Optimal Pricing and Treatment Policies in Health Care

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Abstract

Health care decision makers are faced with many types of uncertainty. There may be uncertainty in the benefit a new treatment provides, uncertainty in the demand for the new treatment, and uncertainty in the drug approval process. In three essays, I study the impact of various types of uncertainty within health care systems.

In the first essay, I study the welfare properties of six practical pharmaceutical pricing and access policies. Using a game theoretic approach, I find that when demand can be influenced by costly marketing effort, there are meaningful differences to the desirability of various pricing and listing agreements, compared to the results in existing literature. I find that all non-value-based agreements result in at least one type of inefficiency. I find that a value-based policy with risk-sharing is preferred by the manufacturer and from a societal perspective, while the payer’s preference depends on the manufacturer’s negotiating power and the risk-sharing rebate rate.

In the second essay, I study the impact of health care fragmentation on treatment policies. I consider the scenario where a patient’s health care is covered by multiple payers across their lifespan. I formulate a multi-decision-maker Markov decision process to capture the payers’ repeated intervention decisions and I partition the optimization problem using a threshold patient age that defines when the patient transitions from one payer to the next. I find that a fragmented health care system always results in a treatment policy that is a subset of the treatment policy in a centralized system and that a simple transfer payment between payers can coordinate the system.

In the third essay, I study the scenario where payers in a fragmented health care system incur different costs for the same treatment. Using a similar multi-decision-maker Markov decision process as in my second essay, I find that when payers incur different costs of treatment, that over- and under-treatment will occur in a fragmented system compared to a centralized system. I find that when pricing is endogenous, the payers will prefer a setting where coordination is not possible and the manufacturer will prefer coordination.

Keywords: Health Care, Pricing, Treatment, Marketing, Game Theory, Markov Decision Process, Optimal Policy, Coordination, Risk-Sharing
Lay Summary

Health care decision makers are faced with many types of uncertainty. There may be uncertainty in the benefit a new treatment provides, uncertainty in the demand for the new treatment, and uncertainty in the drug approval process. In three essays, I study the impact of various types of uncertainty within health care systems.

In the first essay, I study the pros and cons of several common real-world health care systems. For each system, I study how a manufacturer’s marketing efforts can influence the demand for a new pharmaceutical drug (e.g., through advertising) and then the ripple effects on prices and treatment criteria. From a societal perspective, a health care system where prices are linked to the value that a drug provides is always preferred when a manufacturer can influence demand. However, from the health care payer’s perspective, there is no policy that is preferred above all others.

In the second essay, I study health care systems where there is more than one payer over a patient’s lifetime. For example, in the US, many individuals are covered by Medicaid until the age of 65, and then covered by Medicare thereafter. I find that a fragmented health care system will always provide treatment to fewer patients than a system where there is a single health care decision maker. Additionally, I find that if payers can make payments to one another to co-share the cost of treatment, then the inefficiencies from fragmentation disappear.

In the third essay, I study health care systems where there are more than one payer over a patient’s lifetime and these payers incur different prices to provide the same treatment. I find that when payers incur different costs of treatment, that over- and under-treatment will occur when compared to a system with a single health care decision maker. I find that when prices are negotiated in anticipation of coordination between payers, that the payers will prefer a setting where coordination is not possible and the manufacturer will prefer coordination.
Co-Authorship Statement

This thesis includes material that is the result of joint research. Essay 1 is co-authored with Dr. Gregory S. Zaric. Essay 2 is co-authored with Dr. Lauren E. Cipriano, Dr. Gregory S. Zaric, and Dr. Jeremy D. Goldhaber-Fiebert. Essay 3 is co-authored with Dr. Lauren E. Cipriano, Dr. Gregory S. Zaric, and Dr. Jeremy D. Goldhaber-Fiebert. I am the first author on all three essays. As such, I controlled all aspects of the projects including formulating the research questions, conducting the literature reviews, developing the models, performing the analysis, and preparing the manuscripts. I certify that this dissertation, and the research to which it refers, is fully a product of my own work.

This thesis includes 3 original essays. One essay is published in the journal Health Economics. One essay is currently under review at Operations Research.


Dedication

For my wife, Cara.

And my running buddy, Jasper.
Acknowledgements

Thank you to my supervisors, Dr. Gregory S. Zaric and Dr. Lauren E. Cipriano, for their continued support and direction over the past four years. Thank you to my thesis supervisory committee and members of my examination board for their time and commitment. Thank you to my family for their unwavering support from near and far. And thank you to my wife, Cara, without whom this journey could not have been possible.
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Chapter 1

Introduction

Health care decision makers are faced with many types of uncertainty. For example, there may be uncertainty in the benefit that a new treatment provides, uncertainty in the demand for the new treatment, uncertainty in the drug reimbursement approval process, and uncertainty in how the untreated disease progresses. Recently, high pharmaceutical prices have amplified the negative effects of these unknown outcomes. For example, a single unsuccessful treatment of the drug *tisagenlecleucel* (Kymriah) costs a US health care payer $475,000 (US Food and Drug Administration 2017). In response to these uncertainties, health care payers, manufacturers, and policy makers have instituted a variety of creative pricing or treatment policies to share or minimize the risk of unsuccessful expenditures.

One of the first examples of an agreement that shares the risk of uncertain treatment outcomes is the risk-sharing arrangement between the National Health Service (NHS) and manufacturer for the drug *bortezomib* (Velcade), used to treat multiple myeloma (Pollack 2007). The NHS pays the list price for treatments and the drug manufacturer provides a reimbursement for any unsuccessful treatment, defined as a reduction of less than 25% of the serum M protein after the first four weeks of treatment (Neumann et al. 2011). In theory, a risk-sharing arrangement is designed to promote access to treatment for a larger number of patients. However, when pricing or treatment policies are adopted to address an isolated source of uncertainty (e.g., the risk of an unsuccessful treatment), then there may be unintended consequences on other decisions in the health care decision making process. For
example, in anticipation of a risk-sharing arrangement that is intended to provide patients
greater access to treatment, a manufacturer may increase the price of treatment, decreasing
the cost-effectiveness of the drug for all but the most severe cases (Barros 2011). The core
theme of this thesis is to study the effects of health care decisions across multiple stages of
the health care decision making process.

One key stage in health care decision making is the reimbursement approval process for
a new drug, sometimes called the listing process, that defines how the drug will be priced
and outlines any special mechanisms that may be required for approval (e.g., a risk-sharing
arrangement). Health care policy makers, payers, and manufacturers have been particularly
interested in this stage of the process, especially with the significant and growing public
expenditure on pharmaceuticals in Canada, the US, and the UK (Centers for Medicare &
Medicaid Services 2016, Canadian Institute for Health Information 2017, World Health
Organization 2018, Organization for Economic Co-operation and Development 2018a,b).
In an attempt to control these expenditures, health care decision makers have implemented
policies that include risk-sharing arrangements, price caps, and reference pricing (Carlson
et al. 2014, Garrison et al. 2013). However, these arrangements have been difficult to im-
plement and have not resulted in the anticipated benefits; existing discussions as to why
these agreements have been difficult to implement cite high administrative costs, impre-
cise measurements of success, and inadequate infrastructure to track the health care data
(Neumann et al. 2011, Neumann 2013). However, I hypothesize that the impact that these
arrangements have on pricing, treatment policies, and demand may indicate why they have
been less successful than anticipated.

Another key stage in the health care supply chain is the process of defining a treatment
policy. A treatment policy is often defined by a payer, outlining the medical requirements
necessary for the reimbursement of treatment. There is an extensive body of literature
addressing the cost-effectiveness or cost-utility of many new and existing treatments (i.e.,
should the treatment be reimbursed or not?). The Canadian Patented Medicine Prices Re-
view Board (PMPRB), the governing body that ensures affordable pharmaceutical pricing
in Canada, now requires cost-utility analyses for all new or existing Category 1 (‘high
priority’) price reviews (Patented Medicine Pricing Review Board 2019). Guidelines rec-
ommend or require that these analysis use a lifetime horizon when considering the costs and benefits of treatment (e.g., Canadian Agency for Drugs and Technologies in Health 2017). Several studies develop more nuanced treatment policies, beyond a simple yes or no recommendation, including patients’ disease severity and age into the recommendation for treatment. For example, Alagoz et al. (2004) study the optimal time to accept a liver transplant based on the patients disease severity and the quality of the proposed liver. However, compounding the complexity in bringing new treatments to market and defining treatment policies is the fact that most health care systems are composed of multiple payers, each with their own treatment policies. Patients may receive health care coverage from multiple health care payers simultaneously for different health care needs (e.g., hospital care versus prescription drug coverage). Or, patients may be covered by different health care payers at different stages of their life. For example, in the US there is an almost universal shift of health care coverage from private to publicly funded at age 65 (Cebul et al. 2011, Elhauge 2010). I hypothesize that fragmented health care systems may result in treatment inefficiencies because most payers’ treatment horizons are short compared to the recommended lifetime horizon in cost-effectiveness analyses.

In this thesis, I study health care decision making when there is uncertainty in future outcomes. In three essays, I study multi-stage decision making process where prices and treatment policies are determined, and I evaluate the welfare properties for patients, payers, and drug manufacturers. I study the economic outcomes of various pricing and access policies when the benefit of treatment is uncertain and demand for treatment is influenced by a manufacturer’s marketing effort. I study the impact of fragmentation on treatment policies within a health care system where patients’ health progresses with uncertainty and patients receive health care coverage from different payers across their lifetime. Finally, I study the compound effects of pricing and treatment decisions in fragmented health care systems.

Overview of Three Essays

In the first essay, I study the welfare properties of six practical pharmaceutical pricing and access policies from the payer’s, manufacturer’s, and patients’ perspective. Previous
studies have considered the desirability of performance-based risk-sharing arrangements (Barros 2011, Antonanzas et al. 2011, Mahjoub et al. 2018), financial risk-sharing arrangements (Zaric and O’Brien 2005, Zhang et al. 2011, Gavious et al. 2014), uncertain listing processes (Levaggi 2014), and reference pricing (Brekke et al. 2007, Miraldo 2009). In each study, demand for treatment is modeled as either exogenous, as implicitly linked to the price of treatment, or as a portion of a fixed population. However, none of these studies consider the impact of a manufacturer’s efforts to influence demand through marketing. I study these policies when there is uncertainty in the benefit that the treatment provides and when the manufacturer can influence total demand by conducting costly marketing effort. Broadly, I find that when demand can be influenced by marketing effort, there are meaningful differences to the desirability of various pricing and listing agreements compared to the existing literature. Using a game theoretic approach, I formulate a set of multi-stage games, each reflecting a specific real-world pricing and access policy. Under each policy there are three sequential decisions. First, the price of the drug is established, as defined by the pricing or access policy (e.g., through a listing process or through negotiations). Second, the payer decides which portion of the treatment-eligible population will receive treatment based on individuals’ probability of response to the new treatment. Finally, the manufacturer selects its level of marketing investment that influences the total demand for treatment.

I find that all non-value-based policies suffer from at least one form of inefficiency; either access inefficiency, marketing inefficiency, and/or treatment inefficiency. I find that a value-based policy with risk-sharing is preferred by the manufacturer and from a societal perspective, while the payer’s preference depends on the manufacturer’s negotiating power and the risk-sharing rebate rate. Furthermore, the payer and manufacturer never prefer the same pricing and access policy and therefore a policy maker may find it difficult to implement any policy due to resistance from either party. When only considering a listing process and a risk-sharing arrangement, I find that the payer and manufacturer will mutually prefer the risk-sharing arrangement only for low rebate rates. Therefore, this may provide one reason why risk-sharing arrangements that offer full rebates, such as the risk-sharing policy implemented in the UK for bortezomib, has been less successful than
anticipated. To the best of my knowledge, this essay is the first study to evaluate the impact of a manufacturer’s marketing efforts on the desirability of various pricing and access policies. Significantly, I find that policy makers should incorporate the impact of demand-influencing efforts into their pricing and access policy decisions as this effort has a material impact on the economic outcome of these policies.

In the second essay, I study the impact of health care fragmentation on treatment policies for patients. I consider the scenario where a patient’s health care is covered by multiple payers across their lifespan. A payer that only covers a patient’s health care for a portion of the patient’s life may not expect to capture the long-term benefits of preventive care or the benefits of investments in chronic diseases treatments (e.g., reduced future costs). As a result, fragmented health insurance contributes to inadequate investment in the provision of non-acute health care (Helland and Klick 2010). Inadequate long-term-minded health care can result in progression to worse and more expensive health states and in higher costs from preventable disease-related complications (Avraham and Camara 2007, Cebul et al. 2008, Fang and Gavazza 2011). Ultimately, often a publicly funded final payer in a patient’s life will bear the additional cost of expensive delayed treatments because of the insufficient care from a previous health care payer. Fragmented insurance appears to exacerbate the challenges of accessing appropriate care (Herring 2010, Fang and Gavazza 2011). In this essay, I formulate a multi-decision-maker Markov decision process (MDP) to capture the payers’ repeated intervention decisions while the patient’s health state, described using two discrete dimensions, evolves over time. I partition the optimization problem using a threshold patient age (e.g., age 65) that defines when the patient transitions from one payer to the other. Using this model, I establish analytic and practical insights into the treatment and access inefficiencies that result from multiple health care payers over a patient’s lifespan. Using a game theoretic approach, I identify a coordinating contract between payers that results in a lifetime-optimal treatment policy for patients.

I find that the treatment policy in a fragmented system targets a subset of patients compared with a centralized health care system and I identify the conditions such that treatment gaps exist for a period of time in the middle of a patient’s life. These treatment inefficiencies are particularly concerning from a public payer’s perspective. For example, in the US,
Medicaid and Medicare are budgeted separately yet both use public funds to provide health care. Therefore, fewer treatments are provided, more patients progress into severe disease states, and avoidable deaths occur, simply because there is a separation of decision making when establishing treatment policies. Furthermore, even in systems where the final payer in a patient’s life is publicly funded and payers at earlier stages are privately funded (e.g., in the US and Canada), fragmentation results in greater public expenditure as a result of inadequate early preventative and chronic care. I prove that a simple transfer payment can coordinate the system, resulting in lifetime-optimal treatments for patients and improve or maintain welfare for all payers.

In the third essay, I study the multi-stage decision making process where payers in a fragmented health care system each first negotiate the price of treatment and then decide their treatment policy. Similar to the second essay, I consider the scenario where a patient’s health care is covered by multiple payers over their lifespan. It is common that the price of pharmaceuticals varies across health care payers. For example, in the US Medicare pays 88% more, on average, than Medicaid per specialty brand-name prescription ($3,600 versus $1,920) and 182% more than Medicaid per non-specialty brand-name prescription ($155 versus $55) (Anderson-Cook et al. 2019). While different prices for different payers may be profit enhancing from the drug manufacturer’s perspective (Danzon 1998), I find that different prices for different payers exacerbates the treatment inefficiencies already present in multi-payer health care systems. Using a multi-decision-maker MDP model, I find that when payers incur different costs of treatment, that over- and under-treatment will occur in a fragmented system compared to a centralized system. I prove that there always exists a coordinating contract that results in a treatment policy that is the same as if a social planner made treatment decisions for both payers.

In a health care environment where the first payer incurs a lower cost of treatment than the final payer (e.g., in the US, Medicaid generally pays less for treatments than Medicare), I find that coordination between payers will result in over-treatment when compared to a lifetime-optimal treatment policy with lower average prices. Therefore, coordination leads to the counter-intuitive finding that a higher average price results in a treatment policy that targets more patients. In a health care environment where the first payer incurs a higher cost
of treatment than the final payer (e.g., in Canada, private payers generally pay more than government pharmaceutical plans for retirees), I find that coordination between payers will result in under-treatment when compared to a lifetime-optimal treatment policy with higher average prices. Therefore coordination leads to another counter-intuitive finding that lower average prices results in a treatment policy that targets fewer patients. Thus, while payers will prefer coordination given an exogenous set of prices, the coordinated treatment policy is inefficient. From a policy maker’s perspective, a health care system where prices are consistent over time and where decisions are made using a lifetime horizon is preferred.

When prices are endogenous and are negotiated in anticipation of coordination between payers, I find that the price of treatment generally increases unless the payers have significant negotiating power. As a result, I find that payers will generally prefer a scenario where they do not coordinate, and the manufacture will generally prefer a scenario where the payers coordinate. I do not find that the payers and manufacturer ever mutually prefer either the scenario where payers coordinate or not. Significantly, this two-stage model again demonstrates the chain effects of decisions in a health care system; while in my second essay I demonstrate that in isolation, coordination between payers is welfare improving or maintaining for all payers, in my third essay I show that the impact of coordination in the pricing process is significant and changes the payers’ preferences regarding coordination.

To provide applied context to the findings in the second and third essays, I develop a natural history model of the hepatitis C virus (HCV). HCV is a timely example on which to study my framework because of the large absolute number of people affected, the disproportionate impact on individuals aged 45 to 65 (right before many individuals shift their health care coverage to Medicare at age 65), the relatively restricted access to treatment reimbursement being faced by individuals prescribed treatment by their physician, and the variance in the cost of treatment across providers. Using this model, I find that fragmentation results in over 800,000 delayed treatments and approximately 55,000 avoidable deaths.
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Chapter 2

Essay 1: The impact of pharmaceutical marketing on market access, treatment coverage, pricing and social welfare

2.1 Introduction

Pharmaceutical spending in the US, Canada, and the EU\(^1\) grew by 125% between 2000 and 2016, outpacing total GDP growth of 79% (Organization for Economic Co-operation and Development 2018a,b). Public health care payers cover a significant portion of pharmaceutical expenditures (42% in the US (Centers for Medicare & Medicaid Services 2016), 43% in Canada (Canadian Institute for Health Information 2017) and 68% in the EU (World Health Organization 2018)) and have responded to the increase in budgetary pressure by instituting various pharmaceutical pricing and access policies (Adamski et al. 2010, Carone et al. 2012, Le Polain et al. 2011). For example, pricing mechanisms such as reference pricing, price caps, and risk-sharing arrangements are intended to control spending while increasing access to new treatments (Adamski et al. 2010, Carlson et al. 2014, Garrison et al. 2013, 2015).

Two challenges that health care payers face when making pricing and access policy decisions are uncertainty of demand and uncertainty of the benefit of a new drug in the general covered population. Furthermore, policy decisions are rarely reconsidered after

\(^1\)Represents the 15 EU countries in 2000.
a drug has been introduced (Carone et al. 2012). Therefore, pricing and access policy
decisions must incorporate careful estimates of the expected benefit of treatment and post-
approval demand at the time when the drug is first introduced.

Marketing is an effective tool that pharmaceutical manufacturers use to influence de-
mand once a drug is approved (Avery et al. 2012, Iizuka and Jin 2005, Lakdawalla et al.
2013, Narayanan et al. 2004). Nine of the ten largest global pharmaceutical companies
spend more on promotion than on research and development, collectively spending 1.5x
more on marketing (Swanson 2015). This promotional spending includes physician de-
tailing (68% of the total expenditure), direct-to-consumer advertising (21%), professional
meetings (9%; includes conference sponsorship), e-detailing (2%) and journal advertise-
ments (<1%) (Mack 2014). Christopher Wooden, Vice President at Cegedim Strategic
Data, a global health care market research company, summarizes the crucial role that physi-
cian detailing plays: "Nothing can replace the relationship value of face-to-face promotion”
(Cegedim Strategic Data 2013). Other forms of marketing effort include paying for journal
reprints (Sullivan 2018), paying for physicians’ consulting services (Harris 2008), recruit-
ing physicians onto boards of directors, and promoting corporate ownership of pharma-
ceutical companies to physicians (Carroll 2014). Austad et al. (2014) survey over 1,600
medical students and find that students are more likely to recommend brand-name phar-
aceuticals if they perceive positive industry-physician interactions. In a cross-sectional
study, DeJong et al. (2016) find that physician prescribing behavior is influenced by as little
as a single sponsored meal costing less than $20, confirming that the “size of the gift is not
a determinant of its influence” (Rabin 2013).

In this article, we use a game theoretic approach to study the impact of a drug man-
ufacturer’s marketing decision on market access, treatment coverage, pricing and social
welfare under six practical pharmaceutical pricing and access policies: negotiated pricing,
open pricing, controlled pricing, a listing process, a risk-sharing arrangement and a value-
based pricing with risk-sharing arrangement. We analytically characterize the equilibrium
outcomes of each policy and find several compelling results. First, we find that the value-
based pricing with risk-sharing arrangement is preferred by the manufacturer and from a
societal perspective, while no pricing or access policy is universally preferred by a health
care payer. Second, we identify three types of inefficiencies that occur because of pharmaceutical marketing and show that all non-value-based policies suffer from at least one form of inefficiency. Third, we find that the health care payer and the drug manufacturer never mutually prefer a listing process and that a value-based pricing with risk-sharing arrangement is always socially optimal, consistent with existing literature. Among the non-value-based scenarios, we find that a negotiated pricing policy or a risk-sharing arrangement may each result in the highest level of social welfare depending on a drug’s manufacturing cost, the manufacturer’s negotiating power, and the rebate rate.

2.2 Literature Review

Using Hotelling-like models (Hotelling, 1929), Brekke et al. (2007) and Miraldo (2009) show that a manufacturer’s drug price inversely influences the demand for treatment. They find that reference pricing results in lower prices when compared to a scenario with no reference pricing, a finding that has been empirically confirmed (Ghislandi, 2011, Ghislandi et al. 2013, Kaiser et al. 2014, and others). Using a game theoretic approach, Barros (2011), Antonanzas et al. (2011), and Mahjoub et al. (2018) study performance-based risk-sharing arrangements, whereby payers provide treatment based on a patient’s probability of response and manufacturers provide rebates for ineffective treatments. All three studies find that a payer’s prescribing criteria is stricter, thus decreasing demand, when prices are high. In addition, these studies find that if administrative costs are low, then treatment is over-prescribed, and if administration costs are high, then treatment is under-prescribed. Levaggi and Pertile (2016) study treatment decisions in a fixed population of heterogeneous individuals. They show that heterogeneity across patients results in inefficient treatment decisions.

Zhang et al. (2011) use a game theoretic approach to study a financial risk-sharing contract that limits a payer’s total expenditure. They assume that expected demand is exogenous and inelastic to prices and find that higher demand results in lower prices. Gavious et al. (2014) and Zaric and O’Brien (2005) study financial risk-sharing contracts, assuming exogenous prices and exogenous demand. They find that a manufacturer or health care
provider may over- or under-estimate demand depending on price, manufacturing cost, and rebate rate. An empirical study of price-volume agreements in South Korea shows that demand was underestimated for 121 out of 186 drugs (65.1%) under financial risk-sharing agreements (Park et al. 2016).

Levaggi (2014) compares two different pricing mechanisms: a value-based risk-sharing arrangement and a listing process, whereby the probability that a drug is listed for coverage is inversely related to the drug’s price. Assuming exogenous demand, the value-based arrangement always optimizes social welfare whereas the listing process does not. In addition, Levaggi (2014) demonstrates that there is always a value-based risk-sharing arrangement that is preferred to the listing process by the payer, the manufacturer, and from a social welfare perspective.

Much of the related literature considers demand as exogenous (either deterministic (Levaggi 2014) or stochastic (Gavious et al. 2014, Zaric and O’Brien 2005, Zhang et al. 2011)), as implicitly linked to prices (Brekke et al. 2007, Miraldo 2009), or as a portion of a fixed population decided by a health care payer (Barros 2011, Antonanzas et al. 2011, Levaggi and Pertile 2016, Mahjoub et al. 2018), but does not consider the influence of a manufacturer’s marketing effort. Zaric and Xie (2009) compare two performance-based risk-sharing arrangements, based on population-level outcomes, where a manufacturer selects both price and marketing effort. They find that the payer or manufacturer may prefer either arrangement, however they do not consider the implications on social welfare or drug access. Zhang and Zaric (2015) study whether a financial risk-sharing arrangement can reduce off-label use of a drug when a manufacturer decides the level of marketing effort. They segment demand by patients’ disease type, similar to Coyle et al. (2003), and find that the risk-sharing arrangement may be effective at controlling off-label use.

### 2.3 Model

Consider a health care system with a public payer (payer), a pharmaceutical firm (manufacturer), and a set of individuals. The manufacturer has developed a new drug that provides incremental monetary benefit to the payer, \( b \), for each successful treatment compared to the
2.3. Model

next best alternative, exclusive of the cost of treatment. As in Mahjoub et al. (2018), “b could be the quality-adjusted life-years gained relative to the status quo by a successfully treated patient multiplied by the payer’s WTP [willingness-to-pay] per unit gained”. Let \( p \) be the selling price and \( c > 0 \) (\( c < b \)) be the constant marginal manufacturing cost of the drug.

Demand for the new drug depends on both the manufacturer’s and payer’s decisions. The manufacturer influences the number of patients who seek treatment, \( m \geq 0 \), by conducting marketing effort. Let \( \theta(m) \) represent the manufacturer’s cost to generate the demand, including promotional spending and all spending required to establish the drug in the market (e.g., establishing and maintaining distribution channels, package design, etc.).

We assume that \( \theta(m) \) has the following three properties: i) \( \theta(0) = 0 \); ii) \( \theta'(m) > 0 \) (additional demand is costly); and, iii) \( \theta''(m) > 0 \) (diminishing returns). This view of marketing is relatively more broad than what is found in some promotion-only empirical work (e.g., Avery et al. 2012, Iizuka and Jin 2005, Lakdawalla et al. 2013, Narayanan et al. 2004, and others). The payer influences demand by selecting a treatment threshold. We model each individual patient’s successful response to the new drug as a Bernoulli event where the parameter \( \pi \in [0, 1] \) represents an individual’s probability of successful response to treatment.

The Bernoulli parameters for individual patients are distributed with density \( f(\pi) \). We assume that the distribution of patient types is independent of the number of patients that seek treatment (i.e., \( f(\pi) \) does not depend on \( m \)). This assumption is robust for treatments where success depends on unobservable factors (e.g., immunologic, metabolic, or genetic factors) but does not capture treatments where success largely depends on observable factors (e.g., age or weight). A patient’s type is not observable a priori to the manufacturer and is only observed upon seeking treatment, possibly through some diagnostic test or physician observation. Only after seeking treatment does the payer observe each patient’s probability of response and chooses to provide treatment to individuals with a probability of response of at least \( L \). Therefore, individuals with \( \pi \in [L, 1] \) will receive treatment, where \( L \) is the

---

\[^{2}\text{A summary of all notation can be found in Table 2.1 in Section 2.8 on page 37.}\]

\[^{3}\text{Alternately } \theta(x) = 0, \ x > 0. \ \text{However, this alternate formulation implies two assumptions: (1) that the base demand, } x, \ \text{is independent of the selling price; and, (2) that the manufacturer does not have the implicit decision to withhold the drug from the market.}\]
payers’s decision parameter and physicians always follow this policy decision, as in Barros (2011). Thus, the manufacturer can influence the size of the possible treatment pool and the payer has discretion over who in that pool will receive treatment. Let $D$ represent the number of treatments provided (demand), formalized as follows:

$$D = m \int_{L}^{1} f(\pi) \, d\pi \quad (2.1)$$

We assume the public payer maximizes the incremental net monetary benefit (NMB) of treatment, consistent with existing literature (e.g., Barros 2011, Antonanzas et al. 2011, Levaggi 2014, Mahjoub et al. 2018, Zhang et al. 2011) and that the manufacturer maximizes profit, $\Pi$. NMB and profit are defined as follows:

$$NMB = m \int_{L}^{1} (\pi b - p) f(\pi) \, d\pi \quad (2.2)$$

$$\Pi = m \int_{L}^{1} (p - c) f(\pi) \, d\pi - \theta(m) \quad (2.3)$$

We assume that the payer and manufacturer have common knowledge of the parameters $c$ and $b$, as well as the distribution $f(\pi)$ and the function $\theta(m)$. Finally, let social welfare, $W$, be the sum of the manufacturer’s profit and the payer’s NMB:

$$\Pi = m \int_{L}^{1} (\pi b - c) f(\pi) \, d\pi - \theta(m) \quad (2.4)$$

We assume that $\theta(m) = (Km^2)/2$, where $K > 0$ represents an exogenous marketing cost parameter. The quadratic form is common when considering the diminishing returns between demand and cost (e.g., Bala and Bhardwaj 2010, Chintagunta and Desiraju 2005, Ma et al. 2013, Narayanan et al. 2004, Zhang and Zaric 2015) and has the desired properties indicated previously. We discuss the robustness of our results to alternate functional forms of $\theta(m)$ in Section 2.6. We assume that $f(\pi)$ is a uniform distribution on the interval $[0, 1]$, consistent with Levaggi (2014), Brekke et al. (2007), and Miraldo (2009).
There are three sequential decisions. First, the price of the new drug, $p$, is set through some pricing process. Second, the payer decides the threshold, $L$, that defines which patients receive treatment. Finally, the manufacturer exerts marketing effort, $m$. This three-stage process captures the effect of prices on demand, via both the payer’s decision of which individuals will receive treatment and the manufacturer’s marketing decision.

### 2.4 Structural Results

We study one benchmark scenario and six pricing and access policies. We identify the equilibrium price, treatment threshold, and marketing effort under each policy and highlight the key features.

#### 2.4.1 First-Best

The first-best scenario, in which a social planner makes all decisions to maximize social welfare, establishes a benchmark for comparison to other policies. The price of the drug is ignored as it is simply a transfer between the payer and manufacturer and does not impact social welfare. Therefore, the social planner only decides the treatment threshold and the level of marketing effort, introducing the drug only if social welfare is positive.

In equilibrium, the social planner will treat all individuals where the expected benefit of treatment, $\pi b$, exceeds the cost of manufacturing, $c$. Additionally, the social planner will conduct marketing only when the marginal benefit from additional patients outweighs the marginal cost to acquire these patients. This simple result reinforces that if the benefit that a drug provides is greater than the manufacturing cost, then it is socially optimal to introduce the drug into the market (i.e., $m > 0$). However, additional marketing beyond this optimal level results in decreased social welfare. These results are formalized in Proposition 2.4.1.

Let $m^{FB}$, $L^{FB}$, $D^{FB}$, and $W^{FB}$ represent the equilibrium level of marketing, the equilibrium treatment threshold, the equilibrium level of demand, and the equilibrium level of social welfare in the first-best scenario.\footnote{A summary of all scenario-specific notation can be found in Table A.1 in Section A.1 on page 131}
Proposition 2.4.1 In a first-best setting, $L^{FB} = \frac{c}{b}$, $m^{FB} = \frac{(b-c)^2}{2bK}$, $D^{FB} = \frac{(b-c)^3}{2b^2K}$, and $W^{FB} = \frac{(b-c)^4}{6b^2K}$.

One important characteristic to note is that the choice of the treatment threshold does not depend on the level of marketing, only the cost to acquire the drug, $c$. This is because the expected value of treatment for an individual, $\pi b$, is independent of the total number of individuals that seek treatment. In contrast, the equilibrium level of marketing does depend on the optimal treatment threshold because the expected marginal benefit of additional patients results from the treatment threshold that is selected. Throughout the remainder of this paper we use the term socially optimal to refer to the results in the first-best scenario.

The results from Proposition 2.4.1 do not specify a price. A more practical scenario may be one where the social planner also sets the price to ensure the manufacturer receives positive profit and the payer receives positive NMB. We find that it is always possible for a social planner to select a price, treatment threshold and level of marketing that achieves optimal social welfare, positive NMB, and positive profit, as formalized in Proposition 2.4.2. Therefore, optimal social welfare need not be at the expense of the payer or manufacturer.

Proposition 2.4.2 If it is socially optimal to introduce a new drug, then $\exists \ p \leq \bar{p}$, s.t. $\forall \ p \in [\underline{p}, \bar{p}]$, $\Pi \geq 0$ and $NMB \geq 0$.

2.4.2 Negotiated Pricing

Under a negotiated pricing policy, the price that the manufacturer and payer agree upon is the Nash bargaining solution, as in Antonanzas et al. (2011). Let $\alpha \in [0, 1]$ represent the manufacturer’s bargaining power, and therefore $1 - \alpha$ represents the payer’s bargaining power. Formally, the equilibrium price, $p^{NP}$, solves the following,

$$p^{NP} = \arg\max_p \left[ (\Pi^*)^\alpha \cdot (NMB^*)^{(1-\alpha)} \right], \quad (2.5)$$

where $NMB^*$ and $\Pi^*$ represent the net monetary benefit and profit, respectively, given the best response to the payer’s decision of $L$ and the best response to the manufacturer’s decision of $m$. We do not include individual rationality constraints for either the payer
or manufacturer because both parties have the decision making power to force demand to zero (i.e., the payer may select \( L = 1 \), or the manufacturer may select \( m = 0 \)). Thus, the equilibrium decisions are always interior solutions and result in \( p \in [c, b] \), therefore guaranteeing marginal benefit to both the payer and manufacturer.

The equilibrium price is increasing with the manufacturer’s negotiating power. Therefore, payers with more negotiating power will receive lower prices, consistent with empirical findings (Roberts et al. 2017). The payer will always select a treatment threshold, \( L^{NP} \), that is at least as high as the first-best treatment threshold. Therefore, the proportion of the patients that seek and then receive treatment in a negotiated setting is weakly smaller than socially optimal. Only if the payer has complete price setting power (i.e., \( \alpha = 0 \)) is the treatment threshold the same as in the first-best setting. However, the treatment threshold is increasing in the manufacturer’s negotiating power, thus decreasing the proportion of people who receive treatment. The manufacturer will always select a level of marketing, \( m^{NP} \), that is strictly less than socially optimal, with the highest level of marketing occurring when \( \alpha = 1 \). These results are formalized in Proposition 2.4.3.

**Proposition 2.4.3** In a negotiated setting,

a) \( p^{NP} = \frac{b + 3c}{4} + \frac{b - c}{4} \), where \( c < \frac{b + 3c}{4} \leq p^{NP} \leq \frac{2(b + c)}{4} < b \) and \( \frac{dp^{NP}}{d\alpha} > 0 \)

b) \( L^{NP} = \frac{c}{b} + \frac{b - c}{3b} \), where \( 1 \leq \frac{1 - L^{FB}}{1 - L^{NP}} \leq \frac{3}{2} \) and \( \frac{dL^{NP}}{d\alpha} > 0 \)

c) \( m^{NP} = \frac{(b - c)^2(3 - \alpha)(1 + \alpha)}{12bK} \), where \( \frac{3}{2} \leq \frac{m^{FB}}{m^{NP}} \leq 2 \) and \( \frac{dm^{NP}}{d\alpha} \geq 0 \).

Given the equilibrium decisions, the total number of treatments provided, \( D^{NP} \), will always be strictly less than socially optimal. Additionally, the behavior of the equilibrium level of demand is non-monotonic with respect to the manufacturer’s negotiating power (Figure 2.1). In the negotiated setting, demand is maximized when \( \alpha = 1/3 \). Let \( D^{NP} \) represent the maximum level of demand. At this level, demand is just more than half of the socially optimal level. Thus, regardless of the distribution of negotiating power, significantly fewer patients will receive treatment than socially optimal. This result is in line with Cachon and Lariviere (2005) who find that when a product’s demand is influenced by costly marketing effort, the manufacturer’s optimal choice of marketing does not result
in supply chain coordination (i.e., optimal social welfare). Furthermore, this result is consistent with the concept of double marginalization (Spengler 1950) where total welfare is sub-optimal when individual stakeholders in a supply chain (i.e., manufacturers and payers) act independently. Formally,

**Corollary 2.4.4** In a negotiated setting, \( D^{NP} = \frac{(b-c)^3(3-\alpha)^2(1+\alpha)}{36b^2K}, \) \( D^{NP} < D^{FB}, \) and \( D^{NP}/D^{FB} = 128/243. \)

In addition to the case where \( \alpha = 1/3, \) there are two special cases to the negotiated scenario. First, the manufacturer may have complete negotiating power (i.e., \( \alpha = 1 \)). For example, pharmaceuticals in Germany, Denmark and Sweden are launched with relatively minor pricing restrictions (Carone et al. 2012). Alternately, the payer may have complete negotiating power (i.e., \( \alpha = 0 \)). In practice, a payer may determine the price of a new drug by demanding discounts as a condition of listing (Husereau et al. 2014). For example, Bristol-Myers Squibb’s drug *nivolumab* (Opdivo) was recently approved in the UK conditional on a discounted price (Hirschler 2017). We denote these special cases as the **Open Pricing** and **Controlled Pricing** scenarios, respectively.

Under an open pricing policy, it may seem that the manufacturer will always select the highest possible price (i.e., \( p = b \)) to maximize the marginal benefit that it receives from each treatment. However, the payer would respond by restricting the number of treat-
ments provided when prices are high by selecting a high treatment threshold, $L$. Therefore, when the manufacturer selects the price, it balances the trade-off between higher margins or higher volume. In a similar manner, under a controlled pricing scenario, it may seem that the payer will always select the lowest possible price (i.e., $p = c$) to maximize the marginal benefit it receives from each treatment. However, the manufacturer would respond by conducting little marketing effort resulting in a small number of patients treated. Therefore, when the payer selects the price, it also balances the trade-off between higher margins or higher volumes. In these special cases, equilibrium demand is higher when the payer has full negotiating power ($\alpha = 0$, controlled pricing) when compared to when the manufacturer has full negotiating power ($\alpha = 1$, open pricing), as illustrated in Figure 2.1 by the two horizontal grey dashed lines.

**Proposition 2.4.5** When comparing an open pricing policy and controlled pricing policy,

\begin{align*}
& a) \ p^{CP} \leq p^{NP} \leq p^{OP} \\
& b) \ L^{FB} = L^{CP} \leq L^{NP} \leq L^{OP} \\
& c) \ m^{CP} \leq m^{NP} \leq m^{OP} < m^{FB} \\
& d) \ D^{OP} < D^{CP} < D^{NP} < D^{FB}
\end{align*}

### 2.4.3 Listing Process

Through a listing process, the manufacturer sets the price for a new drug and requests that the drug be listed on a public formulary. Then, either the drug is listed or denied by a decision making process that may not be transparent (Carone et al. 2012). For example, to our knowledge, no country explicitly specifies a single cost-effectiveness threshold for evaluating health technology assessments. As in other work (e.g., Gavious et al. 2014, Levaggi 2014, Zaric and O’Brien 2005), we capture the uncertainty in the decision making process by defining the probability that the drug is listed as a linearly decreasing function of price, up to some maximum price $p^{\text{max}}$. We assume that $p^{\text{max}} = b$, ensuring that a drug is denied listing if the price is higher than the monetary benefit that it provides when
successful. Let $Pr(approved) = (b - p)/b$ if $p \in [0, b]$ and $Pr(approved) = 0$ if $p > b$, as in Levaggi (2014). If a drug is approved, then the payer selects the treatment threshold and the manufacturer selects the level of marketing.

We find that the listing process results in equilibrium decisions that are intermediate when compared to the controlled pricing and open pricing scenarios. The equilibrium price, $p^{LP}$, treatment threshold, $L^{LP}$, and level of marketing, $m^{LP}$, are all bounded by the equilibrium decisions in the controlled pricing and open pricing scenarios. For notational convenience we use the ‘hat’ symbol, “$\hat{}$”, to define the equilibrium results of the listing process in expectation with respect to the probability of listing. For example, $\hat{D}^{LP}$ represents the expected demand, where $D^{LP}$ represents the demand conditional upon the drug being approved. We find that the conditional demand under a listing process is higher than it is in both the controlled pricing and open pricing scenarios. However, the expected demand is lower. These results are formalized in Proposition 2.4.6.

**Proposition 2.4.6** Under a listing process,

a) $p^{LP} = \frac{2b+3c}{5}$, where $p^{CP} < p^{LP} < p^{OP}$

b) $L^{LP} = \frac{b+4c}{5b}$, where $L^{FB} = L^{CP} < L^{LP} < L^{OP}$

c) $m^{LP} = \frac{8(b-c)^2}{25bK}$, where $m^{CP} < m^{LP} < m^{OP} < m^{FB}$

d) $\hat{D}^{LP} < D^{OP} < D^{CP} < D^{LP} < D^{NP} < D^{FB}$

**2.4.4 Risk-Sharing**

Under a risk-sharing arrangement, the manufacturer reimburses a portion of the price, $r$, to the payer for each unsuccessful treatment. Risk-sharing arrangements have been implemented in many countries, including the UK, Canada, and the US (Adamski et al. 2010).

Recall that $\pi$ represents an individual’s probability of response to the new treatment and therefore $1 - \pi$ represents the probability that an individual’s treatment will be unsuccessful. Under a risk-sharing arrangement, the manufacturer’s profit and the payer’s NMB are

\[ \hat{D}^{LP} = D^{LP} \cdot Pr(approved) + 0 \cdot (1 - Pr(approved)), \]  

where $\hat{D}^{LP}$ represents the expected demand at the time that the manufacturer makes the pricing decision, before the listing process has occurred.
2.4. Structural Results

defined as follows:

$$\Pi^R = m \int_{L}^{1} \left[ (p - c) - (1 - \pi)(rp) \right] f(\pi) \, d\pi - \frac{Km^2}{2} \quad (2.6)$$

$$NMB^R = m \int_{L}^{1} \left[ (\pi b - p) + (1 - \pi)(rp) \right] f(\pi) \, d\pi \quad (2.7)$$

First, we consider the general case where the price is selected by the manufacturer. Following the three-stage process, the manufacturer selects the price, the payer selects the treatment threshold, and the manufacturer selects the level of marketing effort. The total reimbursement that the manufacturer pays results both from the total number of people who seek treatment, \( m \), and the type of patients that are treated as decided by the payer via \( L \), where both of these decisions are influenced by the price. In the special case where the rebate rate is equal to one and if the manufacturer selects \( p = b \), then the payer is guaranteed to receive zero NMB. Therefore, the payer may select any threshold on the interval \( L \in [(1 - 2(b - c)/b)^+, 1] \) (where \((x)^+ = \max\{x, 0\}\)) , and the manufacturer will select a non-negative level of marketing\(^6\). In this case, we assume that the payer has the secondary objective to maximize social welfare, and therefore the equilibrium treatment threshold is socially optimal, \( L^{RS} = L^{FB} \). As a result, the equilibrium level of marketing is also socially optimal, \( m^{RS} = m^{FB} \) and therefore optimal social welfare is achieved. However, all benefit is captured by the manufacturer.

In the general case (i.e., not \( r = 1 \ & p = b \)), we find that the manufacturer will only participate when the payer is selective regarding which individuals receive treatment (i.e., the manufacturer will only participate if \( L \geq L^{TM} \)). Thus, even though the payer has the incentive to choose a lower treatment threshold when compared to any non-risk sharing scenario, (i.e., because it receives reimbursement for unsuccessful treatments) the payer cannot select an arbitrarily low threshold while still ensuring that the manufacturer will participate. Similarly, the payer will only participate when the manufacturer selects a price

\(^6\)For details, see the proof in Section A.2 on page 132

\(^7\)The socially optimal treatment threshold, \( L^{FB} = c/b \), is always within the interval \([(1 - 2(b - c)/b)^+, 1] \) which can be observed by re-arranging the interval as \([(c/b - (b - c)/b)^+, 1] \)
below some maximum. Otherwise, the payer will select \( L = 1 \) and no treatments will be provided. Therefore, as in all policies previously considered, the manufacturer and payer balance higher margins or higher volumes when making their respective decisions. These results are formalized in Proposition 2.4.7.

Let \( m(L, p) \) and \( L(p) \) represent the manufacturer’s best response with respect to marketing and the payer’s best response with respect to the treatment threshold, respectively. Let \( L_T \) represent a treatment level defined as,

\[
L_T = 1 - \beta - \sqrt{\beta^2 - 2p} / \sqrt{\beta^2 - 32(p-c)(b-p)(b-pr)(pr)) / 4pr(b-pr)} \quad (2.8)
\]

where \( \beta = 3((p-c)(b-pr) + (b-p)(pr)) \). We evaluate the risk-sharing arrangement for \( 0 < r < 2 \).

**Proposition 2.4.7** Under a risk-sharing arrangement,

\[
\begin{align*}
\text{a)} \quad & m(p, L) = \begin{cases} 
 \frac{(p-c)(1-L)}{K} - \frac{pr(1-L)^2}{2K}, & \text{if } 1 - \frac{2(p-c)}{pr} = L^{TM} \leq L \leq 1 \\
\quad 0, & \text{otherwise}
\end{cases} \\
\text{b)} \quad & L(p) = \begin{cases} 
 (L^T)^+, & \text{if } (r < 1 \ AND \ p \leq b) \ OR \ (r \leq 1 \ AND \ p < b) \\
(L^T)^+, & \text{if } (1 < r < 2) \ AND \\
((b \geq 2c \ AND \ p \leq \frac{b}{2-r}) \ OR \ (b \leq 2c \ AND \ p \leq \frac{bc}{b-br+c+cr})) \\
\frac{c}{b}, & \text{if } r = 1 \ AND \ p = b \\
1, & \text{otherwise}
\end{cases}
\end{align*}
\]

As previously discussed, in the special case where \( r = 1 \), we find that the risk-sharing arrangement results in a socially optimal outcome. However, the manufacturer captures all value.

**Proposition 2.4.8** Under a risk-sharing arrangement, if \( r = 1 \) and the payer has the secondary objective to maximize social welfare, then, \( p^{RS} = b \), \( L^{RS} = c/b = L^{FB} \), \( m^{RS} = m^{FB} = (b-c)^4/(8b^2K) \), and therefore \( \Pi^{RS} = W^{RS} = W^{FB} \) and \( NMB^{RS} = 0 \).

We also consider a scenario where the average price that is paid is equal to the average benefit that is received, which we call *value-based pricing with risk-sharing*. For
example, in 2014 the UK proposed launching obligatory value-based pricing on new drugs, although this has not been implemented (Carone et al. 2012). In this special case, the treatment threshold is specified as a function of price, \( L^{VR} = \frac{2p(1-r)}{b-pr} - 1 \), such that \( NMB = 0 \) (i.e., the average price paid equals average benefit received). Following the same sequence as previously, the manufacturer first selects a price, then the treatment threshold is set to ensure price equals benefit, and finally the manufacturer selects the level of marketing.

Consistent with Levaggi (2014), we find that a value-based pricing with risk-sharing scenario always results in a socially optimal outcome. Furthermore, while the price that the manufacturer sets, \( p^{VR} \), is a function of the rebate rate, the average price that is paid by the payer, after any rebates, is always constant. Thus, the equilibrium levels of profit (\( \Pi^{VR} \)), NMB (\( NMB^{VR} \)), and social welfare (\( W^{VR} \)) are always the same, regardless of the rebate rate. As in the special case in the general risk-sharing scenario when \( r = 1 \), the manufacturer captures all value. Let \( L^{VR} \) and \( m^{VR} \) represent the equilibrium treatment threshold and level of marketing, respectively, and let \( p^{VR} \) represent the average price that is paid, after any rebates. The results of the value-based pricing with risk-sharing scenario are formalized in Proposition 2.4.9.

**Proposition 2.4.9** Under a value-based pricing with risk-sharing arrangement,

\[
a) \quad p^{VR} = \frac{b(b+c)}{b(2-r)+cr}, \quad L^{VR} = \frac{c}{b}, \quad \text{and} \quad m^{VR} = \frac{(b-c)^2}{2bK} \\
b) \quad \bar{p}^{VR} = \frac{b+c}{2} \\
c) \quad \Pi^{VR} = W^{VR} = W^{FB} \quad \text{and} \quad NMB^{VR} = 0
\]

Note that \( L^{VR} = L^{FB} \), \( m^{VR} = m^{FB} \), and \( p^{VR} = \bar{p} \), where \( \bar{p} \) is the maximum price such that a socially optimal outcome can occur (Proposition 2.4.2). Also, although the treatment threshold is a function of price, substituting the equilibrium price, \( p^{VR} = \frac{b(b+c)}{b(2-r)+cr} \), into \( L^{VR} = \frac{2p(1-r)}{(b-pr)-1} \) always yields the equilibrium treatment threshold \( L^{VR} = \frac{c}{b} \). Finally, the equilibrium price specifies a relationship between the rebate rate and the optimal price. As long as this relationship is maintained, a value-based pricing with risk-sharing arrangement will always achieve socially optimal outcomes.
2.5 Comparison of Policies

2.5.1 Individual Policy Comparison

Health care policy makers balance many objectives and therefore may value different features of pricing and access policies (Le Polain et al., 2011). One consideration is access to new drugs. We consider access to mean that a socially beneficial drug is introduced to the market. We find that the listing process may result in limited access. Specifically, under a listing process there is always some probability that the drug will not be approved (Figure 2.2). Therefore, this arrangement is *access inefficient*. In contrast, all other policies result in access to socially desirable drugs.

A policy maker may also be interested in the total benefit that a drug can provide in the population. The total benefit depends on the number of patients that receive treatment and therefore the total demand is an important consideration. We distinguish the two components of demand: marketing (i.e., total number of patients that seek treatment) and the treatment threshold (i.e., which patients receive treatment). We find that the negotiated pricing, open pricing, controlled pricing and listing process scenarios always result in a sub-optimal level of marketing when compared to the first best scenario (Figure 2.3). The risk-sharing arrangement also results in a sub-optimal level of marketing except for the special case when $r = 1$ or under a value-based pricing with risk-sharing arrangement where

![Equilibrium probability of listing under a listing process.](image)
2.5. Comparison of Policies

Figure 2.3: Equilibrium marketing effort as a percent of the first-best level of marketing effort \((c = 0.1 \times b; K = 1)\). (a) With respect to the rebate rate, \(r (\alpha = 1/3)\). (b) With respect to the manufacturer’s negotiating power, \(\alpha (r = 0.5)\). The value-based pricing with risk-sharing scenario always results in a first-best level of marketing effort and is omitted.

Additionally, the negotiated pricing, open pricing and listing process scenarios always result in a higher treatment threshold than is socially optimal and therefore these policies are **negative treatment inefficient** (i.e., a restrictive treatment threshold) (Figure 2.4). The risk-sharing arrangement may result in either a higher or lower treatment threshold than is socially optimal and therefore this policy may be either negative treatment inefficient or **positive treatment inefficient** (i.e., a relaxed treatment threshold), respectively. Significantly, positive treatment inefficiency can only occur for rebate rates larger than one. The controlled pricing scenario and the value-based pricing with risk-sharing scenario always result in a socially optimal treatment threshold and are therefore **treatment efficient**.

Except for the special cases of value-based pricing with risk-sharing and risk-sharing where \(r = 1\), the combination of marketing inefficiency and/or treatment inefficiency mean that all policies that we consider result in a sub-optimal level of demand.

The equilibrium decisions produce various levels of NMB and profit (Figure 2.5 and Figure 2.6, respectively). The payer always receives the highest NMB in a controlled pricing scenario, when it has complete price negotiating power. However, the manufacturer
Figure 2.4: Equilibrium treatment threshold \((c = 0.1 \times b; K = 1)\). (a) With respect to the rebate rate, \(r (\alpha = 1/3)\). (b) With respect to the manufacturer’s negotiating power, \(\alpha (r = 0.5)\). The equilibrium treatment threshold in the first-best scenario is the same as in the controlled pricing scenario. The controlled pricing scenario and the value-based pricing with risk-sharing scenario always results in a first-best treatment threshold and are omitted.

does not receive the highest profit when it has complete price negotiating power (open pricing). Under a value-based pricing with risk-sharing scenario, the manufacturer always receives the first-best level of social welfare. When comparing the non-socially-optimal scenarios, the manufacturer always receives the highest profits under a risk-sharing arrangement. This is because, under risk-sharing, the payer will treat more individuals (i.e., select a lower treatment threshold) due to the reimbursement for unsuccessful treatments when compared to the open pricing scenario where there is no reimbursement. Intuitively, the manufacturer and the payer each receive higher profit and NMB, respectively, when their respective negotiating power increases (solid black line in Figure 2.5b and Figure 2.6b). The listing process results in lower expected profits and lower expected NMB when compared with any negotiated scenario (i.e., \(\forall \alpha \in [0, 1]\)). Therefore, poor transparency in the approval process results in inferior outcomes from a social welfare perspective as well.
2.5. **Comparison of Policies**

Figure 2.5: Equilibrium net monetary benefit as a percent of the first-best level of social welfare \((c = 0.1 \times b; K = 1)\). (a) With respect to the rebate rate, \(r\) \((\alpha = 1/3)\). (b) With respect to the manufacturer’s negotiating power, \(\alpha\) \((r = 0.5)\). The value-based pricing with risk-sharing scenario always results in a zero net monetary benefit and is omitted.

Figure 2.6: Equilibrium profit as a percent of the first-best level of social welfare \((c = 0.1 \times b; K = 1)\). (a) With respect to the rebate rate, \(r\) \((\alpha = 1/3)\). (b) With respect to the manufacturer’s negotiating power, \(\alpha\) \((r = 0.5)\).
2.5.2 Social Welfare Comparison

From a policy maker’s perspective, a policy that results in the highest social welfare may be desirable. Because the value-based pricing with risk-sharing scenario always results in a first-best level of social welfare, we exclude it from this comparative analysis. Figure 2.7 illustrates the pricing and access policies that result in the highest level of social welfare with respect to the rebate rate, negotiating power, and manufacturing cost. For drugs with lower manufacturing costs (below \( c \approx 0.25 \) in Figure 2.7a), a risk sharing arrangement is preferred by the social planner when the rebate rate is high and the negotiated scenario is preferred when the rebate rate is low. This occurs because a risk-sharing arrangement becomes less desirable for low rebate rates. Recall that a risk-sharing arrangement is socially optimal when \( r = 1 \) (Proposition 2.4.9) and the risk-sharing arrangement is equivalent to the open pricing scenario when \( r = 0 \) (\( \alpha = 1 \)). Because the negotiated pricing scenario results in higher social welfare than the open pricing scenario there always exists an indifference point between the risk-sharing arrangement and the negotiated pricing scenario for some \( r \in [0, 1] \), \( \forall \alpha \in [0, 1] \) Figure 2.7a & b)\(^8\) For drugs with higher manufacturing costs (above \( c \approx 0.25 \) in Figure 2.7a), a risk-sharing arrangement is preferred by the social planner for intermediate rebate rates (from slightly above one to slightly below one) and the negotiated scenario is preferred for high and low rebate rates. If the rebate rate is large (i.e., \( r \gg 1 \)), then the manufacturer will conduct less marketing under a risk sharing arrangement because ineffective treatments are costly (i.e., high rebate) and therefore the risk-sharing arrangement results in a socially suboptimal number of treatments. Like the scenario when the manufacturing cost is small, if the rebate rate is low (i.e., \( r \ll 1 \)) then the risk-sharing arrangement converges to the open pricing scenario which is socially inferior to the negotiated pricing scenario.

The unique shape of the policy graph in Figure 2.7a for \( r \approx 1.05 \) is the result of small differences between the levels of social welfare under the negotiated pricing scenario and a risk-sharing arrangement. This occurs because the equilibrium price is discontinuous under the risk-sharing arrangement when \( r > 1 \).\(^9\) For low manufacturing costs (below

\[ W^{RS}_{(r=0)} = W^{OP} \leq W^{NP} \leq W^{RS}_{(r=1)} = W^{FB} \]

\(^8\)The equilibrium price under the risk-sharing arrangement is discontinuous for \( r > 1 \). This occurs because
2.5. **Comparison of Policies**

![Figure 2.7: Policy preferences from a social welfare perspective (b = 1; K = 1). (a) With respect to manufacturing cost and rebate rate (\( \alpha = 1/3 \)). (b) With respect to the manufacturer’s negotiating power and rebate rate (c = 0.1 \times b).](image)

When manufacturing costs are low, the manufacturer will select a price that binds the condition \( p = b/(2 - r) > b \), resulting in \( L = 0 \) (Proposition 2.4.7), and when manufacturing costs are high the manufacturer will select an internally optimal price that satisfies \( p < b/(2 - r) \) or \( p < bc/(b - br + cr) \), resulting in \( L = L^T \in (0, 1] \). While this results in a piecewise (although continuous) level of profit, it results in a discontinuous level of NMB and therefore social welfare.

(c \approx 0.25 in Figure 2.7a), the manufacturer will select a price that is slightly higher than the benefit that the drug provides, corresponding to the condition from Proposition 2.4.7 where \( p = b/(2 - r) \). An equilibrium price that is higher than the benefit that the drug provides is possible because the payer receives a rebate greater than 100% for unsuccessful treatments. However, for higher manufacturing costs (above \( c \approx 0.25 \) in Figure 2.7a), the equilibrium price is a non-boundary solution and may be greater than or less than \( b \). This results in either the negotiated pricing policy or the risk-sharing policy having higher social welfare.

Overall, only the value-based pricing with risk-sharing policy is universally optimal from a social welfare perspective. Thus, a social planner should consider the characteristics of the drug (i.e., manufacturing cost), the characteristics of the economic environment (i.e., negotiating power of manufacturers and payers), and the characteristics of a potential risk-sharing arrangement (i.e., rebate rate) when selecting a non-value-based pricing and access policy.
2.5.3 Comparison of a Listing Process and a Value-Based Risk-Sharing Arrangement

In a previous analysis of pricing and access policies without marketing, [Levaggi (2014)] shows that a payer and manufacturer may mutually prefer a value-based risk-sharing arrangement when compared with a listing process. Because the value-based pricing with risk-sharing scenario always results in zero NMB, the payer will never prefer this arrangement.

When we compare a risk-sharing arrangement (non-value-based) and a listing process, we find that the manufacturer always prefers a risk-sharing arrangement to the listing process, while the payer may prefer either arrangement (recall Figure 2.5a and Figure 2.6a). Figure 2.8 illustrates the region where both the manufacturer and the payer prefer the risk-sharing arrangement over the listing process. We find that for any manufacturing cost, there always exists a rebate rate where the manufacturer and the payer mutually prefer a risk-sharing arrangement. However, we find three results that are different than in previous literature. First, when the manufacturing cost is low (below $c \approx 0.3$ in Figure 2.8) we find that the payer will prefer the listing process when the rebate rate is high. Although the payer receives larger reimbursements when rebate rates are high, the manufacturer’s choice of price is increasing in the rebate rate. Therefore, the marginal benefit from each patient deteriorates as the rebate rate increases, thus reducing the payer’s NMB. Second, we find that as the cost to manufacture a drug increases, the range of rebate rates such that the manufacturer and the payer will mutually prefer the risk sharing arrangement increases. Again, this occurs because of the manufacturer’s choice of price. In general, the equilibrium price is increasing, for all arrangements, with respect to the manufacturing cost. However, this increase most negatively impacts the expected NMB under the listing process because high prices reduce both the payer’s marginal benefit of treatment and reduces the probability of listing. These two features make the listing process less desirable to the payer for drugs that are costly to manufacture. Finally, the payer never prefers a risk-sharing arrangement when $r = 1$ because $NMB = 0$ (Proposition 2.4.9). The unique behavior in Figure 2.8 for $r \approx 1.05$ and $c \in [0.3, 0.4]$ occurs due to small differences in the NMB between the
2.5. Comparison of Policies

Figure 2.8: Policy space where the manufacturer and payer agree on the preferred pricing and access policy when comparing the risk-sharing arrangement with the listing process. The white region represents where the payer and manufacturer do not mutually agree. The shaded region corresponds to where both players prefer the risk-sharing arrangement ($b = 1; K = 1$).

Overall, it may be easier to implement a risk-sharing arrangement for a drug that has a high manufacturing cost compared to a drug with a low manufacturing cost. This is because the risk-sharing contract is mutually preferred over a large range of rebate rates for drugs that have high manufacturing costs. This range decreases for drugs that have low manufacturing costs (i.e., risk-sharing is only preferred for $r \in [0, 0.6]$ when the manufacturing cost is 10% of the benefit that the drug provides). To provide some context, biologic drugs are often considered ‘expensive’ to manufacture. However, the manufacturing cost relative to the monetary benefit that a drug provides can be quite small. For example, the biologic drug tisagenlecleucel (Kymriah; Novartis), that costs approximately $20,000 USD per infusion to manufacture (Kleutghen et al. 2018, The Oncologist Journal [Producer] 2013) and has a selling price of $475,000 (US Food and Drug Administration 2017), was recently approved in the US to treat pediatric and young patients with acute lymphoblastic leukemia. If the monetary benefit that the drug provides is equal to its price, then the manufacturing costs is merely 4.2% of the benefit it provides (corresponding to $c = 0.042$ in Figure 2.8). The small-molecule drugs sofosbuvir, daclatasvir and simeprevir that treat hepatitis C, have
an average selling price of $72,000 (Chhatwal et al., 2015, Najafzadeh et al., 2015) and are estimated to have an average manufacturing cost of $54 for a 12-week treatment. If the monetary benefit that these drugs provide is equal to its price, then the manufacturing costs is only 0.08% of the benefit it provides (corresponding to \( c = 0.0008 \) in Figure 2.8). Thus, the range of rebate rates such that a risk-sharing arrangement is mutually preferred in practice is likely to be relatively small and this may be a contributing factor to why these policies have been difficult to implement (Neumann et al., 2011, Neumann, 2013).

### 2.6 Robustness

In the previous sections, we assume \( \theta(m) = Km^2/2 \). Using numerical analysis, we test two alternate functional forms for the cost of marketing effort: exponential \( \theta(m) = e^{\lambda m} - 1 \) and asymptotic \( \theta(m) = \gamma(m/(\tau - m)) \). Each of these functional forms share the three properties outlined in Section 2.3 \( \theta(0) = 0; \theta'(m) > 0; \) and, \( \theta''(m) > 0 \). Qualitatively, there is no difference in the results.

