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Supervisor: Caucutt, Elizabeth M., *The University of Western Ontario* A thesis submitted in partial fulfillment of the requirements for the Doctor of Philosophy degree in Economics © Hyeongsuk Jin 2020

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## Abstract

This thesis studies decisions the family makes regarding household unions, fertility, and child investment. Chapter 2 studies the educational gradient in non-marital fertility and posits that cohabitation is a driver behind the gradient. I build a lifecycle model of fertility and household union choices, featuring a trade-off between quality and quantity of children. Using the model calibrated to the U.S. data, I study implications of introducing common-law marriage, where cohabiting parents are considered to be married couples. I find that the policy leads fewer people to choose cohabitation, and more children are born to married parents. As a result, children receive 20% more parental investment during their childhood compared to the case without the policy.

Chapter 3 studies the trend of household unions. I posit that the trend of delaying marriage arises from a substitution toward cohabitation. I build a bilateral household union decision model that includes cohabitation and marriage as unions. Using the model, I study implications of increasing income uncertainty and closing the gender earnings gap on the trend. I find that increasing income volatility alone can account for the majority of changes in household unions. The result also highlights the importance of considering cohabitation separately from marriage, which is not common in the literature.

In Chapter 4, I study the educational gradients in mothers' time allocation. According to the American Time Use Survey, mothers with higher education spend more time on childcare compared to mothers with lower education. Despite the growing evidence of the significance of parental time on child development, few studies attempt to understand the observed gradients in time allocation. I examine conditions that generate the gradient in a standard time allocation model. I find that the productivity of time input on children and the substitutability of time and goods inputs are the two parameters that determine the gradient in childcare time.

**Keywords:** Household union, Marriage and cohabitation, Fertility, Child investment, Parental time allocation

# **Summary for Lay Audience**

This thesis studies decisions the family makes regarding household unions, fertility, and child investment. In Chapter 2, I document that less-educated women are more likely to have their children outside marriage than more-educated women. I find that cohabitation is a driver behind this pattern. I develop an economic model of fertility and household union choices and use the model to study implications of introducing common-law marriage, where cohabiting parents are considered to be married couples. I find that the policy leads fewer people to choose cohabitation, and more children are born to married parents. As a result, children receive 20% more parental investment during their childhood compared to the case without the policy.

Chapter 3 studies the trend of delaying marriages. I posit that this trend arises from a substitution toward cohabitation. In the labor market, income uncertainty has been increasing, and the gender earnings gap has been closing over time. Using an economic model, I find that increasing income uncertainty is the major cause of the changes in household unions, and the gender gap plays little role.

In Chapter 4, I study the different patterns of mothers' activities by their education level. According to the American Time Use Survey, mothers with higher education spend more time on childcare than mothers with lower education. Despite the growing evidence of the significance of parental time on child development, few studies attempt to understand the observed differences in time allocation. The model suggests that the observed differences can arise if more-educated mothers' time is more valuable to their children's development than less-educated mother's time. Using the model, I study the costs and implications of different policies supporting mothers' childcare.

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To My Parents

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# Chapter 1

## Introduction

My thesis consists of three essays on decisions the family makes. In the first two essays, I study the household union formation including both marriage and cohabitation. The third essay is about the educational gradient in mothers' time allocation.

Chapter 2 studies the educational gradient in non-marital fertility and posits that cohabitation is a driver behind the gradient. Non-marital cohabitation, living with a partner without getting married, is a common practice in the U.S., and increasingly many children are born to cohabiting parents. Despite this prevalence, the literature on family economics typically ignores cohabitation as a form of household union distinct from marriage or being single. I document that there are educational gradients in fertility and union choices. Less educated women are more likely to cohabit and give birth while cohabiting than are more educated women.

I build a lifecycle model of fertility and household union choices, featuring a trade-off between quality and quantity of children. In the model, I assume that married couples pay a cost to divorce. In contrast, cohabitation provides costless separation. I calibrate the model so that agents in the model behave similarly to the observed National Longitudinal Study of Youth 1979 (NLSY79) cohort. Using the calibrated model, I introduce common-law marriage, where cohabiting parents are considered to be married couples. I find that the policy leads fewer people to choose cohabitation, and more children are born to married parents. As a result, children receive 20% more investment during their childhood compared to the case without the policy.

Chapter 3 studies the time trend of household unions. In the 2006–2010 National Survey of Family Growth (NSFG), 11% of 15–44 year old women in the U.S. were cohabiting. The majority of women have cohabited with a partner according to the survey. However, this is a new practice. In the NSFG Cycle 3 (1982), only about 3% of women were cohabiting. In this chapter, I study what has caused this change by building a marriage and cohabitation choice

model. In the model, living in a union provides economic benefits through economies of scale in consumption and a non-economic benefit (or cost) from match quality. Marriage provides greater economies of scale in consumption compared to cohabitation. However, ending a marriage incurs a divorce cost, whereas ending a cohabitation is costless. The uncertainty in the model comes from income shocks and match quality shocks. Couples facing higher chances of ending the relationship choose to cohabit despite the smaller economic benefit, and those with lower chances marry. Using the model calibrated to the U.S. data, I find that increasing income volatility has contributed to the changes in household unions. The closing of the gender earnings gap plays little role in determining this trend.

I study the educational gradients in mothers' time allocation in Chapter 4. According to the American Time Use Survey (ATUS), mothers with higher education spend more time on childcare compared to mothers with lower education. Despite the growing evidence of the significance of parental time on child development, few studies attempt to understand the observed gradients in time allocation. Using a simple time allocation model, I find that the productivity of time input on children and the substitutability of time and goods inputs are two parameters that determine the gradient in childcare time. I calibrate the model parameters to match the time use data. I find that a model with the productivity of time input in the children's human capital production being the same as the wage matches the data well. Using the calibrated model, I study the cost of fiscal policies that target increasing children's human capital. I find that subsidizing the market input in children's human capital is less costly than providing lump-sum transfers to parents.

# Chapter 2

# Cohabitation and educational gradients in non-marital fertility

#### 2.1 Introduction

Non-marital cohabitation, living with a romantic partner outside of marriage, is prevalent. According to the 2013 – 2015 NSFG, 14% of 15 – 44 years old women in the U.S. are currently cohabiting, and 56% of them have ever cohabited. Furthermore, of the respondents' children who were born after 2005, 27% were born to cohabiting mothers. Despite the prevalence, economic analysis of the family typically ignores cohabitation as a type of union distinct from being married or single. In this chapter, I document educational gradients in union and fertility decisions. I find that less-educated women are more likely to cohabit and give birth while cohabiting than more-educated women. Then, I build a simple life-cycle model of union and fertility choices and show it is consistent with the observed data.

Understanding the choice between marriage and cohabitation is important because it is closely related to fertility behaviors and single parenthood. Many studies find that children who grow up with both biological parents show higher achievement along various dimensions compared to those who grow up with a single parent (e.g., Abbott, 2015; Tartari, 2015; Kearney and Levine, 2017). As periods of cohabitation are typically shorter than marriage and do not necessarily end in marriage, children born to cohabiting parents are more likely to live in single-parent households during their childhood than children born to married parents. According to the combined samples from the NLSY79 and its accompanying the Children of the NLSY (CNLSY), 82% of children born to a married mother live with their father at age 6 or 7, whereas only 58% of children born to a cohabiting mother do.

Some developed countries have introduced the institution of common-law marriage to pro-

tect the vulnerable parties in cohabitation. The details vary across jurisdictions, but cohabiting couples who meet certain criteria face legal obligations, which make separations harder. However, cohabiting couples in the U.S. are under no legal regulations.<sup>1</sup> This makes American cohabitation a particularly short-lived relationship compared to cohabitations in other countries (Heuveline and Timberlake, 2004). I study the effect of introducing common-law marriage similar to other countries in the U.S. using a calibrated model.

Only a few studies in the growing literature of family economics consider cohabitation separately from being married or single. To study the volatile nature of cohabitation, Brien *et al.* (2006) build a learning model of match quality. They estimate their model using the National Longitudinal Study of the High School Class of 1972 (NLS72), and find that learning about match quality can explain the choice between marriage and cohabitation. Gemici and Laufer (2012) build a model of cohabitation that provides less commitment because of the lower separation cost than marriage. Adamopoulou (2014) finds that the narrowing gender wage gap is a reason for increasing cohabitation. However, these models on union choice do not consider the impact of union choice on fertility and investment in children. This chapter is also related to Moschini (2020). She finds that the family structure plays a crucial role in child development, and the impact of child care subsidy policies on children differ by their family structure.

I am interested in the choice of household union for its impact on children. In this chapter, in order to have a tractable model that captures salient features of data, I focus on the household union and fertility decisions, and the children are introduced in a reduced-form. My model builds on the life-cycle model of imperfect fertility choice from Choi (2017). I incorporate endogenous union choice, including cohabitation and marriage, in the model. There are two differences between marriage and cohabitation. First, the probability of having an exogenous union dissolution differs between marriage and cohabitation. Second, there is a divorce cost for marriage, but ending a cohabitation is costless. Using the calibrated model, I study the effect of introducing common-law marriage, which treats cohabiting couples who give birth the same as married couples. In the counterfactual case with common-law marriage, fewer people cohabit compared to the baseline case. Furthermore, cohabiting women exert more effort to avoid having children while cohabiting. So fewer children are born to cohabiting parents when common-law marriage is implemented.

This chapter makes two contributions to the literature. First, I study the union decision,

<sup>&</sup>lt;sup>1</sup>Eleven states and the District of Columbia have 'common-law marriage' in their family law. But to be recognized as a common-law married couple in those states, a couple has to have a written contract or evidence that they hold themselves out as spouses, which typically engages lengthy legal disputes in case of their separation. They are different from common-law marriages in other jurisdictions in the sense that no conditions grant the status of common-law married automatically. See Dwyer (2012) for details.

including cohabitation, together with fertility and child investment. This allows the model to generate the educational gradients in union and fertility with the trade-off between quantity and quality of children. This chapter is the first study to understand the choices simultaneously. The literature on household formation and the literature on fertility and child investment have little intersection.<sup>2</sup> Furthermore, as mentioned, the literature on household formation generally focuses on decisions regarding marriage and divorce, without considering cohabitation as an option. To match the observed gradient in union choice, previous studies of cohabitation typically assume people have different preferences toward marriage and cohabitation, and their preferences are correlated with education (Brien *et al.*, 2006; Gemici and Laufer, 2012). The second contribution this chapter makes is understanding the implications of introducing common-law marriage. The model allows me to study the effect of introducing the policy both on union choice and fertility decisions. Importantly, I find that implementing common-law marriage affects fertility and investment in child decisions as well as union choices. More children grow up in two-parent households and receive more investment from their parents compared to the case without the policy.

The rest of the paper is organized as follows. Section 2.2 documents the educational gradients in fertility and household union choice observed in data. Section 2.3 presents the model, and Section 2.4 describes the calibration and the model fit. Using the calibrated model, I conduct quantitative exercises in Section 2.5. And Section 2.6 concludes.

#### 2.2 Evidence of educational gradients

In this section, I document the educational gradients in union and fertility choices observed in samples from the NLSY79, the CNLSY, and the NSFG Cycle 6 (2002). The details of the sample selection are summarized in Appendix A.1. The educational gradients indicate that the choices of union and fertility are interdependent. Further, there is a correlation between the gradients and the intergenerational persistence of educational attainment. These findings motivate studying the relationship between union and fertility decisions, and its impact on children.

Table 2.1 tabulates the percentage of unmarried mothers at a child's birth for each education group using the sample from the NLSY79. Compared to women with some college or more (more-educated women), women with no college education (less-educated women) are more likely to have had non-marital births.<sup>3</sup> About 30% of less-educated women have given birth

<sup>&</sup>lt;sup>2</sup>Caucutt et al. (2002), Greenwood et al. (2003), and Regalia et al. (2011) are the few exceptions.

<sup>&</sup>lt;sup>3</sup>These relative differences are not caused by births to teen mothers (19-year-old or younger at child's birth) or by a particular ethnic group. Although the levels change slightly, the relative differences remain the same if

outside of marriage, and only 11% more-educated women have.

	Less-educated women (1)	More-educated women (2)	Difference (1) - (2)
Any child	30.46	11.18	19.28 (1.74)
First child	26.95	10.38	16.57 (1.65)
Second child	15.89	5.19	10.70 (1.35)
Third child	22.44	4.54	17.89 (2.62)

Table 2.1: Percentage of unmarried mothers at child's birth

Samples from the NLSY79, estimates are computed using provided weights. Standard errors of differences are in parentheses.

'Less-educated women' are those who have no college education at age 22; 'moreeducated women' are those who have at least some college education at age 22.

A driver of this educational gradient of non-marital birth is cohabitation. Table 2.2 shows the percentage of women who have ever cohabited by education group using samples from the NSFG Cycle 6. Less-educated women are more likely than more-educated women to have cohabited. Also, Table 2.3 shows that less-educated women are more likely to give birth during cohabitation. However, as shown in Table 2.4, about half of cohabitations did not result in marriage. Consequently, children born to cohabiting mothers are more likely to grow up without their biological fathers in the household.

Although the NLSY79 did not collect detailed information on the respondents' cohabitation before 1990, the CNLSY surveys whether the child is living with one's father. Using the mother's marital status in the NLSY79, the combined sample reveals the union transition after a child is born. Conditional on living with his or her father when a child is born or one year old, 80% of children in the sample are living with their fathers at age 6 or 7.<sup>4</sup> Table 2.5 tabulates the percentage of children living with their fathers at age 6 or 7 conditional on living with fathers at age 0 or 1. This percentage differs by the educational attainment of mothers. About 74% of children whose mothers do not have any college education by age 22 ('less-educated') live with their fathers at age 6 or 7, and 87% of the children whose mothers have at least some college education by age 22 ('more-educated') do. Conditional on their mothers being married

the sample excludes births to teen mothers (Table A.1). The same is true for ethnic groups. The levels vary substantially across ethnic groups, but the same relative relationships remain (Table A.2).

<sup>&</sup>lt;sup>4</sup>As the CNLSY is a biennial survey, the information is collected every two years. For children who were born in the year when there was no survey, it is not easy to know whether the father was living together. Especially so if the mother was not married. To increase the sample size and ease the analysis, I group the two years' birth cohorts together.

Any cohabitation			No	n-marital cohabitati	on
Less-educated women (1)	More-educated women (2)	Difference (1) - (2)	Less-educated women (3)	More-educated women (4)	Difference (3) - (4)
62.15	49.55	12.60 (3.52)	26.75	16.78	9.96 (3.05)

Table 2.2: Percentage of women ever cohabited before

Samples from the NSFG Cycle 6, born between April, 1957 and December, 1967 (comparable to the NLSY79 cohort, 37 - 44 years old on the day of interview). Estimates are computed using provided weights and survey design variables. Standard errors of differences are in parentheses.

'Any cohabitation' includes pre-marital cohabitation (married with the cohabitation partner) and non-marital cohabitation; 'Non-marital cohabitation' includes current cohabitation and cohabitations that end without marriage.

'Less-educated women' are those who have no college education at age 22; 'more-educated women' are those who have at least some college education at age 22.

at age 0 or 1, children of more-educated mothers are more likely to live with their fathers at age 6 or 7 than children of less-educated mothers.

The literature on child development documents that, on average, children living with both biological parents have better outcomes for various measures.<sup>5</sup> The relationship, regarding children's college attendance rates, is observed in the sample as well. Table 2.6 shows the percentage of children with some college or higher education at age 22 or 23 conditional on their mothers' education. There is an intergenerational correlation of educational attainment. Children of less-educated mothers are less likely to have a college education at age 22 or 23 than children of more-educated mothers. The overall difference between the two groups of children is 20 percentage points (61% versus 81%).

Part of the intergenerational correlation of educational attainment comes from the difference in the parental composition between children of less-educated mothers and more-educated mothers. As shown in Table 2.5, children of less-educated mothers are more likely to grow up without having their biological fathers at home than children of more-educated mothers. When children who lived with their fathers at age 0 or 1 are considered, the gap in children's college enrollment decreases from 20 percentage points to 16 percentage points (68% versus 84%). The difference shrinks further, if children who lived with their father longer are considered. For the children who lived with their fathers at age 6 or 7, the difference reduces to 15 percentage points (72% versus 87%).

<sup>&</sup>lt;sup>5</sup>Abbott (2015) find that imperfect substitutability between parents' time inputs and market purchased alternatives is a reason for the disadvantage of children growing up in single parent households. By examining children whose parents divorced, Tartari (2015) finds that parents' divorce affects child's test score negatively. Kearney and Levine (2017) document a 'marriage premium for children' regarding various aspects of children.

Any cohabitation			Non-marital cohabitation		
Less-educated women (1)	More-educated women (2)	Difference (1) - (2)	Less-educated women (3)	More-educated women (4)	Difference (3) - (4)
26.69	14.69	11.99 (3.51)	31.65	21.30	10.35 (5.97)

Table 2.3: Percentage of women gave birth (pregnancy ended as live birth) during cohabitation among those who have cohabited

Samples from the NSFG Cycle 6, born between April, 1957 and December, 1967 (comparable to the NLSY79 cohort, 37 – 44 years old on the day of interview). Estimates are computed using provided weights and survey design variables.

'Any cohabitation' includes pre-marital cohabitation (married with the cohabitation partner) and non-marital cohabitation; 'Non-marital cohabitation' includes current cohabitation and cohabitations that end without marriage.

'Less-educated women' are those who have no college education at age 22; 'more-educated women' are those who have at least some college education at age 22.

Table 2.4: Percentage of cohabitations that did not end with marriage after giving birth

Less-educated women (1)	More-educated women (2)	Difference (1) - (2)
56.88	48.81	8.07 (11.00)

Samples from the NSFG Cycle 6, born between April, 1957 and December, 1967 (comparable to the NLSY79 cohort, 37 – 44 years old on the day of interview). Estimates are computed using provided weights and survey design variables.

'Less-educated women' are those who have no college education at age 22; 'more-educated women' are those who have at least some college education at age 22.

#### 2.3 Model

The model takes a childless single woman who has finished her education as a utility maximizing decision maker.<sup>6</sup> Over the life-cycle, she meets potential partners and makes union transitions. Her partner provides additional income and match quality. Her utility depends on consumption, match quality, and number and quality of her children. She decides whether or

<sup>&</sup>lt;sup>6</sup>To simplify the model, I abstract from the interaction between household members in this chapter and treat men as completely passive agents. It is challenging to consider both men and women as decision makers as it involves bargaining between them, which is studied in Chapter 3. In addition, as the current model includes fertility and spending on children decisions in an environment where couples can separate and match with other partners, considering the interaction between divorced men and their children living with their ex-partners imposes additional challenges in modeling. By building a unitary model, the possible inefficiency in child investment from the collective decision making and lack of complete insurance against divorce is not considered. See Chiappori and Mazzocco (2017) for a survey of the literature on collective household models and their implications.

	Less-educated	More-educated
All children (regardless of mother's marital status)	74	87
Mother was married at age 0 or 1	77	88
Mother was cohabiting at age 0 or 1	59	56

Table 2.5: Percentages of children who were living with their father at age 6 or 7, by mothers' level of education

Samples from the NLSY79 and the CNLSY.

'Less-educated' represents children whose mothers have no college education at their age 22; 'More-educated' represents those whose mothers have at least some college education at their age 22.

Table 2.6: Percentages of children who have at least some college education at age 22 or 23, by mother's level of education

	Less-educated	More-educated	
All children (regardless of father's presence)	61	81	
Children lived with fathers at age 0 or 1	68	84	
Children lived with fathers at age 6 or 7	72	87	

Samples from the NLSY79 and the CNLSY.

'Less-educated' represents children (young adults) whose mothers do not have any college education at their age 22; 'more-educated' represents those whose mothers have some college education at their age 22.

not to have an additional child, but she has imperfect control over her fertility.

#### **2.3.1** Life-cycle and endowments

At the beginning of her life, the woman has no partner or child. A model period is one year. There are two levels of educational attainment, high and low. I assume that the first model period is age 18 for a less-educated woman and age 21 for a more-educated woman. The model life of a woman ends at age 44. She receives labor income each period, and her wage rate depends on her education level. Children impose time costs on their mother: a woman's income depends on the number of children she has. Conditional on her education level and number of children, a woman's income is given. That is, there is no uncertainty in the income.<sup>7</sup>

<sup>&</sup>lt;sup>7</sup>To simplify the model, I assume no labor-leisure choice, and everyone supplies their entire time endowments to the labor market as a result. However, there is evidence that cohabitants and married couples allocate their time diffrently, even with children (Gemici and Laufer, 2012; Wong, 2016). On average, cohabiting women spend more time in labor market than married women, and the opposite is true for men. Although I do not consider parents' time as an input in children in this chapter, the literature finds that parents' time is a significant input on children's development, and the goods input or market-purchased input is an imperfect substitute of parents' time, as discussed in Chapter 4. Therefore, it is possible that children living with cohabiting parents are worse off than their counterparts with married parents even without their parents' seperation, a channel that is absent in this chapter. In this sense, the model in this chapter may underestimate the disadvantage of cohabiting parents'

Finally, there is no borrowing or saving.

#### 2.3.2 Children

A woman can have up to three children in her lifetime and at most one child per period (i.e., no twins are allowed). Once a child is born, the child stays with the mother, regardless of the mother's union transitions.

Whether a woman has a child is stochastic. Every period, she exerts effort that affects the probability of having an additional child. Her effort can increase or decrease the probability of having a child in the next period. However, due to its stochastic nature, some women cannot have a child when they would like, while others will have one when they were not ready.<sup>8</sup>

A mother's utility depends on her children. In the period a child is born, the mother receives a one-time utility shock, which I call a childbearing cost,  $\theta$ . This childbearing cost depends on the mother's level of education, age, union status, and the number of children she already has. In addition to the childbearing cost, her utility from children depends on the number and the quality of her children in the spirit of Becker and Tomes (1976). I assume that the quality depends on the average amount spent on children in the period. To simplify the model, the past history of spending does not affect the children's quality.<sup>9</sup> In addition to directly impacting her utility, children affect their mother in two ways. First, mothers have lower incomes than childless women, as children reduce their mother's available hours to work (time cost of children). Second, single mothers have a different probability of meeting a potential partner than childless single women.

#### 2.3.3 Union transition

Each period, after the fertility outcome is realized, a single woman meets a potential partner with some probability. The probability of meeting a partner,  $\lambda_t(e, k_t)$ , depends on her age, level

children and the benefit of implementing common-law married on children (2.5.2).

<sup>&</sup>lt;sup>8</sup>Following Choi (2017), I model the effort as an additively separable utility cost. Combined with the functional form assumption, presented in 2.4, this assumption provides a convenient way to calibrate parameters to match the observed fertility rates. See Equations (2.8), (2.9), and the discussion around them. In addition, the unbounded domain of efforts is assumed to ensure avoid having 'corner solutions,' in which everyone has a child or no child in a given period, in counterfactual exercises. The decision of having an abortion is not modeled, but the effort can be viewed as a decision including abortion. The model in Choi (2017) has abortion decisions in addition to imperfect control over fertility as the model in this chapter.

<sup>&</sup>lt;sup>9</sup> This assumption makes spending on children a static decision. As a result, in the model, children receive the same amount of parental spending whether their mother is married or cohabiting. However, as cohabitation is more likely to end than marriage, children of married mothers are expected to have more spending during their childhood than children of cohabiting mothers.

of education, and the number of children she has. Upon meeting, she observes a match quality  $\gamma_t$ , which evolves over time, and decides whether to stay single or to form a union. Living in a union provides her additional income (partner's income) and utility from the match quality. There are two types of union available: marriage and cohabitation. They differ in when and how the union ends. A union can end voluntarily, when she finds it better to be single, or involuntarily, when the couple receives an exogenous divorce or separation shock,  $\delta$ . The probability of a married couple receiving a divorce shock,  $\delta_m$ , is lower than the probability of a separation shock for a cohabiting couple,  $\delta_c$ . A utility cost of divorce,  $\kappa$ , incurs when a married couple divorces either voluntarily. A cohabiting couple separates costlessly.

A couple's match quality is stochastic. A woman in a union can change her union status after observing a new match quality and fertility outcome. A cohabiting woman, if she did not receive the separation shock, can separate, marry her partner, or keep cohabiting. A married woman, if she did not receive the divorce shock, can choose to divorce or stay married.<sup>10</sup> Divorced and separated women stay single for the period and enter the matching market in the next period.

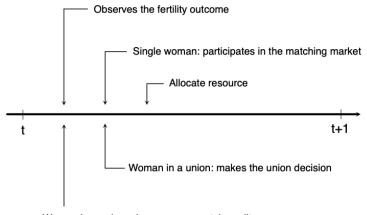
#### 2.3.4 Timing of events within a period

Figure 2.1 summarizes the timing of events within a period.

- 1. The woman observes her fertility outcome.
- 2. The woman makes union transition decisions:
  - (a) If she was single at the end of the previous period, she may meet a potential partner. If she meets a partner, she draws a match quality and makes a union decision.
  - (b) If she was cohabiting at the end of the previous period:
    - i. the separation shock arrives with probability  $\delta_c$ , or
    - ii. if the exogenous separation does not happen, a new match quality is drawn, and she decides whether to keep cohabiting, marry her partner, or separate.
  - (c) If she was married at the end of the previous period:
    - i. the divorce shock arrives with probability  $\delta_m$ , or
    - ii. if the exogenous divorce does not happen, a new match quality is drawn and she decides whether to stay married, or divorce.

<sup>&</sup>lt;sup>10</sup>Starting cohabitation with her husband is a feasible option, but doing so is never an optimal choice as it incurs the divorce cost without altering match quality.

#### Figure 2.1: Timeline of the model





3. She allocates resources to consumption, spending on children, and decides how much fertility/contraceptive effort to make.

#### 2.3.5 Dynamic programs

The problem is solved backwards. Given fertility outcomes and union choice, I solve the resource allocation problem by maximizing utility. Let the final period's continuation value be zero:  $F_{T+1} = 0$ . In period *t*, a single woman has her level of educational attainment, *e*, and the number of children she has,  $k_t$ , as state variables. For a given state, the value function of a single woman at the end of the period, after this period's uncertainties regarding fertility and partner matching are resolved, is given by,

$$F_{t}^{S}(e,k_{t}) = \max_{c_{t},s_{t},x_{t}} \quad u(c_{t},k_{t},s_{t}) - |x_{t}| + \beta \mathbb{E}_{\gamma_{t+1},k_{t+1}} \left\{ F_{t+1}(e,\gamma_{t+1},k_{t+1},k_{t},S_{t}=S)|x_{t} \right\},$$
  
s.t.  $c_{t} + k_{t}s_{t} \le w(e)(1 - \tau k_{t}),$   
 $c_{t} > 0, s_{t} \ge 0,$   
where  $u(c_{t},k_{t},s_{t}) = \begin{cases} (1 - \alpha_{k} - \alpha_{q})\log c_{t} & \text{if no child } (k_{t}=0), \\ (1 - \alpha_{k} - \alpha_{q})\log c_{t} + \alpha_{k}\log k_{t} + \alpha_{q}\log(q_{0} + s) & \text{otherwise.} \end{cases}$   
(2.1)

The woman decides how much to consume,  $c_t$ , how much to spend on her children,  $k_t s_t$ , and how much effort,  $x_t$ , to make to affect her fertility outcome in the next period.<sup>11</sup> Her contraceptive effort,  $x_t$ , can be negative to increase the probability of having a child next period, or it can be positive to reduce the probability. Regardless of its sign, the effort decreases her utility. The parameter  $\tau$  represents the time cost of children.

The value function of a woman in a union at the end of the period, either cohabiting (j = C) or married (j = M), is defined similarly with an additional state variable of the match quality,  $\gamma_t$ . Her utility depends on the match quality,  $\gamma_t$ , as she lives with her partner. Her available resources are increased by her partner's earnings,  $w^*$ , and her consumption is adjusted by an economy of scale parameter,  $\psi$ . The continuation value depends on whether she receives the exogenous separation shock,  $\delta_j$ , or not. If she receives the shock in the next period, she is single regardless of the match quality and pays the divorce cost ( $\kappa$ ) if she is married in this period ( $S_t = M$ ). In addition, if she has an additional child next period ( $k_{t+1} > k_t$ ), her utility is affected by the childbearing cost ( $\theta$ ).<sup>12</sup>

$$F_{t}^{j}(e, \gamma_{t}, k_{t}) = \max_{c_{t}, s_{t}, x_{t}} u(c_{t}, k_{t}, s_{t}) - |x_{t}| + \gamma_{t} + \beta \mathbb{E}_{\gamma_{t+1}, k_{t+1}} \Big[ (1 - \delta_{j}) F_{t+1}(e, \gamma_{t+1}, k_{t+1}, k_{t}, S_{t} = j) \\ + \delta_{j} \Big\{ F_{t+1}^{S}(e, k_{t+1}) - \mathbb{1}_{\{S_{t}=M\}} \kappa + \mathbb{1}_{\{k_{t+1}>k_{t}\}} \theta_{t+1}(e, k_{t+1}, j) \Big\} |\gamma_{t}, x \Big],$$
  
s.t.  $\frac{c_{t}}{\psi} + k_{t} s_{t} \leq w(e)(1 - \tau k_{t}) + w^{*},$   
 $c_{t} > 0, s_{t} \geq 0,$   
where  $j \in \{C, M\},$  and  
 $u(c_{t}, k_{t}, s_{t}) = \begin{cases} (1 - \alpha_{k} - \alpha_{q}) \log c_{t} & \text{if no child } (k_{t} = 0), \\ (1 - \alpha_{k} - \alpha_{q}) \log c_{t} + \alpha_{k} \log k_{t} + \alpha_{q} \log(q_{0} + s_{t}) & \text{otherwise.} \end{cases}$ 
(2.2)

Given the values of each type of union and being single, she makes the union transition that gives her the highest value. This defines the value function at period t after uncertainties are

<sup>&</sup>lt;sup>11</sup>There is no saving or borrowing in the model.

