so understudied. Indeed, it is precisely those theoretical concepts that differ from the rest of the Carolingian tradition that have been least studied, and it is that aspect of the treatise upon which a major part of this dissertation is focussed.

⁷ Huglo, *Les tonaires*, esp. 58–59; Bailey, *Intonation Formulas*, esp. 7 & 9–10.

⁸ Several treatises from the ninth and tenth centuries employ an alternate gamut based on a repeated cycle of the first species of fifth, as will be described in later chapters. The most notable of these are the *Musica enchiriadis*, the *Scolica enchiriadis*, and the *Commemoratio brevis*, all of which are edited in Schmid, *Musica et Scolica enchiriadis una cum aliquibus tractatulis adiunctis*, along with several other, smaller treatises, most of which describe the same gamut. However, the other major treatises of the era are more consistent with the gamut inherited from the classical Greek tradition. It should also be observed at this point that the term “gamut” is derived from the first note of the medieval gamut, labelled *gamma-ut* in the solmization system devised by Guido of Arezzo; it is, therefore, anachronistic to use it to describe the musical systems prior to the eleventh century. Nevertheless, I shall continue to use it here for lack of an appropriate alternative (“scale” is not quite apt, and “tonal system” is rather cumbersome).

There is a famous quotation about quantum mechanics that could be adapted well to describe modes⁹ in the Carolingian period. No one is quite sure who said it, or when or where (it is frequently attributed to either Niels Bohr or Richard Feynman, with probably about as much accuracy as the attribution of nearly all the major Carolingian theory treatises to Hucbald), and it says the following: “If you think you understand quantum mechanics, you don’t understand quantum mechanics.” This quotation works on two levels: first, it conveys the general truth that no one truly understands the concept; but more interestingly, it implies that it is those who are most familiar with it that are also most keenly aware of how much we still do not understand about it.

So, too, with the Carolingian conception of mode, which is frustratingly opaque. It is widely recognized that the ecclesiastical modes arose in the East and were imported to the Latin West and imposed *post hoc* onto a pre-existing chant tradition that had been organized on different terms.¹⁰ The nature of this fusion is less well understood than the fusion of the ecclesiastical modes with ancient Greek harmonics and terminology (which, itself, is not as straightforward as the traditional narrative suggests,

⁹ The term “mode” in this context is also slightly anachronistic. The Latin etymon *modus* is a relatively uncommon label for mode in the Carolingian period (with the interesting exception of the *Musica enchiridis*). More specifically, the term *modus* is used only in a short section of the *Alia* drawn from Boethius (*Alia musica*, §§13–16, ed. Chailley, 105–07); elsewhere, the terms *tonus* and *tropus* are almost universally employed. Nevertheless, the terms “tone” and “trope” generally lack currency as a term for the concept of mode in modern usage and both have alternative meanings that create potential ambiguity; it is, therefore, simplest to continue to use the term “mode” throughout this dissertation, notwithstanding the actual Carolingian usage.

¹⁰ The relationship between the Byzantine and Gregorian chant traditions is widely acknowledged, but often only touched upon quite briefly, especially in sources focussing on Western chant. See, for instance, Apel, *Gregorian Chant*, 37–38; *Grove Music Online*, s.v. “Mode” §II.1.ii; s.v. “Oktōēchos” ¶14; s.v. “Byzantine Chant” §5. A slightly more thorough treatment is given in Hiley, *Western Plainchant*, 525–30, though even here, the discussion of the modes is limited to a single paragraph on p. 527. In scholarship dedicated to the Byzantine tradition, the issue is discussed briefly and intermittently in Wellesz, *A History of Byzantine Music and Hymnography*, especially in the introduction and first chapter (pp. 1–45 in the 2nd ed., especially p. 43), and the origins of the system are explored in Jeffery, “The Earliest Oktōēchoi,” especially pp. 149–52. Charles Atkinson also discussed the topic in a presentation entitled “On Modulation in Byzantine and Early Western Chant: The Treatise of Manuel Chrysaphes, the *Papadikai*, and the *Enchiridis* Complex” at the annual meeting of the American Musicological Society in Boston in 2019, which he has indicated to me that he anticipates publishing in a forthcoming article.

as I shall argue throughout this dissertation); it is difficult to know precisely what characterized each mode or upon which criteria a chant was assigned to a given mode in this period.

Early discussions of mode are largely metaphorical, with little or no technical information. Presumably, Carolingian authors could reasonably expect their target audience to know intuitively what a mode was and which chants were associated with which mode; as a result, the treatises generally do not provide us with this information. Aurelianus addresses the modes extensively in a kind of tonary but says little about their quality. Hucbald hardly discusses the modes at all, except to demonstrate that nearly every pitch is a potential starting note in each mode. The *Musica enchiridis* gives a considerable discussion of the modes, but is framed in an alternative gamut, yielding as many questions as answers – and even here, mode is a relatively minor topic next to the study of the consonances and organum. Of all the Carolingian treatises, the *Alia musica* appears to be the earliest to take technical descriptions of mode as the principal topic and describes characteristics of mode other than the final note (which the *Alia* rarely mentions) and the chants found within the mode.

It is, therefore, in studying the *Alia musica* that we are best able to answer a fundamental question, to which this dissertation is directed: what, precisely, was the nature of a mode in the Carolingian period? This question brings with it a variety of corollaries. How is the mode of a chant determined? Is the mode of a chant evident throughout the chant, or only at key moments (such as the end)? How does the concept of mode in the *Alia musica* compare to the Byzantine conception of mode? How does it relate to the gamut as described by Hucbald? Is it compatible with the gamut described in the *Musica enchiridis*?

This dissertation, then, takes the form of a commentary on the *Alia musica*; more specifically, this commentary focusses upon the question of what the *Alia musica* says about mode and how this conception of mode compares to those of ancient Greek theory, Byzantine theory, and the other treatises

of the Carolingian Renaissance, in order to increase our understanding of precisely what mode was in the ninth century.

Chapter 02 – Historiography of the *Alia musica*

Significant Studies of the *Alia musica* and theories of authorship

In the introduction to his 1966 dissertation, Edmund Heard summarized the scholarship on the *Alia musica* up to his time by noting that “almost every music historian of the nineteenth century refers to it.”¹¹ Fifty years later, one must also observe that the *Alia musica* is also frequently mentioned throughout the twentieth and into the twenty-first century. However, most of these various writings provide only brief or sporadic discussions of the *Alia musica*. Besides the work of Gerbert and Heard, only Wilhelm Mühlman and Jacques Chailley devote entire large-scale works to the topic; in addition, Charles Atkinson dedicates an entire chapter to the *Alia musica*, and Michel Huglo a large subsection of a chapter, while Michael Bernhard and Karl-Werner Gümpel each edit texts closely related to, but distinct from the *Alia musica*. Since this dissertation builds upon and will engage frequently with the work of these authors, a brief introduction to their studies appears below.

Gerbert

Scholarship on the *Alia musica* began with Martin Gerbert, who included a Latin edition in the first volume of his *Scriptores ecclesiastici de musica* in 1784;¹² he attributes the treatise to Hucbald of St. Amand, implying a date in the late ninth or early tenth century. Hucbald is well known for other writings in this period (not exclusively musical), and a wide variety of treatises originating in his milieu have apparently been erroneously attributed to him over the subsequent centuries, including the *Musica*

¹¹ Heard, *Alia musica*, 1–2. Heard goes on to acknowledge references to the *Alia musica* in the work of Hugo Riemann, Willi Apel, Gustav Reese, Otto Gombosi. Other small-scale studies have been done by Antoine Auda, Henri Poitiron and Gustav Jacobsthal (See Chailley, *Alia musica*, 5 & 10); David Russel Williams and C. Matthew Balensuela, Christian Meyer and Shin Nishimagi (Balensuela, “Anonymous Theoretical Writings,” *Grove Music Online*, §2); Nancy Phillips and Cynthia Cyrus (Atkinson, *Critical Nexus*, 171 & 175 fn. 12); and Lawrence Gushee (Gushee, “Questions of Genre,” *Gattungen der Musik in Einzeldarstellungen*, 365–433.).

¹² Gerbert, *Scriptores ecclesiastici I*, 125b–47b.

enchiriadis, the *Inchiriadon*, and the *Commemoratio brevis*; only in the last century or so have scholars systematically refuted most of these attributions. Most of these refutations are beyond the scope of this dissertation,¹³ and I shall address the authorship of the *Alia musica* below and in the manuscript study in Chapter Four. Of all the theoretical treatises, only the *Musica* (formerly called *De armonica institutione*) is now widely considered to have been written by Hucbald.¹⁴

Although Gerbert's edition is not without its flaws, modern scholarship is indebted to Gerbert not only because of his pioneering role in early musicology but also, specifically regarding the *Alia musica*, because his edition was principally based upon a manuscript from Strasbourg that has since been lost, thus indirectly preserving a rare complete copy of the text (there are only four other known complete copies).

Mühlmann

Wilhelm Mühlmann published a significant study of the treatise in 1914, including a German translation, and rejected Gerbert's attribution to Hucbald, suggesting that there may have been five or more authors; he dated the work in the tenth century, near the supposed date of the Munich manuscript, the oldest extant source (a date which has since been revised to the early eleventh century).¹⁵ Since his opinion of authorship is no longer generally accepted, his hypothesis will not be presented in detail here, but a table summarizing and comparing hypotheses of authorship will be presented in the next chapter. His opinion of authorship is no longer generally accepted, as it has been supplanted by Chailley's opinion

¹³ Most are dealt with effectively in Hans Müller, *Hucbalds echte und unechte Schriften über Musik*, Leipzig: R. G. Teubner, 1884; the *Alia musica* is addressed on p. 21. I shall address Müller's comments in Chapter 4.

¹⁴ However, Hucbald also wrote a considerable amount of non-theoretical material and writings on other topics (see Chartier, *L'Œuvre musicale d'Hucbald*, 11–45).

¹⁵ Atkinson, *Critical Nexus*, 171 fn. 2. Atkinson cites Bernhard Bischoff, *Literarisches und künstlerisches Leben in St. Emmeram*, 80–82; Bischoff is a noted paleographer, and he is also cited in the RISM article for this manuscript (http://musmed.fr/RISM/d.htm#D_Mbs s.v. clm 14272), which Atkinson also cites (in printed edition).

(with modifications by Atkinson), but there are still questions remaining about authorship, and some of these bear striking resemblances to Mühlmann's original proposals.

Chailley

In 1965, Jacques Chailley published a critical edition of the Latin text. This edition does not include a translation but incorporates an extremely close commentary (in French) at nearly a sentence-by-sentence level, resulting in effectively a paraphrase of the treatise; he also included some seventy pages of introductory material.

Chailley concluded that there were only three authors, representing three sub-treatises already described in Chapter One. The source treatise, which Chailley described as the "*premier quidam*" (in a mixture of French and Latin, the "first somebody"), is named for the introduction that the revisor gives to his revision after his introductory comments: "*Tandem ad cuiusdam expositionem de praefatis tropis uel modis ueniamus*"¹⁶ [emphasis added; *cuiusdam* is the genitive form of *quidam*] ("Finally we come to someone's exposition of the previously mentioned tropes or modes"). To this author Chailley also ascribes a few philosophical passages comparing octave equivalence to periodic cycles of larger and smaller waves on the shore, or louder and quieter thunder, and so on. Chailley describes the revision of this source treatise, together with the commentary upon it and the supplementary material as the "*Alia proprement dite*"¹⁷ (the *Alia* proper) or the "*traité principal*" (the principal treatise). Chailley refers to the tonary as the *Nova expositio*, after the rubric with which the compiler of the *Alia* routinely announces these interpolations, such as "*Item cuiusdam de eadem re noua expositio*"¹⁸ ("Likewise, concerning the same subject, someone's new exposition"); on account of the recurrence of the term *cuiusdam* in this rubric, Chailley also refers to this author as the *deuxième quidam* (the "second somebody"). Chailley considers

¹⁶ *Alia musica* §30, ed. Chailley, 121.

¹⁷ Chailley, *Alia musica*, 7.

¹⁸ Chailley, *Alia musica*, 126.

this to be the last of the treatises to have been composed, containing a little bit of additional material that he believes to have been composed by the same author, whom he also believes to have been the compiler of the finished treatise. Since he considered these materials to be effectively three independent treatises, he decided to separate the work of each author, originally intermingled, into three discrete blocks, so that the reader could read each treatise independently. (Common criticisms of this decision will be discussed in greater detail later in this chapter).

Heard

The year after Chailley's edition was published, Edmund Heard completed his doctoral dissertation on the *Alia musica*. Heard's dissertation takes the form of a translation into English and a commentary (he also produces an edition in the form of a transcription of the Munich manuscript of the *Alia*). Both the translation and the commentary are problematic (and I shall address these problems periodically throughout this dissertation, as they become relevant to the topic at hand), but they still constitute the most accessible treatment of the entire treatise for an English audience.

Heard indicates that his dissertation was nearly complete when Chailley published the critical edition. He acknowledges the considerable agreement on most points, such as tripartite authorship and approximate dating, but also notes some difference of interpretation between the two works. He argues that the differences in interpretation ensure the value of both works.¹⁹ However, Chailley's commentary resolves difficulties that still confound Heard's translation (and, to a lesser degree, his commentary). Heard seems to have chosen to revise his work only minimally in response to Chailley's edition; thus, while Heard published more recently and had access to Chailley's work, Chailley's remains the superior source.

¹⁹ Heard, *Alia musica*, 2 fn. 2.

Huglo

Michel Huglo's examination of the treatise has been somewhat more limited. Huglo reviewed Chailley's edition of the *Alia musica* when it appeared in 1965,²⁰ contributing an alternate hypothesis for the origin of the tonary portion to the composite treatise (see Geographical Origin, below). Then, in his dissertation, Huglo discusses the tonary elements of the treatise, not only in the *Nova expositio*, but also in the less systematic tonary found in the source text and revision, as well as a set of marginal glosses in the Munich (St. Emmeram) manuscript that Huglo describes (unsurprisingly, given the source of the manuscript) as including antiphons conforming to the German tradition.²¹

Bernhard

Bernhard's contributions to the study of the *Alia musica*, though more limited in scope, have been twofold. First, Bernhard published a supplement to Gerbert's *Scriptores ecclesiastici* that provides useful information to scholars studying the treatises of the *Scriptores*, including lists of manuscripts and editions, and in some cases, including revised texts directly in the supplement. In the case of the *Alia musica*, Bernhard identifies the sources that Gerbert used (perhaps erroneously, as he lists the Cesena manuscript; although Gerbert was aware of it, he seems not to have actually used it – see Chapter Four), cites Chailley's edition, and identifies several manuscripts that Chailley did not consult, citing sources also for these.²²

Second, Bernhard published an edition (with light commentary) of another composite treatise, the *Dulce ingenium*, that independently includes the tonary that Chailley calls the *Nova expositio* (though there is no justification for this title found within the *Dulce ingenium*; it begins without any kind of rubric

²⁰ Huglo, Review of Chailley (*Alia musica*), 230–32. Chailley also served as Huglo's doctoral supervisor, which, however, was not published until six years later, in 1971 (Huglo, *Les tonaires*, 6).

²¹ Huglo, *Les Tonaires*, 58–59.

²² Bernhard, *Clavis Gerberti*, 16–17.

or transition). Although he says little about the rest of the *Alia*, his critical apparatus and commentary provide some insights into the tonary.

Gümpel

Like Bernhard, Gümpel's principal contribution concerns the *Nova expositio*, which exists in two recensions. The principal recension is found almost identically in the *Alia musica* and the *Dulce ingenium*. The second is heavily revised and appears in only one manuscript, which shortly afterwards also contains fragments of the *Alia musica* (including fragments of the principal recension of the *Nova expositio*). Gümpel edited and published this revised recension in a side-by-side comparison of the text against the standard recension.²³

Atkinson

Probably the best discussion of the *Alia musica*, though limited in scope, is the treatment by Charles Atkinson in his 2009 book, *The Critical Nexus*. This book treats the development of mode, gamut and notation from the heritage of the ancient Greeks through the early medieval theorists up to Guido of Arezzo. Although this broad program necessarily limits his attention to some aspects of the treatises, he treats in detail those parts of the treatise relevant to his topic, and for critical passages, he provides translations superior to those of Heard. Atkinson partly disagrees with Chailley's division into three authors, suggesting that the author of the *Nova expositio* (the apparent youngest of the three major constituent treatises) is not, in fact, the compiler and that the true compiler is a fourth author who also contributed sections within the principal treatise (generally in the form of commentary), an hypothesis that corresponds in most respects to my own opinion (and partly to Mühlmann's theory, which also includes a commentator who loosely corresponds to Atkinson's, though in every instance where the

²³ Gümpel, "Die Nova expositio," 129–43.

commentator takes over from the revisor, Mühlmann consistently places the transition earlier than Atkinson).

Atkinson's principal interest in the *Alia musica* lies in the place it occupies in an overarching narrative of the gradual incorporation of ancient Greek theory into the theory of ecclesiastical chant. He begins his discussion of this issue with Hucbald, then moves on to describe how the *Alia* progresses from Hucbald's descriptions of the Greek gamut to the *Alia's* merging of modes and octave species.²⁴ Like Hucbald himself, Atkinson does not mention the octave species at all in his chapter on Hucbald, thus avoiding the error previously made by both Chailley and Chartier (see below) of reading a later theoretical doctrine backwards into an earlier theorist.

Nowacki

With the writing of this dissertation, a coincidental pattern has emerged in the historiography of the *Alia musica*. Just as Chailley's edition was released shortly before the completion of Heard's dissertation, obliging Heard to take the brand-new edition into account, so, shortly before the completion of this present dissertation, Edward Nowacki released his *Greek and Latin Music Theory: Principles and Challenges*, which includes a chapter on the *Alia musica*. In this chapter, Nowacki favours the perspective that the source treatise appended to the end of the *Alia* represents the core of the treatise,²⁵ and provides a translation of Chailley's text for just that portion, along with a commentary on the concepts presented therein.

Unfortunately, Nowacki's reading of Chailley's edition is often uncritical, and one is left with the impression that he has not paid sufficient attention to Chailley's critical apparatus, commentary, or cross-references, causing him to replicate several of Chailley's errors and unjustified decisions. Even so,

²⁴ Atkinson, *Critical Nexus*, 149–201.

²⁵ Nowacki, *Greek and Latin Music Theory*, 112.

Nowacki's examination of the principles behind the unusual numerical procedures in the *Alia* is sometimes substantially different from Chailley's opinion and provides a few interesting new insights. Even here, however, Nowacki's interpretations are often no more compelling than Chailley's. These insights (as well as the above-mentioned errors) will be addressed from time to time throughout this dissertation as they become relevant to the topic at hand.

Origin of the *Alia musica*

Chailley's Hypothesis

Chailley dated his interpretation of the three layers of the treatise at about 875, 880, and 890. He based his dating on an assumed distribution between Hucbald's *Musica*,²⁶ which Chailley believed to have been dated around 870, and the *Music enchiridis*, which Chailley believed to have been written around 895;²⁷ Chailley supposed, therefore, that all layers of the *Alia* must have been composed and compiled within this twenty-five-year window. Subsequent work on these other treatises necessarily impacts upon these assumptions, with the effect that the chain of reasoning is no longer self-consistent, prompting Huglo to remark that "*la datation de Jacques Chailley n'est plus tenable.*"²⁸ ("The dating of Jacques Chailley is no longer tenable"). However, the assumptions underlying Chailley's hypothesis were always somewhat weak, more effectively setting an approximate era than true *termini*; thus, while Chailley's dating should be taken with a grain of salt, the recent research only favours but does not necessarily force a later date.

²⁶ This treatise was formerly known as the *De harmonica institutione*; Chartier, 46.

²⁷ Chailley, 60; cf. Heard, 16.

²⁸ Huglo, "*Notes sur la reproduction*," 3.

Relationship to Hucbald

Chailley supposed that the *Alia musica* must have been written after Hucbald's *Musica*, as part of a broader hypothesis about the trajectory of the gradual fusion of interval species theory with the ecclesiastical modes: he says that “dans la genèse des octaves modales, Hucbald présente un premier stade dont le *Quidam* est logiquement le second et l'*Alia* le troisième.”²⁹ (“In the genesis of the modal octaves [octave-species concept of mode], Hucbald presents a first stage to which the *Quidam* [source treatise] is logically the second and the *Alia* [revision] is the third.”) Chailley cites, in particular, a table in Hucbald's *Musica*, which is reproduced here as Figure 1.

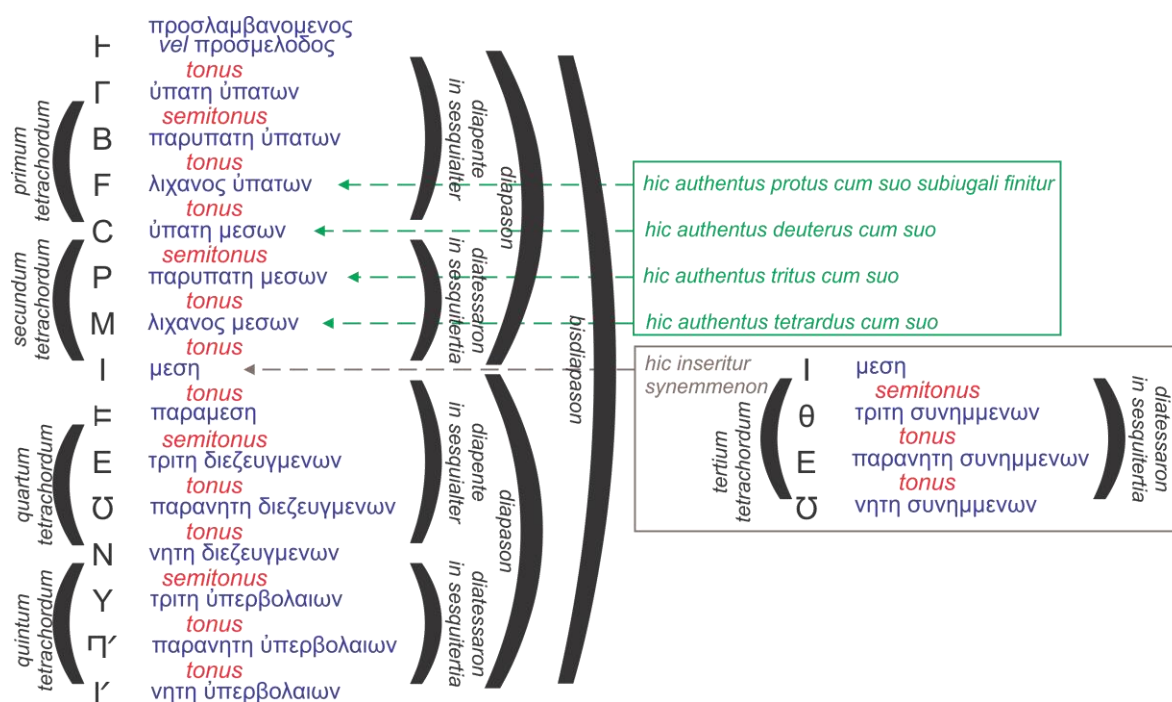


Figure 1 – Recreated from Hucbald's *Musica*

This table is found in §38 of Chartier's edition.³⁰ The figure is quite inconsistent across manuscripts, and the present reproduction is a normalized composite of Chartier's diagram and the manuscript reproductions in Chartier's appendix.³¹ Chailley's hypothesis revolves around the indications of modal *finales*, highlighted here in green text, which, however, are found only in manuscripts copied from later models.

²⁹ Chailley, *Alia musica*, 59; 38–39.

³⁰ Chartier, *L'Œuvre musicale de Hucbald*, 184.

³¹ Chartier, *L'Œuvre musicale de Hucbald*, 451, 453, 455.

Chailley's hypothesis is founded upon the presence of four lines of text in this diagram, indicating the location of modal *finales*, thus: "*hic authenticus protus cum suo subiugali finitur*" ("here is ended the *protus* authentic mode, along with its plagal"), and so on. He then identifies these *finales*, on the fourth through seventh pitches of the gamut, with the roots of the fourth through seventh octave species, and proposes that the remaining species would be easily associated with the plagals because of the closely related structures of octave species separated by a perfect fourth;³² this association is then loosely equivalent to the association of octave species and modes in the *Alia musica*.³³

However, there are several problems with Chailley's analysis. First, the "*hic authenticus protus*" lines appear only in a small subset of manuscripts of Hucbald's treatise, all of which Chartier considers to have been copied from a later model,³⁴ and therefore are probably later additions. Second, Hucbald says nothing at all about octave species and makes no distinction here (and very little distinction elsewhere) between authentic and plagal modes (to say nothing of the octave species shared by the first and eight modes, which Chailley ignores here). Third, the introductory material in the *Alia musica* identifies octave species according to their highest note, rather than their lowest note (as in Chailley's interpretation of Hucbald). Finally, it is not entirely clear whether the *Alia musica* ever even discusses the notion of a modal *finalis* (there are a few ambiguous references that will be discussed later, all of which belong to the youngest layers of the treatise). Thus, it is difficult to support the notion that the *Alia musica* directly develops an idea first hinted at in Hucbald.

In addition, even if Chailley were correct in his reasoning, his argument would not quite support the entirety of his hypothesis: Chailley argues that the entirety of the composite treatise must postdate Hucbald, including the source text (his "first *Quidam*"). However, the source treatise makes no clear use

³² Chailley, *Alia musica*, 38.

³³ Chailley, *Alia musica*, 107.

³⁴ Chartier, *L'Œuvre musicale de Hucbald*, 184, crit. app.; 117.

of the concept of octave species (only one possible reference, ambiguous at best); thus, the only justification for assuming that the source treatise could not predate Hucbald is Chailley's weakly justified assumption that the source treatise and revision were written in relatively quick succession:

*Des trois traités qui la composent, il est évident que le 1er Quidam est le plus ancien, sans que l'écart de temps puisse être considérable : l'auteur principal parle du Quidam comme d'un maître, qu'il se permet du reste de critiquer à l'occasion : les deux auteurs semblent se correspondre dans la situation d'aîné à cadet, mais vivant à peu près dans le mêmes temps.*³⁵

Of the three treatises that compose [the *Alia musica*], it is evident that the [source treatise] is the oldest, but that the time gap would not be considerable: the [revisor] speaks of the author as a master, whom, moreover, he permits himself to criticize on occasion; the two authors seem to correspond in the situation of elder to junior, but living at nearly the same time.

Chailley's assessment of the "not considerable time gap" between the source treatise and the revision is a dubious assumption, as he provides no particular evidence to support his assumption.³⁶

Nevertheless, Chartier agrees with Chailley's assessment that Hucbald is anterior to the *Alia musica*, based on a simpler justification: Hucbald describes the modes according to their manerial names³⁷ but does not associate them with the Greek ethnic names. Chartier argues that "*si Hucbald avait connu cette terminologie nouvelle, on conçoit mal qu'en bon pédagogue il ne l'eût pas évoquée, fût-ce pour la rejeter, fût-ce pour l'accepter, comme le fait l'auteur de la Musica Enchiriadis.*"³⁸ ("If Hucbald had been acquainted with this new terminology, one would hardly suppose that, as a good pedagogue, he would not have mentioned it, be it to reject it, be it to accept it, as did the author of the *Musica enchiriadis*.")

This line of reasoning, however, engages in a certain amount of question-begging: one can hardly take for granted that, had the *Alia musica* been written before Hucbald's *Musica*, Hucbald would necessarily have been familiar with it. And by the same logic, one may well ask the same question regarding Guido: while

³⁵ Chailley, *Alia musica*, 59.

³⁶ I shall compare the differing interpretations of the source treatise and the revision especially in the second half of Chapter 16.

³⁷ Hucbald does not use the term *maneria*, which did not come into use until the twelfth century. I shall continue to use the term as a matter of convenience, to describe the four pairs of modes, grouped according to their *finales*; thus, the terms *protus*, *deuterus*, and so forth, without the specification of either authentic or plagal, are the names of *maneriae*.

³⁸ Chartier, *L'Œuvre musicale de Hucbald*, 46.

there is no evidence that Guido knew the *Alia musica*, he was certainly familiar with the *Musica enchiriadis* (or at least the doctrines therein),³⁹ which also associates the Greek ethnic names with the ecclesiastical modes, yet he does not mention them at all in any of his surviving works.

And again, as with Chailley's argument, even if Chartier's argument were sound, it still would not support the hypothesis that the entire composite treatise must postdate Hucbald: the source treatise does not describe the modes according to the Greek ethnic names; this novelty does not appear until the revision, and there is thus no reason why the source treatise could not predate Hucbald.

Atkinson, by contrast, disagrees slightly with Chailley; although Atkinson accepts Chailley's hypothesis that the revised treatise and the tonary postdate Hucbald's *Musica*, he feels that the source treatise probably predates Hucbald.⁴⁰ Atkinson feels that the source treatise predates Hucbald on account of significant similarities to the work of Aurelianus (which will be described later) and because it lacks the innovations found in Hucbald and any links to the *Musica* and *Scolica enchiriades*.⁴¹

As regards the date of Hucbald's treatise, which is supposed to predate so much of the *Alia musica*, Chartier notes that there are three hypotheses, based on the locations where Hucbald was working at different points in his life:⁴² at the cloister school of Elnone (Saint-Amand) in the 870s, at Saint-Bertin in the 880s, or Reims in the 890s. Edmond de Coussemaker preferred the earliest possibility,⁴³ and his

³⁹ See fn. 85, below.

⁴⁰ Actually, Atkinson says that he agrees with Chailley, but in fact, his opinion is slightly different from Chailley's. Immediately following the passage in Chailley cited above (see fn. 35, above), Chailley says, "*Leurs deux traités sont certainement postérieurs à celui de Hucbald,*" (Chailley, *Alia musica*, 59) ("Their two treatises [the source treatise and the revision] are certainly posterior to that of Hucbald"). Atkinson, by contrast, explains that he "agree[s] [*sic*] with Chailley that the First Quidam is anterior to Hucbald's *De harmonica institutione* [a.k.a. *Musica*], and that the Principal Treatise and *Nova expositio* are posterior to it." (Atkinson, *Critical Nexus*, 173, fn. 6.).

⁴¹ Atkinson's justification for this dating is not given in detail in the *Critical Nexus*, but he providing this explanation in the written feedback to this dissertation following the defence.

⁴² Chartier, *Œuvres musicales*, 76. This analysis assumes, of course, that the *Musica* was, in fact, written by Hucbald; this attribution does not appear in the earliest manuscripts, nor in any other contemporaneous source; however, there is evidence that Hucbald did write a treatise, and Chartier feels that the style of this treatise is consistent with other writings by Hucbald (*op. cit.*, 44–45; 300 n.7).

⁴³ Coussemaker, *Mémoire sur Hucbald*, 48–49.

assessment was still dominant when Chailley made his estimate of the dates of the *Alia musica* in the 1960s. However, Chartier rejects this hypothesis (though working from false, or at least weak, premises); he assumes that Hucbald was, at that time, too young to write the sophisticated treatise (though he would have been at least in his mid-twenties or even late-thirties)⁴⁴ and notes that no manuscript of the treatise is known to survive in locations with which he was associated at that time. Likewise, he rejects the latest possibility (Reims), because there is no manuscript associated with Reims, while Hucbald's other work is preserved there.⁴⁵ Thus, Chartier prefers the intermediate date, at Saint-Bertin, a date at which Hucbald is known to have had acquaintances that could explain the transmission of the treatise to England.⁴⁶

This analysis of the date of Hucbald's *Musica* is, itself, somewhat problematic for the present purposes. His most substantial evidence is the lack of surviving manuscripts at two of the three locations. But manuscript survival has always been a haphazard affair, and Chartier's conclusion, while plausible, does not inspire a great deal of confidence. His objections about Hucbald's age are also unconvincing

⁴⁴ Hucbald's death is relatively securely dated to 930, and the *Acta sanctorum* holds that he was about ninety years old at death (Weakland, "Hucbald", 68); he would therefore have been born around 840. *Hucbald* was at Elnone until 882 (Chartier, *Œuvres musicales*, 76). If Hucbald had indeed been born in 840, then he would have been forty-two when he left, which is certainly not too young to have written his *Musica*. However, Chartier argues in favour of a later birth, closer to 850, apparently on no greater evidence than skepticism of such a long lifespan (Chartier, *Œuvres musicales*, 1–2). It should be remembered, however, that the oft-repeated "short lifespan" of the Middle Ages results from averaging lifespans cut prematurely short primarily by infant and childhood mortality, warfare and violence, and periodic catastrophes; those who died of old age, rather than violence or illness, could live quite long lives (Enrico Dandolo, a Doge of Venice in the twelfth century, famously lived to about the age of ninety-eight); a lifespan of ninety years or so for Hucbald would certainly have been remarkable, but by no means unrealistic, and hardly seems sufficient reason to arbitrarily invent a later date.

⁴⁵ Chartier, *Œuvres musicales*, 76. By contrast, Charles Atkinson states (*Critical Nexus*, 149 fn. 1) that Michel Huglo suggested the period while Hucbald was at Reims; this is not quite accurate. Rather, Huglo was commenting upon his understanding of Chartier's opinion: in a 1976 article, Huglo says (*Les instruments*, 184 fn. 22) that Chartier had preferred the Saint-Bertin hypothesis in his doctoral dissertation (1973), but that he had come to prefer the Reims hypothesis as a result of arguments made at his doctoral dissertation defence by R. H. Bautier; Huglo goes on to say that Chartier's change of opinion would be reflected in the coming edition. However, Chartier's edition, with a greatly expanded scope, was not published until 1995, and there is no indication in that edition that Chartier has indeed changed his opinion (cf. Chartier, *Œuvres musicales*, 76–77). Unfortunately, Huglo does not describe Bautier's arguments, but they may perhaps be reflected in Chartier's comment that the Reims hypothesis "*n'est pas dénuée d'intérêt*" ("is not devoid of interest"), since it would help to explain how the treatise came to be known further east.

⁴⁶ Chartier, *Œuvres musicales*, 76–77.

(and somewhat patronizing). For Chartier's purposes, of course, it is entirely appropriate to have identified a date for Hucbald's *Musica* that seems more likely than the others; for the purposes of dating the *Alia musica* in relation to Hucbald, however, the only date that might to be taken as a *terminus* (if, indeed, the *Alia musica* must postdate Hucbald, which is not certain) would be the earliest date, 873 (the year of Hucbald's arrival at Elnone), but even this date should not be taken with confidence.

Relationship to the *Musica enchiriadis* (etc.)

Chailley proposed that the *Alia musica* must predate the *Musica enchiriadis* on two grounds. The more straightforward to deal with is the argument that the *Alia musica* must predate the *Enchiriades* because it makes no use of either of the two musical notations systems introduced in the *Musica enchiriadis*⁴⁷ (the Daseian notation and the line-based graphical notation based on Daseian notation). Indeed, the *Alia musica* does not seem even to be aware of any practical notation system, nor any note-naming system more recent than the Greek system. However, as Nancy Phillips notes, this argument from Chailley is inconsistent, since Chailley places the *Alia musica* after Hucbald, who presents notation systems remarkably similar to those in the *Enchiriades* (a fully diastematic letter notation and a line-based graphical notation – see Table 1).⁴⁸ In fairness, however, the *Enchiriades* were among the most widely dispersed treatises of the Middle Ages, while Hucbald's *Musica* was not, and so the argument may not be as inconsistent as it at first seems.

Chailley also assumes that the *Alia musica* preceded the *Enchiriades* because the *Musica enchiriadis* employs (sparingly) the Greek ethnic names for modes, but does not bother to explain the usage.⁴⁹ The rationale, then, is that the *Alia musica* created the association between Greek ethnic names and ecclesiastical modes and that this association had become sufficiently widespread by the time of the

⁴⁷ Chailley, *Alia musica*, 59.

⁴⁸ Phillips, *Musica and Scolica*, 220–21.

⁴⁹ Chailley, *Alia musica*, 59.

because it contains terminology borrowed from Martin of Laon's commentary on Martianus Capella, written in the first half of the ninth century (and falling out of fashion in the early tenth century, when Remigius of Auxerre's commentary seems to have become preferred).⁵⁰ This argument is interesting but quite weak, as there is no reason to expect that an author would simply forget about an old favourite commentary just because a new one has become fashionable.

Additionally, there is an alternative version of the *Musica enchiridis*, called the *Inchiriadon*, and although the ultimate form of the *Inchiriadon* may be a later recension, the majority of the text (excepting some new material on organum) appears to be based on an older form of the *Musica enchiridis* than the standard recension of the *Musica enchiridis* itself.⁵¹ Consequently, the base text of the *Inchiriadon* is probably older even than the *Musica enchiridis*, and this text already includes the Greek ethnic names.⁵² Indeed, in a thorough review of the sources referenced in the *Musica* and *Scolica* tradition, Phillips argues that nothing found in the *Enchiridis* tradition precludes the possibility that the treatises were first written as early as the middle of the ninth century, or perhaps a little earlier⁵³ (this date would be earlier than Hucbald!). Thus, if the presence of these ethnic names without justification truly did indicate that the principle had been introduced by the *Alia musica* and had become widely accepted by the time the *Musica enchiridis* was written, then the *Alia musica* would not merely have to have been written first (by sufficient time to allow the concepts to spread), but to have been written much earlier than generally supposed.

Phillips rejects Chailley's assumption that the author of the *Musica enchiridis* uses the ethnic names as equivalent to the ecclesiastical modes.⁵⁴ The passage in question says:

⁵⁰ Phillips, *Musica and Scolica*, 78.

⁵¹ Phillips, *Musica and Scolica*, 45.

⁵² Schmid, *Musica et scolica*, 187; 204.

⁵³ Phillips, *Musica and Scolica*, 516.

⁵⁴ Phillips, *Musica and Scolica*, 188–89.

*Modi uel tropi sunt species modulationum, de quibus supradictum est, ut protos autentus uel plagis, deuterus autentus uel plagis, siue modus Dorius, Frigijs, Lidijs, et ceteri, qui ex gentium uocabulis sortiti sunt nomina.*⁵⁵

The modes or tropes are species of modulation [melodic progression], about which has been spoken above, such as the first authentic and plagal, [or] the second authentic and plagal, or the modes Dorian, Phrygian, [or] Lydian, etc., the names of which are drawn from the designations of peoples.

At question in this passage is the signification of the word *siue*, which means “or,” “or if,” or “or rather.” As a result, this passage can be read in two ways: “the first authentic, or Dorian, and the second authentic, or Phrygian, etc.”; or “the first authentic and the second authentic – or, instead, the Dorian and the Phrygian, etc.” That is to say, it is plausible to interpret *siue* to mean here that the names Dorian, Phrygian, and Lydian are synonyms for the first, second and third authentic modes (*siue* is commonly used in this manner to introduce subtitles), or that they represent separate concepts (but that both concepts are called *modus*). Phillips prefers the latter explanation, explaining that “[t]he author is pointing to the existence of other kinds of ‘species modulationum’ [...], and he is underlining that the procedures for determining the mode of a chant are identical in principle to those used in the Boethius *Musica*, not that the modes are the same.”⁵⁶

In favour of Phillips’ hypothesis, the word *modus* before *Dorius*, *Frigijs*, *Lidijs* is superfluous and may be meant to emphasize that a new category is being presented under that rubric. Additionally, *modus* is widely recognized as the term that Boethius coined to describe his modes, while most Carolingian treatises almost universally prefer *tonus* or *tropus* for the ecclesiastical modes (not, however, the *Musica enchiriadis*, itself, which begins to use the word *modus* regularly beginning in chapter 9, though the term *tonus* is preferred in earlier chapters); thus, the author may be deliberately trying to call to mind Boethius’ modes, which are distinctly different from the ecclesiastical modes (more on this later).

⁵⁵ *Musica enchiriadis*, ch. 9, ed. Schmid, 22, trans. Erickson, 12.

⁵⁶ Phillips, *Musica and Scolica*, 189.

However, if *siue* is meant to imply a complete separation of concepts, it must be acknowledged that there are other words for “or” that would be much less ambiguous (such as *autem*) – though, of course, the availability of better diction is a rather weak argument upon which to ground an interpretation. More substantially, according to this interpretation, this second, unrelated concept of mode also must fit the description “species of modulation [...], about which has been spoken above,” and this is not the case for the Boethian form of mode – at least, not in the *Musica enchiridis* or the *Scolica enchiridis*.

However, while Phillips notes that the above passage is the only use of the ethnic names in either of these two principal treatises, they do, in fact, appear again in the *Inchiradon*, and the usage does not support Phillips’ thesis. This reference recurs in a diagram connected to a passage⁵⁷ that blends the modal theory of the *Enchirides* with a passage modelled upon Boethius’ *Musica*, Book IV, Chapter 15.⁵⁸ Both authors describe a disposition of notes that is identified as the first mode, which is then made higher throughout its entire course by one tone, which then becomes the second mode; then, if it is raised again, it becomes the third mode, and if raised yet again, becomes the fourth mode. The *Inchiradon* says:

[...] *dorio, si per totos flexionis ductus epogdoo uel tono feceris acutiorem, moxque in modum mutatur frigium.*⁵⁹

The Dorian, if you will have made it higher through all of the turning of its course by an ἐπόγδοος (or tone), is thereupon transformed into the Phrygian mode.

The author makes it quite clear that the entire disposition of notes is raised by a whole tone – ἐπόγδοος, from *epi-* (upon) and *ogdoos* (eighth), a ratio of an eighth part greater than a whole (i.e., $1\frac{1}{8} : 1$, or 9:8) – and thus implies transposition, which is consistent with the kind of mode that Boethius describes. This interpretation is reinforced by the diagram, reproduced here as Figure 2.

⁵⁷ Schmid, *Musica et scolica*, 204.

⁵⁸ Boethius, *De institutione musica*, bk. 4, ch. 15, ed. Friedlein, 342, trans. Bower, 153–54.

⁵⁹ *Inchiradon*, ed. Schmid, 204.

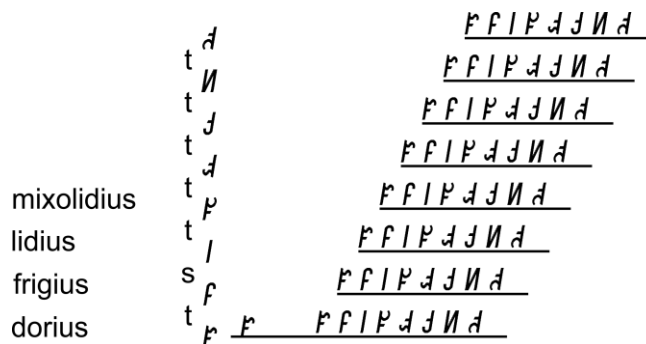


Figure 2 - Disposition of Modes in the Inchiadon. Reproduced from Schmid's descr. 14

It is important to note that the daseian notation used in Figure 2 is fully diastematic, and so implies real transposition, not modal transposition, so that the octave species presented in all of the modes in the diagram is identical, precisely as is the case in Boethius' explanation of the Greek modes. However, the *Inchiadon* goes on to say that, "*si quattuor [modum tono feceris acutiorem], quinta denuo regione primus erit. Similiter et in ceteris constat, et in infinitum semper a quinto prior ordo redit.*"⁶⁰ ("If [you will have made] the fourth [mode higher by a tone], it will be the first [mode] anew at the fifth region. It stands likewise also in the other [modes]: the original order always returns at the fifth [place], out to infinity.") This is distinctly not a characteristic of the Boethian modes, for which there is no defined relationship between modes separated by fifth;⁶¹ rather, it is characteristic of the unusual gamut of the *Enchiriadis* treatises, which does, indeed, repeat itself perpetually at the interval of a perfect fifth – but the modal theory for the *Enchiriades* is based around interval structures, not transposition, which, contrastingly, does imply modal transposition. Thus, this diagram displays elements of both Greek and ecclesiastic modes, and while it is difficult to come to a consistent interpretation (the two perspectives are distinctly contradictory), it is clear that the author considered the two forms of mode somehow to be the same. (It must be acknowledged, however, that this passage is not present in the *Musica enchiriadis*, and thus may

⁶⁰ *ibid.*

⁶¹ Indeed, the Greeks, whose theory Boethius is describing, generally felt some connection between modes related at the fourth, not the fifth; this relationship is something like the modern notion of closely related keys, and extends to the Greek nomenclature of hypo- and hyper- modes, but these modes are certainly not identical to each other, as the *Inchiadon* suggests.

be a later addition to the *Inchiriadon*, perhaps influenced by a later conflation of Greek and ecclesiastical modes – perhaps even by the *Alia musica* itself).

By contrast, there is good reason to believe that at least the youngest layers of the composite treatise were written after the *Musica enchiriadis*. In one very brief passage (§134)⁶² that both Chailley and Atkinson consider to be part of the youngest layer⁶³ (though they disagree on who wrote it), a doctrine is presented and never used again in the *Alia* that is otherwise exclusive to the *Musica enchiriadis* and its associated treatises: that not only are the modes named *protus*, *deuterus*, *tritus*, and *tetrardus*, but so also are the *finale*s and *affinale*s of these modes.⁶⁴ Because of the brevity of the passage and the lack of justification or development of the idea, it seems likely that the doctrine was already well established when the passage was written, while the *Musica enchiriadis* appears to be the earliest treatise to describe it fully.

In sum, the justifications presented in previous considerations of the dating of the *Alia* relative to the *Musica enchiriadis* do little to establish either treatise as older. In particular, it is not at all clear that the *Musica enchiriadis* follows the *Alia*'s example in associating the ecclesiastical modes with the Greek modes, and while the youngest layers of the *Alia* most likely postdate the *Enchiriadis* (because of the use of *protus* and the like to name pitches as well as modes), there is no particular evidence placing the rest of the *Alia* after the *Enchiriadis*.

⁶² Chailley, *Alia musica*, 196.

⁶³ Atkinson, *Critical Nexus*, 177; in the table presented there, Atkinson's four authors are labelled chronologically by ascending letters in the Greek alphabet, and Chailley's opinion of dating is represented by increasing degrees of indentation.

⁶⁴ To be precise, the passage directly specifies only that the pitch a fifth above the *finalis* of a mode (which I have called the *affinalis* based on later usage) shares the name of the mode; however, the doctrine of the *Musica enchiriadis*, which uses these names for the *finale*s and all pitches in successive fifths outwards from the *finale*s, is the only other doctrine of which I am aware that is compatible with the passage in the *Alia*.

A Fresh Consideration of Dating

Phillips considers the *Alia musica*, as a part of a more extensive suite of treatises frequently transmitted together in the manuscript tradition called the *Enchiriadis* complex, to belong to the period between the end of the ninth century and the beginning of the eleventh, and so feels that it was probably completed in the later tenth century, rather than the ninth.⁶⁵ Atkinson observes that the revised dating for the manuscript considered to be the earliest extant copy creates a *terminus ante quem* for the *Alia* potentially as late as the early eleventh century.⁶⁶

But while the inclusion of the treatise within a complex of other treatises and (even more importantly) the dating of the oldest extant manuscripts can provide a *terminus post quem non*, they are not sufficient grounds for assuming a late tenth-century date for the complete treatise. A simple thought experiment should illustrate the problem; the thought experiment admittedly rests upon a couple of problematic assumptions, but not so much as to invalidate the basic principle (I shall address these problems after presenting the thought experiment).

The reasoning begins by assuming (for the sake of argument) that Chailley was correct in his assessment of the age of the *Alia musica* as comparable to, and perhaps even slightly older than, the *Musica enchiriadis*. The *Musica enchiriadis* is an excellent benchmark not only because of its age (which is confidently assigned to at least the late ninth century because manuscripts of comparable age have survived) but also because of its apparent value to medieval musicians, as measured not only by the many surviving manuscripts but also by the care taken in correcting copies against each other⁶⁷ and the frequency with which later theorists engage with it in their own treatises.⁶⁸ The thought experiment

⁶⁵ Phillips, *Musica and Scolica*, 67 fn. 40.

⁶⁶ Atkinson, *Critical Nexus*, 171 fn. 2; Atkinson cites Phillips, "Notationen und Notationslehren," 343.

⁶⁷ Phillips, *Musica and Scolica*, 3–4.

⁶⁸ Erickson, "Musica enchiriadis, Scolica enchiriadis," *Grove Music Online*, §2 ¶1.

continues by assuming (admittedly problematically) that the *Alia musica* had a similar reproductive profile to the *Musica enchiridis*, but scaled according to some linear measure of popularity (as, for instance, one copy of the *Alia musica* was created for every ten copies of the *Musica enchiridis* – a ratio not very different from the ratio of surviving manuscripts). Finally, one assumes (also problematically) that the likelihood of a manuscript of either treatise surviving is to a considerable degree a function of age, such that a similar percentage of tenth-century manuscripts of each treatise would have been lost, and likewise a similar percentage of eleventh-century manuscripts, and so on.

If all of these assumptions were to hold, even (and necessarily) as loose approximations, then one would be able to estimate the number of tenth-century manuscripts of the *Alia musica* that would be expected to have survived as a proportion of the total number of surviving manuscripts, using the pool of surviving *Musica enchiridis* manuscripts as a guide. In his edition of the *Musica* and *Scolica enchirides*, Hans Schmid lists forty-six manuscripts containing the *Musica enchiridis*, of which only four are believed to be older than the end of the tenth century,⁶⁹ a ratio of $11\frac{1}{2} : 1$. The number of surviving manuscripts that contain any part of the *Alia musica* is twelve, and (as will be discussed in greater detail in Chapter 4) three of these are not actually part of the *Alia musica*, but rather a different treatise that borrowed some of the same material as the *Alia* (and some of the others may be copies of the sources that were incorporated into the *Alia*, rather than excerpts of the *Alia* itself). Thus, the number of manuscripts that ought to be counted as the *Alia* is considerably less than eleven and a half; therefore, the number of tenth-century manuscripts that would be expected to have survived (by comparison to the *Musica enchiridis*) is less than one.

Of course, one cannot merely assume that the pattern of manuscript reproduction or survival for the *Alia musica* should in any way resemble that of the *Musica enchiridis*. Both reproduction and survival

⁶⁹ Schmid, *Musica et Scolica*, VII–X.

of the *Enchiridis* would have benefited from its popularity, and those of the *Alia* would have suffered from its apparent lesser relevance to the later theoretical tradition, while the transmission of the *Alia* within the *Enchiridis* complex would benefit immensely from its association with that more popular treatise (as compared to any hypothetical copies of the *Alia* outside the *Enchiridis* complex, of which no complete manuscripts have survived). (Interestingly, these factors would seem to strengthen rather than weaken the argument.) Much more importantly, though, owing to the tiny sample size of surviving manuscripts, sheer random happenstance probably becomes the most significant factor in the survival of manuscripts, making these kinds of comparisons extremely unreliable. Nevertheless, the logic holds, as the preceding thought experiment is not meant to argue in favour of an older date for the *Alia musica* (which it could not do in any case, as comparable logic could argue for an infinitely older date, an obvious absurdity), but simply to illustrate the insufficiency of the dating of the oldest surviving manuscript or the appearance of the *Alia* within the *Enchiridis* complex as indices of the age of the treatise. The takeaway is that even if the *Alia* were to be as old as Chailley supposed, with the tiny sample size of surviving manuscripts, there is no reason to expect that an early manuscript or one outside of the *Enchiridis* complex ought to have survived.

Superior evidence for the dating of the manuscript ultimately still comes from the same principles that Chailley employed, problematic though they are: the relationships amongst the theoretical concepts addressed by the various theoretical treatises of the era. There is, however, a critical caveat to this procedure to which Chailley pays little heed: information does not travel instantaneously from its source to all interested audiences. Even in the digital age, many of the most ground-breaking ideas can take a few years (or sometimes even many years) to become widely known throughout the field; all the more so in the Carolingian period, when even the most influential treatises may have taken many years to become widely known throughout the empire. Thus, dating treatise A prior to Treatise B because Treatise A does not account for an essential development in Treatise B must be taken within a threshold; Treatise A may

well have been written a few years after Treatise B if the dissemination of the ideas from Treatise B has not yet reached the author of Treatise A (to say nothing of the fact that the author of Treatise A could be conservative and reject the innovations of Treatise B). Thus, Chailley's dating is probably too narrow, attempting, as it does, to squeeze all the major phases of the composition of the treatise into the short span between Hucbald and the *Musica enchiridis* (a span that must now be reckoned all the narrower – or even negative! – with more recent estimates of the dates of these two treatises).

As has already been noted, the estimated date for Hucbald's theory is now generally assumed to be later than was believed when Chailley published his edition of the *Alia musica*; regardless, there is no particular reason why the earliest layers of the *Alia musica* must postdate Hucbald. The primary justification for this assumption has always been simply that Hucbald seems to have been the first theorist to attempt a cautious reconciliation between ancient Greek theory and ecclesiastical chant. However, Hucbald is hardly unusual amongst Carolingian theorists in addressing Greek theory in general, and there is no reason why it could not have independently occurred to both Hucbald and the author of the *Alia* to attempt to reconcile the two systems to some degree. Even more importantly, the earliest phase of the *Alia musica* contains no apparent references to Greek theory (Chailley supposes that there is one reference to modal octaves,⁷⁰ but this occurs in a passage on the third mode that is extremely difficult to interpret and may well not mean that at all; the relevant passage will be discussed in detail in a later chapter).

However, coincidentally, there are different reasons for supposing that Chailley's date for the earliest layer of the *Alia* might be reasonable. As will be described more thoroughly in later chapters, the principal topic of all but the *Nova expositio* layer of the treatise is a numerology relating modes to intervals that is almost unique; the only other treatise known to contain any similar concept is Aurelianus' *Musica*

⁷⁰ Chailley, *Alia musica*, 90.

disciplina. This treatise is dated with a considerable degree of confidence to very near the middle of the ninth century.⁷¹

The correspondence between intervals and modes appears to be in a less fully-developed state in Aurelianus than in the *Alia musica*, which might suggest that the oldest layers of the *Alia* represent a development of the basic notion sketched out by Aurelianus. However, Aurelianus is also known to have incorporated a considerable amount of older material into his treatise – indeed, he says so himself in his preface.⁷² Thus, one may equally suppose that the *Alia musica* developed an idea found in a (now lost) earlier source that Aurelianus also incorporated into his treatise, or even that the theory as it is presented in the *Alia musica* was already more fully developed when Aurelianus encountered it, and he, seeing the contradictions within it (which will be addressed in considerable detail in the latter portions of this dissertation), simplified the system.

There is some slight evidence for the hypothesis that Aurelianus borrowed his discussion of this concept from an older source. As Atkinson points out, while no other treatise besides Aurelianus and the *Alia musica* discuss these associations directly in the treatise, the associations are added as marginal glosses to two other treatises.⁷³ The first, and most relevant, is Boethius' *De musica*. Aurelianus'

⁷¹ Gushee, *Aureliani Reomensis Musica disciplina*, 15–16. Gushee bases his argument on Aurelianus' reference to his former abbot, Bernardus, who is elsewhere recorded to have been abbot of Aurelianus' former abbey of Réôme at about that time, and whom Aurelianus calls "archbishop in name and soon to be in fact." This Bernardus may possibly correspond to a certain Bernus who was recommended at about that time to be raised to the see of Autun, which while not technically an archbishopric, was accorded some of the traditional privileges of an archbishopric (notably, the right to wear the *pallium*). Gushee's arguments are uncertain, as his identification of Abbot Bernardus with Bernus disagrees in both name and rank, and these issues of rank tended to be rather important in medieval society (though Gushee cites H. Joly for the idea that the title *archbishop* was new in the West in the ninth century and was not used consistently until the tenth century). Even so, a Bishop Bernardus is listed in a catalogue of the abbots of St. Jean de Réôme in 856 (or perhaps 846, as the timeline of his other postings would conflict 856; *ibid*), whether or not this Bernardus can be associated with Bernus. Atkinson also reviews the dating issue (*Critical Nexus*, 93, fn. 17), and notes that Barbara Hagg, at the conference "*Musiktheorie im Mittelalter*" in Munich in 2000, dated the earliest copy to ca. 860, and called it a revised version, implying an earlier date for the original form. Either way, the date is quite near to the middle of the century.

⁷² Gushee, *Aureliani Reomensis Musica disciplina*, 11.

⁷³ Atkinson, *Critical Nexus*, 94, including fn. 22.

discussion of the intervals is clearly a slight expansion of Boethius' retelling of the story of Pythagoras and the hammers that create the basic consonances,⁷⁴ to which Aurelianus adds the association of modes and example chants. In at least three manuscripts of Boethius from the tenth century, these additional details are also present as glosses;⁷⁵ they are also added as glosses in two manuscripts of Regino of Prüm from the eleventh century at the point where Regino also quotes the passage from Boethius.⁷⁶ Calvin Bower and Michael Bernhard propose that these passages in Aurelianus originated as glosses upon Boethius that then became folded into the principle text of Aurelianus; if so, the same glosses in Boethius could also be the source of the doctrine as it is used in the *Alia musica*. This hypothesis, both in regard to Aurelianus and to the *Alia musica*, is entirely plausible, but as Atkinson observes, it suffers from the fact that none of the manuscripts containing the glosses is older than the tenth century, while the oldest surviving copy of Aurelianus' treatise dates from the ninth century⁷⁷ (it is, thus, also possible that the glosses were added to Boethius from Aurelianus, or from some other common source).

The origin of the doctrine, then, remains an open question, but in any case, it is unlikely that Aurelianus borrowed it from the *Alia musica*. Aurelianus is generally accepted as the author of the first substantial theoretical treatise of the Middle Ages (excluding Boethius, Cassiodorus, and Isidore, who, despite their relatively late dates, clearly wrote in the old Greco-Roman tradition); as a compiler of the most relevant portions of other texts, it is likely that had he known of the *Alia musica*, he would have

⁷⁴ *ibid*, and also Atkinson's Example 3.2 on p. 95; cf. Boethius, *De institutione musica*, bk. I ch. 10, ed. Friedlein, 197–98, trans. Bower, 19; Aurelianus, *Musica disciplina*, ch. 2, ed. Gushee, 62–63, trans. Ponte, 8. I shall say more about Aurelianus' retelling of this story in Chapter 14.

⁷⁵ The relevant manuscripts are Chicago F.9, Paris 7297, and Rome 1638 (see Atkinson, *Critical Nexus*, 94 fn. 22 and Gushee, *Aureliani Reomensis Musica disciplina*, 62 critical apparatus).

⁷⁶ Regino, *De harmonica institutione*, ed. & trans. Chartier, 79 & 105–06. The relevant manuscripts are Montpellier 159 and Metz 494 (see Atkinson, *Critical Nexus*, 94, fn. 23; cf. Bernhard, *Clavis Gerberti*, 57–58, esp. critical apparatus).

⁷⁷ Atkinson, *Critical Nexus*, 94, fn. 22. Atkinson cites Bernhard and Bower, eds., *Glossa maior in institutionem Boethii*, I: I–lvi & 199–201; I have been unable to obtain a copy to confirm the citation.

incorporated more of it into his treatise.⁷⁸ There are, of course, no guarantees in this kind of speculation, but it nevertheless seems likely that the first stages of the *Alia* either postdate Aurelianus (quite likely by a reasonable span, so as to have had time to develop the theory fully) or at least to not predate him by very long. Thus, the mid-ninth century does not seem too early for this portion of the treatise.

By definition, the second layer of the *Alia musica*, which is irrefutably a revision of the first (the author of the second layer tell us so), must postdate the first layer. Whether it must postdate Hucbald, as Chartier supposes, is a different matter. Chartier argues that if the *Alia* were to have predated Hucbald, Hucbald would likely have made at least passing reference to the Greek ethnic names with which the *Alia* identifies the modes. However, this assessment involves much question-begging. Since the *Alia* seems to have had a relatively limited impact on music theory in the Middle Ages, the *Alia* could easily have predated him without his having been aware of it. Furthermore, the association of these ethnic names with the ecclesiastical modes is commonly held to be based on a fundamental misunderstanding of Boethius (more on this later), a misunderstanding that Hucbald was unlikely to have shared, given how much of his treatise is simply a rehashing of Boethius. Sarah Fuller has argued persuasively that Hucbald's intended audience, too, would likewise have been well informed;⁷⁹ thus, Hucbald could have been well aware of the *Alia*'s use of the Greek ethnic names and considered it too obviously incorrect to have been worthy of comment, particularly if Fuller is correct in her belief that it was never Hucbald's intention to present a thorough description of the modes (with which he assumed his audience already to be thoroughly familiar).⁸⁰

⁷⁸ I have previously argued against precisely this kind of deduction regarding Hucbald. The difference in this case stems from the different motivations of the two authors. Hucbald seems to have specific goals in mind, and does not appear to be aiming for comprehensiveness, while Aurelianus is explicitly compiling the best information available to him.

⁷⁹ Fuller, "Interpreting Hucbald on Mode," 24.

⁸⁰ Fuller, "Interpreting Hucbald on Mode," 31.

The relationship to the *Musica enchiriadis*, however, is more significant. The *Musica enchiriadis* was very influential; this popularity is revealed not only by the number of surviving manuscripts and the amount of energy later treatises expend refuting it but also by the sheer number of small tenth- and eleventh-century treatises that proclaim themselves to be homages to it.⁸¹ Even many of the treatises that do not directly cite the *Enchiriades* make use of the Daseian notation system that it introduced,⁸² which does not seem ever to have been used to notate the repertory, but was used exclusively in theoretical treatises. Although this notation has potentially problematic implications for the gamut (see Table 2), it was still probably superior to the long-winded Greek nomenclature used in parts of the *Alia musica* (such as the description of the octave species in the introduction)⁸³ and the awkward species-based notation used in the *Nova expositio* (more about this in Chapter 12), and it is certainly superior to the complete absence of any notation or pitch nomenclature in the source treatise.

This last fact, that the source treatise does not use any kind of pitch nomenclature at all, is unexpected. The closest thing that the source treatise has to pitch nomenclature is its own system of using the numbers from the relation 6:8:9:12 (and, perhaps, associated letters A–D) to generate intervals, which it occasionally relates across modes (even though these do not seem to be consistent from one mode to another; this issue will be discussed at length in the final portion of this dissertation). One is left to wonder why some concrete pitch nomenclature was not used. Certainly, the Greek version was

⁸¹ In Schmid's edition of the *Enchiriades*, several of the minor treatises in the appendix fit this description. *Dimensio monocordi* (Schmid, *Musica et Scolica*, 179–81), which will be important in chapter 4 of this dissertation, includes the transition, "*His praemissis armonicam regulam enchiriadis describere placuit.*" ("These things having been set forth, it is acceptable to describe the harmonic rule of the *Enchiriadis*.") Another, untitled treatise (Schmid, 241) begins "*Si vis mensurare monocordum, quod dicitur Enchiriadis...*" ("If you want to measure the monochord, as is said in the *Enchiriadis*..."). Additionally, no less than four other short treatises signal their homage directly in the titles: "*Compositio monocordi secundum Enchiriadem*" (Schmid, 233–35; "The Composition of the Monochord According to the *Enchiriadis*"); "*Monocordum domni Enchiriadis*" (Schmid, 236–37; "The Monochord of the Master of the *Enchiriadis*"); "*De Enchiriadis monocordi divisione*" (Schmid, 239; "On the Division of the Monochord of the *Enchiriadis*"); and "*Divisio monocordi secundum Enchiriadem*" (Schmid, 240; "The Division of the Monochord According to the *Enchiriadis*").

⁸² viz. *De organo* (Schmid, *Musica et scolica*, 205–13); *Dialogus de organo I* (Schmid, 214–16); *Dialogus de organo II* (Schmid, 217).

⁸³ *Alia musica* §15, ed. Chailley, 107.

available, and the choice not to use it may perhaps imply that the author felt that the Greek gamut did not correspond well to the gamut in use at that time and place for ecclesiastical chant, a possibility potentially also implied by the gamut of the *Enchirides* (though the degree to which the *Enchiridis* gamut might reflect actual plainchant usage in the late Carolingian period is still disputed).⁸⁴

Table 2 - Greek Gamut vs. Enchiridis Gamut

Pitch	Greek Gamut	Enchiridis Gamut
ξ#		↵ <i>Deuterus residuorum</i>
ε		↶ <i>Protus residuorum</i>
α	< νήτη υπερβολαίων	h <i>Tetrardus excellentium</i>
g	∟ παρανήτη υπερβολαίων	t <i>Tritus excellentium</i>
f#		ε <i>Deuterus excellentium</i>
f	λ τρίτη υπερβολαίων	≠
e	∩ νήτη διεξευγμένων	k <i>Protus excellentium</i>
d	Σ παρανήτη διεξευγμένων νήτη συνημμένων	d <i>Tetrardus superiorum</i>
c	Ν τρίτη διεξευγμένων παρανήτη συνημμένων	∩ <i>Tritus superiorum</i>
b	∏ παραμέση	j <i>Deuterus superiorum</i>
b	>	(≠)
a	< μέση	d <i>Protus superiorum</i>
G	∟ λίχανος μέσων	f <i>Tetrardus finalium</i>
F) παρυπάτη μέσων	l <i>Tritus finalium</i>
E	∪ υπάτη μέσων	f <i>Deuterus finalium</i>
D	F λίχανος υπάτων	f <i>Protus finalium</i>
C	∩ παρυπάτη υπάτων	γ <i>Tetrardus gravium</i>
B	∟ υπάτη υπάτων	≠
β		N <i>Tritus gravium</i>
A	∏ προσλαμβανόμενος	γ <i>Deuterus gravium</i>
Γ		γ <i>Protus gravium</i>

If this hypothesis is correct, one may even suppose that the source treatise's gamut was the same as the *Enchiridis* gamut but that the associated Daseian notation and terminology had not yet been introduced (or, at least, had not yet proliferated). Consequently, one would place the source treatise

⁸⁴ Opinions vary, including that the gamut was theoretical only, and not actually employed in practice; that it was used in organum, either in both voices or only in one voice; or that it actually represented the medieval plainchant usage (at least in some regions) until the gamut was standardized to the more familiar medieval gamut. cf. Phillips, *Musica and Scolica*, 472–74; Maloy, “*Scolica enchiridis* and the ‘Non-Diatonic’ Plainsong Tradition”, 73–76; Grier, “Early Polyphony,” 803–07; Atkinson, *Critical Nexus*, 128–34.

prior to or not much later than the *Musica enchiriadis*. In fact, the lack of Daseian notation and terminology would significantly increase the likelihood that nearly all of the parts of *Alia musica* were written either before the rise of the *Musica enchiriadis* (i.e., not much later than the beginning of the tenth century) or after the rise of competing approaches in the later tenth century.⁸⁵ But the fact that the latest layer (viz. the commentary) contains the one single passage to include the doctrines of the *Musica enchiriadis* (specifically, the naming not only of modes but also of pitches according to the pseudo-Greek ordinal numbers – *protus, deuterus*, etc.), suggests that even the latest layer is less likely to have been written especially late in the tenth century, as this doctrine, too, largely falls out of use by this period.

Perhaps an even better indication of the age of the source treatise (and perhaps the revision) is the modal attribution of the chants cited. For each mode, the source treatise identifies anywhere from four to seven (and usually six) chants, distributed evenly amongst introits and antiphons, that the author considers to display the characteristic intervals associated with that mode; the revision generally repeats

⁸⁵ The earliest direct and explicit rejections of the *Enchiriadis* gamut of which I am aware occur in the eleventh century. The first is with Guido, in his *Micrologus*, ch.5, where he says, “*Hac nos de causa omnes sonos secundum Boetium et antiquos musicos septem litteris figurauimus, cum moderni quidam nimis incaute quattuor tantum signa posuerint, quintum et quintum uidelicet sonum eodem ubique caractere figurantes, cum indubitanter uerum sit quod quidam soni a suis quintis omnino discordent nullusque sonus cum suo quinto perfecte concordet. Nulla enim uox cum altera praeter octauam perfecte concordat.*” (Guido, *Micrologus*, ch. 5, ed. Smits van Waesberghe, 112–13, trans. Babb, 62) (“For this reason, we figure all sounds, following Boethius and the ancient musicians, with seven letters, while certain contemporaries exceedingly incautiously set out only four signs, evidently figuring every fifth sound everywhere with the same character, while it is true beyond doubt that certain sounds discord altogether with their fifths and no sound concords with its fifth perfectly. Indeed, no pitch concords perfectly with another except the octave.”) The second direct refutation is in Hermannus Contractus’ *Musica*, ch. 17, where he says, “*Unde longe a ueritate discordant, qui fere ubique in quintis locis eadem signa quasi ibi perfecta concordia sit ponunt, quibus etiam hoc vitio contingit, ut contra communem omnium musicorum consensum immo contra ipsius iura naturae eadem signa in nona potius quam in octaua regione veniant, sicque quod nimis absurdum est caracteres tantum non uoces aequalitatem habeant.*” (Hermannus, *Musica*, ch. 17, ed. & trans. Ellinwood, 126–27) (“Whence they disagree broadly from the truth who place entirely the same signs always at the fifth place as though there were a perfect consonance there, and to whom it even extends, through this vice, that against the common consensus of all musicians – rather, against the laws of nature itself – the same signs come at the ninth rather than the eighth region, and thus – which is exceedingly absurd – only the characters (and not the pitches) have equality.”) Nevertheless, although earlier theorists do not explicitly call out the *Enchiriadis* for its gamut, it is clear that all of the treatises written from the late tenth century onwards employ a gamut that corresponds to the diatonic genus of the Greek Greater Perfect System and also the more conventional medieval gamut (the nomenclature for which is attributed to the early eleventh century dialogue by Pseudo-Odo) – or, at the very least, they take octave equivalence for granted (which is not consistent with the *Enchiriadis* gamut).

these, with few omissions and a few supplemental examples (generally from other genres). For a handful of these chants, the modal assignment given in the source treatise differs from the modal assignment accepted today – and even from that cited throughout much of the Middle Ages. Nevertheless, as will be described in greater detail in the analyses that form the final part of this dissertation, these discrepancies do not seem to be erroneous attributions; instead, it appears that something fundamental about modal doctrine regarding these chants changed sometime during the tenth century because in general, when these chants also appear in other early tonaries (especially that of Regino, and sometimes Metz or Aurelianus), the placement in the *Alia musica* tends to agree with the early tonaries, and differ from later tonaries (such as that of Bern); *Urbs fortitudinis nostrae Sion* and *Malos male perdet* are notable examples.

This observation, unfortunately, does not point to a very precise date, since there are no mid-tenth-century tonaries against which to compare. Willi Apel suggests an explanation for *Urbs fortitudinis* (to be discussed in greater detail later) that relies on a shift from strict alternation between psalm and antiphon to continuous recitation of the psalm bookended by the antiphon, as well as an associated shift from identifying the mode of a chant from the way it begins to the way that it ends; Apel considers this process to have occurred in the mid-tenth century, and to be fully established by the time of Odo,⁸⁶ an assessment that also agrees with differing tonary assignments of these ambiguous chants.

Of course, this consideration has little impact on the source treatise, for which there are sufficient other reasons to assume that it was written in the ninth century. For the revision, the case is somewhat less clear. The revision retains most of the same examples (in the same modes), including *Malos male perdet*; however, the revision omits *Urbs fortitudinis* altogether – a surprising decision, in view of how closely the revisor usually follows the source treatise and that there are only three chants from the source treatise omitted in the revision (and both of the others seem to be due to issues of chant

⁸⁶ Apel, *Gregorian Chant*, 173–78.

misidentification).⁸⁷ It may be that *Urbs fortitudinis* was omitted from the revision because the revisor considered its first-mode attribution to be incorrect, or more likely, at least ambiguous. This hypothesis would place the revision after Regino (around the turn of the tenth century) – but since the other chants are not omitted or reclassified, probably not too close to Odo at the end of the tenth century. Interestingly, *Urbs fortitudinis* reappears in the completed *Alia* in the commentary (though Chailley misattributes the passage to the *Nova expositio*)⁸⁸ in Mode VIII, a mode that does not agree with either the earlier (I) nor the later (VII) attributions but displays some characteristics of each.

Taken together, all of these considerations point to a likely date for the source treatise in the mid- to late-ninth century, with the revision near or not too long after the turn of the century, when the attribution of *Urbs fortitudinis* was considered ambiguous enough to be omitted, and not yet definitively reassigned. The presence of the *Nova expositio* tonary in both the *Alia* and the *Dulce ingenium* also suggests a date not too late into the tenth century, though with a somewhat broader range of possibility than the revision. The final stages of the treatise, including the commentary, seem to have been written at a point of transition in the attribution of modal assignment of antiphons, sometime in the middle of the tenth century. Figure 3 presents a timeline reflecting my opinion of the most likely dates for the major phases in the development of the *Alia musica* compared to the most significant treatises and tonaries of the era.

⁸⁷ The other two chants omitted are *Iohannes autem*, which has been replaced by *Iesus autem*, which Chailley hypothesizes to be the result of the misreading of an abbreviation (Chailley, *Alia musica*, 123–24); and *Homo quidam fecit*, which also appears under the title *Quidam homo fecit*, and which can be easily confused with a responsory by the same name, and may thus have been omitted from the revision to avoid confusion.

⁸⁸ *Alia musica* §144(b), ed. Chailley, 203. This appears in Chailley's edition in a section subtitled "Octaves modales," after the *Nova expositio* tonary, but it actually precedes the *Nova expositio* descriptions of the seventh and eighth modes in the composite treatise, and does not appear in the *Dulce ingenium* (not even in the longer version of the *Dulce*, which contains additional passages from the *Alia musica*) nor in the revised recension of the *Nova expositio*, and is therefore almost certainly not part of the *Nova expositio*.

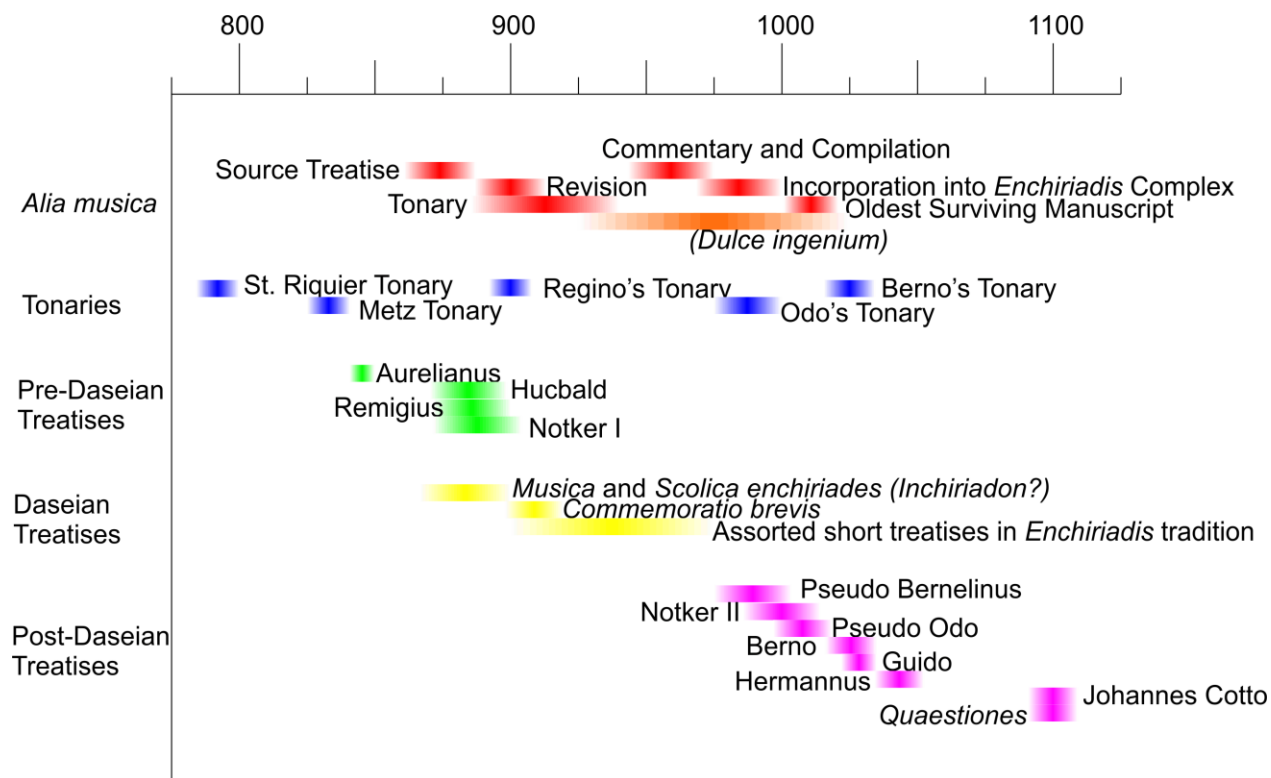


Figure 3 - Timeline of the constituents of the *Alia musica* compared to other medieval theoretical writings

Geographical Origin

Chailley acknowledged that there is little evidence to suggest a geographical origin for the *Alia musica*. It seems reasonable to treat the source text and its revision as part of a single tradition, and the tonary as a separate tradition. For the source text and revision, the only evidence of origin, weak though it is, is the traditional association of the text with Hucbald's *Musica* and the *Enchirades*,⁸⁹ amongst which it is often found in the manuscripts. There are also clear similarities to the *Musica disciplina* of Aurelianus of Réôme, containing concepts found in only these two treatises, suggesting some degree of proximity. This similarity would presumably place the origin of the *Alia musica* somewhere in the North-East of Francia or Lotharingia, but this hypothesis is certainly not reliable. If the date of the *Alia musica* is much

⁸⁹ Chailley, *Alia musica*, 60.

later than the *Enchiridae*s, a more distant origin would be implied, in view of the awkwardness of terminology that could have been avoided through the use of the *Enchiridae*s nomenclature. (It is worth observing, in regard to this latter possibility, that the provenance of those extant manuscripts that contain fragments of the *Alia musica* but do not contain any part of the *Nova expositio* is in the south of Europe, in Madrid, Barcelona, and Florence; the manuscripts will be discussed in detail in Chapter Four.)

For the tonary, the evidence is somewhat better, but contradictory. Chailley suggests an Irish or Anglo-Saxon origin on the basis of an unusual abbreviation in the manuscripts. The mark in question is hr , an abbreviation for *autem* typically found in Irish or Anglo-Saxon manuscripts that is a symbol from Tironian Shorthand.⁹⁰ (This hypothesis may also be supported by the occasional use of the Tironian Note γ for *et* in some manuscripts, though this symbol is considerably more widespread than that for *autem*). Ordinarily, this would simply suggest an Irish or Anglo-Saxon origin for the manuscript, not the treatise, as scribes copy meaning rather than spelling and might be expected to substitute their own preferred abbreviations (or resolve the abbreviations altogether). However, as Chailley notes, the use of this abbreviation is restricted to the tonary portion of the treatise (though it is not used consistently, as other variants of the letter h are also used).⁹¹ Additionally, this abbreviation is not restricted to a single manuscript; it can be found in the tonary sections of the Munich manuscript and both complete Paris manuscripts (though not in the Cesena or Barcelona manuscripts, and it is, of course, no longer possible to check the now lost Strasbourg manuscript). By contrast, outside the tonary sections, the word *autem* is consistently abbreviated $\text{au}\bar{c}$, or occasionally $\text{au}\bar{c}\bar{e}$. Furthermore, the abbreviation hr can also be found in Paris 8663, a manuscript not of the *Alia musica* tradition, but of the *Dulce ingenium* tradition (I have

⁹⁰ Cf. Prou, *Manuel de paléographie*; Chailley cites p. 155 (Chailley, *Alia musica*, 66), but he does not provide a bibliography or publication data for the edition that he consulted; James Grier informs me that Chailley's citation agrees with his copy of the fourth edition, of which appears to be out of print and not readily available; I find the citation on p. 67 of the first edition, freely available online [<https://gallica.bnf.fr/ark:/12148/bpt6k9785654w>]. cf. Bischoff, *Latin Paleography*, 86; Cappelli, *Lexicon abbreviaturarum*, 140 & 360.

⁹¹ Chailley, *Alia musica*, 66.

not been able to check the Prague and Bruges manuscripts of the *Dulce ingenium*). The implication is that the abbreviation *hr* was preserved from the *Nova expositio* source when the *Alia musica* was initially compiled and that the source of this tonary was, therefore, Anglo-Saxon or Irish.

Huglo, however, offers a contradictory assessment.⁹² He notes that the insular *hr* abbreviation could equally signal an origin at a continental centre frequented by insular scholars. The examples that he cites, however – Laon, Cambrai, and Saint-Amand – are all in the north-east of France, close to locations associated with other major Carolingian treatises, thus not accounting particularly well for the absence of the *hr* in the rest of the composite treatise – and, additionally, contradicting his next assumption: the tonary cites the antiphon *O quam clarus est*, which Huglo believes to be unique to the regions of Aquitaine and Toulouse, implying an origin in the south-west of France.⁹³ The accuracy of his assessment is suspect: the CANTUS database lists this chant in four manuscripts, two of which, F-Pnm lat. 1085 and F-Pnm lat. 1240, come from St. Martial de Limoges, which would support Huglo's opinion; but the other two, F-Pnm n.a.lat. 1535 and F-R 248, are from Sens (not far from Paris) and Jumièges (in Normandy), respectively, which does not support Huglo's hypothesis.⁹⁴ (It should be observed, however, that the presence of an antiphon in a manuscript kept in a particular location does not necessarily imply that the antiphon was part of the local usage, as manuscripts did occasionally travel, and they were not always copied in the locations in which they were intended to be used).

If Huglo is correct, then two reasonable hypotheses must be considered: either an Aquitanian treatise, citing an antiphon unique to that region, passed through the hands of an insular (or insular-influenced) copyist, picking up the unusual *hr* abbreviation, probably travelling north, to serve as the

⁹² Huglo, Review of Chailley (*Alia musica*), 231.

⁹³ *ibid.*, 232.

⁹⁴ <http://cantus.uwaterloo.ca/search?op=starts&t=o+quam+clarus+est&genre=All&cid=&mode=&feast=&volpiano=All>; <http://cantus.uwaterloo.ca/source/123663>; <http://cantus.uwaterloo.ca/source/123629>; <http://cantus.uwaterloo.ca/source/123633>; <http://cantus.uwaterloo.ca/source/123662>.

prototype to eventually be copied into both the *Alia musica* and the *Dulce ingenium*; or a northern treatise, already containing the b abbreviation, made its way far enough south to acquire the reference to the Aquitanian antiphon, before being copied into the *Alia musica* and the *Dulce ingenium* – which, if it occurred in the north, would imply that the Aquitanian revision was integrated into both treatises to the detriment of the native northern version. Since the *Alia musica* is so frequently associated with other northern treatises, the former hypothesis seems more likely (though the latter cannot be ruled out).

One further observation must be made before leaving this topic. As has already been observed, the *Alia musica* is generally believed to have been the first western treatise to associate the ancient Greek ethnic names for modes with the ecclesiastical modes. The use of these terms in later centuries is most commonly seen in German treatises.⁹⁵ One could, perhaps, hypothesize from these considerations a German origin for at least the revision, which is the portion of the composite treatise that uses these ethnic names. However, the use of this nomenclature would be relatively weak evidence for such an hypothesis, as it is equally compelling to suppose that the *Alia* introduced a new idea that primarily caught on in Germany. (The Munich – formerly St. Emmeram – manuscript could well have been the means by which this doctrine entered Germany, and was evidently well used, as demonstrated by the many glosses, including an entire additional tonary, added to its margins, though it must also be acknowledged that the doctrine could just as easily have been adopted in Germany from the *Musica enchiriadis*).

Restructuring the *Alia musica*?

⁹⁵ In the eleventh century, the list of treatises employing these terms includes most notably Hermannus Contractus, as well as Aribo, possibly Notker (though Notker's description is Boethian, and it is not clear whether he conflates the Greek and ecclesiastical modes), the anonymous *Quomodo de arithmetica procedit musica*, Theoger of Metz, and (slightly later) Johannes Cotto, all of which fall into the German tradition. Guido, however, while clearly familiar with treatises using this terminology (*viz. Musica enchiriadis*), makes no reference to them at all.

Chailley's critical edition is certainly open to critique for a variety of dubious decisions, including unjustified corrections and inconsistent interpretations (which will be described in later chapters). However, his decision to restructure the treatise has been particularly (and perhaps unfairly) controversial, with other scholars expressing concerns that reorganizing the material in this manner changes the treatise in a significant way – the reader effectively is not reading the same treatise.⁹⁶

This objection is probably overstated; while shuffling the chapters in a novel certainly creates a very different (and generally incomprehensible) novel, shuffling the order of contributed articles in an academic journal – even a special issue in which all articles are dedicated to a single theme – surely does not significantly change the journal. The *Alia musica* is much more like the special edition journal than the novel; it is necessary only to show that each of the three treatises is independently comprehensible to conclude that the order of presentation should not present a substantial barrier to understanding.

We can dismiss certain objections immediately. There can no longer be any reasonable objection to reading the tonary as a single contiguous unit, as it appears in that form in just as many surviving manuscripts (especially in the *Dulce ingenium* composite treatise) as it appears in interleaved form (in the *Alia musica*). It would also be difficult to argue that the rest of the *Alia musica* would not work without the tonary; although there is no surviving copy of the revised treatise without the tonary, the revised text is based upon the source text, which does stand alone in one manuscript. Both the source treatise and the revision include rudimentary tonaries, and thus, while the treatise certainly benefits from the more advanced tonary, it is certainly not a necessity – and with the exception of the additional antiphon citations and *differentiae*, there is nothing in the tonary not already present in the rest of the *Alia* (with one possible, but now unlikely, exception to be discussed later).

⁹⁶ Heard, *Alia musica*, 16; Atkinson, *Critical Nexus*, 173.

Likewise, it can hardly be maintained that the source treatise is not self-sufficient, as it appears in a single contiguous block at the end of the composite treatise, and appears independently, in substantially the same form, in at least one manuscript. It also is not especially integral to the composite treatise, since a revision of the source treatise serves as the central portion of the overall composite; the source treatise is thus, to some degree (though not entirely), redundant. It is, therefore, not unreasonable to present it separately. (In presenting it as he has, though, Chailley has missed an opportunity: perhaps the most useful presentation for modern musicologists would have been to present the source treatise and the revision in adjacent columns, facilitating comparison between the two very similar texts.)

As for the revision itself, it has already been observed that it does not lack anything substantial without the tonary or the source text, though unlike those two constituent treatises, there is no known surviving manuscript in which the revision appears independently; it is possible that it never existed in that form – that the treatise was compiled by the revisor. It is also possible, though by no means certain, that Chailley would have been justified in separating out the summary section (which he did not do) since that section appears independently in two manuscripts. (By contrast, however, even had Chailley considered the commentator and the revisor to be separate individuals, it would have made little sense to try to separate out the commentary, which relies heavily upon the revised text, and could not remain coherent if separated from the text upon which it comments.)

Yet the objection to rearrangement is not entirely without merit, either, insofar as reorganizing the treatise at least frustrates the compiler's (presumably) carefully crafted pedagogical program; the compiler was a medieval scholar who combined several existing treatises in the manner that he believed was the most effective way for a medieval reader to come to understand the material. Since a medieval theorist believed the treatise's original order was the best way to understand the concepts, reading it in its original order may be the best way for a modern reader to seek a genuinely medieval understanding. Thus, Chailley's choice to separate the source treatise from the revision is not particularly problematic,

but his decision to place it first, instead of last, is clearly contrary to the judgment of the compiler. Nevertheless, it should also be observed that Chailley's intended audience – modern musicologists – is rather different from the audience of the compiler of the treatise in its original form; Chailley's audience is typically quite interested in the development of theory over time, and Chailley's intention to present different stages of theory in chronological order makes sense in this context.

Moreover, it must be acknowledged that Chailley included additional passages in the sections attributed to his two *Quidams* (the sections that he considers to be independent of the Principal Treatise) that are of uncertain authorship. If the other manuscripts that have been discovered since the publishing of Chailley's edition are any indication, these extra passages may not be the work of these two authors. Chailley probably attributed some of this additional material (§§154–56) to the first *Quidam* because there seemed to be two different authors contributing similar passages in the Principal Treatise (and perhaps because the end of the passage that he selects is repeated at the end of the summary, implying that §156, too, functions as an ending); Chailley seems to have assumed that §§154–56 represent the comments of the *Quidam*, and the subsequent passages, that of the author of the Principal Treatise. (Mühlmann, too, considered these passages to be the work of two separate authors, but felt that the first author completed descriptions of only the first six modes, as well as §§154–56, and a second author completed the remaining modes and added the commentary following §156). Atkinson seems to accept Chailley's attribution,⁹⁷ even though his proposal of a fourth author, a commentator, could also permit the two passages to be split up between the Principal Treatise and the Commentary. In fact, Atkinson splits the extended passage up between all three authors. Whatever the case, the existence of the Karlsruhe

⁹⁷ Atkinson, *Critical Nexus*, 177.

manuscript containing only the First *Quidam* and not including §§154–56 (more on this in Chapter Four) tends to weaken this hypothesis (though it does not disprove it).⁹⁸

Concerning the Second *Quidam*, a section that is usually more convenient to refer to by Chailley's other label, the *Nova expositio*, Chailley includes a passage here that describes the octave species (§§133–45). This attribution almost certainly is not correct. Considerably more so than the First *Quidam*, which exists apart from the *Alia* in only a single, atypical and incomplete manuscript, the *Nova expositio* appears apart from the *Alia* in at least four manuscripts, in a contiguous form, and the passage in question appears in only one of these four manuscripts (and then, only as far as §141), separated from the *Nova expositio* by some additional material that is not related to the *Alia musica* (moreover, the *Nova expositio* in this manuscript differs from that in the other manuscripts in a few systematic ways). Atkinson, instead, considers this passage to be the work of his fourth author.⁹⁹ This passage will be addressed in greater detail later, and as it will need to be cited repeatedly, it shall be referred to throughout this dissertation as the Disputed Passage.

There is another respect in which Chailley's presentation of the treatise, combining both his reorganization of the treatise and his incorporation of details from atypical manuscripts, disguises an important detail. Chailley's presentation is organized chronologically, and consequently begins with his First *Quidam*, which is presumably the source upon which the rest of the treatise was constructed. Within his presentation of the source treatise, he includes a series of letters superscripted over Roman numerals; these letters appear in the source treatise in only in a single, idiosyncratic manuscript that contains only

⁹⁸ It needs to be acknowledged, however, that the manuscript in question ends mid-sentence, approximately one line before the presumed end of the description of the eighth mode. It is possible that more than just the end of this one sentence is missing.

⁹⁹ Atkinson, *Critical Nexus*, 177. Atkinson separates this passage into two subsections: §§133–42, on the octave species, and §§143–45 on the eighth mode; ultimately, however, the passage on the eighth mode principally explains the difference between Modes I and VIII, which share the same octave species, and is therefore a continuation of the description of the octave species.

the source treatise. In all of the other manuscripts, they do not occur in the source treatise or the revision, but only in the summaries that appear near the end of the treatise. In spite of this fact, Chailley invokes these letters to explain an unclear passage about the third mode, without acknowledging that these numbers do not appear at any point prior to the passage in question, nor for a considerable distance afterwards. This question will be taken up in greater detail in Chapter 16. This detail could easily be missed by a casual reader not paying sufficient attention to the critical apparatus, since Chailley selects the idiosyncratic manuscript as his base text for the source treatise, rather than the much more consistent text found in all the other manuscripts, thus giving the impression that these letters are part of the standard recension for this portion of the treatise, which they are not. Likewise, as a result of the same odd choice of base text, the same casual reader would be led to believe that the source treatise in the standard recension includes systematic references to the *enēkhēmata* (representative modal melodies) and the manerial modal designations (*authentus protus*, etc.) and their translations, which are also exclusive to this one manuscript.

What does *Alia musica* mean?

A Latin Interpretation

One final consideration before moving on to the content of the treatise: what does the title *Alia musica* mean? As a first observation, it should be noted that the title is more than a little suspicious: neither the Cesena manuscript nor either of the Paris manuscripts contains any title at all for this treatise. The title *Alia musica* does appear high in the upper margin of the Munich manuscript, in what does not resemble the hand of the original scribe (alongside the attribution “*Auctor Hucbaldus Elnonensis*” in a hand that more closely resembles the hand of the primary scribe, but still with some notable differences, and that appears in the margin along with many glosses – this issue will be taken up again in the

manuscript study in Chapter 4). The title, therefore, was probably added by a later scribe.¹⁰⁰ The presence or absence of the title in the Strasbourg manuscript can no longer be directly ascertained, but since Gerbert cites the title as coming from the St. Emmeram (*i.e.*, Munich) manuscript, it is unlikely that the Strasbourg manuscript (which was his primary source) contained the title. Thus, the title appears in only one manuscript, probably as a later addition. It is therefore unlikely to have been the original title – if, indeed, the prototype had even been given a title (which is not necessarily to be expected).

Nevertheless, the title has become attached to the treatise, and it is worth knowing what it means, because the title may help to understand how medieval musicians understood the treatise. Unfortunately, the title is something of a puzzle. The word *Alia* is a Latin adjective of the first and second declension with the forms *alius*, *alia*, *aliud*, meaning *other*, *another*, or *different*. Given its form, it must be either feminine singular (either nominative or ablative) or neuter plural (nominative or accusative). *Musica* can be one of two things: a first-declension, feminine noun meaning *music*, or a first- and second-declension adjective of the form *musicus*, *musica*, *musicum*, meaning musical (the forms of which vary in the same ways as *alia*). Thus, the title *Alia musica* seems to suggest something like “Other Music,” which, as both Chailley and Heard acknowledge, seems to be an unlikely translation.¹⁰¹ This interpretation is not, of course, entirely implausible. If feminine, rendered as “Another Music,” ellipsis could be assumed, yielding the translation “Another Music [Book]”;¹⁰² it would then be “another” music book, by reference to the previous books, such as Aurelianus and Hucbald – or by reference to preceding texts in the same manuscript, especially if a preceding text was entitled *Musica*, a reasonably common title for a music treatise (unfortunately for this latter possibility, none of the preceding texts in the Munich manuscript are

¹⁰⁰ For more on this, see the discussion of the Munich manuscript in the manuscript studies in Chapter Four.

¹⁰¹ Chailley, *Alia musica*, 3; Heard, *Alia musica*, 19.

¹⁰² Compare the title of the modern on-line resource *Thesaurus Musicarum Latinarum* (“Treasury of Latin Musics [*sic*, *i.e.* Music Treatises]”), which employs the same ellipsis; certainly there are no shortage of historical treatises on music that are simply referred to as *Musica*.

directly entitled *Musica*, with the closest contenders being *Musica enchiridis* and Boethius' *De institutione musica*, neither of which is given precisely those titles in the manuscript).

Alternatively, both words could be assumed to be neuter, with *musica* functioning as a substantive adjective (“musical things”) standing for music treatises; with the word *alius* potentially meaning *different*, this could then be translated as “Differing Musical Treatises,” which is a reasonably good description of the contents.

A Greek Interpretation?

Heard proposes an alternative explanation of the title, which also seems plausible. He believes that *Alia musica* is a Latinization of Greek ἄλῖα μουσική.¹⁰³ According to Heard, ἄλῖα describes an assemblage, and μουσική means music (which Heard treats as an adjective); ἄλῖα μουσική, then, would mean something like *A Musical Assemblage*.

The suggestion that the title is Greek is plausible in principle. As Heard notes, the authors of the *Alia musica* make considerable use of Greek, and the typical transliteration throughout the treatise is consistent with the way that the title would be transliterated.¹⁰⁴ However, his interpretation takes liberty with the Greek, and so is not to be uncritically accepted.

The word ἄλῖα is a word in the Dorian dialect, similar to the Attic ἐκκλησία, and technically describes a formal assembly of people, such as a political assembly or religious congregation (just as the Latin cognate, *ecclesia*, refers to the Church). I have been unable to verify whether it was ever used in the more general sense of assembling things (such as treatises) or ideas. That said, as an assembly of people, it might well describe an assembly of authors, which is an apt description of the treatise.

¹⁰³ Heard, *Alia musica*, 19.

¹⁰⁴ *Ibid.* Heard particularly notes the lack of an ‘h’ to transliterate the Greek rough breathing.

Μουσική is the less problematic word. Μουσική is a noun, describing any art presided over by the Muses, and particularly lyric poetry sung to music, or music more generally; it is also an adjective of the form μουσικός, μουσική, μουσικόν, describing musical things. These “musical things” could include musical people (but generally as an adjective, and only a noun as a substantive); μουσική, however, is feminine, singular, and nominative, which makes it more likely a direct adjective describing ἄλεια, rather than describing an assembly of musicians. Thus, if *Alia musica* is a transliteration of ἄλεια μουσική, the more accurate translation, based on correct use of Greek and the characteristics of the treatise, would be more like *A Musical Council*, or something similar.

Chapter 03 – The Course of the Treatise

As already described in the last chapter, the complete treatise seems to consist of at least three layers, and perhaps several more that are difficult to distinguish from one another. Mühlmann suspected as many as seven authors: a *Theoretiker der Acht Modi* (“Theorist of Eight Modes”); a *Theoretiker der Principales und Subjugales* (“Theorist of *Principales* and *Subjugales*”, i.e., of authentics and plagals); a *Theoretiker der Sechs Töne* (“Theorist of Six Tones”); an *Überarbeiter* (“Reviser”); a *Kommentar* (“Commentary”); the *Zahlenauszug* and *Tabelauszug* (“Numerical Summary” and “Tabular Summary”); and the *Tonarius* (“Tonary”, i.e., the *Nova expositio*).¹⁰⁵

Chailley identified only three authors (with Heard accepting his analysis): a first “*Quidam*,” a second “*Quidam*” (which he also calls “the tonary”), and a principal treatise (which he identifies as the *Alia* proper). Atkinson, while agreeing with Chailley in broad outline, considers the principal treatise to be a compilation of two separate authors, making four authors that he describes simply as α , β , γ , and δ . In addition to all these names, the *Alia* itself refers to some of its own authors under the names *expositor* and *praemissus expositor*.

All of these naming conventions result in considerable confusion, especially since the names are not particularly descriptive of the material that each author covers (excepting Mühlmann’s names, which are quite descriptive but probably reflect an over-proliferation of authors). Although it is tempting to keep out of the argument altogether, it is impossible to make sense of the modal theory presented in the treatise in its historical context and address the issues of the development of modal theory without a model of the divisions of the treatise. Thus, I shall essentially adopt Atkinson’s hypothesis in most

¹⁰⁵ Mühlmann, *Die Alia musica*, 50–51; 51–55; 55–60; 60; 60–69; 69–71; 71–74. As with Chailley after him, Mühlmann chose to reorganize his German translation into the various authors that he believed composed each passage; Chailley provides a table summarizing Mühlmann’s reorganization (Chailley, *Alia musica*, 11), which is reproduced in the Appendix to this dissertation.

respects, but I shall replace Atkinson's nondescript labels with names that more clearly reflect the function of each section, and which will allow, after some consideration of each section, for a clear and concise summary of the contents and progression of the treatise.

For the present purposes, the material will be described according to the way that the structure of the treatises has been traditionally described. In the next chapter, alternate hypotheses will be presented for certain sections.

The deepest layer(s)

The majority of the treatise is organized around a single original treatise, which Chailley refers to as *le premier quidam*, a blend of French and Latin meaning approximately "the first somebody."¹⁰⁶ Much of the larger treatise serves as a commentary upon this source text, which the commentator refers to as the *praemissus expositor*¹⁰⁷ ("the author sent forth"). Atkinson refers to the author of this treatise as "author α ";¹⁰⁸ in view of the function of this text within the composite treatise, I shall refer to it as the source treatise.

Although it serves as the organizing principle for the treatise, this source treatise is, in fact, quite short, spanning slightly less than ten percent of the total length of the treatise. The author consistently refers to the modes as *toni*,¹⁰⁹ which he labels with Latin ordinal numbers (*primus*, *secundus*, and so on, as opposed to the pseudo-Greek *protus*, *deuterus*, etc.).¹¹⁰ For each *tonus*, the author identifies the consonances of the mode according to numbers that represent the proportional string lengths that create them (this topic will be addressed in greater detail later). After identifying the numbers that represent

¹⁰⁶ Chailley, *Alia musica*, 7.

¹⁰⁷ Chailley, *Alia musica*, 6.

¹⁰⁸ Atkinson, *Critical Nexus*, 175.

¹⁰⁹ Note that the use of the terms *tonus*, *tropus*, and *modus*, all frequently translated as "mode," is problematic in this treatise, as will become clear later. For this reason, I shall normally retain the Latin terms, but will usually normalize to the nominative case.

¹¹⁰ Atkinson, *Critical Nexus*, 175.

the consonances, the author provides a list of introits and antiphons that exemplify the *tonus* and the intervals just described.

It is important to observe that in most manuscripts, the source text only describes the first six *toni* in detail. The seventh *tonus* is described much less thoroughly, and for the eighth *tonus*, the author says only, “*tonum octavum require supra*”¹¹¹ (“seek the eighth *tonus* above” – that is, presumably, the details of the eighth *tonus* are effectively the same as those of the seventh *tonus*). This observation is important in the study of the blending of the Greek and medieval modal systems for which the *Alia musica* is best known (a topic that I shall treat in greater detail later). Chailley, in his edition, incorporates an adjoining passage that would complete the layout of eight modes, but it is my opinion that the manner of presentation of this material is more consistent with a different section of the treatise; it is out of sequence, and represents the numbers corresponding to the consonances in a manner typical of a different section in all manuscripts. This manner of representation is found in the source text only in the incomplete Karlsruhe manuscript, and this issue will be addressed more fully in a discussion of that manuscript in the next chapter. For the moment, it is worth observing that Mühlmann recognized the different style of the discussions of Modes VII and VIII compared to the others, and attributed the source text to two different authors – the first six modes to the *Theoretiker der Sechs Töne* and the completion of it to the *Überarbeiter*.¹¹²

Although the composite treatise is clearly organized around the source text, which is the oldest portion of the treatise,¹¹³ it appears in its original form only at the very end of the compiled treatise; we may suppose, then, either that the source text was added on to the end of the *Alia* as a kind of appendix or that the rest of the *Alia* was intended as a program of study to prepare the reader to read and

¹¹¹ *Alia musica* §188, ed. Chailley, 95.

¹¹² Mühlmann, *Die Alia musica*, 55–57; 60.

¹¹³ Chailley, *Alia musica*, 60; cf. Heard, *Alia musica*, 16; Atkinson, *Critical Nexus*, 175.

understand the source text. (Since much of the commentary seems more erroneous than helpful,¹¹⁴ I feel that the interpretation of the source text as an appendix is more likely).

In addition to the descriptions of the numbers corresponding to consonances in each mode and the examples of chants in each mode, Chailley feels that the *premier Quidam* [source treatise] includes a short section comparing the musical consonances to other natural phenomena (Chailley's §§ 154–56); Mühlmann also attributed these passages to the *Theoretiker der Sechs Töne* (to whom Mühlmann attributes the first six modes of the source treatise),¹¹⁵ and Atkinson also accepts this attribution.¹¹⁶ This interpretation is supported by the fact that the heading in the *Alia* immediately before this section reads *sequitur praemissus expositor*,¹¹⁷ a rubric by which the commentator routinely identifies the source text. However, it is notable that this section does not appear at the end of the treatise with the rest of the source text; it also does not appear in the Karlsruhe manuscript, which contains essentially the entire source text but without this additional section and does not contain any other part of the composite treatise¹¹⁸ (it may be an exemplar of an independent manuscript tradition of the source text separate from the *Alia*, though it is also possible that it is merely an incomplete *Alia*, particularly in view of the fact that it includes the suspicious description of the eighth tone – more on this in the next chapter). Chailley acknowledges the attribution of this section to the source text;¹¹⁹ Atkinson likewise defends the attribution on the grounds of the rubric *praemissus expositor* and also acknowledges Mühlmann's comparable attribution.¹²⁰ It should be observed, though, that there is no difference in Latin between “the *praemissus expositor*” and “a *praemissus expositor*”; it is reasonable to consider, then, that the commentator means merely that he will proceed with the work of a previous author, and not necessarily

¹¹⁴ Chailley, *Alia musica*, 20–21.

¹¹⁵ Mühlmann, *Die Alia musica*, 57–58.

¹¹⁶ Chailley, *Alia musica*, 95–98; Atkinson, *Critical Nexus*, 177.

¹¹⁷ Gerbert, *Scriptores ecclesiastici* Vol. 1, 141.

¹¹⁸ Atkinson, *Critical Nexus*, 173.

¹¹⁹ Chailley, *Alia musica*, 95.

¹²⁰ Atkinson, *Critical Nexus*, 177 fn. 16.

the one that he has thus far been describing. It is unlikely, given the rubric of *praemissus expositor*, that this section is merely another part of the commentator's commentary, but it may represent his incorporation of commentary made by a previous commentator. It is also important to acknowledge that the opinions of Atkinson, Chailley, and Mühlmann on this topic barely amount to more than a single opinion, since both Chailley and Atkinson cite their predecessors in a single chain of citation, and even Chailley finds cause to question this attribution, introducing the relevant passage with the caveat "*Cette partie n'est connue que par la citation du commentateur [...], citation dont la littéralité est sujette à caution.*"¹²¹ ("This part is only known by the citation of the commentator, a citation for which the literality is subject to caution.")

The Middle Layer(s)

Chailley refers to the middle layers, which make up the bulk to the treatise, as *Le traité principale* ("The Principal Treatise"), and sometimes the *Alia* or the *Alia proprement dite* ("The *Alia* proper").¹²² In the text, this author is referred to as the *expositor* (not to be confused with the *praemissus expositor*, who wrote the source text). Atkinson considers it to be the work of two authors, whom he labels authors β and δ , whose functions differ. The first of these, author β , presents a revision of the source text (indeed, Mühlmann attributes it directly to his *Theoretiker der Sechs Töne*, to whom he attributes the source text); I shall call this section the revision, and its author the revisor. In most respects, the text of this section follows the source text in a rather close paraphrase; however, it is interesting to note that the revisor occasionally substitutes different chants to exemplify each mode; Chailley points out that while the source text's modal assignments seem consistently accurate, the revisor's substitutions often seem to be errors,

¹²¹ Chailley, *Alia musica*, 95.

¹²² Chailley, *Alia musica*, 7.

leading Chailley to the conclusion that the edition was not written by a practising musician, but instead was written to serve a general curriculum in the liberal arts.¹²³

Following each section of the revision (usually describing a single mode), commentary upon the mode is provided. Chailley considered this commentary also to be the work of the revisor. Atkinson, however, feels that the commentary represents an additional, youngest layer of the treatise, added by whoever compiled the composite treatise, whom he labels author δ . It is not obvious, but is quite likely, that the commentator should be a separate author from the revisor, in light of certain inconsistencies between the revision and commentary, some of which will be described in Chapter 16). By contrast, however, it is much more difficult to identify the compiler of the treatise, who is as likely to be author γ (Chailley's second *Quidam* – the author of the *Nova expositio* – whom Chailley believed to have been the compiler) as to be author δ – or could even be a separate person altogether (perhaps an author ϵ ?), who left minimal fingerprints on the work, other than its order and headings (and perhaps the occasional transitional phrase). Thus, I shall identify Atkinson's author δ not as the compiler, but simply as the commentator, who is most likely not the same person as the revisor. If this author is, indeed, the compiler, as Atkinson suggests, then the material that he contributed would constitute the chronologically latest portions of the treatise, while Chailley attributes it all to the middle layer.

¹²³ Chailley, *Alia musica*, 20–21. In fact, as described in the last chapter and as to be more fully detailed in the analyses in Chapter 17, even the chants listed in the source treatise are not entirely straightforward in their modal assignments. Furthermore, Chailley's criticism of the revisor is overstated. He says, "*Presque tous les exemples qu'il ajoute de son cru à ceux du QUIADAM (lesquels sont corrects) tombe à faux (§104, 106, 120 etc.)*" (Chailley, *Alia musica*, 21) ("Nearly all the examples that he adds of his own accord to those of the Source Treatise (whose [attributions] are correct) fall falsely.") However, his first example, §104, cites the offertory *Immittit angelus* in a discussion of Mode V, which Chailley identifies as a Mode VIII chant. While Chailley's opinion is supported by the *CANTUS* database (<http://cantus.uwaterloo.ca/search?op=starts&t=immittit+angelus&genre=All&cid=&mode=&feast=&volpiano=All>) this chant is also assigned to Mode V in Aurelianus' *Musica disciplina*, Ch. XIV (Gushee, *Aureliani Reomensis Musica disciplina*, 102; cf. 148), and therefore fits a pattern, to be discussed more fully later, of agreement of older sources against newer sources – a pattern of which Chailley seems not to be aware. The same may be said of his third example, §120, the offertory *Stetit angleus* (compare Gushee, *Aureliani Reomensis Musica disciplina*, 104 & 150 to <http://cantus.uwaterloo.ca/search?op=starts&t=stetit+angelus&genre=149&cid=&mode=&feast=&volpiano=All>; Chailley, *Alia musica*, 160).

In addition to the descriptions of and commentary upon each mode, Chailley feels that the *expositor* is also responsible for the additional material that appears at the beginning and after the end of the edition and commentary (but before the appendix of the source text). Atkinson considers most of this material to be the work of his author β [the revisor], with his author δ [the commentator] contributing a little extra commentary near the end.¹²⁴

The portion of the additional material that appears at the beginning serves as a primer on Greek harmonic theory, divided broadly into three sections. The first is drawn quite closely from Boethius' *De institutione arithmetica II*, chapter 48 (with a brief excerpts from chapters 54, 47 and 49),¹²⁵ concerning the arithmetic and harmonic means and the ratios corresponding to perfect consonances. The second section is drawn more loosely from Boethius' *De institutione musica*, especially book II, chapters 14–15 and 17,¹²⁶ and treats the species of octave and the connection between them and the modes. The third section is less directly drawn from Boethius (with the exception of a single brief excerpt from *De institutione musica II*, chapter 9, a rare case in which the author directly provides a source for his citation),¹²⁷ and sets up the manner in which the preceding ideas will be applied to the modes in the rest of the treatise. Atkinson considers all of this to be the work of the revisor.¹²⁸ Mühlmann sees the material from the *De arithmetica* as the work of the *Theoretiker der Principales und Subiugales*, who contributes another, arguably more important section in the later chunk of added material (near the end of the treatise); Mühlmann then supposes that the next section, from the *De Musica*, is the work of the *Theoretiker der Acht Töne*, while the remaining material belongs to the *Kommentar* (loosely, our

¹²⁴ Atkinson, *Critical Nexus*, 176–77.

¹²⁵ Boethius, *De institutione arithmetica*, ed. Friedlein, p. 155 ln. 7 – p. 158 ln. 10; p. 171 ln. 16–20; p. 153 ln. 29 – p. 154 ln. 2; p. 158 ln. 22–25; also included are more general paraphrases from ch. 42–43 (pp. 139–44). The material is slightly abridged, rearranged, and modified. In particular, most of Boethius' numbers are doubled, as in the relations 3:4:6 and 2:3:6, which are converted into 6:8:12 and 4:6:12, to bring them into line with the usage in the *Alia musica*, which focusses heavily on the relation 6:8:9:12.

¹²⁶ *ibid.*, 337–348.

¹²⁷ *Alia musica* §22(b)–(c), ed. Chailley, 113; Boethius, *De institutione musica*, p. 237 ln. 29 – p. 238 ln. 3.

¹²⁸ Atkinson, *Critical Nexus*, 177.

commentator, or Atkinson's author δ).¹²⁹ Chailley, of course, considers the entire principal treatise to be the work of a single author.

The latter section of additional material, which appears after the end of the revised edition of the source text, is a little less straightforward. It begins with a brief section (already described above) that both Chailley and Atkinson attributed to the source text because the heading describes it as the work of a *praemissus expositor*; Mühlmann ascribes all of this to the *Theoretiker de Sechs Töne* [source treatise].¹³⁰ Following this passage is an expansion and commentary upon this material (the analogy between musical harmonics and natural phenomena, along with some more general observations on harmonic theory) that Atkinson considers to be the work of the revisor;¹³¹ most of this Mühlmann continues to see as the work of the *Theoretiker der Sechs Töne*, with a brief interpolation by the *Überarbeiter* (who elsewhere fills in the gaps to create eight tones); however, Mühlmann attributes the latter part of this, which leaves natural phenomena and returns to strictly harmonic theory, to the *Kommentar*¹³² (our commentator, though Atkinson still considers these sections to be the work of his author β [revisor], rather than author δ [Commentator]). This passage is then followed by another brief section that appears to be even further commentary upon a part of the preceding commentary; Atkinson agrees with Mühlmann that this portion is the work of author δ [Commentator].¹³³

Following this commentary, all of the modal numbers are summarized in a chart, followed by a prose description, which Mühlmann attributes to an independent author, while both Chailley and Atkinson accept these as the work of the revisor;¹³⁴ it is the final few sentences of this section that Chailley

¹²⁹ Mühlmann, *Die Alia musica*, 51–53; 50–51; 60–62.

¹³⁰ *Alia musica*, §§154–65, ed. Chailley, 95–97; cf. Atkinson, *Critical Nexus*, 177, Table 5.3; Mühlmann, *Die Alia musica*, 57–58.

¹³¹ Atkinson, *Critical Nexus*, 177, Table 5.3.

¹³² Mühlmann, *Die Alia musica*, 58–59; 60; 69.

¹³³ Atkinson, *Critical Nexus*, 177, Table 5.3.

¹³⁴ Mühlmann, *Die Alia musica*, 69–71; Chailley, *Alia musica*, 173–74; Atkinson, *Critical Nexus*, 177, Table 5.3.

includes in his edition of the source text as providing the information about the eighth *tonus* that would complete the system of eight *toni* in the source text. To accept this interpretation, however, is perhaps to separate the eighth *tonus* from the summary of the chart, leaving that summary with only seven *toni* – that is, to solve one problem by creating another, less likely problem: the eighth *tonus* would then be present in the chronologically earlier source text, but not in the later prose description of the chart (while the chart itself includes the eighth *tonus*); moreover, this is the last material before the appending of the source text itself, implying that if this interpretation were correct, the eighth *tonus* in the source text would be presented before the other seven *toni*, all the rest of which are presented in numerical order.

If there is any one detail that might tend to support this interpretation, it is that the heading in the manuscript that introduces the source text, *Item expositio eorundem tonorum* (“Likewise, an exposition of those same tones”), is placed later than one would expect, after the first sentence that introduces the first *tonus*; if one takes the liberty of shifting the heading backwards by one sentence, to precede the introduction of the first *tonus* (as seems reasonable), one could perhaps accept also the shifting of the heading slightly further back, to precede the description of the eighth *tonus* (and thereby incorporate that *tonus* in the source text).

Proceeding along this argument, the final line of the source text in the manuscripts says “*Tonum octavum require supra*,”¹³⁵ (“Seek the eighth *tonus* above”), which I have elsewhere interpreted to mean *directly* above – that is, that the eighth *tonus* shows the same properties as the seventh *tonus*; alternatively, however, it could mean that the reader is meant to look back to the description of the eighth *tonus* found in the section immediately above the presentation of the source text, thus saving the scribe from recopying identical text. If this interpretation were correct, one could argue that the passage originally appeared in the source text, was copied verbatim into the summary accompanying the chart,

¹³⁵ *Alia musica* §188, ed. Chailley, 95.

and then omitted from the appended source text to avoid repetitiveness (or perhaps to save space in the manuscript). Nevertheless, this interpretation seems considerably less likely, given the inherently repetitive nature already displayed by the very act of appending the source text to the edition (other passages are duplicated without the subsequent copy being cross-referenced and removed).

The Top Layer(s)

The remaining layer of the composite treatise consists primarily of a tonary, which describes each *tropus* as arising from a specific octave species. The author then describes the *ēkhēma* (intonation formula) for the *tropus*, especially in terms of where it begins and where it ends, as well as notable intervals traversed in the formula. From here, the author identifies the *differentiae* (psalm-tone cadences) and *locae* (the possible first pitches for the antiphons associated with each *differentia*) used in both the daytime and nighttime services and provides examples of chants employing each transition. Each section then ends with a description of the doxologies; since the text of the doxology is the same for each mode, it is clear that notation was intended to be included, showing the melody appropriate to the doxology of each *tropus*; the neumes, however, are presented in only a single manuscript.¹³⁶

This material does not appear all together in a single section (as the appended source text does); instead, the description of a *tropus* is inserted into the principal treatise after the edition and commentary on that *tonus* (the significance of the differing terminology, *tonus* vs. *tropus*, will be explored later), forming the final section pertaining to that mode (excluding the final summaries and appendix). However, in two instances (Modes III and VII), the tonary section is not inserted at the end of its own mode but is held back and inserted together with the following *tropus*. Chailley supposes that this is evidence of the compilation process, showing that the compiler forgot to include the tonary in the expected place, and

¹³⁶ Chailley, *Alia musica*, 27.

simply slipped it in at the next appropriate opportunity.¹³⁷ While this explanation is certainly not impossible, it seems unwise to assume that the arrangement was not a deliberate aspect of the compiler's pedagogical programme – particularly since the pairs of modes that end up being presented together belong to the same *maneria*. Indeed, such an ordering is beneficial, since the *ēkhēma* of the plagal mode of a *maneria* always begins where the *ēkhēma* of the corresponding authentic mode ends. (This fact is both beneficial and detrimental to the present argument, insofar as if this property is, indeed, the reason for grouping the third and fourth *tropi*, and likewise the seventh and eighth *tropi*, one would likewise expect the same procedure for the *protus* and *tritius maneriae*, which are not organized in this way.)

Mühlmann, Chailley, Heard, and Atkinson all agree that this tonary is the work of a single author, an assumption well supported by the fact that it is preserved independently from the rest of the *Alia* in three manuscripts. Mühlmann identified him simply as the Author of the Tonary, while Chailley calls him the *deuxième Quidam* (“second Somebody”); Chailley also calls this material the *Nova expositio* (“A New Exposition”), referring to the heading routinely used to announce the tonary material in the composite treatise.¹³⁸ For the first *tropus*, the heading is “*Item cuiusdam de eadem re noua expositio*”¹³⁹ (“Likewise, somebody’s new exposition of that same thing”); for all the remaining *tropi*, the heading typically follows the formula “*Item de [ordinal number] tono noua cuiusdam expositio*” (“Likewise, of the [ordinal number] *tonus*, somebody’s new exposition”).¹⁴⁰ The word *cuiusdam*, found in this heading, is the genitive form of the word *quidam* and is the source of Chailley’s label for the author of this section (and also for the author of the source text). Atkinson identifies the author of the tonary as author γ.¹⁴¹

¹³⁷ Chailley, *Alia musica*, 9.

¹³⁸ Chailley, *Alia musica*, 8–11; Atkinson, *Critical Nexus*, 178.

¹³⁹ Gerbert, *Scriptores ecclesiastici* Vol. 1, 130.

¹⁴⁰ Curiously, although the heading is consistently the same, Heard translates it differently in each case (Heard, *Alia musica*, 155–56; 169–70; 177–78; 183–84; 197–98).

¹⁴¹ Atkinson, *Critical Nexus*, 178.

The material in this section is considerably different from the material in all the other sections. The other sections are deeply (and often abstractly) theoretical, with little connection to practical music-making. The exception is occasional references to specific chants belonging to specific *toni* in the source text and its revision (and, as already noted, these references in the revision appear to be frequently problematic). By contrast, the material in the tonary is eminently practical. The *ēkhēmata*, *differentiae*, and doxology melodies were absolutely essential knowledge for members of a medieval church choir; this section, therefore, functions as a kind of practical supplement, taking the knowledge of the modes out of the rarified academic discourse and placing it squarely in the hands of the singer, which is probably the reason why this material is added to the treatise. Notwithstanding the presence of tonary-like elements in the other parts of the treatise, I shall generally refer to this section as the tonary.

There is an additional section not yet accounted for that Chailley ascribes to his second *Quidam*. It is the passage that I have referred to as the Disputed Passage, a critical passage in the study of the transition from residual Greek theory to distinctly medieval theory, describing the relationship between the ecclesiastical modes and the old Greek species of octave, and in particular, explaining how modes I and VIII can both correspond to the same octave species and still be different modes (more on this later). Unlike Chailley, Mühlmann considered most of this material to be the work of the *Theoretiker der Principales und Subiugales*, whose only other contribution is the introductory material from Boethius' *Arithmetica* at the beginning of the composite treatise; Mühlmann attributes the rest of this material to the *Kommentar*;¹⁴² Atkinson also attributes this material to the his author δ [Commentator].¹⁴³

Atkinson does not explicitly explain his reasoning for assigning this material to the commentator, rather than grouping it with the tonary as Chailley had done (though, as will become clear throughout this

¹⁴² Mühlmann, *Die Alia musica*, 53–55; 68–69.

¹⁴³ Atkinson, *Critical Nexus*, 177, Table 5.3.

paper, many of his assignments seem to be based on distinctive use of vocabulary).¹⁴⁴ It is suggestive to note that this passage does not seem to appear in those manuscripts in which the tonary enjoys independent transmission.¹⁴⁵

There is one possible justification (which should not necessarily be assumed to be the justification of either Atkinson or Chailley) that ought not to be accepted at face value. The line of inquiry in which this section of the treatise is frequently implicated aims to show the progressive adoption of pseudo-Greek theory into the ecclesiastic modal system. It is therefore tempting to assume that this passage, which seems closer than any other part of the treatise to the later medieval and Renaissance conception of modes, ought naturally to belong to the most recent layer of the treatise (thus, for Atkinson, this passage would have been written by author δ , who is the commentator and perhaps compiler of the composite treatise; for Chailley, it would have been written by the second Quidam, whom Chailley believes to have been the compiler of the treatise).

This assumption may well be true, but it is by no means a given. Theory does not always develop linearly; new ideas may well be initially rejected by contemporary theorists or by the next generation, only to be picked up again by later generations if the intellectual climate becomes more favourable – and such ideas might then carry the additional weight often attributed to historical documents (after objections to them have been forgotten). There are, therefore, many possible ways in which the passage concerning the relationship between octave species and the ecclesiastical modes might belong to an earlier layer without necessarily influencing the later layers.

¹⁴⁴ Although Atkinson does not explain his assignment of the material to the commentator in his book, he provided me with a brief explanation in the feedback following the defence of this dissertation. He confirms that linguistic considerations were among his reasons, but notes his author δ (the commentator) seems to him to have been far more sophisticated than the other authors.

¹⁴⁵ This is a slight overstatement; part of the passage is present in one of the four manuscripts in which the tonary appears separated from the rest of the treatise, but it is not directly connected to the tonary even in that manuscript, and the most important part of the passage is not included. This issue will be more fully addressed in the next chapter.

Perhaps Chailley was correct in attributing this passage to the author of the tonary, and Atkinson also correct in assuming that the commentator was also the compiler; if the commentator felt that the description given by the author of the tonary was already sufficiently clear, he may have felt no need to comment further upon it; alternatively, he may indeed have commented upon it, contributing the last portion of this passage, as Mühlmann supposed.

By contrast, Atkinson could be correct in attributing this passage to the commentator, while the compiler might still be the author of the tonary (as Chailley supposed) or even a separate person altogether; this deduction would not necessarily invalidate Chailley's belief that the tonary came later than the edition and commentary that he identified as the principal treatise. The author of the tonary need not necessarily have described this doctrine in his tonary if, for example, he disagreed with it; or perhaps he simply felt that the information was overly theoretical for what is otherwise a highly practical treatise. More interestingly, if the compiler were a separate person altogether, then the author of the tonary may simply have been unaware of the new doctrine; while there is clearly a chain of succession between source text, revisor and commentator – each later author must necessarily have been familiar with the work of the previous – there is not necessarily any such relationship implied between these authors and the author of the tonary; at the very least, the *ēkhēmata* of the tonary are considerably different from those provided by the other authors. The compiler may then have taken two unrelated treatises – a proto-*Alia* (up to the stage of the commentary) and the tonary – the authors of which may have been entirely unfamiliar with each other – and combined these two treatises. In this case, the author of the tonary could easily have written later than the commentary about the octave species and yet not have commented upon this doctrine in his treatise.

Thus, the authorship of this passage should be hypothesized on stylistic grounds or from its position within the overall layout of the treatise, as well as from manuscript evidence, and not from any assumption about the extent of the development of modal theory. On the grounds of the placement of

this passage within the overall text and its absence from the independently transmitted tonaries, I tend to agree with Atkinson that it was probably added by the commentator (though I suspect, as I shall describe in the next chapter, that it may have been borrowed from yet another source); however, I am not confident in concluding from this attribution that this passage (and by extension, the work of the commentator) must necessarily represent the latest layer of the treatise.

Summary

With the contents of each section now described in some detail and the functional contributions of each of the proposed authors fully described, it is now possible to present a more succinct description of the contents and progression of the treatise. The revision of the older text is preceded by a brief introduction to Greek harmonic theory, drawn from Boethius, and probably provided by the revisor; the revision itself is then presented, with commentary provided at the end of each mode by a commentator who may or may not be the same person as the revisor. Following each section (more or less) of commentary, the corresponding section of the tonary is added as a supplement before moving on to the next mode in the revised edition. A brief digression on the nature of the octave species and the difference between modes I and VIII with respect to the octave species is presented just prior to the commentary on the eighth mode. Following the completion of the eighth mode, a brief afterword is provided, including discussion of the relationship between musical harmonies and other natural phenomena (which may be a part of the source text or may have been drawn from some other older source, but probably also includes some commentary from the commentator) as well as some additional mathematical calculations that could be attributable to the revisor, commentator, or compiler. At the end of the treatise, a chart is added and described in prose that could easily be the work of the revisor, commentator, or compiler, or perhaps some other, unidentified person. Finally, the source text is appended to the end of the treatise. Table 3 summarizes the organization of the composite treatise:

Table 3 – Summary of treatise contents and layout by the presumed authors. Read left-to-right, then top-to-bottom.

Source Text	Revisor	Commentator	Tonary	Unclear Authorship
	Boethian theory			
	<i>Tonus I</i>	<i>Tropus I</i>	<i>Tropus I</i>	
	<i>Tonus II</i>	<i>Tropus II</i>	<i>Tropus II</i>	
	<i>Tonus III</i>	<i>Tropus III</i>		
	<i>Tonus IV</i>	<i>Tropus IV</i>	<i>Tropi III & IV</i>	
	<i>Tonus V</i>	<i>Tropus V</i>	<i>Tropus VI</i>	
	<i>Tonus VI</i>	<i>Tropus VI</i>	<i>Tropus VI</i>	
	<i>Tonus VII</i>	<i>Tropi VII & VIII</i>		Octave Species
		<i>Tropus VIII, cont'd</i>	<i>Tropi VII & VIII</i>	
Allusions to nature	Allusions to nature			
	Additional math	Additional math		
				Table & Summary
Toni I–VIII				

Table 4 summarizes the relationships between the various authors described in the treatise itself and among the modern musicologists that have studied the problem.

Table 4 – Approximate Correspondences Among Theories of Authorship. Roman numerals indicate modes. Brackets indicate functional divisions beginning to the right; underlined titles are Mühlmann's authors.

<i>Alia musica</i>	<i>Praemissus expositor</i>	<i>Expositor</i>			<i>Nova expositio</i>	
Chailley	<i>Premier Quidam</i>	<i>Traité Principale / Alia proprement dite</i>			<i>Deuxième Quidam</i>	
Atkinson [Nace]	Author α [Source Treatise]	Author β [Revision]		Author δ [Commentary]	Author γ [Tonary]	
Mühlmann	<u>[Introduction]</u>	§§1–12		§§13–20	<u>Kommentar</u>	<u>Tonarius</u>
	<u>[Modes]</u>	I. §§31 35–36		§§21–29, 30 31–34		
	<u>Theoretiker der Sechs Töne</u>	II. 45–48	<u>Theoretiker der Principales und Subiugales</u>	37–38	§§39–40	§§41–44
		III. 58–61	<u>Theoretiker der Acht Modi</u>	48–52	53	54–57
		IV. 77–83		61–67	68–76	90–94
		V. 99–100		84–86	87–89	95–98
		VI. 112–13		101–02	103–07	108–11
				114–18	119–21	122–25
	<u>[Disputed Passage]</u>		§§133–42	VII.126–28	VIII.129–32	VII.146–49
[Symbolic Considerations]	§§154–56	157–61	§162	143–45		VIII.150–53
		163–64		165–67	168–70	
			<u>[Summaries]</u>	<u>Tabelauszug</u> §171		
			<u>Überarbeiter</u>	<u>Zahlenauszug</u> §§172–80		
[Source Treatise]	I–VI 181–86					
	VII–VIII §§187–88					

Chapter 04 – The Manuscripts

As described in earlier chapters, the *Alia musica* is a composite music theory treatise that appears to have been compiled from three distinct theory treatises, probably by three separate authors (and probably a separate commentator for the largest treatise); opinions vary as to whether the compiler was one of these authors or someone else entirely. The three basic parts are the *source treatise* (Chailley's first *Quidam*), the *revision* (Chailley's "Principal Treatise"), and the *tonary* (the *Nova expositio*); Atkinson feels that Chailley's Principal Treatise is actually the work of two authors, the second of whom I call the *commentator*. These parts are all interleaved with one another, so that the resulting composite remains organized by topic, skipping back and forth between authors as the various component texts treat the same topics. The source treatise and the tonary each exist independently in the manuscript tradition, as do some of the supplemental parts of the revision. A couple of manuscripts contain parts of multiple treatises (generally the tonary and supplemental parts of the revision), and five contain the entire treatise (one of which has been lost but has been indirectly preserved as the primary source of Gerbert's edition).

In total, thirteen manuscripts survive (including the lost manuscript preserved in Gerbert's edition) that are known to contain part or all of the *Alia musica*. They can be arranged loosely into three broad groups, with three residual manuscripts not falling into these categories. The first group is the Principal Group, containing the five known copies of the complete treatise. The second group may be termed the *Dulce ingenium* Group; *Dulce ingenium* is another composite treatise presumed to date from the tenth or eleventh centuries that shares the *Nova expositio* with the *Alia musica* (and one of the three manuscripts in this group also contains other excerpts from the *Alia musica*). The final group is a pair of manuscripts that contain only the summary material from the *Alia musica*, and will therefore be referred to as the

Summary Group. Table 5 summarizes the contents of the manuscripts,¹⁴⁶ and Table 6 shows what is known of their origins. Figure 4 shows the geographical distribution of the manuscripts, which spreads across most of Western Europe.

Table 5 – Distribution of treatise parts throughout the manuscripts.

	Source Treatise	Revision & Commentary	Tonary
Complete		<u>Principal Group</u> Strasbourg 926 (A) Cesena XXVI (C) Munich 14272 (M) Paris 7212 (P ₁) Paris 7211 (P ₁)	
	Karlsruhe 504 (K) {§§ 181-87, 180}		<u>Alternate Recension</u> Barcelona 42 (R)
			<u>Dulce ingenium Group</u> Bruges 532 (Br) Paris 8663 (Pa)
			Prague 26 (Pr)
Fragments		{§§ 134-41} {§§ 166-68, 162a, 170}	
		Barcelona 42 (R) {§§1-2a}	{§§122-25, 150-53}
		<u>Summary Group</u> Florence 565 (F ₁) Florence 652 (F ₂) {§§ 173-80}	

¹⁴⁶ A more detailed comparison of the contents of each manuscript appears in the appendix.

Table 6 – Bibliographic details of manuscripts.
Reorganized from Atkinson, *Critical Nexus*, Table 5.2, and supplemented

Manuscript	Folia	Siglum	Provenance
Barcelona 42	ff.65v–69v	R	ca. 1018–46
Bruges 532	ff.3–4	Br	13 th C.
Cesena XXVI	ff. 179v–94	C	1425–50, based on a model from Lorraine (11 th C.?)
Florence 565	ff. 76v–77	F ₁	Late 11 th C., Florence, Santa Maria Novella
Florence 652	ff. 83v–85	F ₂	13 th C., unknown provenance
Karlsruhe 504	f. 34r–v	K	11 th – 12 th C., Bamberg or St. Gall
Madrid 9088	ff. 124v–25r	D	Early 11 th C., Northern Italy
Munich 14272	ff. 175–81	M	11 th C., St. Emmeram
Paris 7211	ff. 54–71	P ₂	Late 11 th C., St. Pierre de Luxeuil
Paris 7212	ff. 39v–50	P ₁	Early 12 th C., Bourgogne
Paris 8663	51	Pa	Early 11 th C., Fleury-sur-Loire
Prague 26	ff. 17v–18v	Pr	ca. 1100, region of Liège
Strasbourg 926		A	(15 th C.?) Gerbert's primary source, lost

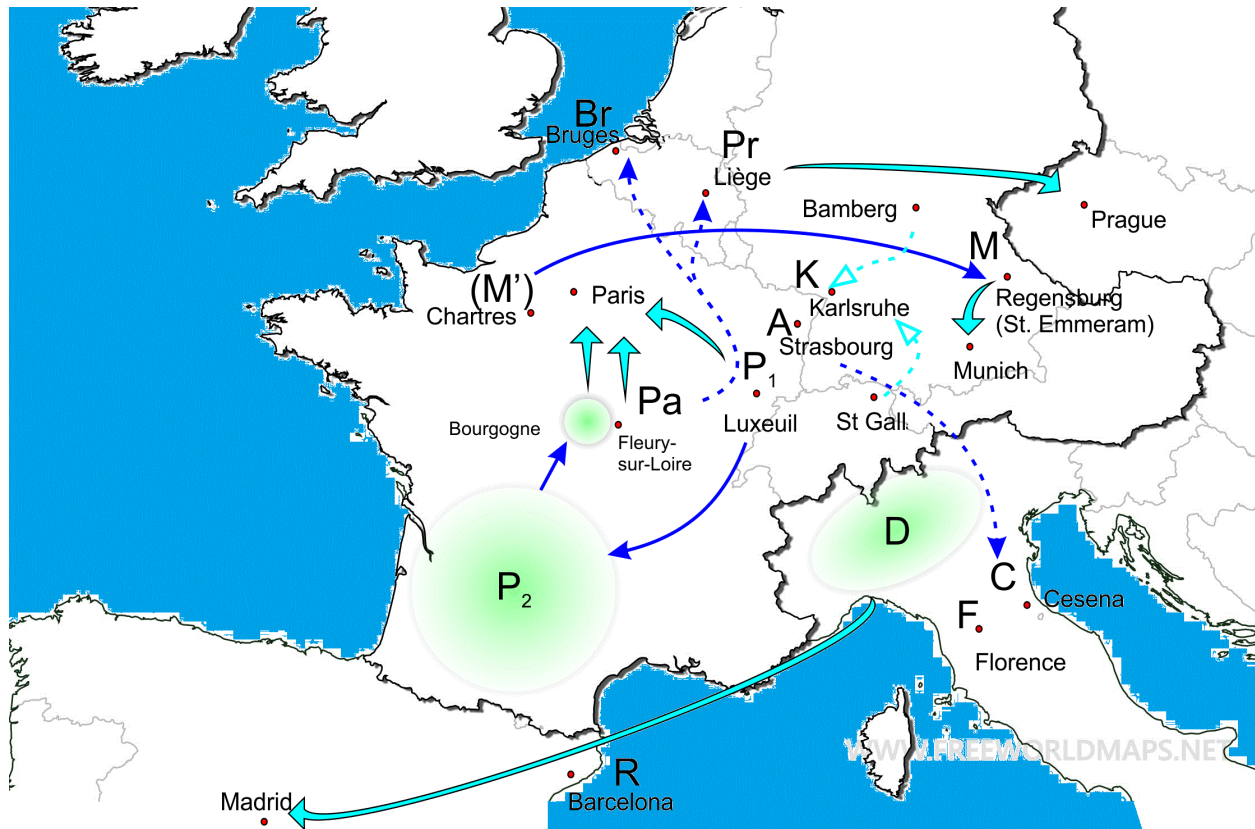


Figure 4 – Geographical distribution of sources. Where the point of origin is known or reasonably suspected, manuscripts are positioned at their presumed point of origin; otherwise, they are positioned at their repositories. Known relationships between sources are given in simple blue arrows and suspected relationships in broken blue arrows. Open cyan arrows show movement of manuscripts to later repositories.

The Principal Group:

The manuscripts of the Principal Group are the best studied. These are, by and large, the sources known to Gerbert, Mühlmann, Heard and Chailley, and all four authors provide at least partial descriptions of the other contents of the manuscripts (the discussions by Chailley and Heard are the most thorough).¹⁴⁷ Mühlmann provides comparisons of the textual variants for a few of these manuscripts, and Chailley for all except Cesena.¹⁴⁸ Chailley selected *sigla* for these manuscripts, and I have retained these *sigla* here.

Sources A and G (Strasbourg 926 and Gerbert)

Gerbert's principal source for his 1784 edition was a manuscript formerly held in Strasbourg that was lost during the Franco-Prussian War, when the Strasbourg Municipal Library burned down on 24 August 1870 during the Siege of Strasbourg. The Roman name for Strasbourg was *Argentoratum*, whence comes the traditional name of the manuscript, *Codex Argentoratensis*, the name by which Gerbert referred to it, and the source of Chailley's siglum, A. However, since the manuscript itself has been lost, and since Gerbert's edition includes comparisons from at least one other manuscript (Munich), Chailley instead cites Gerbert's edition in its own right, under the siglum G, which may nevertheless be taken as loosely equivalent to A except where noted in Gerbert's footnotes. Heard notes that the manuscript was "said to be from the early tenth century,"¹⁴⁹ but does not provide a citation, and I cannot find a source supporting this date. Terence Bailey, by contrast, hypothesizes that the manuscript could not be older than the fifteenth century because it was made of paper,¹⁵⁰ which did not become common in Western Europe until that time; Atkinson, too, puts it in the fifteenth century (see Table 6, above), following Bailey's

¹⁴⁷ Gerbert, *Scriptores ecclesiastici I*, 103; Mühlmann, *Die Alia musica*, 4; Heard, *Alia musica*, 34–36; Chailley, *Alia musica*, 63–65.

¹⁴⁸ Mühlmann, *Die Alia musica*, 4–10; Chailley, *Alia musica*, 85–204.

¹⁴⁹ Heard, *Alia musica*, 34.

¹⁵⁰ Bailey, "De modis musicis," 47; the manuscript is known to have been made of paper from Gerbert's description, "ex Mjc. papyraceo bibliothecæ civicæ Argentoratensis." (Gerbert, *Scriptores ecclesiastici I*, 103) ("from a papyrus [*i.e.*, paper] manuscript from the civic library of Strasbourg.")

hypothesis.¹⁵¹ Even if Bailey is correct, A must certainly have been copied from a much earlier source; since it is believed to share a common model with source C (see below) – which, in turn, is believed to have been copied from an eleventh-century model – A would therefore presumably also have an eleventh-century model; this is still considerably later than Heard’s highly unlikely presumption of early tenth century, a date that may very well be earlier than the completion of the treatise.

According to Gerbert, in this codex, the *Alia* is preceded by Hucbald’s *De harmonica institutione* and followed by a treatise that begins “*musica [dicitur] ἀπὸ τοῦ μῶσθαί*”¹⁵² (“Music is named from ‘to seek’ ”); this latter treatise is elsewhere entitled *Argumentatio cuiusdam de musica*, which Gerbert notes also occurs in Aurelianus’ treatise (Gerbert apparently does not recognize it as an extract from Isidore de Seville), and it is followed in A by *De mensuris organicarum fistularum*.

However, Gerbert appears to include two other brief treatises, *Cita et vera divisio monochordi* and *Dimensio monochordi*, as part of Hucbald’s *De harmonica institutione*. This inference is based on his introductory notes¹⁵³ and the succession of treatises over the following pages,¹⁵⁴ which, with the exception of the said two brief treatises (which Gerbert does not describe in his introduction) and the Greek treatise (which Gerbert does not include in his edition) are presented in the order that Gerbert indicates that they appear in the manuscript. In fact, Gerbert’s treatment causes related problems with the *Alia musica*, as several later treatments of the treatise include within the *Alia musica* several of the

¹⁵¹ Atkinson does not explicitly indicate in his book that he is following Bailey’s assessment, but he confirmed this in his written feedback following the defence of this dissertation.

¹⁵² Gerbert, *Scriptores ecclesiastici I*, 103; the word *dicitur* is not included in Gerbert, but is found in other manuscripts, including Cesena (source C; see below), and I have standardized the Greek orthography. The Greek is complicated and prone to misunderstanding. It is given differently in virtually every source, but the treatise is apparently drawn from Aurelianus (cf. Aurelianus, *Musica disciplina*, ch. 2, ed. Gushee, 61, trans. Ponte, 7), where it was borrowed from Isidore’s *Etymologiarium III.xv*, which, in turn, was borrowed from Plato’s *Cratylus*, 406 (cf. Jowett, *The Dialogues of Plato*, 270). The verb in question is μῶσθαί, the infinitive of μῶμαι, itself a derivative μάω (II), “to seek after” (Liddell & Scott, *A Greek-English Lexicon* 8th Ed., 989; 926) – a Greek etymology for music rather similar to the Latinate etymology for the medieval *troubadour*.

¹⁵³ Gerbert, *Scriptores ecclesiastici I*, 103.

¹⁵⁴ Gerbert, *Scriptores ecclesiastici I*, 122–25 and 147–52.

shorter treatises that follow it in Gerbert but not in most of the other manuscripts. These shorter treatises include the *Argumentio cuiusdam* also used in Aurelianus, a *De modis musicis*,¹⁵⁵ and a passage beginning *Beatus Augustinus perhibet*.¹⁵⁶ Excluding the *Argumentio cuiusdam*, which Gerbert mentions but does not include in his edition, these remaining short treatises are cited as part of the *Alia musica* in the *Grove Music Online* article on Anonymous Theoretical Writings¹⁵⁷ and are all included with the *Alia* in two editions on the *Thesaurus Musicarum Latinarum*, transcribed from Gerbert¹⁵⁸ and from the *Patrologia cursus completus*,¹⁵⁹ the *Patrologia* itself transcribes into its own Hucbald section the entire Hucbald section from Gerbert, from his intro through the end of the *Commemoratio brevis* (the only part of Gerbert's Hucbald section not included in manuscript A).¹⁶⁰

In addition to including *Cita et vera* and *Divisio monochordi* into Hucbald's treatise, Gerbert does not seem to recognize that the last portion of the text preceding the *Alia musica* in his edition is not part of the *Dimensio monochordi*, but is an incomplete copy of yet another treatise, *Ecce modorum sive tonorum*.¹⁶¹ Thus, it is most likely that the *Alia* was actually preceded by *Ecce modorum sive tonorum* in A, as in nearly all the other manuscripts in the Principal group (Chailley and Heard are apparently not aware of the existence of this treatise in A or P₁, since it immediately follows another monochord treatise; consequently, the fact that this treatise precedes the *Alia* in most manuscripts in the Principal Group is obscured in their descriptions).

¹⁵⁵ Edited by Terence Bailey in *Kirchenmusikalisches Jahrbuch* Vol. 61–62, 47–60.

¹⁵⁶ Edited by Bernhard in *Clavis Gerberti*, 33–34.

¹⁵⁷ Balensuela, "Anonymous theoretical writings," *Grove Music Online*.

¹⁵⁸ <http://www.chmtl.indiana.edu/tml/9th-11th/ALIMUS>

¹⁵⁹ <http://www.chmtl.indiana.edu/tml/9th-11th/ALIMU>

¹⁶⁰ Gerbert, *Scriptores ecclesiastici I*, 103–229 = Migne, *Patrologia cursus completes* 132, 905–1042.

¹⁶¹ This treatise is edited in Schmid, *Alia et scolica*, 182–84. Gerbert's footnotes indicate that he is aware that something is missing in his text (Gerbert, *Scriptores ecclesiastici I*, 124b & fn.(a)) but he does not indicate an awareness that a new treatise has begun. The text as it is found in Gerbert begins at line 6 in Schmid. Chailley describes this treatise as a fragment of *Dimensio monochordi* (Chailley, *Alia musica*, 65), which it follows in Gerbert, but this description is unlikely, in view of the large number of texts in which the treatise appears, as indicated in Schmid's critical apparatus.

A manuscript from Cesena is generally thought to have been copied from the same source as A (more about this under Source C, below), and may thus serve as a partial substitute for determining the readings in A where Gerbert's edition is doubtful or unclear. In addition, there may also be a copy directly descended from A, though I have not been able to access this source. The manuscript Sankt Paul im Lavanttal, Archiv des Benediktinerstiftes, ms. 224₂ (olim XXI.c.229) is a manuscript prepared for Gerbert containing excerpts of other manuscripts that he used in the preparation of the *Scriptores ecclesiastici*,¹⁶² Lawrence Gushee, in his edition of *Aurelianus*, gives it the siglum StP. There is, in fact, a large block included in this manuscript that was copied from the Strasbourg manuscript, spanning fols. 302–35, which is described, in a combination of Latin and German, in Oswald Koller's catalogue as:

*Incipit liber Ubaldi peritissimi musici de Armonica institutione. E cod. Argentor. coll. cum Ms. bibl. Cesaenae apud minores Conventuales. Abgedruckt bei Gerbert I, 104 – 125. Über die beiden Hds. vgl. I, 103.*¹⁶³

Koller signifies that this material includes all the material that Gerbert printed between pp. 104–25, which means everything preceding the *Alia musica*, which begins on p. 125b. However, Gushee indicates that *Argumentum cuiusdam de musica* appears on fols. 328v–29v,¹⁶⁴ and this treatise is not included in the indicated pages of Gerbert's edition; rather, it appears immediately after the *Alia musica*, where it is described to appear in the Strasbourg manuscript (as noted above). Yves Chartier states that the Hucbald *De harmonica institutione* ends at fol. 313.¹⁶⁵ By comparison to folia counts for other manuscripts of the *Alia*, there could very well be sufficient space between these treatises for the *Alia musica*. By contrast, however, another copy of the *Alia musica* (transcribed from a manuscript that is still extant – see Source M, below) also appears in StP immediately before the block from S and occupies more space

¹⁶² Koller, "Aus dem Archive des Benediktinerstiftes," 23; cf. Gushee, *Aureliani Reomensis Musica disciplina*, 49.

¹⁶³ Koller, "Aus dem Archive des Benediktinerstiftes," 24.

¹⁶⁴ Gushee, *Aureliani Reomensis Musica disciplina*, 49.

¹⁶⁵ Chartier, *Œuvres musicales d'Hucbald*, 115.

(perhaps because of all the marginalia present in M). Without access to the manuscript, it has not been possible for me to verify whether or not StP contains a copy of the *Alia musica* from A.

Before moving on to the other sources, it is worth noting here that there is evidence that there may be a significant error in Gerbert's description of the manuscript contents (which has been perpetuated in the various manuscript studies in which A is implicated, including Chailley's). Gerbert says:

Primo loco ponimus eius librum de harmonica institutione ex Msc. papyraceo bibliothecæ civicæ Argenteratensis collato cum Msc. bibliothecæ Cesenatensis ord. Minor. Conventualium. Sequitur continenter in cod. Argentor. de harmonica consideratione BOETIUS &c. ut in Msc. San-Emeramensi sub tit. Alia Musica, usque: Tonum octavum require ut supra. Sequitur porro in codice Argenteratensi Musica ἀπὸ τοῦ μῶσαι &c. quæ sparsim ex Aureliano sunt excerpta. Tum De mensuris organicarum fistularum, de Cymbalorum ponderibus, de modis musicis, ac quinque symphoniis seu consonantiis. Ad quorum finem legitur Explicit musica Ubaldi. Eiusdem musicam enchiriadialem edimus ad fidem Mss. Einsidlensis, Tegernseensis, Salemitani, San-Blasiani, Casinensis, Argentonensis cum Scholiis in tres partes divisas, quæ tamen desiderantur in codice Casinensi, Salemitano, San-Blasiano & Argenteratensi.¹⁶⁶

In the first position, we place his [Hucbald's] *De harmonica institutione* from a paper manuscript from the civic library of Strasbourg, collated with a manuscript from the Cesena library [etc.]. There follows continuously in the Strasbourg codex, "*De harmonica consideratione Boethius*," etc., as in the St. Emmeram manuscript under the title *Alia musica*, as far as, "*Tonum octavum require ut supra*." There follows afterwards in the Strasbourg codex "*Musica ἀπὸ τοῦ μῶσαι*," etc., which is sparsely excerpted from Aurelianus. Then *De mensuris organicarum fistularum*, *De cymbalorum ponderibus*, *de modis musicis* and *quinque symphoniis seu consonantiis*, at the end of which reads *Explicit musica Ubaldi* [End of Hucbald's *Musica*]. We edit the *Musica enchiriadis* of the same person [sic], as given in the manuscripts of Einsiedeln, Tegernsee, Salemi, Saint Blasien, Cesena, and Strasbourg, with the *Scolica* divided into three parts, which, however, are wanting in the manuscripts from Cesena, Salemi, Saint Blasien, and Strasbourg.

From this description, it is more or less possible to reconstruct the contents and arrangement of the lost manuscript. Gerbert lists the contents in order from Hucbald's *De harmonica institutione* (it is important to note, though, that he does not say that this is the first treatise in the manuscript, only that it is where he began copying). At the end of the sequence of treatises attributed to Hucbald in the manuscript, he then says that he additionally copies the *Musica enchiriadis* (which he likewise attributes to Hucbald) that also appears in the same manuscript. However, since Gerbert has already reached the end of the Hucbald section of the manuscript (*Explicit musica Ubaldi*), this implies that the *Musica*

¹⁶⁶ Gerbert, *Scriptores ecclesiastici*, 103.

enchiriadis does, in fact, appear in the manuscript *before* the Hucbald *De harmonica institutione* (as will be seen below to be the usual pattern). Finally, Gerbert notes that the *Scolica enchiriadis* is missing in this manuscript.

However, it is also routinely observed that the contents of the Strasbourg manuscript, as described by Gerbert, are nearly identical to those of the Cesena manuscript, with which it is assumed to share a common model.¹⁶⁷ In fact, with the *Musica enchiriadis* listed in its probably correct place *before* Hucbald's *De harmonica institutione*, the only difference between Gerbert's description and the contents of the Cesena manuscript is that Gerbert indicates that the *Scolica* is absent in A. However, Gerbert also indicates that the *Scolica* is absent in Cesena, which it is not; it begins on fol. 143v, with a heading style indistinguishable from the chapter headings of the *Musica enchiriadis*, and with only a simple drop-cap initial instead of the elaborately illuminated versals with which the *Musica enchiriadis* that precedes it and the Hucbald *De harmonica institutione* that follows it begin (see Facsimile 1).

It is likely that Gerbert (or perhaps an assistant), flipping through the manuscript, missed the rubric announcing the beginning of the *Scolica*, assuming it to be just another chapter heading. The *Musica* and *Scolica* are often treated as a unit; the listings of contents of manuscripts in modern editions and manuscript studies are often quite inconsistent about listing the *Musica* and *Scolica* as separate items or as a single item,¹⁶⁸ and sometimes even the manuscripts themselves cover both treatises with a single rubric.¹⁶⁹ At a glance, it is often easy to miss where one ends and the next begins. Since C corresponds

¹⁶⁷ Chartier, *L'Œuvres musicales d'Hucbald*, 114 : "L'ordre et le contenu de cette copie correspond presque exactement à ceux du manuscrit de Césène." ("The order and the content of this copy corresponds almost exactly to those of the Cesena manuscript.") cf. Atkinson, *Critical Nexus*, 171, fn. 2. Similar comments, but somewhat less *à propos* (owing to a different subject matter), are made in Gushee, *Aureliani Reomensis Musica disciplina*, 41, and Huglo, *Les tonaires*, 59 (inc. fn. 2).

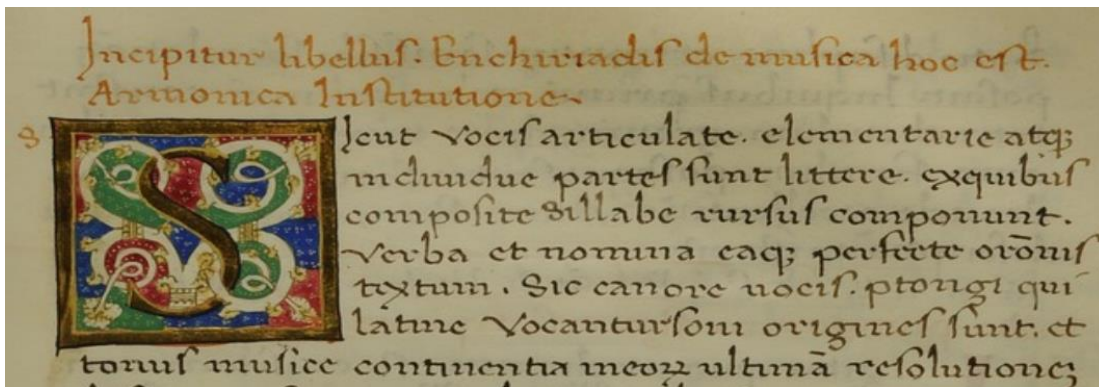
¹⁶⁸ Compare Heard's descriptions of Munich 14272 and Paris 7211, which list them independently, to Paris 7212, which lists them together (Heard, *Alia musica*, 35). The same is (probably not coincidentally) true in Chailley, *Alia musica*, 65.

¹⁶⁹ For instance, Munich 14649 announces both at once with the rubric *Incipiunt quaedam utiliora de arte musica scolica enchiriadis* [emphasis added] (Bailey, *Commemoratio brevis*, 2).

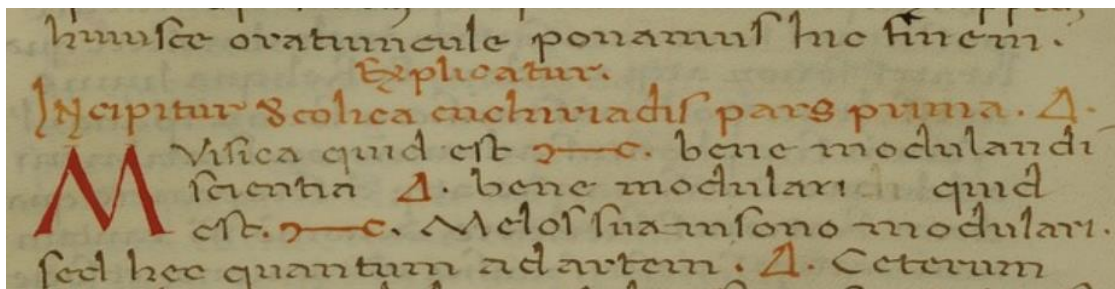
so closely to A in every other respect (see Table 7), it is likely that they correspond in this respect also, and that Gerbert (or an assistant) missed the *Scolica* in A for the same reason as in C. With the manuscript destroyed, it will probably never be possible to confirm this hypothesis, and so Gerbert can only be taken at his word, but his word should perhaps be taken with a grain of salt.

Facsimile 1 – Comparison of beginnings of Musica and Scolica Enchiriades and Hucbald's Musica in Cesena. The transition to the Scolica is much less obvious, and could be easily missed.

