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Characterizing the Value and Effect of Perceptiveness in Various Game-Theoretic Settings

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Abstract

This thesis investigates the value and effect that perceptiveness has in three game-theoretic settings. I consider a player to be *expert* if they know the value of a particular payoff-relevant parameter in the models I study. If the player does not know such value, I consider the player to be *inexpert*. A player is *perceptive* if they know with certainty whether their opponent is expert. Otherwise, the player is *imperceptive*. The goal of this thesis is to provide insight regarding the potential value and effect that perceptiveness has in the game-theoretic settings I study.

The first model I consider emulates a two-player, one-round game of poker. The second model I investigate is a two-player market-entry game. The third model I study depicts a two-player market-entry game that is influenced by an information designer who aims to maximize producer surplus. In each model, I consider distinct information structures, which vary in terms of the players' levels of expertise and perceptiveness. In the first two models, I solve for the Bayesian Nash equilibria of each game and compute each agent's expected payoff. Then, by comparing the equilibrium action and expected payoff of an agent when perceptive to that when imperceptive, holding all else constant, I determine the agent's value of perceptiveness and the effect that perceptiveness has on the agent's equilibrium strategy. In the third model, I solve for the information designer's attainable decision rules, then determine which of the attainable decision rules maximizes producer surplus.

Among other insights, I find that perceptiveness is generally valuable, whether that be from the perspective of a poker player, a player considering market entry, or an information designer in a market-entry game. Furthermore, under an equilibrium that treats the market-entry players as symmetrically as possible, the value of perceptiveness is positive when both players have a sufficiently high probability of being expert; whereas, the value of perceptiveness is zero when either player is inexpert with a sufficiently high probability. Also, perceptiveness is generally less beneficial to players considering market entry than it is to players playing poker.

Keywords: Perceptiveness, game theory, market-entry, information design, poker, value of information, Bayesian games, reading opponents

Summary for Lay Audience

This thesis studies the value and effect that perceptiveness has in three game-theoretic settings. I consider a player to be *expert* if they can discern their likelihood of realizing a high payoff in a strategic setting. If the player cannot discern such, I consider the player to be *inexpert*. A player is *perceptive* if they know with certainty whether their opponent is expert. Otherwise, the player is *imperceptive*. The goal of this thesis is to provide insight regarding the potential value and effect that perceptiveness has in the game-theoretic settings I study.

The first model I consider emulates a two-player, one-round game of poker. The second model I investigate is a two-player market-entry game. The third model I study depicts a two-player market-entry game that is influenced by a third player that can signal to the other two players whether they should enter the market. The third player aims to maximize the combined well-being of the two other players. In each model, I consider distinct endowments of information between the players. These endowments vary in terms of the players' expertise and perceptiveness. By obtaining the solutions and expected payoffs of a player when they are perceptive, then comparing such to that when the player is imperceptive, I determine the player's value of perceptiveness and the effect that perceptiveness has on the player's strategy. In the third model, I solve for the attainable signals that the third player can send, then determine which signal maximizes the combined well-being of the other two players.

Among other insights, I find that perceptiveness is generally valuable in all three models. Furthermore, using a solution that treats the market-entry players as symmetrically as possible, the value of perceptiveness is positive when both players have a sufficiently high probability of being expert; whereas, the value of perceptiveness is zero when either player is inexpert with a sufficiently high probability. I also find that perceptiveness is generally less beneficial to players considering market entry than it is to players playing poker.

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Chapter 1

Introduction

In my thesis, I investigate the value and effect that perceptiveness has in various game-theoretic settings. I consider a player to be *expert* if they know the value of a particular payoff-relevant parameter in the models I study. If the player does not know such value, I consider the player to be *inexpert*. A player is *perceptive* if they know with certainty whether their opponent is expert. Otherwise, the player is *imperceptive*.

To motivate the potential value and effect of perceptiveness, consider the following example. Suppose an entrepreneur develops a novel idea and is considering whether to take it to market. The cost associated with entering the market will surely affect the entrepreneur's market-entry decision. Similarly, the entrepreneur's competitor's market-entry decision will be affected by their own market-entry cost. Since the entrepreneur's expected payoff from entering the market will likely be affected by their competitor's market-entry decision, the competitor's information regarding their own market-entry cost will likely affect the entrepreneur's decision as well. As a result, perceptiveness will likely affect the entrepreneur's market-entry decision. Also, the entrepreneur could be better (or worse) off by being perceptive. The goal of my thesis is to provide insight regarding the potential value and effect that perceptiveness has in three different game-theoretic settings.

The first setting I consider, which is presented in Chapter 2, is a game-theoretic model that emulates a two-player, one-round game of poker. I consider six distinct information structures, which differ in terms of the players' levels of expertise and perceptiveness. I solve for the Bayesian Nash equilibria of each game and compute each agent's expected payoff. By comparing the equilibrium action and expected payoff of an agent when perceptive to that when imperceptive, holding all else constant, I determine the agent's value of perceptiveness and the effect that perceptiveness has on the agent's equilibrium strategy. Among other insights, I find that perceptiveness generally has significant value when the players' chip endowment is sufficiently high.

The second setting I consider, which is presented in Chapter 3, studies the value and effect that perceptiveness has in a market-entry setting. I consider a continuum of information structures, which (similar to the first setting) differ in terms of the players' expertise and perceptiveness. Upon deriving and refining the Bayesian Nash equilibria and computing each agent's expected payoff, I determine the sign and magnitude of each agent's value of perceptiveness. I find that, under an equilibrium that treats the players as symmetrically as possible, the value of perceptiveness is always non-negative. Furthermore, the value of perceptiveness is always zero for an inexpert agent whose opponent is perceptive. Also, when both players have a sufficiently high probability of being expert, the value of perceptiveness is positive; whereas, if either agent is inexpert with a sufficiently high probability, the value of perceptiveness is zero. Additionally, even when the value of perceptiveness is zero, perceptiveness still affects the players' equilibrium actions. I also find that perceptiveness is generally less beneficial in this model as opposed to the model I consider in Chapter 2.

The third setting I consider, which is presented in Chapter 4, studies the effect that perceptiveness has on an information design problem in a market-entry setting. Like the previous two settings, I consider information structures that differ in terms of the players' expertise and perceptiveness. However, this setting also features an information designer that aims to maximize producer surplus and can send action recommendations to each player prior to the players' market-entry decision. I find that perceptiveness provides positive value, in terms of producer surplus, when the difference between the high and low state market-entry fees is sufficiently small, the high state market-entry fee is sufficiently low, and the low state market-entry fee is sufficiently high. Furthermore, perceptiveness inflicts negative value, in terms of producer surplus, when both the high and low market-entry fees are sufficiently high or when the difference between the high and low state market-entry fees is sufficiently small and the low state market-entry fee is sufficiently high.

Chapter 2

Perceptiveness in a Game-Theoretic Model of Poker

"Once you've mastered the basic elements of a winning poker formula, psychology becomes the key ingredient in separating break-even players from players who win consistently. The most profitable kind of poker psychology is the ability to read your opponents." (Caro, 2003, p. 8)

2.1 Introduction

This quotation highlights the presence of two distinct skills required to be a successful poker player. The first is expertise, which in poker can be thought of as a player's ability to gauge how strong their hand is relative to any competitior's hand. The second is perceptiveness, which can be thought of as a player's ability to "read their opponents".

There are several interpretations as to what "reading your opponents" entails. For instance, it could refer to a player's ability to discern their opponents' level of expertise, or it could refer to a player's ability to accurately gauge their opponents' hand strength based on a signal they receive from such opponent. In this chapter, I focus on studying the effect and value that a player's ability to discern their opponents' level of expertise has in my game-theoretic model of poker.

My research in this chapter relates best to literature regarding the value of information[1] and literature regarding poker. Poker has been a topic of discussion among academic economists for years, with works including von Neumann and Morgenstern (1944), Kuhn (1950), and Nash and Shapley (1950). Poker has also been studied in various other fields of research including computer science,[2] psychology,[3] and statistics.[4] In addition to this, poker has intrigued many casual players and non-academic authors who have discussed game theory optimal, exploitative, and conventional poker strategy.

A small subset of influential poker literature includes Acevedo (2019), Harrington and Robertie (2005), Little et al. (2015), and Snyder (2008).[5] Acevedo (2019) presents an in-depth analysis of game theory optimal play and how it applies to various situations in a No-Limit Hold'em poker game. More specifically, Acevedo (2019) teaches readers how to apply game theory in order to develop a non-exploitable poker strategy. Harrington and Robertie (2005) teach fundamental poker strategy and establish *Harrington's M-ratio*, which is a simple ratio calculation that helps inform players how aggressively they should play in a particular poker tournament situation. Little et al. (2015) provides a comprehensive review of modern, expert poker strategy. Some of the broad range of topics covered in Little et al. (2015) include range analysis, satellite play, game theory optimal play, and mental toughness. Snyder (2008) establishes the importance of chip utility, which is founded upon the fact that the more chips a player has in a poker tournament, the more strategies that player can utilize to accumulate even more chips.

Most of the non-academic poker literature, including the literature I reference here, focuses on developing a poker strategy that can be applied while playing poker. My research in this chapter departs from this, but still contributes to poker literature, since I develop a game-

[1] I address the value of information literature in Chapter 3.
[2] For instance Billings et al. (2003), Korb et al. (1999), Shi and Littman (2000), and Southey et al. (2012).
[3] For instance Griffiths et al. (2010), McCormack and Griffiths (2012), and Rapoport et al. (1997).
[4] For instance Borm and van der Genugten (2000, 2001), and Crosen et al. (2008).
[5] I listed these four poker books in particular since they have had the most substantial influence on the poker strategy I use when playing.

theoretic model of poker that incorporates perceptiveness. I also provide insight regarding the value of perceptiveness and the effect that perceptiveness has on a player's equilibrium strategy in my model of poker. Although it is not my intention to provide an applicable poker strategy, I believe that my model and results confirm conventional poker wisdom that recommends aggressive play when facing an opponent who you know is inexpert.

In addition to the poker literature from academic fields outside of economics that I cite above, many studies have focused on examining the behavioural tendencies of players or empirically determining whether poker is a game of luck or skill. Siler (2010) finds that a tight-aggressive strategy (which is where a player plays their hands aggressively, but only plays hands that have an expectation above a particular threshold) tends to be the most lucrative strategy for expert players. Levitt and Miles (2014), Hannum and Cabot (2009), and Meyer et al. (2013) empirically test the relationship between luck and skill in poker. The two former papers find that poker is a game of skill, whereas the latter finds that poker is a game of luck. This is a common debate among poker enthusiasts. From my personal experience, I believe poker to be both a game of luck and a game of skill. Bad luck can ultimately destroy any good player's chance at success. However, in order to win, a highly skilled poker player needs far less good luck than a lesser skilled poker player. Therefore, I believe that both luck and skill are important to achieving poker success.

The model I develop is also related to literature on higher-order beliefs, games of incomplete information, and epistemic game theory. Aumann and Heifetz (2002) acknowledges the importance, and details various methods, of incorporating players' beliefs of the other players' beliefs into game-theoretic models. I do this in my model by constructing perceptiveness as a player's ability to identify whether their opponent knows their own relative hand strength. Jehiel and Koessler (2008) studies the effect of analogy-based expectations in static two-player games of incomplete information by assuming that players understand the average behavior of their opponent over bundles of states, then act on the best responses to their opponent's average behaviour. My work differs from Jehiel and Koessler (2008) as the players in my model act using best responses that are derived from a probabilistic distribution over different levels of their opponent's expertise, whereas Jehiel and Koessler (2008) derive best responses based on a probabilistic distribution over a bundle of states. I also investigate the value of perceptiveness, whereas Jehiel and Koessler (2008) focuses solely on the effect of analogy-based expectations. Friedenberg et al. (2016) studies rationality (a player's tendency to act using a best response) and cognition (a player's tendency to apply some alternative rationale, such as using their best response or using a strategy based on their lucky numbers, to playing a game). They find that rationality is important when determining player behaviour, especially for cognitive players. My work departs from this since I assume all players to be rational and study a player's information about how much their opponent knows, as opposed to studying a player's information about whether their opponents' simply have a rational method behind their actions.

Additionally, my work in this chapter relates to economics literature pertaining to overconfidence and other personality traits in strategic settings. Ando (2004) investigates an economic contest featuring two players that are each overconfident with their relative abilities, then specifically studies two unique sources of overconfidence: a player's overestimation of their own ability and a player's underestimation of their opponent's ability. Ando (2004) finds that a player's overestimation of their own ability always induces the player to become more aggressive, whereas a player's underestimation of their opponents' ability sometimes induces the player to become more passive; thus implying that overconfidence may not always lead to aggressive tendencies. Ludwig et al. (2011) uses a model that features a two-player Tullock contest to show that modest overconfidence can improve a player's performance relative to an unbiased opponent, and thus leads to an absolute advantage for the overconfident player. My work provides an example of how a player's overestimation of their opponent's ability leads to behaviour that is relatively more passive.

The rest of this chapter is organized as follows. Section 2.2 briefly discusses poker and how it is played. Section 2.3 presents the model. Section 2.4 provides details regarding the strategies, best responses, equilibria, and expected payoffs. Section 2.5 reports the value of expertise and the value of perceptiveness, as well as a discussion on the value and effect that perceptiveness has in this chapter's model. Section 2.6 concludes. The supplemental appendix for this chapter is located in Appendix A.

2.2 Preliminaries

Poker is a collection of card games that combine gambling, skill, and strategy. A game of poker begins with each player having a certain number of chips, which can be thought of as a player's capital. A series of hands are then dealt among the players. In each hand, players receive cards and try to make the highest-ranking card combination possible. Throughout each hand, players can place bets to either increase the number of chips the winning player will receive or to attempt to win the hand immediately. Players must also periodically decide whether to remain in the hand by matching the bets made by other players. When a player decides not to match an opponent's bet, the player is eliminated from the hand. A player wins a hand and collects all of the wagered chips if all other players have been eliminated or if they have the highest-ranking card combination after the final betting round.

2.3 Model

2.3.1 Inspiration

To induce action in a game of poker, players may have to pay "blinds" and/or "antes" at the start of each hand. A common, and almost inevitable, occurrence in any poker tournament is when these forced bets are a high proportion of the players' total number of chips. In this situation, a popular and effective strategy is to either "fold" or go "all-in" in the first betting round. When a player folds they relinquish their hand and forfeit their chance of winning all chips wagered throughout the hand. When a player goes all-in, they maintain their chance of winning the hand (and all of the chips wagered), but also risk all of their chips in the process. This approach is effective since going all-in negates the other players' chance of winning the hand by betting. This means that the winning player will win the hand by matching all bets and having the highest-ranking card combination of all players that matched such bets. Furthermore, if no opponent decides to match the player's all-in bet, the reward the player receives from winning the hand is relatively large since the forced bets are a high proportion of the player's total number of chips. The model I develop in this chapter emulates the first betting round in a two-player poker hand, in which each player can either go all-in or fold.

2.3.2 Players, Actions, States

Suppose there are two risk-neutral players, A and B, that are both endowed with chips and receive a hand that is drawn by Nature. Upon receiving their hand, each player must choose whether to "fold" ($a_i = F$) or go "all-in" ($a_i = A$). Player i's payoff function is[6]

$$u_i(a_i, a_j, h_i, h_j) = \begin{cases} 0, & \text{if } (a_i, a_j) = (F, F) \\ 1, & \text{if } (a_i, a_j) = (A, F) \\ -1, & \text{if } (a_i, a_j) = (F, A) \\ 0, & \text{if } (a_i, a_j) = (A, A) \ \& \ h_i = h_j \\ K, & \text{if } (a_i, a_j) = (A, A) \ \& \ h_i > h_j \\ -K, & \text{if } (a_i, a_j) = (A, A) \ \& \ h_i < h_j. \end{cases}$$

I let $\{K \geq 1 | K \in \mathbb{R}\}$ and $h_i \sim i.i.d.\ U[0,1]$. In this setting, K represents the players' chip endowment, and h_i represents the value of player i's hand. The set of states is $[0,1]^2$, which corresponds to the possible hand combinations that can be drawn between the two players.

[6] j will always denote i's opponent.

2.3.3 Types, Information, & Beliefs

In this chapter, I consider six distinct information structures. I will construct the Bayesian Nash equilibria and expected payoffs for each information structure separately, then use the results to determine the value and effect of perceptiveness. The six information structures I consider are

$$(\epsilon_A, \epsilon_B) \in \{(0,0), (0,1), (1,1), (0,1/2), (1,1/2), (1/2,1/2)\}, \tag{2.1}$$

where ϵ_i represents player j's probabilistic belief of player i knowing the value of h_i. Each (ϵ_A, ϵ_B) ordered pair is common knowledge to both players and corresponds to a specific information structure. For instance, suppose $(\epsilon_A, \epsilon_B) = (1/2,1/2)$. Here, $\epsilon_j = 1/2$, which implies that player i knows that player j knows the value of h_j with a probability of $1/2$.[7] For a second example, suppose $(\epsilon_A, \epsilon_B) = (0,1)$. Here, player A knows that player B knows the value of h_B; whereas, B knows that A merely knows that $h_A \sim iid\ U[0,1]$. Furthermore, both players know the probability that the other player believes them to know the value of their hand.

Definition 2.3.1 *Player i is perceptive if player i knows with certainty whether player j knows the value of h_j.*

Definition 2.3.1 classifies player i as *perceptive* if and only if $\epsilon_j \in \{0,1\}$. Hence, player i is *imperceptive* if and only if $\epsilon_j \in (0,1)$. As shown by Reza (1994), uncertainty is maximized when all potential outcomes occur with equal probability. Hence, the most imperceptive player i can be occurs when $\epsilon_j = 1/2$. For this reason, I have chosen $\epsilon_i = 1/2$ as the imperceptive class of games I study in this chapter. Since (ϵ_A, ϵ_B) is common knowledge, each player's perceptiveness is common knowledge as well.

Definition 2.3.2 *Player i is expert if player i knows the value of h_i prior to deciding whether to go all-in or fold.*

[7] I consider i's belief to be an accurate gauge of j's probability of knowing the value of h_j. That is, if i believes j knows h_j with probability ϵ_j, then ex-ante j knows h_j with probability ϵ_j.

Definition 2.3.2 classifies player i as *expert* if and only if player i knows the value of h_i prior to deciding whether to go all-in or fold. If player i is not expert, I classify player i as *inexpert*.

The type space, t_i, for each player i varies depending on the value of ϵ_i. If $\epsilon_i = 0$, the type space for i is $t_i \in \{I_i\}$. If $\epsilon_i = 1$, the type space for i is $t_i \in [0,1]$. If $\epsilon_i \in (0,1)$, the type space for i is $t_i \in [0,1] \cup \{I_i\}$. When $\epsilon_i = 0$, j is perceptive and i merely knows that $h_i \sim iid\ U[0,1]$. As a result, i only has one type, which I denote as I_i. When $\epsilon_i = 1$, j is perceptive and i knows the value of h_i. As a result, i has a continuum of types, each corresponding to a particular value of h_i. Finally, when $\epsilon_i \in (0,1)$, j is imperceptive so it is conceivable for i to be either expert or inexpert, and hence have a type space of $[0,1] \cup \{I_i\}$.

By considering the six information structures listed in Expression (2.1), I cover every player configuration possibility, given that any imperceptive player i has an opponent j such that $\epsilon_j = 1/2$. Since my model features two levels of expertise and two levels of perceptiveness, there are four configuration possibilities for each player. These are: 1) expert, perceptive; 2) inexpert, perceptive; 3) expert, imperceptive; and 4) inexpert, imperceptive. Therefore, there are sixteen total configurations, which can be reduced to six by excluding symmetric and redundant[8] configurations.

2.3.4 Timeline

The timeline for the game in this chapter is as follows. First, each player observes (ϵ_A, ϵ_B) and learns whether they are expert or inexpert. Second, each player i receives their draw of h_i, which player i observes if i is expert. Third, players simultaneously choose to either fold or go all-in. Fourth, the hand values are revealed to both players and payoffs are realized.

[8]By redundant, I refer to configurations that have the existence of multiple agents for one player. That is, when player i is imperceptive, there will be two agents for player j, one for each level of expertise. In my analysis, I group these configuration possibilities together into one game.

2.4 Strategies, Best Responses, & Equilibria

2.4.1 Strategies

A strategy maps each type to a probability distribution over actions. Therefore, I must define a strategy for each level of expertise since the type space for an expert agent is [0,1] and the type space for an inexpert agent is $\{I_i\}$. Since an expert agent i knows the value of h_i, this information should affect their strategy since i's expected payoff depends on h_i. Since i's expected payoff is increasing in h_i,[9] I restrict attention to cut-off strategies for an expert agent i. I let χ_i, where $\chi_i \in [0,1]$, represent an expert agent i's cut-off, such that i chooses to fold for all $h_i < \chi_i$ and chooses all-in for all $h_i \geq \chi_i$.[10]

An inexpert agent i does not know the value of h_i prior to deciding whether to fold or go all-in, which is depicted by an inexpert i's type space of $\{I_i\}$. Therefore, an inexpert i's strategy should not depend on h_i. I let α_i, where $\alpha_i \in [0,1]$, represent the probability that an inexpert agent i chooses to fold.[11]

2.4.2 Best Responses

Before constructing the best response functions, I derive each agent's expected payoff from choosing fold and from choosing all-in. I denote agent i's expected payoff from choosing a_i as $E[u_{i,q}^{a_i}(\chi_j, \alpha_j)]$, where $q \in \{EX, IX\}$ denotes agent i's expertise.[12] Since an agent i that chooses fold has a payoff that is independent of h_i, an agent's expected payoff from choosing fold is independent of their expertise. Hence an expert or inexpert agent i's expected payoff from choosing fold is

[9] To see this, consider the expression for u_i listed in Section 2.3.2. Player i's expected payoff from choosing fold is constant in h_i, whereas i's expected payoff from choosing all-in increases in h_i, given some arbitrary h_j.

[10] I restrict attention to equilibria where an expert i chooses all-in if they are indifferent between the two actions.

[11] Hence, an inexpert i chooses all-in with probability $1-\alpha_i$.

[12] EX indicates that i is expert, whereas IX indicates that i is inexpert.

$$\begin{aligned}E[u_i^F(\chi_j,\alpha_j)] &= \epsilon_j[Pr(a_j{=}F \mid j{=}EX)(0) + Pr(a_j{=}A \mid j{=}EX)(-1)] \\ &\quad + (1-\epsilon_j)[Pr(a_j{=}F \mid j{=}IX)(0) + Pr(a_j{=}A \mid j{=}IX)(-1)] \\ &= -\epsilon_j(1-\chi_j) - (1-\epsilon_j)(1-\alpha_j) \\ &= \epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - 1. \end{aligned} \tag{2.2}$$

An inexpert agent i's expected payoff from choosing all-in is[13]

$$\begin{aligned}&E[u_{i,IX}^A(\chi_j,\alpha_j)] = \\ &\quad \epsilon_j[Pr(a_j{=}F \mid j{=}EX)(1) + Pr(a_j{=}A \mid j{=}EX) \\ &\qquad \cdot [Pr(h_i{>}h_j \mid h_j \geq \chi_j)K + Pr(h_i{<}h_j \mid h_j \geq \chi_j)(-K) + Pr(h_i{=}h_j \mid h_j \geq \chi_j)(0)]] \\ &\quad + (1-\epsilon_j)[Pr(a_j{=}F \mid j{=}IX)(1) \\ &\qquad + Pr(a_j{=}A \mid j{=}IX)[Pr(h_i{>}h_j)K + Pr(h_i{<}h_j)(-K) + Pr(h_i{=}h_j)(0)]] \\ &= \epsilon_j[\chi_j + (1-\chi_j)[\chi_j(-K) + \tfrac{1}{2}(1-\chi_j)(K) + \tfrac{1}{2}(1-\chi_j)(-K)]] + (1-\epsilon_j)[\alpha_j + (1-\alpha_j)K(0)] \\ &= \epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - \epsilon_j\chi_j(1-\chi_j)K. \end{aligned} \tag{2.3}$$

Lastly, an expert agent i's expected payoff from choosing all-in is

[13] Appendix A.1 provides a breakdown of the possible states and the corresponding payoff that an inexpert agent i receives when choosing all-in against an expert agent j, given that j uses a cut-off strategy, χ_j.

$$
\begin{aligned}
&E[u^A_{i,EX}(\chi_j, \alpha_j)] = \\
&\quad \epsilon_j[Pr(a_j{=}F \mid j{=}EX)(1) + Pr(a_j{=}A \mid j{=}EX)[Pr(h_i > h_j \mid h_j \geq \chi_j)K \\
&\qquad + Pr(h_i < h_j \mid h_j \geq \chi_j)(-K) + Pr(h_i = h_j \mid h_j \geq \chi_j)(0)]] \\
&\quad + (1-\epsilon_j)[Pr(a_j{=}F \mid j{=}IX)(1) \\
&\qquad + Pr(a_j{=}A \mid j{=}IX)[Pr(h_i > h_j)K + Pr(h_i < h_j)(-K) + Pr(h_i = h_j)(0)]] \\
&= \epsilon_j(\chi_j + [Pr(h_i < \chi_j)(-K) + Pr(h_i \geq \chi_j)[(h_i - \chi_j)K + (1-h_i)(-K)]]) \\
&\quad + (1-\epsilon_j)[\alpha_j + (1-\alpha_j)(2h_i - 1)K] \\
&= \epsilon_j\chi_j + \epsilon_j K[Pr(h_i \geq \chi_j)(2h_i - \chi_j - 1) - Pr(h_i < \chi_j)] + (1-\epsilon_j)[\alpha_j + (1-\alpha_j)(2h_i-1)K] \\
&= \epsilon_j\chi_j + \epsilon_j K[Pr(h_i \geq \chi_j)(2h_i - \chi_j) - (Pr(h_i \geq \chi_j) + Pr(h_i < \chi_j))] \\
&\quad + (1-\epsilon_j)[\alpha_j + (1-\alpha_j)(2h_i-1)K] \\
&= \epsilon_j\chi_j + Pr(h_i \geq \chi_j)(2h_i - \chi_j)\epsilon_j K - \epsilon_j K + (1-\epsilon_j)[\alpha_j + (1-\alpha_j)(2h_i-1)K] \qquad (2.4) \\
&= \begin{cases} \epsilon_j\chi_j - \epsilon_j K + (1-\epsilon_j)[\alpha_j + (1-\alpha_j)(2h_i-1)K] + \epsilon_j(2h_i - \chi_j)K & \text{if } h_i \geq \chi_j \\ \epsilon_j\chi_j - \epsilon_j K + (1-\epsilon_j)[\alpha_j + (1-\alpha_j)(2h_i-1)K] & \text{if } h_i < \chi_j \end{cases} \qquad (2.5)
\end{aligned}
$$

To solve for the Bayesian Nash equilibria for the six information structures listed in Expression (2.1), there are six best response functions I require. These six best response functions are for an

i inexpert & perceptive agent i versus an inexpert agent j;

ii expert & perceptive agent i versus an inexpert agent j;

iii inexpert & perceptive agent i versus an expert agent j;

iv expert & perceptive agent i versus an expert agent j;

v inexpert & imperceptive agent i;

vi expert & imperceptive agent *i*.

Any agent *i*'s best response function is independent of *j*'s perceptiveness. This is because *j*'s perceptiveness will affect *j*'s best response function, which will in turn affect *j*'s strategy variables, χ_j and α_j. Although these variables substitute into *i*'s best response function, the function itself will remain unchanged if *j*'s perceptiveness were to change. Additionally, since an imperceptive agent *i* is unable to discern *j*'s expertise, *i*'s best response function is independent of *j*'s expertise.

Inexpert & Perceptive vs. Inexpert

Lemma 2.4.1 *Suppose an inexpert, perceptive agent i is facing an inexpert opponent j. Agent i's best response is to choose all-in,* $\alpha_i^{BR} = 0$*, for all* $K \geq 1$.

Proof Suppose *i* is inexpert, perceptive and *j* is inexpert. This implies that $\epsilon_j = 0$. By Equations (2.2) and (2.3), *i*'s expectation from choosing all-in will be greater than or equal to *i*'s expectation from choosing fold if and only if

$$E[u_{i,IX}^A(\chi_j, \alpha_j)] \geq E[u_i^F(\chi_j, \alpha_j)]$$

$$\epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - \epsilon_j\chi_j(1-\chi_j)K \geq \epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - 1.$$

This expression, when $\epsilon_j = 0$, simplifies to $0 \geq -1$. Therefore, agent *i*'s expected payoff from choosing all-in is greater than *i*'s expected payoff from choosing fold for all $K \geq 1$.

Expert & Perceptive vs. Inexpert

Lemma 2.4.2 *Suppose an expert, perceptive agent i faces an inexpert opponent j. Agent i's best response is to, for all* $K \geq 1$*, choose the cut-off*

$$\chi_i^{BR}(\alpha_j) = Max\{\tfrac{1}{2}(1-\tfrac{1}{(1-\alpha_j)K}), 0\}.$$

Proof Suppose i is expert, perceptive and j is inexpert. This implies that $\epsilon_j = 0$. By Equations (2.2) and (2.5), i should select χ_i such that i is indifferent between actions.[14] If no indifference point exists, i should select whichever action yields the highest expected payoff. Therefore, if $h_i \geq \chi_j$,

$$E[u_i^F(\chi_j, \alpha_j)] = E[u_{i,EX}^A(\chi_j, \alpha_j)|h_i \geq \chi_j]$$
$$\epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - 1 = \epsilon_j\chi_j - \epsilon_j K + (1-\epsilon_j)[\alpha_j + (1-\alpha_j)(2h_i-1)K] + \epsilon_j(2h_i-\chi_j)K$$
$$\chi_i(\alpha_j) = \tfrac{1}{2}(1-\tfrac{1}{(1-\alpha_j)K}),$$

while if $h_i < \chi_j$,

$$E[u_i^F(\chi_j, \alpha_j)] = E[u_{i,EX}^A(\chi_j, \alpha_j)|h_i < \chi_j]$$
$$\epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - 1 = \epsilon_j\chi_j - \epsilon_j K + (1-\epsilon_j)[\alpha_j + (1-\alpha_j)(2h_i-1)K]$$
$$\chi_i(\alpha_j) = \tfrac{1}{2}(1-\tfrac{1}{(1-\alpha_j)K}).$$

Since $\chi_i \in [0,1]$, the lowest value that i can select for χ_i is zero. Moreover, $\frac{1}{2}(1-\frac{1}{(1-\alpha_j)K})$ is bounded above by $1/2$. Therefore,

$$\chi_i^{BR}(\alpha_j) = Max\{\tfrac{1}{2}(1-\tfrac{1}{(1-\alpha_j)K}), 0\},$$

for all $K \geq 1$ when i is expert, perceptive and facing an inexpert opponent j.

In this situation, agent i's expected payoff from choosing all-in may be greater than their expected payoff from choosing fold for all $K \geq 1$. This depends on the inexpert j's strategy, α_j. $\chi_i^{BR}(\alpha_j)$ is bounded above by $1/2$, but diverges to $-\infty$ as α_j approaches 1 (which occurs when j always chooses fold). The intuition behind this is that if an inexpert j chooses to fold with a sufficiently high frequency, the expert i should always choose all-in in order to receive the ante that j often relinquishes by choosing fold.

[14] When i is indifferent between choosing all-in and fold, $h_i = \chi_i$.

Inexpert & Perceptive vs. Expert

Lemma 2.4.3 *Suppose an inexpert, perceptive agent i is facing an expert opponent j. Agent i's best response correspondence, for all $K \geq 1$, is*

$$\alpha_i^{BR}(\chi_j) = \begin{cases} \{1\} & if\, \chi_j^2 - \chi_j + \frac{1}{K} < 0 \\ \{0\} & if\, \chi_j^2 - \chi_j + \frac{1}{K} > 0 \\ [0,1] & if\, \chi_j^2 - \chi_j + \frac{1}{K} = 0. \end{cases}$$

Proof Suppose i is inexpert, perceptive and j is expert. This implies that $\epsilon_j = 1$. By Equations (2.2) and (2.3), i's expectation from choosing all-in will be greater than or equal to i's expectation from choosing fold if and only if

$$E[u_{i,IX}^A(\chi_j, \alpha_j)] \geq E[u_i^F(\chi_j, \alpha_j)]$$

$$\epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - \epsilon_j\chi_j(1-\chi_j)K \geq \epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - 1$$

$$\chi_j^2 - \chi_j + \tfrac{1}{K} \geq 0.$$

Therefore, for all $K \geq 1$, when i is inexpert, perceptive and facing an expert opponent j, i should always choose all-in if $\chi_j^2 - \chi_j + \frac{1}{K} > 0$ and always choose fold if $\chi_j^2 - \chi_j + \frac{1}{K} < 0$. Furthermore, if $\chi_j^2 - \chi_j + \frac{1}{K} = 0$, then i will be indifferent between the two actions.

Expert & Perceptive vs. Expert

Lemma 2.4.4 *Suppose an expert, perceptive agent i faces an expert opponent j. Agent i's best response is to, for all $K \geq 1$, choose the cut-off*

$$\chi_i^{BR}(\chi_j) = \tfrac{1}{2}(1-\tfrac{1}{K}+\chi_j),$$

when $h_i \geq \chi_j$, and to choose fold ($\chi_i = \chi_j$) when $h_i < \chi_j$.

