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COMPARATIVE EDUCATION WITH HISTORICAL SOCIOLOGY: A STUDY OF MATHEMATICS EDUCATION OF CANADA AND PAKISTAN

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Abstract

The recent curricular reforms in mathematics education in Punjab, Pakistan and Ontario, Canada are studied in this work. The countries are first studied individually and then compared against each other. Using historical sociology, differences between traditional and modern education as well as the underlying theories of mathematics learning are incorporated to explain the shifts in the mathematics curricula in both the cases. We also discuss the various aspects of shifts in learning theories as mainly a tension between the traditional and progressive modes of learning.

In the case of Punjab, Pakistan, it is found that the revised reform documents as well as the mathematics textbooks, do not displace the traditional views of learning mathematics as passive reception, rote memorization and reproduction of textbook questions in assessment.

In the case of Ontario, Canada, shifts in Ontario mathematics curricular reforms over a decade (1995-2005) indicate adjustment and implementation towards newer learning theories based upon constructivism. This shift is claimed to be based on the 1989 NCTM reforms in the USA. This learning theory shifted considerably in the 1997 document, which is argued to be a mixture of traditionalist skill-based mathematics and the newer problem-solving based constructivist approach. The latest 2005 curriculum document, however, firmly re-focuses on problem-solving as a central feature as well as placing substantial emphasis on "mathematical processes" like communication and information technology for learning and teaching mathematics.

Keywords: Mathematics Education, Comparative Education, Historical Sociology, and Curriculum Studies.

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To my mother

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Chapter 1 Introduction

The Research Problem

The nature and status of Mathematics education in different nations is inextricably bound to the various cultural and social factors present in these countries. The identification and origin of these factors is crucial in understanding the present status of the system. This step is also crucial in attempts regarding educational development and progress for each country. In this work, I shall address these factors in the context of mathematics education of two countries, namely, Canada and Pakistan.

Comparative education utilizes various areas of social sciences to understand the dynamics of educational systems in different cultures. A standard problem, for example, is that of curricular reforms moving from one culture to another, and their implementation involving more than simple translation of language. The dominant cultural and social traditions interact with these reforms, often resulting in their being transformed by the ideas and worldview of the host. Thus, for a comparative educationist, it is useful to have both the knowledge of such factors and their origin, in order to probe deeper into the differences in educational systems.

Historical sociology has emerged as an interdisciplinary field, which combines both history and the social sciences in a common framework. The object of this thesis is to investigate the structures of mathematics curricula and texts in two countries. The frame of historical analysis will be that of the *Annales* school of thought. I seek to do this in the context of mathematics education in Canada and Pakistan. The choice of the two countries is due to the presence of seemingly opposite dynamics and assumptions, which may disclose the range of the viability and applicability of historical sociology to comparative education. Finally, the personal connection in choosing these countries is that I have been formally educated in both Pakistan (high school) and North America (university education in United States and Canada).

Context and Background: Pakistan and Mathematics Education

Pakistan is located geographically in that part of the world in which the "Great Powers" of the nineteenth century played out their "great game" of establishing spheres of political and economic influence. From the days of the Raj, and Crimea, through the maneuvering following the First World War to the Soviet invasion of Afghanistan and the American-led overthrow of the Iraqi regime of Saddam Hussein, the region has been in the fulcrum of profound political, economic and ideological pressures.

Despite its original inception as a secular state (Talbani, 1996) in 1947, Pakistani education has been infused with the belief that the " ... system should be inspired by Islamic ideology" (Jalil, 1998, p. 36), a view stated explicitly in the 1998 policy document: "Pakistan is an ideological Muslim State" and "Pakistan is not a secular country" (Ministry of Education, 1998, p. 9). A question which is begged in such pronouncements is the one concerning the nature of Islamic thought implied: as with other great religions, Islam is rent with sectarian divisions as well as great diversity with respect to the levels of orthodoxy practised by its adherents. The conflict between the competing pressures can be seen in the fundamentalist view of science and mathematics: "science is seen as an ideological tool in which Western civilization extends its hegemony over Muslim societies" (Talbani, 1996). Further, "modern science is guided by no moral values but naked materialism and arrogance"(Engineer, 1986, quoted in Talbani, 1996). Thus, this study explores a combination of long held cultural beliefs, governmental decisions concerning Islamisation, and their effects upon the structure of courses, texts and in particular external examinations that drive mathematics teachers to a mode of teaching reliant on drill, repetition and a close knowledge of the text as authority. Hence, the main role of mathematics education in Pakistan is to consciously build religious and national identity, which is naturally in line with the aim of the full educational system.

Context and Background: Canada and Mathematics Education

The European contact with North America resulted in an Anglo/French colonization of Canada starting in early 15th century. The establishment of New France (1604-1763) and the period of British imperial rule (1764-1867) reflected periods which led to development of a largely Christian population. Gaining independence from the British rule led towards a shift from a primarily agricultural/rural to largely urban populations most significantly between 1908-1914, and the trend continues to this day. Due to shifts in immigration policies notably in 1945 and 1960, large sections of population from Europe and Asia migrated to Canada.

In direct contrast with Pakistan, education in Canada is not overtly specific to any religion or ideology. This is stated clearly in Ontario Education Act, which talks about religious instruction section 51 as the choice of the pupil and the parents " ...a pupil shall be allowed to receive such religious instruction as the pupil's parent or guardian desires..." It is also stated "(2) No pupil in a public school shall be required to read or study in from a book, or to join in an exercise of devotion or religion as the pupil's parent or guardian desires or, where the pupil is an adult, as the pupil desires." (R.S.O. 1990, c. E. 2, s.51 (1,2)). One of the duties of a teacher is "to inculcate by precept and example respect for religion and the principles of Judeo-Christian morality and the highest regard for truth…"(s. 264). These statements in the general educational policy indicate that the purpose of education is not to fabricate the entire nation under a single ideology or religion. Rather, there is an ambiguity relating to the Christian traditions of the country and its changing populations.

Located north of United States of America, Canada is by and large driven to compete and collaborate with its powerful neighbor in most arenas one can imagine. The dominant ideas about education are secular and driven by economic needs. The primary aim of the educational system is to provide individuals with lasting economic and financial security, which translates into an economically well-situated nation.

Recent reforms in educational curricula state the drive to remain competitive and viable in today's global economy as one of the main goals of education. This is particularly true for the Mathematics curriculum, especially in the Northern American sphere, where both in US and Canada reforms were implemented based on the changing workplace and economy. In 1989, the National Council of Teachers of Mathematics (in the U.S.) published the standards of the "new" math in the form of a document called "Curriculum and Evaluation Standards for School Mathematics". In view of the 1997 Ontario mathematics curriculum as well as the latest 2005 document, the influence of the Standards by USA is quite evident.

When one studies both of these mathematics curricula, one sees quite clearly that the changing global economy is the main reason for the required change as in reforms in USA. Students, in order to maintain employability in the new economy must be able to develop problem-solving and communication skills in mathematics. Both the documents introduce these "new" ideals, their importance both in mathematics and real life. Thus, the motivation for teaching mathematics (according to both the documents) is, apart from the lifelong personal enrichment for students, mainly economical.

> Students in schools across Ontario require consistent, challenging programs that will capture their interest and prepare them for a lifetime of learning. They require knowledge and skills that will help them compete in a global economy....(1997, p.3)

This document is completely in line with the Employability Skills Profile (1999) expectations of "employable" graduates in Ontario. This, however, strikes at the heart of constructivism which requires those specific skills to be avoided and a broader range of learning abilities to be explored by teachers and students. The minimal requirement of repetition of calculation skills without context or content provides little for development of problem solving and critical skills in students.

The 1997 Ontario document is an example of the merging of the prevalent view of educators and Mathematics teachers in Canada, with this "new" way of approaching mathematics and its curriculum as required by the involvement of corporate ideals into education reform. The real driving change and the need to adapt the curriculum to model the U.S. version was mainly due to economical reasons. There is tension between educators and businesses, as seen in the "middle of the road" juxtaposition of required skills and constructivist ideals. The role of mathematics curriculum in Ontario and Canada in general, reflects more of a "forced" measure caused by global economy and business demands on the educators and the teachers into a curriculum change, to whom such a change seems uncomfortable yet required.

Thus, the background and role of Mathematics education and curriculum in Pakistan and Canada apparently have contrasting dynamics. In the next section, I discuss how historical sociology can be used to provide deeper reasons and comparison of such recent dynamics.

Historical Sociology as methodology: Expanding the horizons of the Annales School

Over the last fifty years or so, History has become increasingly interdisciplinary. An obvious example is its fruitful collaboration with the Social Sciences over these years. This has resulted in the field of Historical Sociology, which enjoys a great deal of success due to the combined strengths of both fields. In view of such success, the idea of incorporating History with Comparative Educational research (which borrows much from the research of the Social Sciences) is not far-fetched. The aim of this chapter is to elaborate on using historical analysis as a tool for educational research, by the study of history of events (*l'histoire événementielle*) as expounded by Braudel (1980) and Le Roy Ladurie (1981). I explain the primary concept of *longue durée* and emphasize the secondary but unavoidable nature of the short-term history. Further, the idea of grasping and keeping abreast with the *mentalité*, or the relative social perception of the historical events is crucial in the link of history with social sciences. The vital link of comparative education with the social sciences, leads us to discuss the importance of learning from historical sociology when applying historical analysis to research in diverse subject areas like comparative education.

The Annales School of Historical Analysis

One aspect of history is to study the past in the context of events (*l'histoire événementielle*). Our discussion will be centered on two extreme of such historical time of events: the instant time and the very long time span also referred to the *longue durée*. This long span of time usually is related to the slow moving, almost fixed events that are in sharp contrast to the daily or "instant" events, which are markedly more variable.

The best approach to the history of events, according to the Annales school of historians, is to locate the "source event" within its context, which is leading to the currently observed phenomena. The veracity of a certain "source event" is accomplished by actually looking forward and backward in time in order to see the effect of the particular event in question. According to Ladurie (1981, p.2), "…the best studies of *l'histoire événementielle* in the scientific and non-pejorative sense, are precisely those which, in order to gauge the impact of any given event, seek to locate it firmly within its context, looking both backward and especially forward in time, to find out whether the event in question really 'made any difference' or not".

This type of analysis necessarily requires study of historical events in the long time span, as the idea is to reach a point in time, where, the event is as still as possible. Of course, the idea that one can totally isolate an event or period in time is idealistic at best. As stated by Ladurie (1981, p.21) "The theorists of the 'stationary state' have defined it as one of perpetual movement, and where there is a tendency for the final, median state to reproduce what are essentially some of the fundamental features of the original state."According to the Annales school of thought, the study of the long time span or the '*longue durée*' leads to explanations that are decidedly deeper than the strictly short time span or event oriented history. According to Ladurie (1981, p.3), "As for strictly event-centred or instant history – whether written by reactionaries and revolutionaries – it seems to cater to me for a demand which may be legitimate but is often trivial." "...the mass media sometimes confuse the message with the massage, annihilating the logic of books and thought, it is not surprising that "instant history" easily slips back into the habits of undistanced narrative, characteristic of the old medieval chroniclers before the Gutenberg galaxy." However, Ladurie (1981), indicates that such writings, "convey a very good impression of contemporary attitudes – should be regarded, apart from a very few and exceptions, as anything other than they are: namely eye-witness accounts, with all the advantages and disadvantages of the genre." (p.3)

Link of analysis with Sociology

The link of historical events with social sciences is vital. Braudel (1980, p.69) states this to the extent that, "History seems to me a dimension of the social science; they are aspects of one and the same thing. In fact, time, the passage of time, history impose them, or should impose themselves on all the human sciences. Their tendency is not to oppose, but to coalesce." This visionary statement, led to a new branch of study known as historical sociology. According to Smith (1991, p.2), "To oversimplify, historical sociology is the study of the past to find out how societies change and work and change." The concept of *mentalité* or the relative social perception of the historical events is thus crucial in the link of history with social sciences. He further clarifies the application of this branch by "historians and sociologists who investigate the mutual interpretation of the past and the present, events and processes, acting and structuration. They try to marry conceptual clarification, comparative generalization and empirical exploration."

Since the study of comparative education is often done in the context of society, hence the link to Social Sciences is oftentimes unavoidable. Thus, the historical analysis outlined in the previous section cannot work as an exclusive study of historical events. One must fully keep the social perspective of all historical events studied, as they are inextricably bound to the reasons for the social phenomena studied at hand.

Educational policy, Curricula, Textbooks and Teaching practice: Successive Gateways to the Past

A vital question that remains is the educational material amenable for the above analysis. The comparative educationist is usually questioning educational systems and their structure with respect to other systems. The first level of entry and complexity is the education policy for a system. This usually reflects the overall scheme of things and structure of the educational system. This policy can be a very revealing guideline in terms of the political and social outlook of both the authority that designed it and the people which it is intended for. In view of my analysis, one could probe, for example search the "source event" and the historical series of events that lead to such a document. One could gain a deeper insight into the next levels of implementation of the educational process if such fundamental questions are probed into such a "source document". Thus, I will study all relevant historical documents and educational policy documents available for Ontario, Canada and Punjab, Pakistan, in the context of the above analysis.

