2006

BASIC LOGICAL KNOWLEDGE AND ITS JUSTIFICATION

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BASIC LOGICAL KNOWLEDGE AND ITS JUSTIFICATION

(Spine title: Basic Logical Knowledge and its Justification)

(Thesis format: Monograph)

by

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Graduate Program
in
Philosophy

A thesis submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy

Faculty of Graduate Studies
The University of Western Ontario
London, Ontario, Canada

THE UNIVERSITY OF WESTERN ONTARIO
FACULTY OF GRADUATE STUDIES

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entitled:

Basic Logical Knowledge and its Justification

is accepted in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

Date____________________

Chair of the Thesis Examination Board

ABSTRACT

This thesis contributes to the area within the philosophy of logic that concerns epistemological questions about fundamental logical laws. The state of the literature has been dominated by meaning-theoretic approaches that attempt to answer such questions in terms of conditions for understanding words or for possessing concepts, and empiricist approaches that attempt to characterize logical knowledge as a posteriori. The theoretical contribution made by my work amounts to the presentation of an alternative theory that does not appeal to the notion of rational insight, and which is not psychologistic, conventionalist, or inductivist. The theory that I propose involves a philosophical analysis of logical concepts which facilitates a transcendental argument in support of basic logical knowledge. I arrive at this theory by radically readapting methods and insights from the theories of geometry of Kant and Helmholtz, and from Frege's logicist explanation of the basis of our knowledge of arithmetic.

I argue that the meaning-theoretic approaches of Christopher Peacocke and Paul Boghossian overestimate the epistemological power of the conditions for understanding logical constants. I argue that the naturalistic approach of Penelope Maddy is psychologistic. To lay the groundwork for my own proposal I investigate the role of Kant's transcendental method in his theory of geometry, and the role of the theory of analytic definitions in a reconstruction of Frege's logicism. My own proposal posits that logical laws are analytic of logical concepts, and that logical concepts are conditions of the possibility of possessing a minimally reasonable conceptual scheme.

Keywords: a priori, justification, logic, philosophy of logic, logical concepts, conditions of possibility, transcendental argument, rationalism, naturalism, Kant, Frege.

DEDICATION

This thesis is dedicated to my grandmother Mary Perepeltza.

ACKNOWLEDGEMENTS

I wish to express my sincere appreciation to Professor William Demopoulos and Professor Robert DiSalle for their assistance in the preparation of this thesis. For helpful discussions and encouragement during the early stages of this project I would like to thank Bob Hale, Richard Heck, Peter Koellner, and Penelope Maddy. I would like to thank John Bell, Erin Eaker, and Scott Woodcock for their friendship and their advice. I would like to thank my parents Robert and Elsie Boutillier, my sister Shay Boutillier, my grandparents John and Mary Perepeltza, and the rest of the Boutillier clan for their love and support. I would like to thank Gurpreet Rattan, Imogen Dickie, and Philip Kremer for making me feel welcome in the department at the University of Toronto and for many beneficial discussions. For useful conversations over the years at various philosophical events, I would like to thank Dave DeVidi, Tim Kenyon, Philip Hanson, Elaine Landry, Ann Levey, John Heintz, Richard Zach, Anjan Chakravartty, Timothy Childers, Roy Cook, Steven Savitt, Michael Hymers, Rhonda Martens, Janet Folina, Patricia Blanchette, Michael Detlefsen, Joongol Kim, Geoffrey Hellman, and Darko Sarenac. For their camaraderie during graduate school I would like to thank Todd Calder, David Laverty, Ryan Robb, Greg Lavers, Carolyn McLeod, Elana Geller, Sona Gosh, Gillian Crozier, Marika Hadzipetros, Jonathan Tsou, Giovanni Grandi, Gerry Callahan, Darcy Cutler, and the rest. For their friendship from outside of philosophy I would like to thank Sabina Foofat, Gabrielle Zezulka-Mailloux, Peter Moersch, Neelam Sandhu, Sebastian Stewart, Ian Folinsbee, Kyle Bruce, Dee Hon, Jason Anderson, Desmond Lee, the Kelsang family, Carol and John Jepson, Marjorie and Roger Macqueen, and the Hunt family.

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Introduction:
THE NATURE OF THE PROBLEM

> "After we have convinced ourselves that a boulder is immovable, by trying unsuccessfully to move it, there remains the further question, what is it that supports it so securely?"
>
> —Gottlob Frege, *Die Grundlagen der Arithmetik*

1 The problem of basic logical knowledge and the existing state of the literature

This thesis is about the following two epistemological questions in the philosophy of logic:

The acquisition question
How is basic logical knowledge possible?

The justification question
How are fundamental logical laws justified?

By "basic logical knowledge" I mean knowledge that a fundamental logical law is necessarily truth-preserving. The fundamental logical laws that I have in mind are simple underived rules of inference such as *modus ponens*. So the acquisition question can be put by asking, "How can a thinker come to know that *modus ponens* is necessarily truth-preserving?". And the justification question can be put by asking, "What is it in virtue of which it is true that *modus ponens* is necessarily truth-preserving?", or simply, "What makes logical truths true?".

Let me begin by saying a few things about the existing theoretical landscape that one finds when one starts to think about these two questions. Contemporary attempts to answer the acquisition question typically fall into one of two familiar camps: they are either rationalist or empiricist. Broadly speaking, rationalist proposals set out to provide an answer that is independent of sense experience; reason alone, it is held, is sufficient to deliver basic logical knowledge. So construed, the rationalist approach is very broad. The most prevalent type of rationalist proposal to be found in the existing literature starts from the idea that an adequate answer to the acquisition question must be founded on the conditions for understanding the logical connectives. A merit of this sort of "meaning-theoretic" rationalist approach is that it appeals to the nature of logical concepts in order to try to explain how we can come to have basic logical knowledge. A limitation is that its account of what is constitutive of one's understanding of a logical connective is open to serious counter-examples.

Empiricist approaches to the acquisition question posit that basic logical knowledge must be gained with the aid of sense experience. A simple form of empiricist proposal would say that we can come to know that a logical law is sound because we have applied it in a myriad of cases and have come to see that in every instance it is confirmed by experience. Perhaps the most immediate difficulty with this sort of proposal is that it tries to arrive at knowledge that is general and necessary on the basis of sense experience which can deliver only knowledge that is particular and contingent. An arguably more sophisticated defence of the empiricist approach says that we can come to have basic logical knowledge by coming to have access to an explicit empirical account of the ground of logical truth, one which says that logical truths are true of very general structural features of the world, and that human beings cannot help finding them obvious because the logical forms that they express are biologically built-in to our innate representational capacities. A merit of this approach is that its answer to the acquisition question shows some appreciation of how the two questions are related to one another. The limitation of this approach is that its answer to the justification question makes fundamental logical laws contingent, subjective, and lacking in generality.

2 The relevance of the foundations of geometry and the foundations of arithmetic to the problem of basic logical knowledge

My strategy with regard to the acquisition question will be to temporarily set it aside in order to first develop a "moderately Kantian" type of rationalist answer to the justification question. The idea is that if we can first find an answer to the justification question, we will be in a good position to go from there directly to being able to give an answer to the acquisition question. On a straightforward and traditional view of how it is possible to gain knowledge, a thinker can come to know a proposition *p* by coming to recognise, on reflection, the factors that make *p* justified. So, whereas the meaning-theoretic rationalist approach proposes that one can come to know that *modus ponens* is valid simply on the basis of understanding 'if', the view that I propose suggests that one can come to know that *modus ponens* is valid by acquiring an explicit appreciation of what makes a proposition to that effect justified. That is, on the view that I propose, a substantive explanatory justification can provide a route by which someone who does not know whether or not they have basic logical knowledge can come to know that they have it.

My strategy with regard to the justification question will be to use methods and insights drawn from previous work in the foundations of physical geometry and the foundations of arithmetic in order to give an answer to it. For example, a method that I find instructive which is employed in the foundations of geometry is Kant's "transcendental method." Kant uses this method to provide a justification for Euclid's postulates which takes them to be necessary conditions for the possibility of "the successive synthesis of the productive imagination." It turns out that Kant was wrong about the dependence of geometrical concepts on pure intuition, but this drawback leaves the core idea behind the method unaffected.

The core idea behind the transcendental method is that a fundamental constitutive principle can be shown to be justified with the help of a "transcendental argument." Broadly speaking, a transcendental argument is simply one that involves a claim of the form, 'For *Y* to be possible, *X* must be the case', where *Y* represents an undeniable fact about certain cognitive capacities of thinkers (such as that they possess the capacity to have experiences, form certain judgments, or carry out certain knowledge-seeking activities), and where *X* can be expressed by a principle that captures the content of a particular concept. For example, in Helmholtz's theory of geometry, we find what amounts to the following "transcendental conditional": In order for it to be possible to construct figures in the productive imagination, it must be the case that the movements that constitute spatial changes can be done, undone, and combined arbitrarily. The consequent of this conditional is expressed by "the principle of free mobility."

A method that I find instructive comes from the philosophy of arithmetic. According to it the theory of "analytic definitions" lies behind a reconstruction of Frege's explanation of the basis for our knowledge of the basic laws of arithmetic. An analytic definition is one that, in a technical sense, "captures" the pre-theoretic sense of a term that is already in use. What makes an analytic definition correct, according to the theory that I present, is i) that it brings to light something that an ordinary thinker tacitly knows if he or she (perhaps only partially) understands the term defined, and ii) that it reflects the outcome of a correct philosophical analysis of the concept expressed by that term. A correct philosophical analysis of a concept reveals the conditions on which its application depends; it reveals what is "analytic of" a concept in use. So, a correct analytic definition captures the pre-theoretic sense of a term in the sense that it reveals what is analytic of the concept that is expressed by it . An example of a correct analytic definition is Frege's partial contextual definition of the cardinality operator (which states that the sentence 'There is a one-one relation *R* whose domain consists of the *F*s and whose range consists of the *G*s' is equivalent to the sentence 'The number of *F*s is equal to the number of *G*s'), this is a correct analytic definition of cardinal equality.

These two closely connected methods from the philosophy of geometry and the philosophy of arithmetic are among those that I combine and readapt to the context of the philosophy of logic in order to suggest how an account of how fundamental logical laws are justified might be presented.

3 Four constraints

Fundamental logical laws are general and objective. Four historically prominent philosophical methodologies are unable to account for knowledge that is either general or objective. Therefore, an account of basic logical knowledge that appeals to any one of these methodologies will fail to be successful, because it will fail to preserve the generality or the objectivity of fundamental logical laws. These four methodologies therefore suggest constraints on the success of an account of basic logical knowledge. In the course of my investigation, I will evaluate existing accounts in terms of these constraints, and I will use these constraints to guide the design of my own account.

The first constraint is the prohibition of an appeal to intuition, or rational insight. One view is that we can come to have knowledge of certain basic propositions on the basis of intuition. On this view, intuition is thought to be a non-inferential non-sensory faculty whereby a thinker is somehow simply able to "see" the truth of certain propositions. It is hard to see how intuition is not essentially subjective; indeed, Frege explains objectivity partially in terms of the independence of intuition.[1]

The second constraint is the prohibition of an appeal to psychologism. A psychologistic justification of a piece of knowledge would say that it is grounded on the basis of psychological facts. Here again, it is hard to see how psychological facts could ground objective truths. And here again we can refer to Frege, who put it best: "Logic is concerned with the laws of truth, not with the laws of holding something to be true, not with the question of how men think, but with the question of how they must think if they are not to miss the truth" (Frege 1879: p.149).

The third constraint is the prohibition of an appeal to conventionalism. A conventionalist view of logic might say that logical laws are a matter of conventional stipulation, since they are no more than implicit definitions of their constituent terms. Then the truth, and our knowledge of the truth, of logical truths depends on decisions about how words are to be used. On a view such as this, we are free to adopt any set of logical laws that we like, on the basis of pragmatic considerations, as long as we do not thereby create inconsistencies with established fact. It is hard to see how free choices about the use of words can result in objective truths.

[1] Frege writes: "It is in this way that I understand objective to mean what is independent of our sensation, intuition and imagination, and of all construction of mental pictures out of memories of earlier sensations, but not what is independent of the reason,—for what are things independent of the reason? To answer that would be as much as to judge without judging, or to wash the fur without wetting it." (Frege 1884: §26)

The fourth constraint is the prohibition of an appeal inductivism. On this view we learn logical truths by experience. The naïve empiricist view that I discussed earlier in this chapter is a prime example of an inductivist view. And the reason that I gave above for why it faces overwhelming difficulties is that it cannot deliver knowledge that is general.

4 Plan of the study

The plan then for the positive proposal is first to provide an answer to the justification question by carrying out the operation of readapting methods from the philosophy of geometry and the philosophy of arithmetic in a way that does not breach the constraints that help ensure that the generality and objectivity of fundamental logical laws is preserved, and second, to provide an answer to the acquisition question on the basis of the answer to the justification question. The answer that I give to the justification question involves the claim that logical laws (such as *modus ponens*) are analytic of logical concepts such as *if*.[2] I argue that there must be logical concepts if it is possible for a thinker to engage in rudimentary propositional thinking of the kind needed to carry out the simplest forms of mathematical and empirical scientific inquiry.

In the next chapter I reconstruct and discuss four "moderate rationalist" views that have been proposed in the literature by Christopher Peacocke, Paul Boghossian, and Bob Hale. Two important topics that I will develop in this chapter are that of accounting for invalid rules such as those for the connective 'tonk', and that of accounting for errant thinkers who reject valid rules. In chapter 2 I critically assess the naturalist view of logic

[2] When I am referring to concepts, I adopt the convention of putting them in italics; e.g. *if* refers to the truth-functional logical operation that is the semantic value of 'if'.

proposed by Penelope Maddy. I find this view problematic because, despite machinations to the contrary, it is thoroughly psychologistic. In chapter 3 I interpret the views of Kant and Helmholtz on the foundations of geometry. In chapter 4 I present a reconstruction of Frege's account of our knowledge of arithmetic. And in the final chapter I present my own theory of basic logical knowledge.

Chapter 1:
RATIONALISM AND LOGIC

> "But still, I must only infer what really *follows*!—Is this supposed to mean: only what follows, going by the rules of inference; or is it supposed to mean: only what follows, going by *such* rules of inference as somehow agree with some (sort of) reality?"
>
> —Ludwig Wittgenstein, *Remarks on the Foundations of Mathematics*

To have "basic logical knowledge" a thinker must have a justified true belief about the validity of a simple underived rule of inference. An interesting and important problem that we have identified is that of explaining how basic logical knowledge is possible. The question is, What explains how a thinker can come to know that a basic rule of inference such as *modus ponens* is necessarily truth-preserving? The most prominent sort of rationalist strategy that is currently on offer in the literature aims to provide a solution to this problem by framing the sought-after explanation in terms of the conditions for understanding the logical connectives. I will refer to this strategy as the "meaning-theoretic" strategy; it is the focus of this chapter.

1 Rule-circularity, bad company, and dealing with the sceptic.

In this section I describe three problems that any account of basic logical knowledge must confront.

To appreciate the first problem, we need to bear in mind a distinction. Basic logical knowledge is either inferential or non-inferential. Someone who is inclined to say that it is inferential knowledge will want to say something along the following lines: There is an inference to the conclusion that some rule R is sound that follows from premises that are necessarily true and a priori knowable; and the rule according to which this inference proceeds preserves truth and knowledge, and a thinker can depend on it to do so in some kind of reflectively appreciable way without having a justified belief that it does. Also, since *R* is a primitive rule, the rule according to which the justificatory inference proceeds must be *R* itself. So, for example, suppose that the meaning of 'if' is implicitly defined by stipulating that its semantic value is the truth-function that validates *conditional proof* and *modus ponens*. An argument that an inferential approach might rely on to explain a thinker's knowledge of these two rules might look something like this:

(1) If 'if' has the meaning assigned to it by its implicit definition, then *modus ponens* and *conditional proof* are valid.
(2) 'If' has the meaning assigned to it by its implicit definition.
Therefore,
(3) Modus ponens and *conditional proof* are valid.

This argument is "rule-circular". It relies on the validity of *modus ponens* to ensure that its conclusion must be true if its premises are true. Prima facie, this feature leads to the vitiation of an inferential justification of an underived inference rule. The first obstacle, then, that stands in the way of a straightforward solution to our problem is what has come to be called "the problem of rule-circularity."[1] The problem is to explain how an argument can have this feature without presupposing what it is intended to show.

In addition to the problem of rule-circularity there is the problem of capturing only the right rules. If an explanation of the possibility of basic logical knowledge justifies spurious connectives, then it is incorrect. Consider, for example, Arthur Prior's connective tonk.[2] The tonk rules licence inferences from *A*, to *A* tonk *B*; and from *A* tonk

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[1] Cf. e.g. (Boghossian 2000: p. 246).
[2] Cf. (Prior 1960).

B, to B. The tonk rules therefore make it possible to infer $\neg A$ from A. Thus, they do not preserve truth. So it is bad if a purported explanation of the possibility of basic logical knowledge works just as well for 'tonk' as it does for the usual sentential connectives. This is had come to be called "the problem of bad company."[3] An inferential justification runs into the problem of bad company like this:

(1) The connective 'tonk' has the meaning assigned to it by its implicit definition.

Therefore,

(2) The connective 'tonk' has that meaning, tonk Tonk-Introduction and Tonk-Elimination are valid.

Therefore,

(3) Tonk-Introduction and Tonk-Elimination are valid.

Alternatively, one might be incline towards saying that basic logical knowledge is non-inferential. Such a view avoids having to deal with the problem of rule-circularity, but not the problem of bad company. The intuitive idea behind most[4] non-inferential accounts is that of finding and explaining the workings of a "source" of a priori entitlement. For example, an extreme kind of rationalist might say that the source of a thinker's entitlement to the belief that *modus ponens* is valid, lies in that thinker's possession of a special faculty of rational intuition which is analogous to visual perception. The problem with this proposal is that it creates more problems than it resolves.

Rational insight is not, however, the only available source of non-inferential entitlement. For example, a philosopher might draw on Wittgenstein's remarks *On*

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[3] Cf. e.g. (Wright 2001: p.49).
[4] Two notable exceptions are (Field 2000, 2005).

Certainty, having to do with the unavoidability of presuppositions in the acquisition of warrant, to develop a non-inferential notion of so-called "entitlement of cognitive project".[5] But in general a philosopher taking a non-inferential approach tries to identify and flesh out something that can serve as a "source" of (or a condition for) a thinker's entitlement to basic logical beliefs. A non-inferential account suffers from the problem of bad company if its proposed source of entitlement entitles spurious inferential transitions, such as those that involve contonktion. What is needed, in order to address the problem, are considerations that disqualify spurious connectives from receiving justification.

The third and final standard obstacle is called "the problem of dealing with the sceptic."[6] To understand this problem properly it is important to note the distinction between proposals aimed at achieving certainty, and proposals aimed at explicating tacit knowledge. The project of explaining how basic logical knowledge is possible is, as the name suggests, explanatory; that is to say, it is not geared toward providing logic with a firm epistemological foundation. It is not concerned with the pursuit of certainty. Rather, it presupposes that there is such a thing as basic logical knowledge and it seeks to elucidate or explicate how that knowledge can be gained and how it is justified. Nonetheless, it is necessary to deal with a sceptic. Since adequate explanations yield sufficient conditions for what it is that they set out to explain, an explanation of how a thinker might come to know that a logical rule is valid must be capable of being used by someone who is not sure whether they know that that rule is valid to quell his or her doubts. It is easy to imagine a sceptic who doubts whether a logical rule is valid because he or she cannot think of reasons on which to base the belief. If this sceptic contemplates

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[5] Cf. (Wright 2004).
[6] Cp.(Hale 2002: pp.287,89).

the proposal, and can satisfy the conditions that it lays down while sincerely doubting that the rule is valid, then the proposal has not succeeded at doing its job. The problem of dealing with the sceptic, then, is that of showing that it would be unintelligible or incoherent to doubt that a rule is valid while satisfying the conditions laid down by an explanatory proposal.

2 The referentially constrained meaning-theoretic rationalist proposal.

Let us now turn our attention to the first positive proposal. It is called the referentially constrained meaning-theoretic rationalist proposal.[7] This proposal is a meaning-theoretic and rationalist proposal because it maintains the idea that understanding is a condition for justification, and it rejects the idea that coming to have basic logical knowledge depends on experience. It is referential because it rejects the idea that logical truths are true by convention, or true purely in virtue of meaning. It is a moderate (as opposed to extreme) rationalist proposal because it does not rely on the notion that a thinker possesses a faculty of rational intuition which is analogous to perception and which enables a thinker somehow simply to "see" the truth of a proposition. The proposal is also moderate in the sense that it does not require the knowledge that it pretends to be infallible or immune from revision. And lastly, the proposal is referentially constrained because it is subject to substantive constraints at the level of semantic value. For brevity, I will refer to the referentially constrained meaning-theoretic proposal as the "referentialist proposal".

