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Renewable-energy resources, economic growth and their causal link

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Abstract

This thesis examines the presence and strength of predictive causal relationship between renewable energy prices and economic growth. We look for evidence by investigating the cases of Norway, New Zealand, and Canada's two provinces of Alberta and Ontario. The usual vector autoregressive model (VAR) and its various improved versions still assume constant parameters over time. We devise a Markov-switching VAR (MS-VAR) model in order to accommodate the observed time-dependent causal relation changes. Our proposed modelling approach is induced by the hidden Markov model methodologies in terms of an online parameter estimation through recursive filtering. The parameters of the MS-VAR model are governed by a hidden Markov chain, which in turn allows causal relationship to vary amongst different economic regimes. A unidirectional causal link, going from economic growth to the prices of renewable energy in New Zealand, Alberta, and Ontario, is demonstrated by our empirical findings. In particular, the causality emerges in the cases of New Zealand and Ontario during periods of high economic growth while it appears in the case of Alberta during periods of low economic growth.

Keywords: Regime-switching dynamics, Hidden Markov model, Vector autoregressive model, Economic growth, Renewable energy, Causality

Lay Summary

The importance of energy is underscored by its continuing demand to power up economic activities, enable technology performance and meet other energy-dependent needs of the population's households. Non-renewable-energy sources are limited in supply as it would take a considerably long time for them to be replenished. As a consequence, many developed, emerging and transitioning economies have undergone a relatively rapid shift of tapping renewable energy-source alternatives. With a nation's competitiveness and prosperity go hand in hand with sustainable economic growth, this thesis investigates the causal relationship between renewable energy prices and economic growth in Norway, New Zealand, and Canada's two provinces of Alberta and Ontario. Our results show that economic growth brings more renewable-energy investments that strengthen renewable-energy technologies, eventually lowering renewable-energy prices. Thus, policy makers could introduce incentives that will attract more renewable-energy investments. Although the impact of renewable-energy prices to economic growth is not seen within the examined data sets, policies supporting renewable energy may help in achieving a stable energy sector as fossil-based fuels, with considerable price risk, are gradually phased out.

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Chapter 1

Introduction

1.1 Background of world energy

Energy is one of the most essential components of economic infrastructure. It fuels the economy and sustains many of the services that people depend on. There are various sources of energy but they could be classified into two main categories: non-renewable and renewable. Non-renewable energy sources have the advantages of affordability, storage, and consistency. Examples of non-renewable sources are oil, natural gas, coal, and nuclear fuels (i.e., uranium and plutonium). However, they produce greenhouse gases such as carbon dioxide, methane, and nitrous oxide, which are causing climate change and global warming, which in turn affect ecosystems, economies, and human health. In contrast, renewable-energy sources based on hydro, wind, and solar emit no greenhouse gases into the air; although we do acknowledge that there are debates highlighting their associated ecological conservation issues.

To mitigate the drawbacks brought about by non-renewable sources, many countries began adopting renewable energy and others are progressively shifting from conventional-energy systems to renewable-energy systems. According to the *World Energy Outlook 2018* by *International Energy Agency* (IEA) [25], the demand for renewable energy has doubled from 662 million tonnes (Mtoe) of oil equivalent in 2000 to 1334 Mtoe in 2017. There has been no change on the share of fossil-based energy in the world's primary demand over the past two decades.

With the development of new technologies, the cost of installing renewable-energy systems has continued to decrease and the low cost potentially makes the transition to renewable-energy systems more accessible. The IEA [26] reported that renewable energy will have the fastest growth in the electricity sector and would provide 30% of power demand by 2023; this power demand was 24% in 2017. Certainly, the future landscape of the energy industry is altering, and most likely renewable energy will replace fossil fuel and dominate the worldwide-energy mix.

1.2 Previous related studies

The relationship between economy and energy has pre-occupied economists for the past decades because it has considerable implications to governments, policy makers and other stakeholders. There have been numerous studies on the nexus between economic growth and energy consumption in the past. A large number of these studies probed the connection between economic growth and electricity consumption. Some researchers analysed the link between economic growth and consumption of various energy such as non-renewable, renewable, and nuclear types. Nuclear energy is renewable because there is no emission of greenhouse gasses, hence no air pollution. Nonetheless, the by-products of nuclear fission, which is the process in the creation of nuclear energy, are radioactive and toxic waste materials. Also, to generate this type of energy, power plants need radioactive metals (Plutonium-239 or Uranium-235) whose supply is extremely limited, making it non-renewable.

There was, however, no consensus on the causal relationship between economic growth and energy consumption in these previous studies. Specifically, there are four hypotheses of energy-growth nexuses that were previously established: growth hypothesis, conservation hypothesis, feedback hypothesis, and neutrality hypothesis.

The growth hypothesis posits a unidirectional causality from energy consumption to economic growth. This implies that economic consumption has an important impact on the economic growth; in other words, an increase (fall) in energy consumption leads to increase (fall) in the gross domestic product (GDP). The conservation hypothesis presupposes a unidirectional causality from economic growth to energy consumption. This means that energy consumption has no effect on economic growth, but the GDP growth causes energy consumption growth. The feedback hypothesis conjectures a bidirectional causality from economic growth to energy consumption. This indicates that both energy consumption and economic growth influence each other simultaneously. Lastly, the neutrality hypothesis postulates that there is no causality running from either direction of economic growth and energy consumption. That is to say that there is an absence of relationship between energy consumption and economic growth.

The literature on the link between renewable energy and economic growth is expanding. The main focus though has been on the question of whether renewable-energy consumption causally relates to economic growth. Some of the empirical studies concentrate predominantly on the nexus in one country. Utilising the vector error correction model (VECM), Pao and Fu [40] investigated the causal relationships between real GDP and four energy consumption (non-hydro renewable, total renewable, non-renewable, and total primary energy) in Brazil for the period 1980-2010. They found evidence that buttressed the feedback hypothesis on the linkage between total renewable energy and economic growth in the long run. Dogan [8] explored the causal relationships between renewable-/non-renewable-energy consumption and economic growth in Turkey in the years 1988-2012 by using an autoregressive distributed lag (ARDL) model with a structural break. The findings in [8] supported the conservation hypothesis in the short run and the feedback hypothesis in the long run. Most recently, Lee and Jung [30] analyzed the causal relationship between renewable-energy consumption and economic growth in South Korea for the period 1990-2012 by employing the ARDL bounds test and VECM. The study in [30] revealed manifestations in favour of the conservation hypothesis in both the short and long run.

Some researchers tested the causality on panel data sets. Using the data from 1985 to 2005 on 20 member countries of the Organisation for Economic Co-operation and Development, Apergis and Payne [1] explored the presence of causal relationship of renewable-energy consumption and the economic growth within a multivariate framework; it was shown that a bidirectional causality between the two variables existed, which in turn reinforced the feedback hypothesis. Tugcu et al. [49] analyzed the causal relationships between renewable, non-renewable energy, and GDP of the G7 countries covering the 1980-2009 period via the ARDL model. The neutrality hypothesis was supported for the cases of France, Italy, Canada and the US; the feedback hypothesis was validated by the cases of England and Japan; and the conservation hypothesis was bolstered by Germany's case.

Bhattacharya et al. [3] used the panel technique and the fully modified ordinary-least-squares model in the analysis of the top 38 renewable-energy consumption countries, which were looked into separately, for the period 1991-2012. It was concluded that renewable-energy consumption has a positive effect on the economic growth for 57% of the countries included in their analysis. Employing the panel ECM, Fotourehchi [16] addressed the causal-relationship question between the renewable-energy consumption and the economic growth of 42 developing countries based on a 1990-2012 data set. The outcome of the examination in [16] beefed up the growth hypothesis in the said 42 developing countries.

As the results of empirical studies vary depending on methodologies applied and data sets collected, there is no consensus on the findings and implications of the above empirical studies. To a large extent, related research in this area shed light on the causalities between economic growth and renewable energy (e.g., [1, 3, 8, 16, 27, 30, 37, 49]); aggregate energy consumption (e.g., [2] [5], [42]); and CO_2 emission (e.g., [24, 35]). A quite exhaustive survey in [47], which encompassed a 36-year period and covered many regions and countries worldwide, concentrated on research progress that probed causality (a) between economic growth and energy-use variables (electrical, nuclear, renewable and non-renewable); (b) between economic growth and environment; and (c) between two variables that can be formed as a pair from the three variables of economic growth, energy use and environment. The results were conflicting as to which energy consumption could boost economic growth; furthermore, environmental drawbacks could be precipitated by some of these energy-use variables. Such causal relations in this survey were closely monitored because of their immediate relevance to policies that impact energy alongside ecological and economic initiatives by the government and other stakeholders. Another comprehensive survey is [39] on the energy-growth nexus. It stressed that most empirical studies aimed to find what role energy (electricity) does have in stimulating economic growth and if there is also a reverse relation between these two variables.

It is important to note that most of the studies hitherto conducted did not consider the structural breaks in the data sets under a dynamic parameter-estimation setting. Many structural-form models do include regime-switching characteristics but they still depend on static-parameter estimation. Economies have gone through several structural changes over the past decades due to, for example, energy crises, financial turmoils, and technological change, amongst other reasons. Thus, empirical studies that do not take into account the dynamic evolution of parameters' dynamic structural changes in data sets might lead to inadequate or even flawed conclusions.

1.3 Motivation

As sources of renewable energy expand and in an effort to mitigate negative environmental effects, the production of conventional energy has been gradually shrinking. Conventional-energy resources are being replaced progressively by hydropower and ocean resources, wind, solar, geothermal, solid biomass, biogas, and liquid biofuels as basic inputs to economic activities. Nonetheless, as pointed out by the Natural Resources Canada, biomass resources are renewable only if its rate of regeneration surpasses the rate of consumption.

The price of renewable energy hugely impacts the outputs of the manufacturing sector and also affects directly the demand for energy of the residential sector. Thus, investigating the causal relationship between the price of renewable energy and economic growth is of paramount importance. Our investigation will be facilitated by a regime-switching model. As previously acknowledged, structural breaks could be problematic; and so, Dogan [8] utilized the modified unit root, cointegration, and causality tests with a structural break to deal with this issue. However, the power of the causality test, when there are structural break manifestations, depends on the data size. The results of the test could be misleading if the data size is small. A natural way to resolve this structural-break issue is by incorporating regime-switching dynamics into the vector autoregressive (VAR) model. The model's regime-switching capability would enable us to identify the economic states in different regimes. For example, an economy may have two regimes (bust and boom), or have three regimes (bust, normal, and boom). Fallahi [14] analysed the causal relationship between energy consumption and GDP by a Markov-switching model combined with vector autoregressive models (MS-VAR) in the US with the aid of a data set spanning the years 1960-2005. In the MS-VAR setting, the relationship between energy consumption and GDP is time-dependent. The Markov-switching model detected the economic regimes, and VAR was used to delve into the causal relationships in each regime. The investigation's outcome sustained the feedback hypothesis in one regime that includes the energy crisis in the 1970s, and the neutrality hypothesis in the other regime. Similarly, by using the VAR, Kilic and Cankaya [28] studied the Granger causality between oil prices and economic activities in the BRICS-member countries (Brazil, Russia, India, China, and South Africa characterized as major emerging national economies) and the G7 (Group of Seven countries consisting of Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States constituting - in a worldwide scale - the largest IMF-advanced economies). The VAR results in [28] were robustified with the use of an MS-VAR model, and such results showed that Granger causalities vary by different countries and different economic activities.

This thesis extends the MS-VAR model under the hidden Markov model (HMM) set up designed primarily to process evidence of the causal relationship between the price of renewable energy and economic growth using the data sets as study cases for Norway, New Zealand, and two provinces of Canada (Alberta and Ontario). Our inference approach in the examination of causality follows similar idea to Fallahi's [14], i.e., checking each individual parameter's level of significance.

There are two main reasons for selecting the three above-mentioned countries. Firstly, they have relatively high shares of renewable electricity output with respect to total electricity production. Norway is the top-world-ranking nation in terms of share of renewable energy in electricity generation, and it is followed by New Zealand. In 2018, the proportions of electricity generated from renewable energy in Norway, New Zealand, and Canada are 97.9%, 83.1%, and 65.9%, respectively. With electricity's making up the bulk (i.e., high shares) of renewable energy produced, it is reasonable to proxy the price of renewable energy by the price of electricity. Secondly, data on electricity prices in Norway, New Zealand, Alberta, and Ontario are readily available as these four regions all have market-based systems. For these markets, the supply and demand largely dictate the electricity price, and such a price is more likely to be influenced quite freely by economic forces. Although some other countries or regions have a higher share capacity than Canada for renewable energy, they, unfortunately, still operate under regulated markets with massive oversight and control by their governments or very few organisations. We shall evaluate how both the price and share of renewable energy in electricity generation (i.e., energy mix) could affect economic growth.

In the existing literature, the efficient and dynamic estimation procedure for MS models has relied heavily on the the Expectation-Maximisation (EM) algorithm proposed by Hamilton [22]. In Cappé [4], the idea of online EM algorithm for HMMs was explored further. However, such an online EM approach was not tailored to dovetail with the technique of reference probability measure. So, we followed Elliott et al. [9], who introduced the change-of-measure method in combination with the EM algorithm. Under the HMM setting, the dynamic-estimation technique in [9] leads to a self-tuning model; that is, when new information arrives, parameter values are updated automatically. Hidden Markov model found important applications in the fields of finance (e.g., Date et al. [7], Rydén et al. [41], Srivastava et al. [44], Tenyakov and Mamon [45]), insurance (e.g., Elliott et al. [10], Frees and Wang et al. [15], Gao et al. [19], Mamon et al. [34]); economics (Grimm et al. [18], Gregoir and Lenglart [20], Song [43], Xi and Mamon [52]), related areas (Erlwein et al. [12], Xiong and Mamon [51]); and other fields of the natural and social sciences (e.g., Netzer et al. [36], Kundu et al. [29]).

The remaining parts of this thesis is organized as follows. Chapter 2 presents the formulation of our HMM-modulated vector autoregression. The HMM-EM estimators of our proposed model, harnessing the power of the change-of-measure technique, are also given. In Chapter 3, we describe the salient features of our data sets in the context of our empirical analysis. The numerical implementation is executed in Chapter 4 with the results laid out. Chapter 5 discusses policy implications and pertinent insights gained from our results. Finally, Chapter 6 concludes.

Chapter 2

Model description

In this Chapter, we work out the details of constructing an unrestricted VAR model with exogenous variables. The VAR model is the generalisation of the univariate AR model. In the VAR model, each variable is a linear function of the past lags of itself and the past lags of the other variables; this is a common econometric tool to find out causal relationships between variables.

The VAR model has two inherent assumptions:

(a) All variables have to be of the same order of integration.

(b) The error terms have

- (b.i) a zero mean,
- (b.ii) a positive semi-definite covariance matrix, and
- (b.iii) no serial correlation.

The order of integration, denoted by $I(d)$, is the minimum number d of differences needed to produce a covariance-stationary time series.

The goal of this research is to investigate the causality between economic growth and certain aspects of renewable energy via the VAR model. More specifically, we shall explore dynamic causalities under different economic statuses. Enabling the VAR model's coefficients to change under different economic regimes will reflect different causalities. Our mechanism to do this is the MS-VAR model whose advantages include easy interpretation and a handy online-parameter estimation compatible with our formulation. In particular, a hidden Markov chain is embedded into the unrestricted VAR model to yield an MS-VAR model. Following the convention in linear algebra, vectors will be denoted by bold lowercase English or Greek letters, and matrices will be denoted by bold capitalized English or Greek letters.

2.1 Unrestricted VAR model with exogenous variables

In some circumstances, the value of a variable is not only dependent on the variables inside the model. The variables outside the model (called exogenous variables) may also have an impact on the variable under scrutiny. A system of unrestricted VAR model with exogenous variables (VARX(p,q)) has the representation

$$\mathbf{y}_t = \boldsymbol{\mu} + \sum_{k=1}^{p} \boldsymbol{\Psi}_k \mathbf{y}_{t-k} + \sum_{k=0}^{q} \boldsymbol{\Phi}_k \mathbf{x}_{t-k} + \boldsymbol{\sigma}\boldsymbol{\epsilon}_t, \tag{2.1}$$

where $\mathbf{y}_t = (y_{1,t}, y_{2,t}, ..., y_{g,t})^\top$; $\mathbf{x}_t = (x_{1,t}, x_{2,t}, ..., x_{w,t})^\top$; $\boldsymbol{\mu} = (\mu_1, \mu_2, ..., \mu_g)^\top$; $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, ..., \sigma_g)^\top$; $\top$ stands for the transpose of a vector; and $\boldsymbol{\Psi}_k$ and $\boldsymbol{\Phi}_k$ are the respective $g \times g$ and $w \times w$ coefficient matrices. In this formulation, the variables in $\mathbf{y}$ are not only affected by the variables in the VAR model, but also by other variables $\mathbf{x}$ outside the model. A model with two variables and two exogenous variables VARX(p,q) model is written in matrix form as

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \sum_{k=1}^{p} \begin{bmatrix} \psi_{11,k} & \psi_{12,k} \\ \psi_{21,k} & \psi_{22,k} \end{bmatrix} \begin{bmatrix} y_{1,t-k} \\ y_{2,t-k} \end{bmatrix} + \sum_{k=0}^{q} \begin{bmatrix} \phi_{11,k} & \phi_{12,k} \\ \phi_{21,k} & \phi_{22,k} \end{bmatrix} \begin{bmatrix} x_{1,t-k} \\ x_{2,t-k} \end{bmatrix} + \begin{bmatrix} \sigma_1 \epsilon_{1,t} \\ \sigma_2 \epsilon_{2,t} \end{bmatrix}. \tag{2.2}$$

Remark 1. *In a statistical model, an endogenous variable has its value determined by other variables within the model. On the other hand, an exogenous variable has its value determined by other variables outside of the model. More specifically, in Equation (2.1), the vector* $\mathbf{y}$ *is an endogenous variable because its value is dependent on both* $\mathbf{x}$*'s and* $\mathbf{y}$*'s. However,* $\mathbf{x}$ *is an exogenous variable since its value cannot be obtained from factors inside the model. A control variable is a factor, which could have an impact on the outcome of the regression and could be either endogenous or exogenous.*

In this research, the vector $\mathbf{y}$ in Equation (2.2) consists of economic-growth and renewable-energy components. The exogenous variables are the oil price and short-term interest rate, which will be encapsulated in vector $\mathbf{x}$. The following are considerations when applying the VAR model.