What is to be taught in an educational system is found in curriculum documents, which mediate between policy and classroom activity. Since the subject matter becomes involved with the policy, hence I consider this as the next level of complexity in terms of our analysis. These documents outline the mathematics education plan for a certain time period, and are inextricably linked to the social, cultural and political issues of the system. Again, such vital relationships can be probed in the context of the above analysis, for example in the form of the following questions. What historical events, explored fully in their social context lead to the making such a document? Are the reasons due to a series of a traceable long term historical events or are the short term effects more dominant in the document? One has to realize the amount of condensed information that exists and make up a certain document to fully understand the complexity of the problem at hand. Historical sociology, along with subject specific questioning of assumptions, will certainly help clarify the questions about the origin of such documents, and will help attempts to reduce and decipher their complexity.

As one approaches educational textbooks, one actually sees a higher level of complexity due to the "expansion in words" of the curriculum document outlined for a particular system. The assumptions and ideologies present in the curriculum are seen in subject form, and the form of expression is yet again a key to many social, political and cultural issues. The same form of questions amenable to our analysis can be asked and probed as in the case of the curriculum document; however, these will be facing an additional level of complexity in the form of subject specific assumptions, limitations and comparisons with the curriculum document. I will look at recent textbooks and material linked to mathematics for both Canada and Pakistan in the context mentioned earlier.

This study is limited to the textual material. A most difficult level is the analysis of the teaching implementation of the textbooks and curricula. This will be left to later work. This level obviously involves greatest human interaction and hence questioning the historical basis for such actions requires a lot more interdisciplinary work. The questions stated could be asked, for example, as to why a teacher prefers to teach a specific method or not. Such a simple question would require amongst other sources, a good helping of psychology. However, if one starts historically analyzing in increasing levels of complexity, as stated above, and then one would certainly have a better idea about how to approach this level. Thus, one can infer some reasons (from the historical and sociological analysis of earlier levels) regarding the current approaches to Mathematics curriculum in both the countries.

The methodology outlined above was done in two stages. The first involved the application of historical sociology to educational policies, curricular documents, textbooks and examination papers to each country separately. In the second stage, I analyzed the findings in the first stage by doing a comparative study of both the countries. At the outset, the dynamics of both countries are quite dissimilar, but a thorough analysis about the historical origins of these dynamics revealed further whether this superficial view is truly valid. Further, the reason for incorporating a comparative aspect to this study allows some idea of the validity, range and applicability of historical sociology to education.

Traditional versus Modern Education

As a result of time and evolution of various educational paradigms, another yardstick pertinent to our analysis is given by noting the primary differences between traditional and modern approaches to education. In both cases studies, such an additional approach will contrast the findings done with historical sociology in the background. Such a comparative approach is compatible with the Annales approach to historical sociology in that it accounts, in a broad sense the major changes in educational systems through the course of large scales of time.

In the following table, Hoodbhoy (1991, p. 123) summarizes these differences between what he terms as Traditional Education and Modern Education:

Traditional Education	**Modern Orientation**
(1) Other-worldly orientation	**Modern orientation**
(2) Aims at socialization into Islam	**Aims at the development of individuality**
(3) Curricula unchanged since medieval times	**Curricula respond to changes in subject**
(4) Knowledge is revealed and unchangeable	**Knowledge is obtained through empirical and deductive processes**
(5) Knowledge is acquired because of a divine command	**Knowledge is needed as a problem-solving tool**
(6) Questioning of precepts and assumptions not welcomed	**Questioning of precepts and assumptions welcomed**
(7) Teaching style basically authoritarian	**Teaching style involves student participation**
(8) Memorization is crucial	**Internalization of key concepts is crucial**
(9) Mind set of student is passive-receptive	**Mind-set of pupils is anti-positivistic**
(10) Education is largely undifferentiated Modern Education	**Education can be very specialized**

According to this table, a traditional view of schooling is seen as based around the belief in the centrality of revelation, particularly as found through authority, including the text. Knowledge is externally real to the recipient and to be learnt with accuracy.

Outline of work

On the surface, Pakistan and Canada have different aims in education. The form of Mathematics education in both countries stays true to the aims of the full educational system. These differing aims have been briefly outlined in this chapter to appreciate a need for contrasting these dynamics.

I have outlined guidelines for approaching comparative education from a historical sociology perspective, as well as from a traditional versus modern viewpoint, as outlined by Hoodbhoy. These will be methodologically applied to educational documents and curricula in Canada and Pakistan specifically in the context of Mathematics education. The reasons for employing such a method are simply the vital connectivity of both history and comparative education with the Social Sciences with the rise of historical sociology as a genuine field of intellectual merit. The method of finding the "source event" in the *longue durée* is emphasized. I contend that "instant history" is important to keep one's perspective in check, however is very unlikely to give any deep probe, due to its rapidly fluctuating nature. The emphasis on social context of the historical events studied is emphasized, and historical sociology is given as a guideline for such approaches. I further propose a systematic approach of this analysis to increasing levels of complexity in educational research. I contend that if one applies historical analysis (usually in the form of fundamental questioning of the origin of such levels/documents) in such an ordering, then a comprehensive picture is more readily obtained at different levels and apply it in the case of the two countries mentioned earlier. The choice of seemingly opposing dynamics present in mathematics education will give us a better idea of the viability and applicability of historical sociology to comparative education. Furthermore, it is hoped that this approach allows a clearer approach to more complex multi-disciplinary levels like teaching methodology.

In Chapter 2, a discussion of ideas related to different types of mathematics learning, education and recent mathematics reforms in both countries is presented. In Chapter 3, I discuss the case of Pakistan in view of its recent educational reforms. In Chapter 4, a study of the Canadian mathematics education system exclusively is presented. In the concluding chapter of the thesis, I bring these ideas together and form a cogent comparison of both these countries in view of mathematics reforms and education.

Chapter 2 Doing, learning and teaching Mathematics

The prevalent ideas regarding the nature of a subject, as well as associated learning theories have a great deal of influence on the curriculum, textbooks and teaching in that area. Mathematics, being no exception to this link and emerging as a discipline in both "Pure" and "Applied" aspects, as well as employed in both traditional and constructivist modes of learning theories surfaces in a particular form of education by drawing from these sources. In this chapter, I explore what it means to do mathematics, what learning theories align with the various aspects of its nature and their implications on the textbooks, curriculum and teaching methods involved.

What does "doing Mathematics" mean?

Mathematics, in its teaching and curriculum, is very much influenced by the view of its nature and purpose, both to educators and students alike. There are two ideologies that have emerged in recent times are based on "doing mathematics" in a certain way.

The first notion regarding the nature of Mathematics, or the Bourbakist view (established in 1934-35) is based on defining sets of rules, which once determined for a specific mathematical system, will yield a solution to a problem through the application of formal deductive and mathematical inductive processes. Thus, this view places emphasis on proof, operational and formally based mathematics. One can safely say that the roots of such thought process comes from the primacy of abstraction in the school of Pure Mathematics. Many pure mathematicians have viewed the application of Mathematics to real life problems as 'impure' or a departure from the tenets of the exactness of Pure Mathematics. Hence, approximations and a posteriori assumptions about open-ended problems are discouraged in such a process.

On the opposing end of this dogma, mathematics is thought more of an exploratory endeavour that minds 'construct' out of various social, personal, problem-based and other interrelated interactions. This thought process borrows and is influenced largely from the discipline of Applied Mathematics (and Theoretical Physics), which place a greater emphasis on the phenomena studied at hand. The role of mathematics is thus a much more flexible one, and allows for the introduction of approximations, specific (and often non-proof based) solutions which aim at understanding the phenomena in the most open minded way as possible.

Theories in learning Mathematics

Learning mathematics is derived from the ontological assumptions as where the knowledge is situated: either as both external and in the real world, or as internal and constructed within the individual. From these assumptions arise two major theories regarding the teaching and education of Mathematics. The first thought process is the more conventional and supports unidirectional teaching methods in Mathematics. This passive-receptive view of approaching mathematics education presupposes that students learn mathematics via reception and carry out formal operations to execute the subject successfully. Thus, the basis of mathematical knowledge is dominated by the notion that it is external to the individual and revealed by the teacher or the textbook. In this approach, mathematical ideas often center on getting the correct proofs or following of correct set of rules; thus mathematical operations at hand are emphasized with little encouragement in learning the motivation for the techniques used. This passive-receptive view has been interpreted from a tradition of using Pure Mathematics as the underpinning of mathematics education and reinforced by the influence on curriculum development of a Bourbakist view of mathematics activity by its supporters.

The opposing (and current) view in pre-university education of Mathematics in much of the Western world is based on the constructivist thought process. Here mathematical learning is presupposed to be mostly internal and hence "constructed" by the learner. Piaget has been credited to have given impetus to this modern theory from his work in genetic epistemology. Hence he is considered to be a "constructivist epistemologist" (von Glaserfeld, 1982; Richardson, 1982). According to him construction of knowledge and learning happens through cognitive adaptation via 'assimilation' as well as 'accommodation' of experience into 'action schemes'. According to Piaget, "All knowledge is tied to action, and knowing an object or an event is to use it by assimilating it into an action scheme" (Piaget, 1967). This view however implies the learner as an individual without having any social or cultural influences. Further, it casts a negative influence on schooling in that "Each time one prematurely teaches a child something he could have discovered himself, the child is kept from inventing it and consequently from understanding it completely" (Piaget, 1970). This leads to an extreme or "radical" view of constructivism, which was confronted, by a socially motivated view put forth by Vygotsky. According to him, "Human learning presupposes a special social nature and a process by which children grow into the intellectual life of those around them." (1978, p. 88). The disagreement between the two thought processes regarding constructivist theories has important implications for the role of the teacher.

According to Vygotsky (1962, p. 116),

> Our disagreement with Piaget centers on one point only, but an important point. He assumes that development and instruction are entirely separate, incommensurate processes, that the function of instruction is merely to introduce adult ways of thinking, which conflict with the child's own and eventually supplant them. Studying child's thoughts apart from instruction, as Piaget did, excludes a very important source of change and bars the researcher posing the question of the interaction of development and instruction peculiar to each age level. Our own approach focuses on this interaction.

In other words, the Vygotsky school of constructivist thought does not isolate teachers, but places them in an important role in learning as well as emphasizing the great role of social and linguistic factors in the process. With our current view of school based Mathematics learning, the original constructivist thought is thus made pliable for institution-based knowledge. The current context for 'constructivist' learning leans heavily towards the Vygotsky approach, therefore, it means that mathematics learning is more of an endeavour that the learner's mind 'constructs' out of various social, personal, problem-based and other interrelated interactions. The teacher plays a crucial role in its development for the learner by facilitating the "zone of proximal development" (ZPD). According to Vygotsky, ZPD is defined as,

> the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers. (Vygotsky, 1978, p. 86).

This is in contrast to the traditional theory of mathematics learning, which, as described above, views the field as rigid limits of absolute information, one which could be absorbed totally if given in the right doses in school.

Implications for the Curriculum, Textbooks and Teaching

The thrust of the constructivist view in mathematics curriculum comes relatively recently, culminating in the National Council of Teachers of Mathematics, NCTM (1989) (in the USA) publishing the standards of a "new" vision of Mathematics in the form of a document called "Curriculum and Evaluation Standards for School Mathematics". The thrust of this view materialised recently (1989), when the National Council of Teachers of Mathematics, NCTM published the standards of a "new" vision of Mathematics in the form of a document called "Curriculum and Evaluation Standards for School Mathematics".

This curriculum following the constructivist thought process introduces open-ended and real life problems to students and encourage novel solutions which are not immediately proof based, but allow for the student to value specific solutions also. In terms of mathematics teaching, the role of the student has been elevated from the passive receiver of "correct" methods and solutions to that of an active participant. Hence, "mathematics" communication and novel alternate strategies are to be encouraged and allowed by the teacher at hand. It is quite natural that the underlying curriculum assumptions also affect the implications for textbooks and assessment strategies. The constructivist thought-based curriculum would demand textbooks that involve the child in communication, problem solving and contextualize the mathematics in social and cultural sense. The assessment strategies should also be focused on these qualities rather than just looking at the end result. The development of the learner therefore, is judged on various levels and thought processes. On the other hand, the traditional view of Mathematics requires textbooks that are formal and proof-based, with the minimal need to connect with the social or cultural norms. Further, the assessment is likely to be based on correct form of the solution, or even obtaining just the correct answer without looking or evaluating factors considered in the constructivist approach earlier.

In terms of mathematics teaching in the constructivist sense, the role of the student has been elevated from the passive receiver of "correct" methods and solutions to that of an active participant. Hence, "mathematical communication" and novel alternate strategies are to be encouraged and allowed by the teacher at hand. The student is therefore facilitated into developing communication with the teacher so that both of them work towards a clearer understanding and development towards the subject matter.

All of the implications stated above justify the rationale for the study of mathematics curriculum, text and policy documents for identification of the predominant nature of Mathematics and the type of learning theories invoked. Of course, such documents do not inform us as to what actually occurs in the classrooms or how students and teachers are affected as a result. Since I have limited myself to these documents, the implications on teaching will not be studied in this thesis.