The basic idea behind the referentialist proposal is that it tries to explain the possibility of basic logical knowledge in terms of the nature of logical concepts.

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[7] Cf. (Peacocke 1987, 1992, 1993, 2000, 2004).

Concepts, on this proposal, are thought of as the constituents of propositional contents. Thus, for example, the singular concept *Socrates* and the predicative concept *wise* combine to form the thought content *Socrates is wise*. A propositional content may be the content of an intentional mental state. If it is, the mental relation that the intentional mental state bears toward its content is called a propositional attitude. Thus, for example, a thinker may stand in the "believes that" relation to the content *Socrates is wise*. A belief is true if the truth-conditions for the content are satisfied. The nature of a logical concept, on this proposal, is revealed by the conditions that must obtain if a thinker is to possess it, and also by the contribution that the concept makes to the truth-conditions of contents in which it occurs. The referentialist strategy, then, is to identify the explanatory properties of the nature of logical concepts and to use these properties in an explanation of the possibility of basic logical knowledge.

The referentialist proposal has two parts. The first part is called the theory of concept-possession, and it elucidates what it is for a thinker to understand a logical constant. The theory of concept-possession, applied to the concept of the conditional, consists of the following two principles:

> **Conditional-Possession**
> *S* possesses the concept of the conditional just in case *S* finds inferential transitions whose forms are instances of *modus ponens* and *conditional proof* to be primitively compelling.
>
> **Constitutivity**
> Satisfying the possession condition for a concept is partially constitutive of understanding the expression corresponding to that concept.

The principle that I have called Conditional-Possession states what the conditions are that must obtain if a thinker is to possess the concept of the conditional. The Constitutivity principle links concept-possession with understanding. Conditional-Possession and Constitutivity together imply that *S* understands 'if' only if *S* has a compelling primitive impression towards *modus ponens* and *conditional proof*.

The idea, that a thinker understands a logical constant only if primitively compelled to accept the rules that govern it, provides the referentialist with what seems to be a response to the problem of dealing with the sceptic. For the idea implies that the sceptic is unwilling to accept the rule only if she fails to understand the constant. And it is not possible coherently or intelligibly to doubt a rule governing a constant while failing to understanding what that constant means.

The theory of concept-possession says what has to happen if a thinker is to understand a logical constant. By itself the theory of concept-possession is not enough to explain how basic logical knowledge is possible. It is not enough for two reasons. First, a thinker's mere impression that an inference rule is valid is insufficient for it to be the case that the rule is valid. After all, merely having a belief does not entail that that belief is true. And secondly, it is possible for a thinker to have a favourable impression towards a logical rule without being entitled to the belief that the rule is valid. Thus it is necessary that the theory of concept-possession should be supplemented.

The supplementary second part to a referentialist proposal has been referred to in the literature as "the determination theory."[8] Given the two abovementioned reasons for why the theory of concept-possession is not enough, the function of determination theory is likewise twofold. First, it must account for the contribution that a logical constant makes to the truth-conditions of contents in which it appears. And secondly, it must

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[8] Cf. e.g. (Peacocke 1987: p.181).

provide an ingredient (an external condition) that makes it possible to explain how and when having a primitively compelling impression towards an inferential transition is internally sufficient for a thinker to be in possession of basic logical knowledge.

The determination theory, for the concept of the conditional, consists of the following principle:

> **Conditional-Semantic Value**
> The semantic value of the material conditional is the truth-function that makes transitions of the forms mentioned in Conditional-Possession truth-preserving under all assignments to their sentence letters.

The determination theory enables the referentialist proposal to handle the problem of bad company. The condition imposed by the determination theory requires that the semantic value of a connective figuring in a justified logical belief be one that validates the connective's introduction- and elimination-rules. At least in the case of tonk, the problem of bad company is dealt with by the fact that there is no truth-function that validates both Tonk-Introduction and Tonk-Elimination. Inferential transitions involving 'tonk' thus do not satisfy all of the conditions for justification that are set by the referentialist proposal.

With both parts of the referentialist proposal on the table, it is easy to see how it answers our central question. By Conditional-Possession and Constitutivity, a thinker understands 'if' only if a thinker is primitively compelled to accept inferential transitions that have the form of *modus ponens*. By Conditional-Semantic Value, *modus ponens* is valid. The internalist conditions provided by Conditional-Possession and Constitutivity, and the externalist condition provided by Conditional-Semantic Value, are intended to be individually necessary and jointly sufficient for a thinker to be justified in believing that *modus ponens* is valid. Thus the referentialist answer may be expressed by saying: if a thinker understands 'if', and if 'if' means what it means, then the thinker is justified in believing that *modus ponens* is valid.

Using the example of the conditional, the conditions laid down by the referentially constrained meaning-theoretic proposal can be concisely represented in the following way.

A thinker *S* has basic logical knowledge if,

(1) *S* understands 'if'.

(2) *S* understands 'if', only if *S* possesses the concept *if*.

(3) *S* possesses the concept *if* only if *S* has the primitively compelling impression that *modus ponens* and *conditional proof* are truth-preserving.

(4) S's belief that *modus ponens* is valid, is caused by the satisfaction of conditions (1)-(3).

(5) The semantic value of 'if' is the truth-function that validates *modus ponens* and *conditional proof.*

(6) Modus ponens is valid, because of (5)

(7) *S* is justified in believing that *modus ponens* is valid, because of (4) and (6).

If this account is correct, then any thinker who understands 'if' will have tacit knowledge of the validity of *modus ponens*. If this account is correct, then a thinker who is aware of and who grasps the account will know explicitly that *modus ponens* is valid. But is it correct?

There are at least two problems with the referentialist proposal. Let us start with condition (3). It is not clear that (3) is credible. It is easy to imagine a small child who understands the word 'if', but who does not understand the question: What is your primitive impression regarding inferential transitions in the form of *modus ponens* and *conditional proof*? To put the problem differently, it is possible for a small child to engage in various inferential practices, and thereby possess various logical concepts, without knowing anything about inference rules or the relation of logical implication. A question that I shall not endeavour to answer here is whether a small child would have the capacity to acquire a compelling impression regarding the validity of a set of inference rules. We might call this *the small child objection.*

The second problem facing the referentialist proposal has to do with condition (5), and its relationship to condition (6). Condition (5) is intended as a substantive explanation of why it is the case that *modus ponens* is valid. The explanation says that *modus ponens* is valid because the semantic value of the conditional is a truth-function that makes *modus ponens* valid. The problem with this explanation is that it begs the question. And if it begs the question, then it certainly is not substantive. In order to see why it begs the question, we may start by considering a minimalist analysis of truth: A statement A is true iff the semantic value of A makes it true. Along the same lines, a minimalist analysis of validity would be this: An inference rule R is valid iff the semantic value of the logical concept governed by R is the truth-function that makes R valid. Given this analysis, the explanation provided by the determination theory yields an explanans that is equivalent to its explanandum; and therefore it begs the question. It tells us little more than that R is valid if R is valid. Hence, the determination theory does not provide a substantive explanation for how we know e.g. that *modus ponens* is valid.

The explanation put forward by the determination theory fails in a way that is analogous to the failure of the explanation involved in the following situation: Suppose I claim to have a triangle in my pocket. And suppose you want me to give you a reason to believe that what I claim is true. You would be right to be unsatisfied if the reason that I give you in support of my claim is that the thing in my pocket is three-sided. Being told this information does not give you any greater reason to believe my claim; my reason stands in just as much need of support as my original claim. My "explanation" is therefore inadequate. Similarly, the explanation provided by the determination theory fails to give any reason to think that *modus ponens* is valid. The question "How do we know that there exists a semantic value that validates *modus ponens*?" is equivalent to the question "How do we know that *modus ponens* is valid?". For this reason Conditional-Semantic Value is problematic.

At best, the determination theory manages to shift the bump in the rug: instead of having to ask what makes it true that *modus ponens* is valid, we now have the option of asking "Is there a truth-function that validates *modus ponens*?" Hence, the referentialist proposal does not succeed in providing a *substantive* explanation of what makes it true that *modus ponens* is valid; and so it does not actually have an external condition to rely on in order to be able to explain why, when a thinker has certain primitively compelling impressions, the thinker is justified in believing that an inference rule is valid. We can call this the *inadequate explanation objection.* What it shows is that the referentialist proposal needs an adequate explanation of the validity of fundamental logical laws; that is, it requires an account of how logical concepts are given to us.[9]

We are left with the following situation. The small child objection undermines the theory of concept-possession, and with it, the referentialist's reply to the problem of dealing with the sceptic. And the inadequate explanation objection undermines the

[9] Cp.: "To enquire how disputes over the validity of some principle of reasoning may be resolved, if at all, thus affords a particularly vivid way of enquiring how the meanings of the logical constants should be regarded as being given to us" (Dummett 1991a: p. 192).

determination theory, and the referentialist's reply to the problem of bad company. Let us turn now turn to the second positive proposal.

3 The inferential proposal

There are two main differences between the inferential proposal[10] and the referentialist proposal. The first is that the inferential proposal does not attempt to impose substantive constraints at the level of semantic value. It says only that it is something like "the world" that makes it true that a certain logical rule is valid. This difference appears to enable an inferential proposal to sidestep the inadequate explanation objection. If the inferential account of justification has nothing to do with the determination theory (or any other attempt at a substantive explanation of what makes a logical rule is valid), then *a fortiori* there is no need to ensure that that explanation is adequate.

The second main difference is that the theory of concept-possession advanced by the inferential proposal is different than the one put forward by the referentialist proposal. This difference makes it possible for the inferentialist to avoid the small child objection. One way of interpreting the small child objection is to say that it shows that Conditional-Possession is too strong. If this is so, then a natural way to remedy the problem is to replace Conditional-Possession with a weaker principle. The possession-conditions for the conditional suggested by the inferential proposal amount to a weaker version of Conditional-Possession. The principle put forward by the inferential proposal looks like this:

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[10] See (Boghossian 1996, 1997, 2000, 2001, 2003).

> **Weak-Conditional-Possession**
> The conditional is a concept whose possession requires a thinker to have the fundamental inferential disposition to reason in accordance with *modus ponens* and *conditional proof.*

The small child objection does not arise for this principle because it does not put demands on the thinker that involve having the capacity to appreciate inference rules.

But the same feature that makes it possible for the inferentialist to avoid the small child objection, makes it difficult to see how the inferentialist can give a non-inferential justification of basic logical knowledge. A non-inferential justification needs a particular belief about a particular rule that is caused in a particular way; and this is not something that is guaranteed by being in possession of a disposition to reason in accordance with a particular set of inference rules. All that is guaranteed by Weak-Conditional-Possession, together with Constitutivity, is that a thinker understands 'if' only if the thinker has the fundamental inferential disposition to reason in accordance with *modus ponens* and *conditional proof.* Thus the proponent of the inferential proposal pursues the idea that basic logical knowledge is inferential and tries to find a solution to the problem of rule-circularity.

As we have seen, a rule-circular argument uses the rule that it is designed to justify. This is not necessarily a bad thing. It is bad only if the argument's use of the rule causes the argument to be grossly circular. A rule-circular argument for a rule *R* will be grossly circular if that which entitles the thinker to use *R* to confer warrant on its conclusion is a justified belief that *R* is valid. Thus, whether or not a rule-circular argument is acceptable depends on what entitles a thinker's use of a rule. A way to make perspicuous the entitlement to the use of a rule is to set out the conditions that must be satisfied in order for an inference to transmit warrant from its premises to its conclusion. The following set of conditions would make a rule-circular argument grossly circular:

> A thinker *S* succeeds in inferentially transferring warrant from a set of statements Γ to a statement θ just in case,
>
> (1) *S* is justified in believing the premises in Γ.
>
> (2) The inference from Γ to θ is in the form of a valid inference-rule *R*.
>
> (3) *S* is justified in believing that *R* is valid.
>
> (4) *S* is entitled to infer θ from Γ, according to *R*, because of (3).

When the inference from Γ to θ is a rule-circular argument for the inference-rule *R*, θ is replaced by the proposition that the introduction and elimination rules for *R* are valid. Thus, when the inference from Γ to θ is a rule-circular argument for *R*, condition (3) presupposes what the inference is intended to show—namely, that *R* is a valid rule.

The most obvious solution is to find a replacement, for condition (3) that will not lead to gross circularity. According to the inferentialist, the replacement can be supplied by appeal to Weak-Conditional-Possession. It is possible to have a fundamental inferential disposition without having a belief that a particular rule holds, but the inferentialist argues that having such a disposition *is* sufficient for being entitled to use a rule of inference to transmit warrant. The resulting alternative model of warrant transfer replaces conditions (3) and (4) with the following three conditions:

(3′) S grasps the logical concept governed by R.

(3″) S has the primitive inferential disposition to reason according to R, on the basis of (3′) and Weak-Conditional-Possession.

(4′) S is entitled to infer θ from Γ, according to R, because of (3″).

This inferential model generates rule-circular arguments that are not grossly circular. For it is not a presupposition of grasping the logical concept governed by R, or of having the fundamental inferential disposition to reason according to R, that R be valid.

In order for the inferential proposal to work—in order for the inferentialist to be able to give an inferential justification of the belief that *modus ponens* is valid—it must be true that (3″) implies (4′). But this implication is not obvious. How is it that having a primitive inferential disposition is sufficient to be entitled to use a rule? This question cannot be answered without addressing the problem of bad company. For it is clearly not the case that *any* disposition to reason in accordance with a rule implies that the bearer of the disposition is entitled to use it. People commonly have the disposition to reason according to argument forms to which they are not entitled—for example, people tend on occasion to affirm the consequent. Nor is it impossible that someone should have the disposition to reason according to the tonk rules. Yet extending entitlement to the use of these rules in the transmission of warrant would not be desirable. So we need to rephrase the question: How is it that having the disposition to reason according to *the right sort of rule* is sufficient for being entitled to use it? To answer this question, we first need to know which rules are the "right" rules.

It is tempting to appeal to substantive constraints at the level of semantic value—i.e. to an account of how logical concepts are given to us—in order to go about identifying the right rules. Such an appeal would yield a convenient criterion. But such an appeal would also have the effect of stripping the inferential proposal of what makes it inferential. It would transform the inferential proposal into a referentialist proposal that embraces the inferential horn in the inferential knowledge/non-inferential knowledge dilemma. Of the three options: i) appeal to substantive constraints at the level of semantic value by adopting the referentialist's determination theory; ii) appeal to substantive constraints at the level of semantic value by adopting an adequate substantive explanation of what makes an inference rule is valid; and iii) provide a criterion of the correctness of a rule that does not appeal to substantive constraints at the level of semantic value, our inferentialist picks option iii), and puts forward the idea that the right rules are rules that are "meaning-constituting". An introduction or elimination rule is meaning-constituting just in case a thinker's being disposed to reason according to it is a necessary condition for the thinker to possess the concept that it partially defines. To put it differently, a meaning-constituting rule is a rule that is built into the weak-possession-conditions for a concept. Thus, according to the inferentialist, the right rules—the rules that make good company—are the rules that are meaning-constituting. Let us see how having a disposition to reason according to a meaning-constituting rule is thought to be sufficient for entitlement.

Here is the claim that expresses what the relationship between (3″) and (4′) is supposed to be: Having a primitive inferential disposition to reason according to a meaning-constituting inference rule is a sufficient condition for being entitled to use it. When this claim is unpacked, it says that if one has the disposition to use *R*, then when one has that disposition *if* one has the concept *C* that *R* defines, then it is enough to be entitled to use *R*. Here is an argument for the claim: Let us say that a disposition to use *R* is "ill gotten" if a thinker is not entitled to act on it. Next, we may safely assume that a thinker *S* cannot have a belief that involves *C* (justified or otherwise) without having the concept *C*. Now by Weak-Conditional-Possession, if *R* is meaning-constituting for *C*, then *S* possesses *C* only if *S* has the disposition to reason according to *R*. *S* cannot so much as entertain *R* unless *S* is willing to make inferences according to *R*. But this implies that, if *S* has a belief involving *C*, then *S* must have the disposition to reason according to *R*. Which is just to say, that, without the disposition to reason according to *R*, there could be no beliefs about whose justification *S* can intelligibly raise a question. There clearly are beliefs about whose justification *S* can intelligibly raise a question; namely, the belief that *R* is valid. So it follows that *S* cannot but have the disposition to reason according to *R*; *S* has no option. But if *S* must have the disposition to reason according to *R*, then we cannot say that that disposition is ill gotten. And if the disposition is not ill gotten, then *S* is entitled to act on it. In particular, *S* is entitled to act on it to transmit warrant in a rule-circular argument for *R*.

Using the example of the conditional, the conditions laid down by the inferential proposal can be summarised as follows:

A thinker *S* has basic logical knowledge if,

(1) It is true that *modus ponens* is valid (because of the way "the world" is).

(2) *S* believes that *modus ponens* is valid.

(3) *S* believes that *modus ponens* is valid only if *S* possesses the concept *if*.

(4) *S* possesses the concept *if* only if *S* has the primitive inferential disposition to reason according to *modus ponens* and *conditional proof*.

(5) *S* has the primitive inferential disposition to reason according to *modus ponens* only if *S* is entitled to reason according to *modus ponens*.

(6) If *S* believes that *modus ponens* is valid, then *S* is entitled to reason according to *modus ponens* without having reflectively appreciable warrant (from (3) - (5)).

(7) *S* uses *modus ponens* with entitlement in a rule-circular argument to inferentially justify the belief that *modus ponens* is valid (from (6)).

These conditions are intended to be individually necessary and jointly sufficient for a thinker to have basic logical knowledge.

There are at least three problems with the inferential proposal. Let us start with condition (4). Condition (4) encapsulates the theory of concept-possession of the inferential proposal; it says what has to happen if a thinker is to possess a logical concept. It is what replaced the referentialist's theory of concept-possession, which was undermined by the small child objection. The problem with condition (4) is that it suffers from a ramped up version of the small child objection: reasoning according to an inference rule is not a precondition for possessing the concept that the rule partially defines. To see why, first recall that to possess a logical concept it is sufficient to understand the constant that expresses it. Next, note that a thinker cannot reject a pattern of inference because she thinks that the rule that governs it is invalid without failing to have the disposition to reason according to the rule. Condition (4) fails because it is possible for someone who understands 'if' to reject *modus ponens* because they have been convinced by a complex theoretical argument that it is invalid. Let us call this *the errant thinker objection*. It applies just as readily to the referentialist proposal. The example that is often mentioned is that of the logician Vann McGee who understands 'if' if anyone does, but who once proposed a counter-example to *modus ponens*, claiming that it fails to preserve truth when its major premise has a consequent that is itself a conditional.[11]

In response to the errant thinker objection, a proponent of the existence of a constitutive relationship between understanding and the acceptance of inference patterns might charge that the objector begs the question by supposing that someone who takes there to be a counter-example to *modus ponens* can fully understand 'if'. Put differently, a proponent might ask: How can an errant thinker know that 'if' means *if*, if the errant thinker rejects *modus ponens*? If the meaning of 'if' is not constituted by the usual inference rules, then the objector owes an account of how it *is* constituted. And whatever that account is, it must not preclude the possibility of intelligibly regarding something as a counter-example to *modus ponens*. But, the proponent will urge, there is no such account. The account that says that the meaning of an expression is constituted by its contribution to the truth-conditions (or assertion-conditions) of sentences in which it occurs will be unavailable to the objector, since whatever makes that contribution will surely validate *modus ponens*.[12] This ends the description of the defence that can be put up by the proponent. If this defence against the errant thinker objection is not completely successful, and the objector is able to produce an alternate account of how the meaning of

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[11] (McGee 1985).
[12] Cf. (Hale 2002: p. 290).

'if' is to be construed, then we are left with a standoff. In order to negotiate such a standoff, what is needed is an account of how the meanings of the logical constants ought to be regarded as being given to us.

The second problem has to do with condition (5). The argument in support of condition (5) is the same as the argument in support of the claim expressing the relationship between (3″) and (4′) that I described earlier. The problem is that this argument is invalid. It has the same form as the following argument concerning moral entitlement: Psychopath *S* has no option but to slash person *T*. Therefore, *S* is entitled to slash *T*.