<sup>&</sup>lt;sup>12</sup>The childbearing cost is included in the continuation value  $F_{t+1}$ , defined below, in case she does not receive the separation shock.

resolved:

$$F_{t}(e, \gamma_{t}, k_{t}, k_{t-1}, S_{t-1})$$

$$= \max \begin{cases} F_{t}^{M}(e, \gamma_{t}, k_{t}), \\ F_{t}^{C}(e, \gamma_{t}, k_{t}) - \mathbb{1}_{\{S_{t-1}=M\}}\kappa, \\ F_{t}^{S}(e, k_{t}) - \mathbb{1}_{\{S_{t-1}=M\}}\kappa \end{cases} + \mathbb{1}_{\{k_{t} > k_{t-1}\}}\theta_{t}(e, k_{t}, S_{t-1})$$

$$\equiv \tilde{F}_{t}(e, \gamma_{t}, k_{t}, k_{t-1}, S_{t-1}) + \mathbb{1}_{\{k_{t} > k_{t-1}\}}\theta_{t}(e, k_{t}, S_{t-1}).$$
(2.3)

Compared to the union specific value functions defined above  $(F_t^S, F_t^C, \text{ and } F_t^M)$ , the value function has two additional state variables: the number of children she had in the previous period,  $k_{t-1}$ , and the union status in the previous period,  $S_{t-1}$ . If the woman had an additional child  $(k_t > k_{t-1})$ , the value function is affected by the childbearing cost  $(\theta)$ , which depends on the union status in the previous period  $(S_{t-1})$  as well as her age, level of education, and the number of children. Her utility is affected further by the divorce cost  $(\kappa)$  if she was married in the previous period  $(S_{t-1} = M)$  but chooses another union type.

#### 2.3.6 Discussion

In the model, living in a union impacts a woman economically and non-economically compared to living alone. The economic aspect of the difference comes from the partner's income and economies of scale in consumption. That is, the household has two earners, and disposable resources increase even more than the sum of the two incomes effectively by pooling their income together. The non-economic differences arise from the match quality, as a coupled woman derives direct utility from her match quality with her partner.

The divorce cost for marriage and the separation shocks give an intuitive relationship between match quality and union choice. Given the level of education, the number of children has, and the union status in the previous period, there is a threshold match quality where a couple whose match quality is above marries and below does not. There is another threshold above which a couple cohabits and below which separates or stays single.<sup>13</sup> A couple with high enough match quality chooses marriage over cohabitation to avoid the higher separation shock. If the match quality is not high enough, it is better to cohabit to avoid the divorce cost. Once a couple marries, their threshold of ending the marriage is lower than that of a cohabiting couple due to the divorce cost. The duration of marriage is longer than cohabitation in the model for three reasons. First, the selection on match quality means that marriage starts with

<sup>&</sup>lt;sup>13</sup>Depending on parameter values and state variables, the thresholds may collapse into one. For example, there is only one threshold for a married couple above which the couple stays married and below which divorces.

higher match quality than cohabitation. Second, the divorce cost ensures married couples stay in unions with lower match quality. Finally, the lower probability of the exogenous divorce for married couples than cohabiting couples yields a longer duration.

#### 2.4 Matching the model to the data

I solve the model numerically. In this section, I present the parametric assumptions and discuss how to calibrate model parameters. The purpose of the calibration is to have agents in the model behave similarly to the observed NLSY79 cohort. In matching the model to the data, parameters regarding income are set outside of the model, and the remaining parameters are calibrated to match various moments of fertility and union transitions of the NLSY79 cohort. Then, I use the calibrated model to study the effects of decreasing gender wage gaps on union decisions and the expected impacts of introducing a common-law marriage policy.

#### **2.4.1** Wage rates and the time cost of children

Wage rates and the time cost of children are estimated outside of the model. I assume that women have different wage rates depending on their education level and that there is one wage rate for men.<sup>14</sup> I also assume that wage rates do not change over time. This assumption is made to have a match quality process with a constant mean over time. If wage rates increase over time, the match quality should also increase in mean to have a stable fraction of women in unions. Otherwise, increasingly many women will divorce or separate toward the end of their life cycle, which is inconsistent with the data. To simplify the calibration by reducing the number of parameters to be calibrated, I assume both the wage rates and the mean of the match quality process are constant over time.

Using annual earnings data from the NLSY79, I estimate the differences in hourly wages of less-educated women, more-educated women, and men after controlling for their ages.<sup>15</sup> I normalize the men's wage rate to one, and used the relative wages of less-educated women (0.6943) and more-educated women (0.8384) in the calibration.

The time costs of children are also estimated using the NLSY79 data. Specifically, I regress women's annual hours of work on their number of children. On average, the hours of work of a woman decreases by 14% for each child she has. This estimate is used as the time cost of a child ( $\tau$ ) in the calibration.

<sup>&</sup>lt;sup>14</sup>To simplify the model, I assume there is only one wage rate for men for both less-educated and more-educated women. Instead, they face different processes affecting unions, such as partner meeting rates, probabilities of having the exogenous separation shock.

<sup>&</sup>lt;sup>15</sup>Their ages are controlled using a fourth order polynomial of age.

#### 2.4.2 Fertility decisions

The probability of having an additional child next period depends on this period's age (t) and the level of contraceptive effort (x). I assume a scaled complementary error functional form for the probability:

$$p(x;q_t) = q_t \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-z^2) dz.$$
 (2.4)

The age dependent parameter  $q_t$  denotes the probability of having an additional child when zero effort (x = 0) is made. I assume that its value starts from 1/2 and decreases linearly to reach 0 in the last model period:  $q_t = \frac{1}{2} \frac{T-t}{T-1}$ .

Given the parameter  $q_t$ , the level of contraceptive effort, x, affects the probability, p. The probability approaches 0 in the limit as a woman exerts more positive effort  $(x \to \infty)$ . She can also exert negative effort to increase the probability  $(p \to 2q_t \text{ as } x \to -\infty)$  if she wants to have a child.

#### 2.4.3 Partner meeting and match quality

A woman who was single at the end of the previous period participates in the matching market, and the probability of meeting a match ( $\lambda$ ) depends on her level of education, age, and the number of children she has:  $\lambda_t(e, k_t)$ .

An initial match quality ( $\gamma_t$ ) is drawn upon meeting. I assume that the negative of the initial match quality follows a log-normal distribution:  $\log(-\gamma_t) \sim N(0, \sigma_0^2)$ . For an existing couple who was cohabiting or married in the previous period and did not receive the separation shock, the match quality evolves following a random walk:  $\gamma_t = \gamma_{t-1} + \varepsilon_t$ ,  $\varepsilon_t \sim N(0, \sigma_{\varepsilon}^2)$ . The match quality process is approximated as a 30-state Markov process following the discrete approximation method described in Kennan (2006).<sup>16</sup> First, the discrete grid points are set to give each point the same probability of being drawn, following the initial match quality distribution. Second, the Markov transition matrix is calculated by approximating the random walk process of the match quality for an existing couple on the grid points.

#### 2.4.4 Calibration

Some model parameters are set without solving the model. As a model period is one year, the discount factor ( $\beta$ ) is set to 0.96, a standard value in the literature. The parameter for

<sup>&</sup>lt;sup>16</sup>A large number of discrete points are required for the match quality process to match data moments on cohabitation, which lasts for a short period of time and relatively a small number of women do at a given time.

economies of scale,  $\psi$ , is set to 1.7 using the OECD equivalence scale.<sup>17</sup>

Table 2.7 summarizes the parameters that are set without solving the model, including parameters that determine income, as described in Section 2.4.1.

	Value	Description
β	0.96	discount factor
Ψ	1.7	economies of scale in consumption
w(l)	0.6943	wage rate of less-educated women
w(h)	0.8384	wage rate of more-educated women
$w^*$	1.0	wage rate of men
τ	0.14	time cost of a child

Table 2.7: Parameters set outside the model

The remaining parameters are calibrated so that model moments match corresponding moments from the NLSY79 and NSFG Cycle 6 datasets. The calibration is done in three steps.

In the first step, values of three preference parameters,  $\alpha_k$ ,  $\alpha_q$  and  $q_0$ , are set by solving a simplified problem. Model outcomes, such as union decision and fertility outcomes, are affected by uncertainties as well as preferences. As a result, it is not easy to discern the impact of preferences from uncertainties in the outcomes. Instead of calibrating all of the parameters together, I calibrate preference parameters before the parameters regarding uncertainties in a simplified model without any uncertainty. The problems (2.1) and (2.2) are converted into the canonical form of the quantity-quality of children trade-off model, where a parent decides how many children to have and how much to spend on them, in a static deterministic environment.<sup>18</sup>

I target to match the average number of children and the average household expenditure on child-related items using a simplified problem.

$$\max_{c,k,s} \quad (1 - \alpha_k - \alpha_q) \log c + \alpha_k \log k + \alpha_q \log(q_0 + s),$$
  
s.t. 
$$\begin{cases} c + ks \le w(e)(1 - \tau k) & \text{if single }, \\ \frac{c}{\psi} + ks \le w(e)(1 - \tau k) + w^*, & \text{if married }, \end{cases}$$
$$(2.5)$$
$$c > 0, \ k > 0, \ s \ge 0.^{19}$$

The fertility and resource allocation decisions are made simultaneously. Again following the

<sup>&</sup>lt;sup>17</sup>The OECD equivalence scale is commonly used in the family economics literature (for example, Regalia *et al.*, 2011; Santos and Weiss, 2016; Choi, 2017).

<sup>&</sup>lt;sup>18</sup>For example, Becker and Tomes (1976), Moav (2005), and Jones *et al.* (2010), among many others, used the canonical form.

<sup>&</sup>lt;sup>19</sup>The constraint sets are not convex due to the term ks. Jones *et al.* (2010) solved a transformed problem, by letting the term ks serve as a choice variable, instead of *s*. Equivalently, the problem can be turned into a two-stage decision problem. When allocating resources between between consumption, *c*, and spending on children,

canonical model, I assume that women have perfect control over their fertility outcomes, and the number of children, k, can be any positive real number within their budget set.<sup>20</sup>

There are four budget constraints and corresponding solutions depending on the woman's level of education and her union status, which I refer to as household types: single less-educated woman, married less-educated woman, single more-educated woman, and married more-educated woman.<sup>21</sup>

The average number of children women of each type had by age 44 in the NLSY79 sample are used as targets. Lino *et al.* (2017) estimate that households with two children spend 39% of household expenditures on child-related items, on average. The expenditure shares of the problem (2.5) are weighted using the shares of the four household types from the NLSY79, and the weighted average is targeted to match 39%.<sup>22</sup> Table 2.8 presents the calibrated value of preference parameters, and the targeted and calibrated moments are in Table A.6 in Appendix A.

 Table 2.8: Preference parameters

	Value	Description
$\alpha_k$	0.2827	utility weight on the number of children $(k)$
$\alpha_q$	0.2192	utility weight on quality of children $(q)$
$q_0$	0.0230	the innate ability of children (the level of the quality with no spending)

In the second step of the calibration, eight parameters governing the stochastic processes in the model are calibrated to match various moments from the data, taking the income and preference parameters as given. The parameters are the variance of initial match quality ( $\sigma_0^2$ ), the variance of innovation to the match quality ( $\sigma_{\varepsilon}^2$ ), the probabilities of the separation shock to cohabiting women, by their level of education ( $\delta_c^l$ , and  $\delta_c^h$ ), the probabilities of the divorce shock to married women ( $\delta_m^l$  and  $\delta_m^h$ ), and the divorce costs ( $\kappa^l$  and  $\kappa^h$ ). These parameters are set by minimizing the distance between data moments and simulated model moments regarding union and fertility choices. I use the following sets of moments:

• The age profile of the fraction of married women from age 18 to age 44 for less-educated women and from age 21 to age 44 for more-educated women (51 moments from the NLSY79).

s, take the number of children as given. Choosing the number of children, k, given resource allocation rules solves the problem 2.5.

<sup>&</sup>lt;sup>20</sup>In the original model, the number of children can be one of the integers, 0, 1, 2, and 3.

<sup>&</sup>lt;sup>21</sup>In the static environment, there is no difference between marriage and cohabitation.

<sup>&</sup>lt;sup>22</sup>When  $q_0 = 0$ , the share of utility weights,  $\frac{\alpha_q}{1-\alpha_k}$ , is the expenditure share on children. However, when  $q_0 > 0$ , the relationship does not hold. As explained in Jones *et al.* (2010),  $q_0 > 0$  is a necessary condition to have a negative relationship between the mother's wage rate and the number of children, observed in data.

- The age profile of the fraction of women who had ever married from age 18 to age 44 for less-educated women and from age 21 to age 44 for more-educated women (51 moments from the NLSY79).
- The age profile of the fraction of women who had divorced from age 18 to age 44 for less-educated women and from age 21 to age 44 for more-educated women (51 moments from the NLSY79).
- The age profile of the fraction of single women from age 18 to age 37 for less-educated women and from age 21 to age 37 for more-educated women (37 moments the NSFG Cycle 6).
- The average duration of first marriage at age 33 by the level of education (2 moments from the NSFG Cycle 6).
- The average duration of first cohabitation at age 33 by the level of education (2 moments from the NSFG Cycle 6).

In total, there are 194 moments to match.

The variance of initial match quality distribution  $(\sigma_0^2)$  determines the overall fraction of women in unions. The variance of match quality innovation  $(\sigma_{\varepsilon}^2)$  determines transition between union statuses. The average duration of marriage and cohabitation provide information about the divorce cost ( $\kappa$ ) and the exogenous separation shock ( $\delta$ ), respectively. Table 2.9 summarizes the calibrated values of the parameters. Targeted moments are summarized in Appendix A.

	Value	Description
$\sigma_0^2$	1.6195	variance of initial match quality distribution
$\sigma_{\varepsilon}^2$	0.0927	variance of innovation to the match quality
$\delta_{c}^{l}$	0.1254	probability of separation shock to less-educated cohabiting women
$\delta^h_c$	0.2224	probability of separation shock to more-educated cohabiting women
$\delta_m^l$	0.0299	probability of divorce shock to less-educated married women
$\sigma_0^2 \sigma_arepsilon^2 \delta_c^l \delta_c^h \delta_m^l \delta_m^h \kappa^l$	0.0150	probability of divorce shock to more-educated married women
$\kappa^l$	0.8398	divorce cost of less-educated married women
$\kappa_h$	2.7045	divorce cost of more-educated married women

Table 2.9: Calibrated parameter values

In the third step, the probability of meeting a match,  $\lambda_t$ , and the childbearing cost,  $\theta_t$ , are calibrated. These parameters vary over time, and I refer to them as life cycle parameters. This step takes place inside the second step. Given parameters set in the previous steps, these parameters control model moments that can be targeted to corresponding data moments directly. The parameters are calibrated using their explicit relationships to the moments.

From the NSFG Cycle 6 sample, I estimate the annual transition rates from single to either cohabitation or marriage,  $\hat{\mu}_t(e, k_t)$ . By comparing the estimates against the policy function of single women's union decision, after integrating over the distribution of initial match quality,  $G_0(\gamma_t)$ , the required probability of meeting a match ( $\lambda_t$ ) can be derived:

$$\lambda_t(e,k_t) = \max\left\{\frac{\hat{\mu}_t(e,k_t)}{\int_{\gamma_t} P[F_t^S(e,k_t) < \max\{F_t^M(e,\gamma_t,k_t), F_t^C(e,\gamma_t,k_t)\}] dG_0(\gamma_t)}, 1\right\}.$$

The distribution  $G_0$  is given as its parameter ( $\sigma_0^2$ ) is calibrated in the second step.

Also using the NSFG Cycle 6 sample, I estimate the age profile of fertility rates of single women and married women,  $\hat{p}_{t+1}(e, k_t, S_t)$ , conditional on their level of education, the number of children they have, and their union status. I calibrate the childbearing cost ( $\theta_t$ ) so that the solutions of the fertility decision problems are the same as the estimates. I assume the childbearing cost of cohabiting women is the same as that of married women.<sup>23</sup>

The structures of the decision problems, (2.1) and (2.2), separate fertility decisions from resource allocation decisions. Effectively, a woman with  $k_t$  children chooses the level of contraceptive effort,  $x_t$ , by solving the following problem:

$$\max_{x_t} - |x_t| + \beta \left[ p(x;q_t) \left\{ \Delta_k F_{t+1} + \theta_t(e,k_t,S_t) \right\} \right],$$
(2.6)

where  $\Delta_k F_{t+1}$  stands for the difference in expected values of having an additional child and not having a child:

$$\Delta_{k}F_{t+1} = \begin{cases} \mathbb{E}_{\gamma_{t+1}}\{F_{t+1}(e,\gamma_{t+1},k_{t+1}=k+1,k_{t}=k,S) \\ -F_{t+1}(e,\gamma_{t+1},k_{t+1}=k,k_{t}=k,S)|x\} & \text{if single,} \end{cases}$$

$$\mathbb{E}_{\gamma_{t+1}}\left[(1-\delta_{j})\{F_{t+1}(e,\gamma_{t+1},k_{t+1}=k+1,k_{t}=k,S_{t}) \\ -F_{t+1}(e,\gamma_{t+1},k_{t+1}=k,k_{t}=k,S_{t})\} \\ +\delta_{j}\{F_{t+1}(e,\gamma_{t+1},k_{t+1}=k+1,k_{t}=k,S) \\ -F_{t+1}(e,\gamma_{t+1},k_{t+1}=k,k_{t}=k,S)\}|x,\gamma_{t}\right] \\ & \text{if cohabiting } (S_{t}=C) \text{ or married } (S_{t}=M). \end{cases}$$

$$(2.7)$$

<sup>&</sup>lt;sup>23</sup>The sample size for cohabiting women is not big enough to estimate the fertility rates. For many points on the state space (the level of education and the number of children they had before), there are no or few observations. As a result, the estimated fertility rates are highly noisy. Instead of calibrating separate childbearing costs of cohabiting women using the noisy estimates of fertility rates, I assume they have the same childbearing costs with married women.

Assuming a non-zero solution, the optimal level of contraceptive effort  $(x^*)$  is given as

$$x_t^*(\theta_t; e, \gamma_t, k_t, S_t) = \begin{cases} \sqrt{-\log\{-\frac{\sqrt{\pi}}{2q_t(\Delta_k F_{t+1} - \theta_t)}\}} & \text{if } x > 0 \iff p_t < q_t, \\ -\sqrt{-\log\{\frac{\sqrt{\pi}}{2q_t(\Delta_k F_{t+1} - \theta_t)}\}} & \text{if } x < 0 \iff p_t > q_t. \end{cases}$$
(2.8)

And by definition, the fertility rate is given by

$$p_{t+1}^{*}(\theta_{t}; e, \gamma_{t}, k_{t}, S_{t}) = q_{t} \frac{2}{\sqrt{\pi}} \int_{x^{*}(\theta_{t}; e, \gamma_{t}, k_{t}, S_{t})}^{\infty} \exp(-z^{2}) dz.$$
(2.9)

The childbearing  $\cot(\theta_t)$  for single women can be calibrated directly by equating equation (2.9) to the estimate from data ( $\hat{p}_t(e, k_t, S_{t-1})$ ). As an analytical form of the inverse of the function  $p(x;q_t)$  does not exist, I approximate the inverse function using the approximation of the inverse error function in Winitzki (2008). Let  $\tilde{p}^{-1}$  be the approximated inverse function of  $p(x;q_t)$ . Setting the value of childbearing cost as

$$\theta_{t} = \begin{cases} \Delta_{k} F_{t+1} + \frac{\sqrt{\pi}}{2q_{t}} \exp\left[\{\tilde{p}^{-1}(\hat{p}_{t})\}^{2}\right] & \text{if } \hat{p}_{t} < q_{t} \\ \Delta_{k} F_{t+1} - \frac{\sqrt{\pi}}{2q_{t}} \exp\left[\{\tilde{p}^{-1}(\hat{p}_{t})\}^{2}\right] & \text{if } \hat{p}_{t} > q_{t} \end{cases}$$
(2.10)

matches the fertility rates.

The childbearing cost of married women cannot be calibrated directly from equation (2.9) as the optimal fertility rate of a woman in a union depends on the current match quality, which is unobservable in the data.<sup>24</sup> As a result, the calibration requires an integration with respect to the distribution of married women over the match quality,  $\Omega_t^M(\gamma_t; e, k_t, S_{t-1})$ . The distribution of match quality for existing couples is an endogenous object. Instead of using the exact distribution, I use the distribution from the previous iteration.<sup>25</sup>

Let the distribution from the previous iteration be  $\tilde{\Omega}_t^M(\gamma_t; e, k_t)$ . By finding  $\theta_t$  that satisfies the following non-linear equation, I calibrate the childbearing cost of married women:

$$\hat{p}_{t+1}(e,k_t,S_t=M) = \int_{\gamma_t} p_{t+1}^*(\theta_t;e,\gamma_t,k_t,S_t=M) \frac{\tilde{\Omega}_t^M(\gamma_t;e,k_t)}{\int_{\gamma_t'} \tilde{\Omega}_t^M(\gamma_t';e,k_t)d\gamma_t'}d\gamma_t.$$

<sup>&</sup>lt;sup>24</sup>The estimate from the data,  $\hat{p}_t(e, k_t, S_{t-1})$ , does not depend on match quality.

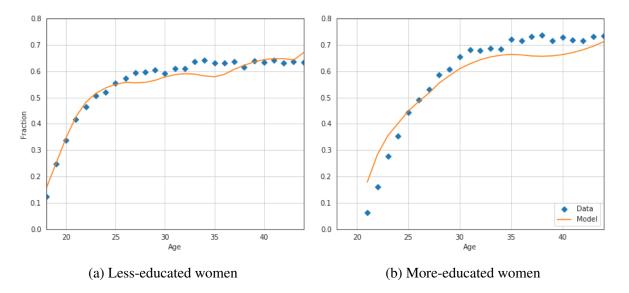
<sup>&</sup>lt;sup>25</sup>For a given set of parameters, which are set in the previous steps of calibration, the distribution of match quality for married couples depends on the union decisions of couples as well as the distribution of initial match quality ( $G_0$ ) and the match quality process. The distribution needs to be calculated from the first period using the union decision rules to use the exact distribution  $\Omega_t$ , and the union decision rules depend on continuation (future) values. Instead of finding the fixed point for every set of parameters (iterations), which is a costly computation, I used the distribution derived in the previous set of parameters. An initial guess of the distribution is used for the first iteration.

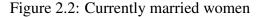
Calibrated values of life cycle parameters and their target moments are summarized in Appendix A.

#### 2.4.5 Results

This section discusses the results of the calibration. The calibrated model is able to match many salient features of the data.

Figure 2.2 plots the fractions of currently married women over the life cycle, by education level. The model matches the patterns for less-educated women well. Earlier in the life cycle, before age 26 for more-educated women, there are more married women in the model than in the data. After age 26, the model closely follows the fraction for more-educated women in the data. The lower fraction of currently married women among the less-educated than the more-educated later in the life cycle is a result of a higher likelihood of divorce (Table 2.10). By age 33, 80% of less-educated women had ever married in the model versus 82% in the data, and 34% of them have ever divorced versus 32% in the data. In comparison, 72% of more-educated women had ever married by age 33 in the model compared to 77% in the data), and 9% of them have divorced versus 13% in the data.





As cohabitation is a more transient union compared to marriage, Figure 2.3 shows that the model has more difficulty matching the fraction of cohabiting women in the data. In both educational levels, the model has more cohabiting women earlier in the life cycle and fewer of them later than the data. Table 2.10 shows how the model replicates the educational gradient in cohabitation. By age 33, more than half (57%) of less-educated women have ever cohabited in

	Less-educated women		More-educated women	
	Data	Model	Data	Model
Average duration of first marriage at age 33 (years)	7.94	8.64	6.67	7.62
Average duration of first cohabitation at age 33 (years)	3.34	2.27	2.25	2.61
Percentage of women who have ever married by age 33	82.07	75.25	76.80	72.55
Percentage of women who have divorced by age 33	32.17	30.50	13.18	10.15
Percentage of women who have cohabited by age 33	50.82	55.45	38.75	37.20

Table 2.10: Union experiences

Data source: NLSY79, NSFG Cycle 6

the model compared to 51% in the data, and 38% of more-educated women have in the model versus 39% in the data.

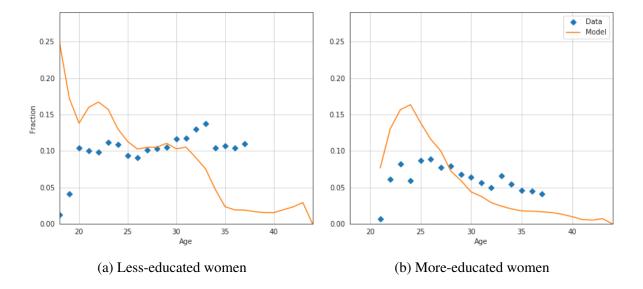


Figure 2.3: Currently cohabiting women

Figure 2.4 shows that, on average, women in the model have more children than the data. This is a result of having more women in a union earlier in their life cycle in the model than in the data. In the model, the childbearing cost is the key parameter shaping the fertility rates. As explained in the previous section, the parameter is calibrated by matching fertility rates in the data conditional on union status. By the design of the calibration strategy, the model could match the number of children if it matches union decisions.<sup>26</sup> However, as the model has more married and cohabiting women, whose fertility rates are higher than single women, earlier in the life cycle, when the fertility rates are higher, women in the model have more children than in the data. Between the two education groups, less-educated women have more children than more-educated women.

<sup>&</sup>lt;sup>26</sup>The fertility rates are matched conditional on union choices.

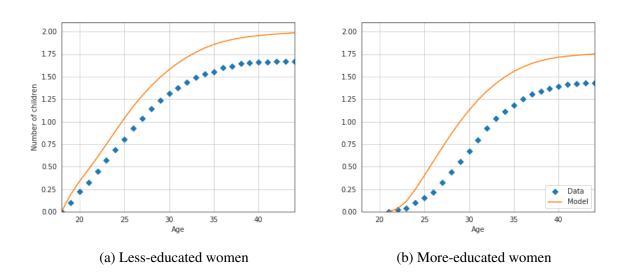


Figure 2.4: Average number of children

#### 2.5 Quantitative exercises

Next, I conduct two quantitative exercises using the calibrated model. First, I investigate whether decreasing the gender wage gap can explain the trend of delaying marriages and increasing cohabitation observed in the data over time. Closing the gender gap leads to marriage delay and increased cohabitation in the model. The model can explain about a half of observed delays in marriage of less-educated women with the gender gap. However, little of the trends in unions of more-educated women can be explained. Second, I investigate the effect of introducing common-law marriage. Under this new institution, cohabiting parents are treated as married couples. In the counterfactual simulation with common-law marriage, fewer people cohabit, and fewer children are born to unmarried parents.

#### 2.5.1 Gender wage gap

This exercise studies the relationship between the decreasing gender earnings gap and changes in the household unions. Over the past few decades, people have been delaying marriage and cohabiting more. Table 2.12 compares union experiences of the NLSY79 cohort (in columns labeled '1990') and the NLSY97 cohort (in columns labeled '2010'). These cohorts are about 20 years apart.<sup>27</sup> More than 80% of the earlier cohort married by age 33, whereas less than half of the later cohort married by the same age. In addition, more of the later cohort cohabited by age 33.

<sup>&</sup>lt;sup>27</sup>The NLSY79 cohort is born between 1957 – 1964, and the NLSY97 cohort is born between 1980 – 1984.

Becker (1991) considers the division of labor the main benefit of household and views a marriage as a contract protecting women who typically specialize in childbearing and domestic activities. Under this view, which is inherited by most of the family economics literature, the decreasing gender earnings gap reduces the benefit of marriage from the division of labor.<sup>28</sup> In their study of the increasing fraction of single households from the early 1970s to the late 2000s, Regalia *et al.* (2011) find that the closing of the gender wage gap alone can explain more than half of the observed changes. Adamopoulou (2014) also finds that the decreasing gender wage gap leads to fewer marriages and more cohabitations.

To study how changes in wages affect union decisions in the model, I estimate the relative wages using the NLSY97 sample. In the NLSY79 sample, the average wage rate of less-educated women is 31% lower than the men's wage rate, and the wage rate of more-educated women is 16% lower (see Table 2.7). The gaps decreased in the NLSY97 sample. In the NLSY97, the wage rate of less-educated women is 17% lower than the men's wage rate, and more-educated women had a wage rate that is 5% lower than men.<sup>29</sup> I solve the model by keeping the value of other parameters the same as the baseline and changing the relative wage rates to the estimates from the NLSY97.