Two Mathematics Curricula: Punjab and Ontario

Punjabi Curriculum: Assumption of Mathematics in textbooks and exams

In the case of Pakistan, I will restrict the focus of study to the province of Punjab. In the systematic study of Punjabi mathematics textbooks (at both the primary and secondary level) and matriculation exams, one finds that the nature of mathematics and the problems constructed are abstract, formalistic and rule/technique based. Euclidean geometry and its reasoning in the proof form are emphasized throughout the secondary level. Geometry introduced in the later grades (9/10) is totally based on axioms and postulates. These are examined by a direct reproduction of proofs in the matriculation assessment. Thus, with the presence of such abstraction and requirements, the bulk of the exercises required from the students seek direct application of rules and theorems presented in the text. Hence one can safely conclude the dominance of an implementation of the Bourbakist ideology in the Punjabi curriculum and its interpretation by teachers in a narrow, rote-driven way.

Perhaps the most remarkable observation is the exact reproduction of the problems in the matriculation examinations. One cannot escape the observation of such a unique extreme form of formalism and its tight, unyielding adherence to the rules. It can be questioned whether the Bourbakist system of Mathematics education provides a certain framework which is amenable to create such a rule oriented system. These anomalies are definitely not entirely due to the assumptions of the Mathematics involved. The cultural and political assumptions and intrusions certainly play a dominant role in shaping such educational thought processes and ultimately, policies. To unveil the current phenomenon of prevalence of such formal views of Mathematics education, I shall return to the case in Chapter 3.

Ontario curriculum: Assumptions regarding Mathematics

The emergence of constructivism as a driving force in education in the U.S. and Canada has led to change in curricula. In this study, I will focus on a recent curriculum in Mathematics for Ontario (Ministry of Education, 1997) to illustrate a "mixed" form of a curriculum as well as the recent 2005 document, which tilts more towards constructivist thought.

In its simplest interpretation, the 1997 document seems to be a marriage of both the constructivist and the formal views of mathematics. This ambiguity poses a serious challenge to teachers and administrators, as a union of two philosophies or ideals often leads to a new and unique entity. One can understand both the constructivist and the traditional view in isolation; however, the combination can lead to effects and/or problems, which are entirely unique and unanticipated. It could be argued that one is simply choosing the best each ideology has to offer, however, one could suffer serious inconsistencies and contradictions by formulating such combinations. The details of this will be analyzed in Chapter 4.

Chapter 3 Punjab, Pakistan

The state of mathematics education in Pakistan is studied in two different time periods. I first summarize the published findings regarding the study of the 1997-1998 textbooks and curricula specifically for the state of Punjab, which were prevalent during the period of Nawaz Sharif's government. In this period a new national education policy was formulated for the period 1998-2010. After the toppling of this government by General Pervaiz Musharraf in 1999, this educational policy was further met with reforms. I also examine the period to this date. Hence, the country while experiencing a great deal of political flux in this time period also reflected a great deal of change in educational policies as well as shift in outlook. The lens through which I will focus is the mathematics curriculum, with emphasis upon the content of Punjabi student texts of 1996 and 2001 (recently implemented due to educational reforms 2001-2005) and the related Punjab matriculation examination papers.

1996-1998 Analysis

In the findings of the study, there is ample religious and political content in the textbooks for grades 1 to 8, and then in the grades 9/10 text which are written exclusively for a cumulative external examination. Hence, in primary grades, it is observed that explanatory text and problem exercises are used to reflect or actively promote particular aspects of social, religious and political imperatives in Pakistani society. In the case of the secondary school material, the nature of the mathematics in the student texts stands out in particular to the relation of the content of the matriculation paper to those books.

The textbooks for Punjab are produced in two language versions, one in Urdu and the other in English. They are similar in nature, but differences are also noted. In our findings,

> In their organizational structures, there is an extremely high level of correlation between the mathematical content of the two sets of books, but there are also subtle differences. There are clear similarities, but there are a number of contrasts which strike us as important. In both language editions the short-term "event" of governmental policy determination is evident. (Pitman & Chishtie, p. 40)

Explicit promotion of nationalism and Islam are present in both versions of the languages of the texts. The outside back cover of student texts carries the Pakistani National Anthem, in Urdu, for all grades and editions. Inside front covers feature a message from Major (Ret.) Iqbal Ahmad, the Chairman of the Punjabi Book Board in 1996. The English edition has the following message: "... the Board also takes care, through these books to inculcate in the students a love for the Islamic values and an awareness to guard the ideological frontiers of your home land."

A similar statement is contained in the Urdu books. Each book contains on the inside back cover the same green drawing of trees. In the Urdu books the caption translates as "The caretaking of trees is equivalent to free prayer to God." In English, the corresponding caption reads "Greenery is the evidence of Allah".

At the high school level, the English text starts with a quote from the Quaid (that is, Jinnah): "You must devote yourself wholeheartedly to your studies, for that is your first obligation to yourselves, your parents and to the state." The Urdu text carries a markedly different quote from Jinnah: "Your attention should only be toward the acquisition of education. It is only through this that you will attain glory in making your country the biggest, the most powerful and the most technologically advanced in the world." It follows with the Islamic quotation, not provided in the English version: "In the name of Allah the most merciful and beneficent." This use of stronger nationalistic language in the Urdu texts occurs consistently from the beginning of primary school.

Some simple problems involving numeration shed some light on the mix of nationalism and religion:

In the Grade 2 Urdu text (p.5), questions 1-4 relate to facts about Islam; questions 5-7, 13 and 14 refer to national history and geography; question 9 refers to height of a Lahore mosque. Examples include:

q. 1 There are ninety nine famous names of Allah (write the numeral), and

q. 4 Muslims pray five times a day (write the numeral);

q. 5 Pakistan was founded on the fourteenth of August.

q. 6 Quaid-e-Azam Mohammed Ali Jinnah [the founder of the nation] was born on the twenty fifth of December.

The extent to which the use of the mathematics texts is used to form national identity as Islamic can be seen in the ways in which religion is incorporated into the materials. Identifying three major strands, termed "Islam as identity", "Islamic ethics", and "Islamic law" (Pitman & Chishtie, 2004), these provide an insight into the official interpretations of religious life as it is to be inculcated through the schools (p.43-46). These include, for purposes of identity, for example, the Islamic and Christian months in Grade 2 (p.111) in Chapter 10. The Islamic calendar in terms of lunar months are outlined, and the solar year is called 'Eiswy' months, that is the months of the followers of

Jesus; it has 12 months, counting from the year the prophet Eisa (Jesus) was born. The English Grade 7 text refers to Greece, Egypt and the mathematician "Al-Khawarzmi".

In terms of "Islamic ethics", 'Zakat' is mentioned as an application in the textbooks with percentages as the mathematical topic. The inclusion of discussion and problems relating to the responsibilities of the rich to provide for the poor (Zakat) and the exclusion of any reference to interest problems. Section 5.8 of the English text (p. 56) begins:

> ZAKAT Islam has laid down fundamental principles for all aspects of life. In order to establish equity and fraternity in social and economic life, the rich have been exhorted to look after the economic need of the poor Muslim brothers. Accordingly in Islam, the rich are duty bound to deposit every year 2½% of their savings in a Fund meant for meeting the needs of the poor. This amount is called "Zakat". It is the religious duty of every Muslim who saves in a year 6 hectograms of silver or 8.7 decagrams of gold or any amount equal to the value of these metals to give 2½ % of that amount as "Zakat".

There is also a complete absence of calculation of interest on funds, as Islamic Law prohibits usury. Methods are also introduced to systematically calculate wills according to the local interpretation of scripture. Further discussion on the differences between the Urdu and English are discussed in Pitman & Chishtie (2004, p. 41-54).

Secondary school mathematics curriculum

The nature of the mathematics present in the secondary school grades in relation to the contents of the external examination suggest strong influences of deeply ingrained cultural/religious beliefs regarding notions of knowledge and its application. Thus, the more subtle effects of *mentalité* are seen in the secondary school mathematics curriculum.

There is a variety of means through which the idea of mathematics is developed and handed over to students. In the texts studied, two dominant modes of conceiving and imparting mathematics seem to be the characteristic of secondary level mathematics. These are noted to be (Pitman & Chishtie, 2004),

> Mathematics as a rigorous application of skills within an abstract math conception (arithmetic, algebra), and mathematics as a set of formal structures based upon sets and the application of defined operations (sets). The view of mathematics presented is quite formalistic, particularly in the texts for the higher grades. Thus, sets are introduced formally in Grade 7; the Grade 9/10 book starts the chapter on algebra (algebra) building directly on the first chapter's treatment of set theory. (p. 46-47)

Hence, the Bourbakistic reform of the new mathematics of the 1960's maintains a strong presence in the 1996 textbooks. The increasing level of formalistic mathematics is seen to fully set in at the higher grad levels. According to Pitman & Chishtie (2004, p.47), "The view of mathematics as a means of reasoning (Euclidean geometry) is pronounced at the secondary level. Geometry is introduced in Grades 9/10 through its postulates and axioms, with definitions and proofs which may be required for examination purposes. About 60 pages are devoted to proof-driven deductive geometry. Only later is there a chapter dealing with practical geometry. It is clear that the relation between the theoretic and structural nature of geometry and its application is one based upon the primacy of the abstract system."

While it is not just work in this particular direction, but the type of assessment, which reproduces problems directly from the 1996 textbooks in examination papers that emphasizes rote memorization. This is stated in Pitman & Chishtie (2004), where explicit examples are also given (p. 48-51),

> Given the very formal view of mathematics, the nature of the tasks given students is one of the correct application of the appropriate rules in order to satisfactorily reach solutions. Thus, the inflexible nature of mathematical tasks is one of application within the realm of known mathematics-that is, known to the students. In general, problems are abstract with no-or highly artificial- relation to application. The examination papers are very much in line with the nature of the problems in the texts: in fact, while we have not taken the time to trace every one of the items in Sections 2 to 4 of the paper, all those which we have, we have been able to find the exact question in the student text either as a worked example, as a given proof or as an exercise question. This is true for the two language versions.

Teaching of mathematics in terms of content and process is also noted to be heavily affected by the presence of central examinations. It is noted that in Pitman & Chishtie, (2004, p. 50) in the 1996 textbooks,

> In the student text, students are given some guidance to the final examination, although it must be noted that the type of questions provided as examples account for only ten percent of the paper. These objective questions, provided at the end of the textbooks, are preceded by the following warning:
>
> Note 1:The questions which follow are being set as samples. In the examination questions different from these may be asked.

Note 2:Answers written after cancellation of answers originally written will not be accepted. (p. 316)

The nature of the questions which are provided for each chapter are: True/False, fill in the blank, matching of items from given lists and multiple choice. A characteristic of some interest is the introduction of English terminology, written in Urdu; for example, in the problems which require students to match mathematical terms in two columns and indicate their answers in a third column, the phonetic reading of the column headings are "column", rather than the Urdu word. The answers are to be given as a, b, c, etc.

In the examination itself, there are three sections. Section 1, worth 10 per cent of the paper, is compulsory, about half of which is devoted to objective questions. Sections 2 and 3 cover a wide range of algebra, numbers and trigonometry. Section 4 is dedicated to geometry, principally involving reconstruction of textbook proofs. Students must answer three of six questions in Section 2, two of four in Section 3 and three of six from Section 4.

This ultimately promotes a particular view of mathematics propagated throughout the primary and secondary school, and emerges as one based around mathematics as abstract, skills-based and memorization oriented. The study of the 1996 texts as well as the examination questions concludes that this particular form produces the effect of reinforcing a narrow view of mathematical accomplishment that actively discourages students from going beyond the prescribed book and curricula; instead, memorization and reproduction are rewarded, as Hoodbhoy (1991, p. 39) observes. One of the more puzzling findings to emerge from the study involved the differences in problem difficulty between the Urdu and English texts at the primary level. The explanation as to why, where such differences occurred, the Urdu version was the more challenging, is not obvious to us. The stronger patriotic and religious flavors of the Urdu texts do seem to reflect a stronger desire to mobilize the loyalties and beliefs of those schooled in that language. The degree to which this is related to rural/urban and to class factors is a matter for further study, although Talbani (1996) does point to the tendency of those "who can afford it [to] send their children to private English-medium schools" (p. 82). The confirmation of these findings as well as providing deeper insights regarding the differences between the two language editions is left for a future study.

Educational Reforms 2000-2006

The Musharraf government, which took hold in 1999, responded to a great deal of pressure, both from within and foreign to improve upon the level and quality of education. The 1998-2010 educational policy coined by Nawaz Sharif's government, while maintained, however has taken a backseat to the Educational Sector Reforms (ESR) by General Musharraf's government. The 1998-2010 document uses verses from the Holy Quran, and is decidedly much more religious and nationalistic in its approach. As the 1998 Policy document puts it, "Islam emphatically requires the quest for knowledge for all individuals and makes provision of education an obligation for the State" (Ministry of Education, 1998, p. ix).

The move towards "reforms" was partly due to the increased significance of Pakistan's role in dealing with terrorism issues and playing the lead role in the "war against terror" with the USA. Hence, the language of these ESR reforms is decidedly much more secular, international and inclusive in their tone.

On page 9, the mission, vision and objectives are given as:

2.1. Mission Statement:

- Developing human resources in Pakistan is a pre-requisite for global peace, progress, and prosperity.