The third problem with the inferential proposal has to do with the criteria that determine whether or not a rule is meaning-constituting. As we have seen, a meaning-constituting rule is a rule that a thinker who possesses the concept figuring in it must be willing to reason in accordance with. But what is it about a rule that makes a thinker willing to reason according to it if she possesses the concept that figures in it? We know what it means for a rule to be meaning-constituting, but what is the general principle that tells us whether a given rule fits the definition? Why, for example, is it the case that the tonk rules are not meaning-constituting? According to the inferential proposal, it is due to the fact that the tonk rules do not generate a coherent meaning for 'tonk'. In order for a set of inference rules to generate a coherent meaning for a constant, it must be possible to use them to say under which interpretations a sentence containing the constant is true. But the tonk rules cannot be used to do this: under the interpretation that makes *A* true and *B* false, tonk-introduction makes '*A* tonk *B*' true, but tonk-elimination makes '*A* tonk *B*' false. For this reason, the tonk rules are not meaning-constituting. Thus, a criterion, for a set of rules to be meaning-constituting, is that it be possible to use the set to determine under which interpretations a complex statement, which contains the constant whose meaning is fixed by the set, is true.

But this cannot be the only criterion according to which a rule is meaning-constituting. What about someone who has been convinced by a complicated theoretical argument that certain forms of reasoning, which are conventionally accepted, are devoid of justification? If there were only the one criterion, then the inferential proposal would work just as well to justify the rules of, say, intuitionistic logic, as it does to justify the rules of classical logic. If the rules of intuitionist logic were just as justified as the rules of classical logic, then it should be possible for a thinker S to be justified both in asserting P, and in refusing to assert P, e.g. P is the proposition expressed by the statement: 'Either there are infinitely many twin primes or there aren't'. If S is justified in refusing to assert P, then S is not justified to assert P, and this, together with the claim that S is justified in asserting P, implies that S would both be justified and not justified in asserting P, which would be inconsistent. So either the inferential proposal is incorrect because it leads to an inconsistency, or there is at least one additional criterion[13] that governs whether or not a rule is meaning-constituting. Let us call this *the absent criterion objection.*

What could the additional criterion be? It cannot be that meaning-constituting rules are those that we freely stipulate to be valid. For if we were to allow such a criterion, we would be assenting to the view that the truth of the sentence '*Modus ponens* is valid' depends entirely on a decision about linguistic meaning, and not at all on the fact

[13] In (Boghossian 2003) it is argued that the rules for so-called "defective concepts" cannot be meaning-constituting, but this additional criterion does not bear on the case that we are considering here; neither the classical nor the intuitionistic introduction- and elimination-rules for the logical constants are defective in the sense discussed.

(if it is one) that *modus ponens* is valid. Such a view has the consequence that facts themselves are dependent upon linguistic decisions: there would be no fact or true proposition that, e.g., *it is not the case that snow is white and snow is not white*, without decisions about what certain words are to express. It is evident, though, that facts (that are not about linguistic decisions) do not depend on linguistic decisions. So a view that implies that they do is incorrect; and it is incorrect to try to employ a conventionalist criterion for deciding whether or not a rule is meaning-constituting.

It cannot be that meaning-constituting rules are those that thinkers happen to reason in accordance with in their ordinary everyday linguistic practices. For if we allow that a rule's status as meaning-constituting depended on the ways in which people happen to think, a statement that a rule was meaning-constituting would be an empirical generalisation. As such, it would be probable at best. The existence somewhere of one unreasonable individual would be enough to strip a logical truth of its status as necessarily true. On the resulting psychologistic view, it would be merely probable that *modus ponens* is valid. Secondly, an attempt to employ a psychologistic criterion would cause a reversal of the correct order of explanation: the reason why people tend to reason according to certain rules on the inferential proposal should be that they are meaning-constituting, not that certain rules are meaning-constituting because people tend to reason according to them.

4 The concept constitution account of blameless blindness.

(Boghossian 2003) attempts to steer a course between "Simple Inferential Externalism", which says that deductive reasoning transfers justification for a belief by virtue of the fact that the inference is valid, and "Simple Inferential Internalism", which says that in addition to being reliable, a thinker is required to be able to know that the inference is deductively good. The problem with simple inferential externalism is that there are cases in which a thinker is epistemically blameworthy for using a reliable belief-forming process because the thinker is not aware that the process that she is using is reliable. The problems with simple inferential externalism is that it generally tends to invoke a faculty of rational insight, and that it generates circular justifications.

The proposed inferential alternative is as follows. First, consider the following three definitions. Let F be the condition that one carries out the process of inferring according to a deductively valid pattern P. F is a *blameworthy* condition for the transmission of justification for belief by deductive reasoning just in case :

(1) There is a case α such that F obtains in α and one is completely in the dark in α about the reliability of F.

A condition D permits F to be *blind* and *blameless* just in case (2) and (3) hold:

(2) There exists a case α in which F and D obtain, and in which one is without any reflective appreciation of the epistemic status of P.
(3) There exists a case α such that F obtains in α, and if D obtains in α, then one is not in a position to know that F is reliable.

On the basis of these two definitions, the concept constitution account of blameless blindness is this: Let D be the condition that F's obtaining is a precondition for having one of the concepts ingredient in P. If F is blind and blameless, then it would seem that justification can be transmitted by deductive reasoning.

Like the meaning-theoretic proposals that have already been considered, the blameless blindness proposal suffers from the errant thinker objection. Recall that the errant thinker objection purports to yield a counter-example to the idea that accepting a rule if one has the concept that it corresponds to. The reason why, again, is that accepting a rule is not a precondition for understanding the expression that corresponds to it, but understanding the expression suffices for having the concept. Thus, when Vann McGee refused to accept *modus ponens* on the basis of an initially plausible albeit erroneous counter-example, he understood 'if', and therefore had the concept *if* without accepting *modus ponens*.

A proponent of the idea that accepting a rule constitutes an understanding of the relevant expression might argue that the counter-example to *modus ponens* is not in fact genuine, and point out that, if it is, only a restricted class of inferences by *modus ponens* are affected—those which have major premises which are themselves conditionals. The objector may respond, however, by saying that it is not the case that a precondition for having the concept *if* is a willingness to make only some restricted class of inferences by *modus ponens*. If a willingness to make inferences by *modus ponens* is a precondition for having the concept *if*, then a thinker must be willing to make any particular inference by *modus ponens*. If this were not the case, and it would be enough to have the concept for a thinker to be willing to make merely most inferences by *modus ponens*, then the claim would be too weak for the blameless blindness proposal which needs to explain why any given inference by *modus ponens* is blameless.

A proponent might attempt to ward off the objection by invoking a distinction between *fully* understanding 'if' and merely having a partial understanding of it: an errant thinker who only partially understands 'if' may find him or herself unwilling to accept an inference by *modus ponens*; but making any given inference by *modus ponens* *is* a precondition for a full understanding, the kind of understanding characteristic of an expert, and, by invoking Putnam's notion of a division of linguistic labour, we can say that that is enough to establish the blamelessness of carrying out basic deductive inferences. But the objector can reply by pointing out that Vann McGee is an expert if anyone is: it is possible that any given inference by *modus ponens* might be rejected by some expert on the basis of a complex theoretical argument. Even though the expert would be mistaken, his or her mistake is compatible with possessing the concept *if*.

Along the lines of (Williamson 2003: p.254), the errant thinker problem may be generalised as follows. Let *C* be any concept, '*C*' any word that means *C*, and *P* any deductive pattern of inference of which *C* is an ingredient. Let *Inst* be an instance of *P*. Consider an expert on *C* and *P*, who thus fully understands '*C*'. It is possible that the expert becomes convinced by a complex theoretical argument that *Inst* is invalid, and therefore rejects it. It is possible that the complex theoretical argument does not generalise to more than a small portion of the instances of *P*. By ordinary standards, the expert continues to fully understand '*C*'; in conversation the expert uses '*C*' appropriately, and therefore still has the concept despite the expert's unorthodoxy over *Inst*. Thus, a willingness to make the inference *Inst* is not a precondition for having *C*, even if willingness to infer in most instances according to *P* is such a precondition. Thus, we cannot explain why one is blameless in making the inference *Inst* by saying that a willingness to make it is a precondition for having *C*.

If any pattern that one must reason according to as a precondition for possessing a concept, is entitling, then, if reasoning according to the tonk rules is a precondition for having the concept *tonk*, then anyone who has the concept *tonk* can blamelessly infer anything from anything. This is a statement of one manifestation of the problem of bad company. The concept constitution account of blameless blindness attempts to deal with the problem of bad company by restricting its explanation to "non-defective" concepts. A quick way to declare the defectiveness of *tonk* is to deny that there is any concept that 'tonk' expresses.

An assumption, however, of the blameless blindness way of approaching the problem of bad company is this: a thinker possesses the concept only if the thinker is willing to infer according to the rules. And this assumption is implausible. I possess the concept *tonk* because I understand what 'tonk' means; if I did not possess *tonk*, I would not be able to talk about it to say what is wrong with it. But I am not in the least inclined to use *tonk* in inference, or to assent to the tonk-rules, because they obviously lead to contradictions. When we hear someone mention the word 'tonk', we understand what that person is talking about, and therefore, we possess the concept; yet we are unwilling to infer according to the introduction and elimination rules for 'tonk'. It is perfectly possible for one to have a concept while rejecting its rules. A willingness to reason according to an objectionable inferential pattern is not a necessary condition for knowing what any given word means. A slightly less quick way of saying what is wrong with 'tonk', without making the implausible assumption, is to say that it does not refer. On one approach to concepts, the introduction- and elimination-rules for an expression play a constitutive role in determining its reference. So, the view goes, if exactly one assignment of reference makes the rules truth-preserving, then the assignment is correct; if more than one assignment, then it is indeterminate which assignment is correct; and if no assignment, then the expression does not refer. On this view, since there is no truth-function that makes the tonk rules truth-preserving, there is no assignment of reference to 'tonk': 'tonk' does not refer, and complex statements that use 'tonk' lack a truth-value. Thus, since a thinker can have a concept while rejecting its rules, *tonk* does not need to be defective in the sense suggested by the blameless blindness proposal.

5 The underlying worry, and the methodology of rational reconstruction

The underlying worry[14] is this: the proponent of the inferential proposal, by seeking a condition which is necessary (and sufficient) for knowing what a word means—hence, possessing a given concept, asks too much of a theory of concept-possession. No necessary condition is ever to be had because understanding words in natural language involves the ability to use them in whatever ways facilitate fruitful communication, and it will always be possible for this ability to outstrip the sort of necessary condition that the inferentialist wants to give. The alternative picture of understanding is one on which a thinker can compensate for his or her unorthodoxy on one point by demonstrating orthodoxy on others, by having the ability to predict the reactions of orthodox speakers, and by being willing to have his or her practice evaluated by intersubjective standards.

A crucial assumption of the underlying worry is that the object of concern is understanding or concept-possession *in natural language*. It is surely true that Peacocke and Boghossian are concerned to explain concept-possession as it relates to natural language; hence their proposals do seem to fall prey to the underlying worry. However, the underlying worry does not hold in general. There is room for a view that is able to

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[14] Cf. (Williamson 2003: p. 276).

sidestep Williamson's underlying worry if one accepts the idea that concepts and expressions can have precise definitions and holds the view that the meaning of certain statements determine and are determined by their use. Such a theoretician is able to make the Carnapian move of using *the method of rational construction*[15] to approach the problem of explaining the possibility of basic logical knowledge. Rather than becoming lost in the chaotic buzz of natural language, one may construct an artificial formal system which involves the apparatus of (at least) ordinary first-order logic and in which the concepts forming the subject-matter of the system are introduced by means of precise definitions. Perhaps the best example of such a system is elementary logic itself, which can be regarded as just such a rational reconstruction of the set of concepts expressed by the logical constants of daily life. In contrast to this method, one may wish to attempt to describe the complex patterns of behaviour which the concepts of daily life exhibit.

The method of rational reconstruction bears on Williamson's underlying worry in the following way. His criticism is that "there is no litmus test for understanding" (Williamson 2003: p. 276). Note that this criticism is not essential in order to criticize the inferential treatment of 'tonk'; it is possible, contra the proponent of the inferential view, to know what 'tonk' means without having to reason in lockstep with its rules, and this does not imply that there is no litmus test. Even if we suppose that it is true of natural language that there is no litmus test for understanding, it does not follow that there is no litmus test for a rational reconstruction of one or another fragment of natural language.

It is one thing to say that a given condition for understanding is implausible (as is the case with the condition that says that a thinker who understands an expression must

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[15] For more on the method of rational reconstruction, see the introduction and §72 of (Carnap 1937) See also ch. 1 of (Carnap 1962) And (Schilpp 1963) pp. 67-71.

reason in lockstep with its rules), but to say that, in general, there can never be a condition for understanding is incorrect. Contra Williamson, under certain conditions, if a thinker doesn't know what the definition of a term is, then the thinker doesn't know what the term means. This, however, is of course not to say that not knowing the definition is, in general, sufficient for one to fail to know what the word means. And this second claim is probably what the proponent of the inferential view requires in order to make his or her treatment of the second type of bad company problem work.

6 A Cartesian-like model of basic logical knowledge.

The meaning-theoretic efforts, to find a way to explain the possibility of basic logical knowledge, appear to fail. Of the several objections raised, the weakest is against the application of the principle of Concept-Possession. Might there be a viable proposal that starts from the principle of Concept-Possession, but which does not follow the path of either the referentialist or the proponent of the inferential proposal? Bob Hale (2002) has a proposal that starts from this starting point, but which differs from the other meaning-theoretic proposals considered so far. Hale's proposal takes as its main focus the problem of dealing with a skeptic who doubts whether basic inferential transitions are sound.

The proposal can be interpreted as having three main steps. The first step is to explain why it is impossible, rationally, to doubt that basic inference-rules are sound. The explanation starts by granting that the acceptance of basic patterns of inference is constitutive of understanding the logical constants featured in those patterns. It is then observed that if understanding is constituted in this way, a thinker cannot doubt the soundness of one of these inferences without being guilty of misunderstanding the featured constant. For example, if the meaning of 'if, then' is constituted by an acceptance of *modus ponens* and *conditional proof*, then a thinker who understands 'if, then' will accept the patterns. A thinker who doubts one of the patterns, by genuinely believing that it is possible for 'p' and 'if p, then q' to be true while 'q' is false, does not *properly* grasp what 'if, then' means. So, by virtue of what a logical constant means, its rules cannot coherently be doubted in the sense of supposing that there might be an inference exemplifying one of the rules, whose premises are true and whose conclusion is false.

The second step is motivated by the problem of bad company. Why can't the skeptic apply the argument described above to the tonk rules to show that they are beyond doubt? It could then be argued that there must be something wrong with the argument at step one, since we must be able to doubt the soundness of the tonk rules. The reply to this is to concede that it is not possible to imagine a counter-example to either of the tonk rules, but to note that we can still doubt that they satisfy Belnap's conservativeness criterion. The conservativeness criterion says that sound rules for an operator *O* do not allow the derivation of *O*-free conclusions from *O*-free premises that could not already be derived using only the rules for the operators that are present in those *O*-free premises and conclusions.

But if we can doubt that the tonk rules are conservative, what is there to prevent the skeptic from blocking the first step, in another way, by doubting that the conditional rules are conservative? To answer this, Hale observes that if it is possible to doubt the conservativeness (or more generally, the soundness) of a pair of rules, it must be possible to demonstrate that they are non-conservative (or unsound). For example, one may doubt the soundness of ZF with some exotic axiom without having a proof of unsoundness in hand, but the doubter would think that such a proof is possible. If it was impossible to demonstrate that the pair of doubted rules are non-conservative, the doubt would fail to be well-founded. Next, it is noted that you can't use a pair of rules, without discharging them, to conclude that they themselves are non-conservative, because if it were true that the rules didn't work, then the argument wouldn't work either. Since a rule-circular argument aimed at vindicating a doubt would be self-refuting, a well-founded doubt about a rule is essentially relative: it assumes the validity of the rules that could be appealed to if it were to be vindicated.

What rules are these that must be assumed by a demonstration of the non-conservativeness of a rule? Here Hale points to the general and conditional character of explicit formulations of inference-rules to suggest that any reasoning about their (un)soundness will involve steps governed by the introduction- or elimination-rules for the conditional and the universal quantifier. This "minimal kit" of rules is immune from doubt, as required by the first step; and whether they possess the property of conservativeness cannot be doubted either.

The third step is about how we might go from this explanation of how certain rules are indubitable, to an explanation of how they can be known to be sound. The suggestion is that beliefs about the soundness of rules from the minimal kit fit neatly into a Cartesian-like model, according to which believing truly something that is impossible rationally to doubt is sufficient for knowledge. The model looks something like this:

The Cartesian-like model of knowledge that *p*

S knows that *p* if,

(1) *S* believes that *p*.

(2) It is true that *p*.

(3) It is impossible rationally to doubt that *p*.

(4) *S*'s belief that *p* is appropriately caused.

(5) *S* is entitled to believe that *p* because of (3) and (4).

Without (4), one could say that *S* knows that *p* even if *S* believed it for Gettier reasons. But it is not clear what it takes to satisfy (4). Hale (p. 303) observes that, on pain of regress, it could not be another one of *S*'s beliefs (that (3), say), or the fact that *S* has arrived at *p* by inferring it. Rather, I arrive at my belief that *p* in the right way when "I cannot see how it might intelligibly be doubted that *p*".

If the belief that *p* must be caused by our rationally intuiting, that is, by just "seeing" that *p* is beyond intelligible doubt, a certain problem threatens to sneak back in. It is tempting to object that the appeal to rational insight, in order to explain how (4) would be satisfied, merely shifts the bump in the rug back to the original horn of the trilemma consisting of intuition, conventionalism, and psychologism. It could be insisted that until the various difficulties having to do with the view that rational insight is a genuine basis for knowledge can be resolved, this approach to basic logical knowledge must be placed on hold. As well, one might wonder what advantage is gained by the detour through the issue of indubitability. A response to this would be to say that (4) alone is insufficient for (5), since merely believing something on the basis of introspection doesn't make it so, and that this is why we need the external condition (3). So the detour through indubitability addresses what would be the fatal problem for a psychologistic account. This seems plausible, but there remains the question, What is it about the combination of (3) and (4) that makes it able to play the role of a source of entitlement?

We have seen that meaning-theoretic moderate rationalist proposals avoid many problems inherent in the reliance on intuition, convention, and experience. They do this by trying to answer what, in the Introduction, I called the acquisition question in terms of the nature of concepts. But because these accounts do not avail themselves of a substantive account of the origins of logical concepts, they either try to make possession-conditions bear more weight than they can, or they end up contravening one of the four constraints, mentioned in the Introduction. The type of approach that we examine in the next chapter does at least attempt to give a substantive answer to the justification question, by asking "What makes logical truths true?".

Appendix: summary of Williamson's attack on blameless blindness

Williamson's attack focuses on the central principle of the theory of concept-possession:

> **Concept-Possession**
> A thinker possesses a concept *C* only if *X*,

where *X* is a condition that is manifest in the use made of *C* as an instrument of communication between individuals. The principle, Concept-Possession, is attacked with the use of this principle:

> **Have**
> A thinker possesses a concept *C* if the thinker understands the word '*C*'.

The principle Have was used to infer, from observations about the nature of natural language, that various substitution instances of *X* are not necessary conditions for understanding a word, and thus, for having a concept. For example, one substitution instance for X is the condition that a thinker accept a given inference.

The weakness of Williamsons attack is that the observations about the nature of natural language, on which it relies, can be ruled out if one is employing the method of rational reconstruction.

Concluding that there can be no necessary condition for understanding a word or having a concept, Williamson suggests an alternative, called "normative inferentialism", based on the distinction between conditions that are normative and conditions that are psychological. He offers the following condition:

> **Williamson-Normativity**
> One understands $\rightarrow$ if one *ought* to use it in reasoning by *conditional proof* and *modus ponens*, whether or not one actually so reasons (Williamson 2003: p.291).

This principle however appears to be wildly implausible; reasoners who do not understand $\rightarrow$ still *ought* to use it in reasoning by *conditional proof* and *modus ponens*. Williamson-Normativity implies that virtually everyone who reasons understands $\rightarrow$. And if the principle can be fixed so that it makes sense, Normative inferentialism does not explain how deductive inference can transfer justification any more than the inferential proposal does. So it represents a big rain check. Furthermore, it prompts a pressing demand that one explain why one ought to reason one way or another. If we could explain that, then we should have already overcome the main obstacle to justifying logical reasoning.