Proof Suppose i is expert, perceptive and j is expert. This implies that $\epsilon_j = 1$. By Equations (2.2) and (2.5), i should select χ_i such that i is indifferent between actions. If no indifference point exists, i should select whichever action yields the highest expected payoff. Therefore, if $h_i \geq \chi_j$, then

$$E[u_i^F(\chi_j, \alpha_j)] = E[u_{i,EX}^A(\chi_j, \alpha_j)|h_i \geq \chi_j]$$

$$\epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - 1 = \epsilon_j\chi_j - \epsilon_j K + (1-\epsilon_j)[\alpha_j + (1-\alpha_j)(2h_i-1)K] + \epsilon_j(2h_i-\chi_j)K$$

$$\chi_i(\chi_j) = \tfrac{1}{2}(1-\tfrac{1}{K}+\chi_j),$$

while if $h_i < \chi_j$,

$$E[u_i^F(\chi_j, \alpha_j)] = E[u_{i,EX}^A(\chi_j, \alpha_j)|h_i < \chi_j]$$

$$\epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - 1 = \epsilon_j\chi_j - \epsilon_j K + (1-\epsilon_j)[\alpha_j + (1-\alpha_j)(2h_i-1)K]$$

$$K = 1.$$

Since $K \geq 1$, this implies that if $h_i < \chi_j$, agent i will be at least weakly better off by choosing to fold when $h_i < \chi_j$, given that i is expert, perceptive and j is expert. Since, in this case, $h_i < \chi_j$, by setting $\chi_i = \chi_j$, i will choose fold when $h_i < \chi_j$.

Inexpert & Imperceptive

Lemma 2.4.5 *Suppose i is inexpert and imperceptive. Agent i's best response correspondence, for all $K \geq 1$, is*

$$\alpha_i^{BR}(\chi_j, \alpha_j) = \begin{cases} \{1\} & \text{if } \chi_j^2 - \chi_j + \frac{2}{K} < 0 \\ \{0\} & \text{if } \chi_j^2 - \chi_j + \frac{2}{K} > 0 \\ [0,1] & \text{if } \chi_j^2 - \chi_j + \frac{2}{K} = 0. \end{cases}$$

Proof Suppose i is inexpert and imperceptive. This implies that $\epsilon_j = 1/2$. By Equations (2.2) and (2.3), i's expectation from choosing all-in will be greater than or equal to i's expectation from choosing fold if and only if

$$E[u^A_{i,IX}(\chi_j, \alpha_j)] \geq E[u^F_i(\chi_j, \alpha_j)]$$
$$\epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - \epsilon_j\chi_j(1-\chi_j)K \geq \epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - 1$$
$$1 \geq \tfrac{1}{2}\chi_j(1-\chi_j)K$$
$$\chi_j^2 - \chi_j + \tfrac{2}{K} \geq 0.$$

Therefore, for all $K \geq 1$, when i is inexpert and imperceptive, i should always choose all-in if $\chi_j^2 - \chi_j + \frac{2}{K} > 0$ and always choose fold if $\chi_j^2 - \chi_j + \frac{2}{K} < 0$. Furthermore, if $\chi_j^2 - \chi_j + \frac{2}{K} = 0$, then i will be indifferent between the two actions.

Expert & Imperceptive

Lemma 2.4.6 *Suppose i is expert and imperceptive. Agent i's best response is to, for all $K \geq 1$, choose the cut-off*

$$\chi_i^{BR}(\chi_j, \alpha_j) = \begin{cases} Max\{\frac{1}{2} + \frac{1}{2(2-\alpha_j)}(\chi_j - \frac{2}{K}), 0\}, & \textit{if } h_i \geq \chi_j \\ Min\{Max\{\frac{1}{2} + \frac{1}{2(1-\alpha_j)}(1 - \frac{2}{K}), 0\}, 1\}, & \textit{if } h_i < \chi_j. \end{cases}$$

Proof Suppose i is expert and imperceptive. This implies that $\epsilon_j = 1/2$. By Equations (2.2) and (2.5), i should select χ_i such that i is indifferent between actions. If no indifference point exists, i should select whichever action yields the highest expected payoff. Therefore, if $h_i \geq \chi_j$,

$$E[u^F_i(\chi_j, \alpha_j)] = E[u^A_{i,EX}(\chi_j, \alpha_j)|h_i \geq \chi_j]$$
$$\epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - 1 = \epsilon_j\chi_j - \epsilon_j K + (1-\epsilon_j)[\alpha_j + (1-\alpha_j)(2h_i-1)K] + \epsilon_j(2h_i-\chi_j)K$$
$$\chi_i(\chi_j, \alpha_j) = \tfrac{1}{2} + \tfrac{1}{2(2-\alpha_j)}(\chi_j - \tfrac{2}{K}),$$

while if $h_i < \chi_j$,

$$E[u_i^F(\chi_j,\alpha_j)] = E[u_{i,EX}^A(\chi_j,\alpha_j)|h_i < \chi_j]$$

$$\epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - 1 = \epsilon_j\chi_j - \epsilon_j K + (1-\epsilon_j)[\alpha_j + (1-\alpha_j)(2h_i-1)K]$$

$$\chi_i(\chi_j,\alpha_j) = \tfrac{1}{2} + \tfrac{1}{2(1-\alpha_j)}(1-\tfrac{2}{K}).$$

Since $\chi_i \in [0,1]$, the lowest (highest) value that i can select for χ_i is zero (one). Therefore,

$$\chi_i^{BR}(\chi_j,\alpha_j) = \begin{cases} Max\{\frac{1}{2}+\frac{1}{2(2-\alpha_j)}(\chi_j-\frac{2}{K}), 0\}, & \text{if } h_i \geq \chi_j \\ Min\{Max\{\frac{1}{2}+\frac{1}{2(1-\alpha_j)}(1-\frac{2}{K}), 0\}, 1\}, & \text{if } h_i < \chi_j, \end{cases}$$

for all $K \geq 1$ when i is expert and imperceptive.[15]

In this situation, agent i's expected payoff from choosing all-in may be greater than or less than their expected payoff from choosing fold for some $K \geq 1$. This depends on j's inexpert strategy (α_j) and j's expert strategy (χ_j). If $h_i \geq \chi_j$, then χ_i^{BR} is bounded above by 1, but $\chi_i^{BR} < 0$ when $1-\frac{1}{K} < \frac{1}{2}(\alpha_j-\chi_j)$. Hence, if $h_i \geq \chi_j$ and $1-\frac{1}{K} < \frac{1}{2}(\alpha_j-\chi_j)$, an expert, imperceptive agent i's best response is to always choose all-in. Similarly, if $h_i < \chi_j$, then $\chi_i^{BR} > 1$ when $\alpha_j > \frac{2}{K}$, and $\chi_i^{BR} < 0$ when $\alpha_j > 2(1-\frac{1}{K})$. This implies that if K is high enough to make $\alpha_j > \frac{2}{K}$, an expert, imperceptive agent i's best response is to always fold. Whereas, if K is low enough to make $\alpha_j > 2(1-\frac{1}{K})$, an expert, imperceptive agent i's best response is to always go all-in. The intuition behind this is that if the number of chips that i can lose is sufficiently high, then i is better off by taking a guaranteed loss of one chip, as opposed to risking all of their chips by going all-in when $h_i < \chi_j$. Similarly, if the number of chips that i can lose is sufficiently low, then i is better off risking all of their chips when $h_i < \chi_j$, on the off chance that j folds.

[15] The expression $\frac{1}{2}+\frac{1}{2(2-\alpha_j)}(\chi_j-\frac{2}{K})$ is bounded above by 1 and can be less than 0. Additionally, the expression $\frac{1}{2}+\frac{1}{2(1-\alpha_j)}(1-\frac{2}{K})$ can be greater than 1 or less than 0.

2.4.3 Equilibria

Using the best responses given by Lemmas 2.4.1-2.4.6, I derive the Bayesian Nash equilibria for each of the six information structures listed in Expression (2.1). I restrict attention to symmetric equilibria when two agents are symmetric.[16] Solving for each equilibrium is tedious, but straightforward. Each agent will have a best response, given by one of Lemmas 2.4.1-2.4.6. When there are x agents, such that $x \in \{2,3,4\}$, there will be x best response functions and x variables to solve for. As a result, the Bayesian Nash equilibrium can be found by setting up and solving the applicable system of equations.

Information Structure: $(\epsilon_A, \epsilon_B) = (0,0)$

Theorem 2.4.7 *Suppose* $(\epsilon_A, \epsilon_B) = (0,0)$. *This implies that both players are inexpert and perceptive. Furthermore, for all* $K \geq 1$*, the unique equilibrium is*

$$(\alpha_A^*, \alpha_B^*) = (0,0).$$

Proof This proof follows directly from Lemma 2.4.1.

Information Structure: $(\epsilon_A, \epsilon_B) = (0,1)$

Theorem 2.4.8 *Suppose* $(\epsilon_A, \epsilon_B) = (0,1)$. *This implies that player A is inexpert and perceptive, whereas player B is expert and perceptive. Furthermore, the unique equilibrium is*

$$(\alpha_A^*, \chi_B^*) = \begin{cases} (0, \frac{1}{2}[1-\frac{1}{K}]), & \textit{if } K \in [1, 2+\sqrt{5}] \\ (1-\frac{1}{\sqrt{K(K-4)}}, \frac{1}{2}[1-\sqrt{1-\frac{4}{K}}]), & \textit{if } K > 2+\sqrt{5}. \end{cases}$$

Proof This proof follows directly from Lemmas 2.4.2 and 2.4.3.

[16] For instance, in the game $(\epsilon_A, \epsilon_B) = (1/2, 1/2)$, the two inexpert agents will be symmetric and the two expert agents will be symmetric. Whereas, in the game $(\epsilon_A, \epsilon_B) = (0, 1/2)$, the two inexpert agents will not be symmetric since $\epsilon_A \neq \epsilon_B$.

Information Structure: $(\epsilon_A, \epsilon_B) = (1,1)$

Theorem 2.4.9 *Suppose* $(\epsilon_A, \epsilon_B) = (1,1)$. *This implies that both players are expert and perceptive. Furthermore, for all* $K \geq 1$*, the unique equilibrium is*

$$(\chi_A^*, \chi_B^*) = (1-\tfrac{1}{K}, 1-\tfrac{1}{K}).$$

Proof This proof follows directly from Lemma 2.4.4.

Information Structure: $(\epsilon_A, \epsilon_B) = (0,1/2)$

Theorem 2.4.10 *Suppose* $(\epsilon_A, \epsilon_B) = (0,1/2)$. *This implies that player A is inexpert and imperceptive, whereas player B is perceptive. Furthermore, the unique equilibrium is*

$$(\alpha_A^*, \chi_B^*, \alpha_B^*) = \begin{cases} (0, \tfrac{1}{2}[1-\tfrac{1}{K}], 0), & \text{if } K \in [1, 4+\sqrt{17}] \\ (1-\tfrac{1}{\sqrt{K(K-8)}}, \tfrac{1}{2}[1-\sqrt{1-\tfrac{8}{K}}], 0), & \text{if } K > 4+\sqrt{17}. \end{cases}$$

Proof This proof follows directly from Lemmas 2.4.1, 2.4.2, and 2.4.5.

Information Structure: $(\epsilon_A, \epsilon_B) = (1,1/2)$

Theorem 2.4.11 *Suppose* $(\epsilon_A, \epsilon_B) = (1,1/2)$. *This implies that player A is expert and imperceptive, whereas player B is perceptive. Furthermore, a Bayesian Nash equilibrium is*

$$(\chi_A^*, \chi_B^*, \alpha_B^*) = \begin{cases} (\tfrac{5}{7}[1-\tfrac{1}{K}], \tfrac{6}{7}[1-\tfrac{1}{K}], 0), & \text{if } K \in [1, 1.7+0.7\sqrt{11}] \\ (\tfrac{1}{2}[1+\sqrt{1-\tfrac{4}{K}}], \tfrac{1}{4}[3-\tfrac{2}{K}+\sqrt{1-\tfrac{4}{K}}], \tfrac{1}{4}[7-\tfrac{3K-10}{\sqrt{K(K-4)}}]), & \text{if } K \in (1.7+0.7\sqrt{11}, \tfrac{25}{6}] \\ (1-\tfrac{5}{3K}, 1-\tfrac{4}{3K}, 1), & \text{if } K > \tfrac{25}{6}. \end{cases}$$

Proof This proof follows directly from Lemmas 2.4.3, 2.4.4, and 2.4.6.

The equilibrium listed in Theorem 2.4.11 is not unique since

$$(\chi_A^*, \chi_B^*, \alpha_B^*) = (1-\tfrac{1}{K}, 1-\tfrac{1}{K}, 0)$$

also exists as an equilibrium, for all $K \geq 1$. I restrict attention to the equilibrium listed in Theorem 2.4.11 since it captures a change, from the $(\epsilon_A, \epsilon_B) = (1,1)$ information structure, in the expert agents' equilibrium strategy.

Information Structure: $(\epsilon_A, \epsilon_B) = (1/2, 1/2)$

Theorem 2.4.12 *Suppose* $(\epsilon_A, \epsilon_B) = (1/2, 1/2)$*. This implies that both players are imperceptive. Furthermore, a symmetric Bayesian Nash equilibrium is*

$$(\chi_A^*, \alpha_A^*, \chi_B^*, \alpha_B^*) = \begin{cases} (\frac{2}{3}[1-\frac{1}{K}], 0, \frac{2}{3}[1-\frac{1}{K}], 0), & \text{if } K \in [1, 4+3\sqrt{2}] \\ (\frac{1}{2}[1+\sqrt{1-\frac{8}{K}}], \frac{1}{2}[3-\frac{K-4}{\sqrt{K(K-8)}}], & \\ \quad \frac{1}{2}[1+\sqrt{1-\frac{8}{K}}], \frac{1}{2}[3-\frac{K-4}{\sqrt{K(K-8)}}]), & \text{if } K > 4+3\sqrt{2}. \end{cases}$$

Proof This proof follows directly from Lemmas 2.4.5 and 2.4.6.

The equilibrium listed in Theorem 2.4.12 is not unique since

$$(\chi_A^*, \alpha_A^*, \chi_B^*, \alpha_B^*) = (1-\tfrac{1}{K}, 0, 1-\tfrac{1}{K}, 0)$$

also exists as an equilibrium, for all $K \geq 1$. I restrict attention to the equilibrium listed in Theorem 2.4.12 since it captures a change, from the $(\epsilon_A, \epsilon_B) = (1,1)$ information structure, in the expert agents' equilibrium strategy. Figures 2.1-2.6 give visual depictions of each agent's equilibrium strategy for $K \in [1, 20]$, for each of the six information structures I consider.

2.4.4 Ex-Ante Expected Payoffs

The ex-ante expected payoff for agent i can be determined by integrating i's best response expected payoff for all possible draws of h_i. Lemmas 2.4.13 and 2.4.14 formalize these functions.

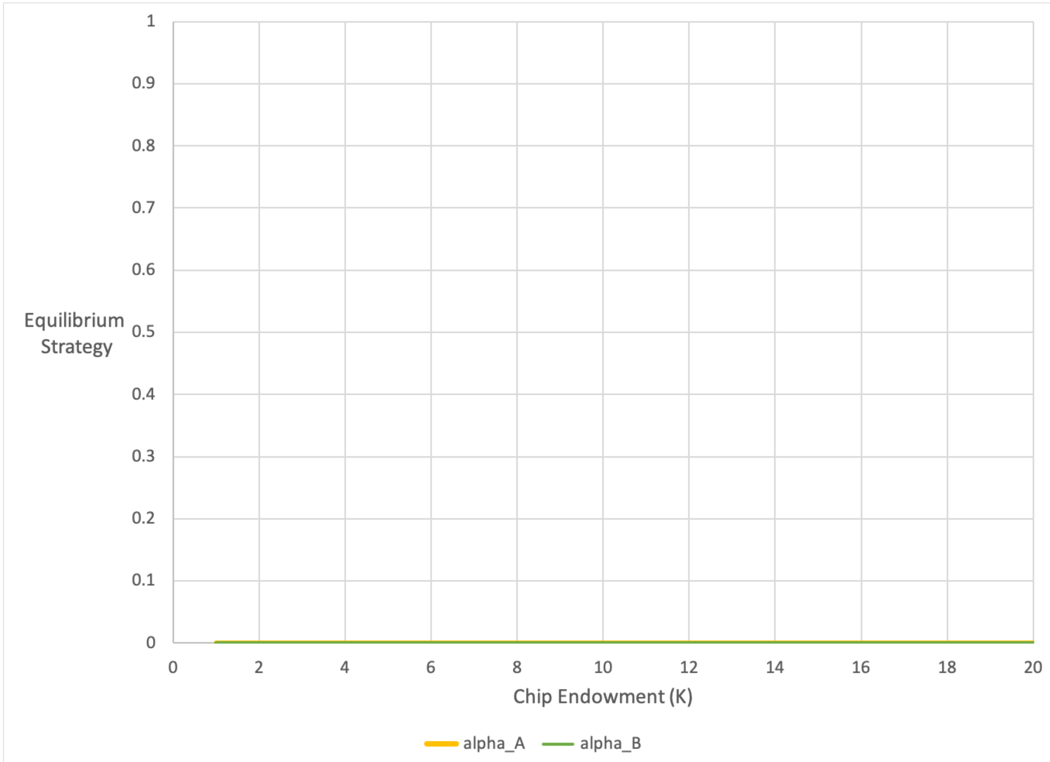


Figure 2.1: Equilibrium strategies when $(\epsilon_A, \epsilon_B) = (0,0)$.

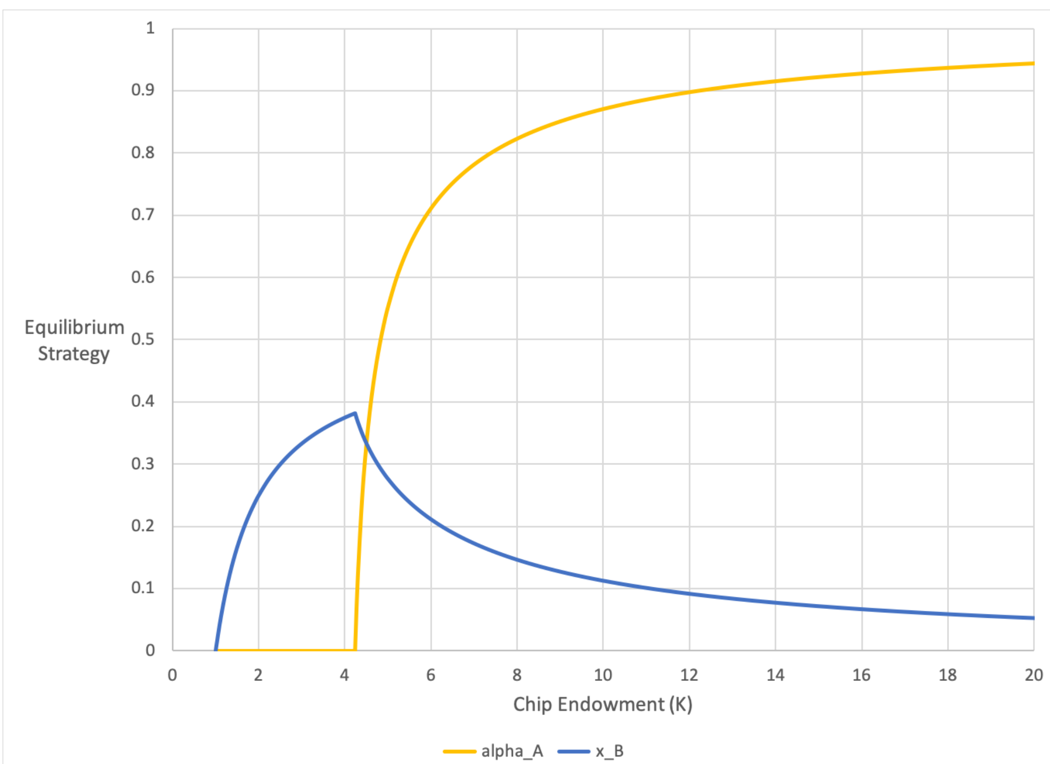


Figure 2.2: Equilibrium strategies when $(\epsilon_A, \epsilon_B) = (0,1)$.

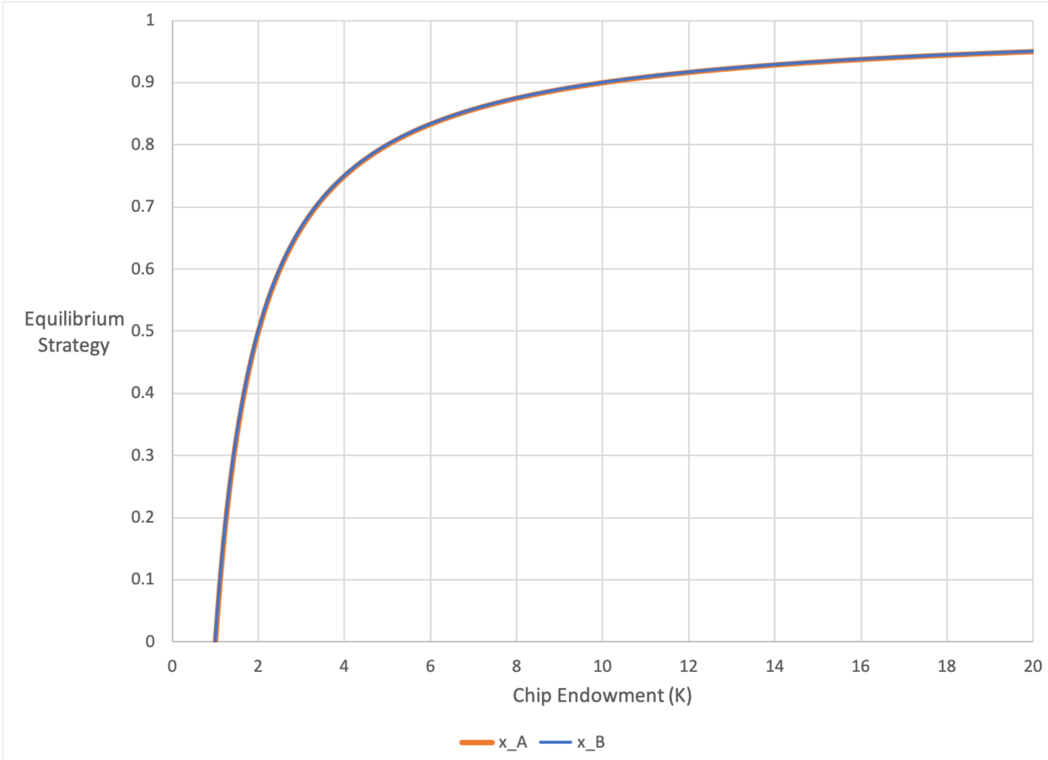


Figure 2.3: Equilibrium strategies when $(\epsilon_A, \epsilon_B) = (1,1)$.

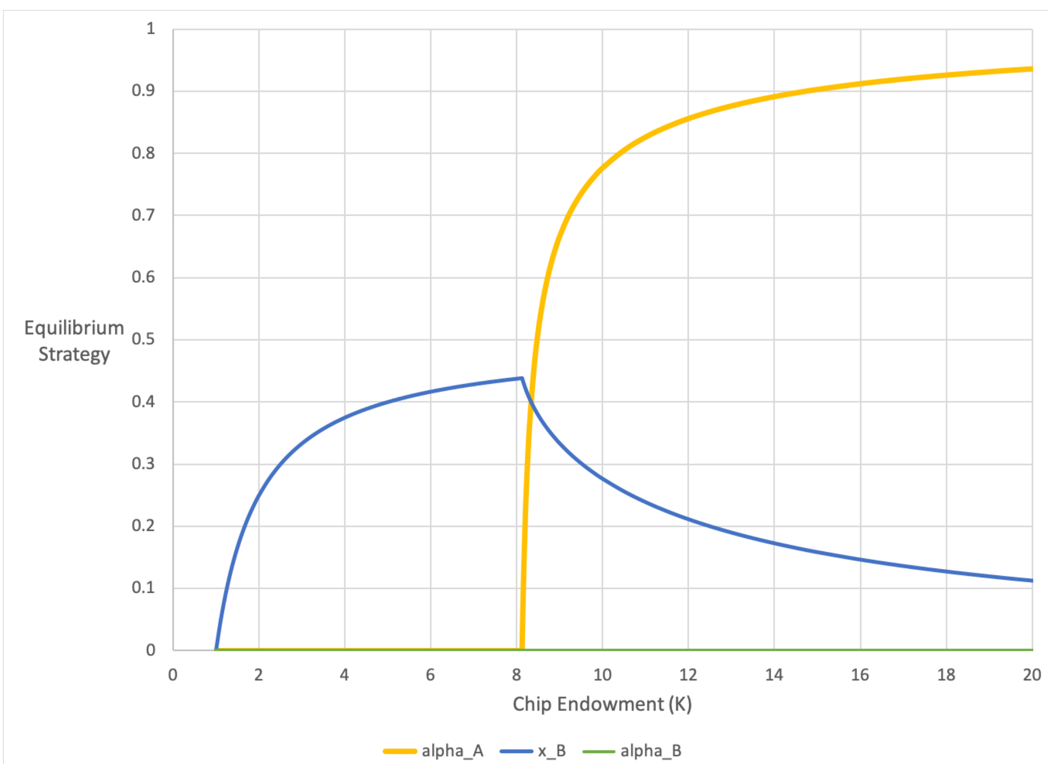


Figure 2.4: Equilibrium strategies when $(\epsilon_A, \epsilon_B) = (0, {}^1\!/_2)$.

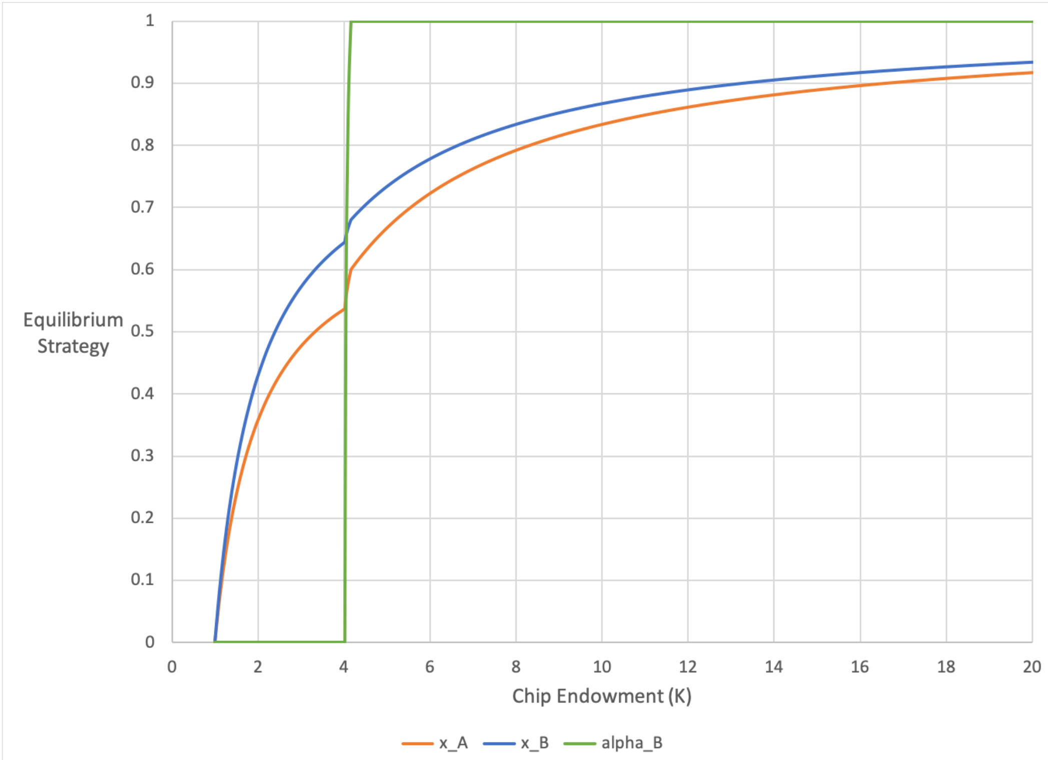


Figure 2.5: Equilibrium strategies when $(\epsilon_A, \epsilon_B) = (1, {}^1/_2)$.

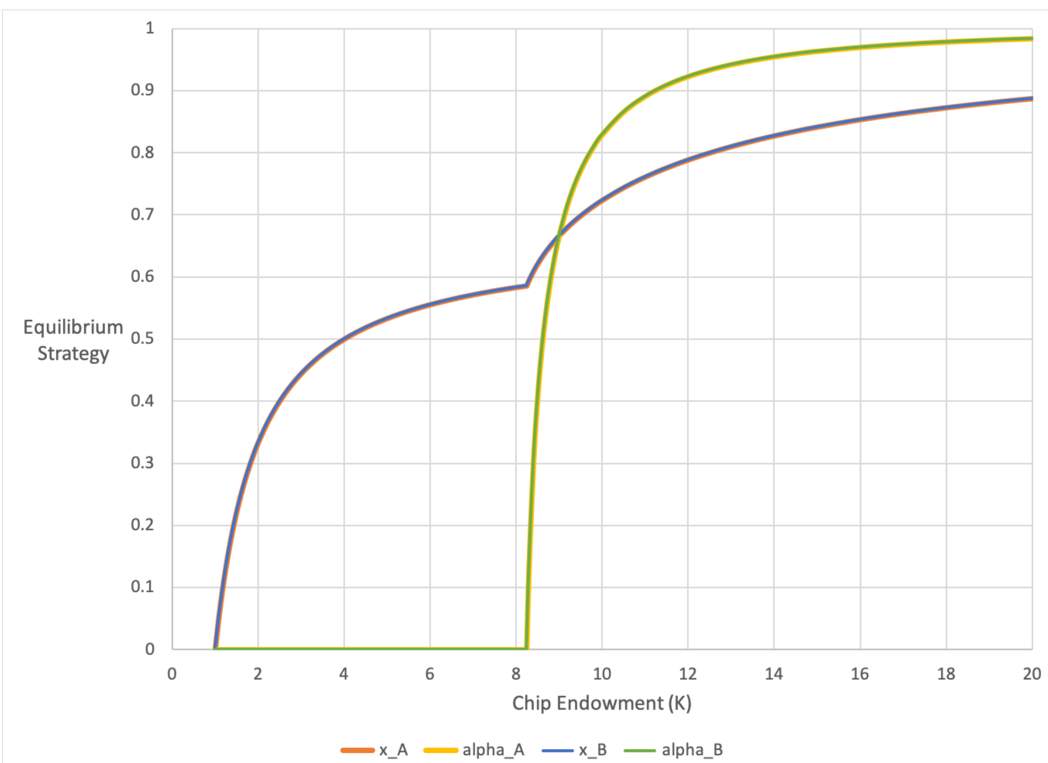


Figure 2.6: Equilibrium strategies when $(\epsilon_A, \epsilon_B) = ({}^1/_2, {}^1/_2)$.