### 2.7 Discussion

We study the impact of pharmaceutical marketing on six practical pricing and access policies. We show that each policy may result in various forms of inefficiencies and we demonstrate that the value-based pricing with risk-sharing policy is preferred by the manufacturer and from a social-welfare perspective while there is no universally preferred policy from the payer’s perspective. Our results demonstrate meaningful differences from the findings of the existing literature on risk-sharing arrangements when marketing is considered. Therefore, our work establishes the importance of incorporating pharmaceutical marketing into pricing and access policy decisions.

We find that a listing process may result in sub-optimal access to new medicines (access inefficiency). We find that a negotiated pricing policy, a controlled pricing policy, an open pricing policy, a listing process and a risk-sharing arrangement all result in a suboptimal volume of patients seeking treatment (marketing inefficiency) and a suboptimal treatment
2.7. Discussion

threshold (treatment inefficiency). Consistent with previous literature ([Levaggi 2014]), we find that a value-based pricing with risk-sharing policy always results in a first-best level of social welfare.

From a social welfare perspective, a value-based pricing with risk-sharing policy is always preferred. From the non-value-based scenarios, we find that a negotiated pricing policy or a risk-sharing arrangement may each result in the highest level of social welfare depending on a drug’s manufacturing cost, the manufacturer’s negotiating power, and the rebate rate. Significantly we find that the listing process never results in the highest level of expected social welfare.

There are limitations to our work. First, we assume collective information about the benefits to patients, $b$, whereas knowledge of a drug’s benefit may be asymmetric between the manufacturer and payer. However, we have demonstrated that even under the assumption of shared information, all pricing and access policies suffer from some form of inefficiency. If asymmetric information is included into a similar analysis, then the social desirability of each policy cannot increase. An extension to our work that incorporates asymmetric information may provide further insight into the magnitude of the misalignment of incentives across policies and possibly change policy preferences. A second limitation is the assumption that the type of individuals (i.e., defined by their probability of response, $\pi$) who are reached through marketing cannot be controlled. Specifically, we consider the scenario where the manufacturer cannot observe an individual’s probability of response a priori. While this approach may accurately reflect the scenario where an individual’s probability of response is only observed only after medical testing (e.g., a genomic diagnostic test), it may not capture the nuances of a scenario where an individual’s probability of response may be observable prior to seeking medical consultation (e.g., if treatment success is correlated with observable characteristics such as age or level of physical activity). In particular, we believe that a manufacturer may be motivated to conduct targeted marketing towards patients that have a higher probability or response under a risk-sharing arrangement because the manufacturer receives greater benefit from successful treatments. However, it is not clear what, if any, impact this would have on prices or the treatment threshold and therefore the net result of targeted marketing is uncertain. A future analysis that includes
targeted marketing may provide insight into a manufacturer’s incentives to target patients with high or low probabilities of response.

We also recognize several paths for future research. First, we have not studied the rebate rate as a decision parameter. While our analysis captures the manufacturer’s, payer’s and social welfare preferences with respect to the rebate rate only if the rebate rate was the first decision to be made, it may be interesting to study various scenarios with respect to the decision of the rebate rate (e.g., the timing and different decision makers). Second, we have captured social welfare as the summation of the payer’s and manufacturer’s objectives. However, it may be that a social planner values these objectives with different weights. While we have implicitly studied the three cases where a social planner only values NMB, only values profit, or equally values NMB and profit, a further analysis of social welfare preferences may identify different policy preferences. Finally, we study the scenario where the manufacturer may influence the size of the market. However, a future study that considers the payer’s ability to influence the size of the market may result in some interesting insights as to when a payer may benefit from promoting a treatment program.
### 2.8 Summary of Notation to Essay 1

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Decision Variables</strong></td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>Unit price of treatment</td>
</tr>
<tr>
<td>$L$</td>
<td>Treatment threshold (payer’s decision)</td>
</tr>
<tr>
<td>$m$</td>
<td>Number of individuals that seek treatment because of marketing effort (manufacturer’s decision)</td>
</tr>
<tr>
<td><strong>Objective Functions and Outcomes</strong></td>
<td></td>
</tr>
<tr>
<td>$NMB$</td>
<td>Net monetary benefit (payer’s objective)</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>Profit (manufacturer’s objective)</td>
</tr>
<tr>
<td>$W$</td>
<td>Social welfare</td>
</tr>
<tr>
<td>$D$</td>
<td>Demand</td>
</tr>
<tr>
<td><strong>Modelling Parameters</strong></td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>Monetary benefit of a successful treatment</td>
</tr>
<tr>
<td>$c$</td>
<td>Unit cost of manufacturing</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Individual patient’s probability of successfully responding to treatment</td>
</tr>
<tr>
<td>$K$</td>
<td>Marketing cost coefficient</td>
</tr>
<tr>
<td>$\theta(m)$</td>
<td>Marketing cost required for $m$ individuals to seek treatment</td>
</tr>
<tr>
<td>$f(\pi)$</td>
<td>Distribution of individuals’ probabilities of success for those who seek treatment</td>
</tr>
</tbody>
</table>

Table 2.1: Summary of notation.
2.9 Bibliography


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Park SY, Han E, Kim J, Lee EK (2016) Factors influencing the difference between forecasted and actual drug sales volumes under the price-volume agreement in South Korea. *Health Policy* 120(8):867–874, ISSN 0168-8510, URL http://dx.doi.org/10.1016/j.healthpol.2016.06.004.


Chapter 3

Essay 2: Coordinating Multi-Payer Health Care Systems

3.1 Introduction

Multi-payer health care systems, where different insurers are responsible for health care coverage over the life of an individual or when different insurers are responsible for different aspects of an individual’s health care, are common. In the US there is an almost universal shift of health care coverage from private to publicly funded coverage at age 65; in Canada there is a similarly timed shift from privately to publicly funded prescription drug coverage at age 65 (Cebul et al. 2008, Elhauge 2010). When patients are covered by multiple health payers over their lifespan, a payer may not expect to capture the long-term benefits of preventive care or investments in treatment for chronic disease (e.g., reduced future costs). As a result, fragmented health insurance contributes to inadequate investment in the provision of non-acute health care (Helland and Klick 2010).

Deficient investments in long-term health care can result in progression to worse and more expensive health states and in higher costs from preventable disease-related complications (Avraham and Camara 2007, Cebul et al. 2008, Fang and Gavazza 2011). Ultimately, the final payer, typically a publicly funded payer, may bear the additional cost of expensive delayed treatments because of the insufficient care from a previous health care payer. Fragmented insurance appears to exacerbate the challenge of accessing appropriate care with
empirical studies finding correlations between high employee turnover (a proxy for insurance turnover) and low utilization of preventive services (Herring 2010, Fang and Gavazza 2011). Recognizing that there are poor incentives for payers to reimburse preventive and chronic care, regulators have mandated coverage for some disease management and preventive health services (Bitler and Carpenter 2016). However, mandates that do not ensure that the incentives of payers align with government objectives only inspire programs that satisfy the minimum legal requirements (Avraham and Camara 2007, Cebul et al. 2008).

For some particularly expensive treatments with long payback periods, such as bariatric surgery and treatment for the hepatitis C virus (HCV), health care payers impose barriers to access (Barua et al. 2015, Phelan 2018) or simply deny treatment (Avraham and Camara 2007, Marcus et al. 2018). Three studies of insurance claims data find that Medicaid (generally, covering individuals under age 65 in the US) denies reimbursement for physician-prescribed HCV treatment for 35% to 46% of patients in contrast to Medicare (generally, covering individuals at least 65-years old) which denies between 2.5% and 14% of reimbursement requests (Re et al. 2016, Younossi et al. 2016, Gowda et al. 2018). While patient age is not specifically listed as a barrier to access, reimbursement may be contingent on the patient overcoming time-consuming obstacles that delay treatment and, for patients who are near age 65, may delay care until after the patient changes insurance providers. During treatment delays, a patient’s health can deteriorate. Therefore, it is prudent to consider coordinating mechanisms that promote optimal health care treatment across a patient’s entire lifespan, increasing patient health and reducing lifetime health care costs.

In this paper, we seek to establish analytic and practical insights into the treatment and access inefficiencies that result from multiple health care payers over a patient’s lifespan and to identify and characterize a coordinating contract that provides economic incentives for all payers to provide social welfare maximizing access to treatment. We consider a progressive health condition with a one-time costly intervention that improves future health. As a result of the high cost, the intervention is only provided if reimbursed by the patient’s health insurance payer. We formulate the problem as a Markov decision process (MDP) to capture a payer’s repeated intervention decision while the patient’s health state, described using two discrete states (age and disease severity), evolves over time. We partition the
optimization problem using a threshold patient age (e.g., age 65), known to all parties, that defines when the patient transitions from one payer to the other. We use a game-theoretic approach to identify a coordinating mechanism between the two payers and numerically compare the optimal solutions of the coordinated and uncoordinated multi-payer problem for the case study of access to HCV treatment.

We provide several meaningful contributions. From a methodological perspective, we develop a multi-decision-maker MDP model that captures the real-world complexities of the impact of each decision maker’s choices on the other decision maker’s outcomes. Using this general multi-decision-maker framework, we prove the characteristics of the equilibrium outcomes and optimal treatment policies in single- and multi-payer systems. Our formulation is flexible in that it can include endogenous coordinating mechanisms that link the payers’ decision making processes and provides tractable insights into the characteristics of these coordinating contracts. Extending the extensive single-decision-maker MDP literature, we provide valuable insights into the impact of decision fragmentation on patients, payers, and total welfare. Finally, we directly apply our decision making framework to a case study to demonstrate the robustness of our analytical results, illustrate the effects, and quantify the impact of fragmentation (e.g., unnecessary deaths).

From an applied perspective, we demonstrate features of the disease process and treatment outcomes that magnify the consequences of fragmentation and inefficient treatment policies. When compared to the literature that studies health care decision making using a lifetime horizon (centralized decision making), we highlight the important difference between the optimal treatment policy from a patient’s perspective and the reimbursement policy that may be enforced by a health care payer. Specifically, we prove that the optimal treatment policy in a fragmented system will target a subset of patients when compared to a coordinated system, and therefore that fragmentation leads to under-treatment. We also prove the conditions such that treatment gaps will exist for patients in the periods immediately prior to switching between payers. This occurs because the next-to-last payer has a decreasing incentive to provide treatment as a patient nears the age of transfer. Furthermore, we provide both analytical and applied insights into the health care problem of stakeholder coordination (i.e., between payers) when compared to the existing body of literature on
inter-stakeholder coordination (e.g., patients and payers, or physicians and hospitals). We prove that the cost to coordinate the fragmented system is bounded by the cost of treatment, and therefore a simple cost sharing mechanism can coordinate the system. We prove that the characteristics of this cost sharing mechanism adopts the structure of the one-step benefit function, as defined in [Oh and Özer (2016)].

In the broader context of multi-agent stochastic decision making, our work addresses the issue of coherence (the desire to work together) in contrast to the majority of multi-agent MDP literature that addresses the issue of competence (the ability to work together). We show that given a simple cost sharing mechanism, payers will work together to coordinate their health care decision making.

3.2 Literature Review

Health Care Decision Making

MDPs have been widely applied to identify the optimal time to initiate treatment, generally, with the objective of maximizing patient health (reviewed in [Alagoz et al. (2010)]), but not necessarily health-economic trade-offs over a lifetime horizon as is recommended for health technology investment decision making ([Neumann et al. 2016], [Canadian Agency for Drugs and Technologies in Health 2017]). For example, [Alagoz et al. (2004, 2007a,b)] identify the optimal timing of liver transplants from both living and cadaveric donors. [Shechter et al. (2008)] study the optimal timing to initiate of HIV treatment. [Chhatwal et al. (2010)] and [Alagoz et al. (2013)] study the optimal timing for breast cancer biopsies and breast cancer screening, respectively. [Ayer et al. (2015)] use a partially observable MDP framework to study the impact of uncertainty in adherence to the optimal timing of breast cancer screening. These studies consider the optimal timing of treatment using a lifetime horizon and assume that the patient will select treatment at the optimal time for their health. Our work extends the optimal timing of treatment literature by incorporating the additional real-world complexity of multiple payers with different decision making horizons and by incorporating endogenous interactions between the payers.
Multi-agent Stochastic Processes

Stochastic problems with multiple decision makers, sometimes called multi-agent MDPs, are prevalent in artificial intelligence research but, to our knowledge, have not been applied to health policy management problems. Often, these problems are formulated with multiple independent agents (e.g., swarm robots) with a common goal (Wolpert and Tumer 2002, Brambilla et al. 2013). Decentralized decision making towards a common objective has been studied in the context of robot soccer (Coradeschi et al. 2000), distributed control of a power grid (Schneider et al. 1999), and in networking problems (Altman et al. 2001). In the stream of research in which agents are not working towards a common goal (e.g., Littman 1994), problems are generally analyzed as repeated games consistent with the traditional game theory literature (Boutilier 1999, Roger 1991, Shapley 1953). Multi-agent MDPs are notoriously hard to solve due to high dimensionality (Bernstein et al. 2002).

Generally, distributed problems face two challenges: coherence, agents’ desire to work together; and, competence, agents’ knowledge of how to work together. Artificial intelligence research on distributed problem solving has generally focused on competence (Durfee 2001). However, decision making within a health care setting does not benefit from an exogenous desire to coordinate (compared to robots that can be programmed to coordinate) nor are the objectives of multiple payers necessarily shared. In our fragmented health insurance model, we address a multi-agent problem where agents know how to work together but may not have the desire to do so.

Coordination

Coordination, the design of contracts that create an incentive to cooperate towards greater utility, has been studied extensively in the supply chain and economics literature. Specifically, numerous contract structures have been identified that can coordinate fragmented systems under various settings with complex product features, uncertain demand, and marketing investments (e.g., Bernstein and Federgruen 2005, Cachon and Lariviere 2005, Tang and Kouvelis 2014, Fu et al. 2018).

In health care settings, coordination studies have predominantly focused on aligning
patient and insurer incentives through financial contracts or aligning physician and hospital incentives through employment or compensation contracts. Handel et al. (2015) study insurance exchange design and focus on two, thoroughly studied, economic phenomena: adverse selection and reclassification risk. Specifically, when regulation mandates that patient insurance premiums are independent of health status, only individuals expecting high health care expenses will seek insurance (adverse selection) and when regulation permits insurance premiums to be fully informed by health status the price of insurance can change dramatically following an acute health event or diagnosis (reclassification risk). Using an empirically-based simulation study, Handel et al. (2015) find that the welfare cost of reclassification risk is five times higher than the welfare cost from adverse selection, providing evidence to support a ‘community’ rating system (i.e., fixed insurance premiums for all individuals in a geography, independent of all demographic characteristics) (Jones et al. 1993). Similarly, Koch (2014) demonstrates that risk-pooling when information is asymmetric between the patient and payer is socially preferred compared to fair pricing, where insurance premiums depend on health status, when information is symmetric. Salas-Lopez et al. (2014) and Suelflow (2016) study four coordination mechanisms between physicians and hospitals with varying degrees of financial commitment. They show that arrangements with the most predictable (controlled) financial outcomes for hospitals result in the highest alignment with desired clinical outcomes. However, these arrangements are less flexible and therefore may not always be preferred by physicians.

Coordination between payers has received less attention. Cochrane (1995) proposes two solutions to coordinate care across patients’ lifetimes to address the issue of inadequate life-long care. In one solution, health care risks are effectively pooled across insurers through individual non-creditable patient health savings accounts. All insurers over a patient’s life withdraw regular premiums and deposit or withdraw additional amounts based on any changes in the patient’s health. In the second solution, insurers pay premiums to future insurers when unhealthy patients transition between providers, effectively forcing the original insurer to cover the expected lifetime costs of all their patients. While both of these solutions are theoretically welfare increasing, they require significant policy interventions and maintenance to enforce. Yoshida and Tsuruta (2013) empirically explore health
3.3. Model

Consider a patient diagnosed with a progressive disease and who is medically eligible for curative treatment. Treatment provides health benefits for the patient and is recommended by the patient’s physician but is costly. The patient cannot afford the treatment on their own and so will only receive treatment if the cost is reimbursed by their health care payer (i.e., health insurance). The patient is covered by two payers over their life: the first payer ($X$) provides health care coverage for the patient until they are $\mu$-years old and the final payer ($Y$) provides health care coverage thereafter. At regular intervals (e.g., annually), the payer covering the patient’s care decides whether or not to reimburse treatment in order to maximize its utility. A payer’s utility may reflect profits, costs (negative), life-years or quality-adjusted life-years (QALYs) gained by patients, or the net monetary benefit (NMB) of care. We formulate the patient’s health and the payers’ repeated treatment decisions as a multi-decision-maker, discrete-time, finite-horizon, discounted MDP.

System states

The patient’s untreated health state, $(f, \omega)$, is characterized by two dimensions: disease severity ($f \in \mathbb{F} = \{0, 1, \ldots, F\}$, where $F$ represents the most severe living disease state) and age ($\omega \in \Omega = \{0, 1, \ldots, N\}$, where $N$ represents the maximum possible age). In practice, disease severity may reflect a variety of patient characteristics (e.g., a vector of lab results). As in prior work (e.g., Sandıkçı et al. 2008, Shechter et al. 2008, Chhatwal et al. 2010, Alagoz et al. 2013), we assume an ordering of health states. Therefore, without

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1A summary of all notation can be found in Table 3.2 in Section 3.7 on page 75.
a loss of generality, we model $f$ as a scalar. Let $\Gamma$ represent the full set of health states including the absorbing death state $F + 1$ (i.e., $\Gamma = \mathbb{F} \cup \{F + 1\}$).

Patients in a treatment-eligible state ($\mathbb{S} = \mathbb{F} \times \Omega$) may receive treatment. A patient who receives treatment transitions into an age- and disease-severity-specific absorbing treated state, $T_{(f,\omega)}$ (where $T_{(f,\omega)}$ is a single state in the set of all treated states $\mathbb{T}$). Let $\mathbb{S}^i$ represent the set of health states where payer $i \in \{X, Y\}$ may provide treatment. Thus, $\mathbb{S}^X = \{(f, \omega) | \omega \in [0, \mu), f \in \mathbb{F}\}$ and $\mathbb{S}^Y = \{(f, \omega) | \omega \in [\mu, N], f \in \mathbb{F}\}$. $\mathbb{S}^X$ and $\mathbb{S}^Y$ are mutually exclusive and collectively exhaustive subsets of $\mathbb{S}$. Overall, the system state space is defined as $\mathbb{S} = \mathbb{S} \cup \mathbb{T} \cup \{F + 1\}$ with individual state $s$.

**Actions and system dynamics**

The patient transitions between states according to the transition probability matrix $P$, where individual probabilities depend on the patient’s current state and the action taken by the decision maker, defined as $p(s'|s, a(s))$, where $s$ is the current state, $a(s)$ is the action taken, and $s'$ is the future state. Each decision period, the decision making payer will select an action, $a'(s)$, from its set of possible actions, $A'(s)$. If a patient is in a treatment-eligible state, then the payer may either reimburse treatment ($1 = \text{treat}$) or not ($0 = \text{wait}$). For all other states, treatment is not an available decision (i.e., $A'(s) = \{0\}, \forall s \in \mathbb{T} \cup \{F + 1\}$). If a payer chooses not to reimburse treatment for a treatment-eligible patient, then the patient transitions according the probabilities $p(s'|s = (f, \omega), a(s) = 0)$. An individual $N$-years old progresses to death with probability 1 (i.e., $p(F + 1|(f, N), 0) = 1, \forall f \in \mathbb{F}$). Once the patient dies, the system remains in the death state indefinitely (i.e., $p(s' = F + 1|s = F + 1, 0) = 1$). If a payer chooses to reimburse treatment, then the patient transitions into the absorbing age- and disease-severity-specific treated state $T_{(f,\omega)}$ with probability 1. Due to the frequent use of the individual transition probabilities when the action taken is wait ($a(s) = 0$), we make two notational modifications: age progresses with certainty over time (i.e., $\omega' = \omega + 1$), therefore we suppress the ‘$\omega + 1$’ notation; and, we suppress the explicit action notation. Therefore, $p((f',\omega + 1)|(f, \omega), 0) \equiv p(f'|(f, \omega))$. 

Rewards

Payers receive rewards (utility) for their decisions depending on the current state and the action taken, which we define as \( r'(s, a'(s)), i, i' \in \{X, Y\} \) (where \( i \) represents the payer receiving the reward and \( i' \) represents the decision making payer). If a payer chooses to reimburse treatment, then both payers may receive a reward. If the first payer provides treatment, then the first payer receives the present value of all expected future utility from the treated patient for the years up until \( \mu \), minus a fixed disutility of treatment, \( c \) (e.g., drug cost plus any treatment or monitoring cost). The final payer receives the present value of all expected future utility from the treated patient for the years from \( \mu \) onward. For each payer, the utility received for treatment is equal to the present value of the expected reward from an embedded Markov process. Let \( \delta \in [0, 1] \) represent the one-period discount factor. Let \( r_{T(f,\omega)}(\tilde{\omega}) \) and \( p_{T(f,\omega)}(T(f,\omega)|\tilde{\omega}) \) represent the one-period reward and one-period probability of living, respectively, for a patient who received treatment when they were in health state \((f, \omega)\) and is currently \( \tilde{\omega} \)-years old. Let \( \theta^i \) represent an indexing variable that represents the future time periods until payer \( i \)'s coverage period ends, conditional on the patient's current age (i.e., \( \forall \omega < \mu, \theta^X = (\mu - 1) - \omega \) and \( \theta^Y = N - \omega \); and, \( \forall \omega \geq \mu, \theta^Y = N - \omega \)). The payers' rewards for any patient treated in state \((f, \omega)\in S_X \) (i.e., \( a^X(f, \omega) = 1 \)) are,

\[
r^X((f, \omega), 1) = r_{T(f,\omega)}(\omega) + \sum_{t=1}^{\theta^X} [\delta^t \cdot r_{T(f,\omega)}(\omega + t) \cdot \prod_{j=0}^{t-1} p_{T(f,\omega)}(T(f,\omega)|\omega + j)] - c \tag{3.1}
\]

and,

\[
r^Y((f, \omega), 1) = \prod_{j=0}^{\theta^X} \delta^j \cdot p_{T(f,\omega)}(T(f,\omega)|\omega + j) \cdot \left[ r_{T(f,\omega)}(\mu) + \sum_{t=1}^{N-\mu} [\delta^t \cdot r_{T(f,\omega)}(\mu + t) \cdot \prod_{k=0}^{t-1} p_{T(f,\omega)}(T(f,\omega)|\mu + k) \right] \tag{3.2}
\]

In contrast, if the final payer provides treatment, the first payer does not receive any reward because all rewards are accrued after the first payer’s coverage has ended. The payers’ rewards for any patient treated in state \((f, \omega)\in S_Y \) (i.e., \( a^Y(f, \omega) = 1 \)) are,

\[
r^X((f, \omega), 1) = 0 \tag{3.3}
\]
and,
\[
    r^Y((f, \omega), 1) = r_{\mathcal{T}(f, \omega)}(\omega) + \sum_{t=1}^{\rho} \delta^t \cdot r_{\mathcal{T}(f, \omega)}(\omega + t) \cdot \prod_{j=0}^{t-1} p_{\mathcal{T}(f, \omega)}(\mathcal{F}(f, \omega)|\omega + j) - c \tag{3.4}
\]

If any payer chooses not to reimburse treatment for a treatment-eligible patient, then
that payer receives a one-period age- and disease-severity-specific reward of waiting, \(r(s, 0)\)
and the other payer receives no reward. The one-period reward is zero in all absorbing
states.

**Objective**

Each decision period, the payer that covers the patient’s health care decides whether to
provide treatment or not. Let \(\Psi^i(s_0)\) represent payer \(i\)’s utility when considering a patient
in initial state \(s_0 \in \mathcal{S}\). Each payer chooses a treatment policy to maximize their expected
utility as in Equation \(3.5\). Let \(\bar{a}^i\) represent payer \(i\)’s treatment policy that represents a
vector of actions, one for each time interval in that payer’s decision period. Therefore, \(\bar{a}^i = \{a^i_0(s_0), a^i_1(s_1), \ldots, a^i_{\rho}(s_{\rho})\} \in \mathcal{A}^i\), where \(a^i_t(s_t)\) represents payer \(i\)’s action in time period
t and \(\mathcal{A}^i\) represents the set of all possible policies available to payer \(i\). Let \(\gamma_t\) represents
a state progression with probability distribution \(P\). Payer \(i\)’s optimization problem is as
follows,
\[
    \max_{a \in \mathcal{A}^i} \mathbb{E}[\Psi^i(s_0)] = \max_{a \in \mathcal{A}^i} \mathbb{E} \left[ \sum_{t=0}^{\rho} \delta^t \cdot [r^i(s_t, a^i_t(s_t))] \bigg| s_0 \right] \tag{3.5}
\]
subject to \(s_{t+1} = \gamma_t\),
for any \(s_0 \in \mathcal{S}^i\)

3.4 **Structural Results**

We solve the two-payer optimal treatment problem for a single patient presented in Equation \(3.5\)
as a state-partitioned, two-decision-maker discounted MDP. The optimal solutions
to Equation \(3.5\) for the first and the final payers, \(a^*^X(f, \omega)\) and \(a^*^Y(f, \omega)\), respectively, are obtained by solving two sets of recursive value function equations (Puterman 1994):
3.4. Structural Results

\[ V^X(f, \omega) = \begin{cases} \max \{ r^X((f, \omega), 1), \quad \text{if } (f, \omega) \in \mathbb{S}^X \\ r^X((f, \omega), 0) + \delta \sum_{f' \in \Gamma} p(f'(f, \omega)) \cdot V^X(f', \omega + 1) \} \end{cases} \]

\[ V^Y(f, \omega) = \begin{cases} \max \{ r^Y((f, \omega), 1), \quad \text{if } a^Y(f, \omega) = 1 \\ \delta \sum_{f' \in \Gamma} p(f'(f, \omega)) \cdot V^Y(f', \omega + 1) \}, \quad \text{if } (f, \omega) \in \mathbb{S}^Y \\ \max \{ r^Y((f, \omega), 0) + \delta \sum_{f' \in \Gamma} p(f'(f, \omega)) \cdot V^Y(f', \omega + 1) \} \}, \quad \text{if } (f, \omega) \in \mathbb{S}^X \\ \end{cases} \]

The value functions associated with states \( T \) and \( F + 1 \) are zero by construction and are omitted.

The remainder of this section is divided into three parts. First, we consider the special case where a patient’s health care is covered by only one payer (i.e., the single-payer scenario). This scenario corresponds to the cases where \( \mu = N \) (i.e., \( \mathbb{S}^Y = \emptyset \)) or \( \mu = 0 \) (i.e., \( \mathbb{S}^X = \emptyset \)) and represents the benchmark or ‘first-best’ scenario where health care decisions are made considering costs and benefits accrued over the patient’s entire lifespan. In the second part, the patient’s health care is covered by two payers (i.e., the multi-payer scenario; \( 0 < \mu < N \), \( \mathbb{S}^X \) and \( \mathbb{S}^Y \) are both non-empty). We analytically characterize the structural form of these policies and discuss the inefficiencies that result from the fragmented system. In the third part, we study a contract between payers that can coordinate the system (i.e., the coordinated scenario).

Throughout this paper we make the following three assumptions regarding rewards:

**Assumption 3.4.1** \([A3.4.1]\): \( r((f, \omega), 0) \) is non-negative and non-increasing in both \( f \) and \( \omega \).

**Assumption 3.4.2** \([A3.4.2]\): \( r_{T(f, \omega)}(\hat{\omega}) \) and \( p_{T(f, \omega)}(T(f, \omega)|\hat{\omega}) \) are both non-increasing in \( f \), \( \omega \), and \( \hat{\omega} \).

**Assumption 3.4.3** \([A3.4.3]\): The immediate one-period reward and mortality from treatment is no worse than the one-period reward and mortality from waiting, respectively.
Specifically,
\[ r_{T(f,\omega)}(\omega) \geq r((f, \omega), 0) \text{ and } p_{T(f,\omega)}(T_{(f,\omega)}|\omega) \geq \left(1 - p(F + 1|(f, \omega))\right). \forall (f, \omega) \in \mathcal{S}. \]

\(A3.4.1\) and \(A3.4.2\) require that younger and healthier patients result in higher utility and are consistent with other MDP models (e.g., Alagoz et al. 2004, 2007a,b, Sandikçı et al. 2008, Shechter et al. 2008, Alagoz et al. 2013). \(A3.4.3\) requires that treatment does not result in an immediate decrease in the one-period reward nor a decrease in one-period probability of mortality, although recall that treatment does result in a fixed disutility \(c\).

### 3.4.1 Optimal Treatment Policy: Single-Payer Scenario

The single-payer optimal treatment problem is a special case of the two-payer problem where \(\mu\) is set to either 0 or \(N\). As a result, the solution to only one set of recursive equations is necessary to identify the optimal set of actions, \(\hat{a}^*(f, \omega)\), suppressing the superscript indicating whether it is payer X or payer Y. Throughout the remainder of this paper, we use the ‘\(^\wedge\)’ symbol to represent the single-payer scenario. The value function equations identifying the single payer’s maximum total discounted expected utility when the current state is \((f, \omega)\) is an algebraic simplification of Equation 3.6. Let \(r((f, \omega), 0)\) and \(r((f, \omega), 1)\) be the single-payer rewards for deciding to wait or treat, respectively. Therefore,

\[
\hat{V}(f, \omega) = \max \left\{r((f, \omega), 1), \ r((f, \omega), 0) + \delta \sum_{f' \in \Gamma} p(f'|((f, \omega))) \cdot \hat{V}(f', \omega + 1)\right\}, \quad \forall (f, \omega) \in \mathcal{S}, \tag{3.7}
\]

The single-payer problem has been studied in previous work (e.g., Alagoz et al. 2004, 2007a,b, 2013, Sandikçı et al. 2008, Shechter et al. 2008, Chhatwal et al. 2010). The following 3 propositions re-present findings from these works in our notation with minor modifications and are necessary steps towards our main findings in the propositions that follow.

Proposition 3.4.1 verifies the intuitive property that healthier and younger patients generate higher expected lifetime utility. One property that may characterize a patient’s tran-

\(^2\)Detailed proofs for all formal statements can be found in Appendix B.3 on page 156.
3.4. Structural Results

3.4.1

sitions between states, and a necessary condition for Proposition 3.4.1 is the property of increasing failure rate (IFR) (Barlow and Proschan 1965). Throughout this paper, we identify technical definitions with the symbol † and formally define these terms in Appendix C. In a health care setting, the IFR property means that a patient’s health deteriorates more quickly (including into the death state) when the patient is in a more severe health state. In a two-dimensional state space, we define IFR$_f$† and IFR$_\omega$† to represent the increasing failure rate with respect to disease severity and age, respectively. Therefore, we use the term IFR$_{f\omega}$ to describe transition rates that are both IFR$_f$ and IFR$_\omega$.

Proposition 3.4.1

a) For any $\omega \in \Omega$, if $P$ is IFR$_f$, then $\hat{V}(f, \omega)$ is non-increasing in $f$.

b) For any $f \in \mathbb{F}$, if $P$ is IFR$_{f\omega}$, then $\hat{V}(f, \omega)$ is non-increasing in $\omega$.

Next, we establish the conditions for a control-limit policy. The transition probability matrix property of increasing disease rate (IDR) characterizes transitions between states and is common in MDP literature (e.g., Chhatwal et al. 2010, Alagoz et al. 2007a,b). The IDR property means that a patient’s health deteriorates more quickly when the patient is in a more severe disease state. In contrast to the IFR property, which applies to transitions among living states and into the absorbing death state, the IDR property applies only to transitions among living health states. IDR is neither a necessary nor sufficient condition for IFR, and vice versa. Proposition 3.4.2 establishes the conditions such that age- and severity-based control-limit treatment policies are optimal.

Proposition 3.4.2

a) If $P$ is both IFR$_f$ and IDR, and the following holds:

$$\frac{r((f, \omega), 1) - r((f + 1, \omega), 1)}{r((f + 1, \omega + 1), 1)} < \delta \left[ p(F + 1|(f + 1, \omega)) - p(F + 1|(f, \omega)) \right]$$

for $f = 0, 1, \ldots, F - 1$, $\omega = 0, 1, \ldots, N - 1$, (3.8)

then there exists an optimal severity-based control-limit policy for each $\omega \in \Omega$. The severity-based policy is such that for each age, $\omega$, there exists a health state, $f^T(\omega)$, such that if $f < f^T(\omega)$, then $\hat{a}^*(f, \omega) = 0$ and if $f \geq f^T(\omega)$, then $\hat{a}^*(f, \omega) = 1$. 

b) If \( P \) is both IFR\( _{f,\omega} \) and IDR, and the following holds:

\[
\frac{r((f,\omega), 1) - r((f,\omega + 1), 1)}{r((f,\omega + 1), 1)} < \delta \left[ p(F + 1|(f,\omega + 1)) - p(F + 1|(f,\omega)) \right],
\]

\( f = 0, 1, \ldots, F, \ \omega = 0, 1, \ldots, N - 1 \) \quad (3.9)

then there exists an optimal age-based control-limit policy for each \( f \in \mathbb{F} \). The age-based policy is such that for each health state, \( f \), there exists an age, \( \omega^T(f) \), such that if \( \omega < \omega^T(f) \), then \( \hat{a}^*(f,\omega) = 0 \) and if \( \omega \geq \omega^T(f) \), then \( \hat{a}^*(f,\omega) = 1 \).

Conditions 3.8 and 3.9 require that when a patient’s disease severity or age increases, that the percent decrease in the treatment reward is less than the discounted increase in the probability of death (i.e., increases in mortality outweigh decreases in rewards). In Appendix B.2 we find the maximum violation of these conditions, and all other conditions, using a case study of HCV treatment.

A control-band policy (Oh and Özer 2016) captures the possibility for two decision thresholds. For example an age-based control-band policy is as follows: if a patient’s age is below some threshold, \( \omega^T \), then the optimal action is to wait; if the patient’s age is between the thresholds \( \omega^T \) and \( \omega^T \), then the optimal action is to provide treatment; and, if the patient’s age is greater than \( \omega^T \), then the optimal action is to wait.

Proposition 3.4.3 establishes the conditions such that the optimal treatment policy is a control-band policy. To establish these conditions, we require the following three definitions. First, stochastic convexity\( ^\dagger \) (SC) describes the increasing rate of change of a patient’s probability of transitioning into a more severe health state. Similar to the IFR property, stochastic convexity may be expressed with respect to both disease severity (SC\( _f \)) or age (SC\( _\omega \)). Second, the one-step benefit function\( ^\dagger \), \( M(f,\omega) \) (Oh and Özer 2016), reflects the incremental discounted benefit of waiting one period and then providing treatment versus providing treatment immediately while in state \( (f,\omega) \). Third, the benefit function\( ^\dagger \), \( B(f,\omega) \) (Oh and Özer 2016), reflects the incremental discounted benefit of waiting one period and then selecting the optimal treatment decision. The benefit function is the difference between the payer’s two decisions in Equation 3.7. We also make the following assumption to characterize a unique action in the scenario where a payer is indifferent between deci-
3.4. **Structural Results**

**Assumption 3.4.4** [A3.4.4]: *If a payer is indifferent between providing treatment and waiting, then the payer will choose to provide treatment.*

A3.4.4 is a tie-breaking assumption, requiring that payers act benevolently towards patients when indifferent to their own utility of treatment. Oh and Özer (2016) prove that the benefit function fully defines the optimal action, formalized within our model as,

\[
B(f, \omega) > 0 \iff \hat{a}^*(f, \omega) = 0 \\
B(f, \omega) \leq 0 \iff \hat{a}^*(f, \omega) = 1
\]

\[\forall (f, \omega) \in \mathcal{S}\]  \hspace{1cm} (3.10)

**Proposition 3.4.3** [Proposition 2 in Oh and Özer (2016)]:

a) *If P is SC\_f and M(f, \omega) is convex in f, then there exists an optimal severity-based control-band policy for each \( \omega \in \Omega \). The severity-based control-band policy is such that for each age, \( \omega \), there exists a set of disease severity states, \( \Gamma^T(\omega) = \left[ f^T(\omega), \bar{f}^T(\omega) \right] \), such that if \( f \in \Gamma^T(\omega) \), then \( \hat{a}^*(f, \omega) = 1 \), otherwise \( \hat{a}^*(f, \omega) = 0 \).*

b) *If P is SC\_\omega and M(f, \omega) is convex in \( \omega \), then there exists an optimal age-based control-band policy for each \( f \in \Gamma \). The age-based control-band policy is such that for each severity state, \( f \), there exists a set of ages, \( \Omega^T(f) = \left[ \omega^T(f), \bar{\omega}^T(f) \right] \), such that if \( \omega \in \Omega^T(f) \), then \( \hat{a}^*(f, \omega) = 1 \), otherwise \( \hat{a}^*(f, \omega) = 0 \).*

Figure [3.1] illustrates the shape of the optimal treatment policies that correspond to the four scenarios covering our state parameters \( (f \text{ and } \omega) \) and the two control policies (limit and band). Within the main text of this paper, we analytically prove the shape characteristics of the scenario in which there is a control-limit policy in \( f \) and control-band policy in \( \omega \) (Figure [3.1b]) because this scenario is consistent with the numerical example in Section 3.5.

Proposition [3.4.4] proves the properties of the treatment boundaries associated with the optimal treatment policy.

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Footnote 3: We provide complete proofs and characterization of the optimal policies for the other three scenarios in Appendix B.1.
Figure 3.1: Optimal single-payer treatment policies. a) control-limit in both disease severity, $f$, and age, $\omega$; b) control-limit in $f$ and control-band in $\omega$; c) control-band in $f$ and control-limit in $\omega$; d) control-band in both $f$ and $\omega$.

Proposition 3.4.4 Suppose the conditions of Proposition 3.4.2a and 3.4.3b are satisfied. Then,

a) $\omega^T(f)$ is non-increasing in $f$.

b) $\bar{\omega}^T(f)$ is non-decreasing in $f$.

c) $|\Omega^T(f)|$ is non-decreasing in $f$.

d) $\exists \bar{\omega}^T$ s.t. $f^T(\omega)$ is non-increasing in $\omega$, $\forall \omega < \bar{\omega}^T$, and $\exists \bar{\omega}$ s.t. $f^T(\omega)$ is non-decreasing in $\omega$, $\forall \omega > \bar{\omega}$.

Before closing the single-payer scenario, we note two properties regarding the size of the treatment region. First, we prove the intuitive property that the size of the treatment region is decreasing in the cost of treatment. Significantly, this finding does not require any
assumptions or additional conditions. Let \( \hat{T} \) represent the set of states where it is optimal to provide treatment.

**Proposition 3.4.5** Let \( \hat{T}(c) \) represent the set of states where it is optimal to provide treatment parameterized by \( c \). Then, \( \frac{d\hat{T}(c)}{dc} \leq 0 \).

Second, Proposition 3.4.6 proves that, all else equal, the optimal treatment region for a slowly progressing disease will be a strict subset of the optimal treatment region for a more rapidly progressing disease.

**Proposition 3.4.6** Let \( \Upsilon_1 \) and \( \Upsilon_2 \) represent two problem instances with disease progression matrices \( P_1 \) and \( P_2 \), and value functions \( \hat{V}_1(f, \omega) \) and \( \hat{V}_2(f, \omega) \), respectively. Let \( \hat{T}_1 \) and \( \hat{T}_2 \) represent the set of states where treatment is optimal in \( \Upsilon_1 \) and \( \Upsilon_2 \), respectively. If \( \Upsilon_1 \) and \( \Upsilon_2 \) have the same reward functions, \( r(s, a) \), and disutility of treatment, \( c \), and \( P_1 \succeq P_2 \), then \( \hat{T}_1 \subseteq \hat{T}_2 \).

### 3.4.2 Optimal Treatment Policy: Multi-Payer Scenario

Recall that \( V^X(f, \omega) \) and \( V^Y(f, \omega) \) represent the value functions for the first and final payer, respectively. For comparison with the single-payer scenario, let \( \hat{V}^X(f, \omega) \) and \( \hat{V}^Y(f, \omega) \) represent the value that the first and final payers would receive, respectively, if each payer acted optimally with respect to the optimal treatment policy from the single-payer scenario. Therefore by construction, \( \hat{V}(f, \omega) = \hat{V}^X(f, \omega) + \hat{V}^Y(f, \omega) \). Similarly, let \( V(f, \omega) \) represent the total value in the multi-payer system, such that \( V(f, \omega) = V^X(f, \omega) + V^Y(f, \omega) \). Let \( \hat{T}_X \) and \( \hat{T}_Y \) represent the sets of states where the first and final payers provide treatment in the multi-payer scenario, respectively.

Proposition 3.4.7 proves that the total value in a multi-payer system is lower than the value in a single-payer system. Significantly, this result holds without any assumptions about the reward functions or transition probabilities. We also prove that the first payer is always better off and the final payer is always worse off in a multi-payer system compared to the single-payer scenario. Therefore, fragmentation results in less total value and a redistribution of value, favoring the first payer. This is important from a public policy perspective when the final payer is the publicly funded.
Proposition 3.4.7 In a multi-payer health care system the total value is lower, the final payer receives less value, and the first payer receives more value compared to a single-payer system. Formally,

\[ a) \ V(f, \omega) \leq \hat{V}(f, \omega) \]

\[ b) \ V^X(f, \omega) \geq \hat{V}^X(f, \omega) \]

\[ c) \ V^Y(f, \omega) \leq \hat{V}^Y(f, \omega) \]

Corollary 3.4.8 Compared to the single-payer scenario, in the multi-payer scenario, the magnitude of the final payer’s loss is always larger than the magnitude of first payer’s gain. Formally,

\[ \hat{V}^Y(f, \omega) - V^Y(f, \omega) \geq V^X(f, \omega) - \hat{V}^X(f, \omega) \]

While Proposition 3.4.7 defines the impact of a fragmented system on the value that the payers receive, we also consider the impact on patients. By Proposition 3.4.7a, it is clear that the treatment policy in the multi-payer scenario is different than in the single-payer scenario, however the characteristics of this difference are unknown. For example, it is unclear whether a larger or smaller set of patients would receive treatment in the multi-payer scenario versus the single-payer scenario.

First, the optimal treatment policy for a patient in state \((f, \omega) \in S_Y\) is the same for the single-payer scenario or the multi-payer scenario, as formalized in Corollary 3.4.9. Significantly, this does not require any assumptions or additional conditions.

Corollary 3.4.9 Let \(\hat{T}^Y = \hat{T} \cap S^Y\) represent the set of states where treatment would be provided in the single-payer scenario within the final payer’s coverage period. Then, \(T^Y = \hat{T}^Y\).

Next, we establish the conditions such that the set of patients who receive treatment in a multi-payer system is always a subset of the patients who would receive treatment in single-payer system (see Figure 3.2). Given these conditions there are no inefficient beneficiaries, patients who receive treatment at the cost of social efficiency. However, there are patients who will receive treatment in the single-payer scenario but not in the multi-payer scenario.
Let $f(f, \omega)$ represent the *healthiest transition state* that a patient can transition into in a single decision period from state $(f, \omega)$. The healthiest transition state is a disease-specific characteristic. For example, if a disease is ‘weakly degenerative’ (i.e., if it is not possible to naturally recover), then $f(f, \omega) = f$, $\forall \omega \in \Omega$. Or, if it is possible for a patient to naturally recover, then $f(f, \omega) < f$, $\forall \omega \in \Omega$. We make the following assumption regarding $f(f, \omega)$.

**Assumption 3.4.5 [A3.4.5]**: For any $f \in \mathbb{F}$, $f(f, \omega)$ is non-decreasing in $\omega$.

A3.4.5 requires that as an individual ages, their ability to recover cannot increase.

Let $TP(f, \omega_1, \omega_2) \in [0, 1]$ represent the *total probability of living* from age $\omega_1$ until age $\omega_2$ (where $\omega_1 < \omega_2$), starting in severity state $f$.

![Figure 3.2: Optimal multi-payer treatment policies. a) control-limit in both disease severity, $f$, and age, $\omega$; b) control-limit in $f$ and control-band in $\omega$; c) control-band in $f$ and control-limit in $\omega$; d) control-limit in both $f$ and $\omega$. Black region represents the states where treatment is optimal in the multi-payer scenario. Shaded region represents the additional states where treatment is optimal in the single-payer scenario.](image-url)
Proposition 3.4.10 Let \( \hat{T}^X = \hat{T} \cap \mathcal{S}^X \) represent the set of states where treatment would be provided in the single-payer scenario within the first payer’s coverage period. Suppose that \( P \) is IFR\(_f\) and the following holds:

\[
\frac{r_{T_{s_2}}(\mu) - r_{T_{s_1}}(\mu)}{r_{T_{s_1}}(\mu)} < \frac{TP(T_{s_1}, \omega, \mu) - TP(T_{s_2}, \omega + 1, \mu) \cdot (1 - p(F + 1|s_1))}{TP(T_{s_2}, \omega + 1, \mu) \cdot (1 - p(F + 1|s_1))} \quad , \quad \forall (f, \omega) \in \mathcal{S}^X
\]

where \( s_1 = (f, \omega) \) and \( s_2 = (f(s_1), \omega + 1) \). Then, \( \mathcal{T}^X \subseteq \hat{T}^X \).

Condition 3.11 has an intuitive explanation. Consider two states, \( s_1 \) and \( s_2 \), where \( s_1 \) represents the current state \((f, \omega)\) and \( s_2 \) represents the healthiest possible state that could be entered into by waiting one period (i.e., \( s_2 = (f(s_1), \omega + 1) \)). Condition 3.11 requires that the percent difference in the one-year rewards from treatment between these two states is less than the percent difference in the total probability of living. A sufficient condition for this to be satisfied is if natural recovery from the disease is not possible.

Corollary 3.4.11 Suppose that \( P \) is IFR\(_f\) and \( f(f, \omega) \geq f, \forall (f, \omega) \in \mathcal{S}^X \). Then, Condition 3.11 holds and \( \mathcal{T}^X \subset \hat{T}^X \).

Together, Corollary 3.4.9 and Proposition 3.4.10 imply that for any state \((f, \omega)\) for which it is optimal for either the first or final payer to provide treatment in the multi-payer scenario, it would also be optimal to provide treatment in the single-payer scenario.

Corollary 3.4.12 Suppose \( P \) is IFR\(_f\) and Condition 3.11 holds. Given any state \((f, \omega) \in \mathcal{S}\), if \( \alpha^i(f, \omega) = 1, i \in \{X, Y\} \), then \( \hat{\alpha}(f, \omega) = 1 \) and therefore \( V^X(f, \omega) = \hat{V}^X(f, \omega), V^Y(f, \omega) = \hat{V}^Y(f, \omega), \) and \( V(f, \omega) = \hat{V}(f, \omega) \).

The structure of the differences in treatment policies between the single- and multi-payer scenarios define the inefficiencies that result from fragmentation. For example, as shown by Corollary 3.4.13 it is possible that there are treatment gaps in the middle of the optimal policy space as a result of the multi-payer system (see Figure 3.2b).
Corollary 3.4.13 Suppose the conditions for Proposition 3.4.2 and 3.3 hold for the first payer and in the single-payer scenario. Then, for any \( f \in \mathbb{F} \), if \( \mu \in \Omega^T(f) \) and \( c > r_{T(f,\mu-1)}(\mu - 1) - r((f,\mu - 1),0) \), then the optimal multi-payer age-based control-band policy is a Wait-Treat-Wait-Treat-Wait (WTWTW) policy.

While sub-optimal treatment is undesirable, regardless of its form, treatment gaps are particularly concerning for patient equity. If a WTWTW policy is optimal in a multi-payer system, an individual of age \( \omega \in (\omega^T_X(f),\mu) \) would not receive treatment, while older and younger patients of the same severity would receive treatment. To the best of our knowledge, treatment policies that explicitly discriminate for intermediate ages do not exist in any clinical guidelines. However in a multi-payer system, some common reimbursement requirements implicitly limit the ages of patients who can receive care. For example, without medical justification, some HCV treatments require multiple confirmatory tests, spaced six months apart, and then a prescription from a specialist, often with long wait lists (Lawitz et al. 2014, Nelson et al. 2015, Afdhal et al. 2014). These restrictions mean that individuals within one to two years of transitioning to another payer’s coverage may have their access to treatment delayed until after they change insurers.

The final result in this section is that the magnitude of the first payer’s benefit, when comparing the single-payer scenario and the multi-payer scenario, is bounded by the disutility of treatment, \( c \).

Proposition 3.4.14 Suppose \( P \) is IFR, and Condition 3.11 and the following holds:

\[
\frac{r^X((f, \omega + 1), 1) + c}{r^X((f(f), \omega + 1), 1)} > \frac{1 - p(F + 1|(f, \omega))}{p_{r_{T(f,\omega)}(f,\omega)|\omega}} ,
\]

\[
\forall f \in \mathbb{F}, \forall \omega \in \{0, 1, \ldots, \mu - 2\}
\]

then, the maximum incremental benefit that the first payer can receive by acting independently is bounded by the disutility of treatment. That is, \( V^X(f,\omega) - \hat{V}^X(f,\omega) \leq c \), \( \forall (f,\omega) \in \mathbb{S}^X \).
3.4.3 Optimal Treatment Policy: Coordinated Scenario

In the previous section, we show that multi-payer health care systems are inefficient. Compared to a single-payer system, only a subset of patients will receive treatment in a multi-payer system. Therefore, we consider if a transfer payment, from the final payer to the first payer, can promote optimal treatment and increase both payers’ utility. We use the ‘~’ symbol to identify the actions and outcomes within a system where transfer payments can be made.

The sequence of decision events is as follows: first, the final payer offers a state-dependent transfer payment to the first payer, $I(f, \omega) \geq 0$; second, the first payer decides if it will accept the payment and provide treatment, or reject the payment and decline to provide treatment in this period. The first payer includes the expectation of future transfer payments into its treatment decision.

Let $I$ represent an incentive payment policy that represents a vector of incentive payments, one for each time interval in the first payer’s decision period. Therefore, $I = \{I_0(s_0), I_1(s_1), \ldots, I^{\theta_X(s_{\theta_X})}\} \in \mathcal{I}$, where $I_i(s_i) \geq 0$ represents the incentive payment in time period $i$ and $\mathcal{I}$ represents the set of all possible policies. The final payer’s objective is to select an incentive payment policy that maximizes its expected utility. Formally,

$$\max_{I \in \mathcal{I}} E[\tilde{\Psi}(s_0)] = \max_{I \in \mathcal{I}} E\left[\sum_{t=0}^{\theta_X} \delta^t \cdot [r^X(s_t, \tilde{a}^X(s_t)) - I_t(s_t) \cdot \tilde{a}^X(s_t)] + \sum_{t=\theta_X+1}^{\theta_Y} \delta^t \cdot [r^Y(s_t, \tilde{a}^Y(s_t))] | s_0]\right]$$

subject to

$$\tilde{a}^X = \arg\max_{\tilde{a}^X \in \mathcal{A}^X} E\left[\sum_{t=0}^{\theta_X} \delta^t \cdot [r^X(s_t, \tilde{a}^X(s_t)) + I_t(s_t) \cdot \tilde{a}^X(s_t)] | s_0]\right]$$

$$\tilde{a}^Y = \arg\max_{\tilde{a}^Y \in \mathcal{A}^Y} E\left[\sum_{t=\theta_X+1}^{\theta_Y} \delta^t \cdot [r^Y(s_t, \tilde{a}^Y(s_t))] | s_0]\right]$$

$$s_{t+1} = \gamma_t,$$

for any $s_0 \in \mathcal{S}^X$

Let $\tilde{V}(f, \omega)$ represent payer i’s value function in the coordinated scenario. The equilibrium solution to the multi-stage problem is obtained by solving the following sets of
3.4. **Structural Results**

equations:

\[
\tilde{V}^Y(f, \omega) = \max_{I(f, \omega) \geq 0} \left\{ \begin{array}{ll}
I^Y(f, \omega) - I^Y(f, \omega), & \text{if } \tilde{a}^X(f, \omega) = 1 \\
\delta \sum_{f' \in \Gamma} p(f'(f, \omega)) \cdot \tilde{V}^Y(f', \omega + 1), & \text{if } \tilde{a}^X(f, \omega) = 0
\end{array} \right.
\]

subject to

\[
\tilde{a}^X(f, \omega) = \arg\max_{a^X(f, \omega) \in \{0, 1\}} \left\{ \begin{array}{ll}
r^X(f, \omega) + I^Y(f, \omega), & \text{if } a^X(f, \omega) = 1 \\
r^X(f, \omega) + \delta \sum_{f' \in \Gamma} p(f'(f, \omega)) \cdot \tilde{V}^X(f', \omega + 1), & \text{if } a^X(f, \omega) = 0
\end{array} \right.
\]

\[
\forall (f, \omega) \in S^X
\]

and,

\[
\forall (f, \omega) \in S^Y
\]

Significantly, there exists a state dependent transfer payment that can coordinate the system.

**Proposition 3.4.15** Suppose \(P\) is IFR\(_f\) and Condition 3.11 holds. Then, there exists a state dependent transfer payment from the final payer to the first payer that coordinates the system and results in at least as high expected outcomes for both payers.

The characteristics of the optimal incentive function are surprising. For example, if the conditions are satisfied such that the optimal policy is a control-limit policy with respect to disease severity, then the optimal incentive offer is decreasing in disease severity. That is, the final payer would pay less to coordinate treatment for patients with a more severe disease. However, this is not because more severe patients provide less value to the final payer, instead it is because there is a small difference in value to the first payer between treating and waiting for more severe patients. Therefore, it takes less incentive for the first payer to switch its treatment decision. If the conditions are satisfied such that the optimal policy is a control-band policy with respect to patients’ age, then the optimal incentive may be both decreasing and increasing with respect to age. For individuals who are at
the younger end of the optimal treatment band, the incentive offer decreases in age, while for individuals who are at the older end of the optimal treatment band, the incentive offer increases in age.

Overall, the optimal incentive payment is equal to the positive component of the first payer’s one-step benefit function (i.e., \( I^*(f, \omega) = (M^X(f, \omega))^+ \)). Therefore, the properties of the optimal incentive payment will adopt the structural properties of the one-step benefit function. From Oh and Özer (2016), the optimal treatment policy also adopts its structure from the one-step benefit function. Therefore, the structural characteristics of the control policy and the incentive payment can be determined without requiring the solution to the dynamic optimization problem. Finally, if the conditions of Proposition 3.4.14 are satisfied, then the incentive payment that coordinates the system will always be less than the cost of treatment.

### 3.5 Case Study: Access to Hepatitis C Virus Treatment

In this section, we demonstrate the analytical results from Section 3.4 by applying our framework to the case of access to chronic HCV treatment in the US. HCV is a slowly progressing blood-borne infection affecting the liver. Despite widespread recognition that HCV treatment is an important element of reducing the burden of HCV morbidity and mortality and the evidence that HCV treatment is cost-effective, access to treatment remains restricted by, for example, requiring significant liver fibrosis, requiring drug and/or alcohol abstinence, or limiting prescriber types (Barua et al. 2015, Chidi et al. 2016, Gowda et al. 2018, Re et al. 2016, Younossi et al. 2016, Waters and Broder 2018, Lawitz et al. 2014, Nelson et al. 2015, Afdhal et al. 2014).

HCV is a timely example on which to demonstrate the utility of our framework because of the large absolute number of people affected, the disproportionate impact on individuals aged 45 to 65 (right before many individuals shift their health care coverage to Medicare at age 65), and the relatively restricted access to treatment reimbursement being faced by individuals prescribed treatment by their physician. We identify the optimal single-payer treatment policy and compare this policy to the optimal treatment policy identified under a
multi-payer health system to quantify the differences between the policies in terms of the number of people treated, the number of people whose disease progresses due to delayed treatment access, and overall net monetary benefit (a combined health-economic measure of health benefits and costs). Finally, we identify the age- and disease-severity-specific incentive payment that will coordinate the system.

### 3.5.1 HCV model and parameter estimation

We develop a state-transition model of chronic HCV infection and treatment similar in structure to several previously published models (Cipriano et al. 2018, Liu et al. 2012, 2016, Salomon et al. 2003). In the model, treatment-naïve individuals progress through stages of liver fibrosis defined by METAVIR score (Bedossa and Poynard 1996) denoted F0 (mild) through F4 (severe) in one-year transition cycles until death (or age 100). Individuals with F4 liver fibrosis (compensated liver cirrhosis) may develop HCV-related hepatocellular carcinoma or decompensated liver cirrhosis; treatment for these health states may include liver transplant. Nearly all individuals in stages F0 through F4 are medically eligible for HCV treatment under current guidelines (>95%) (Stepanova and Younossi 2015). We assume individuals receive treatment only if it is reimbursed by their insurance provider. Treatment transitions patients to a lower cost and higher quality-of-life health state. Detailed information about the parameter estimates are presented in Appendix D.

We assume both payers measure the utility from providing cost-efficient health care in net monetary benefit (NMB). NMB is a single metric that captures both the monetary value of health outcomes and the financial benefits (costs) of health care. We measure health benefits in QALYs, we discount health benefits accrued and costs incurred in the future at an annual discount rate of 3%, and we assume the payer’s value for a marginal QALY is $100,000 per QALY-gained, consistent US-based health economic analyses (Neumann et al. 2016). We implement our HCV model and our optimization framework in the R programming language (R Core Team 2013) and present the results of deterministic and probabilistic sensitivity analysis (PSA). In the PSA, we identify the optimal policy (and all other system outputs) for 1000 randomly generated input sets drawn from the parameter
3.5.2 Results

Single-payer system

The solution to the single-payer system is illustrated in Figure 3.3. In the figure, the dotted black line illustrates the treatment boundary; to the left of the boundary, the optimal policy is to provide treatment in all states; to the right of the boundary, the optimal policy is to not provide treatment. PSA illustrates the sensitivity of the optimal action/inaction regions to the uncertainty in input parameters. In Figure 3.3, we represent a higher probability that treatment is the optimal policy with darker shading.

Consistent with Proposition 3.4.4d, the action/inaction boundary is increasing with respect to age. While the conditions of Proposition 3.4.4 are not strictly satisfied in this numeric example, the resulting policy is of control-limit type with respect to liver fibrosis.

![Figure 3.3: Single payer optimal treatment policy. Black dashed line indicates the treatment boundary. For states to the left of the boundary, treatment is the optimal strategy. For states to the right of the boundary, waiting is the optimal strategy. Shading indicates the probability that treatment is optimal based on probabilistic sensitivity analysis with black representing 100% probability and solid white indicating a 0% probability that treatment is the optimal policy.](image_url)
Table B.1 in Appendix B.2 summarizes the maximum violations of the conditions from Section 3.4.

Figure 3.3 also demonstrates the greater uncertainty in the location of the action boundary at higher ages and lower disease severity. With the base case input parameters, the control-band policy with respect to age at each severity level is not apparent. However, we observe a control-band policy with respect to age in deterministic sensitivity analysis with an increase in the cost of treatment or a decrease in the willingness to pay threshold. It is also observed in Figure 3.3 which identifies higher uncertainty for younger patients with lower disease severity indicating that for some selected input sets in the PSA, the control-band policy with respect to age is optimal. Furthermore, the maximum violations of the conditions required for a control-band policy with respect to age (i.e., $SC_\omega$ and the convexity of $M(f, \omega)$ with respect to $\omega$) are small. Some studies observe a control-limit policy with respect to age (e.g., Alagoz et al. 2013). In contrast, we observe a control-band policy with respect to age. Treatment for HCV may not be optimal for young ages and low severity levels because HCV progresses very slowly and has limited adverse effects for young patients.

**Multi-payer system**

The solution to the multi-payer scenario ($\mu = 65$) is illustrated in Figure 3.4 and is consistent with the results of Section 3.4.2. In the figure, the dotted and dashed lines indicate the first and final payers’ treatment policy boundaries, respectively; the first payer will treat all individuals of age 0 up to the dotted line; and, the final payer will provide treatment for individuals of age 65 up to the dashed line. Similar to Figure 3.3, we illustrate the results of the PSA in the two-payer system such that a higher probability that treatment is the optimal policy is represented with darker shading.

The final payer’s optimal treatment policy is exactly the same as in the single-payer scenario and, consistent with Proposition 3.4.10, the first payer’s optimal treatment policy is strictly a subset of the single-payer optimal treatment policy. This later result follows directly because the conditions of Proposition 3.4.10 are strictly satisfied (i.e., $P$ is IFR and Condition 3.11 is satisfied). Additionally, notice that the structure of the first payer’s
Figure 3.4: Multiple payer optimal treatment policy. Black dotted line represents the first payer’s treatment boundary. Treatment is provided for patients 0-years old until the dotted line. Black dashed line represents the final payer’s treatment boundary. Treatment is provided for patients 65-years old until the dashed-line. Shading indicates the probability that treatment is optimal based on probabilistic sensitivity analysis with black representing 100% probability and solid white indicating a 0% probability that treatment is the optimal policy.