Chapter 2:
NATURALISM AND LOGIC

"Frege said of the laws of logic that they are not the laws of nature but the laws of the laws of nature. It makes no sense to try to observe the world to discover whether or not it obeys some given logical law. Reality cannot be said to obey a law of logic; it is our thinking about reality that obeys such a law or flouts it."
—Michael Dummett, *The Logical Basis of Metaphysics*

Quine (1953) subverted the empiricism of the logical positivists and replaced it with a form of empiricism that has come to be known as "philosophical naturalism." Today, philosophical naturalism is possibly the most widespread form of empiricism that is applied to metaphysical and epistemological questions. It is therefore worthwhile to focus on philosophical naturalism and to examine what sort of answers philosophical naturalism provides to what in the Introduction I called the acquisition problem and the justification problem.

One of the main ideas behind philosophical naturalism is its rejection of the notions of analyticity and apriority. All knowledge, including knowledge in logic and mathematics, is conceived of as having the same status as empirical natural sciences.[1] Furthermore, there is only one way of coming to know something; it is the empirical way that underlies the empirical natural sciences. The motivation for this view of how knowledge can be gained lies in Quine's influential holistic picture of the relationship between knowledge and experience. On this picture, the totality of human knowledge is a "man-made fabric which impinges upon experience only along the edges" (1953: p. 42).

[1] This is the way in which naturalism is typically characterised, cf. e.g. (Friedman 1997). It is worth noting, however, that this characterization presupposes a particularly naïve understanding of the nature of empirical natural science; an understanding e.g. that fails to appreciate that Newton and Einstein believed that fundamental concepts of physical theories, such as *space* and *time*, had a special a priori status.

This leads to a view of logic on which it lies at the center of the fabric that represents our overall system of beliefs. Although its distance from the edge makes it resistant to revision in the face of recalcitrant experience, it remains a posteriori contingent.

A more recent naturalistic view of logic, which has been suggested by Penelope Maddy, attempts to do for Kant what Quine did for Carnap.[2] This view centers around two questions: "What makes logical truths true?" and, "How do we come to know that logical truths are true?". The sort of logical truths that are of concern are those of the simplest and most uncontroversial variety; for example, the sentence: 'If it's either red or green and it's not red, then it must be green.' The view promises to answer these two questions, about this variety of logical truth, in a way that does not rely on psychologism, inductivism, or conventionalism. It is this view that I critically examine in the present chapter.

I begin by briefly characterising naturalism generally, and mention two obvious difficulties with the view as a whole. In the next three sections of this chapter, I consider a naturalistic view of logic with three stages. The first stage has to do with interpreting Kant's position on logic, the second with developing a theory by naturalizing the interpretation of Kant's position. And the third stage has to do with explaining the relationship between the elements of the theory of the second stage and the laws of logic. I conclude with a re-assessment of the theory.

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[2] Cf. (Maddy 2000, 2002, forthcoming).

1 Naturalism

The expressed essential feature of naturalism is the "abandonment of the goal of a first philosophy" (Quine 1975: p. 72), that goal being one of uncovering an extra-scientific foundation for our knowledge.[3] The essential constraint imposed by naturalism is this:

> **Naturalist constraint 1**
> Extra-scientific methods of justification are to be excluded in favour of the methods of science, which are considered to be the best means that we have for answering traditional philosophical questions.

Thus, naturalism emerges as a type of foundationalism; one that replaces the metaphysical foundations of old-fashioned forms of rationalism, such as the foundations defended by Descartes, with a foundation furnished by our "inherited world theory" (p.72). *Our inherited world theory comprises the disciplines that one might expect: physics, botany, biology, astronomy, physiology, psychology, linguistics, chemistry,* geology, sociology, etc.

An objection arises straightaway. Is naturalism itself a scientific thesis? It is hard to see how it could be. But then, does it not cut off the branch on which it stands? If it is not on the basis of scientific methods that naturalistic constraint 1 is justified, then is it not the case that naturalistic constraint 1is unjustified? The naturalist may attempt to respond to this objection by distinguishing between the notion of "a thesis" and that of "an approach". Naturalism, it may be said, is an approach that works "within" science to understand, clarify, and improve it. Whenever possible, the naturalistic approach treats philosophical questions in the same way as a scientist would treat a scientific question.

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[3] Quine writes: "I am of that large minority or small majority who repudiate the Cartesian dream of a foundation for scientific certainty firmer than scientific method itself." (1990: p.19).

And finally, it would be an approach that rejects any other approach that calls for "extra-scientific methods."[4]

This response does not alleviate the problem; it merely depicts naturalism as a arbitrarily adopted metaphysical framework that is biased toward "scientific methods". The question remains, How is the approach justified? If it is not justified by scientific methods, then, assuming that it is grounded at all, whatever methods in virtue of which it is justified would have to be extra-scientific. And if they are extra-scientific, then it would seem that they would have to be rejected on the authority of naturalistic constraint 1. Thus, drawing the distinction between a thesis and an approach does not suffice to deflect the worry that naturalism is self-refuting. We may call this the *problem of self-refutation* or more briefly, *self-refutation.*

Naturalism distinguishes itself from other philosophical approaches in terms of the distinction between scientific and extra-scientific methods. It is therefore important for the naturalist to say precisely what is meant by a scientific method. Otherwise one might claim that Leibniz used scientific methods to criticize the views of Galileo and Newton concerning the nature of things such as space, time, motion, and force.[5] The Leibnizian tradition in metaphysics, after all, did closely model its own methods and practices after the science of mathematics.

According to Maddy it is somehow unscientific to be asked to give a precise account of the distinction between science and extra-science; the only answer that is

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[4] Maddy writes: "Faced with first philosophical demands—that is, with questions and solutions that require extra-scientific methods—[the naturalist] will respond with befuddlement, she knows no such methods, but she has no a priori argument that there are such methods, but she has no a priori argument that there are none; until such methods are explained and justified, she will simply set aside the challenges of first philosophy and get on with her naturalistic business." (Maddy 2000: p.108).

[5] For a discussion of the debate between Newton and Leibniz cf. (DiSalle 2006b).

required is something extremely vague, such as, that science is what scientists do, and that we find out what they do by doing the sociology of science. This response begs the question of whether the scientific methods of justification favoured by the naturalist are exclusively empirical.

The naturalist's quandary can be put in the form of a dilemma: Either non-empirical methods of justification are to be excluded as extra-scientific, or some of them are not. If some of them are not to be excluded, then the naturalist falls prey to the first dogma of naturalism problem. If non-empirical methods are to be excluded, the naturalist falls prey to the it cuts of the branch on which it stands problem.

It is plausible to suppose that the naturalist would respond to this quandary by surveying her reactions to particular cases, and adding at least one further constraint to the naturalistic approach to metaphysical and epistemological questions. On the basis of one reaction that is to be found in the text, an additional constraint might be this:

> **Naturalist constraint 2**
> Non-empirical methods of justification are scientific only if they do not posit non-natural mental powers such as rational insight.[6]

This constraint cannot be sufficient for a philosophical approach to be naturalistic, but it does put the approach on the right side of one of the four guiding constraints of this thesis. In conclusion, it is not clear what constitutes naturalism. We are told that for "a naturalist, ... epistemological questions—how do these humans, as described by

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[6] Textual evidence in support of this constraint is provide by the following comment that Maddy makes about Reichenbach: "Finally, on the methods available, he writes, 'modern science ... has refused the authority of the philosopher who claims to know the truth from intuition, from insight into a world of ideas or into the nature of reason or the principles of being, or from whatever super-empirical source. There is no separate entrance to truth for philosophers. The path of the philosopher is indicated by that of the scientist.' (Reichenbach 1949, p. 310). This is clearly a version of proto-naturalism." (2000, p. 102ff)

physiology, psychology, biology, etc., come to know about this world, as described by physics, chemistry, botany, etc?—... are treated as broadly scientific questions" (Maddy 2002: p.62). I turn now to carefully consider a view which, from this perspective on epistemological questions, addresses the two central questions of this thesis.

2 Stage one: The interpretation of Kant

Let us now return to the questions that this chapter opened with: "What makes logical truths true?" and "How do we come to know that they are true?". The strategy adopted, in order to provide answers to these two questions, has two stages. The first stage is to present an interpretation of Kant's position on logic. The second stage involves "naturalizing" this interpretation. In this section I discuss the first stage of the strategy.

We need to begin with a brief sketch of how the naturalist views Kant's transcendental philosophy, the goal of which is to explain how a priori knowledge is possible.[7] The naturalist is particularly interested in emphasizing the distinction, made on behalf of the Kant's transcendental analysis, between empirical and transcendental levels of inquiry. This interest is partially due to the fact that the empirical level is in keeping with the naturalist's outlook.[8]

The transcendental level is characterised by two faculties of the mind: the sensibility and the understanding. The sensibility operates by applying the forms of

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[7] There is an ambiguity here that the reader needs to be alerted to. *Qua* Kantian question, the question how is a priori knowledge possible, is a question about the "conditions of possibility" of a priori knowledge. This question is not meant to be understood, in the sense of what in Chapter 1 I call "the acquisition question", as asking about how someone could come to arrive at a piece of a priori knowledge.

[8] Maddy writes: "At the empirical level, we function as ordinary natural scientists, guided by theory, observation, and experiment; such things as space, time, and ordinary physical objects related by causal interactions are objectively real features of the world." (2002, p. 66)

intuition (space and time) to raw sensations to produce intuitions. The understanding applies concepts to intuitions to produce judgements. Since a thinker cannot enjoy judgments or experiences without already having intuitions and concepts, some concepts will be prior to experience. The concepts that come before experience are the pure concepts of the understanding, or the pure categories, which, upon being schematized, can be applied to spatiotemporal intuitions. The pure unschematized categories include the concept of an individual object with properties, and the dependence of a consequent on its ground.

Kant's transcendental analysis of our capacity to judge is seen to provide us with an account of our a priori knowledge, by describing the necessary presuppositions of human knowledge. In this way it explains how we can know a priori certain facts about how thinkers like us (in the sense that they share our forms of intuition) must think if we are to think at all. What is important to the naturalist is that Kant's transcendental analysis is not an empirical analysis of actual human cognition of the sort that might be carried out by a cognitive or neuro-psychologist.

The naturalist interprets the two perspectives that the two levels of inquiry provide as introducing a range of ambiguities. For example, depending on what perspective we are considering it from, what our knowledge is knowledge of is ambiguous.[9] It is suggested that from the transcendental perspective, our a priori

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[9] Maddy writes: "[At the transcendental level], our a priori knowledge is of the world of experience, as opposed to the world as it is in itself, but this is a transcendental distinction. If we drop back to the empirical level, the world we experience simply is the world, and our a priori knowledge of space, time and causation is knowledge of that world. Kant's position combines the two perspectives. Space and time, the forms of intuition, are transcendentally ideal; that is, viewed transcendentally, they are present in the world of experience only because of how we cognize, not because of how the world is in itself. But space and time are also empirically real; that is viewed empirically, they are objective features of the world." (2002, p. 66)

knowledge is knowledge of the world of experience, and that from the empirical perspective, our a priori knowledge is knowledge of the world as it is in itself.

Here a confusion in the naturalist's interpretation emerges. The naturalist claims that at the empirical level, our a priori knowledge is knowledge of the world as it is in itself. The problem, for the naturalist, is that there is no theory of the a priori, for Kant, at the empirical level of inquiry. There is no Kantian concept of "the a priori" or a "thing as it is in itself" at the empirical level. For example, Kant's explanation of our a priori knowledge of the geometry of space proceeds by saying that Euclid's axioms are grounded by the form of human intuition. What could the counterpart explanation be at the empirical level? What is the empirical level counterpart to "the form of intuition"? There is none. So the naturalist's claim, that we can drop back to the empirical level and have a priori knowledge of the world as it is in itself, seems to be to be plain wrong. Perhaps what the naturalist is trying to say is that synthetic a priori knowledge is objective.

But let us press on to see how logic is represented as fitting into the Kantian framework of the sensibility and the understanding. Our knowledge is mediated by those concepts that we must use in order to make judgments. Among the concepts that mediate our knowledge are the logical forms which are presented in the Table of Judgments(Kant 1998: A67-76), and the pure categories which are presented in the Table of categories(Kant 1998: A81). The two tables are connected by Kant's so-called "Metaphysical deduction". The pure category of object-with-properties corresponds to the logical form subject/predicate, and the logical form if/then corresponds to the pure category ground/consequent. A thinker must judge according to these logical forms and pure concepts. These judgements are themselves interpreted to be logical rules, of which valid inferences are applications (Maddy 2002: p.67).

2.1 on what makes logical truths true

According to the naturalistic interpretation that we are considering, logical truth is seen as being grounded in the pure understanding similarly to the way that geometry is thought of as being grounded in the forms of intuition.[10] Kant's view of logic is that it "abstracts … from all content of cognition, i.e. from any relation of it to the object, and considers only the logical from in the relation of cognitions to one another, i.e., the form of thinking in general" (Kant 1998: A55).[11] Logical truths are true in virtue of the logical forms and pure categories that Kant posits,[12] a claim the naturalist interprets as being ambiguous between the transcendental level and the empirical level: transcendentally, logical truths are true in virtue of the nature of the discursive understanding, and they are true of the world of experience; empirically, logical truths are true of the world.[13] It is held that, at the transcendental level, what makes logical truths true are the logical forms and pure concepts of the understanding; and at the empirical level, these forms and concepts are interpreted to be features of how the world is in itself. Hence, the naturalist's answer comes down to this, that certain formal features of the world as it is in itself are what make logical truths true.

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[10] Maddy writes: "This means that logical truth is grounded in the pure understanding just as geometry is grounded in the forms of intuition and the law of causality is grounded in the forms of intuition and (the schematized concepts of) pure understanding. (Maddy 2002: p.67).

[11] For discussions of Kant's view of logic as contentless cf. (Linnebo 2003; MacFarlane 2002).

[12] Maddy writes: Logical truths are "true, we're assuming, by virtue of the logical forms of judgment it involves" (Maddy forthcoming: III.2-21).

[13] Maddy writes: "Speaking transcendentally, … [logic] is true of the world of experience because of the structure of the discursive understanding, not because of the features of the world as it is in itself. … Empirically, though, logical truth is as robustly objective as spatiotemporality and causality; logic is true of the world." (2002, p. 67).

2.2 on how we know logical truths

How we come to have knowledge of logical truths on the naturalist's interpretation of Kant's transcendental analysis is ambiguous. On the one hand it is held that we have logical knowledge because we cannot help but believe logical truths. It is not explained how not being able not to believe something entails knowledge. On the other hand, we come to know that logical truths are true by coming to a reflective understanding of transcendental analysis,[14] a claim which seems to me to be straightforward and correct. The idea that we know logical truths by coming to acquire a priori a reflective appreciation of their justification, is similar to the form of answer that I shall give to the "acquisition question" in my own proposal; my answer however has nothing to do with the claim that we cannot but believe logical truths.

The naturalist's answer elides the distinction between psychological necessity and transcendental necessity. Kant has been interpreted as having established that logical truths are psychologically necessary: we are told that, according to Kant, "we can't help believing them". But the naturalist also appears to rely on Kant's having established that the logical forms of judgment and the pure categories are transcendentally necessary. If we rewrite the answer with the distinction being made explicit, it sounds implausible: Reflecting on our transcendental analysis we see that logical truths are determined by the forms and categories which are transcendentally necessary, and therefore we cannot help believing them.

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[14] Maddy writes: "According to Kant, we can't help believing [logical truths], as they are determined by the most general forms of our capacity to judge: the logical forms of judgment and the pure (unschematized) categories. After our transcendental analysis, we see that these forms and pure concepts are necessary to us as discursive intellects, and thus, that the world of our experience will necessarily conform to them. Thus Kant's combined transcendental/empirical analysis secures our a priori knowledge of these facts about the world." (2002, p. 67).

3 Stage two: Naturalizing Kant

Recall that the naturalist's goal is to say what makes simple logical truths true, and to explain how we can come to know them to be true, in a way that does not rely psychologism, inductivism, or conventionalism.

The first step in the process of naturalization is to modify the Kantian view of logic so that it can accommodate Fregean quantificational logic. This is accomplished by replacing the subject/predicate form from the Table of Judgments with the Fregean notion of argument and function. This replacement triggers a replacement of the category of object-with-properties with that of objects-in-relation, which would in turn become schematized as 'spatiotemporal objects in relations'.

The second step is to somehow amalgamate Kant's two levels of inquiry into one "scientific inquiry". The naturalist canvases three ways in which this might be accomplished: The first way interprets the analysis of the transcendental level as being straightforwardly scientific; i.e., as being an ordinary psychological theory of cognition. The central claim of such a theory is that any thinker whose knowledge is mediated by concepts must think using the logical forms and the pure categories, and in this way is "bound by the laws of logic." This option is found to be unappealing because it raises the problem of needing to explain how from the fact that a thinker judges using concepts it is supposed to follow that a thinker's judgment forming activity is governed by the principles that are mentioned in the Table of Judgments and the Table of Categories. Furthermore a psychological theory of cognition of this sort would not yield satisfying answers to the two main questions that we are interested in: logic would be grounded in the structure of human cognition. Such an answer would go against the naturalist's self-professed goal of providing an account that is not psychologistic.

The second way of naturalizing the Kantian position is to adopt Kant's empirical level all by itself. If we did this, we would have a theory that described logical truths as empirical generalizations.[15] On this Millian type of view, the only way that we could come to know that logical truths are true would be by induction; and this contravenes the naturalist's self-professed goal of providing an account that is not inductivist.

The third and final way to naturalize the Kantian position is to collapse Kant's two levels into one, keeping what is considered naturalistically acceptable and rejecting what is not. The result is a theory that makes the following three claims:[16]

> **Psychological Necessity**
> Psychologically, humans are so constructed that they conceptualize the world using the updated Kantian forms of judgment and categories, and for this reason, their thinking is bound by the laws of logic (perhaps only in the sense that they cannot help but believing these laws).

The naturalist believes that Psychological Necessity salvages what is deemed useful from the psychological theory of cognition that would be the result of a naturalization of Kant's transcendental analysis.

Psychological Necessity merely restates the central empirical claim of the first way of naturalizing Kant's position. If the naturalist tries to answer either of the two main questions of this chapter on the basis of this principle, then the answer that is being given is psychologistic. If the principle is put to this use the naturalist cannot avoid the charge

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[15] Maddy writes: "The second approach would be to settle for the empirical level unadorned: logic is true of the world; it is self-evident and universal." (2002, p. 69)
[16] Cf. (2002, p. 69)

of psychologism. If on the other hand the naturalist tries to use Psychological Necessity to furnish a reason for claiming that humans must think in a certain way, then it would merely be the case that a bold and implausible psychological generalization is being made, one which stands in need of significant experimental support.

The second principle is:

> **Structural Correspondence**
> Objectively, the world has very general structural features that in fact correspond to the logical forms and (unschematized) categories—that is, the world consists of objects in relations, with ground/consequent dependencies (not necessarily causal) between various of its aspects—and for this reason, the laws of logic are truths about the world. Logic is true of the world independently of the world's spatiotemporal aspects.

Structural Correspondence restates the main claim of the second option described above. It treats logical laws as empirical generalizations, and thereby provides an answer to the question "What makes logical truths true?". They are true because the world is a certain way. But by making logical truths empirical generalisations, the naturalist fails to extend to logic a sufficient degree of generality. By making logical truths dependent on the structural features of the actual world, Structural Correspondence seems to make logical truths contingent: if the actual world was empty except for one thing, then on view suggested by Structural Correspondence, logical truths would be false.

The answer to the question about how we come to have knowledge of logical laws is suggested in the third claim, which is what is intended to make this third way of naturalizing the Kantian view more than just the conjunction of the first two claims. The third claim accounts for the additional content that the third way contributes. It is:

> **Empirical Veridicality**
> Humans believe the laws of logic because they are dictated by their fundamental conceptual machinery (Psychological Necessity), but they come to know that those laws are true by coming to know the veridical conceptualizations on which they are based by empirical investigation.

Empirical Veridicality embodies an account of logical knowledge that clearly does not characterise logical knowledge as a priori. This view of logic requires that, in order to acquire logical knowledge, one discover which conceptualizations are psychologically necessary, and what these conceptualizations map onto. By induction, every psychologically necessary generalization maps onto a structural feature of the world.

This concludes the second step of the naturalist's process of naturalizing Kant's position on logic. We have seen that, by including Psychological Necessity as part of the account, the naturalist does not appear to have succeeded in providing an account that is independent of psychologistic elements. By making logical laws empirical generalizations, it fails to make logic sufficiently general; and finally, the account requires that in order to acquire logical knowledge one must engage in an extensive empirical investigation that would seem to require reasoning by induction from particular contingent facts to the generalizations of interest.