2.2. Vision:

- Quality education enabling all citizens to reach their maximum potentials;
- Produce responsible, enlightened, and skilled citizens;
- Integrate Pakistan into the global framework of human–centered economic development.

2.3. Objectives:

- Universalization of primary education and adult literacy.
- Mainstreaming *Madaris* for diversifying employment opportunities for their graduates.
- Improvement in the quality of education at all levels through better teachers, upgraded training options, curriculum & textbook reforms, and competency based examination system for promoting Pakistan as a knowledged-based society.
- Introducing a third stream of gender and area specific technical and vocational education at secondary level with innovative approaches for students' counseling.
- Empowerment to district education authorities
- Promote Public Private-Partnerships.
- Diversification of General Education. (Ministry of Education, 2002)

The ESR takes a five-fold approach to improvement in quality of education (Ministry of Education, 2002, p.5),

> Quality improvement is a complex question, unlike improvement in infrastructure, appointment of teachers or even of equipping schools with better academic facilities. Learning from past efforts, it is realized that there is a multi-

pronged approach to the challenge of quality. With this in view, the Education Sector Reforms has pursued a five-fold strategy consisting of (a) improvement in provision of infrastructure and human resources for primary education; (b) provision of improved curriculum and teaching-learning materials; (c) improving the quality of teaching-learning process through the introduction of learner-centered pedagogy; (d) attention to continuous professional development; and (e) increased focus on specification and measurement of learner achievement levels. The goal for quality education is designed to create compatibilities with growing global pressure for knowledge-based societies, an area which received little attention during 1990s.

The strategies for quality improvement specifically in the case for assessment are outlined as well (p. 5).

In the Education Sector Reforms Action Plan 2001-2005, the strategies of quality improvement and assurance at all levels have been identified and they are: (a) Benchmarking competencies; (b) Continuous improvement of curricula; (c) Professional development of teachers, planners, managers and staff at all levels; (d) Establishment of National Educational Assessment System (NEAS); (e) Strengthening and upgradation of Teacher Training institutions; (f) Setting Academic Audit through linkage of grants / incentives with quality; (g) Increase of non-salary budget for provision of conductive educational environment and learning materials; (h) District based educational planning and implementation under the Devolution Plan and (i) Developing a National Strategy for Information Communication Technologies (ICTs) and its concurrent implementation at the

> macro and micro levels through innovative initiatives, mobilizing public, private sectors and international corporate and development partners.

The language and the writing of the ESR is decidedly linked towards goals that are not only time bound, but also connect to the international community and funding agencies requirements. On page 9, these are spelled out as,

> The ESR is based on long-term framework linked to EFA goals by 2015. The main features include (a) sector-wide approach for reinforcement of linkages between sub-sectors (primary, elementary, secondary, technical, higher education, non-formal literacy, madrassah education) to eliminate gender and access gaps and ensure optimum utilization of facilities; (b) macro-level reforms in planning and procedures; (c) institutional reforms at all tiers of the government engaged in planning and service delivery; (d) commencement of vocational/technical education stream at secondary level; (e) quality assurance; (f) public private partnerships. The ESR is fully integrated with relevant Millennium Development Goals (MDGs) It aims at achieving the sub-sector targets by 2005-06: Literacy from 49 per cent in 2000-01 to 60 percent by 2005-06, gross primary enrolment rate from 83 to 100 percent, net primary enrolment rate from 66 to 76 percent, middle school gross enrolment rate from 47.5 to 55 percent, secondary school enrolment from 29.5 to 40 percent; and higher education enrolment from 2.6 to 5 percent.

To achieve these goals, Mrs. Zubaida Jalal was named the Minister of Education in 1999 and under her guidance, the curriculum as well the textbooks were reformed. However, there was considerable textbook controversy during the time the revisions happened. This was mainly due to objections raised at revisions taken into effect for the Islamic Studies texts. This ultimately resulted in replacing Mrs. Zubaida Jalal from heading the education ministry to that of Special Education and Women's development. The current Minister of Education is Lieutenant General (Retired) Mr. Javed Ashraf. In the process, most notably the texts appearing in 2003 as a result of this reform had Ms. Fouzia Saleemi as the Punjab Textbook Chairperson during the 2001-2004 timeframe. She also resigned as a result and now the current textbook chairperson is Mr. Saleem A. Kayani. The textbooks examined here are the latest released in 2005, which are latest reformed versions along with some comparison with those released in 2003 as well.

Mathematics Curricular Reform

While the mathematical content is specified in each grade, the philosophy and objectives for the Mathematics curriculum reform for Grades 9-10 (the matriculation exam years) are,

"Philosophy of Teaching Mathematics at the Secondary School Level

The philosophy of mathematics programme in classes IX and X is reflected in its goals, which are to provide the students with the information and skills necessary for advanced work in Mathematics and the sciences, and the information and skills necessary to become sensible and responsible individuals in a highly technological society of the 21st century.

Objectives of Teaching Mathematics (Classes IX-X)

1. To enable students to acquire understanding of concepts of Mathematics and to apply them to the problems of the world they live in.

2. To provide the students with a sound basis for specialization in Mathematics at higher

stages or to apply it in scientific and technical fields.

3. To enable the students to reason consistently, to draw correct conclusions for given hypotheses; and to inculcate in them a habit of examining any situation critically and analytically.

4. To enable the students to communicate their thoughts through symbolic expressions and graphs.

5. To develop sense of distinction between relevant and irrelevant data.

6. To give the students basic understanding and awareness of the power of Mathematics in generalization and abstraction.

7. To foster in students the spirit of exploration and discovery." (Ministry of Education, p.1, 2000)

Findings for reformed primary and secondary Mathematics textbooks

The Punjab textbooks, which underwent extensive reforms, carried different set of slogans on inside covers. The message of President Musharraf, is notably present in almost all grades and the message by the Mohammad Ali Jinnah is taken out. Here, the message to students is to gain back the earlier glory of Islam.

"It is a historical fact that the Muslims ruled the world for hundreds of years on the basis of the knowledge acquired by their intellectuals, philosophers and scientists. The books written to them were of such a high standard that they served as reference books in the western universities for centuries. As long as the Muslims acted upon the Hadith, "to acquire knowledge is the duty of each Muslim male and female, they ruled the world."

There is emphasis on the more computer science and IT fields, in line with the newer education policy statement (2001-2010), "... it is incumbent upon all Pakistanis to devote their energies for acquiring knowledge with special emphasis on computer and IT education." It is also stated that "Our curriculum was not in concert with the requirements of modern times…Keeping in view these requirements, the textbooks have been developed, revised and developed accordingly." The message ends with "The interest of Pakistan should be the first and foremost objective of your life. May the blessings of God be with us all (Amin!)"

The chief minister of Punjab, Chaudhry Pervez Elahi also has a message, which is present in almost all books. Here, the minister indicates, "Placing main emphasis on modernization of curriculum and textbooks proves beyond any doubt the priority given by my government to the education sector." The need to meld nationalistic and religious identitiy with modern times is again emphasized at the end by the Minister.

"To achieve a better standard of life, quality education plays a pivotal role. My government is trying hard to provide this base. Now it is your duty to make use of these textbooks and play your full role in the development of Pakistan. I pray to God almighty that the new generation, keeping in view the modern trends of education, may make progress by leaps and bounds."

There is an Appeal at the inside back cover by Mr. Saleem Akhtar Kayani, where it is clearly stated that, "Punjab Textbook Board is your own organization and has the mandate of providing high quality but less expensive textbooks, with the approval of Federal Ministry of Education, Government of Pakistan in accordance with national curricula. These books project Islamic values, defend ideological boundaries of the country and help acquaintance with other fields of life." In contrast, the textbooks appearing in 2003 had a rather different tone, where Dr. Fouzia Saleemi, states that, "The

Punjab Textbook Board is your own organization. It produces quality and cost effective books for you. These textbooks, which carry the Board's insignia, are produced under the supervision of experience educationists. They are developed to enhance your creative abilities." The tension and difference of objectives stated and the related events of controversy again indicate conflicts between different ideologies that constitute what education really means in Pakistan. The back cover of the textbooks all contain the national anthem in Urdu.

Minimal Mathematical content changes in 2005 textbooks

The most outstanding difference that appears in the reformed books is "Activities" and "Word Problems" sections for the children with specific instructions to teachers. These are however, mathematics paper and pencil exercises. The Grade 3 book, asks the following under an Activity section in the "Measurements" Unit (p. 81),

150 gm + 100 gm = ___________

In the same unit, "Word Problems" are given. On page 76, question 1 states that "Faizan from his house went to his friend's house at a distance 5km 50m and then from there he went to his school at a distance of 4 km and 675 m. How much distance has he traveled?"

Immediately following this is an "Activity" section where a figure is given where three houses ("School", "Raza's home" and "Ali's home") are shown, separated by indicated distances. The questions asked are:

1. How much nearer Ali's House is from school as compared with Raza's house?
2. What is the shortest distance between Ali and Raza's House?
3. What distance Ali shall have to travel if he goes to Raza's house from school?

4. If Ali goes to Raza's House through school, then how much distance more Ali has to travel? (p. 76).

Religious and Nationalistic content

The religious and nationalistic content has been toned down; however, it varies from the years of reform. A few samples from various grades indicate snapshots of such material.

In grade 1, 2003 edition, "On which day the Prophet of Islam (PBUH) was born?" (p. 82). The end of this textbook has teacher instructions as well. The Unit on Money elaborates on figures of national stature, like Jinnah on the notes. In Grade 2 (2005) (p.78), the "Units of Measurement" shows pictures of children in Army clothes using measuring tapes. Grade 2 has the solar and "Lunar or Islamic" months are given along with exercises (pp. 109-112). In grade 3 (2003), (pp. 94-96), there are chief calendar days, like Eid-ul-Fitr, Quaid's birthday, Christmas etc and emphasis on lunar calendar. In grade 4 (2005) word problems related to everyday situations like land measurement and a presence of small summary type boxes titled, "Remember that", "Be careful" , "Explanation" and "Keep in Mind" that persist across till grade 8. In grade 5 (2004) there are word problems, based on applied situations; however, they contain no religious or nationalistic material. These textbooks were written by Ms. Fouzia Saleemi who was replaced by Mr. Saleem Kayani in 2005. In Grade 6 (2005) p.62 there is a Zakat section, while on p.81 Al-Khwarizmi is declared as the founder of Algebra. In Grade 7 (2005) p. 2, the first chapter "Sets" indicates the onset of Bourbakist thought. These sets, which contain national and religious identities, like months of lunar year and Pakistani leaders, etc. are prominent in examples and exercises. In the Grade 8 (2005) textbook, p.66, there is yet again taxes, more specifically a whole example and section on calculating Income Taxes. The Grade 9 (2005) textbook introduces the full Bourbakist type of Mathematics, as well as "Remember that" boxes persists from earlier texts. Additionally, "Note:" type boxes lay out procedural means of completing a mathematical operation. The mathematical chapters and topics are Sets, operational algebra, real number systems, proof based geometry as well as "practical" geometry purely based on proof based operations. There is, however, mention of European mathematicians like Napier, etc. in addition to historical Muslim mathematicians like Al-Khwarizmi etc. The Grade 10 (2005) textbook continues the same trends as in Grade 9. There are lots of "Remember that" types of boxes. The mathematical topics include Algebraic operations like Elimination, Variation as well as other topics like Information Handling and Trigonometry.

Assessment methods: No change

As before, the years of examination, that is Grades 9 and 10, are filled with formal constructions of Mathematics. There is very little departure from the previously mentioned structure of the assessment as can be seen in the latest 2004 and 2005 papers for these grades respectively.

The ninth grade 2004 paper is divided into sections A (Objective Type, compulsory, 15 marks), B (Subjective, 36 marks, choice of 9 out of 15 questions) and Section C (24, 3 questions out of 5). Most questions from the paper can be found verbatim or marginally different from the Grade 9 textbook, thus indicating little departure from previous assessment methodology.

For example, Question 1 (i) (Section A) in the examination paper, asks the following:

"If the number of elements in set A is 3 and in set B is 4, then the number of elements in A x B is ________"

A slightly different version of the question appears on p.22 of the Grade 9 textbook, as, "5. If there are four elements in a set A and five elements in set B, then what will be the number of elements in A x B and B x A".

Sections B and C of the examination, which features the bulk of the marks, happen to contain questions that are copied verbatim from the textbook. For example, Question 2 (iv) states, "Find the value of x from $\log_{64} x = -2/3$". This appears as question 11, exercise 3.2, page 48 of the Grade 9 text. Similarly, in Section C, Question 6 states, "Prove that the bisector of the angles of a triangle are concurrent. OR Use matrices to solve the linear equations: $3x = 3 - 4y$ and $2y = 2x + 2/3$".

The first question appears as Theorem 19 in the text (p. 168) indicative of the retention of not only the proof based thought, but also the need to reproduce directly from the text. The second question appears on page 126, as exercise 3 (iv) in the text.

Such findings are in stark contrast with the objectives for the Mathematics Curriculum for grades 9-10 stated above. In particular, objectives 3 and 7 are in direct conflict with the assessment style presented. It is further noted that the same patterns and assessment style is found in Grade 10 papers as well, indicating that the reforms have changed practically nothing in order to stop rote learning or assure quality assessment as stated in the latest ESR reforms.