<table>
<thead>
<tr>
<th>Result</th>
<th># of People</th>
<th>95% Range</th>
<th>/100,000*</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Size of optimal treatment population</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single-payer system (i)</td>
<td>3,193,419</td>
<td>[3,013,146 – 3,202,590]</td>
<td>99,293</td>
</tr>
<tr>
<td>Multi-payer system (ii)</td>
<td>2,289,735</td>
<td>[2,392,325 – 1,865,872]</td>
<td>71,195</td>
</tr>
<tr>
<td>Treatments withheld due to fragmentation (i) - (ii)</td>
<td>903,684</td>
<td>[620,821 – 1,336,718]</td>
<td>28,098</td>
</tr>
<tr>
<td>F0 (treatments withheld by severity)</td>
<td>325,924</td>
<td>[162,618 – 556,569]</td>
<td>51,277b</td>
</tr>
<tr>
<td>F1</td>
<td>205,739</td>
<td>[121,649 – 345,480]</td>
<td>26,525b</td>
</tr>
<tr>
<td>F2</td>
<td>172,778</td>
<td>[120,101 – 262,688]</td>
<td>22,327b</td>
</tr>
<tr>
<td>F3</td>
<td>108,795</td>
<td>[77,225 – 145,050]</td>
<td>19,588b</td>
</tr>
<tr>
<td>F4</td>
<td>90,448</td>
<td>[73,114 – 126,287]</td>
<td>19,016b</td>
</tr>
<tr>
<td><strong>Effects of coordination</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Delayed treatments avoided</td>
<td>814,401</td>
<td>[560,191 – 1,180,818]</td>
<td>25,322</td>
</tr>
<tr>
<td>Disease progressions avoided</td>
<td>573,348</td>
<td>[333,218 – 952,545]</td>
<td>17,827</td>
</tr>
<tr>
<td>Early deaths avoided</td>
<td>54,380</td>
<td>[32,020 – 114,843]</td>
<td>2,083</td>
</tr>
</tbody>
</table>

*expressed per 100,000 HCV+ individuals. Population estimates from Cipriano et al. (2018)

bexpressed per 100,000 HCV+ severity-specific individuals.

Table 3.1: US Health Outcomes: Population level results with 95% confidence bands from probabilistic sensitivity analysis.
Figure 3.5: a) Equilibrium incentive payments \((I(f, \omega))\) with respect to age and disease severity. b) Sensitivity analysis on threshold age, \(\mu\). Proportion of HCV-infected population impacted by inefficiency (delayed or withheld treatment access) by threshold age.

optimal treatment policy is consistent with the structure of the single-payer’s optimal treatment policy. Significantly, there is a set of individuals with ages immediately prior to age 65 where it is optimal for the first payer to withhold treatment (i.e., the white region immediately to the left of age 65 in Figure 3.4). In the uncoordinated system, these individuals will be left untreated and may progress into a more severe disease state (or die) prior to reaching age 65 when it becomes optimal for the final payer to provide treatment. The single-payer optimal treatment policy identifies that of the 3.22 million chronically HCV-infected individuals in the US, 3.19 million individuals should receive reimbursement for HCV treatment. Significantly, under the multi-payer system the optimal policy does not provide immediate reimbursement for 28% (903,684) of these people (Table 3.1). Of the untreated individuals, 63.4% will progress into a more severe state (incl. death) prior to reaching the age of 65 and 54,380 unnecessary deaths could be avoided through coordination. Economically, fragmentation results in $64.9 billion of lost NMB (an increase of $31.2 billion NMB for the first payer and a decrease of $96.1 billion NMB for the final payer).
Multi-payer coordinated system

Figure 3.5a illustrates the magnitude of the incentive payments with respect to disease severity and age. There are three important features to note. First, consistent with Proposition 3.4.14, the incentive payments are always less than the cost of treatment. Second, for all severity levels, the incentive payments increase with age. This means that the final payer will pay more for patients that are older. While this may seem counter-intuitive (i.e., if early treatment is desirable, shouldn’t it be that larger incentives earlier are optimal?), the reason for this result is that the first payer has a decreasing incentive to provide treatment as a patient nears the age of transfer. Thus, it takes a larger incentive to encourage the first payer to provide treatment. And third, the incentive payment is decreasing with severity; the final payer must pay a larger incentive for low severity patients because the first payer has the lowest incentive to provide treatment to these individuals.

One alternative mechanism that has been discussed to reduce the economic burden on Medicare is to change the threshold age (Congressional Budget Office 2013). To explore the consequences of this idea on the magnitude of the inefficiency caused by a fragmented system, we re-solve the multi-payer scenario for a variety of threshold ages (see Figure 3.5b). With a threshold age of 65, treatment access is withheld or delayed for 28% of HCV-infected individuals. Changing the threshold age to a slightly lower age (e.g., 60) would increase the inefficiency in the system. For the particular case of access to HCV treatment, the threshold age would need to be lowered to at least 50 years old to begin realizing efficiencies. Alternately, increasing the threshold age increases the number of years the first payer would obtain the benefits of reduced health care costs and so more people would have access to treatment reimbursement immediately.

3.6 Discussion

In this study, we characterize the effects of an uncoordinated health care system, across a patient’s lifespan, on the optimal treatment policy for a generalized disease. We develop a multi-decision-maker MDP and analytically characterize the optimal treatment decisions in both a single- and multi-payer scenario. We integrate a game-theoretic model into the
3.6. Discussion

MDP to study the impact of a coordination mechanism on the optimal treatment policies. To verify and then illustrate our analytical findings, we conduct a robust numeric analysis where we develop a natural history model for HCV, solve the single-payer, the uncoordinated multi-payer, and the coordinated multi-payer scenarios. This numerical case study verifies our analytical findings and demonstrates the magnitude of the impact fragmented health insurance can have on individuals and payers.

Our results highlight the importance of coordinated health care across a patient’s lifespan, particularly from the perspective of patients and the final payer, which is often publicly funded. We find that an uncoordinated health care system in the US impacts 28% of the HCV-infected population. Furthermore, this inefficiency will result in 54,380 more early deaths (<65-years old) and 573,348 individuals progressing into a more severe disease state. We find that transfer payments can coordinate the system, and demonstrate that the equilibrium transfer payments are always bounded by the cost of treatment. Therefore, the transfer payments can be considered as a treatment cost-sharing mechanism between health care payers.

This study has limitations. We model consistent utility across all payers. For example, we do not capture the possibility that the first payer is a cost minimizer while the final payer is a NMB maximizer. However, our results are reflective of a system where both payers are public although administered and budgeted separately. For example, Medicaid and Medicare in the US. Additionally, all analytical results hold when comparing each individual payer with a coordinated system having the same objective. For example, a cost minimizing first payer will always provide treatment to a subset of the optimal treatment population from a cost minimization standpoint, regardless of the final payer’s objective.

Coordination between health care payers would result in a significant increase to the number of patients that receive immediate treatment. However, we do not consider any constraints to providing these increased treatments. For example, there may exist constraints on the infrastructure required to provide additional treatments, including human resources constraints to administer the treatments, manufacturing constraints to produce the treatments, and logistical constraints to distribute the treatments.

We model a constant cost of treatment across payers. In practice health care payers may
pay different prices for drugs. For example, in the US, Medicare is restricted by law from negotiating on pharmaceutical prices, while Medicaid and private insurers often receives price discounts. In future work, we aim to (1) explore the impact of different prices across payers on the characteristics of the optimal treatment policies and (2) study the ability of a coordinating mechanism to address inefficiencies if they continue to exist. To address the first question, we have shown that the optimal treatment region is decreasing in the price of the drug. Therefore all of our analytical results hold when comparing each individual payer with a coordinated system having the same cost of treatment. However, the second question requires the inclusion of an additional player (drug manufacturer) into the dynamic game. While we think a multi-decision-maker MDP with endogenous price negotiations with a third-party is an interesting model to formulate, we leave this for future research.

Finally, we consider a non-infectious disease. As a result of this consideration, the first payers utility is not influenced by the final payer’s treatment decisions. However, for an infectious disease, the first payer may benefit from the final payer providing treatment due to reduced transmission of the disease to patients under the first payer’s care. Furthermore, there may be an enhanced incentive for the first payer to provide treatment to patients nearing the age or transition due to the possibility that these patients transmit the disease to other patients under the first payers care. It is unclear how this will specifically impact each payer’s treatment policy and the coordinating transfer payments.
### 3.7 Summary of Notation to Essay 2

#### Table 3.2: Summary of notation.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Payers</strong></td>
<td></td>
</tr>
<tr>
<td>$X$</td>
<td>First payer</td>
</tr>
<tr>
<td>$Y$</td>
<td>Final payer</td>
</tr>
<tr>
<td><strong>Age-based notation</strong></td>
<td></td>
</tr>
<tr>
<td>$\omega$</td>
<td>Age</td>
</tr>
<tr>
<td>$N$</td>
<td>Maximum age</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Set of all treatment-eligible ages</td>
</tr>
<tr>
<td>$\mu$</td>
<td>First age within the final payer’s coverage</td>
</tr>
<tr>
<td><strong>Severity-based notation</strong></td>
<td></td>
</tr>
<tr>
<td>$f$</td>
<td>Disease severity state</td>
</tr>
<tr>
<td>$F$</td>
<td>Maximum living disease severity state</td>
</tr>
<tr>
<td>$\mathcal{F}$</td>
<td>Set of all treatment-eligible severity states</td>
</tr>
<tr>
<td>$F + 1$</td>
<td>Absorbing death states</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Set of all disease severity states (includes death)</td>
</tr>
<tr>
<td><strong>Treatable states</strong></td>
<td></td>
</tr>
<tr>
<td>$\mathcal{S}$</td>
<td>Set of all treatment-eligible states</td>
</tr>
<tr>
<td>$\mathcal{S}^X$</td>
<td>First payer’s set of treatment-eligible states</td>
</tr>
<tr>
<td>$\mathcal{S}^Y$</td>
<td>Final payer’s set of treatment-eligible states</td>
</tr>
<tr>
<td><strong>Treated states</strong></td>
<td></td>
</tr>
<tr>
<td>$\mathcal{T}_{(f, \omega)}$</td>
<td>Treated state</td>
</tr>
<tr>
<td>$\mathcal{T}$</td>
<td>Set of all treated states</td>
</tr>
<tr>
<td><strong>System states</strong></td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td>System state</td>
</tr>
<tr>
<td>$\mathcal{S}$</td>
<td>Set of all system states</td>
</tr>
<tr>
<td><strong>Transitions</strong></td>
<td></td>
</tr>
<tr>
<td>$P$</td>
<td>Transition probability matrix</td>
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<td><strong>Actions</strong></td>
<td></td>
</tr>
<tr>
<td>$a(s)$</td>
<td>Individual action</td>
</tr>
<tr>
<td>$A(s)$</td>
<td>Set of available actions</td>
</tr>
<tr>
<td><strong>Rewards</strong></td>
<td></td>
</tr>
<tr>
<td>$r(s, a(s))$</td>
<td>Reward</td>
</tr>
<tr>
<td><strong>Other</strong></td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>Disutility of treatment</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Discount Rate</td>
</tr>
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(continued . . .)
Table 3.2: (continued)

<table>
<thead>
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<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>( f^T(\omega) )</td>
<td>Disease severity control-limit threshold for age ( \omega )</td>
</tr>
<tr>
<td>( \Gamma^T(\omega) )</td>
<td>Disease severity control-band, set</td>
</tr>
<tr>
<td>( f^T(\omega) )</td>
<td>Minimum bound on ( \Gamma^T(\omega) )</td>
</tr>
<tr>
<td>( \overline{f^T(\omega)} )</td>
<td>Maximum bound on ( \Gamma^T(\omega) )</td>
</tr>
<tr>
<td>( \omega^T(f) )</td>
<td>Age control-limit threshold for severity ( f )</td>
</tr>
<tr>
<td>( \Omega^T(f) )</td>
<td>Age control-band, set</td>
</tr>
<tr>
<td>( \omega^T(f) )</td>
<td>Minimum bound on ( \Omega^T(f) )</td>
</tr>
<tr>
<td>( \overline{\omega^T(f)} )</td>
<td>Maximum bound on ( \Omega^T(f) )</td>
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</tbody>
</table>
3.8 Bibliography


Chapter 4

Essay 3: Pharmaceutical Pricing in Multi-Payer Health Care Systems

4.1 Introduction

The prices of pharmaceutical drugs varies across health care payers within multi-payer national health care systems (Lichtenberg 2011). In multi-payer systems, variation in prices may occur between competing payers (e.g., between private health care insurance providers) and also across payers who cover patients at different stages of their lifespan. For example, Medicare (generally, covering individuals at least 65-years old) and Medicaid (generally, covering low-income individuals under age 65) are the largest government purchasers of pharmaceutical drugs in the US (Anderson-Cook et al. 2019). On average, Medicare pays 88% more than Medicaid per specialty brand-name prescription ($3,600 versus $1,920) and 182% more than Medicaid per non-specialty brand-name prescription ($155 versus $55) (Anderson-Cook et al. 2019).

A health care payer’s negotiating power influences the price that the payer will incur and is one reason for differences in prices between payers (Grennan 2014, Kouvelis et al. 2015). Payers with more negotiating power will incur lower prices (Morgan et al. 2013). While discriminatory pricing (Ramsay pricing) may be profit-enhancing from the drug manufacturer’s perspective (Danzon 1998), we hypothesize that different prices for different payers exacerbates the inefficiencies already present in multi-payer health care.
systems where payers may not have the incentive to invest in expensive treatments with long-term outcomes (Helland and Klick 2010). For example, in Chapter 3, we prove that fragmentation alone always results in sub-optimal treatment policies when both payers incur the same price for treatment. In Chapter 3, we define the general disease conditions that result in either control-limit and/or control-band policies across multiple patient state dimensions and prove that the treatment policy in a fragmented health care system will target a subset of patients when compared to a single-payer system. We show, in a two-payer system, that a simple transfer payment from the final payer in a patient’s life to the first payer will result in a coordinated treatment policy that is equivalent to what a single payer would decide, maintaining or increasing welfare for both payers and increasing the number of treatments provided. However, if payers attempt to reduce inefficiencies by coordinating treatment coverage, then manufacturers may change their pricing decisions in anticipation of the payers’ strategic behavior. We hypothesize that strategic pricing decisions will impact the benefits of coordination for patients and payers.

Fragmented health care systems are inefficient due to poor incentives to invest in treatments that provide lifelong health benefits (Herring 2010, Fang and Gavazza 2011). These inefficiencies occur because the payer that covers a patient’s health care early in the patient’s lifetime does not capture the lifetime value of costly treatments. For example, Medicaid restricts treatment for sofosbuvir, a cost-effective treatment for the hepatitis C virus (HCV), in many ways not consistent with FDA labelling (Lawitz et al. 2014, Nelson et al. 2015, Afdhal et al. 2014). These restrictions include minimum liver disease severity, alcohol and/or drug abstinence, or required prescriber type. Medicaid denies reimbursement for physician-prescribed HCV treatment for 35% to 46% of patients, in contrast to Medicare which denies between 2.5% and 14% of reimbursement requests (Re et al. 2016, Younossi et al. 2016, Gowda et al. 2018). Intuitively, payers that incur higher costs of treatment will provide treatment to fewer patients (Alagoz et al. 2007a) and Proposition 5.4.5 in Chapter 5, possibly compounding the inefficiency due to fragmentation.

Therefore, there are (at least) two types of inefficiencies in multi-payer systems: fragmentation inefficiency, resulting from inadequate incentives to invest in lifelong care; and, pricing inefficiency, resulting from discriminatory prices across payers over a patient’s
lifespan. In the absence of pricing inefficiency, fragmentation inefficiency can be reduced by simple transfer payments between payers, as demonstrated in Proposition 3.4.15 in Chapter [3]. However, it is unclear whether this is the case in the presence of pricing inefficiency and what impact (if any) these transfer payments have on pricing inefficiencies. When a transfer payment is offered, the payers effectively share the cost of treatment. Due to this coordination, additional patients will receive treatment, beyond what may have been anticipated by a manufacturer during price negotiations. These additional treatments are the result of the final payer’s strategic behavior to seek less expensive treatments (both directly, in regards to the cost of treatment, or indirectly, due to mitigating a patient’s costly progression into a more severe disease state) and are similar to the additional (unauthorized) demand for products that have been purchased in low-cost regions and re-distributed into higher-cost regions, called parallel importing.

Parallel importing, sometimes referred to as ‘grey markets’, where an unauthorized dealer purchases a product from a low-price region and resells it into a high-price region, often occurs in environments where different prices exist for the same product (Inman, 1993; Matsushima and Matsumura, 2010). In a health care setting, parallel importing occurs when pharmaceutical drugs are purchased from regions with lower prices and are distributed into regions with higher prices (Danzon, 1998; Altug and Sahin, 2019). Alternately, rather than the transfer of physical drugs between regions, patients may travel to regions where health care prices are lower to receive treatments that they may not have been able to afford at home (Scherer et al., 2004). Recently, US Democratic candidate Bernie Sanders traveled to Canada, alongside US diabetic patients purchasing inexpensive Canadian insulin, to highlight drug price differences between Canada and the US (Rauhala, 2019). Parallel importing will ultimately reduce price differentials between regions (Altug, 2017; Altug and Sahin, 2019; Ahmadi et al., 2015). However, it is less clear that this same result will occur in a pharmaceutical market if different payers incur different prices for treatments over a patient’s lifetime attempt to coordinate care (i.e., prices are different over time in contrast to differences over geography).

In this essay, we analyze fragmentation and pricing inefficiencies that occur within health care systems with multiple health care payers across a patient’s lifespan, where each
payer may incur a different price for treatment. We seek to identify if a mechanism exists that may reduce or eliminate these inefficiencies and then study the strategic response from drug manufacturers, regarding pricing, in anticipation of these mechanisms. We consider a progressive health condition with a one-time costly intervention where the treatment is only provided if reimbursed by the patient’s health insurance payer. We formulate the problem as a multi-decision-maker Markov decision process (MDP) to capture each payer’s repeated treatment decision while the patient’s health evolves over time. We divide the optimization problem using a threshold patient age that defines when the patient transitions from one payer to the other (e.g., age 65). Considering the cases where prices may be higher or lower for each payer, we use a game-theoretic approach to identify a coordinating mechanism between the two payers. We include the negotiated pricing decision between the manufacturer and each payer into the treatment policy decision sequence. Finally, we numerically compare the optimal solutions of the coordinated and the uncoordinated multi-payer problem, including endogenous pricing decisions, for the case study of access to HCV treatment.

4.2 Literature Review

Pharmaceutical Pricing

Game theoretic models have been widely used to study pharmaceutical pricing decisions and often incorporate a trade-off between higher unit margins (high prices) and higher volumes (low prices). For example, Brekke et al. (2007) and Miraldo (2009) use Hotelling-like models to study the change in the price of pharmaceuticals under reference pricing. Both studies assume an exogenous, linear relationship between the price of the drug and demand. In contrast, several studies have incorporated a payer’s treatment decision into their analysis of price. Barros (2011) uses a two-stage game to analyze the sequential decisions of price and then treatment threshold. The treatment threshold controls the demand for treatment and is used to define which patients will receive treatment based on their random probability of response to the intervention. Antonanzas et al. (2011), Mahjoub et al. (2018), and Critchley and Zaric (2019) use similar multi-stage models to capture price and treatment
decisions and study various pricing arrangements (e.g., risk-sharing arrangements, listing processes, or negotiated pricing scenarios). Overall, the pricing literature has considered demand as exogenous (either deterministic (Levaggi 2014) or stochastic (Gavious et al. 2014, Zaric and O’Brien 2005, Zhang et al. 2011)), as implicitly linked to prices (Brekke et al. 2007, Miraldo 2009), or as a portion of a population decided by a health care payer (either a fixed population (Antonanzas et al. 2011, Barros 2011, Levaggi and Pertile 2016, Mahjoub et al. 2018) or a population size that is influenced by a manufacturer’s marketing effort (Critchley and Zaric 2019)). Additionally, this literature has considered prices as fixed (Lilico 2003), exogenous (Mahjoub et al. 2014), decided by the manufacturer (Barros 2011, Levaggi 2014, Mahjoub et al. 2018), or decided through Nash bargaining with the payer (Antonanzas et al. 2011, Critchley and Zaric 2019).

Optimal Treatment Policies

MDPs are often used to identify the optimal treatment policies for a treatment where the benefit of treatment depends on a patient’s current health status (reviewed in Alagoz et al. (2010)). Alagoz et al. (2004, 2007a,b) study the optimal time to initiate a liver transplant when a patient’s health state and the quality of the offered liver may change over time. Using a similar model, Shechter et al. (2008) study the optimal treatment policy for HIV. Alagoz et al. (2013) study the optimal breast cancer screening policy and Chhatwal et al. (2010) study the the optimal time to conduct breast cancer biopsies. While these studies may provide detailed treatment policies, some do not consider the effects of the cost (or disutility) of treatment (e.g., Alagoz et al. 2004, 2007b, Shechter et al. 2008). Of the studies that do include a disutility of treatment (e.g., Alagoz et al. 2007a, Chhatwal et al. 2010, Alagoz et al. 2013), only Alagoz et al. (2007a) provide any analysis into the impact of exogenous disutility on treatment policies.

Health Care Coordination

There is an extensive body of research on supply chain coordination, identifying numerous contract structures to coordinate fragmented systems under various settings with complex product features, uncertain demand, and marketing investments (e.g., Bernstein and Fed-
Coordination studies that address health care specific issues mainly focus on patient-insurer relationships and physician-hospital relationships. The key problems facing patient-insurer coordination are adverse selection and reclassification risk (Handel et al. 2015, Jones et al. 1993, Koch 2014) and the key problem facing physician-hospital coordination is the alignment of incentives (Salas-Lopez et al. 2014, Suelflow 2016). Aligning incentives is the key issue also faced when considering coordination between payers, which has received less attention. Cochrane (1995) proposes two solutions to coordinate care across patients’ lifetimes. In one solution, insurers incur the cost of inadequate long-term care by paying premiums to future insurers when an unhealthy patient transitions from their care. In the second solution, patient health care risks are pooled among all insurers through patient-level non-creditable health savings accounts. Insurers that cover a patient’s health care costs contribute to or withdraw from these accounts based on changes in the patient’s health. Both of these solutions require significant policy interventions and maintenance to enforce. Yoshida and Tsuruta (2013) empirically study the impact of mandatory contributions to the Health Service Systems for the Elderly. They find that any changes to these contributions, intended to coordinate care across individuals’ lifespans, are incurred largely by employees through increases or decrease to health care premiums.

**Parallel Importing**

Price differences between health care payers may result in strategic behavior to acquire treatments from low-cost regions or from lower-cost payers. Thus, coordination among payers that incur different prices appears to emulate the parallel importing phenomenon where one payer ‘crosses the border’ to co-purchase treatments for patients under a different payer’s coverage. Due to this similarity, it may seem that the insights gained from the parallel importing literature may hold when considering coordination among payers.

Altug and Sahin (2019) study the effects of parallel imports on market entry and pricing decisions in the pharmaceutical industry where prices are different across regions. They find that parallel importing reduces price difference between regions. Similarly, Altug (2017) study domestic grey markets and find that when demand for each genuine retailer
is independent and when all uncaptured demand is available to grey market products, that authorized retailers will drop their retail prices, reducing the retail price gap between authorized and unauthorized products. In a study of parallel imports in the EU, Ganslandt and Maskus (2004) also find that retail prices converge in the presence of parallel imports. Jelovac and Bordoy (2005) use a two-stage game, where first authorized dealers’ retail prices are determined and then unauthorized dealers’ retail prices are determined, to study the impact of parallel importing when regions either differ by health care systems or by health care requirements. They find that when health care systems are different between regions that parallel importing hurts social welfare and when health care requirements are different between regions that parallel importing improves social welfare. Both Ahmadi and Yang (2000) and Chen (2009) find that parallel importing can increase the total demand for a manufacturer’s product, and therefore may increase the manufacturer’s profit, however profit will not always increase as a result of potential decreases in wholesale prices. Hu et al. (2013), Ahmadi et al. (2015), and Ahmadi et al. (2017) all study parallel imports or grey markets and find that manufacturers will strategically respond to these phenomenon by changing their wholesale prices, in contrast to restricting product availability or by investing in enforcement of unauthorized dealers. This strategic response has been referred to as a ‘tolerance of violation’ (Dutta et al. 1994, Bergen et al. 1998, Antia et al. 2004).

4.3 Contribution

Our work provides several meaningful contributions. Compared to the existing research on health care pricing, treatment policies, and coordination, we integrate the treatment policy effects from multiple payers in a fragmented health care systems with endogenous pricing decisions. We are the first pharmaceutical pricing study to consider the effects of multiple payers and also the scenario where each payer receives a different value from the treatment due to differences in patient characteristics under their care (e.g., patient age). We extend the existing literature that studies fragmented health care systems when treatment prices are fixed and constant across payers. We are the first to consider optimal treatment policies that analyze the dynamics of pricing decisions on treatment policies, a manufacturer’s profit,
and payers’ utility.

Consistent with the existing literature, we find that health care systems with multiple payers result in treatment policies that target a subset of the population that would be targeted by a centralized decision maker (social planner). In the scenario where the payer at earlier stages of a patient’s life incurs a lower cost of treatment than the payer at later stages of a patient’s life (e.g., Medicaid and Medicare in the US), we find that coordination results in over-treatment when compared to the optimal treatment policy in a lower-cost system when treatment decisions are made on a lifetime horizon. This may result in larger total budgetary expenditures which may significantly impact systems where both payers are publicly funded. In the scenario where the payer at earlier stages of a patient’s life incurs a higher cost of treatment than the payer at later stages of a patient’s life (e.g., private and public payers in Canada), we find that coordination still provides fewer treatments, even when compared to a scenario with higher costs when treatment decisions are made on a lifetime horizon. Therefore, in these systems a lower average cost may counter-intuitively result in fewer patients receiving treatment.

Extending our model where prices are fixed and different between payers, we study the two-stage process where first, prices are set through negotiations between the health care payers and the manufacturer, and then second, the payers decide their treatment policies. We find that when prices are negotiated in anticipation of coordination between payers, that the benefits of coordination are low for health care payers. In general, we find that payers will prefer an uncoordinated health care system and that manufacturers will prefer a coordinated system. This occurs because prices in equilibrium are significantly higher in a coordinated system. Using a case study of access to HCV treatment, we verify our analytical findings and illustrate our results.

4.4 Model

We study a two-stage health care system: first, the prices of treatment are decided for each health care payer through negotiations with the drug manufacturer; second, each payer determines its treatment policy that defines which patients will receive treatment. We solve
this two-stage game by backwards induction, and therefore we first introduce the treatment model and then introduce the pricing model. For existing treatments, where the price has already been established between the manufacturer and the payer, we use only the treatment model to study the impact of price differences and fragmentation on the outcomes for patients and payers. It is uncommon for pricing policies to be reconsidered after a drug has been introduced (Carone et al. 2012). For new treatments, where both the price and treatment policies have not been decided, we use both the treatment model and the pricing model to study the impact on patients, payers, and the manufacturer.

4.4.1 Treatment Model: Exogenous Pricing

We consider a patient diagnosed with a progressive disease and who is medically eligible for curative treatment. Treatment is costly, provides health benefits for the patient, and is recommended by the patient’s physician. The patient cannot afford the treatment on their own and so will only receive treatment if the cost is reimbursed by their health care payer (i.e., health insurance). The patient is covered by two payers over their life: the first payer (X) provides health care coverage for the patient until they are \( \mu \)-years old and the final payer (Y) provides health care coverage thereafter. The first and final payers incur the cost \( c_i \), \( i \in \{X, Y\} \) when they provide treatment.\(^1\) At regular intervals (e.g., annually), the payer currently covering the patient’s care decides whether or not to reimburse treatment in order to maximize its utility. A health care payer’s utility may reflect profits, costs (negative), life-years or quality-adjusted life-years (QALYs) gained by patients, or the net monetary benefit (NMB) of care. We formulate the patient’s evolving health and the payers’ repeated treatment decisions as a multi-decision-maker, discrete-time, finite-horizon, discounted MDP.

System states

The patient’s untreated health state, \((f, \omega)\), is characterized by two dimensions: disease severity \((f \in \mathbb{F} = \{0, 1, \ldots, F\})\), where \(F\) represents the most severe living disease state).

\(^1\)A summary of all notation can be found in Table 4.1 in Section 4.7 on page 116
and age \((\omega \in \Omega = \{0, 1, \ldots, N\}, \text{ where } N \text{ represents the maximum possible age})\). In practice, disease severity may reflect a variety of patient characteristics (e.g., a vector of lab results). As in prior work (e.g., Sandıkçı et al. 2008, Shechter et al. 2008, Chhatwal et al. 2010, Alagoz et al. 2013), we assume that there exists an ordering of health states that remains the same for all ages. Therefore, without a loss of generality, we model \(f\) as a scalar. Let \(\Gamma\) represent the full set of health states including the absorbing death state, \(F + 1\) (i.e., \(\Gamma = F \cup \{F + 1\}\)).

Patients in a treatment-eligible state, \(\mathcal{S} = \mathcal{F} \times \Omega\), may receive treatment. A patient who receives treatment transitions into an age- and disease-severity-specific absorbing treated state, \(\mathcal{T}_{(f, \omega)}\) (where \(\mathcal{T}_{(f, \omega)}\) is a single state in the set of all treated states \(\mathcal{T}\)). Let \(\mathcal{S}_i\) represent the set of health states where payer \(i\) may provide treatment. Thus, \(\mathcal{S}_X = \{(f, \omega) | \omega \in [0, \mu), f \in \mathcal{F}\}\) and \(\mathcal{S}_Y = \{(f, \omega) | \omega \in [\mu, N], f \in \mathcal{F}\}\). \(\mathcal{S}_X\) and \(\mathcal{S}_Y\) are mutually exclusive and collectively exhaustive subsets of \(\mathcal{S}\). Overall, the system state space is defined as \(\mathcal{S} = \mathcal{S} \cup \mathcal{T} \cup \{F + 1\}\) with individual state \(s\).

**Actions and system dynamics**

The patient transitions between states according to the transition probability matrix \(P\), where individual probabilities depend on the patient’s current state and the action taken by decision maker \(i\), defined as \(p(s'|s, a^i(s, c^X, c^Y))\), where \(s'\) is the future state, \(s\) is the current state, and \(a^i(s, c^X, c^Y)\) is the action taken. Each decision period, the decision making payer will select an action, \(a^i(s, c^X, c^Y)\), from its set of possible actions, \(A^i(s) \in \{0, 1\}\) (where 0 = wait and 1 = treat). Let \(-i\) represent the non-decision-making payer (e.g., \(i = X \Rightarrow -i = Y\), and vice versa). For non-treatment-eligible states, and for the non-decision-making payer, treatment is not an available decision (i.e., \(A^i(s) = \{0\}, \forall s \in \{\{F + 1\} \cup \mathcal{T} \cup \mathcal{S}^X\}\)). An individual \(N\)-years old progresses to death with probability 1 (i.e., \(p(F + 1|(f, N), 0) = 1, \forall f \in \mathcal{F}\)). Once the patient dies, the system remains in the death state indefinitely. If a payer chooses to reimburse treatment, then the patient transitions into the absorbing age- and disease-severity-specific treated state with probability 1 (i.e., \(p(\mathcal{T}_{(f, \omega)}|(f, \omega), 1) = 1, \forall f \in \mathcal{F}\)). Due to the frequent use of the individual transition probabilities when the action taken is wait, we make three notational modifications: age
4.4. Model

progresses with certainty over time (i.e., $\omega' = \omega + 1$), therefore we suppress the ‘$\omega + 1$’ notation; we suppress the explicit action notation; and, we drop the explicit reference to system state $s$. Therefore, $p(s' = (f', \omega + 1)|s = (f, \omega), d'(s, c^X, c^Y) = 0) \equiv p(f'|f, \omega))$.

Rewards

Payers receive rewards (utility) for their decisions depending on the current state and the action taken, which we define as $r^i(s, d'(s, c^X, c^Y))$, $i, i' \in \{X, Y\}$ (where $i$ represents the payer receiving the reward and $i'$ represents the decision making payer). If a payer chooses to reimburse treatment, then both payers may receive a reward. If the first payer provides treatment, then the first payer receives the present value of all expected future utility from the treated patient for the years up until $\mu$, minus the disutility of treatment, $c^X$. The final payer receives the present value of all expected future utility from the treated patient for the years from $\mu$ onward. For each payer, the utility received for treatment is equal to the present value of the expected reward from an embedded Markov process. Let $\delta \in [0, 1]$ represent the one-period discount factor. Let $r_{T_s}(\hat{\omega})$ and $p_{T_s}(T_s|\hat{\omega})$ represent the one-period reward and one-period probability of living, respectively, for a patient who received treatment when they were in health state $s \in S$ and is currently $\hat{\omega}$-years old. Let $\theta^i$ represent an indexing variable that represents the number of future periods until payer $i$’s coverage ends (i.e., $\forall \omega < \mu$, $\theta^X = \mu - \omega - 1$ and $\theta^Y = N - \omega$; and, $\forall \omega \geq \mu$, $\theta^Y = N - \omega$). Formally, the payers’ rewards for any patient treated in state $s \in S^X$ are,

$$r^X(s, 1) = r_{T_s}(\omega) + \sum_{t=1}^{\theta^X} \left[ \delta^t \cdot r_{T_s}(\omega + t) \cdot \prod_{j=0}^{t-1} p_{T_s}(T_s|\omega + j) \right] - c^X \quad (4.1)$$

and,

$$r^Y(s, 1) = \left[ \prod_{j=0}^{\theta^Y} \delta^j \cdot p_{T_s}(T_s|\omega + j) \right] \cdot \left[ r_{T_s}(\mu) + \sum_{t=1}^{N-\mu} \left[ \delta^t \cdot r_{T_s}(\mu + t) \cdot \prod_{k=0}^{t-1} p_{T_s}(T_s|\mu + k) \right] \right]$$

(4.2)

In contrast, if the final payer provides treatment, then the final payer incurs the disutility of treatment, $c^Y$, and the first payer does not receive any reward because all rewards are accrued after the first payer’s coverage has ended. The payers’ rewards for any patient
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treated in state $s \in S^i$ are,

$$r^X(s, 1) = 0$$  \hspace{1cm} (4.3)

and,

$$r^Y(s, 1) = r_{T^i}(\omega) + \sum_{t=1}^{\theta} \delta^t \cdot r_{T^i}(\omega + t) \cdot \prod_{j=0}^{t-1} p_{T^i}(T^i|\omega + j) - c^Y$$  \hspace{1cm} (4.4)

If payer $i$ chooses not to reimburse treatment for a treatment-eligible patient, then payer $i$ receives a one-period age- and disease-severity-specific reward of waiting and the other payer receives no reward (i.e., $r^i(s, a^i(s, c^X, c^Y) = 0) = r(s, 0)$ and $r^j(s, a^j(s, c^X, c^Y) = 0) = 0, \forall s \in S^j$). The one-period reward is zero in all absorbing states.

**Objectives**

We consider three scenarios, $j \in \{A, B, C\}$. The first scenario ($j = A$) represents a benchmark scenario where a social planner makes treatment decisions for both payers across a patient’s entire lifespan. Specifically, the social planner makes the decision to provide treatment incurring the cost $c^X$ for patients in $S^X$ and cost $c^Y$ for patients in $S^Y$. In contrast to the single-payer scenario in Chapter 3, the social planner scenario (i.e., $j = A$) captures the best possible treatment policy in a two-payer system where costs can be different between payers. The second scenario ($j = B$) represents a two-payer uncoordinated scenario where payers act independently when making all treatment decisions. In the third scenario ($j = C$), the final payer has the option to provide an incentive payment to the first payer to encourage the first payer to provide treatment and possibly coordinate the system. We refer to this scenario as the coordinated scenario.

Each decision period, the decision making payer (or social planner on behalf of the payer) decides whether to provide treatment or not. Let $\Psi^i_j(s_0, c^X, c^Y)$ represent payer $i$’s utility, in scenario $j$, when considering a patient in initial state $s_0 \in S$. Each payer chooses a treatment policy to maximize their utility. Let $\overline{a}^i$ represent payer $i$’s treatment policy that represents a vector of actions, one for each time interval in payer $i$’s decision period. Therefore, $\overline{a}^i = \{a^i_0(s_0, c^X, c^Y), a^i_1(s_1, c^X, c^Y), \ldots, a^i_\theta(s_\theta, c^X, c^Y)\} \in \mathcal{A}^i$, where $a^i_t(s_t, c^X, c^Y)$ represents payer $i$’s action in time period $t$ and $\mathcal{A}^i$ represents the set of all possible policies available to payer $i$. Let $\gamma_t$ represent a state progression with probability distribution $P$. 

4.4. Model

Social Planner Scenario \((j = A)\): The social planner’s objective is to maximize the sum of payer \(X\) and \(Y\)’s utilities, as follows,

\[
\max_{\pi^X \in \mathcal{A}^X} \max_{\pi^Y \in \mathcal{A}^Y} \mathbb{E} \left[ \Psi^X_A(s_0, c^X, c^Y) + \Psi^Y_A(s_0, c^X, c^Y) \right]
\]

\[
= \max_{\pi^X \in \mathcal{A}^X} \max_{\pi^Y \in \mathcal{A}^Y} \mathbb{E} \left[ \sum_{t=0}^{\theta} \delta^t \cdot \left[ r^X(s_t, a'_t(s_t, c^X, c^Y)) + r^Y(s_t, a'_t(s_t, c^X, c^Y)) \right] \right] | s_0, c^X, c^Y \]

subject to \(s_{t+1} = \gamma_t\)

\[
\text{for any } s_0 \in \mathbb{S}
\]

Uncoordinated Scenario \((j = B)\): Payer \(i\)’s optimization problem is as follows,

\[
\max_{\pi^i \in \mathcal{A}^i} \mathbb{E} \left[ \Psi^i_B(s_0, c^X, c^Y) \right] = \max_{\pi^i \in \mathcal{A}^i} \mathbb{E} \left[ \sum_{t=0}^{\theta} \delta^t \cdot \left[ r^i(s_t, a'_t(s_t, c^X, c^Y)) \right] \right] | s_0, c^X, c^Y
\]

subject to \(s_{t+1} = \gamma_t\),

\[
\text{for any } s_0 \in \mathbb{S}^i
\]

Coordinated Scenario \((j = C)\): In states \(s \in \mathbb{S}^X\), the final payer may offer an incentive payment to the first payer to encourage treatment. Due to the structure of the state space, the coordinated treatment problem is equivalent to the uncoordinated scenario in Equation 4.6 for any state \(s \in \mathbb{S}^Y\). However, for states \(s \in \mathbb{S}^X\) the treatment problem is significantly different. The sequence of decision events is as follows: first, the final payer offers a state dependent transfer payment to the first payer of the amount \(I(s, c^X, c^Y) \geq 0\); second, the first payer decides if it will accept the payment and provide treatment, or reject the payment and decline to provide treatment in the current period. We assume that the first payer includes the expectation of future transfer payments into its treatment decision.

Let \(\overline{I}\) represent a vector of incentive payments, one for each time interval in the first payer’s decision period. Therefore, \(\overline{I} = \{I_0(s_0, c^X, c^Y), I_1(s_1, c^X, c^Y), \ldots, I_{\theta^X}(s_{\theta^X}, c^X, c^Y)\} \in\)
\( I_t, \) where \( I_t(s_t, c^X, c^Y) \geq 0 \) represents the incentive payment in time period \( t \) and \( I \) represents the set of all possible policies. The final payer’s objective is to select an incentive payment policy that maximizes its utility. Formally,

\[
\max_{I \in I} \mathbb{E} \left[ \Psi_C^Y(s_0, c^X, c^Y) \right] = \max_{I \in I} \mathbb{E} \left[ \sum_{t=0}^{\theta^X} \delta^t \cdot \left[ r^Y(s_t, a^X_t(s_t, c^X, c^Y)) - I_t(s_t, c^X, c^Y) \cdot a^X_t(s_t, c^X, c^Y) \right] + \sum_{t=\theta^X+1}^{\theta^Y} \delta^t \cdot \left[ r^Y(s_t, a^Y_t(s_t, c^X, c^Y)) \right] | s_0, c^X, c^Y \right]
\]

subject to

\[
\bar{a}^X = \arg \max_{a^X \in \mathcal{A}^X} \mathbb{E} \left[ \sum_{t=0}^{\theta^X} \delta^t \cdot \left[ r^X(s_t, a^X_t(s_t, c^X, c^Y)) + I_t(s_t, c^X, c^Y) \cdot a^X_t(s_t, c^X, c^Y) \right] | s_0, c^X, c^Y \right]
\]

\[
\bar{a}^Y = \arg \max_{a^Y \in \mathcal{A}^Y} \mathbb{E} \left[ \sum_{t=\theta^X+1}^{\theta^Y} \delta^t \cdot \left[ r^Y(s_t, a^Y_t(s_t, c^X, c^Y)) \right] | s_0, c^X, c^Y \right]
\]

\[
s_{t+1} = \gamma_t,
\]

for any \( s_0 \in \mathcal{S}^X \)

\( (4.7) \)

### 4.4.2 Pricing Model: Endogenous Pricing

In this section, we extend the treatment model and consider more broadly a health care system with two payers, a set of treatment-eligible patients (normalized to one with discrete distribution \( g(s) \) such that \( \sum_{s \in S} g(s) = 1 \)), and a drug manufacturer. In a two-step process, first, drug prices \( c^X \) and \( c^Y \) are independently negotiated with the manufacturer (pricing model), and second, each payer decides its treatment policy as per scenario \( j \) (treatment model). We solve this game by backwards induction establishing subgame perfect equilibrium for each stage. Let \( V^i_j(s, c^X, c^Y) \) and \( a^*_j(s, c^X, c^Y) \) represent the value function and optimal action from the treatment model (i.e., the optimal solution values of \( \Psi^i_j(s_0, c^X, c^Y) \)).

\( ^2 \) represents the value that payer \( i \) receives when acting optimally with respect to the social planner scenario.
Similarly, let \( T^i_j(c^x, c^y) \) represent the set of states where payer \( i \)'s optimal policy is to provide treatment in scenario \( j \) (i.e., \( T^i_j(c^x, c^y) = \{ s \in S^i \mid a^i_j(s, c^x, c^y) = 1 \} \)). For completeness, let \( V_j(s, c^x, c^y) \) represent the total value that both payers receive in scenario \( j \) (i.e., \( V_j(s, c^x, c^y) = V^x_j(s, c^x, c^y) + V^y_j(s, c^x, c^y) \)), let \( a^*_j(s, c^x, c^y) \) represent the optimal action by the decision making payer (i.e., \( a^*_j(s, c^x, c^y) = a^*_j(s, c^x, c^y) \)), if \( s \in S^i \)), and let \( T^j(c^x, c^y) \) represent the complete treatment policy in scenario \( j \) (i.e., \( T^j(c^x, c^y) = T^X_j(c^x, c^y) \cup T^Y_j(c^x, c^y) \)).

**Objectives**

**Payers:** In all scenarios, when negotiating prices, each payer’s objective is to maximize the total expected value that it receives, \( \forall^i_j \). Formally,

\[
\forall^i_j = \sum_{s \in S} g(s) \cdot V^i_j(s, c^x, c^y) \tag{4.8}
\]

Equation \( 4.8 \) captures both the immediate rewards from treatment and the future expected rewards of treatment implicit in each payer’s value function.

**Manufacturer:** Given the optimal treatment policy from the treatment model, some patients receive treatment immediately, and for patients who do not receive treatment immediately there is some probability of receiving treatment in a future period as a result of progressing into a different disease state. Let \( p(T^i_j \mid s, c^x, c^y) \) represent the total discounted probability of receiving treatment from payer \( i \), ever, for a patient in state \( s \) for scenario \( j \). The discounted probability that the first payer provides treatment is,

\[
p(T^X_j \mid s, c^x, c^y) = a^*_X(s, c^x, c^y) + (1 - a^*_X(s, c^x, c^y)) \cdot \delta \cdot \sum_{f' \in F} p(f' \mid (f, \omega)) \cdot p(T^X_j \mid (f', \omega + 1), c^x, c^y) \tag{4.9}
\]

\( \forall s \in S \)

Equation \( 4.9 \) captures the discounted probability that a treatment-eligible patient re-
ceives treatment from payer X. By definition, the probability that payer X provides treatment does not depend on the treatment decisions of payer Y, although it may depend on payer Y’s incentive payment decision in the coordinated scenario, via \( a^X_c(s, c^X, c^Y) \). In contrast, the probability that payer Y provides treatment does depend on the treatment decisions of payer X. For example, for any state \( s \in \mathcal{S}^X \), if the first payer does not provide treatment, then there is a non-negative probability that the final payer will provide treatment. However, if the first payer provides treatment, then \( p(T^y_j | s, c^X, c^Y) = 0 \) by definition. The discounted probability that the final payer provides treatment is,

\[
p(T^y_j | s, c^X, c^Y) = a^y_j(s, c^X, c^Y) \left[ (1 - a^y_j(s, c^X, c^Y)) \cdot (1 - a^X_j(s, c^X, c^Y)) \cdot \delta \cdot \sum_{f' \in \mathcal{F}} p(f' | (f, \omega)) \cdot p(T^y_j | (f', \omega + 1), c^X, c^Y) \right]
\]

Let \( D^i_j(c^X, c^Y) \) represent the expected discounted total number of treatments (demand) provided to the current set of infected patients, by payer \( i \) in scenario \( j \), defined as,

\[
D^i_j(c^X, c^Y) = \sum_{s \in \mathcal{S}} g(s) \cdot p(T^i_j | s, c^X, c^Y)
\]

The manufacturer’s objective is to maximize profit, \( \Pi_j \), in scenario \( j \), defined as,

\[
\Pi_j = (c^X - \sigma) \cdot D^X_j(c^X, c^Y) + (c^Y - \sigma) \cdot D^Y_j(c^X, c^Y)
\]

where \( \sigma \) represents the manufacturer’s constant marginal cost of production.

**Nash Bargaining:** We consider a setting where each payer and the manufacturer set prices through Nash bargaining. Let \( \alpha^i \in [0, 1] \) represent payer \( i \)’s negotiating power relative to the manufacturer. Therefore \( 1 - \alpha^i \) represents the manufacturer’s negotiating power when negotiating with payer \( i \). Throughout this essay, we present the results when the first payer and the manufacturer negotiate to set \( c^X \) first. Then, the final payer and the manufacturer negotiate to set \( c^Y \) second. We also study the scenario where the sequence of determining \( c^X \) and \( c^Y \) is reversed. However, there are no structural differences and there-
fore these results are omitted.

The equilibrium prices, \( c_{ij}^* \), for scenario \( j \), solve the following,

\[
\arg\max_{c_{ij} \geq 0} \left[ \sum_j P_i \cdot \Pi_j \right]^{(1-\alpha)}
\]

(4.13)

### 4.4.3 Analysis Plan

Overall, the two-stage process is illustrated in Figure [4.1] The pricing process is the same for all scenarios. However, the treatment stage is different for each scenario. First, we solve the treatment model sub-game (exogenous prices). We compare the optimal treatment policies between the uncoordinated scenario and the social planner scenario to identify any treatment inefficiencies. We compare the optimal treatment policies between the uncoordinated scenario and coordinated scenario to identify if coordination is welfare improving for patients and/or payers. And, we compare the coordinated scenario and the social planner scenario to identify if inefficiencies continue to exist in a coordinated system. Second, we solve the full two-stage game (endogenous prices) and compare the equilibrium results between scenarios. We compare the number of treatments provided, the utility that the payer’s receive, and the manufacturer’s profit to evaluate the desirability of coordination from each stakeholders perspective.
4.5 Structural Results

We solve the two-stage model, from Section 4.4, by backwards induction. First, we solve for the optimal treatment actions, $a^*_{ij}(s, c^X, c^Y)$, that define the treatment policies in each scenario, $j$, for a given set of prices (exogenous pricing). Second, we solve for the equilibrium prices, $c^*_{jX}$ and $c^*_{jY}$ (endogenous pricing).

To aid in the presentation of our findings, we apply our two-stage framework to the case of access to chronic HCV treatment in the US. HCV is a timely example on which to study our framework because of the large absolute number of people affected, the disproportionately impact on individuals aged 45 to 65 (right before many individuals shift their health care coverage to Medicare at age 65), the relatively restricted access to treatment reimbursement being faced by individuals prescribed treatment by their physician, and the variance in the cost of treatment across providers. We populate the model using the same parameters as in Chapter 3. A thorough discussion and description of how this natural history model was generated can be found in Appendix D. The disease characteristics of HCV satisfy several common assumptions used in MDP literature (e.g., increasing failure rate or that the utility from providing treatment decreases as a patient’s health deteriorates). However, a majority
of our results do not require any assumptions regarding a disease’s characteristics. Where necessary, we state our assumptions regarding disease characteristics. For illustrative purposes, we provide several figures for visual aids in the presentation of our findings. While these figures are based on the HCV case-study, a majority of our results are generalizable to any disease, and the few results that require basic assumptions are generalizable to diseases with similar characteristics to HCV.

4.5.1 Treatment Model: Exogenous Pricing

Social Planner Scenario:

We solve the social planner scenario (i.e., $j = A$) for a single patient (as presented in Equation 4.5) as a two-price, finite-horizon, discounted MDP. The optimal solution to the social planner scenario is obtained by solving the following set of recursive value function equations (Puterman 1994):

$$
V_A(s, c^X, c^Y) = \max \left\{ r^X(s, 1) + r^Y(s, 1), r^X(s, 0) + r^Y(s, 0) + \delta \sum_{f \in \Gamma} p(f^*|s = (f, \omega)) \cdot V_A((f^* + 1), c^X, c^Y) \right\}
$$

(4.14)

The value functions associated with states $T$ and $F + 1$ are zero by construction and are omitted.

There are several interesting characteristics of the treatment region in the social planner scenario when payers incur different costs. Consider two costs, $c^l$ and $c^h$, that represent a low cost of treatment and a high cost of treatment, respectively, such that $c^l < c^h$. For example in the US, Medicaid incurs a lower cost of treatment than Medicare, on average (Anderson-Cook et al. 2019). Therefore, the US setting inspires the case where $c^X = c^l < c^h = c^Y$. In contrast, for example in Canada, private payers that provide health care coverage for non-retired individuals generally incur higher costs of treatment than government pharmaceutical plans that provide health care coverage for individuals of at least age 65. These price differences may occur because provincial governments often negotiate price discounts with drug manufacturers while private insurers generally do not negotiate
Figure 4.2: Equilibrium treatment regions in the social planner scenario. (a) Case where $c^X > c^Y$. (b) Case where $c^X < c^Y$. The grey shaded region represents where treatment is optimal in the social planner scenario. The large dashed grey line and tight dashed grey line represents the treatment boundaries in the social planner scenario where costs are high or low, respectively, across a patient’s lifespan.

Therefore, the Canadian setting inspires the case where $c^X = c^h > c^l = c^Y$. Figures 4.2a and b illustrate the optimal treatment regions with respect to treatment-eligible states, defined by age and disease severity, for the cases where $c^X > c^Y$ and $c^X < c^Y$, respectively. The schematic shape of the treatment regions are consistent with the treatment policies we find in using a case study of HCV. The two dashed lines in Figure 4.2a and b represent cases where the cost is constant across all states in the social planner scenario and replicate the single-payer findings from Chapter 3.

From Proposition 3.4.5 in Chapter 3, when prices are constant across a patient’s entire lifespan, the treatment region when the cost of treatment is high is a subset of the treatment region when the cost of treatment is low (i.e., $\overline{T}_A(c^h, c^h) \subseteq \overline{T}_A(c^l, c^l)$), as illustrated by comparing the regions bounded by the two dashed lines in Figure 4.2a and b).

For both cases where prices are not constant across a patient’s lifespan (i.e., $c^X > c^Y$ and $c^X < c^Y$), we compare the two-price treatment policy with a one-price treatment policy where the price is constant at either $c^X$ or $c^Y$. Using this simple comparison, we show that the intuition that a health care systems with lower average prices results in more treatments does not always hold. Although $\overline{T}(c^h, c^h) \subseteq \overline{T}(c^l, c^l)$, neither of the following orderings are true: $\overline{T}_A(c^h, c^h) \subseteq \overline{T}_A(c^l, c^l) \subseteq \overline{T}_A(c^l, c^l)$ and $\overline{T}_A(c^h, c^h) \subseteq \overline{T}_A(c^h, c^l) \subseteq \overline{T}_A(c^l, c^l)$. 

For similar discounts (Morgan et al. 2013).
The treatment region when prices are initially high \((c^X = c^h)\) and then low \((c^Y = c^l)\;\text{e.g., private versus public payers in Canada}\) is a subset of the treatment region when prices are low across the patient’s entire lifespan. However, the treatment region when prices are high across the patient’s lifespan is not a subset of the treatment region when prices are initially high and then low. Instead, the social planner will strategically withhold treatment for some patients younger than \(\mu\), knowing that the cost of treatment will drop once patients reach \(\mu\)-years old, at which point it will treat a larger set of older patients as a result of the lower cost. Explicitly, when the social planner incurs a lower cost for treatment in states \(S^Y\), it will treat fewer patient states in states \(S^X\). Therefore, depending on the distribution of the infected population, a lower cost of treatment may decrease the number of immediate treatments provided. This finding is in contrast to previous work (e.g., [Barros 2011] [Antonanzas et al. 2011] [Mahjoub et al. 2018]) that find that a payer’s treatment policy will always target more patients when prices are lower. These results are formalized in Proposition 4.5.1 and are illustrated in Figure 4.2a.

**Proposition 4.5.1**

a) \(\overline{T}_A(c^h, c^l) \subseteq \overline{T}_A(c^l, c^l)\)

b) \(\overline{T}_A(c^h, c^l) \subseteq \overline{T}_A(c^h, c^h)\)

c) \(\overline{T}_A(c^h, c^l) \subseteq \overline{T}_A(c^h, c^l)\)

The treatment region when prices are high across the patient’s entire lifespan is a subset of the treatment region when prices are initially low \((c^X = c^l)\) and then high \((c^Y = c^h)\;\text{e.g., Medicaid versus Medicare in the US}\). However, the treatment region when prices are initially low and then high is not a subset of the treatment region when prices are low across the patient’s entire lifespan. Instead, the social planner will strategically provide treatment to additional patients who are younger than \(\mu\), knowing that the cost of treatment will increase once patients reach \(\mu\)-years of age, at which point it will treat a smaller set of older patients as a result of the higher cost. Explicitly, when the social planner incurs a higher cost for treatment in states \(S^Y\), it will treat more patient states in states \(S^X\). Therefore, depending on the distribution of the infected population, a higher cost of treatment may
increase the number of immediate treatments provided. These results are formalized in Proposition 4.5.2 and are illustrated in Figure 4.2b.

**Proposition 4.5.2**

\[ a) \overline{T}_A(c^h, c^h) \subseteq \overline{T}_A(c^l, c^h) \]

\[ b) \overline{T}_A^X(c^l, c^l) \subseteq \overline{T}_A^X(c^l, c^h) \]

\[ c) \overline{T}_A^Y(c^l, c^h) \subseteq \overline{T}_A^Y(c^l, c^l) \]

Proposition 4.5.1b and Proposition 4.5.2b characterize the counter-intuitive decision making behavior when prices are different across a patient’s lifespan. In any scenario where the price for patients in states \( S^Y \) is high \((c^h)\), if the social planner were to decrease the price for these patients to \( c^l \), then fewer patients in states \( S^X \) would be treated.

**Uncoordinated Scenario:**

We solve the two-payer uncoordinated scenario (i.e., \( j = B \)) for a single patient (as presented in Equation 4.6) as a state-partitioned, two-decision-maker discounted MDP. The optimal solutions to the uncoordinated scenario for the first and final payers are obtained by solving two sets of recursive value function equations (Puterman 1994):

\[
V_B^X(s, c^X, c^Y) = \begin{cases} 
\max \left\{ r^X(s, 1), r^X(s, 0) + \delta \sum_{f' \in \Gamma} p(f'|s = (f, \omega)) \cdot V_B^X((f', \omega + 1), c^X, c^Y) \right\}, & \text{if } s \in S^X \\
0, & \text{if } s \in S^Y
\end{cases}
\]

\[
V_B^Y(s, c^X, c^Y) = \begin{cases} 
\max \left\{ \sum_{f' \in \Gamma} p(f'|s = (f, \omega)) \cdot V_B^Y((f', \omega + 1), c^X, c^Y) \right\}, & \text{if } a^X_B(s, c^X, c^Y) = 1 \\
\max \left\{ r^Y(s, 1), r^Y(s, 0) + \delta \sum_{f' \in \Gamma} p(f'|s = (f, \omega)) \cdot V_B^Y((f', \omega + 1), c^X, c^Y) \right\}, & \text{if } a^X_B(s, c^X, c^Y) = 0
\end{cases}
\]

\forall s \in \mathcal{S}

The value functions associated with states \( \mathcal{T} \) and \( F + 1 \) are zero, for both payers, by construction and are omitted.
Proposition 4.5.3 proves that the total utility in the uncoordinated scenario is always less than in the social planner scenario. The first payer will always be better off and the final payer will be worse off in a fragmented system.

**Proposition 4.5.3** *In an uncoordinated health care system, the total value is lower, the first payer will receive more value, and the final payer will receive less value, compared to a system with a centralized social planner. Formally, *

\[ V_B(s, c^X, c^Y) \leq V_A(s, c^X, c^Y) \]

\[ V^X_B(s, c^X, c^Y) \geq V^X_A(s, c^X, c^Y) \]

\[ V^Y_B(s, c^X, c^Y) \leq V^Y_A(s, c^X, c^Y) \]

Additionally, the optimal treatment policy for a patient in the final payer’s coverage period (i.e., \( s \in S^Y \)) is the same in the uncoordinated scenario as in the social planner scenario.

**Proposition 4.5.4** \( \overline{T}^Y_B(c^X, c^Y) = \overline{T}^Y_A(c^X, c^Y) \)

Next, we establish the conditions such that the set of states in \( S^X \) where treatment is optimal in a multi-payer system is always a subset of the states where treatment is optimal in social planner system. When comparing the treatment regions for the first payer, we require a few common assumptions and technical definitions. We assume the same five assumptions as in Chapter 3 (Assumptions 3.4.1, 3.4.2, 3.4.3, 3.4.4, and 3.4.5, on pages 53, 57, and 61). Briefly, these assumptions require that the utility from treatment diminishes over time and with disease severity, and that a payer will provide treatment when indifferent between actions. Additionally, we make reference to four properties of the probability transition matrix, \( P \): *increasing failure rate, increasing disease rate, healthiest transition state*, and *total probability of living*. Definitions for these terms can be found in Appendix C. Significantly, all prior findings have not required any assumptions regarding the rewards or transition probabilities.
Let $TP(f, \omega_1, \omega_2) \in [0, 1]$ represent the total probability of living from age $\omega_1$ until age $\omega_2$ (where $\omega_1 < \omega_2$), starting in severity state $f$, and let $\bar{f}(f, \omega)$ represent the healthiest transition state that a patient can transition into in a single decision period from state $(f, \omega)$.

**Proposition 4.5.5** Suppose that $P$ is IFR$_f$ and the following holds:

$$\frac{r_{T_2}(\mu) - r_{T_1}(\mu)}{r_{T_1}(\mu)} < \frac{TP(T_{s_1}, \omega, \mu) - TP(T_{s_2}, \omega + 1, \mu) \cdot (1 - p(F + 1|s_1))}{TP(T_{s_2}, \omega + 1, \mu) \cdot (1 - p(F + 1|s_1))},$$  \hspace{1cm} (4.16)

where $s_1 = (f, \omega)$ and $s_2 = (\bar{f}(s_1), \omega + 1)$.

Then, $\overline{T}_B^X(c^X, c^Y) \subseteq \overline{T}_A^X(c^X, c^Y)$.

Condition (4.16) has an intuitive explanation. Consider two states, $s_1$ and $s_2$, where $s_1$ represents the current state $(f, \omega)$ and $s_2$ represents the healthiest possible state that could be entered into by waiting one period (i.e., $s_2 = (\bar{f}(s_1), \omega + 1)$). Condition (4.16) requires that the percent difference in the one-year rewards from treatment between these two states is less than the percent difference in the total probability of living until age $\mu$. A sufficient condition for this to be satisfied is if natural recovery from the disease is not possible.

**Corollary 4.5.6** Suppose that $P$ is IFR$_f$ and natural recovery is not possible (i.e., $\bar{f}(f, \omega) \geq f$, $\forall (f, \omega) \in S^X$). Then, Condition (4.16) holds and $\overline{T}_B^X(c^X, c^Y) \subseteq \overline{T}_A^X(c^X, c^Y)$.

Therefore, if $P$ is IFR and Condition (4.16) holds, then the total treatment region in the multi-payer scenario is a subset of the treatment region in the social planner scenario. In the case of HCV, these two conditions hold (see Appendix B.2 for the maximum violation of these conditions, both zero). Figure 4.3 illustrates the treatment region in the uncoordinated scenario (black shaded region).