The Kantian position on logic has now been naturalized, and the naturalistic view of logic is now out in the open. But there is an issue that remains to be addressed: How are fundamental modes of thought and certain very general structural features of the world related to the laws of modern logic? Addressing this issue constitutes the third stage of the naturalistic view of logic.

4 Stage three: From fundamental modes of conceptualization and structural features of the world to the laws of logic

How do the naturalized Kantian categories ground the laws of logic as correct norms of thought? And how do the very general structural features of the world that correspond to the categories make the laws of logic true?

In order to begin to answer the first question, we need to say in more detail what the naturalist takes to be the logical content of the categories of objects-in-relation and ground/consequent-dependence.

Let us start with the category of objects-in-relation. This category covers a thinker's thinking about individual objects that possess properties. The structural feature of the world that corresponds to this category is the notion of an object being a member of a class of similar things. A thinker can have thoughts, in terms of the category of objects-in-relation, about aspects of the world that have the structural feature of being a member of a class of similar things. Such thoughts are represented by atomic open sentences such as '*Pa*'.

Next, among the pairs, triples, and so on, that there are, there are those which are tuples of objects grouped together by their relational similarities. In this case, the mode of conceptualization is objects-in-relation, and we represent thinking in terms of it through the use of polyadic open sentences such as '*Rab*'.

The naturalist then supposes that a thinker sees the world in terms of the category of objects-in-relation only if that thinker also understands that some objects might enjoy more than one property, or might stand in more than one relation to more than one other object, thereby possessing a rudimentary concept of conjunction. Such a thinker will be able to see the world in terms of objects that might enjoy one or another of several properties, or that stand in relation to one or another of several objects, and thus will be able to use a minimal form of the concept of disjunction. Something similar to negation corresponds to thinking about an object lacking a property, or failing to stand in a relation to something else. And a thinker who can run down through a series of objects, identifying those that possess a certain property, can be said to be using a primitive form of quantification. Finally, thinking in terms of the ground/consequent-dependence corresponds to the concept of the conditional.[17]

We now have a better idea of what concepts are supposed to fall under the two most salient Kantian categories. But the step from thinking of the world in terms of objects-in-relation to having a whole host of rudimentary logical concepts is abrupt and unexplained. Perhaps at some level of evolutionary or developmental sophistication a creature can think in terms of the category of objects-in-relation without being able to grasp the relationships required to possess rudimentary concepts corresponding to quantification or negation. Any lizard or insect that can hunt, one might suppose, can make sense of what is stimulating its sense organs according to the category of objects-in-relation, without being able to understand that some objects might stand in more than one relation to more than one other object. So, whatever relation it is that the naturalist wishes to establish between fundamental modes of conceptualization and logical laws, it is at least unclear that thinking in terms of fundamental modes of conceptualization stands in the relation of being sufficient for possessing rudimentary counterparts to logical concepts.

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[17] Cf. (Maddy 2002: p.70).

But let us suppose that we have these rudimentary logical concepts. The next question is, What logical principles do they satisfy? The naturalist's scientific way of thinking about philosophical questions leads her to conclude that our most fundamental modes of conceptualization do not commit us to the principle of bivalence,[18] but rather lead us to interpret conjunction and disjunction in terms of the (strong) Kleene 3-valued logic[19]—and likewise for existential and universal quantification. Furthermore, a consideration of the data indicates that our ordinary language uses of 'not' indicate that we do not all share the same fundamental conceptual machinery when it comes to negation.

	(a)	(b)	(c)
A	$\neg A$	$\sim A$	$-A$
1	0	0	0
i	1	i	0
0	1	1	1

A thinker may be inclined to employ any of the three concepts, (a), (b), or (c), governed by the truth tables above. While our fundamental modes of thought apparently dictate that the negation of a truth is false, and that the negation of a falsehood is true, they do not

[18] Maddy writes: "We find it quite easy to think, for example, that it's neither true nor false that a given person is bald." (2002, p. 71)

[19] Maddy writes: "Let me continue my conjecturing about our updated and naturalized category of objects-in-relations so far as to suggest that we understand conjunctions of such sentences to be true if both conjuncts are true, false if one or the other is false, and to lack truth value otherwise, and disjunctions to be true if one of the other disjunct is true, false if both are false, and to lack truth value otherwise." (p. 71)

dictate how our thinking should proceed when we negate a proposition that seems to lack a truth value.

When it comes to 'if, then', it seems that we can't construct a complete truth table for the conditional that is based on the category of ground/consequent-dependence. The best that we can do is to say that if the antecedent of a conditional is true and its consequent is false, then the conditional sentence as a whole is false. But we will be able to preserve *modus ponens*.

Now that we have a sketch of some of the primitive logical principles that are satisfied by the rudimentary logical concepts, which are apparently part of our most fundamental modes of conceptualization, the next question is, which familiar inference rules are validated by these concepts? A few pairs of introduction- and elimination-rules will be validated. The law of the excluded middle, will not be valid. There will be no logical truths since there is no formula that cannot be assigned the value *i*. But some equivalences, such as the distributive law,[20] will hold. And detachment inferences will preserve truth.

We now have a thoroughly psychological story about how psychologically necessary modes of conceptualization are related; the story is based on speculation about the inferential behaviour of ordinary thinkers to a small stock of logical laws. This small stock of laws could be said to be psychologistically justified in terms these necessary modes of conceptualization: the laws are validated by concepts that are dictated by our

[20] Maddy writes: "we get some logical equivalences, in the sense that some pairs of statements will have the same truth-value in every situation; for example, the distributive pair—'It's blue and it's either round or square' and 'Either it's blue and round or it's blue and square'—are equivalent in this sense." (p. 72)

most rudimentary conceptual machinery. The problem is that this is just a view of logical laws as laws of thought.

This psychologistic story establishes a relationship between our fundamental modes of conceptualization, and a small stock of logical laws. The gap between the small stock of non-classical rules that are grounded psychologically and the rest of modern logic is bridged by the naturalist by adopting a series of three "idealizations". The first idealization to be adopted is the principle of bivalence.[21] By adopting this idealization the naturalist is able to claim that "the standard truth-functional negation produces the full store of propositional tautologies involving conjunction, disjunction, and negation"(Maddy 2002: p.73).

If the principle of bivalence is strictly speaking false, then what is the status of the logical "truths" that are "recovered" on the basis of it? It seems that we would have to say that they are idealizations too. But then it looks like we are in the awkward position of having to say that the propositional tautologies of classical logic are false, even though there may be something about the world that they reflect and that supports their occasional effectiveness. The naturalistic view of the logical truths of classical logic appears to be that they are inspired by the facts, but are actually false; moreover they are known to be false. Hence there is no logical knowledge—basic or derived—to account for!

The second idealization to be adopted in order to fill in the gap between rudimentary logic and full classical logic is the classical truth-table for the conditional.

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[21] Maddy writes: "Here bivalence in our classical logic is regarded as an idealization of sorts, or a restriction of our attention, for logical purposes, to cases in which it holds, rather than a universal claim that it holds in all cases." (p. 73)

This enables the recovery of the remainder of the propositional statements that are classically valid. And the third idealization is that the domain of quantification is non-empty and well-behaved. This makes it possible to recover first-order quantification. But here too we have explained only idealizations, not logical truths.

Turning finally to the second question posed at the beginning of this section, "How do the very general structural features of the world that correspond to the categories make the laws of logic true?". The naturalist's answer is that they do not. The very general structural features that correspond to the categories are that the world consists of objects in relation, and that there are ground/consequent dependencies between various aspects of the world. The world provides counter-examples to each of these structural features.

With regard to the category of objects in relations, the naturalist notes that the world appears to be made up objects that bear various properties and stand in various relations until we reach the realm of quantum mechanics. At this point, "it becomes difficult to regard electrons, for example, as objects with properties in the familiar categorical sense" (Maddy 2002: p.74). With regard to the category of ground/consequent dependencies, again, in the realm of quantum mechanics, the ordinary notions of causation seem no longer to apply since "quantum mechanics seems incorrigibly non-local" (Maddy 2002: p.89).

Although, in a wide range of cases the world has the structure that is suggested by the Kantian categories, there are exceptions at the level of quantum mechanics. The naturalist concludes that logical laws cannot be considered to be universal empirical generalizations.

5 Naturalism revised

The naturalist's proposal that was presented as naturalizing the Kantian position on logic consisted of three claims: Psychological Necessity, Structural Correspondence, and Empirical Veridicality. This proposal led to two questions. One question about the relationship between fundamental modes of conceptualization and logical laws, and another question about the relationship between very general structural features of the world, and logical laws. In answering the first question, it was found that only a small fragment of classical logic is related in the desired way to fundamental modes of conceptualization. In answering the second question, it was found that logical laws cannot be considered as universal empirical generalizations because the world does not always exhibit the very general structural features that associated with the Kantian categories. Thus the three-principle theory that was initially presented by the naturalist would have to be modified as follows:

> **Psychological Necessity***
> Humans are so constructed that they conceptualize using the Kantian forms and categories, and for this reason, their thinking is bound by a rudimentary logic, but the complete laws of modern logic result only after some significant idealizations and additional assumptions.
>
> **Structural Correspondence***
> To a large extent, the world has general structural features corresponding to the forms and categories, but there are exceptions, in which case, even the rudimentary logic loses its foundation.
>
> **Empirical Veridicality***
> Humans believe the rudimentary parts of logic because they are dictated by their fundamental conceptual machinery, but they come to know those laws only to the extent that those fundamental conceptualizations can be shown to be veridical.

From the point of view of this modified version of the naturalist's original theory, the necessity, apriority, analyticity, and normativity of logic can be reconsidered. Metaphysically, on the naturalistic view, logical truth is contingent. The logical truths from the rudimentary part of logic are contingent on it being possible to describe the world as consisting of stable objects which fall into definable groupings and relations, and on it being possible that some of these situations so described have systematic interdependencies between them. Logical truth is however not contingent on the spatiotemporal structure of the world, and it is independent of the causal nature of the interdependencies of spatiotemporal objects. This outcome is perfectly acceptable to the naturalist: where our fundamental categories coöperate with the world, logic applies, where they do not, logic no longer applies. The logical truths that make up the gap between the rudimentary logic and classical logic are also contingent. The reliability of classical logic depends on whether the idealizations distort the phenomena to the point that their idealized description is no longer useful for the purposes of science.[22]

It is hard to see, however, how this picture of the metaphysics of logic could help decide whether to adopt an intuitionist or classical logic as the underlying logic of a mathematical theory. The possibility of being able to describe the world as consisting of stable objects that fall into definable groupings does not seem to be relevant to making such a decision. Nor does the issue of whether or not our categories cooperate with the world enter into it. Epistemologically, the naturalistic view of our rudimentary logical knowledge seems to be that it is a posteriori. The account of the relationship between our

[22] Maddy writes: "The reliability of logic depends on whether on not our idealizations and assumptions are benign in the particular context of their application, that is, on whether or not they distort the underlying phenomena to the point of leading us astray." (p. 77)

most fundamental modes of conceptualisation and the logical laws that are psychologically connected to them seems to be meant as providing nothing more than an account of why ordinary thinkers virtually cannot help but believe that certain logical laws hold. Such laws are believed because they appear obvious; and they appear obvious because they reflect our fundamental modes of conceptualisation. This account is not taken to show that such beliefs count as knowledge.[23]

This account does not explain why most ordinary thinkers believe those logical truths that are not rudimentary but are thought to be idealized on the naturalist's account. (Indeed, the laws that turn out to be idealizations on the naturalist's theory tend to be more obvious than the psychologically necessary that involving the Kleene connectives.) Even if classical laws are not more obvious than the suggested psychologically necessary laws, the naturalist's account does not explain why ordinary thinkers find those logical truths that are based on idealizations to be equally as obvious as those that are based on our fundamental modes of conceptualisation. The naturalist's account introduces an unnatural distinction among at least equally obvious logical laws.

Psychologically necessary logical beliefs become knowledge, according to the naturalist when, by empirical investigation, we discover their veridicality. And as for beliefs about idealized logical truths, we know them when we have verified that they are "benign".[24]

But what does 'verification' mean in this context? A verification of veridicality is not an enumerative induction, not a holistic inference to items at the center of a web of

[23] Maddy writes: "Such belief doesn't count as knowledge until we verify that, in a given context, these modes of conceptualization are veridical and the various idealizations and assumptions are benign, so our knowledge is not a priori." (2002, p. 77)
[24] See the above footnote.

belief, and it does not apply equally in all contexts. A falsification of veridicality is not a recalcitrant observation or failed experiment. To falsify a logical law that issues from a fundamental category would require that the underlying mode of conceptualisation be replaced in one's thinking.[25] We are told what the naturalist takes it to mean to falsify a logical law,[26] but there is no clear example of what a verification of a logical truth would be. Hence, on the naturalist's view, we don't really know what it takes to be said to know a logical truth in a given context. It is difficult to see how we are supposed to verify that we are really thinking about objects in relations, when we can't help but think in terms of objects in relations.

Semantically, the naturalist position is that logic is not analytic in the sense of being true by virtue of purely linguistic characteristics, although it might be analytic in the Kantian sense of not being synthetic. For the naturalist, a logical truth has much the same semantic status as a fundamental physical truth.[27]

[25] Regarding the example of the law of non-contradiction, Maddy writes: "The conviction that a statement can't be both true and false is part of the underlying conception of objects standing in relations, and thus revising it would also involve revision part of our fundamental categorical underpinning." (2002, p. 90)
[26] Maddy writes: "My naturalist's justification of our logical beliefs begins from the scientific observation that the world consists (largely) of objects in relations with ground/consequent dependencies, and proceeds through a story of how we humans have come to see it in this way." (p. 77-8) But her naturalist's justification couldn't end there or else it would be psychologistic. Then, in the second stage, the naturalist's "naturalist traces the difficulty of revising our rudimentary logic to its grounding in our most basic modes of conceptualizing the world." (p. 78)
[27] Maddy writes: "Except for its independence of the spatiotemporal features of the world, logical truth differs from fundamental physical truths only in matters of degree, not kind." (p. 78)

Chapter 3: KANT AND GEOMETRY

> "Euclid's axiomatic construction ... had reduced the science [of geometry] to a system of axioms. But now arose the epistemological question how to justify the truth of those first assumptions."
>
> —Hans Reichenbach, *The Philosophy of Space and Time*

My goal in the remainder of this thesis is to lay the groundwork for, and to present a sketch of, my own positive proposal with regard to the justification question and the acquisition question which were introduced in the Introduction. My strategy will be to develop my own answers to these questions by radically readapting insights and methods from earlier work in the foundations of physical geometry and the foundations of arithmetic. In the present chapter I discuss Kant's philosophy of geometry, and the philosophy of geometry of Herman von Helmholtz.

In this chapter I do three main things. First, I discuss how Kant broke with the epistemological tradition his time, and how this break yielded a "transcendental method" for achieving knowledge a priori. Kant's transcendental method works by looking at the conditions of the possibility of knowledge that we already have. It is this method that I readapt in Chapter 5 to provide an explanation of the possibility of basic logical knowledge. The second thing that I do is to examine the first major application that Kant makes of his transcendental method; that is, I examine his theory of geometry which provides a justification for Euclid's postulates on the basis of "the successive synthesis of the productive imagination." Thirdly, I examine Helmholtz's discovery of a more general transcendental principle which permits arbitrary continuous motions of rigid bodies, and from which can be derived the postulates of geometries of constant curvature.

1 The "Copernican revolution" and the transcendental method

Kant's *Critique of Pure Reason* is a reaction to the rationalist and empiricist traditions in metaphysics and epistemology that came before it. Traditional pre-Kantian rationalism says roughly that the pronouncements of science, for example, can be shown to follow from a small group of basic principles, knowledge of which can be gained directly through a faculty of rational intuition that is analogous to sense experience. Traditional pre-Kantian empiricism, on the other hand, says roughly that scientific propositions rest on basic sentences that report or describe certain sorts of directly evident sense experience. From this second point of view, it is the self-supporting nature of experience that serves as the ground for basic knowledge.

These pre-Kantian forms of rationalism and empiricism are designed to provide an answer to what might be called,

> **The pre-Kantian question**
> Which beliefs are justified and which are not?

Although the character of the particular foundation is different in the two cases, the goal and the method behind both positions is the same. Namely, to justify beliefs about things "in themselves" by providing them with a firm epistemological foundation. The goal is to give an answer to the pre-Kantian question, and the method is to provide certain beliefs with a foundation that either involves the senses or rational insight.

Kant's rejection of this goal and method is an fundamental component of his Copernican revolution in philosophy.[1] Kant replaces the idea that metaphysics can provide us with knowledge of things in themselves with the idea that the conditions of the

[1] For further commentary see (Brittan 1978).

possibility of knowledge that we have are features of our own cognition. If metaphysics has anything to say, according to Kant, it is about these conditions that constrain our experience and our ability to imagine rather than about the world of things in themselves.

If Kant's ultimate goal is not to arrive at an answer to the pre-Kantian question, then what role does it play in his thinking? The answer is that an answer to it, rather than serving as the goal, serves as a starting point. Kant's transcendental approach is based on the idea that we can point to certain uncontroversial bodies of judgements, our confidence in which is independent of any question we might ask about their foundation. This is Kant's Copernican revolution, it stands the traditional state of affairs on its head. From the assumption that the truths of certain sciences are known, Kant goes on to ask what is presupposed by the possibility of that knowledge. In particular he asks the following two questions: "How is pure mathematical science possible?" and "How is pure natural science possible?" (Kant 1998: B21). Therefore, Kant's question is not "Is knowledge of *X* possible?", but rather "*How* is knowledge of *X* possible?". The pre-Kantian question is replaced by,

The Kantian question
What is presupposed by the possibility of knowledge of *X*?

Where the variable *X* is to be replaced by the name for a particular domain of knowledge such as physical geometry or arithmetic.

By seeking an answer to the Kantian question Kant's approach yields an interesting method for gaining knowledge a priori. This "transcendental method" is used by Kant to provide a solution to the problem of explaining the possibility of synthetic a priori knowledge. The method works by uncovering the necessary presuppositions underlying the possibility of one or another kind of knowledge-seeking activity. Knowledge of a proposition expressing a necessary condition for the possibility of a practice of knowledge acquisition (or judgment formation) which is justified on the basis of the transcendental method is called "transcendental"[2] knowledge. To get a clearer picture of how the transcendental method works, I turn to consider the example provided by the first major use that Kant makes of it.

2 The transcendental method in Kant's theory of geometry

In the present section I address Kant's treatment of the general epistemological problem of space and time.[3] The general epistemological problem of space and time (that is addressed by Kant) is that of explaining whether Newtonian physics does more than merely scratch the sensible surface of the world.[4] How do we know that the fundamental concepts of Newtonian physics do not merely refer to phenomena with no basis in intelligible things and their intelligible causal relations?

The way in which this epistemological problem is broached depends on how the relationship between scientific and metaphysical principles is understood. There are three views. On what may be called the "Cartesian view," metaphysics and physics are competing sources of claims about the world. This view results in irresolvable fruitless disputes about which set of principles has a greater claim to the truth, and is exemplified

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[2] Kant writes: "I call transcendental all knowledge which is concerned, not so much with objects, as with our mode of knowledge of objects in general, in so far as this [knowledge] is to be possible a priori" (Kant 1998: B25).

[3] For a recent discussion of this problem see (DiSalle 2006b: Ch. 03).

[4] Kant writes: "What, then, are space and time? Are they real existences? Are they only determinations or relations of things, yet such as would belong to things even if they were not intuited? Or are space and time such that they belong only to the form of intuition, and therefore to the subjective constitution of our mind, apart from which they could not be ascribed to anything whatsoever?" (Kant 1998: B38).

by the dispute between Newton and Leibniz. The second view may be called the "foundationalist view," which holds that physics requires a foundation in metaphysics. This view is exemplified by Kant's own position during his "dogmatic" phase, and it holds that scientific principles are justified if they can be shown to be derivable from deeper, supposedly intrinsically intelligible, metaphysical principles. The third view (which Kant firmly held by the time of the *Critique*) may be called the "dialectical view;" it holds of physics and metaphysics that they stand in a relationship of "dialectical engagement". On this third view, physical theories are able to inform metaphysical issues through dialectical arguments. A dialectical argument applies the transcendental method of starting with a body of knowledge that is taken to be uncontroversial and revealing the assumptions implicit in it and the transcendental conditions that underlie it. The dialectical view of the relationship between physics and metaphysics is an example of Kant's transcendental method in action.