Chapter 4 Ontario, Canada

Curriculum Analysis: Short Term Historical Reasons for current reforms

The emergence of constructivism as a driving force in education in the U.S. and Canada has forced changes in curricula accordingly. Certainly this new adaptation creates a new set of tensions between theory and the practice of Mathematics teaching. This work first highlights such adaptations to constructivist thought, specifically in the light of articles by Windschitl (1999) and Hirsch (1996). I focus on a recent curriculum revision in Mathematics for Ontario (Ministry of Education, 2005) which is based on the original Ontario curriculum document (Ministry of Education, 1997), to illustrate the transition of a pedagogically "mixed" form of curriculum, towards a constructivist based approach to the subject. The "mixed" form of the earlier document can be described as the union of two learning theory models: here constructivism in contrast with a transmission-based algorithmic skills learning. The identification of the philosophy behind the curriculum is critical as far as understanding and implementation (teaching practices) is concerned. It is argued that obstacles in clarity and implementation could arise quite naturally when a "best of both worlds" curriculum is designed and that the recent revision clears some of this by taking problem solving as a frontline approach.

I further look into the possible short-term causes for this earlier adaptation and consequent revision. Due to the political nature of education, it is not surprising that curricular reforms are likely a result of short-term political aims of a particular government in power. In summary, I look at three recent Mathematics curriculum documents in Ontario to explore this aspect of curriculum reform. On comparing two documents from the mid-1990s, *Provincial Standards Mathematics grades 1-9* (1995) and the more recent *The Ontario Curriculum: Mathematics Grades 1-8* (1997), I find similar observations as McNay (2000), who studies *The Ontario Curriculum: Science and Technology, Grades 1-8* (1997) in her work.

The newer 1997 Mathematics curriculum document is designed as a mixture of problem-solving and algorithmic skills (or 'expectations') making it more assessment based and well controlled. The expectations or skills required of students are much more specific and rigorous in the later Ontario document (1997) than the one written in 1995. This is observed to be in line with the conservative political agenda of the government of the day, which explicitly aimed to have a greater control of education. This, I contend, along with inherent contradictions in the curriculum, will affect the learning and implementation of Mathematics in a problematic way. The newest document of 2005, which is based on more emphasis on constructivism, lines up well with the philosophies of a liberal government in power.

Background: 1995 and 1997 Documents

The drive to remain competitive and viable in today's global economy is certainly one of the main reasons behind recent reforms. This is particularly true for the Mathematics curriculum, especially in the Northern American sphere, where both in US and Canada reforms were implemented based on the changing workplace and economy. In 1989, the National Council of Teachers of Mathematics (NCTM) (in the U.S.) published the standards of the "new" math in the form of a document called 'Curriculum and Evaluation Standards for School Mathematics'. In view of the new Ontario mathematics curriculum document (1997), the influence of the Standards (NCTM, 1989) is quite evident. The purpose of the next section is to study the 1997 Ontario document and compare it to the Standards in the relevant contexts of employability skills (Taylor, 1998) present in these mathematics curricula, and the motivation of teaching mathematics in Ontario. I contend that, in possibly conforming to the demands of employers and businesses, the document is sacrificing consistency to the constructivist view of Mathematics. Furthermore, I find evidence that equal access is not completely outlined in the 1997 Canadian document, and this shows the presence of hierarchical views of business prevalent in the curriculum document. This is dealt with in greater scope and detail in the 2005 document.

As an example of one the "newer" demands by businesses on Mathematics teaching, I give a closer look to the requirement of inculcating critical thinking in students. Recent reforms in curriculum place an emphasis on teaching critical thinking "skills" to students. In mathematics, the most striking aspect of this change is the focus on developing problem-solving skills in students, a consequence of the Ontario mathematics curriculum (grades 1-8) (1997) requirement of introducing critical thinking skills in the curriculum. Furthermore, in view of the skills or "expectations" required at each grade level, and as suggested by Bailin et al (1997), the view of critical thinking is that of transferable heuristic mental procedures gained in the long term via skills and the development of algorithmic skills in the earlier grades. This "fragmented" view of critical thinking is contradictory and reflects the contradictions between constructivism and the formal school of mathematics education. It is however, not seen as serious as the former, due to the need for context building (via algorithmic approach) that is vital to critical thinking. However, an inherent feature of Mathematics, knowledge of exact answers, is considered to be rather problematic in introducing relevant standards in critical thinking skills within students (Bailin, 1997).

Ontario Mathematics curriculum: Changing from a "mixture of different philosophies" towards "constructivism"

The curriculum of 1997

In the Ontario Mathematics curriculum document, (Ministry of Education, 1995), the influence of the original NCTM Standards (1989) is quite evident. From the study of the Standards the most evident view of the mathematics that is implied, is that it is 'constructivist' in nature. When one studies the 1997 Ontario document one starts to see that the view of mathematics implied here is, on the surface, just the same as in the Standards document (1989). The Ontario document encourages problem solving, its importance both in mathematics and real life as does the United States document. The 1997 Ontario document states that, "Students also need to be able to use mathematics in connection with technology and in their daily lives, and, eventually in the workplace" (p.5). Also it is claimed that,"Students engage in problem solving in all strands of the mathematics curriculum" (p.6).

However, as one starts to study the syllabus, one starts to notice the presence of the formal nature of mathematics that is emphasised throughout the syllabus. While the emphasis is placed on problem solving, certain "skills" are to be mastered, however, before that stage is approached. Though it is stated, "When students learn mathematics, they do more than master basic skills...." (p. 5), it is also written that "Students should be expected to work on purely mathematical problems as well on problems that relate mathematics to the real world." (p. 72).

Each grade level contains a section called "Expectations", which are nothing but a set of skills, the least to be fulfilled. This part of the curriculum seems to be in good degree of congruence with the ideas of Hirsch (1996), who proposes that "once basic underlying skills have been automated, the almost universal feature of reliable higher-order thinking about any subject or problem is the possession of a broad, well-integrated base of background knowledge relevant to the subject." (p. 160)

The "constructivist" or the "exploratory" stage is approached in the latter stages of learning, which is contradictory to the whole idea of constructivism theories. If the child "constructs" the view of mathematics, then why not introduce it as a conscious practice in the very early stages, as the Standards (1989) postulate? Furthermore, Windschitl (1999) points out that "…constructivism cannot make an appearance in the classroom as a set of isolated instructional methods grafted on to otherwise traditional teaching techniques." (p. 151)

By Grades 7 and 8, the students are expected to have the "...freedom to explore, and the process of exploration itself are essential elements..." by carefully mastering skills and exploring mathematics at the same time. Thus, it seems that "constructivism" is approached or evolved into (along with traditional ideas) rather than being present at the start.

Thus, the implied idea about the view of Mathematics that is given in the Ontario counterpart is ambiguous, to say the least. In its simplest interpretation, it seems to be a marriage of both the constructivist and the formal views of mathematics. This ambiguity poses a serious challenge to teachers and administrators, as a union of two philosophies or ideals often leads to a new and unique entity. One can understand both the constructivist and the traditional view in isolation; however, the combination can lead to effects and/or problems, which are entirely unique and unanticipated. It could be argued that one is simply choosing the best each ideology has to offer; however, one could suffer serious inconsistencies and contradictions by formulating such combinations. Hence, the temptation to "manufacture" a curriculum based on differing philosophies should be avoided, just on the basis of our lack of knowledge of its true nature.

Political Intrusions

The 1995 Ontario standards issued by the Ministry of Education bears a close resemblance to the 1989 NCTM document. This is evident in the emphasis of problem-solving as the basis behind the learning of Mathematics in all the grades of students. This is stated as one of the key components of Mathematics learning, "Problem solving, communication, reasoning, connections, and technology are central and essential components..." as well as "Problem solving should be the central focus of the curriculum" (p.11). Furthermore, the assessments called for are extended from regular paper and pencil tests to those which require a broader view of the student abilities ranging from students doing "open-ended" problems, writing mathematics journals and students doing self-assessments. Thus, this curriculum, although geared towards the changing economic trends, is very much based on constructivist ideals. This document was used to give mathematics expectations and assessments for Grades 3, 6 and 9.

Again, when one studies the more recent document (1997), one starts to see that the mathematics implied here is similar to the previous document. Thus, it seems that "constructivism" is approached or evolved into (along with traditional ideas) rather than being present at the start.

At the outset, the later document (1997) states the reasons for replacing the earlier document (1995). "The mathematics curriculum set out in this document is significantly more rigorous and demanding that the previous curricula". "...includes a broader range of knowledge and skills and introduces many skills...Expectations for pencil-and-paper skills in mathematical operations have been raised..."(p.3). Assessment is related to province wide testing, and the resulting consistency is cited as the virtue of the reform. Furthermore, this consistency is further emphasised by the provision of detail of 'expectations' at all grade levels, hence eliminating the need for schools to have their individual expectations and "facilitate province-wide testing" (p.3).

All of these new requirements are similar to those outlined by McNay (2000) when she studied the recent Science and Technology document. The need for greater control and intervention are clear and are explicitly stated in the later curriculum document. The greater amount of rigour and skill testing are similar in vein to that of the new Science and Technology curricula. Since the issuing authorities for both these documents are the same, this is not surprising, and as McNay (2000) points out, both are a "...brilliant example of how a conservative political agenda can be inserted into classrooms in the name of curricular reforms" (p. 750). Furthermore, as stated above, since the latest Ontario curriculum is seemingly a forced marriage of two opposite ideologies, the inherent inconsistency in the demands leads to, as stated by Barlow et al (1994) "...little motivation for students to develop the skills of problem-solving or to hone their abilities to think critically, if their success is to be measured exclusively by the extent of their personal possession of easily measured facts and competencies." (pp.119-120). The issue of critical thinking in the form of problem-solving will be explored in detail in a later section. A striking side-note observation is that "*Rich Tasks for Mathematics Learning*" (Flewelling & Higginson, 2001) (a document produced in Canada) does not cite the 1997 Ontario document/guideline at all. In fact, it only points to the original NCTM document as a source.

The 2005 Curriculum Document

With fundamental issues as the contradictory nature of the curriculum, compounded by the conservative political aims to control education by requiring consistency via rigorous skills-based 'expectations' and testing, the ground was set for fundamental problems in consistent learning and teaching policies for Mathematics education in Ontario schools. The revised and latest edition of this document (Ministry of Education, 2005) makes up for such contradictions by identifying problem solving and mathematical understanding as the key features of learning the subject.

> An information- and technology-based society requires individuals who are able to think critically about complex issues, analyse and adapt to new situations, solve problems of various kinds, and communicate their thinking effectively. The study of mathematics equips students with knowledge, skills, and habits of mind that are essential for successful and rewarding participation in such a society. To learn mathematics in a way that will serve them well throughout their lives, students need classroom experiences that help them develop mathematical understanding; learn important facts, skills, and procedures; develop the ability to apply the processes of mathematics; and acquire a positive attitude towards mathematics (p. 3).

Problem solving which lies at the heart of constructivist thought is given a higher priority in the revised document with operational skills as a part of the entire process of learning mathematics, as in the earlier 1995 document.

> This curriculum is designed to help students build the solid conceptual foundation in mathematics that will enable them to apply their knowledge and further their learning successfully. It is based on the belief that students learn mathematics most effectively when they are given opportunities to investigate ideas and concepts through problem solving and are then guided carefully into an understanding of the mathematical principles involved. At the same time, it promotes a balanced program in mathematics. The acquisition of operational skills remains an important focus of the curriculum (p. 4).

There are seven "mathematical processes" that are identified over and above the Skills Expectations found in the 1997 curriculum document.

> Attention to the processes that support effective learning of mathematics is also considered to be essential to a balanced mathematics program. Seven mathematical processes are identified in this curriculum document: **problem solving**, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing, and communicating. The curriculum for each grade outlined in this document includes a set of "**mathematical process** expectations" that describe the practices students need to learn and apply in all areas of their study of mathematics.

Intrusions from Businesses

The Ontario document clearly states that the changing global economy is a principal rationale offered for the required change, as it was with the USA Standards (NCTM, 1989) initiative. Students, in order to maintain employability in the new economy, must be able to develop problem-solving and communication skills in mathematics. Both documents introduce these "new" ideals, and stress their importance both in mathematics and real life. Thus, the motivation for teaching mathematics (according to both the documents) is, apart from the lifelong personal enrichment for students, mainly economical.

> Students in schools across Ontario require consistent, challenging programs that will capture their interest and prepare them for a lifetime of learning. They require knowledge and skills that will help them compete in a global economy....(p. 3)

It further states that,

> Students also need to be able to use mathematics in connection with technology and in their daily lives, and, eventually in the workplace (p. 5).

There is some difference between the two documents in terms of viewing employability skills. The 1997 Ontario document places emphasis on problem solving; however, certain mathematical "skills" are to be mastered before that stage is approached. By Grades 7 and 8, the students are expected to have the "...freedom to explore, and the process of exploration itself are essential elements..." by carefully mastering skills and exploring mathematics at the same time. These are completely in line with the Employability Skills Profile (The Conference Board of Canada, 1999) expectations of "employable" graduates in Ontario. This, however, strikes at the heart of constructivism which requires that specific skills be avoided and a broader range of learning abilities to be explored by teachers and students. Secondly, as Hyslop-Margison (2000) states, there is obvious confusion between the physical skills and mental skills (by job skills proponents): this ambiguity is also exposed in this document. The minimal requirement of repetition of calculation skills without context or content provides little for development of problem solving and critical skills in students. These contradictions are smoothed out somewhat in the later 2005 document which recognizes problem solving as the central theme in learning: the centrality of problem-solving is based on two rationales, one economic, the other a theory of learning.