Figure 2.5 shows the changes in relative wages affect less-educated women's union allocations substantially. In contrast, the predicted changes in more-educated women's union allocations are relatively small. With the smaller gender gap, there are fewer married women. Compared to the baseline, women have higher incomes, and potential partners' have relatively lower incomes. As the economic benefit of a union has decreased, so does the benefit of marriage. The economic benefit of a union is more important for less-educated women than for more-educated women due to their lower income and higher marginal utility of consumption. Thus, the decrease in marriage is greater among less-educated women.

As the fraction of married women decreases with the smaller gender gap in wages, some women who were married in the baseline economy choose to cohabit. Figure 2.6 compares the fractions of cohabiting women in the baseline model and with smaller gender gap counterfactual. The increase in cohabitation is clearly demonstrated among less-educated women. Although it is not clear in the figure for more-educated women, Table 2.12 shows that there is

<sup>&</sup>lt;sup>28</sup>This chapter also employs the view of Becker (1991) and uses the model to quantitatively test the hypothesis. However, it is possible that the causal relationship holds in the opposite direction. That is, exogenous changes, such as changes in technology, culture, social norms, or legal system, led to changes in household unions, and these in turn caused changes in the labor market. Greenwood and Guner (2010) study a model where changes in contraceptive technology led to changes in collective behaviors. Guvenen and Rendall (2015) study the role of education as an insurance against divorce for women.

<sup>&</sup>lt;sup>29</sup>The gender wage gap is estimated using the sample of full-year full-time workers as the model abstracts from labor-leisure choice. Although changes in labor supply are not considered in this chapter, it is known that the labor force participation rates of women has increased in the last couple of decades. See, for example, Attanasio *et al.* (2008).

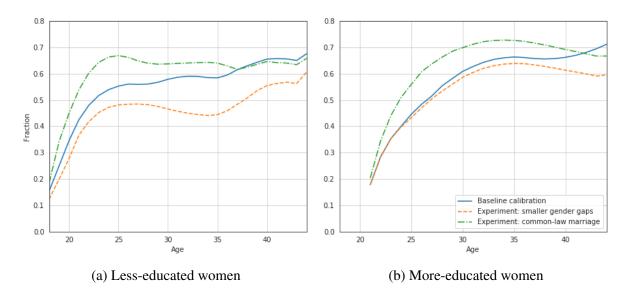


Figure 2.5: Fraction of currently married women by age and education

a slight increase in the fraction of women who have ever cohabited.

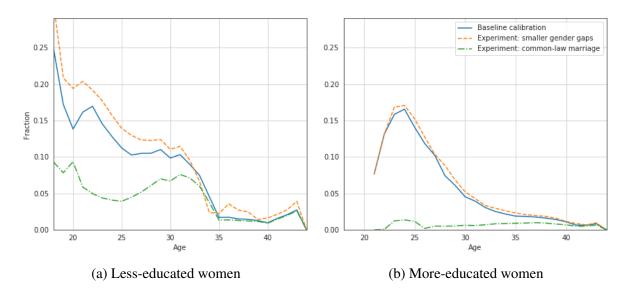


Figure 2.6: Fraction of currently cohabiting women by age and education

The childbearing cost parameter,  $\theta$ , controls fertility directly, but changes in income also affect fertility in two opposing directions.<sup>30</sup> Rising income increases the time cost of children, which is the substitution effect, lowering the incentive to have children. In contrast, the rising income makes having children less costly, which is the income effect, encouraging women to

 $<sup>^{30}</sup>$ As a result, Figure 2.7 shows that the average number of children a woman has moves little from the baseline with the changes in wages only.

reduce their contraceptive efforts. For less-educated women, the income effect is greater than the substitution effect, and they have slightly more children on average. For more-educated women, the two effects offset each other, and the average number of children remains virtually the same.

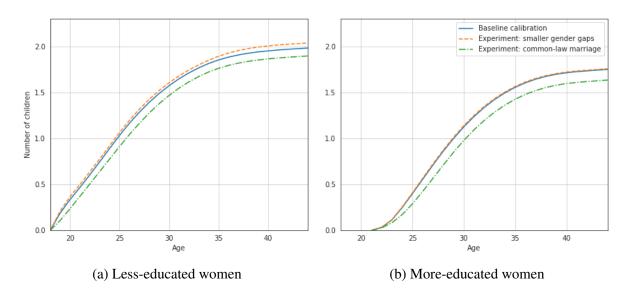


Figure 2.7: Average number of children by age and education

Table 2.11 shows that, as marriage declines, more children have lived with a single mother than in the baseline, and the average amount of spending on a child increase slightly. There are two opposing channels affecting investment as a result of the decreased gender wage gaps. Household income has increased compared to the baseline as the wage rates of women have increased. The increased household income increases spending on children directly. In addition, changes in union decisions also indirectly affect the amount. As more women stay single for a longer period of time, the spending decreases. For less-educated mothers, the two channels have similar impacts, and their children receive about the same amount spending, on average, as in the baseline (0.174 versus 0.173 in the baseline). For more-educated mothers, the direct channel has quantitatively larger impacts than the indirect channel, and their children receive about 5% more spending when the gender gap decreases than in the baseline calibration (0.221 versus 0.210 in the baseline).

To examine the contribution of each channel on spending, I give women the same union decisions as in the baseline with the decreased gender wage gaps.<sup>31</sup> Without changing union decisions (no impacts from the indirect channel), the increased women's wages increase the amount of spending by 10% for less-educated women (from 0.173 to 0.190) and by 7% for

<sup>&</sup>lt;sup>31</sup>I decrease the gender wage gaps and solve the model using the union decision rules from the baseline calibration.

more-educated women (from 0.210 to 0.225). However, the positive effect of the closing gender wage gaps on children is partly offset by the increased prevalence of single motherhood (the indirect channel). The negative effect almost entirely removes the increase for children of less-educated women: the average amount ends up being 0.174, which is less than 1% increase from its value in the baseline calibration, 0.173. For children of more-educated women, the negative effect cancels about a quarter of the increase making the total effect an increase of 5%.

Table 2.11: Experience of children

	Less-educ	ated women	More-educ	cated women
_	Baseline	Smaller gap	Baseline	Smaller gap
Have lived with a single mother (%)	33.57	36.92	15.39	16.22
Average investment per child per period	0.173	0.174	0.210	0.221

Columns labeled 'Baseline' are the results of baseline calibration, which is calibrated to match the NLSY79 cohort. Columns labeled 'Smaller gap' are the results from the model with the relative wages of the NLSY97 cohort.

Table 2.12 compares the changes in union experiences from the data and the model. With the decline in gender gaps only, the model moments move in the same direction as the data. The model explains the changes in unions of less-educated women relatively well, capturing about a third to half of movements observed in the data. In contrast, union experiences of more-educated women changed substantially changed over time in the data, but the model moments change little from the baseline. These results are consistent with the findings of Regalia *et al.* (2011) and Adamopoulou (2014) that the decline in gender gaps can explain the decrease in marriages. However, quantitatively, the estimated impacts of gender gaps are much smaller than Regalia *et al.* (2011), where the authors find that the gender gap alone can explain about 60% of the decrease in marriage.<sup>32</sup>

### 2.5.2 Introducing common-law marriage

Lastly, I study the effect of introducing a policy that treats certain cohabiting couples as married couples on fertility and union decisions. In many European countries and Canada (excluding Québec), cohabiting couples are deemed as *de facto* married couples if they meet certain criteria ('common-law marriage' or 'civil union'). The criteria vary across jurisdictions, but they are typically the length of cohabitation period or existence of children between the couple. For example, since 1978, the Ontario Family Law Act treats a cohabiting couple as a married cou-

<sup>&</sup>lt;sup>32</sup>Adamopoulou (2014) finds that the decline of gender gap can explain the changes in household union but does not provide a quantitative results.

		Less	-educa	ited wor	nen			More	e-educa	ated wo	men	
		Data			Model			Data			Model	
	1990	2010	Δ	1990	2010	Δ	1990	2010	Δ	1990	2010	Δ
Currently cohabiting (%)	14	14	0	7	7	0	8	11	+38	2	2	0
Currently married (%)	62	27	-56	59	42	-29	68	44	-35	66	64	-3
Ever cohabited (%)	55	67	+22	55	62	+13	46	57	+30	38	39	+3
Ever married (%)	82	42	-49	75	68	-9	84	49	-42	76	74	-3

Table 2.12: Union experience at age 33, with smaller gender wage gaps

For the data moments, columns '1990' are for the birth cohort of 1957 - 1964 (the NLSY79 cohort) using the NSFG Cycle 6 sample and columns '2010' are for the NLSY97 sample. The NLSY79 cohorts were 33 years old around the year 1990, and the NLSY97 cohorts were 33 years old around the year 2010. For the model moments, columns '1990' represents the baseline calibration, which targets the NLSY79 cohort, and columns '2010' represents the counterfactual results with the relative wage of the NLSY97. Columns ' $\Delta$ ' represent percentage changes from '1990' to '2010.'

ple if they have lived together longer than three years or have had children together.<sup>33</sup> However, cohabitants become *de facto* married couples in no states of the U.S.<sup>34</sup>

In the baseline, no cohabiting woman becomes married unless she chooses to. In this counterfactual, simulating a feature of the common-law marriage in other jurisdictions, I treat a cohabiting woman who gives birth in the cohabitation as a married woman. That is, once a child is born, she faces the divorce shock, instead of the separation shock of cohabiting couples, and has to pay the divorce cost to end the union. The parameter values remain the same as the baseline calibration.

Cohabitation serves as an intermediate form of union in the model. Due to the higher separation shock and costless exit, a couple whose match quality is not good enough to marry chooses to cohabit as they are more likely to pay the divorce cost if they marry. The risk involving cohabitation in the baseline economy is losing a partner due to the separation shock. In the economy with common-law marriage, as people have imperfect control over fertility, there is an additional risk of getting married to a cohabiting partner if she gives birth, even if the match quality is not high enough to want to get married. If there were no divorce cost, most of these new couples would divorce immediately. However, due to the divorce cost, some of these couples married by the common-law stay in the relationship. As a result, there are more married women and fewer cohabiting women in the counterfactual economy compared to the baseline.

<sup>&</sup>lt;sup>33</sup>By exploiting different timing of introduction of common-law marriage in different Canadian provinces, Lafortune *et al.* (2017) examines the impact of introduction on various cohabiting couples' decisions, and how the impact differs between pre-existing couples and couples formed after the changes. They find that introducing common-law marriage lowered the labor market participation rate of already cohabiting women but had little impact on couples start cohabitation after the change.

<sup>&</sup>lt;sup>34</sup>See Footnote 1 for the detail.

	Less-educa	ated women	More-educ	ated women
	Baseline	Common- law	Baseline	Common- law
Average duration of first marriage at age 33 (years)	8.64	8.37	7.62	8.00
Average duration of first cohabitation at age 33 (years)	2.27	2.04	2.61	1.41
Percentage of women who have ever married by age 33	75.25	88.45	72.55	81.50
Percentage of women who have divorced by age 33	30.50	44.90	10.15	13.80
Percentage of women who have cohabited by age 33	55.45	35.50	37.20	5.95

Table 2.13: Union experiences, with common-law marriage

Columns 'Baseline' represent the baseline calibration results, and columns 'Common-law' represent results of the model with common-law marriage.

Figure 2.5 shows that there are more married women, including those who are living in common-law marriage, early in their lifecycle with the introduction of common-law marriage. Although more women marry earlier, Table 2.13 shows that there are more divorces than the baseline as couples who married by common-law tend to have lower match quality on average. Both for less-educated and more-educated women, the fractions of married women are lower than that in the baseline calibration toward the end of the life.

The additional risk of cohabitation due to common-law marriage lowers the benefit of cohabitation. Compared to the baseline, Figure 2.6 shows that fewer women are cohabiting with common-law marriage, and the decrease is more pronounced among more-educated women. Although there are more married women, the average number of children women have decreases as in Figure 2.7.

	Less-educ	cated women	More-edu	cated women
	Baseline	Common-law	Baseline	Common-law
Have lived with a single mother (%) Average investment per child per period	33.57 0.173	22.77 0.184	15.39 0.210	2.40 0.229

Table 2.14: Experience of children

Columns 'Baseline' are the results of baseline calibration, which is calibrated to match the NLSY79 cohort. Columns 'Common-law' are the results from the model with the common-law marriage.

Table 2.14 shows that the introduction of common-law marriage reduces the percentage of children living with a single mother, as children born to cohabiting mothers are living in common-law marriages which tend to last longer than cohabitation. This change increases the amount of investment a child receives by 6% for children of less-educated women and by 9% for children of more-educated women.

The common-law marriage policy lowers the expected value of lifetime utility of women, as it restricts their choices. In other words, the women in the model prefer the baseline to the economy with common-law marriage. Less-educated women consider having common-law marriage implemented equivalent to having 1.52% lower wages over their life time without the policy. The policy even more costly for more-educated women. The decrease of more-educated women's utility from the policy is equivalent to 2.61% decrease in their wages over their life time from the baseline.<sup>35</sup>

# 2.6 Conclusion

Despite its increasing prevalence and its tight linkage to non-marital childbearing, the literature of family economics typically overlooks cohabitation as a distinct form of union. From the samples from NLSY79 and NSFG Cycle 6, I find that there are educational gradients in fertility and union choice: less-educated women are more likely to cohabit and more likely to give birth while cohabiting than more-educated women. Based on these observations, I build a life-cycle model of union and fertility choices to understand those choices. The model features a partial equilibrium partner search and quantity-quality trade-off of children with imperfect control over fertility. The calibrated model captures the observed educational gradients in cohabitation and non-marital fertility.

Using the calibrated model, I conduct two quantitative exercises. First, I show that the decline of the gender earnings gap leads to a delay of marriage, as observed in data in the past couple decades. According to the model, the changes in gender gap alone can explain about a third to half of the changes in union allocation of less-educated women observed in the data.

Second, I study the effect of introducing common-law marriage on fertility and union decisions. By treating cohabiting couples who give birth while cohabiting as married couples, union allocations change substantially in the model. There are more married couples and fewer cohabitants. As a result, fewer children live with single mothers and receive more spending from their mothers during childhood, as compared to the baseline where no common-law marriage exists.

The current model abstracts from many important features of union decisions, including the interaction of the two individuals of a union. In the next chapter, I study the bilateral household union decision.

<sup>&</sup>lt;sup>35</sup>I include the welfare analysis of the policy to measure the policy's cost, but it does not capture the complete cost or benefits as the model misses two important margins. First, the model treats men as passive agents, and their welfare is not considered in the analysis. Second, children are considered only through a component of mother's utility function. As a result, their own benefits (or losses) from the common-law marriage is not considered in the calculation.

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# Chapter 3

# Income uncertainty, marriage and cohabitation

# 3.1 Introduction

Over the past five decades, Americans have been delaying their first marriage. According to the Current Population Survey (CPS) Annual Social and Economic Supplement (ASEC), in 1970, 80% of 18 – 44 year-old women had been married (either currently married, separated, divorced, or widowed). In 2017, according to the same survey, only 53% of the women of the same age had been married. I posit that the trend of delaying marriage arises from a substitution toward cohabitation, living with a partner without getting legally married. If I count both marriage and cohabitation, the fraction of people living in a union has remained remarkably stable around 0.6 since the 1980s. I propose that increasing income volatility and the closing of the gender earnings gap are key drivers of the substitution. To assess the hypothesis, I build an equilibrium model of household union choice, including both cohabitation and marriage as options, and conduct quantitative exercises with income volatility and the gender earnings gap.

In the model, living in a union provides economic benefits through economies of scale in consumption and a non-economic benefit (or cost) from match quality. I assume that marriage provides greater economies of scale in consumption compared to cohabitation. However, ending a marriage (divorce) incurs a divorce cost, whereas ending a cohabitation is costless. The uncertainty in the model comes from income shocks and match quality shocks. Couples expecting a higher chance of the relationship ending choose to cohabit despite the smaller economic benefit, and those with lower chances marry. In the model, increasing income volatility increases the risk of a relationship ending due to an income shock. It leads couples to be more cautious about getting married and to cohabit instead.

There are papers in the literature that study the causes behind the trend of delaying marriage using structural models. Greenwood and Guner (2008) argue that technological progress in household production, such as improved refrigerators and washing machines, has reduced the value of having spouses specializing in market work and home production, and this lowers the value of marriage. Regalia *et al.* (2011) find the closing gender wage gap is responsible for the increase in single households. As women have more income, the value of living alone increases relative to the value of marriage, and there are fewer married households. Closely related to the delay in marriage, the literature also studies the delay of fertility. Caucutt *et al.* (2002) find that increasing returns to experience in the labor market for women make having a child, especially early in their career, more costly for women, and this increasing opportunity cost makes women wait longer to have children. More recently, Sommer (2016) links increasing income volatility with the decrease in fertility. She models children as a consumption commitment: parents must provide a certain level of consumption to their children. Increasing idiosyncratic income shocks makes it harder to meet the level of consumption commitment, which effectively increases the cost of having children, and delays their fertility decision.

There is empirical evidence suggesting a link between increasing inequality and the delay in marriage in the literature. Oppenheimer *et al.* (1997) argue that increasing inequality makes the career-entry process harder for young men, and these young men delay their marriage until their careers mature. Using the samples from the NLSY79 and developed measures of career maturity, they find the difficulty of the career-entry process is correlated with the timing of marriage. Using the U.S. Census, Gould and Paserman (2003) find that higher income inequality in a city encourages women in the city to search longer for a husband. Ahn and Mira (2001) find that people delay marriage during periods of higher unemployment rates in Spain and show that men's employment status affects their timing of marriage. Based on this empirical evidence, Santos and Weiss (2016) build an equilibrium model of marriage and demonstrate a mechanism for how increasing income inequality delays marriage by assuming children are a consumption commitment.

This chapter differs from Santos and Weiss (2016) and the literature by assuming trichotomous household union status including (non-marital) cohabitation. Most models of household formation in the economics literature assume dichotomous household union status, either married or being single, without considering cohabitation as a distinct option. Considering cohabitation separately is not a mere sophistication in modeling. The spousal insurance of income shocks is considered to be an important role of household union. Kotlikoff and Spivak (1981) argue that families provide insurance against uncertain life when the annuity market is incomplete. The authors estimate that the benefit of marriage for a single person to be 10 - 20% of personal wealth. Hess (2004) finds that couples with higher correlation in income are more likely to divorce, suggesting the insurance against income shocks plays a role in marriage. In models without cohabitation, increasing income volatility increases the benefit from the income insurance of marriage, and models produce more married couples when volatility increases. To break this relationship, which appears contradictory to empirical observations, Santos and Weiss (2016) introduce children as a consumption commitment in a model similar to Sommer (2016). In contrast, I introduce cohabitation as an intermediate household union form to produce the delay in marriage as income volatility increases.

This chapter contributes to the literature by building a model of household union choice including both marriage and cohabitation. As mentioned, most household formation models do not consider cohabitation as a separate form of household union. A few existing models that include cohabitation assume unilateral decision making (Brien *et al.*, 2006; Adamopoulou, 2014) or that individuals have preferences for marriage or cohabitation (Gemici and Laufer, 2012) to avoid solving bargaining problems. I provide a simple way to solve the bargaining problem of a couple by assuming that part of match quality is private information. This assumption leads to an agreement between the two parties of a couple on the type of household union to choose and simplifies the bargaining problem. Calibrated the model to U.S. data, I find that the model can explain about 70% of the delay in marriage with changes in income volatility and the gender earnings gap.

In the next section, I present empirical observations of changes in household unions, income volatility, and the gender earnings gap in the past three decades in the U.S. Section 3.3 presents the model, and the calibration procedure is discussed in Section 3.4. Section 3.5 reports the quantitative exercises using the calibrated model, and Section 3.6 concludes this chapter.

## **3.2** Trends in household union

Figure 3.1 shows union status of 18 - 44 year-old women in the U.S. from 1982 to 2015 using the NSFG. According to the NSFG Cycle 3, which was conducted in 1982, 57% of 18 - 44 year-old women were married, and only about 3% of women were cohabiting on the day of the interview. About 30 years later, in the 2013 - 2015 NSFG, 42% of women of the same age were married, and 16% were cohabiting. The delay in marriage is well known, but if women are counted as living in a household union, either marriage or cohabitation, the fraction remains remarkably stable, around 60%, over the past three decades.

People appear to delay their marriage over time, choosing to cohabit instead. Figure 3.2 plots the mean age at first marriage (solid blue line), first cohabitation (dashed green line), and first union, either first marriage or first cohabitation (dotted red line) of 35 - 44 year-old women. The age at first cohabitation is computed from 1988, as questions on previous cohabitation

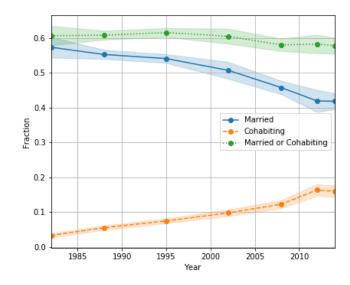


Figure 3.1: Union status of 18 – 44 year-old women

*Notes.* Using samples from the NSFG Cycle 3 (for 1st marriage), Cycles 4, 5, and 6, 2006 – 2010 NSFG, 2011 – 2013 NSFG, and 2013 – 2015 NSFG; Weights and survey design variables are used to compute the point estimates and confidence intervals. Colored areas denote 95% confidence intervals of the estimates.

experience are first included in the NSFG Cycle 4. The average age at first marriage increased by about four years from 1988 to 2015, but the age at first union increased by only two years during the same period. As Figure 3.2 shows, this is because increasingly more people have cohabited at an earlier age.

Another way to see this trend is examining the experience of cohabitation. The solid blue line in Figure 3.3 shows the fraction among all 18 - 44 year-old women who have cohabited before the day of the interview in the different cycles of the NSFG. Similar to the fraction of currently cohabiting women summarized in Figure 3.1, more people have experienced cohabitation. For example, in 1988, about a third of 18 - 44 year-old women have cohabited before the day of the interview. The percentage steadily increased over time, with more than 60% of women having cohabited according to the 2013 – 2015 NSFG. Furthermore, increasingly more people now cohabit before their first marriage. The dashed green line in the figure shows the fraction of women who have ever cohabited among those who never married before. In the most recent survey, about half of never-married women have lived with their partner in cohabitation.

During this period, income inequality increased (e.g., Juhn *et al.*, 1993; Heathcote *et al.*, 2010). Figure 3.4 shows variances of white prime age (25 - 54 year-old) full-time and full-year workers' log weekly wages (solid blue line) and residual variances (dotted red line) in the CPS ASEC from 1970 to 2017. For the residual (or within group) variance, I control for

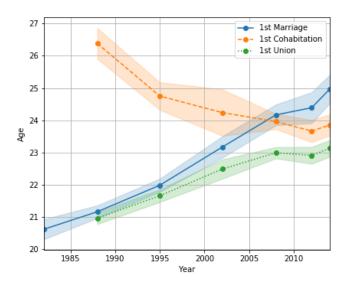


Figure 3.2: Age at first union among 35 – 44 year-old women

*Notes.* Using samples from the NSFG Cycle 3 (for 1st marriage), Cycles 4, 5, and 6, 2006 - 2010 NSFG, 2011 - 2013 NSFG, and 2013 - 2015 NSFG, conditional on have lived in a union before; Weights and survey design variables are used to compute the point estimates and confidence intervals. Colored area round lines denotes 95% confidence interval of the estimates.

potential experience (age - 6 - years of schooling) and education following a standard log wage regression. Over this period, as the figure presents, wage inequality increased, and the majority of the increase was not driven by changes in educational attainment or the age distribution. In other words, compared to the past, it has become harder to guess someone's income after observing characteristics such as age and level of education.

In addition to increasing inequality, the gender gap in earnings has shrunk over the period (Blau and Kahn, 1995). Figure 3.5 plots the gender earnings gap as given by the average weekly earnings of full-time and full-year female white prime age (25 - 54 year-old) workers relative to male workers' weekly earnings after controlling for potential experience and education. In the 1970s, the weekly earnings of female workers were about 60% of male workers with similar experience and education. The gap shrinks over time, with female workers receiving 80% of male workers' wages in 2010. From the perspective of women, this decreasing gender earnings gap reduces the economic benefit of a union.

In summary, people are delaying marriage, and more people are cohabiting over time. At the same time, income inequality has been increasing, and the gender earnings gap has been closing. I argue that there is a link between these changes in the labor market and changes in household unions. In the next section, I propose a model of household union choice and explain the link using the model.

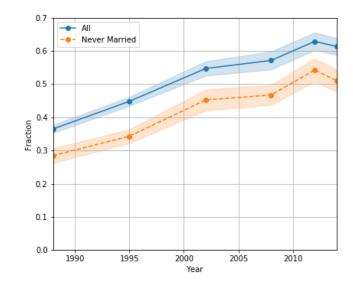


Figure 3.3: Fraction of 18 – 44 year-old women who have cohabitated before

*Notes.* Using samples from the NSFG Cycle 3 (for 1st marriage), Cycles 4, 5, and 6, 2006 – 2010 NSFG, 2011 – 2013 NSFG, and 2013 – 2015 NSFG; Weights and survey design variables are used to compute the point estimates and confidence intervals. Colored area round lines denotes 95% confidence interval of the estimates.

# 3.3 Model

#### **3.3.1** Environment

There are unit measures of men and women. Each period a person receives a random amount of goods. The endowment cannot be saved.

Figure 3.6 summarizes the timing of events within a period. There is no market to trade the endowment, but there is a matching market to form a household union with a person of the opposite sex. Each participant meets a potential partner and observes the partner's endowment and a match quality, which is also a random variable. The matching market has a search friction. Participants meet each other randomly. After meeting each other, the pair can either remain single or form a household union of two types: marriage or cohabitation. By living with a partner in a union, the couple receives utility from the match quality and the economic benefit from economies of scale in consumption. The two types of union, marriage and cohabitation, have different degrees of economies of scale and exit costs. Marriage provides higher economies of scale than cohabitation: a couple with the same aggregate endowments can consume more if they are married than cohabiting. However, if a couple wants to end the union, a married couple pays a divorce cost,  $\kappa$ , whereas a cohabiting couple separates without incurring any cost. These assumptions stand in for the different levels of commitment and specialization of a couple arising from institutional differences in union types. To simplify the problem, I

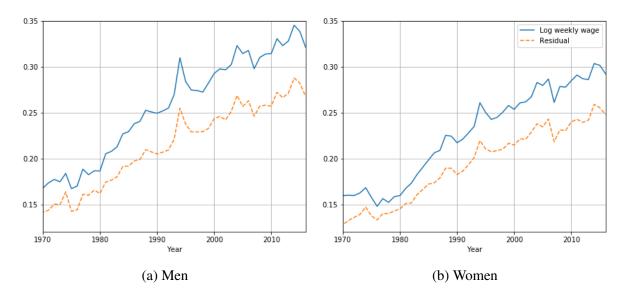


Figure 3.4: Variances of log weekly wages of white prime age workers

*Notes.* Using the CPS ASEC samples from 1970 to 2017. 'Residual' is the variance of residual (within group variance) after controlling for the potential experience and years of education.

assume that only people who are single at the beginning of the period can participate in the period's matching market.

Each period a person exits the model with probability  $\delta$  and is replaced by a single person with the same gender and the same endowment.

#### 3.3.2 Preference

Every period, people get utility from consumption,  $u(c_t)$ , and match quality,  $\eta_t$ , which provides direct utility additively:  $u(c_t) + \eta_t$ . I assume that single people receive no match quality ( $\eta_t \equiv$ 0 if single), so the match quality can be interpreted as a non-economic benefit (or cost) of living with a partner in addition to the economic benefit through the economies of scale in consumption. People living in a union receive a match quality which changes over time.

#### **3.3.3** Match quality

The match quality is composed of a component that both share,  $\gamma_t$ , and a component that is specific to each individual,  $v_{t,W}$  and  $v_{t,H}$ :  $\eta_{t,W} = \gamma_t + v_{t,W}$ ,  $\eta_{t,H} = \gamma_t + v_{t,H}$ . The individual specific components,  $v_{t,W}$  and  $v_{t,H}$ , are random variables that have zero means and are independent of each other and over time. The component is unobservable to the partner. As it is independent over time, neither person learns or infers their partner's v. That is, both know

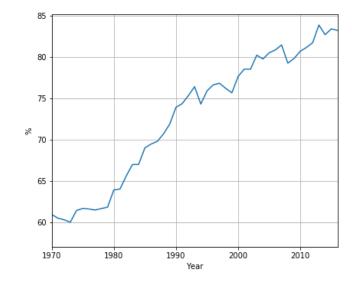


Figure 3.5: Gender earnings gap: female workers' average weekly earnings relative to males'

*Notes.* Using the CPS ASEC samples from 1970 to 2017. Using the log weekly wage of full-time and full-year white prime age (25 - 54 year-old) workers after controlled for potential experience and education for each gender.

nothing more about their partner's match quality beyond the common component,  $\gamma_t$ .