(a) Beginning of the *Musica enchiriadis*, with an illuminated versal. f. 133r.



(b) Transition from the *Musica* to the *Scolica*, with a simple drop-cap initial. f. 143v.



(a) Beginning of Hucbald's *Musica*, with an illuminated versal. f. 167v.

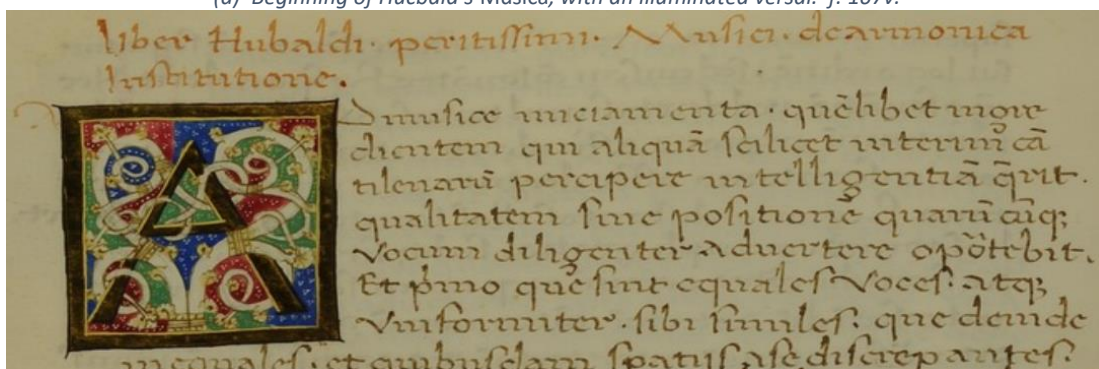


Table 7 - Comparison of the contents of the lost Strasbourg manuscript to the Cesena manuscript. Excluding the additional material at the beginning and end of the manuscript, and aside from the placement of the *Musica enchiriadis* (which is not directly specified by Gerbert) and the presence or absence of the *Scolica enchiriadis* (about which Gerbert is in error regarding Cesena and therefore probably also regarding Strasbourg), the only difference is the presence of a gloss on Boethius in Cesena that Gerbert does not mention that occupies on complete folio, front and back, and may simply have been misplaced.

GSI	Strasbourg 926	Cesena XXVI	
		<i>De institutione arithmetica</i> (Boethius)	01r
		Non-musical material	57v
		<i>De institutione musica</i> (Boethius)	61v
		<i>Musica enchiriadis</i>	133r
		<i>Scolica enchiriadis</i> , <i>definitely present, though Gerbert says no</i>	143v
	[<i>Musica enchiriadis</i>]		
	[<i>Scolica enchiriadis</i> , <i>probably, though Gerbert says no</i>]		
	<u>Beginning of Gerbert's description</u>		
	104a <i>De institutione musica</i> (Hucbald's <i>De musica</i>)	<i>De institutione musica</i> (Hucbald's <i>De musica</i>)	167v
	121a <i>In primo diapason</i>	<i>In primo diapason</i>	177r
	122a <i>Cita et vera divisio</i>	<i>Cita et vera divisio</i>	177v
	122b <i>Dimensio monochordi</i>	<i>Dimensio monochordi</i>	178r
	124b <i>Ecce modorum sive tonorum</i> (title & beginning absent, uninterrupted from previous)	<i>Ecce modorum sive tonorum</i> (title & beginning absent, uninterrupted from previous)	179r
	125b <i>Alia musica</i> (<i>De armonica consideratione</i>)	<i>Alia musica</i> (<i>De armonica consideratione</i>)	179v
	† <i>Argumentatio cuiusdam</i> (<i>Musica ἀπὸ τοῦ μῶσθαι</i>)	<i>Argumentatio cuiusdam</i> (<i>Musica ἀπὸ τοῦ μῶσθαι</i>)	194r
	147b <i>De mensuris organicarum</i>	<i>De mensuris organicarum</i>	195v
	149a <i>De cymbalorum ponderibus</i>	<i>De cymbalorum ponderibus</i>	196v
	149a <i>De modis musicis</i>	<i>De modis musicis</i>	196v
	149b <i>De quinque symphoniis</i>	<i>De quinque symphoniis</i>	197r
	150a <i>De consonantiis tribus</i>	<i>De consonantiis tribus</i>	197v
	151a <i>Beatus Augustinus perhibet</i> <i>included by Gerbert as part of previous</i>	Gloss on Boethius <i>not mentioned or included by Gerbert</i>	198r
	151a <i>Beatus Augustinus perhibet</i> <i>included by Gerbert as part of previous</i>	<i>Beatus Augustinus perhibet</i>	199r
	<u>Gerbert: "also included"</u>		
	152a <i>Musica enchiriadis</i> <i>in Gerbert's description, listed last but probably actually first in manuscript.</i>	End of musical portion (rest of manuscript is treatises on geometry)	200r
	173a <i>Scolica enchiriadis</i> <i>according to Gerbert, not included [!]</i>		

† Gerbert acknowledges this treatise but omits it from his Hucbald section because he recognizes it as an excerpt from Aurelianus. In a footnote at the end of the *Alia musica* where this treatise should begin (Gerbert, *Scriptores ecclesiastici I*, 147 fn. A), he indicates that the text appears earlier in the volume, in the Aurelianus section. However, he does not provide the independent text; he clearly intends for the form in Aurelianus to be sufficient. The text, however, is not contiguous in Aurelianus: in his edition of Aurelianus, Gushee indicates that it is distributed among the chapters 20, 5, and 10–18 (Gushee, *Aureliani Reomensis Musica disciplina*, 44). Thus, providing page numbers is not practical.

Source M (Munich, Bayerische Staatsbibliothek, cIm 14272)

This manuscript comes originally from St. Emmeram of Regensburg¹⁷⁰ and is on that account identified by Gerbert as *Codex San-Emeramensi*. This source serves as a second witness against which Gerbert compared A in the production of his edition. Most of Gerbert's footnotes present alternate readings drawn from M, and Gerbert also indicates that he drew the tabular summary near the end of the treatise from this source (it is clear from Gerbert's footnotes that the table was present in A; perhaps Gerbert drew the table from M because he felt it to be a superior reading). It is from this manuscript that the title *Alia musica* has been adopted; the title is added here in the upper margin, in what appears to be a different hand,¹⁷¹ while the treatise is untitled in the other sources. It is also this source that (incorrectly) identifies Hucbald as the author of the treatise, with the marginal note "*Auctor Hucbaldus Elnonensis*" ("Author Hucbald of Saint Amand"),¹⁷² also in a later hand (as evidenced not only by the visual difference, but also by the use of the later habit of distinguishing between medial (long) s and terminal (round) s, a convention not observed elsewhere in the manuscript, and not in common use in medieval Latin scripts until the development of Gothic scripts in the twelfth century.¹⁷³ There are also a few differences in the letter forms themselves that mark the glosses as belonging to different hands without necessarily revealing their provenance, such as the descender on the long S in the title, while the long S in the text usually sits on the baseline.

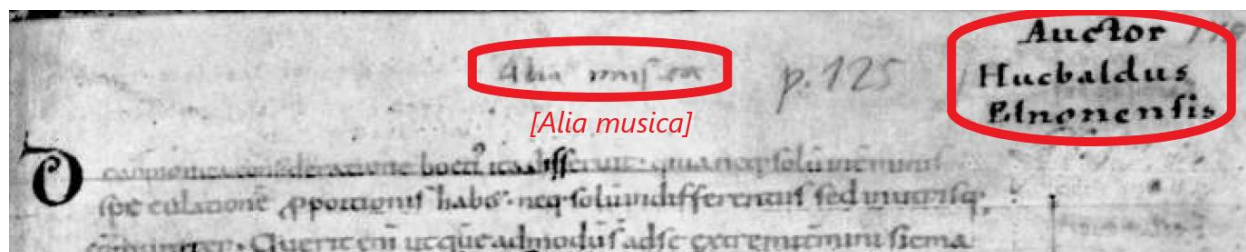
¹⁷⁰ Chailley, *Alia musica*, 64.

¹⁷¹ I make this assertion with caution because of the low resolution of my facsimile of this manuscript.

¹⁷² The term *Elnonensis* ("from Elnon") refers to a brook by that name that marks the frontier between France and Belgium near which Hucbald's Saint-Amand Abbey was located (and after which it was once named).

¹⁷³ Drogan, *Medieval Calligraphy*, 64; cf. Bischoff, *Latin Palaeography*, 130. The majuscules also do not resemble earlier scripts.

Facsimile 2 – Incipit of the Munich manuscript (fol. 175r), showing the title *Alia musica* and the attribution to Hucbald "Elnonensis" [i.e., of Saint-Amand]. This is the only manuscript containing either of these details.



Hans Müller, the scholar who, in the late nineteenth century, most comprehensively challenged the attribution of most Carolingian theory treatises to Hucbald, describes the treatise and these glosses in a single concise statement, thus:

Auf einer neuen Blätterlage beginnt fol. 178 De armonica consideratione, darüber hat Aventin geschrieben „Alia musica“, was zum Titel geworden ist, dann Sanftl, der um 1800 einen vorzüglichen Katalog der Handschriften von St. Emmeram verfasst hat: Auctor Hucbaldus Elnonensis. fol. 181 v in der Mitte „tonum octavum require supra“ (nicht ut supra).¹⁷⁴

On a new leaf (fol. 178r) begins *De armonica consideratione*, above which Aventin wrote "*Alia musica*," which has become the title; then Sanftl, who, around 1800, wrote an excellent catalogue of the manuscripts of St. Emmeram, [added] "*Auctor Hucbaldus Elnonensis*"; [the treatise ends] in the middle of fol. 181v, "*Tonum octavum require supra*" (not "*ut supra*").¹⁷⁵ [Emphasis added]

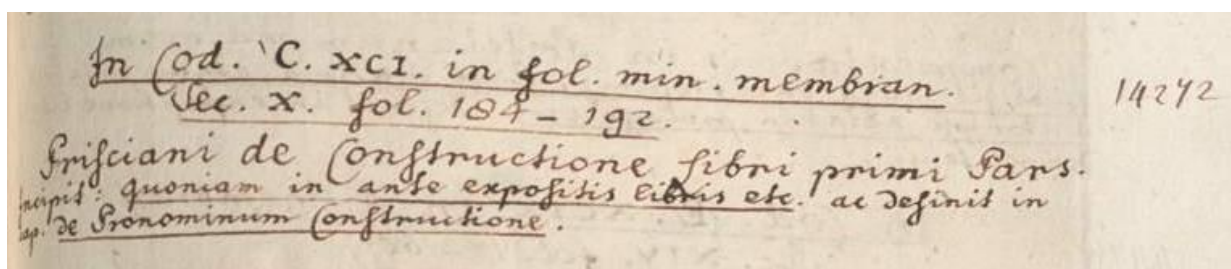
Although he does not give full names or discuss them anywhere else, the individuals to whom Müller attributes these glosses are most likely Johannes Aventinus and Kolomon Sanftl, both of whom are known to have worked with the manuscript. Aventinus was a sixteenth-century historian; the Bayerische Staatsbibliothek catalogue description of the manuscript (by Friedrich Helmer) indicates that the

¹⁷⁴ Müller, *Hucbalds echte und unechte Schriften*, 21.

¹⁷⁵ Müller's afterthought was clearly intended as a correction to Gerbert, as Gerbert's text does indeed end "*require ut supra*" (Gerbert, *Scriptores ecclesiastici I*, 147a), while M ends "*require supra*" (Munich, clm 14272, f. 181v); so, for that matter, do the two Paris manuscripts (Paris 7212, f. 50v; Paris 7211, 71r). From the perspective of most scholars who have studied the *Alia musica* until recently, then, Gerbert's edition was the only one to read "*ut supra*," and Gerbert would thus seem to be in error. However, it is likely that A did, indeed, say "*ut supra*," as that is the form found also in Cesena (Cesena XXVI.1, fol. 194r), which (as discussed above) was probably copied from the same source as A. Gerbert is thus likely vindicated, though it is disappointing that he fails to acknowledge the differences between his two sources at this point with a footnote. (A footnote does actually appear at this point, but it only announces the omission in his edition of the immediately following treatise in the manuscript because it has already been reproduced elsewhere – see the note to Table 7 above).

manuscript contains “häufiger Randvermerke in Griechisch [...] von Johannes Aventinus”¹⁷⁶ (“frequent marginal notes in Greek [...] by Johannes Aventinus”). However, later in the catalog description, in the section describing the *Alia musica*, Helmer acknowledges the late date of the title without repeating Müller’s hypothesis that the title was added by Aventinus; Helmer speculates only that the title may date from the fifteenth century, a date not necessarily incompatible the Aventinus, who was active in the very early sixteenth century.

Facsimile 3 - Sanftl's catalogue entry for Munich 14272 (Sanftl, *Catalogus veterum*, 1679). The description makes no reference to the musical treatises present in this manuscript; it is of Priscian's *Institutiones grammaticae*, book XVII, *De constructione uel syntaxi*, which begins at fol. 184r, a few folia after the end of the *Alia musica*.



Regarding Müller’s other attribution, Koloman Sanftl was the librarian at St. Emmeram who was a somewhat later contemporary of Gerbert. The catalogue to which Müller refers is the *Catalogus veterum codicum mancriptorum ad S. Emmeram Ratisbonae* (1809), a handwritten catalogue of the manuscripts kept at St. Emmeram. The entry for Munich 14272 is given as Facsimile 3 (the entry, interestingly, ignores the music treatises altogether). However, as with Aventinus, Sanftl is referenced elsewhere in Helmer’s catalog description, but not in the description of the *Alia musica*.

Both of Müller’s attributions are plausible and have interesting implications. If Aventinus did, indeed, provide the title, the fact that he provided “frequent marginal notes in Greek” might provide some small support for Heard’s hypothesis that the title is Greek (though there is no reason why he might not also have given it a Latin title, despite his habit of glossing in Greek). In the case of Sanftl, his career being

¹⁷⁶ Helmer, *Katalog der Handschriften*, clm. 14272, p. 1; available online: <http://www.manuscripta-mediaevalia.de/hs/projekt-BSB-Emmeram-pdfs/Clm%2014272.pdf>.

contemporary to and continuing well after the publication of Gerbert's edition, if Sanftl did, indeed, add the attribution to Hucbald, this attribution may well not have been present when Gerbert consulted the manuscript, and Sanftl may thus have added the attribution on the strength of Gerbert's opinion.

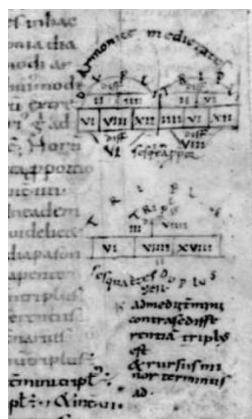
Unfortunately, there is no way to verify Müller's attributions. Both Aventinus and Sanftl were long gone by the time Müller completed his study (Aventinus especially so, and even Sanftl by nearly a century); Müller could not have had direct knowledge in this matter, and he does not bother to provide sources or explain the reasons for his opinion. Perhaps his opinion comes from comparing handwriting, but from the facsimiles presented above, Sanftl's handwriting in the catalogue hardly resembles the attribution to Hucbald in M (though this might not be expected in any case, as the level of calligraphy appropriate to each document is not the same). In all likelihood, his opinion is either speculation or based upon oral history at St. Emmeram. It is worth observing that while Müller's study is routinely cited in studies of all the treatises formerly ascribed to Hucbald (including Chailley's),¹⁷⁷ no author since Müller has repeated Müller's opinion. As with Helmer, Chailley attributes the title to a fifteenth-century hand but not directly to Aventinus, and unlike Müller, Chailley speculates that the attribution to Hucbald, which he estimates to be eighteenth-century, may have been added by Gerbert himself,¹⁷⁸ rather than Sanftl.

¹⁷⁷ Chailley, *Alia musica*, 3 fn. 2.

¹⁷⁸ *ibid.* Chailley's discussion of the title contains an interesting error. He states that according to Gerbert, the title appears in the lost Strasbourg manuscript, and not exclusively in the Munich manuscript. If so, then the title could have been added to the Munich manuscript in the fifteenth century by someone familiar with the Strasbourg manuscript, itself presumed to have been copied in the fifteenth century. However, Chailley's statement probably actually stems from a misreading of a slightly ambiguous statement in Gerbert's introduction. Gerbert says that "*Sequitur continenter in cod. Argentor. de harmonica consideratione BOETIUS &c. ut in Msc. San-Emeramensi sub tit. Alia Musica*" (Gerbert, *Scriptores ecclesiastici I*, 103) ("There follows continuously in the Strasbourg codex, '*De harmonica consideratione Boethius,*' etc., as in the St. Emmeram manuscript under the title *Alia musica*"). The last clause can be read in two ways: that the treatise appears under the title *Alia musica* in A, just as it does in M (Chailley's interpretation); or that the treatise appears in M as well, where it is given the title *Alia musica* (my interpretation). Chailley's interpretation, formerly at least plausible (if less than convincing) is now much less so, since the title does not appear in the Cesena manuscript (also from the fifteenth century) that appears to have been copied from the same source (or another source in the same lineage) as A.

Whatever the origin of the title and the attribution, they represent only a small part of what makes this manuscript of the *Alia musica* unique. For instance, exclusively in this manuscript, the *Alia musica* is preceded not by *Ecce modorum sive tonorum*, but a different monochord treatise, *De dimensione monochordi*¹⁷⁹ (not to be confused with *Dimensio monorchordi*), which begins “*Super unum concavum lignum.*” This latter treatise appears in many of the manuscripts immediately before *Ecce modorum sive tonorum*, but in M, it directly precedes the *Alia musica* without *De dimensione monochordi*. The treatise following the *Alia musica* in M does not pertain to music.

This manuscript is also unique amongst the surviving sources in at least two other respects. First, it is the only manuscript to provide neumes (chiefly, but not exclusively, for the *ēkhēmata* and doxologies of each mode in the *Nova expositio*), though these cannot be assumed to be original; and second, the margins are full of notes, chiefly consisting of an entire additional tonary unrelated to the tonaries already present in the *Alia musica*.¹⁸⁰ The margins also contain a handful of glosses and a couple of diagrams not found in other manuscripts that help to explain the concepts being described.

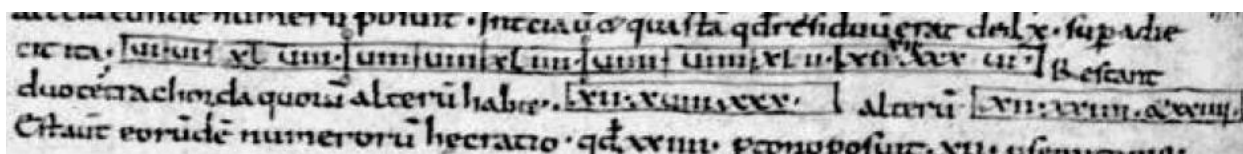


Facsimile 4 – Diagrams and glosses in the margins of M (f. 175r)

Another minor variant unique to M is the enclosure of lists of numbers in boxes Facsimile 5).

¹⁷⁹ This treatise is edited in Schmid, *Musica et scolica*, 179–81. The title *De dimensione monochordi* is the title in Schmid; the treatise is untitled in the sources containing the *Alia musica*.

¹⁸⁰ Edited in Chailley, *Alia musica*, 205–209.



Facsimile 5 - Lists of numbers enclosed in boxes in the Munich manuscript (f. 177v). These boxes are unique to this source.

It may be observed that StP, the manuscript containing Gerbert's preparatory copies, and which I have discussed above with respect to the possibility that it may contain a copy of the *Alia musica* from A (since it contains so much else from the same manuscript), definitely contains a copy of the *Alia musica* from M. However, as the original is still extant and StP was copied some seven centuries after the other copies of the *Alia* (and then almost immediately incorporated into an annotated edition), consulting this additional copy of the text would be of quite limited value. It is, however, circumstantial evidence against Chailley's hypothesis that the attribution to Hucbald may have been added to M by Gerbert himself, as Gerbert likely consulted M primarily through StP (since StP was created for that purpose).

Source P₁ (Paris, Bibliothèque nationale de France, f. lat. 7212)

This manuscript was not available to Mühlmann,¹⁸¹ it is unclear whether it was known to Gerbert.¹⁸² As in most of the other manuscripts of the Principal Group, the *Alia* is here preceded by *Ecce modorum sive tonorum* and is the final treatise in this manuscript.

Source P₂ (Paris, Bibliothèque nationale de France, f. lat. 7211)

Gerbert and Mühlman apparently were not aware of this manuscript. The first half of this manuscript is almost certainly a copy of P₁ and contains a considerable number of additional errors and

¹⁸¹ Mühlmann, *Die Alia musica*, 4.

¹⁸² Gerbert mentions a *codex Parisiensi* in his introduction to the works of Hucbald (within which he grouped the *Alia musica*), but he does so while discussing the *Musica enchiridis*, and there are three manuscripts from Paris containing the *Musica enchiridis* (see Schmid, *Musica et scolica*, viii): Paris 7212 (here, P₁), Paris 7211 (here, P₂), and Paris 7210 (which does not contain the *Alia musica*); it is not possible to be sure to which manuscript Gerbert was referring. In any case, Gerbert indicates that he did correct his editions against the said Paris manuscript, due to abundant interpolations (Gerbert, *Scriptores ecclesiastici I*, 103).

corrupted readings.¹⁸³ A brief comparison of the two manuscripts shows that P₂ is particularly corrupted in the *Scolica enchiridis*. For example, one may compare P₁ fols. 16v–17r to P₂ fols. 15v–16r; in P₂, just below the diagram on fol. 16r, the *Scolica* breaks off at exactly the point where P₁ reaches the bottom of the page (P₁ fol. 16v), and begins mid-sentence with excerpts from other treatises, none of which are separated from one another. Then, near the bottom of fol. 19v, an excerpt from Aurelianus breaks off mid-sentence, and resumes mid-sentence in the *Scolica*, beginning at exactly the word where P₁ begins on fol. 17r. One may hypothesize that at one point, P₁ had several folios (perhaps three or four) inserted between fols. 16 and 17 (which have since been removed), containing excerpts from other treatises (presumably one treatise per folio). Frequently (though not always), at the end of a treatise, the scribe of P₁ leaves the rest of the page blank and begins the next treatise at the top of the next page, but the scribe of P₂ does not. The scribe copying P₂ presumably reached the point of the added folia mid-page, and simply continued copying without recognizing that these folia did not belong in that location. This pattern, alone, is very strong evidence that P₂ was copied from P₁. In addition, the *Scolica enchiridis* in P₁ contains blank spaces apparently intended for later insertion of diagrams that were never completed; most of these blank spaces are preserved in P₂.

Yet, for reasons that are not clear, Chailley's descriptions of the contents of P₁ and P₂ are presented differently, and therefore, the close relationship between these two manuscripts is obscured. (The contents of the two manuscripts are given in Table 8, and are identical – except for a few interpolated fragments – up to the end of the *Alia musica*, though P₂ continues with another block of music treatises of approximately equal length to the materials copied from P₁). Despite the different presentation in Chailley's manuscript study, he recognizes the relationship between them and uses the siglum P for readings in which P₁ and P₂ agree.

¹⁸³ Chailley, *Alia musica*, 66.

Table 8 - Comparison of the contents of P₁ and P₂¹⁸⁴

	P ₁	P ₂
<i>Musica enchiriadis</i>	1v–12v	1r–12r
<i>Scolica enchiriadis</i>	12v–37v	12r–51r
[Excerpt from <i>Musica enchiriadis</i> , ch. 14–16]		[16r–16v]
[Excerpt from Aurelianus, ch. 8]		[17r]
[Unidentified treatise]		[17r–17v]
[Unidentified tonary with <i>Parapteres</i>]		[17v–19r]
[Excerpt from Aurelianus, ch 18]		[19r–19v]
<i>De dimensione monochordi</i>	38r–38v	51r–52v
<i>Ecce modorum sive tonorum</i>	39r–39v	52v–54r
<i>Alia musica</i>	39v–50v	54r–71r
<i>Guido Exercitationes in cantandis intervallis</i> ¹⁸⁵		71v–72v
<i>Guido Epistola ad Theodaldum episcopum</i>		73r–73v
<i>Guido Micrologus</i>		73v–89r
<i>Guido De sex motibus vocum ad se invicem et divisione earum</i>		89r–90r
<i>Guido Regulae rhythmicae in antiphonarii sui prologum prolatae</i>		90r–95r; 72v
<i>Guido Prologus in musica</i>		95r–97v
<i>Guido Prologus ad Michaellem</i>		97v–105r
<i>Odo Dialogus de musica</i>		105v–115r
<i>Liber argumentorum</i>		115v–116r
<i>Liber specierum</i>		116v–124r
<i>Fragmenta musica</i> ¹⁸⁶		124r–125v
<i>Liber specierum</i>		125v–127r
Unidentified Tonary		127v–131v
Guidonian Hand		132r
<i>Fragmenta musica</i>		132v
Excerpt from Matianus Capella <i>De musica</i>		133r–134r
Excerpt from <i>Cassiodorus</i>		134r–134v
Excerpt from Aurelianus, ch. 8; 10–20.		134v–144v
Unidentified Tonary		144v–145r
<i>Fragmenta musica</i>		145r–146r
<i>De octo tonis</i> (Aurelianus, ch. 8)		146r–146v
Excerpt from <i>Cassiodorus</i>		146v
<i>Fragmenta musica</i>		146v–147r
Monochord division diagram in three genera		147v
Unidentified Tonary		148r–149r
Hand-based mnemonic diagram (not Guidonian hand)		149v
<i>Instrumenta Hieronymi</i>		150r–151r
Excerpts from Vincent of Beauvais, <i>Speculum doctinale XVII.xxxi-xxxiv</i>		151r–151v
Excerpt from <i>De numeris musicis et de consonantiis</i>		151v
Excerpt from Macrobius, <i>Commentarium in somnium Scipionis</i>		151v

¹⁸⁴ A facsimile of this manuscript was published in 1991 by Alma Santosuosso (*Paris, Bibliothèque Nationale, fonds Latin 7211: Analysis, Inventory, and Text*, Ottawa: Institute of Medieval Music). Unfortunately, due to the library closures resulting from the COVID–19 epidemic during the last six months of the completion of this dissertation, I have not been able to consult this facsimile; I have compiled this table from my own observations of the digital facsimile on *Gallica* (<https://gallica.bnf.fr/ark:/12148/btv1b8432471z>).

¹⁸⁵ This excerpt is untitled in the manuscript. This is the title as given by Chailley (*Alia musica*, 65).

¹⁸⁶ cf. <http://www.chmtl.indiana.edu/tml/12th/ANOFRA2>; the relationship between these two texts is obvious, though they differ dramatically in form. They both appear to be an extension of the *Liber specierum*. More of these same fragments return later in the manuscript.

Source C (Cesena, Biblioteca Comunale Malatestiana, Pluteus S. XXVI.1)

This manuscript was not consulted by Gerbert, Mühlmann or Chailley. It is part of a set of one hundred commissioned by the founder of the Malatestiana Library and is believed to be a copy of an unknown, eleventh-century manuscript that was also the source of the lost Strasbourg manuscript (Source A, above).¹⁸⁷ Atkinson notes that the model manuscript was from Lorraine, but does not explain this hypothesis.¹⁸⁸ Atkinson also observes that Gerbert consulted this manuscript,¹⁸⁹ but the evidence of his use of it for the *Alia musica* is unclear: Gerbert does, indeed, refer to this manuscript (under the name *Cesenatensis*), which he claims to have compared against A with his edition of Hucbald's *De harmonica institutione*,¹⁹⁰ and there are footnotes to that effect throughout his edition of that treatise;¹⁹¹ however, his discussion of the *Alia musica* (a few sentences earlier) does not mention Cesena, and though the two discussions are very close to each other (and so the mention of Cesena may perhaps be assumed to apply equally well to the *Alia musica*), there is no reference to Cesena in the footnotes to the *Alia musica*. Additionally, there are instances of discrepancies between C and G that are not acknowledged in Gerbert's footnotes,¹⁹² while discrepancies between A and M are noted. Hence, it is likely that he did not consult Cesena for his edition of the *Alia*.

Descriptions of the contents of this manuscript appear in Chartier's edition of Hucbald¹⁹³ and Gushee's edition of Aurelianus;¹⁹⁴ the content of the manuscript corresponds very well with Gerbert's

¹⁸⁷ Chartier, *L'Œuvre musicale d'Hucbald*, 101.

¹⁸⁸ Atkinson, *Critical Nexus*, 174. It may perhaps come from Chartier's observation that "*presque tous les exempla sont pourvus de neumes lorrains*," (Chartier, *L'Œuvre musicale d'Hucbald*, 101) ("Almost all the *exempla* are provided with Lorrainian [Messin] neumes"); interestingly, the *Alia musica* is an exception to this comment; no neumes are provided within the *Alia musica* in this manuscript, though neumes do appear in M.

¹⁸⁹ Atkinson, *Critical Nexus*, 174, Table 5.2.

¹⁹⁰ Gerbert, *Scriptores ecclesiastici I*, 103.

¹⁹¹ *ibid. passim; inter alia*, pp. 104, 106, 107, etc.

¹⁹² *e.g.*, the numbers (but not the interval labels) in the tabular summary for Mode III in C (f. 192r) are miscopied, presenting instead the numbers appropriately belonging to Mode II (*cf.* Gerbert, *Scriptores Ecclesiastici I*, 143b).

¹⁹³ Chartier, *L'Œuvre musicale d'Hucbald*, 101.

¹⁹⁴ Gushee, *Aureliani Reomensis Musica disciplina*, 43.

description of A (excepting the observations made above in the discussion of A about the *Musica* and *Scolia enchiridae*), and C may consequently serve (along with G) as a reasonable substitute for A. As with most other treatises in the Principal Group, the *Alia musica* is preceded by *Ecce modorum sive tonorum* and, as in A, it is followed by *Argumentatio cuiusdam de musica*.

The close relationship between C and A may potentially answer an outstanding question about A. Gerbert based his edition (G) primarily upon A, but included the tabular summary from M, with no explanation. Significantly, though, the table in C is riddled with errors.¹⁹⁵ It contains all the errors found in P₁ and quite a few more, as well as a muddled layout (especially in for Modes VII and VIII; the same headings appear as in the other manuscripts, but attached to the wrong sets of numbers – see Figure 5). The arrangement is unusual both because one heading is missing (*Item de eodem*), with another heading shifted over into its place, and also because the chart spreads across two pages, which has the effect of structuring the chart in a different, less intuitive order than is found in the other manuscripts. Figure 6 compares the order of presentation in C against that found in M and P (G fits fewer cells in a column, but still conforms to the same pattern as M and P in principle). Modes VII and VIII are presented multiple times in different ways, and only one each is labelled with the mode's number, while the others are presented as "*Item de eodem*" ("likewise, concerning the same [mode]"); the arrangement in C obscures the connection between an *item* cell and the cell presenting the mode number.

Since the table is also present in A (as evidenced by Gerbert's footnotes),¹⁹⁶ and since it appears that the table in A was in the same corrupted state as in C (from which one must conclude that the same corrupted table was present in the common model of both A and C), it is likely that Gerbert used the table from M because of its dramatically superior reading (the table in M contains fewer errors than in any other manuscript).

¹⁹⁵ A comparison of the numbers presented throughout the treatise in each manuscript is included in the appendix.

¹⁹⁶ Gerbert, *Scriptores ecclesiastici I*, 143a–44b.

- diapason

<p><i>Tonus septimus</i></p> <p>a a XII XII</p> <p>c c VI VI</p> <p>b b b VIII VIII VIII</p> <p>d d VIII VIII</p> <p style="text-align: right;">=VIIa</p>	<p>Tonus <i>Item propria consonantia</i>^Λ</p> <p>a a a a XII XII XII XII</p> <p>c c c c c c c c VI VI VI VI VI VI VI VI</p> <p>b b b b b b b b VIII VIII VIII VIII VIII VIII</p> <p style="text-align: right;">=VIIc</p>	<p><i>Item ex eodem</i></p> <p>a a a XII XII XII</p> <p>c c c c VI VI VI VI</p> <p>b b b b VIII VIII VIII VIII</p> <p>d d d d VIII VIII VIII VIII</p> <p style="text-align: right;">=VIIb</p>
<p><i>Tonus octavus</i></p> <p>a a a XII XII XII</p> <p>c c c c VI VI VI VI</p> <p>b b b b VIII VIII VIII VIII</p> <p>d d d d VIII VIII VIII VIII</p> <p style="text-align: right;">=VIIb</p>	<p>[Heading absent: <i>Item de eodem</i>]</p> <p>a a XII XII</p> <p>c c c VI VI VI</p> <p>b b b b VIII VIII VIII VIII</p> <p>d d VIII VIII</p> <p style="text-align: right;">=VIIIa</p>	

Figure 5 – The layout issues in the Tabular Summary in Source C

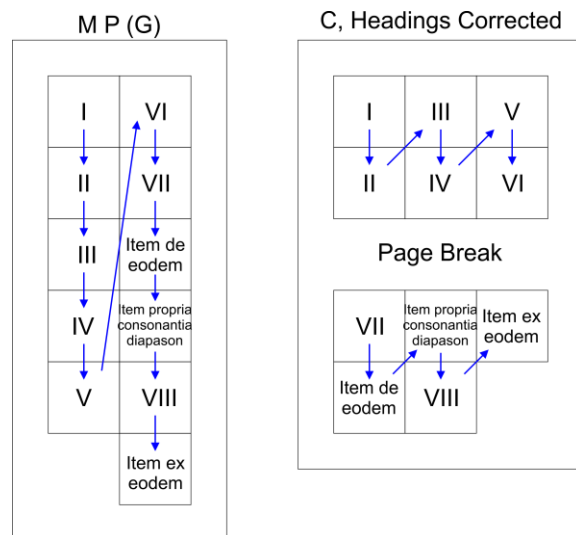


Figure 6 – Layout of the Tabular Summary in C relative to M and P

Relationships Amongst the Manuscripts of the Principal Group

Chailley observes that G, M, and P₁ form a coherent group, with a set of shared errors and no significant divergences.¹⁹⁷ This observation is barely a grouping, since from the entire set of complete manuscripts that Chailley consulted, only P₂ is absent from this list, and P₂ is clearly a copy of P₁. Thus,

¹⁹⁷ Chailley, *Alia musica*, 65.

Chailley very nearly says that all the extant complete manuscripts form a coherent group. There are two caveats to this observation, however. First, despite P_2 's being a copy of P_1 , Chailley considers P_2 to be distinct from the GMP_1 group because of the numerous additional errors introduced in P_2 ; and second, Chailley also compares against the Karlsruhe manuscript (Source K, below), which does not contain the complete treatise, and varies from the GMP_1 group much more substantially.

Chailley's observations about GMP_1 would almost certainly apply to C as well, since A (the principal source for G) and C were presumably copied from the same source (or another source in the same lineage), on account of the near-perfect agreement of manuscript contents. Grouped according to this criterion, the manuscripts may be subdivided amongst three streams of transmission: the A(G)C stream, in which the *Alia* is preceded by *Ecce modorum sive tonorum* and followed by *Argumentatio cuiusdam de musica*; the P (*i.e.*, P_1P_2) stream, in which the *Alia* is preceded by *Ecce modorum sive tonorum*, and in which the *Alia* ends the manuscript or ends a major partition in the manuscript; and the M stream (of which M is the sole surviving representative), in which the *Alia* is preceded directly by *De dimensione monochordi* and is the end of the musical portion of the manuscript.

A particularly interesting indication of copying history for these manuscripts is in the series of numbers presented for each mode, especially in the tabular summary and the prose summary. For each mode, the base numbers 12, 9, 8, and 6 are multiplied by varying coefficients, producing a unique signature for that mode. Interpreting these modal numbers will form a principal goal for the final portion of this dissertation. For now, it suffices to observe that the multiplication is usually indicated by rewriting each factor a specified number of times, as indicated by the coefficient. So, the complete list of modal numbers for Mode VI, presented as a direct list (as they are given in the prose summary), appears as follows: "xii·xii·vi·vi·vi·vi·vi·vi·viii·viii·viii·viii·viii·viii." It is, of course, not difficult to see how a number might be accidentally omitted from (or occasionally even added to) such a string of numbers. These variants are much more reliable than other variants in wording because wording variants often do not

alter the fundamental meaning of the passage, and it can be challenging to determine which wording was original; by contrast, altered modal numbers dramatically change the underlying doctrine of a concept that is fundamental to the treatise, and each of these numbers is presented, confirmed, and used in mathematical procedures well over a dozen times throughout the treatise, leaving no doubt about the correct values. And while two different scribes might independently make the same type of error (*i.e.*, adding or omitting one repetition of a number that is repeated many times in a long sequence), it is not especially likely that the specific details of that error would be the same; one scribe might omit an viii, while another scribe might omit a viiii. These number sequences thus form an interesting category of evidence that is more reliable in one direction than the other. There is a particularly high probability of making these errors while copying (indeed, this manner of presentation – a long list of consecutive numbers – appears only in the prose summary and in manuscript K, and these sections contain far more errors than any of the other discussions of the modal numbers, as may be confirmed from the tables in the appendices to this dissertation), and the substantial change in meaning makes the error very easy to distinguish from the correct reading, thus presumably making the copy easy to distinguish from the source; but the many repetitions and obvious contradictions would also have made these errors easy to spot by a medieval copyist, which could easily have led to corrections that would reverse a modern scholar's impression of which is the model and which the copy. By contrast, however, the likelihood of independently making precisely the same error of this kind is low enough that two manuscripts sharing the same such error are particularly likely to be closely related.

The P stream and the A(G)C stream seem closer to each other than to M on two accounts: in both streams, the *Alia* is preceded by *Ecce modorum sive tonorum*, and the tabular summary in both streams share a common set of errors (though C adds considerably more), while M contains a nearly perfect table (only one error).

Despite the potential described above for scribes to recognize errors easily in these number sequences, the copyists largely seem to have copied these erroneous sequences uncritically, passing on the errors into the new manuscripts, without comparing the sequences to other descriptions in the treatise (often only a sentence or two away). There are also a few instances where the manuscripts show signs of having been corrected in such a way that even these supposed corrections also turn out to be errors and can also aid in the tracing manuscript descent (though the number of corrections is dwarfed by the number of errors that remain uncorrected.) Tables showing the variant readings amongst the surviving manuscripts in the modal numbers throughout the treatise are presented in the appendix.

A particularly interesting example of both of these effects (uncritically copied errors and erroneous corrections) occurs with Mode V in the prose summary. §177 says, “*Quintus tonus est ·xii·^a·xii·^a·xii·^a·vi·^c·vi·^c·vi·^b·viii·^b·viii·^d·viii·^d·viii· et est diapenticus per ter ·viii· ad ter ·xii· qui sunt ·xxxvi· et quater ·vi· ad quater ·ix· qui sunt ·lx·*”¹⁹⁸ (“The fifth tone is $3 \times_{12}^a$, $3 \times_6^c$, $3 \times_8^b$, $2 \times_9^d$, and is of a fifth through $(3 \times 9):(3 \times 12) = 36$ and $(4 \times 6):(4 \times 9) = 60$.”) This statement contains several errors but is fairly consistent in most manuscripts.¹⁹⁹ In this passage, the factor $\overset{c}{vi}$ has been omitted once from the listing at the beginning of the sentence in every manuscript, a consistent error that is contradicted routinely throughout the treatise but has been faithfully reproduced in every surviving manuscript. Additionally, the factor $\overset{d}{viii}$ has been omitted either once or twice, depending on the manuscript: most manuscripts omit it once, leaving a list that shows each base number occurring three times in this mode; M, however, omits this factor twice, leaving only two

¹⁹⁸ *Alia musica* §177, ed. Chailley, 178; cf. Gerbert, *Scriptores ecclesiastici I*, 145a; I have restored the original orthography (excluding abbreviations) because Chailley’s simplified presentation, while much easier to read, obscures the very characteristic at the heart of the present discussion: the long sequence of numbers that is easily miscopied. Additionally, I have walked back Chailley’s corrections for the sake of demonstration.

¹⁹⁹ It also reveals an interesting divergence of editorial procedure between Chailley and Gerbert. Chailley makes corrections directly within the body of his edited text, noting the error only in the critical apparatus (thereby giving anyone but the most careful reader the impression that at least one of the manuscripts provides the correct number, which none does), while Gerbert faithfully reproduces his model text and makes the correction in a footnote (*Alia musica* §177, ed. Chailley, 178; Gerbert, *Scriptores ecclesiastici I*, 145a).

viii^ds. Gerbert also omits two, adding a footnote that two ought to be added, and does not indicate any discrepancy between his two sources, suggesting that A also only had two viii^ds. C, however, a manuscript much closer to A than M, also contains three viii^ds, and because of Gerbert's inconsistency in noting differences between his two sources (I shall return to this shortly), the reading in C is sufficient cause to question Gerbert's edition on this point.

Regardless, what is most interesting about this passage is that in the second half of the very same sentence, the ratios presented clearly reflect the correct coefficients. No copyist for any of the surviving manuscripts noticed and corrected the discrepancy between the coefficients of the first half of the sentence and the ratios in the second half. Even more interestingly, the arithmetic accompanying the ratios in the second half is flawed. The second comparison, $(4 \times 6) : (4 \times 9) = 60$ is correct (in so far as the colon separating the terms of the ratio is treated as an addition sign – this will be discussed extensively in a later chapter), but the first is incorrect: $(3 \times 8) + (3 \times 12)$ should also equal sixty, not thirty-six; in fact, thirty-six is the product of just the final part, (3×12) . This error, too, is repeated in every single manuscript; however, the scribe of C seems to have noticed the error and attempted to correct it, but in the wrong way: he omits the coefficient for twelve altogether in the ratios in the second half of the sentence – *i.e.*, $(3 \times 8) : 12 = 36$. The arithmetic is now correct, but he has introduced yet another error by omitting two of the xii^s, even though he does not omit them in the list at the beginning of the sentence. That is to say, the scribe noticed the faulty arithmetic, but instead of recognizing that the product was faulty, the scribe assumed that the product was correct and the factors were faulty, incorrectly adjusting a factor without updating the list at the beginning of the sentence. Moreover, in the ratios, an additional marking is added above the line that is difficult to read but that may be an attempt to squeeze in an abbreviation for *ter* (“three times”) just before the twelve (see Figure 7); if so, this marking would imply that someone noticed the discrepancy between the three twelves in the list at the beginning and only a single twelve in the

ratios and attempted a correction, but once again breaking the arithmetic in the process and failing to recognize the reason for the faulty correction (*i.e.*, the faulty arithmetic) – and strangely, the scribe also does not seem to have noticed the discrepancy in the other coefficients (perhaps the twelve only stood out because it was lacking a coefficient altogether, rather than merely having had the wrong coefficient).

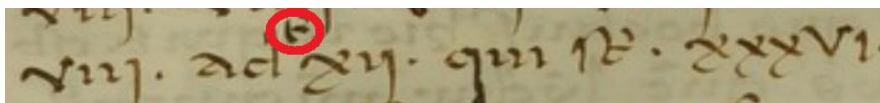


Figure 7 - A possible attempted correction in Cesena XXVI, f. 192v that creates some consistency between the number list and the ratios, but misses other discrepancies and also breaks the arithmetic. The marking may be an abbreviation for *ter*, as follows: \bar{c} viij. ad xij. qui ft' xxxvi. The small size needed to fit between the lines could easily make the abbreviation \bar{c} difficult to write in subscript without using a smaller pen, while the line thickness clearly shows that a comparable pen was used.

Meanwhile, F₁, a manuscript that contains only the prose summary (more about this manuscript below), altogether lacks the list of numbers at the beginning of the sentence, but includes the ratios and includes yet another coefficient for twelve (*bis*, or “two times”), which may also be an attempt to correct the arithmetic, but still does not succeed in doing so.

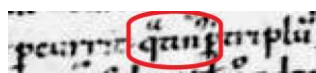
These same chains of numbers also provide another characteristic that is potentially informative of the copying history of the manuscripts. As the reader will no doubt have noticed in the preceding discussion, the numbers are joined to letters, which are usually written directly above the number, thus: $xii^a \cdot vi^c \cdot viii^b \cdot viiiiii^d$. The theoretical aspects of these letters will be discussed in a later chapter, but their application in the manuscripts can be even stronger evidence than the omission of a number. These letters, too, are occasionally omitted even where the number is not, and these omissions are also sometimes passed on to subsequent copies of the flawed manuscript – and, once again, while this type of omission could easily arise independently in multiple manuscripts, it would be extraordinarily unlikely for precisely the same letters to be independently omitted from these long sequences. Another minor detail of particular interest is the letter associated with VIII, which is given as majuscule ($viii^B$) in C (and apparently in A), but as minuscule ($viii^b$) in M and P (and these usages are consistent through each individual

manuscript). Although this distinction would be relatively weak evidence of filiation by itself, it can help to confirm the pathways of manuscript descent already noted for other reasons. It also speaks to Gerbert's methodology: his usage in this respect is inconsistent; his choice of viii^{b} or viii^{b} probably indicates where he is reproducing A and where he is borrowing from M. This otherwise silent borrowing reveals that despite the apparently liberal footnotes in his edition, he is not always entirely transparent about his use of manuscripts, suggesting that G might not quite as close to A as it appears.

A correction needs to be made to Chailley's observations on the relationship between manuscripts. Chailley notes an error in §168 that he interprets as resulting from the difficulty of interpreting an unclear abbreviation, possibly reduplicated. Source M has the abbreviation qan^{b} ; Chailley hypothesizes that although it is the abbreviation for *quatinus*, it could be mistaken for an abbreviation of *quintus*, and indeed, G reads "*quatenus quintus*," which he indicates also occurs in P;²⁰⁰ however, neither P₁ nor P₂ have this reading (both using the abbreviation quatin . Thus, it seems that the misreading appears only in G (and, by extension, presumably A); the misreading does not even appear in C, which writes quatinu out in full, though Chailley could not have known this (as he did not consult C).

Figure 8 – Chailley's "quatinus quintus" example in all five sources

M, f.180v



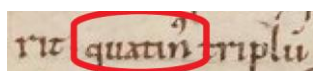
G, p. 143a



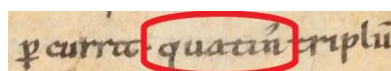
C, f. 192r



P₁, f. 48v



P₂, f. 68v



²⁰⁰ Chailley, *Alia musica*, 66; 171.

Chailley's hypothesis in this case is unlikely. As shown in Figure 8, there is a superscript letter in the abbreviation in M that Chailley seems to have interpreted as a ^u (which would be unorthodox in an abbreviation for *quintus*; a u after a q is generally assumed, and is far more likely to be omitted altogether than to be superscripted; while I am not aware of a standard abbreviation for *quintus*, such an abbreviation would likely contain a superscript i, as $q\overset{i}{n}c$ or $q\overset{i}{c}$, not a u). Instead, the superscript letter is an ^a, in the open form α (a letterform that is not unusual in earlier hands, and by contrast to q^u as an abbreviation for *qu*, which is not typical, q^a is a very typical abbreviation for *qua*²⁰¹), and this reading is not at all consistent with an interpretation as *quintus*, which does not have an a. For Chailley's hypothesis to hold, it would require that source A (or the model from which it was copied) contained this abbreviation in an illegible form that might introduce such a misreading (perhaps lacking a superscript altogether).

Even so, M does seem to be an outlier for a variety of other reasons, including the other contents of the manuscript (a consideration that I shall discuss extensively later in this chapter). In combination with these other factors, this orthographic difference may serve as additional (though not particularly strong) evidence of a common model for P and A(G)C, distinct from M. Thus, accounting for the presumed dates of the various manuscripts, the similarity of the text, and the contents of the manuscripts, the stemma given in Figure 9 for the Principal Group of manuscripts may be proposed (Greek letters represent presumed sources no longer extant: γ for the common model for both A and C; β for the antecedent that these two manuscripts seem to share with P₁; α for the original form of the complete *Alia musica*;²⁰² σ for the source treatise before the *Alia* was compiled; and ν for the *Nova expositio* prior to its incorporation

²⁰¹ Bischoff, *Latin Palaeography*, 160.

²⁰² I have chosen to eschew the traditional label of Ω for the archetype. Although there must certainly have been a first copy of the treatise to appear in the same form as all the manuscripts of the principal group, it is not clear that this state of the developing treatise (or, due to the gradual accretion of material and questions about how much of the *Alia musica* has been borrowed from other sources, any other particular state) should appropriately be accorded the status implied by Ω (as, for instance, in precedence to a form otherwise complete but not yet including the *Nova expositio*, which is, for all intents and purposes, an independent treatise).

into the complete *Alia*; μ reflects the fact that the *Nova expositio* as found in both the *Alia musica* and the *Dulce ingenium* appears to be corrupted – more on this later. Throughout this chapter, a simple line indicates simple successions of manuscripts of the same text, while an open-ended arrow indicates that a text was absorbed into or served as the inspiration for a different text; due to both uncertainty and space constraints, the position of each source relative to the vertical timeline is a loose approximation).

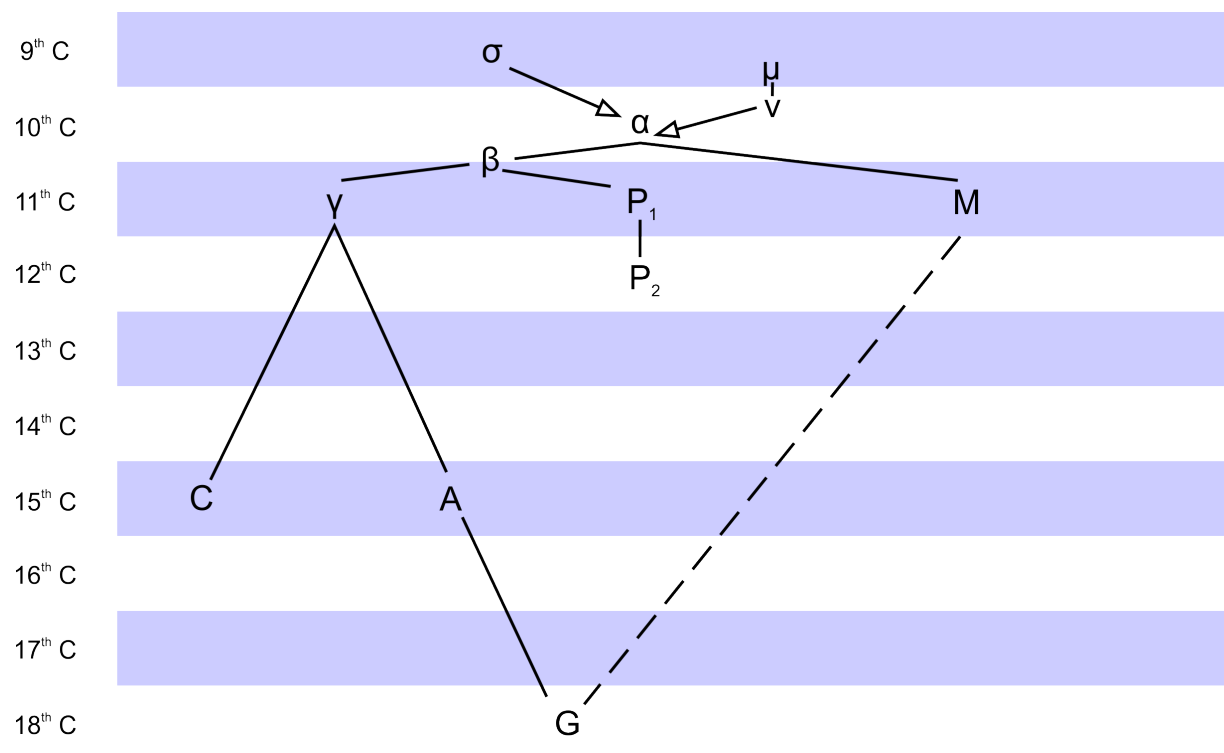


Figure 9 - Stemma for Principal Group

The *Dulce ingenium* Group

Dulce ingenium is another composite treatise from the tenth or eleventh century, edited by Michael Bernhard.²⁰³ Like the *Alia musica*, this treatise also contains the *Nova expositio*. However, unlike the *Alia*, in which this tonary is separated into sections for each mode and dispersed throughout the treatise, in

²⁰³ Bernhard, Michael, *Anonymi saeculi decimi vel undecimi tractatus de musica "Dulce ingenium musicae,"* Veröffentlichungen der Musikhistorischen Kommission, Band 6, Munich: Verlag der Bayerischen Akademie der Wissenschaften, 1987.

Dulce ingenium, the *Nova expositio* is presented in a single, contiguous block at the end of the treatise. There are three manuscripts containing this treatise, divided into two substantially different variants, with two extant copies of the shorter version and one copy of the more extended version. Bernhard selected *sigla* for these three manuscripts, which fortunately do not overlap with the *sigla* that Chailley selected for the *Alia*; thus, Bernhard's *sigla* shall be retained here.

Regarding this group of manuscripts, Bernhard says,

*Bei der Kollation der Handschriften zeigt sich, daß in Einzelheiten eine andere Textfassung vorliegt, die vom Hauptkorpus der Quellenhandschriften abweicht und für einige Stellen überzeugendere Lesarten bietet als der von Chailley gedruckte Text.*²⁰⁴

During the collation of the manuscripts, it becomes apparent that, in the details, another recension [of the *Nova expositio*] exists that deviates from the main body of the source manuscripts and, for some places, offers more compelling readings than the text published by Chailley.

Bernhard goes on to provide a short list of emendations present in the *Dulce ingenium* manuscripts that he considers to be inescapable [*zwingend*].²⁰⁵ (The complete list of these will be presented later in this chapter.) The most interesting emendation is the final one, which provides evidence that both the *Alia musica* and the *Dulce ingenium* share a critical error, and consequently, suggests that neither is likely to present the original form of the text. The passage arises in §151(a) of the *Alia* and in the *Dulce ingenium* at §83 of the short version or §127 of the long version. The passage indicates that the *differentia* under discussion at that moment has five *loca* (starting points of antiphons relative to the *differentia*),²⁰⁶ and provides the interval between the *differentia* and each *locum*. In every manuscript of the Principal Group and of the *Dulce ingenium* Group (except Pa – see below – which ends immediately before the passage in question), the fourth and final [!] locus is described as a fourth below the *differentia*, while the pitch cited

²⁰⁴ Bernhard, *Dulce ingenium*, 1.

²⁰⁵ *ibid.*, 1–2.

²⁰⁶ This concept will be described in greater detail in Chapter 11.

is a fourth above. Chailley corrects the interval direction based on the specified note,²⁰⁷ without remarking upon the fact that there are only four *loca* given when five were announced. Bernhard, by contrast, hypothesizes a lacuna, in which a *locum* below is properly identified and an example given, and then the upper fourth is announced, leading back into the note given in the passage,²⁰⁸ this solution not only reconciles the specified pitch with the specified direction but also reconciles the number of *loca* given with the number of *loca* announced. If Bernhard's hypothesis is correct, then both versions must ultimately descend from a common source already containing the error.

Source Pa (Paris, Bibliothèque nationale de France, f. lat. 8663)

This manuscript contains the shorter version of the *Dulce ingenium*. It is, however, incomplete at the end,²⁰⁹ breaking off at the end of §82 of the shorter *Dulce ingenium*, which is §150 of the *Alia musica*, two sentences from the end of the treatise as it appears in the other manuscripts.

Source Br (Bruges, Stadsbibliotheek, 532)

This manuscript contains the shorter version of the *Dulce ingenium*. Bernhard describes it as more or less a poor copy of Pa, with some of the examples incomplete (but with space reserved for them); but, Pa is incomplete at the end, while Br is not. That is, both copies are incomplete, but with each copy omitting different parts; thus, neither can be the model for the other, and a common model must be hypothesized for them both²¹⁰ (furthermore, Br is also believed to have been copied substantially later than Pa). Interestingly, the *Dulce ingenium* is here split into two sections. Nearly the entire first half is

²⁰⁷ “*intensum correximus* : remissum, GMP.” Chailley, *Alia musica*, 195 §151(a), critical apparatus. All the manuscripts containing the *Nova expositio* that Chailley consulted [GMP] describe the fourth as *remissum* (low – strictly, “slackened,” as a low-pitched string is looser than a high pitched string), and Chailley has corrected it to say *intensum* (high – strictly, “stretched,” as a high-pitched string is stretched tight).

²⁰⁸ Bernhard's critical apparatus for this passage accepts Chailley's “correction” (see fn. 207, above) as the form given in the *Alia musica*, without acknowledging that Chailley's reading is not present in any of the manuscripts.

²⁰⁹ Bernhard, *Dulce ingenium*, 2.

²¹⁰ Bernhard, *Dulce ingenium*, 2

presented, leaving out only the last few sections, which are diagrams of the species of fourth and fifth, demonstrating the position of the semitone in each (the incomplete examples to which Bernhard alludes); after this, though, is interpolated a passage about proportions, which Bernhard identifies as belonging to a later hand. Only after this passage does the *Nova expositio* appear.

Source Pr (Prague, Národní knihovna České Republiky, CZ, XIX.C.26)

This manuscript contains the longer version of the *Dulce ingenium*. Despite the substantial differences between the short and long versions, Bernhard observes an interesting set of variant readings that he believes could have resulted from an unclear abbreviation, which leads him to propose that Pr could well have had the same model as Pa and Br.²¹¹ The additional material in this version of the treatise comes from a series of other treatises, most notably Regino (making the treatise probably no older than the early tenth century) and additional excerpts from the *Alia musica* (these are not part of the *Nova expositio* tonary), including part of the disputed passage and a few sections, contiguous in Pr but not in the *Alia musica*, distributed throughout the supplementary material leading up to the summaries. Since these three sections are separated from one another in both treatises and differ substantially from one another, it will occasionally be useful to treat them as three discrete sources. For that purpose, I propose the following sub-sigla: P^α for the *Nova expositio*; P^β for the disputed passage; and P^γ for the supplemental material.

Relationships Amongst the Manuscripts of the *Dulce ingenium* Group

According to Bernhard, Pa and Br likely stem from a common model (neither can be the model for the other); Pr could also potentially be drawn from the same source as Pa and Br, but the exact relationship is uncertain.

²¹¹ Bernhard, *Dulce ingenium*, 3.

The three manuscripts containing the *Dulce ingenium* have very little in common. In Pa, the treatise appears near the end of the codex, while it appears very near the beginning of Br. The only notable similarity between them is that the *Dulce ingenium* is preceded in both manuscripts by a *Horologium* (which is, therefore, likely also to be true of the common model that Bernhard proposes). This *Horologium* is not, however, present in Pr, and the manuscripts have little else in common either amongst themselves or with the other manuscripts containing parts of the *Alia musica*.

In fact, even within the *Dulce ingenium* itself, the manuscripts do not have as much in common as the simple descriptions “short” and “long” versions would typically imply. The long recension (found only in Pr) by no means consists of the entire short recension (Pa and Br) with additional material. In general, both versions begin with the incipit *Dulce ingenium* and a half-dozen shared sentences, contain another dozen or so shared sentences distributed throughout the treatise, and end with the *Nova expositio*. Excluding the *Nova expositio*, the material in the short recension consists mostly of paraphrases of Boethius and Martianus Capella, but only about a third of this material is retained in the long recension (all in the same order, but separated into smaller chunks and interspersed with other material). These shared passages amount to only about one-sixth of the material in the long recension (excluding the *Nova expositio*); the rest is made up primarily of extended excerpts and paraphrases of other treatises, including a treatise beginning *In primo diapason* (associated in Gerbert with [Pseudo-]Bernelinus),²¹² a considerable amount of material based on Regino’s *Epistola*, and another short excerpt of unknown provenance that is

²¹² In fact, as Bernhard notes (*Anonymi saeculi decimi vel undecimi tractatus*, 28), this material appears twice in Gerbert, *Scriptores Ecclesiastici I*. It occurs within two large blocks of treatises attributed respectively to Hucbald (pp. 121a–22a) and Bernelinus (p. 329a–30a); however, the two texts in Gerbert differ slightly, as the text in the *Bernelinus* section contains a two-octave scale built on F at the end, which is also present in the *Dulce ingenium*, but is lacking from the version connected to the Hucbald texts. Thus, the text in *Dulce ingenium* seems a closer reference to the text connected to *Bernelinus* than to that connected to *Hucbald*.

Interestingly, the text that immediately precedes *In primo diapason* in the Bernelinus section, which is the text attributed directly to Bernelinus, is an elaborated version of an excerpt also found in the Hucbald section immediately following *In primo diapason*, where it is known as *Cita et vera divisio monochordi in diatonico genere*, which immediately precedes *Dimensio monochordi*, and as such, is also common to several of the manuscripts in the Principal Group of *Alia musica* manuscripts.

notable for exclusively identifying the ecclesiastical modes by the old Greek ethnic names (Dorian, Phrygian, *etc.*).

The most likely conclusion of these observations is that Pr, at least, must have a somewhat later date than is traditionally attributed to the *Alia musica*. It must almost certainly be post-Regino,²¹³ and probably later still on account of presumed associations with Pseudo-Bernelinus. Furthermore, if the traditional assumption is correct that the *Alia musica* first directly associated the ecclesiastical modes with the Greek ethnic names, then the long version of *Dulce ingenium* must be post-*Alia musica* since it contains an excerpt that uses these ethnic names exclusively (and with no doubt that it is referring to the ecclesiastical modes).

As for the relationship to the *Alia musica*, there are two crucial distinctions between Pr on the one hand and Pa and Br on the other. Firstly, Pr (and only Pr) contains the aforementioned additional passages shared with the *Alia musica* (one of which has been significantly rearranged). Secondly, the final section of each mode in the *Nova expositio*, which was clearly intended to describe the doxology (but fails to do so in most manuscripts, owing to the lack of neumes), has been preserved in Pa and Br, but omitted from Pr; thus, Pr contains both more of the *Alia musica* (additional excerpts) and less (missing doxologies). In the other passages shared with the *Alia musica*, the *Alia* contains more fully developed versions; thus, neither recension of *Dulce ingenium* nor the *Alia musica* seems likely to be the model for any of the others.

In contrast to these observations, it is worth mentioning now an issue that I shall describe in greater detail later in this chapter: the Disputed Passage, which is shared between the *Alia musica* and the long recension of the *Dulce ingenium*, contains references to the *emmelis* and the relation 6:8:9:12 that are much more characteristic of the *Alia* than of any other passage in the *Dulce* (or any other Carolingian

²¹³ Unless Regino's *Epistola* is instead assumed to be a rearrangement of the material in *Dulce ingenium*, which does not seem likely.

treatise). While this observation quite strongly implies the *Alia* as the model for the *Dulce*, the observations above reduce the likelihood of this conclusion; one would have to suppose that one recension of the *Dulce* was created while drawing upon the *Alia*, and then that another, quite different recension was created that drops elements originally borrowed from the *Alia* but then returns to the *Alia* to borrow other passages previously not considered significant enough to warrant inclusion. This scenario is certainly not impossible, but it seems to me less likely than that the two composite treatises were written within a shared musical subculture, both drawing upon a common tradition and shared stock of shorter treatises.

Assuming that the *Nova expositio* did, in fact, exist prior to the compilation of both the *Alia musica* and the *Dulce ingenium*, the stemma for this group is presented in Figure 10, where the source of the *Nova expositio* for both the *Alia* and the *Dulce* is represented by ν , and a presumed original form (lacking the error described above in the introduction to this manuscript group) is represented by μ .

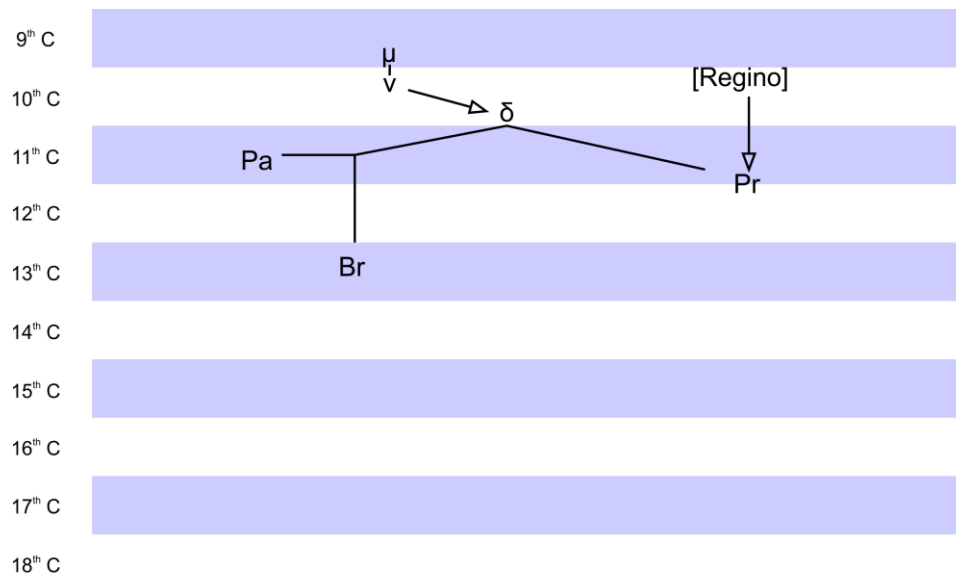


Figure 10 – Stemma for the *Dulce ingenium* Group

The Summary Group

These two manuscripts contain only a portion of the composite treatise, a summary that appears near the end and is, to a certain degree, complete in and of itself. Since these manuscripts have not been considered in other studies of the *Alia musica* (except Atkinson, who does not discuss manuscripts in detail and does not use *sigla*), I have selected *sigla* for them.

Source F₁ (Florence, Biblioteca Nazionale Centrale, Conventi soppressi, F.III.565)

This appears to be an important manuscript, as it seems to be at least a partial model for several later manuscripts, including F₂ (see below).²¹⁴ It contains significant discrepancies against the Principal Group; on this account, it is appropriate to consider the possibility that F₁ was not a copy of the summary from the complete *Alia musica*, but rather a copy of a text that predates the compilation of the treatise (perhaps as a summary of the source treatise, or even as the inspiration for the source treatise). A more detailed discussion of the implications of the independent transmission of the summary, together with a textual comparison of F₁ against Chailley's edition, will be presented later in this chapter.

Source F₂ (Florence, Biblioteca Riccardiana, Fr. 652)

This is the only manuscript known to contain any part of the *Alia musica* that I have not been able to consult in either edition or facsimile. However, it is generally believed to be at least partly a copy of F₁, and although the exact relationship between the two manuscripts is a matter of some debate,²¹⁵ the exact correspondence of the sections of the *Alia* covered in the manuscript²¹⁶ strongly implies that at least this portion of the manuscript is copied from F₁; therefore, aside from any additional errors that may have been introduced by the scribe, it is unlikely that access to F₂ would significantly impact the present study.

²¹⁴ Meyer, *Review of Firenze*, 287–88.

²¹⁵ Gushee, *Aureliani Reomensis Musica disciplina*, 38.

²¹⁶ Atkinson, *Critical Nexus*, 174.

On that account, wherever there is no need to distinguish between these two manuscripts, a common siglum F will be used.

Relationships Amongst the Manuscripts of the Summary Group

As has already been described, F_2 is presumed to be a copy of F_1 , at least with respect to the excerpt relevant to the *Alia musica*. Additionally, as will be described in greater detail later in this chapter, there is reason to suspect that F could have been written before the compilation of the complete *Alia musica*, either as a summary of the source treatise or as the inspiration for it. Incorporating the presumed dates implies the stemma in Figure 11, where ϕ represents a proposed source independent of the *Alia* (and σ again represents the source treatise before amalgamation into the completed *Alia*).

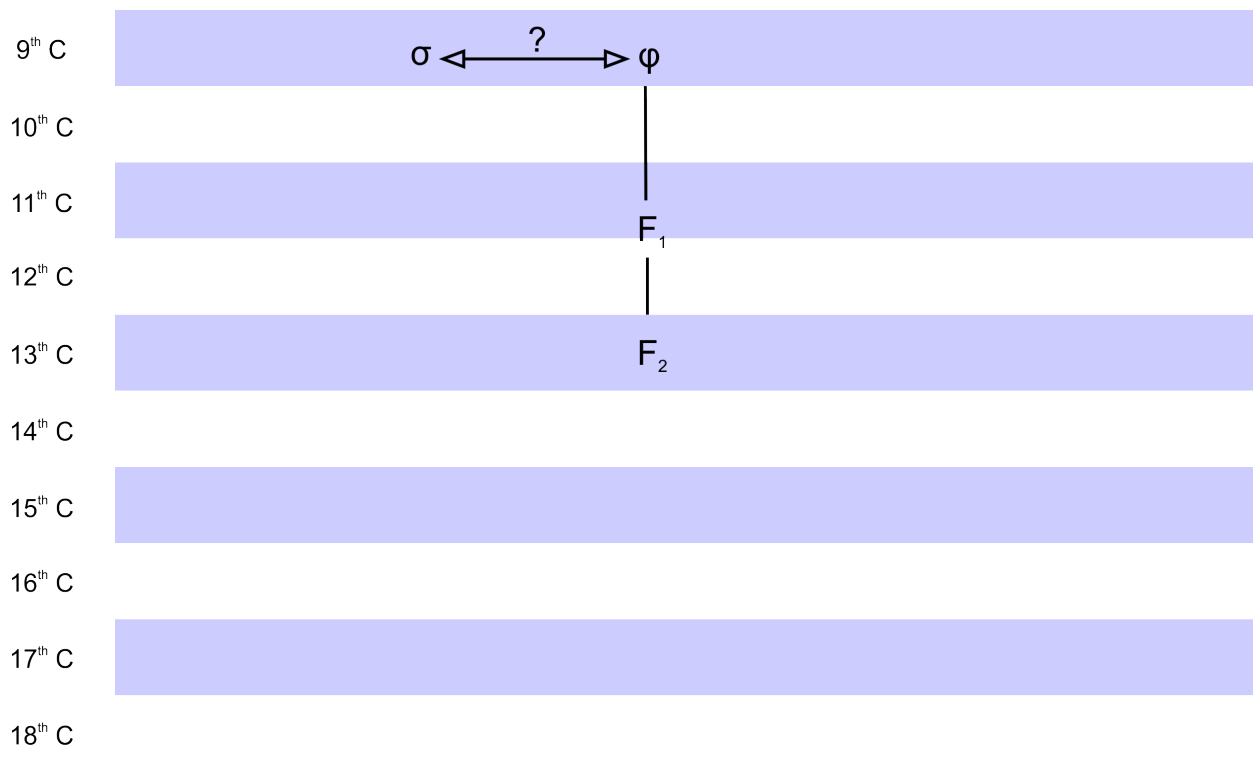


Figure 11 – Stemma for the Summary Group Manuscripts

Sui generis Manuscripts

The remaining manuscripts are all unique in the portions of the treatise that they incorporate, which often are not extensive. It is these manuscripts (and also those of the summary group above) to which Atkinson refers when he says that:

Although one cannot rule out the possibility that all of these were extracted from the *Alia musica* as a whole, it would seem more likely that at least some were circulating independently or as parts of small *libelli* before being combined in the late tenth or early eleventh century into the complete work we know as the *Alia musica*.²¹⁷

Source K (Karlsruhe, Badische Landesbibliothek, K. 504)

This source is the only manuscript that Chailley consulted outside the Principal Group. It contains an independent transmission of exclusively the final portion of the *Alia musica*: the source treatise; the siglum is Chailley's. It is idiosyncratic in several ways, which led Chailley to conclude that it may descend directly from the source treatise, rather than being an excerpt from the completed *Alia*.²¹⁸ The treatise is untitled in this manuscript (as in all manuscripts except M), and it includes the (usually misplaced) opening statement about the first mode in its appropriate place (which helps to support the case that the rubric given for it at the end of the composite *Alia musica* is misplaced). It also contains a description of the eighth mode that, though brief compared to the other modes, is considerably superior to the note found in the Principal Group manuscripts to find a description of the eighth mode earlier in the treatise. There is also a connection between this description of Mode VIII and that in the prose summary; unfortunately, the ending is incomplete, and so it is difficult to explain this relationship conclusively; it is likewise also not sufficient to rule out the additional passages – what Chailley calls “*Considérations symboliques*” – as part of the source treatise conclusively.

²¹⁷ Atkinson, *Critical Nexus*, 174.

²¹⁸ Chailley, *Alia musica*, 8; 67.

Manuscript K is also the only manuscript to use the *oktōēkhos* labels (*autentos protos*, etc.) and to explain their meanings; also, to give the *enēkhēmata* (*NONANOEANE*, etc.) in the source treatise; and likewise to present the modal numbers as a list, similar to the way that they appear in the tabular summary. Unlike the tabular summary, however, and exclusively in K, the letters associated with the modal numbers are not presented above the numbers, but before them, exclusively in majuscules, and most interesting of all, substituting out A (corresponding to twelve) in favour of CC (where a single C corresponds to six).

Chailley's discussion of these idiosyncrasies is worth reproducing here:

Ces divergences s'explique aisément, dans cette hypothèse, par le fait que l'auteur de l'Alia n'avait à s'intéresser, dans ce traité, qu'aux propositions qu'il commente : il était inutile, pour lui, de copier pour chaque ton la définition des noms (authentos protos etc.), dont il n'emploie pas la nomenclature, ni les coefficients des nombres de base, qu'il réunit en un tableau §177 et présente lui-même dans son propre style §28. La description du 8ème ton ne l'intéressait pas, puisqu'il ne pouvait le rattacher à Boèce, préoccupation essentielle de son travail. Enfin, il supprime les apêkhēmata, différents des siens propres. On objectera ici que ce scrupule n'a pas arrêté le rédacteur en ce qui concerne la Nova expositio du 2ème QUIDAM. C'est oublier que cette partie est une interpolation de scribe, et que l'auteur ne tient aucun compte de cette Nova expositio alors qu'il s'intéresse personnellement au texte du 1^{er} QUIDAM. D'où on déduit qu'il dut rédiger lui-même l'abrégé de K, qui figure dans ses propres manuscrits, ou du moins que cette partie a été copiée en tenant compte de son propre traité, alors que l'interpolation du 2ème QUIDAM reste étrangère à sa rédaction.²¹⁹

These divergences are easily explained in this hypothesis [that K is a copy of the source, not an excerpt from the *Alia*] by the fact that the author of the *Alia* [revision] had nothing to interest him in this treatise except the propositions upon which he comments: for him, it was pointless to copy, for each mode, the definition of the names (*authentus protus*, etc.), which nomenclature he does not use, nor the coefficients of the base numbers, which he reunites in a table (§171) and presents, himself, in his own style (§28). The description of the eighth mode does not interest him, since he could not connect it to Boethius, an essential preoccupation of his work. Finally, he suppresses the *enēkhēmata*, different from his own. One will object, here, that this scruple has not stopped the editor concerning the *Nova expositio* of the second *Quidam*; this is to forget that this part is a scribal interpolation and that the author takes no account of this *Nova expositio*, whereas he is personally interested in the text of the first *Quidam* [the source treatise]. Whence one deduces that he needed to draft, himself, the abridgement of K, which figures in his own manuscripts [*i.e.*, the Principal Group], or at least that this part was copied while taking account of his own treatise, whereas the interpolation of the second *Quidam* remains estranged from his redaction.

²¹⁹ Chailley, *Alia musica*, 67.

Source D (Madrid, Biblioteca Nacional, 9088)

Chailley did not consult this source, and he therefore did not select a siglum. However, this source does appear in Schmid's edition of the *Musica enchiridis* and related treatises under the siglum D, and since M is already taken for Munich, D seems as good a choice as any. The source is remarkable for several reasons. It does not contain any part of the modal number theory with which the majority of the *Alia musica* is chiefly concerned, nor any part of the *Nova expositio*. Instead, it contains only an excerpt (roughly the middle third) of the introductory material placed at the beginning of the composite treatise. This material is based upon Boethius' *De institutione musica*, and concerns interval species theory. It is the first passage in the *Alia*, both in reading order and chronology (and perhaps the first passage ever), to associate the octave species directly with the ecclesiastical modes, and it does so in a different manner from the *Nova expositio*. Regarding the passage found in D, Atkinson says:

These [paragraphs, and perhaps also those immediately surrounding them in the *Alia*] may have been the "kernel" that was then expanded by Author β [revisor]. He presents harmonic theory from Boethius's *De arithmetica* and *De musica*, probably introduced in order to provide a rationale for the intervallic ratios in the First Quidam [source treatise]. It is in this section that the diatonic scale is segmented progressively to yield *species*, which Author β [revisor] equates with *modi*, and which are given *ancient* Greek ethnic names (Dorian, Phrygian, etc.).²²⁰

It is also noteworthy that the contents of this manuscript, particularly surrounding the *Alia musica* excerpt, bear a striking resemblance to that of manuscripts A and C. On the one hand, this could be interpreted as evidence that this material was, indeed, copied from a manuscript in the same lineage as A and C. This hypothesis would require the assumption that the scribe considered this short passage from the *Alia musica* as the only passage worth copying from the treatise, an assumption that is not impossible but not entirely convincing; this passage is one of two from the *Alia musica* that treat the octave species and their relationship to the ecclesiastical modes, and the other passage (the Disputed Passage) seems, at least to a modern musicologist, to be the superior treatment, since it addresses the problem of the

²²⁰ Atkinson, *Critical Nexus*, 177.

eight mode, while the passage that appears in D does not (the two passages are otherwise quite similar). If one were to excerpt a passage from the *Alia musica* specifically to discuss the octave species, one would be far more likely to pick the Disputed Passage than this excerpt from the introduction.

On the other hand, this portion of D could also plausibly be a copy of an earlier state of the same manuscript tradition, preceding the *Alia musica*; the model from which D was copied might thus not only provide the kernel upon which the revised *Alia musica* was constructed (as Atkinson suggests) but possibly also serve as the source that first drew together the surrounding complex of treatises. There are, in addition, textual variations that might tend to support this latter hypothesis; a complete examination of these variants will be presented later in this chapter. However, it must also be acknowledged that the *Musica enchiridis* and *Scolica enchiridis* (and also the *Inchiridon*, which is a divergent and possibly earlier recension of the *Musica enchiridis*) all appear earlier within the same manuscript; thus, accepting this hypothesis requires either that the first part of the manuscript (containing the *Enchirides*) was copied from a different source from the latter portion or that the completed form of the *Alia musica* was assembled after the composition of the *Musica enchiridis* (an hypothesis contrary to Chailley's opinion, but now supported by most other scholars, as described in Chapter 2).

Source R (Barcelona, Arxiu de la Corona d'Aragó, Ripoll 42)

This manuscript was also not consulted by Chailley, and thus not given a siglum. However, both Schmid (in his edition of the *Musica enchiridis*) and Chartier (in his edition of Hucbald) use the siglum R (in reference to the Ripoll Monastery north of Barcelona, the point of origin of the manuscript). This source is the final manuscript currently known to contain any part of the *Musica enchiridis*, and is probably the most complex. Unlike the other fragmentary sources, it is relatively clear that R descends

from a manuscript that contained the complete *Alia musica*. Chartier believed it to be related to manuscripts A and C.²²¹

From contents alone, it is evident that the manuscript was not copied from a single source. There are at least two treatises that appear, in whole or in part, twice within the larger manuscript, both of which are familiar from the discussions above. Fols. 6–64 present a very familiar sequence of treatises, including Boethius, both the *Enchirides*, and the *Commemoratio brevis*, skips over Hucbald and goes straight to *De dimensione monochordi* and *Ecce modorum*, then skips the *Alia musica* and returns to Hucbald and *In primo diapason* (which would typically occur before *De dimensione*). Here, the sequence breaks off. There is a brief interpolation of a treatise not common to the rest of the tradition, followed by an extremely idiosyncratic (but essentially complete) copy of the *Nova expositio*, which has been edited by Karl-Werner Gumpel.²²² This revised form is followed by a related text that Gumpel considers to be part of this alternative recension. Following this revised *Nova expositio*, however, the *Ecce modorum*, already appearing earlier in the manuscript, returns, and this time (as usual), it is immediately followed by the opening of the *Alia musica*. This treatise is interrupted, however, after only a few sentences, and then moves on to yet another copy of the *Nova expositio* – but only two modes are presented (specifically, Modes VI and VIII, in their entirety). They are followed by a table entitled *Littere designantes directim nervos*, which describes the monochord notation (based on Boethius' division of the monochord) used in the *Nova expositio* and which also appears in the margins of P₁. The rest of the manuscript does not resemble the other sources.

²²¹ Chartier, *L'Œuvre musicale d'Hucbald*, 114.

²²² Karl-Werner Gumpel, "Die 'Nova expositio' der Handschrift Ripoll 42: Text und Kommentar," in *Miscellània litúrgica catalana* 15, 2007: 125–86.

Table 9 – Order of treatises in portion of R relevant to the *Alia musica* and its associated complex of treatises. The usual is followed, with minor exceptions, up to the *Nova expositio*, which appears in an alternate recension, followed by a second set of treatises also in the correct order that begins partway through the sequence, repeating some material.

First Set	Second Set
Boethius <i>Musica</i>	
<i>Musica enchiridis</i>	
<i>Scolica enchiridis</i>	
<i>Commemoratio brevis</i>	
[Hucbald <i>Musica</i>] usual position	
[<i>In primo diapason</i>] usual position	
<i>De dimensione monochordi</i>	
<i>Ecce modorum sive tonorum</i>	<i>Ecce modorum sive tonorum</i>
Hucbald <i>Musica</i> actual position	
<i>In primo diapason</i> actual position	
<i>Aforismus artis musicae</i> (not part of complex)	<i>Alia musica</i> introduction fragment
Alternate <i>Nova expositio</i>	Standard <i>Nova expositio</i> fragments
	<i>De tonis</i>

The impression left by the ordering of this manuscript is that it has been copied from multiple sources, or (less likely) that it was copied from a source whose binding had separated, resulting in parts being copied out of order. Gumpel, in fact, considers the manuscript to consist of three layers, with the third layer beginning at the Hucbald treatise.²²³ Gumpel's prose is not quite clear on the point of division between the first and second layers, as either immediately before or (more likely) immediately after

²²³ Gumpel, "Die 'Nova expositio,'" 125.

Boethius.²²⁴ Regardless, the second layer ends, as suggested above, at the end of the *Ecce modorum*, right where the *Alia musica* ought to (but does not) appear. The third layer begins with Hucbald and *In primo diapason*, then the short treatise not found in the rest of the *Alia's* manuscript tradition, and the alternate recension of the *Nova expositio*, which is then followed again by the *Ecce modorum*, followed by fragments of the *Alia musica* in the expected place.

This description explains why the *Ecce modorum* appears twice in the manuscript. It does not, however, explain the presence of the *Nova expositio* twice in a single layer (once in revised form, and once as fragments of the *Alia musica*). One could suppose that there is an additional, heretofore unobserved split between the third and a fourth layer, or perhaps a single third layer copied from a manuscript that was itself made up of separate layers, but there is not sufficient evidence available to have confidence in such an hypothesis.

Chartier, citing R. Beer, notes that another manuscript from the library at Ripoll (which may be designated R') served as the model for this manuscript; the contents of R' have been reconstructed to include "*le De Harmonica Institutione (=Musica) d'Hucbald et d'autres traités de musique préservés dans R'*"²²⁵ ("the *Musica* of Hucbald and other music treatises preserved in R"), which presumably includes the fragments of the *Alia musica* and quite possibly also the revised form of the *Nova expositio*. It is not clear from this description whether Hucbald was the first music treatise in R' (*i.e.*, if R' was only the source of the third layer, or whether it was also the source of the second layer). Like A, R' is believed to have been destroyed in a fire (in this case, when the Abbey's library and much of the archives burned down on 9 August 1835).²²⁶

²²⁴ Gumpel, "*Die 'Nova expositio,'*" 126.

²²⁵ Chartier, *L'Œuvre musicale d'Hucbald*, 87.

²²⁶ *ibid.*

Gümpel, however, suspects the place of origin of R to have been the Abbey of Saint-Benoît-sur-Loire in Fleury, a site with known ties to Ripoll.²²⁷ These two hypotheses (Ripoll or Fleury) need not be mutually exclusive, as it could well be that R' (or an exemplar upon which it was based) came from Fleury. Gümpel notes a similarity between the manuscripts R and P₂ and hypothesizes that the changes in the text of the *Nova expositio* in R reflect the practices of a French abbey²²⁸ (for which reason it is interesting that the only extant copy of this revised recension survives in Spain, while all the surviving copies in French regions, both in the *Alia musica* and the *Dulce ingenium* – and even the repeated sections later in R that are undoubtedly fragments of the *Alia* – retain the principal recension).

In terms of the text, the differences between the two recensions are considerable. In addition to occasional added commentary throughout and a substantial passage of commentary added to the end, in each mode, the statement of the *enēkhēma* for the mode is moved from the beginning of the section to the end. Additionally, note names, given in both monochord division letters and Greek lyre string names (more about both of these in later chapters) in the principal recension are given only in monochord letters in the revised recension. By contrast, in the principal recension, modes are identified only by the Latin ordinals (*primus, secundus, etc.*), while in the revised recension, additional passages are added to include manerial names (*authentus protus, etc.*).

Also of potentially considerable significance is that the ordering of the octave species in this revised recension has changed. As will be described in greater detail in Chapter 12, the *Nova expositio* makes an idiosyncratic use of the octave species that can be counterintuitive to interpret. The ascending octave species numbers are replaced in the revised recension with a model of descending octave species numbers, but without ultimately solving the difficulties (which, under a conventional interpretation of octave species, includes Modes I and III sharing a species, as well as a mismatch in the relationships

²²⁷ Gümpel, "Die 'Nova Expositio,'" 126; cf. Chartier, *L'Œuvre musicale d'Hucbald*, 87.

²²⁸ Gümpel, "Die 'Nova Expositio,'" 126.

between authentic and plagal modes), and the references to octave species are removed from Modes V and VII altogether.

In terms of content, the number of *differentiae* and *loca* for each mode does vary a little bit (presumably updated to reflect the usage where revision was composed), and the list of examples is also partly changed (most of the examples remain the same, but the new examples are not limited to new *loca*; even where a *locum* is the same, the example is sometimes changed). Additionally, each example in the revised recension is preceded by the formula *saeculorum amen*; although this might seem redundant, it serves a useful purpose: in some cases, neumes and monochord letters are provided above this formula and the antiphon *incipit*, which give to each *differentia* and *locum* a melodic profile, and not just a single note each. Unfortunately, these cannot be assumed to be the original melodies known to the author, as they could just as easily have been added by whoever created the revised recension, or even added by a later hand (though there are no visible indications of this latter possibility). Additionally, the neumes and monochord letters are only present for the first mode; however, the presence of the *saeculorum amen* formula in the rest of the modes implies that neumes were probably provided for all modes in the model manuscript.

Since this manuscript contains multiple (and sometimes conflicting) fragments of the *Alia musica*, it will be convenient to assign to it a set of sub-*sigla*, as shown in Table 10.

With the survey of the manuscripts now complete, Table 10 summarizes the *sigla* for all the manuscripts known to contain any portion of the *Alia musica*.

Table 10 – Summary of Sigla

	C	Cesena XXVI.1		
Chailley	A	Strasbourg 926		
	G	Gerbert		
	P {	P ₁	Paris 7212	
		P ₂	Paris 7211	
	K	Karlsruhe 504		
Bernhard (e.e.)	Pa	Paris 8663		
	Br	Bruges 532		
	Pr {	P _r ^α	Prague 26 {	<i>Nova expositio</i>
		P _r ^β		Disputed Passage
		P _r ^γ		Supplemental material
F {	F ₁	Florence 565		
	F ₂	Florence 652		
D		Madrid 9088		
R {	R ^α	Barcelona 42 {	Introduction	
	R ^β		Incomplete standard <i>Nova expositio</i>	
	R ^γ		Complete revised <i>Nova expositio</i>	

The Broader Manuscript Tradition and the *Enchiridis* Complex

At this point, it should be clear that there is a remarkable similarity amongst the manuscripts containing the *Alia musica* in terms of the ordering of their contents. A similar observation has long been made with respect to the *Musica enchiridis*. The *Musica enchiridis* is probably the best known and certainly the most widely disseminated treatise of the Carolingian period (and among the most widely disseminated treatises of the entire Middle Ages), and Nancy Phillips says of the principal recension of it that:

This standard recension was often transmitted as a part of a larger collection of treatises on music. When the Boethius *De institutione musica* was included, it most often preceded the *Enchiridis* treatises. The collections of treatises following the *Scolica* were extremely varied in content, but generally limited to treatises of the ninth to early eleventh centuries. Occasionally the collection included one or more of the treatises of Hucbald, Aurelian, or *Alia musica*, and the latter is known in its entirety only by its copies in the collection following the *Enchiridis* treatises.²²⁹

²²⁹ Phillips, *Musica and Scolica Enchiridis*, 66

In fact, Phillips' observation about Hucbald, Aurelianus, and the *Alia musica* misses a fascinating nuance. While perhaps not quite true of Aurelianus,²³⁰ the presence of either Hucbald or the *Alia musica* is surprisingly likely to imply the presence of the other – and even more so, to imply the presence of the half-dozen shorter treatises like *De dimensione monochordi* (indeed, these smaller treatises usually appear in conjunction with either Hucbald or the *Alia* or both). This set of treatises is so consistent that Lawrence Gushee described it as a “superwork” that may have formed a “comprehensive program of studies.”²³¹ Consequently, evidence of the relationships between the different copies of the *Alia musica* can be determined not only from the texts of the *Alia* itself, but also from the relationships among the texts of the other treatises. Detailed, partially overlapping manuscript studies for this group of manuscripts appear in Terence Bailey's edition of the *Commemoratio brevis*,²³² Yves Chartier's edition of Hucbald,²³³ Lawrence Gushee's edition of Aurelianus²³⁴ (and also in a separate article by Gushee),²³⁵ and in editions and studies of the *Musica enchiridis* by Hans Schmid,²³⁶ Nancy Phillips,²³⁷ and (to a lesser degree) Raymond Erickson,²³⁸ as well in Michel Huglo's extensive study of tonaries.²³⁹

Not all of the manuscripts containing parts of the *Alia musica* belong to this complex. The manuscripts of the *Dulce ingenium* group and the Florence group are exceptions, as is Karlsruhe. However, all of the extant manuscripts containing any part of the introduction to the *Alia musica* belong

²³⁰ Gushee considers that only five of the two dozen or so manuscripts seeming to contain portions of Aurelianus' treatise actually preserve Aurelianus' text, while the remaining manuscripts preserve copies of the sources upon which Aurelianus drew when compiling his treatise (Gushee, *Aureliani Reomensis Musica disciplina*, 47). A complete consideration of this issue is beyond the scope of this dissertation, but the principle is certainly consistent with the fact that Aurelianus himself describes his treatise as a compilation of the best material that was available to him (*ibid.*, 11).

²³¹ Gushee, “Questions of Genre,” 398.

²³² Bailey, *Commemoratio brevis*, 1–5.

²³³ Chartier, *L'Œuvre musicale d'Hucbald*, 82–117.

²³⁴ Gushee, *Aureliani Reomensis Musica disciplina*, 19–49.

²³⁵ Gushee, “Questions of Genre,” 398–401.

²³⁶ Schmid, *Musica et scolica*, VII–XII.

²³⁷ Phillips, *Musica and scolica*, 44–119; esp. 66–80.

²³⁸ Erickson, *Musica enchiridis*, xlii–xlix; Erickson's edition is, however, based principally on a different set of manuscripts.

²³⁹ Huglo, *Les tonaires*, 66–71.

to this “superwork” group of treatises. For that reason, I shall refer to this subgroup of the *Enchiriadis* complex as the *Alia musica* subcomplex. (The term, however, is not likely to find much use outside this study, as there are a few manuscripts in the group that do not contain the *Alia musica*.) There are two in particular that routinely appear in manuscript studies of the other treatises (and can, therefore, serve as useful points of comparison): Krakow, Biblioteka Jagiellńska, Rpis BJ 1965 (BB XXIII 8), given the siglum J (in reference to the Jagiellonian Library) in Schmid, Chartier, and Bailey; and Oxford, Bodleian Library, Ms. Canon. Misc. 212, given the siglum O in Chartier.

The treatises in the complex form up into four distinct blocks (plus a few individual treatises), which may be termed the *Enchiriadis* block, the Hucbald block, the monochord block, and the *Argumentatio* block. Within each block, the individual treatises (to the extent that they are each present) always appear in the same order. The blocks, themselves, also appear consistently in the same order, with only one notable point of variation: the Hucbald block may appear either before or after the monochord block.

Table 11 shows the contents of the principal members of the *Alia musica* manuscript complex. It is visually apparent that the manuscripts come together into groups of similar contents. There is not a single consistent pattern across the table (it would be remarkable, indeed, if such a clear pattern did emerge), but there are clear subgroups of manuscripts that contain the same treatises, with only minor deviations, and the order of those elements present varies only in the position of the Hucbald group either before or after the monochord group. Interestingly, the position before the monochord group only occurs in the related manuscripts A and C, which also – probably not coincidentally – lack the *De dimensione monochordi* (a.k.a. *Super unum concavum lignum*) and the beginning of *Ecce modorum sive tonorum*. The implication is that the correct position for the Hucbald treatise is after the *Ecce modorum* and that in the presumed common model for both A and C, the Hucbald block was accidentally shifted, and in the process, the beginning of *Ecce* was lost. It is also worth noting that no manuscript containing the Hucbald block after *Ecce modorum* includes the traditional recension of the *Alia musica* (and the *Ecce modorum* is always

directly followed by either Hucbald or the *Alia musica*); thus, one may alternatively suppose that there was some causal connection between the presence of both Hucbald and the *Alia* (or not) and the position of Hucbald.

Table 11 – Contents of the principal manuscripts of the *Alia musica* subcomplex. Items missing from groups are given in grey text; entire missing groups are omitted.

Key:	Boet. = Boethius' <i>Musica</i>	Inch. = <i>Inchiriadon</i>	Mus. En = <i>Musica enchiriadis</i>
	Scol. En = <i>Scolica enchiriadis</i>	Comm. Br. = <i>Commemoratio brevis</i>	Huc. = Hubald's <i>De harmonica</i>
	Primo = <i>In primo diapason</i>	Cita = <i>Cita et vera divisione</i>	Primis = <i>Dimensio monochordi</i>
	Super = <i>De dimensione monochordi</i>	Ecce = <i>Ecce modorum</i>	Alia = <i>Alia musica</i>
	Nova = <i>Revised Nova expositio</i>	Argum. = <i>Argumentio cuiusdam</i>	Mens. = <i>De mensuris organicarum</i>
	Cymb. = <i>De cymbalorum ponderibus</i>	Modis = <i>De modis</i>	Quinque = <i>De quinque symphoniis</i>

D	C	A	O	J	R	P ₁	P ₂	M
Boet.	Boet.				Boet.			Boet.
Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.
Mus.En.	Mus.En.	Mus.En.	Mus.En.	Mus.En.	Mus.En.	Mus.En.	Mus.En.	Mus.En.
Scol. En.	Scol. En.	Scol. En.	Scol. En.	Scol. En.	Scol. En.	Scol. En.	Scol. En.	Scol. En.
Comm. Br.	Comm. Br.	Comm. Br.	Comm. Br.	Comm. Br.	Comm. Br.	Comm. Br.	Comm. Br.	Comm. Br.
	Huc. Primo Cita Primis	Huc. Primo Cita Primis						
Super Ecce	Super Ecce	Super Ecce	Super Ecce	Super Ecce	Super Ecce	Super Ecce	Super Ecce	Super Ecce
			Huc. Primo Cita Primis	Huc. Primo Cita Primis	Huc. Primo Cita Primis			
Alia (frag.)	Alia	Alia			[Nova]	Alia (frag.)	Alia	Alia
	Argum. Mens. Cymb. Modis Quinque	Argum. Mens. Cymb. Modis Quinque	Argum. Mens. Cymb. Modis Quinque					

*Primo

If the groupings of manuscripts manifest in Table 11 agree with the relationships noted in the studies of the constituent treatises (especially the *Musica* and *Scolica enchiriades*, the *Commemoratio brevis*, and Hucbald), this agreement would substantially increase confidence in the use of manuscript contents as indices of manuscript relationships in the *Alia musica* – and, indeed, this does appear to be the case. (It should be noted, however, that all of these sources cite each other liberally; thus, there is a

certain risk that agreement amongst them results as much from referring to each other as from actual relationships among manuscripts.)

The best indicator is probably the *Musica enchiriadis*, for which an extensive manuscript study appears in Nancy Phillips' doctoral dissertation. Although Phillips does not produce a stemma, she describes the manuscripts and groupings in detail. The larger manuscript tradition of the *Musica enchiriadis* is divided into three recensions: a principal recension, the *Inchiriadon* recension, and a rare rearranged version of the *Musica enchiriadis*.²⁴⁰ The manuscripts containing multiple treatises of the *Enchiriadis* complex exclusively contain the principal recension of the *Musica enchiriadis*.²⁴¹ This principal recension is further subdivided into three subgroups, which are identified according to three different forms of a word (which Phillips suspects to be the result of three different resolutions of an abbreviation, *simpl*): *simpla*, *simplex*, and *simplum*.²⁴² These subgroups are somewhat less stable, as scribes sometimes compared multiple manuscripts when copying or correcting their own manuscripts;²⁴³ even so, the manuscripts of the *Alia musica* subcomplex contain only the *simpla* form of the *Musica enchiriadis*.²⁴⁴ The *simpla* variant is also subdivided into two smaller subgroups; in one group, the word *duplicaveris* is replaced by the word *multiplicaveris* (among other variants), and these two words have become labels for the two subgroups.²⁴⁵ Phillips places only one manuscript from the *Alia musica* subcomplex in the *simpla-duplicaveris* family; this is manuscript M, which has already been observed above to differ notably from the treatises of the *Alia musica* Principal Group (however, it is also possible that this manuscript ought also to belong to the *simpla-multi* family, but with the *Musica enchiriadis* having been corrected against

²⁴⁰ Phillips, *Musica and Scolica*, 44.

²⁴¹ *ibid.*, 66. Manuscript D also contains the *Inchiriadon* in what appears to be a separate layer (as I have already described above), but also contains the principal recension of the *Musica enchiriadis*, and none of these treatises contains the rearranged recension.

²⁴² Phillips, *Musica and Scolica*, 47.

²⁴³ *ibid.*, 61–62.

²⁴⁴ *ibid.*, 67–68.

²⁴⁵ *ibid.*, 52.

a *simpla-dupli* manuscript; St. Emmeram, where the manuscript was formerly kept, had two other copies of the *Enchiridis*, with all three manuscripts falling into separate subgroups).²⁴⁶

Phillips associates the *simpla-multi* family with the Chartres school, which seems to attempt to bring the text more closely in line with Boethius, an inclination this group shares with the revisor of the *Alia musica*.²⁴⁷ There are frequent mentions of the Chartres school throughout the manuscript studies for this entire complex of treatises. Since the *Alia musica* appears exclusively in this tradition, it is possible that the origins of the *Alia musica* may have some connection to Chartres (though much more evidence would be needed to support this hypothesis).

Most of the remaining manuscripts of this complex can be organized according to the state of the *Commemoratio brevis* found in each. None of the manuscripts in the *Alia musica* subcomplex contain the complete text of the *Commemoratio brevis*, which is present only in two extant manuscripts.²⁴⁸ One subgroup of manuscripts ends early, with the words *in deuterum excellentem*; the *excellentem* subgroup includes M, P₁, and P₂.²⁴⁹ Yet another subgroup ends even earlier, at the words *apud nos habet*; the *habet* subgroup includes R and J²⁵⁰ (it is interesting, and perhaps not coincidental, that R, the manuscript containing the most heavily truncated form of the *Commemoratio brevis*, also contains the least complete text of the *Alia musica*). Manuscripts D and O are part of the *simpla-multi* family, but do not contain any part of the *Commemoratio brevis*; A and C also lack the *Commemoratio brevis*, but Phillips does not identify the family to which they belong. Phillips suggests that the absence of the *Commemoratio brevis* in these and other manuscripts of the *Enchiridis* tradition may reflect the fact that the *Commemoratio*

²⁴⁶ *ibid.*, 59.

²⁴⁷ *ibid.*, 52.

²⁴⁸ *ibid.*, 71.

²⁴⁹ Phillips, *Musica and Scolica*, 71 (inc. fn. 44); cf. Bailey, *Commemoratio brevis*, 4.

²⁵⁰ Phillips, *Musica and Scolica*, 71 (inc. fn. 45); cf. Bailey, *Commemoratio brevis*, 4.

brevis was no longer practical;²⁵¹ the same hypothesis may explain why the *Alia musica* is not present in all sources. Figure 12 shows the relationships between manuscripts, as described by Phillips.

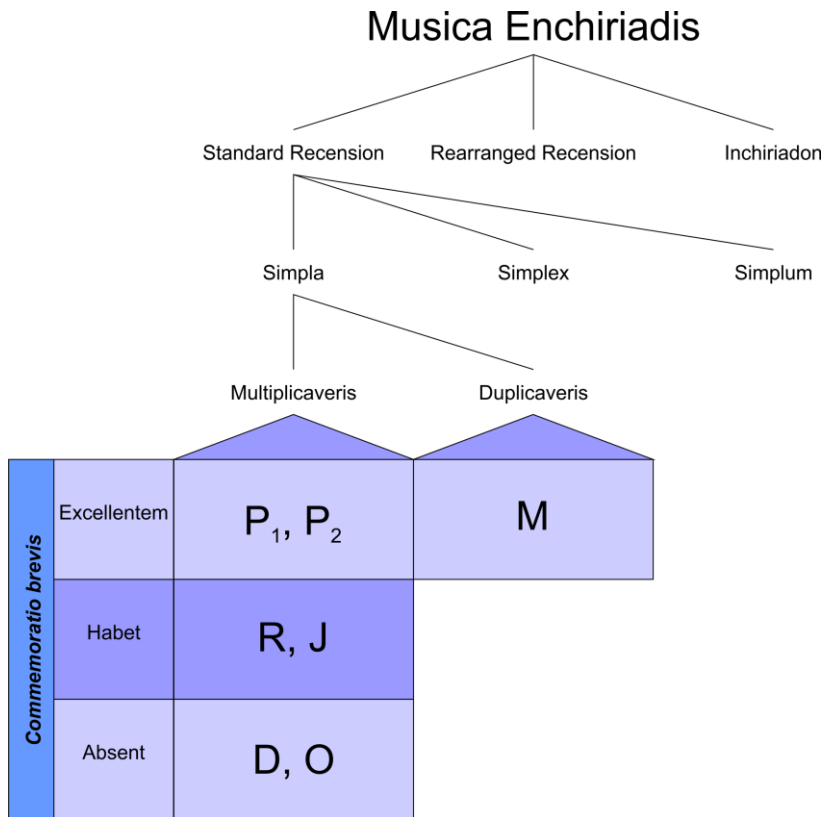


Figure 12 – Relationships among manuscripts as described by Phillips

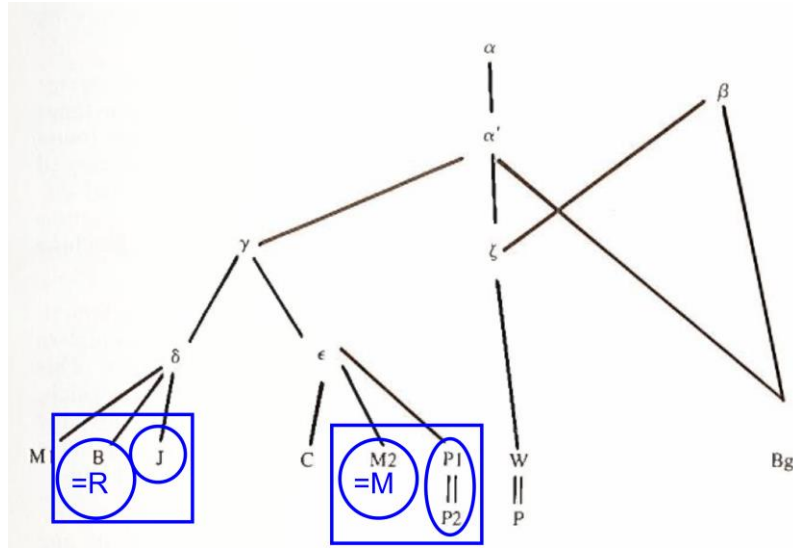
Phillips' description also agrees well with Bailey's stemma of the *Commemoratio brevis*²⁵² (Facsimile 6) and Schmid's stemma for the *Musica enchiriadis*²⁵³ (Facsimile 7). Schmid's stemma adds a small point of additional information regarding the placement of C (and presumably, by extension, A). However, this information is suspect, since he believes C to have been copied from T, a Munich manuscript formerly from Tegernsee, another source used by Gerbert but definitely not the model ordinarily supposed for it (C is traditionally supposed to have been copied from a model from Lorraine), and also with a quite different manuscript content compared to C and A (including, most notably, lacking the *Alia musica*).

²⁵¹ Phillips, *Musica and Scolica*, 72–75.

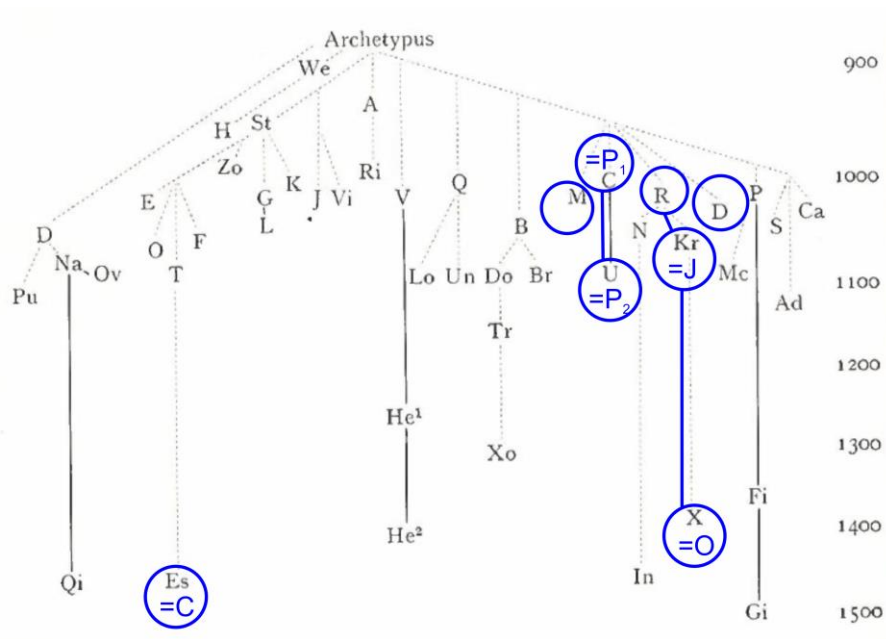
²⁵² Bailey, *Commemoratio brevis*, 5.

²⁵³ Schmid, *Musica et Scolica*, X.

Schmid does not explain his stemma, but if his analysis is based on solid premises, it implies that C may have been corrected against T during copying.

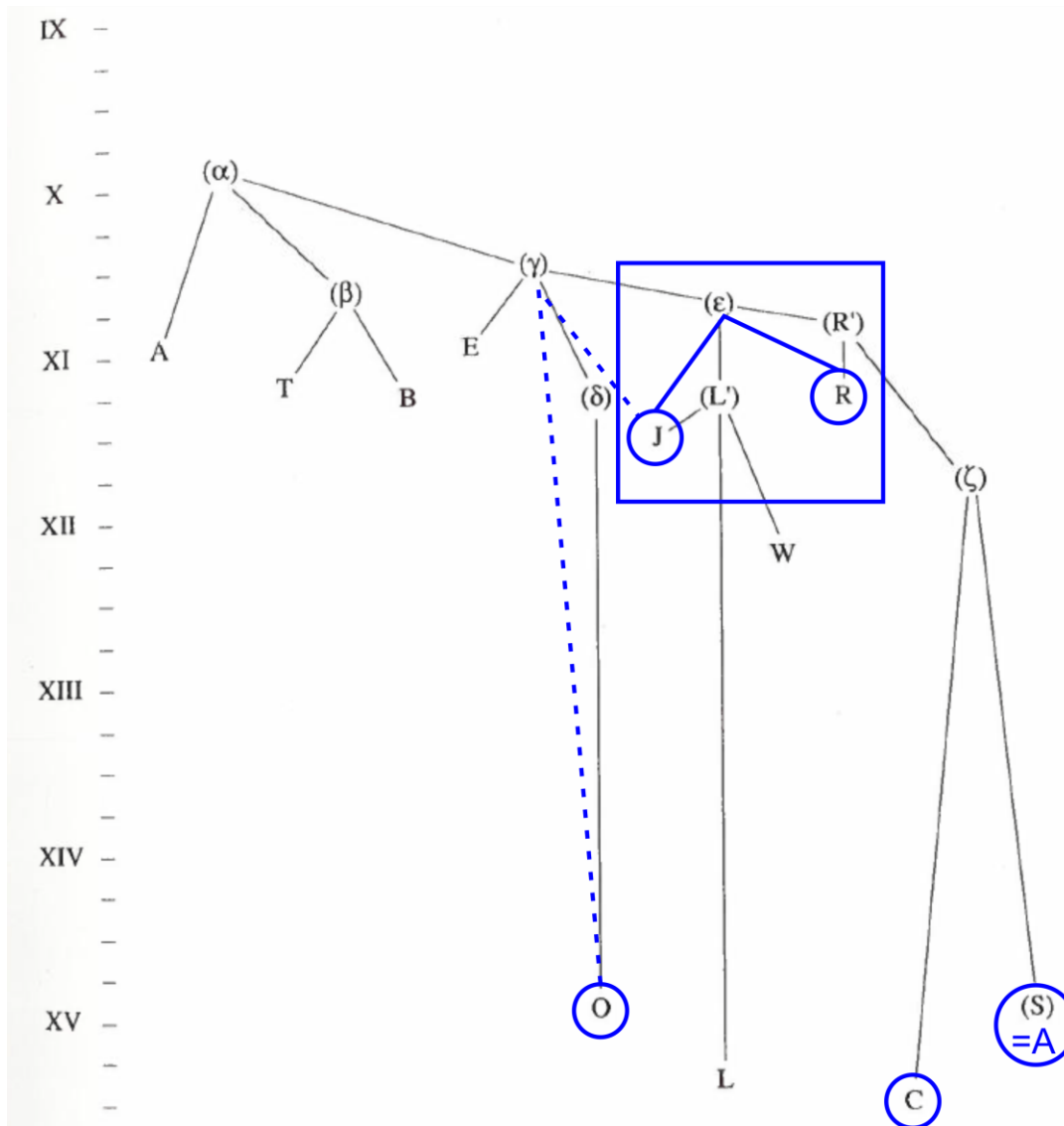


Facsimile 6 – Bailey’s stemma for the Commemoratio brevis, showing siglum equivalence and consistent groupings



Facsimile 7 – Schmid’s stemma for the Musica echiadis, showing siglum equivalence and consistent groupings

Chartier's stemma for Hucbald (Facsimile 8) also agrees reasonably well with the posited groups, though the relationships are somewhat less direct (though this difference primarily concerns the two manuscripts not containing the *Alia musica*). Helpfully, Chartier's stemma acknowledges R', a known model for R, also from Ripoll, and hypothesizes R' as an antecedent to the common model of C and A; this hypothesis seems much more likely than the placement of C in Schmid.



Facsimile 8 – Chartier's stemma for Hucbald, showing siglum equivalence and consistent groupings

In general, these various *stemmata* agree reasonably well with each other. To the degree that there is some disagreement between them, the disagreement generally appears in Schmid as a direct succession between two manuscripts that the other *stemmata* consider to be independently descended from a common model. Schmid does not explain his stemma, but this kind of disagreement is not especially surprising; where manuscript succession is not directly attested, it is often not possible to distinguish between these two possibilities. The techniques for establishing manuscript descent generally rely upon common differences and errors. If one were to suppose a hypothetical common source X, with hypothetical copies Y and Z descending from it, if Y is a perfect copy of X with no errors, and Z is a copy of X with errors, and X is lost, a researcher would be left to suppose that Z was copied instead from Y (and the distinction would not be particularly important). When one takes into account the additional complexity that copies of the *Musica enchiridis* were sometimes corrected against copies from different branches, such minor discrepancies among *stemmata* become almost inevitable.

Setting aside O and J, which were included simply as landmarks for the purpose of comparing *stemmata* for consistency, the various *stemmata* presented above provide additional details about the relationships among manuscripts containing the *Alia musica*. This comparison is particularly valuable with respect to R, the fragments of which are so brief that it would be difficult to use textual evidence to assess its relationship to the Principal Group. Chartier's and especially Bailey's *stemmata* show that R not only contains a far less complete form of the *Alia musica* than do the manuscripts of the Principal Group, but also that it is less closely related to them. Chartier's stemma shows that the shared model of A and C is closely related to R, and probably descends not from R itself, but from the source R' – or perhaps from a

source one generation earlier than R'.²⁵⁴ Phillips has also noted a close relationship between R and P₁²⁵⁵ (or, more likely, R' and P₁), while Bailey shows P₁ and M stemming from a common source (M, however, in addition to textual variants in the *Alia*, also belongs to a different text family of the *Musica enchiridis*, and so is probably slightly less closely related). Taking these stemmata together implies that all three streams of the Principal Group (AC, P, and M) descend from a common source that was also the source of R'.

The case of D is more complex. This manuscript lacks both the *Commemoratio brevis* and Hucbald's *Musica*; hence, this manuscript does not appear on either Bailey's or Chartier's *stemmata* (though it does appear on Schmid's). Phillips indicates that the manuscript contains an "ideal representative" of the *simpla-multi* text family of the *Musica enchiridis*²⁵⁶ (like nearly all of the other *Alia musica* manuscripts), and she also notes that M contains a marginal gloss that also appears, attributed to Fulbertus of Chartres, in D (and that this gloss is unique to these two manuscripts).²⁵⁷ However, D also contains the third major recension of the *Musica enchiridis*, the *Inchiriadon*, which is not found in any other manuscript in the *Alia musica* subcomplex; that said, Phillips also notes that the Boethius and *Inchiriadon* appear in a different hand from the *Musica*, and with a different page preparation, suggesting that these two sections may once have been independent manuscripts later bound together;²⁵⁸ presumably, the fragment of the *Alia musica* belongs to the same portion of the manuscript as the *simpla-multi* form of the *Musica enchiridis*.

²⁵⁴ Chartier's stemma does not show this latter possibility, but Chartier does note (citing R. Beer) that Ripoll had standing relationships with French scriptoria "*pour l'obtention des mss dont ils tiraient des copies destinées à leur bibliothèque*" (Chartier, *L'Œuvre musicale d'Hucbald*, 87) ("for the obtaining of manuscripts from which they drew the copies destined for their library.") From the direction of travel of manuscripts implied by this observation, it seems more likely that A and C descend from the model from which R' was copied than that A and C were descended from R' itself.

²⁵⁵ Phillips, *Musica and Scolica*, 68 fn. 41.

²⁵⁶ Phillips, *Musica and Scolica*, 52.

²⁵⁷ Phillips, *Musica and Scolica*, 60.

²⁵⁸ Phillips, *Musica and Scolica*, 96.

There is little in this description (or on Schmid's stemma) to refute Atkinson's hypothesis that the fragment of the *Musica* in this manuscript formed the basis for the revision portion of the *Alia musica*. There is one possible slight contradiction: the *Musica enchiridis* in D belongs to the *simpla-multi* family, a family that Phillips identifies as having been altered to agree better with Boethius (and thus, presumably, a later family), and while these circumstances might apply to nearly all of the *Alia musica* manuscripts, there is one exception: M belongs to the *simpla-dupli* family (presumably an older form). The existence of the complete *Alia musica* in a manuscript in both the older and younger text families would suggest that the complete *Alia* had already been composed before the two families split apart from each other; thus, it would seem unlikely that a member of the younger family would contain the model for a portion of the completed *Alia*. However, the history of collating and correcting the *Musica enchiridis* against manuscripts in multiple families complicates this analysis. It has already been noted that M was kept in a library also containing copies of the *Enchiridis* from two other text families, and also that M and D share common glosses present in no other manuscripts. Thus, the existence of M in an apparently older text family is hardly a refutation.

But there is, correspondingly, little in this analysis to support Atkinson's hypothesis, either. From Schmid's stemma, one can see only that all of the manuscripts of the *Alia musica* subcomplex stem from a single common exemplar, which may be little more than to say that they all stem from a manuscript containing the *Alia musica* (a rather obvious conclusion – and, since Schmid does not explain his stemma, quite possibly part of the reason why Schmid grouped them as he did); one can also observe that Schmid considers D to exist alone on a separate sub-branch from all the other manuscripts in the subcomplex, which certainly prevents D itself from serving as the archetype for the revision portion of the *Alia*, but the hypothesis is not that D was the archetype, but rather that D was a copy of the archetype in a succession branching off before the completion of the *Alia*. Hence, there is still too little evidence either to accept or reject Atkinson's hypothesis with confidence.

Textual Variants and Key Passages

The Disputed Passage on Modal Octaves (and other material shared between the *Alia* and the *Dulce*)

The longer recension of the *Dulce ingenium* includes, in addition to the *Nova expositio*, a few other fragments also shared with the *Alia musica*, part of which is the disputed passage on modal octaves. Although Bernhard's edition of the *Dulce ingenium* already contains a critical apparatus, this passage is of such importance to the study of the *Alia musica*, and the differences between the two versions is so significant, that it is worth presenting the textual variants again here.

§	Chailley	§	Bernhard
133			<i>totum deest</i>
134	(a)	80	<i>etiam deest</i>
	graeca lingua dicitur		grece dicitur
	protus ²⁵⁹		prothus
	pentachordo		pentacordo
	diapente		dyapente
	(b) tetrachordum		tetracordum
	diatessaron		dyatesseron
	requirunt		requirit
	diapason		dyapason
	evagando		evaganda
	(c) Cui scilicet diapason		Cui dyapason
	exterius additur		exterius ad acutam vel gravem partem additur
	qui emmelis, id est aptus melo, vocatur		aptus melo.

²⁵⁹ This spelling variant may be spurious. According to Chailley's critical apparatus, *protus* is found only in G (Gerbert's edition based on the lost source A), while M and P contain *prothus*, just as does the *Dulce ingenium*. However, given Gerbert's penchant for silent correction, it is entirely reasonable to suppose that A may well have read *prothus* as well. Unfortunately, C is no help in this instance, as it reads *tropus*, a clear misreading.

§	Chailley	§	Bernhard
135	(a) Sciendum quoque quod maxime proto similiter mixolydius (b) proti tangat protum (c) Et id fas est experiri	81	<i>deest</i> autem protho maxime <i>deest</i> mixolidius prothi prothum Quod fas est probari
136	(a) ecclesiasticis diapason a graviore	82	aecclesiasticis dyapason in graviori
137	(a) tertia lydii (b) diapason ab o in a vel ab a in o	83	tercia lidii dyapason ab o in a et ab a in o
138	(a) e, h, i, m (b) hypodorium, hypophrygium, hypolydium, hypermixolydium (c) finiunt ex graviore parte phrygium	84	e, b, i, m ypodorium, ypophrygium, ypolidium, ypermixolidium ex graviori parte finiunt frigium
139	(a) hypodorii dorii est. (b)- (d) Eodem modo infra speciem hypophrygii sua est, superius ascendens phrygii est. Infra quoque speciem hypolydii sua est, superius ascendens lydii est. (Et hypermixolydio similiter intelligendum est.)	85	ypodorii dorii. Sic et de ypophrygio et ypolydio, Pypermixolydioque [sic] intellegendum est.
140	(a) modulationem phrygii finit dd modulationem lydii finit ff modulationem mixolydii finit nn	86	phrygii dd lydii ff mixolydii nn

§	Chailley	§	Bernhard
(b)	hosque superius vel inferius habeat limites phrygius lydius ab y aut descendit nihilominus ab cc		hosque habeat limites supra vel infra frigius lydius ab y aut descendat nichilominus a cc
(c)	diapente diatessaron		dyapente dyatesseron
(d)	chorda diapente vero diatessaron		corda dyapente quoque dyatesseron
141	emmelim tetingirit diapente diatessaron		emmelin contingerit dyapente dyatesseron

Discussion

As my name for it suggests, the disputed passage is probably the segment with the least consistency of attributed authorship amongst the scholars who have studied the *Alia musica*. In terms of a timeline for the development of modal species theory, there is a great deal riding on this attribution, as it is the passage within the *Alia musica* that most clearly describes not only the association of octave species with modes, but also the significance of the *media* (mean, or average) note – the middle tone of particular prominence that has either a fourth above and a fifth below (authentic modes, using the harmonic mean) or vice versa (plagal modes, using the arithmetic mean); the author uses this principle to explain the difference between Modes I and VIII, which share the same octave species. Atkinson identifies this passage as a critical passage in the development of modal species theory, as the first time that the “problem of the eighth mode” had been solved, and as the source of a distinction that would continue to

be central right through the Renaissance.²⁶⁰ (I shall address the “problem of the eighth mode” in greater detail in a later chapter. For the present, it is sufficient to know that there are eight modes that must be accommodated to only seven octave species, and that in the Greek system, after the first seven modes were associated with an octave species, the eighth mode, called Hypermixolydian, was associated with the only remaining unoccupied octave of the Greater Perfect system a tone beyond the Mixolydian – hence the name **Hypermixolydian** – with an interval structure redundant to that of the Hypodorian, while in the later medieval system, the eighth mode was placed a fourth below the Mixolydian – hence, **Hypomixolydian** – with an interval structure shared with the Dorian. During the Carolingian period, the term Hypomixolydian was not yet in use, and the term Hypermixolydian might be used to represent either possibility; furthermore, it had not yet been made clear why two modes should exist that shared the same octave species – precisely the problem that the Disputed Passage is famous for “solving,” though it is far from the last word on the subject.)

Concerning the evidence that the more recently discovered manuscripts provide regarding the origin of this passage, the sentence in §139(d) is of particular interest. Chailley places the entire subsection in parentheses, because he believes that it was not present in the original form of the text. In his commentary, he says:

Le §d) semble une glose intercalée : l’auteur devait avoir passé sous silence l’hypermixolydien, à qui il consacre un développement à part ; un lecteur surpris de l’omission aura noté en marge : « On doit comprendre de même pour l’hypermixolydien », ce qui est partiellement faux et rompt la symétrie de la démonstration.²⁶¹

Subsection (d) seems to be an interpolated gloss: the author must have passed silently by the hypermixolydian, to which he consecrates a separate development; a reader surprised by the omission will have noted in the margin: “One must understand the Hypermixolydian the same way,” which is partially false and breaks the symmetry of the demonstration.

²⁶⁰ Atkinson, *Critical Nexus*, 196–99.

²⁶¹ Chailley, *Alia musica*, 200.

The context is a discussion of relative ranges of melodies between authentic and plagal modes, which are consistent among the first three *maneria*, but cause difficulties for *tetrardus*, as the composite *Alia* wavers between the conceptual Hypermixolydian (which is understood to be a tone above the Mixolydian) and the conceptual Hypomixolydian (which is understood to be a fourth below the Mixolydian, sharing its octave species with the Dorian, and which the Disputed Passage – and, within the *Alia musica*, only the Disputed Passage – defines in principle but never describes by that name).

Chailley argues that explaining the Hypermixolydian as comparable to the other plagal modes in this way is incorrect, an objection that is only valid if one assumes that the Hypermixolydian corresponds to the octave species a tone beyond the Mixolydian, as it does in Greek theory²⁶² and the middle layer of the treatise (*viz.*, §20, part of the Boethian introduction to the revision). However, as Chailley himself observes, the mode is explained shortly afterwards in a manner that makes it a proto-Hypomixolydian, corresponding to the octave species D – d; the description of the mode in §139(d) is entirely consistent with this manifestation of the Hypermixolydian.

Chailley continues by assuming that subsection (d) must therefore have been a marginal addition to an early copy of the text (he leaves unsaid that it would then have been incorporated directly into the text in another intermediate copy, as it appears in the body of the text, not in the margins, in every surviving copy of the complete treatise). In light of the *Dulce ingenium*, this possibility, already hanging on only very weak evidence, becomes even less likely, since the *Dulce ingenium* also includes the Hypermixolydian.

There are only four possibilities in this case. The first is that the text always included the Hypermixolydian, the direct antithesis of Chailley's hypothesis. The second would be that the reference to the Hypermixolydian was added initially to the *Dulce ingenium* and then copied thence into the *Alia*

²⁶² *cf.* Boethius, *De institutione musica*, bk. IV, ch. 17, ed. Friedlein, 345–47, trans. Bower, 158–59.

musica, which agrees with Chailley's belief that the reference to Hypermixolydian was not original to the text, but invalidates the arguments through which he supports his opinion – specifically, the belief that the reference to the Hypermixolydian began as a marginal gloss and that the author must have originally skipped the Hypermixolydian because he addresses it elsewhere; the later passage in which it is discussed in the *Alia musica* is not present in the *Dulce ingenium*, and it therefore could not have influenced the decision to include or omit the Hypermixolydian under this hypothesis. The third possibility is that Chailley is correct, and that the correction was originally made in the *Alia musica* and then copied into the *Dulce ingenium*; as I have already discussed (and as I shall reiterate below), the flow of ideas does not seem particularly likely to have moved in this direction. With respect to the Disputed Passage, the form in the *Alia* tends to be the more elaborated form, and such is certainly true for the excerpt in question, in which subsections (b) – (d) in the *Alia* correspond to a single, concise statement in the *Dulce ingenium*. The final possibility is independent correction in both treatises, a hypothesis that is not impossible but compounds the continuing accrual of coincidences concerning the passages shared between the *Alia musica* and the *Dulce ingenium* to the point of straining credulity. The truth is almost certainly the first possibility, that the reference to the Hypermixolydian was always part of the text – and, by extension, that whichever contributing author ultimately incorporated the Disputed Passage into the composite treatise saw no contradiction in this description of the Hypermixolydian.

Mühlmann divided this passage amongst two hypothetical authors. He felt that §§133–42 were the work of a *Theoretiker der Principales und Subjugales*, to whom he also attributes the first half of the introduction of the revised treatise²⁶³ (a passage based on Boethius that conveys the basics of proportion

²⁶³ Mühlmann, *Die Alia musica*, 51–55. Note that Chailley erroneously claims that Mühlmann omits §142 from the passages attributed to this author (Chailley, *Alia musica*, 10–11), probably due to that fact that between §§141–42, Mühlmann interpolates a brief passage from §§26–27 (Mühlmann, *Die Alia musica*, 55), from a section otherwise attributed to a *Kommentator des Principales*, which describes the *echemata* of the *protus* modes in terms of species of fourth. Mühlmann describes this passage with the footnote “Vom Kommentator erweitertes und an eine andere Stelle versetztes Stück des Theoretikers der Principales” (*ibid.*) (“A piece of the ‘Theorist of the *Principales*”

and perfect consonances, and – importantly for the attribution of both passages to one author – also describes the harmonic and arithmetic means). Mühlmann attributes the remainder of the passage (§§143–45) to the *Kommentar*,²⁶⁴ the supposed contributions of which essentially correspond to Atkinson’s author δ ²⁶⁵ [Commentator].

Unlike Mühlmann, both Chailley and Atkinson consider the entire passage from §133 to §145 to be the work of a single author, but they disagree on which one. Chailley attributes this passage to the author of the *Nova expositio* tonary, his so-called “second *Quidam*.”²⁶⁶ All three scholars posit a single author for the entire tonary of the *Nova expositio*, who more or less correspond to one another; however, only Chailley (and presumably Heard, following Chailley’s lead)²⁶⁷ also attributes the Disputed Passage to this author. Atkinson prefers to attribute this entire passage to his author δ [Commentator].

However, in view of evidence from the *Dulce ingenium*, which was not available to Mühlmann, Chailley or Heard, it is possible that none of these proposals is correct, though each scholar appears to have seen part of the picture. By comparison to the *Dulce*, it is possible to attribute greater or lesser confidence to the varying hypotheses concerning which passages properly belong to the *Nova expositio*.

and *Subiugales*’ expanded upon by the commentator and moved to another place” – *i.e.*, the commentator took this brief passage from its presumed proper place between §§141–42 and elaborated upon it, then moved the newly enlarged section to near the beginning of the treatise, following the introduction, while Mühlmann has restored it to its presumed original place). However, this passage does not appear in any manuscript not containing the entire *Alia musica*, while the preceding passage in Mühlmann’s ordering does, and so there does not seem to be sufficient evidence to justify the belief that it was originally part of the passage in question.

²⁶⁴ Mühlmann, *Die Alia musica*, 60–69, esp. 68.

²⁶⁵ Except that in each passage where Mühlmann sees the hand of the *Kommentator*, Atkinson does not see his author δ beginning until slightly later.

²⁶⁶ Chailley, *Alia musica*, 196–204.

²⁶⁷ Heard, *Alia musica*, 15. Heard does not explicitly identify the passages that he believes belong to each author, but he agrees with Chailley that there were three authors, and he agrees about the approximate dating of the three phases represented by those authors; his only point of disagreement with Chailley seems to be a philosophical point about the relationship between the authors and the appropriate final form of the composite treatise.

It is important to remember that there are two different forms of *Dulce ingenium*, and what each has to say about the disputed passage is inconsistent, but this inconsistency, too, is revealing. The disputed passage is entirely absent from the shorter version of the treatise, which appears in the Paris and Bruges manuscripts.²⁶⁸ The more extended version, from the Prague manuscript, however, is a different matter. In this manuscript, after a discussion of the diatonic, chromatic, and enharmonic genera, follows the majority of the disputed passage, from §§134–41.²⁶⁹ §§142–45 are absent from this manuscript (and Atkinson is therefore probably correct to attribute them to the Commentator). This section is then followed by passages found elsewhere in the *Alia musica*, drawn from §162, §§166–68, and §170, all of which Chailley attributes to the principal treatise, and which Atkinson distributes among his authors β and δ [Revisor and Commentator]. There are then several more passages shared with the shorter version of the *Dulce ingenium* before the tonary common to both the *Dulce ingenium* and the *Alia musica* begins.

There are, of course, three possible explanations for the shared material in these two treatises. The first is that the *Alia musica* served as the model for these passages in the *Dulce ingenium*. Atkinson appears to support this hypothesis when he observes that at least in the case of Pr, “we can be reasonably certain [...] that the compiler had the complete *Alia musica* before him, and not just one or more of its constituent parts,”²⁷⁰ though he does not make the same observation for the shorter recension of the *Dulce ingenium*. It is my opinion that this first hypothesis, while plausible, is not so certain, for a variety of reasons. The *Nova expositio* was divided up and distributed in many places throughout the *Alia musica* – and more importantly, so, too, were the additional shared passages; it is not clear why the author of the *Dulce ingenium* would include these specific additional passages from the *Alia musica*, extracted from the

²⁶⁸ cf. Bernhard, *Dulce ingenium*, 13; 14–26.

²⁶⁹ *Dulce ingenium*, §§80–87 [long recension], ed. Bernhard, 35–37.

²⁷⁰ Atkinson, *Critical Nexus*, 174.

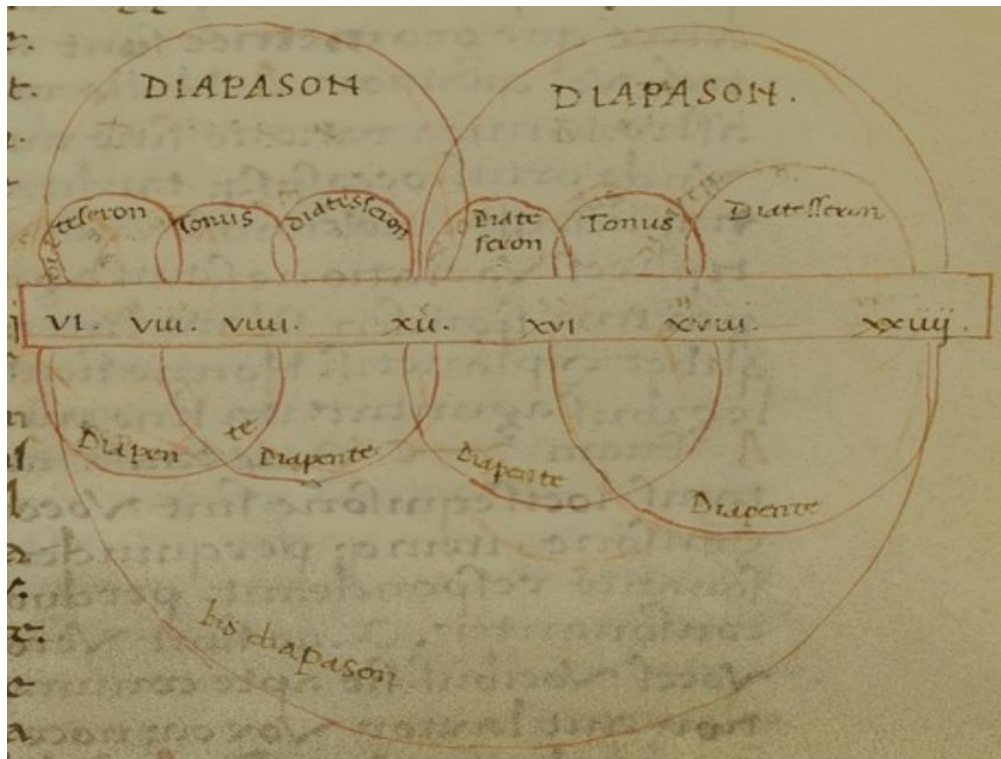
other material around and between them. It is also noteworthy that in some of these passages, the *Alia* continues to use its framing device, clearly indicating that the author is reporting on the ideas of another. A good example is §166, which begins, “*Tandem, quia dixit...*”²⁷¹ (“Finally, since he says that...”). These framing statements are not present in the *Dulce ingenium*. Thus, the *Alia musica* acknowledges citation from another author, while the *Dulce ingenium* does not.

By contrast, one possible piece of evidence might argue in favour of this model: §140 (c) – (d) compare the relationship between the two boundaries of a mode’s range and its mean pitch (*finalis* for the plagal modes and *affinalis* for the authentic modes – more on this in a later chapter) to the arithmetic and harmonic means, specifically in the form of the relations 6:9:12 and 6:8:12, which is very much in accordance with the general approach of the *Alia musica* and is not common in other Carolingian treatises. This passage is reproduced faithfully in the *Dulce ingenium*, which otherwise does not use these relations. This presentation is somewhat surprising because both of these relations could be expressed in lower terms (2:3:4 and 3:4:6, as in Boethius),²⁷² and are instead expressed in the very same terms as in the *Alia*. But while these relations are unusually prominent in the *Alia musica*, they are by no means unknown in other treatises. In particular, these numbers occur in comparable demonstrations in the *Scolica enchiridis* (see Facsimile 9) and are also the lowest terms in which both means can be expressed within the same boundary terms (the basic arithmetical procedures for deriving these larger versions of the relations are found in Boethius), so the possibility of independent derivation is also not entirely unlikely (to say nothing of the many Greek treatises that use these numbers, as the relation 6:8:9:12 is fundamental to Greek theory).

²⁷¹ *Alia musica* §166(a), ed. Chailley, 170.

²⁷² Boethius, *De institutione musica*, bk. 2, ch. 15, ed. Friedlein, 245–46, trans. Bower, 68–70.

Facsimile 9 - Diagram from the *Scolica enchiriadis* from which the relations 6:8:12 and 6:9:12 could be easily derived. Cesena XXVI, f. 155v.



Additional evidence supporting the view that the *Dulce ingenium* borrowed the disputed passage from the *Alia musica* comes from §141, which describes the doctrine of the *emmelis* in a manner entirely consistent with its use in the *Alia musica* (which is quite idiosyncratic, and will be discussed at length in the second half of this dissertation); it is not used in this manner elsewhere in the *Dulce ingenium*.²⁷³ Because of this passage, it becomes difficult to imagine that the *Alia musica* was not, in some way, the source of this passage. However, this conclusion does not require that the *Dulce ingenium* borrowed the passage directly from the completed form of the *Alia musica*, for if so, one is left with a vexing question: immediately after the passage just cited, the *Dulce ingenium* breaks with the *Alia* in its discussion of modal octaves, and skips to a different set of excerpts also shared with the *Alia musica*, but separated in the *Alia*

²⁷³ The term *emmelis* does appear elsewhere in the *Dulce ingenium*, in a passage drawn from Regino which is in turn based on Boethius, but this use corresponds clearly to the conventional use of the term as an adjective describing pitches that are carefully tuned (and therefore suitable for melodies), rather than the idiosyncratic use in the *Alia musica* as a noun describing a pitch one step beyond the theoretically correct boundaries of a melody which may nevertheless be used in the melodies, much in the way that later writers describe a modes as consisting of an octave plus a tone “by license.”

by a considerable amount of other material. Meanwhile, the *Alia* continues to discuss the modal octaves, and two sections later (§143) finally lays out explicitly the difference between Modes I and VIII with respect to their shared modal octaves. This passage is very short and very clear, and at least to a modern reader, feels very much like the apotheosis toward which the entire preceding discussion of modal octaves was building. It is very difficult to explain why the author of the *Dulce ingenium* would go to the trouble to copy the entire preceding passage, only to quit an inch from the finish line (the solution to what Atkinson calls “the problem of the eighth mode”).²⁷⁴

²⁷⁴ Atkinson feels strongly that the material shared between the *Alia* and the longer *Dulce* was borrowed into the *Dulce* from the *Alia*. He raised this issue at my dissertation defence, and also in the written feedback that he provided after the defence. He noted that the longer version of the *Dulce* is essentially a patchwork of several earlier treatises, joined together in a not particularly smooth or consistent manner. He proposes that the reason that the *Dulce* stops copying the *Alia* at the point where it does is because immediately afterwards, the *Alia* mentions the synemmenon and the associated b, a concept not otherwise employed in the *Dulce*, and in particular, not needed to prepare for the *Nova expositio* that follows in the *Dulce*. Atkinson’s argument is compelling, especially since, as he points out, if the *Alia* had borrowed the passage from another treatise and then added the conclusion about the eighth mode, then to the same extent that I ask why one might copy the passage but omit the explanation of the difference between modes I and VIII, one might just as easily ask why a hypothetical original form of the passage also did not follow through to explain this difference. Additionally, one may also observe that, like the *Dulce*, the *Alia* also makes no significant use of the synemmenon (outside its use in the Disputed Passage, there is only one passing mention of the synemmenon, in the discussion of the fifth mode in the revision, which happens earlier in the treatise; the synemmenon is certainly not in the *Nova expositio* or at any later point in the *Alia* any more than it is used at any later point in the *Dulce*, so that its introduction in the Disputed Passage serves no more purpose in the *Alia* than in the *Dulce*). Furthermore, since the longer recension of the *Dulce* also shares other sections with the *Alia* in which brief sections are missing from the *Dulce* compared to the *Alia* and are substantially rearranged (see pages 141–142 below), it is clear that the author of the *Dulce* had no problem manipulating the text that he was borrowing; thus, while Atkinson’s hypothesis could explain why the author of the *Dulce* stopped before the very brief passage describing the synemmenon, it would not explain why he would not simply skip over that passage, as he does elsewhere, and continue onward to explain the difference between modes I and VIII – an explanation in which the synemmenon is not implicated. Atkinson further proposes that the author of the *Dulce* did not bother to explain the difference between modes I and VIII because “for him it wasn’t a problem” (dissertation written feedback). While I acknowledge that the problem may seem more significant to a modern musicologist than to a medieval musician, it is not clear to me why the distinction between modes I and VIII should have seemed more of a problem to the author of the ostensibly earlier *Alia musica* than the apparently later *Dulce ingenium* when the association of modes with octave species tends to be a later phenomenon, nor how any treatise, including the *Dulce*, could conceive of defining modes according to octave species without clarifying the difference between modes I and VIII if the author himself had a clear understanding of the difference (and in this case, if the author of the *Dulce* borrowed this passage from the completed *Alia*, he had access to such an explanation in the *Alia*). I certainly do not intend to suggest by any of this argumentation that Atkinson’s hypothesis is not correct. Indeed, I think that it is quite compelling, especially in light of the carelessness with which the longer recension of the *Dulce* seems to have been assembled. I only disagree that it is “reasonably certain,” because none of his arguments rule out the possibility that both treatises borrowed the shared material from a common source.

Although the problem just described may be anachronistic (what appears to be a very important passage to a modern musicologist may have seemed rather less important to a medieval theorist), there is a fairly straightforward solution. Theory treatises do not come into being *ex nihilo*. At the very least, they come into being as the culmination of extensive study and deliberation on the part of the author, but by extension, they also usually represent the doctrines not only of the authors themselves but also of their local musical subcultures. That is, whenever and wherever the *Alia musica* was originally written, it likely represents the teachings not only of its authors but of the community where it was written. If so, then the emphasis on the relation 6:8:9:12 and the idiosyncratic use of the term *emmelis* might well be common to that whole community, and the source of the Disputed Passage might well also originate from that community. (In fact, similar origins for these two treatises would not be greatly surprising, since the Disputed Passage seems to be a logical next step compared to the passage in §15 that first associates the modes with octave species in the Boethian introduction). It is also not unreasonable to wonder whether the common source might even have been the *Alia* itself, but at an intermediate state that did not yet contain the sections not included in the *Dulce* (this is a state for which no manuscripts have survived but that seems highly likely to have once existed, if the assumption that the revisor and commentator are separate authors is correct). Interestingly, Mühlmann, though he lacked any information about the *Dulce ingenium*, came to exactly this conclusion by different means, assigning the shared portion of the Disputed Passage to his *Theoretiker der Principales und Subiugales* and the culmination of the argument to the *Kommentar*. This possibility also would be more or less consistent with Atkinson's opinion that the compiler of Pr "had the complete *Alia musica* before him."²⁷⁵

The second possible explanation for the material shared between the *Dulce* and the *Alia* is that the *Dulce ingenium* served as the model for the *Alia musica*, and the third is that both the *Dulce ingenium* and

²⁷⁵ Atkinson, *Critical Nexus*, 174.

the *Alia musica* both borrowed from another, no longer extant model (or perhaps two or three models, though each additional proposed model decreases the likelihood of this hypothesis, as both authors would have to have independently had access to and found value in each source; thus, a single source for all of these excerpts is more likely). Both of these possibilities have effectively the same repercussions for the *Alia musica*: none of the shared material is likely to be the work of either the Revisor or the Commentator, but the possibility that the Disputed Passage originated in the *Dulce ingenium* can probably be dismissed in view of the remarkable fit between the doctrines of this passage and those of the *Alia* and the lack of fit with the rest of the *Dulce*.

The fact that the disputed passage (and the material that immediately follows it in the *Dulce ingenium*) only appears in one version of the *Dulce ingenium* and not the other adds additional context. As noted, the disputed passage is a passage of considerable consequence in the development of modal species theory, and as such, is a passage that seems less likely to be dropped in the act of abridging a treatise (particularly since interval species and modes are both treated in the shorter recension); it therefore seems more likely that the longer treatise is an elaboration than that the shorter version is an abridgement. This assessment is certainly not conclusive, based as it is upon a modern assessment of the importance of the passage, rather than any evidence that the importance of the passage was recognized in its own time. But the assessment is compatible with Bernhard's opinion that "*Trotzdem konnte Pr dieselbe Vorlage wie PaBr gehabt haben*"²⁷⁶ ("Still, Prague could have had the same model as Paris and Bruges").

Since both versions of *Dulce ingenium* contain the tonary, while only one version contains the other passages shared with the *Alia musica*, there is also cause to question whether the source of both sets of passages (*i.e.*, the tonary vs. the other two passages) is the same; it seems less likely that the original

²⁷⁶ Bernhard, *Dulce ingenium*, 2.

version of *Dulce ingenium* would borrow some material from a source, and then that a later elaboration would return to the same source and borrow more material, especially when the disputed passage, which deals with octave species, presents such a useful supplement to the tonary, each section of which begins with the identification of a corresponding octave species. Thus, Chailley's opinion that the passage is the work of the author of the *Nova expositio* is probably also incorrect.

Instead, it seems more likely that the disputed passage was borrowed from a separate source by a different author. If one accepts this hypothesis, it becomes largely immaterial whether the Prague version of the *Dulce ingenium* served as the source for the *Alia musica*, or whether both treatises borrowed from another source. But as a matter of probability, it would be a substantial coincidence that the very same two treatises (*Alia musica* and *Dulce ingenium*) that already borrowed the *Nova expositio* would also borrow the same passages from another source, without at least one treatise having been aware of the other.

This coincidence suggests that the tonary of the *Nova expositio* may have already been incorporated into either a proto-*Alia* or proto-*Dulce* and then borrowed into the other treatise. The remaining material shared between the two treatises was then borrowed, probably not from the same source as the tonary (though it is also possible that the elaborator of the long recension of the *Dulce* wrote this material himself), and included in the expanded version of *Dulce ingenium* as given in the Prague manuscript, and also in the *Alia musica*. There are several reasons to prefer the *Dulce* as the source of the *Nova expositio*, rather than the *Alia*, including the fact that the *Alia* splits the tonary up and introduces it as "someone's new exposition." But neither the *Alia* nor the *Dulce* in any extant form is likely to be the original source of the treatise, since both contain the same errors. As for the other shared material, these passages were then distributed throughout the *Alia musica* in locations in which they seemed most appropriate; and finally, the Commentator, who may or may not have been the person who incorporated

the passages into the treatise, expanded upon the disputed passages, providing the otherwise unaccounted for §133, §§142–45, and perhaps at least some of §§154–61, as well as §§163–65 and §169.

This model helps to explain some oddities in §§161–70 of the *Alia musica*. Amongst these ten sections, only five (and sometimes only parts of each) are found in *Dulce ingenium*, and not in the same order as in the *Alia*. The order in *Dulce ingenium* is §§166–68(a), §170(a), §162(a), §170(d)–(e). Omissions notwithstanding, the most significant change here is that in the *Dulce ingenium*, §162 stands almost at the end of the passage, functioning as a kind of summary of the ideas previously presented, while in the *Alia musica*, it appears before these ideas, requiring a new introduction to be written to explain the five perfect intervals, so that the material in §162 makes sense. However, even with the introduction, the passage does not quite make sense: the introduction explains only the five perfect consonances, but says nothing about the eleventh as compound fourth, which the authors of the *Alia* do not list as a perfect consonance (this is consistent with many medieval treatments, in which the compound fourth is not a perfect consonance because it cannot be expressed as multiple or superparticular ratio);²⁷⁷ however, §162 nevertheless attributes it to the seventh mode, although it had not been mentioned elsewhere in the treatise.²⁷⁸ This numerology (which does not seem to reflect any known modal phenomenon) is also in partial contradiction to the numerology displayed throughout the rest of the *Alia musica* (especially the Source Treatise). By contrast, the interval of a compound fourth is mentioned (though not approved as a perfect interval) a few pages earlier in the *Dulce ingenium*,²⁷⁹ and so, would need no further introduction

²⁷⁷ Boethius describes this as the opinion of the Pythagoreans in *De institutione musica*, Book II Chapter 27 (Friedlein, *Anicii Manlii Torquati Severini Boetii*, 259–60; Bower, *Fundamentals of Music*, 81–82); however, he also presents Ptolemy's dissenting opinion in Book V Chapter 9 (Friedlein, 358–60; Bower, 169).

²⁷⁸ With the exception of §156(c), a section that is probably corrupted (Chailley, *Alia musica*, 97) and whose wording borrows so thoroughly from §162 that it is probably best explained as an interpolation by the Commentator to try to tie things together.

²⁷⁹ Bernhard, *Dulce ingenium*, 21, §48.

(and there is little other numerology in the *Dulce ingenium* with which to conflict). The two versions of this material, omitting the additional commentary in the *Alia musica*, read as follows:

Alia musica:²⁸⁰

[§162(a)] On this account, the first trope will be in the double proportion [2:1, octave], the second in triple [3:1, twelfth], the third in quadruple [4:1, double octave], the fourth in sesquitercian [4:3, fourth], the fifth in sesquialter [3:2, fifth], the sixth – like the third – in the octave plus a fifth [twelfth]; the octave plus a fourth [eleventh(!)] in the seventh trope; and a double octave in the eighth, together with the double [octave] and triple [twelfth].

[§166] Finally, he says that some of the tones are composite and others simple, it is to be known that the composition of such a kind of these [the composite ones?] is just as among grammatical things, a whole out of two parts, as is “*magister equitum*” [lit. “master of the horse,” but here simply standing as an example of a compound word – a unified concept named in two parts]. [§167] While the first tone is the noble consonance of an octave, and moreover, the fifth [tone] is made of a singular [perfect] fifth, the second is composed of both consonances, as it spans a triple proportion [a twelfth, or an octave plus a fifth]. [§168(a)] Since indeed, as he has said, the gradual *Universi qui te expectant* runs through the entire octave species of the Hypodorian, and a fifth above, it is able to complete the triple [proportion].²⁸¹

[§170(a)] Truly, simple tropes are those that are contained in one *symphonia* [perfect consonance]. [§170(d–e)] About [the fifth trope], Boethius said this: [the mode that] is first and simplest we name Lydian.²⁸² Whence Plato instructed that it does not at all behoove boys to be taught all the modes, but rather the strong and simple ones.²⁸³

Dulce ingenium:²⁸⁴

⁸⁸It is to be known that some of the tones are composite and others simple. And the composition of such a kind of these is just as among grammatical things, a whole out of two parts, as is “*magister equitum*.” ⁸⁹While the first tone is the noble consonance of

²⁸⁰ cf. Chailley, *Alia musica*, 168–72.

²⁸¹ It should be noted that the gradual *Universi qui te expectant, non confundentur, Domine* does not freely range over the entire twelfth throughout the whole chant. The respond spans only the modal octave of the plagal *protus*, while the verse, *Vias tuas, Domine, notas fac mihi: et semitas tuas edoce me*, spans the complete modal octave of the corresponding authentic mode, a common enough situation for graduals (cf. Apel, *Gregorian Chant*, 167). It is the union of the ranges of both the respond and the verse that collectively spans a twelfth.

²⁸² This citation is a nonsensical attempt to lend the authority of Boethius to a post-Boethian concept. The sense in which the Lydian mode (which Boethius would have understood as a transposition-scale) is first and simplest in ancient Greek theory is equivalent to the sense in which C-Major is first and simplest in the modern tonal system. It has nothing to do with the Lydian mode being associated with a simple (*i.e.*, non-compound) characteristic interval.

²⁸³ Here, again, a citation is made to lend spurious authority, as the modes known to Plato (and discussed without any kind of theoretical detail in the *Republic*) are entirely unrelated to those of the *Alia musica* – and Plato explicitly rejects the Lydian mode that he knew because it was too soft; he approved only of the Dorian and Phrygian (Strunk, *Source Readings*, 10–11; cf. Chailley, *Alia musica*, 172).

²⁸⁴ cf. Bernhard, *Dulce ingenium*, 37.

an octave, [and] the fifth is made of a singular fifth, the second is composed of both consonances, as it spans a triple proportion. ⁹⁰Since indeed, the gradual *Universi qui te expectant* runs through the entire octave species of the Hypodorian, and a fifth above, it is able to complete the triple. ⁹¹Truly, simple tropes are those that are contained in one *symphonia*, such as are the fourth and the fifth. ⁹²Of these, they individually have thus: the first trope is in the double proportion, the second in triple, the third in quadruple, the fourth in sesquitercian, the fifth in sequialter, the sixth – like the third – in the octave plus a fifth. The octave together with a fourth in the seventh trope, and a double octave in the eighth, together with the double and triple. ⁹³Of the fifth, Boethius says this: this is first and simplest that we name Lydian. Whence Plato instructed that it does not at all behoove boys to be taught all the modes, but rather the strong and simple ones.

It is likely that the presentation in the *Dulce ingenium* is a more faithful copy of the source of this passage, or perhaps even was the original source of the passage. In addition to the apparent better flow of the text as it appears in the *Dulce ingenium*, Chailley also acknowledges at several points in his commentary to these passages that the text does not make sense in the context of the *Alia musica*. In regards to §162, he says, “*On aura naturellement noté le manque total de méthode de toutes ces recherches numériques sans valeur musicale*”²⁸⁵ (“One will have naturally noticed the total lack of method to all these numerical investigations without musical value”). Regarding §169, a short elaboration on §168, Chailley notes that “*aucune de ces proportions ne figure dans les nombres du 2ème ton (12, 16, 18) ce qui montre l’incohérence des exégèses de l’Alia*”²⁸⁶ (“not one of these proportions [24:8 or 18:6, which might represent the triple proportion in Mode II] figures in the numbers of the second tone (12:16:18), which shows the incoherence of the exegeses of the *Alia*”). By contrast, there is no concern with modal numbers in the *Dulce ingenium*, and therefore, no contradiction.

This kind of inconsistency lends further support to the hypothesis that all of these passages, including the critical Disputed Passage regarding octave species, have been borrowed from the long recension of the *Dulce ingenium* itself – or, more likely, that both treatises independently borrowed these

²⁸⁵ Chailley, *Alia musica*, 168.

²⁸⁶ Chailley, *Alia musica*, 171.

passages from a third source. Perhaps the significance of the *Alia musica* in the development of modal species theory ought to be reconsidered, as the passages on which this pride of place is established seems to have come from another treatise (though the *Alia* still merits a place in the narrative; the Disputed Passage is certainly the most innovative passage about modal species theory in the *Alia musica*, but it is not the only such passage, as Atkinson notes²⁸⁷).

The Prose Summary

If one were to remove the tonary of the *Nova expositio* and the appendix-like source treatise (and ignore the distinction between the revision and the commentary), as Chailley did, then the overall form of the remainder of the treatise – what Chailley calls the “Principal Treatise” or the “*Alia* proper”²⁸⁸ – can be outlined as follows: Boethian Introduction, Revision of Source Treatise, Supplement, Tabular Summary, and Prose Summary. This form is a very standard and expected rhetorical form in all respects except one.

The core of the treatise is, of course, the Revision. In such an edition, it is to be expected that the revisor would add an introduction to provide some context for the complex number theory of the Source Treatise. It is also not surprising to see the supplement, which explores some of the ramifications of the number theory. Finally, because the number theory is widely dispersed throughout the composite treatise, it would have been surprising if the revisor had not included some form of summary.

However, the “Principal Treatise” contains not one, but two summaries, one immediately following the other. The first is a table that effectively speaks for itself; the second is a prose summary that very much resembles a transcript of someone reading out the table to an audience (with the exception of the arithmetic sums that are included in the prose summary but not the table, the significance of which is not

²⁸⁷ Atkinson, *Critical Nexus*, 186f.

²⁸⁸ Chailley, *Alia musica*, 7.

clear). But there is another critical fact about the prose summary that needs to be considered: like the source treatise and the tonary, both of which are considered to be independent units, the prose summary also appears independently in the manuscript tradition.