> An information- and technology-based society requires individuals who are able to think critically about complex issues, analyse and adapt to new situations, solve problems of various kinds, and communicate their thinking effectively. (Ministry of Education, 2005, p. 3)

The learning-theory rationale is stated as follows,

> Problem solving is central to learning mathematics. By learning to solve problems and by learning *through* problem solving, students are given numerous opportunities to connect mathematical ideas and to develop conceptual understanding. Problem solving forms the basis of effective mathematics programs and should be the mainstay of mathematical instruction. It is considered an essential process through which students are able to achieve the expectations in mathematics, and is an integral part of the mathematics curriculum in Ontario, for the following reasons. (p. 11)

The salient features of these "reasons" for problem solving are increased confidence in students, application to real world problems, increased communication of ideas to peers and teachers leading to greater collaboration and development of critical skills (the full list is given in Appendix A).

The document does keeps a broader outlook than just focusing on problem solving and states that,

> Not all mathematics instruction, however, can take place in a problem-solving context. Certain aspects of mathematics need to be taught explicitly. Mathematical conventions, including the use of mathematical symbols and terms, are one such aspect, and they should be introduced to students as needed, to enable them to use the symbolic language of mathematics. (p.12)

As far as the accessibility of mathematics and the need for it to be taught, the 1997 Ontario document seems similar in its orientation to the US Standards. The access that is implied, and stated in the Standards, is that all students should have equal access to mathematics. The 1997 Ontario document does the same; however in one section of the document, it states that exceptional students should be exposed to a program, which maximizes their potential. It is stated that, the document is "...intended for use with all students, including exceptional students...Some exceptional students may need to be given the opportunity to participate in special programs that will help them achieve at the highest possible level." (p. 7) Although this is stated in the best of language, the mere mention of exceptional students and a suggested separate program for these students may imply the presence of the corporate school of thought as expounded by Taylor (1998). This situation is addressed and again ameliorated in the 2005 edition. Under the title, "Antidiscrimination Education in Mathematics", the following is stated.

> Learning activities and resources used to implement the curriculum should be inclusive in nature, reflecting the range of experiences of students with varying

backgrounds, abilities, interests, and learning styles. They should enable students to become more sensitive to the diverse cultures and perceptions of others, including Aboriginal peoples. For example, activities can be designed to relate concepts in geometry or patterning to the arches and tile work often found in Asian architecture or to the patterns used in Aboriginal basketry design. By discussing aspects of the history of mathematics, teachers can help make students aware of the various cultural groups that have contributed to the evolution of mathematics over the centuries...To achieve their mathematical potential, however, different students may need different kinds of support. Some boys, for example, may need additional support in developing their literacy skills in order to complete mathematical tasks effectively. For some girls, additional encouragement to envision themselves in careers involving mathematics may be beneficial. For example, teachers might consider providing strong role models in the form of female guest speakers who are mathematicians or who use mathematics in their careers. (Ministry of Education, 2005, p. 28-29)

This indicates that the 2005 document has also been extended to cover cultural diversity as well as issues of gender equity. The issue of exceptional students is dealt more comprehensively in the 2005 reform document as well. In an earlier section which makes their distinction and individual treatment clearer for practitioners. Under "Planning Mathematics Programs for Exceptional Students", it is stated:

In planning mathematics programs for exceptional students, teachers should begin by examining both the curriculum expectations for the appropriate grade level and the needs of the individual student to determine which of the following options is appropriate for the student:

- no accommodations or modifications; or
- accommodations only; or
- modified expectations, with the possibility of accommodations.

If the student requires either accommodations or modified expectations, or both, the relevant information, as described in the following paragraphs, must be recorded in his or her Individual Education Plan (IEP). (Ministry of Education, 2005, p. 26-27)

The Ontario document of 1997 is yet again an example of the merging of the prevalent view of educators and Mathematics teachers, with this "new" way of approaching mathematics and its curriculum as required by the involvement of corporate ideals into education reform. It can be argued on a firm basis that the real driving change and the need to adapt the curriculum, modeling the portion of the U.S. document was mainly due to economical reasons. However, there is obvious tension between educators and businessmen, as seen in the "middle of the road" juxtaposition of required skills and constructivist ideals. The change of mathematics curriculum in Ontario as seen in the 1997 document, therefore, reflects more of a "forced" measure caused by global economy and business demands on the educators and the teachers into a curriculum change, to whom such a change seems uncomfortable yet required. The later revision in 2005 clears and releases most of these tensions by placing problem solving as the key to mathematics learning, with an emphasis for its rationale in an implied theory of learning.

CRITICAL THINKING ON TAP: Great "Expectations" from Businesses

Although the 1997 Ontario document states that, "Students engage in problem solving in all strands of the mathematics curriculum" (p. 6), it is only after certain mathematical "skills" are attained before this stage is approached.

In view of the article by Bailin (1999), this implies that the view of critical thinking in this framework is that of "mental procedures". Bailin states that "Glaser (1984: 96) notes that 'most of these programs place emphasis on the teaching of general processes, general heuristics and rules for reasoning and problem solving, that might be acquired as transferable habits of thinking' " (p. 277). Although students are required to perform algorithmic procedures at the early grades, the overall push is to foster a heuristic approach to these procedures, as seen in the requirements of the later grades. At best, one can then say that students (in later grades) are being taught to develop critical thinking in mathematics, via problem solving, which is seen as following a set of mental procedures, which allow for alternative solutions and the development of strategies.

While one can criticize the fragmented approach to teaching problem-solving skills, it can easily be countered as following. The algorithmic approach (in the early grades) can be argued as nothing more than building of mathematical knowledge or context, which is viewed as vital for critical thinking skills, as stated by the Ontario document. Thus, one can argue that the overall goal in the curriculum is not particularly problematic, as according to the Ontario curriculum, Bailin states that "We are not claiming that teaching about general procedures is a completely inappropriate way to promote critical thinking. Rather we emphasize that the effectiveness of any procedure depends on its efficacy in helping students meet the relevant standards for good thinking:

there are no inherent or highly reliable connections between learning to think well and performing particular operations." (Bailin, p.279). So, after all, it can be stated that there is overall consistency in the critical thinking aspects of the Ontario mathematics curriculum, if properly supplemented with the right standards of thinking. However, there is one feature of Mathematics, which provides a formidable obstacle to setting such standards. Mathematics has correct answers to problems, and that can skew the whole idea of critical thinking. If a student knows that he or she has the correct answer (or conversely if they are wrong), critical thinking and reflection is invariably biased by this unavoidable fact. Thus, there is a great tendency by students to justify the "gold-plated" end product (usually found in the Answers section at the end of a textbook) by a skewed reflection, which is certainly not critical thinking in all its glory. This bias towards a certain mode of reflection or "critical thought" process is fostered in earlier grades by the algorithmic skill approach to mathematics in the Ontario curriculum. As a consequence, students at higher grades will likely find it harder to develop the focus not to get distracted by the habit of obtaining the right answer using a standardized procedure, and then reflecting critically on it later.

Thus, with justification, there apparently exists little contradiction between the two different types of "mental procedures" suggested in the 1997 Ontario curriculum; however, there exists a considerable challenge in the imposition of setting good standards in critical thinking, due to the early inculcation of algorithmic skills in students and the very nature of Mathematics itself. I also contend that the constructivist thought also promotes a humanistic approach to the subject, as a greater emphasis on obtaining "correct answers" with mere operational skills can stultify mathematics learning for students failing to do so in the earlier grades and result in repeated cognitive failures. Hence, their entire interest and confidence in the subject can be sustained by introducing a holistic approach towards mathematics learning which respect and guide a child "constructing" mathematics with skills, as contained in the latest 2005 Ontario document.

The 2005 document clearly sticks with problem solving as its center and as cited earlier, considers learning algorithmic skills as a part of the entire process. This is indeed an improvement over the 1997 version, as well as clarification on the type of learning and critical thinking required of the students.

The influence of NCTM Standards (2000) on the latest Ontario curricula

The 1989 NCTM document set the precedent for change in both the 1995 and 1997 Mathematics curricula, the 1995 document was much more aligned with the original 1989 NCTM document as stated earlier. The new NCTM document (NCTM, 2000) seemingly introduces a considerable influence in the latest Ontario Mathematics Curriculum (2005). The trend to a problem-solving centered curriculum in Ontario may well have been strengthened by NCTM's firm stance on this issue.

Maintaining the 1989 NCTM stance that change is required in mathematics learning and sustaining this vision is a challenge, the 2000 document builds upon its predecessor. In the section "A vision for School Mathematics" (NCTM, 2000), it is clearly stated that:

> The vision for mathematics education described in *Principles and Standards for School Mathematics* is highly ambitious. Achieving it requires solid mathematics curricula, competent and knowledgeable teachers who can integrate instruction with assessment, education policies that enhance and support learning, classrooms

> with ready access to technology, and a commitment to both equity and excellence. The challenge is enormous and meeting it is essential. Our students deserve and need the best mathematics education possible, one that enables them to fulfill personal ambitions and career goals in an ever-changing world. (p. 2)

Here the NCTM reiterates the broad ranging aims of the mathematics curriculum, which are also echoed in the latest Ontario curricula.

The 2000 NCTM document is based on certain "Principles",

> The principles are statements reflecting basic precepts that are fundamental to a high-quality mathematics education...The Principles should be useful as perspectives on which educators can base decisions that affect school mathematics. NCTM's commitment to mathematics for all is reaffirmed in the Equity Principle. In the Curriculum Principle, a focused curriculum is shown to be an important aspect of what is needed to improve school mathematics. The Teaching Principle makes the case that students must have opportunities to learn important mathematics under the guidance of competent and committed teachers. The view of learning that is the basis for the document is taken up in the Learning Principle. The important roles of assessment and technology in school mathematics programs are discussed in the Assessment and Technology Principles. (NCTM, 2000, p. 5)

All of these principles are covered in the Ontario document, with special focus on the Equity principle, which implies equal access as well as high performance expectations from all the students in both the curriculum documents.

The most distinctive feature of the latest Canadian document is the introduction of "mathematical processes" alongside content "Expectations". The 2000 NCTM document introduces this clearly, as cited above.

> The first five Standards describe mathematical content goals in the areas of number and operations, algebra, geometry, measurement, and data analysis and probability. The next five Standards address the processes of problem solving, reasoning and proof, connections, communication, and representation. In each grade-band chapter, a set of "expectations" is identified and discussed for each Content Standard (p. 5).

The NCTM document puts the content expectations as well as discussion of the nature of "Process Standard" for each grade in separate chapters. This occurs in the Ontario document as well. It should be noted that there is no difference between the content topics of the two curricula, and the 2005 Ontario document lists two additional "mathematical processes" to those stated above, namely, "reflecting" and "Selecting tools and computational strategies" (p.11). The NCTM clearly establishes the link between the mathematical processes and content material (p. 6), a feature that appears quite prominently in the Ontario document as well.

> The mathematical Content and Process Standards discussed in chapters 3–7 are inextricably linked. One cannot solve problems without understanding and using mathematical content. Establishing geometric knowledge calls for reasoning. The concepts of algebra can be examined and communicated through representations (p. 6).

The precedence of mathematical "process" as well as intertwining with the content expectations in the NCTM document, which preceded the Ontario document, is indeed a remarkable and noticeable occurrence. The 2005 Ontario document introduces "mathematical processes" as a new item of improvement for the 1997 document and shares this feature most prominently with the NCTM 2000 document.

Assessment outlined in 1997 Ontario curriculum

The assessment outlined by the curriculum is used in all the various aspects of education in Ontario (p.9). A reproduction of the table below (given in the 1997curriculum) serves as a steady reference when one looks at textbooks, provincial testing and exemplars.

Knowledge/Skills	Level 1	Level 2	Level 3	Level 4
Problem Solving	**The student solves problems:**			
	-with assistance -with a limited range of appropriate strategies -rarely accurately	-with limited assistance -with appropriate strategies -frequently accurately	- independently - by choosing the most appropriate strategies - usually accurately	- independently - by modifying known strategies or creating new strategies - almost always accurately
Understanding of Concepts	**The student shows understanding of concepts:**			
	-with assistance -by giving partially complete but inappropriate explanations -using only few of the required concepts	- independently - by giving appropriate but incomplete explanations - using more than half of the required concepts	- independently - by giving both appropriate and complete explanations - using most of the required concepts	- independently - by giving both appropriate and complete explanations, and by showing that he or she can apply the concepts in a variety of contexts - using all of the required concepts
Application of mathematical procedures	**The student applies mathematical procedures:**			
	-with assistance -that are considered to be basic in solving problems -with major errors and/or omissions	-with limited assistance -that are considered to be appropriate in solving problems -with several minor errors and/or omissions	- independently - that are considered to be the most appropriate in solving problems - with a few minor errors and/or omissions	- independently - that are considered to be the most appropriate in solving problems, and justifies the choice - with practically no minor errors and/or omissions

Communication of the required knowledge related to concepts, procedures, and problem solving	The student communicates the required knowledge			
	-with assistance -unclearly and imprecisely -rarely using appropriate mathematical knowledge	- independently - with some clarity and some precision - sometimes using appropriate mathematical terminology and symbols	- independently - clearly and precisely - usually using appropriate mathematical terminology and symbols	- independently - clearly, precisely, and confidently - always using appropriate mathematical terminology and symbols

Topical Structure of the Ontario Curriculum: Grades 1-8

The Ontario curriculum divides the mathematical topics into five different content sections relevant for grades one to eight. These are:

- Number Sense and Numeration
- Measurement
- Geometry and Spatial Sense
- Patterning and Algebra
- Data Management and Probability

Each of these sections starts with an overview of the topic at hand. Then, for each grade level, the relevant sub-topics are detailed in the form of "Overall Expectations" and "Specific Expectations".