Lemma 2.4.13 *Suppose agent i is expert and $\chi_i \leq \chi_j$. Agent i's ex-ante expected payoff is*

$$EU_{i,EX}(\chi_i,\chi_j,\alpha_j) = \epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - \chi_i + K(1-\chi_i)[\chi_i-(1-\epsilon_j)\alpha_j\chi_i] - \epsilon_j\chi_jK + \epsilon_jK\chi_i^2.$$

Additionally, suppose agent i is expert and $\chi_i \geq \chi_j$. Agent i's ex-ante expected payoff is

$$EU_{i,EX}(\chi_i,\chi_j,\alpha_j) = \epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - \chi_i + K(1-\chi_i)[\chi_i-(1-\epsilon_j)\alpha_j\chi_i] - \epsilon_j\chi_jK + \epsilon_jK\chi_i\chi_j.$$

Proof Suppose agent i is expert and that $\chi_i \leq \chi_j$. Agent i's ex-ante expected payoff is

$$\begin{aligned}
EU_{i,EX}(\chi_i,\chi_j,\alpha_j) &= \int_0^{\chi_i} E[u^F_{i,EX}(\chi_j,\alpha_j)]\,dh_i + \int_{\chi_i}^{\chi_j} E[u^A_{i,EX}(\chi_j,\alpha_j)]\,dh_i + \int_{\chi_j}^{1} E[u^A_{i,EX}(\chi_j,\alpha_j)]\,dh_i \\
&= \int_0^{\chi_i} (\epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - 1)\,dh_i \\
&\quad + \int_{\chi_i}^{\chi_j} (\epsilon_j[\chi_j-K] + (1-\epsilon_j)[\alpha_j + (1-\alpha_j)(2h_i-1)K])\,dh_i \\
&\quad + \int_{\chi_j}^{1} (\epsilon_j[\chi_j-K+(2h_i-\chi_j)K] + (1-\epsilon_j)[\alpha_j+(1-\alpha_j)(2h_i-1)K])\,dh_i \\
&= \epsilon_j\chi_j\chi_i + (1-\epsilon_j)\alpha_j\chi_i - \chi_i + \epsilon_j(\chi_j-K)(\chi_j-\chi_i) + (1-\epsilon_j)\alpha_j(\chi_j-\chi_i) \\
&\quad - (1-\epsilon_j)(1-\alpha_j)K(\chi_j-\chi_i) + (1-\epsilon_j)(1-\alpha_j)K[\chi_j^2-\chi_i^2] \\
&\quad + \epsilon_j(\chi_j-K)(1-\chi_j) - \epsilon_j\chi_jK(1-\chi_j) + \epsilon_jK(1-\chi_j^2) \\
&\quad + (1-\epsilon_j)\alpha_j(1-\chi_j) - (1-\epsilon_j)(1-\alpha_j)K(1-\chi_j) + (1-\epsilon_j)(1-\alpha_j)K[1-\chi_j^2] \\
&= \epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - \chi_i + K\chi_i(1-\chi_i)[1-(1-\epsilon_j)\alpha_j] - \epsilon_j\chi_jK + \epsilon_jK\chi_i^2.
\end{aligned}$$

Now suppose agent i is expert and that $\chi_i \geq \chi_j$. Agent i's ex-ante expected payoff is

$$\begin{aligned}
EU_{i,EX}(\chi_i,\chi_j,\alpha_j) &= \int_0^{\chi_i} E[u^F_{i,EX}(\chi_j,\alpha_j)]\,dh_i + \int_{\chi_i}^{1} E[u^A_{i,EX}(\chi_j,\alpha_j)]\,dh_i \\
&= \int_0^{\chi_i} (\epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - 1)\,dh_i \\
&\quad + \int_{\chi_i}^{1} (\epsilon_j[\chi_j-K+(2h_i-\chi_j)K] + (1-\epsilon_j)[\alpha_j+(1-\alpha_j)(2h_i-1)K])\,dh_i \\
&= [\epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - 1]\chi_i \\
&\quad + \epsilon_j\chi_j(1-\chi_i) - \epsilon_jK(1-\chi_i) - \epsilon_j\chi_jK(1-\chi_i) + \epsilon_jK(1-\chi_i^2) \\
&\quad + (1-\epsilon_j)\alpha_j(1-\chi_i) - (1-\epsilon_j)(1-\alpha_j)K(1-\chi_i) + (1-\epsilon_j)(1-\alpha_j)K(1-\chi_i^2) \\
&= \epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - \chi_i + K\chi_i(1-\chi_i)[1-(1-\epsilon_j)\alpha_j] - \epsilon_j\chi_jK + \epsilon_jK\chi_i\chi_j.
\end{aligned}$$

Lemma 2.4.14 *Suppose agent i is inexpert. Agent i's ex-ante expected payoff is*

$$EU_{i,IX}(\alpha_i, \chi_j, \alpha_j) = \epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - \alpha_i - \epsilon_j(1-\alpha_i)\chi_j(1-\chi_j)K.$$

Proof Suppose agent i is inexpert. Agent i's ex-ante expected payoff is

$$\begin{aligned}
EU_{i,IX}(\alpha_i, \chi_j, \alpha_j) &= \alpha_i \int_0^1 E[u^F_{i,IX}(\chi_j, \alpha_j)]\, dh_i + (1-\alpha_i) \int_0^1 E[u^A_{i,IX}(\chi_j, \alpha_j)]\, dh_i \\
&= \alpha_i \int_0^1 (\epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - 1)\, dh_i \\
&\quad + (1-\alpha_i) \int_0^1 (\epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - \epsilon_j\chi_j(1-\chi_j)K)\, dh_i \\
&= \epsilon_j\chi_j + (1-\epsilon_j)\alpha_j - \alpha_i - \epsilon_j(1-\alpha_i)\chi_j(1-\chi_j)K.
\end{aligned}$$

2.5 Value of Expertise & Perceptiveness

Substituting the equilibria listed in Theorems 2.4.7-2.4.12 and applying Lemmas 2.4.13 and 2.4.14, I compute player i's value of expertise as

$$VoE^{\omega}_i(\chi_i, \alpha_i, \chi_j, \alpha_j) = EU^{\omega}_{i,EX}(\chi_i, \chi_j, \alpha_j) - EU^{\omega}_{i,IX}(\alpha_i, \chi_j, \alpha_j), \tag{2.6}$$

where $\omega \in \{(P, IP), (P, IM), (P, EP), (P, EM), (M, P), (M, M)\}$ represents the specific situation in terms of the players' expertise and perceptiveness. Each ordered pair in ω corresponds to a viable situation for player i. For instance, (P, IP) corresponds to when i is perceptive and facing an inexpert, perceptive opponent j. Similarly, (P, IM) corresponds to when i is perceptive and facing an inexpert, imperceptive opponent j, (P, EP) corresponds to when i is perceptive and facing an expert, perceptive opponent j, (P, EM) corresponds to when i is perceptive and facing an expert, imperceptive opponent j, (M, P) corresponds to when i is imperceptive and facing a perceptive opponent j, and (M, M) corresponds to when i is imperceptive and facing an imperceptive opponent j. Figure 2.7 depicts i's value of expertise in each of these situations for $K \in [1, 20]$.

Likewise, by substituting the equilibria listed in Theorems 2.4.7-2.4.12 and applying Lemmas 2.4.13 and 2.4.14, I compute player i's value of perceptiveness as

$$VoP_i^{\rho}(s_i, \chi_j, \alpha_j) = \tfrac{1}{2}[EU_{i,x}^{\rho}(s_i, \chi_j | \epsilon_j = 1) + EU_{i,x}^{\rho}(s_i, \alpha_j | \epsilon_j = 0)] - EU_{i,x}^{\rho}(s_i, \chi_j, \alpha_j | \epsilon_j = 1/2), \tag{2.7}$$

where $\rho \in \{(E, P), (I, P), (E, M), (I, M)\}$ represents the specific situation in terms of the players' expertise and perceptiveness, $x \in \{EX, IX\}$ represents i's expertise, and $s_i \in \{\chi_i, \alpha_i\}$ represents i's equilibrium strategy.[17] Each ordered pair in ρ corresponds to a viable situation for player i. For instance, (E, P) corresponds to when i is expert and facing a perceptive opponent j. Similarly, (I, P) corresponds to when i is inexpert and facing a perceptive opponent j, (E, M) corresponds to when i is expert and facing an imperceptive opponent j, and (I, M) corresponds to when i is inexpert and facing an imperceptive opponent j. Figure 2.8 depicts i's value of perceptiveness in each of these situations for $K \in [1, 20]$.

In Figure 2.7, each unique colour corresponds to a specific amount of information that i has regarding j's expertise. Also, the solid lines in Figure 2.7 represent the instance when i is against a perceptive opponent j, whereas the dashed lines represent the instance when i is against an imperceptive opponent j. Given these results, Figure 2.7 shows that the value of expertise is positive for all $K \in (1, 20]$. This actually holds for all $K > 20$ as well. Furthermore, as the players' chip endowment increases, i's value of expertise converges to 1 when both i and j are perceptive and converges to $5/6$ when i is imperceptive and j is perceptive. Contrarily, as the players' chip endowment increases, i's value of expertise converges to 0 when j is imperceptive.

The intuition for this is that, as the players' chip endowment increases, each player i tends to select fold with a weakly increasing frequency unless they are certain that j is inexpert. That is, as K increases, i will select fold with a weakly increasing frequency unless i is perceptive and

[17] This notation assumes that $s_i = \chi_i$ if and only if $x = EX$.

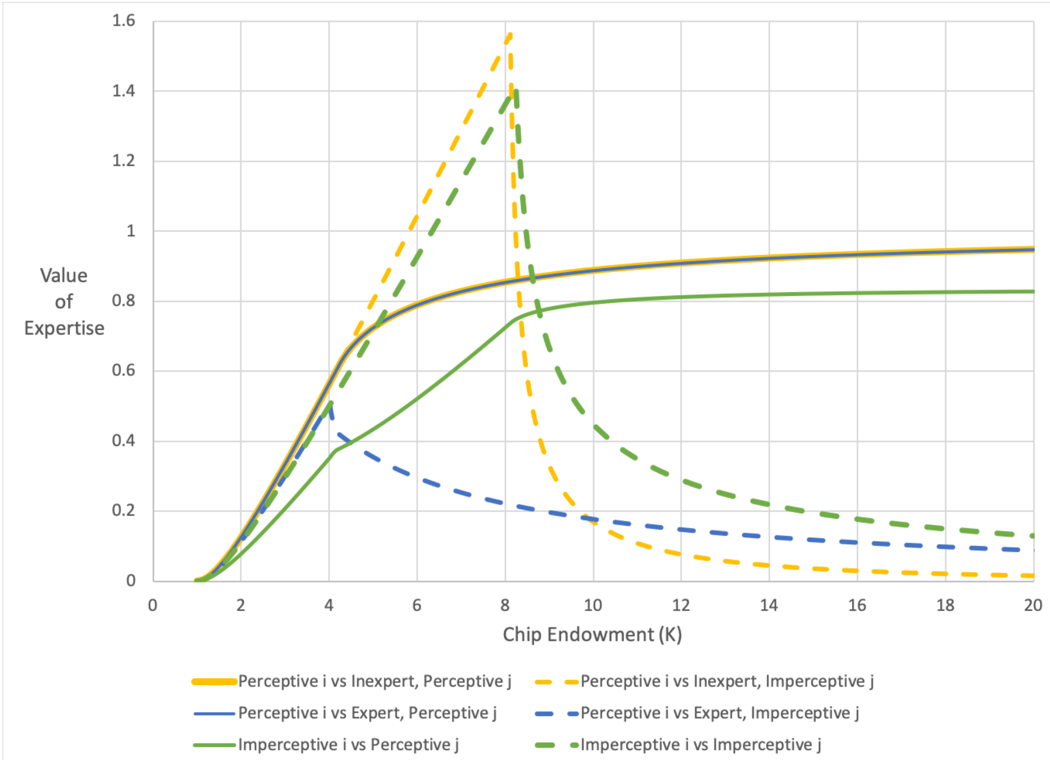


Figure 2.7: Player i's value of expertise in various situations.

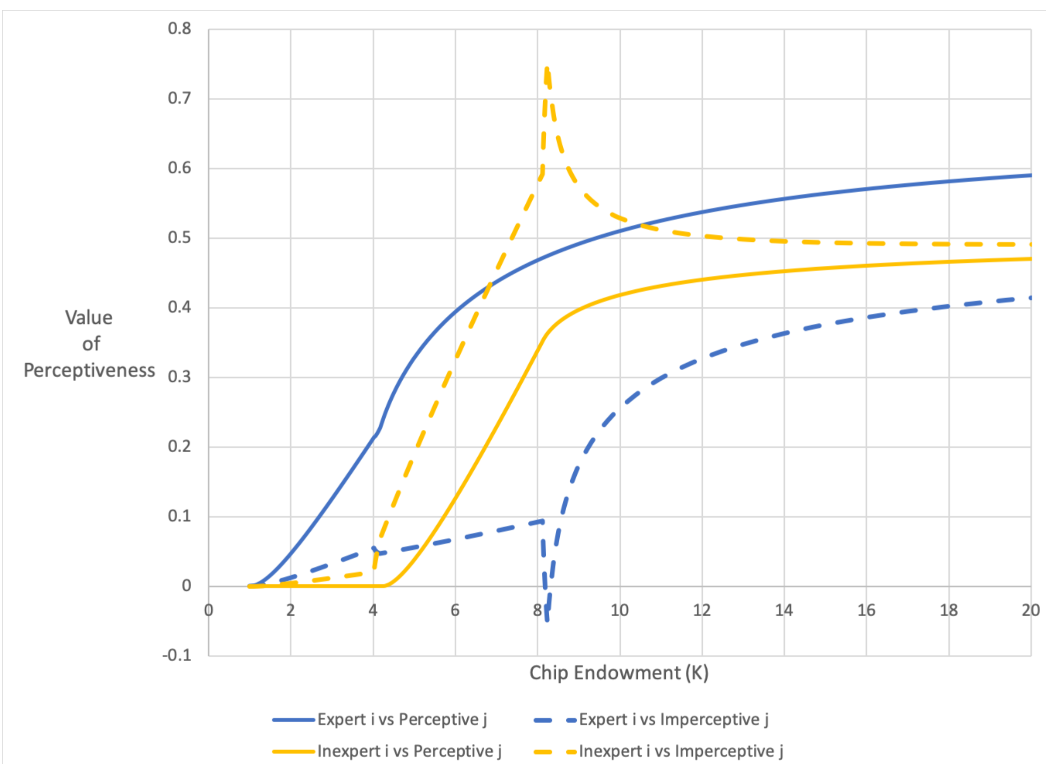


Figure 2.8: Player i's value of perceptiveness in various situations.

j is inexpert. In this case, as K increases, once the inexpert j switches from always choosing all-in to choosing fold with some positive probability, the expert i will lower their cut-off and effectively select all-in more frequently (as shown in Figures 2.2 and 2.4). As the frequency of an imperceptive opponent j choosing fold converges to 1, the expertise of player i matters progressively less since h_i becomes increasingly more irrelevant. Contrarily, when player i is facing a perceptive opponent j, j will choose all-in with increasing probability beyond a certain chip endowment threshold (as shown in Figures 2.2 and 2.4) when i is inexpert, but will continue choosing fold with increasing probability as K increases when i is expert. The influence that i's expertise has over j's equilibrium strategy is what drives i's value of expertise when j is perceptive.

In Figure 2.8, the blue lines correspond to an expert i, while the yellow lines correspond to an inexpert i. Furthermore, the solid lines correspond to an instance when j is perceptive, while the dashed lines correspond to an instance when j is imperceptive. By Equation (2.7), an expert i's value of perceptiveness when j is perceptive is

$$VoP_i^{(E,P)} = \begin{cases} \frac{37}{392}(K+\frac{1}{K}-2) & \text{if } K \in [1, \frac{17}{10}+\frac{7\sqrt{11}}{10}] \\ \frac{K}{4} + \frac{3}{8K} - \frac{7}{8} - \frac{(K-3)}{8}\frac{\sqrt{K-4}}{\sqrt{K}} & \text{if } K \in (\frac{17}{10}+\frac{7\sqrt{11}}{10}, 25/6] \\ \frac{K}{8} - \frac{7}{8K} - \frac{1}{12} & \text{if } K \in (25/6, 2+\sqrt{5}] \\ \frac{5}{12} + \frac{\sqrt{K-4}}{4\sqrt{K}} - \frac{1}{K} & \text{if } K > 2+\sqrt{5}. \end{cases}$$

Furthermore, an expert i's value of perceptiveness when j is imperceptive is

$$VoP_i^{(E,M)} = \begin{cases} \frac{85}{3528}(K+\frac{1}{K}-2) & \text{if } K \in [1, \frac{17}{10}+\frac{7\sqrt{11}}{10}] \\ \frac{11K}{144}-\frac{5}{18}(1-\frac{1}{2K})-\frac{(K-2)\sqrt{K-4}}{16\sqrt{K}} & \text{if } K \in (\frac{17}{10}+\frac{7\sqrt{11}}{10}, \frac{25}{6}] \\ \frac{1}{72}(K-2+\frac{5}{K}) & \text{if } K \in (\frac{25}{6}, 4+\sqrt{17}] \\ \frac{17}{36} + \frac{K-6}{4\sqrt{K(K-8)}} - \frac{1}{9}(K+\frac{1}{2K}) & \text{if } K \in (4+\sqrt{17}, 4+3\sqrt{2}] \\ \frac{1}{18K} + \frac{1}{4} + \frac{K-10}{4\sqrt{K(K-8)}} & \text{if } K > 4+3\sqrt{2}. \end{cases}$$

Additionally, an inexpert i's value of perceptiveness when j is perceptive is

$$VoP_i^{(I,P)} = \begin{cases} 0 & \text{if } K \in [1, 2+\sqrt{5}] \\ \frac{1}{8}(K+\frac{1}{K}) - \frac{1}{2} - \frac{\sqrt{K-4}}{4\sqrt{K}} & \text{if } K \in (2+\sqrt{5}, 4+\sqrt{17}] \\ \frac{1}{2} - \frac{\sqrt{K-4}-\sqrt{K-8}}{4\sqrt{K}} & \text{if } K > 4+\sqrt{17}. \end{cases}$$

Lastly, an inexpert i's value of perceptiveness when j is imperceptive is

$$VoP_i^{(I,M)} = \begin{cases} \frac{4}{441}(K+\frac{1}{K}-2) & \text{if } K \in [1, \frac{17}{10}+\frac{7\sqrt{11}}{10}] \\ \frac{1}{4}\sqrt{\frac{K-4}{K}} + \frac{1}{9}(K+\frac{1}{K}) - \frac{17}{36} & \text{if } K \in (\frac{17}{10}+\frac{7\sqrt{11}}{10}, \frac{25}{6}] \\ \frac{1}{9}(K-2) - \frac{13}{18K} & \text{if } K \in (\frac{25}{6}, 4+\sqrt{17}] \\ \frac{5}{18} - \frac{1}{2\sqrt{K(K-8)}} + \frac{1}{9}K - \frac{13}{18K} & \text{if } K \in (4+\sqrt{17}, 4+3\sqrt{2}] \\ \frac{1}{2}[1 - \frac{5}{3K} + \frac{1}{\sqrt{K(K-8)}}] & \text{if } K > 4+3\sqrt{2}. \end{cases}$$

The main takeaway from Figure 2.7 is that the value of perceptiveness is generally positive in all four situations. As the players' chip endowment increases, i's value of perceptiveness converges to $2/3$ when i is expert and j is perceptive. Otherwise, as the players' chip endowment increases, i's value of perceptiveness converges to $1/2$. Furthermore, for all $K \geq 1$ when i is expert, i's value of perceptiveness is higher when j is perceptive than it is when j is imperceptive. Contrarily, for all $K \geq 1$ when i is inexpert, i's value of perceptiveness is higher when j is imperceptive than it is when j is perceptive.

The intuition for why perceptiveness generally provides positive value in all four situations is similar to why expertise provides positive value for i when j is perceptive. Perceptiveness allows a player i to identify, with certainty, a situation where j is inexpert. When i is perceptive and j is inexpert, as the players' chip endowment increases, i's equilibrium strategy allows i to select all-in with an increasing probability. Whereas, when i is imperceptive or when j is expert, as the players' chip endowment increases, i's equilibrium strategy causes i to select fold with an increasing probability. To summarize this point, as K increases beyond a certain level, when i is imperceptive, i will always fold more often; whereas, when i is perceptive, i will fold more often when j is expert, but go all-in more often when j is inexpert. When i is in an equilibrium that has i going all-in with a relatively high frequency compared to j, i is able to realize value from winning the forced bets uncontested a higher proportion of the time. This is similar to the benefit that an aggressive poker player experiences when facing a passive poker player that tends to fold too often.

2.6 Conclusion

In this chapter, I develop and study a model that features six distinct information structures and emulates a two-player, one-round game of poker. Player i is expert if they know the value of their hand, h_i, prior to deciding whether to go all-in or fold. Player i is perceptive if they know whether their opponent j is expert. The six information structures I consider vary in terms of the players' expertise and perceptiveness.

The main result that I find in this chapter is that when the players' chip endowment is sufficiently high, perceptiveness always provides value. The intuition for this is that perceptiveness allows a player to identify an inexpert opponent. This allows the player to effectively utilize an aggressive strategy to take advantage of their opponent's inexpertise. Whereas, had the player been imperceptive, the player would utilize a more passive equilibrium strategy, taking into consideration that their opponent may actually be expert. The effectively aggressive (all-in with a high probability) strategy against an inexpert opponent allows the player to capture the forced bets, uncontested, a high percentage of the time. Whereas, the passive (fold with a high probability) strategy mitigates the player's risk, but also causes the player to forgo the opportunity of capturing any forced bets.

Chapter 3

Perceptiveness in a Market-Entry Game

3.1 Introduction

This chapter of my thesis studies the value and effect that perceptiveness has in a market-entry setting. A player is expert if they know their market-entry fee prior to making their market-entry decision. Whereas, a player is perceptive if they know whether their opponent is expert. Under an equilibrium refinement that treats the players as symmetrically as possible, I find that the value of perceptiveness is always non-negative. Furthermore, the value of perceptiveness is always zero for an inexpert agent whose opponent is perceptive. Also, when both players have a sufficiently high probability of being expert, the value of perceptiveness is positive; whereas, if either player is inexpert with a sufficiently high probability, the value of perceptiveness is zero. Moreover, even when the value of perceptiveness is zero, perceptiveness can still affect the players' equilibrium actions.

I also find that a player's value of perceptiveness is minimized when their competitor enters with a specific probability regardless of their expertise. As the difference in the competitor's market-entry probability increases, with respect to their level of expertise, the player's value of perceptiveness increases. This is because perceptiveness allows the player to more accurately gauge their competitor's propensity to enter the market, which is what ultimately influences the player's payoff. When the competitor enters the market with a specific probability regardless of their expertise, the value of knowing the competitor's expertise is minimized since the player can already infer the competitor's probability of entering the market.

Studying perceptiveness in a market-entry setting is beneficial since doing so provides theoretical results that show whether perceptiveness has tangible value. My goal for this chapter is to provide general insight towards determining when perceptiveness is beneficial in a market-entry setting and how perceptiveness affects the market-entry strategies of potential entrants.

Perceptiveness is directly linked to the information players have in games, so this chapter is closely related to the value of information literature. This literature is vast, but still growing, with many influential papers[1] along with new developments.[2] Despite this, the notion of studying a player's information about their opponents' information is quite novel. However, it is becoming more prominent in recent years with the development of papers such as Mekonnen and Leal Vizcaíno (2018), Denti (2019), and Tirole (2016).

Mekonnen and Leal Vizcaíno (2018) studies how information quality about an uncertain state affects the induced distribution of an agent's optimal action. They primarily focus on a single agent setting where the agent's action and the payoff-relevant state are complements, but extend their results to supermodular games with incomplete information in order to understand how information quality affects the equilibrium in games with strategic complementarities. They find that as one player's information quality increases, the other player is indirectly more informed about the former player's signal. Mekonnen and Leal Vizcaíno (2018) also investigates a two-player Bayesian game with one-sided information acquisition of a payoff-relevant state. They find that a player's value of acquiring information is always positive when their opponent does not know that they acquired such. However, they also find that a player's value of information acquisition may be negative if their opponent knows such information was acquired. My research departs from Mekonnen and Leal Vizcaíno (2018) since I investigate a market-entry game with strategic substitutes. I also allow for informational symmetry and fo-

[1] Such as Hirshleifer (1971), Milgrom and Stokey (1982), Milgrom and Weber (1982), and Vives (1984).
[2] For instance Myatt and Wallace (2015), Pęski (2008), and Ui and Yoshizawa (2015).

cus my efforts toward determining when perceptiveness provides value and the effect that such information has on the players' equilibrium strategies.

Denti (2019) examines a model of endogenous information acquisition in coordination games where players can choose how much information to acquire, where such information pertains to what the other players know about the value of a common payoff-relevant state. Tirole (2016) considers a framework where players can choose their information structure, and subsequently play a game that contains linear-quadratic payoffs and binary information structures using the information structure chosen by the agent. The key distinction between my research and that of Denti (2019) and Tirole (2016) is that I study the value of information regarding another player's information in a market-entry game. I also focus my attention on how such information influences the outcome of a market-entry game, whereas both of these papers study how much information should be acquired in their respective settings.

This chapter also relates to market-entry literature. To the best of my knowledge, my research is the first to investigate the value and effect that a player's information regarding another player's information about a payoff-relevant state has in a market-entry setting. Market-entry literature includes papers that have used experiments to study how to determine which of the multiple equilibria that exist in market-entry games agents are likely to coordinate upon,[3] as well as papers that have conducted market-entry empirical studies.[4] I contribute to this literature by providing theoretical results which indicate that a player's information regarding another player's information of a payoff-relevant state should be considered when modeling a market-entry game.

The rest of this chapter is comprised as follows. Section 3.2 describes the model and the information structures I consider. Section 3.3 develops the strategies, best responses, equilibria, and expected payoffs used to compute the value and effect of perceptiveness. Section 3.4 derives the value of perceptiveness and discusses key insights. Section 3.5 concludes. Appendix B provides all supplemental appendices for this chapter.

[3]Including Camerer and Lovallo (1999), Duffy and Hopkins (2005), and Erev and Rapoport (1998).

[4]Including Berry and Tamer (2006), and Bresnahan and Reiss (1990, 1991).

3.2 Model

Players, Actions, States

I study a Bayesian game that features two players, A and B, that each produce the same product. Both players must consider whether to *"enter"* ($a_i = E$) or *"not enter"* ($a_i = N$) the market in which this product is sold. Player i's payoff function is[5]

$$u_i(a_i, a_j, \phi_i) = \begin{cases} 0 & \text{if } a_i = N \\ 1 - \phi_i & \text{if } (a_i, a_j) = (E, N) \\ \pi_D - \phi_i & \text{if } (a_i, a_j) = (E, E). \end{cases}$$

I let $\phi_i \sim i.i.d.\ U[0,1]$ and $\pi_D \in [0, 1/2]$, where ϕ_i represents the market entry fee, and π_D represents a duopolist's post-entry profit. In this chapter, I normalize a monopolist's post-entry profit to equal 1.

Microfoundations

I consider a linear inverse demand curve and a situation such that each player can produce with a marginal cost of zero. Furthermore, I assume that the players' products are identically-perceived by consumers. Bertrand competition arises when $\pi_D = 0$, since under Bertrand competition both players would continually undercut the other player's price. Cournot competition arises when $\pi_D = 4/9$.[6]

Types

In this chapter, I consider a continuum of information structures. Similar to the last chapter, I construct the Bayesian Nash equilibria and expected payoffs for each information structure, then use the results to determine the value and effect of perceptiveness. The parameter space

[5] j will always denote i's opponent.

[6] A derivation showing that Cournot competition arises when $\pi_D = 4/9$ is shown in Appendix B.1.

for the information structures I consider is $(\epsilon_A, \epsilon_B) \in [0,1]^2$, where ϵ_i represents the probability of player i knowing the value of ϕ_i. Each (ϵ_A, ϵ_B) ordered pair corresponds to a specific information structure. Furthermore, (ϵ_A, ϵ_B) is common knowledge to both players.

Definition 3.2.1 *Player i is perceptive if player i knows with certainty whether player j knows the value of ϕ_j.*

By Defintion 3.2.1, player i is perceptive if and only if $\epsilon_j \in \{0,1\}$. Otherwise, player i is *imperceptive*. Since (ϵ_A, ϵ_B) is common knowledge, each player's perceptiveness is common knowledge as well.

Definition 3.2.2 *Player i is expert if player i knows ϕ_i prior to making their market-entry decision.*

Definition 3.2.2 classifies player i as expert, in this chapter, if and only if i knows the value of ϕ_i prior to deciding whether to enter the market. If player i does not know ϕ_i prior to making such decision, I classify i as being *inexpert*.

The type space, t_i, for each player i depends on their opponent's perceptiveness. If $\epsilon_i = 1$, player j is perceptive, and i's type space is $[0,1]$ since i is expert. If $\epsilon_i = 0$, player j is perceptive, and i's type space is $\{I_i\}$ since i is inexpert. If $\epsilon_i \in (0,1)$, player j is imperceptive, and i's type space is $[0,1] \cup \{I_i\}$ since i may be expert or inexpert.

Timeline

The timeline for the game in this chapter is as follows. First, each player i observes (ϵ_A, ϵ_B) and learns whether they are expert or inexpert. Second, each player i receives their draw of ϕ_i, which player i observes if i is expert. Third, both players simultaneously decide whether to enter the market. Fourth, payoffs are realized.

3.3 Strategies, Best Responses & Equilibria

Player Strategies

Similar to the last chapter, an expert agent i knows the value of ϕ_i. This should affect an expert i's strategy since i's expected payoff depends on ϕ_i. Since i's expected payoff is decreasing in ϕ_i, I restrict attention to cut-off strategies for an expert agent i. I let χ_i, where $\chi_i \in [0,1]$, represent an expert agent i's cut-off, such that i chooses to enter for all $\phi_i \leq \chi_i$ and chooses to not enter for all $\phi_i > \chi_i$.[7]

An inexpert agent i does not know the value of ϕ_i prior to making their market-entry decision. Thus, an inexpert i's expected payoff does not depend on ϕ_i. I henceforth define η_i, such that $\eta_i \in [0,1]$, as the probability that an inexpert agent i chooses "enter". Thus, an inexpert agent i chooses "not enter" with probability $1-\eta_i$.

Best Responses

I obtain the best response for an expert agent i by finding the value of ϕ_i that makes i indifferent between entering and not entering the market. The payoff i receives from not entering is zero regardless of ϕ_i. Contrarily, the payoff i receives from entering is a probability distribution over j's potential expertise and corresponding strategies, multiplied by the payoff i would receive given j's action. More specifically, an expert i's payoff from entering the market is

$$\begin{aligned} u^E_{i,EX}(\chi_j, \eta_j) &= \epsilon_j[\chi_j(\pi_D - \phi_i) + (1-\chi_j)(1-\phi_i)] + (1-\epsilon_j)[\eta_j(\pi_D - \phi_i) + (1-\eta_j)(1-\phi_i)] \\ &= 1 - \phi_i - (1-\pi_D)(\epsilon_j\chi_j + (1-\epsilon_j)\eta_j). \end{aligned} \tag{3.1}$$

An expert i is indifferent between entering and not entering the market when $\chi_i = \phi_i$. Hence, I obtain an expert i's best response function by equating i's payoff from entering the market to i's payoff from not entering the market, while setting $\chi_i = \phi_i$. As a result, an expert i's best response function is

[7] I restrict attention to equilibria where an expert i chooses enter if they are indifferent between the two actions.

$$\chi_i^{BR}(\chi_j, \eta_j) = 1 - (1-\pi_D)(\epsilon_j\chi_j + (1-\epsilon_j)\eta_j). \tag{3.2}$$

I obtain the best response function for an inexpert agent i by determining when each of i's actions result in a higher expected payoff than i's other action. An inexpert i's expected payoff from not entering the market is zero. Whereas, an inexpert i's expected payoff from entering the market is

$$\begin{aligned} E[u_{i,IX}^E(\chi_j, \eta_j)] &= \epsilon_j[\chi_j(\pi_D - E[\phi_i]) + (1-\chi_j)(1-E[\phi_i])] \\ &\quad + (1-\epsilon_j)[\eta_j(\pi_D - E[\phi_i]) + (1-\eta_j)(1-E[\phi_i])] \\ &= 1 - E[\phi_i] - (1-\pi_D)(\epsilon_j\chi_j + (1-\epsilon_j)\eta_j). \end{aligned} \tag{3.3}$$

I obtain an inexpert i's best response correspondence by comparing i's expected payoff for each action and simplifying.[8] Therefore, an inexpert i's best response correspondence is

$$\eta_i^{BR}(\chi_j, \eta_j) = \begin{cases} \{1\} & \text{if } \epsilon_j\chi_j + (1-\epsilon_j)\eta_j < \frac{1}{2(1-\pi_D)} \\ \{0\} & \text{if } \epsilon_j\chi_j + (1-\epsilon_j)\eta_j > \frac{1}{2(1-\pi_D)} \\ [0,1] & \text{if } \epsilon_j\chi_j + (1-\epsilon_j)\eta_j = \frac{1}{2(1-\pi_D)}. \end{cases} \tag{3.4}$$

The expression $\epsilon_j\chi_j + (1-\epsilon_j)\eta_j$ represents j's aggregate probability of entering the market. Consequently, i will be less (more) inclined to enter the market as j's aggregate probability of entering increases (decreases). As j enters the market with a higher probability, the incentive for i entering lessens since the probability of realizing the duopoly profit, as opposed to the monopoly profit, increases. Furthermore, the best responses illustrate that as a duopolist's post-entry profit increases, both players should be equally or more inclined to enter the market, given their opponent's aggregate probability of entering remains constant.

[8] Since $\phi_i \sim i.i.d.\ U[0,1]$, $E[\phi_i] = 1/2$.

Equilibria

I use the Bayesian Nash solution concept to solve my model. By applying the best responses depicted by Expressions (3.2) and (3.4), I develop Propositions 3.3.1-3.3.6.

Proposition 3.3.1 *Suppose* $(\epsilon_A, \epsilon_B) = (0,0)$. (η_A^*, η_B^*) *is an equilibrium if and only if for all* $i \in \{A,B\}$

$$\eta_i^* = 1 \quad \textit{if } \eta_j^* < \frac{1}{2(1-\pi_D)}, \tag{3.5a}$$

$$\textit{and } \eta_i^* = 0 \quad \textit{if } \eta_j^* > \frac{1}{2(1-\pi_D)}, \tag{3.5b}$$

where $j \in \{A,B\}$ *is such that* $i \neq j$.

Proposition 3.3.2 *Suppose* $(\epsilon_A, \epsilon_B) \in \{(0,1),(1,0)\}$. (η_i^*, χ_j^*) *is an equilibrium if and only if for all* $i \in \{A,B\}$

$$\chi_j^* = 1 - (1-\pi_D)\eta_i^*, \tag{3.6}$$

$$\eta_i^* = 1 \quad \textit{if } \chi_j^* < \frac{1}{2(1-\pi_D)}, \tag{3.7a}$$

$$\textit{and } \eta_i^* = 0 \quad \textit{if } \chi_j^* > \frac{1}{2(1-\pi_D)}, \tag{3.7b}$$

where $j \in \{A,B\}$ *is such that* $i \neq j$.