Specifically, the prose summary appears entirely independent of any other part of the *Alia musica* in two manuscripts: F₁ and F₂. These two Florence manuscripts contain the same sections, and as I have noted above (see the description of F₂ above), it is generally believed that F₂ was copied from F₁. Mühlmann, Chailley, and Heard were apparently unaware of these manuscripts. Atkinson does note the independent transmission of the prose summary in these two manuscripts – and also notes that a part of the introduction that describes the octave species appears independently in D – but stops short of directly concluding that these passages were borrowed into the *Alia musica* from earlier sources (though he does acknowledge the likelihood that at least some of the fragments were).²⁸⁹

Caution about this possibility is warranted, as it is also possible that these passages were simply excerpted from the *Alia musica*. But caution notwithstanding, it is possible to compare the relative likelihoods of the two possibilities. The Prose Summary seems very much out of place in the composite treatise, appearing as it does immediately after the tabular summary, in relation to which it is entirely redundant. It provides no new information, and merely does in long form what is already done far better by the table: it provides a quick reference of all the modal numbers and the intervals that they represent. The question then becomes, how likely is it that a single author of a unified work would deliberately structure his own contributions in this manner?

While both the table and the prose summary are independently valuable additions to the *Alia musica*, it seems unlikely that a single author would feel the need to summarize the work in both ways. But even if an author were to wish to summarize in both manners, it is far more likely that he would

²⁸⁹ Atkinson, *Critical Nexus*, 174.

choose to do the prose summary first, and place the table after it, proceeding in the direction of ever more compressed summary. Instead, in the final form of the treatise, they appear in the reverse order. This is by no means conclusive; authors will do as they wish. But the prose summary feels quite superfluous after the tabular summary, while the table would not seem so after the prose summary.

The placement of the prose summary does make sense in a different light, however – especially in view of the fact that it enjoys independent transmission in surviving manuscripts. It appears very near the end of the composite treatise, just before the source treatise, which also enjoys independent transmission (and almost certainly existed as an independent entity before the creation of the principal treatise). Additionally, like the prose summary, the presence of the source treatise is highly redundant. Its presence at the end of the composite treatise is fortuitous from a research perspective because it implies much about the development of the treatise, but pedagogically, it contributes even less than the prose summary, being nothing more than a contiguous repetition of passages already given (and scarcely altered) earlier in the treatise. The function of the source treatise at the end of the composite treatise appears to be little more than an appendix. Perhaps, then, the prose summary, placed immediately before this appendix, also functions as a kind of appendix, revealing the textual lineage of the treatise. At any rate, this hypothesis seems more likely than that the tabular summary was insufficient, and a prose transcription of the same material was also deemed necessary.

If true, this hypothesis raises a critical question in the history of the treatise: what is the relationship between the prose summary and source treatise? There are at least three possibilities. Perhaps the least interesting is that the prose summary and the source treatise represent two independently written accounts of the same modal doctrine, possibly issuing from disciples of the same teacher (or, at least, the same cultural centre, an hypothesis that I have already proposed in connection with the *Dulce ingenium*). However, the very close similarity in some of the wording between these two sections probably makes this possibility the least likely hypothesis.

A more interesting possibility is that the prose summary is not actually a summary at all. It is possible that the “summary” came first, as a simple listing of the modal numbers for each mode, which would be sufficient for anyone who already understands the system, and merely needs to record the numbers themselves. In this possibility, the “source treatise” would then become an elaboration upon the “summary” (which, itself, was then elaborated upon again by the principal treatise). The position of the prose summary in the composite treatise immediately before the source treatise, rather than after, provides a small measure of support for this hypothesis.

The other likely possibility is that the prose summary originally served as a summary to the source treatise, not the revision, and was then included in the composite treatise for the same reason that the source treatise was. If this possibility is correct, it is slightly odd that it should appear before the source treatise, rather than after, but the hypothesis does have one significant piece of evidence in its favour. The source treatise enjoys independent transmission in exactly one surviving manuscript: Karlsruhe 504. What is most interesting about this manuscript is that the end of the source treatise is slightly different in this manuscript compared to the others.

The source treatise proceeds through most modes with brief commentary in only a few places, but generally only listing the modal numbers and providing examples of two to three introits and two to four antiphons (in that order) that demonstrate the qualities of each of the first six modes. However, the seventh and eighth modes lack any examples in the source treatise in any of the manuscripts. In fact, in most manuscripts, there is not even a description of the modal numbers of the eighth mode. After the description of the modal numbers of the seventh mode, which ends with §187(e), all manuscripts except Karlsruhe proceed immediately to §188, which reads, “*Tonum octavum require supra*”²⁹⁰ (“Seek the eighth tone above”). Chailley interprets this passage to mean that the original text after §187(e) was lost; he

²⁹⁰ Chailley, *Alia musica*, 94–95.

therefore supplies examples for the seventh mode based on those provided in the revision, using the same phrasing that the author of the source treatise used in the other modes, completely ignoring the fact that the chants that he calls antiphons are, in fact, responds.²⁹¹

However, the Karlsruhe manuscript ends differently. Following §187(e) in the Karlsruhe manuscript appears a passage that seems to be a slightly adapted and incomplete version of the passage about mode VIII in the prose summary. Here, too, Chailley accepts this passage as part of the original form (or reasonable facsimile thereof) of the source treatise. Mühlmann does not make this assumption, attributing the first six modes of the source treatise (as well as the revision of that material) to a *Theoretiker der 6 Töne*,²⁹² and the last two modes to an *Überarbeiter*²⁹³ (Revisor).

Mühlmann may be correct in his assessment. It is not clear whether the source treatise originally contained only six modes; because of the dramatically different nature of these two modes, assuming that the source treatise only included the first six would help to explain the discrepancy. In that case, the Prose Summary may also be credited with supplying the modal numbers of the last two modes (which are otherwise quite confusing, since they are presented as sets of alternatives, which does not occur in any other mode). Regardless, if the Prose Summary was initially drafted as a summary of the source treatise, it is entirely possible that the state of the treatise in the Karlsruhe manuscript reflects a revision of the source treatise either prior to or independent of the revision that led to the primary recension of the *Alia musica*, adapting the final passages from the prose summary. If this explanation of the state of the text

²⁹¹ Regarding this, his critical apparatus states: “ f) *totum deest in omnibus codd.; restituimus ut in §131*” (“The entirety [of subsection (f)] is absent in all manuscripts; we reconstruct as in §131.” But the end of §187(f) reads “... *et antiphonae Iste est frater vester minimus et Dixit Judas fratribus suis*” (Chailley, *Alia musica*, 94), clearly labelling these two chants as antiphons, which they are not. Here, Chailley has introduced the error himself, as §131(b) reads, “*Et nocturnalia responsoria eodem modo incipiunt R/ Iste est frater vester minimus et Dixit Judas fratribus suis*,” (Chailley, *Alia musica*, 164) (“And the evening responsories of the same mode begin *R/Iste est frater vester minimus* and *R/Dixit ludas fratribus suis*”), which are clearly identified as responds.

²⁹² Mühlmann, *Die Alia musica*, 55–60.

²⁹³ Mühlmann, *Die Alia musica*, 60.

in Karlsruhe is correct, it would favour the possibility that the prose summary came after the source treatise, with the author of the prose summary completing the last two modes (though the hypothesis that the source treatise was an elaboration of the prose summary with the ending lost is not impossible).

Of course, it is also possible that the Karlsruhe manuscript was excerpted from the completed *Alia musica*. In such a case, though, it is reasonable to question why a scribe would choose to copy specifically these sections from the entirety of the *Alia musica*. More specifically, this hypothesis would raise two questions. First, why copy the source treatise? If the goal were a complete understanding of the concept, would not the revision be superior?²⁹⁴ By contrast, if the goal were instead for brevity, would not the table (or, at least, the prose summary) be superior? (A similar question may lend support for the more fundamental question of whether the prose summary is truly independent of the composite treatise: if the Florence manuscripts were excerpted from the completed *Alia* with the intent to be brief, would not the table have been preferable?) Second, why complete the missing information from the Prose Summary, instead of from the Revision? At the very least, why not include the examples provided by the Revisor? For a scribe who carefully adapts the passages from the prose summary for consistency of presentation compared to the other modes, it would seem that examples, which are presented for every other mode, would also be a valuable addition.

In light of all of the above considerations, it would seem that the balance of probabilities favours an explanation for the prose summary as having begun as an independent entity, either as the inspiration for or a summary of the source treatise, which was then appended to the completed composite treatise along with the source treatise.

²⁹⁴ Note that this statement does not reflect the principles behind the present study. My focus will, indeed, be chiefly upon the source treatise because my goal is to understand the origin of the theory, and to attempt to discover something of the earlier nature of the modes hidden in the subtext. However, a typically coeval reader of the text would presumably be more interested in the most fully-developed and mature state of the theory available to him, rather than the earliest form and gradual development of the theory.

Florence 565 Variant Readings

§	Chailley	Florence 565
172		<i>totum deest</i>
173	in 18 diapason, 6 ad 12; diapente, 8 ad 12; Diatessaron viiii	<i>est deest</i> in duple. Diapason, vi ad xii. Diapason per octo ²⁹⁵ ad xii. Diatessaron per viiii
174	et in 28	<i>expositio numerorum modi (inter nominem modi et consummatio diastematis primi) deest.</i> diapente <i>additus est inter "xxx" [30] et "per ter"</i> ; in quibus est diapente <i>deest.</i> et per xxviii
175		<i>tonus deest.</i> <i>expositio numerorum modi (inter nominem modi et consummatio diastematis primi) deest.</i>
176	tonus est	<i>expositio numerorum modi (inter nominem modi et consummatio diastematis primi) deest.</i> tonus totus est
177	tonus est ter 12 qui sunt 36 36 et quater	<i>expositio numerorum modi (inter nominem modi et consummatio diastematis primi) deest.</i> tonus totus est bis xii id est ²⁹⁶ xxxvi. Quater

²⁹⁵ Aside from the incorrect interval, given as an octave, instead of a fifth (an error that also occurs in Mode VI), this is the only occurrence of a modal number being written out as a word, instead of as a Roman numeral. I propose that the model from which F₁ was copied read "diapason [or diapente] proportio viii ad xii," probably with the word *proportio* abbreviated with the symbol p for *pro* (which the scribe of F₁ interprets here as p, meaning *per*), giving pporcio (or pporcio); the scribe misread this as "porcio;" then the Roman numeral viii would become redundant, and the scribe omitted it.

²⁹⁶ The abbreviation used here is id, which does not appear in any of the reference material that I consulted regarding medieval Latin abbreviations, but is used frequently in the *Alia* manuscripts, and context suggests *id est* as a likely resolution.

§	Chailley	Florence 565
	60	xl ²⁹⁷
178	est diapason qui sunt 48, aut diapente per quater 9 ad bis 12 qui sunt 60	<i>expositio numerorum modi (inter nomenclam modi et consummatio diastematis primi) deest.</i> est aut diapason <i>totum deest usque ad secundum "qui sunt"</i> xl ²⁹⁸
179	diapente qui sunt 144 octies	<i>expositio numerorum modi (inter nomenclam modi et consummatio diastematis primi) in tribus partibus deest, una cum commemoratione ad modos quartum et quintum, usque ad "sua proportione."</i> aut in <i>praeter</i> diatessaron <i>additus est.</i> dia pentic qui est xcliiii ²⁹⁹ ocies
180	(a)–(c) (e) 60 et 60 habens 120	<i>totum deest</i> xl et lx cxx habens

General Observations:

There are several textual reasons to suspect that F₁ was not copied from a complete copy of the *Alia*, but rather was copied from a model that existed prior to the complete *Alia* and was adapted and incorporated into it. First, the introductory statement, beginning, "*Hic continentur proportiones octo*

²⁹⁷ Chailley notes that this error also occurs in P, but the correct numeral (lx) is present in M and G. The error is also given in C, but if it is present in A, Gerbert gives no indication.

²⁹⁸ This is the same error that occurs in §177; but unlike §177, the error in §178 is unique to F.

²⁹⁹ Note that the Roman numeral given as XCLIIII is properly ordered CXLIIII, and is thus in the other manuscripts.

tonorum,” is absent from this manuscript, which implies that the statement was composed for the complete *Alia* as a transition.

Similarly, a substantial passage is missing from the treatment of the eighth mode. This passage bears upon a point of confusion in the structure of the treatise. The end of the summary in the complete *Alia* contains two sentences in a row that begin “*Tonus octavus*” or “*Octavus tonus*.” Immediately thereafter is the source treatise, with the heading “*Expositor eorundem tonorum*” misplaced to after the first sentence of the source treatise. As already discussed, the end of the source treatise does not include a complete description of Mode VIII, but merely the comment, “*Tonum octavum require ut supra*” (“Look for the eighth mode above”). Chailley assumed that the indication to look above meant that the description of the eighth mode, which belongs at the end of the source treatise, appears in the wrong place, at the end of the prose summary. To complete the picture, it is necessary to take into account manuscript K (containing only the idiosyncratic form of the Source Treatise), which ends with Chailley’s §180 (a)–(c), but is cut off in the middle of subsection (c); F₁ lacks subsections (a)–(c) altogether, but includes subsections (d)–(e).

Unfortunately, since K ends mid-sentence, the case that K was only meant to go as far as the end of subsection (c) is less clear than it would have been had subsection (c) been complete. Nevertheless, the following hypothesis is consistent with the sources: subsections (a)–(c) belong to the end of the source treatise, while subsections (d)–(e) belong to the end of the summary in F₁; when the compiler of the composite treatise added the summary to the end, he felt that the description of the eighth mode in the summary was insufficient, and so he borrowed from the end of the source treatise to clarify Mode VIII at the end of the summary (immediately before the Mode VIII passage that properly belongs to the summary), accidentally misplacing the heading in the process (though this may have occurred in a subsequent copying); finally, at the end of the source treatise, he did not feel the need to rewrite the

passage in its proper place, and so he simply directed the reader to the same passage at the end of the summary.

Aside from the substantial differences at the beginning and at the end, there are also a considerable number of smaller variants (and a somewhat larger dropped clause in Mode VI that could very easily have occurred in the process of copying F_1 from its model – F_1 is assumed to be late eleventh century, and so if the text found therein does predate the completion of the *Alia*, it must still be a copy of the model that was incorporated into the *Alia*, not the model itself), there is one other notable difference. None of the modes contains the usual listing of modal numbers at the beginning of the mode, which makes the passage substantially shorter in F than in any other manuscript. Each other manuscript contains the complete listing of modal numbers before identifying the intervals made from them, but F jumps immediately to the intervals themselves. Since, in the summary, the letters associated with the modal numbers are only presented in the listings of modal numbers, and not in the discussions of intervals, the absence of these lists from F_1 also means the complete absence of these letters from this source – a point of no small interest, since in all of the manuscripts of the Principal Group, the letters are only present in the prose summary and in the table. This absence may indicate that the state of the text copied into this manuscript is older than association of these letters with the modal numbers, though there is not sufficient evidence to be confident in this hypothesis.

Madrid 9088 Variant Readings

§	Chailley	Madrid 9088
13	(a)	<i>totum deest</i>
	(b)	Primoque <i>deest</i>
	Sciendum quod	Sciendum est quod
	tropos de graeco in latinum conversio dicitur	de graecis tropos nominant quos nos tonos dicimus

§	Chailley	Madrid 9088
	idcirco quod...convertitur	<i>totum deest</i>
	(c) Toni vero	Toni autem ³⁰⁰
	(d) troporum proprium	eorum proprium
14	intexatur	texatur
	sesquialteris	sesqualteris
	sesquiteritiis	sesquiterciis
	interjectis	<i>deest</i>
	designari	signari
15	(a)	<i>et deest</i>
	eo qiu	eo quod
	(b) paramesen	paramese
	finitur	finit ³⁰¹
	(c) Tertium ... tertia	tercium ... tertia
	diapason determinat	diapason factum et terminatur
		eum quem vocant <i>deest</i>
	in [...] triten	intrite [sic]
		nervum <i>deest</i>
	(d) finit	finitur
	(e) phrygius	frigius
	quinta specie	est ex quinta specie
		finitur <i>deest</i>
	cui nete diezeugmenon nervus est ultimus	cui est nete diezeugmenon nervus ultimus
	(g) quam paranete ³⁰²	qui in paranete
16	(a) pluresque	plusque
	(b) hypermixolydium	hypermixolidium
	tertii	tercii

³⁰⁰ The different choice of discourse markers between the two versions may stem from a misinterpretation of an abbreviation, perhaps confusing *h* for *l* or vice versa.

³⁰¹ Chailley chooses *finitur*, as given in P, but *finit* is found in G and M (as well as Madrid 9088) and is more likely to be the original reading.

³⁰² In G, *eum paranete*.

§	Chailley	Madrid 9088
	(c) diatessaron chordarum [<i>bis</i>]	diatesseron cordarum [<i>bis</i>]
17	(a) diatessaron tertio ... tertia (b) tetrachordum species redit	diatesseron tercio ... tertia tetracordum species edit ³⁰³ redit
18	(a) (b) diatessaron tres prima species dipente (c) (d) diezeugmenon sumit initium	locis <i>deest</i> diatesseron tres species primas διὰ πεντ̄ ³⁰⁴ <i>vero deest</i> diezeugmenon ∙ sumit Inicium
19	(b) tertio et sexto loco utitur semitonio (d) tertia (f) tertio (i) tertio	Tercio loco et sexto ∙ utitur semitonio tercia tercio tercio
20	(a) lichanos (b) similiter phrygium et hypophrygium lydium et hypolydium	lychanos similis prhigium [sic] et hypofrigium lydium et hypolidium

³⁰³ This word between *species* and *redit* is unclear and appears as though an attempt may have been made to correct it. It resembles the word that follows. It is likely that the reading in the model manuscript is unclear at this point, as there is also considerable variety in the manuscripts that Chailley consulted.

³⁰⁴ The abbreviation here is uncertain. The marking over the t usually implies an ending with an m (or sometimes n), which would imply a reading as *diapentem*. However, *diapente*, a Greek loanword, is usually considered to be invariable. It is possible that the abbreviation simply indicates the dropped e, but the scribe does not generally abbreviate word-final e in this way (as, for instance, *diapente* is written out in full in §16(c); in this instance, however, the abbreviated word appears at the end of the line, and already runs closer to the edge of the page than any other line near it; perhaps the scribe used the unusual abbreviation here simply to avoid running further into the margin.

§	Chailley	Madrid 9088
(c)	mixolydius ab hypermixolydio	mixolidius ab hypermixolidio

General Observations :

It seems unlikely that D is a copy of any extant manuscript of the *Alia musica*. A number of elements of the *Alia* version are missing, including not only the introductory passage (§13a), but also a clarifying comment in §13(b), as well as a handful of individual words, here and there, that were obviously added to improve clarity (e.g., *interjectis*, §14); only in a very few instances are words added to D compared to the *Alia*. In addition, there are several cases in which the same stem word appears in the *Alia musica* and in Madrid 9088, but with an additional, intensifying prefix in the *Alia musica* (*signari* vs. *designari*, *texatur* vs. *intexatur*); it seems more likely that such prefixes would be added into a copy than that they would be omitted from a copy.

It also appears that the scribe of D is not familiar with the Greek terminology. He frequently breaks off at the end of a line in the middle of a Greek word, but very rarely does so in the middle of a Latin word. Additionally, he is fairly consistent about spacing between words, so that it is nearly always obvious where one word ends and the next begins, except in Greek words, which he routinely splits in half (even away from line breaks) and sometimes joins part of them to adjacent words. For instance, on f.125r, the second line ends “ $\kappa\upsilon\iota\tau\tau\iota \leftarrow \tau\epsilon$ hÿpboleon” for “*cui trite hyperbolaeon*,” and the following line ends “ $\kappa\upsilon\iota\eta\pi\alpha\rho\alpha \leftarrow \eta\epsilon\tau\epsilon$ hÿpboleon” for “*qui in paranete hyperbolaeon*.”

His spelling of Greek terms is interesting. It is relatively consistent for a single term, but not so across compounds. For instance, he spells *Phrygius* (in any grammatical case) with ph, but spells *Hypophrygius* with f instead of ph; similarly, he spells *Lydius* with a y, but *Hypolydius*, *Mixolydius*, and *Hypermixolydius* with an i after the l.

The end of §15(c) is a particularly interesting case, demonstrating both the additions to the *Alia* compared to D and the latter's awkwardness with Greek. It is likely that the original form of the text reads "*terminat in trite diezeugmenon.*" In D, "*in trite*" appears as one word, "*intrite*" (and the "ite" is so close together, and the loop of the e so squashed, that it appears to read "*intrux*" – one of only two places in this passage where the handwriting is unusually difficult to read). This elision of words would not be possible if Madrid 9088 had been copied from any extant manuscript of the complete *Alia musica*, because in those manuscripts, the clarifying phrase "*eum quem vocant*" has been interjected between these two words, giving "*in (eum quem vocant) triten*" (the n has presumably been added to *trite* to agree with the case of *eum quem*, through *trite* ought to be invariable in Latin).

In view of the late date of all of the manuscripts containing any portion of the *Alia musica* (none of which is presumed to have been copied before the eleventh century) compared to the presumed date in which the *Alia musica* is thought to have been compiled (late-ninth or tenth century) and the unlikelihood that Madrid 9088 was copied from the *Alia musica*, it may be hypothesized with some confidence that this excerpt, forming the second part of the introduction to the *Alia musica* (the section that draws upon Boethius' *Musica*, rather than the *Arithmetica*), was written prior to the compilation of the *Alia musica* as a whole. This hypothesis is important, because it is the only section of the so-called "Principal Treatise" to connect the modes directly to species theory, and the species theory presented here differs from that presented elsewhere in the treatise (particularly in the numbering of octave species).

Barcelona – Ripoll 42 Variant Readings

The variance between the revised recension of the *Nova expositio* and the standard recension in the *Alia musica* has already been published by Gumpel³⁰⁵ and will not be presented here. The variants in

³⁰⁵ Gumpel, "*Die Nova expositio*," 129–43.

the *Alia* fragments appear below. It is perhaps worth noting that while both R and its presumed model, R', are associated with the Spanish monastery at Ripoll, the manuscript uses a Carolingian minuscule hand without obvious influences of the Spanish Visigothic script. By contrast, the manuscript does include the wedged ascenders that are the hallmark of the insular scripts,³⁰⁶ and also makes liberal (though by no means exclusive) use of the insular d. These characteristics support the supposition of a link to Fleury in Paris, a scriptorium noted for insular influence.³⁰⁷ This characteristic implies that if R was indeed copied from R', then both R and R' may have been copied at Fleury before being sent to Ripoll; alternatively, R may have been copied at Ripoll by a scribe trained at Fleury (or, at least, in imitation of the Fleury style).

1. Introduction: §§1–2(a), Paraphrase on Boethius' *Arithmetica* (R^α)

This excerpt begins with the last two lines at the bottom of fol. 68v, immediately following the familiar treatise *Ecce modorum sive tonorum* (a member of the *Enchiridis* complex); it runs through the first two lines of f. 69r and is followed by the orphaned excerpts of the *Nova expositio* tonary (discussions of only the sixth and eighth modes).

§	Chailley	Barcelona, Ripoll 42
	[In Boethio <i>Arithmetica II</i> , cap. 48, "Considerandum ... consideratione", ³⁰⁸ <i>totum deest in Alia musica</i>]	
1 (a)	consideratione	consideracione
(b)	speculationem communiter	speculacionem comuniter
(c)	Quaerit differentia contineat differentiam	Queriter differentia contra differentiam

³⁰⁶ Drogin, *Medieval Calligraphy*, 40; 51.

³⁰⁷ Bischoff, *Latin Palaeography*, 114.

³⁰⁸ Boethius, *De institutione arithmetica*, bk. 2, ch. 48, ed. Friedlein, 155.

§	Chailley	Barcelona, Ripoll 42
	[In Boethio, “Ad aliquid autem considerationem armonicae proprie esse, in primi libri rerum omnium divisione monstravimus”; ³⁰⁹ <i>totum deest in Alia musica.</i>]	
2 (a)	symphonias medietate	simphonias medietati

General Observations :

The differences in this text are exclusively differences in orthography; nevertheless, in spite of the brevity of the fragment, there are a few clues to the relationships among manuscripts. Most notably, the in §1(c) *differentia contra differentiam* reading in this manuscript is correct, potentially distancing it from M and (presumably) A – and also from P₂, despite the supposed relationship between these two manuscripts noted by Gumpel. One could argue that the correct reading could easily have been verified by referring to Boethius; however, there is a potential (weak) contradiction in this argument, as both R^α and P₂ contain the §2(a) reading *medietati*, erroneous by comparison to Boethius;³¹⁰ it would be surprising (though certainly not impossible) for the scribe of R to correct an erroneous reading in §1(c) and then leave behind an erroneous reading in §2(a) (particularly when the two sentences are also very close together in Boethius).

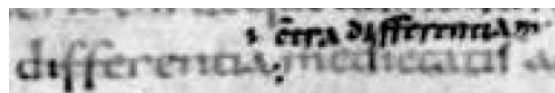
Chailley’s handling of the *contra/contineat* variant in §1(c) is both ambiguous and faulty. The reading he selects (including the word *contineat*) is found in G – and, according to Chailley, also in M. However, M does not actually contain this reading; in M, there is only one word “*differentia*,” with the expected following words “*contra differentiam*” missing, and then corrected interlinearly as “*contra differentiam*” (see Facsimile 10). His note regarding the Paris variant reads “*diff. contra differentiam cont.*,”

³⁰⁹ *ibid.*

³¹⁰ *ibid.*

P.”³¹¹ This is not a redundancy found in the manuscripts, but rather two different variants presented in the two related Paris manuscripts. P₁ reads, “*differentia contra differentiam*,” just as is found in R^α; P₂, which was probably copied from P₁, contains an error, giving “*differentiam conta differentiam*.” Strangely, Chailley accepts the reading in G (“*contineat*”), despite the fact that it does not occur in any other manuscript, and is easily shown to be a corrupted reading by comparison against Boethius’ *Arithmetica* II, chapter 48, which also gives “*differentia contra differentiam*.”³¹² The Cesena manuscript, which Chailley did not consult, also gives the correct “*contra*” reading.

Facsimile 10 - Interlinear correction in Munich Staatsbibliothek clm 14272, f. 175r.



2. Fragments of the *Nova expositio*: §§122–25; 150–53 (R^β)

§	Chailley	Barcelona, Ripoll 42
122 (b)	o ad e	o et e ³¹³
123 (a)	enarmonium memetipsum	enarmonicum ³¹⁴ memedipsum
124	In diurnis ipsa Qui manducat carnem	In diurnis enim ipsa Qui manducat carnem meam
125		<i>Doxologia secunda abbreviatus est, sic</i> : Gloria patri· Seculorum amen·
150 (b)	m ad cc remissa	in ad cc remissa ³¹⁵

³¹¹ Chailley, *Alia musica*, 99.

³¹² Boethius, *De institutione arithmetica*, bk. 2, ch. 48, ed. Friedlein, 155, trans. Masi, 177.

³¹³ This reading is shared with M (Chailley, *Alia musica*, 192 §122b, critical apparatus) from the Principal Group and Pa and Br (Bernhard, *Anonymi saeculi decimi vel undecimi*, 24 §74) from the *Dulce ingenium* Group.

³¹⁴ This reading is shared with G (Chailley, *Alia musica*, 192 §123a, critical apparatus).

³¹⁵ While Chailley gives the word inflection *remissa*, he does so as a correction, where the manuscripts variously give the inflections *remissam* or *remissum* (always abbreviated with a mark over the vowel – i.e., remi{̃}ā, etc. – or

§	Chailley	Barcelona, Ripoll 42
151	(a) diatessarum intensum in y	diatessarum ad accutum y ³¹⁶
152	autem ³¹⁷ habet 2 loca	de est habet ·+· locum ³¹⁸
153		<i>Ambo doxologiae abbreviati sunt, sic: Gloria patri· Seculorum amen·</i>

General observations:

This excerpt contains errors otherwise unique to particular manuscripts, but not always the same manuscripts, and does not contain the complete set of errors of any known manuscript, nor does any single manuscript contain all the errors in R. In other words, it is unlikely that R was copied from any known manuscript or that any known manuscript was copied from R. As already mentioned, it is believed that R was copied from R', which, if not actually copied at Fleury itself, was probably in turn copied from a manuscript from Fleury; thus, there are enough generations between R and the most recent antecedent shared with the Principal Group that a variety of changes could have arisen. However, the fact that errors from multiple manuscripts are present suggests the possibility (if not necessarily the probability) that multiple copies of the treatise were collated in the copying of others (a process Phillips noted in the copying of the *Musica enchiridis*, which also appears in all the relevant manuscripts). Alternatively, it is

in the case of C, with the form $\text{remi}\{\{\lambda_3\}$; no manuscript in the Principal Group contains the reading suggested by Chailley, but it is attested here in R^β (a manuscript that Chailley apparently did not consult).

³¹⁶ The wording here is problematic in every manuscript except this one. Although Chailley gives the word *intensum*, he indicates it as a correction, where the Principal Group manuscripts say *remissum*, which is the wrong direction. In R, the direction is correct, but the word given is *accutum*, a word not used anywhere else in the treatise, while *intensum* is used repeatedly; it is, therefore, probably a scribal correction.

³¹⁷ Here, Chailley indicates *autem* by the abbreviation H in manuscripts M and P, while the reading in G is *enim* (Chailley, *Alia musica*, 195 §152, critical apparatus). In fact, the abbreviation in M and P is h , a sign usually interpreted as *hoc* (Bischoff, *Latin Palaeography*, 161; Cappelli, *Elements of Abbreviation*, 21), which would be a nonsensical reading here; instead, the abbreviation h here is probably a variant of the abbreviations ·n· or H, which stand for *enim* (Bischoff, 161; 86; Cappelli, 22), rather than the form h that is used elsewhere in the manuscripts for *autem*. If this interpretation is correct, then all the manuscripts of the Principal Group agree on the reading *enim* – except R^β, in which the discourse marker is absent altogether. (However, Pa and Br give *vero*.)

³¹⁸ The strikethrough on the i is in red ink, and many (though not all) numbers are struck through in this fashion, so it is probably just a means of distinguishing numbers from words. The relevant observation is the change from two *loca* to only one; nevertheless, two *loca* are then identified, with two examples. This error is also present in P.

also entirely possible that an astute scribe simply corrected the errors that one might have expected in R; or perhaps these additional differences simply result from the number of generations, not only back from R to the common antecedent, but also from the common antecedent forward to the Principal Group manuscripts (during which time, those other manuscripts may have accumulated additional errors).

Probably the most useful observation to be made about this fragment is that it belongs very clearly to the principal recension of the *Nova expositio*; there does not seem to be any influence whatsoever upon these fragments from the revised recension found earlier in the same manuscript (not even glosses demonstrating any awareness that the two versions differed). Nor, for that matter, does it show any connection to the only slightly different *Dulce ingenium* versions (which was not to be expected, in any case).

Another unique feature in this manuscript is that three of the four times that the doxology is presented in these excerpts (excepting only the first time), it is heavily abbreviated, as though the scribe did not see a reason to recopy the same text verbatim four times. This abridgment implies that the model manuscript, like R itself, lacked neumes for these formulae (which only appear in M), as neumes were presumably the justification for including these otherwise superfluous formulae, and their presence in the model would have suggested the reason for copying them out in full. However, R is the only surviving manuscript to abbreviate the doxologies in this manner.

The selection of excerpts in this manuscript is curious. On the surface, there is nothing particularly special about the content of these excerpts that would predispose a scribe to copy only these passages from a complete copy of the treatise. This state of affairs might lead to the conclusion that portions of the model had been damaged or lost, and that the scribe copied whatever was left. However, the passages from the *Nova expositio* are exactly complete with respect to the two modes presented, neither lacking any part of a passage nor including any additional text before or after them. It is possible that a

scribe, copying only from what survived of a damaged or lost fascicle of a manuscript, might have chosen to copy only those surviving sections that seemed complete in themselves; however, the fragment of the introduction is an incomplete citation from Boethius that continues into §2(b) in a complete copy of the *Alia musica* but is left incomplete in R.

Another hypothesis that could explain the selection of just Modes VI and VIII from the *Nova expositio* would be that the scribe believed that the revised recension of the *Nova expositio*, found only slightly earlier in the same manuscript, did not accurately reflect local usage concerning specifically these two modes, and that the principal recension of these two modes was a better fit; thus, these excerpts were copied as a supplement – a kind of *errata* section. In this scenario, the incomplete opening passages of the *Alia musica* may have served as an *incipit*, as a reminder of the larger text from which these alternate passages of the tonary were excerpted. This hypothesis could perhaps be confirmed or refuted by an examination of the liturgical usages in Fleury and Ripoll (which, however, is beyond the scope of the present study).

If the previous hypothesis is not correct, there is one other characteristic of note that might help to explain why Mode VI is followed by Mode VIII, even if it would not explain why the rest of the modes are missing. The ordering of the octave species used in the *Nova expositio* runs in ascending order by pitch. Since the ordering of modes alternates between authentic and plagal modes (in which authentic modes are conceptually a fourth higher than the corresponding plagal modes), the number of the octave species for each consecutive mode does not run in strictly ascending order, but alternates between two ascending series (see Table 12).

Table 12 - Octave Species Associations in the *Nova expositio*

Mode		I	II	III	IV	V	VI	VII	VIII
Species #	Authentic	1		1(!)		3		4	
	Plagal		4		5		6		7

As already mentioned, the meaning of the octave species in the *Nova expositio* does not reflect how they are usually described (either in other treatises or elsewhere in the *Alia musica*), and this usage will be discussed in greater detail in Chapter 12. For present purposes, it is sufficient and interesting to note that the two passages excerpted in R^b are Modes VI and VIII, which do present consecutive species numbers, unlike the usual presentation of the *Nova expositio* (because Mode VII, which would have broken the pattern, is omitted in R). It is possible that the model from which R (or R') was copied once contained the complete *Nova expositio* consolidated into a single block and rearranged so that all of the authentic modes were grouped together and all of the plagal modes likewise grouped together, resulting in a strictly ascending order of octave species. However, this is an extraordinarily large hypothesis to support with such slight evidence, and it is far less likely than the preceding hypothesis.

Putting it All Together: A summary of the Constituents, Dating, and Manuscripts

With all the above information, it is possible to attempt to assemble a more comprehensive picture of the genesis of the *Alia musica* and the approximate relationships amongst the sources. As a caveat, it is important to remember that several of the assumptions built into this picture rely upon weak evidence, as more substantial evidence is unfortunately not available, and many of the hypotheses presented above

F₁ in the later eleventh century, which was then copied into F₂ in the thirteenth century, while the source treatise was independently copied into K in the eleventh or twelfth centuries.

The revision that forms the largest part of the composite treatise was necessarily completed after the source treatise, but not necessarily as soon afterwards as Chailley supposed (especially since, as Chailley noted repeatedly, there are passages that the revisor apparently did not understand, which implies that he lacked access not only to the author of the source treatise but also to anyone else with direct knowledge of the doctrine). Still, it was probably completed before the *Musica enchiridis* became widespread, since the *Enchiridis* would be likely to have had a much greater effect on a later treatise (as it appears to have had on a single passage that is probably from a later layer); the revision, therefore, was probably completed either in the late ninth century or more likely in the early tenth century. As Atkinson suggests, the revision may have coalesced around a previously written précis of basic harmonic theory drawn from Boethius (δ) that provides sufficient background to understand the principals underlying the source treatise. This précis, then, would presumably date from the ninth or early tenth centuries, and it is presumably this précis that was copied into D in the early eleventh century (though it remains possible that D is just a fragment of the completed *Alia*).

The *Nova expositio* tonary was almost certainly an independent treatise (μ), which may even have been inspired by the earlier layers of the *Alia*, as Chailley supposed,³¹⁹ but the evidence for this is not as strong as Chailley believed; if the précis of Boethian theory was, indeed, originally an independent tract, the tonary could just as easily have been inspired by that précis as by the revision of the *Alia*, but even this hypothesis is highly questionable, as the tonary does not use the octave species in the same way as the rest of the *Alia*.

³¹⁹ Chailley, *Alia musica*, 27.

The tonary is present in both the *Alia musica* and the *Dulce ingenium*, already in an apparently corrupted state (v). Within the *Dulce ingenium* group, Pa and Br were apparently copied from the same model, and contain the *Dulce ingenium* introduction, some material from Boethius and Martianus, and the tonary; Pr could also be from the same model but omits about half of the material from Boethius and Martianus, and supplements with far more from Regino, and also contains a few other passages shared with the *Alia musica* – most notably, the disputed passage on modal octaves. This passage is the only one in the *Alia* showing evident influence from the same school of thought as the *Musica enchiridis*, and therefore was most likely not written until the tenth century. It is unclear how these passages relate to the *Nova expositio*, but they are unlikely to have been copied directly from the *Alia musica* since the *Alia musica* contains the more extensive version of the disputed passage; if these passages were instead copied into the *Alia* from the *Dulce ingenium*, it is unlikely to have occurred until the tenth century because some passages are adapted from Regino, whose treatise is thought to have been compiled around the turn of the century. But since neither the *Alia* nor the *Dulce* seems to have been the direct model for the other, the passages shared probably stem from a common model that would have to predate them both, and would, therefore, have been written not too late in the tenth century. Since the use of octave species in the tonary differs substantially from the usage in the disputed passage (and in any other treatise), these excerpts likely do not come from a single treatise, as Chailley supposed, but separate treatises either in the same manuscript source or at least in the same library, which might suggest a common or related point of origin for both treatises; if so, questions remain as to why the shorter version of the *Dulce ingenium* lacks the additional passages. It is also unclear whether the revised form of the *Nova expositio* found in R was revised from the *Alia musica*, the *Dulce ingenium*, or from the shared model for both. In any case, it was apparently copied in the eleventh century from another Ripoll manuscript, R' (subsequently lost in a fire), and these manuscripts have connections with the scriptorium at Fleury, and so were probably copied from a Fleury model (ρ).

The complete treatise was probably assembled in the tenth century (α), incorporating the *Nova expositio* and the remainder of the supplemental content (including, most likely, the Disputed Passage on modal octaves, or at least the commentary upon it that does not appear in the *Dulce ingenium*) and commentary most likely penned by the compiler (in addition to some inconsistencies within the text, this commentary accounts for a relatively small number of passages within the main text that resemble the style of the *Nova expositio*).

At some point in the late tenth or early eleventh century, the composite treatise was incorporated into the *Enchiridis* complex of theory treatises that were routinely transmitted together; it is unclear whether the archetype for the completed *Alia musica* was written directly to be placed within this “superwork” or if it was added to the group afterwards, but every known complete copy belongs to this complex. Of the extant eleventh-century manuscripts of the *Alia* subcomplex, none is likely to be the model for the others, but P_1 seems to have more in common with A and C than does M; thus, it is likely that all the manuscripts except M stem from another common source (β). P_2 is a twelfth-century copy of P_1 , while A and C were both copied in the fifteenth century, almost certainly from a common, apparently eleventh-century source (γ). Also apparently in the fifteenth century, the title *Alia musica*, attested nowhere else, was added to manuscript M, possibly (but by no means conclusively) by Aventinus.

In the later eighteenth century, Gerbert prepared his edition based on A and compared against M. Shortly afterwards, possibly on Gerbert’s authority, the attribution to Hucbald was added to manuscript M (perhaps by the St. Emmeram librarian Sanftl, though this cannot be confirmed, and the evidence for it, besides Müller’s opinion, is unknown).

Both A and R' were lost in fires in the nineteenth century. Mühlmann then completed a German translation in the early twentieth century and Heard produced an edition and English translation in the

mid-twentieth century at about the same time that Chailley produced the critical edition and a very close French commentary.

Section II: The Theoretical Context

5. Greek Theory (171)

6. Medieval Theory up to Aurelianus (209)

7. Carolingian Theory (223)

8. The Development of Species Theory (236)

Chapter 05: Greek Theory

As I have already suggested, the *Alia musica* differs from the other large-scale music theory treatises of the Carolingian Renaissance in a fundamental way: unlike any of the others, the vast majority of the *Alia musica* is dedicated to the study of the modes. Several other treatises of the era examine the modes, sometimes at considerable length, but no others (tonaries excepted) dedicate so much space to mode or make it the central or nearly exclusive topic of the treatise.

The description of mode in the *Alia musica* is unlike the description of mode in any of these other treatises – or, for that matter, in any treatise in any other era. The *Alia* is well known for superimposing Greek terminology and Greek harmonic ideas onto its modal system, but the modal concept of the *Alia* remains dramatically different from the Greek system. It exists within a context of an ecclesiastical modal system that borrows much from Byzantine theory, but it is also quite a different system from the Byzantine modes. It is routinely cited as a milestone in the development of the later medieval modal system with which most musicologists are much more familiar, a lens through which it is all too easy to project anachronistic concepts onto the treatise. This section of the dissertation will review these different conceptions of mode – the ancient Greek, the Byzantine, the other Carolingian treatises, and the later medieval models – to ascertain which elements of the older theories were or were not incorporated in the *Alia musica* and which elements of later modal theory are or are not already present in the *Alia*. This chapter will begin with the ancient Greek system.

The earliest descriptions of mode in Greek theory appear in philosophical writings not primarily about music, such as Plato's *Republic* and Aristotle's *Politics*, and the details are unclear. In these writings, the modes are called ἁρμονίαι (*harmoniai*), and the principal value to these passages is that they clearly specify what makes a mode a mode: the emotive character generated. Aristotle, in his *Politics*, explains

that each ἀρμονία tends to draw forth from the listener specific emotions (often with moral subtones), called ἦθος (*ēthos*) and loosely comparable to the Baroque concept of affect:

Ἐν δὲ τοῖς μέλεσιν αὐτοῖς ἔστι μιμήματα τῶν ἠθῶν (καὶ τοῦτ' ἐστὶ φανερόν: εὐθὺς γὰρ ἡ τῶν ἀρμονιῶν διέστηκε φύσις, ὥστε ἀκούοντας ἄλλως διατίθεσθαι καὶ μὴ τὸν αὐτὸν ἔχειν τρόπον πρὸς ἐκάστην αὐτῶν).³²⁰

And in melodies themselves are imitations of character (and this is manifest: for the nature of the *harmoniai* stands directly apart, so that those hearing are differently influenced and do not have the same reaction towards each of them).

He goes on to identify which ἀρμονία implies which emotion, such as the Mixolydian, which evokes grief and anxiety, the Dorian, associated with firmness, and the Phrygian with excitement. Plato, in the *Republic*, describes how, in his ideal society, only those ἀρμονία that call forth virtuous emotional states would be permitted;³²¹ the *Alia musica* directly cites this passage in §170(e), saying “*Unde Plato praecepit minime oportere pueros ad omnes modos erudiri, sed potius ad valentes et simplices.*”³²² (“Whence Plato instructed that it does not at all behoove boys to be taught all the modes, but rather the strong and simple ones.”)

Neither Plato nor Aristotle provides any detail on the structure of these ἀρμονία, nor gives any other clue as to how this emotive character is achieved, but Andrew Barker considers it “quite clear that the *harmoniai* of the late fifth and early fourth centuries [...] were distinguished from one another primarily by being constituted out of different sequences of intervals.”³²³ The first theorist to claim to describe these old modes in detail is Aristides Quintilianus, in his *On Music*.³²⁴ The date for this treatise is unknown, but could not be older than the first century B.C. (since he cites Cicero),³²⁵ and is probably considerably later (perhaps as late as the third century A.D., but no later than the fourth century, as he is

³²⁰ Aristotle, Πολιτικῶν, bk. 8, ed. Ross, *Aristotle's Politica*, 1340a–b, trans. Barker, *Greek Musical Writings I*, 175–76.

³²¹ Barker, *Greek Musical Writings I*, 130–32.

³²² *Alia musica* §170(e), ed. Chailley, 172.

³²³ Barker, *Greek Musical Writings I*, 164.

³²⁴ Barker, *Greek Musical Writings II*, 420–21.

³²⁵ Barker, *Greek Musical Writings II*, 465–66.

a principal source for Martianus Capella).³²⁶ Aristides claims that the modes that he is describing are those mentioned by Plato,³²⁷ but this claim is problematic at best since he probably wrote centuries after Plato and does not precisely identify the source of his information. Barker proposes that his source was probably a copy of a lost work by Aristoxenus (more about him shortly) describing the work of his predecessors;³²⁸ he also notes that the structures that Aristides presents are all plausible antecedents of the succeeding structures that would bear the same names; he thus feels that it is reasonable cautiously to accept Aristides' descriptions as at least more or less accurate.³²⁹

The structures presented by Aristides manifest two fascinating characteristics: they are not uniformly bounded by the octave (the Dorian spans a ninth, while the Syntonolydian spans only a sixth); and while including quartertones between many notes (as with later Greek modes), they also contain surprisingly large gaps between other notes (the Mixolydian leaps a tritone between its last two notes). But if these modes are, indeed, the modes cited by Plato (as Aristides claims), they would not last much longer. According to Barker, "at a date probably not far from that of the *Republic*, theorists were engaged on the project of organising the structures of the existing *harmoniai* into the framework of a single, co-ordinated system."³³⁰

The system in question is more or less comparable to the octave species concept of mode in the Middle Ages, though the details differ in a few crucial ways. In the first place, the Greek modal system coexists with and operates within an entirely different dimension of interval variation called *genus*, such

³²⁶ Barker, *Greek Musical Writings II*, 392; cf. Mathiesen, *Aristides Quintilianus*, 10–14.

³²⁷ Barker, *Greek Musical Writings II*, 419–20.

³²⁸ Barker, "Aristides Quintilianus," *passim*. This entire article is dedicated to the question, and the reasoning is beyond the scope of the present discussion. Put very briefly, Aristides' other remarks about the theory of the early theorists ("οἱ ἀρχαῖοι," the ancients; "οἱ πάνυ παλαιότατοι," the very old ones) agree well with comments in surviving works by older theorists, including Aristoxenus, and may even help to explain some of the more cryptic references in Aristoxenus' surviving works. See also Barker, *Greek Musical Writings I*, 165.

³²⁹ Barker, *Greek Musical Writings I*, 165–68.

³³⁰ *ibid.*, 164.

that the tuning of most of the notes in each mode could vary quite substantially according to genus, apparently without affecting the mode. In the medieval system, by contrast, with the exception of the ability to alternately use \flat or \natural , each pitch has a fixed place (at least in theory). The *finalis* of the first mode is characterized by having a whole tone beneath it, and a whole tone, then a semitone, and two more whole tones above it. Not so in the Greek system. In the Greek system of fifteen notes spanning two octaves (called the Greater Perfect System), only seven pitches have a fixed position; the other eight have fuzzy regions within which they can appear; consequently, in any mode, three to four pitches will have a fixed position and the remaining four or five pitches can vary – though not freely, as there are rules that govern their placement. There is also only one place in each mode where two fixed pitches occur directly adjacent to each other, so that there is only one interval between directly adjacent scale steps that always retains a constant size, a whole tone.

In principle, these variations do not have much to do with modes, which maintain their identities irrespective of how these moveable notes are tuned. However, the *Alia musica* appropriates (and abuses) the conception of genus that underlies this system, so it is worth briefly examining this concept. The Greek scale system is built around tetrachords. In each tetrachord, the two outside pitches must be tuned to a perfect fourth, with frequencies in the ratio 4:3; these are the fixed pitches. The complete scale spans two octaves, with the two tetrachords that make up each octave overlapping at a shared pitch in the middle (referred to as conjunction); these pairs of tetrachords actually only span a minor seventh, but the two pairs are connected to each other by a whole tone of disjunction, completing the upper octave, and an additional tone of disjunction is added to the bottom of the entire system to complete the lower octave. This lowest tone is called the $\pi\rho\omicron\sigma\lambda\alpha\mu\beta\alpha\nu\omicron\mu\epsilon\nu\omicron\varsigma$ (*proslambanomenos*, lit. “taken in addition”), and it is this pitch, along with its associated etymology, that Chailley believes the authors of the *Alia*

musica conflate with the concept of *emmelis*,³³¹ an idea that will be examined extensively in the final section of this dissertation.

The remaining two pitches of each tetrachord can appear in a variety of places, measured from the lowest pitch of the tetrachord, and usually relatively close to it. This clustering of notes around the bottom is called the *πυκνόν* (*pyknon*, a substantive from *pyknos*, “crowded”). The possible intervals between the three notes of the *πυκνόν* are divided into three genera, traditionally described as follows: the *enharmonic* (two quartertones), the *chromatic* (two semitones), and the *diatonic* (a semitone below and a wholetone above).³³² However, this description is oversimplified, as it provides only one form of each genus. Strictly speaking, the genera are actually defined by the interval left over between the highest note of the *πυκνόν* and the upper fixed pitch, which are a ditone (*i.e.*, a doubly-augmented second) for the enharmonic, anywhere from a trihemitone (*i.e.*, an augmented second) up to but not including a ditone for the chromatic, and anywhere from a tone up to but not including a trihemitone for the diatonic (this distinction becomes important for the *Alia musica*).



Figure 14 – The fixed pitches of the Greek Greater Perfect System.

Note that pitch names refer to strings placement on a lyre, so high pitches are labelled as low and vice-versa.

³³¹ Chailley, *Alia musica*, 20.

³³² Strictly speaking, the diatonic genus does not have a *πυκνόν*; since the word means “crowding,” Aristoxenus explains that a *πυκνόν* occurs only if the two intervals within it are collectively smaller than the remaining interval above it in the tetrachord. A more thorough examination of the *πυκνόν* may be found in Barker, *Science of Harmonics*, 178–80. However, this distinction is not important for the present discussion, and as the term is useful, the distinction will be ignored in the interest of concision.

The enharmonic genus includes only a single form: two quartertones and a ditone. However, the $\pi\upsilon\kappa\nu\acute{o}\nu$ in the other two genera can be arranged in multiple ways, which are termed $\chi\rho\acute{o}\alpha$ (*chroai*, which is related to the word *chrōma*); Barker translates $\chi\rho\acute{o}\alpha$ as “shade.”³³³ The list of shades differs somewhat amongst the Greek theorists according to their various theories of what makes an acceptable shade, but Aristoxenus gives three chromatic shades and two diatonic shades, as shown in Table 13.³³⁴

Table 13 – Aristoxenus’ Genera and Shades. Intervals are given in fractions of a whole tone.

Genus	Enharmonic	Chromatic			Diatonic	
Shade		Soft	Hemiolic	Tonic	Soft	Tense
Highest Interval	2	$1\frac{5}{6}$	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{1}{4}$	1
Middle Interval	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{3}{4}$	1
Lowest Interval	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

It is these divisions that the revisor of the *Alia musica* borrows at the beginning of his discussion of the fourth mode.³³⁵ However, from the nature of his presentation, it is clear that he did not get them from Aristoxenus himself, but rather from Boethius’ presentation of Aristoxenus’ divisions (*De institutione musica* V.17),³³⁶ which is itself a translation of Ptolemy’s presentation of the same (*Harmonics* I.12).³³⁷ Ptolemy, after describing the divisions in fractions (as Aristoxenus had done), presents a table in which he divides the tone into twenty-four equal parts so that it is possible to present each fraction of a tone in Aristoxenus’ divisions as a whole number of twenty-fourths (see Table 14):

³³³ Barker, *Greek Musical Writings II*, 142.

³³⁴ cf. Barker, *Greek Musical Writings II*, 164–65.

³³⁵ *Alia musica* §§77–78, ed. Chailley, 143.

³³⁶ Boethius, *De institutione musica*, bk. 5, ch. 17, ed. Friedlein, 365–68, trans. Bower, 174–77.

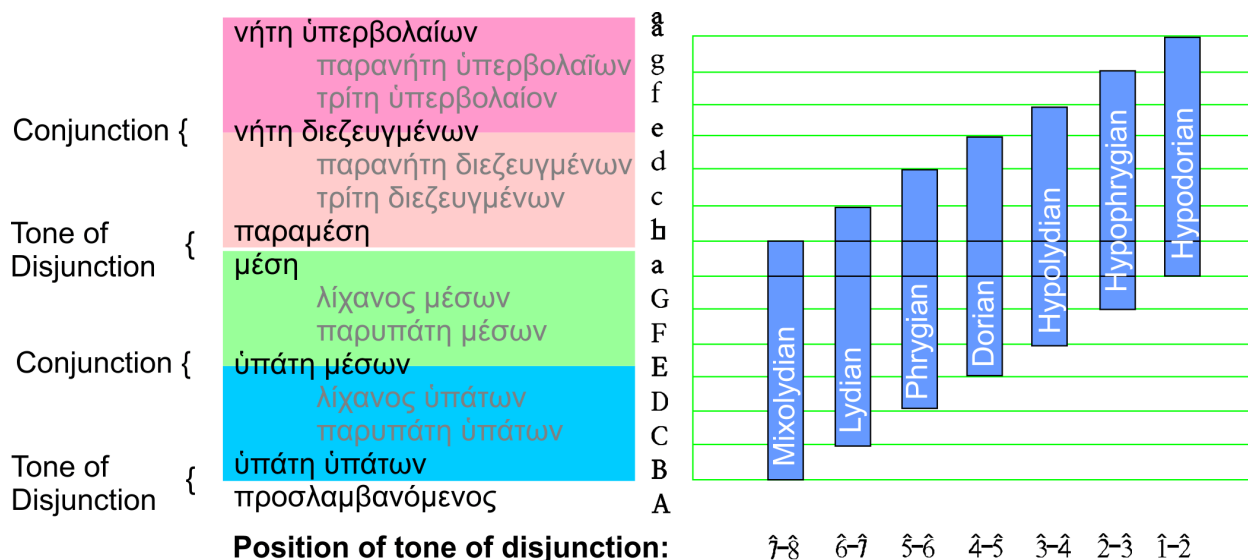
³³⁷ Ptolemy, Ἀρμονικά, bk. 1, ch. 12, ed. Düring, 28–30, trans. Barker, 301–03.

Table 14 – Aristoxenus' Genera and Shades as presented by Ptolemy and Boethius.
Intervals are given in twenty-fourths of a tone.

Genus	Enharmonic	Chromatic			Diatonic	
		Soft	Hemiolic	Tonic	Soft	Tense
Highest Interval	48	44	42	36	30	24
Middle Interval	6	8	9	12	18	24
Lowest Interval	6	8	9	12	12	12
	60	60	60	60	60	60

Although the revisor of the *Alia musica* presents all of the numbers in Table 14, it is only the numbers of the top row that he misincorporates into his own theorizing, attempting to show how each corresponds to one of the perfect consonances (notwithstanding that in Aristoxenus' divisions, they represent intervals ranging only from a tone to a ditone). This matter will be taken up again in the discussion of the numerology of the *Alia musica*.

Table 15 – Greek Modes as Octave Species, based on the description of Aristides Quintilianus³³⁸
(Medieval note equivalents for movable notes are accurate only within the Tense Diatonic)



³³⁸ Aristides Quintilianus, *On Music*, bk. I, ch. 8, trans. Barker, 416–17 & fn. 100; cf. trans. Mathiesen 83 & fn. 92. Although no Greek treatise prior to Ptolemy and Aristides directly applies modal names to octave species, the existence of such a system in the early stages of Greek modal theory is generally inferred from Aristoxenus' remark that theorists anterior to himself described a set of seven octachords (presumably, the octave species) as ἀρμονία (Aristoxenus, *Ἀρμονικά*, bk. II, trans. Barker, 153; cf. Barker, *Greek Musical Writings II*, 15).

To return to the modes, it should be reiterated that these genera and shades do not appear to affect the modes, but they do make the modes slightly more complicated to describe. As Barker suggests, at a time probably not too distant from Plato, theorists systematized the old ἀρμονία into a coherent set of one-octave scales that could be described as one-octave segments of the Greater Perfect System.³³⁹ This framework is quite similar to the octave species (in Greek, ἔιδος or sometimes σχῆμα, both meaning “form”) concept of mode familiar from medieval theory post-Hermannus Contractus, though with some crucial differences. Most notably, because the genera and shades changed the sizes of most of the intervals, it is not practical to describe the modes as a sequence of intervals, as, for instance, by describing the position of semitones, as was common in the Middle Ages. Instead, the best way to describe the species of fourth and fifth is by specifying the location of the πυκνά, and the species of octave according to the position of the tone of disjunction (the only interval between consecutive scales steps in the Greater Perfect System not affected by the genera, excluding the tone above the προσλαμβανόμενος). It was also common to specify the position of the species within the Greater Perfect System.

In Table 15, the reader will certainly recognize the use of the Greek ethnic names for the modes. As with medieval theory, the prefix *hypo-* indicates a similarity between the *hypo-* mode and the mode of the same name without the prefix; unlike the medieval system, however, it does not indicate anything about pitch centricism, and they also appear in the reverse order and beginning from a different point within the gamut. Two of the three basic names, Phrygian and Lydian, refer not to Greek cultures *per se*, but cultures within the Greek sphere of influence, in Anatolia. The Dorians were the people of the Peloponnese, while the later terms Iastian (a.k.a. Ionian) and Aeolian refer to the peoples of Attica and Thessaly, respectively (see Figure 15). Of course, two and a half millennia later, it is no longer possible to verify whether these modes (or their antecedents, described above) at all reflected the gamuts used by

³³⁹ See fn. 330 above.

the people for whom they are named. As for the Mixolydian, the meaning of the word is unclear. Musicologists have suggested a variety of derivations, including that it mixed the characteristics of the Lydian with some other scale, but the only explanation in the ancient sources appears to be in Ptolemy, who says it simply reflects the proximity to the Lydian (depending on the genus, between a quartertone and a semitone away).³⁴⁰

Unfortunately, the model presented above comes with a significant caveat: like the old ἀρμονία, of which these modes appear to be descendants, no coeval descriptions of the system have survived; all the writings describing this system as presented above come from substantially later, with the potential exception of Aristoxenus.

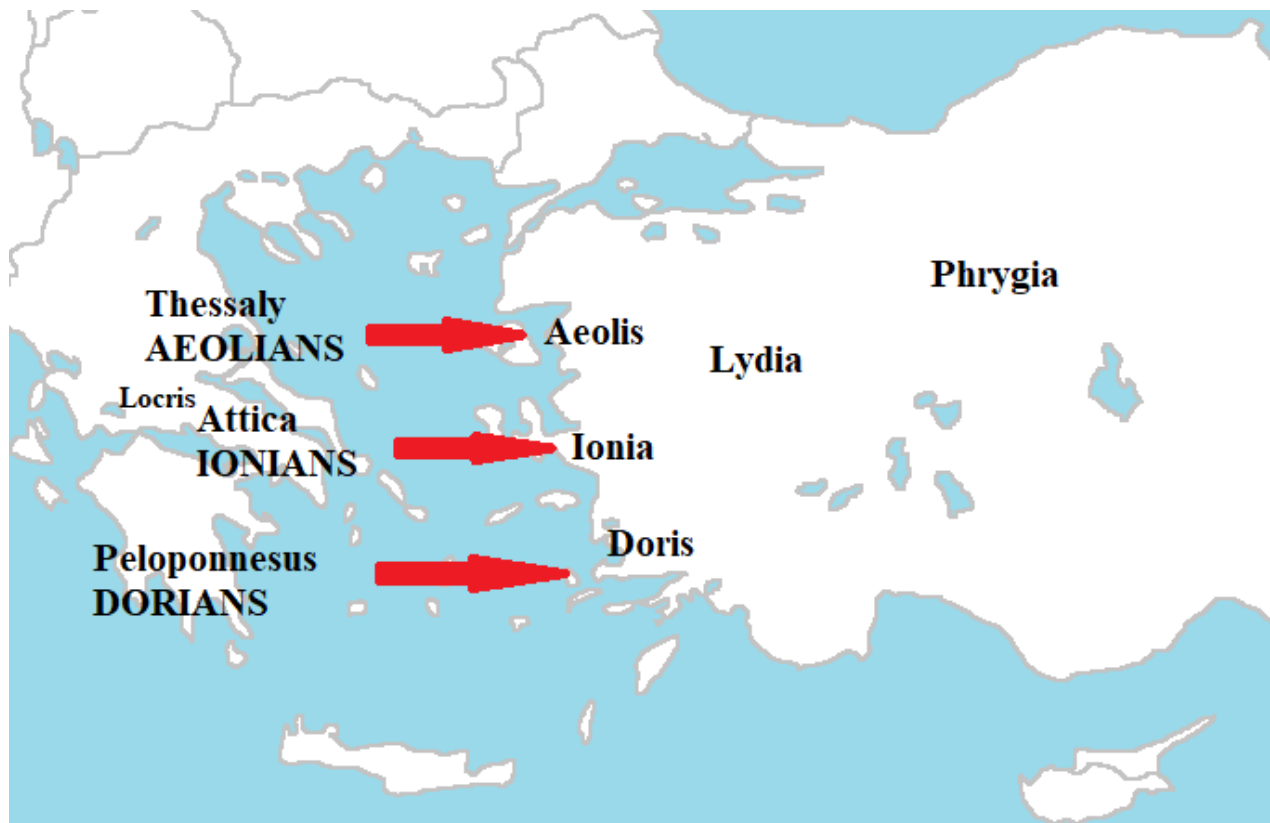


Figure 15 – Map of Ancient Greece and Anatolia, showing the origins of the modal names. The cultures from the Greek mainland also had colonies on the coast of Anatolia sharing their names.

³⁴⁰ Barker, *Greek Musical Writings II*, 336.

Greek Theory After Aristoxenus

Up to this point, it has been convenient to maintain the assumption that the variation created by the three *genera* and associated shades had no impact upon the modes. This assumption, however, seems to conflict with the notion that the ἦθος of a mode is determined by its interval structure, because only part of the interval structure of a mode is actually consistent; if the boundary notes held anything like the significance in Greek modes that they hold in later medieval modes, then the contradiction with concept of ἦθος is particularly significant for about half of the modes because the boundary pitches of these modes are themselves moveable notes. This fact is surprising, but since about half of the pitches in the gamut are moveable because of the *genera* and shades, the modes that run between these moveable notes are also similarly affected by the *genera*.

For example, the Phrygian mode spans from λίχανος ὑπάτων to παρνήτη διεzeugμένων, both of which are movable notes within the gamut. Depending on the genus and shade, the lowest interval can be as small as a tone or as large as a ditone (or one of several possible fractions in between). It might be natural to suppose that the two outer pitches of the Phrygian mode would remain in the same place regardless of genus and the entire scale structure would rise and fall within the bounds of the two outer pitches, but Ptolemy is quite particular that the correct manner of relating modes to one another is to maintain a constant interval amongst the fixed pitches.³⁴¹ For example, the lower boundary pitches of the Phrygian and Dorian modes are the λίχανος ὑπάτων and ὑπάτη μέσων, respectively; in the enharmonic genus, the interval between these two pitches is a ditone, while it is only a tone in the diatonic genus. For the same reason, the first interval in the Phrygian mode varies from a tone to a ditone, and yet, the ἦθος, apparently determined by interval structure, is unaffected.

³⁴¹ Ptolemy, Ἀρμονικά, bk. 2, ch. 10, ed. Düring, 62–64, trans. Barker, 336–38.

Furthermore, since the largest interval between the lowest note of a tetrachord and its second-lowest pitch is a semitone (in the chromatic and diatonic *genera*) and the smallest pair of intervals from the bottom up through the next two pitches is a pair of quartertones in the enharmonic (which also make a semitone), it is possible for two different pitches to occupy the same position relative to the bottom of the tetrachord in different *genera*. Any pitch called *παρυπάτη* in its own tetrachord occupies the same position in the tonic chromatic and both shades of the diatonic that is occupied by the *λίχανος* in the enharmonic; the same relationship occurs between pitches called *τρίτη* and *παρηνήτη*. This overlap creates the interesting by-product that the positions within the gamut of the boundary pitches of Phrygian and Hypophrygian in the enharmonic genus coincide with the positions of the boundary pitches of Lydian and Hypolydian in the diatonic and the tonic chromatic (Figure 16), yet they remain distinct modes because of their differing internal interval structures, however flexible those structures might be.

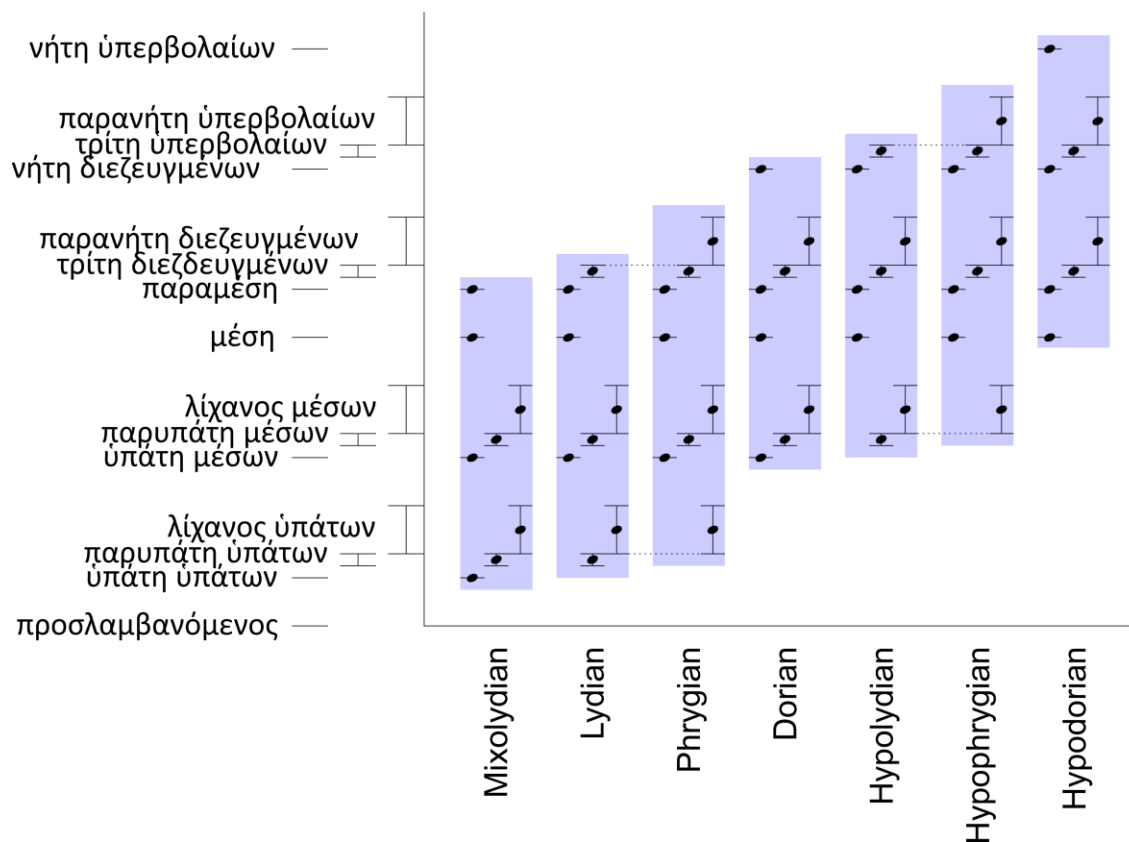


Figure 16 – Variability in interval structure within the modes caused by the genera

However, the evidence that the modes were not affected by *genera* is largely evidence from absence: if *genera* had any effect on mode, the ancient sources do not say so. It may well be that the modes were not actually defined outside the enharmonic genus, as the earliest sources only describe them in that context. This system was no longer the dominant form of mode by the time of Aristoxenus (more about this shortly), and Aristoxenus himself tells us that the theorists before him concerned themselves only with the enharmonic genus.³⁴² Even within the enharmonic genus, Barker has remarked that “we do not know precisely how artificial [the octave-species modal system] was, how carefully or how cavalierly it treated the facts of real performance.”³⁴³ The system may well have been an abstract theoretical construction that systematizes principles that were never so regular in practice (an observation that should, perhaps, be applied to more modal paradigms than is usually acknowledged).

A significant change in the conception of modes was already underway by the time of the first truly extensive Greek music treatise, that of Aristoxenus of Tarentum (fourth century B.C.), leading to what appears, on the surface, to be a dramatically different modal paradigm. Rather than defining modes as octave species within the Greater Perfect System, the modes take on a form traditionally described as “transposition-scales,” though I prefer to describe them by analogy to the modern concept of key signatures; however, the reader should not necessarily infer from this analogy the various trappings of modern keys (such as hierarchical pitch relations and functions). Several related phenomena are probably implicated in the paradigm shift, including the declining relevance of the enharmonic genus (notably in favour of the chromatic),³⁴⁴ which might have weakened the ἦθος by which the modes were distinguished, to the point at which Aristoxenus and his followers scarcely concern themselves with ἦθος.³⁴⁵ Certainly,

³⁴² Barker, *Greek Musical Writings II*, 152.

³⁴³ Barker, *Greek Musical Writings I*, 164.

³⁴⁴ Hagel, *Ancient Greek Music*, 10, 43–45; cf. West, *Ancient Greek Music*, 165.

³⁴⁵ Winnington-Ingram, *Mode*, 48–49.

it seems that the distinctions between the modes began to breakdown,³⁴⁶ perhaps (though not necessarily) resulting in a single octave species (or perhaps a few) becoming dominant.³⁴⁷ At that point, the only remaining difference between modes, which is the relative pitches between them, became the defining characteristics.

The two different conceptions of mode are not entirely unrelated. Using, for convenience, the terminology of modern major and minor modality as an analogy, the shift from major to minor may be accomplished in two ways: relative and parallel. The relative minor shares the same pitches as the major (notwithstanding the complexities of the harmonic and melodic minor) but begins two scale-degrees lower; this is approximately how the modes as octave species are conceptualized. However, shifting to the parallel minor involves the creation of the minor scale's characteristic interval pattern beginning from the same pitch as the major, and thus not using the same pitches – that is, creating a different key signature. Analogously, in the medieval modal system, moving from the third mode to the fifth can, loosely speaking, be accomplished either by remaining within the same gamut (*i.e.*, using only white keys) and shifting upwards in pitch by one scale step or by retaining the same overall pitch level and raising the second, third, fourth, sixth, and seventh scale degrees;³⁴⁸ one might anachronistically describe this process as implying a key signature (which would, in this case, be a relative of B–Major). Moreover, it is

³⁴⁶ Hagel, *Ancient Greek Music*, 39. Hagel makes the observation in the context of a different discussion, about the notating of music within a given mode within the corresponding key; *cf.* West, *Ancient Greek Music*, 185.

³⁴⁷ Winnington-Ingram, *Mode*, 77–78.

³⁴⁸ For simplicity, I have here ignored the flexibility in the medieval system between \flat and \natural (which correspond very loosely to an analogous concept in the Greek theory that I shall describe at the end of this chapter). In this particular example, it might not always be necessary to raise the fourth scale degree because the fifth mode frequently employs the flexibility between \flat and \natural to obtain the perfect fourth above the *finalis* rather than the augmented fourth implied by the *finalis*' usual position on F. It also should be acknowledged that the medieval system lacks nomenclature for many of these altered pitches; that is, if we assume that the third mode starts on E, raising the third scale degree (as described in the example) would produce G-sharp, a note for which there is no medieval name. This is really a non-issue, because the pitches of the medieval system do not have absolute frequencies; medieval musicians could quite happily shift from the third mode to the fifth while starting both at the same frequency and changing the name of the centric pitch from E to F, so that the conceptual G-sharp would be named A.

important to observe that the shift to a relative mode occurs in the opposite direction as the shift in resulting key signature: in modern terms, to shift from C-Ionian to E-Phrygian is a shift upwards by two tones, but the shift from C-Ionian to C-Phrygian implies the same key signature as A \flat -Ionian, a shift downwards by two tones; this reciprocity probably explains the reversal of direction in the naming of the Greek octave-species modes and the Greek key-signature modes (which, unlike the octave species, run in the same direction as the modern use of the ethnic names).

The shift in modal paradigm also brought with it a shift in terminology, from ἀρμονία to τόνος (*tonos*), though it is probably not the case that the two terms represent the two paradigms. Instead, the word ἀρμονία probably gradually fell out of favour as the word τόνος rose to prominence, much in the same way that the term *mode* has come to dominate for the same concept for which *tone* and *trope* were once far more common. Additionally, the paradigm shift was probably not a new paradigm arising to replace the old, but a gradual shift in one's understanding. R. P. Winnington-Ingram suggests that:

The conception of τόνος passed through two phases. In the first the τόνοι were the means of relating modal octaves in the same range of pitch by representing them as segments of a uniform scale repeated at different degrees of pitch. In the second these repetitions of the uniform scale took on an independent existence as keys in the modern sense. The second phase was clearly reached as a development of the first.³⁴⁹

What Winnington-Ingram means by his “first phase” is that one way of conceptualizing the relationship between octave species and keys is that transposing the gamut by a specified distance (for instance, down by step) brings a different octave species (in this case, the next higher octave species) into the same range as the previous. Thus, if one were to select a specific pitch range upon which to focus one's attention (perhaps conceptualized as a tessitura) – for instance, the highest octave of the lowest key, then cycling through successively higher keys would bring each successive octave species into the tessitura (see Figure 17).

³⁴⁹ Winnington-Ingram, *Mode*, 71.

Hypodorian		a	b	c	d	e	f	g	ȧ						
Hypophrygian		G	a	b	c	d	e	f	g						
Hypolydian		F	G	a	b	c	d	e	f						
Dorian		E	F	G	a	b	c	d	e						
Phrygian		D	E	F	G	a	b	c	d						
Lydian		C	D	E	F	G	a	b	c						
Mixolydian		B	C	D	E	F	G	a	b						
Mixolydian (e \flat -)	E \flat	F	G \flat	a \flat	b \flat	c \flat	d \flat	e \flat	f	g \flat	a \flat	b \flat	c \flat	d \flat	e \flat
Lydian (d-)	D	E	F	G	a	b	c	d	e	f	g	ȧ	ḃ	ċ	ḋ
Phrygian (c-)	C	D	E \flat	F	G \flat	a \flat	b \flat	c	d	e \flat	f	g	ȧ	ḃ	ċ
Dorian (b \flat -)	ḃ	C	D \flat	E \flat	F	G \flat	a \flat	b \flat	c	d \flat	e \flat	f	g \flat	ȧ	ḃ
Hypolydian (a-)	A	B	C	D	E	F	G	a	b	c	d	e	f	g	ȧ
Hypophrygian (g-)	Γ	A	ḃ	C	D	E \flat	F	G	a	b	c	d	e \flat	f	g
Hypodorian (f-)	Φ	Γ	A \flat	ḃ	C	D \flat	E \flat	F	G \flat	a \flat	b \flat	c	d \flat	e \flat	f

Figure 17 – The relationship between modes as octave species and modes as transposed gamuts. The upper chart shows the octave species in the Greek gamut in the Tense Diatonic; in the lower chart, the box shows these same octave species transposed into the range F–f; these are then extended outwards to show the entire two-octave gamut implied by these species. The accidentals imply key signatures that ascend (lower chart) as the corresponding octave species descend within the gamut (upper chart), resulting in a reversal of direction for the order of modal names. Note that while I am describing the Greek modes, I have, for simplicity, employed the usual practice for medieval music of identifying B-flats with the symbols \flat and \flat , while B-natural is indicated by B and \natural .

These two paradigms would almost certainly have operated in tandem when a pair of musicians attempted to play a duet on lyre and aulos, the two most popular instruments of the era. The closest comparison for the aulos today is probably an oboe, but originally without mechanical keys and with relatively limited ability to play in different key areas. Like modern harmonicas and especially pennywhistles, they would have come pre-tuned to a variety of keys, and like the pennywhistle, varying in size while maintaining a consistent fingering pattern (to a lesser degree, this also resembles families of transposing wind instruments like the saxophone). The lyre, however, was a harp-like instrument, and was too large to make it practical to carry around a lyre in several different keys (as an aulos player could), but unlike an aulos, it could be easily retuned. Like all string instruments, however, the ability to retune

any given string is limited to within a relatively small range; if tuned too low, the tone quality becomes unacceptable, and if tuned too high, strings begin to break too easily. Thus, if the aulos-lyre duo wished to change keys between songs, the aulos player would most likely put down one aulos and pick up a different one (*i.e.*, transpose the fundamental pitch of the instrument), while the lyre player would retune some of the lyre strings to achieve the corresponding octave species on the lyre (thus staying within the same range, but achieving all the necessary accidentals, to use an anachronistic term).

The exact position of Aristoxenus in the development of this modal system over time is uncertain. Winnington-Ingram believes that “he stands at the point of transition from phase to phase, and his τόνοι hover uneasily between the two conceptions.³⁵⁰ Unfortunately, it is difficult to know for certain what Aristoxenus himself thought about mode because the surviving manuscripts end immediately after his enumeration of the species of fourth, just before the expected discussion of octave species (and then, presumably, keys).³⁵¹ What is known about the rest of Aristoxenus’ theory comes from the writings of his followers, most notably Cleonides, who, as Barker puts it, “follows Aristoxenus slavishly.”³⁵²

Although all of this attention to the finer distinctions of mode is rather far afield from the modality of the *Alia musica*, it is necessary background towards a caveat that ought to be made and is often missed regarding the interpretation of the revisor’s fusion of Greek and ecclesiastical modes. This caveat still requires a little more background in the development of the modes through Ptolemy and Boethius.

Before moving on to Ptolemy, however, it is important to note that with the final shift away from octave species to modes conceptualized exclusively as keys, modes were no longer limited to the maximum of seven (sometimes eight; more about this in the discussion of Ptolemy) for which there are unique octave species. The Aristoxenians initially filled in the gaps between the original seven with six

³⁵⁰ Winnington-Ingram, *Mode*, 74.

³⁵¹ Barker, *Greek Musical Writings II*, 184.

³⁵² Barker, “Aristides Quintillianus,” 187.

more. At first, the names for these additional keys were repetitions of the existing names, distinguished by labels like “upper Lydian” and “lower Lydian” (a completely separate distinction from Lydian and Hypolydian, as the Hypolydian also had “upper” and “lower” forms). The meaning of these labels is easily explained in the context, again, of the lyre. If a lyre is tuned to the Dorian octave species (regardless of absolute pitch, equivalent to E – e) and one wishes to retune to the Lydian octave species (equivalent to C – c, but maintaining the same overall pitch), there would be two possible ways to do so that would be equally efficient. The difference between the two modes is on four of the eight notes of the octave; thus, four strings could be tuned up by semitone, giving the upper Lydian, or the opposite four strings could be tuned down by semitone, giving the lower Lydian; the overall pitch of these two modes differs by a semitone (Figure 18).

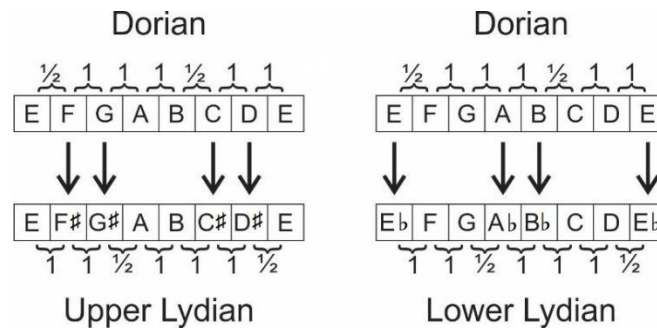


Figure 18 – Retuning the lyre from the Dorian octave species to the Lydian.

There are two possible ways to retune; half of the strings can be tuned up a semitone, or the opposite four strings can be tuned down a semitone. Both retunings result in the Lydian octave species, but the first converts the overall C-Major key signature up a major third to E-Major, while the second converts up a minor third to E_b-Major; thus, the first is the upper Lydian and the second is the lower Lydian.

The expanded set of modes also included two additional modes added to the top, referred to as the upper and lower *Hyperphrygian*, standing a fourth above the Phrygian, by analogy to the *Hypophrygian* a fourth below the Phrygian. In the process, the Mixolydian was also renamed the Hyperdorian. Eventually, the new modes were given unique names, based on the additional ethnic names *Iastian* (a variant of *Ionian*, and the preferred form throughout antiquity, the Middle Ages, and the Renaissance) and *Aeolian*. Yet another pair of modes was also added to the top of the system so that each of the five ethnic names had their own Hypo- and Hyper- forms, giving a total of fifteen modes (see

Table 16), three of which would be considered octave equivalents by modern standards, creating a circle of keys remarkably similar to the modern cycle of fifths with its three overlapping pairs of keys; however, it is not entirely clear that the Greeks would have considered them to be equivalent.

Table 16 – Two alternate nomenclatures for the Greek modes (as keys).³⁵³

Upper Hyperphrygian (Hypermixolydian)	Hyperlydian
Lower Hyperphrygian	Hyperaeolian
Hyperdorian (Mixolydian)	Hyperphrygian
Upper Lydian	Hyperaastian
Lower Lydian	Hyperdorian
Upper Phrygian	Lydian
Lower Phrygian	Aeolian
Dorian	Phrygian
Upper Hypolydian	Iastian
Lower Hypolydian	Dorian
Upper Hypophrygian	Hypolydian
Lower Hypophrygian	Hypoeolian
Hypodorian	Hypophrygian
	Hypoastian
	Hypodorian

Ptolemy

The system of fifteen *τόνοι* described above remained essentially the standard paradigm right through the rest of the Greco-Roman era, with a few refinements. These refinements were proposed by the second-century Alexandrian theorist Claudius Ptolemy.

By Ptolemy's time, not only the enharmonic genus but also the chromatic genus had largely fallen out of use (Ptolemy says that only the shades of the diatonic would sound familiar to his audience).³⁵⁴ It seems likely that just as the rise in the popularity of the Chromatic genus guided the change in modal

³⁵³ The dual nomenclature for the new *tonoi* ("lower Hypophrygian" vs. "Hypoastian," etc.) is described in Aristides Quintilianus, *On Music*, ch. 10, trans. Barker, 421–22, and is also touched on (and criticized) in Ptolemy, *Ἀρμονικά*, bk. II, ch. 11, ed. Düring, 65, trans. Barker, 339; the terms Ptolemy uses for "lower" and "higher," respectively, are βαρύτερον and ὀξύτερον.

³⁵⁴ Barker, *Greek Musical Writings II*, 311.

conception in Aristoxenus' time, so the rise of the diatonic influenced Ptolemy's formulations. Ptolemy's treatise manifests two, mutually reinforcing changes to the system.

First, Ptolemy argues that since moveable pitches imply a multiplicity of available notes, and since keys are built on top of available notes, if just any old note could be the basis of a key, there would be a proliferation of keys, many of which would be indistinguishable by ear³⁵⁵ (as, for instance, two keys based upon the second note from the bottom of any tetrachord in the soft and hemiolic chromatics, which would differ from each other only by a twelfth of a semitone – slightly less than eight and one-third cents). He argues against the placement of keys on just any pitch, and particularly on pitches that are evenly spaced, and notes that the traditional placement of a τόνοϛ on each semitone (a) becomes redundant because two different modes can project the same octave species onto the central octave (see Figure 18, above); and (b) is also inconsistent, because the projection of these same two modes would cause the overall pitch of the central octave to fluctuate by semitone.³⁵⁶ He also incorporates octave equivalence into his argument, rejecting those modes that create the same octave species.³⁵⁷ Thus, he argues that the only acceptable keys are the seven traditional modes – those that project, in turn, the seven octave species onto the central octave.³⁵⁸ These keys, he says, should be positioned relative to each other by comparing the positions only of the fixed notes (especially the μέση), which should relate to one another by perfect fourth or fifth.³⁵⁹ Because of the close relationship between the octave species and the Greater Perfect System, the result is a system in which pairs of fixed pitches separated by an octave (such as the μέση and

³⁵⁵ Barker, *Greek Musical Writings II*, 331–32; 335.

³⁵⁶ Barker, *Greek Musical Writings II*, 335; 338–40.

³⁵⁷ Barker, *Greek Musical Writings II*, 333–34.

³⁵⁸ Barker, *Greek Musical Writings II*, 334–36.

³⁵⁹ Barker, *Greek Musical Writings II*, 336–38.

νήτη ὑπερβολαίων) of each mode in succession collectively project the interval structure of the Greater Perfect System in the ditonic diatonic.³⁶⁰

Ptolemy's second refinement is easily missed if one reads only his discussion of the τόνοι, which he seems to describe as being raised up or down, as had his predecessors. Ptolemy's actual view is not entirely clearly articulated and requires a little reading between the lines. It comes from his discussion of the structure of the gamut and the naming of notes by both position and function. As Ptolemy explains, the names of the notes in the Greek system are derived from their positions in the gamut; for instance, μέση literally means "middle [note]," while ὑπάτη ὑπάτων means "high [note] of the high [notes]." Thus, regardless of how the intervals are rearranged within the gamut, it still makes sense to use these names for the notes that appear in these positions – what Ptolemy called the name by θέσις (*thesis*, position). However, each note also has a function, derived from the interval structure surrounding that note; the μέση, for instance, has fixed notes a perfect fourth below, a major second above, and a perfect fifth above. Any pitch meeting that description may appropriately be called the μέση, regardless of where it appears within the overall system; this is note naming by δύναμις (*dynamis*, faculty or function).³⁶¹

It is important to acknowledge that these two note-naming systems presuppose a particular conception of the modes; they do not differ from each other in any way in a system in which a key is understood as a wholesale transposition of the gamut. If one begins with, for instance, the Lydian τόνος, with the μέση in its usual place as the eighth of fifteen pitches, and then transposes the gamut up a tone to reach the Phrygian τόνος, as in the traditional understanding of mode, the interval structure transposes

³⁶⁰ Ptolemy's shades are slightly different than Aristoxenus' because he bases them on ratios rather than equal divisions of a tone, but his ditonic shade of the diatonic genus is loosely equivalent to Aristoxenus' tense diatonic. For this division, cf. Ptolemy, Ἀρμονικά, bk. 1, ch. 16, ed. Düring, 40, trans. Barker, 314.

³⁶¹ Barker, *Greek Musical Writings II*, 325–27.

with it, so that interval structure that implies the function of μέση continues to appear at the eighth of fifteen pitches.

Ptolemy's actual conception of mode is revealed in his description of note-naming by function: he says that one begins by finding the higher of the two tones of disjunction (the only two places in the gamut where two fixed notes appear side-by-side) and labelling the lower pitch μέση and the higher pitch παραμέση. One then proceeds to the other tone of disjunction and labels the higher pitch ὑπάτη ὑπατων, but importantly, labels the lower pitch as *both* προσλαμβανόμενος (the lowest pitch of the standard system) *and* νήτη ὑπερβολαίων (the highest pitch of the standard system).³⁶² That is, in Ptolemy's conception, the lowest pitch and the highest pitch are equivalent to each other. The gamut is thus not linear but circular, and modes are created not simply by successively raising the entire gamut, as it is furthermore necessary to rotate any intervals that rise above the standard range down to the bottom of the system to fill the gap left behind at the bottom end of the range. Put another way, Ptolemy's conception of mode may be better described not as the transposition of an entire gamut upward to project successive octave species upon a central octave, but rather as a circular gamut rotated so as to project successive octave species upon a specified half of the circle (see Figure 19). In this sense, Ptolemy's keys might equally be considered to be species of the double-octave held within a fixed range.³⁶³

³⁶² cf. Barker, *Greek Musical Writings II*, 325–26. Barker also address Ptolemy's characterisation of the modes extensively in *Scientific Method*, 159–91 (especially 179–82).

³⁶³ Ptolemy's discussion of the issue is, as I have mentioned, focussed on the two systems of pitch nomenclature; he never actually describes the modes in this manner. Nevertheless, my interpretation of Ptolemy's conception is not controversial. Barker speaks of Ptolemy's conception of modes in this manner in *Greek Musical Writings II*, 19, as does Winnington-Ingram in *Mode*, 62–71. See also Solomon, *Ptolemy Harmonics*, 74 fn. 77, though I do not agree with the assertion of Ernest McClain, cited by Solomon, that the bending of the gamut into a circle implies equal temperament, a concept essentially refuted by Ptolemy's procedure for relating τόνοι to one another through chains of pure fourths and fifths, as described above.

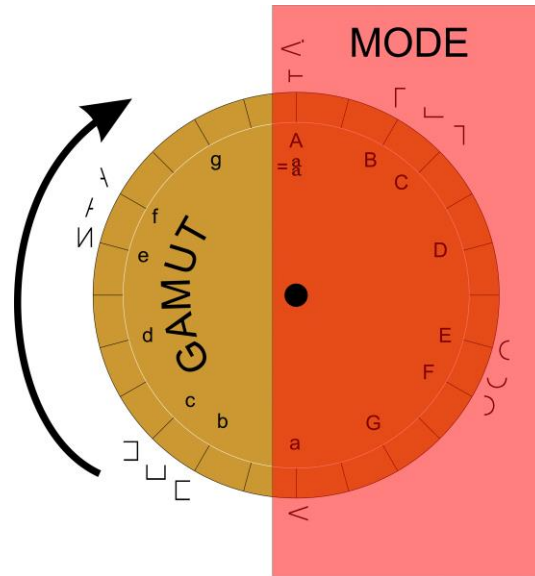


Figure 19 – A representation of the conception of mode implied by Ptolemy’s description of the note-naming system.

The gamut is effectively circular, with the highest and lowest pitches of the standard position being equivalent to each other. Mode arises from the rotation of the system so that a different octave species arises on a static half of the system.

Taken together, the effect of Ptolemy’s refinements to the modal concept is that mode has effectively returned from a key signature paradigm to an octave species paradigm. Once again, the purpose of this detailed examination of Ptolemy’s conception is in preparation for a caveat regarding the *Alia musica*, to be made after a consideration of Boethius. Before leaving Ptolemy, however, there are two direct references to Ptolemy in the *Alia musica* that ought to be addressed.

The references in question are in §16(b) and §130(c); both say essentially the same thing, and make the same error – but one that scarcely deserves to be so identified, as it is a repetition of an error in Boethius (and verifies the general assumption that the authors of the *Alia musica* never read Ptolemy, but learned of his doctrines via Boethius).

As may be verified by reviewing either Figure 17 or Table 15 above, the seven octave species occupy only seven of the eight possible octave segments of the Greater Perfect System. The final octave segment, that spanning A – a (and thus including the προσλαμβανόμενος, the note “taken in addition” to complete the second octave) does not receive a mode of its own. The reason for this exception is that the interval structure of such a mode would be equivalent to the interval structure for the Hypodorian, and the mode

is therefore redundant. This logic, however, did not always stop theorists from describing this mode for the sake of completion. The extra mode was called the Hypermixolydian since it stood one tone beyond the Mixolydian. (The redundancy also does not apply to the modes-as-keys paradigm, and when the Mixolydian was renamed Hyperdorian, the Hypermixolydian was renamed Hyperphrygian, thus preserving the threefold symmetry of the system).

The Hypermixolydian mode also appears in the *Alia musica*, where it appears as the eighth and final mode, the later term Hypomixolydian for an eighth mode not occurring in the *Alia*.³⁶⁴ In §16(b), the *Alia* says, “*octauum modum hypermixolydium Ptolomaeus adiecit*,”³⁶⁵ (“Ptolemy added the eighth mode, the Hypermixolydian); in §130(c), the *Alia* says “*hypermixolydus sane dicitur, quod mixolydium transcendit, qui iuxta Ptolomaeum octauam speciem diapason*.”³⁶⁶ (“[That mode] is clearly called the Hypermixolydian that transcends the Mixolydian, which in accordance with Ptolemy is the eighth octave species.”)

In fact, contrary to the statements in the *Alia musica*, Ptolemy did not admit of the Hypermixolydian or an eighth octave species in general. He devotes Book II, Chapter 9 to the proposition that there are only seven τόνοι, equal to the number of octave species,³⁶⁷ and he begins Chapter 10 with a criticism of “οἱ μέχρι τῶν ὀχτῶ τόνων προσελθόντες διὰ τὸν ἕνα τὸν περισσῶς τοῖς ἑπτὰ συναριθμούμενον”³⁶⁸ (“those going as far as the eighth τόνος by counting the superfluous one with the seven”). As I have already suggested, however, the erroneous attribution of this eighth mode to Ptolemy is not original to the *Alia musica* but comes by way of Boethius’ *De institutione musica*, Book IV, Chapter 17. Boethius says,

³⁶⁴ As far as I can tell, the earliest uses of “Hypomixolydian” for the eighth mode are from Hermannus Contractus, Theoger of Metz, Aribo, and Johannes Affligemensis, all late eleventh or early twelfth century. The conception of this eighth mode is significantly different in these later works than in Ptolemy and Boethius, but the *Alia musica* wavers indecisively between the two conceptions, as I will describe in chapter 09.

³⁶⁵ *Alia musica* §16(b), ed. Chailley, 107. Chailley, *Alia musica*, 107. cf. Atkinson, *Critical Nexus*, 188; 190.

³⁶⁶ *Alia musica* §130(c), ed. Chailley, 163.

³⁶⁷ Barker, *Greek Musical Writings II*, 334–36.

³⁶⁸ Ptolemy, Ἀρμονικά, bk. 2, ch. 9, ed. Düring, 62, trans. Barker, 336.

“*Cur autem octauus modus, qui est hypermixolydianus, adiectus sit, hinc patet.*”³⁶⁹ (“Why, however, the eighth mode, which is the Hypermixolydian, has been added is exposed next”). He then sets out a linear representation of the fifteen notes of the gamut (in this instance ignoring distinctions of genus) and labels the notes with the first fifteen letters of the Latin alphabet (A–P, remembering that J is not an independent letter in Latin); after describing the standard seven modes as spanning segments from A–H through G–O, he remarks that “*relinquitur igitur extra ·HP·, quae ut totus ordo inpleretur, adiecta est. Atque hic est octauus modus, quem Ptolomaeus superadnexuit.*”³⁷⁰ (“There remains, besides, H–O, which is added so that the complete order might be filled. And this is the eighth mode, which Ptolemy joined above [the others].”)

There is widespread agreement that Boethius’ fifth book is a loose translation of Ptolemy, and so Boethius must certainly have known that Ptolemy disagreed with, rather than added, the eighth octave species. It is therefore strange that Boethius should have directly attributed the Hypermixolydian to Ptolemy. However, Boethius’ treatment is incomplete, ending effectively at the end of Ptolemy’s Book I (though chapter headings survive for additional chapters), and the relevant discussion in Ptolemy occurs in the middle of Book II. Calvin Bower, who wrote the standard English translation of Boethius, suggests that the comment should be interpreted as “a gloss concerning the *position* [emphasis added] of the eighth mode,” rather than attributing the very existence of the eighth mode to Ptolemy,³⁷¹ an explanation that is only partly convincing; on the one hand, the term *superadnexuit* does signify the *positioning* of something at the top, rather than *inventing* something (as the *Alia* would seem to have interpreted it), but on the other hand, it also does not mean merely that Ptolemy *described* it as being customarily placed

³⁶⁹ Boethius, *De institutione musica*, bk. 4, ch. 17, ed. Friedlein, 347, trans. Bower, 159.

³⁷⁰ Boethius, *De institutione musica*, bk. 4, ch. 17, ed. Friedlein, 347–48, trans. Bower, 159–60.

³⁷¹ Bower, *Fundamentals of Music*, xxvi; 160, fn. 91.

at the top, which would be a much more accurate description (and well within Boethius' command of the language). Stefan Hagel's opinion seems more likely:

It is therefore hardly possible that Boethius derived his erroneous attribution from a reading of Ptolemy's work, however cursory it might have been. Presumably he had not yet studied [Ptolemy's] *Harmonics* closely when working on the fourth book of his *De institutione musica*. At any rate, the present chapters are certainly not based directly on Ptolemy. Both the seven-key and eight-key systems were older than Ptolemy, and the association of *tónoi* and octave species might date back even to pre-Aristoxenian theory. Apparently, therefore, Boethius' source for his tables of *modi* adheres to the tradition Ptolemy criticises. Nicomachus, whose work stands behind the initial books of *De institutione musica*, is a likely candidate. On the other hand, there are good arguments that Nicomachus is probably dependent on Ptolemy. In this case, Boethius' attribution of the eighth key to Ptolemy might have been induced by some ambiguous phrasing in Nicomachus' text.³⁷²

Boethius

Anicius Manlius Torquatus Severinus Boethius was almost certainly the most important pre-medieval source of music theory for medieval theorists. He was a Roman functionary in the late fifth and early sixth centuries who took it upon himself “to record in Latin the sources and background of his exceptional Greek education.”³⁷³ Bower describes him as “the unique source [to the Carolingians] for the

³⁷² Hagel, *Ancient Greek Music*, 100. Unfortunately, it would be impossible at present to confirm the final element of Hagel's hypothesis – that there was some ambiguity in Nicomachus' text; Hagel leaves unsaid that while Nicomachus' *Enchiridion* (“handbook”) has survived (translated in Barker, *Greek Musical Writings II*, 247–69), he promised in it to write a longer *Eisagoge* (“elementary treatise, introduction”), which, if it was ever completed, has not survived, and his description what it would contain corresponds quite well with the pre-Ptolemaic part of Boethius' treatise (Bower, *Fundamentals of Music*, xxvi–xxvii); thus, if such an ambiguity were present in Nicomachus, it would be in the lost *Eisagoge*, and so the hypothesis cannot be evaluated. One point in Hagel's hypothesis is suspicious (though it is not a critical point, and probably does not invalidate the hypothesis): it seems unwise to suppose that Boethius had not yet studied Ptolemy in detail when writing the fourth book of his treatise. Throughout the earlier books, Boethius frequently acknowledges where Ptolemy would disagree with the arguments being presented and indicates that he will describe Ptolemy's dissenting opinion later. A particularly notable case occurs in Book I, Chapter 32: “*Non uero eodem modo hoc Ptolomaeus, cuius omnem sententiam posterius explicabo.*” (Boethius, *De institutione musica*, bk. 1, ch. 32, ed. Friedlein, 222, trans. Bower, 49) (“Ptolemy, all of whose opinions I shall explain later, does not truly [rank the consonances] in this same manner”). Additional examples of Boethius' continual awareness of Ptolemy's opinions appear in Book I, Chapters 5 & 6; and Book II, Chapters 18 & 27 (Bower, *Fundamentals of Music*, 14, 15, 72, & 82). In fact, Hagel, himself, acknowledges that Boethius' initial presentation of only seven modes in the preceding chapter (Book IV, Chapter 16) reflects Ptolemy's opinion (as opposed to that of the Aristoxenians), though without providing Ptolemy's rationale (Hagel, *Ancient Greek Music*, 99). Hagel's final speculation in this excerpt, that Nicomachus may have drawn upon Ptolemy, comes from Bower, *Boethius and Nicomachus*, 42; this hypothesis conflicts with the usual assumption about the relative dates of the two theorists, and though the dating for Nicomachus, in particular, is very uncertain, the hypothesis should be taken as speculative at best.

³⁷³ Bower, *Fundamentals of Music*, xix.

thorough mathematical underpinning of Western musical theory.”³⁷⁴ His two most important treatises for musical purposes are the *De institutione arithmetica*, a translation of Nicomachus’ *Eisagōgē arithmētikē*, and the *De institutione musica*, at least the first three books (and perhaps the fourth) of which are presumed to be a translation of a non-extant *Eisagōgē mousikē* (also by Nicomachus), while the fifth book is a loose translation of Ptolemy’s *Harmonics*.³⁷⁵ Although Boethius’ treatise is ultimately the source of many elements of the *Alia musica*, I shall here confine myself to a discussion of modal theory and will address the remaining issues later, in a chapter describing the conventional theoretical elements of the *Alia musica*.

Despite Ptolemy’s return to the octave species paradigm of modality and Boethius’ use of Ptolemy as a principal source, there can be little question that Boethius described mode in his *Musica* as a raising or lowering of the gamut – that is, in the manner of keys signatures.³⁷⁶ The presentation is given in Book IV, Chapter 15, in a passage that Atkinson calls “a model of clarity.”³⁷⁷ In fairness to Atkinson, the particular excerpt that he cites, drawn from the middle of the chapter, is relatively clear, but it follows closely on the heels of a confounding introduction that may easily prime the reader to interpret the subsequent passage differently. He says:

*Ex diapason igitur consonantiae speciebus existunt, qui appellantur modi, quos eosdem tropos uel tonos nominant.*³⁷⁸

Out of the species of the octave consonance, therefore, emerge what are called modes, the same which they name tropes or tones.

He goes on to say that the modes are systems (in the Greek sense of *συστήματα*, a scale segment spanning a perfect consonance, of which the Greater Perfect System is one example) that vary according to overall pitch. It is easily overlooked that this statement applies equally well both to keys and to octave

³⁷⁴ Bower, *Fundamentals of Music*, xx.

³⁷⁵ Bower, *Fundamentals of Music*, xxvi–xxvii.

³⁷⁶ Boethius, *De institutione musica*, bk. 4, chs. 15–16, ed. Friedlein, 341–43, trans. Bower, 153–56.

³⁷⁷ Atkinson, *Critical Nexus*, 22.

³⁷⁸ Boethius, *De institutione musica*, bk. 4, ch. 15, ed. Friedlein, 341, trans. Bower, 153.

species. Of course, the keys are, according to the pre-Ptolemaic conception, the entire Greater Perfect System raised or lowered. However, the octave species equally are systems, spanning an octave in this case, which in their original conception (and in Boethius' description in the preceding chapter) are demonstrated as higher or lower one-octave segments of the Greater Perfect System (with one minor distinction: Boethius leaves the προσλαμβανομένος out of this discussion, thus omitting the eighth, superfluous octave species).

For a reader well versed in the Aristoxenian conception of mode, and likewise for a careful reader lacking any preconceptions about mode, the description that follows the citation given above (and after a few examples of systems) is, indeed, as Atkinson describes it, "a model of clarity." However, a reader approaching the passage with a preconception of mode as defined principally by interval structure, such as the authors of the *Alia musica*, it would be easy to interpret Boethius' explanation in that light. Boethius takes the Hypodorian mode as his starting point, and then says:

*Si quis igitur proslambanomenon in acumen intendat tono hypatenque hypaton eodem tono detenuet ceterasque omnes tono faciat acutiores, acutior totus ordo proveniet, quam fuit priusquam toni susciperet intentionem. Erit igitur tota constitutio acutior effecta hypophrygius modus.*³⁷⁹

If someone, therefore, would raise the προσλαμβανόμενος in height by a tone [*i.e.*, by a major second; here, he uses *tonus* to mean an interval, while he describes modes as *modus*] and would increase the ὑπάτη ὑπάτων by the same tone and would make all the rest higher by a tone, the whole order would rise higher than it was before it took up the raising of the tone. The entire constitution, having been made higher, will, therefore, be the Hypophrygian mode.

The interpretation of *cetera* can affect the understanding of this passage. Almost certainly, Boethius means that all the other notes should individually be raised by a tone. However, if the reader has preconceptions of modes as interval structures, *cetera* could be interpreted to mean the rest of the structure, collectively, being raised by a tone; within this interpretation, some notes could rise by tone while others rise by semitone, while the overall pitch of the collective structure (that is, its range) rises by tone – or, put another way, the frame of reference rises by a tone, so that the pitches seem to rise either

³⁷⁹ Boethius, *De institutione musica*, bk. 4, ch. 15, ed. Friedlein, 342, trans. Bower, 154.

by tone or semitone; such a reader would thereby be able to hold on to the preconception.³⁸⁰ Though this interpretation of *cetera* seems problematic in light of the better fit of the key signature paradigm of modes, it seems that it may be the interpretation favoured by Bower, who translates the critical clause (“*ceterasque omnes tono faciat acutiores*”) as “thereby making the whole disposition higher by a tone.”³⁸¹

Boethius in the *Alia musica*

Enough background has now been presented to present the promised discussion of the *Alia musica*'s appropriation of the Greek ethnic names. Despite the long unquestioned medieval narrative of continuity between Greek modes and the ecclesiastical modal system, it is now usually held that the two systems are entirely unrelated. Thus, it is commonly accepted that the application of the Greek modal nomenclature to the ecclesiastical modes in the *Alia musica* resulted from the author of the *Alia* misreading Boethius, and the discussion above presents a possible mechanism by which that misreading could have occurred. Chailley comments:

*[C]hez celui-ci, l'Alia retrouvera mention des 8 octaves, assorties de noms topiques, sans savoir qu'il s'agit des tons de transposition; on comprend dès lors aisément que l'auteur, ignorant totalement la musique grecque en général et les tons de transposition en particulier, ait cru qu'il s'agissait d'un renseignement complémentaire sur les aspects d'octave, et ait pensé agir en commentateur habile bien qu' aventureux [...] en mélangeant ces deux séries d'information.*³⁸²

In the work of this author [Boethius], the *Alia* finds mention of eight octaves, matched with “topic” names [*i.e.*, Greek ethnic names, Dorian, Phrygian, etc.], without knowing that they pertain to the modes of transposition; we understand easily from this that the author, being totally ignorant of Greek music in general and the modes of transposition in particular, had believed that [the passage in question] pertained to supplementary information on the species of the octave, and had thought to act as a commentator both able and adventurous [...] while mixing these two series of information.

³⁸⁰ In his written feedback following the defence of this dissertation, Atkinson suggested to me that Boethius may well have intended *tota constitutio* as a translation of σύστημα τέλειον, the Perfect System, which would therefore require the entire interval structure to be transposed by a tone. I suspect that Atkinson is probably correct in this hypothesis. However, the question at hand is not what Boethius meant, but what he said and how it could reasonably have been interpreted by medieval readers lacking other sources of Greek theory and with strong preconceptions about the nature of mode.

³⁸¹ Bower, *Fundamentals of Music*, 154.

³⁸² Chailley, *Alia musica*, 39.

I do not disagree with this hypothesis in principle; I consider it to be the most likely explanation. However, it is not the only possible explanation for the *Alia's* use of these names. As has been discussed at length over the preceding pages, the notions of mode as key and mode as octave species are intimately linked to each other. It is not at all clear where Aristoxenus stood on this point, and while his successors certainly treated mode as key, Winnington-Ingram notes that for Ptolemy, “the two conceptions were indistinguishable,”³⁸³ (though he also feels that “parts of Ptolemy’s polemic are unintelligible except as a polemic against pitch-keys”;³⁸⁴ presumably he means against pitch-keys dissociated from octave species; thus, Ptolemy’s description leans in the direction of octave species).

Here, it must be reiterated that the last book of Boethius’ *De institutione musica* is a loose translation of Ptolemy, and much of the rest is considered to be a loose translation of a lost treatise by Nicomachus. And while the sources of Boethius’ Book IV (and thus, the description of the modes in Chapter 16) are the least reliably established, Hagel considers Boethius’ description of the modes to be at least “related to Ptolemy’s approach.”³⁸⁵ Indeed, there is practically nothing in Boethius’ Book IV, Chapter 16 that is inconsistent with Ptolemy’s conception of the modes; it is true that Boethius says nothing about the rotation of excessively high intervals back down to the bottom of the system as modes are transposed upwards, but neither does Ptolemy in his discussion of the τόνοι; that detail must be read between the lines in his discussion of pitch names. In every other respect, Boethius’ description agrees with Ptolemy’s: the modes are derived from the octave species; they can also be described as transposing a fixed sequence of intervals up by a constant interval; and at least initially, Boethius only describes Ptolemy’s seven.

The principle distinction between Boethius’ presentation and Ptolemy’s is that Ptolemy explicitly describes the manner in which the keys reflect the octave species. Boethius, too, states that they are

³⁸³ Winnington-Ingram, *Mode*, 20.

³⁸⁴ *ibid.*, 51.

³⁸⁵ Hagel, *Ancient Greek Music*, 99.

related. In fact, he says so twice; in addition to the passage cited above, from the introductory paragraph of his discussion, he also says, just before describing the intervals by which the modes are transposed, that:

*Has igitur constitutiones si quis totas faciat acutiores, uel in grauius totas remittat secundum supradictas diapason consonantiae species, efficiet modos ·VII·.*³⁸⁶

If someone, therefore, would make these entire constitutions higher, or return them to a lower position, *according to the above-mentioned species of the octave consonance* [emphasis added], it would produce the seven modes.

But beyond these two references, Boethius never explains what it means to transpose “in accordance with the octave species.” He goes on to demonstrate the transposition, but even after he is finished, the reader is still no wiser about how his transpositions reflect the octave species. Bower notes that “the explanation is sparse. Nevertheless, the principle determining the transposition of a system to derive a mode is clear: it is the species.”³⁸⁷ Bower then directs the reader to a footnote to the subsequent chapter, in which Boethius demonstrates the modes in Alypian notation (which Boethius elsewhere describes, but which is not easily internalized and thus is not immediately intelligible to the average reader), where he says that “visual perception of species as the governing principle in modal transpositions is not explained in the text and thus requires imagination on the part of the reader!”³⁸⁸

The diagram is presented twice: first as a blocky table of each of the (now eight) modes directly side-by-side, displayed only in Alypian notation, which would require cross-referencing to interpret; this first table would likely have been ignored by most readers in favour of the second, more intuitive diagram (the so-called “Wing Diagram”) in which each column is offset from its neighbours to show relative pitch and in which empty cells are included to show the difference between tones and semitones. Experienced musicologists can easily underestimate how unintuitive this table can be to a reader unfamiliar with Greek

³⁸⁶ Boethius, *De institutione musica*, bk. 4, ch. 15, ed. Friedlein, 343, trans. Bower, 153.

³⁸⁷ Bower, *Fundamentals of Music*, 154 fn. 84.

³⁸⁸ *ibid.*, fn. 87.

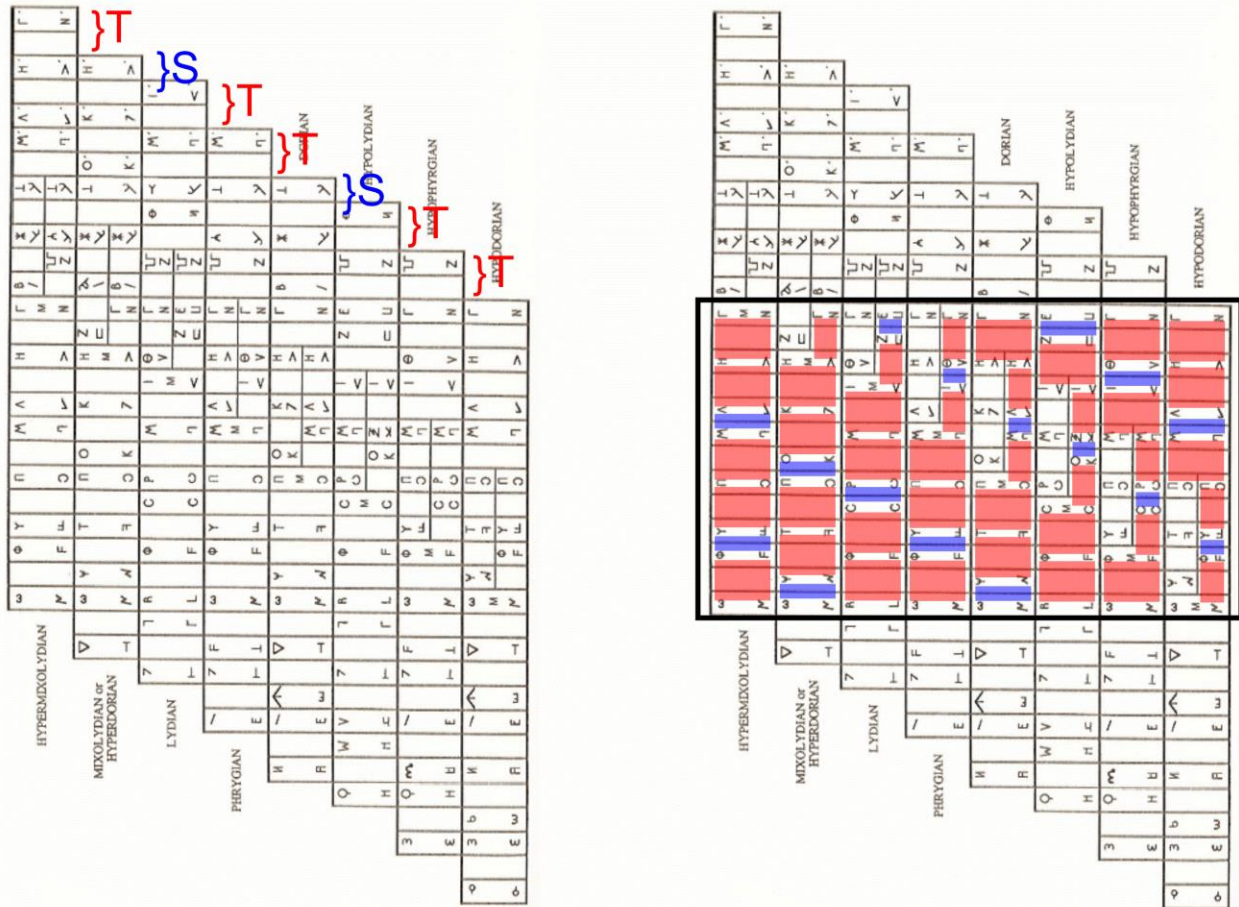
theory. The inclusion of the *συνεμμένων* adds considerable complexity to the supposedly simple diagram, and a reader not well versed in Alypian notation might well miss the point that rows represent consistent pitches because they do not always employ consistent notation symbols (these symbols are the Greek version of enharmonic equivalents, but the nature of the notation does not lend itself towards easy recognition of this fact, as the modern system does).

Confronted with this diagram, the reader attempting to ascertain the way in which the transposition depends upon the species of the octave would most likely draw one of two conclusions (see Facsimile 11). First, the reader might examine the spacing manifested by the stair-like upper and lower edges of the table and recognize the pattern of tones and semitones as one of the species of the octave – but only one, the Hypophrygian, and not the one that manifests the basic structure of the gamut (which would be, from the top down, two conjunct tetrachords of the form Tone-Tone-Semitone with a disjunct tone below – the actual distribution in the diagram is, in that respect, upside-down). The alternative, if the reader were able to ignore the *συνεμμένων*, would be to see the various octave species arising in those rows of the table common to every column of the table – that is, the central octave.

This latter interpretation would almost certainly seem the more compelling of the two since all of the octave species are evident within it. It is also the traditional relationship between keys and octave species in Greek theory and the foundation of Ptolemy's interpretation – which, as I have repeatedly observed, is one in which the octave species are primary.

Facsimile 11 – Two copies of the "Wing Diagram" (Bower, Fundamentals of Music, 156), marked up to show the two likely ways to see octave species in the diagram. The first, representing tones and semitones with the letters T and S, is more immediately apparent but shows only a single octave species, and not the one that is paradigmatic of the Greater Perfect System. The second represents tones in red and semitones in blue, as well as by height (width, however, is meaningless, and results only from the

avoidance of the synemmenōn, which is a complicating factor), and demonstrates all the octave species, which appear in the central octave in the same manner as in traditional Greek theory.



What I hope to have demonstrated with all of this lengthy background and argumentation is that there is nothing in Boethius to preclude the interpretation that the keys and octave species are two sides of the same coin, and that mode is, in fact, not one or the other, but both simultaneously, just as in Ptolemy’s conception, which is not surprising since Ptolemy is one of Boethius’ principal sources. Moreover, all the necessary information is presented in Boethius for any careful reader, willing to read between the lines, to arrive at an understanding of this dual nature of modes.

It is not, therefore, necessarily the case that the revisor of the *Alia musica* arrived at the conclusion that Boethius’ modes were octave species through a *misreading* of Boethius. It is certainly possible, even likely. However, it is also possible that he could have reached the same conclusion through a closer than

average reading of the same passages. If a modern musicologist could come to that understanding from a reading of Boethius (whether or not it was the interpretation intended by Boethius), it would be arrogant to assume that a medieval theorist could not. It is my opinion that Chailley's assertion that the revisor did not understand Boethius in general³⁸⁹ may be overstated and is probably predicated on the more specific assumption that he misinterpreted Boethius' presentation of mode. The revisor demonstrates his understanding of Boethius in other passages. However, even if Chailley is correct that the revisor himself did not understand Boethius, it would not necessarily follow that his understanding of mode comes from a misreading of Boethius. I have argued in Chapter Four, building upon an observation by Atkinson that the passage in which the *Alia* first relates the modes to octave species may have formed the "kernel" upon which the revisor constructed his revision,³⁹⁰ that the said passage may have predated the revision and have been borrowed from another source. In this case, it is not necessary that the revisor himself understand Boethius well, but only that the source from which he drew this passage be written by someone who understood Boethius well.

I wish to reiterate at this point that I do not present this argument as necessarily the most likely explanation for the *Alia*'s interpretation of mode. The explanation as a misreading is at least as likely, and probably more so. I present the argument only as a caveat that such a misreading of Boethius is not a given – that the hypothesis of a misreading should be accepted only with caution and should be applied judiciously to the question of whether the revisor understood Boethius.

Odds and Ends

³⁸⁹ "*Ignorant la musique grecque, hors ce que lui en dit Boèce, qu'il lit sans le comprendre...*" (Chailley, *Alia musica*, 20) ("Being ignorant of Greek music, aside from that which Boethius tells him, which he reads without understanding it...").

³⁹⁰ Atkinson, *Critical Nexus*, 177.

Before moving on from Greek theory, there are two brief matters to which to attend: the other Latin sources of Greek theory that survived into the Carolingian period; and the *συνεμμένων* and the Lesser Perfect System.

As regards the other Latin theorists, there are three of significance. The first is Martianus Capella, who predated Boethius by about a century. Martianus' treatise, *De nuptiis Philologiae et Mercurii* ("On the Wedding of Philology and Mercury"), is an extended allegory on the Seven Liberal Arts, personified as Philology's maidservants. One of these maidservants is, of course, Harmony, and one chapter is dedicated to that topic. This material is derived mainly from Aristides Quintilianus' treatment of harmonics and rhythmic.³⁹¹ As such, it contains descriptions of octave species³⁹² and the keys,³⁹³ the keys, here, are separated from the discussion of octave species and are presented first. Martianus describes the five ethnic names and how each exists in a triplet of unmarked, hypo-, and hyper- forms that are related to one another in some way that he does not describe. He also does not describe the relationships among the five ethnic names themselves, nor could the correct order be intuited from the order in which he presents them. Finally, in a passage that shows clear evidence of having been copied by one who does not understand the system (be that Martianus, himself, or a later copyist), he describes the three pairs of enharmonically equivalent keys at opposite ends of the cycle.³⁹⁴ Though the passages in Martianus are perfectly intelligible to those already familiar with the Greek system, Martianus does not present all the necessary information for a beginner to understand the system, even only as a system of keys, and it is

³⁹¹ Barker, *Greek Musical Writings*, 399.

³⁹² Martianus Capella, *De nuptiis*, ed. Willis, 367–68, trans. Stahl, 368–69.

³⁹³ Martianus Capella, *De nuptiis*, ed. Willis, 359–60, trans. Stahl, 362.

³⁹⁴ That is, Hypodorian=Hyperphrygian; Hypoionian=Hyperaeolian; Hypophrygian=Hyperlydian. In the text, the three hyper- keys are incorrectly labelled as hypo- keys (Martianus Capella, *De nuptiis*, ed. Willis, 359, trans. Stahl, 362), which renders his description nonsensical. However, his subsequent statement, that the μέση of the lower mode is the προσλαμβανόμενος of the higher mode, clarifies the meaning.

unlikely that a reader already familiar with Boethius would be able to make sense of all of Martianus' additional keys.

The second remaining important Latin writer is Boethius' successor as *magister officiorum*,³⁹⁵ Cassiodorus Senator. Cassiodorus' discussion of music spans a single, brief chapter (about eight pages) in his *Institutiones divinarum*, nearly a third of which is dedicated to mode³⁹⁶ (the only portion relevant to the current discussion). Unfortunately, his presentation of mode is not so much informative as verbose and pedantic. Cassiodorus names the fifteen modes, and then goes on – and on, and on – to describe their positions relative to one another. The presentation takes a format familiar from the Christmas carol “The Twelve Days of Christmas,” in that his descriptions become increasing long and repetitive at each stage; with each new mode that he describes, he indicates not only its position a semitone higher than the previous but also the interval between it and each of the other modes that he has already presented, in order, so that for the Hypophrygian, he describes it as a semitone above the Hypoionian *and* (which is redundant) a tone above the Hypodorian; by the time that he reaches the Hyperlydian, he relates its position relative to all fourteen lower modes. Perhaps the most impressive fact about this list is that Aurelianus had the patience to reproduce it in its entirety in his *Musica disciplina*.³⁹⁷

In all this extensive description, Cassiodorus fails to explain what a mode actually is. The definition he provides is, “*Tonus est totius constitutionis armonicae differentia et quantitas, quae in vocis accentu siue tenore consistit.*”³⁹⁸ (“*Tonus* is the quantity and difference of the entire harmonic system, which consists in the accent or course of the voice”). This description is fascinating both for the way in which it borrows from Gaudentius and Donatus and is, in turn, incorporated into later medieval treatises³⁹⁹ and

³⁹⁵ Bower, “Cassiodorus,” *Grove Music Online*, ¶1.

³⁹⁶ Cassiodorus, *Institutiones*, bk.2, ch. 5, §8, ed. Mynors, 145–48, trans. Goode & Drake, 6–9.

³⁹⁷ Aurelianus, *Musica disciplina*, ch. 6, ed. Gushee, 75–76, trans. Ponte, 17–19.

³⁹⁸ Cassiodorus, *Institutiones*, bk.2, ch. 5, §8, ed. Mynors, 145, trans. Goode & Drake, 6.

³⁹⁹ This relationship is beyond the scope of the present dissertation, but Atkinson treats it extensively throughout his *Critical Nexus*.

for the opacity that it presents to anyone who does not already understand what a mode (in the key signature sense) is. It is clear that something is being transposed up or down, but since he does not describe the gamut, it is not clear precisely what is being so transposed. By contrast to Martianus' discussion, a reader already familiar with Boethius could also certainly understand where Cassiodorus' additional modes would fit in between Boethius' modes but would be baffled as to the manner in which they could have arisen from the species of octave (as Boethius says that they must).

The final remaining Latin source of Greek theory is Isidore of Seville. As with Cassiodorus, who preceded him by three-quarters of a century (though, owing to Cassiodorus' very long life, their lives overlapped substantially), he writes only a chapter on music within a more substantial work, the *Etymologiae* ("Etymologies"); his chapter is about the same length as Cassiodorus', but his description of mode is much briefer and is the last important discussion of mode for almost two centuries. He begins by repeating Cassiodorus' definition and then says only that there are fifteen modes, of which the Hyperlydian is the highest (and newest), while the Hypodorian is the lowest.⁴⁰⁰

The Synemmenōn and the Lesser Perfect System

The final matter to be described in this chapter is the *συνεμμένων*. I have largely ignored it heretofore because it has limited influence on mode. The Greek gamut is constructed by combining instances of a single type of tetrachord (with the *πυκνόν* at the bottom), which may be joined either conjunctly (the highest note of one is the lowest note of the other) or disjunctly (the two tetrachords are separated from each other by a tone). In the Greater Perfect System, the first two tetrachords, from the bottom up, are connected conjunctly, but the third is connected disjunctly, and is therefore called the disjunct tetrachord (*διεξευγμένων*). In the Lesser Perfect System, the third tetrachord is also joined

⁴⁰⁰ Isidore, *Etymologies*, bk. 3, ch. 20, ed. Lindsay, under opening header "LIB. III. xx, xxi," left page; trans. Goode & Drake, 15–16.

conjunctly, and is called the conjunct tetrachord (συνεμμένων); it is also the last tetrachord in this system, which lacks the ὑπερβολαίων. The two systems, Greater Perfect and Lesser Perfect, coexist side-by-side in the Ametabolic System (Table 17), which allows for a kind of soft modulation; one can shift key areas by switching from the disjunct system into the conjunct system.

Table 17 – The Ametabolic System, with the Lesser Perfect System on the Right and the Greater Perfect System on the Left

νήτη ὑπερβολαίων		
παρανήτη ὑπερβολαίων		
τρίτη ὑπερβολαίων		
νήτη διεzeugμένων		
παρανήτη διεzeugμένων		
τρίτη διεzeugμένων		
παραμέση		νήτη συνεμμένων
		παρανήτη συνεμμένων
		τρίτη συνεμμένων
	μέση	
	λίχανος μέσων	
	παρυπάτη μέσων	
	ὑπάτη μέσων	
	λίχανος ὑπάτων	
	παρυπάτη ὑπάτων	
	ὑπάτη ὑπάτων	
	προσλαμβανόμενος	

With the modern predisposition towards the diatonic genus, it is easy to underestimate just how different the διεzeugμένων and συνεμμένων can be. In the enharmonic genus, though conceptually sharing the same relative position immediately above the μέσων tetrachord, they do not share a single pitch. Because of the quartertones of the enharmonic genus, the διεzeugμένων would be loosely equivalent to the notes B, C half-flat, D double-flat, E, while the συνεμμένων would be A, B flat-and-a-half, C double-flat, D.

However, in the tense diatonic (Ptolemy's ditonic diatonic), which is comparable to the medieval gamut, the two systems, up to the top of the συνεμμένων, differ by only a single note: B natural on the disjunct side in place of B flat on the conjunct side. It is probably no coincidence that these two notes are

the same two notes that are variable in the medieval system (b and b̄, respectively). Although the medieval gamut is seldom described in such terms (Aurelianus, for instance, makes no mention of the *συνεμμένων*, and it has no place in the gamut of the *Enchirides* or the theories of Guido onwards), it is important to observe that medieval musicians recognized their own fluctuation between b and b̄ in the *συνεμμένων*, and used the term *συνεμμένων* in their own writings to describe this fluctuation; this occurs in several of the treatises already discussed in this dissertation, including Hucbald's *De musica*, the *Ecce modorum sive tonorum*, and not least, in the *Alia musica* itself.⁴⁰¹ Consequently, one must be suspicious of Chailley's hypothesis that the awkward language in the description of the third mode stems from a lack of vocabulary to describe the B-flat and its associated semitone⁴⁰² (more on this in Chapter 16).

⁴⁰¹ *Alia musica* §§150, 133, 142, ed. Chailley, 154, 196, 201.

⁴⁰² Chailley, *Alia musica*, 89.

Chapter 06: Medieval Theory up to Aurelianus

It is, of course, true that the fall of the Western Empire did not immediately bring about the end of theorizing about music in the Greek tradition. Of the four principal Latin sources, only Martianus Capella wrote before the fall of Rome. Boethius and Cassiodorus were both high-ranking administrators in the post-imperial government of Theodoric, the Ostrogothic king of Italy, and Isidore wrote considerably later again, somewhat more than a century after the fall of Rome.

However, after Isidore, writing about music theory effectively stopped, and it is not known what became of the concept of mode during the next two centuries. What is certain is that when the concept reappears in the historical record in the late eighth century, it has nothing to do with transposed gamuts and key signatures, but it also is not initially defined as octave species; the new conception of mode was something else again. The formal structure is thought to have arisen in the Middle East (perhaps in Jerusalem),⁴⁰³ and then to have been adopted by the Byzantines, then by the Latin West, where authors like Aurelianus identified it as essentially Greek in origin. There was, of course, also a system already in place in the West upon which this structure was superimposed. This chapter will review what is known or believed about these systems as the immediate predecessors of the system of the ninth century.

It is now commonly held that these new systems, be it the Byzantine tradition (known as the *oktōēkhos* – “eight sounds”), or the pre-Carolingian system of the Latin West, are independent traditions, not derived from the ancient Greek system. However, little is known about either the end of the ancient Greek modal system or the origins of the *oktōēkhos* or the Franko-Roman systems, and the *oktōēkhos*, on the one hand, does seem to have arisen in a part of the world that had long been subject to Greek (and later, Roman) cultural domination, while the Franko-Roman tradition rose in the remains of the Western

⁴⁰³ Jeffery, “Earliest Oktōēkhoi,” *passim*.

Empire; both, therefore, arose in the spheres of influence of the old system. Furthermore, later generations of theorists in many different regions employing the *oktōēkhos* (including the West, after the superimposition of the *oktōēkhos* upon the existing structures) were eventually to “reconcile” the two systems – a task that is not possible with other, definitively independent systems, like the east Indian ragas, or the Chinese scales. It therefore seems at least plausible that the *oktōēkhos* could have evolved from the ancient Greek system. If this hypothesis is correct, it need not necessarily imply that the split away from the ancient Greek system occurred during the centuries between the end of the Roman age and the beginning of the Carolingian era; the split might well be much earlier, with the *oktōēkhai* developing in one location while the ancient Greek system continued in use elsewhere.

Whatever its provenance, the *oktōēkhos* as it was practiced at the end of the eighth century was undeniably quite different from any previous conception of mode. The *oktōēkhos* is defined principally by a set of melodic patterns associated with each mode, and while it is possible to derive an interval structure for each mode from these melodic patterns, it is the melodic patterns that are primary and the interval structures that are secondary. Miloš Velimirović explains the *ēkhai* in this way:

There is considerable difference between the Eastern and Western European understanding of modality. In the West, the term ‘mode’ most often means a scale or ‘octave species’; but an *ēchos* depends rather on a ‘mood’, which is in turn dependent on the types of melody found in that *ēchos*. When systematized by theorists, these melody-types do produce different ‘octave species’ or scales; this is of secondary significance, however, compared to the melodies themselves. An *ēchos* in fact consists primarily of a repertory of melodic formulae together with some melodic motifs and even melody-types.⁴⁰⁴

Velimirović is, of course, describing the Byzantine form of the *oktōēkhos*. As with so many other aspects of late eighth- and early ninth-century theory, the degree to which this description applies to the system as it was used in the West is not entirely clear. Willi Apel, for instance, rejects at least the last notion – that of melody-types – as a reasonable description of medieval ecclesiastical modes.⁴⁰⁵

⁴⁰⁴ Velimirović, “*Ēchos*,” *Grove Music Online*, §1¶1.

⁴⁰⁵ Apel, *Gregorian Chant*, 136–37.

Nevertheless, the Western modal system shows clear evidence of the importance of melodic formulae (particularly in the genre of antiphons) in the definition of mode.

It is possible, though not certain, that antiphonal psalmody represents the impetus for the Western modal system; it certainly forms a principal concern in the treatises about mode. Throughout the course of the various Hours of the Divine Office, more than two dozen psalms are sung per day (the exact number varies with the date and season). These are intoned to simple recitation formulae known as psalm tones, the principal feature of which is the tenor – the recitation tone itself, a note that is repeated as necessary to accommodate the varying lengths of the psalms so that the many hundreds of psalm verses do not require a similar number of independent melodies. For each day, a selection of psalms is specified, each with an accompanying antiphon that functions as a kind of refrain. It is likely that the antiphon was originally sung between every single verse of the psalm,⁴⁰⁶ but the practice was streamlined, probably already by the time of the first Carolingian treatises, to singing the antiphon only at the beginning and end of the complete psalm (or complete section of a psalm; a few of the psalms are so lengthy that they are split into sections, to be recited in parts over multiple days). The same practice also applied to three chants in the Mass: the introit, the communion, and the offertory; over time, the communion and the offertory lost their psalms, retaining only the antiphon, but the introit retains a single psalm verse (plus the doxology). The *Alia musica*, and the source treatise in particular, devote considerable attention to the antiphons of the office and the introit.

The key to understanding mode in relation to psalms and antiphons is that the psalm recitation needed to accord with the range and *ēthos*⁴⁰⁷ of the accompanying antiphon, and the transitions between psalm and antiphon, in each direction, must be smooth. Simplistically, this is accomplished by

⁴⁰⁶ Apel, *Gregorian Chant*, 188–89.

⁴⁰⁷ Here, and throughout the rest of this dissertation, I shall adopt the term *ēthos*, as used by the ancient Greeks and as described in the last chapter, to refer to the essential emotive quality of a mode, while I acknowledge that the Latin writers did not use this term themselves.

determining the mode of the antiphon, which in the fully-developed medieval system is determined by checking the final note and the range of the antiphon; the psalm would be recited to the psalm tone corresponding to that mode.

However, the actual practice is much more complex. In the first place, ninth-century theorists, in contrast to all later theorists, do not uniformly define a mode by its ending. Aurelianus says:

*Notandum sane quia in offertoriis et responsoriis atque inuitatoriis non aliubi requirendi sunt toni nisi ubi fines uersuum intromittuntur, maximeque seruandus est sensus litterature quam modulationis. In introitis uero, antiphonis necne communionibus semper in capite requirantur.*⁴⁰⁸

It is to be clearly noted that in the offertories and responsories, and also the invitatories, the modes are not to be sought anywhere except where the ends of the verses are inserted, and where the sense of the wording [as much] as the melody is to be especially preserved. *In the introits, however, the antiphons, and no less the communions, [the modes] are always to be sought at the beginning.* [Emphasis added]

Thus, Aurelianus singles out chants of the Office and Mass that included psalm verses and says that unlike other chants, the modes of these should be determined from the way that the antiphons begin, rather than the way that they end. Similarly, Regino says:

*Illud autem summopere prudens cantor observare debet, ut semper magis principium antiphonae, introitus uel communionis attendat in toni sonoritate, quam finem. Et e contrario in responsoriis magis consideret finem, et exitum in toni consonantia, quam initium.*⁴⁰⁹

That, however, which the prudent singer ought to observe with greatest diligence is that one always *attend more to the beginning of the antiphon, introit, or communion than to the end.* [Emphasis added] And, on the contrary, in the responsories, one should consider the end, and the egress in the consonance of a tone, more than the beginning.

Regino selects the same three genres as Aurelianus and observes that they, unlike responsories, are assigned to modes according to their beginnings. The context for Regino's remark is a discussion of the *nothi*, chants that seem to begin in one mode, carry on in another, and perhaps even finish in yet

⁴⁰⁸ Aurelianus, *Musica disciplina*, ch. 89, ed. Gushee, , trans. Ponte, 28.

⁴⁰⁹ To the best of my knowledge, no critical edition of this text has even been published, though a list of emendation with respect to Gerbert's edition is provided in Bernhard, *Studien zur Epistola*, 31–33; cf. Gerbert, *Scriptores ecclesiastici I*, 231b; LeRoux, *The 'De harmonica institutione'*, 26–27.

both begin on D, *Ecce nomen Domini* rises quickly back to the vicinity of its *differentia* (which ended on G), while *Euge serve bone* remains for much longer in the vicinity of D (where its *differentia* ends).

Thus, a more thorough description of antiphonal psalmody is as follows: an antiphon is sung, taking particular account of how it begins. The psalm is then recited to the formula appropriate to the mode of the antiphon, for a varying number of verses. Since in the majority of antiphons, the ending is correlated with the beginning, and the chant ends on the modal *finalis*, the transition from the antiphon into the psalm is generally smooth by default, as each psalm tone begins with a fixed intonation formula that presumes the previous antiphon to have ended on the *finalis*. For simplicity, the exceptional cases, where the antiphon begins and ends in different modes, will be discussed later, as such chants are cited in the *Alia musica*.

The final two verses are usually the doxology, *Gloria Patri, et Filio, et Spiritui Sancto, sicut erat in principio, et nunc, et semper, et in saecula saeculorum. Amen*. The antiphon is to be sung again immediately after the doxology, but while an antiphon will usually end on its *finalis*, it may begin in a variety of ways, and a single ending to the psalm tone for each mode is, therefore, insufficient to ensure a smooth transition. Instead, the melody for the last two words of the doxology, *saeculorum amen*, is selected from a list of alternative possibilities called *differentiae* in order to effect a smooth transition into the particular *locum* of the antiphon. Each mode may include several *differentiae*, and most *differentiae* are associated with several *loca* (though each *locum* implies only a single *differentia*). Interestingly, while the antiphon only recurs after the doxology, and not between every psalm verse, there is no default cadence for each psalm tone between the remaining verses; instead, the *differentia* required to make the smooth transition from doxology to antiphon is also applied to the end of every psalm verse (Apel takes this as additional evidence that the antiphons were once sung between every verse).⁴¹² See Figure 20.

⁴¹² Apel, *Gregorian Chant*, 189.

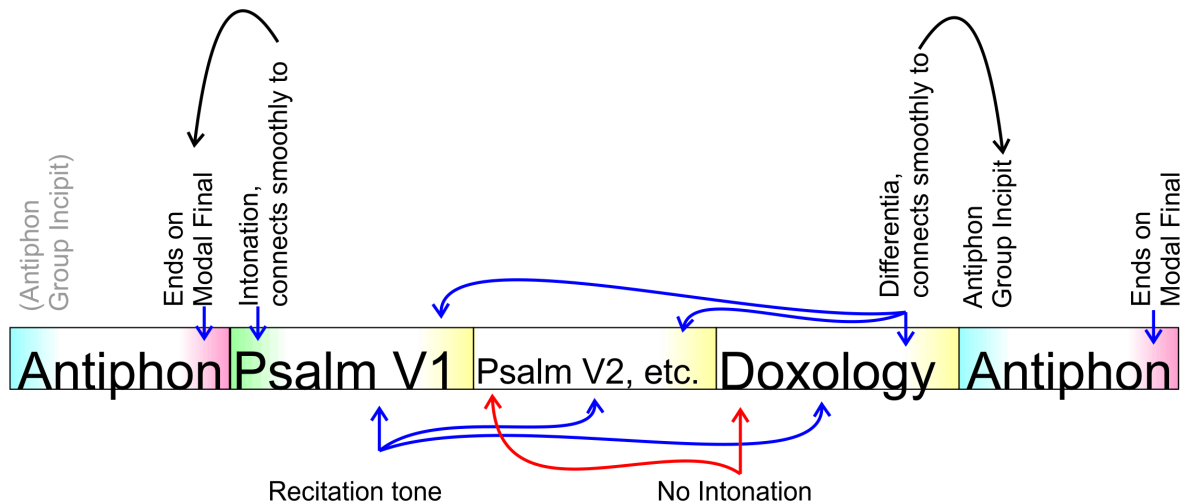


Figure 20 – The structure of antiphonal psalmody

Although I have described this system in terms of mode, as is convenient, the system does not actually require a theory of mode; it requires only a set of *differantiae*, each of which can be associated with an antiphon theme group. In such a model, the *differantia* would include not only the cadential formula, but also the tenor and an intonation – that is, each *differantia* could represent, in and of itself, an independent psalm tone (intonation, tenor, and cadence) associated with a single antiphon theme group. Keith Falconer has proposed that just such a system may have been the norm in Western Europe prior to the adoption of the *oktōēchos*. Building upon observations about alternative psalm tones such as the *tonus peregrinus*, a special psalm tone with two different tenors used with a handful of modally ambiguous chants, and the *parapteres*, a group of supplemental tones associated in a handful of treatises with small numbers of antiphons (and which may be related to Regino’s *nothi*),⁴¹³ Falconer says:

Perhaps [they] came into being in order to preserve older methods of singing psalms with antiphons that resisted the octoechos, for by organizing the irregular antiphons into categories that resembled modes, the psalm tones associated with them could escape the bounds of the octoechos, at least for a time. If this is true it would imply that, before the octoechos came into existence, the earlier method of choosing a psalm tone for an antiphon resembled the later method of choosing a *differantia* within a mode, based on a variety of

⁴¹³ Atkinson addresses this issue in “The *Parapteres*: *Nothi* or not?,” *Musical Quarterly* 68 (1982), 32–59.

‘surface’ melodic and other considerations rather than on more abstract modal qualities such as range final [*sic*].⁴¹⁴

Thus, the *differentiae* may have been the ancestors of the modal system in Western Europe. When the *oktōēkhos* was then adopted and adapted from the East, the various *loca* (and with them, their associated *differentiae*) would have been categorized according to the *finale*s of the majority of the chants in their theme groups. This hypothesis would go a long way toward explaining the modally ambiguous chants, which would have been unproblematic prior to adoption of the *oktōēkhos*.

Another chant tradition that may suggest what modes were like prior to the ninth century is Ambrosian chant.⁴¹⁵ Unlike Gregorian chant, Ambrosian chant was never subsumed under the *oktōēkhos*, and it retains much of the complexity that may have been smoothed out of other Western chant traditions when the *oktōēkhos* was adopted. For simplicity, I shall here use corresponding terminology from Gregorian chant, rather than the terminology native to Ambrosian chant.

Ambrosian chant is not usually spoken of as being divided into modes, though the antiphons are grouped into categories according to their *finale*s, which are recognizably equivalent to *protus*, *deuterus*, *tritus*, and *tetrardus*, without distinction between authentic and plagal. The Ambrosian psalm tones contain all the elements described above for Gregorian psalm tones,⁴¹⁶ but with considerably more variety. Each *finalis* is not associated with a single tenor, but a variety of tenors from which one is to select the tenor closest to the pitch that dominates the antiphon. Each tenor (not *finalis*) then supports a variety of *differentiae* from which one is selected to make the smoothest connection to the beginning of the antiphon; each tenor also has its own intonation (and sometimes more than one).

⁴¹⁴ Falconer, “The Modes Before the Modes,” 142.

⁴¹⁵ My discussion of Ambrosian chant relies heavily upon Roy Jesson’s contributed chapter in Apel’s *Gregorian Chant* (pp. 465–483, esp. 470–72 & 480).

⁴¹⁶ However, the Ambrosian psalm tones lack the mediation present at the middle of verses in the Gregorian chant, an element I have ignored in my presentation of Gregorian psalmody because it does not affect the argument at hand.

The most interesting characteristic of the Ambrosian system is that despite being grouped according to *finales* (a trait that could easily represent the influence of other chant traditions), the psalm tone is actually driven by the selection of tenor, which does not correspond in a systematic, one-mode–one-tenor relationship, as it does in the Gregorian chant tradition. The tenor is determined by the dominant pitch in perhaps the same way that Gregorian *differentiae* are selected not exclusively from the first pitch of the antiphon but from the general sweep of the intonation formula. Regarding the selection of dominant (and the relationship between Ambrosian chant and Gregorian chant in general), Roy Jesson says:

The choice of the “dominant” note of the Antiphon seems at times rather arbitrary. Nevertheless, the underlying principle is of great interest, representing as it does, an attempt to weld Antiphon and psalm tone into even closer “tonal unity” than is the case in the Roman system.

It is quite possible that the flexibility and variability of the Ambrosian psalm tone system is characteristic of a rather irregular practice common to all the western liturgies before the Gregorian system was devised to conform with the theory of the eight church modes.⁴¹⁷

Jesson’s hypothesis regarding the chant tradition before the adoption of the *oktōēkhos* is not necessarily inconsistent with Falconer’s. Both suggest that the modes formerly lacked an eightfold division based on *finalis* and range, a function instead performed by tenor and *differentiae* . If they differ at all, it is in the priority assigned to either the tenor or the *differentia* .

Although this exploration of the state of mode in the West prior to the *oktōēkhos* is ultimately inconclusive, it is extremely important, because the number of new treatises arising in the ninth century tends to give the false impression that modality of Carolingian music is now well understood. To the extent that modern scholars understand medieval modality, the system that is well understood cannot be shown to predate the late tenth century. It is possible that the same modal paradigm was already established at or near the beginning of the ninth century, but the ninth-century treatises do not provide enough detail to confirm this hypothesis (except in the *Enchirides* , regarding which debate continues

⁴¹⁷ Jesson, “Ambrosian Chant,” in Apel, *Gregorian Chant* , 471–72.

over the degree and manner in which their peculiar gamut may or may not have reflected actual musical practice).

However, I believe that it is more likely that, with the *oktōēkhos* still being a novelty imposed from without upon a system with which it bore only an approximate resemblance, modal theory in the early ninth-century was still in the process of settling down into its new configuration. The chants had been sorted into groups, partly in accordance with their *finales* and range, but also in accordance with their intonations; none of these characteristics alone can be responsible for the modal *ēthos*, and the modal assignments of chants were not always stable over time. In all likelihood, the theorists of the ninth century were themselves still grappling with the question of what, precisely, the chants of a given mode have in common – that is, what makes a mode a mode, and what is responsible for its *ēthos*?

If this hypothesis is correct, it would explain much about the treatises of the ninth century. For instance, it would explain why Aurelianus, who thoroughly describes intervals, says nothing whatsoever about the interval structure even of the gamut, much less the modes. It would explain why Hucbald provides only examples of the pitches upon which phrases of a chant may begin in each mode, without providing a theory to explain them (or other aspects of mode), and pays little attention to the distinction between authentic and plagal modes. It might help to explain why the gamut of the *Musica enchiriadis* differs so much from the other medieval presentations of mode and the gamut (such as why the *Enchirades* differ from later theorists on the appropriate range of an authentic mode) and to clarify whether the lack of octave equivalence impacts upon mode (more about this in Chapter 07). Above all, it might explain why the numerology of the *Alia musica* resembles nothing else described by any other treatise in the history of music theory (except, perhaps, a few brief comments by Aurelianus).

Whatever the status of mode before the adoption of the *oktōēkhos*, the *oktōēkhos* was already a going concern in Western chant by the late eighth century, when it appears as the organizing principle of the incomplete St. Riquier Tonary, also called Charlemagne's Psalter.⁴¹⁸ The tonary only goes as far as the *tritus* authentic and lists only about twenty chants for each mode. It is particularly interesting to note that, however much the original purpose of the modal system may have been to join psalms appropriately to antiphons (and other related genres), this earliest surviving tonary includes Alleluias, a genre that does not include a recitation tone. It appears, therefore, that while modes may have originated as a mechanism for joining appropriate chants, they were felt to possess some kind of innate quality (presumably *ēthos*) that was sufficiently perceptible that it was found valuable to classify non-antiphonal chants within the modal system.

In addition to the St. Riquier Tonary, another early description of the *oktōēkhos* occurs in the brief treatise *De octo tonis*, traditionally attributed to Alcuin.⁴¹⁹ Gushee outlines a few reasons for not accepting the attribution to Alcuin; his justification, however, merely comes from the weakness of the arguments in favour of the attribution, rather than any meaningful arguments against the attribution, while noting that "it is ironic that Alcuin is chronologically quite appropriate for the *De octo tonis*, and that [Gerbert's source, which contains the attribution to Alcuin] is more primitive than any of the others."⁴²⁰ Questions of attribution aside, this treatise is principally interesting for the fact that its text was incorporated into several Carolingian treatises. There are a variety of versions, but as printed in Gerbert's edition, it begins with an analogy between music and grammar that is famously developed in the *Musica enchiridis*. This analogy is followed by Cassiodorus' definition of mode (but no description of the Greek

⁴¹⁸ Edited in Huglo, *Les tonaires*, 25–29.

⁴¹⁹ Gerbert, *Scriptores ecclesiastici* Vol. 1, 26–27; Gushee, *Aureliani Reomensis Musica disciplina*, 40–41.

⁴²⁰ Gushee, *Aureliani Reomensis Musica disciplina*, 40–41; for an alternate opinion supporting Alcuin as author, see Möller, "De octo tonibus" (thanks to Charles Atkinson for bringing this paper to my attention).

modes). The rest of the text is devoted exclusively to etymologies for the pseudo-Greek manerial nomenclature for the ecclesiastical modes (*protus* authentic, etc.).

The entire text of *De octo tonis* forms the beginning of Chapter VIII of Aurelianus' *Musica disciplina*, the first medieval source to discuss the modes in any detail, and also the earliest large-scale treatise of the Carolingian music theory tradition (a tradition of which the *Alia musica* is also a part).

Aurelianus describes the old Greek modes in an early chapter, exactly following the model of Cassiodorus, right down to the pedantic repetition of the intervals between every possible combination of modes.⁴²¹ Like Cassiodorus, he describes fifteen modes that are all separated from each other by a semitone but never really clarifies that each simply represents a transposed gamut (thus, any reader expecting modes to be intervallic patterns is never directly contradicted or enlightened in this passage, but would be at a loss to explain the extra modes beyond the usual seven).

Later in the same treatise (and making up the bulk of the treatise), Aurelianus describes the ecclesiastical modes, the Latin form of the *oktōēkhos*, including a famous passage about the meaning of the *ēkhēmata* syllables. He describes having once asked a Greek person what the syllables meant (since he knew the system to have been Byzantine in origin), only to be told that they were untranslatable – simply interjections conveying joy.⁴²²

Aurelianus' description of mode also includes another crucial element, absent from most later descriptions of mode, but critical for the understanding of the *Alia musica*: according to Aurelianus, each of the four modal *maneriae* is associated with a particular type of interval;⁴²³ this matter will be addressed extensively in Chapter 14.

⁴²¹ Aurelianus, *Musica disciplina*, ch. 6, ed. Gushee, 75–76, trans. Ponte, 17–19.

⁴²² Aurelianus, *Musica disciplina*, ch. 9, ed. Gushee, 84, trans. Ponte, 25. Strictly speaking, Aurelianus' text calls the *ēkhēmata* adverbs, but the context makes clear that interjection is a superior label.

⁴²³ Aurelianus, *Musica disciplina*, ch. 2, ed. Gushee, 62–63, trans. Ponte, 8.

The rest of Aurelianus' discussion of the ecclesiastical modes seems rudimentary by later standards. He describes the *ēkhēmata* and *differentiae* of the modes but describes neither intervallic patterns in general, nor octave species in particular, and beyond the *ēkhēmata*, there is nothing particularly Greek about any part of his description. And yet, it is not entirely unreasonable to wonder how Aurelianus understood the relationship between Greek and Latin modes. On the one hand, Aurelianus never directly equates the Greek and ecclesiastical modes, and the descriptions of them are separated by an intervening chapter concerning the difference between a musician and a singer; one also should not read too deeply into the fact that he uses the word *tonus* to describe both systems.

On the other hand, in the very chapter in which Aurelianus introduces the *oktōēkhos*, he also describes the consonances, quoting from Boethius,⁴²⁴ whose treatise, whatever his intentions may have been, could potentially inspire an interpretation of his modal system based on interval structure (as I have argued in the previous chapter), and thus similar to the fully-developed ecclesiastical system. This argument, of course, is not in any way convincing by itself.

Much more significantly, though, Aurelianus borrows from Cassiodorus both the definition of mode and the description of the Greek modes. In Cassiodorus, these two elements are directly consecutive. However, Aurelianus separates the definition of mode from the passage describing the Greek modes and shifts it to the beginning of the passage describing the ecclesiastical modes. This decision is intriguing. In Cassiodorus, there is no question that the definition he gives for mode is meant to reflect the Greek modes that he immediately proceeds to enumerate; the context is so clear that no reader could ever suppose otherwise, including Aurelianus. Thus, the decision to define the ecclesiastical modes with a definition

⁴²⁴ Aurelianus, *Musica disciplina*, ch. 2, ed. Gushee, 62–63, trans. Ponte, 9. Aurelianus' familiarity with Boethius is also demonstrated in a variety of other places in his treatise, including direct citations in chapters III, V, and VII (Ponte, 9–11, 14, & 19–20).

clearly intended for the Greek modes perhaps implies the perception of some kind of connection between the two systems.

There is, of course, an alternative explanation for this shift: the place in which Aurelianus repeats Cassiodorus' definition in connection with the ecclesiastical modes is the very place in which *De octo tonis* is incorporated into the *Musica disciplina*, and the *De octo tonis* had already applied Cassiodorus' definition to the ecclesiastical modes. Thus, Aurelianus was not so much copying Cassiodorus' definition of mode as copying that of the *De octo tonis*. Nevertheless, it is most unlikely that Aurelianus did not notice the presence of this same definition in both of these sources in connection to two different modal systems. Furthermore, one must also contend with the use of the definition in the *De octo tonis* itself, which, being considerably older, may imply that a connection between the Greek modes and the ecclesiastical modes was supposed much earlier than the *Alia musica*.

Chapter 07: Carolingian Theory Treatises

The purpose of this chapter is to examine the modal theories of the large scale treatises most directly contemporaneous to the presumed dates of (at least the earliest layers of) the *Alia musica*, in order to illustrate how different these treatises are from one another (but equally, the ways in which they are not different) – and to demonstrate, as I have previously asserted, that the concept of mode was not yet clearly and consistently defined in the period in which the *Alia musica* was written. Furthermore, since the *Alia musica* does not describe many aspects of modes and the gamut, the doctrines from these treatises may perhaps supply valuable missing information. The treatises in question are principally the *De musica* of Hucbald and the anonymous *Musica enchiridis* (along with its associated works). As these are the two treatises most commonly consulted in the attempt to determine the date of the *Alia musica*, many of the basic ideas in these treatises have already been addressed in Chapter Two. The purpose of this brief chapter, then, is to fill in the gaps.

Hucbald

The modal theory of Hucbald's *De musica* is relatively limited. Hucbald describes the gamut several times. The gamut that he is describing is clearly the Greater Perfect System; although he does not call it that, he does cite both Boethius and Martianus as his authorities.⁴²⁵ He describes the system exclusively in the diatonic genus (no longer divided into shades, but consisting exclusively of standard tones and semitones), and initially describes it in the manner of the Greeks, as two pairs of conjunctly joined tetrachords separated by a tone and with a single disjunct tone added to the bottom, with each tetrachord in the structure semitone-tone-tone (from bottom up).⁴²⁶ He then describes what is effectively an

⁴²⁵ Hucbald, *Musica*, §42, ed. Chartier, 192, trans. Babb, 35.

⁴²⁶ Hucbald, *Musica*, §23, ed. Chartier, 162, trans. Babb, 24.

alternate species of the gamut, used for instruments such as the hydraulis; where the Greater Perfect System corresponds in modern terms to two octaves of the A-minor scale, this alternate gamut corresponds to two octaves of the C-major scale.⁴²⁷

Hucbald proceeds to describe the original gamut (*i.e.*, Greater Perfect System) again, citing segments of familiar chants to illustrate the tetrachord structure, both ascending and descending, first avoiding the disjunct tone at the bottom of the system. However, he then repeats the exercise, beginning from the absolute bottom of the system and redefining the tetrachord's internal structure to tone-semitone-tone – a significant change that will characterize most medieval theory. The structure of the complete gamut remains two pairs of conjunct tetrachords joined by tone, but the additional disjunct tone now appears at the top of the system. He frames this alternative derivation as though it were merely elaboration: “*Sin autem penitus ab ipsa prima seriem tetrachordorum cupias aggregare...*”⁴²⁸ (“But if, however, you would wish to bring together a series of tetrachords entirely from the very beginning [*i.e.*, from the very bottom of the system, including the προσλαμβανόμενος] ...”). However, the motivation for this reframing becomes clear later in the treatise.

First, though, Hucbald presents a discussion of the συνεμμένων and demonstrates how it arises in chant, as an explanation for the alternation between *b* and *b̄*. I have previously discussed the significance of this usage at the end of Chapter 05, in that it demonstrates the ready availability of vocabulary to describe this fluctuation and the two alternative semitones that it creates, vocabulary that Chailley supposed the authors of the *Alia musica* to lack. Hucbald then describes the complete nomenclature of the pitches of the Greater Perfect System (with etymologies) and presents his customized variant of the Alypian notation (modified, as he explains, from the symbols for the Lydian key only),⁴²⁹ the use of which

⁴²⁷ Hucbald, *Musica*, §§25–26, ed. Chartier, 164, trans. Babb, 24–25.