The tradition espousing the foundationalist view of the relationship between physics and metaphysics proposes to respond to the challenge posed by the general epistemological problem of space and time by putting forward a "metaphysics of nature," or a foundation of supposedly certain metaphysical principles. This foundation is intended to serve as the ground for rational belief concerning the principles of physics. The decisive critique of this view however is that it cannot account for the objectivity of physics. A metaphysical foundation can be shown to be rational, but rationality is insufficient for objectivity.

To say that a system of principles is rational is to say little more than that it is clearly expressed and systematic. The rationality of Descartes' mechanical philosophy, for example, comes down to the fact that its picture of the universe has the feature that it can reduce every natural process to characteristic kinds of entity and interaction which can in turn be understood in mechanical terms. Descartes' system of principles is completely open to dispute from the next system of freely adopted metaphysical principles that comes along. And so, while it is eminently rational, Descartes' metaphysics is also entirely subjective.

Kant's dialectical perspective on the other hand allows for a dialectical argument which recognises Newton's principles as conditions of the possibility of comprehending things in space and time under a system of causal interactions.[5]

2.1 Constructive definitions and the axioms of geometry

To ensure that a fundamental principle is not arbitrary, the concepts that are the constituents of the proposition that it expresses must not be mere inventions of the mind. What Kant had already realised by the time of (1992: esp. pp. 248-56) is that mathematics and physics have a non-arbitrary way of formulating fundamental concepts; in the natural sciences fundamental concepts are "constructed" under particular objective constraints. Kant went on to argue that these constraints are imposed by the nature of sensible intuition, and that the transcendental principles that express the constraints are justified because they express necessary conditions for the possibility of thinking about the physical world.

[5] For example, DiSalle writes: "Motion belongs to the group of fundamental concepts for which Leibniz had thought to provide a purely metaphysical understanding, but whose definition is possible only within the framework of space and time. ... As Kant's analysis shows, force in Leibniz's sense is something that cannot be comprehended, or even represented, independently of geometry. ...[Kant's analysis] is the recognition of physics in its transcendental role, as the source of constructive definitions for metaphysical concepts." (DiSalle 2006b: 70-71)

When, taking geometrical reason as given, Kant inquires in (1992) into the way in which concepts are defined in the mathematical sciences he finds that mathematical definitions in geometry are *constructive*. For example, the proof procedures in Euclid's *Elements* introduce concepts by employing a process of construction with straight edge and compass to iteratively or successively generate instances of those concepts. From an initial object (e.g., a given pair of points, or a line segment) the existence of another, possibly infinite, geometrical object can be iteratively generated by a given, possibly infinitary, initial operation. Constructive definitions capture geometrical concepts in a non-arbitrary way, because the procedures of construction underlying them ensure that what is proper to the concept is what is contained in the definition. It is in this way that Kant avoids the problem of lack of objectivity for fundamental principles involving geometrical concepts.

Kant explains the possibility of the constructive procedures that are reflected in constructive definitions in terms of the activity of our own a priori imagination, whose nature is dictated by the a priori structure of pure spatial intuition.[6] By this fundamental imaginative activity a thinker might "draw" or describe a straight line in thought, and she then might rotate this straight line around a fixed point. When one engages in this fundamental transcendental activity, one adopts a particular point of view and a particular perspective on the imaginative space that is apprehended from this point of view.

The axioms of geometry express the conditions of the possibility of any constructions whatsoever. They assert the possibility of the constructions that occur within the space of the productive imagination. And they express general conditions on

[6] For a recent discussion of how the role of intuition in Kant's theory of geometry ought to be interpreted see (Friedman 2000).

constructive manipulation of spatial objects. So they are the conditions of the possibility, not only of doing geometry, but of having any objective conception of the things located around us in space. So in this sense it could be said that they are justified by the possibility of perceptual experience.

The application of the transcendental method in Kant's account of constructive definitions occurs where, by taking constructive geometrical reasoning as given, Kant is able to find its transcendental principles. In this case, the axioms of geometry that assert either the possibility of certain constructions, or the global features that certain constructions will have.

One thing that Kant was wrong about however is the indispensability of intuition for the definition of geometrical concepts. Advancements in the 19^{th} century, such as the rigorization of analysis brought about by Bernhard Bolzano, that made it possible to construct mathematical concepts, by means of mere concepts, without appeal to intuition. This does not undermine the interest in the transcendental method since it is clearly separable from intuitive construction. What is essentially valuable about the method is the idea that a concept can be defined objectively by looking into what must be the case in order for a certain kind of reasoning, or knowledge-seeking activity, involving the concept to be possible, and that the principle expressing that definition can be seen as being justified in terms of the possibility of that activity. I turn now to consider the way in which the transcendental method is applied in the work of Hermann von Helmholtz.

3 The transcendental method in Helmholtz's theory of geometry

Helmholtz used the transcendental method to uncover a more general necessary presupposition underlying the possibility of the successive synthesis of the productive imagination. Helmholtz thereby shows the postulates of geometries of constant curvature to be equally grounded on a more general principle, viz., "the principle of the free mobility of rigid bodies."[7] Implicit in the possibility of performing kinematical geometrical constructions in the productive imagination is the fact that the movements of one's own point of view can be done, undone, and combined arbitrarily.[8] In other words, our use of the concept of spatial displacement depends on the assumption that these displacements allow us to treat them as forming the "group of rigid motions". And by deriving the general form of the Pythagorean metric from the mathematical formulation of the principle of free mobility, Helmholtz showed that it expresses what is implicit in our theoretical and practical judgments about geometrical measurement.

Helmholtz himself, however, holds that the principle of free mobility expresses a fact of experience; he claims that it indicates a "fact which lies at the foundations of geometry" (cf. 1868). Whereas Poincaré and the logical empiricists (notably (Reichenbach 1957)) hold it to be a conventional stipulation. From the point of view of a kind of Kantian apriorist position, the principle of free mobility is neither a straightforwardly empirical claim, nor a conventional stipulation. It is not straightforwardly empirical because it is presupposed by all empirical claims about space;

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[7] Cf. (DiSalle 2006a).
[8] Cf. (DiSalle 2002: 179; 2006b: 77).

that is, it expresses a condition of the possibility of our experience of space.[9] So it does not seem correct to say that the principle of free mobility expresses the simple contingent empirical fact that, in the world, there are bodies that are sufficiently rigid and which can be moved in a certain way to allow us to perform measurements of distance.

On the other hand, from the point of view of the Kantian apriorist position, the principle of free mobility is not a convention. If it were a convention, it would be an analytic arbitrarily chosen definition of length. Seen as a convention, the principle suggests a certain interpretation of Helmholtz's famous thought experiment involving a mirrored sphere (cf. 1876): Reflected in a spherical mirror, rigid measuring rods appear curved, and their rectilinear motions appear distorted. But all measurements will agree. And, from the point of view of the people on the sphere, it is our motions and rods that appear distorted and curved. If, in accordance with this thought experiment, we attributed the appropriate distortions to the bodies we take to be rigid, and treat as rigid those measuring instruments that we see on the sphere, then we would be able to treat our own space as pseudo-spherical. Thus, if the principle of free mobility is thought of as a convention, the thought experiment seems to show that there are objectively equivalent different descriptions of our experience, and that the choice between them is conventional.

Seen from a non-conventionalist non-naïve-empiricist point of view, however, Helmholtz's spherical mirror thought experiment shows that, for the inhabitants of the sphere-world, the bodies, that we take to be distorted, satisfy the criteria for being rigid.

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[9] Cp. the argument of (Torretti 1977: p. 168), where he writes, "The notion of rigid body must … be regarded, if Helmholtz is right, as a concept constitutive of physical experience, that is, as a transcendental concept in the proper Kantian sense."

And thus, that these rigid bodies, that appear to us to be distorted, enable us to attribute a geometry to the space of the that world. From this point of view, the principle of free mobility is constitutive of the meaning of the terms of classical homogeneous geometries. And so it is not empirical. But it is analytic of our concept of measurement; that is, it expresses what is implicit in our theoretical and practical judgements about geometrical measurement. And so it is not conventional. It is not arrived at by stipulation, because it is discovered by asking after what is implicit in the practice of performing Euclidean constructions in the productive imagination. And it acquires empirical content by virtue of the fact that it captures what is implicit in our correct practice of making empirical judgments about spatial measurement.

Chapter 4:
FREGE ON ARITHMETIC

"His principal object in *Grundlagen* was to determine the *justification* of the propositions of number theory, and of others involving the natural numbers."
—Michael Dummett, *Frege: Philosophy of Mathematics*

If basic logical knowledge is not explained in terms of a non-natural faculty of rational intuition, not explained on the basis of psychological dispositions, and not explained on the basis of arbitrary stipulations, then what explains how it comes about? At least one kind of logicist account, in the context of the foundations or arithmetic, is able to provide an explanation of arithmetical knowledge that steers clear of the three abovementioned positions. It does so by appeal to the notion of an analytic definition and the methodology of conceptual analysis. In this chapter I reconstruct the logicist explanation of arithmetical knowledge.

I start out by presenting a theory of analytic definitions that bypasses the paradox of analysis, and that allows one to account for their epistemic justification. I then present an account of the epistemic justification of Frege's partial contextual definition of numerical identity, and I explain how on the basis of this definition, and what is known as Frege's theorem, a logicist is able to provide a basis for our knowledge of the basic laws of arithmetic. I conclude by suggesting how the logicist account, which relies on the methodology of conceptual analysis, might be readapted to the context of the foundations of logic.

1 Logicism, analytic definitions, and the paradox of analysis.

Frege's goal in *Die Grundlagen der Arithmetik* (*Gl*) is to show that "the laws of arithmetic are analytic judgments and consequently a priori" (Frege 1980 §87).[1] In the first sentence of his Introduction to *Grundgesetze der Arithmetik* the goal of *Gl* stated with a slightly different emphasis: "In my *Grundlagen der Arithmetik* I sought to make it probable that arithmetic is a branch of logic and that no ground of proof needs to be drawn either from experience or from intuition" (Frege 1964).

To achieve this goal, his method is to demonstrate that arithmetical truths can be proved, with recourse to definitions, entirely from unprovable general logical laws. By the phrase 'general logical laws' Frege intends general laws that are topic-neutral—laws that exclusively involve terms of universal applicability and which are not restricted to any particular domain of knowledge. By the word 'definitions' Frege has in mind definitions that are analytic as opposed to stipulative.[2] He is not interested in stipulative definitions because his aim is not to lay down meanings for newly introduced terms, nor does he wish to merely provide abbreviations for terms already understood. His intention is rather to settle the epistemic status of ordinary arithmetical laws that are already in use and which contain arithmetical concepts that we already possess; for this purpose analytic definitions are the appropriate tool. An analytic definition 'captures' the pre-theoretic sense of a term that is in use. Furthermore, an analytic definition is one that can be seen as the articulation of the outcome of a philosophical analysis of a concept.

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[1] For a discussion of how the views of Kant and Frege differ regarding the analytic-synthetic distinction and the a posteriori-a priori distinction, see (Dummett 1991c: Ch. 03).

[2] For a discussion of Frege's view of the distinction between stipulative and analytic definitions, see (Frege 1979).

A crucial issue arising in relation to Frege's method is that of articulating the conditions under which an analytic definition is correct. This issue is crucial because, without the means of determining the correctness of the definitions to which appeal is made in the course of the justificatory proofs of basic arithmetical truths, the logicist is unable to claim to have decisively demonstrated that the basic laws of arithmetic are analytic and hence a priori.[3] Put differently, the issue is crucial because without analytic definitions that can be assured of being correct, the logicist cannot be said to have explained how basic arithmetical knowledge is possible.

The main obstacle to accounting for the correctness of an analytic definition is the so-called paradox of analysis. The paradox of analysis may be expressed in the form of a dilemma as follows. Regarding a definition that is the result of an analysis, there seem to be two alternatives: Either the analysis is successful and the definition is correct, or the definition is incorrect and the analysis is unsuccessful. But if the analysis is successful (in the sense that it yields a definition that reproduces the preanalytic sense of the term defined) then the resulting definition is uninformative because its definiendum will have exactly the same sense as the definiens. And if the definition is informative (in the sense that it has the capacity to extend knowledge), then it is incorrect in the sense that the analysed sense is not the same as the sense that the term originally had. In short: If the analysis is correct, the resulting analytic definition has no point; and if it has a point, it is incorrect.

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[3] Frege writes: "If we now try to meet this demand [to prove the basic propositions of arithmetic], we very soon arrive at propositions a proof of which remains impossible so long as we do not succeed in analyzing the concepts that occur in them into simpler ones or in reducing them to what has greater generality. Number itself is what, above all, has either to be defined or to be recognized as indefinable. This is the problem to which this book is addressed. On its solution the decision on the nature of arithmetical laws depends" (Frege 1980: §4).

The characterisation of the correctness of an analytic definition that the paradox of analysis presupposes is this.

> **The provisional characterisation of correctness**
> An analytic definition is correct just in case it succeeds in exactly reproducing the sense that the defined expression bears in its ordinary use.

The paradox cannot arise unless the correctness of an analytic definition depends entirely on whether or not it succeeds in exactly reproducing the sense that the defined expression bears in its ordinary use. On this characterisation, what it means for an analytic definition to capture the pre-theoretic sense of an expression already in use is for the definition to provide a synonymous expression (possibly in the context of a sentence).

There is a worry that accompanies the provisional characterisation. If a correct analytic definition is one that exactly reproduces the sense of an expression as it is ordinarily used, then we run into a problem if the ordinary way in which the expression is used is inconsistent or otherwise incorrect. Such a circumstance would cast serious doubt on the epistemic status of the principles (the basic laws of arithmetic in Frege's case) that are shown to follow from the definitions. If the practice according to which a concept is ordinarily used is incorrect, the provisionally characterised analytic definitions based on that practice would produce justificatory derivations which would sometimes fail to correctly determine the truth or falsity of basic propositions.

It is entirely possible that a familiar concept is used in a way that turns out to be inconsistent. The obvious example is the concept *set*; understood in terms of naïve set theory, the concept of a set leads to Russell's contradiction.[4] And even if the everyday way in which a concept is used is not inconsistent, it is rare that it is grasped completely and clearly by ordinary thinkers. Rigorous definitions, especially when logical proofs are concerned, are required for philosophical purposes. If analytic definitions are completely beholden to ordinary use, then the justifications on which they are based will be faulty when the ordinary way in which the concept is understood is incomplete.

This worry about the fallibility of ordinary practice is reason to reject the provisional characterisation. Frege, who addresses the worry thirty years after writing *Gl*,[5] reacted to it by abandoning the method of conceptual analysis that leads to the paradox of analysis. His recommendation was this: when there is a discrepancy between the confused original sense of a term in use and its precise post-analytic sense, we should abandon the old term and adopt the new rigorous definition as a stipulation. In other words, if the ordinary sense of an expression is incorrect, then resort to convention and introduce a new concept.[6] By introducing a new concept, there is no longer any issue of the analyticity of a definition but only of the utility of the new concept.

We may conclude, then, that analytic definitions, understood in terms of the provisional characterisation, have no philosophically useful role to play when the defined expression is ordinarily understood in an inconsistent or confused way. In other words,

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[4] We might put forward the hypothesis that Frege understood the nature of analytic definitions in terms of the provisional characterisation. If this hypothesis were correct, it would go some way toward explaining why, by the time of 'Logik in Der Mathematik' in 1914, he came to deny that conceptual analyses are possible save in rare and unproblematic cases.

[5] Cf. (Frege 1979: 211).

[6] Note that the maneuver that Frege recommends at this point does not have anything to do with the objective content of the expression under consideration; nor is it based on the intrinsic features of his method. The maneuver relies on the capacity of the philosopher not to be confused about the correct sense of an expression. It is not a good thing for a method of justification to depend on the ability of the philosopher employing it to "get it right".

on the provisional characterisation, the resulting justification of a particular collection of fundamental principles would be incorrect if the senses borne by the expressions contained in those principles in their ordinary use were somehow incorrect; analytic definitions and hence the demonstrations of analyticity and thus apriority on which they are based would only be as justified as our pre-analytic practices. For example, if the inhabitants of the fictional world of George Orwell's *1984* were to use the method of conceptual analysis, understood in terms of the provisional characterisation (and without resorting to the conventionalist option (as Frege does) of falling back on stipulative definitions) to evaluate the epistemic status of the statement '2+2=5', they would be led to the conclusion that that statement is analytic and hence a priori. We thus ought to reject the provisional characterisation as the correct account of the correctness of analytic definitions.

2 Frege's strategy for explaining the foundations of arithmetic

So far we have rejected one possible characterisation of the correctness of analytic definitions. The reasons for rejecting the provisional characterisation are (i) that it leads to the paradox of analysis, (ii) that it causes a method of justification that appears to be faulty, and (iii) that it precludes useful and needed criticisms of incorrect conceptual practices.

It is necessary to find something with which to replace the provisional characterisation if we are to reject it without entirely abandoning the methodology of conceptual analysis. In connection with Frege's logical construction of number theory, the task of finding a replacement characterisation is made more difficult by the fact that

Frege's writings do not contain an explicit statement of what such a characterisation might be. However, in the absence of an explicit statement, we can still scrutinize the approach that he actually takes toward defining arithmetical notions in order to elicit what his view should have been. Therefore, in the present Section, I investigate Frege's strategy of definition, from the portion of *Gl* entitled "To obtain the concept of Number, we must fix the sense of a numerical identity", which extends from §62 to §69.

Frege begins §62 by asking 'How, then, are numbers given to us, if we cannot have any ideas or intuitions of them?'. This is a Kantian question. Frege is asking after an explanation of the arithmetical knowledge that it is possible for a thinker to possess; he is not asking whether, but how, such knowledge is possible. Frege uses the term 'ideas' to mean mental images and the like, and he uses the term 'intuition' in the same sense as Kant. Thus, by asking the question, he is also declaring his opposition to psychologism and to any account of the epistemic status of fundamental arithmetical propositions that grounds arithmetical knowledge on intuition.

The answer that Frege provides invokes what has come to be referred to as the "context principle." The context principle states that only in the context of a sentence does a word have meaning. By invoking the context principle Frege converts the epistemological problem of explaining how arithmetical knowledge is possible into the problem of explaining how the senses of sentences that contain terms for numbers ought to be regarded as being fixed.[7] This manoeuvre has come to be called the 'linguistic turn' in philosophy. It signals the dawn of analytical philosophy for the reason that, through it, Frege was the first philosopher to return a linguistic answer to a non-linguistic question.

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[7] Frege writes: "Since it is only in the context of a proposition that words have any meaning, our problem becomes this: To define the sense of a proposition in which a number word occurs." (1980: §62).

How is it that the senses of sentences that contain terms for numbers are to be settled, and which sentences should have their sense settled first? Since Frege takes number words to stand for objects, and given that he maintains that terms for objects must be provided with criteria of identity,[8] a fundamental class of sentences that contain terms for numbers that must have senses are those that express our recognition of a number as the same again. Since Frege takes the fundamental type of terms standing for numbers to be those that are formed by the operator 'the number of…', the identity-statement for which a truth-condition must be provided is a "recognition judgment" of the form:

(1) the number of *F*s = the number of *G*s.

The term-forming operator 'the number of…' will be referred to as the 'cardinality operator'. The explanation of the cardinality operator involves providing an informative condition for the truth of (1). The specification of the truth-conditions begins by noting that

(2) the number of *F*s = the number of *G*s iff there are just as many *F*s as *G*s.

The statement 'There are just as many *F*s as *G*s' is in turn explained by Frege in terms of the equivalence:

(3) There are just as many *F*s as *G*s iff there is a one-one function between the *F*s and the *G*s.

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[8] Frege's principle of criteria of identity states: "If the symbol *a* is to designate an object for us, we must have a criterion that will in every case decide whether *b* is the same as *a*, even if it is not always within our power to apply this criterion." (Frege 1980: §62).

Equivalence (3) can be referred to as Frege's definition of cardinal equivalence. Combining (2) and (3) yields what in the secondary literature is called the partial contextual definition of number or of the cardinality operator and more recently "Hume's principle":

(*HP*) The number of *F*s = the number of *G*s iff there is a one-one function between the *F*s and the *G*s.

Which gives Frege's informative condition for the truth of a recognition judgment. Frege deduces the essential laws of arithmetic, or what amounts to the Peano axioms from *HP*.