Proposition 3.3.3 *Suppose* $(\epsilon_A, \epsilon_B) = (1,1)$. (χ_A^*, χ_B^*) *is an equilibrium if and only if for all* $i \in \{A,B\}$

$$\chi_i^* = 1 - (1-\pi_D)\chi_j^*, \tag{3.8}$$

where $j \in \{A,B\}$ *is such that* $i \neq j$.

Proposition 3.3.4 *Suppose $\epsilon_i = 0$ and $\epsilon_j \in (0,1)$, for all $i \in \{A,B\}$ where $j \in \{A,B\}$ is such that $i \neq j$. $(\eta_i^*, \chi_j^*, \eta_j^*)$ is an equilibrium if and only if for all $i \in \{A,B\}$*

$$\eta_i^* = 1 \quad \text{if } \epsilon_j\chi_j^* + (1-\epsilon_j)\eta_j^* < \tfrac{1}{2(1-\pi_D)}, \tag{3.9a}$$

$$\eta_i^* = 0 \quad \text{if } \epsilon_j\chi_j^* + (1-\epsilon_j)\eta_j^* > \tfrac{1}{2(1-\pi_D)}, \tag{3.9b}$$

$$\chi_j^* = 1 - (1-\pi_D)\eta_i^*, \tag{3.10}$$

$$\eta_j^* = 1 \quad \text{if } \eta_i^* < \tfrac{1}{2(1-\pi_D)}, \tag{3.11a}$$

$$\text{and } \eta_j^* = 0 \quad \text{if } \eta_i^* > \tfrac{1}{2(1-\pi_D)}, \tag{3.11b}$$

where $j \in \{A,B\}$ is such that $i \neq j$.

Proposition 3.3.5 *Suppose $\epsilon_i = 1$ and $\epsilon_j \in (0,1)$, for all $i \in \{A,B\}$ where $j \in \{A,B\}$ is such that $i \neq j$. $(\chi_i^*, \chi_j^*, \eta_j^*)$ is an equilibrium if and only if for all $i \in \{A,B\}$*

$$\chi_i^* = 1 - (1-\pi_D)(\epsilon_j\chi_j^* + (1-\epsilon_j)\eta_j^*), \tag{3.12}$$

$$\chi_j^* = 1 - (1-\pi_D)\chi_i^*, \tag{3.13}$$

$$\eta_j^* = 1 \quad \text{if } \chi_i^* < \tfrac{1}{2(1-\pi_D)}, \tag{3.14a}$$

$$\text{and } \eta_j^* = 0 \quad \text{if } \chi_i^* > \tfrac{1}{2(1-\pi_D)}, \tag{3.14b}$$

where $j \in \{A,B\}$ is such that $i \neq j$.

Proposition 3.3.6 *Suppose $(\epsilon_A, \epsilon_B) \in (0,1)^2$. $(\chi_A^*, \eta_A^*, \chi_B^*, \eta_B^*)$ is an equilibrium if and only if for all $i \in \{A,B\}$*

$$\chi_i^* = 1 - (1-\pi_D)(\epsilon_j\chi_j^* + (1-\epsilon_j)\eta_j^*), \tag{3.15}$$

$$\eta_i^* = 1 \quad \text{if } \epsilon_j\chi_j^* + (1-\epsilon_j)\eta_j^* < \tfrac{1}{2(1-\pi_D)}, \tag{3.16a}$$

$$\text{and } \eta_i^* = 0 \quad \text{if } \epsilon_j\chi_j^* + (1-\epsilon_j)\eta_j^* > \tfrac{1}{2(1-\pi_D)}, \tag{3.16b}$$

where $j \in \{A,B\}$ is such that $i \neq j$.

The proof of Propositions 3.3.1-3.3.6 follows from the definition of a Bayesian Nash equilibrium and the best response derivations I completed earlier in Section 3.3. I find all Bayesian Nash equilibria by using Propositions 3.3.1-3.3.6 and solving the resulting system of equations.

Theorem 3.3.7 *Suppose* $(\epsilon_A, \epsilon_B) = (0,0)$. $(\eta_A^*, \eta_B^*) \in \{(\frac{1}{2(1-\pi_D)}, \frac{1}{2(1-\pi_D)}), (0,1), (1,0)\}$ *is an equilibrium. Furthermore, if* $\pi_D = 1/2$, *then* $(\eta_i^*, \eta_j^*) = (1, \bar{\eta}_j)$ *such that* $\bar{\eta}_j \in [0,1)$ *is an equilibrium as well, for all* $i \in \{A,B\}$ *where* $j \in \{A,B\}$ *is such that* $i \neq j$.

Proof Theorem 3.3.7 follows directly from Proposition 3.3.1.

Theorem 3.3.8 *Suppose* $(\epsilon_A, \epsilon_B) \in \{(0,1),(1,0)\}$. $(\eta_i^*, \chi_j^*) \in \{(\frac{1-2\pi_D}{2(1-\pi_D)^2}, \frac{1}{2(1-\pi_D)}), (0,1), (1, \pi_D)\}$ *is an equilibrium for all* $i \in \{A,B\}$ *where* $j \in \{A,B\}$ *is such that* $i \neq j$.

Proof Theorem 3.3.8 follows directly from Proposition 3.3.2.

Theorem 3.3.9 *Suppose* $(\epsilon_A, \epsilon_B) = (1,1)$. $(\chi_A^*, \chi_B^*) = (\frac{1}{2-\pi_D}, \frac{1}{2-\pi_D})$ *is an equilibrium.*

Proof Theorem 3.3.9 follows directly from Proposition 3.3.3.

Theorem 3.3.10 *Suppose* $\epsilon_i = 0$ *and* $\epsilon_j \in (0,1)$, *for all* $i \in \{A,B\}$ *where* $j \in \{A,B\}$ *is such that* $i \neq j$. $(\eta_i^*, \chi_j^*, \eta_j^*) \in \{(0,1,1), (1, \pi_D, 0)\}$ *is an equilibrium. Furthermore, if* $\epsilon_B > 1 - \frac{\pi_D}{1-\pi_D}$, *then* $(\eta_i^*, \chi_j^*, \eta_j^*) = (\frac{1-2\pi_D}{2\epsilon_B(1-\pi_D)^2}, 1 - \frac{1-2\pi_D}{2\epsilon_B(1-\pi_D)}, 1)$ *is an equilibrium. Additionally, if* $\epsilon_B \leq 1 - \frac{\pi_D}{1-\pi_D}$, *then* $(\eta_i^*, \chi_j^*, \eta_j^*) = (\frac{1}{2(1-\pi_D)}, 1/2, \frac{1}{2} + \frac{\pi_D}{2(1-\epsilon_B)(1-\pi_D)})$ *is an equilibrium. Lastly, if* $\pi_D = 1/2$, *then* $(\eta_i^*, \chi_j^*, \eta_j^*) = (1, \pi_D, \bar{\eta}_j)$ *such that* $\bar{\eta}_j \in [0,1]$ *is an equilibrium as well.*

Proof Theorem 3.3.10 follows directly from Proposition 3.3.4.

Theorem 3.3.11 *Suppose* $\epsilon_i = 1$ *and* $\epsilon_j \in (0,1)$, *for all* $i \in \{A,B\}$ *where* $j \in \{A,B\}$ *is such that* $i \neq j$. $(\chi_i^*, \chi_j^*, \eta_j^*) = (\frac{\pi_D}{1-\epsilon_j(1-\pi_D)^2}, 1 - \frac{\pi_D(1-\pi_D)}{1-\epsilon_j(1-\pi_D)^2}, 1)$ *is an equilibrium. Additionally, if* $\epsilon_j \leq 1 - (\frac{\pi_D}{1-\pi_D})^2$, *then* $(\chi_i^*, \chi_j^*, \eta_j^*) \in \{(\frac{1-(1-\pi_D)\epsilon_j}{1-\epsilon_j(1-\pi_D)^2}, \frac{\pi_D}{1-\epsilon_j(1-\pi_D)^2}, 0), (\frac{1}{2(1-\pi_D)}, \frac{1}{2}, \frac{1}{2} - \frac{\pi_D^2}{2(1-\epsilon_j)(1-\pi_D)^2})\}$ *is an equilibrium as well.*

Proof Theorem 3.3.11 follows directly from Proposition 3.3.5.

Theorem 3.3.12 *Suppose* $(\epsilon_A, \epsilon_B) \in (0,1)^2$.

$$(\chi_A^*, \eta_A^*, \chi_B^*, \eta_B^*) = (1-\frac{(1-\pi_D)(1-\epsilon_B(1-\pi_D))}{1-\epsilon_A\epsilon_B(1-\pi_D)^2}, 1, 1-\frac{(1-\pi_D)(1-\epsilon_A(1-\pi_D))}{1-\epsilon_A\epsilon_B(1-\pi_D)^2}, 1)$$

is an equilibrium if and only if $1-(\frac{\pi_D}{1-\pi_D})^2 \leq (2-\epsilon_i)\epsilon_j$, *for all* $i \in \{A,B\}$ *where* $j \in \{A,B\}$ *is such that* $i \neq j$. *Furthermore,*

$$(\chi_A^*, \eta_A^*, \chi_B^*, \eta_B^*) \in \{(1-\frac{\epsilon_B\pi_D(1-\pi_D)}{1-\epsilon_A\epsilon_B(1-\pi_D)^2}, 1, \frac{\pi_D}{1-\epsilon_A\epsilon_B(1-\pi_D)^2}, 0), (\frac{\pi_D}{1-\epsilon_A\epsilon_B(1-\pi_D)^2}, 0, 1-\frac{\epsilon_A\pi_D(1-\pi_D)}{1-\epsilon_A\epsilon_B(1-\pi_D)^2}, 1)\}$$

is an equilibrium if and only if $\epsilon_A\epsilon_B \leq 1-(\frac{\pi_D}{1-\pi_D})^2$. *Additionally,*

$$(\chi_i^*, \eta_i^*, \chi_j^*, \eta_j^*) = (1-\frac{1-2\pi_D}{2\epsilon_i(1-\pi_D)}, 1, 1/2, \frac{1-2\pi_D-\epsilon_i\epsilon_j(1-\pi_D)^2}{2\epsilon_i(1-\epsilon_j)(1-\pi_D)^2})$$

is an equilibrium, for all $i \in \{A,B\}$ *where* $j \in \{A,B\}$ *is such that* $i \neq j$, *if and only if*

$$\epsilon_i\epsilon_j \leq 1-(\frac{\pi_D}{1-\pi_D})^2,$$

$$\epsilon_i \geq 1-\frac{\pi_D}{1-\pi_D}$$

$$\text{and } \epsilon_i(2-\epsilon_j) \geq 1-(\frac{\pi_D}{1-\pi_D})^2.$$

Lastly,

$$(\chi_A^*, \eta_A^*, \chi_B^*, \eta_B^*) = (1/2, \frac{1}{2}+\frac{\pi_D}{2(1-\pi_D)(1-\epsilon_A)}, 1/2, \frac{1}{2}+\frac{\pi_D}{2(1-\pi_D)(1-\epsilon_B)})$$

is an equilibrium if and only if $\epsilon_A \leq 1-\frac{\pi_D}{1-\pi_D}$ *and* $\epsilon_B \leq 1-\frac{\pi_D}{1-\pi_D}$.

Proof Theorem 3.3.12 follows directly from Proposition 3.3.6.

Figures 3.1-3.5 depict the equilibrium existence regions when $\pi_D = 4/9$ and $(\epsilon_A, \epsilon_B) \in (0,1)^2$. Corollary 3.3.13 formalizes equilibrium existence for all $(\epsilon_A, \epsilon_B) \in [0,1]^2$.

Corollary 3.3.13 *There exists an equilibrium for all* $(\epsilon_A, \epsilon_B) \in [0,1]^2$.

Proof Corollary 3.3.13 follows directly from Theorems 3.3.7-3.3.12.

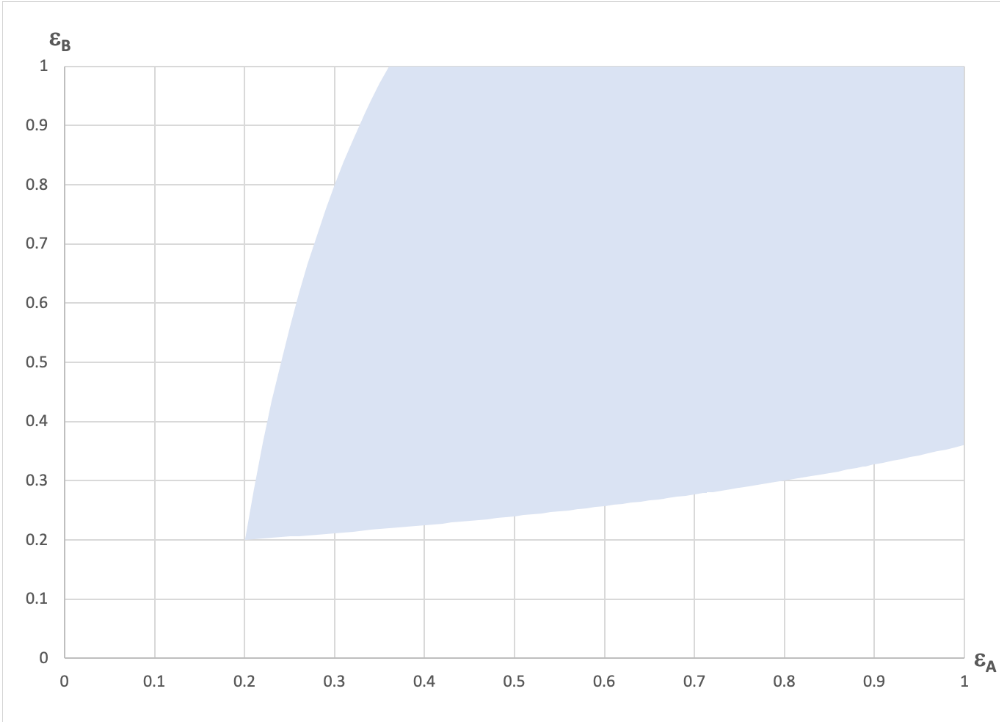


Figure 3.1: Equilibrium existence when both inexpert agents always enter.

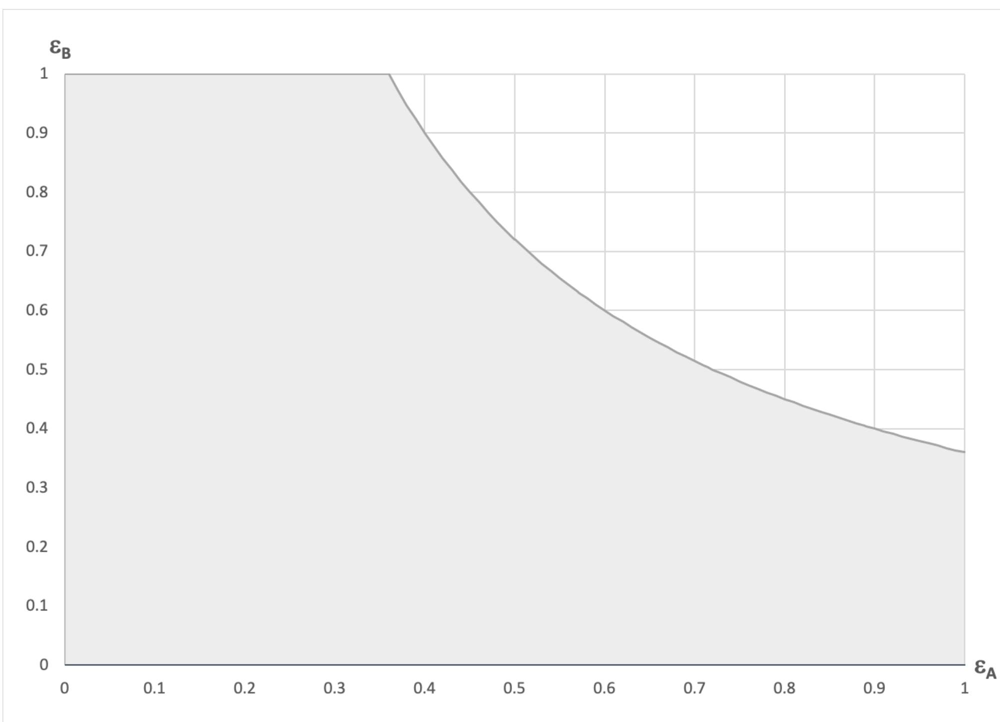


Figure 3.2: Equilibrium existence when one inexpert agent always enters, whereas the other inexpert agent never enters.

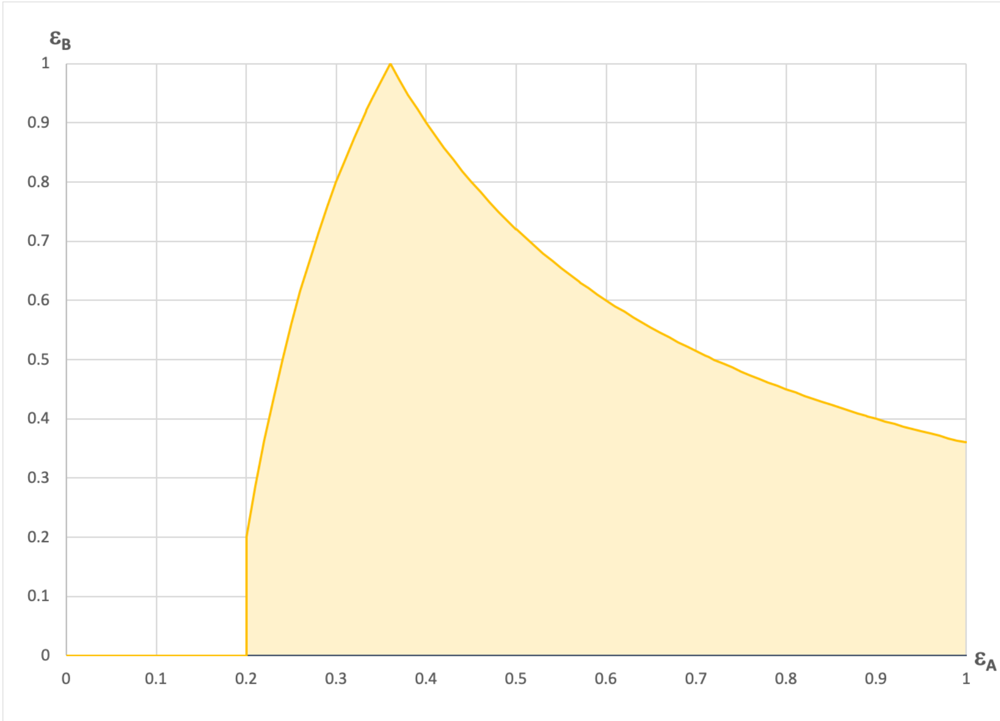


Figure 3.3: Equilibrium existence when inexpert agent *A* always enters, whereas inexpert agent *B* mixes between entering and not entering.

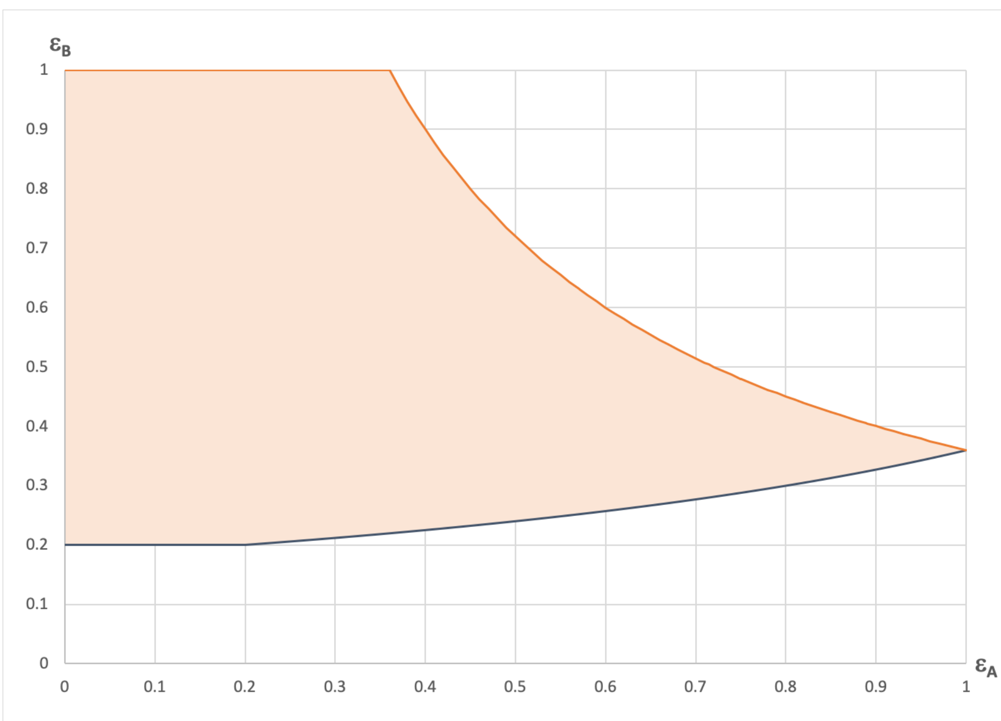


Figure 3.4: Equilibrium existence when inexpert agent *B* always enters, whereas inexpert agent *A* mixes between entering and not entering.

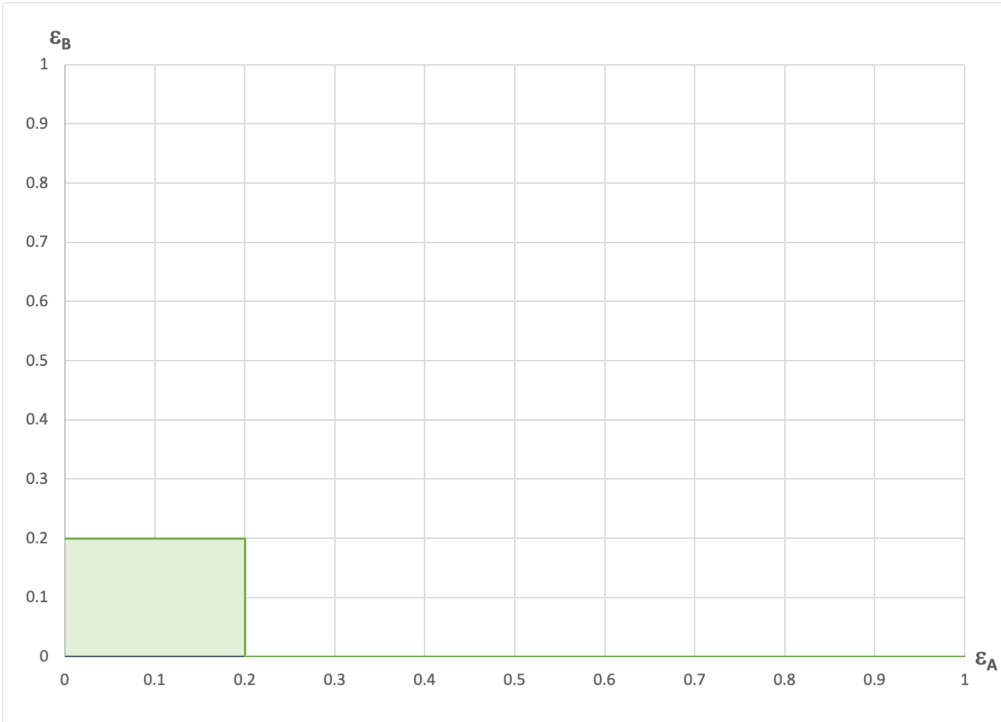


Figure 3.5: Equilibrium existence when both inexpert agents mix between entering and not entering.

Equilibrium Refinement

As illustrated by Figures 3.1-3.5 and Theorems 3.3.7-3.3.12, multiple equilibria exist for many (ϵ_A, ϵ_B) information structures. Because of this, I refine the equilibria when multiple exist. The equilibrium selection rule I follow can be thought of as the "middle equilibrium". For all $(\epsilon_A, \epsilon_B) \in [0,1]^2$, the equilibria that exist can be ranked in terms of each player's aggregate entry probability. As shown by expressions (3.2) and (3.4), i's best response is decreasing in j's aggregate entry probability. Hence, i's rank of equilibria in terms of aggregate entry probability will be the reverse of j's rank.

The "middle equilibrium" selection rule selects a symmetric equilibrium when $\epsilon_A = \epsilon_B$, whereas it treats the players as symmetrically as possible, in terms of aggregate entry probability, when $\epsilon_A \neq \epsilon_B$. For this rule to work nicely, there must be an odd number of equilibria that exist for any (ϵ_A, ϵ_B) coordinate. This property holds generically for my model.

Based on the middle equilibrium selection rule and the aggregate entry probability rankings, the refined equilibrium can be segmented into four regions on the ϵ_A-ϵ_B plane. The four regions are as follows. First, if $\epsilon_A(2-\epsilon_B) \geq 1-(\frac{\pi_D}{1-\pi_D})^2$ and $\epsilon_B(2-\epsilon_A) \geq 1-(\frac{\pi_D}{1-\pi_D})^2$, then an equilibrium where any inexpert agent always enters arises. Second, if $\epsilon_A \leq 1-\frac{\pi_D}{1-\pi_D}$ and $\epsilon_B \leq 1-\frac{\pi_D}{1-\pi_D}$, then an equilibrium where any inexpert agent mixes between entering and not entering the market arises. Third, if $\epsilon_A \geq 1-\frac{\pi_D}{1-\pi_D}$ and $\epsilon_B(2-\epsilon_A) < 1-(\frac{\pi_D}{1-\pi_D})^2$, then an equilibrium where any inexpert agent for A always enters, whereas any inexpert agent for B mixes between entering and not entering the market arises. Fourth, if $\epsilon_B \geq 1-\frac{\pi_D}{1-\pi_D}$ and $\epsilon_A(2-\epsilon_B) < 1-(\frac{\pi_D}{1-\pi_D})^2$, then an equilibrium where any inexpert agent for B always enters, whereas any inexpert agent for A mixes between entering and not entering the market arises. Figure 3.6 gives an image of the refined equilibrium regions and how they adjust for different values of π_D.

Ex-Ante Expected Payoffs

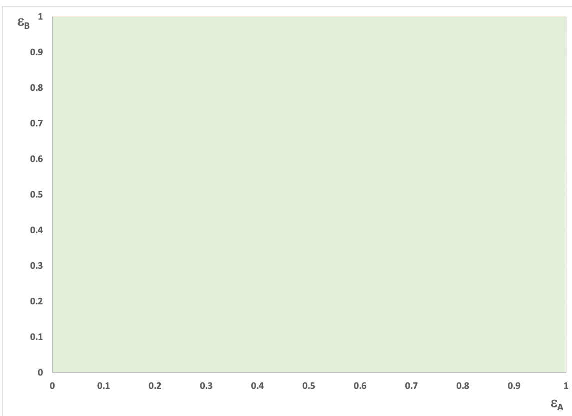
In order to determine a player's value of perceptiveness, I must derive a player's ex-ante expected payoff given such player's expertise. I focus attention towards expectations given expertise since a player's market-entry decision is made after the player has observed their expertise.

I obtain i's ex-ante expected payoff functions by integrating over all of i's possible draws of ϕ_i with respect to i's optimal action for each specific draw. An expert agent i's ex-ante expected payoff is

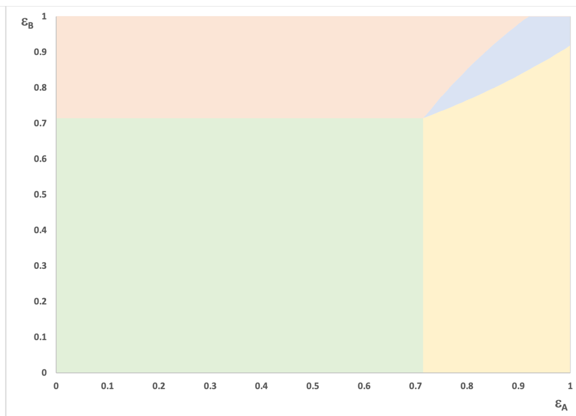
$$\begin{aligned} EU_i^{EX}(\chi_j,\eta_j) &= \int_0^{\chi_i} u_{i,EX}^{E}(\chi_j,\eta_j)d\phi_i + \int_{\chi_i}^1 u_{i,EX}^{N}(\chi_j,\eta_j)d\phi_i \\ &= \int_0^{\chi_i} [1-\phi_i-(1-\pi_D)(\epsilon_j\chi_j+(1-\epsilon_j)\eta_j)]d\phi_i + \int_{\chi_i}^1 [0]d\phi_i \\ &= \chi_i(1-(1-\pi_D)(\epsilon_j\chi_j+(1-\epsilon_j)\eta_j)-\tfrac{1}{2}\chi_i) \\ &= \chi_i(\chi_i^{BR}(\chi_j,\eta_j)-\tfrac{1}{2}\chi_i). \end{aligned} \tag{3.17}$$

If an expert agent i plays a best response strategy, then Equation (3.17) can be rewritten as

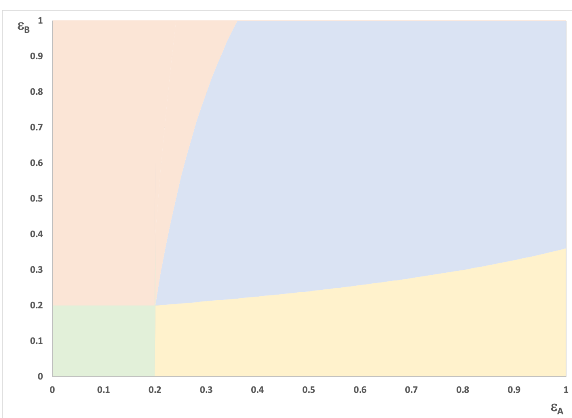
$$EU_i^{EX}(\chi_j,\eta_j) = \tfrac{1}{2}(\chi_i^{BR}(\chi_j,\eta_j))^2. \tag{3.18}$$



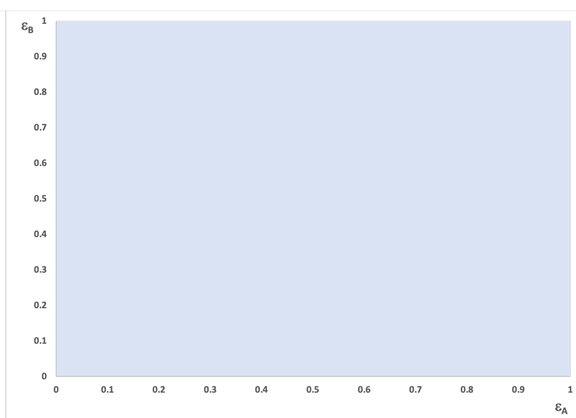
(a) Refined equilibrium when $\pi_D = 0$.



(b) Refined equilibrium when $\pi_D = 2/9$.



(c) Refined equilibrium when $\pi_D = 4/9$.



(d) Refined equilibrium when $\pi_D = 1/2$.

Figure 3.6: Refined equilibrium regions for various levels of π_D.

Moreover, an inexpert agent i's ex-ante expected payoff is

$$\begin{aligned} EU_i^{IX}(\chi_j,\eta_j) &= \eta_i \int_0^1 E[u_{i,IX}^E(\chi_j,\eta_j)]d\phi_i + (1-\eta_i)\int_0^1 u_{i,IX}^N(\chi_j,\eta_j)d\phi_i \\ &= \eta_i \int_0^1 [1-E[\phi_i]-(1-\pi_D)(\epsilon_j\chi_j+(1-\epsilon_j)\eta_j)]d\phi_i + (1-\eta_i)\int_0^1 [0]d\phi_i \\ &= \eta_i(\tfrac{1}{2}-(1-\pi_D)(\epsilon_j\chi_j+(1-\epsilon_j)\eta_j)). \end{aligned} \tag{3.19}$$

Equation (3.18) shows that an expert agent's ex-ante expected payoff strictly increases in their best response cutoff. This implies that any equilibrium where an expert agent i enters the market with a higher cutoff than in some other equilibrium, will have a higher ex-ante expected payoff for i than the other equilibrium had.

Equations (3.2), (3.18), and (3.19) together show that regardless of i's expertise, i's ex-ante expected payoff is weakly decreasing in j's aggregate entry probability. This implies that any player, regardless of their expertise, that is using a best response, is weakly better off by their competitor entering the market less frequently. An interesting insight that arises from this is that if a player could perform an action that somehow deters (or at least makes deterrence more likely) their opponent from entering the market, the player would strictly benefit from doing so if they enter the market with positive probability. Therefore, there is tangible value in deterring a competitor's entry into the market.

3.4 Value of Perceptiveness

Identifying the value of perceptiveness using the ex-ante expected payoffs when i is perceptive ($\epsilon_j \in \{0,1\}$) and when i is most imperceptive ($\epsilon_j = 1/2$) is problematic in this chapter since multiple equilibria exist and the existence regions vary across the ϵ_i-ϵ_j plane. Moreover, these existence regions change as the duopoly profit changes. Also, there is no combination of equilibria that connects the existence regions across the ϵ_i-ϵ_j plane uniformly.[9] Therefore, in

[9] To clarify, by saying that "there is no combination of equilibria that connects. . . uniformly", I mean that across the entire ϵ_i-ϵ_j plane, there is at least one equilibrium region boundary that will not be continuous, in terms of the

order to obtain an accurate gauge of the value of perceptiveness, I must restrict my focus to analyzing its value within specific equilibrium regions.