⁴²⁸ Hucbald, *Musica*, §32, ed. Chartier, 174, trans. Babb, 27.

⁴²⁹ Hucbald, *Musica*, §47, ed. Chartier, 198, trans. Babb, 37–38.

he proceeds to demonstrate. It is this system previously cited (Chapter 02) for revealing the inconsistency in the proposal that the *Alia* arose before the *Enchiriadis* (but after Hucbald) because the *Alia* makes no use of the *Enchiriadis*' fully diastematic notation; Hucbald's system, too, is fully diastematic, and the *Alia* likewise makes no use of it.

After he has established all this background, Hucbald concludes his treatise with a brief discussion of the modes, and one begins to suspect why he redefined the gamut to be constructed from tetrachords of the structure tone-semitone-tone, with the additional disjunct tone at the top: in this manner, the tetrachords all shift down by one scale-degree to span the pitch-classes A – D and D – G. He explains that by ignoring the first three pitches from the bottom (that is, the first tetrachord, excluding the top note, which is shared with the second tetrachord), the next four pitches (*i.e.*, the next tetrachord) govern the four modes (*i.e.*, the four *maneriae*), and identifies these four pitches as *finales*.⁴³⁰

I offer the editorial comments regarding tetrachords in the preceding paragraph with some caution: Hucbald does not, himself, frame this description in terms of tetrachords; to identify the four *finales* as members of a single tetrachord is a doctrine present in the *Enchiriades*, and later, in Hermannus Contractus, and one must be cautious not to assume that the doctrine was universal. However, Hucbald then proceeds to frame the rest of the discussion in terms of tetrachords, explaining that the remaining tetrachords “*Ad quarum exemplum [...] spatia uel qualitates deducunt sonorum.*”⁴³¹ (“[They] draw out the spacing and qualities of the sounds from this example”).

Hucbald approaches the end of his treatise with a discussion of the *affinales* (though he does not use the term), explaining that the notes a fifth above each *finalis* are so similar to the *finales* that chants may end upon them without objection. He says, further, that the pitches a fourth below, and sometimes

⁴³⁰ Hucbald, *Musica*, §49, ed. Chartier, 200, trans. Babb, 38–39.

⁴³¹ Hucbald, *Musica*, §50, ed. Chartier, 202, trans. Babb, 39.

a fifth below each *finalis* also display similar characteristics, but that they are not suitable for beginnings, only for endings. In fact, he explains that these pitches, above and below, displaying these affinities, serve as the boundaries of the range within which a chant may begin or end, and the final section of his treatise consists of tables identifying chants that begin on each pitch with the range so defined for each mode.⁴³²

The tables display a few notable characteristics. First, the tables correspond not to the modes in the usual sense but the *maneriae*. In fact, Hucbald makes very little distinction throughout his treatise between authentic and plagal modes. He acknowledges that the modes come in pairs and that the plagals are considered subordinate to the authentics, but says nothing about the plagals not ascending as high as the authentics. Second, despite combining the authentics and plagals into combined tables, in a few cases, he uses the beginning of a phrase from within a chant, rather than the beginning of a chant itself. And finally, despite both of these concessions, there remain pitches in some *maneria* for which he cannot find an example (foreshadowing the much later tendency of modal theorists to draw up lists of notes within each mode upon which it is acceptable to begin a phrase).

In all of this discussion, Hucbald never precisely defines what a mode is. His comment on the affinity of pitches a fifth above the *finales* is as close as he comes to describing mode as a product of interval structure, but he does not directly explain the nature of this affinity, only the locations at which it may be found; the reader is perhaps left to infer that if the affinity is in quality and spacing, that the quality is derived from the spacing (but Hucbald does not reinforce this inference anywhere else). It is true that the intervals between pitches are included in the tables at the end, but they cannot be interpreted as implying interval structure as a characteristic of mode: they are limited to the range within which a chant may begin – sometimes an octave, sometimes a ninth – and are inevitably centred on the *finalis*, and would

⁴³² Hucbald, *Musica*, ed. Chartier, 202–11, trans. Babb, 39–44.

reflect, at best, only the interval structure of the plagal modes, not the authentics. Instead, the intervals are probably provided in the tables only out of a desire to be thorough.

Musica enchiridis

The most salient characteristic of the *Musica enchiridis* is its idiosyncratic gamut, though it is not unique: the gamut and its associated notation appear in numerous treatises in the late ninth and much of the tenth century. The *Enchiridis* begins by making an axiom of a premise only implicit in Hucbald: that the four *finales* are grouped into a single tetrachord with the interval structure tone-semitone-tone. Naturally, they are said to govern the four *maneriae*: *protus*, *deuterus*, *tritus*, and *tetrardus*. Importantly, in the *Enchirides*, these pseudo-Greek names describe not only the modes, but also the *finales* themselves; the note later to be called D is called the *protus* of the *finales* tetrachord, and so on. The character (*ēthos*?) of a mode is said to derive from the character of the pitches sharing its name.

Also as in Hucbald (and the Greeks before him), the rest of the gamut is constructed by repeating the same tetrachordal structure. Within each tetrachord, since each shares the same interval structure, each pitch also receives the names *protus*, *deuterus*, and so on. However, unlike the Greater Perfect System, the gamut of the *Enchiridis* joins tetrachords exclusively through a tone of disjunction. Although the system may theoretically spread outwards *ad infinitum*, in practice, only one tetrachord below the *finales* (called the *graves*) and two above (in order, the *superiores* and the *excellentes*), as well as two additional pitches added to the top (the name of which varies from treatise to treatise) are used, and only these are given symbols in the notation system.

Three of the four notes in each tetrachord are given a symbol based on the *daseia* (the Greek symbol Ϝ, representing rough breathing at the beginning of a word), with rotated variants of the letters S or Sigma (in an alternate form that resembles the Latin letter C) affixed to the top, creating symbols that to the modern eye resemble the letter F. These same symbols are repeated, in order, in every tetrachord,

rotated or flipped (or both) to indicate the tetrachord. (The remaining symbol in each tetrachord alternates between variants of I or N). See Table 18.

Table 18 – The gamut of the *Enchiriades*

The rightmost column indicates the equivalent pitch in conventional pitch names. The symbol \flat is used here to represent B-flat in the lowest octave, where B-flat is not usually permitted in later systems.

	τ	<i>deuterus residuorum</i>	$\xi\#$
	τ	<i>protus residuorum</i>	ξ
Excellente	h	<i>tetrardus excellentium</i>	α
	t	<i>tritus excellentium</i>	g
	\flat	<i>deuterus excellentium</i>	f#
	k	<i>protus excellentium</i>	e
Superiore	d	<i>tetrardus superiorum</i>	d
	l	<i>tritus superiorum</i>	c
	j	<i>deuterus superiorum</i>	b
	d	<i>protus superiorum</i>	a
Finales	F	<i>tetrardus finalium</i>	G
	l	<i>tritus finalium</i>	F
	f	<i>deuterus finalium</i>	E
	F	<i>protus finalium</i>	D
Graues	ψ	<i>tetrardus grauium</i>	C
	N	<i>tritus grauium</i>	\flat
	η	<i>deuterus grauium</i>	A
	η	<i>protus grauium</i>	Γ

The notion that the modes and the corresponding pitches in each tetrachord share a fundamental character is central to the system and is reflected in the notation; although the *Enchiriadis* does not say outright that the intervals are responsible for the character of the modes, it would difficult to interpret the text in any other way. The structure of the *Enchiriades'* gamut is fascinating and has significant repercussions for the concept of mode. It is so structured as to lack any diminished fifth; it is commonly supposed that the presence of a perfect fifth both above and below every pitch is expressly created to facilitate parallel organum at this interval,⁴³³ a point to which I shall return shortly. But a gamut lacking diminished fifths comes at the cost of additional augmented fourths – one above every pitch named *tritus*.

⁴³³ Maloy, "Scolica enchiriadis," 74 & fn. 15; Maloy argues against this viewpoint, but provides a useful listing of the relevant literature supporting the viewpoint in her footnote.

In addition, the system also lacks complete octave equivalence: the same *tritus* pitches that have augmented fourths also have augmented octaves.

The presence of augmented fourth above *tritus* pitches is entirely to be expected: the *tritus* modes are well known for this scale degree (though the perfect fourth is also frequently employed through the introduction of *b*). However, the augmented octave is unexpected in conventional modal theory, and seems particularly surprising in the authentic mode, since the *Enchiriadis* tradition defines all authentic modes as able to span an octave or even a ninth.⁴³⁴ In fact, though, the theories presented in the *Alia musica* may help to explain why this interval does not seem to affect the modal theory of the *Enchiriades*. The *Alia musica* identifies the perfect consonances most characteristic of each mode. This theory will form the central concern of the final section of this dissertation; for now, it suffices to note that the *tritus* modes are both characterized by perfect fifths, not the fourths or octaves that characterize the other modes. There may, therefore, be no reason why the *Enchiriadis* should be concerned that the fourth and octave are augmented.

This observation, that the *Alia musica* can help to explain a surprising characteristic in the *Enchiriades*, can also be turned in the opposite direction: there does not seem to be anything in the theory of the *Alia musica* that would preclude the possibility that at least the source treatise of the *Alia*, too, operates within the gamut of the *Enchiriades*. It is by no means a certainty; the theory of the *Alia* also functions equally well within the Greater Perfect System – and, in fact, in some layers of the *Alia*, the Greek string-based nomenclature is used, though this need not invalidate the possibility that even those parts of the *Alia* operate within the *Enchiriadis* gamut, as I shall demonstrate shortly.

⁴³⁴ This definition is presented in slightly different terms: the *Musica enchiriadis* says that each authentic mode may ascend to the second pitch of the same name – that is, the note of corresponding name two tetrachords above the *finalis* (*Musica enchiriadis*, chs. 4–5, ed. Schmid, 8–9, trans. Erickson, 4–5); however, the simpler definition is quite common in the other treatises of the *Enchiriadis* tradition, including the *Compositio monochordi secundum Enchiriadem* (Schmid, *Musica et Scolica*, 235) and *Mensura fistularum* (Schmid, *Musica et Scolica*, 232).

First, it is worth observing that in at least one passage, in the younger layers of the treatise, displays a likely connection to the doctrines of the *Enchiridae*: §133 – 34 say:

*Sunt autem a mese superius quatuor chordae, quibus synemmenae sunt proximae et e latere uicinae, quae dant ab excellentiori parte exordium quatuor primis speciebus diapason, atque melodiis quatuor troporum. Quarum videlicet troporum, siue etiam sonorum, primus graeca lingua dicitur protus, secundus deuterus, tertius tritus, quartus tetrardus, qui singuli a suis finalibus deorsum pentachordo different.*⁴³⁵

There are, however, four strings upward from the μέση, to which the συνεμμέναι are close and neighbours to the side, which give (from the higher end) the beginning to the first four species of diapason and to the melodies of four tropes. Wherefore one may see that of the tropes, or even of the sounds, the first is called, in the Greek language, *protus*, the second *deuterus*, the third *tritus*, and the fourth *tetrardus*, which each differ from their *finale*s below by a pentachord.

The first four notes from the μέση are a, b, c, and d, which do not directly correspond to single tetrachord in the Greater Perfect System; the διεzeugμένων begins from b. The notes are, however, very close (“neighbours”) to the notes of the συνεμμένων, which differ only by the presence of b instead of B. In the *Enchiridae*s gamut, these notes do form a single tetrachord. Furthermore, as the *Alia* says, the melodies (ἡχήματα) of the four authentic modes do begin, more or less, from these pitches, as do the first four octave species (corresponding to the four plagal modes). The *Alia* says that not only the modes are called *protus*, *deuterus*, and so on, but so, also, are these four pitches (“*sonorum*”). Actually, this term *sonorum* could be interpreted as another name for a mode, as the word *sonus* is sometimes used in this way; however, the final clause clarifies the interpretation, stating that these four pitches are a pentachord above the *finale*s (this is also the only passage in the *Alia* describing the *finale*s by name).

Interestingly, the *Alia* describes these pitches in the vocabulary of the Greater Perfect System – μέση and συνεμμένων. It also does not describe the names of the *maneriae* applying to the other tetrachords. This passage, therefore, seems to represent a compromise position between the Greater Perfect System and the gamut of the *Enchiridae*s.

⁴³⁵ *Alia musica* §134(a), ed. Chailley, 196. cf. Atkinson, *Critical Nexus*, 195.

The question becomes, then, how compatible are the two gamuts? As mentioned above, it is frequently suggested that the structure of the *Enchirades'* gamut, lacking diminished fifths, was created to facilitate parallel organum at the fifth, organum being a principal topic of the *Musica enchiradiadis*. However, it is also commonly objected that this interpretation ignores the fact that the fourth is generally the favoured interval for organum in the early sources, rather than the fifth, and that the *Enchirades'* gamut does not lend itself to parallel organum at the fourth. Yet, examples supporting this notion that organum at the fourth was preferred are not unambiguous (and generally come from significantly later sources, and thus are poor evidence of ninth-century practice).

The only evidence that one could draw from the treatise itself regarding the preference for organum at the fourth or fifth is that considerably more ink and parchment is occupied with organum at the fourth than at the fifth; however, this argument is easily dismissed: organum at the fifth poses no difficulties in this gamut and therefore may be explained briefly, while organum at the fourth causes difficulties and therefore requires much more discussion to explain. Organum at the fourth in the *Enchirades* (and in other ninth- through eleventh-century treatises, such as Guido's *Micrologus*) is not parallel, it is oblique: whenever parallel motion by fourth within the gamut would result in a tritone, the added voice instead holds obliquely to a single pitch. Since the added voice is generally below the original voice, the result is that whenever the lowest voice descends as far as any pitch called *tetrardus*, it must stay put as the upper voice continues to descend, because the next pitch (*tritius*) has a tritone above it; if the upper voice continues to fall, the two voices will eventually come together to a unison, as they always do at the end of a chant. The two voices also often begin on a unison and diverge, with the lower voice remaining oblique until they reach a perfect fourth.⁴³⁶

⁴³⁶ *Musica enchiradiadis*, chs. 13–14 & 17–18, ed. Schmid, 37–41 & 47–56, trans. Erickson, 21–23 & 26–31. My description is a bit oversimplified; if the upper voice descends far enough or skips over the *deuterus* pitch with which the lower fourth would clash, the added voice may skip down over the offending *tritius* pitch to achieve a

This description implies that oblique organum evolved in response to the structure of the *Enchiriadis* gamut, though it is also possible that the oblique endings (and to a slightly lesser degree, oblique beginnings) arose as desirable characteristics on their own, and that the description in the *Enchiriadis* is a *post hoc* systematization of this practice. Regardless, the extra attention devoted to organum at the fourth in the *Enchiriadis* is clearly explainable as the result of the complexities of the oblique procedures compared to the simplicity of parallel organum at the fifth, and cannot be taken to imply, by itself, that organum at the fourth was preferred in the ninth century. Both forms of organum described in the treatise clearly fit well with the structure of the *Enchiriadis* gamut, and an increasing number of scholars have come to consider the possibility that the gamut of the *Enchiriadis* may actually represent the standard gamut of the ninth century.⁴³⁷

The aforementioned difficulties in octave equivalence need not be impediments to this hypothesis; although the *tritus* pitches lack perfect octaves above, octave equivalence is, nevertheless, presupposed and maintained by a “*mutatione mirabili*”⁴³⁸ (“a marvellous transformation”). The system of the *Enchiriades* also allows for other kinds of pitches outside the gamut, described more fully in the *Scolica enchiriadis*. The master of the *Scolica* describes these pitches as *uitia*, a word that means “vices” or “errors.” However, he then goes on to say that they are not always the result of mistakes: “*Uitia nimirum sunt, sed sicut barbarismi et soloecismi metris plerumque figuraliter intermiscentur, ita limmata interdum de industria cantibus inseruntur.*”⁴³⁹ (“Errors they doubtless are, but just as barbarisms and solecisms are

new perfect fourth; also, contrary motion and parallel unisons are permitted in certain circumstances, especially to rejoin the original voice on a unison at a cadence. cf. Guido, *Micrologus*, chs. 18–19, ed. Smits van Waesberghe, 196–214, trans. Babb, 77–82; Guido’s description varies slightly in its details (not least because the gamut is different), but the overall procedure is essentially the same.

⁴³⁷ cf. Phillips, *Musica and Scolica enchiriadis*, 471–497; Erickson, *Musica enchiriadis*, xxxi.

⁴³⁸ *Musica enchiriadis*, ch. 11, ed. Schmid, 34, trans. Erickson, 19.

⁴³⁹ *Scolica enchiriadis*, pt. 1, ed. Schmid, 70, trans. Erickson, 41.

commonly mixed into verses as poetic devices, so *limmata* (incomplete tones) are sometimes deliberately inserted into chants.”)

Meanwhile, the flexibility in the Greater Perfect System may well be understated. In its original Greek form, a considerable amount of modulation was possible. The Ametabolic system encodes one form of that modulation, from *διεξευγμένων* to *συνεμμένων*, which is loosely equivalent to modulating down to the hypo- form of the same key; however, modulations to other keys were also possible (not least modulating upwards by fourth, which is implicit in modulating back up from the *συνεμμένων*). While music would necessarily have changed between the fall of Rome and the Carolingian period, people certainly did not stop singing during that time, and it is not clear that the practice of modulating should have disappeared (the concept of *uitia* in the *Scolica enchiridis* supports this suggestion). As Phillips notes, however these additional pitches may have been understood, they were indisputably present in the chants.⁴⁴⁰

In fact, it is entirely possible, even likely, that the actual state of the gamut in the ninth century is best described as the superposition of both the Greater Perfect System and the gamut of the *Enchirides* – or, probably more accurately, the superposition of the governing principals of both. That is, the organizing principle was the use of a single tetrachordal structure (tone-semitone-tone), but with the ability to connect those tetrachords either conjunctly or disjunctly at will, and with different theorists subscribing to different interpretations of the default manner of connecting them. Both of these two systems, then, become essentially theoretical constructs approximating, rather than defining, the gamut. If so, it makes little difference which notes were considered central and which were considered “vices” used for effect. If this hypothesis is correct, it would then not even be necessary to invoke the characteristic intervals of the *tritus* modes to explain away the augmented octaves; the perfect octave is

⁴⁴⁰ Phillips, *Musica and Scholica*, 477.

available via a “marvellous transformation” or as a *uitia*, and the augmented octave itself may be entirely appropriate within the mode, just as the fluctuation between *b* and *b* is not considered to affect the mode.

A final characteristic of mode in the *Enchiriadis* tradition that bears consideration is the question of modal ambitus. Modes will later be defined as octave species, and even those treatises that do not define mode in this fashion tend to define the appropriate ambitus of a mode as being more or less an octave (sometimes with an additional tone above or below) – in the authentic modes, an octave rising from the *finalis*, and in the plagal modes, centred upon the *finalis*. However, there is an essential difference in the definition of the ranges in the *Enchiriades*. The *Musica enchiriadis* says that each mode may descend to the next note of the same name (*i.e.*, one tetrachord, or a perfect fifth). It goes on to say:

At uero in acumine a quocumque finali sono usque in tertium eiusdem nominis sonum efferri ualet, id est usque in excellentes.

Praeterea cum eodem sono autentus quisque tonus et qui sub ipso est regantur et finiantur, unde et pro uno habentur tono, in hoc tamen differunt, quod minoribus tonis minora in elevando sunt spacia, et inferior quisque tonus non nisi ad quintum usque sonum a finali sono ascendit, sed et hoc raro.⁴⁴¹

But truly, it is possible to rise in pitch from any particular final sound as far as the third sound of the same name – that is, as far as the *excellentes* [*i.e.* two tetrachords above, or a major ninth].

Besides this, each authentic mode and that which is beneath the same [*i.e.*, the plagal mode] is ruled by and finished with the same sound, whence also they are taken for one mode; they differ, however, in this: that the spacing in ascension is lesser in the lesser modes [*i.e.*, the plagals], and each lesser mode ascends no more than to the fifth sound from the final sound (but even this only rarely).

Thus, while the upper range of each mode is essentially the same as in later formulations, the lower range corresponds to the later formulations only for the plagal modes; in the *Enchiriades*, the authentic modes may descend just as far as the corresponding plagals. Presumably, this definition should reflect the range of chants in each mode in the late ninth century, the same period in which the oldest layers of the *Alia musica* were probably written. However, range is implicated in the interpretation of the characteristic intervals of each mode, and in the *Alia musica*, several of the modes do not span an entire

⁴⁴¹ *Musica enchiriadis*, chs. 4–5, ed. Schmid, 8–9, trans. Erickson, 4–5.

octave. The issue of range and characteristic intervals will be examined in detail in the final section of this dissertation.

There is one other treatise that needs to be addressed at this point: the *Commemoratio brevis*. The *Commemoratio* gives little in the way of hard theory. Instead, it presents the necessary information to accompany a tonary. It describes the ἦχημα for each mode, as well as the psalm tone, notated in the *daseian* notation of the *Enchirides*. It also includes precise notation for the *differentiae* and *loca*, and as the only treatise of the ninth or early tenth century to notate most of these precisely, it will serve as a useful reference for comparison in later chapters of this dissertation.

Chapter 8: The Development of the Octave-Species Concept of Mode

I have observed several times throughout the course of this dissertation that it is an easy trap to fall into to read later theory into earlier treatises. Concepts sometimes develop rapidly and appear fully formed in the first treatise to contain them, but sometimes they develop slowly, over many years, and are only partially developed in the earliest treatises in which they appear. (And even the description “partially developed” is problematic, as it presupposes that the “fully developed” state was, in some way, inevitable.) But for a reader already familiar with the later state of the theory, it is easy to read an earlier form and recognize the later form, assuming that the later form is already present in the earlier treatise.

An example of this effect has already been discussed in Chapter 02, where it was noted that Chailley assumed the octave-species model of mode found in the *Alia musica* built upon a “first stage” presented in Hucbald, though Hucbald says nothing at all about octave species. The purpose of this chapter is to examine the development of modal theory in the treatises of the later tenth century and the first half of the eleventh century, most notably in terms of species theory, paying particular attention to the elements of the theory that are not present in the *Alia musica*, in order to avoid reading later theory into the *Alia*.

The fully developed theory of mode as octave species extends beyond the octave species and incorporates species of fourth and fifth in order to solve what Atkinson called “the problem of the eighth mode”: the fact that the *protus* authentic mode and the *tetrardus* plagal mode share the same octave species, D – d. The solution to this problem stems from the fact that an octave may be created from the concatenation of a perfect fourth and a perfect fifth, and that by extension, a species of octave may be created through the concatenation of a species of fourth and a species of fifth.

The developments to be described here have been described very well already, by Charles Atkinson. For this reason, the presentation here will be relatively brief and primarily limited to only the most relevant points; readers wishing for greater detail may find it in Chapter 6 of Atkinson’s *Critical Nexus*.

That said, Atkinson begins his discussion of this thread with Medieval theorists, and this concept is not, in fact, entirely new in the Middle Ages. The principle appears in the writings of Gaudentius, an early fourth-century theorist whose Greek treatise Cassiodorus recommends at the beginning of his chapter on music, saying also that Gaudentius' treatise had been translated into Latin by Cassiodorus' friend Mutianus.⁴⁴² It is, therefore, possible that the Latin theorists who first described the modes as concatenations of the species of fourth and fifth may have gotten the idea from Gaudentius (perhaps via Mutianus), but this hypothesis cannot be verified.

Gaudentius begins his discussion of species by describing the species of fourth and fifth in terms of where each begins in the Greater Perfect System. He describes the species of fourth as beginning from ὑπάτη ὑπάτων (B) – the lowest note of the lowest tetrachord (a tetrachord that is joined to the next tetrachord conjunctly) – and rising upwards from there; he describes the species of fifth as beginning from ὑπάτη μέσων (E) – the lowest note of the second tetrachord (a tetrachord that is joined to the next higher tetrachord through a tone of disjunction). The basic structure of the species of fifth would seem as though it should be the same as for the species of fourth, since the ὑπάτων and the μέσων tetrachords, like all the tetrachords of the Greater Perfect System, share the same structure; the tone of disjunction, however, converts the second set of species from fourths to fifths and adds one additional species (see Figure 21). It is an interesting consequence that the first three species of fifth begin where the three species of fourth end; it is unclear whether this is a coincidence or deliberate in Gaudentius, but it becomes important in the medieval form of this theory.

⁴⁴² Cassiodorus, *Institutiones*, bk.2, ch. 5, §1, ed. Mynors, 142, trans. Goode & Drake, 3.

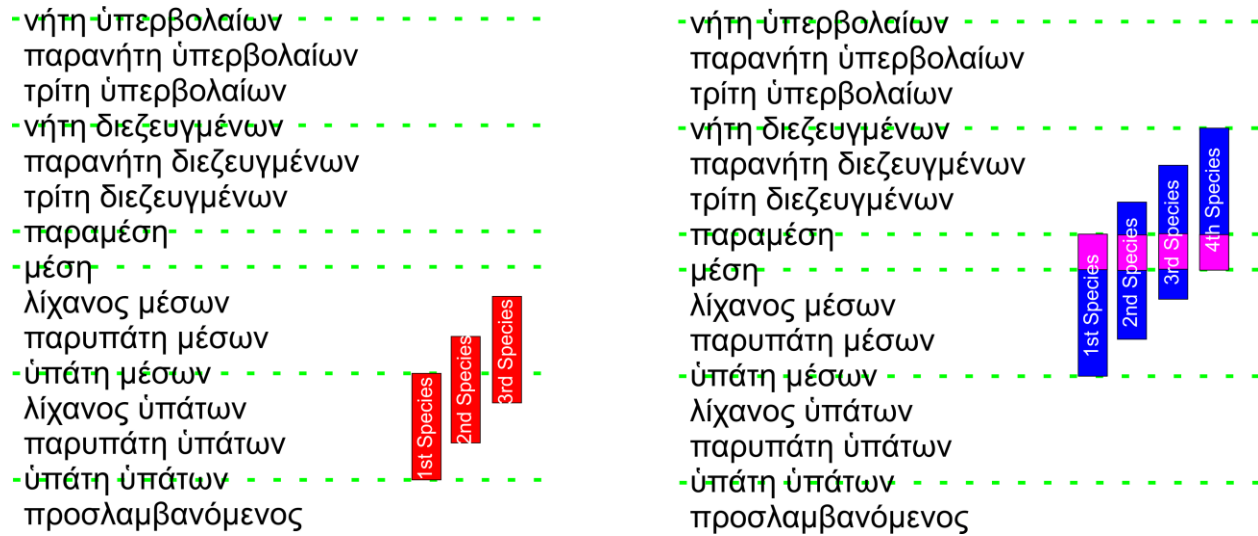


Figure 21 - The species of fourth (left) and fifth (right), according to Gaudentius

Gaudentius then proceeds to say:

Τοῦ δὲ διὰ πασῶν ὀχταχόρδου συνάγεται μὲν εἶδη ἦτοι σχήματα ἰβ, διὰ τὸ τοῦ μὲν διὰ τεσσάρων εἶναι σχήματα τρια, τοῦ δὲ διὰ πέντε δεδειχθαι σχήματα τέσσαρα, ἐξ ἀμφοῖν δὲ συντίθεσθαι τὸ διὰ πασῶν. οὐ μὴν ἀλλὰ τὰ γε ἐμμελῆ χαι σύμφωνα αὐτοῦ εἶδη ἐστὶν ἦτοι σχήματα ἐπτὰ· τὴν δὲ αἰτίαν ὕστερον ἀποδώσομεν.⁴⁴³

Of the diapason [octave] of the octachord are brought together twelve forms [εἶδη, = species] or schemata, since of the diatessaron [fourth], on the one hand, there are three schemata, of the diapente [fifth], on the other hand, are admitted four schemata, and the diapason is synthesized from both. But indeed, there are not other than seven forms or schemata of these suited to melody [ἐμμελῆ, with the same root as the “*emmelis*” that the *Alia* misuses] and consonant [σύμφωνα; “*symphonies*” refers to the perfect consonances]; I shall give an account of the cause later [ὕστερον].

Thus, there are twelve forms of the octave, because one may combine each of the three species of fourth with each of the four species of fifth, resulting in $4 \times 3 = 12$ species. In fact, Gaudentius has underestimated by half, because he has not accounted for the fact that the fourth and fifth could be combined in two different orders (fourth and then fifth or fifth and then fourth), so that there should be three species of fourth times four species of fifth times two arrangements equals twenty-four potential species; a good number of these would be duplicates, but not all of them.

⁴⁴³ Gaudentius, *Ἀρμονικὴ εἰσαγωγή*, ed. von Jan, 46, trans. Treitler, 78.

Unfortunately, as Ch.–Émile Ruelle notes, Gaudentius never seems to return to the promised explanation for why only seven of the species are suited to melody.⁴⁴⁴ He probably could not have meant to limit the field to combinations of equivalently numbered species (first-species fourth with first-species fifth, and so on), as in some descriptions of medieval theory (such as Hermannus Contractus), because this would produce, at most, six species (and only three, if order is not taken into account). One could achieve the correct set of species by requiring that the $\pi\upsilon\kappa\upsilon\acute{\alpha}$ of the species of fourth or fifth must always be either a fourth or a fifth apart (though this, too, would require both possible orders of fourth and fifth).

Ultimately, the most likely explanation is simply that the resulting species of octave must naturally arise somewhere in the gamut, and indeed, immediately after this passage, Gaudentius proceeds to identify the seven species by their location within the Greater Perfect System. Perhaps this is the explanation that he meant; the belief that the explanation should come “later” depends on the translation of ὕστερον. Treitler translates ὕστερον as “later,” and Ruelle translates into French as *plus tard*, which also means “later.” However, in some cases, ὕστερον can also be translated as “next.” I am not confident of my translation here, but Gaudentius may simply mean that he will “give an account of the cause next.”

To return to the question of medieval species theory, the mature form of the theory may be described as follows: there are three species of fourth, which may be defined according to the position of the semitone, but which are more readily illustrated by the positions in which they naturally arise in the gamut, with the first rising from A, the second from B, and the third from C – or, equally, the first rising from a, the second from b, and the third from c; there are four species of fifth, defined and illustrated in the same manner, with the first rising from D, the second from E, the third from F, and the fourth from G (each species of fifth begins where the lower species of fourth of the same number ends, and ends where the equivalent higher species of fourth begins); the species of octave are created by joining equivalently

⁴⁴⁴ Ruelle, *Alypius et Gaudence*, 81, fn. 2: “Cette explication ne se retrouve ni dans Gaudence, ni chez aucun autre musicographe.” (“This explanation is not found in Gaudentius, nor in any other author about music.”)

numbered species of fourth or fifth (leaving aside, for the moment, that there is one fewer species of fourth than fifth) in either order; and the *manerial* numbers (*protus*, *deuterus*, etc.) of the modes reflect the number of the species of fourth and fifth, with the plagal modes corresponding to the species in which the fourth is below the fifth, and the authentic modes corresponding to the species in which the fourth is above the fifth (see Figure 22).

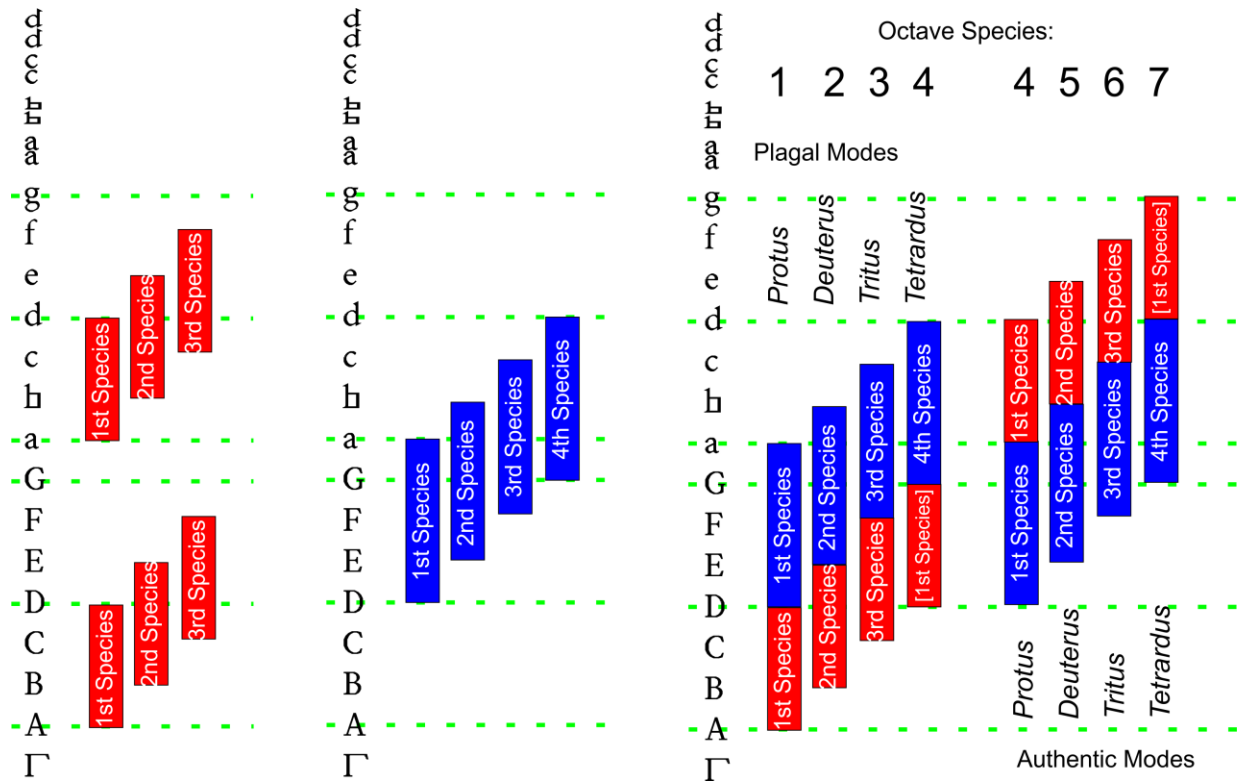


Figure 22 - The mature medieval mode as octave species theory

The leftmost column shows the species of fourth situated in two places in the gamut; the central column shows the sole position in which the species of fifth naturally arise in order in the gamut; the final column shows how these species are combined to produce the seven octave species (with the fourth species arising in two different ways) and correspondence of these octave species to the modes.

As described by Atkinson, the stages in the development of this theory progressed after the *Alia musica* as follows:

1. Pseudo-Bernelinus (*Prima species*) defined the species of fourth divorced from the gamut, according to the position of semitones, then defined the first two species of fifth by adding a tone to the top of the first two species of fourth, and the last two species of fifth by adding a tone to the bottom of the

last species of fourth and then the first species again (see Figure 23). The modes are created by combining corresponding species, with fourths below for plagals and above for authentics, and with the fourth species of fifth taking the first species of fourth; interestingly, pseudo-Bernelinus does not refer to these as octave species, only modes.⁴⁴⁵

2. Bern takes pseudo-Bernelinus' description and attempts to anchor it in the gamut, but anchors the species of fourth a perfect fourth higher than the lower set presented in Figure 22 above, in the same position as the fifths; the complications introduced by the tone of disjunction between G and a force him to separate the third species from the first two, not placing it one step higher than the second species, but two steps higher. He creates his first two species of fifth in the same place, adding a tone above the species of fourth, and the third by adding a tone below the second species of fourth; for the final species of fifth, he must transpose the first species of fourth up by a perfect fourth in order to follow pseudo-Bernelinus' instruction to add a tone beneath it (see Figure 23). He then repeats the instructions of pseudo-Bernelinus for creating the modes, and notes that the first authentic mode uses the transposed version of the first species of fourth, while the first plagal mode uses yet another transposition an octave lower; he does not situate the remaining three *maneriae* in the gamut, leaving it to the reader to infer. Unlike pseudo-Bernelinus, Bern does describe the octave species, but he does not equate them to the modes nor describe them as concatenations of fourth and fifth.⁴⁴⁶

⁴⁴⁵ Atkinson, *Critical Nexus*, 203–04.

⁴⁴⁶ *ibid.*, 204–08.

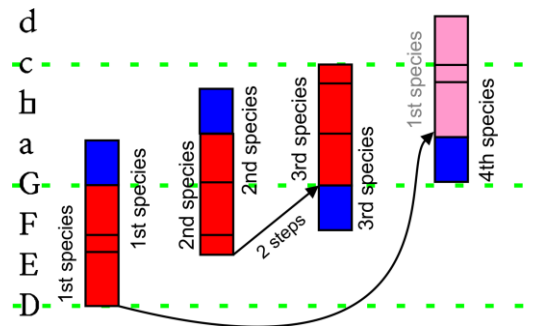


Figure 23 – Relationship between species of fourth and fifth in pseudo-Bernelinus and Bern (pseudo-Bernelinus does not situate them within the gamut).

3. An unknown interpolator revised Bern's treatise so that the species of fourth appear in their more intuitive place in the lowest tetrachord.⁴⁴⁷

For all intents and purposes, Atkinson ends his discussion of species theory here. However, there is one further state worthy of discussion: Hermannus Contractus. Hermannus, writing around the middle of the eleventh century, describes the gamut in the manner of the Greater Perfect System, but appropriates the nomenclature of the *Musica enchiriadis* (though rejecting the structure of its gamut), particularly in describing the tetrachords with the names *graues*, *finale*, *superius*, and *excellens*.⁴⁴⁸ He then describes each species of fourth as spanning from the correspondingly numbered pitch in the *graues* tetrachord to the corresponding pitch in the *finales* tetrachord (in some instances, he even uses the terms *protus*, *deuterus*, and so on, but in others, he uses the pure Latin terms *prima*, *secunda*, and so on).⁴⁴⁹ The species of fifth are described in the same manner, but between the *finales* and the *superiores*.⁴⁵⁰ This refinement produces a very elegant manner of describing the species that relates the nomenclature of the gamut to the nomenclature of species and modes.

The second refinement that Hermannus adds is a fourth species of octave. This species is effectively redundant, in much the same way that the Hyperphrygian species of octave in the Greek system is

⁴⁴⁷ *ibid.*, 208–11.

⁴⁴⁸ Hermannus, *Musica*, ch. 3, ed. & trans. Ellinwood, 68–69.

⁴⁴⁹ Hermannus, *Musica*, ch. 5, ed. & trans. Ellinwood, 76–81.

⁴⁵⁰ Hermannus, *Musica*, ch. 6, ed. & trans. Ellinwood, 80–83.

redundant. However, because he treats the species as spanning similarly named pitches in consecutive tetrachords, the span from D – G counts as a fourth species because it spans the fourth pitches of the two lowest tetrachords, and because – as in the *Enchiridis* – the qualities of the modes (and species) are drawn from the qualities of the pitches themselves (which in turn, presumably, are drawn from their context within the gamut). The fourth species contains the same internal interval structure as the first, but its external context is different, and therefore the species is different. Because of this refinement, the fourth species of fifth is no longer joined with the first species of fourth, but with the fourth species of fourth, so that species of fourth and fifth are always joined together in corresponding numbers.

However, as I suggest at the beginning of this chapter, the principal purpose for this examination is to clarify which elements of the theory are present in the *Alia musica* and which are not, in order not to assume the presence of elements familiar from later theory that are not actually present in the *Alia*. To begin with, unlike pseudo-Bernelinus and Bern, the *Alia musica* explicitly associates the modes with the octave species. This fact, in and of itself, gives cause for caution in supposing a linear development of the octave species model with the *Alia* preceding pseudo-Bernelinus and Bern; if one were to assume that they were building upon the doctrine presented in the *Alia*, it would be strange that they do not equate the modes to the octave species, but only to concatenations of the fourths and fifths.

On that topic, the *Alia* does not define the octave species or modes as concatenations of fourths and fifths, and thus, the distinction between the first mode and the last does not come from the order of species of fourth and fifth, but rather from the position of the *median* (average) pitch, a pitch held to be important to the character of the mode, which is a fourth above the bottom in plagal modes and a fifth above the bottom in authentic modes. (The mechanism by which the median pitch is defined will be described in Chapter 09).

Guido

There is one more theorist whose doctrines need to be addressed in this chapter, not because he continues the development of species theory (which he largely does not), but because his formulation of the gamut and mode is probably the best known medieval formulation, and because his formulation responds to tensions within the system that reveal unspoken exceptions.

Guido is ultimately responsible for an impressive number of staple concepts in later medieval theory, though the degree to which he developed them is easy to overestimate. For instance, he is credited with inventing staff notation, but he merely adapted a graphical system of notation already used in Hucbald's *Musica* and in the *Enchirides* (see Table 1 in Chapter 2 above). The notation that he presented in the *Epistola ad Michahelem*⁴⁵¹ used only a single reference line – a significant improvement upon traditional neumes, but still challenging to read and write accurately. In the *Regulae rithmicae*, he describes other people fitting multiple pitches between two lines or leaving out the lines altogether. All that Guido contributes to this system is to advocate for exactly one pitch between lines and a colour code for the lines where a clef would appear;⁴⁵² the first of these recommendations has, of course, become standard, but the second was applied only sporadically and the only traces of it that remain today are a comparable colour code for harp strings. Similarly, the Guidonian hand, a well-known mnemonic device that bears his name, did not arise in its traditional form until the twelfth century.⁴⁵³

Guido also invented solmization and laid the foundation for the theory of hexachords.⁴⁵⁴ A hexachord is a six-note structure that can be described as an expansion upon the standard tetrachord of

⁴⁵¹ cf. Pesce, *Guido d'Arezzo*, 466.

⁴⁵² Guido, *Regulae musicae*, ed. & trans. Stephenson, 25–27; cf. Guido, *Prologus in antiphonarium*, ed. & trans. Pesce, 418–31.

⁴⁵³ Palisca and Pesce, "Guido of Arezzo," *Grove Music Online*, §2.v, ¶12.

⁴⁵⁴ Guido, *Epistola*, ed. & trans. Pesce, 465–75. I suspect that more misinformation has been disseminated about this issue than nearly any other in medieval music theory. It should be noted that Guido did not create the

medieval theory (tone-semitone-tone) by adding an additional tone on each side; its most basic form corresponds to the *finalis* tetrachord (D – G), expanded in each direction so that it spans C – a.

Guido principally describes the interval structure found in a hexachord in the *Epistola ad Michahelem*, where he describes the chant *Ut queant laxis*, for which the first note of each phrase begins with the next higher pitch in the hexachordal structure. Meanwhile, his modal theory is most typically cited in the *Micrologus*, a treatise that makes no direct reference to the structure of the hexachord; nevertheless, the hexachords are heavily implicated in his system of mode. As I have already suggested, Guido does not define his modes directly as octave species. Even so, his conception responds to a similar motivation: to define the modes in relation to the intervals within which they move. Unlike the octave species formulation, however, Guido defines the modes in relation to the constellation of intervals surrounding the *finalis*. For the first three *maneriae*, this constellation of intervals is identical to the

hexachords themselves. He created the six solmization syllables that identify the notes within a hexachord, but he never uses the term “hexachord” to describe these notes, and he certainly never uses the terms *natural*, *hard*, or *soft* to describe the three familiar versions on C, G, and F. He also defines the modes according to the position of the *finalis* within an interval structure equivalent to that formed by these six notes (as I shall describe below), and allows the modes to appear wherever the appropriate interval structure arises (*ibid.*, 492–95; cf. *Micrologus*, ch. 7, ed. Smits van Waesberghe, 117–19, trans. Babb, 63). Guido does not precisely describe the hexachord in the *Micrologus*, despite the indication to the contrary in Hershberg’s “Hexachord” article on *Grove Music Online*, nor does he define the “relationships between different six-note groups” in the *Epistola*, as Hershberg also asserts; in the *Epistola*, he defines only one six-note group (anything else must be inferred). To the extent that one recognizes the interval structures that he describes around the *finalis* of the modes as a hexachord, his description of the affinities between pitches sharing these interval structures implies a hexachord on both C and G (the natural and soft), but the clear description of these structures had to wait for later theorists. Unfortunately, Gerbert has muddled this issue, because in his edition of Guido’s *Regulae rhythmicae*, Gerbert has included solmization syllables for the natural and hard hexachords above the letter notation in the description of the gamut (Gerbert, *Scriptores ecclesiastici II*, 25); according to Pesce’s critical edition, they appear only in a single manuscript, M1 (Pesce, *Guido d’Arezzo*, 334), an incomplete copy in a German manuscript from ca. 1100 (*ibid.*, 243 & 246). They do not appear in any other manuscript, nor do they appear in any manuscript on the other, similar diagram later in the treatise; they are, however, taken for granted in much scholarship about him (cf. Stephenson, *Guido’s Rhythmic Rules*, 3, and Allaire, *Theory of Hexachords*, 16, both of which reproduce Gerbert’s diagram), leading to the impression that Guido defined these hexachords, when in fact, he only implied them. One might also observe that his teachings could imply the hexachord on F (the soft hexachord), which correspondingly would be necessary to account for the *b*, the provision of which, as is explained below, seems to be the driving force behind Guido’s novel formulation of mode; however, Guido disliked the *b* (Guido, *Epistola*, ed. & trans. Pesce, 510–17), and even had he developed the natural and hard hexachords himself, he probably still would not have approved of the soft hexachord.

hexachord C – a,⁴⁵⁵ so that the modes effectively are defined according to the position of the *finalis* within the hexachord.

It is important that the only note not included in the hexachord is *b* (or *♭*); this can hardly be a coincidence. Consequently, the modes are defined according to all the intervals except the one interval that can fluctuate. This definition is actually not that different from associating modes with the octave species, except that it responds to two crucial tensions between actual modal practice and theory. First, and most significantly, if a mode is defined by its octave species, then *b* and *♭* cannot be flexible, because switching to *♭* changes the interval structure (and thus, the octave species). Previous theorists seem to have been content simply to accept the tension between the rigidity of octave species and the flexibility of *b* and *♭*; Guido defines the modes so that the intervals are mostly defined, but equally so that the flexibility is encoded into the system. (Interestingly, though, Guido did not like *♭*,⁴⁵⁶ and seems not so much to have defined the modes in this way to allow the flexibility of *b* and *♭* in modes based on D, E, and F as to allow the same modes to arise on a, *♭*, and c, so that the flexibility between *b* and *♭* gets replaced by the single interval difference between the interval structures above these two sets of *finales* – *i.e.*, the ambiguous *♭* in the modes on D, E, and F gets replaced by the unambiguous *f* in the modes on a, *♭*, and c.)⁴⁵⁷

Unlike the first three *maneriae*, however, the fourth is not defined within the hexachord, but rather, it is defined as containing a tone below and two tones and a semitone above⁴⁵⁸ (see Figure 24). This definition is important because it is the only mode whose definition directly incorporates the *♭*, and

⁴⁵⁵ Guido, *Micrologus*, ch. 7, ed. Smits van Waesberghe, 117, trans. Babb, 63.

⁴⁵⁶ Guido, *Epistola ad Michaelem*, ed. & trans. Pesce, 510–17.

⁴⁵⁷ Guido, *Micrologus*, ch. 8, ed. Smits van Waesberghe, 124–26, trans. Babb, 64.

⁴⁵⁸ *ibid.* cf. Guido, *Epistola ad Michaelem*, ed. & trans. Pesce, 494–95; in the *Epistola*, Guido defines the lower intervals of *tetrardus* all the way down to the bottom of the hexachord, as with the other three *maneria*, but he still defines the upper intervals as two tones and a semitone, thus unambiguously prescribing the *♭* and describing this *maneria* (and only this *maneria*) within the full compass of an octave (though not corresponding to the octave species associated with either the authentic or plagal mode *tetrardus*).

Alia musica needed to respond to and the newer concepts that have been described as developments of the theory in the *Alia musica*. The remaining section of this dissertation will turn to the theory of the *Alia musica* itself.

Section III: The Doctrines of the *Alia musica*

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Chapter 9: The Conventional Theory of the *Alia musica*

The first section of this dissertation placed the *Alia musica* treatise and its manuscripts in their historical context; the second outlined the historical context for the most important theoretical concepts in the *Alia musica*. This final section of the dissertation examines the theoretical concepts as presented in the *Alia* itself and attempts to make sense of the complicated discussions and incomplete presentations of the central issues.

This chapter will begin the process by briefly outlining the elements of theory in the *Alia musica* that are conventional (or nearly so). This material principally appears in the supplemental portions of the revision, especially the introductory passages based on Boethius (though a few elements are presented as interludes between the discussions of particular modes).

Musical Proportion

The first and central issue is the numerical relation 6:8:9:12. In an interesting departure from the usual manner of presenting this relation, it is not presented first as a whole, nor as a synthesis of each of the possible two-term ratios of which it is composed; instead, it is presented as the superposition of two means (more on this shortly), consisting of three terms each. This introduction ultimately leads to the same understanding gained through more traditional presentations, but it will be more productive here to present the concepts in the more traditional fashion.

It has become traditional to introduce these concepts through a “just so” story about Pythagoras. The story is probably apocryphal, as the earliest known account is in Chapter 6 of the handbook of Nicomachus, written around the turn of the second century.⁴⁶⁰ Although Boethius’ principal source for

⁴⁶⁰ Levin, “Nicomachus of Gerasa,” *Grove Music Online*, ¶¶1–2; Nicomachus, *Ἀρμονικὸν ἐνχειρίδιον*, ch. 6, ed. von Jan, 245–48, trans. Barker, 256–58.

most of his *De institutione musica* is believed to be a different, lost treatise by Nicomachus, this story nevertheless appears in Boethius treatise (either supplementarily or because Nichomachus repeated it in the other treatise) in Book I, Chapters 10 and 11,⁴⁶¹ whence it became widely known to medieval theorists.

Put very briefly, the story says that Pythagoras, while walking passed a smithy, recognized the sounds of the perfect consonances in the din of fours hammers striking the anvils; he borrowed the hammers and performed measurements of them and experiments with them to discover that the perfect consonances resulted from the relationship among the weights of the hammers, for which the three smaller hammers were in the proportions 12:6, 12:8, and 12:9 against the largest hammer, producing the perfect consonances of the octave, fifth, and fourth, respectively. Similarly, the three larger hammers were in the proportions 8:6, 9:6, and 12:6 against the smallest, producing the consonances of a fourth, fifth, and octave, respectively. Finally, the two middle hammers were in the proportion 8:9, which produces the tone.

In the introduction to his translation of Boethius, Calvin Bower describes these numbers as an instance of a *tetraktys* (he calls it the “Pythagorean tetractys”),⁴⁶² a set of four numbers amongst which every possible ratio between any two of these four numbers is meaningful – in this case, each produces a musically important interval (all but one of which is a perfect consonance). Amongst these various ratios, 8:6 and 12:9 reduce to 4:3, the prototypical ratio of the perfect fourth; 9:6 and 12:8 reduce to 3:2, the prototypical perfect fifth; and 12:6 reduces to 2:1, the perfect octave. These relationships are crucial to the understanding of the theory of the *Alia musica*.

⁴⁶¹ Boethius, *De institutione musica*, bk. 1, ch. 10, ed. Friedlein, 196–98, trans. Bower, 17–19.

⁴⁶² Bower, *Fundamentals of Music*, xxii.

However, this relation is not the set of numbers most commonly described as a *tetraktys*, which is the *tetraktys of the dekad*,⁴⁶³ a relation that is not directly presented in the *Alia* but is useful for understanding the theory. The *tetraktys of the dekad* is the relation 1:2:3:4, which while seemingly mundane, produced an enormous amount of hidden meaning for the Pythagoreans, especially as represented by geometrical divisions and relationships found within a triangular arrangement of points in which each row contains the number of points corresponding to each consecutive term of the *tetraktys* (it is known as the *tetraktys of the dekad* because there are a total of ten points). As with the relation 6:8:9:12, every possible ratio between two terms of the *tetraktys of the dekad* produces a musically important interval – in this case, exclusively perfect consonances, including some that are not found in 6:8:9:12. In addition to 2:1, 3:2, and 4:3, already described above as the octave, fifth, and fourth, respectively, the ratio 3:1 gives the perfect twelfth (compound fifth), and 4:1 gives the double octave (while 4:2 reduces to 2:1, another octave). The *tetraktys of the dekad* thus produces all the perfect consonances within the span of two octaves (except the perfect eleventh, the status of which was controversial). These two *tetraktyes* together present all the prototypical ratios from which the *Alia* derives the rest of its number theory (though the *Alia* does not present them in this manner).

Superparticularity

One concept that is frequently described in old treatises that is not directly described in the *Alia* but is useful for understanding the concepts that follow is superparticularity. According to both Greek and medieval theory, all consonant intervals must be either multiple (2:1, 3:1, and 4:1, which correspond

⁴⁶³ This observation should not be taken to mean that Bower is wrong to identify the relation 6:8:9:12 as a *tetraktys*. Nowacki also describes the series 6:8:9:12 as the Pythagorean *tetraktys*, though he cites Bower's discussion and is therefore not an independent source (Nowacki, *Greek and Latin Music Theory*, 112 & 200). More usefully, Theon of Smyrna, in his *Mathematics Useful for the Understanding of Plato*, describes a total of eleven *tetraktyes* (Theon, *Των κατα το μαθηματικον*, bk. 2, chs. 37–38, ed. Dupuis, 152–62, trans. Lawlor & Lawlor, 62–66), and while 6:8:9:12 does not appear in his list, the list nevertheless demonstrates that a *tetraktys* is a more general concept.

to the octave, the twelfth, and the double octave) or superparticular. *Superparticularis* is the Latin term (Greek ἐπιμόριος, epimoric) for a ratio containing one part greater than the whole – that is, $1 + \frac{1}{n} : 1$, which can also be expressed as $n + 1 : n$; the ratios 3:2 and 4:3, corresponding to the perfect fifth and perfect fourth, are superparticular. One of the principal reasons why the perfect eleventh (3:8) is not usually considered to be consonant is because it is not superparticular. The octave, 2:1, is not usually described as superparticular because it is multiple, and superparticulars are subordinate to multiple; however, by the strict definition, it is the first superparticular ratio.

The Mathematical Means

Instead of producing the relation 6:8:9:12 from the synthesis of the ratios representing each perfect consonance as in other treatises, the *Alia* produces the relation by superimposing the arithmetic and harmonic means within the same bounds.

In mathematics, a *mean* is one of the three traditional types of averages (the other two being the *median* – the middle number in an ordered list of values, regardless of how it relates to the other values – and the *mode* – the most common value in a list of values). The other two types of averages only make sense for lists of three or more numbers and do not try to take account of all of the values in the list; the mean accounts for all of the values in the list and makes sense even for only two values – in fact, there are a considerable number of different types of mean, and most of them are most readily defined for only two values.

The means are concepts from ancient Greek mathematics and music theory, and the three classic means, the arithmetic, geometric, and harmonic, are traditionally associated with the Pythagoreans; the arithmetic and harmonic means also appear in Plato.⁴⁶⁴ The geometric mean was considered to be the

⁴⁶⁴ Burnet, *Platonis opera*, 991a.

first mean and is also the mean value of the other two means. Three more means are traditionally ascribed to Eudoxus, and the complete list of these six appears in Theon of Smyrna's *Mathematics Useful for the Understanding of Plato* (he also notes that there are six more that are subcontraries to the others, but he does not define them).⁴⁶⁵ Boethius describes these six in his *De arithmetica institutione*, and further notes that yet four more were added to bring the total number of means to ten;⁴⁶⁶ the significance of having ten, which Boethius leaves unstated, is that the system thus developed from one (geometric) to add two (arithmetic and harmonic), then to add three more (all described as contraries), and then finally to add four more; the development of the system therefore proceeded by 1:2:3:4, the *tetraktys of the dekad*.

To understand the means, it is useful to define five terms: let A and B represent the two terms for which the mean value is being sought, and μ represent the mean itself; additionally, let α represent the difference between A and the mean ($\alpha = |A - \mu|$) and β the difference between B and the mean ($\beta = |B - \mu|$). The first six means are most readily described by comparing the ratio of $\alpha:\beta$ to another ratio, and in fact, all the possible comparisons are exhausted in these six means (though some can be described more intuitively by comparing these numbers in a different way). Since all the possible comparisons are thereby exhausted, the final four require one more term: the difference between A and B; let δ represent this difference ($\delta = |A - B|$). The complete set of means is given in Table 19.

While Boethius describes all of these means in his *De institutione arithmetica* (which is the source of the first several sections of the *Alia musica*), his discussion in the *De institutione musica* (primarily in Book II, Chapter 12)⁴⁶⁷ is limited to only the first three. These three are readily expandable to cover more than two pitches, and thus have more complete definitions and applications in traditional mathematics.

⁴⁶⁵ Theon, *Των κατα το μαθηματικον*, bk. 2, ch. 50, ed. Dupuis, 174, trans. Lawlor & Lawlor, 70.

⁴⁶⁶ Boethius, *De institutione arithmetica*, bk.2, chs. 42–54, ed. Friedlein, 139–73, trans. Masi, 166–88.

⁴⁶⁷ Boethius, *De institutione musica*, bk. 2, ch. 12, ed. Friedlein, 241–42, trans. Bower, 65–66.

Table 19 – The Ten Means

Only the first three are described in Boethius' *De institutione musica*, and only the second and third in the *Alia musica*.

Name	Standard Description ($\alpha:\beta = ?$)	Alternate Description
1. Geometric Mean	$\alpha:\beta = A:\mu, \alpha:\beta = \mu:B$	$A:\mu = \mu:B$
2. Arithmetic Mean	$\alpha:\beta = A:A, \alpha:\beta = B:B$	$\alpha:\beta = 1:1, \alpha = \beta$
3. Harmonic Mean	$\alpha:\beta = A:B$	
4. Contrary to the Harmonic Mean	$\alpha:\beta = B:A$	
5. 1 st Contrary to the Geometric Mean	$\alpha:\beta = B:\mu$	
6. 2 nd Contrary to the Geometric Mean	$\alpha:\beta = \mu:A$	
7. [1 st Supplementary Mean]		$B:A = \delta:\alpha$
8. [2 nd Supplementary Mean]		$B:A = \delta:\beta$
9. [3 rd Supplementary Mean]		$\mu:A = \delta:\alpha$
10. [4 th Supplementary Mean]		$\mu:A = \delta:\beta$

The Arithmetic Mean

The arithmetic mean is the familiar average used in most contexts in everyday life. It can be loosely defined as the mean that evenly splits the difference between two numbers (or tries to minimize the distance of all of the numbers from the average). The usual procedure for calculating the arithmetic mean is to sum all of the terms and then divide the sum by the number of terms; since there will typically only be two terms in music, the formula for the mean is $\mu = \frac{A+B}{2}$. As an example, if A is 10 and B is 40, then the arithmetic mean is $\mu = \frac{A+B}{2} = \frac{10+40}{2} = \frac{50}{2} = 25$. The completed relation $A:\mu:B$ is then 10:25:40, where the differences between each original term and the mean are both 15.

The Geometric Mean

The geometric mean behaves much the same way as the arithmetic mean, except on a geometric scale – a scale for which the increments become continually larger as the scale goes up (as is the case for

frequency in musical pitch). Another reasonably familiar analogy would be compound interest, where if the principal and return after one year are known, the geometric mean would give the return at six months. Instead of taking the sum of all the values and dividing by the number of values, the geometric mean is calculated by taking the product of all the values and taking the n th root of the product (where n is the number of values). So, for two terms, the formula is $\mu = \sqrt{A \cdot B}$. For example, again taking A as 10 and B as 40, the harmonic mean is $\mu = \sqrt{A \cdot B} = \sqrt{10 \cdot 40} = \sqrt{400} = 20$. The completed relation is then 10:20:40, where the ratio 10:20 is equivalent to the ratio 20:40 (both reduce to 1:2).

The Harmonic Mean

The harmonic mean is the least intuitive of the three classical means, most readily understood by comparison to another analogy for the arithmetic mean. If a vehicle travels for a fixed time at one speed and then for the same amount of time at a different speed, the average speed is the arithmetic mean; but if a vehicle travels for a fixed distance, instead of a fixed time, at one speed, and then the same distance at another speed, the average speed is the harmonic mean. The harmonic mean is calculated by dividing the number of terms by the sum of the reciprocals of all the terms. Thus, for two terms, the formula is $\mu = \frac{2}{A^{-1} + B^{-1}}$. Where A is 10 and B is 40, the harmonic mean is $\mu = \frac{2}{A^{-1} + B^{-1}} = \frac{2}{10^{-1} + 40^{-1}} = \frac{2}{1/10 + 1/40} = \frac{2}{5/40} = \frac{2 \cdot 40}{5} = \frac{80}{5} = 16$. The completed relation is then 10:16:40, where the difference between 10 and 16 is 6, the difference between 16 and 40 is 24, and the proportion between them (6:24) is equivalent to the original proportion 10:40 (both reduce to 1:4).

The Means in Music

One thing that the *Alia musica* does better than probably any other music treatise is to clarify the manner in which these means relate to music. Even in the *De institutione musica*, Boethius focusses on the math and barely describes the musical significance of the concept. The point of these means is that

musical pitches are represented as numbers, such that it is possible to find the mean pitch between two given pitches – a pitch that is “halfway” between the original two pitches in some meaningful sense. For example, the octave is represented by the ratio 2:1. It is possible to find the middle pitch between the two pitches of the octave by taking the mean, though the ratio must first be raised to higher terms so that there is room between them for a mean. To find the arithmetic mean, the terms should be doubled to give 4:2; the arithmetic mean is then $\mu = \frac{4+2}{2} = \frac{6}{2} = 3$, producing the relation 2:3:4. To find the harmonic mean, the terms should be tripled to produce 6:3; the harmonic mean is then $\mu = \frac{2}{\frac{1}{6} + \frac{1}{3}} = \frac{2}{\frac{1}{2}} = \frac{2 \cdot 2}{1} = 4$, producing the relation 3:4:6. These two means produce the perfect fourth and the perfect fifth, both of which could meaningfully be said to be the note halfway between an octave, though in different ways.

It is possible to combine these two means into a single relation by raising the terms of each so that the outer two terms are the same (*i.e.*, so that the means are calculated for not just any octave, but the same octave). The relation 2:3:4 must be tripled and the relation 3:4:6 must be doubled, so that both relations begin with 6 and end with 12. The resulting relations are 6:9:12 (arithmetic) and 6:8:12 (harmonic). Put together, these relations produce the combined relation 6:8:9:12, the same relation described by Pythagoras and upon which the number theory of the *Alia* is constructed. This relation appears in Boethius only at the very end of the *Arithmetica*. It is fascinating to observe that the opening sections of the *Alia musica* borrow earlier chapters of the *Arithmetica* to describe the concepts of the arithmetic and harmonic means, but the *Alia* substitutes the numbers from Boethius’ original presentations (which gives the lower series of numbers) with the increased terms used in his final chapter so that both means are immediately presented in common terms.

While the arithmetic and harmonic means may be expressed in common terms in this manner, the geometric cannot be. The octave, represented by 2:1, while usually referred to as a multiple proportion, is also technically a superparticular proportion, and no superparticular proportion has a whole-number

geometric mean. In a musical application, the lack of a geometric mean for the octave makes sense: musical frequency is geometric in nature, with higher scale-steps becoming ever larger. The geometric mean provides the precise middle on just such a scale, such that the geometric of the octave is an equal-tempered tritone, an interval that has no place in early music. It is perhaps for this reason that the *Alia* makes no use of the geometric mean.

The geometric mean can, however, have musical significance for dividing other intervals, just not perfect consonances. For instance, the geometric mean can produce the middle pitch between the boundaries of a major ninth. The major ninth is an octave (2:1) plus a tone (9:8), and adding intervals is accomplished by multiplying the ratios, giving $(2 \times 9) : (1 \times 8) = 18 : 8 = 9 : 4$. The geometric mean of the major ninth (9:4) is therefore $\mu = \sqrt{9 \times 4} = \sqrt{36} = 6$; the complete relation is then 9:6:4, with the 6 representing the perfect fifth, which is indeed the midpoint of a major ninth. (The arithmetic and harmonic means likewise have other applications outside the octave, such as producing the major and minor thirds as the midpoints of a perfect fifth).

There is perhaps another reason why the *Alia* focusses on the arithmetic and harmonic means. They have a special relationship to each other, forming a reciprocal pair. The harmonic mean is the subcontrary to the arithmetic mean, and it is calculated by taking the reciprocal of the arithmetic mean of the reciprocals of the terms: instead of summing the terms and dividing the sum by the number of terms, one must first flip the terms to their reciprocals (put them in a fraction under a 1, so that A becomes $\frac{1}{A}$), take the arithmetic mean of those terms, and then perform the same flipping procedure upon that mean. This reciprocal nature is also expressed in music, in that the arithmetic and harmonic means become inverses of each other. Not inversions, quite; although this appears to be true for the means of the octave (perfect fourth and fifth), it is not true for the means of the fifth (major and minor third). Rather, they are inverses of each other in the sense that the relations created by placing the means

between the two original pitches create two intervals, one from the mean down to the lower pitch and one from the mean up to the upper pitch, and the arithmetic and harmonic means produce the same two intervals in reverse order.

There is another way in which the inverse relationship may be understood in music, and it is the reason why I have thus far been vague as to which mean produces the fifth and which the fourth. The application of numbers to music may be accomplished in two ways that function as reciprocals to each other: the numbers may represent frequencies, in which case, a higher number corresponds to a higher pitch, or the numbers may represent the length of a string (or air column, etc. – a proxy for wavelength) needed to produce the pitch, in which case, a lower number (shorter string) represents a higher pitch. Frequency and wavelength are reciprocals of each other, and thus, the interpretation of the arithmetic and harmonic means invert between the two paradigms. In the frequency paradigm, which is the paradigm most commonly employed in modern discussions of harmonics, where pitch numbers are typically derived from term indices in the overtone series, the arithmetic mean produces the fifth, and the harmonic mean produces the fourth. However, in the wavelength paradigm, which is employed in the *Alia musica*, the interpretation is reversed: the harmonic mean produces the fifth, and the arithmetic mean produces the fourth.

The introduction to the *Alia musica* describes the arithmetic and harmonic means and demonstrates the fourth and fifth as the mean pitches of the octave, then runs to considerable length with simple numberplay that seems divorced from musical significance (which is not surprising, since much of it is drawn from Boethius' *Arithmetica* rather than the *Musica*). This section ends by explaining why the harmonic mean is sometimes called a cubic proportion: the relation 6:8:12, produced by taking the harmonic mean of 6 and 12, corresponds to the members of a cube: six faces, eight vertices, and twelve edges. The *Alia* then ends its discussion of mathematics and begins a discussion of the octave species without clearly explaining the relevance of the mathematics, but in view of the rest of the treatise,

it seems likely that the purpose of the mathematics was to legitimize 6:8:9:12 (which is central especially to the source treatise) as the superposition of the harmonic and arithmetic means.

Species Theory

The introduction to the *Alia* already demonstrates the significance of the arithmetic and harmonic means to music in a much clearer way than in most other treatises. However, the *Alia* ultimately uses these principles to explain a far more important musical concept, though the reader must wait for it. The rest of the introduction describes the octave species, initially according to the position of the upper boundary pitch of the octave within the Greater Perfect System using the Greek string-based nomenclature⁴⁶⁸ – importantly, according to the position of the top note only, not according to the position of the entire octave (this issue will be revisited in Chapter 12). These octave species are described here explicitly as *modus*, a word that seems to equate the octave species with the ecclesiastical modes, but a word not frequently used elsewhere in the treatise, where the terms *tonus* and *tropus* are generally employed (this terminology will be examined in detail in Chapter 12). The modes are not named according to the *maneriae* here, but according to the Greek ethnic names (Dorian, Phrygian, etc.), largely in the expected places; they are also numbered *primus* through *octauus*, but not in the usual order for numbering the ecclesiastical modes; instead, these numbers reflect the traditional numbering for the octave species, and may not be meant to be names so much as simple tallies (not so much “First Mode,” but simply “first mode”). For the purposes of the present discussion, I shall temporarily accept the going assumption that this passage is associating the octave species with the ecclesiastical modes;⁴⁶⁹ however, I shall examine this interpretation more closely shortly.

⁴⁶⁸ *Alia musica* §15, ed. Chailley, 107.

⁴⁶⁹ cf. Atkinson, *Critical Nexus*, 186–89.

It is at this point that the *Alia* repeats Boethius' misleading statement that Ptolemy added the eighth mode to the top of the system.⁴⁷⁰ The result is that the Hypermixolydian is described here as above the Mixolydian, and consequently has the same interval structure as the Hypodorian – that is, the *protus* plagal mode, rather than the *protus* authentic mode. This state of affairs is not revised until much later in the treatise.

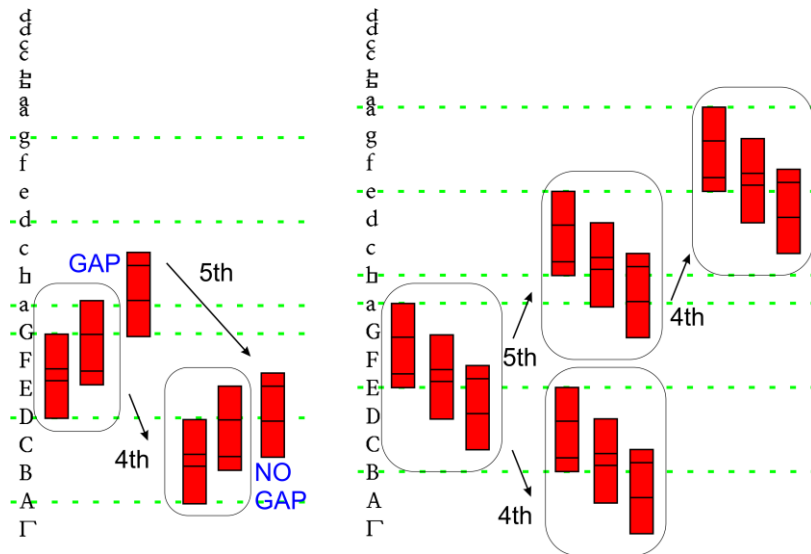


Figure 25 – The species of fourth in Bern (left) and the *Alia musica* (right)

Because there is a gap in Bern's presentation, the species do not repeat exactly at the fourth. There is no gap in the *Alia*'s presentation, and so the species of fourth repeat perfect at fourths or fifths.

The *Alia* then proceeds to describe the species of fourth and fifth, though the treatise ultimately does nothing with them. He does not describe them as pseudo-Bernelinus and those after him will do, with the semitone at the middle of the first species (and thus rising from D or A); instead, the *Alia* enumerates the species of fourth as Boethius does, with the semitone at the bottom of the first species (thus rising from B or E). The distinction between these two approaches may seem small, but it has a very important consequence for relationships between the species of fourths and fifths and the gamut. The *Alia* says that the species of fourth always repeat themselves, in order, either at the fourth or the fifth. However, because Bernelinus placed the semitone of the first species in the middle (and Bern placed it

⁴⁷⁰ *Alia musica* §16(a)–(b), ed. Chailley, 107.

rising from D), the three species of fourth cannot simply be transposed down by fourth; the first two are transposed by fourth, but the third is transposed down by fifth. In the *Alia*'s description, though, all three of the species of fourth do, in fact, repeat in order a fourth below and fifth above (and also a fourth above that).

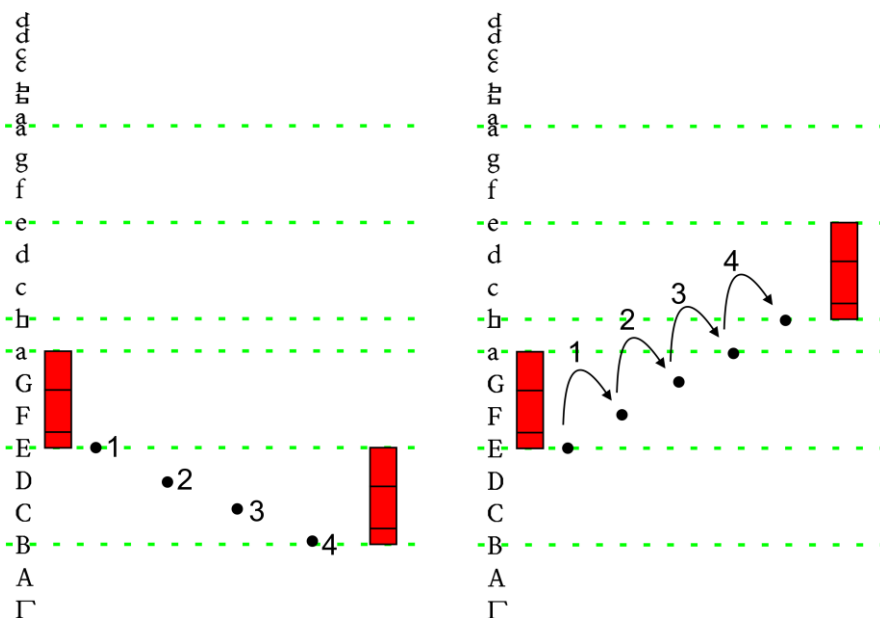


Figure 26 – Measuring four places by conjunction or disjunction, producing a fourth or a fifth

As Chailley notes, the precise wording of the *Alia* is slightly ambiguous. The *Alia* says, “*semperque siue per disiunctum siue per coniunctum tetrachordum quartis locis eadem species redit.*”⁴⁷¹ (“And always, either by disjunction or conjunction, the same species of tetrachords return at the fourth place.”) Chailley calls this a “*sottise*”⁴⁷² (“nonsense”), but it seems more likely that Chailley is simply reading the phrase “fourth place” too narrowly, since the phrase “either by disjunction or by conjunction” clarifies the intention (and would be redundant if the species always recurred at the fourth). Thus, the pure fourth is “a fourth by conjunction,” and the fifth is “a fourth by disjunction.” Another way to say this is that the species of fourth repeat themselves in order always four notes away, either counting the original position

⁴⁷¹ *Alia musica* §17(b), ed. Chailley, 108.

⁴⁷² Chailley, *Alia musica*, 109.

as a note (“by conjunction”) or not counting the original position as a note (“by disjunction”) – see Figure 26.

Chailley also says that *semperque* is likewise nonsense because the correspondence disappears if the tetrachord incorporates a tritone.⁴⁷³ However, the *Alia* is not describing any possible segment of four scale steps, but only the three species of fourth, none of which contains a tritone.

The *Alia* then proceeds to define the first three species of fifth as being created from the three species of fourth plus a disjunct tone at the bottom; the fourth species of fifth contains a semitone at the bottom, and the first species begins from the νήτη διεζευγμένων (e) (Figure 27).⁴⁷⁴

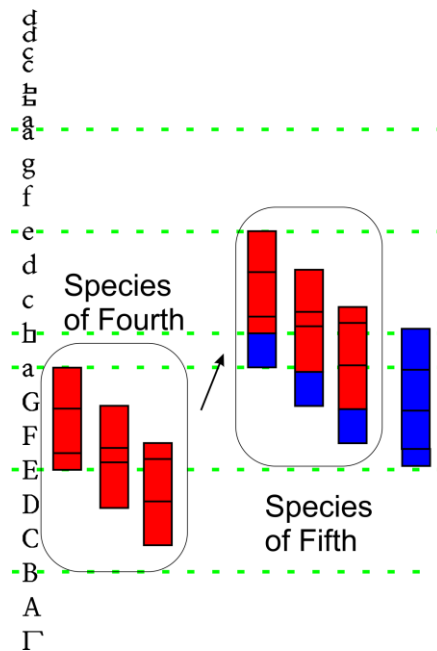


Figure 27 – The relation between species of fourth and fifth in the *Alia musica*

The *Alia* notes that unlike the fourths, the fifths do not recur every four or five places. (The *Alia* does not acknowledge that the fifths recur every octave.) It then proceeds to describe the octave species

⁴⁷³ *ibid.*

⁴⁷⁴ *Alia musica* §18, ed. Chailley, 109.

again, this time in terms of the position of the semitones, and without any reference to the species of fourth and fifth.⁴⁷⁵

<u>Hypodorian</u>	<u>Dorian</u>
-νήτη ὑπερβολαίων - - - - -	-νήτη ὑπερβολαίων - - - - -
παρανήτη ὑπερβολαίων	παρανήτη ὑπερβολαίων
τρίτη ὑπερβολαίων	τρίτη ὑπερβολαίων
-νήτη διεζευγμένων - - - - -	-νήτη διεζευγμένων - - - - -
παρανήτη διεζευγμένων	παρανήτη διεζευγμένων
τρίτη διεζευγμένων	τρίτη διεζευγμένων
-παραμέση - - - - -	-παραμέση - - - - -
-μέση - - - - -	-μέση - - - - -
λίχανος μέσων	λίχανος μέσων
παρυπάτη μέσων	παρυπάτη μέσων
-ὑπάτη μέσων - - - - -	-ὑπάτη μέσων - - - - -
λίχανος ὑπάτων	λίχανος ὑπάτων
παρυπάτη ὑπάτων	παρυπάτη ὑπάτων
-ὑπάτη ὑπάτων - - - - -	-ὑπάτη ὑπάτων - - - - -
προσλαμβανόμενος	προσλαμβανόμενος

Figure 28 – The relationships between Hypodorian and Dorian modes

The passage that follows is another example of a passage that demonstrates that the *Alia*'s understanding of Boethius' modal theory may not be as confused as it has sometimes been interpreted to be. The *Alia* compares the position of pitches within the Hypodorian mode to those in the Dorian mode. Chailley acknowledges that this section is modelled on Boethius, but is not a direct copy of Boethius' presentation; rather, it is an entirely new comparison using the same methodology.⁴⁷⁶ The comparison is made using three different tetrachords, spanning the διεζευγμένων to the ὑπάτων (and also the προσλαμβανόμενος) – well beyond the range of any single octave species, regardless of how it is defined, but the author of the *Alia* explicitly states that he is still speaking of the octave species: “*Ut ad ipsas*

⁴⁷⁵ *Alia musica* §19, ed. Chailley, 110; cf. Atkinson, *Critical Nexus*, 191.

⁴⁷⁶ Chailley, *Alia musica*, 111–12.

*species diapason redeamus*⁴⁷⁷ (“So that we return to those species of octave”). The comparisons are summarized in Figure 28.

The *Alia* says:

*Lichanos hypaton hypodorii est proslambanomens dorii, meses dorii, quae est paranete diezeugmenon hypodorii, integra consonantia diatessaron acutior est a mese eiusdem hypodorii.*⁴⁷⁸

The Hypodorian’s λίχανος ὑπάτων is the Dorian’s προσλαμβανόμενος; the Dorian’s μέση, which is the Hypodorian’s παρανήτη διεzeugμένων, is a full consonance of a fourth higher than that same Hypodorian’s own μέση.

The *Alia* follows up this description by explaining that the same kind of relationship exists between the Phrygian and Hypophrygian, and also between the Lydian and Hypolydian, but reiterates that the Hypermixolydian is not separated from the Mixolydian by fourth, but by a tone (and therefore does not fit the same pattern).

As suggested, this passage is entirely consistent with Boethius, and in light of it, it is necessary to reconsider the earlier passage that apparently associates the modes with the octave species and to read it carefully, without assumptions. The passage says:

Erit ergo primus modus omnium gravissimus videlicet hypodorius ex prima specie diapason, et terminatur eo qui meses dicitur, medio neruo. Secundum modum hypophrygium secunda species diapason efficit, quae in paramesen finitur. Tertium modum hypolydium tertia species diapason determinat in eum quem uocant triten diezeugmenon neruum. Quartum modum dorium quarta species diapason reddit, quae finit in paranete diezeugmenon. Quintus modus phrygius quinta specie diapason finitur, cui nete diezeugmenon neruus est ultimus. Sextum nihilominus modum lydium sexta species diapason exerit, cui trite hyperbolaeon est finis. Septimum quoque modum mixolydium septima species diapason informat, quam paranete hyperbolaeon determinat.

*Verum quia unus duplus, hoc est una diapason, octo vocibus pollens, plures species no recipit, quandoquidem omnis symphonia unam uocem pluresque species admittit, octauum modum hypermixolydium Ptolomaeus adiecit, quem secundi ac tertii modi proprietatibus informauit. Est enim diatessaron 4 chordarum et trium specierum, diapente quoque 5 chordarum et 4 specierum, quapropter et diapason 8 chordarum et 7 specierum.*⁴⁷⁹

The first mode, therefore, will be the lowest of all, that is the Hypodorian, from the first species of octave, and it is ended at that which is called μέση, the middle string. The second species of octave brings about a second mode, Hypophrygian, which is ended at the παραμέση. The third species of

⁴⁷⁷ *Alia musica* §20(a), ed. Chailley, 111.

⁴⁷⁸ *Alia musica* §20(a), ed. Chailley, 111.

⁴⁷⁹ *Alia musica*, §§15–16, ed. Chailley, 107.

octave limits a third mode, Hypolydian, at that string that they call τρίτη διεzeugμένων. The fourth species of octave renders a fourth mode, Dorian, which ends at the παρανήτη διεzeugμένων. A fifth mode, Phrygian, is ended at the fifth species of octave, to which the νήτη διεzeugμένων is the last string. Just so, the sixth species of octave puts forth the sixth mode, Lydian, to which the τρίτη υπερβολαίων is the end. The seventh species of octave forms also a seventh mode, which the παρανήτη υπερβολαίων limits.

Truly, since one double, which is one octave, flourishing with eight pitches, does not receive more species (since all symphonies [perfect consonances] receive one pitch more than there are species), Ptolemy added an eighth mode, Hypermixolydian, which he forms from the properties of the second and third modes. For there are four strings and three species in a fourth, also five strings and four species in a fifth, wherefore also eight strings and seven species in the octave.

Like Boethius, the *Alia* does not say that the first mode actually *is* the first octave species, but only that it *comes from* the first octave species. In fact, there is nothing in this passage that cannot be understood as a paraphrase of Boethius' description of the modes, usually interpreted to mean the system of keys. Since the *Alia* identifies only the top note of each mode, it need not be the octave species, but potentially a transposition of the entire octave, measured against a prototypical gamut (see Figure 29).

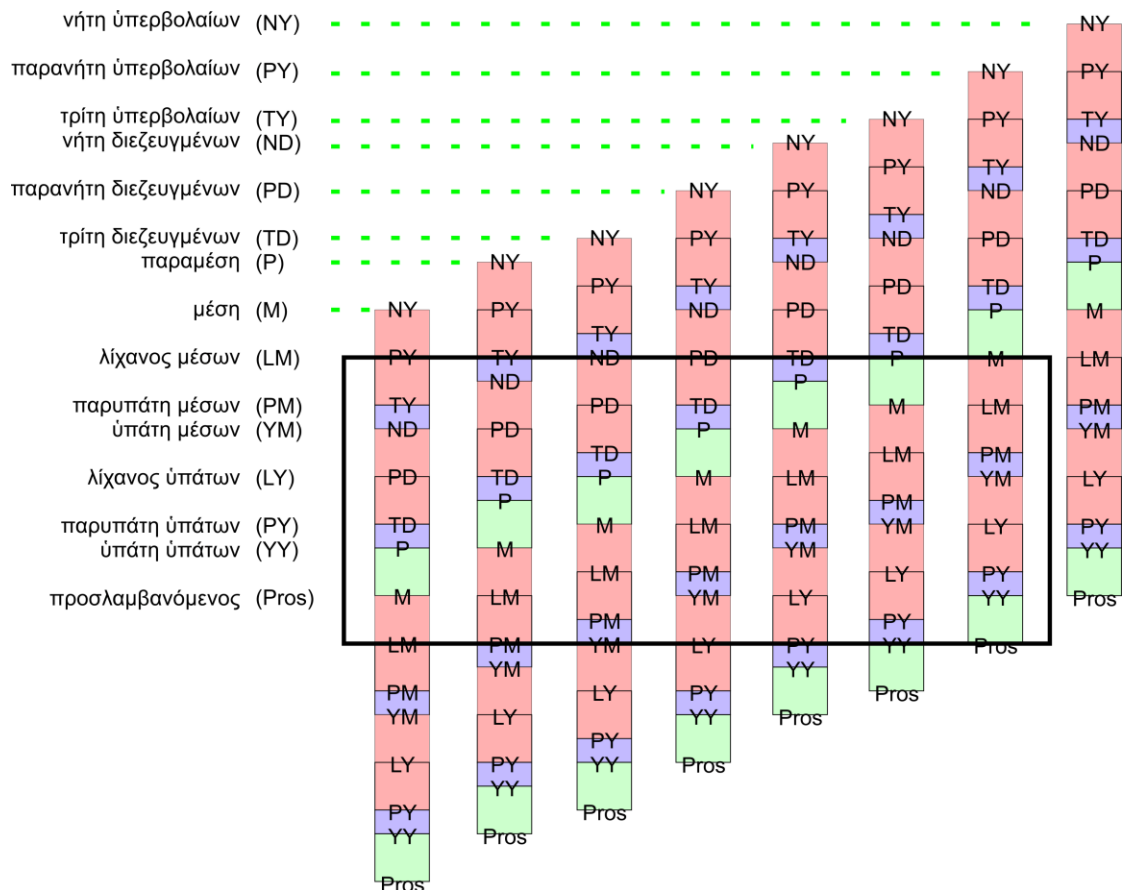


Figure 29 – Interpretation of *Alia musica*'s first description of octave species as a paraphrase of Boethius. The box shows the octave species (as defined in the *Alia* by the position of semitones) corresponding to the transposed gamuts.

As in Boethius, the octave species appear in the same range while the modes transpose around them. It is somewhat surprising that the octave species in this interpretation, as described a few sections later in terms of the placement of semitones (shown in Figure 29 in blue), appear one tone too low to be able to describe them all in the terminology of the Greater Perfect System, which begins from A (though not too low to describe them in the medieval gamut beginning from Γ). Consequently, the eighth octave species falls out of the specified range. However, it is not clear that this description fits much better for the assumption that the *Alia* is here associating the octave species with the ecclesiastical modes, because the position of semitones would require that intervals be counted from top down while the octaves species are counted from bottom up.

Regardless, if the development of the octave-species paradigm of mode was built upon these passages, the development is certainly not linear. The species of fourth presented in the *Alia* are those of the old Greek system (with the semitone at the bottom of the first species), not the newer medieval system (with the semitone in the middle of the first species) hinted at in Hucbald and dominant from the *Enchirides* onwards. The first three species of fifth are defined by adding a tone always to the bottom of a species of fourth, unlike pseudo-Bernelinus, who adds sometimes to the bottom and sometimes to the top, and the *Alia* does not define the fourth species of fifth as in any way related to a species of fourth. Furthermore, the *Alia* does not define either the octave species or the modes as concatenations of species of fourth or fifth; yet, as is evident in Figure 27, though the species of fifth are defined in terms of the species of fourth, they are transposed in such a way that they are already conjunct with each other in the *Alia*, which was not even true for Bern (who put pseudo-Bernelinus' species into the gamut) and was finally established only by the anonymous revisor of Bern's treatise. But the *Alia* definitely does associate the modes directly with the species of octave, which pseudo-Bernelinus and Bern do not (they use only the species of fourth and fifth for this purpose).

Most of the material that follows the description of octave species until relatively near the end of the treatise is dedicated to an idiosyncratic application of the numbers from basic harmonic theory to the modes. The system is extremely complex, and several chapters at the end of this dissertation are dedicated to that topic. Heard's insistence on translating *tropus* throughout these passages as "scallic pattern" gives the reader the faulty impression that the octave species are invoked far more frequently in the revision or commentary than is actually the case. In fact, throughout the explanations of the modal numbers, the phrase *species diapason* appears in only a few places: in §62, in a description of the third mode, where the first note of the fifth species is invoked only as a landmark to help clarify the identity of a pitch (because the author is comparing string names – μέση, etc. – across two different modes, where a single note is given two different names); a general remark in §67 that a species of octave is defined by the position of its semitones; and in §105, explaining how the second species is unusual in having three consecutive tones beneath its semitone, and therefore cannot contain an 8 (Chailley unjustifiably corrects as 9)⁴⁸⁰ in the octave 6:12. In none of these cases does the author ever identify an octave species directly with a mode.

There is also relatively minimal use of the species of fourth and fifth after they are described. There is an interesting exception: in the third portion of the introduction, which Atkinson attributes to the

⁴⁸⁰ Chailley, *Alia musica*, 154. Since the *Alia* defines the second species as B– \flat , 12 is B, 9 is E, 8 would be F \sharp , and 6 is \flat ; it is, therefore, the 8, not the 9, that does not fit into the gamut. Chailley is troubled by the subsequent statement that the synemmenon tetrachord often assists at this position. Since the context of the discussion is the fifth mode, Chailley naturally supposes that the *Alia* is describing the use of \flat in the fifth mode (which is probably true), but in his explanation, he inexplicably shifts from the second octave species on B to an octave species on F (where the offending note would, indeed, be represented by 9). The *Alia* is not directly describing the octave species of F, with a tritone upwards from the bottom, but is instead describing this same tritone more generally within the gamut, extending downwards from the top, by analogy to the octave species in which it appears as the first few notes. It is therefore not at this position *in the mode* but at this position *in the gamut* where the synemmenon arises to solve the tritone problem. Chailley's explanation is a simpler explanation of the same concept, but the *Alia*'s explanation is correct as it stands.

commentator, the intonation formulae for the first two modes are described according to the species of fourth that they outline:

Itaque melodiam primi tropi, quae est NONANOEANE, 8 et 12 videntur claudere ; sed eidem clausulae, quae constat diapente, quidam addunt tonum, ut prius in eadem melodia resonat prima species Diatessaron, deinde secunda, ut postmodum intendatur tertia; ad extremum ex ordine remittitur ab 8 ad 12 prima species diapente. Hinc incipit melodia secundi tropi, quae est NOEAGIS, et non ad 8 sed ad 9 pertingit, sicut omnes cantilenae eiusdem tropi. Clauduntur enim duabus diatessaron consonantiis.⁴⁸¹

And so 8 and 12 are seen to enclose the melody of the first trope, which is NONANOEANE; but to the same *clausula* [phrase?], which consists of a perfect fifth, some add a tone, so that first in that melody resonates the first species of fourth, then the second, as afterwards is extended the third; from this order to the end is returned the first species of fifth from 8 [a] to 12 [D]. Here begins the melody of the second trope, which is NOEAGIS, and it does not reach to 8 [a] but to 9 [G], just as one chants of the same trope. It, indeed, is enclosed by two consonances of a fourth.

Thus, the commentator says that the intonation formula for the first mode runs through the first two species of fourth, the union of which makes the interval structure of the first species of fifth (since they overlap by three of their four pitches), though this is not the place in the gamut where the *Alia* originally described the first species of fifth (compare Figure 27, above). Indeed, the formula descends through the entire first species (a–E), then returns to the top of the second species and descends entirely through that (G–D) to end on the *finalis*. This fact is demonstrated in Figure 30. In addition, he says that some people add one more tone below. Indeed, if one considers the extended form of the intonation formulae (more about this in Chapter 11), the next pitch is C, and adding this note would make the final four pitches the third species of fourth. He goes on to say that the D at the bottom of the first species of fifth, where the first formula ended, is where the second formula begins. It does not reach as high as the a (which rules out the first species of fourth), but only as far as the G, just like all the other chants of the second mode, and it is enclosed by two fourths. Indeed, notwithstanding the neighbour motion with which it begins, the second formulae rises through the entire third species and then descends through the

⁴⁸¹ *Alia musica*, §§26–27, ed. Chailley, 116–17.

second species to arrive at the *finalis*. The commentator leaves unsaid that these two species of fourth would combine to make the second species of fifth.

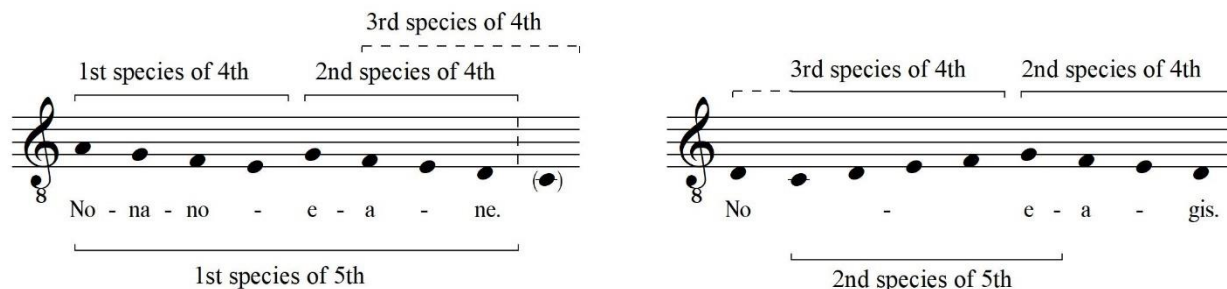


Figure 30 - The commentator's analysis of the first two intonation formulae as species of fourth and fifth⁴⁸²

Atkinson describes this manner of analysis as an innovation anticipating the sophisticated analytical methodologies of later theorists such as Marchetto of Padua and Johannes Tinctoris, marking the commentator as “one of the finest music-theoretical minds of the early Middle Ages.”⁴⁸³

The Mathematical Means and Species Theory Collide

In the passage that I have described as the Disputed Passage, near the end of the treatise, octave species theory returns, and the placement of this passage is intrinsically revealing. It occurs after the discussion of the seventh mode, but immediately before the discussion of the eighth mode, and it is in this passage that “the problem of the eighth mode” is addressed, though in terms unrelated to the number theory presented throughout the majority of the treatise. The passage begins with a section that I have already cited several times that identifies the fifth above the *finalis* as sharing the same name as the mode (and presumably the *finalis* as well).⁴⁸⁴ This note is described as being a pentachord above the *finalis*, and the *Alia* further specifies that the modes then need a tetrachord to complete the octave. It should be

⁴⁸² The melodies for these formulae are drawn from *Commemoratio brevis*, ed. Bailey, 10–11 & 34–35; cf. Bailey, *Intonation Formulas*, 48–49.

⁴⁸³ Atkinson, *Critical Nexus*, 199.

⁴⁸⁴ *Alia musica* §133–34(a), ed. Chailley, 196.

noted that a pentachord and a tetrachord are general concepts and should not be confused with the species of fourth and fifth, which are more specific concepts; the author of the *Alia* may have had the species in mind, but he does not say so. It is also important to note, with respect to the pentachord only, that the author draws attention to the two notes that govern the mode: the upper fifth and the *finalis*, pitches that the *Alia* describes as *metae*, a simple noun referring to a boundary, but it will be convenient to treat the word as a proper noun, a name for these important pitches; in the same way that a *finalis* is literally only the last note of a chant but is also a name for the pitch upon which a chant typically ends, a *meta* may be understood not only as a generic boundary but also as a name for the pitch upon which this boundary is fixed.

The subsequent passage is the first time in the composite treatise that the octave species are indisputably associated with the ecclesiastical modes. Unlike the first description of the octave species in §15, which simply says that the first mode (not specifying what kind of mode that is) comes from the first octave species in some way and that it is called Hypodorian, §135 in the disputed passage explicitly states that the modes, as identified by the Greek ethnic names (Dorian, Phrygian, etc.), are governed by the pitches with the above-mentioned *manerial* names (*protus*, *deuterus*, etc.) in phrasing remarkably similar to that of the *Musica enchiriadis*.⁴⁸⁵

Musica enchiriadis, ch.III

*Etenim primi toni melum et subiugalis suo sono
P archoo regitur et finitur.*

*Secundus tonus cum subiugali suo sono F
deutero regitur et finitur.*

*Tertius eiusque subiugalis sono I trito regitur et
finitur.*

*Quartus cum suo subiugali sono P tetrardo
regitur et finitur.*

Alia musica, §135(a)

Sciendum quoque quod dorius maxime proto regitur,

similiter phrygius deutero,

lydius trito,

mixolydius tetrardo.

⁴⁸⁵ *Musica enchiriadis*, ch. 3, ed. Schmid, 7–8, trans. Erickson, 4; *Alia musica*, §135(a), 197.

<p>And indeed, the melody of the first tone and its plagal are ruled by and finished on <i>archos</i> [a synonym for <i>protus</i>], ♯ [D].</p> <p>The second tone, with its plagal, is ruled by and finished on <i>deuterus</i>, ♯ [E].</p> <p>The third tone and its plagal are ruled by and finished on <i>tritus</i>, I [F].</p> <p>The fourth tone, with its plagal, is ruled by and finished on <i>tetrardus</i>, ♯ [G].</p>	<p>It is to be known also that the Dorian is greatly ruled by <i>protus</i>,</p> <p>similarly the Phrygian by <i>deuterus</i>,</p> <p>the Lydian by <i>tritus</i>,</p> <p>the Mixolydian by <i>tetrardus</i>.</p>
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The *Alia* then addresses the *metae*, saying, “*Quarum uidelicet specierum metas principalium troporum superius et inferius obseruantium prima dorii est, secunda phrygii, tertia lydii, quarta mixolydii.*”⁴⁸⁶ (“One may see that of these species observing the boundaries [*metae*] of the principal tropes, above or below, the first is of the Dorian, the second of the Phrygian, the third of the Lydian, and the fourth of the Mixolydian.”)

The Disputed Passage then proceeds by presenting the octave species again, but this time, only seven of them. In this instance, there can be no doubt the *Alia* is defining the octave species, because the author provides both the bottom and the top of each species (where the first description of the octave species in §15 only defined them according to their top note). (This passage can probably not be used to settle the interpretation of the earlier passage, because Mühlmann, Chailley, and Atkinson, the three scholars who have previously addressed authorship, while disagreeing on the specific authors of these two passages, all agree that the two were not written by the same author). The *Alia* then explicitly states that an octave species can originate from either end, a point that will become important in the subsequent section, where the *Alia* returns to the *metae* (using note names drawn from a diagram of monochord divisions in Boethius – more about these in Chapter 12):

Sunt igitur quatuor superiores, id est o, x, y, cc; et quatuor inferiores, id est e, h, i, m. Et superiores quidem excellentiori parte finiunt, hypodorium, hypophrygium, hypolydium,

⁴⁸⁶ *Alia musica* §137(a), ed. Chailley, 198.

*hypermixolydium. Inferiores uero finiunt ex graviore parte, dorium, phrygium, lydium, mixolydium, unde et finales dictae sunt.*⁴⁸⁷

There are four higher [sc. *metae*], that is *o, x, y, cc* [a, b, c, d], and four lower [sc. *metae*], that is *e, h, i, m* [D, E, F, G]. And the higher [*metae*] end the Hypodorian, Hypophrygian, Hypolydian, and Hypermixolydian at the upper end; the lower [*metae*] end the Dorian, Phrygian, Lydian, and Mixolydian at the lower end, wherefore they are also called *finales*.

There is, then, a set of upper *metae*, which are “upper” not only because they are the upper boundaries of the pentachord, but also because they are the upper boundaries of the plagal modes; likewise, the lower *metae* are the lower boundaries of the pentachord and the authentic modes.

The *Alia* continues that when a melody rises above the octave species assigned to a plagal mode, the melody becomes authentic;⁴⁸⁸ this limit is presumably subject to the exception for an occasional tone above the *meta*, described as an *emmelis*.⁴⁸⁹ It does not say that an authentic mode that extends below the *meta* becomes plagal, and so it differs from later theory, but it is entirely consistent with the descriptions in other coeval treatises, including the *Musica enchiridis* (as described in Chapter 07). It does say, however, that “*hypermixolydio similiter intelligendum est*”⁴⁹⁰ (“It is to be understood likewise for the Hypermixolydian”). I have elsewhere addressed (and rejected) Chailley’s belief that this final sentence was a later gloss. For the present purposes, the relevant observation is that for the first time, the Hypermixolydian is understood to be below the Mixolydian, just as the other plagal modes are below their authentic counterparts.

After reiterating the ranges of the four authentic modes, the *Alia* makes a crucial observation, effectively repeated twice, which constitutes the *Alia*’s answer to “the problem of the eighth mode”:

Et semper unusquisque principalis tropus inferius habet diapente a media chorda, superius diatessaron, ac si 8 sint inter 6 et 12. Subiugalis uero unusquisque tropus a finali chorda superius habet diapente, inferius uero diatessaron, ac si 9 sint inter 6 et 12.

[...]

⁴⁸⁷ *Alia musica* §138, ed. Chailley, 199.

⁴⁸⁸ *Alia musica* §139, ed. Chailley, 200.

⁴⁸⁹ cf. *Alia musica* §134(c), ed. Chailley, 197.

⁴⁹⁰ *Alia musica* §139(d), ed. Chailley, 200.

*Tandem octauus tropus tenet eandem speciem diapason quam et primus ; tamen eo differt quod ille habet m mediam chordam suam qualitatis custodem : hic uero o sub proti nomine.*⁴⁹¹

And always, any particular principal trope [authentic mode] has a fifth below the median string and a fourth above, as if eight between six and twelve. Truly, any particular subordinate trope [plagal mode] has a fifth above its final string, and below, truly, a fourth, as if nine between six and twelve.

[...]

Finally, the eighth trope holds the same species of octave as also does the first; however, it differs from that one in that it has *m* [G] as its median string keeper of quality; the latter, truly, [has] *o* [a] under the name of *protus*.

Thus, each mode, conceptualized as an octave species, is divided at a median pitch [*media*] – these are the *metae* – which are a fourth above the bottom of the plagal modes (where the median pitch is equivalent to the *finalis*) and a fifth above the bottom of authentic modes. Consequently, while the eighth mode seems to share the octave species of the first mode, they are different because the median pitch is different. It is important to note that while in the first excerpt, the term *media* is used only for the authentic modes (where the plagal modes are defined in relation to the *finalis*), the second excerpt describes the eighth mode – a plagal mode – in relation to a *media*; thus, while the *mediae* of the plagal modes are equivalent to the *finales*, they are still *mediae*.

The choice of words in this description is interesting. The phrase that is used to describe the important central pitch is *media chorda*. Heard translates this simply as “middle tone” or “middle note,”⁴⁹² but I feel that “median string” better reflects the implications of the passage because I agree with Atkinson that the *mediae* are a reference to the arithmetic and harmonic means.⁴⁹³

The term used to describe the arithmetic and harmonic means in the introduction to the treatise is *medietas*;⁴⁹⁴ the word *media* is an adjective meaning middle, and *medietas* is a noun derived from *media* using the generative suffix *-tas*, a relationship similar to that in English between the words “central” and

⁴⁹¹ *Alia musica* §§140(c)–(e) & 143, ed. Chailley, 200–02.

⁴⁹² Heard, *Alia musica*, 196.

⁴⁹³ Atkinson, *Critical Nexus*, 196.

⁴⁹⁴ *Alia musica* §12(a)–(b), ed. Chailley, 104.

“centrality,” the connection between which is immediately apparent. Alone, this etymological argument is a bit weak, and one would be perfectly justified in asking why, if the author wished to refer to the arithmetic and harmonic means, he would not simply have written *medietas* instead of *media chorda*. Nevertheless, the hypothesis is reinforced by the explicit analogy of the median string of the authentic modes to the eight that falls between six and twelve in the relation 6:8:12, the very relation used to demonstrate the harmonic mean in the introduction to the *Alia*, and likewise by the analogy of the median string of the plagal modes to the nine that falls between six and twelve, which is the relation used to demonstrate the arithmetic mean.

In this instance, it may not even be entirely relevant whether the author intended to describe these *mediae chordae* as the arithmetic and harmonic means; what matters is that the similar wording and the analogies to the familiar number sequences are sufficient to invoke the arithmetic and harmonic means to the reader, regardless of whether the author intended that interpretation. This principle also reappears in the modal theories of Renaissance theorists.⁴⁹⁵

And ultimately, whether the reference to the means was intended or not, it brings additional value to the description of the means in the introduction to the treatise. The *Alia* thus provides superior justification for the discussion of the means compared to other treatises, and it does so twice over: they are not merely used to demonstrate the intervals within them, as in other treatises, but are used to generate the complete relation 6:8:9:12 by the superposition of the two means within the same boundaries, and they are also used to explain the difference between authentic and plagal modes (and, by extension, to solve the problem of the eighth mode).

⁴⁹⁵ Atkinson, *Critical nexus*, 199.

Metae and Mediae

There is a fascinating relationship between the *metae* [boundaries] and the *mediae* [medians]. The *metae* are endpoints, first and foremost for the pentachord that is shared between an authentic mode and its plagal, but secondarily for the modes themselves. Since, as the *Alia* points out, the octave species can be constructed either from the bottom up or from the top down, the species for the plagal modes are constructed from the top down, beginning at the upper *metae*, while the authentic modes are constructed from the bottom up, beginning at the lower *metae* (which are also the *finales*). The *mediae* are the middle notes governing each mode, such that the *mediae* of the authentic modes are the fifth above the *finales*, while the *mediae* of the plagal modes are equivalent to the *finales* themselves. Thus, the *metae* and *mediae* are the same two sets of four notes, but they swap sets between plagal and authentic modes: the *metae* of plagal modes are the *mediae* of authentic modes and vice versa (Figure 31).

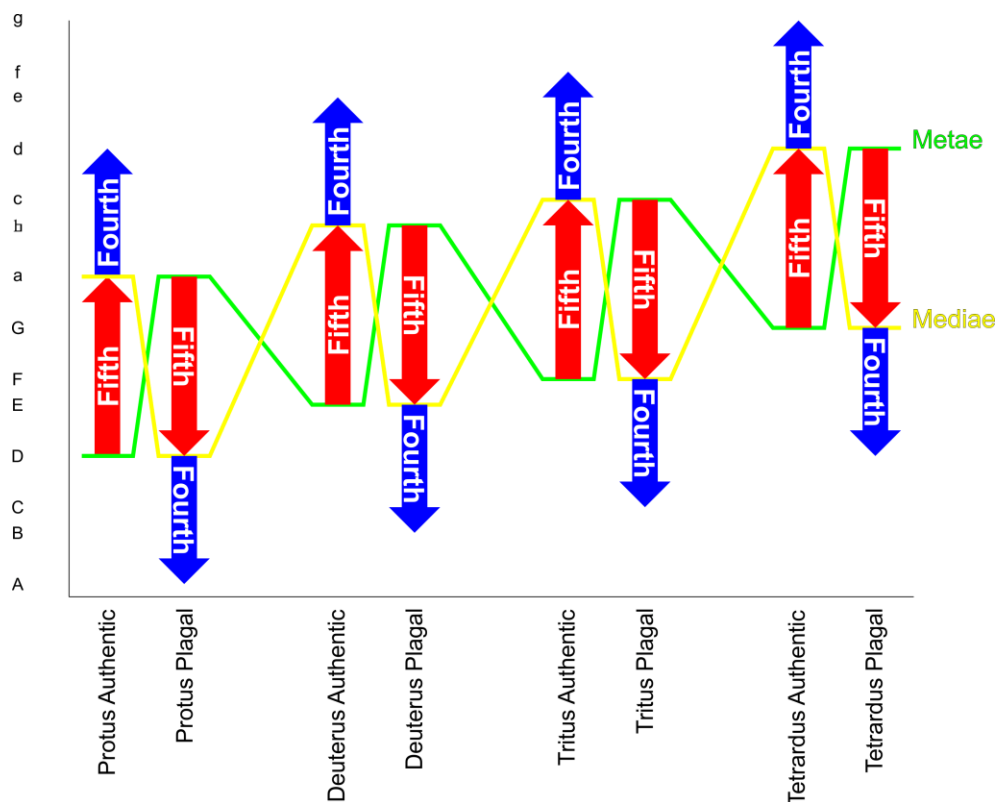


Figure 31 – The relationship between the *metae* and the *mediae* in plagal and authentic modes.

Authentic modes ascend from their *metae*, while plagal modes descend from their *metae*; the *mediae* are the middle pitches of each mode and are the same pitches as the *metae* of the counterpart modes.

The discussion above presents essentially the entirety of the conventional theory in the *Alia* (and little bit of what is novel but closely connected to the conventional). The only other theoretical concept of note is a very brief discussion of the τρίτη συνεμμένων (*i.e.*, b) which the *Alia* notes exists between a and h but is not permitted to be sung directly after h; it must be sung after a or c. The remainder of the theory presented in the *Alia* forms part of the complex numerology of the treatise, which will occupy the last few chapters of this dissertation.

Chapter 10: The *Alia musica* and Philosophy

After the discussion of all eight modes, the *Alia* takes a short digression into philosophical analogies between music and nature. This chapter will briefly summarize and comment upon these analogies, and where possible, will identify the classical sources from which these analogies are drawn.

For the first analogy, the author of the *Alia* directly identifies the source, Vitruvius' *De architectura*. The *Alia* says the philosophers identify no more than eight winds, four principal and four subordinate, an analogy to the four principal [authentic] modes and the four subordinate [plagal] modes. The reference appears to be to Book I of *De architectura*, Chapter 6, §§4–5, where Vitruvius says that some identify only four winds, associated with the four cardinal compass points, while others acknowledge four more associated with the ordinal compass points.⁴⁹⁶

The *Alia* then says that four are added to eight to give twelve, just as four semitones are added to eight tones. This passage cannot be interpreted with confidence. It no longer appears to be a reference to Vitruvius or the winds, unless as a misunderstanding. Vitruvius says that some identify four winds (and he names them), and then that others identify eight winds; he then makes some additional comments and then proceeds to name the winds that he had not already named above; it is possible that the author of this section of the *Alia* mistook these four new names for yet another set of four winds. These four winds are half-directions, in the sense that they appear midway between the cardinal directions, and might make a suitable analogy to a semitone. As for the music, though, it is not clear in what manner four semitones are added to eight tones. Chailley tentatively proposes that they could represent the semitones created by \flat added to each of the modes (*tonoi*).⁴⁹⁷ Another possibility is that these semitones might be references to four additional modes described by Aurelianus, who attributes them to

⁴⁹⁶ Vitruvius, *De architectura*, ed. Krohn, 21.

⁴⁹⁷ Chailley, *Alia musica*, 96, (b).

Charlemagne.⁴⁹⁸ It is not clear what these four modes in Aurelianus are meant to be, but they could perhaps be the *parapteres*, or perhaps analogues to “middle modes” sometimes described in Byzantine theory; either might perhaps be described as a semi-mode, and thus, a “semitone.” If, however, the semitone must be an interval (which seems more likely), it might also refer to the four semitones added to the eight normal tones found within the four tetrachords of the Greater Perfect System (thereby excluding the two disjunct tones), a solution favoured by a later author of the *Alia* (discussed below). Ultimately, none of these explanations is particularly convincing.

The next analogy is to the waves of the seas and rivers, the first of which is always greater than the following seven. In actual fact, the periodicity of waves in a sea is a consequence of interference patterns as waves are reflected and refracted according to the perimeter of the sea and the contours of the sea floor (to say nothing of the impact of wind conditions); as a result, the periodicity is not consistent and varies from place to place. Nevertheless, the *Alia* compares the repetition of the wave cycle every eighth wave to the return of an equivalent pitch every octave; the author does not cite a source for the periodicity of the waves, but the concept is well known in folklore, and a similar statement is made in Ovid’s *Metamorphoses*, Book XI, ln. 530 (though in this instance, it is the tenth wave that is largest, and the periodicity is only implied, not directly stated).

The *Alia* follows this analogy with a related analogy to periodicity in thunderclaps, which apparently inspired Pythagoras to discover the consonances of the eight modes of the harmony of the spheres while visiting Mt. Atlas (which is near to the heavens). I have been unable to find any other reference to this story. The *Alia* then compares these consonances themselves, which are the fourth, fifth, octave, twelfth (but not the eleventh), and double octave, to the five zones of the heavens in Pythagorean thought; this

⁴⁹⁸ Aurelianus, *Musica disciplina*, ch. 8, ed. Gushee, 82, trans. Ponte, 24.

element of Pythagorean doctrine is well attested and is attributed to Thales and Pythagoras in pseudo-Plutarch, *Placita philosophorum*, Book II, Chapter 12.⁴⁹⁹

All the preceding analogies appear in §§154–56, which both Chailley and Atkinson attribute to the source treatise. These passages do not, however, appear at the end of the composite treatise with the rest of the source treatise, nor do they appear in source K, which contains only the source treatise (though, in fairness, K ends just before the end of the source treatise, and it may be that these passages ought to have appeared in K and were lost along with the end of the description of the eighth mode). Chailley and Atkinson attribute the subsequent passages to the revision, and indeed, they do appear to be commentary upon these analogies. Regardless of the precise attribution of these passages, there can be no question that a different author writes them because he passes judgment on the analogies that the previous author presented.

The author of this new section, be it the revisor or the commentator, first notes that it was entirely appropriate to analogize tones to the winds since musical sounds are produced either by means of a plectrum or, more relevantly, by the breath (which is comparable to the wind), though he says nothing about the fourfold and eightfold divisions of either the modes or the winds.

This author also attempts to interpret the previous author's comments about the semitones by noting that there are four tetrachords in the Greater Perfect System (which excludes the *συνεμμένων*) and that there is one semitone in each, a solution that I proposed above (though without confidence); there is no explanation here, however, of the manner in which these four semitones would constitute four added to eight. Eight of what? There are not eight tetrachords, and this author says nothing about eight tones.

⁴⁹⁹ Pseudo-Plutarch, *Περὶ τῶν ἀρεσκοντῶν*, ed. Bernardakis, 303.

The author then comments upon the previous analogy to the waves, for which he also does not provide a citation, but simply acknowledges that previous philosophers have addressed the issue. He then adds another analogy, this time to the repetition of years, months, and days in cycles of seven, noting that years and days return to their starting condition on the eighth year or day, which illustrates the excellence of the number eight, into which the heavenly spheres also are divided in the equinoctial cycle, four parts of which are superior and the other four inferior, a clear allusion to the four authentic modes and the four plagal modes. The author cites his discussion of years and days to Hyginus' *Astrologia*; the reference appears to be Hyginus' *Astronomia*, Book IV, Chapter 2, whence it is possible to clarify that when the author says that years return to their original condition, he means with reference to the movements of the stars.⁵⁰⁰

Here, the author remarks that there is an apparent contradiction, where the periodicity of the cycle in these analogies is sometimes said to be seven and sometimes eight (*i.e.*, repeating on the eighth position or on the ninth position). The contradiction disappears, however, when one accounts for the distinction between the heptachord and the octachord. The author does not explain what is meant by the heptachord and octachord. One viable explanation would be that the heptachord represents all the notes of a single octave (of course, without its final pitch, because the final pitch begins a new octave) without accounting for the two possible conditions of \flat and \flat , which therefore repeats at the eighth note, while the octachord represents the same collection of pitches but including both \flat and \flat , which therefore repeats at the ninth note.

The author then returns to his critique, picking back up at the analogy to thunder, of which he approves, and about which he says nothing more, directing the reader to his comments about the waves; like the first author, he does not provide a citation for the connection to Pythagoras. He then also

⁵⁰⁰ *Alia musica* §158, ed. Chailley, 166; Hyginus, *Astronomica*, ed. Bunte, 99–101.

underlines the importance of the analogy of the five consonances to the five zones of the heavens but declines to comment further. Instead, the author effects a smooth transition into a discussion of the five consonances and thereby returns to the application of numerology to the modes.

There are, in summary, five principal analogies presented in these sections, of which four are presented by the first author and the fifth presented by a second author, commenting upon the first:

- Eight winds, four principal and four subordinate, analogous to the authentic and plagal modes.
 - References Vitruvius, *De architectura*, Book I, Chapter 6.
 - Particularly apt because musical sounds are produced by the breath.
 - Reference to four semitones added to eight tones is not clear; possible reference to one semitone in each of four tetrachords (which each have two tones)
- Waves travel in cycles of eight so that the ninth is the same as the first, analogous to repetition of pitches at the octave.
 - No direct citation, but well established in folklore.
 - *cf.* Ovid, *Metamorphoses*, Book XI, but not a perfect comparison.
- Thunder repeats in cycles of eight, another analogy to the octave.
 - Connected to a story about Pythagoras; no apparent source.
- Five zones of the heavens are analogous to the five perfect consonances.
 - The zones also attributed to Pythagoras.
 - No direct citation, but *cf.* pseudo-Plutarch, *Platcita philosophorum*, Book II, Chapter 12.
- Years and days repeat in cycles of seven, another analogy to the octave.
 - Reference to Hyginus, *Astronomia*, Book IV, Chapter 2.

Chapter 11: The Tonaries of the *Alia musica*

A tonary is a highly practical book that, unlike most theory treatises, provides little in the way of actual theory, but instead functions as a quick reference, especially for the practice of antiphonal psalmody. Within a tonary, a list of chants is sorted among the eight modes, especially antiphons, as well as other genres of chants that are combined with psalm recitations, such as introits (though some tonaries contain chants that would not need to be paired with psalms; for instance, the St. Riquier tonary contains alleluias). Most tonaries then subcategorize the chants according to the *differentiae* and *loca*; a singer needing to perform a psalm and antiphon could then look up the antiphon in the tonary to confirm the correct *differentia* (cadence pattern) to sing at the end of the psalm recitation in order to create the desired smooth transition between psalm and antiphon.

The principal characteristic of the third of Chailley's major divisions of the *Alia musica*, the second *Quidam* (a.k.a. *Nova expositio*), is a tonary. In fact, in view of the other manuscripts containing this tonary and lacking the other passages that Chailley attributes to this author (the Disputed Passage), there seems to be little reason to believe that this author contributed anything but the tonary.

The *Nova expositio* tonary, however, is not the only tonary in the *Alia musica*. Michel Huglo, in his study of tonaries, says that "*Il faut remarquer que les trois auteurs avaient sûrement sous les yeux un tonaire.*"⁵⁰¹ ("It is necessary to observe that the three authors surely had a tonary in front of them.") The source treatise contains some elements of a tonary (though it lacks discussion of the *differentiae* and *loca*) and the revision expands upon these elements. Additionally, manuscript M contains an entire additional tonary in the margins (which likewise lacks *differentiae* and *loca*). This chapter will examine the tonaries of the *Alia musica*.

⁵⁰¹ Huglo, *Les tonaires*, 58.

Of the three tonaries presented in the treatise, the least significant is the supplemental tonary in the margins of manuscript M. This tonary is not present in any other manuscript and thus is not really a part of the *Alia musica* beyond the coincidence of appearing on the same folia in one manuscript. Furthermore, this tonary is already edited in a contiguous block in Chailley;⁵⁰² there is, therefore, no value in reproducing this tonary here.

The Tonaries of the Source Treatise and Revision

To a certain extent, the source treatise itself could be considered to be a tonary. Each section begins with an identification of the mode not entirely different from the kind of descriptions provided in typical tonaries but including the numbers that describe the characteristic intervals of the mode (this very complex issue will be the principal topic of Chapters 13–19). After some number manipulation that is not typical of a tonary, each section concludes with a short list of chants that exemplify each mode. For the first six modes, two to three introits are provided, followed by two to four antiphons; the source treatise appears to be incomplete at the end, and no examples are provided for Modes VII or VIII.

One characteristic that differentiates the source treatise from a typical tonary is that the organizing principle for chants within a mode is not *differentiae* or *loca*, it is the characteristic intervals of the mode. For example, in the first mode, the author explains that the chants of this mode may be characterized by the octave, fifth, and fourth, and that many chants will have all three, but that some will have only the fifth or only the fourth, and he selects one antiphon to exemplify each situation.

The revision retains the examples from the source treatise to a considerable degree. Of the thirty-five chants cited in the source treatise, all but three reappear in the revision, and it is not difficult to understand why the other three were omitted (more on this shortly). In addition, the revisor and

⁵⁰² Chailley, *Alia musica*, 205–209.

especially the commentator add several more examples to each mode except the second, bringing the total to sixty-four chants. A half-dozen of these are additional introits and antiphons, but most are from other genres, including graduals, offertories, alleluias, and responsories, not all of which would strictly require modal assignment, since they are not paired with psalms.

This fact is of no small import. It is generally supposed that the original purpose of the modal system was to facilitate the selection of an appropriate recitation formula for a psalm to agree well with the characteristics of the antiphon that accompanies it. However, the classification of non-antiphonal chants into modes implies a much broader conception of modes. It suggests that for the commentator, at least, the concept of mode was much more pervasive. The idea is not without other evidence in other treatises. For example, Guido observes that *“Horum quidam troporum exercitati ita proprietates et discretas ut ita dicam, facies extemplo ut audierint, recognoscunt.”*⁵⁰³ (“Indeed, those well versed in the properties and, as I should say, the separate aspects of these tropes recognize them as soon as they hear them.”)

In fact, this observation is not substantially more than what is implied by the basic observation that the objective is to match the characteristics of the antiphon and the psalm recitation: that the chants manifest sufficiently distinct characteristics derived from their modes that to fail to match the mode of the antiphon with the corresponding psalm tone would create an undesirable effect. That being the case, it would be surprising if musicians did not recognize these characteristics when they appeared in other kinds of chants.

However, the modal classification of non-antiphonal chants implies at least two steps beyond this general observation. In the first place, it suggests that characteristics encoded by the modes are not limited to describing some small subset of the musical materials employed in Gregorian chant – that is, there is not something special about antiphons and psalm recitations that causes them to manifest modal

⁵⁰³ Guido, *Micrologus*, ch. 14, ed. Smits van Waesberghe, 158, trans. Babb, 69.

characteristics while other genres don't (such an hypothesis does not appear to be widespread in any case, but it is valuable to have this issue confirmed). In the second place, since the purpose of the relevant passages in the *Alia musica* is presumably to identify particularly good examples of the modal characteristics that the treatise is describing, it suggests that at least in the mind of the commentator, the specific modal characteristics described in the *Alia* are of precisely the kind that do arise in other types of chants. That is, the commentator considered the properties that much of the rest of this dissertation will be dedicated to explaining to be properties of chants in general, and not merely properties of antiphons.

The tonary of the source treatise and revision (including the commentary) may be summarized as follows:

Protus Authentic

Source Treatise:	Revision:
<p><u>Introits:</u> <i>Rorate caeli desuper</i> <i>Gaudete in Domino semper</i> <i>Justus es Domine</i></p>	<p><u>Introits:</u> <i>Rorate caeli desuper</i> <i>Gaudete in Domino semper</i> <i>Justus es Domine</i> <i>Inclina Domine</i></p>
<p><u>Antiphons:</u> <i>Traditor autem dedit eis</i> <i>Urbs fortitudinis nostrae Sion</i> <i>Ioannes autem cum audisset</i></p>	<p><u>Antiphons:</u> <i>Traditor autem dedit eis</i> — <i>*Iesus autem cum ieiunasset</i> <i>Posuerunt super caput</i></p>
	<p><u>Alleluias:</u> <i>Fulgebunt iusti</i> <i>Tanquam scintillae</i> <i>Arundineti discurrent in aeternum</i></p>
	<p><u>Graduals:</u> <i>Posuisti Domine</i> <i>Universi qui te expectant</i></p>
	<p><u>Responsories:</u> <i>Circumdederunt me</i></p>

Two of the three chants that are omitted from the revision are in this mode. The first is *Urbs fortitudinis*, which is notoriously difficult to assign to a mode, as it seems to begin in Mode I and end in Mode VII; this difficulty is probably the reason that the revisor chose not to include it in his revision. The second is *Ioannes autem cum audisset*, which has been replaced by *Iesus autem cum ieiunasset*; Chailley suspects that the change results from a misinterpretation of an abbreviation, as the names of the chants are quite similar (it is interesting that both forms appear in their appropriate places in all the complete manuscripts, but consistency is not a characteristic of the *Alia musica*).

The last two chants added, *Universi* and *Circumdederunt*, are unique, in that they are not described in the section on the first mode. Instead, they are described in the section on the second mode as examples of chants that appear to be in the second mode but are actually first mode chants because they ultimately rise beyond the range of the plagal mode (in both cases, the upper range occurs in the verses that follow the antiphon or respond). The responsory *Circumdederunt me* should not be confused with the introit of the same name cited for Mode V.

Protus Plagal

Source Treatise:	Revision:
<p>Introits: <i>Ecce aduenit</i> <i>Ueni et ostende nobis faciem tuam</i></p>	<p>Introits: <i>Ecce aduenit</i> <i>Ueni et ostende nobis faciem tuam</i></p>
<p>Antiphons: <i>Omnipotens sermo tuus Domine</i> <i>Dominus Deus auxiliator meus</i></p>	<p>Antiphons: <i>Omnipotens sermo tuus Domine</i> <i>Dominus Deus auxiliator meus</i></p>

Deuterus Authentic

Source Treatise:	Revision:
<p>Introits: <i>Confessio et pulchritudo</i> <i>Dispersit dedit pauperibus</i> <i>Cognovi Domine</i></p>	<p>Introits: <i>Confessio et pulchritudo</i> <i>Dispersit dedit pauperibus</i> <i>Cognovi Domine</i> <i>Dum clarmarem ad Dominum</i></p>

Source Treatise:

Revision:

<p><u>Antiphons:</u> <i>Qui de terra est</i> <i>Quando natus es</i> <i>Malos male perdet</i> <i>Homo quidam fecit cenam magnam</i></p>	<p><u>Antiphons:</u> <i>Qui de terra est</i> <i>Quando natus es</i> <i>Malos male perdet</i> — <i>Peccaui super numerum</i></p>
	<p><u>Alleluias:</u> <i>Fulgebunt iusti</i> <i>Tanquam scintillae</i> <i>Arundineto discurrunt in aeternum</i></p>
	<p><u>Graduals:</u> <i>Posuist Domine</i> <i>Uniuersi qui te expectant</i></p>

The final chant omitted from the revision appears in this mode. The chant is the antiphon *Homo quidam fecit cenam magnam*, which also appears under the name *Quidam homo fecit*, and which shares its name with a responsory; it is probably for this reason that this chant has been omitted from the revision, to avoid ambiguity.

Deuterus Plagal

Source Treatise:

Revision:

<p><u>Intros:</u> <i>Resurrexi</i> <i>Misericordia Domini</i> <i>In uoluntate tua Domine</i></p>	<p><u>Intros:</u> <i>Resurrexi</i> <i>Misericordia Domini</i> <i>In uoluntate tua Domine</i></p>
<p><u>Antiphons:</u> <i>Rubum quem uiderat Moyses</i> <i>Turba multa</i> <i>Tria sunt munera</i></p>	<p><u>Antiphons:</u> <i>Rubum quem uiderat Moyses</i> <i>Turba multa</i> <i>Tria sunt munera</i></p>
	<p><u>Alleluias:</u> <i>Pascha nostrum</i></p>
	<p><u>Graduals:</u> <i>Haec dies</i></p>

Source Treatise:

Revision:

Responsories:*Rex noster**Tritus Authentic*

Source Treatise:

Revision:

Introits:*Domine refugium
Circumdederunt me
Domine in tua misericordia***Introits:***Domine refugium
Circumdederunt me
Domine in tua misericordia***Antiphons:***Soluite templum hoc
Salve crux
Exultet spiritus meus***Antiphons:***Soluite templum hoc
Salve crux
Exultet spiritus meus***Alleluias:***Beatus uir***Graduals:***Immitti angelus***Responsories:***Obsecro Domine**Tritus Plagal*

Source Treatise:

Revision:

Introits:*Os justii
Omnes gentes
Quasimodo geniti***Introits:***Os justii
Omnes gentes
Quasimodo geniti***Antiphons:***O admirabile commercium
Uade Satana
Virgo hodie fidelis***Antiphons:***O admirabile commercium
Uade Satana
Virgo hodie fidelis
O quam gloriosum est***Offertories:***Stetit angelus***Responsories:***Aspiciebam
Esto nobis*

Tetrardus Authentic

Revision:

Intros:

*Puer natus est nobis*⁵⁰⁴

Audiuit Dominus

Responsories:

Iste est frater uester minimus

Dixit ludas fratribus suis

Ecce dies ueniunt

Nascetur nobis

The source treatise does not provide examples for the *tetrardus* modes. The revisor adds examples; the first two are identified as *gradales antiphonae*⁵⁰⁵ – antiphons of the gradual (that is, of the Mass), which is another way of saying intros. The last two responsories are not announced in a list with the first two but are added in the subsequent subsection, almost as a kind of commentary. Chailley attempts to “reconstruct” the examples in the source treatise (which are missing from all manuscripts, including source K, which contains only the source treatise),⁵⁰⁶ perhaps from the assumption that the examples as given in the revision were copied from the source treatise and then somehow lost from the source treatise but retained in the revision. This assumption is probably not correct, as the examples in this section of the revision lack any office antiphons, thus breaking the pattern employed throughout the source treatise of citing approximately equal numbers of intros and antiphons (and nothing else). Chailley, however, follows the pattern precisely, copying the intros and the first two responsories but labelling the responsories as antiphons.

⁵⁰⁴ This chant is incorrectly cited as *Puer natus est uobis* in Chailley’s “reconstruction” of the source treatise (Chailley, *Alia musica*, 94 §187(f)), but the title is correct in the revision (Chailley, *Alia musica*, 164 §131(a)).

⁵⁰⁵ *Alia musica* §131(a), ed. Chailley, 164.

⁵⁰⁶ *Alia musica*, §187(f), ed. Chailley, 94, trans. Nowacki, 126; cf. Chailley, *Alia musica*, 94, critical apparatus to §187(f): “*totum deest in omnibus codd.; restitutum ut in §131.*” (“The entirety is absent in all manuscripts; we reconstruct as in §131.”)

Nowacki repeats Chailley's error,⁵⁰⁷ apparently not noticing the note in Chailley's critical apparatus that clearly indicates that the passages are not present in any extant manuscript but are reconstructed from the corresponding passages in the central part of the treatise;⁵⁰⁸ had he followed Chailley's cross-reference, he would have discovered that the chants are correctly labelled as responsories in the corresponding passages.⁵⁰⁹ Mühlmann, by contrast, did not attempt to reconstruct a list of chants for this mode, but instead assumes that the source treatise ended after Mode VI and that Modes VII and VIII were added by another author.⁵¹⁰

Tetrardus Plagal

Because the source treatise is incomplete, there are no chants listed for either of the *tetrardus* modes; unlike the authentic mode, however, the revisor does not suggest examples for the plagal mode.

The Nova expositio

All evidence suggests that the *Nova expositio* is entirely unrelated to the rest of the *Alia musica*, and that characteristic carries over into the selections of chants for each mode. In general, for each mode, there is exactly one chant in common between the source treatise and the *Nova expositio*; the exceptions are obviously the *tetrardus* modes, for which there are no examples in the source treatise, and also Mode IV, for which there are two chants in common (the shared chants are usually introits, except in Mode III and the additional common chant in Mode IV). Only four of the chants added in the revision are also cited in the *Nova expositio*, and one of them, *Ecce dies ueniunt*, is listed in a different mode (it is *tetrardus* in both, but the revision assigns it to authentic, while the *Nova expositio* assigns it to the plagal).

⁵⁰⁷ Nowacki, *Greek and Latin Music Theory*, 126; Chailley, *Alia musica*, 94 §187(f).

⁵⁰⁸ See fn. 506 above.

⁵⁰⁹ Chailley, *Alia musica*, 164 §131(b).

⁵¹⁰ Mühlmann, *Die Alia musica*, 60.

The focus of the *Nova expositio* is different from that of the source treatise. Like most tonaries, the *Nova expositio* is focussed on the issue of *differentiae* and *loca*. For each mode, the *Nova expositio* begins with a description of the ἤχημα for the mode, then presents a list of *differentiae*; for each *differentia*, it then presents a list of *loca*, and for each *locum*, a chant. Unlike the better-known tonaries, such as Regino's tonary, there is no attempt to classify all the chants that the singer might expect to need to sing. Instead, only a single example is provided for each *locum*, evidently to stand as a model (Huglo refers to this kind of tonary as educational or didactic, as opposed to practical⁵¹¹). As I have already noted in an earlier chapter, only a single starting note is identified for each *locum*, but there are frequently multiple *loca* in a single mode beginning on the same pitch but belonging to different *differentiae*. This characteristic implies that the reader was already meant to be familiar with the antiphons cited, and thus, to be able to recognize the rest of the *locum*, not only in the cited chant but also in other chants that begin the same way, thereby being able to determine the correct *differentia* for any chant.

There is one more way in which the *Nova expositio* appears to differ from the tonaries of the rest of the *Alia* (though the appearance is misleading): for each mode, the *differentiae* and *loca* are presented in two different sets, one for chants used "*in nocturnis*" and one for chants used "*in diurnis*." These labels do not seem to mean what the words literally imply, which is chants for the nighttime and the daytime.

Liturgical practice could suggest a reasonable interpretation for these labels. Most psalm singing takes place within the Hours of the Divine Office. There are eight Hours: four greater Hours and four lesser Hours (it is interesting that in the passage of the *Alia* devoted to analogies, these Hours are not compared to the authentic and plagal modes, as similar analogies are made). The lesser Hours are simply named for the hour of the day when they are observed: prime, terce, sext, and nones – that is, one o'clock, three o'clock, six o'clock, and nine o'clock according to the old manner of time keeping (where the twelve

⁵¹¹ Huglo, *Les tonaires*, 29.

hours of the day count upwards from dawn and are divided evenly until dusk at twelve o'clock), which is approximately six o'clock, nine o'clock, twelve o'clock, and three o'clock in modern timekeeping. The four greater Hours are Lauds (dawn), Vespers (dusk), Compline (bedtime), and Matins. This last Hour, Matins, literally means "morning," but is celebrated very early in the morning, several hours before dawn. Consequently, Matins is frequently called the nighttime Office, while the other seven Hours are considered the daytime Office. Matins was considered to be the most important Hour of the Office, and the observance is more elaborate than the other Hours, so one might very well suppose that a special set of *differentiae* could have been used.

Unfortunately, a consideration of the chants listed as nocturnal chants does not support this interpretation. While the majority of the chants cited for modes V through VII are Matins chants, they are responsories, a characteristic that is very curious since responsories are a different kind of recitation from antiphonal psalmody and do not use *differentiae*. It is true that many texts were applied to both antiphons and responds, and the chants cited for these modes could, therefore, be either, but a search on the CANTUS database fails to find antiphons amongst these chants with such consistency that this hypothesis is difficult to credit; even so, it is difficult to understand how these *differentiae* should be understood, since every responsory verse in a given mode always ends in the same way. However, despite the consistency in these three modes, the "nocturnal" chants listed in other modes are mostly not responsories.⁵¹² This problem appears to have been acknowledged by the unknown author who revised

⁵¹² Apel notes that "attempts in the direction toward different endings can be traced in some of the earliest *Antiphonaries*, e.g., in the twelfth-century *Antiphonal of St. Maur-des-Fossées* (Paris, B. N. 12044)" (Apel, *Gregorian Chant*, 239). It is possible that the usage in the *Nova expositio* may also fit into this category, in which case it would be older than Apel's example by perhaps two centuries. However, such an interpretation would still not explain why the responsories in the *Nova expositio* seem to be concentrated in three modes or why the *differentiae* of the antiphons and responsories would be intermixed in the *Nova expositio's* presentation (since the character of the antiphons and responds differed, one would not expect them to share the same *differentiae*). A more extensive treatment of *differentiae* in one particular antiphoner may be found in Davis, *The Gottschalk Antiphony*, 73–77. The endings of responsory verses could also vary in ways not consistent with *differentiae*; a detailed discussion of a specific instance of this practice may be found in Grier, *The Musical World*, 136–54.

the *Nova expositio* to form the alternate recension found in source R. While the chants cited for Mode I in R include essentially all the chants presented in the standard recension, with a good number of supplementary examples added but only two (*Sint lumbi uestri* and *Statuit*) removed, almost every single chant from Mode V onward has been changed, and most of those included in R are antiphons (though a few responsories remain).

The chants for the rest of the modes are predominantly antiphons (with a few exceptions), but almost none are for Matins; they are for the other greater Hours (Lauds is particularly well represented).

Setting aside the puzzling issue of the responsories, one might suspect that the division was based on a looser definition of daytime and nighttime, with all four greater Hours considered to be nighttime, a definition that would not be unreasonable, since compline is observed after dark and lauds and vespers are observed at the transitions between day and night. However, the chants in the lists for the daytime are not chants for the lesser hours. Instead, they are all antiphons for the Mass. The majority are introits, with only a couple of communions and a tract.

Ultimately then, while it is not clear why the terms *nocturnus* and *diurnus* should be used for these categories, the actual distinction between the two lists for each mode is essentially the same as the two types of chants given in the source treatise, though in reverse order: the first list (*nocturnus*) is for the antiphons of the Office and the second list (*diurnus*) is for the antiphons of the Mass.

Below, I present a summary of the chants identified in each mode. To be of greatest use, it would be ideal to provide the notation for the recitation formulae, the *differentiae*, and the first several notes of each *locum*. Unfortunately, there are several challenges that would be encountered in the attempt to do so. In the first place, the *Nova expositio* does not list the reciting tones for these modes. There is a real question about the reciting tones in this period: ordinarily, the reciting tone should be the fifth above the *finalis* in authentic modes and the third above the *finalis* for plagal modes; however, the reciting tones for

modes III, IV, and VIII are all one step higher than this description would suggest, which is generally interpreted as reflecting an avoidance of h .⁵¹³ However, the best source for the psalm tones during this period is the *Commemoratio brevis*, and in that treatise, the reciting tone for Mode III is on h . There is no way to know whether this would be true for the *Nova expositio*. The *Commemoratio brevis* also contains two sets of psalms tones, and one would have to select the correct set, all of which ignores the larger problem that many of the chants cited appear to be responsories, which have their own, much more complex recitation formulae.





















Then, after selecting the recitation formula, one would have to reconstruct the *differentiae* themselves. Although the *Nova expositio* provides the final note for each *differentia*, the *differentiae* are cadence formulae spanning the last six syllables of the verses or the doxology. However, even the number of *differentiae* for each mode is not consistent from treatise to treatise, and so it is not at all clear that the melodies for the *differentiae* should be consistent, even for those *differentiae* whose final pitch is consistent with a *differentia* in the *Commemoratio*. And again, it is definitely not clear how to interpret the *differentiae* for the responsories in Modes V through VII.

It is, therefore, not practical at present to attempt a reconstruction of the psalmodic practice presented in the *Nova expositio*. Instead, I present below only a summary of the chants cited, along with the final pitch of the *differentia* and the first pitch of the *locum*, as it is presented in the standard recension of the *Nova expositio*.⁵¹⁴ (Letters in brackets above staves give the pitch as it is identified in the treatise, based on a series of division points on a monochord diagram in Boethius – more about this in Chapter 12).

⁵¹³ Apel, *Gregorian Chant*, 210–11. My explanation is slightly oversimplified. In Modes III and VIII, the raised reciting tone can be an avoidance of h , but in Mode IV, the usual interpretation would be that it is raised to maintain a relationship a third below the reciting tone of Mode III. However, as Apel notes, in the *Commemoratio brevis*, Mode IV has a raised reciting tone, but Mode III does not. Thus, the explanation for Mode IV, at least, is not entirely clear. Most likely, the oddities result from the messy process of adapting a much more complex older tradition of recitation without the modes to suit the new *oktōēkhos* (see Chapter Six).









⁵¹⁴ A thorough summary of the chants in the alternate recension is found in Gumpel, *Die Nova expositio*, 143–78.

Protus Authentic









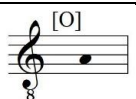

Observance	Differentia	Locum	Example
Nocturnis			<i>Ueniet dominus</i>
			<i>Apertis thesauris suis</i>
			<i>Canite tuba</i>
			<i>Ecce nomen domini</i>
			<i>Intempesta nocte</i>
			<i>O beatum pontificem</i>
			<i>Inclinans se Iesus</i>
			<i>Euge serve bone</i>
			<i>Sint lumbi uestri</i>
Diurnis			<i>Inclina Domine</i>
			<i>Statuit</i>
			<i>Gaudete</i>

†This *locum* is listed as being both “*in se*” (equal to the *differentia*) and “*in diatessaron remissum*” (down a fourth from the *differentia*); the latter is correct, as verified against the CANTUS database.






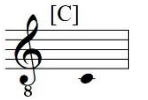


Protus Plagal

Observance	Differentia	Locum	Example
Nocturnis			<i>Laetentur caeli</i>
			<i>Igitur</i>
			<i>Iuste et pie</i>
Diurnis			<i>Ecce aduenit</i>
			<i>Sitientes</i>
			<i>De necessitatibus</i>

Deuterus Authentic



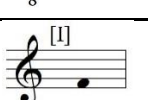

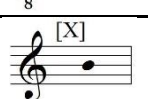
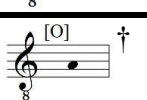
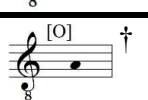




Observance	Differentia	Locum	Example
Nocturnis			<i>Qui odit animam</i>
			<i>Qui de terra est</i>
			<i>Et respicientes</i>
Diurnis			<i>Ego autem cum iustitia</i>
			<i>Dum sanctificatus</i>

Deuterus Plagal

Observance	Differentia	Locum	Example
Nocturnis		 [H]	<i>O quam clarus est</i>
		 [I]	<i>Ierusalem</i>
		 [E]	<i>Rubum quem uiderat</i>
	 [E]	 [C]	<i>Bethleem, non es minima</i>
Diurnis	 [M]	 [E] †	<i>Resurrexi</i>




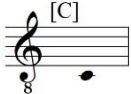



†The position of the *locum* is not listed in the text; the appropriate position has been verified against the CANTUS database.

Tritus Authentic











Observance	Differentia	Locum	Example
Nocturnis		 [O]	<i>Obsecro domine</i>
		 [I]	<i>Uox clamantis</i>
	 [Y]	 [X]	<i>Hodie nobis</i>
Diurnis	 [O] †	 [O] †	<i>Exaudi deus</i>
	 [M]	 [I]	<i>Circumdederunt me</i>
	 [Y]	 [Y]	<i>Iustus dominus</i>

†Neither the *differentia* nor the *locum* is described in the text, beyond the note “se” (“itself”). The *locum* has been verified against the CANTUS database; the *differentia* is assumed by virtue of the indication “se,” but must be taken as tentative, as the same indication was incorrect for *O beatum pontificem* in Mode I.









Tritus Plagal

Observance	Differentia	Locum	Example
Nocturnis		 [I]	<i>Aspiciebam</i>
		 [O]	<i>Modo ueniet</i>
		 [C]	<i>Per memetipsum</i>
Diurnis		 [I]	<i>Os iusti</i>
		 [C]	<i>Qui manducat carnem</i>

Tetrardus Authentic

Observance	Differentia	Locum	Example
Nocturnis		 [M]	<i>Summae trinitati</i>
		 [Y]	<i>Dixit Iudas</i>
		 [CC]	<i>Aspiciens</i>
		 [O]	<i>Attende Domine ad me</i>
		 [M]	<i>Missus est Gabriel</i>
Diurnis		 [M]	<i>Puer natus est</i>

Tetrardus Plagal

Observance	Differentia	Locum	Example
Nocturnis			<i>Ecce dies ueniunt</i>
			<i>Doceam iniquos</i>
			<i>Quodcumque</i>
			<i>Ecce ancilla domini</i>
Diurnis			<i>In uirtute tua</i>
			<i>Domine ne longe</i>

The Intonation Formulae

There is one more characteristic of tonaries to be considered here. Most tonaries, in their introduction to each mode, cite an intonation formula that helps to situate the singer within the mode. The most familiar of these are a set of biblical verses from the gospels (except Mode VII, which is from Revelations) that begin their texts with the number of the mode.⁵¹⁵ For instance, the formula from the first mode (*tonus primus*) sets the text “*primum quaerite regnum dei.*” However, these are not the formulae in common use in the Carolingian period. Instead, the formulae used in Carolingian treatises are the ῥηγήματα, the nonsense syllables (NOEANE and the like) that are first described by Aurelianus.⁵¹⁶

As Aurelianus explains, the syllables to which these formulae are sung have no specific meaning. They are based upon similar formulae in use in Byzantine music, and Aurelianus describes having asked a

⁵¹⁵ Bailey, *Intonation Formulas*, 27.

⁵¹⁶ Bailey, *Intonation Formulas*, 5.

Greek what they meant and having been told that they were joyful interjections.⁵¹⁷ It is unclear if Aurelianus' information is correct in this matter. In Byzantine sources, etymologies for the Byzantine ἠχήματα are sometimes given. For instance, the *Ἀγιοπολίτης* says:

Δεῖ δὲ ἐν τῷ μέλλειν ἡμᾶς ψάλλειν ἢ διδάσκειν ἄρχεσθαι μετὰ ἐνηχήματος. ἐνήχημα δὲ ἐστὶν ἢ τοῦ ἤχου ἐπιβολή, οἷον τι λέγω “ἄνα, ναὶ ἄνες”· ὅπερ ἐστὶν “ἄναξ, ἄνες.”⁵¹⁸

And when we are about to sing or instruct, one begins with an *enēkhēma*. For an *enēkhēma* is a casting on of the *ēkhos* [mode], such as when I say “*ananeanes*” – that is, “O Lord, forbear.”

However, the *Ἀγιοπολίτης* was written in the first half of the fourteenth century,⁵¹⁹ and there are no surviving Byzantine descriptions of the ἠχήματα older than the twelfth or thirteenth century.⁵²⁰ Consequently, there is no way to know whether Aurelianus' source simply did not know the etymologies or whether the etymologies are spurious, foisted upon the system by later authors.

Whatever their origins and etymologies, the ἠχήματα were the standard intonation formulae of the Carolingian period, and reference to them is made in every single major Carolingian treatise implicated in discussions of the *Alia musica*. Aurelianus provides a complete set in Chapter XIX;⁵²¹ Hucbald does not provide a complete list, but mentions two in passing (NOEANE for *tritus* authentic and NONENOEANE for *protus* authentic);⁵²² the *Musica enchiridis* mentions two in passing (NOANNOEANE and NOEAGIS) but does not specify to which modes they correspond.⁵²³ The complete set also appears in Regino's tonary, distributed throughout at the beginning of each mode,⁵²⁴ as well as in the *Commemoratio brevis*, again distributed throughout the first half of the treatise.⁵²⁵

⁵¹⁷ See Chapter Six.

⁵¹⁸ *Βιβλίου Ἀγιοπολίτης*, ed. & trans. Raasted, 11.

⁵¹⁹ Raasted, *The Hagiopolites*, 8.

⁵²⁰ Bailey, *Intonation Formulas*, 13.

⁵²¹ Aurelianus, *Musica disciplina*, ch. 19, ed. Gushee, 118–28, trans. Ponte, 45–53.

⁵²² Hucbald, *Musica*, §§36 & 46, ed. Chartier, 182 & 196, trans. Babb, 31 & 37.

⁵²³ *Musica enchiridis*, ch. 8, ed. Schmid, 20, trans. Erickson, 12.

⁵²⁴ Rausch, *Die Musiktraktate*, 201–24.

⁵²⁵ *Commemoratio brevis*, ed. & trans. Bailey, 31–45.

The *Alia musica*, itself, contains two, and after a fashion, three complete sets of ἤχηματα. The first set may be distributed amongst the revision and the commentary, and it is thus not certain that it represents a cohesive set: the first two appear in a section attributed to the revision, and all the rest are added by the commentator; however, the commentator does not see fit to comment upon those provided in the revision, and it is, therefore, reasonable to suppose that they were the same as the ἤχηματα he knew. Interestingly, while Atkinson attributes the passage surrounding the first two ἤχηματα to his author β [revision], he ascribes just the sections containing the ἤχηματα to author δ [commentary], presumably because of the presence of the ἤχηματα, and perhaps also because of the species analysis applied to them (described in Chapter 9 above), a methodology Atkinson also attributes to the commentator when it reappears in Mode VI.⁵²⁶ If Atkinson is correct (as seems reasonable, but not certain), then these ἤχηματα would then represent a coherent set. The second set is the set from the *Nova expositio*. The final set, if it ought to be appropriately considered to be part of the *Alia musica*, is the set added to the source treatise in manuscript K.

In addition to the ἤχηματα and the *primum quaerite* formulae, there is one other type of intonation formula. Terence Bailey, in his authoritative study on intonation formulas, describes them thus:

The third set is textless, made up of characteristic melismas which are usually appended to the other formulas just mentioned. These melismas – or *neumae*, as we will call them – are very seldom found detached, and then only in late sources.⁵²⁷

Bailey indicates that the ἤχηματα of both the revision and the *Nova expositio* seem to include these additional *neumae*. He does not justify the entirety of this hypothesis, but with respect to the *Nova expositio*, he points to the description of the third mode:

*A prima specie diapason et primus, propter quamdam naturam, inchoatur et tertius tropus, finiturque in diatessaron : in hoc scilicet concluditur eius forma intra diapente o ad e et diatessaron o ad cc.*⁵²⁸

⁵²⁶ Atkinson, *Critical Nexus*, 176, Table 5.3; 199.

⁵²⁷ Bailey, *Intonation Formulas*, 3.

⁵²⁸ *Alia musica* §90, ed. Chailley, 187.

From the first species of octave, as in the first [mode] (according to a certain nature), also begins the third trope, and it ends a fourth away; in this is enclosed its form, within the fifth *o* [a] to *e* [D] and the fourth *o* [a] to *cc* [d].

Bailey explains that the range specified here substantially exceeds the range of the short ἤχημα for the third mode, but perfectly corresponds to the range of the *neuma*.

Table 20 compares the three sets of ἤχηματα to those in the *Commemoratio brevis*, Aurelianus, and Regino, as well as a set of the Byzantine ἤχηματα presented in Bailey's study.

Table 20 – The ēkhēmata in Carolingian sources

	Aurelianus	Comm. Brev.	Alia musica MS K	Alia musica commentary	Nova expositio	Regino	Byzantine (Bailey p.12)
Protus authentic	NONANNOEANE	NOANOEANE	NONANNOEANE	NONANNOEANE	NOANNOEANE	NONANNOEANE	ANANEANES
Protus plagal	NOEANE	NOEAIS	NONEAYS	NOEAGIS	NOEAGIS	NOEAIS	ANEANES
Deuterus authentic	NOIOEANE	NOEANOEANE	AYNOEANE	AIANOEANE (NOEOEANE, MS. M margin)	NOEOEANE	NOENDANE	NEANES
Deuterus plagal	NOEANE	NOEAIS	NOEAIS	NOEAGIS	NOEAGIS	NOEAIS	NEANES
Tritus authentic	NOIOEANE	NOEANE	NOEANE	NOEOEANE	NOEOEANE	NOEOEANNE	ANEANES
Tritus plagal	NOEANE	ANNE	NOEAIS	AANNES	NOEAGIS	NOEOEAIS	ANES
Tetrardus authentic	NOIOEANE	NOEANE	NOEANE	NOEOEANE	NOEOEAGIS	NOIOEANNE	AGIA
Tetrardus plagal	NOEANE	NOEAIS	NOEAYS	NOEAGIS	NOEAGIS	NOEAIS	NEAGIE

Chapter 12: Terminological Issues

There are three sets of terminology for which a variety of overlapping vocabulary is used through the *Alia musica*: modal labels, the word used to describe the concept of mode, and the manner of identifying specific pitches. In general, there are at least three possible explanations (not necessarily mutually exclusive) for this wealth of terminology: (1) the terms are entirely synonymous, and the variety is merely for variety's sake; (2) the terms are synonymous, but authors have preferred terminology, so that the chosen terminology provides clues to which author wrote a passage; or (3) the different terms carry slightly different meanings, at least for any given author. This chapter will describe this terminology and examine the way that it is used in the *Alia musica*.

Tonus, Tropus, and Modus

Charles Atkinson provides a good summary of the use of this terminology in his *Critical Nexus*,⁵²⁹ and Chailley also discusses the issue in various places in the introduction to his edition. Atkinson notes that his author α [source treatise] routinely refers to modes as *toni*. By extension, the revisor also uses the term *tonus* through much of the treatise; however, he also uses the term *modus* in the introduction to Boethian theory. The use of *modus* in connection with Boethian theory is no surprise since Boethius is generally credited with coining this term (or at least popularizing it, since it appears in Martianus Capella, but with a different meaning).⁵³⁰ It is also consistent with the hypothesis that the Boethian introduction might have been borrowed from an earlier source, but it is certainly also quite plausible that an author otherwise preferring the term *tonus* might choose to use Boethius' preferred term when paraphrasing Boethius' treatise. Additionally, Chailley notes that the author of his "Principal Treatise" also sometimes

⁵²⁹ Atkinson, *Critical Nexus*, 175–78.

⁵³⁰ Atkinson, *Critical Nexus*, 20; 67, fn. 56.

uses the term *tropus* “*lorsqu’il parelera en son nom propre*”⁵³¹ (“when he speaks in his own name”); it should be noted, however, that the specific examples that he cites of this usage are generally ascribed by Atkinson to his author δ [commentator], not the revisor⁵³² (indeed, this usage of *tropus* is at least part of the reason why Atkinson attributes these passages to a separate author).⁵³³ The most consistent use of the term *tropus* is in the tonary, which does not use the term *tonus*.

However, there is an alternative explanation for these terms, as proposed by Heard. Heard suggests that the term *tonus* refers to a mode in all of its aspects, comprising every attribute that contributes to the *ēthos* of the mode.⁵³⁴ This definition would seem to include the interval structure (and by extension, the octave species), the melodic formulae, and the characteristic intervals of each mode, as well as the important structural pitches (*finales*, *mediae*, and *metae*).

In contrast, Heard suggests that *tropus* refers specifically to the various species of intervals associated with each mode – and not exclusively the octave species, but also the species of fifth and fourth; he says that “it refers to the pattern of whole and half steps within a single scalar unit, having the range of a fourth (tetrachord), a fifth (pentachord), or on occasion an octave consisting of both tetrachord and pentachord.”⁵³⁵ Heard’s entire definition cannot be accepted wholesale, as a careful reading of the *Alia* shows that the species of fourth and fifth are not implicated in the nature of the modes in this treatise, nor are they combined to create species of octave; that doctrine is a later formulation that Heard reads into an earlier treatment with which it would be consistent but is not stated.

⁵³¹ Chailley, *Alia musica*, 23.

⁵³² Chailley explicitly cites §§ 26, 27, 67, 76, and 89 (Chailley, *Alia musica*, 23). Atkinson’s Table 5.3 clearly shows all but one of these sections as the work of author δ . The exception, § 67, Atkinson identifies as the last sentence in a section by author β before another passage by author δ . In the written feedback following the defence of this dissertation, he clarified to me that he considers §67 to be the end of author β ’s commentary on the source treatise’s discussion of the third tone, particularly on the phrase *cuius diapason suum semitonium explicat*.

⁵³³ Atkinson is not explicit on this point in the *Critical Nexus*, but he confirmed my supposition in the written feedback following the defence of this dissertation.

⁵³⁴ Heard, *Alia musica*, 72–3.

⁵³⁵ *ibid.*, 74.

However, the interpretation of *tropus* as implying the interval structure of a mode through relation to the octave species is at least plausible; the use of this term in the Disputed Passage is certainly consistent with this interpretation, and the use in the *Nova expositio* may be as well, though as will be discussed shortly, the use of the octave species in the *Nova expositio* is unconventional. However, the hypothesis is more complex than the simpler assumption that the use of terminology simply reflects the preferences of the individual authors; thus, the hypothesis that the words have different meanings requires correspondingly stronger evidence. Yet there is considerable evidence that the terminology is associated with the differing authors (the *Alia* regularly explicitly tells the reader when the author is changing, and the terminology does seem to be consistent within the sections obviously attributable to a single author), while there is very little evidence that the terms are associated with specific meanings, perhaps resulting only from the coincidence that each author emphasizes different aspects of the modes. There is, therefore, insufficient evidence to be confident of Heard's analysis (though it should not necessarily be discounted out of hand).