For example, the "Number Sense and Numeration" section starts with "It is important for students to develop the mathematical competence that comes from understanding numbers, number systems, and their related operations. Number is a complex and multifaceted concept. A well-developed understanding of number includes a grasp not only counting and numeral recognition but also of a complex system of more-or-less relationships, part-whole relationships, the role of special numbers such as five and ten, connections between numbers and real quantities and measures in the environment, and much more...It is the teacher's job to provide students with a broad range of activities that will help them develop many of these ideas about number." (p.10). This leads into sections about "Mental Mathematics and Estimation", "Pencil-and-Paper Computation", "Fractions and Rationals", "Calculators" and "Computers". In these sections the teachers are advised on how to develop and foster skills and understanding amongst students regarding these aspects about numbers and numeration.

As an illustrative example, let us look at Grade 3 "Number Sense and Numeration" expectations for students. The Overall Expectations state that,

"By the end of Grade 3, students will:

- represent whole numbers using concrete materials, drawings, numerals, and number words;
- compare and order whole numbers using concrete materials, drawings and ordinals;
- represent common fractions and mixed numbers using concrete materials;
- understand and explain basic operations (addition, subtraction, multiplication, division) involving whole numbers by modelling and discussing a variety of problem situations (e.g., show divisions as sharing, show multiplication as repeated addition);.... (p. 16)

The Specific Expectations are stated under "Understanding Number", "Computations" and "Applications". For example students will required to "read and print numerals from 0 to 1000" (p. 16) amongst others, under the "Understanding Number" expectation. Under the "Computations" section various tasks are expected, one of them being, "add and subtract money amounts and represent the answer in decimal notation (e.g., 5 dollars and 75 cents plus 10 cents is 5 dollars and 85 cents, which is $5.85)." (p. 17). In the "Applications" section real world problems are expected to be solved by these tasks, such as, "pose and solve problems involving more than one operation (e.g. if there are 24 students in our class and five boys and 9 girls wore boots, how many students did not wear boots?)..." (p. 17). For completeness, I have reproduced the entire section as a sample illustration of the type of "Expectations" present in the curriculum in Appendix B.

Mathematics Textbooks in Ontario classrooms

The Ontario curriculum lays out a definite set of mathematical topics, skills and tasks for textbooks to cover in the classrooms. The textbook publishers need to fulfill criteria set by the Ministry of Education to be named on the "Trillium List" in the "Guidelines for Approval of Textbooks" (Ministry of Education, 2006). I find this tight integration very much present in the sample of textbooks that were studied. This is done with the help of teachers and educators who evaluate the quality of the textbooks according to the requirements laid out by the document "Guidelines for Approval of Textbooks" available at the website of the Education Ministry (http://www.edu.gov.on.ca/trilliumlist).

From the point of view of the curriculum, this document states "eligibility requirements" (for full requirements see Appendix C) for texts to fulfill in which the "Provision of a Teacher's Resource Guide" section states one of such requirements as:

> Textbooks must be accompanied by a teacher's resource guide. This guide must be provided in both languages of instruction (English and French) if the textbook is translated. (Ministry of Education, 2006, p. 7)

The textbooks are also required to be oriented towards Canadian content. It is stated that,

> The content must have a Canadian orientation. It must acknowledge Canadian contributions and achievements and use Canadian examples and references wherever possible…The vocabulary and examples should be familiar to Canadians. (p. 8)

Also, these textbooks are to be produced in the country.

> Textbooks must be manufactured in Canada and, wherever possible, are to be written, adapted, or translated by a Canadian citizen or citizens or by a permanent resident or residents of Canada. (p. 8)

Additionally, there is a list of "evaluation criteria" for the textbooks, which is stated under sections titled, "Content Quality", "Reference to Use of Technology", "Health and Safety", "Environmental Responsibility", "Language Level", "Instructional and Assessment Strategies" and "Bias" (full list given in Appendix C). The "Content Quality" eligibility requirement states that,

> The content must be of sound scholarship and must have contemporary relevance. The information must be presented in adequate depth and sophistication for the grade or learning area/subject/course and build on students' previous knowledge and skills. Graphics, such as charts, diagrams, and illustrations and photos should be used where appropriate to support students' understanding of the content. (p. 8)

The inclusion of technology and its effective usage must also form a part of the eligibility standards. It is thus stated that,

> The content must reflect uses of technology related to the Kindergarten learning

> area, elementary subject, or secondary course, where appropriate, and allow students to use and develop these skills. (p.8)

The textbooks must also have various strategies in employing learning and testing incorporated.

> The content must support a broad range of instructional strategies and learning styles. The activities must be appropriate for the skills and knowledge described in the curriculum or learning expectations. The activities must also provide opportunities for students to engage in higher-order thinking and problem solving, to apply concepts and procedures, and to communicate their understanding. (p. 9)

Finally the multicultural aspect of the Canadian society, gender and religious equity is to be respected, as well any other form of bias is prohibited in the texts.

> The content must be free from racial, ethnocultural, religious, regional, gender-related, or age-related bias; bias based on disability, sexual orientation, socioeconomic background, occupation, political affiliation, or membership in a specific group; and bias by omission. The material should present more than one point of view, and be free from discriminatory, exclusionary, or inappropriately value-laden language, photographs, and illustrations. (p. 9)

All of these criteria are amply met in a Grade 3 mathematics textbook published by Nelson in 2004. The cover of this text playfully sports a picture of a sea otter. Most of the text is quite colourful and supported by children from various cultural and ethnic backgrounds, thus fulfilling the image of equal access to Mathematics education. As an example, the grade 3 Numeration and Number Sense requirement is well supported by seven chapters in the textbook, each fulfilling them comprehensively as follows:

Patterns in Mathematics; Numeration; Data Management; Addition and subtraction; Adding and Subtracting with Greater Numbers; Multiplication; and Division. Each chapter is divided into colour coded sections with green indicating “Guided Activity”, purple indicating “Direct Instruction” and blue denoting “Exploration”.

As an example in Chapter 2 (Numeration) the Numeration chapter starts with a cover page with a subtitle “Out to the ball game” showing children attending a baseball game, with a bill drawn as

Popcorn	75 cents
Drink	$1.00
Nuts	$1.25
Pretzels	$1.50

It also shows a baseball score and indicates the money needed for admittance.

The “Goals” of the chapter are stated explicitly:

“You will be able to

- represent 3-digit numbers in a variety of ways
- round, compare, and order 3-digit numbers
- solve problems using organized lists
- use ordinal numbers
- count money”

The subsections of the chapter are worthy of note, given the stated role of constructivist learning theory and sensitivity to gender and cultural diverts in the provincial curriculum documents.

- Getting started: Numbers
- Lesson 1: Representing Numbers (this is coloured purple or termed as "Direct Instruction")
- Lesson 2: Renaming Numbers (coloured as "Guided Activity")
- Lesson 3: Place-Value Patterns (coloured as "Exploration")
- Mental Math: Adding Tens
- Lesson 4: Rounding to Estimate Numbers ("Direct Instruction")
- Lesson 5: Comparing and Ordering ("Guided Activity")
- Mid-Chapter Review
- Math Game: Duelling Digits
- Lesson 6: Solving Problems Using Organized Lists ("Direct Instruction")
- Lesson 7: Ordinal Numbers ("Guided Activity")
- Lesson 8: Counting and Trading Coins ("Exploration")
- Math Game: Race for 2 Toonies
- Lesson 9: Trading Bills ("Exploration")
- Skills Bank
- Problem Bank
- Chapter Review
- Chapter Task: Palindromes

These subsections fulfil the numeration content as well as cultural and gender requirements of the Ontario 1997 curriculum document. The texts incorporate multiculturalism, gender equity and secularism as part of mathematics education. Keeping in line with the curriculum, there are multiple instances of children of Asian, African and American backgrounds displayed throughout the texts. For example, in Chapter 5 "Measuring Length, Time and Temperature" (p. 118) titled "Telling Analog Time", Nisha (a girl of South East Asian origin) is shown wearing an ornate and traditional Indian dress with hands raised in a cultural dance move. The human hands are associated with the "minute hand' and the "hour hand", both used to locate time on a clock. This is called "Nisha's Analog Clock", in which Nisha concludes the utility of clocks are to make sure that "I'm not late for class!". The Canadian flag as represented by eighteen geometrical shapes is shown on p. 172. A picture of the Royal Canadian Mint is shown in Chapter 6 titled "Adding and Subtracting with greater numbers" (p. 143); thus showing the texts meeting the requirement to indicate Canadian specific content. Thus, multicultural, secular and economic themes persist in the textbooks, which regard keeping time, data application to real life situations and money handling as means of utilizing mathematics in the society. These books also follow the content and assessment requirements from the curriculum as required by the Ministry of Education for eligibility for the "Trillium List".

Assessment of quality of Mathematics: the EQAO

Aims and Tests

The Ontario system of education is monitored by The Education Quality and Accountability Office (EQAO), in the form of assessments that occur at Grades 3, 6, 9 and 12. This is an "independent, arm's-length agency established by the Government of Ontario" (p.1, Grade 9 Assessment of Mathematics). The EQAO is led by a board of directors and a Chief Executive Officer. According to the EQAO (p.1, Grade 9 Assessment of Mathematics), their role is to

- design and implement a comprehensive program of student assessment within government-established guidelines;
- advise the Minister of Education on assessment policy;
- develop and implement a system for measuring the quality of education in Ontario;
- lead Ontario's participation in national and international assessments;
- promote research in best practices in assessment and accountability;
- conduct quality reviews in cooperation with school boards; and
- report to the Minister of Education, the public and the education community on system quality issues and make recommendations for improvement.

Thus, the role of EQAO is quite broad and serves as a dynamical entity that monitors curriculum and adapts from the evaluations accordingly. The feedback from the evaluations is not only provided to teachers and parents, but also to policy makers and researchers.

Although the EQAO mandate covers the entire curriculum, its tests are geared towards ascertaining the performance level of student achievement and understanding in the areas of reading, writing and mathematics only. The Mathematics evaluation is made only in Grades 3, 6 and 9. As stated these are meant to assure quality of education amongst students and their grades do not count towards future entry into universities, colleges and employment. In the sections below I shall sample a few tests and remark on the type of mathematics involved in these assessment exams.

Structure of the Mathematics tests: Some sample questions

For the purposes of this work, I shall only work with the Mathematics material of the tests at Grades 3, 6 and 9. The most common form of questions present in all grades is multiple-choice questions. These are followed in frequency by questions that require written responses. In Grade 9, however, students are additionally expected to complete mathematical tasks in two of their mathematics classes.

Multiple-choice questions are usually based on "real life" scenarios that involve mathematical usage of skills as outlined in the Expectations of the Ontario Curriculum. In Grade 3, a sample task involves using of money to buy a newspaper. There are three related questions. The first one asks, "You have $1.00 to buy a newspaper. The newspaper costs 64 cents. How much change would you get back?" The possible answers are illustrated with pictures of Canadian currency coins and are 30, 31, 36 and 35 cents respectively. Hence, there is real life "contact" with usage of mathematics. The second builds into "The cost of a different newspaper is $1.25. What is the total of three of these newspapers?" Here the answers are $37.50, $375, $3.75 and $3.00 respectively. The final question falls away from the cost of newspapers to their thickness. It states, "If one newspaper is 3 cm thick, how many newspapers would there be in a bundle that is 12 cm thick?" All of the required work would involve basic arithmetic skills as required by the curriculum. This theme is repeated in Grade 6 and Grade 9 multiple choice questions as well. Problems which represent the subject matter in EQAO can be found in the sample Grade 3 text studied (Nelson, pp. 42-43). Here Lyn, a child of African descent, provides a solution to the question, "What coins make $4.83?" under Chapter 2 (pp.42-43), section 8 of the text. A piggy bank is shown, denoting Lyn's saving and the currency is graphically represented as coins in the text. The right hand corner of the text also points to the entire Canadian currency in coinage, with the decimal value attached to each.