3 Frege's definition of cardinal equivalence, the paradox of analysis, and conceptual analysis

Let us now return to the issue, left in abeyance, of the problem highlighted by the paradox of analysis: the problem of establishing what it is in virtue of which an analytic definition counts as being correct. Frege's definition of cardinal equivalence is an analytic definition *par excellence*. The paradox of analysis arises in relation to it in the following way: An analytic definition of a relation is assumed to be *correct* only if it expresses what a thinker knows when he or she knows that the defined relation obtains. An analytic definition of a relation is informative-for-a-thinker *S* if it extends *S*'s knowledge. An analytic definition is *informative* if there exists a thinker *S* for which it is informative-for-*S*. The paradox of analysis, as it arises in relation to (3), may then be stated as follows: If (3) is informative, then it is incorrect; and if it is correct, it is uninformative. For, as the intuition goes, surely a thinker (e.g. Edmund Husserl[9] or a small child) can understand the phrase 'just as many' and, for some reason, reject its definition in terms of one-one correspondence.

There are two cases. A thinker might reject the definition because he or she is unfamiliar with the notion of a one-one mapping; we may call this *the small child case*. Or a thinker might reject the definition on the basis of the view that cardinal equivalence should be defined in terms of the cardinality operator (as opposed to the other way around); we may call this *the Husserl case*. These two cases suggest that the left- and right-hand sides of (3) do not coincide in meaning. The challenge, then, is to articulate the sense in which cardinal equivalence is indeed analysed in terms of the notion of a one-one mapping.

In response to this challenge, there are at least two options: (i) we can say that (3) brings to light what an ordinary thinker tacitly (instead of explicitly) knows when she (perhaps only partially) understands the phrase 'just as many', or (ii) we can concede that (3) merely lays down truth-conditions. In order to make option (i) work, we need to explain whether the left- and right-hand sides of (3) can be synonymous even though someone who understands the right-hand side of (3) may fail to understand the left-hand side.

Along the lines of the account of sameness of content that Frege gives in 'Logik in der Mathematik',[10] one can say that two sentences are synonymous if anyone who understands both of them immediately recognises that, in every situation, they will

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[9] For discussions of the relationship between the views of Frege and Husserl see (Dummett 1991c: Ch. 12) and (Dummett 1991b).
[10] (Frege 1979: p. 197).

coincide in truth-value. This partial characterisation leaves open the possibility of two sentences being synonymous even though a thinker fails to recognise them as synonymous. By adopting this characterisation of synonymy, it is possible to meet the challenge posed by the objector who agrees that the two sides of (3) are extensionally equivalent, but who denies that the left-hand side captures the sense of the right-hand side. In other words, it is possible to explain away the Husserl case. In this case the friend of analytic definitions can say that (3) is indeed correct since, by recognising both sides to be extensionally equivalent, the Husserlian satisfies the condition for the two sides to *be* synonymous even though the objector fails to recognise them *as* synonymous.

The above proposal, however, is powerless to meet the more difficult challenge posed by an objector who initially rejects a definition because she does not understand its definiens. We can imagine a case in which a child or an ordinary speaker seems to understand the definiendum well enough, but does not understand the definiens, and for this reason rejects the definition. The small child case tempts the philosopher to conclude that the definition is incorrect because it shows that the two sides of the definition are homonymous. If the definition is meant to reproduce exactly the sense that the defined expression bears in its ordinary use—if it is meant to articulate what an ordinary thinker knows when she understands the defined expression—then the two sides of the definition must be synonymous, and there is a problem. If instead, however, analytic definitions are governed by the following principle, the allegation of incorrectness can be deflected.

> **The tacit knowledge criterion**
> An analytic definition is correct if it reveals what a thinker must tacitly know in order to be said to understand the defined expression.

On this different criterion of correctness, the homonymy of the two sides of (3) is not a cause for its incorrectness. For this reason the tacit knowledge criterion makes it possible to bypass the paradox of analysis. From the point of view of the tacit knowledge criterion, an analytic definition does not simply attempt to record a preanalytic pattern of use, rather, it attempts to reconstruct the tacit knowledge which guides a pattern of use. If the function of an analytic definition is to reveal tacit knowledge, then there is simply no need for its two sides to share the same sense; although, to use Frege's analogy, its two sides would carve up one and the same *content* in two different ways.

On the view provided by the tacit knowledge criterion, the child's rejection of (3) is merely a manifestation of an inability to perceive the truth-functional connections between the definiendum and nearby sentences in the web of language. This inability in turn is an indication of the child's partial grasp of the sense of the definiendum. The inability indicates a partial understanding for the reason that a sentence has a meaning only in the context of a range of sentences which makes up its truth-functional neighbourhood.[11] This reason (the idea that Gareth Evans called the 'generality constraint'[12]) can be motivated with the use of an example: suppose that the child knows how to count, and correctly counts the same number of hats as dolls. Now imagine that the child is asked whether there are just as many hats as dolls, and responds in the affirmative. If the child does not then go on to agree that there is one hat for each doll, the child cannot be said to have understood the sentence 'There are just as many hats as dolls'. If someone does not understand 'There is one hat for each doll' by failing to assent

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[11] For more details on this view of meaning, see (Dummett 1991a: p. 222).
[12] For more details on this see (Evans and McDowell 1982: pp. 100-04).

to it when it is true, then they do not count as understanding the sentence 'There are just as many hats as dolls'.

The thought that there is one hat for each doll is of course part of what is involved in the thought that the hats and dolls can be put in one-one correspondence. Hence, from the point of view of the tacit knowledge criterion, the child's acceptance or rejection of (3) does not determine its status one way or another. Its status as correct is determined by whether or not it succeeds at revealing tacit knowledge.

But what is it that one must tacitly know in order to be able to understand an expression? The tacit knowledge criterion can be unpacked a little further in the following way. An analytic definition succeeds in revealing tacit knowledge if (i) any ordinary speaker can be brought to agree that it provides necessary and sufficient conditions for the application of the preanalytic concept, (ii) it is couched in terms of actual linguistic practice, and (iii) there is no other definition waiting in the wings that does a better job of capturing the necessary conditions for ordinary linguistic practice. The correctness of an analytic definition is not, however, fully characterised solely on the basis of the tacit knowledge criterion. It follows from (i) (as well as (iii)) that a correct analytic definition makes explicit the presuppositions underlying the ordinary application of the preanalytic concept. This consequence prompts the following second criterion for the correctness of an analytic definition:

> **The conceptual analysis criterion**
> An analytic definition is correct if it expresses the result of a correct analysis of the pre-existing concept associated with the expression defined.

A correct conceptual analysis reveals the presuppositions on which the concept, as it is ordinarily used, depends.[13]

Since the existence of the ordinary everyday analysed concept is a necessary condition of the possibility of certain basic mental activities involving it (such as the act of having a belief, forming a judgment, or possessing conceptual scheme), the necessary conditions revealed by a correct conceptual analysis are necessary conditions for certain basic experiences involving the concept analysed. For this reason, the methodology of conceptual analysis is directly connected with a way of providing an epistemic justification, i.e., with the possibility of providing an explanation of the basis of a certain sort of knowledge. The nature of this connection is as follows. A correct conceptual analysis expresses the condition(s) necessary for the ordinary everyday application of the concept analysed. Such conditions are presupposed by the very possibility of making ordinary judgments involving the analysed concept. And we can then go on, on the basis of this principle, to explain those assumptions of our knowledge that rest on it.[14]

On the basis of the tacit knowledge criterion and the conceptual analysis criterion we have an illuminating and useful characterisation of the correctness of an analytic definition. The tacit knowledge criterion says that a correct analytic definition must reveal what a thinker must tacitly know in order to be said to understand the expression defined. And the conceptual analysis criterion says what that tacit knowledge must

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[13] Cp. "We may take the practice of recovering a central feature of a concept in use by revealing the assumptions on which our use of the concept depends as a characterization of what traditionally passes for a conceptual analysis." (Demopoulos 2003: p. 16).

[14] Cp. "If the criterion of identity [i.e. Hume's principle] is what our applications of the numbers rest upon, then the very possibility of making applied arithmetical judgments explains [the Dedekind infinity] of pure arithmetic. The importance of Frege's theorem is that it vindicates the philosophical program of explaining *pure* arithmetical knowledge on the basis of its account of the *application* of numbers in ordinary judgments of cardinality." (Demopoulos 2005: p. 155).

consist in. It is a characterisation on which the paradox of analysis is seen simply as a symptom of the acceptance of an oversimplified and mistaken view of the nature of analytic definitions. It is also a characterisation that illuminates the role played by analytic definitions in the philosophical program of explaining pure scientific knowledge.

4 Frege's definition of the cardinality operator and its epistemic justification

There are two particularly important distinctions that have been mentioned so far in this chapter. The first is between two kinds of definition: stipulative definitions which lay down how a new expression is to be used, and analytic definitions which, in a technical sense, capture the pre-existing sense of an expression already in use. The second distinction is between two senses in which a definition can be seen as analytic: an analytic definition may be thought of as a principle that reproduces exactly the ordinary sense of an expression already in use, or it may be thought of as a substantive truth that reveals conditions that constrain the application of a concept.

Frege's partial contextual definition of the cardinality operator (which is designated above as (*HP*)) is an analytic definition which is a substantive truth. The fact that (*HP*) reveals conditions that constrain the application of a concept, accounts for its epistemic justification. Here is how: If our conceptual scheme is rich enough to enable the formation of judgments involving cardinal identity, then it must be possible to represent that conceptual scheme by a model that contains sortal concepts, the cardinality operator, and the relation of numerical identity which satisfies (*HP*). It must be possible to extend such a model so as to include the result of the cardinality operator's unrestricted application to all sortal concepts. Such a family of concepts is referred to as a 'Frege structure'.[15] (*HP*) is true in any model that contains a Frege structure, regardless of whether or not such a model models the facts of experience.[16]

Since any model that is capable of representing such a conceptual scheme is a model that represents our absolute notion of truth, we may assert the following "transcendental" claim:

> **The cardinality conditional**
> Any conceptual scheme rich enough to represent the formation of judgments involving the cardinality operator, is one in which (*HP*) is true.

Ours is such a conceptual scheme. Hence (*HP*) is true in any model which represents it. In this way we can claim to have established that (*HP*) is epistemically justified.

By deriving the basic laws of arithmetic from (*HP*) (from an epistemically justified substantive truth that is analytic of our ordinary notion of numerical identity) the logicist establishes the philosophical basis on which our knowledge of arithmetic rests. This derivation constitutes the logicist's explanation of the philosophical basis of arithmetical knowledge.

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[15] For a detailed specification of the notion of a Frege structure, and its relation to Peano arithmetic see (Demopoulos 2005: p.153). See also (J. Bell 1999).

[16] This series of relationships is described in more detail in (Demopoulos 2003: Section 5) See also (Demopoulos 2000: p. 219).

Chapter 5: BASIC LOGICAL KNOWLEDGE AND ITS JUSTIFICATION

In this final chapter I propose an alternative to the theories of basic logical knowledge investigated earlier. The theory that I have in mind is "moderately Kantian". It is moderately Kantian because, without making any appeal to the notion of pure intuition, it seeks to explain the possibility of basic logical knowledge by reference to an analysis of the notion of a logical concept. The theory adapts the methodology used by Frege in his logicist explanation of the possibility of arithmetical knowledge, which was discussed in Chapter 4, and extends Helmholtz's explanation of geometrical knowledge, discussed in Chapter 3. The Helmholtz analysis of the concept of space shows geometry to rest on the observation that spatial displacements can be carried out, cancelled, and arbitrarily combined. The Fregean analysis of cardinal identity shows the basic laws of arithmetic to rest on the observation that, for the number of two sortal concepts *F* and *G* to be the same, the *F*s must be able to be paired one-one with the *G*s. The analysis of logical concepts that I present in this chapter shows logical laws to rest on the fact that they express conditions on the possibility of a "minimally reasonable conceptual scheme". This notion will be defined below.

The main point of this chapter is to demonstrate how it is possible to provide a theory of the justification of logical laws on the basis of an analysis of the general notion of a logical concept. To undertake this demonstration is to address two fundamental epistemological questions in the philosophy of logic: "What does our knowledge of fundamental logical laws rest upon?" and, "How is deductive reasoning justified?". The deceptively simple and moderately Kantian answer that can be given to these two questions is this:

> **Foundation**
> Logical laws are analytic of logical concepts.

What this means is that, by systematically reflecting on the nature of basic logical concepts, we find that logical laws are the principles that express the conditions on which the application of logical concepts depend. Putting forward an answer of this sort however requires an answer to the following further question: "What explains (without appeal to rational insight, psychology, induction, or convention) how logical concepts are given to us?". [1] In other words, "What is the source of our logical concepts?", and "What exactly is their nature?". The bulk of the work that goes into getting the moderately Kantian strategy up and running involves providing a satisfactory answer to this subsequent question—that is to say, it involves providing an analysis of truth-functional logical concepts. The analysis that I propose is this,

> **Analysis**
> Logical concepts are given to us as conditions on the possibility of the existence of a minimally reasonable conceptual scheme.

1 A sketch of a moderately Kantian explanatory justification of logical laws

The overall positive goal of this thesis has been to arrive at a successful explanation of the possibility of basic logical knowledge. This goal is achieved if an explanatory

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[1] I intend this question to be understood in the same sense as the question posed by Frege in §62 of *Gl.* Cf. (Frege 1980)

justification of a system of fundamental logical laws can be articulated and presented. The point of this section is to present and articulate such a justification.

In order to provide a justification of a system of fundamental logical laws it suffices to present an argument that the laws constituting that system are necessarily truth-preserving. For the argument to be explanatory, it must proceed from an explanation of the role of logical concepts in our thinking. In order for the argument to be "successful", in the relevant sense, it must abide by the methodological constraints that have guided my investigation thus far and be free of any appeal to illegitimate methodologies such as psychologism, conventionalism, and inductivism. In outline, the explanatory argument that can be presented on behalf of the moderately Kantian strategy is as follows:

The moderately Kantian transcendental argument

(i) For it to be possible for there to be a minimally reasonable conceptual scheme, it must be the case that certain logical laws are necessarily truth-preserving.

(ii) It is possible for there to be a minimally reasonable conceptual scheme.

Therefore,

(iii) There are some logical laws that are necessarily truth-preserving.

This argument is fairly straightforward; it provides an answer to what in the introduction I called "the justification question". The fact that it proceeds in accordance with *modus ponens* does not pose a problem because the main aim of the moderately Kantian strategy is not to put forward an argument aimed at persuading someone of the truth of (iii) but only to explain what its truth rests upon.[2] Premise (ii) is secure since people do actually possess conceptual schemes, and since it is not possible to attempt to reject premise (ii) without guaranteeing that premise (ii) is true. In order to be able to deny or reject a claim, a thinker must be in possession of a "minimally reasonable conceptual scheme". Once again, I will say what I mean by a minimally reasonable conceptual scheme shortly. Premise (i) however (which we might call "the transcendental conditional") stands in need of support and explanation. This is what I provide in the next section.

1.1 The subordinate argument for the transcendental conditional

The subordinate argument for the transcendental conditional is somewhat less straightforward than the moderately Kantian transcendental argument itself. Let us start with the notion of a conceptual scheme.[3] There are three kinds of conceptual scheme that we need to talk about: reasonable conceptual schemes, minimally reasonable conceptual schemes, and unreasonable conceptual schemes. These three kinds of conceptual scheme are related to the following three plausibly necessary conditions that are required by rationality:[4]

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[2] Cf. e.g. (Dummett 1973: p. 296).

[3] For a discussion of the idea of a conceptual scheme see (Strawson 1959: esp. Pt. I). E.g., at p. 15 he writes: "We think of the world as containing particular things some of which are independent of ourselves, we think of the world's history as made up of particular episodes in which we may or may not have a part, and we think of these particular things and events as included in the topics of our common discourse, as things about which we can talk to each other. These are remarks about the way we think of the world, about our conceptual scheme. A more recognizably philosophical, though no clearer, way of expressing them would be to say that our ontology comprises objective particulars."

[4] For a discussion, from a perspective different than my own, of the relationship between various plausible necessary conditions on rationality and the possession of logical concepts see (Evnine 2001).

Rationality requires,

C1) that a thinker have the capacity to reflectively entertain a theory about some aspect of the world (where, by 'theory', I mean a set of propositions),

C2) that a thinker have the capacity to deliberate about alternative possibilities, and

C3) that a thinker have the capacity to contemplate various unspecified mutually exclusive pairs of concepts such as up/down, in/out, alive/dead, true/false, and so on.

These are plausibly necessary conditions on rationality; they are individually necessary for a thinker satisfying them to engage in the most basic forms of mathematical and empirical scientific thinking. An example of a most basic form of mathematical thinking is counting. An example of a most basic form of empirical scientific thinking is making predictions about future states of affairs.

It is possible to extend this framework of three conditions in the following way. There are a number of more primitive capacities that must be satisfied if conditions C1) – C3) are satisfied.

If it is possible to grasp rudimentary theories, then it is possible to,

C4) identify objective particulars (i.e. objects),

C5) distinguish one object from another, and

C6) track objects over time or over modes of presentation (i.e. recognise an object as the same again).

Conditions C4) – C6) can be thought of as individually necessary for C1).

If it is possible to deliberate about alternative possibilities and contemplate binary oppositions, the it is possible to recognise,

C7) a particular object as being a member of a given type of thing (i.e. be able to see that a given object has a certain property),

C8) that sometimes having one property excludes having another property, and

C9) that in addition to a subtype of a type of things there is the subtype consisting of everything else of that type.

Conditions C7) – C9) can be thought of as individually necessary for C2) and C3). And finally:

If it is possible to satisfy C7) – C9), then it is possible to,

C10) recognise that one type of thing can be included in another,

C11) recognise that one object may be a member of more than one type of things, and

C12) recognise that sometimes one type of things is nothing more than an aggregate of some other types of things.

It seems safe, if not trivial, to suppose that C7) – C12) are individually necessary for conditions C2) together with C3).

Conditions C7) – C12) involve properties and various structural relationships that hold among properties, so we can refer generally to the practice of exercising capacities C7) through to C12) as "the primitive practice of making judgments about structural relationships between properties".

At this point we can define a "reasonable" conceptual scheme to be one the possession of which is sufficient for the satisfaction of conditions C4) – C12). It is likely that there are a variety of conceptual schemes, which differ from culture to culture and from one period of history to another, that are sufficiently rich to enable their possessor to satisfy C4) – C12). There is however only one conceptual scheme that is minimally necessary in order to enable its possessor to do basic math and science; we can call this scheme the "minimally reasonable" conceptual scheme. And although "unreasonable" conceptual schemes (those that lack one or more of C4) – C12)) are not beyond the realm of possibility, it is hard to see how a being in possession of such a conceptual scheme would be able to get by in the world. How such a being would stack up against a Neanderthal, or an eight-month-old human infant, or an African Grey parrot, it is hard to say, but it is highly doubtful that a thinker in possession of an unreasonable conceptual scheme would be able to participate in philosophical inquiry.

We are now in a position to assert the first premise of the subordinate argument for the transcendental conditional. The first premise states that it is possible to possess a minimally reasonable conceptual scheme only if it is possible to engage in the primitive practice of making judgments about structural relationships between properties.

At this point a worry can be anticipated and addressed. The worry is that the moderately Kantian argument for the transcendental conditional might appear to fail to heed Frege's dictum that the psychological ought always to be sharply separated from the logical.[5] The surest way of preventing any conflation is to eschew all reference to mental activity. But the argument being presented here depends on a number of psychological notions, not the least of which is the notion of a conceptual scheme. So there is the potential for confusion. It is not the case however that every account that makes mention of psychological ideas is psychologistic; it would be a crude oversimplification to suggest that that was so, and it would be an inhibiting overreaction to prohibit any mention whatsoever of psychological ideas. A justification is psychologistic if it makes the content of what is being justified mentalistic, or, if it makes the warrant for a proposition

[5] Cf. (Frege 1884: p. x).

dependent on facts about a creature's psychology. Now, there are creatures whose psychologies exemplify the connection between conceptual schemes and cognitive abilities that is described by the first premise of the subordinate argument, but the first premise is not simply a statement of fact. The first premise would remain true even if there were no thinkers whose psychologies exemplified the connection. If it turned out that no one's thinking had the structure of a minimally reasonable conceptual scheme, it would make no difference to the argument being presented here. The first premise of the subordinate argument is the result of an analysis of the concept of a conceptual scheme, it is not a psychological law. The first premise reveals what is constitutively associated with the notion of a conceptual scheme. If it were a psychological law, it would be able to be falsified by some form of potential psychological behaviour; but since no potential behaviour could possibly falsify the first premise, it is not a psychological law. And so the first premise cannot imply that logical laws are warranted because of how people happen to think. And so, while the first premise does make mention of psychological ideas, it does not conflate the logical and the psychological.