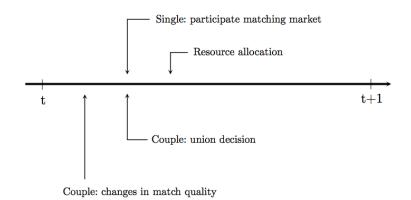
I assume that a couple decides how to allocate their pooled endowment to individual consumption *before* observing the individual specific components, and they commit to that allocation. After each person observes his or her own realization of the individual specific component, the couple stays in the union only if both parties receive greater utility together than being single and after paying the divorce cost, if married. Because of the timing of the resource allocation decision, there may be ex-post inefficient dissolutions of couples: the sum of the surplus is strictly positive, but a couple separates because one party ends up having a negative surplus. I assume that there is no renegotiation of the allocation.<sup>1</sup>

#### **3.3.4 End-of-period value for singles**

Because of the assumptions of perishable endowment and lack of exchange market, the problem of someone who stays single is trivial: he or she consumes what he or she is endowed with. Let W and H denote value functions and x and y be endowments of women and men,

<sup>&</sup>lt;sup>1</sup>Alternatively, the model can include an additional stage where couples renegotiate the resource allocation after observing individual specific components to maximize their joint surplus à la Myerson and Satterthwaite (1983). This additional renegotiation complicates the computation and changes the result quantitatively. The results will be similar *qualitatively*: there still will be ex-post inefficient dissolutions, but the measure of these dissolutions will be smaller compared to the present model without the renegotiation. Furthermore, the quantitative difference will be muted in the calibration procedure, which will be discussed in Section 3.4 (the calibrated parameter values will be different to match the same moments).





respectively. Union status is noted using a superscript (S for single in this case).

$$W^{S}(x_{t}) = u(x_{t}) + \beta(1-\delta) \mathbb{E} \left[ W(x_{t+1}, y_{t+1}, \gamma_{t+1}, s_{t} = S) | x_{t} \right],$$
  

$$H^{S}(y_{t}) = u(y_{t}) + \beta(1-\delta) \mathbb{E} \left[ H(x_{t+1}, y_{t+1}, \gamma_{t+1}, s_{t} = S) | y_{t} \right].$$
(3.1)

The discount factor is denoted by  $\beta$ . Because they survive to the next period with the probability  $1 - \delta$ , the effective discount factor is  $\beta(1 - \delta)$ . They form expectations for the continuation value that depend on own endowment, partner type, and the current union status,  $s_t$ , which is single, S. The current union status affects the continuation value as it affects whether or not the person participates in the matching market.

#### **3.3.5** End-of-period value for couples

If a union forms, or a couple from last period decides to stay together, they pool their endowments and decide how much each individual consumes. In order to define end-of-period values for couples, which depend on how they share the surplus which is the value net of the value of being single, I need to make an assumption about their bargaining protocol. I assume that each person consumes a share of the pooled resource so that he or she has the same surplus *in expectation* with respect to the individual specific component of the match quality (v's).<sup>2</sup> A couple who is in union type *j* decides the pair of individual consumption,  $c_W^j$  and  $c_M^j$ , to maximize the sum of their utility. The solution must satisfy the budget constraint, the non-negativity

<sup>&</sup>lt;sup>2</sup>Because I assume that the individual specific components of the match quality have zero means and are independent over time, the expectations effectively remove them: they are 'integrated out.' As a result, the pair of individual consumption can be derived simply by equating the surpluses of men and women ignoring the individual components of the match quality.

condition, and the surplus parity in expectation condition:

$$\begin{split} \max_{c_{W}^{j}, c_{H}^{j}} & W^{j}(c_{W}^{j}; x_{t}, y_{t}, \gamma_{t}, s_{t-1}) + H^{j}(c_{H}^{j}; x_{t}, y_{t}, \gamma_{t}, s_{t-1}) \\ \text{s.t.} & \frac{c_{W}^{j} + c_{H}^{j}}{\psi^{j}} \leq x + y, \\ & c_{W}^{j}, c_{H}^{j} > 0, \\ & \mathbb{E}_{v_{t,W}} W^{j}(x_{t}, y_{t}, \gamma_{t}, s_{t-1}) - W^{S}(x_{t}) = \mathbb{E}_{v_{t,M}} H^{j}(x_{t}, y_{t}, \gamma_{t}, s_{t-1}) - H^{S}(y_{t}), \\ \text{where} & W^{j}(v_{t,W}; x_{t}, y_{t}, \gamma_{t}, s_{t-1}) = u(c_{W}) - \kappa \mathbb{1}_{\{s_{t-1} = M \text{ and } j \neq M\}} + \gamma_{t} + v_{t,W} \\ & + \beta(1 - \delta) \mathbb{E} \left[ (1 - \delta) W(x_{t+1}, y_{t+1}, \gamma_{t+1}, s_{t} = j) + \delta W^{S}(x_{t+1}) | x_{t}, y_{t}, \gamma_{t} \right], \\ H_{j}(v_{t,H}; x_{t}, y_{t}, \gamma_{t}, s_{t-1}) = u(c_{H}) - \kappa \mathbb{1}_{\{s_{t-1} = M \text{ and } j \neq M\}} + \gamma_{t} + v_{t,H} \\ & + \beta(1 - \delta) \mathbb{E} \left[ (1 - \delta) H(x_{t+1}, y_{t+1}, \gamma_{t+1}, s_{t} = j) + \delta H^{S}(y_{t+1}) | x_{t}, y_{t}, \gamma_{t} \right], \\ \text{for} & j \in \{C, M\} \quad \text{(Cohabiting or Married).} \end{split}$$

$$(3.2)$$

The function  $\mathbb{1}_{\{\cdot\}}$  is an indicator function, which takes a value of one if the argument condition in the bracket is true and is zero otherwise. Parameter  $\psi^j$  denotes the economies of scale in consumption, which depends on the union type, *j*. A couple can consume  $\psi^j$  times their combined endowments. If a couple was married last period  $(s_{t-1} = M)$ , the couple has to pay the divorce cost ( $\kappa$ ) to cohabit, which makes starting cohabitation with a spouse never an optimal choice for a married couple. Each person exits the model with probability  $\delta$  every period. This applies to both parties of a couple independently, so one may leave the model earlier than the other. The continuation value takes this into account. If both survive to the next period, then the couple retains the current union status in the next period and makes decisions given this period's union status ( $s_t = j$ ). However, if a person survives but his or her partner dies, then this person is single in the next period and participates in the matching market in the period after that.

#### 3.3.6 Union decision

Given the values of being single from Section 3.3.4 and the expected values of living in a union defined in Section 3.3.5, a couple chooses the type of union which gives the greatest utility. The assumption on the timing and the commitment to resource allocation makes the union decision a two-step process. Before observing the individual component of match quality, based on the expected values, a couple decides the type of union (marriage or cohabitation). Then each person decides whether to stay in the union or not after observing the individual match quality. Starting cohabitation with a current spouse is a feasible option for a married couple. However,

as it incurs the divorce cost without changing the couple's match quality or endowments, it always gives a smaller value than staying married. As a result, no married couple chooses to start cohabitation with the spouse.

Before observing the individual component of match quality, as the couple shares the surplus equally *in expectation*, both persons always agree on what type of union (marriage or cohabitation) to form. Formally, as

$$\mathbb{E}_{\mathbf{v}_{t,W}} W^{M}(\mathbf{v}_{t,W}; x_{t}, y_{t}, \gamma_{t}, s_{t-1}) - W^{S}(x_{t}) = \mathbb{E}_{\mathbf{v}_{t,H}} H^{M}(\mathbf{v}_{t,H}; x_{t}, y_{t}, \gamma_{t}, s_{t-1}) - H^{S}(y_{t}), \text{ and}$$
  
$$\mathbb{E}_{\mathbf{v}_{t,W}} W_{C}(\mathbf{v}_{t,W}; x_{t}, y_{t}, \gamma_{t}, s_{t-1}) - W^{S}(x_{t}) = \mathbb{E}_{\mathbf{v}_{t,H}} H_{C}(\mathbf{v}_{t,H}; x_{t}, y_{t}, \gamma_{t}, s_{t-1}) - H^{S}(y_{t}),$$

by subtracting both sides,

$$\mathbb{E}_{\mathbf{v}_{t,W}}\left[W^{M}(\mathbf{v}_{t,W};x_{t},y_{t},\gamma_{t},s_{t-1})-W^{C}(\mathbf{v}_{t,W};x_{t},y_{t},\gamma_{t})\right]$$
$$=\mathbb{E}_{\mathbf{v}_{t,H}}\left[H^{M}(\mathbf{v}_{t,H};x_{t},y_{t},\gamma_{t},s_{t-1})-H_{C}(\mathbf{v}_{t,H};x_{t},y_{t},\gamma_{t})\right].$$

In other words, if a woman prefers marriage over cohabitation, her partner also prefers marriage over cohabitation, and vice versa.

However, they may disagree about whether to stay together *after* observing the individual specific component of the match quality. I assume that union dissolution is a unilateral decision: to stay in a union, both individuals must agree.<sup>3</sup> As a result, conditional on state variables  $(x_t, y_t, \gamma_t, s_{t-1})$ , a couple plays a mixed strategy whether to stay together. Each person in a couple wants to stay in the union, if the value of staying exceeds the value of being single after paying the divorce cost if the couple was married. The probability that a woman wants to stay in a union of type *j* is,

$$P_{W}^{j}(x_{t}, y_{t}, \gamma_{t}, s_{t-1}) = Pr\left[W^{j}(v_{t,W}; x_{t}, y_{t}, \gamma_{t}, s_{t-1}) > W_{t}^{S}(x_{t}) - \kappa \mathbb{1}_{\{s_{t-1}=M\}}\right]$$
  
=  $Pr\left[v_{t,W} > W_{t}^{S}(x_{t}) - \kappa \mathbb{1}_{\{s_{t-1}=M\}} - \{u(c_{W}) + \gamma_{t} + \beta(1-\delta) \mathbb{E}\left[(1-\delta)W(x_{t+1}, y_{t+1}, \gamma_{t+1}, s_{t}=j) + \delta W_{S}(x_{t+1})|x_{t}, y_{t}, \gamma_{t}\right]\}\right],$   
(3.3)

<sup>&</sup>lt;sup>3</sup>This is also true in the United States since the 1980s. See, for example, Gruber (2004) and Wolfers (2006).

and similarly, her partner wants to stay together with probability

$$P_{H}^{j}(x_{t}, y_{t}, \gamma_{t}, s_{t-1}) = Pr\left[v_{t,H} > H_{t}^{S}(y_{t}) - \kappa \mathbb{1}_{\{s_{t-1}=M\}} - \left\{u(c_{H}) + \gamma_{t} + \beta(1-\delta) \mathbb{E}\left[(1-\delta)H(x_{t+1}, y_{t+1}, \gamma_{t+1}, s_{t}=j) + \delta H_{S}(y_{t+1})|x_{t}, y_{t}, \gamma_{t}\right]\right\}\right].$$
(3.4)

By the unilateral separation assumption, the product of the two probabilities gives the probability that the couple stays together in union of type *j*. Let the probability that the couple cohabits be  $P^C = P^C_W \times P^C_H$  and the probability they marry be  $P^M = P^M_W \times P^M_H$ . The couple chooses to separate with the complementary probability:  $P^S = 1 - P^M - P^C$ . This provides the union decision rules for a couple (or a match who has just met),  $P^j(x_t, y_t, v_t, s_{t-1})$ .

#### **3.3.7** Value functions

Value functions are defined using the end-of-period value functions and the union decision rules:

$$W(x_{t}, y_{t}, \gamma_{t}, s_{t-1}) = P^{S}(x_{t}, y_{t}, \gamma_{t}, s_{t-1}) \left\{ W^{S}(x_{t}) - \kappa \mathbb{1}_{\{(s_{t-1}=M)\}} \right\} + P^{C}(x_{t}, y_{t}, \gamma_{t}, s_{t-1}) W^{C}(x_{t}, y_{t}, \gamma_{t}, s_{t-1}) + P^{M}(x_{t}, y_{t}, \gamma_{t}, s_{t-1}) W^{M}(x_{t}, y_{t}, \gamma_{t}, s_{t-1}), H(x_{t}, y_{t}, \gamma_{t}, s_{t-1}) = P^{S}(x_{t}, y_{t}, \gamma_{t}^{M}, s_{t-1}) \left\{ H^{S}(y_{t}, s_{t-1}) - \kappa \mathbb{1}_{\{(s_{t-1}=M)\}} \right\} + P^{C}(x_{t}, y_{t}, \gamma_{t}^{M}, s_{t-1}) H^{C}(x_{t}, y_{t}, \gamma_{t}^{M}, s_{t-1}) + P^{M}(x_{t}, y_{t}, \gamma_{t}^{M}, s_{t-1}) H^{M}(x_{t}, y_{t}, \gamma_{t}^{M}, s_{t-1}).$$
(3.5)

#### **3.3.8 Distributions**

Distributions of single men,  $\Omega_H^S$ , single women,  $\Omega_W^S$ , cohabiting couples,  $\Omega^C$ , and married couples,  $\Omega^M$ , are determined endogenously. These distributions are defined using the union decision rules from Section 3.3.6.

Let  $G_x(x_{t+1}|x_t)$  and  $G_y(y_{t+1}|y_t)$  be conditional distributions of the endowments of women and men, respectively. Let  $\Gamma(\gamma_{t+1}|\gamma_t)$  be the conditional distribution of the match quality, and let  $\Gamma_0(\gamma)$  be the initial distribution of the match quality for a newly matched couple. As mentioned, I assume that each person exits the model with probability  $\delta$ . To have a stationary aggregate endowments distribution, I assume that, when a person exits the model, a new single person enters the model with the same amount of endowment. Let  $\Omega'_W^S$  be next period's distribution of single women. Regardless of their union status,  $\delta$  of current period's women leave the model and are replaced by single women,  $\Omega'_{\Delta,W}^S$ . Among women in the current period who stay in the model next period, with total measure of  $1 - \delta$ , the measure of single women is the sum of this period's single women who stay single after participating the matching market,  $\Omega'_W^{S,S}$ , this period's cohabiting women who separate from their partner,  $\Omega'_W^{C,S}$ , and married women who divorce  $(\Omega'_W^{M,S})$ . Formally, the next period's distribution of single women is:

$$\begin{aligned} \Omega_{W}^{lS}(x_{t+1}) &= \Omega_{W}^{lS,S}(x_{t+1}) + \Omega_{W}^{lC,S}(x_{t+1}) + \Omega_{W}^{lM,S}(x_{t+1}) + \Omega_{W}^{l\Delta,S}(x_{t+1}), \\ \Omega_{W}^{lS,S}(x_{t+1}) &= \int_{\gamma_{t+1}} \int_{y_{t+1}} P^{S}(x_{t+1}, y_{t+1}, \gamma_{t+1}, s_{t} = S) \\ &\times \int_{x_{t}} (1 - \delta) \Omega_{W}^{S}(x_{t}) G_{x}(x_{t+1} | x_{t}) dx_{t} \int_{y_{t}} \frac{\Omega_{H}^{S}(y_{t})}{\int_{y_{t}} \Omega_{H}^{S}(y_{t}) dy_{t}} G_{y}(y_{t+1} | y_{t}) dy_{t} \Gamma_{0}(\gamma_{t+1}) dy_{t+1} d\gamma_{t+1}, \\ \Omega_{H}^{lC,S}(x_{t+1}) &= \int_{\gamma_{t+1}} \int_{y_{t+1}} P^{S}(x_{t+1}, y_{t+1}, \gamma_{t+1}, s_{t} = C) \\ &\times \int_{\gamma} \int_{y_{t}} \int_{x_{t}} (1 - \delta) \Omega^{C}(x_{t}, y_{t}, \gamma_{t}) G_{x}(x_{t+1} | x_{t}) G_{y}(y_{t+1} | y_{t}) \Gamma(\gamma_{t+1} | \gamma_{t}) dx_{t} dy_{t} d\gamma_{t} dy_{t+1} d\gamma_{t+1}, \\ \Omega_{W}^{lM,S}(x_{t+1}) &= \int_{\gamma_{t+1}} \int_{y_{t+1}} P^{S}(x_{t+1}, y_{t+1}, \gamma_{t+1}, s_{t} = M) \\ &\times \int_{\gamma} \int_{y_{t}} \int_{x_{t}} (1 - \delta) \Omega^{M}(x_{t}, y_{t}, \gamma_{t}) G_{x}(x_{t+1} | x_{t}) G_{y}(y_{t+1} | y_{t}) \Gamma(\gamma_{t+1} | \gamma_{t}) dx_{t} dy_{t} d\gamma_{t} dy_{t} d\gamma_{t} dy_{t+1} d\gamma_{t+1}, \\ \Omega_{W}^{l\Delta,S}(x_{t+1}) &= \int_{x_{t}} \delta \left[ \Omega_{W}^{S}(x_{t}) + \int_{\gamma} \int_{y_{t}} \left\{ \Omega^{C}(x_{t}, y_{t}, \gamma_{t}) + \Omega^{M}(x_{t}, y_{t}, \gamma_{t}) \right\} dy_{t} d\gamma_{t} \right] G_{x}(x_{t+1} | x_{t}) dx_{t}. \end{aligned}$$

$$(3.6)$$

The distributions of single men, of cohabiting couples, and of married couples are defined analogously in Appendix B.

#### 3.3.9 Equilibrium

An equilibrium of the model is a set of decision rules for consumption  $(c_W^S, c_H^S, c_W^C, c_H^C, c_W^M)$ , and  $c_H^M$ , union choice  $(P_W^S, P_H^S, P_W^C, P_H^C, P_W^M)$ , and  $P_H^M$ , value functions  $(W^S, W^C, W^M, W, H^S, H^C, H^M)$ , and H, and distributions of people,  $(\Omega_W^S, \Omega_H^S, \Omega^C)$ , and  $\Omega^M$ , such that:

- 1. The consumption  $(c_W^S \text{ and } c_H^S)$  and the end-of-period value functions  $(W^S \text{ and } H^S)$  of single women and men are as defined in Equation (3.1).
- 2. The consumption  $(c_W^j \text{ and } c_H^j)$  of couples solves (3.2) and the end-of-period value functions  $(W^j \text{ and } H^j)$  are defined as the indirect value functions, for  $j \in \{C, M\}$  (cohabitation and marriage).

- 3. The individual union choice rules for staying together  $(P_W^C, P_H^C, P_W^M, \text{ and } P_H^M)$  are as defined in Equations (3.3) and (3.4), and the union choice rules for separation  $(P_W^S \text{ and } P_H^S)$  are defined as their complementary probabilities.
- 4. The value functions (W and H) are as defined in Equation (3.5).
- 5. The distributions of people  $(\Omega_W^S, \Omega_H^S, \Omega^C, \text{ and } \Omega^M)$  are defined to be consistent with respect to the union choice rules in the way described in (3.6).

A steady state is an equilibrium that does not change over time. In the quantitative analysis of the model, I restrict my attention to a steady state equilibrium.

## **3.4** Matching the model to the data

In this section, I solve the model numerically and calibrate model parameters to replicate key data observations in the U.S. data.

#### **3.4.1** Parametric assumptions

To solve the model numerically, I make parametric assumption on the utility function and distributions of random variables. I use a CRRA utility function:<sup>4</sup>

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \quad \sigma > 0.$$

I assume that both men and women's endowments are log-normally distributed, following AR(1) processes in logs:

$$\log x_{t+1} \sim N\left((1-\phi)\log \bar{x} + \phi\log x_t, \sigma^2\right) \equiv G_x(x_{t+1}|x_t),$$
  
$$\log y_{t+1} \sim N\left((1-\phi)\log \bar{y} + \phi\log y_t, \sigma^2\right) \equiv G_y(y_{t+1}|y_t).$$

I approximate both processes as discrete state Markov chains with 11 states using the method of Rouwenhorst (1995) following Kopecky and Suen (2010).

<sup>&</sup>lt;sup>4</sup>As this chapter considers insurance against income uncertainty as a key benefit of a household union, I assume a CRRA utility function, a common specification of risk-averse preference. As a result, the model falls into the non-transferable utility environment. The transferable utility (TU) environment, where agents have (quasi-)linear utility functions, are also commonly used in the literature of family economics, and the literature of bargaining in general, due to its theoretical convenience. See, for example, Chiappori *et al.* (2015) for a discussion of TU in the context of household decision making. In TU, agents are risk-neutral, and there is no benefit from risk-sharing as long as income shocks are independent across individuals. However, inefficient separations can happen in TU if couples have private information, as shown in Myerson and Satterthwaite (1983).

The shared component of match quality ( $\gamma$ ) is normally distributed. Depending on whether it is for a couple who just met or a couple from the previous period, it follows one of two distributions:

$$\gamma_{t+1} \sim \begin{cases} N\left(\bar{\gamma}_{0}, \sigma_{0}^{2}\right) \equiv \Gamma_{0}(\gamma_{t+1}), & \text{for a couple who just met,} \\ N\left((1 - \phi_{\gamma})\bar{\gamma} + \phi_{\gamma}\gamma_{t}, \sigma_{\gamma}^{2}\right) \equiv \Gamma(\gamma_{t+1}|\gamma_{t}), & \text{for an existing couple with match quality } \gamma_{t} \end{cases}$$

The AR process of  $\Gamma(\gamma_{t+1}|\gamma_t)$  is also discretized using Rouwenhorst (1995) as an 11-state Markov chain. The distribution  $\Gamma_0(\gamma_{t+1})$  is discretized following Kennan (2006) on the same grid points of  $\gamma_{t+1}(\gamma_t)$ .

Lastly, the individual specific components of the match quality,  $v_W$ ,  $v_H$ , are normally distributed with zero mean and the same variance:  $v_W$ ,  $v_H \sim N(0, \sigma_v^2)$ .

#### **3.4.2** Parameters set outside of the model

I set values of some parameters outside of the model. One model period is a year, so the discount factor,  $\beta$ , is set to 0.98, a standard value in the literature. The curvature parameter of the utility function,  $\sigma$ , (the inverse of relative risk aversion) is set to 2. As the model is calibrated to match moments of 18 - 44 year-old women in the U.S., the probability of leaving the model,  $\delta$ , is set to  $\frac{1}{27}$ . The parameter for economies of scale in marriage,  $\psi^M$ , is set to 1.1765 using the OECD equivalence scale, which is used typically in the family economics literature (for example, Regalia *et al.* (2011); Santos and Weiss (2016); Choi (2017)).<sup>5</sup> I assume that cohabitation provides half of the economies of scale compared to marriage, and the value for the economies of scale in cohabitation,  $\psi^C$ , is set to 1.0883.<sup>6</sup>

Santos and Weiss (2016) estimate a labor income process using the Panel Study of Income Dynamics (PSID) that is similar to the endowment processes in this chapter, and I take their point estimates for the persistence and variance parameters in the year 2000 as baseline values.

<sup>&</sup>lt;sup>5</sup>It is commonly assumed that due to the economies of scale, when an additional adult is added to an household, the two adults can maintain the same level of consumption with only 70% (not 100%) increase of the household's expenditure. Following the assumption, in Chapter 2, the economies of scale in consumption parameter is set to 1.7. To a woman in a union consume a unit, and her partner also consumes the same amount, she needs 1.7 units of income. In this chapter, although it is based on the same assumption, the parameter value differs as each person of a couple can consume different amount. The parameter value, 1.1765, is derived as 2 divided by 1.7: if each of a couple consumes the same amount,  $\bar{c}$ , they need combined endowment of  $\frac{2}{1.1765}\bar{c} = 1.7\bar{c}$ .

<sup>&</sup>lt;sup>6</sup>Both the economies of scale in cohabitation (relative to that in marriage) and the divorce cost affect couples' choice between marriage and cohabitation. To separately identify these two parameters, I need both income and consumption data, but data on consumption is not available in the data sets I am using. Instead, I set the relative economies of scale in cohabitation exogenously and calibrate the divorce cost given the economies of scale. The overall findings of the quantitative exercises remain robust to different normalization. In Appendix B, I recalibrate the model with two different values of the economies of scale of cohabitation,  $\psi^{C}$ . Although the exact results vary, both versions of calibration results are similar to the findings in this section.

Following Santos and Weiss (2016), I assume the same process for men and women. The persistence parameter,  $\phi$ , is set to 0.98, and the variance,  $\sigma^2$ , are set to 0.034. The mean of the men's endowment level,  $\bar{x}$ , is normalized to 1. Using the gender earnings gap in the CPS, the mean for women's endowment,  $\bar{y}$ , is set to 0.74. Lastly, the variance of individual specific match quality,  $\sigma_v^2$ , is set to 0.1. This parameter value is a normalization, and other parameter values regarding the match quality process are set relative to this normalization in the calibration. Table 3.1 summarizes parameter values set outside of the model.

	Value	Description
β	0.98	Discount factor
σ	2.0	Curvature of the utility function
δ	1/27	Probability of leaving the model
$\psi^M$	1.1765	Economies of scale in marriage
$\psi^{C}$	1.0883	Economies of scale in cohabitation
$\phi$	0.98	Persistence parameter of endowment processes
$\sigma^2$	0.034	Variance of endowment shocks
$\bar{x}$	1.0	Mean of men's endowment
ÿ	0.74	Mean of women's endowment
$\sigma_v^2$	0.1	Variance of individual specific match quality

Table 3.1: Parameters set a priori

#### 3.4.3 Calibration

Six parameters are left to be calibrated: three parameters regarding the conditional match quality process  $(\bar{\gamma}, \phi_{\gamma}, \sigma_{\gamma}^2)$ , two parameters for the initial match quality distribution  $(\bar{\gamma}_0, \sigma_0^2)$ , and the divorce cost ( $\kappa$ ). These parameter values are chosen to minimize the distance between moments in the model and moments in the data. I pick the following 12 moments to match:

- the fraction of married women, among 18 44 year-old women,
- the fraction of cohabiting women, among 18 44 year-old women,
- the following moments on union experience by their 38th birthday, among 38 44 yearold women,
  - conditional on having married, the mean age at first marriage,
  - conditional on having married, the variance of age at first marriage,
  - conditional on having married, the fraction of women who are divorced from first husbands,

- conditional on the first marriage having ended in divorce, the mean duration of first marriage,
- conditional on the first marriage having ended in divorce, the variance of the duration of first marriage,
- conditional on having cohabited, the mean age at first cohabitation,
- conditional on having cohabited, the variance of age at first cohabitation,
- conditional on having cohabited, the fraction of women whose first cohabitation ended (including by marrying the partner),
- conditional on the first cohabitation ending, the mean duration of first cohabitation,
- conditional on the first cohabitation ending, the variance of the duration of first cohabitation.

Parameters regarding the conditional match quality process,  $\phi_{\gamma}$  and  $\sigma_{\gamma}^2$ , are closely related to the durations. The mean of the match quality,  $\mu_{\gamma}$ , determines the fraction of people in unions. Parameters of the initial match quality distribution,  $\phi_0$  and  $\sigma_0^2$ , affect the timing of first marriage and cohabitation. Given the difference in economies of scale, the differences in the fractions of people and durations of marriage and cohabitation are providing information to pin down the divorce cost,  $\kappa$ .<sup>7</sup>

I target the model to match the U.S. around year 2000. Data moments are computed using combined samples from the NSFG Cycle 6 and the 2006 – 2010 NSFG. In the model, the two cross-sectional moments, 'the fraction of married women' and 'the fraction of cohabiting women,' are computed directly from the equilibrium distribution. The remaining ten moments are calculated by simulating the model at the steady state. Specifically, I simulate model agents' lives for 20 periods, and analyze the history of union choice, and compare them to data.

Table 3.2 shows the targeted moments in the data and the model, and Table 3.3 summarizes the values of calibrated parameters. Though it generates more first marriages ending in divorce compared to the data, the model successfully replicates the targeted moments in general. As in the data, there are more married women than cohabitants, and marriages last much longer than cohabitations. The model also closely matches the timing of first marriages and cohabitations.

<sup>&</sup>lt;sup>7</sup>Parameters for the economies of scale are set outside of the model.

	Data	Model
Married women (%)	47.96	48.56
Cohabiting women (%)	11.08	9.30
Average age at 1st marriage	24.10	24.54
Standard deviation of age at 1st marriage	4.72	5.41
Divorced from first husband by 38th birth day (%)	28.02	41.30
Average duration of 1st marriage (years)	7.12	6.85
Standard deviation of duration of 1st marriage	4.43	4.61
Average age at 1st cohabitation	24.57	24.45
Standard deviation of age at 1st cohabitation	5.13	5.40
First cohabitation has ended by 38th birth day (%)	90.02	91.11
Average duration of 1st cohabitation (years)	2.35	2.92
Standard deviation of duration 1st cohabitation	2.72	2.56

Table 3.2: Model and data moments

Table 3.3: Calibrated parameters

	Value	Description
к	0.9144	Divorce cost
$\bar{\gamma}$	-0.3071	Mean of conditional match quality (match quality for existing couples)
$\phi_{\gamma}$	0.9945	Persistence parameter of conditional match quality
$\sigma_{\gamma}^2$	0.0100	Variance of conditional match quality
Ϋ́ο	-2.3031	Mean of unconditional match quality (match quality for new matches)
$\sigma_0^2$	2.2310	Variance of unconditional match quality

# **3.5 Quantitative analysis**

As summarized in Section 3.2, income volatility has increased, and the gender earnings gap has been closing in the labor market. Also, fewer people are getting married or people are delaying marriage, and more people are living in cohabitation. Using the calibrated model, I explore how the changes in the labor market affect household union choices. Specifically, I solve the model with changed parameter values for income volatility,  $\sigma^2$ , and the gender earnings gap,  $\bar{y}$ , with other parameter values fixed at the calibrated level. Using the PSID, Santos and Weiss (2016) estimate that the variance of shocks are 0.0113 in the 1970s, and I use this value as an alternative level of income volatility. For the gender earnings gap, I change the mean for women's endowment to 0.6, which is the estimated gender gap from the CPS for the 1970s.