(a) All variables must be of the same order of integration; this could be checked via the unit-root test for stationarity in time series. If all variables are non-stationary, a cointegration test is performed. If the variables are cointegrated, a vector error correction model (VECM), which is a restricted VAR model, is used to deal with the non-stationary series. If the variables are not cointegrated, differencing the variables is undertaken. Then, the unit-root tests are employed to check the stationarity of the differenced series. If the differenced series is still non-stationary, we repeatedly difference the series until stationarity is achieved. VAR variables must be stationary; otherwise, differenced variables should be employed instead. Having non-stationary time series as inputs in VAR will end up in getting spurious-regression relation.

(b) The optimal lag length of the VAR model is chosen based on some information criteria (e.g., Akaike information criterion (AIC) or the Hanan-Quinn criterion).

(c) There are two ways to assess the presence of a causal relation. The first one is through a significance test of the individual parameters. The second strategy is by checking the joint effects of all lags in the variables of interest with the utility of a likelihood-ratio test.

2.2 A Markov-chain driven VAR model

As mentioned above, it would be realistic for the parameters of the VAR model to be time-dependent. Hamilton [23] pointed out that, by and large, economic variables have time-dependent behaviour. For example, the growth rate of GDP fluctuates at various phases of a business cycle (expansion, peak, contraction, and trough). Moreover, many empirical studies took into consideration structural breaks characterized by parameters of economic-variable models that keep changing through time; see Dogan [8] in the context of causal-relation between economic growth and energy use. Similarly, Fallahi [14] utilized the MS-VAR model and showed that causality between economic growth and energy use could change as time progresses.

To equip a modelling approach that can adapt to time dependence, a hidden Markov chain is incorporated into the VAR model. Our use of an HMM-embedded algorithm is an improvement to Fallahi's approach [14] of adopting the Expectation-Maximization (EM) algorithm proposed by Hamilton in estimating the MS-VAR model. The essential technique used throughout this thesis is the change of reference-probability measure. Under the real-world probability P, the observations are not independent and identically distributed (IID). So, the calculations of the filters under P are difficult. A new probability measure, say $\tilde{P}$, equivalent to P is constructed such that under the new measure $\tilde{P}$, the observations are IID. The calculations of the filters are then facilitated by this ideal setting of $\tilde{P}$, where with the use of Fubini's Theorem, the interchange of expectations and summations are permitted. The optimal filters under the real-world P are obtained via the reverse change of probability measure. The visualization of these concepts and their interconnection are schematically diagrammed in Figure 2.1

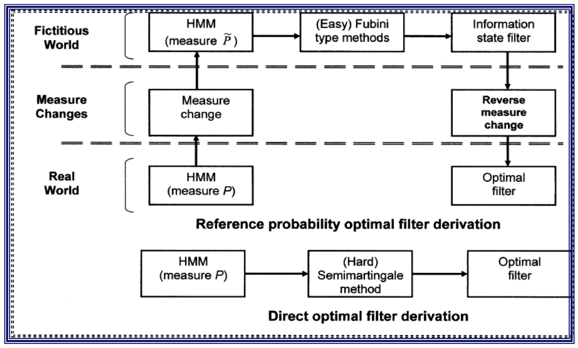

Figure 2.1: A portrayal of the idea behind the change of reference probability measure in filtering

The HMM algorithm in this thesis is more customizable than other HMM algorithms built-in software packages. For example, our HMM algorithm can work in high dimensions, an important functionality that is lacking in some packages. In addition, our change-of-measure methodology uses previously estimated parameter as initial values to process the next batch of data. Compared with other filtering algorithms, which rely on forward-backward algorithm, our approach requires much less computing memory; this is beneficial in the analysis of big data sets. Tenyakov et al. [46] pointed out that the Hamilton algorithm needs to be rerun whenever there are new data. In contrast, the HMM algorithm using the change-of-measure technique is computationally less costly; there is no need to store past data and prior estimates. The calculation is quick because processing targets only the newly available information. In addition, our HMM-recursive filtering algorithms naturally yields an online estimation of parameters. Each time a new point data or a batch of data arrives the parameters are automatically updated instantly.

In particular, the parameters of the VAR model are governed by a Markov chain $\mathbf{z}_t$, with a finite-state space, in discrete time t, for t=0, 1, $\dots T$. The state of the Markov chain reflects the regime of an economy. To facilitate subsequent algebraic computations, we set a one-to-one correspondence between the state space and the canonical basis $\{\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_n\}$, where $\mathbf{e}_i = (0, \dots, 0, 1, 0, \dots, 0)^\top \in \mathbb{R}^n$ with 1 in its i-th position and 0 elsewhere. The Markov chain $\mathbf{z}_t$ has a semi-martingale representation

$$\mathbf{z}_{t+1} = \mathbf{\Pi}\mathbf{z}_t + \mathbf{v}_{t+1}, \tag{2.3}$$

where $\mathbf{\Pi} = (\pi_{ij})$ is a transition matrix, $\pi_{ij} = P(\mathbf{z}_{t+1} = \mathbf{e}_i | \mathbf{z}_t = \mathbf{e}_j)$; $\mathbf{v}_{t+1}$ is a martingale increment with $E[\mathbf{v}_{t+1} | \mathscr{F}_t^z] = 0$; and $\mathscr{F}^z$ is the filtration generated by $\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_t$.

Note that $\boldsymbol{\mu}_g = (\mu_g^{(1)}, \mu_g^{(2)}, \dots, \mu_g^{(n)})^\top$, $\boldsymbol{\psi}_{ij,k} = (\psi_{ij,k}^{(1)}, \psi_{ij,k}^{(2)}, \dots, \psi_{ij,k}^{(n)})^\top$, $\boldsymbol{\phi}_{ij,k} = (\phi_{ij,k}^{(1)}, \phi_{ij,k}^{(2)}, \dots, \phi_{ij,k}^{(n)})^\top$, and $\boldsymbol{\sigma}_g = (\sigma_g^{(1)}, \sigma_g^{(2)}, \dots, \sigma_g^{(n)})^\top$ are all vectors in $\mathbb{R}^n$. With our formulated bijection between $\mathbf{z}_t$'s state space and the basis of $\mathbb{R}^n$, we obtain a simple representation of the time-dependent parameters as $\mu_g(\mathbf{z}_t) = \langle \boldsymbol{\mu}_g, \mathbf{z}_t \rangle$, $\psi_{ij,k}(\mathbf{z}_t) = \langle \boldsymbol{\psi}_{ij,k}, \mathbf{z}_t \rangle$, $\phi_{ij,k}(\mathbf{z}_t) = \langle \boldsymbol{\phi}_{ij,k}, \mathbf{z}_t \rangle$, and $\sigma_g(\mathbf{z}_t) = \langle \boldsymbol{\sigma}_g, \mathbf{z}_t \rangle$. Here, $\langle \cdot, \cdot \rangle$ denotes the Euclidean scalar product in $\mathbb{R}^n$. Also, the subscripts g, i, and j are the position locations of the parameters as described in Equation (2.2) with g, i, and $j \in \{1, 2\}$; the subscript k refers to the k-th lag with $k \in \{1, 2, \ldots, p\}$; and the symbol $^{(m)}$ stands for the parameter in the m-th regime for $m = 1, \ldots, n$. The VAR model in Equation (2.2), enriched by $\mathbf{z}_{t-1}$, is expressed as

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} \mu_1(\mathbf{z}_{t-1}) \\ \mu_2(\mathbf{z}_{t-1}) \end{bmatrix} + \sum_{k=1}^{p} \begin{bmatrix} \psi_{11,k}(\mathbf{z}_{t-1}) & \psi_{12,k}(\mathbf{z}_{t-1}) \\ \psi_{21,k}(\mathbf{z}_{t-1}) & \psi_{22,k}(\mathbf{z}_{t-1}) \end{bmatrix} \begin{bmatrix} y_{1,t-k} \\ y_{2,t-k} \end{bmatrix} + \sum_{k=0}^{q} \begin{bmatrix} \phi_{11,k}(\mathbf{z}_{t-1}) & \phi_{12,k}(\mathbf{z}_{t-1}) \\ \phi_{21,k}(\mathbf{z}_{t-1}) & \phi_{22,k}(\mathbf{z}_{t-1}) \end{bmatrix} \begin{bmatrix} x_{1,t-k} \\ x_{2,t-k} \end{bmatrix} + \begin{bmatrix} \sigma_1(\mathbf{z}_{t-1})\epsilon_{1,t} \\ \sigma_2(\mathbf{z}_{t-1})\epsilon_{2,t} \end{bmatrix}. \quad (2.4)$$

Remark 2. *As a clarification, each regression equation in the VAR model assumes that inputs (variables' original or differenced levels) are stationary. Due to economic structural changes, the Markov chain drives the random switching between regression equations having dynamic parameter values as time unfolds.*

Our objective is to detect the causality between the renewable-energy price (EP) and economic growth (EG). The variables EP and EG are endogenous to the VARX(p,q) system. Two variables, namely the oil price (OP) and short-term interest rates (IR) are included as the system's control variables. We would like to see the effects of OP and IR on EG. Control variables are introduced to avoid spurious causality if we concentrate on EP and EG. It may be possible that the relationship found between EP and EG is actually caused by the other variables not yet considered. In microeconomics, EG is often linked with IR. In general, when IR level is low, the economy will tend to grow; and conversely, when IR level is high, the economy will tend to slow down. As argued in Elliott and Mamon [11], this could be attributed to the mean reversion of interest rates, which is supported by the supply-and-demand analysis. It is observed that when interest rates are low, there is more borrowing that stimulates economic activities. The high demand for funds will cause interest rates to rise. When interest rates are high, demand for funds will decrease and with less spending the economy will tend to slow down. The variable OP is used as a proxy for the price of conventional-energy sources. Evidently, OP is employed for the purpose of comparison with EP.

Thus, it is reasonable to include OP and IR in EG's dynamics in Equation (2.4). However, as far as we are aware, there is no empirical finding that could vouch for the impact of IR on EP. Excluding IR, being viewed as an insignificant causal factor for EP, enables us to work on a manageable VAR system. That being said, we shall ignore the effects of IR on EP but focus instead on the effects of OP on EP. Therefore, $\phi_{22,k}$ is assigned the value 0 for all k. Furthermore, σ will be treated as constant. To confirm that this is a reasonable assumption, a univariate model with switching intercept and sigma was fitted to our four data sets. The regime-switching characteristic of the data is significantly pinned down by the switching intercept and not by the sigma. For many economic variables, it is the intercept that drives the regime-switching movement. The standard deviation stays the same even in different regimes. For example, the economic-growth rate has high and low growth periods. Yet, the variations (measured by the standard deviation) would be almost identical for both high and low periods even for a time horizon of two years.

Consequently, the model that we shall identify simplifies to

$$\begin{bmatrix} EG_t \\ EP_t \end{bmatrix} = \begin{bmatrix} \mu_1(\mathbf{z}_{t-1}) \\ \mu_2(\mathbf{z}_{t-1}) \end{bmatrix} + \sum_{k=1}^{p} \begin{bmatrix} \psi_{11,k}(\mathbf{z}_{t-1}) & \psi_{12,k}(\mathbf{z}_{t-1}) \\ \psi_{21,k}(\mathbf{z}_{t-1}) & \psi_{22,k}(\mathbf{z}_{t-1}) \end{bmatrix} \begin{bmatrix} EG_{t-k} \\ EP_{t-k} \end{bmatrix} + \sum_{k=0}^{q} \begin{bmatrix} \phi_{11,k}(\mathbf{z}_{t-1}) & \phi_{12,k}(\mathbf{z}_{t-1}) \\ \phi_{21,k}(\mathbf{z}_{t-1}) & 0 \end{bmatrix} \begin{bmatrix} OP_{t-k} \\ IR_{t-k} \end{bmatrix} + \begin{bmatrix} \sigma_1 \epsilon_{1,t} \\ \sigma_2 \epsilon_{2,t} \end{bmatrix}. \tag{2.5}$$

Let $(\Omega, \mathscr{F}_t, \mathbb{P})$ be the probability space providing the background for all stochastic processes in our modelling set up. In this framework, $\mathscr{F}_t = \mathscr{F}_t^{\mathbf{z}} \vee \mathscr{F}_t^{EP} \vee \mathscr{F}_t^{EG}$ is the global filtration; and $\mathscr{F}_t^{EP}$ and $\mathscr{F}_t^{EG}$ are the filtrations generated by $\{EP_t\}$ and $\{EG_t\}$, respectively. In Equation (2.5), $\{\epsilon_{1,t}\}$ and $\{\epsilon_{2,t}\}$ are sequences of IID standard normal random variables. Consequently,

$$EG_t|_{\mathscr{F}_t} \sim N(\mu_{EG},\ \sigma_{EG})$$

with

$$\begin{cases} \mu_{EG} = \mu_1(\mathbf{z}_{t-1}) + \sum_{k=1}^{p} \left(\psi_{11,k}(\mathbf{z}_{t-1})EG_{t-k} + \psi_{12,k}(\mathbf{z}_{t-1})EP_{t-k}\right) + \sum_{k=0}^{q} \left(\phi_{11,k}(\mathbf{z}_{t-1})OP_{t-k} + \phi_{12,k}(\mathbf{z}_{t-1})IR_{t-k}\right) \\ \sigma_{EG} = \sigma_1 \end{cases}$$

and

$$EP_t|_{\mathscr{F}_t} \sim N(\mu_{EP},\ \sigma_{EP})$$

with

$$\begin{cases} \mu_{EP} = \mu_2(\mathbf{z}_{t-1}) + \sum_{k=1}^{p} \left(\psi_{21,k}(\mathbf{z}_{t-1})EG_{t-k} + \psi_{22,k}(\mathbf{z}_{t-1})EP_{t-k}\right) + \sum_{k=0}^{q} \phi_{21,k}(\mathbf{z}_{t-1})OP_{t-k} \\ \sigma_{EP} = \sigma_2 \end{cases}$$

2.3 Change of reference probability measure

With their observed values as sample path's realisations, neither $\{EG_t\}$ nor $\{EP_t\}$ is a sequence of independent random variables under the real-world probability measure P. We shall change our reference-probability measure to $\overline{P}$ so that under this new measure, both $\{EG_t\}$ and $\{EP_t\}$ are sequences of IID random variables; $\overline{P}$ coincides with the generic $\widetilde{P}$ measure in Figure 2.1. This will, in turn, make the evaluation of conditional expectations of a product easy. Furthermore, the calculations of filters, which are merely conditional expectations, will be straightforward under $\overline{P}$. The filters computed under $\overline{P}$ are related back to the measure P, without any difficulty, through the use of Bayes' theorem for conditional expectation. To define a measure $\overline{P}$ equivalent to P, we construct a Radon-Nikodym derivative

$$\Lambda_t = \frac{\overline{P}}{P}\bigg|_{\mathscr{F}_t} = \prod_{k=1}^{t} \lambda_k^{(EG)} \lambda_k^{(EP)},$$

for $t \geq 1,\ \Lambda_0 = 1$,

$$\lambda_t^{(EG)} = \sigma_{EG} \exp\Big\{ -\frac{1}{2}\Big(EG_t{}^2 - \big(\frac{EG_t - \mu_{EG}}{\sigma_{EG}}\big)^2\Big)\Big\}$$

and

$$\lambda_t^{(EP)} = \sigma_{EP} \exp\Big\{ -\frac{1}{2}\Big(EP_t{}^2 - \big(\frac{EP_t - \mu_{EP}}{\sigma_{EP}}\big)^2\Big)\Big\}.$$

