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Rethinking Individuality in Quantum Mechanics

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Abstract

One recent debate in philosophy of physics has centered on whether quantum particles are individuals or not. The received view is that particles are not individuals and the standard methodology is to approach the question via the structure of quantum theory. I challenge both the received view and the standard methodology. I contend not only that the structure of quantum theory is not the right place to look for conditions of individuality that quantum particles may or may not satisfy, but also that there is an important role for traditional metaphysics to play. Consequently, my work brings together the philosophy of physics and traditional metaphysics literatures to shed new light on the debate over the individuality of quantum particles. I defend a set of conditions of individuality and argue that quantum particles satisfy these conditions thereby defending the view that particles are individuals in opposition to the received view. I also challenge a second feature of the standard methodology insofar as I challenge the significance of the Principle of the Identity of Indiscernibles in terms of which much discussion in the philosophy of physics literature is framed. My work is significant in a number of additional ways as well. My work implies that the dominant explanation for quantum statistics in terms of non-individuality is incorrect, and it also undermines the ontic-structural realists metaphysical underdetermination challenge to the scientific realist.

Keywords: Individuality; Non-Individuality; Identity; Discernibility; Quantum mechanics; Quantum particles; Identity of Indiscernibles

Lay Summary

One recent debate in philosophy of physics has centered on whether quantum particles are individuals or not. It is commonly thought that particles are not individuals because the formulas governing their behavior dont care which particle is which when we have multiple particles of the same kind. The standard way to determine whether particles are individuals or not is to assume that being an individual requires possessing identity (in some sense) and looking at the structure of quantum theory to try to determine whether quantum particles have some form of identity. I challenge the common view by rethinking what it means for something to be an individual. I also challenge the standard approach of looking at the structure of quantum theory because quantum theory gives us both reasons to think that particles have some form of identity as well as reasons to think that particles dont have any sort of identity. So, the structure of quantum theory underdetermines the answer to our question of whether particles are individuals or not. By rethinking what it means to be an individual we can avoid this underdetermination by providing an account of individuality in which the structure of quantum theory gives us a clear answer. I also think that there is an important role for traditional metaphysics to play given that we cant extract a unified account of individuality from quantum theory itself. Consequently, my work brings together the philosophy of physics and traditional metaphysical literatures to shed new light on the debate. I defend a set of conditions of individuality independently of any considerations regarding quantum theory and argue that quantum particles satisfy those conditions and are, therefore, individuals. My work challenges several additional positions as well. It challenges the dominant explanation for certain aspects of the behavior of quantum particles where those aspects are explained by viewing quantum particles as non-individuals. It also allows the scientific realist (the view that the theoretical entities posited by science exist) to respond to a challenge from ontic-structural realists (the view that we should only accept the existence of structural features of our scientific theories).

Acknowledgements

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Introduction

My goal in this dissertation is to defend the claim that a new view of individuality should be adopted in discussions of individuality in non-relativistic quantum mechanics (NRQM); one that is more metaphysically robust than the accounts currently on offer. I have a number of motivations for this. My primary motivation is that I think the philosophical questions asked by philosophers of science can often benefit from the work of metaphysicians. Discussions of individuality in NRQM is just one instance of where the work of metaphysicians can prove useful. A second motivation is that focusing on the issue of individuality in NRQM allows this work to provide tools that impact on another debate of interest; that between scientific realists and ontic-structural realists. It would be well beyond the scope of this dissertation to focus on both debates, however, a defense of a conception of individuality that isn't meant to be grounded in physical theory provides tools to counter the ontic-structural realists metaphysical underdetermination argument.[1] So this dissertation serves to show how the work of metaphysicians can play a useful role in the debate over the individuality of quantum particles and how that same work has implications for a second debate.[2]

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[1]The metaphysical underdetermination argument is essentially that NRQM doesn't tell us whether quantum particles are individuals or not and that this sort of underdetermination is problematic for the scientific realist. We can accept that NRQM doesn't settle the issue of whether quantum particles are individuals, that not being able to determine whether quantum particles are individuals or not would be a problem for the scientific realist, and still counter the argument by arguing that the physical theory isn't the right place to look for conditions on individuality that quantum particles may, or may not, satisfy.

[2]Obviously, those implications are not worked out, but they could be in a future expansion of this project.

My goal in chapter two is to show that NRQM does indeed fail to determine whether quantum particles are individuals or not, paving the way to look elsewhere to settle the issue. This will be accomplished by looking at the four major arguments in the literature in favor of or against viewing quantum particles as individuals. Two of these arguments conclude that quantum particles are not individuals whereas two of them conclude that quantum particles are individuals. I conclude the chapter with a brief discussion of metaphysical underdetermination to situate my work with respect to the debate between scientific realists and ontic-structural realists.

In chapter three I outline four challenges that must be kept in mind in discussions of individuality in NRQM. The first three of these challenges are for those who want to claim that quantum particles are non-individuals. The challenges mostly take the form of pointing to ways in which entertaining the idea of non-individuals requires revisions to more than just the notion of individuality and how these further revisions point to problems that may be missed by focusing too narrowly on the concept of an individual. The challenge to those who think that quantum particles are individuals comes from the existence of various proofs in the literature that demonstrate ways in which states in NRQM violate various versions of the Principle of the Identity of Indiscernibles. These violations put restraints on the sorts of accounts of individuality quantum particles could satisfy.

I begin chapter four by distinguishing between a number of questions one might ask about individuality that have sometimes been conflated. I then move on to consider different possible conditions on individuality. In each case I consider whether the condition in question may be either necessary, sufficient, or both. There are two main results of this chapter. The first is that I settle on a set of conditions that I take to be individually necessary and jointly sufficient for something to count as an individual. The second is that

I show how one might respond to the first three challenges discussed in chapter three.

In chapter five I consider the possible metaphysical grounds of individuality. That is, where chapter four was about what it means for something to be an individual chapter five is about what metaphysical features of the world could serve to underpin individuality. In both chapters four and five my evaluations of the positions under consideration are novel insofar as I adopt a new evaluative criterion. That criterion is whether the position in question allows for the *possible* existence of non-individuals.

Finally, in chapter six I wrap up by considering how quantum particles fare with respect to the conditions of individuality defended in chapter four and the metaphysical grounds of individuality defended in chapter five. I conclude that quantum particles are individuals.

Individuality in Quantum Mechanics

My broader goal in this dissertation is to defend the claim that a new view of individuality should be adopted in discussions of individuality in non-relativistic quantum mechanics (NRQM); one that is more metaphysically robust than the accounts currently on offer.[1] As a first step I will show that there is good reason to think that quantum mechanics itself cannot provide us with an account of individuality, otherwise that account of individuality should be the one we adopt. This will be accomplished by a review of the existing arguments regarding what quantum mechanics tells us, or doesn't tell us, about individuality.

I will begin with two arguments that conclude that quantum objects are not individuals. The first is an argument due to Redhead and Teller according to which the most ontologically perspicuous formalism of quantum mechanics is the Fock space formalism. The second will be a much-discussed argument that I shall refer to as "the argument from quantum statistics." Next will be two arguments to the effect that quantum objects are individuals. The first will be a two-part argument that quantum objects are weakly discernible, and that weak discernibility is sufficient for individuality. The second is due to Dorato and Morganti who argue that quantum mechanics may endorse a *primitive thisness* view of individuality according to which quantum objects are individuals. Finally, I will conclude with a brief discussion of metaphysical underdetermination. That is, a

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[1]There are a number of reasons why I think it appropriate to limit myself to NRQM. I discuss these reasons in chapter six.

discussion of how we should understand the fact that NRQM is compatible with particles-as-individuals and particles-as-non-individuals interpretations.

2.1   Labels, Individuality, and Fock Space

Redhead and Teller provide an argument to the effect that quantum mechanics tells us that particles are not individuals (Redhead and Teller 1991, Redhead and Teller 1992, and Teller 1995).[2] They begin by arguing that the standard formalism of quantum mechanics (which they call the labeled tensor product Hilbert space formalism LTPHSF) combined with the assumption that distinct rays represent distinct physical situations forces us to accept *label transcendental individuality* (described below). They then argue that the need to symmetrize or antisymmetrize multiparticle wave functions for systems of identical particles is an artifact of the use of particle labels in the LTPHSF.[3] Next, they claim that the need to (anti)symmetrize creates surplus formal structure. Finally, they argue that the Fock space formalism does not contain this surplus structure and so is a more appropriate formalism for answering metaphysical questions. Because the Fock space formalism lacks particle labels they conclude that particles are not individuals.

Principles of individuation can be divided into two types. *Transcendental individuality* is individuality granted by something over-and-above an object's properties (for example, individuality granted by haecceities, primitive thisness, or bare particulars).[4] Redhead and Teller further divide transcendental individuality into *property transcendental individuality* and *label transcendental individuality* (LTI). Property transcendental

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[2]I will use the term "particle" to refer to quantum particles except when I am discussing both classical and quantum particles.

[3]I will follow the practice of using the phrase "identical particles" to refer to particles that share all the same state independent properties.

[4]I will discuss candidates for conferring transcendental individuality in detail in chapter five.

individuality refers to that in virtue of which an object bears properties (a bare particular would be a candidate for conferring property transcendental individuality).[5] Label transcendental individuality refers to that in virtue of which an object is an individual (haecceities or Dorato and Morganti's primitive thisness would be candidates for conferring label transcendental individuality). For Redhead and Teller an object is an individual if it can, *in principle*, be labeled, distinguished (from other objects of the same kind), and reidentified over time. So, label transcendental individuality refers to that in virtue of which an object can be labeled, distinguished, and reidentified over time.

The other type of principle of individuation is that of *bundle individuality*. Bundle individuality is individuality conferred by some bundle of individuating properties. These could be either intrinsic or extrinsic properties. However, since I will only be concerned with collections of particles that share the same intrinsic properties, I will only be concerned with the latter. Bundle individuality is an analog of label transcendental individuality. The bundle of individuating properties is that in virtue of which the object is an individual. A good example from classical physics is spatiotemporal properties. Classical objects are taken to be impenetrable and so every classical object will have a unique spatiotemporal trajectory. Note that I will use the phrase "label individuality" when I want to refer to label transcendental individuality and bundle individuality together.

With the above in mind, we can get a better idea of what Redhead and Teller mean when they say that particles are not individuals. To say that particles are not individuals is to say they cannot be labeled even in principle. This in turn means that particles cannot, in general, be distinguished from other particles of the same kind and that particles cannot, in general, be reidentified over time. However, it is important to note that this is not an

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[5]But note that something that confers property transcendental individuality needn't also confer label transcendental individuality as a bare particular would.

epistemic failure. It's not merely that we can't tell which particle is which, but that there is no fact of the matter as to which is which. Switching the positions of two identical particles doesn't create a distinct, but empirically identical, state of affairs, but instead amounts to doing nothing at all (so it's questionable whether it's even coherent to consider switching the positions of two identical particles if particles are not individuals). Similarly, there's no fact of the matter as to whether an electron you detected in the lab on Tuesday is the same electron you detected last Friday.

It is important to note, to stave off counterexamples, that even if particles are not individuals it can still be possible to distinguish them from others of the same kind or reidentify them over time in *some* cases. This is because even particles of the same kind can differ in their extrinsic properties (e.g. the property of having been detected by detector 1) or their state dependent properties (e.g. being spin-up). To take a simple example, consider sending a single electron through a series of Stern-Gerlach magnets (so doing a series of spin measurements). Then you know that it was the same electron setting off each detector. Or consider Priscilla, a positron kept isolated in a Penning trap for three months by Nobel Laureate Hans Dehmelt. According to Dehmelt "[t]here can be little doubt about the identity of Priscilla during this period, since in ultrahigh vacuum she never had a chance to trade places with a passing antimatter twin" (Dehmelt 1990). Or consider doing a Bell type experiment where you send each member of a pair of entangled electrons to opposite sides of the lab for spin measurements. After the measurements, it makes sense to say, "this electron on the left side of the lab is spin-up whereas that electron on the right side of the lab is spin-down". Consequently, the ability to distinguish between particles of the same kind or reidentify particles over time based on state dependent or extrinsic properties does not constitute a counterexample to Redhead and Teller's claim that particles are not individuals.

2.1.1 The Problem with the Hilbert Space Formalism

The first part of Redhead and Teller's argument is meant to show how the LTPHSF forces us to accept LTI beginning with the standard assumption that each distinct ray in a Hilbert space represents a distinct physical situation. To see how this leads us to LTI consider a pair of electrons represented in Hilbert spaces $\mathcal{H}(1)$ and $\mathcal{H}(2)$. The state of the joint system is represented by a ray in the tensor product space $\mathcal{H}(1) \otimes \mathcal{H}(2)$. The joint state in which one electron has property a and one electron has property b can be written as the tensor product $|a(1)\rangle |b(2)\rangle$. However, $|a(1)\rangle |b(2)\rangle$ and $|b(1)\rangle |a(2)\rangle$ are distinct rays in $\mathcal{H}(1) \otimes \mathcal{H}(2)$. Consequently, the standard assumption that distinct rays represent distinct physical situations requires us to treat these states as representing distinct physical situations. So, we have a state in which electron 1 has property a and electron 2 has property b as well as a distinct state in which electron 1 has property b and electron 2 has property a. Combining the LTPHSF with the assumption that distinct rays represent distinct physical situations requires us to treat the electrons as though they have some individuating feature over-and-above their properties. A feature in virtue of which it makes a difference whether it is electron 1 or electron 2 that has property a (or b).

We now have a problem. Our state space seems to include states that it shouldn't. Considering again a two-particle system in which the particles can have either property a or property b. The joint system states in the LTPHSF should be the tensor products of single system states. The possible joint states should be

$$|a(1)\rangle |a(2)\rangle \tag{2.1}$$

$$|b(1)\rangle |b(2)\rangle \tag{2.2}$$

$$|a(1)\rangle |b(2)\rangle \tag{2.3}$$

$$|b(1)\rangle |a(2)\rangle . \tag{2.4}$$

But that can't be right. These four states should be equiprobable in equilibrium which would give us Maxwell-Boltzmann statistics (MB statistics).[6] To get the appropriate Bose-Einstein (BE) and Fermi-Dirac (FD) statistics, we need to impose a symmetrization requirement. More specifically, we rule out any states that are neither symmetric nor antisymmetric with respect to the particle labels. We are left with the symmetric states

$$|a(1)\rangle |a(2)\rangle \tag{2.1}$$

$$|b(1)\rangle |b(2)\rangle \tag{2.2}$$

$$\frac{1}{\sqrt{2}}(|a(1)\rangle |b(2)\rangle + |b(1)\rangle |a(2)\rangle), \tag{2.5}$$

which yield BE, and the antisymmetric state

$$\frac{1}{\sqrt{2}}(|a(1)\rangle |b(2)\rangle - |b(1)\rangle |a(2)\rangle), \tag{2.6}$$

which yields FD. States (2.3) and (2.4) must be considered distinct in the LTPHSF because they are represented by distinct rays. So, we impose symmetrization requirements to rule that those states are never realized, although they are still mathematically part of the state space. It is these states that are never realized that constitute the surplus structure we are left with after imposing the symmetrization requirement.

Redhead and Teller note that we have two interpretive options if we want to keep the LTPHSF. The first is to say that non-symmetric states (states that are neither symmetric nor antisymmetric) have a physical interpretation according to which it does matter which particle is which. But this leaves us to answer the question of why those states never occur. The second is to say that non-symmetric states have no physical interpretation so

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[6]Here I'm assuming the standard measure which assigns equal probabilities to all of the microstates compatible with the initial macrostate of the system. The microstate is a specification of the positions and velocities of the particles whereas the macrostate is given by the temperature, pressure, and volume.

they do not correspond to unactualized physical possibilities at all. The second option requires rejecting LTI which, in turn, requires rejecting the standard view that distinct rays represent distinct physical situations. But rejecting LTI amounts to accepting that particles are not the kinds of things that can bear labels and so this option leaves us with the question of how the particle labels in the LTPHSF are to be understood.

While we could choose one of these interpretative options to hold onto the LTPHSF, we have the option of avoiding the issue altogether by adopting the Fock space formalism. The Fock space formalism does not use particle labels. Instead it simply tells us how many particles of a given type are occupying a certain state without concern for keeping track, either conceptually or ontologically, of which particle is which. The Fock space formalism is empirically adequate and avoids the surplus structure that the LTPHSF formalism must remove using a symmetrization requirement. The surplus structure in the LTPHSF formalism is the set of states that are neither symmetric nor antisymmetric with respect to particle labels; that is, states where the particle labels make a difference. But it is not even possible to represent these states in the Fock space formalism due to the absence of particle labels. As Teller says of the Fock space formalism, "[t]he description uses no particle labels, and so the issue of which particle has which property never comes up" (Teller 1995, 28). The need to impose symmetrization in the LTPHSF is merely an artifact of the use of particle labels. It seems clear that the Fock space formalism is to be preferred and its lack of particle labels should, per Redhead and Teller, lead us to conclude that particles are not individuals.

2.1.2   Hilbert and Fock Space Formalisms: Not so Different After All

There are several issues with Redhead and Teller's argument all stemming from relations between the LTPHSF and the Fock space formalism. Teller himself notes (Teller 1995,

27-28) that the symmetric states in the LTPHSF form a subspace of the larger Hilbert space. The symmetric subspace contains no surplus elements, so one option for defenders of the LTPHSF is to take that as the state space for bosons. However, Teller doesn't see a difference between (i) leaving the surplus elements uninterpreted or (ii) taking the symmetric subspace as the state space for bosons (Teller 1995, 28). The symmetric subspace is still constructed by taking a Hilbert space with surplus structure and then discarding that structure. Similar comments hold for the antisymmetric states and fermions.

The problem for Redhead and Teller arises when we note that the Fock space formalism has a similar issue. The full Fock space (that is, the most general Fock space one can construct) is the direct sum of all n-particle Hilbert spaces

$$F = \bigoplus_{n=0}^{\infty} \mathcal{H}^{\otimes n} = \mathbb{C} \oplus \mathcal{H} \oplus (\mathcal{H} \otimes \mathcal{H}) \oplus \cdots .^{7} \qquad (2.7)$$

Except for the first two terms (corresponding to states with zero or one particle, respectively), the terms all contain tensor products of single particle Hilbert spaces. Consequently, the most general Fock space includes all the surplus structure included in the LTPHSF. Constructing symmetric or antisymmetric Fock spaces for bosons and fermions, respectively, also involves constructing a space containing surplus structure which is then discarded. The symmetric Fock space can be written as

$$F = \bigoplus_{n=0}^{\infty} \mathcal{H}^{\odot n} = \mathbb{C} \oplus \mathcal{H} \oplus (\mathcal{H} \odot \mathcal{H}) \oplus \cdots , \qquad (2.8)$$

where $\odot$ represents the symmetric tensor product. Taking the symmetric tensor product is equivalent to taking the ordinary tensor product then removing all the states that are not

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[7] This result, as well as those that follow, relating the LTPHSF to the Fock space formalism is well known in NRQM. For an argument that they extend to quantum field theory see (Huggett 1994).

symmetric. The antisymmetric Fock space can be written the same way with the exterior (or antisymmetric) product $\wedge$ instead of the symmetric tensor product.

It seems then, by Teller's own lights, that restricting our attention to the symmetric or antisymmetric Fock spaces should be just as unappealing as focusing on the symmetric or antisymmetric subspaces in the LTPHSF. Or, equivalently, that holding onto the LTPHSF and taking the symmetric (or antisymmetric) subspaces as our state spaces should be just as acceptable as adopting the Fock space formalism.

Not only that, but the fact that the full Fock space contains the same surplus structure as the LTPHSF makes it unclear why the other interpretive option for saving the LTPHSF (as well as LTI) is problematic. Even in the Fock space formalism we must decide whether the surplus structure has a physical interpretation or not. If not, we can just take the symmetric (or antisymmetric) subspaces in either the LTPHSF or Fock space formalism as our state space (bracketing any worries about the role of the surplus structure in their construction). If so, then we face the issue of why our theory allows for physically intelligible states that are never realized regardless of which formalism we choose.

Others have also objected that the LTPHSF and Fock space formalisms are more similar than Redhead and Teller suggest. Huggett, for example, has demonstrated that the LTPHSF and Fock space formalisms are mostly equivalent even in the relativistic regime (Huggett 1994). The Fock space of quantum field theory (QFT) contains all the structure of the LTPHSF, but, unlike the LTPHSF, it also allows us to represent coherent states (that is, states of indefinite particle number).[8] Huggett argues that the ability to represent states of indefinite particle number is logically posterior to the issue of what metaphysical significance we should attach to labels. If the lack of labels in the Fock space formalism is what's metaphysically salient, then even free field states of definite particle number in

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[8]Note that coherent states are only found in QFT.

QFT would be incompatible with individuality (due to the lack of labels). However, free field states of definite particle number have identical state spaces to those found in NRQM, which are compatible with individuality. The point is, that QFT has a structure that is compatible with viewing particles as individuals. The fact that the Fock space formalism can represent coherent states may be a reason to prefer that formalism, but the formalism itself doesn't tell us whether particles are individuals or not. Once we've settled the issue of what metaphysical significance labels should have, we can then look at the consequences for coherent states. Consequently, the Fock space of QFT is equivalent to the LTPHSF in all the ways that matter for Redhead and Teller.

But, as Huggett also demonstrates, the equivalence is not merely structural. He notes that "if we map n particle momentum wave functions into field wave functions in n-excitation subspaces, [that is, Fock subspaces,] the observables and equations of motion agree" (Huggett 1994, 74). Therefore, Huggett concludes, the formalisms also have the same physical interpretation. The formalisms are mathematically and physically equivalent (except for coherent states in QFT). If Fock space is both mathematically and physically equivalent to the LTPHSF in every respect that matters for discussing metaphysical questions about the relation between individuality and labels, then it is hard to see how the fact that one formalism has labels and one doesn't could be metaphysically salient. So, considering Huggett's argument, my demonstration that the LTPHSF and the Fock space formalism have the same interpretive issues (at least the issues Redhead and Teller identified with the former) is somewhat unsurprising.

Another similarity, noted in (Baker 2013) and (Krause and Arenhart 2015b), is that the Fock space formalism is, in a sense, dependent on labels as well as compatible with the use of labels. Krause and Arenhart note that the construction of Fock spaces begins with labeled single particle Hilbert spaces (Krause and Arenhart 2015b, 8). The labels are necessary to distinguish between the non-symmetric states even if those states will be

discarded. Baker, on the other hand, provides an example of labeling in the Fock space formalism by asking us to consider the state (Baker 2013, 266)

$$0 \oplus \frac{1}{\sqrt{2}} \left| \psi \right\rangle \oplus \frac{1}{\sqrt{2}} \left(\left| \psi \right\rangle_1 \left| \phi \right\rangle_2 + \left| \phi \right\rangle_1 \left| \psi \right\rangle_2 \right) \oplus 0 \oplus \cdots. \tag{2.9}$$

This describes a coherent state. A measurement of particle number would yield either one or two, each with a 50% chance. The two-particle state is symmetric, but there is no reason we can't, as Baker has done, label it the same way we would in the LTPHSF. We can, if we choose, understand the two-particle term as describing two distinct physical situations depending on which particle (1 or 2) is in which state (ψ or ϕ).

Baker goes so far as to say that "[t]he apparent difference between Fock space and the labeled tensor-product Hilbert spaces of QM is simply an artifact of the occupation number basis" (Baker 2013, 266). Baker's accusation here is that Redhead and Teller's claim that the Fock space formalism doesn't use labels is a result of them combining the Fock space formalism with the occupation number representation. In the occupation number representation, a state is specified by the occupation numbers of the single particle states as well as the symmetry of the state. Suppose we have two bosons divided between two boxes *a* and *b*. Equations (2.1) to (2.3) give the states as represented in the LTPHSF. However, in the occupation number representation, these states could be written as $\left| a_2, b_0 \right\rangle$, $\left| a_0, b_2 \right\rangle$, and $\left| a_1, b_1 \right\rangle$, where the subscripts represent the number of particles in each box. Note that these states are not written as tensor products of single particle states. The occupation number representation cannot be used with the LTPHSF, but it can be used with the Fock space formalism. However, the converse does not hold. The Fock space formalism needn't use the occupation number representation. The Fock space formalism can easily be represented with direct sums of LTPHSF states with labels present. Consequently, Redhead and Teller's identification of the Fock space formalism with the use of occupation numbers is misleading. It is not the Fock space formalism itself that

tells us to focus only on how many particles of a given type are occupying a certain state, but merely one way of representing that formalism.

There is one final issue, that I will merely mention, that does not depend on similarities between the LTPHSF and the Fock space formalism. The issue is simply that if paraparticles exist, then Redhead and Teller's surplus structure isn't really surplus structure.[9] At least not all of it. That the Fock space formalism, on Redhead and Teller's understanding of it, has no non-symmetric states would then be a drawback not a benefit.

2.2   The Argument from Quantum Statistics

The argument from quantum statistics is the primary argument for the conclusion that particles are not individuals. Its seemingly wide acceptance has led French and Krause, for example, to refer to the idea that particles are not individuals as the "Received View" (French and Krause 2006, 115). The argument has been presented in a variety of ways, although the core of the argument remains the same.[10] The differences are mostly differences in making the argument precise. Different authors will have slightly different formulations of the Indistinguishability Postulate or different notions of individuality in mind. However, it is possible to extract a single unified argument from the various presentations albeit at the expense of precision.

The argument starts by noting the differences between classical and quantum statistics, which were noted in section 2.1.1. Suppose we have two objects, 1 and 2, that can be in

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[9]Paraparticles are particles that are neither bosons nor fermions; particles that obey different statistics.

[10]See, for instance, (Dieks 1990), (van Fraassen 1991), (Reichenbach 1998), (French and Rickles 2003), (French and Krause 2006), (Ladyman and Ross 2007), and (Morganti 2009) for various presentations.

one of two boxes *a* and *b*. Classically we have four equally probable configurations corresponding to (2.1) through (2.4). We can have both in box *a*, both in box *b*, 1 in *a* and 2 in *b*, or 1 in *b* and 2 in *a*. But quantum statistics are different. In BE we have three equally probable configurations corresponding to (2.1), (2.2), and (2.5) (both in box *a*, both in box *b*, or one in each box). In FD we just have one possible configuration corresponding to (2.6) (one in each box).

Classical statistics treats the situation in which 1 is in *a* and 2 is in *b* as distinct from the situation in which 1 is in *b* and 2 is in *a*. Quantum statistics, however, does not treat these cases as distinct. Permuting particles is treated as making a difference in classical statistics but not in quantum statistics. It is at this point that different authors will diverge a little in offering different formulations of the Indistinguishability Postulate (also called "permutation invariance" or "permutation symmetry"). The Indistinguishability Postulate is a statement of the fact that permuting particles makes no difference in quantum statistics. How this statement is made precise is unimportant for my purposes as none of the criticisms of the argument depend on it.

The argument continues by noting that classical particles of the same kind cannot be distinguished on the basis of their state-independent, or intrinsic, properties. Consequently, the fact that classical statistics treats (2.3) and (2.4) as distinct arrangements must involve something beyond the (classical) particles intrinsic properties. So, classical particles have some form of label individuality in virtue of which states (2.3) and (2.4), which only differ in the labels, are considered distinct. The important point isn't merely that classical particles are individuals, but that it seems to make an empirical difference that they are individuals. That classical particles are individuals is meant to explain why (2.3) and (2.4) are treated as distinct and, consequently, why we have four possible configurations rather than three.

The final part of the argument involves looking for the source of the difference between classical and quantum statistics. In the classical case, we are looking at collections of classical particles sharing all the same intrinsic properties in addition to having some form of label individuality. In the quantum case, we are looking at collections of quantum particles that also share all the same intrinsic properties. The only apparent explanation for the difference in statistics is then that quantum particles lack the label individuality of classical particles. Consequently, quantum particles are not individuals. I now turn to several arguments against inferring that quantum particles are not individuals on the basis of their statistical behavior.

2.2.1 The Gibbs Paradox: Permutability of Classical Particles

Saunders argues that the Gibbs paradox can be taken to show that classical particles are permutable (Saunders 2006b). Saying that a particle is permutable amounts to saying that switching particle labels for particles of the same kind doesn't change the state. If Saunders is correct, then the argument from quantum statistics fails because it takes permutability as a key difference between classical and quantum statistics; a difference that is to be explained in terms of individuality. Saunders agrees that permutability has empirical consequences and that it is connected to the difference between classical and quantum statistics. However, permutability is not the whole story. According to Saunders, an important part of the explanation for the difference between classical and quantum statistics is that phase space is continuous whereas Hilbert space is discrete. I will not be concerned with this last part of Saunders' argument since the argument from quantum statistics fails as long as Saunders is correct about the permutability of classical particles.

The Gibbs paradox refers to the fact that MB statistics does not predict the correct entropy when mixing samples of the same gas. Consider a box divided into two halves each containing identical samples of the same gas. Each sample has entropy S, so the total entropy of the system is $2S$. The system is in equilibrium and would remain in equilibrium if the divider were removed. Consequently, mixing the two samples of gas should involve no change of entropy. However, not only does MB predict an increase in entropy if the divider is removed, it also predicts that the entropy will drop back down to $2S$ if the divider is put back. To get the correct result the entropy predicted by MB must be divided by $N!$, where N is the number of particles in the gas. This, however, is exactly what we would do if classical particles were permutable. It is equivalent to identifying states related by a permutation to avoid over-counting the number of distinct states.

There are two primary objections to Saunders' explanation of the corrective factor $N!$. The first, an argument Saunders calls the dispensability argument, is that classical permutability is unnecessary in the sense that alternative explanations exist. This argument originated in (Ehrenfest and Trkal 1920) and was later defended in (van Kampen 1984). Both provide derivations of the correct result in which classical particles are explicitly assumed to be permutable. Saunders' response is that these derivations are only applicable in certain domains. Ehrenfest and Trkal assume that total particle number remains constant. However, Saunders points out that this assumption would not hold in a system subject to nuclear processes. Even though nuclear processes are not covered by statistical mechanics, Saunders thinks that "[s]o long as initial and final states are thermodynamically describable, classical thermodynamical principles should apply, however violent (and non-classical) the transformations that connect them" (Saunders 2006b, 197). In the case of van Kampen, Saunders' concern is that his derivation is inapplicable to closed systems.

The other objection, the incoherence, or unintelligibility, argument, is found in (van Kampen 1984) and (Bach 1997). Van Kampen offers just a few brief comments rather than worked out arguments, see (Saunders 2006b, 196-97) for discussion and response. The more serious incoherence argument comes from Bach who argued that classical particles cannot be permutable because they would have no spatiotemporal trajectories. Bach offers a formal argument for this conclusion in addition to pointing out that particles, of any sort, that have spatiotemporal trajectories can't be permutable because the trajectories distinguish them. Saunders takes Bach's argument to be question-begging. First, the formal argument takes configuration space to be the appropriate space for representing the possible configurations of the particles. But a defender of permutable classical particles would take the reduced configuration space (that obtained by identifying points in configuration space that only differ by a permutation) as the appropriate space. Second, the claim that particles with trajectories can't be permutable because the trajectories distinguish them is not defended. It is just assumed that the idea of permutable particles with trajectories is nonsensical. It is nonsensical if we use the full configuration space because every particle then does have a unique spatiotemporal trajectory. However, if we use the reduced configuration space the spatiotemporal trajectories aren't necessarily unique. The cost of this approach, however, is that it makes no sense to ask which particle has which spatiotemporal trajectory. Spatiotemporal trajectories, on Saunders' view, do not individuate classical particles meaning that classical particles are not individuals unless they have some sort of LTI.

One important thing to note here is that classical particles, even if they are permutable under some circumstances, aren't always permutable. They wouldn't, for example, obey MB statistics (outside of the Gibbs paradox) if they were always permutable.[11] This

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[11]This is simply the result of the fact that treating classical particles as permutable means that we identify states (2.3) and (2.4), which gives us something other than MB statistics.

means that we should qualify the claim that classical particles are permutable in some way. We could agree with Saunders that the reason we have to identify permutations of particles to avoid the Gibbs paradox is because there is no fact of the matter as to which particle has which trajectory. However, we could instead say that we have to identify permutations of particles because which particle has which trajectory simply isn't physically relevant to the entropy of a system when the particles are all of the same kind. So, classical particles are permutable only if information about which particle has which trajectory is not physically relevant. Then we can understand the issue with calculating the entropy of mixing similar gases as one of including irrelevant information in the calculation that we need to remove. My proposal allows us to understand why identical classical particles obey MB statistics most of the time, but do not in the case of the Gibbs paradox. It also allows us to retain the idea that classical particles have unique spatiotemporal trajectories that serve to individuate them.

Saunders' position is clearly controversial. However, the idea that classical particles are permutable has been little discussed and Saunders makes a good case that we have no particularly compelling reasons to reject it (even if an examination of the issue would find such reasons). Again, if Saunders is correct then the argument from quantum statistics fails because permutability is not a feature unique to quantum particles. The argument from quantum statistics takes a lack of individuality to explain permutability and permutability to explain quantum statistics. Saunders' argument challenges the latter claim. Permutability can't explain quantum statistics because classical particles are also permutable and obey different statistics. On the other hand, one can accept Saunders' position and still accept that a lack of individuality is what explains permutability so that quantum particles are not individuals (ideally with some account of what's wrong with LTI).

My position, on the other hand, challenges both aspects of the argument from quantum statistics. Permutable particles can obey MB statistics, in addition to BE or FD statistics, and can be individuals. Still, my view on the Gibbs paradox is compatible with the view that quantum particles are not individuals (it merely questions the support for this view that the argument from quantum statistics is alleged to provide).

2.2.2   The Fundamental Postulate of Statistical Mechanics

While Saunders argues that even classical particles are permutable, Belousek argues that quantum particles are only permutable if the Fundamental Postulate of Statistical Mechanics (FPSM) is correct (Belousek 2000). Or, equivalently, Belousek argues that statistical indistinguishability is not needed to derive quantum statistics. Belousek defines FPSM as follows: "[e]very distinct equilibrium microstate configuration of identical particles, or distribution of identical particles over single-particle microstates, is assigned the same statistical weight" (Belousek 2000, 2). Identical particles are *statistically distinguishable* if particle permutations result in distinct states that are assigned their own statistical weight. In other words, particles are statistically indistinguishable if they are permutable.

Belousek first demonstrates that assuming FPSM and statistical distinguishability is sufficient to derive MB. Similarly, assuming FPSM and statistical indistinguishability allows one to derive BE. FD can be derived in the same way as BE with the additional assumption of an exclusion principle that forbids particles being in the same state. Clearly, given FPSM, permutability is *sufficient* to derive quantum statistics (with the added exclusion principle for fermions). However, Belousek notes that if we want to conclude that quantum particles are permutable based on quantum statistics, we want to know if the assumption of permutability is *necessary*.

If we are willing to reject FPSM, then one way we can get BE statistics for a system of two identical particles with properties a and b is to assign probabilities of 1/3, 1/3, 1/6, and 1/6 to states (2.1) through (2.4). Similarly, we can get FD statistics by assigning a probability of 1/2 each to (2.3) and (2.4). We can also get MB statistics by assigning a probability of 1/4 each to states (2.1) through (2.4). That we can derive quantum statistics for statistically distinguishable particles in general has been demonstrated in (Tersoff and Bayer 1983). Instead of using the FPSM, they assign each discrete state i an arbitrary probability w_i and then average over every possible w_i.[12] The resulting probability distribution for a given occupancy n_i is given by

$$P\{n_i\} = N! \left[\prod_{i=1}^{M} \frac{1}{n_i!} \int_0^1 dw_i w_i^{n_i} \right] \delta \left(1 - \sum_{i=1}^{M} w_i \right), \tag{2.10}$$

where M is the number of states available to particles (for example, two in the case of particles divided between two boxes). The equation can be solved recursively to obtain BE statistics

$$P\{n_i\} = \frac{N!(M-1)!}{(N+1-M)!}. \tag{2.11}$$

FD statistics can be obtained in the usual way by adding an exclusion principle, which, in this case, amounts to the restriction that $n_i = 0$ or $n_i = 1$. Tersoff and Bayer provide no justification for using an exclusion principle to obtain FD statistics. Presumably, it is either meant to be empirically justified (the occupation number for fermions is either 0 or 1) or justified by the spin statistics theorem. The question now becomes whether there is any compelling reason to accept one approach over the other. Whether and how FPSM is justified is an issue on which much has been written. I don't have the space to get into the issue in any detail so I will confine myself to just a few brief remarks to the effect that it would not be unreasonable to accept Tersoff and Bayer's approach.

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[12]Note that Tersoff and Bayer have no additional justification for this procedure beyond that it gives the correct statistics.

Firstly, Tersoff and Bayer claim that the assumptions that go into (2.10) are logically weaker than the assumption of uniform probabilities (Tersoff and Bayer 1983, 554). Consequently, the former assumption is to be preferred in the absence of any information that warrants making the latter assumption. Secondly, FPSM is notoriously hard to justify. Teller, for example, remarks that the FPSM is "an assumption noted both for its astonishing power in classical statistical mechanics and for its resistance to justification other than its empirical success" (Teller 1995, 24). One could take the difficulty in justifying FPSM as itself reason to be open to alternative principles that can reproduce the empirical successes attributed to FPSM. The justification here is very weak. It's simply that in the absence of any reasons to favor one approach over the other, Tersoff and Bayer's approach at least requires less substantive assumptions then assuming the standard measure.

Since the concern here is with equilibrium statistical mechanics it is clear that any appropriate probability distribution must be invariant in time. Any probability distribution is going to assign zero probability to some states (this is unavoidable to ensure the probabilities sum to 1). Consequently, the standard measure assigns zero probability to certain states. There are other measures that also assign zero probability to the same states. However, if we assume that systems in equilibrium are ergodic it can be shown that, of these measures that agree on which states have probability zero, only the standard measure is time invariant.[13][14] The problem with justifying the standard measure in equilibrium statistical mechanics amounts to the problem of justifying (i) the claim that equilibrium systems are ergodic, and (ii) the claim that we can ignore states that are assigned zero probability in the standard measure. However, these claims are very hard to

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[13]Ergodicity refers to the fact that any initial state, with the exception of the initial states assigned probability zero by the standard measure, will eventually pass through any state that has non-zero probability. Roughly, given enough time, a system in equilibrium will have spent some time in every microstate compatible with its macrostate.

[14]See (Werndl 2013).

justify.[15] It is an open question whether realistic systems in equilibrium are ergodic and an open question whether states assigned zero probability by the standard measure should be ignored. In light of these issues, Tersoff and Bayer's justification isn't as weak as it first appears.

At this point it seems reasonable to accept that one cannot claim that quantum particles are permutable on the basis of quantum statistics. Defenders of the non-individuality of quantum particles need to combine the argument from quantum statistics with an independent defense of the permutability of quantum particles. However, there is an objection van Fraassen has raised against Tersoff and Bayer. Teller and Redhead have claimed that this objection undermines Belousek's argument (Teller and Redhead 2000).

Van Fraassen claims that Tersoff and Bayer's derivation only holds in the special case of maximal ignorance. He asks to us to suppose that we prepare a pure state of the form (2.5). When dealing with such states we get deviations from classical probabilities because we have interference terms. Van Fraassen asserts that "[i]f at this point someone were to suggest that we could continue to look for a classical model . . . he would run into the usual obstacle for all hidden-variable interpretations. For we would ask him to let the interpretation cover various divisions into cells, to correspond to non-commuting observables. The 'no hidden variable' theorems would show that this could not be done, with any single probability function" (van Fraassen 1991, 418). Teller and Redhead rephrase van Fraassen's claim as follows: "to assume that bosonic statistics could be reproduced for families of non-commuting operators by assuming a distribution of exact values given by priors, uniform or otherwise, assumes precisely the exact values which set

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[15]See, for example, (Frigg and Werndl 2013), (Emch and Liu 2002), (Earman and Rédei 1996), and (Sklar 1993) for some back-and-forth on whether realistic statistical mechanical systems are ergodic.

into motion both the algebraic and Bell inequality no-go theorems" (Teller and Redhead 2000, 954).

I think that Teller and Redhead have misapplied van Fraassen's reasoning. The abstract of Tersoff and Bayer's paper began by stating "[q]uantum statistics can be reconciled with such classical ideas as distinguishable particles" (Tersoff and Bayer 1983, 553). Van Fraassen's objection is that Tersoff and Bayer's result "does not amount to a reconciliation of quantum statistics with classical concepts" (van Fraassen 1991, 414). Although Tersoff and Bayer are only focused on showing that statistical distinguishability is compatible with quantum statistics, they do appear to be making a grander claim about the (alleged) similarity between classical and quantum statistics. It seems to be the grander claim van Fraassen is responding to. Otherwise van Fraassen's argument would be question begging. Tersoff and Bayer explicitly note in their paper that their work could provide the foundation for a statistical interpretation of quantum mechanics in the form of a stochastic hidden variable theory and state that "[s]ince there is no reason why such a theory should be factorizable, it need not have the undesirable features of deterministic hidden-variable theories (such as satisfying the Bell inequalities)" (Tersoff and Bayer 1983, 554). But note that van Fraassen's criticism depends on an assumption factorizability (divisions into cells). But also note that van Fraassen is criticizing the idea of finding a classical model that yields BE rather than criticizing the more specific claim that BE is compatible with statistical distinguishability. Furthermore, the section of van Fraassen's book in which he discusses Tersoff and Bayer is concerned with the question of whether a classical reconstruction of quantum statistics is possible. If what van Fraassen is objecting to is the claim that Tersoff and Bayer have provided a classical reconstruction of quantum statistics, then van Fraassen's argument seems to be correct and is not question begging.

The quotation from Teller and Redhead seems to provide an accurate characterization of van Fraassen's position. However, Teller and Redhead's mistake is that they take van

Fraassen's comments as undermining Belousek's argument by undermining the specific claim that BE is compatible with statistical distinguishability. Recall that what Tersoff and Bayer are doing is assigning not one probability distribution, but all possible arbitrary probability distributions before averaging over them to get BE. Assuming an exact distribution given by priors is exactly what the no-go theorems state that one can't do with a *single probability function*. They do not rule out the possibility of averaging over a family of probability functions to get correct results, which is exactly what Tersoff and Bayer are doing (hence their claim that their work could provide the foundation for a *statistical* interpretation of quantum mechanics).

2.2.3   Holistic Properties and Individuality

Morganti argues that the only properties that are statistically relevant for many particle systems are inherent properties, which are insensitive to permutations of particles because particle identity plays no role in the determination of these properties (Morganti 2009). Consequently, the explanation of the permutability of particles is to be found in the kind of properties with which quantum statistics is concerned rather than in a lack of individuality of particles.