**Coordinated Scenario:**

We solve the two-payer optimal treatment problem in the coordinated scenario (i.e., $j = C$) for a single patient (as presented in Equation 4.7) by solving the following sets of equations:
\[
V_C^Y(s, c^X, c^Y) = \max \left\{ \frac{r^Y(s, 1) - I(s, c^X, c^Y)}{\delta} \sum_{f' \in \Gamma} p(f'|s = (f, \omega)) \cdot V_C^Y((f', \omega + 1), c^X, c^Y) \mid \begin{array}{l}
\text{if } a^C_C(s, c^X, c^Y) = 1, \\
\text{if } a^C_C(s, c^X, c^Y) = 0
\end{array} \right., \\
\text{subject to} \\
a^X_C(s, c^X, c^Y)
\]

\[
= \arg\max_{a^X_C(s, c^X, c^Y)} \left\{ \begin{array}{l}
\frac{r^X(s, 1) + I(s, c^X, c^Y)}{\delta} \sum_{f' \in \Gamma} p(f'|s = (f, \omega)) \cdot V_C^Y((f', \omega + 1), c^X, c^Y) \mid \begin{array}{l}
\text{if } a^C_C(s, c^X, c^Y) = 1, \\
\text{if } a^C_C(s, c^X, c^Y) = 0
\end{array} \right.
\right.
\]

\[
V_C^X(s, c^X, c^Y)
\]

\[
= \max \left\{ r^X(s, 1) + I(s, c^X, c^Y), r^X(s, 0) + \delta \sum_{f' \in \Gamma} p(f'|s = (f, \omega)) \cdot V_C^Y((f', \omega + 1), c^X, c^Y) \right\}
\]

\[
\forall s \in S^X
\]

\[
V_C^Y(s, c^X, c^Y)
\]

\[
= \max \left\{ r^Y(s, 1) - c^Y, r^Y(s, 0) + \delta \sum_{f' \in \Gamma} p(f'|s = (f, \omega)) \cdot V_C^Y((f', \omega + 1), c^X, c^Y) \right\}
\]

\[
\forall s \in S^Y
\]

\[4.17\]

Figure 4.3: Equilibrium treatment regions. (a) Case where \( c^X > c^Y \). (b) Case where \( c^X < c^Y \). The black region represents where treatment is optimal in the uncoordinated scenario. The grey region represents the additional region where treatment is optimal in the coordinated scenario. The large dashed grey line and tight dashed grey line represents the treatment boundaries in the social planner scenario where costs are high or low, respectively, across a patient’s lifespan.
In Proposition 4.5.7 we show that there exists a state dependent transfer payment that can coordinate the multi-payer treatment policy to be equivalent to the treatment policy in the social planner scenario.

**Proposition 4.5.7** Suppose that $P$ is IFR$_f$ and Condition 4.16 holds. Then, there always exists a state dependent transfer payment from the final payer to the first payer that coordinates the system (i.e., $\overline{T}_C(c^X, c^Y) = \overline{T}_A(c^X, c^Y)$) and results in at least as high expected outcomes for both payers (i.e., $V_i^C(s, c^X, c^Y) \geq V_i^B(s, c^X, c^Y)$).

Because the coordinated treatment policy is equivalent to the treatment policy in the social planner scenario, we are able to fully characterize the strategic behavior of the final payer as it makes incentive payments. Figure 4.3 illustrates the treatment regions in the coordinated scenario (and uncoordinated scenario). As in Figure 4.2, the shape of the treatment regions are consistent with the treatment policies we find in using the case study of HCV treatment.

There are two significant findings from the coordinated scenario.

**Corollary 4.5.8** Suppose that $P$ is IFR$_f$ and Condition 4.16 holds. If $c^l < c^h$, then,

a) $\overline{T}_C(c^h, c^l) \subseteq \overline{T}_A(c^h, c^l)$

b) $\overline{T}_A(c^l, c^l) \subseteq \overline{T}_C(c^l, c^l)$

First, if $c^X > c^Y$ (e.g., as is the case in Canada), then the coordinated scenario will result in a smaller treatment region for patients in states $S^X$ compared to a social planner scenario in which prices are high across a patient’s entire lifespan (i.e., $\overline{T}_C(c^h, c^l) = \overline{T}_A(c^h, c^l) \subseteq \overline{T}_A(c^h, c^h) \subseteq \overline{T}_A(c^l, c^l)$, Corollary 4.5.8a, see Figure 4.3a). Therefore, given an initial scenario where the first and final payer incur the same cost, if the final payer is able to secure a lower cost and the payers coordinate, then fewer patients will receive treatment. Therefore, in this scenario, if a large portion of the treatment-eligible population is in states $S^X$, it may be desirable (from a ‘number of treatments’ perspective) to incur a higher cost of treatment for the final payer, and thus increase the size of the treatment region in states $S^X$. In contrast to Chapter 3 where we show that simple transfer payments are effective at eliminating
4.5. Structural Results

treatment inefficiencies due to fragmentation, here we find that similar transfer payments when prices are different may result in counter-intuitive relationships between prices and demand (i.e., pricing inefficiencies).

Second, if $c^X < c^Y$ (e.g., as is the case in the US), then the coordinated scenario will result in a larger treatment region for the patients in states $S^X$ compared the social planner scenario when prices are low across patients’ entire lifespans (i.e., $\overline{T}_A^X(c^h, c^h) \subseteq \overline{T}_A^X(c^l, c^l) \subseteq \overline{T}_A^X(c^l, c^h) = \overline{T}_C^X(c^l, c^h)$, Corollary [4.5.8b], see Figure [4.3b]). Therefore, transfer payments may result in over-treatment when compared to the low-cost social planner policy. While large treatment regions are appealing, these over-treatment regions are not the result of practical health-economic treatment decisions using a lifetime horizon (as recommended in health technology decision making ([Neumann et al. 2016](#), [Canadian Agency for Drugs and Technologies in Health 2017](#))), and are instead the result of strategic gamesmanship in a health care environment with discriminatory pricing.

The total demand (i.e., the total number of patients that receive treatment) in the coordinated scenario will always be higher than the total demand in the uncoordinated scenario, given an exogenous set of treatment costs (Propositions [4.5.4](#) and [4.5.5](#) combined with Proposition [4.5.7](#)). Due to the discrete state space, the demand for treatment as a function of cost is a decreasing step-wise function, where steps are defined by threshold prices. Each treatment-eligible state, $(f, \omega)$, has a corresponding threshold price, $c_{j}^{(f, \omega)}$, that defines the maximum price such that treatment is optimal in that state (e.g., for $(f, \omega) \in S^X$, $c^X \leq c_{j}^{(f, \omega)} \iff a_j^{X}((f, \omega), c^X, c^Y) = 1$). Let $C_j$ represent a vector of threshold prices for scenario $j$ across states $(f, \omega) \in S^i$, with individual elements $c_{j}^{(f, \omega)}$. Threshold prices are solved using Algorithm 1 in the Section [4.8](#).

**Remark** Algorithm 1 solves for all threshold prices exactly.

For any disease, given the complete set of threshold prices, $C^i_j$, the optimal treatment policy for any cost of treatment can be immediately identified without resolving the MDP. For example, given the costs $c^X$ and $c^Y$, then the first payer’s treatment policy is $\overline{T}_j^X(c^X, c^Y) = \{(f, \omega) \in S^X \mid c^X \leq c_{j}^{(f, \omega)}\}$. Significantly, the threshold price for every treatment-eligible state is higher in the coordinated scenario versus the uncoordinated sce-
nario.

**Proposition 4.5.9** Suppose that \( P \) is IFR\(_{f} \) and Condition [4.16] holds. Then, \( c_{B}^{(f, \omega)} \leq c_{C}^{(f, \omega)} \), \( \forall (f, \omega) \in \mathcal{S} \).

These higher threshold prices are the reason that the optimal treatment policy in the coordinated scenario targets a larger number of states (treatment region) than the uncoordinated scenario, at the same set of prices, and therefore a larger number of patients will receive treatment. Figure [4.4] illustrates the impact of coordination on the proportion of HCV-infected individuals that fall within the treatment region in the coordinated and uncoordinated scenarios with respect to \( c^{X} \) (Figure [4.4a]) and \( c^{Y} \) (Figure [4.4b]). Figure [4.4a] shows that the number of individuals that receive treatment in both the coordinated and uncoordinated scenarios is decreasing with respect to \( c^{X} \), given a fixed \( c^{Y} \) (i.e., in Figure [4.4a], \( c^{Y} = 85,000 \) to illustrate the impact of \( c^{X} \) on the proportion of the population treated). For any \( c^{X} \), the number of individuals that receive treatment is always higher in the coordinated scenario. For significantly high \( c^{X} \) (\( \sim > 600,000 \)) the total number of treatments converge, because the first payer will not provide treatment to any patients, in either scenario (i.e., the first payer’s treatment regions reduce to zero states at \( c^{X} \)). The total number of treatments does not fall to zero because the final payer still provides treatment, independent of \( c^{X} \). Figure [4.4b] also illustrates that for any \( c^{Y} \), the number of individuals that receive treatment in the coordinated scenario is larger than in the uncoordinated scenario, given a fixed \( c^{X} \) (i.e., in Figure [4.4b], \( c^{X} = 85,000 \) to illustrate the impact of \( c^{Y} \) on the proportion of the population treated). Significantly, the number of treatments provided in the coordinated scenario increases with respect to \( c^{Y} \) at low costs. This is because as the cost for the final payer increases, the final payer will provide larger transfer payments to the first payer to encourage early treatment. Due to the distribution of the HCV-infected population, the increase to the number of treatments provided by the first payer (due to the transfer payments) outweighs the decrease in the number of treatments provided by the final payer (due to the increased costs). As in Figure [4.4a], the number of treatments provided plateaus at high values of \( c^{Y} \), however the two scenarios do not converge because the number of treatments provided by the first payer, at a fixed cost of \( c^{X} = 85,000 \), is
greater in the coordinated scenario compared to the uncoordinated scenario as a result of the higher threshold prices in the coordinated scenario.

4.5.2 Pricing Model: Endogenous Pricing

In this section, we extend our model to consider the case where prices are set through negotiations between the manufacturer and the payers, prior to the payers’ establishing their treatment policies. We study the-two stage process where, first, prices are established through negotiations between payers and the manufacturer, and second, treatment policies are defined (see Figure 4.1 for the decision making timelines). The previous section provides the solution to the final stage, establishing treatment policies (refer to Figures 4.2 and 4.3 for illustrations). Thus, by backwards induction, we solve the equilibrium prices by Equation 4.13, given these the optimal treatment policies.

Intuitively, if a payer has complete bargaining power (i.e., $\alpha^i = 1$), then the payer will
set the price to the minimum level such that the manufacturer’s incentive compatibility constraint is binding (i.e., \( c^i = \sigma \)), because each payer’s utility is decreasing in its price of treatment. In contrast, if the manufacturer has complete bargaining power (i.e., \( \alpha^i = 0 \)), then the manufacture sets the price, making the standard trade-off between margins and volume. Total demand is not a differentiable function, and therefore the equilibrium price cannot be solved by standard differentiation techniques. Instead, we assume that the set of threshold prices, \( C^i_j \), is the set of available prices to the payers and manufacturer during price negotiations.

The equilibrium price for the first payer, \( c^X_j^* \), solves,

\[
c^X_j^* = \arg \max_{c^X \in C^X_j} \left[ \sum_{s \in S} g(s) \cdot V^X_j(s, c^X, c^Y) \right]^{\alpha^X} \cdot \\
\left[ (c^X - \sigma) \cdot D^X_j(c^X, c^Y) + (c^Y - \sigma) \cdot D^Y_j(c^X, c^Y) \right]^{(1 - \alpha^X)}
\]

Similarly, the equilibrium price for the final payer, \( c^Y_j^* \), solves,

\[
c^Y_j^* = \arg \max_{c^Y \in C^Y_j} \left[ \sum_{s \in S} g(s) \cdot V^Y_j(s, c^X, c^Y) \right]^{\alpha^Y} \cdot \\
\left[ (c^X - \sigma) \cdot D^X_j(c^X, c^Y) + (c^Y - \sigma) \cdot D^Y_j(c^X, c^Y) \right]^{(1 - \alpha^Y)}
\]

Because the individual elements of \( C^i_j \) are calculated exactly using Algorithm 1, \( c^X_j^* \) and \( c^Y_j^* \) are the maximizers over a discrete set of possible prices.

In contrast to Chapter 3, we find that when prices are set in anticipation of coordination between payers, that the payers generally receive less total utility in a coordinated scenario compared to the uncoordinated scenario (see Figure 4.5a & b). This occurs because, for all but extreme levels of negotiating power (\( \sim > 0.9 \)), the equilibrium prices in the coordinated scenario are significantly higher than in the uncoordinated scenario. Equilibrium prices in the coordinated scenario are higher because the threshold prices in the coordinated scenario are higher. For example, if the manufacturer is the price setter (i.e., \( \alpha^i = 0 \)), then the manufacturer can set a higher price in the coordinated scenario and achieve the same demand as in the uncoordinated scenario due to the higher threshold prices (i.e., higher margins with
equal demand). Given this insight, it may not be surprising that the manufacturer generally receives higher utility when the payers coordinate (see Figure 4.5c). Because coordination is generally worse for the payers, it may seem that the final payer should simply offer an incentive payment of zero to achieve the same utility as in the uncoordinated scenario. However, it is not sub-game optimal for the final payer to offer zero incentive (recall that the incentive payment decision is made after prices are set). Instead, given any pair of prices, $c^x$ and $c^y$, there exists an incentive payment that is welfare improving for both payers (c.f., Proposition 4.5.7).

**Figure 4.5:** Equilibrium outcomes with respect to the payers’ negotiating power, when payers have equal negotiating power, for the (a) First Payer, (b) Final Payer, and (c) Manufacturer. Solid line represents the equilibrium utility in the coordinated scenario. Dashed line represents the equilibrium utility in the uncoordinated scenario. In all figures, $\alpha^x = \alpha^y$.

Figure 4.6 illustrates the policy space such that each stakeholder’s utility is higher in the coordinated scenario versus the uncoordinated scenario. Each panel of the figure illustrates the preference for a different stakeholder. Shaded regions represent combinations of $\alpha^x$ and $\alpha^y$ where the stakeholder prefers the equilibrium outcome in the coordinated scenario over the uncoordinated scenario. The payers generally prefer the uncoordinated scenario and the manufacturer prefers the coordinated scenario. Significantly, the manufacturer and the payers never mutually prefer coordination given any combination of $\alpha^x$ and $\alpha^y$. Overall, while coordination is welfare increasing for payers given a fixed set of prices (exogenous prices), when prices are included in the decision making process (endogenous prices), payer’s will
generally prefer the uncoordinated scenario due to severe changes in prices in anticipation of coordination.

![Figure 4.6: Stakeholder preference for the coordinated versus the uncoordinated scenario. Shaded regions illustrate where each stakeholder’s utility is higher in the coordinated scenario. (a) First payer’s preference; (b) Final payer’s preference; and (c) Manufacturer’s preference.](image)

4.6 Discussion

In this study, we analyze the effects discriminatory pricing in a multi-payer health care system for a generic disease. We develop a two-stage model that includes a negotiated price-setting stage and a dynamic multi-decision-maker treatment-policy stage. We analytically characterize the optimal treatment decisions for various scenarios where payers incur different prices for the same treatment.

For existing treatments where prices are already established, we study three scenarios. First, we study a scenario where a social planner makes all treatment decisions on behalf of both payers. When the price for treatment is different for each payer, we find that the social planner will strategically under- or over-treat portions of the population. Significantly, we find that the relationship between prices and demand is not monotonic. For example, we find that price increases for payers that cover patients at late stages of life may increase the total number patients that receive treatment. Second, we study a scenario where each health care payer makes treatment decisions independently. We find that the treatment
policy in this uncoordinated scenario will target a subset of the patients that would receive treatment in the scenario where the social planner makes all treatment decisions. Third, we study a scenario where the final payer may make an incentive payment to the first payer to encourage the first payer to provide treatment. We find that a state-dependent incentive payment *always* exists that is incentive compatible and is welfare improving or maintaining for both payers. In this coordinated scenario, the equilibrium treatment policy is the same as in the scenario where the social planner makes all treatment decisions.

In the case where the first health care payer (e.g., covering patients under age 65) incurs a lower cost than the final health care payer (e.g., covering patients at least age 65; Medicaid and Medicare in the US), we find that coordination will promote over-treatment when compared to the optimal treatment policy generated using a lifetime horizon at a lower average cost. Therefore coordination between payers that incur different prices may increase the total pharmaceutical spending beyond what is cost-effective over a lifetime horizon. In the case where the first health care payer incurs a higher cost than the final health care payer (e.g., private payers and public payers in Canada), we find that coordination will still result in under-treatment when compared to the optimal treatment policy generated using a lifetime horizon at a higher average cost.

For new treatments, where prices have not yet been set, we analyze the two-stage process where first, prices are set, and second, treatment policies are established. When prices are established through negotiations between each payer and the drug manufacturer, we find that the benefits of coordination, from the payers’ perspective, are reduced. The equilibrium price in the scenario where payers can coordinate is generally higher than the equilibrium price in the scenario where coordination is not possible. Therefore payers will generally prefer a scenario where coordination is not possible while the manufacturer will prefer the scenario where payers coordinate.

Using a detailed natural history model of HCV, we numerical verify our analytical findings and demonstrate the impact of exogenous price differences, and endogenous price setting, on the desirability of coordination from both the payers’ and manufacturer’s perspectives.

From a policy maker’s perspective, it initially may seem appealing to allow, encourage,
or facilitate incentive payments between payers to increase the number of treatments provided. We find that coordination, in a setting where prices are fixed, will increase size of the treatment region, increase the number of treatments provided, and improve or maintain the welfare to all payers. However, it may be difficult to realize these benefits for future drugs where the prices have not yet been established. For example, when manufacturers anticipate that coordination will occur, we find that generally the payers will receive less net monetary benefit compared to the scenario where the payers negotiate prices and establish treatment policies independently. Our results therefore show the importance of considering the impact of endogenous pricing when evaluating the desirability of policies aimed at increasing treatment volumes.

In contrast to the literature on parallel imports that find that prices generally decrease when there is parallel importing, we find that prices generally increase when payer’s coordinate. While Danzon (1998) points out that discriminatory pricing encourages higher levels of research and development than would occur if all prices were equal and therefore that parallel importing may limit research and development, we find that coordination generally increases prices and treatment volumes, and therefore a drug manufacturer may actually conduct more research and development if payers coordinate.

This study has limitations. We model consistent utility across all payers. For example, we do not capture the possibility that the first payer is a cost minimizer while the final payer is a NMB maximizer. However, our results are reflective of a system where both payers are public although administered and budgeted separately. For example, Medicaid and Medicare in the US. Additionally, all analytical results hold when comparing each individual payer with a coordinated system having the same objective. For example, a cost minimizing first payer will always provide treatment to a subset of the optimal treatment population from a cost minimization standpoint, regardless of the final payer’s objective. We also assume that the date when patients transition between payers is fixed and known to all payers. We do not consider the scenario where payers are uncertain of when a patient leaves their coverage, nor do we consider the scenario where a payer is uncertain that a patient will enter their coverage (e.g., individuals that change jobs may leave one payer’s coverage and enter another payer’s coverage at any time). Furthermore, we do not consider that a patient
may be able to decide the date that they transition coverage. Incorporating uncertainty into the timing of the transition between insurers may increase the inefficiencies due to fragmentation and price discrimination, but the magnitude and the relative impact on different patient groups is unknown and an interesting area for future study. Finally, we study a coordinated scenario where the final payer offers a state-dependent (severity and age) incentive payment. In practice, it may be difficult to implement such a detailed schedule of payments and instead a fixed, severity-based, or age-based incentive payment may be more practical. While we prove that a age- and severity-based incentive payment always exists, it is not clear that coordination can occur with less nuanced incentive programs. Therefore, we believe a valuable direction of further study may be to explore various permutations of the state-dependent incentive payments.
4.7 Summary of Notation to Essay 3

Table 4.1: Summary of notation.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agents</td>
<td>First payer</td>
</tr>
<tr>
<td>X</td>
<td>Final payer</td>
</tr>
<tr>
<td>Y</td>
<td>Index for payers (i.e., $i \in {X, Y}$)</td>
</tr>
<tr>
<td>i</td>
<td>Index for payers (i.e., $i \in {X, Y}$)</td>
</tr>
<tr>
<td>Scenarios</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>Social planner scenario</td>
</tr>
<tr>
<td>B</td>
<td>Uncoordinated scenario</td>
</tr>
<tr>
<td>C</td>
<td>Coordinated scenario</td>
</tr>
<tr>
<td>j</td>
<td>Index for scenarios (i.e., $j \in {A, B, C}$)</td>
</tr>
<tr>
<td>Objective Functions</td>
<td></td>
</tr>
<tr>
<td>$\gamma_i^j$</td>
<td>Payer i’s objective function in scenario j</td>
</tr>
<tr>
<td>$\Pi_f$</td>
<td>Manufacturer’s objective function in scenario j</td>
</tr>
<tr>
<td>Price of Treatment</td>
<td></td>
</tr>
<tr>
<td>$c_i$</td>
<td>Price that payer i pays the manufacturer</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Manufacturing cost for manufacturer</td>
</tr>
<tr>
<td>Age-based notation</td>
<td></td>
</tr>
<tr>
<td>$\omega$</td>
<td>Age</td>
</tr>
<tr>
<td>N</td>
<td>Maximum age</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Set of all treatment-eligible ages</td>
</tr>
<tr>
<td>$\mu$</td>
<td>First age within final payer’s coverage period</td>
</tr>
<tr>
<td>Severity-based notation</td>
<td></td>
</tr>
<tr>
<td>$f$</td>
<td>Disease severity state</td>
</tr>
<tr>
<td>$F$</td>
<td>Maximum living severity state</td>
</tr>
<tr>
<td>$\mathbb{F}$</td>
<td>Set of all treatment-eligible severity states</td>
</tr>
<tr>
<td>$F + 1$</td>
<td>Absorbing death states</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Set of all severity states (includes death)</td>
</tr>
<tr>
<td>Treatable states</td>
<td></td>
</tr>
<tr>
<td>$\mathbb{S}$</td>
<td>Set of all treatment-eligible states</td>
</tr>
<tr>
<td>$\mathbb{S}^i$</td>
<td>Payer i’s set of treatment-eligible states</td>
</tr>
</tbody>
</table>

(continued . . .)
### Table 4.1: (continued)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Treated states</strong></td>
<td></td>
</tr>
<tr>
<td>$T_{(f,\omega)}$</td>
<td>Treated state</td>
</tr>
<tr>
<td>$\mathbb{T}$</td>
<td>Set of all treated states</td>
</tr>
<tr>
<td><strong>System States</strong></td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td>System state</td>
</tr>
<tr>
<td>$S$</td>
<td>Set of all system states</td>
</tr>
<tr>
<td><strong>Transitions</strong></td>
<td></td>
</tr>
<tr>
<td>$P$</td>
<td>Transition probability matrix</td>
</tr>
<tr>
<td><strong>Actions</strong></td>
<td></td>
</tr>
<tr>
<td>$a^i(s, c^X, c^Y)$</td>
<td>Payer $i$’s action</td>
</tr>
<tr>
<td>$A^i(s)$</td>
<td>Payer $i$’s set of available actions</td>
</tr>
<tr>
<td><strong>Rewards</strong></td>
<td></td>
</tr>
<tr>
<td>$r^i(s, a(s, c^X, c^Y))$</td>
<td>Payer $i$’s reward</td>
</tr>
<tr>
<td><strong>Treatment Policies</strong></td>
<td></td>
</tr>
<tr>
<td>$\overline{T}^j(c^X, c^Y)$</td>
<td>Set of states where payer $i$ provides treatment in scenario $j$</td>
</tr>
<tr>
<td>$\overline{T}_j(c^X, c^Y)$</td>
<td>Total set of states where treatment is provided in scenario $j$</td>
</tr>
<tr>
<td><strong>Population Distribution</strong></td>
<td></td>
</tr>
<tr>
<td>$g(s)$</td>
<td>Discrete population distribution</td>
</tr>
<tr>
<td><strong>Probability of Treatment</strong></td>
<td></td>
</tr>
<tr>
<td>$p(\overline{T}^j \mid s, c^X, c^Y)$</td>
<td>Probability that a patient in state $s$ receives treatment from payer $i$ in scenario $j$</td>
</tr>
<tr>
<td><strong>Demand</strong></td>
<td></td>
</tr>
<tr>
<td>$D^j(c^X, c^Y)$</td>
<td>Demand for treatments from payer $i$ in scenario $j$</td>
</tr>
<tr>
<td><strong>Other</strong></td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>Discount rate</td>
</tr>
</tbody>
</table>
4.8 Algorithm 1: Threshold Prices

Algorithm 1 solves for the exact vector of threshold prices for the first payer, $C^X$. Equivalent calculations for the final payer’s and social planner’s threshold prices follow a similar format and are omitted. This algorithm takes advantage of the property that the benefit function equals zero at the threshold price (i.e., the payer is indifferent between treatment and waiting). Throughout this algorithm, two vectors are used: $C^X_j$ represents the vector of threshold prices; $B$ represents the vector of benefit function values, with one element for each threshold price in $C^X_j$. By definition, the benefit function is increasing in the cost of treatment. Therefore, the ordering of elements in $C^X_j$ and $B$ will be the same. The following set of comments supplement Algorithm 1 and relate to the lines marked with ‘//’.

**Line 1.7:** For the current iteration, $(f, \omega)$, populate a vector of benefit function values, one for each threshold price.

**Line 1.12:** $c^+$ defines the threshold price that corresponds to the smallest positive benefit function value from the vector $B$.

**Line 1.13:** $c^−$ defines the threshold price that corresponds to the largest non-positive benefit function value from the vector $B$.

**Line 1.14:** If $0 \leq b$, then the threshold price for the current iteration, $(f, \omega)$, must be lower than all existing threshold prices currently in the vector $C^X_j$ (see Figure 4.7a).

**Line 1.16:** If $b \leq 0 \leq \bar{b}$, then the threshold price for the current iteration, $(f, \omega)$, must be intermediate with respect to the existing threshold prices currently in vector $C^X_j$ (see Figure 4.7b).

**Line 1.20:** If $\bar{b} \leq 0$, then the threshold price for the current iteration, $(f, \omega)$, must be higher than all existing threshold prices currently in the vector $C^X_j$ (see Figure 4.7a).

**Line 1.26:** Initially, element $c_0$ was used to initialize the vector $C^X_j$. Once through the first iteration, this element is no longer necessary.
Let $B^X_j((f, \omega), c^X, c^Y)$ represent the first payer’s benefit function. For any $c^Y$:

**Data:** $r^X((f, \omega), 1)$ and $r^X((f, \omega), 0), \forall (f, \omega) \in S^X$, and $P$

**Result:** $C^X_j$, vector of threshold prices for states $(f, \omega) \in S^X$

1.1 begin
1.2 Initialize $C^X_j \leftarrow \{c_0 = 0\}$
1.3 foreach $\omega \in \{\mu - 1, \mu - 2, \ldots, 0\}$ do
1.4    foreach $f \in F$ do
1.5      Solving the critical price for state $(f, \omega)$:
1.6        Initialize $B \leftarrow \emptyset$
1.7        foreach $c^X \in C^X_j$ do
1.8          $B \leftarrow B \cup B^X_j((f, \omega), c^X, c^Y)$ \hfill //
1.9        end
1.10       $B^+ = \{b \in B : b > 0\}$ and $B^- = \{b \in B : b \leq 0\}$
1.11       $\bar{b} = \max[B]$ and $\underline{b}^+ = \min[B^+]$
1.12       $\underline{b} = \max[B^-]$ and $\bar{b} = \min[B]$ \hfill //
1.13       $\bar{c} = \max[C^X_j]$ and $\underline{c}^+ = \{c^X \in C^X_j \ | \ B^X_j((f, \omega), c^X, c^Y) = \bar{b}^+\}$ \hfill //
1.14       $\underline{c}^− = \{c^X \in C^X_j \ | \ B^X_j((f, \omega), c^X, c^Y) = \underline{b}^-\}$ and $\underline{c} = \min[C^X_j]$ \hfill //
1.15       if $0 < b$ then \hfill //
1.16          $c^j_{(f,\omega)} = \max[0, \underline{c} = \frac{b}{1−\delta·(1−p(F+1|f,\omega))}]$
1.17       else if $\underline{b} \leq 0 \leq \bar{b}$ then \hfill //
1.18          $\gamma = \frac{(0-\underline{b})}{(\bar{b}−\underline{b})} \in [0, 1]$
1.19          $c^j_{(f,\omega)} = \bar{c} + \gamma \cdot (\underline{c}^+−\bar{c}^−)$
1.20          Clear $\gamma$
1.21       else if $\bar{b} \leq 0$ then \hfill //
1.22          $c^j_{(f,\omega)} = \bar{c} + |\bar{b}|$
1.23       end
1.24       $C^X_j \leftarrow C^X_j \cup \{c^j_{(f,\omega)}\}$ \hfill //
1.25       Clear $B, B^+, B^−, \bar{b}, \underline{b}, \bar{c}^+, \underline{c}^+, \bar{c}^−, \underline{c}^−$
1.26       end
1.27       if $\omega = \mu - 1$ then $C^X_j \leftarrow C^X_j \setminus \{c_0\}$ \hfill //
1.28 end

**Algorithm 1:** Defining Threshold Prices
Figure 4.7: Algorithm scenarios. Black dots represent pairs of threshold prices and benefit function values from the vectors $C^X_j$ (x-values) and $B$ (y-values). a) New threshold price will be below lowest existing threshold price. b) New threshold price will be intermediate compared to existing threshold prices. c) New threshold price will be higher than largest existing threshold price. The new threshold is always where the benefit function equals zero.
4.9 Bibliography


Chapter 5

Conclusion

Discussion

In this thesis, I examine various decision making challenges faced by health care decision makers. I study multi-step decision processes that relate to pricing, access, and treatment policies for new drugs. First, I integrate a manufacturer’s ability to influence demand through marketing effort in an analysis of pricing and access policies. Second, I study the impact of fragmented health care systems on treatment policy decisions. And finally, I study the multi-stage process where prices are determined through negotiations and then treatment policies are determined by health care payers, in a fragmented health care system.

In my first essay, I build on the existing literature that considers pricing and listing processes from the payer’s, manufacturer’s and a societal perspective. In contrast to previous work, I capture the manufacturer’s ability to influence demand through marketing. I study a three-stage decision process for each pricing and access policy: first, prices are determined through some price setting mechanism (e.g., through negotiations or a listing process); second, the payer decides the treatment criteria to treat new patients; and third, the manufacturer decides its investment into marketing activities that impacts the demand for the treatment. Significantly, I find that the manufacture’s ability to influence the demand for treatments has a meaningful impact on the manufacturer’s, payer’s, and societal preferences. Therefore, this work establishes the importance of incorporating pharmaceutical marketing into pricing and access policy decisions and highlights the ripple effects of pricing and access decisions throughout a health care system. I find that all non-value-based
policies result in either restricted access or suboptimal treatment coverage. I find that marketing is the highest in the first-best setting where all decisions are made by a social planner. I also find that the value-based pricing with risk-sharing arrangement is preferred by the manufacturer and from a societal perspective whereas no policy is universally preferred by the health care payer.

In my second essay, I study treatment policies when there is more than one payer in a health care system. Single-decision-maker treatment policies have been thoroughly studied in previous work. However, multi-payer treatment policies have not been considered in the operations research literature. Significantly, this work more accurately reflects the current health care systems in Canada and the US where there is an almost universal shift of health care coverage when patients reach age 65. I develop a multi-decision-maker Markov decision process to capture the payers’ repeated decision processes, incorporating patients’ uncertain disease progressions. I find general disease conditions that result in control-limit and control-band treatment policies with respect to patients’ health states. I find that a fragmented system always results in a treatment policy that targets a subset of patients when compared to a centralized system and I find the conditions such that treatment gaps exist for intermediate-aged aged patients. To address these inefficiencies, I prove that simple transfer payments between payers can coordinate the system to be equivalent to a single-payer system and increase or maintain the welfare for all payers. This study demonstrates that payers are intrinsically motivated to provide lifetime optimal treatments when a mechanism exists to coordinate over a patient’s lifetime. Therefore, structured policies that are intended to coordinate care across a patient’s lifetime (e.g., mandatory contributions to the Health Service Systems for the Elderly in Japan [Yoshida and Tsuruta (2013)]) may be more costly, more time-consuming and more difficult to implement compared to the incentive compatible transfer payments between payers.

In my third essay, I study the compound process of pricing and treatment decisions in fragmented health care systems. Payers may have different abilities to negotiate prices with drug manufactures and therefore it is common that payers incur different costs for the same treatment. Using a game theoretic approach, I formulate a two-stage problem where first each payer and the manufacturer negotiate to determine the price of the new
treatment. Then, each payer defines its own treatment policy. I find that pricing differences exacerbates the inefficiencies that already exist as a result of fragmentation. Significantly, I find that both over- and under-treatment can occur when payers incur different prices. Furthermore, when coordination between payers is anticipated during the pricing process, prices will increase. As a result of these increased prices, I find that the manufacturer will generally prefer when payers coordinate where payers prefer a system where coordination is not possible.

Using the case study of treatment for the hepatitis C virus, I verify the analytical findings from my second and third essays. I find that the treatment policies in a multi-payer system are suboptimal when compared to the treatment policy in a centralized decision making scenario. I find that prices will generally increase when coordination between payers is anticipated during the pricing process and as a result, manufacturers will prefer coordination.

Overall, I provide two significant contributions in this thesis. First, I find that the structure of a health care environment has a significant impact on the number of treatments provided and the economic outcomes for payers and manufactures. Significantly, I find that the optimal treatment policies that are recommend in lifetime-optimal cost-effectiveness studies are different than the treatment policies that individual payers have the incentive to implement in a fragmented health care system. I find that a fragmented health care system always results in treatment inefficiencies. Therefore, this thesis provides support for implementing coordinating mechanisms between health care payers, single-payer health care systems, or health care systems where payers are responsible for providing care across a patient’s entire life.

Second, and most broadly, I provide analytical and empirical evidence that supports considering multiple stages of health care decision making into policy level decisions. In my first essay, I demonstrate that a drug manufacturers ability to influence demand has a meaningful impact on the desirability of various pricing and access policies. For example, on its own a risk-sharing sharing arrangement appears to benefit all stakeholders (payers, by reducing the economic burden of unsuccessful treatments; manufacturers, by providing a market for drugs that would otherwise not receive reimbursement approval; and, patients,
who may receive life-saving treatments). However, a risk-sharing arrangement is not mutually preferred by payers and drug manufacturers when demand is influenced by a manufacturers marketing effort. Significantly, a less regulated environment where manufacturers and payers negotiate for drug prices generally results in better outcomes for all stakeholders. In my third essay, I find that the benefits of coordinating contracts are diminished when price setting is included in the decision making process. Given exogenous treatment prices, I show (in my second essay) that coordination improves or maintains the welfare for all payers and increases the number of patients that receive treatment, increasing demand for the drug from the manufacturer. However, when prices are negotiated in anticipation of coordination, I find that payers and patients may be worse off compared to when coordination is not possible. Therefore, this thesis provides evidence that policy makers must consider the impact of multiple health care decisions on policies that may only be intended to address an isolated health care problem.

**Future Research**

The research included in this thesis has established the foundation to a number of interesting streams of future work. In the first essay, I assume shared information regarding the expected benefit that the new treatment provides. However, there may be asymmetries between the payer and the drug manufacturer about the expected effectiveness of the drug. While I demonstrate that all pricing and access policies suffer from some form of inefficiency, even with shared information, further work that explores asymmetries may provide further insight into the misalignment of incentives between payers and manufacturers. Additionally, I do not consider the possibility that the manufacturer may target their marketing towards individuals that have the highest probability of response to the drug. I only consider the scenario where a manufacturer cannot observe patient characteristics that indicate higher probabilities of successful treatment (e.g., treatment that depends on a specific genotype). I believe that a manufacturer may have the incentive to target individuals with high probabilities of response, especially when there is a pay-for-performance component of the pricing or listing policy, and leave this to future work.
Most significantly, I provide a foundation for research into multi-payer health care systems. I am the first to explicitly model the effects of fragmentation on treatment policies. In my second and third essays I consider a constant measurement of utility for all payers (i.e., all payer’s value net monetary benefit, or all payers value quality-adjusted life-years). In practice, payers may have different values. For example, a private payer may be profit driven, balancing the cost of future care with health care premiums earned, while a public payer may value additional life-years gained. The results that I provide are reflective of a system where both payers are, for example, publicly funded although budgeted separately, like Medicaid and Medicare in the US. While all of my results hold when comparing each individual payer with a centralized system where the centralized system shares the same utility objective, I believe an interesting avenue for future research would be to consider the impact of different, possibly opposing, objectives across payers. Furthermore, I consider a system where patients transition between payers at a pre-defined, perfectly observable age. However, it may be interesting to explore the scenario where patients may transition between payers at any time (unknown age), possibly more than once in their lifetime. This scenario would reflect the system where, for example, individuals switch jobs a number of times throughout their lifetime and each employer provides group coverage using a different health care insurer. Finally, I study coordinating contracts between payers that depend on a patient’s age and disease severity. However, in practice these nuanced contracts may be difficult to implement due to the large number of possible patient health states. Instead, fixed, age-specific, or severity-specific contracts may be more realistic to implement, although it is unclear that these contracts could provide similar benefit compared to the individual state-specific contracts from my second and third essays. Overall, there are a number of real-world-motivated avenues of future research that would address the challenges within fragmented health care systems.
5.1 Bibliography

## Appendix A

### Appendix: Essay 1

### A.1 Summary of Equilibrium Notation

<table>
<thead>
<tr>
<th>Policy</th>
<th>Price</th>
<th>Treatment Threshold</th>
<th>Marketing</th>
<th>Demand</th>
<th>Profit</th>
<th>NMB</th>
<th>Social Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>(FB) First-Best</td>
<td>$p^{FB}$</td>
<td>$L^{FB}$</td>
<td>$m^{FB}$</td>
<td>$D^{FB}$</td>
<td>$\Pi^{FB}$</td>
<td>$NMB^{FB}$</td>
<td>$W^{FB}$</td>
</tr>
<tr>
<td>(NP) Negotiated Pricing</td>
<td>$p^{NP}$</td>
<td>$L^{NP}$</td>
<td>$m^{NP}$</td>
<td>$D^{NP}$</td>
<td>$\Pi^{NP}$</td>
<td>$NMB^{NP}$</td>
<td>$W^{NP}$</td>
</tr>
<tr>
<td>(OP) Open Pricing</td>
<td>$p^{OP}$</td>
<td>$L^{OP}$</td>
<td>$m^{OP}$</td>
<td>$D^{OP}$</td>
<td>$\Pi^{OP}$</td>
<td>$NMB^{OP}$</td>
<td>$W^{OP}$</td>
</tr>
<tr>
<td>(CP) Controlled Pricing</td>
<td>$p^{CP}$</td>
<td>$L^{CP}$</td>
<td>$m^{CP}$</td>
<td>$D^{CP}$</td>
<td>$\Pi^{CP}$</td>
<td>$NMB^{CP}$</td>
<td>$W^{CP}$</td>
</tr>
<tr>
<td>(LP) Listing Process</td>
<td>$p^{LP}$</td>
<td>$L^{LP}$</td>
<td>$m^{LP}$</td>
<td>$D^{LP}$</td>
<td>$\Pi^{LP}$</td>
<td>$NMB^{LP}$</td>
<td>$W^{LP}$</td>
</tr>
<tr>
<td>(RS) Risk-Sharing</td>
<td>$p^{RS}$</td>
<td>$L^{RS}$</td>
<td>$m^{RS}$</td>
<td>$D^{RS}$</td>
<td>$\Pi^{RS}$</td>
<td>$NMB^{RS}$</td>
<td>$W^{RS}$</td>
</tr>
<tr>
<td>(VR) Value-Based Pricing with Risk-Sharing</td>
<td>$p^{VR}$</td>
<td>$L^{VR}$</td>
<td>$m^{VR}$</td>
<td>$D^{VR}$</td>
<td>$\Pi^{VR}$</td>
<td>$NMB^{VR}$</td>
<td>$W^{VR}$</td>
</tr>
</tbody>
</table>

Table A.1: Summary of equilibrium notation.
A.2 Proofs

Proof of Proposition 2.4.1 (first-best solution)

Social welfare is equal to:

\[ W = \Pi + NMB = m(1 - L)\left[\frac{b}{2} \frac{1 + L}{2} - c\right] - \frac{Km^2}{2} \tag{A.2a} \]

Solving the first-order conditions (FOCs) (i.e., \( \frac{dW}{dL} = 0 \) and \( \frac{dW}{dm} = 0 \)), there are three candidate solutions \((m, L)\): \((0, 1)\), \((0, 1 - \frac{2(b-c)}{b})\), \((\frac{(b-c)^2}{2bK}, \frac{c}{b})\). The first and second candidate solutions result in zero social welfare because of the property that \( \theta(0) = 0 \). The third candidate solution results in positive social welfare for all \( 0 < c < b \) and \( K > 0 \). Using the second-partial derivative test, the third candidate point is a local maximizer. Thus, \( L^{FB} = \frac{c}{b} \) and \( m^{FB} = \frac{(b-c)^2}{2bK} \). It follows from (2.1) that \( D^{FB} = \frac{(b-c)^3}{2bK} \) and from (A.2a) that \( W^{FB} = \frac{(b-c)^4}{8b^2K} \).

Proof of Proposition 2.4.2 (incentive compatible first-best solution)

Price must satisfy two conditions: (1) \( p \geq \frac{c + (Km)}{2(1 - L)} \) (to satisfy the manufacturer’s rationality constraint, \( \Pi \geq 0 \)), and (2) \( p \leq \frac{b(1 + L)}{2} \) (to satisfy the payer’s rationality constraint, \( NMB \geq 0 \)). Let \( p = \frac{c + (Km)}{2(1 - L)} \) and \( \overline{p} = \frac{b(1 + L)}{2} \). Note that when it is social optimal to introduce the drug (i.e., if \( b > c \)), then the optimal treatment threshold is \( L^{FB} = \frac{c}{p} \) and the optimal marketing effort is \( m^{FB} = \frac{(b-c)^2}{2bK} \). Substituting \( L^{FB} \) and \( m^{FB} \) into \( p \) and \( \overline{p} \) and then evaluating \( \overline{p} - p \), it follows that,

\[ \overline{p} - p = \frac{b + c}{2} - \frac{b + 3c}{4} = \frac{b - c}{2} \geq 0 \tag{A.2b} \]

Thus, if it is socially optimal to introduce the drug, then there will exist a price, \( p \in [p, \overline{p}] \), that satisfies both rationality constraints.

Proof of Proposition 2.4.3 (negotiated pricing)

We first solve the equilibrium solutions \( m^{NP} \), \( L^{NP} \), and \( p^{NP} \). Then we prove the relationships in Proposition 2.4.3a, b, & c. Observe that from Proposition 2.4.2, the equilibrium price
must be less than \( \bar{p} \leq b \), \( \forall L \in [0, 1] \) and the equilibrium price must be greater than \( p \geq c \), \( \forall K \geq 0 \), \( L \in [0, 1] \). Thus, we restrict our analysis to prices in the range \([c, b]\).

This proof is by backwards induction. First, we solve for \( m^{NP} \). Solving the FOC (i.e., \( \frac{d\Pi}{dm} = 0 \)), there is one candidate solution, \( m = \frac{(p-c)(1-L)}{K} \). Using the second-derivative test, this point is a maximizer (i.e., \( \frac{d^2\Pi}{dmc^2} \leq 0 \), \( \forall K > 0 \)). Thus, \( m^{NP} = \frac{(p-c)(1-L)}{K} \). Next, we solve for \( L^{NP} \). Substituting \( m^{NP} \) into \( NMB \) and solving the FOC (i.e., \( \frac{dNMB}{dl} = 0 \)), there are two candidate solutions, \( L = 1 \) and \( L = 1 - \frac{4(b-p)}{3b} \). If \( L = 1 \), then demand is zero and therefore \( NMB = 0 \) and \( \Pi = 0 \). However, if \( L = 1 - \frac{4(b-p)}{3b} \) then \( NMB \geq 0 \), \( \forall K > 0 \), \( 0 < c \leq p \leq b \).

Using the second-derivative test, this point is a maximizer (i.e., \( \frac{d^2\Pi}{dl^2} \leq 0 \), \( \forall K > 0 \), \( 0 < c \leq p \leq b \)). Thus, \( L^{NP} = 1 - \frac{4(b-p)}{3b} \), resulting in \( m^{NP} = \frac{4(p-c)(b-p)}{3bK} \). Finally, we solve for \( p^{NP} \). Let \( \Omega = (\Pi)^{\alpha} \cdot (NMB)^{1-\alpha} \). Substituting \( m^{NP} \) and \( L^{NP} \) into \( \Omega \) and solving the FOC (i.e., \( \frac{d\Omega}{dp} = 0 \)), there are three candidate solutions: \( p = c \), \( p = b \), \( p = \frac{b+3c}{4} + \frac{\alpha(b-c)}{4} \). The first two candidate solutions result in \( \Omega = 0 \), while the third candidate solution always results in \( \Omega \geq 0 \). Using the second-derivative test, the third candidate solution is a maximizer (i.e., \( \alpha \leq \frac{1}{3} \), \( 0 < c \leq p \leq b \)). Thus \( p^{NP} = \frac{b+3c}{4} + \frac{\alpha(b-c)}{4} \) (observe that \( c < p^{NP} < b \)). Therefore, \( L^{NP} = \frac{c}{b} + \frac{\alpha(b-c)}{3b} \) (observe that \( 0 < \frac{c}{b} \leq L^{NP} \leq 1 - \frac{2(b-c)}{3b} < 1 \)) and \( m^{NP} = \frac{(b-c)^3(3-\alpha)(1+\alpha)}{12bK} > 0 \).

\( p^{NP} \) is strictly increasing in \( \alpha \) (i.e., \( \frac{dp^{NP}}{d\alpha} = \frac{4(b-c)}{3b} > 0 \), \( \forall 0 < c < b \)) and therefore it follows directly that \( p^{NP} \geq \frac{b+3c}{4} \) (\( \alpha = 0 \)) and \( p^{NP} \leq \frac{2(b+c)}{4} \) (\( \alpha = 1 \)). Furthermore, \( \frac{b+3c}{4} \geq c \) and \( \frac{2(b+c)}{4} \leq b \) and therefore \( c < \frac{b+3c}{4} \leq p^{NP} \leq \frac{2(b+c)}{4} < b \).

\( L^{NP} \) is strictly increasing in \( \alpha \) (i.e., \( \frac{dL^{NP}}{d\alpha} = \frac{b-c}{3b} > 0 \), \( \forall 0 < c < b \)) and therefore it follows directly that \( \frac{1-L^{FB}}{1-L^{NP}} = \frac{3}{3-\alpha} \). Observe that \( \frac{1-L^{FB}}{1-L^{NP}} \geq 1 \) (\( \alpha = 0 \)) and also \( \frac{1-L^{FB}}{1-L^{NP}} \leq \frac{3}{2} \) (\( \alpha = 1 \)).

\( m^{NP} \) is increasing in \( \alpha \) (i.e., \( \frac{dm^{NP}}{d\alpha} = \frac{(b-c)^2(1-\alpha)}{6bk} \geq 0 \), \( \forall b > 0 \), \( K > 0 \), \( \alpha \in [0, 1] \)) and therefore it follows directly that \( \frac{m^{NP}F_{P}}{m^{NP}F_{P}} = \frac{(b-c)^2(3-\alpha)(1+\alpha)}{12bK} = \frac{6}{(3-\alpha)(1+\alpha)} \). Observe that \( \frac{m^{NP}F_{P}}{m^{NP}F_{P}} \geq \frac{3}{2} \) (\( \alpha = 1 \)) and \( \frac{m^{NP}F_{P}}{m^{NP}F_{P}} \leq 2 \) (\( \alpha = 0 \)).

**Proof of Corollary 2.4.4 (negotiated demand)**

From (2.1) it follows that \( D^{NP} = \frac{(b-c)^3(3-\alpha)(1+\alpha)}{36b^2K} \). \( D^{NP} \) is concave and quadratic in \( \alpha \) and \( \alpha = \frac{1}{3} \) solves the FOC (i.e., \( \frac{dD^{NP}}{d\alpha} = 0 \)) and is therefore the maximizer. Observe that \( \frac{dD^{NP}}{dp^{FB}} \)

\[
\frac{(b-c)^3(3-\alpha)(1+\alpha)}{36b^2K} \frac{2b^3K}{(b-c)^3} = \frac{(3-\alpha)(1+\alpha)}{18} < 1, \forall \alpha \in [0, 1].
\]

Thus \( D^{NP} < D^{FB} \). It follows that for \( \alpha = \frac{1}{3} \),

\[
\frac{dD^{NP}}{dp^{FB}} = \frac{128}{243}.
\]
Proof of Proposition 2.4.5 (open and controlled pricing)

Substituting $\alpha = 1$ (open pricing) into $m^{NP}$, $L^{NP}$, $p^{NP}$, and $D^{NP}$ it follows that $m^{OP} = \frac{(b-c)^2}{36K}$.

$L^{OP} = \frac{c}{b} + \frac{b-c}{3b}$, $L^{OP} = \frac{b+c}{2}$, and $D^{OP} = \frac{2(b-c)^3}{9b^2K}$. Substituting $\alpha = 0$ (controlled pricing) into $m^{NP}$, $L^{NP}$, $p^{NP}$, and $D^{NP}$ it follows that $m^{CP} = \frac{(b-c)^2}{4bK}$, $L^{CP} = \frac{c}{b}$, $p^{CP} = \frac{b+c}{4}$, and $D^{CP} = \frac{b-c^3}{4b^2K}$.

Observe that:

a) $\frac{b+3c}{4} \leq \frac{b+3c}{4} + \frac{a(b-c)}{3b} \leq \frac{b+c}{2}$ and therefore $p^{CP} \leq p^{NP} \leq p^{OP}$

b) $\frac{c}{b} \leq \frac{c}{b} + \frac{a(b-c)}{3b} \leq \frac{c}{b} + \frac{b-c}{3b}$ and therefore $L^{FB} = L^{CP} \leq L^{NP} \leq L^{OP}$

c) $\frac{(b-c)^2}{4bK} \leq \frac{(b-c)^2(3-\alpha)(1+\alpha)}{12bK} \leq \frac{(b-c)^2}{3bK} \leq \frac{(b-c)^2}{2bK}$ and therefore $m^{CP} \leq m^{NP} \leq m^{OP} < m^{FB}$

d) $\frac{2(b-c)^3}{9b^2K} < \frac{(b-c)^3}{4b^2K} < \frac{6(b-c)^3}{243b^2K} < \frac{(b-c)^3}{2b^2K}$ and therefore $D^{OP} < D^{CP} < D^{NP} < D^{FB}$

Proof of Proposition 2.4.6 (listing process)

This proof is by backwards induction. First, we solve for $m^{LP}$. Solving the FOC (i.e., $\frac{d\Pi}{dm} = 0$), there is one candidate solution, $m = \frac{(p-c)(1-L)}{K}$. Using the second-derivative test, this point is a maximizer (i.e., $\frac{d^2\Pi}{dm^2} \leq 0$, $\forall K > 0$). Thus, $m^{LP} = \frac{(p-c)(1-L)}{K}$. Next, we solve for $L^{LP}$. Substituting $m^{LP}$ into $NMB$ and solving the FOC (i.e., $\frac{dNMB}{dL} = 0$), there are two candidate solutions, $L = 1$ and $L = 1 - \frac{4(b-p)}{3b}$. If $L = 1$, then demand is zero and therefore $NMB = 0$ and $\Pi = 0$. However, if $L = 1 - \frac{4(b-p)}{3b}$ then $NMB \geq 0$, $\forall K > 0$, $0 < c \leq p \leq b$. Using the second-derivative test, this point is a maximizer (i.e., $\frac{d^2NMB}{dL^2} \leq 0$, $\forall K > 0$, $0 < c \leq p \leq b$). Thus, $L^{LP} = 1 - \frac{4(b-p)}{3b}$, resulting in $m^{LP} = \frac{4(p-c)(b-p)}{3bK}$. Finally, we solve for $p^{LP}$.

Let the manufacturer’s expected profit with respect to the probability that the drug is listed be represented by $\hat{\Pi} = \Pr(approved) \cdot (\Pi) + (1 - \Pr(approved)) \cdot 0 = \frac{b-p}{b} \cdot \Pi$. Substituting $m^{NP}$ and $L^{NP}$ into $\hat{\Pi}$ and solving the FOC (i.e., $\frac{d\Pi}{dp} = 0$) we find three candidate solutions: $p = b$, $p = c$, $p = \frac{2b+3c}{5}$. The first and second candidate solutions result in $\hat{\Pi} = 0$, while the third candidate solution always results in positive expected profits. Using the second derivative test, the third candidate solution is a maximizer. Therefore, $p^{LP} = \frac{2b+3c}{5}$ and it follows that $L^{LP} = \frac{c}{b} + \frac{b-c}{5b}$ (observe that $0 < \frac{c}{b} \leq L^{LP} = 1 - \frac{4(b-c)}{5b} < 1$) and $m^{LP} = \frac{8(b-c)^2}{25bK}$. 

Observe that:
a) \( \frac{b+3c}{4} < \frac{2b+3c}{5} < \frac{b+c}{2} \) and therefore \( P_{CP} < P_{LP} < P_{OP} \)

b) \( \frac{c}{b} < \frac{c}{b} + \frac{b-c}{3b} < \frac{c}{b} \) and therefore \( L^{FP} = L^{CP} < L^{LP} < L^{OP} \)

c) \( \frac{(b-c)^2}{4b^2K} < \frac{8(b-c)^2}{25b^2K} < \frac{(b-c)^2}{3b^2K} < \frac{(b-c)^2}{2b^2K} \) and therefore \( m^{CP} < m^{LP} < m^{OP} < m^{FP} \)

Let \( \hat{D}L^P \) represent the expected demand with respect to the probability that the drug is approved. Formally, \( \hat{D}L^P = \text{Pr(approved)} \cdot D^L + (1-\text{Pr(approved)}) \cdot 0 \). Substituting \( L^P \) and \( m^L \) into \( D^L \) (conditional demand) and \( \hat{D}L^P \) (expected demand), it follows that \( D^L = \frac{32(b-c)^3}{125b^2K} \) and \( \hat{D}L^P = \frac{24(b-c)^3(22b^2-9bc+12c^2)}{3125b^4K} < \frac{2(b-c)^3}{9b^2K} \) and therefore \( \hat{D}L^P < D^OP < D^CP < D^LP < D^SNP < D^FB \). 

Proof of Proposition 2.4.7 (risk-sharing, p is a decision variable)

This proof is by backwards induction. First, we solve for \( \hat{m}(p, \hat{L}) \), the best response level of marketing given \( p \) and \( \hat{L} \). Solving the FOC (i.e., \( \frac{d\Pi^P}{dm} = 0 \)), there is one candidate solution, \( m = \frac{(p-c)(1-L)}{K} - \frac{pr(1-L)^2}{2K} \). Using the second-derivative test we verify that this point is a maximizer (i.e., \( \frac{d^2\Pi^P}{dm^2} \leq 0 \), \( \forall K > 0 \)). However, it must be that marketing is non-negative. Solving where the candidate solution equals zero, we find that the candidate level of marketing is positive for all \( L \in [1-\frac{2(p-c)}{pr}, 1] \). Let \( L^{TM} = 1-\frac{2(p-c)}{pr} \) and observe that \( L^{TM} \) may be less than zero. Let \( (L^{TM})^+ = \max[0, L^{TM}] \). Thus, for feasibility with the problem constraints \( L \in [0, 1] \) and \( m \geq 0 \),

\[
m^{RS} = \begin{cases} \frac{(p-c)(1-L)}{K} - \frac{pr(1-L)^2}{2K}, & L \in [(L^{TM})^+, 1] \\ 0, & \text{otherwise} \end{cases} \tag{A.2c} \]

Next, we solve for \( \hat{L}(p) \), the best-response treatment threshold given \( p \). Substituting \( m^{RS} \) into \( NMB^R \), we have that \( NMB^R \) is piecewise with respect to \( L \):

\[
NMB^R = \begin{cases} \frac{(1-L)^2(2(p-c)-pr(1-L))(2(b-p)-(b-pr)(1-L))}{4K}, & L \in [(L^{TM})^+, 1] \\ 0, & \text{otherwise} \end{cases} \tag{A.2d} \]

Observe that if the payer selects \( L \notin [(L^{TM})^+, 1] \), then \( NMB = 0 \) because \( m^{RS} = 0 \), and therefore the second case from (A.2c) always results in \( NMB^R = 0 \). If the payer is restricted
to select $L \in [(L^TM)^+, 1]$, then there always exists an $L$ (in this range) such that $NMB^R = 0$ (i.e., $L = 1$). Therefore the payer may always select $L \in [(L^TM)^+, 1]$ (i.e., that satisfies the first case from (A.2c)) and achieve at least as high NMB than in the second case from (A.2c). Therefore, we only consider the non-trivial case where the payer is restricted to select $L \in [(L^TM)^+, 1]$. For notational convenience, let $\beta = 3((p - c)(b - pr) + (b - p)(pr))$. Solving the FOC (i.e., $\frac{dNMB^R}{dL} = 0$) there are three candidate solutions: $L = 1$, $L = 1 - \frac{\beta - \sqrt{\beta^2 - 32(p-c)(b-p)(b-pr)(pr)}}{4pr(b-pr)}$, and $L = 1 - \frac{\beta + \sqrt{\beta^2 - 32(p-c)(b-p)(b-pr)(pr)}}{4pr(b-pr)}$. If $L = 1$, then demand is zero and therefore $NMB = 0$. However, the result at the other candidate solutions is less obvious.

There are 45 total scenarios that correspond to the following seven cases (See Table A.2 for Case 0A; Case 1A, B & C; and, Case 2A, B, & C), defined by the exogenous parameters $b$, $c$, and $r$:

<table>
<thead>
<tr>
<th>Case 0</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) A</td>
<td>$r = 1$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 1 ($b \geq 2c$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2) A</td>
</tr>
<tr>
<td>(3) B</td>
</tr>
<tr>
<td>(4) C</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 2 ($b &lt; 2c$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5) A</td>
</tr>
<tr>
<td>(6) B</td>
</tr>
<tr>
<td>(7) C</td>
</tr>
</tbody>
</table>

Table A.2: Summary of seven cases used to identify equilibrium solution in the risk-sharing arrangement scenario.

Within each case, a scenario is defined by the relationship of the manufacturer’s decision variable, $p$, to various thresholds. We proceed with an exhaustive examination of all scenarios.

Case 0A: In the special case where $r = 1$, $NMB = 0, \forall L$. Therefore, the payer chooses
A.2. Proofs

L with the secondary objective to maximize social welfare, thus selecting \( L = L^{FB} = \frac{c}{b} \geq L^{TM} \).

**Cases 1 & 2 (A, B, C):** Each case (e.g., case 1A) corresponds to an ordering of several price thresholds. For example, case 1A corresponds to the following:

\[
\frac{2c}{2 - r} \leq \frac{b}{2 - r} \leq b < \frac{b}{r}
\]

Each scenario corresponds to the relationship of \( p \) with these thresholds. For example, the five scenarios in case 1A are: (1) \( p \leq \frac{2c}{2 - r} \); (2) \( \frac{2c}{2 - r} \leq p \leq \frac{b}{2 - r} \); (3) \( \frac{b}{2 - r} \leq p \leq b \); (4) \( b \leq p \leq \frac{b}{r} \); and, (5) \( \frac{b}{r} \leq p \) (notice that these scenarios simply correspond evaluating (A.2e) for \( p \) at different levels within the ordering). The price thresholds that define each scenario define characteristics of the \( NMB^R \) function. Observe that \( NMB^R \) is quartic in \( L \) from (A.2d). To establish the functional form, we define the roots that solve \( NMB^R = 0 \) (although \( NMB^R \) is quartic in \( L \) there are only three distinct roots, \( L = 1 \) is a repeated root). Let \( L^1_0 = 1 \), \( L^2_0 = 1 - \frac{2(b-p)}{b-pr} \), and \( L^3_0 = L^{TM} = 1 - \frac{2(p-c)}{pr} \) represents the roots that solve \( NMB^R = 0 \) (although \( NMB^R \) is quartic in \( L \) there are only three distinct roots, \( L = 1 \) is a repeated root). Let \( L^1_1 \), \( L^2_1 \), and \( L^3_1 \) represent the candidate solutions that solve the FOC (i.e., \( \frac{dNMB}{dL} = 0 \)). And, let \( L^1_2 = 1 - \frac{\beta + \sqrt{\beta^2 - 24pr(b-pr)(p-c)(b-p)}}{6pr(b-pr)} \) and \( L^2_2 = 1 - \frac{\beta - \sqrt{\beta^2 - 24pr(b-pr)(p-c)(b-p)}}{6pr(b-pr)} \) represent the roots that solve \( \frac{d^2NMB}{dL^2} = 0 \). For each scenario, we demonstrate the functional characteristics of \( NMB^R \) to determine the optimal choice of \( L \), for all possible values of \( p \), \( b \), \( c \), and \( r \).

Each scenario (summarized in Table [A.3]) corresponds to a specific ordering of the roots, and therefore defines the equilibrium treatment threshold. To illustrate, we provide a full explanation of the scenario in row 2 of Table [A.3] Throughout the following discussion we define, and then refer to, properties of the roots using the notation ‘(i)’ (i.e., \( i \in \{i, ii, iii, vi, \ldots \} \).