Suppose the vector $\widetilde{\mathbf{p}}_t$ is the conditional expectation of $\mathbf{z}_k$ under P. To be concise,

$$\widetilde{\mathbf{p}}_t = (\widetilde{p}_t^{(1)}, \widetilde{p}_t^{(2)}, \ldots, \widetilde{p}_t^{(n)})^\top \in \mathbb{R}^n \ \text{ and } \ \widetilde{p}_t^{(i)} = P(\mathbf{z}_t = \mathbf{e}_i | \mathscr{F}_t) = E[\langle \mathbf{z}_t, \mathbf{e}_i \rangle | \mathscr{F}_t].$$

Write

$$\overline{\Lambda}_t := \Lambda_t^{-1}$$

for the inverse of Λ_t. With the aid of Bayes' theorem,

$$\widetilde{\mathbf{p}}_t = E[\mathbf{z}_t | \mathscr{F}_t] = \frac{\overline{E}[\overline{\Lambda}_t \mathbf{z}_t | \mathscr{F}_t]}{\overline{E}[\overline{\Lambda}_t | \mathscr{F}_t]}. \tag{2.6}$$

Write $\boldsymbol{\xi}_t := \overline{E}[\overline{\Lambda}_t \mathbf{z}_t | \mathscr{F}_t]$. Noting that $\displaystyle\sum_{i=1}^{n} \langle \mathbf{z}_t, \mathbf{e}_i \rangle = 1$,

$$\begin{aligned}\overline{E}[\overline{\Lambda}_t | \mathscr{F}_t] &= \overline{E}\Big[\overline{\Lambda}_t \Big(\sum_{i=1}^{n} \langle \mathbf{z}_t, \mathbf{e}_i \rangle \Big) \Big| \mathscr{F}_t \Big] = \sum_{i=1}^{n} \overline{E}[\langle \overline{\Lambda}_t \mathbf{z}_t, \mathbf{e}_i \rangle \big| \mathscr{F}_t] \\ &= \sum_{i=1}^{n} \langle \overline{E}[\overline{\Lambda}_t \mathbf{z}_t | \mathscr{F}_t], \mathbf{e}_i \rangle = \sum_{i=1}^{n} \langle \boldsymbol{\xi}_t, \mathbf{e}_i \rangle.\end{aligned}$$

Thus, Equation (2.6) has the compact form

$$\widetilde{\mathbf{p}}_t = \frac{\boldsymbol{\xi}_t}{\sum_{i=1}^{n} \langle \boldsymbol{\xi}_t, \mathbf{e}_i \rangle}. \tag{2.7}$$

2.4 Filters and parameter estimates

In this subsection, we outline the derivation of the optimal estimators of the model parameters. Although the maximum-likelihood-estimation (MLE) method is a common approach to compute the parameters of a probability distribution, such a method is not straightforward to apply for a more elaborate model. We invoke the EM algorithm as an iterative technique to obtain the local maxima. We first consider and calculate the following Markov-dependent quantities:

$$J_t^{(sj)} = \sum_{l=1}^{t} \langle \mathbf{z}_{l-1}, \mathbf{e}_j \rangle \langle \mathbf{z}_l, \mathbf{e}_s \rangle, \tag{2.8}$$

$$O_t^{(j)} = \sum_{l=1}^{t} \langle \mathbf{z}_{l-1}, \mathbf{e}_j \rangle, \tag{2.9}$$

$$\text{and } T_t^{(j)}(f) = \sum_{l=1}^{t} \langle \mathbf{z}_{l-1}, \mathbf{e}_j \rangle f(\cdot, \cdot). \tag{2.10}$$

The respective scalar quantities in Equations (2.8), (2.9) and (2.10) refer to the number of jumps from state j to state s in time t; the amount of time $\mathbf{z}$ spent in state j up to time t; and an auxiliary process that depends on the function $f(\cdot, \cdot)$, where f is a function taking the forms $x_{t-k}, x_{t-k}x_{t-h}$, and $x_{t-k}y_{t-h}$; the x and y represent the variables EG, EP, IR, and OP; and the k and h denote the lags with $k, h = 0, 1, ..., p$. Define the diagonal matrix $\mathbf{D}_t$ as

$$\mathbf{D}_t = \begin{bmatrix} d_{t,1} & & & \\ & d_{t,2} & & \\ & & \ddots & \\ & & & d_{t,n} \end{bmatrix},$$

with diagonal elements

$$d_{t,j} = \overline{\lambda}_{t,j}^{(EG)} \overline{\lambda}_{t,j}^{(EP)}. \tag{2.11}$$

In Equation (2.11),

$$\begin{aligned}
\overline{\lambda}_{t,j}^{(EG)} &= \frac{1}{\sigma_1} \exp\Big(-\frac{1}{2}(\beta_{t,j}^{(EG)2} - EG_t^{\,2}) \Big), \\
\beta_{t,j}^{(EG)} &= \frac{EG_t - \mu_{1,j} - \sum_{k=1}^{p} \Big(\psi_{11,k,j} EG_{t-k} + \psi_{12,k,j} EP_{t-k} \Big) - \sum_{k=0}^{q} \Big(\phi_{11,k,j} OP_{t-k} + \phi_{12,k,j} IR_{t-k} \Big)}{\sigma_1}, \\
\overline{\lambda}_{t,j}^{(EP)} &= \frac{1}{\sigma_2} \exp\Big(-\frac{1}{2}(\beta_{t,j}^{(EP)2} - EP_t^{\,2}) \Big), \\
\text{and } \beta_{t,j}^{(EP)} &= \frac{EP_t - \mu_{2,j} - \sum_{k=1}^{p} \Big(\psi_{21,k,j} EG_{t-k} + \psi_{22,k,j} EP_{t-k} \Big) - \sum_{k=0}^{q} \phi_{21,k,j} OP_{t-k}}{\sigma_2},
\end{aligned}$$

where j is the parameter at the j-th regime. For any process C_t, by the Bayes' theorem again,

$$E[C_t|\mathscr{F}_t] = \frac{\overline{E}[C_t\overline{\Lambda}_t|\mathscr{F}_t]}{\overline{E}[\overline{\Lambda}_t|\mathscr{F}_t]} = \frac{\overline{E}[C_t\overline{\Lambda}_t|\mathscr{F}_t]}{\sum_{i=1}^n\langle\boldsymbol{\xi}_t, \mathbf{e}_i\rangle}. \tag{2.12}$$

If we define $\gamma(C_t) := \overline{E}[C_t\overline{\Lambda}_t|\mathscr{F}_t]$ then $\gamma(C_t) = \gamma(C_t\langle\mathbf{z}_t, \mathbf{1}\rangle) = \gamma(\langle C_t\mathbf{z}_t, \mathbf{1}\rangle) = \langle\gamma(C_t\mathbf{z}_t), \mathbf{1}\rangle$. Equation (2.12) has the usable representation

$$E[C_t|\mathscr{F}_t] = \frac{\overline{E}[C_t\overline{\Lambda}_t|\mathscr{F}_t]}{\sum_{i=1}^n\langle\boldsymbol{\xi}_t, \mathbf{e}_i\rangle} = \frac{\langle\gamma(C_t\mathbf{z}_t), \mathbf{1}\rangle}{\langle\boldsymbol{\xi}_t, \mathbf{1}\rangle}.$$

By taking advantage of the semi-martingale representation in (2.3), it may be shown (see Mamon et al. [32] and Elliot et al. [9]) that the vector processes $\boldsymbol{\xi}_t$, $J_t^{(sj)}\mathbf{z}_t$, $O_t^{(j)}\mathbf{z}_t$, and $T_t^{(j)}(g)\mathbf{z}_t$ have the recursions

$$\begin{aligned}
\boldsymbol{\xi}_t &= \mathbf{\Pi}\boldsymbol{D}_t\boldsymbol{\xi}_{t-1}, & (2.13)\\
\gamma(J_t^{(sj)}\mathbf{z}_t) &= \mathbf{\Pi}\boldsymbol{D}_t\gamma(J_{t-1}^{(sj)}\mathbf{z}_{t-1}) + d_{t,j}\langle\boldsymbol{\xi}_t, \mathbf{e}_j\rangle\pi_{sj}\mathbf{e}_s, & (2.14)\\
\gamma(O_t^{(j)}\mathbf{z}_t) &= \mathbf{\Pi}\boldsymbol{D}_t\gamma(O_{t-1}^{(j)}\mathbf{z}_{t-1}) + d_{t,j}\langle\boldsymbol{\xi}_t, \mathbf{e}_j\rangle\mathbf{\Pi}\mathbf{e}_j, & (2.15)\\
\text{and}\quad \gamma(T_t^{(j)}(f)\mathbf{z}_t) &= \mathbf{\Pi}\boldsymbol{D}_t\gamma(T_{t-1}^{(j)}(f)\mathbf{z}_{t-1}) + f(\cdot)d_{t,j}\langle\boldsymbol{\xi}_t, \mathbf{e}_j\rangle\mathbf{\Pi}\mathbf{e}_j. & (2.16)
\end{aligned}$$

To estimate the parameters of our MS-VAR model, the EM algorithm is utilized. In the E-step, the conditional expectation of the log-likelihood function is formulated, which is $E\left[\log\frac{\mathrm{d}P^{\widehat{\theta}}}{\mathrm{d}P^{\theta}}\middle|\mathscr{F}_t\right]$, where θ is the parameter of interest. In the M-step, we maximize the expression obtained in the E-step by differentiating with respect to the parameter of interest. Appendices 1 and 2 demonstrate how the EM algorithm is executed. The EM-based estimators for the model parameters are

$$\begin{aligned}
\widehat{\pi}_{sj} &= \frac{\widehat{J}_t^{(sj)}}{\widehat{O}_t^{(j)}}, & (2.17)\\
\widehat{\psi}_{11,k,j} &= \frac{\begin{bmatrix}\widehat{\Gamma}_{t,j}(EG_{t-k}EG_t) - \sum_{l\neq k}^p\widehat{\psi}_{11,l,j}\widehat{\Gamma}_{t,j}(EG_{t-k}EG_{t-l}) - \sum_{l=1}^p\widehat{\psi}_{12,l,j}\widehat{\Gamma}_{t,j}(EG_{t-k}EP_{t-l})\\ - \sum_{l=0}^q\widehat{\phi}_{11,l,j}\widehat{\Gamma}_{t,j}(EG_{t-k}OP_{t-l}) - \sum_{l=0}^q\widehat{\phi}_{12,l,j}\widehat{\Gamma}_{t,j}(EG_{t-k}IR_{t-l}) - \widehat{\mu}_{1,j}\widehat{\Gamma}_{t,j}(EG_{t-k})\end{bmatrix}}{\widehat{\Gamma}_{t,j}(EG_{t-k}^2)}, & (2.18)\\
\widehat{\psi}_{12,k,j} &= \frac{\begin{bmatrix}\widehat{\Gamma}_{t,j}(EP_{t-k}EG_t) - \sum_{l=1}^p\widehat{\psi}_{11,l,j}\widehat{\Gamma}_{t,j}(EP_{t-k}EG_{t-l}) - \sum_{l\neq k}^p\widehat{\psi}_{12,l,j}\widehat{\Gamma}_{t,j}(EP_{t-k}EP_{t-l})\\ - \sum_{l=0}^q\widehat{\phi}_{11,l,j}\widehat{\Gamma}_{t,j}(EP_{t-k}OP_{t-l}) - \sum_{l=0}^q\widehat{\phi}_{12,l,j}\widehat{\Gamma}_{t,j}(EP_{t-k}IR_{t-l}) - \widehat{\mu}_{1,j}\widehat{\Gamma}_{t,j}(EP_{t-k})\end{bmatrix}}{\widehat{\Gamma}_{t,j}(EP_{t-k}^2)}, & (2.19)\\
\widehat{\phi}_{11,k,j} &= \frac{\begin{bmatrix}\widehat{\Gamma}_{t,j}(OP_{t-k}EG_t) - \sum_{l=1}^p\widehat{\psi}_{11,l,j}\widehat{\Gamma}_{t,j}(OP_{t-k}EG_{t-l}) - \sum_{l=1}^p\widehat{\psi}_{12,l,j}\widehat{\Gamma}_{t,j}(OP_{t-k}EP_{t-l})\\ - \sum_{l\neq k}^q\widehat{\phi}_{11,l,j}\widehat{\Gamma}_{t,j}(OP_{t-k}OP_{t-l}) - \sum_{l=0}^q\widehat{\phi}_{12,l,j}\widehat{\Gamma}_{t,j}(OP_{t-k}IR_{t-l}) - \widehat{\mu}_{1,j}\widehat{\Gamma}_{t,j}(OP_{t-k})\end{bmatrix}}{\widehat{\Gamma}_{t,j}(OP_{t-k}^2)}, & (2.20)
\end{aligned}$$

$$\widehat{\phi}_{12,k,j} = \frac{\begin{bmatrix}\widehat{\Gamma}_{t,j}(IR_{t-k}EG_t) - \sum_{l=1}^{p}\widehat{\psi}_{11,l,j}\widehat{\Gamma}_{t,j}(IR_{t-k}EG_{t-l}) - \sum_{l=1}^{p}\widehat{\psi}_{12,l,j}\widehat{\Gamma}_{t,j}(IR_{t-k}EP_{t-l}) \\ - \sum_{l=0}^{q}\widehat{\phi}_{11,l,j}\widehat{\Gamma}_{t,j}(IR_{t-k}OP_{t-l}) - \sum_{l\neq k}^{q}\widehat{\phi}_{12,l,j}\widehat{\Gamma}_{t,j}(IR_{t-k}IR_{t-l}) - \widehat{\mu}_{1,j}\widehat{\Gamma}_{t,j}(OP_{t-k})\end{bmatrix}}{\widehat{\Gamma}_{t,j}(IR_{t-k}^2)}, \tag{2.21}$$

$$\widehat{\psi}_{21,k,j} = \frac{\begin{bmatrix}\widehat{\Gamma}_{t,j}(EG_{t-k}EP_t) - \sum_{l\neq k}^{p}\widehat{\psi}_{21,l,j}\widehat{\Gamma}_{t,j}(EG_{t-k}EG_{t-l}) - \sum_{l=1}^{p}\widehat{\psi}_{22,l,j}\widehat{\Gamma}_{t,j}(EG_{t-k}EP_{t-l}) \\ - \sum_{l=0}^{q}\widehat{\phi}_{21,l,j}\widehat{\Gamma}_{t,j}(EG_{t-k}OP_{t-l}) - \widehat{\mu}_{2,j}\widehat{\Gamma}_{t,j}(EG_{t-k})\end{bmatrix}}{\widehat{\Gamma}_{t,j}(EG_{t-k}^2)}, \tag{2.22}$$

$$\widehat{\psi}_{22,k,j} = \frac{\begin{bmatrix}\widehat{\Gamma}_{t,j}(EP_{t-k}EP_t) - \sum_{l=1}^{p}\widehat{\psi}_{21,l,j}\widehat{\Gamma}_{t,j}(EP_{t-k}EG_{t-l}) - \sum_{l\neq k}^{p}\widehat{\psi}_{22,l,j}\widehat{\Gamma}_{t,j}(EP_{t-k}EP_{t-l}) \\ - \sum_{l=0}^{q}\widehat{\phi}_{21,l,j}\widehat{\Gamma}_{t,j}(EP_{t-k}OP_{t-l}) - \widehat{\mu}_{2,j}\widehat{\Gamma}_{t,j}(EP_{t-k})\end{bmatrix}}{\widehat{\Gamma}_{t,j}(EP_{t-k}^2)}, \tag{2.23}$$

$$\widehat{\phi}_{21,k,j} = \frac{\begin{bmatrix}\widehat{\Gamma}_{t,j}(OP_{t-k}EP_t) - \sum_{l=1}^{p}\widehat{\psi}_{21,l,j}\widehat{\Gamma}_{t,j}(OP_{t-k}EG_{t-l}) - \sum_{l=1}^{p}\widehat{\psi}_{22,l,j}\widehat{\Gamma}_{t,j}(OP_{t-k}EP_{t-l}) \\ - \sum_{l\neq k}^{q}\widehat{\phi}_{21,l,j}\widehat{\Gamma}_{t,j}(OP_{t-k}OP_{t-l}) - \widehat{\mu}_{2,j}\widehat{\Gamma}_{t,j}(OP_{t-k})\end{bmatrix}}{\widehat{\Gamma}_{t,j}(OP_{t-k}^2)}, \tag{2.24}$$

$$\widehat{\mu}_{1,j} = \frac{\begin{bmatrix}\widehat{\Gamma}_{t,j}(EG_t) - \sum_{l=1}^{p}\widehat{\psi}_{11,L,j}\widehat{\Gamma}_{t,j}(EG_{t-l}) - \sum_{l=1}^{p}\widehat{\psi}_{12,l,j}\widehat{\Gamma}_{t,j}(EP_{t-l}) \\ - \sum_{l=0}^{q}\widehat{\phi}_{11,l,j}\widehat{\Gamma}_{t,j}(OP_{t-l}) - \sum_{l=0}^{q}\widehat{\phi}_{12,l,j}\widehat{\Gamma}_{t,j}(IR_{t-l})\end{bmatrix}}{\widehat{O}_{t,j}}, \tag{2.25}$$

$$\text{and } \widehat{\mu}_{2,j} = \frac{\widehat{\Gamma}_{t,j}(EP_t) - \sum_{l=1}^{p}\widehat{\psi}_{12,L,j}\widehat{\Gamma}_{t,j}(EG_{t-l}) - \sum_{l=1}^{p}\widehat{\psi}_{22,l,j}\widehat{\Gamma}_{t,j}(EP_{t-l}) - \sum_{l=0}^{q}\widehat{\phi}_{21,l,j}\widehat{\Gamma}_{t,j}(OP_{t-l})}{\widehat{O}_{t,j}}. \tag{2.26}$$

The proof of (2.17) is similar to that in Mamon et al. [32]. The derivations of Equations (2.18)-(2.26) are given in Appendices 1 and 2. As established in van der Vaart [50], the MLE estimators are consistent and follow an asymptotically normal sampling distribution.