Having established a connection between the possibility of possessing a minimally reasonable conceptual scheme and the possibility of engaging in a certain kind of judgement forming practice, we can point to some obvious examples of the sorts of judgments that would be produced by engaging in the practice. Some examples are: the judgment that all *A*s are *B*s, the judgment that some *A*s are *B*s, the judgment that no *A*s are *B*s, and so on. There is an aspect of the content of these judgments that has nothing to do with any particular features of the objects that they are about. In addition to the concept of being *A* and being *B*, these judgements essentially involve concepts that are purely structural in nature. For example, the judgement that all *A*s are *B*s essentially involves an application of the concept of one property being related to another by inclusion. And, indeed, each type of judgment about structural relationships between properties necessarily involves a concept that relates one property to another solely in terms of the structural features of the properties involved. Thus, the practice of recognising structural relationships between properties necessarily involves concepts such as *property inclusion* and *property intersection.*

There is an observation about concepts such as *property inclusion* and *property intersection* that is very important to emphasize: such concepts have the property of being topic-neutral. More precisely, such concepts are independent of the features of the specific objects, if any, which possess the properties that they relate. This form of insensitivity is captured in a mathematically precise way by the notion of permutation invariance.[6] That is to say, permuting the objects that possess the properties related by the concept of e.g. *property inclusion* does not make any difference to the concept's extension. We can refer generally to permutation invariant concepts such as *property inclusion* and *property intersection*, which are necessarily implicitly applied in judgements of structural relationships between properties, as "latent logical concepts". On the basis of the observation that the practice of recognising structural relationships between properties necessarily involves certain concepts, and the observation that these concepts are permutation invariant, we can formulate the second premise in the

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[6] The formal notion of permutation invariance can be defined as follows: A permutation is a one-one function whose domain and range coincide which preserves only cardinality. Let D_e be a nonempty universe of individuals, D_τ the set of all classes, relations, or functions in a hierarchy of types with D_e at its base, and π a permutation of D_e such that it induces permutations of the elements of D_τ. An object $f \in D_\tau$ is permutation invariant if $\pi(f) = f$ for all π of D_e.

subordinate argument for the transcendental conditional: the primitive practice of making judgments about structural relationships between properties requires "latent logical concepts" such as *property inclusion* and *exclusion.*

The concept of *property inclusion* partially orders the set of properties falling under it. The set S of properties partially ordered by *property inclusion* naturally constitutes a lattice. Since S is a lattice, every pair of elements of S has a "meet" (or greatest lower bound) and a "join" (or least upper bound). In terms of the operations of meet and join, it is possible to say when a pair of elements of S are "complements" of one another: two elements of S are complements of one another just in case their join is the property holding of everything and their meet is the property holding of nothing. Designating the property holding of everything as 1 and the property holding of nothing as 0, we can define a stock of truth-functional logical concepts in the usual way terms of the lattice S.[7]

Hence, a set of properties partially ordered by *property inclusion* supplies the necessary presuppositions needed in order to formulate a characterisation of a stock of truth-functional logical operations. More specifically, given a structure that can play the role of a set of truth-values and a domain of basic individuals, it is possible to characterise a collection of permutation invariant truth-functions of the sort referred to by words like 'and', 'or', and 'not'. And given this collection of truth-functions, it is straightforward to derive the truth-function that is referred to by the word 'if'. And if you have the concepts that serve as the semantic values of the sentential connectives, then you

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[7] Cf. (J. L. Bell and Slomson 1969: Ch.2 Sec.1).

have the concepts in virtue of which the logical laws that govern those connectives are valid.

We can now state the third premise in the subordinate argument: if there are latent logical concepts, then, by appeal to the characterisation mentioned above, there must be the truth-functional logical concepts *and*, *or*, *if*, and *not*. The fourth premise is essentially a restatement of the principle that I referred to toward the beginning of this chapter as **Foundation**: logical laws such as *modus ponens* are constitutively associated with logical concepts such as *if*, in such a way that the concept *if* validates *modus ponens*, i.e., forces it to be valid. And finally, logical laws such as *modus ponens* are expressive of basic logical knowledge.

We can summarize the subordinate argument for the transcendental conditional as follows:

The subordinate argument for the transcendental conditional

(i) If it is possible to possess a minimally reasonable conceptual scheme, it is possible to recognize that various structural relations obtain between properties.

(ii) Only if there are latent logical concepts can there be the primitive practice of making judgments about structural relationships between properties.

(iii) If there are latent logical concepts, then, by the lattice-theoretic characterization, there must also be a stock of truth-functional logical concepts.

(iv) The truth-functional logical concepts *and*, *or*, *if*, and *not* validate certain logical laws (e.g. *modus ponens*).

Therefore,

(v) If it is possible to possess a minimally reasonable conceptual scheme, then there are some logical laws that are necessarily truth-preserving.

The subordinate argument shows what is at stake if certain logical laws do not hold. What is at stake in the laws of logic is the possibility of the practice of elementary propositional thinking.

If a thinker has the capacity to recognise structural relationships among properties, then she implicitly has basic logical knowledge. The subordinate argument constitutes an explanation of what we know implicitly. It is an explanation that does not depend on rational insight, psychologism, inductivism, or conventionalism. By reflecting on the capacities that are the conditions that underlie the possibility of thinking at a level sufficient to do rudimentary mathematics and science, and reflecting the conditions that underlie the possibility of the concepts that play an essential role within those capacities, a thinker can have an explicit reflective understanding of his or her own basic logical knowledge. This is my answer to what in the Introduction I called “the acquisition question”.

2 Objectivity and justification

Supported by the subordinate argument for the transcendental conditional, the moderately Kantian transcendental argument proposes an answer to the justification question which hinges on the fact that logical laws are analytic of logical concepts, and equally of the notion of a minimally reasonable conceptual scheme. In this section I hope to intimate the sense in which the justification provided by the moderately Kantian proposal captures all that there is to the content of fundamental logical concepts. I approach the task by considering a second worry that might be raised against the moderately Kantian approach.

The worry is similar to what in earlier chapters I have called the "small child" objection. Surely, the objector might say, a thinker can be justified in the belief that *modus ponens* is necessarily truth-preserving without ever having heard of any of the host of obscure technical notions employed in the subordinate argument. This objection can be answered by observing clearly the distinction between the objective and subjective points of view from which the epistemic status of a belief can be evaluated—between evaluating the objective fit between a belief and the world, and evaluating the subjective appropriateness of a belief. The moderately Kantian strategy is not first and foremost attempting to account for the subjective appropriateness of basic logical beliefs. The factors that are relevant to the sort of justification of basic logical beliefs being put forward by the moderately Kantian strategy are not essentially internal to a thinker's perspective. In order to have a conceptual scheme, for example, a thinker does not need to know that she has one; and it is certainly possible for thinkers to engage in basic practices of inferential reasoning without understanding what they are doing from a post-analytic perspective. Such an understanding requires an objective meta-theoretical viewpoint. From the required correct meta-viewpoint, the factors that determine the epistemic status of a basic logical belief do not necessarily constitute a part of a thinker's mental life. So the objection is out of place. The moderately Kantian strategy seeks to explain, from an objective point of view, how it is possible for there to be a good fit between a thinker's fundamental inferential dispositions or primitive compelling impressions, and the structures on which logical truths depend for their truth.

The explanation that the moderately Kantian strategy puts forward starts from careful reflection on ordinary rudimentary practices of reasoning, and reveals conditions on the possibility of such practices. The conditions that it reveals have to do with the necessarily implicit involvement in such practices of certain intuitive concepts—of concepts such as *property inclusion* and *property intersection*. The explanation then proceeds to turn its attention to the concepts themselves, and, again through careful reflection, reveals that a certain property bears necessary connections to the concepts. The property of permutation invariance is revealed to be constitutively associated with these intuitive logical concepts, as well as with their truth-functional counterparts, such as *if*, and *and*. Indeed, permutation invariance is constitutively associated with all logical concepts. If something is a logical concept, then it must possess the property of permutation invariance.

It is important to emphasize that it is this association that endows our basic logical beliefs with a certain kind of objective status. Without this feature, our beliefs about what is logical would be entirely personal. Consequently, there would be no way for two thinkers to explain to each other why they have the primitively compelling impressions that they have. To carry out such an intersubjective dialogue requires that the two parties to the discussion apply the same criteria by which to recognise an instance of the concept. And shared criteria require that there be necessary conditions on the application of the concept. Such necessary conditions are revealed by the moderately Kantian explanation. The latent logical concepts essentially involved in the most primitive forms of reasoning about the world are discovered to necessarily exhibit the property of permutation invariance. In general, we can say that it is essential to a thinker's so much as having any logical concept that the concept exhibits this property. The following principle captures the condition:

> **Tarski's thesis**
> Something is a logical concept iff it is permutation invariant.[8]

By expressing the condition, the principle captures the objective content of the notion of *logicality*. What is more, the principle is in an important sense *justified* due to the fact that it expresses what is exhibited by anything that is recognisable as an application of a logical concept. Any possible use of a logical concept exhibits the feature that is expressed by Tarski's thesis.

Tarski's thesis is not telling us what people's primitively compelling impressions are. People do not in general share, or even have any, primitively compelling impressions that lead them to recognise certain expressions or concepts as logical. And among those who do have primitively compelling impressions, we not know in general know what

[8] I call the principle Tarski's thesis in reference to Tarski's extension of the methodology of Klein's Erlanger Programm to the domain of logic which yields an explication of the general concept of a "logical notion". Generalizing from the class of structures consisting of geometrical spaces to the class of structures containing all structures of classes and relations of finite type over a basic domain of individuals, Tarski takes logical notions to be those that are invariant under every permutation of the basic domain of individuals. Cf. (Tarski and Corcoran 1986).

they are. In order to know what it means for a concept to be logical, however, knowledge of what people's impressions are is entirely unnecessary. What it means for a concept to be logical depends not on what primitive inferential dispositions or fundamental compelling impressions one has. Nor can it; because if it did, there would be no way to understand how logical concepts function together with other concepts within coherent frameworks of principles that constitute theories about the world. Nor is Tarski's Thesis telling us something new about a previously understood conception of logicality. Tarski's Thesis is a principle that is partially constitutive of any understanding of logicality that one might have.[9] Thus, rather than resting on convention or intuition, what it means for a concept to be logical depends on there being a formally presentable and conceptually graspable principle that encompasses all practical applications of the concept. This principle, which says what is constitutive of logicality, provides it with an objective foundation. If someone applies a concept that is not encompassed by Tarski's Thesis, then that person is applying a concept that neither fulfills the aim nor performs the function of a logical concept—they are applying a concept that is not achieving the purpose of a logical concept. What is made precise in Tarski's thesis is implicitly assumed in any use of a concept, if that concept is logical.

I would also suggest that certain logical laws governing individual logical operations enjoy the same epistemic status as Tarski's Thesis, for reasons parallel to those just mentioned. The application of truth-functional logical operations cannot

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[9] Cp. the exchange between Russell and Poincaré concerning (Russell 1897), discussed in (DiSalle 2006b: p. 85). E.g., Poincaré is at pains to explain that propositions such as the principle of free mobility, which says that bodies can be moved in space without change of shape, is really a "definition in disguise" that expresses the conditions of possibility of the practice of spatial measurement. He writes, "in order for measurement to be possible, it is necessary that figures be susceptible of certain movements, and that there be a certain thing that will not be altered by those movements and that we will call 'shape'" (Poincaré 1899: p. 259).

possibly depend on the intuitions that people have about what follows from what. Again, if it did, there would be no explaining what the point is of applying them. We explain the point of using truth-functional logical concepts by carefully reflecting on the practices that they are essentially involved in. In this case, the practice is that of primitive propositional inferential thinking. And we then determine what elementary properties these concepts must have if they are to perform their allotted function within the practices that they are constitutively associated with. For example, the practice of primitive propositional inferential thinking essentially involves the concept of *negation*. In classical logic, *negation* has the property that it makes valid the formula

(DNE) $\neg\neg A \leftrightarrow A$

In intuitionistic logic, *negation* does not have the property that it validates (DNE), but instead it validates

(TNE) $\neg\neg\neg A \leftrightarrow \neg A$.

Any possible application of *negation* however has the property that it conforms to the following principle:

(Minimal Negation) A implies $\neg B$ iff B implies $\neg A$.

(Minimal Negation) captures the objective content of *negation* by isolating a property that any possible application of the concept must exhibit; it expresses the essential point of using the concept. We would not acknowledge a logical operation as the operation of *negation* if it did not exhibit the property that is essential to any possible application of it. This proposal is not conventionalist. The primitive logical concepts, such as *property exclusion*, that are used to construct a structured set of truth-values in terms of which truth-functional logical concepts, such as *not*, are defined are not chosen arbitrarily. They are motivated by the fact that the practice of thinking about objects with properties depends on them. Anyone's capacity to think about objects and properties depends on the intuitive notions implicitly involved in practices satisfying conditions C4) – C9).

It is also possible to anticipate a form of "errant thinker" objection that could be raised against the moderately Kantian strategy. The objector might say, You cannot argue that certain logical laws are justified, because there are people (Vann McGee for instance, or an extreme relativist) who for whatever reason do not, at least some of the time, reason according to the logical laws that you claim to be justified. Of course, people do make mistakes. We can say that certain inferential behaviours constitute mistakes because logical laws are not descriptive. From the point of view of the moderately Kantian strategy, logical laws say how one ought to think if one possesses a minimally reasonable conceptual scheme—the laws express conditions on the possibility of its possession. So, if someone possesses a reasonable conceptual scheme, and does not reason in accordance with the laws that are constitutively associated with it, whether it be due to carelessness, flippancy, or being mislead by complex theoretical considerations, then that person is simply in error. If, on the other hand, a being systematically and honestly makes mistakes concerning practices closely linked to conditions C4) – C12), then we do not have to worry about such a being disagreeing with us, or posing a competing alternative way of doing things that follows different rules, because such a being will not have satisfied the necessary conditions on the possibility of coherent thought; such a being will not have the capacity to use the word for the concept that we are interested in in a way that facilitates smooth and fruitful communication. Errant thinkers do not pose a problem for the moderately Kantian strategy because, unlike the strategies of Peacocke or Boghossian, the moderately Kantian strategy does not rely on links between understanding expressions or explicitly grasping concepts, and believing or being justified in believing or knowing certain propositions. A thinker can of course implicitly grasp a concept without assenting to or understanding a sentence that expresses a proposition that involves it. From the point of view of the moderately Kantian strategy, Vann McGee and the extreme relativist implicitly grasp *if* and *not*. Constitutive concepts such as *if* and *not* are such that a failure to implicitly grasp them impels a failure to have the capacity to engage in practices that are necessary for the possibility of smooth and fruitful communication. So, being able to engage in smooth and fruitful communication implies that one implicitly grasps constitutive concepts such as *if* and *not*.

If someone uses 'not' to apply a concept that does not have the property expressed by (Minimal Negation), then that person is not using a concept that performs the function of the operation of *negation*. That there is a concept that performs that function is essential to the possibility of doing science, mathematics, and philosophy in the broadest senses of these terms. Hence the kind of justification that has been extended to logical laws by the moderately Kantian account is this: a principle is justified if it says what must be the case in order for a concept to perform the function that must be performed in order for a practice to be possible. If (Minimal Negation) did not hold, we would not be able to contemplate or doubt whether it held. Similarly, a thinker would not be able to say what it is for a conjunct to follow from a conjunction, except to the extent that *and* exhibits the property expressed by the principle that explicates *property intersection*. And so on for other individual truth-functional logical concepts and the logical laws that are associated with them. These laws are expressive of basic logical knowledge. What has been shown then is that basic logical knowledge is possible if elementary deductive thinking is possible.

3 Concluding reflections

The moderately Kantian framework provides answers to the two leading questions of this thesis: the acquisition question, and the justification question. The answer that it provides to the justification question proceeds via an account of the origins of logical concepts. The concept *if*, for example, is objectively given because it is reducible to primitive logical concepts that are essentially involved in the practice of making judgments about structural relationships between properties, and because this practice is partially constitutive of the possibility of possessing a minimally reasonable conceptual scheme.

This account of the origins of logical concepts can be taken in two directions: an empiricist direction, and a rationalist direction. If we take the account in the empiricist direction, we suppose that fundamental properties of individuals can be "read off" the world. Then, what these properties turn out to be determines which primitive logical concepts correctly capture the structural relationships between them. These primitive logical concepts are used to define certain truth-functional logical concepts; and hence, the way the world is determines which logical concepts are given. The laws expressive of the world-determined concepts are those that are justified. This direction has its merits, but it also has some problems associated with inductivism, e.g., logic does not have the requisite generality that we think it ought to have.

If one does not take the moderately Kantian account of the origins of logical concepts in the empiricist direction, then one must take it in the rationalist direction. Taking it in the rationalist direction however does not make it a moderately rationalist inferential proposal that is the same in its essentials as that of Boghossian. Boghossian's inferential proposal provides a psychologistic answer to the justification question; Boghossian holds that logical concepts are given by primitive inferential dispositions. People's primitive inferential dispositions can differ however, so this view does not account for the requisite objectivity that we think logic ought to have.

Furthermore, the answer provided to the acquisition question by Boghossian's moderate rationalist inferential proposal does not provide an *explanation* of how one may come to know that an inference rule *R* is valid. An explanation of how someone may come to know that *R* is valid ought to indicate a route by which someone who was not sure whether *R* is valid could come to know that it is so.[10] The kind of rule-circular justificatory argument that is deployed in Boghossian's account says only: if 'if' has the meaning assigned to it by its implicit definition then *modus ponens* and *conditional proof* are valid, 'if' does have that meaning, therefore *modus ponens* and *conditional proof* are valid. This rule-circular argument does not explain why *modus ponens* is valid; someone who was not sure whether *modus ponens* was valid could not use this argument to gain an assurance that it is valid. Unlike Boghossian's rule-circular argument, the moderately Kantian transcendental argument and the subordinate argument for the transcendental conditional do explain why *modus ponens* is valid, and hence they do provide an inferential explanation of how one may come to know that an inference rule *R* is valid.

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[10] Cp. (Hale 2002: p. 287).

Taken in the rationalist direction, the moderately Kantian proposal provides the following answer to the justification question: *modus ponens* e.g. is justified because it depends for its validity on the nature of *if*, which is itself dependent on the nature of the practice of making judgments about structural relationships between properties, which in turn is partially constitutive of rationality. On the rationalist direction, the moderately Kantian answer to the acquisition question says that a thinker can come to know that *modus ponens* is valid by reflecting a priori on the nature of primitive reason and by being impelled from one step of the moderately Kantian justification to the next until she reaches the conclusion that *modus ponens* is valid. This answer is rationalist in the sense that the knowledge in question can be attained by reason alone.

On the moderately Kantian proposal, a thinker must carefully reflect on the nature of rudimentary propositional thinking in order to become explicitly aware of the conceptual content that is implicit in it, and in order to see that the logical laws that are analytic of that content hold. So, it is possible on the moderately Kantian proposal to have only an implicit grasp of the concept *if* and hence implicit knowledge of the validity of *modus ponens*.

On a moderate rationalist proposal such as Peacocke's, basic logical knowledge is immediate: if a thinker understands 'if', then she knows that *modus ponens* is valid. Both Peacocke's and Boghossian's proposals turn on the claim that possessing *if* implies that a thinker must accept *all* instances of *modus ponens*. These proposals imply that if someone makes a mistake and rejects one single instance of *modus ponens* then they do not possess *if*. As I have argued, I find this implication implausible. On the moderately Kantian proposal, if a thinker makes many undisclosed inferences according to *modus* *ponens* and uses 'if' appropriately in communication, then she can be fooled into rejecting an instance of *modus ponens* and still have *if.*

On the moderately Kantian proposal, if a thinker believes that *modus ponens* is valid because she understands the meaning of 'if', then she is entitled to that belief on the basis of the externalist justification that the moderately Kantian proposal provides. A thinker can know explicitly that *modus ponens* is valid by knowing its justification; that is by carefully reflecting on the practices that are necessary in order to have a minimally reasonable conceptual scheme, finding that they essentially involve certain concepts, recognising that those concepts can be used to define *if*, and observing that *if* validates *modus ponens*.

Finally, with regard to correctness, not just any practice is constitutively associated with having a minimally reasonable conceptual scheme. Not just any logical concept figuring in an ostensible basic logical law is definable in terms of primitive latent logical concepts.

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