I determine i's value of perceptiveness by taking the difference between i's expected payoff with information pertaining to j's expertise and i's expected payoff without such information. Hence, the value of perceptiveness for a player i with expertise x, such that $x \in \{EX, IX\}$, is given by

$$VoP_i^x(\epsilon_i, \epsilon_j, \mu) = \tfrac{1}{2}[EU_i^x(\epsilon_i, \epsilon_j{+}\mu) + EU_i^x(\epsilon_i, \epsilon_j{-}\mu)] - EU_i^x(\epsilon_i, \epsilon_j), \tag{3.20}$$

where μ is some arbitrary positive number such that $[\epsilon_j{-}\mu, \epsilon_j{+}\mu] \subset [LB_{region}, UB_{region}]$.[10]

Plotting ϵ_j on the x-axis and $EU_i^x(\epsilon_i, \epsilon_j)$ on the y-axis, $VoP_i^x(\epsilon_i, \epsilon_j, \mu)$ can be interpreted as the vertical distance between the weighted average of i's ex-ante expected payoff using $\bar{\epsilon}_j = \epsilon_j + \mu$ and $\underline{\epsilon}_j = \epsilon_j - \mu$, and i's ex-ante expected payoff using ϵ_j. The weighted average of i's ex-ante expected payoff, using $\underline{\epsilon}_j$ and $\bar{\epsilon}_j$, provides a notion to compare mean-preserving spreads of the same player. Computing the vertical distance identifies how much i values being at a weighted-average of two different ϵ_j points, at least one of which results in i being more perceptive,[11] as opposed to a middling ϵ_j point. Switching from ϵ_j with certainty to a weighted-average of $\underline{\epsilon}_j$ and $\bar{\epsilon}_j$ could hypothetically occur if i were granted some additional information regarding j's expertise. Conditional on the weighted-average of $\underline{\epsilon}_j$ and $\bar{\epsilon}_j$ being a mean-preserving spread of ϵ_j, the vertical distance between the resulting ex-ante expected payoffs captures i's value of perceptiveness. In Appendix B.2, I include an example that illustrates the intuition behind how I measure the value of perceptiveness, as well as why this method is credible.

By taking the second derivative of i's ex-ante expected payoffs with respect to ϵ_j, I am able to determine whether the value of perceptiveness is positive, zero, or negative. If the second derivative of i's ex-ante expected payoff with respect to ϵ_j is strictly greater (less) than zero, i's

players' equilibrium strategies, as either ϵ_i or ϵ_j changes.

[10] LB_{region} and UB_{region} represent the lower and upper ϵ_j bounds for the specific equilibrium existence region being considered.

[11] As shown by Reza (1994), uncertainty is maximized when all potential outcomes occur with equal probability. Hence, the most imperceptive player i can be occurs when $\epsilon_j = 1/2$. Consequently, i becomes more perceptive as $|\epsilon_j - \frac{1}{2}|$ becomes larger. From here it is easy to show that for all valid ϵ_j and μ combinations, at least one of $\underline{\epsilon}_j$ and $\bar{\epsilon}_j$ results in i being more perceptive than i is with ϵ_j.

value of perceptiveness is positive (negative). If the second derivative of i's ex-ante expected payoff with respect to ϵ_j equals zero, i's value of perceptiveness is zero. This follows from the properties of second derivatives, convexity, and concavity.

To determine the magnitude of i's value of perceptiveness, different values of μ can be applied within Equation (3.20). This allows me to determine the resulting vertical distance, which captures i's value of perceptiveness, for the specific value of μ that I consider. Whether searching for the sign or magnitude of i's value of perceptiveness, it is imperative that I use mean-preserving spreads of ϵ_j in equation (3.20). Doing so allows for a direct comparison to be made between two different levels of perceptiveness for player i.

3.4.1 Application to Market-Entry Setting

Being perceptive for i can be thought of as having information pertaining to j's expertise. As such, the steps for determining the value of perceptiveness are similar to those taken to determine the value of information in the oil investment decision problem outlined in Appendix B.2, where p is now the ex-ante probability of j being expert (ϵ_j) and the expected profit is now i's ex-ante expected payoff ($EU_i^x(\epsilon_i, \epsilon_j)$).

Without loss of generality, in this section I restrict attention to the $(\epsilon_i, \epsilon_j) \in (0,1)^2$ information structures. By substituting the refined "middle equilibria" listed in Theorem 3.3.12 into the ex-ante expected payoff equations, (3.18) and (3.19), I determine the ex-ante expected payoffs for the refined equilibria. If $\epsilon_A(2-\epsilon_B) \geq 1-(\frac{\pi_D}{1-\pi_D})^2$ and $\epsilon_B(2-\epsilon_A) \geq 1-(\frac{\pi_D}{1-\pi_D})^2$,

$$EU_i^{EX}(\epsilon_i,\epsilon_j) = \tfrac{1}{2}(1-\tfrac{(1-\pi_D)(1-\epsilon_j(1-\pi_D))}{1-\epsilon_i\epsilon_j(1-\pi_D)^2})^2$$
$$\text{and } EU_i^{IX}(\epsilon_i,\epsilon_j) = \tfrac{\epsilon_j(2-\epsilon_i)(1-\pi_D)^2-(1-2\pi_D)}{2(1-\epsilon_i\epsilon_j(1-\pi_D)^2)}.$$

If $\epsilon_i \leq 1-\frac{\pi_D}{1-\pi_D}$ for $i \in \{A,B\}$,

$$EU_i^{EX}(\epsilon_i,\epsilon_j) = 1/8$$
$$\text{and } EU_i^{IX}(\epsilon_i,\epsilon_j) = 0.$$

If $\epsilon_i \geq 1-\frac{\pi_D}{1-\pi_D}$ and $\epsilon_j(2-\epsilon_i) < 1-(\frac{\pi_D}{1-\pi_D})^2$ for $i \in \{A,B\}$,

$$EU_i^{EX}(\epsilon_i,\epsilon_j) = \tfrac{1}{2}(1-\tfrac{1-2\pi_D}{2\epsilon_i(1-\pi_D)})^2,$$

$$EU_i^{IX}(\epsilon_i,\epsilon_j) = \tfrac{1}{2}-\tfrac{1-2\pi_D}{2\epsilon_i(1-\pi_D)},$$

$$EU_j^{EX}(\epsilon_i,\epsilon_j) = 1/8,$$

$$\text{and } EU_j^{IX}(\epsilon_i,\epsilon_j) = 0.$$

When either $\epsilon_A(2-\epsilon_B) < 1-(\frac{\pi_D}{1-\pi_D})^2$ or $\epsilon_B(2-\epsilon_A) < 1-(\frac{\pi_D}{1-\pi_D})^2$, perceptiveness has zero value for player i regardless of i's expertise since $EU_i^x(\epsilon_i,\epsilon_j)$ is linear with respect to ϵ_j for $x \in \{EX,IX\}$ and $i \in \{A,B\}$. However, if $\epsilon_A(2-\epsilon_B) \geq 1-(\frac{\pi_D}{1-\pi_D})^2$, $\epsilon_B(2-\epsilon_A) \geq 1-(\frac{\pi_D}{1-\pi_D})^2$, and i is expert,

$$\frac{dEU_i^{EX}(\epsilon_i,\epsilon_j)}{d\epsilon_j} = \frac{(1-\epsilon_i(1-\pi_D))(\pi_D+\epsilon_j(1-\epsilon_i)(1-\pi_D)^2)(1-\pi_D)^2}{(1-\epsilon_i\epsilon_j(1-\pi_D)^2)^3}$$

$$\text{and } \frac{d^2EU_i^{EX}(\epsilon_i,\epsilon_j)}{d\epsilon_j{}^2} = \frac{(1-\epsilon_i(1-\pi_D))(1-\epsilon_i+\epsilon_i(3\pi_D+2\epsilon_j(1-\epsilon_i)(1-\pi_D)^2))(1-\pi_D)^4}{(1-\epsilon_i\epsilon_j(1-\pi_D)^2)^4}; \quad (3.21)$$

whereas, if $\epsilon_A(2-\epsilon_B) \geq 1-(\frac{\pi_D}{1-\pi_D})^2$, $\epsilon_B(2-\epsilon_A) \geq 1-(\frac{\pi_D}{1-\pi_D})^2$, and i is inexpert,

$$\frac{dEU_i^{IX}(\epsilon_i,\epsilon_j)}{d\epsilon_j} = \frac{(1-\epsilon_i(1-\pi_D))(1-\pi_D)^2}{(1-\epsilon_i\epsilon_j(1-\pi_D)^2)^2}$$

$$\text{and } \frac{d^2EU_i^{IX}(\epsilon_i,\epsilon_j)}{d\epsilon_j{}^2} = \frac{2\epsilon_i(1-\epsilon_i(1-\pi_D))(1-\pi_D)^4}{(1-\epsilon_i\epsilon_j(1-\pi_D)^2)^3}. \quad (3.22)$$

Suppose $\epsilon_A(2-\epsilon_B) \geq 1-(\frac{\pi_D}{1-\pi_D})^2$ and $\epsilon_B(2-\epsilon_A) \geq 1-(\frac{\pi_D}{1-\pi_D})^2$. Equations (3.21) and (3.22) depict the convexity of i's ex-ante expected payoff functions, with respect to ϵ_j, for expert and inexpert agents of i respectively. These equations show that perceptiveness has positive value for i given that $\epsilon_A(2-\epsilon_B) \geq 1-(\frac{\pi_D}{1-\pi_D})^2$ and $\epsilon_B(2-\epsilon_A) \geq 1-(\frac{\pi_D}{1-\pi_D})^2$, since $\frac{d^2EU_i^{EX}(\epsilon_i,\epsilon_j,\pi_D)}{d\epsilon_j{}^2} > 0$ and $\frac{d^2EU_i^{IX}(\epsilon_i,\epsilon_j,\pi_D)}{d\epsilon_j{}^2} > 0$. This implies that both ex-ante expected payoff functions are convex in ϵ_j. Theorems 3.4.1 and 3.4.2, as well as Corollary 3.4.3, formalize these results.

Theorem 3.4.1 *Suppose i is expert. Under the "middle equilibrium" refinement, i's value of perceptiveness is positive if and only if*

$$i)\ \epsilon_i(2-\epsilon_j) \geq 1-(\tfrac{\pi_D}{1-\pi_D})^2,$$

$$ii)\ \epsilon_j(2-\epsilon_i) \geq 1-(\tfrac{\pi_D}{1-\pi_D})^2,$$

$$\text{and } iii)\ (\pi_D, \epsilon_i) \neq (0,1).$$

Otherwise, i's value of perceptiveness is zero.

Theorem 3.4.2 *Suppose i is inexpert. Under the "middle equilibrium" refinement, i's value of perceptiveness is positive if and only if*

$$i)\ \epsilon_i(2-\epsilon_j) \geq 1-(\tfrac{\pi_D}{1-\pi_D})^2,$$

$$ii)\ \epsilon_j(2-\epsilon_i) \geq 1-(\tfrac{\pi_D}{1-\pi_D})^2,$$

$$\text{and } iii)\ \epsilon_i \neq 0.$$

Otherwise, i's value of perceptiveness is zero.

Corollary 3.4.3 *Under the "middle equilibrium" refinement, i's value of perceptiveness is never negative.*

The proof for Theorems 3.4.1 and 3.4.2 follow from the preceding discussion combined with determining when the second derivatives, depicted in Equations (3.21) and (3.22), equal zero.[12] Corollary 3.4.3 can be established by showing that there is no (ϵ_A, ϵ_B) ordered pair that results in i's ex-ante expected payoff being strictly concave in ϵ_j under the "middle equilibrium" refinement. As it turns out, the non-negativity property listed in Corollary 3.4.3 does not hold in general. The value of perceptiveness for i can actually be negative if a change in i's perceptiveness causes the equilibrium to shift from an equilibrium where i enters frequently to one where i enters rarely.[13] However, under the "middle equilibrium" selection rule, i's value of perceptiveness is either positive or zero.

[12]Thereby indicating when, in the corresponding equilibrium region, the value of perceptiveness is zero.

[13]Recall that i's ex-ante expected payoff is weakly increasing in their probability of entering the market. Hence, if an increase in i's perceptiveness causes the equilibrium to shift in a way that i's market-entry frequency sufficiently decreases, i's value of perceptiveness will be negative.

Theorems 3.4.1 and 3.4.2 establish whether i's value of perceptiveness is positive, zero, or negative within each equilibrium region. Based on these theorems, given the "middle equilibrium" refinement, the only equilibrium region where the value of perceptiveness may differ from zero is the region where $\epsilon_A(2-\epsilon_B) \geq 1-(\frac{\pi_D}{1-\pi_D})^2$ and $\epsilon_B(2-\epsilon_A) \geq 1-(\frac{\pi_D}{1-\pi_D})^2$.

Theorem 3.4.4 *Suppose* $\epsilon_i(2-\epsilon_j) \geq 1-(\frac{\pi_D}{1-\pi_D})^2$ *and* $\epsilon_j(2-\epsilon_i) \geq 1-(\frac{\pi_D}{1-\pi_D})^2$*. An expert i's value of perceptiveness is*

$$VoP_i^{EX}(\epsilon_i, \epsilon_j, \mu) = \frac{(fk+gh)^2 + (fh+gk)^2}{2(k^2-h^2)^2} - \frac{f^2}{2k^2}; \tag{3.23}$$

whereas, an inexpert i's value of perceptiveness is

$$VoP_i^{IX}(\epsilon_i, \epsilon_j, \mu) = \frac{lh}{k(k^2-h^2)}, \tag{3.24}$$

such that

$$k = 1 - \epsilon_i\epsilon_j(1-\pi_D)^2,$$

$$f = \pi_D + (1-\epsilon_i)\epsilon_j(1-\pi_D)^2,$$

$$g = (1-\epsilon_i)(1-\pi_D)^2\mu,$$

$$h = \epsilon_i(1-\pi_D)^2\mu,$$

$$\text{and } l = (1-\epsilon_i(1-\pi_D))(1-\pi_D)^2\mu.$$

Theorem 3.4.4 summarizes the magnitude of i's value of perceptiveness when such value is non-zero under the "middle equilibrium" refinement. The proof of Theorem 3.4.4 follows from substituting i's ex-ante expected payoffs, when $\epsilon_A(2-\epsilon_B) \geq 1-(\frac{\pi_D}{1-\pi_D})^2$ and $\epsilon_B(2-\epsilon_A) \geq 1-(\frac{\pi_D}{1-\pi_D})^2$, into Equation (3.20). Equation (3.24) shows that when i is inexpert and j is perceptive, i's value of perceptiveness is always zero (since $\epsilon_i = 0$). Furthermore, an inexpert agent i's value of perceptiveness is positive when j is imperceptive, $\epsilon_i \in (0,1)$. Also, for both levels of expertise, the magnitude of i's value of perceptiveness increases as the quality of such information (μ) increases.

Corollary 3.4.5 *Suppose* $(\epsilon_i,\epsilon_j) \in (0,1)^2$ *and* $\chi_j = \eta_j$*, for all* $i \in \{A,B\}$*, where* $j \in \{A,B\}$ *such that* $i \neq j$*. Player i's value of perceptiveness is zero. Moreover, perceptiveness will not effect i's equilibrium strategy.*

Proof See Appendix B.3.

The intuition for Corollary 3.4.5 comes from each player knowing that their opponent will enter the market with a particular probability regardless of their expertise. As shown by Equations (3.2), (3.4), (3.17), and (3.19), each agent i's best response and ex-ante expected payoff, with respect j's equilibrium strategies, depends solely on j's aggregate entry probability, $\epsilon_j\chi_j + (1-\epsilon_j)\eta_j$. Given that j enters the market with the same probability regardless of j's expertise, i's best response and ex-ante expected payoff will be independent of i's perceptiveness since ϵ_j drops out of Equations (3.2), (3.4), (3.17), and (3.19), when substituting in $\chi_j = \eta_j$. Therefore, when j enters the market with a specific probability regardless of j's expertise, i's value of perceptiveness will be zero and perceptiveness will not affect i's equilibrium strategy. Moreover, given the "middle equilibrium" refinement, under Bertrand competition (when $\pi_D = 0$), when $(\epsilon_A,\epsilon_B) \in (0,1)^2$, player i will enter the market with a probability of $1/2$ regardless of i's expertise, for all $i \in \{A,B\}$. That is, an expert agent i will use a cut-off strategy of $\chi_i = 1/2$, whereas an inexpert agent i will use a mixing strategy of $\eta_i = 1/2$.[14] This implies that perceptiveness will not have any value or effect to players under Bertrand competition.

3.4.2 Comparing Results Between Chapters 2 & 3

Comparing the results between Chapter 2 and Chapter 3 can be done by fixing ϵ_i, such that $\epsilon_i \in \{0, 1/2, 1\}$, while varying ϵ_j, then applying i's expected payoffs to Equation (3.20) in order to obtain i's value of perceptiveness. In Chapter 2, I obtain i's value of perceptiveness using $\mu = 1/2$. Whereas, in Chapter 3, I must consider the bounds of the equilibrium region of

[14]This result can be obtained by substituting $\pi_D = 0$ into Theorem 3.3.12.

interest. Hence, there will be an upper bound for the μ-value I consider in Chapter 3. For the following comparison, I set $\pi_D = 4/9$. Based on the equilibrium region bounds in Chapter 3, when $(\epsilon_i, \epsilon_j) = (0, 1/2)$, I can consider $\mu \in [0, 3/10]$; when $(\epsilon_i, \epsilon_j) = (1/2, 1/2)$, I can consider $\mu \in [0, 13/50]$; and when $(\epsilon_i, \epsilon_j) = (1, 1/2)$, I can consider $\mu \in [0, 7/50]$.[15] Figures 3.7-3.10 present agent i's value of perceptiveness for the aforementioned parameter values and (ϵ_i, ϵ_j) ordered pairs. Figures 3.7 and 3.9 correspond to my results in Chapter 2, whereas Figures 3.8 and 3.10 correspond to my results in Chapter 3.

As illustrated by Figures 3.7-3.10, perceptiveness generally provides positive value to players in both models. However, the magnitude of benefit that arises from perceptiveness varies substantially between the models. Figures 3.7-3.10 show that perceptiveness has a more substantial benefit in Chapter 2 than it has in Chapter 3. The value of perceptiveness in Chapter 2 (Figures 3.7 and 3.9) is measured in terms of chips (K); whereas, the value of perceptiveness in Chapter 3 (Figures 3.8 and 3.10) is measured in terms of monopolistic profit ($\pi_M = 1$). For instance, suppose $(\epsilon_i, \epsilon_j) = (1/2, 1/2)$ and $K = 18$ in the Chapter 2 model, while $(\pi_D, \mu) = (4/9, 1/5)$ in the Chapter 3 model. In Chapter 2, perceptiveness approximately provides an added expected value of 0.4 chips for an expert agent i and 0.5 chips for an inexpert agent i. In this poker setting, the added value is actually quite substantial given that a player effectively only loses 1 chip by folding and can only win or lose a maximum of 18 chips. In Chapter 3, perceptiveness approximately provides an added expected value of 0.24% of monopolistic profit for an expert agent i and 0.175% of monopolistic profit for an inexpert agent i. Although these calculations involve a strictly larger μ-value in the Chapter 2 model than the Chapter 3 model, the difference in the magnitude of the value of perceptiveness in both models is enough for me to infer that perceptiveness, despite generally having positive value in both games, is much more beneficial in poker than it is in a market-entry setting. This is likely due to j's payoff-relevant realiza-

[15]These bounds can be determined by identifying which equilibrium region the corresponding (ϵ_i, ϵ_j) coordinate lies within, then subsequently determining how far the regional boundary is from (ϵ_i, ϵ_j). For instance, suppose $(\epsilon_i, \epsilon_j) = (1/2, 1/2)$. By Theorem 3.3.12, the equilibrium is such that $1-(\frac{\pi_D}{1-\pi_D})^2 \leq (2-\epsilon_i)\epsilon_j$ and $1-(\frac{\pi_D}{1-\pi_D})^2 \leq (2-\epsilon_j)\epsilon_i$. Substituting $\pi_D = 4/9$ and $\epsilon_i = 1/2$ yields equilibrium region conditions of $\epsilon_j \leq \frac{32}{25}$ and $\frac{6}{25} \leq \epsilon_j$. Therefore, the maximum value for μ is $\frac{1}{2} - \frac{6}{25} = \frac{13}{50}$.

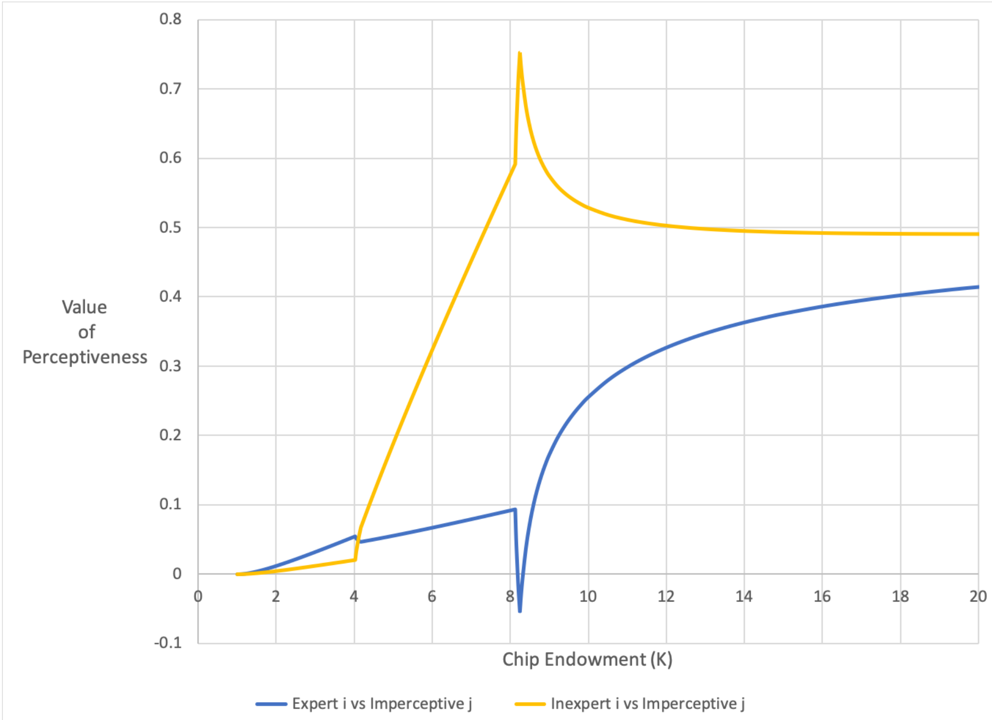


Figure 3.7: Agent i's value of perceptiveness in Chapter 2, when $(\epsilon_i, \epsilon_j) = (^1/_2, ^1/_2)$ and $\mu = {}^1/_2$.

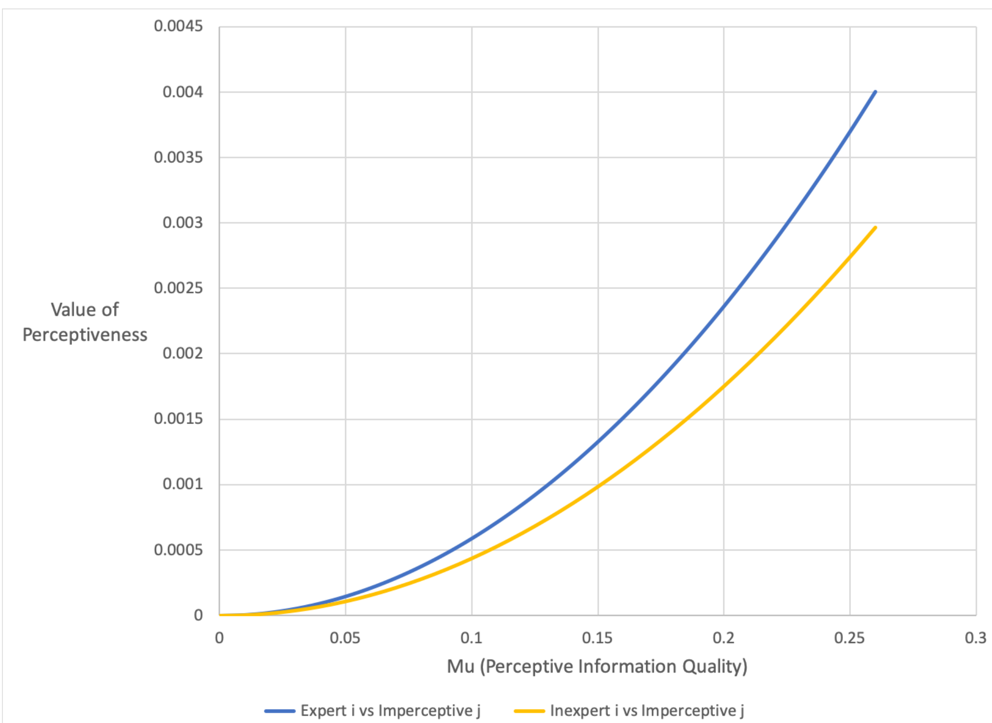


Figure 3.8: Agent i's value of perceptiveness in Chapter 3, when $(\epsilon_i, \epsilon_j) = (^1/_2, ^1/_2)$ and $\pi_D = {}^4/_9$.

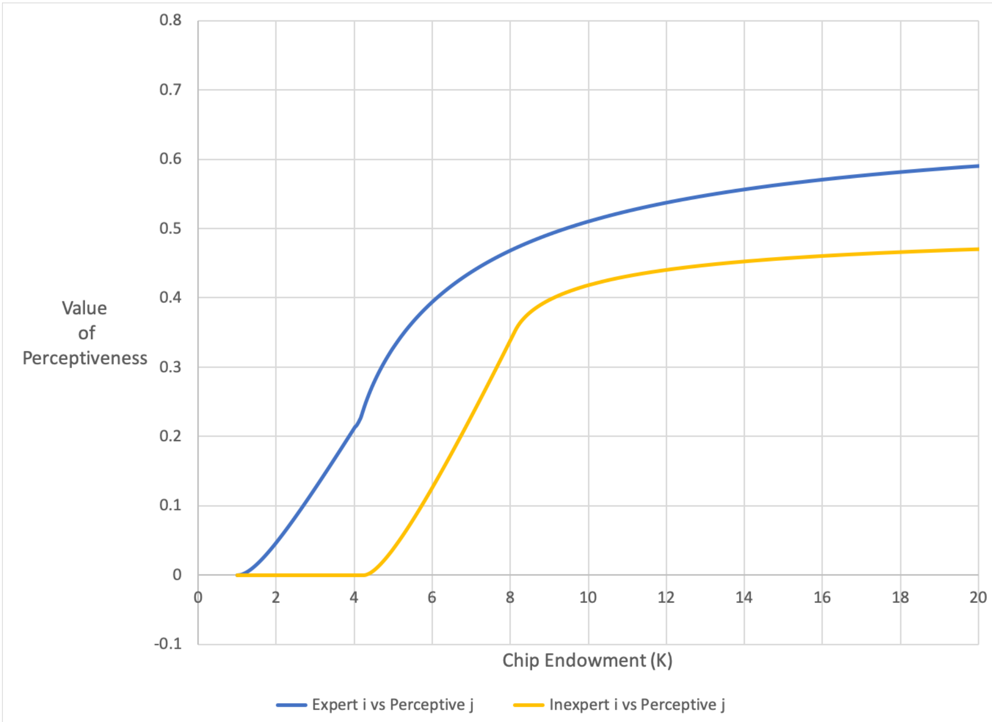


Figure 3.9: Agent i's value of perceptiveness in Chapter 2, when $\epsilon_i \in \{0,1\}$, $\epsilon_j = 1/2$, and $\mu = 1/2$.

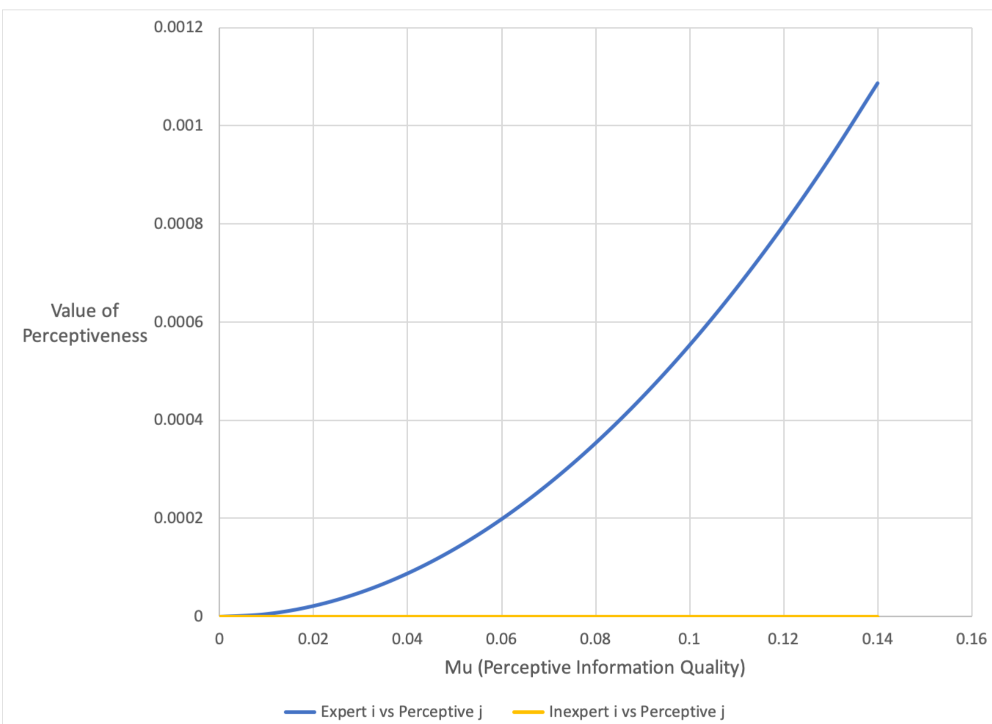


Figure 3.10: Agent i's value of perceptiveness in Chapter 3, when $\epsilon_i \in \{0,1\}$, $\epsilon_j = 1/2$, and $\pi_D = 4/9$.

tion (h_j) having a direct effect on i's payoff in the poker setting, whereas j's payoff-relevant realization (ϕ_j) merely had an indirect effect on i's payoff in the market-entry setting.

3.5 Conclusion

In this chapter, I develop and study a model that depicts a two-player market-entry game featuring a continuum of information structures. Player i is expert if they know their market-entry fee, ϕ_i, prior to deciding whether to enter the market. Player i is perceptive if they know whether their competitor j is expert. The information structures I consider vary in terms of the players' expertise and perceptiveness.

The main results that I find in this chapter are as follows. Under an equilibrium refinement that treats the players as symmetrically as possible, when both players have a sufficiently high probability of being expert, the players' value of perceptiveness is positive; whereas, if either player is inexpert with a sufficiently high probability, the players' value of perceptiveness is zero. Furthermore, the value of perceptiveness is always non-negative under the equilibrium refinement I consider. Lastly, even when the value of perceptiveness is zero, perceptiveness can still affect the players' equilibrium strategies.

Chapter 4

Perceptiveness in a Market-Entry, Information Design Setting

4.1 Introduction

In this chapter, I apply the same notion of perceptiveness, which I investigated in Chapters 2 and 3, to a market-entry setting with an information designer. The information designer will have the ability to influence the players' market-entry decision by sending action recommendation signals to the players. Doing so will subsequently influence the players' individually optimal behaviour, which will in turn help the information designer achieve a particular objective.

As noted in Bergemann and Morris (2019), information design literature has been covered more extensively over the past decade. Furthermore, information design has spanned across several distinct bodies of literature, such as Bayesian persuasion, Bayesian games with communication, and literature pertaining to various economic applications of information design. Bayesian persuasion literature, notably Kamenica and Gentzkow (2011), typically features an interaction between a receiver and a sender, who has an informational advantage over the receiver. The sender selects an informational signal to send to the receiver, who then chooses an action, which subsequently determines the payoffs that each player receives. Bayesian games with communication, notably Myerson (2013), typically features multiple players, including a mediator that has no informational advantage over the other players. The mediator, in this literature, can be viewed as an information designer. Additionally, some of the economic applications of information design include voter persuasion (Alonso & Câmara (2016)), the welfare consequences of price discrimination (Bergemann, Brooks, & Morris (2015)), and matching markets between schools and job placements (Ostrovsky & Schwarz (2010)).

In general, information design literature pertains to the instance of when the information designer has an informational advantage over multiple players. Recent developments include Bergemann and Morris (2019), Mathevet, Perego, and Taneva (2020), and Taneva (2019). As titled, Bergemann and Morris (2019) provides a unified perspective of information design. In this paper, Bergemann and Morris (2019) provides an extensive review and comparison of several bodies of literature, including Bayesian persuasion and communication in Bayesian games, that incorporate information design. In addition to this, Bergemann and Morris (2019) discuss the distinction between literal and metaphorical information design, information design's relationship to mechanism design, and various applications of information design. Most importantly, Bergemann and Morris (2019) outlines the general information design setting and provides an investment example to illustrate how an information design problem can be solved in four different scenarios: 1) Single player without prior information; 2) Single player with prior information; 3) Many players without prior information; and 4) Many players with prior information. Detailed solutions for this investment example are provided and explained in Bergemann and Morris (2013, 2016, 2019). The explanations of the investment example outlined by these three papers provide a clear description of how to structure and solve an information design problem, which proved invaluable as I developed this chapter.