Morganti defines an inherent property as a property possessing three features. An inherent property is a property (i) of a complex whole (a whole made up of multiple simpler parts), (ii) that is not reducible to separate intrinsic properties of the parts. Inherent properties are then holistic properties of complex systems that do not supervene on the intrinsic properties of the parts. The third feature is that if P is an inherent property of a whole composed of parts a and b, then (iii) "it is not necessary for P to contain information about a and b only if it also contains 'non-trivial' specific information about a and b separately" (Morganti 2009, 227). This last feature calls for some explanation.

To explain the third condition Morganti asks us to consider a situation in which a pair of fair coins will be tossed, and a powerful demon will make the coins land on the same side, making a decision as to which side just after the toss. Note that the demon's decision is about the pair of coins and not about the individual coins. Morganti says we can attribute a disposition for the system to evolve in such a way that the coins will land on the same side. This disposition is an inherent property. The property says something about the coins, but the only specific information about each coin (that coin 1 will land on the same side as coin 2 and vice versa) trivially follows from a fact about the whole (that the pair will be made to land on the same side). Our inherent property contains no specific information about the individual coins except that implied by facts about the whole. So, the third feature says that some inherent properties contain no specific information about the individual parts of the system except for anything implied by facts about the whole. Morganti says that this is the key to understanding the difference between classical and quantum statistics. The information "missing" from these kinds of inherent properties is the sort of information about the specific parts of the system that is always available, at least in principle, in a classical setting. For example, facts about the coins to be flipped by means of which one could, in principle, use classical mechanics to calculate how they will land.

Consider a pair of particles in the state

$$\frac{1}{\sqrt{2}} \left(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle \right). \tag{2.12}$$

This is analogous to the coin example in that a measurement of spin will find both particles to be spin-up or both particles to be spin-down. On Morganti's view, the system has an inherent property in the form of a disposition for the system to evolve in such a way that the particles are found to have matching spin if a spin measurement is made. However, there is no specific information about the particles beyond that which follows

from facts about the overall state of the system. More specifically, the individual particles do not have spin at all prior to measurement. The permutability of the particles is not a result of a lack of individuality but because "*the particles' identities do not play any role in the determination of the states*" (Morganti 2009, 228). Which particle has which spin is irrelevant, not because there is no fact of the matter as to which particle is which but because there is no fact of the matter as to the spin of the particles until a measurement is made. There is a fact of the matter as to which particle is which, it simply isn't physically salient to the statistical behavior of the particles. This is similar to my proposed explanation of the Gibbs paradox. We can say that particles are permutable because they lack individuality (as in the argument from quantum statistics) or we can say that they are permutable because their individuality isn't physically relevant to their statistical behavior (as Morganti does).[16]

While some may object to Morganti's characterization of inherent properties preferring, perhaps, another definition, this is a relatively standard way to think of entangled systems. Where Morganti's position is unique, and where it provides a response to the argument of quantum statistics, is in the fact that Morganti takes the same view of non-entangled states. States, for example, described by (2.1) and (2.2) are, on Morganti's view, also states in which there is an inherent property attached to the whole system and the individual particles are lacking definitive properties of their own. In state (2.1) the particles do not both have property *a* prior to measurement although they are guaranteed to both have property *a* post measurement. It is only if all quantum states have inherent properties that Morganti's explanation of the permutability of particles can work. Particles are permutable because the question of which particle has which property is never physically relevant. Not because the particles aren't individuals but because the properties in question aren't present until a measurement is made. However, Morganti's only defense

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[16]The difference between these two options is purely theoretical.

of his extension of holism to non-entangled systems is to say that "once holism has been acknowledged to hold for some quantum systems, one seems justified in accepting it also for other quantum systems insofar as doing so solves existing conceptual problems" (Morganti 2009, 227).

I agree with Morganti here. We have a conceptual problem, how to explain the difference between classical and quantum statistics, and we have a number of explanations on offer. In each case the justification for accepting the explanation is just that the explanation succeeds at its goal. There are negative arguments against some of these explanations, that is, arguments that purport to demonstrate a problem with the explanation. However, the positive argument, if we can even call it that, for accepting any of the explanations is simply that they do the explanatory work they are intended to do. We can, and ought, to take the explanations as equally good to begin with and then adjust our view by looking at the strength of negative arguments against each position. Naively following this advice, however, would lead one to conclude that Morganti's position is preferable simply due to its recent appearance in the literature. I don't intend, or need to, provide an account of how alternative explanations ought to be assessed in this kind of situation. I only want to stress that even though Morganti has provided no negative argument against the argument from quantum statistics, we currently have no compelling reason to favour one view over the other. It is perfectly reasonable to accept Morganti's view given the explanations on offer, and even more so if one finds Saunders' or Belousek's arguments that permutability isn't a difference between classical and quantum particles compelling (since permutability is where the argument from quantum statistics locates the difference).

2.3 Weak Discernibility

The primary case for the weak discernibility of particles is made in a series of four papers by Saunders, Muller, and Seevinck. Two objects are weakly discernible if they stand in a symmetric non-reflexive relation to one another.[17] For example, in the earliest two papers (Saunders 2003) and (Saunders 2006a), Saunders claimed that a pair of fermions in the singlet state are weakly discernible because they have opposite spin. The relation of *having opposite spin* is a two-place non-reflexive relation; a particle cannot have opposite spin to itself, but it can have opposite spin to a second particle. A major criticism of Saunders claim was that fermions in the singlet state are generally not considered to have spin values at all and so the relation of *having opposite spin* cannot be used to claim that fermions are weakly discernible.[18] In response (Muller and Saunders 2008) shows how fermions can be weakly discerned in any state using only non-probabilistic (or categorical) properties. The final paper, (Muller and Seevinck 2009) shows how any particles can be weakly discerned in any state using only categorical properties. I will limit my discussion to the latter two papers.

Muller and Saunders begin by demonstrating how a pair of fermions is weakly discernible before extending this result to any number of fermions. First, consider an arbitrary physically meaningful operator A with eigenbasis $|\phi_1\rangle, |\phi_2\rangle, \ldots, |\phi_d\rangle \in \mathcal{H}$. We can then define a complete set of one-dimensional projectors P_m that project the state they act on onto the ray containing $|\phi_m\rangle$. Now let $P_{lm} \equiv P_l - P_m$, $P_{lm}^{(1)} \equiv P_{lm} \otimes I$, and $P_{lm}^{(2)} \equiv I \otimes P_{lm}$,

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[17]A relation is non-reflexive if the relation cannot apply to a single object. A relation between two objects, a and b is symmetric if $a's$ standing in that relation to b implies that b also stands in that relation to a. The relation of being *opposite spin to* is a symmetric non-reflexive relation.

[18]Examples of this criticism can be found in (Dieks and Versteegh 2008) and (Morganti 2008).

where I is the identity operator. Finally, we can define the relation

$$R_t(a, b) \quad \text{iff} \quad \sum_{l,m=1}^{d} P_{lm}^{(a)} P_{lm}^{(b)} |\psi\rangle = t|\psi\rangle \tag{2.13}$$

where $t \in \mathbb{R}$. The relation $R_t(a, b)$ holds between objects a and b iff the system is in an eigenstate of the operator $\sum_{l,m=1}^{d} P_{lm}^{(a)} P_{lm}^{(b)}$ with eigenvalue t.

The relation is parameterized by t and may be reflexive or non-reflexive for different values of t. In order to use this relation to weakly discern fermions it must be the case that the eigenvalue when $a \neq b$ is distinct from the eigenvalue when $a = b$. First, consider the case where $a = b$. We have

$$\sum_{l,m=1}^{d} P_{lm}^{(1)} P_{lm}^{(1)} = \sum_{l,m=1}^{d} P_{lm}^{(2)} P_{lm}^{(2)} = \sum_{l,m=1}^{d} (P_{lm} \otimes I)^2 = \sum_{l,m=1}^{d} (P_{lm})^2 \otimes I$$

$$= \sum_{l,m=1}^{d} \left(P_l^2 - P_l P_m - P_m P_l + P_m^2 \right) \otimes I = \sum_{l,m=1}^{d} (2I - 2I\delta_{lm}) \otimes I \tag{2.14}$$

$$= (2dI - 2I) \otimes I = 2(d-1)I \otimes I,$$

so that

$$\sum_{l,m=1}^{d} P_{lm}^{(1)} P_{lm}^{(1)} |\psi\rangle = \sum_{l,m=1}^{d} P_{lm}^{(2)} P_{lm}^{(2)} |\psi\rangle = 2(d-1)|\psi\rangle, \tag{2.15}$$

where d is the dimension of the state space. For $a \neq b$ we have

$$\sum_{l,m=1}^{d} P_{lm}^{(2)} P_{lm}^{(1)} = \sum_{l,m=1}^{d} P_{lm}^{(1)} P_{lm}^{(2)} = \sum_{l,m=1}^{d} (P_{lm} \otimes I)(I \otimes P_{lm}) = \sum_{l,m=1}^{d} P_{lm} \otimes P_{lm}$$

$$= \sum_{l,m=1}^{d} (P_l - P_m) \otimes (P_l - P_m) = \sum_{l,m=1}^{d} ((P_l - P_m) \otimes P_l - (P_l - P_m) \otimes P_m)$$

$$= \sum_{l,m=1}^{d} (P_l \otimes P_l - P_m \otimes P_l - P_l \otimes P_m + P_m \otimes P_m) \tag{2.16}$$

$$= \sum_{l,m=1}^{d} (P_l \otimes P_l + P_m \otimes P_m - 2\delta_{lm}I \otimes I)$$

$$= \sum_{l=1}^{d} 2P_l \otimes P_l - 2I \otimes I = 2\left(\sum_{l=1}^{d} P_l \otimes P_l - I \otimes I\right).$$

For an antisymmetric state we find that

$$\sum_{l,m=1}^{d} P_{lm}^{(1)} P_{lm}^{(2)} |\psi_-\rangle = 2 \sum_{l=1}^{d} P_l \otimes P_l |\psi_-\rangle - 2I \otimes I |\psi_-\rangle = -2 |\psi_-\rangle. \qquad (2.17)$$

Since d is a positive number, the eigenvalues in the two cases are distinct. Consequently, the relation $R_{-2}(a, b)$ weakly discerns a pair of fermions in any state because the relation is non-reflexive, symmetric, and applies to any pair of fermions. The result does not, however, extend to bosons, since $2\sum_{l=1}^{d} P_l \otimes P_l$ does not act on a symmetric state as a multiple of the identity. Consequently $R_t(a, b)$, for some t, cannot serve as a weakly discerning relation for any arbitrary pair of bosons since the eigenvalue in the case where $a \neq b$ depends on the particular state. Note that Muller and Saunders' result for two particles generalizes to any number of particles with the redefinition $P_{lm}^{(n)} = I \otimes \ldots \otimes P_{lm} \otimes \ldots \otimes I$, where n indicates the nth place in the n-fold tensor product.

Muller and Saunders also show that bosons can be weakly discerned by non-categorical properties. However, Muller and Seevinck have demonstrated that fermions, bosons, and even paraparticles, can be weakly discerned by categorical properties. Consequently, I will now turn to their argument. Muller and Seevinck provide two distinct arguments. One that applies to all particles and one that applies to only particles with non-zero spin. Both arguments take the same approach as that of Muller and Saunders.

The argument that all particles are weakly discernible only involves position, X, and momentum, P, operators. They define operators $P^{(1)} = P \otimes I$, $P^{(2)} = I \otimes P$, $Q^{(1)} = Q \otimes I$,

and $Q^{(2)} = I \otimes Q$ with which they define the relation

$$C(a, b) \quad \text{iff} \quad \left[P^{(a)}, Q^{(b)} \right] |\psi\rangle = c \, |\psi\rangle \tag{2.18}$$

for some non-zero c. The eigenvalues are easy to compute using the standard commutation relations between position and momentum. When $a = b$ we find that $\left[P^{(1)}, Q^{(1)} \right] |\psi\rangle = \left[P^{(2)}, Q^{(2)} \right] |\psi\rangle = -i\hbar \, |\psi\rangle$. When $a \neq b$ the eigenvalue is zero since the momentum operator of one particle commutes with the position operator of another; $\left[P^{(1)}, Q^{(2)} \right] |\psi\rangle = \left[P^{(2)}, Q^{(1)} \right] |\psi\rangle = 0$. It is clear that $C(a, b)$ is a symmetric reflexive relation that does not hold between distinct particles in any state. On the other hand, the negation of $C(a, b)$ is a symmetric non-reflexive relation that applies to any pair of particles in any state. Consequently, the relation $\neg C(a, b)$ serves to weakly discern any particles.

The second argument demonstrates that all particles with non-zero spin can be weakly discerned by a relation constructed out of spin operators. First define operators $S^{(1)} = S \otimes I$ and $S^{(2)} = I \otimes S$ and the relation

$$T(a, b) \quad \text{iff} \quad \left| S^{(a)} + S^{(b)} \right|^2 |\psi\rangle = 4s(s + 1)\hbar^2 \, |\psi\rangle . \tag{2.19}$$

When $a = b$ we find that

$$\left| S^{(a)} + S^{(a)} \right|^2 |\psi\rangle = 4 \, |S|^2 \, |\psi\rangle = 4s(s + 1)\hbar^2 \, |\psi\rangle . \tag{2.20}$$

Whereas when $a \neq b$ we find that

$$\left| S^{(a)} + S^{(b)} \right|^2 |\psi\rangle = \left(S^{(a)^2} + S^{(a)} S^{(b)} + S^{(b)} S^{(a)} + S^{(b)^2} \right) |\psi\rangle$$

$$= \left(2s(s + 1)\hbar^2 + 2(S \otimes S) \right) |\psi\rangle \leq \left(2s(s + 1)\hbar^2 + 2s\hbar^2 \right) |\psi\rangle \tag{2.21}$$

$$= 2s(2s + 1)\hbar^2 \, |\psi\rangle < 4s(s + 1)\hbar^2 \, |\psi\rangle \, .$$

Similarly with $C(a, b)$, $T(a, b)$ is a symmetric reflexive relation that fails to hold between distinct particles with non-zero spin in any state. Its negation, however, is a symmetric non-reflexive relation that holds between any pair of particles with non-zero spin in any state. Consequently, the relation $\neg T(a, b)$ weakly discerns any particles with non-zero spin.

Before moving on to some alternative proposals for weakly discerning particles there is one important thing to mention noted by Bigaj (Bigaj 2015, 46). In the case of relations $R_t(a, b)$, $C(a, b)$, and $\neg C(a, b)$ every state is an eigenstate and the discernment relies purely on the difference in eigenvalues depending on whether $a = b$ or not. As a result, the arguments involving these relations only rely on assuming the less controversial part of the eigenvector-eigenvalue link (that a system in an eigenstate of an operator possesses the associated eigenvalue). On the other hand, systems that satisfy $\neg T(a, b)$ are systems that are not in eigenstates of the operator $\left| S^{(a)} + S^{(b)} \right|^2$. Consequently, an additional assumption needed in this case in order to claim that the eigenvalues are different is that a system that is not in the eigenstate of a given operator does not possess the corresponding eigenvalue. In other words, using $\neg T(a, b)$ to weakly discern particles requires assuming that being in an eigenstate of an operator is a *necessary* condition for having the corresponding eigenvalue rather than merely being a *sufficient* condition.

2.3.1   Physically Inadmissible Relations

Two alternatives to the Muller-Saunders-Seevinck approach have been proposed; one by Caulton and the other by Huggett and (Joshua) Norton. The motivation in each case is that the weakly discerning relations defined by Muller, Saunders, and Seevinck are thought to

be physically inadmissible. To understand the criticisms, I will first explain some conditions Muller and Saunders put on *physically admissible* relations.

The first condition is that of *Physical meaning*: "All properties and relations should be transparently defined in terms of physical states and operators that correspond to physical magnitudes" (Muller and Saunders 2008, 527-528). Muller and Saunders point out that predicates that express membership or non-membership in a set and predicates that express names or labels are not physically meaningful. However, they say nothing else about what might count as a physically meaningful operator. It therefore appears that any Hermitian operator is acceptable.

The second is *Permutation invariance*: "Any property of one particle is a property of any other; relations should be permutation-invariant, so *binary* relations are symmetric *and* either reflexive or irreflexive" (Muller and Saunders 2008, 528). This condition only requires that any candidate for a weakly discerning relation be permutation invariant. However, Caulton, Huggett, and Norton think this requirement is too weak. They prefer the stronger requirement that both the weakly discerning relation and the operators out of which it is constructed are permutation invariant (Caulton 2013, Huggett and Norton 2014).

The first criticism of the physical admissibility of the weakly discerning relations proposed by Muller, Saunders and Seevinck is that $R_t(a, b)$ and $C(a, b)$ are not permutation invariant. I will begin with Huggett and Norton's criticism of $R_t(a, b)$. The crux of their criticism is that $\sum_{l,m=1}^{d} P_{lm}^{(a)} P_{lm}^{(b)}$ is not permutation invariant if the number of particles is greater than two. This makes no difference in the case of fermions since, for n fermions, $\sum_{l,m=1}^{d} P_{lm}^{(a)} P_{lm}^{(b)} |\psi_-\rangle = -2I^{\otimes n} |\psi_-\rangle = -2 |\psi_-\rangle$ (see (2.17)). The problem is that space of bosonic states, $\mathcal{H}_+$, is not closed under the action of the 'operator' (so it's not an operator on $\mathcal{H}_+$ at all). Huggett and Norton demonstrate this (Huggett and Norton 2014,

45) by showing that

$$\sum_{l,m=1}^{d} P_{lm}^{(1)} \otimes P_{lm}^{(2)} \otimes I \cdot \frac{1}{\sqrt{3}} \left[|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle \right]$$

$$= \frac{1}{\sqrt{3}} \left[2(d-1) |\uparrow\uparrow\downarrow\rangle - 2 \left(|\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle \right) \right] \notin \mathcal{H}_+.$$

(2.22)

Performing a measurement of the physical quantity associated with $\sum_{l,m=1}^{d} P_{lm}^{(a)} P_{lm}^{(b)}$ on a system of bosons has a non-zero probability of producing non-symmetric bosons.

There are two options for responding to this criticism. One, very implausible, option would be to say that standard measurement theory doesn't apply to $\sum_{l,m=1}^{d} P_{lm}^{(a)} P_{lm}^{(b)}$ in order to avoid finding a non-zero probability of producing non-symmetric bosons (Huggett and Norton 2014, 45-46). The second option is to reject the standard practice of requiring any physically meaningful operator to be defined on both $\mathcal{H}_+$ and $\mathcal{H}_-$. However, I'm not sure how to make sense of a physical quantity that's only defined for fermions. Even spinless particles are eigenstates of the total spin operator (with eigenvalue zero). Similarly, if there are any physical quantities that are only possessed by fermions, I would expect bosonic states to be eigenstates of the relevant operator with eigenvalue zero. If an operator is only defined on $\mathcal{H}_+$ then it implies neither that bosons have nor lack the property in question. It would imply something else and it's not clear what that something else might be.

Caulton's criticism is similar, however, he takes aim at the building blocks of $R_t(a, b)$ and $C(a, b)$. The criticism is very straightforward. It is simply that the physical admissibility of $\sum_{l,m=1}^{d} P_{lm}^{(a)} P_{lm}^{(b)}$ and $\left[P^{(a)}, Q^{(b)} \right]$ is supposed to be grounded in the physical meaning of $P_{lm}^{(a)}$, $P^{(a)}$, and $Q^{(a)}$ which are themselves not permutation invariant (Caulton 2013, 55-56). Caulton is perfectly fine with physically admissible operators being constructed out of components that are not permutation invariant. The problem is that in such cases the physical admissibility of the operator must be demonstrated directly since it can't 'inherit' physical admissibility from its physically inadmissible components. Consequently,

Caulton's criticism is not that Muller and Saunders' relations are physically inadmissible but just that they haven't succeeded in showing them to be physically admissible.

The second criticism, made by Caulton, Huggett, and Norton, is that $R_t(a, b)$ and $C(a, b)$ are physically inadmissible because they are trivial. $\sum_{l,m=1}^{d} P_{lm}^{(a)} P_{lm}^{(b)}$ reduces to multiples of the identity when applied to non-symmetric states (see 2.17) whereas $\left[P^{(a)}, Q^{(b)} \right]$ reduces to multiples of the identity regardless of the state to which it is applied (see 2.14). $R_{-2}(a, b)$ can be restated, based on the equations just referenced, as (Caulton 2013, 56; Huggett and Norton 2014, 47)

$$R_{-2}(a, b) \quad \text{iff} \quad (a = b \text{ and } 2(d-1)I \left| \psi \right\rangle = -2 \left| \psi \right\rangle) \text{ or } (a \neq b \text{ and } -2I \left| \psi \right\rangle = -2 \left| \psi \right\rangle) \quad (2.23)$$

which can be further simplified to (Caulton 2013, 56)

$$R_{-2}(a, b) \quad \text{iff} \quad (a = b \text{ and } -2 = 2(d - 1)) \text{ or } (a \neq b \text{ and } -2 = -2). \quad (2.24)$$

This second form makes it clear that $R_{-2}(a, b)$ is not a physical relation. The rhs contains no references to physical properties or physical states. Caulton levels the same triviality charge at C(a,b) which (Bigaj 2015, 47) shows can be rewritten as

$$C(a, b) \quad \text{iff} \quad (a = b \text{ and } -i\hbar \neq 0) \text{ or } (a \neq b \text{ and } 0 \neq 0). \quad (2.25)$$

Again, the rhs contains no references to physical properties or physical states.

2.3.2 Alternative Weakly Discerning Relations

I will begin with Caulton's proposed alternative as he takes the same approach as Muller, Saunders, and Seevinck, merely choosing a different operator to define the weakly discerning relation. Caulton defines the variance of an operator A as

$$\Delta_A^2 = \frac{1}{4}(A \otimes I - I \otimes A)^2 \tag{2.26}$$

and the statistical mean of A as

$$\bar{A} := \frac{1}{2}(A \otimes I + I \otimes A). \tag{2.27}$$

Note that these are both operators and they are both permutation invariant. Finally, the statistical mean of A^2 is

$$\overline{A^2} = \frac{1}{2}(A^2 \otimes I + I \otimes A^2). \tag{2.28}$$

Caulton shows that the variance operator can be rewritten as

$$\Delta_A^2 = \frac{1}{4}(A \otimes I - I \otimes A)^2 = \frac{1}{4}(A^2 \otimes I + I \otimes A^2 - 2A \otimes A)$$

$$= \frac{1}{2}(A^2 \otimes I + I \otimes A^2) - \frac{1}{4}(A^2 \otimes I - 2A \otimes A + I \otimes A^2) \tag{2.29}$$

$$= \overline{A^2} - \bar{A}^2,$$

justifying the term "variance." The variance operator can also be rewritten as

$$\Delta_A^2 = \frac{1}{4}(A^2 \otimes I + I \otimes A^2 - 2A \otimes A) = \frac{1}{2}(\overline{A^2} - A \otimes A), \tag{2.30}$$

which helps us make physical sense of the variance operator. It measures the anticorrelation between the particles' A eigenstates (Caulton 2013, 62).

Caulton, following Muller, Saunders, and Seevinck, defines the operators $A^{(1)} = A \otimes I$ and $A^{(2)} = I \otimes A$ and the relation

$$R'(A, x, y) \quad \text{iff} \quad \frac{1}{4}\left(A^{(x)} - A^{(y)}\right)^2 |\psi\rangle \neq 0. \tag{2.31}$$

It may initially seem like an odd choice of operator for weakly discerning particles since, for some operator A, bosons may be perfectly correlated. However, Caulton claims the solution is just to change to a basis in which anticorrelations are present and use "the quantity associated with this new basis" (Caulton 2013, 61). A basis in which some variance operator or other (associated with some observable or other) has a non-zero eigenvalue is only unavailable in the case of a single particle (given Caulton's definition of variance). Consequently, some variance operator or other takes different eigenvalues for single- (zero) or multi-particle states (non-zero) just as those proposed by Muller, Saunders, and Seevinck. $R'(A, x, y)$ serves to weakly discern particles and the (permutation invariant) variance operator has clear physical meaning as a measure of anticorrelations between eigenstates.

While the variance operator has clear physical meaning, it's not clear that $R'(A, x, y)$ has physical meaning. Bigaj points out that $R'(A, x, y)$ can be rewritten as (Bigaj 2015, 49)

$$R'(A,\ x,\ y) \quad \text{iff} \quad (x \neq y \text{ and } \Delta_A^2 \ket{\psi} \neq 0) \text{ or } (x = y \text{ and } 0 \ket{\psi} = 0). \tag{2.32}$$

Bigaj's complaint is that the "physical" conditions $0 \ket{\psi} = 0$ and $\Delta_A^2 \ket{\psi} \neq 0$ don't represent the same physical property of the system. The former condition doesn't appear to be associated with a physical property at all let alone the same physical property as that associated with the latter condition. Furthermore, Bigaj points out that when we want to measure variance for a single particle, we use the standardly defined variance operator $(\Delta A)^2 = (A - \langle A \rangle)^2$. Not only is this not equivalent to the zero operator, but $(A - \langle A \rangle)^2 \ket{\psi}$ is not equal to zero for even single particle states if the particle is not in an eigenstate of A. So while it may make sense to understand $\Delta_A^2 \ket{\psi} \neq 0$ as saying that the variance for A is not equal to zero, it doesn't look like $0 \ket{\psi} = 0$ is a claim about variance at all. Consequently, it looks like Caulton's relation $R'(A,\ x,\ y)$ doesn't represent a single physical relation. Instead, each disjunct is concerned with a different relation. The first is variance but the second is something else. Ironically, the second disjunct is trivial and this, by Caulton's own lights, calls in to question whether it's about a physical relation at all.

Bigaj's final criticism is that Caulton appears to be using two different conceptions of "variance." Consider a two boson state $\ket{\phi} \ket{\phi}$ that is perfectly correlated. The only reason it's possible to find an operator A such that $\Delta_A^2 \ket{\phi} \ket{\phi} \neq 0$ is that (i) $\ket{\phi} \ket{\phi}$ is not an eigenstate of all observables and (ii) any state that is not an eigenstate of A has non-zero variance for A. Since $\ket{\phi}$ also will not be an eigenstate of all observables, it should have non-zero variance for some observables. However, the condition $0 \ket{\psi} = 0$ means that all states have zero variance, including those that are not eigenstates of A. We have a conception of variance in the $x \neq y$ case according to which states that are not eigenstates of A have non-zero variance for A. We have a second conception of variance in the $x = y$ case according to which eigenstates of A have zero variance for A. Applying the second

conception consistently would mean that all states have zero variance whereas applying the first conception consistently would mean that $R'(A, x, y)$ is neither reflexive nor non-reflexive. Either way, $R'(A, x, y)$ doesn't weakly discern particles. With the first conception of variance the relation isn't weakly discerning because it isn't non-reflexive. With the second conception of variance the relation isn't weakly discerning because the result of applying the operator to any state is zero; so, there's no distinction between states in which $x = y$ and states in which $x \neq y$.

Huggett and Norton's proposal is also very similar to that of Muller, Saunders, and Seevinck, with the major difference being their insistence that the operator in the weakly discerning relation be permutation invariant. Huggett and Norton use symmetrized operators that are similar in form to symmetrized versions of $P_{lm}^{(n)} = I \otimes \ldots \otimes P_{lm} \otimes \ldots \otimes I$. The symmetric operators will not contain any reference to specific particles in which case we have to consider two cases $a = b$ and $a \neq b$ separately. When $a = b$ the symmetrized operator is

$$\frac{1}{n} \sum_{p \in \mathbb{P}} (AB \otimes I \otimes \cdots \otimes I), \tag{2.33}$$

where the sum is over all the permutations of the n particles, the division by n is to avoid over-counting, and A and B are operators representing observables. When $a \neq b$ we have instead

$$\frac{1}{2 \ {}^{n}C_2} \sum_{p \in \mathbb{P}} (A \otimes B \otimes I \otimes \cdots \otimes I), \tag{2.34}$$

where ${}^{n}C_2$ is the binomial coefficient $\binom{n}{2}$. They then define the relation

$$\mathcal{R}_t(a, b) \quad \text{iff} \quad \left(a = b \text{ and } \frac{1}{n} \sum_{p \in \mathbb{P}} (AB \otimes I \otimes \cdots \otimes I) \, |\psi\rangle = t \, |\psi\rangle \right)$$

$$\text{or } \left(a \neq b \text{ and } \frac{1}{2 \, {}^n C_2} \sum_{p \in \mathbb{P}} (A \otimes B \otimes I \otimes \cdots \otimes I) \, |\psi\rangle = t \, |\psi\rangle \right). \tag{2.35}$$

Since we want the eigenvalues to be different in the two cases and we want the weakly discerning relation to be non-reflexive, this relation weakly discerns when t is such that the first disjunct is false (so non-reflexive) and the second disjunct is true.

Huggett and Norton's approach has a severe drawback compared to the other approaches. As Huggett and Norton acknowledge, the scope of their result is very limited. They have not shown that there is a value t such that $\mathcal{R}_t(a, b)$ weakly discerns all fermionic states. In particular, they have only shown that there exists such a t for fermionic states of the form

$$|\psi\rangle = \frac{1}{\sqrt{n!}} \sum_{p \in \mathbb{P}} c^S \, |a_1\rangle |a_2\rangle \cdots |a_n\rangle \qquad (\langle a_i | a_j \rangle = \delta_{ij}), \tag{2.36}$$

which does not include, for example, the two fermion state (Huggett and Norton 2014, 53)

$$|\psi\rangle = \frac{1}{2} (|a_1\rangle |a_2\rangle - |a_2\rangle |a_1\rangle + |a_3\rangle |a_4\rangle - |a_4\rangle |a_3\rangle). \tag{2.37}$$

Furthermore, there exists no t such that $\mathcal{R}_t(a, b)$ weakly discerns all bosonic states. Huggett and Norton show that $\mathcal{R}_t(a, b)$ can't weakly discern bosons that are all in the same state, that is, bosonic states of the form $|\psi\rangle = \otimes^n |\phi\rangle$.

2.3.3 Weak Discernibility and Individuality

Discernibility is often taken as a sufficient condition for individuality. However, the sort of discernibility people tend to have in mind is absolute discernibility. Two objects are *absolutely discernible* when they differ in their intrinsic properties. When objects differ in their intrinsic properties it is always possible to use those properties to individuate them. When objects are merely weakly discernible, it is not possible to individuate them purely on the basis of their properties. Weakly discerning relations are symmetric and so objects that are merely weakly discernible have all the same properties (that is, properties that make no explicit reference to identity). The idea that a weakly discerning relation could provide grounds for individuality at all is based on the fact that a weakly discerning relation can only hold between numerically distinct objects. In other words, numerical distinctness is a necessary condition for a weakly discerning relation to obtain. So, the question is whether the existence of a relation between particles that requires numerical distinctness to obtain implies that the particles are individuals.

There are no positive arguments in the literature for the idea that objects that are weakly discernible are individuals. Indeed, most of those involved in the debate over the weak discernibility of particles agree that weak discernibility does not ground individuality. However, there are two major arguments to the effect that weak discernibility cannot ground individuality. I will look at these arguments first, before providing my own reasons in favor of accepting weak discernibility as grounds for individuality.

One complaint is that there is a circularity in claiming that weak discernibility grounds individuality (Hawley 2006, Hawley 2009, French and Krause 2006). Following (Hawley 2009) the circularity charge is as follows. First, weak discernibility implies that weakly discerned objects *a* and *b* have the distinct non-qualitative properties *bears relation R to b* and *bears relation R to a*, respectively. These properties are non-qualitative because they

reference identity in virtue of the fact that they include object labels. Possessing these different relational properties is then what grounds the claim of the numerical distinctness of *a* and *b*. But, in order to say that *a* has the property *bears relation R to b* presupposes that *a* and *b* are numerically distinct. The complaint, then, is that the numerical distinctness required to conclude that a weakly discerning relation obtains in the first place must be put in by hand. It is important to note that this is an epistemic worry. How are we to actually demonstrate that a weakly discerning relation obtains without assuming numerical distinctness?

There are two problems with the circularity criticism that I will address, the latter of which applies only to the weak discernibility of particles. The first problem, noted in (Muller 2015), is that the circularity objection misunderstands the way in which weakly discerning relations are meant to discern particles. The reason weakly discerning relations discern is entirely because the relations are non-reflexive. The reason objects *a* and *b* are weakly discernible isn't because they have distinct non-qualitative properties but because they stand in a relation that no object can bear to itself. Allowing non-qualitative properties is problematic in that it collapses the distinction between different kinds of discernibility. For instance, if we allow Hawley's relations *bears relation R to a* and *bears relation R to b*, then weak discernibility collapses to extrinsic absolute discernibility; that is, discernibility by a specific relation to a specific object that is also extrinsically absolutely discernible (Muller 2015, 215). This doesn't strike me as a suitable response. Even if Hawley has misrepresented how weakly discerning relations actually discern objects, it still seems to be the case that we need to know whether or not we have numerical distinctness in order to say whether a weakly discerning relation applies or not.

The second issue with the circularity criticism in the context of the weak discernibility of particles is that in NRQM there is never any question as to how many particles are in a given state. Consequently, claims about the weak discernibility of particles in NRQM are

always claims about whether or not *a known number of particles* obeys a weakly discerning relation. So, we can accept Hawley's claim that we need to know whether we have numerical distinctness in order to apply a weakly discerning relation and still avoid the circularity objection because we have a means to determine particle number.

A second circularity criticism comes from (Bigaj 2015). Bigaj claims that the proposals for weakly discerning relations discussed above depend on assuming numerical distinctness in order to determine the correct form of the operator. The operators are all either indexed by particle labels or, in the case of Huggett and Norton, the operators for when we have either one or more than one particle are explicitly different. Consequently, we *must* know whether we have one or more than one particle before applying any of the proposed weakly discerning relations for particles. However, Bigaj's concern is that showing that a weakly discerning relation applies when we already know the objects in question are numerically distinct doesn't appear to have any interesting metaphysical implications.

I find Bigaj's claim somewhat puzzling. The assumption seems to be that those who want to connect weak discernibility to individuality want to do it via numerical distinctness by endorsing the following claims: (i) If objects *a* and *b* are weakly discernible then they are numerically distinct, and (ii) if objects *a* and *b* are numerically distinct then they are individuals. However, for reasons discussed above, we do not need weak discernibility to determine whether we have one or more than one particle in a state in NRQM. Anyone that wants to endorse (ii) can start there directly without worrying about whether particles are weakly discernible.

The second major criticism of a connection between weak discernibility and individuality is that weak discernibility "is a condition which entails that there is more than one entity of a given kind, without implying that there are specifiable differences that distinguish them" (van Fraassen and Peschard 2008, p. 19). Ladyman, Bigaj, and Arenhart all raise

this criticism as well (Ladyman and Bigaj 2010, Arenhart 2013a). The majority of the discussion is concerned with what kind of connection, if any, weak discernibility has to the Principle of the Identity of Indiscernibles (PII). More specifically, the concern is whether "weakly discernible" objects count as discernible in the sense relevant for PII. We can separate the issue from PII by simply asking whether weak discernibility is a *physically* interesting kind of discernibility. The above authors all answer this question in the negative. Ladyman and Bigaj, for example, require that any physically interesting discernibility relation allow for a physical procedure that would distinguish the objects in question. But objects that are merely weakly discernible do not differ in their physical properties (monadic or relational).

At this point I should probably explain why I'm avoiding talking about PII since this avoidance means I can't go into detail regarding Ladyman, Bigaj, and Arenhart's criticisms of the connection between weak discernibility and individuality. I have avoided talking about PII up to this point largely because I think the issue of whether some version of PII holds for particles is a much more complicated issue than whether particles are individuals or not. If one wants to argue from the truth or falsity of PII to claims about individuality, there are only two ways to go. If PII is true, then particles are individuals and if PII is false then particles are not individuals. However, these are not the only possibilities. If, for example, particles are weakly discernible, weak discernibility grounds individuality, and weak discernibility is not discernibility in the sense relevant to PII, then particles are individuals that violate PII. My point is that the issue of whether PII is true depends on more than objects being individuals or not and so brings inappropriate baggage into the debate over the individuality (or non-individuality) of particles. The issues involved in answering the questions of whether particles are individuals and whether particles violate PII are related, but the answers needn't be the same. The key to making sense of the idea of individuals that violate PII is to note that identity and individuality are

two different things and there are different proposals for how they are connected.[19] Framing the arguments without reference to PII avoids obscuring this option. It also immunizes the arguments from criticisms that cannot be reformulated without reference to PII (two of which I will discuss later) and, in doing so, strengthens the arguments. Of course, PII is such an important part of the current debate that I would be negligent were I not to spend a fair bit of time on its discussion. However, for the reasons given above I think its discussion is best separated from the arguments for and against the individuality of particles. Consequently, I will save discussion of PII for chapters three and five.

I have the same response to both Bigaj's circularity objection and the objection that appropriate weakly discerning relations should allow for particles to be distinguished (at least in principle). I think these objections both misunderstand the way in which weak discernibility is supposed to ground individuality. I've already explained why I think this is the case with regard to the circularity objection. The latter objection can be viewed as either a criticism of the idea that weak discernibility can ground individuality or a challenge to provide an explicit explanation of how that grounding is supposed to work. Viewing the objection as a criticism, it is straight-forwardly question begging since weak discernibility, by definition, doesn't provide for a difference of physical properties (either monadic or relational) by which we could distinguish the particles.

If the objection is viewed as a challenge, the challenge can be answered. Ladyman, Linnebo, and Pettigrew have proven that weak discernibility is the weakest form of non-trivial discernibility (Ladyman, Linnebo, and Pettigrew 2012). They also show that weak discernibility is more discerning than mere numerical distinctness. It is therefore possible for a collection of numerically distinct objects to not even be weakly discernible. So, we can ask, what is the metaphysical difference between a collection of numerically

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[19]These proposals will be discussed in chapter four.

distinct objects that are weakly discernible and a collection that are not? What do weakly discernible objects have, over-and-above merely numerically discernible objects, in virtue of which they are weakly discernible? Since weak discernment is discernment in the absence of qualitative difference, it would be odd if the answer were that weakly discerned objects have some qualitative property (or set of properties) that merely numerically discernible objects lack. So, unless we want to take the difference as an unexplained brute fact the only option seems to be that weakly discerned objects have a non-qualitative property (identity or individuality of some sort), that merely numerically discernible objects lack. The connection between weak discernibility and individuality is neither via the fact that weakly discernible objects are numerically distinct nor by virtue of possessing qualitative properties that ground a physically detectable (at least in principle) difference. It is in virtue of the fact that a physically meaningful difference (the holding or not of a physically meaningful relation) *prima facie* seems best explained by appeal to a property that makes explicit reference to identity (that is, a non-qualitative property).

2.4   Primitive Thisness

The second argument to the effect that particles are individuals comes from Dorato and Morganti. They claim that particles should be taken as individuals because (i) this allows us to explain why particles are formally countable and (ii) the primitive thisness account of individuality is the most ontologically parsimonious view (Dorato and Morganti 2013). Formal countability simply refers to the fact that states in NRQM always have well-defined particle number. According to Dorato and Morganti, we should understand the fact that particles are formally countable as having "a direct ontological counterpart, so that it can be concluded that those particles are *n* individuals *independently of their*

qualities" (Dorato and Morganti 2013, p. 606). Consequently, we should take particles to be primitively individuated in order to account for their formal countability.

Dorato and Morganti use the phrase "primitive thisness" to refer to primitively individuated objects. But they are advocating a metaphysically minimal notion of primitive thisness. They don't want primitive thisness to "constitute 'mysterious metaphysical additions' to the qualities of things based on full-blown 'properties'" (Dorato and Morganti 2013, p.598). On this account, primitive thisness does not refer to any ontological component of an object. It is simply a brute fact, which cannot be further analyzed or explained, that particles are individuals. Because the claim that particles are individuals in this sense doesn't attribute any additional ontological component to particles, the view is metaphysically conservative.

Arenhart has raised objections to both points Dorato and Morganti offer in favor of their view. First, Arenhart claims that numerical diversity is compatible both with transcendental individuality and non-individuality (Arenhart 2013b). But this objection misses a key feature of Dorato and Morganti's position. Their argument claims that reasons for revising our ontology in terms of non-individuals are inconclusive and so it's reasonable to take labels as attaching to individuals. Another line of response to Arenhart would be to say that primitive individuality is meant to provide an explanation for how we know how many particles are in a given state not as an explanation for the metaphysical fact that there are that many particles.

Arenhart's second objection is very straightforward. Even bracketing worries about whether metaphysical simplicity should be considered an epistemic virtue, it's not obvious that primitive thisness is metaphysically simpler than non-individuality. *Prima facie* non-individuality is simpler because the primitive thisness account introduces an additional brute fact. It's possible that Dorato and Morganti simply mean that primitive thisness is the most metaphysically economical way of treating particles as individuals.

It's clearly more economical than transcendental accounts of individuality in which case, accepting metaphysical economy as an epistemic virtue, primitive thisness seems to win out over transcendental individuality in spite of the previous objection.

2.5   States versus Particles

It is commonly assumed that the labels in NRQM indicate quantum particles. This is why we have a connection between the Indistinguishability Postulate and the requirement that states be symmetric or antisymmetric with respect to particle labels. The labels represent particles and their indistinguishability is captured by the requirement that their states be symmetric or antisymmetric with respect to those labels.

Redhead and Teller argued that particles are not individuals because they lack labels. But this was because they thought the Fock space formalism itself didn't include labels, not because they didn't associate labels, when present, with particles. The argument from quantum statistics depends on the labels indicating particles. Weak discernibility also depends on labels indicating particles insofar as it's the entities to which the labels attach that are merely weakly discernible. Finally, Dorato and Morganti's primitive thisness account also depends on labels indicating particles since the particle number operator returns the value of the number of unique particle labels in an expression. But what if it's the case that we shouldn't be associating labels with particles at all? What if it is something other than the labels that we should be associating with particles? Dieks and Lubberdink have offered an such an alternative for understanding particles in NRQM (Dieks and Lubberdink 2011).

Dieks and Lubberdink are offering a solution to what I'll refer to as the problem of indistinguishable classical particles. The issue, as they describe it, is that the

symmetrisation postulates, which are taken to imply the indistinguishability of quantum particles, are valid even in the classical limit where we would expect to see classical particles emerge. Consequently, classical particles that emerge from NRQM are just as indistinguishable as quantum particles. Their solution is to identity quantum particles with localized states rather than with labels. So, consider again the state

$$\frac{1}{\sqrt{2}}(|a(1)\rangle\,|b(2)\rangle + |b(1)\rangle\,|a(2)\rangle) \tag{2.38}$$

where $|a\rangle$ and $|b\rangle$ are localized states. According to Dieks and Lubberdink, this state describes two distinguishable quantum particles. One of them is at the position where $|a\rangle$ is localized and the other where $|b\rangle$ is localized. Consequently, classical particles emerging in the classical limit will also be distinguishable.