Table 3.4 summarizes the results of the exercise. Column (1) reports the baseline, which targets to replicate the year 2000. Column (4) contains the model's prediction around the year 1970, with decreased income volatility and increased the gender earnings gap. To examine impacts of each channel separately, two additional counterfactual exercises are conducted. Column (2) shows the results with income volatility of 1970 and the gender earnings gap of

2000. Conversly, column (3) has the results with the gender earnings gap of 1970 and income volatility of 2000.

By comparing columns (1) and (2), the model shows how income volatility affects household union choices. With decreased income volatility, there are more married women, more women have ever married by 38, the average age at their first marriage decreases by about a year, and the duration of first marriage is longer. The statistics regarding cohabitation move in the opposite way. There are fewer women cohabiting, and fewer women have ever cohabited. The average age at their first cohabitation increases, and the average duration of first cohabitation decreases. In sum, as income volatility decreases, people are more likely to marry and less likely to cohabit. In the model, a couple divorces or separates when one's income is too low so that the other person is better off living alone, or when its match quality is low enough. Decreased income volatility implies that the risk of divorce due to the income disparity has decreased, and this leads more people who did not marry in the baseline to marry and marriages to last longer. It also reduces the benefit of cohabitation. In the model, couples whose expected cost of divorce exceeds the expected benefit of marriage cohabit. As income volatility decreases, the probability of divorce also decreases, which increases the net expected benefit of marriage relative to cohabitation.

The difference between columns (1) and (3) is the effect of increasing the gender earnings gap on household unions. Relative to income volatility, the gender earnings gap has a smaller impact on union decisions in the model. As the gender gap increases, or women's average income decreases relative to men's, without changing income volatility, there are small increases in the fraction of married and cohabiting women. From women's perspective, the benefit of living in unions increases. At the margin between marriage and cohabitation, some couples who cohabited in the baseline marry. The fraction of married women increases slightly, and the average age at first marriage falls. Similarly, at the margin between living alone and cohabitation, some people who decided not to cohabit in the baseline cohabit now. The fraction of cohabiting women increases, and the average age at first cohabitation falls. Because of the intermediate nature of cohabitation in the model, the changes in cohabitation are determined by the relative changes in the two margins, the margin between marriage and cohabitation and the margin between cohabitation and living alone. The increased fraction of cohabiting women indicates that there are more single women who decide to cohabit than cohabiting women who decide to marry. As income volatility is still at its 2000 level, not enough cohabiting couples marry due to the divorce risk from income shocks.

Combining the two changes together, decreasing income volatility and increasing the gender gap, there are more married women and fewer cohabiting women in column (4) compared to the baseline, which is consistent with the trends observed in the data. Compared to col-

	2000	Counte	Counterfactual	1970	
	Baseline (1)	Smaller volatility (2)	Bigger gender gap (3)	Model $(2) + (3)$ (4)	Data
Married women (%)	48.56	64.09	48.77	65.60	72.41
Fraction of ever married by 38 (%)	74.10	88.50	74.60	91.20	95.44
Average duration of 1st marriage	6.85	7.02	6.79	7.05	
Average age at 1st marriage	24.54	23.60	24.48	23.52	
Cohabiting women (%)	9.30	4.29	10.11	3.81	3.32
Fraction of ever cohabited by 38 (%)	45.00	21.80	45.50	18.60	
Average duration of 1st cohabitation	2.92	2.88	3.14	2.93	
Average age at 1st cohabitation	24.45	24.52	24.31	24.51	
Fractions of married women and ever married women by 38 for the year 1970 in data (the rightmost column) are from the CPS. Neither the CPS nor the NSFG contain information on cohabitation in 1970. The fraction of cohabiting women for the column is computed using the NSFG 1982, which is the earliest wave of the survey containing information on cohabitation status. In the baseline calibration, Column (1), parameters for income volatility ( $\sigma_x$ , $\sigma_y$ ) are set to 0.1849, and they are set to 0.1 in the counterfactuals, Columns (2) and (4). The gender gap, the average earnings of women relative to mens', is 0.74 in the baseline, and it is set to 0.6 in the counterfactuals, Columns (3) and (4).	r married won on cohabitatio ne survey cont to 0.1849, and to 0.74 in the	nen by 38 for the year 1970 on in 1970. The fraction o aining information on cohe d they are set to 0.1 in the e baseline, and it is set to 0	0 in data (the rightmost colum of cohabiting women for the c abitation status. In the baselin counterfactuals, Columns (2) 0.6 in the counterfactuals, Col	<ul> <li>un) are from the CPS. Neith column is computed using e calibration, Column (1), and (4). The gender gap, unnus (3) and (4).</li> </ul>	her the CPS the NSFG parameters the average

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Table 3.4

umn (2), where income volatility alone decreases, there are even more married women, more women have ever married, and first marriages last even longer because of the effect of the increasing gender gap. In contrast to column (3), the increasing gender gap reduces the fraction of cohabiting women. Decreased income volatility lowers the divorce risk, and more women choose to marry over cohabitation as marriage provides the greater economic benefit because of the greater economies of scale than cohabitation. Compared to the data, which is in the last column of Table 3.2, the model predicts about 70% of the decrease in the fraction of married women from 1970 to 2000. The fraction of cohabiting women in 1970 is not available in the NSFG 1982, which is the earliest available estimate. The model also predicts the fraction of cohabiting women that is close to the 1982 estimate. Although there were likely fewer cohabiting women in 1970 than in 1982, the model is still able to explain a large part of the increase in cohabitation over time.

# 3.6 Conclusion

In this chapter, I study the trend of delaying marriage and increasing cohabitation in the U.S. Over time, in the labor market, the gender earnings gap has been decreasing and income volatility has been increasing. I posit that these changes in the labor market have contributed to the observed changes in household unions. I build a model of bilateral household union choice, which includes cohabitation as a distinct form of union. The model differs from the model in Chapter 2 in that there are two people who make a joint decision on household union status. The model is tractable, because the bargaining problem of a couple has a simple solution when assuming a part of a couple's match quality is private information.

According to the model calibrated to the U.S. data on the household union allocations and the labor market in 2000, increasing income volatility is the main driver of changes in household union observed over time. The decreasing gender earnings gap plays little role in the model.

In the model, people delay their marriage due to the increasing risk of divorce as income volatility increases. This relationship highlights the role of cohabitation as an intermediate form of household union between marriage and remaining single. In models without cohabitation, which are typical in the literature, increasing income volatility encourages people to marry more as the insurance benefit dominates the divorce risk. By ignoring the third option, the literature overlooks the impacts of increasing income volatility on changing household unions in favor of other mechanisms to explain the trends of delaying marriage.

Although it has much smaller impacts than income volatility, the decreasing gender earn-

ings gap reduces the economic benefit from living together and causes fewer people to marry. According to the counterfactual exercise, the direction of the gender gap's effect on cohabitation depends on the level of income volatility. If the gender gap falls without increasing income volatility, it increases cohabitation. With decreased income volatility, the decreasing gender gap lowers the fraction of cohabiting women. With the two changes in the labor market, the model can explain 70% of the decrease in married women over the past three decades in the U.S.

There are several possible extensions of the model. I abstract from some important household decisions. Incorporating fertility and child investment decisions into the model and comparing the decisions to the decisions in the unilateral model developed in Chapter 2 are left for future research.

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# Chapter 4

# **Educational gradient in time allocation**

# 4.1 Introduction

According to the ATUS, mothers with higher education (or wages) spend more time on childcare compared to mothers with lower education (or wages) (Kimmel and Connelly, 2007; Guryan *et al.*, 2008). Despite the growing evidence of the significance of parental time on child development (Del Boca *et al.*, 2014; Baker *et al.*, 2008; Agostinelli and Sorrenti, 2018), few studies attempt to understand the observed gradients in time allocation. This chapter studies a standard time allocation model and finds conditions that generate the gradients.

Guryan *et al.* (2008) find that more-educated mothers spend more time in childcare compared to less-educated mothers on average using the ATUS. The gradient is robust to different employment rates across education levels, parents' age, marital status, and number and age of children. The same gradient is found in time use data from other countries. They find gradients of the same direction for market work and of the opposite direction in home production (non-market work) and leisure: more-educated mothers work more but spend fewer hours on home production and leisure than less-educated mothers. Using the same survey, Kimmel and Connelly (2007) find the same relationships also hold for wages: mothers with a higher wage work more and spend more time in childcare, but less time in home production and leisure compared to mothers with a lower wage.

These gradients can be an important channel of intergenerational transmission of socioeconomic statuses, such as educational attainment and earnings given evidence supporting the significance of parental time in child's development. Del Boca *et al.* (2014) estimate a child's cognitive development production function nested in a structural model of household behavior. Their estimates indicate parents' time inputs are more important than monetary inputs for cognitive development, particularly when the child is young. Baker *et al.* (2008) and Agostinelli and Sorrenti (2018) find that increasing mother's labor supply has a negative impact on children's non-cognitive development, suggesting that parental (maternal) time is a crucial input in the children's non-cognitive development and that increasing income does not completely offset the negative impact from reduced time.

In this chapter, I study a simple time allocation model where a parent sees her children's human capital as a consumption good. I find that to generate a positive wage gradient of labor supply and a negative gradient of leisure, the elasticity of substitution between utility components must be greater than 1. If the productivity of time spent on children does not depend on the wage, time and market purchased inputs must be complementary to maintain a positive wage gradient of childcare time. If the productivity of time increases in the wage, then the model is able to generate the positive gradient even when the two inputs are substitutes in production. I calibrate the model parameters to match time use data from the ATUS and find that the model in which the productivity of childcare time is the same as the wage matches the data moments very well.

Despite the significance of parental time on child development and its potential role as a channel of intergenerational transmission, only a few papers attempt to explain the observed gradient in childcare time. Ramey and Ramey (2010) examine various U.S. time use datasets and confirm that the positive educational gradient in childcare is observed in the samples from earlier surveys as well as the ATUS sample. Also, they find that the level of time spent in childcare has increased substantially for all mothers, but by a greater extent for more-educated mothers. That is, the gradient becomes greater over time. They argue that increased competition among parents, especially among college-educated parents, to send their children to elite universities makes parents spend more time with their children. They present a theoretical model that generates the gradient and show how the increasing demand for college education makes the gradient steeper.

Closely related to this chapter, Molnar (2018) also studies the educational gradients in mothers' time allocation. Using multiple sources of Canadian data, Molnar (2018) finds that similar conditions to those in this chapter are necessary in order for the mothers' childcare time to increase with mothers' education. This chapter differs in two ways. First, I consider the educational gradients in labor supply and leisure as well. Second, I study the effects of changes in wages and policies on the distribution of children's human capital.

This chapter is also related to Moschini (2020), who studies the child skill production and the role of family structure in it. She estimates and finds that single mothers and married couples use different child skill production technology. According to the estimates, non-parental child care plays a bigger role in child development for single mothers than married couples, and children of single mothers benefit more from a child care subsidy.

In the next section, I summarize facts on time allocation from the ATUS sample follow-

ing Kimmel and Connelly (2007) and Guryan *et al.* (2008). In Section 4.3, I present a simple model of time and resource allocation, and study conditions that generate the observed educational gradient by examining special cases of the model. I calibrate the model parameters by matching moments on time allocation from the ATUS sample in Section 4.4, and then I study the effects of changes in wage distribution and policy using the calibrated model in Section 4.5. Conclusions are given in Section 4.6.

# 4.2 Time allocation

In this section, I reproduce the gradients reported in Kimmel and Connelly (2007) and Guryan *et al.* (2008) using the pooled samples from the ATUS 2003 - 2017 and the Annual Social and Economic Supplement of the Current Population Studies (CPS-ASEC) 2003 - 2017.<sup>1</sup> Following the studies, mothers activities are classified into four groups: market work, non-market work, childcare, and leisure. The education levels are grouped into five based on the highest level of education: those without high school diploma, high school graduates, those who had college education but did not receive a bachelor's degree<sup>2</sup>, those with bachelor's degree, and those with more than bachelor's degree. Table 4.1 reports time allocations of the four activities and the average hourly wage rates of the five education groups.

On average, Table 4.1 shows that mothers with more education spend more time in market work and childcare but less time in non-market work (home production) and leisure compared to mothers with less education. Compared to mothers with less than 12 years of schooling, mothers with 16 years of schooling spend about 11 hours and about 4 hours more per week on average in market work and childcare, respectively.<sup>3</sup> These increases in time spent on market work and childcare come from decreases in non-market work time and leisure. Mothers with 16 years of schooling spent 9 hours less on non-market work and have 5 hours less leisure compared to mothers with less than 12 years of schooling.

Mothers with more years of schooling have a higher wage on average. The last column of Table 4.1 lists the average hourly wage of full-time, full-year working married women with two children by the education group in 2016 dollars using samples from the CPS-ASEC. Mothers with 16 years of schooling earn \$27 per hour on average, 2.5 times the average hourly wage of

<sup>&</sup>lt;sup>1</sup>See Appendix C.1 for the details of the data source and sample section.

<sup>&</sup>lt;sup>2</sup>This includes those who dropped out of college education without receiving any degree and those who received occupational degrees.

<sup>&</sup>lt;sup>3</sup>These time allocations include mothers who are not working as well as working mothers. Market work hours for non-working mothers are included as (close to) zeros in the average market work hours. See Table C.2 for time allocations conditional on work status. Conditional on working, the gradient in market work hours is much smaller and statistically insignificant. I cannot reject the null hypothesis that there is no gradient in the market work hours.

Years of Schooling	Market Work	Non-market Work	Childcare	Leisure	Hourly Wage
		(Hours p	er week)		(In 2016\$)
< 12	13.4	36.3	10.6	104.9	10.40
	(1.30)	(1.14)	(0.75)	(1.26)	(0.210)
12	20.6	30.2	11.8	103.0	15.02
	(0.78)	(0.56)	(0.36)	(0.64)	(0.090)
13 – 15	24.6	26.5	11.9	101.8	18.65
	(0.72)	(0.42)	(0.29)	(0.58)	(0.078)
16	24.3	26.9	14.4	99.5	26.92
	(0.65)	(0.39)	(0.29)	(0.49)	(0.077)
16 +	27.8	25.0	15.1	97.1	34.78
	(0.90)	(0.50)	(0.43)	(0.65)	(0.096)

Table 4.1: Gradients in time allocations and hourly wage of married mothers with two children

Time allocations are estimated using the pooled sample of the ATUS 2003 - 2017, and the hourly wage rates are estimated using the samples from the CPS-ASEC 2003 - 2017. Estimates are computed using provided weights, and robust standard errors are in parentheses. Samples are restricted to 21 - 55 (inclusive) year-old married women with two children under the age of 18 in the household. The average wage rates are estimated for full-time and full-year workers.

mothers with less than 12 years of schooling, \$10.

In summary, mothers with more education work more time in the labor market and spend more time with children by reducing time in home production and leisure compared to mothers with less education despite having higher wage rates.

### 4.3 Model

In this section, I present a static time allocation model and study under what conditions the model generates the observed gradients in the data. A mother receives utility from consumption of market goods, c, home-produced goods, g, children's human capital, h, and leisure, l. I consider an additively separable CRRA utility function where a single parameter  $\sigma$  captures the common curvature:

$$u(c,g,h,l) = \mu_c \frac{c^{1-\sigma}}{1-\sigma} + \mu_g \frac{g^{1-\sigma}}{1-\sigma} + \mu_h \frac{h^{1-\sigma}}{1-\sigma} + \mu_l \frac{l^{1-\sigma}}{1-\sigma},$$

with  $\sigma > 0$  and  $\mu_c + \mu_g + \mu_h + \mu_l = 1$ . Because of the common curvature, the utility function exhibits constant elasticity of substitution (CES) between each component, and the parameter  $\sigma$  determines the elasticity,  $\frac{1}{\sigma}$ . Log utility is a special case when  $\sigma = 1$ .

To produce the home good, g, her own time,  $t_g$ , and market purchased inputs, d, are used.

Similarly, producing her children's human capital, *h*, also requires her own time,  $t_h$ , and market purchased inputs, *m*. The productivity of her time investment in children may depend on her wage: A(w). For home production and the child's human capital, I consider CES production functions, which take the Cobb-Douglas production function as a special case when  $\rho_x = 0$ . A mother decides how to allocate her time endowment, which is normalized to 1, and her labor income:

$$\max_{c,n,d,t_g,m,t_h,l} \quad \mu_c \frac{c^{1-\sigma}}{1-\sigma} + \mu_g \frac{g^{1-\sigma}}{1-\sigma} + \mu_h \frac{h^{1-\sigma}}{1-\sigma} + \mu_l \frac{l^{1-\sigma}}{1-\sigma}, \\
\text{s.t.} \quad g = [\alpha_g d^{\rho_g} + (1-\alpha_g) t_g^{\rho_g}]^{\frac{1}{\rho_g}}, \\
h = [\alpha_h m^{\rho_h} + (1-\alpha_h) \{A(w)t_h\}^{\rho_h}]^{\frac{1}{\rho_h}}, \\
c + p_d d + p_m m \le wn, \\
n + t_g + t_h + l = 1.$$
(4.1)

The analytical solution to equation (4.1) is in Appendix C.2. It does not permit a general condition for the observed gradients, as the solution depends on many parameters non-linearly. In the following subsections, I study some special cases of the model, focusing on the wage gradients of time investment in children's human capital,  $t_h$ , and labor supply, n. The curvature of the utility function,  $\sigma$ , and the substitutability of the two inputs of human capital production, which is controlled by  $\rho_h$ , are the two key determinants of the wage gradients of labor supply, n, and time on childcare,  $t_h$ .<sup>4</sup> To help understand how these two parameters affect behavior in the model, I parameterize the remaining parameters, which are later calibrated to match data moments in Section 4.4.<sup>5</sup>

For the home production function parameters,  $\alpha_g$  and  $\rho_g$ , I use the point estimates of Mc-Grattan *et al.* (1997):  $\alpha_g = 0.206$ , and  $\rho_g = 0.186$ . The weight on the market input in the human capital production,  $\alpha_h$ , is set to 0.3. As the utility weights,  $\mu_c$ ,  $\mu_g$ ,  $\mu_h$ , and  $\mu_l$ , affect the levels of time and resource allocations, not the gradients, the weights are all set to 0.25 in the following exercise. The relative prices of market inputs,  $p_d$  and  $p_m$ , are both set to 1.

As a special case, I first consider a log utility function,  $U(c,g,h,l) = \log(c) + \log(g) + \log(h) + \log(l)$ , and a productivity of time input in children's human capital production which is independent of the wage: A(w) = 1. Figure 4.1 plots time invested in children,  $t_h$ , and leisure,

<sup>&</sup>lt;sup>4</sup>For a given parameter  $\rho_h$ , the elasticity of substitution between the two inputs is  $\frac{1}{1-\rho_h}$ .

<sup>&</sup>lt;sup>5</sup>As in Chapter 2, it is common to extend this class of model to include non-labor income in its budget constraint and interpret the income as income from an additional household member (e.g., husband). Non-labor income shifts the labor supply curve, and the results of exercises in this section hold for the extension as long as the non-labor income does not depend on wage rates. However, if the non-labor income increases with the wage rate, non-labor income affects the gradients of labor supply curve. As a result, the results may not hold in such an environment depending on the size of the correlation.

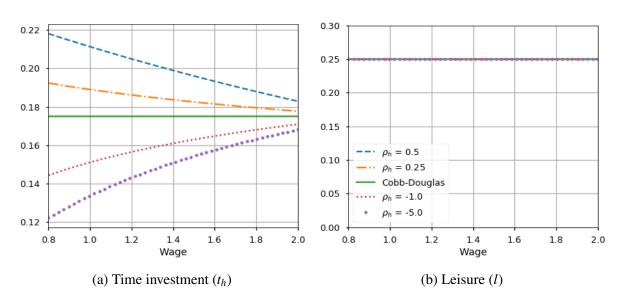


Figure 4.1: Time allocations with log utility and A(w) = 1.

*Note.* Using log utility with the equal weights:  $U(c, g, h, l) = \log c + \log g + \log h + \log l$ . The productivity of time investment does not depend on the wage: A(w) = 1.

*l*, as a function of the wage for different parameter values of  $\rho_h$ . In this case, in order for the child investment time to be increasing in the wage, the parameter  $\rho_h$  must be negative, which implies that the elasticity of substitution between time and market purchased input is smaller than unity. That is, they are complements in production. Because of the income effect, the mother prefers for her children to have more human capital, *h*, as her wage increases. When the two inputs are complementary in human capital production, she increases both inputs although the effective price of the time input increases in order to produce more human capital for her children.

With log utility, the income effect of the increased wage increases leisure demand, but the substitution effect decreases its demand by the same amount. As a result, leisure does not change with respect to changes in the wage, regardless of the parameter  $\rho_h$  or the productivity of time investment, A(w). Furthermore, if time investment increases in the wage ( $\rho_h < 0$ ), labor supply generally decreases in the wage.<sup>6</sup>

In order for a mother to choose less leisure as her wage increases, the substitution effect must dominate the income effect. This happens in equation (4.1) when the parameter  $\sigma$ , which captures the substitutability between different components of the utility, is smaller than one.

<sup>&</sup>lt;sup>6</sup>Because I assume that time and goods inputs are substitutable in home production ( $\rho_g$  is set to 0.186), time spent on home production decreases in wage. Therefore, if the increase in time investment ( $t_h$ ) as the wage increases is smaller than the decrease in the home production hours ( $t_g$ ), the labor supply can increase in wage. With the current parameter values, this can happen if the complementarity between time and market inputs in the human capital production is small ( $\rho_g$  is negative, but close to 0).

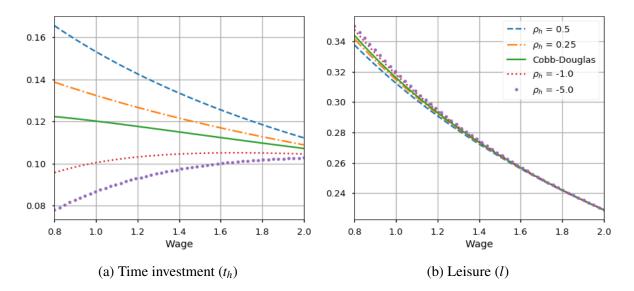


Figure 4.2: Time allocations with CRRA utility ( $\sigma = 0.5$ ) and A(w) = 1.

*Note.* Using a CRRA utility with equal weights:  $U(c,g,h,l) = \frac{1}{2} (c^{1/2} + g^{1/2} + h^{1/2} + l^{1/2})$ . The productivity of time investment does not depend on wage: A(w) = 1.

Figure 4.2 plots time investment and leisure with  $\sigma = 0.5$  and different parameter values of  $\rho_h$ . Regardless of the value of  $\rho_h$ , leisure is decreasing in the wage.

Regarding time investment,  $t_h$ , the figure delivers a similar message as the log utility case. Time investment increases in the wage, if there is enough complementarity between the inputs in human capital production ( $\rho_h < 0$ ). However, the threshold of  $\rho_h$  that generates the positive wage gradient of time investment is smaller (negative, but greater in absolute value) than in the log utility case, for which the threshold was 0 as shown in Figure 4.1. Compared to the log utility case, as the substitution effect dominates the income effect, spending time in human capital production is more costly in utility terms. Therefore, time investment is increasing in the wage only if the complementarity in the human capital production is strong enough. This reduces the threshold value of the parameter  $\rho_h$ .

The productivity of time investment, A(w), also affects the gradient. I consider a simple functional form for the productivity:  $A(w) = w^k$ , k > 0. Figure 4.3 plots time spent on children's human capital production,  $t_h$ , with different specifications of the productivity, A(w), and different values of  $\rho_h$ . Compared to the case where the productivity does not depend on the wage, A(w) = 1, the gradient of time investment becomes larger if the human capital production productivity of time is increasing in the wage (i.e.,  $\frac{dA(w)}{dw} > 0$ ). When it is the same as the wage, A(w) = w, the parameter  $\rho_h$  does not affect the gradient of time investment but affects its level. As a result, two parameters of the human capital production function,  $\alpha_h$  and  $\rho_h$ , affect the level of time input, and they cannot separate be separated. This is shown in Figure 4.4.

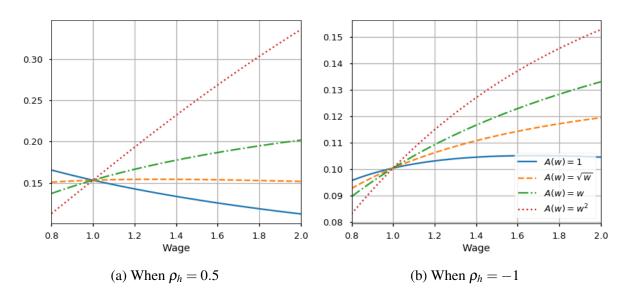


Figure 4.3: Time investment with CRRA utility ( $\sigma = 0.5$ ).

Note. Using a CRRA utility with the equal weights:  $U(c,g,h,l) = \frac{1}{2} (c^{1/2} + g^{1/2} + h^{1/2} + l^{1/2}).$ 

To understand why the parameter  $\rho_h$  does not affect the gradient, consider the first order conditions of equation (4.1). From the first order conditions and the functional form assumptions, the relationship between  $t_h^*$  and  $m^*$  is given by:

$$t_{h}^{*} = \frac{m^{*}}{A(w)} \left\{ \frac{\alpha_{h}}{1 - \alpha_{h}} \frac{w}{A(w)p_{m}} \right\}^{\frac{1}{p_{h}-1}}.$$
(4.2)

Observe that, when the productivity is the same as wage, A(w) = w, the substitution between time and the market input with respect to changes in wage is independent of the parameter  $\rho_h$ . Intuitively, when the productivity does not depend on the wage, A(w) = 1, increases in the wage make time investment more expensive compared to the market input. This relative price change can lead the parent to reduce or increase her time investment depending on the substitutability between the two inputs, which is controlled by the parameter  $\rho_h$ . However, if the productivity is the same as the wage, A(w) = w, changes in the wage do not affect the effective relative price of the two inputs. As a result, the parameter  $\rho_h$  no longer affects the substitution between time and the market input.

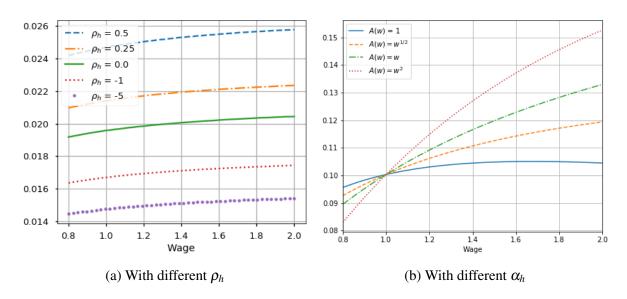


Figure 4.4: Time investment with CRRA utility ( $\sigma = 0.5$ ) and A(w) = w.

*Note.* Using a CRRA utility with the equal weights:  $U(c, g, h, l) = \frac{1}{2} (c^{1/2} + g^{1/2} + h^{1/2} + l^{1/2})$ . The productivity of time investment is the same as wage: A(w) = w. When changing  $\rho_h$ , (a),  $\alpha_h$  is set to 0.33. When changing  $\alpha_h$ , (b),  $\rho_h$  is set to 0 (the production function is a Cobb-Douglas form).

### 4.4 Calibration

I calibrate the model parameters to match time use data of married women.<sup>7</sup> From the data estimates on time use in Table 4.1, I deduct 70 hours from leisure and regard the rest as 'discretionary time use.' I normalize the total discretionary time (98 hours per week) to one. I also normalize the high school graduate's average hourly wage to one.

Some parameters are set outside of the model. Prices of both market inputs, *d* and *m*, are set to one. The productivity of time investment is set equal to the wage: A(w) = w.<sup>8</sup> As discussed previously, in this case, the parameter  $\rho_h$  does not affect the gradient and cannot be separately identified from  $\alpha_h$ . As a normalization, I set the parameter  $\rho_h$  to 0, which sets the children's human capital production function to be Cobb-Douglas:  $h = m^{\alpha_h} (wt_h)^{1-\alpha_h}$ .

The curvature of utility functions,  $\sigma$ , three utility weights,  $\mu_g$ ,  $\mu_h$ , and  $\mu_l$ , two parameters of the home production function,  $\alpha_g$  and  $\rho_g$ , and the weight on the market input in the human capital production,  $\alpha_h$ , are calibrated using time use data. The curvature of the utility function,  $\sigma$ , determines the gradients of labor supply and leisure. Given the parameter  $\sigma$ , the CES parameter of the home production function,  $\rho_g$ , controls the gradient of time spent on home

<sup>&</sup>lt;sup>7</sup>Although the ATUS includes male respondents (see Table C.1), it does not provide time allocation of a couple. In addition, as Table C.1 shows, women still spend considerably more time on child care than men. For these reasons, I consider only women in the calibration.