Taneva (2019) details a general approach to deriving the information designer's optimal information structure in static finite environments. Taneva (2019) then applies this approach to a symmetric binary environment, derives the corresponding constraint set, and solves for the optimal information structure for an information designer wishing to miscoordinate actions between players. Following this, Taneva (2019) gives a complete characterization of the optimal information structure for all possible symmetric information designer payoff functions in a symmetric binary environment. Mathevet, Perego, and Taneva (2020) investigates information design from a belief manipulation perspective by characterizing the feasible distribution of players' beliefs that an information designer can induce by their choice of information structure. Then, using their results, Mathevet, Perego, and Taneva (2020) develop a novel approach to solving the information designer's problem. Their approach features the information designer first optimizing over private information, then sending out an optimal public signal.

My research in this chapter contributes to and departs from information design literature by incorporating perceptiveness into an information design problem. I also determine the effect that perceptiveness has on an information designer's ability to maximize producer surplus in a market-entry setting. I find that if a governing body subsidizes market-entry by a sufficiently high amount (indicated by a sufficiently small, negative market-entry fee), perceptiveness will not affect producer surplus nor the information designer's producer-surplus-maximizing decision rule. Moreover, when both the high and low state entry fees are sufficiently large, perceptiveness will not affect the information designer's producer-surplus-maximizing decision rule, but will inflict negative value in terms of producer surplus.

Additionally, I find that when there is a sufficiently small difference between the high and low state market-entry fees, perceptiveness will affect the information designer's producer-surplus-maximizing decision rule. However, despite this, the maximized value of producer surplus will only be affected for market-entry fee differences that are sufficiently small, within this already sufficiently small difference. Furthermore, when perceptiveness affects the information designer's producer-surplus-maximizing decision rule and the maximized value of producer surplus, perceptiveness will provide positive value, in terms of producer surplus, when the high state market-entry fee is sufficiently low. Contrarily, when perceptiveness affects the information designer's producer-surplus-maximizing decision rule and the maximized value of producer surplus, perceptiveness will inflict negative value, in terms of producer surplus, when the high state market-entry fee is sufficiently high.

The remainder of this chapter is comprised as follows. Section 4.2 describes the model, specifically detailing the game I study. Section 4.3 develops the Bayes correlated equilibria that the information designer can attain, taking into consideration any applicable constraints. Section 4.4 highlights the information designer's objective of maximizing producer surplus, then outlines the decision rule that achieves this for all market-entry fee regions I consider. Section 4.5 discusses the effect that perceptiveness has on this information designer problem, and subsequently discusses whether perceptiveness provides positive, zero, or negative value, in terms of producer surplus, in each market-entry fee region. Section 4.6 concludes. Appendix C provides the supplemental appendix for this chapter.

4.2 Model

4.2.1 Players, Actions, States

I study a Bayesian game that features two players, A and B, that each produce the same product. Both players must consider whether to *"enter"* ($a_i = E_i$) or *"not enter"* ($a_i = N_i$) the market in which this product is sold. Player i's payoff function is[1]

$$u_i(a_i, a_j, \phi) = \begin{cases} 0 & \text{if } a_i = N_i \\ \pi_M - \phi & \text{if } (a_i, a_j) = (E_i, N_j) \\ \pi_D - \phi & \text{if } (a_i, a_j) = (E_i, E_j). \end{cases}$$

I let $(\pi_M, \pi_D) \in \mathbb{R}^2_+$ and $\phi \in \{\phi_H, \phi_L\}$ where $(\phi_H, \phi_L) \in \mathbb{R}^2$ such that $\phi_H \geq \pi_M > \pi_D > \phi_L$. In this setting, π_M and π_D respectively represent the post-entry profit that a monopolist and duopolist would receive by operating in the market. Furthermore, ϕ represents the market-entry fee that each player must pay to enter the market.[2] There are two possible states of the world, H and

[1] j will always denote i's opponent.

[2] In this chapter, my model features a market-entry fee that is common for both players. Whereas, in Chapter

L, which represent the high and low market-entry fees respectively. The high state occurs with probability $h \in (0,1)$, whereas the low state occurs with probability $(1-h)$. All of the above is common knowledge to both players. Throughout my analysis, I focus attention towards a parameterization of $\pi_M = 1$, $\pi_D = 4/9$, and $h = 1/2$. By doing so, I study a Cournot competition market-entry setting with two states that are equally likely to occur. The microfoundations of my model are discussed in Appendix C.1.

In addition to the two players, there is an omniscient information designer who can provide the players with additional information in order to induce them to make particular action choices. Bergemann and Morris (2019) refers to the omniscient case as the case where the information designer faces no constraints on their ability to condition the signals on the payoff-relevant states of the world and all players' prior information. In this chapter, I focus my attention to this case as opposed to cases where the players have some prior information that the information designer is not privy to. Additionally, the objective of the information designer will be to maximize producer surplus, given the state and the actions of each player. I will address the information designer's objective in more detail in Section 4.4.

4.2.2 Types

The parameter space for the information structures I consider is $(\epsilon_A, \epsilon_B) \in (0,1)^2$, such that $\epsilon_A = \epsilon_B$, where ϵ_i represents the probability of player i knowing ϕ. Each (ϵ_A, ϵ_B) ordered pair is common knowledge to both players and corresponds to a specific information structure. For instance, suppose $(\epsilon_A, \epsilon_B) = (1/2, 1/2)$. Here, $\epsilon_j = 1/2$, which implies that player i knows that player j knows the state of the world with a probability of $1/2$. Additionally, player i knows that, with a probability of $1/2$, player j believes the state of the world is H with probability h and L with probability $1-h$. For a second example, suppose $(\epsilon_A, \epsilon_B) = (1/4, 1/4)$. Here, player i knows that player j knows the state of the world with a probability of $1/4$. Hence, player i also knows that with a probability of $3/4$, player j believes the state of the world is H with

3, each player had a separate draw for their market-entry fee.

probability h and L with probability $1-h$. Since (ϵ_A, ϵ_B) is common knowledge, both players know the probability that the other player believes them to know the state of the world with. Since I focus on symmetry between ϵ_A and ϵ_B, I henceforth let $\epsilon = \epsilon_A = \epsilon_B$.

Definition 4.2.1 *Player i is perceptive if player i knows with certainty whether player j knows the value of ϕ.*

Definition 4.2.1 classifies player i as *perceptive* if and only if $\epsilon_j \in \{0,1\}$. Since I focus my study on $(\epsilon_A, \epsilon_B) \in (0,1)^2$, neither player will truly be perceptive in this setting. However, some classes of players will be more perceptive than others. As shown by Reza (1994), uncertainty is maximized when all potential outcomes occur with equal probability. Hence, the most *imperceptive* player i can be occurs when $\epsilon_j = 1/2$. Consequently, i becomes more perceptive as $|\epsilon_j - \frac{1}{2}|$ increases. Therefore, for instance, the players in the game featuring $(\epsilon_A, \epsilon_B) = (1/4, 1/4)$ are more perceptive than the players in the game featuring $(\epsilon_A, \epsilon_B) = (1/2, 1/2)$. Similarly, the players in the game featuring $(\epsilon_A, \epsilon_B) = (1/2, 1/2)$ are more imperceptive than the players in the game featuring $(\epsilon_A, \epsilon_B) = (1/4, 1/4)$. Consequently, since (ϵ_A, ϵ_B) is common knowledge, each player's perceptiveness is common knowledge as well.

Definition 4.2.2 *Player i is expert if player i knows ϕ prior to making their market-entry decision.*

Definition 4.2.2 classifies a player as *expert* if and only if such player knows the market-entry fee prior to making their market-entry decision. If player i does not know the value of ϕ prior to making such decision, I classify player i as being *inexpert*.

Since both players are at least a little imperceptive, the type space for each player i is $t_i \in \{H_i, L_i, I_i\}$, since j is uncertain whether i is expert and knows the state with certainty or inexpert and does not know the state with certainty. Player i is type H_i if i is expert and the state is H; player i is type L_i if i is expert and the state is L; and, player i is type I_i if i is inexpert. Even as ϵ_i approaches 1 (or 0), there is still an ex-ante chance that i is inexpert (or expert). Thus, player i's expertise is private information to i, since j will always believe there to be some chance that i is expert and some chance that i is inexpert. Consequently, even if both players know ϕ_s, where $s \in \{H, L\}$ represents the state, the state will not be common knowledge to the players since both players believe there to be some positive probability that the other does not know the true value of ϕ.

4.2.3 Timeline

The timeline for each particular game is as follows. First, the information designer chooses and commits to a state-contingent and expertise-contingent decision rule regarding the signal recommendations they will send to each player. Second, the state s and the players' expertise is realized. Third, the information designer sends each player an action recommendation signal corresponding to the information designer's decision rule. Fourth, each player simultaneously chooses whether to enter the market, taking into consideration the prior information and their recommended action from the information designer. Fifth, payoffs are realized.

4.3 Equilibria & Obedience Constraints

I consider the Bayes correlated equilibrium solution concept developed by Bergemann and Morris (2016) when determining the information designer's decision rule,

$$\sigma((a_i, a_j)|(t_i, t_j), s).$$

Specifically, the decision rule is a probability distribution over actions, given each player's type and the state, that the information designer bases their recommendations on. That is, the decision rule $\sigma((a_i, a_j)|(t_i, t_j), s)$ represents the probability that the information designer chooses to recommend the market-entry outcome (a_i, a_j), given the state and each player's type. The Bayes correlated equilibrium solution concept is founded upon having the players' actions constitute a Bayesian Nash equilibrium.

Bergemann and Morris (2019) shows that an omniscient information designer can attain a decision rule if and only if it is a Bayes correlated equilibrium. Additionally, Bergemann and Morris (2016) shows that a Bayes correlated equilibrium only requires that the players be *obedient*, which is defined by Definition 4.3.1.

Definition 4.3.1 *Decision rule $\sigma((a_i, a_j)|(t_i, t_j), s)$ is obedient if for each i, $t_i \in \{H_i, L_i, I_i\}$, and $a_i \in \{E_i, N_i\}$,*

$$\sum_{a_j \in \{E_j, N_j\},\ t_j \in \{H_j, L_j, I_j\},\ s \in \{H, L\}} u_i((a_i, a_j), s)\sigma((a_i, a_j)|(t_i, t_j), s)\rho((t_i, t_j)|s)\psi(s)$$
$$\geq \sum_{a_j \in \{E_j, N_j\},\ t_j \in \{H_j, L_j, I_j\},\ s \in \{H, L\}} u_i((a_i', a_j), s)\sigma((a_i, a_j)|(t_i, t_j), s)\rho((t_i, t_j)|s)\psi(s) \quad (4.1)$$

for all $a_i' \in \{E, N\}$, where ρ represents the probability distribution of types conditional on the state, and ψ represents the probability distribution over the possible states.

A decision rule, is obedient if each player i, after receiving their action recommendation a_i, has no other action a_i' that could provide them with a strictly higher payoff. Definition 1 in Bergemann and Morris (2019) provides the technical definition of obedience in a general information design setting. Definition 4.3.1 is an application, to my model, of Bergemann and Morris (2019)'s definition of obedience. As shown in Bergemann and Morris (2016), a decision rule that satisfies obedience is a Bayes correlated equilibrium. Similarly, a decision rule, $\sigma((a_i, a_j)|(t_i, t_j), s)$, that satisfies obedience, in terms of Definition 4.3.1, is a Bayes correlated equilibrium. Therefore, to solve for the Bayes correlated equilibria, I must determine which decision rules are attainable for the information designer, based on the obedience constraints I derive using Definition 4.3.1. Upon identifying all attainable Bayes correlated equilibria, I will determine which attainable decision rule maximizes producer surplus.

4.3.1 Obedience Constraints for Expert Agents

Using Definition 4.3.1, I can generate obedience constraints for an expert agent of type H, an expert agent of type L, and an inexpert agent of type I. Each agent will have two obedience constraints, one for entering the market ($a_i = E$) and one for not entering the market ($a_i = N$). I henceforth denote each obedience constraint for i as $OC^{t_i}_{a_i}$.

Lemma 4.3.1 *If* $\sigma((a_i, a_j)|(t_i, t_j), s)$ *is obedient, then*

$$
\begin{aligned}
&1)\ \sigma((E_i, E_j)|(H_i, t_j), H) = 0;\\
&2)\ \sigma((E_i, N_j)|(H_i, t_j), H) = 0;\\
&3)\ \sigma((N_i, E_j)|(L_i, t_j), L) = 0;\\
&4)\ \sigma((N_i, N_j)|(L_i, t_j), L) = 0;\\
&5)\ \sigma((N_A, N_B)|(H_A, H_B), H) = 1;\\
\textit{and }&6)\ \sigma((E_A, E_B)|(L_A, L_B), L) = 1.
\end{aligned}
$$

Proof Suppose i is expert and $s = H$. This implies that i's type will be H_i. By Definition 4.3.1, i's obedience constraint for entering the market ($a_i = E_i$), for all $t_j \in \{H_j, L_j, I_j\}$, is

$$
\begin{aligned}
&u_i((E_i, E_j), H)\sigma((E_i, E_j)|(H_i, t_j), H)\rho((H_i, t_j)|H)\psi(H)\\
&\quad + u_i((E_i, N_j), H)\sigma((E_i, N_j)|(H_i, t_j), H)\rho((H_i, t_j)|H)\psi(H)\\
&\geq u_i((N_i, E_j), H)\sigma((E_i, E_j)|(H_i, t_j), H)\rho((H_i, t_j)|H)\psi(H)\\
&\quad + u_i((N_i, N_j), H)\sigma((E_i, N_j)|(H_i, t_j), H)\rho((H_i, t_j)|H)\psi(H).
\end{aligned}
\tag{4.2}
$$

Since $u_i = 0$ whenever $a_i = N_i$, Inequality (4.2) simplifies to

$$
\begin{aligned}
&u_i((E_i, E_j), H)\sigma((E_i, E_j)|(H_i, t_j), H)\rho((H_i, t_j)|H)\psi(H)\\
&\quad + u_i((E_i, N_j), H)\sigma((E_i, N_j)|(H_i, t_j), H)\rho((H_i, t_j)|H)\psi(H) \geq 0.
\end{aligned}
$$

Moreover, $u_i((E_i, E_j), H) = \pi_D - \phi_H$ and $u_i((E_i, N_j), H) = \pi_M - \phi_H$. Hence, i's obedience constraint for entering the market further simplifies to

$$
\begin{aligned}
&(\pi_D - \phi_H)\sigma((E_i, E_j)|(H_i, t_j), H)\rho((H_i, t_j)|H)\psi(H)\\
&\quad + (\pi_M - \phi_H)\sigma((E_i, N_j)|(H_i, t_j), H)\rho((H_i, t_j)|H)\psi(H) \geq 0.
\end{aligned}
\tag{4.3}
$$

Recall that $\phi_H \geq \pi_M > \pi_D > \phi_L$. This implies that Inequality (4.3) will only be satisfied when $\sigma((E_i, E_j)|(H_i, t_j), H) = 0$ and $\sigma((E_i, N_j)|(H_i, t_j), H) = 0$, which must be the case since $\rho((H_i, t_j)|H)\psi(H) \in (0,1)$.[3] Furthermore, by the laws of probability,

$$\sigma((E_A, E_B)|(H_A, H_B), H) + \sigma((E_A, N_B)|(H_A, H_B), H) \\ + \sigma((N_A, E_B)|(H_A, H_B), H) + \sigma((N_A, N_B)|(H_A, H_B), H) = 1.$$

This implies that $\sigma((N_A, N_B)|(H_A, H_B), H) = 1$.

Now suppose i is expert and $s = L$. This implies that i's type will be L_i. By Definition 4.3.1, i's obedience constraint for not entering the market ($a_i = N_i$), for all $t_j \in \{H_j, L_j, I_j\}$, is

$$\begin{aligned} &u_i((N_i, E_j), L)\sigma((N_i, E_j)|(L_i, t_j), L)\rho((L_i, t_j)|L)\psi(H) \\ &\quad + u_i((N_i, N_j), L)\sigma((N_i, N_j)|(L_i, t_j), L)\rho((L_i, t_j)|L)\psi(L) \\ &\geq u_i((E_i, E_j), L)\sigma((N_i, E_j)|(L_i, t_j), L)\rho((L_i, t_j)|L)\psi(L) \\ &\quad + u_i((E_i, N_j), L)\sigma((N_i, N_j)|(L_i, t_j), L)\rho((L_i, t_j)|L)\psi(L) \end{aligned} \tag{4.4}$$

Since $u_i = 0$ whenever $a_i = N_i$, Inequality (4.4) simplifies to

$$\begin{aligned} 0 &\geq u_i((E_i, E_j), L)\sigma((N_i, E_j)|(L_i, t_j), L)\rho((L_i, t_j)|L)\psi(L) \\ &\quad + u_i((E_i, N_j), L)\sigma((N_i, N_j)|(L_i, t_j), L)\rho((L_i, t_j)|L)\psi(L) \end{aligned}$$

Moreover, $u_i((E_i, E_j), L) = \pi_D - \phi_L$ and $u_i((E_i, N_j), L) = \pi_M - \phi_L$. Hence, i's obedience constraint for entering the market further simplifies to

$$\begin{aligned} 0 &\geq (\pi_D - \phi_L)\sigma((N_i, E_j)|(L_i, t_j), L)\rho((L_i, t_j)|L)\psi(L) \\ &\quad + (\pi_M - \phi_L)\sigma((N_i, N_j)|(L_i, t_j), L)\rho((L_i, t_j)|L)\psi(L) \end{aligned} \tag{4.5}$$

Recall that $\phi_H \geq \pi_M > \pi_D > \phi_L$. This implies that Inequality (4.5) will only be satisfied when $\sigma((N_i, E_j)|(L_i, t_j), L) = 0$ and $\sigma((N_i, N_j)|(L_i, t_j), L) = 0$, which must be the case since $\rho((H_i, t_j)|H)\psi(H) \in (0,1)$. Furthermore, by the laws of probability,

$$\sigma((E_A, E_B)|(L_A, L_B), L) + \sigma((E_A, N_B)|(L_A, L_B), L) \\ + \sigma((N_A, E_B)|(L_A, L_B), L) + \sigma((N_A, N_B)|(L_A, L_B), L) = 1.$$

This implies that $\sigma((E_A, E_B)|(L_A, L_B), L) = 1$.

[3]When $\pi_M = \phi_H$, I restrict attention to decision rules where $\sigma((E_i, N_j)|(H_i, t_j), H) = 0$.

Lemma 4.3.1 implies that the information designer has no ability to influence the market-entry decision of an expert agent. Since an expert agent knows the state prior to making their market-entry decision, they will always enter in the low entry fee state and never enter in the high entry fee state. As a result, for a decision rule σ to be obedient, it is necessary that the information designer always recommends entry to expert agents in the low entry fee state and never recommends entry to expert agents in the high entry fee state. As such, I will henceforth restrict attention to decision rules where the information designer always recommends entry to expert agents in the low entry fee state, but never in the high entry fee state.

It can easily be verified that never recommending entry to an expert agent in the high entry fee state satisfies the obedience constraint for not entering the market. Likewise, it can easily be verified that always recommending entry to an expert agent in the low entry fee state satisfies the obedience constraint for not entering the market. It is interesting to note that an expert agent's market-entry decision is made independent from their opponent's expertise. Additionally, if one player happens to be expert, a duopoly will never occur in the high entry fee state; whereas, entry by at least one player will occur in the low entry fee state.

4.3.2 Obedience Constraints for Inexpert Agents

Since an inexpert agent does not know the state, the information designer will be able to influence their market-entry decision more substantially than they could with an expert agent. Let δ^s represent the information designer's probability of recommending an inexpert agent i to enter the market in state s, given that player j is expert. Let η^s represent the information designer's probability of recommending an inexpert agent i to enter the market in state s, given that player j is inexpert. Let η_D^s represent the information designer's probability of recommending both inexpert agents to enter the market in state s, given that both players are inexpert. These action recommendation variables are defined such that

$$(\delta^L, \delta^H, \eta^L, \eta_D^L, \eta^H, \eta_D^H) \in [0,1]^6,$$

where $2\eta^L - \eta_D^L \leq 1$ and $2\eta^H - \eta_D^H \leq 1$.

When both players are inexpert, the information designer must decide whether or not to coordinate entry between the players. Because of this, I am required to define two separate variables, η^s and η_D^s, for each state in order to fully characterize the instance when both players are inexpert. I restrict attention to symmetric decision rules,[4] which is consistent with information design literature, like Bergemann and Morris (2019). Tables 4.1-4.8 summarize the probabilities associated with each market-entry outcome for the four possible configurations of the players' expertise.

Tables 4.9 and 4.10 provide a breakdown of the components used to derive an inexpert agent's obedience constraint for entering the market, OC_E^I. Table 4.9 provides the components for the left side of Definition 4.3.1, whereas Table 4.10 provides the components for the right side. An inexpert agent's obedience constraint for entering the market is

$$\begin{aligned} OC_E^I: \quad & h(1-\epsilon)^2[\eta_D^H(\pi_D-\phi_H) + (\eta^H-\eta_D^H)(\pi_M-\phi_H)] \\ & + (1-h)(1-\epsilon)^2[\eta_D^L(\pi_D-\phi_L) + (\eta^L-\eta_D^L)(\pi_M-\phi_L)] \\ & + h(1-\epsilon)\epsilon\delta^H(\pi_M-\phi_H) \\ & + (1-h)(1-\epsilon)\epsilon\delta^L(\pi_D-\phi_L) \geq 0. \end{aligned} \tag{4.6}$$

Tables 4.11 and 4.12 provide a breakdown of the components used to derive an inexpert agent's obedience constraint for not entering the market, OC_N^I. Table 4.11 provides the components for the right side of Definition 4.3.1, whereas Table 4.12 provides the components for the left side. An inexpert agent's obedience constraint for not entering the market is

$$\begin{aligned} OC_N^I: \quad 0 \geq \; & h(1-\epsilon)^2[(\eta^H-\eta_D^H)(\pi_D-\phi_H)+(1-2\eta^H+\eta_D^H)(\pi_M-\phi_H)] \\ & + (1-h)(1-\epsilon)^2[(\eta^L-\eta_D^L)(\pi_D-\phi_L)+(1-2\eta^L+\eta_D^L)(\pi_M-\phi_L)] \\ & + h(1-\epsilon)\epsilon(1-\delta^H)(\pi_M-\phi_H) \\ & + (1-h)(1-\epsilon)\epsilon(1-\delta^L)(\pi_D-\phi_L). \end{aligned} \tag{4.7}$$

[4] That is, symmetric for agents with the same expertise.

Table 4.1: Probability distribution of market-entry outcomes in the high entry fee state when both players are expert.

	B enters	*B* does not enter
A enters	0	0
A does not enter	0	1

Table 4.2: Probability distribution of market-entry outcomes in the low entry fee state when both players are expert.

	B enters	*B* does not enter
A enters	1	0
A does not enter	0	0

Table 4.3: Probability distribution of market-entry outcomes in the high entry fee state when *A* is inexpert and *B* is expert.

	B enters	*B* does not enter
A enters	0	δ^H
A does not enter	0	$1-\delta^H$

Table 4.4: Probability distribution of market-entry outcomes in the low entry fee state when *A* is inexpert and *B* is expert.

	B enters	*B* does not enter
A enters	δ^L	0
A does not enter	$1-\delta^L$	0

Table 4.5: Probability distribution of market-entry outcomes in the high entry fee state when *A* is expert and *B* is inexpert.

	B enters	*B* does not enter
A enters	0	0
A does not enter	δ^H	$1-\delta^H$

Table 4.6: Probability distribution of market-entry outcomes in the low entry fee state when *A* is expert and *B* is inexpert.

	B enters	*B* does not enter
A enters	δ^L	$1-\delta^L$
A does not enter	0	0

Table 4.7: Probability distribution of market-entry outcomes in the high entry fee state when both players are inexpert.

	B enters	*B* does not enter
A enters	η_D^H	$\eta^H-\eta_D^H$
A does not enter	$\eta^H-\eta_D^H$	$1-2\eta^H+\eta_D^H$

Table 4.8: Probability distribution of market-entry outcomes in the low entry fee state when both players are inexpert.

	B enters	*B* does not enter
A enters	η_D^L	$\eta^L-\eta_D^L$
A does not enter	$\eta^L-\eta_D^L$	$1-2\eta^L+\eta_D^L$

Table 4.9: Summary of possible contingencies for an inexpert player (A), as well as the information designer's decision rule for recommending such player to enter the market.

State	Types		Probability	Actions			Decision Rule	$u_A(a'_A, a_B)$
	t_A	t_B	$\rho((t_i,t_j)\|s)\psi(s)$	a'_A	a_A	a_B	$\sigma((a_i,a_j)\|(t_i,t_j),s)$	
H	I_A	I_B	$(1-\epsilon)^2h$	E_A	E_A	E_B	η_D^H	$\pi_D-\phi_H$
H	I_A	I_B	$(1-\epsilon)^2h$	E_A	E_A	N_B	$\eta^H-\eta_D^H$	$\pi_M-\phi_H$
L	I_A	I_B	$(1-\epsilon)^2(1-h)$	E_A	E_A	E_B	η_D^L	$\pi_D-\phi_L$
L	I_A	I_B	$(1-\epsilon)^2(1-h)$	E_A	E_A	N_B	$\eta^L-\eta_D^L$	$\pi_M-\phi_L$
H	I_A	H_B	$(1-\epsilon)\epsilon h$	E_A	E_A	E_B	0	$\pi_D-\phi_H$
H	I_A	H_B	$(1-\epsilon)\epsilon h$	E_A	E_A	N_B	δ^H	$\pi_M-\phi_H$
L	I_A	L_B	$(1-\epsilon)\epsilon(1-h)$	E_A	E_A	E_B	δ^L	$\pi_D-\phi_L$
L	I_A	L_B	$(1-\epsilon)\epsilon(1-h)$	E_A	E_A	N_B	0	$\pi_M-\phi_L$

Table 4.10: Summary of possible contingencies for an inexpert player (A), as well as the information designer's decision rule for recommending such player to enter the market.

State	Types		Probability	Actions			Decision Rule	$u_A(a'_A, a_B)$
	t_A	t_B	$\rho((t_i,t_j)\|s)\psi(s)$	a'_A	a_A	a_B	$\sigma((a_i,a_j)\|(t_i,t_j),s)$	
H	I_A	I_B	$(1-\epsilon)^2h$	N_A	E_A	E_B	η_D^H	0
H	I_A	I_B	$(1-\epsilon)^2h$	N_A	E_A	N_B	$\eta^H-\eta_D^H$	0
L	I_A	I_B	$(1-\epsilon)^2(1-h)$	N_A	E_A	E_B	η_D^L	0
L	I_A	I_B	$(1-\epsilon)^2(1-h)$	N_A	E_A	N_B	$\eta^L-\eta_D^L$	0
H	I_A	H_B	$(1-\epsilon)\epsilon h$	N_A	E_A	E_B	0	0
H	I_A	H_B	$(1-\epsilon)\epsilon h$	N_A	E_A	N_B	δ^H	0
L	I_A	L_B	$(1-\epsilon)\epsilon(1-h)$	N_A	E_A	E_B	δ^L	0
L	I_A	L_B	$(1-\epsilon)\epsilon(1-h)$	N_A	E_A	N_B	0	0

Table 4.11: Summary of possible contingencies for an inexpert player (A), as well as the information designer's decision rule for recommending such player to not enter the market.

State	Types		Probability	Actions			Decision Rule	$u_A(a'_A, a_B)$
	t_A	t_B	$\rho((t_i,t_j)\|s)\psi(s)$	a'_A	a_A	a_B	$\sigma((a_i,a_j)\|(t_i,t_j),s)$	
H	I_A	I_B	$(1-\epsilon)^2 h$	E_A	N_A	E_B	$\eta^H - \eta^H_D$	$\pi_D - \phi_H$
H	I_A	I_B	$(1-\epsilon)^2 h$	E_A	N_A	N_B	$1-2\eta^H + \eta^H_D$	$\pi_M - \phi_H$
L	I_A	I_B	$(1-\epsilon)^2(1-h)$	E_A	N_A	E_B	$\eta^L - \eta^L_D$	$\pi_D - \phi_L$
L	I_A	I_B	$(1-\epsilon)^2(1-h)$	E_A	N_A	N_B	$1-2\eta^L + \eta^L_D$	$\pi_M - \phi_L$
H	I_A	H_B	$(1-\epsilon)\epsilon h$	E_A	N_A	E_B	0	$\pi_D - \phi_H$
H	I_A	H_B	$(1-\epsilon)\epsilon h$	E_A	N_A	N_B	$1-\delta^H$	$\pi_M - \phi_H$
L	I_A	L_B	$(1-\epsilon)\epsilon(1-h)$	E_A	N_A	E_B	$1-\delta^L$	$\pi_D - \phi_L$
L	I_A	L_B	$(1-\epsilon)\epsilon(1-h)$	E_A	N_A	N_B	0	$\pi_M - \phi_L$

Table 4.12: Summary of possible contingencies for an inexpert player (A), as well as the information designer's decision rule for recommending such player to not enter the market.

State	Types		Probability	Actions			Decision Rule	$u_A(a'_A, a_B)$
	t_A	t_B	$\rho((t_i,t_j)\|s)\psi(s)$	a'_A	a_A	a_B	$\sigma((a_i,a_j)\|(t_i,t_j),s)$	
H	I_A	I_B	$(1-\epsilon)^2 h$	N_A	N_A	E_B	$\eta^H - \eta^H_D$	0
H	I_A	I_B	$(1-\epsilon)^2 h$	N_A	N_A	N_B	$1-2\eta^H + \eta^H_D$	0
L	I_A	I_B	$(1-\epsilon)^2(1-h)$	N_A	N_A	E_B	$\eta^L - \eta^L_D$	0
L	I_A	I_B	$(1-\epsilon)^2(1-h)$	N_A	N_A	N_B	$1-2\eta^L + \eta^L_D$	0
H	I_A	H_B	$(1-\epsilon)\epsilon h$	N_A	N_A	E_B	0	0
H	I_A	H_B	$(1-\epsilon)\epsilon h$	N_A	N_A	N_B	$1-\delta^H$	0
L	I_A	L_B	$(1-\epsilon)\epsilon(1-h)$	N_A	N_A	E_B	$1-\delta^L$	0
L	I_A	L_B	$(1-\epsilon)\epsilon(1-h)$	N_A	N_A	N_B	0	0

4.3.3 Equilibria

Combining Lemma 4.3.1 with both obedience constraints, OC_E^I and OC_N^I, for an inexpert agent, provides the necessary existence conditions for a Bayes correlated equilibrium, given that $\epsilon_A = \epsilon_B = \epsilon$ and

$$(\delta^L, \delta^H, \eta^L, \eta_D^L, \eta^H, \eta_D^H) \in [0,1]^6,$$

such that $2\eta^H - \eta_D^H \leq 1$ and $2\eta^L - \eta_D^L \leq 1$. Theorem 4.3.2 formalizes how to determine whether a decision rule σ constitutes a Bayes correlated equilibrium.

Theorem 4.3.2 *Suppose $\epsilon_A = \epsilon_B = \epsilon$. The decision rule*

$$(\delta^L, \delta^H, \eta^L, \eta_D^L, \eta^H, \eta_D^H)$$

is a Bayes correlated equilibrium if and only if

$$\begin{aligned}
&1)\ (\delta^L, \delta^H, \eta^L, \eta_D^L, \eta^H, \eta_D^H) \in [0,1]^6;\\
&2)\ 2\eta^L - \eta_D^L \leq 1;\\
&3)\ 2\eta^H - \eta_D^H \leq 1;\\
&4)\ h(1-\epsilon)^2[\eta_D^H(\pi_D - \phi_H) + (\eta^H - \eta_D^H)(\pi_M - \phi_H)]\\
&\qquad + (1-h)(1-\epsilon)^2[\eta_D^L(\pi_D - \phi_L) + (\eta^L - \eta_D^L)(\pi_M - \phi_L)]\\
&\qquad + h(1-\epsilon)\epsilon(\pi_M - \phi_H)\delta^H\\
&\qquad + (1-h)(1-\epsilon)\epsilon(\pi_D - \phi_L)\delta^L \geq 0;\\
\textit{and }&5)\ 0 \geq h(1-\epsilon)^2[(\eta^H - \eta_D^H)(\pi_D - \phi_H) + (1 - 2\eta^H + \eta_D^H)(\pi_M - \phi_H)]\\
&\qquad + (1-h)(1-\epsilon)^2[(\eta^L - \eta_D^L)(\pi_D - \phi_L) + (1 - 2\eta^L + \eta_D^L)(\pi_M - \phi_L)]\\
&\qquad + h(1-\epsilon)\epsilon(\pi_M - \phi_H)(1 - \delta^H)\\
&\qquad + (1-h)(1-\epsilon)\epsilon(\pi_D - \phi_L)(1 - \delta^L).
\end{aligned}$$

The proof of Theorem 4.3.2 follows from Lemma 4.3.1, Inequality (4.6), Inequality (4.7), and the proof from Bergemann and Morris (2016) that shows that a Bayes correlated equilibrium only requires that each player be obedient. Moving forward, I use Theorem 4.3.2 to identify all attainable Bayes correlated equilibria. I then determine which of these equilibria the information designer should select in order to maximize producer surplus.