Dieks and Lubberdink solve the problem of indistinguishable classical particles at the cost of creating the problem of distinguishable quantum particles. If we define classical and quantum particles in the same way, then the validity of quantum mechanics in both the classical and quantum domains, ensures that classical and quantum particles are either both distinguishable or both indistinguishable. Is the problem of distinguishable quantum particles a genuine problem?

In general, on Dieks and Lubberdink's definition of 'particle,' a quantum state will not include any particles. If $|a\rangle$ and $|b\rangle$ overlap in the above state, then there is no unique decomposition of the position distributions into localized states. But whenever a quantum state can be uniquely decomposed in terms of localized single-particle states, there are quantum particles and those particles are distinguishable. So, the 'indistinguishability of quantum particles' is an artifact of the fact that most quantum states don't describe particles at all.

Dieks and Lubberdink can perhaps explain away the indistinguishability postulate on the basis that the postulate is based on the assumption that labels indicate particles. But how are they to explain the difference between classical and quantum statistics? Shouldn't states that have a unique decomposition in terms of localized single particle states obey the same statistics? If what the classical limit does is ensure that we have a unique decomposition in terms of localized single particle states and that those states remain localized, then this should play some explanatory role in why classical statistics differs from quantum statistics. But then shouldn't a state that has a unique decomposition in terms of localized single particle states outside of the classical limit obey the same statistics for as long as that decomposition remains applicable? This strikes me as an empirical consequence of the view. Quantum particles should, even if briefly, sometimes obey MB statistics. It's difficult to see how the difference between classical and quantum statistics could be divorced from indistinguishability, and so any view that proposes that quantum particles are sometimes distinguishable (in a metaphysical rather than epistemic sense) needs to explain how it upholds the difference between classical and quantum statistics.

There are at least two other issues. One is that the physical meaning of the particle number operator seems to vary depending on the state to which it's applied. It gives us the number of quantum particles in states with unique decompositions in terms of localized single-particle states, but it doesn't give us the number of quantum particles for other states (where all other states have zero quantum particles). I think Dieks and Lubberdink need to provide us with an explanation of how we are to understand the particle number operator on their view.

The other issue is how to understand states like $|a\rangle|a\rangle$. If the number of particles is equal to the number of distinct localized single particle states, then this is a one-particle state. But there are empirical differences in behavior between the state $|a\rangle$ and the state $|a\rangle|a\rangle$.

Note that the decomposition into localized single particle states here *is* unique. The (alleged) two bosons are each in state $|a\rangle$, they aren't each in state $|b\rangle$ or $|c\rangle$. And this is despite having overlapping wave functions. When we have two distinct overlapping wave functions, then we can't determine a unique decomposition in terms of localized single particle states. But if $|a\rangle \neq |b\rangle$ then $|a\rangle |a\rangle \neq |b\rangle |b\rangle$. It seems to me that situations we would normally describe as involving *n* bosons in the same state would be described by Dieks and Lubberdink as involving a single particle in that state. But the value of *n* makes an empirical difference. Dieks and Lubberdink need to explain how to account for this difference, but it's not obvious that they could explain it.

All of this suggests that it would be problematic to associate quantum particles with localized single particle states rather than with labels. But there is a way to solve the problem of indistinguishable particles without creating these problems. Classical particles and quantum particles are very different things and I think it's misleading to use the term "particle" to describe the latter (but it is the term generally used). Instead we have classical particles and quantum wavicles (for lack of a better term) and we should not be trying to define them in the same way. In light of this perhaps what we should do is accept Dieks and Lubberdink's definition for classical particles (and so apply that definition in the classical limit) but view labels as indicating particles outside of the classical limit. Then classical particles are distinguishable and quantum particles are indistinguishable; standard explanations for the difference between classical and quantum statistics still apply; the particle number operator always gives us the number of particles; and there's no mystery about understanding the difference between having 1, or 2, or *n* bosons in the same state. Consequently, for the remainder of this dissertation I will assume that labels in NRQM indicate particles.

2.6   Conclusion

The forgoing should make it clear that we have a case of metaphysical underdetermination here. However, it's not merely that NRQM is compatible with particles-as-individuals and particles-as-non-individuals metaphysical packages, but that there is no clear winner between the two. On the side of non-individuality, we have, in the first instance, Redhead and Teller whose arguments depend on treating the LTPHSF and Fock space formalism as more different than they actually are. Secondly, we have the argument from quantum statistics where Saunders and Belousek question whether permutation invariance is actually a difference between classical and quantum mechanics and Morganti questions whether permutation invariance is a *relevant* difference. On the side of individuality, we first have weak discernibility where it's not only questionable whether particles are weakly discernible in a *physically meaningful* sense but also whether weak discernibility implies anything about individuality. Finally, we have Dorato and Morganti's primitive thisness account where we have little positive argument. What should we do in light of this metaphysical underdetermination?

Ontic structural realists, such as Ladyman and French, have cited this metaphysical underdetermination as a cause for adopting ontic structural realism (OSR).[20] For example, Ladyman writes

> Even if we are able to decide on a canonical formulation of our theory, there
> is a further problem of metaphysical underdetermination [...] In the case of
> individuality, it has been shown [...] that electrons may be interpreted either as
> individuals or as non-individuals. We need to recognize the failure of our best

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[20]Roughly, ontic structural realism is the view that the only things in our scientific theories that we should be realist about are structural features.

theories to determine even the most fundamental ontological characteristic of the purported entities they feature. It is an *ersatz* form of realism that recommends belief in the existence of entities that have such an ambiguous metaphysical status. What is required is a shift to a different ontological basis altogether, one for which questions of individuality simply do not arise.[21]

Ladyman sees scientific realism as incompatible with this sort of metaphysical underdetermination. I am not going to be concerned with whether there is such an incompatibility, nor will I be concerned with OSR itself.[22] However, for the purposes of this dissertation I will be assuming that scientific realism, of some sort, is correct.

I have two options here. I can agree with Ladyman that metaphysical underdetermination is a problem for scientific realism or I can disagree. I will look at both options in turn. First, suppose I agree with Ladyman. Is there a way to save scientific realism? Since this is a compatibility issue, there is clearly another potential source of the problem. Perhaps it is not scientific realism that needs to be revised but our notion of individuality. Quantum theory has a habit of challenging our classical conception of the world and perhaps in this instance it's our concept of individuality that needs revision. So even accepting Ladyman's challenge to scientific realism, there is another, as yet unexplored, option besides accepting OSR. I will be developing this option in chapters four through six.

On the other hand, suppose I disagree with Ladyman and don't see the metaphysical underdetermination here as a problem to be resolved. Even in this case I can still ask the question of whether this a genuine case of metaphysical underdetermination or an artifact of an inappropriate concept of individuality. I needn't be concerned with OSR or whether

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[21](Ladyman 1998, pp. 419-420).

[22]Though I should note that more recent defenses of OSR don't rely on underdetermination arguments. Instead they say that NRQM indicates that quantum objects have relationally determined identity. For more information on OSR see (Landry and Rickels 2012).

there is genuine incompatibility between scientific realism and the metaphysical underdetermination of the individuality, or lack thereof, of particles. My project makes sense regardless of whether Ladyman is correct.

I do not actually see this metaphysical underdetermination as a serious problem. If the world is such that we can't know whether particles are individuals or not, then so be it. There is no need to remove the underdetermination and the removal of this underdetermination is not an epistemic virtue that should count in favor of accepting OSR and against the position I will develop in chapters three through five. However, I do not think that the world is such that we can't know whether particles are individuals or not. We can answer this question, and we can do so in a way that has its own independent merits.

Issues with Individuality in NRQM

The second step in my project is to lay out some challenges that must be kept in mind in discussions of individuality in quantum mechanics. I will begin with three challenges the defender of non-individuality must address. The first two challenges are tied together by Jantzen, the originator of these challenges, in the form of a dilemma for anyone who wants to speak of non-individuals (Jantzen 2011, Jantzen 2014). The challenges can be separated, indeed the second doesn't appear in (Jantzen 2011), and I will treat them separately. The first challenge is the idea that it is nonsensical to claim that particles are not individuals because (i) collections of particles have cardinality and, (ii) cardinality presupposes identity. The second challenge is the idea that terms like "electron" function, semantically, as mass nouns although we treat them, syntactically, as count nouns. Once we realize that terms like "electron" are, semantically, mass nouns there is no longer any reason to claim that particle are not individuals. The third challenge is the claim, defended in (Bueno 2014), that the concept of identity is fundamental in such a way that it makes the most sense, pragmatically, to treat particles as possessing identity. These challenges all have the same basic form. The idea is that identity is related to other concepts and so if identity goes so do these other concepts. The reason I want to present these arguments merely as challenges rather than as arguments to the effect that particles are individuals is that the arguments can't be generalized in a non-question-begging way. I stated in chapter two that NRQM has a habit of challenging our classical concepts, so one way a defender of non-individuality can respond is by proposing revised versions of these concepts (such as cardinality). These proposals will have to be evaluated on a case-by-case basis.

The rest of this chapter will be a challenge to the defender of individuality in the form of a discussion of the Principle of the Indiscernibility of Identicals (PII). PII is appealing to many philosophers of physics because it provides empirical meaning to the claim that objects are individuals by connecting identity with distinguishability. So, a challenge for anyone who wants to claim that particles are individuals, and thereby separate individuality from distinguishability, is to explain the difference between individuals and non-individuals. If the difference isn't that the former are distinguishable and the latter are not, then what is it?

3.1   Classical Cardinality

While a detailed discussion of what it means for something to be an individual or non-individual will wait until chapter four, understanding Jantzen's argument requires noting one particular feature of non-individuals. Everyone agrees that part of what it means to be an individual is to have identity in some sense. An individual a stands in the relation $a = a$ to itself. Consequently, one way to deny that an object is an individual is to say that the identity relation, denoted by "=," does not apply to the object. This is, in fact, what all the defenders of the claim that particles aren't individuals do say. I will look more at how we should understand the claim that some objects lack identity in chapter four. All that matters here are some of the consequences of denying the identity relation.

The problem for the defender of non-individuality, according to Jantzen, is that cardinality presupposes identity. Modern set theory uses the von Neumann cardinal assignment according to which the cardinal number of a set S is the smallest ordinal number n such that there is a bijection from n to S. Jantzen points out two ways in which this definition of cardinality presupposes identity. The first is that one cannot define ordinals without defining asymmetric relations and, as noted in (French and Krause 2006, p. 284), one

cannot define asymmetric relations without identity. The other is that a bijection is a function and "the notion of a function depends upon the primitive identity of elements in a set" (Jantzen 2011, p. 442). This is easy to see when noting that functions have one, and only one, output for a given input. If we have a function $f : S \rightarrow T$ we require that for all $s \in S$ and $t, t' \in T$ that if $f(s) = t$ and $f(s) = t'$ then $t = t'$. Finally, Janzten also provides a nice non-technical way in which we can understand the relation between cardinality and identity: "we understand the 'size' of a finite set A to correspond to what we get by 'counting' the elements of A in the normal, intuitive sense of counting. Counting is really just indexing, affixing to each element of A a unique label as we do when we point to each element and say "one," "two," "three," etc. But indexing requires that we be able to pick out individual elements of A without reference to their properties" (Jantzen 2011, p. 442). The point is that what it means for a collection of objects to have a cardinality, on the standard ways of thinking about it, "is for the entities to be identical with themselves and no others in the collection" (Jantzen 2014, p. 3).

The technical details as to why classical set-theoretic cardinality presupposes identity aren't actually what's important here. Defenders of the view that particles are non-individuals recognize that their view implies that collections of particles can't be appropriately described in classical set theory. Hence the development of quasi-set theory. The issue is whether defenders of non-individuality can adequately explain, in a sensible way, why collections of particles in NRQM are eigenstates of the particle number operator. The particle number operator gives us something which it makes sense to refer to as a cardinality. It does the primary thing that cardinality does, namely indicate how many objects there are. Consequently, the defender of non-individuality owes us a definition of cardinality that makes sense as such. That is, the defender of non-individuality can't merely attach the word "cardinality" to any definition. The definition must be such that we can recognize it as a plausible formalization of what we mean when we say such things as "there are two bosons."

Jantzen takes things a little further than this. He claims that, as a result of the success of classical set theory, very strong motivation is needed to warrant even searching for, let alone adopting, an alternative. Jantzen sees the fact that an interpretation of NRQM in which it is compatible with classical set theory (the particles-as-individuals interpretation) in conjunction with the success of classical set theory undermines any motivation for developing alternative set theories. We needn't go as far as Jantzen and I don't agree that the project of developing alternative set theories is unmotivated. However, I do agree that we ought to carefully examine the analogs of classical cardinality these set theories provide and ensure they are acceptable. I will now proceed to do just that.

3.1.1 Cardinality in Quasi-Set Theory

Quasi-set theory is a conservative extension of Zermelo-Fraenkel set theory with urelements (ZFU). To say that theory T_2 is a conservative extension of theory T_1 is to say (i) that all the theorems of T_1 are theorems of T_2 and (ii) that any theorem of T_2 that can be entirely expressed in the language of T_1 is a theorem of T_1. This essentially means that quasi-set theory "encompasses a 'classical' counterpart which coincides with ZFU" (French and Krause 2006, p. 284). Urelements are objects that are not themselves sets but can be elements of sets; an urelement may denote a physical object. The urelements in quasi-set theory are M-atoms, which denote classical objects, and m-atoms, which denote the basic entities of quantum theory. Making quasi-set theory a conservative extension of ZFU ensures that classical objects are treated in the same way in both theories.

The basic differences between quasi-set theory and ZFU are as follows.[1] First, the axioms of quasi-set theory are expressed in a first-order language *without* identity. Note that

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[1] See (French and Krause 2006) for an extensive account of quasi-set theory.

French and Krause do not spell out the logic behind quasi-set theory (i.e. the logic of the metalanguage used to describe quasi-set theory), leaving it open as to whether or not it is identical to classical first-order predicate logic without identity. Second, quasi-set theory contains a primitive binary relation of indistinguishability denoted by '$\equiv$'. Because there are two types of urelements there are two monadic predicates, $m(x)$ and $M(x)$, to specify, for some urelement x, its type. Finally, there is a monadic predicate $Z(x)$ indicating that x is a set in the sense of ZFU. The basic components of quasi-set theory are quasi-sets (qsets), M-atoms, and m-atoms where (i) anything that isn't an M-atom or an m-atom is a qset and (ii) some qsets are sets.

One peculiar feature of quasi-set theory to note is that expressing quasi-set theory in a first-order language without identity means that the best one can do to represent identity is to define a relation of extensional identity. French and Krause define extensional identity as follows:

$$x =_E y =_{df} [(Z(x) \wedge Z(y) \wedge \forall z(z \in x \leftrightarrow z \in y))$$

$$\vee (M(x) \wedge M(y) \wedge \forall_Q z(x \in z \leftrightarrow y \in z))],$$

(3.1)

where $\forall_Q$ is the universal quantifier restricted to qsets (French and Krause 2006, p. 277). This says that x and y are extensionally identical if they (i) are sets with all the same members or (ii) are M-atoms that are elements of all the same qsets.

Extensional identity and numerical identity are logically distinct, and one might be concerned that quasi-set theory only has the former. To see that they're logically distinct consider the standard example that every species that has a heart also has kidneys. The phrase "has a heart," understood in the sense we mean when we say things like "members of the human species have hearts," and the phrase "has a kidney," understood in the same sense, are extensionally equivalent.[2] However, they clearly aren't equivalent in the sense

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[2]Note that people temporarily lacking a kidney during a transplant, for example, is not a counterexample to the claim that every species that has a heart has kidneys when that claim is understood as intended.

of having a heart being the very same thing as having a kidney. It's a theorem of quasi-set theory that extensional identity has the properties of first-order (Hilbert-Bernays) identity (French and Krause 2006, p. 279). What this claim amounts to is that for any claim in ZFU that involves numerical identity the corresponding claim using extensional identity is also true. So, French and Krause aren't disputing that extensional identity and numerical identity are logically distinct, but they think the difference doesn't matter because it makes no practical difference in the context of quasi-set theory. In the context of quasi-set theory, it's perfectly safe to use a relation of extensional identity without fear that it will identify distinct things. Still, one might be concerned by the fact that quasi-set theory lacks the ability to say that the sets {1} and {2} have numerically the same cardinality. To put it more strikingly, quasi-set theory can't articulate the claim that 1 is numerically identical to itself. Instead, it can only articulate the claim that 1 is extensionally identical to itself.

The few features of quasi-set theory I have described above are sufficient to understand its claims about cardinality. So, I'll leave my misgivings about extensional identity behind and turn, first, to what French and Krause say about cardinality. French and Krause define the quasi-cardinal of a qset as "The quasi-cardinal of a qset is a cardinal (defined in the 'classical part' of the theory) and coincides with its cardinal itself when this qset is a set" (French and Krause 2006, p. 286). What this means is that a quasi-cardinal is a von Neumann cardinal and that in the case of a qset that is also a set in ZFU the quasi-cardinal assignment from quasi-set theory and the cardinal assignment from ZFU agree. They state that this definition doesn't commit them to the claim that a qset consisting only of non-individuals (a pure qset) can be ordered because "the associated ordinal of a quasi-set of indistinguishable *m*-atoms cannot be something that belongs to the 'classical' part of the theory" (French and Krause 2006, p. 286). The idea is that we can use either a different definition of 'ordinal' for *m*-atoms or, instead of defining ordinals for *m*-atoms, we could simply assign ordinals to them. The latter option would mean that "the

cardinality of pure qsets is always an ad hoc addendum" (Jantzen 2011, p. 447). French and Krause say little more about the cardinality of pure qsets saying only that it should be further investigated.

The only attempt, so far, to provide a satisfactory account of cardinality for pure qsets can be found in (Domenech and Holik 2007) and further developed in (Arenhart 2011) and (Arenhart 2012). Jantzen gives a very nice brief summary of Domenech and Holik's approach: "If we call the qset in question X then—informally speaking—their approach is to construct qsets from X which resemble classical singletons, argue that these 'quasi-singletons' should be assigned a cardinality of 1, and then to count how many such quasi-singletons can be extracted from X" (Jantzen 2011, p. 445). Let X be a non-empty qset and let $x \in X$. We can construct the quasi-singleton in two steps. The first is to construct the following qset

$$A_x =_E [a \in P(X) : x \in a],$$ (3.2)

where $P(X)$ is the power qset of X. The power set of a set S is the set of all subsets of S. Similarly, the power qset of a qset X is the qset of all sub-qsets of X. A_x, then, is the qset of all sub-qsets of X that contain x. The quasi-singleton $\langle x \rangle$ can then be written as

$$\langle x \rangle =_E \bigcap_{a \in A_x} a.$$ (3.3)

$\langle x \rangle$ is the intersection of A_x; that is, the elements of $\langle x \rangle$ are all and only those elements that are members of every sub-qset of X that contains x. If X is a set then $\langle x \rangle =_E \{x\}$ and the cardinality of $\langle x \rangle$ is uncontroversially 1.

To show that we should take the cardinality of $\langle x \rangle$ to be 1 for any qset X, including qsets whose members are all m-atoms, Domenech and Holik show that if $Q\alpha$ and $\alpha \subseteq \langle x \rangle$, then $\alpha =_E \emptyset$ or $\alpha =_E \langle x \rangle$, where $Q\alpha$ means that α is a qset (Domenech and Holik 2007, p. 865).

In other words, they show that the only sub-qsets of $\langle x \rangle$ are itself and the empty set. There is no smaller non-empty subset of $\langle x \rangle$. This result, which they call "Proposition 4.4", seems like a good reason to call $\langle x \rangle$ a quasi-singleton.

However, Jantzen shows that Proposition 4.4, in conjunction with the definitions of a quasi-singleton and a power qset, provides the resources to construct a first-order identity relation for m-atoms. Specifically, Jantzen defines the relation $x \sim y =_E y \in \langle x \rangle$ and shows that '$\sim$' has the properties of first-order identity just as '$=_E$' does (Jantzen 2011, p. 446). The proof begins with the assumption that $y \in \langle x \rangle$ where X is a pure qset. The proof can then be divided into four steps. Showing that $\forall_{z \in P(X)}(x \in z \rightarrow y \in z)$ is the first step. This follows straightforwardly from $y \in \langle x \rangle$ and the definition of $\langle x \rangle$.

The second step is to establish that $\forall_{z \in P(X)}(y \in z \rightarrow x \in z)$. Jantzen provides a proof by contradiction and so begins by asking us to suppose that there exists a qset $z \in P(X)$ such that $y \in z$ and $x \notin z$. Then, by the definition of $\langle y \rangle$, it follows that $x \notin \langle y \rangle$. Since $x \in \langle x \rangle$ it must be the case that $\neg(\langle x \rangle =_E \langle y \rangle)$. Now, $y \in \langle x \rangle$ implies that $\langle x \rangle \in A_y$. This is because A_y is the qset of all sub-qsets of X that contain y and $\langle x \rangle$ is such a sub-qset. Next note that it follows from $\langle x \rangle \in A_y$ that $\langle y \rangle \subseteq \langle x \rangle$. To see this, note first that if all the elements of $\langle x \rangle$ are in the intersection of A_y then $\langle y \rangle =_E \langle x \rangle$ and if only some of the elements of $\langle x \rangle$ are in the intersection of A_y then $\langle y \rangle \subset \langle x \rangle$. However, from Proposition 4.4 if $\langle y \rangle$ is a non-empty subset of $\langle x \rangle$, then $\langle y \rangle =_E \langle x \rangle$. The assumption $\neg\forall_{z \in P(X)}(y \in z \rightarrow x \in z)$ has led to a contradiction and so it is the case that $\forall_{z \in P(X)}(y \in z \rightarrow x \in z)$.

The third step begins by noting that $y \in \langle x \rangle$ implies $\forall_{z \in P(X)}(y \in z \leftrightarrow x \in z)$. Since m-atoms are indistinguishable from one another we can change this to $y \in \langle x \rangle$ implies $\forall_{z \in P(X)}((y \in z \leftrightarrow x \in z) \wedge (x \equiv y))$. Consequently, '$\sim$' functions as a first-order indistinguishablity relation on $P(X)$. The final step comes from (Ketland 2006, p. 307) who shows that for a language with finitely many predicates first-order identity and first-order indistinguishability have all the same properties. So, "if we restrict the range of

our quantifiers to the qsets in $P(X)$ then '~' is a first-order identity relation for m-atoms just as '$=_E$' is an identity relation for everything else" (Jantzen 2011, p. 446). The relation '~' is an identity relation in exactly the same way as extensional identity (that is, it has the properties of first-order Hilbert-Bernays identity). Consequently, defenders of non-individuality require a different definition of cardinality for m-atoms. However, the very fact that it's possible to define an identity relation for m-atoms that is compatible with the axioms of quasi-set theory is itself cause for concern. If particles are not individuals in the sense of lacking self-identity, then an appropriate theory presumably wouldn't be compatible with defining an identity relation for them.

Arenhart endorses and builds on Domenech and Holik's results. Consequently, if Jantzen's criticisms are correct, they apply equally well to Arenhart. Arenhart does, however, have a reply to Jantzen. Arenhart challenges the claim that '~' has been shown to be an indistinguishability formula. He points out that the relation only shows that "m-atoms x and y, *under the conditions of the definition*, belong to the same q-sets, which is not the same as an indistinguishability formula" (Arenhart 2012). The reason, according to Arenhart, is that an indistinguishability formula must apply to *all* predicates of the language. But Jantzen's proposed indistinguishability relation isn't defined on the full domain of non-individuals. Jantzen has responded that the fact that '~' has a restricted domain is irrelevant because "[w]e wouldn't worry whether electrons in a collection are in some global sense *really* non-individuals if it were the case that in any given collection of electrons, there is a property that distinguishes them from each other" (Jantzen 2014, p. 4). I agree with Jantzen; if we can define an identity relation for any collection of particles of the same type why isn't that enough to say that the particles have identity? Granted it isn't enough to conclude that particles have identity in the sense of possessing some sort of metaphysical label (e.g. haecceity). However, it's not clear why that would be a drawback and Arenhart certainly wouldn't view it as such.

Arenhart's second criticism is that even granting that Jantzen has defined some sort of individuality for *m*-atoms it seems to be *contextual individuality*. Contextual individuality is a kind of individuality that we might attribute Priscilla or electrons in a Bell type experiment. We might capture the fact that we can individuate the particles in those contexts by asserting that the particles have contextual individuality. However, there is a big difference between Jantzen's definition and contextual individuality. Namely that we can define an identity relation in Jantzen's sense for any collection of particles of the same type whatsoever. The importance of this was seen in my discussion of weak discernibility. Perhaps the claim that Jantzen's individuality is contextual individuality could be defended, especially since unlike the case of weak discernibility the individuality granting relation depends on the collection of particles under consideration. However, it cannot be defended merely by pointing to the case of Priscilla or Bell type experiments as Arenhart does.

If we accept Jantzen's indistinguishability relation '$\sim$', then we can pose the following dilemma for defenders of non-individuality. Either distinguishability is sufficient for identity or it isn't. If it is, then the ability to define a distinguishability relation for any collection of quantum particles implies that they satisfy an identity relation.[3] If it isn't then the normal reason for taking classical particles to be self-identical (distinguishability in virtue of having distinct spatiotemporal trajectories) is no longer available. In that case, defenders of non-individuality for quantum particles must explain what grounds their differential treatment of classical and quantum particles. But if identity isn't just a matter of distinguishability, then the identity that classical particles have in virtue of which quasi-set theory defines an identity relation for them, must be a form of transcendental identity. So this route requires defenders of non-individuality of quantum particles to

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[3]Even if this is a case of contextual identity, that doesn't appear to be problematic if identity reduces to distinguishability, as I'm supposing here, and when some relation of contextual identity or other is always available.

either (i) defend the claim that identity doesn't reduce to distinguishability and drop identity for classical particles or (ii) defend the claim that identity doesn't reduce to distinguishability and the claim that classical particles have transcendental identity whereas quantum particles do not. The former route requires either (iii) accepting that identity reduces to distinguishability and accepting that all particles are individuals or (iv) accepting that identity reduces to distinguishability and finding a way to undermine Jantzen's relation '~'. Most defenders of non-individuality of quantum particles would only find (iv) appealing. But the prospects for (iv) do not appear particularly good. The formal results, including that the relation *functions* as an identity relation in the same sense as extensional identity, makes it difficult to argue that identity should apply to, and only to, classical particles without adopting option (ii).

3.2   Mass Nouns

The next challenge also comes from Jantzen who suggests that particle terms, such as 'electron,' are actually mass nouns although we treat them, syntactically, as count nouns (Jantzen 2014). In general, the only kinds of things we can say about quantum systems do not make reference to any specific particle. The only exceptions are cases such as that of Priscilla or Bell type experiments where the context allows us to individuate the particles. Jantzen notes that any sentence that doesn't make reference to specific particles is a sentence in which particle terms can be understood as mass nouns. For example: "(S1) One electron in the system is spin-up, the other is spin-down" (Jantzen 2014, p. 7) is similar to "(S5) Half of the liquid is vinegar, the other half is water" (Jantzen 2014, p. 8). We could rephrase (S1) as "Half of the electron-stuff is spin-up and half is spin-down" (Jantzen 2014, p. 8). Or "(S4) Lithium atoms have 3 electrons" (Jantzen 2014, p. 7) is similar to "(S8) A shot of whiskey has 0.6 oz. of alcohol" (Jantzen 2014, p. 8). We might

rephrase (S4) as "A standard unit of lithium atom has 3 units of electron" (Jantzen 2014, p. 8).

If Jantzen is correct that we can *always* replace 'electron(s)' with either 'electron-stuff' or 'unit of electron' in a *sensible* way, then there would seem to be no question that 'electron' *functions* as a mass noun. Consequently, an explanation of why 'electron' is not a mass noun despite functioning as such would be required from anyone who wants to treat 'electron' as a count noun or sortal. Note that the feature Jantzen points out as ensuring that we can always sensibly replace 'electron(s)' with 'electron-stuff' or 'unit of electron' is exactly the feature that is supposed to lead us to the view that particles are non-individuals. It is permutation symmetry that, except in some special cases, prevents us from making reference to specific particles in our assertions about quantum systems. But this is a general feature of mass nouns such as 'water.' If we want to refer to specific molecules of water in making assertions about some body of water, then we have to tag those molecules; there is no way to identify them otherwise. That just amounts to attaching something else to those molecules and we can easily do that for electrons as well. For example, we can attach alpha particles to some electrons to form helium atoms.

The idea that particle terms are mass nouns makes a lot of sense when considering the wave nature of particles. We could view the wave function of, say, an electron as representing a spatial distribution of a single unit of electron-stuff. This is, in fact, essentially what the GRW interpretation with mass density ontology says.[4] But the question of whether particles terms are mass nouns can, presumably, be answered without reference to any particular interpretation. If we have a bunch of units of electron-stuff, then it would be difficult to distinguish them just as it's difficult to distinguish molecules

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[4]GRW refers to the Ghirardi-Rimini-Weber wave function collapse interpretation of quantum mechanics. On the mass density view of this interpretation, wave functions represent mass density waves.

of water in a body of water. Is the inability to distinguish the units of electron-stuff due to their being metaphysically indistinguishable or is it just an epistemic matter? That quantum particles are not impenetrable complicates the issue. Water molecules are always distinguishable because they won't overlap, so let's say that water molecules are individuals. Now suppose that water molecules could overlap. Would that make a difference to their individuality? That depends on whether distinguishability is necessary for individuality, something I will discuss in chapters four and five. But for now, I will just say that I think the fact that quantum particles are not impenetrable raises a question as to whether it makes sense to connect distinguishability with individuality. Adding the possibility that particle terms are mass nouns makes the issue even murkier.

Jantzen views this as a problem for the defender of non-individuality as he takes mass nouns to refer to individuals. However, there is room for debate. Even if 'electron' is a mass noun, that doesn't settle the issue of individuality. It's not obvious that mass nouns must refer to individuals. 'Furniture' seems to refer to individuals (at least when used in the singular rather than the plural) whereas it's not obvious that 'water' does. If some mass nouns refer to individuals and others do not, then even if 'electron' is a mass noun we haven't determined whether electrons are individuals. The defender of non-individuality could say that 'water' doesn't refer to individuals. In chapter four when looking at various potential components of an account of individuality we will see that some accounts of individuality would say that 'water' doesn't refer to individuals. So, I see the challenge for defenders of non-individuality as not coming from Jantzen's claim that 'electron' is a mass noun alone. Rather, it will be Jantzen's claim in conjunction with considerations relevant to defining 'individuality.' And, depending on the particular considerations, we could have a challenge for defenders of individuality as well. Consequently, I will return to this issue in chapter four.

I will finish this section with two brief comments on reasons why one might reject the claim that 'electron' is a mass noun. The first is that, since mass nouns are contrasted with count nouns, one might take the ability to count electrons (at least in the sense of assigning a cardinality to a collection of them in NRQM) as a relevant difference. However, ordinary mass nouns like 'water' have minimal parts that we could count. Responding to Jantzen's proposal by saying that we can count electrons would be like saying 'water' isn't a mass noun because we can count water molecules. If we use the term 'electron-stuff' for a collection of electrons and 'electron' for a single electron, the response would amount of the claim that 'electron-stuff' isn't a mass noun because we can count electrons.

The second reason is similar, if not identical, to the first. It is that French and Krause claim that terms like 'proton' can't be understood as mass nouns because they don't divide their reference (French and Krause 2006, p. 347). That is to say, that protons can't be divided into smaller parts that are also protons. However, Jantzen notes that 'electron' in the phrase 'electron system' does divide its reference. You can split electron systems into other electron systems, up to a point. Jantzen takes French and Krause's mistake to be of the same sort as that mentioned in the previous paragraph. 'Proton' doesn't divide its reference when referring to a single proton just as 'water' wouldn't divide its reference if we used the term 'water' to refer to a single water molecule. "That [electron systems] have minimal parts—that some parts of an electron system are not themselves electron systems—is not a worry; the same is true of many ordinary mass-nouns" (Jantzen 2014, p. 8).

3.3   Identity as Fundamental

The final challenge to the idea that particles are not individuals comes from Bueno. He argues that we have good pragmatic reasons to treat particles as possessing identity

regardless of whether they have some form of metaphysical identity or not. He provides four reasons why this is the case. These reasons are:

> (a) Identity is presupposed in every conceptual system: without identity, it is unclear that any conceptual system can be formulated. (b) Identity is required to characterize an individual: nothing can be an individual unless it has well-specified identity conditions. (c) Identity cannot be defined: even in systems that allegedly have the resources to define identity. (d) Identity is required for quantification: the intelligibility of quantification presupposes the identity of the objects that are quantified over.[5]

I will look at each of these in turn. However, I will save discussion of (b) for chapter four.

3.3.1   Identity is Presupposed in Every Conceptual System

Bueno's first argument is an indispensability argument. Concepts are needed to do metaphysics. Concepts presuppose, and therefore require, numerical identity. Therefore, metaphysics requires numerical identity. The reason concepts require numerical identity is in order to serve their classificatory purpose. Objects falling under the *same* concept are grouped together, which presupposes numerical identity of the concept in question. Objects falling under different (numerically distinct) concepts are differentiated in virtue of *not being qualitatively identical* in the relevant respect.

Bueno doesn't clearly distinguish between the identity of concepts and the identity of the objects falling under the concepts. Krause and Arenhart seize on this to argue that a weak

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[5](Bueno 2014, p. 325).

notion of distinguishability would serve to differentiate objects that fall under different concepts (Krause and Arenhart 2015a). I agree with Krause and Arenhart, and I suspect that Bueno does as well.

The more problematic part of Krause and Arenhart's criticism is their discussion of objects falling under the same concept. They describe the following dilemma. If concepts are extensional then the identity of a concept would be defined in terms of the identity of the objects falling under it. In that case, the identity of the concept wouldn't be fundamental. On the other hand, "[o]n an intensional understanding of concepts ... it is notoriously difficult to account for the identity of concepts" (Krause and Arenhart 2015a, p. 7). The latter part of the dilemma is surprising. If identity is fundamental, and undefinable as Bueno claims, one would not expect to find an account of identity. Being charitable to Bueno I expect he was thinking of concepts in intensional terms and the lack of an account of identity is because there can, on his view, be no such thing.

Finally, Krause and Arenhart note that because the same linguistic entity applies to multiple objects it doesn't follow that there is any ontological counterpart that applies to both objects. That is, the identity of concepts doesn't imply the existence of, say, universals bearing a metaphysical identity relation. One could, they point out, be a trope theorist where two objects that have property P have it in virtue of having numerically distinct tropes of the same type. I have one problem with the objection. Bueno makes it clear that he's offering pragmatic arguments that could be turned into metaphysical arguments with some additional steps. So it isn't really a criticism when Krause and Arenhart point out that additional steps are needed to reach a metaphysical conclusion.

3.3.2 Identity Cannot be Defined

The next argument, (c), is that numerical identity is fundamental because it cannot be defined. We already saw that French and Krause accept this and have therefore opted for extensional identity instead. But, as Bueno points out, it has been claimed that numerical identity can be defined. For example, there is the well-known Whitehead-Russell definition of identity in terms of Leibniz's Law: $x = y$ if, and only if, $\forall P(Px \leftrightarrow Py)$. However, McGinn has noted that the variables 'x' and 'y' must be the same on either side of the bi-conditional (McGinn 2000). For it cannot be the case that 'x' and 'y' denote the same object if some other objects 'w' and 'z' share all the same properties. Consequently, the Whitehead-Russell definition presupposes, rather than defines, identity.

Krause and Arenhart respond that "this is not a problem of questioning identity. The two exes in the Leibniz Law are instances of the same abstract object (a variable)" (Krause and Arenhart 2015a, p. 16). I'm not sure what the objection is supposed to be. What do Krause and Arenhart mean by 'same' and why does having two instances of the *same* abstract object in the definition of identity not amount to presupposing identity? Krause and Arenhart don't elaborate. Instead they immediately follow the above statement with a discussion of how we build up conceptual schemes in a constructive way, noting that we can assume identity initially and still end up with a system that lacks identity. I'm also not sure what that has to do with whether or not identity can be defined. It should be clear that Bueno is only claiming that numerical identity cannot be defined and, as far as I can tell, Krause and Arenhart agree. But if that is the case, I'm not sure what Krause and Arenhart are objecting to with the above quoted statement and the subsequent discussion.

3.3.3   Identity is Required for Quantification

The final reason offered for the fundamentality of identity is that identity is required for quantification in classical logic. For example, consider universal quantifier introduction. If we know that Fa for arbitrary a we can infer that $\forall x F x$. That is, "if *each* object in the domain of quantification is F, then *every* object in the domain is F. However, this holds only if each [numerically] *distinct* object in the domain is in the range of the universal quantifier" (Bueno 2014, p. 329). Since we are saying that all the objects in the domain have property F the possibility of differentiating the objects qualitatively isn't even on the table. Universal quantifier introduction doesn't require that I appeal to some distinguishing feature of the objects in question as a means to ensure that all the objects that should be in the domain are in the domain. Asserting that all electrons have electric charge $-e$, for example, on Bueno's view, requires that the electrons have some form of numerical identity. If we were repeatedly quantifying over the (numerically) same electron, "then there would be no support for the conclusion that *every* [electron has electric charge $-e$]" (Bueno 2014, p. 329).

Krause and Arenhart object that alternative interpretations of the universal quantifier are possible. For example, we can "call $|F|$ the class of objects of the domain that have F, and let D be the domain of interpretation. The interpretation for $\forall x F x$ can now be stated simply as saying that D is a subset of $|F|$. For instance, we may say that $|F|$ is the class of all ... Oxygen atoms in a molecule of O_2 without need of identifying them" (Krause and Arenhart 2015a, p .19). As far as I can tell, Bueno's comments apply just as well to $|F|$. How are we to make sure that all of the objects that are F are in $|F|$. We can't qualitatively distinguish them, and we also can't quantitatively distinguish them if they don't have identity. We still appear to need a non-qualitative way of distinguishing between objects,

at least in principle, to make sense of the idea that all and only the objects that should be within the domain of the universal quantifier are in fact in the domain.

I take the first two of these arguments as arguments in favor of the idea that identity is primitive. That, by itself, does not tell us that we should take particles to be individuals. However, it does suggest that we take numerical identity as primitive, and so, for example, should make Dorato and Morganti's *primitive thisness* view more appealing. Bueno's argument also provides us reason to question any argument for the non-individuality of particles that is based on the idea that numerical identity must be grounded in qualitative facts. Consequently, I see these arguments as providing a challenge to defenders of non-individuality. In particular, they need to better explain why primitive identity isn't acceptable for particles.

On the other hand, the final argument provides reason to question the coherence of various statements about particles if those particles are taken to lack identity. Still, this can be treated as a challenge; defenders of non-individuality must explain why quantification over non-individuals makes sense. Bueno notes that if collections of particles have cardinality, even without an ordinal, a coherent explanation of quantification over non-individuals may be possible. However, he agrees with Jantzen that Domenech and Holik's attempt to define cardinality for particles presupposes identity. Still, if the defenders of non-individuality can provide a satisfactory account of cardinality for non-individuals that may also deal with Bueno's quantification objection.

There are two ways we can read Bueno's arguments. Bueno himself treats them as pragmatic arguments. We should treat particles as individuals because it is convenient and useful to do so. Even reading them this way we can still understand them as establishing that identity, if it is some metaphysical component of objects, is primitive. We could also treat the arguments as metaphysical arguments. In this case, the first two arguments would have to be supplemented with reasons to draw a connection between fundamentality or

primitiveness and metaphysical reality. However, the last argument can stand on its own as a metaphysical argument as long as substantive worries about defining cardinality for collections of non-individuals remain.

3.4 The Principle of the Identity of Indiscernibles

The final challenge I want to discuss is a challenge for those who claim that particles are individuals. If NRQM violates PII, as many claim, then on what grounds can particles be said to be individuals? I will divide my examination of this issue into three parts. In the first part I will provide a historical overview of the debate surrounding PII and NRQM in order to get a sense of why it is thought that discussion of PII is important. The latter two parts will wait until chapter five after I've discussed the relation between identity, individuality, and distinguishability as well as the distinction between metaphysical and conceptual kinds of identity.

In second-order logic with identity PII can be expressed as

$$\forall F(Fx \leftrightarrow Fy) \to x = y. \tag{3.4}$$

This says that if 'two' objects share all the properties over which F ranges, then 'they' are one and the same object. We can recognize three different versions of PII depending on the kinds of properties we allow F to range over. The logically weakest form of PII, PII(1), is obtained if F ranges over all qualitative properties and relations. So PII(1) says that 'two' objects that share *all* of their qualitative properties and relations in common are one and the same object. The next strongest form, PII(2), excludes spatiotemporal properties and relations from the range of F. So PII(2) says that 'two' objects that share all of their non-spatiotemporal, qualitative properties and relations in common are one and

the same object. Finally, the strongest form, PII(3), excludes all relational properties. So PII(3) says that 'two' objects that share all or their qualitative monadic properties in properties in common are one and the same object. Most authors leave out the word 'qualitative' although most clearly have only qualitative properties in mind. So, for example, two particles that only differ in that they have different metaphysical labels (like primitive thisness) violate even PII(1). PII, understood in terms of qualitative properties, links identity with distinguishability in principle. Metaphysical properties that aren't discernible under any circumstances are excluded.

Since particles seem to possess all their monadic properties in common, NRQM violates PII(3). This is unimportant for our purposes since PII(3) isn't a suitable criterion for identity anyway as it identifies even classical particles that possess unique spatiotemporal trajectories. Whether NRQM violates PII(2) depends on whether particles obey weakly discerning relations. Whether NRQM violates PII(1) depends on whether particles either obey weakly discerning relations or can be individuated by spatiotemporal properties. I will not be concerned with spatiotemporal trajectories since whether these can serve as individuators for particles depends on the correct interpretation of quantum mechanics. For example, each particle has a unique spatiotemporal trajectory in the de Broglie-Bohm pilot wave theory since "[i]n effect the impenetrability of the particles is built into the guidance equations" (French and Rickles 2003, p. 224).[6] Consequently, for my purposes the question of whether NRQM violates PII collapses to the question of whether particles are weakly discernible.

It should be clear why I included PII as a challenge for those who claim that particles are individuals rather than as an argument in favor of non-individuality. If NRQM does violate PII it doesn't follow that particles are not individuals because this is compatible

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[6]See (Pylkkänen, Hiley, and Pättiniemi 2016) for an extended discussion of individuality and the de Broglie-Bohm pilot wave theory.

with individuality on metaphysical grounds (e.g. on the basis of primitive thisness). Additionally, it is not clear that NRQM does violate PII due to the possibility of weakly discerning relations. I have already discussed weakly discerning relations in chapter two and will not discuss them further here.