**Case 1A (b \geq 2c \& r \in [0, 1]) - Scenario:** \( \frac{2c}{2 - r} \leq p \leq \frac{b}{2 - r} < b < \frac{b}{r} \): First, note that at \( L = 1 \), \( NMB^R = 0 \) (i) and \( \frac{dNMB^R}{dL} = 0 \) (ii) in every scenario (i.e., \( L^1_0 = 1 \) and \( L^1_1 = 1 \) are independent of all parameters). Specific to this scenario, \( NMB^R \) is convex at \( L = 1 \) (iii) (i.e., \( \frac{d^2NMB^R}{dL^2}\big|_{L=1} > 0 \)).
Because \( p < \frac{b}{r}, \) \( NMB \) is upward-quartic (\( iv \)) (i.e., increasing for \( L \to \infty \) and \( L \to -\infty \)). Additionally, because \( p \geq \frac{2c}{2r}, L_0^3 \leq 0 \) (\( v \)), and because \( p \leq \frac{b}{2r} < \frac{b}{r}, L_0^2 \leq 0 \) (\( vi \)). Let \( L_0 = \min(L_0^1, L_0^2) \) and \( \overline{L}_0 = \max(L_0^1, L_0^2) \). Therefore, because of (\( i - vi \)) \( NMB^R \) must be positive (strict) for all \( L \in (\overline{L}_0, 1) \) (\( vii \)) and further that \( NMB^R \) must be positive for all \( L \in [0, 1) \) because \( \overline{L}_0 \leq 0 \) from (\( v \)) and (\( vi \)). It can be shown that \( L_1^2 > L_1^3 \) (\( viii \)) and therefore because of (\( i \)), (\( v \)), (\( vi \)), and (\( vii \)), \( L_1^2 \in [\overline{L}_0, 1] \) (\( ix \)) (i.e., the candidate solution \( L_1^2 \) is on the interval \([\overline{L}_0, 1]\)) and \( L_1^3 \in [L_0, \overline{L}_0] \) (\( x \)). Because of (\( vii \)) and (\( ix \)), \( NMB^R|_{(L=L_1^2)} > 0 \) and because of (\( i - iv \)), (\( vii \)), and (\( ix \)), \( \frac{\partial^2 NMB^R}{\partial L^2}|_{(L=L_1^2)} < 0 \) and therefore \( L_1^2 \) is a maximizer on the interval \( L \in [\overline{L}_0, 1] \). Recall that \( L_3^0 = L^{TM} \) and therefore \((L_1^2)^+ = \max(0, L_1^2) \geq L^{TM} \) is the maximizer on the interval that results in non-zero marketing from (A.2c). Let \( L^T = L_1^2 \) as defined in (2.8) and therefore, in this scenario, \( \overline{L}(p) = (L^T)^+ \).

By contrast the scenario that corresponds to row 3 in Table A.3 results in \( L_0^2 \in [0, 1] \) and \( L_0^3 \leq 0 \) and therefore, in this scenario, \( \overline{L}(p) = L^T \) (i.e., strictly positive).

While we have provided an illustration of the behavior of \( NMB \) for each scenario to provide a convenient interpretation of the best response, a formal proof of the relationships among \( L_0^1, L_0^2, L_0^3, L_1^1, L_1^2, L_1^3, L_2^1, \) and \( L_2^2 \) may be provided on request from the authors.
Table A.3: Summary of the payer’s best response with respect to the treatment threshold. In the third column, the figure illustrates the functional form of $NMB^R$ (heavy black solid line). The maximum $NMB^R$ that satisfies the problem constraints (i.e., $L \in [0, 1]$) is illustrated with a solid dot.

<table>
<thead>
<tr>
<th>Case</th>
<th>Scenario</th>
<th>Illustration</th>
<th>$L(p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A</td>
<td>$b \geq 2c$ and $r \in [0, 1)$</td>
<td>$p \leq \frac{2c}{x-\gamma} \leq \frac{b}{x-\gamma} \leq b \leq \frac{b}{r}$</td>
<td><img src="image1" alt="Illustration" /></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$L_T$</td>
<td></td>
</tr>
<tr>
<td>1A</td>
<td>$b \geq 2c$ and $r \in [0, 1)$</td>
<td>$\frac{2c}{x-\gamma} \leq p \leq \frac{b}{x-\gamma} \leq b \leq \frac{b}{r}$</td>
<td><img src="image2" alt="Illustration" /></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(L_T)^+$</td>
<td></td>
</tr>
<tr>
<td>1A</td>
<td>$b \geq 2c$ and $r \in [0, 1)$</td>
<td>$\frac{2c}{x-\gamma} \leq \frac{b}{x-\gamma} \leq p \leq b \leq \frac{b}{r}$</td>
<td><img src="image3" alt="Illustration" /></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$L_T$</td>
<td></td>
</tr>
<tr>
<td>1A</td>
<td>$b \geq 2c$ and $r \in [0, 1)$</td>
<td>$\frac{2c}{x-\gamma} \leq \frac{b}{x-\gamma} \leq b \leq p \leq \frac{b}{r}$</td>
<td><img src="image4" alt="Illustration" /></td>
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</tr>
<tr>
<td>1A</td>
<td>$b \geq 2c$ and $r \in [0, 1)$</td>
<td>$\frac{2c}{x-\gamma} \leq \frac{b}{x-\gamma} \leq b \leq \frac{b}{r} \leq p$</td>
<td><img src="image5" alt="Illustration" /></td>
</tr>
<tr>
<td></td>
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</tbody>
</table>

(continued ...)
Table A.3: (continued)

<table>
<thead>
<tr>
<th>Case</th>
<th>Scenario</th>
<th>Illustration</th>
<th>$\bar{L}(p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1B</strong></td>
<td>$b \geq 2c$ and $r \in \left(1, \frac{2b - \sqrt{2b^2 - 4c}}{2c}\right]$</td>
<td>$p \leq \frac{2c}{2c - r} \leq \frac{b}{2} \leq b \leq \frac{b}{2c - r}$</td>
<td>$L_T$</td>
</tr>
<tr>
<td><strong>1B</strong></td>
<td>$b \geq 2c$ and $r \in \left(1, \frac{2b - \sqrt{2b^2 - 4c}}{2c}\right]$</td>
<td>$\frac{2c}{2c - r} \leq \frac{b}{r} \leq b \leq \frac{b}{2c - r}$</td>
<td>$(L_T)^r$</td>
</tr>
<tr>
<td><strong>1B</strong></td>
<td>$b \geq 2c$ and $r \in \left(1, \frac{2b - \sqrt{2b^2 - 4c}}{2c}\right]$</td>
<td>$\frac{2c}{2c - r} \leq \frac{b}{2c - r} \leq b \leq \frac{b}{2c - r}$</td>
<td>$(L_T)^r$</td>
</tr>
<tr>
<td><strong>1B</strong></td>
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<td>$(L_T)^r$</td>
</tr>
<tr>
<td><strong>1B</strong></td>
<td>$b \geq 2c$ and $r \in \left(1, \frac{2b - \sqrt{2b^2 - 4c}}{2c}\right]$</td>
<td>$\frac{2c}{2c - r} \leq \frac{b}{2c - r} \leq b \leq \frac{b}{2c - r} \leq p$</td>
<td>1</td>
</tr>
</tbody>
</table>

(continued . . .)
Table A.3: (continued)

<table>
<thead>
<tr>
<th>Case</th>
<th>Scenario</th>
<th>Illustration</th>
<th>$\bar{L}(p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1C</td>
<td>$b \geq 2c$ and $r \in \left[ \frac{2b}{3c+b}, 2 \right]$</td>
<td><img src="image" alt="Net Monetary Benefit" /></td>
<td>$L^T$</td>
</tr>
</tbody>
</table>

\[ \frac{b}{r} \leq \frac{2c}{2-r} \leq b \leq \frac{b}{2-r} \]

\[ L^T \]

\[ (L^T)^+ \]

\[ (L^T)^+ \]

\[ \frac{b}{r} \leq \frac{2c}{2-r} \leq b \leq \frac{b}{2-r} \]

\[ 1 \]

---

\(^1\)There are 10 scenarios for case 1C: (5) $\frac{2c}{2-r} \leq b$ (current page); (5) $b \leq \frac{2c}{2-r}$ (following page).
Table A.3: (continued)

<table>
<thead>
<tr>
<th>Case</th>
<th>Scenario</th>
<th>Illustration</th>
<th>$\bar{L}(p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1C</td>
<td>$b \geq 2c$ and $r \in \left[\frac{2b}{2c+b}, 2\right)$</td>
<td>$\frac{b}{r} \leq p \leq \frac{b}{2c-r} \leq \frac{b}{2-r}$</td>
<td>$L^T$</td>
</tr>
<tr>
<td>1C</td>
<td>$b \geq 2c$ and $r \in \left[\frac{2b}{2c+b}, 2\right)$</td>
<td>$\frac{b}{r} \leq p \leq \frac{2c}{2c-r} \leq \frac{b}{2-r}$</td>
<td>$L^T$</td>
</tr>
<tr>
<td>1C</td>
<td>$b \geq 2c$ and $r \in \left[\frac{2b}{2c+b}, 2\right)$</td>
<td>$\frac{b}{r} \leq p \leq \frac{2c}{2c-r} \leq \frac{b}{2-r}$</td>
<td>$L^T$</td>
</tr>
<tr>
<td>1C</td>
<td>$b \geq 2c$ and $r \in \left[\frac{2b}{2c+b}, 2\right)$</td>
<td>$\frac{b}{r} \leq p \leq \frac{2c}{2c-r} \leq \frac{b}{2-r}$</td>
<td>$(L^T)^r$</td>
</tr>
<tr>
<td>1C</td>
<td>$b \geq 2c$ and $r \in \left[\frac{2b}{2c+b}, 2\right)$</td>
<td>$\frac{b}{r} \leq p \leq \frac{2c}{2c-r} \leq \frac{b}{2-r}$</td>
<td>1</td>
</tr>
</tbody>
</table>

(continued . . .)

\[2\text{There are 10 scenarios for case 1C: (5) } \frac{2c}{2c-r} \leq b \text{ (previous page); (5) } b \leq \frac{2c}{2c-r} \text{ (current page).} \]
Table A.3: (continued)

<table>
<thead>
<tr>
<th>Case</th>
<th>Scenario</th>
<th>Illustration</th>
<th>$\bar{L}(p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2A</td>
<td>$b &lt; 2c$ and $r \in \left[0, \frac{2b}{2c+b}\right]$</td>
<td>$p \leq \frac{b}{2c} \leq b \leq \frac{2c}{2c-r} \leq \frac{b}{r}$</td>
<td>$L^T$</td>
</tr>
<tr>
<td>2A</td>
<td>$b &lt; 2c$ and $r \in \left[0, \frac{2b}{2c+b}\right]$</td>
<td>$b \frac{bc}{b-2b+c-r} \leq \leq \frac{2c}{2c-r} \leq \frac{b}{r}$</td>
<td>$L^T$</td>
</tr>
<tr>
<td>2A</td>
<td>$b &lt; 2c$ and $r \in \left[0, \frac{2b}{2c+b}\right]$</td>
<td>$\frac{b}{2c-r} \leq \leq \frac{bc}{b-2b+c-r} \leq \leq \frac{2c}{2c-r} \leq \frac{b}{r}$</td>
<td>$L^T$</td>
</tr>
<tr>
<td>2A</td>
<td>$b &lt; 2c$ and $r \in \left[0, \frac{2b}{2c+b}\right]$</td>
<td>$\frac{b}{2c-r} \leq \leq \frac{2c}{2c-r} \leq \frac{b}{r}$</td>
<td>$L^T$</td>
</tr>
<tr>
<td>2A</td>
<td>$b &lt; 2c$ and $r \in \left[0, \frac{2b}{2c+b}\right]$</td>
<td>$\frac{b}{2c-r} \leq \leq \frac{2c}{2c-r} \leq \frac{b}{r}$</td>
<td>$L^T$</td>
</tr>
</tbody>
</table>

(continued . . .)

There are 12 scenarios for case 2A: (6) $b \leq \frac{2c}{2c-r}$ (current page); (6) $\frac{2c}{2c-r} \leq b$ (following page).
<table>
<thead>
<tr>
<th>Case</th>
<th>Scenario</th>
<th>Illustration</th>
<th>$\bar{L}(p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2A</td>
<td>$b &lt; 2c$ and $r \in \left[0, \frac{2b}{2c} \right]$</td>
<td>$p \leq \frac{b}{2c - r} \leq \frac{2c}{2c - r} \leq b \leq \frac{b}{r}$</td>
<td>$L^T$</td>
</tr>
<tr>
<td>2A</td>
<td>$b &lt; 2c$ and $r \in \left[0, \frac{2b}{2c} \right]$</td>
<td>$\frac{b}{2c - r} \leq p \leq \frac{bc}{b - br + cr} \leq \frac{2c}{2c - r} \leq b \leq \frac{b}{r}$</td>
<td>$L^T$</td>
</tr>
<tr>
<td>2A</td>
<td>$b &lt; 2c$ and $r \in \left[0, \frac{2b}{2c} \right]$</td>
<td>$\frac{b}{2c - r} \leq \frac{bc}{b - br + cr} \leq p \leq \frac{2c}{2c - r} \leq b \leq \frac{b}{r}$</td>
<td>$L^T$</td>
</tr>
<tr>
<td>2A</td>
<td>$b &lt; 2c$ and $r \in \left[0, \frac{2b}{2c} \right]$</td>
<td>$\frac{b}{2c - r} \leq \frac{2c}{2c - r} \leq b \leq \frac{b}{r}$</td>
<td>$L^T$</td>
</tr>
<tr>
<td>2A</td>
<td>$b &lt; 2c$ and $r \in \left[0, \frac{2b}{2c} \right]$</td>
<td>$\frac{b}{2c - r} \leq \frac{2c}{2c - r} \leq b \leq p \leq \frac{b}{r}$</td>
<td>1</td>
</tr>
<tr>
<td>2A</td>
<td>$b &lt; 2c$ and $r \in \left[0, \frac{2b}{2c} \right]$</td>
<td>$\frac{b}{2c - r} \leq \frac{2c}{2c - r} \leq b \leq \frac{b}{r} \leq p$</td>
<td>1</td>
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</tbody>
</table>

(continued . . .)

There are 12 scenarios for case 2A: (6) $b \leq \frac{2c}{2c - r}$ (previous page); (6) $\frac{2c}{2c - r} \leq b$ (current page).
<table>
<thead>
<tr>
<th>Case</th>
<th>Scenario</th>
<th>Illustration</th>
<th>( \bar{L}(p) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2B</td>
<td>( b &lt; 2c ) and ( r \in \left[ \frac{2b}{bc + b}, 1 \right] )</td>
<td>( \frac{b}{bc + b} \leq \frac{b}{b - br + cr} \leq \frac{b}{r} \leq \frac{2c}{2c - r} )</td>
<td>( L^T )</td>
</tr>
<tr>
<td>2B</td>
<td>( b &lt; 2c ) and ( r \in \left[ \frac{2b}{bc + b}, 1 \right] )</td>
<td>( \frac{b}{bc + b} \leq \frac{b}{b - br + cr} \leq \frac{b}{r} \leq \frac{2c}{2c - r} )</td>
<td>( L^T )</td>
</tr>
<tr>
<td>2B</td>
<td>( b &lt; 2c ) and ( r \in \left[ \frac{2b}{bc + b}, 1 \right] )</td>
<td>( \frac{b}{bc + b} \leq \frac{b}{b - br + cr} \leq \frac{b}{r} \leq \frac{2c}{2c - r} )</td>
<td>( L^T )</td>
</tr>
<tr>
<td>2B</td>
<td>( b &lt; 2c ) and ( r \in \left[ \frac{2b}{bc + b}, 1 \right] )</td>
<td>( \frac{b}{bc + b} \leq \frac{b}{b - br + cr} \leq \frac{b}{r} \leq \frac{2c}{2c - r} )</td>
<td>( 1^\dagger )</td>
</tr>
<tr>
<td>2B</td>
<td>( b &lt; 2c ) and ( r \in \left[ \frac{2b}{bc + b}, 1 \right] )</td>
<td>( \frac{b}{bc + b} \leq \frac{b}{b - br + cr} \leq \frac{b}{r} \leq \frac{2c}{2c - r} )</td>
<td>( 1^\dagger )</td>
</tr>
<tr>
<td>2B</td>
<td>( b &lt; 2c ) and ( r \in \left[ \frac{2b}{bc + b}, 1 \right] )</td>
<td>( \frac{b}{bc + b} \leq \frac{b}{b - br + cr} \leq \frac{b}{r} \leq \frac{2c}{2c - r} )</td>
<td>( 1 )</td>
</tr>
</tbody>
</table>
Table A.3: (continued)

<table>
<thead>
<tr>
<th>Case</th>
<th>Scenario</th>
<th>Illustration</th>
<th>$\bar{L}(p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2C</td>
<td>$b &lt; 2c$ and $r \in (1, 2)$</td>
<td>$p \leq \frac{b}{\tau} \leq b \leq \frac{b}{2-\tau} \leq \frac{2c}{2-\tau}$</td>
<td>$L^T$</td>
</tr>
<tr>
<td>2C</td>
<td>$b &lt; 2c$ and $r \in (1, 2)$</td>
<td>$\frac{b}{\tau} \leq p \leq b \leq \frac{b}{2-\tau} \leq \frac{2c}{2-\tau}$</td>
<td>$L^T$</td>
</tr>
<tr>
<td>2C</td>
<td>$b &lt; 2c$ and $r \in (1, 2)$</td>
<td>$\frac{b}{\tau} \leq b \leq \frac{bc}{b-2br+cr} \leq \frac{b}{2-\tau} \leq \frac{2c}{2-\tau}$</td>
<td>$L^T$</td>
</tr>
<tr>
<td>2C</td>
<td>$b &lt; 2c$ and $r \in (1, 2)$</td>
<td>$\frac{b}{\tau} \leq b \leq \frac{bc}{b-2br+cr} \leq p \leq \frac{b}{2-\tau} \leq \frac{2c}{2-\tau}$</td>
<td>$1^\dagger$</td>
</tr>
<tr>
<td>2C</td>
<td>$b &lt; 2c$ and $r \in (1, 2)$</td>
<td>$\frac{b}{\tau} \leq \frac{b}{2-\tau} \leq p \leq \frac{2c}{2-\tau}$</td>
<td>$1^\dagger$</td>
</tr>
<tr>
<td>2C</td>
<td>$b &lt; 2c$ and $r \in (1, 2)$</td>
<td>$\frac{b}{\tau} \leq \frac{b}{2-\tau} \leq \frac{2c}{2-\tau} \leq p$</td>
<td>1</td>
</tr>
</tbody>
</table>

$^\dagger$ In the instances where $L = 1$ and $L = L^TM$ are possible solutions, demand is zero in both cases. We use $L = 1$ without a loss of generality.
In summary,

$$L(m) = \begin{cases} (L^T)^+ & \text{if } (r < 1 \text{ AND } p \leq b) \text{ OR } (r \leq 1 \text{ AND } p < b) \\ (L^T)^+ & \text{if } (1 < r < 2) \text{ AND } \\ ((b \geq 2c \text{ AND } p \leq \frac{b}{2-r}) \text{ OR } (b \leq 2c \text{ AND } p \leq \frac{bc}{b-b\epsilon+\epsilon})) \\ \frac{c}{b} & \text{if } r = 1 \text{ AND } p = b \\ 1 & \text{otherwise} \end{cases} \text{ (A.2f)}$$

Given the payer’s best response with respect to the treatment threshold and the manufacturer’s best response to the level of marketing, it is feasible to derive the manufacturer’s best-response with respect to price. However, due to the significant number of cases required to evaluate the three-stage game (note there are already 2 cases to the best response $m(p, L)$ and 45 cases to the best response $\bar{L}(p)$), we illustrate the full game solution using numeric analysis. Here, we provide one instance of the solution procedure.

Given: $r = 0.8$, $b = 1$, $c = 0.1 \times b$, and $K = 1$:

From (A.2f) we know that the payer’s best response to the manufacturer’s choice of price will be $(L^T)^+$ if the manufacturer selects any $p \leq b$ (because $r < 1$). If the manufacturer selects $p > b$, then the payer’s best response is to select $L = 1$ and therefore demand is zero. Substituting $(L^T)^+$ into $\bar{m}(L, p)$, and then both $(L^T)^+$ and $\bar{m}(p)$ into $\Pi_R$, observe the payer’s profit with respect to price in Figure A.1. We use the golden search algorithm (Kiefer 1953) to identify the maximizer, denoted on the graph as a solid dot at $p = p^{RS}$. 
Figure A.1: Manufacturer’s profit with respect to price, given the payer’s best response for the treatment threshold and the manufacturer’s best response for marketing effort. The solid dot indicates the maximum profit corresponding to the price \( p^{RS} \). (\( r = 0.8, b = 1, c = 0.1 \times b, K = 1 \)).

Proof of Proposition 2.4.8 (risk sharing, \( p \) is a decision variable, (\( r = 1 \)))

Set \( r = 1 \). This proof is by backwards induction. First, we solve for \( m^{RS} \). Solving the FOC (i.e., \( \frac{d\Pi}{dm} = 0 \)), there is one candidate solution, \( m = \frac{(p-c)(1-L)}{K} - \frac{p(1-L)^2}{2K} \). Using the second-derivative test, this point is a maximizer (i.e., \( \frac{d^2\Pi}{dm^2} \leq 0, \forall K > 0 \)). However, it must be that marketing is non-negative. Solving where the candidate solution equals zero, we find that the candidate level of marketing is positive for all \( L \in \left[ 1 - \frac{2(p-c)}{p}, 1 \right] \). Let \( L_{TM}^2 = \frac{1 - 2(p-c)}{p} \) and observe that \( L_{TM}^2 \) may be less than zero. Let \( (L_{TM}^2)^+ = \max[0, L_{TM}^2] \). Thus, for feasibility with the problem constraints \( L \in [0, 1] \) and \( m \geq 0 \),

\[
m^{RS} = \begin{cases} 
\frac{(p-c)(1-L)}{K} - \frac{p(1-L)^2}{2K}, & \text{if } L \in [(L_{TM}^2)^+, 1] \\
0, & \text{otherwise}
\end{cases}
\quad (A.2g)
\]

Substitute \( m^{RS} \) into NMB. Recall the payer’s secondary objective to maximize social welfare when indifferent to choices in \( L \). Thus, the payer will select \( L = \frac{c}{b} \) if \( p = b \). Otherwise, observe that if the payer selects \( L \notin [L_{TM}^2, 1] \) then \( NMB = 0 \), because \( m^{RS} = 0 \), and therefore the second case from \( (A.2g) \) always results in \( NMB^R = 0 \). If the payer is
restricted to select \( L \in [(L^{TM2})^+, 1] \), then there always exists an \( L \) (in this range) such that \( NMB^R = 0 \) (i.e., \( L = 1 \)). Therefore the payer may always select \( L \in [(L^{TM2})^+, 1] \) (i.e., that satisfies the first case from \((A.2g)\)) and achieve at least as high NMB than in the second case from \((A.2g)\). Therefore, we only consider the non-trivial case where the payer is restricted to select \( L \in [(L^{TM2})^+, 1] \). Solving the FOC (i.e., \( \frac{dNMB^R}{dL} = 0 \)) there are three candidate solutions, \( L_1 = 1, L_2 = 1 - \frac{3(2p-c)-\sqrt{(2p-c)^2+8c^2}}{4p}, \) and \( L_3 = 1 - \frac{3(2p-c)+\sqrt{(2p-c)^2+8c^2}}{4p} \). If \( L = 1 \), then demand is zero and therefore \( NMB = 0 \) and \( \Pi = 0 \). Observe that \( 0 \leq L_2 \leq 1, L^{TM2} \leq L_2, \) and \(-1 \leq L_3 \leq 0, \) and therefore \( L_2 \) is the only feasible solution. Using the second derivative test, \( L_2 \) maximizes \( NMB \) \( p \in [c, b] \) (i.e., \( \frac{d^2NMB}{dL^2}|_{(L=L_2)} \leq 0 \)). Therefore,

\[
L^{RS} = \begin{cases} 
L_2 & , \text{if } p \in [c, b) \\
\frac{c}{b} & , \text{if } p = b \\
1 & , \text{otherwise}
\end{cases} \tag{A.2h}
\]

Substitute \( L^{RS} \) and \( m^{RS} \) into \( \Pi \). Due to the piecewise nature of \( L^{RS} \) and \( m^{RS} \) we separate the optimal choice of \( p \) into two parts. First, we find the optimal \( p \) given \( L^{RS} = L_2 \), constraining \( p \in [c, b) \) and then we compare the equilibrium result with the scenario where \( p = b \) and therefore \( L^{RS} = \frac{c}{b} \). It can be shown that the manufacturer’s profit given \( p = b \) is higher than the profit given any \( p \in [c, b) \). Therefore, the manufacturer will always choose \( p = b \).

Thus, \( p^{RS} = b, L^{RS} = \frac{c}{b} \), and \( m^{RS} = \frac{(b-c)^3}{8b^2K} \). Therefore, \( \Pi^{RS} = W^{RS} = W^{FB} \), and \( NMB^{RS} = 0 \). \[\square\]

**Proof of Proposition 2.4.9** (value-based pricing with risk-sharing, \( NMB = 0 \))

Set \( NMB = 0 \) and solve for \( L \) to find \( L^{VR} = \frac{2p(1-r)}{b-pr} - 1 \). This proof is by backwards induction. First, we solve for \( m^{VR} \). Substituting \( L^{VR} \) into \( \Pi^R \) and then solving the FOC (i.e., \( \frac{d\Pi^R}{dm} = 0 \)), there is one candidate solution, \( m = \frac{2(b-p)(bp(1-r)-c(b-pr))}{K(b-pr)^2} \). Using the second-derivative test, this point is a maximizer (i.e., \( \frac{d^2\Pi^R}{dm^2} \leq 0, \forall K > 0 \)). However, it must be that marketing is non-negative. Solving where the candidate solution equals zero, we find that the candidate level of marketing is positive for all \( p \in \left[\frac{bc}{p-br+c}, b\right] \) if \( r \leq 1 \) and for all \( p \in \left[b, \frac{bc}{b-br+cr}\right] \) if \( r > 1 \). Now, solving for \( p^{VR} \) (recall that \( L^{VR} \) is defined as per the value-based risk-sharing
arrangement). Substituting \( m^{VR} \) into \( \Pi^R \) and then solving the FOC (i.e., \( \frac{d\Pi^R}{dp} = 0 \)), there are three candidate solutions, \( p_1 = b \), \( p_2 = \frac{bc}{b-br+cr} \), and \( p_3 = \frac{b(b+c)}{b(2-r)+cr} \). \( \Pi^R = 0 \) for the first and second candidate solutions, while \( \Pi^R > 0 \) at the third candidate solution. Using the second derivative test, the third candidate solution is a maximizer (i.e., \( \frac{d^2\Pi^R}{dp^2}\mid_{p=p_3} < 0, \forall K > 0 \)). Therefore \( p^{VR} = \frac{b(b+c)}{b(2-r)+cr} \). Substituting \( p^{VR} \) into \( L^{VR} \) and \( m^{VR} \) it follows that \( L^{VR} = \frac{c}{b} \) and \( m^{VR} = \frac{(b-c)^2}{2bK} \). The equilibrium outcomes are therefore \( W^{VR} = \Pi^{VR} = \frac{(b-c)^2}{8b^2K}, NMB^{VR} = 0 \), and \( D^{VR} = D^{FB} = \frac{(b-c)^2}{2b^2K} \). Let \( \overline{p}^{VR} \) represent the average price that is paid, defined as follows:

\[
\overline{p} = \theta \cdot p^{VR} + (1 - \theta) \cdot p^{VR} \cdot (1 - r) \tag{A.2i}
\]

where \( \theta \) represents the portion of the treated population whose treatment is successful, calculated by:

\[
\theta = \frac{1}{L^{VR}} \int_{L^{VR}} \pi f(\pi) d\pi
\]

Substituting \( L^{VR} \) into \( \theta \), we find that \( \theta = \frac{b+c}{2b} \). Substituting \( p^{VR} \) and \( \theta \) into \( \overline{p}^{VR} \), we find that \( \overline{p}^{VR} = \frac{b+c}{2} \).
A.3 Bibliography

Appendix B

Appendix: Essay 2

B.1 Treatment Policy Sub-Cases

Proposition B.1.1 Suppose $P$ is $\text{IFR}_f$, and that both Conditions 3.8 and 3.9 hold. Then,

a) $f^T(\omega)$ is non-increasing in $\omega$.

b) $\omega^T(f)$ is non-increasing in $f$.

Proposition B.1.2 Suppose $P$ is $\text{IFR}_f$ and $\text{SC}_f$, $M(f, \omega)$ is convex in $f$, and Condition 3.9 holds. Then,

a) $\underline{f}^T(\omega)$ is non-increasing in $\omega$.

b) $\overline{f}^T(\omega)$ is non-decreasing in $\omega$.

c) $|\Gamma^T(\omega)|$ is non-decreasing in $\omega$.

d) $\exists f^T$ s.t. $\forall f \leq f^T$, $\omega^T(f)$ is non-increasing in $f$, and $\exists \overline{\overline{f}}^T$ s.t. $\forall f \geq \overline{\overline{f}}$, $\omega^T(f)$ is non-decreasing in $f$.

Proposition B.1.3 Suppose $P$ is $\text{SC}_f$ and $\text{SC}_\omega$, and that $M(f, \omega)$ is convex in both $f$ and $\omega$. Then, the treatment region is convex.

\footnote{The proofs of Propositions B.1.1, B.1.2 and B.1.3 can be found in Appendix B.3 on page 156}
B.2 Maximum Violation of Conditions

<table>
<thead>
<tr>
<th>Index</th>
<th>Condition</th>
<th>Maximum Violation (\epsilon)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\epsilon_1)</td>
<td>(IFR_f)</td>
<td>(\omega \in [0, 100])</td>
</tr>
<tr>
<td>(\epsilon_2)</td>
<td>(IFR_\omega)</td>
<td>0.009</td>
</tr>
<tr>
<td>(\epsilon_3)</td>
<td>(IFR_f\omega)</td>
<td>0.009</td>
</tr>
<tr>
<td>(\epsilon_4)</td>
<td>(IDR)</td>
<td>0.165</td>
</tr>
<tr>
<td>(\epsilon_5)</td>
<td>(SC_f)</td>
<td>0.962</td>
</tr>
<tr>
<td>(\epsilon_6)</td>
<td>(SC_\omega)</td>
<td>0.000</td>
</tr>
<tr>
<td>(\epsilon_7)</td>
<td>Convexity of (M(f,\omega)) in (f) &amp; (B.1a)</td>
<td>39,206</td>
</tr>
<tr>
<td>(\epsilon_8)</td>
<td>Convexity of (M(f,\omega)) in (\omega) &amp; (B.1b)</td>
<td>95</td>
</tr>
<tr>
<td>(\epsilon_9)</td>
<td>Condition 3.8 &amp; (B.1c)</td>
<td>5.464</td>
</tr>
<tr>
<td>(\epsilon_{10})</td>
<td>Condition 3.9 &amp; (B.1d)</td>
<td>3.035</td>
</tr>
<tr>
<td>(\epsilon_{11})</td>
<td>Condition 3.11 and 4.16 &amp; (B.1e)</td>
<td>0</td>
</tr>
<tr>
<td>(\epsilon_{12})</td>
<td>Condition 3.12 &amp; (B.1f)</td>
<td>0.002</td>
</tr>
</tbody>
</table>

For example, \(\epsilon_2 = 0.009\) indicates that the maximum decrease in failure rate with respect to age (i.e., a violation of increasing failure rate) is 0.9%, corresponding to a decrease in the probability of progressing into a more severe disease state of 0.9%.

Expressed as USD.

†For example, \(\epsilon_2 = 0.009\) indicates that the maximum decrease in failure rate with respect to age (i.e., a violation of increasing failure rate) is 0.9%, corresponding to a decrease in the probability of progressing into a more severe disease state of 0.9%.

‡Expressed as USD.

Table B.1: Maximum violation of conditions. Unless indicated otherwise, values are expressed as percentages.†

Formulas

Let \((x)^+ = \max[0, x]\).

\[
\epsilon_1 = \max_{k,f,\omega} \left( \left( \sum_{f' = k}^{F+1} \left[ p(f'\mid f+1,\omega) - p(f'\mid f,\omega) \right] \right)^+ \right), 
\]

for \(k = 0,\ldots,F+1; f = 0,\ldots,F; \) and \(\omega = 0,\ldots,N\)

\[
\epsilon_2 = \max_{k,f,\omega} \left( \left( \sum_{f' = k}^{F+1} \left[ p(f'\mid f,\omega+1) - p(f'\mid f,\omega) \right] \right)^+ \right),
\]

for \(k = 0,\ldots,F+1; f = 0,\ldots,F+1; \) and \(\omega = 0,\ldots,N - 1\)

\[
\epsilon_3 = \max \{\epsilon_1, \epsilon_2\}
\]

\[
\epsilon_4 = \max_{k,f,\omega} \left( \left( \sum_{f' = k}^{F} \left[ p(f'\mid f+1,\omega) - p(f'\mid f,\omega) \right] \right)^+ \right),
\]

for \(k = 0,\ldots,F+1; f = 0,\ldots,F+1; \) and \(\omega = 0,\ldots,N\)

(B.1a)
for \( k = 0, \ldots, F; \ f = 0, \ldots, F - 1; \) and \( \omega = 0, \ldots, N \)
\[
\epsilon_5 = \max_{k,f,\omega} \left\{ \left( \sum_{f'=k}^{F+1} [2 \cdot p(f'|f + 1, \omega) - p(f'|f, \omega) - p(f'|f + 2, \omega)] \right)^+ \right\}, \tag{B.1e}
\]

for \( k = 0, \ldots, F + 1; \ f = 0, \ldots, F - 1; \) and \( \omega = 0, \ldots, N \)
\[
\epsilon_6 = \max_{k,f,\omega} \left\{ \left( \sum_{f'=k}^{F+1} [2 \cdot p(f'|f, \omega + 1) - p(f'|f, \omega) - p(f'|f, \omega + 2)] \right)^+ \right\}, \tag{B.1f}
\]

for \( k = 0, \ldots, F + 1; \ f = 0, \ldots, F + 1; \) and \( \omega = 0, \ldots, N - 2 \)
\[
\epsilon_7 = \max_{f,\omega} \left\{ \left( 2 \cdot M(f + 1, \omega) - M(f, \omega) - M(f + 2, \omega) \right)^+ \right\}, \tag{B.1g}
\]

for \( f = 0, \ldots, F - 1; \) and \( \omega = 0, \ldots, N \)
\[
\epsilon_8 = \max_{f,\omega} \left\{ \left( 2 \cdot M(f, \omega + 1) - M(f, \omega) - M(f, \omega + 2) \right)^+ \right\}, \tag{B.1h}
\]

for \( f = 0, \ldots, F + 1; \) and \( \omega = 0, \ldots, N - 2 \)
\[
\epsilon_9 = \max_{f,\omega} \left\{ \left( \frac{r((f, \omega), 1) - r((f + 1, \omega), 1)}{r((f + 1, \omega + 1), 1) - r((f, \omega + 1), 1) - c} \right) \right\}, \tag{B.1i}
\]

for \( f = 0, \ldots, F - 1; \) and \( \omega = 0, \ldots, N - 1 \)
\[
\epsilon_{10} = \max_{f,\omega} \left\{ \left( \frac{r((f, \omega), 1) - r((f, \omega + 1), 1)}{r((f, \omega + 1), 1) - c} \right) \right\}, \tag{B.1j}
\]

for \( f = 0, \ldots, F; \) and \( \omega = 1, \ldots, N - 1 \)

Let \( s_1 = (f, \omega) \) and \( s_2 = (f(f, \omega), \omega + 1) \). Then,
\[
\epsilon_{11} = \max_{f,\omega} \left\{ \left( \frac{r_{\mathcal{T}_2}(\mu) - r_{\mathcal{T}_1}(\mu)}{r_{\mathcal{T}_1}(\mu)} \right) \right\}, \tag{B.1k}
\]

for \( f = 0, \ldots, F; \) and \( \omega = 0, \ldots, \mu - 2 \)
\[ \epsilon_{12} = \max_{f,\omega} \left\{ \left( \frac{p_X(s_2, T) - p_X(s_1, T)}{r_X(s_2, T)} - \frac{p(F + 1|s_1) - (1 - p_{T,s}(T_{s_1}|\omega))}{p_{T,s}(T_{s_1}|\omega)} \right)^+ \right\}, \quad (B.11) \]

for \( f = 0, \ldots, F; \) and \( \omega = 0, \ldots, \mu - 2 \)

### B.3 Proofs

**Lemma B.3.1** Let \( \{x_j\} \) and \( \{x'_j\} \) be real-valued non-negative sequences satisfying:

\[ \sum_{j=k}^{\infty} x_j \geq \sum_{j=k}^{\infty} x'_j, \]

\( \forall k \in \{0, 1, \ldots\} \), with equality holding for \( k = 0 \). Suppose \( v_j \geq v_{j+1} \) (\( j = 0, 1, \ldots \)), then,

\[ \sum_{j=0}^{\infty} v_j x_j \geq \sum_{j=0}^{\infty} v_j x'_j \]

**Proof of Lemma B.3.1** An equivalent proof of Lemma B.3.1 is provided to Lemma 4.2.7 in Puterman (1994) and is omitted.

**Proof of Proposition 3.4.1** Proof of a: This proof is by backwards induction. By construction, an individual cannot live beyond \( N \)-years of age. Therefore, \( V(f, \omega) = 0, \forall \omega > N \).

Starting with \( \omega = N \):

\[ V(f, N) = \max \left\{ r((f,N), 1), \ r((f,N), 0) + \delta \sum_{f' \in \mathbb{F}} p(f'|f, N) \cdot V(f', N + 1) \right\} \]

\[ = \max \left\{ r((f,N), 1), \ r((f,N), 0) \right\} \quad (B.2a) \]

**B.2a** results because \( V(f, N + 1) = 0, \forall f \in \mathbb{F} \). By A3.4.1 and A3.4.2 both \( r((f, \omega), 0) \) and \( r((f, \omega), 1) \) are non-increasing in \( f \), respectively, and therefore \( V(f, N) \) is non-increasing in \( f \).

For \( \omega = N - 1 \):
By the property that $P$ is IFR and by Lemma B.3.1, the result holds. Therefore,

$$
\sum_{f' \in \mathcal{F}} p(f'|(f, N - 1)) \cdot V(f', N) \geq \sum_{f' \in \mathcal{F}} p(f'|(f + 1, N - 1)) \cdot V(f', N) \quad (B.2b)
$$

Combining B.2b with A3.4.1,

$$
r((f, N - 1), 0) + \delta \sum_{f' \in \mathcal{F}} p(f'|(f, N - 1)) \cdot V(f', N) \geq
r((f + 1, N - 1), 0) + \delta \sum_{f' \in \mathcal{F}} p(f'|(f + 1, N - 1)) \cdot V(f', N) \quad (B.2c)
$$

Separately, by A3.4.2,

$$
r((f, N - 1), 1) \geq r((f + 1, N - 1), 1) \quad (B.2d)
$$

Combining B.2c and B.2d,

$$
\max \left\{ r((f, N - 1), 1), r((f, N - 1), 0) + \delta \sum_{f' \in \mathcal{F}} p(f'|(f, N - 1)) \cdot V(f', N) \right\} \geq
\max \left\{ r((f + 1, N - 1), 1), r((f + 1, N - 1), 0) + \delta \sum_{f' \in \mathcal{F}} p(f'|(f + 1, N - 1)) \cdot V(f', N) \right\}
$$

(B.2e)

By the definition of the value function in (3.7), B.2e can be rewritten as:

$$
V(f, N - 1) \geq V(f + 1, N - 1)
$$

Therefore, $V(f, N - 1)$ is non-increasing in $f$. Repeating the same process for $\omega = N - 2, N - 3, \ldots, 0$ (omitted), the result follows; $V(f, \omega)$ is non-increasing in $f$, $\forall \omega \in \Omega$. □
Proof of \( \text{b} \): This proof is by backwards induction. By construction, an individual cannot live beyond \( N \)-years of age. Therefore, \( V(f, \omega) = 0, \forall \omega > N \).

Starting with \( \omega = N \):

\[
V(f, N) = \max \left\{ r((f, N), 1), r((f, N), 0) + \delta \sum_{f' \in \mathcal{F}} p(f'|(f, N)) \cdot V(f', N + 1) \right\} \\
= \max \left\{ r((f, N), 1), r((f, N), 0) \right\} \\
\geq 0 \\
= V(f, N + 1) \tag{B.3a}
\]

\( \text{B.3a} \) follows from A[3.4.1]. Therefore, \( V(f, N) \geq V(f, N + 1), \forall f \in \mathcal{F} \).

For \( \omega = N - 1 \):

By the property that \( P \) is IFR\(_f\), the result of Proposition 3.4.1a holds. Combining the result of Proposition 3.4.1a with the property that \( P \) is IFR\(_\omega\),

\[
\sum_{f' \in \mathcal{F}} p(f'|(f, N - 1)) \cdot V(f', N) \geq \sum_{f' \in \mathcal{F}} p(f'|(f, N)) \cdot V(f', N) \tag{B.3c}
\]

Combining B.3b and B.3c

\[
\sum_{f' \in \mathcal{F}} p(f'|(f, N - 1)) \cdot V(f', N) \geq \sum_{f' \in \mathcal{F}} p(f'|(f, N)) \cdot V(f', N + 1) \tag{B.3d}
\]

Combining A[3.4.1] with B.3d

\[
r((f, N - 1), 0) + \delta \sum_{f' \in \mathcal{F}} p(f'|(f, N - 1)) \cdot V(f', N) \geq \\
r((f, N), 0) + \delta \sum_{f' \in \mathcal{F}} p(f'|(f, N)) \cdot V(f', N + 1) \tag{B.3e}
\]
Separately, by A3.4.2,

$$r((f, N - 1), 1) \geq r((f, N), 1)$$  \hspace{1cm} (B.3f)

Combining B.3e and B.3f,

$$\max \left\{ r((f, N - 1), 1), r((f, N - 1), 0) + \delta \sum_{f' \in \mathcal{P}} p(f'| (f, N - 1)) \cdot V(f', N) \right\} \geq$$

$$\max \left\{ r((f, N), 1), r((f, N), 0) + \delta \sum_{f' \in \mathcal{P}} p(f'| (f, N)) \cdot V(f', N + 1) \right\}$$  \hspace{1cm} (B.3g)

By the definition of the value function in (3.7), B.3g can be rewritten as,

$$V(f, N - 1) \geq V(f, N)$$  \hspace{1cm} (B.3h)

Therefore $V(f, N - 1) \geq V(f, N), \forall f \in \mathcal{F}$.

Repeating the same process for $\omega = N - 2, N - 3, \ldots, 0$ (omitted), the result follows; $V(f, \omega)$ is non-increasing in $\omega, \forall f \in \mathcal{F}$.

**Lemma B.3.2** (A and B: Alagoz et al. (2004, 2007)): Let $P$ be an $N \times N$, IFR transition probability matrix and $V(h)$ be a non-increasing function in $h$, $h \in S, |S| = N$. Then, the following hold:

**a)** $\sum_{h' \leq h} [p(h'|h) - p(h'|h + 1)] \cdot V(h') \geq \sum_{h' \leq h} [p(h'|h) - p(h'|h + 1)] \cdot V(h)$

**b)** $\sum_{h'' > h} [p(h''|h) - p(h''|h + 1)] \cdot V(h'') \geq \sum_{h'' > h} [p(h''|h) - p(h''|h + 1)] \cdot V(h + 1)$

**c)** $\sum_{h'' > h} [p(h''|h) - p(h''|h + 1)] \cdot V(h'') \geq \sum_{h'' > h} [p(h''|h) - p(h''|h + 1)] \cdot V(m), \forall m \leq h + 1$

**d)** $\sum_{h' \in S} [p(h'|h) - p(h'|h + 1)] \cdot V(h') \geq \sum_{h' \in S} [p(h'|h) - p(h'|h + 1)] \cdot V(h + 1)$

**Proof of Lemma B.3.2** Proof of **a**: Lemma B.3.2a is equivalent to Lemma 1a in Alagoz et al. (2004) and therefore the proof is omitted.
Proof of b: Lemma B.3.2b is equivalent to Lemma 1b in Alagoz et al. (2004) and therefore the proof is omitted.

Proof of c: From Lemma B.3.2b we know,
\[
\sum_{h'' > h} \left[ p(h''|h) - p(h''|h + 1) \right] \cdot V(h'') \geq \sum_{h'' > h} \left[ p(h''|h) - p(h''|h + 1) \right] \cdot V(h + 1) \quad (B.4a)
\]
For any \( m \leq h + 1 \), \( V(m) \geq V(h + 1) \) because \( V(h) \) is non-increasing in \( h \), by definition. As result of the condition that \( P \) is IFR, \( \sum_{h'' > h} \left[ p(h''|h) - p(h''|h + 1) \right] \leq 0 \), \( \forall h \in S \), the result follows.

Proof of d: From the left-hand side of d),
\[
\sum_{h'' \in S} \left[ p(h'|h) - p(h'|h + 1) \right] \cdot V(h')
\]
\[
= \sum_{h'' \leq h} \left[ p(h'|h) - p(h'|h + 1) \right] \cdot V(h') + \sum_{h'' > h} \left[ p(h''|h) - p(h''|h + 1) \right] \cdot V(h'')
\]
\[
\geq \sum_{h'' \leq h} \left[ p(h'|h) - p(h'|h + 1) \right] \cdot V(h + 1) + \sum_{h'' > h} \left[ p(h''|h) - p(h''|h + 1) \right] \cdot V(h + 1)
\]
\[
= \sum_{h'' \in S} \left[ p(h'|h) - p(h'|h + 1) \right] \cdot V(h + 1)
\]
(B.5a)
\]
\)
B.5a follows from Lemma B.3.2b and Lemma B.3.2c.

Proof of Proposition 3.4.2 Proof of a: The result of Proposition 3.4.2a is equivalent to,
\[
a^*(f, \omega) = 1 \Rightarrow a^*(f + 1, \omega) = 1
\]
(B.6a)
This proof is by contradiction. Assume the converse of B.6a Therefore,
\[
a^*(f, \omega) = 1 \Rightarrow a^*(f + 1, \omega) = 0, \text{ uniquely.}
\]
Therefore, the following must be true,

\[ r((f, \omega), 1) \geq r((f, \omega), 0) + \delta \sum_{f' \in \mathbb{F}} p(f'(f, \omega)) \cdot V(f', \omega + 1) \]  

(B.6b)

and,

\[ r((f + 1, \omega), 1) < r((f + 1, \omega), 0) + \delta \sum_{f' \in \mathbb{F}} p(f'(f + 1, \omega)) \cdot V(f', \omega + 1) \]  

(B.6c)

Subtracting B.6c from B.6b,

\[ r((f, \omega), 1) - r((f + 1, \omega), 1) > \delta \sum_{f' \in \mathbb{F}} p(f'(f, \omega)) \cdot V(f', \omega + 1) - \delta \sum_{f' \in \mathbb{F}} p(f'(f + 1, \omega)) \cdot V(f', \omega + 1) \]

By A3.4.1, \( r((f, \omega), 0) \geq r((f + 1, \omega), 0) \). Therefore,

\[ r((f, \omega), 1) - r((f + 1, \omega), 1) > \delta \sum_{f' \in \mathbb{F}} p(f'(f, \omega)) \cdot V(f', \omega + 1) - \delta \sum_{f' \in \mathbb{F}} p(f'(f + 1, \omega)) \cdot V(f', \omega + 1) \]

\[ = \delta \left[ \sum_{f' = 0}^{f} p(f'(f, \omega)) \cdot V(f', \omega + 1) + \sum_{f'' = f + 1}^{F} p(f''(f + 1, \omega)) \cdot V(f'', \omega + 1) \right] - \]

\[ \delta \left[ \sum_{f' = 0}^{f} p(f'(f + 1, \omega)) \cdot V(f', \omega + 1) + \sum_{f'' = f + 1}^{F} p(f''(f + 1, \omega)) \cdot V(f'', \omega + 1) \right] \]

\[ = \delta \left[ \sum_{f' = 0}^{f} p(f'(f, \omega)) \cdot V(f', \omega + 1) - \sum_{f' = 0}^{f} p(f'(f + 1, \omega)) \cdot V(f', \omega + 1) \right] + \]

\[ \delta \left[ \sum_{f'' = f + 1}^{F} p(f''(f, \omega)) \cdot V(f'', \omega + 1) - \sum_{f'' = f + 1}^{F} p(f''(f + 1, \omega)) \cdot V(f'', \omega + 1) \right] \]
By the condition that $P$ is IFR, and Lemma B.3.2a, we can substitute $V(f, \omega + 1)$ for $V(f', \omega + 1)$ and the inequality holds. And, by the condition that $P$ is IDR and Lemma B.3.2a, we can substitute $V(f+1, \omega+1)$ for $V(f'', \omega+1)$ and the inequality holds. Therefore,

$$r((f, \omega), 1) - r((f + 1, \omega), 1)$$

$$> \delta V(f, \omega + 1) \left[ \sum_{f'=0}^{f} \left( p(f'(f, \omega)) - p(f'(f + 1, \omega)) \right) \right] +$$

$$\delta V(f + 1, \omega + 1) \left[ \sum_{f''=f+1}^{F} \left( p(f''(f, \omega)) - p(f''(f + 1, \omega)) \right) \right]$$

$$= \delta V(f, \omega + 1) \left[ \sum_{f'=0}^{f} \left( p(f'(f, \omega)) - p(f'(f + 1, \omega)) \right) \right] +$$

$$\delta V(f + 1, \omega + 1) \left[ \sum_{f''=f+1}^{F} p(f''(f, \omega)) \right] -$$

$$\delta V(f + 1, \omega + 1) \left[ \sum_{f''=f+1}^{F} p(f''(f + 1, \omega)) \right]$$

$$= \delta V(f, \omega + 1) \left[ \sum_{f'=0}^{f} \left( p(f'(f, \omega)) - p(f'(f + 1, \omega)) \right) \right] +$$

$$\delta V(f + 1, \omega + 1) \left[ 1 - \sum_{f'=0}^{f} p(f'(f, \omega)) - p(F + 1|(f, \omega)) \right] -$$

$$\delta V(f + 1, \omega + 1) \left[ 1 - \sum_{f'=0}^{f} p(f'(f + 1, \omega)) - p(F + 1|(f + 1, \omega)) \right]$$

$$= \delta [V(f, \omega + 1) - V(f + 1, \omega + 1)] \cdot \left[ \sum_{f'=0}^{f} \left( p(f'(f, \omega)) - p(f'(f + 1, \omega)) \right) \right] +$$

$$\delta V(f + 1, \omega + 1) \cdot \left[ p(F + 1|(f + 1, \omega)) - p(F + 1|(f, \omega)) \right]$$

By Proposition B.3.4.1a, $V(f, \omega + 1) - V(f + 1, \omega + 1) \geq 0$. By the property that $P$ is IFR,
\[ \sum_{f'=0}^f \left[ \frac{p(f'|(f, \omega))}{p(f'|(f + 1, \omega))} - \frac{p(f'|(f + 1, \omega))}{p(f'|(f, \omega))} \right] \geq 0. \]

And, \( \delta \in [0, 1] \). Therefore, the first term in \( B.6d \) can be dropped because all elements are positive and the inequality holds. Therefore,

\[ r((f, \omega), 1) - r((f + 1, \omega), 1) \]

\[ > \delta V(f + 1, \omega + 1) \cdot \left[ p(F + 1|(f + 1, \omega)) - p(F + 1|(f, \omega)) \right] \]

By the definition of the value function in (3.7), \( V(f + 1, \omega + 1) \geq r((f + 1, \omega + 1), 1) \).

Therefore,

\[ r((f, \omega), 1) - r((f + 1, \omega), 1) \]

\[ > \delta \left[ r((f + 1, \omega + 1), 1) \right] \cdot \left[ p(F + 1|(f + 1, \omega)) - p(F + 1|(f, \omega)) \right] \]

Which implies,

\[ \frac{r((f, \omega), 1) - r((f + 1, \omega), 1)}{r((f + 1, \omega + 1), 1)} > \delta \left[ p(F + 1|(f + 1, \omega)) - p(F + 1|(f, \omega)) \right] \]

Which contradicts (3.8). Therefore, it must be that \( a^*(f + 1, \omega) = 1 \), and the result follows.

**Proof of b:** The result of Proposition 3.4.2b is equivalent to,

\[ a^*(f, \omega) = 1 \Rightarrow a^*(f, \omega + 1) = 1 \quad (B.7a) \]

This proof is by contradiction. Assume the converse of \( B.7a \). Therefore,

\[ a^*(f, \omega) = 1 \Rightarrow a^*(f, \omega + 1) = 0, \text{ uniquely.} \]

Therefore, the following must be true,

\[ r((f, \omega), 1) \geq r((f, \omega), 0) + \delta \sum_{f' \in \mathbb{P}} p(f'|(f, \omega)) \cdot V(f', \omega + 1) \quad (B.7b) \]
and,

\[ r((f, \omega + 1), 1) < r((f, \omega + 1), 0) + \delta \sum_{f' \in \mathcal{F}} p(f'|f, \omega + 1)) \cdot V(f', \omega + 2) \]  \hspace{1cm} (B.7c)

Subtracting \[ B.7c \] from \[ B.7b \]

\[ r((f, \omega), 1) - r((f, \omega + 1), 1) > r((f, \omega), 0) - r((f, \omega + 1), 0) + \delta \sum_{f' \in \mathcal{F}} p(f'|f, \omega)) \cdot V(f', \omega + 1) - \delta \sum_{f' \in \mathcal{F}} p(f'|f, \omega + 1)) \cdot V(f', \omega + 2) \]

By A3.4.1 \[ r((f, \omega), 0) \geq r((f, \omega + 1), 0). \] Therefore,

\[ r((f, \omega), 1) - r((f, \omega + 1), 1) > \delta \sum_{f' \in \mathcal{F}} p(f'|f, \omega)) \cdot V(f', \omega + 1) - \delta \sum_{f' \in \mathcal{F}} p(f'|f, \omega + 1)) \cdot V(f', \omega + 2) \]

By the property that \( P \) is IFR, the result of Proposition 3.4.1b holds. Therefore,

\[ r((f, \omega), 1) - r((f, \omega + 1), 1) > \delta \sum_{f' \in \mathcal{F}} p(f'|f, \omega)) \cdot V(f', \omega + 1) - \delta \sum_{f' \in \mathcal{F}} p(f'|f, \omega + 1)) \cdot V(f', \omega + 1) \]

\[ = \delta \left[ \sum_{f' = 0}^{f} p(f'|f, \omega)) \cdot V(f', \omega + 1) + \sum_{f'' = f + 1}^{F} p(f''|f, \omega)) \cdot V(f'', \omega + 1) \right] - \]

\[ \delta \left[ \sum_{f' = 0}^{f} p(f'|f, \omega + 1)) \cdot V(f', \omega + 1) + \sum_{f'' = f + 1}^{F} p(f''|f, \omega + 1)) \cdot V(f'', \omega + 1) \right] \]

\[ = \delta \left[ \sum_{f' = 0}^{f} p(f'|f, \omega)) \cdot V(f', \omega + 1) - \sum_{f' = 0}^{f} p(f'|f, \omega + 1)) \cdot V(f', \omega + 1) \right] + \]

\[ \delta \left[ \sum_{f'' = f + 1}^{F} p(f''|f, \omega)) \cdot V(f'', \omega + 1) - \sum_{f'' = f + 1}^{F} p(f''|f, \omega + 1)) \cdot V(f'', \omega + 1) \right] \]
B.3.2c, we can substitute $V$.

By the condition that $P$ is IFR and Lemma B.3.2a, we can substitute $V(f, \omega + 1)$ for $V(f', \omega + 1)$ and the inequality holds. And, by the condition that $P$ is IDR and Lemma B.3.2c, we can substitute $V(f + 1, \omega + 1)$ for $V(f'', \omega + 1)$ and the inequality holds. Therefore,

$$r((f, \omega), 1) - r((f, \omega + 1), 1)$$

$$> \delta V(f, \omega + 1) \left[ \sum_{f'=0}^{f} \left[ p(f'(f, \omega)) - p(f'(f, \omega + 1)) \right] \right] +$$

$$\delta V(f + 1, \omega + 1) \left[ \sum_{f''=f+1}^{F} \left[ p(f''(f, \omega)) - p(f''(f, \omega + 1)) \right] \right] -$$

$$\delta V(f + 1, \omega + 1) \left[ \sum_{f''=f+1}^{F} p(f''(f, \omega + 1)) \right]$$

$$= \delta V(f, \omega + 1) \left[ \sum_{f'=0}^{f} \left[ p(f'(f, \omega)) - p(f'(f, \omega + 1)) \right] \right] +$$

$$\delta V(f + 1, \omega + 1) \left[ 1 - \sum_{f'=0}^{f} p(f'(f, \omega)) - p(F + 1|(f, \omega)) \right] -$$

$$\delta V(f + 1, \omega + 1) \left[ 1 - \sum_{f'=0}^{f} p(f'(f, \omega + 1)) - p(F + 1|(f, \omega + 1)) \right]$$

$$= \delta \left[ V(f, \omega + 1) - V(f + 1, \omega + 1) \right] \left[ \sum_{f'=0}^{f} \left[ p(f'(f, \omega)) - p(f'(f, \omega + 1)) \right] \right] +$$

$$\delta V(f + 1, \omega + 1) \left[ p(F + 1|(f, \omega + 1)) - p(F + 1|(f, \omega)) \right] \quad \text{(B.7d)}$$

Proposition 3.4.1b. $V(f, \omega + 1) - V(f + 1, \omega + 1) \geq 0$. By the property that $P$ is IDR,
\[ \sum_{f'=0}^{f'} \left[ p((f'|(f, \omega))-p(f'|f, \omega+1)) \right] \geq 0. \] And, \( \delta \in [0, 1] \). Therefore the first term in \( B.7d \) can be dropped because all elements are positive and the inequality holds. Therefore,

\[
\begin{align*}
    r((f, \omega), 1) - r((f, \omega + 1), 1) \\
    > \delta V(f + 1, \omega + 1) \left[ p(F + 1|(f, \omega + 1) - p(F + 1|(f, \omega)) \right]
\end{align*}
\]

By the definition of the value function in (3.7), \( V(f + 1, \omega + 1) = r((f + 1, \omega + 1), 1) \). Therefore,

\[
\begin{align*}
    r((f, \omega), 1) - r((f, \omega + 1), 1) \\
    > \delta \left[ r((f + 1, \omega + 1), 1) \left[ p(F + 1|(f, \omega + 1) - p(F + 1|(f, \omega)) \right] \right]
\end{align*}
\]

Which implies,

\[
\frac{r((f, \omega), 1) - r((f, \omega + 1), 1)}{r((f + 1, \omega + 1), 1)} > \delta \left[ p(F + 1|(f, \omega + 1) - p(F + 1|(f, \omega)) \right]
\]

Which contradicts (3.9). Therefore, it must be that \( a^*(f, \omega + 1) = 1 \), and the result follows.

Proof of Proposition 3.4.3  
Proof: Proposition 3.4.3 is equivalent to Proposition 2 in Oh and Özer (2016) and therefore the proof is omitted.

Proof of Proposition 3.4.4  
Proof of a: We want to show that \( \omega^T(f) > \omega^T(f + 1), \forall f \in \Gamma \). It is equivalent to show that,

\[ a^*(f, \omega^T(f)) = 1 \Rightarrow a^*(f + 1, \omega^T(f)) = 1 \]

By the definition of \( \omega^T(f) \), \( a^*(f, \omega^T(f)) = 1 \). The conditions of Proposition 3.4.2a hold, and therefore \( a^*(f, \omega) = 1 \Rightarrow a^*(f + 1, \omega) = 1, \forall \omega \in \Omega, \forall f \in \{0, 1, \ldots, F - 1\} \). Therefore, \( a^*(f, \omega^T(f)) = 1 \Rightarrow a^*(f + 1, \omega^T(f)) = 1 \). By the definition of \( \omega^T(f + 1) \), if \( a^*(f + 1, \omega^T(f)) = 1 \)
1, then \( \omega_T(f) > \omega_T(f + 1) \). Therefore, \( a'(f, \omega_T(f)) = 1 \Rightarrow a'(f + 1, \omega_T(f)) = 1 \).  

**Proof of b:** We want to show that \( \omega_T(f) < \omega_T(f + 1) \), \( \forall f \in \Gamma \). It is equivalent to show that, 

\[
a'(f, \omega_T(f)) = 1 \Rightarrow a'(f + 1, \omega_T(f)) = 1
\]

By the definition of \( \omega_T(f) \), \( a'(f, \omega_T(f)) = 1 \). The conditions of Proposition 3.4.2\( \text{b} \) hold, and therefore \( a'(f, \omega) = 1 \Rightarrow a'(f + 1, \omega) = 1, \forall \omega \in \omega, \forall f \in \{0, 1, \ldots, F - 1\} \). Therefore, 

\[
a'(f, \omega_T(f)) = 1 \Rightarrow a'(f + 1, \omega_T(f)) = 1.
\]

By the definition of \( \omega_T(f) + 1 \), if \( a'(f + 1, \omega_T(f)) = 1 \), then \( \omega_T(f) < \omega_T(f + 1) \). Therefore, \( a'(f, \omega_T(f)) = 1 \Rightarrow a'(f + 1, \omega_T(f)) = 1 \).  