<sup>&</sup>lt;sup>8</sup>I also tried to calibrate the parameters with A(w) = 1, but the model performs poorly in matching the data.

Parameter	Description	Value
σ	Curvature of utility functions	0.6032
$\mu_{g}$	Utility weight on home production goods	0.3347
$\mu_h$	Utility weight on child's human capital	0.2500
$\mu_l$	Utility weight on leisure	0.2940
$\alpha_{g}$	Weight on the market input in home production	0.2011
$\rho_{g}^{\circ}$	CES parameter of home production	0.5607
$\alpha_h$	Weight on the market input in human capital production	0.1001

Table 4.2: Calibrated parameter values

production,  $t_g$ . The three utility weights,  $\mu_g$ ,  $\mu_h$ , and  $\mu_l$ , determine the levels of labor supply, leisure, home production, and human capital. Given the utility weights, the weights on market inputs,  $\alpha_g$  and  $\alpha_h$ , control the levels of time spent on home production and child investment.

Table 4.2 summarizes the calibrated parameter values. The calibrated value for the parameter  $\sigma$  is 0.6032, indicating that the substitution effect of the increasing wage dominates the income effect. For the home production function parameters, the weight on the market input,  $\alpha_g$ , is calibrated to 0.2011, which is close to the point estimate of McGrattan *et al.* (1997) of 0.206. The CES parameter of the home production function,  $\rho_g$ , is calibrated to 0.5607, indicating that the goods and time input in home production is highly substitutable with an implied elasticity of substitution of 2.28. This value is higher than the corresponding parameter of McGrattan *et al.* (1997), 0.186, but it is close to the point estimate of Gemici and Laufer (2012) of 0.51. The calibrated value for the weight on the market input in children's human capital production,  $\alpha_h$ , is 0.1. This indicates that mothers' time has much greater impact on their children's human capital than the market input. Although the parameter is not directly comparable due to the different specification, it is in line with the finding of Del Boca *et al.* (2014) that parents' time input is more important than monetary inputs in children's cognitive development.

Figure 4.5 plots the time allocations from the calibrated model. The model matches the target moments very well. It successfully replicates the overall gradients of all four time uses. Compared to the data points, the labor supply and the home production hours in the model show smaller curvature, but time investment in children and leisure are mostly within the 95% confidence interval of the data estimates. Figure 4.6 plots the demand of market inputs, for home production, d, and child's human capital, m, of the calibrated model. As expected, people purchase more market inputs as their wage increases.

Lastly, Figure 4.7 plots the production outputs of the calibrated model. People with higher wages have children with higher human capital, h.<sup>9</sup> In contrast, the increase in the wage

<sup>&</sup>lt;sup>9</sup>It is well known that children of higher income parents perform better than their counterparts of lower income

reduces the home production output, g. Although people with higher wages purchase more market inputs, d, they spend less time in home production,  $t_g$ . The calibrated parameter value,  $\alpha_g = 0.2011$ , indicates that time has greater weight than the market input in home production. As a result, according to the calibrated model, people with higher wages consume less home-produced goods compared to people with lower wages. Instead, as each utility component is highly substitutable, people with higher wages substitute the reduction in utility from home-produced goods with utility from increased consumption and children's human capital.

### 4.5 Quantitative analysis

In this section, I conduct two quantitative exercises using the calibrated model. First, I explore how changes in the wage distribution affect the human capital distribution of children. Second, I introduce a government in the model and study how fiscal policies that are targeted to increase children's human capital affect mothers' time and resource allocation.

#### 4.5.1 Mothers' wages and children's human capital

Using the calibrated model, I explore how changes in the mothers' wage distribution affect the distribution of children's human capital. I assume that the wage follows a log-normal distribution and estimate its mean and variance from the sample. Given the wage distribution, I compute the mean and variance of children's human capital implied by the model. I examine how the human capital distribution changes when the wage distribution changes by varying its mean,  $\mu_w$ , and variance,  $\sigma_w^2$ . Table 4.3 reports the results of this exercise.

An interesting finding from this exercise is that changes in the children's human capital distribution are greater than the changes in the wage distribution. For example, a percent increase in the mean wage increases the mean of human capital by 1.3%. Also, when the variance of the wage increases by one percent, the variance of human capital increases by 1.14%. The relationship holds for different magnitudes of change in the wage distribution. This result indicates that increasing inequality of parental wages has greater impacts on children.<sup>10</sup>

As mothers with different wages also have different productivity in their children's human capital production, their investment behaviors are different. Table 4.4 summarizes child investment decisions of mothers with different wage rates in the wage distribution. When the mean

parents along many dimensions, and there is a large literature studying the underlying mechanisms. Caucutt *et al.* (2017) study four leading mechanisms in the literature and their implications. The model in this chapter generates the achievement gap by 'consumption value of investment,' one of the mechanism they study.

<sup>&</sup>lt;sup>10</sup>Income inequality has been rising over time. The wage distribution in this chapter is estimated using the CPS-ASEC sample for the period from 2003 to 2017. The mean and variance of hourly wage have increased by 20% and 63%, respectively, from their levels in the period from 1990 to 1999 in the same survey.

	Wage Di	stribution	Human C	Capital (h)
	$\mu_w$	$\sigma_w^2$	$\mu_h$	$\sigma_h^2$
Baseline (Data)	1.5234	0.6765	0.1867	0.0211
Increase $\mu_w$ by 1%	1.5386	0.6765	0.1891	0.0211
	(+1%)		(+1.29%)	(+.30%)
Increase $\mu_w$ by 10%	1.6758	0.6765	0.2111	0.0217
	(+10%)		(+13.09%)	(+3.04%)
Increase $\mu_w$ by 50%	2.2851	0.6765	0.3162	0.0243
• •	(+50%)		(+69.39%)	(+15.44%)
Increase $\sigma_w^2$ by 1%	1.5234	0.6833	0.1868	0.0213
		(+1%)	(+.05%)	(+1.13%)
Increase $\sigma_w^2$ by 10%	1.5234	0.7442	0.1875	0.0235
		(+10%)	(+.46%)	(+11.38%)
Increase $\sigma_w^2$ by 50%	1.5234	1.0148	0.1907	0.0335
		(+50%)	(+2.17%)	(+58.94%)
Increase $\mu_w$ and $\sigma_w^2$ by 1%	1.5386	0.6833	0.1892	0.0214
	(+1%)	(+1%)	(+1.33%)	(+1.43%)
Increase $\mu_w$ and $\sigma_w^2$ by 10%	1.6758	0.7442	0.2119	0.0241
- · · · · · · · ·	(+10%)	(+10%)	(+13.52%)	(+14.53%)
Increase $\mu_w$ and $\sigma_w^2$ by 50%	2.2851	1.0148	0.3193	0.0374
	(+50%)	(+50%)	(+71.02%)	(+77.61%)

Table 4.3: Child's human capital (*h*) distribution

The mean and variance of the child's human capital from the calibrated model is computed by Gauss-Hermite quadrature with 9 points assuming the wage is log-normally distributed. Numbers in parenthesis indicate the relative change from the baseline in percent. The average hourly wage rate of mothers with 12 years of schooling is normalized to 1.

of the wage distribution is increased by 50% the size of the relative wage increase is greater at the lower end of the distribution and is smaller at the upper end.<sup>11</sup> As a result, when there is an increase in the mean wage, the human capital of children of mothers with lower wages increases more in relative terms than children of mothers with higher wages.

<sup>&</sup>lt;sup>11</sup>This is because of the assumption of log-normal wage distribution.

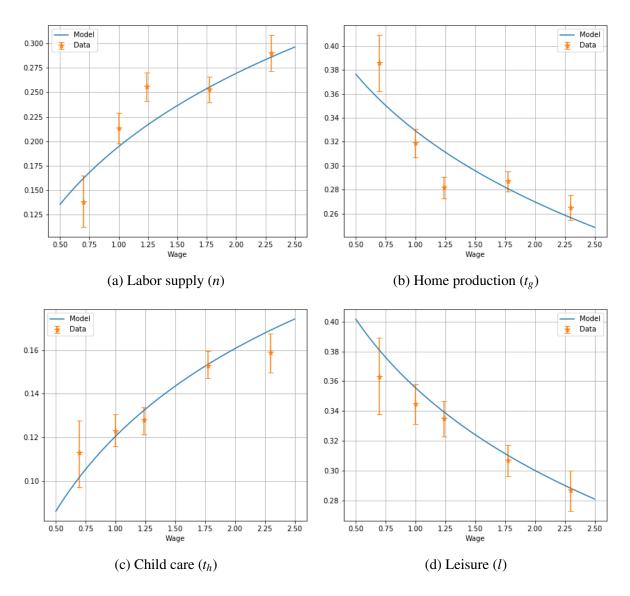


Figure 4.5: Time allocations of the calibrated model

Note. Data moments and their 95% confidence intervals are computed using provided weights.

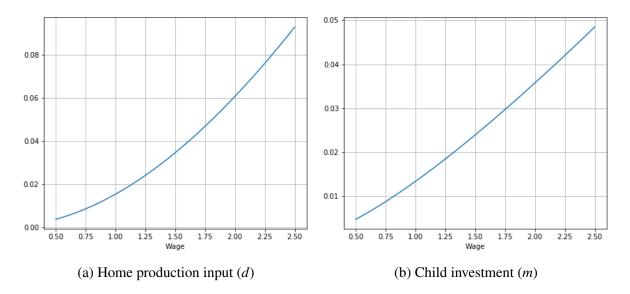
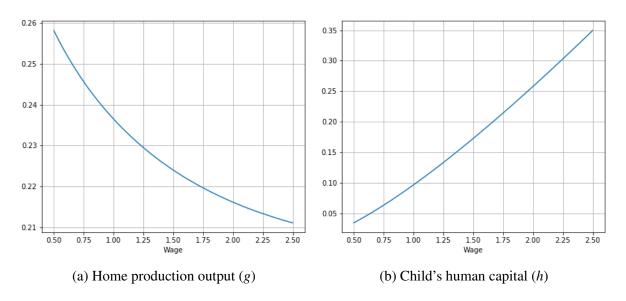


Figure 4.6: Resource allocations of market inputs of the calibrated model

Figure 4.7: Production outputs of the calibrated model



wage percentile	10th	25th	50th	75th	90th
		E(w) = 1.5234, Var(w) = 0.6765	w) = 0.6765		
Wage, <i>w</i>	0.7011	0.9531	1.3405	1.8855	2.5631
Child's human capital, h	0.0575	0.0902	0.1474	0.2377	0.3614
Market input, <i>m</i>	0.0080	0.0125	0.0205	0.0330	0.0502
Mother's time, $t_h$	0.1021	0.1180	0.1370	0.1571	0.1757
	E(w) = 2.285	E(w) = 2.2851, Var(w) = 0.6765	(increase $E(w)$ by 50%)	y 50%)	
Wage, w	1.3747	1.6991	2.1501	2.7208	3.3629
)	(96.1%)	(78.3%)	(60.4%)	(44.3%)	(31.2%)
Child's human capital, <i>h</i>	0.1527	0.2057	0.2847	0.3915	0.5184
	(165.8%)	(128.0%)	(93.2%)	(64.7%)	(43.5%)
Market input, <i>m</i>	0.0212	0.0286	0.0396	0.0544	0.0720
	(165.8%)	(128.0%)	(93.2%)	(64.7%)	(43.5%)
Mother's time, $t_h$	0.1385	0.1509	0.1650	0.1793	0.1921
	(35.6%)	(27.9%)	(20.5%)	(14.1%)	(9.3%)
	E(w) = 1.5234	1, Var(w) = 1.0148	(increase $Var(w)$ by 50%)	y 50%)	
Wage, w	0.5873	0.8465	1.2707	1.9075	2.7495
	(-16.2%)	(-11.2%)	(-5.2%)	(1.2%)	(7.3%)
Child's human capital, <i>h</i>	0.0441	0.0759	0.1366	0.2416	0.3970
	(-23.2%)	(-15.9%)	(-7.3%)	(1.6%)	(0.9%)
Market input, <i>m</i>	0.0061	0.0105	0.0190	0.0336	0.0552
	(-23.2%)	(-15.9%)	(-7.3%)	(1.6%)	(0.9%)
Mother's time, $t_h$	0.0936	0.1117	0.1339	0.1578	0.1800
	(-8.3%)	(-5.3%)	(-2.2%)	(0.4%)	(2.4%)

Table 4.4: Child investment decisions of mothers with different wage

#### 4.5.2 Effects of fiscal policies on children's human capital

In this subsection, I study the effects of fiscal policies targeted to increase the overall level of children's human capital. To do so, I introduce a government that has three policy instruments affecting children's human capital: it can subsidize the price of the market input,  $\tau_m$ , directly provide the market input,  $m_0$ , or make a lump-sum transfer to mothers,  $\tau$ .<sup>12</sup> The government runs a balanced budget. The policies must be fully funded either by a proportional labor income tax ( $\tau_n$ ) or by a consumption tax ( $\tau_c$ ).

Mothers take the tax and subsidy schedules  $(\tau_n, \tau_c, m_0, \tau_m, \tau)$  as given and solve the maximization problem:

$$\max_{c,n,d,t_g,m,t_h,l} \quad \mu_c \frac{c^{1-\sigma}}{1-\sigma} + \mu_g \frac{g^{1-\sigma}}{1-\sigma} + \mu_h \frac{h^{1-\sigma}}{1-\sigma} + \mu_l \frac{l^{1-\sigma}}{1-\sigma},$$
s.t.  $g = [\alpha_g d^{\rho_g} + (1-\alpha_g) t_g^{\rho_g}]^{\frac{1}{\rho_g}},$   
 $h = (m+m_0)^{\alpha_h} \{A(w)t_h\}^{1-\alpha_h},$   
 $(1+\tau_c)c + p_d d + (1-\tau_m)p_m m \le (1-\tau_n)wn + \tau,$   
 $n+t_g+t_h+l = 1.$ 
(4.3)

In addition to mothers, I assume that there is a measure ( $\mu_n = 1 - \mu_m = 0.54$ ) of nonmothers.<sup>13</sup> They are also subject to the same tax schedule ( $\tau_n$  and  $\tau_c$ ) as mothers, but they do not receive any benefits. They solve a similar problem:

$$\begin{split} \max_{c,n,d,t_g,l} & \frac{1}{\mu_c + \mu_g + \mu_l} \left( \mu_c \frac{c^{1-\sigma}}{1-\sigma} + \mu_g \frac{g^{1-\sigma}}{1-\sigma} + \mu_l \frac{l^{1-\sigma}}{1-\sigma} \right), \\ \text{s.t.} & g = [\alpha_g d^{\rho_g} + (1-\alpha_g) t_g^{\rho_g}]^{\frac{1}{\rho_g}}, \\ & (1+\tau_c)c + p_d d + \le (1-\tau_n)wn, \\ & n+t_g+l = 1. \end{split}$$

As they do not have children, they do not receive utility from children's human capital (h). The utility weights of non-mothers are adjusted so that they sum up to one.

Given the wage distributions of mothers,  $F_m(w)$ , and non-mothers,  $F_n(w)$ , the government takes the subsidy schedule,  $m_0$ ,  $\tau_m$ , and  $\tau$ , as given and sets the labor income tax rate,  $\tau_n$ , or the

<sup>&</sup>lt;sup>12</sup>Typically, primary and secondary education is mandatory and fully funded by tax in most jurisdictions. These public education system can be viewed as a direct provision of the market inputs,  $m_0$ . In addition, many jurisdictions have subsidized public childcare system for children before attending primary school such as 'Universal Child Care' in Quebec. 'Canada Child Benefit,' which pays certain amount to parents, is similar to a lump-sum transfer to mothers,  $\tau$ , considered in the exercise.

<sup>&</sup>lt;sup>13</sup>In the ATUS sample, 46% of 21 – 55-year-old women have 18-year-old or younger children in their household.

consumption tax rate,  $\tau_c$ , to achieve the balanced budget:

$$\mu_{m} \int (\tau_{n} n_{m}^{*} + \tau_{c} c_{m}^{*}) dF_{m}(w) + (1 - \mu_{m}) \int (\tau_{n} n_{n}^{*} + \tau_{c} c_{n}^{*}) dF_{n}(w)$$

$$= \mu_{m} \left\{ \int \tau_{m} m^{*} dF_{m}(w) + m_{0} + \tau \right\}.$$
(4.4)

Variables with subscript *m* denote mothers' allocations, whose measure is  $\mu_m$ , and variables with subscript *n* denote non-mothers' allocations. I assume that the distribution of non-mothers' wages ( $F_n(w)$ ) also follows a log-normal distribution and estimate its mean and variance from the CPS-ASEC sample. The mean is estimated to be 1.4217, and the variance is 0.5384.

		Human capital (h)		Time input $(t_h)$		Market input $(m + m_0)$			
Subsidy	Tax rate	Mean	Variance	Mean	Variance	Mean	Variance		
% % change from the baseline (no policy intervention)									
	Subsidize the market input in children's human capital production $(\tau_m)$								
10	0.57	1.65	3.32	0.64	1.26	11.18	23.60		
20	1.30	3.52	7.17	1.36	2.73	25.06	56.41		
30	2.23	5.69	11.73	2.20	4.49	42.75	103.83		
40	3.48	8.24	17.25	3.19	6.64	66.01	175.81		
50	5.24	11.33	24.17	4.40	9.40	97.89	292.30		
	Provide the market input in children's human capital production $(m_0)$								
1	0.05	0.02	0.02	0.02	-0.03	-0.03	-0.08		
10	0.52	0.16	0.15	0.25	-0.31	-0.30	-0.79		
25	1.31	0.44	0.27	0.67	-1.71	-0.40	-2.95		
50	2.65	1.27	-0.28	1.61	-6.32	2.79	-13.01		
50	5.41	4.26	-2.68	3.87	-13.57	25.15	-44.88		
		L	ump-sum tran	sfer to moth	ers $(\tau)$				
1	0.78	0.24	0.23	0.37	-0.41	-0.46	-1.17		
2.5	1.97	0.60	0.58	0.93	-1.03	-1.18	-2.95		
5	4.04	1.22	1.20	1.91	-2.07	-2.46	-6.05		
7.5	6.24	1.88	1.83	2.94	-3.25	-3.86	-9.31		
10	8.57	2.56	2.49	4.05	-4.58	-5.38	-12.77		

Table 4.5: Impact of policies on child's human capital through labor income tax

The amount of the market input provided by the policy is denoted  $(m_0)$  by its amount relative to the average demand of the good in the baseline. The lump-sum transfer  $(\tau)$  are denoted as its amount relative to the average labor income. The tax rate (second column) denotes required labor income tax rate to implement the subsidy. Changes in allocations are denoted as percentage changes from their respective values in the baseline. When the market input is provided  $(m_0)$ , the amount is added to mothers' demand to compute the changes in mean and variance.

First, I explore how these types of policies affect the children's human capital distribution. Table 4.5 reports how mothers' behavior and the distribution of children's human capital change as the policy schedule changes, if the policies are funded through labor income tax.<sup>14</sup> All three policies increase the average human capital of children. With a similar level of the labor income tax, the increase in the average is the greatest when the market input is subsidized.

However, these policies have different impacts on the dispersion of the children's human capital distribution. Both of subsidizing the market input price,  $\tau_m$ , and making lump-sum transfers to mothers,  $\tau$ , increase the variance of children's human capital. The increase in inequality is smaller for the same level of increase in the mean of children's human capital when the government implements transfers compared to paying the subsidy. For example, the first row of Table 4.5 shows that when the government subsidizes 10% of the market input, the mean of children's human capital increases by 1.65% and the variance increases by 3.31%. The mean of human capital increases more (1.72%) when the government transfers 7.5% of the average baseline income as shown in the second row from the bottom of Table 4.5, whereas the variance increases by 1.59%. This difference arises because the two policies have different impacts on mothers' labor supply, especially for mothers with lower wages.

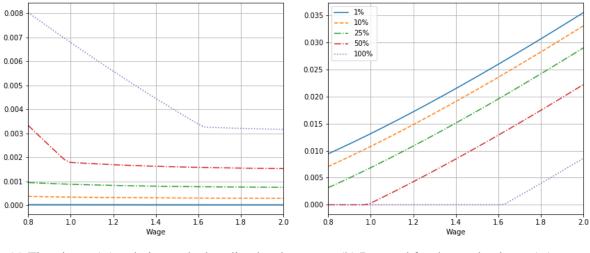
When the market input is subsidized, mothers purchase more, and the increase is relatively larger among those who have higher wages, as they were buying more before the subsidy. When mothers are given lump-sum transfers, mothers reduce their labor supply, and mothers with lower wages adjust their labor supply on the extensive margin, increasing their time with children. As mothers' time has greater weight in the children's human capital production function than the market input, children's human capital increases more at the bottom of mothers' wage distribution compared to the subsidy case. As a result, when the mean of human capital increases by the same amount, human capital inequality increases by a lesser amount when lump-sum transfers are made.

In contrast to the two other policies, directly providing the market input in children's human capital production affects the variance of children's human capital distribution nonmonotonically. The variance increases moderately for a small amount of public provision, and it decreases as the amount increases further. The public provision of the market input decreases mothers' demand for the market input and their labor supply as they need less income to purchase the market input. The decreased labor supply allows mothers to spend more time in other activities, including children's human capital production. Similar to the other policies, these changes increase the variance of children's human capital distribution initially. As mothers with higher wages are also more productive at children's human capital production, the human capital of their children increases more than children of mothers with lower wages. As the amount of public provision increases, it crowds out mothers' private provision. From the lower

<sup>&</sup>lt;sup>14</sup>The overall implication on children's human capital distribution is similar when the government funds these policies through the consumption tax. See Table C.3 in Appendix C.

end of the wage distribution, mothers' demand for the market input, m, becomes 0, and their time in children's human capital production increase faster than mothers who still demand positive amount of the market input.<sup>15</sup> Figure 4.8 plots the responses of mothers' time and demand for the market input as the amount of public provision,  $m_0$ , changes.

Figure 4.8: Response of inputs in children's human capital production to different amounts of public provision of the market input  $(m_0)$ 



(a) Time input  $(t_h)$ , relative to the baseline level

(b) Demand for the market input (*m*)

*Note.* The amount of public provision of the market input,  $m_0$ , is denoted relative to the average demand in the baseline, where there is no public provision.

Second, I analyze the tax rates needed to implement fiscal policies targeted to increase the average child's human capital by 2%. As before, I consider the three fiscal policies, subsidizing the market input,  $\tau_m$ , public provision of the market input,  $m_0$ , and lump-sum transfers to mothers,  $\tau$ . All policies must be fully funded either by a labor income tax,  $\tau_n$ , or by a consumption tax,  $\tau_c$ . In this exercise, the policies can be targeted based on mothers' wage rates. Specifically, I compare the cases where the policies are targeted to mothers in the lower end of the wage distribution.

Table 4.6 summarizes the results of the exercise. The required tax rate is lower for the consumption tax than the labor income tax to achieve the same goal in all policies, because the labor income tax affects mothers' labor-leisure choice. The subsidy reaches the goal with the lowest tax rate among the three policies, because it mostly affects mothers' child investment behavior. The public provision policy requires a higher tax rate than the subsidy, because the

<sup>&</sup>lt;sup>15</sup>I do not allow mothers to have negative demand for the market input. If they were allowed to have negative demand (sell), mothers with lower wages sell some of the public provision amount,  $m_0$ , and their time in children's human capital production does not increase more than mothers with higher wages. As a result, the variance of children's human capital keeps increasing as the cases of other policies.

market input is less important than mothers' time in children's human capital production. The transfer policy is the most expensive in terms of the required tax rate to reach the goal, as it causes mothers to adjust their allocation in other activities as well as child investment.

When subsidizing the market input, the required tax rates increase monotonically as more mothers become eligible for the subsidy. Interestingly, to achieve the goal of increasing the average human capital by 2%, it is less costly to pay a lump-sum transfer to all mothers than to target mothers at the bottom of the wage distribution. Further, the required tax rates change non-monotonically. This is because the lump-sum transfer reduces the labor supply of mothers who receive it, but the negative effect diminishes as mothers' wages increase.<sup>16</sup> Starting from the bottom of the wage distribution, as the transfer is made to more mothers, the tax base is decreasing, and the required tax rate increases to fund the transfer. However, the required tax rates start to decrease when mothers whose wages are above the median of the distribution receive the transfers. Mothers with higher wages invest more into their children's human capital, and their children have greater human capital stock than children of mothers with lower wages. As a result, providing transfers to mothers with higher wages makes achieving the goal of increasing the *average* human capital by 2% easier, and this effect lowers the required tax rates as more mothers with higher wages become eligible for the transfer.

## 4.6 Conclusion

In this chapter, I study the educational gradients in mothers' time allocation. Using a simple static model of time allocation, I find that when the productivity of mothers' time in childcare is correlated with education and wages, the model is able to generate the positive educational gradient observed in the data. The calibrated model matches the data very well assuming the productivity is the same as the wage.

Using the calibrated model, I study the effects of changes in the wage distribution of mothers on children's human capital. Because the increasing wage also increases mothers' productivity in childcare, changes in the distribution of children's human capital are greater than underlying changes in the wage distribution. A one percent increase in the mean of wages increases the mean of children's human capital by 1.29%, and a one percent increase in the variance of wages increases the variance of children's human capital by 1.14%.

I also study the effects of three fiscal policies that target an increase in children's human capital. Of the three policies considered, subsidizing the market input in children's human capital production is the most cost effective to increase the average human capital stock of

<sup>&</sup>lt;sup>16</sup>As people receive utility from leisure, they reduce their labor supply and increase leisure when they were given the transfers.

	Beneficiary	Bottom 25%	Bottom 50%	Bottom 75%	All mothers
Funded through		Subsidize	the market input in	children's human c	capital $(\tau_m)$
Income tax	Tax rate	0.40%	0.54%	0.68%	0.70%
	Subsidy Rate	74.77%	39.88%	22.50%	11.98%
Consumption tax	Tax rate	0.28%	0.39%	0.52%	0.51%
-	Subsidy rate	74.78%	39.89%	22.51%	11.99%
		Provide th	ne market input in c	children's human ca	apital $(m_0)$
Income tax	Tax rate	2.05%	2.66%	3.50%	3.46%
	Subsidy Rate	735.01%	167.98%	80.61%	27.51%
Consumption tax	Tax rate	1.45%	1.93%	2.67%	2.49%
	Subsidy rate	741.15%	169.41%	81.42%	27.84%
			Lump-sum paym	ent to mothers $(\tau)$	
Income tax	Tax rate	6.83%	9.54%	8.91%	6.66%
	Subsidy Rate	363.16%	75.14%	23.57%	7.96%
Consumption tax	Tax rate	5.23%	7.55%	7.28%	5.00%
*	Subsidy rate	393.31%	83.37%	25.92%	8.56%

Table 4.6: Costs of fiscal policies increasing the average human capital of children by 2%

The subsidy rates when the government provides the market input in children's human capital production,  $m_0$ , are calculated as the amount of the required provision relative to the average demand of the beneficiaries in the baseline. Similarly, the subsidy rates when the government makes lump-sum payments to mothers,  $\tau$ , are calculated as the amount of the transfer relative to the average labor income of the beneficiaries in the baseline.

children, but it also increases inequality in the children's human capital distribution more than other policies. Public provision of the market input is more expensive than the subsidy in increasing children's human capital, but it can decrease inequality.

Although the current model helps to understand and highlight a possible mechanism underlying the observed gradient in time allocation, it has a number of limitations that are left for future research. First, the current model is for mothers with children, and I treat mothers as if they are all single mothers. I expect the general messages of the study to remain the same, but the results of the quantitative exercises will differ if a second parent is added to the household. Second, only moments on time use are used in the current calibration. Using the data on household expenditure in calibration would help to identify some model parameters, especially those governing the elasticity of substitution between goods and time inputs. Third, the model is static, and mothers view their children's human capital as a consumption good. Comparing the results of this chapter with a dynamic model with altruistic parents, where parents care about the utility of their children, would be an interesting exercise. The dynamic model would also provide a framework for studying intergenerational transmission of socioeconomic status.

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# Chapter 5

# Conclusion

Despite many changes the family has experienced over time, it still serves a significant role in shaping future generations. This thesis studies decisions the family makes regarding house-hold unions, fertility, and child investment. Differences in family background during child-hood affect the socio-economic status of children when they grow up. Chapter 2 studies the educational gradient in non-marital fertility and posits that cohabitation is a driver behind the gradient. I document that less-educated women are more likely to cohabit and give birth while cohabiting than are more-educated women. I build a lifecycle model of fertility and house-hold union choice, featuring a trade-off between quality and quantity of children. Using the model calibrated to the U.S. data, I find that introducing the common-law marriage policy leads more children are born to married parents. As a result, children receive more investments from parents with the policy implemented.

In the past few decades, the fraction of married individuals has decreased, and more people are cohabiting with their partners in the U.S. Despite empirical evidence suggesting the link between the trends in household unions and increasing income inequality, few studies provide a mechanism behind the link. Chapter 3 suggests a mechanism, using the idea of private information on match quality between partners. I build a bilateral household union decision model that includes cohabitation and marriage as unions. Using the model calibrated to the U.S. data, I find that increasing income uncertainty accounts for more than half of the decrease in the fraction of married women from 1970 to 2000. The decreasing gender earnings gap has much smaller impacts than income uncertainty. The model in Chapter 3 abstracts from fertility and child investment decisions highlighted in Chapter 2. Incorporating them into the model is an important area of future research.