The estimation of the parameter σ is not calculated through the EM-based procedure. The number of variables included in the MS-VAR system is already too many, and this number becomes overwhelmingly large particularly when the lags of the MS-VAR system are long into the past. As a result, the estimation of σ is not only unwieldy but also unstable. A simple device to circumvent this issue is through the use of the estimators

$$\widehat{\sigma}_1 = \sqrt{\frac{\sum_{j=1}^{N}\sum_{t=1}^{T}\left(EG_t - \widehat{\mu}_{1,j} - \sum_{k=1}^{p}\widehat{\psi}_{11,k,j}EG_{t-k} - \sum_{k=0}^{q}\left(\widehat{\psi}_{12,k,j}EP_{t-k} + \widehat{\phi}_{11,k,j}OP_{t-k} + \widehat{\phi}_{12,k,j}IR_{t-k}\right)\right)^2}{NT-1}},$$

and

$$\widehat{\sigma}_2 = \sqrt{\frac{\sum_{j=1}^{N}\sum_{t=1}^{T}\left(EP_t - \widehat{\mu}_{2,j} - \sum_{k=1}^{p}\widehat{\psi}_{22,k,j}EP_{t-k} - \sum_{k=0}^{q}\left(\widehat{\psi}_{21,k,j}EG_{t-k} + \widehat{\phi}_{11,k,j}OP_{t-k}\right)\right)^2}{NT-1}}.$$

The rationale for the above estimator is consistent with the assumption in Equation (2.5) that σ is assigned a constant value; that is, σ is the same in all regimes. Each equation in the VAR system is a linear regression, with σ being the residual term. Once the coefficients are estimated through the EM-based procedure, the residuals, together with their means and squares, are readily generated for each individual state.

In order to determine the causality between EG_t and EP_t, it is essential to compute the Fisher information of every parameter. In Equation (2.5), if any of the coefficients $\widehat{\psi}_{12,k}$ are significantly different from zero, then we conclude that EP_{t-k} causes EG_t. If any of the coefficients of $\widehat{\psi}_{21,k}$ is significantly different from zero, then we conclude that EG_{t-k} causes EP_t. The same principle applies to the causality between OP and IR. To test the significance of a variable in the regression, a hypothesis test is required. The null hypothesis, which asserts that the coefficient of the variable is not different from zero, is paired with the alternative hypothesis stating that the coefficient is different from zero.

There are various ways to compute the Fisher information for EM algorithm. Louis [31], under an incomplete-data setting, developed a procedure for extracting the observed Fisher-information matrix when the EM algorithm is applied to find the MLEs. In this study, the Fisher information $\mathcal{I}(\theta)$ is calculated with the use of the HMM algorithm. For a generic parameter θ, the expressions for the pertinent $\mathcal{I}(\widehat{\theta})$ are

$$\begin{aligned} &\mathcal{I}(\widehat{\psi}_{11,k,j}) = \frac{\widehat{\Gamma}_{t,j}(EG^2_{t-k})}{\widehat{\sigma}^2_{1,j}},\ \mathcal{I}(\widehat{\phi}_{12,k,j}) = \frac{\widehat{\Gamma}_{t,j}(IR^2_{t-k})}{\widehat{\sigma}^2_{2,j}},\ \mathcal{I}(\widehat{\psi}_{12,k,j}) = \frac{\widehat{\Gamma}_{t,j}(EP^2_{t-k})}{\widehat{\sigma}^2_{1,j}},\ \mathcal{I}(\widehat{\psi}_{22,k,j}) = \frac{\widehat{\Gamma}_{t,j}(EP^2_{t-k})}{\widehat{\sigma}^2_{2,j}}, \\ &\mathcal{I}(\widehat{\phi}_{11,k,j}) = \frac{\widehat{\Gamma}_{t,j}(OP^2_{t-k})}{\widehat{\sigma}^2_{1,j}},\ \mathcal{I}(\widehat{\phi}_{21,k,j}) = \frac{\widehat{\Gamma}_{t,j}(OP^2_{t-k})}{\widehat{\sigma}^2_{2,j}},\ \mathcal{I}(\widehat{\phi}_{12,k,j}) = \frac{\widehat{\Gamma}_{t,j}(IR^2_{t-k})}{\widehat{\sigma}^2_{2,j}}. \end{aligned} \tag{2.27}$$

The derivations of the expressions in Equation in (2.27) are relegated to Appendices 3 and 4.

As per Toda and Peter [48], the examination of causality in a VAR model makes use of the Wald test, which involves the Fisher information of each parameter through the p-value

$$2\left(1 - \Phi\left(\left|\frac{\widehat{\theta}}{1/\sqrt{\mathcal{I}(\theta)}}\right|\right)\right), \tag{2.28}$$

where Φ is the cumulative distribution function of the standard normal distribution. In this case, $\Phi(\cdot)$ has the range $[0.5, 1.0]$ due to its argument constrained by the absolute value. The $p-$value is our tool in gauging whether or not the distance between the coefficient estimate $\widehat{\theta}$ and 0 is statistically significant.

Chapter 3

Data overview

The data sets used in this study consist of five variables (viz. wholesale electricity price (EP), share of renewable energy in electricity generation (REG), volume index of real GDP, Brent oil price (OP), and short-term interest rate (IR)); three of these (i.e., EP, OP and REG) are economic indicators from the energy sector and the remaining two (i.e., GDP and IR) measure economic performance. In this study, two MS-VAR models are estimated using the data sets for each of the four regions. The first model deals with two endogenous variables (GDP and EP) and two exogenous variables (IR and OP). The second model focuses on two endogenous variables (GDP and REG) and two exogenous variables (IR and OP).

The central concern of this thesis is the examination of the nexus between the price of renewable energy and economic growth. Unlike non-renewable resources (e.g., coal, petroleum, natural gas, etc), there is no tangible price or direct index for renewable sources. Given that a huge portion of renewable sources are harnessed to primarily produce electricity, especially in the four regions of our case study, it makes the electricity price to be a rational substitute for the price of renewable energy. In the past, a majority of the empirical investigations were devoted to probing, by and large, the effects of renewable-energy output on economic growth. Therefore, for the purpose of comparison, we also delve into the share of renewable energy in electricity generation (REG). The volume index of real GDP is used as a proxy for economic growth. Furthermore, it is plausibly deemed that the price of conventional-energy sources, to some extent, impacts economic growth along with renewable energy. Thus, the Brent oil price (OP) is taken to assume the role of conventional energy. Oil price is a macroeconomic-activity variable and akin to this, the short-term interest rate is part of our data set to boot.

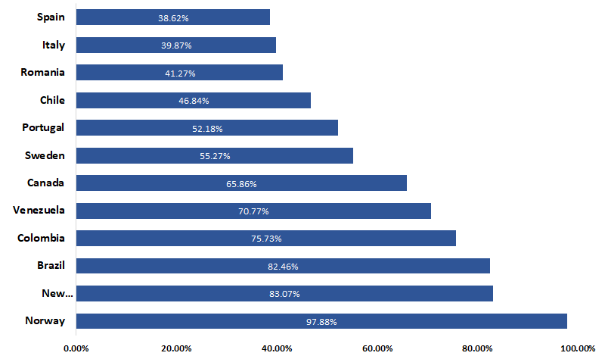

Figure 3.1: Proportion of renewable sources used in electricity generation in year 2018 for the top 10 countries

Our data sets include the time series data of three countries that belong to the top 10 in terms of the share of renewable sources in electricity generation. Figure 3.1 depicts the top ten countries in the world in reference to their renewable resources utilisation to produce electric power.

We selected three countries (Norway, New Zealand, and Canada) in our analysis because these countries have very high shares of renewable energy in the production of electricity. On account of this, the electricity price is regarded as a satisfactory proxy for renewable-energy price. The said three countries were also specifically chosen in view of data availability and reliability. Each country has their own regulatory framework for the electricity market. Norway, New Zealand, and the two provinces of Canada (Alberta and Ontario) deregulated their electricity markets at different levels of liberalisation in the last decade, allowing a healthy market competition in the electricity-generation sector. These regions operate their own electricity markets: Norway's Nord Pool, New Zealand Electricity Market (NZEM), Alberta Electric System Operator (AESO), and Ontario Independent Electricity System Operator (IESO), whereby firms buy and sell wholesale electricity. Whereas, the electricity markets in Brazil, Colombia, and others are heavily regulated and do not allow market competition; as such, there are no wholesale electricity price data for these countries. The variables in each country included in our data sets together with their descriptions are listed in Table 3.1.

The GDP volume index data for Norway and New Zealand were collected from the OECD statistics. For Alberta, the annual real GDP data (in \$ billion) were compiled by the Government of Alberta, while for Ontario the quarterly real GDP data were gathered by the Ontario Ministry of Finance. The Canadian real GDP data are publicly accessible. The GDP series is transformed into a GDP volume index by using 2015 as the base year. The monthly average wholesale electricity prices for Norway, New Zealand, Alberta, and Ontario were obtained from Statistics Norway, Electricity Authority, AESO, and IESO, respectively. The price data are then homogenized into prices on a quarterly basis by recording only the last value for each quarter. The time series data on the shares of renewable energy utilized for electricity generation were acquired from Enerdata Global Energy Yearbook for Norway and New Zealand. The Alberta's and Ontario's REG data were separately obtained from Statistics Canada and IESO. The short-term interest rates, proxied by appropriate yield rates of T-bill instruments in each region, were taken from the OECD statistics for all countries; for emphasis, Alberta and Ontario share the same Canadian short-term interest rates.

We sourced out the quarterly Brent oil prices (in $ per barrel), keeping only the last value of each quarter, from the U.S. Energy Information Administration (EIA). For certain variables on a yearly basis, they were converted into quarterly series by using the quadratic match method noted for its convenient operational scheme.

Table 3.1: Description of variables for the four regions

Region	Variable	Definition	Time	Source
Norway	GDP	GDP volume index with 2015 as base year	1998Q1-2019Q3	OECD statistics
	EP	Wholesale electricity price in $ per KWh	1998Q1-2019Q3	Statistics Norway
	REG	Share of renewable energy in electricity generation (%)	1998-2018	Enerdata Global Energy Statistical Yearbook
	IR	Short-term interest rates	1998Q1-2019Q3	OECD statistics
New Zealand	GDP	GDP volume index with 2015 as base year	1996Q4-2019Q3	OECD statistics
	EP	Wholesale electricity price in $ per MWh	1996Q4-2019Q3	Electricity Authority
	REG	Share of renewable energy in electricity generation (%)	1996-2018	Enerdata Global Energy Statistical Yearbook
	IR	Short-term interest rates	1996Q4-2019Q3	OECD statistics
Alberta	GDP	GDP volume index with 2015 as base year	2000-2018	Government of Alberta
	EP	Wholesale electricity price in $ per MWh	2000Q1-2018Q4	Alberta Electric System Operator
	REG	Share of renewable energy in electricity generation (%)	2005-2016	Statistics Canada
	IR	Short-term interest rates	2000Q1-2018Q4	OECD statistics
Ontario	GDP	GDP volume index with 2015 as base year	2002Q3-2019Q3	Ontario Ministry of Finance.
	EP	Wholesale electricity price in $ per MWh	2002Q3-2019Q3	Independent Electricity System Operator
	REG	Share of renewable energy in electricity generation (%)	2003-2019	Independent Electricity System Operator
	IR	Short-term interest rates	2002Q3-2019Q3	OECD statistics
All	OP	Brent oil price	1996Q1-2019Q3	U.S. Energy Information Administration

The original data sets for each region are shown in Figures 3.2-3.5. The GDP time series in the four regions have general upward trends. The EP dynamics in the four regions as well as the movements of the OP keep fluctuating. With the exception of Norway, the REG data in New Zealand, Alberta, and Ontario have progressively increased over time. For the IR numbers in all regions, they remained at a high level before 2010 but were relatively stable at a low level thereafter.

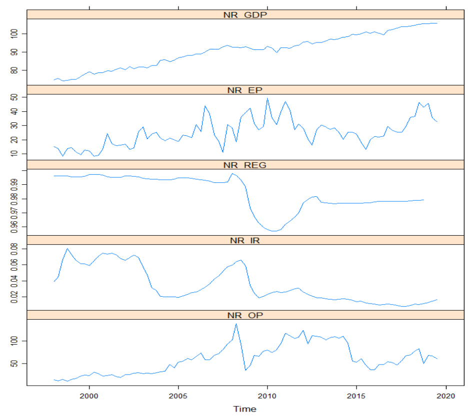

Figure 3.2: Original time-series data for Norway

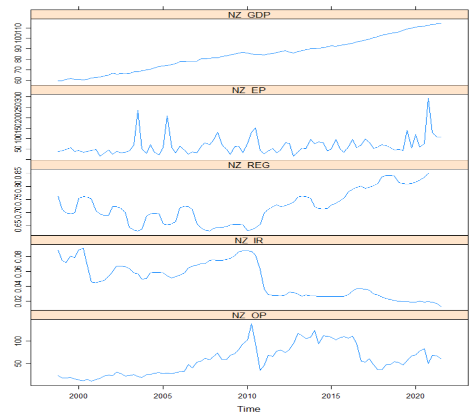

Figure 3.3: Original time-series data for New Zealand

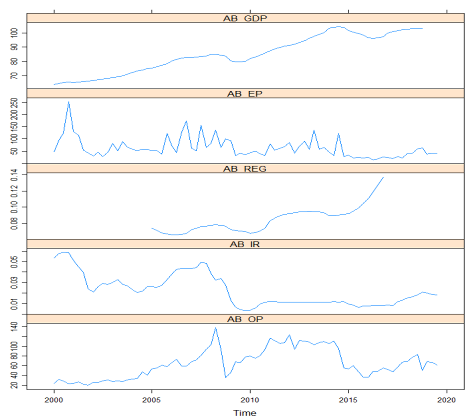

Figure 3.4: Original time-series data for Alberta

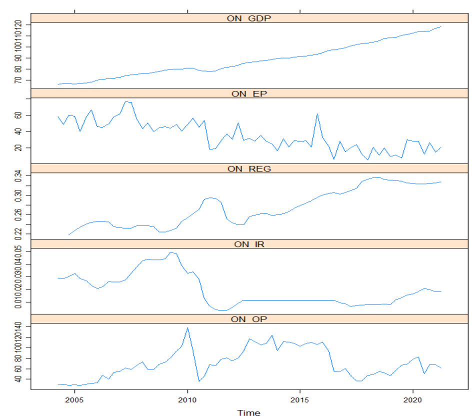

Figure 3.5: Original time-series data for Ontario

Since we use the MS-VAR model, it is assumed that all the variables in the model are stationary. For compatibility with this assumption, all the variables are converted to percentage change over the previous quarter. After this data transformation, the magnitudes of some transformed variables are no longer comparable with the other transformed variables. The transformed GDP and REG scales are very small; therefore, a multiplier of 100 is initiated to make them at comparable levels with others. Figures 3.6-3.9 display the the percentage-change time-series plots for each region.