Corollary 4.3.3 *The decision rule*

$$(\delta^L, \delta^H, \eta^L, \eta^L_D, \eta^H, \eta^H_D) = (1, 0, 1, 1, 0, 0)$$

is a Bayes correlated equilibrium for all $(\pi_M, \pi_D) \in \mathbb{R}^2_+$ *and* $(\phi_H, \phi_L) \in \mathbb{R}^2$ *such that* $\phi_H \geq \pi_M > \pi_D > \phi_L$.

Corollary 4.3.3 follows from substituting the

$$(\delta^L, \delta^H, \eta^L, \eta^L_D, \eta^H, \eta^H_D) = (1, 0, 1, 1, 0, 0)$$

decision rule into Theorem 4.3.2. This corollary establishes that the set of attainable decision rules for the information designer to select from is nonempty.

4.4 Maximizing Producer Surplus

I assume the market to follow a linear inverse demand curve of the form $P = a - bQ_T$, where P represents the market price and Q_T represents the total quantity sold. I also assume that each player produces identical products that can be produced with a marginal cost of zero and that there are no fixed costs besides ϕ. In this chapter, I consider an information designer whose objective is to maximize producer surplus under Cournot competition. The calculations for the equilibrium quantity, equilibrium price, consumer surplus, producer surplus, and total surplus for each market outcome are listed in Appendix C.1. Additionally, in Appendix C.1, I include two diagrams that illustrate the regions of consumer and post-entry producer surplus, along with the equilibrium outcome, for a duopoly and a monopoly. It is important to note that the producer surplus the information designer maximizes accounts for the market-entry fees.[5]

Using the variables defined in Section 4.3.2 and the producer surplus computations in Appendix C.1, I obtain an expression for producer surplus, which is given by

[5]The regions of producer surplus shown in Appendix C.1 do not account for the market-entry fee(s). As such, the producer surplus I measure for the information designer's objective will be the corresponding shaded regions less the applicable market-entry fee(s).

$$\begin{aligned}PS(\delta^L,\delta^H,\eta^L,\eta^L_D,\eta^H,\eta^H_D) =& \\ & 2\epsilon^2(1-h)(\tfrac{4}{9}\pi_M-\phi_L) \\ & + 2\epsilon(1-\epsilon)[(1-h)(\pi_M-\phi_L)-(1-h)(\tfrac{1}{9}\pi_M+\phi_L)\delta^L-h(\phi_H-\pi_M)\delta^H] \\ & + 2(1-\epsilon)^2[(1-h)(\pi_M-\phi_L)\eta^L-h(\phi_H-\pi_M)\eta^H-\tfrac{5}{9}(1-h)\pi_M\eta^L_D-\tfrac{5}{9}h\pi_M\eta^H_D].\end{aligned} \tag{4.8}$$

As mentioned in Section 4.2.1, I focus attention towards a situation where $h = 1/2$, $\pi_M = 1$, and $\pi_D = 4/9$. As such, Equation (4.8) reduces to

$$\begin{aligned}PS(\delta^L,\delta^H,\eta^L,\eta^L_D,\eta^H,\eta^H_D) =& \\ & \epsilon^2(\tfrac{4}{9}-\phi_L) + \epsilon(1-\epsilon)[(1-\phi_L)-(\tfrac{1}{9}+\phi_L)\delta^L-(\phi_H-1)\delta^H] \\ & + (1-\epsilon)^2[(1-\phi_L)\eta^L-(\phi_H-1)\eta^H-\tfrac{5}{9}\eta^L_D-\tfrac{5}{9}\eta^H_D].\end{aligned} \tag{4.9}$$

I segment my analysis into four regions of the "ϕ_H–ϕ_L" plane, where $\phi_H \geq 1 > 4/9 > \phi_L$. These regions are[6]

1) $(\phi_H,\phi_L) \in [1,\infty) \times (-\infty,-1/9]$;

2) $(\phi_H,\phi_L) \in [14/9,\infty) \times [-1/9, 4/9)$;

3) $(\phi_H,\phi_L) \in [1,14/9] \times [-1/9, 4/9)$ and $\phi_H \geq 1+\frac{1}{2}(1+\epsilon)(\frac{4}{9}-\phi_L)$;

and 4) $(\phi_H,\phi_L) \in [1,14/9] \times [-1/9, 4/9)$ and $\phi_H \leq 1+\frac{1}{2}(1+\epsilon)(\frac{4}{9}-\phi_L)$.

I further segment the fourth region into four separate subregions. Given that

$$(\phi_H,\phi_L) \in [1,14/9] \times [-1/9, 4/9) \text{ and } \phi_H \leq 1+\tfrac{1}{2}(1+\epsilon)(\tfrac{4}{9}-\phi_L),$$

these four subregions are

1) $\phi_H-\phi_L \geq \frac{10}{9}$ and $\phi_H \in [1+\frac{1}{2}(1-\epsilon)(\frac{4}{9}-\phi_L), 1+\frac{1}{2}(1+\epsilon)(\frac{4}{9}-\phi_L)]$;

2) $\phi_H-\phi_L \geq \frac{10}{9}$ and $\phi_H \in [1, 1+\frac{1}{2}(1-\epsilon)(\frac{4}{9}-\phi_L)]$;

3) $\phi_H-\phi_L \leq \frac{10}{9}$ and $\phi_H \in [\frac{2}{(1+\epsilon)}(\frac{4}{9}+\epsilon)-\phi_L, 1+\frac{1}{2}(1+\epsilon)(\frac{4}{9}-\phi_L)]$;

and 4) $\phi_H-\phi_L \leq \frac{10}{9}$ and $\phi_H \in [1, \frac{2}{(1+\epsilon)}(\frac{4}{9}+\epsilon)-\phi_L]$.

[6]The section and subsection headings for Regions 1, 3, 4, and 4.1-4.4 (on the following pages) should have a square bracket "]" instead of a rounded bracket ")" wherever a square bracket "]" occurs when I list the regions and subregions in the text. There seems to be a "Table of Contents" macro in the Western Thesis LaTeX template that restricts me from using the square bracket "]" in section and subsection headings.

4.4.1 Region 1: $(\phi_H, \phi_L) \in [1, \infty) \times (-\infty, {}^{-1}/_9)$

Theorem 4.4.1 *If $(\phi_H, \phi_L) \in [1, \infty) \times (-\infty, {}^{-1}/_9]$, then the producer-surplus-maximizing decision rule is*

$$(\delta^L, \delta^H, \eta^L, \eta^L_D, \eta^H, \eta^H_D) = (1, 0, 1, 1, 0, 0). \tag{4.10}$$

Proof This solution can be verified by showing that the gradient of the producer surplus is a linear combination of a non-negative weighted average of the gradients for the binding constraints. Here, the binding constraints are $2\eta^L - \eta^L_D \leq 1$, the non-negativity constraints for δ^H, η^H, and η^H_D, as well as the upper bound constraints for δ^L, η^L, and η^L_D. The weights

$$\begin{aligned}&(\lambda_{2\eta^L - \eta^L_D \leq 1}, \lambda_{\delta^L \leq 1}, \lambda_{\delta^H \geq 0}, \lambda_{\eta^L \leq 1}, \lambda_{\eta^L_D \leq 1}, \lambda_{\eta^H \geq 0}, \lambda_{\eta^H_D \geq 0})\\ &= (\tfrac{5}{9}(1-\epsilon)^2, \epsilon(1-\epsilon)(-\tfrac{1}{9}-\phi_L), \epsilon(1-\epsilon)(\phi_H - 1), \epsilon(1-\epsilon)(-\tfrac{1}{9}-\phi_L), 0, (1-\epsilon)^2(\phi_H - 1), \tfrac{5}{9}(1-\epsilon)^2)\end{aligned}$$

satisfy the expression

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} \lambda_{2\eta^L - \eta^L_D \leq 1} \\ \lambda_{\delta^L \leq 1} \\ \lambda_{\delta^H \geq 0} \\ \lambda_{\eta^L \leq 1} \\ \lambda_{\eta^L_D \leq 1} \\ \lambda_{\eta^H \geq 0} \\ \lambda_{\eta^H_D \geq 0} \end{bmatrix} = \begin{bmatrix} -\epsilon(1-\epsilon)(\frac{1}{9}+\phi_L) \\ -\epsilon(1-\epsilon)(\phi_H - 1) \\ (1-\epsilon)^2(1-\phi_L) \\ -\frac{5}{9}(1-\epsilon)^2 \\ -(1-\epsilon)^2(\phi_H - 1) \\ -\frac{5}{9}(1-\epsilon)^2 \end{bmatrix}, \tag{4.11}$$

where the columns of the 6×7 matrix represent the binding constraints, while each row represents a particular decision variable.[7] Furthermore, the 6×1 matrix represents the gradient of producer surplus. Additionally, since $\epsilon \in (0,1)$, $\phi_L \leq {}^{-1}/_9$, and $\phi_H \geq 1$, the weights can each be shown to be non-negative.

[7] Throughout this chapter, the order of rows for each matrix that does not contain weights is $\delta^L, \delta^H, \eta^L, \eta^L_D, \eta^H$, and η^H_D. The order of columns in each left-most matrix begins with any applicable obedience constraints, followed by any applicable market-entry outcome probability constraints, followed by any binding non-negativity or upper bound constraints listed in the order of $\delta^L, \delta^H, \eta^L, \eta^L_D, \eta^H$, and η^H_D.

4.4.2 Region 2: $(\phi_H, \phi_L) \in [14/9, \infty) \times [-1/9, 4/9)$

Theorem 4.4.2 *If $(\phi_H, \phi_L) \in [14/9, \infty) \times [-1/9, 4/9)$, then the producer-surplus-maximizing decision rule is*

$$(\delta^L, \delta^H, \eta^L, \eta^L_D, \eta^H, \eta^H_D) = (0, 0, 1/2, 0, 0, 0).$$

Proof This solution can be verified by showing that the gradient of the producer surplus is a linear combination of a non-negative weighted average of the gradients for the binding constraints. Here, the binding constraints are $2\eta^L - \eta^L_D \leq 1$, as well as the non-negativity constraints for δ^L, δ^H, η^L_D, η^H, and η^H_D. The weights

$$\begin{aligned}
&(\lambda_{2\eta^L-\eta^L_D\leq 1}, \lambda_{\delta^L\geq 0}, \lambda_{\delta^H\geq 0}, \lambda_{\eta^L_D\geq 0}, \lambda_{\eta^H\geq 0}, \lambda_{\eta^H_D\geq 0}) \\
&\quad = (\tfrac{1}{2}(1-\epsilon)^2(1-\phi_L), \epsilon(1-\epsilon)(\tfrac{1}{9}+\phi_L), \epsilon(1-\epsilon)(\phi_H-1), \\
&\qquad\qquad \tfrac{1}{2}(1-\epsilon)^2(\tfrac{1}{9}+\phi_L), (1-\epsilon)^2(\phi_H-1), \tfrac{5}{9}(1-\epsilon)^2)
\end{aligned} \tag{4.12}$$

satisfy the expression

$$\begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} \lambda_{2\eta^L-\eta^L_D\leq 1} \\ \lambda_{\delta^L\geq 0} \\ \lambda_{\delta^H\geq 0} \\ \lambda_{\eta^L_D\geq 0} \\ \lambda_{\eta^H\geq 0} \\ \lambda_{\eta^H_D\geq 0} \end{bmatrix} = \begin{bmatrix} -\epsilon(1-\epsilon)(\frac{1}{9}+\phi_L) \\ -\epsilon(1-\epsilon)(\phi_H-1) \\ (1-\epsilon)^2(1-\phi_L) \\ -\frac{5}{9}(1-\epsilon)^2 \\ -(1-\epsilon)^2(\phi_H-1) \\ -\frac{5}{9}(1-\epsilon)^2 \end{bmatrix} \tag{4.13}$$

where the columns of the 6×6 matrix represent the binding constraints, while each row represents a particular decision variable. Furthermore, the 6×1 matrix on the right side of Equation (4.13) represents the gradient of producer surplus. Additionally, since $\epsilon \in (0,1)$, $\phi_L \in [-1/9, 4/9)$, and $\phi_H \geq 14/9$, the weights can each be shown to be non-negative.

4.4.3 Region 3: $(\phi_H, \phi_L) \in [1, {}^{14}/_9) \times [{}^{-1}/_9, {}^{4}/_9)$ & $\phi_H \geq 1 + \frac{1}{2}(1+\epsilon)(\frac{4}{9}-\phi_L)$

Theorem 4.4.3 *If* $(\phi_H, \phi_L) \in [1, {}^{14}/_9] \times [{}^{-1}/_9, {}^{4}/_9)$ *such that* $\phi_H \geq 1 + \frac{1}{2}(1+\epsilon)(\frac{4}{9}-\phi_L)$*, then the producer-surplus-maximizing decision rule is*

$$(\delta^L, \delta^H, \eta^L, \eta^L_D, \eta^H, \eta^H_D) = (0, 0, {}^{1}/_{2}, 0, 0, 0).$$

Proof This solution can be verified by using an identical process, matrix equation, and weights as those used in the proof of Theorem 4.4.2. Furthermore, since $\epsilon \in (0,1)$, $\phi_L \in [{}^{-1}/_9, {}^{4}/_9)$, and $\phi_H \in [1, {}^{14}/_9]$, the weights can each be shown to be non-negative.

4.4.4 Region 4: $(\phi_H, \phi_L) \in [1, {}^{14}/_9) \times [{}^{-1}/_9, {}^{4}/_9)$ & $\phi_H \leq 1+\frac{1}{2}(1+\epsilon)(\frac{4}{9}-\phi_L)$

Region 4.1: $\phi_H - \phi_L \geq \frac{10}{9}$ & $\phi_H \in [1+\frac{1}{2}(1-\epsilon)(\frac{4}{9}-\phi_L), 1+\frac{1}{2}(1+\epsilon)(\frac{4}{9}-\phi_L))$

Theorem 4.4.4 *If* $(\phi_H, \phi_L) \in [1, {}^{14}/_9] \times [{}^{-1}/_9, {}^{4}/_9)$ *such that*

$$\begin{aligned} \phi_H - \phi_L &\geq \tfrac{10}{9}, \\ \phi_H &\leq 1+\tfrac{1}{2}(1+\epsilon)(\tfrac{4}{9}-\phi_L), \\ \textit{and } \phi_H &\geq 1+\tfrac{1}{2}(1-\epsilon)(\tfrac{4}{9}-\phi_L), \end{aligned}$$

then the producer-surplus-maximizing decision rule is

$$(\delta^L, \delta^H, \eta^L, \eta^L_D, \eta^H, \eta^H_D) = (\frac{(1+\epsilon)(\frac{4}{9}-\phi_L) - 2(\phi_H-1)}{2\epsilon(\frac{4}{9}-\phi_L)}, 0, {}^{1}/_{2}, 0, 0, 0).$$

Proof This solution can be verified by showing that the gradient of the producer surplus is a linear combination of a non-negative weighted average of the gradients for the binding constraints. Here, the binding constraints are the OC^I_N, $2\eta^L - \eta^L_D \leq 1$, as well as the non-negativity constraints for δ^H, η^L_D, η^H, and η^H_D. The weights

$$\begin{aligned} &(\lambda_{OC^I_N}, \lambda_{2\eta^L-\eta^L_D\leq 1}, \lambda_{\delta^H\geq 0}, \lambda_{\eta^L_D\geq 0}, \lambda_{\eta^H\geq 0}, \lambda_{\eta^H_D\geq 0}) \\ &\quad = (\frac{(1-\epsilon)(\frac{1}{9}+\phi_L)}{(\frac{4}{9}-\phi_L)}, \frac{25(1-\epsilon)^2}{81(\frac{4}{9}-\phi_L)}, \frac{5\epsilon(1-\epsilon)(\phi_H-1)}{9(\frac{4}{9}-\phi_L)}, 0, \frac{5(1-\epsilon)^2[(\phi_H-1)-(\frac{1}{9}+\phi_L)]}{9(\frac{4}{9}-\phi_L)}, \frac{25(1-\epsilon)^2}{81(\frac{4}{9}-\phi_L)}) \end{aligned} \tag{4.14}$$

satisfy the expression

$$\begin{bmatrix} -\epsilon(\frac{4}{9}-\phi_L) & 0 & 0 & 0 & 0 & 0 \\ \epsilon(\phi_H-1) & 0 & -1 & 0 & 0 & 0 \\ -(1-\epsilon)(\frac{14}{9}-\phi_L) & 2 & 0 & 0 & 0 & 0 \\ \frac{5}{9}(1-\epsilon) & -1 & 0 & -1 & 0 & 0 \\ -(1-\epsilon)(\frac{14}{9}-\phi_H) & 0 & 0 & 0 & -1 & 0 \\ \frac{5}{9}(1-\epsilon) & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} \lambda_{OC^I_N} \\ \lambda_{2\eta^L-\eta^L_D\leq 1} \\ \lambda_{\delta^H\geq 0} \\ \lambda_{\eta^L_D\geq 0} \\ \lambda_{\eta^H\geq 0} \\ \lambda_{\eta^H_D\geq 0} \end{bmatrix} = \begin{bmatrix} -\epsilon(1-\epsilon)(\frac{1}{9}+\phi_L) \\ -\epsilon(1-\epsilon)(\phi_H-1) \\ (1-\epsilon)^2(1-\phi_L) \\ -\frac{5}{9}(1-\epsilon)^2 \\ -(1-\epsilon)^2(\phi_H-1) \\ -\frac{5}{9}(1-\epsilon)^2 \end{bmatrix} \quad (4.15)$$

where the columns of the 6×6 matrix represent the binding constraints, while each row represents a particular decision variable. Furthermore, the 6×1 matrix on the right side of Equation (4.15) represents the gradient of producer surplus. Additionally, since $\epsilon \in (0,1)$, $\phi_L \in [^{-1}/_9, ^4/_9)$, $\phi_H \in [1, ^{14}/_9]$, and $(\phi_H-1)-(\frac{1}{9}+\phi_L) \geq 0$, the weights can each be shown to be non-negative.

Region 4.2: $\phi_H-\phi_L \geq \frac{10}{9}$ & $\phi_H \in [1, 1+\frac{1}{2}(1-\epsilon)(\frac{4}{9}-\phi_L))$

Theorem 4.4.5 *If* $(\phi_H, \phi_L) \in [1, ^{14}/_9] \times [^{-1}/_9, ^4/_9)$ *such that*

$$\phi_H-\phi_L \geq \frac{10}{9}$$
$$\textit{and } \phi_H \leq 1+\frac{1}{2}(1-\epsilon)(\frac{4}{9}-\phi_L),$$

then the producer-surplus-maximizing decision rule is

$$(\delta^L, \delta^H, \eta^L, \eta^L_D, \eta^H, \eta^H_D) = (1, 0, 1-\frac{(\phi_H-1)}{(1-\epsilon)(\frac{4}{9}-\phi_L)}, 1-\frac{2(\phi_H-1)}{(1-\epsilon)(\frac{4}{9}-\phi_L)}, 0, 0)$$

Proof This solution can be verified by showing that the gradient of the producer surplus is a linear combination of a non-negative weighted average of the gradients for the binding constraints. Here, the binding constraints are the OC^I_N, $2\eta^L-\eta^L_D \leq 1$, the upper bound constraint for δ^L, and the non-negativity constraints for δ^H, η^H, and η^H_D. The weights

$$(\lambda_{OC_N^I}, \lambda_{2\eta^L-\eta_D^L\leq 1}, \lambda_{\delta^L\leq 1}, \lambda_{\delta^H\geq 0}, \lambda_{\eta^H\geq 0}, \lambda_{\eta_D^H\geq 0})$$
$$= (\frac{(1-\epsilon)(\frac{1}{9}+\phi_L)}{(\frac{4}{9}-\phi_L)}, \frac{25(1-\epsilon)^2}{81(\frac{4}{9}-\phi_L)}, 0, \frac{5\epsilon(1-\epsilon)(\phi_H-1)}{9(\frac{4}{9}-\phi_L)}, \frac{5(1-\epsilon)^2[(\phi_H-1)-(\frac{1}{9}+\phi_L)]}{9(\frac{4}{9}-\phi_L)}, \frac{25(1-\epsilon)^2}{81(\frac{4}{9}-\phi_L)}) \tag{4.16}$$

satisfy the expression

$$\begin{bmatrix} -\epsilon(\frac{4}{9}-\phi_L) & 0 & 1 & 0 & 0 & 0 \\ \epsilon(\phi_H-1) & 0 & 0 & -1 & 0 & 0 \\ -(1-\epsilon)(\frac{14}{9}-\phi_L) & 2 & 0 & 0 & 0 & 0 \\ \frac{5}{9}(1-\epsilon) & -1 & 0 & 0 & 0 & 0 \\ -(1-\epsilon)(\frac{14}{9}-\phi_H) & 0 & 0 & 0 & -1 & 0 \\ \frac{5}{9}(1-\epsilon) & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} \lambda_{OC_N^I} \\ \lambda_{2\eta^L-\eta_D^L\leq 1} \\ \lambda_{\delta^L\leq 1} \\ \lambda_{\delta^H\geq 0} \\ \lambda_{\eta^H\geq 0} \\ \lambda_{\eta_D^H\geq 0} \end{bmatrix} = \begin{bmatrix} -\epsilon(1-\epsilon)(\frac{1}{9}+\phi_L) \\ -\epsilon(1-\epsilon)(\phi_H-1) \\ (1-\epsilon)^2(1-\phi_L) \\ -\frac{5}{9}(1-\epsilon)^2 \\ -(1-\epsilon)^2(\phi_H-1) \\ -\frac{5}{9}(1-\epsilon)^2 \end{bmatrix} \tag{4.17}$$

where the columns of the 6×6 matrix represent the binding constraints, while each row represents a particular decision variable. Furthermore, the 6×1 matrix on the right side of Equation (4.17) represents the gradient of producer surplus. Additionally, since $\epsilon \in (0,1)$, $\phi_L \in [-1/9, 4/9)$, $\phi_H \in [1, 14/9]$, and $(\phi_H-1)-(\frac{1}{9}+\phi_L) \geq 0$, the weights can each be shown to be non-negative.

Region 4.3: $\phi_H-\phi_L \leq \frac{10}{9}$ & $\phi_H \in [\frac{2}{(1+\epsilon)}(\frac{4}{9}+\epsilon)-\phi_L, 1+\frac{1}{2}(1+\epsilon)(\frac{4}{9}-\phi_L))$

Theorem 4.4.6 *If* $(\phi_H, \phi_L) \in [1, 14/9] \times [-1/9, 4/9)$ *such that*

$$\phi_H-\phi_L \leq \frac{10}{9},$$
$$\phi_H \leq 1+\frac{1}{2}(1+\epsilon)(\frac{4}{9}-\phi_L),$$
$$\text{and } \phi_H \geq \frac{2}{(1+\epsilon)}(\frac{4}{9}+\epsilon)-\phi_L,$$

then the producer-surplus-maximizing decision rule is

$$(\delta^L, \delta^H, \eta^L, \eta_D^L, \eta^H, \eta_D^H) = (0, 0, 1/2, 0, \frac{\frac{1}{2}(1+\epsilon)(\frac{4}{9}-\phi_L) - (\phi_H-1)}{(1-\epsilon)(\frac{14}{9}-\phi_H)}, 0)$$

Proof This solution can be verified by showing that the gradient of the producer surplus is a linear combination of a non-negative weighted average of the gradients for the binding constraints. Here, the binding constraints are the OC_N^I, $2\eta^L - \eta_D^L \leq 1$, as well as the non-negativity constraints for δ^L, δ^H, η_D^L, and η_D^H. The weights

$$\begin{aligned}
&(\lambda_{OC_N^I}, \lambda_{2\eta^L-\eta_D^L\leq 1}, \lambda_{\delta^L\geq 0}, \lambda_{\delta^H\geq 0}, \lambda_{\eta_D^L\geq 0}, \lambda_{\eta_D^H\geq 0}) \\
&= (\frac{(1-\epsilon)(\phi_H-1)}{(\frac{14}{9}-\phi_H)}, \frac{5(1-\epsilon)^2(\phi_H-\phi_L)}{18(\frac{14}{9}-\phi_H)}, \frac{5\epsilon(1-\epsilon)[(\frac{1}{9}+\phi_L)-(\phi_H-1)]}{9(\frac{14}{9}-\phi_H)}, \\
&\qquad \frac{5\epsilon(1-\epsilon)(\phi_H-1)}{9(\frac{14}{9}-\phi_H)}, \frac{5(1-\epsilon)^2[(\frac{1}{9}+\phi_L)-(\phi_H-1)]}{18(\frac{14}{9}-\phi_H)}, \frac{25(1-\epsilon)^2}{81(\frac{14}{9}-\phi_H)})
\end{aligned} \tag{4.18}$$

satisfy the expression

$$\begin{bmatrix}
-\epsilon(\frac{4}{9}-\phi_L) & 0 & -1 & 0 & 0 & 0 \\
\epsilon(\phi_H-1) & 0 & 0 & -1 & 0 & 0 \\
-(1-\epsilon)(\frac{14}{9}-\phi_L) & 2 & 0 & 0 & 0 & 0 \\
\frac{5}{9}(1-\epsilon) & -1 & 0 & 0 & -1 & 0 \\
-(1-\epsilon)(\frac{14}{9}-\phi_H) & 0 & 0 & 0 & 0 & 0 \\
\frac{5}{9}(1-\epsilon) & 0 & 0 & 0 & 0 & -1
\end{bmatrix} \times \begin{bmatrix}
\lambda_{OC_N^I} \\ \lambda_{2\eta^L-\eta_D^L\leq 1} \\ \lambda_{\delta^L\geq 0} \\ \lambda_{\delta^H\geq 0} \\ \lambda_{\eta_D^L\geq 0} \\ \lambda_{\eta_D^H\geq 0}
\end{bmatrix} = \begin{bmatrix}
-\epsilon(1-\epsilon)(\frac{1}{9}+\phi_L) \\ -\epsilon(1-\epsilon)(\phi_H-1) \\ (1-\epsilon)^2(1-\phi_L) \\ -\frac{5}{9}(1-\epsilon)^2 \\ -(1-\epsilon)^2(\phi_H-1) \\ -\frac{5}{9}(1-\epsilon)^2
\end{bmatrix} \tag{4.19}$$

where the columns of the 6×6 matrix represent the binding constraints, while each row represents a particular decision variable. Furthermore, the 6×1 matrix on the right side of Equation (4.19) represents the gradient of producer surplus. Additionally, since $\epsilon \in (0,1)$, $\phi_L \in [^{-1}/_9, {}^4/_9)$, $\phi_H \in [1, {}^{14}/_9]$, $\phi_H > \phi_L$, and $(\frac{1}{9}+\phi_L)-(\phi_H-1) \geq 0$, the weights can each be shown to be non-negative.

Region 4.4: $\phi_H-\phi_L \leq \frac{10}{9}$ & $\phi_H \in [1, \frac{2}{(1+\epsilon)}(\frac{4}{9}+\epsilon)-\phi_L)$

Theorem 4.4.7 *If* $(\phi_H, \phi_L) \in [1, {}^{14}/_9] \times [^{-1}/_9, {}^4/_9)$ *such that*

$$\phi_H-\phi_L \leq \frac{10}{9}$$
$$\text{and } \phi_H \leq \frac{2}{(1+\epsilon)}(\frac{4}{9}+\epsilon)-\phi_L,$$

then the producer-surplus-maximizing decision rule is

$$(\delta^L, \delta^H, \eta^L, \eta^L_D, \eta^H, \eta^H_D) = (\frac{(\frac{4}{9}-\phi_L)-(\phi_H-1)-(1-\epsilon)(1-\frac{1}{2}\phi_L-\frac{1}{2}\phi_H)}{\epsilon(\frac{4}{9}-\phi_L)}, 0, 1/2, 0, 1/2, 0)$$

Proof This solution can be verified by showing that the gradient of the producer surplus is a linear combination of a non-negative weighted average of the gradients for the binding constraints. Here, the binding constraints are the OC^I_N, $2\eta^L - \eta^L_D \leq 1$, $2\eta^H - \eta^H_D \leq 1$, as well as the non-negativity constraints for δ^H, η^L_D, and η^H_D. The weights

$$\begin{aligned}
&(\lambda_{OC^I_N}, \lambda_{2\eta^L-\eta^L_D\leq 1}, \lambda_{2\eta^H-\eta^H_D\leq 1}, \lambda_{\delta^H\geq 0}, \lambda_{\eta^L_D\geq 0}, \lambda_{\eta^H_D\geq 0}) \\
&= (\frac{(1-\epsilon)(\frac{1}{9}+\phi_L)}{(\frac{4}{9}-\phi_L)}, \frac{25(1-\epsilon)^2}{81(\frac{4}{9}-\phi_L)}, \frac{5(1-\epsilon)^2[(\frac{1}{9}+\phi_L)-(\phi_H-1)]}{18(\frac{4}{9}-\phi_L)}, \\
&\qquad \frac{5\epsilon(1-\epsilon)(\phi_H-1)}{9(\frac{4}{9}-\phi_L)}, 0, \frac{5(1-\epsilon)^2(\phi_H-\phi_L)}{18(\frac{4}{9}-\phi_L)})
\end{aligned} \tag{4.20}$$

satisfy the expression

$$\begin{bmatrix} -\epsilon(\frac{4}{9}-\phi_L) & 0 & 0 & 0 & 0 & 0 \\ \epsilon(\phi_H-1) & 0 & 0 & -1 & 0 & 0 \\ -(1-\epsilon)(\frac{14}{9}-\phi_L) & 2 & 0 & 0 & 0 & 0 \\ \frac{5}{9}(1-\epsilon) & -1 & 0 & 0 & -1 & 0 \\ -(1-\epsilon)(\frac{14}{9}-\phi_H) & 0 & 2 & 0 & 0 & 0 \\ \frac{5}{9}(1-\epsilon) & 0 & -1 & 0 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} \lambda_{OC^I_N} \\ \lambda_{2\eta^L-\eta^L_D\leq 1} \\ \lambda_{2\eta^H-\eta^H_D\leq 1} \\ \lambda_{\delta^H\geq 0} \\ \lambda_{\eta^L_D\geq 0} \\ \lambda_{\eta^H_D\geq 0} \end{bmatrix} = \begin{bmatrix} -\epsilon(1-\epsilon)(\frac{1}{9}+\phi_L) \\ -\epsilon(1-\epsilon)(\phi_H-1) \\ (1-\epsilon)^2(1-\phi_L) \\ -\frac{5}{9}(1-\epsilon)^2 \\ -(1-\epsilon)^2(\phi_H-1) \\ -\frac{5}{9}(1-\epsilon)^2 \end{bmatrix} \tag{4.21}$$

where the columns of the 6×6 matrix represent the binding constraints, while each row represents a particular decision variable. Furthermore, the 6×1 matrix on the right side of Equation (4.21) represents the gradient of producer surplus. Additionally, since $\epsilon \in (0,1)$, $\phi_L \in [-1/9, 4/9)$, $\phi_H \in [1, 14/9]$, $\phi_H > \phi_L$, and $(\frac{1}{9}+\phi_L)-(\phi_H-1) \geq 0$, the weights can each be shown to be non-negative.

4.5 The Effect of Perceptiveness

As shown by Theorem 4.4.1, when $(\phi_H, \phi_L) \in [1, \infty) \times (-\infty, {}^{-1}/_9]$ the information designer's producer-surplus-maximizing decision rule is

$$(\delta^L, \delta^H, \eta^L, \eta^L_D, \eta^H, \eta^H_D) = (1, 0, 1, 1, 0, 0).$$

Since this decision rule is independent of ϵ, perceptiveness does not affect the information designer's producer-surplus-maximizing decision rule when $(\phi_H, \phi_L) \in [1, \infty) \times (-\infty, {}^{-1}/_9]$. Additionally, this decision rule represents the instance of when the information designer completely reveals the state to both players, regardless of their expertise. Because of this, perceptiveness does not affect producer surplus nor does it affect the obedience constraints for each player.