I have three concerns about appealing to PII in this context. The first is that most authors engaged in the debate do not discuss the relationship between distinguishability and identity and so take a close relationship between the two for granted. The second is that the relationship between identity and individuality is generally not discussed and, again, it is assumed that there is a close relationship between the two. The third issue is that I have a concern that discussions involving PII sometimes conflate metaphysical and conceptual notions of identity. I will discuss these further in chapter five.

There are a number of points where I will claim that a conflation has been made. I want to say something about this because it's not always clear that a conflation is being made. When I accuse someone of a conflation it is because they are writing as if A implies B for conceptually distinct concepts A and B (i) without demonstrating any recognition that there is a conceptual distinction and (ii) without explicitly mentioning or defending the connection between A and B (where failing to do the latter involves failing to do the former). In some cases, A and B are used interchangeably and in some cases instances of A are replaced by B. But it is often difficult to tell whether A and B are being used interchangeably or if instances of A are being replaced by B. But even in the latter case, if (i) and (ii) are true we can't tell whether it's only coincidental that instances of A are replaced by B without the opposite also occurring.

In such cases there is either (iii) a conflation or (iv) the author is begging the question against those that dont accept the connection. But why would the author not say something about the connection between A and B if they recognize that A and B are conceptually distinct? They are either making a conflation or begging the question and the

former charge seems to be the more charitable one in these instances (where nothing at all is said about a connection between A and B). But the important point is that nothing turns on whether a given instance is a case of conflation or a case of begging the question. At least not as far as anything I will say goes. In each either case the author has failed to establish a connection between A and B and it's that failure that will be important.

3.4.1 Historical Overview

The earliest philosophical discussion of PII in relation to NRQM seems to be that in (Margenau 1944). Margenau argues that the Exclusion Principle (EP) implies that PII is false. He defines EP qualitatively as "*no two [fermions] (of the same kind) can be in the same state of motion*" (Margenau 1944, p. 188) and more rigorously as "*state functions representing several similar [fermions] must be antisymmetrical*" (Margenau 1944, p. 195). Margenau asks us to consider two fermions in the antisymmetric state

$$a(1)b(2) - b(1)a(2), \tag{3.5}$$

where I have dropped the ket notation for sake of convenience as well as to match Margenau's notation. The probability of finding particle 1 at position (1) is found by squaring this equation and integrating over the space of particle 2. So, the probability of finding particle 1 at position (1) is given by

$$a^2(1) \int b^2(y)dy - 2a(1)b(1) \int a(y)b(y)dy + b^2(1) \int a^2(y)dy = a^2(1) - b^2(1), \tag{3.6}$$

where I have invoked the normalization condition and that the inner product of orthogonal functions is zero. Similarly, the probability of finding particle 2 at (2) is $a^2(2) - b^2(2)$. So, the probability of finding one particle at a particular location is the same as the probability for finding the other particle at the same location. Additionally, "[w]hat has here been

proved for the observable position holds for all other observable properties of the two particles, and in general for all constituents of an *n*-particle system" (Margenau 1944, pp. 201-202). It follows that electrons are indiscernible and therefore that PII is violated.

PII can be read as stating that qualitative identity, with respect to the relevant properties and relations, implies numerical identity. So if all electrons, for example, have all of the relevant properties and relations in common it follows that they violate the corresponding version of PII because there are numerous electrons.

PII has often been defended as a principle of logic and, in light of that, claims that PII is false are sometimes met with the question ''how can entities that are indiscernible be different with respect to number?'' Margenau's answer is that number is an observable property. The number of particles in a quantum mechanical system is the number needed to correctly describe the system in NRQM (under the assumption that what we usually refer to as an *n*-particle wave function actually describes a system of *n* particles). Number of particles does make an empirical difference, however, since number is a property of systems rather than particles, it is still the case that particles themselves are indiscernible. Margenau's response is ultimately a holistic one that grants that there is something strange about the idea of indiscernible objects that differ in number. There's nothing about the 'individual' particles that tells us how many there are but there is something about the behavior of collections of particles in virtue of which we can determine the number.

If we accept that number of particles is an observable, then we might try to argue that this implies that PII isn't violated. The proposal is similar to that of individuation on the basis of weak discernibility and so faces the same sorts of issues. Why should we accept that number counts as an observable in the relevant sense when it cannot serve as a basis for distinguishing between the particles? Weak discernibility seems to fare better since we can appeal to individuality as an explanation of the difference between particles that obey

physically meaningful weakly discerning relations and those that are merely numerically diverse.

Next comes van Fraassen who claims that PII is often incorrectly understood (van Fraassen 1972). Van Fraassen claims that even two objects that share all properties and relations in common at a time satisfy PII if they have different histories. This is an important point that is often, though not always, left aside in more recent discussions of PII. There is at least one good reason to leave it aside, namely that whether particles can be individuated by history is tied to one's interpretation of quantum mechanics. History could serve to individuate particles on the de Broglie-Bohm pilot wave interpretation and may work on some versions of other interpretations. However, particles wouldn't even have histories in anything like the ordinary sense of the term if GRW with the flash ontology is the correct interpretation.[7] On that interpretation we would need something other than history in order to even determine which flashes constitute the historical progression of a single particle. And that's assuming it even makes sense to connect separate flashes as historical progressions in the first place.

After van Fraassen comes Cortes who criticizes the claim that PII is a theorem of second-order logic with identity (Cortes 1976). Cortes presents two 'proofs' of PII. The first is a very simple informal proof of PII. We assume that objects a and b share all properties in common so that $\forall F(Fa \leftrightarrow Fb)$. Then since a has the property of being identical to a, it follows from our assumption that b also has the property of being identical to a. So it follows from $\forall F(Fa \leftrightarrow Fb)$ that $a = b$.

The other is a formal proof beginning with the assumption $\forall F(Fx \leftrightarrow Fy)$, where x and y are arbitrary. Next, we instantiate a particular property; namely, the property $(\lambda z)[z = x]$.

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[7]On the flash ontology view of GRW there are no actual waves. Instead, the ontology consists only of brief "flashes" when wave function collapse occurs.

This expression can be read as 'the property of those z such that $z = x$ and it indicates the property of being identical to x.[8] Instantiating this property, $\forall F(Fx \leftrightarrow Fy)$ becomes $(\lambda z)[z = x]x \leftrightarrow (\lambda z)[z = x]y$, which reduces to $x = x \leftrightarrow y = x$. Since $x = x$ is a theorem of classical logic we can invoke it to conclude that $y = x$ and further use it to establish that $x = y$. Consequently $\forall F(Fx \leftrightarrow Fy)$ implies that $x = y$.

Cortes says that these proofs are question-begging because they both involve naming the objects in question and then *using* those names to say something about the objects. This amounts, according to Cortes, to assuming the objects are individuals. We conceptually treat the objects as individuals when we suppose that we can name the objects and use those names to talk about the objects separately. Cortes explains the problem as follows:

> [I]f I talk about a and then about b, and then again about a, I must suppose that conceptually I will not confuse a with b. That is, it would have to be *logically* possible to identify and reidentify a and b in order for me *validly* to make the assumption that a and b are conceptually distinguishable. But if it is logically possible to identify and reidentify a and b, then it must be the case that a and b differ in some respect which means that they do not share all properties in common. In other words if I know nothing whatsoever about two objects, then if the mere fact that I can name one 'a' and other 'b' allows me to infer that they are indeed distinguishable from each other, then it must be the case that the assumption that I can name one 'a' and the other 'b' implies that they are distinguishable. Thus, I have shown that *to suppose that the*

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[8] The λ operator allows us to abstract over a variable. The expression $(\lambda x)[f(x)]$ is understood as an expression waiting for a value to input for x. The λ operator binds the variable. We can then apply the expression $(\lambda x)[f(x)]$ to an argument a to get a value. So $(\lambda x)[f(x)]a = f(a)$.

> *object(s) under consideration are nameable is equivalent to assuming that*
>
> *they are INDIVIDUALS.*[9]

There is an important lesson here that has been noted by many authors since. In labeling objects for our convenience, we must be careful not to inadvertently draw metaphysical conclusions from the mere labeling itself.

Cortes also considers van Fraassen's proposal that particles with different histories would thereby be individuated. Cortes claims that if history individuates objects then it should be logically possible to determine the history of the objects. For example, consider bosons that come from distinct spacetime points A and B, completely overlap for a time, and then go off to distinct spacetime points C and D. Cortes claims that it is not logically possible to determine whether the boson at C originally came from A or B. Cortes does not elaborate and it is therefore unclear why this is not logically possible. Barnette understands Cortes as confusing epistemological with metaphysical issues (Barnette 1978), which would make it clear why Cortes thinks there is a problem. Although it is a metaphysical fact that the bosons in the example have different histories, once they overlap we are no longer in an epistemic position to identify the boson that arrives at C as the one that came from A (or B). We can describe one of the bosons as "the object which is identical with the object having history A" (Barnette 1978, p. 469). Whether the boson at C or D satisfies this description is a metaphysical issue. If Barnette is right, then Cortes' criticism fails. If not, then Cortes needs to clearly spell out why the temporary overlap creates a problem viewing bosons as individuals in a metaphysical sense. We can understand Barnette as claiming that all that matters is that nature is able to keep track of the bosons. We can then understand Cortes as claiming that there must be some

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[9](Cortes 1976, p. 498).

distinguishing feature during the period of overlap in order to make sense of the idea that nature is able to keep track of the bosons.[10]

From 1984 to 1991 there are three discussions of Margenau's arguments that draw attention to an important feature missed in the original discussion. Van Fraassen pointed out in 1984 that the separate states assigned to the fermions in Margenau's argument are mixed states (van Fraassen 1984, see also van Fraassen 1991). It was then noticed by French and Redhead that the mixed states are improper rather than proper mixtures (French and Redhead 1988). Noting that the states assigned to the fermions are mixed states allowed van Fraassen to characterize Margenau's argument as a dilemma: either PII is violated or quantum mechanics is incomplete. PII is violated if the mixtures are interpreted ontologically. However, PII is saved if we take the fermions to have distinct states and understand the fact that NRQM assigns them the same mixed state as a matter of ignorance/incompleteness. If we assume scientific realism, as I am, and also take quantum mechanics to be complete, then Margenau's argument does appear to establish that fermions violate PII.[11]

Margenau's argument can be viewed as a first attempt to provide a formal proof that NRQM violates PII. The proof was extended in (French and Redhead 1988), (Butterfield 1993), and reached its maximum level of generality in (Huggett 2003). Margenau's argument only applies to fermions and he only demonstrated that two fermions in an antisymmetric state have the same expectation values for monadic properties. So Margenau has, at best, only shown that fermions violate PII(3). French and Redhead extended the proof to show the same result is obtained for all observables and for both

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[10]See (Ginsberg 1981) for a criticism of Barnette on the basis of QFT. Also see (Teller 1983) for a suggestion that the issue is more complicated than Cortes, Barnette, and Ginsberg realize.

[11](Massimi 2001) raises an interesting challenge to Margenau's argument. Massimi argues that PII(3) isn't applicable to fermions in the first place and so cannot be considered to be violated. This is unimportant for my purposes as (i) PII(3) isn't a suitable candidate for individuating particles anyway and (ii) if Massimi is correct PII(3) still couldn't be used as an individuating principle even if it were suitable.

symmetric and antisymmetric states. Their proof that fermions are indiscernible with respect to their monadic properties is the same as that provided by Margenau.

Extending the proof to relational properties is straightforward. The probabilities we want to compare here are conditional properties. Consider an observable Q. I will write $P(a(Q = x)|b(Q = y))$ to mean the probability that a has value x for property Q conditional on b having value y for property Q. If we can show that

$$P(a(Q = x)|b(Q = y)) = P(b(Q = x)|a(Q = y)) \tag{3.7}$$

for fermions, then we have shown that fermions cannot be distinguished on the basis of relations. This is because it amounts to demonstrating that any relations the fermions can have with respect to measurable observables are symmetric. Conditional probability is defined as

$$P(A|B) = P(A \cap B)/P(B). \tag{3.8}$$

We can rewrite this as

$$P(A \cap B) = P(A|B)P(B). \tag{3.9}$$

Noting that $P(A \cap B) = P(B \cap A)$ we can write

$$P(B \cap A) = P(A|B)P(B). \tag{3.10}$$

We can therefore write

$$P(B|A) = P(B \cap A)/P(A) = P(A|B)P(B)/P(A). \tag{3.11}$$

Writing this in the same notation as above we have

$$P(b(Q = x)|a(Q = y)) = P(a(Q = x)|b(Q = y))P(b(Q = y))/P(a(Q = y)). \tag{3.12}$$

Now $P(b(Q = y)) = P(a(Q = y))$ by Margenau's original proof since these are the

probabilities that b and a have value y for the monadic property Q. So, we find that

$$P(b(Q = x)|a(Q = y)) = P(a(Q = x)|b(Q = y)). \qquad (3.13)$$

Therefore, fermions cannot be discerned on the basis of either monadic or relational

properties.

As might be expected, the calculation for bosons is even simpler. We just consider a pair

of bosons that are both in the same state $a(1)a(2)$. The numbers '1' and '2' are just to

distinguish the arguments for the two functions and have no physical import. With no

difference between the states obtaining different values for the expectation values of

observables isn't even a possibility. French and Redhead also demonstrate that there are

even paraparticle states that violate PII.

French and Redhead's results were then extended by Butterfield in 1993. Butterfield

considers the possibility of relations that are based on two observables. That is, instead of

looking at probabilities of the form $P(a(Q = x)|b(Q = y))$ for some observable Q, he

considers probabilities of the form $P(a(Q = x)|b(Q' = y'))$ for observables Q and Q'. He

then shows that $P(a(Q = x)|b(Q' = y')) = P(b(Q = x)|a(Q' = y'))$ and

$P(a(Q' = x')|b(Q = y)) = P(b(Q' = x')|a(Q = y))$ for both bosons and fermions.

Additionally, Butterfield also proved the same result while supposing the presence of a

third particle of different type. That is, Butterfield considered the possibility that two

particles of the same type might be distinguished by their relation to a third particle of a

different type. He demonstrated that this is not possible for either bosons or fermions in

certain states.[12] Finally, Huggett extends the proof to cover systems of *n* 'identical' particles.

I will not go into detail on any of these other calculations as the important point to note is that the conclusion that relations do not distinguish particles in the states in question is because the particles are only related by symmetric relations (at least as far as observables are concerned). Looking at the nature of proofs it is clear why they do not rule out the options discussed in chapter two for taking particles to be individuals. The proposal of weak discernibility is that even symmetric relations can serve to individuate as long as those relations are irreflexive. On the other hand, proposals involving primitive thisness are not ruled out because the proofs only consider discernment on the basis of observable properties. The challenge, then, from PII is simply that accounts of individuality not run afoul of the various proofs discussed above.

3.5   Conclusion

In this chapter we have seen that there are a number of issues to keep in mind when considering whether particles are individuals or not. The first three issues present challenges for the view that particles are not individuals. The first issue is that non-individuals, defined in terms of lacking self-identity, don't line up with our classical notion of cardinality. That in itself doesn't imply that particles must be individuals since we can understand this as just another case where quantum mechanics requires we revise our classical concepts. But defenders of non-individuality owe us an account of this revised concept of cardinality. The second issue is that there is a connection between the

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[12]Recall that I mentioned in chapter two that no one is claiming that particles are never discernible. They only seek to establish that particles sometimes violate PII in which case PII cannot be appealed to as a *general* individuating principle for particles.

features of particles that are taken to suggest non-individuality and the features of mass nouns. This raises the possibility that the features of particles that are taken to suggest non-individuality aren't even in need of explanation in the first place given that such features are expected for mass nouns. Finally, the third issue is that identity is so tied up with how we think that talk of non-individuals requires a large overhaul of our conceptual scheme. Consequently, we have pragmatic reason not to bother with talk of non-individuals even if there are such things. Pragmatic considerations aside this can also be viewed as spelling out some of the things defenders of non-individuality must do. They must provide a sensible conceptual scheme for thinking about non-individuals. One that explains why identity is ordinarily presupposed, why identity appears to be undefinable, and how to make sense of quantification in the absence of identity.

The challenges are primarily for the defender of non-individuality. But that is to be expected. Talk of non-individuals does require a substantial change to the way we tend to conceptualize things. Although this doesn't mean that the language of particle physics is in need of any change. As we saw in chapter two quantum theory appears to be neutral on whether particles are individuals or not. The conceptual revisions are only needed at the level of interpretation of the theory. It will take some time to identify all of the concepts that need revision to talk about non-individuals as well as time to provide suitable revisions. In the next chapter, when I start looking at what metaphysicians have had to say about individuality, it is likely that more challenges will arise. At the same time, looking at the work of metaphysicians will also likely provide some resources for the defenders of non-individuality to respond to those challenges as well as those discussed in the present chapter.

What does it mean to be an Individual?

We've seen that NRQM doesn't tell us whether particles are individuals or not let alone provide us with an account of what it means to be an individual. Redhead and Teller argue that the (allegedly metaphysically appropriate) Fock space formalism provides us no reason to think that particles are individuals due to the lack of labels. This isn't a positive argument for viewing particles as non-individuals unless combined with an argument to the effect that NRQM would tell us if particles were individuals. The argument from quantum statistics, on the other hand, can be understood as a positive argument in favor of non-individuals. A lack of individuality is taken to explain quantum statistics. However, this argument still cannot be understood as NRQM telling us that particles aren't individuals because there are other explanations of quantum statistics on the table compatible with the view that particles are individuals. Whether particles are individuals or not cannot simply be read off the theory.

How then are we to determine whether particles are individuals or not? The only option seems to be to find a way to define what it means for something to be an individual and see if the definition applies. However, since we are concerned with whether particles are individuals in a metaphysical sense, we cannot merely stipulate a definition. Instead we must look at clear cases of individuals and non-individuals and attempt to determine what it is that accounts for their individuality or lack thereof. That will be the focus of this chapter. But first I should spell out more precisely what the metaphysical issue is. One thing to note first though is that there is no guarantee that there is a universal concept of an

individual. However, in that case there would be no way to develop a definition of 'individual' that applies to all and only clear cases of individuals. In short, if there is no universal notion of individual then any attempt to find such a notion would be faced with unavoidable counterexamples. My focus on clear cases is, of course, because unclear cases are cases that can be contested. If something that isn't clearly an individual is claimed to be a counterexample to my definition of 'individuality' it would be reasonable for me to deny that it is a counterexample (although it may also be reasonable for me to accept it as a counterexample).

Gracia distinguishes between six philosophical issues involving individuality (Gracia 1988). Most of them won't be important for my purposes but I want to at least briefly mention each one. The reason for this is that much work on individuality, particularly prior to Gracia's work but also in subsequent works, conflates some of the issues Gracia identifies. It will be easier to point out such conflations if I at least outline each of the six issues. It will also help my readers avoid making such conflations themselves as well as arm my readers to identify any such conflations I may have made.

The first issue involves determining what it means to be an individual, that is, determining the intension of 'individuality'. As Gracia points out, it's unorthodox to use the terms 'intension' and 'extension' with respect to 'individuality,' but their use fits well enough and I will follow Gracia's terminology.[1] Gracia characterizes this first issue as a logical issue of conceptual clarification. However, I see it as a partially metaphysical issue. 'Individual' is defined ostensively. To determine what it means for something to be an individual we look at the things we refer to as individuals and attempt to figure out what they all possess that non-individuals lack. What are the necessary and sufficient conditions for us to refer to something as an individual? This certainly sounds like it's just a

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[1]The intension of a term is essentially just its definition where the extension of a term is the set of things to which the term applies.

conceptual issue. However, whether that's the case depends in part on whether our use of the term individual picks out, roughly, a metaphysical category of objects. It also depends on whether the project of conceptual clarification is revisionary or merely descriptive.

If our use of the term 'individual' were to exactly pick out a metaphysical category of objects then even a purely descriptive approach to conceptual clarification will tell us something about what it means, metaphysically, for something to be an individual. This is because we are finding necessary and sufficient conditions for objects to be a member of a metaphysical group. If, more realistically, our use of the term 'individual' only roughly maps onto a metaphysical group of objects then our project of conceptual clarification ought to be revisionary to some extent. At least if what we are really interested in is the conditions for an object's membership in that metaphysical group rather than our linguistic practices.

Does our use of the term 'individual' map on to a metaphysical category of objects? The best way to answer this question is to actually undertake the project of conceptual clarification and see if the necessary and sufficient conditions we come up with do pick out a metaphysical category of objects.

The philosophers of physics engaged in the debate take 'identity' as a necessary condition for 'individuality' in a metaphysical sense. Consequently, they accept that there is a metaphysical class of objects that is at least partially defined by possessing 'identity' (and another partially defined by a lack of 'identity'). Occasionally, philosophers of physics involved in the debate do offer a definition of individuality. For example, Krause and Arenhart define an individual as "something that is considered as *one*, distinct from any other individual, and which at least in principle can be reidentified in a different situation (within the same context) as being *that* same item" (Krause and Arenhart 2016, p. 62). Identity, unity, and the possibility of reidentification over time are all commonly proposed

as necessary components of individuality. The focus on identity, the fact that it is very rare to see a definition of individuality appear in the debate, and the fact that the definitions that do appear match the sorts of definitions offered by metaphysicians via 'conceptual analysis' indicate that the debate is about whether particles are individuals in the ordinary sense of the term. Given also that the debate is explicitly metaphysical, that is the debate is proceeding under the assumption that 'individual' does pick out, at least roughly, a metaphysical class of objects. Consequently, it is clearly worth paying attention to the metaphysical literature and examining possible definitions of 'individual'. That, as I already mentioned, will be the task of this chapter.

The next three issues that Gracia describes are more clearly metaphysical. First is the extension of 'individuality'. This involves determining if there are any individuals and distinguishing them from any non-individuals. How we would go about this depends on the relation between intension and extension. Does intension determine extension or vice versa? Or is the relation between the two more complicated than one simply fixing the other? I don't think we need to answer these questions to proceed. Regardless of the specific details of the relation between intension and extension, we can still learn a lot about the extension of 'individuality' from a study of the intension of 'individuality'. Gracia also agrees, noting the obvious point that anything that doesn't meet the necessary conditions for individuality can't be in the extension of 'individual'. Similarly, anything that meets the sufficient conditions for individuality must be in the extension. However, if something that we have good reason to consider an individual fails to be in the extension of 'individuality' then we have reason to reconsider our set of necessary conditions. Similarly for an apparent non-individual that appears to satisfy our sufficient conditions for individuality.

The next issue is the ontological status of individuality. This involves determining whether individuality is a property, a relation, a mental construct, or maybe something else. For

my purposes, this issue can be set aside. My main concern is with whether particles are in the extension of 'individuality' and with the intension of 'individuality' insofar as it helps us answer that question. However, the intension of 'individuality' may shed some light on this issue as well.

The last of the metaphysical issues is concerned with principles of individuation. This is concerned with what it is that makes something an individual (for example, this could be having a unique spatiotemporal trajectory or possessing primitive thisness). The main question here is "in virtue of what do individuals satisfy the necessary and sufficient conditions of 'individuality'?" I portrayed Gracia's first issue as a partially metaphysical one because the issue isn't merely one of picking a definition of 'individuality' that we like. That definition will be informed by features of the class of objects we call 'individuals'. However, finding the necessary and sufficient conditions for individuality doesn't tell us how individuals actually satisfy those conditions. That is what principles of individuation are for.

Finally, the last two issues Gracia identifies are discernibility of individuals (an epistemic issue), and reference to individuals (a semantic issue). I will not be concerned with either of these. Note that here when I say 'discernibility of individuals' the issue is specifically how knowers are able to distinguish between individuals. There are principles of individuation that are couched in terms of discernibility understood in a metaphysical sense. Discernibility in this case refers to objects possessing different properties or standing in different relations than other objects regardless of whether anyone could actually distinguish the objects on the basis of those properties or relations. Discernibility in this metaphysical sense will be discussed extensively.

The focus of this chapter will be on the intension of individuality. What does it mean to be an individual? What are the logical constraints on the notion of an individual? By asking these questions I am expecting to find a minimal notion of individuality that is universal.

But note that this is not incompatible with there being a variety of meanings of 'individual' in different fields. The reason for this has to do with ambiguity surrounding the word 'meaning'. I am not asking two questions here but only one worded in two different ways. What it means for something to be an individual can involve more than just logical constraints. For example, what it means for something to be an individual can involve metaphysical details about why the logical constraints are satisfied. I am assuming that the logical constraints on individuality are universal, but I am not assuming that that a complete answer to the question 'what does it mean to be an individual?' is universal. The definition of 'individuality' that I will be defending will be compatible with different fields of inquiry providing different answers to the question of what it means for something to be an individual *in that field*. The distinction is between logical and non-logical conditions. What I aim to provide is a set of logically necessary and sufficient conditions for individuality that specific scientific disciplines may augment with non-logical conditions. For example, what it means for something to be an individual in biology will be to satisfy the logical constraints on individuality that I will be defending *and* satisfying some, presumably, biological conditions.

My main result in this chapter is to conclude that relative indivisibility, formal immiscibility, and ontological autonomy are each necessary, and jointly sufficient, for individuality. However, in examining different possible requirements for individuality responses to the first three challenges raised in chapter three arise. First, since I conclude that identity isn't a necessary or sufficient condition for individuality, there is no reason why a definition of cardinality that presupposes identity would be unsuitable for individuals.[2] Second, if identity isn't a necessary or sufficient condition for individuality then Bueno's arguments based on the fundamentality of identity lose some of their force.

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[2]However, note that the presupposition of identity is still a problem for French and Krause's particular view since, for them, a non-individual is something that lacks identity.

Third, since I conclude that things referred to by mass nouns cannot be individuals it is no challenge to defenders of non-individuality that particle terms may be mass nouns rather than count nouns.

Before beginning to look at proposed candidates for necessary conditions of individuality, I want to quickly note two distinctions. First is the distinction between the problem of individuation and the problem of accounting for numerical difference. These are often taken to be the same problem. However, there is reason to think they are different issues. It seems logically possible for there to be a universe containing exactly one object and for that object to be an individual. It makes sense to ask in virtue of what is this object an individual despite the lack of need to account for numerical difference.[3] It is possible for a solution to the problem of individuation to provide an account of numerical difference. Indeed, in most cases where an author conflates individuation with numerical difference their proposal serves to account for both. Still, some of what is said for or against various accounts of individuality depends on conflating individuation and numerical difference. Consequently, it will be important to keep the distinction in mind. What's ultimately important here is that numerical distinctness is sometimes taken to explain individuality when instead it could be the case that either (i) individuality explains numerical distinctness or (ii) something else explains both individuality and numerical distinctness.

The second distinction is that between individuality and distinguishability. In our one object universe, there is nothing to distinguish our individual from. That something is an individual does not imply that it is distinguishable, even in a metaphysical sense, from other things. That isn't to say that there isn't a relation between individuality and distinguishability or that an account of individuality *can't* imply that individuals are

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[3]This point has been made by others, see, for example, (Castañeda 1975).

distinguishable under most circumstances. Even French and Krause state that "To say what an entity's individuality *consists in ...* is first to say that it consists in something having to do—metaphysically speaking—with that entity and no other and secondly to say what precisely that something is" (French and Krause 2006, p. 7). This is, in part, why they claim that particles lack self-identity rather than merely claiming that permutation invariance, and the associated indistinguishability, directly imply a lack of individuality. Still, in light of the distinction between individuality and distinguishability we can question the inference from indistinguishability to lack of self-identity.

4.1 Identity

The first candidate for a necessary component of individuality I will look at is identity. Since we're interested in metaphysical questions, we want a criterion of identity that tells us what the identity of objects consists in. As Lowe puts it,"[an identity criterion] purports to state ... the truth-conditions of canonical identity statements concerning individual Ks—statements such as 'This K is identical with that K'. Such statements may concern identity *over* time—diachronic identity—or identity *at* a time—synchronic identity" (Lowe 2005, p. 90). Our interest is in the metaphysical question of what makes identity statements true rather than the epistemological question of how we have access to that information.

There are, at least, two questions we have to address here before getting started. One is whether to adopt the metalinguistic view that the identity relation holds between labels (or names) or the metaphysical view that the identity relation holds between objects. On the former view, the identity relation is *only* a relation between labels, and it holds between two labels when the labels refer to the same object. On the latter view, the identity relation holds between 'two' objects when they are the same object. On the metalinguistic view,

then, when you learn that 'Hesperus' and 'Phosphorus' refer to the same object you've learned that there is a relation between the labels 'Hesperus' and 'Phosphorus'. But you haven't learned that Hesperus and Phosphorus are the same object. This is because a consequence of requiring that the identity relation only hold between labels, i.e. only holds at the level of language, is that we cannot engage in semantic descent to assert anything about the objects the labels represent. The metalinguistic view doesn't license any metaphysical conclusions including the obvious one that if two labels co-refer the labeled object is just one object. It was such bizarre features of the metalinguistic view that led to the metaphysical view.[4] Besides the issues with the metalinguistic view, the metaphysical view seems to be assumed by everyone involved in the debate. Indeed, the metaphysical view is required if one is going to conclude that particles lack individuality on the basis of their lacking identity. So I will set the metalinguistic view aside.

The second question is what kinds of identity there are that we might use to formulate conditions on individuality. On the one hand, we have the distinction between numerical, qualitative, and extensional identity. On the other hand, we have the distinction between synchronic, diachronic, and self-identity. Of the former distinctions, clearly numerical identity is the kind of identity at issue. Whether qualitative identity is important as well depends on the status of PII. With respect to the latter distinctions, it's unclear how many kinds of identity we actually have here. Is there an important metaphysical difference between synchronic identity and self-identity or are they the same kind of identity? While there is a clear epistemological difference between synchronic identity and diachronic identity it isn't obvious that there is an important metaphysical difference. If there is no difference between synchronic identity and self-identity and if synchronic identity accounts for diachronic identity, then there may be no interesting metaphysical distinction

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[4]This is most easily seen in (Frege 1948) where Frege discusses his reasons for abandoning the metalinguistic view in favor of the metaphysical view.

here. I will treat self-identity first since there are reasons to think it importantly different from synchronic or diachronic identity. Then I will treat synchronic and diachronic identity together.

4.1.1 Self-Identity

The reasons to think of self-identity as different from synchronic and diachronic identity primarily have to do with a distinction between merely logical relations of identity and metaphysical relations of identity. Lowe is one of the defenders of this distinction and he states that "[t]he [logical] relation of identity may be exhaustively characterized ... by two logical properties of that relation: its *reflexivity* and its governance by *Leibniz's law*" (Lowe 2016, p. 52). So, the logical identity relation is such that for any object x, $x = x$. And for any two objects x and y, if $x = y$ then x and y have *all* of their properties in common (in other words, numerical identity implies qualitative identity). Clearly it is numerical identity that is being characterized.

It seems to me that if we understand '=' as indicating numerical identity, then the claim that for any x, $x = x$ is the claim that for any object x (of kind K) it is *one* object (of kind K).[5] Reflexivity and Leibniz's law together imply that any object has the properties it in fact has. None of these claims are particularly interesting as they are logical truths. Hence why this is a merely logical relation.

Now, everyone agrees that claims of self-identity can appropriately be represented in the form $x = x$. In which case, everyone has to either accept that claims of self-identity are, at least in part, claims that the object in question is one object (of a particular kind) or they need to understand '=' as indicting something other than numerical identity. The other

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[5]Obviously, any composite object isn't *one* object in an unqualified sense, hence the restriction on kind.

options are, as noted before, qualitative identity or extensional identity. Anyone who wants to deny that particles have self-identity, like French and Krause, given that single-particle states have *one* particle (as in the particle number operator returns the value 1 when acting on these states), is going to have to go with one of the other options. It would undermine their position to accept that particles have self-identity in single-particle states, even if they maintain that they lack self-identity in multi-particle states. In that case, particles could oscillate back and forth between being individuals or non-individuals depending on whether they satisfy whatever other conditions on individuality one might want to pair with self-identity, if any.

Understanding '=' in terms of qualitative identity won't help French and Krause because then the claim that $x = x$ is the claim that x has all the properties it in fact has. It appears that $x = x$, for any x, understood in terms of qualitative identity is still a logical truth. In terms of extensional identity, $x = x$ is the claim that x has the very extension that it has. Again, there seems to be no reasonable prospect for denying that except in the case where x is something that doesn't have an extension. Since x will be denoting *a* particle in the cases of interest it is the case that $x = x$ is not true when '=' denotes extensional identity. But this is because the claim, and its denial, don't make sense for x's for which the concept of extension simply doesn't apply. Kind terms, like 'boson,' have extension but a particular particle doesn't have an extension.

If there is a way to understand self-identity such that its denial makes sense, French and Krause (and probably most of the other defenders of the non-individuality of particles) need to explain it. As it stands, French and Krause do take '=' to represent numerical identity in self-identity claims and so are committed to denying that the particle in a single-particle state is one particle (of whatever type it is); at least if they want to claim that single quantum particles are non-individuals (which they do). They could instead claim that particles are only non-individuals when in multi-particle states. Arguably, that's

the conclusion the argument from quantum statistics should reach anyway because it only concerns multi-particle states. But if single particles are individuals why conclude that they are not individuals in multi-particle states rather than claiming that their individuality isn't physically relevant to quantum statistics?

It is entirely unclear how to turn self-identity claims into metaphysical claims rather than trivial logical claims. Without showing how that can be done, there is no prospect for including a requirement of self-identity in a metaphysical account of individuality. While most defenders of non-individuality characterize it in terms of objects lacking self-identity, they have said virtually nothing about what it means to have (or lack) self-identity. The most that has been said is that French and Krause have claimed that it is a relation and that "the existence of the individual and the establishment of self-identity are conceptually on a par in that we cannot envision the possibility of one without the other" (French and Krause 2006, p. 14).This latter claim is made in the context of discussing whether individuals must exist before 'entering into' relations, including relations of self-identity. In that case, the claim means that the relata (the individual) isn't ontologically prior to the relation of self-identity (otherwise self-identity would be a consequence of, rather than a condition for, individuality). None of what French and Krause say here provides reason to think that self-identity makes sense understood as a metaphysical relation. This is especially problematic given that paying attention to the possible meanings of '=' in attempting to understand the possible meanings of claims of the form '$a = a$' results in tautologies.

One possible way to make sense of the idea that self-identity might not apply to particles is to say that it is indeterminate whether $a = b$ or $a \neq b$. There are at least two ways in which we could understand what such a claim means. One is to take it as meaning that the concept of self-identity is inapplicable here such that particles neither have nor lack self-identity. The other option is to say that it is metaphysically indeterminate whether the

particles have or lack self-identity. But neither of these approaches will save the Received View. If self-identity is a necessary condition for individuality, then the former approach implies that particles are neither individuals nor non-individuals whereas the latter approach implies that it is indeterminate whether particles are individuals or non-individuals. The former approach seems to favor a version of OSR in which there are no individuals or non-individuals whereas the latter would give ammunition to underdetermination arguments in favor of OSR.

A second issue with a metaphysical understanding of self-identity arises when we consider what we're looking for in giving a metaphysical account of identity. Identity, in the metaphysical sense, is what makes a *particular* object the very object that it is. It is something that we could reasonably call an 'individual essence,' and note that we needn't understand this in the metaphysically robust way normally associated with this terminology. An individual essence, in the sense of being that which an objects identity consists in, could easily just be a set of properties (I'll talk about bundle theories of individuality in the next chapter). How would we understand an individual essence as relational? In the logical sense, identity is a relation between labels. But in the metaphysical sense it seems strange to think of that in virtue of which an object is the very object it is in relational terms. The problem here is the same as that which motivates the distinctions between individuality, distinguishability, and numerical difference. A universe with a single individual is logically possible, so how could its identity (and its individuality) be grounded in something relational?

I don't mean to suggest that relational accounts of individuality are non-starters. But the relation in question must be able to obtain in a single object universe. This issue is likely part of the reason why the only relational accounts of individuality, prior to the debate about particles, are spatiotemporal accounts. In a single object universe, the object has a unique spatiotemporal trajectory. Since spacetime and the object are the only 'things' in

these single object universes it's unclear how we could have a relational non-spatiotemporal condition on identity (or individuality). The only option seems to be to say that the object bears some sort of relation to itself, but that just seems to be an inaccurate way of talking about monadic properties or some other sort of non-relational feature. Of course, composite objects can have relations among their parts, but we can imagine a universe with just a single classical point particle.

Note that because what we are concerned with here is logical constraints on a notion of individuality any suitable conditions must be compatible with any logically possible individuals. Hence why the mere logical possibility of a universe containing only a single classical point particle presents a problem for the possibility of a relational non-spatiotemporal account of individuality.

4.1.2 Synchronic and Diachronic Identity

From the preceding section one might already guess how synchronic identity (could) differs from self-identity. If one thinks that objects possess metaphysical identity (that is, identity conferred by some ontological component of an object) of some sort at a time, then that will be a form of synchronic identity but not self-identity. So, the question of this section is whether possessing some form of metaphysical identity (either at a time or over time) is a necessary and/or sufficient condition for individuality.

The claim that identity is a necessary and/or sufficient condition for individuality is sometimes made but it seems to have never been explicitly defended.[6] In the absence of such a defense I will confine myself to a brief discussion of some reasons to think that identity is neither necessary nor sufficient for individuality. Of course, one can conceive of

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[6]Gracia noted the same thing in 1988.

identity in a metaphysical sense in terms of bare particulars or haecceities. Certainly, views in which bare particulars or haecceities provide necessary (or necessary and sufficient) conditions for individuality have been defended. But I will discuss those views in the next chapter. Here my focus is on the view that metaphysical identity is required for individuality independently of any particular details about the nature of that metaphysical identity.

The first objection to the view that metaphysical identity is necessary for individuality that I want to look at comes from Woods (Woods 1968). Woods is one of the few to consider synchronic identity rather than just diachronic identity. Woods distinguishes between two types of questions. Type 1 questions are of the form 'How many Xs are there?' and type 2 questions are of the form 'Is this X the same X as the X which...?' These kinds of questions are related. To avoid double-counting in answering type 1 questions information relevant to answering some type 2 questions is needed. On the other hand, answering type 2 questions requires information relevant to answering type 1 questions since you want to know if there are one or two objects.

Type 2 questions can be reformulated as type 1 questions, and the most natural way to do so involves making use of what Woods calls a 'frame of enumeration'. For example, if one is looking at a church that is on the same site as a church their great-grandfather attended in the early 1940s, they might ask 'is this church the same church as the church which my great-grandfather attended?' This could, in this instance, be reformulated as 'How many churches have stood on this site since 1940?' This reformulation uses, in Woods terminology, spatial and temporal frames of enumeration.

Woods also distinguishes principles of individuation (P.I.) from criteria of individuation (C.I.). He claims that "[t]o possess a C.I. for a kind of object is to have a rule for answering Type 1 questions within a temporal frame of enumeration; to possess a P.I. is to have a rule for answering Type 1 questions within a spatial frame of enumeration" (Woods

1968, p. 127). On the face of it, type 1 questions are about individuation and type 2 questions are about identity. So, as Woods points out, it may seem counterintuitive to understand a criterion of identity in terms of answering type 1 questions. But the intuition is explained by the fact that we don't generally recognize that we can reformulate type 2 questions as type 1 questions.

Woods doesn't explain why he associates temporal frames of enumeration with criteria of identity and spatial frames of enumeration with principles of individuation. However, type 1 questions with only a temporal frame of enumeration are about diachronic identity. On the other hand, type 1 questions with only a spatial frame of enumeration are not about identity but about whether it is possible to individuate the objects in question in order to enumerate them. This has nothing to do with whether a given object has some sort of metaphysical identity. No one, for example, is arguing that particles must have metaphysical identity because we can enumerate them (ordinal counting, rather than mere enumeration, would, of course, be a different story).

With all of this in place we can ask whether a criterion of identity is either a necessary or sufficient condition for a principle of individuation. The answer is straightforward. A temporal frame of enumeration isn't needed to answer the question "[h]ow many animals are there in this cage now?" (Woods 1968, p. 128), consequently, a criterion of identity isn't necessary for a principle of individuation. We may dispute whether this question lacks a temporal frame of enumeration given the presence of the word 'now', but I don't think it matters. The question is about individuation (how many individuals are there are in the cage) not synchronic identity (do these individuals possess some form of metaphysical identity at a time).

A criterion of identity is, however, a sufficient condition for a principle of individuation. This is because in order to determine whether a is identical with b requires that we be able to individuate a and b. If we can't individuate a and b then we wouldn't be able to tell that

there are two objects even if there were. Suppose that clouds are individuals in a metaphysical sense such that when two clouds merge there are still two clouds. In order for someone who didn't see two clouds merge to determine that there are two clouds, they must be able to individuate the two clouds. They must be able to determine that this part (or these parts) belong to one cloud and this other part (or these other parts) belong to a distinct cloud. I don't mean to suggest that clouds are individuals, but it helps to illustrate that the ability to individuate is necessary to answer identity questions. But what is important here is the possibility of individuating the clouds in principle, not whether the epistemic issue of whether someone could actually do so.

Since Woods is concerned with the ability to individuate or make identifications, one might wonder what all this has to do with whether identity in a metaphysical sense is necessary for individuality. This has to do with the fact that Woods is concerned with what is logically required in order for individuation or identification rather than with how we might actually individuate or identify things. If a criterion of identity isn't needed for individuation, then we have reason to doubt that there is a metaphysical role for identity to play in grounding an objects individuality. In particular, if a criterion of identity isn't needed to *correctly* individuate objects, then it's possible to correctly individuate things that lack metaphysical identity. But if that's possible, then it's contingent whether all actual individuals possess metaphysical identity. Metaphysical identity can't be necessary for individuality (even if it happens that all actual individuals have metaphysical identity).

Because the claim that identity is a logically necessary and/or sufficient condition for individuality has been taken as obvious (hence the lack of explicit defense of this claim in the literature) there is also very little critical discussion. Other than Woods, the only other detailed criticism seems to be found in (Gracia 1988). But there is at least one more reason for the lack of critical discussion in the literature. The vast majority of discussion

in the literature is about metaphysically necessary and/or sufficient conditions tied to specific proposals for the metaphysical grounds of individuality. Those discussions are about principles of individuation rather than the intension of 'individuality' and so belong in the next chapter. It is also possible that it was assumed that logical conditions relating to identity were irrelevant to the question of whether something is an individual in a metaphysical sense. Lowe, for example, argues that self-identity is not the right sort of identity for such discussions precisely because he takes it to be a logical relation (Lowe 2016). Regardless of the reasons, however, there is virtually no discussion of the claim that identity is a *logically* (as opposed to metaphysically) necessary and/or sufficient condition for individuality.

Gracia is completely focused on diachronic identity because, prior to the idea that self-identity is important for individuality, the focus was always on diachronic identity. In particular, the ability for individuals to persist through various kinds of change. Gracia offers two reasons to think that diachronic identity isn't necessary for individuality and another two reasons as to why it isn't sufficient for individuality.