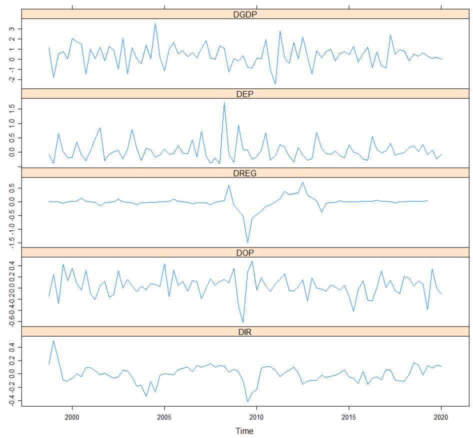

Figure 3.6: Time-series data (in % change) for Norway

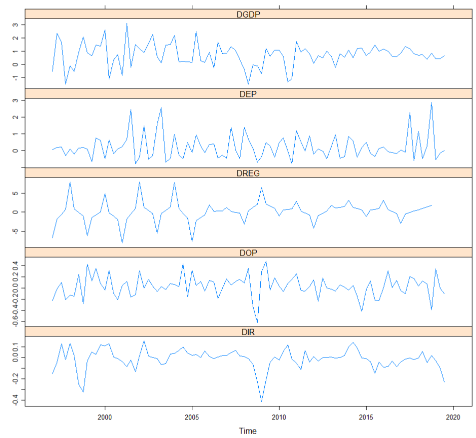

Figure 3.7: Time-series data (in % change) for New Zealand

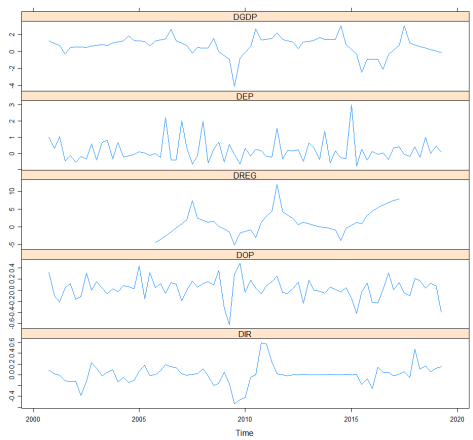

Figure 3.8: Time-series data (in % change) for Alberta

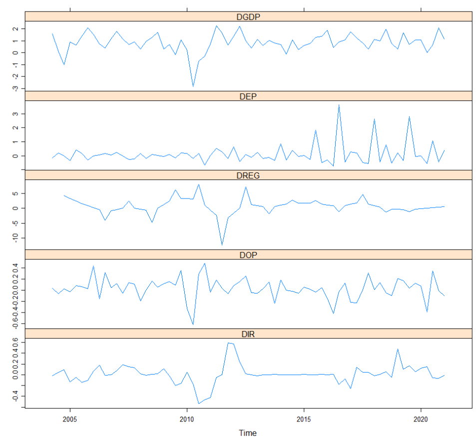

Figure 3.9: Time-series data (in % change) for Ontario

Chapter 4

Empirical investigation

As discussed in Chapter 3, two different models are estimated with one of the endogenous variables being replaced. The two models are:
Model I

- Endogenous variable: ΔGDP and ΔEP
- Control variable: ΔOP and ΔIR

Model II

- Endogenous variable: ΔGDP and ΔREG
- Control variable: ΔOP and ΔIR

The use of the difference operator Δ above is necessary to achieve stationary of input variables. As noted above, we now deal with the variables' percentage change and the resulting series are stationary at this level.

4.1 Unit root tests

As previously emphasized, one important consideration for the unrestricted VAR is that all series in the VAR should be stationary. Before fitting our MS-VAR models, the augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests are performed first to ascertain the stationarity of the original variables. Table 4.1 reports the results of the unit-root tests applied to the differenced time-series data. For the ADF test of each differenced series, the lag length p must be assigned when applying the test to allow for higher-order AR processes. The optimal lag lengths were selected with the use of the AIC.

An AR model of order p (AR(p)) can be written as

$$y_t = \alpha + \beta t + \sum_{i=1}^{p} \phi_i y_{t-i} + \epsilon_t, \tag{4.1}$$

where α is the intercept term and βt is the time trend. To perform the ADF unit-root test on an AR(p) process, we re-write Equation (4.1) as

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + \epsilon_t,$$

where $\gamma = 1 - \phi_1$. The p lagged-difference terms capture the serial correlation . If $\gamma = 0$ (or $\phi_1 = 1$), the AR(p) process is a random walk process, which is a non-stationary series. The existence of non-stationarity causes the potential issue of spurious regression.

A time series is trend-stationary if it has no unit root ($\gamma < 0$) and $\beta \neq 0$. With other terms remaining unchanged, y_t increases with time t at the rate β. The Dickey-Fuller (DF) test statistic is computed as

$$\mathrm{DF} = \frac{\widehat{\gamma}}{s_{\widehat{\gamma}}}.$$

The coefficient $s_{\widehat{\gamma}}$ represents the standard deviation of the estimated γ. The DF statistic follows an asymptotic t-distribution.

Unlike the ADF test, which uses a parametric autoregressive structure in capturing serial correlation, the PP test uses a non-parametric method to deal with serial correlation. The PP test involves fitting the regression

$$y_t = \alpha + \beta t + \phi y_{t-1} + \epsilon_t.$$

The null hypothesis is $\phi = 1$, which means the series is non-stationary. For the alternative hypothesis, $\phi < 1$ indicating stationarity. A modification to account for serial correlation leads to the test statistic

$$\begin{aligned}
\mathrm{ADF} &= \sqrt{\frac{\widehat{\sigma}^2}{\widehat{\lambda}^2}}\frac{\widehat{\phi}-1}{s_{\widehat{\phi}}} - \frac{1}{2}\left(\frac{\widehat{\lambda}^2 - \widehat{\sigma}^2}{\widehat{\lambda}^2}\right)\left(\frac{n s_{\widehat{\phi}}}{\widehat{\sigma}^2}\right),\\
\widehat{\lambda}^2 &= \widehat{\gamma}_0 + 2\sum_{j=1}^{q}\left(1 - \frac{j}{q+1}\right)\widehat{\gamma}_j, \quad \text{where}\\
\widehat{\gamma}_j &= \frac{1}{n}\sum_{i=j+1}^{n} \widehat{\epsilon}_i \widehat{\epsilon}_{i-j},\\
\widehat{\sigma}^2 &= \frac{1}{n}\sum_{i=1}^{n} \widehat{\epsilon}_i^2.
\end{aligned} \tag{4.2}$$

In Equations (4.2), n is the number of observations of the series and q is the number of lags determined by the Newey-West estimator introduced by Newey and West [38]. It is advisable to make several tests and see if the results reconcile. Before we test for the stationarity of the time series, we recall that there are three specifications for the unit-root test:

(a) Trend and intercept components in the test equation (i.e., $\alpha \neq 0$ and $\beta \neq 0$);

(b) An intercept only in the test equation (i.e. $\alpha \neq 0$ and $\beta = 0$); and

(c) Neither trend nor intercept is in the test equation (i.e. $\alpha = 0$ and $\beta = 0$). The steps to choose the appropriate specification is outlined below.

  (c.i) Plot the data and check the resulting graph for presence of deterministic regressor. If there is a time trend, both trend and intercept are included in the test equation. If no time trend is observed, we estimate the model with the intercept only.

  (c.ii) After the model with intercept is estimated, we check whether the intercept coefficient is significant. If the intercept is significant, we include intercept in the model. If the intercept is not significant, we estimate the model with no trend and no intercept.

From Figures 3.6-3.9, there is no clear time trend in all time series. So, the model with intercept is estimated for all time series. The results show that the intercepts of GDP and EP in all four regions are significant. REG and IR in all four regions have no trend and no intercept. OP has intercept in Alberta and Ontario, but has no trend and no intercept in New Zealand and Norway.

The conclusions of both the ADF and PP tests are in agreement. The null hypothesis of non-stationarity is rejected for all variables at the 5% significance level, and hence also at the 1%-significance level. All variables are, therefore, stationary, i.e. $I(0)$ as well. We can now proceed to the identification or parameter estimation of the MS-VAR model.

4.2 Numerical implementation

In implementing our HMM algorithm to find the parameters of the MS-VAR model, we must decide on the number of lags and the number of regimes of the MS-VAR model. Note that the number of coefficients increase dramatically with the increase in the number of lags and the number of regimes. As each country's data set used here is of relatively small size, it is reasonable to keep the lag length and number of regimes small to make the estimation manageable. We choose $p = 2$ and $q = 1$ and the number of regimes is set to 2. Hence, we include GDP_{t-1}, GDP_{t-2}, EP_{t-1}, and EP_{t-2} for the endogenous variables and OP_t, OP_{t-1}, IR_t, and IR_{t-1} for the control variates.

The implementation of the HMM algorithm requires initial values. There are several methods to find the initial parameters. These methods are discussed in Erlwein and Mamon [13], Erlwein et al. [12], and Date and Ponamareva [6]. Unlike the aforesaid techniques for initial-

Table 4.1: Outcome of the ADF and PP unit-root tests. The values reported are the ADF and PP test statistics. The symbols ** and *** signifies rejection of the null hypothesis of a unit root at the 0.05- and 0.01-significance levels, respectively.

	ΔGDP	ΔEP	ΔREG	ΔIR	ΔOP
Norway data					
ADF	-11.781***	-9.783***	-4.327***	-4.657***	-7.908***
PP	-11.821***	-13.194***	-4.405***	-4.455***	-10.420***
New Zealand data					
ADF	-9.812***	-10.312***	-3.431***	-4.902***	-8.057***
PP	-9.186***	-12.088***	-6.801***	-5.077***	-10.489***
Alberta data					
ADF	-4.122**	-8.796***	-2.024**	-4.537***	-8.439***
PP	-4.200***	-13.172***	-2.109**	-4.599***	-8.452***
Ontario data					
ADF	-5.775***	-7.036***	-2.215***	-4.560***	-7.077***
PP	-5.830***	-11.353***	-4.669***	-4.292***	-8.447***

value determination under some stochastic process modelling set up, our initial-value search is conducted in the context of regression models. In particular, we estimate the parameters of the Markov-switching regression, and use these parameters as initial-value benchmarks; they are presented in Appendix 6.

As previously stated, the data sets were collected and quoted on a quarterly basis. A window size of 2 points is used for each data set of the four regions. In other words, the model parameters and filtered probabilities are updated every half a year. The HMM algorithm starts by running the recursive Equations (2.13)-(2.16) on the data set. The processing of one batch of data constitutes the so-called one algorithm step or an algorithm pass. In every pass, 2 vectors of data points, i.e., the data in $t-1$ and t, are processed to estimate the Markov-chain-dependent processes (2.8)-(2.10). When each pass is completed, updated parameter estimates are generated by Equations (2.17)-(2.26). The filtered probabilities are updated by Equation (2.7). This is followed by the next algorithm pass, which processes the next batch of data via the filtering recursions, and the results are used as inputs for the initial values of the subsequent data processing. The HMM estimation continues through the passes until the last batch of data is processed exhaustively. The Fisher information could be calculated through Equation (2.27), which is useful not only for the computation of the standard errors (SE) accompanying each estimate but also - as Equation (2.28) shows - for the p-value in testing the null hypothesis that the coefficient is equal to zero. A low p-value indicates that the coefficient is different from zero.

4.3 Empirical analysis

The estimates of the HMM-driven parameters based on the data for all four regions are reported in Tables 4.2-4.9. Regime 1 is identified by the set of parameters with a higher intercept term, and regime 2 is associated with the set of parameters with a lower intercept term. On this account, regime 1 corresponds to a high-economic-growth period, and regime 2 is assigned to the period of low-economic-growth period. The numbers in parentheses are the p-values of the corresponding estimated parameters. If the p-value is very small, there is strong evidence to reject the null hypothesis that the coefficient is equal to zero. The filtered probabilities for each model are presented in Appendix 6.

Remark 3. *To avoid clutter of numbers, the standard errors (SEs) of the estimated parameters in Tables 4.2-4.9 are not shown. The histogram of the SEs are rightly skewed and almost all of them lie in the interval [0.034, 0.473].*

Remark 4. *To address the adequacy of the estimated models, a residual analysis is conducted. We calculate the pooled standardized residuals. Quantile-quantile plots are obtained to evaluate the normality of the pooled residuals. The autocorrelation and partial autocorrelation functions of the residuals are also analysed to assess the independence of residuals. The results show that the residuals are independent and normally distributed.*

Now, the models with the estimated parameters are ready for use to find out the causal relationship between the variables. We customize the regression equation (2.2) to our study, and consider

$$\begin{bmatrix} EG_t \\ EP_t \end{bmatrix} = \begin{bmatrix} \mu_1(\mathbf{z}_{t-1}) \\ \mu_2(\mathbf{z}_{t-1}) \end{bmatrix} + \sum_{k=1}^{2} \begin{bmatrix} \psi_{11,k}(\mathbf{z}_{t-1}) & \psi_{12,k}(\mathbf{z}_{t-1}) \\ \psi_{21,k}(\mathbf{z}_{t-1}) & \psi_{22,k}(\mathbf{z}_{t-1}) \end{bmatrix} \begin{bmatrix} EG_{t-k} \\ EP_{t-k} \end{bmatrix} + \sum_{k=0}^{1} \begin{bmatrix} \phi_{11,k}(\mathbf{z}_{t-1}) & \phi_{12,k}(\mathbf{z}_{t-1}) \\ \phi_{21,k}(\mathbf{z}_{t-1}) & 0 \end{bmatrix} \begin{bmatrix} OP_{t-k} \\ IR_{t-k} \end{bmatrix} + \begin{bmatrix} \sigma_1 \epsilon_{1,t} \\ \sigma_2 \epsilon_{2,t} \end{bmatrix}, \tag{4.3}$$

and

$$\begin{bmatrix} EG_t \\ REG_t \end{bmatrix} = \begin{bmatrix} \mu_1(\mathbf{z}_{t-1}) \\ \mu_2(\mathbf{z}_{t-1}) \end{bmatrix} + \sum_{k=1}^{2} \begin{bmatrix} \psi_{11,k}(\mathbf{z}_{t-1}) & \psi_{12,k}(\mathbf{z}_{t-1}) \\ \psi_{21,k}(\mathbf{z}_{t-1}) & \psi_{22,k}(\mathbf{z}_{t-1}) \end{bmatrix} \begin{bmatrix} EG_{t-k} \\ REG_{t-k} \end{bmatrix} + \sum_{k=0}^{1} \begin{bmatrix} \phi_{11,k}(\mathbf{z}_{t-1}) & \phi_{12,k}(\mathbf{z}_{t-1}) \\ \phi_{21,k}(\mathbf{z}_{t-1}) & 0 \end{bmatrix} \begin{bmatrix} OP_{t-k} \\ IR_{t-k} \end{bmatrix} + \begin{bmatrix} \sigma_1 \epsilon_{1,t} \\ \sigma_2 \epsilon_{2,t} \end{bmatrix}. \tag{4.4}$$

Remark 5. *We shall call Equations (4.3) and (4.4) Models I and II, respectively.*

As an illustration, if the estimated coefficients of a variable other than those of the GDP are statistically different from zero, a causal relationship is established from such a variable to the GDP. Likewise, a causal relationship could be examined between other endogenous variables and their control variables.

In this thesis, our analysis also categorically focuses on Granger causality, which is a statistical concept of causality drew upon prediction. Through the utility of linear-regression modelling of stochastic processes, Granger [17] laid out the mathematical formulation for this type of causality. To be precise, a signal A "Granger-causes" a signal B means that the past values of A contain information that enables the prediction of B above and beyond the information contained in the past values of B alone. The null hypothesis of the Granger causality is "variable/signal A does not Granger-cause variable/signal B", which means the coefficients of the model for A and its lag terms are all equal to zero. The alternative hypothesis is "at least one coefficient of the model for A and its lag terms is not equal to zero". The distribution of the Granger-causality test statistic follows an F-distribution. If the p-value of the F-statistic is small (at most 5%), the null hypothesis of no Granger-causality is rejected; and so A Granger-causes B. The strength of the Granger causality is measured by the p-value of the F-statistic. If the p-value is 1% or below (10% or above), the causality is very strong (very weak).

4.3.1 The case of Norway

Tables 4.2 and 4.3 give the respective Model I's and II's parameter estimates in the case of Norway. These estimated parameters, as per Equations (4.3) and (4.4), are the coefficients of the regressors (ψ's and ϕ's) and the intercepts (μ_i's). In Model I, except for the intercept and GDP terms, the differences between zero and the estimated coefficients of other variables are not statistically significant. This evinces no causal relationship between any variables in Model I. To that end, there is no significant relationship between the price of renewable energy and the economic growth in Norway.

In Model II, no causality is found between the GDP and REG in either regime. The neutrality hypothesis is supported in both regimes in the case of Norway. However, we observe that the two control variables OP and IR have an impact on GDP and REG. There is a unidirectional causality running from IR to GDP in the first regime. There is a unidirectional causality as well running from OP to REG in the first regime. In regime 2, there exists a unidirectional causality running from OP to GDP. The results of the linear Granger-causality test for Norway are summarized in Table 4.4, showing no causal relation between the variables in both models.

Table 4.2: Parameter estimates based on the Norway data under Model I. Numbers in parentheses are p-values of the corresponding estimated parameters. Each parameter is indicated as significant at the 0% level (with the symbol ***), at the 1 % level (with the symbol **), at the 5% level (with the symbol *) and at the 10% level (with the symbol *).

	Regime 1		Regime 2	
	ΔGDP	ΔEP	ΔGDP	ΔEP
Intercept	1.093*** (0.000)	0.125* (0.098)	0.615*** (0.000)	0.123** (0.005)
ΔGDP_{t-1}	-0.518* (0.038)	-0.070 (0.281)	-0.211 (0.167)	-0.032 (0.418)
ΔGDP_{t-2}	-0.754** (0.003)	-0.048 (0.466)	-0.214 (0.155)	-0.057 (0.143)
ΔEP_{t-1}	-0.169 (0.849)	-0.124 (0.594)	-0.416 (0.374)	-0.199 (0.102)
ΔEP_{t-2}	-0.217 (0.811)	-0.070 (0.767)	-0.747 (0.109)	-0.189 (0.118)
ΔOP_{t}	1.703 (0.281)	-0.252 (0.54)	0.332 (0.692)	0.044 (0.838)
ΔOP_{t-1}	0.267 (0.864)	0.037 (0.927)	0.682 (0.416)	-0.003 (0.988)
ΔIR_{t}	0.197 (0.934)		2.047 (0.118)	
ΔIR_{t-1}	-3.768 (0.106)		-1.230 (0.349)	
p11	0.805			
p22	0.939			
σ_{GDP}	1.340			
σ_{EP}	0.348			

Table 4.3: Parameter estimates based on the Norway data under Model II. Numbers in parentheses are p-values of the corresponding estimated parameters. Each parameter is indicated as significant at the 0% level (with the symbol ***), at the 1 % level (with the symbol **), at the 5% level (with the symbol *) and at the 10% level (with the symbol *).