Given that more educated people have higher opportunity costs, the positive educational gradient in mothers' childcare time observed in the data appears puzzling. Using a standard time allocation model, Chapter 4 studies the educational gradient and finds that increasing time

productivity of mothers' childcare in education can generate the gradient. Studying conditions for the gradient in a dynamic dynastic environment and implications for the intergenerational transmission of socio-economic status are left for future research.

# Appendix A

# **Chapter 2** Appendix

### A.1 Data sources and sample selection

#### **NLSY 79**

The National Longitudinal Survey of Youth 1979 (NLSY 79) consists of 12,686 individuals who are born between January 1, 1957, and December 31, 1964. The survey began in 1979 and interviewed the cohort annually until 1994 and biennially since then through 2013. I made several sample restrictions to use in this paper. I exclude the military subsample; respondents who missed interviews more than one time to reduce issues of imputation; respondents who had more than three children. These restrictions produce the sample of 2,775 females and 2,297 males.

I divide the sample by their level of educational attainment. I group respondents with some college experience, regardless whether they have degree, at age 22 and label them as 'more-educated,' and group the rest and label them as 'less-educated.'

#### CNLSY

The National Longitudinal Survey of Youth 1979 Children and Young Adults (CNLSY) is a biennial survey that follows all of the biological children of the women in the NLSY 79 since 1986. As of 2014, there are 11,521 children are born to NLSY 79 mothers. I combine the CNLSY sample with the sample of NLSY 79. I exclude children whose mothers are dropped in the sample restriction described above. The CNLSY provides information whether the child lived with his/her biological father. As the information is provided since 1984, I drop children born before 1983 from the sample. These restrictions leave the sample of 4,741 children. For Table 2.6, as I need information on the education at the age 22 or 23, the sample is further reduced to those who born before 1992 (who turned age 22 in the 2014 survey).

#### NSFG

The National Survey of Family Growth (NSFG) gathers information on family life, marriage and divorce, pregnancy, infertility, use of contraception, and general and reproductive health. It is designed to be nationally representative of 15-44 years of age in the civilian, noninstitutionalized population of the United States (household population). I use the survey of the year 2002 (Cycle 6) to have the comparable birth cohorts of NLSY79, born between 1957 and 1964.<sup>1</sup> The survey includes 7,643 females and 4,928 males.

I use the female sample and restrict to the sample of respondents who born before December 31, 1964, a comparable birth cohorts to NLSY79<sup>2</sup>. The sample contains 1,951 respondents. Although it is a cross-sectional survey,<sup>3</sup>, it asks questions regarding past union experiences including cohabitation and pregnancy. I exploit this retrospective aspect of the survey.

The survey includes survey design variables and weights to provide estimates at the national level. The estimates in tables of the main body are computed using those variables.

<sup>&</sup>lt;sup>1</sup>The sample from Cycle 5, which is surveyed in 1995, also contains the birth cohorts of NLSY79. However, I do not use the sample as they are relatively young in the Cycle 5.

<sup>&</sup>lt;sup>2</sup>The oldest respondent in NSFG 2002 is born on April 1957.

<sup>&</sup>lt;sup>3</sup>The respondents in different survey cycles are not connected.

## A.2 Additional tables on educational gradients

	Less-educated women	More-educated women	Difference
	(1)	(2)	(1) - (2)
Any child	24.46	10.01	14.45
			(1.66)
First child	20.68	8.60	12.08
			(1.73)
Second child	15.06	5.18	9.88
			(1.37)
Third child	22.05	4.54	17.50
			(2.62)

Table A.1: Percentage of unmarried mothers at child's birth: mothers are at least 20 years old at child's birth

Samples from the NLSY 79, estimates are computed using provided weights. Standard errors of differences are in parentheses.

		Less-educated women (1)	More-educated women (2)	Difference (1) - (2)
White	Any child	21.29	4.03	17.26 (1.85)
	First child	17.69	3.55	14.14 (1.72)
	Second child	7.48	1.35	6.13 (1.29)
	Third child	15.08	1.69	13.39 (2.99)
Black	Any child	77.97	54.35	23.63 (4.08)
	First child	75.00	51.19	23.81 (4.14)
	Second child	58.66	34.84	23.82 (4.93)
	Third child	51.75	23.96	27.79 (7.27)
Hispanic	Any child	34.90	19.31	15.59 (5.08)
	First child	31.15	19.31	11.85 (4.99)
	Second child	16.80	3.33	13.47 (3.46)
	Third child	17.75	2.04	15.71 (4.90)

Table A.2: Percentage of unmarried mothers at child's birth by race

Samples from the NSFG 2002, born between April, 1957 and December, 1967 (comparable to the NLSY79 cohort, 37 – 44 years old on the day of interview). Estimates are computed using provided weights and survey design variables. Standard errors of differences are in parentheses.

'Any cohabitation' includes pre-marital cohabitation (married with the cohabitation partner) and non-marital cohabitation; 'Non-marital cohabitation' includes current cohabitation and cohabitations that end without marriage.

_	Any cohabitation			Non-marital cohabitation		
Race	Less- educated women (1)	More- educated women (2)	Difference (1) - (2)	Less- educated women (3)	More- educated women (4)	(3) - (4)
White	63.95	49.37	14.58	21.71	15.92	5.79
Black	65.13	54.69	(4.34) 10.44 (9.46)	41.50	28.32	(3.77) 13.18 (7.68)
Hispanic	55.17	55.20	(9.10) -0.03 (9.82)	31.28	15.61	(7.00) 15.67 (5.90)

Table A.3: Percentage of women ever cohabited before, by race

Samples from the NSFG 2002, born between April, 1957 and December, 1967 (comparable to the NLSY79 cohort, 37 – 44 years old on the day of interview). Estimates are computed using provided weights and survey design variables. Standard errors of differences are in parentheses.

'Any cohabitation' includes pre-marital cohabitation (married with the cohabitation partner) and non-marital cohabitation; 'Non-marital cohabitation' includes current cohabitation and cohabitations that end without marriage.

'Less-educated women' are those who have no college education at age 22; 'more-educated women' are those who have at least some college education at age 22.

Table A.4: Percentage of women gave birth (pregnancy ended as live birth) during cohabitation, by race

_		Any cohabitation	n	Non-marital cohabitation		
Race	Less- educated women (1)	More- educated women (2)	Difference (1) - (2)	Less- educated women (3)	More- educated women (4)	(3) - (4)
White	17.90	9.87	8.03 (4.79)	19.03	16.95	2.08 (9.25)
Black	50.46	25.93	(4.79) 24.53 (7.89)	49.05	33.91	(9.23) 15.14 (12.41)
Hispanic	44.59	47.30	(1.05) -2.71 (12.35)	50.17	41.56	8.61 (24.25)

Samples from the NSFG 2002, born between April, 1957 and December, 1967 (comparable to the NLSY79 cohort, 37 - 44 years old on the day of interview). Estimates are computed using provided weights and survey design variables. Standard errors of differences are in parentheses.

'Any cohabitation' includes pre-marital cohabitation (married with the cohabitation partner) and non-marital cohabitation; 'Non-marital cohabitation' includes current cohabitation and cohabitations that end without marriage.

_		Any cohabitation	n	Non-marital cohabitation		
Race	Less- educated women (1)	More- educated women (2)	Difference (1) - (2)	Less- educated women (3)	More- educated women (4)	(3) - (4)
All	18.76	8.96	9.80 (2.55)	9.57	4.40	$5.18 \\ (1.75)$
White	13.38	5.96	7.42 (3.29)	4.83	3.30	1.53 (2.02)
Black	35.81	17.70	18.11	22.18	11.99	10.19
Hispanic	25.99	31.38	(6.90) -5.39 (8.16)	16.58	7.80	(6.41) 8.78 (5.19)

Table A.5: Percentage of women gave birth (pregnancy ended as live birth) during cohabitation among those who have had child

Samples from the NSFG 2002, born between April, 1957 and December, 1967 (comparable to the NLSY79 cohort, 37 - 44 years old on the day of interview). Estimates are computed using provided weights and survey design variables. Standard errors of differences are in parentheses.

'Any cohabitation' includes pre-marital cohabitation (married with the cohabitation partner) and non-marital cohabitation; 'Non-marital cohabitation' includes current cohabitation and cohabitations that end without marriage.

# A.3 Additional information on calibration and the results

Description	Data	Model
Number of children less-educated single women had by age 44	0.93	0.75
Number of children less-educated married women had by age 44	1.77	1.84
Number of children more-educated single women had by age 44	0.28	0.72
Number of children more-educated married women had by age 44	1.77	1.57
Average expenditure share on child-related items (%)	39	40.0

Table A.6: Targeted moments for preference parameters

Table A.9: Lifecycle parameter: the probability of meeting a match

		Target n	noments		Model parameter			
Age	$\hat{\mu}_t(e,0)$	$\hat{\mu}_t(e,1)$	$\hat{\mu}_t(e,2)$	$\hat{\mu}_t(l,3)$	$\lambda_t(e,0)$	$\lambda_t(e,1)$	$\lambda_t(e,2)$	$\lambda_t(e,3)$
			Ι	Less-educa	ted wome	n		
18	0.403	0.523	0.517	0.614	0.961	1.000	1.000	1.000
19	0.356	0.477	0.471	0.567	1.000	1.000	1.000	1.000
20	0.314	0.434	0.428	0.525	1.000	1.000	1.000	1.000
21	0.274	0.394	0.388	0.485	1.000	1.000	1.000	1.000
22	0.239	0.359	0.353	0.450	0.925	0.926	1.000	1.000
23	0.206	0.326	0.320	0.417	0.798	0.722	1.000	1.000
24	0.177	0.297	0.291	0.388	0.782	0.613	1.000	1.000
25	0.150	0.270	0.264	0.361	0.664	0.523	1.000	1.000
26	0.126	0.246	0.240	0.337	0.558	0.477	1.000	1.000
27	0.105	0.225	0.219	0.316	0.463	0.410	1.000	1.000
28	0.085	0.205	0.200	0.296	0.378	0.375	1.000	1.000
29	0.068	0.189	0.183	0.279	0.303	0.344	1.000	1.000
30	0.053	0.173	0.168	0.264	0.237	0.316	1.000	1.000
31	0.040	0.160	0.154	0.251	0.156	0.292	1.000	0.973
32	0.029	0.149	0.143	0.239	0.111	0.288	1.000	0.928
33	0.018	0.138	0.132	0.229	0.071	0.268	1.000	0.888
34	0.009	0.129	0.123	0.220	0.035	0.267	1.000	0.853
35	0.001	0.121	0.115	0.212	0.004	0.268	1.000	0.822
36	0.001	0.114	0.108	0.205	0.003	0.252	1.000	0.794
37	0.001	0.107	0.101	0.198	0.003	0.277	0.786	0.683
38	0.001	0.101	0.095	0.192	0.003	0.314	0.591	0.662
39	0.001	0.095	0.089	0.186	0.003	0.329	0.462	0.642
40	0.001	0.090	0.084	0.181	0.004	0.309	0.371	0.622
41	0.001	0.084	0.078	0.175	0.006	0.289	0.268	0.542
42	0.001	0.078	0.072	0.169	0.006	0.268	0.223	0.523

43	0.001	0.071	0.065	0.162	0.006	0.245	0.202	0.503
44	0.001	0.064	0.058	0.155	0.006	0.220	0.164	0.480
			Ν	Iore-educa	ated wome	n		
21	0.255	0.374	0.268	0.248	0.792	0.610	1.000	1.000
22	0.239	0.358	0.252	0.232	0.618	0.584	1.000	1.000
23	0.223	0.341	0.235	0.215	0.531	0.557	1.000	0.954
24	0.206	0.325	0.219	0.199	0.456	0.530	0.968	0.880
25	0.189	0.307	0.201	0.182	0.418	0.502	1.000	0.804
26	0.172	0.290	0.184	0.164	0.380	0.500	1.000	0.728
27	0.155	0.273	0.167	0.147	0.342	0.498	1.000	0.571
28	0.138	0.256	0.150	0.130	0.328	0.467	0.931	0.505
29	0.121	0.240	0.134	0.114	0.289	0.464	0.690	0.440
30	0.105	0.223	0.117	0.097	0.250	0.462	0.606	0.336
31	0.089	0.208	0.102	0.082	0.213	0.429	0.450	0.282
32	0.074	0.193	0.087	0.067	0.177	0.427	0.384	0.230
33	0.060	0.179	0.073	0.053	0.143	0.395	0.281	0.181
34	0.047	0.165	0.059	0.039	0.112	0.366	0.230	0.122
35	0.035	0.153	0.047	0.027	0.083	0.339	0.163	0.085
36	0.024	0.142	0.036	0.016	0.057	0.315	0.113	0.051
37	0.014	0.133	0.027	0.007	0.034	0.294	0.075	0.021
38	0.006	0.125	0.019	0.001	0.016	0.276	0.048	0.003
39	0.001	0.118	0.012	0.001	0.003	0.282	0.034	0.003
40	0.001	0.113	0.007	0.001	0.003	0.293	0.026	0.003
41	0.001	0.111	0.005	0.001	0.005	0.381	0.016	0.003
42	0.001	0.110	0.004	0.001	0.010	0.425	0.013	0.003
43	0.001	0.111	0.005	0.001	0.010	0.430	0.017	0.003
44	0.001	0.114	0.008	0.001	0.010	0.444	0.029	0.003

Table A.10: Lifecycle parameter: child bearing costs

		Target moments	М	lodel paramet	er	
Age	$\hat{p}_{t+1}(e,0,S_t)$	$\hat{p}_{t+1}(e,1,S_t)$	$\hat{p}_{t+1}(e,2,S_t)$	$\theta_t(e,0,S_t)$	$\theta_t(e,1,S_t)$	$\theta_t(e,2,S_t)$
		Le	gle women			
18	0.035	0.082	0.070	0.674	1.595	32.943
19	0.049	0.082	0.070	-0.999	1.807	31.958
20	0.061	0.096	0.070	-1.664	1.607	30.978
21	0.070	0.106	0.076	-1.958	1.715	29.712
22	0.077	0.112	0.081	-2.108	2.037	28.561
23	0.081	0.115	0.085	-2.130	2.443	27.490
24	0.084	0.115	0.088	-2.084	2.933	26.477

25	0.085	0.113	0.089	-1.939	3.482	25.508	
26	0.084	0.108	0.090	-1.749	4.109	24.573	
27	0.082	0.101	0.090	-1.469	4.779	23.665	
28	0.079	0.093	0.089	-1.110	5.520	22.780	
29	0.075	0.083	0.088	-0.649	6.350	21.918	
30	0.069	0.072	0.085	-0.066	7.301	21.077	
31	0.063	0.061	0.082	0.688	8.439	20.271	
32	0.056	0.049	0.078	1.644	9.911	19.517	
33	0.049	0.038	0.074	2.894	11.961	18.830	
34	0.042	0.027	0.068	4.534	15.221	18.234	
35	0.035	0.017	0.063	6.755	21.486	17.766	
36	0.028	0.008	0.056	9.885	32.456	17.479	
37	0.021	0.000	0.050	14.367	32.775	17.496	
38	0.014	-0.005	0.042	21.617	33.236	17.915	
39	0.008	-0.009	0.035	31.034	33.911	19.005	
40	0.003	-0.010	0.027	32.756	34.925	21.346	
41	-0.001	-0.008	0.019	34.976	36.511	26.538	
42	-0.004	-0.003	0.010	38.116	39.162	41.226	
43	-0.005	0.005	-0.002	43.423	43.957	44.947	
44	-0.005	0.018	-0.019	57.744	57.744	57.744	
		Les	s-educated mai	ried women			
18		0.032	0.014	-6.912	5.592	48.437	
19		0.032	0.014	-6.028	5.717	47.376	
20		0.069	0.014	-5.334	1.574	46.302	
21	0.228	0.099	0.027	-4.735	0.575	37.493	
22		0.122	0.036	-4.191	0.251	34.021	
23	0.191	0.139	0.043	-3.681	0.192	31.954	
24		0.151	0.047	-3.190	0.270	30.494	
25	0.159	0.158	0.050	-2.706	0.437	29.376	
26		0.160	0.050	-2.219	0.668	28.490	
27		0.158	0.049	-1.719	0.956	27.789	
28		0.153	0.047	-1.197	1.300	27.266	
29		0.145	0.043	-0.642	1.708	26.940	
30	0.099	0.134	0.039	-0.041	2.193	26.863	
31	0.090	0.121	0.033	0.620	2.778	27.135	
32		0.107	0.028	1.360	3.497	27.947	
33		0.092	0.022	2.206	4.408	29.677	
34		0.076	0.016	3.194	5.606	33.165	
35		0.060	0.011	4.378	7.263	40.654	
36		0.045	0.006	5.843	9.586	41.289	
37		0.031	0.001	7.732	13.263	40.576	
38		0.018	0.002	10.320	20.437	39.996	
39	0.026	0.008	-0.002	14.128	33.699	39.610	

40	0.019	-0.001	-0.003	20.350	34.940	39.526
41	0.010	-0.006	-0.002	34.473	36.663	39.944
42	0.002	-0.007	0.000	38.501	39.285	41.309
43	-0.007	-0.005	0.005	43.628	44.050	44.910
44	-0.017	0.002	0.012	57.744	57.744	52.493
		Me	ore-educated sin	ngle women		
21	0.018	0.010	0.001	11.416	20.342	52.413
22	0.010	0.010	0.001	9.516	21.065	51.099
23	0.021	0.023	-0.037	8.558	9.471	49.755
24	0.026	0.032	0.003	8.102	7.005	48.382
25	0.027	0.039	0.032	8.027	6.153	30.308
26	0.027	0.044	0.053	8.203	5.885	25.759
27	0.027	0.047	0.067	8.583	5.908	23.657
28	0.026	0.048	0.077	9.227	6.076	22.321
29	0.024	0.048	0.081	10.081	6.433	21.334
30	0.023	0.046	0.081	11.182	6.951	20.627
31	0.020	0.043	0.077	12.635	7.651	20.103
32	0.018	0.039	0.070	14.532	8.596	19.787
33	0.016	0.035	0.062	17.106	9.857	19.705
34	0.013	0.030	0.052	20.705	11.558	19.989
35	0.011	0.025	0.041	25.948	13.950	20.855
36	0.008	0.019	0.030	27.907	17.462	22.850
37	0.006	0.014	0.019	28.745	22.958	27.438
38	0.004	0.010	0.010	29.857	32.101	39.730
39	0.002	0.006	0.003	31.206	33.208	39.388
40	0.000	0.003	0.001	32.890	34.569	39.351
41	-0.001	0.001	0.001	35.077	36.390	39.823
42	-0.001	0.000	0.001	38.188	39.092	41.246
43	-0.001	0.001	0.001	43.460	43.929	44.907
44	0.000	0.004	0.001	57.744	57.744	57.744
		Мо	re-educated ma	rried women		
21	0.068	0.165	0.002	-0.096	-5.028	53.437
22	0.118	0.165	0.002	-1.738	-4.447	52.290
23	0.158	0.200	-0.021	-2.187	-4.107	51.139
24	0.189	0.226	-0.005	-2.272	-3.561	49.987
25	0.211	0.244	0.009	-2.186	-2.892	48.834
26	0.225	0.255	0.019	-1.999	-2.300	35.944
27	0.233	0.260	0.028	-1.740	-1.741	30.939
28	0.234	0.259	0.034	-1.424	-1.193	28.188
29	0.229	0.252	0.039	-1.059	-0.644	26.330
30	0.220	0.241	0.041	-0.645	-0.085	24.946
31	0.207	0.226	0.043	-0.180	0.495	23.870

32	0.190	0.208	0.043	0.345	1.108	23.032
33	0.171	0.188	0.041	0.943	1.773	22.410
34	0.150	0.166	0.039	1.639	2.513	22.021
35	0.129	0.142	0.036	2.473	3.366	21.915
36	0.106	0.118	0.032	3.512	4.391	22.197
37	0.084	0.095	0.027	4.873	5.692	23.064
38	0.064	0.072	0.022	6.775	7.467	24.911
39	0.045	0.051	0.017	9.672	10.149	28.627
40	0.029	0.032	0.011	14.616	14.890	36.684
41	0.016	0.016	0.006	24.408	25.882	39.973
42	0.008	0.004	0.001	38.534	39.221	41.348
43	0.004	-0.005	-0.005	43.642	43.988	44.959
44	0.006	-0.008	-0.012	57.744	57.744	57.744

Fertility rates are estimated from the samples of the NSFG 2002. By the level of education, number of children, and marital status, separate OLS regression models are estimated using a fourth order polynomial of age, and the fitted values of the regressions are used as the calibration targets,  $\hat{p}_{t+1}(e, k_t, S_t)$ . The fitted values can have negative values, but the model does not allow negative fertility rates (the model fertility rate approaches to zero in the limit.) When the target is smaller than 0.001, 0.001 is used as target.

	L	ess-educa	ted wom	en	More-educated women				
	Marrie	d women	Ever r	narried	Married women		Ever married		
Age	Data	Model	Data	Model	Data	Model	Data	Model	
18	0.124	0.155	0.124	0.155					
19	0.249	0.250	0.264	0.267					
20	0.339	0.346	0.374	0.372					
21	0.418	0.426	0.466	0.456	0.063	0.179	0.063	0.179	
22	0.464	0.481	0.536	0.524	0.162	0.285	0.166	0.301	
23	0.506	0.515	0.599	0.569	0.276	0.355	0.285	0.382	
24	0.521	0.536	0.649	0.606	0.355	0.402	0.374	0.426	
25	0.555	0.549	0.697	0.631	0.444	0.448	0.478	0.471	
26	0.572	0.556	0.731	0.658	0.492	0.484	0.544	0.514	
27	0.594	0.552	0.765	0.674	0.530	0.516	0.579	0.556	
28	0.595	0.553	0.777	0.687	0.586	0.554	0.640	0.598	
29	0.604	0.561	0.789	0.699	0.607	0.582	0.679	0.624	
30	0.592	0.576	0.807	0.717	0.655	0.609	0.730	0.660	
31	0.608	0.586	0.817	0.725	0.682	0.627	0.754	0.686	
32	0.609	0.590	0.821	0.737	0.679	0.642	0.768	0.710	
33	0.636	0.588	0.842	0.750	0.686	0.653	0.780	0.725	
34	0.641	0.581	0.845	0.772	0.683	0.659	0.787	0.739	
35	0.632	0.577	0.847	0.796	0.720	0.662	0.807	0.754	
36	0.631	0.585	0.834	0.813	0.716	0.660	0.819	0.760	
37	0.636	0.605	0.868	0.827	0.730	0.656	0.824	0.763	
38	0.616	0.620	0.852	0.839	0.736	0.655	0.835	0.766	
39	0.639	0.633	0.876	0.858	0.715	0.656	0.833	0.768	
40	0.634	0.642	0.874	0.869	0.728	0.661	0.858	0.770	
41	0.641	0.645	0.878	0.879	0.718	0.669	0.853	0.772	
42	0.630	0.645	0.876	0.881	0.715	0.680	0.858	0.774	
43	0.636	0.640	0.885	0.886	0.731	0.694	0.856	0.775	
44	0.634	0.668	0.886	0.903	0.734	0.711	0.859	0.783	

Table A.7: Targeted moments in calibration: age profile of fraction of married women and of women who had ever married

	L	ess-educa	ited wom	More-educated women				
	Dive	orced	Sir	ngle	Divorced		Single	
Age	Data	Model	Data	Model	Data	Model	Data	Model
18	0.001	0.000	0.835	0.597				
19	0.005	0.006	0.647	0.578				
20	0.017	0.017	0.560	0.516				
21	0.026	0.027	0.483	0.414	0.000	0.000	0.876	0.745
22	0.041	0.038	0.424	0.352	0.002	0.004	0.756	0.585
23	0.071	0.058	0.384	0.328	0.003	0.009	0.665	0.488
24	0.106	0.080	0.385	0.334	0.012	0.013	0.558	0.434
25	0.138	0.104	0.355	0.338	0.019	0.026	0.467	0.413
26	0.169	0.128	0.327	0.341	0.036	0.036	0.430	0.399
27	0.196	0.147	0.302	0.340	0.053	0.044	0.390	0.383
28	0.222	0.164	0.300	0.340	0.063	0.054	0.347	0.373
29	0.244	0.193	0.279	0.340	0.070	0.060	0.329	0.358
30	0.262	0.224	0.290	0.369	0.086	0.069	0.289	0.347
31	0.289	0.256	0.261	0.372	0.106	0.078	0.269	0.335
32	0.308	0.279	0.253	0.392	0.105	0.090	0.255	0.328
33	0.322	0.305	0.260	0.413	0.132	0.102	0.260	0.322
34	0.326	0.327	0.252	0.417	0.132	0.113	0.271	0.320
35	0.356	0.351	0.264	0.416	0.160	0.122	0.235	0.319
36	0.336	0.364	0.259	0.395	0.143	0.134	0.242	0.322
37	0.398	0.378	0.261	0.376	0.182	0.144	0.243	0.327
38	0.363	0.388		0.363	0.160	0.153		0.335
39	0.416	0.401		0.351	0.205	0.163		0.344
40	0.394	0.413		0.342	0.182	0.175		0.354
41	0.430	0.426		0.335	0.235	0.182		0.365
42	0.420	0.438		0.331	0.203	0.192		0.372
43	0.438	0.452		0.330	0.253	0.199		0.377
44	0.442	0.464		0.332	0.203	0.209		0.382

Table A.8: Targeted moments in calibration: age profile of fraction of women who had ever divorced and single women

# **Appendix B**

# Chapter 3 Appendix

### **B.1** Definition of distributions

Similar to the definition for single women, in Equation (3.6), the measure of single men in the next period,  $\Omega_{H}^{\prime S}$ , is the sum of single men who did not form a union in the matching market,  $\Omega_{H}^{\prime S,S}$ , those who separate with their cohabiting partners,  $\Omega_{H}^{\prime C,S}$ , those who divorce with their spouses,  $\Omega_{H}^{\prime M,S}$ , and those who enter the model next period replacing those who left,  $\Omega_{H}^{\prime \Delta,S}$ . Formally, they are defined as the following:

$$\begin{aligned} \Omega_{H}^{\prime S}(y_{t+1}) &= \Omega_{H}^{\prime S,S}(y_{t+1}) + \Omega_{H}^{\prime C,S}(y_{t+1}) + \Omega_{H}^{\prime M,S}(y_{t+1}) + \Omega_{H}^{\prime \Delta,S}(y_{t+1}), \\ \Omega_{H}^{\prime S,S}(y_{t+1}) &= \int_{\gamma_{t+1}} \int_{x_{t+1}} P^{S}(x_{t+1}, y_{t+1}, \gamma_{t+1}, s_{t} = S) \\ &\times \int_{y_{t}} (1 - \delta) \Omega_{H}^{S}(y_{t}) G_{y}(y_{t+1}|y_{t}) dy_{t} \int_{x_{t}} \frac{\Omega_{W}^{S}(x_{t})}{\int_{x_{t}} \Omega_{W}^{S}(x_{t}) dx_{t}} G_{x}(x_{t+1}|x_{t}) dx_{t} \Gamma_{0}(\gamma_{t+1}) dx_{t+1} d\gamma_{t+1}, \\ \Omega_{H}^{\prime C,S}(y_{t+1}) &= \int_{\gamma_{t+1}} \int_{x_{t+1}} P^{S}(x_{t+1}, y_{t+1}, \gamma_{t+1}, s_{t} = C) \\ &\times \int_{\gamma} \int_{y_{t}} \int_{x_{t}} (1 - \delta) \Omega^{C}(x_{t}, y_{t}, \gamma_{t}) G_{x}(x_{t+1}|x_{t}) G_{y}(y_{t+1}|y_{t}) \Gamma(\gamma_{t+1}|\gamma_{t}) dx_{t} dy_{t} d\gamma_{t} dx_{t+1} d\gamma_{t+1}, \\ \Omega_{H}^{\prime M,S}(y_{t+1}) &= \int_{\gamma_{t+1}} \int_{x_{t+1}} P^{S}(x_{t+1}, y_{t+1}, \gamma_{t+1}, s_{t} = M) \\ &\times \int_{\gamma} \int_{y_{t}} \int_{y_{t}} \int_{x_{t}} (1 - \delta) \Omega^{M}(x_{t}, y_{t}, \gamma_{t}) G_{x}(x_{t+1}|x_{t}) G_{y}(y_{t+1}|y_{t}) \Gamma(\gamma_{t+1}|\gamma_{t}) dx_{t} dy_{t} d\gamma_{t} dy_{t+1} d\gamma_{t+1}, \\ \Omega_{H}^{\prime \Delta,S}(y_{t+1}) &= \int_{y_{t}} \delta \left[ \Omega_{H}^{S}(y_{t}) + \int_{\gamma_{t}} \int_{x_{t}} \left\{ \Omega^{C}(x_{t}, y_{t}, \gamma_{t}) + \Omega^{M}(x_{t}, y_{t}, \gamma_{t}) \right\} dx_{t} d\gamma_{t} \right] G_{y}(y_{t+1}|y_{t}) dy_{t}. \end{aligned} \tag{B.1}$$