**Proof of c:** From \( a \) and \( b \) the result follows directly.  

**Proof of d:** Let \( \hat{f} \) represent the healthiest severity state such that it is optimal to provide treatment for some age (i.e., \( \hat{f} = \min \{f \mid f \in \Gamma, \Omega^T(f) \neq \emptyset\} \)). Let \( \omega_T^T = \omega_T^{\hat{f}} \) and \( \omega_T^\hat{f} = \omega_T^{\hat{f}} \). 

First, we show that \( f^T(\omega) \) is non-increasing in \( \omega \) for all \( \omega < \omega_T^T \). It is sufficient to show that, 

\[
a'(f^T(\omega - 1), \omega - 1) = 1 \Rightarrow a'(f^T(\omega - 1), \omega) = 1, \forall \omega < \omega_T^T
\]

By the definition of \( f^T(\omega - 1) \), \( a'(f^T(\omega - 1), \omega - 1) = 1 \). The conditions of Proposition 3.4.3\( \text{a} \) hold, and therefore \( a'(f, \omega - 1) = 1 \Rightarrow a'(f, \omega) = 1, \forall \omega < \omega_T^{\hat{f}} \). Therefore, \( a'(f^T(\omega - 1), \omega - 1) = 1 \Rightarrow a'(f^T(\omega - 1), \omega) = 1, \forall \omega < \omega_T^{\hat{f}} \). Therefore, \( a'(f^T(\omega - 1), \omega - 1) = 1 \Rightarrow a'(f^T(\omega - 1), \omega) = 1, \forall \omega < \omega_T^{\hat{f}} \). 

Next, we show that \( f^T(\omega) \) is non-decreasing in \( \omega \) for all \( \omega > \omega_T^\hat{f} \). It is sufficient to show that, 

\[
a'(f^T(\omega + 1), \omega + 1) = 1 \Rightarrow a'(f^T(\omega + 1), \omega) = 1, \forall \omega > \omega_T^\hat{f}
\]

By the definition of \( f^T(\omega + 1) \), \( a'(f^T(\omega + 1), \omega + 1) = 1 \). The conditions of Proposition 3.4.3\( \text{b} \) hold, and therefore \( a'(f, \omega + 1) = 1 \Rightarrow a'(f, \omega) = 1, \forall \omega > \omega_T^\hat{f} \). Therefore, \( a'(f^T(\omega + 1), \omega + 1) = 1 \Rightarrow \).
1), \omega + 1) = 1 \Rightarrow a^*(f^T(\omega + 1), \omega) = 1, \forall \omega > \overline{\omega}^T. Therefore, a^*(f^T(\omega + 1), \omega + 1) = 1 \Rightarrow a^*(f^T(\omega), \omega) = 1, \forall \omega > \overline{\omega}^T and thus \( f^T(\omega) \) is non-decreasing in \( \omega \), \( \forall \omega > \overline{\omega}^T \).

Finally, \( f^T(\omega) \) is constant for all \( \omega \in [\underline{\omega}^T, \overline{\omega}^T] \), by definition.  

**Lemma B.3.3** Let \( \Upsilon_1 \) and \( \Upsilon_2 \) represent two problem instances of the single-payer scenario, sharing the same parameters with the exception of the disutilities of treatment, which are \( c_1 \) and \( c_2 \), respectively, such that \( c_2 - c_1 = k \geq 0 \). Let \( V_1(f, \omega) \) and \( V_2(f, \omega) \) represent the value functions in these two problem instances. Then,

\[
V_1(f, \omega) - V_2(f, \omega) \in [0, k], \forall (f, \omega) \in \mathcal{S}.
\]

**Proof of Lemma B.3.3** Proof: This proof is by backwards induction. Consider \( V_2(f, \omega) \) for \( \omega = N \),

\[
V_2(f, \omega) = \max\{r_2((f, N), 1), r_2((f, N), 0)\}
\geq \max\{r_1((f, N), 1), r_1((f, N), 0)\} - k  \tag{B.8a}
= V_1(f, \omega) - k
\]

**B.8a** follows because \( r_1((f, \omega), 1) - r_2((f, \omega), 1) = k \) and \( r_1((f, \omega), 0) = r_2((f, \omega), 0) \). Also,

\[
V_2(f, \omega) = \max\{r_2((f, N), 1), r_2((f, N), 0)\}
\leq \max\{r_1((f, N), 1), r_1((f, N), 0)\} \tag{B.8b}
= V_1(f, \omega)
\]

**B.8b** follows because \( r_1((f, \omega), 1) - r_2((f, \omega), 1) = k \geq 0 \) and \( r_1((f, \omega), 0) = r_2((f, \omega), 0) \). Therefore,

\[
V_2(f, N) - V_1(f, N) \in [0, k], \forall k \geq 0, \forall f \in \Gamma \tag{B.8c}
\]
For $\omega = N - 1$, 

$$V_2(f, N - 1)$$

(B.8d) 

$$= \max \left\{ r_2((f, N - 1), 1), r_2((f, N - 1), 0) + \delta \sum_{f' \in \Gamma} p(f'|f, N - 1) \cdot V_2(f', N) \right\}$$

$$\geq \max \left\{ r_2((f, N - 1), 1), r_1((f, N - 1), 0) + \delta \sum_{f' \in \Gamma} p(f'|f, N - 1) \cdot (V_1(f', N) - k) \right\}$$

(B.8e) 

$$= \max \left\{ r_2((f, N - 1), 1), r_1((f, N - 1), 0) + \delta \sum_{f' \in \Gamma} p(f'|f, N - 1) \cdot V_1(f', N) - \delta k \right\}$$

$$\geq \max \left\{ r_1((f, N - 1), 1), r_1((f, N - 1), 0) + \delta \sum_{f' \in \Gamma} p(f'|f, N - 1) \cdot V_1(f', N) \right\} - k$$

(B.8f) 

$$= V_1(f, N - 1) - k$$

(B.8g) 

[B.8e] results because of [B.8c] and because $r_1((f, \omega), 0) = r_2((f, \omega), 0)$. [B.8f] results because $r_1((f, \omega), 1) - r_2((f, \omega), 1) = k \geq 0$ and because $\delta \in [0, 1]$.

Also,

$$V_2(f, N - 1)$$

(B.8h) 

$$= \max \left\{ r_2((f, N - 1), 1), r_2((f, N - 1), 0) + \delta \sum_{f' \in \Gamma} p(f'|f, N - 1) \cdot V_2(f', N) \right\}$$

$$\leq \max \left\{ r_1((f, N - 1), 1), r_1((f, N - 1), 0) + \delta \sum_{f' \in \Gamma} p(f'|f, N - 1) \cdot V_1(f', N) \right\}$$

(B.8i) 

$$= V_1(f, N - 1)$$

[B.8i] results because of [B.8c] because $r_1((f, N - 1), 1) \geq r_2((f, N - 1), 1)$, and because $r_1((f, N - 1), 0) = r_2((f, N - 1), 0)$. Therefore,

$$V_2(f, N - 1) - V_1(f, N - 1) \in [0, k], \ \forall k \geq 0, \ \forall f \in \Gamma$$

(B.8j)
Repeating the process for \( \omega = N - 2, N - 3, \ldots, 0 \), we find that,

\[
V_2(f, \omega) - V_1(f, \omega) \in [0, k], \forall k \geq 0, \forall (f, \omega) \in \mathbb{S}
\]  

(B.8k)

\[\]

**Proof of Proposition 3.4.5**  
Proof: We want to show that, all else equal, if the disutility of treatment increases, then the number of states where treatment is optimal decreases (i.e., the size of the treatment region decreases).

Let \( \mathcal{T}_1 \) and \( \mathcal{T}_2 \) represent two problem instances sharing the same parameters, with the exception of the disutilities of treatment which are \( c_1 \) and \( c_2 \), respectively, such that \( c_1 \leq c_2 \). Notationally, we use the subscript ‘1’ and ‘2’ to denote these two problem instances throughout the following proof. Note, \( r_1((f, \omega), 1) \neq r_2((f, \omega), 1) \) due to the difference in disutility. However, \( r_1((f, \omega), 1) - r_2((f, \omega), 1) = c_2 - c_1 = k \), where \( k \geq 0 \) represents the difference in disutility. We want to show that for any \( k \geq 0 \),

\[ a^*_2(f, \omega) = 1 \Rightarrow a^*_1(f, \omega) = 1 \]

From the definition of the benefit function in Appendix C, it is optimal to provide treatment when the benefit function is negative. Therefore it is sufficient to show that,

\[ B_2(f, \omega) - B_1(f, \omega) \geq 0, \forall k \geq 0 \]

Note, the result of Lemma B.3.3 holds.

Consider the conditional benefit function, \( B_2(f, \omega) \),

\[
B_2(f, \omega) = \left[ r_2((f, \omega), 0) + \delta \sum_{f' \in \Gamma} p(f'|f, \omega) \cdot V_2(f', \omega + 1) \right] - r_2((f, \omega), 1)
\]

(B.9a)

\[
\geq \left[ r_1((f, \omega), 0) + \delta \sum_{f' \in \Gamma} p(f'|f, \omega) \cdot [V_1(f', \omega + 1) - k] \right] - [r_1((f, \omega), 1) - k]  
\]  

(B.9b)

\[
= \left[ r_1((f, \omega), 0) + \delta \sum_{f' \in \Gamma} p(f'|f, \omega) \cdot V_1(f', \omega + 1) \right] - [r_1((f, \omega), 1)] + [k - \delta k]
\]
\[ B_1(f, \omega) + k[1 - \delta] \] (B.9c)

\[ B_2(f, \omega) - B_1(f, \omega) \geq 0, \ \forall k \geq 0, \ \forall (f, \omega) \in S \] (B.9d)

Thus, the size of the treatment region decreases when the disutility of treatment increases.

**Lemma B.3.4** Let \( \Upsilon_1 \) and \( \Upsilon_2 \) represent two problem instances with disease progression matrices \( P_1 \) and \( P_2 \), and value functions \( \hat{V}_1(f, \omega) \) and \( \hat{V}_2(f, \omega) \), respectively. If \( \Upsilon_1 \) and \( \Upsilon_2 \) have the same reward functions, \( r(s, a) \), and disutility of treatment, \( c \), and \( P_1 \succeq P_2 \), then,

\[ \hat{V}_1(f, \omega) \geq \hat{V}_2(f, \omega), \ \forall (f, \omega) \in S \]

**Proof of Lemma B.3.4** Proof: This proof is by backwards induction.

Consider \( V_1(f, \omega) \) for \( \omega = N \),

\[ V_1(f, N) = \max\{r((f, N), 1), \ r((f, N), 0)\} \]

\[ = V_2(f, N) \]

Therefore,

\[ V_1(f, N) = V_2(f, N), \ \forall f \in \Gamma \] (B.10a)

For \( \omega = N - 1 \),

\[ V_1(f, N - 1) \]

\[ = \max \left\{ r((f, N - 1), 1), r((f, N - 1), 0) + \delta \sum_{f' \in \Gamma} p_1(f'|f, N - 1) \cdot V_1(f', N) \right\} \]
\[
\begin{align*}
\text{B.10b} & \geq \max \left\{ r((f, N - 1), 1), r((f, N - 1), 0) + \delta \sum_{f' \in \Gamma} p_1(f'(f, N - 1)) \cdot V_2(f', N) \right\} \\
\text{B.10c} & = \max \left\{ r((f, N - 1), 1), r((f, N - 1), 0) + \delta \sum_{f' \in \Gamma} p_2(f'(f, N - 1)) \cdot V_2(f', N) \right\} \\
\text{B.10d} & = V_2(f, N)
\end{align*}
\]

B.10b follows directly from B.10a. B.10c follows because the result of Lemma B.3.1 holds. Lemma B.3.1 holds because (1) \( V_2(f, \omega) \) is decreasing in \( f \) as a result of Proposition 3.4.1a, which holds because \( P_2 \) is IFR, and (2) because \( P_1 \geq P_2 \). Therefore,

\[
V_1(f, N - 1) \geq V_2(f, N - 1), \quad \forall f \in \Gamma
\]

(B.10d)

Repeating the process for \( \omega = N - 2, N - 3, \ldots, 0 \), we find that,

\[
V_1(f, \omega) \geq V_2(f, \omega), \quad \forall (f, \omega) \in \mathbb{S}
\]

(B.10e)

**Proof of Proposition 3.4.6**

Proof: Let \( a_1^*(f, \omega) \) and \( a_2^*(f, \omega) \) represent the optimal action and \( B_1(f, \omega) \) and \( B_2(f, \omega) \) represent the benefit function given problem instances \( \Upsilon_1 \) and \( \Upsilon_2 \), respectively. We want to show that,

\[
a_1^*(f, \omega) = T \Rightarrow a_2^*(f, \omega) = T, \quad \forall (f, \omega) \in \mathbb{S}
\]

From the definition of the benefit function, we know that it is optimal to provide treatment when the benefit function is negative. Therefore it is sufficient to show that,

\[
B_1(f, \omega) - B_2(f, \omega) \geq 0, \quad \forall (f, \omega) \in \mathbb{S}
\]

Note that the result of Lemma B.3.4 holds.
Consider,

\[ B_1(f, \omega) - B_2(f, \omega) \]

\[ = \left[ r((f, \omega), 0) + \delta \sum_{f' \in \Gamma} p_1(f'(f, \omega)) \cdot V_1(f', \omega + 1) \right] - r((f, \omega), 1) \]

\[ \quad - \left[ r((f, \omega), 0) + \delta \sum_{f' \in \Gamma} p_2(f'(f, \omega)) \cdot V_2(f', \omega + 1) \right] + r((f, \omega), 1) \]

\[ = \delta \sum_{f' \in \Gamma} p_1(f'(f, \omega)) \cdot V_1(f', \omega + 1) - \sum_{f' \in \Gamma} p_2(f'(f, \omega)) \cdot V_2(f', \omega + 1) \]

\[ \geq \delta \sum_{f' \in \Gamma} p_1(f'(f, \omega)) \cdot V_2(f', \omega + 1) - \sum_{f' \in \Gamma} p_2(f'(f, \omega)) \cdot V_2(f', \omega + 1) \]

\[ = \delta \sum_{f' \in \Gamma} [p_1(f'(f, \omega)) - p_2(f'(f, \omega))] \cdot V_2(f', \omega + 1) \]

\[ \geq V_2(f + 1, \omega + 1) \cdot \delta \sum_{f' \in \Gamma} [p_1(f'(f, \omega)) - p_2(f'(f, \omega))] \]

\[ = 0 \]  

(B.11a)

(B.11b)

(B.11c)

\[ \text{B.11a results from Lemma B.3.4. B.11b results from Lemma B.3.2d. And, B.11c results because } \]

\[ \sum_{f' \in \Gamma} [p_1(f'(f, \omega)) - p_2(f'(f, \omega))] = 0. \]

Therefore,

\[ B_1(f, \omega) - B_2(f, \omega) \geq 0, \forall (f, \omega) \in \mathcal{S} \]  

(B.11d)

Proof of Proposition 3.4.7: First, notice that for any state \((f, \omega) \in \mathcal{S}^Y\), the optimal treatment policy that the final payer selects will be the same as in the single-payer scenario. This follows by construction, because the final payer receives all the value for treatment from individuals in \(\mathcal{S}^Y\).

Therefore, for any state \((f, \omega) \in \mathcal{S}^Y\),

\[ V^X(f, \omega) + V^Y(f, \omega) = V^Y(f, \omega) = \hat{V}(f, \omega) \]  

(B.12a)

For any state \((f, \omega) \in \mathcal{S}^X\), we demonstrate that \(V^X(f, \omega) + V^Y(f, \omega) \leq \hat{V}(f, \omega)\). This proof
is by backwards induction.
Starting with $\omega = \mu - 1$:
The first payer receives,

$$V^X(f, \mu - 1) = \max \left\{ r^X((f, \mu - 1), 1), \ r^X(f, \mu - 1, 0) \right\} \quad (B.12b)$$

and the final payer receives,

$$V^Y(f, \mu - 1) = \begin{cases} 
  r^Y((f, \mu - 1), 1), & \text{if } a^X(f, \mu - 1) = 1 \\
  \delta \sum_{f' \in \Gamma} p(f'(f, \mu - 1)) \cdot V^Y(f', \mu), & \text{if } a^X(f, \mu - 1) = 0
\end{cases} \quad (B.12c)$$

Therefore, the total value is,

$$V^X(f, \mu - 1) + V^Y(f, \mu - 1) = \begin{cases} 
  r^X((f, \mu - 1), 1) + r^Y((f, \mu - 1), 1), & \text{if } a^X(f, \mu - 1) = 1 \\
  r^X((f, \mu - 1), 0) + \delta \sum_{f' \in \Gamma} p(f'(f, \mu - 1)) \cdot V^Y(f', \mu), & \text{if } a^X(f, \mu - 1) = 0
\end{cases} \quad (B.12d)$$

$$= \begin{cases} 
  r((f, \mu - 1), 1), & \text{if } a^X(f, \mu - 1) = 1 \\
  r((f, \mu - 1), 0) + \delta \sum_{f' \in \Gamma} p(f'(f, \mu - 1)) \cdot V(f', \mu), & \text{if } a^X(f, \mu - 1) = 0
\end{cases} \quad (B.12e)$$

$$\leq \max \left\{ r((f, \mu - 1), 1), \ r((f, \mu - 1), 0) + \delta \sum_{f' \in \Gamma} p(f'(f, \mu - 1)) \cdot \hat{V}(f', \mu) \right\} \quad (B.12f)$$

$$= \hat{V}(f, \mu - 1) \quad (B.12g)$$

\[\text{B.12d} \] does not include $V^X(f, \mu)$ because $V^X(f, \mu) = 0$ (i.e., the first payer does not cover patients $\mu$-years old). \[\text{B.12e} \] results because $r((f, \mu-1), 1) = r^X((f, \mu-1), 1) + r^Y((f, \mu-1), 1)$ and $V(f, \mu) = V^Y(f, \mu)$. \[\text{B.12f} \] results because the first payer’s decision may not be the same as in the single-payer scenario (i.e., $\hat{a}^*(f, \mu - 1) = 1 \Rightarrow a^X(f, \mu - 1) = 1$). Therefore,
\[ V^X(f, \mu - 1) + V^Y(f, \mu - 1) \leq \hat{V}(f, \mu - 1) \]  \hspace{1cm} (B.12h)

For \( \omega = \mu - 2 \):

The first payer receives,

\[
V^X(f, \mu - 2) = \max \left\{ r^X((f, \mu - 2), 1), \ r^X((f, \mu - 2), 0) + \delta \sum_{f' \in \Gamma} p(f'|(f, \mu - 2)) \cdot V^X(f', \mu - 1) \right\}
\]  \hspace{1cm} (B.12i)

the final payer receives,

\[
V^Y(f, \mu - 2) = \begin{cases} 
  r^Y((f, \mu - 2), 1) & \text{, if } a^X(f, \mu - 2) = 1 \\
  \delta \sum_{f' \in \Gamma} p(f'|(f, \mu - 2)) \cdot V^Y(f', \mu - 1) & \text{, if } a^X(f, \mu - 2) = 0
\end{cases}
\]  \hspace{1cm} (B.12j)

Therefore, the total value is,

\[
V^X(f, \mu - 2) + V^Y(f, \mu - 2) = \begin{cases} 
  r^X((f, \mu - 2), 1) + r^Y((f, \mu - 2), 1) & \text{, if } a^X(f, \mu - 2) = 1 \\
  r^X((f, \mu - 2), 0) + \delta \sum_{f' \in \Gamma} p(f'|(f, \mu - 2)) \cdot (V^X(f', \mu - 1) + V^Y(f', \mu - 1)) & \text{, if } a^X(f, \mu - 2) = 0
\end{cases}
\]  \hspace{1cm} (B.12k)
\[ \leq \max \left\{ r((f, \mu - 2), 1), r(f, \mu - 2), 0 \right\} + \delta \sum_{f' \in \Gamma} p(f')(f, \mu - 2) \cdot \hat{V}(f', \mu - 1) \]  

(B.12l)

\[ = \hat{V}(f, \mu - 2) \]  

(B.12m)

(B.12k) results because \( r((f, \mu - 2), 1) = r^X((f, \mu - 2), 1) + r^Y((f, \mu - 2), 1), r^X((f, \mu - 2), 0) = r((f, \mu - 2), 0) \), and \( V^X(f', \mu - 1) + V^Y(f', \mu - 1) = V(f', \mu - 1) \). (B.12l) results from (B.12h).

(B.12m) results because the first payer’s decision may not be the same as in the single-payer scenario (i.e., \( \hat{a}^*(f, \mu - 2) = 1 \Rightarrow a^X(f, \mu - 2) = 1 \)). Therefore, \( V^X(f, \mu - 2) + V^Y(f, \mu - 2) \leq \hat{V}(f, \mu - 2) \).

Repeating the same process for \( \omega = \mu - 3, \mu - 4, \ldots, 0 \), we find that,

\[ V^X(f, \omega) + V^Y(f, \omega) \leq \hat{V}(f, \omega), \forall (f, \omega) \in \mathbb{S}^X \]  

(B.12o)

Combined with (B.12a),

\[ V^X(f, \omega) + V^Y(f, \omega) = V(f, \omega) \leq \hat{V}(f, \omega), \forall (f, \omega) \in \mathbb{S} \]  

(B.12p)

---

**Proof of b**: First, notice that for any state \((f, \omega) \in \mathbb{S}^Y\), the first payer receives no value in either the single- or multi-payer scenarios. Therefore,

\[ \hat{V}^X(f, \omega) = V^X(f, \omega) = 0, \forall (f, \omega) \in \mathbb{S}^Y \]  

(B.13a)

For any state \((f, \omega) \in \mathbb{S}^X\), we demonstrate that \( V^X(f, \omega) \geq \hat{V}^X(f, \omega) \). This proof is by backwards induction.

Starting with \( \omega = \mu - 1 \):
\( \hat{V}^X(f, \mu - 1) \)

\[
\begin{align*}
\hat{V}^X(f, \mu - 1) & = \begin{cases} 
 r^X((f, \mu - 1), 1) & \text{if } \hat{a}^*(f, \mu - 1) = 1 \\
 r^X((f, \mu - 1), 0) & \text{if } \hat{a}^*(f, \mu - 1) = 0
\end{cases} \\
& \leq \max \left\{ r^X((f, \mu - 1), 1), r^X((f, \mu - 1), 0) \right\} \\
& = V^X(f, \mu - 1)
\end{align*}
\]

(B.13b)

\[\text{B.13b results because the choice that the first payer makes is at least as large as what it would receive if choosing } \hat{a}^*(f, \mu - 1). \text{ Therefore,} \]

\[
V^X(f, \mu - 1) \geq \hat{V}^X(f, \mu - 1), \forall f \in \Gamma
\]

(B.13c)

For \( \omega = \mu - 2 \):

\[ \hat{V}^X(f, \mu - 2) \]

\[
\begin{align*}
\hat{V}^X(f, \mu - 2) & = \begin{cases} 
 r^X((f, \mu - 2), 1) & \text{if } \hat{a}^*(f, \mu - 2) = 1 \\
 r^X((f, \mu - 2), 0) + \delta \sum_{f' \in \Gamma} p(f'|f, \mu - 2) \cdot \hat{V}^X(f', \mu - 1) & \text{if } \hat{a}^*(f, \mu - 2) = 0
\end{cases} \\
& \leq \begin{cases} 
 r^X((f, \mu - 2), 1) & \text{if } \hat{a}^*(f, \mu - 2) = 1 \\
 r^X((f, \mu - 2), 0) + \delta \sum_{f' \in \Gamma} p(f'|f, \mu - 2) \cdot V^X(f', \mu - 1) & \text{if } \hat{a}^*(f, \mu - 2) = 0
\end{cases} \\
& \leq \max \left\{ r^X((f, \mu - 2), 1), r^X((f, \mu - 2), 0) + \delta \sum_{f' \in \Gamma} p(f'|f, \mu - 2) \cdot V^X(f', \mu - 1) \right\} \\
& = V^X(f, \mu - 2)
\end{align*}
\]

(B.13d)

(B.13d) results because the choice that the first payer makes is at least as large as what it would receive if choosing \( \hat{a}^*(f, \mu - 2) \). Therefore, \( V^X(f, \mu - 2) \geq \hat{V}^X(f, \mu - 2), \forall f \in \Gamma \).

Repeating the same process for \( \omega = \mu - 3, \mu - 4, \ldots, 0 \), we find that,
\[ V^X(f, \omega) \geq \hat{V}^X(f, \omega), \ \forall (f, \omega) \in \mathbb{S}^X \]  
\text{(B.13e)}

Combined with \textbf{B.13a},

\[ V^X(f, \omega) \geq \hat{V}^X(f, \omega), \ \forall (f, \omega) \in \mathbb{S} \]  
\text{(B.13f)}

\begin{proof}
\textbf{Proof of c:} By the definition of \( \hat{V}^X(f, \omega) \) and \( \hat{V}^Y(f, \omega) \),

\[ \hat{V}(f, \omega) = \hat{V}^X(f, \omega) + \hat{V}^Y(f, \omega) \]  
\text{(B.14a)}

Subtracting \( V^X(f, \omega) \) and \( V^Y(f, \omega) \) from both sides,

\[ \hat{V}(f, \omega) - V^X(f, \omega) - V^Y(f, \omega) = \hat{V}^X(f, \omega) - V^X(f, \omega) + \hat{V}^Y(f, \omega) - V^Y(f, \omega) \]  
\text{(B.14b)}

Rearranging,

\[ \hat{V}^Y(f, \omega) - V^Y(f, \omega) = [\hat{V}(f, \omega) - V^X(f, \omega) - V^Y(f, \omega)] - [\hat{V}^X(f, \omega) - V^X(f, \omega)] \]  
\text{(B.14c)}

The first term in brackets on the right-hand-side is always positive, from Proposition \textbf{3.4.7a}.

And, the second term in brackets on the right-hand-side is always negative, from Proposition \textbf{3.4.7b}.

Therefore,

\[ \hat{V}^Y(f, \omega) - V^Y(f, \omega) \geq 0 \]  
\text{(B.14d)}

And it follows that,

\[ V^Y(f, \omega) \leq \hat{V}^Y(f, \omega), \ \forall (f, \omega) \in \mathbb{S} \]  
\text{(B.14e)}
Proof of Corollary 3.4.8 \[ \text{Proof:} \] The result follows directly from Proposition 3.4.7a, 3.4.7b, and 3.4.7c.

Lemma B.3.5 Let \( r^{X}_{\mu^{'}}((f, \omega), 1) \) represent the reward that the first payer receives from providing treatment, given the final age of patients that the first payer provides coverage for is \( \mu' \in \Omega \) (i.e., \( \mu' = \mu - 1 \)). Then,

a) \[ r^{X}_{\mu^'+1}((f, \omega), 1) - r^{X}_{\mu^{'}}((f, \omega), 1) = r_{T(f,\omega)}(\mu' + 1) \cdot \delta^{\mu'+1-\omega} \cdot T P(\mathcal{L}|T_{(f,\omega)}, \omega, \mu'+1) \]

b) \( r^{X}_{\mu^'+1}((f, \omega), 1) - r^{X}_{\mu^{'}}((f, \omega), 1) \) is non-increasing in \( f \).

Proof of Lemma B.3.5 \[ \text{Proof of a:} \] From the definition of the first payer’s reward from treatment in (3.1),

\[ r^{X}_{\mu^'+1}((f, \omega), 1) - r^{X}_{\mu^{'}}((f, \omega), 1) \]

\[ = r_{T(f,\omega)}(\omega) + \sum_{t=1}^{\mu'-\omega} \left[ \delta^{t} \cdot r_{T(f,\omega)}(\omega + t) \cdot \prod_{j=0}^{t-1} p_{T(f,\omega)}(T_{(f,\omega)}|\omega + j) \right] \]

\[ - r_{T(f,\omega)}(\omega) - \sum_{t=1}^{\mu'-\omega} \left[ \delta^{t} \cdot r_{T(f,\omega)}(\omega + t) \cdot \prod_{j=0}^{t-1} p_{T(f,\omega)}(T_{(f,\omega)}|\omega + j) \right] \]

\[ = \delta^{\mu'+1-\omega} \cdot r_{T(f,\omega)}(\mu' + 1) \cdot \prod_{j=0}^{\mu'-\omega} p_{T(f,\omega)}(T_{(f,\omega)}|\omega + j) \]

\[ = \delta^{\mu'+1-\omega} \cdot r_{T(f,\omega)}(\mu' + 1) \cdot T P(\mathcal{L}|T_{(f,\omega)}, \omega, \mu'+1) \]

(B.15a)

B.15a results from the definition of the total probability of life.

**Proof of b:** From A3.4.2, \( r_{T(f,\omega)}(\mu' + 1) \) and \( p_{T(f,\omega)}(T_{(f,\omega)}|\omega + j) \) are non-increasing in \( f \) for any \( \omega \in \Omega \). Therefore, the result follows directly from B.15a.

Proof of Proposition 3.4.10 \[ \text{Proof:} \] We want to show that,

\[ a^{*X}(f, \omega) = 1 \Rightarrow \hat{a}^{*}(f, \omega) = 1 \]

\( \forall (f, \omega) \in \mathbb{S}^{X} \)

(B.16a)
From the definition of the benefit function, we know that treatment is provided when the benefit function is negative. Therefore, it is equivalent to show that,

\[ B_X(f, \omega) \leq 0 \Rightarrow B(f, \omega) \leq 0 \quad \forall (f, \omega) \in S_X \]  

(B.16b)

And therefore it is sufficient to show that,

\[ B_X(f, \omega) - B(f, \omega) \geq 0 \quad \forall (f, \omega) \in S_X \]  

(B.16c)

Let \( B^X_{\mu'}(f, \omega) \) represent the first payer’s benefit function, parameterized by \( \mu' \) which represents the maximum patient age under the first payer’s coverage (i.e., \( \mu' = \mu - 1 \)). Therefore, \( B^X_N(f, \omega) \) represents the special case where the first payer provides coverage over a patient’s entire lifespan. For completeness, let \( S^X_{\mu'}, r^X_{\mu'}((f, \omega), 1), V^X_{\mu'}(f, \omega), \) and \( M^X_{\mu'}(f, \omega) \) represent the set of states where the first payer provides coverage, the first payer’s reward from treatment, the first payer’s value function, and the first payer’s one-step benefit function, each parameterized by \( \mu' \). Note that \( B^X_N(f, \omega) = B(f, \omega), \forall (f, \omega) \in S_X^N \equiv S. \)

Therefore, it is sufficient to show that,

\[ B^X_{\mu'}(f, \omega) - B^X_{\mu'+1}(f, \omega) \geq 0 \quad \forall \mu' \in \Omega, \forall (f, \omega) \in S^X_{\mu'} \]  

(B.16d)

This proof is by backwards induction with respect to \( \omega \), with each step a proof by contradiction.

Starting with \( \omega = \mu' \).

For clarity, we provide Condition 3.11 from Proposition 3.4.10 here and simplify for \( \omega = \mu' \) (recall that \( s_1 = (f, \omega) \) and \( s_2 = (\underline{f(s_1)}, \omega + 1) \)),

\[
\frac{r_{T_{s_2}}(\mu) - r_{T_{s_1}}(\mu)}{r_{T_{s_1}}(\mu)} < \frac{TP(L|T_{s_1}, \omega, \mu) - TP(L|T_{s_2}, \omega + 1, \mu) \cdot (1 - p(F + 1|s_1))}{TP(L|T_{s_2}, \omega + 1, \mu) \cdot (1 - p(F + 1|s_1))}
\]  

(3.11)
Adding 1 to each side,
\[
\frac{r_{T_{2z}}(\mu)}{r_{T_{1z}}(\mu)} < \frac{TP(L|T_{s1}, \omega, \mu)}{TP(L|T_{s2}, \omega + 1, \mu) \cdot (1 - p(F + 1|s1))}
\]

Evaluating at \(\omega = \mu'\), and substituting \(\mu = \mu' + 1\),
\[
\frac{r_{T_{(f,\mu')}_{\mu'+1}}(\mu' + 1)}{r_{T_{(f,\mu')}_{\mu'+1}}(\mu')} < \frac{TP(L|T_{(f,\mu')}, \mu', \mu' + 1)}{TP(L|T_{(f,\mu')}, \mu' + 1, \mu' + 1) \cdot (1 - p(F + 1|(f,\mu'))} \]

where, \(TP(L|T_{(f,\mu')}, \mu', \mu' + 1) = 1\) from the definition of total probability of life (i.e., the probability of living from year \(\mu' + 1\) until \(\mu' + 1\) is 1). Therefore,
\[
\frac{r_{T_{(f,\mu')}_{\mu'+1}}(\mu' + 1)}{r_{T_{(f,\mu')}_{\mu'+1}}(\mu')} < \frac{TP(L|T_{(f,\mu')}, \mu', \mu' + 1)}{(1 - p(F + 1|(f,\mu'))} \]

Rearranging, setting the left-hand-side equal to zero, Condition 3.11 is equivalent to,
\[
0 < r_{T_{(f,\mu')}_{\mu'+1}}(\mu') \cdot TP(L|T_{(f,\mu')}, \omega, \mu) - r_{T_{(f,\mu')}_{\mu'+1}}(\mu' + 1) \cdot (1 - p(F + 1|(f,\mu'))) \quad (B.16e)
\]

By the definition of the benefit function,

\[
B^X_{\mu'}(f, \mu') - B^X_{\mu'+1}(f, \mu')
= \left[ r^X((f, \mu'), 0) + \delta \sum_{f' \in \mathbb{F}} p(f'|f, \mu') \cdot V^X_{\mu'}(f', \mu' + 1) \right] - r^X_{\mu'}((f, \mu'), 1)
- \left[ r^X((f, \mu'), 0) - \delta \sum_{f' \in \mathbb{F}} p(f'|f, \mu') \cdot V^X_{\mu'+1}(f', \mu' + 1) \right] + r^X_{\mu'+1}((f, \mu'), 1)
= r^X_{\mu'+1}((f, \mu'), 1) - r^X_{\mu'}((f, \mu'), 1) - \delta \sum_{f' \in \mathbb{F}} p(f'|f, \mu') \cdot V^X_{\mu'+1}(f', \mu' + 1) \quad (B.16f)
\]

results because \(V^X_{\mu'}(f', \mu' + 1) = 0, \forall f' \in \Gamma\). Substituting the result from Lemma B.3.5 gives

\[
B^X_{\mu'}(f, \mu') - B^X_{\mu'+1}(f, \mu')
\]
\[= \delta \cdot r_{T,(f')}(\mu' + 1) \cdot TP(L|T_{(f',\mu')}, \mu', \mu' + 1) - \delta \cdot \sum_{f' \in \mathbb{P}} p(f'|(f, \mu')) \cdot V^X_{\mu' + 1}(f', \mu' + 1)\]

Suppose the converse of (B.16d) is true (i.e., the contradiction assumption). Therefore,

\[0 \geq \delta \cdot r_{T,(f')}(\mu' + 1) \cdot TP(L|T_{(f',\mu')}, \mu', \mu' + 1) - \delta \cdot \sum_{f' \in \mathbb{P}} p(f'|(f, \mu')) \cdot V^X_{\mu' + 1}(f', \mu' + 1)\]

\[\geq \delta \cdot r_{T,(f')}(\mu' + 1) \cdot TP(L|T_{(f,\mu')}, \mu', \mu' + 1) - \delta \cdot \sum_{f' \in \mathbb{P}} p(f'|(f, \mu')) \cdot V^X_{\mu' + 1}(f, \mu', \mu' + 1)\]

\[= B_{\mu'}^{X}(f, \mu') - B_{\mu' + 1}^{X}(f, \mu') \geq 0, \forall f \in \Gamma\]  (B.16h)
Next, for $\omega = \mu' - 1$:

Again, for clarity, we provide Condition 3.11 from Proposition 3.4.10 here and simplify for $\omega = \mu' - 1$ (recall that $s_1 = (f, \omega)$ and $s_2 = (f(s_1), \omega + 1)$).

\[
\frac{r_{T_2}(\mu) - r_{T_1}(\mu)}{r_{T_1}(\mu)} < \frac{TP(L|T_{s_1}, \omega, \mu) - TP(L|T_{s_2}, \omega + 1, \mu) \cdot (1 - p(F + 1|s_1))}{TP(L|T_{s_2}, \omega + 1, \mu) \cdot (1 - p(F + 1|s_1))} \quad (3.11)
\]

Adding 1 to each side,

\[
\frac{r_{T_2}(\mu) - r_{T_1}(\mu)}{r_{T_1}(\mu)} < \frac{TP(L|T_{s_1}, \omega, \mu)}{TP(L|T_{s_2}, \omega + 1, \mu) \cdot (1 - p(F + 1|s_1))}
\]

Evaluating at $\omega = \mu' - 1$, and substituting $\mu = \mu' + 1$,

\[
\frac{r_{T_{(f, \mu'-1)(\mu'+1)}}(\mu' + 1)}{r_{T_{(f, \mu'-1)(\mu'+1)}}(\mu' + 1)} < \frac{TP(L|T_{(f, \mu'-1)}, \mu' - 1, \mu' + 1)}{TP(L|T_{(f, \mu'-1)(\mu'+1)}, \mu', \mu' + 1) \cdot (1 - p(F + 1|f, \mu' - 1))}
\]

Rearranging, setting the left-hand-side equal to zero, Condition 3.11 is equivalent to,

\[
0 < r_{T_{(f, \mu'-1)(\mu'+1)}}(\mu' + 1) \cdot TP(L|T_{(f, \mu'-1)}, \mu' - 1, \mu' + 1)
- r_{T_{(f, \mu'-1)(\mu'+1)}}(\mu' + 1) \cdot TP(L|T_{(f, \mu'-1)(\mu'+1)}, \mu', \mu' + 1) \cdot (1 - p(F + 1|f, \mu' - 1))
\]

(B.16i)

By the alternate definition of the benefit function in Appendix C,

\[
B^X_{\mu'}(f, \mu' - 1) - B^X_{\mu'+1}(f, \mu' - 1)
= M^X_{\mu'}(f, \mu' - 1) - M^X_{\mu'+1}(f, \mu' - 1)
+ \delta \cdot \sum_{f' \in \mathbb{F}} p(f'||(f, \mu' - 1)) \cdot [(B^X_{\mu'}(f', \mu'))^+ - (B^X_{\mu'+1}(f', \mu'))^+]
\]

Suppose the converse of B.16d is true (i.e., the contradiction assumption). Therefore,

\[
0 \geq M^X_{\mu'}(f, \mu' - 1) - M^X_{\mu'+1}(f, \mu' - 1)
+ \delta \cdot \sum_{f' \in \mathbb{F}} p(f'||(f, \mu' - 1)) \cdot [(B^X_{\mu'}(f', \mu'))^+ - (B^X_{\mu'+1}(f', \mu'))^+]
\]
\[ \geq M_{\mu'}^X(f, \mu' - 1) - M_{\mu'}^{X+1}(f, \mu' - 1) \]  

Equation (B.16j)

Equation (B.16j) results directly from Lemma B.16h By the definition of the one-step benefit function in Appendix C.

\[ 0 \geq r^X((f, \mu' - 1), 0) + \delta \sum_{f' \in \mathcal{F}} p(f'| (f, \mu' - 1)) \cdot \left[ r_{\mu'}^X((f', \mu'), 1) \right] - r_{\mu'}^X((f, \mu' - 1), 1) \]

\[ \delta \sum_{f' \in \mathcal{F}} p(f'| (f, \mu' - 1)) \cdot \left[ r_{\mu'}^X((f', \mu'), 1) + r_{\mu'}^{X+1}((f, \mu' - 1), 1) \right] \]

Substituting the result from Lemma B.3.5a (twice),

\[ 0 \geq \left[ r_{T(f, \mu' - 1)}(\mu' + 1) \cdot \delta^2 \cdot TP(L|T(f, \mu' - 1), \mu' - 1, \mu' + 1) \right] \]

\[ - \delta \sum_{f' \in \mathcal{F}} p(f'| (f, \mu' - 1)) \cdot \left[ \delta \cdot TP(L|T(f', \mu'), \mu' + 1) \right] \]

Equation (B.16k)

Equation (B.16k) follows because \( r_{T(f', \mu')} \cdot (\mu' + 1) \cdot \delta \cdot TP(L|T(f', \mu'), \mu' + 1) \) is decreasing in \( f' \) (from Lemma B.3.5a). Therefore,

\[ 0 \geq \delta^2 \left[ r_{T(f, \mu' - 1)}(\mu' + 1) \cdot TP(L|T(f, \mu' - 1), \mu' - 1, \mu' + 1) \right] \]

\[ - \delta^2 \left[ r_{T(f, \mu' - 1)}(\mu' + 1) \cdot TP(L|T(f, \mu' - 1), \mu' - 1, \mu' + 1) \right] \cdot \left[ 1 - p(F + 1 | f, \mu' - 1) \right] \]

\[ = \delta^2 \left[ r_{T(f, \mu' - 1)}(\mu' + 1) \cdot TP(L|T(f, \mu' - 1), \mu' - 1, \mu' + 1) \right] \]

\[ - r_{T(f, \mu' - 1)}(\mu' + 1) \cdot TP(L|T(f, \mu' - 1), \mu' - 1, \mu' + 1) \cdot \left[ 1 - p(F + 1 | f, \mu' - 1) \right] \]

Equation (B.16l)
B.3. Proofs

which is a contradiction to Condition [3.11] as easily observed in comparison with Equation [B.16i]. Therefore,

\[ B_{\mu'}^X(f, \mu' - 1) - B_{\mu'+1}^X(f, \mu' - 1) \geq 0, \forall f \in \Gamma \]

Following the same process for \( \omega = \mu' - 2, \mu' - 3, \ldots, 0 \), the result follows. Therefore,

\[ B_{\mu'}^X(f, \omega) - B_{\mu'+1}^X(f, \omega) \geq 0, \forall (f, \omega) \in S^X \]

And thus,

\[ B^X(f, \omega) - B(f, \omega) \geq 0 \]
\[ \forall (f, \omega) \in S^X \]

\[ \square \]

**Proof of Corollary 3.4.11**

Proof: We demonstrate that if \( f(f, \omega) \leq f \), \( \forall (f, \omega) \in S \), then Condition [3.11] holds, and therefore the result of Proposition 3.4.10 holds.

From the right-hand-side of Condition [3.11],

\[ \frac{r_{T(f, \omega)}(\mu) - r_{T(f, \omega)}(\mu)}{r_{T(f, \omega)}(\mu)} \leq \frac{r_{T(f, \omega+1)}(\mu) - r_{T(f, \omega)}(\mu)}{r_{T(f, \omega)}(\mu)} \]
\[ \leq 0 \] (B.17a)

(B.17b)

B.17a results from the condition that \( f(f, \omega) \leq f \), \( \forall (f, \omega) \in S \) and A3.4.2. B.17b results directly from A3.4.2.

Separately, from the left-hand-side of Condition [3.11],

\[ TP(L|T_{(f, \omega)}, \omega, \mu) - TP(L|T_{(f, \omega), \omega+1}, \omega + 1, \mu) \cdot \left[ 1 - p(F + 1|(f, \omega)) \right] \]
\[ \geq TP(L|T_{(f, \omega), \omega+1}, \omega + 1, \mu) \cdot \left[ 1 - p(F + 1|(f, \omega)) \right] \] (B.17c)
\[\begin{align*}
&= TP(L|T_{f,\omega}, \omega, \mu) - TP(L|T_{f,\omega}, \omega + 1, \mu) \cdot \left[1 - p(F + 1|f, \omega)\right] \\
&\geq \frac{TP(L|T_{f,\omega}, \omega + 1, \mu) \cdot \left[1 - p(F + 1|f, \omega)\right]}{TP(L|T_{f,\omega}, \omega, \mu) - TP(L|T_{f,\omega + 1}, \omega, \mu)} \\
&\geq 0
\end{align*}\] (B.17d)

B.17c results because the total probability of life is decreasing in \(f\), from A3.4.2. B.17d results because the total probability of life is decreasing in \(\omega\), from A3.4.2. B.17e results because \(p_{T_{f,\omega}}(T_{f,\omega}|\omega) \geq \left(1 - p(F + 1|f, \omega)\right)\), from A3.4.3. Finally, B.17f results because the total probability of life is decreasing in \(\omega\), from A3.4.2.

Therefore, the right-hand-side in Condition 3.11 is always negative and the left-hand-side is always positive. Thus, (3.11) holds when \(f(\omega) \leq f\), \(\forall(f, \omega) \in \mathfrak{S}\), and therefore the result of Proposition 3.4.10 holds.

Proof of Corollary 3.4.12  
Proof: Given that \(P\) is IFR\(_f\) and Condition 3.11 hold, the result of Proposition 3.4.10 holds. Therefore, if it is optimal under a multi-payer scenario for treatment to be provided, then it is also optimal under a single-payer scenario for treatment to be provided. Thus, in the states where treatment is optimal in the multi-payer scenario, the value that the first payer and the final payer receives is equivalent under both scenarios.

Lemma B.3.6

a) Let \(c_T^f(\omega) = r_{T_{f,\omega}}(\omega) - r((F, \omega), 0)\) represent a treatment cost threshold with respect to age. If the conditions in Proposition 3.4.3d are satisfied and \(\Gamma^T(\omega)\) is non-empty, then \(c > c_T^f(\omega) \Leftrightarrow \overline{f}^T(\omega) < F\).

b) Let \(c_T^\omega(f) = r_{T_{f,N}}(N) - r((f, N), 0)\) represent a treatment cost threshold with respect to disease severity. If the conditions in Proposition 3.4.3d are satisfied and \(\Omega^T(f)\) is non-empty, then \(c > c_T^\omega(f) \Leftrightarrow \overline{\omega}^T(f) < N\).

Proof of Lemma B.3.6  
Let \(r_c((f, \omega), 1)\) represent the reward from treatment excluding the fixed disutility, i.e., \(r_c((f, \omega), 1) \equiv r((f, \omega), 1) + c\).
Proof of a: For any $\omega \in \Omega$, consider the benefit function at $f = F$,

$$B(F, \omega) = r((F, \omega), 0) - [r_{\tilde{f}}((F, \omega), 1) - c]$$  \hfill (B.18a)

Treatment is not provided to patients in state $(f, N)$ if and only if $B(F, \omega) > 0$ (Oh and Özer 2016). The benefit function is positive, and therefore treatment is not provided, at severity $F$ when,

$$c > r_{\tilde{f}}((F, \omega), 1) - r((F, \omega), 0) = r_{T,\omega}(\omega) - r((F, \omega), 0) = c_f^T(\omega)$$  \hfill (B.18b)

Therefore, if $c > c_f^T(\omega)$, then treatment is not provided at severity $F$ (i.e., $B(F, \omega) > 0$). Given that there exists a disease severity where treatment is optimal (i.e., $B(f, \omega) \leq 0$ for some $f \in \mathbb{F} \setminus F$), then by the intermediate value theorem there must exist some severity, $\tilde{f}^T(\omega) < F$, such that $B(f, \omega) > 0$, $\forall f > \tilde{f}^T(\omega)$. 

Proof of b: For any $f \in \mathbb{F}$, consider the benefit function at $\omega = N$,

$$B(f, N) = r((f, N), 0) - [r_{\tilde{f}}((f, N), 1) - c]$$  \hfill (B.19a)

Treatment is not provided if and only if $B(f, N) > 0$ (Oh and Özer 2016). The benefit function is positive, and therefore treatment is not provided, at age $N$, when,

$$c > r_{\tilde{f}}((f, N), 1) - r((f, N), 0) = r_{T,f,N}(N) - r((f, N), 0) = c_{\omega}^T(f)$$  \hfill (B.19b)

Therefore, if $c > c_{\omega}^T(f)$, then treatment is not provided at age $N$ (i.e., $B(f, N) > 0$). Given that there exists age where treatment is optimal (i.e., $B(f, \omega) \leq 0$ for some $\omega \in \Omega \setminus N$), then by the intermediate value theorem there must exist some age, $\omega^T(f) < N$, such that $B(f, \omega) > 0$, $\forall \omega > \omega^T(f)$. 

Proof of Corollary 3.4.13 Proof: The conditions of Proposition 3.4.2a and Proposition 3.4.3b are satisfied and therefore the first payer’s optimal treatment policy and the single-payer’s optimal treatment policy are both of control-limit type with respect to disease severity and of control-band type with respect to age. The condition that $\mu \in \Omega^{\tilde{T}}(f)$ implies
that the optimal single-payer treatment band (for disease severity $f$) includes treatment for individuals $\mu$-years old. However, the condition that $c > r_{f,\mu-1}(\mu - 1) - r(f, \mu - 1, 0)$ satisfies the conditions for Lemma [B.3.6b] and therefore the first payer will stop treatment prior to age $\mu$. Thus, for a given disease severity $f$, the first payer’s optimal policy is to provide treatment over a band of ages, stopping treatment prior to age $\mu$. And, the optimal single-payer policy (and therefore the optimal final payer’s policy) is to provide treatment to individuals from age $\mu$ until $\bar{\omega}(f) = \max(\Omega^T(f)) > \mu$. Thus, for the given severity $f$, the optimal policy with respect to age will be of Wait-Treat-Wait-Treat-Wait (WTWTW) type.

**Proof of Proposition 3.4.14** This proof is by backwards induction. Because $P$ is IFR$_f$ and Condition 3.11 holds, the result of Proposition 3.4.10 holds (i.e., the first payer will treat a subset of the optimal patients) and therefore $a^*_X(f, \omega) = 1$ and $\hat{a}^*(f, \omega) = 0$ is not a possible outcome. Therefore, there are three possible scenarios for each $(f, \omega)$:

a) $a^*_X(f, \omega) = 1$ and $\hat{a}^*(f, \omega) = 1$

b) $a^*_X(f, \omega) = 0$ and $\hat{a}^*(f, \omega) = 0$

c) $a^*_X(f, \omega) = 0$ and $\hat{a}^*(f, \omega) = 1$

Let $r_f((f, \omega), 1) = r((f, \omega), 1) + c$ represent the reward from treatment excluding the fixed disutility. Starting with the final year of the first payer’s coverage, $\omega = \mu - 1$

From scenario a),

$$V^X(f, \mu - 1) - \hat{V}^X(f, \mu - 1) = r^X((f, \mu - 1), 1) - r^X((f, \mu - 1), 1) = 0 \quad (B.20a)$$

From scenario b)

$$V^X(f, \mu - 1) - \hat{V}^X(f, \mu - 1) = r^X((f, \mu - 1), 0) - r^X((f, \mu - 1), 0) = 0 \quad (B.20b)$$
From scenario c)

\[ V^X(f, \mu - 1) - \hat{V}^X(f, \mu - 1) = r^X((f, \mu - 1), 0) - r^X((f, \mu - 1), 1) \]
\[ \leq r_{T(f, \omega - 1)}(\mu - 1) - r^X((f, \mu - 1), 1) \]  
\[ = r_{T(f, \omega - 1)}(\mu - 1) - r_{T(f, \omega - 1)}(\mu - 1) + c \] (B.20c)
\[ = c \] (B.20d)

where B.20c follows from A3.4.3 and B.20d follows from the definition of the first payer’s terminal reward in Equation (3.1). Therefore,

\[ V^X(f, \mu - 1) - \hat{V}^X(f, \mu - 1) \leq c, \forall f \in F \] (B.20e)

For \( \omega = \mu - 2 \),

From scenario a),

\[ V^X(f, \mu - 2) - \hat{V}^X(f, \mu - 2) \]
\[ = r^X((f, \mu - 2), 1) - r^X((f, \mu - 2), 1) \]
\[ = 0 \]

From scenario b),

\[ V^X(f, \mu - 2) - \hat{V}^X(f, \mu - 2) \]
\[ = r^X((f, \mu - 2), 0) + \delta \sum_{f' \in F} p(f'|(f, \mu - 2)) \cdot V^X(f', \mu - 1) \]
\[ - r^X((f, \mu - 2), 0) + \delta \sum_{f' \in F} p(f'|(f, \mu - 2)) \cdot \hat{V}^X(f', \mu - 1) \]
\[ = \delta \sum_{f' \in F} p(f'|(f, \mu - 2)) \cdot [V^X(f', \mu - 1) - \hat{V}^X(f', \mu - 1)] \]
\[ \leq \delta \cdot c \cdot [1 - p(F + 1|(f, \mu - 2))] \] (B.20f)
\[ \leq c \] (B.20g)
Appendix: Essay 2

B.20f follows directly from B.20d and because \( V^X(F + 1, \mu - 1) = \hat{V}^X(F + 1, \mu - 1) = 0 \).

B.20g follows because \( \delta \in [0, 1] \) and \( [1 - p(F + 1(f, \mu - 2))] \in [0, 1] \).

From scenario c),

The proof of this iteration of scenario c), and those that follow, is by contradiction. For clarity, we provide Condition 3.12 from Proposition 3.4.14 here and simplify for \( \omega = \mu - 2 \).

\[
\frac{r^X((f, \omega + 1), 1) + c}{r^X((f(f, \omega), \omega + 1), 1)} > \frac{1 - p(F + 1(f, \omega))}{p_{T(f, \omega)}(T(f, \omega) | \omega)}
\]

(3.12)

Rearranging and substituting \( \omega = \mu - 2 \), therefore Condition 3.12 is equivalent to,

\[
0 > r^X((f(f, \mu - 2), \mu - 1), 1) \cdot (1 - p(F + 1(f, \mu - 2)))
\]  
\[
- \left(r^X((f, \mu - 1), 1) + c\right) \cdot p_{T(f, \mu - 2)}(T(f, \mu - 2) | \mu - 2)
\]

(B.20h)

By scenario c),

\[
V^X(f, \mu - 2) - \hat{V}^X(f, \mu - 2)
\]
\[
= r^X((f, \mu - 2), 0) + \delta \sum_{f' \in \Gamma} p(f'|(f, \mu - 2)) \cdot V^X(f', \mu - 1) - r^X((f, \mu - 2), 1)
\]

Now, suppose the converse of \( V^X(f, \mu - 2) - \hat{V}^X(f, \mu - 2) \leq c \) is true (i.e., the contradiction assumption). Then,

\[
0 \leq \left[r^X((f, \mu - 2), 0) \right] - \left[r^X((f, \mu - 2), 1) \right] + \left[\delta \sum_{f' \in \Gamma} p(f'|(f, \mu - 2)) \cdot r^X((f', \mu - 1), 1) \right] - c
\]

(B.20i)

\[
\leq \left[r_{T(f, \mu - 2)}(\mu - 2) \right]
\]
\[
- \left[r_{T(f, \mu - 2)}(\mu - 2) + \delta \cdot p_{T(f, \mu - 2)}(T(f, \mu - 2) | \mu - 2) \cdot r^X((f, \mu - 1), 1) - c \right]
\]

\[
+ \left[\delta \sum_{f' \in \Gamma} p(f'|(f, \mu - 2)) \cdot r^X((f', \mu - 1), 1) \right] - c
\]

(B.20j)
B.20 results because $V^X(f, \omega) \geq r^X((f, \omega), 1)$, $\forall (f, \omega) \in \mathbb{S}^X$ by the definition of the value function and because $V^X(F + 1, \mu - 1) = 0$. B.20 results from A3.4.3 and because $r^X((f, \mu - 2), 1) = r_{T(f, \mu - 2)}(\mu - 2) + \delta \cdot p_{T(f, \mu - 2)}(T(f, \mu - 2)|\mu - 2) \cdot r^X(f, \mu - 1, 1) - c$ from the definition of the first payer’s terminal reward. Therefore,

$$
0 \leq \delta \left[ \sum_{f' \in \mathbb{R}} p(f'|f, \mu - 2)) \cdot r^X((f', \mu - 1), 1) - p_{T(f, \mu - 2)}(T(f, \mu - 2)|\mu - 2) \cdot r^X(f, \mu - 1, 1) \right]
$$

$$
\leq \delta \left[ \sum_{f' \in \mathbb{R}} p(f'|f, \mu - 2)) \cdot r^X((f(f, \mu - 2), \mu - 1), 1) - p_{T(f, \mu - 2)}(T(f, \mu - 2)|\mu - 2) \cdot r^X(f, \mu - 1, 1) \right]
$$

$$(B.20k)$$

B.20k results because $r^X((f, \mu - 1), 1)$ is decreasing in $f$ from A3.4.2 and because $P$ is IFR$^f$ (i.e., severity state $f(f, \mu - 2)$ is the best possible next-period state). By definition, $r^X(f, \mu - 1, 1) = r^X((f, \mu - 1, 1) + c$, therefore,

$$
0 \leq \delta \left[ r^X((f(f, \mu - 2), \mu - 1), 1) \cdot \left( 1 - p(F + 1|(f, \mu - 2)) \right) - r^X((f, \mu - 1, 1) + c) \cdot p_{T(f, \mu - 2)}(T(f, \mu - 2)|\mu - 2) \right]
$$

which is a contradiction to Condition 3.12 as easily observed in comparison with Equation [B.20h]. Therefore,

$$
V^X(f, \mu - 2) - \hat{V}^X(f, \mu - 2) \leq c, \ \forall f \in \Gamma
$$

(B.20i)

Following the same process for $\omega = \mu - 3, \ \mu - 4, \ldots, 0$, the result follows. Therefore,

$$
V^X(f, \omega) - \hat{V}^X(f, \omega) \leq c, \ \forall (f, \omega) \in \mathbb{S}^X
$$

Proof of Proposition 3.4.15 Proof: Because $P$ is IFR$^f$ and Condition 3.11 holds, the treatment region in the multi-payer scenario is a subset of the treatment region in the single-
payer scenario. In this proof, we will demonstrate that an incentive payment exists that can coordinate the multi-payer scenario. This requires showing that an incentive payment exists when the first payer’s actions are sub-optimal and that an incentive payment does not exist that is incentive compatible for the final payer and that causes the first payer to deviate from an optimal decision to a sub-optimal decision.

This proof is by backwards induction.

For each step in this backwards induction proof, there are three possible scenarios. The scenarios are defined by the optimal action and the first payer’s action in an uncoordinated multi-payer system. The scenarios are:

a) The first payer acts optimally and provides treatment, \( a^X(f, \omega) = 1 \) and \( \hat{a}^*(f, \omega) = 1 \)
b) The first payer acts sub-optimally and does not provide treatment, \( a^X(f, \omega) = 0 \) and \( \hat{a}^*(f, \omega) = 1 \)
c) The first payer acts optimally and waits, \( a^X(f, \omega) = 0 \) and \( \hat{a}^*(f, \omega) = 0 \)

For each step in this backwards induction proof, if the first payer acts optimally and provides treatment, then the final payer will make an incentive payment of zero as \( V_Y(f, \omega) = \hat{V}_Y(f, \omega) \) if \( a^X(f, \omega) = 1 \), \( \forall (f, \omega) \in \mathbb{S}_X \), where \( \hat{V}_Y(f, \omega) \) represents the maximum possible value that the final payer can achieve. Thus, in scenario [a], the optimal incentive payment is \( I^*(f, \omega) = 0 \) and the outcome is the same in the single-payer scenario and the multi-payer scenario. For \( \omega = \mu - 1 \):

Consider scenario [b]. Because \( a^{X}(f, \mu - 1) = 0 \), it must be that,

\[
r^X((f, \mu - 1), 0) > r^X((f, \mu - 1), 1)
\]  \hspace{1cm} (B.21a)

Therefore, let \( I(f, \omega) = r^X((f, \mu - 1), 0) - r^X((f, \mu - 1), 1) \) represent the minimum incentive payment required for the first payer to provide treatment. Therefore,

\[
I(f, \mu - 1) = V^X(f, \mu - 1) - \hat{V}^X(f, \mu - 1)
\]  \hspace{1cm} (B.21b)
The first payer’s optimal action, can be summarized as,

\[ a^*(f, \mu - 1) = \begin{cases} 
    r^X((f, \mu - 1), 1) + I(f, \mu - 1) & \text{if } I(f, \omega) \geq I(f, \mu - 1) \\
    r^X((f, \mu - 1), 0) & \text{if } I(f, \omega) < I(f, \mu - 1)
\end{cases} \]  

(B.21c)

Therefore, the final payer will select an incentive level that solves the following problem,

\[
\hat{V}^Y(f, \mu - 1) = \max_{I(f, \mu - 1) \geq 0} \left\{ \begin{array}{l}
    r^Y((f, \mu - 1), 1) - I(f, \mu - 1) \quad \text{if } I(f, \omega) \geq I(f, \mu - 1) \\
    \delta \sum_{f' \in \Gamma} p(f'|(f, \mu - 1)) \cdot \hat{V}^Y(f', \mu) \quad \text{if } I(f, \omega) < I(f, \mu - 1) 
\end{array} \right. 
\]

(B.21d)

Let \( \bar{I}(f, \mu - 1) = r^Y((f, \mu - 1), 1) - \delta \sum_{f' \in \Gamma} p(f'|(f, \mu - 1)) \cdot \hat{V}^Y(f', \mu) \) represent the maximum incentive payment that the final payer would be willing to provide. Observe that \( \hat{V}^Y(f, \mu) = \hat{V}^Y(f, \mu) = V(f, \mu) \). Therefore,

\[
\bar{I}(f, \mu - 1) = \hat{V}^Y(f, \mu - 1) - V^Y(f, \mu - 1)
\]

(B.21e)

From Corollary 3.4.8 we know that \( \hat{V}^X(f, \mu - 1) - V^X(f, \mu - 1) \leq V^Y(f, \mu - 1) - \hat{V}^Y(f, \mu - 1) \) and therefore \( I(f, \mu - 1) \leq \bar{I}(f, \mu - 1) \). Thus, there always exists a range of incentive payments, \([I(f, \mu - 1), \bar{I}(f, \mu - 1)]\), such that coordination is incentive compatible for both payers. The final payer’s objective is decreasing in the size of the incentive. Therefore, the final payer will offer the smallest possible incentive, \( I^*(f, \omega) = I(f, \mu - 1) \).