	Regime 1		Regime 2	
	ΔGDP	ΔREG	ΔGDP	ΔREG
Intercept	1.021*** (0.000)	-0.036 (0.382)	0.662** (0.004)	-0.015 (0.765)
ΔGDP_{t-1}	-0.440** (0.005)	0.051 (0.136)	-0.387* (0.079)	0.014 (0.772)
ΔGDP_{t-2}	-0.538** (0.001)	0.052 (0.118)	-0.268 (0.224)	0.037 (0.431)
ΔREG_{t-1}	1.891 (0.136)	0.658* (0.016)	0.458 (0.486)	0.302* (0.033)
ΔREG_{t-2}	-0.805 (0.560)	1.203*** (0.000)	-0.369 (0.567)	0.200 (0.149)
ΔOP_{t}	0.007 (0.995)	0.006 (0.981)	0.661 (0.540)	-0.029 (0.899)
ΔOP_{t-1}	-0.724 (0.509)	-0.491* (0.037)	1.799* (0.088)	0.019 (0.932)
ΔIR_{t}	-3.657* (0.026)		2.025 (0.204)	
ΔIR_{t-1}	0.853 (0.596)		-0.290 (0.861)	
p11	0.920			
p22	0.873			
σ_{GDP}	1.318			
σ_{REG}	0.284			

Table 4.4: Results of the Granger's linear-causality test for the Norway data

Null hypothesis	F-statistic	p-value
Model I		
ΔEP does not Granger cause ΔGDP	3.042	0.219
ΔOP does not Granger cause ΔGDP	0.698	0.706
ΔIR does not Granger cause ΔGDP	0.942	0.624
ΔGDP does not Granger cause ΔEP	2.523	0.283
ΔOP does not Granger cause ΔEP	0.067	0.967
Model II		
ΔREG does not Granger cause ΔGDP	3.120	0.210
ΔOP does not Granger cause ΔGDP	1.304	0.521
ΔIR does not Granger cause ΔGDP	1.190	0.552
ΔGDP does not Granger cause ΔREG	3.291	0.193
ΔOP does not Granger cause ΔREG	3.216	0.200

4.3.2 The case of New Zealand

The respective sets of parameter estimates under Models I and II in the case of New Zealand are displayed in Tables 4.5 and 4.6. From the results using Model I, we find evidence of a unidirectional causality going from GDP to EP in regime 1. Nonetheless, the same causality does not hold in regime 2. In the high-economic-growth period, the GDP has a significant effect on the price of renewable energy. Furthermore, a unidirectional causality is discernible from IR to GDP in regime 2.

Based on Model II's results, no link is found between the GDP and REG. Accordingly, there is no prevailing relationship between the share of renewable energy and economic growth. In both regimes, the New Zealand data gives credence to the neutrality hypothesis. The results of the Granger's linear-causality test for the New Zealand data are shown in Table 4.7. A unidirectional causality running from OP to EP is unveiled in Model I while a unidirectional causality is seen in Model II running from REG to GDP. Finally, each of IR and OP individually causes GDP in the second regime under Model II.

Table 4.5: Parameter estimates based on the New Zealand data under Model I. Numbers in parentheses are p-values of the corresponding estimated parameters. Each parameter is indicated as significant at the 0% level (with the symbol ***), at the 1 % level (with the symbol **), at the 5% level (with the symbol *) and at the 10% level (with the symbol *).

	Regime 1		Regime 2	
	ΔGDP	ΔEP	ΔGDP	ΔEP
Intercept	0.866*** (0.000)	0.089 (0.467)	0.377** (0.008)	0.228* (0.018)
ΔGDP_{t-1}	-0.093 (0.542)	0.225* (0.030)	0.143 (0.259)	0.112 (0.193)
ΔGDP_{t-2}	-0.090 (0.547)	0.028 (0.785)	0.233* (0.072)	-0.064 (0.466)
ΔEP_{t-1}	0.011 (0.960)	-0.100 (0.503)	-0.063 (0.738)	-0.291* (0.023)
ΔEP_{t-2}	0.194 (0.38)	-0.127 (0.396)	-0.072 (0.703)	-0.283* (0.027)
ΔOP_{t}	1.070 (0.295)	0.747 (0.281)	0.623 (0.369)	0.532 (0.258)
ΔOP_{t-1}	-0.141 (0.892)	-0.176 (0.803)	-0.23 (0.742)	0.364 (0.442)
ΔIR_{t}	-0.722 (0.762)		-2.622* (0.060)	
ΔIR_{t-1}	-1.503 (0.544)		0.474 (0.725)	
p11	0.852			
p22	0.906			
σ_{GDP}	1.051			
σ_{EP}	0.713			

Table 4.6: Parameter estimates based on the New Zealand data under Model II. Numbers in parentheses are p-values of the corresponding estimated parameters. Each parameter is indicated as significant at the 0% level (with the symbol ***), at the 1 % level (with the symbol **), at the 5% level (with the symbol *) and at the 10% level (with the symbol *).

	Regime 1		Regime 2	
	ΔGDP	ΔREG	ΔGDP	ΔREG
Intercept	0.906*** (0.000)	0.045 (0.896)	0.435* (0.051)	0.980* (0.035)
ΔGDP_{t-1}	0.008 (0.955)	0.000 (0.999)	0.198 (0.290)	-0.475 (0.222)
ΔGDP_{t-2}	-0.137 (0.334)	0.051 (0.862)	0.400* (0.041)	-0.107 (0.793)
ΔREG_{t-1}	-0.103 (0.114)	0.237* (0.079)	-0.129 (0.120)	0.206 (0.233)
ΔREG_{t-2}	0.034 (0.606)	0.268* (0.053)	0.094 (0.202)	0.211 (0.166)
ΔOP_{t}	-0.065 (0.942)	-1.131 (0.542)	-2.255* (0.041)	-2.146 (0.351)
ΔOP_{t-1}	-0.784 (0.366)	-1.796 (0.320)	-1.799 (0.123)	2.241 (0.356)
ΔIR_{t}	-1.237 (0.513)		-2.871 (0.160)	
ΔIR_{t-1}	-0.506 (0.76)		6.912** (0.007)	
p11	0.888			
p22	0.782			
σ_{GDP}	1.229			
σ_{REG}	2.559			

Table 4.7: Results of the Granger's linear-causality test for the New Zealand data

Null hypothesis	F-statistic	p-value
Model I		
ΔEP does not Granger cause ΔGDP	0.590	0.744
ΔOP does not Granger cause ΔGDP	1.100	0.577
ΔIR does not Granger cause ΔGDP	0.193	0.908
ΔGDP does not Granger cause ΔEP	2.680	0.262
ΔOP does not Granger cause ΔEP	0.089	0.957
Model II		
ΔREG does not Granger cause ΔGDP	6.929	0.031
ΔOP does not Granger cause ΔGDP	1.966	0.374
ΔIR does not Granger cause ΔGDP	0.548	0.760
ΔGDP does not Granger cause ΔREG	3.389	0.184
ΔOP does not Granger cause ΔREG	0.802	0.670

4.3.3 The case of Alberta

For the Alberta data, the model-parameter estimates are recorded in Tables 4.8 and 4.9. There exist, under the framework of Model I but in regime 2 only, two unidirectional casual relationships running from GDP and OP to EP. Contrary to what was discovered in the New Zealand data, the effect of economic growth on the price of renewable energy is present on the low economic growth period. Additionally, the GDP is caused by two control variables, namely, the IR and OP, in the first regime.

In Model II, a unidirectional causality running from REG to GDP is substantiated under regime 1. This signifies that the share of renewable energy has a significant effect on the economic growth in the high-economic-growth period. Both the growth and neutrality hypotheses are sustained in regimes 1 and 2. Moreover, the same casualties in Model I are also observed. The two control variables, IR and OP, in the first regime cause the GDP. As per the results of the linear-Granger-causality test for the Alberta data, depicted in Table 4.10, there is no causality between the variables for either model.

Table 4.8: Parameter estimates based on the Alberta data under Model I. Numbers in parentheses are p-values of the corresponding estimated parameters. Each parameter is indicated as significant at the 0% level (with the symbol ***), at the 1 % level (with the symbol **), at the 5% level (with the symbol *) and at the 10% level (with the symbol *).

	Regime 1		Regime 2	
	ΔGDP	ΔEP	ΔGDP	ΔEP
Intercept	0.486* (0.023)	0.230* (0.040)	0.251 (0.295)	0.273* (0.030)
ΔGDP_{t-1}	0.674*** (0.000)	0.071 (0.406)	0.252 (0.168)	-0.262** (0.007)
ΔGDP_{t-2}	1.221*** (0.000)	0.048 (0.578)	0.374* (0.036)	0.358*** (0.000)
ΔEP_{t-1}	0.136 (0.628)	-0.562*** (0.000)	-0.209 (0.554)	-0.798*** (0.000)
ΔEP_{t-2}	0.057 (0.835)	-0.501*** (0.000)	-0.425 (0.237)	-0.759*** (0.000)
ΔOP_{t}	-1.391 (0.179)	0.362 (0.506)	1.229 (0.415)	1.479* (0.062)
ΔOP_{t-1}	-5.321*** (0.000)	0.591 (0.284)	0.378 (0.797)	0.561 (0.468)
ΔIR_{t}	-6.891*** (0.000)		-0.036 (0.978)	
ΔIR_{t-1}	1.265 (0.295)		0.397 (0.759)	
p11	0.928			
p22	0.917			
σ_{GDP}	1.352			
σ_{EP}	0.712			

Table 4.9: Parameter estimates based on the Alberta data under Model II. Numbers in parentheses are p-values of the corresponding estimated parameters. Each parameter is indicated as significant at the 0% level (with the symbol ***), at the 1 % level (with the symbol **), at the 5% level (with the symbol *) and at the 10% level (with the symbol *).

	Regime 1		Regime 2	
	ΔGDP	ΔREG	ΔGDP	ΔREG
Intercept	2.071* (0.079)	0.022 (0.977)	0.067 (0.927)	0.516 (0.272)
ΔGDP_{t-1}	-5.269*** (0.000)	0.652 (0.201)	-0.018 (0.971)	-0.030 (0.927)
ΔGDP_{t-2}	3.379*** (0.000)	-0.091 (0.856)	0.703 (0.159)	0.118 (0.713)
ΔREG_{t-1}	0.180 (0.515)	0.217 (0.224)	-0.071 (0.766)	0.458** (0.003)
ΔREG_{t-2}	-0.822** (0.002)	0.386* (0.027)	0.066 (0.797)	0.647*** (0.000)
ΔOP_{t}	8.542 (0.154)	3.142 (0.417)	4.067 (0.263)	1.784 (0.448)
ΔOP_{t-1}	23.409*** (0.000)	-3.390 (0.391)	1.966 (0.600)	-0.386 (0.873)
ΔIR_{t}	21.277** (0.001)		1.666 (0.645)	
ΔIR_{t-1}	-7.418 (0.213)		-1.415 (0.700)	
p11	0.827			
p22	0.946			
σ_{GDP}	4.105			
σ_{REG}	2.653			

Table 4.10: Results of the Granger's linear-causality test for the Alberta data

Null hypothesis	F-statistic	p-value
Model I		
ΔEP does not Granger cause ΔGDP	0.778	0.678
ΔOP does not Granger cause ΔGDP	1.205	0.547
ΔIR does not Granger cause ΔGDP	0.044	0.978
ΔGDP does not Granger cause ΔEP	3.364	0.186
ΔOP does not Granger cause ΔEP	3.077	0.215
Model II		
ΔREG does not Granger cause ΔGDP	0.030	0.985
ΔOP does not Granger cause ΔGDP	3.906	0.142
ΔIR does not Granger cause ΔGDP	0.217	0.897
ΔGDP does not Granger cause ΔREG	0.400	0.819
ΔOP does not Granger cause ΔREG	3.297	0.192

4.3.4 The case of Ontario

Tables 4.11 and 4.12 set out the estimates of model parameters for the Ontario data. In Model I, we found evidence of a unidirectional causality from GDP to EP in regime 1; but this causality does not, however, materialize in regime 2. Economic growth has a significant impact on the price of renewable energy in the high-economic-growth period. A unidirectional causality running from OP to GDP appears in regime 2.

Under Model II, the causality from GDP to REG in regime 1 is unidirectional. But, a bidirectional causality running from GDP to REG exists in regime 2. The above results affirm the conservation and feedback hypotheses in regimes 1 and 2, respectively. Added to that, the GDP is also affected by the IR in regime 2. The standard linear-Granger-causality test's results are tabulated in Table 4.13, indicating a bidirectional causality between GDP and REG.

Table 4.11: Parameter estimates based on the Ontario data under Model I. Numbers in parentheses are p-values of the corresponding estimated parameters. Each parameter is indicated as significant at the 0% level (with the symbol ***), at the 1 % level (with the symbol **), at the 5% level (with the symbol *) and at the 10% level (with the symbol *).

	Regime 1		Regime 2	
	ΔGDP	ΔEP	ΔGDP	ΔEP
Intercept	$0.428^{\star}$ (0.056)	0.624^{**} (0.008)	0.322^{**} (0.010)	0.146 (0.268)
ΔGDP_{t-1}	0.118 (0.551)	-0.461^{*} (0.027)	0.316^{**} (0.003)	0.048 (0.668)
ΔGDP_{t-2}	0.314 (0.118)	0.448^{*} (0.034)	0.380^{***} (0.000)	-0.039 (0.729)
ΔEP_{t-1}	0.182 (0.361)	-0.611^{**} (0.004)	0.105 (0.598)	-0.522^{*} (0.013)
ΔEP_{t-2}	0.297 (0.134)	-0.721^{**} (0.001)	0.069 (0.732)	-0.531^{*} (0.013)
ΔOP_{t}	0.038 (0.977)	-0.710 (0.618)	$-1.038^{\star}$ (0.096)	0.802 (0.221)
ΔOP_{t-1}	-0.334 (0.808)	-1.764 (0.221)	-0.478 (0.455)	0.342 (0.611)
ΔIR_{t}	0.126 (0.945)		-0.774 (0.231)	
ΔIR_{t-1}	-0.112 (0.949)		0.317 (0.625)	
p11	0.889			
p22	0.965			
σ_{GDP}	0.890			
σ_{EP}	0.936			

Table 4.12: Parameter estimates based on the Ontario data under Model II. Numbers in parentheses are p-values of the corresponding estimated parameters. Each parameter is indicated as significant at the 0% level (with the symbol ***), at the 1 % level (with the symbol **), at the 5% level (with the symbol *) and at the 10% level (with the symbol *).

	Regime 1		Regime 2	
	ΔGDP	ΔREG	ΔGDP	ΔREG
Intercept	0.860^{***} (0.000)	-0.001 (0.999)	-1.199^{***} (0.000)	-0.176 (0.715)
ΔGDP_{t-1}	-0.027 (0.890)	-0.848^{*} (0.020)	0.075 (0.734)	$0.697^{\star}$ (0.094)
ΔGDP_{t-2}	0.222 (0.260)	1.136^{**} (0.002)	1.654^{***} (0.000)	-0.244 (0.566)
ΔREG_{t-1}	-0.085 (0.224)	0.268^{*} (0.042)	0.012 (0.924)	$0.398^{\star}$ (0.098)
ΔREG_{t-2}	-0.057 (0.433)	0.478^{**} (0.001)	$0.177^{\star}$ (0.096)	0.147 (0.466)
ΔOP_{t}	1.725 (0.101)	-2.777 (0.162)	0.891 (0.567)	1.218 (0.678)
ΔOP_{t-1}	-0.643 (0.532)	-1.488 (0.444)	0.244 (0.881)	-0.924 (0.765)
ΔIR_{t}	-0.291 (0.787)		$-3.337^{\star}$ (0.078)	
ΔIR_{t-1}	-0.533 (0.628)		$3.044^{\star}$ (0.089)	
p11	0.906			
p22	0.858			
σ_{GDP}	1.369			
σ_{REG}	2.583			

Table 4.13: Results of the Granger's linear-causality test for the Ontario data

Null hypothesis	F-statistic	p-value
Model I		
ΔEP does not Granger cause ΔGDP	2.010	0.366
ΔOP does not Granger cause ΔGDP	0.595	0.743
ΔIR does not Granger cause ΔGDP	0.156	0.925
ΔGDP does not Granger cause ΔEP	0.545	0.761
ΔOP does not Granger cause ΔEP	0.606	0.739
Model II		
ΔREG does not Granger cause ΔGDP	5.456	0.065
ΔOP does not Granger cause ΔGDP	0.918	0.632
ΔIR does not Granger cause ΔGDP	1.810	0.405
ΔGDP does not Granger cause ΔREG	5.436	0.066
ΔOP does not Granger cause ΔREG	2.468	0.291

With the various aspects of our empirical investigation taken into consideration, common features emerge from the four data sets. It is important to note that economic growth possesses more impact on the energy variables, whereas the energy variables have less impact on economic growth. This is the causality that came out for New Zealand, Alberta, and Ontario, that is, a unidirectional causal relationship running from the economic growth to the price of renewable energy albeit in one of the regimes only. For New Zealand and Ontario, the causality arises in the high-economic-growth period. For Alberta, the causality surfaces in the low-economic-growth period. This upshot bespeaks the economic growth's impact on the price of renewable energy, in consonance with economic performance. The causality encountered in the Norway and Ontario data is unidirectional from economic growth to the share of renewable energy; this causality occurs in different regimes for Norway and Ontario.