Proceeding further along his hypothesis, Heard suggests that the term *modus* also carries a distinct meaning, as a sub-specification within the concept of *tropus* that refers exclusively to the octave species⁵³⁶ – this definition would potentially imply that there would be eight *tropi* but only seven *modi*, depending on how one interprets the Hypermixolydian mode; the editor of the *Alia* would not have seen this to be an issue, accepting the Hypermixolydian as a proper, non-redundant mode, but the Disputed Passage describes only seven octave species. In principle, it is at least plausible that the editor of the *Alia* indeed reserved the term *modus* to refer to the octave species, believing this to have been Boethius' usage. Nevertheless, the term *modus* is used in this manner only in the introductory material drawn from Boethius and is avoided elsewhere throughout the rest of the treatise (which assists comprehensibility,

⁵³⁶ Heard, *Alia musica*, 74.

since the term *modus*, like much of the terminology discussed in this chapter, has other common and important, non-musical meanings in Latin). Furthermore, in view of the problems with Heard's description of *tropus*, which, if it can be salvaged at all, ultimately reduces to the octave species, the hypothesis that *modus* also refers only to the octave species makes these two terms exact synonyms, the very conclusion Heard wishes to avoid. Hence, the hypothesis that the term *modus* is used in deference to Boethius while discussing his work is both simpler and more likely than that the term was reserved for the octave species, separate from the other characteristics of modes.

To return briefly to the term *tropus*, there is an alternative possibility that would contradict Heard's hypothesis. The author of the tonary begins,

*A prima quoque specie diapason, quae est mese, id est o, inchoatur primus tropus : finiturque in diapente remissum, quod est e, et haec est forma NOANNOEANE.*⁵³⁷

And so, from the first species of octave, which is the *mese* – that is, O [the symbol denoting that pitch *a* in monochord divisions] – begins the first *tropus*; and it is ended on a slacked [*i.e.*, lower pitched] fifth, which is E [the pitch D]. And this is the form: NOANNOEANE.

This description clearly states that the first *tropus* begins on *a* and ends on D; however, while the first mode is generally understood to end on D, it is not clear how it could be said to begin on *a*, unless the term *tropus* does not mean the mode (nor the species, as Heard suggests), but rather the ἤχημα that is supposed to embody the fundamental character of the mode. This interpretation is quite consistent with the descriptions of the ἠχήματα, both in the tonary of the *Alia* and as more precisely notated (using Daseian notation) in the *Commemoratio brevis*;⁵³⁸ the interpretation is also reinforced by the clause immediately following the description of the beginning and ending pitches, which clearly states that it has the form NOANNOEANE, the ἠχημα syllables for the first mode. It seems, then, that at least some of the time, the author of the tonary uses the word *tropus* to refer not to a mode in the usual sense, but rather

⁵³⁷ *Alia musica* §41(a), ed. Chailley, 183. cf. Atkinson, *Critical Nexus*, 186.

⁵³⁸ *Commemoratio brevis*, ed. & trans. Bailey, 30–31.

to the ἤχημα (or, perhaps, that the author of the tonary considers the ἤχημα to be in some sense the defining feature of the mode in the same way that the Byzantine ἤχοι are defined more by recurring melodic formulae than by the interval structures that arise from them).

Enumeration and Naming of Modes

Another set of terminology on which the various authors differ is the manner of identifying each mode; as with the term chosen for the concept of mode itself, the terminology for identifying a mode seems to be connected most closely with specific authors. As Atkinson notes, the source treatise labels the *toni* with the eight Latin ordinal numbers (*i.e.*, *primus*, *secundus*, etc.).⁵³⁹

Chailley's edition gives considerably more information. In Chailley, the first mode (which sets the pattern that the other modes follow) begins thus: "*Tonus primus NONANOEANE, qui graece dicitur autentos protos, id est auctoritas prima, CC·12, C·6, B·8, D·9.*"⁵⁴⁰ ("The *primus* mode, [with the ἤχημα] NONANOEANE, which in Greek is called *authentos protos* – that is, the first authentic, CC=12, C=6, B=8, D=9"). Here, the mode is named by its Latin ordinal number and also by its pseudo-Greek manerial designation, as well as by the syllables of its recitation formula. It is important to note, however, that all of this additional information is found only in manuscript K, which contains only the source treatise, while everything after the Latin ordinal designation is absent from all the other manuscripts (as Chailley notes in his critical apparatus).⁵⁴¹ In view of the presence of this additional information in only source K and its absence from the rest of the sources, and because the formulaic phrase appears at the beginning of each mode's section, and not merely in one place, it seems more likely to have been added as supplementary information by a knowledgeable scribe than to have been the original form with the information routinely

⁵³⁹ Atkinson, *Critical Nexus*, 175.

⁵⁴⁰ *Alia musica* §181(a), ed. Chailley, 85, trans. Nowacki 113. Chailley gives the letter A as equivalent to 12, as in the table and the prose summary, but only source K includes the letters in the source treatise, and source K uses CC instead of A, I have restored the original text here.

⁵⁴¹ Chailley, *Alia musica*, 86.

excised from the source of the other manuscripts during copying. Thus, the only terminology actively used in the source treatise in the principal group manuscripts is the set of Latin ordinals, *primus*, *secundus*, and so on.

For obvious reasons, the editor also describes the *toni* with Latin ordinal numbers; it is also in this section of the treatise, however, in which the Greek ethnic names (*Dorian*, *Phrygian*, etc.) are connected directly to the ecclesiastic modes. The phrasing of this section is less consistent, but loosely follows the model, “*modus primus, quem dorium dicimus*”⁵⁴² (“the first mode, which we call Dorian”). It is interesting to note that the author says “we call” (*dicimus*), and not “in Greek is called” (*graece dicitur*), implying (though by no means proving) that the ethnic names had at least some degree of currency in the time and place where he was writing.

As another point of interest, Chailley feels that this same author was also responsible for the introductory material that discusses the octave species, drawn from Boethius (and Atkinson concurs, though noting that the passage could have been the “kernel” around which the introduction was constructed).⁵⁴³ In that section, the octave species are enumerated in two different ways. The first time he discusses them, he enumerates them in ascending order, thus:⁵⁴⁴

Table 21 – Seven Octave Species

- | |
|--------------------|
| 8. Hypermixolydian |
| 7. Mixolydian |
| 6. Lydian |
| 5. Phrygian |
| 4. Dorian |
| 3. Hypolydian |
| 2. Hypophrygian |
| 1. Hypodorian |

⁵⁴² *Alia musica* §31, ed. Chailley, 121.

⁵⁴³ Atkinson, *Critical Nexus*, 177.

⁵⁴⁴ *Alia musica* §§15–16, ed. Chailley, 107.

The author repeats this order again when discussing the octave species, noting the locations of the semitones in each species, including the eighth, redundant species.⁵⁴⁵ However, when he returns to the modes themselves, he uses the conventional numbering system, in which the numbers alternate between authentic and plagal modes. Mühlmann, of course, attributes the former enumeration to a distinct author, the *Theoretiker der Acht Modi* (who contributed nothing else to the treatise), eliminating the contradiction.⁵⁴⁶ The contradiction might also lend support to Heard's hypothesis that the *modi* are octave species (thus, enumerated in ascending order), while the *toni* are modes (thus, enumerated in manerial pairs); against this argument, however, it should be observed that the phrasing of the initial enumeration explicitly applies these numbers to both the octave species and the modes:

*Erit ergo primus modus omnium gravissimus videlicet hypodorius ex prima specie diapason, et terminatur eo qui meses dicitur, media nervo. Secundum modum hypophrygium secunda species diapason efficit, quae in paramesen finitur. [etc.]*⁵⁴⁷

The first mode, therefore, will be the lowest of all, that is the Hypodorian, from the first species of octave, and it is ended at that which is called μέση, the middle string. The second species of octave brings about a second mode, Hypophrygian, which is ended at the παραμέση. [etc.]

The case endings here ("*primus modus [...] hypodorius*," "*secundum modum hypophrygium*") make it clear that both the ethnic names and the ordinal numbers are being ascribed to the *modi* separately from the octave species.

Atkinson holds that his author δ [commentator], too, employs the Greek ethnic names, but also employs the manerial designations.⁵⁴⁸ In particular, he employs this nomenclature in the section that relates octave species and their median pitches to the modes,⁵⁴⁹ these passages are the primary section about which Atkinson and Chailley disagree on authorship (the Disputed Passage), with Chailley

⁵⁴⁵ *Alia musica* §19, ed. Chailley, 110.

⁵⁴⁶ Mühlmann, *Die Alia musica*, 50–51.

⁵⁴⁷ *Alia musica* §15(a)–(b), ed. Chailley, 107.

⁵⁴⁸ Atkinson, *Critical Nexus*, 178.

⁵⁴⁹ *Alia musica* §§133–45, ed. Chailley, 196–204.

attributing this section to the author of the tonary; Chailley also does not distinguish between the editor and the commentator, considering the terminological differences between them to reflect two different roles performed by the same person. However, outside the aforementioned section regarding octave species, the manerial nomenclature is used twice in the principal treatise: once in a section that both Chailley and Atkinson identify as commentary (even if disagreeing on the identity of the commentator),⁵⁵⁰ but also in one passage that is copied from the source text, and is thus the work of the revisor.⁵⁵¹ Although the revisor does not ordinarily use this terminology, neither does he avoid it or substitute his own preferred terms for it (as he had consistently done in the introduction to each mode, by relabelling the modes with the Greek ethnic names); thus, the use or avoidance of the manerial terms is of limited reliability in attributing authorship of each section.

The author of the tonary, like the author of the source treatise and the editor, refers to each mode first and foremost by the Latin ordinal number; Atkinson is surprised to note that he does not employ the manerial designations.⁵⁵² He does, however, employ yet another set of Greek designations, the ἤχηματα. According to tradition extending back to Aurelianus, the syllables of the ἤχηματα are effectively nonsense syllables that are expressive only of emotion generally, rather than having any concrete meaning.⁵⁵³ In some formulations, in principle at least, each mode should have a unique set of syllables; thus, the syllables themselves can act as a kind of name for the mode. In practice, however, many of the modes came to share syllable sets, with a common variant having all four plagal modes set to the syllables

⁵⁵⁰ *Alia musica* §68, ed. Chailley, 138; cf. Atkinson, *Critical Nexus*, 176.

⁵⁵¹ *Alia musica*, §61, ed. Chailley, 135.

⁵⁵² Atkinson, *Critical Nexus*, 178.

⁵⁵³ This issue is addressed in Chapter Six.

NOEAGIS,⁵⁵⁴ and this is the case for the *Nova expositio* tonary (which also merges modes III and V on the syllables NOEOEANE).⁵⁵⁵

While this duplication of syllables would seem to make these syllables unsuitable for identification of the mode, there is one manuscript that does not manifest this problem: source M. This manuscript includes unheighted neumes over the ἠχῆματα, which would allow the reader to distinguish between even the modes that share the same syllables. An alternative interpretation of this manuscript might propose that since the neumes appear only in a single manuscript, they might perhaps have been added to that manuscript, rather than having been omitted from the others (indeed, I used a similar argument when discounting the additional information added to the introduction to each mode in the source text in manuscript K). However, the presence of these neumes also potentially solves another problem in the tonary: each section of the tonary concludes with the formula for the doxology for both the daytime and evening services. Since the text of the doxology is invariable for both daytime and evening and across all eight modes, the text repeats verbatim sixteen times – and this is a text that every reader in any conceivable target audience would be expected to have memorized. There is thus no reason for these lines to be present at all unless they originally included neumes (a conclusion apparently shared by the scribe of Pr, who abbreviated these passages) – and indeed, these lines also are provided with neumes in the Munich manuscript. It is, of course, still entirely possible that the original neumes were lost prior to the copying of this manuscript (or never included for some unexpected reason) and then supplied independently by the copyist, but this is a more complicated hypothesis, and complicated hypotheses should generally not be favoured over simpler hypotheses, except with correspondingly stronger evidence.

⁵⁵⁴ cf. Aurelianus, *Musica disciplina*, ch. 9, ed. Gushee, 84, trans. Ponte, 25. “*In plagis autem eorum consimilis est litteratura, scilicet NOEANE, sive secundum quosdam NOEAGIS.*” “In the plagal [modes] however, their text is all alike, which is NOEANE or, according to some, NOEAGIS.”

⁵⁵⁵ Chailley, *Alia musica*, 62.

It should also be noted here that the author of the tonary is not the only author to make use of the ἠχήματα as labels for the modes, though the exact formulae differ amongst the authors. In the introduction to his critical edition, Chailley constructs a chart showing the ἠχήματα syllables employed by each author.⁵⁵⁶ As already described, the source text presents the ἠχήματα quite systematically at the beginning of the section for each mode, but only in manuscript K, which probably reflects supplemental information added by the scribe, and not the formulae known to the original author of the source text.⁵⁵⁷ The revisor and commentator use these less systematically, but between the two of them, each formula is given. For modes III-VIII, the formula is given in the commentary; for the first two modes, the formula is given in the introduction, as an application of Boethian species theory to ecclesiastic chant, occurring in the only two passages in the entire introduction that Atkinson attributes to the commentator, rather than the editor. There is also a marginal gloss in the Munich manuscript that provides an alternative formula for the third mode, bringing the syllables for this mode into closer agreement with those of the tonary (though there is no such gloss to bring the formula of the sixth mode into agreement with the tonary, despite the two contradictory formulae appearing rather close to each other on the page, nor for Mode VII).

Identifying Pitches

The third area in which the authors differ from each other in preferred terminology is the identification of pitches. The source treatise does not identify specific pitches at all, except through the numbers representing the consonances. In general, the rest of the authors employ two other methods to identify specific pitches, both of which may be found in Boethius. The first is the Greek instrumental

⁵⁵⁶ Chailley, *Alia musica*, 62.

⁵⁵⁷ Bailey, *Intonation Formulas*, 7.

nomenclature, which has its origin in the names of the strings on the lyre where the pitches could be found (discussed in Chapter Five); this is the method favoured by the revisor.

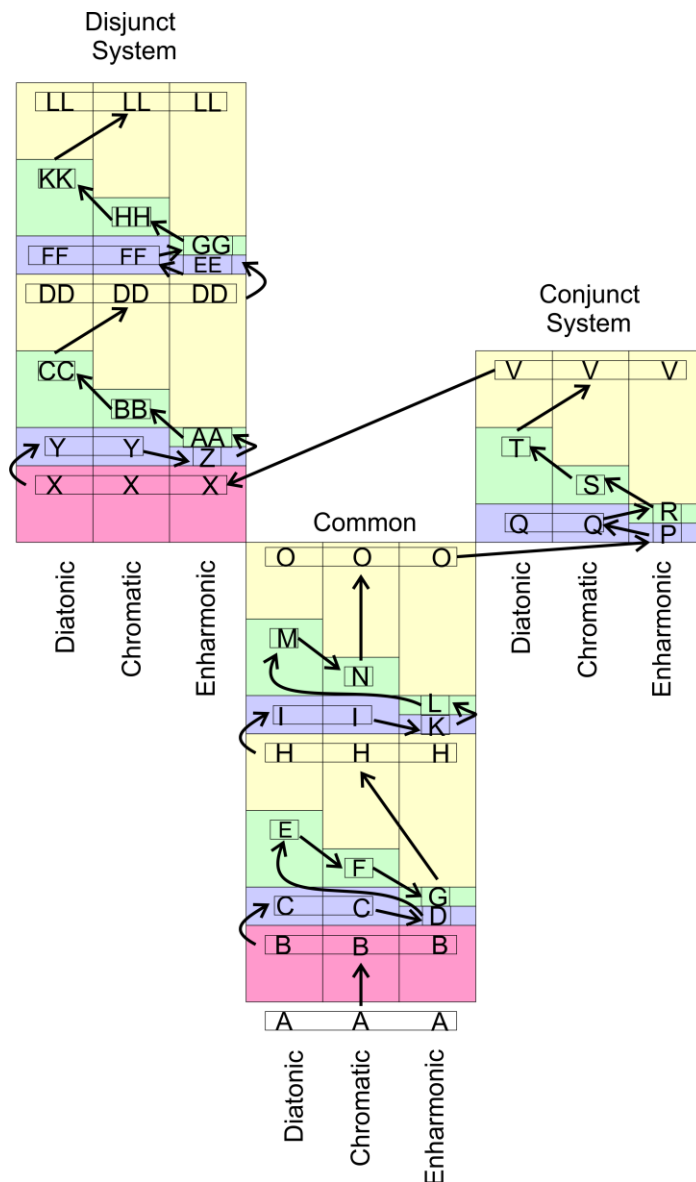
Table 22 – The Ametabolic System in all three genera

Note that intervals are not drawn to scale in quartertones; note-names are bottom-aligned to the equivalent notes in the later medieval gamut (with the addition of quartertones, and excepting that the distinction between square and round b is not employed, because it does not combine well with quartertones). Bolded names are fixed notes. Common colours identify notes of equivalent types across genera.

hyperbolaiōn	nētē hyperbolaiōn			ā			
	enharmonic paranētē hyperbolaiōn	chromatic paranētē hyperbolaiōn	diatonic paranētē hyperbolaiōn	g			
		chromatic tritē hyperbolaiōn	diatonic tritē hyperbolaiōn	g [♯]			
	enharmonic tritē hyperbolaiōn			g [♯] / f			
				f [♯]			
			e				
diezeugmenōn	nētē diezeugmenōn			d	nētē synēmnenōn		
	enharmonic paranētē diezeugmenōn	chromatic paranētē diezeugmenōn	diatonic paranētē diezeugmenōn	d [♯]	diatonic paranētē synēmnenōn		
		chromatic tritē diezeugmenōn	diatonic tritē diezeugmenōn	d [♯] / c			
	enharmonic tritē diezeugmenōn			c [♯]	diatonic tritē synēmnenōn	chromatic paranētē synēmnenōn	enharmonic paranētē synēmnenōn
	paramesē			b / c [♯]			
			c [♯] / b [♯]	chromatic tritē synēmnenōn	chromatic paranētē synēmnenōn	enharmonic tritē synēmnenōn	
			b [♯]				
mesē			a	mesē			
mesōn	enharmonic likhanos mesōn	chromatic likhanos mesōn	diatonic likhanos mesōn	G			
		chromatic parhypatē mesōn	diatonic parhypatē mesōn	G [♯]			
	enharmonic parhypatē mesōn			G [♯] / F			
				F [♯]			
				E			
hypatōn	hypatē mesōn			D			
	enharmonic likhanos hypatōn	chromatic likhanos hypatōn	diatonic likhanos hypatōn	D [♯]			
		chromatic parhypatē hypatōn	diatonic parhypatē hypatōn	D [♯] / C			
	enharmonic parhypatē hypatōn			C [♯]			
	hypatē hypatōn			B			
proslambanomenos			A				

Table 23 – Boethius' Monochord Notation, adapted from Bower, 144–45.

Boethius shared a common set of letters amongst all three genera. The notation ascends alphabetically, running from A–Z, then AA–LL. (Note that J is omitted because it is identical to I in Latin; likewise, U and W are omitted, being identical to V in Latin). Although the pitches differ among genera, they retain the same pitch names (shown here by maintaining common colours). Boethius always names all the disparately-tuned pitches of the same name together, though he is not systematic in the order in which he labels the equivalent pitches amongst the genera. It is not clear why Boethius omitted II; one may, perhaps, surmise that he felt the label II to be ambiguous in some way.



The second method is a letter notation drawn from the second division of the monochord presented by Boethius in his *Musica*.⁵⁵⁸ Contrary to Chailley's complicated explanation,⁵⁵⁹ the letters employed in the tonary correspond almost exactly to the letters with which Boethius labelled the pitches

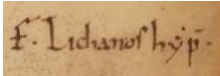
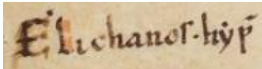
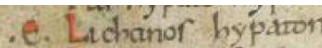
⁵⁵⁸ Boethius, *De institutione musica*, bk. 4, chs. 6–12, ed. Friedlein, 318–35, trans. Bower, 131–46.

⁵⁵⁹ Chailley, *Alia musica*, 181–2.

of the diatonic genus in his tetrachord division, excepting only NN, which ought to be labelled KK⁵⁶⁰ (though B, V, and LL are not used in the *Alia*).

The complete set of notation runs alphabetically through the Latin alphabet, A–Z, and then starts again with doubled letters, AA – LL; however, this alphabetical scheme is not apparent in its applications to medieval music because it was initially used to label all the notes of the diatonic, chromatic, and enharmonic genera, which includes sets of notes that had the same name (in the old Greek instrumental names, referring to the strings of the lyre), but different tunings (and thus, different locations on the monochord). The complete system, modelled on Boethius’ presentation, is given in Table 23, above. After the pitches of the enharmonic and chromatic genera are removed, the notation runs thus:

⁵⁶⁰ Boethius, *De institutione musica*, bk. 4, ch. 11, ed. Friedlein, 333b, trans. Bower, 145. According to Atkinson, *kk* is used in its proper place (instead of NN) in the *Dulce ingenium* (Atkinson, *Critical Nexus*, 184, Example 5.4). There is a contradiction in Bernhard’s edition on this point: in his commentary after the editions of the two recensions of the treatise, he provides a key to the monochord notation (Bernhard, *Anonymi saeculi*, 47) with only the comment “*Die Buchstaben entsprechen folgenden Tonbezeichnungen*” (“The letters correspond to the following pitch names”) and the section numbers of the *Nova expositio* in the short recension (§§53–85). His table gives the letters in standard form, with *kk*, rather than *nn*. However, none of the pitches described in the *Nova expositio* itself ever reach this high. The only place in the rest of either treatise that uses this system is the Disputed Passage, which reaches this high only once; this passage appears in the *Dulce ingenium* only in the other, longer recension in Pr, which appears to give *nn*, just as in the *Alia* (*Alia musica*, §137(b), ed. Chailley, 198; *Dulce ingenium*, long recension §83, ed. Bernhard, 36). There is no indication in Bernhard’s critical apparatus that the manuscript originally contained *kk*, and I have not been able to access the manuscript myself to verify. Both Chailley (*Alia musica*, 180–82) and Atkinson (*Critical Nexus*, 184) also describe an alternative form of this series that appears in the margins of P₁ (ff. 41v–42r) and P₂ (ff. 57r–v), as well as directly below the fragmentary excerpts of the *Nova expositio* in R (ff. 69r–v). This alternate series differs primarily in using majuscule letters instead of the minuscules used in the text of the *Nova expositio*. Chailley and Atkinson both describe the *e* of the *Nova expositio* being replaced by an *F* in this alternate series, but in fact, it is quite clearly an *E* in both P₂ and R; in P₁, the letter resembles an *F* with a rather large lower serif, but in all likelihood, it, too, is simply a poorly formed *E*.

P₁, f. 42r:  P₂, f. 57v:  R, f. 69r: 

Additionally, Atkinson seems to show *b* being used instead of *h* in the *Nova expositio*; in all likelihood, this is no more than a typo, since the *b* does not occur in any part of the *Alia*. There would be no reason for a square *b* in this context and a round *b* is already in use earlier in the series, and Atkinson has confirmed to me that the *b* was not intended.

Table 24 – Monochord notation as used in the *Alia musica*.

Modern Pitch	A	B	C	D	E	F	G	a	b	h	c	d	e	f	g	g
Monochord Label	a	b [†]	c	e	h	i	m	o	q	x	t, y [‡]	v [†] , cc [‡]	dd	ff	kk, nn [*]	ll [†]

* Identifies a note labelled differently in the *Alia musica* than in Boethius

† Identifies a note used by Boethius, but not referenced (nor contradicted) by the *Alia musica*

‡ Identifies a note that has two names because the first refers to the note in the *synemmenōn* tetrachord, while the second refers to a note with the same tuning in the *diazeugmenōn* tetrachord

Excepting the substitution of NN for KK, this table can be generated by reading only the leftmost column (the diatonic column) of each group of three columns in Table 23.

This notation is the method by which the author of the tonary usually labels his pitches. However, he also occasionally labels the pitches using the Greek string names, especially the *mesē*. In addition to these two methods of naming pitches, he also uses a third method, which reflects an idiosyncratic understanding of the octave species: he associates each octave species with a single pitch and uses the first octave species synecdochically as an alternative name for that pitch. For instance, he begins his description of the first *tropus* thus: “*A prima quoque specie diapason, quae est mese, id est o, inchoatur primus tropus*”⁵⁶¹ (“And so, from the first species of octave, which is the μέση – that is, O [a] – begins the first *tropus*”). Here, the author associates the first octave species with the pitch a and then uses this new label to identify the first pitch in the ἤχημα for the first mode. There is a slight parallel here to the way in which the octave species are presented at the beginning of the treatise, in §15, since the modes in that section were identified according to a single pitch, the upper boundary, rather than the entire span. The pitches by which the species are identified only partly match the usage in the tonary, insofar as the pitch-

⁵⁶¹ *Alia musica* §41(a), ed. Chailley, 183.

classes agree, but the octaves do not; additionally, the correspondence to the modes is flipped with respect to plagal and authentic modes.

Table 25 – Octave Species, Associated Pitches, and Tropus

Octave Species	Pitch	Tropus
1	a	I, III
2	Not cited, presumably \flat	
3	c	V
4	d	VII
	D	II
5	E	IV
6	F	VI
7	G	VIII

The exact nature of the association between these pitches and the octave species – and, by extension, perhaps also the ordering of the octave species – is idiosyncratic. The pitches identified with each octave species and each *tropus* are given in Table 25.

This table is unusual in a few ways. First, the order of pitches across the seven species does not move in a consistent direction, either up or down. Instead, it begins in the middle, on a, and proceeds upwards to d, drops down to the lower octave D, and continues to rise back up to G, one step below where the series started. Second, the fourth octave species is identified in two places, both d and D, implying that the use of the octave species as a synecdochal name for a pitch may actually label something more like a pitch-class, incorporating some degree of octave equivalence, which is why the pitches do not quite agree with the pitches that define the modes in the species introduction in §15. Third, the order of

octave species ascends linearly through all four authentic *tropi*, then drops down an octave and ascends through all four plagal *tropi*. The single exception is *tropus* III, which is described as sharing the octave species of *tropus* I. This is not an error: the author is not here associating the octave species with modal octaves, but rather with pitches, and using them to identify the first pitches of the ἠχρήματα. The ἠχρήματα of all the plagal modes both begin and end on the modal final, and the *ēkhēmata* of the authentic modes all end on the final but begin a fifth above the final – except for *deuterus*, which begins on the fourth above the final, avoiding the pitch \mathfrak{b} ;⁵⁶² this avoidance of \mathfrak{b} as a structurally important pitch is quite characteristic of ecclesiastical modal theory in general.⁵⁶³

The first conclusion to draw from Table 25 is that the order of octave species as envisioned in the *Nova expositio* (as compared to the introduction) is drawn, *post hoc*, from the relative ranges of the modes, as though the modes had predated the octave species. However, it still remains to explain the choice of pitch that characterizes each octave species. If Atkinson is correct in his belief that the Disputed Passage is the work of his author δ (the commentator), rather than the work of the author of the tonary (as Chailley believed), it follows that the author of the tonary never explicitly defines the octave species, which means that we do not know his preferred ordering of the species, and we cannot thence infer the relationship between the octave species and the pitches that he identifies with them.

Since the pitches are taken from the central octave of the two-octave gamut, these pitches can neither be uniformly the highest pitches of each octave species (D, E, F, and G are too low to be the top of an octave in the gamut) nor the lowest pitches (a, \mathfrak{b} , c, and d are too high in a two-octave gamut, and

⁵⁶² This can be confirmed from the *Commemoratio brevis* (ed. Bailey, 36–37 *et passim*).

⁵⁶³ Consider, for instance, that the selection of reciting tones for each psalm tone follows a clear pattern, interrupted only by the avoidance of \mathfrak{b} (the authentic modes use the fifth above the final, except *deuterus*, which ought to be \mathfrak{b} , but is instead raised to c; the plagal modes use the third below the reciting tone of the corresponding authentic – including the raised *deuterus* – except from *tetrardus*, which ought to be \mathfrak{b} , but is likewise raised to c).

while one might argue for $\frac{1}{2}B$ and ξ create complete octaves, $\frac{d}{d}$ probably was not yet available as an upper octave for d).

It is possible that the specified pitches are the top pitches of the octave species for the first four species and the bottom pitches of the last four species (leaving the fourth species identified at both the bottom and top); this seems somewhat arbitrary but would be consistent with the presentation of the *metae* (boundary notes) given in the Disputed Passage, except that the connection of the *metae* to the modes is reversed with respect to authentic and plagal modes. The better correspondence, then, is that these pitches are *mediae* of the octave species, so that the pitches cited give a central pitch around which the species is spread. Although I suspect the tonary and the Disputed Passage to have been borrowed separately into the *Alia musica*, this correspondence is perhaps the most persuasive argument in favour of Chailley's opinion that the Disputed Passage is part of the *Nova expositio*. It is interesting here, though, that the *mediae*, if they are, indeed, *mediae*, are here understood as properties of the octave species themselves, not the modes. This interpretation is supported particularly by the fact that the *deuterus* authentic trope is defined from the first species, just as the *protus* authentic was. As has already been explained, this overlap occurs because the *deuterus* authentic ἦχημα begins a tone lower (relative to its *finalis*) than the other authentic modes; thus, the third mode ἦχημα is described as beginning from the *media* of the octave species proper to the first mode, and the *media* is therefore not a property of the mode, only of the species.

It is also worth remarking at this point that use of the octave species as proxies for their *mediae* when describing the top pitch of an ἦχημα appears not to have been understood by the unknown author who created the alternate recension of the *Nova expositio* in source R. One of the most significant revisions in this alternative recension is the re-enumeration of the octave species in all four plagal modes and the complete omission of any reference to the octave species in Modes V and VII. If one assumes

that in this alternate revision, the octave species represent the standard octave ranges of each mode, then the species identifications for all the plagal modes and for *deuterus* authentic could be consistent with each other, with the first octave species spanning E–e and the enumeration running downwards, as shown in Table 26.

Table 26 – The re-enumeration of octave species in the revised recension of the *Nova expositio in source R*.

Ranges of the octave species are speculative but are consistent across all plagal modes and Mode III; Mode I is inconsistent, and species numbers are not provided for Modes V and VII, perhaps because the appropriate species would be in the wrong octave (species identified for these modes are speculative).

Mode	Octave Species Number	Octave Species Range?
VII	[6']	[Γ–G → G–g?]
V	[7']	[Φ–F → F–f?]
III	1	E–e
I	1 [! – should be 2?]	E–e [D–d?]
VIII	2	D–d
VI	3	C–c
IV	4	B–b
II	5	A–a

However, the first mode would be incorrect, as it is still identified as sharing an octave species with the third mode, which only makes sense if the octave species represent, in some way, the first note of the ἤχημα; this interpretation does not work with the species enumeration of the alternate recension because it would cast the first mode as the mode whose starting pitch does not fit the usual pattern, rather than the third mode. Furthermore, it is not at all clear why the author would not have completed the system by specifying species for Modes V and VII (perhaps he was simply uncomfortable with assigning them to the appropriate octave species, which, following in the sequence used by the plagal modes, would be an octave lower than the correct range for these modes).

Chapter 13: Preliminary Numerology

Although the great bulk of the scholarship surrounding the *Alia musica* has been dedicated either to sorting out the thornier issues of authorship or to illuminating the place of the *Alia* in the development of a Greek-influenced model of ecclesiastical modality, there is another aspect of the treatise that is probably more distinctive than either of these, and perhaps unique in the entire corpus of medieval theory treatises: the numerology. Oddly, this element of the treatise has been underexamined in the scholarship on the treatise, which is particularly unexpected because the discussion of the numerology occupies more of the treatise than any other topic.

I use the term “numerology” instead of “harmonics” to describe the numerical descriptions of mode in the *Alia* because the system, while rooted in traditional harmonics, is not consistent with conventional harmonic approaches. This incompatibility will be demonstrated in part in this chapter, which provides an overview of the system and an examination of how previous scholars have interpreted the system; the incompatibility will also be addressed in Chapter 17, which presents analyses of the chants that the *Alia* cites as exemplifying the principles that it describes. A justification of the term “numerology” will be presented in Chapter 19, where I present my own interpretation of the *Alia*’s peculiar treatment of harmonics.

The numerology is already present in (and is the principal concern of) the source text and is treated again in the revised edition and the commentary, though it is absent from the tonary. However, as Chailley repeatedly notes,⁵⁶⁴ there is good reason to question the editor’s understanding of the source text (and perhaps of practical music theory in general, though as I have already noted, his understanding of Boethius may not be as problematic as Chailley believes). Chailley supposes that the editor was probably not even

⁵⁶⁴ Chailley, *Alia musica*, 20–21 *et passim*.

a musician; if this hypothesis is correct, then the editor would instead most likely be a liberal arts generalist, writing about music as an element of the Quadrivium. In this case, it would make little sense to treat the editor's presentation of modal numerology as an accurate presentation of the theory as the author of the source text understood it. And while it is not implausible that an idiosyncratic form of a theory could, if widely disseminated, become the standard form of the theory for a subsequent generation, the numerology of the *Alia musica* (unlike its fusion of ancient Greek and medieval concepts) seems not to have had any particular influence on the subsequent theoretical tradition. Hence, wherever the revised edition deviates from the presentation of the source text in its understanding of the numerology, the editor's version is less likely to provide any additional insights into Carolingian theory. Therefore, for reasons of scope, the present examination will focus primarily on the presentation of modal numbers in the source text and will address the contributions of the editor and commentator only secondarily (though a short section will be dedicated to that purpose in Chapter 16).

In most manuscripts of the source text, the very first piece of information presented about each mode is an explanation of the derivation of a set of numbers associated with the mode; the revision follows almost the same pattern, with the numbers usually being preceded only by a short list of alternative naming conventions. This material is then elaborated in the commentary, sometimes obfuscating more than clarifying. It is no exaggeration to say that these numbers are treated as the most defining characteristic of the modes, at least in the source text, revision, and commentary (though not in the tonary).

But the numbers associated with each mode are puzzling. The arithmetical derivation of the gamut (particularly with the use of the monochord) is quite common in medieval theory treatises, but they generally did not use these numbers to describe the modes, except by extension (since the modes were generally situated at specified positions within the gamut). The method used in the *Alia musica* is quite different from this and is not well explained in the text.

The numbers associated with the modes are based upon the relation 12:9:8:6, a relation that underlies much Greek harmonic theory and brought historical authority to any medieval treatise that derives its doctrine therefrom. As explained in Chapter 09, the significance of this relation is that the standard consonances are all to be found amongst the possible ratios in the relation, and also that all the possible ratios within the relation are standard consonances except one (that one being a tone, which in Greek theory was thought to be akin to the consonances). The ratio 12:6, which simplifies to 2:1, is a perfect octave; both the ratios 12:8 and 9:6, which simplify to 3:2, are perfect fifths; the intervals 12:9 and 8:6, which simplify to 4:3, are perfect fourths; and the ratio 9:8 (already in lowest terms) is a tone.

The challenge in understanding the numerology of the *Alia musica* is that it is not immediately clear what kind of relationship these numbers could have with the modes that would, in some way, clarify the differences between them. The relation 12:9:8:6 taken as a single chord consists of an octave filled in with both its fourth and fifth. If the *finalis* of an authentic mode is set at the bottom of the octave, the remaining notes in the chord are always potential notes in the authentic modes (even *tritus*, which appears to lack a perfect fourth above the *finalis* in the octave-species definition, but the perfect fourth is provided by the use of *b*). Similarly, if the *finalis* of a plagal mode is set a fourth above the bottom of the octave, the rest of the notes generally exist as possible pitches within the mode, except *deuterus*, which lacks a perfect fifth above the lowest note (*i.e.*, it lacks a tone above the final). Thus, it would be accurate to suggest that seven of the eight modes could be described with the relation 12:9:8:6, with Mode IV lacking one number. However, this proposal would mean that seven of the eight modes would be described by an identical set of four numbers, and this is not at all how the intervals are described in the *Alia musica*.

Instead, a different set of numbers is derived for each mode by multiplying the numbers by different coefficients – but, once again, not in an expected way. One intuitive possibility that would have been quite consistent with the way that harmonic theory was used in other treatises would have been to situate

all of the modes within a shared gamut. Ptolemy illustrated the modes in this manner,⁵⁶⁵ as did Boethius,⁵⁶⁶ whose treatment is derived from Ptolemy's. Each proceeded to identify the frequencies of each pitch within the octave species of each mode.⁵⁶⁷ Musicians focussed on the importance of the relation 12:9:8:6 might well limit their calculations to only those pitches corresponding to this relation within the overall structure of the mode, maintaining a consistent number across modes for shared pitches that correspond to these important numbers in more than one mode. The entire gamut ultimately does get calculated as a side effect, but the calculations have nevertheless identified the numbers corresponding to 12, 9, 8, and 6 in each mode. The smallest set of whole numbers that can be used to complete these calculations are given in Table 27.

While the numbers presented in Table 27 are entirely consistent with familiar medieval theory, they do not at all resemble the numbers used in the *Alia musica*. In fact, they differ quite dramatically: the smallest number present in the table is 648, while the largest number present in the *Alia musica* appears to be 96 (and this is only by extension; the highest number presented without complex manipulation is 48).⁵⁶⁸ In addition, the *Alia musica* presents numbers as string lengths, rather than frequencies, and so the higher pitches should correspond to lower numbers (as is suggested by the hypothetical table below) – and so they do *within* each mode in the *Alia*; however, *between* modes, as the pitch of the final rises (in conventional descriptions), the numbers increase, counterintuitively implying downward progression of the pitches of the *finales*. Furthermore, the numbers representing subsequent *finales* do not merely

⁵⁶⁵ Ptolemy, Ἀρμονικά, bk. 2, ch. 10, ed. Düring, 62–64, trans. Barker, 336–38.

⁵⁶⁶ Boethius, *De institutione musica*, bk. 4, ch. 17, ed. Friedlein, 343–48, trans. Bower, 156–60.

⁵⁶⁷ Ptolemy's presentation holds the overall range more or less constant and calculates the frequencies for the octave species within that range, loosely assigned between the numbers 60 and 120 (Ptolemy, Ἀρμονικά, bk. 2, ch. 15, ed. Düring, 76–80, trans. Barker, 352–55). Boethius instead calculates all the frequencies for the gamut (Boethius, *De institutione musica*, bk. 4, chs. 6–11, ed. Friedlein, 318–34, trans. Bower, 131–45); he then defines the octave species within this gamut (Boethius, *De institutione musica*, bk. 4, ch. 14, ed. Friedlein, 339, trans. Bower, 150), which effectively provides the calculations for each octave species in ranges spanning across the gamut, though he does not list these explicitly.

⁵⁶⁸ The larger number 144 also appears in the *Alia*, but it is a sum of three modal numbers, rather than a modal number in its own right.

increase, but increase considerably, well beyond the tones and semitones that are usually held to lay between them.

Table 27 – Numbers corresponding to 12:9:8:6 in each mode in a shared gamut, in lowest terms.

Numbers in the *Alia musica* reflect string length, not frequency, and so smaller numbers represent higher pitches. The b and especially F# are noted in brackets as potentially problematic.

Mode	I	II	III	IV	V	VI	VII	VIII
g							648	
f					729			
e			768					
d	864						864	864
c					972	972	972	
b			1024	1024	[1093.5=b]			
a	1152	1152	1152					1152
G	1296					1296	1296	1296
F				[1365 ≈ F#]	1458	1458		
E		1536	1536	1536				
D	1728	1728						1728
C						1944		
B				2048				
A		2304						

An alternative approach that could produce numbers much more similar to those actually found in the treatise would be to assign the *finales* of the *maneriae* to consecutive multiples of twelve. Such an approach might produce results similar to Table 28; as will be described later in this chapter, this hypothesis is a simplified form of Mühlmann’s and Heard’s interpretation.

Table 28 – Hypothetical harmonic numbers if each maneria begins on the next multiple of twelve

Protus (12 × 1)	Deuterus (12 × 2)	Tritus (12 × 3)	Tetrardus (12 × 4)
6	12	18	24
8	16	24	32
9	18	27	36
12	24	36	48

This chart has much in common with the numbers presented in the *Alia musica*, and this fact may potentially imply some rather interesting things about the general understanding of modal theory in the

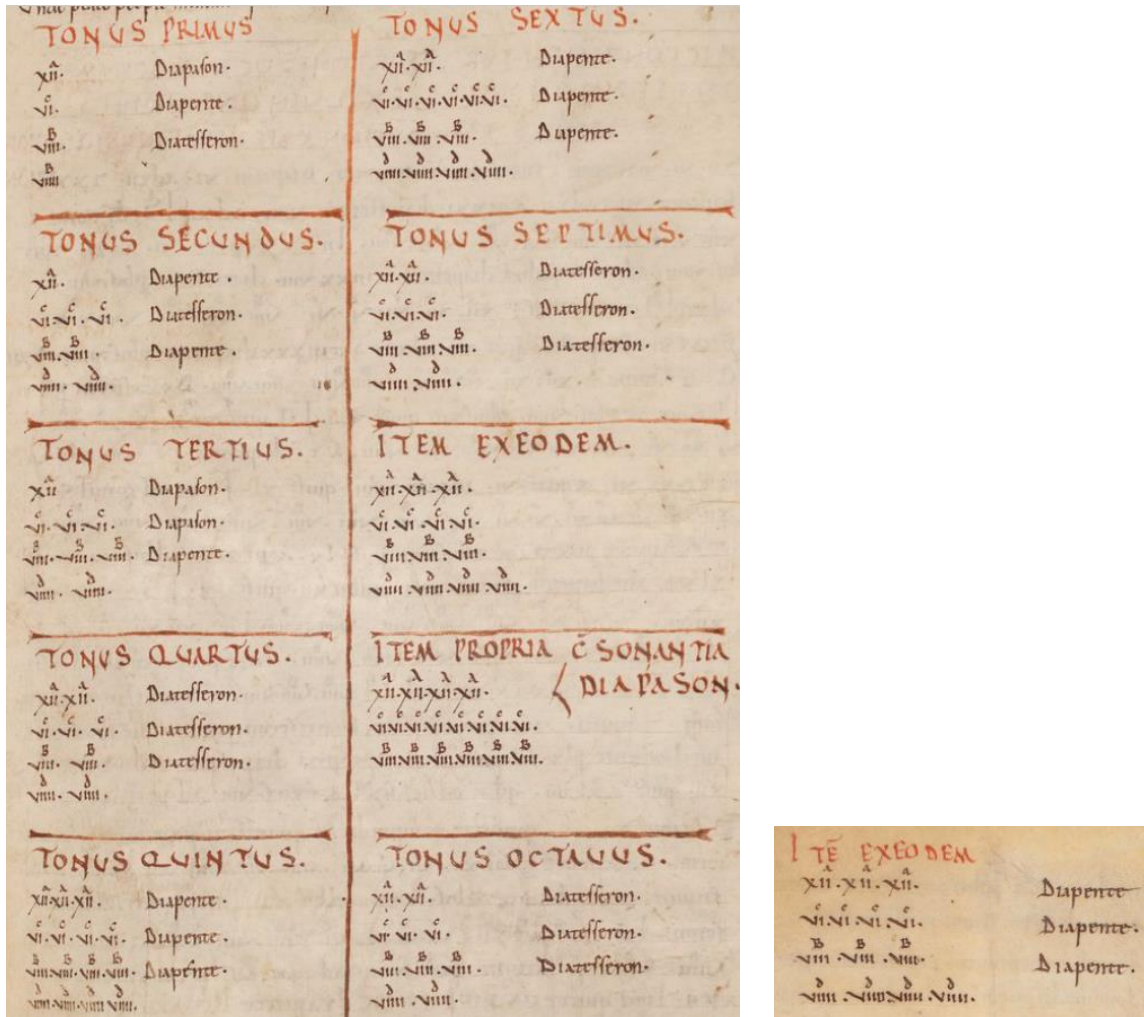
Alia musica's historical setting, such as the possibility that the author of the source treatise, at least, may not have understood the modes to have fixed positions relative to one another within the gamut; these implications will be taken up after a more sophisticated understanding of the numbers has been reached. For now, however, it must also be acknowledged that this chart still ultimately does not reflect the numbers as they appear in the *Alia*.

The most significant way in which the actual numbers of the *Alia musica* differ from the hypothetical numbers presented in Table 28 is that the coefficients by which the numbers are multiplied are not constant even within a single mode (except for Mode I). Instead, two or three different coefficients might be used in each mode, with no immediately apparent pattern to explain which numbers might share a coefficient. These coefficients are presented several times throughout the treatise – not only in the source text and its revision (and forming the object of much of the commentary) but also summarized in a table near the end, and then summarized again, following the table, in prose form. The most concise treatment is in the table, reproduced here from the P₁ as Facsimile 12.

In general, each mode has its own little table within the larger chart, though the *tetrardus* modes have duplicates. Within each little table (with only one notable exception), there are four rows, each corresponding to the fundamental numbers of the relation 12:9:8:6. They do not appear in this standard order, however, but in the order 12, 6, 8, 9 (given in the manuscript as Roman numerals).

As may be seen in Facsimile 12, above each number is a letter that corresponds perfectly with the number. These letters may be redundant, but they may also have additional significance since the order of letters alphabetically does not correspond to the order of numbers, either numerically or in the order in which they are presented in the chart; these letters may, then, provide a clue as to how medieval

theorists thought that these numbers should relate to each mode. The correspondence between numbers and letters is given in Table 29.⁵⁶⁹



Facsimile 12, Paris Bibliothèque Nationale, MS Lat. 7212, ff. 49r-v.

Table 29 – The relationship between numbers and letters in the chart of numbers corresponding to each mode. Presented in varying orders, demonstrating the lack of direct correspondence, which may be suggestive of a deeper significance to the letters.

Ascending Numerical Order				Ordered as in Manuscript				Ascending Alphabetical Order			
c	b	d	a	a	c	b	d	a	b	c	d
6	8	9	12	12	6	8	9	12	8	6	9

⁵⁶⁹ There is an error in Mode 1 in the Paris MS Lat. 7212 (whence comes Facsimile 12) whereby both viii and viiii are labelled as B; this error does not appear in the other sources that I have consulted, and so it is corrected in my chart here.

As Table 29 shows, if one rearranges the numbers into alphabetical order, the ordering is twelve, eight, six, nine. As pitches, taking twelve as a fundamental, these numbers would produce a fundamental, a fifth, an octave, and a fourth. It is possible that this ordering might reflect the order in which the pitches were thought to be important to each mode; such a ranking would be intuitive for the authentic modes, but not the plagals, and would require the assumption that the contradiction would be ignored because plagal modes were subordinate. There is, at any rate, far too little evidence to be confident of this hypothesis, but it does not seem likely that the correspondence of numbers and letters is purely arbitrary. Another possibility is that the ordering of the first three numbers reflects the order of numbers in the paradigmatic presentation of the harmonic mean (presented in the introduction of the treatise), while the nine from the arithmetic mean is tacked onto the end of the series (one does get the impression from the introduction that the harmonic mean was considered to be more important than the arithmetic mean).

Returning to the tables in Facsimile 12, each number appears on its own line in each little table, but generally is written several times in a row, such as $\overset{B}{viii} \cdot \overset{B}{viii} \cdot \overset{B}{viii} \cdot \overset{B}{viii}$. This presentation indicates serial addition and is generally converted to multiplication in modern treatments; thus, the example just given is interpreted as 4×8 , which is equal to 32. Throughout this dissertation, I refer to the repeated numbers 12, 9, 8, and 6 as base numbers and the number of repetitions as coefficients; as traditional for multiplication, I call the result the product. Since the coefficients for each base number in a mode are not consistent, the result is that some of the products of this multiplication result in duplicate numbers; hence, Mode I, which presents the base numbers unmultiplied, is the only mode that ultimately has four unique numbers.

Table 30 – The harmonic numbers of the Alia musica, adapted from Facsimile 12 ⁵⁷⁰

Mode I ^a $12 \times 1 = 12$ Octave ^c $6 \times 1 = 6$ Fifth ^b $8 \times 1 = 8$ Fourth ^d $9 \times 1 = 9$	Mode II ^a $12 \times 1 = 12$ Fifth ^c $6 \times 3 = 18$ Fourth ^b $8 \times 2 = 16$ Fifth ^d $9 \times 2 = 18$	Mode III ^a $12 \times 1 = 12$ Octave ^c $6 \times 4 = 24$ Octave ⁵⁷¹ ^b $8 \times 3 = 24$ Fifth ^d $9 \times 2 = 18$	Mode IV ^a $12 \times 2 = 24$ Fourth ^c $6 \times 3 = 18$ Fourth ^b $8 \times 3 = 24$ Fourth ⁵⁷² ^d $9 \times 2 = 18$
Mode V ^a $12 \times 3 = 36$ Fifth ^c $6 \times 4 = 24$ Fifth ^b $8 \times 3 = 24$ Fifth ⁵⁷³ ^d $9 \times 4 = 36$	Mode VI ^a $12 \times 2 = 24$ Fifth ^c $6 \times 6 = 36$ Fifth ^b $8 \times 3 = 24$ Fifth ^d $9 \times 4 = 36$	Mode VII ^a $12 \times 2 = 24$ Fourth ^c $6 \times 3 = 18$ Fourth ^b $8 \times 3 = 24$ Fourth ^d $9 \times 2 = 18$ Likewise from the same ^a $12 \times 3 = 36$ ^c $6 \times 4 = 24$ ^b $8 \times 3 = 24$ ^d $9 \times 4 = 36$ Likewise, the Properties and Consonances of the Octave ^a $12 \times 4 = 48$ ^c $6 \times 8 = 48$ ^b $8 \times 6 = 48$ ^d Missing from table	Mode VIII ^a $12 \times 2 = 24$ Fourth ^c $6 \times 3 = 18$ Fourth ^b $8 \times 3 = 24$ Fourth ^d $9 \times 2 = 18$ Likewise from the same ^a $12 \times 3 = 36$ Fifth ^c $6 \times 4 = 24$ Fifth ^b $8 \times 3 = 24$ Fifth ^d $9 \times 4 = 36$

As may be seen in Facsimile 12, above each number is a letter that corresponds perfectly with the number. These letters may be redundant, but they may also have additional significance since the order

⁵⁷⁰ The interval labels in this table are difficult to interpret as they are presented here (which is consistent with the way that the table is given in the two Paris manuscripts, the Munich and Cesena manuscripts, and Gerbert's edition, which is based primarily on the lost Strasbourg manuscript). Chailley, correctly in my estimation, hypothesizes that the interval labels are uniformly placed one line higher than the number to which they correctly apply. It is easy to see how such a change could occur in a manuscript; if there is nothing written in the top row of a column, a scribe might well assume that the source from which he was copying simply hadn't correctly aligned the columns; I have chosen not to alter the chart, since with awareness of the issue, the chart is easy enough to read as given. Chailley, with minimal explanation, also omits several of the interval labels; the reason appears to be that the numbers where he omits labels are identical to the product of twelve given (which seems to function as the reference pitch), and so properly represent unisons, rather than the intervals given. However, this suggestion assumes that the author of the table was consistent about labelling intervals compared to the single reference pitch (the multiple of twelve), rather than against the pitch in the previous row; certainly, this assumption is true in many cases, but both possibilities are intuitive, and so the possibility of inconsistency of methodology must be allowed, particularly since it is much less obvious how superfluous labels might be introduced to all manuscripts (which otherwise include notable differences from each other), compared to simply shifting columns upwards. Chailley also provides interval labels for the second and third little tables for Mode VII, which are not presented in any manuscript.

of letters alphabetically does not correspond to the order of numbers, either numerically or in the order in which they are presented in the chart; these letters may, then, provide a clue as to how medieval theorists thought that these numbers should relate to each mode. The correspondence between numbers and letters is given in Table 29.⁵⁷⁴

Next to most of the numbers are Greco-Latin interval labels (*diapente*, *diatessaron*, *diapason*), which are translated here as fifth, fourth, and octave. The chart, reproduced in simplified form, is given here as Table 30.

As already suggested, the modern scholarship on the *Alia musica* has generally not paid substantial attention to this aspect of the treatise. Unfortunately, Atkinson, whose work on the treatise has generally been the most helpful on other aspects of the treatise, is rather brief in his description of this numerology (likely because it falls outside the scope of his project, which deals primarily with the relationship between Greek and medieval concepts of mode and gamut). Heard does attempt to explain the source of these numbers, but his attempts are unhelpful and sometimes obfuscating. Chailley's explanation is much better, but still ultimately incomplete.

⁵⁷¹ As given in Facsimile 12, the coefficient for six in Mode III is three, not four; however, this number is not consistent with the coefficient elsewhere in the treatise; the expected coefficient of four is given in the Munich manuscript and in Gerbert's edition. The coefficient is corrected here.

⁵⁷² As given in Facsimile 12, the coefficient for eight in Mode IV is only two, not three; however, this number is not consistent with the coefficient elsewhere in the treatise (*cf.* Chailley, *Alia musica*, 120). The coefficient is corrected here to reflect the rest of the text.

⁵⁷³ As given in Facsimile 12, the coefficient for eight in Mode V is four, not three; this is not only inconsistent with the rest of the text, but is also inconsistent with the chart given in other manuscripts. The coefficient is corrected here.

⁵⁷⁴ There is an error in Mode 1 in the Paris MS Lat. 7212 (whence comes Facsimile 12) whereby both viii and viiii are labelled as B; this error does not appear in the other sources that I have consulted, and so it is corrected in my chart here.

The Multiplex Hypothesis

The finer details of how the numbers are presented will be discussed in Chapter 14. For the moment, the principal question is what the product numbers represent (and, by extension, how the coefficients are derived).

Both Heard and Chailley propose related (but slightly different) hypotheses that the range of numbers for each *maneria* reflects a multiplication of the range of numbers from a preceding *maneria*. Heard proposes, based on an analysis by Mühlmann, that the *maneriae protus, deuterus, tritus*, and *tetrardus* relate to each other as simplex, duplex, triplex, and quadruplex.⁵⁷⁵ This proposal makes a certain amount of linguistic sense since the names of the *maneriae* are (more or less) the first four Greek numbers (but ordinal numbers, not multiplicative numbers).

Table 31 – Heard’s interpretation of the relationship between maneria numbers and modal product numbers

Greek	<i>Protos</i>	<i>Deuterus</i>	<i>Tritos</i>	<i>Tetartos</i>
Translation	“First”	“Second”	“Third”	“Fourth”
Interpretation	Single	Double	Triple	Quadruple

What Heard means by this proposal is that the twelve in *protus* corresponds to twenty-four (double twelve) in *deuterus*, thirty-six (triple twelve) in *tritus*, and forty-eight (quadruple twelve) in *tetrardus*. In terms of the numbers presented in Table 30, Heard’s analysis is also consistent with the general observation that the largest number in each mode reflects the desired multiple of twelve (not to be confused with the product of twelve and its coefficient, which is not always the largest product in the mode) – with the interesting exception of Mode VIII, which lacks the expected forty-eight, and also,

⁵⁷⁵ Heard, *Alia musica*, 61.

inexplicably, Mode II. There is no consistency about which base number should be the source of these multiples of twelve, but neither is it obvious that there ought to be a consistent relationship.

Table 32 – Consistencies and Contradictions in Heard's Hypothesis

Each *maneria* contains a corresponding multiple of twelve; Mode VIII lacks this number, and it is not the largest number in Mode II.

Base Numbers	<i>Protus</i>	<i>Deuterus</i>	<i>Tritus</i>	<i>Tetrardus</i>		
Authentic	12:	12	36	24	36	48
	6:	6	24	18	24	48
	8:	8	24	24	24	48
	9:	9	18	36	18	36
Plagal	12:	12	24	24	36	
	6:	18	18	36	18	24
	8:	16	24	24	24	48?
	9:	18	18	36	18	36

The idea that each *maneria* should be linked to a particular multiple of twelve has an interesting side effect. The revisor and commentator routinely speak of these numbers as though they represent specific pitches relative to the *finales* of the modes, and not merely as abstract representations of intervals that could appear somewhere within the mode. If these numbers truly were understood to represent pitches, it is difficult to understand how all the numbers across all eight modes could be understood to overlap in a single gamut. Thus, this interpretation might imply that the modes were not understood to overlap in a single gamut, even if the octave species associated with them did so. That is, the association of octave species with modes may, at least in the older parts of the *Alia musica*, have been understood as a mere convenience for the purpose of demonstrating interval structures, rather than any understanding that the finals of modes were literally higher or lower than each other.

The Octaves Hypothesis

Table 33 – Chailley’s interpretation of the relationship between maneriae and modal product numbers

Greek	Protos	Deuterus	Tritos	Tetartos
Translation	“First”	“Second”	“Third”	“Fourth”
Interpretation	First octave	Second octave	Third octave	Fourth octave
	6 12	12 24	24 48	48 96

In contrast to Heard’s hypothesis, Chailley proposes that each *maneria* occupies a distinct octave. He suggests that *protus* occupies the octave between six and twelve, *deuterus* the octave between twelve and twenty-four, *tritus* between twenty-four and forty-eight, and *tetrardus* between forty-eight and ninety-six.⁵⁷⁶ Chailley’s interpretation does not reflect characteristics of Table 30; nearly each mode includes one of the two numbers that define its range, but only Modes I and III actually include both, and Modes II, VII, and VIII include numbers that do not fall within the expected range; Mode VIII does not include any numbers within the expected range. Nowacki apparently accepts Chailley’s interpretation,⁵⁷⁷ without acknowledging these inconsistencies.

Table 34– Consistencies and Contradictions in Chailley’s Hypothesis

Base Numbers	Protus	Deuterus	Tritus	Tetrardus		
Authentic	12: 12	12	36	24	36	48
	6: 6	24	24	18	24	48
	8: 8	24	24	24	24	48
	9: 9	18	36	18	36	96?
Plagal	12: 12	24	24	24	36	
	6: 18	18	36	18	24	
	8: 16	24	24	24	24	
	9: 18	18	36	18	36	
	6?: 6?	12?	48?			48? 96?

⁵⁷⁶ Chailley, *Alia musica*, 17.

⁵⁷⁷ Nowacki, *Greek and Latin Music Theory*, 112.

Chailley also proposes that the plagal modes must share at least one of either the largest or the smallest number with their corresponding authentic modes.⁵⁷⁸ He draws this conclusion from the description of the plagal modes as “*a latere*”⁵⁷⁹ (“at the side of”) the authentic modes. There is nothing in the text of the *Alia* that would confirm this reading, and *a latere* is a very common description of the plagal modes that is usually understood to refer to their shared characteristics with the corresponding authentic modes. It is true that for each *maneria*, either the largest number or the smallest exists in the counterpart mode. However, there is no consistency about whether it is the highest of the lowest number, which makes it difficult to assess the significance of this correspondence; it is not even consistent within a single *maneria*: in *protus*, for instance, it is the largest number in the authentic that is shared as the smallest number in the plagal (Table 35).

Table 35 – Shared extreme numbers across maneriae. Note that some of these numbers arise more than once in each mode.

		Protus	Deuterus	Tritus	Tetrardus
Authentic	Lowest			^b 8 × 3 = 24	^c 6 × 3 = 18
	Highest	^a 12 × 1 = 12	^b 8 × 3 = 24	^d 9 × 4 = 36	^d 9 × 4 = 36
Plagal	Lowest	^a 12 × 1 = 12		^b 8 × 3 = 24	^c 6 × 3 = 18
	Highest		^b 8 × 3 = 24	^d 9 × 4 = 36	^d 9 × 4 = 36

Meanwhile, if one were to assert only that the two modes of each *maneria* must share any common product, and not exclusively the highest or lowest, the same shared products would arise, with the addition of the number eighteen, which would be present in *deuterus* and *tetrardus*. Not only that, but each common number between modes can arise from the very same coefficient-base pair. Moreover,

⁵⁷⁸ Chailley, *Alia musica*, 17.

⁵⁷⁹ Chailley, *Alia musica*, 87.

most of the *maneriae* share more than one such commonality (*tetrardus* shares almost everything) with the exception of *protus*. More specifically, *deuterus*, *tritus*, and *tetrardus* (but not *protus*) all maintain a constant coefficient for both eight and nine within the *maneria*. Additionally, if one were to simplify the principle to maintain a consistent ratio between coefficients (that is, the same interval), rather than holding to the exact same coefficients, even the *protus* would share this commonality between the products of the base numbers 8 and 9 (the coefficients are both one in the authentic mode and both two in the plagal mode, making a ratio of 1:1 in either case). This relationship even holds for the first two presentations of *tetrardus* (it cannot, however, be true for the third presentation of Mode VII, since there is no corresponding presentation for Mode VIII). See Table 36.

Table 36 – Fixed ratios between multiples of eight and nine within each *maneria*.

		<i>Protus</i>	<i>Deuterus</i>	<i>Tritus</i>	<i>Tetrardus</i>
Authentic	b	$8 \times 1 = 8$	$8 \times 3 = 24$	$8 \times 3 = 24$	$8 \times 3 = 24$
	d	$9 \times 1 = 9$	$9 \times 2 = 18$	$9 \times 4 = 36$	$9 \times 2 = 18$
					$8 \times 3 = 24$
				$9 \times 4 = 36$	
Plagal	b	$8 \times 2 = 8$	$8 \times 3 = 24$	$8 \times 3 = 24$	$8 \times 3 = 24$
	d	$9 \times 2 = 9$	$9 \times 2 = 18$	$9 \times 4 = 36$	$9 \times 2 = 18$
					$8 \times 3 = 24$
				$9 \times 4 = 36$	
Ratio		1:1	3:2	3:4	3:2, 3:4

Ultimately, the preceding observations are effectively the result of data mining. There is nothing in the text of the *Alia musica* to suggest that the relationship shown in Table 36 is to be expected. This relationship is, however, more consistent and systematic than the one suggested by Chailley.

But what about Chailley's principal hypothesis, that the *maneriae* each get their own octave ranges, which does not even seem to reflect the values in the tabular summary (Table 30)? The key to

understanding this hypothesis is that the modal numbers are presented several times within the *Alia musica*. They are presented in the source text and its revised edition; they are also discussed in the commentary and are summarized in a table, and then again in prose. These presentations are mainly consistent with each other, but there are minor differences, particularly in presentation. The most idiosyncratic of the presentations is the prose summary after the table, which generates additional numbers for each mode by summing together the numbers already present in the chart (more will be said about this procedure, which also appears in the source treatise, in Chapter 14). One such number is presented for Mode VII, which is one hundred forty-four; this is expressed as the sum of forty-eight (as a multiple of twelve) against the sum of the two other instances of forty-eight (as multiples of six and eight); Chailley interprets this phrasing to imply that the multiples of six and eight should be taken together to imply the number ninety-six (*i.e.*, $4 \times 12 : [\{6 \times 8\} + \{8 \times 6\}] = 48 : 96 \Rightarrow 48 + 96 = 144$), which he sees as one of the boundary numbers of the *tetrardus* modes, even if this number is not explicitly cited.⁵⁸⁰

Both Heard's and Chailley's hypotheses have their merits, but neither is entirely consistent with the complete set of numbers presented in the *Alia musica*. In view of these problems, I shall present an alternative hypothesis in Chapter 19, after a closer look at how the numbers are presented and demonstrated in the treatise and how they relate to the chants cited in the treatise as good examples of the modal numbers in context.

⁵⁸⁰ Chailley, *Alia musica*, 179.

Chapter 14: The Numerology of the Oldest Layers

This chapter will examine the numerology of the *Alia musica* more closely, focussing on the presentation in the oldest layers of the treatise, especially the source treatise that is appended to the end of the composite treatise. Heard and Mühlmann have previously presented examinations of this topic, but their presentations have problems, and so this chapter will begin by following their discussions, critiquing as necessary.

Heard's discussion of the numerology of the *Alia musica*

Heard begins his explanation with the observation that the “basic numbers” are drawn from Boethius' *De institutione arithmetica*, chapter 48.⁵⁸¹ These basic numbers to which he refers are not yet 6:8:9:12, but rather, 2, 3, 4, and 6. So far, so good; these numbers are, indeed, central to that chapter of Boethius' work⁵⁸² – the numbers drawn from Boethius' original demonstration of the harmonic mean, by presenting the two smallest relations than can be divisible at the harmonic mean: 2:3:6 and 3:4:6. The *Alia musica* does not present these relations directly, however, but doubles them to 4:6:12 and 6:8:12; I shall return to this point shortly.

Heard goes on to note that these numbers (2, 3, 4, and 6) also comprise four of the first six numbers of the harmonic series. This second observation is problematic. In the first place, the harmonic series is most formally described in the form $\frac{1}{n}$; it would then be more accurate to say that these basic numbers comprise the reciprocals of four of the first six numbers of the harmonic series; this, however, is a minor technicality. More importantly, the reciprocals of the first n terms of the harmonic series are simply the first n natural numbers; that is, saying that these numbers comprise four of the first six terms of the

⁵⁸¹ Heard, *Alia musica*, 53.

⁵⁸² Boethius, *De institutione arithmetica*, bk. 2, ch. 48, ed. Friedlein, 155–58, trans. Masi, 177–79.

harmonic series is equivalent to saying that they comprise four of the first six numbers of our counting system. To ascribe significance to such a fact is equivalent to ascribing significance to the fact that the name Becca contains four of the first five letters of the alphabet: accurate, but not meaningful.

Heard explains that these basic numbers are then multiplied by two and three to produce the additional numbers 4, 6, 8, 9, 12, and 18; these are then multiplied again to achieve even larger numbers.⁵⁸³ This description, of course, is just basic number theory; the vast majority of all small non-prime numbers can be created by multiplying iteratively by twos and threes. More importantly, everything that Heard has described up to this point has been speculation into the procedures that the author of this passage has already taken before beginning to write; as previously noted, the *Alia* does not give the numbers 2, 3, 4, and 6 and then double them, it begins with them already doubled.

Furthermore, while this presentation provides a plausible explanation for how a theorist might get from the chapters in *De institutione arithmetica* to the numbers presented in the *Alia*, in this discussion, Heard makes no mention of the harmonic mean that these numbers are used to illustrate in the introduction to the *Alia*. The harmonic mean appears to be the reason why the *Alia* invokes these numbers in the first place, and in combination with the arithmetic mean, is, as I have already argued, the principal means by which the *Alia* justifies the importance of the relation 6:8:9:12.

Next, Heard describes the five perfect intervals whose ratios can be created from the numbers one through four (see Chapter 09), then passes to one of the most unusual aspects of the arithmetical procedures in the *Alia musica*: the *Alia* usually identifies all of the multiplex ratios – those that take the form $n:1$ – using exclusively the antecedent term (sometimes multiplied by six); thus, 2 represents the octave (2:1), 3 represents the twelfth (3:1), and 4 represents the double octave (4:1). However, the *Alia* refers to the two superparticular ratios – those of the form $n + 1:n$ – by the sums of their terms (*i.e.*, the

⁵⁸³ Heard, *Alia musica*, 53.

perfect fifth is identified by the number 5, which is the sum of the two terms in the ratio 3:2, while the perfect fourth is identified by the number 7, the sum of the terms of 4:3).⁵⁸⁴

At a glance, the method of identifying an interval by the sum of the terms in its ratio is an odd and unexpected one. There is no procedure in conventional mathematics that calls for adding together the terms of a ratio. However, in a musical context, there is a way in which these numbers can, in some sense, represent their intervals, in that these numbers reflect the procedures for demonstrating the intervals abstractly on the monochord.

Although the word “monochord” literally means “one string,” the use of a monochord to produce specific pitches, intervals, or melodies presupposes multiple strings,⁵⁸⁵ or less directly, a single string in differing states at multiple points in time. That is to say that to produce a particular octave on a monochord, one must, in fact, have two strings, with the bridges set so that one string is twice as long as the other (or, on a single string, one must first play one note, then move the bridge to make the string half as long and play the second note); this process will produce an octave with respect to any particular first pitch (such as with respect to the entire available string length of the monochord, or an octave above any particular specific pitch previously constructed on the monochord). In this procedure, the octave is produced by playing two different notes in succession on the same side of the bridge whose string lengths are in the ratio 2:1. When Guido criticizes the use of the monochord to learn to sing songs that one has never before heard,⁵⁸⁶ this is likely the kind of procedure he means, and it was certainly the procedure that Boethius used to map the entire gamut on a monochord.⁵⁸⁷

⁵⁸⁴ Heard, *Alia musica*, 53–54.

⁵⁸⁵ Ptolemy, for instance, criticizes the “monochord canon” and describes the use of an “octochord canon” (Ptolemy, Ἀρμονικά, bk. 2, chs. 12–13 & bk. 1, ch. 11, ed. Düring, 66–70 & 25–28, trans. Barker, 340–45 & 298–301).

⁵⁸⁶ Guido, *Epistola ad Micahalem*, ed. & trans. Pesce, 458–61.

⁵⁸⁷ Boethius, *De institutione musica*, bk. 4, chs. 4–5, ed. Friedlein, 314–17, trans. Bower, 126–31.

However, if the intention is merely to demonstrate the abstract concept of an octave, without reference to any particular starting pitch, this objective can most easily be accomplished on a single string by dividing the string into three equal parts and placing the bridge at one of the two dividing nodes, leaving the lengths of string on opposite sides of the bridge in the ratio 2:1. The octave is then played by plucking the string on both sides of the bridge, producing an octave whose lower pitch is a perfect fifth higher than the lowest pitch that can be produced on the monochord. Under this procedure, it makes a certain amount of sense to label the interval according to the number of parts into which the string must be divided. To produce a perfect fourth, one must divide the string into seven equal parts, so that three parts can be on one side of the bridge and four parts on the other side. Likewise, to produce a perfect fifth, one must divide the string into five parts, so that two parts can be on one side and three parts on the other (Figure 32).⁵⁸⁸

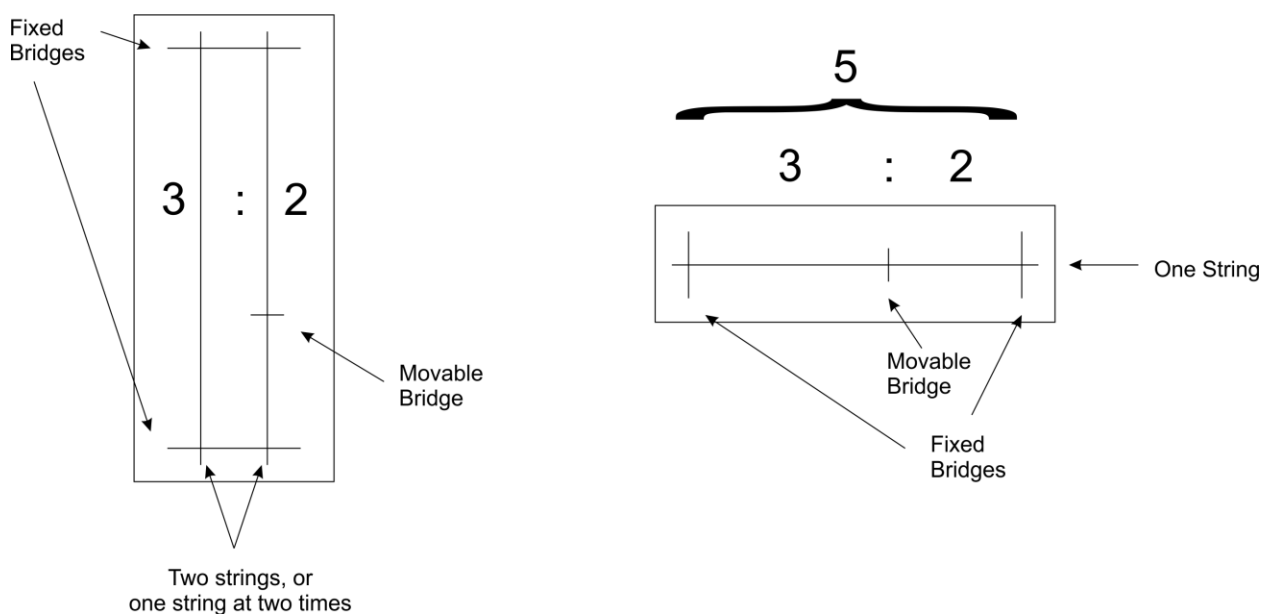


Figure 32 – Two ways to use a monochord or similar canon to produce a perfect fifth.

A fifth can be produced on two strings (not a literal monochord) or the same string at two separate times, by playing first the entire length of the string, and then only two-thirds of the length of the string; alternatively, a fifth can also be produced by dividing the string into five parts, placing a bridge with three parts on one side and two parts on the other, and playing the string on both sides of the bridge. This second manner of using the monochord provides some support for using the sum of the two terms of the ratio as a label for the interval since the string must be divided into that many parts.

⁵⁸⁸ Ptolemy describes this procedure for using the monochord (Ptolemy, Ἀρμονικά, bk. 1, ch. 8, ed. Düring, 18–19, trans. Barker, 292–93), though he does not use the total number of parts into which the string is divided as a label for the interval created.

As Nowacki points out, this procedure of demonstrating consonances outside the context of any scale is presented in Boethius' *De institutione musica*, Book IV, Chapter 18.⁵⁸⁹ Boethius does not, however, label the resulting intervals according to the number of string segments needed for the demonstration.

Of course, this manner of labelling intervals neglects the fact that dividing the string into five equal parts, as when creating a perfect fifth, can also produce a double octave by placing only one part on one side of the bridge and the remaining four parts on the other side. Thus, the number five might justifiably represent the double octave just as appropriately as the perfect fifth (Figure 33). However, as already suggested, the *Alia* does not use this method to label multiplex ratios. As a result, each of the five perfect intervals can be represented by unique single numbers: the octave (2); the twelfth (3); the double-octave (4); the fifth ($3 + 2 = 5$); and the fourth ($4 + 3 = 7$). Curiously, though, the perfect fifth is sometimes represented by the number ten (double the usual number five); the fourth, which is otherwise treated much like the fifth, does not get this special treatment.

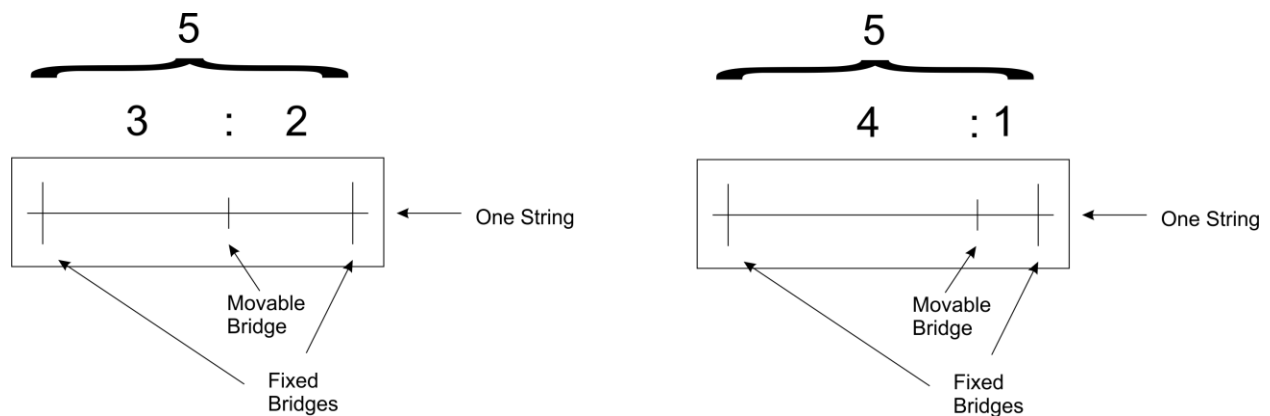


Figure 33 – Two different intervals that can be represented by the number five.

When using the monochord in the second manner presented in Figure 32, the string is divided into five units (and the resulting interval may, therefore, be represented by the number five). Since the movable bridge may be placed either one or two units from a fixed bridge, the ratios 3:2 (perfect fifth) and 4:1 (double octave) are both possible; thus, both intervals may be represented by the same number. The *Alia musica*, however, reserves the number five for the perfect fifth only.

⁵⁸⁹ Nowacki, *Greek and Latin Music Theory*, 111; cf. Boethius, *De institutione musica*, bk. 4, ch. 18, ed. Friedlein, 348–49, trans. Bower 160–61.

To return to Heard's examination of the numerology of the *Alia*, Heard proceeds with a description of Aristoxenus' six divisions of the tetrachord⁵⁹⁰ (which divide the tetrachord not only in the three standard genera – diatonic, chromatic, and enharmonic – but into multiple shades of each genus; see Chapter 05). As slightly adapted by Ptolemy (and transmitted to the Carolingians by Boethius), Aristoxenus divided the perfect fourth into sixty perceptually equal increments (in modern terms, about 8.3 cents each, with twelve increments per semitone). He then outlined six shades of genera, as shown in Table 37.

From here, as Heard explains, the *Alia* takes the largest numbers from each division (*i.e.*, the rightmost column of Table 37) and assigns each to a particular perfect consonance, excepting only the number forty-four (from the Soft Chromatic), which is not divisible by six.⁵⁹¹ Of course, the result is to assign a perfect interval to each multiple of six from twenty-four to forty-eight. Interestingly, although the *Alia* assigns five numbers to perfect intervals, and there are traditionally five perfect intervals, the correspondence presented in the *Alia* does not assign a number to the perfect twelfth, and in Heard's summary, the double-octave is used twice (Table 38).

Table 37 – Aristoxenus' divisions of the tetrachord across three genera and six shades

Genus	Shade	Interval Size (increments of 8.3 cents)		
		Lowest	Middle	Highest
Enharmonic	n/a	6	6	48
	Soft	8	8	44
Chromatic	Hemiolic	9	9	42
	Tonic	12	12	36
Diatonic	Soft	12	18	30
	Tense/Sharp/Syntonic	12	24	24

Heard's description is a mostly accurate summary of a passage in the *Alia* that directly precedes the description of the fourth *tonus* in the revision. However, Heard seems to ignore the fact that the *Alia*

⁵⁹⁰ Heard, *Alia musica*, 54.

⁵⁹¹ Heard, *Alia musica*, 55.

makes no use of these numbers outside this passage, and these numbers, therefore, have no particular relevance to the way in which numbers are applied to the mode. Indeed, Chailley observes of this passage that “*ces spéculations dénuées de toute valeur musicale ont sans doute pour seul but de fournir à l’auteur des bases symboliques plus importantes à ses yeux que la réalité.*”⁵⁹² (“These speculations, stripped of all musical value, doubtless have for their sole objective to furnish the author with symbolic bases more important to his eyes than reality.”) While this passage says much about the revisor’s willingness to bolster his arguments with the prestige and authority of Greek theory using elements of Greek theory that are otherwise irrelevant to medieval music (or, perhaps, elements of Greek theory that the author has not correctly understood),⁵⁹³ it yields minimal insight into the application of harmonic numbers to modes.

Table 38 – Intervals vs. multiples of six, perhaps in relation to the largest intervals of each of Aristoxenus’ divisions of the tetrachord (compare the leftmost column of this table to the rightmost column of Table 37).

Number	As a multiple of 6	As derived in the <i>Alia musica</i>	Assigned interval
48	8×6	$48 = 2 \times 24 = 4 \times 12$; $48 = 6 \times 8$; $48 = 8 \times 6$ ⁵⁹⁴	n/a ⁵⁹⁵
42	7×6	$(2 \times 9) : (4 \times 6) = 18 : 24$ ↳ $18 + 24 = 42$	Perfect Fourth
36	6×6	$(2 \times 6) : (4 \times 6) = 12 : 24$ ↳ $12 + 24 = 36$	Perfect Octave
30	5×6	$(2 \times 6) : (2 \times 9) = 12 : 18$ ↳ $12 + 18 = 30$	Perfect Fifth
24	4×6	$24 = 2 \times 12$; $24 = 4 \times 6$; $24 = 3 \times 8$	Double Octave

⁵⁹² Chailley, *Alia musica*, 146.

⁵⁹³ Chailley, *Alia musica*, 144–45.

⁵⁹⁴ The presentation here appears redundant to the modern eye based on the familiar commutative property of multiplication (order is irrelevant in multiplication). However, in the original Latin in which the passage is written, the two numbers are not perceived as equivalent: the second number is an ordinal number, the value to be replicated, and the first is an adverbial (or multiplicative) number, specifying the number of iterations in which the second number should be added to itself: “*sexies 8 [= 8 + 8 + 8 + 8 + 8 + 8] et octies 6 [= 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6]*” (*Alia musica* §81(a), ed. Chailley, 145).

⁵⁹⁵ According to Heard, this interval is assigned to the double octave (Heard, *Alia musica*, 55), resulting in both twenty-four and forty-eight corresponding to the double octave. However, the passage in question reads, “*48 sunt bis 24, id est bisdiapason*” (*Alia musica* §81(a), ed. Chailley, 145); (“48 is twice 24 – that is, the double octave”). I agree with Chailley’s assessment that “*id est bisdiapason*” is intended to modify the number twenty-four, not forty-eight.

At this point in Heard's analysis, he begins to lean heavily on Mühlmann, and so it will be useful to examine Mühlmann's analysis. Mühlmann says that the old Greek Dorian mode was characterized by the tones e, b, a, and E;⁵⁹⁶ he neither explains nor supports this assertion. Certainly, the pitches E and e make sense, as they are the boundaries of ancient Greek Dorian octave species. However, although the relation 6:8:9:12 is important in Greek theory and could correspond to a fundamental, fourth, fifth, and octave, I am not aware of any Greek treatise that identifies the fourth and fifth degrees as particularly important within an octave species (or any other Greek formulation of mode). The closest of which I am aware is found in Gaudentius, who described octave species as concatenations of species of fourth and fifth; however, even in this case, only one of the two middle pitches would be the boundary point between these two species. The most that one can say is that the four pitches that Mühlmann identifies are the four fixed pitches that appear within the Dorian octave species. It should be noted that if this correspondence is the intended interpretation, this manner of characterizing a mode is not easily applicable to other modes: most of the Greek octave species are not bounded by fixed pitches. It is entirely possible that Mühlmann is simply projecting the procedures implied in the *Alia musica* backward onto the old Greek modes.

Regardless, Mühlmann proceeds to say that a particular presentation of these pitches was derived "*in der Spätantike*"⁵⁹⁷ ("in late antiquity") from an undertone series originating two octaves and a tone above the top of the Greater Perfect System (as Mühlmann puts it, four octaves above *ὑπάτη ὑπάτων*). Once again, Mühlmann does not identify his source, and the dating to "late antiquity" is far too vague to suggest where to look. It is possible that Mühlmann does not mean that he finds this sequence in any particular treatise, but that he surmises from the treatment in the *Alia* and other Carolingian sources that

⁵⁹⁶ Mühlmann, *Die Alia musica*, 30. Mühlmann uses a different convention for octaves and note naming, and gives the sequence e' : h : a : e; here and throughout, I shall convert to Odonian notation.

⁵⁹⁷ Mühlmann, *Die Alia musica*, 30.

such a sequence must have been derived in late antiquity. As for the sequence itself, the undertone series is less well known today, but is the reciprocal of the more familiar overtone series: while the overtone series generates pitches for whole-number multiples of a frequency, the undertone series generates pitches for whole-number multiples of wavelength (for instance, string length). Although the sequence is conceptually produced from the pitch assigned to the number 1 (two octaves above the top of the Greater Perfect System), the position of this pitch appears to be *post hoc*: there is no apparent reason why the mathematical treatment should begin two octaves above the system; in fact, if the goal is to identify all fixed pitches with whole numbers, one would need to start another octave higher again. Instead, it seems likely that the governing principle behind this series is for the prototypical tone in the middle of the relation 6:8:9:12 to correspond to the tone between the pitches \flat and a (in Greek terms, the tone of disjunction between μέση and παραμέση). Consequently, \flat would correspond to the number 8, which would be successively halved until the number 1 is reached, and then the rest of the sequence can be derived from that point. The sequence that Mühlmann presents then provides every undertone in the pitch-classes of A, B, and E (and omitting those in other pitch-classes) down as far as the number 16 (interestingly, he stops just short of the end of the Greater Perfect System on A, which would be given as 18). The complete sequence is given in Figure 34:

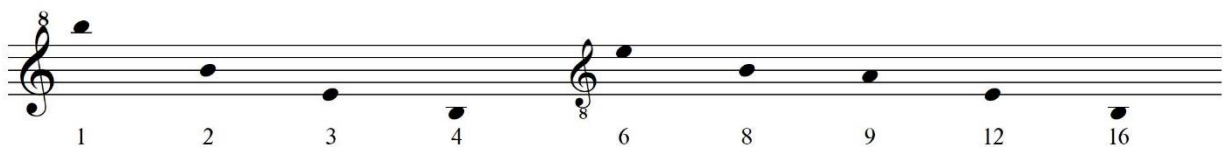


Figure 34 – The undertone series presented by Mühlmann as representing the Greek Dorian mode

Mühlmann next argues that this sequence must have survived into the Carolingian period because a table survives in manuscript M that derives the gamut numerically in a way that is consistent with these numbers. The table to which he refers appears on fol. 173v in the margins of the end of the *Scolica enchiridis* and the beginning of the *Commemoratio brevis*, a few *folia* before the *Alia musica*. The top portion of this table is reproduced here as Facsimile 13.

It is true that the sequence of undertones presented in Figure 34 from eight onwards corresponds perfectly with the whole numbers presented in Facsimile 13, though the latter also provides fractions in order to include every available pitch. However, the correspondence is probably nothing more than a coincidence resulting from the fact that both sequences begin from B. In the case of the undertone series presented in Figure 34, B is an ideal starting point because it places the fixed pitches E, a, b, and e that characterize the Dorian mode (according to Mühlmann) in the relation 12:9:8:6 in the undertone series. In the case of the gamut presented in Facsimile 13, B may have been selected as the starting point simply because it is represented by G in its own notation (the instrumental gamut), and G is the last letter in the seven-letter series by which notes are named (an intuitive place to start or end); there is no indication in either source that it has anything to do with the Dorian mode. The table presented in pseudo-Bernelinus adds yet another complication because it labels the position of νήτη ὑπερβολαίων and μέση but places both of them one pitch higher than their correct positions by function (on Bs rather than As – that is, G rather than F in the instrumental gamut nomenclature).

Mühlmann then explains that when the octave species are first described in the *Alia* (§15), the application of Greek ethnic names to octave species becomes reversed, making the Hypodorian the lowest rather than the highest within effectively the same gamut.⁶⁰⁰ Because of its new position in the gamut, 12:9:8:6 should switch from representing E:a:b:e to representing D:G:a:d. However, Mühlmann indicates

⁶⁰⁰ I discuss the progression from Greek octave species through to the medieval modes in extensive detail in Section II, especially Chapter 05. To reiterate in brief, Mühlmann's explanation is oversimplified. The octave species names originally ran in the opposite direction, with Hypodorian at the top. However, each octave species can be brought into a common tessitura by transposing the entire gamut. These transpositions came to be the principal characteristic of a mode and began to behave like keys signatures. These key signatures are a reciprocal concept to the octave species; as a result, the names of keys ascend as the corresponding names for the octave species descend. Thus, the order of the ethnic names has already reversed in Greek times. When Boethius describes these concepts, he gives the names only to the keys, but describes the keys as originating from the octave species. Consequently, when the names become applied to the octave species in the *Alia* (or any source that it may have drawn upon for this passage), Boethius' order for the ethnic names of the keys gets mapped upon the octaves species within the same gamut in the same order, this time without accounting for the reciprocal nature of the paradigm.

instead that it came to represent $\frac{3}{2}:e:d:a$ due to a kind of conflation of the Dorian and Hypodorian, producing a comparable undertone series over two octaves, from 6 through 24, in which the lower octave gives the fundamental, fourth, fifth, and octave of the Hypodorian octave species (accurately, with respect to later conceptions) and the upper octave gives the fundamental, fourth, fifth, and octave of the Dorian octave species (differing from later theory), thus separating the plagal and authentic modes by an octave, rather than a fourth,⁶⁰¹ see Figure 35.

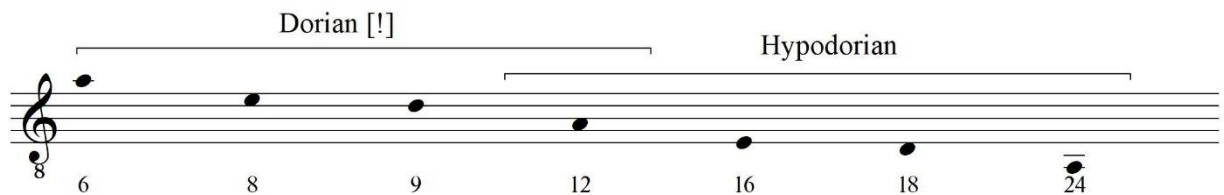


Figure 35 – A hypothetical undertone series hypothesized by Mühlmann to explain the numbers in the *Alia musica*. In this hypothesis, the authentic and plagal modes are conceptualized as an octave apart, rather than a fourth.

According to Mühlmann, his *Theoretiker der Sechs Töne* [loosely, the source treatise] then did a corpus study of the chant repertoire and recognized that with exceptions made for the *emmelis* (more about this later), the range of chants in the second mode never exceeds the perfect fifth (though the first mode does reach the octave). He therefore retains the numbers 6:12 for Dorian but redefines Hypodorian as spanning the fifth 12:18; however, he reassigns these numbers to different pitches in the gamut, placing the Dorian in its proper place, spanning D–d, with the Hypodorian now spanning Γ –D⁶⁰² (as will be demonstrated in Chapter 17, the Hypodorian actually seems to span the fifth C–G, but as these two fifths have the same interval structure, this distinction might not invalidate the argument). Finally, Mühlmann argues that the *Theoretiker* did not know of ratios for intervals smaller than the tone (8:9 or 16:18) that would need to use numbers larger than 18, and therefore, when describing modes whose boundary tones did not correspond to the Ds, Gs, and As in his new series, he simply multiplied the base numbers up into

⁶⁰¹ Mühlmann, *Die Alia musica*, 31.

⁶⁰² Mühlmann, *Die Alia musica*, 31–32.

higher ranges so that they would not interfere with each other (though he does not explain why Modes II and III overlap at the ratio 18:12).

Mühlmann's argument is not entirely implausible, but it rests upon extremely limited evidence, particularly because of his lack of citations, and much of it is almost certainly speculative. To the best of my knowledge, there is no evidence that the Greeks felt the relation 6:8:9:12 to characterize the Dorian mode, nor that the undertone series beginning from 1=B (or equivalent) was described in antiquity. The series does correspond to the table in pseudo-Bernelinus (and accompanying the *Scolica enchiridis* in M), but the correspondence is likely a coincidence from the nature of the instrumental gamut. His characterization of the flipping of direction of the order of the ethnic names is oversimplified but essentially accurate; however, he provides no explanation for why the Dorian should have appeared an octave above the Hypodorian, instead of a fourth. His characterization of the change in position of the plagal relative to the authentic reflects the numbers presented in the *Alia* but is silent about the potential discrepancy between the ranges Γ–D and C–G, and does not explain how the range of the Dorian shifted from a– $\frac{3}{2}$ to D–d. And ultimately, none of this extensive speculation is necessary to explain how the numbers 6:8:9:12 became connected to the Dorian octave, as they effectively characterize any octave in which the fourth and fifth are significant. He also ignores the fact that the most detailed illustration of the meaning of these numbers in the *Alia* (in the chant *Rorate caeli*) cannot perfectly conform to these numbers, an issue that will be described in detail later.

Perhaps the most useful observation that Mühlmann makes in this discussion is his hypothesis for the multiplication of the modal numbers into higher ranges in subsequent modes, in which he demonstrates an awareness that the various modal numbers cannot be interpreted consistently within a single gamut and that the selection of a number to represent the first pitch in each *maneria* appears to be arbitrary (with the other pitches defined in relation to that pitch).

Heard presents the key points of Mühlmann's argument. It is not entirely clear that he understands it, as he repeats it uncritically, including remarking that the *Theoretiker der Sechs Töne* must have only worked out the numbers for the first three *maneriae*, with another theorist supplying the numbers for the last two modes;⁶⁰³ this remark is consistent with Mühlmann's theory of authorship but contradictory to Heard's own (which is effectively the same as Chailley's), and Heard apparently sees no reason to tailor the explanation for consistency with his own view of authorship.

The rest of Heard's discussion of this section consists of a table that compares the numbers associated with each mode to pitches.⁶⁰⁴ This table is inconsistent about the association of a single pitch to a single number; it has already been noted here that this must be so, but unlike Mühlmann, Heard passes over this inconsistency without comment and does not explain the derivation. The table also has notable gaps on the pitch side, where pitches implied by the numbers are omitted (even where there is no reason to suppose that those numbers might be problematic) and is inconsistent with respect to octaves. Table 39 is an annotated recreation of Heard's Table IX, demonstrating the inconsistencies.

Table 39 – Mühlmann's Table IX, marked up to demonstrate inconsistencies.

Red lines connect different letter names indicated by the same number; green lines connect the same letter names created by different numbers. Blue letters are absent notes implied by numbers; orange figures are numbers from the *Alia* not included in chart; orange octagons indicate numbers in chart not cited in *Alia*.

Dorian	d' a? g d	6 8 9 12
Hypodorian	d A G	12 16 18
Phrygian	e' a? e	12 18 24
Hypophrygian	a? e	18 24
Lydian	(f') c' f	(18) 24 36
Hypolydian	c' f c	24 36 48
Mixolydian	g' d' c' g	18? 24 32 36 48
Hypomixolydian	d' c' g	18? 24? (32) 36 48

⁶⁰³ Heard, *Alia musica*, 57.

⁶⁰⁴ Heard, *Alia musica*, 58; Heard neglects to acknowledge that this table is reproduced from Mühlmann, *Die Alia musica*, 39.

The numbers presented in Table 39 are not the numbers identified in the *Alia*. Instead, these numbers are calculated anew from the intervals identified in the *Alia*, standardized against the hypothesis that the *finalis* of each *maneria* is always the next multiple of twelve. Although in the first four modes the numbers are the same as those presented by the *Alia*, there are discrepancies in the other modes. In addition, most of the numbers recur in multiple *maneriae* and represent different pitches in each *maneria*, sometimes with three different pitches for one number; furthermore, several pitches recur that correspond to different numbers. This table helps to demonstrate how inconsistent the numerology is but is also compromised by Heard's manipulation of the numbers.

Heard ends his discussion with another table summarizing another set of numbers associated with each mode, which is borrowed from Mühlmann,⁶⁰⁵ and Heard again does not clearly explain the derivation. Furthermore, the numbers presented are different from the previous table and again do not reflect the numbers presented in the *Alia*; these numbers turn out to be nothing more than Mühlmann's final undertone series segment from 6 to 18 multiplied in turn by one, two, three, and four to give a single combined series for each of *protus*, *deuterus*, *tritus*, and *tetrardus* (Figure 36). In the case of Figure 36, there are considerably more numbers than are actually cited in the *Alia musica* for these modes, but at least it is true that nearly all of the numbers cited in the *Alia* appear in this figure (the one notable exception is the 18 from *tetrardus*). Heard also notes that Mühlmann had removed the B-flats from *tritus*, but he quite reasonably argues that there is no apparent justification for doing so since the frequent use of b is a noted characteristic of *tritus* chants. Heard, therefore, restores the B-flats. Both Mühlmann and Heard also extend the *tetrardus* series one position further to the fourth multiple of 24, without explaining why; this is particularly odd, since the numbers cited in the *Alia* do not even extend to the last two numbers already presented in this series, so that there would appear to be no reason to extend the series

⁶⁰⁵ Heard, *Alia musica*, 62; Mühlmann, *Die Alia musica*, 40.

yet further. Admittedly, the number 96 does arise in the bizarre mathematical procedures of the *Alia*, but never directly as a modal number.

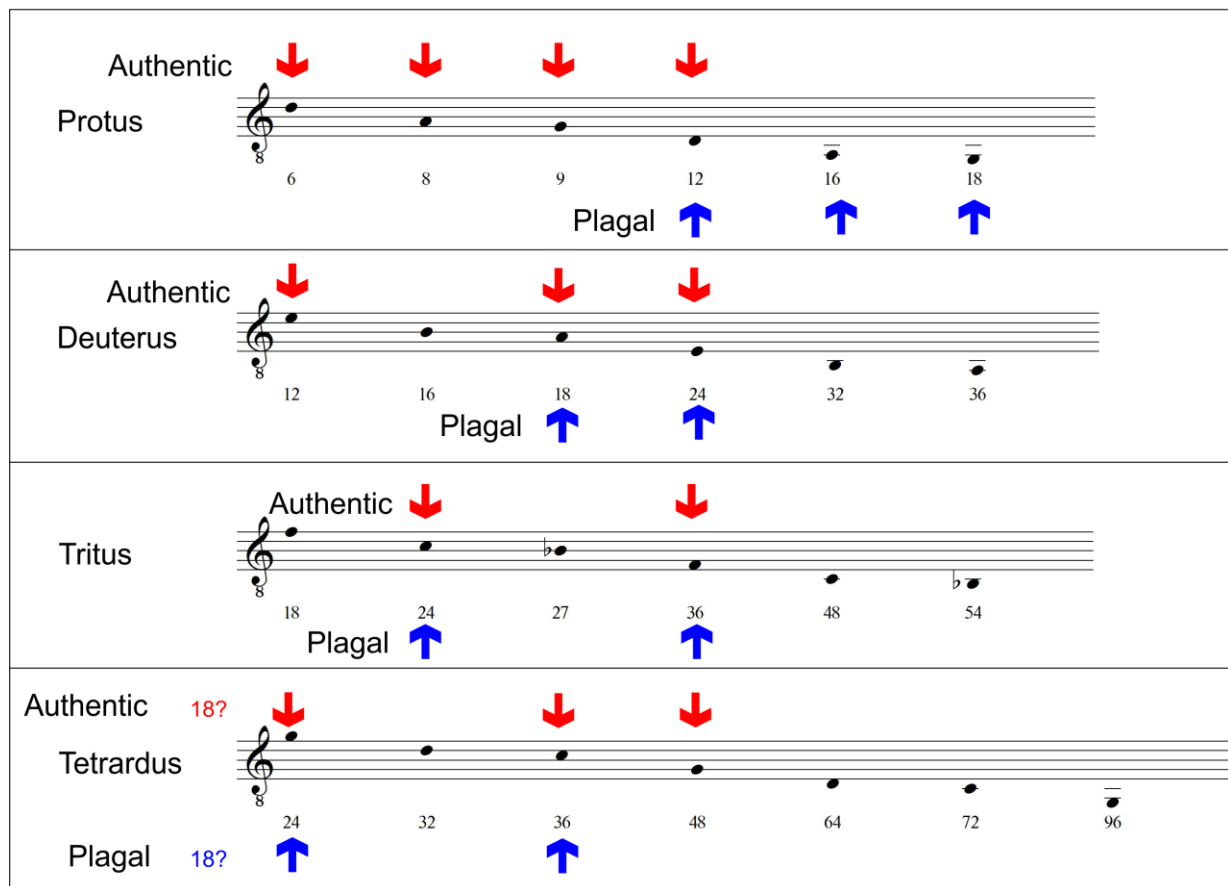


Figure 36 – Annotated recreation of Heard's Table 12

The original table is given in letter notation, and lacks any acknowledgement of which pitches are actually cited in the *Alia*. Each consecutive series is the next multiple of the first series. Heard has restored the B-flats in *tritus* relative to Mühlmann's original presentation. No justification is given for extending *tetrardus* one position further.

Unfortunately, although this presentation seems to be a reasonable description of modes in principle (except for the relations between modes, dominated as it is by the assumption that the numbers of each *maneria* must be corresponding multiples of the initial series), nearly half of these numbers are not derivable from the *Alia musica* itself. Ultimately, Heard's discussion leaves the reader largely unenlightened about the significance of these modal numbers.

Towards an explanation for the source of the modal numbers of the *Alia musica*

From where, then, are the modal numbers actually derived? An important clue may be found in Aurelianus' *Musica disciplina*. In his discussion of the ecclesiastical modes, he makes an observation that is not common in later medieval discussions of mode: each mode is associated with a particular interval from amongst the intervals found in the relation 12:9:8:6. Aurelianus, drawing upon the famous story about Pythagoras and the hammers, says:

Sint uerbi causa quattuor mallei, qui subter insertos contineant numeros: xii, viiii, viii, vi. Hi igitur mallei qui xii et vi ponderibus uergebant diapason in duplo consonantium concinebant ut hic :

Ā Inclina Domine aurem tuam,

et omnia quae in primo inueniuntur tono.

Malleus xii ponderum ad malleum viiii et malleus viii ad malleum vi ponderum, secundem epitritam proportionem, diatessaron consonantiam perficiebant. Adest exemplum:

Ā Confessio et pulchritudo,

et cuncta que in tono autentici deuteri conscribuntur.

Viii uero ponderum ad vi, et xii ad viii, diapente consonantiam permiscebant, ueluti hic:

Ā Circumdederunt me,

et cetera que in autentico trito inueniuntur.

Viii uero ad viii, in sesquioctava proportione resonabat tonum, iuxta illud:

Ā Puer natus est nobis,

et omnia quae autentici tetrardi adscribuntur norme.⁶⁰⁶

Let there be, for example, four hammers, which comprise the numbers inserted below: twelve, nine, eight, and six. Those hammers, then, that were in the weights of twelve and six would sing together an octave, in the duplex consonance, as thus: the antiphon *Inclina domine aurem tuam*, and all those that are found in the first *tonus*.

The hammers of weights twelve to nine and the hammers of weights eight to six would bring about the consonance of a perfect fourth, according to the sesquitercian [4:3] proportion. Here is an example: the antiphon *Confessio et pulchritudo*, and all that were composed in the second authentic *tonus*.

⁶⁰⁶ Aurelianus, *Musica disciplina*, ch. 2, ed. Gushee, 62–63, trans. Ponte, 8. Note that Aurelian attributes these intervals to the four authentic modes. The succeeding passage introduces the plagal modes, but neither attributes these same intervals nor any others to the plagal modes, nor does he explicitly reject association of these intervals equally to the corresponding plagal modes.

Truly, those of weights nine to six, and twelve to eight, would mix together in the consonance of a perfect fifth, as thus: the antiphon *Circumdederunt me*, and the others that are found in the third authentic.

Truly, nine to eight would resound at the tone, in the sesquioctave proportion, like this: the antiphon *Puer natus est nobis*, and all those that are written to the fourth authentic pattern.

Aurelian, then, is telling us that all of the antiphons in each of the four authentic *toni* bear a meaningful relationship to an interval associated with each *tonus*, as summarized in Table 40:

Table 40 – Intervals associated with *toni*, according to Aurelianus

<i>Protus</i>	<i>Deuterus</i>	<i>Tritus</i>	<i>Tetrardus</i>
Perfect Octave	Perfect Fourth	Perfect Fifth	Tone
2:1 (12:6)	4:3 (12:9, 8:6)	3:2 (12:8, 9:6)	(9:8)

It is, of course, important to consider whether or not these associations have real validity, or whether they are merely the result of a quasi-mysticism that wishes to justify the four *maneriae* by associating them with the authority of the relation 12:9:8:6. However, if the latter possibility were true, and the association of intervals with *toni* were to have symbolic rather than concrete applications, then it is likely that the assignment of intervals to modes would be made in a more direct order (either ascending or descending); there is no reason why this *must* be the case, but it is the more probable procedure.

By contrast, there is good reason to suppose that these intervals had real meaning to the early concept of modality. It should be remembered that the *oktōēkhos* modal system used in Byzantine music, upon which the Latin ecclesiastical modal system was based, defined modes principally in terms of characteristic melodic gestures, and that any potential associations with octave species are seen to be implied by the gestures, rather than the gestures being constrained by the octave species. Hence, it should come as no surprise to discover that intervals, as a kind of melodic gesture, might be a defining part of the ecclesiastical modal system.

Furthermore, if the intervals were meaningfully related to the modes, one would expect the intervals to reflect other properties of the modes. Such an analysis is noticeably on less firm ground, but an argument could be made to justify some of these assignments. Such an argument might reasonably suppose that the intervals are measured upwards from the final – a supposition that requires justification and that will be challenged later in this paper, but a useful assumption for the present discussion. Any expected arbitrary ordering that associates the octave with *protus* and the tone with *tetrardus* would most likely associate the fifth with *deuterus* and the fourth with *tritus* for sheerly numerical reasons (*i.e.*, decreasing size of the interval). Alternatively, any arbitrary ordering could be proposed that associates the fourth with *deuterus* and the fifth with *tritus*, as would seem to be a better fit to characters of those two modes (because in the other solution, the fifth above the final of *deuterus* [E] and the fourth above the final of *tritus* [F] are both \flat , which would not only emphasize the same pitch in both modes, but would also increase ambiguity within a mode because of the potential for the use of \flat); in this case, a purely arbitrary numerical association would tend to identify the tone with *protus* and the octave with *tetrardus* (this time, based on increasing size of the intervals). Aurelianus' associations, however, do not fit either model, which supports the interpretation that these associations are more than merely symbolic, and instead reflect actual perceived characteristics of the modes.

As an important caveat, it should be noted that while it is tempting to argue also that the decreasing size of the intervals overall as the *finalis* ascends in the first model proposed above might reflect practical performance considerations (*i.e.*, it would be more difficult to sing an octave above a higher *finalis* than above a lower *finalis*), this particular argument is not valid: fixed frequency for pitches is not a characteristic of medieval ecclesiastical chant, and thus, the notion that the *finalis* of *tetrardus* is literally a fourth higher than the final of *protus* is misleading; any given choir singing chants in either mode would be likely to sing the *finales* of any particular mode at whatever frequency places a particular chant within the most comfortable range of the choir.

None of the preceding should be taken to argue that the association is purely practical, devoid of ideology. Indeed, it is likely that a few practical observations, when combined with confirmation bias, could easily lead to a system of half real, half spurious associations. In spite of the fact that Aurelianus' associations for *deuterus* and *tritus* seem quite reasonable, one is forced to question whether it makes any sense to say that any mode is associated with a tone. The disagreement between Aurelianus and the *Alia musica* on the appropriate assignment for *tetrardus* might also imply a tension between ideology and practical observation in this manner.

Whatever the case, the *Alia musica*, itself, would also seem to lend support to the notion that Aurelianus' association of interval and mode were more than merely symbolic, as the *Alia musica* seems to do very much the same thing. In fact, as Atkinson notes, the *Alia* borrows three of Aurelianus' four examples when selecting a list of chants to exemplify each mode.⁶⁰⁷ The only exception is Aurelianus' example for the first mode, *Inclina Domine* (perhaps because it extends down as far as the low A, while none of the *Alia*'s other examples for this mode extend below the C). Instead, the *Alia* begins the list of first-mode examples with a different introit, *Rorate caeli*, which begins very similarly to *Inclina domine* (this similarity be discussed in more detail later), which has the benefit of being able to quickly demonstrate all three of the intervals that the *Alia* associates with Mode I in a chant that is half as long and has fewer complications.

The *Alia* carries on Aurelianus' practice of characterizing modes with specific intervals, but does so in a much more complex way, and this is key to understanding the significance of the modal numbers that occupy so much of the authors' attention throughout the *Alia*. The *Alia* does several things that Aurelianus does not. The first and most obvious difference is that the *Alia musica* differentiates between the intervals associated with the authentic and plagal modes. Aurelianus does not mention the plagal modes at all in

⁶⁰⁷ Atkinson, *Critical Nexus*, 179.

his discussion of this phenomenon, leaving the reader uncertain whether the plagal modes share the associations of their corresponding authentic modes, or carry different associations that Aurelianus neglects to mention, or perhaps lack associations altogether. The *Alia musica* defines the associations of authentic and plagal modes separately, and even where they do not differ in outcome, they generally differ in derivation.

Another essential difference between Aurelianus and the *Alia musica* is in philosophical approach. For Aurelianus, the intervals are those taken from the relation 12:9:8:6; importantly, each mode is associated with only one of the four possible intervals, and consequently, two numbers, separated from the rest of the relation, suffice to define the intervallic association of some modes (*i.e.*, the first mode can be defined using the numbers twelve and six, without reference to eight or nine). For the *Alia musica*, though, it is clearly ideologically important that all four numbers in the fundamental relation be employed in the derivation of each mode (perhaps to increase the prestige and authority of the derivation). Thus, for the first mode, all four numbers are presented unmodified; this procedure allows twelve to be compared to each of nine, eight, and six, and results in the association of the octave, fifth, and fourth (not exclusively the octave, as in Aurelianus) with the first mode.

Throughout the rest of the modes, each of the four base numbers in the fundamental relation is multiplied by some coefficient, but Mode I is the only mode for which the product of all four multiplications is unique; in all other modes, at least one pair of multiplications produce identical products from different factors. Thus, in Mode IV, six is multiplied by three and nine is multiplied by two, both producing eighteen; likewise, eight is multiplied by three and twelve by two, both producing twenty-four. All four numbers of the fundamental relation are used, but only two products result, producing the ratio 24:18; this ratio corresponds to the perfect fourth, which is precisely the ratio Aurelianus tells us ought to be associated with *deuterus* (though, as already suggested, Aurelianus does not clarify whether this interval ought to be associated with the plagal mode).

However, this procedure also dissociates the intervals from any concrete relation to the base numbers 6:8:9:12 (associations taken for granted in Mühlmann's and Heard's analyses), since it allows for two base numbers to be compared in such a way that their products may produce an interval different from the interval produced by the base numbers themselves. For instance, in Mode VI, twelve is doubled and six is sextupled, yielding twenty-four and thirty-six, which relate to each other as a perfect fifth while the unmodified base numbers (six and twelve) produce an octave.

As a first approximation, then, the answer to the question about the origin and justification of the coefficients is that they are chosen to ensure that all four terms of the fundamental relation 12:9:8:6 are used in the definition of each mode, while selectively filtering out the intervals that ought not to be associated with any given mode. I shall present an hypothesis of the mechanism behind the selection of coefficients in Chapter 19, after an analysis of how the intervals created by these number may or may not relate to the chants in each mode.

Chapter 15 – Interpreting the Intervals

What, ultimately, do the modal numbers represent? The answer is not clear. At least a partial answer can be given because the author of the source treatise (and by extension, the revisor and the commentator) explicitly states that the ratios between these numbers produce intervals. The authors are somewhat less clear about how these abstract intervals relate to the chants, but by comparing the chants to the listed intervals, it is possible to infer the relationship with reasonable confidence.

The most valuable passage in this regard is found near the beginning of the source treatise, in which the author summarizes the three intervals characteristic of the first mode and identifies the location of all three intervals within the chant *Rorate caeli*,⁶⁰⁸ as well as citing a few other chants demonstrating the same intervals. The text reads:

Omnis igitur primus tonus aut ter 6 habet in dupla proportione diapason, ut est Rorate celi desuper, aut quater 5, id est 2 de 8 et 3 de 12 in sesquialtera proportione qui faciunt diapente, quod est 20, ut est et nubes pluant iustum, aperiatur ; aut ter 7, id est 3 de 9 et 4 de 12 in sesquitertia proportione, qui faciunt diatessaron, ut est terra et germinet Salvatorem. Item introitum Gaudete in Domino semper, et Iustus es Domine ; similiter antiphonae Urbs fortitudinis nostrae Sion, Iohannes autem cum audisset in uinculis opera Christi, Traditor autem dedit eis. Et hoc uidendum, quod saepe euenit, ut bis aut ter aut totum etiam in antiphonis, aut quocumque cantu primi toni fit, aut per 6 et 12 quod est diapason, aut per 5 aut 10 quod est diapente, aut per 7 totum quod est diatessaron, decurrit ; ut Urbs fortitudinis ; Iohannes autem diapente.⁶⁰⁹

Accordingly, every first mode [chant] has either three 6s in the double proportion of the octave, as is “*Rorate caeli desuper...*,” or four 5s — that is, 2 from 8 and 3 from 12 — in the sesquialter proportion, which make the fifth, which is 20, as is “*...et nubes pluant iustum, aperiatur...*”; or three 7s — that is, 3 from 9 and 4 from 12 — in the sesquitertian proportion, which make the fourth, as is “*... terra et germinet*

⁶⁰⁸ Throughout this discussion, I shall maintain the spelling of chant titles found in Chailley’s critical edition.

⁶⁰⁹ *Alia musica* §181(e)–(h), ed. Chailley, 85–86, trans. Nowacki 113–15. This passage in Chailley’s edition contains the first of many errors in his critical apparatus. He says that in the first line, “*aut ter 6 habet*” appears only in manuscript K, while “*aut 3 habebit*” appears in all other manuscripts (Chailley, *Alia musica*, 86). However, “*aut 3*” does not, in fact, appear in any manuscript. As Chailley suggests, *habebit* appears in every manuscript except K, which contains the abbreviation *hāb*. Chailley then says that in the subsequent clarification “*dupla proportione diapason*,” the word *diapason* is present only in K and is absent from all other manuscripts; in fact, all other manuscripts do contain the word *diapason*, preceded by “*id est*” (in all manuscripts except C and G – thus, presumably also A – abbreviated as *id*).

Saluatorem.” Likewise, the introit *Gaudete in Domino semper*, and *Iustus es Domine*; similarly, the antiphons *Urbs fortitudinis nostrae Sion*; *Iohannes autem cum audisset in uinculis opera Christi*; and *Traditor autem dedit eis*. And this is to be seen, that it often happens that either two or three times, or even in the entire antiphon, or in whatever chant is made of the first tone, it runs either by 6 and 12 (which is the octave), or by 5 or 10 (which is the fifth), or by 7 entirely (which is the fourth), as in *Urbs fortitudinis*; *Iohannes autem* [through] the fifth.

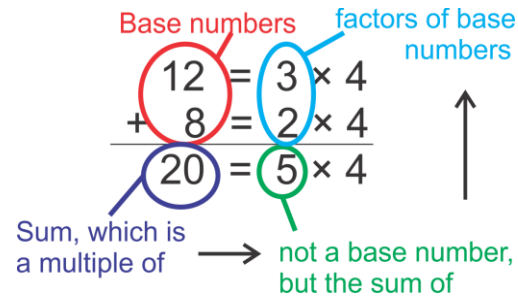
This passage is somewhat complicated and is worth parsing. Having just finished identifying the modal numbers for the first mode (which are identical to the four base numbers, 12, 9, 8, and 6), the author explains that each chant in the first mode contains the six three times, the five four times, and the seven three times. The author is not changing his mind about the appropriate coefficients for this mode (all the base numbers in this mode are multiplied by one); rather, the author is reverting to his unusual manner of describing ratios according to their sums. Thus, the double proportion 2:1 is represented in this mode by the ratio 12:6, and the sum of the two terms is eighteen, which can also be expressed as 6×3 , which is why the author says that there are three sixes (six also being one of the base numbers).

$$\begin{array}{r} \text{Base numbers} \\ \textcircled{12} \\ + \textcircled{6} \\ \hline \textcircled{18} = \textcircled{6} \times 3 \end{array}$$

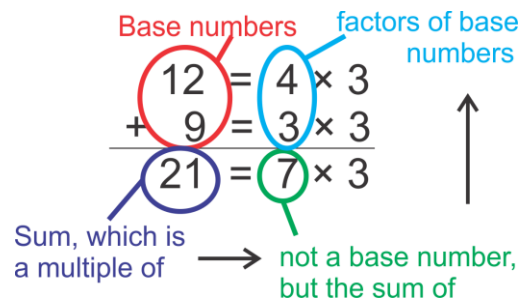
Sum, which is a multiple of → a base number

Similarly, the sesquialter proportion 3:2 is represented by the ratio 12:8; the sum of these two terms is twenty, which can be expressed as 5×4 , and although the number five is not one of the base numbers, it can be formed by adding together factors of each of the two base numbers twelve and eight — that is, dividing each by the common factor four gives the numbers three and two, respectively, which sum to five.⁶¹⁰

⁶¹⁰ This description is essentially tautological: the sum of two numbers sharing a common factor — as both nine and eight independently share a common factor with twelve — can always be expressed as a multiple of the sum of those same numbers reduced to lowest terms. This is the distributive property of multiplication: If $A + B = C$, all of which are multiples of x (so that $A = ax$, etc.), then $x(a + b) = cx$ and $ax + bx = cx$.



Likewise, the sesquitercian proportion 4:3 is represented by the ratio 12:9; the sum of these two terms is twenty-one, which can be expressed as 7×3 , and although the number seven is not one of the base numbers, it can be formed by adding together factors of each of the two base numbers twelve and nine — that is, dividing each by the common factor three gives the numbers four and three, respectively, which sum to seven:



All of this numberplay adds nothing of consequence to the description, serving as it does simply to represent the same information in multiple forms and show off the author's knowledge of number theory. What is most important about this passage is that it clearly associates the three important intervals — octave, fifth, and fourth — with three subdivisions of a specific chant: *Rorate caeli*.

The author tells us that the portion of the chant to which are set the words "*Rorate caeli desuper*" demonstrates an octave; the portion setting the words "*et nubes pluant et iustum aperiatur*" demonstrates the fifth; and the ending, setting "*terra et germinet saluatorem*" demonstrates the fourth. Unfortunately, the text *Rorate caeli* is quite popular; it appears in many different types of chants, and the author does specify which chant he intends. Of these, three are Mode I chants (identified on the CANTUS

database by the identification numbers 007177a, 006641c, and 501007);⁶¹¹ the first two are responsory verses with very similar melodic profiles, while the third is an introit. It is highly likely that the *Alia* is describing the introit, both because all of the chants that he identifies in each mode are either introits or antiphons (and he routinely begins with the introits) and because there is an immediately apparent pattern in this introit corresponding to the author's description of intervals while such a pattern is not immediately apparent in the other chants carrying this text.

CH-P 18
f. 17

CH-P 18
f. 296

D-Gsta
AB III 9
f. 10r

Sarum
#331

8ve

Ro - ra - te ce - li de - su - per, —

5th

et nu - bes plu - ant ius tum; a - pe - ri - a tur

Example 1 (cont'd on next page) – Rorate caeli, comparing several extant variations of the melody,⁶¹² and showing considerable consistency. Each system contains a discrete portion of the chant identified by the author of the *Alia musica*, and the highlighting shows that with the exception of a subtone below the tonic in the first and last systems, the three systems are each confined within (and range throughout) the three intervals associated with them.

⁶¹¹ <http://cantus.uwaterloo.ca/search?op=starts&t=rorate&genre=All&cid=&mode=1&feast=&volpiano=All>

⁶¹² The source sigla are those of the Cantus Index (<http://cantusindex.org/>), except the last, the melody of which has been transcribed from Frere, *Antiphonale Sarisburiense*. I shall use these *sigla* consistently throughout the rest of this dissertation.

ter - ra et ger - mi - net sal - va - to - rem.

In the introit *Rorate caeli*, the intervals associated with each section appear to represent the range of the musical phrase. The middle phrase quite perfectly spans a fifth, while the first and last phrase span an octave and a fourth (respectively) – provided that one ignores the subtone.

Digression: The subtone and the concept of the *emmelis*

The use of this subtone is justified elsewhere in the treatise — not in the source text, but in the revision. The revisor says, in his introduction, that:

*Denique sciendum quod post symphonias ex arbitrio musici propositas, id est diapason, diapente ac diatessaron, aliquando unus tonus ad grauem uel acutam partem additur, qui emmelis, id est aptus melo uocatur.*⁶¹³

Finally, it is to be understood that after the symphonies, which have been set forth out of the judgement of the musician⁶¹⁴ — that is, the octave, the fifth, and the fourth — sometimes one tone in the low end or the high end is added, which is called the *emmelis*, i.e., ‘suited to melody.’

In his revision of the discussion of *Rorate caeli*, he reinforces this usage, saying:

*Inter 6 et 12 qui faciunt 18 senarius est differentia, quam melodia primae incisionis ascendendo totam percurrit, a lichanos hypaton uidelicet in paraneten diezeugmenon, praemisso ad grauem partem tono qui emmelis dicitur, id est melo aptus.*⁶¹⁵

There is a different of six between 6 and 12 (which make 18), through which the first clause of the melody ascends entirely — that is, from the λιχανος ὑπάτων [D] to the

⁶¹³ *Alia musica* §29(a), ed. Chailley, 120.

⁶¹⁴ Since no particular musician has been specified, the would *musicus* here should probably be taken to mean a prototypical scholar of music, as Boethius defines the term (Boethius, *De institutione musica*, bk. 1, ch. 34, ed. Friedlein, 223–25, trans. Bower, 50–51), as contrasted against a performer.

⁶¹⁵ *Alia musica* §32(b), ed. Chailley, 120.

παρνήτη διεzeugμένων [d] — a tone having been sent forth at the low end, which is called *emmelis* (i.e., ‘suited to melody’).

Thus, while the author of the source text ignores the subtone, the revisor has noticed the problem and explained it away. He justifies this decision by reference to the authority of Greek theory, citing the *emmelis* — a term that he is, however, misusing. He has lifted the term and its gloss (*aptus melo*) from Boethius’ *Musica*, Book I, Chapter 8, where Boethius says, “*Sonus igitur est uocis casus emmelis, id est aptus melo, in unam intensionem.*”⁶¹⁶ (“*Sonus*, then, is an *emmelic* falling of a note — that is, one suited to melody — within a tuning system.”) Boethius, then, is using the term *emmelis* as an attribute that separates musical pitches from non-musical sounds or noise, which has nothing to do with the use of pitches beyond an expected range.

In fact, even Boethius used the term *emmelis* in a much broader sense than did Ptolemy (who served as one of Boethius’ principal sources). Ptolemy defines the term, similarly to “*aptus melo*,” as “ὄσοι συναπτόμενοι πρὸς ἀλλήλους εὐφοροὶ τυγχάνουσι πρὸς ἀχοίην”⁶¹⁷ (“when joined together with one another, they befall well upon the hearing.”) However, while Boethius uses the term to describe notes, Ptolemy consistently uses it to describe intervals, with *emmelic* intervals taking a place along a spectrum in his interval classification system, beneath homophones and consonances, as epimoric (i.e., superparticular) ratios beyond 4:3.⁶¹⁸ The *emmelic* intervals, then, would include intervals such as 5:4 and 6:5, which are present in some of the tuning systems that Ptolemy explores,⁶¹⁹ and even in just intonation; but among the intervals present in the Pythagorean intonation in common use in medieval theory, only the wholetone (9:8) is *emmelic*.

Nevertheless, the author of the *Alia* almost certainly learned the term through Boethius, and

⁶¹⁶ Boethius, *De institutione musica*, bk. 1, ch. 8, ed. Friedlein, 195, trans. Bower, 16.

⁶¹⁷ Ptolemy, Ἀρμονικά, bk. 1, ch. 4, ed. Düring, 10, trans. Barker, 284.

⁶¹⁸ Ptolemy, Ἀρμονικά, bk. 1, ch. 7, ed. Düring, 16, trans. Barker, 290. Barker translates *emmelic* as *melodic*, a translation that is linguistically accurate but obscures something of the technical way that Ptolemy uses the term; cf. Solomon, *Ptolemy Harmonics*, 24.

⁶¹⁹ cf. Ptolemy, Ἀρμονικά, bk. 1, ch. 15, ed. Düring, 35, trans. Barker, 309.

because of the relatively vague meaning of Boethius' gloss, it is not difficult to see how one might adapt the term as a more general adjective. Chailley suggests that the principle the author is trying to invoke under the name of *emmelis* is the προσλαμβανόμενος.⁶²⁰ As I have described in Chapter 05, in the Greek string-name system for note names, all notes of the Greater Perfect System but one fall within one of four identically structured tetrachords; however, the resulting system is incomplete, forming only an octave plus a minor seventh. Also, as Boethius notes, the μέση — the so-called “middle note” — does not lie at the very middle of the system, but rather falls slightly closer to the low end.⁶²¹ Thus, the προσλαμβανόμενος is added to the bottom⁶²² to complete the system, even though it falls outside the theoretical principles according to which the rest of the system is constructed. This function is reflected in the name of the pitch, which signifies “taken in addition” (whence Boethius also calls it “*adquisitus*,” or “added”).⁶²³ Since Boethius also calls it *prosmelodos*,⁶²⁴ it is perhaps possible to see how the author might have associated the term with the *emmelis* that is *aptus melo*. If Chailley's suggestion is, indeed, correct, then the revisor has apparently extended the logic permitting the addition of an appended note at the bottom of the four-tetrachord Greek gamut to permit an extra note at either the bottom or the top of any system spanning a perfect consonance. However, it must be acknowledged that this association is pure speculation, since the author (and revisor) consistently use the term *emmelis* to refer to the added pitches, and the term *proslambanomenos* to refer to the lowest pitch in the gamut (there is thus no direct evidence for the association in the text).

In any case, musicologists familiar with modal theory of somewhat later eras would not be surprised to learn that the intervals held to be characteristic of each mode correspond to ranges, as range is

⁶²⁰ Chailley, *Alia musica*, 120.

⁶²¹ Boethius, *De institutione musica*, bk. 1, ch. 20, ed. Friedlein, 211, trans. Bower, 38.

⁶²² Note that Boethius says that it is added above (*ibid*) because Boethius uses terminology related to string position on a lute, which runs in the opposite direction to pitch.

⁶²³ Boethius, *De institutione musica*, bk. 4, ch. 3, ed. Friedlein, 309, trans. Bower, 123.

⁶²⁴ Boethius, *De institutione musica*, bk. 1, ch. 20, ed. Friedlein, 211, trans. Bower, 38.

frequently a defining factor in modal paradigms (though a certain skepticism is warranted against teleological thinking and confirmation bias), and it is likewise familiar doctrine in the various modal paradigms of the Middle Ages that a chant may exceed the expected range of the mode (the octave) by one tone in either direction. It will be seen shortly, however, that this *emmelis* is somewhat counterintuitive to a modern scholar in two respects: first, since the ranges expected of a mode in this treatise are different from the ranges expected in later medieval theory, the identity of the added tone is correspondingly different; and second, the added tone appears not always to be the note directly beyond the interval boundary, sometimes occurring a minor third away, instead (probably to avoid a semitone between a boundary pitch and the *emmelis*).

Other Interpretations of Intervals

Before returning to the *Alia's* association of chants with intervals, it should be acknowledged that the discussion so far has taken it for granted that the intervals indicate ranges. There are a few other possible interpretations of what these intervals could mean, and although the case for the intervals as ranges is quite strong, it is worth taking a moment to consider the merits of the other possibilities. First, the interval could represent a specific pitch within the modal structure, a pitch that receives particular emphasis – something like a dominant or a reciting tone. Of course, the *Alia* specifies intervals, not pitches. To interpret these intervals as pitches requires a reference pitch against which these intervals should be measured. There are only a handful of candidate pitches against which to measure: the *finalis*, the reciting tone, and the highest or lowest notes in the chants or the accepted ranges of the modes.

Most of these possibilities can be dismissed immediately. The reciting tone is a characteristic of chants sung to a recitation formula, such as the psalms to which the antiphons are connected, but they are not particularly prominent in the antiphons themselves. They do sometimes arise as secondary tonal centres, but in some modes, some of the chants identified in the *Alia* as characteristic of the mode contain

extended passages that do not even contain the reciting tone, much less emphasize it; for example in *Rorate caeli*, which the *Alia musica* uses as a model for how these intervals work, the final *incisio*, clearly spelled out by the author, never rises above G, and the interval therefore could not be measured against the reciting tone, a. There are also supposed to be entire chants of the first mode said to emphasize the interval of a fourth; if this fourth were to be measured against the reciting tone a fifth above the *finalis*, then the *finalis* itself would be extraneous to this range.

The boundary pitches of the accepted ranges of the modes are also unlikely. There are no accepted ranges specified by the *Alia* (if the characteristic intervals, themselves, are not ranges), and the other treatises of the time period are not always consistent about the ranges of the modes. The boundary pitches of specific chants are easier to determine but are even less consistent, varying as they do not only from chant to chant but sometimes from version to version of the same chant. Only the *finalis* is actually consistent, but in any case, none of these works even for *Rorate caeli*, since the middle *incisio* does not share a boundary pitch with the other two *incisiones*.

The other possibility is that a particular interval is somehow made prominent within the chant. That is to say that the interval does not represent a range, but simply a prominent melodic figure. Although Atkinson does not clearly describe his interpretation of what these intervals mean, his diagram for *Rorate caeli* indicates that he interprets the intervals in this manner (Facsimile 14). He also provides similar analyses for a pair of the *Alia musica*'s Mode VI chants (*Omnes gentes* and *O admirabile commercium*) that follow the same procedure; in these chants, he finds both a fifth and an octave, just as the *Alia* suggests, but in this case, he finds these intervals to overlap with each other,⁶²⁵ an idea more consistent with the interpretation of prominent intervals than ranges.

⁶²⁵ Atkinson, *Critical Nexus*, 183. Atkinson confirmed my supposition about his approach in the written feedback following the defence of this dissertation.

Facsimile 14 – Atkinson’s Example 5.2 (p. 181): *Rorate caeli* from *Graduale Romanum*, marked up with brackets and interval names

Diapason

Diapente

Intr. I.

R

ORA-TE * coe-li dé-su-per, et nu-bes plu-ant

Diatessaron

ju- stum: a-pe-ri-á-tur ter-ra, et gér-mi-net Sal-va-

Atkinson’s fifth and fourth agree with the intervals that I have selected, though his octave is a tone lower, as he has selected the opening gesture as his significant interval; here, he includes the low C that I have interpreted as an *emmelis* and does not include the high d that occurs near the end of the *incisio*.

Atkinson is, at this point in his discussion, directly analyzing the source treatise in the context of continuity with Aurelianus, and in that regard, his analysis is quite compelling, as I shall describe momentarily, but it is interesting to observe that his selection of C–c as the octave differs from the revisor’s interpretation:

*Inter 6 et 12 qui faciunt 18 senarius est differentia, quam melodia primae incisionis ascendendo totam percurrit, a lichanos hypaton uidelicet in paraneten diezdeugmenon, praemisso ad grauem partem tono qui emmelis dicitur, id est melo aptus.*⁶²⁶

Between 6 and 12 (which make 18) is a difference of six, through which the melody of the first *incisio* runs entirely, which one may see is from λίχανος ὑπάτων [D] to παρανήτη διεζευγμένων [d], preceded at the low end by a tone [C] that is called *emmelis* – that is, suited to melody.

⁶²⁶ *Alia musica* §32(b), ed. Chailley, 121.

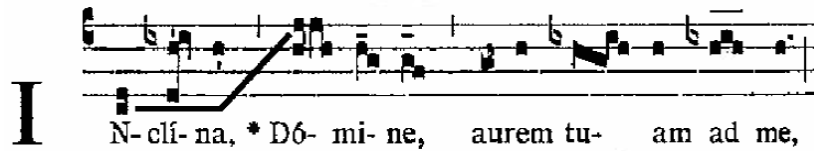
The revisor's opinion certainly does not invalidate Atkinson's analysis;⁶²⁷ as I have noted from time to time throughout this dissertation, it is not always clear that the revisor has correctly understood the source treatise. Furthermore, the function that the revisor and the commentator perform is effectively the same function performed by Atkinson himself: to attempt to make sense of the source treatise for the sake of future readers. While the revisor and commentator are closer to the musical culture of the *Alia*, their comments often obfuscate more than they clarify (as I shall describe in the second half of Chapter 16), and it seems appropriate for a subsequent commentator like Atkinson (or, for that matter, Chailley or Heard) to draw their conclusions principally from the source treatise, without unduly privileging the opinions of the earlier interpreters.

In support of Atkinson's interpretation, he sees the *Alia* as continuing in the same tradition as Aurelianus, and in his discussion of the same concept in Aurelianus, he analyzes the prominent intervals of the openings of the four chants with which Aurelianus demonstrates the interval associations of each mode. As mentioned above, the second, third, and fourth, are all reused in the *Alia* for the authentic *deuterus*, *tritus*, and *tetrardus* modes (the *tetrardus* case is particularly interesting, since Aurelianus and the *Alia* consider *tetrardus* to be characterised by different intervals), but the first has been changed. Aurelianus' *protus* example is *Inclina Domine*, which begins very similarly to *Rorate caeli*, with the same opening formula from C to c (see Facsimile 15), and in this case, there is no way to describe the opening as D–d, as *Inclina Domine* never reaches above c, as with many other antiphons in the same family (though one wonders if this might not be part of the reason why the *Alia* changes this example to *Rorate*, a relatively uncommon example of an antiphon in this family that does rise to the d). If both Aurelianus and the *Alia musica* are correct, and if the characteristic intervals of each mode must always take a consistent

⁶²⁷ Indeed, in the written feedback following the defence of this dissertation, Atkinson comments that in his view, the revisor is in contradiction to Aurelianus, and also to the general trends of the repertoire.

position within the mode, then the octave in *Rorate caeli* must be C–c; however, as I shall continue to observe throughout this dissertation, there is good reason to question the latter condition.

Facsimile 15 – Atkinson's analysis of Inclina Domine, Aurelianus' example for protus, on a facsimile from the Graduale Romanum.



However, Atkinson's interpretation of *Rorate caeli*, with his chosen octave of C–c rather than D–d, fares no better (though no worse) than the alternative in terms of consistent application of a modal number to a pitch. The second *incisio* now shares a boundary pitch with the first, but it is the wrong boundary pitch (the top, rather than the bottom), and the third *incisio* no longer shares a boundary pitch with either of the others.

In all three intervals that Atkinson labels, he selects the first pitch of the *incisio* as the first pitch of the interval (the fact that all three intervals are also ascending in this case is probably a coincidence resulting from the overall arch shape of most phrases; one might expect a descending interval to be reasonably common for final *incisones*). There is, however, no apparent reason why the interval should always be measured at the beginning of an *incisio*; it happens to work well with the description of *Rorate caeli* in the *Alia*, but there are plenty of other chants in which the first pitch does not appear to be a boundary pitch or in which the melody does not quickly reach the other expected boundary pitch.

But if the interval need not appear at the very beginning of an *incisio*, such an interval would be a great deal more difficult to identify than a range (or, alternatively, much easier to find unjustifiably, because of confirmation bias). However, it should also be observed that a given interval can only be prominent if the range of the *incisio* is at least as large as that interval; thus, there is little distinction between an octave as a range and an octave as a prominent interval. Potentially, a range of an octave

could include a prominent fifth, but in fact, octaves are not especially common within the cited chants in the first place; fourths and fifths are much more common. The possibility that the range is a fifth but that the prominent interval is a fourth is not unreasonable, but it is effectively indistinguishable from the case of a range of a fourth with an *emmelis*.

Thus, there is minimal distinction in most cases between an interval as a range and an interval as a melodic gesture. And in any case, any alternative interpretation of the characteristic interval would require an alternative reading of the word *decurrit* (runs through) with which the author of the *Alia* describes *incisiones* and entire chants “running through” these intervals.⁶²⁸

Another problem with the characteristic-interval interpretation is that the *Alia*'s presentation of *Rorate caeli* spreads the intervals across more than the single word or two implied in Atkinson's diagram.⁶²⁹ The entire text of *Rorate caeli* is used in the *Alia*'s demonstration, and Nowacki notes that “this analysis is intended to give an account of the entire melodic stream without leaving any gaps.”⁶³⁰ Having said this, Nowacki's analyses seem to fluctuate between these two different interpretations of the intervals, which is aptly demonstrated in his commentary on the chants for Mode II:

The example of the diapente [perfect fifth], the introit antiphon “Ecce advenit,” exhibits only one prominent fifth. It occurs in the first four notes, which **span the interval** from the fourth below the final to the step above it (A to E). The example of the Diatessaron [perfect fourth], the introit antiphon “Veni et ostende,” is of the same melody-type as “Ecce advenit,” but has, in addition, a **prominent descending fourth** from D to A, which was probably the reason for its selection. Both antiphons also have **prominent ascending fourths** from C to F. For good measure, the author adds one more example of each interval. The “diapentic” antiphon “Omnipotens sermo tuus” has a **total compass** of a fifth from C to G. The other example, the antiphon “Dominus deus auxiliator,” has the same **compass** as “Omnipotens sermo” but also has two melodically **prominent fourths**, C–D–F and D–F–G. We note that all of the fourths and fifths cited as tokens of their modal type, except A–D, differ from the ones that came to define the modes categorically in the eleventh century.⁶³¹ [Emphasis added]

⁶²⁸ *Alia musica* §181(g), ed. Chailley, 86, trans. Nowacki, 115.

⁶²⁹ Atkinson does, in fact, acknowledge that the intervals also appear in places other than those that he has bracketed in his diagram (Atkinson, *Critical Nexus*, 181, fn. 28).

⁶³⁰ Nowacki, *Greek and Latin Music Theory*, 114.

⁶³¹ Nowacki, *Greek and Latin Music Theory*, 118.

In this description, Nowacki cannot seem to decide whether the intervals represent prominent segments within the chant or the overall compass of a chant. This ambiguity may, perhaps, be consistent with Nowacki's suggestion that the author of the source treatise "is guided instead by an empirical approach that permits him to detect perfect intervals wherever the melodic design may place them."⁶³² However, it is not consistent with his assertion, cited above, that the analysis of the intervals should be completed "without leaving any gaps."

Intervals Revisited

To return to the analysis of the modal numbers, the author goes on to say that any of the specified intervals may be spanned several times throughout a chant, as in *Urbs fortitudinis*, or perhaps only one interval will be used, as in *Iohannes autem*, which indeed spans only a fifth if one ignores the subtones (I use the term to refer to the pitch one full tone below the *finalis*) and a single upper auxiliary tone that is not present in every source (see Example 2). (Actually, the final *incisio* does not reach up to the fifth, reaching only a fourth; this will be discussed later).

Jo han - nes au - tem cum au - dis - set, in vin - cu - lis,
 18 o - pe - ra Chris - ti, mit tens du os de dis ci pu lis su is,
 37 ait il li: "Tu es qui ven - tu - rus es,
 55 an a - li - um ex - pec - ta - mus?" 5th

Example 2 - Iohannes autem as given in the Sarum antiphoner, showing a melody largely confined to the fifth, excluding subtones and a single upper auxiliary.

⁶³² Nowacki, *Greek and Latin Music Theory*, 114.

Unfortunately, *Urbs fortitudinis* is more difficult to analyze than *Iohannes autem*, because the author lists it as an example in which multiple intervals recur, and it is more difficult than *Rorate caeli* because the author does not break down the divisions of the chant, highlighting a curious characteristic of the author's analysis of *Rorate caeli*: the division of the chant does not seem to follow any apparent systematic principles. The revisor states that "*Quam gradalem antiphonam per singulas incisiones singulis symphoniis subditam hoc modo pandimus.*"⁶³³ ("In this manner, we explain how, step by step, the antiphon is set through individual clauses to individual symphonies.") But in fact, the sections into which the author has divided the chant *Rorate caeli* are not clauses [*incisiones*]. A clause, in the traditional grammatical definition, is a unit expressing a more or less complete idea containing a verb (generally an action) and a subject (someone or something to perform the action) along with any additional words that directly modify the subject or verb and any grammatical objects (recipients or beneficiaries of the action). The chant *Rorate caeli* contains four verbs, and may, therefore, be divided into the following four clauses (with verbs underlined):

<i><u>Rorate</u> caeli desuper</i>	<u>Drip</u> , O ye Heavens, from above,
<i>et nubes <u>pluant</u> iustum;</i>	and let the clouds <u>rain</u> righteousness;
<i><u>aperiatur</u> terra</i>	let the earth <u>be opened</u>
<i>et <u>germinet</u> salvatorem.</i>	and may it <u>sprout</u> a saviour.

There are, then, four clauses. The first clause, *Rorate caeli desuper*, does indeed match the first of the three sections the author identifies, spanning an octave. The second section, however, includes the entire second clause and half of the third clause, spanning a fifth, while the third section incorporates the rest of the third clause and all of the fourth, spanning the interval of a fourth. Even if the word *incisiones* is taken loosely, there is no way to reinterpret the passage by combining the entire third clause with either

⁶³³ *Alia musica* §32(a), ed. Chailley, 121.

the second or the fourth (giving three grammatically complete units), since *aperiatur* would then take the final *incisio* well above its sanctioned range of a fourth, while *terra* would take the second *incisio* well below the bottom of its fifth. Even accepting a four-part division does not solve the problem, since the combined range of *aperiatur terra* is a seventh, and one that is not well suited to being reduced to a fifth by invoking the *emmelis*.

Of course, it hardly seems likely that the chant is divided arbitrarily, because if so, it would be possible to divide the chant into segments spanning virtually any interval. But if it is not clear why *Rorate caeli* was divided as it was, then it is certainly not clear how one would correctly divide other chants purporting to demonstrate multiple intervals (or to demonstrate the same interval multiple times), such as *Urbs fortitudinis*. If the rationale is not grammatical, it must be musical, and one must then identify musical principles according to which the divisions should be made, a proposition that is not as straightforward as it may initially appear.

All of the preceding discussion still fails to answer the most fundamental question asked at the beginning of this chapter: yes, the ratios appear to represent interval ranges of chant segments (*incisiones* that may not be quite the same as clauses), but what does each number independently represent? The obvious answer would seem to be that the numbers should represent the individual pitches bounding the interval. Unfortunately, though, every reasonable method for assigning pitches to numbers across the entire system invariably results in contradictions. In a previous chapter, it has already been demonstrated that the numbers cannot be taken to represent consistent pitches in the gamut, with a single number always representing the same pitch in every mode. Figure 37 shows the pitches that would result from interpreting the numbers of the tabular summary in this manner, taking the number 12 as the pitch d. Interpreted in this fashion, every pitch, regardless of mode, would be selected exclusively from the pitch classes D, G, and A — an interpretation that cannot be squared with any known modal paradigm.

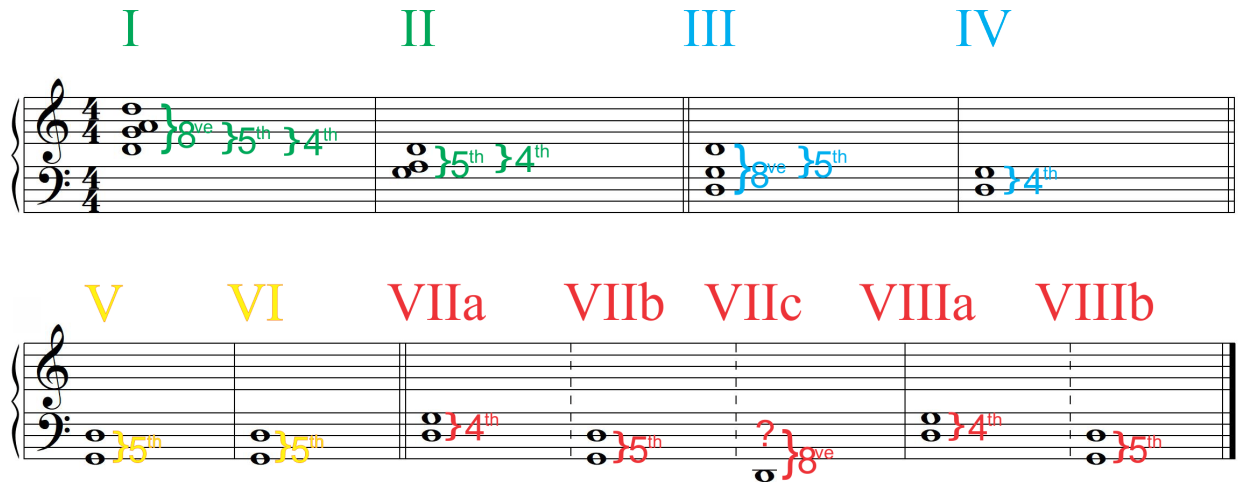


Figure 37 – The numbers of the tabular summary of the *Alia musica*, interpreted as consistent pitches within a gamut. This interpretation does not correspond with any known modal paradigm.

Thus, it is unlikely that the numbers could have a consistent meaning throughout the entire gamut. Perhaps, then, the numbers take an independent meaning for each maneria, as Mühlmann suggested. This interpretation would require that numbers in common between authentic and plagal modes of a given maneria should represent the same pitch. Table 41 shows the modal numbers rearranged in numerical order, showing the common numbers within each maneria.

Table 41 – Common numbers between authentic and plagal modes in each maneria.

I	II	III	IV	V	VI	VII	VIII
6							
8							
9		12				18	18
12	12	18	18	24	24	24	24
	16	24	24	36	36	36	36
	18					48	

A few brief examples are sufficient to undermine this interpretation substantially. It has already been shown, based on *Rorate caeli* and *Iohannes autem*, that the number 12 in the *protus* maneria represents (if anything) the *finalis*, D. (In fact, *Rorate caeli* is a bit problematic in this respect, but consideration of this issue will be deferred briefly to a more apt discussion). By extension, then, the 16

and 18 of the plagal *protus* mode should correspond to A and Γ,⁶³⁴ a fourth and fifth below the final, respectively. In regard to this mode, the author of the source treatise says:

*Omnis enim melodia secundi toni aut per diapente denarium numerabit totum, ut est antiphona Ecce aduenit, in qua sunt 10 uel 5, aut per diatessaron totum, id est 7, ut est Ueni et ostende nobis faciem tuam. Item antiphona diapentica Omnipotens sermo tuus Domine, et per diatessaron ut est Dominus Deus auxiliator meus.*⁶³⁵

Indeed, all melodies of the second tone will number through either the entire fifth that is 10, as is the antiphon *Ecce aduenit*, in which are 5 or 10, or through an entire fourth — that is, 7 — as is *Ueni et ostende nobis faciem tuam*.⁶³⁶ Likewise for the fifth-bound antiphon *Omnipotens sermo tuus Domine*, and through the fourth, as is *Dominus Deus auxiliator meus*.

Here, again, the numbers given represent the sums of the two terms of the ratios representing each interval (the 2:3 of the fifth becoming $2 + 3 = 5$, with 10 being its double, and the 4:3 of the fourth becoming $4 + 3 = 7$). More importantly, the author tells us that the chants *Ecce aduenit* and *Omnipotens sermo tuus Domine* should span a fifth, while the chants *Ueni et ostende* and *Dominus deus auxiliator* should span a fourth. *Ecce aduenit* provides a good test case (Example 3).

According to the author of the source text, *Ecce aduenit* ought to span a fifth between the numbers 18 and 12. If these numbers were shared with Mode I, this fifth would be the fifth Γ—D. Not only does *Ecce aduenit* entirely lack the expected Γ, but in fact, the majority of the introit falls outside the range of this fifth. By contrast, however, the entire chant (except a single A that is not present in all extant sources) falls within the perfect fifth C — G. It should be noted that in this case, the subtone C does not behave as an occasional extraneous note (*i.e.*, it does not serve as an *emmelis*, as it did in Mode I), but rather, it forms a core pitch in this melody, appearing much more frequently and forming an intrinsic part of the

⁶³⁴ There is probably no reason to be concerned about the possibility of Γ in this mode, as its presence in the Carolingian gamut is well attested in the *Musica enchiridis* and related treatises; nevertheless, as will become clear presently, the note is, in fact, not present in any of the chants described in this passage.

⁶³⁵ *Alia musica* §182(g)–(i), ed. Chailley, 87, trans. Nowacki, 117.

⁶³⁶ The author here refers to the chants *Ecce aduenit* and *Veni et ostende* as antiphons; this is probably an error, as there do appear to be any extent Mode II antiphons to these texts, while both texts have Mode II introits that appear to display the appropriate intervals; it is possible that the author simply includes introits under the heading of antiphon — a practice Chailley describes elsewhere (Chailley, *Alia musica*, 149), but the author is otherwise usually rather precise about this.

defining range of the mode (while the aforementioned A might serve as an *emmelis*, though a minor third below the bottom of the range, rather than the expected wholetone – perhaps justified by the fact that the expected wholetone below the range would be a B-flat).

Sarum #393

CH-P 18

D-Gsta AB III 9

Ec - ce ad - ve - nit do - mi - na - tor do - mi - nus:

31

31

et reg - num in ma - nu e - - - ius

59

59

59

et po - tes - tas, et im - per - ri - um.

Example 3 – Ecce advenit, showing that the chant does, indeed, span a fifth, but not the fifth Γ —D that would be implied if Mode II shares numbers with Mode I, but rather the fifth C-G.

It is clear, then, that even the plagal and authentic modes of the same maneria do not use common modal numbers to represent common pitches. Indeed, this fact is quite intuitive on the basis of mature modal theory, in that an interpretation of the shared modal numbers in the *Alia* as being common across

maneria would imply that Mode VII chants could descend a fourth lower than Mode VIII chants, which is the opposite of what is conventionally expected (refer back to Table 41).

The interpretation of the numbers, then, appears to be unique to each mode. Thus, if each individual number is to have any objective meaning separate from the intervals that they can produce (and so, reveal any kind of underlying logic behind the mathematical procedures), then one must look to find a consistent relationship between base numbers and products (or perhaps coefficients), perhaps in terms of highest or lowest pitches, or pitches that could reasonably serve as *finales*.

In the search for such a pattern, certain possibilities can be quickly ruled out. There is, for example, no consistent direct relationship between base numbers and products arranged in numerical order — that is, no single base number is consistently multiplied to get the highest product, or the lowest product, or any other particular product. For example, twelve is the highest base number and is multiplied to become the highest product in Modes I, IV, V, VII, and VIII but is multiplied to yield the lowest product in Modes II, III, and VI; the opposite circumstance is true for the base number six; the two groupings of modes produced in this analysis would appear to have nothing particular in common. Similarly, each of the two middle base numbers 8 and 9 may become the highest or lowest products, or remain as a middle product.

The relationships between base numbers and products is shown in Figure 38; although a few apparent relationships arise in this figure, the appearance of pattern is probably partly illusory, since many of the products appear more than once and there is no obvious way to distinguish between these multiple versions of the same number.

I 12 — 12 9 — 9 8 — 8 6 — 6	II 12 — 18 9 — 18 8 — 16 6 — 12	III 12 — 24 9 — 24 8 — 18 6 — 12	IV 12 — 24 9 — 24 8 — 18 6 — 18
V 12 — 36 9 — 36 8 — 24 6 — 24	VI 12 — 36 9 — 36 8 — 24 6 — 24	VIIa 12 — 24 9 — 24 8 — 18 6 — 18	VIIIa 12 — 24 9 — 24 8 — 18 6 — 18
		VIIb 12 — 36 9 — 36 8 — 24 6 — 24	VIIIb 12 — 36 9 — 36 8 — 24 6 — 24
		VIIc 12 — 48 9 — 8 — 48 6 — 48	

Figure 38 – Base numbers vs. products. Note that the limited number of patterns found in this example may be partly illusory, as many products appear more than once.

Interestingly, the author of the source text indicates that there *is* a distinction between the multiple copies of the same product that can arise from separate base numbers. In his description of Mode IV, the author says that:

*Idcirco ter 6 non ad bis 12 comparatur, quoniam per 3 non diuiditur, sed ter 8 ad ter 6 comparatur, et bis 9 ad bis 12 per sesquiterciam proportionem.*⁶³⁷

Therefore, three 6s are not compared to two 12s, since [two 12s] is not divisible by three,⁶³⁸ but three 8s are compared to three 6s, and two 9s to two 12s, in the sesquitercian proportion.

Thus, although Mode IV contains two copies of the product 18 and two copies of the product 24, these apparent duplicates are not interchangeable, since each should only be combined to form an interval with the product of a commensurable coefficient. This passage provides one of several tantalizing

⁶³⁷ *Alia musica* §184 (c), ed. Chailley, 92, ed. Nowacki, 121.

⁶³⁸ Two twelves, which give twenty-four, are, of course, divisible by three (as is twelve, itself); what the author means here is that the coefficient two is not divisible by three, and so does not compare to the three sixes. In this case, the argument would have been better phrased in terms of letters instead of numbers. Substituting the variables A for twelve and C for six, as the author does elsewhere in the treatise, the observation would read as “3C is not compared to 2A, since 2A is not divisible by three, but 3B is compared to 3A, and 2D to 2A.”

hints that the mathematical procedures may have some underlying logic — as, for instance, that the particular base number used to generate a particular product may have a bearing on identifying the pitch within the mode that the product should represent, though it is still not clear how this principle should be applied.

The Mode V introit *Domine in tua misericordia* might be seen as supporting this hypothesis. Sections of chants in Mode V ought to span a perfect fifth, but the range of this chant substantially exceeds this span. As previously discussed, it is not clear how chants are meant to be divided, but if the prepositional phrase *in salutari tuo* is taken as an *incisio*, then the chant may correctly reflect the author's description by being composed of *incisiones* reflecting two different fifths (see Example 4).

<i>Domine,</i>	O Lord,
<i>in tua misericordia speravi;</i>	I have trusted in your mercy;
<i>exultavit cor meum</i>	my heart has exulted
<i>in salutari tuo;</i>	in your salvation;
<i>cantabo Domino,</i>	I shall sing to the Lord
<i>qui bona tribuit mihi.</i>	who has bestowed good things upon me.

D-Gsta AB III 9
f. 164v

Sarum
#503

CH-P 18
f. 240

5th
F-c

5th
F-c

e - xul - ta - vit cor me - um

in sa - lu - ta - ri tu - o;

can - ta - bo do - mi - no

qui bo - na tri - bu - it mi - hi.

Example 4 – *Domine in tua misericordia*. While Mode V has only two modal numbers (36 and 24), corresponding to a fifth, the fourth system spans a different, higher fifth than the other five, suggesting that derivation of modal numbers may be significant.

Alternatively, it is also possible that this *incisio* is merely the result of transposition, and that in the version known to the author of the source treatise, it appeared within the span of the same fifth as the rest of the chant. This possibility is unlikely, because of the agreement between extant sources and the lack of a surviving source giving the passage in transposition. However, a similar hypothesis is essentially required of the paradigmatic case of *Rorate caeli* if a modal number is, indeed, to be interpreted as representing a consistent pitch within that mode. Although it has been convenient to ignore the inconsistency thus far, it should be noted that the middle *incisio* of *Rorate caeli*, while appropriately spanning a fifth (as described by the author of the source treatise), spans specifically the fifth between F

and c, and therefore does not share either boundary pitch with either the octave or the fourth of the other two *incisiones* (see Example 5).

The image shows three staves of musical notation for the chant 'Rorate caeli'. Each staff is highlighted with a different color and has a label on the right indicating a range and a ratio.

- Staff 1 (Green):** Range D - d, Ratio 12:6. The notes are mostly on a high line, with a few lower notes.
- Staff 2 (Yellow):** Range F - c, Ratio 12:8. The notes are mostly on a middle line, with a few lower notes.
- Staff 3 (Blue):** Range D-G, Ratio 12:9. The notes are mostly on a low line, with a few higher notes.