Consider scenario [c].

From the proof of [b] above, the first payer will provide treatment if \( I(f, \mu - 1) \geq I(f, \mu - 1) = r^X((f, \mu - 1), 0) - r^X((f, \mu - 1), 1) \). And, the final payer could provide an incentive \( I(f, \mu - 1) \leq \bar{I}(f, \mu - 1) = r^Y((f, \mu - 1), 1) - \delta \sum_{f' \in \Gamma} p(f'|(f, \mu - 1)) \cdot \hat{V}^Y(f', \mu) \).

In this scenario it is socially optimal to wait. Therefore, by the definition of the benefit function,

\[
0 < B(f, \mu - 1)
\]

\[
= r((f, \mu - 1), 0) + \delta \sum_{f' \in \mathcal{X}} p(f'|(f, \mu - 1)) \cdot \hat{V}(f', \mu) - r((f, \mu - 1), 1)
\]
\[ I(f, \mu - 1) > \bar{I}(f, \mu - 1) \]

Therefore, there is no incentive payment that is incentive compatible for both payers. Thus, the final payer cannot induce the first payer to act sub-optimally using an incentive payment.

For \( \omega = \mu - 2 \):

Consider scenario [5]. The minimum incentive payment that would change the first payer’s action to treat is,

\[ I(f, \mu - 2) = r^X((f, \mu - 2), 0) + \delta \sum_{f' \in \mathcal{F}} p(f'|(f, \mu - 2)) \cdot \hat{V}^X(f', \mu - 1) - r^X((f, \mu - 2), 1) \]

The maximum incentive that the final payer would be willing to provide is,

\[ \bar{I}(f, \mu - 2) = r^Y((f, \mu - 2), 1) - \delta \sum_{f' \in \mathcal{F}} p(f'|(f, \mu - 2)) \cdot \hat{V}^Y(f', \mu - 1) \]

The optimal action is to provide treatment. Therefore,

\[ 0 \geq B(f, \mu - 2) \]

\[ = r((f, \mu - 2), 0) + \delta \sum_{f' \in \mathcal{F}} p(f'|(f, \mu - 2)) \cdot \hat{V}(f', \mu - 1) - r((f, \mu - 2), 1) \]

\[ = r^X((f, \mu - 2), 0) + \delta \sum_{f' \in \mathcal{F}} p(f'|(f, \mu - 2)) \cdot [\hat{V}^X(f', \mu - 1) + \hat{V}^Y(f', \mu - 1)] \]

\[ - r^X((f, \mu - 2), 1) - r^Y((f, \mu - 2), 1) \]
In this scenario, it is socially optimal to wait. Therefore, from the benefit function, 
\[ 2) \]
would provide an incentive \[ I \]
Therefore, there exists a range of incentive payments, \( \mathbb{I} \).
Consider scenario \( c \). The first payer will provide treatment if \( I \)
the final payer’s reward is decreasing in the size of the incentive. Therefore, the final payer will offer the smallest possible incentive, \( \mathbb{I} \).
Consider scenario \( c \). The first payer will provide treatment if \( I \) \( \mathbb{I} \). And, the final payer would provide an incentive \( I \) \( \mathbb{I} \). In this scenario, it is socially optimal to wait. Therefore, from the benefit function, 
\[ 0 < B(f, \mu - 2) \]
\[ = r((f, \mu - 2), 0) + \delta \sum_{f' \in \mathcal{F}} p(f'| (f, \mu - 2)) \cdot \tilde{V}(f', \mu - 1) - r((f, \mu - 2), 1) \]
\[ r^X((f, \mu - 2), 0) + \delta \sum_{f' \in F} p(f'(f, \mu - 2)) \cdot \left[ \hat{V}^X(f', \mu - 1) + \hat{V}^Y(f', \mu - 1) \right] \]
\[ = r^X((f, \mu - 2), 0) - r^X((f, \mu - 2), 1) \]
\[ \geq r^X((f, \mu - 2), 0) + \delta \sum_{f' \in F} p(f'(f, \mu - 2)) \cdot \left[ V^X(f', \mu - 1) + V^Y(f', \mu - 1) \right] \]
\[ - r^X((f, \mu - 2), 1) \]
\[ = r^X((f, \mu - 2), 0) + \delta \sum_{f' \in F} p(f'(f, \mu - 2)) \cdot \left[ V^X(f', \mu - 1) + I'(f', \mu - 1) \right] \]
\[ - r^Y((f, \mu - 2), 1) \]
\[ \geq I(f, \mu - 2) - \tilde{I}(f, \mu - 2) \]

Equivalently,
\[ I(f, \mu - 2) > \tilde{I}(f, \mu - 2) \]

Therefore, there is no incentive payment that is incentive compatible for both payers. Thus, the final payer would offer zero incentive.

Repeating the iterative process for \( \omega = \mu - 3, \mu - 4, \ldots, 0 \), there always exists an incentive that coordinates the system to achieve a first-best treatment policy, as in the single-payer scenario.

**Proof of Proposition B.1.1**

We want to show that \( f^T(\omega) \geq f^T(\omega + 1) \). It is equivalent to show that,
\[ a^*(f^T(\omega), \omega) = 1 \Rightarrow a^*(f^T(\omega), \omega + 1) = 1 \]

Because \( P \) is IFR and \( (3.9) \), the result of Proposition 3.4.2 holds (i.e., \( a^*(f, \omega) = 1 \Rightarrow \)
\[ a^*(f, \omega + 1) = 1, \ \forall \omega \in \Omega. \] Therefore, \[ a^*(f^T(\omega), \omega) = 1 \Rightarrow a^*(f^T(\omega), \omega + 1) = 1. \]

By the definition of \( f^T(\omega + 1) \), if \( a^*(f^T(\omega), \omega + 1) = 1 \), then \( f^T(\omega) \geq f^T(\omega + 1) \).

**Proof of b.** We want to show that \( \omega^T(f) \geq \omega^T(f + 1) \). It is equivalent to show that,

\[ a^*(f, \omega^T(f)) = 1 \Rightarrow a^*(f + 1, \omega^T(f)) = 1 \]

Because \( P \) is IFR\(_f\) and (3.8), the result of Proposition 3.4.2b holds (i.e., \( a^*(f, \omega) = 1 \Rightarrow a^*(f + 1, \omega) = 1, \ \forall f \in \Gamma \)). Therefore, \( a^*(f, \omega^T(f)) = 1 \Rightarrow a^*(f + 1, \omega^T(f)) = 1 \).

By the definition of \( \omega^T(f + 1) \), if \( a^*(f + 1, \omega^T(f)) = 1 \), then \( \omega^T(f) \geq \omega^T(f + 1) \).

**Proof of Proposition B.1.2**

**Proof of a:** We want to show that \( \overline{f}^T(\omega) \geq \overline{f}^T(\omega + 1) \). It is equivalent to show that,

\[ a^*(\overline{f}^T(\omega), \omega) = 1 \Rightarrow a^*(\overline{f}^T(\omega), \omega + 1) = 1 \]

By the definition of \( \overline{f}^T(\omega) \), \( a^*(\overline{f}^T(\omega), \omega) = 1 \). Because \( P \) is IFR\(_{f\omega}\) and (3.9), the result of Proposition 3.4.2b holds (i.e., \( a^*(f, \omega) = 1 \Rightarrow a^*(f, \omega + 1) = 1, \ \forall f \in \Gamma \)). Therefore, \( a^*(\overline{f}^T(\omega), \omega) = 1 \Rightarrow a^*(\overline{f}^T(\omega), \omega + 1) = 1 \).

By the definition of \( \overline{f}^T(\omega + 1) \), if \( a^*(\overline{f}^T(\omega), \omega + 1) = 1 \), then \( \overline{f}^T(\omega) \geq \overline{f}^T(\omega + 1) \).

**Proof of b:** We want to show that \( \overline{f}^T(\omega) \leq \overline{f}^T(\omega + 1) \). It is equivalent to show that,

\[ a^*(\overline{f}^T(\omega), \omega) = 1 \Rightarrow a^*(\overline{f}^T(\omega), \omega + 1) = 1 \]

By the definition of \( \overline{f}^T(\omega) \), \( a^*(\overline{f}^T(\omega), \omega) = 1 \). Because \( P \) is IFR\(_{f\omega}\) and (3.9), the result of Proposition 3.4.2b holds (i.e., \( a^*(f, \omega) = 1 \Rightarrow a^*(f, \omega + 1) = 1, \ \forall f \in \Gamma \)). Therefore, \( a^*(\overline{f}^T(\omega), \omega) = 1 \Rightarrow a^*(\overline{f}^T(\omega), \omega + 1) = 1 \).

By the definition of \( \overline{f}^T(\omega + 1) \), if \( a^*(\overline{f}^T(\omega), \omega + 1) = 1 \), then \( \overline{f}^T(\omega) \leq \overline{f}^T(\omega + 1) \).

**Proof of c:** From a) and b) the result follows directly.

**Proof of d:** Let \( \hat{\omega} \) represent the youngest age such that it is optimal to provide treatment for
some severity (i.e., \( \hat{\omega} = \min \{ \omega \mid \omega \in \Omega, \Gamma^T(\omega) \neq \emptyset \} \)). Let \( f^T = f^T(\hat{\omega}) \) and \( \bar{f}^T = f^T(\hat{\omega}) \).

First, we show that \( \omega^T(f) \) is non-increasing for all \( f < f^T \). It is sufficient to show that,

\[
a^*(f - 1, \omega^T(f - 1)) = 1 \Rightarrow a^*(f, \omega^T(f - 1)) = 1, \ \forall f < f^T.
\]

By the definition of \( \omega^T(f - 1) \), \( a^*(f - 1, \omega^T(f - 1)) = 1, \ \forall f < f^T \). Because \( P \) is SC\(_f\) and \( M(f, \omega) \) is convex in \( f \), the result of Proposition [3.4.3] holds (i.e., \( a^*(f, \omega) = 1 \Rightarrow a^*(f + 1, \omega) = 1, \ \forall f < f^T, \ \forall \omega \in \Omega \)). Therefore, \( a^*(f - 1, \omega^T(f - 1)) = 1 \Rightarrow a^*(f, \omega^T(f - 1)) = 1, \ \forall f < f^T \).

Next, we show that \( \omega^T(f) \) is non-decreasing for all \( f > \bar{f} \). It is sufficient to show that,

\[
a^*(f + 1, \omega^T(f + 1)) = 1 \Rightarrow a^*(f, \omega^T(f + 1)) = 1, \ \forall f > \bar{f}^T.
\]

By the definition of \( \omega^T(f + 1) \), \( a^*(f + 1, \omega^T(f + 1)) = 1, \ \forall f > \bar{f}^T \). Because \( P \) is SC\(_f\) and \( M(f, \omega) \) is convex in \( f \), the result of Proposition [3.4.3] holds (i.e., \( a^*(f + 1, \omega) = 1 \Rightarrow a^*(f, \omega) = 1, \ \forall f > \bar{f}, \ \forall \omega \in \Omega \)). Therefore, \( a^*(f + 1, \omega^T(f + 1)) = 1 \Rightarrow a^*(f, \omega^T(f + 1)) = 1, \ \forall f > \bar{f}^T \).

Finally, \( \omega^T(f) \) is constant for all \( f \in [\bar{f}^T, \bar{f}] \), by definition.

**Proof of Proposition B.1.3**

Proof: Because \( M(f, \omega) \) is convex in both \( f \) and \( \omega \), and because \( P \) is SC\(_f\) and SC\(_\omega\), the benefit function is also convex in both \( f \) and \( \omega \). The shape of the surface created by the intersection of \( B(f, \omega) \) with any plane on \( S \) is also convex. Thus, the shape of the surface where \( B(f, \omega) \) intersects with the plane \( g(f, \omega) = 0 \) is convex (i.e., \( \{(f, \omega) \mid B(f, \omega) \leq 0, (f, \omega) \in S\} \) is convex).
B.4 Bibliography


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Appendix C

Technical Definitions

**Definition C.1.1** Increasing Failure Rate (1-dimension): \cite{Barlow and Proschan [1965]} An $N \times N$ transition matrix is said to be IFR if its rows are in increasing stochastic order, that is:

$$b(i) = \sum_{h=k}^{N} p(h|i)$$

is non-decreasing in $i$ for all $k = 0, \ldots, N$.

**Definition C.1.2** Increasing Failure Rate (2-dimensions): Let

$$g_k(f, \omega) = \sum_{f'=k}^{F+1} p(f'|f, \omega)$$

Then,

a) $P$ is IFR in $f$ (IFR$_f$) if for each $\omega \in \Omega$, $g_k(f, \omega)$ is non-decreasing in $f$, $\forall k \in \Gamma$.

b) $P$ is IFR in $\omega$ (IFR$_\omega$) if for each $f \in \Gamma$, $g_k(f, \omega)$ is non-decreasing in $\omega$, $\forall k \in \Gamma$.

c) $P$ is IFR in both $f$ and $\omega$ (IFR$_{f,\omega}$) if \(a\) and \(b\)

**Definition C.1.3** Increasing Disease Rate: Let:

$$y_k(f, \omega) = \sum_{f'=k}^{F} p(f'|f, \omega)$$

Then, $P$ is IDR if for any $\omega \in \Omega$, $y_k(f, \omega)$ is non-decreasing in $f$, $\forall k \in \Gamma$
**Definition C.1.4** Stochastic Convexity *(Oh and Özer 2016)*: Let

\[ g_k(f, \omega) = \sum_{f'=k}^{F+1} p(f'|f, \omega) \]

Then,

a) \( P \) is stochastically convex in \( f \) (SC\(_f\)) if:

\[ g_k(f + 2, \omega) - g_k(f + 1, \omega) \geq g_k(f + 1, \omega) - g_k(f, \omega) \]
\( \forall f \in \{0, 1, \ldots, F - 1\}, \forall \omega \in \Omega, \forall k \in \Gamma \)

b) \( P \) is stochastically convex in \( \omega \) (SC\(_\omega\)) if:

\[ g_k(f, \omega + 2) - g_k(f, \omega + 1) \geq g_k(f, \omega + 1) - g_k(f, \omega) \]
\( \forall f \in \Gamma, \forall \omega \in \{0, 1, \ldots, N - 2\}, \forall k \in \Gamma \)

**Definition C.1.5** One-Step Benefit Function: *(Oh and Özer 2016)* Let \( M(f, \omega) \) represent the one-step benefit function such that:

\[ M(f, \omega) = \left[ r((f, \omega), 0) + \delta \sum_{f' \in \mathcal{F}} p(f'|f, \omega) \cdot r((f', \omega + 1), 1) \right] - \left[ r((f, \omega), 1) \right] \]

**Definition C.1.6** Benefit Function: *(Oh and Özer 2016)* Let \( B(f, \omega) \) represent the benefit function such that:

\[ B(f, \omega) = \left[ r((f, \omega), 0) + \delta \sum_{f' \in \mathcal{F}} p(f'|f, \omega) \cdot V(f', \omega + 1) \right] - \left[ r((f, \omega), 1) \right] \]

or, equivalently:

\[ B(f, \omega) = M(f, \omega) + \delta \sum_{f' \in \mathcal{F}} p(f'|f, \omega) \cdot (B(f', \omega + 1))^+ \]

where \((x)^+ = \max[0, x]\).

**Definition C.1.7** Total Probability of Life: Let \( TP(f, \omega_1, \omega_2) \in [0, 1] \) represent the total probability of living from age \( \omega_1 \) until age \( \omega_2 \) (where \( \omega_1 < \omega_2 \)), starting in severity state \( f \),
defined as:

$$TP(f, \omega_1, \omega_2) = \sum_{f' \in \mathbb{F}} p(f'(f, \omega_1)) \cdot TP(f', \omega_1 + 1, \omega_2)$$

$$\forall f \in \mathbb{F}, \forall \omega_1, \omega_2 \in \Omega \text{ s.t. } \omega_1 < \omega_2$$

In the special case where there is only one severity state (i.e., living, \(\mathcal{L}\)), then,

$$TP(\mathcal{L}, \omega_1, \omega_2) = \prod_{\omega' = \omega_1}^{\omega_2 - 1} p(\mathcal{L}|(\mathcal{L}, \omega'))$$

$$\forall \omega_1, \omega_2 \in \Omega \text{ s.t. } \omega_1 < \omega_2$$

Trivially, if \(\omega_1 = \omega_2\), then the total probability of life is 1.

**Definition C.1.8 Healthiest Transition State:** Let \(\underline{f}(f, \omega)\) represent the healthiest possible severity state that a patient in state \((f, \omega)\) can transition into, defined as:

$$\underline{f}(f, \omega) = \min \{ f' \in \Gamma \mid p(f'(f, \omega)) > 0 \}$$
C.2 Bibliography


Appendix D

Hepatitis C Virus: Natural History Model

D.1 Overview

We develop a state-transition model of chronic hepatitis C virus (HCV) infection that is similar in structure to previously published models of HCV progression and treatment (Cipriano et al. 2018, Liu et al. 2012, 2016, Salomon et al. 2003). In the model, treatment-naïve individuals progress through stages of chronic HCV infection defined by META VIR score (Bedossa and Poynard 1996) (see Figure D.1 for the state-transition diagram). All individuals in stage F0 through F4 are medically eligible for HCV treatment under current guidelines. Individuals with F4 liver fibrosis (compensated liver cirrhosis) may develop HCV-related hepatocellular carcinoma or decompensated liver cirrhosis; treatment for these health states may include liver transplant. Our model does not stratify by race or sex. Detailed information about the parameter estimates for state-specific mortality, disease progression, costs, and quality-adjusted life-years (QALYs) are presented in the remainder of this appendix.
Figure D.1: Hepatitis C virus state transition diagram. For illustrative purposes, we have suppressed all age-based notation. Within our model, the states in Figure D.1 are replicated 100 times, capturing ages 1 through 100. SVR: Sustained virologic response. F0-F4: METAVIR fibrosis score. DC: Decompensated cirrhosis. HCC: Hepatocellular carcinoma. LT: Liver transplant.
D.2 Mortality

Mortality rates depend on both age and disease severity. We estimate age- and disease-severity-specific mortality rates in two steps. First, we estimate baseline non-liver related age-specific mortality rates. Then, we calculate hazard ratios for each severity state to adjust for increases to mortality correlated with chronic HCV infection.

We estimate non-liver age-specific mortality rates by subtracting deaths attributed to liver-related causes from all-cause mortality rates. Our HCV model captures liver-related mortality directly through the mortality hazard ratios for HCV health states, as well as the health states for decompensated cirrhosis (DC), hepatocellular carcinoma (HCC), and liver transplant (LT).

We estimate all-cause age-specific mortality rates using the 2014 US lifetable [Arias et al. 2017]. We then subtract liver-related mortality rates, derived from 2014 US person-level cause of death data [Kochanek et al. 2016] (ICD-10 codes: K70, K73, and K74 [Classifications Download 2008]) and the 2014 US population census estimates [National Center for Health Statistics 2015a,b]. For example, for a 50-year-old individual the all-cause mortality rate is 412 per 100,000 individuals and the liver-related death rate is 23 per 100,000 individuals (1,024 deaths out of 4,488,666 individuals). Therefore, the non-liver death rate is 389 per 100,000 individuals, or equivalently 0.39% annually. All annual rates are converted to probabilities using Equation D.1

\[
p(D|\text{age}) = 1 - e^{-\frac{\text{rate}}{100,000}},
\]

where \(r_{\text{age}}\) represents the age-specific mortality rate per 100,000 individuals. See Table D.1 for age-specific non-liver mortality rates.
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<td>67 - 68</td>
<td>1,421</td>
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Table D.1: Non-liver mortality rates by age for a patient in F0 fibrosis (per 100,000 individuals).
Non-liver mortality is higher for HCV positive individuals due to relatively higher rates of comorbidities, correlation with risky health behaviors, correlation with low socioeconomic status, and decreased access to medical care (Liu et al. 2016). El-Kamary et al. (2011) and Liu et al. (2012) estimate hazard ratios for mortality for HCV-infected individuals using NHANES III data. A similar study in Australia also finds higher mortality in HCV-infected individuals (Amin et al. 2006). In a Swiss study, Prasad et al. (2009) find a significant, although small, increase in non-liver mortality among HCV-infected individuals who have low alcohol consumption (≤ 40 g/d). Ultimately, we estimate liver fibrosis-level-specific hazard ratios using all-cause and liver-related data from a Trent, England study by Neal (2007), due to the severity-level detail in their estimates. We convert Ishak score levels used in Neal (2007) to META VIR scores (Bedossa and Poynard 1996) using the conversion from Shiha and Zalata (2011).

Individuals who receive successful treatment will benefit from lower liver-related mortality, although there is no consensus as to the magnitude of the change to non-liver mortality. Liu et al. (2016) use the same non-liver mortality rates for infected and treatment-successful individuals and posit that non-liver related comorbidities would remain unchanged after treatment. Cipriano et al. (2018) decrease the health-state specific hazard ratios by 10% for treatment-successful individuals after calibrating their model with Backus et al. (2011). We use the same non-liver mortality rates for infected and treatment successful individuals, as in Liu et al. (2016).

Table D.2 summarizes the hazard ratios applied to age-specific mortality rates for each health state in the model.
### Table D.1: Mortality rates: severity-specific hazard ratios.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Range</th>
<th>Distribution</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline non-liver mortality by age</td>
<td>§</td>
<td></td>
<td>Bootstrap</td>
<td>(i)</td>
</tr>
<tr>
<td>Disease-severity-based hazard ratios</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HCV-infected (non-liver related)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F0</td>
<td>1</td>
<td>(1.00 - 1.19)</td>
<td>Log-Normal</td>
<td>(i) &amp; (ii)</td>
</tr>
<tr>
<td>F1</td>
<td>1.18</td>
<td>(1.09 - 1.41)</td>
<td>Log-Normal</td>
<td></td>
</tr>
<tr>
<td>F2</td>
<td>1.37</td>
<td>(1.24 - 1.72)</td>
<td>Log-Normal</td>
<td></td>
</tr>
<tr>
<td>F3</td>
<td>1.55</td>
<td>(1.38 - 1.96)</td>
<td>Log-Normal</td>
<td></td>
</tr>
<tr>
<td>F4</td>
<td>1.74</td>
<td>(1.54 - 2.25)</td>
<td>Log-Normal</td>
<td></td>
</tr>
<tr>
<td>Decompensated Cirrhosis</td>
<td>24.9</td>
<td>(12.5 - 42.9)</td>
<td>Log-Normal</td>
<td>(i) &amp; (iii)</td>
</tr>
<tr>
<td>Hepatocellular Carcinoma</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st Year</td>
<td>78.02</td>
<td>(66.4 - 90.4)</td>
<td>Log-Normal</td>
<td>(iv)</td>
</tr>
<tr>
<td>2nd Year +</td>
<td>30.86</td>
<td>(27.1 - 35.2)</td>
<td>Log-Normal</td>
<td>(v)</td>
</tr>
<tr>
<td>Liver Transplant</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transplant mortality (prob.)</td>
<td>0.14</td>
<td>(0.12 - 0.16)</td>
<td>Normal</td>
<td>(vi)</td>
</tr>
<tr>
<td>Post-transplant</td>
<td>4.68</td>
<td>(4.17 - 5.21)</td>
<td>Log-Normal</td>
<td>(vii)</td>
</tr>
<tr>
<td>Post-treatment (SVR)</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treatment state F0</td>
<td>1</td>
<td>(0.91 - 1.15)</td>
<td>Uniform</td>
<td>(i), (ii), &amp; (vi)</td>
</tr>
<tr>
<td>Treatment state F1</td>
<td>1.18</td>
<td>(1.03 - 1.37)</td>
<td>Uniform</td>
<td></td>
</tr>
<tr>
<td>Treatment state F2</td>
<td>1.37</td>
<td>(1.17 - 1.63)</td>
<td>Uniform</td>
<td></td>
</tr>
<tr>
<td>Treatment state F3</td>
<td>1.55</td>
<td>(1.30 - 1.88)</td>
<td>Uniform</td>
<td></td>
</tr>
<tr>
<td>Treatment state F4</td>
<td>1.74</td>
<td>(1.46 - 2.16)</td>
<td>Uniform</td>
<td></td>
</tr>
</tbody>
</table>

§ Table D.1
† Ordering maintained in sensitivity analysis.
‡ 0 - 10% decrease to fibrosis specific hazard ratio in sensitivity analysis.
(i) Arias et al. (2017), Kochanek et al. (2016), Classifications Download (2008)
(ii) National Center for Health Statistics (2015a,b)
(iii) Neal (2007), Shiha and Zalata (2011)
(iv) Salomon et al. (2003)
(v) National Cancer Institute (2013b)
(vi) Liu et al. (2012)
(vii) American Liver Foundation (2018)
D.3 Progression Rates

Chronic HCV infection fibrosis stages F0 - F4:

Similar to Salomon et al. (2002), we use constant progression rates between fibrosis states (i.e., the same rate between state F0 to F1 as between F2 to F3). This assumption is supported by the thorough meta-analysis presented in Thein et al. (2008). This analysis found that the difference between progression rates between states F0 to F4 were small and that only the progression rate between states F1 to F2 was statistically different than the progression rates between the other states. The empirically-calibrated age-specific rates estimated by Salomon et al. (2002) are consistent with the meta-analysis of Thein et al. (2008).

To create rates that increase continuously by age, we fit a logistic model to estimate age-specific progression rates. Age-specific confidence intervals are calculated using the bootstrap method (1000 iterations).

The parameter values are estimated by non-linear optimization, minimizing the sum of squared errors. The logistic model has the form:

\[
Progression \ Rate(age) = H - \frac{H-L}{1+e^{-K(age-a_0)}},
\]

where \(H\) and \(L\) represent the highest and lowest possible progression rates, \(a_0\) represents the midpoint (inflection point), and \(K\) represents a steepness coefficient. The fitted coefficients are summarized in Table D.3, the annual HCV disease progression rates indicated by the logistic model are summarized in Table D.4, and these rates are illustrated with error bands in Figure D.2.

<table>
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<th>Parameter</th>
<th>Description</th>
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<th>Bootstrap Mean</th>
<th>Bootstrap SE</th>
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<td>Maximum progression rate</td>
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<td>29,373</td>
<td>7,764</td>
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<td>(L)</td>
<td>Minimum progression rate</td>
<td>3,877</td>
<td>3,710</td>
<td>1,649</td>
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<td>(a_0)</td>
<td>Midpoint (age; years)</td>
<td>57</td>
<td>58</td>
<td>4.5</td>
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<tr>
<td>(K)</td>
<td>Steepness coefficient</td>
<td>(0.16)</td>
<td>(0.17)</td>
<td>0.08</td>
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Table D.3: Parameters for the logistic model used to estimate age-specific progression rates.
Table D.4: Annual HCV fibrosis progression rate (F0 - F4) (per 100,000 individuals). Progression rates are given as one-stage rates (i.e., F0 $\rightarrow$ F1). All one-stage rates are equal (i.e., F0 $\rightarrow$ F1 is the same as F1 $\rightarrow$ F2, etc.).

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<th>Age</th>
<th>Rate</th>
<th>Age</th>
<th>Rate</th>
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</table>
Figure D.2: Annual fibrosis progression rate by age (per 100,000 individuals). Fitted logit model with bootstrapped error bands.
Advanced HCV health states:

In our model, individuals are eligible for interferon-free direct acting antiviral treatment in META VIR fibrosis stages F0 through F4. Transitions among treatment ineligible health states, including decompensated cirrhosis, hepatocellular carcinoma, and individuals who have received a liver transplant, are estimated from prior empirical studies. Table D.5 provides a full summary of all progression rates between these states.

<table>
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<th>Parameter</th>
<th>Value</th>
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<th>Distribution</th>
<th>Source</th>
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<td>±</td>
<td>Bootstrap</td>
<td>(i)</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td>(i) &amp; (ii)</td>
</tr>
<tr>
<td>F4 (cirrhosis) to DC †</td>
<td>0.32</td>
<td>(0.25 - 0.40)</td>
<td>Log-Normal</td>
<td></td>
</tr>
<tr>
<td>Hepatocellular carcinoma (HCC)</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F4 (cirrhosis) to HCC †</td>
<td>0.16</td>
<td>(0.12 - 0.20)</td>
<td>Log-Normal</td>
<td></td>
</tr>
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<td>DC to HCC</td>
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<td>(1,951 - 2,103)</td>
<td>Log-Normal</td>
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<tr>
<td>Liver Transplant</td>
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<td></td>
<td></td>
<td>(iii)</td>
</tr>
<tr>
<td>DC to liver transplant</td>
<td>8,785</td>
<td>(8,508 - 9,085)</td>
<td>Log-Normal</td>
<td></td>
</tr>
<tr>
<td>HCC to liver transplant ‡</td>
<td>8,785</td>
<td>(8,508 - 9,085)</td>
<td>Log-Normal</td>
<td></td>
</tr>
<tr>
<td>Probability of achieving SVR</td>
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<td>(0.90 - 0.98)</td>
<td>Normal</td>
<td>(iv)</td>
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</table>

| § Table D.4                                      |
| ⌹ Parameter is expressed as a hazard ratio to the rate of fibrosis progression by age. |
| ‡ We assume that an individual can only receive a liver transplant during the first year after progressing to hepatocellular carcinoma. |

Note: Transitions from decompensated cirrhosis and hepatocellular carcinoma are constant with respect to age.

(i) Liu et al. (2012), Salomon et al. (2003)
(ii) Salomon et al. (2002), Fattovich et al. (1997), Grieve et al. (2006)
(iii) Kochanek et al. (2016), Salomon et al. (2003)
(iv) Organ Procurement and Transplantation Network (2018)

Table D.5: Annual transition rates between advanced HCV health states (per 100,000 individuals)
D.4 Quality-Adjusted Life-Years

Quality-of-life (QOL) utility weights in each health state depend on both age and disease severity. We estimate baseline age-specific utility weights for healthy individuals using age-specific estimates from the general population. Then, we lower the utility based on disease-severity state utility weight ratios.

Table D.7 summarizes QOL utility weight estimates by age for various survey methods and across different geographical regions. Each of these studies find that QOL is decreasing in age. We estimate smoothly decreasing age-specific QOL utility weights by fitting a logistic model to the US general population estimates. Age-specific confidence intervals are calculated using the bootstrap method (1000 iterations).

The parameter values are estimated by non-linear optimization, minimizing the sum of squared errors. The logistic model has the form:

\[
QALY(age) = H - \frac{H - L}{1 + e^{-K(age - a_0)}},
\]

where \(H\) and \(L\) represent the highest and lowest possible QALYs, \(a_0\) represents the midpoint (inflection point), and \(K\) represents a steepness coefficient. The fitted coefficients are summarized in Table D.6, the annual QOL weights indicated by the logistic model are summarized in Table D.6, and these weights and error bands are illustrated in Figure D.3.

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<th>Parameter</th>
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<th>Bootstrap SE</th>
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<td>(H)</td>
<td>Maximum QALY</td>
<td>1.00</td>
<td>1.00</td>
<td>0.013</td>
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<tr>
<td>(L)</td>
<td>Minimum QALY</td>
<td>0.62</td>
<td>0.63</td>
<td>0.087</td>
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<tr>
<td>(a_0)</td>
<td>Midpoint (age; years)</td>
<td>67</td>
<td>67</td>
<td>134</td>
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<tr>
<td>(K)</td>
<td>Steepness coefficient</td>
<td>0.03</td>
<td>0.03</td>
<td>0.011</td>
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Table D.6: Parameters for the logistic model used to estimate age-specific quality-adjusted life-year weights for healthy individuals.
### Table D.7: Age-specific quality-of-life utility weights presented in the literature.

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<tr>
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<th>(iv)</th>
<th>(v)</th>
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<td>UK†</td>
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<td>US‡</td>
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<td>(local)</td>
<td>(local)</td>
<td>(national)</td>
<td>(national)</td>
<td>(national)</td>
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<tr>
<td>≤ 24</td>
<td>–</td>
<td>–</td>
<td>0.878</td>
<td>0.904</td>
<td>0.920</td>
<td>0.922</td>
<td>0.925</td>
</tr>
<tr>
<td>24 - 34</td>
<td>–</td>
<td>–</td>
<td>0.870</td>
<td>0.907</td>
<td>0.920</td>
<td>0.912</td>
<td>0.912</td>
</tr>
<tr>
<td>35 - 44</td>
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<td>–</td>
<td>0.860</td>
<td>0.882</td>
<td>0.880</td>
<td>0.886</td>
<td>0.887</td>
</tr>
<tr>
<td>45 - 54</td>
<td>0.941</td>
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<td>0.820</td>
<td>0.847</td>
<td>0.853</td>
<td>0.857</td>
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<td>55 - 64</td>
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<td>65 - 74</td>
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<td>0.778</td>
<td>0.790</td>
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<td>0.740</td>
<td>0.724</td>
<td>+</td>
<td>0.763</td>
<td>0.755</td>
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<tr>
<td>85 +</td>
<td>0.817</td>
<td>0.800</td>
<td>0.730</td>
<td>+</td>
<td>+</td>
<td>0.736</td>
<td>+</td>
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</tbody>
</table>

† EQ-5D results converted to TTO utilities using UK population derived tariff (Dolan 1997)
‡ EQ-5D results converted to TTO utilities using US population derived tariff (Shaw et al. 2005)
* Where the age range in the source material is different than listed here, we calculate a weighted average quality-of-life weight.

− No data
+ Highest age data point applies to all higher ages (e.g., Petrou and Hockley (2005) ’75+’ = 0.724)
(i) Fryback et al. (1993) (Men)
(ii) Fryback et al. (1993) (Women)
(iii) Kind et al. (1998)
(iv) Petrou and Hockley (2005)
(v) Sullivan et al. (2005)
(vi) Sullivan and Ghushchyan (2006)
(vii) Nyman et al. (2007)
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<th>Age</th>
<th>QALY</th>
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<td>1 - 2</td>
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<td>0.897</td>
<td>69 - 70</td>
<td>0.807</td>
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<td>0.895</td>
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<td>0.785</td>
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<td>0.776</td>
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<td>14 - 15</td>
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<td>17 - 18</td>
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<td>21 - 22</td>
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<td>26 - 27</td>
<td>0.916</td>
<td>60 - 61</td>
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<td>94 - 95</td>
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<td>27 - 28</td>
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<td>61 - 62</td>
<td>0.830</td>
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<td>28 - 29</td>
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<td>62 - 63</td>
<td>0.827</td>
<td>96 - 97</td>
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<td>29 - 30</td>
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<td>30 - 31</td>
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<td>31 - 32</td>
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<td>33 - 34</td>
<td>0.901</td>
<td>67 - 68</td>
<td>0.813</td>
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</table>

Table D.8: Quality-of-life weights by age (healthy individuals).
Figure D.3: Quality-of-life weights by age. Fitted logit model with bootstrapped error bands.
Several empirical studies have found that chronic HCV infection decreases health-related QOL compared to perfect health and that QOL decreases as disease severity increases (Nyman et al. 2007, Sullivan and Ghushchyan 2006, Sullivan et al. 2005, Younossi et al. 2001, Gutteling et al. 2006, Foster et al. 1998). Additionally, several studies reporting QOL estimates from a panel of experts also estimate that QOL decreases in disease severity (Kim et al. 1997, Dusheiko and Roberts 1995, Cotler et al. 2001, Bennett et al. 1997, Wong et al. 1998, Sinha and Das 2000). Table D.9 summarizes disease-severity-specific QOL utility weights identified in prior work.

We use an average of four empirically-based studies to estimate the QOL adjustment for different disease severities due to the consistency and reliability of their methods (Siebert et al. 2001, Chong et al. 2003, Sherman et al. 2004, Wright et al. 2006). We apply the estimate for ‘Mild Chronic HCV’ to the state F0, ‘Moderate Chronic HCV’ to the state F2, and ‘Cirrhosis’ to state F4. We estimate F1 as the average of F0 and F2, and F3 as the average between F2 and F4. See Table D.10 for a summary of the QOL estimates used.
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**Age specific weights**

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<th>(vi)</th>
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<td>0.82</td>
<td>0.93</td>
<td>0.90</td>
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<td>0.76</td>
<td>0.85</td>
<td>0.77</td>
<td>0.75</td>
</tr>
<tr>
<td>Moderate Chronic HCV</td>
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<td>0.78</td>
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<td>0.59</td>
<td>0.76</td>
<td>0.76</td>
<td>–</td>
<td>0.66</td>
<td>0.75</td>
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<tr>
<td>Compensated Cirrhosis</td>
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<td>0.85</td>
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<td>0.74</td>
<td>0.79</td>
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<td>Hepatocellular Carcinoma</td>
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<td>–</td>
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<td>–</td>
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<td>Liver Transplant</td>
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<td>0.86</td>
<td>–</td>
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</tr>
<tr>
<td>‡ EQ-5D tariff not stated</td>
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<tr>
<td>– No data</td>
</tr>
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</tr>
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<td>(ii) \cite{Wong1998}</td>
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<td>(iii) \cite{Sinha2000}</td>
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<td>(iv) \cite{Cotler2001}</td>
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Table D.9: HCV health-state-specific quality-of-life weights presented in the literature.
D.4. **Quality-Adjusted Life-Years**

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<th>Distribution</th>
<th>Source</th>
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<td></td>
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<td>Severity-based QOL weights*</td>
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<td>(ii)</td>
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<tr>
<td>F4</td>
<td>0.705</td>
<td>(0.68 - 0.72)</td>
<td>Beta</td>
<td></td>
</tr>
<tr>
<td>Decompensated cirrhosis</td>
<td>0.638</td>
<td>(0.61 - 0.67)</td>
<td>Beta</td>
<td>(ii)</td>
</tr>
<tr>
<td>Hepatocellular carcinoma</td>
<td>0.550</td>
<td>(0.52 - 0.58)</td>
<td>Beta</td>
<td>(iii)</td>
</tr>
<tr>
<td>Liver transplant</td>
<td>0.717</td>
<td>(0.61 - 0.75)</td>
<td>Beta</td>
<td>(iii)</td>
</tr>
<tr>
<td>Sustained virologic response</td>
<td>0.825</td>
<td>(0.80 - 0.85)</td>
<td>Beta</td>
<td>(iii)</td>
</tr>
</tbody>
</table>

§ Table D.8

* To calculate the QALY weight applied to a specific individual at a specific age and fibrosis level, we assumed a multiplicative model. Specifically, we multiplied the age-specific weight by the health state specific weight.

† Ordering maintained in sensitivity analysis.

‡ Ordering maintained in sensitivity analysis.

(i) Nyman et al. (2007), Sullivan et al. (2005), Younossi et al. (2001)

(ii) Siebert et al. (2001), Chong et al. (2003), Sherman et al. (2004), Wright et al. (2006)

(iii) Chong et al. (2003), Wright et al. (2006)

Table D.10: Health state specific quality-of-life utility weights.
D.5 Health Care Costs

Health care costs in each health state depend on both age and disease severity. We estimate baseline age-specific health care costs using average US population data. Then, we add an HCV-severity-specific premium.


Similar to Goldhaber-Fiebert et al. [2015] and Liu et al. [2016], we fit a restricted cubic spline to the MEPS cost data. Unlike Goldhaber-Fiebert et al. [2015] we do not inflate all MEPS cost data (by 10%) for nursing home costs as these costs only apply to the oldest population, and within this group only account for a maximum of 7.1% of costs (Meara et al. [2004]). The range of parameter values that we use in the sensitivity analysis is large relative to this specific source of uncertainty. Cubic splines are used to estimate trends in health care costs due to their ability to capture non-monotonic features (Durrleman and Simon [1989]). Typically, fewer knots are recommended (Stone [1986]) because selecting a large number of knots can result in overfitting. Durrleman and Simon [1989] suggests knot placement at quantiles (in our study, age), bounded by the 5% and 95% quantiles, with knots evenly spaced (by quantiles) between. We fit 13 different models (3-15 knots) and estimate out-of-sample testing error using 5-fold cross validation to measure overfitting. We replicate the cross-validation process 200 times and find that a 12-knot model minimizes the testing error. Using the 1-standard deviation rule (Hastie et al. [2009]), we select a 6-knot model as our final choice (see Figure D.4a).

The restricted cubic spline model is linear in its predictors and is estimated using ordinary least squares. Let \( K \) represent the number of knots (\( K = 6 \)) and let \( t_j \) represent the \( j \)th knot, \( j \in \{1, 2, \ldots, K\} \). Let \( i \in \{1, 2, \ldots, n\} \) represent the \( i \)th observation. To estimate the cubic splines, \( K - 2 \) additional independent variables are necessary (Durrleman and Simon [1989]).
D.5. Health Care Costs

The additional variables are defined as in Equation [D.4]:

\[
x_{i,j} = (age_i - t_j)^3 - (age_i - t_{K-1})^3 \cdot \frac{t_{k-t_j}}{t_{k-t_{K-1}}},
\]

where \((x)_+ = \max[0, x]\).

The fitted coefficients of the six-knot model are summarized in Table D.11, the annual age-based health care costs indicated by the restricted cubic model are summarized in Table D.12, and these costs and error bands are illustrated in Figure D.4b.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>SE</th>
<th>p-value</th>
<th>Bootstrap Estimate</th>
<th>Bootstrap SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>1575.10</td>
<td>128.22</td>
<td>0.000 ***</td>
<td>1577.44</td>
<td>99.17</td>
</tr>
<tr>
<td>Age (39.89)</td>
<td>15.95</td>
<td>0.012 *</td>
<td>(40.26)</td>
<td>12.36</td>
<td></td>
</tr>
<tr>
<td>(x_1)</td>
<td>0.21</td>
<td>0.05</td>
<td>0.000 ***</td>
<td>0.21</td>
<td>0.04</td>
</tr>
<tr>
<td>(x_2)</td>
<td>(0.56)</td>
<td>0.13</td>
<td>0.000 ***</td>
<td>(0.57)</td>
<td>0.10</td>
</tr>
<tr>
<td>(x_3)</td>
<td>0.75</td>
<td>0.15</td>
<td>0.000 ***</td>
<td>0.75</td>
<td>0.12</td>
</tr>
<tr>
<td>(x_4)</td>
<td>(0.64)</td>
<td>0.11</td>
<td>0.000 ***</td>
<td>(0.65)</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Table D.11: Fitted parameters for the cubic spline model used to estimate age-specific baseline health care costs.
<table>
<thead>
<tr>
<th>Age</th>
<th>Cost</th>
<th>Age</th>
<th>Cost</th>
<th>Age</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 1</td>
<td>1,575</td>
<td>34 - 35</td>
<td>2,351</td>
<td>68 - 69</td>
<td>8,289</td>
</tr>
<tr>
<td>1 - 2</td>
<td>1,535</td>
<td>35 - 36</td>
<td>2,388</td>
<td>69 - 70</td>
<td>8,488</td>
</tr>
<tr>
<td>2 - 3</td>
<td>1,495</td>
<td>36 - 37</td>
<td>2,434</td>
<td>70 - 71</td>
<td>8,686</td>
</tr>
<tr>
<td>3 - 4</td>
<td>1,455</td>
<td>37 - 38</td>
<td>2,493</td>
<td>71 - 72</td>
<td>8,883</td>
</tr>
<tr>
<td>4 - 5</td>
<td>1,416</td>
<td>38 - 39</td>
<td>2,566</td>
<td>72 - 73</td>
<td>9,081</td>
</tr>
<tr>
<td>5 - 6</td>
<td>1,377</td>
<td>39 - 40</td>
<td>2,655</td>
<td>73 - 74</td>
<td>9,279</td>
</tr>
<tr>
<td>6 - 7</td>
<td>1,341</td>
<td>40 - 41</td>
<td>2,763</td>
<td>74 - 75</td>
<td>9,477</td>
</tr>
<tr>
<td>7 - 8</td>
<td>1,309</td>
<td>41 - 42</td>
<td>2,888</td>
<td>75 - 76</td>
<td>9,675</td>
</tr>
<tr>
<td>8 - 9</td>
<td>1,282</td>
<td>42 - 43</td>
<td>3,029</td>
<td>76 - 77</td>
<td>9,873</td>
</tr>
<tr>
<td>9 - 10</td>
<td>1,261</td>
<td>43 - 44</td>
<td>3,185</td>
<td>77 - 78</td>
<td>10,070</td>
</tr>
<tr>
<td>10 - 11</td>
<td>1,248</td>
<td>44 - 45</td>
<td>3,354</td>
<td>78 - 79</td>
<td>10,268</td>
</tr>
<tr>
<td>11 - 12</td>
<td>1,244</td>
<td>45 - 46</td>
<td>3,534</td>
<td>79 - 80</td>
<td>10,466</td>
</tr>
<tr>
<td>12 - 13</td>
<td>1,249</td>
<td>46 - 47</td>
<td>3,725</td>
<td>80 - 81</td>
<td>10,664</td>
</tr>
<tr>
<td>13 - 14</td>
<td>1,266</td>
<td>47 - 48</td>
<td>3,923</td>
<td>81 - 82</td>
<td>10,862</td>
</tr>
<tr>
<td>14 - 15</td>
<td>1,296</td>
<td>48 - 49</td>
<td>4,129</td>
<td>82 - 83</td>
<td>11,059</td>
</tr>
<tr>
<td>15 - 16</td>
<td>1,339</td>
<td>49 - 50</td>
<td>4,340</td>
<td>83 - 84</td>
<td>11,257</td>
</tr>
<tr>
<td>16 - 17</td>
<td>1,394</td>
<td>50 - 51</td>
<td>4,555</td>
<td>84 - 85</td>
<td>11,455</td>
</tr>
<tr>
<td>17 - 18</td>
<td>1,458</td>
<td>51 - 52</td>
<td>4,772</td>
<td>85 - 86</td>
<td>11,653</td>
</tr>
<tr>
<td>18 - 19</td>
<td>1,530</td>
<td>52 - 53</td>
<td>4,991</td>
<td>86 - 87</td>
<td>11,851</td>
</tr>
<tr>
<td>19 - 20</td>
<td>1,607</td>
<td>53 - 54</td>
<td>5,208</td>
<td>87 - 88</td>
<td>12,049</td>
</tr>
<tr>
<td>20 - 21</td>
<td>1,687</td>
<td>54 - 55</td>
<td>5,424</td>
<td>88 - 89</td>
<td>12,246</td>
</tr>
<tr>
<td>21 - 22</td>
<td>1,769</td>
<td>55 - 56</td>
<td>5,638</td>
<td>89 - 90</td>
<td>12,444</td>
</tr>
<tr>
<td>22 - 23</td>
<td>1,849</td>
<td>56 - 57</td>
<td>5,850</td>
<td>90 - 91</td>
<td>12,642</td>
</tr>
<tr>
<td>23 - 24</td>
<td>1,927</td>
<td>57 - 58</td>
<td>6,060</td>
<td>91 - 92</td>
<td>12,840</td>
</tr>
<tr>
<td>24 - 25</td>
<td>1,999</td>
<td>58 - 59</td>
<td>6,269</td>
<td>92 - 93</td>
<td>13,038</td>
</tr>
<tr>
<td>25 - 26</td>
<td>2,064</td>
<td>59 - 60</td>
<td>6,476</td>
<td>93 - 94</td>
<td>13,236</td>
</tr>
<tr>
<td>26 - 27</td>
<td>2,119</td>
<td>60 - 61</td>
<td>6,681</td>
<td>94 - 95</td>
<td>13,433</td>
</tr>
<tr>
<td>27 - 28</td>
<td>2,164</td>
<td>61 - 62</td>
<td>6,885</td>
<td>95 - 96</td>
<td>13,631</td>
</tr>
<tr>
<td>28 - 29</td>
<td>2,200</td>
<td>62 - 63</td>
<td>7,089</td>
<td>96 - 97</td>
<td>13,829</td>
</tr>
<tr>
<td>29 - 30</td>
<td>2,228</td>
<td>63 - 64</td>
<td>7,291</td>
<td>97 - 98</td>
<td>14,027</td>
</tr>
<tr>
<td>30 - 31</td>
<td>2,252</td>
<td>64 - 65</td>
<td>7,492</td>
<td>98 - 99</td>
<td>14,225</td>
</tr>
<tr>
<td>31 - 32</td>
<td>2,274</td>
<td>65 - 66</td>
<td>7,692</td>
<td>99 - 100</td>
<td>14,423</td>
</tr>
<tr>
<td>32 - 33</td>
<td>2,296</td>
<td>66 - 67</td>
<td>7,892</td>
<td></td>
<td></td>
</tr>
<tr>
<td>33 - 34</td>
<td>2,321</td>
<td>67 - 68</td>
<td>8,091</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table D.12: Annual health care cost by age (healthy individuals)(2017 USD).
Figure D.4: Annual health care cost by age, model selection and estimates. a) Health care cost by age model selection using n-knot restricted cubic spline model. Estimated testing root-mean-squared-error by 5-fold cross-validation (200 iterations). Average error and standard deviation bands. Star indicates the most simplified model within one standard deviation of the model with the lowest estimated testing error. b) Base annual health care cost by age (2017 USD). Estimated using a 6-knot restricted cubic spline. Shading represents bootstrapped 95% population confidence interval. Vertical dashed lines represent knot placement.
Table D.13 summarizes incremental, severity-specific health care cost premiums (adjusted to 2017 USD) used in previous work. We use the inflation adjusted estimates from Liu et al. (2016). See Table D.14 for a complete summary of the health care costs used in our model.

<table>
<thead>
<tr>
<th>Condition</th>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
<th>(iv)</th>
<th>(v)</th>
<th>(vi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCV + (F0 - F3)</td>
<td>1,573</td>
<td>3,961</td>
<td>12,255</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mild Chronic HCV</td>
<td></td>
<td></td>
<td></td>
<td>3,320</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moderate Chronic HCV</td>
<td></td>
<td></td>
<td></td>
<td>8,300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Severe Chronic HCV (F4)</td>
<td>4,699</td>
<td>3,961</td>
<td>14,490</td>
<td>9,719</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decompensated Cirrhosis</td>
<td>12,448</td>
<td>28,453</td>
<td>42,835</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hepatocellular Carcinoma</td>
<td>49,554</td>
<td>48,657</td>
<td>61,698</td>
<td>43,976</td>
<td>37,546</td>
<td></td>
</tr>
<tr>
<td>Liver Transplant</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st year following transplant</td>
<td>163,194</td>
<td>193,125</td>
<td>209,289</td>
<td>155,520</td>
<td>134,340</td>
<td></td>
</tr>
<tr>
<td>2nd + years following transplant</td>
<td>28,495</td>
<td>42,053</td>
<td>60,918</td>
<td>23,310</td>
<td>19,612</td>
<td></td>
</tr>
<tr>
<td>Sustained Virologic Response</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percent of pre-treatment cost</td>
<td>50%</td>
<td>50%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(i) Liu et al. (2012)
(ii) Liu et al. (2016)
(iii) McAdam-Marx et al. (2011)
(iv) Davis et al. (2011)
(v) Salomon et al. (2003)
(vi) Bennett et al. (1997)

Table D.13: Annual health care premiums (over age-specific health care costs) by HCV disease severity presented in the literature (2017 USD, adjusted for inflation using GDP price deflator).
### D.5. Health Care Costs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Range</th>
<th>Distribution</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Health care costs by age</td>
<td>§</td>
<td></td>
<td>Bootstrap</td>
<td>(i)</td>
</tr>
</tbody>
</table>

#### Severity-based premiums

- **HCV+**
  - F0: 3,320 (2,987 - 3,631) Normal
  - F1: 5,810 (5,222 - 6,377) Normal
  - F2: 8,300 (7,566 - 9,172) Normal
  - F3: 9,010 (8,268 - 9,978) Normal
  - F4: 9,719 (8,924 - 10,692) Normal

- ** Decompensated Cirrhosis**: 28,453 (25,697 - 31,225) Normal (iii)
- **Hepatocellular Carcinoma**: 48,657 (43,820 - 53,327) Normal (iii)

- **Liver Transplant**
  - 1st Year: 193,125 (173,719 - 212,372) Normal (iii)
  - 2nd Year +: 42,053 (38,310 - 46,189) Normal (iii)

- **Sustained Virologic Response**
  - F0: 1,660 (1,440 - 1,896) Normal (iv)
  - F1: 2,905 (2,535 - 3,314) Normal (iv)
  - F2: 4,150 (3,614 - 4,756) Normal (iv)
  - F3: 4,505 (3,941 - 5,152) Normal (iv)
  - F4: 4,860 (4,265 - 5,564) Normal (iv)

§ Table D.12

(ii) Davis et al. (2011), US Bureau of Economic Analysis (2018b)
(iii) Liu et al. (2016), US Bureau of Economic Analysis (2018b)
(iv) Liu et al. (2016, 2012)

Table D.14: Annual health care costs (2017 USD).
D.6 Terminal Rewards

We estimate terminal rewards with respect to QALYs and health care costs as the total present value of future QALYs and health care costs. Let $TQ(f, \omega)$ represent the total QALYs received when treatment is provided from state $(f, \omega)$, as calculated in Equation D.5

$$TQ(f, \omega) = q(T(f, \omega), \omega) + \sum_{j=\omega+1}^{N} \left[ \delta^{(j-\omega)} \cdot q(T(f, \omega), j) \cdot \prod_{k=\omega}^{j-1} TP(L|T(f, \omega), k, k+1) \right], \quad (D.5)$$

where $q(T(f, \omega), j)$ represents the one-year QALY received from a year in treated state $T(f, \omega)$ at age $j$, and where $TP(L|T(f, \omega), k, k+1)$ represents the one-year probability of living while in treated state $T(f, \omega)$ between ages $k$ and $k + 1$.

Similarly, let $TC(f, \omega)$ represent the total cost incurred when treatment is provided from state $(f, \omega)$, as calculated in Equation D.6

$$TC(f, \omega) = c(T(f, \omega), \omega) + \sum_{j=\omega+1}^{N} \left[ \delta^{(j-\omega)} \cdot c(T(f, \omega), j) \cdot \prod_{k=\omega}^{j-1} TP(L|T(f, \omega), k, k+1) \right], \quad (D.6)$$

where $c(T(f, \omega), j)$ represents the one-year cost incurred from a year in treated state $T(f, \omega)$ at age $j$, and where $TP(L|T(f, \omega), k, k+1)$ represents the one year probability of living while in treated state $T(f, \omega)$ between ages $k$ and $k + 1$. 
D.7 Bibliography


National Cancer Institute (2013b) Relative survival by survival time by cancer site all ages, all races, both sexes 1988-2013. [Technical Report], National Cancer Institute: Surveillance Epidemiology and End Results Program.


Appendix E

Appendix: Essay 3

E.1 Proofs

**Lemma E.1.1** Let \( \{x_j\} \) and \( \{x'_j\} \) be real-valued non-negative sequences satisfying:

\[
\sum_{j=k}^{\infty} x_j \geq \sum_{j=k}^{\infty} x'_j ,
\]

\( \forall k \in \{0, 1, \ldots\} \), with equality holding for \( k = 0 \). Suppose \( v_j \geq v_{j+1} \) (\( j = 0, 1, \ldots \)), then,

\[
\sum_{j=0}^{\infty} v_j x_j \geq \sum_{j=0}^{\infty} v_j x'_j
\]

Proof of Lemma [E.1.1] *Proof:* An equivalent proof of Lemma [E.1.1] is provided to Lemma 4.2.7 in [Puterman (1994)](1994) and is omitted.

**Lemma E.1.2** For any \( \omega < \mu \), if \( P \) is IFR, then \( V^X_B((f, \omega), c^X, c^Y) \) is non-increasing in \( f \).

Proof of Lemma [E.1.2] *Proof:* This proof is by backwards induction. By construction, \( V^X_B((f, \omega), c^X, c^Y) = 0 \), \( \forall \omega \geq \mu \). Starting with \( \omega = \mu - 1 \):

\[
V^X_B((f, \mu - 1), c^X, c^Y)
= \max \left\{ r^X((f, \mu - 1), 1), \ r^X((f, \mu - 1), 0) + \delta \sum_{f' \in \mathcal{F}} p(f' | f, \mu - 1) \cdot V^X_B((f', \mu), c^X, c^Y) \right\}
= \max \left\{ r^X((f, \mu - 1), 1), \ r^X((f, \mu - 1), 0) \right\}
\]

(E.1a)
E.1a results because $V_B^X((f, \mu), c^X, c^Y) = 0$, $\forall f \in \mathbb{F}$. By A3.4.1 and A3.4.2, both $r^X((f, \omega), 0)$ and $r^X((f, \omega), 1)$ are non-increasing in $f$, respectively, and therefore $V_B^X((f, \mu - 1), c^X, c^Y)$ is non-increasing in $f$.

For $\omega = \mu - 2$:

By the property that $P$ is IFR and by E.1a, the result of Lemma E.1.1 holds. Therefore,

$$\sum_{f' \in \mathbb{F}} p(f'|f, \mu - 2) \cdot V_B^X((f', \mu - 1), c^X, c^Y) \geq \sum_{f' \in \mathbb{F}} p(f'|f + 1, \mu - 2) \cdot V_B^X((f', \mu - 1), c^X, c^Y)$$

(E.1b)

Combining E.1b with A3.4.1

$$r((f, \mu - 2), 0) + \delta \sum_{f' \in \mathbb{F}} p(f'|f, \mu - 2) \cdot V_B^X((f', \mu - 1), c^X, c^Y) \geq r((f + 1, \mu - 2), 0) + \delta \sum_{f' \in \mathbb{F}} p(f'|f + 1, \mu - 2) \cdot V_B^X((f', \mu - 1), c^X, c^Y) \quad \text{(E.1c)}$$

Separately, by A3.4.2

$$r((f, \mu - 2), 1) \geq r((f + 1, \mu - 2), 1) \quad \text{(E.1d)}$$

Combining E.1c and E.1d

$$\max \left\{ r((f, \mu - 2), 1), r((f, \mu - 2), 0) + \delta \sum_{f' \in \mathbb{F}} p(f'|f, \mu - 2) \cdot V_B^X((f', \mu - 1), c^X, c^Y) \right\} \geq \max \left\{ r((f + 1, \mu - 2), 1), r((f + 1, \mu - 2), 0) + \delta \sum_{f' \in \mathbb{F}} p(f'|f + 1, \mu - 2) \cdot V_B^X((f', \mu - 1), c^X, c^Y) \right\}$$

(E.1e)

By the definition of the value function, E.1e can be rewritten as:

$$V_B^X((f, \mu - 2), c^X, c^Y) \geq V_B^X((f + 1, \mu - 2), c^X, c^Y)$$

Therefore, $V_B^X((f, \mu - 2), c^X, c^Y)$ is non-increasing in $f$. 
Repeating the same process for $\omega = \mu - 3, \mu - 4, \ldots, 0$ (omitted), the result follows; $V_B^X((f, \omega), c^X, c^f)$ is non-increasing in $f$, $\forall \omega \in \Omega$. 

**Lemma E.1.3** If $c^l < c^h$, then,

\begin{enumerate}
  \item $V_A(s, c^l, c^l) \geq V_A(s, c^l, c^h)$, $\forall s \in \mathcal{S}$
  \item $V_A(s, c^l, c^l) \geq V_A(s, c^h, c^l)$, $\forall s \in \mathcal{S}$
  \item $V_A(s, c^l, c^h) \geq V_A(s, c^h, c^h)$, $\forall s \in \mathcal{S}$
  \item $V_A(s, c^h, c^l) \geq V_A(s, c^l, c^h)$, $\forall s \in \mathcal{S}$
\end{enumerate}

**Proof of Lemma E.1.3**

**Proof of a:** We want to show that,

$$V_A((f, \omega), c^l, c^l) \geq V_A((f, \omega), c^l, c^h), \forall (f, \omega) \in \mathcal{S}$$

This proof is by backwards induction.

Starting with $\omega = N$.