The first reason that diachronic identity isn't necessary for individuality is that the concept of an instantaneous individual doesn't appear contradictory (Gracia 1988, p. 40). An instantaneous individual can be understood either as a being that comes and goes out of existence in an instant or a non-temporal being. The Abrahamic God would, presumably, be a non-temporal individual (barring a way to make sense of God's eternal and unchanging nature in such a way that He could exist in time).

The other reason to think that diachronic identity isn't necessary for individuality is finding principles of individuation and finding criteria of identity are different issues. Gracia demonstrates this by noting that having a unique spatiotemporal trajectory can function as a principle of individuation, but it doesn't seem suitable as a criterion of (diachronic) identity (Gracia 1988, p. 40). An object isn't identical over time because it

has a unique spatiotemporal trajectory. It seems to be more the other way around. An object having a unique spatiotemporal trajectory is partly explained by its diachronic identity. If diachronic identity were a requirement for individuality, then it shouldn't be possible to formulate an acceptable (and, in this case, popular) principle of individuation that doesn't also serve as, or include as a component something that serves as, a criterion of identity.

Diachronic identity is also not sufficient for individuality because some putative non-individuals are identical over time, such as universals. Gracia's arguments in this section are all extremely brief, presumably because he has no particular defense of identity as a condition of individuality to critique. I'm not sure there's a non-controversial example of something that has diachronic identity but would generally be agreed to be a non-individual. Some other (potential) examples would be clouds or puddles, but some people, perhaps Jantzen, would see these as individuals. Universals might be the best example since all that is needed is that it be possible that universals exist and acceptance that universals aren't, or wouldn't be, individuals. Gracia seems to take it for granted that universals are not individuals (but note that he doesn't dismiss claims to the contrary as contradictory and so recognizes that there is a distinction between particularity and individuality). But anyone that accepts that there could be non-instantaneous non-individuals will have to accept that diachronic identity is not sufficient for individuality.

The final reason that diachronic identity isn't sufficient for individuality is also extremely brief. Gracia just notes that "[i]t is altogether possible to answer the question, "What makes S to be the same S through time?" by saying that S has not changed or has not changed much. But certainly that answer is not sufficient or even appropriate to answer

the question, "What makes S to be the individual it is?"" (Gracia 1988, pp. 40-41).[7] The first question isn't about why S is the very S it is, but just why it at a later time is the same S as at an earlier time. An explanation of an instance of diachronic identity isn't necessarily an explanation of individuality (even if it sometimes could be).

With a number of considerations against, and no considerations for, taking either synchronic or diachronic identity as necessary and/or sufficient conditions for individuality, it seems appropriate to conclude that identity, despite the undefended claims to the contrary, isn't important for individuality.

This is why I cautioned against reading Jantzen's cardinality argument and Bueno's fundamentality arguments as objections to the general view that particles are not individuals. If identity isn't a sufficient condition for individuality, then a definition of cardinality that presupposes identity isn't a problem for defenders of non-individuality. However, it is still an issue for French and Krause's view, which is shared by many others in the debate. French and Krause must explain how to understand self-identity as a metaphysical relation and, even if they do, they still have to address Jantzen's argument. Additionally, if identity isn't a sufficient condition for individuality then Bueno's fundamentality arguments don't imply that particles are individuals. However, Bueno's arguments do still challenge French and Krause's view.

In light of problems taking self-identity as a metaphysical relation necessary for individuality, the lack of positive reasons to take either synchronic or diachronic identity as either necessary or sufficient for individuality, the issues formulating a definition of cardinality that doesn't presuppose identity, and Bueno's arguments which draw attention to the fact that even concepts unrelated to the debate over individuality (e.g. logical quantifiers) need to be revised to support a view in which some objects aren't

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[7]This point is also made in (Lowe 2005, pp. 91-92)

self-identical, it seems that defenders of non-individuality should be looking for another way to understand non-individuality. Certainly, there seems to be nothing about permutation invariance that suggests that understanding it in terms of non-individuality *requires* understanding non-individuality in terms of a lack of self-identity.

4.2 Unity

The proposals for conditions of individuality that I will discuss in this section all have in common that they can be understood as claiming that individuals must be unified in some way. Either they must be indivisible in some sense or they must be distinct from everything else, I will look at each proposal in turn.

The view that individuals must be distinct from everything else is sometimes expressed in terms of individuals each counting as one object (in the sense that individuals are countable). This version of the distinctness view is never explicitly defended, the closest thing to a defense is provided by Lowe when he argues that number is a property of objects (Lowe 2009, pp. 42-51). While it does need to be the case that number be a metaphysical component of objects, or a consequence of metaphysical aspects of an object, in order for a criterion of countability to provide a metaphysical grounding for individuality, we are provided no reason to think this feature of objects has anything to do with individuality. Additionally, the primary objections (discussed in section 4.2.2) to viewing distinctness as a necessary condition for individuality applies equally well in its countability guise. Consequently, I will not explicitly discuss the countability version of the distinctness requirement.

4.2.1 Indivisibility

First, we should distinguish between two kinds of indivisibility. Gracia refers to them as 'absolute' and 'relative' indivisibility (Gracia 1988, 30). Something is absolutely indivisible if it cannot be divided. For physical objects this means that they can't be physically divided. For things like universals or tropes it's not clear how it should be understood. But a universal or trope is absolutely indivisible if it cannot be metaphysically divided, however we come to understand 'metaphysically divided' here. This isn't important for my purposes. It's clear that some individuals (e.g. people) aren't absolutely indivisible, so absolute indivisibility isn't a necessary condition for individuality.

On the other hand, if absolute indivisibility were a sufficient condition for individuality then, presumably, universals would be individuals. Restricting our attention to physical objects, if absolute indivisibility were a sufficient condition for individuality then particles would (likely) be individuals. It's not clear why this kind of indivisibility would be important for individuality, and it would be question begging to merely assert it as a sufficient condition. I mention it merely for a contrast with relative indivisibility.

Something is relatively, or formally, indivisible if it cannot be divided into other entities of the same kind. So, if an entity of type X can't be divided into two, or more, other entities of type X then it is relatively indivisible. Living creatures are, generally, relatively indivisible (exceptions would be creatures that satisfied the myth that splitting an earthworm in two results in two earthworms). Examples of entities that are relatively divisible includes bodies of water, clouds, and piles of rice.

Gracia considers two objections to the view that relative indivisibility is a necessary condition for individuality. The first objection is that we might consider a pile of stones to be an individual (Gracia 1988, p. 31). No reasons are offered for why we might think this,

and I have no linguistic intuitions one way or the other about whether a collection of individuals is itself an individual. A pile of stones is relatively divisible in that we can divide it into two piles of stones, so this would be a counterexample to the view that relative indivisibility is a necessary condition for individuality.

Gracia also considers a response to the objection. It could be that it is the particular quantity of stones that is an individual rather than the pile itself. That is, maybe the relevant type is, for example, 'pile of ten stones' rather than 'pile of stones'. A pile of ten stones can't be divided into two piles of ten stones, so this view allows piles of stones to be individuals and to be relatively indivisible. That there are ten stones is part of what makes a pile of ten stones the very pile of stones that it is. When considering whether an object is an individual one must be careful not to illegitimately abstract away from particularizing details. If something were merely an individual pile of stones, and not an individual pile of n stones, then the individual pile of stones would remain the same individual pile of stones if we removed a stone. But sets, collections, piles, etc... are defined in terms of their members. Removing a member results in a different set. There seems to be nothing else about the pile of stones we could point to as making it the very pile of stones that it is. The only other option is to characterize the individuality of the pile of stones in terms of its spatiotemporal trajectory in which case an individual pile of stones could remain the same individual pile of stones after losing (or gaining) a member. I will discuss such views of individuality in the next chapter. But, without moving to another view of individuality, there appears to be no alternative to the usual view of collections being defined by their members. Consequently, even if I accept that piles of (n) stones are individuals, it doesn't pose a problem for the view that relative indivisibility is necessary for individuality. The issue here has to do with whether the relevant kind is 'pile of stones' or 'pile of n stones'. If we take piles of stones to be individuals and take their individuality to involve which stones are in the pile, then the relevant kind with respect to concerns of individuality is 'pile of n stones'.

The other objection, which Gracia accepts, is that an infinite collection of things is both an individual and is divisible into other infinite collections (Gracia 1988, p. 31). A counterexample to the idea that relative indivisibility is necessary for individuality needn't be something that actually exists, but it must be something that could exist. So, to accept this objection we would need (i) reasons to think that an infinite collection could exist and (ii) reasons to think that an infinite collection of things would be an individual. As I stated previously, I think we need to have a worked-out account of individuality before we can answer questions of the latter sort. The former issue is controversial, and I can't devote the space to discuss it properly. But I will point out that to say that there are an infinite number of objects is, in part, to say that it is indeterminate how many objects there are. Infinity isn't a number and if there were a definite number of objects then there couldn't also be an infinite number of them. It is unclear whether it even makes sense for there to be an indefinite number of objects. And while an infinite number of objects implies an indeterminate number of objects, it's not clear that an indeterminate number of objects implies an infinite number of objects. So if we, for example, understood quantum states that aren't eigenstates of the particle number operator as states of indeterminate particle number, it's not obvious that Gracia could use this to claim that infinite collections could, and in fact do, exist.

Chauvier defends the view that relative indivisibility is a necessary condition for individuality (Chauvier 2016). However, he combines it with the requirement of formal immiscibility. An entity is formally immiscible if mixing two entities of the same kind together does not yield a larger entity of the same kind. Piles of rice are formally miscible since mixing them together yields a larger pile of rice. Rabbits are not formally miscible since 'mixing', or aggregating, them just yields an aggregation of rabbits rather than a larger rabbit.

The reason we need both conditions, according to Chauvier, is that being an individual has to do with 'the fact that [an individual] has its *own or proper boundaries* that separate it both from beings with which it coexists and from beings of lower levels (of which it can be composed) as well as from beings of higher levels (of which it may be a physical part)' (Chauvier 2016, p. 35). Chauvier disagrees with Gracia that piles of stones are individuals because piles of stones are relatively divisible. Note that my response to Gracia's first objection was premised on the assumption that piles of stones are individuals. It was because a pile of stones was being considered an individual that it was important to focus on particularizing details like the number of stones and which particular stones are part of the pile. If we don't think that piles of stones are individuals, then we have no reason to think that the description 'pile of stones' is in some way unsuitable.

Thinking of individuality in terms of individuals possessing boundaries that separate them from other entities gives us reason to be suspicious of the idea of relatively divisible individuals. If I divide a pile of rice into two piles of rice, then recombine them, how many piles of rice are there? Is there just one large pile of rice or does the large pile of rice have two smaller piles of rice as parts? If piles of rice are individuals, then presumably the answer is that we have three piles of rice after the recombination. The individuality of an entity shouldn't depend on where it is or how it is spatially related to other objects. I shouldn't be able to destroy an individual pile or rice by combining it with another one. I'm inclined to agree with Chauvier that a pile of rice isn't an individual and that the reason why is piles of rice don't have suitable boundary conditions for individuating them from other piles of rice. So I'm inclined to agree that an individual can't be both relatively divisible and formally miscible. So then piles of stones are also not individuals.

At this point it's open whether an individual could be relatively divisible but not formally miscible or the other way around. It is necessary that at least one of the conditions of relative indivisibility and formal immiscibility be satisfied in order for something to be an

individual, I think, but are both conditions necessary? Could there be an individual that is relatively divisible but not formally miscible or vice versa. Any mere collection of objects will be both relatively divisible and formally miscible. So finding a counterexample requires finding an entity that is not a mere collection, but that can be divided into two entities of the same kind without recombination being possible. Or finding an entity that is not a mere collection, but that can be combined into one larger entity of the same kind without redivision being possible.

If a given entity of type *X* is not a mere collection of *X*s, how could it be divided into *X*s? If an entity of type *X* doesn't have entities of type *X* as parts, then dividing it into parts would only yield entities of type *X* if the parts transformed into *X*s after being separated. But then that involves more than mere division. That is, it's not the mere act of dividing it that creates *X*s but an additional transformation event that occurs after the division. Consequently, this type of scenario doesn't seem to be a counterexample.

Is there an entity of type *X* that is not a mere collection but is made up of *X*'s? A stone seems to be an example. A stone can be broken into two stones which cannot be recombined in a single stone (at least not without adding an adhesive). Stone can be either a count noun and a mass noun, and the reason a stone can be broken into other stones is because of the connection between the two. Being a stone amounts to having a collection of stone that is held together by the appropriate chemical bonds. It's not a mere collection of stone because of this additional requirement. So, a stone is an example of something that is relatively divisible without being formally miscible. Is a stone an individual? If so, it's a counterexample to my (and Chauvier's) position.

Suppose I glue a bunch of rice together. Once the glue has dried the resulting object is relatively divisible without being formally miscible (additional adhesive is needed for recombination). It seems to me that this object only has contextual individuality. It's a mere collection that has been turned into something that behaves as an individual by

sticking the individual grains of rice together. It would become a non-individual again if the glue was removed. And note that the bonds in this case are chemical bonds as well. In order for the stone to have more than the contextual individuality of the glued pile or rice, there has to be a relevant different between the two cases. Since they both involve chemical bonds, the only difference one could appeal to is that a stone is naturally occurring where a glued together pile of rice is not. We normally think that artifact kinds, such as laptops, can be individuals. So, at minimum, artifact kinds that are not mere collections (or mere collections that have been glued together) can be individuals. So it's not clear why a stone being naturally occurring would be a relevant difference here. So either the stone and the glued together pile of rice are both individuals or are both non-individuals. I would say that they are both non-individuals but that they both possess a form of contextual individuality that explains why we would intuitively view the stone as an individual (and perhaps why one might view the glued together pile of rice as an individual as well).

What about an entity of type *X*, that isn't a mere collection, that could be combined with other entities of type *X* to create a larger *X*? Since *X* isn't a mere collection, combining two of them just results in two *X*s side-by-side. They would have to merge in a more intimate and transforming way than merely pushing two piles of rice together. Again, just putting the *X*s together doesn't create a larger *X*. It appears that there can be no such thing as an entity that is formally miscible but relatively indivisible since there can be no entity that is formally miscible that is not a mere collection.

It seems the only possible counterexample to Chauvier's position would be an individual collection (accepting my claims that things like stones only have contextual individuality).[8] I've already provided reasons above why an individual collection would

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[8]Of course this doesn't mean the Chauvier's position is immune to criticism. Criticisms needn't rely on counterexamples. One could, for example, argue that Chauvier's position gets the order of explanation

be relatively indivisible; they would be formally immiscible, on my view, for the same reasons. If one views it as necessary that individual collections are, or would be if mere collections were individuals, relatively indivisible and formally immiscible, then it doesn't even seem possible to object to Chauvier's dual requirement of relative indivisibility and formal immiscibility by providing an example of an individual that fails to meet it. That is, if one views my argument that individual collections would be relatively indivisible and formally immiscible as a purely logical argument (for example, by holding the view that abstracting away from particularizing details like the number of stones amounts to moving away from talking about the individuality of *that* pile of stones), then it looks like it may be impossible to formulate a coherent counterexample to Chauvier's position.

The apparent impossibility of finding a counterexample to Chauvier's requirements suggests that they are purely logical constraints that provide no metaphysical insight into individuality. In other words, the requirements of relative indivisibility and formal immiscibility seem to be constraints on the meaning of 'individual' such that it is incoherent to suppose that the term 'individual' could apply to anything that fails to satisfy these requirements. But note that it is not merely a failure to find a counterexample but the apparent *impossibility* of doing so that suggests that Chauvier's requirements are logical requirements.[9] It's unclear whether Chauvier views them this way. Although he does also require that individuals be ontologically autonomous (I will discuss this criterion in the next section), which could serve as the metaphysical grounds of individuality.

I myself am inclined to view the requirements of relative indivisibility and formal immiscibility as merely logical constraints. Whether one agrees with such a strong

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wrong. I.e. they could claim that individuality explains relative indivisibility and formal immiscibility rather than the other way around.

[9]It may be more appropriate for me to speak of conceptual requirements rather than logical requirements. But the point remains that the force of the argument is that one would be asserting something contradictory to suppose that there are individuals that are relatively divisible or formally miscible.

reading of the requirements, they do provide strong reasons to think that a *mere* collection of *X*s isn't an individual. Consequently, any entity we'd correctly refer to with mass nouns also wouldn't be an individual, contrary to Jantzen's claims. The members of a collection may still be individuals and collections that have additional individuating features may still be individuals. Any composite material object is an example of a collection that isn't merely a collection. Persons, laptops, and stones, for example, aren't merely collections of particles. Jantzen's argument would still cause some trouble for French and Krause's view as they do explicitly reject the idea that particle terms are mass nouns. Since fundamental particles aren't collections at all and composite particles aren't necessarily mere collections of their components, Chauvier's requirements are compatible with a particles-as-individuals view.

I should note that the requirements of relative indivisibility and formal immiscibility cut across the distinction between count nouns and mass nouns. While mass nouns will always refer to things that are relatively divisible and formally miscible (e.g. you can divide a body of water into two smaller bodies of water or combine two bodies of water in to one larger body of water), whether something is referred to by a count nouns won't tell us whether it's relatively indivisible or formally immiscible. For example, 'cloud' is a count noun, but clouds are relatively divisible and formally miscible. However, 'laptop' is also a count noun and laptops are not relatively divisible or formally miscible.

Finally, there is an ambiguity regarding the conditions of relative indivisibility and formal immiscibility. Take the condition of relative indivisibility. What is required to satisfy this condition? On the one hand we have the consequences of 'division' and on the other hand we have what the process of division involves. Another way to ask the question is to ask whether what is important for this condition as a condition of individuality is the consequences (one entity of a given kind becoming two entities of that same kind) or if what is important is something about the nature of the process of division.

The conditions of relative indivisibility and formal immiscibility are both kind relative. Chauvier prefers the phrase 'formal indivisibility' because both conditions are about whether the 'form' of a given kind is indivisible or immiscible. Consequently, it's not the nature of the process of division that's important. Since the conditions are kind relative 'division' can mean different things for different kinds. And while the word 'division' makes sense for the sorts of objects I've been using for examples the word 'division' may not be appropriate for other kinds of objects. However, that's not important. The conditions are about capturing the idea that individuals have clear boundaries that separate them from objects of the same kind rather than fuzzy boundaries that get blurred when 'division' or 'mixing' happens. So it's the consequences these conditions associate with 'division' and 'mixing' that matter.

For the condition of relative indivisibility, then, the issue is whether there is a way for an object to become two objects of the same kind in a way that blurs the boundaries that metaphysically separate the objects. For particles the issue is whether there is any physical process that starts with one particle of a given kind and ends with two particles of the same kind. To determine whether the process blurs boundaries in a problematic way we can consider whether the question 'which particle is identical to the original particle' makes sense or if that question is confused in the same way that asking that question about a cloud that's been divided is confused. Similarly, for the condition of formal immiscibility, the issue is whether there is any physical process that starts with two particles of a given kind and ends with one particle of the same kind. Again, we can determine whether the process blurs boundaries in a problematic way by considering whether it makes sense to ask which of the original particles is identical to the final particle. Whether these questions make sense will depend on the nature of the physical processes, if any, that take us from one particle to two or two particles to one. What isn't important is whether the words 'division' or 'mixing' are appropriate ways of describing the process. But, more importantly, the nature of the physical process isn't important for

determining whether the conditions of relative indivisibility or formal immiscibility are applicable (even though the nature of the physical process is relevant for determining whether the conditions are satisfied or not).

4.2.2   Distinctness

Another condition that has sometimes been proposed as a necessary, or necessary and sufficient, condition for individuality is distinctness. Views according to which particular properties, sets of properties, or distinguishing relations ground individuality will be discussed later. Here the claim is that mere distinction, aside from the details of any particular distinguishing features, is necessary, or necessary and sufficient, for individuality.

Gracia considers three reasons why one might think that distinctness is not a necessary condition for individuality. The first is simply that there appears to be no logical connection between individuality and distinctness (Gracia 1988, p. 34). Gracia provides no reason to think this. However, it seems clear insofar as the individuality of an object is something that pertains to that particular object whereas its distinctness is not.

The second reason Gracia offers, which I would take as explaining the first, is that the same considerations I provided at the beginning of the chapter in favor of distinctions between individuality, numerical difference, and distinguishability (Gracia 1988, p. 35). Distinctness is what is meant when people speak of metaphysical distinguishability. In our single object universe, that object can be an individual even though it cannot be distinguished (epistemically) from any other object and even though it is not distinct from any other object. For increased clarity in future sections, I will reserve 'distinguishability' for epistemic issues and 'distinctness' for metaphysical issues.

There are some objections to the use of this thought experiment, which, if correct, would also undermine the reasons I offered in favor of differentiating between individuality, numerical difference, and distinguishability. The first objection is that "arguments based on imaginary examples do not prove anything because we tacitly introduce in them the features they set out to prove" (Gracia 1988, p. 35). I agree with Gracia's response, which is simply that such examples are meant to show logical possibility rather than to prove anything.

The second objection is that "although the answer is supposed to operate under a condition that prescribes the non-existence of other entities in the universe, it functions under the tacit assumption of the possible existence of other entities" (Gracia 1988, p. 35). Gracia accuses these objectors of conflating psychological necessity with logical necessity. The idea seems to be that we can legitimately consider a universe that necessarily only has one object in it, but the objector is suggesting that we can't psychologically imagine such a universe. That is, we always imagine the universe could have more objects in it, even if we don't intend to. Gracia's point is that since we're trying to show that it's logically possible to have an individual that isn't distinct from anything else, all that matters is the logical possibility of a universe that necessarily contains one object. It doesn't seem to matter if we are psychologically capable of imaging such a universe unless logical necessity is, or reduces to, psychological necessity.

However, we could understand this objection in a different way. The idea could be that the object in our single object universe could be an individual in virtue of the fact that it would be numerically distinct from other objects were there any other objects. So the conclusion of the thought experiment is reached by ignoring the possibility of a modal understanding of the proposed distinctness requirement. This certainly weakens the argument. However, what matters here is whether it's reasonable to separate individuality

from distinctness. We have some admittedly weak considerations in favor of separating the two. However, we also have little but intuitions in favor of connecting the two.

The final objection is that the conditions of the example are violated when we introduce an observer to note that the solitary object isn't distinct from anything else (Gracia 1988, p. 36). Again, I agree with Gracia's response which is simply that this objection confuses "being in a universe" for "thinking about a universe".

The final reason for thinking that distinctness isn't a necessary condition of individuality is that distinctness seems to presuppose individuality. The alleged reason for this is that if *X* and *Y* are distinct then this is because *X* is not *Y* and if *X* is not *Y* then they are individuals. Gracia takes the argument to be question begging since defenders of the view that distinctness is necessary for individuality would presumably say that the distinctness of *X* and *Y* explains why *X* is not *Y* (Gracia 1988, p. 36). I agree but think there is another issue as well. Why think that if *X* is not *Y* then they are individuals? This would follow if distinctness were a sufficient condition for individuality, but what we're assuming is that distinctness is a necessary condition. The objection seems better understood as an objection to the view that distinctness is necessary and sufficient for individuality, although Gracia's charge that the objection is question begging still stands.

Gracia also offers one reason to think that distinctness isn't sufficient for individuality. This is simply that universals are, presumably, not individuals and yet they are distinct from one another. This isn't particularly compelling without reason to think that universals wouldn't be individuals. One could still claim that distinctness is a sufficient condition for the individuality of material objects, however, it would be question begging to do so here without some positive reasons in favor of doing so. If distinctness were a sufficient condition for individuality, then particles would be individuals at least some of the time since electrons, say, are sometimes both distinct and distinguishable from each

other. I think Gracia is right that distinctness has to do with different issues than individuality and that any view that intimately connects them would be confused.

4.3   Ontological Autonomy

As I mentioned above, Chauvier adds a requirement of ontological autonomy in addition to relative indivisibility and formal immiscibility. An object is ontologically autonomous when it has its own being. The standard example is that Socrates' nose, unlike Socrates, is not ontologically autonomous. The example already gives us one reason to think that Chauvier's other requirements are not sufficient for individuality since they imply that Socrates' nose would be an individual. Noses are relatively indivisible and formally immiscible and yet they don't seem to be individuals, at least not when part of someone.

Ontological autonomy is important because individuals should be "*ontologically* separated from the rest of the things, in contrast with simply being *cognitively* separated from the rest of the world by an act of demonstrative focusing" (Chauvier 2016, p. 32). The logical requirements of relative indivisibility and formal immiscibility, in addition to not being jointly sufficient for individuality, don't provide us the resources to find a metaphysical ground for individuality. However, given Chauvier's characterization of individuality in terms of things having their own proper boundaries, ontological autonomy makes sense as the metaphysical counterpart to his logical requirements. Consequently, the claim that ontological autonomy is a condition for individuality receives some support from the defense of the logical requirements.

Chauvier's claim that ontological autonomy is a necessary condition for individuality inherits the main problem any sort of claims about ontological autonomy must deal with.

The primary issue is explaining what ontological autonomy amounts to. When is an object ontologically autonomous? Chauvier distinguishes ontological autonomy from existential independence. The latter applies to objects whose existence is not connected in any way whatsoever to other objects. But individuals are connected in a number of ways to other objects including existentially. Paradigm examples of individuals, such as a given person, depend on their parents for their existence. Any requirement of ontological autonomy can't, therefore, mean being causally or physically isolated. Chauvier defines ontological autonomy as "not needing to be a proper part of something in order to exist" (Chauvier 2016, p. 33).

If we now distinguish between attached and detached noses, the requirement of ontological autonomy implies that an attached nose can't be an individual whereas an unattached nose could be as long as it meets the other requirements of relative indivisibility and formal immiscibility (which it does). The requirement of ontological autonomy, then, prevents us from counting arbitrary undetached parts as individuals, while allowing us to count detached parts as individuals.

Chauvier's view then, is that relative indivisibility, formal immiscibility, and ontological autonomy are each necessary for individuality. They are also jointly sufficient for individuality. This is my view as well. Although, in the case of ontological autonomy, unlike the other requirements, I have little to add to Chauvier's discussion. I think that defending the requirements of relative indivisibility and formal immiscibility provides some support for ontological autonomy, so my additional defense of the other requirements, then, adds additional support for this aspect of Chauvier's view.

4.4   Non-instantiability

The final candidate for a condition on individuality is the one that Gracia defends. There have been two other proposals that have not been defended by any contemporary writers, but I'm not going to discuss them in detail. The first of these is that it's a necessary and/or sufficient condition for individuality to belong to a species consisting of multiple individuals. It's not clear why somethings being the very individual it is would require that there be other individuals of the same kind. It's also unclear why we should think that any member of a kind that has multiple instances would thereby be an individual. Gracia notes that this view had been defended in the medieval period, but there seem to be no defenders since and the view seems to confuse individuality with numerical difference.[10]

The latter of the two proposals is the view that impredicability is a necessary and/or sufficient condition for individuality. If impredicability is a feature of language, then it can't provide a condition for individuality if individuality is understood as a metaphysical feature of the world. If impredicability is instead a metaphysical relation between properties and objects, then this condition on individuality implies that objects are bare particulars in which properties inhere (mirroring the linguistic subject-predicate distinction). I will consider the bare particular view in the next chapter, but for now I will just say that proposing a criterion of individuality that implies a very specific metaphysical view is suspicious. Rather than providing a logical condition on what it means for something to be an individual the position smuggles in metaphysical assumptions.[11]

The view that Gracia holds is that non-instantiability is a necessary and sufficient condition for individuality. Instantiability is a feature of properties. If an object possesses

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[10]See (Gracia 1988, pp. 37-38) for additional discussion.
[11]See (Gracia 1988, pp. 41-43) for further discussion.

property X then that object is said to instantiate property X. The ability of X to be instantiated can be called 'instantiability'. Most properties can be instantiated multiple times (e.g. there are multiple red objects). However, some properties can only be instantiated once, such as ordinal properties (e.g. the property of being the first person on the moon). Objects or substances cannot be instantiated since the relationship, say, a book has with its redness is not the kind of relationship a book could have with another object.

On Gracia's view, the fundamental feature of individuals is non-instantiability whereas individuals themselves are non-instantiable instances (of instantiables). Universals, on the other hand, are instantiables. Gracia characterizes individuals as non-instantiable instances rather than merely instances because 'instance' is sometimes used to refer to instantiables. For example, "human being" is instantiable but might also be considered an instance of "animal." (Gracia 1988, p. 47). This view, according to Gracia, overcomes the problems we saw with the other proposals. Gracia's view allows for both instantaneous and atemporal individuals, unlike views that take diachronic identity to be a necessary condition for individuality. Gracia's view allows for the possibility of individual infinite collections that can be further divided into individual infinite collections, unlike the indivisibility view. Gracia's view allows for universes containing only a single individual unlike the distinctness and species views. Finally, Gracia's view doesn't imply that objects are bare particulars unlike the impredicability view. Gracia doesn't offer any reasons in favor of his view beyond his claim that his view avoids the problems discussed above. Non-instantiability is a feature of everything other than properties and so Gracia's view implies that everything that isn't a property is an individual. Gracia also doesn't appear to offer any reason as to why non-instantiability explains individuality as opposed to individuality explaining non-instantiability. His argument for why we should accept his proposal seems to be (i) all individuals are non-instantiable, (ii) viewing non-instantiability as a necessary and sufficient condition for individuality avoids the

problems faced by alternative proposals, therefore (iii) we should accept that non-instantiability is a necessary and sufficient condition for individuality.

I would like to point out that Chauvier's view also avoids all of the issues that Gracia's proposal avoids with the exception of the possibility of individual infinite collections. However, I have provided reasons to think that *mere* collections are not individuals. If a given infinite collection of *X*s forms an individual it is not merely because it is a collection of *X*s. Rather it would be because the infinite collection falls under a kind whose individuation conditions render objects of that kind relatively indivisible and formally immiscible.

Gracia considers two objections to the view that non-instantiability is a necessary and sufficient condition for individuality. The first is that contradictory properties such as "round-squareness" are non-instantiable and yet not individuals (Gracia 1988, p. 46). Gracia's response is that a theory can't be considered a failure merely because it fails to apply to contradictions. He points out that the property of "round-squareness' can't be an individual because it is composed of two universals and that it cannot be a universal because it's non-instantiable. If it's not an individual (particular) or universal, then what is it? Gracia says that the puzzle arises due to the illegitimate attempt to "apply to contradictions categories that do not apply to them" (Gracia 1988, p. 47). While Gracia views this objection as making a category mistake insofar as it attempts to treat contradictions as being either individuals or universals, we can go further and say that the problem, ultimately, is trying to treat contradictions as components of our ontology. At any rate, I agree with Gracia that this isn't a problem for his view.

The second objection is that non-instantiability can't be a necessary condition for individuality because cloned organisms are both instantiable and individual. Gracia's response is that "[c]loning is not a case of instantiation but rather a case of reproduction" (Gracia 1988, p. 47). It seems pretty clear that this is the case; the objection seems to have

been raised for the purposes of clarification rather than as an objection someone might actually raise. For a discussion of particular ways in which cloning and instantiation differ see (Gracia 1988, p. 47).

Gracia distinguishes non-instantiability from multiple non-instantiability. The latter allows for something to be instantiated once and only once. For example, if the Abrahamic religions are correct, then divinity would be multiply non-instantiable, but not simply non-instantiable, if it only applies to God (Gracia 1988, p. 48). If multiple non-instantiability were the necessary and sufficient condition for individuality, then divinity, in this case, would be an individual rather than a universal. However, since divinity is instantiable (though limited to one instance), divinity in this scenario still counts as a universal on Gracia's view. Similarly, one shouldn't view the negation of multiple instantiability as the necessary and sufficient condition for individuality because, for example, "the last Dodo bird" is not multiply instantiable and is a universal rather than an individual. Problematically, Gracia's arguments regarding the distinction between non-instantiability and multiple non-instantiability rely entirely on his undefended claim that universals are not individuals.

Gracia's proposal does seem to face one of the issues already mentioned. I claimed that one reason to reject the impredicability view is because it smuggled in a metaphysical viewpoint. Gracia doesn't actually view that as a problem. His issue is rather that the impredicability view is unacceptable because the bare particular view is unacceptable (as an account of individuality). I however, think that a discussion of the meaning of individuality should be at least somewhat limited in its metaphysical commitments since it's supposed to be a logical exercise. Gracia's proposal seems to depend on the view that properties can only be understood as instances of universals. As long as it is possible to understand properties as tropes, then it is possible to have instantiable individuals. Tropes are particulars and, as such, are candidates for individuality. Not only that, proposals for

necessary and sufficient conditions of individuality shouldn't simply commit us to a particular metaphysical position. I'm not claiming that we couldn't have necessary and sufficient criteria of individuality that do settle the metaphysics. But they shouldn't settle it so straightforwardly and simply. We need to assess whether tropes are individuals independently and only accept Gracia's view if it turns out that tropes aren't (or wouldn't be) individuals.

So, ruling out Gracia's view in light of its unsupported metaphysical commitments, we are left with only Chauvier's conditions of relative indivisibility, formal immiscibility, and ontological autonomy.

4.5 Conclusion

We might think that conditions of individuality might vary by kind of object or might depend on features of the universe in which the object exists. For example, Gracia claims that "[d]istinction is a necessary condition of individuals in a universe where there are more individuals than one. Division is a necessary condition of individuals that belong to a species where more than one individual is possible. Identity is a necessary condition of individuals that endure in a world of time and change... Finally, relative indivisibility is a necessary condition of material individuals" (Gracia 1988, p. 50). I think this gets things backwards. Rather than these being necessary conditions for individuality under certain circumstances, we should view them as features of individuals under those circumstances. That is, instead of viewing diachronic identity as a necessary condition of individuals in a world like ours, we should view diachronic identity as a necessary *consequence* of the fact that material individuals in our world exist in spacetime. We could say the same thing regarding numerical distinctness.

Division also seems better understood as a necessary consequence of the existence of individuals that are part of a species. It is because there are multiple individuals that form a given species that the species is divisible into multiple individuals. It is not because a species is divisible into multiple members that those members are individuals.

Diachronic identity is also not a necessary condition of individuality for material objects in a world like ours. First, nothing about our world seems to preclude the existence of instantaneous individuals. Second, it is because a given material object is an individual that exists in spacetime that it has diachronic identity rather than the other way around. It is because a given material object is relatively indivisible and formally immiscible that we can distinguish it from other objects of the same type and assign them unique spatiotemporal trajectories. If material objects weren't relatively indivisible and formally immiscible, then we couldn't assign them unique spatiotemporal trajectories under *all* circumstances any more than we could do so with clouds.

My view, then, is that the *logically* necessary and sufficient conditions for individuality don't vary by context or by kind of object. What does vary, however, is the kinds of inferences one can make on the basis of combining these conditions with other information. For example, if one knows that a given object is both an individual and has a unique spatiotemporal trajectory, then one can conclude that the object has diachronic identity. As a result, different fields of inquiry could reach different conclusions about what features individuals in those fields have beyond satisfying the logical constraints. For example, it could be the case that certain biological facts imply that everything in biology that satisfies the constraints on individuality also has certain additional features in common. However, it would not be the case that biological individuals have these features in common because they are constraints on individuality but rather because being an individual partly explains (in conjunction with certain biological facts) why they have those features. Consider a group of objects that are (i) all individuals and (ii) are all

non-instantiable. They could all be individuals because they are all non-instantiable, or they could all be non-instantiable because they are all individuals, or it could be a coincidence. My point is, if all members of a group have two things in common there are two ways the order of explanation could go (assuming one feature does explain the other). Something that all biological individuals have in common could be interpreted as a condition on what it means for something to be a biological individual or it could be taken to be something that follows from being an individual with certain biological features. I will say more about this point in chapter five, but my view allows individuals in different fields to have different features. It only denies that those features are, strictly speaking, defining features of individuality.

Individuals in biology, for example, may all have certain features that individuals in other fields do not. Strictly speaking, those features wouldn't have anything to do with why the individuals in question are individuals. In other words, the differences in the features of individuals in different fields would be differences in what follows from individuality in that field. Additionally, the logical constraints can be supplemented with metaphysical constraints. So while something may be an individual because it is relatively indivisible, formally immiscible, and ontologically autonomous it being an individual of a specific, say, biological kind could involve additional metaphysical conditions. Regardless of the kind of object or the sort of universe in which it exists, the necessary and jointly sufficient conditions for individuality are relative indivisibility, formal immiscibility, and ontological autonomy.

One final thing before moving on to the next chapter. I spent a lot of time here discussing other people's objections to the various positions I examined here. In some cases, I had additional points of my own to make or responses to those objections. But in a number of cases I had nothing to add. As a result, it may be unclear precisely what my contributions are. Consequently, I will briefly list them.

The discussion of self-identity is almost entirely original. In particular the examination of the possible ways to understand what it means for something to be self-identical and why none of those ways seem to support the view articulated by French and Krause (and accepted by many others) that a lack of self-identity is a sufficient condition for non-individuality (and a necessary condition for individuality).

The responses I outlined to the three challenges to non-individuality discussed in chapter three are my own. These are, again, that discounting identity as a necessary or sufficient condition for individuality implies that non-individuals could possess identity in which case a definition of 'cardinality' that assumes identity isn't thereby unsuitable for non-individuals. Additionally, the lack of role for identity in defining individuality takes some of the force out of Bueno's arguments. Finally, the conclusion that relative indivisibility and formal immiscibility are requirements of individuality implies that mass nouns do not refer to individuals undermining Jantzen's argument that quantum particles may be individuals in virtue of particle terms functioning as mass nouns.

The Metaphysical Grounds of Individuality

Having looked at what it means for something to be an individual there is still one final thing to do before I can examine whether particles are individuals. I must consider the question "in virtue of what do individuals satisfy the necessary and sufficient conditions of 'individuality'?" One possibility is that it's merely a brute fact that anything that satisfies the necessary and sufficient conditions for being an individual is an individual. In that case, I could simply argue that particles do satisfy these conditions and are, therefore, individuals. However, the problem with that approach is that we can give metaphysical accounts of why various kinds of individuals do satisfy the conditions of 'individuality.' For example, it is commonly argued that classical particles are individuals whose individuality is grounded in their unique spatiotemporal trajectories. If I can provide a metaphysical explanation of why some objects count as individuals and some other objects are not individuals, then it appears ad hoc to claim some objects are or are not individuals without also providing an explanation of why that is the case. If it were a brute fact whether an object was an individual or not, we would not expect to be able to provide explanations as to why some objects are individuals.

A second reason why the 'brute fact' approach might be undesirable is that what makes something an individual may depend on the kind of thing it is. What makes a biological organism an individual may not be the same as what makes a particle an individual. For

example, the most common principles of individuation appealed to regarding biological

organisms involve physiology or natural selection rather than uniqueness of

spatiotemporal trajectories (Guay and Pradeu 2016). Providing a general account of the

necessary and sufficient conditions of 'individuality' is compatible with a variety of

metaphysical explanations as to why objects of a given kind do or do not satisfy those

conditions.

I will look at three general sorts of principles of individuation in this chapter. First, I will

consider principles that appeal to relations as individuators. My primary focus here will be

on certain kinds of spatiotemporal accounts as these are the only well-developed

(non-structuralist) relational accounts of the metaphysical grounds of individuality. Next, I

will look at accounts that take qualitative monadic properties as individuators. These will

be bundle theory accounts (objects are individuated by some set, or bundle, of monadic

properties). Finally, I will examine the possibility that some objects are individuated by

non-qualitative properties. Unlike the first two sorts of principles of individuation, there is

more internal variety in this last category. In particular, there are accounts in terms of bare

particulars, haecceities, matter, form, and existence. Finally, I should note that this chapter

isn't meant to be exhaustive. One could write an entire dissertation, and more, on the

metaphysical grounds of individuation. My goal here is to highlight problems with the

sorts of metaphysical grounds empiricists are most likely to favor and defend the sorts of

grounds they are likely to dismiss. The purpose is to highlight the need to pay attention to

sections of the metaphysical literature that have been largely ignored in the debate over

the individuality of particles.

As stated at the beginning of chapter four in my discussion of the various sorts of

questions one can ask about individuality, questions about principles of individuation are

questions about the metaphysical grounds of individuality. These questions are meant to

answer what it is about the world that confers individuality on an individual. In evaluating

the various proposals for metaphysical grounds of individuality I will, of course, be considering traditional sorts of objections. Additionally, I will be considering whether they are compatible with the conditions I defended in the previous chapter.

But there is also a third criterion of evaluation I will be using that has not been considered in previous debates. That is whether a given account of the metaphysical grounds of individuality is compatible with the existence of non-individuals (including composite non-individuals). The recent literature is almost devoid of any discussion of the possibility of non-individuals prior to the idea that quantum particles may be non-individuals. Many accounts of the possible metaphysical grounds of individuality simply assume that everything is an individual or, at least, that every composite, physical object is an individual. As a result, many of the proposals do not even allow for the possibility of composite, physical objects that are not individuals. In the context of the question of whether quantum particles are individuals or non-individuals these accounts are question-begging. To avoid this charge these accounts would need to be paired with an argument to the effect that the existence of physical non-individuals is *impossible*. With serious and sensible proposals for the possibility of composite, physical non-individuals on the table we must consider whether the accounts of the possible metaphysical grounds of individuality in the literature treat these proposals fairly or dismiss them out of hand.[1]

With all of this in mind I now turn to the first category of proposals: individuation on the basis of spatiotemporal relations.

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[1] I realize this is a somewhat anachronistic way of putting of the point. It would be unfair of me to characterize the authors who have defended the various positions I'll be looking at as treating proposals for the existence of physical non-individuals unfairly when there were either no such proposals at the time of their writing or, at least, no such proposals that they could reasonably have been expected to be aware of.

5.1   Are Objects Individuated by Spatiotemporal Relations?

I mentioned in the preceding chapter that the possibility of a universe with a single, non-composite individual seems to limit relational accounts of individuation to spatiotemporal accounts. There is nothing other than the individual and the background spacetime (or space and time) to be related. Consequently, the focus of this section will be on spatiotemporal relations.[2]

The idea that spatiotemporal relations individuate quantum particles is definitely compatible with Bohmian mechanics. However, it is unclear if spatiotemporal relations could serve to individuate quantum particles on other interpretations. At the very least, existing accounts of spatiotemporal relations would need some modification to apply them to quantum particles. Since it is beyond the scope of this work to concern myself with the various interpretations of quantum mechanics, I will not be considering the possibility that particles are individuated by spatiotemporal relations. However, it is still worthwhile considering how spatiotemporal relations individuate. If spatiotemporal relations aren't good candidates for metaphysical individuators then we might conclude that whether particles have well-defined spatiotemporal trajectories isn't relevant to determining whether they are individuals or not. On the other hand, if spatiotemporal relations are good candidates for metaphysical individuators then we might conclude either (i) that particles cannot be individuals unless Bohmian mechanics is the correct interpretation or (ii) that something else serves to individuate particles. If it turns out that a sensible spatiotemporal account could be developed for non-Bohmian interpretations, then a third

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[2]If spacetime is relational, then a universe with a single, non-composite individual seems to rule out spatiotemporal accounts as well. At least if such a world wouldn't have a spacetime. But if spacetime is relational there is a circularity concern about appealing to spacetime relations to individuate objects.

option would be that particles are individuals individuated by spatiotemporal relations independently of the correct interpretation of quantum mechanics.