On the energy side, there is no causal relation running from the price of renewable energy to economic growth. The effect of the share of renewable energy on economic growth is evident only in Alberta and Ontario. The two control variates, interest rate and oil price, have their importance in certain circumstances. In all four regions studied, there is a presence of causality from the interest rate to economic growth, confirming the critical role of interest rate on the economic growth. This reconciles with the monetary policy that economic managers and regulators typically would adjust short-term interest rate in response to varying economic developments and conditions. It has to be noted that oil price, for the most parts, also affects economic growth; but its effect on the renewable-energy variables is only seen in Norway and Alberta. For the four regions that we investigated, there are only two Granger-based causalities: a unidirectional causal relationship running from the share of renewable energy to economic growth and reverse of this causal relation in the case of Ontario.

Remark 6. *In some regimes, the MS-VAR model's result of no causal relation jibes with those of the linear VAR model. However, the MS-VAR model captures causality in some regimes that a linear VAR model simply cannot.*

An increase in economic growth rate could imply a rise in renewable-energy infrastructure investments. New technology development is then strengthened and this could lower down the cost of adopting renewable energy. Consequently, there may be a reduction in the price of renewable energy and a continuing enlargement in the share of renewable energy for power generation. On the other side, the transition from non-renewable energy to renewable energy is typically a slow progress. Therefore, the impact of the change in the share of renewable energy on economic growth is not observable in the short-run (e.g., 1 or 2 years). This explains why the renewable-energy price may not have much impact on economic growth.

Chapter 5

Policy implications

The results of the unit-root test signified that all the variables are integrated of order zero, $I(0)$. Considering the small sample size of the data set, the lag length for all the variables and the number of regimes are set to 2. The results from the MS-VAR models were fairly mixed. There is no causality relationship between the price of renewable energy and the economic growth in Norway in any economic regime. There are unidirectional causalities running from economic growth to the price of renewable energy in the high-economic-growth period in New Zealand and Ontario. The same causality appears in Alberta, but in the low-economic-growth period. The results on the connection between the share of renewable energy and economic growth are also mixed. There exists a unidirectional causality going from economic growth to the share of renewable energy in Norway and Ontario in the low- and high-economic-growth periods, respectively. A bi-directional causality is found in the low regime in Ontario. The effect of the share of renewable energy on economic growth only manifested in the high-economic growth in Alberta. In many cases, there exists causalities running from the short-term interest rate and oil price to economic growth. However, oil price has causal effect only on the price of renewable energy and the share of renewable energy in Alberta and Norway, respectively. The empirical findings of this study have several far-reaching implications.

First, the price of renewable energy and the share of renewable energy play little role on enhancing economic growth. Despite the share of renewable energy having impact on the economic growth in the two Canadian provinces, it is impossible to increase further the share of renewable energy in the short run. To stimulate economic growth, the short-term interest rate is an efficient and practical tool for the policy makers to use. The GDP is determined by consumption, investment, government outlays, and net exports. From these determiners, investment is the most sensitive to changes in interest rate as most investment purchases are based on bank borrowing. If interest rates fall, the cost of borrowing decreases and more economic activity will ensue. As a result, more goods are being created. Also, lower interest rates lead to lower returns from savings. That is, this situation encourages people to spend more on goods and services, thereby increasing consumption and GDP. The combined effects of investment and consumption show that lower interest rates can stimulate economic growth efficiently.

Second, the fossil-fuel price has a significant impact on economy. The potential episodes of instability in the conventional energy market make an economy vulnerable. It is, therefore, prudent for the government to keep in mind the importance of reducing dependency on conventional energy and working on energy-conservation policy.

Third, the share of renewable-energy sources in electricity generation improves economic growth in different regimes for both provinces of Canada. This could be explained by the fact that their individual share is still relatively small compared to those of New Zealand and Norway; and the change in share has a bigger impact on economic growth.

A firm recommendation to the Canadian government is to encourage the society to look for more renewable energy sources. It only not eases the devastating consequences of climate change, but also fortifies economic stability. Policy makers could design systems, guidance, and mechanisms to achieve such a goal with the aid of certain financial tools (e.g., increasing direct investment to renewable-energy sources, providing loans to the private sector, and feed-in tax incentives).

Chapter 6

Conclusion

Using quarterly data, this research empirically examined the price of renewable energy and its linkage to the economic growth of the top three countries in terms of their share of renewable energy in electricity generation. Given the high value of the share of renewable energy in electricity generation in these three countries, the wholesale electricity price is used as a proxy for the price of renewable energy. In addition, the causal relation between the share of renewable energy and economic growth is investigated in these countries. To explore the causality, an MS-VAR model is employed with two control variables, namely, the short-term interest rate and Brent oil price. The percentage change of the variables are used in the MS-VAR model. In contrast to the usual estimation procedure in the EM algorithm proposed by Hamilton, we developed the HMM algorithm tailored to MS-VAR model in estimating all the parameters. Using the technique of measure change along with the EM algorithm, the HMM algorithm spawned a self-calibrating model. The optimal estimates of the model parameters - established through the EM algorithm - were presented in terms of the various filters involving the Markov chain.

The causal relation from renewable energy's consumption and production to economic growth was examined in various studies. The results of these prior studies vary across different countries. Our findings support the heterogeneity in the causal linkage between renewable energy and economic growth across the four regions. In addition, the causal relation within a region is not constant, and it changes with different characterisations of economic status.

We recognize the limitation of this study. When the lag length of MS-VAR model increases, the model could be very unstable to estimate. Also, the parameters of all variables are governed by one Markov chain only. An immediate enrichment of our model setting could be considered with the two endogenous variables being governed by two different Markov chains as put forward in [33]. Although the estimation is expected to be involved, it is envisaged that this doable with the current computing technologies. In general, this could reduce the number of lags and permit the scenario of two variables being in different regimes for a given time period. Such a new setting will give more insights on how the regimes for each variable are also related.

By applying a modified HMM algorithm, the parameters and filtered probabilities could be estimated. To this end, the regimes of the two endogenous variables are determined based on their filtered probabilities. The resulting filtered regimes of the endogenous variables may be used as dummy variables in a linear model, fortifying a new estimation procedure to be attainably implementable.

Finally, this study concentrated on renewable-energy development within the industrial sector only. However, renewable-energy use in the industrial sector for countries like Norway and New Zealand is already approaching their maximum limits. There is almost no room for more industrial capacity in these countries in catering for additional renewable-energy initiatives. With developments in technology, the use of renewable energy in the household sectors starts increasing; for example, trend in installing rooftop solar panels, procurement of small wind turbines, and exploring the option for electric cars. As in the industrial segment of the economy, there is a large potential for renewable-energy sources that could be developed and primarily tailored to the household sector. Future research should also consider the advancements of renewable energy, purpose-built for the household sector taking into account sustainable economy and environment.

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Appendices

These Appendices provide supplementary materials such as proofs of the main results, values for the initialisation of the parameter estimation, and additional Figures and Tables to support empirically the findings of this research. Appendices 1-2 present the proofs of the optimal HMM-based parameters. The derivation of the pertinent Fisher information is contained in Appendices 3-4 whilst the dynamic behaviour of filtered probabilities are depicted in Appendix 5. Lastly, we report in Appendix 6 the initial values for the estimation of the two MS-VAR models.

Appendix 1. Optimal estimate for ψ in Subsection 2.4

In this Appendix, we detail the estimation procedure for the optimal estimate of $\psi_{11,h,j}$. The estimation steps for the other ψ's and ϕ's are similar. Consider $\boldsymbol{\psi}_{11,h} := (\psi_{11,h,1}, \psi_{11,h,2}, \ldots, \psi_{11,h,n})^\top \in \mathbb{R}^n$. To solve for the estimate $\widehat{\boldsymbol{\psi}}_{11,h} := (\widehat{\psi}_{11,h,1}, \widehat{\psi}_{11,h,2}, \ldots, \widehat{\psi}_{11,h,n})^\top \in \mathbb{R}^n$, define a new measure $P^{\widehat{\psi}}$ through

$$\left.\frac{P^{\widehat{\psi}}}{P^{\psi}}\right|_{\mathscr{F}_t} = \Lambda_t^{\psi} = \prod_{l=1}^{t} \lambda_l^{\psi},$$

where

$$\begin{aligned}
\lambda^{\psi_l} = \exp\Big\{\frac{1}{2}\sigma_1{}^{-2}(\mathbf{z}_{l-1})\Big[\Big(&EG_l - \mu_1(\mathbf{z}_{l-1}) - \sum_{k=1}^{p} \psi_{11,k}(\mathbf{z}_{l-1})EG_{l-k} \\
&- \sum_{k=1}^{p} \psi_{12,k}(\mathbf{z}_{l-1})EP_{l-k} - \sum_{k=0}^{q} \phi_{11,k}(\mathbf{z}_{l-1})OP_{l-k} - \sum_{k=0}^{q} \phi_{12,k}(\mathbf{z}_{l-1})IR_{l-k}\Big)^2 \\
&- \Big(EG_l - \widehat{\mu}_1(\mathbf{z}_{l-1}) - \sum_{k\neq h}^{p} \psi_{11,k}(\mathbf{z}_{l-1})EG_{l-k} - \sum_{k=1}^{p} \psi_{12,k}(\mathbf{z}_{l-1})EP_{l-k} \\
&- \sum_{k=0}^{q} \phi_{11,k}(\mathbf{z}_{l-1})OP_{l-k} - \sum_{k=0}^{q} \phi_{12,k}(\mathbf{z}_{l-1})IR_{l-k} - \widehat{\psi}_{11,h}(\mathbf{z}_{l-1})EG_{l-h}\Big)^2\Big]\Big\}.
\end{aligned}$$

This means that

$$
\begin{aligned}
L(\widehat{\psi}_{11,h}) &= E\Big[\log\frac{P^{\widehat{\theta}}}{P^{\theta}}\Big|\mathscr{F}_t\Big]\\
&= E\Big[\sum_{l=1}^{t}\frac{1}{2}\sigma_1^{-2}(\mathbf{z}_{l-1})\Big[\psi_{11,h}^2(\mathbf{z}_{l-1})EG_{l-h}^2-\widehat{\psi}_{11,h}^2(\mathbf{z}_{l-1})EG_{l-h}^2+2\big(\psi_{11,h}(\mathbf{z}_{l-1})EG_{l-h}\\
&\quad-\widehat{\psi}_{11,h}(\mathbf{z}_{l-1})EG_{l-h}\big)\Big(EG_l-\mu_1(\mathbf{z}_{l-1})-\sum_{k\neq h}^{p}\psi_{11,k}(\mathbf{z}_{l-1})EG_{l-k}-\sum_{k=1}^{p}\psi_{12,k}(\mathbf{z}_{l-1})EP_{l-k}\\
&\quad-\sum_{k=0}^{q}\phi_{11,k}(\mathbf{z}_{l-1})OP_{l-k}-\sum_{k=0}^{q}\phi_{12,k}(\mathbf{z}_{l-1})IR_{l-k}\Big)\Big]\Big|\mathscr{F}_t\Big]\\
&= E\Big[\sum_{l=1}^{t}\sum_{i=1}^{n}\langle\mathbf{z}_{l-1},\mathbf{e}_i\rangle\frac{1}{2}\sigma_{1,i}{}^{-2}\Big(-\widehat{\psi}_{11,h,i}^2EG_{l-h}^2-2\widehat{\psi}_{11,h,i}EG_{l-h}EG_l+2\hat{\psi}_{11,h,i}\mu_{1,i}EG_{l-h}\\
&\quad+2\sum_{k\neq h}^{p}\hat{\psi}_{11,h,i}\psi_{11,k,i}EG_{l-h}EG_{l-k}+2\sum_{k=1}^{p}\hat{\psi}_{11,h,i}\psi_{12,k,i}EG_{l-h}EP_{l-k}\\
&\quad+2\sum_{k=0}^{q}\hat{\psi}_{11,h,i}\phi_{11,k,i}EG_{l-h}OP_{l-k}+2\sum_{k=0}^{q}\hat{\psi}_{11,h,i}\phi_{12,k,i}EG_{l-h}IR_{l-k}+\mathrm{R}\Big)\Big|\mathscr{F}_t\Big]\\
&= \sum_{i=1}^{n}\frac{1}{2}\sigma_{1,i}{}^{-2}\Big(-\widehat{\psi}_{11,h,i}^2\widehat{\Gamma}_{t,i}(EG_{t-h}^2)-2\widehat{\psi}_{11,h,i}\Big(\widehat{\Gamma}_{t,i}(EG_{t-h}EG_t)-\mu_{1,i}\widehat{\Gamma}_{t,i}(EG_{t-h})\\
&\quad-\sum_{k\neq h}^{p}\psi_{11,k,i}\widehat{\Gamma}_{t,i}(EG_{t-h}EG_{t-k})-\sum_{k=1}^{p}\psi_{12,k,i}\widehat{\Gamma}_{t,i}(EG_{t-h}EP_{t-k})\\
&\quad-\sum_{k=0}^{q}\phi_{11,k,i}\widehat{\Gamma}_{t,i}(EG_{t-h}OP_{t-k})-\sum_{k=0}^{q}\phi_{12,k,i}\widehat{\Gamma}_{t,i}(EG_{t-h}IR_{t-k})+\mathrm{R}\Big)\Big),
\end{aligned}
$$

where R is a reminder that does not contain $\widehat{\psi}_{11,h,i}$. Then, we differentiate $L(\widehat{\psi}_{11,h})$ with respect to $\widehat{\psi}_{11,h,j}$ and set the result equal to zero. Thus,

$$
\widehat{\psi}_{11,h,j}=\frac{\begin{bmatrix}\widehat{\Gamma}_{t,j}(EG_{t-h}EG_t)-\sum_{k\neq h}^{p}\psi_{11,h,j}\widehat{\Gamma}_{t,j}(EG_{t-h}EG_{t-k})-\sum_{k=1}^{p}\psi_{12,k,j}\widehat{\Gamma}_{t,j}(EG_{t-h}EP_{t-k})\\-\sum_{k=0}^{q}\phi_{11,k,j}\widehat{\Gamma}_{t,j}(EG_{t-h}OP_{t-k})-\sum_{k=0}^{q}\phi_{12,k,j}\widehat{\Gamma}_{t,j}(EG_{t-h}IR_{t-k})-\mu_{1,j}\widehat{\Gamma}_{t,j}(EG_{t-h})\end{bmatrix}}{\widehat{\Gamma}_{t,j}(EG_{t-h}^2)}.
$$

Appendix 2. Optimal estimate for μ in Subsection 2.4

The derivation of the optimal estimate for $\mu_{1,j}$ is provided below. Similar steps apply for the derivation of the optimal estimate for $\mu_{2,j}$. Consider $\boldsymbol{\mu}_1=(\mu_{1,1},\mu_{1,2},\ldots,\mu_{1,n})^\top\in\mathbb{R}^n$. Solving for the estimate $\widehat{\boldsymbol{\mu}}_1 = (\widehat{\mu}_{1,1}, \widehat{\mu}_{1,2}, \ldots, \widehat{\mu}_{1,n})^\top \in \mathbb{R}^n$, we define a new measure $P^{\widehat{\mu}}$ via

$$\frac{P^{\widehat{\mu}}}{P^{\boldsymbol{\mu}}}\bigg|_{\mathscr{F}_t} = \Lambda_t^{\boldsymbol{\mu}} = \prod_{l=1}^{t} \lambda_l^{\boldsymbol{\mu}},$$

where

$$\begin{aligned}\lambda^{\boldsymbol{\mu}_l} = \exp\Big\{\frac{1}{2}{\sigma_1}^{-2}(\mathbf{z}_{l-1})\Big[\Big(EG_l - \mu_1(\mathbf{z}_{l-1}) - \sum_{k=1}^{p}\psi_{11,k}(\mathbf{z}_{l-1})EG_{l-k} - \sum_{k=1}^{p}\psi_{12,k}(\mathbf{z}_{l-1})EP_{l-k}\\ - \sum_{k=0}^{q}\phi_{11,k}(\mathbf{z}_{l-1})OP_{l-k} - \sum_{k=0}^{q}\phi_{12,k}(\mathbf{z}_{l-1})IR_{l-k}\Big)^2 - \Big(EG_l - \widehat{\mu}_1(\mathbf{z}_{l-1}) - \sum_{k=1}^{p}\psi_{11,k}(\mathbf{z}_{l-1})EG_{l-k}\\ - \sum_{k=1}^{p}\psi_{12,k}(\mathbf{z}_{l-1})EP_{l-k} - \sum_{k=0}^{q}\phi_{11,k}(\mathbf{z}_{l-1})OP_{l-k} - \sum_{k=0}^{q}\phi_{12,k}(\mathbf{z}_{l-1})IR_{l-k}\Big)^2\Big]\Big\}.\end{aligned}$$