By the definition of the value function,

$$V_A((f, N), c^l, c^l) = \max \{ r((f, N), 1) - c^l, r((f, N), 0) \}$$

$$\geq \max \{ r((f, N), 1) - c^h, r((f, N), 0) \}$$

$$= V_A((f, N), c^l, c^h) \quad \text{(E.2a)}$$

\[ \text{E.2a results because } c^l < c^h. \] Therefore,

$$V_A((f, N), c^l, c^l) \geq V_A(f, N, c^l, c^h) \quad \text{(E.2b)}$$

For $\omega = N - 1$, By the definition of the value function,

$$V_A((f, N - 1), c^l, c^l)$$

$$= \max \left\{ r((f, N - 1), 1) - c^l, r((f, N - 1), 0) + \delta \sum_{f' \in \mathcal{F}} p(f'|f, N - 1) \cdot V_A((f', N), c^l, c^l) \right\}$$
\[ \geq \max \left\{ r(f, N - 1, 1) - c^h, r(f, N - 1, 0) + \delta \sum_{f' \in \Gamma} p(f'|f, N - 1) \cdot V_A((f', N), c^l, c^h) \right\} \]

\[ = V_A((f, N - 1), c^l, c^h) \quad \text{(E.2c)} \]

E.2c results because \( c^l < c^h \) and from E.2b. Therefore,

\[ V_A((f, N - 1), c^l, c^l) \geq V_A((f, N - 1), c^l, c^h) \quad \text{(E.2d)} \]

Repeating the process for \( \omega = N - 2, \ N - 3, \ldots, 0 \), the result holds. Therefore,

\[ V_A(s, c^l, c^l) \geq V_A(s, c^l, c^h), \ \forall s \in S \]

\[ \Box \]

**Proof of b, c, and d:** The proofs of Lemma E.1.3b, c, and d follow the same format as the proof for Lemma E.1.3a and are omitted. \[ \Box \]

**Proof of Proposition 4.5.1**

**Proof of a:** We want to show that, \( \overline{T}_A(c^h, c^l) \subseteq \overline{T}_A(c^l, c^l) \).

Let \( B_A(s, c^X, c^Y) \) represent the benefit function in the social planner scenario. From, the definition of the benefit function in \( \text{Oh and Özer (2016)} \), it is equivalent to show that,

\[ B_A((f, \omega), c^h, c^l) \leq B_A((f, \omega), c^l, c^l) \quad \text{(E.3a)} \]

For all \( (f, \omega) \in S^\gamma \), from the left-hand-side of (E.3a),

\[ B_A((f, \omega), c^h, c^l) = r((f, \omega), 0) + \delta \sum_{f' \in \Gamma} p(f'|f, \omega) \cdot V_A((f', \omega + 1), c^h, c^l) - [r((f, \omega), 1) - c^l] \]

\[ \leq r((f, \omega), 0) + \delta \sum_{f' \in \Gamma} p(f'|f, \omega) \cdot V_A((f', \omega + 1), c^l, c^l) - [r((f, \omega), 1) - c^l] \]

\[ \leq B_A((f, \omega), c^l, c^l) \quad \text{(E.3b)} \]

E.3b results because \( V_A((f, \omega), c^h, c^l) \leq V_A((f, \omega), c^l, c^l) \) from Lemma E.1.3b.
And, for all \((f, \omega) \in \mathcal{S}^X\), from the left-hand-side of (E.3a),

\[
B_A((f, \omega), c^h, c^l) = r((f, \omega), 0) + \delta \sum_{f' \in \Gamma} p(f'|f, \omega) \cdot V_A((f', \omega + 1), c^h, c^l) - [r((f, \omega), 1) - c^h]
\]

\[
\leq r((f, \omega), 0) + \delta \sum_{f' \in \Gamma} p(f'|f, \omega) \cdot V_A((f', \omega + 1), c^l, c^l) - [r((f, \omega), 1) - c^l]
\]

\[
= B_A((f, \omega), c^l, c^h)
\]

(E.3d)

(E.3e)

(E.3d) results because \(V_A((f, \omega), c^h, c^l) \leq V_A((f, \omega), c^l, c^l)\) from Lemma E.1.3b, and because \(c^l < c^h\). Therefore,

\[
B_A((f, \omega), c^h, c^l) \leq B_A((f, \omega), c^l, c^l), \forall (f, \omega) \in \mathcal{S}
\]

And it follows that,

\[
\overline{T}_A(c^h, c^l) \subseteq \overline{T}_A(c^l, c^l)
\]

\[
\text{Proof of b:} \text{ Consider the two scenarios. In both scenarios, the social planner’s price for treating patients in states } \mathcal{S}^X \text{ is high and the same, } c^h. \text{ However, in the first scenario, the social planner incurs a low price of treatment, } c^l, \text{ for patients in states } \mathcal{S}^Y \text{ and in the second scenario the social planner incurs a high price of treatment, } c^h, \text{ for patients in states } \mathcal{S}^Y. \text{ We want to show that } \overline{T}_A^X(c^h, c^l) \subseteq \overline{T}_A^X(c^h, c^h). \text{ It is equivalent to show that,}
\]

\[
a^*_A(s, c^h, c^l) = 1 \Rightarrow a^*_A(s, c^h, c^h) = 1, \forall s \in \mathcal{S}^X
\]

(E.4a)

This proof is by contradiction. Assume the converse of (E.4a). That is,

\[
a^*_A(s, c^h, c^l) = 1 \Rightarrow a^*_A(s, c^h, c^h) = 0, \text{ uniquely}
\]

(E.4b)
Then, the following must be true,

\[ r(s, 1) - c^h \geq r(s, 0) + \delta \sum_{f' \in \Gamma} p(f'|s) \cdot V_A((f', \omega + 1), c^h, c^l) \]  
(E.4c)

and,

\[ r(s, 1) - c^h < r(s, 0) + \delta \sum_{f' \in \Gamma} p(f'|s) \cdot V_A((f', \omega + 1), c^X, c^X) \]  
(E.4d)

Subtracting (E.4d) from (E.4c)

\[ r(s, 1) - c^h - r(s, 1) + c^h > r(s, 0) + \delta \sum_{f' \in \Gamma} p(f'|s) \cdot V_A((f', \omega + 1), c^h, c^l) - r(s, 0) - \delta \sum_{f' \in \Gamma} p(f'|s) \cdot V_A((f', \omega + 1), c^h, c^h) \]  
(E.4e)

The left-hand-side of (E.4e) reduces to zero. And, simplifying the right-hand-side,

\[ 0 > \delta \sum_{f' \in \Gamma} p(f'|s) \cdot \left[ V_A((f', \omega + 1), c^h, c^l) - V_A((f', \omega + 1), c^h, c^h) \right] \]  
(E.4f)

which is a contradiction to Lemma E.1.3d. Therefore, it must be that \( a^*_A(s, c^h, c^h) = 1 \) and thus,

\[ a^*_A(s, c^h, c^l) = 1 \Rightarrow a^*_A(s, c^h, c^h) = 1, \ \forall s \in \mathbb{S}^X \]  
(E.4g)

Proof of c: Consider two scenarios: In one scenario, the social planner incurs a low price of treatment, \( c^l \), for patients in states \( \mathbb{S}^Y \); In the second scenario the social planner incurs a high price of treatment, \( c^h \), for patients in states \( \mathbb{S}^Y \). Because the treatment policy for patients in states \( \mathbb{S}^Y \) is independent of the price of treatment for patients in states \( \mathbb{S}^X \), by Proposition 3.4.5 in Chapter 3 the treatment region when the cost of treatment is high is a subset of the treatment region when the cost of treatment is low.

Proof of Proposition 4.5.2: Proof of a: The proof of Proposition 4.5.2a follows a similar
format to the proof of Proposition 4.5.1a and is omitted.

Proof of [E.1.3a] Consider the two scenarios. In both scenarios, the social planner’s price for treating patients in states $S^X$ is low and the same, $c_l$. However, in the first scenario, the social planner incurs a low price of treatment, $c_l$, for patients in states $S^Y$ and in the second scenario the social planner incurs a high price of treatment, $c_h$, for patients in states $S^Y$.

We want to show that $T_A^X(c_l, c_l) \subseteq T_A^X(c_l, c_h)$. It is equivalent to show that,

$$a^*_A(s, c_l, c_l) = 1 \Rightarrow a^*_A(s, c_l, c_h) = 1, \forall s \in S^X \quad (E.5a)$$

This proof is by contradiction. Assume the converse of (E.5a). That is,

$$a^*_A(s, c_l, c_l) = 1 \Rightarrow a^*_A(s, c_l, c_h) = 0, \text{ uniquely} \quad (E.5b)$$

Then, the following must be true,

$$r(s, 1) - c_l \geq r(s, 0) + \delta \sum_{f' \in \Gamma} p(f'|s) \cdot V_A((f', \omega + 1), c_l, c_l) \quad (E.5c)$$

and,

$$r(s, 1) - c_l < r(s, 0) + \delta \sum_{f' \in \Gamma} p(f'|s) \cdot V_A((f', \omega + 1), c_l, c_h) \quad (E.5d)$$

Subtracting (E.5d) from (E.5c),

$$r(s, 1) - c_l - r(s, 1) + c_l > r(s, 0) + \delta \sum_{f' \in \Gamma} p(f'|s) \cdot V_A((f', \omega + 1), c_l, c_l) -$$

$$r(s, 0) - \delta \sum_{f' \in \Gamma} p(f'|s) \cdot V_A((f', \omega + 1), c_l, c_h) \quad (E.5e)$$

The left-hand-side in (E.4e) reduces to zero. And, simplifying the right-hand-side,

$$0 > \delta \sum_{f' \in \Gamma} p(f'|s) \cdot \left[ V_A(f', \omega + 1, c_l, c_l) - V_A(f', \omega + 1, c_l, c_h) \right] \quad (E.5f)$$

which is a contradiction to Lemma E.1.3a. Therefore, it must be that $a^*_A(s, c_l, c_h) = 1$ and
thus,

\[ a^*_A(s, c', c') = 1 \Rightarrow a^*_A(s, c', c^b) = 1, \forall s \in S^X \]  \hspace{1cm} (E.5g)

\[ \square \]

**Proof of c.** The proof of Proposition 4.5.2c is the same as the proof of Proposition 4.5.1c and is omitted. \[ \square \]

**Proof of Proposition 4.5.3**

**Proof of a.** First, notice that for any state \( s \in S^Y \), the optimal treatment policy that the final payer selects will be the same as in the social planner scenario. This follows by construction, because the final payer receives all value for treatment from individuals in \( S^Y \). Therefore, for any state \( s \in S^Y \),

\[ V^Y_B(s, c^X, c^Y) + V^Y_B(s, c^X, c^Y) = V^Y_B(s, c^X, c^Y) = V^Y_B(s, c^X, c^Y) = V_A(s, c^X, c^Y) \]  \hspace{1cm} (E.6a)

For any state \( s \in S^X \), we demonstrate that \( V^X_B((f, \mu - 1), c^X, c^Y) \leq V_A(s, c^X, c^Y) \). This proof is by backwards induction.

Starting with \( \omega = \mu - 1 \):

The first payer receives,

\[ V^X_B((f, \mu - 1), c^X, c^Y) = \max \left\{ r^X((f, \mu - 1), 1) - c^X, r^X((f, \mu - 1), 0) \right\} \]

and the final payer receives,

\[ V^Y_B((f, \mu - 1), c^X, c^Y) = \begin{cases} r^Y((f, \mu - 1), 1), & \text{if } a^*_X((f, \mu - 1), c^X, c^Y) = 1 \\ \delta \sum_{f' \in f} p(f' | f, \mu - 1) \cdot V^Y_B((f', \mu), c^X, c^Y), & \text{if } a^*_X((f, \mu - 1), c^X, c^Y) = 0 \end{cases} \]

Therefore, the total value is,

\[ V^X_B((f, \mu - 1), c^X, c^Y) + V^Y_B((f, \mu - 1), c^X, c^Y) \]
Therefore, social planner scenario (i.e., a from E.6a. E.6e results because the first payer’s decision may not be the same as in the 
V E.6b results because \(r((f, \mu - 1), 1) = r^X((f, \mu - 1), 1) + r^Y((f, \mu - 1), 1) - c^X\), if \(a^X_B((f, \mu - 1), c^X, c^Y) = 1\)
\[
E.6b
\begin{align*}
= \begin{cases} 
  r^X((f, \mu - 1), 1) + r^Y((f, \mu - 1), 1) - c^X, & \text{if } a^X_B((f, \mu - 1), c^X, c^Y) = 1 \\
  r^X((f, \mu - 1), 0) + \\
  \delta \sum_{f' \in \mathcal{F}} p(f'|f, \mu - 1) \cdot V^Y_B((f', \mu), c^X, c^Y), & \text{if } a^X_B((f, \mu - 1), c^X, c^Y) = 0
\end{cases}
\end{align*}
\]

(E.6b)

\[
E.6c
\begin{align*}
= \begin{cases} 
  r((f, \mu - 1), 1) - c^X, & \text{if } a^X_B((f, \mu - 1), c^X, c^Y) = 1 \\
  r((f, \mu - 1), 0) + \\
  \delta \sum_{f' \in \mathcal{F}} p(f'|f, \mu - 1) \cdot V_B((f', \mu), c^X, c^Y), & \text{if } a^X_B((f, \mu - 1), c^X, c^Y) = 0
\end{cases}
\end{align*}
\]

(E.6c)

\[
E.6d
\begin{align*}
\leq \max \left\{ r((f, \mu - 1), 1) - c^X, \\
  r((f, \mu - 1), 0) + \delta \sum_{f' \in \mathcal{F}} p(f'|f, \mu - 1) \cdot V_A((f', \mu), c^X, c^Y) \right\}
\end{align*}
\]

(E.6d)

\[
E.6e
V_A((f, \mu - 1), c^X, c^Y)
\]

\[
= V_A((f, \mu - 1), c^X, c^Y)
\]

\[E.6f\] results because \(V^X_B((f, \mu), c^X, c^Y) = 0\) (i.e., the first payer does not cover patients \(\mu\)-years old). \(E.6c\) results because \(r((f, \mu - 1), 1) = r^X((f, \mu - 1), 1) + r^Y((f, \mu - 1), 1), r((f, \mu - 1), 0) = r^X((f, \mu - 1), 0), \) and \(V_B((f, \mu), c^X, c^Y) = V^Y_B((f, \mu), c^X, c^Y)\). \(E.6d\) results from \(E.6a\) \(E.6e\) results because the first payer’s decision may not be the same as in the social planner scenario (i.e., \(a^*_A((f, \mu - 1), c^X, c^Y) = 1 \Rightarrow a^X_B((f, \mu - 1), c^X, c^Y) = 1\)). Therefore,

\[
V^X_B((f, \mu - 1), c^X, c^Y) + V^Y_B((f, \mu - 1), c^X, c^Y) \leq V_A((f, \mu - 1), c^X, c^Y)
\]

For \(\omega = \mu - 2\):
The first payer receives,

\[ V_B^X((f, \mu - 2), c^X, c^Y) = \max \left\{ r^X((f, \mu - 2), 1) - c^X, \\
\quad r^X((f, \mu - 2), 0) + \delta \sum_{f' \in \Gamma} p(f'|f, \mu - 2) \cdot V_B^X((f', \mu - 1), c^X, c^Y) \right\} \]

the final payer receives,

\[ V_B^Y((f, \mu - 2), c^X, c^Y) = \begin{cases} 
  r^Y((f, \mu - 2), 1) & \text{if } a_B^X((f, \mu - 2), c^X, c^Y) = 1 \\
  \delta \sum_{f' \in \Gamma} p(f'|f, \mu - 2) \cdot V_B^Y((f', \mu - 1), c^X, c^Y) & \text{if } a_B^X((f, \mu - 2), c^X, c^Y) = 0
\end{cases} \]

Therefore, the total value is,

\[ V_B^X((f, \mu - 2), c^X, c^Y) + V_B^Y((f, \mu - 2), c^X, c^Y) = \begin{cases} 
  r^X((f, \mu - 2), 1) + r^Y((f, \mu - 2), 1) - c^X & \text{if } a_B^X((f, \mu - 2), c^X, c^Y) = 1 \\
  r^X((f, \mu - 2), 0) + \delta \sum_{f' \in \Gamma} p(f'|f, \mu - 2) \cdot \left[ V_B^X((f', \mu - 1), c^X, c^Y) + V_B^Y((f', \mu - 1), c^X, c^Y) \right] & \text{if } a_B^X((f, \mu - 2), c^X, c^Y) = 0
\end{cases} \]

\[ = \begin{cases} 
  r((f, \mu - 2), 1) - c^X & \text{if } a_B^X((f, \mu - 2), c^X, c^Y) = 1 \\
  r((f, \mu - 2), 0) + \delta \sum_{f' \in \Gamma} p(f'|f, \mu - 2) \cdot V((f', \mu - 1), c^X, c^Y) & \text{if } a_B^X((f, \mu - 2), c^X, c^Y) = 0
\end{cases} \]

\[ \leq \begin{cases} 
  r((f, \mu - 2), 1) - c^X & \text{if } a_B^X((f, \mu - 2), c^X, c^Y) = 1 \\
  r((f, \mu - 2), 0) + \delta \sum_{f' \in \Gamma} p(f'|f, \mu - 2) \cdot V_A((f', \mu - 1), c^X, c^Y) & \text{if } a_B^X((f, \mu - 2), c^X, c^Y) = 0
\end{cases} \]

(E.6g)
\[ \leq \max \left\{ r((f, \mu - 2), 1) - c^X, r((f, \mu - 2), 0) + \delta \sum_{f' \in \Gamma} p(f'|f, \mu - 2) \cdot V_A((f', \mu - 1), c^X, c^Y) \right\} \]

(E.6j)

\[ = V_A((f, \mu - 2), c^X, c^Y) \quad \text{(E.6k)} \]

\[ \text{E.6h results because} \ r((f, \mu - 2), 1) = r^X((f, \mu - 2), 1) + r^Y((f, \mu - 2), 1), \ r((f, \mu - 2), 0) = r((f, \mu - 2), 0), \text{ and } V_B^X((f', \mu - 1), c^X, c^Y) + V_B^Y((f', \mu - 1), c^X, c^Y) = V_B((f', \mu - 1), c^X, c^Y). \]

\[ \text{E.6i results from E.6h. E.6j results because the first payer’s decision may not be the same as} \]

in the social planner scenario (i.e., \( a^*_A((f, \mu - 2), c^X, c^Y) = 1 \Rightarrow a^*_B((f, \mu - 2), c^X, c^Y) = 1 \)). Therefore,

\[ V_B^X((f, \mu - 2), c^X, c^Y) + V_B^Y((f, \mu - 2), c^X, c^Y) \leq V_A((f, \mu - 2), c^X, c^Y) \]

Repeating the same process for \( \omega = \mu - 3, \mu - 4, \ldots, 0 \), we find that,

\[ V_B^X(s, c^X, c^Y) + V_B^Y(s, c^X, c^Y) \leq V_A(s, c^X, c^Y), \ \forall s \in S^Y \]

Combined with [E.6a]

\[ V_B^X(s, c^X, c^Y) + V_B^Y(s, c^X, c^Y) = V_B(s, c^X, c^Y) \leq V_A(s, c^X, c^Y), \ \forall s \in S \]

Proof of \( b \): First, notice that for any state \( s \in S^Y \), the first payer receives no value in either the social planner or multi-payer scenarios. Therefore,

\[ V_A^X(s, c^X, c^Y) = V_B^X(s, c^X, c^Y) = 0, \ \forall s \in S^Y \quad \text{(E.7a)} \]

For any state \( s \in S^X \), we demonstrate that \( V_B^X(s, c^X, c^Y) \geq V_A^X(s, c^X, c^Y) \). This proof is by backwards induction.

Starting with \( \omega = \mu - 1 \):

\[ V_A^X((f, \mu - 1), c^X, c^Y) \]
\[
\begin{align*}
&= \begin{cases} 
  r^X((f, \mu - 1), 1) - c^X & \text{if } a^*_A((f, \mu - 1), c^X, c^Y) = 1 \\
  r^X((f, \mu - 1), 0) & \text{if } a^*_A((f, \mu - 1), c^X, c^Y) = 0 
\end{cases} \\
&\leq \max \left\{ r^X((f, \mu - 1), 1) - c^X, \ r^X((f, \mu - 1), 0) \right\} \\
&= V^X_b((f, \mu - 1), c^X, c^Y)
\end{align*}
\] (E.7b)

Results because the choice that the first payer makes is at least as large as what it would receive if choosing \(a^*_A((f, \mu - 1), c^X, c^Y)\). Therefore,

\[
V^X_b((f, \mu - 1), c^X, c^Y) \geq V^X_a((f, \mu - 1), c^X, c^Y), \ \forall f \in \Gamma \quad (E.7c)
\]

For \(\omega = \mu - 2\):

\[
\begin{align*}
V^X_{A_b}(f, \mu - 2, c^X, c^Y) & = \begin{cases} 
  r^X((f, \mu - 2), 1) - c^X & \text{if } a^*_A((f, \mu - 2), c^X, c^Y) = 1 \\
  r^X((f, \mu - 2), 0) & \text{if } a^*_A((f, \mu - 2), c^X, c^Y) = 0 
\end{cases} \\
&\leq \begin{cases} 
  r^X((f, \mu - 2), 1) - c^X & \text{if } a^*_A((f, \mu - 2), c^X, c^Y) = 1 \\
  r^X((f, \mu - 2), 0) & \text{if } a^*_A((f, \mu - 2), c^X, c^Y) = 0 
\end{cases} \\
&\leq \max \left\{ r^X((f, \mu - 2), 1) - c^X, \ r^X((f, \mu - 2), 0) + \delta \sum_{f' \in \Gamma} p(f'|f, \mu - 2) \cdot V^X_b((f', \mu - 1), c^X, c^Y) \right\} \\
&= V^X_b((f, \mu - 2), c^X, c^Y)
\end{align*}
\] (E.7d)

Results from (E.7c) (E.7e) results because the choice that the first payer makes is at least as large as what it would receive if choosing \(a^*_A((f, \mu - 2), c^X, c^Y)\). Therefore,

\[
V^X_b((f, \mu - 2), c^X, c^Y) \geq V^X_a((f, \mu - 2), c^X, c^Y), \ \forall f \in \Gamma
\]
Repeating the same process for \( \omega = \mu - 3, \mu - 4, \ldots, 0 \), we find that,

\[
V^X_B(s, c^X, c^Y) \geq V^X_A(s, c^X, c^Y), \; \forall s \in \mathbb{S}^X
\]

Combined with E.7a,

\[
V^X_B(s, c^X, c^Y) \geq V^X_A(s, c^X, c^Y), \; \forall s \in \mathbb{S}
\]

Proof of \( c \): By the definition of \( V^X_A(s, c^X, c^Y) \) and \( V^Y_A(s, c^X, c^Y) \),

\[
V^A(s, c^X, c^Y) = V^X_A(s, c^X, c^Y) + V^Y_A(s, c^X, c^Y)
\]

Subtracting \( V^X_B(s, c^X, c^Y) \) and \( V^Y_B(s, c^X, c^Y) \) from both sides,

\[
V^A(s, c^X, c^Y) - V^X_B(s, c^X, c^Y) - V^Y_B(s, c^X, c^Y)
= V^X_A(s, c^X, c^Y) - V^X_B(s, c^X, c^Y) + V^Y_A(s, c^X, c^Y) - V^Y_B(s, c^X, c^Y)
\]

Rearranging,

\[
V^Y_A(s, c^X, c^Y) - V^Y_B(s, c^X, c^Y)
= [V^X_A(s, c^X, c^Y) - V^X_B(s, c^X, c^Y) - V^Y_B(s, c^X, c^Y)] - [V^X_A(s, c^X, c^Y) - V^X_B(s, c^X, c^Y)]
\]

The first term in brackets on the right-hand-side is always positive, from Proposition 4.5.3a. And, the second term in brackets on the right-hand-side is always negative, from Proposition 4.5.3b. Therefore,

\[
V^Y_A(s, c^X, c^Y) - V^Y_B(s, c^X, c^Y) \geq 0
\]

And it follows that,

\[
V^Y_B(s, c^X, c^Y) \leq V^Y_A(s, c^X, c^Y), \; \forall s \in \mathbb{S}
\]
Proof of Proposition 4.5.4  
Proof: By construction, the social planner scenario and the uncoordinated multi-payer scenario are equivalent for all states \( s \in S^Y \). Therefore, the treatment policies are equivalent.  

Lemma E.1.4  
Let \( r^X_{\mu'}((f, \omega), 1) \) represent the reward that the first payer receives from providing treatment, given the final age of patients that the first payer provides coverage for is \( \mu' \in \Omega \) (i.e., \( \mu' = \mu - 1 \)). Then,

\[
a) \quad r^X_{\mu'+1}((f, \omega), 1) - r^X_{\mu'}((f, \omega), 1) = r_{T(f,\omega)}(\mu' + 1) \cdot \delta^{\mu'+1-\omega} \cdot TP(L|T(f,\omega), \omega, \mu' + 1)  \\
b) \quad r^X_{\mu'+1}((f, \omega), 1) - r^X_{\mu'}((f, \omega), 1) \text{ is non-increasing in } f.
\]

Proof of Lemma E.1.4  
Proof of a: From the definition of the first payer’s reward from treatment in (4.1),

\[
r^X_{\mu'+1}((f, \omega), 1) - r^X_{\mu'}((f, \omega), 1) = r_{T(f,\omega)}(\omega) + \sum_{t=1}^{\mu'-1} \left[ \delta^t \cdot r_{T(f,\omega)}(\omega + t) \cdot \prod_{j=0}^{t-1} p_{T(f,\omega)}(T(f,\omega)|\omega + j) \right] \\
- r_{T(f,\omega)}(\omega) - \sum_{t=1}^{\mu'-\omega} \left[ \delta^t \cdot r_{T(f,\omega)}(\omega + t) \cdot \prod_{j=0}^{t-1} p_{T(f,\omega)}(T(f,\omega)|\omega + j) \right]  \\
= \delta^{\mu'+1-\omega} \cdot r_{T(f,\omega)}(\mu' + 1) \cdot \prod_{j=0}^{\mu'-\omega} p_{T(f,\omega)}(T(f,\omega)|\omega + j)  \\
= \delta^{\mu'+1-\omega} \cdot r_{T(f,\omega)}(\mu' + 1) \cdot TP(L|T(f,\omega), \omega, \mu' + 1)  
\]

(E.8a) results from the definition of the total probability of life.  

Proof of b: From A3.4.2, \( r_{T(f,\omega)}(\mu' + 1) \) and \( p_{T(f,\omega)}(T(f,\omega)|\omega + j) \) are non-increasing in \( f \) for any \( \omega \in \Omega \). Therefore, the result follows directly from E.8a.  

Proof of Proposition 4.5.5  
Proof: We want to show that,

\[
a^X_B(s, c^X, c^Y) = 1 \Rightarrow a^X_A(s, c^X, c^Y) = 1, \quad \forall s \in S^X
\]
From the definition of the benefit function, we know that treatment is provided when the benefit function is negative. Let $B_X^P(s, c^X, c^Y)$ represent the first payer’s benefit function in the uncoordinated scenario and let $B_A(s, c^X, c^Y)$ represent the social planner’s benefit function. Therefore, it is equivalent to show that,

$$B_X^P(s, c^X, c^Y) - B_A(s, c^X, c^Y) \geq 0, \forall s \in \mathbb{S}^X$$

Let $B_{X'\mu'}^P(s, c^X, c^Y)$ represent the first payer’s benefit function, parameterized by $\mu'$, which represents the maximum patient age under the first payer’s coverage (i.e., $\mu' = \mu - 1$). Therefore, $B_{X\mu}^P(s, c^X, c^Y)$ represents the special case where the first payer provides coverage over a patient’s entire lifespan. For completeness, let $\mathbb{S}_{\mu'}^X$, $r_{\mu'}^X(s, 1)$, $V_{X'\mu'}^X(s, c^X, c^Y)$, and $M_{X'\mu'}^X(s, c^X, c^Y)$ represent the set of states where the first payer provides coverage, the first payer’s reward from treatment, the first payer’s value function, and the first payer’s one-step benefit function, each parameterized by $\mu'$. Note that $B_{X\mu}^P(s, c^X, c^Y) = B_A(s, c^X, c^Y)$, $\forall s \in \mathbb{S}_N \equiv \mathbb{S}$.

Therefore, it is sufficient to show that,

$$B_{X'\mu'}^P(s, c^X, c^Y) - B_{X'\mu' + 1}^P(s, c^X, c^Y) \geq 0$$

$$\forall \mu' \in \Omega, \forall s \in \mathbb{S}_{\mu'}^X$$

(E.9a)

This proof is by backwards induction with respect to $\omega$, with each step a proof by contradiction.

Starting with $\omega = \mu'$,

For clarity, we provide Condition (4.16) from Proposition 4.5.5 here and simplify for $\omega = \mu'$ (recall that $s_1 = (f, \omega)$ and $s_2 = (f(s_1), \omega + 1)$),

$$\frac{r_{T_2}^T(\mu) - r_{T_1}^T(\mu)}{r_{T_1}^T(\mu)} < \frac{TP(T_{s_1}, \omega, \mu) - TP(T_{s_2}, \omega + 1, \mu) \cdot (1 - p(F + 1|s_1))}{TP(T_{s_2}, \omega + 1, \mu) \cdot (1 - p(F + 1|s_1))}$$

(4.16)
Adding 1 to each side,
\[
\frac{r_{T_{\mu}}(\mu)}{r_{T_{\mu}}(\mu)} < \frac{TP(T_{\mu}, \omega, \mu)}{TP(T_{\mu}, \omega + 1, \mu) \cdot (1 - p(F + 1|s_{\mu}))}
\]
Evaluating at \( \omega = \mu' \), and substituting \( \mu = \mu' + 1 \),
\[
\frac{r_{T_{(f,\mu')},\mu'+1}}{r_{T_{(f,\mu')},\mu'+1}} < \frac{TP(T_{(f,\mu')}, \mu', \mu' + 1)}{TP(T_{(f,\mu')}, \mu', \mu' + 1) \cdot (1 - p(F + 1|(f,\mu')))}
\]
where, \( TP(T_{(f,\mu')}, \mu' + 1, \mu' + 1) = 1 \) from the definition of total probability of life (i.e., the probability of living from year \( \mu' + 1 \) until \( \mu' + 1 \) is 1). Therefore,
\[
\frac{r_{T_{(f,\mu')},\mu'+1}}{r_{T_{(f,\mu')},\mu'+1}} < \frac{TP(T_{(f,\mu')}, \mu', \mu' + 1)}{(1 - p(F + 1|(f,\mu')))}
\]
Rearranging, setting the left-hand-side equal to zero, Condition (4.16) is equivalent to,
\[
0 < r_{T_{(f,\mu')},\mu'+1} \cdot TP(T_{(f,\mu')}, \omega, \mu) - r_{T_{(f,\mu')},\mu'+1} \cdot (1 - p(F + 1|(f,\mu')))
\] (E.9b)
By the definition of the benefit function,
\[
B_{B_{(f,\mu')},\mu'+1}((f,\mu'), c^X, c^Y) = B_{B_{(f,\mu')},\mu'+1}((f,\mu'), c^X, c^Y)
\]
\[
= \left[ r^X((f,\mu'), 0) + \delta \sum_{f' \in F} p(f'|f,\mu') \cdot V_{B_{(f,\mu')},\mu'+1}((f',\mu' + 1, c^X, c^Y)) \right] - r_{\mu'}^X((f,\mu'), 1)
\]
\[
- \left[ r^X((f,\mu'), 0) - \delta \sum_{f' \in F} p(f'|f,\mu') \cdot V_{B_{(f,\mu')},\mu'+1}((f',\mu' + 1, c^X, c^Y)) \right]
\]
\[
+ r_{\mu'+1}^X((f,\mu'), 1)
\]
\[
= r_{\mu'+1}^X((f,\mu'), 1) - r_{\mu'}^X((f,\mu'), 1) - \delta \sum_{f' \in F} p(f'|f,\mu') \cdot V_{B_{(f,\mu')},\mu'+1}((f',\mu' + 1, c^X, c^Y))
\] (E.9c)
\[ (E.9d) \] results because \( V_{B_{\mu'}}^X((f', \mu' + 1), c^X, c^Y) = 0, \forall f \in \Gamma. \) Substituting the result from Lemma \( \text{E.144} \),

\[
B_{B_{\mu'}}^X(f, \mu', c^X) - B_{B_{\mu'} + 1}^X(f, \mu', c^X)
= \delta \cdot r_{T(f, \mu')}(\mu' + 1) \cdot TP(L|T(f, \mu'), \mu', \mu' + 1)
- \delta \cdot \sum_{f' \in \mathbb{F}} p(f'|f, \mu') \cdot V_{B_{\mu'} + 1}^X((f', \mu' + 1), c^X, c^Y)
\tag{E.9d}
\]

Suppose the converse of \( (E.9a) \) is true (i.e., the contradiction assumption). Therefore, from \( (E.9d), \)

\[
0 \geq \delta \cdot r_{T(f, \mu')}(\mu' + 1) \cdot TP(L|T(f, \mu'), \mu', \mu' + 1)
- \delta \cdot \sum_{f' \in \mathbb{F}} p(f'|f, \mu') \cdot V_{B_{\mu'} + 1}^X((f', \mu' + 1), c^X, c^Y)
\geq \delta \cdot r_{T(f, \mu')}(\mu' + 1) \cdot TP(L|T(f, \mu'), \mu', \mu' + 1)
- \delta \cdot \sum_{f' \in \mathbb{F}} p(f'|f, \mu') \cdot V_{\mu' + 1}^X(f(f, \mu'), \mu' + 1, c^X)
\tag{E.9e}
\]

\( (E.9e) \) follows because \( V_{B_{\mu'} + 1}^X((f', \mu' + 1), c^X, c^Y) \) is decreasing in \( f' \) (i.e., the result of Lemma \( \text{E.12} \) holds, because \( P \) is IFR \( f \)), and therefore \( V_{\mu' + 1}^X(f(f, \mu'), \mu' + 1, c^X) \) represents the largest possible next-period value function.

Separately, by \( A3.4.2 \) and \( A3.4.5 \), \( r_{T(f, \mu' - 1)}((f, \mu' + 1)) \geq r_{T(f, \mu' - 1)}((f, \mu' + 1)) \) and \( r_{T(f, \mu' - 1)}((f, \mu' + 1)) \geq r_{T(f, \mu' - 1)}((f, \mu' + 1), 1) \) (i.e., the reward from treatment is less than \( r_{T(f, \mu' - 1)}((f, \mu' + 1)) \)).

And, by \( A3.4.2 \), \( A3.4.3 \) and \( A3.4.5 \), \( r_{T(f, \mu' - 1)}((f, \mu' + 1)) \geq r_{T(f, \mu' - 1)}((f, \mu' + 1)) \geq r_{T(f, \mu' - 1)}((f, \mu' + 1), 0) \) (i.e., the reward from waiting is less than \( r_{T(f, \mu' - 1)}((f, \mu' + 1)) \)). Therefore, by the definition of the value function, \( r_{T(f, \mu' - 1)}((f, \mu' + 1)) \geq V_{\mu' + 1}^X((f, \mu'), \mu' + 1, c^X) \).

Therefore,

\[
0 \geq \delta \cdot r_{T(f, \mu')}(\mu' + 1) \cdot TP(L|T(f, \mu'), \mu', \mu' + 1) - \delta \sum_{f' \in \mathbb{F}} p(f'|f, \mu') \cdot r_{T(f, \mu' - 1)}((f, \mu' + 1))
= \delta \cdot r_{T(f, \mu')}(\mu' + 1) \cdot TP(L|T(f, \mu'), \mu', \mu' + 1) - \delta \cdot r_{T(f, \mu' - 1)}((f, \mu' + 1)) \cdot \sum_{f' \in \mathbb{F}} p(f'|f, \mu')
\]
Rearranging, setting the left-hand-side equal to zero, Condition (4.16) is equivalent to,

\[ \omega = \delta \cdot r_{T(f,\mu')}(\mu' + 1) \cdot TP(L|T_{(f,\mu')}, \mu', \mu' + 1) \]

\[ - \delta \cdot r_{T(f(f,\mu'-1),\mu')}(\mu' + 1) \cdot [1 - p(F + 1)(f, \mu')] \]

\[ = \delta \left[ r_{T(f,\mu')}(\mu' + 1) \cdot TP(L|T_{(f,\mu')}, \mu', \mu' + 1) - r_{T(f(f,\mu'-1),\mu')}(\mu' + 1) \cdot [1 - p(F + 1)(f, \mu')] \right] \]

which is a contradiction to Condition (4.16) as easily observed in comparison with (E.9b). Therefore,

\[ B^X_{B,\mu'}(f, \mu') - B^X_{B,\mu'+1}(f, \mu') \geq 0, \forall f \in \Gamma \]  

(E.9f)

Next, for \( \omega = \mu' - 1 \):

Again, for clarity, we provide Condition (4.16) from Proposition 4.5.5 here and simplify for \( \omega = \mu' - 1 \) (recall that \( s_1 = (f, \omega) \) and \( s_2 = (f(s_1), \omega + 1) \)),

\[ \frac{r_{T_{s_2}}(\mu) - r_{T_{s_1}}(\mu)}{r_{T_{s_1}}(\mu)} < \frac{TP(T_{s_1}, \omega, \mu) - TP(T_{s_2}, \omega + 1, \mu) \cdot (1 - p(F + 1|s_1))}{TP(T_{s_2}, \omega + 1, \mu) \cdot (1 - p(F + 1|s_1))} \]  

(4.16)

Adding 1 to each side,

\[ \frac{r_{T_{s_2}}(\mu)}{r_{T_{s_1}}(\mu)} < \frac{TP(T_{s_1}, \omega, \mu)}{TP(T_{s_2}, \omega + 1, \mu) \cdot (1 - p(F + 1|s_1))} \]

Evaluating at \( \omega = \mu' - 1 \), and substituting \( \mu = \mu' + 1 \),

\[ \frac{r_{T(f(f,\mu'-1),\mu')}(\mu' + 1)}{r_{T(f,\mu'-1),\mu'}(\mu' + 1)} < \frac{TP(T_{(f,\mu'-1)}, \mu' - 1, \mu' + 1)}{TP(T(f(f,\mu'-1),\mu'), \mu', \mu' + 1) \cdot (1 - p(F + 1)(f, \mu' - 1))} \]

Rearranging, setting the left-hand-side equal to zero, Condition (4.16) is equivalent to,

\[ 0 < r_{T(f,\mu')}(\mu' + 1) \cdot TP(T_{(f,\mu'-1)}, \mu' - 1, \mu' + 1) \]

\[ - r_{T(f(f,\mu'-1),\mu')}(\mu' + 1) \cdot TP(T(f(f,\mu'-1),\mu'), \mu', \mu' + 1) \cdot (1 - p(F + 1)(f, \mu' - 1)) \]  

(E.9g)
By the alternate definition of the benefit function (see Technical Definitions),

\[ B^X_{B,B'}(f, \mu' - 1, c^X, c^Y) - B^X_{B,B' + 1}(f, \mu' - 1, c^X, c^Y) \]

\[ = M^X_{B,B'}(f, \mu' - 1, c^X, c^Y) - M^X_{B,B' + 1}(f, \mu' - 1, c^X, c^Y) \]

\[ + \delta \sum_{f' \in \mathbb{P}} p(f'|(f, \mu' - 1)) \left[ (B^X_{B,B'}(f', \mu'), c^X, c^Y) - (B^X_{B,B' + 1}(f', \mu'), c^X, c^Y) \right] \]

(E.9h)

Suppose the converse of (E.9a) is true (i.e., the contradiction assumption). Therefore, from (E.9h),

\[ 0 \geq M^X_{B,B'}(f, \mu' - 1, c^X, c^Y) - M^X_{B,B' + 1}(f, \mu' - 1, c^X, c^Y) \]

\[ + \delta \sum_{f' \in \mathbb{P}} p(f'|(f, \mu' - 1)) \left[ (B^X_{B,B'}(f', \mu'), c^X, c^Y) - (B^X_{B,B' + 1}(f', \mu'), c^X, c^Y) \right] \]

\[ \geq M^X_{B,B'}(f, \mu' - 1, c^X, c^Y) - M^X_{B,B' + 1}(f, \mu' - 1, c^X, c^Y) \]  

(E.9i)

(E.9i) results directly from (E.9h). By the definition of the one-step benefit function,

\[ 0 \geq r^{X}(f, \mu' - 1, 0) + \delta \sum_{f' \in \mathbb{P}} p(f'|(f, \mu' - 1)) \left[ r^{X}_{\mu'}(f', \mu'), 1 \right] - r^{X}_{\mu'}(f, \mu' - 1, 1) \]

\[ - r^{X}(f, \mu' - 1, 0) - \delta \sum_{f' \in \mathbb{P}} p(f'|(f, \mu' - 1)) \left[ r^{X}_{\mu' + 1}(f', \mu'), 1 \right] + r^{X}_{\mu' + 1}(f, \mu' - 1, 1) \]

= \[ r^{X}_{\mu' + 1}(f, \mu' - 1, 1) - r^{X}_{\mu'}(f, \mu' - 1, 1) \]

\[ - \delta \sum_{f' \in \mathbb{P}} p(f'|(f, \mu' - 1)) \left[ r^{X}_{\mu' + 1}(f', \mu'), 1 \right] - r^{X}_{\mu'}(f', \mu'), 1 \]

Substituting the result from Lemma [E.1.44](twice),

\[ 0 \geq \left[ r_{T(f, \mu' - 1)}(\mu' + 1) \cdot \delta^2 \cdot TP(L|T(f, \mu' - 1), \mu' - 1, \mu' + 1) \right] \]

\[ - \delta \sum_{f' \in \mathbb{P}} p(f'|(f, \mu' - 1)) \left[ r_{T(f', \mu')}(\mu' + 1) \cdot \delta \cdot TP(L|T(f', \mu'), \mu', \mu' + 1) \right] \]
\[ (E.9j) \]

\[
0 \geq \delta^2 \left[ r_{T(f,\mu'-1)}(\mu' + 1) \cdot TP(L|T_{f,f',\mu'-1},\mu',\mu' + 1) \right]
- \delta \cdot \sum_{f' \in F} p(f'|f,\mu'-1) \cdot \left[ r_{T(f,f',\mu'-1)}(\mu' + 1) \cdot TP(L|T_{f,f',\mu'-1},\mu',\mu' + 1) \right]
\]

(E.9k)

which is a contradiction to Condition (4.16) as easily observed in comparison with (E.9g). Therefore,

\[
0 \geq \delta^2 \left[ r_{T(f,\mu'-1)}(\mu' + 1) \cdot TP(L|T_{f,f',\mu'-1},\mu',\mu' + 1) \right]
- \delta^2 \left[ r_{T(f,f',\mu'-1)}(\mu' + 1) \cdot TP(L|T_{f,f',\mu'-1},\mu',\mu' + 1) \right] \cdot \left[ 1 - p(F + 1|f,\mu' - 1) \right]
\]

(E.9l)

Proof of Corollary 4.5.6

Proof: The proof of Corollary 4.5.6 follows similar form to Corollary 3.4.11 in Chapter 3 and is omitted.

Proof of Proposition 4.5.7

Proof: Because \( P \) is IFR and Condition (4.16) holds, the
treatment region in the multi-payer uncoordinated scenario is a subset of the treatment region in the social planner scenario (i.e., \( \mathcal{T}_B(c^X, c^Y) \subseteq \mathcal{T}_A(c^X, c^Y) \)). In this proof, we will demonstrate that an incentive payment exists that can coordinate the multi-payer scenario such that \( \mathcal{T}_C(c^X, c^Y) = \mathcal{T}_A(c^X, c^Y) \). This requires showing that an incentive payment exists when the first payer’s actions are sub-optimal, compared to the social planner scenario, and that an incentive payment does not exist that is incentive compatible for the final payer and that causes the first payer to deviate from an optimal decision, compared to the social planner scenario, to a sub-optimal decision.

This proof is by backwards induction.

For each step in this backwards induction proof, there are three possible cases. The cases are defined by the optimal action in the social planner scenario and the first payer’s action in the multi-payer uncoordinated scenario. The three cases are:

a) The first payer acts optimally compared to the social planner scenario and provides treatment,
\[
a^*_B(s, c^X, c^Y) = 1 \quad \text{and} \quad a^*_A(s, c^X, c^Y) = 1.
\]

b) The first payer acts sub-optimally compared to the social planner scenario and does not provide treatment,
\[
a^*_B(s, c^X, c^Y) = 0 \quad \text{and} \quad a^*_A(s, c^X, c^Y) = 1.
\]

c) The first payer acts optimally compared to the social planner scenario and waits,
\[
a^*_B(s, c^X, c^Y) = 0 \quad \text{and} \quad a^*_A(s, c^X, c^Y) = 0.
\]

For each step in this backwards induction proof, if the first payer acts optimally and provides treatment (case a) above), then the final payer will make an incentive payment of zero as \( V^*_B(s, c^X, c^Y) = V^*_A(s, c^X, c^Y) \) when \( a^*_A(s, c^X, c^Y) = 1 \), \( \forall s \in \mathcal{S}^X \), as a result of Proposition 4.5.5. Thus, in case a), the optimal incentive payment is \( I^*(s, c^X, c^Y) = 0 \) and the outcome is the same as in the social planner scenario.

For cases b) and c) from above. Starting with \( \omega = \mu - 1 \),

Consider case b). Because \( a^*_B((f, \mu - 1), c^X, c^Y) = 0 \), it must be that,
\[
r^X((f, \mu - 1), 0) > r^X((f, \mu - 1), 1) - c^X
\]
Therefore, let \( I(s, c^X, c^Y) = r^X((f, \mu - 1), 0) - \left[ r^X((f, \mu - 1), 1) - c^X \right] \) represent the minimum incentive payment required for the first payer to provide treatment. Therefore,

\[
I((f, \mu - 1), c^X, c^Y) = V^X_B((f, \mu - 1), c^X, c^Y) - V^X_A((f, \mu - 1), c^X, c^Y)
\]

The first payer’s optimal action in the coordinated scenario, can be summarized as,

\[
a^X_C((f, \mu - 1), c^X, c^Y) = \begin{cases} 
    r^X((f, \mu - 1), 1) - c^X + I((f, \mu - 1), c^X, c^Y) & \text{if } I((f, \mu - 1), c^X, c^Y) \\
    \geq I((f, \mu - 1), c^X, c^Y) \\
    r^X((f, \mu - 1), 0) & \text{if } I((f, \mu - 1), c^X, c^Y) \\
    < I((f, \mu - 1), c^X, c^Y)
\end{cases}
\]

Therefore, the final payer will select an incentive level, \( I((f, \mu - 1), c^X, c^Y) \geq 0 \), that solves the following problem,

\[
V^Y_C((f, \mu - 1), c^X, c^Y) = \max \begin{cases} 
    r^Y((f, \mu - 1), 1) - I((f, \mu - 1), c^X, c^Y) & \text{if } I((f, \mu - 1), c^X, c^Y) \\
    \geq I((f, \mu - 1), c^X, c^Y) \\
    \delta \sum_{f' \in \Gamma} p(f'|f, \mu - 1) \cdot V^Y_C((f', \mu), c^X, c^Y) & \text{if } I((f, \mu - 1), c^X, c^Y) \\
    < I((f, \mu - 1), c^X, c^Y)
\end{cases}
\]

Let \( \bar{I}((f, \mu - 1), c^X, c^Y) = r^Y((f, \mu - 1), 1) - \delta \sum_{f' \in \Gamma} p(f'|f, \mu - 1) \cdot V^Y_C((f', \mu), c^X, c^Y) \) represent the maximum incentive payment that the final payer would be willing to provide. Recall that \( V^Y_C((f, \mu), c^X, c^Y) = V^Y_B((f, \mu), c^X, c^Y) = V^Y_A((f, \mu), c^X, c^Y) \). Therefore,

\[
\bar{I}((f, \mu - 1), c^X, c^Y) = V^Y_A((f, \mu - 1), c^X, c^Y) - V^Y_B((f, \mu - 1), c^X, c^Y) - V^Y_C((f, \mu - 1), c^X, c^Y)
\]

An incentive payment that is incentive compatible for both payers will exist if,

\[
0 \leq \bar{I}((f, \mu - 1), c^X, c^Y) - I((f, \mu - 1), c^X, c^Y)
\]
\[ V_Y^Y((f, \mu - 1), c^X, c^Y) - V_Y^Y((f, \mu - 1), c^X, c^Y) \]
\[ - V_X^X((f, \mu - 1), c^X, c^Y) - V_X^X((f, \mu - 1), c^X, c^Y) \]

Which is true because the term in the first set of brackets is positive by Proposition 4.5.3c, and the term in the second set of brackets is negative, by Proposition 4.5.3b. Thus, there always exists a range of incentive payments, \([I((f, \mu - 1), c^X, c^Y), \tilde{I}((f, \mu - 1), c^X, c^Y)]\), such that coordination is incentive compatible for both payers. The final payer’s objective is decreasing in the size of the incentive. Therefore, the final payer will offer the smallest possible incentive, \(I^*(f, \omega), c^X, c^Y) = I((f, \mu - 1), c^X, c^Y)\).

Consider case (c).

From the proof of (b) above, the first payer will provide treatment if,

\[ I((f, \mu - 1), c^X, c^Y) \geq I((f, \mu - 1), c^X, c^Y) = r^X((f, \mu - 1), 0) - \left[ r^X((f, \mu - 1), 1) - c^X \right] \]

And, the final payer could provide an incentive

\[ I((f, \mu - 1), c^X, c^Y) \leq \tilde{I}((f, \mu - 1), c^X, c^Y) \]
\[ = r^Y((f, \mu - 1), 1) - \delta \sum_{f' \in \Gamma} p(f'|f, \mu - 1)) \cdot V_Y^X((f', \mu), c^X, c^Y) \]

In this case it is optimal to wait in the social planner scenario. Therefore, by the definition of the benefit function,

\[ 0 < B_A((f, \mu - 1), c^X, c^Y) \]
\[ = r((f, \mu - 1), 0) + \delta \sum_{f' \in \Xi} p(f'|f, \mu - 1)) \cdot V_A((f', \mu), c^X, c^Y) \]
\[ - [r((f, \mu - 1), 1) - c^X] \]
\[ = r^X((f, \mu - 1), 0) + \delta \sum_{f' \in \Xi} p(f'|f, \mu - 1)) \cdot V_A((f', \mu), c^X, c^Y) \]
\[ - [r^X((f, \mu - 1), 1) - c^X + r^Y((f, \mu - 1), 1)] \]
\[ (E.10a) \]
\[
= \left[ r^X((f, \mu - 1), 0) - r^X((f, \mu - 1), 1) - c^X \right] \\
- \left[ r^Y((f, \mu - 1), 1) - \delta \sum_{f' \in \mathbb{F}} p(f'|f, \mu - 1)) \cdot V_Y^C((f', \mu), c^X, c^Y) \right] \\
= I((f, \mu - 1), c^X, c^Y) - \overline{I}(f, \mu - 1), c^X, c^Y) \\
\]

(E.10b)

(E.10a) results because \( r((f, \mu - 1), 0) = r^X((f, \mu - 1), 0) \) and \( r((f, \mu - 1), 1) = r^X((f, \mu - 1), 1) - r^Y((f, \mu - 1), 1) \). (E.10b) is equivalent to,

\[
I((f, \mu - 1), c^X, c^Y) > \overline{I}(f, \mu - 1), c^X, c^Y) \\
\]

Therefore, there is no incentive payment that is incentive compatible for both payers (i.e., the minimum payment that the first payer would require to deviate from its current decision is larger than the maximum payment that is incentive compatible for the final payer). Thus, the final payer cannot induce the first payer to act sub-optimally using an incentive payment.

For \( \omega = \mu - 2 \):

Consider case 5. The minimum incentive payment that would change the first payer’s action to treat is,

\[
I((f, \mu - 2), c^X, c^Y) \\
= r^X((f, \mu - 2), 0) + \delta \sum_{f' \in \mathbb{F}} p(f'|f, \mu - 2)) \cdot V_Y^C((f', \mu - 1), c^X, c^Y) \\
- \left[ r^X((f, \mu - 2), 1) - c^X \right] \\
\]

The maximum incentive that the final payer would be willing to provide is,

\[
\overline{I}((f, \mu - 2), c^X, c^Y) = r^Y((f, \mu - 2), 1) - \delta \sum_{f' \in \mathbb{F}} p(f'|f, \mu - 2)) \cdot V_Y^C((f', \mu - 1), c^X, c^Y) \\
\]

The optimal action in the social planner scenario is to provide treatment. Therefore, from the definition of the benefit function,

\[
0 \geq B_A((f, \mu - 2), c^X, c^Y) \\
\]
\[ r((f, \mu - 2), 0) + \delta \cdot \sum_{f' \in \mathbb{F}} p(f'|(f, \mu - 2)) \cdot V_A((f', \mu - 1), c^X, c^Y) - [r((f, \mu - 2), 1) - c^X] \]

\[ = r^X((f, \mu - 2), 0) \]

\[ + \delta \cdot \sum_{f' \in \mathbb{F}} p(f'|(f, \mu - 2)) \cdot [V^X_A((f', \mu - 1), c^X, c^Y) + V^Y_A((f', \mu - 1), c^X, c^Y)] \]

\[ - [r^X((f, \mu - 2), 1) - c^X + r^Y((f, \mu - 2), 1)] \]

\[ \geq r^X((f, \mu - 2), 0) \]

\[ + \delta \cdot \sum_{f' \in \mathbb{F}} p(f'|(f, \mu - 2)) \cdot [V^X_B((f', \mu - 1), c^X, c^Y) + V^Y_B((f', \mu - 1), c^X, c^Y)] \]

\[ - [r^X((f, \mu - 2), 1) - c^X + r^Y((f, \mu - 2), 1)] \]

\[ = r^X((f, \mu - 2), 0) \]

\[ + \delta \cdot \sum_{f' \in \mathbb{F}} p(f'|(f, \mu - 2)) \cdot [V^X_B((f', \mu - 1), c^X, c^Y) + V^X_B((f', \mu - 1), c^X, c^Y)] \]

\[ - [r^X((f, \mu - 2), 1) - c^X + r^Y((f, \mu - 2), 1)] \]

\[ = r^X((f, \mu - 2), 0) \]

\[ + \delta \cdot \sum_{f' \in \mathbb{F}} p(f'|(f, \mu - 2)) \cdot V^X_C((f', \mu - 1), c^X, c^Y) - [r^X((f, \mu - 2), 1) - c^X] \]

\[ + \delta \cdot \sum_{f' \in \mathbb{F}} p(f'|(f, \mu - 2)) \cdot V^X_C((f', \mu - 1), c^X, c^Y) - r^Y((f, \mu - 2), 1) \]

\[ = I((f, \mu - 2), c^X, c^Y) - \bar{I}((f, \mu - 2), c^X, c^Y) \]

(E.10c) results because \( r^X((f, \mu - 2), 0) = r((f, \mu - 2), 0) \), \( r((f, \mu - 2), 1) = r^X((f, \mu - 2), 1) + r^Y((f, \mu - 2), 1) \), and \( V_A((f', \mu - 1), c^X, c^Y) = V^X_A((f', \mu - 1), c^X, c^Y) + V^Y_A((f', \mu - 1), c^X, c^Y) \).

(E.10d) results from Proposition 4.5.3a. (E.10e) is equivalently to,

\[ I((f, \mu - 2), c^X, c^Y) \leq \bar{I}((f, \mu - 2), c^X, c^Y) \]

Therefore, there exists a range of incentive payments such that coordination is incentive compatible for both payers. The final payer’s reward is decreasing in the size of the incentive. Therefore, the final payer will offer the smallest possible incentive, \( I((f, \mu - 2), c^X, c^Y) \).
Consider case \( c \). The first payer will provide treatment if,

\[
I((f, \mu - 2), c^X, c^Y)
\]

\[
\geq I((f, \mu - 2), c^X, c^Y)
\]

\[
= r^X((f, \mu - 2), 0) + \delta \cdot \sum_{f' \in \mathbb{F}} p(f'|(f, \mu - 2)) \cdot V_C^X((f', \mu - 1), c^X, c^Y)
\]

\[- \left[ r^X((f, \mu - 2), 1) - c^X \right]
\]

And, the final payer could provide an incentive,

\[
I((f, \mu - 2), c^X, c^Y)
\]

\[
\leq \bar{I}(f, \mu - 2), c^X, c^Y)
\]

\[
= r^Y((f, \mu - 2), 1) - \delta \cdot \sum_{f' \in \mathbb{F}} p(f'|(f, \mu - 2)) \cdot V_C^Y((f', \mu - 1), c^X, c^Y)
\]

In this case, it is optimal to wait in the social planner scenario. Therefore, from the benefit function,

\[
0 < B_A((f, \mu - 2), c^X, c^Y)
\]

\[
= r((f, \mu - 2), 0) + \delta \sum_{f' \in \mathbb{F}} p(f'|(f, \mu - 2)) \cdot V_A((f', \mu - 1), c^X, c^Y) - \left[ r((f, \mu - 2), 1) \right]
\]

\[
= r^X((f, \mu - 2), 0)
\]

\[
+ \delta \cdot \sum_{f' \in \mathbb{F}} p(f'|(f, \mu - 2)) \cdot \left[ V_A^X((f', \mu - 1), c^X, c^Y) + V_A^Y((f', \mu - 1), c^X, c^Y) \right]
\]

\[- \left[ r^X((f, \mu - 2), 1) - c^X + r^Y((f, \mu - 2), 1) \right]
\]

\[
= r^X((f, \mu - 2), 0)
\]

\[
+ \delta \cdot \sum_{f' \in \mathbb{F}} p(f'|(f, \mu - 2)) \cdot V_C^X((f', \mu - 1), c^X, c^Y) - \left[ r^X((f, \mu - 2), 1) - c^X \right]
\]

\[
+ \delta \cdot \sum_{f' \in \mathbb{F}} p(f'|(f, \mu - 2)) \cdot V_C^Y((f', \mu - 1), c^X, c^Y) - r^Y((f, \mu - 2), 1)
\]

\[
= I((f, \mu - 2), c^X, c^Y) - \bar{I}(f, \mu - 2), c^X, c^Y)
\]

(E.10f) results because \( r^X((f, \mu - 2), 0) = r((f, \mu - 2), 0) \), \( r((f, \mu - 2), 1) = r^X((f, \mu - 2), 1) + \)
$r^Y((f, \mu - 2), 1)$, and $V_A((f', \mu - 1), c^X, c^Y) = V_A^X((f', \mu - 1), c^X, c^Y) + V_A^Y((f', \mu - 1), c^X, c^Y)$.

(E.10g) results from the previous iteration of $\omega$ (i.e., the coordinated scenario results in the same treatment policy as in the social planner scenario). (E.10h) is equivalent to,

$$I((f, \mu - 2), c^X, c^Y) > \bar{I}((f, \mu - 2), c^X, c^Y)$$

Therefore, there is no incentive payment that is incentive compatible for both payers. Thus, the final payer would offer zero incentive.

Repeating the iterative process for $\omega = \mu - 3, \mu - 4, \ldots, 0$, there always exists an incentive that coordinates the system to achieve the equivalent treatment policy as in the social planner scenario. 

Proof of Corollary 4.5.8 The proof of Corollary 4.5.8 follows directly from Propositions 4.5.1b and Propositions 4.5.2b, because the treatment region in the coordinated scenario is equivalent to the treatment region in the social planner scenario, from Proposition 4.5.7.

Proof of Proposition 4.5.9 Proof: The threshold prices that correspond to states $s \in S^Y$ are the same in the uncoordinated and coordinated scenario because the final payer’s treatment choices are by definition, equivalent. Therefore, we want to show that,

$$c^s_B \leq c^s_C, \forall s \in S^X$$

(E.11a)

This proof is by contradiction. Suppose, the converse of (E.11a) is true. Specifically, assume,

$$c^s_B > c^s_C, \forall s \in S^X$$

(E.11b)

Then, by the definition of the threshold price, the following must be true,

$$a^{s_X}_B(s, c^s_B, c^Y) = 1$$

(E.11c)

$$a^{s_X}_C(s, c^s_C, c^Y) = 1$$

(E.11d)
By Proposition 4.5.5

\[ a^X_C(s, c^s_B, c^y) = 0 \]  \hspace{1cm} (E.11e)

which is a contradiction to (E.11e). Therefore,

\[ c^s_B \leq c^s_C, \ \forall s \in S \]
E.2 Bibliography


Appendix F

Curriculum Vitae

Gregory Critchley

Post-Secondary Education and Degrees

- HBA - Western University, Ivey Business School, London, ON 2009
- MBA - Western University, Ivey Business School, London, ON 2013
- PhD - Western University, Ivey Business School, London, ON 2019

Honours and Awards

- Dean’s Scholarship, Ivey Business School, Western University 2018
- C.B. (Bud) Johnston Ontario Graduate Scholarship, Ivey Business School, Western University 2018
- Ontario Graduate Scholarship, Province of Ontario, Canada 2018
- Dean’s Scholarship, Ivey Business School, Western University 2017
- C.B. (Bud) Johnston Ontario Graduate Scholarship, Ivey Business School, Western University 2017
- Ontario Graduate Scholarship, Province of Ontario, Canada 2017
- Dean’s Scholarship, Ivey Business School, Western University 2016
- John F. Rankin Doctoral Scholarship, Ivey Business School, Western University 2016
- Ontario Graduate Scholarship, Province of Ontario, Canada 2016
- Dean’s Scholarship, Ivey Business School, Western University 2015
Plan for Excellence Scholarship, Ivey Business School, Western University 2015
Graduation with Distinction, Master of Business Administration, Ivey Business School, Western University 2013
Accelerated Master of Business Administration Scholarship 2012
Deans Honor List, Honors of Business Administration, Ivey Business School, Western University 2009
Graduation with Distinction, Honors of Business Administration, Ivey Business School, Western University 2009
HBA Jon and Nancy Love Scholarship 2007
Professor Mel Poucher Award in Civil Engineering 2006
Canadian Science Fair Scholarship (Gold) 2005
Western’s Continuing Admissions Scholarship for Academic Achievement, Western University 2005

Related Work Experience

Lecturer
Western University, Ivey Business School, London, ON
Business 2257 2013 - 2015

Teaching Assistant
Western University, Ivey Business School, London, ON
Introduction to Programming (MSc) 2019
Decision Making with Analytics (HBA, MSc, MBA, AMBA) 2019
Introduction to Data Science (MBA) 2016
Simulation (MSc) 2016

Publications
Critchley, G. J. & Zaric, G. S., The impact of pharmaceutical marketing on market access, treatment coverage, pricing and social welfare. 2019
Health Economics, 28(8), 1035-1051