There isn't much to say directly in favor of spatiotemporal relations as individuators. It seems intuitively obvious that unique spatiotemporal trajectories, at least, can serve to individuate objects. Such accounts are defended primarily, or, as far as I can tell, entirely, by responding to criticism. As a result, this section will proceed by looking at a series of criticisms of spatiotemporal accounts of individuation and possible responses. But first, for reference, I will briefly describe two spatiotemporal accounts of individuation.

The first account is from Karl Popper who states that "[i]f the body A lies within region P, and the body B lies with region Q, and if P and Q are separated by a gap, then A ≠ B" (Popper 1953, p. 110). Popper defines being "separated by a gap," or being "disconnected," and being 'connected' as follows: "The regions P and Q are called 'connected' or 'overlapping' if and only if they have at least one sub region (consisting of one point at least) in common; otherwise they are called "disconnected" or "separated by a gap" (Popper 1953, p. 110). A and B are individuated if they occupy disconnected spatial regions and, in virtue of being individuated, are also non-identical. Popper views this as a sufficient condition for individuality. Poppers terminology is a bit misleading as it suggests a connection between distinguishability and individuality that Popper doesn't endorse. In particular, he isn't suggesting that spatial separation is a sufficient condition for distinguishing between individuals, but only that it is a sufficient condition for something *being* an individual.

Before arriving at his final criterion of individuation Popper begins with a simpler criterion that he modifies in order to respond to a criticism. The criticism is that in attempting to formulate a criterion of spatiotemporal individuation one may end up simply replacing "the problem of the *difference of bits of matter* by that of the *difference of spatial regions*" (Popper 1953, p. 106). The criterion provided above does not require that

spatial regions be individuated as we can sensibly talk about whether spatial regions are connected or disconnected regardless of whether it even makes sense to talk about spatial regions in terms of individuation.

Understanding Popper's criterion as a metaphysical criterion of individuality implies that any objects that occupy disconnected spatial regions at a given time are individuals. This implies that clouds, for example, are sometimes individuals and sometimes not. It implies the same for quantum particles on non-Bohmian interpretations of quantum mechanics. Although Popper's proposal can be viewed as properly metaphysical in the sense that he isn't conflating epistemology and metaphysics, it does imply that anything we can distinguish on the basis of spatial location is an individual. It leaves little room for the possibility that there are things that we can individuate (at least some of the time) that are not individuals. In particular, this spatiotemporal account isn't a threat to the idea that quantum particles are individuals (at least some of the time) even if it is correct.

Although this view was first defended almost 70 years ago, it was defended as recently as 1989 by Legenhausen (Legenhausen 1989). Legenhausen defines a property F as "$Fx =_{df}$. There is a location l and an object y such that x occupies l and y does not occupy l" (Legenhausen 1989, p. 632). Legenhausen does not explicitly define separation or disconnectedness. He does, however, note that his definition also avoids the criticism Popper considered, saying "[u]nlike other accounts which explain non-identity in terms of difference in location, there is no appeal in this proposal to antecedently defined thisnesses or individual locations" (Legenhausen 1989, p. 632). I should point out that both Popper and Legenhausen accept that this criticism is a good one. In fact, it is their acceptance that accounts in terms of objects possessing unique spatiotemporal trajectories presuppose the individuality of spacetime points that motivates them to develop their spatiotemporal separation accounts. It's not entirely clear that Legenhausen's version of the view does avoid the criticism. What individuates objects is supposed to be their spatial separation

hence the important part of property F is that *x* and *y* are spatially separated. However, Legenhausen is significantly less careful than Popper who explicitly defines spatial separation in such a way as to avoid reference to locations.

The second account states that "A at t1 at p1 is identical with B at t2 at p2 provided that there is a continuous spatial path between p1 and p2 on every pair of neighboring points of which there are closely qualitatively similar things at neighboring moments of the interval from t1 to t2 [and provided that all of the objects on that path are of the same kind]" (Quinton 1973, p. 67). The requirement that the objects at 'adjacent' points on the trajectory be qualitatively similar is meant to allow for the possibility of gradual change over time while disallowing significant instantaneous changes. The restriction on kind is there because the criterion would otherwise imply that, for example, a log that's been burned is identical to the pile of ashes that it becomes. Quinton is offering a criterion of diachronic identity and defining individuals in terms of objects that satisfy that criterion. Quinton's view implies that quantum particles are not individuals.

The first sort of spatiotemporal account of individuation in terms of separation implies that quantum particles are individuals at least some of the time (or all of the time on the Bohmian interpretation). Whereas the second sort of account in terms of the possession of a unique spatiotemporal trajectory may imply that quantum particles are not individuals unless Bohmian mechanics is the correct interpretation of quantum mechanics. So while it may seem intuitively obvious that spatiotemporal accounts of individuation would imply that quantum particles are not individuals this is not the case.

There are other spatiotemporal accounts, but they are similar enough to the ones described above. They either provide an identity condition in terms of possessing a unique spatiotemporal trajectory wherein any two objects that satisfy the condition are metaphysically individuated and are, therefore, metaphysical individuals. Or they use spatial separation at a time as an individuator. There are accounts that include a

spatiotemporal condition in conjunction with some other conditions. For example, Reichenbach's account defines what he calls 'material genidentity' in terms of objects possessing a unique spatiotemporal trajectory in conjunction with a condition that it be possible in principle to label the objects in some way (Reichenbach 1956).

However, such accounts are still open to the primary criticisms leveled at spatiotemporal accounts. The accounts in terms of spatial separation are too permissive insofar as they count things like clouds as individuals (at least most of the time). The accounts in terms of unique trajectories presuppose the individuality of spacetime points in order to talk about *unique* spatiotemporal trajectories and so are open to the criticism that Popper and Legenhausen touted their proposals as avoiding.[3] The arguments in the preceding chapter imply that clouds aren't individuals and seem to imply that spacetime points aren't individuals (at least in the absence of an explanation of how the condition of formal immiscibility could possibly be applied to, and therefore satisfied by, spacetime points). Consequently, most of my work for this section has already been completed insofar as the account of 'individuality' provided in the previous chapter supports these two criticisms of the two spatiotemporal accounts. Therefore, I will set these two criticisms aside. There is one remaining criticism to consider.

That criticism contends that spatiotemporal accounts get things backwards. As Hausman puts it, "[t]wo particulars are two and not one not just because they stand in certain spatial and temporal relations which fulfill certain axioms, but simply because they *are* two. They are not two because they are spatially diverse. They are spatially diverse because they are two" (Hausman and Wilson 1967, pp. 42-43). Here Hausman is conflating the problem of accounting for numerical diversity with the problem of identifying a principle of

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[3]It may be the case that spatiotemporal accounts of individuality could be developed which allow for individuation by unique spatiotemporal trajectory without presupposing the individuality of spacetime points. It's beyond the scope of this work to develop such an account, hence I'm restricting my attention to existent accounts of spatiotemporal individuation.

individuation: a conflation that leaves no room for the possibility of distinct non-individuals. However, we can simply replace the talk of numerical diversity with talk of individuation. In that case, the criticism is that objects need to be individuated in order to stand in certain spatial and temporal relations.

Allaire makes the same point when he says, "[r]elations ... *presuppose* numerical difference; they do not account for it. The *thisness* and the *thatness* of things is presupposed in saying that *the one* is to the left of *the other*. Were it not, then in at least some cases we would be forced to say what we all know to be false; namely, that the same *thing* is to the left of itself' (Allaire 1970, p. 254). Allaire is ultimately arguing for bare particulars as individuators, however, he also makes the mistake of conflating numerical difference with individuality. This conflation is particularly ironic given that he goes on to say:

> The mistaken belief that relations individuate derives from confusing numerical difference—call it simply difference—with qualitative (relational or nonrelational) difference—call it simply nonidentity. Difference is primary; nonidentity is not. This may be shown by considering a representation ... built in accord with the rule that there is but one sign for one entity. In such a representation, the difference of two entities, say, *a* and *b*, shows itself in the difference of '*a*' and '*b*.' The nonidentity of *a* and *b* shows itself in the occurrence of at least two sentences, one true and the other false, which are the same except for the one containing '*a*,' and the other '*b*.'[4]

Allaire's conflation isn't important. And I suspect it to be true that some accounts of relations as individuators are based on confusing numerical difference with qualitative difference. However, I think it's often the case that it is assumed that two things can be

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[4](Allaire 1970, pp. 254-255).

related to one another even if they aren't individuals. Quinton states the problem very succinctly: '[p]osition individuates parasitically. To put the point in linguistic fashion, a language can individuate only if it contains singular terms' (Quinton 1973, p. 18).

This criticism is similar to the problem of the individuality of spacetime points faced by the 'unique trajectory' accounts of individuation. Not only do spacetime points need to be individuals in order for an object to be related to a unique set of them, but the object needs to be an individual in order to be the other relatum. In other words, both relata need to be individuals in order to be relata in the first place. So even if spacetime points are individuals, they can't serve to individuate objects. I don't find this criticism convincing since it's not obvious why only individuals could enter into relations. The conflation between numerical difference and individuality may be important here insofar as while the argument can be changed to avoid the conflation, it doesn't seem like a good argument once reference to numerical difference is removed. Put another way, the idea that relations precede numerical difference is potentially incoherent in a way in which the idea of relations preceding individuality (or just relations between non-individuals) is not.

If spacetime points are not individuals, we can ask why a given object is associated with a given spacetime trajectory and not another since the spacetime trajectories aren't individuated (in virtue of spacetime points not being individuated).[5] If spacetime points are not individuals, then we can't talk about *this* trajectory or *that* trajectory at all. Although, we can still talk about trajectories of objects as long as we realize that we aren't associating a unique set of spacetime points with that object. But if a given object isn't an individual, we can ask why a given spacetime trajectory is associated with one object and

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[5]Note that the issue here is that of which spacetime trajectory token is which. Even if there isn't a distinction between two spacetime trajectory tokens of the same kind there is still a distinction between types of spacetime trajectories. That may serve to individuate objects in different states of motion, but there needs to be a distinction between tokens, not just types, if the proposal is to individuate two objects that are both in the same state of motion (and so have the same type of spacetime trajectory).

not another since the objects *aren't yet* individuated. The objects need to be individuated in some way before we can talk about which object is which. There are responses to this criticism, and Reichenbach's account already avoids it through his 'possibility of labeling' criterion. If we can, in principle, label the objects, then we can talk about which is which and we can associate a given spacetime trajectory with *this* object rather than *that* object.

I'm not convinced that this is a satisfying response. First of all, is having a unique spatiotemporal trajectory doing any work in individuating objects if we have to be able to label them first? And what is it that allows us to label the objects in the first place? Reichenbach's account doesn't strike me as a spatiotemporal account insofar as the actual metaphysical ground of individuation seems to be whatever feature of the objects allows us to label them. I see no problem with the idea that there could be non-individuals which could be labeled.[6] But it's unclear what role the possibility of labeling could play in an account of individuation that would leave any room for further conditions. Unfortunately, as (Reichenbach 1956) was published posthumously, there was no opportunity for the role of labels to be further clarified. This issue seems that it would be a general feature of most responses to the criticism: To say that we need to be able associate a spacetime trajectory with *this* object rather than *that* object in order to individuate *this* object is to say that we need to individuate the objects prior to associating them with spacetime trajectories. Consequently, I don't think that unique spacetime trajectories could serve as individuators even if I did accept spacetime points as individuals.

There are, at least, two responses that could avoid attributing individuality to objects prior to their entering into relations. One response is to say that spacetime relations don't obtain

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[6]This depends on one's account of individuality. If labeling is associated with some form of identity (self-identity, synchronic identity, or diachronic identity), then an account of individuality on which these forms of identity play no role could be compatible with labeling of non-individuals.

between spacetime and objects but between spacetime and properties. Although this response just shifts the problem. The properties need to be individuated to associate *this* property with a given spacetime trajectory rather than *that* property.

The other response is found in OSR. If OSR is correct that relations ontologically precede relata, then it can't be the case that objects must be individuated before entering into relations. This is, according to OSR, to get things backward or, in other words, to make the same kind of mistake I'm attributing to defenders of spatiotemporal trajectory accounts of individuation.

One upshot of this discussion is that it leads to a more general problem for defenders of non-individuality. If only individuals can enter into relations, and since any physical object clearly enters into spatiotemporal relations, then there can't be any physical non-individuals. This is far too quick, and it would be problematic if my discussion had this implication as then clouds, for example, would be individuals. For composite entities, such as clouds, we can say that they are not individuals but that they can enter relations in virtue of being composed of individuals. If only individuals can enter into relations, then as long as everything is composed of individuals then we can say that composite objects, even when they aren't individuals, can 'enter into relations' insofar as their components can enter into relations. But we obviously can't say the same thing for non-composite individuals. Consequently, if an object either needs to be an individual or needs to be composed of individuals in order to enter into relations, then there can't be non-composite non-individuals. And so fundamental non-composite particles cannot be non-individuals, as these particles do enter into relations.

5.2   Are Objects Individuated by Monadic Properties?

According to the bundle theory of individuation, objects are individuated by their set of monadic, or non-relational properties. Bundle theories of individuation are always paired with bundle theories of objects which assert that objects just are bundles of properties. To use a bundle theory of individuation without a bundle theory of objects would be to accept the existence of bare particulars to which properties attach while denying that those themselves serve as individuators. So the justification for the bundle theory of individuation is that, after ruling out relational accounts of individuation, there is nothing other than monadic properties to serve as individuators.

Existent bundle theories of individuality would not serve the defender of non-individuality without modification. This is because these theories don't specify any particular properties as individuators. What individuates any given object is the complete set of monadic properties that it has. On such a view, every object is an individual. It is easy to see how a bundle theory of individuation could allow for the possibility of non-individuals by considering one standard objection to the view. It is commonly objected that bundle theories of individuation cannot account for multiple individuals possessing the same sets of monadic properties. If what makes an object an individual is the specific set of monadic properties it has then in the case of 'two' qualitatively identical objects it's not clear whether we have two individuals or one scattered individual. But an explanation as to what *individuates* objects shouldn't leave this sort of question unanswered. One way to respond to this objection is to focus on particular sets of properties in conjunction with a principle that ensures that no two objects have the same set of such properties. For example, French and Krause point out that one response to the objection is to focus on spatiotemporal properties and a principle of impenetrability (French and Krause 2006, p. 8). This particular response would also allow for the possibility of non-individuals if there

are any objects that don't satisfy the principle of impenetrability. So, quantum particles could be non-individuals on the bundle view (as long as Bohmian mechanics isn't the correct interpretation of quantum mechanics).

It is well beyond the scope of this project to work out a specific bundle theory of individuation that would allow for non-individuals. Additionally, focusing on spatiotemporal monadic properties opens the view up to some of the objections that face relational spatiotemporal accounts. Consequently, I will simply assume that it is possible to construct a bundle theory of individuation on which quantum particles would count as non-individuals. As a result of this assumption, my focus here will be on general objections to bundle theories of individuation.

One final note before I begin looking at problems with bundle theories of individuation. A bundle theorist can view properties as either universals or tropes and some of the objections to bundle theories will depend on which view of properties is taken. I am not going to get into the debate over whether properties are universals or tropes, but I will differentiate between the two in what follows.

The first, and most common, objection is the one I already mentioned. That a bundle theory of individuation can't make sense of qualitatively identical yet numerically distinct objects. This objection is raised in (Allaire 1963), (Hochberg 1969), and (Oaklander 1977). The objection is also raised in a slightly different, but equivalent, form in (Gracia 1988), and (Lowe 2005). The second form of the objection states that the bundle theory of individuation makes the strongest form of PII an analytic truth. That is, the bundle theory of individuation implies that it is not possible for numerically distinct objects to share all of their monadic properties.

Allaire's version of the objection is aimed specifically at bundle theories that take properties to be universals. Two qualitatively identical objects will instantiate exactly the

same set of universals. But then that set of universals can't individuate those two objects. The objection still holds if we focus on subsets of properties. However, the objection is avoided if properties are tropes. Then two qualitatively identical objects have their own distinct properties. Each has, say, their own redness: red_1 and red_2.

Moore's version of the objection also only targets bundle theories that take properties to be universals. I will follow Hochberg's description of Moore's argument (Hochberg 1969, p. 157-165).

> Consider four things: a white square, a white circle, a black square, a black circle. Assume, for simplicity, they have no other non-relational properties. Call them Peter, Paul, Mary, and Joan. To say that Peter is white, on the view Moore wishes to refute, is to say that the predicate white is related by predication to the predicate square, the latter being the point of difference with the white circle, Paul. Moreover, Peter is identified with the predicate square. But to say that Mary is black is to say that black is related to square, by predication, and to identify Mary with the predicate square, since that is the point of difference with the black circle, Joan. We thus identify Peter with Mary. Moreover, they cannot be differentiated, even by introducing relations, since it will always be the same universal square that is involved in any relation to anything else.[7]

In other words, the problem with the bundle of universals view is that the only thing we can attach predicates to is another predicate. What grounds the difference between Peter and Paul such that the predicate "is white" has two distinct subjects? To say that Peter is white is to treat "being square" as the subject to which the predicate "is white" attaches. So, Peter is the name for the property of "being square." To treat "is white" as the subject,

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[7](Hochberg 1969, pp. 158-159).

given that Paul is also white, would result in Peter and Paul both being names for the property of "being white". This leads to a contradiction since Peter and Paul are numerically identical yet qualitatively distinct. So, on pain of contradiction, we conclude that Peter is the name for the property of "being square." Now, to say that Mary is black is to treat "being square" as the subject to which the predicate "is black" attaches. So, Mary is the name for the property of "being square." This leads to the contradiction that Peter is numerically identical to Mary even though Peter and Mary are qualitatively distinct. Without bare particulars the only thing to act as a subject for a predication of color is being square or being round. But treating these as the subjects of predication leads to identifying a white square with a black square or a white circle with a black circle.

Moore's objection is more general in that it points to a problem that comes up when you have multiple objects that share properties even if they don't share all of their properties. And note that the response I mentioned earlier isn't applicable here. That response would work for Allaire's objection since it would rule out cases of numerically distinct objects that share all monadic properties in which case there's no need to explain how there could be such objects. But Moore's objection isn't limited to objects that share all the same properties. As I see it, the only way to avoid Moore's objection is to reject the claim that properties are universals. If properties are tropes than Peter is the name for "being square$_1$" whereas Mary is the name for "being square$_2$", so there is no contradiction.

Finally, the other form of the objection is also only limited to views according to which properties are universals. If properties are tropes, then there can be numerically distinct objects that share all of their monadic properties (in the sense in which two, say, red objects share the property of being red in virtue of having the properties red$_1$ and red$_2$). In other words, the strongest form of PII is only an analytic truth on a bundle theory of individuation if properties are universals. Strictly speaking, the objection does still hold for tropes since qualitatively identical objects will have distinct tropes. But this isn't

something a trope theorist would be concerned with and the puzzles that arise in the case of universals don't arise in the case of tropes. Or, in other words, the question of how it's possible for there to be numerically distinct but qualitatively identical objects has a straightforward answer if properties are tropes. There are no such objects whose existence must be explained, in the strictest sense, since "qualitatively identical" objects differ in the specific tropes they possess.

A second problem facing the universals version of bundle theory was initially raised by Gracia. This problem is that a bundle theory of individuation undermines the distinction between individuals and universals. Gracia states the problem as "[t]hey have to worry also about the obliteration of the very distinction between individual and universal that their theory aims to support, for a bundle of universals is a complex universal and cannot be regarded as an individual just in virtue of being complex" (Gracia 1988, p. 148). The problem is that of explaining why certain complex universals count as individuals and other complex universals, such as those that are never instantiated, are not individuals. The worry is that if what makes something an individual is its bundle of universals and if objects are nothing but bundles of universals, then any bundle of universals is an individual. But then any bundle of universals is an individual including complex universals that 'have' other universals as 'parts'. For example, *being a methane molecule* is a complex universal that involves the universals *being a hydrogen atom*, *being a carbon atom*, and *being a chemical bond*. As long as one accepts the logical possibility of uninstantiated complex universals, then the universals version of bundle theory seems to imply that these would be individuals. It's not clear to me that this would really undermine the distinction between individuals and universals since simple universals wouldn't be individuals. It depends on whether the idea of an individual universal makes sense. *Prima facie* the idea of an individual universal doesn't make sense so, at minimum, Gracia's objection highlights an explanatory burden defenders of bundle theory need to discharge. Either explain how it's possible for objects to be nothing more than bundles of universals

that are individuated without implying that uninstantiated complex universals are individuals *or* explain how a universal could be an individual. While Gracia has, in places, recognized a difference between particularity and individuality, it does appear that he's conflating them here. I'm not sure this is a problem insofar as I've seen no bundle theorists respond to this objection by defending the idea that universals are individuals.

There are two ways bundle theorists attempt to respond to this objection. One response is to characterize properties as tropes, view the tropes as individuals, and claim that objects inherit their individuality from the individuality of the tropes that compose them. This response implies that all objects are individuals, so this isn't an option for defenders of non-individuality. Additionally, this response only seems to shift the question from what makes an object an individual to what makes a trope an individual.

The second response is to claim that there is a metaphysical relation that ties features together into a bundle. This relation is sometimes referred to as a "tie," "nexus," or "operator" (Gracia 1988, p. 148). This response does not allow for the existence of complex non-individuals (neither complex in the sense of being aggregates with multiple parts nor complex in the sense of possessing more than one property). Any object that possesses more than one property will necessary possess the metaphysical relations that, on this account, grounds individuality (i.e. the relation that ties the properties together). Additionally, as Gracia points out, if the metaphysical relation is not unique then the question is just being shifted from what makes an object an individual to what individuates this metaphysical relation. If the metaphysical relation is unique to an individual, then we've abandoned bundle theory and adopted a bare particular or haecceity based account of individuation.

The trope version of bundle theory has a similar problem with explaining what individuates tropes. This objection is discussed primarily by Rosenkrantz (Rosenkrantz 1993, p. 102) and Lowe (Lowe 2005, p. 83). The issue with the trope version of bundle

theory is that it seems to be circular. If, on the one hand, we say that tropes are partly individuated by the objects to which they belong then it's viciously circular to say that the objects are also individuated by their tropes. To avoid the circularity, one could say that tropes aren't individuated, even partly, by the objects to which they belong. But then it's unclear why a particular trope must be associated with a particular object. A trope could belong to different objects at different times or a trope could potentially overlap with multiple objects at the same time (Lowe 2005, p. 83). In other words, if tropes aren't individuated by the objects to which they belong, then we need an explanation as to why objects can't exchange, or possibly share, tropes. If the circularity were avoided by providing an account of individuation for tropes and claiming that objects inherit their individuality from their tropes then, as stated above, all objects would be individuals, so this response is of no use to defenders of non-individuality.

Finally, there are two major objections that are independent of whether one views properties as universals or as tropes. The first of these is that bundle theory implies that all of an object's properties are essential properties. "If an individual is the same as the bundle of features it has, then any true proposition that says that any such and such individual has such and such a feature must be analytic and/or necessary and the features that it specifies are essential to the individual in question" (Gracia 1988, p. 145). This objection is entirely based on the intuition that at least some properties of objects are not essential properties. It at least seems obvious that many macroscopic objects have properties that are not essential properties. It's not so clear in the case of particles. However, any attempt to utilize a bundle theory of individuation for some objects and not others will have to explain why it's an appropriate account of the individuation of objects for the former set of objects and not for the latter. A bundle theorist could claim that only non-composite objects are individuals, but then they would need to explain away the many paradigm cases of composite individuals.

The second and, in my opinion, more problematic objection is that bundle theories of individuation aren't really concerned with individuation at all. Rather, bundle theories conflate some of the issues that Gracia carefully pulls apart. In particular, "[b]undle theorists suffer from two errors: the confusion of the problem of individuation with the problem of discernibility and the confusion of individuality with distinction or difference. It is for this reason that their view aims to account for difference, primarily epistemically, and not for individuality understood metaphysically" (Gracia 1988, p. 150). According to every version of bundle theory there are, strictly speaking, no numerically distinct yet qualitatively identical objects. There is always something unique to each object whether it's the metaphysical tie that binds a bundle of universals together or that one object has the trope red_1 whereas its 'qualitatively identical' partner has the trope red_2. Every object has a uniquely identifying feature that an epistemically perfect being would have access to even if we do not. In other words, the criteria for being an individual on bundle theories of individuation is being discernible to an epistemically perfect being. As for the other confusion; accounting for difference between two objects is not the same as accounting for individuality. First of all, if particles are non-individuals then there are non-individuals with different properties. Defenders of non-individuality could not accept accounting for difference as sufficient for accounting for individuality because it undermines their position. But there's also the more general issue that while objects that are different must be distinct individuals (assuming they are individuals) it is not the case that distinct individuals must be different. There is nothing impossible about qualitatively identical yet numerically distinct objects. Or, put another way, the strongest version of PII cannot be an analytic truth and my primary opponents (e.g. French, Krause, and other defenders of the 'Received View') don't view the strongest version of PII as even contingently true (French 1989).

From the preceding discussion it's not clear that my initial assumption that a bundle theory of individuation could allow for non-individuals is correct. But if bundle theory

ultimately does not allow for the existence of non-individuals, then defenders of non-individuality would be in agreement with me that the correct account of individuation cannot be a bundle theory.

5.3 Bare Particulars and Non-Qualitative Properties

The remaining accounts of the metaphysical grounds of individuality all take those grounds to be provided by entities that are often considered metaphysically problematic in similar ways, particularly by empiricists. So I want to explain why we should take such accounts seriously before moving on to the accounts themselves. What these accounts all have in common is that the metaphysical grounds of individuality are taken to be something non-qualitative. There are different accounts of the distinction between the qualitative and the non-qualitative and it is beyond the scope of this work to wade into that debate. For my purposes, it is sufficient that we have a general understanding of what sorts of things count as qualitative or not. The simplest way to provide that is to work with a simplified definition. A non-qualitative property is a property that concerns a particular entity whereas a property is qualitative if and only if it is not non-qualitative. A haecceity is a non-qualitative property because haecceities uniquely identify the possessing object. The property of existing, understood in Gracia's sense as an individuator, also counts as a non-qualitative property. There are two classes of non-qualitative properties, intrinsic and extrinsic, and only the former are considered metaphysically problematic. We will here be concerned with the former. But to give a quick example of the latter, the relational property of owning a *particular* piece of land is a non-qualitative property because it makes reference to a specific piece of land. In addition to properties being qualitative or non-qualitative, other things can be non-qualitative provided they, in some sense, concern a particular entity. Any property, substance, or whatever else, whose sole purpose is to

individuate the specific entity of which it is a component will count as non-qualitative.

We already saw one example of using a non-qualitative feature as an individuator in Dorato and Morganti's account of primitive thisness in chapter two. I use the term 'feature' here as they don't take primitive thisness to be an ontological component of an object such as a property. However, even Dorato and Morganti's primitive thisness is still something that empiricists may view as problematic for the same sorts of reasons they might view bare particulars or intrinsic non-qualitative properties as problematic. So I will begin here with Dorato and Morganti's defense of the idea that naturalism is compatible with the existence of non-qualitative features. Their defense takes the form of a criticism of the alleged inference from naturalism to Leibnizian reductionism.

Dorato and Morganti claim that there is a conflation "between *two* different ways of determining what is supported by, or to be deemed meaningful on the basis of, science and what is not" (Dorato and Morganti 2013, pp. 597-598). The conflation is between (i) the idea that what is naturalistically acceptable for scientific and philosophical theorizing is qualitative and (ii) the idea that what is naturalistically acceptable is whatever contributes to the success of our best scientific theories. Dorato and Morganti favor the second way of understanding what is naturalistically acceptable and go on to point out how it is incompatible with restricting one's focus to the qualitative. They offer three reasons to explain this incompatibility. The first reason is that empirical differences can arise out of non-qualitative facts. It matters empirically how many qualitatively identical particles are present even if the only empirical difference in some circumstances is the mass. As long as the objects in question have additive properties, like mass, it will always make an empirical difference how many numerically distinct yet qualitatively identical objects are present. If qualitative uniqueness is not necessary for empirical significance, why should we view the non-qualitative as naturalistically unacceptable? The second reason is that

"the possibility that facts of numerical distinctness might be as fundamental as, or even *more fundamental* than, facts about qualities appear directly suggested by some scientific theories" (Dorato and Morganti 2013, p. 598). As an example, they point out that in NRQM we start with particle number in constructing a Hilbert space with the correct number of dimensions for the task at hand. The point is that in NRQM particle number is *always* a starting assumption for "the construction of the right kind of model for the physical problem at hand" (Dorato and Morganti 2013, p. 598).

Interestingly, the idea that particles are not individuals arguably strengthens the claim that non-qualitative facts about number are empirically significant. Putting these two reasons together, we can say that if the number of qualitatively identical particles present is empirically significant and if particles are not individuals, then there seems to be nothing beyond the number of particles to point to in order to explain the empirical differences. If particles are individuals, then there's the possibility of explaining the empirical difference in terms of whatever grounds that individuality (and construing facts about number as non-primitive facts that depend on those grounds of individuality). I would, however, make a claim even stronger than Dorato and Morganti and say that if number of qualitatively identical particles makes an empirical difference then particles *must* have a non-qualitative feature or property that explains that difference. But note that a non-qualitative feature or property needn't imply that the objects possessing that property are individuals.[8] So I agree with Dorato and Morganti that naturalism is compatible with non-qualitative features or properties, I would go further and say that naturalism requires that we accept non-qualitative features or properties. But, as I said, that doesn't imply anything about whether particles are individuals or not. That depends on what non-qualitative features we take to be important.

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[8]Presumably any object that possesses a feature or property that makes specific reference to *that* object has some form of identity. However, since I've claimed that identity isn't a sufficient, or even necessary, condition for individuality I allow for the possibility of non-individuals that have non-qualitative properties.

The third and final reason Dorato and Morganti offer for the compatibility of naturalism with the non-qualitative is that we can take there to be brute non-qualitative metaphysical facts that do not constitute *additions* to our ontology. We can get a handle on what this means by distinguishing between two different aspects of ontology. On the one hand, we can talk about components of our ontology in the sense of building blocks that make things up. On the other hand we can talk about features of those building blocks. The distinction is between being and *ways of being* or between what exists and what features they have. Consider Bohmian mechanics where particles follow the guidance equation, which gives rise to wavelike behavior. One possible ontology for Bohmian mechanics includes both particles and physical waves that 'guide' the particles. Another possible ontology doesn't include physical waves at all and just says that the guidance equation just describes how the particles happen to behave. The former makes guiding waves a component of our ontology whereas the latter doesn't add any components to our ontology. But the latter is still making an ontological claim about the way in which particles in Bohmian exist. This is how one can make ontological claims that don't add any additional components to our ontology.

While I agree that Dorato and Morganti are right about what most naturalists would prefer, I don't think it's a preference that naturalists should have. Dorato and Morganti's first two reasons in favor of a compatibility between naturalism and non-qualitative facts are not restricted to brute non-qualitative metaphysical facts. If those reasons are good reasons to accept brute non-qualitative metaphysical facts, under the right conditions, then they are just as good reasons for accepting non-qualitative properties (or other ontological components) under the right conditions. Having said that, I am sympathetic to the idea that there are some brute non-qualitative metaphysical facts, I just don't think they should be inherently preferable to things like haecceities or bare particulars.

5.4 Are Objects Individuated by Bare Particulars?

A bare particular is a propertyless, in the sense of having no intrinsic properties, substance that serves as a bearer of properties. Bare particulars do not differ from one another qualitatively because they lack intrinsic qualitative properties. Bare particulars are individuals, but this is a brute fact that is not grounded in any non-qualitative property that bare particulars possess. I will use the term 'feature' here as I did with respect to Dorato and Morganti's primitive thisness. However, the metaphysical grounds for the individuality of a bare particular is the bare particular itself. The individuality of a bare particular is so tightly tied to what it means to be a bare particular that it would be incorrect to view bare particulars as possessing a property that grants individuality (and not merely because bare particulars, by definition, don't have intrinsic properties). A bare particular "is just that entity which is a constituent of one and only one ordinary thing. Thus it accounts for the difference between any one ordinary thing and all others" (Hausman and Wilson 1967, p. 42).[9] In short, bare particulars are self-individuating.

The bare particular view is similar to bundle theory except that it adds one additional ontological component to objects. Instead of objects being a bundle of properties with, perhaps, some metaphysical tie that binds them together, objects are bundles of properties bound, possibly by a metaphysical tie, to a bare particular. Still, the addition of the bare particular allows this view to avoid some of the objections against bundle theories of individuation. On the bare particular view, the difference between two qualitatively identical objects "is accounted for by each containing a different individual; the sameness by each containing literally the same characters" (Allaire 1963, p. 3). Thus one, alleged, advantage of the view is that it grounds both the difference and sameness of qualitatively

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[9]Here 'ordinary thing' means anything other than a bare particular. It's just a, perhaps oddly phrased, way of indicating that a bare particular doesn't have other bare particulars as constituents.

identical objects. So the universals version of the bare particulars view avoids the first objection against the universals version of bundle theory; it can make sense of qualitatively identical yet numerically distinct individuals. Moore's version of the objection makes it clear why that is the case. The problem was that the only thing predicates (or properties) can attach to is other predicates on the bundle theory. But on the bare particular view predicates attach to a bare particular, so we don't get the contradictory identifications we saw with the bundle theory (e.g. identifying a white square with a black square).

The universals version of the bare particulars view also avoids the need to explain why certain complex universals count as individuals whereas others do not. All and only complex universals that are instantiated (i.e. are associated with a bare particular) are individuals. However, the bare particulars view faces a very similar problem. On the one hand, we could say that all objects are individuals or, equivalently, that all instantiated properties must be attached to a bare particular. On the other hand, we could allow for the existence of non-individuals by allowing that some bundles of instantiated properties are not attached to bare particulars. But then, we require an explanation of why some bundles of instantiated properties are associated with bare particulars whereas others are not. Note that the objections to bundle theory wouldn't reoccur in this case because those objections all stemmed from, in one way or another, the idea that bundles of properties are themselves individuating. So these objections would not apply to the idea that there are non-individuals that are merely bundles of properties. Defenders of non-individuality would, of course, prefer the latter option. But then we get a new form of Gracia's objection: why do some properties attach to bare particulars whereas others do not? So the bare particulars view avoids Gracia's objection to bundle theory only by either insisting that all objects are individuals or by creating a new, and similar, explanatory need.

The bare particulars view also avoids the remaining three objections to bundle theories as well. It has no problem with, because it has no need to, explaining what individuates

tropes and it doesn't imply that all of an object's properties are essential properties because the particular properties an object has have nothing to do with why it's an individual. Finally, bare particular views do not confuse individuation with discernibility, nor do they confuse individuality with distinction or difference. Bare particulars are not discernible and do not render qualitatively identical objects discernible. And while bare particulars can account for difference they need not be viewed as individuated as a result of accounting for difference, and so needn't conflate individuality with difference.[10]

I will now turn to a number of objections to the bare particulars view. The first objection is an epistemic worry. If the bare particulars view were correct, then we would not know if there were any cases for which it was needed to account (Meiland 1970, p. 261). Meiland is understanding the bare particulars view as aimed at explaining numerical difference and is claiming that the view makes it impossible to have a criterion for determining numerical difference in the case of qualitatively identical objects. Note that Meiland isn't conflating the alleged metaphysical grounds of numerical difference with the epistemic issue of how we would determine numerical difference. The problem is that we can't, according to Meiland, know whether the philosophical problem the bare particulars view is trying to solve is actually a genuine problem because we never have any grounds for treating qualitatively identical objects as numerically distinct. Even in the case of spatially separated qualitatively identical objects we have no evidence that they are two objects rather than one scattered object because what makes two objects two objects is that they are numerically distinct bare particulars; spatial separation isn't relevant. Note that Meiland isn't relying on any specific view of composition in making his criticism. The problem is that those who want to explain numerical difference by way of bare particulars aren't in a position to reject the possibility of scattered objects. A scattered object, on their

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[10]For some recent defenses of bare particulars see (Sider 2006), (Pickavance 2014), (Garcia 2014), (Connolly 2015), and (Perović 2017).

view, is a composite object with spatially separated parts unified by a single bare particular. Without epistemic access to bare particulars there is no way to rule out the existence of such objects. In short, on the view Meiland is criticizing, how many objects are present is entirely a matter of how many bare particulars are present independently of any principles of composition one might accept.

If we can't determine whether numerically distinct yet qualitatively identical objects exist in the first place, then we can't determine whether the explanatory need the bare particulars view is meant to satisfy even exists. So, the bare particulars view is, in a sense, self-defeating in that its truth makes it impossible to know whether its explanatory target (accounting for numerical difference of qualitatively identical objects) is actually something that needs explaining in the first place.

While I agree with this criticism, it is important to note that it only targets bare particulars views that, strictly speaking, don't address the problem of individuation. As we saw in the last chapter, the problem of accounting for numerical difference and the problem of individuation are not the same. It is possible for a solution to the problem of individuation to also account for numerical difference. However, there is no need for the problems to be connected in that way. Adopting a bare particulars solution to the problem of individuation while denying that bare particulars ground numerical difference allows one to avoid Meiland's objection. In fact, a defender of non-individuality who wanted to adopt a bare particulars view to explain the metaphysical grounds of individuality would have to deny that bare particulars grounds numerical difference. If they didn't then there could be no more than one of any given non-individual since non-individuals, on this view, are not associated with bare particulars.

A second objection to bare particular views of individuation is that they are circular (Rosenkrantz 1993, pp. 99-100). Rosenkrantz also talks about the bare particular view in terms of diversity and provides the following criterion: "(P2) At time *t*, a particular *x* is

diverse from a particular *y* =df. There exists a substratum S such that: (i) at *t x* is supported by S, and (ii) at *t y* is not supported by S" (Rosenkrantz 1993, p. 84). Here '*x* is supported by S' means that S serves as the bearer of properties of *x*. Or, more succinctly, that S is the bare particular for *x*. He goes on to point out that because a bare particular is a particular, it must also be individuated by a substratum.

> But such a state of affairs is incoherent. An ordinary particular is alleged to be some sort of combination of a substratum and properties. However, since a substratum is "bare", it seems that there could not be a substratum which is itself some sort of combination of a substratum and properties. Thus, there could neither be a substratum which is supported by *another* substratum, nor be a substratum which is supported by *itself*.[11]

I must admit I can't see how the difference between ordinary and bare particulars is supposed to imply the conclusion. Although I'm not sure that matters as there's a problem that prevents this line of reasoning from getting off the ground in the first place. The problem is that Rosenkrantz has mischaracterized the bare particulars view. The circularity arises because, according to Rosenkrantz, diverse bare particulars themselves must be individuated by something else. According to (P2) what it means for particulars to be diverse is for them to each have their own substratum. Consequently, diverse bare particulars must each have their own substratum. But defenders of bare particulars reject (P2) as a description of their view. Bare particulars are self-individuating. It is only non-bare particulars (i.e. bundles of properties attached to a bare particular) that require a substratum. One could perhaps criticize the idea that bare particulars are self-individuating. But that's not what Rosenkrantz is doing. He's claiming, incorrectly, that defenders of bare particulars accept that diverse bare particulars are individuated by

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[11](Rosenkrantz 1993, p. 99).

their own substratum. Recognizing that bare particulars are self-individuating the circularity objection can't get off the ground.

A third objection is that the bare particular view "solves the problem of the individuation of objects by fiat, but at the expense of generating impenetrable mysteries" (Lowe 2005, p. 86). This is a common objection, but the objection itself is somewhat mysterious in that there is a lack of details about (i) why the bare particulars view only solves the problem by fiat and (ii) why the alleged mysteries are mysteries at all. Lowe doesn't go into any detail regarding (i) but he, unlike most authors, does offer some insight into (ii). It is largely taken as intuitively obvious that the bare particulars view amounts to nothing more than a statement that the problem of individuation has an ontological solution and that the name of the ontological component that solves the problem is 'bare particular.' The bare particulars view differs from the previous views we've looked at insofar as it relies on transcendental argument. The idea is that in order for the world to be the way it in fact is, bare particulars are necessary. If we accept that there are individuals and that there must be some metaphysical grounds of individuality, and if we accept that neither relations nor qualitative monadic properties can individuate objects, we're already committed to some sort of non-qualitative metaphysical grounds of individuality. It certainly seems incorrect to say that the bare particulars view solves the problem of individuation by fiat if we have reasons to think that there is a non-qualitative metaphysical solution. That solution must either be in terms of non-qualitative properties such as haecceities, of must be in terms of something that's both non-qualitative and not a property. Furthermore, as I already explained, the bare particular view is essentially just bundle theory with an ontological addition that addresses all of the major objections to bundle theory. So, one reason to accept the bare particulars view is by accepting both the motivations for accepting bundle theory and the objections to bundle theory. Whatever one thinks of the bare particulars view it is has plenty of principled motivation.

It is also not clear what the alleged mysteries are supposed to be, or why they are supposed to be insoluble. We saw two 'mysteries' above that are both based on misconceptions. The first based on conflating the problem of individuation with the problem of accounting for numerical difference and the second based on misrepresenting the view such that bare particulars need their own substrata for individuation. But beyond that the literature contains brief unexplained claims about the mysteriousness of 'bare particulars'. I'm very unsympathetic to mere claims that a view is unacceptable merely because it is unintuitive. Consequently, I'm very unsympathetic to mere claims (as opposed to arguments) that bare particular views of individuation are unacceptable because bare particulars are mysterious. Such objections should clearly state what mystery needs to be explained and provide some reasons as to why an explanation is required in the first place. Explanations have to stop somewhere so any calls for explanations regarding bare particulars (such as asking for some further explanation of what individuates bare particulars) should be paired with an argument for why such an explanation is necessary. In the absence of such an explanation we are in no position to determine whether the potential problem (i) is actually a problem and (ii) is a problem with the bare particulars view or a problem with the intuitions of the objector.

Lowe claims that since bare particulars lack properties "it is something of a mystery how they manage to 'support' the properties of the objects whose subtratra they are' (Lowe 2005, p. 86). So this is a little better than merely saying that bare particulars are unacceptable because they're mysterious. It points to a specific explanatory need although doesn't provide any reasons as to why an explanation is required. There are two parts to the objection. One of them is simply that the notion of 'support' needs to be clarified. The other part is that there is something about a bare particulars lack of intrinsic properties that creates a concern about the prospects for clarifying the notion of 'support'. This second part of the objection isn't spelled out and it strikes me as strange to suggest that something that is meant to serve as a property-bearer would require properties to serve that purpose.