The lyrics are: Ro - ra - te ce - li de - su - per, et nu - bes plu - ant ius tum; a - pe - ri - a tur ter - ra et ger - mi - net sal - va - to - rem.

Example 5 – Rorate caeli. Although the three *incisiones* correspond to the specified ranges sizes, the range of the middle *incisio* is a third higher than would be expected if the modal number twelve reflects a consistent pitch throughout a chant.

Thus, if a modal number does represent a common pitch, it would require that the passage *et nubes pluant iustum aperiatur* was known to the author of the source treatise transposed down a third from the version found in extant manuscripts, to fit the span from D-a; such a transposition would be consistent with the span of a fifth in *Iohannes autem*, but it is not attested in any extant manuscript of *Rorate caeli*. The fact that no known manuscript gives the chant in that form may stand as evidence against a consistent association between modal number and pitch. This lack of association is probably the best explanation for both *Rorate caeli* and *Domine, in tua misericordia*.

However, such an association should not be dismissed too quickly; there are a number of passages in the treatise that seem to be making comparisons between numbers across multiple modes, a procedure that makes little sense unless the numbers represent consistent pitches. For instance, in his description of the second mode, the author of the source treatise says that there are three sixes (that is, eighteen), and that “*Ad 6 uero de primo tono isti ter 6 de secundo triplo proportio est, quia triplicatus eum superat.*”⁶³⁹ (“Truly, to the six of the first tone, these three sixes of the second tone make a triple proportion, since it

⁶³⁹ *Alia musica* §182(c), ed. Chailley, 87, trans. Nowacki 116.

exceeds it by a factor of three.”) This passage implies that the three sixes that make one of the two eighTEENS found in Mode II stand at the interval of a twelfth (the interval corresponding to 3:1) to the d that is the six in Mode I – that is, the eighteen is the note Γ. Unfortunately, none of the chants specified for Mode II contains a Γ, which might stand against the hypothesis of the numbers being consistent between modes. Interestingly, though, an analysis of the Mode II chants in extant sources suggests that the eighteen might represent C, and there is an affinity between C and Γ in terms of the intervals above these notes, such that the expected fourth and fifth of the second mode built on C are indistinguishable from a fourth and fifth built on Γ (in terms of systems or species of intervals – notwithstanding the B-flat that could possibly appear in the G systems, but which the *Alia musica* scarcely mentions).

As usual, this possibility brings one no closer to a definitive answer. To come to a conclusion on this question, it will be necessary to analyze all of the examples explicitly identified in the source treatise, to identify the intervals prevalent in each. In order to undertake such analyses, however, it will be necessary to divide the chants into *incisiones*, and as has already been demonstrated, it is not at all clear how this task should be accomplished. The grammatical approach does not seem to work consistently; other possibilities must therefore be considered.

An Alternative Approach to Segmentation?

I demonstrate above that segmenting a chant into *incisiones* based on rhetorical or grammatical principles fails even to account appropriately for *Rorate caeli*, the chant that the author of the source treatise offers as the paradigmatic case. In that chant, the author identifies three *incisiones* that, except for the first one, do not correspond to conventional grammatical or rhetorical units. However, I have also argued that the segmentation could hardly be arbitrary, lest it be possible to emphasize any arbitrary interval. But if the segmentation is not to be arbitrary, and yet does not correspond to grammatical or

rhetorical units, then the only likely alternative is for the *incisiones* to be divided according to musical principles.

Finding a consistent manner of analyzing early music has been a critical unresolved problem for generations of musicologists. Several possibilities have been suggested, but no consensus has ever been reached for analytical principles comparable to those used for the analysis of tonal or post-tonal music. Instead, the manner of analysis is usually designed by the analyst to suit the particular objectives of the analysis; in order to complete the analysis of the chants of the *Alia musica*, it will be necessary to decide on a consistent manner of dividing the chants into *incisiones*.

One obvious suggestion would be to divide the *incisiones* by cadences. Unfortunately, this possibility generally fails to bring any improvement, since cadences in plainchant usually correspond very closely to grammatical and rhetorical units. The only obvious alternative, then, is that *incisiones* in this context may correspond to stock musical formulae. It must be admitted, before proceeding, that a “stock musical formula” is a nebulous concept; although recurring ideas clearly exist through the repertory, no two musicologists are likely to agree about how to define and catalogue them. It is also not clear that the results of this manner of analysis are ultimately more consistent with the descriptions in the *Alia musica*. However, since the authors of the *Alia* do not specify a guiding principle for segmentation, all reasonable possibilities must be considered.

The notion that ecclesiastical chant consists of stock musical formulae has had a reasonably long but somewhat controversial history. No one who has spent any substantial time studying or performing this repertory could fail to notice the similarities between certain groups of chants, and medieval monks singing them on a regular basis as part of the Divine Office and Mass (including, most likely, the author of the source treatise) would undoubtedly have been aware of them; it is, therefore, not unreasonable to suppose that a medieval theorist might segment a chant according to these formulae. By contrast,

modern studies describing these similarities are routinely met with objections describing the richness and variety of the repertoire and demonstrating both extensive variation of the formulae and significant amounts of apparently freely composed material. There is, of course, truth in both perspectives. However, for these formulae to serve as a basis for segmentation, it must be shown that they are sufficiently common and systematic (and then that their use does not correspond directly to the rhetorical and grammatical units so that such a segmentation would provide new information).

Probably the first study to discuss this issue at length was conducted by François-Auguste Gevaert,⁶⁴⁰ a Belgian composer who was also one of the great musicologists of the nineteenth century. Gevaert's study proceeds from the premise that ecclesiastical chant is modelled on ancient Greek precedent, not only in its modal and harmonic underpinnings but even down to the melodies themselves. Such a premise, were it true, would have explained why so many chants seem to have so much in common; however, with increasing understanding over the last century and a half of the nature of both ancient Greek music and early ecclesiastical music (and the obvious disconnect between them), it has become clear that no such close relationship exists between Greek music and ecclesiastical chant. Consequently, Gevaert's presentation of his observations now appears rather idiosyncratic.

Nevertheless, the second half of Gevaert's book presents an extremely valuable study of the corpus of antiphons in the chant repertory. Antiphons are a fascinating genre of chant, occupying a middle ground between the formulaic and the wholly original. Unlike the psalms that they accompany, antiphons are not recited to formulaic patterns; neither, however, are they entirely original compositions, freely composed. Because of their close relationship to the psalms and the need for a smooth transition between the two, many antiphons share a substantial amount of melodic material amongst themselves, especially in their opening phrases. Gevaert, working from the tonary of Regino of Prüm, assigned well

⁶⁴⁰ Gevaert, *La Mélodie antique*, esp. 227–381.

over one thousand antiphons into forty-seven families according to their opening phrases, which he called “*thèmes*.” Of the eighteen Office antiphons explicitly cited by the author of the source treatise of the *Alia musica* as representative of each mode, all but two appear in Gevaert’s index of antiphons⁶⁴¹ (and two even belong to the same theme type). These opening phrases, then, could certainly form recognizable segments in which the author of the source treatise might see a characteristic interval of a mode. However, Gevaert identified only shared openings,⁶⁴² to be useful for segmentation, it would be necessary to identify such formulae throughout a chant.

Dom Paolo Ferretti made a significant development in this line of inquiry in 1934.⁶⁴³ Ferretti argued that the compositional process for Gregorian chant resembles a kind of “centonization.” Historically, “cento” is a word with roots in Latin and Greek⁶⁴⁴ referring to a patchwork garment and, by analogy, to a poem comprised of assembled fragments of other poems. Ferretti applied the term to a musical composition assembled from stock phrases, which he argues was one of the principal methods of composing Gregorian chant,⁶⁴⁵ alongside the wholesale recycling of pre-existing chant melodies to suit a new text⁶⁴⁶ (which melody might, itself, have been formed through centonization, though Ferretti does acknowledge instances of free composition⁶⁴⁷). Ferretti’s analysis has been criticized, not least by Leo Treitler, who objects to the overstatement of the effect, the suitability of the term “centonization,” and the implications for the manner of composition; none of his arguments, however, ultimately refutes Ferretti’s fundamental observations. He argues primarily that the term *centonization* implies a more systematic form of composition than the extempore reconstruction that Treitler envisions as the

⁶⁴¹ The two antiphons missing from Gevaert’s index are *Salve crux* and *Uade Satanas*, both of which present difficulties of modal attribution.

⁶⁴² It is worth noting that the opening phrases sometimes recur later in the antiphon, and so these phrases need not necessarily be thought of as openings by definition.

⁶⁴³ Ferretti, *Estetica gregoriana ossia Trattato delle forme musicali del canto gregoriano* (Rome, 1934).

⁶⁴⁴ See Lewis and Short, *cento*, *-onis*; Liddell and Scott, κεντρων, -ωνος.

⁶⁴⁵ Ferretti, *Estetica gregoriana*, esp. 114–31.

⁶⁴⁶ *ibid.*, 111–13.

⁶⁴⁷ *ibid.*, 96–110.

mechanism by which the formulae entered the chants,⁶⁴⁸ and also implies a kind of intertextuality characteristic of many centonate poems but not so much the melodies of Gregorian chant,⁶⁴⁹ even so, he does not deny the presence of the melodic formulae.

The most important criticism that Treitler offers is that it can be quite problematic to identify the formulae.⁶⁵⁰ He notes:

We want to be able to say with some confidence, here is that formula, there it is not, and here again is a variant of it. If we cannot do these things with confidence and with criteria that are demonstrable, we will have lost the point of the analysis, for then we would have no reason to think that the formula was any more distinct in the mind of the composer or the singer than in that of the analyst.⁶⁵¹

This point is well taken, though it risks running afoul of the fallacy of intentionality; the ability to carefully and precisely articulate a formula is not a prerequisite for the ability to recognize or to habitually compose (or especially improvise) substantially similar melodic ideas. Nevertheless, Treitler has identified the fundamental problem with any theory based on assembling formulae: how is the concept of the formula defined? How is a specific formula identified and described? Where is the division point between related formulae? And who gets to decide about each of these issues? Whatever one may ultimately conclude about the aptness of Ferretti's characterization of the chant repertoire, this problem represents a significant drawback to any attempt to apply his theory to the analysis of the chants cited in the *Alia musica*. Ferretti does not provide a table of formulae (though he does provide some partial tables for some individual modes and genres) or a clear methodology;⁶⁵² while he provides the analytical philosophy, he does not provide all the tools.

⁶⁴⁸ Treitler, "'Centonate' Chant," 10–11.

⁶⁴⁹ *ibid.*, 13–14.

⁶⁵⁰ *ibid.*, 15.

⁶⁵¹ *ibid.*, 16–17.

⁶⁵² For instance, Ferretti provides a partial table of formulae for Antiphons in the first mode. Ferretti, *Estetica gregoriana*, 113–14.

Ferretti's discussion of the manner of composition of Gregorian chant does suggest another possible explanation for the bizarre segmentation of *Rorate caeli*. This explanation is not particularly likely, but it needs to be considered before it is dismissed. As observed above, Ferretti discusses three different possible ways in which Gregorian chant was composed: through centonization, through free composition, and by fitting a new text to a pre-existing melody, producing a type of composition that Ferretti calls a "*melodia-tipo*"⁶⁵³ (but in later musical contexts, came to be called a *contrafact*). If Gregorian chants were, indeed, sometimes composed as contrafacts, then there is a possibility that *Rorate caeli* is, itself, a contrafact of an older chant, and that the unusual segmentation of the chant in the *Alia musica* reflects a more logical segmentation in the chant that served as the source of the melody.

There are several problems with this explanation. First, there is no obvious source chant from which the melody could have been drawn. Although there are many chants with similar openings (the offertory *Benedicam dominum qui mihi tribuit* being a particularly good example), a similarity close enough to call a contrafact seems never to last beyond the word "*caeli*"; indeed, few of the chants sharing the same intonation reach as high as the *d*, a note that is critical to the *Alia's* description of the intervals found within the chant (which is probably the reason why the author of the *Alia* chose *Rorate caeli* instead of any of the myriad other chants in the same theme group). The absence of a known source melody does not disprove the possibility that *Rorate* could be a contrafact, as the source chant may simply not have survived, but at present, the only evidence supporting this hypothesis is the unexpected segmentation of the chant in the *Alia*, a relatively weak argument.

Secondly, it would be surprising for the author of the *Alia musica* to select *Rorate caeli* as the exemplar using segmentation based on a different chant compared to which *Rorate* is a contrafact. As evidenced by the discussion of the segmentation of *Rorate* here, such a procedure obfuscates far more

⁶⁵³ Ferretti, *Estetica gregoriana*, 95.

than it elucidates. Instead, if a source chant were originally to have been the inspiration for the segmentation, it would have been far more logical simply to have used the source chant for the demonstration of intervals, rather than using *Rorate*.

Thirdly, Ferretti speaks against the idea of contrafacts containing a different number of phrases from the model chant. He says that:

*Chi volesse adattare a una melodia-tipo di due membri, un testo che ne avesse tre o quattro, riuscirebbe o a fare un pasticcio, o a introdurre formule melodiche estranee al tipo stesso. In questi secondo caso si avrebbe una specie di melodia centonizzata.*⁶⁵⁴

A person that wished to adapt to a melody-type of two members a text that had three or four of them would succeed either in making a mess or in introducing melodic formulae extraneous to that type itself. In that second case, one would have a kind of *centonized* melody.

Finally, Ferretti also cites *Rorate caeli*, itself, not as an example of a contrafact, but as an example of a freely-composed melody. Although Ferretti could be mistaken on this point (a contrafact for which the source of the melody is not known is indistinguishable from free composition), he does not argue from the lack of a known model, but from what he considers to be evidence of word-painting.⁶⁵⁵ While Ferretti's assessment is debatable (analyses of word-painting in composition are problematic; they often reveal more about the analyst than about the composer – much like seeing images in clouds), the combined weight of all the above arguments greatly reduces the likelihood that the problematic segmentation of *Rorate caeli* is a side effect of being a contrafact.

Ferretti's observations about chant as assembled formulae, however, remain essentially valid, even if overstated. Walter Howard Frere further developed these same principles in his editions of the liturgical usages of Salisbury.⁶⁵⁶ Frere analyzes the responds, invitatories, and antiphons in use at Salisbury, and groups them into families, much as Gevaert had done for antiphons (and agreeing with Gevaert in most

⁶⁵⁴ Ferretti, *Estetica gregoriana*, 112.

⁶⁵⁵ Ferretti, *Estetica gregoriana*, 103–04.

⁶⁵⁶ See especially Frere, *Antiphonale Sarisburiense*, 3–76; and to a lesser degree, Frere, *The Use of Sarum II*, i–lxxiv, which is necessary for the understanding of some of his notations in the *Antiphonale*.

cases), but he grouped them according to the entire progression of the chant, rather than merely the opening phrases. Frere's index classifies every antiphon in the Sarum Antiphonary into a mode and a modal variant, according to the groupings found in the Sarum Tonary.

Though much later than Regino's tonary (from which Gevaert drew his observations), the Sarum Tonary was a beneficial source from which to work, as it is much more detailed than any of the early tonaries. Many of the simplest tonaries, such as the so-called "Tonary of Charlemagne" (a.k.a. the St. Riquier Tonary) – and, for that matter, the tonary-like passages in the source treatise portions of the *Alia musica* – contain little more than a list of chants belonging to each mode. More complicated tonaries, such as the Metz tonary – and, in a much abbreviated form, the largest tonary section of the *Alia musica*, the *Nova expositio* – further subdivide the chants according to the *differentia* used with them (the *differentiae*, again, being a set of alternative cadences for each psalm tone to ensure a smooth connection between the end of the psalm recitation and the beginning of the antiphon); sometimes, these *differentiae* are identified only by number or perhaps final pitch, and sometimes by the notes upon which the associated antiphons begin (the "loca" of the *Nova expositio*). In others, the exact melody of the *differentia* is given, as in the *Commemoratio brevis*. In addition to providing all these things, the Sarum Tonary also gives a precise description of the intonation formulae (a more detailed version of the *Alia's* "loca") of antiphons belonging to each *differentia*, as in the following passage, describing the third variation of the first *differentia* of Mode I (the variation to which *Rorate caeli* belongs), noted as I¹₃:

*Tercia uariacio est hec. Omnis antiphona primi toni, que incipit in desolre uel in Cefaut deinde desolre bis percuciens saliendo ad alamire, finem uero in befabemi et in alamire faciens, hoc modo:*⁶⁵⁷

The third variation is this: all antiphons of the first tone that begin at D, or at C and then striking D twice, by leaping to a, and truly making an end on b/ b̄ and [then] on a, in this manner:

⁶⁵⁷ Frere, *The Use of Sarum*, iii.

uel sic:
or thus:

In general, there are usually fewer than ten “variations” (intonation formulae) for each differentia, though in the most extreme case, there are thirty-nine variations to the first differentia of the eighth mode, which Frere notates as VIII¹₃₉. The fact that most differentiae correspond to a relatively small number of intonation formulae, most of which are used for many antiphons, provides some support for Falconer’s hypothesis that *differentiae* served as a kind of modal classification before the adoption of the *oktōēkhos* system in the West.⁶⁵⁸

This much of Frere’s system is not substantially different from Gevaert’s. However, Frere then proceeds to classify as many of these as he can to larger families in which sets of stock phrases appear together, often in a predictable order. In general, Frere is able to classify just under two-fifths of the antiphons in his index into one of these families. Moreover, unlike Ferretti, Frere provides an extensive (if, perhaps, not exhaustive) reference of the stock formulae in each mode. Here, at last, is a model of stock formulae that is practical to use as a framework for segmenting chants, to see whether it will allow the modern theorist to analyze the chants of the *Alia musica* in a non-arbitrary way to find the characteristic intervals that the author of the source treatise says ought to be present in chants in each mode.

Does this framework result in an improved analysis? An ideal way to answer this question is to consider the case of *Rorate caeli*, the chant that the author of the source treatise uses to demonstrate the concept. Unfortunately, this particular chant is not so straightforward. Ferretti examines this chant, not as an example of centonization, but of an original melody, more or less freely composed (and even

⁶⁵⁸ Described above in Chapter 06.

employing word-painting).⁶⁵⁹ Meanwhile, both Gevaert and Frere analyze a different melody altogether for *Rorate caeli*, a Mode IV⁴ chant that Gevaert classifies as belonging to Theme 29; this alternate melody certainly cannot be reconciled with the comments that the author of the source treatise makes about *Rorate caeli*.

Nevertheless, it is possible, using Frere's tables, to attempt an analysis of the Mode I¹₃ melody that conforms so well to the author's description (notwithstanding that the chant in question is, in fact, an introit, rather than an antiphon of the Divine Office).⁶⁶⁰ In this case, the melody begins in the manner of the Mode I theme group (b).⁶⁶¹ This Theme group includes chants in variations I¹₂, I¹₃, I⁹, and I⁴₃, all of which have similar opening phrases, incorporating a prominent leap from D up to a. Frere notes that it is a quite popular theme, but not entirely stable.⁶⁶² He provides the following examples of group (b) themes:



Steph. XL⁴ 66. 180. Iste est Io-han-nes qui ... domini in cena re-cu-bu-it. Be-a-tus a-pos-to-lus, cui re-ue-la-ta sunt se-cre-ta ce-le-sti-a. Si du-o ex ... ter-ram de-om-ni re, quamcumque pete-riat fiet il-lis ... meo ... est, dicit do-mi-nus.

Adv.⁴ 39. Erunt prava in-di-rec-ta et aspera in vias planas. Veni, do-mi-ne, et noli tar-dare, al-le-lu-ia.

Cross Innoc. Pasch. 304. 71. A-do-ra-mus te, chri-ste, et benedicimus ti-bi, quia per crucem tuam re-de-mi-sti mundum. Hi sunt qui cum mulieribus non sunt coinqui-na-ti vir-gi-nes enim sunt, et sequuntur agnum, quocumque i-e-rit. Mu-li-e-res flentes ... tum lamenta-(bantur) flen-(tes) dominum.

Cena 220. Traditor autem dedit eis signum, dicens; Quem osculatus fuero, ipse est, tene-te eum.

Facsimile 16 - Frere, *Antiphonale Sarisburiense*, 66. Several examples of antiphons exemplifying Mode I theme group (b).

⁶⁵⁹ Ferretti, *Estetica gregoriana*, 103–04.

⁶⁶⁰ It is, of course, true that introits are, themselves, a form of antiphonal psalmody – in fact, the early sources often call them *Antiphona ad Introitum* (Apel, *Gregorian Chant*, 189) – but they belong to the mass, and the author of the source treatise of the *Alia musica* routinely and carefully distinguishes between introits and antiphons of the office.

⁶⁶¹ Frere, *Antiphonale Sarisburiense*, 66–67.

⁶⁶² *ibid.*, 66.

Such an analysis would be meaningfully different from the grammatical analysis already presented, as the intonation formula covers only the single opening word, *Rorate*; the word *caeli* and the beginning of *desuper* partly resemble the second segment of Frere's first example, though the ending of *desuper* is higher than any part of any of these examples, reaching the octave above the final – which, in the interpretation of the revisor, is presumed to be the characteristic to which the author of the source treatise is drawing attention, though the source treatise itself is not specific on this point. The next portion of the chant, beginning with *et nubes*, bears only a passing resemblance to the third unit of theme group (b), but does not descend to the final, as Frere indicates to be the usual course of this theme group;⁶⁶³ instead, this phrase probably best represents the freely composed elements that Ferretti describes. The phrase *aperiatur terra*, by contrast, does manifest a certain resemblance back to the third unit of the theme group (particularly in comparison to the first and second of Frere's examples and his observation that this unit usually descends to the final); and the remaining material, while not particularly formulaic, represents a reasonable cadential passage. The introit therefore manifests a combination of formulaic and freely composed elements.

Rorate celi

Unit 1 Unit 2? Free

D - d
12:6

Ro - ra - te ce - li de - su per, ___

Free Unit 3

F - c
12:8

et nu - bes plu - ant ius ___ tum; a - pe - ri - a ___ tur

Unit 3, sort of?
Cadence

D-G
12:9

ter - ra et ___ ger - mi - net ___ sal - va - to - rem.

Figure 39 – Analysis of *Rorate caeli* based on Frere's formulae

⁶⁶³ Frere, *Antiphonale Sarisburiense*, 66.

Thus, there is a degree to which *Rorate caeli* can be seen to be formulaic, but this analysis fails to solve the problem that initially led to the search for an alternate method of segmentation: here, as in the grammatical interpretation, the phrase *aperiatur terra* seems to form a single melodic formula sufficiently consistent to have found its way into Frere's discussion – a unit that the author of the *Alia musica* breaks apart in order to get the interval ranges that he is trying to demonstrate. It is difficult to know whether a medieval musician would have agreed with this assessment, given that no medieval treatise presents a catalogue of melodic formulae. Interestingly, though, every example that Frere gives of the Mode I theme group (b) does loosely reflect the *Alia's* description of *Rorate caeli*: only the opening formula and the immediately following unit ever reach above the a and thus approach an octave (though none actually reaches to the octave, and perhaps this is why the author of the source treatise chose *Rorate*); the middle units generally span a fifth (and unlike the surprising case of *Rorate caeli*, it is the fifth above the final, where one would expect it to be); and the remaining units all outline a descent of a fourth down to the final, all with minimal use of the subtone.

	8ve (D-d)?	5th D-a	4th D-G
Steph. XL ⁴	66. 180. Iste est Io-han-nes qui ... domini Si du-o ex- ... ter-ram	in cena re-cu-bu- it. Be-a-tus a-pos-to- lus, de om- ni re, quamcumque petie- rint	cui re-ue-la-ta sunt se-cro-ta ce-le-sti-a. fiet il-lis ... meo ... est, dicit do-mi-nus.
Adv. ⁴	39. Erunt prava in-di-rec-ta	et aspera in vias planas. Veni, do-mi-ne,	et noli tar-dare, al-le-lu-ia.
Cross Innoc. Pasch.	304. 71. A-do-ra-mus te, chri-ste, 234. Hi sunt qui cum mulieribus Mu-li-e-res flentes ... tum	et benedicimus ti-bi, quia per crucem tuam non sunt coinqui-na-ti vir- gi-nes enim sunt, lāmentā- (bantur)	re-de-mi-sti mundum. et sequuntur agnum, quocumque i- e- rit. sēn- (tes) dominum.
Cena	220. Traditor autem	dedit eis signum, dicens; Quem osculatus fuero,	ipse est, tene-te eum.

Figure 40 - Relationship of Mode I Theme Group (b) antiphons to the *Alia musica's* description of *Rorate caeli*. The chants of this theme group generally display the same intervals in the same order, excepting only that most do not reach the octave in the opening phrases.

To decide, then, whether this method of segmentation is valid, one would need to apply it to another case in which it is problematic to segment the chant grammatically to find the expected intervals.

These conditions would arise only in modes expecting more than one characteristic interval since segmentation becomes irrelevant where only one characteristic interval is expected. Thus, one would be looking for a problematic chant in Modes I, II, III, or VI (no examples are given for Mode VIII, and none of the examples for Mode VII are Office antiphons), and ideally, one for which Frere has identified a theme family.

Few of the antiphons cited in the source treatise meet these criteria. The first antiphon listed in the treatise for Mode I is *Urbs fortitudinis nostrae Sion*, which is generally classified as a Mode VII chant,⁶⁶⁴ and while Gevaert classifies it under Theme 3 (and most Theme 3 antiphons belong to Mode I), Frere does not assign it to a theme group.⁶⁶⁵ The next antiphon, *Iohannes autem*, fits into Frere's Mode I theme group (a), but the source treatise identifies it as exemplifying only the interval of a fifth, making segmentation irrelevant.

The last antiphon listed for Mode I, *Traditor autem dedit eis*, is a possible candidate; it belongs to Theme group (b), and in fact, is one of the examples cited by Frere (and so, has already been loosely analyzed in Figure 40, above). In this case, this melody-formula analysis does provide a segmentation that one might not predict from a grammatical analysis alone, as the words "*Traditor autem*" do not immediately imply an independent unit. However, the opening formula hardly exceeds the fifth (except what the revisor of the *Alia* might describe as *emmelis*), and so the analysis of intervals would not change significantly if this opening formula were kept together with "*dedit eis signum*," as a grammatical analysis might do.

Neither of the Mode II antiphons listed in the source treatise conforms to one of Frere's theme groups, and so neither is of much help in this question. However, Frere identifies theme groups for all

⁶⁶⁴ *cf.*

<http://cantus.uwaterloo.ca/search?op=starts&t=urbs+fortitudinis&genre=All&cid=&mode=&feast=&volpiano=All>

⁶⁶⁵ The peculiarities of this chant will be discussed more fully in Chapter 18.

four of the antiphons that the source treatise identifies for Mode III. *Qui de terra est* provides for a much more promising case. This antiphon belongs to Mode III theme group (a), a group that, according to Frere, demonstrates considerable variety beyond its opening formula; fortunately, the opening formula recurs in this antiphon, making it easier to segment into formulae than would otherwise be the case in such a loose theme group.

The text of the antiphon may be grammatically segmented as follows:

<i>Qui de terra est</i>	<i>de terra loquitur,</i>	He who is of the Earth	speaks of the Earth;
<i>qui de caelo uenit</i>	<i>super omnes est;</i>	he who comes from Heaven	is above everyone;
<i>et quod uidit et audiuit,</i>	<i>hoc testatur,</i>	and what he saw and he heard,	he bears witness to this,
<i>et testimonium eius</i>	<i>nemo accepit ;</i>	and his testimony	no one has accepted ;
<i>qui autem acceperit eius testimonium,</i>		but the one who will have accepted his testimony	
<i>signauit</i>	<i>quia Deus verax est.</i>	has signified	that God is true.

Under this segmentation, it is difficult to recognize the intervals that the *Alia musica* says ought to be present: the octave (possibly two different octaves, as the octave is derived twice) and the fifth (only derived in one way). As an example, the clause *qui autem acceperit eius testimonium* fits within the span of a fifth, but it is the fifth G–d, while *et quod uidit et audiuit, hoc testatur* spans a different fifth, D–a; these are two different fifths, while there is only one derivation of the fifth for this mode. However, segmenting the chant according to its melodic formulae, which do not align with the grammatical segments, better reflects the expected intervals. Frere provides only one example (Facsimile 17) of a Mode III theme group (a) chant, and says that “nearly all the thirty antiphons assigned [...] to III¹ may be said to correspond with the above model though they differ from it very widely.”⁶⁶⁶

⁶⁶⁶ Frere, *Antiphonale Sarisburiense*, 69.

Pent. 280. Dum com-ple- ren- tur di- es Pen- te- cos- tes, e-rant om-nes pa- ri- ter di- cen- tes al- le- lu- ia.

Facsimile 17 - Frere's paradigm for Mode III theme group (a)

The passage as far as *loquitur* conforms relatively closely to the first two melodic formulae in Frere's example, which can probably be taken as a single unit (on account of brevity) that spans an octave. The next stretch of music somewhat resembles the third formula, as far as the beginning of *omnes*, and thereafter, does not particularly resemble anything in Frere's model until the opening phrase returns; regardless, all of this material spans a fifth. The opening formula returns, interestingly, between *testimonium* and *eius*, which is an unexpected place to segment the chant grammatically. The next stretch of the chant, as far as *acceperit*, again resembles the first two formulae and spans an octave. The next segment resembles the third formula, perhaps as far as the beginning of *quia*. Very shortly thereafter, in the middle of the word *Deus*, the cadence formula begins, quite closely resembling the cadence of the model; it spans only a fourth, but is quite short, and might be appropriately grouped with the preceding material, just as the very brief opening formula is grouped with the material that follows it.

A-Wn 1799, f. 022v
 CDN-Hsmu M2149.L4, f. 043r
 D-KNd 1161, f. 024r

F-Pnm lat. 12044, f. 021v

F-Pnm lat. 15181, f. 157v

CH-E 611, f. 032v
 NL-Uu 406 (3 J 7), f. 036r
 D-KA Aug. LX, f. 028v

Qui de terra est de terra lo- qui - tur, 8ve

qui de cae - lo ve - nit su - per om - nes est;

The first system of music shows a vocal line on a treble clef staff with a yellow highlight. The lyrics are "qui de cae - lo ve - nit su - per om - nes est;". Below the vocal line are three staves of piano accompaniment. The first staff has a treble clef, and the second and third staves have bass clefs. The music is in a simple, homophonic style.

et quod vi - dit et au - di - vit, hoc tes - ta - tur, et tes - ti - mo - ni - um

The second system of music shows a vocal line on a treble clef staff with a yellow highlight. The lyrics are "et quod vi - dit et au - di - vit, hoc tes - ta - tur, et tes - ti - mo - ni - um". Below the vocal line are three staves of piano accompaniment. The first staff has a treble clef, and the second and third staves have bass clefs. The music continues in the same homophonic style.

e - ius ne - mo ac - ci - pit qui au - tem, ac - ce - pe - rit

The third system of music shows a vocal line on a treble clef staff with a green highlight. The lyrics are "e - ius ne - mo ac - ci - pit qui au - tem, ac - ce - pe - rit". Below the vocal line are three staves of piano accompaniment. The first staff has a treble clef, and the second and third staves have bass clefs. The music continues in the same homophonic style.

e - ius tes - ti - mo - ni - um sig - na - vit qui - a De - us ve - rax est.

The fourth system of music shows a vocal line on a treble clef staff with a yellow highlight. The lyrics are "e - ius tes - ti - mo - ni - um sig - na - vit qui - a De - us ve - rax est.". Below the vocal line are three staves of piano accompaniment. The first staff has a treble clef, and the second and third staves have bass clefs. The music concludes in the same homophonic style.

This analysis is promising in some ways but does not come without its own challenges. In the first place, while it is fairly straightforward to decide which segments correspond to an octave and which to a fifth, it is more difficult to decide whether the octave in question spans D to d, which fills a complete octave – in which case, it shares its range with the octave of Mode I (which may or may not be a problem) – or the octave E to e, which intuitively seems more appropriate to this mode, but the phrases lack the high e (the presence of the subtone D, however, poses no problem as an *emmelis*). The fifth in the second and fifth segments straightforwardly must correspond to the range E to b, with occasional use of both upper and lower *emmeles*. The third segment, though, does not fill this entire span, reaching only as high as the a. If the D is taken as an *emmelis*, then the span is only a fourth, which is not supposed to be characteristic of this mode. Alternatively, the fifth could be interpreted as spanning D to a, but this analysis would require two different fifths because the other phrases exceed the a by two steps, an anomaly that is difficult to explain as an *emmelis*.

Or, alternatively again, the second and third segments could be taken as a single segment, which would correctly span the fifth E to b. Such a segmentation, though, highlights another problem. While the analysis presented is not implausible, a certain amount of cherry-picking has already gone into this segmentation, in terms of which smaller segments might be taken together as longer segments. There is considerable risk of confirmation bias in this procedure. If, for example, the intonation formula is not combined with the following segment, the following segment can no longer correspond to an octave, and would instead correspond to yet another fifth, from G to d. Additionally, the division between the third and fourth segments, where the opening formula returns and where the grammatical and musical segmentations most obviously differ, is somewhat ambiguous, as some of the surviving melody variants may lead to a slightly later segmentation at this moment, after *eius*, which is more in line with a plausible grammatical segmentation.

The other Mode III antiphons do not offer better evidence in this regard. One of them, *Quando natus es*, is of the same theme group as *Qui de terra est* and behaves very similarly. Another, *Homo quidam fecit cenam magnam*, belongs to theme group (b), the chants of which Frere says share a common opening, but little else, which makes Frere's survey of themes much more challenging to apply to this chant. Similarly, the remaining antiphon, *Malos male perdet*, also belongs to a theme group sharing only an opening – and, in this case, it is a theme group generally associated with a different mode. Of the antiphons cited for Mode VI, two (*O admirabile commercium* and *Uirgo hodie fidelis*) are assigned to a theme group – Mode VI group (a) in both cases – but they are no help, because their melodic formulae conform well to their grammatical divisions.

Whatever the successes or failings of this manner of analysis, the above treatment of Mode III chants does have significant repercussions for an important discussion that will be undertaken in greater detail in Chapter 16 of the characteristics of Mode III. The author of the source treatise explains that Mode III has “two A's” (presumably, two interpretations of the base number 12). Chailley interprets all of this as periphrasis for the place of semitones. Within that discussion, Chailley notes that the fifth found within the mode is not the lower fifth, E–b, but rather, the upper fifth, a – e. This interpretation certainly seems correct if the numbers have consistent meaning within a single mode, as the modal numbers for the octave, 24:12, and for the fifth, 12:18, share their highest pitch (lowest number), twelve.

However, none of the chants cited by the author of the source treatise emphasizes the fifth a – e. Thus, it is unlikely that the *Alia* truly intends the characteristic interval to be this particular fifth. Perhaps, if the octave is interpreted as D – d, instead of E – e, then it could represent the fifth G – d, an interval that is potentially present in *Qui de terra est*. But in light of such an interpretation, Chailley's suggestion that the convoluted explanation of Mode III reflects the placement of semitones comes into question. Furthermore, the fifth G – d, while present in *Qui de terra est*, is not particularly common in the chants cited by the author of the source treatise. Instead, even a cursory glance at the chants cited (a more

thorough analysis will be undertaken in Chapter 17) seems to place considerable emphasis on the interval G to c – a fourth, an interval not emphasized by the *Alia*. Furthermore, the fifth above the *finalis* clearly is significant, but it cannot be interpreted as corresponding to 18:12 unless the interpretation of the numbers differs between the interpretation of the octave and the interpretation of the fifth. Of course, it could work if the numbers reflect frequency, instead of string length (*i.e.*, the lower numbers represent lower pitches instead of higher pitches); such a solution, though, would cause more contradictions in other modes than it solves in this mode.

To return to the broader question of modal analysis, there is another potential model for analysis that blends melodic and grammatical principles. Calvin Bower suggested this model, and he draws upon a *topos* common to many early theory treatises – importantly for the current study, nearly all the substantial theory treatises of the ninth century, including Aurelianus, Hucbald, the *Musica enchiriadis* (and the closely related *Inchriadon*), the *Scolica enchiriadis*, and the *Commemoratio brevis*, thus being absent only from the *Alia musica*.⁶⁶⁷ This *topos* is an analogy between music and grammar. It should be noted that this analytical process is rather different from a grammatical analysis of chant text because the grammatical principles are applied directly to the melodic material, not the text.

The classic analogy between grammar and music occurs in the *Musica enchiriadis* and its half-sibling, the *Inchriadon*. It runs thus:

Sicut uocis articulatae elementariae atque indiuiduae partes sunt litterae, ex quibus compositae syllabae rursus componunt uerba et nomina eaque perfectae orationis textum, sic canorae uocis ptongi, qui Latine dicitur soni, origines sunt et totius musicae continentia in eorum ultimam resolutionem desinit. Ex sonorum copulatione diastemata, porro ex diastematibus concrescunt systemata.

[...]

Particulae sunt sua cantionis cola uel commata, quae suis finibus cantum distingunt. Sed cola fiunt coeuntibus apte commatibus duobus pluribusue, quamuis

⁶⁶⁷ Bower, “The Grammatic Model,” 135 (Table I). The topic is also missing from Regino, who probably wrote in the very early tenth century, and from the *Dulce ingenium*, which is closely related to the *Alia musica* but appears to borrow from Regino (Bernhard, *Dulce ingenium*, 2), and therefore can be no earlier than tenth century.

interdum est, ubi indiscrete seu comma siue colon dici potest. At ipsa commata per arsin et thesin fiunt, id est eleuationem et remissionem. Sed alias simplici arsi et thesi uox in commate semel erigitur ac deponitur, alias sepius. Discrimen autem inter summam et infimam uocem commatis appellatur diastema. Quae diastemata nunc quidem minora sunt ut est illud, quod uocamus tonum, nunc maiora, ut duum triumue ac deinceps aliquot tonorum habentia interuallum. Porro autem sicut cola commatibus constant, sic commatum spacia dicimus diastemata. Quae in colis uero spacia fuerint uel in integro quodlibet melo, sistemata nominamus. Ex sonorum copulatione diastemata et ex diastematibus sistemata concrescunt.⁶⁶⁸

Just as the individual and elementary, distinct parts of speech are letters, out of which syllables are brought together, and in turn, verbs and nouns, and from them, the fabric of a complete speech, thus the sources of song are the φθόγγοι, which in Latin are called *sonus*, and the content of all music gives over in the ultimate resolution of them. From the joining of *soni* grow *diastemata*, and from the *diastemata*, in turn, the *systemata*.

[...]

The *particulae* of a song are its *commas* and *colons*, the bounds of which divide the chant. But *colons* are made by the apt coming together of two or more *commata*, although it occasionally happens that it can be indifferently called either *comma* or *colon*. And *commata*, themselves, are made through *arsis* and *thesis* – that is, through elevation and return. But the voice is raised and set down through *arsis* and *thesis* sometimes once in the *comma*, sometimes repeatedly. The distance, meanwhile, between the top *uox* of the *comma* and the bottom we call a *diastema*. These *diastemata* are indeed now smaller than that which we call a tone, now larger, as having the interval of two or three and even several tones in succession. Then, moreover, just as *colons* consist of *commata*, so we say that the spans of *commata* are *diastemata*. Truly, those spans that will have been in *colons* or any complete melody we name *systemata*. *Diastemata* grow out of the conjoining of *sonis*, and *systemata* out of *diastemata*.

The *Scolica enchiridis* adds slightly to that description:

In colis uel commatibus diastemata dicimus, sistemata in particulis perfectioribus seu toto periodo. Nam diastema est spatium quodlibet sonorum, quo particula complectitur, id est quo acuta et grauior uox includitur, sistema totius spatium meli. Item sistemata sunt species tetracordorum, pentacordorum, ogdocordorum, quae modis singulis suas dant species.⁶⁶⁹

We say that *diastemata* are in *cola* and *commata*, *systemata* in more complete *particula* or an entire *period*. For a *diastema* is the span of any particular *soni* by which a *particula* is encompassed – that is, enclosed by a higher and lower *uox*; a *systema* is the span of an entire melody. Likewise, *systemata* are *species* of tetrachords, pentachords, and octochords, which grant their *species* to the individual *modes*.

⁶⁶⁸ *Musica enchiridis*, chs. 1 & 9, ed. Schmid, 3 & 22–23, trans. Erickson, 1 & 12–13; *Inchiridon*, ed. Schmid, 188 & 187–88. The two passages are nearly identical; the passage given here is primarily the form given in the *Musica enchiridis*, with a few minor additions from the *Inchiridon* (including, in particular, the entirety of the last sentence).

⁶⁶⁹ *Scolica enchiridis*, pt. 1, ed. Schmid, 85–86, trans. Erickson, 50.

These passages define an assortment of terminology from grammar and Greek theory. The terms in italics are words that have very particular meanings in the context of Carolingian theory; though most have English cognates by which they are most readily translated, the treatises impart them with greater meaning than their English cognates imply. The first several terms exist more or less at the note-to-note level. *Uox* (pl. *uoces*) is a pitch, by analogy to a sustained vocalization. *Sonus* (pl. *soni*) refers only to musical sound, not sound in general, and is used as a loose Latin equivalent to the Greek word φθόγγος (pl. φθόγγοι; *phthongos*, *phthongoi*);⁶⁷⁰ however, the *Enchiridies* note that *sonus* is sometimes taken to be more precise: “*Ptongi autem non quicumque dicitur soni, sed qui legitimis ab inuicem spaciis melo sunt apti.*”⁶⁷¹ (“Not just any φθόγγοι are called *soni*, but only those that, through successively appropriate spacings, are suited to melody.”)⁶⁷² This definition is, of course, familiar, as it is a clear paraphrase of Boethius’ definition: “*Sonus*, then, is an *emmelic* pitch event – that is, one suited to melody – within a

⁶⁷⁰ “*In quibus uocibus quia plerumque sonos et ptongos indifferenter accipimus,*” (*Musica enchiridies*, ch. 9, ed. Schmid, 20, trans. Erickson, 12; *Inchiridon*, ed. Schmid, 187). (“Since, for the most part, we treat pitches [*uoces*] indifferently as *soni* or φθόγγοι.”)

⁶⁷¹ *Musica enchiridies*, ch. 1, ed. Schmid, 3, trans. Erickson, 1–2; *Inchiridon*, ed. Schmid, 188.

⁶⁷² This passage, explaining the difference between a *sonus* and a φθόγγος is somewhat problematic to translate. The principal clause is simply a copulative, with both nouns being nominative; in isolation, it is difficult to determine which is the subject and which the predicate nominative – it is clear that one of the two terms is more specific than the other, and why, but it is not clear which is which, as *quicumque* (“whatever kind”) could equally modify either one. Erickson, in his translation, assumes that φθόγγος is the more specific term (Erickson, *Musica enchiridies*, 1–2), perhaps because the placement of *quicumque* in relation to *soni* is consistent with the placement of the other modifiers in relation to their nouns throughout the sentence. However, this interpretation is inconsistent with Boethius’ definition of *sonus*, (Boethius, *De institutione musica*, bk. 1, ch. 8, ed. Friedlein, 195, trans. Bower, 16) which shares much of the same phrasing, but does not mention φθόγγοι until the following sentence, when he says that “*sonum uero non generalem nunc uolumus definire, sed eum qui graece dicitur phthongos, dictus a similitudine loquendi, id est φθεγγεσθαι*” (*ibid.*) (“We do not now wish to define sound in general, but only that which, in Greek, is called φθόγγος, so called from the similarity to speaking – that is, φθεγγεσθαι [to voice].”) One could conclude from this follow-up statement that it is a rephrasing of the first – that *sonus* in Boethius’ treatment is not just anything called *sonus*, but just those that are equivalent to the Greek φθόγγος, which is to say, those that are suited to melody; however, this interpretation ignores two problems: first, Boethius explicitly cited the etymological relationship to speech, and not all speech sounds are suited to melody; second, on this interpretation, Boethius directly equates *sonus* and φθόγγος, while the author of the *Musica enchiridies* is making a distinction. Thus, regardless of how Boethius himself understood this definition (or how the Greek theorists understood φθόγγος), the better interpretation from the perspective of the *Musica enchiridies* is that Boethius restricts *sonus* first to vocalised sounds (φθόγγος), and then furthermore, to those that are suited to melody because they maintain a fixed frequency that corresponds to a valid position within a tuning system.

tuning system.”⁶⁷³ (This is the same passage cited above as the likely source of the term *emmelis* in the *Alia musica*).⁶⁷⁴ Finally, although the *Musica enchiriadis* does not list it as a musical term, but merely a grammatical one, Bower includes *syllable* in his chart of terms, as there is no musical equivalent listed in the analogy at the beginning of the excerpt: if *uoces* [itches] are analogous to letters, and several letters make a syllable, then a musical “syllable” would be a “melodic gesture,”⁶⁷⁵ made up of multiple *uoces*.

The next four terms are the most important for the grammatical model: *period*, *particula*, *colon*, and *comma*. With the exception of *particula*, these are cognate with modern punctuation marks, but these terms historically referred to the divisions (*incisiones?*) of a sentence comparable to phrases that today would be bounded by such marks. A *comma* (pl. *commata*) is the smallest division, perhaps comparable to a noun phrase or a prepositional phrase.⁶⁷⁶ A *colon* (pl. *cola*) is somewhat larger, perhaps comparable to a dependent clause in a complex sentence.⁶⁷⁷ The distinction between these is fuzzy, and there are sometimes divisions that might equally be called *comma* or *colon*; Bower notes that these two divisions are sometimes even reversed in the hierarchy.⁶⁷⁸ *Particula* (pl. *particulae*) appears to be a more general term encompassing both *commata* and *cola*, but perhaps also larger units. The *period* appears to be the largest unit, encompassing a complete sentence, and perhaps even a complete melody.

The remaining definitions are less obviously relevant to the grammatical model but are worth a brief examination for the insight that they can provide to the more general enterprise of relating intervals to modes. These terms are *diastema* and *systema*. In traditional Greek theory, a *diastema* is an interval,

⁶⁷³ Boethius, *De institutione musica*, bk. 1, ch. 8, ed. Friedlein, 195, trans. Bower, 16; see previous note.

⁶⁷⁴ cf. p. 360, above.

⁶⁷⁵ Bower, “The Grammatical Model,” 134.

⁶⁷⁶ Lewis and Short give the definition of a *comma* as “a division of a period,” or “in verse, the *cæsura*.” (Lewis and Short, *A Latin Dictionary*, 375); they define a *caesura* as “in metre, a pause in a verse [...], called also *incisio*.” (*ibid.*, 265). This last term is, of course, the same term used in the *Alia musica*.

⁶⁷⁷ Lewis and Short define the *colon* as “a member of a verse.” (*ibid.*, 370).

⁶⁷⁸ Bower, “The Grammatical Model,” 134. Bower does not provide a specific example of the reversal of the hierarchy.

and a *systema* is a scale or a segment of a scale. However, as used in the ninth century, the relationship between these may be a little more complicated. *Diastemata* and *systemata* can coincide, as, for example, the diatessaron (a *diastema*) and the tetrachord (a *systema*) both span a fourth. However, the master of the *Scolia enchiriadis* says that *diastemata* are found in *commata* and *cola*, while the *systemata* are found in the larger *particula* and whole melodies. This statement may imply that in the medieval conception, *diastemata* and *systemata* both contain an internal interval structure since it makes no sense that a *colon* should have only an interval, but not a scale segment. If so, then the medieval *diastema* will have taken on something of the definition of the Greek *systema*; the medieval *systema*, then, might have something of the concept of the Greek “perfect” or “complete” *systema*: a *systema* of *systemata*, a large *systema* that contains a full complement of smaller *systemata* within it (especially the those of the perfect fourth and fifth). This property is responsible for the name of the Greater Perfect System, within which it is possible to find all seven species of octachord, all four species of pentachord, and all three species of tetrachord. If this hypothesis (that the medieval *diastema* is what Greeks called a *systema* and the medieval *systema* is what Greeks called a perfect *systema*) is correct, then it is possible that intervals like the fourth and fifth in the Carolingian period were generally understood to contain internal interval structures (certainly, interval species do receive discussion in the *Alia musica*⁶⁷⁹). If so, the modal numbers of the *Alia musica* may represent not merely intervals, but entire interval species – not necessarily octave species, as in later theory, but equally species of fourth and fifth. This interpretation might help to explain the *Alia*’s description of the seventh mode, which carries the same modal numbers as Mode IV, and is said to be characterized by the fourth in a passage that may mean “just like Mode IV” or may mean “the same fourth as Mode IV” (this passage will be examined more closely in Chapter 16). At the least, this interpretation might serve to reinforce the possibility that the intervals represent ranges of *incisiones*, rather than simple intervallic leaps or quick intervallic outlines. Unfortunately, it is difficult to confirm this

⁶⁷⁹ *Alia musica* §15, ed. Chailley, 107.

hypothesis, since the *Alia musica* is the one substantial treatise of the era that does not include any of the terminology presented in the above citations.

Speculation aside, the concepts of *syllable*, *comma*, *colon*, and *period* have particular meaning within Bower's hypothesis. According to Bower, this analogy is more profound than it seems at first glance. The *commata* and *cola* here are not divisions of the lyrics, but divisions of the melody itself. Importantly, Bower notes that the theorists always invoke the grammatical analogy as part of the discussion of modes.⁶⁸⁰ In Bower's analyses, some of the larger *particulae*, such as the *colon*, are obvious, since they match the comparable divisions of the text. However, smaller divisions – especially the *syllable*, which is part of the analogy, but does not coincide with the grammatical syllable – are harder to find. However, Bower argues that these, too, can be identified because all of the *particulae* of whatever size always end on one of the structurally important pitches, which Bower suggests are usually the *finalis*, upper and lower fourths and fifths, and the subtone, though he notes that any pitch might become important by being repeated;⁶⁸¹ this last observation makes the identification of important pitches both subjective and heavily dependent upon context.

It is important to keep in mind that much of the support for his hypothesis and the specific emphasis on these particular intervals comes from somewhat later sources, like Johannes Cotto (a.k.a. Johannes Affligemensis), Guido, and Aribo,⁶⁸² but the groundwork is laid in the *Scolica enchiridis*,⁶⁸³ and the principles may be older still. At any rate, it is certainly possible to identify the structural pitches based on the pitches found at the ends of *cola*, which are easier to identify because they correspond to textual divisions; from there, any point in the text where a word comes to rest on one of these structural intervals may perhaps be identified as the dividing point of a syllable or perhaps even a comma, even where there

⁶⁸⁰ Bower, "The Grammatical Model," 138.

⁶⁸¹ Bower, "The Grammatical Model," 142–43.

⁶⁸² Bower, "The Grammatical Model," 136–37; 140–41.

⁶⁸³ Bower, "The Grammatical Model," 139–40.

is not another obvious grammatical reason to divide the text; that is, it is the grammar of the melody, not the text, that defines the smaller *incisiones*.

The question remains, does Bower's model help to clarify how to segment a chant in the quest to find the characteristic intervals? One way to attempt to answer this question would be to segment *Rorate caeli* according to Bower's principles and verify whether the *Alia's* analysis of *Rorate caeli* becomes more consistent. The process to undertake such a segmentation begins by dividing the chant into traditional textual clauses with a verb and a subject (which I have already done above); these clauses are the *cola*, and the ends of these segments are marked with a colon. The separation of *cola* into *commata* is a little more subjective, but I shall take the approach that the verb and subject, when they are adjacent, will form a single *comma* and any remaining nouns with their associated modifiers will form their own *commata*; I shall mark the ends of these with a comma. Finally, I shall mark with an 's' the end of any other word (except minor function words like *et*) that ends on the structurally important pitches that Bower describes (since *Rorate* is a typical authentic chant, these will be the fourth, fifth, and subtone).

CH-P 18
f. 17

Ro - ra - te ce - li de - su - per,

et nu - bes plu - ant ius - - - tum;

a - pe - ri - a - tur ter - - - ra

et ger - mi - net sal - va - to - rem.

Figure 41 – Segmentation of *Rorate caeli* based on Bower’s model of musical grammar

An initial observation of this segmentation is that nearly every word in this chant constitutes the end of a syllable, *comma*, or *colon*. The only exceptions are the word *et* at the beginning of the second and fourth systems. The fact may perhaps be explained by the simplicity of the text itself, as there is not a single clause in this text that is longer than four words. It may also be a quirk of this particular melody that such a large proportion of the words end on structurally important pitches.

There is one noticeable benefit to this segmentation: the *Alia*’s odd division of *Rorate caeli* in the middle of the third clause might be consistent with this model; the division occurs at the end of a musical syllable (according to Bower’s definition). Additionally, the intervals spanned by several of the syllables and *commata* agree with the intervals cited in the *Alia*, including both syllables of the third system and the first syllable of the last system. The final syllable of each of the second and last systems possesses too narrow a range to classify as a perfect consonance (unless, in the last system, the subtone is considered part of the interval, rather than an *emmelis*).

However, the beginning of the chant is more complicated. The first syllable spans only a fifth, and the second, to the end of the first *comma*, spans the fourth G–c, and the final *comma*, to the end of the first *colon*, spans a different fourth, A–d. The only way to achieve the complete octave specified in the *Alia* is to take the entire *colon* as a single unit. Similarly, the second *colon* begins with a syllable that spans an augmented fourth, and the second syllable, to the end of the *comma*, is too narrow to classify as a perfect consonance; only the complete *comma* spans the fifth indicated by the *Alia*. If Bower’s model of segmentation is appropriate for use with the doctrines of the *Alia musica*, then it is not clear how one should decide when to analyze the interval of an individual syllable and when to analyze a complete *comma* or even a complete *colon*. It is also interesting to note that the first two *cola* do not end on one of Bower’s structurally important pitches. The ending of the second *colon* might be interpreted as explaining why the *Alia* does not end the second *incisio* here, but the same cannot be said for the first

colon, which corresponds perfectly with the *Alia's* description of the first *incisio*. And in any case, both *cola* end instead on a pitch that has been emphasized within the syllable by repetition.

In addition to the specific issues cited above, the model still contains a few more general difficulties in application. First, there is still a great deal of ambiguity in terms of how to identify musical syllables. Although Bower always ends his syllables at the end of a word, landing on the *finalis*, an upper or lower fourth or fifth, or the subtone, he does not identify a syllable every time a word ends on these pitches (as I have); there is clearly some subjective judgment in the process. Additionally, Bower's model is effectively hierarchical. Consequently, it is reasonable to wonder whether it is consistent with the model that the first syllable of a colon should manifest a different interval from the rest of the colon (continuing the same interval as the previous colon), as in the case of the third *colon* of *Rorate caeli*.

Most importantly, though, the model presupposes the importance of a subset of pitches within the intervallic structure around the *finalis*. Fortunately, these pitches do not necessarily have to be the same as the boundary pitches of the characteristic intervals (in the case of *Rorate caeli*, the first syllable of the third *colon*, which is the end of the *Alia's* second *incisio*, ends on the upper fourth, G, but the range of the *incisio* is F–c). Nevertheless, since the purpose of analyzing the introits and antiphons cited by the source treatise is to try to identify boundary pitches of intervals, it is problematic to presuppose the importance of any pitches.

Ultimately, then, it is unclear whether segmentation by melodic formula, whether modelled on Gevaert's, and later, Frere's theme groups, or upon Bower's grammatical model (with its musical "syllables"), is more consistent with the *Alia musica* than segmentation purely according to the grammatical structure of the text. In most cases, the segmentation is effectively the same; in some cases, such as *Qui de terra est*, segmentation by melodic formula seems to be superior, but the case is never entirely clear; and in some cases, such as *Rorate caeli*, segmentation by melodic formula or grammatical

segmentation are equally inconsistent. Nevertheless, since these methods offer improvement in some cases, they will be taken into account in the analyses of chants in Chapter 17.

Chapter 16: Complications in the Numerology

Before an analysis of the chants cited for each mode can be undertaken, it is necessary to explore a few specific statements made in the *Alia musica*, especially in the source treatise, that complicate the interpretation of the numerology.

Direct comparison of modal numbers between Modes I and II

The first of these issues has been mentioned in passing already but is worth reviewing here. The first mode, uniquely, is characterized by the four base numbers, not multiplied by any coefficient. Not just any combination of these four numbers produces a meaningful interval, however; the intervals cited are octave 12:6, the fifth 12:9, and the fourth 12:8. It is tempting to define these ratios as corresponding to the intervals D–d, D–a, and D–G, but as I have demonstrated, even the basic test case of *Rorate caeli* contradicts this assumption. This contradiction leads to the conclusion that the ratios do not reflect specific pitches, but only abstract intervals, prompting Nowacki to observe that the author of the source treatise “is guided instead by an empirical approach that permits him to detect perfect intervals wherever the melodic design may place them.”⁶⁸⁴ Nowacki soon contradicts himself on this point.

The complication arises in the description of Mode II. The products for this mode are twelve (unmultiplied), sixteen (two eights), and eighteen (derived twice, from three sixes or two nines). The author then observes that “*ad 6 uero de primo tono isti ter 6 de secundo tripla proportio est*” (“Truly, to the 6 of the first tone, this triple 6 of the second is the triple proportion”).⁶⁸⁵ The conclusion of the description of the second mode develops this principle further: “*Si autem in diapason euenerit proportio secundi toni, uel 6 ad 12, ad primum est, ut ter 6 ad 6 primi, et bis 8 ad 8 et bis 9 ad 9 primi toni semitonium*

⁶⁸⁴ Nowacki, *Greek and Latin Music Theory*, 114.

⁶⁸⁵ *Alia musica* §182(c), ed. Chailley, 87, trans. Nowacki, 116.

*efficit.*⁶⁸⁶ (“If, however, the proportion of the second tone should come forth to the octave, or 6:12, it is [assigned] to the first mode, as the three 6s to the 6 of the first [mode] and the two 8s to 8 and the two 9s to 9 of the first mode produce the semitone.”) The final clause may also be interpreted in another way: “the two 8s to 8 and the two 9s to 9 produce the semitone of the first mode.” Nowacki’s translation is compatible with the first interpretation, while Chailley’s paraphrase implies the latter.⁶⁸⁷ What the author seems to say is that the comparison of the products of corresponding base numbers (except twelve) between the two modes produce octaves, or in one case, a twelfth (Table 42), and that these intervals are beyond the scope of the second mode, which is characterized by fourths and fifths; thus, if a chant apparently in Mode II reaches such extreme intervals, it becomes a Mode I chant and produces the semitone (presumably, the semitone characteristic of Mode I chants – I shall return to this shortly).

Table 42 – Comparison of modal numbers between the first and second modes

Base number	Mode I Product	Mode II Product	Ratio	Interval
6	6	$3 \times 6 = 18$	$18:6 = 3:1$	Twelfth
8	8	$2 \times 8 = 16$	$16:8 = 2:1$	Octave
9	9	$2 \times 9 = 18$	$18:9 = 2:1$	Octave

Here, Nowacki contradicts his earlier assessment that the numbers do not reflect specific pitches, suggesting that the ratios between the modal numbers across these two modes suggest the spans A–a, Γ–G, and an unspecified twelfth that he surely intends to associate with Γ–d,⁶⁸⁸ thus, chants that exceed the a, or perhaps even the G, become Mode I chants. This interpretation does agree with the four chants cited for this mode in the *Alia*, none of which rise above G, but relies upon the assumption that the numbers represent consistent pitches, an idea that has already been shown to be quite problematic.

⁶⁸⁶ *Alia musica* §182(j), ed. Chailley, 87, trans. Nowacki, 118.

⁶⁸⁷ Nowacki, *Greek and Latin Music Theory*, 118; Chailley, *Alia musica*, 88 (j).

⁶⁸⁸ Nowacki, *Greek and Latin Music Theory*, 119. Nowacki also gives the octave D–d, implied by the earlier reference to the octave 12:6.

Chailley is more careful here, saying only that the chants become first mode chants if their ranges reach an octave, without specifying which octave.⁶⁸⁹

Both Chailley and Nowacki agree that the reference to the semitone means that if the range rises beyond the identified pitches, it thereby includes the semitone characteristic to the first mode. This statement, too, is problematic. The semitone native to the octave species associated with the first mode in the *Alia musica* appears between the sixth and seventh scale degrees (that is, between \flat and c). Exceeding the ranges Γ – G or A – a will not necessarily cause a chant to include that semitone. It is also not clear that this semitone can really be considered characteristic of the first mode, as many first mode chants do not rise so high (*Iohannes autem*, for instance, does not rise above A), and the same semitone between the sixth and seventh scale degrees also occurs in *tetrardus* (though, of course, the placement of the other semitone differs).

Nowacki suggests that the characteristic semitone is the one created by the use of \flat . This interpretation is somewhat consistent, because it is quite likely that an apparent Mode II chant that rises above a would incorporate the semitone a – \flat (since it would likely be a neighbour tone, and the use of \flat in such a position would tend to draw attention to the tritone between F and \flat ; it must be acknowledged, however, that the prescription to avoid such figures comes from much later treatises and cannot be assumed to be operative in the Carolingian period). However, if Nowacki is correct in assuming that 18:9 represents Γ – G , then a chant that exceeds G will still not reach this semitone. Furthermore, the *synemmenon* and the associated \flat are mentioned only in the latest layers of the *Alia*, and it is problematic to invoke them to explain comments made in the source treatise. Chailley, again, is more careful, not specifying the pitch of the characteristic semitone.

⁶⁸⁹ Chailley, *Alia musica*, 88.

The fundamental problem posed by this passage, then, is how to interpret it if numbers do not correspond consistently to pitches, which all signs suggest that they do not. There is another possible interpretation for the numbers of Mode II that would be partly consistent with this passage, though it would be inconsistent with other treatises of the time, and it is not clear how to apply it to the other *maneriae*. If one were to suppose that the 12 of both Modes I and II must be D, then 16 and 18 in Mode II must be A and Γ. The ranges A–D and Γ–D are inconsistent with the way that Mode II chants are conventionally notated in a notation system that originates much later, but they are intervallically indistinguishable from the spans C–G and D–G that appear in the chants of this mode in later manuscripts.⁶⁹⁰ However, to be consistent with those later spans, in the spans A–D and Γ–D, A must be interpreted as the *finalis* – that is, the plagal mode must be understood to end a fourth lower than the corresponding authentic mode. According to this interpretation, the typical notation for Mode II chants must be transposed down by fourth to be consistent with the way that the *Alia* describes them. This procedure is possible from *protus* chants without affecting the interval structure because Mode II chants never exceed a fifth, and the pentachords Γ–D and C–G are the same species. However, if one were to attempt to apply this procedure to *tritus* or *tetrardus*, intervallic distortion would result (not so much in *deuterus*, however, because although the fifth above the *finalis* would be diminished if the *finalis* were transposed down to ♭, the characteristic interval of Mode IV chants is only a fourth and doesn't reach F, and even with the invocation of an *emmelis*, the perfect fifth scale degree does not occur in any of the Mode IV chants cited in the *Alia*).

Such an intervallic distortion may or may not be a problem. Some of these distortions could be remedied with the judicious use of ♭. Furthermore, transpositions of a similar nature do occur in the repertory (though the transpositions are usually a fifth upwards rather than a fourth downwards) that are

⁶⁹⁰ See the analyses for Mode II in Chapter 17.

frequently described as being done deliberately to take advantage of these intervallic distortions (because the interval that is present in the transposition that differs from the standard position is actually used in the chant instead of the interval found in the standard position).⁶⁹¹ An example of such a transposition occurs in the introit *Uenite adoremus*, a Mode II chant that is typically noted with its *finalis* on a. The incipit for this chant, on “*Uenite*,” begins c-c-d-a-G-a-F.⁶⁹² It is the last pitch, F, which justifies the transposition; without transposing, this pitch would be a low B-flat, a pitch not admitted in the later medieval system, though it appears in place of B in the gamut of the *Enchiriades*. Thus, if Phillips is correct that the *Enchiriadis* gamut more closely represented the Carolingian gamut than did the Greater Perfect System, the hypothetical systematic placement of plagal *finales* a fourth lower than the *finales* of the authentic chants could be more consistent than it actually appears.

Unfortunately, this hypothesis does not improve the situation much. In the first place, authentic chants are as likely to be transposed as plagal chants, and so the hypothetical placement of plagal *finales* a fourth below the authentic *finales* probably could not result from any meaningful difference in intervallic structure between authentic and plagal modes; ultimately, the difference between authentic and plagal chants in the *Alia* is probably still primarily range, as it is in all the other treatises. Furthermore, if A were to be understood as the *finalis*, then it is difficult to imagine an apparent Mode II chant rising high enough to exceed the g or a that would have it reclassified as Mode I, because these notes would be a seventh or octave above the *finalis*, rather than the fourth or fifth usually specified as the limiting point in other treatises. Thus, it is highly unlikely that the modal numbers 18, 16, and 12 can be associated with the pitches Γ, A, and D, even by invoking transposition. The comparison of modal numbers between Modes I and II here is probably more symbolic than literal. Even so, it is difficult to understand how the range

⁶⁹¹ cf. Apel, *Gregorian Chant*, 157–59.

⁶⁹² cf. CANTUS database, <http://cantus.uwaterloo.ca/chant/643137>; melodies are given from manuscripts CH-P 18 and D-Gsta AB III 9.

beyond which a plagal chant becomes authentic could be defined using such symbolism; it would have been much more practical for the author to have constrained this discussion to the numbers of the second mode itself, explaining that an apparent Mode II chant becomes a Mode I chant if the range rises above the twelve of Mode II itself, or perhaps if it reaches nine with respect to the other Mode II numbers.

An interesting problem not addressed by the *Alia* itself, nor by either Nowacki or Chailley, is how this doctrine interacts with the antiphon families. This issue is not addressed in the *Musica enchiridis*, which likewise holds that a plagal chant should not rise beyond the fifth note above the final. However, the *Enchiridis* does not say that an apparently plagal chant becomes authentic if it rises above the fifth; it only says that plagal chants don't rise above the fifth. The difference, though subtle, is one of description and prescription. It is important to remember that a plagal antiphon is not differentiated from the authentic exclusively by its range; it is also differentiated according to the intonation formula by which it is sorted into *loca* and *differentiae*. But the *Alia* says that if a chant that appears to be second mode (presumably because it begins with a second-mode *locum*, an intonation formula associated with a second-mode theme group) but then rises above a specified range (however that is defined), it becomes a first-mode antiphon.⁶⁹³ The question becomes, how would one select an appropriate *differentia*? By way of demonstration, the very first *locum* identified for Mode II in the *Nova expositio* is A. What would happen if an antiphon in this theme group were to rise above the range of a plagal chant and become authentic? The first mode does not have a *locum* on A; there is, therefore, no *differentia* associated with a chant that begins this way in Mode I. In all likelihood, this question is purely academic. Although it would be arrogant to put words in the mouths of the authors of the *Alia*, one suspects that they would answer that there is no such chant in that theme group that rises to such heights, and to compose one

⁶⁹³ *Alia musica*, §182(j), ed. Chailley 87, trans. Nowacki 118; the interpretation of this passage in the source treatise is a little bit unclear, and more definitive expressions of the principle are left to the latter authors.

that does would be improper. Nevertheless, the question is fascinating because it reveals the inherent tensions in a new modal system with one foot still firmly in the old paradigm.

The special case of Mode III

Mode III is a special case amongst the modes in the *Alia musica*. The explanation of this mode is very complex: it employs four unique coefficients (though it does not yield four unique products) and is connected in the *Nova expositio* to an octave species other than the E–e species that later theorists would expect (though it is likely that this contradiction is the result of octave species being used as a paraphrastic way of naming pitches).⁶⁹⁴ Due to the complexity of the passage, it is worth taking a close look at what the *Alia* says about this mode. The source treatise says:

Tonus tertius AYNOEANE, qui graece dicitur autentos deuterus, id est auctoritas secunda, qui ideo duo A habet, primum propter mensuram autenti deuteri, et secundem A propter plagis autenti deuteri, in cujus diapason suum semitonium explicat, et secundi toni, qui ambo sunt semitonia, propter triplam ut dictum est et quadruplam propter quater 6 ad unum 6 de primo, A 12, CCCC 6.6.6.6, BBB 8.8.8.8, DD 9.9. Quater enim 6 ad quater 3 de 12 quater 9 sunt, qui sunt 36 diapason, id est sexies 6 et ter 12; similiter ter 8 ad 12, quia duplae proportionis sunt. Bis 9 ad 12, quia sesquialtera proportio est, 30 diapente fiunt. Unde iste tertius tonus aut per diapason erit per 6 ad 12, aut per diapente, id est 30, ut in introitibus Confessio et pulchritudo et Dispersit dedit pauperibus et Cognovi Domine, et antiphonis Qui de terra est et Quando natus es et Malos male perdet et Homo quidam fecit cenam magnam.⁶⁹⁵

The third *tonus* [with the ἄληθα] AYNOEANE, which in Greek is called the *authentos deuterus* – that is, the second authentic [lit., second authority] – which on that account has two A's, the first on account of the measure of the *deuterus* authentic [*i.e.*, Mode III itself], and the second A on account of [the measure] of the plagal of the *deuterus* authentic [*i.e.*, Mode IV, the plagal counterpart to Mode III (*deuterus authentic*)], in the octave of which it releases its semitone, and of the second *tonus*, both of which are semitones, on account of the tripling, as has been said, and the quadrupling on account of the four 6's to the one 6 of the first. A 12; CCCC 6·6·6·6 [= 4 × 6 = 24]; BBB 8·8·8 [= 3 × 8 = 24]; DD 9·9 [= 2 × 9 = 18]. Indeed, four 6's to the four 3's (from 12) are four 9's [*i.e.*, 4(6) + 4(3) = 4(9)], which is the octave, 36 – that is, six 6's and three 12's [*i.e.*, 6(6) = 36, and also 3(12) = 36]; likewise, [36 is also] three 8's to 12 [= 3(8):12 ⇔ 3(8) + 12 = 36], since they are of the double proportion. Two 9's to 12, since they are a sesquialter proportion [*i.e.*, 3:2], they become the fifth, 30 [*i.e.*, 2(9):12 ⇔ 2(9) + 12 = 30]. Whence this third *tonus* will be either through the octave 6:12 or through the fifth, which is 30, as in the introits *Confessio et pulchritudo*, *Dispersit dedit pauperibus*, and *Cognovi*

⁶⁹⁴ This issue is addressed in Chapter 12 above.

⁶⁹⁵ *Alia musica* §183, ed. Chailley, 88, trans. Nowacki, 119–20.

Domine, and the antiphons *Qui de terra est*, *Quando natus es*, *Malos male perdet*, and *Homo quidam fecit cenam magnam*.

Parts of this passage are clear enough. The opening of the passage identifies different names for the mode, and the end provides examples of introits and antiphons that exemplify the mode. The author also identifies the four products of the numbers from 12:9:8:6 with their respective coefficients according to the pattern used throughout the treatise.⁶⁹⁶ Much of the middle of the passage is devoted to comparing the resultant products to each other, to identify the intervals formed by these products. The early part of the passage, however, is less clear. The author says that Mode III:

[...] on that account has two As, the first on account of the measure of the *deuterus* authentic, and the second A on account of the plagal of the *deuterus* authentic, in the octave of which it releases its semitone, and of the second *tonus*, both of which are semitones, on account of the tripling, as has been said, and the quadrupling on account of the four 6's to the one 6 of the first.

What does this passage mean? At first glance, it seems very likely that the “two As” refers to the use of the letter A to represent the number twelve in the proportion 12:9:8:6. However, it is unlikely that this statement should mean that the number twelve should be multiplied by the coefficient two because such an interpretation would contradict the statement of coefficients that immediately follows this passage (as well as every other statement of the coefficients throughout the treatise). There are at least two plausible alternative explanations. It could mean that the number twelve can potentially correspond to two different pitches in this mode, or it could mean that while the number twelve corresponds to only one pitch, it is used in two different ways.

The first twelve, the author seems to say, is the one that would naturally be expected to occur in the third mode; the twelve that one would expect to find here is presumably either the *finalis* itself (if one

⁶⁹⁶ It should be noted, however, that the *ēkhēma* and *manerial* designation in the first sentence, as well as the listing of modal numbers and letters are absent from every manuscript in the Principal Group, and are present only in source K, the manuscript that contains only a copy of the source treatise.

were to assume that the multiple of twelve always represents the *finalis*) or the octave above the *finalis* (if one were to assume that the largest number represents the *finalis*), though in fact, as the analyses in Chapter 17 will demonstrate, neither assumption seems to agree well with the chants that the source treatise cites for this mode. The second twelve, by contrast, is somehow connected to the way that the number twelve is used in the fourth mode (the plagal counterpart of the third mode); the number twelve itself does not actually appear in Mode IV, and the base number twelve is doubled to get to twenty-four, which seems to be the lower tone of a perfect fourth.

The next clause states that the semitone of the octave of the fourth mode is released. Chailley proposes that the octave referred to here is the “modal octave”⁶⁹⁷ (more or less synonymous with octave species).⁶⁹⁸ He notes that this passage makes the only reference in the source text (his “first Quidam”) to the modal octaves, which is nevertheless sufficient to show that the concept of modal octaves predates the source text.⁶⁹⁹ His hypothesis is certainly plausible, but it should also be noted that elsewhere in the *Alia musica*, the octave species are consistently identified as *species diapason*, and not merely *diapason* (as in this passage); the term *diapason*, taken alone, usually refers simply to the interval of an octave. Of course, the numbers assigned to the fourth mode do not create an octave, but then, the numbers seem to define *characteristic* intervals, not the entire set of *available* intervals. Thus, the octave of the fourth mode might well correspond to a particular octave that could occur in the fourth mode, of which the only plausible candidate (*i.e.*, an octave that fits within the acceptable range of the mode) is presumably the octave B–b (though the analyses in Chapter 17 do not provide any support for this interpretation; no

⁶⁹⁷ Chailley, *Alia musica*, 90.

⁶⁹⁸ Chailley seems to mark a distinction between modal octaves and octave species (pp. 32–37), but the distinction seems more practical and linguistic than semantic: he uses the term “*octave modale*” when describing medieval theory and “*aspect d’octave*” when describing Greek theory, in order to avoid implying an equivalence (much less a continuity) between Greek octave species and medieval modes, while the principle described by both (that each species or modal octave consists of a unique distribution of five tones and two semitones between the boundaries of the octave) is effectively the same.

⁶⁹⁹ Chailley, *Alia musica*, 90.

Mode IV chant cited in the source treatise spans an entire octave). If B is the intended octave, it does, indeed, participate in a semitone; in this way, it is possible that the author of the source text could well be describing the semitone at the bottom of the octave species without having a fully-developed theory of octave species – this, in fact, is precisely the kind of paraphrase one might expect in a theory of mode in which the octave species are secondary structures derived from characteristic gestures, rather than being primary structures that constrain the melody.

Regardless of whether the reference to the octave of the fourth mode necessarily implies a theory of octave species, it seems very likely that the semitone described in the passage is a semitone at the bottom of an octave. The next two clauses collectively imply that the second mode also includes such a semitone. This statement is odd because, under the usual assumptions about the second mode, the lowest step of the corresponding octave is A–B, which is a tone, not a semitone. However, with the flexibility of B-natural and B-flat,⁷⁰⁰ a semitone could appear at the bottom of the octave A–a (which would then be converted into the same octave species as E–e). Chailley, therefore, interprets these two clauses simply to clarify the meaning of the previous clause, that there should be a semitone at the bottom.

Yet, having established that these clauses describe a semitone at the bottom, the question remains of what the second usage of the number twelve means. The answer probably lies in the complete set of numbers for the mode. Coefficients aside, the complete set of products is twelve, eighteen, and twenty-four. The first usage of the number twelve is the expected octave between twelve and twenty-four. The remaining possible intervals that could exist amongst these numbers are the fifth (18:12) and the fourth

⁷⁰⁰ In most medieval theory, the flexibility between B-natural and B-flat is held to exist in the upper octaves (*viz.* b and b), but not in the lowest octave; this is explained in later theory in terms of hexachords, with no soft hexachord beginning low enough to provide the necessary pitch. However, the low B-flat is present in the gamut of the *Musica enchiridis*, and so the presence of this pitch need not pose a serious problem for this interpretation, though it might provide additional support for the possibility that the theory in the source treatise is rooted in the *Enchiridis'* gamut. But if Chailley is correct in the rest of his interpretation of this passage, the point may ultimately be moot, as the low B-flat is not used as a literal pitch, but simply by analogy.

(18:24). The *Alia* does not cite the fourth above the final, and this omission is initially somewhat surprising, since Aurelianus considers the fourth to be a defining interval in the mode. Instead, the *Alia* cites the fifth. The fifth that the *Alia* cites, however, is unexpected in another way: it is not the fifth above the lowest note (twenty-four), but the fifth below the highest note (twelve). This second interval, then, would be another use of the number twelve. But if twenty-four and twelve are understood to represent E and e, respectively (as Chailley supposes), then the ratio 18:12 represents the fifth a–e, beginning a fourth above the final and reaching up to the octave above the final.

Chailley interprets the fact that the fifth runs a–e to be the reason why the author invoked the second tone (A–a). He explains that the emphasis on the semitone at the bottom applies equally to this fifth, which should also have a semitone at the bottom, which requires the pitch b.⁷⁰¹

The final clause of the shorter excerpt holds that these usages arise on account of the tripling and the quadrupling. This clause relates backwards, beyond the references to octaves and semitones, to the fact that the two uses of twelve relate to Mode III itself and to Mode IV. The statement is further clarified by the clause “on account of the four 6’s to the one 6 of the first.” The “one 6 of the first” is probably a reference to the first mode, which indeed has only one six. It is unlikely that this is a reference to the pitch that might correspond to this six (d) because this pitch does not seem to have any kind of meaningful relationship to the rest of the argument; instead, invoking the single six of the first mode is probably another way of invoking the base number, rather than any product of it. If this interpretation is correct, then the “quadrupling” refers to the quadrupling of the base number six in Mode III, which yields the twenty-four against which the first use of twelve creates the octave “on account of the *deuterus* authentic”; the “tripling,” then, refers to the tripling of the same base number six, not in Mode III, but in

⁷⁰¹ Chailley, *Alia musica*, 89.

Mode IV (and also Mode II), which yields the eighteen against which the second use of twelve creates the fifth “on account of the plagal of the *deuterus*.”⁷⁰²

The complexity of this explanation, however, raises an obvious question: why invoke Mode IV to explain the relationship of 18:12, when the numbers eighteen and twelve are both part of Mode III in the first place? The eighteen is described as the tripling of six in Mode IV, but it also exists in Mode III as the doubling of nine. Perhaps this complexity here is merely the result of needing to explain the semitone issue, the explanation of which seems to require reference to other modes. Chailley hypothesizes that the author lacks the vocabulary to describe the semitone at the bottom of the range appropriately,⁷⁰³ though I have made several observations throughout this dissertation of passages in other coeval or earlier treatises that seem to have no such difficulties; thus, his argument is not especially convincing.

Additionally, why is it necessary to state explicitly that the number twelve is used in two different ways in this mode? No such statement is made in other modes, so why here? This second concern may perhaps be clarified by the way that the fourth mode is described. The fourth mode is characterized by the products $12 \times 2 = 24$, $9 \times 2 = 18$, $8 \times 3 = 24$, and $6 \times 3 = 18$; thus, the products twenty-four and eighteen each occur twice – once as the double of a base number and once as the triple of a base number. One possible way to look at these numbers is to say that the multiple of eight creates a unison with the multiple of twelve, while the multiples of six and nine each create a fourth with the multiple of twelve. However, the *Alia* explicitly rejects this interpretation, saying that “*ter 6 non ad bis 12 comparatur, quoniam per 3 non dividitur, sed ter 8 ad ter 6 comparatur, et bis 9 ad bis 12 per sesquiterciam proportionem.*”⁷⁰⁴ (“Three 6’s [=18] is not compared to two 12’s [24], since it [*i.e.* two 12’s] is not divided by three,⁷⁰⁵ but three 8’s [=24] is compared to three 6’s [=18], and two 9’s [=18] is compared to two 12’s [=24] in the sesquiterian

⁷⁰² Cf. Chailley, *Alia musica*, 90.

⁷⁰³ Chailley, *Alia musica*, 89.

⁷⁰⁴ *Alia musica* §184(c), ed. Chailley, 91, trans. Nowacki, 121–22.

⁷⁰⁵ Twenty-four is, of course, divisible by three, but it is not here being divided by three, but rather by two.

proportion [4:3].”) That is, although either instance of the number twenty-four would form a 4:3 ratio with either instance of the number eighteen, these comparisons are only to be made between products of the same coefficient, creating two and only two distinct pairs of products that create a perfect fourth. On the basis of this doctrine, it might well deserve comment that the number twelve gets used twice in Mode III since in many other modes, each number is used only once. This interpretation, however, fails to explain why no comment is made in Modes I or II about the repeated use of the number twelve.

Nowacki’s interpretation of this passage differs from Chailley’s in several respects but maintains some of the same assumptions. Nowacki agrees that the reference to two As probably means two twelves. He supposes, however, that the remark is purely symbolic: “The remark appears to have no purpose beyond a play on the word *deuterus* and the number two.”⁷⁰⁶ This interpretation has no particular evidence in its favour, but it is certainly plausible and is much simpler than the contortions required to justify Chailley’s interpretation.

Nowacki also suggests an alternative interpretation, that the two As, one for the authentic and one for the plagal, might simply be an acknowledgement that both the authentic and plagal modes contain a twelve in their basic ratios. The authentic does, indeed, have a twelve in the ratio 24:12, but the plagal has the ratio 24:18, and it is not the twelve, but the twenty-four that is common to these two ratios. Nowacki observes that twenty-four is a multiple of twelve, and therefore, the twelve is still present in the fourth mode. Unfortunately, this explanation is tautological: each mode uses all of the base numbers, and twelve is one of the base numbers, such that every single mode contains a twelve in this manner. Nowacki’s alternate interpretation is not impossible, but the statements in the *Alia* are usually more substantive.

⁷⁰⁶ Nowacki, *Greek and Latin Music Theory*, 119.

Nowacki also interprets the two semitones differently. Chailley supposes that the two semitones are a–b and b–c. Nowacki suggests, instead, that the author means that both Modes II and III share the same semitone, a–b, which, in the second mode, is the semitone that appears when a Mode II chant crosses into the range of the first mode. This interpretation has the advantage of linking back to the interpretation of the second mode, but it seems self-contradictory in that sense because the *Alia* specifically said that a chant crossing into that range is no longer a Mode II chant; thus, it is difficult to imagine how this particular semitone could be characteristic of the second mode (and indeed, in the Mode II discussion, the author of the source treatise called it the semitone of the first mode, not the second).

Nowacki defends his interpretation based on “the principle of markedness.”⁷⁰⁷ He says that b–c is the unmarked form (*i.e.*, the default state of the gamut) and a–b is the marked form (the form that must be specified in distinction to the default form). It is not clear that an unmarked form should be preferable to a marked form in this instance, but even if that were to be true, Nowacki’s interpretation of a–b as a marked form might be making an unjustified assumption. The use of b certainly seems to be a marked form in later theory, but it is not necessarily so for the *Alia musica*. As has been observed at multiple points in this dissertation, the structure of the gamut in the *Alia* is not clear. There are passages in the later layers of the composite treatise that would seem to be most consistent with the Greater Perfect System and the later medieval gamut, but there is very little evidence about the structure of the gamut in the source treatise. More specifically, there is nothing in the source treatise to preclude the possibility that the gamut of this part of the treatise was comparable to that of the *Musica enchiridis*, which uses a low B-flat by default, or the Lesser Perfect System, which uses the b by default. And since the gamut is not specified, it may not even be clearly defined; there is no evidence of any kind of note-naming system in the source treatise. Thus, the source treatise’s gamut may not have a consistent marked and unmarked

⁷⁰⁷ Nowacki, *Greek and Latin Music Theory*, 120.

form, so that the marked form may be dependent upon mode: \flat does seem to be a reasonable unmarked form for Mode VII, but it is not at all clear that it should be considered the unmarked form in Mode VI.

Nowacki goes on to agree with Chailley that the fifth identified in the third mode is the upper fifth, $a-e$. He says that this observation is confirmed by the prominence of a and the scarcity of \flat in the chants cited for this mode.⁷⁰⁸ In this interpretation, though, the ratio 18:12 not only does not represent a range, it does not even represent an interval, just a pitch at the end of an interval. This interpretation also flies in the face of Nowacki's own general observation that the author of the source treatise happily identifies the specified intervals at whichever pitch he can find them. Nowacki returns to this same observation in a slightly different context a paragraph later, when he cautions the reader not to assume that "the mode's characteristic diatessaron [fourth] [...] is complementary to the diapente [fifth] and spans the notes E to a ." He says that "in the light of the author's habit of placing the lesser perfect consonances at various places in the modal compass and the prominence of other fourths in the musical examples ($D-G$ and $G-c$ alongside $E-a$), we cannot draw that conclusion with any confidence."⁷⁰⁹ Quite so – except that according to the *Alia*, the third mode is not characterized by a fourth in any case; it is characterized by the octave and the fifth.

Nowacki also observes that the description seems to reveal a tension between an older tradition in which Mode III chants were defined in the sixth $E-c$ (a minor sixth) with an inflection point at a and a newer tradition that defines modes in relation to octaves. As has been routinely noted throughout this dissertation, it is not at all clear that the author of the source treatise considered the modes to have anything to do with octave species; there are a few passages (the beginning of the Mode III excerpt being one of them) that are suggestive on this issue, but they are hardly conclusive. More directly, while I agree with Nowacki's general characterization of a and c as important inflection points in Mode III chants, there

⁷⁰⁸ Nowacki, *Greek and Latin Music Theory*, 121.

⁷⁰⁹ *ibid.*

is no trace of the interval of a sixth anywhere in the text of the *Alia*, nor any indication that the pitch c should be important (unless it be as one of the boundary pitches of the characteristic fifth, which, as the analyses in Chapter 17 will demonstrate, does seem to occur with some frequency).

The preceding discussions show that Chailley and Nowacki differ considerably in their interpretations of the description of Mode III in the source treatise. One point upon which Chailley and Nowacki agree is that the letter A is a proxy for the base number twelve in the statement that Mode III has two As (despite having only one amongst its modal numbers). There is a critical caveat to this interpretation – and this caveat is one of the few cases in which Chailley’s reorganization of the treatise could potentially disguise the issue: unlike the order in which Chailley presents the *Alia* in his edition (in which the source treatise comes first), the actual ordering of the composite treatise places the source treatise at the end, along with the tabular summary and the prose summary; these are the three sections that associate the letters with the numbers (and, in fact, the letters do not appear in the prose summary in source F, which presents only the prose summary). The revision and commentary use only the modal numbers without the letters, and as a result, in the proper ordering of the treatise, these letters never appear prior to the first passage (in the revision) that says that Mode III has two As.

This fact does not make Chailley’s interpretation impossible, as it is possible that the letters were once included in an early copy of the complete treatise and were lost during or prior to the copying of the common archetype of all the extant manuscripts, or even that the revisor simply did not notice that the letters had not been presented yet when he reproduced this passage (in fact, the revisor explicitly clarifies that the two As represent two twelves; however, the description of the third mode is one of the very passages that Chailley cites as evidence that the revisor does not completely understand the meaning of the source treatise⁷¹⁰). Even though the source treatise appears at the end of the composite treatise, it

⁷¹⁰ Chailley, *Alia musica*, 133.

was written first, and in that treatise, the modal numbers do appear (and therefore appear prior to the passage in question) – but only in manuscript K (which seems to have been a copy of the source treatise from before it was incorporated into the composite treatise); in the rest of the manuscripts, the letters appear only in the table and the prose summary.

Interestingly, though, K uses the letters in a different and surprising way. The use of B, C, and D is unchanged, but A (which represents the number twelve) is not used in this manuscript. In its place is a double C; since C represents the modal number six, a doubled C therefore represents the number twelve. (Chailley, in his edition, blithely substitutes A for CC, as he systematically does elsewhere in the treatise, thus presenting a reading not supported by any manuscript).⁷¹¹ Consequently, in K, when the relevant passage on the third mode arrives, the letter A has never been presented in that manuscript (nor is it ever presented afterwards), so a reader of this manuscript would never be in a position to interpret the “two As” statement as Chailley or Nowacki propose, even after reading the manuscript multiple times. It is unclear whether the double C was the original state of the text, or whether it was changed in a subsequent copying. If the former possibility is correct, then Chailley’s interpretation becomes all but impossible; in the latter possibility, Chailley’s hypothesis would require that the scribe who changed A to CC did so consistently everywhere except in this passage (perhaps because he, too, did not understand the passage’s meaning); this latter possibility is easier to accept than the former (notwithstanding the relative likelihood that A or CC was the original state of the manuscript), especially the part where the scribe who changed A to CC did not understand the treatise that he was copying; this last hypothesis, however, becomes even more difficult to trust in light of the fact that the revisor also did not notice that the letters had not been presented prior to this passage – or likewise did not understand the text that he was copying.

⁷¹¹ “(a) ex K ; totum deest in GMP ; [...] Pro A semper ante XII adscribitur CC in cod. K ; ex tabellis et §3 restituimus.” (Chailley, *Alia musica*, 86 §1 critical apparatus). (“[Subsection] (a) [the Greek names and list of modal numbers, comes] from K; the whole is lacking from GMP [the Principal Group manuscripts]; [...] CC is always written before XII instead of A in manuscript K; we have restored [A] from the tables and from §3 [the passage in question above].”

It is, in fact, Chailley's opinion that the revisor did not understand the broader passage surrounding this issue,⁷¹² but it must be acknowledged at this point that Chailley's interpretation requires an error or lack of understanding on the part of not only the revisor himself (and perhaps also of every scribe that copied the *Alia*, though fidelity to the model might explain these cases) but also the scribe who changed A to CC in manuscript K (or its model); the multiple errors that need to have been made in independent manuscripts makes Chailley's hypothesis that much more difficult to accept with confidence.

It is also interesting to note that in two manuscripts, A and C, the passage in question gives the letter A in majuscule. Although there is a disagreement between sources A and C, on one hand, and M and P, on the other, about the correct case for the letter B, all of the scribes in each manuscript appear to be carefully consistent about their usage, never substituting majuscule for minuscule (or vice versa) within a single manuscript.⁷¹³ However, in sources A and C, the letter A in the passage in question is given in majuscule, and everywhere else, it is consistently given in minuscule. This, again, would imply that for Chailley's interpretation to be correct, the scribes of A and C (or, at least, the scribe of the common model from which these two manuscripts were copied) must not have understood the passage; otherwise, the A would have been given in minuscule, as it is everywhere else in the manuscript.

Furthermore, for the letter A, taken in isolation, to be referring to the number twelve would, in itself, be unique within the *Alia*. Throughout the treatise, modal numbers routinely appear without the associated letters, and only occasionally appear with them (generally only in the table and in the first clause of each mode in the prose summary, though not in F), but never do the letters appear without the numbers; if Chailley's interpretation is correct, then the passage in question would be the only time that such a presentation occurs in the entire *Alia musica*.

⁷¹² Chailley, *Alia musica*, 133.

⁷¹³ Gerbert's edition would seem to be an exception to this observation, but I have speculated in Chapter 4 that Gerbert's usage reflects which of the two sources he used (M and A) he is copying in any particular passage.

In addition to these concerns, one must also take into account that the rest of Chailley's argument relies on the assumption that the convoluted description of Mode III is ultimately attempting to describe the location of the semitone above the *finalis*, which assumes that the author would not have had the vocabulary to describe this semitone more directly, an assumption that is difficult to accept. Furthermore, Chailley's argument also relies upon the fifth present in Mode III alongside the octave to correspond to the upper fifth of the octave, rather than the lower fifth, an assumption about which analysis of the model chants provided in the *Alia* (Chapter 17) provides contradictory evidence.

There is an alternative possibility, though it, too, is far from clear. In every single copy of the manuscript, the passage describing two As always places the A between a pair of dots, thus: ·A· or ·a·. This procedure is somewhat unusual within the *Alia musica*, but not entirely so. In general, Roman numerals in the *Alia musica* manuscripts are almost always followed by a dot (sometimes mid-line, sometimes at the baseline, like a period), but they are not often preceded by a dot. As for the letters, they generally appear above the numbers, and little can be said about their relationships to these dots. It is possible that these dots are present simply to ensure that the letter is read as a discrete entity, rather than being read as a word or part of a word. However, surrounding a letter with a pair of dots is also a common way of identifying an abbreviation (as, for instance, ·n· is a common abbreviation for *enim*). In truth, this form of abbreviation is not prominent in the extant manuscripts; however, abbreviation practices are quite inconsistent both within and among the manuscripts, and there is no reason to assume that this form of abbreviation was not used in an earlier, lost archetypal manuscript.

Manuscript K provides ambiguous evidence on this hypothesis. On the one hand, it is the only manuscript in which the letters are not placed above the Roman numerals, but before them (and exclusively as majuscules, where minuscules predominate in all the other manuscripts) – and, indeed, they are generally both preceded and followed by a dot. But, as already described, K does not use the letter

A, it uses a doubled C. Furthermore, K does include prominent examples of abbreviations marked by two dots.

Thus, it may be hypothesized that ·A· is not a reference to the base number twelve, but an abbreviation, and perhaps one not recognized by the scribes making the copies. It is even possible that the abbreviation was not originally the letter A, but rather some other symbol that resembles the letter A, and that the scribes, not knowing the symbol, mistook it for an A. However, against this hypothesis, I am not able to suggest an appropriate resolution of an abbreviation that would render this passage intelligible; I offer this hypothesis only because the interpretation of this passage is so problematic that all possibilities ought to be considered.

Mode IV: Numbers that are not to be compared

In the fourth mode, there is a brief but complex passage that I have already cited regarding numbers that are not to be compared to each other. It reads as follows:

Hic, quia bis 12 ad ter 8 et ad ter 6 bis 9 una consonantia numerorum in diapason 6 ad 12, id est 24, conueniunt. Idcirco ter 6 non ad bis 12 comparatur, quoniam per 3 non diuiditur, sed ter 8 at ter 6 comparatur, et bis 9 ad bis 12 per sesquiterciam proportionem.⁷¹⁴

Here, since twice 12 to thrice 8, and to thrice 6 twice 9 come together in one consonance of numbers in the octave 6:12, therefore, three 6s are not compared to two 12s, since [two 12s] is not divisible by three, but three 8s are compared to three 6s, and two 9s to two 12s, in the sesquitercian proportion.

This passage has been interpreted in several ways. Chailley says,

2 × 12 se confound avec 3 × 8 et 2 × 9 avec 3 × 6 en une même consonance de nombres ou se retrouve 18 qui est l'octave 6 + 12. Mais l'octave 6. 12 n'apparaît dans ce ton, car on ne peut comparer

⁷¹⁴ *Alia musica* §184(b)–(c), ed. Chailley, 90–91, trans. Nowacki 121–22. It should be noted that Chailley “corrects” the 24 to 18, though every single manuscript is unanimous on this point. Chailley has interpreted the number here as the sum of 6:12, while I interpret it as the result of both two twelves and three eights from earlier in the sentence, as the author is making the point that both come out to the same number (see below); however, it is odd under this interpretation that the comparable product of 18 is not given for three sixes and two nines, also from the beginning of the sentence. Regardless, this comment is redundant, and does not affect the interpretation of the passage.

3 × 6 et 2 × 12 qui n'ont pas le même coefficient, tandis que l'on peut comparer 24 et 18 qui s'écrivent soit 3 × 8 et 3 × 6, soit 2 × 9 et 2 × 12, faisant la proportion sesquiterce 4/3 (quarte).⁷¹⁵

2 × 12 gets confused with 3 × 8 et 2 × 9 with 3 × 6 in a common consonance of numbers where is found 18, which is the octave 6 + 12. But the octave 12:6 does not appear in this mode, for one cannot compare 3 × 6 and 2 × 12, which do not have the same coefficient, while one can compare 24 and 18, which are written either as 3 × 8 and 3 × 6 or 2 × 9 and 2 × 12, making the sesquitercian proportion 4:3 (a fourth).

Thus, Chailley has interpreted this passage to mean that in order to compare modal numbers, the coefficients must be the same. This interpretation is strongly implied by the clause "*quonian per 3 non diuiditur*" ("since it is not divided by three"). Thus, one would understand that although the numbers 24 and 18 each appear twice, the instances of each are different from one another in some way, so that only those produced from the same coefficients may be compared against each other. However, this interpretation does not quite reflect the beginning of the passage, which does not compare incompatible versions of 18 and 24 against each other (as for instance, two twelves and three sixes). Instead, the beginning of the passage rejects the comparison of the two twenty-fours against each other and the two eighTEENS against each other, which come together into the octave, represented by the number eighteen.

Chailley does not explain how these numbers come together into the octave, but merely notes that there is not an octave in this mode because it does not arise from comparing numbers with like coefficients. Nowacki, however, does try to explain how the forbidden comparisons result in the octave. He notes that both twenty-four and eighteen are multiples of three, and that three is the sum of the terms of the ratio 2:1 that represents the octave.⁷¹⁶ The mathematical procedure that Nowacki is invoking is common in the *Alia musica*, and his explanation initially seems reasonable. For example, the *Alia* notes in the first mode that 12:8 sums to 20, and that 20 is four times the number that represents the perfect fifth (which is five, the sum of the terms of the ratio 3:2) because twelve and eight sum to four times the terms of the ratio. Nowacki, then, seems to be interpreting that both eighteen and twenty-four are

⁷¹⁵ Chailley, *Alia musica*, 91.

⁷¹⁶ Nowacki, *Greek and Latin Music Theory*, 125.

respectively six and eight times the sum of the terms of 2:1, which therefore makes them representatives of the octave in the same way that twenty represents the fifth. However, the *Alia* does not appear to treat multiplex intervals, such as the octave (2:1), twelfth (3:1), and double octave (4:1) in this manner. Furthermore, Nowacki is ignoring (or, at least, does not acknowledge) that in this case, twenty-four and eighteen are not sums of modal numbers, but the modal numbers themselves.

What this distinction may mean is that the *Alia* is trying to have it both ways. The modal numbers ordinarily represent intervals through the comparison of two modal numbers, but occasionally, a single modal number seems to represent an interval all by itself. Or perhaps, the *Alia* also represents an octave by comparing a single number against itself. That is, the octave might be represented by the ratio 1:1 in addition to 2:1, perhaps by double-dipping, comparing a single copy of the number with the sum of both instances of the same number. This interpretation seems problematic, but there is additional evidence of this procedure in every mode after Mode IV, as will be described later in this chapter. Thus, it can hardly be a universal rule that like numbers must not be compared against one another.

Instead, perhaps, the *Alia* means to say that comparing like numbers produces a kind of octave, but since the fourth mode lacks the octave (as Chailley notes), they should not be compared against each other in this instance. However, one must still contend with the clause “since it is not divided by three.” As Chailley suggested, that clause seems to imply that the coefficients must be the same. Unfortunately, the *Alia* does not seem to follow this rule consistently. In Mode VI, six sixes and four nines are both compared to two twelves. There are several other such examples. The only similar statement that can be made that appears to be consistently true is that the coefficients of products that are compared to each other are always commensurable: the one coefficient is always the same as *or a factor of* the other; this solution accounts for the above-cited discrepancy in Mode VI (and the other discrepancies), but is not directly articulated by the *Alia* itself; it must be read between the lines (especially from the discussion of Mode VI – see below) and must be treated as speculative.

It is worth observing, here, that Nowacki remarks upon the “noncongruence between description and evidence,”⁷¹⁷ noting that he does not find a consistent interval of a fourth either as a prominent interval or as the overall range in the chants cited by the *Alia*; speculating about the author of the source treatise’s decision, he hypothesizes that “instead of yielding to confirmation bias by citing a falsely uniform sample, he deliberately selected examples that expose the mode’s full range of variability.”⁷¹⁸ I come to precisely the opposite conclusion from the analyses in Chapter 17, as I find the examples in this mode to be among the most consistent presented for any mode in the *Alia*. The distinction between Nowacki’s interpretation and my own is that I follow the *Alia*’s model (from the discussion of *Rorate caeli* in Mode I) by dividing the chant into *incisiones* and looking for the range of each *incisio*, not the range of the entire chant; Nowacki also seems to be neglecting the impact of the *emmelis*.

Modes V and VI

The descriptions of Modes V and VI simply provide additional examples of phenomena already described in other modes. Both modes contain references to octaves that result from comparing like numbers. In the case of Mode V, the *Alia* says that the chants always display the fifth, because there are only two modal numbers, each derived twice: twenty-four (three eights and four sixes) and thirty-six (three twelves and four nines). As in Mode IV, the author carefully compares only those that share a coefficient. The *Alia* then says, “*quorum consonantia diapason est, id est in duodenarium quarter 9 et ter 12, et quarter 6 et ter 8*”⁷¹⁹ (“The consonance of these is the octave, that is containing twelve, four 9s and three twelves [both sum to thirty-six], and four sixes and three eights [both sum to twenty-four].” Here, again, the *Alia* compares two identical numbers to each other and calls them an octave. Nowacki again

⁷¹⁷ Nowacki, *Greek and Latin Music Theory*, 123.

⁷¹⁸ *ibid.*

⁷¹⁹ *Alia musica* §185(c), ed. Chailley, 92, trans. Nowacki 123–24.

explains that these sums are divisible by three, which is the sum of the ratio 2:1,⁷²⁰ an explanation that I have already questioned in connection with Mode IV. In addition to my previous objections, it is worth observing that with the exception of eight and sixteen, each of which occurs as a modal number in only one mode, every other modal number cited in the *Alia* is divisible by three (because the base numbers six, nine, and twelve are all divisible by three themselves, which makes their multiples also divisible by three, and while eight is not divisible by three, it is directly multiplied by the coefficient three in most modes); Nowacki's logic could therefore be applied to get an octave out of just about any modal number.

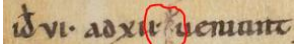
The sixth mode also displays this characteristic, though in a more complex way. The modal numbers for the sixth mode are two instances of twenty-four and two instances of thirty-six, just as in the fifth mode; however, they are derived differently. The *Alia* says:

*Sexies enim 6 ad sexies 4 de bis 12 et quater 9 ad quater 6 de bis 12, quia sesquialter proportio est in diapente, et sexies 4 de ter 8 et sexies 4 de bis 12 in consonantia diapason, id est 6 et 12 ; ueniunt per 8 qui sunt 48, sicut sexies 6 et quater 9 in 72.*⁷²¹

Indeed, six 6s to six 4s (from twice 12) and four 9s to four 6s (from twice 12), because the sesquialter proportion is in a fifth, and six 4s (from three 8s) and six 4s (from twice 12) are in the consonance of the octave, which is 12:6, come by way of 8, which is 48, just as six 6s and four 9s are in 72.

There is a lot to unpack in this excerpt, and brief digression is in order. The author makes frequent reference to six fours in this passage, but of course, four is not one of the base numbers. As in Modes IV and V, there are only two modal numbers in this mode, each derived twice. However, unlike Modes IV and V, there are not only two coefficients (whereby, in Modes IV and V, each instance of a modal number has a counterpart instance of the other modal number that shares the same coefficient). Instead, there are four unique coefficients: 2×12 , 6×6 , 3×8 , and 4×9 . The author, however, has made a rule that two

⁷²⁰ Nowacki, *Greek and Latin Music Theory*, 124.

⁷²¹ *Alia musica* §186(b)–(c), ed. Chailley, 92, trans. Nowacki 124–25. The semicolon before *ueniunt* is given in Chailley's edition, but is probably incorrect. The only justification for it in any of the manuscripts is a smudge in P₂ (fol. 63r, ln. 13: ) that might once have been punctuation; the only punctuation in the other manuscripts is the dot that routinely follows numerals. A semicolon here would leave the principal clause without a verb.

modal numbers should only be compared to each other if they share a coefficient, and now he is stuck with this rule. There are six sixes, and in order to compare anything to them, he must have six of something else. He has two instances of twenty-four (from two twelves and three eights), which can be expressed as six fours. He also has four nines, and in order to compare anything to them, he must have four of something else. If twenty-four can be six fours, it can equally be four sixes.

The first statement in the excerpt, then, is straightforward. The author produces the perfect fifth in the ratio 36:24 twice, and he follows his rule of comparing only products of like coefficients by redefining twenty-four as necessary to match the coefficients of the two instances of thirty-six. Against the six sixes, he compares the six fours, and against the four nines, he compares the four sixes. In both cases, he gets the twenty-four originally from two twelves and not from three eights, which is interesting because it means that the three eights are not used in the production of the perfect fifth. What differentiates two twelves from three eights? It is difficult to say for sure, but it may perhaps be because both four sixes and six fours are commensurable with twelve but not with eight (that is, if one counts by fours or sixes up to twenty-four, one will pass through twelve *en route*; the same may be true of eight when counting by fours, but not when counting by sixes – see Table 43).

Table 43 – Commensurabilities of four and six with twelves but not eights

Both sixes and fours can be used to even measure out any multiple of twelve (upper table). However, only fours can evenly measure out all multiples of eight; sixes do not (lower table). Thus, four sixes and six fours are both commensurable with two twelves, but not with three eights.

0	6		12	18		24
	12			24		
	4	8	12	16	20	24

0	6		?	12	?	18	24
	8		16		24		
	4	8	12	16	20	24	

The second statement in the excerpt says that the two instances of twenty-four, each of which is equivalent to six fours, create an octave by way of eight, which creates forty-eight. Here, Nowacki's translation and interpretation differ dramatically from mine. Nowacki translates, "They come to 72 through [six times] 8, which is 48, as [do] the six 6s and the four 9s."⁷²² Here, I believe that Nowacki has completely missed the mark. If he is correct, he must bend over backwards to explain in what sense two twenty-fours create seventy-two. It is not impossible, combining procedures employed by the *Alia*, but now not only double-dipping but triple-dipping: (1) the two twenty-fours sum together to create forty-eight; (2) either twenty-four creates an octave compared to this same forty-eight (in which they are both already implicated; and (3), twenty-four to forty-eight sums together to create seventy-two.

However, Nowacki's translation overcomplicates the analysis, based on his reading of *sicut* ["just as" or "in the same way that"]. He believes that *sicut* here means that the combination of the twenty-fours must somehow come to seventy-two, *just as* the combination of the thirty-sixes does. However, in all likelihood, *sicut* is meant to indicate a simpler analogy: the combination of the twenty-fours gives forty-eight *in the same way that* the combination of thirty-sixes gives seventy-two. Thus, the combination of the two twenty-fours creates an octave in forty-eight, and the combination of two thirty-sixes create seventy-two (and perhaps also create an octave, but the *Alia* is not explicit on this point).

However, it still remains to explain in what manner forty-eight arises "by way of eight." I believe that Nowacki was correct in a general sense in supplying the editorial note "through [six times] eight." Indeed, $6 \times 8 = 48$. However, there are not six eights among the modal numbers of Mode VI, there are only three. Where do the six eights come from? The two twenty-fours are interpreted in this passage as six fours. When one combines six fours with another six fours, one gets twelve fours, not six eights.

⁷²² Nowacki, *Greek and Latin Music Theory*, 125; in this instance, the notes in brackets are Nowacki's, not mine.

Arguably, one could say that one gets six pairs of fours, which gives six eights, but this explanation is convoluted.

It would be much simpler if the *Alia* had instead interpreted the twenty-fours as four sixes instead of six fours. These two interpretations are not the same. Although the commutative property of multiplication says that the product is the same regardless of the order of the terms, this property applies to abstract multiplication; it does not mean that the numbers are literally interchangeable in practical applications: four six-pound hammers (to borrow the Pythagorean metaphor) cannot simply be redistributed among six baskets weighing four pounds each, even though the resulting total weight would be the same. In six fours, four is a base number and six is a coefficient, and these are not interchangeable with each other.

However, six fours is not one of the procedures for creating a modal number in Mode VI; it is derived from two twelves or three eights to suit the needs of the author. Thus, why did he not simply derive four sixes instead? This derivation would have the advantage that six is actually one of the standard base numbers (though there are more than four of them in this mode), and it would be more obvious how forty-eight is attained “by way of eight”: four sixes combined with another four sixes would give eight sixes (if six is substituted with x , then $4x + 4x = 8x$). So why not use four sixes, which provide the simple solution, instead of six fours, which require a convoluted explanation?

Again, the only answer that suggests itself appears to be commensurability. In this case, the author must use both instances of twenty-four – both the two twelves and the three eights. This procedure is different from that in the first statement in the excerpt, where he derives the perfect fifth. In that instance, he derived the fifth twice, by independently comparing each of the two instances of thirty-six with the same instance of twenty-four; he was able to use two twelves to the exclusion of three eights because he was making two separate comparisons. However, to produce his octave, he must have both

copies of twenty-four in a single comparison; consequently, he must include the three eights. And unfortunately, three eights is not commensurable with four sixes (see Table 43 above); thus, he must use six fours, which is commensurable with both derivations of twenty-four.

Mode VI, then, provides additional evidence of both the hypotheses suggested by Mode IV: the author continues to compare only modal products that share a common coefficient (as in Modes IV and V), and must go to considerable trouble to make it possible to do so in a mode in which all the coefficients are unique, thus implying commensurability (rather than equality) of coefficients as the real defining characteristic; and the author continues the practice of identifying the octave in comparisons of two instances of the same modal number. In this latter characteristic, however, there is one important difference compared to Modes IV and V: in Mode VI, this octave is actually considered to be a characteristic interval of the mode. The author went out of his way in Modes IV and V to reject this possibility, explaining that while the octave could, in theory, be achieved by comparing the same two numbers to each other, they should not be compared to each other in these modes because the coefficients are incompatible. In Mode VI, though, the author employs a sleight of hand to make the coefficients commensurable with each other, so that it becomes acceptable to compare these duplicate twenty-fours to produce the octave.

Interestingly, this octave does not appear in the tabular summary at the end of the *Alia* (which contains all the correct modal numbers, but the fifth is the only interval cited). It also corresponds to a curious anomaly in the prose summary. There, the author acknowledges both the octave and fifth. The fifth is correctly cited as four nines to two twelves (36:24). The octave is cited twice: once as three eights to two twelves (24:24, which is 48), and once as six sixes to two twelves (36:24), which ought to be a fifth.⁷²³ There are two likely explanations for this discrepancy. According to the conventional

⁷²³ *Alia musica* §178, ed. Chailley, 178.

interpretation that the prose summary belongs to a later layer of the *Alia*, it is possible that the author of the prose summary simply confused the numbers that should create each interval. However, if the prose summary was not a summary of the *Alia*, but rather served as the inspiration for the source treatise, this very passage may have been the origin for the unusual notion that two identical modal numbers compared against each other should produce the octave, which would have caused a chain reaction whereby the author of the source treatise would have felt the need to explain why the same procedure is not followed in Modes IV and V, each of which contains duplicated modal numbers in this manner.

It is worth noting, at this point, that Nowacki interprets this passage to mean that the fifth that should be present in Mode VI ought to be the upper fifth of the modal octave for the same reason that he and Chailley both draw the same conclusion in Mode III. He says that the mode is represented by the relation 48:36:24, and that 36:24 is therefore the upper fifth. As a plagal mode, it is deceptively intuitive to suspect that the upper fifth of the traditional modal octave should be more prominent than the lower fifth. Because of the way that Nowacki interprets the intervals within the chants, he feels that this characterization agrees with the examples provided in the *Alia*, which seem to emphasize the octave C:F:c. However, he must then acknowledge that his reading also implies the complementary fourth C:F, and the fourth is not identified as important in the *Alia*; he therefore hypothesizes that the author of the source treatise considered the fifth to imply its counterpart.⁷²⁴ This interpretation is difficult to accept. If it is correct, then the same procedure could well be available in every other mode, and each of the three possible perfect consonances would be present in each mode (and notably in Mode III, where precisely this procedure must be rejected). However, Nowacki includes the number 48 in his reckoning, and that number is not given as one of the modal numbers for this mode; it is a sum only, and sums are not treated as equivalent to modal numbers for the other modes in the *Alia*. His interpretation that the fifth belongs

⁷²⁴ Nowacki, *Greek and Latin Music Theory*, 126.

to the top of this apparent octave also once again requires that the modal numbers imply specific pitches, not abstract intervals, a notion he has elsewhere rejected.

Modes VII and VIII

The *tetrardus* modes are unique within the *Alia musica* in that there are multiple presentations of modal numbers for both the authentic and plagal modes, allowing, in the case of Mode VII, for all three perfect intervals, just as Mode I contains all three perfect intervals. Mode VII could just as easily include all three intervals by sharing a single coefficient for all four base numbers, as Mode I does, but instead derives each interval independently. Similarly, Mode VIII includes both fourths and fifths but no octaves, which could easily be achieved by maintaining a shared coefficient for each of twelve, nine, and eight, eliminating the octave by using a different coefficient for six (twice the coefficient used for the other three numbers would work nicely); alternatively, all four coefficients could be a common multiple of the coefficients of Mode II, which likewise has the fourth and fifth without the octave. Again, Mode VIII instead derives each interval separately. In addition to these two shared proportions, Mode VII also gets a new proportion of its own, but Mode VIII does not.

Actually, it is not entirely clear how Mode VIII is defined in the source treatise. As has been described multiple times in this dissertation, the source treatise, functioning as an appendix to the complete *Alia*, ends “*Tonum octauum require supra*,”⁷²⁵ an indication that the necessary information is to be found above. It is not clear whether this indication means far above (*i.e.*, in a previous section that described Mode VIII, in which case, it would be worth considering whether it could be significant that this procedure is not employed elsewhere in this heavily repetitive treatise) or immediately above (*i.e.*, the properties of the eighth mode are effectively the same as those of the seventh mode).

⁷²⁵ *Alia musica* §188, ed. Chailley, 95.

Regardless, both Modes VII and VIII are said to consist of the proportions of Mode IV (24:18) and Mode V (36:24). For Mode VIII, Nowacki observes that these intervals could be combined in the composite relation 36:24:18, which could imply a lower fifth and an upper fourth within the bounds of a shared octave; however, he rejects this explanation because it does not seem credible for a plagal mode to be divided in this fashion. He is, of course, thereby assuming that the modal numbers correspond to pitches and that the *finalis* should be the dividing point.⁷²⁶ He makes a related argument, but differing substantially in its details, for Mode VII: he expects this mode to fit in the span 96:48, and since the ratios for the fourth and fifth fall entirely outside this range, he argues that they are not meant to reflect the position of the intervals within Mode VII's range; instead, they imply their position by analogy, since both the fourth and fifth seem to be situated upwards from the *finalis* in their own modes, and therefore should be so situated in Mode VII.⁷²⁷ However, it is not clear that his conclusions about these other modes are correct, and consequently, it is not clear that his conclusions about this mode are correct. More specifically, he once again assumes that the intervals have a fixed position at precisely the same time that he acknowledges that the author happily finds any given perfect interval "wherever it may occur in the melodic design."⁷²⁸ Nowacki describes this practice as "following an empirical approach, not entirely beholden to theoretical dogma," but I take it as additional evidence that the modal numbers do not represent specific pitches. Nowacki also takes for granted the importance of octaves in the definition of modes in this source treatise, an interpretation that is only very weakly supported and far from conclusive.

The third presentation of modal numbers for Mode VII is particularly interesting, and it is unique in at least two ways: in the first place, only three of the base numbers are used (nine is omitted); and in the second place, coefficients for all three remaining base numbers are selected to produce three instances

⁷²⁶ Nowacki, *Greek and Latin Music Theory*, 128.

⁷²⁷ Nowacki, *Greek and Latin Music Theory*, 127.

⁷²⁸ *ibid.*

of the same product; this third proportion is characterized exclusively by the product forty-eight: twelve is multiplied four times, while six and eight, in commutatively equivalent procedures, are multiplied by eight and six, respectively. Even these procedures, though, are not specified in the source treatise (except in K), as no modal numbers are given at all, only the composite product of one hundred forty-four.

Because there is only one product in this proportion, it is not clear to what it ought to be compared, but the *Alia* does make it clear that the comparison produces an octave:

*Hic tonus in proportione quarti toni habet totum diatessaron, et in proportione quinti habet totum diapente, et in propria proportione habet duodecies 12 diapason, quod est 144.*⁷²⁹

This mode has a complete fourth in the proportion of the fourth mode, and a complete fifth in the proportion of the fifth mode, and in its own proportion, it has the octave twelve twelves, which is 144.

Unlike the other modes, the treatment of this mode differs dramatically across the different sections of the treatise. Nowacki's and Chailley's interpretations of this passage clearly privilege the form of the source treatise in manuscript K. The source treatise in all of the manuscripts of the Principal Group contains complete descriptions only of Modes I – VI; these descriptions are then followed exclusively by the passage just cited and the indication to find Mode VIII above; manuscript K replaces these passages with more thorough descriptions similar to the descriptions of Modes VII and VIII in the prose summary (which also contains a slightly elaborated version of the passage cited above). That version alone contains all the modal numbers.

Chailley and Nowacki interpret the reference to the number one hundred forty-four not as a product, as it is given in the text (they presumably consider the multiplication to be mere numberplay of the kind found in Mode I), but as a sum. Since the modal number proper to this mode is forty-eight and the interval is an octave, the sum must be of terms in the ratio 2:1, one of which must be forty-eight: thus, the ratio must be 96:48. This ratio supports Chailley's interpretation that the fourth *maneria* is

⁷²⁹ *Alia musica* §187(e), ed. Chailley, 93–94.

characterized by modal numbers in the range of the fourth octave above the octave of the first *maneria* (96:48 compared to 12:6).

Or, perhaps the reverse is true, and Chailley's interpretation of the *manerial* octaves supports his interpretation of one hundred forty-four as representing the sum of the terms of 96:48. Nowacki, too, supposes that the number ninety-six is implied in this passage. However, it must be acknowledged that *the number ninety-six never appears* anywhere in the *Alia musica*. The one place in which the number ninety-six may be implied is in the prose summary, which elaborates that one hundred forty-four "*est octies 6 et sexies 8 ad quarter 12*"⁷³⁰ ("is eight 6s and six 8s to four 12s").

This passage is the only occurrence in the entire *Alia* of a procedure of comparing the sum of two modal numbers against yet another modal number. It is not remotely clear how this passage from the prose summary should be interpreted. It does provide yet additional evidence that numbers do not represent specific pitches. If one were to suppose that forty-eight were to represent a specific pitch in the seventh mode – perhaps G, for the sake of argument – then the prose summary is asking the reader to compare G (eight sixes) to G (six eights) and then compare this result to G (four twelves) to find an octave. This explanation verges on the nonsensical, but it is consistent with the pattern in the *Alia* of treating a number compared against itself as an octave: here, a number is being compared against two other copies of itself and still produces the octave. This passage is also consistent with the tabular summary, which provides three instances of forty-eight; unlike all of the other miniature tables that make up the tabular summary, the interval label (an octave in this case) does not appear next to the numbers but in the title of the miniature table,⁷³¹ perhaps to clarify that the octave in this instance is fundamentally

⁷³⁰ *Alia musica* §179, ed. Chailley, 179.

⁷³¹ *Alia musica* §171, ed. Chailley, 173–74.

different from the intervals in the other tables – that is, as an acknowledgement that there is something unusual about deriving an octave from the comparison of two identical numbers.

The revision, however, treats Mode VII entirely differently. The revisor says:

*In proportione autem propria habet diapason, quod est quater 6 ad quater 12, et sexies 4 de bis 12 ad sexies 8, qui numeri simul iuncti faciunt 48.*⁷³²

In its own proportion, however, it has the octave, which is four 6s to four 12s [48:24], and six 4s (from two 12s) to six 8s [48:24], which numbers joined together make 48.

The revisor, then, compares the product twenty-four against forty-eight. But twenty-four is not one of the modal numbers that properly belong to Mode VII itself; instead, it is the number that is common to both of the borrowed ratios from Mode IV (24:18) and Mode V (36:24). The revisor follows the example of the source treatise in comparing only products with a shared coefficient, so that the twenty-four is given both as four sixes (to compare against four twelves) and six fours (to compare against six eights); he does not make any use of the eight sixes that also produce forty-eight. It is interesting to note, here, that while he claims to be borrowing the twenty-four from Modes IV and V, his divisions (four sixes and six fours) are not those of Modes IV and V, they are the special divisions introduced in Mode VI. It might seem that he has selected these simply to follow the rule of maintaining the same coefficient, but in the immediately preceding passage, he makes the same mistake, confusing the modal numbers of Mode VI for those of Mode V; as described above, they share the same products, but the derivations are radically different.

The Revision and Commentary

In the last section, it was useful to invoke the revisor to demonstrate the uneven treatment of Mode VII throughout the *Alia*. However, this chapter has otherwise been relatively silent about the treatment of the numerology in the revision and commentary. There is good reason for this silence. As Chailley has

⁷³² *Alia musica* §128, ed. Chailley, 162.

observed (and as I have cited elsewhere in this dissertation), there is evidence to suggest that the revisor and commentator do not completely understand the source treatise. They function less as authors than as interpreters, attempting to help readers of the source treatise to understand the underlying doctrines, and sometimes to enrich the study by adding additional discussions of related concepts. In the attempt to clarify what the previous authors have said, they sometimes contradict the previous authors instead, and much of the “enrichment” seems rather stream-of-consciousness, squeezing in any fact for which they can make a smooth connection, whether it fits the context or not.

I have already argued that undue weight should not be given to their opinions relative to a fresh consideration of what is said directly in the source treatise; what a modern musicologist does in attempting to interpret the source treatise in its own terms is no different from what the revisor and commentator do (though admittedly, the revisor and commentator do have the advantage of being much closer in time to the source treatise and being immersed in a much more similar musical culture, but for the same reason, they may be more likely to have a blind spot towards their biases than a modern musicologist trained to avoid such biases). Because of the concerns about the accuracy of the revisor’s and commentator’s interpretations of the source material, it would be misleading to present their interpretations on an equal footing with the presentation in the source treatise itself. Nevertheless, the revisor and commentator contribute a considerable amount of material to the composite treatise, and it would also be inappropriate to ignore them entirely. I shall, therefore, take a middle-ground and examine their contributions to the study of the *protus* modes as an example of the kinds of contributions they make.

The revisor’s description of the first mode begins nearly identically to the text of the source treatise:⁷³³

⁷³³ *Alia musica* §§181(a)–(e) & 31, ed. Chailley, 85 & 121, §181 trans. Nowacki, 113–14.

<i>Alia musica, §181(a)–(e)</i>	<i>Alia musica, §31</i>
<p><i>Tonus primus</i></p> <p><i>NONANOEANE, qui grace dicitur autentos protos, id est auctoritas prima, A 12, C 6, B 8, D 9.</i></p> <p><i>6 ad 12 18 sunt, qui sunt ter 6, quae proportio dupla diapason dicitur. Item 8 ad 12 20 sunt, quae proportio sesquialtera dicitur ad 12, ideo quia 12 habet 8 in se et alteram eius partem, id est 4, et facit diapente. Item 9 ad 12 21 sunt, quae proportio sesquialtera dicitur ad 12, quia 12 habet 9 in se et eius tertiam partem, id est 3, et facit diatessaron, id est ter 7 qui sunt 21.</i></p> <p><i>Omnis igitur primus tonus</i></p> <p><i>aut ter 6 habet in dupla proportione diapason,</i></p> <p><i>ut est Rorate caeli desuper,</i></p> <p><i>aut quater 5, id est 2 de 8 et 3 de 12 in sesquialtera proportione qui faciunt diapente, quod est 20,</i></p> <p><i>ut est et nubes pluant iustum, aperiatur ;</i></p> <p><i>aut ter 7, id est 3 de 9 ad 4 de 12 in sesquitertia proportione, qui faciunt diatessaron,</i></p> <p><i>ut est terra et germinet Salvatorem.</i></p>	<p><i>Omnis, inquit, primus tonus,</i></p> <p><i>quem dorium dicimus,</i></p> <p><i>[Omnis, inquit, primus tonus,]</i></p> <p><i>aut ter 6 habebit in dupla proportione, id est diapason,</i></p> <p><i>ut est Rorate caeli desuper,</i></p> <p><i>aut quater 5 quod est 20, id est 2 de 8 et 3 de 12 in sesquialtera proportione faciens diapente ;</i></p> <p><i>ut est et nubes pluant iustum, aperiatur ;</i></p> <p><i>aut ter 7 quod est 21, id est 3 de 9 ad 4 de 12, qui in sesquitertio faciunt diatessaron,</i></p> <p><i>ut est terra et germinet Salvatorem.</i></p>

Both versions begin by identifying the first mode; the revision actually borrows this identification from slightly later in the source treatise (indicated in blue text). They then provide a Greek name; in the source treatise, this name is the *manerial* designation, and is accompanied by a gloss on the name, its ἦχημα, and a list of modal numbers; all of this material is present only in source K. In the revision, the Greek ethnic name is supplied instead. The next passage in the source treatise is omitted from the revision (text found only in one version is in grey); it describes the basic intervals that appear between the four base numbers 12:9:8:6, and is unnecessary in the revision because all this information was already provided in the introduction of the composite treatise, though some of it returns in the discussion that

follows this excerpt. The remainder of the passage is virtually identical, with only occasional alterations of diction (given in red).

From here, the source treatise lists the other introits and antiphons that exemplify the characteristic intervals and then moves on to Mode II. This material will appear in the revision, but the revisor adds some of his own comments first. He goes on to explain the previous presentation of *Rorate caeli*, indicating that the first *incisio* runs through the range D–d (employing the Greek string names) with an *emmelis* below. He says that this octave is represented by the ratio 12:6 (which sums to eighteen); this remark constitutes the return of some of the material from the omitted section above. But while the source treatise then noted that eighteen was also 3×6 , the revisor instead notes that the difference between six and twelve is also six; thus, the octave span from D–d is, in some sense, represented by the number six itself (the difference between the pitches is represented by the difference between the numbers that represent those pitches).⁷³⁴ He goes on to say that this same difference (six) may be divided equally in half, giving three. He is effectively taking the arithmetic mean (though he does not say so), evenly splitting the difference between the two numbers to produce the intermediate number nine, and he explains that this nine forms a fifth against the number six.⁷³⁵ This explanation is awkward because the source treatise says that the fifth of the first mode ought to be 12:8, not 9:6, though both are representatives of the fundamental proportion 3:2. Indeed, the revisor immediately proceeds to repeat the correct formula, that the five that represents a perfect fifth is the sum of two and three, and that these addends are factors borrowed from the base numbers eight and twelve (each divided by four). At the same time, he notes that a fifth is the concatenation of a fourth and a tone, and that eight is two more than six, and through this fact, the fourth arises.⁷³⁶

⁷³⁴ *Alia musica* §32, ed. Chailley, 121–22.

⁷³⁵ *Alia musica* §33(a), ed. Chailley, 122.

⁷³⁶ *Alia musica* §33(b), ed. Chailley, 122.

The relevance of this last point is not clear, but appears to be stream-of-consciousness: he has just invoked the two as a factor of eight (as one of the two terms that make the fifth), and then derives the fourth as eight which is two more than six – one is tempted to interpolate, “speaking of two and eight, eight is two more than six,” a point that has no particular relevance to the previous discussion, but makes for a smooth connection. Regardless, his explanation of the fourth is also awkward, because the fourth ought to be 12:9, not 8:6 (though they are in the same proportion). When invoking the fact that the fourth is a tone less than a fifth, perhaps he means for the reader to compare it to the eight (representing the fourth) that is one less than nine (representing the fifth); the tone is, indeed, produced by the ratio 9:8, but in creating this analogy, he must describe everything backwards, using the wrong fifth and fourth (this is a side effect of describing harmonics by wavelength rather than frequency).

As before, the revisor almost immediately switches to the correct ratio of 12:9, indulging himself with a little bit of numberplay *en route*. Nine is three more than six, and is also the mid-point of the upper six (*i.e.*, the six that is the difference between six and twelve); three, therefore, is both a quarter of twelve and a third of nine; these two numbers (twelve and nine), therefore, make a perfect fourth. This is a complicated way of saying that twelve and nine are commensurable by three, and therefore, three can be treated as a unit, so that 12:9 becomes 4:3 (the sesquitercian ratio that represents the perfect fourth). In modern terminology, three is a common factor, and therefore 12:9 is reduced to lowest terms as 4:3 by dividing each term by three. The revisor then says that this is the interval of the third *incisio* of *Rorate caeli*.⁷³⁷

It is important to observe that the revisor explicitly provided pitches for the octave, D–d, which circumscribe the first *incisio* of *Rorate caeli*. However, when describing the fourth and fifth, he does not provide pitches. Previous discussions in this dissertation make the reason clear: there is no D (12) in the

⁷³⁷ *Alia musica* §34, ed. Chailley, 123.

second *incisio*, which must share the number twelve. The revisor has identified the pitch to which the number 12 must correspond and cannot reconcile it with the second *incisio*; instead of acknowledging the problem, he simply bypasses it by not identifying the positions of the other pitches.

In the next section, the revisor returns to copying the text of the source treatise, identifying the introits (which he calls “*antiphonae gradales*,” antiphons of the Mass) and antiphons (“*antiphonae nocturnales*,” evening antiphons, or antiphons of the Divine Office), though his list is slightly different (see Chapter 11). He also copies the explanation that some intervals appear throughout an entire chant, again using virtually identical wording to the source treatise.

The revisor next tries to explain why the source treatise describes the perfect fifth by both the number 5 and 10. As Chailley notes, “*L’explication de l’auteur n’est pas très convaincante*”⁷³⁸ (“The [revisor’s] explanation is not very convincing.” The revisor says that in any sesquialter ratio, the smaller term is always divisible by two and the larger by three, and that therefore, the larger number is always half again larger than the smaller, and since this is true of 6:4 just as much as it is true of 3:2, therefore the sum of the terms of 6:4 (which is ten) makes a fifth just as easily as the sum of the terms of 3:2 (which is five).⁷³⁹ This passage demonstrates that ten is the sum of a sesquialter, but so is any multiple of five. The revisor has failed to explain why the source treatise invoked both forms, why it does not invoke other multiples of five, and why it does not do the same for the perfect fourth, which it describes in the following passage only by the sum seven (but not fourteen, which would be an analogous conclusion).⁷⁴⁰

Atkinson suggests that it is at this point that the commentator takes over.⁷⁴¹ The commentator looks ahead, observing that the Hypophrygian is only characterized by the fourth, and therefore is

⁷³⁸ Chailley, *Alia musica*, 124.

⁷³⁹ *Alia musica* §37, ed. Chailley, 124.

⁷⁴⁰ *Alia musica* §38, ed. Chailley, 125.

⁷⁴¹ cf. Atkinson, *Critical Nexus*, 176, Table 5.3.

measured exclusively by the number seven.⁷⁴² However, he indicates that the ten appears in the Hypodorian and the Lydian.⁷⁴³ Finally, he adds a few extra example chants from other chant genres.⁷⁴⁴

Overall, the revisor and commentator do not significantly enlighten the reader; the issues that the revisor clarifies are elementary (6:4 is equivalent to 3:2); other issues that he addresses are tangential at best (that the fourth and fifth differ by a tone) or outright obfuscating (measuring fourths and fifths against six instead of twelve), and the reader remains no wiser about the fundamental challenges in interpreting the source treatise (especially the application of the fifth F–c to the second *incisio* of *Rex caeli* when the numbers would seem to imply a fifth on D).

After this discussion, the *Alia* takes its first digression into the *Nova expositio*, following which the revisor turns to the second mode. As with the first mode, he begins by repeating what the source treatise says about the mode, this time reproducing virtually the entirety of the source treatise’s passage with only very minor variants in diction. The only important differences arise in the comparison of Mode II to Mode I, a matter of that was discussed extensively at the beginning of this chapter. The revisor follows the wording almost exactly, but makes two notable omissions to the last sentence.⁷⁴⁵

<i>Alia musica</i> , §182(j)	<i>Alia musica</i> , §48(a)
<i>Si autem in diapason euenerit proportio secundi toni,</i>	<i>Si autem in diapason uenerit proportio secundi toni,</i>
<i>uel 6 ad 12, ad primum est,</i>	<i>id est 12 ad 6,</i>
<i>ut ter 6 ad 6 primi,</i>	
<i>et bis 8 ad 8 et bis 9 ad 9</i>	<i>et bis 9 ad 9, et bis 8 ad 8,</i>
<i>primi toni semitonium efficit.</i>	<i>primum tonum efficit.</i>

⁷⁴² *Alia musica* §39, ed. Chailley, 125.

⁷⁴³ *Alia musica* §40(a), ed. Chailley, 125.

⁷⁴⁴ *Alia musica* §40(b)–(c), ed. Chailley, 125.

⁷⁴⁵ *Alia musica* §§182 & 45–48, ed. Chailley, 87 & 126–27.

Aside from a few inconsequential changes of wording (red) or order (blue), the revisor has omitted (grey) the comparison of the eighteen (three sixes) of Mode II to the six of Mode I and the vague reference to the semitone; these issues are precisely those that make the source treatise difficult to interpret, and instead of clarifying them, the revisor has simply avoided them; in the next sentence, he fills the gap left behind by omitting the reference to the semitone with a remark that each mode has its own constitution.

The revisor next says that corresponding authentic and plagal modes “look upon” (*respicunt*) one another, the one being sent lower, the other being elevated; he then repeats that the Hypodorian becomes Dorian if it exceeds its own range, this time precisely specifying that the range exceeded is an octave.⁷⁴⁶ Now the revisor returns, after a fashion, to the omitted comparison of eighteen to six, though he does not state the numbers; instead, he merely indicates that the Hypodorian is lower by triple than the Dorian. Without stating the numbers, this remark is problematic; eighteen is, indeed, triple six, but six is the top of Dorian and eighteen is the bottom of Hypodorian. A more apt comparison would be between the two bottom pitches, so that Hypodorian is not really lower by triple, it is lower by a half ($18:12 = 3:2$, or $1\frac{1}{2}:1$). Another reasonable comparison, not connected to the modal numbers but to the conventional relationships between the modes, would be that Hypodorian is lower by third (a perfect fourth, $4:3$, or $1\frac{1}{3}:1$). It is possible but not clear that the revisor may be confusing the triple for a third.

Next, the revisor sets out the entire two-octave gamut of the Hypermixolydian mode (more evidence that the authors of the *Alia* understand the connection between the Greek transposition modes and the octave species), identifying by Greek string-name the positions of the multiples of six:

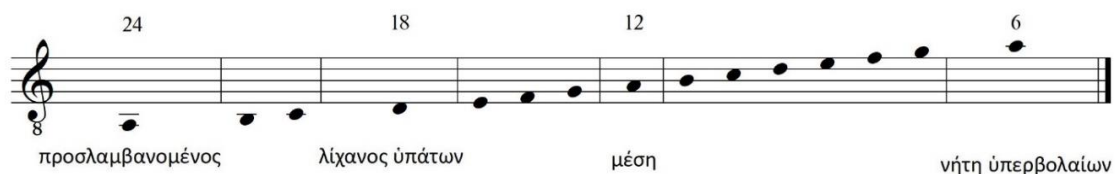


Figure 42 – Multiples of Six in Hypodorian

⁷⁴⁶ *Alia musica* §48(b)–(d), ed. Chailley, 127.

He explains that the difference between twenty-four and twelve is also twelve, and that antiphons in this plagal mode scarcely exceed this difference at the upper end (presumably, he means either the one octave range 24:12, which has a difference of twelve, or perhaps the note identified by the number twelve, which amounts to the same thing), but remains below, measured by the fourth (symbolized by seven) or the fifth (symbolized by ten). He then confirms that the two modes are a perfect fourth apart.⁷⁴⁷

Once again, the revisor's explanation is awkward. It is quite intuitive to a modern musicologist on the basis of later theory that the melodies of the second mode should generally remain within the range A–a. However, the revisor has identified this range through the numbers 24:12, and twenty-four is not one of the defining numbers for Mode II. The source treatise, by contrast, explained this issue relative to the numbers 18:6, against which the revisor's numbers do not make sense: even the first mode does not rise as high as a (6), the second mode routinely goes below D (18) and rarely exceeds a (12). Meanwhile, both the source treatise and the revisor cite the octaves 18:9 (two nines to nine) and 16:8 (two eights to eight); most of these numbers (except eighteen) do not appear in Figure 42, but if they did, they would represent the ranges D–d and E–e, which likewise make little sense as constraining the range of the second Mode.

Here, the revisor's discussion again slips into stream of consciousness. He attempts to justify his previous comment that the Hypodorian is triple the Dorian with a reference to Boethius, who (he says) considers the perfect fourth "to be begotten" (*procreari*) from the double and the triple. He then copies the majority of §3 from the introduction of the treatise (which was based on Boethius), describing two instances of the harmonic mean, 6:8:12 and 4:6:12.⁷⁴⁸ Treating the numbers strictly as corresponding to pitches, the first mean would divide an octave into a fourth and fifth, and the second would divide twelfth into an octave and a fifth, but this is not what the revisor says. Instead, he says that if one collects the

⁷⁴⁷ *Alia musica* §49–51, ed. Chailley, 128–29.

⁷⁴⁸ *Alia musica* §§52 & 3(a)–(d), ed. Chailley, 129 & 100.

differences between the two boundary terms of each relation, one gets $12 - 6 = 6$ and $12 - 4 = 8$, and the comparison of these two differences (8:6) is epitrite (sesquitercian, 4:3), which is the proportion that corresponds to the perfect fourth.

The revisor's observation is true as far as it goes, but it is purely abstract, and does not correspond to any musical phenomenon; in one were to maintain the numbers that the revisor proposed for the gamut of the Hypodorian mode (though any set of pitches would serve the needs of this demonstration), the interval between 12:6 would be the octave $a-\frac{a}{2}$ and the interval between 12:4 would be the twelfth $a-\frac{a}{3}$. It is not entirely clear how one should compare these intervals to one another, but the same result is obtained by either comparing the numbers representing an octave (two, from 2:1) and a twelfth (three, from 3:1), which gives 3:2, or by taking the difference in the size of the two intervals (a twelfth minus an octave); either way, the result corresponds to a fifth, not a fourth. The revisor's arithmetic is essentially correct, but he compares apples to oranges in order to arrive at the result he wants. It is also clear that in this instance, he does not actually need the complete harmonic proportions (6:8:12, 4:6:12) that he cites, because he makes no use of the harmonic mean itself (the middle notes, eight and six, respectively), though interestingly, these, too, could be compared to one another in the sesquitercian proportion. Clearly, what he really wanted was to prop up his earlier, flimsy arguments with the authority of Boethius.

In the final section about Mode II (excluding the *Nova expositio*), the commentator takes another kick at the can, trying to explain what the source treatise meant with the comparison of Modes I and II. He borrows the comparison of these two modes in the description of octave species in the introduction, comparing the different Greek string names for the same note between the two modes. He, too, applies numbers to these pitches, but his numbers are different. Chailley, who considers all this material to be the work of a single author, considers this new set of numbers to be contradictory, but the contradiction

disappears if it was written by a separate commentator, as Atkinson suggests;⁷⁴⁹ and these numbers also do not fit with those of the source treatise, but are slightly better. The commentator puts six at the παρανήτη διεzeugμένων of the Hypodorian [d], which is the same as the μέση of Dorian.⁷⁵⁰ This is the numbering that describes the Dorian mode in terms of the paradigmatic octave, 12:6, where 12 is D, 6 is d, 9 is G, and 8 is a; even so, it still calls the highest note μέση, so that it is clear that in both Hypodorian and Dorian, the mode occupies the lower half of the two-octave gamut (this placement differs from Heard's and Mühlmann's treatment, which puts the authentic mode in the top half and the plagal mode in the lower half).

The commentator then says that if a second-mode chant reaches this μέση (d), it becomes Dorian, and gives the example of the responsory *Uniuersi qui te expectant*. As is commonly the case with responsories, the respond and the verse appear to be in different aspects of the same *maneria*: the respond is in the plagal mode, spanning the complete octave A–a, but the verse moves up to the authentic, spanning the octave D–d. The commentator describes it in approximately this way, but shifts his number system back to that of the revisor (the numbers of the Hypodorian), so that the respond spans 24:12 instead of 18:9, and the verse spans 18:9 instead of 12:6. In this way, the commentator is able to justify the source treatise's comment that the plagal mode becomes authentic if it reaches the six (d, reckoned in the Dorian number series) or the 9 (also d, but reckoned in the Hypodorian number series). He says that the chant *Circumdederunt me* also demonstrates this relationship.⁷⁵¹ It is interesting that according to the commentator, the melody should have to rise quite so high as d to become authentic, while for the

⁷⁴⁹ Chailley, *Alia musica*, 130; cf. Atkinson, *Critical Nexus*, 176, Table 5.3.

⁷⁵⁰ This passage gives yet another example where Heard's translation is problematic. He treats the string names as always corresponding to the same pitches no matter what mode they are in, so that he must supply editorial "corrections" and still does not get two equal pitches (Heard, *Alia musica*, 154); he changes *paranētē diezeugmenōn* to *likhanos hypatōn*, which is D, and he assumes that *mesē* in Dorian is still just a; he therefore must add an editorial comment to the effect that the two notes are not the same, but equivalent. The only way to justify this equivalence would be to say that they are both *mediae* of their respective modes, acknowledging that the definition of *media* is different between authentic and plagal modes.

⁷⁵¹ *Alia musica* §53, ed. Chailley, 130.

revisor, it should only have to exceed a. It should also be observed that the commentator has still bypassed the ratio 16:8 that was cited in the source treatise; in the Hypodorian number series, eight is e, which is clearly too high to serve as a reasonable limit, but in the Dorian, eight is a, which is precisely the limit that the revisor proposes, and yet the commentator passes over this correspondence without comment.

Of course, all of this reckoning requires that the numbers have consistent associations with pitches, an assumption that both the revisor and the commentator take for granted but which cannot be sustained by comparison to *Rorate caeli* and become utterly nonsensical when moving up to higher *maneriae* (for which the larger spans of modal numbers would paradoxically imply lower overall pitches).

For Mode II, then, the contributions of the revisor and the commentator are once again unhelpful. Both authors ignore the reference to the semitone, the most opaque part of the source treatise's description. The revisor dives into pseudo-metaphysical nonsense to try to justify the importance of 18:6 on the authority of Boethius and uses numbers not present in the proper modal numbers of Mode II; the commentator is slightly more successful in using the source treatise's numbers to limit the range of Mode II, but he must jump unsystematically back and forth between two parallel number series to make it work.

As the preceding discussions demonstrate, the revisor and the commentator do not seem to understand the language of the source treatise, and their comments largely demonstrate the extents to which they must go to make the source treatise's explanations conform to their own sense of how modes should behave. It would bring little profit to continue examining the rest of their variants and commentary upon the source treatise; it is sufficient to note that they continue in much the same manner.

Alternate Intervals?

There is one other passage that ought to be considered here. Although it has no numbers attached directly to it, it contains an association of intervals to modes that is somewhat different from the associations that dominate the rest of the treatise. It appears following the initial set of analogies to nature and references to classical authors but precedes the commentary on the same material. The passage reads:

*Unde ex quinque speciebus tonorum : simplicium, id est diapente, diatessaron, diapason ; compositorum, ut diapason simul et diapente, ut tertius tonus et sextus, et diapason ac diatessaron in eodem quinto, et dupla diapason in octavo tono, una cum duplo et triplo.*⁷⁵²

Whence from five species of tones [five sizes of intervals?]: of the simplex [ones], that is the fifth, fourth, and octave; of the composite [ones], as the octave together with the fifth [*i.e.*, the twelfth], as in the third tone and the sixth, and the octave and fourth [*i.e.*, the eleventh] in the same fifth, and the double octave in the eighth tone, together with the double and triple.

This passage is difficult to interpret. Chailley says:

Ce passage est probablement corrompu : la phrase est inachevée grammaticalement, et quinque species tonorum est impropre. [...] En outre, un passage manquant devait faire allusion au 5ème ton, car eodem ne se justifie pas dans le texte actuel.

This passage is probably corrupted: the phrase is grammatically unfulfilled, and “five species of tones” is improper. Besides, a missing passage must have made an allusion to the fifth tone, as “the same” is not justified in the actual text.

This material is revisited a few paragraphs later, after the commentary on the previously cited classical allusions, in a passage that also appears, somewhat modified, in the *Dulce ingenium*:

*Quapropter primus tropus erit in dupla [proportione], secundus in tripla, tertius in quadrupla, quartus in epitrita, quintus in sesquialtera, sextus sicut [et] tertius in diapason simul ac diapente: diapason quoque ac diatessaron in septimo: dupla diapason in octavo, una cum duplo [et] triplo.*⁷⁵³

On that account, the first trope will be in the double proportion [2:1], the second in the triple [3:1], the third in the quadruple [4:1], the fourth in the epitrite [sesquitercian] [4:3], the fifth in the sesquialter [3:2], the sixth, as the third, in the octave together with the fifth; the octave also and the fourth in the seventh; the double octave in the eighth, together with the double and the triple.

⁷⁵² *Alia musica* §156(c), ed. Chailley, 97.

⁷⁵³ *Alia musica* §162(a), ed. Chailley, 168; cf. *Dulce ingenium*, §92 [long recension], ed. Bernhard, 37, whence I have added the notes in brackets.

This passage clarifies somewhat, though it appears immediately after a commentary and may, therefore, be a commentary itself; conclusions drawn from it may, therefore, be an attempt to make a passage that does not relate to the theory presented in the *Alia* seem to be more consistent with the rest of the treatise. At the very least, it should be noted that the original passage did not identify the correct interval (or intervals) for the first two modes or the fourth.

Both passages are immediately preceded by a reminder of the five perfect consonances and the numbers that represent them. Three are multiplex intervals represented by only a single number (the larger number of their ratio, omitting the 1): the octave is two, the twelfth is three, and the double octave is four. The other two are the superparticular ($n + 1 : n$) intervals, represented by the sum of their terms: the perfect fifth is five (3:2) and the perfect fourth is seven (4:3). Each of these numbers is given in basic form before the first passage and given multiplied by six before the second passage.

I disagree with Chailley that the first passage is corrupted. Instead, I feel that this passage is probably taken out of context; it was probably incorporated into the *Alia* from another source and does not directly reflect the doctrines of the *Alia* at all. At the very least, it says that there ought to be an octave and fourth in one of the modes. This “octave and fourth” may perhaps be interpreted as an eleventh, but the eleventh is not only not one of the perfect consonances just described, it also is not present in any mode in the *Alia* (none of which is characterized by an interval larger than an octave). It may also be interpreted to mean that the mode has an octave and also has a fourth, which also does not describe any mode in the *Alia musica* (at least, not without also having a fifth).

I suspect that this first passage seemed, to one of the contributors to the *Alia*, to have had just enough in common with the doctrines of the *Alia* to be worthy of inclusion, even if it didn't quite fit (it certainly does not agree with the theory presented in the source treatise, though Chailley attributes it to

that author). The commentary was then added (perhaps by the same author who included the original passage, perhaps by a later author) to attempt, not entirely successfully, to reconcile the two doctrines.

By multiplying the numbers representing the consonances by six, the second passage labels the multiplex intervals with the numbers twelve, eighteen, and twenty-four. In this way, the first three modes are labelled with continually larger multiplex intervals: the octave, twelfth, and double octave, respectively. The intervals themselves make no sense: the first mode is, indeed, represented by the octave (but also by the four and fifth), but the second mode is not represented by the twelfth, nor is the third mode represented by the double octave. However, as Chailley, points out, these three modes can be represented by the numbers twelve, eighteen, and twenty-four, each of which is the largest modal number in the corresponding mode.⁷⁵⁴ The passage goes on to say that the fourth mode will be in the ratio 4:3 and that the fifth mode will be in the ratio 3:2; both of these statements agree with the intervals associated with those modes, but not the derivation: the numbers multiplied by six suggest the ratios 24:18 (which is fine) and 18:12 (which is not; the derivation in Mode V is double that, 36:24).⁷⁵⁵ It is surely not a coincidence, however, that the first three modes are given as ascending multiplex ratios and the next two as progressively larger superparticular ratios; the ratios of the perfect consonances have simply been gathered together and run out in order, and by coincidence, there is a way that they can be interpreted to match the modal numbers of the first several modes.

In Chailley's opinion, the correspondences cease at Mode V (because the derivation of the fifth in Mode V differs from the rest of the treatise). He says that Mode VI ought to be the same as Mode V, with which it shares modal numbers, and yet this passage considers Mode VI to be the same as Mode III, instead.⁷⁵⁶ Nevertheless, the statement can be true if read from a different perspective. Mode VI is not,

⁷⁵⁴ Chailley, *Alia musica*, 168.

⁷⁵⁵ *ibid.*

⁷⁵⁶ *ibid.*

in fact, the same as Mode V because although the modal products are the same, the derivations are different, and importantly, the interval of an octave is permitted for Mode VI (even though it is not clearly displayed in the modal numbers). Thus, if the intervals are the most important element – as they clearly are in this passage – then Mode VI does agree with Mode III, in that both modes contain a fifth and an octave. The text says that Mode VI is the same as Mode III in having “the octave together with the fifth,” a phrasing that in many contexts commonly refers to the twelfth (it almost certainly does so in §156, the passage describing the perfect consonances directly before the first of these two passages under consideration, where it is identified as one of the three composite intervals), but it may, in this case, be interpreted as meaning that these modes have an octave, and also, separately, a fifth.

In the same way, it could be held to be true that the seventh mode has the octave and fourth, which should almost certainly be interpreted to mean an eleventh in the first passage (though the eleventh is not described as a perfect consonance in the preceding section!), but it is important to note that the first passage does not identify the seventh mode here; instead, there is a cryptic reference to *eodem quinto* (the same fifth). It is not certain what is this fifth to which the author refers, but it is unlikely to be the interval of a fifth, which is usually called a *diapente*; it most likely means the fifth mode, and as Chailley notes, calling it “the same fifth” is not justified by the context. This phrasing could be evidence that the passage was adopted from another source in which the context was more appropriate. It is also possible that a few words are missing; perhaps the intended meaning was “in the same manner as the fifth mode.” This interpretation, however, would conflict with the descriptions of the fifth mode, both in this passage and in the rest of the *Alia*. It is also notable that the description of the seventh mode in this manner is only mostly accurate: both the octave and the fourth are held to be present in this mode, but so, also, is the fifth, of which this passage makes no mention (though this passage is also not exhaustive about the intervals in the first three modes).

Finally, the eighth mode is cited as apparently being in the double octave, together with the double and the triple. Actually, this passage may mean something else entirely. The phrasing “*dupla diapason*” is an unusual way to describe a double octave in Latin. Ordinarily, this interval is described by combining the Latin prefix for double (*bis-*) with the Greek name for the octave, *diapason*, giving *bisdiapason*;⁷⁵⁷ occasionally, the pure Greek form *disdiapason* also appears (the *Enchirides* prefer this form);⁷⁵⁸ or it may simply be identified as a quadruple in the same way that the octave is frequently identified simply as a double. It may be, then, that *dupla diapason* is not a double octave, but a redundancy describing an octave: “the double [proportion] (that is, the octave).” Of course, the octave is not one of the specified intervals in most treatments of Mode VIII in the *Alia*, but it should be remembered that one possible interpretation of *octauus modus require supra* is that the eighth mode is much like the seventh, and would therefore include the octave, at least in the source treatise.

The preceding interpretations repeatedly strain credulity. The point is not to suggest that the interpretations presented above ought to be accepted as the correct interpretations for these passages. Instead, the point is simply that there is a way to interpret these passages so that they appear to be consistent with the doctrines of the *Alia musica*; one of the authors of the *Alia* would then have seen this similarity and incorporated these passages into the composite treatise, though they probably actually represent an entirely different paradigm.

It is also worth noting that the interval description given before the second passage resembles but is ultimately different from the description of the perfect consonances preceding the discussion of the fourth mode, which invokes (and abuses) the Aristoxenian divisions of the tetrachord into genera and shades (see Chapter 14); only the fourth and fifth agree between these two presentations (Table 44).

⁷⁵⁷ cf. *Alia musica* §§8(a) & 81(a)–(b), ed. Chailley, 103 & 145.

⁷⁵⁸ For just the first example in each of the *Enchirides*, cf. Schmid, *Musica et scolica*, 28 (*Musica enchiridis*); 90 (*Scolica enchiridis*); 197 (*Inchiridon*).

Table 44 – Alternate derivations of the perfect consonances

Interval	§81 (after Aristoxenus)	§161 (current section)
Octave (Double, 2:1)	$12 + 24 = 36$	$12 + 6 = 18$
Twelfth (Triple, 3:1)	n/a	$18 + 3 = 24$
Double Octave (Quadruple, 4:1)	$2 \times 12 = 24$ ($12 + 12 = 24?$)	$24 + 6 = 30$
Fifth (Sesquialter, 3:2)	$12 + 18 = 30$	$12 + 18 = 30$
Fourth (Sesquitercian, 4:3)	$18 + 24 = 42$	$18 + 24 = 30$

* 2×24 (no assigned interval)

Chapter 17 – Analyses

This chapter will focus on the analysis of the chants cited in the *Alia musica* as exemplifying the characteristic intervals of each mode, in the attempt to identify any consistent patterns throughout a single mode. There are approximately forty such chants identified in the source treatise, distributed amongst the first six modes (Mode VII is a special case and Mode VIII is not discussed in the source treatise). The revisor omits a few of these and adds a few more, and the commentator adds a few more again, collectively amounting to another two dozen or so; however, given the uncertain reliability of the revision and commentary, the present analyses will be restricted primarily to the chants cited in the source treatise, with the additional chants from the revision reserved for supplemental evidence only where absolutely necessary (as in the case of Mode VII, where the source treatise does not cite any chants). Likewise, the tonary sections of the treatise are effectively entirely independent of the rest of the treatise and should not be assumed to display the same characteristics.

As a review, the *Alia* explains that each mode displays certain characteristic intervals (represented by comparisons of multiples of the base numbers 6:8:9:12). The chants may be divided into *incisiones*, and each *incisio* is said to “run through” a particular interval, which probably means that the range of the *incisio* corresponds to that interval (loosely, because of the possibility of the *emmelis*, one additional step outside the range, whether above or below, which is permitted because it “suits the melody”). For a mode that is represented by several intervals, different *incisiones* of the same chant may display different intervals, but it is also possible for an entire chant to display only one of the characteristic intervals. Even for modes that are characterized by only a single interval, it does not necessarily follow that the entire chant should fit within the designated range; it is entirely possible that the various *incisiones* will span intervals of the same size but at different pitch levels.