The distribution of cohabiting couples,  $\Omega'^{C}$ , is the sum of distributions of couples who are starting their cohabitation,  $\Omega'^{S,C}$ , of couples who keeps cohabiting,  $\Omega'^{C,C}$ , and of married

couples who decided to cohabit with their spouse,  $\Omega'^{M,C}$ :<sup>1</sup>

$$\begin{aligned} \Omega^{\prime C}(x_{t+1}, y_{t+1}, \gamma_{t+1}) &= \Omega^{\prime S, C}(x_{t+1}, y_{t+1}, \gamma_{t+1}) + \Omega^{\prime C, C}(x_{t+1}, y_{t+1}, \gamma_{t+1}) + \Omega^{\prime M, C}(x_{t+1}, y_{t+1}, \gamma_{t+1}), \\ \Omega^{\prime S, C}(x_{t+1}, y_{t+1}, \gamma_{t+1}) &= P^{C}(x_{t+1}, y_{t+1}, \gamma_{t+1}, s_{t} = S) \\ &\times \int_{x_{t}} \Omega^{S}_{W}(x_{t}) G_{x}(x_{t+1}|x_{t}) dx_{t} \int_{y_{t}} \frac{\Omega^{S}_{H}(y_{t})}{\int_{y_{t}} \Omega^{S}_{H}(y_{t})} G_{y}(y_{t+1}|y_{t}) dy_{t} \Gamma_{0}(\gamma_{t+1}), \\ \Omega^{\prime C, C}(x_{t+1}, y_{t+1}, \gamma_{t+1}) &= P^{C}(x_{t+1}, y_{t+1}, \gamma_{t+1}, s_{t} = C) \\ &\times \int_{\gamma} \int_{y_{t}} \int_{x_{t}} (1 - \delta) \Omega^{C}(x_{t}, y_{t}, \gamma_{t}) G_{x}(x_{t+1}|x_{t}) G_{y}(y_{t+1}|y_{t}) \Gamma(\gamma_{t+1}|\gamma_{t}) dx_{t} dy_{t} d\gamma_{t}, \\ \Omega^{\prime M, C}(x_{t+1}, y_{t+1}, \gamma_{t+1}) &= P^{C}(x_{t+1}, y_{t+1}, \gamma_{t+1}, s_{t} = M) \\ &\times \int_{\gamma} \int_{y_{t}} \int_{x_{t}} (1 - \delta) \Omega^{M}(x_{t}, y_{t}, \gamma_{t}) G_{x}(x_{t+1}|x_{t}) G_{y}(y_{t+1}|y_{t}) \Gamma(\gamma_{t+1}|\gamma_{t}) dx_{t} dy_{t} d\gamma_{t}. \end{aligned}$$

Lastly, the distribution of married couples,  ${\Omega'}^M$ , is defined similarly:

$$\Omega^{\prime M}(x_{t+1}, y_{t+1}, \gamma_{t+1}) = \Omega^{\prime S, M}(x_{t+1}, y_{t+1}, \gamma_{t+1}) + \Omega^{\prime C, M}(x_{t+1}, y_{t+1}, \gamma_{t+1}) + \Omega^{\prime M, M}(x_{t+1}, y_{t+1}, \gamma_{t+1}), 
\Omega^{\prime S, M}(x_{t+1}, y_{t+1}, \gamma_{t+1}) = P^{M}(x_{t+1}, y_{t+1}, \gamma_{t+1}, s_{t} = S) 
\times \int_{x_{t}} \Omega^{S}_{W}(x_{t}) G_{x}(x_{t+1}|x_{t}) dx_{t} \int_{y_{t}} \frac{\Omega^{S}_{H}(y_{t})}{\int_{y_{t}} \Omega^{S}_{H}(y_{t})} G_{y}(y_{t+1}|y_{t}) dy_{t} \Gamma_{0}(\gamma_{t+1}), 
\Omega^{\prime C, M}(x_{t+1}, y_{t+1}, \gamma_{t+1}) = P^{M}(x_{t+1}, y_{t+1}, \gamma_{t+1}, s_{t} = C) 
\times \int_{\gamma} \int_{y_{t}} \int_{x_{t}} (1 - \delta) \Omega^{C}(x_{t}, y_{t}, \gamma_{t}) G_{x}(x_{t+1}|x_{t}) G_{y}(y_{t+1}|y_{t}) \Gamma(\gamma_{t+1}|\gamma_{t}) dx_{t} dy_{t} d\gamma_{t}, 
\Omega^{\prime M, M}(x_{t+1}, y_{t+1}, \gamma_{t+1}) = P^{M}(x_{t+1}, y_{t+1}, \gamma_{t+1}, s_{t} = M) 
\times \int_{\gamma} \int_{y_{t}} \int_{x_{t}} (1 - \delta) \Omega^{M}(x_{t}, y_{t}, \gamma_{t}) G_{x}(x_{t+1}|x_{t}) G_{y}(y_{t+1}|y_{t}) \Gamma(\gamma_{t+1}|\gamma_{t}) dx_{t} dy_{t} d\gamma_{t}.$$
(B.3)

## **B.2** Alternative calibration

In Section 3.4, I assume that cohabitation provides half of the economies of scale compared to marriage and set the value of the parameter,  $\psi^{C}$ , to 1.0883 (= 1 + 1/2( $\psi^{M} - 1$ )). Rest of the parameters are calibrated according to the value. To examine the robustness of quantita-

<sup>&</sup>lt;sup>1</sup>As explained in Section 3.3.6, although it is never an optimal decision, starting cohabitation with the current spouse is a feasible option for married couples. Formally,  $P^{C}(x_{t+1}, y_{t+1}, y_{t+1}, s_t = M) = 0$ , for any values of  $x_{t+1}, y_{t+1}, and \gamma_{t+1}$ .

tive exercise results to the assumption, I recalibrate the model with two alternative values the parameter.<sup>2</sup> In first alternative calibration, I assume cohabitation provides two thirds of the economies of scale compared to marriage:  $\psi^C = 1.1177 = 1 + 2/3(\psi^M - 1)$  ('Alternative 1' in Tables B.1, B.2, and B.3). In another calibration, I assume cohabitation provides a third:  $\psi^C = 1.0294 = 1 + 1/3(\psi^M - 1)$  ('Alternative 2' in the tables). Although the exact magnitude of changes vary, both alternative calibration results are similar to the findings from Section 3.4.

Table B.1: Model and data moment	S

	Data		Model	
		Main	Alternative 1	Alternative 2
Married women (%)	47.96	48.56	44.08	41.53
Cohabiting women (%)	11.08	9.30	10.91	9.98
Average age at 1st marriage	24.10	24.54	23.98	25.20
Standard deviation of age at 1st marriage	4.72	5.41	5.23	5.53
Divorced from first husband by 38th birth day (%)	28.02	41.30	43.81	41.16
Average duration of 1st marriage (years)	7.12	6.85	6.97	6.45
Standard deviation of duration of 1st marriage	4.43	4.61	4.80	4.56
Average age at 1st cohabitation	24.57	24.45	24.13	24.83
Standard deviation of age at 1st cohabitation	5.13	5.40	5.40	5.47
First cohabitation has ended by 38th birth day (%)	90.02	91.11	90.20	90.06
Average duration of 1st cohabitation (years)	2.35	2.92	2.69	2.98
Standard deviation of duration 1st cohabitation	2.72	2.56	2.41	2.57

Main is the results from the calibration in Section 3.4. Alternative 1 is the results of calibration setting the economies of scale parameter for cohabitation,  $\psi^{C}$ , to 1.1177. Alternative 2 is the results with  $\psi^{C} = 1.0294$ .

		Value		Description
	Main	Alternative 1	Alternative 2	
к	0.9144	0.8620	1.3200	Divorce cost
$\bar{\gamma}$	-0.3071	-0.2256	-0.2608	Mean of conditional match quality
$\phi_{\gamma}$	0.9945	0.9973	0.9860	Persistence parameter of conditional match quality
$\sigma_{\gamma}^{2}$	0.0100	0.0045	0.0142	Variance of conditional match quality
γ <sub>0</sub>	-2.3031	-2.7022	-3.0349	Mean of unconditional match quality
$\sigma_0^2$	2.2310	3.0018	2.5985	Variance of unconditional match quality

Main is the results from the calibration in Section 3.4. Alternative 1 is the results of calibration setting the economies of scale parameter for cohabitation,  $\psi^{C}$ , to 1.1177. Alternative 2 is the results with  $\psi^{C} = 1.0294$ .

<sup>&</sup>lt;sup>2</sup>If cohabitation provides the same level of economies of scale to marriage,  $\psi^{C} = \psi^{M}$ , there's no benefit of marriage unless the divorce cost is zero.

	2000	Counter	factual	1970
	Baseline	Smaller volatility	Bigger gender	Model $(2) + (3)$
			gap	
	(1)	(2)	(3)	(4)
		Mai	n calibration	
Married women (%)	48.56	64.09	48.77	65.60
Fraction of ever married by 38 (%)	74.10	88.50	74.60	91.20
Average duration of 1st marriage	6.85	7.02	6.79	7.05
Average age at 1st marriage	24.54	23.60	24.48	23.52
Cohabiting women (%)	9.30	4.29	10.11	3.81
Fraction of ever cohabited by 38 (%)	45.00	21.80	45.50	18.60
Average duration of 1st cohabitation	2.92	2.88	3.14	2.93
Average age at 1st cohabitation	24.45	24.52	24.31	24.51
		Al	ternative 1	
Married women (%)	44.08	52.58	49.17	56.61
Fraction of ever married by 38 (%)	63.00	75.60	73.00	83.40
Average duration of 1st marriage	6.97	6.79	6.92	6.75
Average age at 1st marriage	23.98	24.13	24.07	23.97
Cohabiting women (%)	10.91	11.63	8.40	9.97
Fraction of ever cohabited by 38 (%)	55.10	49.90	47.00	42.70
Average duration of 1st cohabitation	2.69	3.36	2.60	3.33
Average age at 1st cohabitation	24.13	23.94	23.98	24.14
		Al	ternative 2	
Married women (%)	41.53	54.63	45.05	57.22
Fraction of ever married by 38 (%)	62.20	78.20	68.90	81.60
Average duration of 1st marriage	6.45	6.61	6.57	6.71
Average age at 1st marriage	25.20	24.74	25.00	24.58
Cohabiting women (%)	9.98	6.09	8.84	5.26
Fraction of ever cohabited by 38 (%)	49.30	30.50	42.70	26.30
Average duration of 1st cohabitation	2.98	2.91	3.07	2.88
Average age at 1st cohabitation	24.83	24.95	24.88	25.17

### Table B.3: Changes in household union

Main calibration is the results from the Section 3.4. Alternative 1 is the results of calibration setting the economies of scale parameter for cohabitation,  $\psi^{C}$ , to 1.1177. Alternative 2 is the results with  $\psi^{C} = 1.0294$ .

# **Appendix C**

# Chapter 4 Appendix

## C.1 The American Time Use Survey

The American Time Use Survey (ATUS) from the American Time Use Survey Data Extract Builder (Hofferth *et al.* (2018)) is used for time allocation. I pooled the samples from 2003 to 2017 survey. Following Guryan *et al.* (2008), I restrict the sample to those who are 21 - 55 years old with at least one child under 18-year-old in the household. I also restrict my analysis to married (spouse present) respondents. I group the individual activities into five categories: market work, non-market work, childcare, leisure, and other. I use the 'BLS Published Tables' aggregate variables, which is available in the the American Time Use Survey Data Extract Builder to group them.

The market work is the same as the 'BLS: work and work-related activities.' For nonmarket work, I sum 'BLS: Household activities,' 'BLS: Purchasing goods and services,' 'BLS: Caring for and helping household members,' and 'BLS: Caring for and helping non-household members.' And subtract 'BLS: Caring for and helping household children' and 'BLS: Caring for and helping non-household children' times from non-market work and use the sum of the two as childcare hours. The leisure is sum of 'BLS: Personal care,' 'BLS: Eat and drinking,' 'BLS: Leisure and sports,' 'BLS: Organizational, civic, and religious activities,' and 'BLS: Telephone calls, mail, and e-mail.' The 'BLS: Educational activities' is considered other activity, which is not reported. And I exclude a few respondents with the sum of time spent on the five groups of activities exceeding 24 hours. As a result of this sample section, the final sample has 21,927 men and 24,522 women.

## C.2 Solution of the model

The model 4.1 has the following solution:

$$\begin{split} d^{*} &= Dt_{g}^{*} \coloneqq \left(\frac{\alpha_{g}}{1-\alpha_{g}} \frac{w}{p_{d}}\right)^{\frac{1}{1-p_{g}}} t_{g}^{*}, \\ g^{*} &= Gt_{g}^{*} \coloneqq \left\{ \begin{pmatrix} (\alpha_{g}D^{\rho_{g}}+1-\alpha_{g})^{\frac{1}{p_{g}}} t_{g}^{*}, & (\text{if } \rho_{g} \neq 0) \\ D^{\alpha_{g}} t_{g}^{*}, & (\text{if } \rho_{g} = 0) \end{pmatrix}, \\ m^{*} &= MA(w)t_{h}^{*} \coloneqq \left(\frac{\alpha_{h}}{1-\alpha_{h}} \frac{w}{p_{m}}\right)^{\frac{1}{1-\rho_{h}}} A(w)t_{h}^{*}, & (\text{if } \rho_{g} \neq 0) \\ M^{\alpha_{h}}A(w)t_{h}^{*}, & (\text{if } \rho_{g} \neq 0) \\ M^{\alpha_{h}}A(w)t_{h}^{*}, & (\text{if } \rho_{h} = 0) \end{pmatrix}, \\ t_{g}^{*} &= T_{g}t_{h}^{*} \coloneqq \left[\frac{\mu_{g}\{p_{m}MA(w) + w\}G}{\mu_{h}\{p_{d}D + w\}HA(w)}\right]^{\frac{1}{\sigma}} \frac{H}{G}A(w)t_{h}^{*}, & (C.1) \\ n^{*} &= N_{0} + N_{1}t_{h}^{*} \\ &\coloneqq \frac{\mu_{l}^{-\frac{1}{\sigma}}}{w(\mu_{c}w)^{-\frac{1}{\sigma}} + \mu_{l}^{-\frac{1}{\sigma}}} \\ &+ \frac{\{p_{d}DT_{g} + p_{m}MA(w)\}(\mu_{c}w)^{-\frac{1}{\sigma}} - (T_{g} + 1)\mu_{l}^{-\frac{1}{\sigma}}}{w(\mu_{c}w)^{-\frac{1}{\sigma}} + \mu_{l}^{-\frac{1}{\sigma}}} \\ t_{h}^{*} &= \frac{1-N_{0}}{\left\{\mu_{l}(1 + \frac{p_{d}D}{w})\right\}^{\frac{1}{\sigma}} \left[GT(\mu_{g}G)^{-\frac{1}{\sigma}} + \left\{\mu_{l}(1 + \frac{p_{d}D}{w})\right\}^{-\frac{1}{\sigma}}(N_{1} + T_{g} + 1)\right]. \end{split}$$

Taxes and fiscal policy variables introduced in Section 4.5.2 affects the solution. Depending on the schedule of taxes and policies, the model can have a corner solution: one or more endogenous choice variable is zero. For example, increasing the amount of public provision of the market input ( $m_0$ ) crowds out mothers' demand for the market input (m), and it eventually reaches 0. Similarly, increasing the amount of transfers to mothers ( $\tau$ ) decreases mothers labor supply (n) to 0. If the model has an interior solution, no choice variable is zero, then the model with taxes and policy variables has a similar solution:

$$\begin{split} d^{*} &= Dt_{g}^{*} \coloneqq \left(\frac{\alpha_{g}}{1-\alpha_{g}}\frac{\tilde{w}}{(1-\tau_{d})p_{d}}\right)^{\frac{1}{1-\rho_{g}}}t_{g}^{*}, & (\text{if }\rho_{g}\neq 0)\\ D^{\alpha_{g}}t_{g}^{*}, & (\text{if }\rho_{g}=0)\\ m^{*} &= MA(w)t_{h}^{*} - m_{0} \coloneqq \left(\frac{\alpha_{h}}{1-\alpha_{h}}\frac{\tilde{w}}{(1-\tau_{m})p_{m}}\right)^{\frac{1}{1-\rho_{h}}}A(w)^{\frac{1}{\rho_{h}-1}}A(w)t_{h}^{*} - m_{0}, \\ h^{*} &= HA(w)t_{h}^{*} \coloneqq \left\{\frac{\{\alpha_{h}M^{\rho_{h}}+1-\alpha_{h}\}^{\frac{1}{\rho_{h}}}A(w)t_{h}^{*}}{(\text{if }\rho_{g}\neq 0)}\right\}^{\frac{1}{\sigma}}A(w)t_{h}^{*}, & (\text{if }\rho_{g}\neq 0)\\ M^{\alpha_{h}}A(w)t_{h}^{*}, & (\text{if }\rho_{h}=0), \\ t_{g}^{*} &= T_{g}t_{h}^{*} \coloneqq \left[\frac{\mu_{g}\{(1-\tau_{m})p_{m}MA(w)+\tilde{w}\}G}{\mu_{h}\{(1-\tau_{d})p_{d}D+\tilde{w}\}HA(w)}\right]^{\frac{1}{\sigma}}\frac{H}{G}A(w)t_{h}^{*}, & (C.2)\\ n^{*} &= N_{0} + N_{1}t_{h}^{*} \coloneqq \frac{\mu_{l}^{-\frac{1}{\sigma}}-\tau(\mu_{c}\tilde{w})^{-\frac{1}{\sigma}}}{\tilde{w}(\mu_{c}\tilde{w})^{-\frac{1}{\sigma}}+\mu_{l}^{-\frac{1}{\sigma}}} \\ &+ \frac{\{(1-\tau_{d})p_{d}DT_{g}+(1-\tau_{m})p_{m}MA(w)\}(\mu_{c}\tilde{w})^{-\frac{1}{\sigma}}-(T_{g}+1)\mu_{l}^{-\frac{1}{\sigma}}}}{\tilde{w}(\mu_{c}\tilde{w})^{-\frac{1}{\sigma}}+\mu_{l}^{-\frac{1}{\sigma}}} \\ t_{h}^{*} &= \frac{1-N_{0}}{\left\{\mu_{l}(1+\frac{(1-\tau_{d})p_{d}D})\right\}^{\frac{1}{\sigma}}\left[GT(\mu_{g}G)^{-\frac{1}{\sigma}}+\left\{\mu_{l}(1+\frac{(1-\tau_{d})p_{d}D})\right\}^{-\frac{1}{\sigma}}(N_{1}+T_{g}+1)\right]}, \\ \tilde{w} \coloneqq \frac{1-\tau_{l}}{1+\tau_{c}}w. \end{split}$$

As the amount of public provision of the market input  $(m_0)$  increases, it crowds out mothers' private provision of the input (m). Once mothers' private provision reaches 0, their time spent on children's human capital production,  $t_h^*$ , is the solution to the following non-linear equation from the first order condition:

$$h(t_h^*) \left\{ \frac{dh(t_h^*)}{dt_h^*} \frac{\mu_h}{\mu_l} \right\}^{-\frac{1}{\sigma}} = l^*$$

Other variables are defined in the same way as in C.2, except demand for the market input (m) is 0 and children's human capital (h) is no longer a linear function of mothers' time  $(t_h)$ .

Mothers decrease their labor supply as the amount of transfers to mothers increases. Observe that once labor supply (n) reaches 0, allocations do not depend on the wage rate. Using the fact, to compute mothers allocations with transfers, first solve for allocations by C.2. Given the solution, identify the wage rate where labor supply is 0, and mothers with wages lower than

the point have the same allocations.

## C.3 Allocations with the CRRA utility, $\sigma = 2$

I repeat the exercises as in 4.3 for the CRRA utility with the curvature parameter,  $\sigma$ , sets to 2. Figure C.1 plots mothers' time investment in children and leisure when the time productivity does not depend on wage (A(w) = 1). Compared to the log utility, where the income and substitution effects cancel each out, with  $\sigma = 2$ , the income effect dominates the substitution effect with respect to changes in wage. As a result, leisure increases in wage. In terms of time investment in children, the overall finding is similar to other cases. As the complementarity between goods and time inputs becomes stronger (the parameter value of  $\rho_h$  becomes smaller), the time investment increases in wage. Note that when the children's human capital production is Cobb-Douglas function,  $\rho_h = 0$ , the time investment is increasing in wage, where as it does not change in wage with the log utility.

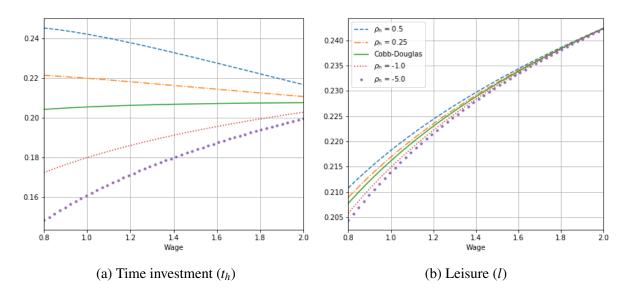


Figure C.1: Time allocations with CRRA utility ( $\sigma = 2$ ) and A(w) = 1.

*Note.* Using a CRRA utility with equal weights:  $U(c,g,h,l) = -(c^{-1}+g^{-1}+h^{-1}+l^{-1})$ . The productivity of time investment does not depend on wage: A(w) = 1.

Figure C.2 plots the time investments when the productivity of time depends on wage. In constrast with the case where  $\sigma = 0.5$ , in Figure 4.3, having the productivity positively correlated with wage (A'(w) > 0) decreases the slope of the time investment. For example, when  $\rho_h = -1$ , mothers time spent on children increases in wage when the productivity of their time does not depend on their wages (the blue solid line in Figure C.2b). As the productivity in-

creases in wage, mothers with higher wages decrease time with their children, and the gradient becomes negative.

A(w) = 10.26 0.17  $A(w) = w^{1/2}$  $-\cdots A(w) = w$  $A(w)=w^2$ 0.24 0.16 0.22 0.15 0.20 0.14 0.18 0.13 1.4 Wage 1.4 Wage 0.8 1.2 1.6 1.8 1.0 1.2 1.6 1.0 2.0 1.8 0.8 2.0 (a) When  $\rho_h = 0.5$ (b) When  $\rho_h = -1$ 

Figure C.2: Time investment with CRRA utility ( $\sigma = 2$ ).

Note. Using a CRRA utility with equal weights:  $U(c,g,h,l) = -(c^{-1}+g^{-1}+h^{-1}+l^{-1})$ .

## C.4 Additional Tables

Years of Schooling	Market Work	Non-market Work	Child Care	Leisure	Hourly Wage
-		(Hours p	er week)		(In 2016\$)
< 12	12.2	35.6	11.3	106.3	10.40
	(0.73)	(0.64)	(0.43)	(0.74)	(0.127)
12	20.6	29.8	11.3	103.7	14.92
	(0.50)	(0.35)	(0.24)	(0.42)	(0.058)
13 – 15	23.8	27.0	11.9	101.6	18.47
	(0.48)	(0.29)	(0.20)	(0.38)	(0.052)
16	24.4	26.5	14.2	99.9	26.44
	(0.44)	(0.27)	(0.21)	(0.34)	(0.052)
16 +	28.6	25.1	14.5	96.9	34.27
	(0.62)	(0.36)	(0.29)	(0.45)	(0.066)
			Married fathers		
< 12	41.0	14.3	4.9	106.5	13.26
	(1.03)	(0.52)	(0.41)	(0.84)	(0.109)
12	41.9	15.8	5.3	103.4	19.11
	(0.59)	(0.29)	(0.15)	(0.50)	(0.061)
13 – 15	42.8	16.5	6.7	99.5	23.75
	(0.56)	(0.29)	(0.18)	(0.45)	(0.064)
16	44.5	15.6	7.7	97.9	36.55
	(0.52)	(0.26)	(0.17)	(0.39)	(0.067)
16 +	45.5	14.4	8.3	97.3	48.72
	(0.62)	(0.28)	(0.20)	(0.48)	(0.085)

Table C.1: Gradients in time allocations and hourly wage of married parents

Time allocations are estimated using the pooled sample of the ATUS 2003 - 2017, and the hourly wage rates are estimated using the samples from the CPS-ASEC 2003 - 2017. Estimates are computed using provided weights, and robust standard errors are in parentheses. Samples are restricted to 21 - 55 (inclusive) year-old men and women with at least one child younger under the age of 18 in the household. The average wage rates are estimated for full-time and full-year workers.

Schooling	% Working	Market Work	Non-market Work	Child Care	Leisure	Market Work	Non-market Work	Child Care	Leisure
		~	Working married mothers	d mother	S	No	Not working married mothers	ried moth	ers
< 12	35.3	33.5	27.6	5.7	98.7	0.6	40.0	14.3	110.4
	(1.42)	(1.43)	(0.86)	(0.40)	(1.13)	(0.18)	(0.81)	(0.59)	(0.92)
12	57.4	35.4	24.7	7.7	98.4	0.7	36.5	16.2	110.8
	(0.85)	(0.64)	(0.41)	(0.24)	(0.51)	(0.13)	(0.52)	(0.42)	(0.64)
13 - 15	66.7	35.2	23.6	9.2	97.4	1.1	33.9	17.4	110.0
	(0.70)	(0.57)	(0.33)	(0.20)	(0.44)	(0.23)	(0.51)	(0.43)	(0.64)
16	68.2	35.2	22.9	11.4	96.1	1.1	34.2	20.1	108.1
	(0.66)	(0.52)	(0.30)	(0.21)	(0.39)	(0.15)	(0.49)	(0.45)	(0.59)
16 +	74.6	37.7	22.2	11.7	94.0	1.9	33.7	22.8	105.5
	(0.84)	(0.69)	(0.37)	(0.27)	(0.51)	(0.31)	(0.73)	(0.73)	(0.86)
		r	Working married father	ed fathers	~	ž	Not working married fathers	rried fathe	rs
< 12	86.3	46.8	12.9	4.1	103.0	4.4	23.2	10.1	128.9
	(1.01)	(1.05)	(0.52)	(0.39)	(0.84)	(1.26)	(1.71)	(1.64)	(2.30)
12	86.9	47.8	14.6	4.8	99.4	2.5	23.8	8.5	129.9
	(0.57)	(0.59)	(0.28)	(0.15)	(0.48)	(0.47)	(1.02)	(0.57)	(1.28)
13 - 15	89.2	47.5	15.2	6.3	97.0	3.9	27.5	9.9	120.2
	(0.52)	(0.55)	(0.29)	(0.18)	(0.45)	(0.62)	(1.14)	(0.76)	(1.47)
16	93.6	47.1	14.8	7.4	96.7	6.7	27.8	13.0	115.1
	(0.38)	(0.52)	(0.25)	(0.17)	(0.40)	(1.11)	(1.38)	(0.96)	(1.52)
16+	93.4	48.2	13.7	8.0	96.1	9.9	24.7	12.5	114.6
	(0.50)	(0.61)	(0.28)	(0.20)	(0.47)	(1.12)	(1.49)	(1.19)	(2.12)

Table C.2: Educational gradients in time allocation of mothers, conditional on work status

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		Human o	capital (h)	Time in	nput $(t_h)$	Market inp	out $(m+m_0)$
Subsidy	Tax rate	Mean	Variance	Mean	Variance	Mean	Variance
ç	%		% change fro	m the baseli	ne (no policy	intervention)	
	Subsidi	ze the mark	et input in chil	dren's huma	n capital proc	luction $(\tau_m)$	
10	0.41	1.65	3.29	0.62	1.18	11.34	23.94
20	0.93	3.52	7.11	1.33	2.53	25.48	57.38
30	1.62	5.68	11.62	2.14	4.14	43.58	106.02
40	2.56	8.23	17.08	3.10	6.09	67.53	180.50
50	3.91	11.33	23.91	4.26	8.54	100.66	302.55
	Provid	le the goods	input in child	ren's human	capital produ	ction $(m_0)$	
1	0.04	0.02	0.01	0.02	-0.04	-0.02	-0.05
10	0.37	0.16	0.13	0.23	-0.39	-0.17	-0.55
25	0.93	0.44	0.21	0.63	-1.90	-0.06	-2.32
50	1.88	1.24	-0.39	1.52	-6.65	3.44	-11.65
100	3.86	4.14	-2.86	3.64	-14.24	26.09	-42.40
		L	ump-sum payı	ment to moth	hers $(\tau)$		
1	0.55	0.23	0.20	0.34	-0.52	-0.26	-0.79
2.5	1.40	0.59	0.49	0.86	-1.30	-0.66	-1.99
5	2.85	1.17	0.98	1.73	-2.58	-1.35	-3.99
7.5	4.35	1.75	1.47	2.60	-3.92	-2.07	-6.01
10	5.91	2.34	1.95	3.48	-5.32	-2.81	-8.05

Table C.3: Impact of policies on child's human capital through consumption tax

The amount of the market input provided by the policy is denoted  $(m_0)$  by its amount relative to the average demand of the good in the baseline. The lump-sum transfer  $(\tau)$  are denoted as its amount relative to the average labor income. The tax rate (second column) denotes required labor income tax rate to implement the subsidy. Changes in allocations are denoted as percentage changes from their respective values in the baseline. When the market input is provided  $(m_0)$ , the amount is added to mothers' demand to compute the changes in mean and variance.

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