The request for an explanation of the notion of 'support' is reasonable and is a version of what French and Krause call the problem of describability which I will discuss at the end of the section.

A fourth objection, often referred to as the 'classic objection,' is that it's nonsensical to suppose that something without properties could exist. While some defenders of bare particulars have claimed that bare particulars have no properties whatsoever, most defenders of bare particulars do not accept that claim. While bare particulars lack intrinsic properties, they have properties in virtue of instantiating properties. A bare particular has no essential properties, but any actually existing bare particular has the properties it instantiates. There are different proposals for explaining precisely in what sense bare particulars are considered to have properties (see (Bailey 2012) for an overview), but what's important is that, with the exception of a few specific versions of the bare particulars view, the classic objection is confused (see (Sider 2006) for more detailed discussion of the confusion). Some people, such as Bailey, still object that the explanations of the ways in which bare particulars are alleged to have properties only avoid the mystery of positing a genuinely propertyless substance at the expense of creating new mysteries (Bailey 2012, p. 35). However, Bailey notes that it's not satisfactory to object in this way and goes on to provide a more detailed objection.

Bailey's updated version of the classic objection grants that bare particulars have properties and asks whether or not they have the properties of 'host substances.' He contends that bare particulars are untenable regardless of the answer. Consider a tomato. Answering 'no' to Bailey's question means that the bare particular associated with the tomato doesn't instantiate the properties of a tomato and, as a result, doesn't exemplify *being a tomato* (Bailey 2012, p. 36). This is a problem because bare particulars are supposed to explain how objects have properties. An object is supposed to have properties because one of its constituents is the bare particular that instantiates those properties. So

answering 'no' to Bailey's question, according to Bailey, leaves the bare particular view as unmotivated. Unfortunately for Bailey, this simply isn't true. Explaining why objects have properties isn't the only thing the bare particulars view is meant to do, one could adopt a bare particulars view as a solution to another problem while maintaining that it doesn't explain why objects have properties. One could adopt bare particulars merely as a means to solve the problem of individuation and could even hold that there is no need to explain why objects have properties. It could be accepted as a (metaphysically grounded) brute fact that objects have properties. So, one horn of Bailey's dilemma only holds against views that claim that the *only* role of bare particulars is to explain why objects have properties.

On the other hand, answering 'yes' to Bailey's question leads to a crowding problem. Consider our tomato again. Our tomato has all the properties that make something a tomato. But the bare particular associated with that tomato also has all the properties that make something a tomato. So we actually have two tomatoes. The bare particular is a tomato and the combination of the bare particular with the bundle of properties it possesses is a second tomato. Bailey considers and responds to a few objections, but he doesn't consider what I take to be the most serious objection to his argument. His argument relies on the idea that what it is to be an object of a certain kind is just to have a certain set of properties. This is why he claims that, in addition to the regular bare particular + properties tomato, we have a second tomato that is simply the bare particular. Recall that properties are not constituents of bare particulars even though, with our 'yes' answer, the bare particular is considered to possess those properties. This is what allows Bailey to take the bare particular by itself as counting as a tomato.

The problem I see with Bailey's argument is that it seems to conflate the bare particulars view with bundle theory. If I accept bare particulars, why would I accept that being a tomato is only a matter of possessing the right set of properties? Instead, I would claim

that being a tomato amounts to having the right set of properties bundled together via instantiation by a bare particular. The whole point of positing bare particulars over-and-above bundles of properties is to solve (alleged) problems that arise from viewing objects merely as bundles of properties. But to take something to be an object of a certain kind merely by possessing the right bundle of properties is to, at least partly, defeat the purpose of proposing bare particulars in the first place. This is true even if we agree with Bailey that the only motivation for bare particulars is to explain how objects can have properties. Being a tomato isn't just about having the right bundle of properties because, according to the bare particular view, we can't have a tomato, or any ordinary object at all, if all we have is a bundle of properties. So while a defender of bare particulars can, and probably should, accept that what makes an object the kind of object it is depends on the set of properties it possesses, there is still more to it in virtue of the fact that free-floating bundles of properties don't exist. To put it another way, being a tomato isn't merely a matter of satisfying the kind conditions for the kind *tomato*, but also satisfying conditions on being a concrete particular. It is satisfying the latter conditions, given the ontology of bare particular theory, that requires that being a tomato amount to more than merely having the right set of properties. If being a concrete particular is a matter of being a bundle of properties instantiated by a bare particular, then we only have one concrete particular. If a tomato is a concrete particular, then we only have one tomato. So, my claim that being a tomato amounts to having the right sets of properties bundled together via instantiation by a bare particular isn't merely a way to avoid Bailey's objection. It's what a defender of bare particulars already accepts in order for the bare particulars to do the metaphysical work they are intended to do. Consequently, Bailey's argument is yet another based on confusion. Answering 'no' doesn't remove all motivation for positing bare particulars because explaining why objects have properties isn't the only metaphysical work bare particulars are meant to do. And answering 'yes'

doesn't lead to Bailey's crowing problem because his argument depends on a claim that that no defender of bare particulars would accept.[12]

But not that responding to Bailey's argument in this way undermines the ability to view non-individuals as bundles of properties lacking a bare particular. If a concrete particular, like a cloud, is a non-individual lacking a bare particular, then being a concrete particular can't require the possession of a bare particular. So, we end up with a dilemma for defenders of bare particulars views. Either Bailey's argument fails for the reasons I described and the existence of concrete, particular non-individuals is impossible or the existence of concrete, particular, non-individuals is possible and Bailey's argument succeeds. Of course, one way to avoid the dilemma is to accept an alternative response to Bailey's argument.

Ultimately, despite the unpopularity of bare particular views, there's a surprising lack of definitive, or even on target, objections to such views. An inclination for metaphysical parsimony may be largely responsible for this state of affairs. It may be viewed as the burden of defenders of bare particulars to explain why bare particulars are a fruitful addition to our ontology. Combining this with a widespread acceptance that the reasons offered for accepting bare particulars fails to satisfy that burden, then we can make sense of at least one possible reason for the lack of worked out objections in the literature. I myself don't favor the bare particular view. However, I see no issues with the transcendental and bundle theory based motivations for positing bare particulars and so see no reasonable way to personally deny that defenders of bare particulars haven't satisfied this burden. On top of that, the only worked out objections to bare particulars that I can find in the literature are based on confusions. Those confusions aren't always on the part of objectors, but also resulting from defenders of bare particulars conflating the

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[12]For another criticism of Bailey's argument see (Pickavance 2014).

problem of individuation with another problem. But, unlike with the metaphysical grounds I've looked at previously, avoiding these conflations, as far as I can tell, answers all existing objections to the bare particular view. That's not to say that there are no legitimate undiscovered objections, including ones that may be more devastating than those we've seen regarding the other views. But in terms of existing objections the bare particular view of individuation stands out as the clear frontrunner at this point regardless of my own intuitions. If I could turn my intuitions into arguments against the view, then perhaps it wouldn't fare nearly as well. But I won't treat my intuitions as arguments. The world is the way it is regardless of what I think about it so mere intuitions, including my own, do not constitute good reason to discount a view. To sum up "[t]he complaint about "bare particulars" is mostly confusion; and in the rest, there is no solid argument against the substratum theory" (Sider 2006, p. 395)

However, there are two major problems with bare particulars views. The first was mentioned at the beginning of this section. On a bare particulars account of individuation either all objects are individuals or we need to explain why only some objects are associated with bare particulars and others (non-individuals) are not. Since I take things such as clouds not to be individuals, and defenders of the 'Received View' think particles are not individuals, we would both require an explanation as to why things such as clouds or particles aren't associated with bare particulars whereas objects such as people and laptops are. In a sense, if both individuals and non-individuals exist then the bare particulars view doesn't actually solve the problem of individuation. A further distinction between individuals and non-individuals is needed to explain why bare particulars are associated with the former and not the latter, but if such a distinction can be provided then what do we need the bare particulars for? The issue here is a metaphysical one and it relates to the additional role as property bearers that bare particulars are meant to have. Why is it that individuals need a property bearer and non-individuals do not? If it's possible for there to be non-individuals and if to be a non-individual is to lack a bare

particular, then there are objects that are just bundles of properties not attached to a bare particular. So then why posit an individuator that also serves as a property bearer as opposed to an individuator that doesn't serve as a property bearer (like a haecceity)? If individuals and non-individuals both exist, then bare particulars have a redundant metaphysical role on top of serving as individuators. The issue here is a matter of simplicity. Even though one could say that bare particulars don't need to serve as property bearers, why accept this addition to our ontology when a property (like a haecceity) will do?

There is at least one response open to the bare particular view. They could claim that all and only non-composite objects are individuals possessing bare particulars. Then composite objects could have their properties in virtue of the properties of their parts.[13] But if allowing for the existence of non-individuals requires claiming that all composite objects aren't individuals, then that is a difficult position to defend. It requires explaining away all paradigm examples of individuals. Perhaps proponents of the bare particulars view could say that composite objects are individuals in virtue of being composed of individuals. But that leads us right back to the objection because again every object is an individual.

The second major problem is what French and Krause call the problem of describability (French and Krause 2006, p. 12). If we accept an account of individuality that counts as a transcendental account (see chapter two), then how are we to explain, in positive terms rather than negative terms, the nature of the individuator? There are no positive descriptions of bare particulars in the literature. They are described by pointing out what they lack; namely, intrinsic, or essential, monadic properties. But that doesn't actually tell us what a bare particular is. I don't think this, by itself, is a good enough reason to set

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[13]Though it is worth noting that this position is incompatible with the existence of holistic properties since the properties of the composite object supervene on the properties of its parts.

aside the bare particular view. However, a non-qualitative account of individuality that avoids the problem of describability would have an advantage over the bare particulars view. However, I think the previous objection is sufficient to dispense with the bare particulars view. As long as both individuals and non-individuals exist, there seems to be little hope for a bare particulars account of individuation.

5.5   Are Objects Individuated by Haecceities?

A haecceity is "the property of being identical with a certain particular individual—not the property that we all share, of being identical with some individual or other, but my property of being identical with me" (Adams 1979, p. 6). A haecceity is a non-qualitative property that is meant to serve as a metaphysical individuator without either postulating a new type of entity, such as a bare particular, or adopting any particular view of objects, such as bundle theory. In addition to a haecceity being different from the property of being identical with something, or some individual, haecceities are distinct from the properties of "(ii) *being an x such that* $(\exists y)(x = y)$, or (iii) *being self-identical*, or (iv) *being an x such that x is identical with x*" (Rosenkrantz 1993, p. 3). None of these properties are non-qualitative because none of them have to do with a specific particular. The haecceity that an object has is the property of being identical with that specific object token. Furthermore, none of these qualitative properties seem like good candidates for individuators because all individuals, and perhaps non-individuals as well, have these properties whereas a haecceity is a property that is unique to each individual.

Before moving on to look at the pros and cons of individuation by haecceities, I should distinguish this view from the doctrine of haecceitism. "If two worlds differ in what they represent *de re* concerning some individual, but do not differ qualitatively in any way, I shall call this a *haecceitistic difference*. *Haecceitism* is the doctrine that there are at least

some cases of haecceitistic difference between worlds. *Anti-haecceitism* is the doctrine that there are not" (Lewis 1986, p. 221). My primary reason for distinguishing the existence of haecceities from the doctrine of haecceitism is that there are reasons to think that the latter is incompatible with quantum mechanics (Huggett 1999). But as Huggett, and Lewis, point out,

> [H]aecceitism is neither necessary nor sufficient for haecceities. For instance, in two identical worlds of individuals that maximally violate the Identity of Indiscernibles—so that all the individuals in both worlds are in identical states—both worlds represent the same of any individual, and so haecceitism fails. But there could still be nonqualitative properties differentiating the individuals. On the other hand, differences in representation de re do not have to be explained in terms of haecceities, for they could be brute metaphysical facts.[14]

Consequently, arguments to the effect that haecceitism is incompatible with quantum mechanics do not rule out the possibility that objects, including particles, are individuated by haecceities. Haecceitism is, essentially, the doctrine that there are, or at least there could be, modal differences that are not grounded in qualitative properties. But haecceities serving as individuators need not imply that there could be any such modal differences. The existence of haecceities is compatible with anti-haecceitism; modal differences may always be grounded in differences in qualitative properties even if some non-qualitative properties, such as haecceities, exist.

The motivations for proposing haecceities as individuators are the same as those behind proposing bare particulars. We accept that being an individual or not is a metaphysical, not merely linguistic, fact that must have some kind of ontological grounds. We then rule

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[14](Huggett 1999, pp. 7-8).

out theories of individuation based on relations and qualitative monadic properties. This forces us to consider non-qualitative ontological grounds for individuality. Haecceities aren't as metaphysically revisionary as bare particulars since there are simple and observable non-qualitative relational properties. So haecceities aren't a new class of entities, at least not in the same way that bare particulars are. Even if one doesn't view non-qualitative relational properties as genuine parts of our ontology over-and-above the qualitative properties that ground them, it's less revisionary to claim that they are genuine parts of our ontology than it is to postulate propertyless substances. The most revisionary part of the haecceities proposal is that it postulates the existence of monadic non-qualitative properties. Additionally, haecceity based views of individuality also avoid the objections against bundle theory for the same reasons as the bare particular view. And, similar to the bare particular view, supposing that there are both individuals possessing haecceities and non-individuals lacking haecceities creates a need to explain why some objects and not others possess hacceities.

None of the views we've looked at seem to work particularly well with the existence of non-individuals save for spatiotemporal theories. With the bundle theory it's not clear that it's coherent to take that as our theory of individuation and also accept the existence of non-individuals. And the bare particulars and haecceities views both have an explanatory burden created when we need to account for individuals. Still, I think the haecceity view is the one defenders of non-individuality should favor. It avoids not only the objections to accounts based on relations or qualitative monadic properties but, more importantly, it actually addresses the problem of individuation. As we saw above, accounts based on relations or qualitative monadic properties address problems that are related to the problem of individuation, but don't actually address the problem of individuation when the problems are carefully distinguished. Additionally, the bare particulars view only seems to address the problem of individuation if either there are no non-individuals or all composite objects are non-individuals. I don't think the explanatory burden belongs to defenders of

the 'Received View' insofar as myself, and many others, also accept the existence of non-individuals (e.g. clouds). So even if I think that particles are individuals, I still have to explain how things such as clouds can be non-individuals and so need to explain either why clouds lack haecceities or why clouds aren't associated with a bare particular.

I think haecceity based views of individuation have two advantages over bare particulars views. The first, which I already mentioned, is that it's less revisionary. Aside from illegitimate or confused calls for explanation from the bare particulars view, there is still a need to explain what bare particulars are. However, while bare particulars are open to the problem of describability, haecceities are not. A haecceity is a non-qualitative property and a haecceity based account of individuation is a transcendental account. However, we can explain what a haecceity is in positive terms. French and Krause treat the problem of describability as a general problem for transcendental accounts, however, it's really only a problem for some transcendental accounts. They phrase the problem in terms of an inability to provide a list of attributes and, as a result, the problem only comes up when the proposed individuator lacks attributes to list. Hence why bare particulars fall prey to the objection and haecceities do not.

The second reason to favor a haecceity based account of individuality over a bare particulars account is that it should be much easier to explain why some objects lack haecceities as opposed to explaining why some objects aren't associated with bare particulars. For one thing, bare particulars are supposed to be what properties attach to. In addition to solving the problem of individuation they are meant to explain a metaphysical distinction underlying the difference between subject and predicate. That explanatory aspect of the bare particulars view is lost if there are subjects (i.e. non-individuals) that are not bare particulars. This is a problem even for one who doesn't think the subject-predicate distinction corresponds to a metaphysical distinction of this sort insofar as it removes some of the motivation for postulating bare particulars in the first place.

Additionally, if the subject-predicate distinction doesn't play any role in how we should interpret our physical theories, then this motivation for positing bare particulars is also lost in considering whether particles are individuated by bare particulars. But, more importantly, there is nothing mysterious that needs explaining in terms of some objects possessing and others lacking a property. There may be an epistemic issue about how we determine what is and what isn't an individual, but we don't normally think that the fact that objects with property X exist and that objects without property X exist calls for an explanation as to why the former objects have X and the latter do not. In short, the explanatory burden of explaining the difference between individuals and non-individuals is a genuine burden for the bare particulars account but can only be seen as a genuine burden for the haecceities account by confusing epistemology for metaphysics.

Furthermore, just because haecceities serve as individuators doesn't preclude the possibility of there being criteria for what is or isn't an individual. I defended such criteria in the previous chapter in the form of conditions that are logically necessary and jointly sufficient for individuality. This actually provides a means for understanding Dorato and Morganti's primitive thisness as something other than a metaphysical claim lacking an ontological truth maker. We could say that to possess a haecceity is just to possess the properties of relative indivisibility, formal immiscibility, and ontological autonomy. I'm not sure they'd want to go this route. While these latter properties would be qualitative it's not clear that they are sensibly understood as actual properties that objects could possess. The first two would be formal modal properties regarding what could or couldn't be done to an object without changing its form (e.g. without changing the kind of object it is). So going this route requires defending the existence of *de re* modal properties. It's also not clear that any of these properties could be understood as intrinsic properties. If they were relational properties, then we'd get into the problems we saw with spatiotemporal accounts of individuation. Consequently, if we're going to take haecceities as

individuators, I think it makes the most sense to treat them as properties rather than as Dorato and Morganti's primitive thisnesses.

The literature on haecceities is much sparser than that on bare particulars and most of the recent literature on haecceities has nothing to do with the problem of individuation. For example, Gary Rosenkrantz's 1993 book *Haecceity: An Ontological Essay* has 65 citations, the vast majority of which have nothing to do with the problem of individuation. Rosenkrantz's positive argument for haecceities takes the form of criticism of alternative views along with, where applicable, demonstrations of how using haecceities as individuators avoids those criticisms. Additionally, similarly to the situation with bare particulars, the few worked out objections to haecceities don't stand up to scrutiny. The few objections in the literature are all discussed by Rosenkrantz and there have been no new objections, as far as I can tell, since then. Consequently, I will discuss the objections in the same order as Rosenkrantz.

The first objection is that there is something problematically trivial about using haecceities as individuators. As Rosenkrantz points out, there is some variation here that all occurs with one particular premise. The general argument, which I will follow Rosenkrantz in referring to as *Argument C*, is as follows:

(C1) The haecceity of a particular, *a*, is the property of being identical with *a*, and to say that *a* has this property is to say that *a is identical with a...*

(C2) If a particular is individuated by its having a haecceity, then the diversity of particulars at a time can be analyzed in terms of a particular's bearing the relation of identity to itself at a time...

(C3) The diversity of particulars at a time cannot be analyzed in this way, since any attempt to do so suffers from (familial) triviality...

(C4) A particular is *not* individuated by its having a haecceity.[15]

In addition to this version of the argument, there are four variations that involve changing the first premise. The problem with this first version of the argument is that either (C1) is false or else we have no reason to think that (C2) follows from (C1). (C1) is ambiguous because we can mean different things when we say '*a* is identical with *a*.' "This expression refers to either a *relational statement* ... or an *attributional statement* (Rosenkrantz 1993, p. 107). The former way of understanding the statement has us ascribing to *a* the qualitative relation ' ＿ being identical with ＿.' Rosenkrantz views this as a qualitative property being ascribed to *a* whereas I don't view it as a metaphysical statement at all (recall my discussion of self-identity in the previous chapter). However, what's important is that understanding (C1) in this way misrepresents haecceities. Haecceities are non-qualitative properties, and the relation of self-identity, property or not, is qualitative. Granted, (C1) understood in this way does seem to imply (C2), but it doesn't matter because (C1), on this understanding, is also false. If instead we understand *a is identical with a* as indicating that *a* possesses the non-qualitative property of *being identical with a*, then it's not clear how (C2) is supposed to follow. How does an object possessing a haecceity imply that diversity of particulars can be analyzed in terms of a relation of self-identity?[16] It's not obvious that it follows on Rosenkrantz's metaphysical understanding of self-identity and less obvious if we view 'a is identical with a' as merely a logical truth with no metaphysical import (as I view it).

I will now turn to the four variations of the argument. My purpose in looking at these variations is to clarify what haecceities are *not* by rejecting a number of problematic attempts to explicate the property of being identical with *a*. What is important for my

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[15](Rosenkrantz 1993, pp. 106-107).

[16]Because (C3) is the rejection of the idea that the diversity of particulars can be analyzed in this way, the analysis referred to in (C2) must be one that is only available due to the presence of haecceities.

purposes is just to note that the property of being identical with *a* is not to be understood in any of the senses proposed below. However, I do include discussion of why the proposals are incorrect rather than merely asserting that the property of being identical with *a* should not be understood in any of these ways. The first variation replaces (C1) with:

"(C1a) The haecceity of a particular, *a*, is the property of being identical with *a*, and this property is identical with the ordered pair of *a* and the identity relation" (Rosenkrantz 1993, p. 108).

This variation avoids the problem of identifying a haecceity with something qualitative since the ordered pair makes reference to a particular object. But it's not clear how the combination in an ordered pair of a qualitative identity relation could have the features of a haecceity. Rosenkrantz notes that haecceities can be exemplified whereas an ordered pair can't be and that ordered pairs have elements whereas haecceities do not. Haecceities and ordered pairs have different features and so can't be identical.

In the next variation we have:

"(C1b) The haecceity of a particular, *a*, is the property of being identical with *a*, and this property is identical with a collection (sum) of *a* and the identity relation" (Rosenkrantz 1993, p. 109).

This proposal claims that haecceities include particulars as parts. The terminology is a bit strange here since a mereological sum of one object is just that object. But setting the terminological point aside, Rosenkrantz claims that the problem with this proposal is that haecceities are abstract entities and, as such, cannot have particulars as parts (Rosenkrantz 1993, p. 109). I disagree with Rosenkrantz that haecceities are abstract entities and I think a response to (C1b) needn't rely on whether haecceities are abstract entities or not. It already seems to be wrong to say that a particular object can be part of a property. More specifically, it seems to be putting the cart before the horse to say that objects are parts of

the properties they possess rather than that properties are parts of objects (assuming parthood is the right way to think of the relation between objects and properties in the first place). Defenders of the (C1b) version of *Argument C* need, at minimum, to explain both how a particular object *could* be part of any of the properties it possesses.

The penultimate version is:

> "(C1c) The haecceity of a particular, *a*, is the property of being identical with *a*,
>
> and this property is identical with the property of being an *x* such that *x*
>
> is identical with *a*" (Rosenkrantz 1993, p. 109).

Rosenkrantz's objection to (C1c) is somewhat involved, and unnecessarily complicated insofar as there's a simpler objection which Rosenkrantz already raised against (C1). (C1c) has left behind any reference to the identity relation that (C2) refers to. Consequently, just as with (C1) understood in the attributional sense, it's not clear how (C2) is supposed to follow. How does identifying a haecceity with the property of *being an x such that x is identical with a* lead to (C2)? If the proper connection can't be established with the identity relation, then (C2) doesn't follow.

Rosenkrantz's objection to (C1c) focuses on a specific argument in its defense. The argument notes that the properties identified in (C1c) are necessarily coinstantiated and claims that any two properties that are necessarily coinstantiated are the same property. Rosenkrantz refers to this latter claim as committing one to a 'coarse grained view of property identity' (Rosenkrantz 1993, p. 110) and goes on to explain why a fine-grained view of property identity is preferable. But we can avoid that debate entirely by noting that we have no reason to think that (C2) follows from (C1c).

The last version of the argument uses the premise:

"(C1d) The haecceity of a particular, *a*, is the property of being identical with *a*,

and this property has *a* and the relation of Identity as logical

constituents" (Rosenkrantz 1993, p. 114).

Here we have a return of the identity relation which makes it less mysterious how (C2) is supposed to follow. This proposal differs from (C1b) in that it does not suggest that haecceities have particulars as actual parts. The proposal is that the properties of __ *being identical with a* and __ *being identical with a* are the same property. Rosenkrantz explains (C1d) as follows:

An advocate of (C1d) maintains that the idea behind this premise can be understood from the Fregean perspectives of linguistic expression, sense, and reference. From the perspective of linguistic expression, a name such as 'being identical with *a*' is created from the predicate '__ is identical with __' by "plugging in" two names '*a*' and '*b*' to produce the sentence '*a* is identical with *b*', "plucking out" '*a*', and nominalizing the result. From the perspective of sense, it is impossible to grasp the sense of 'being identical with *a*' without grasping the senses of the identity predicate and the name '*a*'. From the perspective of reference, the compound relational property of being identical with *a* derives from *a*'s being "plugged" into the right hand variable position in the relation, __ being identical with __, resulting in the monadic attribute __ being identical with *a*.[17]

These three perspectives provide three different approaches to defending (C1d).

The idea from the linguistic expression perspective is that we can read off the metaphysical constituents of properties by looking at the structure of property-designating expressions.

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[17](Rosenkrantz 1993, p. 114).

Rosenkrantz doesn't mention any particular defenders of this line of reasoning; likely because it's too obviously flawed. Otherwise, to use Rosenkrantz's example, the property of *being a believer in Santa Claus* has Santa Claus as a constituent.

The idea from the sense perspective is that if I must grasp A and B in order to grasp C, then A and B must be constituents of C. I'm not sure that's right, but even if it is it seems that we can grasp an objects haecceity without grasping the sense of the identity relation. Rosenkrantz defends this possibility using his distinction between relational and attributional understandings of *a* is *a*. One can have the attributional belief that *a* is *a*, and so attribute a haecceity to *a*, without thinking that *a* stands in a relation of identity to itself. Similarly, one can think that *a* stands in a relation of identity to itself without thinking that it has a haecceity (Rosenkrantz 1993, p. 115). Many people think the latter, including French and Krause, whereas anyone that thinks self-identity isn't a metaphysical relation but accepts the existence of haecceities will think the former. Whatever the relation between the concept of a haecceity and the concept of self-identity, they can come apart. Indeed, it's not clear, as I've mentioned before, how they are supposed to be connected in the first place.

Finally, it's not clear what the idea behind the reference perspective is. I should note, however, that the unclarity isn't so much a feature of Rosenkrantz's discussion as it is that the notion of "plugging" he borrows from Edward Zalta isn't sufficiently explicated. "Plugging" is a technical term that refers to "a putative logical analog of the linguistic operation of partially saturating a multi-place predicate with a name" (Rosenkrantz 1993, p. 114). In order for this approach to differ from the approach from linguistic expression, there has to be a genuine difference between the logical operation of partially saturating a multi-place predicate with a constant and the linguistic operation of partially saturating a multi-place predicate with a name. Not only that, but that difference must be metaphysically salient if it's going to lead to the conclusion that haecceities are to be

identified with the identity relation with a constant plugged in. Rosenkrantz's use of the word 'putative' suggests he isn't convinced that there's a difference and I should note that Rosenkrantz doesn't endorse this argument. He presents it as an alternative way that someone what might defend (C1d) before arguing that this sort of defense of (C1d) leads to a contradiction regardless of how we understand "plugging". However, Rosenkrantz's argument depends on viewing haecceities as abstract entities.

Since I don't view haecceities as abstract entities this leaves me with no counterargument to (C1d). There is the issue of what it means for a non-qualitative property to have the object to which it refers as a logical constituent. I don't think such a view is obviously problematic like the view that haecceities have the object to which they refer as a part. But there still seems to be something backwards about the proposal. At the very least it seems potentially circular to claim that *a* is an individual because it has a haecceity *and* that this individuating haecceity has *a* as a logical part. My arguments from chapter four, however, do allow me to formulate an objection here. That objection is simply that haecceities can't serve to individuate if they are to be understood as (C1d) suggests because the identity relation isn't a sufficient condition for individuation.

These objections have all claimed that haecceities can't serve to individuate (C4). However, there is one more set of objections that claim that haecceities are impossible. Some of these objections are very similar to those we've seen above and fail for similar reasons. For example, one such objection relies on (C1b) in conjunction with the claim that abstract objects can't have concrete objects as parts. It might seem question begging to use the same response to (C1b) here since that response was essentially this argument in reverse. Abstract objects can't have concrete objects as parts and so a haecceity can't have a concrete object as a part. But (C1b) misrepresents the view it is attempting to refute. There's no point postulating non-primitive haecceities to serve as individuators if the qualitative properties that the objectors are proposing as the components of haecceities in

(C1) and its variations could do the work haecceities are meant to do. If a haecceity is non-primitive, it must be something over and above its constituents if it's going to do any metaphysical work that those constituents can't do. Consequently, there's something peculiar about trying to take a reductionist approach to haecceities, or to treat defenders of haecceities as viewing them in a reductionist way. They *couldn't* just be what these objectors are claiming them to be. A haecceity is the property of being identical with *a* or *b*, etc., for objects *a*, *b*.... Rosenkrantz considers a revised form of this objection that avoids this problem (Rosenkrantz 1993, p. 125).

Instead of claiming that haecceities include concrete particulars as parts, the revised claim is just that haecceities, being non-qualitative, must be intimately related to concrete particulars in a way that doesn't make sense. After all a haecceity is always associated with one specific concrete particular. Rosenkrantz notes that the sense in which the haecceity and concrete particular are intimately related isn't made clear. There is also no explanation of why the alleged problematic relation is actually problematic. If there are such things as sets of concrete objects then there are abstract entities with an intimate relation to concrete objects insofar as they have those objects as elements (but not as parts or constituents) (Rosenkrantz 1993, p. 125). And, I mentioned previously, there are completely unmysterious non-qualitative relational properties (e.g. the property of owning a particular piece of land). So, it can't be the case that properties are problematic simply in virtue of making reference to specific particulars. Ultimately, this objection amount to the intuition that there is something strange about intrinsic non-qualitative properties (so I've responded to it previously).

The other objections to the effect that haecceities are impossible are similarly question begging. They rely on claims such as (i) "[n]ecessarily, if *P* is a property, then an individual's conceiving *P* does not entail his conceiving of a particular *concretum*" (Rosenkrantz 1993, p. 126), (ii) "[n]ecessarily, if *P* is a property, then *P* is possibly

exemplified by something, *x*, and *P* is possibly exemplified by something, *y*, which is not identical with *x*" (Rosenkrantz 1993, p. 127), (iii) "[i]t is impossible that an abstract entity has contingent existence" (Rosenkrantz 1993, p. 128), and (iv) "[n]ecessarily, a property is possibly grasped" (Rosenkrantz 1993, p. 128). (i) and (ii) are simply statements that there are no such things as non-qualitative properties. (iii) is the claim that a certain kind of relation between abstract entities and concrete particulars isn't acceptable (although one needn't accept that haecceities are contingent entities even if they are abstract since one can accept the existence of uninstantiated abstract entities). Still, (iii) is question-begging insofar as we're given no reason to think abstract objects couldn't exist contingently. Finally, while one could defend the view that haecceities can all be "possibly grasped," (iv) seems to embody a kind of epistemic arrogance that sometimes lies behind the dismissal of "metaphysically suspect" entities. While one could challenge the use of haecceities as individuators using these four claims, they can't be the starting point of those arguments. Each of these claims would need its own independent defense.

5.6   Are Objects Individuated by Matter or Form?

The next two metaphysical grounds I want to look at rely on a distinction between matter and form. The material theory of individuation states that objects are individuated by the matter that they possess whereas the formal theory of individuation states that objects are individuated by their substantial form. These views have received even less attention than haecceities but, I think, for good reason.

The idea behind the material theory is that "[i]t is in the nature of matter to be unshareable and, therefore, all material things are rendered individual by it" (Gracia 1988, p. 156). Gracia notes a number of problems with this view but leaves aside what I think is the biggest issue. According to the material theory of individuation all material objects are

individuals. There's no distinction between composite and non-composite objects as far as individuality is concerned since being made of matter is what makes something an individual. Clouds, for example, would count as individuals. Gracia doesn't consider this a problem and, as we'll see in the next section, Gracia takes clouds to be individuals. But my discussion in the previous chapter shows that clouds do no satisfy the logically necessary and sufficient conditions for individuality.

Gracia's main concern with the material theory is that it's not clear how matter itself is to be individuated ((Lowe 2005) raises this worry as well). If we have two marble statues it can't be that "marble" accounts for the individuality of each statue; it must be that *this* marble accounts for the individuality of one and *that* marble accounts for the individuality of the other (Gracia 1988, p. 156). To deal with this problem defenders of the material theory have added additional features that play a role in individuating objects. This additional feature has either been the actual dimensions of the object in question or merely the fact that material objects always have particular dimensions. In the first case, what individuates the two statues is the actual dimensions each statue has. There are two obvious problems here. One is that the statues could have the same dimensions. The other, which Gracia mentions, is that the actual dimensions an object has are accidental features (this may not be true for elementary particles, but it seems true for any composite material object–there are no perfectly rigid bonds between particles). It seems to defeat the purpose of a theory of individuation to invoke accidental features as individuators. How can an object be individuated by features that it needn't possess? In the second case, what individuates the statues is that a piece of marble must always have particular dimensions. This isn't an accidental feature, but this doesn't seem to solve the problem it's intended to solve. The marble of both statues has the feature that it must have some physical dimensions or other and so we are still left wondering what it is that individuates the statues.

The idea that objects are individuated by matter plus its actual dimensions demonstrates another major problem with the material theory of individuation. It's not actually a theory of individuation at all but rather a theory of difference. It conflates the problem of individuation with the problem of accounting for difference and tries to provide a solution to the latter. It isn't trying to explain why a given marble statue is an individual rather it is trying to explain why any two marble statues are distinct. And it is reasonable, I think, to explain difference in terms of accidental features. At least for qualitative difference. Explaining numerical difference in terms of accidental features requires PII. The view still fails as an account of difference if the accidental feature in question is the actual dimensions of objects because different objects can have the same dimensions. But if no two objects could have the same dimensions then perhaps one could explain why there are two objects by appealing to their actual dimensions.

The formal theory of individuation locates the metaphysical grounds for individuation in an objects substantial form. "A form is a structural principle, and a substantial form is the structural principle that determines the fundamental (i.e., necessary and sufficient) features of a substance" (Gracia 1988, p. 158). In other words, the formal theory of individuation says that what individuates objects are the necessary and sufficient conditions for being the *kind* of object that it is. The standard, and obvious, objection to this view is that "the very notion of form, as opposed to matter, is the notion of what is shareable by and common to many" (Gracia 1988, p. 158). If we have two identical spheres and we want to know what makes sphere A an individual it can't be its sphericity "for its sphericity cannot be what makes the sphere the very sphere that it is" (Lowe 2005, p. 81). Gracia doesn't view this objection to the formal theory as conclusive because one could appeal to ordinal forms such as "the first man born in space" (Gracia 1988, p. 159). But it strikes me as question begging to, essentially, individuate people based on birth order. More importantly, birth, or creation, order is an accidental feature as well, so this response fails for the same reasons as using matter along with an object's actual dimensions.

The formal theory of individuation can be viewed as a more restricted version of bundle theory that picks some particular subset of features as individuators rather than all of an object's features. But it still leads to the problem that all objects are individuals since every object is an object of a given kind. Some of the objections against bundle theory also apply here. But I still think the most problematic issue is that it tries to locate conditions of individuality in kind criteria. It's understandable in the sense that it's difficult to specify bundles of features to serve as individuators in a non-arbitrary way. One at least can't object that there's something arbitrary about bundles of properties that constitute the essential features of a given kind of object. But the focus on kind suggests that the formal theory is concerned with discernibility rather than individuation, just as the full-blown bundle theory is. What individuates two objects of the same kind if the relevant properties are all and only the essential properties those objects all share?

5.7   Are Objects Individuated by Existence?

The final theory of the metaphysical grounds of individuality that I will look at is the existential theory of individuation developed by Gracia. The theory states that "existence is the principle of individuation for all individuals" (Gracia 1988, p. 170) and that existence is not a property. The reason for the latter claim is that existence, understood as a property, would be something possessed by every existing object leading to the question of what individuates one object's property of existence from another. If, instead, we understand existence as something unique to each object, then, according to Gracia, there's no need to explain what individuates existence. "Existence" here refers to some ontological component of objects that is unique to each object like a haecceity. However, unlike a haecceity, it is not a property. It is also not a substance like a bare particular. The idea that existence is not a property is uncontroversial, however, the idea that existence is

an ontological feature of objects is controversial. Gracia's view is very similar to the bare particulars view with 'bare particular' being replaced by 'existence' and 'substance' being replaced with some new unexplained ontological category. Arguably this is just a modification of the bare particulars view such that it is confusing for Gracia to use the term 'existence'. The difference from the bare particulars view seems to be only that 'existence' doesn't serve as a bearer of properties and doesn't serve to explain the subject-predicate distinction. So it may be best to understand Gracia's proposal as simply a variant of the bare particulars view in which two of the metaphysical roles assigned to bare particulars have been removed leaving only the role of individuator.

As with the regular bare particulars view, Gracia's view faces the problem of describability. Other than saying that existence is not a property of objects, Gracia says nothing else about what sort of thing it is. It is described entirely in negative terms and that description is even more sparse than that given to bare particulars. Consequently, there is nothing more I can say about the kind of thing existence is beyond it being an ontological component of objects that is neither a property nor a substance. That the bare particulars view and Gracia's view both face the problem of describability may explain why it's difficult to distinguish between them aside from the difference in metaphysical roles they are meant to play. At worst, the problem of describability might ensure that there can be no substantive difference between Gracia's view and the bare particular view. Much of what Gracia says implies that he sees what he's referring to by 'existence' as closely related to the ordinary sense of the term as seen, for example, by his concern (discussed below) with the objection that there are possible individuals that don't actually exist. For my purposes it may not matter whether we can get clear on precisely what Gracia's proposal is. If it is just a modification of the bare particulars view, then it is one that doesn't avoid the objections I raised against the view in the preceding section. If, on the other hand, it is not merely a modification of the bare particulars view then it still faces the problem of describability and the issue that my arguments in favor of existence of

non-individuals in chapter four constitute counterarguments (i.e. my conditions on individuality imply that there are non-individuals, such as clouds, and so imply that a view that implies that clouds are individuals, or more generally that there cannot be non-individuals, is incorrect).

The existential theory of individuation shares the same advantages that the non-qualitative approaches have. It avoids the objections leveled at theories that attempt to ground individuation in qualitative properties (either relational or monadic properties). It avoids creating a demand for an explanation of the individuation of the proposed individuator because existence, just like bare particulars and haecceities, isn't shareable. A lot of the objections to views based on qualitative properties arise because such properties are shareable leading to difficulties in explaining the uniqueness that is a feature of individuals. Similar to the bare particulars and haecceities views, the positive case for the existential theory of individuation is that it avoids objections facing alternative views while, allegedly, not facing any devastating objections itself. Although I agree with French and Krause that the problem of describability is a significant problem and it is one faced by both the bare particular and existence-based accounts.

There are three major objections to the existential theory of individuation. The first is that non-existing entities can be individuals as well. The objection is a modal one regarding the possible existence of individuals that don't actually exist. It claims that the existential theory of individuation is incompatible with the idea that it's possible for there to exist individuals that don't actually exist. Gracia provides two answers to this objection. The first is that the existential theory needn't be compatible with possible individuals because even if they would be individuals if they did exist, they are not individuals if they don't exist (Gracia 1988, pp. 172-173). His reasoning behind this is based on claiming that the only candidates for possible individuals are "(1) composites of universal features that as such are instantiable; or (2) mental images of individuals that have actually existed"

(Gracia 1988, p. 172). In the case of (1) there seems to be no reason to think that a collection of uninstantiated universals is an individual even if an individual could instantiate that collection of universals. The same is true even if properties are tropes. Why would a collection of uninstantiated but coinstantiable tropes count as an individual? In the case of (2), existence can explain why the individual was an individual.

Gracia doesn't provide any reasons for thinking that (1) and (2) exhaust everything we might mean by 'possible individual' beyond claiming that his experience suggests that actual references to possible individuals is always about such cases. Although Gracia gives an example of the possible existence of a silver dollar that wasn't actually minted, he leaves aside such cases. In addition to (1) and (2) we have (3) individuals that could exist but don't exist. Gracia must be treating these as instances of (1) which is a bit strange given that he rejects the bundle theory. But I think there is a more straightforward response that works for cases (1) and (3), even if (3) is treated as distinct from (1). This response is simply that there is no such thing as a possible individual. Since what we're talking about here is the metaphysical grounds of individuality a 'possible individual' in this context must possess some actual ontological component that grounds its individuality.

To be a bit more precise, I think this objection conflates two different issues. It conflates the issue of possessing the logically necessary and sufficient conditions for individuality with possessing the metaphysical grounds for individuality. To object that an account of the metaphysical grounds for individuality doesn't allow for possible individuals is to commit oneself to some form of modal realism. Even though the possible individual doesn't actually exist, the objector would have to take the ontological grounds for its individuality to exist in order to object in this way. Any individuals that don't exist on a given account of the metaphysical grounds of individuality fail to exist because the metaphysical grounds for their individuality fails to exist. To object that a given account of the metaphysical grounds of individuality doesn't allow for certain kinds of individuals

is to claim that the metaphysical grounds for the individuality of those individuals does, or at least could, exist. But what would could it mean to say that the metaphysical grounds for the individuality of a given possible individual exist without adopting modal realism?

I wonder if what's going on is that the objector is expecting an explanation of the metaphysical grounds of individuation to set the logically necessary and sufficient conditions of individuality. Or, put another way, I wonder if the objector is conflating the question of what the intension of 'individual' is with the question of what the metaphysical grounds of individuation are. In that case proper target of the objection would be the conditions discussed in preceding chapter rather than the metaphysical accounts being discussed here. Recall on Gracia's account that what it means to be an individual is to be non-instantiable. Possible individuals are non-instantiable (on the assumption that properties aren't individuals as Gracia claims) and so Gracia's position allows for possible individuals. It's just that those individuals don't exist in the ordinary sense of the term and also don't possess existence in Gracia's special sense of the term. To object that Gracia's proposed metaphysical ground of existence doesn't allow for possible existence is to confuse one of the roles given to the condition of non-instantiability for a role meant for the metaphysical grounds of individuality. One could certainly object that Gracia's 'existence' should play the role of determining the conditions of individuality. However, that's a different objection. The objection here isn't that Gracia has misassigned roles but that by his own lights his theory doesn't allow for possible individuals. On Gracia's view, a *merely* possible individual is an individual that satisfies his definition of individuality as non-instantiability but doesn't exist (and so lacks the metaphysical grounds for individuality).[18] An actual individual exists in addition to satisfying the definition.

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[18]But note that Gracia isn't allowing for possible individuals in a modal realist sense.

The additional answer Gracia gives, which I will only briefly mention for the sake of completeness, is that we could distinguish between actual existence, possible existence, and impossible existence and allow the first two both to serve as individuators (Gracia 1988, pp. 173-175). This response applies only to the modal realist version of the objection and not the version that conflates the logically necessary and sufficient conditions for individuation with the metaphysical grounds of individuation. While the idea of possible existence is controversial and would need to be defended, it isn't something a modal realist should take issue with.