Theorems 4.4.2 and 4.4.3 show that when $(\phi_H, \phi_L) \in [{}^{14}/_9, \infty) \times [{}^{-1}/_9, {}^{4}/_9)$ or when $\phi_H \geq 1+\frac{1}{2}(1+\epsilon)(\frac{4}{9}-\phi_L)$, such that $(\phi_H, \phi_L) \in [1, {}^{14}/_9] \times [{}^{-1}/_9, {}^{4}/_9)$, the information designer's producer-surplus-maximizing decision rule is

$$(\delta^L, \delta^H, \eta^L, \eta^L_D, \eta^H, \eta^H_D) = (0, 0, {}^{1}/_2, 0, 0, 0).$$

Similar to when $\phi_L \in (-\infty, {}^{-1}/_9]$, perceptiveness does not affect the information designer's producer-surplus-maximizing decision rule nor does it affect the obedience constraints in these "$\phi_H - \phi_L$" regions. However, with this decision rule, producer surplus simplifies to

$$PS = \tfrac{1}{2}(1-\phi_L) - \tfrac{1}{2}(\tfrac{1}{9}+\phi_L)\epsilon^2$$

which is concave in ϵ, since $\phi_L \in [{}^{-1}/_9, {}^{4}/_9)$. As illustrated by the oil investment example listed in Appendix B.2, non-linearity in ϵ indicates that perceptiveness will have an effect. Since producer surplus is concave in ϵ, the value of perceptiveness, in terms of producer surplus, is negative when either $(\phi_H, \phi_L) \in [{}^{14}/_9, \infty) \times ({}^{-1}/_9, {}^{4}/_9)$, or $(\phi_H, \phi_L) \in [1, {}^{14}/_9] \times ({}^{-1}/_9, {}^{4}/_9)$ such that $\phi_H \geq 1+\frac{1}{2}(1+\epsilon)(\frac{4}{9}-\phi_L)$.

When $(\phi_H, \phi_L) \in [1, {}^{14}/_9] \times [{}^{-1}/_9, {}^{4}/_9)$ and $\phi_H \leq 1+\frac{1}{2}(1+\epsilon)(\frac{4}{9}-\phi_L)$, as shown by Theorems 4.4.4-4.4.7, perceptiveness will affect the information designer's producer-surplus-maximizing decision rule since at least one of the decision rule variables is nonlinear in ϵ. Regardless of the four subregions within this "$\phi_H - \phi_L$" region, the OC^I_N will bind and the OC^I_E will be slack. Hence, in this region, perceptiveness will not affect whether either obedience constraint is binding or slack. However, when $(\phi_H, \phi_L) \in [1, ^{14}/_9] \times [^{-1}/_9, ^4/_9)$, $\phi_H - \phi_L \geq \frac{10}{9}$, and $\phi_H \leq 1 + \frac{1}{2}(1+\epsilon)(\frac{4}{9} - \phi_L)$,[8] substituting the producer-surplus-maximizing decision rule variables into Equation (4.9) finds that producer surplus equals

$$(\tfrac{4}{9} - \phi_L) + (1-\epsilon)\frac{(\frac{1}{9} + \phi_L)(\phi_H - 1)}{(\frac{4}{9} - \phi_L)},$$

which is linear in ϵ. Therefore, when $(\phi_H, \phi_L) \in [1, ^{14}/_9] \times [^{-1}/_9, ^4/_9)$, $\phi_H - \phi_L \geq \frac{10}{9}$, and $\phi_H \leq 1 + \frac{1}{2}(1+\epsilon)(\frac{4}{9} - \phi_L)$, perceptiveness does not affect the maximized value of producer surplus.

When $(\phi_H, \phi_L) \in [1, ^{14}/_9] \times [^{-1}/_9, ^4/_9)$, $\phi_H - \phi_L \leq \frac{10}{9}$, and $\phi_H \leq 1 + \frac{1}{2}(1+\epsilon)(\frac{4}{9} - \phi_L)$, substituting the producer-surplus-maximizing decision rule variables into Equation (4.9) finds that producer surplus is nonlinear in ϵ, so perceptiveness will affect the maximized producer surplus in Regions 4.3 and 4.4. More specifically, given the region specification that begins this paragraph as well as[9]

$$\phi_H \in [\tfrac{2}{(1+\epsilon)}(\tfrac{4}{9} + \epsilon) - \phi_L, 1 + \tfrac{1}{2}(1+\epsilon)(\tfrac{4}{9} - \phi_L)],$$

the second derivative of producer surplus with respect to ϵ, when substituting in the producer-surplus-maximizing decision rule variables, is

$$\frac{5[(\phi_H - \phi_L) - \frac{10}{9}]}{9(\frac{14}{9} - \phi_H)}. \tag{4.22}$$

Additionally, given the same initial region specification, but instead supplementing with[10]

$$\phi_H \in [1, \tfrac{2}{(1+\epsilon)}(\tfrac{4}{9} + \epsilon) - \phi_L],$$

the second derivative of producer surplus with respect to ϵ, when substituting in the producer-surplus-maximizing decision rule variables, is

$$\frac{5[\frac{10}{9} - (\phi_H - \phi_L)]}{9(\frac{4}{9} - \phi_L)}. \tag{4.23}$$

Since $\phi_H - \phi_L \leq \frac{10}{9}$ in Regions 4.3 and 4.4, Expression (4.22) shows that the second derivative

[8] This specification corresponds to Regions 4.1 and 4.2.
[9] This specification corresponds to Region 4.3.
[10] This specification corresponds to Region 4.4.

of producer surplus with respect to ϵ is non-positive in Region 4.3; whereas, Expression (4.23) shows that the second derivative of producer surplus with respect to ϵ is non-negative in Region 4.4. This implies that, if $\phi_H - \phi_L < \frac{10}{9}$, then perceptiveness will provide negative value in terms of producer surplus in Region 4.3. Whereas, if $\phi_H - \phi_L < \frac{10}{9}$, then perceptiveness will provide positive value in terms of producer surplus in Region 4.4.

4.6 Conclusion

In this chapter, I study the effect that perceptiveness has on an information design problem in a market-entry setting. I specifically investigate the effect that perceptiveness has when the information designer wishes to maximize producer surplus.

I find that perceptiveness affects the information designer's producer-surplus-maximizing decision rule when the difference between the high and low market-entry fee states is sufficiently small, as depicted by Regions 4.1-4.4. Additionally, perceptiveness provides positive value, in terms of producer surplus, only when the difference between the high and low state market-entry fees is sufficiently small and the high state market-entry fee is sufficiently low, as depicted by Region 4.4. Contrarily, perceptiveness inflicts negative value, in terms of producer surplus, when the high and low state market-entry fees are both sufficiently high, as depicted by Regions 2, 3, and 4.3. I also find that, despite perceptiveness affecting the information designer's producer-surplus-maximizing decision rule when the difference between the high and low market-entry fee states is sufficiently small, if such difference is still adequately large, the maximized value of producer surplus will not change with perceptiveness, as depicted by Regions 4.1 and 4.2. Finally, I find that perceptiveness has no effect when the low state market-entry fee is a sufficiently small, negative value, as depicted by Region 1.

Chapter 5

Conclusion

Throughout my thesis, I investigate the value and effect of perceptiveness in various game-theoretic settings. The first model, covered in Chapter 2, emulates a two-player, one-round game of poker. The second model, covered in Chapter 3, features a two-player market-entry game. The third model, covered in Chapter 4, depicts a two-player market-entry game that is influenced by an information designer who aims to maximize producer surplus.

A player is *expert* if and only if they know the value of a particular payoff-relevant parameter. Furthermore, a player is *perceptive* if and only if they know whether their opponent is expert. I let ϵ_i represent the probability of player i knowing the value of such payoff-relevant parameter. In my first model, the payoff-relevant parameters of interest are the players' hand values, (h_i, h_j). Here, player i is expert if and only if they know the value of h_i. In my second model, the payoff-relevant parameters of interest are the players' market-entry fees, (ϕ_i, ϕ_j). Here, player i is expert if and only if they know the value of ϕ_i. Lastly, in my third model, the payoff-relevant parameter of interest is the market-entry fee, ϕ, which is the same for both players. Here, player i is expert if and only if they know the value of ϕ.

In each model, I restrict attention to a different subset of (ϵ_A, ϵ_B) ordered pairs. In the first model, I consider

$$(\epsilon_A, \epsilon_B) \in \{(0,0), (0,1), (1,1), (0,1/2), (1,1/2), (1/2,1/2)\},$$

In the second model, I consider $(\epsilon_A, \epsilon_B) \in [0,1]^2$. In the third model, I consider $(\epsilon_A, \epsilon_B) \in (0,1)^2$ such that $\epsilon_A = \epsilon_B$.

In my thesis, I find that perceptiveness generally has value to players, whether that be from the perspective of a poker player, a player considering market-entry, or an information designer in a market-entry game. To draw comparisons between the three models, consider the information structure where $(\epsilon_A, \epsilon_B) = (1/2, 1/2)$. This particular information structure is good for comparison since it represents the information structure in which the perceptiveness of each player is minimized, and is also featured in my analysis of each model. In order to make the three models as comparable as possible, I set the model-specific parameters to be as follows. In Model 1, I set the chip endowment (K) equal to 15. In Model 2, I set the duopoly profit (π_D) equal to $4/9$, and I set the change in player A's perceptiveness (μ) equal to $1/5$. Lastly, in Model 3, I set the probability of the high-state market-entry fee occurring (h) equal to $1/2$, the duopoly profit (π_D) equal to $4/9$, the monopoly profit (π_M) equal to 1, the high-state market-entry fee (ϕ_H) equal to 1, the low-state market-entry fee (ϕ_L) equal to 0, and the change in the players' perceptiveness (μ) equal to $1/5$.

First, consider Model 1 when $(\epsilon_A, \epsilon_B) = (1/2, 1/2)$. Suppose player i could choose between a situation where they must decide whether to go all-in or fold before realizing j's expertise, or a situation where Nature can flip a coin to have the uncertainty of j's expertise realized prior to i deciding whether to go all-in or fold, with j knowing that i has realized this information. The former situation corresponds to when i is imperceptive and the latter situation corresponds to when i is perceptive. My results show that, for instance when $K = 15$, player i would prefer to have the uncertainty of j's expertise realized prior to deciding whether to go all-in or fold. So, in this situation, perceptiveness is beneficial to player i. This result is generally true for all stakes regardless of i's expertise and j's perceptiveness.[1]

Next, consider Model 2 when $(\epsilon_A, \epsilon_B) = (1/2, 1/2)$. As in Model 1, certain qualifications

[1]There exists some stakes where an expert player i has a negative value of perceptiveness when facing a perceptive player j. However, this case seems to be a particular exception. Also, for sufficiently low stakes, i's value of perceptiveness is zero. This occurs when $K = 1$, or when i is inexpert, j is perceptive, and $K \leq 2+\sqrt{5}$.

must be met in order for perceptiveness to have value. For instance, in Model 2, I focus attention to the "middle equilibrium". I also consider different values for duopolistic profit, similar to how I considered different stakes in Model 1. However, in general, starting from $(\epsilon_A, \epsilon_B) = (1/2, 1/2)$, staying close to this, introducing more perceptiveness to i, and maintaining focus on the "middle equilibrium", perceptiveness has value to i. Similar findings can be made at other (ϵ_A, ϵ_B) starting points, deviations, and duopolistic profit values.

Model 3 is different than Models 1 and 2 since Model 3 is normative, whereas Models 1 and 2 are positive. In Model 3, the information designer is trying to determine what could conceivably be possible in society in order to maximize producer surplus. Since this model is much more difficult, as it involves maximizing surplus over the correlated equilibria in a market-entry game, I had to assume that both players have the same market-entry fee and restrict my attention to symmetric decision rules. Hence, I can only consider moving from the $(\epsilon_A, \epsilon_B) = (1/2, 1/2)$ starting point by simultaneously changing both players' probability of being expert, as well as both players' perceptiveness. As it turns out, when $(h, \pi_D, \pi_M, \phi_H, \phi_L, \mu_i, \mu_j) = (1/2, 4/9, 1, 1, 0, 1/5, 1/5)$, while starting at $(\epsilon_A, \epsilon_B) = (1/2, 1/2)$, perceptiveness increases the information designer's maximum attainable producer surplus. So, perceptiveness is once again beneficial, just from a different perspective. Although this comparison has many fixed parameters, the parameters are fixed in a way that makes the results between the three models roughly compatible.

Appendix A

Appendices for Chapter 2

A.1 Payoff Grid: Inexpert Agent i Choosing All-In vs. Expert Agent j

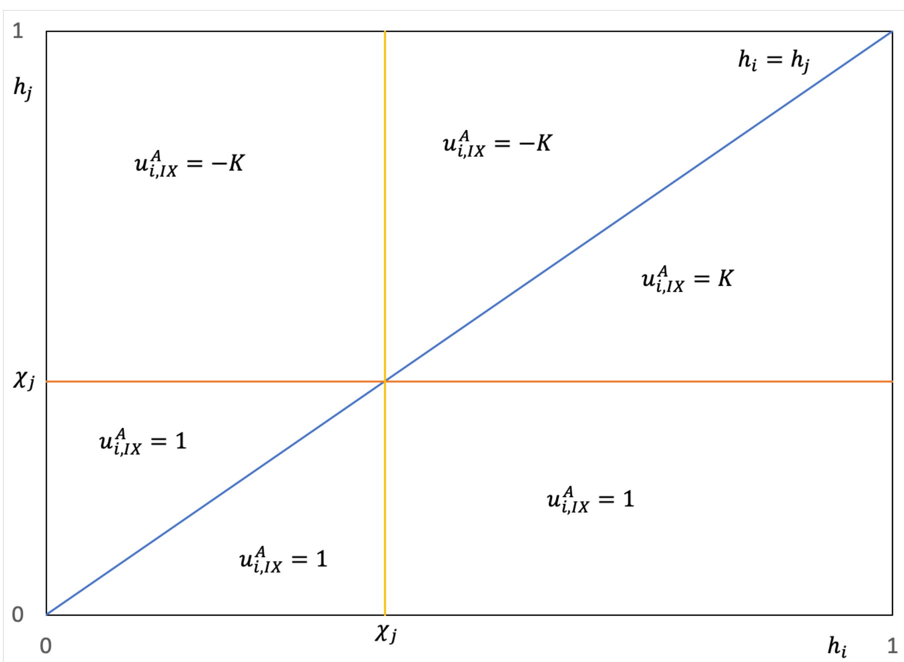


Figure A.1: An inexpert i's payoff from choosing all-in against an expert j using the cut-off strategy, χ_j, across all possible states.

Appendix B

Appendices for Chapter 3

B.1 Cournot Competition: Derivation of $\pi_D = 4/9$

.

I consider a market that follows a linear inverse demand curve and a situation such that the players' products are identically-perceived by consumers. Hence, the inverse market demand curve is

$$P(q_A, q_B) = a - bq_A - bq_B.$$

Suppose that both players enter the market. This implies that i's post-entry profit is given by

$$\pi_{i,D}(q_i, q_j) = P(q_i, q_j)q_i - MC_i q_i. \tag{B.1}$$

I also assume that $MC_i = 0$. Hence, Equation (B.1) simplifies to

$$\pi_{i,D}(q_i, q_j) = (a - bq_i - bq_j)q_i. \tag{B.2}$$

Taking the first-order condition of $\pi_{i,D}(q_i, q_j)$ with respect to q_i, for $i \in \{A,B\}$, yields

$$q_A^{BR}(q_B) = \frac{a - bq_B}{2b} \tag{B.3}$$

$$\text{and } q_B^{BR}(q_A) = \frac{a - bq_A}{2b}. \tag{B.4}$$

Substituting $q_B^{BR}(q_A)$ into Equation (B.3) yields

$$q_A^* = \frac{a}{3b}$$

Similarly, $q_B^* = \frac{a}{3b}$. Substituting $q_A^* = q_B^* = \frac{a}{3b}$ into Equation (B.2) finds that

$$\pi_{i,D}(q_i, q_j) = \frac{a^2}{9b}. \tag{B.5}$$

Now suppose that player i enters the market as a monopolist. This implies that i's post-entry profit is

$$\pi_{i,M}(q_i, 0) = (a - bq_i)q_i. \tag{B.6}$$

Taking the first-order condition of $\pi_{i,M}(q_i, 0)$ with respect to q_i yields

$$q_i^* = \frac{a}{2b}.$$

Substituting $q_i^* = \frac{a}{2b}$ into Equation (B.6) finds that

$$\pi_{i,M} = \frac{a^2}{4b}.$$

Normalizing $\pi_{i,M} = 1$ implies that $\frac{a^2}{b} = 4$. Substituting this into Equation (B.5) yields $\pi_{i,D} = 4/9$.

B.2 Example: Oil Investment Decision Problem

The value of perceptiveness I study is analogous to the value of testing for oil in the following decision problem. Consider a risk-neutral oil company that knows that oil exists in a certain location with a probability of $p \in [0,1]$. The company can choose $x \in \{0,1\}$, where $x = 1$ ($x = 0$) represents when the company decides to drill (not drill) for oil. The state of the world is given by $s \in \{0,1\}$, where $s = 1$ ($s = 0$) indicates that oil is present (absent). The value from striking oil is v, while the cost of building an oil rig is c, such that v and c are positive numbers. The company's payoff function is summarized as

$$\pi(v, c, s, x) = \begin{cases} v - c & \text{if } s = 1 \ \& \ x = 1 \\ -c & \text{if } s = 0 \ \& \ x = 1 \\ 0 & \text{if } x = 0. \end{cases}$$

Given the company's payoff function, the company's expected payoff is

$$E[\pi(v, c, p, x)] = \begin{cases} pv - c & \text{if } x = 1 \\ 0 & \text{if } x = 0. \end{cases}$$

This implies that the company should drill for oil if $p \geq \frac{c}{v}$.

Suppose $v = 100$, $c = 70$, and $p = 4/5$. The oil company should choose to drill ($x = 1$) since $p > 7/10$. By drilling, the company has an expected profit of 10. Now suppose the company is presented with the opportunity to acquire additional information pertaining to the probability of striking oil. There is a 50% chance this information will be favourable, allowing the company to update its beliefs to $p = 1$. However, there is a 50% chance this information will be unfavourable, causing the company to update its beliefs to $p = 3/5$. The company's expected profit with this information is 15. Hence, this information is clearly valuable to the company as it increases the company's expected profit by 5. This example is graphically depicted by Figure B.1.[1]

The value of information in this example comes from the oil company's ability to make its decision contingent on the updated probabilities. Without receiving the additional information, the company will decide to drill. This decision will remain unchanged in the presence of favourable information. However, in the presence of unfavourable information, the company will switch its decision to deciding not to drill. Generally speaking, if one of the post-information states results in the company switching its decision away from what it would have previously chosen, the value of information is strictly positive. Otherwise, the value of

[1] In Figure B.1, the blue solid line represents the company's expected profit without additional information. The yellow dashed line represents the company's expected profit with additional information. The green dotted line represents the value of information when the probability of striking oil is $4/5$.

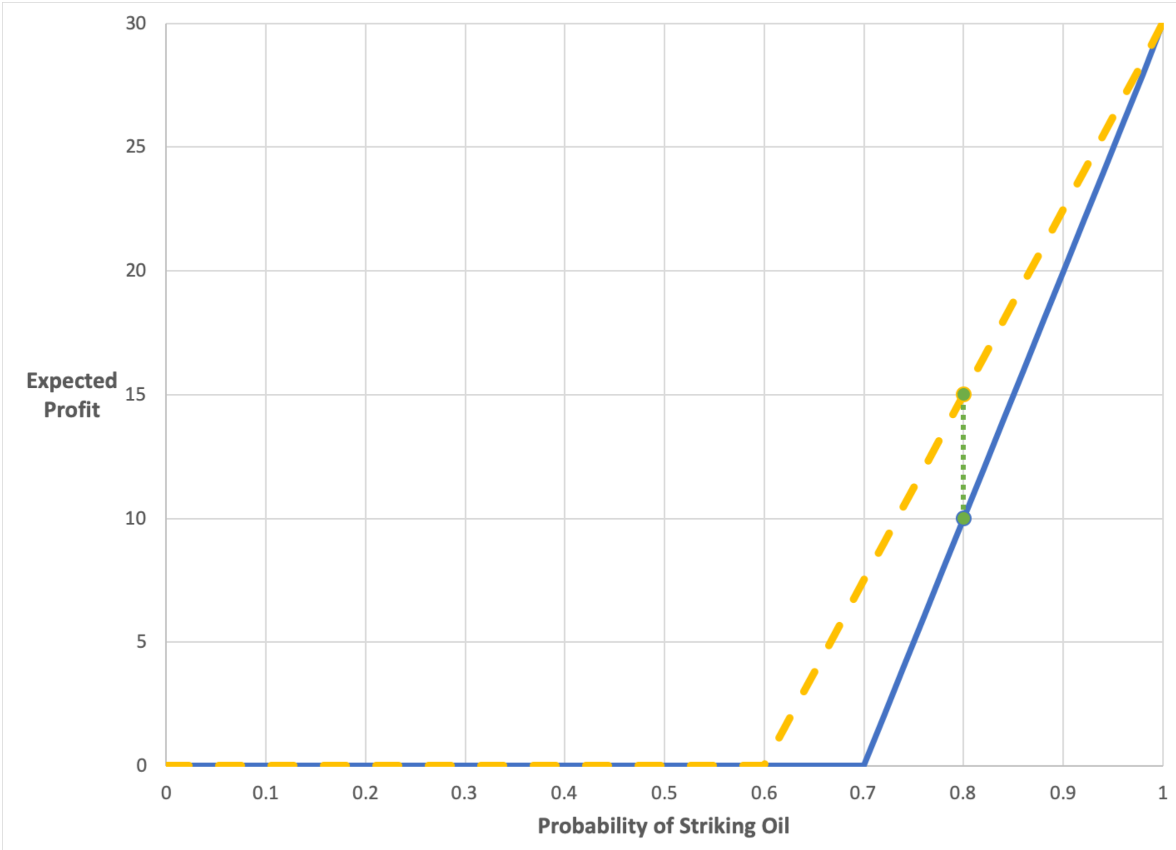


Figure B.1: Oil company's expected profit from striking oil given the probability of striking oil.

information is zero. Proposition B.2.1 claims that the oil company's expected profit function is convex for all p, and strictly convex if and only if a mean-preserving spread of p has bound values p_{Low} and p_{High} such that $0 \leq p_{Low} < \frac{c}{v} \leq p_{High} \leq 1$. In other words, the expected profit function is strictly convex if and only if the oil company makes its decision based on the information it acquires. From this, it follows that the value of information is zero if and only if the information puts the oil company at a mean-preserving spread of p with $p_{Low} \geq \frac{c}{v}$ or $p_{High} < \frac{c}{v}$. Additionally, the value of information is positive if and only if the information puts the oil company at a mean-preserving spread of p with $0 \leq p_{Low} < \frac{c}{v} \leq p_{High} \leq 1$.

Proposition B.2.1 *Suppose $v > c > 0$.*

$$E[\pi(p)] = \begin{cases} pv - c & \text{if } p \geq \frac{c}{v} \\ 0 & \text{if } p < \frac{c}{v} \end{cases}$$

is convex for all $p \in [0,1]$, and strictly convex if and only if a mean-preserving spread of p has bound values p_{Low} and p_{High} such that $0 \leq p_{Low} < \frac{c}{v} \leq p_{High} \leq 1$.

Proof Suppose $(p_1, p_2) \in [0,1]^2$ such that $p_1 < p_2$, $v > c > 0$, and $\alpha \in [0,1]$. If $p_2 < \frac{c}{v}$, then $E[\pi(\alpha p_1 + (1-\alpha)p_2)] = 0 = \alpha E[\pi(p_1)] + (1-\alpha)E[\pi(p_2)]$, which implies that $E[\pi(p)]$ is convex, but not strictly convex for all $\alpha \in [0,1]$. If $p_1 \geq \frac{c}{v}$, then

$$\begin{aligned} E[\pi(\alpha p_1 + (1-\alpha)p_2)] &= (\alpha p_1 + (1-\alpha)p_2)v - c \\ &= \alpha(p_1 v - c) + (1-\alpha)(p_2 v - c) \\ &= \alpha E[\pi(p_1)] + (1-\alpha)E[\pi(p_2)], \end{aligned}$$

which implies that $E[\pi(p)]$ is convex, but not strictly convex for all $\alpha \in [0,1]$. Finally, if $p_1 < \frac{c}{v} \leq p_2$, then

$$\begin{aligned} E[\pi(\alpha p_1 + (1-\alpha)p_2)] &= (\alpha p_1 + (1-\alpha)p_2)v - c \\ &= \alpha p_1 v + (1-\alpha)p_2 v - c \\ &< \alpha c + (1-\alpha)p_2 v - c \\ &= \alpha(0) + (1-\alpha)(p_2 v - c) \\ &= \alpha E[\pi(p_1)] + (1-\alpha)E[\pi(p_2)], \end{aligned}$$

which implies that $E[\pi(p)]$ is strictly convex for all $\alpha \in [0,1]$.

This oil investment decision problem and subsequent analysis to derive the value of information was inspired by Section 12.5 of DeGroot (2005). In such section, DeGroot (2005) develops optimal bounded sequential decision procedures that feature a statistician deciding whether to observe an additional observation at some arbitrary positive cost. By finding where the oil company is indifferent between having the pre-information expected profit in addition to the cost of acquiring such information and the post-information expected profit, I am able to infer the value of this information.

B.3 Proof of Corollary 3.4.5

Proof To prove that perceptiveness has zero value and effect for i when their opponent j enters the market with the same probability regardless of j's expertise, it is sufficient to show that agent i's best response and ex-ante expected payoff is independent of ϵ_j when $\chi_j = \eta_j$.

Suppose $(\epsilon_i, \epsilon_j) \in (0,1)^2$ and that j enters the market with probability ρ_j regardless of j's expertise ($\chi_j = \eta_j = \rho_j$). Further suppose that i is expert. Equation (3.2) shows that i's best response will be $\chi_i^{BR}(\rho_j) = 1 - \rho_j(1-\pi_D)$. Moreover, Equation (3.17) shows that i's ex-ante expected payoff will be $EU_i^{EX}(\chi_i, \rho_j) = \chi_i(1 - \rho_j(1-\pi_D) - \frac{1}{2}\chi_i)$. Now suppose that i is inexpert. Equation (3.4) shows that i's best response will be

$$\eta_i^{BR}(\rho_j) = \begin{cases} \{1\} & \text{if } \rho_j < \frac{1}{2(1-\pi_D)} \\ \{0\} & \text{if } \rho_j > \frac{1}{2(1-\pi_D)} \\ [0,1] & \text{if } \rho_j = \frac{1}{2(1-\pi_D)}. \end{cases}$$

Finally, Equation (3.19) shows that i's ex-ante expected payoff will be $EU_i^{IX}(\eta_i, \rho_j) = \eta_i(\frac{1}{2} - (1-\pi_D)\rho_j)$. Therefore, if $\chi_j = \eta_j$, i's best response and ex-ante expected payoff will be independent of ϵ_j. Since i's perceptiveness is defined by ϵ_j, i's best response and ex-ante expected payoff will be independent of i's perceptiveness if $\chi_j = \eta_j$.[2]

[2]Recall that i's degree of perceptiveness is depicted by $|\epsilon_j - \frac{1}{2}|$, as described in Footnote 11 of Chapter 3.

Appendix C

Appendices for Chapter 4

C.1 Microfoundations

Player i's profit upon entering the market is given by $\pi_i = (a - bq_j - bq_i)q_i - \phi_i$. When i decides to not enter the market, i earns zero profit. Therefore, if neither player enters the market, zero quantity will be produced and no surplus will be realized.

If both players enter, they will each produce a quantity of $Q_{i,D} = \frac{a}{3b}$, thereby making $Q_{T,D} = \frac{2a}{3b}$ and $P_D = \frac{a}{3}$.[1] In this case, consumer surplus is $CS_D = \frac{2a^2}{9b}$, total producer surplus[2] is $PS_{T,D} = \frac{2a^2}{9b} - \phi_i - \phi_j$, and total surplus is $TS_D = \frac{4a^2}{9b} - \phi_i - \phi_j$. Furthermore, player i's producer surplus is $PS_{i,D} = \frac{a^2}{9b} - \phi_i$, so $\pi_D = \frac{a^2}{9b}$. The subscript labelled as "D" indicates the duopolistic case. Figure C.1 graphically depicts the surplus regions arising from a duopoly.

Finally, if only one player enters, the player, i, entering the market will produce a quantity of $Q_{T,M} = \frac{a}{2b}$, while the other player will produce nothing, thereby making $P_M = \frac{a}{2}$.[3] In this case, consumer surplus is $CS_M = \frac{a^2}{8b}$, total producer surplus, which is solely realized by the player entering the market, is $PS_{T,M} = \frac{a^2}{4b} - \phi_i$, hence $\pi_M = \frac{a^2}{4b}$, and total surplus is $TS_M = \frac{3a^2}{8b} - \phi_i$.

[1] These values can be obtained by using standard unconstrained optimization techniques, taking first-order conditions with respect to q_i.

[2] When calculating producer surplus, I include the market-entry fee as opposed to excluding it. Thus, producer surplus here can be considered as earned profit.

[3] Again, these values can be obtained by using standard unconstrained optimization techniques, taking first-order conditions with respect to q_i.

The subscript labelled as "M" indicates the monopolistic case. Figure C.2 graphically depicts the surplus regions arising from a monopoly.

Throughout my analysis, I normalize $\pi_M = 1$. This implies a normalization of $\frac{a^2}{b} = 4$. Subsequently, for Cournot competition, $\pi_D = \frac{4}{9}$, $CS_D = \frac{8}{9}$, $PS_{T,D} = \frac{8}{9} - \phi_i - \phi_j$, and $TS_D = \frac{16}{9} - \phi_i - \phi_j$. Also, for Cournot competition when i enters the market and j does not enter the market, $CS_M = \frac{1}{2}$, $PS_{T,D} = 1 - \phi_i$, and $TS_D = \frac{3}{2} - \phi_i$.

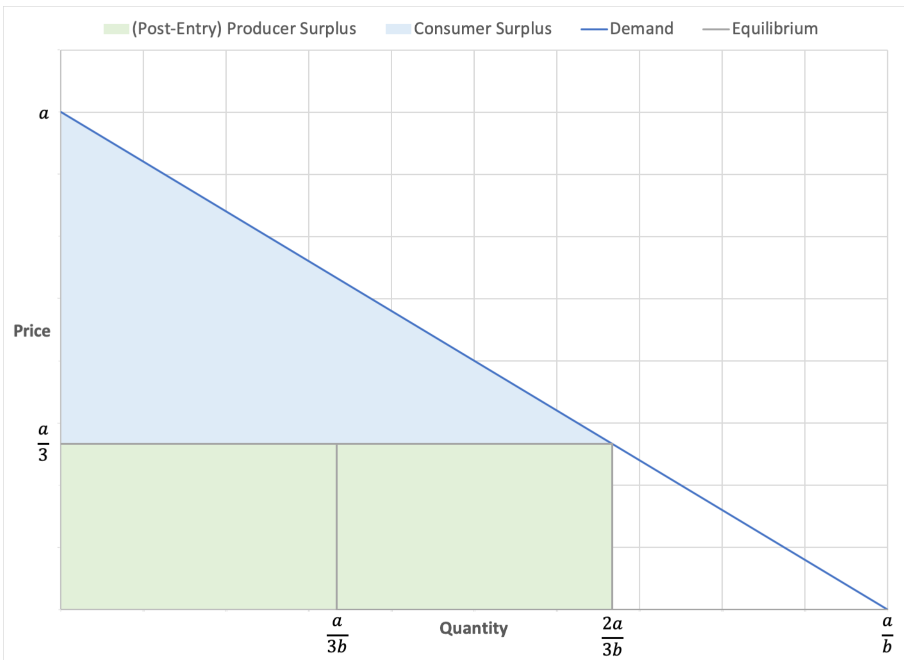


Figure C.1: Diagram displaying the consumer and post-entry producer surplus for a duopoly under Cournot competition.

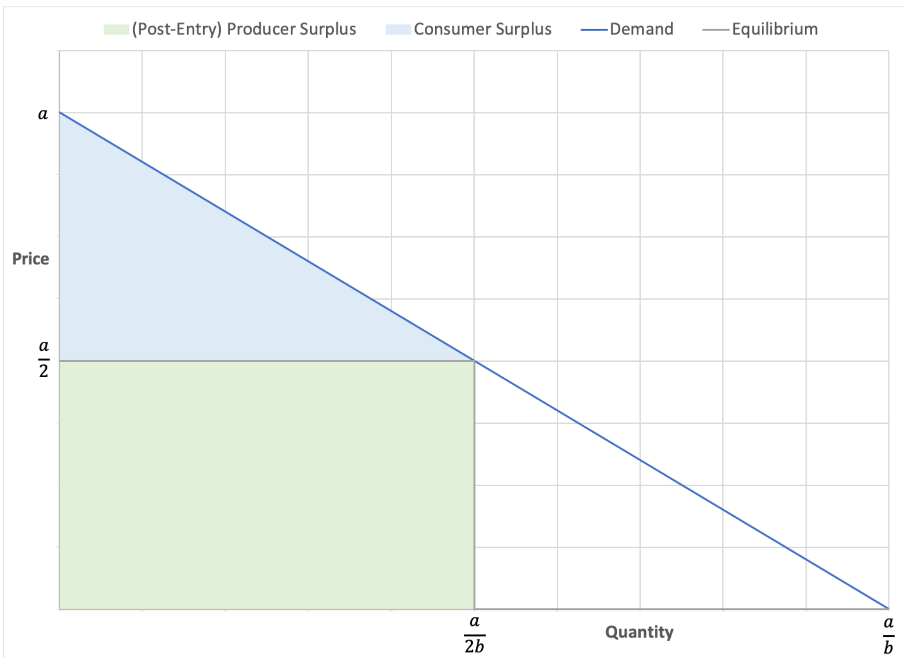


Figure C.2: Diagram displaying the consumer and post-entry producer surplus for a monopoly under Cournot competition.

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