It is not clear how the *incisiones* should be identified. The concept is analogous to grammatical phrases, and on the assumption that music was generally composed to fit the text, one would ordinarily expect there to be considerable overlap between the grammatical divisions of the text and the musical *incisiones*; certainly, cadences tend to correspond well with the ends of grammatical units. In addition to this overlap, Bower has suggested that there are points between cadences that might correspond to meaningful inflection points, down even to the level of a musical “syllable,” which he defines as ending where the end of a word in the text coincides with the arrival of the melody on one of a handful of functional pitches within the melody. He feels that these functional pitches usually correspond to the *finalis*, the fourths and fifths above and below the *finalis*, and the subtone; additionally, other notes may become functional in this way if they are repeated frequently. However, his hypothesis makes assumptions about the nature of mode that are analogous to the very principles that this chapter is attempting to discover (that is, whether the intervals that the *Alia* says should characterize each mode are tied to specific pitches that would, therefore, be functionally important), and so Bower’s approach should be applied only with caution. Additionally, there are recurring musical ideas common to many chants in each mode, and while there is also freely composed material in most chants, it seems unlikely that the formulae that do appear in a chant would be inserted haphazardly; it is likely that recognizable formulae such as those described by Frere also correspond to *incisiones*.

Unfortunately, none of the sources just discussed provides a clear and systematic approach for segmenting a chant, and there is a not inconsiderable risk that unsystematic segmentations would reveal more about my own biases and assumptions about mode than the actual influence of mode upon these chants. As a result, in preparing the transcriptions for analysis, I have attempted to follow procedures that are at least consistent, even if I have not been able to justify every division based on a clearly articulated musical principle. Because of the strong relationship between music and text in most chants,

I have based my segmentations in the first place upon the grammatical segmentation of the text.⁷⁵⁹ However, since even the paradigmatic example of *Rorate caeli* does not correspond well to this kind of grammatical segmentation, I have adjusted these divisions when an obvious musical segmentation presents itself, such as the presence of Frere's formulae (especially if a chant corresponds well with one of his themes) or an obvious melodic repetition. Where I am not confident about melodic segmentation, I have retained the grammatical segmentation in order to avoid biasing the analysis to find those intervals that I expect to find, rather than those that the author of the source treatise intended.

The analyses presented here are in no way intended to be thorough musical analyses. The objective of these analyses is exclusively to identify the characteristic intervals of the mode within the *incisiones* of the chant, and I have restricted the scope of these analyses to that end. Throughout this chapter, *incisiones* identified with confidence to reflect an octave will be highlighted in green; those reflecting a fifth will be highlighted in yellow; and fourths in blue. The boundaries of these intervals will be identified alongside the *incisio*. In a few cases, there will be multiple possibilities listed, owing to the ambiguity created by the concept of the *emmelis*, which allows the occasional use of a note just beyond the boundaries of the interval; if an *incisio* is supposed to span a fifth and it spans a sixth, with both the lowest and highest pitch being used only occasionally, it is difficult to know whether the *emmelis* is the lowest pitch or the highest pitch (in cases like these, and only in such circumstances, I have also taken into account prominent intervals in addition to simple range). Such a sixth could also be a fourth with both upper and lower *emmeles*, an issue I address more fully in the next chapter.

Ideally, the sources of the melodies to be analyzed below would be selected principally for geographic proximity to the origin of the treatise, and secondarily for age. Unfortunately, it turns out to

⁷⁵⁹ The grammatical segmentation that I have used comes first and foremost from *The Gregorian Repertory* database, hosted by the Diocese of Rottenburg-Stuttgart (<https://gregorien.info/en>). In a few instances, I have supplemented these divisions with my own judgment, particularly where a segment appears to be unusually long.

be impractical to select sources in this manner because there is not sufficient evidence to support any hypothesis of the geographical origin of the treatise, and its date, while uncertain, is sufficiently older than the widespread use of diastematic notation that various sources of the chants are likely to be chronologically closer to one another than they are to the *Alia musica*. Instead, where possible, two (and occasionally more) versions of the chant will be compared to one another. The principal access-point for these sources, except where otherwise noted, is the *CANTUS Manuscript Database*, an on-line database hosted by the University of Waterloo,⁷⁶⁰ and chants and sources will be identified according to the ID number and manuscript *sigla* used by that database.

Unless otherwise noted, the antiphons are transcribed from A-Wn 1799,⁷⁶¹ a thirteenth-century Cistercian antiphoner from Reims, and are compared against the Salisbury Antiphoner (which Frere edited and used in his classifications of antiphon theme groups and noted with the *siglum* “Sarum”), while the introits are transcribed from CH-P 18,⁷⁶² a twelfth-century Premonstratensian gradual and compared against Montpellier H.159 (an early eleventh-century antiphoner, ca. 1031).⁷⁶³ In addition to the obvious benefits of using the Sarum antiphoner (even though it is a later source, it is valuable precisely because it is Frere’s source) and Montpellier H.159 (a very early manuscript in alphabetical, and therefore pitch-specific, notation), all of these sources were selected because they contain complete melodies for the substantial majority of the chants cited in the source treatise, thus making for more consistent comparisons.⁷⁶⁴ Where additional sources are needed for comparison purposes, D-Gsta AB III 9, a German

⁷⁶⁰ <http://cantus.uwaterloo.ca/>

⁷⁶¹ Originally transcribed by, Elizabeth Sander as per *CANTUS Database*.

<http://cantus.uwaterloo.ca/source/123667>

⁷⁶² Originally transcribed by Barbara Swanson, as per *CANTUS Database*.

<http://cantus.uwaterloo.ca/source/638308>

⁷⁶³ Hansen, *H159 Montpellier*, 21.

⁷⁶⁴ In his written feedback following the defence of this dissertation, Atkinson pointed out to me that the Cistercians are known to have sometimes modified chants, particularly their ranges, to accord with Guido’s precepts. Consequently, the selection of A-Wn 1799, a Cistercian antiphoner, was perhaps not an ideal choice. It is not practical, as I make these revisions following the defence, to redo all the analyses based on a different

manuscript long held at Göttingen but probably compiled nearby in Hardegsen, has also proved useful for the same reason.

Mode I

The passage identifying the chants and expected intervals of the first mode has already been considered above (p. 361). In summary, all *incisiones* of Mode I chants should display the intervals of an octave, a fifth, or a fourth. Importantly, although the author often goes to considerable lengths to demonstrate his mastery of numberplay (in terms of the various ways of arriving at a number or proportion), he does not identify all six possible intervals between the numbers 12:9:8:6; he identifies the fifth as 12:8, but not 9:6, and he likewise identifies the fourth as 12:9, but not 8:6; and, of course, he identifies the octave as 12:6 (the only possibility), but entirely ignores the ratio 9:8 (understandably, since it produces a tone, which is too small to represent a significant interval or span). Thus, it appears that the ratios 9:6 and 8:6, while producing fourths and fifths, play no part in his understanding of Mode I, and each *incisio* must necessarily include the number 12. Thus, if one were to assume that modal numbers should correspond to specific pitches, one probably should not suppose that the two different spans of a fifth found in *Rorate caeli* and *Iohannes autem*, respectively, represent the two different fifths, 9:6 and 12:8, because there is no justification in the text to expect 9:6 at all.

Below are analyses of the six chants identified by the author of the source treatise as exemplifying Mode I.

***Rorate caeli*, Mode I Introit, Cantus ID 501007**

Text: Isaiah 45:8.

source, but this caveat should be kept in mind throughout these analyses; nevertheless, since the melodies for analysis are drawn from multiple sources, it is likely that the overall analysis will not be excessively influenced by this consideration.

This chant is the only chant cited in the *Alia musica* for which a precise description of the *incisiones* is given, and therefore is the only chant for which the division can be done with complete confidence. However, the second point of division, in the middle of the third clause, is unexpected, though it does agree with the divisions that Frere identifies in other chants of the same theme group (see p. 396, above) and with Bower's conception of a musical "syllable" (see p. 406, above).

The image shows a musical score for a chant, divided into three sections. Each section consists of a treble clef staff with a melody line and a lower staff with a bass clef. The lyrics are written below the lower staff.

- Section 1 (Green highlight):** Labeled "CH-P 18 f. 17" and "8th D-d". The melody is on a high register. The lyrics are "Ro - ra - te ce - li de - su - per,".
- Section 2 (Yellow highlight):** Labeled "5th F-c". The melody is on a medium register. The lyrics are "et nu - bes plu - ant ius - - - tum; a - pe - ri - a - tur".
- Section 3 (Cyan highlight):** Labeled "4th D-G". The melody is on a lower register. The lyrics are "ter - ra et ger - mi - net sal - va - to - rem."

The source is identified as "Montpellier #331".

The final *incisio*, aside from the unusual point of division, is otherwise unremarkable, spanning the fourth D-G. However, the middle *incisio* spans the fifth F-c, whose boundary pitches do not match the boundary pitches of the other two *incisiones*, which are supposed to share the number twelve; this interval is common in Mode V. The first *incisio* is also potentially ambiguous. It is clearly identified in the *Alia* as an octave, but depending upon the identity of either the d or the C as *emmelis*, the octave may span D-d or C-c (the revisor argues for D-d, but his opinion may not be definitive).

A close look at the melody of the second and third *incisiones* may explain the reasoning for the unusual segmentation. The second clause shifts to a higher overall position and creates, at least to a modern ear, a noticeable shift in character; although *aperiatur terra* ought to be a single clause, the

melody for *aperiatur* continues in this higher position, while *terra* returns back to the original pitch level (though it is not clear whether medieval musicians would have heard a shift of character at these moments). Furthermore, the melody from the beginning of *terra* returns at the end of *germinet*, and because of this repetition, one might expect them to belong to the same *incisio*.

***Gaudete in Domino semper*, Mode I Introit, Cantus ID g00501**

Text: Philippians 4:4–6.

CH-P 18
p. 015

Montpellier
#339

5th
D-a

Gau - de - te in do - mi - no sem - per

4th
D-G

i - te - rum di - co gau - de - te

5th
D-a

mo - des - ti - a ves - tra no - ta sit

4th
F-b \flat

om - ni - bus ho - mi - ni - bus

4th
F-b \flat

do - mi - nus pro - pe est

5th
F-c

ni - chil sol - li - ci - ti si - tis

The image displays two systems of musical notation for a chant. Each system consists of a vocal line (treble clef) and a lute line (treble clef).
 The first system is labeled '8th C-c' and '5th D-a'. The vocal line has a green highlight and the lyrics 'sed in om - - - ni o - ra - ti - o - - - ne'. The lute line has a yellow highlight.
 The second system is labeled '4th D-G'. The vocal line has a cyan highlight and the lyrics 'pe - ti - ci - o - nes ves - ter in no - tes - cant a - pud de - um.'. The lute line has a black highlight.

This chant is problematic to analyze according to the precepts of the *Alia musica*. The first and third systems correspond to the fifth D–a (with minimal use of both upper and lower *emmeles*), which is different from the fifth found in *Rorate caeli* (see above), but the same as the fifth that is supposed to be the sole interval in *Iohannes autem* (see below); the sixth system, however, displays the same fifth as *Rorate caeli*, which is also found in many of the fifth-mode chants cited by the *Alia musica*. The second and final systems display the same fourth as *Rorate caeli*; the fourth system, however, displays a fourth one third higher, from F–b, which also appears in many of the chants of Mode V (though not sanctioned there by the *Alia musica*). This same interval also appears in the fifth system in Montpellier, but in CH-P 18, the upper pitch is \flat , creating an augmented fourth (though it is possible that \flat ought to be supplied editorially). The second last system is the most problematic. The two sources differ meaningfully in only a single pitch: the climax tone near the end of the system. If the pitch is a c, as given in CH-P 18, then the span is an octave from C–c, which is the same as one of the two possible interpretations of the octave at the beginning of *Rex caeli*; but if the climax tone is \flat , as in Montpellier, then the span is a fifth from D–a, with both the upper and lower *emmeles*. More will be said about this kind of ambiguity in the next chapter.

Iustus es Domine, Mode I Introit, Cantus ID g01216

Text : Psalms 118 :137 ;124.

CH-P 18
p. 255

Montpellier
#346

5th
D-a

Ius - - - tus es do - mi - ne

10

5th
F-c

et rec - tum iu - di - ci - um tu - um;

28

5th
a-e

fac cum ser - - - vo tu - o

10

5th
F-c

et rec - tum iu - di - ci - um tu - um;

28

5th
a-e

fac cum ser - - - vo tu - o

38

?

se - cun - dum mi - se - ri - cor - di - am tu - am.

This introit contains at least three discrete fifths: D–a, F–c, and a–e. Even if both possible derivations of a fifth (9:6 and 12:8) were to be allowed, there would no way to account for three different fifths if the modal numbers were actually to represent consistent pitches. The final system is a rather long phrase that does not quite span an octave; if it is an incomplete octave, it is not clear whether the octave in question ought to be C–c or D–d. It is also possible that the relatively long phrase should be divided into two *incisiones*; however, the splitting point is not obvious. It seems unlikely for both grammatical and melodic reasons to split between *miserordiam* and *tuam*, and it is also unlikely to split in the middle of a word. According to Bower’s model, the most likely division would be at the end of *secundum*, as this word ends on the fourth above the *finalis*, which makes it a reasonable candidate for a musical syllable, while *miserordiam* is not. In this case, the remaining *comma* (*miserordiam tuam*) would nicely span the expected fifth, D–a; however, *secundum*, by itself, would span a fourth of G–c (yet another new interval not yet seen in previous Mode I chants), and grouping it together with the previous (shorter) *incisio* would cause the *incisio* to end on the *emmelis* – a procedure not explicitly ruled out in the *Alia*, but one not seen elsewhere and one that seems unlikely because of the status of the *emmelis* as extraneous.

Urbs fortitudinis nostrae Sion, Mode I [VII] Antiphon, Cantus ID 005281

Text: Isaiah 26:1-2 (& Isaiah 8:10?)

Gevaert: Theme 3

Frere: VII⁴

In the *Alia musica*, this chant is the quintessential example of a first mode chant that spans only the fourth, but it is routinely classified as Mode VII in most sources after the early tenth century. However, the Mode VII melody is similar to a Mode I melody, particularly in the incipit, and it was routinely classified as a Mode I melody in the Carolingian period. In fact, Gevaert classifies it as Theme 3, a theme that he describes as a *protus* authentic theme. The peculiarities of this chant will be addressed in greater detail in the next chapter. However, due to the problems inherent in this chant, a large number of sources have

been consulted.⁷⁶⁵ Here, it has been transposed down a fourth to end on D, to facilitate comparison to other Mode I chants.

A-Wn 1799 f. 006v * Faulty text setting
 CH-E 611 f. 005v
 CDN-Hsmu M2149.L4 f. 012v
 NL-Uu 406 (3 J 7) f. 009v
 D-KNd 1161 f. 000c*
 D-KA Aug. LX f. 006v

Sarum
 p. 26

I-Rvat SP B.79
 f. 10v
 CH-Fco 2
 f. 012r

F-Pnm lat. 15181
 f. 117v

Urbs for-ti-tu-di-nis nos-trae Si-on

5th
D-a

17

Sal-va-tor po-ne-tur in e-a-mu-rus

5th
D-a

⁷⁶⁵ Except Sarum, all other sources are taken from the *CANTUS* database, listed according to the *sigla* used by that database.

33

et an - te mu - ra - le

4th
a-d

54

A - pe - ri - te por - tas

68

qui - a no - bis - cum de - us al - le - lu - ia.

4th
D-G

Contrary to the observations of the *Alia musica*, the first two systems span not a fourth, but the fifth D–a. The first system includes an *emmelis* that is a minor third above the top note, which is not

common in the other chants specified in this mode in the *Alia musica*; however, in the very common Mode I intonation formula C–D–D–a–b–a, which spans the same fifth D–a (*emmeles* notwithstanding), the b is sometimes replaced by c, even in the same chant in different manuscripts. Is it likely that these two variants of the same intonation formula in the same chant should not be understood to span the same interval? It is likely that this c still functions as an *emmelis*, though in this case, it does not obviously avoid a semitone, as it does in other instances where the *emmelis* is a minor third above the intended range. Perhaps the first system could be reconciled with the idea that the entire chant runs through the fourth if the opening D were to be counted as extraneous, but there seems to be no support for such a large *emmelis* in the *Alia*. In the second system, an analysis as a fourth would require the a to be labeled as an *emmelis*, raising difficult questions about the difference between a fourth with an *emmelis* and a fifth without one.

The final system spans the expected fourth D–G. The remaining systems are more difficult to explain. The third system might span the fifth G–d or only the fourth a–d (with the G as *emmelis*); neither span is common in the other chants specified for this mode. The fourth system is even less clear. The complete span is a minor sixth from E–c. It is unlikely that the E could be an *emmelis* because the next lowest pitch is, in this case, an F-sharp, which would create a diminished fifth against the c. Perhaps the c could be taken as an *emmelis*, leaving the fourth E–a, again requiring the minor third *emmelis*; this interpretation is supported by the fact that approximately half of the sources consulted give b here.

It should also be observed here that if the *Alia*'s discussion of the chant as exclusively manifesting the fourth is correct, then it would effectively require that multiple different fourths be possible, since the total range of the entire chant is a ninth. This observation adds further support to the hypothesis that the modal numbers do not directly represent pitches.

Iohannes autem cum audisset in uinculis opera Christi, Mode I Antiphon, Cantus ID 003496

Text: Matthew 11:2–3.

Gevaert: Theme 6

Frere: I² a

This antiphon is described explicitly in the *Alia* as a Mode I chant that holds exclusively to a single interval, the perfect fifth, and therefore does not include the octave or fourth that could be present in Mode I.

A-Wn 1799
f. 009r

Sarum
p. 32

14

25

46

58

5th
D-a

5th
D-a

5th
D-a

5th
D-a

4th
D-G

Jo - han - nes au - tem cum au - dis - set,
in vin - cu - lis, o - pe - ra Chris - ti,
mit - tens du - os de dis - ci - pu - lis su - is, a - it il - li:
"Tu es qui ven tu rus es,
an a li um ex pec - ta - mus?"

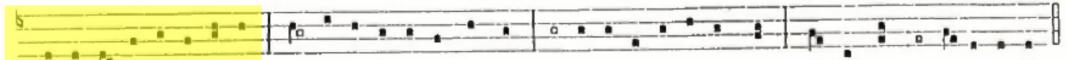
This chant raises difficult questions in regards to segmentation, because while the author of the source treatise clearly describes it as an exemplar of a chant exclusively using the fifth, the final *inciso*,


both in terms of meaningful segments of text and in terms of discrete musical segments, spans only the fourth D–G – unless this final system is combined with the preceding system to create a considerably longer *incisio* (which might imply considerably less flexibility about what counts as an *incisio*, allowing only larger units, making the *Rorate caeli* analysis even more problematic) or reinterpreting the subtone as no longer an *emmelis*, but a fundamental pitch including within the range of the *incisio* (which is not otherwise seen in this mode).


However, there is good reason to be confident of both the segmentation and the intervals selected, since this chant is a reasonably good exemplar of its theme group, as this theme group is described in detail by Frere (Facsimile 18).⁷⁶⁶ The opening *incisio* is a near perfect match for the first phrase in each of Frere's five examples (corresponding to nine chants, some of which are near duplicates of one another), with the exception of a trivial interpolation (an F at the end of "*Iohannes*") that probably only occurs because the opening *incisio* of *Iohannes autem* contains more syllables than any of Frere's examples. The third system begins with an exact repetition of this opening *incisio*, with a few more notes tacked onto the end similar to the way that the third *incisio* of Frere's third example ends. The final *incisio* conforms perfectly to the final *incisio* of Frere's third example, and while there is less consistency within the group about the melodic profile of the cadence, most of the examples span the same fourth D–G, usually without the subtone. The second system represents a slightly shorter version of the second *incisio* of Frere's fourth example. Only the fourth system deviates from the type in any significant way.


⁷⁶⁶ Frere, *Antiphonale Sarisburiense*, 65–66.


§§1, 3

(1) 
 XL⁴ 187. De quinque panibus et du-o-bus pis-ci-bus sa-ti-a-vit do-mi-nus quinque mil-li-a hominum.
 John Evang. 62. Val-de honorandus est be-a-tus Jo-hän-nes, qui supra pectus do-mi-ni (in ce-) na re-cu-bu-it.
 Ferial 121. De-po-su-it po-tën-tes sanc-tos per-se-qüentes, et exal-ta-vit hu-mi-les christo con-fi-ten-tes.

(2) 
 LX 141. Qui verbum de-i re-ti-nent cor-de perfecto et op-ti-mo, fructum af-ferunt in pa-ci-en-ti-a.
 Cap. Jej. 149. Cum facis e-le-mo-si-nam, ne-sci-at (si-nis-)tra tu-a quid fa-ci-at dex-te-ra tu-a. §5

(3) 
 XL³ 179. Cum im-mun-dus spi-ri-tus ex-i-e-rit ab ho-mi-ne, am-bu-lat per lo-ca in-a-quo-sa, querens re-qui-em, et non in-ve-nit.
 Mrr. 640. Qui mi-hi mi-ni-strat më sequä-tur, et u-bi e-go sum, il-lic e-rit et mi-ni-ster meus. §2

(4) 
 Epiph.³ 126. De-fi-ci-en-te vi-no, iussit Iesus im-ple-re hy-dri-as aqua; quae in vinum conver-sa est: al-le-lu-ia.

(5) 
 XL³ 158. Per ar-ma iu-sti-ci-e vir-tu-tis de-i com-men-demus nos-met ip-sos in mul-ta pa-ti-en-ti-a.
 (p. 148) ad cru-ci-fi-gen-dum.

Facsimile 18 – Frere’s exemplars for Mode I Theme Group (a), edited. Highlighting [added] shows relationships between these exemplars and Johannes autem.



Traditor autem dedit eis, Mode I Antiphon, Cantus ID 005169

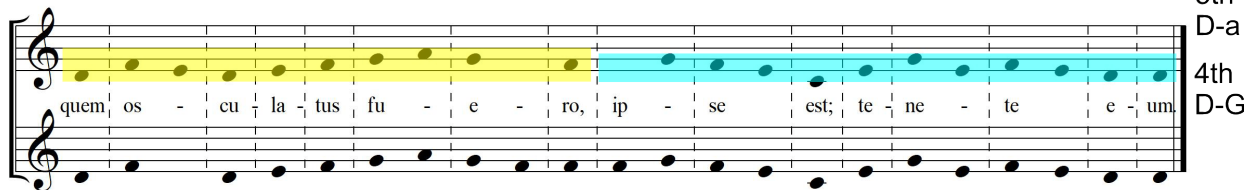
Text: Matthew 64:48

Gevaert: Theme 5

Frere: I¹₂ b

This chant has been analyzed already (p. 396, above) according to the principles described by Frere; it belongs to the same theme group as *Rorate caeli* (and conforms to the type better than *Rorate*, being one of the examples Frere chooses to illustrate the type).

A-Wn 1799 f. 067v  5th D-a
 Tra-di-tor au-tem de-dit e-is sig-num, di-cens:
 Sarum p. 220 

 5th D-a
 quem os-cu-la-tus fu-e-ro, ip-se est; te-ne-te e-um.
 4th D-G

Like every chant in this theme group, the middle *incisiones* span a fifth and the final *incisio* spans the fourth D–G (as well as the lower *emmelis*). The first *incisio* in the theme type is a little less consistent, sometimes spanning the octave (as in *Rorate caeli*), but more often spanning the fifth D–a, as it does here.

Mode I Summary

If the four base numbers are supposed to represent consistent pitches, then there are serious challenges involved in the analysis of the Mode I chants cited by the *Alia musica*. Among these six chants are the three that are described in greatest detail in the *Alia*, and yet all three are difficult to reconcile with the description given. The difficulties involved in analyzing *Rorate caeli* have already been described at length, but the other two – *Iohannes autem*, supposed to conform entirely to the fifth, and *Urbs fortitudinis*, supposed to be confined to the fourth – also do not clearly manifest the expected intervals throughout their entirety, as the *Alia* says that they should. In fact, *Traditor autem* is probably the only Mode I chant cited in the *Alia* that does not pose some analytical problem.

Mode II

Regarding this mode, the author of the source treatise says,

Item bis 8 ad 12 per sesquiterciam proportionem 28 fiunt, id est quater 7 quod est diatessaron. [...] Item bis 9 ad 12, qui sunt ter 6, id est 18 ad 12 per sesquialteram proportionem fiunt ut supra 30 diapente. Omnis enim melodia secundi toni aut per diapente denarium numerabit totum, ut est antiphona Ecce aduenit, in qua sunt 10 vel 5, aut per diatessaron totum, id est 7, ut est Ueni et ostende nobis faciem tuam. Item antiphona diapentica Omnipotens sermo tuus Domine, et per diatessaron ut est Dominus Deus auxiliator meus.⁷⁶⁷

Likewise, twice eight to twelve in the sesquitercian proportion make twenty-eight, that is quadruple seven, which is a fourth. [...] Likewise, twice nine to twelve, which is triple six – that is, eighteen – to twelve in the sesquialter proportion makes, as above,⁷⁶⁸ thirty (a fifth). Indeed, all melodies of the second tone will number through ten [*i.e.*, a multiple of five, which is the sum of the terms of 3:2], the complete fifth, as is the antiphon *Ecce aduenit*, in which are ten or five, or through a complete fourth – that is, seven [the sum

⁷⁶⁷ *Alia musica* §182(d) & (f)–(i), ed. Chailley, 87, trans. Nowacki 116–17.

⁷⁶⁸ In a statement not presented here, the author finds that the triple six that also gives eighteen sums with twelve to thirty.

of the terms of 4:3] – as is *Ueni et ostende nobis faciem tuam*. Likewise, the antiphon of a fifth *Omnipotens sermo tuus Domine*, and through a fourth, as is *Dominus Deus auxiliator meus*.

This passage can be summarized as saying that Mode II chants are characterized either by the fourth 16:12 or the fifth 18:12. Below are analyses of the four chants identified by the author of the source treatise as exemplifying Mode II. This mode is among the simplest to analyze because the *Alia* explicitly identifies the correct interval for each chant; however, those cited for a fifth do not obviously stand out as spanning a fifth.

***Ecce aduenit*, Mode II Introit, Cantus ID g00596**

Text: Related to Malachi 03:01

This chant is cited as spanning a fifth throughout. However, it is not obvious that it should be so.

CH-P 18
f. 046

Montpellier
#393

Ec - ce ad - ve - nit, do - mi - na - tor, do - mi - nus

4th
C-F

et reg - num in ma - nu e - - - ius

4th
C-F

ma - - - nu

et po - tes - tas et im - pe - ri - um.

4th
D-G

The image displays three systems of Gregorian chant notation. Each system consists of a vocal line (treble clef) and a lute line (treble clef). The lyrics are written below the notes. A light blue shaded area highlights the pitch range of the vocal line in each system. The first system is labeled '4th C-F', the second '4th C-F', and the third '4th D-G'. The lyrics are: 'Ec - ce ad - ve - nit, do - mi - na - tor, do - mi - nus', 'et reg - num in ma - nu e - - - ius', and 'et po - tes - tas et im - pe - ri - um.' The lute line provides a rhythmic accompaniment with a consistent eighth-note pattern.

This entire chant clearly falls within the expected range, though there is some room to debate certain fine distinctions. The first system spans the fourth C–F, with an ordinary upper *emmelis* on G and a lower *emmelis* on A, a minor third below the lower boundary at C, which is easily explained as avoiding

the semitone that would be created by using the B. Here, an important distinction is demonstrated between the authentic and plagal *protus* chants: in the authentic mode chants, where C occurs, it is almost invariably an *emmelis*; in the plagal chants, the C is often integral to the range of the interval, as it is in both the first and second systems of *Ecce aduenit*.

It is certainly possible instead to interpret the G upper *emmelis* in the first system as essential to the range, creating the fifth C–G without an upper *emmelis*, though the tone occurs only once and moves like a neighbour tone. Similarly, the final system spans the fourth D–G, lacking any *emmelis*; it could potentially be part of an incomplete fifth at C–G, which would reflect the same range manifest in the first system. The only obvious way in which this chant could be said to correspond definitively to the fifth is as the total range of the entire chant (excluding the lower *emmelis* A).

Ueni et ostende nobis faciem tuam, Mode II Introit, Cantus ID g00514

Test: Psalm 79

This chant is cited as exemplifying the fourth, and analysis as such poses no difficulties.

The image displays three systems of Gregorian chant notation for the Introit 'Ueni et ostende nobis faciem tuam'. Each system consists of a vocal line (treble clef) and a corresponding line of square neumes on a four-line staff. A light blue horizontal bar highlights the interval between the fourth line (C) and the first space (F) in each system, labeled '4th C-F' on the right. The lyrics are: 'Ue - ni et os - ten - de no - bis fa - ci - em tu - am do - mi - ne qui se - des su - per che - ru - bim'. The first system is identified as 'CH-P 18 f. 21' and 'Montpellier #392'.

This chant is quite similar to *Ecce aduenit*, above, but is even more consistent. The entire chant spans the fourth C–F, with only a single use of the upper *emmelis* G (which could potentially make the third system a fifth, C–G). The lower *emmelis* is used occasionally, chiefly at the beginning of the chant and at the beginning of the third system (where the intonation formula from the beginning of the chant returns). Interestingly, the lower *emmelis* occurs once on B, but only in CH-P 18; this note is entirely absent in Montpellier and occurs instead as the expected *emmelis* A in D-Gsta AB III 9 (f. 011r).

***Omnipotens sermo tuus Domine*, Mode II Antiphon, Cantus ID 004144**

Text: *Liber sapientiae* 18:15

Gevaert: Theme 45

Frere: II¹₅

This antiphon is clearly cited by the *Alia* as running through a fifth; however, such an analysis is not entirely clear from the music.

This chant is generally consistent within the usual fourth C–F, rather than a fifth. The second system does contain the upper *emmelis* a couple of times, and the lower *emmelis* only once, so the fifth C–G or

the fourth D–G might also be reasonable analyses for this second system, but the fifth could only be interpreted as the sole governing interval for the chant if the entire antiphon is taken as a single *incisio*.

***Dominus Deus auxiliator meus*, Mode II Antiphon, Cantus ID 002405**

Text: Isaiah 50:07

Gevaert: Theme 47

Frere: II¹₁₀

This antiphon is cited in the *Alia* as exemplifying the fourth.

A-Wn 1799
f. 063v

Sarum
p. 205

Do - mi - nus de - us au - xi - li - a - tor me - us

4th
C-F

et i - de - o non sum con - fu - sus.

4th
D-G

This chant is also very consistent. In this case, however, the first system spans the same fourth that dominates the other chants in this mode, but the second system spans a fourth a step higher at D–G, as at the end of *Ecce aduenit*. Unlike *Ecce aduenit*, however, this chant is not cited as spanning a fifth, though the complete range of the entire chant does span the fifth C–G.

Mode II Summary

The source treatise provides fewer examples of Mode II chants than for any other mode; they are also more consistent than the chants for most other modes (Mode IV excepted). The substantial majority of the *incisiones* of all these Mode II chants span the fourth C–F, and a few span the fourth D–G. However, there are still a few inconsistencies here. The *Alia* identifies two of the chants as spanning the fifth, and two as spanning the fourth. However, neither of the two chants cited for the fifth unambiguously span

the fifth except over the course of their entire lengths, while one of the two chants specified for the fourth also could be said to span the fifth over its entire range, but is not identified as spanning a fifth; it is unclear what makes this latter case different from the former two.

Even so, the remarkable consistency of these four chants strongly suggests that if the modal numbers are supposed to represent specific pitches within a mode, then the intended intervals would be the fourth C–F and the fifth C–G. For this mode, then, it might be possible to make the following associations: 12=C, 16=F, 18=G. Awkwardly, these pitches run in the reverse of the expected order, with lower numbers corresponding to lower pitches. It cannot be interpreted in the other direction, however, since it would require the fourth D–G instead of C–F, and the fourth C–F is the only possible interpretation for *Ueni et ostendi*, both because the G appears only once (presumably as an upper *emmelis*) and because it must start at C or there would be no way to account for the A (which is an *emmelis*), and the chant is cited in the *Alia* as exemplifying a fourth, not a fifth.

Mode III

Regarding this mode, the author of the source treatise says,

Quater enim 6 ad quater 3 de 12 quater 9 sunt, qui sunt 36 diapason, id est sexies 6 et ter 12; similiter ter 8 ad 12, quia duplae proportionis sunt. Bis 9 ad 12, quia sesquialtera proportio est, 30 diapente fiunt. Unde iste tertius tonus aut per diapason erit per 6 ad 12, aut per diapente, id est 30, ut in introitibus Confessio et pulchritudo et Dispersit dedit pauperibus et Cognoui Domine, et antiphonis Qui de terra est et Quando natus es et Malos male perdet et Homo quidam fecit cenam magnam.⁷⁶⁹

Indeed, four sixes to the four threes of twelve are four nines, which make the octave [represented by] thirty-six [the sum of the terms of 24:12], which is [also] six sixes and three twelves; similarly, three eights to twelve [give the same octave], since they are in the double proportion. Twice nine to twelve, since it is the sesquialter proportion, which make the fifth [represented by] thirty [the sum of the terms of 18:12]. Whence this third tone will either be through a diapason ([generically] 12:6 [but strictly 24:12 in this mode]), or through the fifth – that is, thirty – as in the introits *Confessio et pulchritudo* and *Dispersit dedit pauperibus* and *Cognoui Domine*, and in the antiphons *Qui de terra*

⁷⁶⁹ *Alia musica* §183(d)–(h), ed. Chailley, 88, trans. Nowacki 120.

est and *Quando natus es* and *Malos male perdet* and *Homo quidam fecit cenam magnam*.

This passage includes considerable numberplay, but can be summarized as follows: chants in Mode III correspond to the octave of 24:12 (with the twenty-four arising equally from multiples of either six or eight) or to the fifth 18:12. The author does not say whether the two different ways of arriving at the octave would result in two different octaves, and it is unclear why he goes on to refer to the octave by the generic ratio, 12:6, where six is not one of the modal numbers in this mode.⁷⁷⁰ Furthermore, the author does not sanction the appearance of the fourth that would be implied by the numbers 24:18. Below are analyses of the seven chants identified by the author of the source treatise as exemplifying Mode III (more examples than are provided for any other mode, perhaps because of the complexity of the modal description).

The chants of the *deuterus* modes are particularly difficult to analyze according to the precepts of the *Alia musica*. This difficulty is not entirely surprising. The passage immediately prior to the passage cited above describes how the number 12 is interpreted in multiple ways in this mode (see p. 420, above), and how one of these reflects the way that the number is used Mode IV, but also confusingly cites Mode II. Chailley supposes that this reference to both the second and fourth modes – both of whose modal octaves, depending on the status of \flat and \sharp , can potentially contain a semitone above the bottom note – is a complicated way of explaining that there is a semitone above the bottom of the modal octave of the third mode (these are, in fact, the only three places in the usual medieval gamut where a semitone occurs). However, there are no other references to the modal octaves in the source treatise (the discussion of modal octaves occurs only in the supplementary material at the beginning of the composite treatise and in the Disputed Passage); additionally, much simpler terminology for describing a semitone would have

⁷⁷⁰ This presence of 12:6 is particularly interesting, since the author, so fond of numberplay, does not bother to note that the sum of the terms of 12:6 gives eighteen, which is one of the modal numbers of the third mode (though it is arrived at differently).

been well known to all of the contributing authors of the *Alia musica*, and it would have been trivially simple to explain the semitone without such a tortuous circumlocution.

The interpretation of this passage is discussed at length in the previous chapter, but another possibility is that the references to these other modes and other ways of using the twelve are actually a way of describing the modal instability that has often been noted in regard to *deuterus* modes. For instance, Willi Apel says:

Another important trait, noticeable at least in some of the examples, is the tendency toward what may be called tonal (or modal) instability. [...] [This phenomenon] not only occurs frequently in Gregorian chant but actually forms a characteristic trait of some of its modes. [...] as can easily be confirmed by many other examples, the melodies of the *deuterus*, with the final on e (modes 3 and 4) are the ones most liable to tonal instability. There are numerous chants in this group whose opening phrases, through their outline and cadential points, suggest any other tonality than E; among the shorter chants there are not a few in which this tonality is never established until the very last note appears, [...].⁷⁷¹

Since the author of the source treatise seems to be trying to use the modal numbers to describe something like the “tonality” of each mode, it is hardly surprising that he must find some way to deal with this modal instability. The complicated description of the third mode in the *Alia musica*, explaining how the third mode manifests usages from other modes, is probably a way of describing this modal instability. Thus, the analyst should expect some *incisiones* not to reflect the intervals described in the *Alia*.

With all this complexity, it is not surprising that the *Alia musica* cites seven examples of chants in this mode (more than he cites for any other mode). Unfortunately, unlike for Modes I and II, he does not explicitly state whether any of these are confined to a single interval or whether each uses both the octave and the fifth.

⁷⁷¹ Apel, *Gregorian Chant*, 142.

Confessio et pulchritudo, Mode III Introit, Cantus ID g00331

Text: Psalm 95:6.

CH-P 18
p. 086

Montpellier
#436

8th
C-c

5th
F-c

5th
F-c

5th
F-c?

Con - fes - si - o et pul - chri - tu - do

in con - spec - tu e - ius

san - cti - tas et mag - ni - fi - cen - ti - a

in san - cti - fi - ca - ti - o - ne e - ius.

This chant begins by spanning the octave C–c. However, the rest of the chant instead spans the fifth F–c, with the occasional lower *emmelis*; this *emmelis* is unusual because it is only a semitone below the range to which it forms an adjunct, which weakens the argument in favour of *emmeles* of a minor third in other modes in order to avoid the semitone. It would not, however, solve the problem to reinterpret the range as E– \flat , with an upper *emmelis* of c, both because c would also be a semitone *emmelis* and because c is much more prominent in this chant than E. It is also not preferable to interpret the range as an octave, since it generally only reaches a sixth. The final *incisio*, however, reaches down to the D, creating a seventh, but still does not reach a full octave (and would not be well interpreted as the fifth E– \flat with both *emmeles*, since the \flat does not actually occur in this *incisio*).

An unusual characteristic of this analysis is that while the final *incisio* quite naturally ends on the *finalis*, this *finalis* is an *emmelis* compared to the range of the *incisio*. It is worth noting that this chant does agree with Chailley's hypothesis that the fifth of Mode III corresponds to the upper fifth of the octave, not the lower fifth (though both the fifth and the octave are lower than the notes he expected, coming in at F-c and C-c rather than a-e and E-e). However, not all of the chants of this mode are so consistent with Chailley's hypothesis.

***Dispersit dedit pauperibus*, Mode III Introit, Cantus ID g00327**

Text: Psalms 111:09

CH-P 18
p. 324
Montpellier
#443

Dis-per - sit, de - dit pau - pe - ri - bus;

ius - ti - ti - a e - ius, ma - net,

in se - cu - lum se - cu - li.

Cor - nu e - ius e - xal - ta - bi - tur

Cor - nu

5th
F-c

5th
F-c

5th
F-c

5th
F-c

This entire chant spans the fifth F–c, with the occasional use of the *emmelis*, except the final system, which has a much narrower range – potentially a fourth, E–a, which is not one of the intervals sanctioned by the author of the *Alia* (but it should probably just be interpreted as an extension of the previous system, and therefore part of the same fifth).

Cognoui Domine, Mode III Introit, Cantus ID g01396

Text: Psalms 118:78; 118:10

CH-P 18
p. 333
Montpellier
#431

8th
D-d

5th
F-c

5th
F-c

5th
F-c

hu - mi - li - a - - sti - - me

Con - fi - ge ti - mo - re tu - o

car - nes me - as,

a , man - da - tis tu - is non me re - pel - las

This chant is similar to *Confessio et pulchritudo* (above), in that the first system spans an octave and the rest span a fifth – chiefly the fifth F–c. However, in this case, the octave of the first system is D–d, not C–c, and the fifth F–c is therefore not the upper fifth of the octave. In addition, there is also one system that spans the fifth D–a, so that in this chant, there is a fifth that coincides with the lower fifth of the octave (though the other fifth, F–c, is considerably more common); there do not appear to be any compelling reasons for the system spanning the fifth D–a to be combined with the previous or next system to produce a larger interval (which would not, in any case, reach a complete octave).

***Qui de terra est*, Mode III Antiphon, Cantus ID 004464**

Text: John 3:31–33

Gevaert: Theme 35

Frere: III¹ a

This antiphon is an example that is well suited to a melodic analysis corresponding to Frere’s theme groups; the double bar lines are added here to show the divisions between apparent melodic units within that theme group.

A-Wn 1799
f. 022v

Sarum
p. 75

8th
D-d

Qui de ter ra est de ter ra lo - qui - tur,

5th
E-b

qui de cae lo ve - - nit su - per om - nes est;

5th
D-a

et quod vi - dit et au - di - vit, hoc tes - ta - tur, et tes ti - mo ni - um

8th
D-d

e - ius ne - mo ac - ci - pit qui au - tem ac - ce - pe - rit

5th
E-b

e - ius tes - ti - mo - ni - um, sig - na - vit qui - a De - us ve - rax est.

This chant is more variable than those analyzed above. The fourth system is a slightly varied repeat of the first, which assists in the partition into *incisiones*; both systems span the octave D–d. The middle system spans the lower fifth of this octave, D–a. The second and final *incisiones* also resemble each other, especially at the beginnings, and span the fifth E–b.

Quando natus es, Mode III Antiphon, Cantus ID 004441

Text: references Psalms 71:06

Gevaert: Theme 35

Frere: III¹ a

A-Wn 1799
f. 023v

Sarum
p. 79

23

36

67

5th F-c

5th G-d?

5th F-c

Quan-do na-tus es in ce-fa-bi-li-ter ex vir-gi-ne

tunc im-ple-te sunt scrip-tu-re

si-cut plu-vi-a in vel-lus des-cen-dis-ti

ut sal-vum fa-ce-res ge-nus hu-manum

te lau-da-mus de-us nos-ter.

This chant continues the pattern of *incisiones* spanning the fifth F-c (with cadences on the lower *emmelis*) seen in most of the chants above. The third system, however, seems instead to span the fifth G-d, but it could also be interpreted as an extension of the preceding system, in which case the d would simply function as an upper *emmelis* (without the preceding system, however, it lacks the F). The first and last systems are more problematic. They both contain the subtone D, but neither has the octave d,

so an analysis as an octave would be tenuous; yet the analysis as the fifth E–b understates the significance of c in these systems – particularly in the first system; in the final system the c is less prominent, but it is both approached and departed by leap rather than as a neighbour tone (as it usually used elsewhere).

***Malos male perdet*, Mode III Antiphon, Cantus ID 003687**

Text: Matthew 21:41

Gevaert: Theme 12

Frere: VI⁴ d

The modal attribution of this chant is complicated, as it is one of the chants sometimes cited as requiring a *paraptere* – an alternate psalm tone that accords with a small set of highly irregular chants that Regino calls *nothae*.⁷⁷² There is considerable variety in the form of the final cadence, with the most common version appearing in multiple transpositions, ending on either G, F, or E. The *Alia musica* clearly considers the chant to be Mode III – as does Regino – presumably in the form in which it is found in A-Wn 1799, ending on E, at the bottom of its range (and containing only one subtone). On the CANTUS database, perhaps a third of the sources containing this chant list it as Mode III, ending on E, a third below where it begins. However, a slightly greater number of sources on the database – and Bern – list the chant as belonging to Mode VIII, with a slight variation of the cadence, ending on G, where it began, instead of E; an example has been included here, from F-Pnm lat. 12044, a twelfth-century manuscript from St-Maur-des-Fossés.

The modal attribution is not specified in most of the remaining CANTUS sources, but a handful of sources, as well as the Sarum antiphoner, list the chant as Mode VI. Relative to the version in A-Wn 1799, the Mode VI variation is a simple transposition down by step, with consistent use of b. In this form, there can be no distinguishing between Modes VI and VIII on traditional lines (since there is no subtone); the cadence, however, is transposed up a third (relative to the rest of the chant; put another way, the majority

⁷⁷² For more about these, see Atkinson, “The *Parapteres*,” *Malos male* is cited on his list on p. 36 and in an excerpt from *De tonis* on p. 50.

of the chant is transposed down by second, but the cadence is transposed up by second), so that the chant ends where it begins. Placing this chant in Mode VI, as the Sarum antiphoner does, is probably a response to the other musical characteristics of the mode. Indeed, Frere notes that chants beginning in this form conform loosely to a theme group.⁷⁷³ Regardless, the form of the opening is always the same. The relationship between the modal attributions of earlier and later sources as regards the beginning and end of the chant is the opposite of what would be expected (as will be described more fully in the next chapter).

In any case, the *Alia musica* lists the chant in Mode III, so it is this version of the chant that must be analyzed for the present purposes. However, given the complexity of the modal attribution, any conclusions drawn from this chant should be viewed with some skepticism, and all three variants are included for comparison.

A-Wn 1799
f. 051v

F-Pnm lat. 12044
f. 078v

Sarum
p. 168

Ma - - - los ma - le per - det

et vi - ne am su - am lo ca - bit a li - is a - gri co - lis

qui red dant e - i fruc - tum tem - po - ri - bus su - is

8th
D-d

8th
E-e?

⁷⁷³ Frere, *Antiphonale Sarisburiense*, 72.

The variability in this chant makes analysis difficult. The middle system nicely spans the octave D–d, but the first system only spans the fourth G–c, an interval not held to be characteristic of this mode in the *Alia musica*, though it is also present in *Homo quidam fecit* (below). The final system, when it ends on E (as a Mode III chant ought to) spans the octave E–e; in the other forms, however, the final system spans only a fifth.

***Homo quidam fecit cenam magnam*, Mode III Antiphon, Cantus ID 004536**

Text: Luke 14:16–17

Gevaert: Theme 36

Frere: III⁵ b

It should be noted that there are two chants under this title that are likely to be confused with each other. This title often refers to a Mode VI responsory. Almost certainly, the *Alia* intends the similarly titled Mode III Antiphon that is often also called *Quidam homo fecit cenam magnam* (since the *Alia* describes it as an antiphon, and in fact, the title *Homo quidam* appears to be somewhat more common for this antiphon and is used in A-Wn 1799; by contrast, the title *Quidam homo* does not appear to be used for the responsory).

A-Wn 1799
f. 106r

Sarum
p. 323

5th
F-c

Ho - mo qui - dam fe - cit ce - nam mag - nam

4th
G-c

et vo - ca - - - vit mul - tos

5th
F-c?

et mis - sit ser - vum su - um ho - ra ce - ne

4th
G-c

di - ce - re in - vi - ta - tis ut ve - ni - rent

5th
F-c

qui a om - ni - a pa - ra - ta sunt al - le - lu - ia.

This chant does not correspond well to the descriptions of Mode III in the *Alia*. The first and last systems can be interpreted to span the fifth F–c (though in the final system, the fifth E–b̄ is equally compelling). However, in the first system, the F occurs only once and could be seen as an *emmelis* to the fourth G–c, which, while not one of the characteristic intervals specified in the *Alia*, does seem particularly prominent in this chant; it is the span of the second and fourth systems, and potentially also in the third system (if the E is treated as a *emmelis* at the minor third).

Mode III Summary

There is considerable variability in the spans of *incisiones* in the Mode III chants cited by the *Alia musica*. The fifth F–c does seem to be particularly prominent, but the lower *emmelis* E is not only a semitone *emmelis*, which seems to be avoided in other circumstances, but is also often the cadence note (not least at the end of the chant, as it is, of course, the *finalis*), which gives the *emmelis* much more prominence than the term would suggest. There are also a few other fifths that occasionally occur, and perhaps some (such as G–d) could be explained as the influence of other modes; however, the fifth E–b̄, while far less common than F–c, is not characteristic of other modes; perhaps it is E–b̄ that properly belongs to Mode III, while F–c is a result of the modal instability, as it is also the interval most common to Mode V.

As regards the octave, C–c occurs occasionally, and would be consistent, in principle, with Chailley’s interpretation of this mode’s fifth (F–c?) being the upper fifth of the octave (though Chailley supposes the octave and fifth to be E–e and a–e); but the octave D–d is more common, and cannot be reconciled with Chailley’s interpretation.

Mode IV

Regarding this mode, the author of the source treatise says,

Hic, quia bis 12 ad ter 8 et ad ter 6 bis 9 una consonantia numerorum in diapason 6 ad 12, id est 18, conueniunt. Idcirco ter 6 non ad bis 12 comparatur, quoniam per 3 non diuiditur, sed ter 8 ad ter 6 compatatur, et bis 9 ad bis 12 per sesquiterciam proportionem. Totus iste tonus semper erit diatessaron. [...] Quorum proportio, id est comparatio majoris ad minorem aut minoris ad majorem, semper diatessaron id est septenario mensurabitur; ut introitus Resurrexi et Misericordia Domini et In uoluntate tua Domine, et antiphonae Rubum quem uiderat Moyses et Turba multa et Tria sunt munera.⁷⁷⁴

Here, because twice twelve to three eights come together in a consonance of numbers at the octave six to twelve (which is eighteen) – and to three sixes, twice nine – on that account, three sixes are not compared to two twelves, since [two twelves] is not grouped in threes, but three eights are compared to three sixes, and two nines to two twelves, in the sesquitercian proportion. This entire tone will always be a fourth. [...] The proportion of this – that is, the comparison of the greater to the lesser or of the lesser to the greater – is always measured a fourth, which consists of seven [the sum of the terms of the corresponding ratio, 4:3]; such is the introit *Resurrexi*, and *Misericordia Domini*, and *In uoluntate tua Domine*, and the antiphons *Rubum quem uiderat Moyses* and *Turba multa* and *Tria sunt munera*.

The author explains that the fourth mode is based on the fourth 24:18, and takes pains to clarify that this can arise from the combinations $3 \times 8 : 3 \times 6$ or $2 \times 12 : 2 \times 9$, because their coefficients are commensurable,⁷⁷⁵ but not from combinations like $3 \times 8 : 2 \times 9$, for which the coefficients are not commensurable. Concerning the hypothesis that the modal numbers correspond to consistent pitches,

⁷⁷⁴ *Alia musica* §184(b)–(d) & (f)–(g), ed. Chailley, 90, trans. Nowacki 121–22.

⁷⁷⁵ Strictly speaking, the passage says that “ 3×6 cannot be compared to 2×12 because it is not divisible by three”; as I have explained in Chapter 16, the passage can only be reconciled if six and twelve are not part of the consideration, which implies that the coefficients must be the same. This interpretation, however, is violated in other modes, such as Mode VI, in which six sixes are compared to two twelves as “the six fours of the two twelves,” and the only apparent difference between these two cases is that the coefficients, themselves, in Mode VI share common factors, while those in Mode IV do not – hence, the terminology of commensurability.

the author does not clarify whether these alternative derivations should result in two different fourths, or whether the two derivations, being equivalent, would both correspond to the same fourth. Below are analyses of the six chants identified by the author of the source treatise as exemplifying Mode IV.

Resurrexi, Mode IV Introit, Cantus ID g01007

Text: Psalms 138:18; 138:05; 138:06.

CH-P 18
f. 200

Montpellier
#465

4th
D-G

Re sur - re xi, et ad - huc, te - cum sum,

4th
D-G

al - le - lu - ia:

4th
D-G

po - su - i - sti su - per me ma - num tu - am,

4th
D-G

al - le - lu - ia:

4th
D-G

mi - ra - bi - lis fac - ta est sci - en - ti - a tu - a,

tu - ia,

4th
D-G

al - le - lu - ia, al - le - lu - ia.

This chant is quite consistent, maintaining a span of the fourth D–G throughout, with occasional use of the *emmeles* (chiefly the lower *emmelis*).

Misericordia Domini, Mode IV Introit, Cantus ID g01053.

Text: Psalms 32:05–06

CH-P 18
f. 217

Montpellier
#470

4th
D-G

Mi - se - ri - cor - di - a do - mi - ni

4th
D-G

ple - na est ter - ra al - le - lu - ia

4th
D-G

ver - bo de - i cae - li fir - ma - ti sunt

5th
F-c?

al - le - lu - ia al - le - lu - ia.

This chant mostly spans the fourth D–G common to most of the chants of this mode; however, the final system presents a phrase much more at home in this mode’s authentic counterpart, spanning the fifth F–c with the *finalis* as a lower *emmelis*.⁷⁷⁶

⁷⁷⁶ Atkinson has suggested to me, in the written feedback following the defence of this dissertation, that the final alleluia may have been to this chant, which seems reasonably likely and might explain the higher range of this *incisio*.

In uoluntate tua Domine, Mode IV Introit, Cantus ID g01241

Text: Esther 13:09–11

CH-P 18
f. 264

Montpellier
#466

In uoluntate tua domine

4th
D-G

uniuersa sunt posita

4th
D-G

et non est qui possit resistere

4th
D-G

uoluntati tuae

4th
D-G

tu enim fecisti omnia

4th
D-G

caelum et terram

et u - ni - ver - sa que cae - - - li

am - bi - tu con - ti - nen - tur

do - mi - nus u - ni - ver - so - num tu

The entirety of this long chant spans the fourth D–G with regular use of both *emmeles*; each *incisio* spans the entire range except a single *comma* (the sixth system) that is too short to expect it to do so and probably belongs are part of the previous system.

Rubum quem uiderat Moyses, Mode IV Antiphon, Cantus ID 004669

Text: refers to Exodus 3:2–3

Gevaert: Theme 32 Frere: IV³

This chant is not present in the Sarum Antiphoner. It is found transposed up a fifth in A-Wn 1799 but has been transposed back down to E here for ease of comparison against other Mode IV chants.

A-Wn 1799
f. 023v

Ru - bum quem vi - de - rat Mo - y - ses in - com - bus - tum

con - ser - va - tam ag - no - vi - mus tu - am lau - da - bi - lem vir - gi - ni - ta - tem

de - i ge - ni - trix in - ter - ce - de pro no - bis.

4th
E-a

The first *incisio* spans the fourth D–G, as do most other *incisiones* in the model chants for this mode. However, the rest of the chant instead spans the fourth E–a. (These latter *incisiones* could also be interpreted as the fifth D–a, but this interval is not presented in the *Alia* as characteristic of this mode).

Turba multa, Mode IV Antiphon, Cantus ID 005256

Text: John 12:12–13

Gevaert: Theme 30

Frere: IV¹₇

A-Wn 1799
f. 64r

Tur - ba mul - ta quae con - ve - ne - rat ad di - em fes - tum

Sarum
p. 206

4th
E-a

cla - ma - bat do - mi - no

4th
D-G

be - ne - dic - tus qui ve - nit in no - mi - ne do mi ni

4th
D-G

ho - san - na in ex - cel - sis.

4th
D-G

The first *incisio* of this chant, like the later portion of *Rubum quem uiderat* (above), spans the fourth E–a, but the rest of the chant spans the more common fourth, D–G; even the opening might be considered

to span this more common fourth if the single *a* is considered to be an *emmelis*, but it lacks the *D*, and would have to be combined with the following *comma* to get it (which would not be an unreasonable interpretation).

***Tria sunt munera*, Mode IV Antiphon, Cantus ID 005181**

Text: refers to Matthew 2:11

Gevaert: Theme 33

Frere: IV¹₂₃

A-Wn 1799
f. 25r

Sarum
p. 89

Tri - a sunt mu - ne - ra

4th
D-G

quae ob - tu - le - runt ma - gi do - mi - no

4th
D-G

au - rum thus et my - ram fi - li - o de - i re - gi mag - no

4th
D-G

al - - - le - - - lu - - - ia.

This chant, like most of the Mode IV models, spans the fourth D–G throughout, excepting only the final *incisio*, an *alleluia* that is too short to be expected to fill the entire range.

Mode IV Summary

Despite the modal ambiguity that can frequently affect *deuterus* modes, the model chants cited by the *Alia musica* for this mode are among the most consistent models presented. Most of them continuously span the fourth D–G, with a few *incisiones* instead spanning the fourth E–a (and most of these could also potentially be interpreted as the fifth D–a, though this interval is not characteristic of the mode).

Mode V

Regarding this mode, the author of the source treatise says,

Totus iste tonus semper diapenticus per denarium efficitur, quia ter 8 ad ter 12 et quater 6 ad quater 9, id est per sesquialteram proportionem in diapente cadunt, [...] ut introitus Domine refugium, Circumdederunt me et Domine in tua misericordia, et antiphonae Soluite templum hoc, Salve crux, Exultet spiritus meus, et similia.⁷⁷⁷

This entire tone is brought about always by the fifth through ten [a multiple of five, which is, itself, the sum of the terms of 3:2], since three eights to three twelves and four sixes to four nines – that is, through the sesquialter proportion – fall into a fifth, [...] such as the introits *Domine refugium*, *Circumdederunt me*, and *Domine in tua misericordia*, and the antiphons *Soluite templum hoc*, *Salve crux*, *Exultet spiritus meus*, and similar.

Thus, the chants of this mode should always display the fifth; but the ratio 36:24 arises two different ways, independently through multiples of three and four, and so if modal numbers were to represent consistent pitches, it could be reasonable to expect two different fifths to arise. Below are analyses of the six chants identified by the author of the source treatise as exemplifying Mode V.

***Domine refugium*, Mode V Introit, Cantus ID g00696**

Text: Psalms 89:01–02

⁷⁷⁷ *Alia musica* §185(b) & (d), ed. Chailley, 92, trans. Nowacki 123–24.

CH-P 18
f. 80

Montpellier
#505

Do mi ne, re fu - gi um, fac - tus es no - bis

a ge ne - ra ti o - ne et pro ge - ni - e.

A sae - cu - lo, et in sae - cu - lum tu es.

This introit is quite consistent, spanning the fifth F–c with regular use of the upper *emmelis*, but no use of the lower *emmelis*.

Circumdede runt me, Mode V Introit, Cantus ID g00631

Text: adapted from Psalms 17:05–07

CH-P 18
f. 55

Montpellier
#504

Cir - cum de - de - runt me ge - mi - tus mor - tis,

do - lo - res in - fer - ni

cir - cum - de - de - runt me:

et in tri - bu - la - ti - o - ne me - a

in - vo - ca - vi Do - mi - num,

et e - xau - di - vit de tem - plo san - cto su - o

vo - cem me - am.

This chant is also quite regular, with six of the seven systems spanning the fifth F–c, with occasional use of the upper *emmelis*. The remaining (second) system does not reach down to the F, though it might reasonably be grouped together with the following system. (It is also possible that the d above the specified range might not be properly identified as an *emmelis*, and instead should be part of the complete fifth G–d, which, however, is not common in the other chants cited for this mode).

***Domine in tua misericordia*, Mode V Introit, Cantus ID g01122**

Text: Psalms 12:6.

CH-P 18
f. 240
Montpellier
#503

Do - mi - ne

10
in tu - a mi-se - ri - cor - di - a spe - ra - vi

49
e - xul - ta - vit cor me - um

62
in sa - lu - ta - ri tu - o

84
can - ta - bo do - mi - no

99
qui bo - na tri - bu - it mi - hi.

5th F-c

5th F-c

5th a-e

5th F-c

This chant is less regular than the preceding chants. Three complete systems span the fifth F–c; another system spans the fifth a–e, which might possibly support the hypothesis that the two different derivations of 36:24 represent two different fifths. The first system spans only a major third, which is unsurprising, since the system is quite short; it was separated from the following system only because the complete grammatical segment was too long to fit on one system, and so the first system is really part of the second. However, the second last system also spans only the major third, and it is longer. In either case, the span of the third corresponds to the lower third of the fifth F–c.

Soluite templum hoc, Mode V Antiphon, Cantus ID 004982

Text: John 2:19;21

Gevaert: Theme 42

Frere: V¹₁₀

A-Wn 1799
f. 57v

Sarum
p. 188

5th
F-c

Sol - vi - te tem - plum hoc,

di - cit Do - mi - nus; et post tri - du - um

ex - ci - ta - bo il - lud: hoc au - tem di - ce - bat

re - ae - di - fi - ca - bo il - lud:

de tem - plo cor - po - ris su - i.

5th
F-c

This chant is problematic to analyze. On the one hand, it is unusually easy to determine appropriate musical segmentations for the chant, owing to the fact that both the second system and (to a slightly lesser extent) the third system consist of two shorter *incisiones* sharing the same melodic structure (demarcated here by double barlines). This division, however, is somewhat at odds with the grammatical structure of the text, since the strongest grammatical divisions occur in the middle of these two systems. Additionally, neither of these two systems span the expected fifth. The second system spans the major third c–e, which might be interpreted as the upper third of the fifth a–e that also appeared in *Domine in tua misericordia*. The third spans the fourth a–d; but if the d is interpreted as an *emmelis*, this span would

be a minor third a–c, which could be interpreted as the upper third of the fifth F–c; however, the two systems collectively span the fifth a–e that is also present in *Domine in tua misericordia*.

Salve crux, Mode V (or VII ?) Antiphon, Cantus ID 004693

Text: n/a

Gevaert: n/a

Frere: VII²

It should be noted that this chant is almost universally held to be in Mode VII. However, its range and interval structure (in some sources), when coupled with the use of B-flat that is so common in Mode V, could easily correspond to either *tritus* or *tetrardus* modes; without additional information about the differences between modes in the opinion of the author of the source treatise, there is no reason not to assume that the same antiphon usually identified as a Mode VII chant was known to him as a Mode V chant. Here, it has been transposed down a wholetone to end on F, to facilitate comparison to other fifth-mode chants. In A-Wn 1799, the subtone at the beginning of the third system (given as F in the original transposition) becomes an E-flat, a note extraneous to the medieval gamut as it is usually presented, which may explain why it has been transposed to end on G; however, there is a variant reading in Sarum (and, in fact, most of the sources on the Cantus database), which does not use the subtone, and therefore fits nicely into the usual interval structure of Mode V.

A-Wn 1799
f. 183r

Sarum
p. 351

Sal - ve , crux pre - ti - o - sa

sus - ci - pe dis - ci - pu - lum e - ius

5th
F-c

In this chant, only the middle system clearly displays the expected fifth, F–c (plus the upper *emmelis*). The last system is suggestive of this same range but does not reach up to the fifth, ending at the b.

Exultet spiritus meus, Mode V Antiphon, Cantus ID 002820

Text: Luke 1:47

Gevaert: Theme 42 Frere: n/a

This chant is not included in the Sarum antiphoner (and therefore is not classified by Frere). It conforms quite nicely to the expected pattern. It spans the fifth F–c, including the upper *emmelis*.

Mode V Summary

Most of the chants do seem to present the fifth F–c, and occasionally also the fifth a–e. However, there are several *incisiones* that do not display the complete range. There is little consistency amongst these exceptions, except that they generally share one of the two boundary pitches of these two fifths, and could be interpreted as incomplete fifths; some of them, however, are so long that this explanation becomes somewhat dubious.

Mode VI

Regarding this mode, the author of the source treatise says,

Sexies enim 6 ad sexies 4 de bis 12 et quater 9 ad quater 6 de bis 12, quia sesquialtera proportio est in diapente, et sexies 4 de ter 8 et sexies 4 de bis 12 in consonantia diapason, id est 6 et 12; ueniunt per 8 qui sunt 48, sicut sexies 6 et quater 9 in 72. Totus iste tonus aut per diapason consonantiam, id est 6 et 12, aut per diapente, id est denarium, mensurabitur. Ut introitus Os iusti, Omnes gentes et Quasimodo geniti, et antiphonae O admirabile commercium et Uade Satana et Uirgo hodie fidelis.⁷⁷⁸

Indeed, six sixes to the six fours of twice twelve and four nines to the four sixes of twice twelve, since they are in the sesquialter proportion, makes a fifth, and six fours of thrice eight and the six fours of twice twelve are in the consonance of the octave (that is, [prototypically] 6:12), come through eight, which are forty eight, just as six sixes and four nines are seventy-two. This entire tone is measured either through the octave consonance (which is [prototypically] 6:12), or through the fifth (which is [prototypically] ten [a multiple of five, which is the sum of the terms of 3:2]). Such are the introits *Os iusti*, *Omnes gentes*, and *Quasimodo geniti*, and the antiphons *O admirabile commercium* and *Vade Satana* and *Virgo hodie fidelis*.

This passage is tricky to interpret. Chailley suggests that “come through eight” here refers to the division of the octave proportion through the harmonic mean, 6:8:12. If this is correct, the two modal numbers 24 and 36 represent the prototypical numbers 4 and 6, and by extension, the forty-eight would then correspond to eight in this division. However, four is not a prototypical number used in the source treatise; this kind of number theory can be found in the *Alia musica* in the sections borrowed from Boethius, which are probably the work of the revisor, not the source treatise.

This interpretation also ignores a trend throughout the source treatise of treating large proportions as multiples of smaller proportions, where a common coefficient can be found. Thus, six sixes are not compared directly to two twelves, but rather, to the four sixes that make up two twelves (*i.e.*, not $(6 \times 6):(2 \times 12)$, but rather $(6 \times 6):(2 \times (3 \times 4))$, which is equivalent to $(6 \times 6):(6 \times 4)$ in which the coefficients are all six. Likewise, four nines are compared not to two twelves, but to the four sixes that make up two twelves (*i.e.*, not $(4 \times 9):(2 \times 12)$, but rather $(4 \times 9):(2 \times (4 \times 3))$, which is equivalent to $(4 \times 9):(4 \times 6)$ in which the coefficients are all four.

When the author then says that the six fours of three eights and the six fours of two twelves (two equivalent ways of describing twenty four) are forty eight, he is adding the terms of the proportion

⁷⁷⁸ *Alia musica* §186(b)–(e), ed. Chailley, 92–93, trans. Nowacki 124–25.

together, and is essentially saying that $(3 \times 8) + (2 \times 12) = 48$ is equivalent to $(6 \times 4) + (6 \times 4) = (6 \times 8)$, in which the coefficient is always six, and thus, the sum forty-eight represents the number eight (after the coefficient is disregarded). This interpretation is weakly corroborated in the next clause, as the author explains that six sixes and four nines give seventy-two, but he does not equate seventy-two to any other number (as, indeed, he would have no reason to do in this instance, since the two terms do not share a coefficient, and there is therefore no reason to find a common factor in the sum).

The author says, then, that the sixth mode is characterized by the fifth in the proportion 36:24, in which one of the modal products, two twelves, is treated as equivalent to six fours or four sixes in order to ensure that the proportion 36:24 can always be represented as sharing a common coefficient throughout (thus, 36:24 is equal to $[6 \times 6]:[6 \times 4]$ or $[4 \times 9]:[4 \times 6]$). Interestingly, while both derivations of thirty-six (six sixes and four nines) are said to make a fifth against twenty-four, only one derivation of twenty-four (two twelves) is used for this purpose.

The other derivation of twenty-four, which is three eights, is used in a different and surprising way, to generate an additional pseudo modal number, by comparing this twenty-four to the other one, oddly creating an octave (even though it ought to create a unison), a fact that is not reflected in the tabular summary but is noted in the prose summary. It is not certain why the author did not simply take the otherwise unused three eights and instead turn them into six eights to give a forty-eight against which the twenty-four could be compared, but it is likely that in the author's view, since the number thirty-six also appears as a multiple of six, listing forty-eight as a multiple of six would lead to a comparison between thirty-six and forty-eight as a fourth, which is not appropriate for this mode. This issue is discussed in greater detail in the previous chapter.

Whatever the case, below are analyses of the six chants identified by the author of the source treatise as exemplifying Mode VI.

Os iusti, Mode VI Introit, Cantus ID g01349

Text: Psalms 36:30.

CH-P 18
f. 274

Montpellier
#530

5th
D-a

Os ius - ti, me - di - ta - bi - tur sa - pi - en - ti - am,

5th
D-a

et, lin - gua e - ius lo - qui - tur iu - di - ci - um

5th
D-a

lex de - i e - ius in cor - de ip - si - us.

This chant consistently maintains a fifth from D–a, with occasional use of both lower and upper *emmeles* (C and b). However, the D is absent from several *commata* of this chant that might possibly be considered independent *incisiones*; consequently, to treat these *commata* as independent would result in the range of a fourth from E–a, which is not sanctioned by the *Alia*.

Omnes gentes, Mode VI Introit, Cantus ID g01161

Text: Psalms 46:02

CH-P 18
p. 224

Montpellier
#532

5th
D-a

Om - nes gen - tes

4th
D-G

plau - di - te ma - ni - bus

5th
F-c

iu - bi - la - te de - - - o

5th
D-a

in - voc - ce ex - sul - ta - ti - o - - - nis.

This chant is framed by the fifth D–a (with a lower *emmelis* in the first *incisio* only) but is less consistent than others in this mode. The second system never reaches the a, and so spans only the fourth D–G, which is not sanctioned by the *Alia* (but may be viewed simply as an extension of the preceding system, which was only separated from it for reasons of length). More importantly, the third system spans a completely different fifth, from F–c. Relative to the hypothesis that each modal number represents a consistent pitch, these two fifths become problematic: since the *Alia* uses only one derivation of twenty-four, and therefore only one fifth, there is no obvious reason why there should be two separate fifths in this chant.

***Quasimodo geniti*, Mode VI Introit, Cantus ID g01049**

Text: I Peter 02:02

This chant is transposed up a perfect fifth in both sources. It is transposed back down to the paradigmatic position here to facilitate comparison to other Mode VI chants. The transposition from F up to c is a common procedure for Mode VI chants because of the ubiquity of b in this mode, which creates an interval pattern identical to that beginning on c; thus, any Mode VI chant that exclusively uses b instead

of \flat can be notated unambiguously in the higher transposition. However, in this chant, unlike the next several chants (which are also transposed up to c), the notation on c also includes a \flat , and transposing it back down creates an E-flat, a pitch not normally considered to be part of the medieval gamut. It is commonly held that chants of this kind are one of the chief reasons for the “co-finals,” the transposed versions of modes; that is, transposing Mode VI chants up to c allows the \flat / \flat flexibility to notate what would, in the untransposed position, require a comparable flexibility between E and E-flat that is not normally held to exist in the standard medieval gamut. This characteristic is important in this chant; were there not to be any E-naturals in this chant (highlighted here with the use of courtesy accidentals), the interval structure would be indistinguishable from a Mode VIII chant. This issue also highlights one of the chief advantages gained by the source treatise from not directly specifying the nature of the gamut: the nature of each mode may be partially defined in terms of its intervals while still retaining the flexibility to accommodate these E-flats that would cause difficulties if the gamut were to be precisely defined.

CH-P 18
p. 214

Montpellier
#536

Qua - si mo - do ge - ni - ti in - fan - tes

al - le - lu - ia

ra - ti - o - na - bi - les

4th
E \flat -a

5th
D-a

5th D-a

si ne do - lo lac con - cu pis - ci te

5th D-a

al - le - lu - ia al - le - lu - ia al - le - lu - ia.

Most of this chant consistently spans the fifth D–a with regular use of the lower *emmelis* and occasional use of the upper *emmelis*. However, the opening *incisio* differs dramatically between the two sources (and amongst other sources as well), and lacks the D in CH-P 18, while not reaching up to the a in Montpellier (nor in D-Gsta AB III 9 f. 116v, another source that begins like CH-P 18, but otherwise more closely resembles Montpellier). The first *alleluia* spans only a fourth (and an augmented fourth at that); however, if the alleluia is included in the first *incisio*, in Montpellier and D-Gsta, the expected fifth occurs with both *emmeles*.

***O admirabile commercium*, Mode VI Antiphon, Cantus ID 003985**

Text: n/a

Gevaert: Theme 41 Frere: VI₁ a

This chant is also transposed up a perfect fifth in both sources. It is transposed back down to its normative position here to facilitate comparison to other Mode VI chants.

A-Wn 1799 f. 23r

5th D-a

O ad - mi - ra - bi - le com - mer - ci - um!

Sarum p. 79

4th F-b b
 cre a tor ge ne - ris hu - ma ni a ni ma - tum cor pus su - mens,

4th F-b b
 de vir - gi - ne nas - ci dig - na - - - tus est

5th D-a
 et pro - ce - dens ho - mo si - ne se - mi - ne

8th C-c
 lar - gi - tus est no - bis su - am de - i - ta - tem.

The segmentation of this chant is assisted by the fact that it corresponds very nicely to Frere's description of its theme group.⁷⁷⁹ Even so, it is problematic to analyze according to the doctrines of the *Alia*. The opening and the fourth system both maintain the same D-a fifth common to the other chants in this mode, with only one use of a lower *emmelis*. The final system mostly spans the same fifth, but in addition to the lower *emmelis*, there is an upper c, which may or may not qualify as an *emmelis*; although the use of c instead of b is common, it might be unexpected in this case, since the b does appear as an upper auxiliary in both the second and third systems. However, these same two systems do not span the fifth from D-a, but rather a fourth from F-b; this fourth would not be expected according to the *Alia*'s description of the mode, but it does correspond loosely to the higher fifth F-c that appears in *Omnes gentes*.

⁷⁷⁹ Frere, *Antiphonale Sarisburiense*, 71.

Uade Satana, Mode VI Antiphon, Cantus ID 005303

Text: Matthew 4:10;7

Gevaert: n/a

Frere: I³

This chant is classified variously in two different modes. In A-Wn 1799, the chant is given in Mode VI, but transposed up a fifth; here, it has been transposed down to facilitate comparison to other Mode VI chants. However, in many sources (including Sarum), it ends somewhat differently and is classified in Mode I. The *Alia musica* lists it as an example of Mode VI, and so it is that version that needs to be analyzed here.

A-Wn 1799
f. 47v

Sarum
p. 157

Va-de Sa-ta - nas non tem - pta - bis do-mi-num de-um tu-um.

5th
D-a

The image shows two staves of musical notation. The top staff is for A-Wn 1799, f. 47v, and the bottom staff is for Sarum, p. 157. Both staves are in treble clef. The A-Wn 1799 staff has a yellow highlight under the notes, and a vertical line on the right side of the staff is labeled '5th D-a'. The lyrics are written below the notes: 'Va-de Sa-ta - nas non tem - pta - bis do-mi-num de-um tu-um.'

This chant is very short, containing only a single obvious *incisio* that spans the fifth D-a with no *emmeles*.

Uirgo hodie fidelis, Mode VI Antiphon, Cantus ID 005452

Text: n/a

Gevaert: Theme 40

Frere: VI₁₇ a

This antiphon is also transposed up a perfect fifth in A-Wn 1799. It is transposed back down to its normative position here to facilitate comparison to other Mode VI chants.

A-Wn 1799
f. 20r

Sarum
p. 55

Vir - go ho - di - e fi - de - lis,

et - si ver - - - bum ge - nu - it in - car - na - tum,

4th
F-b \flat

The image shows two staves of musical notation. The top staff is for A-Wn 1799, f. 20r, and the bottom staff is for Sarum, p. 55. Both staves are in treble clef. The A-Wn 1799 staff has a blue highlight under the notes. The lyrics are written below the notes: 'Vir - go ho - di - e fi - de - lis,' and 'et - si ver - - - bum ge - nu - it in - car - na - tum,'. On the right side of the bottom staff, there is a vertical line labeled '4th F-b \flat '.

vir - go man - sit et post par - - - tum;

quem lau - dan - tes om - nes di - ci - mus:

be - ne - dic - ta tu in mu - li - e - ri - bus.
in mu - li - e - ri - bus.

The structure of this chant very much resembles *O admirabile commercium* (above); they belong to the same theme and both correspond nicely to Frere's description of the theme.⁷⁸⁰ The most noticeable difference between the two is that the opening *incisio* of *Virgo hodie* spans only a major third from F–a. The final system might be divisible into two *incisiones*, *benedicta tu* and *in mulieribus*, which would each then span only a fifth (C–G and F–C, respectively).

Mode VI Summary

The majority of the *incisiones* in the Mode VI chants cited by the author of the source treatise span the perfect fifth D–a. However, the fifth F–c or the fourth F–b is also reasonably common, and possibly also the octave C–c (though this latter possibility may represent the fifth D–a with *emmeles* or the combination of two different fifths, one rising from C–G and another completing the rise and then descending c–F).

⁷⁸⁰ Frere, *Antiphonale Sarisburiense*, 71.

It is an interesting characteristic that the intervals specified for this plagal mode contain the octave while its authentic counterpart does not. This need not be inconsistent with the usual description of plagal modes in other treatises, as the octave in question is the usual one-octave span C–c that is generally associated with Mode VI. It is somewhat unintuitive that Mode VI should contain an octave when Mode V does not (because treatises like the *Musica enchiridis* indicate that authentic modes generally have wider ranges than plagal modes), but Mode V still effectively spans an octave, more or less, because the two fifths found within that mode collectively span the range F–e (just under an octave), which more or less corresponds to the usual octave described in other treatises.

Mode VII

Regarding this mode, the source treatise says,

*Hic tonus in proportione quarti toni habet totum diatessaron, et in proportione quinti toni habet totum diapente, et in propria proportione habet duodecies 12 diapason, quod est 144. Ut introitus Puer natus est uobis [sic] et Audivit Dominus, et antiphonae Iste est frater uester minimus et Dixit Iudas fratribus suis.*⁷⁸¹

This tone, in the proportion of the fourth tone, has a complete fourth, and in the proportion of the fifth tone, it has a complete fifth, and in its own proportion, it has the octave twelve twelves, which is one hundred forty-four. Such are the introits *Puer natus est nobis* and *Audivit Dominus*, and the antiphons *Iste est frater uester minimus* and *Dixit Iudas fratribus suis*.

The author says, then, that the seventh mode may manifest any of the three basic intervals (fourth, fifth, octave), but not in its own right (as in Mode I), but rather by sharing the proportions of the fourth and fifth modes in addition to its own proportion. As for its own proportion, this is given as an octave that is the product of twelve twelves. This derivation is unusual in several ways. In this case, only three modal numbers are given, and all three are multiplied to the same product: 4×12 , 8×6 , and 6×8 , all

⁷⁸¹ *Alia musica* §187(e)–(f), ed. Chailley, 93–94, trans. Nowacki 126. Note that subsection (f) is a reconstruction provided by Chailley based on the commentary in §131 (see Chailley, p. 94, n. (f)). Chailley has miscopied the pronoun in the introit incipit *Puer natus est nobis*; it appears correctly in the commentary (cf. *Alia musica* §131, ed. Chailley, 164), and I have corrected it in the translation.

three of which give forty-eight. No coefficient is given for nine (*i.e.*, the letter D, which represents the base number nine, is entirely omitted from the derivation of Mode VII's own proportion), perhaps because there is no way to multiply nine evenly to get another forty-eight. The author then says that the sum of the octave is one hundred forty-four. Following the author's usual procedures, this would imply the sum of forty-eight and ninety-six, though ninety-six is nowhere to be found among the numbers of this mode, except by implication.

There is a very important caveat to the four analyses presented below, which are described as exemplifying Mode VII: they are not provided by the author of the source treatise, who does not cite any examples for this mode. Instead, Chailley has supplied these examples in his edition, having drawn them from the commentary.⁷⁸² In principle, therefore, one must be cautious about accepting these chants as exemplifying the characteristics of the mode that the author of the source treatise describes, as the understanding of modality may well have changed by the time the commentator was writing. Nevertheless, the analyses are presented here for lack of preferable examples.

***Puer natus est nobis*, Mode VII Introit, Cantus ID g00553**

Text: adapted from Isaiah 09:06

CH-P 18
p. 33
Montpellier
#568

Pu - er na - tus est no - bis

et fi - li - us da - tus est no - bis

5th
G-d

⁷⁸² *Alia musica* §131, ed. Chailley, 164; Chailley considers this passage to be the work of the revisor (since he does not posit a separate commentator), but *cf.* Atkinson, *Critical Nexus*, 177, Table 5.3.

5th a-e?

cu-ius im-pe-ri-um su-per hu-me-rum e-ius

5th a-e?

et vo-ca-bi-tur no-men e-ius

5th G-d

mag-ni-con-si-li-i an-ge-lus.

The first two systems of this chant are very similar, and span the fifth G–d. The next two systems are less clear. The third system spans nearly, but not quite, an octave, from G–f. Thus, it is probably best to analyze it as the fifth a–e; however, the f that would then be classified as an *emmelis* behaves somewhat awkwardly, with a skip both in and out, while it is, itself, decorated by a lower neighbour tone. The fourth system spans exactly the fifth a–e, but the a and e, themselves, only occur at opposite ends of the phrase, and it might well be equally valid to identify it as a fourth, either b–e or a–d. The final system also spans the fifth G–d.

***Audiuit Dominus*, Mode VII Introit, Cantus ID g00670**

Text: Psalms 29:11

CH-P 18
p. 71

5th G-d

Au-di-vit do-mi-mus,

Montpellier
#571

The image displays three systems of Gregorian chant notation. Each system consists of a vocal line with Latin text and a corresponding melodic line. The first system is labeled '5th c-g', the second '5th G-d', and the third '5th G-d'. The melodic lines are highlighted in yellow.

System 1: 5th c-g
 et mi - ser - tus est mi - hi:

System 2: 5th G-d
 do - mi - nus fac - tus est

System 3: 5th G-d
 a - diu - tor me - - - us.

Except for the second system, the rest of the chant consistently spans the fifth G–d with the upper *emmelis* e (though in both the third and fourth systems, the fifth a–e with a lower *emmelis* G is also plausible; however, this latter possibility seems less likely since the G is also the cadence tone). The second system instead spans the fifth c–g (which shares the same interval structure as G–d, though there is little evidence to suggest that this fact is significant).

***Iste est frater uester minimus*, Mode VII Responsory [!], Cantus ID 006999**

Text: Genesis 43:29–30

Gevaert: n/a

Frere: VII⁶

This chant is not classified as an antiphon, but a responsory; its classification in the source treatise in Chailley’s edition is probably the result of his attempting to follow the pattern presented in the previous six modes when he created his “reconstruction,” without noticing that the chants supplied by the commentator do not fit the pattern. It is not entirely clear whether this discrepancy should have an impact on the present analyses. A modern understanding of modality intuitively expects the characteristics of the modes to be consistent across genres; however, the melodic formulae and clichés that may have formed an essential characteristic of early ecclesiastical modality differ among genres. Unfortunately, the

lack of examples explicitly cited in the source treatise makes it impractical to reject analysis of this chant as too far removed from the rest of the repertoire under consideration (and at any rate, the functions of antiphons and responsories are somewhat similar).

A-Wn 1799
f. 53r

Sarum
p. 175

5th
G-d

Is te est fra ter ves ter mi - ni - mus

5th
G-d

de quo di xe - ra - tis mi - - - hi?

5th
c-g

De - - - us mi - se - re - a - tur ti - bi,

fi - - - li mi.

5th
a-e

Fes ti - na - vit - que in do - - - mum

et plo - - - ra - - - vit,

qui a e - rum pe - bant la - cry - mae,

et non po - te - rat se con - ti - ne - re.

Much of this chant spans the fifth G–d. However, the third system instead spans the fifth c–g (which could equally be the fourths c–f or d–g), and the fifth system spans the fifth a–e with both *emmeles* (therefore actually spanning almost an octave). The fourth system only spans a third but is short and could easily be interpreted as an extension of the third system. The sixth system also only spans the third; it stands alone well enough melodically, but could join grammatically with the previous system, in which case the larger *incisio* would span the octave F–f.

***Dixit Iudas fratribus suis*, Mode VII Responsory [!], Cantus ID 006477**

Text: Genesis 37:26–30

Gevaert: n/a

Frere: VII^ε

The discussion of the classification of *Iste est frater uester minimus*, above, applies equally well to this chant, which manifests the same misclassification.

A-Wn 1799
f. 52r

Sarum
p. 171

Di - xit Iu - das

fra - tri - bus su - is:

5th
c-g
ec-ce Is-ma-e-li-tae tran-se-unt,

8th
F-f
ve-ni-te, ve-num-de-tur,

8th
F-f
et ma-nus nos-trae non pol-lu-an-tur,

4th
c-f
ca-ro e-nim

5th
G-d
et fra-ter nos-ter est.

This chant contains two systems spanning the fifth G–d that is common in the other model chants for this mode. There are also two systems spanning the octave F–f that was suggested as an alternative interpretation for a larger *incisio* in *Iste est frater uester minimus* (above). The opening *incisio* spans the fourth b–e, though if it were combined with the following system (which was separated from it only for reasons of length), the e would serve as an *emmelis* to the overall fifth G–d, which may be a better interpretation. The second last system spans the fourth c–f, which is an interval not seen elsewhere in these analyses, but it does not obviously combine well with either the preceding or succeeding systems

into a larger *incisio*; if combined with the previous system, it would fit the overall octave F–f, but it seems grammatically better connected to the final system, with which it would form an incomplete octave, G–f, or a fifth a–e with both upper and lower *emmeles*, in which the lower *emmelis* is both the cadential note and the *finalis* (an awkward interpretation). However, the fifth c–g also seen in *Audiuit Dominus* and *Iste est frater uester minimus* is also present in this chant, so that the span c–f might also be seen as an incomplete instance of this fifth. Alternatively, it could also be interpreted as the fourth b–e that occurs at the beginning of the chant. Interpreting this system as a fourth, whether it be c–f or b–e, results in an *emmelis* by semitone that potentially causes problems for the explanation of *emmeles* at the minor third as avoidance of the semitone, but there is one important benefit: this chant is the only model chant cited for this mode that contains the fourth, an interval that is supposed to be characteristic of this mode.

Mode VII Analysis

There are fewer examples provided for this mode than for most other modes (Mode II excluded), and the selection of chants is suspicious, coming from the commentary rather than the source treatise and drawing from responsories in place of antiphons. There is some consistency, in that the fifth G–d is fairly common, and the fifth c–g also common (but somewhat less so). The octave F–f might also be considered common. The span of a fourth, which is also supposed to be characteristic of this mode, occurs in only one chant, and there is not enough consistency between the two instances of it to decide on the appropriate position of this fourth, either as b–e or c–f.

Mode VII, as described by the *Alia*, also contains the oddity that the modal numbers that create the three different intervals are derived separately, rather than in a single derivation (as is Mode I). However, there is no obvious difference between the chants cited for Mode VII and those cited for Mode I that would explain this procedural difference. Additionally, the *Alia* specifically compares the fourth and fifth to the intervals of Modes IV and V, but the relationship between them is tenuous at best. The fifth of

Mode V is typically F–c, rather than G–d. These two fifths often share the same interval structure, as the F–c of Mode V often uses b, but this correspondence is not consistent; if anything, it would be much more accurate to say that Mode V sometimes uses the fifth of Mode VII than that Mode VII uses the fifth of Mode V. Meanwhile, Mode IV is extremely consistent about its uses of the fourth D–G, which does not ever occur in that position in Mode VII, and the interval structure does not agree with either the fourths h–e or c–f that occur in Mode VII.

Mode VIII

Regarding this mode, in most manuscripts, the author of the source treatise says only, “*Tonum octauum require supra.*”⁷⁸³ (“Seek the eighth mode above”). That is, the author considers that everything that one would need to know about the eighth mode is given in a previous passage; this reference is not particularly helpful, since this sentence is the very last sentence of the composite treatise in its usual form, and it may well refer to a large number of passages – not only in the source treatise, but quite possibly in the revision, since this sentence may well be the work of the revisor. It is absent from the Karlsruhe manuscript, the only manuscript that transmits the source treatise independently of the rest of the composite treatise.⁷⁸⁴ Instead, the Karlsruhe manuscript contains an entirely different passage, unattested in other manuscripts, that closely resembles the corresponding passage in the prose summary. It is unlikely that this passage was original to the source treatise and removed by a subsequent scribe (perhaps the revisor) as redundant – it is sufficiently different from the preceding material to make it difficult to see as redundant – or that it has perhaps been lost from the end of a prototype manuscript upon which all the other manuscripts (excluding Karlsruhe) were based. More likely, the unique passage

⁷⁸³ *Alia musica* §188, ed. Chailley, 95.

⁷⁸⁴ cf. *Karlsruhe*, Badische Landesbibliothek, K. 504 f.34v.

is the work of a later scribe attempting to complete the eight-mode scheme by borrowing from the prose summary.

Whatever the explanation, no Mode VIII chants are identified here, nor in any part of the composite treatise that is clearly related to the source treatise (only the tonary of the *Nova expositio*, which is clearly a separate tradition from the source treatise, contains Mode VIII chants), and it would thus be highly dubious to undertake any analysis of these Mode VIII chants according to the already dubious description found in Karlsruhe. This observation may provide support for the alternative hypothesis of the constituents of the composite treatise, that the passages generally described as a “prose summary,” which are transmitted independently in the Florence manuscripts, may not be a summary of the source treatise, but may instead be the ultimate source upon which the “source treatise” is an exegesis.

Meta-analysis:

In most modes, Modes II and IV excepted, the model chants do not show a desirable level of consistency in the intervals that they favour. However, each mode does demonstrate at least a favoured interval (or sometimes a few favoured intervals) when analyzed in light of the size of intervals that the *Alia musica* suggests ought to be characteristic of the *incisiones* in each. Assuming that the interval sizes given by the *Alia* are accurate, Table 45 gives a summary of the most common intervals according to their boundary pitches.

Table 45 – Favoured intervals in the cited chants in each mode

	Authentic Modes			Plagal Modes		
	Fourth	Fifth	Octave	Fourth	Fifth	Octave
Protus	D–G	D–a (F–c?)	D–d (C–c?)	C–F (D–G?)	C–G	
Deuterus		F–c (D–a?) (E–h?)	D–d (C–c?) (E–e)	D–G		
Tritus		F–c (a–e?)			D–a (F–c?)	(C–c?)
Tetrardus	c–f? h–e?	G–d (c–g?) (a–e?)	F–f	No information		

Even if one keeps all the various possibilities open (not restricting the comparison to the single most common interval of each type in each mode), there is no way to reconcile these pitches with a consistent set of pitch-to-number relationships. For the purpose of the following analysis, it will not even be assumed whether the numbers represent frequencies or string lengths (*i.e.*, whether larger numbers represent higher or lower pitches), but only that the procedure is consistent throughout the complete system.

The *protus* modes share the number 12 only, and in each mode, it is attached to a fifth (one above 12 and one below it), but their fifths are a wholetone apart (D–a vs. C–G), rather than the implied fifth apart. The *deuterus* modes share the numbers 18 and 24, which are a fourth apart, but the authors of the *Alia* do not consider the fourth to be characteristic of the authentic mode; instead, the relevant numbers should be 12:18 in authentic and 18:24 in plagal; since the only interval in Mode IV is D–G, the fifth in Mode III ought to be G–d, but it is actually F–c – again, a wholetone away from where it should be; needless to say, the correspondences for these numbers in *deuterus* also is not consistent with the correspondences for the same numbers in the *protus* modes.

Both *tritus* modes contain only the numbers 24 and 36, which are a fifth apart, but the most common fifths in each are different (F–c and D–a). While F–c does occur in both modes (as a less common interval in the plagal mode), one of these would have to be represented by the number 24, which is shared with the *deuterus* modes; but the only interval in the *deuterus* modes that has either one of these pitches is the very same fifth F–c, which should therefore have both of the same numbers, while only one modal number is actually shared (and it is the only modal number in Mode III that does not participate in the fifth, while Mode IV is characterized only by fourths). There is also a possible octave C–c in Mode III, but the octave in Mode III is 12:24, and the fifth in *tritus* is 24:36, so that the c that would be represented by 24 in this interpretation would have to be between the two intervals, instead of being the top note of both.

As for *tetrardus*, there is no information about the plagal against which to compare the intervals of the authentic mode. But the fourth ought to be 18:24 and the fifth 24:36, so that the number 24 should represent a single shared pitch that one interval is above and the other below; although there are two possibilities for the fourth (b–e and c–f), there is no way to compare either of these fourths to any of the possible fifths in such a way as to put the common note in the middle of the range. Additionally, the octave is given as being represented exclusively by the number 48; the other, unspecified number can only be 24 or 96. 96 makes no sense since the range of this mode would then become excessive; 24 maintains a more reasonable range, but then must match the 24 in the other intervals; since the octave is F–f, the only possible way for this to work would be if 24 represented the f, which it would share with the fourth c–f, but as already described, this fourth not only does not pair appropriately with a fifth, it also does not agree with the use of 24 in the *tritus* modes, for which no likely interval is bounded by the pitch f.

Here, it is possible to see one important difference between Modes I and VII, which are both supposed to be characterized by all three perfect intervals: in Mode I, the boundary pitches of the most common versions of each interval are shared (notwithstanding *Rorate caeli*, the *Alia*'s chief model, which is an exception), while in Mode VII, there seems to be no common reference point for these intervals; perhaps this distinction is the reason why the *Alia* derives each of the three intervals separately in Mode VII, but all at once for Mode I. If so, this interpretation would provide yet further evidence against any consistency in the way the modal numbers are used from mode to mode.

A number of other possible relationships can now also be rejected. For instance, it is unlikely that the base numbers have any greater relationship to pitches than do the products. The *tritus* modes provide a ready demonstration. Both modes are defined as containing only the fifth in the ratio 36:24 (other than the unconventional octave in Mode VI that does not appear to be especially common), with two versions of each modal number, and in both modes, a single fifth predominates, with another fifth a third higher

also receiving some prominence. From comments in the *Alia musica* about Mode IV⁷⁸⁵ (which has a similar structure), it is clear that these pairs of equal modal numbers are not compared against each other indiscriminately (*i.e.*, each version of 24 being compared against both versions of 36, giving four possible combinations), but rather are compared in two mutually exclusive pairs that have a specific kind of commensurability.

According to the *Alia*, in Mode IV (12×2, 6×3, 8×3, 9×2), the multiples of eight and nine are compared to each other because they are both multiplied by two, and the multiples of twelve and nine are compared to each other because they are both multiplied by three, but a modal number resulting from multiplying by two is not compared to one created by multiplying by three. The *Alia* explains this requirement directly only with respect to Mode IV and not in relation to Modes V and VI, but the influence of this doctrine may be detected in the convoluted numberplay in each mode.

In Mode V, the relevant coefficients are three and four, and so it is clear that 8×3 should be compared to 12×3, and 6×4 should be compared to 9×4 (there is a certain redundancy to all of this reckoning, since these combinations of base numbers, themselves, produce the same intervals – intervals that they continue to produce after multiplication because they share the same coefficients). The author does not explicitly reject the other two possible combinations in his description of this mode, but he does present them in these specific pairings.⁷⁸⁶

In Mode VI, the situation is slightly more complex, because there are four unique coefficients, rather than two pairs of identical coefficients, and the combinations of products that produce the fifth after multiplication are mutually exclusive from the combinations of base numbers that produce the fifth before multiplication. The principle for which pairs may be compared must be extrapolated from the text

⁷⁸⁵ *Alia musica* §184(c), ed. Chailley, 91, trans. Nowacki 121.

⁷⁸⁶ *Alia musica* §185(b), ed. Chailley, 92, trans. Nowacki 123.

of the *Alia* and is described in the previous chapter. The coefficients for Mode VI are two, three, four, and six. Amongst these coefficients, the number three is commensurable only with the number six. Thus, the products of 6×6 and 8×3 could be compared to each other, but in fact, they are not; instead, 12×2 is compared to both 9×4 and 6×6 . Thus, the two fifths present in each of Modes V and VI are summarized in Table 46.

Table 46 - Relationship of base numbers to ratios in Modes V and VI

Mode V	Mode VI
$12 \times 3 = 36 : 8 \times 3 = 24$	$6 \times 6 = 36; 12 \times 2 = 24$
$9 \times 4 = 36 : 6 \times 4 = 24$	$9 \times 4 = 36; 12 \times 2 = 24$

In this arrangement, however, we can see that there is not a common pair of base numbers forming a fifth in both modes. The closest thing to a pattern that can be found is that the base numbers in the bottom row differ by three in both modes (nine and six in Mode V, nine and twelve in Mode VI), but this pattern does not hold in the top row. It is unlikely, therefore, that there exists any kind of meaningful relationship between the base numbers used to derive a ratio and the pitch upon which the resulting interval is rooted.

Certainly, no particular base number or product necessarily represents the *finalis*, as several of the characteristic intervals do not contain the *finalis* as a boundary pitch. It can also be demonstrated, with slightly greater complexity, that a single base number does not always produce, for instance, the lowest pitch of the characteristic interval. Table 47 presents the relationship between base numbers, products, and intervals (including the most likely pitches) in a way that draws particular attention to the base numbers. Mode VIII is excluded here since there are no examples in the *Alia* from which to draw the

necessary information. Also, since the three intervals found in Mode VII do not share a common pitch, each derivation is given its own column; the same is true for the octave and fifth of Modes III and VI, which apparently do not share a common pitch, even though they are derived from a common set of products.

Table 47 – Comparisons between base numbers and pitches.

Pitches may run either upwards or downwards with increasing numbers. The only consistent principle in this table is pitches that are shared between intervals. Even with this much flexibility, no obvious pattern emerges.

Largest number:	Mode I	Mode II		Mode III		Mode IV
	Lowest pitch	Highest Pitch		Highest Pitch		Lowest Pitch
8 ^{ve}	$6 \times 1 = 6$ (d)				$6 \times 4 = 24$ (d) $[8 \times 3 = 24$ (d)] ⁷⁸⁷	
5 th	$8 \times 1 = 8$ (a)	$6 \times 3 = 18$ (G) $9 \times 2 = 18$ (G)		$9 \times 2 = 18$ (c)		
4 th	$9 \times 1 = 9$ (G)	$8 \times 2 = 16$ (F)				$9 \times 2 = 18$ (G) $6 \times 3 = 18$ (G)
1	$12 \times 1 = 12$ (D)	$12 \times 1 = 12$ (C)		$12 \times 1 = 12$ (F)	$12 \times 1 = 12$ (D)	$12 \times 2 = 24$ (D) $8 \times 3 = 24$ (D)
Largest number:	Mode V	Mode VI		Mode VIIa	Mode VIIb	Mode VIIc
	Lowest pitch	Lowest Pitch / Only Pitch		Lowest Pitch	Highest Pitch	(Only Pitch)
8 ^{ve}			$12 \times 2 = 24$ (c) $8 \times 3 = 24$ (c)			$12 \times 4 = 48$ (f) $8 \times 6 = 48$ (f) $6 \times 8 = 48$ (f)
5 th	$8 \times 3 = 24$ (c) $6 \times 4 = 24$ (c)	$12 \times 2 = 24$ (a)			$12 \times 3 = 36$ (d) $9 \times 4 = 36$ (d)	
4 th				$9 \times 2 = 18$ (f) $6 \times 3 = 18$ (f)		
1	$12 \times 3 = 36$ (F) $9 \times 4 = 36$ (F)	$9 \times 4 = 36$ (D) $6 \times 6 = 36$ (D)	48? (C)	$12 \times 2 = 24$ (c) $8 \times 3 = 24$ (c)	$8 \times 3 = 24$ (G) $6 \times 4 = 24$ (G)	24? (F)

⁷⁸⁷ This combination is possible based on the modal numbers presented in the source treatise and is presented in the tabular summary

In this presentation, no assumptions are made even about consistency of direction. I have allowed the largest number to be either the highest or lowest pitch in any given mode, as the numbers require to allow a single number to represent a consistent pitch within a single mode; although it is very clear throughout the *Alia* that the largest numbers are the lowest pitches, there are instances in which a single number could only represent a consistent pitch within the mode if the pitches run in the opposite direction. For example, in Mode II, the two likely intervals are C–F and C–G, which share the lowest note C, but they share the smallest number, 12. In the interest of not missing any possible patterns, both possible directions have been considered. (However, where there is no good way to choose between one direction and the other, the default position is that the largest number is the lowest pitch).

Even in this presentation, deliberately susceptible to data mining, there are no obvious patterns. It is surprisingly common for the multiple of twelve to be the largest number in the mode; however, the pattern breaks down for Mode VI, and only appears to hold for Mode II because the direction of pitch is reversed for that mode. Thus, the fact that the multiple of twelve is usually the largest product is probably a simple coincidence from the fact that twelve is the largest base number.

What, then, can ultimately be concluded from all these analyses? Firstly, there is at least a kernel of truth to the intervals that the source treatise says ought to be present in each mode; the size of intervals that the *Alia* identifies do seem to be prevalent in most of the chants cited. Secondly, however, finding these intervals in the cited chants frequently requires liberal application of the principle of the *emmelis*, a concept that causes the intervals to seem to appear more often than they actually should. Finally, there does not appear to be any consistent relationship between the modal numbers or base numbers and specific pitches; the modal numbers appear to be purely abstract, reflecting the correct interval types in an exclusively symbolic way. On the basis of these conclusions, I shall propose an hypothesis to explain the origin of the modal numbers in Chapter 19. First, however, Chapter 18 will discuss some of the complexities encountered in the analyses presented above.

Chapter 18: Issues in the Analytical Process

The previous chapter presented brief analyses of the introits and antiphons cited by the source treatise; before I present an hypothesis for the logic behind the modal numbers, it will be valuable to take a step back and look at the analytical process itself and the challenges involved in it. The analysis was, of course, not quite undertaken to discover something unknown or unexpected, but to confirm what the source treatise says and thereby, perhaps, to gain deeper insight into both the modal system of the time and place that the source treatise was written and the author's understanding of that system. The Metz tonary, which is believed to date to approximately 830,⁷⁸⁸ and therefore likely predates even the source treatise of the *Alia musica*, contains more than fifteen hundred office antiphons and about one hundred fifty introits, which can probably be assumed to correspond reasonably well with the repertoire of chants known to the author of the source treatise; out of this very large repertoire, he selects some eighteen office antiphons and seventeen introits as exemplifying the modes (actually only the first six modes) that he is describing. This subset represents only about two percent of the chants listed in the Metz tonary for these genres, presumably selected for being particularly good representatives of the characteristics of their respective modes. And yet, even this modest repertoire of presumed prototypical models presents several analytical challenges that may perhaps serve as clues to the relationship between the modal system behind the treatise and the mathematical procedures with which the author describes that system.

⁷⁸⁸ Huglo observes that contents of the tonary seems to agree well with the time period clearly determined for the material that follows it, which contains the names of Pope John VIII, King Louis II (the Stammerer), and a Bishop Wala, which places it at 878 or 879 (Huglo, *Les tonaires*, 30). However, he notes that the archetype from which it was copied, must have dated from the second third of the ninth century, based on the structure of the liturgical calendar to which it conforms.

Anachronistic Assumptions

The first observation does not directly stem from the analysis itself so much as from the process of selecting suitable sources from which to transcribe the chants, and it provides an important reminder and caveat about the hazards of studying an older version of a tradition already understood in a later form. The very first chant that the author cites (and the only one about which the author gives any detail), *Rorate ceali*, together with *Traditor autem* (as well as other chants of the same theme group), has long been observed to have a problematic reading in the opening formula. In his influential book on Gregorian chant, Willi Apel notes that:

The similarity between the Gregorian b-flat and that of early polyphonic music also extends to the modern publications which, in either case, show a tendency to conform to nineteenth-century principles of tonality by introducing numerous chromatic alterations that are, to say the least, of doubtful authenticity. Certainly, the basic Solesmes editions, *Liber usualis*, *Graduale*, and *Antiphonale*, contain numerous b-flats which cannot be justified. In a recent article, [...] J. Gajard reproached the editors for their [...] deference to the habits of the modern ear. The case most frequently noted is the formula c-d-a-bb-a found at the beginning of many Introits and Antiphons of the first mode. According to the best manuscripts this should read: c-d-a-b-a. The faulty version of the standard Solesmes book has indeed been corrected in more recent publications based on more exacting principles of research and scholarship. [...] In these books the Antiphons *Traditor autem* [...] and others appear without a flat. Similar corrections should be made for many Introits, e.g., *Rorate* [...], not only at the beginning but also at other places where the *Liber usualis* has a b-flat.⁷⁸⁹

This observation concerns the opening formula for Gevaert's Theme 5, or Frere's first mode theme group (b). The opening formula is typically given as in Figure 43.

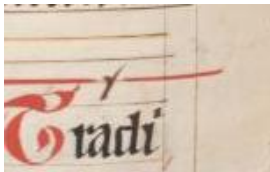


Figure 43 - Intonation formula for Gevaert's Theme 5 or Frere's Mode I theme group (b), as typically given. The notes before the dashed barline are not present in every chant in this group, and the first pitch differs among manuscripts, ranging from F to a.

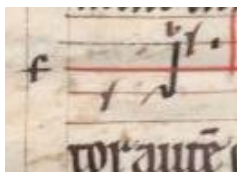
⁷⁸⁹ Apel, *Gregorian Chant*, 153–54. The article cited by Apel is: Gajard, Joseph, "Du rôle des principales familles de manuscrits dans la restauration grégorienne," *Revue grégorienne* Vol. 30/1, 1951, 1–11.

Apel argues that the \flat given by Gevaert, Frere, and Solesmes is erroneous, as “the best manuscripts” (he does not specify how these “best manuscripts” are to be identified) provide \natural instead; he suggests, in accordance with Gajard, that the tendency to supply \flat in modern editions is the result of modern listening habits. Presumably, he means the projection of the interval structure of the modern minor scale upon the *protus* modes. However, it is not only modern editors that have given the theme with the \flat . Of the fifteen melodies or incipits found on the *Cantus* database for *Traditor autem*,⁷⁹⁰ only seven (just under half) give the melody with the \flat ; four more give the note as a c (placing it more properly in Gevaert’s Theme 4, and avoiding the \flat or \natural issue entirely). But three manuscripts, dating from the thirteenth and fourteenth centuries, give the \flat very clearly and unambiguously (see Facsimile 19); similarly, the remaining manuscript transposes the melody up a fifth to end on a , placing the pitch in question on f (see Facsimile 20) thus resulting in the same interval structure created by the use of \flat in the untransposed manuscripts.

A.



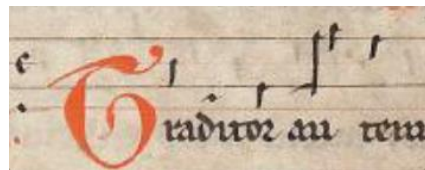
B.



C.



Facsimile 19 - (A) A-Wn 1799 f. 67v; (B) PL-WRu I F 401, f. 74r; (C) F-Pnm lat. 15181, f. 284v. The opening formula for *Traditor autem* in three different manuscripts, each clearly marking the highest pitch as b -flat.



Facsimile 20 - D-KNd 1161, f. 74r. The opening formula for *Traditor autem*, transposed to end on a , which is diastematically equivalent to the usual form on D with the use of b -flat.

⁷⁹⁰ <http://cantus.uwaterloo.ca/chant/544454> §List of Melodies, accessed 20 January 2020.

In fact, musicologists familiar with later medieval and Renaissance theoretical principles might intuitively expect the b , as it would seem to be mandated by the oft-repeated later maxim *una nota super la semper est canendum fa* (“a note above *la* is always to be sung as *fa*,” which is to say that an upper auxiliary note decorating the highest note of the current hexachord – in this case, the natural hexachord, whose highest note is *a* – is always to be sung as a semitone above that highest note – in this case, b). But, of course, the majority of the manuscripts (and perhaps, as Apel would have it, the best manuscripts) do not indicate b , and one even appears to go out of its way to clarify with the otherwise unnecessary step of indicating a square b (Facsimile 21).



Facsimile 21 – F-Pnm lat. 12044, f. 95r. The opening formula for *Traditor autem*, with the *b*-natural explicitly indicated by a square *b*.

In the case of *Rorate caeli*, there are only three melodies given on the CANTUS database,⁷⁹¹ but the three basic possibilities for the upper auxiliary (b , \flat , and c) are all represented.

The point of this discussion is not whether these two chants ought to use b or \flat . The *Alia musica* contains only ambiguous references to b . It is possible that the distinction between b and \flat may have some slight bearing on the identification of intervals within these chants (that is, b might not be an acceptable *emmelis* above the range of an interval ending at *a*, since there is some evidence that a semitone *emmelis* might be avoided), but this is not clear.

The point also is not whether Apel is correct in his judgment about b or \flat ; rather, the point is more generally that later precepts about modality and the structure of the gamut (such as the *nota super la*

⁷⁹¹ <http://cantusindex.org/id/501007>. §Melodies in Sources, accessed 21 January 2020.

rule) cannot be trusted to help explain the nature of the modes in the Carolingian era. Chronologically, the importation of the Byzantine *oktōēkhos* into the Latin west seems to have occurred in the very late eighth century, and while multiple diastematic notations were developed and used in theoretical treatises during the Carolingian period, none was ever used to transcribe any appreciable portion of the repertoire until the very late tenth century – a gap of two centuries. A similar length of time in later music history covers the entire Classical and Romantic periods and the first half of the twentieth century, spanning from Mozart through Stravinsky. One would hardly expect such a long period of time to pass without appreciable changes in musical thought, including concerning the nature of mode. Indeed, the treatises of the tenth century show these changes quite clearly; unfortunately, the treatises of the ninth century, though they often speak extensively about the modes, rarely provide the kinds of details or clarity that would allow a modern musicologist to understand what Carolingian modes were. Even the apparent exceptions, the treatises of the *Enchiriadis* group, contain their share of ambiguity – to say nothing of the idiosyncratic gamut that they employ and the many places where they directly contradict later notions of mode and gamut. The critical point, then, is that one must not assume that modality in the *Alia musica* is necessarily the same as in later treatises.

Unexpected Modal Assignments

Keith Falconer once observed:

In any attempt to construct a system out of human affairs, there will always be grey areas for things that do not quite fit – things that belong yet don't belong. And despite the traditional admonition that 'difficult cases make bad law', the exceptional cases will often figure disproportionately in defining the system's boundaries – occasionally even its central concerns. This is nowhere more true than of attempts to define tonal or modal systems for large bodies of music, such as what we know as Gregorian chant. For even the most comprehensive attempts to classify this music reveal numerous chants that fall both within and without the modes to which they are assigned.⁷⁹²

⁷⁹² Falconer, "The Modes Before the Modes," 132.

This observation describes a number of the chants cited in the *Alia musica* quite well, and the above caveat about anachronism should be borne in mind in consideration of this issue. For instance, the antiphon *Urbs fortitudinis nostrae Sion* is listed as a Mode I chant, but as noted above the analysis in the previous chapter, it is usually classified as a Mode VII chant, and it is one of a handful of chants in the *Alia* that share this characteristic. It is not very surprising that the treatises and tonaries do not all agree about the modal assignment of some chants; this phenomenon is well known. What is surprising is that the author of the source treatise should have felt that such chants, which presumably manifested some kind of modal ambiguity for them to have shifted modes over the years, somehow still manifested the modal characteristics strongly enough to serve as particularly good examples of their modes.

On the CANTUS database, there are more than eighty entries for *Urbs fortitudinis*;⁷⁹³ approximately seventy sources list it as a Mode VII chant, and only one lists it as Mode I, while it is unspecified or uncertain in the rest. However, it is listed as a Mode I chant belonging to the third differentia in the Metz Tonary,⁷⁹⁴ and is also listed in Mode I in Regino of Prüm's Tonary,⁷⁹⁵ both of which may be said to belong to the ninth century.⁷⁹⁶ It does not appear to be listed in any other ninth-century source.⁷⁹⁷ By contrast, a tonary attributed to Bern of Reichenau (whose career spanned the first half of the eleventh century)⁷⁹⁸ lists *Urbs fortitudinis* as a Mode VII chant.

Thus, it may be said that there is perfect agreement amongst all extant ninth-century sources that *Urbs fortitudinis* is a Mode I chant, and near perfect agreement amongst eleventh-century and later

⁷⁹³

http://cantus.uwaterloo.ca/search?op=starts&t=urbs%20fortitudinis&genre=All&cid=&mode=&feast=&volpiano=All&order=field_mode&sort=asc. Accessed 20 January 2020.

⁷⁹⁴ Metz 351, f. 69v.

⁷⁹⁵ Rausch, *Die Musiktraktate*, 201.

⁷⁹⁶ Regino died in 915 (See Chartier, "Regino of Prüm," *Grove Music Online*, §1); the tonary therefore cannot be later than the early tenth century.

⁷⁹⁷ It does appear, in Mode VII, in a tonary attributed to the ninth-century abbot Odo of Cluny (printed in Coussemaeker, *Scriptores de musica II*: 117–49), but this attribution is erroneous, and the tonary must be of a much later date (Huglo, *Les tonaires*, 184).

⁷⁹⁸ Gushee, "Bern of Reichenau", *Grove Music Online*, §1.

sources that it is a Mode VII chant. Should it be assumed that the chant as it was known in the ninth century was not the same melody as the chant known from the eleventh century onwards? This assumption is unlikely, in view of the remarkable stability of the repertoire overall. Perhaps, then, the two eras shared more or less the same melody, but some significant transformation occurred to the melody in the intervening century? This hypothesis is slightly more plausible but is again unlikely: such an occurrence would be expected to show more inconsistency in the sources of the latter era. One might also propose that both the *Alia musica* and Regino were simply perpetuating an error found in the Metz Tonary. This kind of deference to authority is not unexpected in medieval scholarship, but in this case, it does not seem to fit the circumstances: in the first place, medieval theorists occasionally do point out that other sources have misclassified chants; and in the second place, with all the many chants that the author of the source treatise could have selected for the two percent of the repertory that he would use as his examples, it seems unlikely that he would have selected a chant the classification of which did not clearly demonstrate the principles he held to be central to modality.

Instead, the most probable explanation for the sudden complete switch in modal assignment of *Urbs fortitudinis* is that something significant and perhaps fundamental changed in the way that the modes were understood, so that the very same melody displayed the attributes that ninth-century theorists held to be characteristics of the first mode but also those that later theorists held to be characteristics of the seventh mode. It is interesting to note at this point that *Urbs fortitudinis* is one of the very few chants listed in the source treatise that the revisor omits from the revision (and without comment),⁷⁹⁹ suggesting that the ambiguity may already have been causing difficulties at the time of the revision.

⁷⁹⁹ cf. *Alia musica* §35, ed. Chailley, 123. The other missing chant is *Homo quidam* (cf. *Alia musica* §60, ed. Chailley, 134), which probably reflects difficulties identifying the chant. There are two different chants based on the parable of the great feast (Luke 14). The first is called *Homo quidam*, but is a responsory; the second is an

There is precedent to support the hypothesis that the modal paradigm changed between the ninth century and the eleventh. Several of the treatises of the ninth century make statements about mode that are much clearer than those in the *Alia musica* and directly contradict the tenets of the later modal paradigm. For example, in later modal theory, the appropriate range of each mode is approximately one octave, with the authentic modes ranging above the final and the plagal modes centred on the final; but treatises in the *Enchiridis* group routinely state that the authentic modes can span not only their entire upper octave, but also the lower fourth normally associated only with the corresponding plagal mode.⁸⁰⁰ As another example, in later medieval theory, the *maneria* of a chant is determined solely by the final pitch in the chant, irrespective of other important pitches (including the way that the chant begins);⁸⁰¹ however, both Aurelianus and Regino state that this process is valid only for certain types of chants, such as responsories, while the mode should be determined by the beginning for antiphons (including introits and communions) – which agrees well with the fact that antiphons are grouped into families with shared intonation formulae. (This issue was addressed in detail in Chapter 06).

More specifically, Regino cites numerous chants that seem to begin in one mode and end in another.⁸⁰² This observation could be interpreted to mean that Regino felt that a chant ought to begin and end on the same pitch, but since most of the cited chants do begin and end on the same pitch, this interpretation is difficult to credit. Alternatively, Regino could be referring to an intervallic difference

antiphon, but the incipit is often reversed, as *Quidam homo* (the antiphon is actually found under both incipits, but the responsory is always given as *Homo quidam*). The revisor may have omitted this chant because of the ambiguity. It is also true that the revisor omits *Iohannes autem*, but replaces it with *Iesus autem*, which Chailley believes to be the result of a simple misreading (Chailley, *Alia musica*, 123–24).

⁸⁰⁰ *Musica enchiridis*, ch. 4, ed. Schmid, 8–9, trans. Erickson, 4–5.

⁸⁰¹ See, for example, Oddo of Arezzo, *Prooemium tonarii*, ed. Gerbert, 248–50: *Studiositatem autem quicumque habere uoluerit in cantu [...], quando uoluerit antiphonam in ecclesia incipere, non teneat introitum eius, sed ad finem quantocyus currat, et in cuius tono eam inuenerit, in ipsius sono incipiat psalmum.* (p. 45a–b) (“However, whoever would want to have studiousness in chant [...], when they would want to begin a chant in church, let them not hold to its introduction, but as quickly as possible run to the end, and in whichever tone they find it [the ending], let them begin the psalm in that sound.”)

⁸⁰² Regino, *De armonica institutione*, §2, ed. & trans. Protase, 24–27.

created by the use of \flat and \natural . For instance, in the case of Modes I and VII (the two modes to which *Urbs fortitudinis* are assigned), the interval structure of a Mode VII chant that consistently uses \flat is indistinguishable from Mode I (indeed, Apel notes that Mode VII chants rarely use \flat ,⁸⁰³ and it is probably for this reason); in any case, the first several chants that Regino cites are all examples of chants that “*a septimo tono incipiunt, et quaedam in primo, aliae in quarto finiuntur tono*”⁸⁰⁴ (“begin in the seventh mode, and some finish in the first and others in the fourth mode”), and the use of \flat would not explain these chants. It is interesting to note that Regino describes some of these chants as containing elements of both Modes VII and I, the same two modes ascribed to *Urbs fortitudinis*.

Perhaps, then, the observation that some chants begin in one mode and end in another means that during the ninth century there was some other meaningful characteristic that determined the mode, something that transcends questions of initial and final pitch, which became a less significant element of modal doctrine by the eleventh century, that may explain why *Urbs fortitudinis* was consistently classified as Mode I before Bern and as Mode VII afterward. If so, it is not unreasonable to suppose that it was precisely this principle that the author of the source treatise of the *Alia musica* was attempting to articulate.

Apel proposes a different hypothesis for the attribution of *Urbs fortitudinis*.⁸⁰⁵ He describes it as a Mode VII chant that uses \flat in the first half and \natural in the second half. It should be noted here that many of the manuscripts do not include \flat ; however, since the chant is consistently attributed to Mode VII in nearly all the sources containing diastematic notation, it seems more likely that the \flat was ironed out of some manuscripts to reflect the tendency of Mode VII not to have \flat than that \flat was added to a Mode VII chant in some manuscripts against the usual character of the mode. This reason supports the implication

⁸⁰³ Apel, *Gregorian Chant*, 157.

⁸⁰⁴ Regino, *De armonica institutione*, §2, ed. & trans. Protase, 25.

⁸⁰⁵ Apel, *Gregorian Chant*, 177–78.

already created by the presence of *b* in most sources that the use of *b* in this chant is authentic, however unusual it may be in this Mode VII (where Apel puts it).

Apel notes that the *Alia musica* identifies *Urbs fortitudinis* as a chant that begins in the first mode (consistent with the use of *b* in the first half), but ends not in the seventh mode, but in the eighth. This statement does not occur in either the source treatise or the passages confidently ascribed to the revision, but in the Disputed Passage about octave species – not, however, in the part of the passage shared with the *Dulce ingenium*, but in the commentary that follows it, which should probably be attributed to the commentator, and is therefore a relatively late addition. Furthermore, this description has an unconventional relationship to traditional modal theory; the second half of the chant also spans a full octave, but the eighth mode is plagal, and so the chant ought to end in the middle of that range (even according to the *Alia*; although the *Alia* is usually vague about *finalis*, this passage is connected to the only real discussion of *finalis* in the treatise). Instead, it ends at the bottom, on G, like an authentic mode. Chailley explains that it constitutes Mode VIII because the note *c* plays a prominent role in the end of the chant.⁸⁰⁶ This note actually appears to be prominent only at the cadence. But even if it were to be more prominent throughout, the interval structure would be problematic, as the eighth mode should have a whole-tone below the inflection point (the *finalis*) – which is true of the eighth mode in its usual position (D–d with its *finalis* on G), but when transposed up to span G–g to reflect the chant’s actual position as usually described (putting the inflection point on *c*),⁸⁰⁷ it could only retain that subtone with the use of *b*, which explicitly does not occur in the second half of the chant (the only part said to be in this mode).

Instead, Apel suggests that *Urbs fortitudinis* once started on D and ended on G for the end of most repetitions (at a time when the antiphon was regularly alternated with every psalm verse), but had an

⁸⁰⁶ Chailley, *Alia musica*, 203.

⁸⁰⁷ Note that I transposed it down to D–d in my analyses in Chapter 17 to facilitate comparison to other Mode I chants.

alternate ending on D for the end of the final repetition of the antiphon, so that it would end where it began. However, since the intervallic structure required the use of F-sharp, it was eventually transposed up to G, where the distinction between F and F-sharp transposes to become b and \flat .⁸⁰⁸

Apel cites precedent in Aurelianus supporting the general principle, if not the specifics; thus, this explanation is plausible (even compelling). However, with no extant manuscripts showing (or at least describing) the alternate version, this hypothesis is impossible to verify. Moreover, it is worth noting that the cadence in the surviving form of *Urbs fortitudinis* does not resemble Apel's example, *Puer Iesus*. In that example, the standard form of the chant ends where it begins, but with an idiosyncratic cadence from the lower fourth. Even Apel notes that such a cadence is very unusual in the Gregorian repertory. The hypothesis suggests that *Puer Iesus* once had a cadence that did not end on the *finalis* corresponding to the chant's intonation and theme group but that otherwise had a typical cadential melodic profile; because the cadence did not end on the *finalis*, an alternate cadence was constructed to end on the *finalis*, initially only for the final statement of the antiphon, but it eventually became the only cadence in use, and the change to the cadence was minimal, so that the resulting cadence becomes melodically unusual compared to most cadences in the Gregorian repertoire.

But the cadence of *Urbs fortitudinis* seems perfectly normal, and it is not obvious how a typical Mode VII cadence a fourth higher might once have existed without the material leading up to the cadence having been changed much more dramatically than was the case for *Puer Iesus*, which decreases the likelihood that *Urbs fortitudinis* was the result of this kind of procedure; essentially, to propose this procedure for *Urbs fortitudinis* creates the reverse situation from *Puer Iesus*: instead of proposing that

⁸⁰⁸ Apel, *Gregorian Chant*, 177. In the written feedback following the defence of this dissertation, Atkinson suggested to me that Apel's interpretation assumes a fixed matrix of pitches, which may not be a justified assumption for such an early antiphon; thus, he proposes that the change in modal assignment result from the change in paradigm from modal assignment according to locum to modal assignment according to range, final, and octave species.

the original cadence was typical and that the change created an idiosyncratic cadence, one must instead propose that the original cadence was idiosyncratic, and that the change created a typical cadence (or that a considerably longer segment of the chant was changed). It is also worth noting that *Urbs fortitudinis* is still listed as an exclusively Mode I chant in Regino's tonary,⁸⁰⁹ while it is entirely absent from the list of chants beginning in one mode and ending in another in Regino's treatise or in the similar discussion in Aurelianus upon which Apel bases his hypothesis.⁸¹⁰

In any case, such an explanation is not necessarily inconsistent with the idea that repeated musical formulae formed an important part of modal identity (and thus, contributed towards different modal attributions than would be expected from later modal paradigms), since the octave species can often be derived from the interval structure of the melodic formulae; when the interval structure changed in the second half of the chant, the melodic formulae would also change

The situation for *Salve crux* is more complicated but perhaps similar. The *Alia musica* gives only the partial incipit *Salve crux* and calls the chant an antiphon. This identification could refer either to *Salve crux pretiosa suscipe discipulum*, a Mode VII antiphon, or *Salve crux quae in corpore*, a Mode VIII antiphon. In either case, the *Alia* identifies it as a Mode V antiphon. Because both Mode V and Mode VII are authentic modes, while Mode VIII is plagal, it is somewhat more likely that the intended chant is the authentic chant *Salve crux pretiosa*, especially in view of the fact that *Salve crux quae in corpore* makes extensive use of the lower range; however, since the Carolingian rules regarding the acceptable range of an authentic chant (as given in the *Enchirides*) included the lower range, this attribution is by no means definitive.

⁸⁰⁹ Rausch, *Die Musiktraktate*, 201.

⁸¹⁰ Apel, *Gregorian Chant*, 175–78; cf. Regino, *De armonica institutione*, §2, ed. & trans. Protase, 24–27; Aurelianus, *Musica disciplina*, ch. 15, ed. Gushee, 105–06, trans. Ponte, 38.

Salve crux pretiosa resembles *Urbs fortitudinis* in the sense that the use of *b* can cause ambiguity about modal assignment, in this case between Modes V and VII. The status of *b* in Mode V is effectively the reverse of Mode VII; Apel observes that Mode VII is “virtually untouched by the *b*-flat,”⁸¹¹ and also that in Mode V, “not a few melodies employ the flat throughout.”⁸¹² Since the melody of *Salve crux pretiosa* in most manuscripts spans only the first six pitches above the final (*i.e.*, G – e, assuming a Mode VII attribution, as is typical), precisely the same interval structure can be achieved in Mode V with the consistent use of *b* – a situation that, as established above, is not entirely unexpected.

And in fact, one of the odd statements made elsewhere by the author of the source treatise may perhaps be interpreted to be making the same point. In his description of Mode VII, the author provides three different series of modal numbers to produce all three characteristic intervals. This process is odd, because Mode I also has the same three intervals, but achieves them using only a single series. In Mode VII, the author says,

*Hic tonus in proportione quarti toni habet totum diatessaron, et in proportione quinti toni habet totum diapente, et in propria proportione habet duodecies 12 diapason, quod est 144.*⁸¹³

This tone has a perfect fourth in the proportion of the fourth tone, and a perfect fifth in the proportion of the fifth tone, and in its own proportion it has the octave twelve 12s, which is 144.

This statement seems to mean that the seventh tone has the characteristic interval of an octave in its own right but also shares the fourth that is characteristic of the fourth mode and the fifth that is characteristic of the fifth mode (one should not read any kind of wordplay into this description; in the original Latin, the words for the interval of a fourth and the fourth mode are completely different, and likewise for the fifth). This statement may be an indication that the fifth present in the fifth mode and the one in the seventh mode are related to each other in a more meaningful way than simply being the same

⁸¹¹ Apel, *Gregorian Chant*, 157.

⁸¹² Apel, *Gregorian Chant*, 156.

⁸¹³ *Alia musica* §187(e), ed. Chailley, 93–94, trans. Nowacki 126.

size. The analyses seem to bear this out, as the fifths in both modes appear to be rooted on the *finalis*, and the chants cited for Mode V do make frequent use of the b. Not all of the chants cited for Mode V in the *Alia* use the b, but *Salve crux* is a good example of one that does. (However, the same argument for the fourths is not so well supported by the analyses in the last chapter.)

The explanation of the unusual modal assignment of *Salve crux* is, however, less clear than for *Urbs fortitudinis*. Unlike *Urbs fortitudinis*, it cannot be said to have a consistent classification in early sources, though it cannot be said to be inconsistent either: it does not appear in any other early source besides the *Alia musica*. Regardless, since the author of the source treatise is generally quite accurate in his attribution of modes, it is not unreasonable to suppose that the attribution of *Salve crux* to Mode V instead of Mode VII is not an error, but results from placing higher priority upon some other characteristic of modality.

The Problem of the *Emmelis*

(1) Upper or lower *emmelis*?

The *emmelis* concept creates several problems for analysis. If one were to suppose that the characteristic interval of a chant is supposed to be an octave, one could use the *emmelis* to expand that interval range to reach a ninth or a tenth. A tenth presents no problem: the octave must be in the centre, with one additional tone added on each end.

But an *incisio* apparently spanning a ninth presents a problem. If the actual range is a ninth, the interval should be an octave with one *emmelis*, and it could be that the octave is at the bottom, with an additional tone at the top, or it could be that the octave is at the top, with the additional tone at the bottom. This issue arises already with the paradigmatic example of *Rorate caeli*, in which the first *incisio* is clearly identified as spanning an octave, but the absolute range is C–d, which could be interpreted to

be either the octaves C–c or D–d. While intuition might suggest that the octave should originate at the *finalis*, the *Alia musica* does not actually assert this as a general principle, and since the source treatise has nothing to say about *finales* at all, there is little justification for this assumption. Furthermore, on the basis of the chants cited for Mode II (which are said to be characterized by the fourth and fifth), it seems clear that the subtone in this mode is part of the characteristic interval itself, not an *emmelis*. This observation is supported by the fact that not one of the chants cited in Mode II ever reaches up to the a (which would be required to fill out a fifth above the *finalis*), and all of them make frequent, not merely occasional, use of the subtone. In addition, the two introits cited use the low A, itself functioning as a kind of *emmelis* (which, in this case, stands more than a whole tone beneath the C that marks the bottom of the range, but this is expected in order to avoid the \flat or \natural issue).

The analysis of Mode III chants is particularly difficult in this regard, because Mode III chants are supposed to span the perfect fifth (in addition to an octave), and the fifth that seems to appear most frequently is the fifth F–c. It is quite common in most of the chants cited for the absolute range of these *incisiones* (and in the case of *Dispersit dedit*, the entire chant) to be the minor sixth E–c. In this case, the fifth could hypothetically be either E– \flat with an upper *emmelis* on c or F–c with a lower *emmelis* on E. Both possibilities involve a semitone *emmelis*, a situation that most of the other modes seem to avoid. The latter possibility is counterintuitive because the *emmelis* would then also be the *finalis*. However, the former possibility is also problematic in context because the pitch E is actually relatively uncommon in these *incisiones*, despite being the *finalis*, while the c is quite common and is even routinely emphasized by (sometimes extensive) repercussion. Of course, the simplest solution is that the correct range of the *incisio* is the minor sixth E–c, but this interval is not sanctioned by the *Alia*, which discusses only perfect consonances.

(2) The *emmelis* and intervallic ambiguity

A related issue is that it is not always clear what makes an *emmelis* different from any other pitch, other than being outside the sanctioned range. This problem tends not to affect octaves much because octaves are considerably larger than the next largest perfect consonance, the fifth. However, it becomes a considerable problem when differentiating a fifth from a fourth. Any *incisio* that spans exactly a fifth might hypothetically also be described as spanning a fourth with an *emmelis*. In fact, in an extreme case, an *incisio* that spans a sixth, which would most intuitively be described as spanning a fifth with a single *emmelis* might also be described as spanning a fourth with both an upper and lower *emmelis*. I have taken the approach, throughout my analyses, that an *emmelis* should be used in an *incisio* less frequently than the pitches that properly belong to that *incisio*, that it should not be emphasized by repercussion, and that neighbour motion is particularly well suited to being described as an *emmelis*, but these principles are based entirely upon my own intuition regarding the implications of an extraneous pitch that is permitted because it is “suited to the melody.” The *Alia* never provides explicit justification for this interpretation (in fact, the source treatise never describes the *emmelis* at all; it is an addition by the revisor to explain the extraneous notes in *Rorate caeli* that are inconsistent with the source treatise’s explanation).

In its most intuitive incarnation, this problem would be expected to arise in modes in which both the fourth and the fifth are characteristic intervals – specifically, in Modes I, II, VII, and VIII. However, the problem potentially arises in every single mode if one is attempting to verify that the modes actually do favour the specified intervals, rather than taking those intervals for granted and attempting to find them where they occur. In fact, if the intervals are not attached to specific pitches and it does not matter which mathematical procedures are employed in generating them, then the entire distinction between modes can be reduced to two principles. In the first place, some modes have an octave and some do not. But no mode is characterized exclusively by the octave, and even in those modes that have octaves, the

octaves are less common than the other intervals. Thus, the second and principal distinction between modes in such a simplified model would simply be whether the mode is characterized by the fourth or the fifth or both. Because of the considerable overlap between *incisiones* that could be considered to correspond to the fourth and those that could correspond to the fifth, it would be possible to select chants that would demonstrate precisely the reverse of the associations identified in the *Alia*.

As a general example, the fourth mode is supposed to correspond only to the perfect fourth. My analyses respect that correspondence, showing a considerable preference for the fourth D–G, but nearly every *incisio* for each Mode IV chant actually spans the fifth D–a. I have interpreted the a as an *emmelis*, not necessarily because the a behaves in the manner that I would expect from an *emmelis* (though it does in most cases) but because the fourth, and not the fifth, is the interval that *Alia* stipulates that one should expect in this mode. The problem with this approach, of course, is that it represents a confirmation bias: one not only favours, but in fact generates evidence that supports the model that one is trying to test (that the characteristic intervals listed in the *Alia* are actually present in the chants cited) while indirectly destroying evidence that would refute the model. In response to this problem, I have also tried to favour specific intervals, not just in terms of size, but also in terms of position within the gamut, that seem to recur frequently throughout the cited modes, but this, too, presents a confirmation bias in at least two ways: first, it favours the intervals that the analyst encounters first, because each new *incisio* is necessarily interpreted in light of what has already been discovered in previous *incisiones*; and second, this approach may reveal characteristics that are common to these chants but are not necessarily the characteristics that the author of the source treatise intended the reader to recognize in these chants.

A related problem that I have mostly avoided but that must at least be considered is the possibility that some *incisiones* would be best described as spanning an incomplete interval. For example, suppose that the absolute range of an *incisio* is a seventh. In this case, the perfect consonance would presumably be a fifth, with both an upper and lower *emmelis*. However, is this analysis still reasonable if the outside

notes, classified here as *emmeles*, seem to be important pitches, rather than incidental? Or would it perhaps be more appropriate to analyze such and *incisio* as an octave, but one that does not quite reach the full octave? There is no justification for such an analysis in the *Alia*, but as noted, even the concept of the *emmelis* was not described in the source treatise and appears to be a requirement for correctly understanding the analysis of *Rorate caeli*. It is not very different to propose that if an *incisio* is sometimes one tone too large, perhaps it is also sometimes one tone too small.

Summary

With the completion both of the analyses and of the discussion of the problems inherent in the analytical methodology, it is possible to summarize the relationship between the doctrines of the *Alia musica* (especially the source treatise) and the chants cited to exemplify these doctrines. It is possible to make the following observations: first, the intervals described in the source treatise do seem to correspond to the melodies of the cited chants in a general way. However, these intervals are not as prevalent as they appear to be because the doctrine of the *emmelis* creates a system-breaking level of flexibility to analyze *incisiones* as consistent with the theory when they do not clearly reflect the correct interval. This bias is also increased by the lack of a truly clear and systematic procedure for segmenting a chant into *incisiones* in the first place, as described in Chapter 15. And even with all this enormous flexibility biasing the results in favour of consistency with the specified intervals, there seems to be no consistent way to associate either the base numbers involved in the creation of modal products or the modal products themselves directly with specific pitches – not across all eight modes; not across a single *maneria*; not within a single mode; and not even within a single chant. Moreover, there are examples of chants cited in the *Alia* that are presumably selected for containing particularly good examples of the intervals that define each mode, and yet they are assigned to different modes in other (and especially later) sources. The next chapter will present an hypothesis of the source of these modal numbers and the

mathematical procedures that create them, an hypothesis that discards the assumptions violated by the preceding observations.

Chapter 19: An Hypothesis for the Modal Numbers

Over the course of this dissertation, I have challenged the hypotheses forwarded by the various scholars who have attempted to explain the numerology of the *Alia musica*. The particular problems with each hypothesis vary, but the fundamental problem with all of them is that they are trying to find a systematic explanation for a set of doctrines that are ultimately not systematic.

The *Alia musica* presents a theory in which entire chants should give particular prominence to the intervals associated with their modes. Even without the evidence presented here, a moment's careful reflection on the basic premise should be enough to recognize the fundamental implausibility of that idea. It is difficult enough to imagine a system of music in which composers willingly constrain themselves to such principles once they already exist. One might propose that the various characteristics of each mode were so strong that composers would fall into such patterns instinctively, rather than intentionally, but this proposal conflicts with the known ambiguity of some chants that leads them to be attributed to different modes in different sources. But accepting the premise that chants conform to the intervals associated with their modes becomes even more difficult in view of the fact that the repertoire existed long before it was sorted into eight modes. Ultimately, it is likely that the authors of the *Alia* would have recognized and agreed with the observation that not all chants conform to the intervals identified in the treatise, but they seem to have believed that the conformance was greater than it actually is.

The human mind is, among other things, a pattern detection device. One need only look at the sky to see images in the clouds that are not really there except in the mind of the beholder. Even where I have found fault with the *Alia's* methodology and rejected some of its conclusions, I have acknowledged that there does seem to be a certain amount of truth to its observations. The chant repertoire does seem to contain many formulaic elements (as described by Gevaert, Feretti, and Frere), even if it is a

considerable overstatement to call the chants fundamentally formulaic. These formulae do tend to be associated with specific modes, and the intervals that circumscribe these formulae would tend to become particularly well represented in the chants of their respective modes.

I propose that the origin of the complex doctrines of the *Alia musica* lies in the recognition of a partial pattern of this nature – an image in the clouds. It does not necessarily follow that the founder of the doctrine recognized the recurring intervals to be the result of recurring melodic formulae, or even that all of the examples of a given interval that collectively gave rise to the impression of consistency necessarily came from such formulae. It is necessary only that the founder noticed a prevalence of certain intervals in certain modes and began to attempt to confirm the hypothesis that all the chants in a given mode should display the same intervals. However, once primed with a set of intervals that he expected to find, a confirmation bias allowed him to find these intervals in places where he otherwise might not have, such as by segmenting chants in ways that are more convenient than systematic. In this hypothesis, the *emmelis* is easily explained as a simple contrivance to save the system from the myriad counterexamples that do not clearly emphasize the expected intervals. The *emmelis* has the additional side effect of ensuring that any *incisio* with a range of at least a perfect fourth can be described in terms of one of the perfect consonances, even if the absolute range does not directly correspond to one. The importance of these perfect consonances in speculative music theory from the Greeks right through the *Alia* (and beyond) might well have led the founder to believe that he was coming to understand a deeper, even metaphysical truth about the modes, and thus further increase the confirmation bias.

This hypothesis, however, does not distinguish my position from those of the other scholars who have studied the *Alia musica*. The observations that the system of the *Alia* might not be entirely consistent with the repertoire and that the interval associations for each mode probably gave rise to the number system rather than the other way around are hardly inconsistent with Heard's or Chailley's explanations of the number system itself. However, in their models, the numbers always fall into specific one-octave

ranges, and by extension, each number represents a specific pitch, notwithstanding inconsistencies even in the paradigmatic example of *Rorate caeli*. It is understandable that one should wish to interpret the *Alia musica* in this manner because it is the approach of every other treatise that applies harmonics to musical practice. But since the evidence suggests that the numbers probably do not correspond to specific pitches, it is necessary to propose a model that would explain the mathematical procedures that generate these products.

The answer appears to be found in the tension between two conflicting desires. The author wishes to be able to incorporate all four base numbers (12:8:9:6) into the definition of each mode; these numbers had been understood to be important in music theory for centuries, especially as the source of the perfect consonances (which, of course, are central to the definitions of modes in the *Alia*), and their use would lend considerable authority to the system. However, the use of all four numbers necessarily implies all three perfect consonances, and most of the modes should be associated with only one or two consonances. Thus, the mathematical procedures applied to the modal numbers are, at minimum, a kind of filtering algorithm, carefully designed to keep all four base numbers in the definition of each mode in some manner while removing the undesired intervals.

A simplified chart of the intervals that ought to be present in each mode is given in Table 48.

Table 48 – Intervals associated with modes in the *Alia Musica*

	Protus	Deuterus	Tritus	Tetrardus
Authentic	Octave Fifth Fourth	Octave Fifth	Fifth	Fourth alt: Fifth alt: (Octave)
Plagal	Fifth Fourth	Fourth	Fifth (Octave)	Fourth alt: Fifth

As can be seen in Table 48, there is no fourth in the *tritus* modes, or in *deuterus* authentic (contrary to Aurelianus), or in the alternate presentations of the *tetrardus* modes. To create this result, it is necessary to select coefficients so that no two products end up in the ratio 4:3. Since the base numbers come from the relation 12:9:8:6, which contains two instances of 4:3 (12:9 and 8:6), it is necessary that the two terms of each of these ratios must be multiplied by a different coefficient in order to break up the 4:3 ratio. However, these modes must still contain a fifth. The relation 12:9:8:6 contains two instances of the fifth [3:2] (which are 12:8 and 9:6); it is necessary that the two terms of each ratio must be multiplied by the same coefficient in order to preserve the fifths. By combining these two ideas, it can be seen that fourths can be filtered out and fifths preserved by ensuring that twelve and eight share one coefficient and that nine and six share another coefficient; this is the procedure followed in Mode V and the alternate presentations of Modes VII and VIII.

There is an additional condition that must be met here: one must not select the coefficients for each pair in such a way that it would reintroduce the very interval that one has been attempting to filter out. As a result, certain proportions between the two coefficients must be avoided. It is not complicated, mathematically, to determine the relationships between coefficients that would result in reintroducing the undesired interval of a fourth. Since there are four base numbers, there are $\binom{4}{2} = 6$ unique combinations of two base numbers that could be multiplied in such a way as to create the ratio 4:3. Four of these could create this ratio in two different manners, as each base number could come to represent either the four or the three, and therefore would need to be tested twice, once in each direction; however, two pairs of these comparisons can be ignored because they are being deliberately set at 1:1 (to preserve the perfect fifths). Additionally, the two combinations that are already in the ratio 4:3 need only be tested in the opposite direction since the procedure is already designed to break the relationship in one direction (the original relationship).

Let the coefficients by which 12, 9, 8, and 6 will be multiplied be represented by the variables w , x , y and z , respectively. If the base numbers are set as coefficients to these variables, placed in fractions, and set to equal the ratio 4:3 that is to be filtered out (also in the form of a fraction), solving the equations will give the ratios of coefficients that must be avoided (in lowest terms). Since 12:8 and 9:6 are being deliberately multiplied by the same coefficient (*i.e.*, $w : y = 1 : 1, x : z = 1 : 1$), there is no need to test these combinations. Since 12:9 and 8:6 already create the fourth when multiplied by the same coefficient and the procedure is deliberately breaking that relationship, these two ratios only need to be tested in the opposite direction. The other two combinations must be tested in both directions. See Table 49.

Table 49 – Calculating coefficients that would reintroduce fourths after they have been filtered out. Highlighted fractions indicate ratios of coefficients within the range employed by the *Alia musica*.

$\frac{8y}{9x} = \frac{3}{4}$	$\frac{8y}{9x} = \frac{4}{3}$
$\frac{y}{x} = \frac{9 \times 3}{8 \times 4}$	$\frac{y}{x} = \frac{9 \times 4}{8 \times 3}$
$\frac{y}{x} = \frac{27}{32}$	$\frac{y}{x} = \frac{36}{24} = \frac{3}{2}$

$\frac{12w}{6z} = \frac{4}{3}$	$\frac{12w}{6z} = \frac{3}{4}$
$\frac{w}{z} = \frac{6 \times 4}{12 \times 3}$	$\frac{w}{z} = \frac{6 \times 3}{12 \times 4}$
$\frac{w}{z} = \frac{24}{36} = \frac{2}{3}$	$\frac{w}{z} = \frac{18}{48} = \frac{3}{8}$

$\frac{8y}{6z} = \frac{3}{4}$
$\frac{y}{z} = \frac{6 \times 3}{8 \times 4}$
$\frac{y}{z} = \frac{18}{32} = \frac{9}{16}$

$\frac{12w}{9x} = \frac{3}{4}$
$\frac{w}{x} = \frac{9 \times 3}{12 \times 4}$
$\frac{w}{x} = \frac{27}{48}$

Finally, since w and y must share the same coefficient (according to the conditions set above), as must x and z , all these proportions are interchangeable with one another (*i.e.*, $\frac{w}{z} = \frac{w}{x} = \frac{y}{x} = \frac{y}{z}$). Thus, if one ignores the improbably large coefficients (the *Alia* uses coefficients no larger than eight), a perfect

fourth would be reintroduced if the two different coefficients were to be in the ratio of two to three (in either direction) or three (for twelve and eight) to eight (for nine and six).

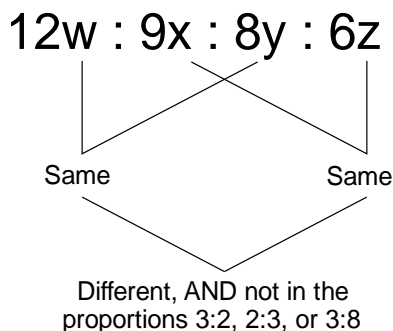


Figure 44 – Coefficients for filtering out fourths while retaining fifths

The opposite situation applies to the plagal *deuterus* and to the first presentation of the *tetrardus* modes. These have fourths, but no fifths. In order to achieve this circumstance, twelve and nine must share a coefficient, while six and eight must share a different coefficient. The same procedure may be followed. It is unnecessary to test the combinations 12:9 or 8:6, since they are being deliberately set to the same coefficient. Similarly, the combinations 12:8 and 9:6 need only be tested once, because their basic position (with coefficients 1:1) is already a fifth, and this fifth is being deliberately broken, so that only the opposite direction need be tested.

Table 50 – Calculating coefficients that would reintroduce fifths after they have been filtered out. Highlighted fractions indicate ratios of coefficients within the range employed by the *Alia musica*.

$\frac{12w}{6z} = \frac{2}{3}$	$\frac{12w}{6z} = \frac{3}{2}$
$\frac{w}{z} = \frac{6 \times 2}{12 \times 3}$	$\frac{w}{z} = \frac{6 \times 3}{12 \times 2}$
$\frac{w}{z} = \frac{12}{36} = \frac{1}{3}$	$\frac{w}{z} = \frac{18}{24} = \frac{3}{4}$

$\frac{9x}{8y} = \frac{2}{3}$	$\frac{9x}{8y} = \frac{3}{2}$
$\frac{x}{y} = \frac{8 \times 2}{9 \times 3}$	$\frac{x}{y} = \frac{8 \times 3}{9 \times 2}$
$\frac{x}{y} = \frac{16}{27}$	$\frac{x}{y} = \frac{24}{18} = \frac{4}{3}$

$$\frac{12w}{8y} = \frac{2}{3}$$

$$\frac{w}{y} = \frac{8 \times 2}{12 \times 3}$$

$$\frac{w}{y} = \frac{16}{36} = \frac{4}{9}$$

$$\frac{9x}{6z} = \frac{2}{3}$$

$$\frac{x}{z} = \frac{6 \times 2}{9 \times 3}$$

$$\frac{x}{z} = \frac{12}{27}$$

Once again, all the proportions within these tables are interchangeable. These coefficients must therefore not be in the proportion four to three (in either direction) or one (for twelve and nine) to three (for eight and six).

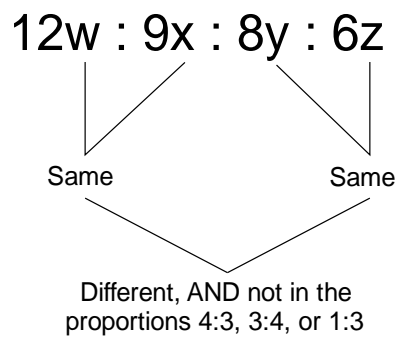


Figure 45 – Coefficients for filtering out fifths while retaining fourths

There is an additional side effect to both of these procedures. Whether one is filtering out fourths or fifths, the coefficients for twelve and six will be different, which will also filter out the perfect octave. In most cases, this is a desirable side effect, since several modes have only a single interval and the octave needs to be filtered out along with the fourth or fifth; in particular, this need arises for Modes IV and V, but the impact is more widespread because these are precisely the two modes whose proportions are borrowed in Modes VII and VIII, so that four of the eight modes will require these procedures. In these cases, there is also a need to avoid reintroducing the octave, which means further restrictions on the possible ratios between coefficients.

Table 51 – Calculating coefficients that would reintroduce octaves after fourths and fifths have been filtered out. Highlighted fractions indicate ratios of coefficients within the range employed by the *Alia musica*.

Filter out fourths $w = y, x = z$

$$\frac{8y}{9x} = \frac{2}{1} \quad \frac{8y}{9x} = \frac{1}{2}$$

$$\frac{y}{x} = \frac{9 \times 2}{8 \times 1} \quad \frac{y}{x} = \frac{9 \times 1}{8 \times 2}$$

$$\frac{y}{x} = \frac{18}{8} = \frac{9}{4} \quad \frac{y}{x} = \frac{9}{16}$$

$$\frac{12w}{6z} = \frac{1}{2}$$

$$\frac{w}{z} = \frac{6 \times 1}{12 \times 2}$$

$$\frac{w}{z} = \frac{6}{24} = \frac{1}{4}$$

$$\frac{8y}{6z} = \frac{2}{1} \quad \frac{8y}{6z} = \frac{1}{2}$$

$$\frac{y}{z} = \frac{6 \times 2}{8 \times 1} \quad \frac{y}{z} = \frac{6 \times 1}{8 \times 2}$$

$$\frac{y}{z} = \frac{12}{8} = \frac{3}{2} \quad \frac{y}{z} = \frac{6}{16} = \frac{3}{8}$$

$$\frac{12w}{9x} = \frac{2}{1} \quad \frac{12w}{9x} = \frac{1}{2}$$

$$\frac{w}{x} = \frac{9 \times 2}{12 \times 1} \quad \frac{w}{x} = \frac{9 \times 1}{12 \times 2}$$

$$\frac{w}{x} = \frac{18}{12} = \frac{3}{2} \quad \frac{w}{x} = \frac{9}{24} = \frac{3}{8}$$

Filter out fifths $w = x, y = z$

$$\frac{12w}{6z} = \frac{1}{2}$$

$$\frac{w}{z} = \frac{6 \times 1}{12 \times 2}$$

$$\frac{w}{z} = \frac{6}{24} = \frac{1}{4}$$

$$\frac{9x}{8y} = \frac{2}{1} \quad \frac{9x}{8y} = \frac{1}{2}$$

$$\frac{x}{y} = \frac{8 \times 2}{9 \times 1} \quad \frac{x}{y} = \frac{8 \times 1}{9 \times 2}$$

$$\frac{x}{y} = \frac{16}{9} \quad \frac{x}{y} = \frac{8}{18} = \frac{4}{9}$$

$$\frac{12w}{8y} = \frac{2}{1} \quad \frac{12w}{8y} = \frac{1}{2}$$

$$\frac{w}{y} = \frac{8 \times 2}{12 \times 1} \quad \frac{w}{y} = \frac{8 \times 1}{12 \times 2}$$

$$\frac{w}{y} = \frac{16}{12} = \frac{4}{3} \quad \frac{w}{y} = \frac{8}{24} = \frac{1}{3}$$

$$\frac{9x}{6z} = \frac{2}{1} \quad \frac{9x}{6z} = \frac{1}{2}$$

$$\frac{x}{z} = \frac{6 \times 2}{9 \times 1} \quad \frac{x}{z} = \frac{6 \times 1}{9 \times 2}$$

$$\frac{x}{z} = \frac{12}{9} = \frac{4}{3} \quad \frac{x}{z} = \frac{6}{18} = \frac{1}{3}$$

Some of the ratios derived in Table 51 that would reintroduce octaves are the same ones that already must be avoided to prevent reintroducing fourths and fifths, but one is new: whether filtering out fourths or fifths, the coefficients for twelve and six, respectively, must not be in the ratio 1:4.

A noticeable pattern emerges from the restrictions on the coefficients: to remove fifths (3:2) while retaining fourths (4:3), the coefficients must themselves not be in a ratio that characterizes the fourth (4:3). To filter out fourths (4:3) while retaining fifths (3:2), the coefficients themselves must not be in a ratio that characterizes a fifth (3:2). In general, the coefficients should not be in the same proportion as the ratio that is retained. However, there is a considerable range of possibilities for coefficients that could have been chosen that would have fulfilled the needs of any given mode. For instance, in Mode V, 12 and 9 must have the same coefficient, as must 8 and 6, and these two coefficients must differ from each other. However, there is no inherent reason why the coefficients could not have been one and two (or vice versa). Instead, the proportion selected for these coefficients is 3:2, precisely the same proportion that, when applied directly to the modal products (rather than the coefficients), represents the very interval to be filtered out. This observation applies equally to both Mode IV and Mode V: each mode filters out the interval kept in the other mode, and each does so by using coefficients in the proportion corresponding to the interval of the other (Figure 46).

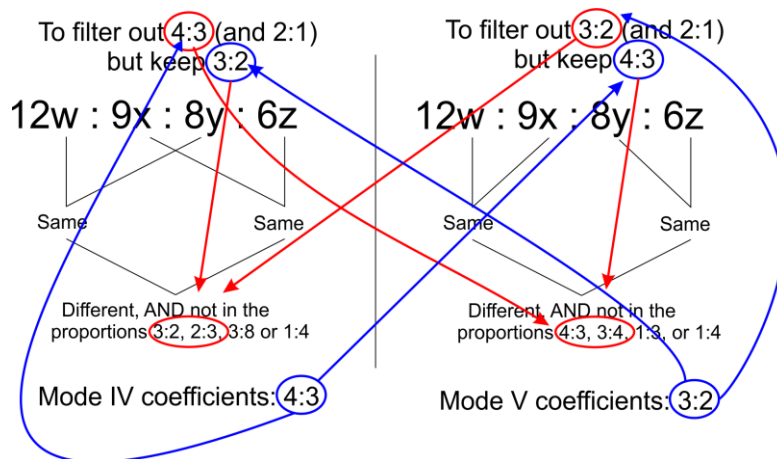


Figure 46 – Figure 44 and Figure 45, slightly expanded and marked up to show the relationships among coefficients, intervals filtered out, and intervals retained among Modes IV and V

It is, in part, for this reason that I use the term *numerology* to describe the mathematical procedures in the *Alia musica*. In the first place, there is no obvious reason why all four base numbers ought to be involved in the definition of a mode that is characterized by intervals that do not require all four numbers. These base numbers are then multiplied in a manner that has no direct interpretation as a specific pitch or as a process that one would apply to a particular pitch. The coefficients selected for this process of multiplication have no direct justification in terms of pitch representation but are selected simply because they successfully filter out the undesired intervals. Where more than one possible set of coefficients would serve the same ends, the *Alia* selects coefficients that relate in proportions that are musically significant in other contexts (*i.e.*, 4:3 and 3:2) but have no inherent meaning when used as coefficients (because the coefficients themselves have no inherent meaning).

This final point is the least well supported. It does agree with the coefficients selected for each mode that is characterized by only a single interval, but there is another way to explain the selection of coefficients in these cases. Although there are a number of pairs of coefficients that would successfully filter out the unwanted intervals (including the octave), the coefficients chosen have one other important virtue. The interval that is retained in each case is retained because the two base numbers that already form this interval before multiplication are multiplied by the same coefficient; these pairs may be referred to as “stable pairs,” because the interval between base numbers is the same as the interval between the products. There are two sets of these stable pairs in each case, and the coefficients chosen for each stable pair are such that they result in both stable pairs producing the same two products. In the case of Mode V, for instance, the stable pair 12:8 is multiplied by three to yield 36:24, and the stable pair 9:6 is multiplied by 4 to likewise yield 36:24, not some other multiple of 3:2. This process works because the ratio of the coefficients is the inverse of the comparison between the two stable pairs; the comparison of the larger number of the two stable pairs gives 4:3, as does the comparison of the smaller number of the two stable pairs, so that the relevant coefficients are in the reverse proportion, 3:4 (Figure 47).