The second objection is that "the same individual may cease to exist at one time and begin to exist later at another time and that in this case it would have two separate existences" (Gracia 1988, p. 175). This objection simply begs the question. Existence is not a property that is multiply instantiable such that the individual in question could have existence$_1$ as a property when it first exists and existence$_2$ as a property when it comes back into existence. Suppose this objection were raised against the haecceity view of individuation. In order for the individual to be the same individual at both times it exists on the haecceity view would be to have the very same haecceity both times. For it to be an individual both times and possess a different haecceity both times would be for it to be two different individuals at those times. For the objector to insist that the same individual would have two different haecceities, on the assumption that the haecceity view of individuation were correct, is to beg the question. The same is true with respect to existence. Why think that the ontological component of an individual to which 'existence' refers would be different? If you combine the idea that existence individuates with the idea that existence is a property, then it makes sense why the individual in the example would have two separate existences. There would either be two individual existence tropes, or two instances of an existence universal being instantiated that are, somehow, individuated. But if existence is

not a property, then it's not clear why an individual that exists for some time, ceases to exist, then begins to exist again would have two separate existences.[19]

The final objection is also based on a confusion. It states that "if existence is responsible for making "man" this *or* that man, then it cannot be the principle that makes "man" this man *rather than* that man" (Gracia 1988, p. 176). The objection is that if existence is what makes an instantiation of "man" into *this* man rather than merely *a* man, then existence can't account for why *this* man is different than *that* man. But this is to conflate the problem of individuation with the problem of accounting for difference (Gracia 1988, p. 176). Existence is what makes a given object an individual regardless of whatever else exists including other instances of the same kind of object. A principle of individuation isn't meant to account for difference, even though some principles of individuation could do so. Making "table" into *this* table is all that a principle of individuation needs to do in order to address the problem of individuation. However, as stated before, a principle of individuation will serve to account for difference, at least among individuals, in a universe that contains more than just a single, simple individual.

5.8 PII Revisited

Before wrapping up this chapter I would like to return to PII and discuss the three concerns I had mentioned in chapter three. The first of these concerns has to do with the relationship between identity and individuality. We saw in chapter four that there doesn't seem to be any sensible metaphysical understanding of self-identity as a relation to ground metaphysical claims of self-identity. There also doesn't seem to be a sensible way of

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[19]I'm assuming here that there is a way to understand Gracia's position without existence collapsing into some other account of individuation, such as a spatiotemporal account.

understanding the denial of self-identity understood as a relation. One could claim that self-identity serves as a logically necessary and sufficient condition for individuality, but I didn't discuss that option (despite such conditions being the focus of that chapter) because that would imply that everything is an individual. One could say that self-identity refers to a non-relational ontological feature of objects, but then we're talking about bare particulars, haecceities, or primitive thisness. We also saw that there is a difference between a criterion of identity and a principle of individuation, and that the former isn't necessary for the latter.

How does this create a problem for PII? Recall that PII states that objects that are qualitatively identical are numerically identical. Not only do we need a connection between identity and individuality for the failure of PII to have implications for individuality, we need the right kind of connection since, as I stated in chapter two, it is possible to conceive of individuals that fail to satisfy PII. But it looks to me like there's no connection at all. Additionally, since the failure of PII implies a lack of numerical identity it is numerical identity or lack thereof that must be relevant to individuality. So, it must be the case that objects that possess numerical identity are individuals and objects that lack numerical identity are not individuals. But the failure of PII doesn't imply that there is any single object that lacks numerical identity. That is, it doesn't imply that our, say, 'two' qualitatively identical bosons 'are' really one boson that lacks numerical identity. It implies that one boson is not numerically identical with a distinct boson that is qualitatively identical. Don Howard puts the point this way, "even if one accepts the conclusion that, owing to their indiscernibility, two bosons are not individuals, in spite of their difference in spatial situation, this lack of individuality does not come in the form of the bosons' being numerically identical" (Howard 2011, p. 228). If bosons aren't individuals the reason for this can't be a matter of numerical identity. Consequently, it's not clear that PII has any role at all in this debate.

Looking at my second concern with PII may help clarify why PII is often invoked. The second concern is that people often conflate the problem of individuation with the problem of accounting for difference. We saw that in the metaphysics literature both with some proposed solutions to the problem of individuation that were really addressing the problem of accounting for difference and with some misguided criticisms of proposed solutions that really do address the problem of individuation. I said before that the problem of individuation still needs to be addressed in a one object world, or in a world in which there are no qualitatively identical objects. That the problem of individuation still arises in these situations makes it clear "both that the principle of the indiscernibility of identicals and that the principle of the identity of indiscernibles are exactly what their names indicate: principles about identity, and, hence, principles belonging to the problem of differentiation. But they are *not* principles pertaining to the problem of individuation" (Castañeda 1975, p. 133). If one conflates the problem of individuation with the problem of accounting for difference, then it makes sense, given how often the conflation seems to occur, why PII is often invoked in discussions about the former. But indistinguishability does not imply a lack of individuality nor does distinguishability imply individuality.

French and Krause, for example, state that distinguishability and individuality are two different things but don't seem to realize the implications for PII. Howard notes the tension here when he says "French and Krause stress the importance of noting a distinction between individuality and distinguishability, so what is assumed cannot be a *definition* of individuality as distinguishability. Instead, what seems to be assumed is something that one might dub the 'Principle of the Non-Individuality of Indistinguishables'" (Howard 2011, pp. 228-229). It seems to me that assuming such a principle would be question begging in two ways. First, it simply asserts that elementary particles are non-individuals and second it merely assumes that there is a relation between distinguishability and individuality that is such that indistinguishables are not individuals.

Consequently, the conflation between distinguishability and individuality results in misguided arguments from a failure of PII to a lack of individuality whose question begging nature is obscured by the fact that defenses of the failure of PII for particles are mistakenly viewed as defenses of the actually undefended Principle of the Non-Individuality of Indistinguishables.

The final issue is that of the conflation of metaphysical and logical notions of identity. Identity understood as a relation that obtains between an object and itself seems to be a purely logical relation that does not correspond to any metaphysical fact. In other words, identity statements of the form '$x = x$' are true in virtue of their logical form not in virtue of the existence of some metaphysical truth maker.[20] The failure of PII, understood as implying that some objects lack the metaphysical grounds for individuality, confuses metaphysical identity with logical identity. In other words, the mistake here is to think that an object's lack of numerical identity (which, as I said, a failure of PII doesn't actually imply) is some sort of metaphysical fact. Note that this issue is distinct from the first issue because the problem there was the lack of connection between numerical identity and individuality. One could provide an argument for a connection between numerical identity and individuality that doesn't explain how to understand the falsity of statements of the form '$x = x$.'

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[20]If one thinks that 'Pegasus=Pegasus' is false because 'Pegasus' doesn't refer, then part of what makes identity statements true are that the objects they refer to exist. In this case, identity statements are not true merely in virtue of their logical form. However, those who want to characterize non-individuals as objects that lack self-identity won't find any assistance here. They need to make sense of how statements of self-identity can fail to apply to existing objects.

5.9   Conclusion

The primary purpose of the previous chapters is to demonstrate that there are sensible ways to understand the logically necessary and sufficient conditions for individuality such that (i) there exist both individuals and non-individuals, and (ii) there are sensible metaphysical grounds that are compatible with (i).

So how does my discussion fare with respect to these goals? Self-identity fails to satisfy (i) since, understood as a logical condition it is incoherent to suppose that something could lack self-identity. So, the only coherent account of individuality based on a criterion of self-identity would imply that all objects are individuals. Synchronic and diachronic identity fail to satisfy (i) insofar as they are unsatisfactory as criteria of individuality. Distinctness also fails to satisfy (i) since two putative non-individuals would be numerically distinct and so would actually be individuals. At best, using distinctness as a criterion is compatible with the existence of one non-individual of any given kind if the condition on distinctness is relativized to kind. Finally, the condition of non-instantiability also fails to satisfy (i). Since any particular object is not instantiable every particular object would be an individual.

The conditions of formal immiscibility and relative indivisibility satisfy (i); they imply that things such as rabbits are individuals whereas things such as clouds and piles of rice are not individuals. The condition of ontological autonomy also satisfies (i), even without being paired with formal immiscibility and relative indivisibility, since some things are ontologically autonomous whereas others aren't. Although, without the other conditions things that shouldn't be counted as individuals, like clouds, are counted as individuals. So although I concluded that these three conditions are the appropriate logically necessary and sufficient conditions independently of criterion (i), they are the only conditions we've looked at that seem to be compatible with the existence of both individuals and multiple

non-individuals of the same kind. Defenders of non-individuality also require that (i) be satisfied by their preferred account of individuality. They couldn't deny that individuals exist altogether because then the metaphysical import of the lack of individuality of particles and its alleged role in explaining quantum statistics would be lost. Put another way, if classical particles aren't individuals then the lack of individuality of quantum particles can't explain the difference between classical and quantum statistics. Consequently, for the sake of the 'Received View' (and its primary defense, the argument from quantum statistics), classical particles had better be individuals. So defenders of the 'Received View' require (i) to be satisfied and that seems to require accepting some combination of the conditions of formal immiscibility, relative indivisibility, and ontological autonomy as the logically necessary and sufficient conditions for individuality.

Spatiotemporal relations are, I think, unsuitable as individuators for the reasons I mentioned previously, but at least spatiotemporal relations satisfy (ii). If spatiotemporal relations individuate then there can be both individuals and non-individuals. And it may even give the correct answers as to what is and what is not an individual. At the very least, there seems to be room on a spatiotemporal account to explain why things like piles of rice and clouds are not individuals due to the object as a whole not possessing a unique spatiotemporal trajectory. Although there would be the worry about the possibility of a pile of rice or cloud that does happen to possess a unique spatiotemporal trajectory. So spatiotemporal relations may not satisfy (ii) in its intended sense if it would end up implying that clouds sometimes are and sometimes aren't individuals dependent on whether they happen to mix with other clouds at some point during their existence.

We saw already that neither the bundle theory nor bare particulars allow for non-individuals. Bundle theory appears to be incompatible with the existence of non-individuals since there seems to be no way to explain what a non-individual would

even be if objects are individuated by the complete set of properties they actually possess. Bare particulars are incompatible insofar as the they wouldn't solve the problem of individuation if we need to further explain why some objects are associated with bare particulars and some other objects aren't. Or, in other words, if we have to explain why some objects are individuals and other objects are non-individuals. All material objects would be individuals on the matter theory of individuation and all objects that are of some kind or other (i.e. all objects besides bare particulars) would be individuals on the formal theory of individuation. Finally, if existence is our individuator then, again, there is no room for non-individuals.

Consequently, if we require that there are both individuals and non-individuals then our options for the metaphysical grounds of individuality seem to be reduced to either spatiotemporal relations or haecceities. If we further require that things such as clouds aren't individuals and if the spatiotemporal account can't avoid treating them as individuals some of the time, then hacceities seem to be the only option.

While philosophers have considered, and endorsed, the possibility of non-individuals in the past, the possible, or actual, existence of non-individuals has been largely absent from contemporary debates over both the logically necessary and sufficient conditions for individuation and the metaphysical grounds of individuation. Allowing for the existence of non-individuals seems to clear away a lot of the views on these issues. This perhaps explains why French and Krause developed a new account of individuation in terms of self-identity. But combining the requirement that our theory allow for both individuals and non-individuals with my criticisms of the coherence, or lack thereof, of self-identity as an individuator, we seem to be left with criteria that seem to imply that particles are individuals as well as a metaphysical ground that is applicable to particles.

As in chapter four it is likely unclear what my contributions are in this chapter. Particularly since I spend significantly more time in this chapter merely describing what

other philosophers have said. I see myself providing three major contributions in this chapter. The first is determining which accounts of the possible metaphysical grounds of individuality are compatible with Chauvier's conditions. This dissertation is just as much a work of analytic metaphysics as it is a work in philosophy of physics. Consequently, while many philosophers of physics may not be concerned with whether there are metaphysical grounds compatible with the conditions of individuality that I defend, it is something metaphysicians would be very much concerned with.

The second major contribution is determining which existent accounts of the possible metaphysical grounds of individuality are compatible with the existence of non-individuals. Not merely because both myself and the defenders of non-individuality would require such metaphysical grounds (assuming that we accept that individuality does need metaphysical grounding), but because as long as there are sensible accounts of non-individuals we should take care that our metaphysics doesn't dismiss such accounts out of hand.

The third, and final, major contribution is my attempt to defend metaphysical grounds that are generally unpopular among philosophers of science. I do this partly by criticizing Dorato and Morganti's attempt to restrict their category of the naturalistically acceptable non-qualitative to metaphysically minimal brute facts. I also do this partly by providing responses to a number of objections to the bare particulars view and supplementing Rosenkrantz's discussion of objections to haecceities with a few responses of my own. Part of the reason for this is simply so that I'm not relying entirely on my analysis of individuality and my acceptance of the existence of non-individuals in evaluating these accounts. Part of it is that I accept one of these unpopular views myself (i.e. the existence of haecceities that serve as individuators). But part of it is that the motivation for this entire work is that I think it both important and worthwhile to pay attention to the work of metaphysicians on these issues. In this work I've provided not only a new argument in

favor of particles being individuals (primarily discussed in the next chapter) but an argument that seems to be different in kind from the arguments we saw in chapter two. An argument that arose from paying attention to the sorts of issues pertaining to individuality that metaphysicians focus on.

Particles as Individuals

6.1 Individuality in Quantum Mechanics

Having looked at the arguments in the philosophy of physics literature for and against particles being individuals; some challenges, again coming from the philosophy of physics literature, against these views; what it means for something to be an individual and the logically necessary and sufficient conditions for individuality from the metaphysics literature; and the possible metaphysical grounds for individuality, I am now in a position to bring everything together.

I want to begin with a small review of some of the things discussed in the previous chapters. First, let us return to the arguments for particles as non-individuals. The first argument was based on an alleged difference between the labeled tensor-product Hilbert space formalism and the Fock space formalism. But, as we saw, the apparent lack of labels in the Fock space formalism is just that; an "apparent" lack resulting from the fact that the Fock space formalism is compatible with the occupation number representation. It's not that the Fock space formalism suggests that particles are not individuals it's merely that the Fock space formalism can, but needn't, be represented in a way in which particles are not explicitly treated as individuals.

The second argument was that the non-individuality of particles explains quantum statistics. One may therefore think that my commitment to the view that particles are

individuals comes with an obligation to provide an alternative explanation for quantum statistics. I don't think I have such a commitment nor do I have an alternative explanation. However, I can provide an additional reason to think that the explanation must be something other than non-individuality. In explaining why quantum statistics differs from classical statistics what is being explained is why quantum particles display different dynamical behavior than classical particles. But this immediately leads to the question: "why would a particle's being an individual or not have any implications for its dynamical behavior?" Any proposed metaphysical underpinnings of the difference between classical and quantum statistics must come along with an explanation of why those underpinnings are something that would have implications for the dynamical behavior of particles. In the case of individuality, it's not clear why being an individual or not should have anything to do with a particle's dynamical behavior. This isn't to say that there could be no such explanation, only that such an explanation would need to be provided. If individuality isn't the sort of thing that influences dynamics, then the sort of explanation being offered isn't the right kind of explanation.

Consequently, it's unclear whether an explanation in terms of non-individuality is even the right kind of explanation for the difference between classical and quantum statistics let alone whether it's a successful explanation of that difference. So while I can't provide an alternative explanation of the difference between classical and quantum statistics, I can at least say that we have reason to think that an explanation in terms of non-individuality is the wrong sort of explanation (absent an explanation of the connection between individuality and dynamics).

Additionally, recall that the permutability that non-individuality is invoked to explain may not be a difference between classical and quantum particles. Saunders provided reason to think that classical particles are permutable as well. And Belousek provided reason to think that the permutability of quantum particles is not necessary to derive quantum

statistics in which case it seems that the explanation for quantum statistics needn't depend on permutability nor the non-individuality invoked to explain it. Similarly, Morganti argues that the only statistically relevant properties for many particle systems are inherent properties that are insensitive to permutations. In short, we have a number of reasons to question whether permutability is relevant to the difference between classical and quantum statistics. Therefore, we have a number of reasons to question whether non-individuality, whose role in the argument from quantum statistics is to explain permutability, is relevant to the difference between classical and quantum statistics.

I provided some defense of the idea of weak discernibility as a transcendental argument for the individuality of particles. If weak discernibility is something over and above mere numerical distinctness and if the weakly discerning relations that apply to all fermions and to all bosons are physically meaningful, then there must be something about the world in virtue of which particles are weakly discernible rather than merely numerically distinct. That something should be non-qualitative since weak discernibility isn't grounded in differences in qualitative properties.[1] So weak discernibility suggests that particles are individuals in virtue of possessing some non-qualitative something or other (a property, a bare particular, etc...). However, there is a problem here. It's the same problem with the appeal to non-individuality to explain quantum statistics. Why would individuality or lack thereof be the sort of thing that could ground a physically meaningful weakly discerning relation? The problem doesn't seem as pressing here since we have reason to think that whatever explains weak discernibility would be non-qualitative and, I think, it's individuality being a non-qualitative matter that creates the explanatory issue. If individuality were a qualitative matter, then we could look at how the relevant qualitative properties function in our theory in order to look for a connection between individuality

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[1]Granted the proposals for weakly discerning relations themselves are qualitative. But if we're looking for further explanation as to why such relations hold in the first place, there doesn't seem to be anything else qualitative that we could appeal to.

and weak discernibility or between individuality and quantum statistics. If individuality is non-qualitative then finding a connection is more difficult. But if what grounds weak discernibility must be non-qualitative then, assuming weak discernibility is something that needs to be grounded, then there must be a connection. Combining my discussion of weak discernibility with my discussion of the possible non-qualitative grounds of individuality, we can view the existence of physically meaningful weakly discerning relations, if there are any that are in fact physically meaningful, as reason to accept hacceities.

6.2   Issues with Individuality in NRQM

In chapter three I looked at a number of issues with viewing particles as individuals or non-individuals. These took the form of question that defenders of non-individuality need to answer, and some proofs related to PII that may cause issues for defenders of individuality.

The first issue for defenders of non-individuality was that they need to develop a notion of cardinality that is appropriate for collections of non-individuals. Domenech and Holik's results may have already accomplished this. Recall that Jantzen's criticism was that Domenech and Holik's definition of cardinality assumed self-identity. Consequently, Domenech and Holik's definition of cardinality would not be suitable for the dominant particles-as-non-individuals view in which non-individuality is defined as lack of self-identity. However, I don't see this as a problem for defenders of non-individuality since I don't think they should be defining non-individuality as lack of self-identity (I explained in chapter four why I think this is incoherent). So Domenech and Holik's definition of cardinality may be suitable for alternative particles-as-non-individuals views although there are currently no such worked out alternatives.

The second issue was that particle terms may be mass nouns and that mass nouns denote individuals. I addressed this issue in chapter four. The conditions of formal immiscibility and relative indivisibility ensure that mass nouns don't denote individuals. Clouds, piles of rice, bodies of water, collections of furniture, etc... may be made up of individuals, but they themselves are not individuals. So, if particle terms are mass nouns, that does not imply that particles are individuals nor does it imply that they are non-individuals. It doesn't even imply that collections of particles are not individuals because the conditions of formal immiscibility and relative indivisibility don't imply that any possible mass nouns we could conceive of would describe non-individuals. Plenty of macroscopic objects satisfy these conditions despite being composed of particles. This may be a reason to think that particle terms are not mass nouns although whether particle terms are mass nouns strikes me as an empirical matter.

The third issue was a collection of arguments to the effect that identity is fundamental and therefore we should at least treat particles as individuals. But I have the same response here as I did to the first issue. There is no problem for defenders of non-individuality here because they shouldn't be defining non-individuality as lack of self-identity. So even if Bueno's arguments from chapter three are correct, it's only defenders of the view that non-individuality is a lack of self-identity that need to be concerned.

So while I ultimately think examining what it means for something to be an individual and looking at the possible metaphysical grounds of individuality favor a particles-as-individuals view, that same examination removes all three of the challenges I discussed in chapter three. Although, again, those challenges do cause problems for the only developed particles-as-non-individuals view in which particles are non-individuals because they lack self-identity.

Finally, the issue for defenders of individuality was not to defend accounts that were incompatible with the various proofs regarding PII I discussed in chapter three. However,

this isn't really an issue because when it comes to individuality, PII is beside the point. PII has to do with distinguishability and so the only accounts of individuality that run into trouble here are those that either conflate individuality and distinguishability or connect them in the right sort of way. Any of the views we looked at that didn't make this conflation didn't connect individuality and distinguishability in any meaningful way (at least not explicitly). And we saw in the preceding chapters why there shouldn't be a meaningful connection between the two. So the accounts of individuality that run into this issue are not really accounts of individuality at all. Rather, they were accounts of distinguishability. This is why I didn't discuss this issue in chapters four and five. Its targets are not accounts of individuality to begin with.

6.3   Particles as Individuals

Finally, I am in a position to address the issue of whether particles are individuals. On the one hand is the question of whether particles satisfy the logically necessary and sufficient conditions for individuality. On the other hand, is the question of what the provides the metaphysical grounds of that individuality. I will look at each condition for individuality in turn starting with relative indivisibility.

As I stated in chapter four, what is important here isn't what it might mean to 'divide' or 'mix' particles since what matters isn't whether the terms 'divisibility' or 'miscibility' are appropriate. What matters for the case of relative indivisibility is whether there is a physical process that takes one particle and turns it into two particles of the same kind. Similarly, what matters for formal immiscibility is whether there is a physical process that takes two particles and turns it into one particle of the same kind.

Because elementary particles are not made of further more elementary parts, they cannot violate the condition of relative indivisibility by being 'broken' into parts. So, leptons, quarks, gauge bosons, and the Higgs boson appear to be relatively indivisible in virtue of having no components to be broken into. One potential source of complication here would be the fact that even elementary particles have decay pathways. Even if an elementary particle decay into two particles of the same type were not, strictly speaking, a case of relative divisibility, it would still be a problem. If a cloud breaks up into two clouds, we can start asking questions like which cloud is identical with the original cloud. This is a confused question, but it is so because clouds aren't individuals. If an individual could become two individuals of the same kind, whether through breakdown into components or a particle decay process, then some of the confused questions we may ask about clouds would demand answers. These are "which individual is which" sorts of questions which is why they're confused questions to ask about non-individuals.

Of course, there's no genuine problem here because energy conservation prevents decay processes in which one particle becomes two particles of the same type. It is, however, possible for a two-step decay process to bring one from particle X to particles Y and Z then back to particle X. For example, an up quark can decay into a down quark and a W^+ boson after which the down quark could decay into an up quark and a W^- boson, the W bosons would then annihilate leaving only an up quark. However, that is not a violation of relative indivisibility because the end result is only one particle of the original kind. Such two-step decay processes may bring up the question of whether the resulting up quark is the same individual as the original up quark or even whether the down quark is the same individual as the original up quark. However, we would answer those questions by looking at the metaphysical grounds of the particles' individuality rather than the logically necessary and sufficient conditions for individuality. The only way that conservation of energy would allow a decay process that violates the condition of relative indivisibility

would be if essential properties of particles were such that one particle could become two particles of the same kind without violating energy conservation.

Conservation of energy ensures that relative indivisibility holds for composite particles as well. No meson will decay into two mesons of the same type and no baryon will decay into two baryons of the same type. Additionally, since mesons are composed of a quark and an anti-quark, breaking them down into their constituents would yield one quark, one anti-quark and no mesons. Similarly with the three quark baryons. Breaking a baryon down into its constituents yields three quarks (since there are no two quark bound states you wouldn't end up with a two-quark particle) and no baryons.

So particles, whether elementary or composite, satisfy the condition of relative indivisibility. They satisfy the condition in the ordinary sense of being indivisible into further particles of the same kind unlike a pile of rice that can be divided into two more piles of rice. They also satisfy a similar condition that takes into account particle decay processes rather than mere breakdown into components.

Moving on to the condition of formal immiscibility we must consider whether two particles can combine into a single particle of the same kind. Since such a process would be the reverse of a process that violates relative divisibility, conservation of energy also prevents violations of formal immiscibility. The only physical situation I'm aware of that might be described as particles of a given kind combining into a single particle of that same kind is in the case of a Bose-Einstein condensate (BEC). A BEC essentially behaves as a single large boson. However, BECs only form under specific conditions. Combining a bunch of bosons of the same type does not, by itself, result in the creation of BEC. To form a BEC a *dilute* gas of bosons must be *cooled* to near absolute zero. Because of these requirements, bosons are not *formally* miscible. The apparent miscibility of bosons in a BEC is not something that follows merely from the kinds of things bosons are (i.e. it does not follow from their form alone) but requires certain physical conditions in addition.

Another way to put the point, physical miscibility is not sufficient for formal miscibility. Being formally miscible is a matter of an object being miscible in virtue of the *kind* of object that it is. Consequently, formally miscible objects should generally be able to 'mix' in the absence of physical conditions that prevent 'mixing.' They have 'permission' to 'mix' as long as nothing is preventing them from doing so, and so shouldn't require very specific physical conditions for 'mixing.' Consequently, any existing physical processes that do result in two (or more) particles becoming one particle of the same or the reverse would not be violations of formal immiscibility or relative indivisibility as long as those processes require specific physical conditions to occur.[2]

However, there are still two additional issues. The first is whether the bosons forming the BEC are combined in the right way to create an issue. A BEC is composed of a large number of bosons whereas a cloud is not composed of a number of clouds. The behavior of BECs suggests that they themselves are not individuals in the same way clouds are not. In order for BECs to challenge the condition of formal immiscibility for bosons, they must somehow blur the boundaries between the bosons that make them up. However, they do not seem to do that as the aggregated bosons don't 'disappear into one another' the way mixing clouds do. Average particle number is conserved in a BEC and so saying that bosons are not individuals because BECs don't behave like individuals would be like saying grains of rice aren't individuals because piles of rice don't behave like individuals. Finally, the other issue is whether the BEC counts as a particle of the same kind as the bosons that make it up. The answer seems to obviously be 'no' as a result of the BECs composition and mass.

Some may find my discussion here unsatisfying insofar as the conditions of relative divisibility and formal immiscibility are trivially fulfilled as a result of conservation of

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[2]So if photon splitting experiments are accurately interpreted as involving splitting photons, then these would still not be counterexamples to the condition of relative indivisibility.

energy. But we need to keep in mind the distinction between the necessary and sufficient conditions for individuality and the metaphysical grounds of individuality. I'm not saying that conservation of energy grounds the individuality of particles, my claim here is an epistemic one. We can tell that particles are individuals, that they satisfy the three conditions of individuality, as a result of conservation of energy. I admit that grounding the individuality of particles in conservation of energy involves a worry of triviality since, arguably, the actual grounds of individuality in that case are just a matter of particles having their mass as an essential, or defining, property (but see below).

The final condition is that of ontological autonomy (which was defined as an objects not needing to be a proper part of another object in order to exist). Neither elementary particles nor composite particles need to be a proper part of another object in order to exist. Consequently, all particles are ontologically autonomous. The only apparent challenges to the idea that particles are ontologically autonomous would, potentially, be (i) the fact that quarks generally only exist in bound states outside of extreme conditions[3] and (ii) the existence of virtual particles. Whether virtual particles actually exist is a controversial issue (see (Fox 2008) for an overview). In the case of quarks, what's important for the condition of ontological autonomy is that it be possible (in some sense of possible) that the objects in question can exist without being parts of other objects. Something that couldn't possibly exist except as a part of another object doesn't seem to be a candidate for individuality. The condition of ontological autonomy is imposed to rule out such cases as they are compatible with the other conditions of relative indivisibility and formal immiscibility. Consequently, what is important is that quarks can exist outside of bound states even if the conditions under which they do so do not generally obtain.

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[3]Quarks haven't existed outside of bound states since the very early universe.

The second challenge may not be much of one insofar as it is generally accepted that virtual particles aren't actual particles. However, let's consider whether the existence of virtual particles would challenge the condition of ontological autonomy. Whatever explanation one may accept for the violation of mass/energy conservation involved in the creation of virtual particles, it is agreed that the violation must be temporary. In other words, whenever virtual particles are created, they *must* very quickly be annihilated. As a result, one may think that there is a special relation of existential dependence between virtual particles and the "real particles" from which they came; a relation that differs from, say, the existential dependence relation between a person and their parents. However, Chauvier's move to distinguish ontological autonomy from the latter form of existential dependence also distinguishes it from any form of existential dependence there might be between virtual particles and the particles from which they come. Virtual particles are not proper parts of other particles. So even if, say, an electron can sometimes exist as a virtual particle it is still the case that electrons, both "real" and "virtual" satisfy the condition of ontological autonomy.

The primary purpose of this dissertation was to defend the view that particles are individuals and I have done so. Particles satisfy the logically necessary and sufficient conditions for individuality and so are individuals whatever the metaphysical grounds for that individuality may be. So, in a sense, I could stop here and leave aside the issue of the metaphysical grounds of their individuality. Of course, I've already done all the work to discuss the metaphysical grounds of particle individuality; all that's left to do is to consider whether there are any problems that arise with attributing haecceities to particles. But it is important to note that criticisms of this part of my discussion will not challenge my claim that particles are individuals (although an argument to the effect that there are no suitable metaphysical grounds for individuality whatsoever would suggest that Chauvier's conditions for individuality are incorrect).

As with the application of the conditions of individuality to particles, the application of haecceities is straightforward. The hard part is defending the idea that there are such things as haecceities that serve as individuators. With that done, the only difficulties in suggesting that particles have haecceities would be (i) providing reasons to think that particles are individuals and (ii) ruling out alternative metaphysical grounds of that individuality. I've taken care of both of these in chapters four and five, respectively. This may seem unsatisfying, but this is the nature of transcendental arguments. I claim that particles are individuated by haecceities because particles are individuals and only haecceities can serve as their individuators. I have no further positive argument for why particles have haecceities.

However, noticing the connection between conservation and laws and the allowance or disallowance of physical processes that violate relative indivisibility and formal immiscibility gives us another option for grounds of individuality. Are there conservation laws that prevent violation of these conditions in which the triviality concern with conservation of energy doesn't arise? Are there conservation laws that prevent violation of these conditions in which either (i) there is no reliance on particles having the specific values that they have for an additive property (such as mass) or (ii) having the specific value that they have for the additive property in question is metaphysically significant?

For leptons we could appeal to conservation of lepton number. A lepton has lepton number 1 and a non-lepton has lepton number 0. A process that violates relative indivisibility for leptons has lepton number 1 in the initial state and lepton number 2 (or greater) in the final state. A process the violates formal immiscibility would be the reverse. Conservation of lepton number rules out such violations. While lepton number functions mathematically as an additive property, it doesn't refer to an actual property that leptons possess. It's just using 1 and 0 to stand for 'yes' or 'no' answers to the question "is

this a lepton?" The conservation law simply tells us that leptons are not the kinds of things that can violate relative indivisibility or formal immiscibility.

For quarks we have conservation of color charge. Color charge is the strong force, or color force, equivalent of electric charge. However, it is interestingly different in that there is no physical magnitude associated with color charge. Electric charge can be positive or negative *and* has a physical magnitude attached. Color charge, on the other hand, can only be 'positive' or 'negative' (i.e. red, green, blue, antired, antigreen, or antiblue). The color charge of a state with one quark will be one of these six colors, whereas a state with *n* quarks will involve a combination of *n* of these color charges. But no combination of color charges is equivalent to a single color charge. A combination of a color with its anticolor (e.g. red and antired) is considered to be colorless whereas the combinations red-green-blue and antired-antigreen-antiblue are also considered to be colorless. So, conservation of color charge gives us a way to ground the individuality of quarks without appeal to specific values for additive properties.

In the case of the gauge bosons, the force mediating particles, there are both massive and massless gauge bosons. The massless gauge bosons are photons (which mediate the electromagnetic interaction) and gluons (which mediate the strong interaction). Because massless particles don't have decay pathways, the only kind of photon (or gluon) 'splitting' is that which occurs in photon 'splitting' experiments. But even if we view that as a matter of actually dividing photons into multiple photons, it only occurs under very specific experimental conditions and so is not a violation of relative indivisibility. The same is true for the reverse process: it would not a violation of formal immiscibility. So, like leptons, it seems to be the case that photons and gluons just are the kinds of things that can't violate the conditions of relative divisibility and formal immiscibility. The way to understand this is the same as understanding the difference between an ontology for Bohmian mechanics that either has or lacks a physical wave associated with the guidance

equation. Not being open to processes that violate relative indivisibility and formal immiscibility is a 'way of being' for leptons, photons, and gluons because there is no actual conserved quantity (no property) involved in explaining why these particles can't violate the conditions of relative indivisibility and formal immiscibility.

The only elementary particles left are the W and Z bosons, which mediate the weak interaction, and the Higgs boson. The only composite particles are mesons or baryons, which are composed to two or three quarks, respectively. Baryon number conservation functions in the same way as lepton number conservation. So it's only the W, Z, Higgs, and mesons left to account for. There do not appear to be any conservation laws I can appeal to here that don't involve conservation of specific values of additive properties. So I could either say that the satisfaction of the conditions of relative indivisibility and formal immiscibility for these particles isn't grounded in a 'way of being' for these particles or I could say that we just lack evidence for such grounds. But, of course, absence of evidence is not evidence of absence. It is still the case that all particles satisfy the conditions of relative indivisibility and formal immiscibility, it's just that for some particles I can't point to a reason why their satisfaction of these conditions doesn't involve a property.

So I can say that satisfaction of the conditions of relative divisibility and formal immiscibility for the remaining particles are grounded in a 'way of being' wherein we simply lack conservation laws in terms of W-boson number, Z-boson number, or Higgs number. This lack could represent a genuine lack of such conservation laws, or it could represent a lack of practical benefit of assigning such numbers to such small groupings of particles. Especially in light of the fact that there is an approximate conservation law for flavor (or kind) or particle that is only violated in weak interactions. Or I could say that these particles have haecceities whereas, say, leptons do not. Or I could say that all particles have haecceities. I prefer the first option.

Lepton number is a flavor quantum number that is conserved in all interactions. There are three more flavor quantum numbers that are also conserved in all interactions: these are baryon number, electric charge and weak isospin. However, the latter two seem to be importantly different in that they seem to represent actual properties that have physical magnitudes connected to the strength with which a particle interacts via the electromagnetic or weak forces.

For composite particles, there are only mesons (composed of two quarks) and baryons (composed of three quarks). Baryon number conservation works for baryons as lepton number works for leptons. So baryons seem to satisfy the conditions of relative indivisibility and formal immiscibility due to a 'way of being.' But, for mesons, I can't point to any specific conservation law that doesn't involve additive properties. So I can either say we should accept that mesons satisfy the conditions of relative indivisibility and formal immiscibility as a result of a 'way of being' based on the fact that this seems to be how most particles satisfy these conditions. Or I could say that mesons have haecceities. As before, I prefer the former option. I don't think accepting something as a 'way of being' rather than accepting an ontological component like a haecceity is inherently preferable (as I mentioned before). But here we have reason to think the former option is the right way to go with respect to leptons, photons, gluons, and baryons. So I don't think it's a stretch to say that quarks, W-bosons, Z-bosons, the Higgs, and mesons have their individuality in the same way (given that conservation laws do ensure that these latter particles never violate the conditions of individuality).

Finally, with respect to the condition of ontological autonomy, I already said in chapter four that it makes sense to view this condition as a metaphysical one. Ontological autonomy is a matter of an object having its own being separate from other things. And if we view the condition of being able to exist without being a proper part of something as a consequence of ontological autonomy (a consequence that is particularly relevant for

considerations of individuality), then we needn't view the metaphysical counterpart of the condition as a *de re* modal property.

Ultimately, particles are individuals because they satisfy the conditions of relative indivisibility, formal immiscibility, and ontological autonomy. And they satisfy those conditions in virtue of having their own being wherein it is simply a feature of the sort of being that they have that they satisfy the conditions of relative indivisibility and formal immiscibility.

The only thing left to do is to say something about whether anything changes when we consider QFT. We saw in chapter one that QFT has the resources for viewing particles as individuals with respect to states that are eigenstates of the particle number operator. As a result, whether QFT has implications about whether particles are individuals or not hinges on the significance of states that are not eigenstates of the particle number operator.

States that are not eigenstates of the particle number operator are often called 'states of indefinite particle number'. However, I think this terminology should be avoided as it suggests a particular metaphysical interpretation of those states that we needn't accept; namely, that they are states in which there is no number which is *the* number of particles in that state. Whether the existence of states that are not eigenstates of the particle number operator has any implications about the individuality, or lack thereof, of particles depends on (i) whether we should understand these states as states of indefinite particle number and (ii) whether a state of indefinite particle number is incompatible with our conditions of individuality.

We saw in chapter two that Huggett argues that the ability to represent states of indefinite particle number is logically posterior to the issue of what metaphysical significance we

should attach to labels. Huggett actually goes so far as to claim that quantum theory introduces no new issues for individuality at all.[4]

Dorato and Morganti also briefly mention QFT after defending their primitive thisness view stating that, "[t]hings, however, are likely to be different as one moves to quantum field theory, where we can have *superpositions of particle number* and, consequently, *countability cannot be expected to play the same role as in the case of non-relativistic quantum mechanics*" (Dorato and Morganti 2013, p. 608). But they caution that "different conclusions may be in order when it comes to even more fundamental physical theories such as quantum gravity, string theory, etc" (Dorato and Morganti 2013, p. 608).

Do coherent states have an indefinite number of particles? I think that's a complicated question that isn't answered merely by noting that it is very difficult to interpret them as statistical mixtures rather than as pure states. The answer to this question may depend on the interpretation of the theory more generally. Would a relativistic extension of Bohmian mechanics treat coherent states as having an indefinite number of particles? I'm not sure and I haven't seen the question addressed. Discussions of relativistic extensions of Bohmian mechanics seem to focus on ensuring the extension is Lorentz invariant and worrying about dependence on a preferred foliation.

Aside from interpretation of quantum theory as a whole there are other interpretative issues as well. Does an answer to the question of whether coherent states are states of indefinite particle number depend on what it means for something to be a particle? I think that it does. For example, if particles are emergent entities then there is a question of whether there even are particles present in a coherent state. It could be the case that coherent states are not eigenstates of the particle number operator because there are no particles in that state rather than because there is an indefinite number of particles in that

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[4]See (Gordon 2002) for discussion.

state. If there are no particles present in a coherent state, then it's not clear why coherent states would have any implications for the question of whether particles are individuals or not.

With Huggett's objection that the issue of coherent states is logically posterior to the issue of whether particles are individuals, Dorato and Morganti's cautioning words that the way we interpret QFT may change depending on the development of more fundamental theories that are currently active areas of research, and the fact that the interpretation of coherent states as states that have an indefinite number of particles seems to be a question that is both open and complicated, it just isn't clear enough whether there would be an objection here when it's all sorted out. Because even if we can agree that coherent states are states in which there is an indefinite number of particles, that, by itself, doesn't necessarily create a problem for the view that particles are individuals. There is something unintuitive about the idea of an indefinite number of individuals, but is that a problem with my position or a problem with the intuition? Is this a case where the oddities of quantum theory lead to the existence of some individuals whose features are in conflict with our intuitions or is it a case where those intuitions point to a significant flaw in my position? Coherent states are bound by conservation of energy just like any other states and so they shouldn't lead to an issue with my condition of relative indivisibility. The condition of formal immiscibility is more complicated here but being a state of indefinite particle number doesn't seem to imply that a particle in that state could become two, or more, particles of the same type. Consequently, even if coherent states are states of indefinite particle number, I can say particles satisfy my independently motivated condition on individuals (conditions that were not designed to have the peculiar result that states with an indefinite number of individuals are possible).

6.4   Conclusion

I have achieved three major goals in this dissertation. The first was to demonstrate significant issues with the primary particles-as-non-individuals view in terms of lack of self-identity. I have done so by (i) arguing that it is incoherent to suppose that there are objects to which identity statements of the form 'x=x' do not apply and (ii) showing that there are challenges to particles-as-non-individuals views that are particularly problematic when non-individuality is understood as a lack of self-identity. As I mentioned before, Domenech and Holik's definition of cardinality for non-individuals may be perfectly fine as long as non-individuality is not understood as a lack of self-identity.

The second goal was arguing that particles are individuals. I defended three conditions that are each logically necessary and are jointly sufficient for individuality. And I defended these conditions independently of any concerns regarding particles. These conditions are, in my view, the logically necessary and sufficient condition for anything to be an individual. Having defended these conditions there was little left to say about their application to particles. Decay pathways, BECs, and virtual particles provided what could serve as challenges to some possible accounts of individuality; but, properly understood, there is no incompatibility between these features of quantum theory and Chauvier's conditions of individuality. As a result, the completion of this goal in this chapter required only ensuring that the conditions of individuality were sufficiently clear.

Finally, the third goal was defending the idea that there is a sensible metaphysical ground for the individuality of particles. However, I went beyond that as a result of the lack of consideration for the possibility of non-individuals in existent accounts of individuality. While I view particles as individuals, I also accept the existence of non-individuals and so require that any suitable account of metaphysical grounds of individuality allow for the existence of non-individuals. Ruling out possible metaphysical grounds of individuality

due to (i) not allowing for the possibility of non-individuals and (ii) not being concerned with individuality at all as a result of conflating individuality with another concept (such as distinguishability) left only haecceities as the remaining candidate.

I've already explicitly mentioned most of my contributions in the preceding two chapters. My major contribution from chapters three and four was to outline and respond to challenges leveled against defenders of non-individuality. The criticisms raised by Jantzen and Bueno are not general criticisms of particles as non-individuals as they were presented. They were only challenges to specific particles as non-individuals views (specifically that largely developed by French and Krause) in which being a non-individual involves a lack of self-identity. Additionally, in chapter four was my criticism of self-identity as a condition of individuality (self-identity being a relatively new proposal for such a condition) as well as my focus on keeping the existence of possible non-individuals in mind while considering the various possible conditions of individuality. There has been an assumption in recent debates that essentially everything was an individual save universals. It's only recently that sophisticated ways of thinking about non-individuals have emerged leading to serious proposals for non-individuals other than universals including physical non-individuals. So chapter four (and chapter five as well) largely focuses on rethinking traditional proposals with a new evaluative criterion in mind.

In addition to evaluating the proposals in chapter five with respect to the possible existence of physical non-individuals, I also evaluated them with respect to the position I had defended in chapter four. But, more importantly, I defended metaphysical views that tend to be controversial among philosophers of science. This is important because my primary motivation for this dissertation as a philosopher of physics with a background in analytic metaphysics was to demonstrate how the work of metaphysicians is not only important but how it can be useful in addressing questions in the philosophy of physics.

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