This tells us that

$$\begin{aligned}L(\widehat{\mu}_1) &= E\Big[\log\frac{P^{\widehat{\theta}}}{P^{\theta}}\Big|\mathscr{F}_t\Big]\\ &= E\Big[\sum_{l=1}^{t}\frac{1}{2}\sigma_1^{-2}(\mathbf{z}_{l-1})\Big[\mu_1^2(\mathbf{z}_{l-1}) - \widehat{\mu_1}^2(\mathbf{z}_{l-1}) + 2(\mu_1(\mathbf{z}_{l-1}) - \widehat{\mu_1}(\mathbf{z}_{l-1}))\Big(EG_l - \sum_{k=1}^{p}\psi_{11,k}(\mathbf{z}_{l-1})EG_{l-k}\\ &\quad - \sum_{k=1}^{p}\psi_{12,k}(\mathbf{z}_{l-1})EP_{l-k} - \sum_{k=0}^{q}\phi_{11,k}(\mathbf{z}_{l-1})OP_{l-k} - \sum_{k=0}^{q}\phi_{12,k}(\mathbf{z}_{l-1})IR_{l-k}\Big)\Big]\Big|\mathscr{F}_t\Big]\\ &= E\Big[\sum_{l=1}^{t}\sum_{i=1}^{n}\langle\mathbf{z}_{l-1}, \mathbf{e}_i\rangle\frac{1}{2}{\sigma_{1,i}}^{-2}\Big(-\widehat{\mu}_{1,i}^2 - 2\widehat{\mu}_{1,i}EG_l + 2\sum_{k=1}^{p}\hat{\mu}_{1,i}\psi_{11,k,i}EG_{l-k}\\ &\quad + 2\sum_{k=1}^{p}\hat{\mu}_{1,i}\psi_{12,k,i}EP_{l-k} + 2\sum_{k=0}^{q}\hat{\mu}_{1,i}\phi_{11,k,i}OP_{l-k} + 2\sum_{k=0}^{q}\hat{\mu}_{1,i}\phi_{12,k,i}IR_{l-k} + \mathrm{R}\Big)\Big|\mathscr{F}_t\Big]\\ &= \sum_{i=1}^{n}\frac{1}{2}{\sigma_{1,i}}^{-2}\Big(-\widehat{\mu}_{1,i}^2\widehat{O}_{t,i} - 2\widehat{\mu}_{1,i}\Big(\widehat{\Gamma}_{t,i}(EG_t) - \sum_{k=1}^{p}\psi_{11,k,i}\widehat{\Gamma}_{t,i}(EG_{t-k})\\ &\quad - \sum_{k=1}^{p}\psi_{12,k,i}\widehat{\Gamma}_{t,i}(EP_{t-k}) - \sum_{k=0}^{q}\phi_{11,k,i}\widehat{\Gamma}_{t,i}(OP_{t-k}) - \sum_{k=0}^{q}\phi_{12,k,i}\widehat{\Gamma}_{t,i}(IR_{t-k}) + \mathrm{R}\Big)\Big),\end{aligned}$$

where the remainder R is free of $\widehat{\mu}_{1,i}$. Differentiating $L(\widehat{\mu}_1)$ with respect to $\widehat{\mu}_{1,j}$ and setting it the resulting derivative to 0, we get

$$\mu_{1,j} = \frac{\widehat{\Gamma}_{t,j}(EG_t) - \sum_{k=1}^{p}\psi_{11,k,j}\widehat{\Gamma}_{t,j}(EG_{t-k}) - \sum_{k=1}^{p}\psi_{12,k,j}\widehat{\Gamma}_{t,j}(EP_{t-k}) - \sum_{k=0}^{q}\phi_{11,k,j}\widehat{\Gamma}_{t,j}(OP_{t-k}) - \sum_{k=0}^{q}\phi_{12,k,j}\widehat{\Gamma}_{t,j}(IR_{t-k})}{\widehat{O}_{t,j}}.$$

Appendix 3. Fisher information for ψ in Subsection 2.4

We show the derivation of the Fisher information of $\psi_{11,h,j}$. The Fisher information of other ψ's and ϕ's follow similar derivation steps. The log-likelihood of $\psi_{11,h,j}$ is written as

$$l(\psi_{11,h,j}) = \sum_{t=1}^{T}\Big[\langle \mathbf{z}_{t-1}, \mathbf{e}_j\rangle\Big(-\frac{1}{2}\log(2\pi) - \log(\sigma_{1,j}) - \lambda^{\psi_l}\Big)\Big].$$

Consequently, the Fisher information of $\psi_{11,h,j}$ is

$$\mathcal{I}(\psi_{11,h,j}) = -E\Big[\frac{d^2 l}{d\psi_{11,h,j}{}^2}\Big|\psi_{11,h,j}\Big] = E\Big[\sum_{t=1}^{T}\langle \mathbf{z}_{t-1}, \mathbf{e}_j\rangle\Big(\frac{EG_{t-h}^2}{(\sigma_{1,j})^2}\Big)\Big|\psi_j^{(1k)}\Big] = \frac{\hat{\Gamma}_{t,j}(EG_{t-h}^2)}{(\sigma_{1,j})^2}.$$

Appendix 4. Fisher information for μ in Subsection 2.4

We provide the Fisher information calculation of $\mu_{1,j}$ below. The same procedure applies in the corresponding computation involving $\mu_{2,j}$. The log-likelihood of $\mu_{1,j}$ can be written as

$$l(\mu_{1,j}) = \sum_{t=1}^{T}\Big[\langle \mathbf{z}_{t-1}, \mathbf{e}_j\rangle\Big(-\frac{1}{2}\log(2\pi) - \log(\sigma_j^{(1)}) - \lambda^{\mu_l}\Big)\Big].$$

Consequently, the Fisher information of $\mu_j^{(1)}$ is

$$\mathcal{I}(\mu_{1,j}) = -E\Big[\frac{d^2 l}{d\mu_j^{(1)2}}\Big|\mu_j^{(1)}\Big] = E\Big[\sum_{t=1}^{T}\frac{\langle \mathbf{z}_{t-1}, \mathbf{e}_j\rangle}{(\sigma_{1,j})^2}\Big|\mu_j^{(1)}\Big] = \frac{\hat{O}_{t,j}}{(\sigma_{1,j})^2}.$$

Appendix 5. Filtered probabilities for Models I and II in Subsection 4.3

This appendix presents the plots of the filtered probabilities implied by each region's data set. The graph in red indicates the filtered probabilities of being in regime 1, and the graph in green traces the filtered probabilities of being regime 2.

Figure 6.1: Evolution of the filtered probabilities under Model I: Norway data

Figure 6.2: Evolution of the filtered probabilities under Model I: New Zealand data

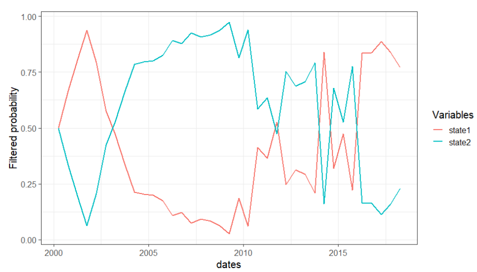

Figure 6.3: Evolution of the filtered probabilities under model I: Alberta data

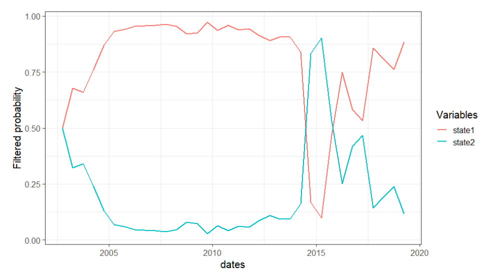

Figure 6.4: Evolution of the filtered probabilities under Model I: Ontario data

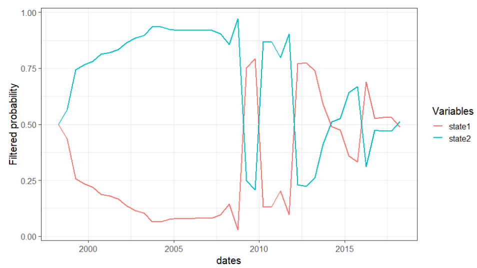

Figure 6.5: Evolution of the filtered probabilities under Model II: Norway data

Figure 6.6: Evolution of the filtered probabilities under Model II: New Zealand data

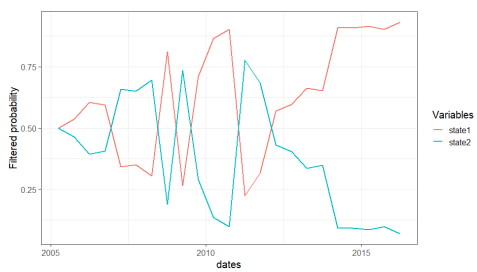

Figure 6.7: Evolution of the filtered probabilities under Model II: Alberta data

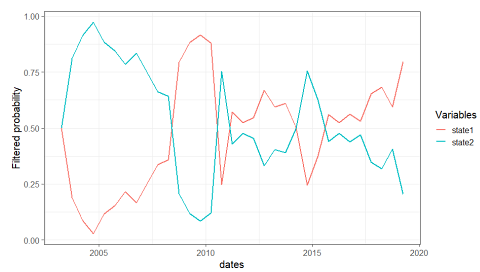

Figure 6.8: Evolution of the filtered probabilities under Model II: Ontario data

Appendix 6. Initial values in the model's implementation described in Subsection 4.3

The Tables in this Appendix display the initial values of the parameter estimation procedure for Models I and II.

Table 6.1: Initial values for the Norway data

Model I	Regime 1		Regime 2	
	ΔGDP	ΔEP	ΔGDP	ΔEP
Intercept	0.80	-0.01	0.50	1.52
ΔGDP_{t-1}	-1.00	-4.50	-0.28	-0.51
ΔGDP_{t-2}	-0.43	-2.50	0.27	0.21
ΔEP_{t-1}	-0.42	0.59	0.01	0.59
ΔEP_{t-2}	-0.19	0.00	-2.00	0.00
ΔOP_t	-6.00	-0.01	5.00	0.15
ΔOP_{t-1}	-2.00	-0.04	3.80	-0.04
ΔIR_t	0.03		-6.61	
ΔIR_{t-1}	3.09		-1.13	

Model II	Regime 1		Regime 2	
	ΔGDP	ΔREG	ΔGDP	ΔREG
Intercept	0.54	-0.03	0.64	-0.26
ΔGDP_{t-1}	-0.34	0.30	-10.00	0.80
ΔGDP_{t-2}	-0.20	0.06	-3.00	0.04
ΔREG_{t-1}	3.50	0.63	0.42	0.63
ΔREG_{t-2}	-3.50	0.00	-0.51	0.00
ΔOP_t	-2.60	0.02	-0.73	0.04
ΔOP_{t-1}	5.40	0.04	-0.53	0.04
ΔIR_t	-10.78		0.94	
ΔIR_{t-1}	-4.30		1.12	

Table 6.2: Initial values for the New Zealand data

Model I	Regime 1		Regime 2	
	ΔGDP	ΔEP	ΔGDP	ΔEP
Intercept	-0.08	-0.01	1.07	0.03
ΔGDP_{t-1}	0.01	20.00	-0.29	6.68
ΔGDP_{t-2}	0.14	-0.01	0.03	-1.00
ΔEP_{t-1}	-0.01	0.40	0.00	0.40
ΔEP_{t-2}	-0.01	0.00	0.00	0.00
ΔOP_t	-0.02	-1.00	-0.01	0.06
ΔOP_{t-1}	0.01	0.07	0.01	0.07
ΔIR_t	-0.16		0.29	
ΔIR_{t-1}	-0.39		0.02	

Model II	Regime 1		Regime 2	
	ΔGDP	ΔREG	ΔGDP	ΔREG
Intercept	0.59	-0.01	0.42	0.03
ΔGDP_{t-1}	0.49	5.00	0.33	8.00
ΔGDP_{t-2}	0.14	-1.00	-0.31	-1.00
ΔREG_{t-1}	-0.75	0.41	-0.50	0.41
ΔREG_{t-2}	-0.20	0.00	0.29	0.00
ΔOP_t	-0.20	-1.20	-0.01	0.06
ΔOP_{t-1}	0.00	0.07	0.01	0.07
ΔIR_t	-0.12		0.40	
ΔIR_{t-1}	0.34		-0.57	

Table 6.3: Initial values for the Alberta data

Model I	Regime 1		Regime 2	
	ΔGDP	ΔEP	ΔGDP	ΔEP
Intercept	0.12	42.69	0.33	-2.89
ΔGDP_{t-1}	0.65	-12.19	4.20	6.05
ΔGDP_{t-2}	0.02	4.69	0.06	-0.87
ΔEP_{t-1}	0.01	0.58	0.00	-0.92
ΔEP_{t-2}	-0.01	0.72	0.03	-0.79
ΔOP_{t}	-0.02	-3.80	0.02	-0.09
ΔOP_{t-1}	0.00	0.00	0.00	0.00
ΔIR_{t}	2.16		-0.10	
ΔIR_{t-1}	0.00		0.00	

Model II	Regime 1		Regime 2	
	ΔGDP	ΔREG	ΔGDP	ΔREG
Intercept	-0.70	0.02	0.23	0.06
ΔGDP_{t-1}	2.82	-0.01	2.14	0.04
ΔGDP_{t-2}	-0.75	0.00	0.34	0.01
ΔREG_{t-1}	1.69	0.35	0.14	1.07
ΔREG_{t-2}	-0.93	0.16	0.09	0.13
ΔOP_{t}	-0.04	0.00	0.02	0.00
ΔOP_{t-1}	0.00	0.00	0.00	0.00
ΔIR_{t}	0.64		-0.35	
ΔIR_{t-1}	0.00		0.00	

Table 6.4: Initial values for the Ontario data

Model I	Regime 1		Regime 2	
	ΔGDP	ΔEP	ΔGDP	ΔEP
Intercept	0.65	0.99	0.54	0.50
ΔGDP_{t-1}	1.21	-3.42	5.00	-0.15
ΔGDP_{t-2}	-2.00	-0.09	5.00	0.09
ΔEP_{t-1}	-0.11	0.39	0.10	0.39
ΔEP_{t-2}	0.28	0.00	0.23	0.00
ΔOP_{t}	2.94	0.75	-0.15	-0.01
ΔOP_{t-1}	2.59	0.14	0.15	0.14
ΔIR_{t}	-0.04		-0.18	
ΔIR_{t-1}	-0.09		0.43	

Model II	Regime 1		Regime 2	
	ΔGDP	ΔREG	ΔGDP	ΔREG
Intercept	0.79	-2.12	0.95	22.27
ΔGDP_{t-1}	-20.00	0.33	1.70	4.41
ΔGDP_{t-2}	-0.30	0.01	0.01	-0.97
ΔREG_{t-1}	-0.24	0.35	-0.05	0.35
ΔREG_{t-2}	0.22	0.00	-0.02	0.00
ΔOP_{t}	4.00	-0.04	0.55	-0.15
ΔOP_{t-1}	0.27	-0.03	0.94	-0.03
ΔIR_{t}	4.91		-1.90	
ΔIR_{t-1}	-0.31		-2.25	

YIYANG CHEN

PROFILE

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- *Gained familiarity with R language and MATLAB software*
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- ***Dean's Honor List (2018)***
- ***Dean's Honor List (2017)***
- ***Dean's Honor List (2016)***
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May. 2019 To Present **University of Western Ontario** **London, ON**
RESEARCH ASSISANT

- Working with Prof R Mamon and N Spagnolo on causality between economic growth and renewable energy prices by employing hidden Markov models (HMMs); Implemented certain HMM algorithms in the R software; Learned statistics and econometrics models for time series data.

Sept. 2018 To Present **University of Western Ontario** **London, ON**
TEACHING ASSISTANT, DEPARTMENT OF APPLIED MATHEMATICS AND STATISTICS

- Performs several duties supporting a positive, inclusive classroom environment assisting with the preparation of tests; holding an "office hour" to answer any student questions and provide greater understanding of the course material; and marking tests and quizzes

May 2017 To Sept. 2017 **University of Western Ontario** **London, ON**
RESEARCH ASSISTANT

- Throughout the summer of 2017, worked with the professor every two weeks – completing research on compound probabilities; researched relevant articles with regards to what others have done; and completed further academic research according to the needs and instruction of the professor