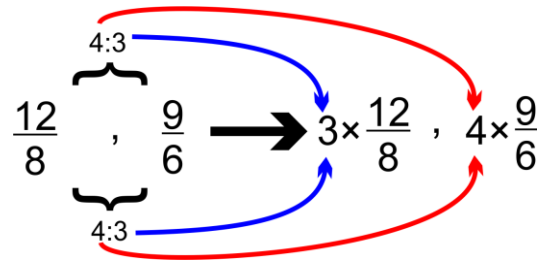


Figure 47 – An interpretation of the relationship between coefficients and base numbers in Mode V

This characteristic ensures that there are only two unique modal products in each of Modes IV and V (as well as the presentations of Modes VII and VIII that are borrowed from them), but there is no strict requirement according to the doctrines of the *Alia* that it should be so. If one were to suppose, for instance, that the coefficients for Mode V had instead been three and five, there would be four unique modal numbers: $12 \times 3 = 36$, $8 \times 3 = 24$, $9 \times 5 = 45$, $6 \times 5 = 30$. Two pairs, 36:24 and 45:30, would still relate in the proportion of a perfect fifth, while the remaining four pairs would not.

There are at least two reasons why these additional comparisons would not pose a problem according to the doctrines presented in the *Alia musica*. In the first place, none of the remaining possible comparisons result in a perfect consonance: 30:24 and 45:36 both correspond to 5:4 (a justly-tuned major third), 36:30 reduces to 6:5 (a justly-tuned minor third), and 45:24 reduces to 8:15 (a justly-tuned major seventh), none of which have any place in medieval harmonics; they can be easily ignored for presumably the same reason that the proportion 9:8 that is implied by the base numbers in Mode I is also ignored. Second, the *Alia* has clearly established that modal products that are derived from incommensurable coefficients are not to be compared to one another, and the coefficients 5:3 are certainly not commensurable with each other. (Coincidentally, the resulting modal numbers also all fit with the span 24–48 that both Heard and Chailley consider to be appropriate to this mode).

But although this proportion between coefficients would be possible under the rules of the *Alia*, it is nevertheless true that the coefficients actually used in the *Alia musica* bring about a symbolically

preferable condition, which is a more streamlined system with fewer unique modal numbers and no need to explain why some should be compared to each other and some not. Ultimately, there is no way to know which of the preceding explanations is correct (or if there is not, perhaps, another explanation altogether). Either way, it seems that the selection of coefficients, at least for Modes IV and V, is more symbolic than meaningful (though still consistent with the requisite intervals).

Explaining the Modal Numbers for the Remaining Modes

Mode II

The preceding precepts are not sufficient to explain all of the selected coefficients for all modes. Mode I, of course, has no coefficients at all, and all three perfect consonances are present; thus, there is little need to account for this mode – other than, perhaps, the ratio 9:8, which is probably ignored because it is not a perfect consonance (it is, interestingly, one of only two potential comparisons amongst all the modes that does not result in a perfect consonance, the other being the same proportion in Mode II that results from doubling both of these two base numbers).

The second mode is an excellent example of a mode with a more complex selection of coefficients. According to the *Alia*, the second mode is defined by both the fourth and the fifth (but not the octave, which differentiates it from Mode I). It is, therefore, necessary to filter out the octave without also filtering out either the fourths or fifths. All that is required, at minimum, to filter out the octave, is to multiply the base numbers six and twelve by two different coefficients other than in the proportion 4:1. However, if the twelve is doubled (the minimum change), it becomes widely separated from the other modal numbers unless they, too, are multiplied. If, instead, the six is multiplied, it cannot be doubled because it would then be equal to twelve, and since the coefficients (2:1) are commensurable, these two twelves could be compared against each other to create an octave (in the idiosyncratic usage of the *Alia*

where two equal numbers compared to each other create an octave). If six is instead multiplied by four or more, the same problem arises as when doubling twelve: the range becomes very wide. Thus, tripling six is the only reasonable possibility.

At that point, however, eighteen becomes comparable to nine, once again producing the octave, which needs to be filtered out. Therefore, nine is doubled to reach eighteen – but this eighteen is not comparable to the eighteen created from tripling six because the coefficients are in the proportion 3:2 and are not commensurable.

It remains only to explain why eight must also be multiplied, which is not clear. One possible explanation is that eight, unmultiplied, would then be too far separated from the eighteens; they would collectively span more than an octave, a relationship among modal products that does not arise in any other mode (except if the separate presentations in Modes VII and VIII are compared against one another, a process that is probably not intended).

It is interesting to note that as a result of this overall process, six and eight have different coefficients, as do nine and twelve, which would ordinarily filter out the fourths; however, twelve and eight do not share a coefficient (as they do in Mode V, which also filters out fourths), and the proportion between their coefficients results in the reintroduction of the fourth 16:12 between them.

Mode III

The third mode is also interesting. It is not difficult to see why Mode III could not follow the usual pattern followed by Modes IV and V. Mode III is characterized by a fifth and an octave, and so must filter out the fourths but not the octaves. As is easily verified by consulting Table 49 and the top half of Table 51 above, if one were to filter out the fourths and lose the octave with them, there are only a few possible ways to get the octave back, all of which would either also reintroduce the fourth or would use awkwardly

large coefficients (or both), with only one exception: quadrupling the six and leaving the twelve alone, which is precisely what the *Alia* does.

As in Mode II, the eight and nine probably cannot then be left alone, because they would be very widely separated from the twenty-four that results from quadrupling six, and they cannot simply both be doubled because they would collectively produce both a fourth and a fifth against the octave between twelve and twenty-four, and the fourth is not supposed to characterize this mode. Instead, as in Mode IV, these two coefficients are in the proportion 3:2, the ratio of a perfect fifth that is to be filtered out of a mode while retaining a fourth; this procedure is perhaps surprising, as it does not produce the desired outcome (the desired interval is a fifth, not a fourth). However, these coefficients interact with the unaltered twelve so that the coefficients of twelve and eight are set in the ratio 1:3; ordinarily, this is the proportion that must be avoided in order to not reintroduce a fifth that was previously filtered out, but in this case, the reintroduction of a fifth accomplishes precisely what is needed for this mode.

As a consequence of this overall process, all four coefficients for Mode III are different, resulting in one of the two fourths being filtered out; the two paradigmatic fifths and the octave are filtered out, but two new octaves and a new fifth are introduced. Interestingly, a fourth is still present between the multiples of eight and nine (which are eighteen and twenty-four); perhaps these are not to be compared to one another because the coefficients three (from three eights) and two (from two nines) are not commensurable, but it is not clear why the other derivation of twenty-four (four sixes) should not be compared to the two nines, as these coefficients are commensurable. The ultimate answer is probably that this fourth is inevitable: Mode III is one of only two modes in which an octave is combined with either of the two smaller perfect consonances without the other; since these two smaller consonances are inversions of each other, the numbers required to produce one of them and to also produce an octave, if one number is shared and they do not span an extremely broad range, necessarily imply the remaining (inverse) perfect consonance.

Mode VI

At first glance, the modal numbers in Mode VI may appear to be surprising, because the modal products and the presentation in the tabular summary both imply that Mode VI should be the same as Mode V, and yet the derivation of the modal numbers is different. In Mode VI, the coefficients for eight and nine are the same as in Mode V, which produces the same two products. Likewise, the other two modal numbers are effectively also the same, but the two base numbers that produce them are reversed, which is possible because six is half of twelve, so that any number that is a multiple of twelve is also a multiple of six, and any even-numbered multiple of six (as in Mode V, which has four sixes) is also a multiple of twelve.

However, what is hidden from this description is that Mode VI also has an octave in addition to its fifth, and so it would potentially run afoul of the same problem as in Mode III: the presence of both an octave and a fifth would imply the complementary fourth, and it is perhaps for this reason that products of twelve and six are reversed. As the coefficients are given in Mode V, only the two products resulting from the shared coefficient three can be compared to each other, and likewise, the two products of the shared coefficient four, because the other comparisons are incommensurable. However, when the products of six and twelve are reversed in Mode VI, there are four distinct coefficients, and the *Alia* allows a larger variety of these to be compared to one another, including allowing the two twenty-fours to be compared to each other and the two thirty-sixes to be compared to each other, which produce the octave.

There are still some questions that remain to be answered about this mode, including why it is permissible to compare equal products, since they do not result from commensurable coefficients; the commensurable coefficients are invoked to explain the fifths, but not the octaves, and the *Alia* offers a rather convoluted explanation for the octave (described in Chapter 16). It is also not clear why the solution to the octave and fifth problem differs between Modes III and VI.

Modes VII and VIII

Modes VII and VIII are handled differently from any of the other modes. They resemble Modes I and II, respectively, in the intervals that characterize them: the perfect fourth and fifth in all four modes, and also the octave for Mode VII as in Mode I. However, the presentation of these intervals is dramatically different: for Modes I and II, all of the intervals that characterize the modes are derived from a single series of products each (*i.e.*, from a single set of coefficients for the four base numbers); for Modes VII and VIII, however, each required interval is derived independently from its own independent series of modal numbers. It is not at all clear what is special about these modes that they should be derived in this manner. Nowacki suggests that these independent derivations are necessary, at least for Mode VIII, because deriving them from a single series would result in the fourth and the fifth being complementary to each other, being positioned as upper and lower halves of an octave; instead, Mode VII is said to have the same fourth as the fourth mode and the same fifth as the fifth mode, both of which rise from the *finalis* of their modes, as the same intervals appear to do in Mode VII.⁸¹⁴ Even leaving aside, for the moment, the fact that my analyses did not find the fourth of Mode VII to rise from its *finalis* (nor does the fourth of Mode IV, and the fourths of these two modes also do not rise from the comparable positions away from the *finalis*), Nowacki's logic is not consistent. The fourth and fifth of the first mode, outside the anomaly of *Rorate caeli*, seem also to rise from the *finalis*, rather than joining together to form an octave, and yet they are derived in the normal fashion (as the *Alia* explicitly defines these intervals against twelve only, and not against six). The same logic would be equally problematic if applied to the eighth mode, as the fourth and fifth in Mode II both arise from the subtone and are again derived from a single series; thus, there is no reason why the same procedure could not be applied to Mode VIII.

⁸¹⁴ Nowacki, *Greek and Latin Music Theory*, 127.

It is difficult to know why Modes VII and VIII are treated differently from the other modes. It is possible that the remark that they have the fifth “in the proportion of the fifth mode” might reflect the similarity in the species of fifth that results from the use of b in Mode V, though in that case, it would be more reasonable to say that the fifth mode has the same fifth as Mode VII. Alternatively, it might mean that the fifth $F-c$, rising from the subtone, is the relevant fifth in Mode VII; this hypothesis is inconsistent with the analyses in Chapter 17, but this inconsistency is not entirely unexpected, since the examples for Mode VII are supplied by the commentator, rather than the source treatise. But it is not clear, under either hypothesis, how the fourth from the fourth mode ($D-G$, which rises from the subtone of Mode IV) could either be literally the same as the fourth ($D-G$) in Mode VII, which does not descend so low in any of the cited chants, or rise from its own subtone ($F-b$), which does not have the same interval structure.

Another possible explanation is that the best way to achieve the desired intervals in a single series has already been done in Modes I and II, and recreating these same intervals in another mode would simply involve using the same coefficients, or perhaps coefficients in the same proportions. Such a duplication does not occur anywhere else in the system of the *Alia musica*, and although the *Alia* does not say so, it is probably important that each mode should have a unique definition (in fact, this probability may also provide an alternate explanation for why Mode VI differs from both Mode V, which shares the same modal numbers, and Mode III, which shares the same intervals).

However, I believe that there may be a simpler explanation for the unusual derivation of Modes VII and VIII, because these two modes share one other characteristic that differs from the other six modes: they are not thoroughly described in the source treatise. To the extent that the seventh mode is described in the source treatise at all, the passage breaks dramatically from the model for the other six modes while it is clearly much more closely related to the description in the prose summary than are the descriptions of any other mode, and the description of the eighth mode is nothing more than a cross-reference. Mühlmann believed that only the first six modes were described by a common author, a *Theoretiker der*

Sechs Töne, and that the descriptions of the last two modes were added by another. However much Mühlmann's model of authorship may represent an over-proliferation of authors, his opinion on this particular matter still seems entirely plausible. In addition, in the source treatise, the intervals borrowed from the fourth and fifth modes are identified by the names of the intervals only, not their modal numbers. I suspect that the unusual description of Modes VII and VIII stems from one theorist's defining the first six modes (and perhaps lost passages on the latter two); a subsequent theorist then cobbled together a description of the final two modes by comparing them to Modes IV and V, so that at least the correct intervals are specified, even if he was not confident of the modal numbers that ought to give rise to these intervals.

As for the final interval of Mode VII, the octave, it has already been observed that only a single product is given in this presentation, and three of the four base numbers are multiplied in such a way as to produce this one product. The coefficients for these three base numbers may be easily explained. In order to achieve only the octave (since the fourth and fifth are already accounted for in the other two presentations), all three modal numbers must be the same, so that the octave is created by comparing equal modal numbers. The nine, however, is not included because the lowest common multiple of nine with all the other modal numbers is seventy-two, which is much larger than any of the other modal numbers.

Summary

Having considered in depth the most unusual descriptions of modal numbers to be found in the source text of the *Alia musica*, I shall now take stock of what is clear about the numerology of the treatise, what may tentatively be proposed, and which questions have yet to be answered.

First, it seems clear that the primary consideration behind the particular coefficients chosen for each of the four base numbers in each mode is the desire to use all four base numbers in each of the eight

modes, yet through them to imply only a limited subset of the intervals suggested by the complete set of base numbers; this subset of intervals to be associated with each mode seems to be a variation on the doctrine found in Aurelianus, but is extended to apply to plagal modes in addition to the authentic modes, and does not agree entirely with Aurelianus' associations.

The most straightforward approach to filtering out unwanted fourths or fifths is to pair base numbers together in groups reflecting the type of interval to be retained, ensuring that the coefficient for each base number in the pair is the same, but that the coefficients differ between the pairs. In so doing, it is necessary to avoid setting the ratio between the coefficients to certain special proportions – especially the ratio of the interval to be retained – so as to avoid reintroducing the filtered-out interval in a different way. At the same time, this procedure also filters out the octave, which is generally also desirable, and maintaining this property also requires the avoidance of another small subset of ratios, mostly overlapping with those already to be avoided, but also including the ratio 4:1.

However, within these conditions, there are many possible ratios between coefficients that could be selected to filter out the undesirable intervals, and there is no clear rationale for making the selection. Heard, drawing on Mühlmann's previous analysis, proposes that each *maneria* is to be understood as a multiple of the *protus* maneria, with the multiple reflecting the Greek ordinal number for which the *maneria* is named, while Chailley proposes that each *maneria* occupies a subsequently lower octave. Each of these hypotheses accurately reflects a considerable subset of the numbers presented in the *Alia*, but ultimately proves inconsistent with the remaining numbers. Neither hypothesis can account, for instance, for the presence of the number eighteen in the *tetrardus* modes. In addition, both hypotheses also assume that specific modal numbers represent specific pitches, and these pitches do not always appear to be unusually prominent in the chants that the *Alia* cites for each mode. Neither of these hypotheses is necessarily ruled out by these inconsistencies because the *Alia* itself has consistency problems.

However, as an alternative to these hypotheses, I have suggested another model that I believe is more consistent both with the statements made in the *Alia* and with the characteristics of the chants cited in the treatise. I have acknowledged a potential pattern for the coefficients selected for filtering out fourths and fifths that seems more likely to have been deliberate and symbolic than to have arisen coincidentally from random or non-symbolic considerations. The pattern seen in the coefficients is that the ratio between pairs of coefficients selected to filter out a specific interval is itself the ratio that represents the very interval to be filtered out (*e.g.*, to filter out the fifth, in the ratio 3:2, the coefficients themselves are set in the ratio 3:2). That there is no particular reason to expect this pattern to have arisen is itself moderately good evidence that it was deliberately created, but it may equally be explained by the fact that the process results in two equal pairs of modal numbers, rather than two unequal pairs that relate internally in the same way but do not interrelate. Either hypothesis has the advantage that it would not presuppose a range of numbers to be associated with each *maneria*, a serious (though not necessarily fatal) weakness in both Heard's and Chailley's hypotheses.

Aloys Schulte once remarked, "*In der geschichtsforschung soll man nicht immer erst warten, bis es möglich ist, eine Aufgabe völlig zu lösen.*"⁸¹⁵ ("In historical research, one should not always wait until it is possible to fully untangle a problem.") Although my hypothesis is capable of explaining the majority of the modal coefficients employed in the *Alia musica*, it still leaves some of the numbers not so much contradictory as incompletely accounted for. In particular, Modes III and VI resemble each other in terms of their intervals but not in terms of their derivations. They share a challenge, which is that numbers that imply both a fifth and an octave also tend to imply the fourth, but each mode takes a different approach to solving this problem. Similarly, Modes VII and VIII share the same intervals as Modes I and II, but derive these intervals independently, instead of in a single series as do the *protus* modes. Both problems may

⁸¹⁵ Schulte, Aloys, *Der hohe Adel im Leben des mittelalterlichen Köln*, Sitzungsberichte der Bayerischen Akademie der Wissenschaften, 1918: 49.

share the same explanation, that each mode should have a unique identification. However, in the case of the *tetrardus* modes, the unusual presentation probably results from a different theorist completing an otherwise incomplete exposition of the modes.

Chapter 20: Conclusions

The *Alia musica* is an enormously complex work. The thorny problem of authorship and the contradictions that the various authors bring with them, the awkward use of language and disparate use of both terminology and note-naming conventions, and the simple fact that the treatise sits at the crossroads of a paradigm shift all add no small amount of complexity to a composite treatise that freely mixes elements of both practical and speculative theory. But even the source treatise itself, the very core of the *Alia*, is far from straightforward.

The principal purpose of this dissertation has been to contribute toward answering a troublesome question: what was the nature of mode in the ninth and tenth centuries? While the date of the final compilation of the *Alia* remains uncertain, the source treatise, at least, seems very likely to be one of the earliest treatises connected to the Gregorian chant tradition, and with its heavy emphasis on mode and harmonics, it is uniquely situated to answer this question.

The division of the chant corpus into four pairs of modes appears to have been a relatively recent innovation shortly before the beginning of the ninth century, and the contradictions within the *Alia musica*, in addition to the quite widely varied presentations among the most important early treatises in this period (Aurelianus, Hucbald, and the *Enchirides*), suggest that the new paradigm had not yet settled. The modal system originated in the need to select appropriate recitation formulae for the psalms to match the accompanying antiphons, but it is clear that the characteristics that made it desirable to match psalms and antiphons in this manner were not limited to the transition points between them (*i.e.*, *differentiae*, *loca*, and the cadences of the antiphons). While other treatises argue over whether the mode is most appropriately revealed at the beginning or the end of a chant, the *Alia* sees some important characteristics of mode as continuously present throughout the chant.

To that end, the *Alia* expands extensively on Aurelianus' brief observations that the modes each favour specific intervals. The *Alia* almost certainly exaggerates the degree to which these intervals permeate the chants and develops an elaborate numerological system to justify these principles and support them on the perceived authority of ancient Greek theory; nevertheless, there is unquestionably a degree of truth to these associations, which are overlooked in subsequent medieval treatises.

What these associations accomplish, in conjunction with a thorough familiarity with the repertoire (which most of the authors of the *Alia* could presumably have expected from their intended audience), is to provide a much closer description of the general shape of the melodies in each mode than is provided by most other treatises, with their more simplified descriptions of approximately one octave of range either above or centered on the *finalis*. In increasing this precision, the *Alia* sacrifices a degree of consistency, in that chants will vary in how closely they conform to the pattern, but it gains something comparable to the notion of a tessitura – a kind of “comfort zone” for each mode, or a sense of the most common ways in which the chants tend to move around within the broader range.

With respect to authorship there are still a considerable number of open questions, but it is possible to suggest a few things. The overall picture suggested by Chailley and refined by Atkinson remains a quite reasonable model of the authorship of the treatise, but as Atkinson suggested, there is good reason to suspect that several parts of the treatise, specifically the parts attributed to the revisor, may have been borrowed from other sources, and in this sense, the revisor, like Aurelianus before him, takes on the role of compiler, though probably not the final compiler of the completed treatise. Among the sections that might reasonably have originated in this manner are the introductory passages about octave species, the Disputed Passage (also about octave species), the prose summary, and perhaps also the section that Chailley called “Symbolic Considerations.”

Chailley's general observation that the revisor does not seem entirely to understand the source treatise remains reasonably likely, and although the commentator seems more sophisticated, he, too, seems not to have entirely understood it, a fact that may be explained by my hypothesis that the source treatise itself seems to be built on exaggeration via confirmation bias. It may, therefore, still be reasonable to wonder whether the revisor might have been more of a general arts scholar than a practicing musician; Atkinson has observed that the revisor seems more attuned to speculative theory than the other authors, who seem more heavily invested in practical theory,⁸¹⁶ and as Chailley notes, he references classical authors (especially Boethius, and through him, Plato, Aristoxenus, and Ptolemy),⁸¹⁷ while the other contributors to the *Alia* do not. It may, then, be the case that the revision was intended as a resource on music as a part of the quadrivium. The numerical and philosophical focus of this layer of the treatise corresponds nicely to the intellectual climate of the Carolingian Renaissance and would likely have appealed to the general intellectuals of the era, while much of the material in this section, especially the supplemental material not found in the source treatise, would have been of limited practical use to singers. The knowledge of the characteristic intervals of each mode would have been of some practical use, but less so the complex mathematical procedures by which they are "justified." The focus seems to have shifted, though, with the addition of the *Nova expositio*, as the extremely practical nature of the latter treatise does not seem especially relevant to the philosophical approach to music typical of a quadrivium program.

But Chailley's suggestion that the revisor badly misunderstands Boethius⁸¹⁸ is at least overstated. His application of the Greek ethnic names for modes to the ecclesiastical system cannot be entirely dismissed as the result of a failure to understand Boethius' description of the modes as transposed gamuts

⁸¹⁶ Atkinson, *Critical Nexus*, 179. Atkinson's preferred terminology for this distinction is harmonic theory and cantus theory, a distinction that he describes in his discussion of the *Musica enchiriadis* (*ibid.*, 127) and continues to develop in his discussion of Regino (*ibid.*, 169–70).

⁸¹⁷ Chailley, *Alia musica*, 20.

⁸¹⁸ *ibid.*

because other passages in the revision clearly demonstrate an understanding of the ways that the Greek string-based note names relate to one another across different modes. Instead, it may well be that a careful consideration of Boethius' wing diagram, in conjunction with Boethius's repeated statement that the modes derive from the octave species, led to the understanding of a correspondence between these two concepts, even if the correspondence at which he arrived is not quite the same as the original Greek correspondence.

The role of the commentary is a little less clear. It carries on the revisor's program of attempting to clarify the numerology, and likely also provides the comments on the classical allusions of the "Symbolic Considerations," and might therefore also be felt to reflect a generalist approach, but the commentator does not add much in the way of new classical references; he also provides relatively sophisticated analyses of melodies through species of fourth and fifth. Both the revisor and the commentator serve to provide commentary on the source treatise, and if Chailley is correct that the revisor was more scholar than musician, then the revisor provides commentary from a scholastic and philosophical perspective and the commentator provides a second commentary, this time from the perspective of a more experienced musician. The *Nova expositio*, too, unquestionably falls into the practical realm, and it seems quite likely that the final form of the treatise, with all of its tonary elements, was intended at least in part for practical use by singers in the monasteries.

This dissertation will certainly not be the last word on the *Alia musica*. At the very least, the many errors in Chailley's edition (to say nothing of his questionable choices) and the more recently discovered manuscripts would all recommend the production of a new critical edition. Furthermore, there are still questions that remain to be answered, and with luck, additional evidence will eventually come to light to resolve them. Yet even without all the answers, the *Alia musica* still stands as one of the best witnesses into the nature of mode in the ninth and tenth centuries and merits its place alongside Aurelianus, Hucbald, and the *Enchiridies* as a monument in early music theory.

Bibliography

I. Manuscripts

Alia musica. Complete. Cessena. S. XXVI-1, ff. 179v–94r. Biblioteca Comunale Malatestiana, *Pluteus*.

———. Munich. Clm 14272, ff. 175r–81v. Bayerische Staatsbibliothek.

———. Paris. F. Lat. 7211, ff. 54r–71r. Bibliothèque nationale de France.

———. Paris. F. Lat. 7212, ff. 39v–50v. Bibliothèque nationale de France.

———. Fragments. Florence. FIII.565, ff. 76v–77r. Bibliotheca nazionale centrale, *Conventi soppressi*.

———. Madrid. 9088. Biblioteca nacional.

———. “First Quidam”. Karlsruhe. K. 504, 34r–34v. Badische Landesbibliothek.

———. “Nova expositio”. Barcelona. Ripoll 42, ff. 66v–69v. Arxiu de la Corona d’Aragó.

———. Paris. F. Lat. 8663, f. 51r. Bibliothèque nationale de France.

St. Riquier Tonary. Paris. F. Lat. 13159, f. 167r–68r. Bibliothèque nationale de France.

Metz Tonary. Metz. 351, f. 66r–75v. Bibliothèques médiathèques de Metz, *Collections patrimoniales*.

II. Music Treatise and Tonary Editions and Translations

Alia musica. In *Alia musica, Traité de musique du IX^e siècle : Edition critique commentée avec une introduction sur l’origine de la nomenclature modale pseudo-grecque du Moyen-Age*, 85–213. Ed. Jacques Chailley. Publications de L’Institute de musicology de l’Universit’e de Paris. Paris : Centre de documentation universitaire, 1965.

———. In *Alia Musica: A Chapter in the History of Medieval Music Theory*, 119–230. Trans. Edmund Brooks Heard. PhD Dissertation: University of Wisconsin – Madison, 1966. ProQuest Dissertations Publishing (6613798).

———. In *Die “Alia musica” (Gerbert, Scriptorum, 1): Quellenfrage, Umfang, Inhalt und Stammbaum*, 50–74. Trans. (German) Wilhelm Mühlmann. PhD Dissertation, Universität Leipzig, 1914. Facsimile reprint, Pranava Books (Printed on demand, 2019).

———. In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 1: 125–47. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.

———. In *Patrologia cursus completus, series latina*, ed. J. P. Migne. Vol. 132: 929–58. Paris: Garnier, 1844–1904.

———. “First Quidam.” In *Greek and Latin Music Theory: Principals and Challenges*, 113–128. Trans. Edward Nowacki. Eastman Studies in Music. Rochester, NY: University of Rochester Press, 2020.

- . “Nova expositio.” In *Anonymi saeculi decimi vel undecimi tractatus de musica “Dulce ingenium musicae”*, 20–26; 39–43. Ed. Michael Bernhard. Munich: Verlag de Bayerischen Akademie der Wissenschaften, 1987.
- . ———. In “Die ‘Nova expositio’ der Handschrift Ripoll 42: Text und Kommentar.” Ed. Karl-Werner Gümpel. *Miscel·lània litúrgica catalana*, Vol. 15 (2007), 129–43. Societat Catalana d’Estudis Litúrgics.
- Aristides Quintilianus. *On music*. In *Greek Musical Writings II: Harmonic and Acoustic Theory*, 399–535. Trans. Andrew Barker. Cambridge Readings in the Literature of Music. Cambridge: Cambridge University Press, 1989.
- . ———. In *Aristides Quintilianus: On Music, in Three Books; Translation, with Introduction, Commentary, and Annotations*. Trans. Thomas J. Mathiesen. Music Theory Translation Series. New Haven: Yale University Press, 1983.
- Aristoxenus. Ἀρμονικά. In *Aristoxenou Harmonika stoicheia: The Harmonics of Aristoxenus*, 95–222. Ed. and trans. Henry S. Macran. Oxford, 1902. Reprint Hildesheim: Olms, 1974.
- . ———. In *Greek Musical Writings II: Harmonic and Acoustic Theory*, 126–184. Trans. Andrew Barker. Cambridge Readings in the Literature of Music. Cambridge: Cambridge University Press, 1989.
- Aurelianus of Réôme. *Musica disciplina*. In *Aureliani Reomensis Musica disciplina*, 58–153. Ed. Lawrence Gushee. Corpus scriptorum de musica, Vol. 21. American Institute of Musicology, 175.
- . In *The Discipline of Music (Musica disciplina)*, 3–58. Trans. Joseph Ponte. Colorado College Music Press Translations, T-3. Colorado Springs: Colorado College Music Press, 1968.
- . In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 1: 28–63. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.
- Beatus Augustinus perhibet*. In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 1: 151–52. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.
- . In *Clavis Gerberti: Eine Revision von Martin Gerberts Scriptores ecclesiastici de musica sacra potissimum*, Teil 1, 33–35. Ed. Michael Bernhard. Munich: Verlag der Bayerischen Akademie der Wissenschaften, 1989.
- Bern of Reichenau. *Prologus in tonarium*. In *Die Musiktraktate des Abtes Bern von Reichenau: Edition und Interpretation*, 31–68. Ed. Alexander Rausch. Musica mediaevalis Europae occidentalis, Band 5. Tutzing: Schneider, 1999.
- . ———. In *Greek and Latin Music Theory: Principals and Challenges*, 133–48. Trans. Edward Nowacki. Eastman Studies in Music. Rochester, NY: University of Rochester Press, 2020.
- . ———. In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 2: 62–79. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.
- . *Tonarius*. In *Die Musiktraktate des Abtes Bern von Reichenau: Edition und Interpretation*, 75–115. Ed. Alexander Rausch. Musica mediaevalis Europae occidentalis, Band 5. Tutzing: Schneider, 1999.
- . ———. In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 2: 79–91. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.

- Βιβλίου Ἀγιοπολίτης. In *The Hagiopolites: A Byzantine Treatise on Musical Theory, Preliminary edition*, 9–94. Ed. and trans. Jørgen Raasted. Université de Copenhague, Cahiers de L'Institut du moeyn-âge grec et latin, 45. Copenhagen: Erik Paludan – International Boghandel, 1983.
- Boethius, Anicius Manlius Torquatus Severinus. *De institutione arithmetica*. In *Anicii Manlii Torquati Severini Boetii: De institutione arithmetica libri duo, De institutione musica libri quinque, accedit Geometria quae fertur Boetii, e libris manu scriptis*, 1–173. Ed. Godofredus Friedlein. Leipzig : B. G. Teubner, 1867.
- . ———. In *Boethian Number Theory: A Translation of De Institutione Arithmetica with Introduction and Notes*, 71–188. Trans. Michael Masi. Amsterdam: Rodopi, 1983.
- . *De institutione musica*. In *Anicii Manlii Torquati Severini Boetii: De institutione arithmetica libri duo, De institutione musica libri quinque, accedit Geometria quae fertur Boetii, e libris manu scriptis*, 175–371. Ed. Godofredus Friedlein. Leipzig : B. G. Teubner, 1867.
- . ———. In *Fundamentals of Music (Anicius Manlius Severinus Boethius)*, 1–79. Trans. Calvin M. Bower. Music Theory Translation Series, edited by Claude V. Palisca. New Haven: Yale University Press, 1989.
- Cassiodorus, Magnus Aurelius (Senator). *Institutiones*. Bk. 2, ch. 5, *De musica*. In *Cassiodori Senatoris Institutiones*, 142–50. Ed. R. A. B. Mynors. Oxford: Clarendon Press, 1937.
- . ———. In *Cassiodorus Institutiones, Book II, Ch. V.*, 3–10. Trans. Helen Dille Goode and Gertrude C. Drake. Colorado College Music Press Translation Series, T-12. Colorado Springs: Colorado College Music Press, 1980.
- . ———. In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 1: 15–19. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.
- Cita et vera divisio monochordi* [Short Recension]. In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 1: 122. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784. See also under Pseudo-Bernelinus.
- Cleonides. *Εἰσαγωγή ἁρμονική*. In *Musici scriptores graeci*, 167–207. Ed. and trans. (Latin) Karl von Jan. Leipzig: B. G. Teubner, 1895.
- . ———. In *Source Readings in Music History*, revised edition, 35–46. Trans. Oliver Strunk, rev. Leo Treitler. New York: W. W. Norton & Company, 1998.
- Commemoratio brevis de tonis et psalmis modulandis*. In *Musica et scolica enchiridis una cum aliquibus tractatulis adiunctis: recensio nova post Gerbertinam altera ad fidem omnium codicum manuscriptorum*, 157–78. Ed. Hans Schmid. Munich: Bayerische Akademie de Wissenschaften, 1981.
- . In *Commemoratio brevis de tonis et psalmis modulandis: Introduction, Critical Edition, Translation*, 26–107. Ed. and trans. Terence Bailey. Ottawa: The University of Ottawa Press, 1979.
- . In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 1: 312–29. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.
- De consonantiis tribus*. In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 1: 150–51. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.

- . In *Clavis Gerberti: Eine Revision von Martin Gerberts Scriptores ecclesiastici de musica sacra potissimum*, Teil 1, 20–32. Ed. Michael Bernhard. Munich: Verlag der Bayerischen Akademie der Wissenschaften, 1989.
- De cymbalorum ponderibus*. In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 1: 149. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.
- De dimensione monochordi [Super unum concavum lignum]*. In *Musica et scolica enchiridis una cum aliquibus tractatulis adiunctis: recensio nova post Gerbertinam altera ad fidem omnium codicum manuscriptorum*, 179–81. Ed. Hans Schmid. Munich: Bayerische Akademie de Wissenschaften, 1981.
- De mensuris organicarum*. In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 1: 147–48. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.
- De modis musicis*. In “De modis musicis: A New Edition and Explanation.” *Kirchenmusikalisches Jahrbuch*, Vol. 61–62 (1977–78): 50–54. Ed. Terence Bailey.
- . In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 1: 149. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.
- De quinque symphoniis*. In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 1: 149–50. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.
- . In *Clavis Gerberti: Eine Revision von Martin Gerberts Scriptores ecclesiastici de musica sacra potissimum*, Teil 1, 19–20. Ed. Michael Bernhard. Munich: Verlag der Bayerischen Akademie der Wissenschaften, 1989.
- Dimensio monochordi*. In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 1: 122–24. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.
- . In *Clavis Gerberti: Eine Revision von Martin Gerberts Scriptores ecclesiastici de musica sacra potissimum*, Teil 1, 14–16. Ed. Michael Bernhard. Munich: Verlag der Bayerischen Akademie der Wissenschaften, 1989.
- Dulce ingenium musicae*. In *Anonymi saeculi decimi vel undecimi tractatus de musica “Dulce ingenium musicae”*, 14–26 [short recension]; 27–43 [long recension]. Ed. Michael Bernhard. Munich: Verlag der Bayerischen Akademie der Wissenschaften, 1987.
- Ecce modorum sive tonorum*. In *Musica et scolica enchiridis una cum aliquibus tractatulis adiunctis: recensio nova post Gerbertinam altera ad fidem omnium codicum manuscriptorum*, 182–84. Ed. Hans Schmid. Munich: Bayerische Akademie de Wissenschaften, 1981.
- . In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 1: 124–25. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.
- Gaudentius. *Ἀρμονικὴ εἰσαγωγή*. In *Musici scriptores graeci*, 317–56. Ed. and trans. (Latin) Karl von Jan. Leipzig: B. G. Teubner, 1895.
- . ———. In *Source Readings in Music History*, revised edition, 66–85. Trans. Leo Treitler. New York: W. W. Norton & Company, 1998.
- . ———. In *Alypius et Gaudence; Bacchius l’ancien*, 53–91. Trans. (French) Ch.-Émile Ruelle. Collection des auteurs grecs relatifs à la musique. Paris: Imprimerie Nationale, 1895.

- Guido of Arezzo. *Epistola ad Michaelem [Epistola de ignoto cantu]*. In *Guido d'Arezzo's Regulae rithmice, Prologus in antiphonarium, and Epistola ad Michaelem: A Critical Text and Translation*, 438–531. Ed. and trans. Dolores Pesce. Ottawa: Institute of Medieval Music, 1999.
- . ———. In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 2: 43–50. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.
- . *Micrologus*. In *Guidonis Aretini Micrologus*, 79–234. Ed. Joseph Smits van Waesberghe. *Corpus scriptorum de musica*, Vol. 4. Rome: American Institute of Musicology, 1955.
- . ———. In *Hucbald, Guido, and John on Music: Three Medieval Treatises*, 57–83. Trans. Warren Babb. Music Theory Translation Series, ed. Claude V. Palisca. New Haven: Yale University Press, 1978.
- . ———. In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 2: 2–24. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.
- . *Prologus in antiphonarium*. *Guido d'Arezzo's Regulae rithmice, Prologus in antiphonarium, and Epistola ad Michaelem: A Critical Text and Translation*, 406–435. Ed. and trans. Dolores Pesce. Ottawa: Institute of Medieval Music, 1999.
- . ———. In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 2: 34–37. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.
- . *Regulae rithmice*. *Guido d'Arezzo's Regulae rithmice, Prologus in antiphonarium, and Epistola ad Michaelem: A Critical Text and Translation*, 328–403. Ed. and trans. Dolores Pesce. Ottawa: Institute of Medieval Music, 1999.
- . ———. In *Guido's Rhythmic Rules: Medieval Fundamentals of Music Theory in Verse Form*. Trans. Ken Stephenson. Musical Theorists in Translation, Vol. 20. Ottawa: Institute of Medieval Music, 2012.
- . ———. In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 2: 25–33. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.
- Hermannus Contractus. *Musica*. In *The Musica of Hermannus Contractus*. Ed. and trans. Leonard Ellinwood. Eastman Studies in Music. Rochester, NY: University of Rochester Press, 2015.
- . ———. In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 2: 124–155. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.
- Hucbald of St. Amand. *Musica [formerly De harmonica institutione]*. In *L'Œuvre musicale d'Hucbald de Saint-Amand: Les compositions et le traité de musique*, 136–213. Ed. and trans. (French) Yves Chartier. Cahiers d'Études Médiévales, Cahier spécial n° 5. Saint-Laurent, Québec : Bellarmin, 1995.
- . In *Hucbald, Guido, and John on Music: Three Medieval Treatises*, 13–44. Trans. Warren Babb. Music Theory Translation Series, ed. Claude V. Palisca. New Haven: Yale University Press, 1978.
- . In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 1: 104–21. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.
- In primo diapason*. In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 1: 121–22 ; 329–30. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.

- Inchiriadon*. In *Musica et scolica enchiriadis una cum aliquibus tractatulis adiunctis: recensio nova post Gerbertinam altera ad fidem omnium codicum manuscriptorum*, 187–213. Ed. Hans Schmid. Munich: Bayerische Akademie de Wissenschaften, 1981.
- Isidore (St.) of Seville. *Etymologies*. Bk. 3, ch. 14–23, *De musica*. In *Isidori Hispalensis episcopi Etymologiarum sive originum libri xx* [page numbers not printed]. Ed. W. M. Lindsay. Scriptorum classicorum bibliotheca Oxoniensis. Oxford: Clarendon Press, 1911.
- . ———. In *Isidore of Seville: Concerning Music*, 13–20. Trans. Helen Dille Goode and Gertrude C. Drake. Colorado College Music Press Translation Series, T-12. Colorado Springs: Colorado College Music Press, 1980.
- . ———. In *The Etymologies of Isidore of Seville*, 95–99. Trans. Stephen A. Barney, W. J. Lewis, J. A. Beach, and Oliver Berghof. Cambridge: Cambridge University Press, 2006.
- . ———. In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 1: 20–25. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.
- Johannes Affligemensis [Johannes Cotto]. *De musica*. In *De musica cum tonario*, 43–200. Ed. J. Smits van Waesberghe. Corpus scriptorum de musica, Vol. 1. Rome: American Institute of Musicology, 1950.
- . ———. In *Hucbald, Guido, and John on Music: Three Medieval Treatises*, 101–87. Trans. Warren Babb. Music Theory Translation Series, ed. Claude V. Palisca. New Haven: Yale University Press, 1978.
- . ———. In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 2: 230–65. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.
- Martianus Capella. *De nuptiis Philologiae et Mercurii*. Bk. 9, *De harmonia*. In *Martianus Capella*, 337–86. Ed. James Willis. Bibliotheca scriptorum graecorum et romanorum Teubneriana. Leipzig: B. G. Teubner Verlagsgesellschaft, 1983.
- . ———. In *Martianus Capella and the Seven Liberal Arts*, Vol. 2: 345–82. Trans. William Harris Stahl and Richard Johnson. New York: Columbia University Press, 1971.
- Metz Tonary*. In *Der Karolingische Tonar von Metz*, 12–100. Ed. Walther Lipphardt. Münster Westfalen: Aschendorffsche Verlagbuchhandlung, 1965.
- Musica enchiriadis*. In *Musica et scolica enchiriadis una cum aliquibus tractatulis adiunctis: recensio nova post Gerbertinam altera ad fidem omnium codicum manuscriptorum*, 1–59. Ed. Hans Schmid. Munich: Bayerische Akademie de Wissenschaften, 1981.
- . In *Musica enchiriadis and Scolica enchiriadis*, 1–32. Trans. Raymond Erickson. New Haven: Yale University Press, 1995.
- . In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 1: 152–73. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.
- Nicomachus. *Ἀρμονικὸν ἐνχειρίδιον*. In *Musici scriptores graeci*, 208–65. Ed. and trans. (Latin) Karl von Jan. Leipzig: B. G. Teubner, 1895.
- . ———. In *Greek Musical Writings II: Harmonic and Acoustic Theory*, 247–69. Trans. Andrew Barker. Cambridge Readings in the Literature of Music. Cambridge: Cambridge University Press, 1989.

- . ———. In *The Manual of Harmonics of Nicomachus the Pythagorean*, 33–176. Trans. Flora R. Levin. Grand Rapids, MI, USA: Phanes Press, 1994.
- Oddo of Arezzo. *Prooemium tonarii*. In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 1: 248–50. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.
- Pseudo-Alcuin. *De octo tonis*. In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 1: 26–27. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.
- Pseudo-Bernelinus. *Cita et vera divisio monochordi* [Long recension]. In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 1: 313–30. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784. See also *Cita et vera divisio monochordi*.
- Ptolemy. *Ἀρμονικά*. In *Die Harmonielehre des Klaudios Ptolemaios*, 3–121. Ed. Ingemar Düring. Göteborg, 1930. Reprint Hildesheim: Georg Olms Verlag, 1982.
- . ———. In *Greek Musical Writings II: Harmonic and Acoustic Theory*, 275–391. Trans. Andrew Barker. Cambridge Readings in the Literature of Music. Cambridge: Cambridge University Press, 1989.
- . ———. In *Ptolemy Harmonics, Translation and Commentary*, 1–166. Trans. Jon Solomon. Leiden: Brill, 2000.
- Regino of Prüm. *De armonica institutione*. In *L'Épistola de armonica institutione de Regino de Prüm : Texte établi, traduit et commenté*, 61–116. Ed. and trans. (French) Yvon Chartier. PhD Dissertation: Université d'Ottawa, 1965. ProQuest Dissertations Publishing (EC56055).
- . ———. In *The 'De harmonica institutione' and 'Tonarius' of Regino of Pruem: Latin Text with English Translation*, 21–84. Ed. and trans. Sister Mary Protase. PhD Dissertation: The Catholic University of America, 1965. ProQuest Dissertations Publishing (6600318).
- . ———. In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 1: 230–47. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.
- . *Tonarius*. In *Die Musiktraktate des Abtes Bern von Reichenau: Edition und Interpretation*, 201–24. Ed. Alexander Rausch. *Musica mediaevalis Europae occidentalis*, Band 5. Tutzing: Schneider, 1999.
- . ———. In *The 'De harmonica institutione' and 'Tonarius' of Regino of Pruem: Latin Text with English Translation*, 137–294. Ed. Sister Mary Protase. PhD Dissertation: The Catholic University of America, 1965. ProQuest Dissertations Publishing (6600318).
- Scolica enchiridis*. In *Musica et scolica enchiridis una cum aliquibus tractatulis adiunctis: recensio nova post Gerbertinam altera ad fidem omnium codicum manuscriptorum*, 60–156. Ed. Hans Schmid. Munich: Bayerische Akademie de Wissenschaften, 1981.
- . In *Musica enchiridis and Scolica enchiridis*, 33–93. Trans. Raymond Erickson. New Haven: Yale University Press, 1995.
- . In *Scriptores ecclesiastici de musica sacra potissimum*, Vol. 1: 173–212. Ed. Martin Gerbert. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.
- St. Riquier Tonary*. In *Les tonaires: Inventaire, Analyse, Comparaison*, 26–28. Michael Huglo. Paris: Société Française de Musicologie, 1971.

Theon of Smyrna. *Των κατά το μαθηματικόν χρησιμῶν εἰς τὴν Πλάτωνος ἀναγωγῶν*. In *Théon de Smyrne, philosophe platonicien: Exposition des connaissances mathématiques utiles pour la lecture de Platon*. Ed. and trans. (French) J. Dupuis. Paris: Librairie Hachette, 1892.

———. ———. In *Θεωνος Σμυρναίου: Mathematics Useful for Understanding Plato*. Trans. Robert and Deborah Lawlor. Secret Doctrine Reference Series. San Diego: Wizards Bookshelf, 1979.

III. Other Ancient Sources

Aristotle. *Πολιτικῶν*. In *Aristotelis Politica, recognovit brevique adnotatione critica instruxit*. Ed. W. D. Ross. Oxford: Clarendon Press, 1957.

Hyginus. *Astronomica*. In *Hygini Astronomica ex codicibus a se primum collatis*. Ed. Bernhardus Bunte. Leipzig: T. O. Weigell, 1875.

Plato. *Ἐπινομίς*. In *Platonis opera, tomus V, 973–92*. Ed. John Brent. Scriptorum classicorum bibliotheca Oxoniensis. Oxford: Clarendon Press, 1903.

———. *Κρατυλος*. In *The Dialogues of Plato: Translated into English with Analyses and Introductions*, Vol. 1, 3rd Ed. & trans. Jowett, B. Oxford: Clarendon Press, 1892.

Pseudo-Plutarch. *Περὶ τῶν ἀρεσκοντῶν φιλοσοφῶν γυσικῶν δογματῶν [De placitis philosophorum]*. In *Moralia*, Vol. 5, 264–372. Ed. Gregorius N. Bernardakis. Leipzig: B. G. Teubner, 1893.

Vitruvius. *De architectura*. In *Vitruvii De architectura libri decem*. Ed. F. Krohn. Leipzig: B. G. Teubner, 1912.

IV. Secondary Sources

Apel, Willi. *Gregorian Chant*. Bloomington, IN: Indiana University Press, 1958.

Atkinson, Charles M. *The Critical Nexus: Tone-System, Mode, and Notation in Early Medieval Music*. Oxford: Oxford University Press, 2009.

———. “The *Parapteres*: *Nothi* or Not?” *The Musical Quarterly*, Vol. 68, no. 1 (Jan. 1982): 32–59. Oxford University Press. <http://www.jstor.com/stable/741868>.

Babb, Warren, trans. *Hucbald, Guido, and John on Music: Three Medieval Treatises*. Music Theory Translation Series, ed. Claude V. Palisca. New Haven: Yale University Press, 1978.

Bailey, Terence. *Commemoratio brevis de tonis et psalmis modulandis: Introduction, Critical Edition, Translation*. Ottawa: The University of Ottawa Press, 1979.

———. “*De modis musicis*: A New Edition and Explanation.” *Kirchenmusikalisches Jahrbuch*, Vol. 61–62 (1977–78): 47–60.

———. *The Intonation Formulas of Western Chant*. Toronto: Pontifical Institute of Medieval Studies, 1974.

- Barker, Andrew. "Aristides Quintilianus and Constructions in Early Music Theory." *The Classical Quarterly*, Vol. 32, no. 1: 184–97. The Classical Association, Cambridge University Press. www.jstor.org/stable/638757.
- . *Greek Musical Writings I: The Musician and His Art*. Cambridge Readings in the Literature of Music. Cambridge: Cambridge University Press, 1984.
- . *Greek Musical Writings II: Harmonic and Acoustic Theory*. Cambridge Readings in the Literature of Music. Cambridge: Cambridge University Press, 1989.
- . *The science of Harmonics in Classical Greece*. Cambridge: Cambridge University Press, 2007.
- . *Scientific Method in Ptolemy's Harmonics*. Cambridge: Cambridge University Press, 2000.
- Barney, Stephen A., W. J. Lewis, J. A. Beach, and Oliver Berghof. *The Etymologies of Isidore of Seville*. Cambridge: Cambridge University Press, 2006.
- Bernhard, Michael. *Anonymi saeculi decimi vel undecimi tractatus de musica "Dulce ingenium musicae."* Munich: Verlag der Bayerischen Akademie der Wissenschaften, 1987.
- . *Clavis Gerberti: Eine Revision von Martin Gerberts Scriptores ecclesiastici de musica sacra potissimum*, Teil 1. Munich: Verlag der Bayerischen Akademie der Wissenschaften, 1989.
- . *Studien zur „Epistola de armonica institutione“ des Regino von Prüm*. Veröffentlichungen der Musikhistorischen Kommission, Band 5, Bayerische Akademie der Wissenschaften. Munich: Verlag der Bayerischen Akademie der Wissenschaften, 1979.
- Bower, Calvin M. *Fundamentals of Music (Anicius Manlius Severinus Boethius)*. Music Theory Translation Series, edited by Claude V. Palisca. New Haven: Yale University Press, 1989.
- . "Boethius and Nicomachus: An Essay Concerning the Sources of *De institutione musica*." *Vivarium*, Vol. 16, no. 1 (1978): 1–45. Brill. <http://www.jstor.com/stable/42569706>.
- . "Cassiodorus." In *Grove Music Online*. Oxford University Press, 2001. <https://doi.org/10.1093/gmo/9781561592630.article.05108>.
- . "The Grammatical Model of Musical Understanding in the Middle Ages." In *Hermeneutics and Medieval Culture*, 133–45. Edited by Patrick J. Gallacher and Helen Damico. New York: State University of New York Press, 1989.
- Chailley, Jacques. *Alia musica, Traité de musique du IX^e siècle : Edition critique commentée avec une introduction sur l'origine de la nomenclature modale pseudo-grecque du Moyen-Age*. Publications de L'Institute de musicology de l'Universit'e de Paris. Paris : Centre de documentation universitaire, 1965.
- Chartier, Yves. "Hucbald of St Amand." *Grove Music Online*, 2001.
- . *L'oeuvre musicale d'Hucbald de Saint-Amand : Les compositions et le traité de musique*. Cahiers d'Études Médiévales, Cahier spécial n° 5. Saint-Laurent, Québec : Bellarmin, 1995.
- Chartier, Yvon. *L'Epistola de armonica institutione de Reginon de Prüm : Texte établi, traduit et commenté*, 61–116. PhD Dissertation: Université d'Ottawa, 1965. ProQuest Dissertations Publishing (EC56055).
- Cyrus, Cynthia J. "Compilation, Synthesis, and Modal Understanding in the *Alia musica*." *Abstracts of Papers Read at the Fifty-Eighth Annual Meeting of the American Musicological Society, Pittsburgh*,

- Pennsylvania, 4-8 November 1992*: 14. American Musicological Society. <https://ams-net.site-ym.com/resource/resmgr/files/abstracts/1992-pittsburgh.pdf>.
- Davis, Lisa Fagin. *The Gottschalk Antiphonary: Music and Liturgy in Twelfth-Century Lambach*. Cambridge Studies in Palaeography and Codicology, 8. Cambridge: Cambridge University Press, 200.
- De Coussemaker, Edmond. *Mémoire sur Hucbald et sur ses traités de musique, suivi de recherches sur la notation et sur les instruments de musique*. Paris: J. Techener, Libraire: 1841.
- Dupuis, J. *Théon de Smyrne, philosophe platonicien: Exposition des connaissances mathématiques utiles pour la lecture de Platon*. Paris: Librairie Hachette, 1892.
- Düring, Ingemar. *Die Harmonielehre des Klaudios Ptolemaios*. Göteborg, 1930. Reprint Hildesheim: Georg Olms Verlag, 1982.
- Ellinwood, Leonard, trans. *The Musica of Hermannus Contractus*. Eastman Studies in Music. Rochester, NY: University of Rochester Press, 2015.
- Erickson, Raymond, trans. *Musica enchiriadis and Scolica enchiriadis*. New Haven: Yale University Press, 1995.
- Falconer, Keith. "The Modes Before the Modes: Antiphon and Differentia in Western Chant." In *The Study of Medieval Chant: Paths and Bridges, East and West; In Honor of Kenneth Levy*, ed. Peter Jeffery, 131–45. Woodbridge, Suffolk: Boydell Press, 2001.
- Ferretti, Paolo (Dom). *Estetica Gregoriana, ossia trattato delle forme musicali del canto Gregoriano*. Rome: Pontificio Istituto de Musica Sacra, 1934.
- . *Esthétique Grégorienne, ou traité de formes musicales du chant Grégorien*. Trans. A. Agaësse. Abbaye Saint-Pierre de Solesmes, 1938.
- Frere, Walter Howard. *Antiphonale Sarisburiense*. London: Plainsong & Mediaeval Music Society, 1901–1924. Reprint, Westmead, UK: Gregg Press Limited, 1966.
- . *The Use of Sarum II: The Ordinal and Tonal*. Cambridge: Cambridge University Press, 1901.
- Friedlein, Godofredus. *Anicii Manlii Torquati Severini Boetii De institutione arithmetica libri duo, De institutione musica libri quinque, accedit Geometria quae fertur Boetii, e libris manu scriptis*. Leipzig: B. G. Teubner, 1867.
- Frøyshov, Stig Simeon R. "The Early Development of the Liturgical Eight-Mode System in Jerusalem." *St. Vladimir's Theological Quarterly*, Vol. 51, no. 2–3 (2007): 139–78.
- Fuller, Sarah. "Interpreting Hucbald on Mode." *Journal of Music Theory* 52, no. 1 (Spring 2008): 13–40. <http://www.jstor.org/stable/40607028>.
- Gevaert, François Auguste. *La melopée antique dans la chant de l'église Latine*. 1895.
- Goode, Helen Dille & Gertrude C. Drake. *Cassiodorus Institutiones, Book II, Ch. V*. Colorado College Music Press Translation Series, T-12. Colorado Springs: Colorado College Music Press, 1980.
- . *Isidore Etymologies, Book III, Ch. 15–23*. Colorado Springs: Colorado College Music Press, 1980.
- Gerbert, Martin. *Scriptores ecclesiastici de musica sacra potissimum*, Vols. 1 & 2. St. Blasien, Black Forest: Saint Blaise Abbey, 1784.

- Grier, James. "Early Polyphony." In *The Cambridge History of Medieval Music*, 801–33. Edited by Mark Everist and Thomas Forrest Kelly. Cambridge: Cambridge University Press, 2018.
- . *The Musical World of a Medieval Monk: Adémar de Chabannes in Eleventh-Century Aquitaine*. Cambridge: Cambridge University Press, 2006.
- Gümpel, Karl-Werner. "Die 'Nova expositio' der Handschrift Ripoll 42: Text und Kommentar." *Miscel·lània litúrgica catalana*, Vol. 15 (2007): 125–86. Societat Catalana d'Estudis Litúrgics.
- . "Musica cum rhetorica: die Handschrift Ripoll 42." *Archiv für Musikwissenschaft*, 34 Jahrgang, H. 4 (1977): 260–86. Franz Steiner Verlag. <https://www.jstor.org/stable/930660>.
- Gushee, Lawrence. *Aureliani Reomensis Musica disciplina*. Corpus scriptorum de musica, Vol. 21. American Institute of Musicology, 1975.
- . "Hermannus Contractus." In *Grove Music Online*. Oxford University Press, 2001. <https://doi.org/10.1093/gmo/9781561592630.article.12864>.
- . "Questions of Genre in Medieval Treatises on Music." *Gattungen der Musik in Einzeldarstellungen: Gedenkschrift Leo Schrade*, edited by Wulf Arlt, Ernst Lichtenhahn, and Hans Oesch, 365–433. Munich: Francke, 1973.
- . *The Musica disciplina of Aurelian of Réôme: A Critical Text and Commentary*. PhD Dissertation: Yale University, 1963. ProQuest Dissertations Publishing (6411873).
- Gushee, Lawrence, and Dolores Pesce. "Berno of Reichenau." In *Grove Music Online*. Oxford University Press, 2001. <https://doi.org/10.1093/gmo/9781561592630.article.02878>.
- Hagel, Stefan. *Ancient Greek Music: A New Technical History*. Cambridge: Cambridge University Press, 2010.
- Hansen, Finn Egeland. *H159 Montpellier: Tonary of St. Bénigne of Dijon*. Copenhagen: Dan Fog Musikforlag, 1974.
- Heard, Edmund Brooks. *Alia Musica: A Chapter in the History of Medieval Music Theory*. PhD Dissertation: University of Wisconsin – Madison, 1966. ProQuest Dissertations Publishing (6613798).
- Hiley, David. *Western Plainchant: A Handbook*. Oxford: Clarendon Press, 1993.
- Huglo, Michel. *Les tonaires: Inventaire, Analyse, Comparaison*. Paris: Société Française de Musicologie, 1971.
- . "Les instruments de musique chez Hucbald." In *Hommages à André Boutemy*, edited by Guy Cambier, 178–96. Collection Latomus, Vol. 145. Wetteren, Belgium: Imprimerie Universa, 1976.
- . "L'auteur du 'Dialogue sur la Musique' attribué a Odon." *Revue de Musicologie*, T. 55, no. 2 (1969): 119–71. Société Française de Musicologie. <https://www.jstor.org/stable/927822>.
- . "L'Inchiriadon Hucbaldi: Son rapport avec la 'Musica enchiriadis' et sa circulation en Italie." *Il Saggiatore musicale*, Vol. 19, No. 2 (2012): 179–98. Casa Editrice Leo S. Olschki s.r.l. <https://www.jstor.org/stable/43030139>.
- . "Notes sur la reproduction de mes Tonaires en PDF." 2001. Archivum de musica medii aevi. Available on-line: http://www.musmed.fr/AdMMAe/Huglo_Tonaires_01.pdf.

- . Review of *Alia musica*, by Jacques Chailley. *Revue de Musicologie*, T. 51, no. 2 (1965): 230–32. Société Française de Musicologie. <https://www.jstor.org/stable/927337>.
- . Review of *Der karolingische Tonar von Metz*, by Walther Lipphardt. *Revue de Musicologie*, T. 51, no. 2 (1965): 228–30. Société Française de Musicologie. <https://www.jstor.org/stable/927336>.
- Jeffery, Peter. “The Earliest *Oktōēchoi*: The Role of Jerusalem and Palestine in the Beginnings of Modal Ordering.” In *The Study of Medieval Chant: Paths and Bridges, East and West; In Honor of Kenneth Levy*, ed. Peter Jeffery, 147–209. Woodbridge, Suffolk: Boydell Press, 2001.
- Jesson, Roy. “Ambrosian Chant.” In *Gregorian Chant*, 456–83. Ed. Willi Apel. Bloomington, IN: Indiana University Press, 1958.
- Jones, H. Stuart. “Gevaert on Ancient Music and Plain-Song.” Review of *La mélopée antique dans le chant de l’Église Latine*, by Francois-Auguste Gevaert. *The Classical Review*, Vol. 10, no. 1 (Feb., 1896): 70–72. The Classical Association, Cambridge University Press. <https://www.jstor.org/stable/690761>.
- Koller, Oswald. “Aus dem Archive des Benedictinerstiftes St. Paul im Lavantthal in Kärnten.” *Monatshefte für Musik-Geschichte*, 22 Jahrgang (1890): 22–29.
- Lawlor, Robert, and Deborah Lawlor. *Θεωρος Συμυρναιου: Mathematics Useful for Understanding Plato*. Secret Doctrine Reference Series. San Diego: Wizards Bookshelf, 1979.
- Levin, Flora R. *The Manual of Harmonics of Nicomachus the Pythagorean*. Grand Rapids, MI, USA: Phanes Press, 1994.
- . “Nicomachus of Gerasa.” In Grove Music Online. Oxford University Press, 2001. <https://doi.org/10.1093/gmo/9781561592630.article.19911>.
- Levy, Kenneth. “On the Origin of Neumes.” *Early Music History*, Vol. 7 (1987): 59–90. Cambridge University Press. <https://www.jstor.org/stable/853888>.
- Lindsay, W. M. *Isidori Hispalensis Episcopi etmologiarum sive originum, libri XX*. Oxford: Oxford University Press, 1911.
- Liphardt, Walther. *Der Karolingische Tonar von Metz*. Münster Westfalen: Aschendorffsche Verlagbuchhandlung, 1965.
- Macran, Henry S. *Aristoxenou Harmonika stoicheia: The Harmonics of Aristoxenus*. Oxford, 1902. Reprint Hildesheim: Olms, 1974.
- Maloy, Rebecca. “*Scolica enchiridis* and the ‘Non-Diatonic’ Plainsong Tradition.” *Early Music History*, Vol. 28 (2009): 61–96. Cambridge University Press. <https://www.jstor.org/stable/40800897>.
- Masi, Michael. *Boethian Number Theory: A Translation of De Institutione Arithmetica with Introduction and Notes*. Amsterdam: Rodopi, 1983.
- Mathiesen, Thomas J. *Aristides Quintilianus: On Music, in Three Books; Translation, with Introduction, Commentary, and Annotations*. New Haven: Yale University Press, 1983.
- Merkley, Paul. “Tonaries and Melodic Families of Antiphons.” *Journal of the Plainsong and Mediaeval Music Society*, Vol. 11 (1988): 13–24. <https://doi.org/10.1017/S0143491800001136>.

- Meyer, Christian. Review of *Firenze, Biblioteca Nazionale Centrale, Conventi Soppressi, F.III.565*, edited by Alma Santosuosso. *Revue de Musicologie*, T. 81, no. 2 (1995): 287–88. Société Française de Musicologie. <https://www.jstor.org/stable/946970>.
- Möller, Hartmut. "De octo tonibus: Ein europäisch-amerikanisches Verwirrspiel und sein Klärung." In *IMS Study Group Cantus Planus: Papers Read at the 6th Meeting, Eger, Hungary, 1993*. Ed. László Dobszay, 697–710. Budapest: Hungarian Academy of Sciences, 1995.
- Mühlmann, Wilhelm. *Die "Alia musica" (Gerbert, Scriptorum, 1): Quellenfrage, Umfang, Inhalt und Stammbaum*. PhD Dissertation, Universität Leipzig, 1914. Facsimile reprint, Pranava Books (Printed on demand, 2019).
- Müller, Hans. *Hucbalds echte und unechte Schriften über Musik*. Leipzig: B. G. Teubner, 1884.
- Mynors, R. A. B. *Cassiodori Senatoris Institutiones*. Oxford: Clarendon Press, 1937.
- Nowacki, Edward. *Greek and Latin Music Theory: Principles and Challenges*. Eastman Studies in Music. Rochester, NY: University of Rochester Press, 2020.
- Pesce, Dolores. *Guido d'Arezzo's Regule rithmice, Prologus in antiphonarum, and Epistola ad Michaelem: A Critical Text and Translation with an Introduction, Annotations, Indices, and New Manuscript Inventories*. Ottawa: Institute of Mediaeval Music, 1999.
- Phillips, Nancy. "Musica" and "Scolica enchiridis": *The Literary, Theoretical, and Musical Sources*. PhD Dissertation, New York University, 1984. ProQuest Dissertations Publishing (8505525).
- . Review of *Das älteste Dokument zur Entstehung der abendländischen Mehrstimmigkeit: eine Handschrift aus Werden an der Ruhr. Das 'Düsseldorfer Fragment,'* by Dieter Torkewitz. *Plainsong and Medieval Music*, Vol. 9, no. 1 (2000): 77–80. Cambridge University Press.
- Planer, John Harris. *The Ecclesiastical Modes in the Late Eighth Century*. PhD Dissertation, University of Michigan, 1970. ProQuest Dissertations Publishing (7115270).
- Ponte, Joseph. *Aurelian of Réome: The Discipline of Music (Musica Disciplina)*. Colorado College Music Press Translations, No. 3. Colorado Springs: Colorado College Music Press, 1968.
- Protase, Mary (Sister). *The 'De harmonica institutione' and 'Tonarius' of Regino of Pruem: Latin Text with English Translation*. PhD. Dissertation: The Catholic University of America, 1965. ProQuest Dissertations Publishing (6600318).
- Raasted, Jørgen. *The Hagiopolites: A Byzantine Treatise on Musical Theory, Preliminary edition*. Université de Copenhague, Cahiers de L'Institut du moeyn-âge grec et latin, 45. Copenhagen: Erik Paludan – International Boghandel, 1983.
- Rausch, Alexander. *Die Musiktraktate des Abtes Bern von Reichenau: Edition und Interpretation*. Musica mediaevalis Europae occidentalis, Band 5. Tutzing: Schneider, 1999.
- Ruelle, Ch.-Émile. *Alypius et Gaudence; Bacchius l'ancien*. Collection des auteurs grecs relatifs à la musique. Paris: Imprimerie Nationale, 1895.
- Sanftl, Koloman. *Catalogus veterum codicum manuscriptorum ad S. Emmeram*. Bayerische Staatsbibliothek, 1809: <https://bildsuche.digitale-sammlungen.de/index.html?c=viewer&bandnummer=bsb00095226&pimage=3&v=100&nav=&l=fr>
- Schmid, Hans. *Musica et scolica enchiridis una cum aliquibus tractatulis adiunctis: recensio nova post Gerbertinam altera ad fidem omnium codicum manuscriptorum*. Munich: Bayerische Akademie de Wissenschaften, 1981.

- Snow, Robert J. "The Old-Roman Chant." In *Gregorian Chant*, 484–505. Ed. Willi Apel. Bloomington, IN: Indiana University Press, 1958.
- Solomon, Jon. *Ptolemy Harmonics, Translation and Commentary*. Leiden: Brill, 2000.
- Stahl, William Harris, and Richard Johnson. *Martianus Capella and the Seven Liberal Arts*. New York: Columbia University Press, 1971.
- Stephenson, Ken. *Guido's Rhythmic Rules: The Medieval Fundamentals of Music Theory in Verse Form*. Musical Theorists in Translation. Ottawa: Institute of Medieval Music, 2012.
- Strunk, W. Oliver. *Source Readings in Music History from Classical Antiquity through the Romantic Era*. Revised edition, ed. Leo Treitler. New York: Norton, 1950.
- . "Antiphons of the *Oktoechos*". *Journal of the American Musicological Society*, Vol. 13, No. 1 (1960): 50–67. University of California Press. <https://www.jstor.org/stable/830246>.
- Tillyard, H. J. W. "The Modes in Byzantine Music." *The Annual of the British School at Athens*, Vol. 22 (1916–18): 133–56. British School at Athens. <https://www.jstor.org/stable/30096514>.
- Torkewitz, Dieter. *Das älteste Dokument sur Entstehung der abenländischen Mehrstimmigkeit: eine Handschrift aus Werden an der Ruhr: Das Düsseldorfer Fragment*. Beihefte zum Archiv für Musikwissenschaft, Band 44. Stuttgart: Franz Steiner Verlag, 1999.
- Treitler, Leo. "'Centonate' Chant: 'Übles Flickwerk' or 'E pluribus unus?'" *Journal of the American Musicological Society*, Vol. 28, no. 1 (Spring, 1975): 1–23. University of California Press. <https://www.jstor.org/stable/830914>.
- Von Jan, Karl. *Musici scriptores graeci*. Leipzig: B. G. Teubner, 1895.
- Velimirović, Miloš. "Échos." In *Grove Music Online*. Oxford University Press, 2001. <https://doi.org/10.1093/gmo/9781561592630.article.08518>.
- Weakland, Rembert. "Hucbald as Musician and Theorist." *The Musical Quarterly*, Vol. 42, no. 1 (Jan. 1956): 66–84. Oxford University Press. <https://www.jstor.org/stable/740475>.
- Wellesz, Egon. *A History of Byzantine Music and Hymnography*. 2nd Edition. Oxford: Clarendon Press, 1961.
- West, M. L. *Ancient Greek Music*. Oxford: Clarendon Press, 1992.
- Willis, James. *Martianus Capella*. Bibliotheca scriptorum graecorum et romanorum Teubneriana. Leipzig: B. G. Teubner Verlagsgesellschaft, 1983.
- Winnington-Ingram. *Mode in Ancient Greek Music*. London: Cambridge University Press, 1936.
- Yampolsky, Asher Vijay. "Carolingian Conceptions of Mode: Exploring Modal Significance and Signification." *Chant: Old and New, Proceedings of the Conference of the Gregorian Institute of Canada, Dalhousie University, 4–7 August 2011*: 137–47. Musicological Studies Vol. 98. Lions Bay, Canada: The Institute of Medieval Music, 2012.

V. Reference Works, Non-Musical Resources, and Databases

- Bischoff, Bernhard. *Latin Palaeography: Antiquity and the Middle Ages*. Trans. Dáibhí Ó Cróinín and David Ganz. Cambridge: Cambridge University Press, 1990.
- Cantus: A Database for Latin Ecclesiastical Chant – Inventories of Chant Sources*. University of Waterloo. <http://cantus.uwaterloo.ca/>.
- Cappelli, Adriano. *The Elements of Abbreviation in Medieval Latin Paleography*. Trans. David Heimann and Richard Kay. Lawrence, Kansas, USA: University of Kansas Libraries, 1982.
- . *Lexicon abbreviatarum quae in paucibus, codicibus et chartis praesertim medii-aevi occurrunt: Dizionario di Abbreviature Latine ed Italiane*. Milan, Editore-Libraio della real casa, 1899.
- Clemens, Raymond, and Timothy Graham. *Introduction to Manuscript Studies*. Ithaca: Cornell University Press, 2007.
- Drogin, Marc. *Medieval Calligraphy: Its History and Technique*. New York: Dover Publications, Inc., 1980.
- Gregorian Repertory*. Diözese Rottenburg-Stuttgart. <http://gregorien.info/>.
- Grier, James. "Lachmann, Bédier and the Bipartite Stemma: Towards a Responsible Application of the Common-Error Method." *Revue d'histoire des textes*, Tome XVIII (1988): 263–78.
- Grove Music Online*. Oxford University Press. <https://www.oxfordmusiconline.com/grovemusic/>.
- Helmer, Friedrich. *Katalog der Handschriften aus dem Benediktinerkloster St. Emmeram in Regensburg*. Munich: Bayerische Staatsbibliothek, 2009.
- Lewis, Charlton T., and Charles Short. *A Latin Dictionary, Founded on Andrews' Edition of Freund's Latin Dictionary*. Oxford: Clarendon Press, 1879.
- Liddell, Henry George, and Robert Scott. *A Greek-English Lexicon*, 8th ed. New York: American Book Company, 1882.
- Perseus Digital Library*. Tufts University. <https://www.perseus.tufts.edu/hopper/>
- Poorter, A. de. *Catalogue des manuscrits de la Bibliothèque publique de la ville de Bruges*. Gembloux, Belgium: Duculot, 1943.
- Prou, Maurice. *Manuel de paléographie latine et française du VI^e au XVII^e Siècle*. Paris: Libraire des Archives nationales et de la Société de l'École des Chartes, 1890.
- Sidwell, Keith. *Reading Medieval Latin*. Cambridge: Cambridge University Press, 1995.
- Sources manuscrites de la théorie de la musique (S. IX–XVI)*. Répertoire International des Sources Musicales. <http://musmed.fr/RISM/rismindex01.htm>
- Thesaurus musicarum latinarum*. Jacobs School of Music, Indiana University. Center for the History of Music Theory and Literature. <http://www.chmtl.indiana.edu/tml/>

Appendix A

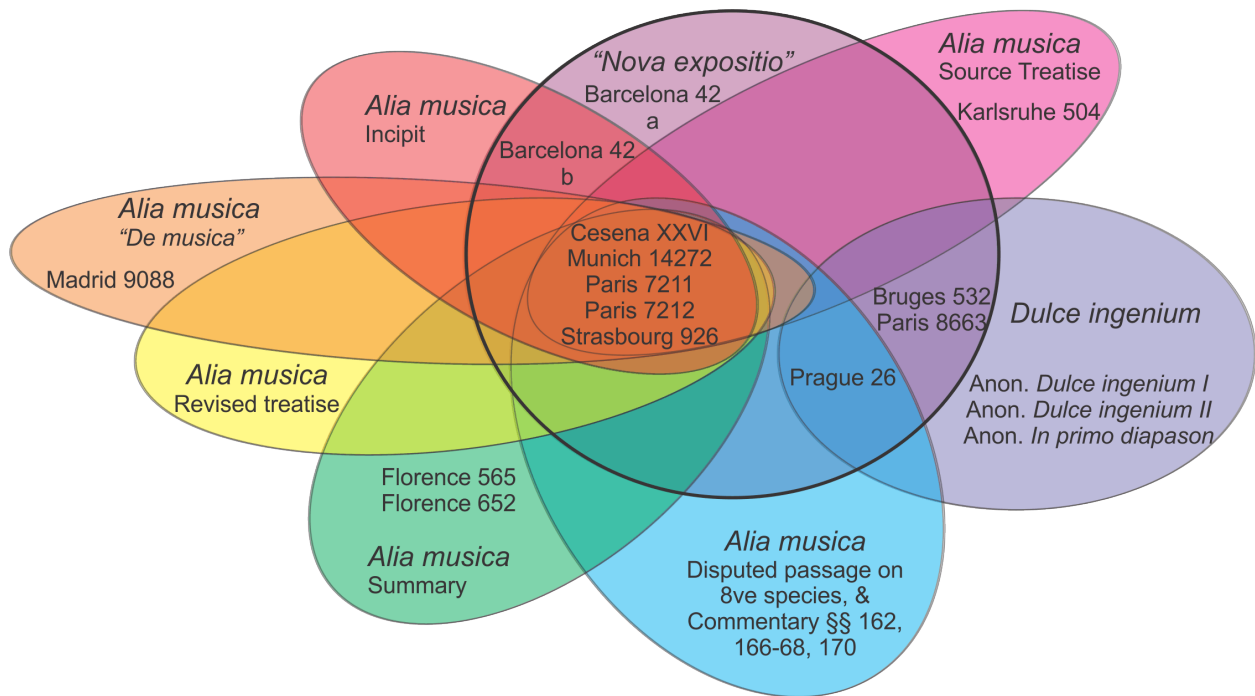
Mühlmann's Segmentation of the *Alia musica*

The following table is reproduced from Chailley, *Alia musica*, 11. I have restored Mühlmann's original German (*e.e.*) descriptions for each section and provided English translations.

<i>Theohretiker der Acht Modi</i>	<i>Theoretiker der Principales und Suiugales</i>	<i>Theoretiker der Sechs Töne</i>	<i>Überarbeiter</i>	<i>Kommentar</i>	<i>Zahlenauszug/ Tabelauszug</i>	<i>Tonarius</i>
Theorist of Eight Modes	Theorist of Authentics and Plagals	Theorist of Six Tones	Revisor	Commentary	Numerical Summary/ Tabular Summary	Tonary <i>Nova Expositio</i>
	§§1–12					
§§13–20				§§21–30		
		§31†		31†–34		
		§§35–36		37–40		§§41–43
		45–48†		48†–53		54–57
		58–61†		61†–76		
		77–83		84–89		90–98
		99–100		101–07		108–11
		112–13		114–21		122–25
				126–32		
	133–41			143–45		146–53
		154–61	§162			
		163–64		165–70	§§171–80	
		181–86	§§187–88			

† Chailley identifies these passages as having been split across two sections in Mühlmann's translation, between the Theorist of Six Tones and the Commentary.

*Variant forms from the revised *Nova expositio*. Where given as ×2, the standard form is also present.



Appendix C

Comparison of the two recensions of the *Dulce ingenium*

Begins	Ends	Pa	Br	Pr	Concordances
<i>Dulce ingenium...</i>	<i>... parte prioris.</i>	1–7	1–7	1–7	Bernhard indicates that some passages are based on Boethius.
<i>Et sicut constat...</i>	<i>...mixolydius, ypermixolydius.</i>	8–9	8–9		
<i>In primo diapason...</i>	<i>... tertium F quadruplo.</i>			8–21	In Gerbert, <i>Scriptores ecclesiastici I</i> , 121a–22a, immediately after Hucbald's <i>De institutione musica</i> ; also, 329a–330a, shortly after Bernelinus. Corresponds better to the latter instance, which shares with <i>Dulce ingenium</i> a two-octave F scale at end of passage missing in early instance.
<i>PROSLAMBANOMEN OS adquisitus...</i>	<i>... se distantes tono</i>			23–50	Excerpts from Regino <i>Epistola</i> . Some sections are pure excerpts, others are not (but Bernhard indicates that they are based upon the <i>Epistola</i>).
<i>Dorius habet melodiam...</i>	<i>... habens in lycanos meson.</i>			51–58	Rekurs in Pr, 18 folia after the end of <i>Nova expositio</i> . Also found in <i>Fragmenta musica</i> . * Notable for the fact that the ecclesiastical modes are named here exclusively by Greek ethnic names.
<i>Dicendum est...</i>	<i>... et ditonium.</i>			59–71	Excerpts from Regino <i>Epistola</i> . Some sections are pure excerpts, others are not (but Bernhard indicates that they are based upon the <i>Epistola</i>).
<i>Quorum videlicet troporum...</i>	<i>... vel dyatesseron et tonus.</i>			80–87	<i>Alia musica</i> §134–41. This is most of the <i>disputed passage</i>.
<i>Tonorum alios...</i>	<i>... valentes ac simplices.</i>			88–93	Excerpts from the supplementary material in the <i>Alia musica</i>, in the following order: §§166–68(a), 170(a), 162(a), 170(d)–(e).
<i>Ita autem oportet... / Oportet igitur ita...</i>	<i>...simphonicas voces informare.</i>	10–13	10–13	94–97	Passages based on Boethius and Martianus Capella.
<i>Haec quoque nomina...</i>	<i>... id est inadunata.</i>	14–17	14–17		

Begins	Ends	Pa	Br	Pr	Concordances
<i>Et est notandum...</i>	<i>... mutantur genera.</i>	18–19	18–19	98–99	
<i>Ypate meson dicitur...</i>	<i>... acutior exurgat.</i>	20–33	20–33		
<i>Quiquid autem...</i>	<i>...emitonia duo. / ...dieses quattuor.</i>	34–38	34–38	100–104	
<i>Symphoniae autem...</i>	<i>... dieses LIII</i>	39–44	39–44		Passage comparable to Martianus Capella.
<i>Igitur ipsas species...</i>	<i>... quarto loco semitonium.</i>	45–52			Bernhard notes that a space is left for this material in Br, but it is not completed.
<i>A prima quoque...</i>	<i>DE OCTAVO... In diurnis autem... seculorum amen. / Domine ne longe.</i>	53–85	53–85	105–128	<i>Nova expositio. This is the entire tonary.</i> It is notable that in Pr, the final section of each mode, describing the doxology, is omitted.

*<http://www.chmtl.indiana.edu/tml/13th/ANOFRA>.

Appendix D

Variations in modal coefficients across manuscripts

Mode I								
		C	A	G	P ₁	P ₂	M	K
Source	^a XII	1		1	1	1	1	1
	^c VI	1		1	1	1	1	1
	^b VIII	1		1	1	1	1	1
	^d VIII	1		1	1	1	1	1
Revision	^a XII	1		1	1	1	1	
	^c VI	1		1	1	1	1	
	^b VIII	1		1	1	1	1	
	^d VIII	1		1	1	1	1	
Table	^a XII	1		1	1	1	1	
	^c VI	1		1	1	1	1	
	^b VIII	1 ⁸¹⁹		1 ⁸²⁰	1	1	1	
	^d VIII	1		1	1	1	1	
Summary	^a XII	1		1	1	1	1	
	^c VI	1		1	1	1	1	
	^b VIII	1		1	1	1	1	
	^d VIII	1		1	1	1	1	

Mode II								
		C	A	G	P ₁	P ₂	M	K
Source	^a XII	1		1	1	1	1	1
	^c VI	3		3	3	3	3	3
	^b VIII	2		2	2	2	2	2
	^d VIII	2		2	2	2	2	2
Revision	^a XII	1		1	1	1	1	
	^c VI	3		3	3	3	3	
	^b VIII	2		2	2	2	2	
	^d VIII	2		2	2	2	2	
Table	^a XII	1		1	1	1	1	
	^c VI	3		3	3	3	3	
	^b VIII	2		2	2	2	2	
	^d VIII	2		2	2	2	2	
Summary	^a XII	1		1	1	1	1	
	^c VI	3		3	3	3	3	
	^b VIII	2		2	2	2	2	
	^d VIII	2		2	2	2	2	

⁸¹⁹ The letter above XII here is B, instead of D.

⁸²⁰ The letter above XII here is B, instead of D, an error copied directly from P₁.

Mode III								
		C	A	G	P ₁	P ₂	M	K
Source	^a XII	1		1	1	1	1	1
	^c VI	4		4	4	4	4	4
	^b VIII	1 ⁸²¹	∅ ⁸²²	∅ ⁸²³	3	3	3	3 ⁸²⁴
	^d VIII	2		2	2	2	2	2
Revision	^a XII	1		1	1	1	1	
	^c VI	4		4	4	4	4	
	^b VIII	3 ⁸²⁵		3	3	3	3	
	^d VIII	2		2	2	2	2	
Table	^a XII	1 ⁸²⁶		1	1	1	1	
	^c VI	3		4	3	3	4	
	^b VIII	2		3	3	3	3	
	^d VIII	2		2	2	2	2	
Summary	^a XII	1		1	1	1	1	
	^c VI	4		4	4	4	4	
	^b VIII	3		3	3	3	3	
	^d VIII	2		2	2	2	2	

Mode IV								
		C	A	G	P ₁	P ₂	M	K
Source	^a XII	2		2	2	2	2	2
	^c VI	3		3	3	3	3	3
	^b VIII	3		3	3	3	3	3 ⁸²⁸
	^d VIII	2		2 ⁸²⁷	2	2	2	2
Revision	^a XII	2		2	2	2	2	
	^c VI	3		3	3	3	3	
	^b VIII	3		3	3	3	3	
	^d VIII	2		2	2	2	2	
Table	^a XII	2		2	2	2	2	
	^c VI	3		3	3	3	3	
	^b VIII	2		3	2	3 ⁸²⁹	3	
	^d VIII	2		2	2	2	2	
Summary	^a XII	2		2	2	2	2	
	^c VI	3		3	3	3	3	
	^b VIII	2		3	2	2	3 ⁸³⁰	
	^d VIII	2		2	2	2	2	

⁸²¹ The coefficient is missing here.

⁸²² Presumably. See fn. 823.

⁸²³ In Gerbert's edition, "*ter viii*" is missing and a single vi appears in its place. This is presumably an attempted correction, as the text says that the three eights to twelve should be a double proportion, which is not correct, while a single six to twelve is a double proportion. However, there is reason to suspect that Gerbert has also had a hand in furthering this correction, as C has a single eight instead of a single 6 (cf. C in fn. 821)

⁸²⁴ In this manuscript, both letters and numbers are presented, in that order; the letter C is correctly given three times, but only two eights follow.

⁸²⁵ The coefficient ("*ter*") is missing from the body of the text and is corrected above the line.

⁸²⁶ All the numbers for Mode III in the table in C are actually the numbers for Mode II.

⁸²⁷ The first presentation of this coefficient contains a typo (acknowledged by a footnote) indicating that it should apply to viii instead of viiii, but subsequent presentations are correct.

⁸²⁸ In subsection (c), "*ter viii*" is given as "*ter viiii*", but this error is contradicted by the opening list.

⁸²⁹ Here, the third VIII contains crossbars above and below (while no other number does), and the letter B is missing; this number is missing altogether in P₁, indicating that this is a correction.

⁸³⁰ The third VIII has no letter over it; it is underlined, and there is a mark resembling an inverted comma placed above it. It would be difficult to explain as a correction made after the fact, since there would not have been room, but it could be the result of copying out a correction that had been made in the model manuscript. It is interesting to note that it is extremely reminiscent of a similar oddity for the same figure in the table in P₂.

Mode V								
		C	A	G	P ₁	P ₂	M	K
Source	^a XII	3		3	3	3	3	2
	^c VI	4		4	4	4	4	4
	^b VIII	3		3	3	3	3	3
	^d VIII	4 ⁸³¹		4	4	4	4 ⁸³²	4 ⁸³³
Revision	^a XII	3 ⁸³⁴	835	3	3	3	3	
	^c VI	4		4	4	4	4	
	^b VIII	3		3	3	3	3	
	^d VIII	4		4	4	4	4	
Table	^a XII	3 ⁸³⁶		3	3	3	3	
	^c VI	4	4	4	4	4	4	
	^b VIII	4	4	3	4	4	4	
	^d VIII	4		4	4	4	4	
Summary	^a XII	3		3	3	3	3	
	^c VI	3		3	3	3	3	
	^b VIII	3		3	3	3	3	
	^d VIII	3		2	3	3	2	

Mode VI								
		C	A	G	P ₁	P ₂	M	K
Source	^a XII	2		2	2	2	2	2
	^c VI	6		6	6	6	6	6 ⁸³⁹
	^b VIII	∅ ⁸³⁷	∅ ⁸³⁸	3	∅	∅	∅	3 ⁸⁴⁰
	^d VIII	4		4	4	4	4	4 ⁸⁴¹
Revision	^a XII	2		2	2	2	2	
	^c VI	6		6	6	6	6	
	^b VIII	3		3	3	3	3 ⁸⁴³	
	^d VIII	4 ⁸⁴²		4	4	4	4	
Table	^a XII	2		2	2	2	2	
	^c VI	6		6	6	6	6	
	^b VIII	3		3	3	3	3	
	^d VIII	3		4	4	4	4	
Summary	^a XII	2		2	2	2	2 ⁸⁴⁴	
	^c VI	6		6	6	6	5	
	^b VIII	3		3	3	3	3	
	^d VIII	4		3	4	4	3	

⁸³¹ The first presentation of this coefficient is applied to an viii rather than a viiii; however, the second presentation is correct. This error appears in every manuscript of the principal group except G, but including C; it is unclear whether the correction was made in A or silently corrected by Gerbert.

⁸³² The error here is of the same kind as in fn. 831, except compounded again, as the base number is given as vii rather than either viii or viiii, with one additional i added as a partial correction above the line, which still does not give the correct value of viiii.

⁸³³ In this manuscript only, both letters and numbers are presented, in that order. While four Ds are given, only three nines follow.

⁸³⁴ The coefficient (“*ter*”) is missing, and is replaced by the letter *c*, presumably a misreading of a illegibly written abbreviation \bar{c} .

⁸³⁵ According to Gerbert’s footnote, the coefficient (“*ter*”) is missing, and is replaced by the letter *c* (*cf.* source C in fn. 834). Gerbert acknowledges the correct that is implied by context but does not acknowledge the correct reading in M.

⁸³⁶ The letter *a* is missing from above the third XII.

⁸³⁷ The base number here is consistently given as viiii instead of viii in every manuscript, including K, but not in Gerbert’s edition, suggesting that it is correct in A; however, Gerbert does not acknowledge the error in M, and it is therefore likely that it is given as viiii in all manuscripts, including A, and that Gerbert has silently corrected this error.

⁸³⁸ Presumably. See fn. 837.

⁸³⁹ Uniquely in this mode, the letters and numbers for this base number are divided into two subgroups: CCC 666 CCC 666, instead of giving all six letters first followed by all six numbers, as is the practice everywhere else.

⁸⁴⁰ This coefficient is given incorrectly in subsection (c), as in the other manuscripts (see fn. 837); however, the correct coefficient is given in the opening list.

⁸⁴¹ In this manuscript only, both letters and numbers are presented, in that order. While four Ds are given, only three nines follow.

⁸⁴² Stated twice; on the second instance, given as four eights instead of four nines.

⁸⁴³ Missing from main text but added in the margin.

⁸⁴⁴ All the letters over the numbers are missing in this mode in the manuscript.

Mode VIIa								
		C	A	G	P ₁	P ₂	M	K
Source	^a XII							2
	^c VI							3
	^b VIII							3
	^d VIII							2
Revision	^a XII	2		2	2	2	2	
	^c VI	3		3	3	3	3	
	^b VIII	3		3	3	3	3	
	^d VIII	2		2	2	2	2	
Table	^a XII	2		2	2	2	2	
	^c VI	2	2	3	3	3	3	
	^b VIII	3		3	3	3	3	
	^d VIII	2		2	2	2	2	
Summary	^a XII	2		2	2	2	2	
	^c VI	3		3	3	3	3	
	^b VIII	3		3	3	3	3 ⁸⁴⁶	
	^d VIII	2 ⁸⁴⁵		2	2	2	2	

Mode VIIb								
		C	A	G	P ₁	P ₂	M	K
Source	^a XII							2 ⁸⁴⁷
	^c VI							4
	^b VIII							3
	^d VIII							4
Revision	^a XII	2 ⁸⁴⁸		2	2	2	2	
	^c VI	6		6	6	6	6	
	^b VIII	∅		∅	∅	∅	∅	
	^d VIII	4		4	4	4	4	
Table	^a XII	3		3	3	3	3	
	^c VI	4		4	4	4	4	
	^b VIII	3	3	3	3	3	3	
	^d VIII	3	3	4	4	4	4	
Summary	^a XII	3		3	3	3	3	
	^c VI	3		4	4	4	4 ⁸⁵⁰	
	^b VIII	3		3	3	3	3	
	^d VIII	2 ⁸⁴⁹		4	4	4	4	

⁸⁴⁵ After the four expected sets of numbers, the manuscript repeats the three eights and the two nines, but does not include letters over them.

⁸⁴⁶ The letters are missing above all the VIIIs in this part of this manuscript.

⁸⁴⁷ This coefficient appears to be an error, but the coefficients for this mode meant to be repeated from Mode V, and indeed, the same error appears in Mode V in this manuscript.

⁸⁴⁸ According to the text, these coefficients should be those of the fifth mode, but instead, the passage from the sixth mode is repeated, and the citation does not continue far enough to reach the coefficient for 8.

⁸⁴⁹ The sequence of coefficients here is dramatically different from any other manuscript. The only other place where this sequence is given is in the prose summary description of Mode V (which the second presentation of Mode VII is supposed to reflect), but interestingly, not in this manuscript; this sequence

appears for Mode V only in M and G (and therefore, presumably A). The implication is that the common model of both A and C noticed the disagreement between Modes V and VIIb (which are supposed to be the same) and copied VIIb from V. Then the scribe of C recognized that the series for Mode V was also incorrect in the model and corrected it in the copy but did not do so when the same sequence reappeared in Mode VIIb. This explanation presents a slight difficulty, as C "corrects" the sequence in Mode V to yet another incorrect sequence that also appears in P (a more distantly related source) and it is still substantially different from the correct coefficients. The implication is that either C or its model were corrected against some other manuscript, rather than against the number series as it is presented elsewhere in the treatise.

⁸⁵⁰ Letters above numbers are present only for the XIIIs in this part of the manuscript.

Mode VIIc								
		C	A	G	P ₁	P ₂	M	K
Source	^a XII	12 ⁸⁵¹		12	12	12	12	4
	^c VI	∅		∅	∅	∅	∅	8 ⁸⁵²
	^b VIII	∅		∅	∅	∅	∅	6
	^d VIII	∅		∅	∅	∅	∅	∅
Revision	^a XII	4		4	4	4	4	
	^c VI	4		4	4	4	4	
	^b VIII	6		6	6	6	6	
	^d VIII	∅		∅	∅	∅	∅	
Table	^a XII	4		4	4	4	4	
	^c VI	8		8	8	8	8	
	^b VIII	6	∅	6	6	6	6	
	^d VIII	∅		∅	∅	∅	∅	
Summary	^a XII	4		4	4	4	4 ⁸⁵⁴	
	^c VI	8		8	8	8	8	
	^b VIII	6 ⁸⁵³		6	6	6	6	
	^d VIII	∅		∅	∅	∅	∅	

Mode VIIIa								
		C	A	G	P ₁	P ₂	M	K
Source	^a XII							2
	^c VI							3
	^b VIII							3
	^d VIII							2
Revision	^a XII							
	^c VI							
	^b VIII							
	^d VIII							
Table	^a XII	2		2	2	2	2	
	^c VI	3		3	3	3	3	
	^b VIII	3		3	3	3	3	
	^d VIII	2		2	2	2	2	
Summary	^a XII	2		2	2	2	2 ⁸⁵⁸	
	^c VI	3		3	3	3	3	
	^b VIII	3 ⁸⁵⁵		3	3	3	3	
	^d VIII	2 ⁸⁵⁶		2 ⁸⁵⁷	2	2	2	

⁸⁵¹ Except in K, the source treatise gives only the sum 144, which is listed as twelves twelves; this presentation is probably not meant to be the coefficient for twelve, but I have included it here since it is the only number given.

⁸⁵² In this manuscript only, both letters and numbers are presented, in that order. While there are, indeed, eight sixes, they appear to be preceded by only six Cs (though I am not certain in this instance because of the proximity of the margin and the low resolution of my facsimile).

⁸⁵³ The final viii in this mode may possibly be a correction. It is in its correct place, but its correct place appears at the end of a line, and the second last viii

extends right up to the margin, placing the final viii entirely in the margin, while the shorter number vi could easily have appeared at the end of the line above without extending so far into the margin but did not, which gives the impression of an afterthought.

⁸⁵⁴ Letters are missing above all the numbers in this mode in this part of the manuscript.

⁸⁵⁵ The final viii was apparently missing here, added above the line as a correction.

⁸⁵⁶ The letters are missing above the two instances of viiii.

⁸⁵⁷ Letters are missing over these.

⁸⁵⁸ All the letters over the numbers are missing in this mode in the manuscript.

Mode VIIIb								
		C	A	G	P ₁	P ₂	M	K
Source	^a XII							3
	^c VI							4
	^b VIII							3 ⁸⁵⁹
	^d VIII							4 ⁸⁶⁰
Revision	^a XII							
	^c VI							
	^b VIII							
	^d VIII							
Table	^a XII	3		3	3	3	3	
	^c VI	4	3	4	4	4	4	
	^b VIII	3		3	3	3	3	
	^d VIII	4 ⁸⁶¹		4	4	4	4	
Summary	^a XII	3		3	3	3	3 ⁸⁶³	
	^c VI	4		4	4	4	4	
	^b VIII	3		3	3	3	3	
	^d VIII	4 ⁸⁶²		4	4	4	4	

Interval labels within the table cells are consistently absent in Mode VIIb and Mode VIIc.

The letter above VI in the Tabular Summary is given as b in C, but as B in M and P. G is inconsistent (b in Modes IV and V, B elsewhere). This inconsistency seems to reflect the fact that all the letters above the numbers are absent in M (Gerbert's principal source for the table) for the last three rows of Mode IV and all of Mode V. Gerbert seems to have substituted for these letters from A, which implies that A also uses the minuscule b, as in source C, instead of the majuscule B as in the remaining sources.

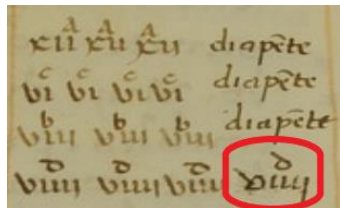
Every single manuscript contains the same error in Mode V in the table, listing VIII four times. Gerbert notes the existence of the error in A but misses the error in M and instead gives the correct value.

The sequence of coefficients for each mode is omitted in F, but is implied by the comparisons and sums for each mode, which are given in the next appendix.

⁸⁵⁹ In this manuscript only, both letters and numbers are presented, in that order. There are three Bs, but only two eights follow.

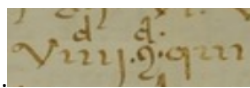
⁸⁶⁰ Uniquely amongst all of the direct lists of coefficients, which in the source treatise appear only in K, this coefficient only is given as *quater* instead of listing the nine four times, however, the preceding letter D is written four times.

⁸⁶¹ The final viiii is written beyond its column and appears to be in a different hand, implying that it may perhaps have been a correction. It corresponds to the same position as the error in the corresponding chart for Mode VIIb, though there is no indication in Gerbert that the same error exists in A.



⁸⁶² This final viiii is followed immediately by an apparent abbreviation that is difficult to interpret in

the form $\frac{d}{2}$ or perhaps $\frac{d}{4}$; it could potentially indicate a word like *quid* or *quod*, but this would not fit with the following word, *qui*. It is also followed by a punctuation point, marking it as the end of an idea, rather than the beginning. It is not clear why it is there, though there is a vague resemblance to the way that the v is drawn in other Roman numerals in this manuscript, implying perhaps an aborted additional number, a possibility supported by the fact that the letter d above the principal symbol is the same letter that would be expected above another viiii; however, the resemblance of the symbol to a V is far too loose to be confident of this hypothesis, and the punctuation point beside the d is also unexpected.



(f. 193r).

⁸⁶³ All the letters over the numbers are missing in this mode in the manuscript.


Appendix E

Variation in sums of modal numbers across manuscripts

Prose Summary										
				C	A	G	P ₁	P ₂	M	F
Mode I	1:1	VI:XII	8 ^{ve}	18		18	18	18	18	19
	1:1	VIII:XII	5 th	20		20	20	20	20	20
	1:1	VIII:XII	4 th	21		21	21	21	21	21
Mode II	3:1	VI:XII	5 th	30		30	30	30	30 ⁸⁶⁵	30
	2:1	VIII:XII	5 th	30		30	30	30	30 ⁸⁶⁶	30
	2:1	VIII:XII	4 th	28 ⁸⁶⁴		28	28	28	28	28
Mode III	4:1	VI:XII	8 ^{ve}	36		36	36	36	36	36
	2:1	VIII:XII	5 th	30		30	30	30	30	30
Mode IV	3:3	VI:VIII	4 th	42 ⁸⁶⁷		42	42	42	42	42
	2:2	VIII:XII	4 th	42		42	42	42	42	42
Mode V	3:3	VIII:XII	5 th	36 ⁸⁶⁸		36!	36!	36!	36!	36! ⁸⁷⁰
	4:4	VI:VIII	5 th	40		60	40	40	60 ⁸⁶⁹	40
Mode VI	6:2	VI:XII	8 ^{ve!}	60		60	60	60	60	60
	3:2	VIII:XII	8 ^{ve!}	871		48	48	48	48	40 ⁸⁷³
	4:2	VIII:XII	5 th			60	60	60	60 ⁸⁷²	

⁸⁶⁴ The comparison is incomplete as it is given in the body of the text, with the final portion, “*ad xii*” added in the margin.

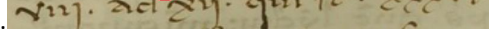
⁸⁶⁵ This number is difficult to read and appears to have originally given as xx , with an additional x squeezed in

between them by way of correction:  (f. 181r).

⁸⁶⁶ See endnote 866, above.

⁸⁶⁷ The comparison is incomplete, with the word *ter* missing before the vi, which is then added as a gloss above the line.

⁸⁶⁸ The comparison is incomplete, with the word *ter* missing before the xii. There is a mark added above the line

which may be an attempt at correction, though it is difficult to read. 

(f. 192v) More importantly, the sum of thirty-six that is given reflects the passage without the corrected coefficient. In all the other manuscripts, the coefficient is present, and yet the sum is still given as thirty-six.

⁸⁶⁹ Here, the scribe gives “*ter vi*,” not the four sixes that are required to give the correct sum. The scribe appears to have noticed that only three sixes are given in the preamble to the mode, and instead of correctly identifying it as an error, “corrects” the comparison without correcting the resulting sum.

⁸⁷⁰ In this manuscript, not only does the sum not agree with the terms, but the coefficient for xii is given as two, rather than the correct value of three or being omitted as in other manuscripts.

⁸⁷¹ The remaining comparisons in this mode are missing in this manuscript; since the first and last comparison give the same sum, it is likely that the scribe simply confused one for the other.

⁸⁷² Here, the scribe gives “*ter viii*,” not the four nines that are required to give the correct sum, making the same kind of “correction” described in endnote 869 above.

⁸⁷³ This sum of forty is given as the sum of three eights plus two twelves, which should come out to forty-eight. However, the final comparison is absent. The Roman numeral for forty, *xl*, is followed by a punctuation dot,

Mode VIIa			4 th	42		42	42	42	42	42
Mode VIIb			5 th	60		60	60	60	60 ⁸⁷⁴	40 ⁸⁷⁵
Mode VIIc	8:6:4	VI:VIII:XII	8 ^{ve} !	144		144	144	144	144	144 ⁸⁷⁶
Mode VIIla	$n:n$ ⁸⁷⁷	VI:VIII	4 th	42	$7x$ ⁸⁷⁸	42 ⁸⁷⁹	42	42	42	42 ⁸⁸⁰
	2:2	VIII:XII	4 th	42		42	42	42	42	
Mode VIIlb	4:4	VIII:VI	⁸⁸¹	60		60	60	60	60 ⁸⁸³	60
	3:3	VIII:XII		<u>60</u> ⁸⁸²		<u>60</u>	<u>60</u>	<u>60</u>	<u>60</u>	<u>60</u>
				120		120	120	120	120	120 ⁸⁸⁴

implying that it was not merely truncated. Instead, it is more likely that the scribe skipped over from the second comparison to the third product, and that the sum forty (xl) is a misreading of sixty (lx) rather than of forty-eight (xlviii).

⁸⁷⁴ Here, there is clear evidence of a correction, as the L from LX is written above the line, where insufficient space was available.

⁸⁷⁵ This is probably not an independent error, but rather a consequence of the fact that Mode VIIb is supposed to be the same as Mode V; most of the manuscripts give forty (xl) instead of the correct value of sixty (lx) for Mode V, but only F copies this incorrect sum over to Mode VIIb.

⁸⁷⁶ The number one hundred forty-four is misspelled here as xcliuii (instead of cxliiii); it does not, however, produce a misreading, as the form given does not produce any correctly-spelled Roman numeral.

⁸⁷⁷ The coefficients here ought to be 3:3, but they are omitted in all extant manuscripts.

⁸⁷⁸ Gerbert gives “*ter viii*” in this passage; this number is not present in any other surviving manuscript, and may be supposed to be given thus in A; however, he provides no footnotes indicating the absence of the coefficient in M, and it is also not present in C; therefore, it is also reasonably likely to have been added by Gerbert himself (though, if he had corrected it, it is not clear why he did not also supply the same coefficient for 8).

⁸⁷⁹ See endnote 878.

⁸⁸⁰ Only half of the comparison is given in F, which does not confirm the correct coefficients, only the product.

⁸⁸¹ The comparison here is given over the course of two lines; in the first, the proportion is usually incorrectly labelled as a *sesquitercia*, which corresponds to a fourth, rather than a fifth, and contradicts the label *denarii* that follows, a label that the *Alia* uses as corresponding to a fifth. In M, the proportion is corrected to *sesquialtera*, (also in G, though Gerbert is not clear whether it is his correction or the result of comparing against M), while in P, the word *dyapente* is added for clarification without correcting the proportion; this correction also is added to the margins in C, in a position in which it would be difficult for someone only consulting C to interpret where it belongs.

⁸⁸² Following the first of two lines that make up the comparison, there is in this manuscript is a marginal correction adding the word *dyapente* without correcting the proportion *sesquitercia*; this correction also occurs in P, directly in the text rather than in the margin, providing more evidence that P (or a related source) was consulted in the copying of C. cf. endnote **Error! Bookmark not defined.**, above.

⁸⁸³ Several words are missing here and are supplied above the text as a correction.

⁸⁸⁴ See endnote 880.

Revision									
				C	A	G	P ₁	P ₂	M
Mode I	1:1	VI:XII	8 ^{ve}	18		18	18	18	18
	1:1	VIII:XII	5 th	20		20	20	20	20
	1:1	VIII:XII	4 th	21		21	21	21	21 ⁸⁸⁵
Mode II	3:1	VI:XII	5 th	30		30	30	30	30
	2:1	VIII:XII	5 th	"		"	"	"	"
	2:1	VIII:XII	4 th	28		28	28	28	28
Mode III	4:1	VI:XII	8 ^{ve}	36		36	36	36 ⁸⁸⁶	36
	3:1	VIII:XII	8 ^{ve}	"		"	"	"	"
	2:1	VIII:XII	5 th	30		30	30	30	30
Mode IV	3:3	VI:VIII	4 th	42		42	42	42	42
	2:2	VIII:XII	4 th	42		42	42	42	42
Mode V	3:3	VIII:XII	5 th	60		60	60	60	60
	4:4	VI:VIII	5 th	60		60	60	60	60
Mode VI	6:2	VI:XII	5 th	-		-	-	-	-
	6:4	VI:VIII	8 ^{ve} !	72		72	72	72	72 ⁸⁸⁷
	3:2	VIII:XII	8 ^{ve} !	44		48	48	48	48
	4:2	VIII:XII	5 th	-		-	-	-	-
Mode VIIa	2:2	VIII:XII	4 th	42		42	42	42	42
	3:3	VI:VIII	4 th	42		42	42	42	42
Mode VIIb	6:2	VI:XII	5 th	60		60	60	60	60
	4:2	VIII:XII	5 th	60		60	60	60	60
Mode VIIc	4:4	VI:XII	8 ^{ve}	48!		48!	48!	48!	48!
	2:6	XII:VIII	8 ^{ve}	48!		48!	48!	48!	48!

In this layer of the treatise, sums are frequently preceded by a subsidiary calculation that expresses the sum as a multiple of smaller numbers (for instance, $6 + 12 = 3 \times 6 = 18$). I do not include the intermediate calculations in the table, but if something is wrong with them, I acknowledge them in the footnotes.

⁸⁸⁵ The twelve here is incorrectly given as xi.

⁸⁸⁶ The subsidiary calculation four nines is missing here, but the sum of 36 is given.

⁸⁸⁷ Given as 82, then corrected.

Source Treatise										
				C	A	G	P ₁	P ₂	M	K
Mode I	1:1	VI:XII	8 ^{ve}	18		18	18	18	18	18
	1:1	VIII:XII	5 th	20		20	20	20	20	20
	1:1	VIII:XII	4 th	21		21	21	21	21	21
Mode II	3:1	VI:XII	5 th	30		30	30	30	30	30
	2:1	VIII:XII	5 th	30		30	30	30	30	30
	2:1	VIII:XII	4 th	28		28	28	28	28	28
Mode III	4:1	VI:XII	8 ^{ve}	36	⁸⁸⁸	36	36	36	36	36
	3:1	VIII:XII	8 ^{ve}	-	-	-	"	"	"	"
	2:1	VIII:XII	5 th	30		30	30	30	30	30
Mode IV rejects	2:3	XII:VIII	8 ^{ve}	-		-	-	-	-	-
	3:2	VI:VIII	8 ^{ve}	24! ⁸⁸⁹		24!	24!	24!	24!	24!
Mode IV	3:3	VI:VIII	4 th	-		-	-	-	-	-
	2:2	VIII:XII	4 th	-		-	-	-	-	-
Mode V	3:3	VIII:XII	5 th	-		-	-	-	-	-
	4:4	VI:VIII	5 th	-		-	-	-	-	-
Mode VI	6:2	VI:XII	5 th	-		-	-	-	-	-
	6:4	VI:VIII	8 ^{ve!}	82	72? ⁸⁹⁰	72	82	82	72	72
	3:2	VIII:XII	8 ^{ve!}	48		48	48	48	48	48
	4:2	VIII:XII	5 th	-		-	-	-	-	-
Mode VIIc	12	XII	8 ^{ve}	144		144	144	144	144	144

In this layer of the treatise, sums are frequently preceded by a subsidiary calculation that expresses the sum as a multiple of smaller numbers (for instance, $6 + 12 = 3 \times 6 = 18$). I do not include the intermediate calculations in the table, but if something is wrong with them, I acknowledge them in the footnotes.

⁸⁸⁸ The correct sum of 36 is given, but is given in a subcalculation as four eights instead of four nines. Gerbert retains this error in his edition but corrects it in a footnote based on M. The same error appears in C and P, but not in K.

⁸⁸⁹ This sum is extremely convoluted. The sum given in each manuscript is 24, which is not actually a sum, but an acknowledgment that the two modal numbers are the same. They should be eighteen, and Chailley corrects it this way, even though it is not so in any manuscript, but 24 would make sense with the previous set in the same sentence, which behaves the same way but for which no sum is given. I refer to this section as rejects as they are the numbers that the *Alia* says must not be compared to one another, but I include them here anyway because the *Alia* does compare them to make a point.

⁸⁹⁰ Most of the manuscripts give 82 instead of 72, but not M. Gerbert gives 72, and makes no footnote to indicate that he has accepted M's reading in preference to A, suggesting that A also gives 72; however, C gives 82, and so this instance is unclear.

Appendix F

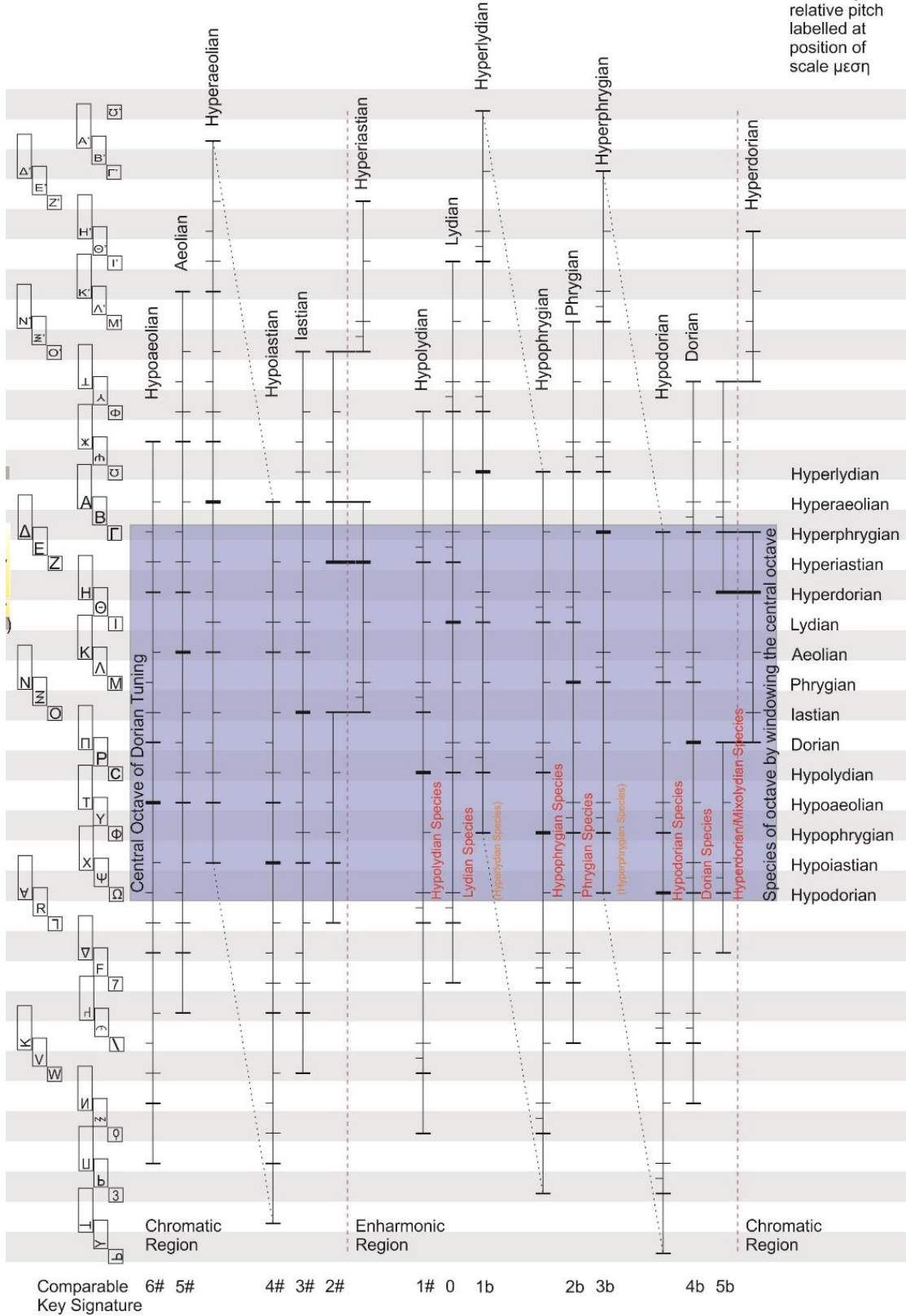
Greek Theory Quick-Reference

Species of Octave with Respect to μεση	Instrumental Notation	Greater Perfect System	Medieval Gamut	Lesser Perfect System
<p>Species of Octave with Respect to μεση</p>		<p>νητη ὑπερβολαιων</p> <p>παρηνητη ὑπερβολαιων</p> <p>τριτη ὑπερβολαιων</p> <p>νητη διεzeugμενων</p> <p>παρηνητη διεzeugμενων</p> <p>τριτη διεzeugμενων</p> <p>μεση</p> <p>λιχανος μεσων</p> <p>παρυπατη μεσων</p> <p>λιχανος ὑπατων</p> <p>παρυπατη ὑπατων</p> <p>ὑπατη ὑπατων</p> <p>προσλαμβανομενος</p>	<p>α</p> <p>β</p> <p>γ</p> <p>δ</p> <p>ε</p> <p>ς</p> <p>ζ</p> <p>η</p> <p>θ</p> <p>ι</p> <p>κ</p> <p>λ</p> <p>μ</p> <p>ν</p> <p>ξ</p> <p>ο</p> <p>π</p> <p>ρ</p> <p>σ</p> <p>τ</p> <p>υ</p> <p>φ</p> <p>χ</p> <p>ψ</p> <p>ω</p> <p>Α</p> <p>Β</p> <p>Γ</p> <p>Δ</p> <p>Ε</p> <p>Ζ</p> <p>Η</p> <p>Θ</p> <p>Ι</p> <p>Κ</p> <p>Λ</p> <p>Μ</p> <p>Ν</p> <p>Ξ</p> <p>Ο</p> <p>Π</p> <p>Ρ</p> <p>Σ</p> <p>Τ</p> <p>Υ</p> <p>Φ</p> <p>Χ</p> <p>Ψ</p> <p>Ω</p>	<p>νητη συνημμενων</p> <p>παρηνητη συνημμενων</p> <p>τριτη συνημμενων (συναφη)</p> <p>μεση</p> <p>α</p> <p>β</p> <p>γ</p> <p>δ</p> <p>ε</p> <p>ς</p> <p>ζ</p> <p>η</p> <p>θ</p> <p>ι</p> <p>κ</p> <p>λ</p> <p>μ</p> <p>ν</p> <p>ξ</p> <p>ο</p> <p>π</p> <p>ρ</p> <p>σ</p> <p>τ</p> <p>υ</p> <p>φ</p> <p>χ</p> <p>ψ</p> <p>ω</p> <p>Α</p> <p>Β</p> <p>Γ</p> <p>Δ</p> <p>Ε</p> <p>Ζ</p> <p>Η</p> <p>Θ</p> <p>Ι</p> <p>Κ</p> <p>Λ</p> <p>Μ</p> <p>Ν</p> <p>Ξ</p> <p>Ο</p> <p>Π</p> <p>Ρ</p> <p>Σ</p> <p>Τ</p> <p>Υ</p> <p>Φ</p> <p>Χ</p> <p>Ψ</p> <p>Ω</p>
		<p>Enharmonic</p> <p>soft</p> <p>Chromatic hemiolic</p> <p>tonic</p> <p>soft</p> <p>Diatonic</p> <p>tense</p>	<p>μεση = middle</p> <p>λιχανος = licking (index) finger</p> <p>ὑπατη = highest (description of string, not pitch)</p> <p>νητη = lowest (description of string, not pitch)</p> <p>τριτη = third (from the end)</p> <p>παρ, παρα = next to</p> <p>ὑπερ = beyond</p> <p>μεσων = of the middle tetrachord</p> <p>ὑπατων = of the higher tetrachord</p> <p>ὑπερβολαιων = of those thrown beyond</p> <p>διεzeugμενων = of the disjunct tetrachord</p> <p>συνημμενων = of the conjunct tetrachord</p> <p>διεzeugξις = disjunction</p> <p>συναφη = conjunction</p> <p>διαπεμπτος = through five</p> <p>εναρμονιος = enharmonic</p> <p>χρωματικη = chromatic</p> <p>διατονος = diatonic</p> <p>προσλαμβανομενος = taken in addition</p>	

Vocal Notation

Modes as Keys

Modes by relative pitch labelled at position of scale $\mu\epsilon\sigma\eta$



Glossary

Greek headwords are given in bold Greek text, with the transliteration in parentheses and unbolded italics, and Latinized forms in bolded italics. English headwords are bolded, but not italicized.

A

affinalis

Also ***cofinalis***, a pitch around which the interval structure for a specified distance (which generally avoids the fluctuating pitches \flat and \natural) is the same as around the *finalis* of a mode. The *affinalis* is usually a fifth above the *finalis*, and it is often possible to transpose a chant up to end on its *affinalis* without affecting the modal quality of the chant. Transposition to the *affinalis* may also be done to avoid certain weaknesses in the medieval pitch nomenclature and notational system.

αίολεύς (*aioleus*)

The Aeolian mode. In Greek theory, principally a transposition of the gamut with μέση on G-sharp. Has no corresponding octave species. Has no equivalent mode in medieval Latin theory, but added by Glarean as *aeolius*, or the eleventh mode (with a *finalis* on a), in the sixteenth century.

άλια (*halia*)

An assembly of people, especially in the Doric Greek dialect, comparable to ἐκκλησία (*ekklēsia*) in the Attic dialect, which is cognate with the Latin *ecclesia*. This term is implicated in Heard's hypothesis for the meaning of the title *Alia musica*.

ἀπήχημα (*apēkhēma*)

See ἦχημα.

arithmetic mean

The average or middle point between two numbers so that the mean is equally distant from both. Between 6 and 12, which represents an octave, the arithmetic mean is 9, since 9 differs from both 6 and 12 by 3. In the *Alia*, which assigns numbers to pitches according to wavelength, the arithmetic mean is a fourth above the bottom.

ἀρμονία (*harmonia*)

In Greek theory, a word for the modes, especially in reference to older systems about which little is known, while later writers tend to prefer the terms τόνος (*q.v.*) and τρόπος (*q.v.*).

authentus

Also *autentus*, *authenticus*, etc; also *principalis*. An authentic mode, which typically rises a fifth or more (up to a ninth) above the *finalis*. In mature medieval theory, authentic modes are not supposed to descend more than a tone below the *finalis*, but in the ninth and tenth centuries, some treatises (notably the *Enchirides*) permit them to descend a fifth below the *finalis*.

B

b

b-rotundus, or round b. Indicates a b-flat in the middle and upper octaves of the medieval gamut (see also **ᵇ**)

ᵇ

An altered form of *b-rotundus* (see **b**). In the medieval gamut, b-flat was traditionally only permitted in the middle and upper octaves, but the chant repertoire sometimes uses a b-flat in the lower octave, and Hermannus Contractus used this symbol to represent that pitch.

ᵇ

b-quadratus, or square b. Indicates a b-natural in the middle and upper octaves of the medieval gamut (see also **ᵇ**).

bisdiapason

A double octave. From the combination of the Latin prefix *bis-* (“double”) with the Greek διαπασῶν (*q.v.*) (“octave”). The pure Greek form uses the equivalent prefix *dis-*, giving *disdiapasōn* (*q.v.*).

C

clausula

A closing, or cadence; also, the musical phrase ending with a cadence.

cofinalis

See ***affinalis***.

colon

A medium-length segmentation in traditional grammar and in Bower’s model of musical grammar. Shorter than a *periodus* (*q.v.*), and usually considered to be longer than a *comma* (*q.v.*). Corresponds approximately to a clause.

comma

A medium-length segmentation in traditional grammar and in Bower’s model of musical grammar. Usually considered to be shorter than a *colon* (*q.v.*).

D/δ

daseian notation

The system of notation used in the *Musica enchiridis*, *Scolica enchiridis*, *Inchiriadon*, *Commemoratio brevis*, and a handful of very short treatises for the late ninth or tenth centuries. So called because most of the symbols are based on the *daseia*, Ϝ, the Greek symbol for rough breathing, with some variant of the letters S or C attached to one end and rotated or flipped through various positions. The system provides precise information about pitch and intervals, which the neumes in widespread use at the time did not, but it does not appear to have been used outside of theoretical treatises.

deuterus

A pseudo-Greek (see ***tetrardus***) ordinal number, meaning “second”; derived from Greek δεύτερος (*deuteros*). Refers to the second *maneria* (*q.v.*) of modes with a *finalis* (*q.v.*) on E, and in the *Musica enchiridis*

tradition, to the *finalis* itself, along with the pitches a fifth below and above and also a ninth above, which are all surrounded by the same interval structure in the *Enchiriadis* gamut.

διαπασῶν (*diapāsōn*),
diapason, dyapason

Lit. “through all [the strings]”; the interval of an octave. Created by the double proportion, 2:1.

διάπεντε (*diapente*),
diapente, dyapente

Lit. “through five [strings]”; the interval of a perfect fifth. Created by the ἡμιόλιος (*q.v.*) proportion (Lat. *sesquialter* – *q.v.*).

διάστημα, **diastema**
(*diastema*)

An interval. In Greek, especially a melodic interval, as opposed to συμφωνία (*q.v.*).

διατεσσάρων
(*diatessarōn*),
diatessaron,
dyatesseron, etc.

Lit. “through four [strings]”; the interval of a perfect fourth. Created by the επίτριτος (*q.v.*) proportion (Lat. *sesquitercius* – *q.v.*).

διάτονον
(*diatonon*)

The diatonic *genus* (*q.v.*). The diatonic *genus* includes all shades (see χρώα) in which the highest interval is smaller than a trihemitone (augmented second) (and is generally not smaller than a tone).

διεzeugμένων
(*diezeugmenōn*)

Lit. “of the disjunct [notes].” The name (given in genitive plural, as it usually appears in the compound-names of pitches) for the second highest tetrachord of notes in the Greater Perfect System (*q.v.*) of Greek instrument-based pitch nomenclature, corresponding to the span from β to e in the medieval gamut. The name reflects the fact that the tetrachord begins immediately above the tone of disjunction in the middle of the system, while the corresponding tetrachord in the Lesser Perfect System (*q.v.*), the συνεμμένων (*q.v.*), begins a tone lower and is conjunct with the lower octave.

differentia

A cadential formula for the end of a psalm tone, identified in the *Nova expositio* by its final pitch, selected from a set of such formulae belonging to a given mode in order to allow for a smooth transition between the psalm recitation and the antiphon that follows; the *differentia* is selected to match the *locum* (*q.v.*), often with several *loca* requiring the same *differentia*.

δισδιαπασῶν
(*disdiapasōn*),
disdiapason

Also *bisdiapason*. A double octave.

ditone

See **δίτονος**.

δίτονος, **ditonus**
(*ditonos*)

An interval the size of two tones, therefore in the ratio $(9:8 \times 9:8 = 81:64)$; a doubly-augmented second, as distinct from the major third (5:4) (which is not used in medieval theory). Used in this paper especially to refer to a single scale-step of this size in the enharmonic (see ἐναρμόνιον) *genus* (*q.v.*) as opposed to the succession of two tones or a skip of two tones.

δώριος, *dorius*
(*dōrios*)

The Dorian mode. In Greek theory, principally a transposition of the gamut rooted on E. In Latin theory from the *Alia musica* onward, corresponds to the first ecclesiastical mode, and in some sources (including the *Alia*) the octave species D to d; in this context, it differs from *hypomixolydius* (*q.v.*), which shares the same octave species, in that Dorian, like all authentic modes, is divided into a lower fifth and upper fourth (according to the **Harmonic mean** – *q.v.*).

E

εἶδή (*eidē*)

Lit. “form.” See ***species***.

emmelis

Glossed in Latin as *aptus melo*, “suited to melody.” Ptolemy uses this term to describe intervals corresponding to epimoric (superparticular, *q.v.*) ratios that are smaller than a perfect fourth, and therefore, too small to count as a perfect consonance, but suitable for use as a scale degree. In the *Alia musica*, this term refers to one additional scale degree beyond the range of a perfect consonance that a melody can use while still being said to run through that perfect consonance.

ἐναρμόνιον
(*enarmonion*)

The enharmonic *genus* (*q.v.*). Defined by the ditone that is the highest interval of its tetrachord. There are no shades in this *genus*; the two lowest intervals are always a quartertone.

ἐνήχημα
(*enēkhēma*)

See **ἦχημα**.

epimoric

See *superparticular*.

ἐπίτριτος, *epitritus*
(*epitritos*)

Lit. “a third over”. A ratio against one of one third greater than the whole (*i.e.*, $1\frac{1}{3}:1$, or 4:3). Lat. *Sesquitertius* (*q.v.*). Corresponds to the interval of a perfect fourth, or διατεσσάρων (*q.v.*).

ἐπόγδοος
(*epogdoos*)

Lit. “an eighth over”. A ratio against one of one eighth greater than the whole (*i.e.*, $1\frac{1}{8}:1$, or 9:8). Lat. *Sesquioctavus* (*q.v.*). Corresponds to the interval of a tone.

excellentes

Lit. “the exceeding [notes].” The highest complete tetrachord of the medieval gamut, especially in the *Enchirides* and Hermannus Contractus’ *Musica*. Corresponds to the span e– $\frac{3}{2}$ in the *Enchirides* and d–g in Hermannus’ *Musica*.

F

finales

Also *termini*. The second tetrachord of the medieval gamut, especially in the *Enchirides* and Hermannus Contractus’ *Musica*. So named because

the *finalis* (*q.v.*) of each mode is in this tetrachord. Corresponds to the span D–G in both systems.

finalis

Strictly speaking, the final note of a chant; however, usually also implies some of the characteristics associated with a tonic in tonal music.

frigius, frygius

See ***phrygius***.

G/γ

Γ

Gamma, traditionally the lowest pitch in the medieval gamut, one octave below G.

γένος, *genus*
(*genos*)

A category of tunings for the moveable notes of the tetrachord in the Greek gamut. There are three *genera*, most usefully defined by the size of the highest interval; except for the enharmonic, the *genera* are subdivided into multiple shades (see **χρόα**).

graves

Lit. “the low [notes].” The lowest tetrachord of the medieval gamut, especially in the *Enchirides* and Hermannus Contractus’ *Musica*. Corresponds to the span Γ–C in the *Enchirides* and A–D in Hermannus’ *Musica*.

Greater Perfect System

The standard gamut of Greek theory, constructed from two identical pairs of conjunctly joined tetrachords (with the **πυκνόν**, *q.v.*, at the bottom) that are separated by a tone of disjunction; an additional tone of disjunction is added to the bottom to bring the range to two complete octaves. Called a “system” because it is a scale segment spanning a perfect interval (see **σύστημα**); called “perfect” because all the species of the smaller systems of fourth, fifth, and octave may be found within it; called “greater” because it contains four tetrachords, by comparison to the Lesser Perfect System (*q.v.*), which contains only three.

H/η

N.B. In Greek script, the H sound is expressed through the rough-breathing mark ‘ over the following vowel, but the H is conventionally included in the transliteration. Look for Greek terms beginning with rough breathing according to the first vowel (*e.g.*, look for *hyperastios* = ὑπεριάστιος under ὑ, etc.).

h̄

An abbreviation for *autem* used chiefly under insular influence, the frequent use of which, in the *Alia musica*, led Chailley to propose that the *Alia musica* had insular connections (Chailley, *Alia musica*, 66).

h̄

An abbreviation traditionally used for *hoc*, but found in some manuscripts in the *Alia musica* where *autem* would be expected, implying confusion with the insular abbreviation h̄ (*q.v.*).

H	An abbreviation for <i>enim</i> that occurs in some manuscripts of the <i>Alia musica</i> where <i>autem</i> appears in others, implying confusion with the insular abbreviation <i>h</i> (<i>q.v.</i>).
hard hexachord	A hexachord (<i>q.v.</i>) in which the lowest note is Γ (<i>q.v.</i>), G, or (in later theory) g. The pitch-class B is natural in this hexachord, while the pitch-class F is missing.
harmonic mean	The average or middle point between two numbers so that the two differences between the original numbers and the mean are in the same proportion to each other as are the original two numbers. Between 6 and 12, which represents an octave, the harmonic mean is 8, because the difference between 6 and 8 is 2, between 8 and 12 is 4, and 4:2 is equivalent to the original ratio 12:6; in the <i>Alia</i> , which assigns numbers to pitches according to wavelength, the arithmetic mean is a fifth above the bottom.
hexachord	A six-note structure in medieval theory that incorporates the standard form (<i>i.e.</i> , first species) tetrachord at its centre with one additional tone on each end, so that the interval structure is tone-tone-semitone-tone-tone. Derived from the doctrines of Guido d'Arezzo in his invention of solmization (<i>q.v.</i>), but he did not actually use the term himself. In later medieval theory, there are three standard varieties (natural on C, hard on G, and soft on F), distinguished from one another by the status of the pitch-class B. By the Renaissance, hexachords could arise on pitches other than C, F, and G, but only these three have traditional names.
ἦθος (<i>ēthos</i>)	The emotional or affective character of a mode, especially in Greek theory.
ἡμιόλιος (<i>hēmīolios</i>), <i>haemiolus</i>, <i>hemiolus</i>, <i>emiolus</i>	A ratio against one of one half greater than the whole (<i>i.e.</i> , $1\frac{1}{2}:1$, or 3:2). Lat. <i>sesquialter</i> (<i>q.v.</i>). Corresponds to the interval of a perfect fifth, or a <i>διάπεντε</i> (<i>q.v.</i>).
<i>hypermixolydius</i>, <i>hypermixolidius</i>	The Hypermixolydian mode. In the <i>Alia musica</i> , corresponds to the eighth ecclesiastical mode. Named after <i>ὑπερμιχολύδοις</i> (<i>q.v.</i>) from Greek theory via Boethius, where it appears as an octave species a tone beyond the Mixolydian: below (as A to a) in traditional Greek theory, where octave species descend; above (as a to A^{\flat}) in <i>Alia musica</i> , where octave species ascend. Soon after the <i>Alia musica</i> , renamed <i>hypomixolydius</i> (<i>q.v.</i>) by analogy to the other plagal modes, and already implying a range a fourth below the Mixolydian in the later layers of the <i>Alia</i> . The usage creates ambiguities in the <i>Alia musica</i> regarding the eighth mode.
<i>hypodorius</i>	The Hypodorian mode. In Latin theory from the <i>Alia musica</i> onward, corresponds to the second ecclesiastical mode, and in some sources (including the <i>Alia</i>) the octave species A to a. Named after the Greek <i>ὑποδώριος</i> (<i>q.v.</i>).

<i>hypofrygius,</i> <i>hypofrigius</i>	See <i>hypophrygius</i> .
<i>Hypolydius, hypolidius</i>	The Hypolydian mode. In Latin theory from the <i>Alia musica</i> onward, corresponds to the sixth ecclesiastical mode, and in some sources (including the <i>Alia</i>) the octave species C to c. Named after the Greek ὑπολύδιος (<i>q.v.</i>).
<i>hypomixolydius,</i> <i>hypomixolidius</i>	The Hypomixolydian mode. In Latin theory after the <i>Alia musica</i> , corresponds to the eighth ecclesiastical mode, and in some sources, the octave species D to d (including in the <i>Alia</i> , where it is described in this form in the later layers, though it is still described as <i>hypermixolydius, q.v.</i>); in this context, it differs from <i>dorius (q.v.)</i> , which shares the same octave species, in that Hypomixolydian, like all plagal modes, is divided into a lower fourth and upper fifth (according to the Arithmetic mean – <i>q.v.</i>). Not derived directly from a Greek term but derived by analogy to the other plagal modes.
<i>hypophrygius,</i> <i>hypophrigius</i>	The Hypophrygian mode. In Latin theory from the <i>Alia musica</i> onward, corresponds to the fourth ecclesiastical mode, and in some sources (including the <i>Alia</i>) the octave species B to b. Named after the Greek ὑποφρύγιος (<i>q.v.</i>).
ἦχημα (pl. ἠχήματα) <i>(ēkhēma, ēkhēmata)</i>	Also ἄχημα, ἀπήχημα, ἐνήχημα. Literally, a sound. In music theory, used to describe an intonation formula representative of the sound of a mode in Byzantine theory (and sometimes used also by modern musicologists to describe the comparable phenomenon in Western chant – NOEANE, etc.).
ἦχος (ēkhos)	Sound, especially the sound of words or an echo. Used in Byzantine theory to describe a mode. See also οκτώηχος .
I/J	
ιάστιος, ἰόνιος <i>(iastios, ionios)</i>	The Iastian mode; also called Ionian (but less commonly before the sixteenth century). In Greek theory, principally a transposition of the gamut with μέση on F-sharp. Has no corresponding octave species. Has no equivalent mode in medieval Latin theory, but added by Glarean as <i>ionicus</i> , or the ninth mode (with a <i>finalis</i> on C), in the sixteenth century, after which Ionian becomes the preferred form.
<i>incisio</i>	Loosely speaking, a clause; imprecisely defined in the <i>Alia</i> as a segment of a chant that should manifest one of the characteristic intervals of its mode.

L/λ

Lesser Perfect System

A form of the Greek gamut, constructed from three conjunctly joined tetrachords, all with the *πυκνόν* in the lowest position, plus one tone of disjunction at the bottom, bringing the total span to an eleventh. Called “lesser” because it contains only three tetrachords, compared to the four tetrachords of the Greater Perfect System (*q.v.*). Displays only some of the characteristics of a perfect system.

λίχανος
(*lichanos*)

Lit. “licking,” the traditional Greek name for the forefinger. The highest note of either the *ὑπάτων* (*q.v.*) or *μέσων* (*q.v.*) tetrachords (excluding the *μέση*, *q.v.*) of the Greater Perfect System (*q.v.*) of Greek instrument-based pitch nomenclature, corresponding to D and G, respectively, in the medieval gamut. The name reflects the finger used to play this note on the simple lyre.

locum

An intonation figure for an antiphon (designated in the *Nova expositio* by its first pitch) that determines which *differentia* (*q.v.*) is appropriate for the end of the psalm recitation in order to facilitate a smooth transition between the end of the psalm and the beginning of the antiphon. There are frequently several *loca* that call for the same *differentia*. In Latin, while the more general term *locus* refers only to a place in general, the more specific term *locum* (especially in the plural, *loca*) refers to places that are connected.

λύδιος, lydius
(*lydios*)

The Lydian mode. In Greek theory, principally a transposition of the gamut rooted on C. In Latin theory from the *Alia musica* onward, corresponds to the fifth ecclesiastical mode, and in some sources (including the *Alia*) the octave species F to f.

M/μ

maneria

Probably best translated as a “tonality,” a grouping of the two modes (one authentic and one plagal) that share the same *finalis*. While the modes are named with Latin ordinal numbers, *maneriae* are named with pseudo-Greek ordinal numbers: *protus*, *deuterus*, *tritus*, *tetrardus* (*q.v.*).

media

Lit. “middle.” In the *Alia musica*, the *media* of a mode is the middle pitch. For authentic modes, the *media* corresponds to the harmonic mean (by wavelength) and is therefore the fifth above the bottom of its octave; for plagal modes, the *media* corresponds to the arithmetic mean, and is therefore the fourth above the bottom. In authentic modes, the interval structure around the *media* within a certain range is the same as the interval structure around the *finalis*, and it is therefore an *affinalis*, while in plagal modes, the *media* is the same pitch as the *finalis*. The *mediae* have the same pseudo-Greek ordinal names (*protus*, etc.) as the modes to which

they belong. The *mediae* have an inverse relationship with the *metae* (*q.v.*), so that the *mediae* of the authentic modes are the *metae* of the plagal modes and vice versa.

μέση
(*mesē*)

Lit. “middle.” The central note of the Greater Perfect System (*q.v.*) of Greek instrument-based pitch nomenclature, corresponding to a in the medieval gamut.

μέσων
(*meson*)

Lit. “of the middle [notes].” The name (given in genitive plural, as it usually appears in the compound-names of pitches) for the second-lowest tetrachord of notes in the Greater Perfect System (*q.v.*) and Lesser Perfect System (*q.v.*) of Greek instrument-based pitch nomenclature, corresponding to span from E to a in the medieval gamut. The name reflects the fact that it is literally the middle (second out of three) tetrachord of the Lesser Perfect System and one of the two middle tetrachords (second out of four) of the Greater Perfect System.

meta

A boundary point. In the *Alia musica*, the fifth shared by corresponding authentic and plagal modes has an upper and lower *meta*, which are also the *metae* of the plagal and authentic modes, respectively, so that the *meta* of the plagal mode appears at the top of its octave, a fifth above the *finalis*, and the *meta* of the authentic mode appears at the bottom of its octave and is the same note as the *finalis*. The *metae* have the same pseudo-Greek ordinal names (*protus*, etc.) as the modes to which they belong. The *metae* have an inverse relationship with the *mediae* (*q.v.*), so that the *metae* of the authentic modes are the *mediae* of the plagal modes, and vice versa.

μιξολύδιος, mixolydius
(*mixolydios*)

The Mixolydian mode. In Greek theory, principally a transposition of the gamut rooted on B; also called ὑπερδώριος. In Latin theory from the *Alia musica* onward, corresponds to the seventh ecclesiastical mode, and in some sources (including the *Alia*) the octave species G to g.

modus

Lit. “a way or manner.” Used in Latin theory for a mode; however, this term is relatively uncommon in medieval usage, where *tonus* (*q.v.*) and *tropus* (*q.v.*) prevail. The term was used by Boethius, probably as a Latin translation of the Greek τρόπος, and its most common use in medieval theory is in citations to or paraphrases of Boethius and related contexts, as appears to be the case in the *Alia musica*.

musica

Both a noun meaning “music” and an adjective meaning “musical”. By extension, a substantive meaning a musical thing, a thing pertaining to music, or a musician; frequently, a music treatise. Most of these meanings could potentially be implicated in the title *Alia musica*.

N/v

natural hexachord

A hexachord (*q.v.*) in which the lowest note is C or c. The pitch-class B is absent from this hexachord.

νήτη, νεάτη
(*nētē, neātē*)

Lit. “lowest.” Counterintuitively, the highest note of the συνημμένων (*q.v.*), διεζευγμένων (*q.v.*), or ὑπερβολαίων (*q.v.*) tetrachords of the Greater Perfect System (*q.v.*) or Lesser Perfect System (*q.v.*) of Greek instrument-based pitch nomenclature, corresponding to d, e and $\sharp a$, respectively, in the medieval gamut. The name reflects the fact that the lyre was held so that the lowest strings were tuned highest.

O

ὀκτωήχος
(*oktōēkhos*)

Lit. “eight sounds.” The system of eight modes, especially as divided into four pairs of authentic and plagal modes. Historically used only to describe the modes in the Byzantine system, where it corresponds to an eight-part cycle within the liturgy, but sometimes used by modern musicologists to describe any comparable modal system, including the eight modes of Western chant.

P/π

παραμέση
(*paramesē*)

Lit. “next to middle.” A central note of the Greater Perfect System (*q.v.*) of Greek instrument-based pitch nomenclature, directly above the μέση, corresponding to b in the medieval gamut.

παρανήτη
(*paranētē*)

Lit. “next to lowest.” Counterintuitively, the second highest note of the συνημμένων (*q.v.*), διεζευγμένων (*q.v.*), or ὑπερβολαίων (*q.v.*) tetrachords of the Greater Perfect System (*q.v.*) or Lesser Perfect System (*q.v.*) of Greek instrument-based pitch nomenclature, corresponding to c, d and g, respectively, in the medieval gamut. The name reflects the fact that the lyre was held so that the lowest strings were tuned highest.

παρυπάτη
(*parhypatē*)

Lit. “next to highest.” Counterintuitively, the second-lowest note of either the ὑπάτων (*q.v.*) or μέσων (*q.v.*) tetrachords of the Greater Perfect System (*q.v.*) and Lesser Perfect System (*q.v.*) of Greek instrument-based pitch nomenclature, corresponding to C and F, respectively, in the medieval gamut. The name reflects the fact that the lyre was held so that the highest strings were tuned lowest.

periodus

One of the largest segmentations in traditional grammar and in Bower’s model of musical grammar. Corresponds to a sentence.

<i>phrygius, phrigius</i>	The Phrygian mode. In Latin theory from the <i>Alia musica</i> onward, corresponds to the third ecclesiastical mode, and in some sources (including the <i>Alia</i>) the octave species E to e. Named after the Greek φρύγιοις (<i>q.v.</i>).
<i>plagis, plagalis</i>	Also <i>subiugalis</i> . A plagal mode, which typically rises no more than a fifth above its <i>finalis</i> and descends by the same distance. Often described as <i>a latere</i> (“to the side”) of the authentic modes.
<i>principalis</i>	See <i>authentus</i> .
προσλαμβανόμενος (<i>proslambanomenos</i>)	Lit. “taken in addition.” The lowest note in the Greater Perfect System (<i>q.v.</i>) and Lesser Perfect System (<i>q.v.</i>) of Greek instrument-based pitch nomenclature, corresponding to A in the medieval gamut. It is the only note in the Greek system not belonging to a defined tetrachord, giving rise to its name.
<i>protus</i>	A pseudo-Greek (see <i>tetrardus</i>) ordinal number, meaning “first”; derived from Greek πρώτος (<i>prōtos</i>). Refers to the first <i>maneria</i> (<i>q.v.</i>) of modes with a <i>finalis</i> (<i>q.v.</i>) on D, and in the <i>Musica enchiriadis</i> tradition, to the <i>finalis</i> itself, along with the pitches a fifth below and above and also a ninth above, which are all surrounded by the same interval structure in the <i>Enchiriadis</i> gamut. In some treatises, the term <i>archos</i> (<i>archoos, archous</i> , from ἀρχός = leader/chief) is treated as a synonym.
πυκνόν (<i>pyknon</i>)	Lit. “a crowding.” In Greek theory, the three lowest pitches of a first-species tetrachord in any <i>genus</i> (<i>q.v.</i>) and shade (see χρώα) in which the two lowest intervals are collectively smaller than the remaining interval (<i>i.e.</i> , any shade except those of the diatonic genus).

S/σ

<i>semiditonus</i>	Also trihemitone. An interval the size of a tone plus a semitone; an augmented second, as distinct from the minor third (which is not used in medieval theory). Used in this paper especially to refer to a single scale-step of this size in the chromatic (see χρωματικόν) <i>genus</i> (<i>q.v.</i>) as opposed to the succession of a tone and a semitone or a skip of the same distance.
<i>sesquialter</i>	A ratio against one of one half greater than the whole (<i>i.e.</i> , $1\frac{1}{2}:1$, or 3:2). Gr. ἡμιόλιος (<i>q.v.</i>). Corresponds to the interval of a perfect fifth, or a διάπεντε (<i>q.v.</i>).
<i>sesquitertius</i>	A ratio against one of one third greater than the whole (<i>i.e.</i> , $1\frac{1}{3}:1$, or 4:3). Gr. επίτριτος (<i>q.v.</i>). Corresponds to the interval of a perfect fourth, or διατεσσάρων (<i>q.v.</i>).
<i>sesquiocavus</i>	A ratio against one of one eighth greater than the whole (<i>i.e.</i> , $1\frac{1}{8}:1$, or 9:8). Gr. επόγδοος. Corresponds to the interval of a tone.

shade	See χρόα.
species	Corresponds to Greek εἰδή (<i>eidē</i>). The form of a system, or scale segment, especially a fourth, fifth, or octave, based on the interval arrangement of intervals. In medieval theory, defined by the position of the semitone (or semitones) compared to the boundary pitches. More complicated in Greek theory because of the interval variation introduced by <i>genus</i> (<i>q.v.</i>); in Greek, the octave species are most readily defined by the position of the tone of disjunction, while the species of fourth and fifth are most readily defined by the position of the πυκνόν (or for fourths, equivalently, by the position of the one interval that does not participate in the πυκνόν).
soft hexachord	A hexachord (<i>q.v.</i>) in which the lowest note is F or f. The pitch-class B is flat in this hexachord, and the pitch-class E is absent.
solmization	A system for learning to sing devised by Guido d'Arezzo based on the hymn <i>Ut queant laxis</i> . Each of the first six subdivisions of the chant begins one scale degree higher than the previous, and the syllable sung on that pitch in this hymn became the name of the pitch, creating the sequence <i>ut-re-mi-fa-sol-la</i> , spanning a hexachord (<i>q.v.</i>). To move outside the range of these six pitches, one must pivot into a different hexachord (much later, a seventh syllable was added as <i>si</i> or <i>ti</i> , and <i>ut</i> was changed to <i>do</i> , for a complete one octave system).
sonus	Lit. "sound." In music, a pitch with a precise tuning and a fixed position within a formally defined tuning system.
subiugalis	See <i>plagis</i> .
superiores	Lit. "the higher [notes]." The second highest tetrachord of the medieval gamut, especially in the <i>Enchiridae</i> s and Hermannus Contractus' <i>Musica</i> . Corresponds to the span a–d in both systems.
superparticular	A ratio against one of one part greater than the whole (<i>i.e.</i> $1\frac{1}{n} : 1$, or $n + 1 : n$). Important because all consonant intervals must either be multiple ($n : 1$) or superparticular, and according to Ptolemy, scale degrees (see <i>emmelis</i>) must also be superparticular.
syllaba	A syllable. Does not appear in the traditional analogy between grammar and music, but added by Bower in his model of musical grammar. In this model, a <i>syllaba</i> ends when the end of a word in the text of the chant coincides with a structurally important pitch, especially the upper and lower fourths and fifths (compared to the <i>finalis</i>) and the subtone.
συμφωνία, symphonia (<i>symphonia</i>)	Lit. "sounding together." In Greek, usually a harmonic interval (as opposed to διάστημα, <i>q.v.</i> , which is usually a melodic interval). In both Greek and Latin, usually a perfect consonance.

συνεμμένων
(*synemmenōn*)

Lit. “of the conjunct [notes].” The name (given in genitive plural, as it usually appears in the compound-names of pitches) for the highest tetrachord of notes in the Lesser Perfect System (*q.v.*) of Greek instrument-based pitch nomenclature, corresponding to span from a to d in the medieval gamut (using b rather than B). The name reflects the fact that the tetrachord begins at the same pitch where the next lower tetrachord ends, and so is conjunct with the rest of the system, while the corresponding tetrachord in the Greater Perfect System (*q.v.*), the διαζευγμένων (*q.v.*), begins a tone higher and is disjunct from the lower octave.

σύστημα, *systema*
(*systēma*)

A scale segment, especially one that spans a perfect consonance. Considered to be a “perfect system” if every species of each smaller system can be found within it.

T

tetrachord

A system, or scale segment, of four pitches spanning a perfect fourth. In Greek theory, the tuning of the two inner pitches could vary not only by *species* (*q.v.*) but by *genus* (*q.v.*), but the standard form (*i.e.*, the first species) places the πυκνόν (*q.v.*) in the lowest position. In medieval theory, the inner pitches vary only by species, always consisting of two tones and a semitone, and the standard form places the semitone in the middle.

tetrardus

A pseudo-Greek ordinal number meaning “fourth.” Refers to the fourth *maneria* (*q.v.*) of modes with a *finalis* (*q.v.*) on G, and in the *Musica enchiriadis* tradition, to the *finalis* itself, along with the pitches a fifth below and above and also a ninth above, which are all surrounded by the same interval structure in the *Enchiriadis* gamut. This word is not merely a transliteration of the Greek, as are the other three *manerial* names; there is no such number as *tetrardos* in Greek, the word for “fourth” being τέταρτος (*tetartos*).

τόνος, *tonus*
(*tonos*)

Lit. “something stretched,” as a lyre string.

(1) A musical sound.

(2) The interval of a major second, in the ratio of the επόγδοος (*q.v.*) or *sesquioctavus* (*q.v.*). In some contexts, perhaps also, less precisely, a generic scale step.

(3) A mode in both Greek and Latin theory. Often synonymous with τρόπος (*q.v.*) and *modus* (*q.v.*); where a distinction is made, this distinction is not consistent amongst theorists, and by comparison to *tropus* and *modus*, medieval theorists frequently label the use of *tonus* for this purpose to be incorrect, though its use is far more common than *modus*. Musicologists do not agree about whether the *Alia musica* distinguishes

between these terms, or whether the preference for one of them is evidence of the authorship of varying passages.

trihemitone

See *semiditonus*.

τρίτη
(*trite*)

Lit. “third.” The third note from the top of the συνημμένων (*q.v.*), διεzeugμένων (*q.v.*), or ὑπερβολαίων (*q.v.*) tetrachords of the Greater Perfect System (*q.v.*) or Lesser Perfect System (*q.v.*) of Greek instrument-based pitch nomenclature, corresponding to \flat , c and f , respectively, in the medieval gamut.

tritus

A pseudo-Greek (see *tetrardus*) ordinal number, meaning “third”; derived from Greek τρίτος (*tritos*). Refers to the third *maneria* (*q.v.*) of modes with a *finalis* (*q.v.*) on F , and in the *Musica enchiridis* tradition, to the *finalis* itself, along with the pitches a fifth below and above and also a ninth above, which are all surrounded by the same interval structure in the *Enchiridis* gamut.

τρόπος, tropus
(*tropos*)

Lit. “a turn”, metaphorically, “a custom or manner.” Used for a mode in both Greek and Latin theory. Often synonymous with τόνος (*q.v.*) and *modus* (*q.v.*); where a distinction is made, this distinction is not consistent amongst theorists. Musicologists do not agree about whether the *Alia musica* distinguishes between these terms, or whether the preference for one of them is evidence of the authorship of varying passages.

υ/ν/γ/υ

υοχ

Generally, the voice. In music, often a musical sound, especially a vocal sound. See also φθόγγος.

ὑπάτη
(*hyratē*)

Lit. “highest.” Counterintuitively, the lowest note of either the ὑπάτων (*q.v.*) or μέσων (*q.v.*) tetrachords of the Greater Perfect System (*q.v.*) and Lesser Perfect System (*q.v.*) of Greek instrument-based pitch nomenclature, corresponding to B and E , respectively, in the medieval gamut. The name reflects the fact that the lyre was held so that the highest strings were tuned lowest.

ὑπάτων
(*hyratōn*)

Lit. “of the high [notes].” Counterintuitively, the name (given in genitive plural, as it usually appears in the compound-names of pitches) for the lowest tetrachord of notes in the Greater Perfect System (*q.v.*) and Lesser Perfect System (*q.v.*) of Greek instrument-based pitch nomenclature, corresponding to span from B to E in the medieval gamut. The name reflects the fact that the lyre was held so that the highest strings were tuned lowest.

ὑπεραιολεύς (<i>hyperaioleus</i>)	The Hyperaolian mode. In Greek theory, principally a transposition of the gamut with μέση on c-sharp. Has no corresponding octave species, nor a mode in Latin theory.
ὑπερβολαίων (<i>hyperbolaiōn</i>)	Lit. “of the [notes] thrown beyond.” The name (given in genitive plural, as it usually appears in the compound-names of pitches) for the highest tetrachord of notes in the Greater Perfect System (<i>q.v.</i>) of Greek instrument-based pitch nomenclature, corresponding to span from e to ḗ in the medieval gamut. The name reflects the fact that the tetrachord appears beyond (<i>i.e.</i> , above) the διεzeugμένων tetrachord (the point where the Lesser Perfect System, <i>q.v.</i> , differs from the Greater). There is no corresponding tetrachord in the Lesser Perfect System.
ὑπερδώριος (<i>hyperdōrios</i>)	The Hyperdorian mode. In Greek theory, principally a transposition of the gamut with μέση on \flat ; also, an octave species from B to \flat . Also called μιξολύδιος (<i>q.v.</i>).
ὑπεριάστιος, (<i>hyperiastios</i>) ὑπεριόνιος (<i>hyperionios</i>)	The Hyperiastian mode; less commonly called Hyperionian. In Greek theory, principally a transposition of the gamut with μέση on \flat . Has no corresponding octave species, nor a mode in Latin theory.
ὑπερλύδιος (<i>hyperlydios</i>)	The Hyperlydian mode. In Greek theory, principally a transposition of the gamut with μέση on d. Has no corresponding octave species, nor a mode in Latin theory.
ὑπερμιχολύδιος (<i>hypermixolydios</i>)	The Hypermixolydian mode. In Greek theory, not traditionally used to represent a transposition of the gamut, but appearing sometimes as an “eight” octave species from A to a in the double-octave Greater Perfect System (<i>q.v.</i>), one step below the Mixolydian. Redundant in most contexts because it is the octave equivalent of (and shares its interval structure with) the Hypodorian.
ὑπερφρύγιος (<i>hyperphrygios</i>)	The Hyperphrygian mode. In Greek theory, principally a transposition of the gamut with μέση on c. Has no corresponding octave species, nor a mode in Latin theory.
ὑποαιολεύς (<i>hyperaioleus</i>)	The Hyperaolian mode. In Greek theory, principally a transposition of the gamut with μέση on D-sharp. Has no corresponding octave species. Has no equivalent mode in medieval Latin theory, but added by Glarean as <i>hypoaelius</i> , or the twelfth mode (with a <i>finalis</i> on E), in the sixteenth century.
ὑπορδώριος (<i>hyperdōrios</i>)	The Hypodorian mode. In Greek theory, principally a transposition of the gamut with μέση on C; also, an octave species from a to ḗ . Namesake of Latin <i>hypodorius</i> .

ὑποιάστιος, ὑποιώνιος (<i>hypoiaastios,</i> <i>hypoionios</i>)	The Hypoiaastian mode; also called Hypoionian (but less commonly before the sixteenth century). In Greek theory, principally a transposition of the gamut with μέση on B. Has no corresponding octave species. Has no equivalent mode in medieval Latin theory, but added by Glarean as <i>hypoionicus</i> , or the tenth mode (with a <i>finalis</i> on Γ), in the sixteenth century, after which Hypoionian becomes the preferred form.
ὑπολύδιος (<i>hypolydios</i>)	The Hypolydian mode. In Greek theory, principally a transposition of the gamut with μέση on E; also, an octave species from F to f. Namesake of Latin <i>hypolydius</i> .
ὑποφρύγιος (<i>hypophrygios</i>)	The Hypophrygian mode. In Greek theory, principally a transposition of the gamut with μέση on D; also, an octave species from A to a. Namesake of Latin <i>hypophrygius</i> .

Φ

φθόγγος, <i>phthongus</i> (<i>phthongos</i>)	In Latin, also <i>ptongus</i> , etc. A sound, usually vocal; a musical pitch. See also <i>uox</i> .
φρύγιος (<i>phrygios</i>)	The Phrygian mode. In Greek theory, principally a transposition of the gamut with μέση on G; also, an octave species from D to d. Namesake of Latin <i>phrygius</i> .

Χ

χρόα (<i>chroa</i>)	A shade. A subdivision of a <i>genus</i> (<i>q.v.</i>) in which the tuning of the moveable notes differs within the overall definition of that <i>genus</i> .
χρωματικόν (<i>chrōmatikon</i>)	The chromatic <i>genus</i> (<i>q. v.</i>). The chromatic <i>genus</i> includes all shades (see <i>χρόα</i>) in which the highest interval of the tetrachord is at least a trihemitone (augmented second) but less than a ditone.