The written questions required after the multiple choice ones carry more of the open-ended nature answers from students. In Grade 6, for example, an "Investigation" is to be carried out for a Grade 6 class on a trip to Algonquin Park. The question starts with "...On the first morning, the class goes on a nature walk with the park naturalist. Each student keeps track of the wildlife they see. Then they graph their data. The students experiment with three different types of graphs:" Following this, a bar graph, a scatter plot and a line graph are given with the x-axis labelled as "Animal" (types of animals) and the y as "Number Seen". The first question is, thus, "Of the three graphs, which BEST represents the data?" Then, a box where a written response is expected asks, "Explain your thinking". In part b, a chart of temperature is tabulated with days. The question states "...The students look over the data for the daily temperature for a two-week period. Here is the data: ...What is the median daily high temperature?" A box is given where students are expected to show their work. The final part c, asks students to "Construct a graph of the daily high temperature". Hence, with this sample question I see that students have to communicate well enough their reasons for applying the skills used in tackling a relatively open ended question.

The testing is consistent with the curriculum in that it reserves the most open-ended type of mathematical "tasks" at Grade 9. The curriculum document states that problem-solving skills will be approached with a backing of mathematical skills and certainly the tests seem to be placed in that order. In the 2000-2001 test, for example, students were required to solve a total of six tasks. Three of these were solved in two 40 minutes mathematics classes. Task 1 was about "Aqua Aquariums", which sold two types of aquaria in the form of rectangular prisms. The earlier questions involve application of formulae to calculate the volume and surface areas of each type of aquarium. Then they delve into calculating costs of such aquariums based on the price/surface area of the materials used. Parts e and f are the most open-ended types of questions asked. Part e) states "The selling price of the small aquarium is $24. The selling price of the large aquarium is $115. Do the selling prices of the aquariums seem appropriate according to your calculations? **Give reasons for your answer**." Part (f) states,

> "Mohammed went into the store to buy an aquarium. After comparing the small and large aquariums, he tells the owner, 'The large aquarium should only cost two times as much as the small aquarium. He gives the following reasons:
>
> - The dimensions of the large aquarium are two times bigger than those of the small aquarium.
> - It takes two times more material to build the larger aquarium.'
>
> **Explain** the mathematical error in Mohammed's reasons."

Then, a box follows which states, "Hint: To answer this question, refer to your previous answers for help." These final questions require additional "opinion" which requires reasoning and integration of previous knowledge of facts to tackle an open-ended problem. Hence, in Grade 9, in addition to multiple choice questions and written type questions students are expected to complete "tasks" as exemplified above. The tests indicate that approaching problem solving by first inculcating algorithmic skills within students in earlier grades is followed as outlined in the 1997 Ontario curriculum document.

Assessment strategies for tests: Criteria

The assessment of the grades 3, 6 and 9 Mathematics exams is based on both quantitative and qualitative information from teachers. It should be remarked that these tests provide a guideline and their outward motive on "quality assurance" places a heavy emphasis on what teachers have to do in their classroom. Hence, the assessment and tasks to be carried out by the teachers are important to study along with the exemplars given as guidelines.

Not surprisingly, the assessment criteria chosen for the EQAO chimes quite well with those of the Ontario Curriculum. The assessment for grades 3 and 6 in the EQAO tests are divided in four "categories/criteria" which are the same that are stated in *Ontario curriculum 1-8* (1997) where these are given as "Knowledge/Skills". These skills/abilities are assessed at four levels where Level 1 indicates least performance whereas Level 4 is the highest level of performance. The criteria are:

1. Problem solving
2. Understanding of concepts
3. Application of mathematical procedures
4. Communication of required knowledge related to concepts, procedures and problem solving.

These are exactly the same in the case of the EQAO performance charts for both Grades 6 and 9. For example, at Level 3 the student performs, (according to *Ontario curriculum 1-8* (1997))

"- independently

–by choosing the most appropriate strategies

– usually accurately."(p. 9). Similarly, a Level 3 EQAO student at both grades 3 and 6 "demonstrates a general understanding of problems by consistently choosing and carrying out appropriate strategies that usually lead to accurate solutions". Additionally, the EQAO achievement chart states the criteria for each stated categories. For example, problem Solving has the following criteria:

- "demonstrates understanding of problems by choosing, carrying out and analyzing appropriate problem solving strategies
- applies strategies correctly"

In Grade 9 the difference is again quite minimal. Both the documents have the same categories, with the same content as explanation. These categories are:

- Knowledge/Understanding
- Thinking/Inquiry/Problem Solving
- Communication
- Application

The *Ontario curriculum 9-10* (1999) document attaches percentages with the different "Levels" in contrast with the *Ontario curriculum 1-8* (1997). These are 50-59% (Level 1), 60-69% (Level 2), 70-79% (Level 3) and 80-100% (Level 4) (p. 9). Not quite significant, but these levels are changed in the EQAO material to "Below Level 1", "Level 1", "Level 2" and "Level 3" in ascending order of assessed student performance level. This would appear to reflect the statement of necessary performance level criteria on one hand and the assessment of actual student performance on the other.

Exemplars as guidelines for Ontario Teachers

An "exemplar" is defined in *The Ontario Curriculum-Exemplars, Grades 1-8* (p. 213, 2001) as "work or performance by a student that demonstrates a particular level of achievement." This document contains samples of student work for the following topics (or mathematical skills) at a given grade.

- Grade 1: geometry and spatial sense
- Grade 2: patterning and algebra
- Grade 3: geometry and spatial sense
- Grade 4: patterning and algebra
- Grade 5: measurement
- Grade 6: measurement
- Grade 7: number sense and numeration
- Grade 8: data management and probability

All these samples are assessed with a scale or "rubric". These rubrics were designed according to the assessment levels in *Ontario curriculum 1-8* (1997) which are mentioned above. According to the Ontario document, each scale describes levels of attainment for a particular task and then guides the scoring of the task according to the task according to applicable criteria.

Thus, each of the grades and topics in the documentation has student samples where these are given at all levels (1- 4) of achievement. Also, it is clear that these exemplars have the similar scales as used in the EQAO tests.

> ...tasks in the exemplars project are similar to the mathematics scales used by the Education Quality and Accountability Office (EQAO) for the Grade 3 and Grade 6 provincial assessments in both the rubrics and the EQAO scales are based on the curriculum expectations and the achievement levels for mathematics in Ontario. (Ministry of Education, 2001, p.5)

This statement reinforces our observation that there is tight integration between the curriculum and assessment. The students are prepared for and in line with the provincial tests by teachers that are encouraged to specifically follow these rubrics in their everyday classes. The other suggested forms of assessment are, "anecdotal records, checklists, tests, quizzes, teacher observation, journals, and teacher-student conferencing." (p. 5)

Exemplars: A detailed look at a sample task

I shall now look at a given exemplar for Grade 3 Mathematics. This particular exemplar is titled "Lots of Coins!". The total time required for this set of tasks is 200 minutes (40 minutes for the pre-task 1, 20 minutes for pre-tasks 2 and 3, 40 minutes for questions 1, 2, 3, 4, 5 and 6). In the Teacher Package, the description of the task is as follows (p.53, 2002 exemplars):

- investigate the use of coins in different situations;
- explore concepts of measurement, data management, and probability;
- investigate the relationship of weeks to months.

Under the heading "Expectations addressed in the Exemplar Task" the package states that the tasks will involve three expectations from the Ontario curriculum (1997) for Grade 3. The areas of expectations are Measurement, Patterning and Algebra, and Data Management and Probability. The stated expectations are fully given in Appendix D.

Measurement appears on page 33, Patterning and Algebra on page 55 and Data Management and Probability on page 64 of the package.

The task document then details the instructions for the teachers in terms of assessment, materials required for the exemplar and details about accommodations. The rubric for this exemplar is based on the Ontario curriculum and this information is instructed to be informed to students. "The rubric is based on the achievement chart given on page 9 of the *The Ontario Curriculum, Grades 1-8*: Before asking students to do the task outlined in this package, review with them the concept of a rubric. Rephrase the rubric so that students can understand the different levels of achievement." In the case of this task it is important to note the type of materials required. These are stated to be (p.54): copies of the student package for each student; picture book about money; coins (pennies, nickels, dimes, quarters, dollar coins); paper or cloth bag; paper for labels; writing instruments (pencils, erasers); calculators.

The students are expected to complete this assessment entirely at their school. They are instructed to work in groups for the pre-tasks and individually on the exemplar tests. "The exemplar test will allow the students to conduct investigations by using their knowledge of money, patterning, probability, and time relationships to make conjectures." (p.54). The purpose of the pre-tasks is to "review and reinforce the skills and concepts that students will be using in the exemplar task and to model strategies useful in completing the task." The pre-tasks 1, 2 and 3 are arranged for 40, 20 and 20 minutes respectively. Explicit instructions are provided, for modeling the intent of the exercise:

Pre-task 1 (40 minutes)

Introduce the tasks with a picture book about money. Pose questions topics that arise in the book.

Have the student suggest different ways of giving change for a sum of money (e.g. fifteen cents).

Record the responses, and ask how students can be sure that they have listed all the possibilities. Elicit from students the method of using a systematic list for recording possibilities.

Pre-task 2 (20 minutes)

Place a nickel, a dime, and a quarter inside a bag. Ask the students:

- "What are the chances of removing the quarter from the bag?"
- "If two coins are removed, what is the probability that the sum is fifteen cents? thirty five cents?"

Discuss the students' different approaches to this task.

Pre-task 3 (20 minutes)

Place the following coins in a linear pattern in front of the students, and label the groupings as shown:

dime, nickel, quarter	dime, nickel, quarter
Step 1	Step 2

[This pattern of coins is an example of an a, b, c, a, b, c…pattern].

Ask:

- "If this pattern continued, what coin would be in the twelfth position? the twentieth position?"
- "How did you arrive at the answer?"

Discuss the strategies that students used in arriving at the answer.

Then ask:

- "What is the total value of the coins at the end of step 1?"
- "What is the combined value of steps 1, 2, and 3? steps 1, 2, 3, 4, and 5?"
- "In how many different ways can you find the answers to these questions?"

Have students discuss their strategies with the rest of the class.

It is quite noticeable that pre-task 1 is far more open-ended for teachers; however the pre-tasks 2 and 3 provide explicit questions to be asked from students.

Exemplar Tasks: Comparison with EQAO provincial tests

The exemplar questions are to be individually attempted by students as stated earlier. Here, students are given 6 questions to be attempted in 50 minutes. The first question has pictures of the nickel, dime, quarter and dollar coins of the Canadian currency. It states,

> Dawson has the following coins. a) What are the possible sums you can get by combining three coins? Present your information in such a way that someone looking at your work will see how you solved your problem.

The second question states that

> Lo said that you can make $9.56 by combining many of the following coins. Do you agree with Lo? Yes No. Explain why you agree or disagree.

There are figures of a nickel, dime and a quarter. Here the student, although expected to give a rather limited multiple-choice answer in the first part of the question is expected to explain his/her reasoning. The third question poses the following,

> Imagine that have these three coins in your pocket:

> If you take two coins out of your pocket, what is the probability that you removed 30 cents? Show your work. The coins drawn are a nickel, dime and a quarter as before.

Question four states,

> Make up another probability question using coins. Show how would solve the problem.

This is perhaps the most open-ended question so far seen in examples stated earlier. It is quite remarkable that students are invited to pose a problem to their liking (within the confines of coins and probability), judge themselves and then explain why they think the solution is correct.

Question five states that

> Here is a pattern using loonies, quarters, and dimes. The value of step 1 is $1.60. The value of step 2 is $2.20. If this pattern were to continue, what would be the value of Step 5? Show how you arrived at your answer. Use the back of the sheet if needed.

In step 1, there is a succession of a nickel, quarter followed by another nickel. Under the quarter there is a dime. In step 2, there are two nickels followed by a quarter and then by two nickels. Placed vertically under the quarter are two dimes in step 2.

Question six requires students to

> ESTIMATE how much money you would have if you were given $0.55 each week, and you saved the money for three months. My estimate is:_______________

Show how you estimated your answer. Organize your work here, and explain your thinking.

It must be remarked that all the questions asked require written responses hence there is quite a contrast to the multiple choice questions asked in the EQAO exams. This is the striking difference in the type of mathematics required in the exemplars and on the provincial tests. The multiple-choice questions are essentially application of algorithmic skills versus a great deal of open-endedness required in the exemplars. This is seen in the example presented above for the EQAO test presented above involving coins in comparison with the problem presented in the Exemplars document. All these questions required a combination of verbal/mathematical reasoning for justification of the results. Hence, there is a marked difference in EQAO testing with respect to the Exemplars, both of which place their source as the Ontario Curriculum for standards, expectations, content and rubrics.

The 2005 change in Assessment Strategy

The greater shift towards problem solving also induces a complete change in assessment strategy. There is certainly a shift from focus from the previous strategy to fulfill "Expectations" towards "mathematical process expectations" in the revised 2005 edition. The document states such new expectations at the beginning of each Grade. In Grade 3, for example, it is stated, (p.54)

Grade 3: Mathematical Process Expectations

The mathematical process expectations are to be integrated into student learning associated with all the strands.

PROBLEM SOLVING	• apply developing problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;
REASONING AND REFLECTING	• apply developing reasoning skills (e.g., pattern recognition, classification) investigate conjectures (e.g., through discussion with others);
PROVING	• demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by explaining to others why they think their solution is correct);
SELECTING TOOLS AND COMPUTATONAL STRATEGIES	• select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;
CONNECTING	• make connections among simple mathematical concepts and procedures, and relate mathematical ideas to situations drawn from everyday contexts;
REPRESENTING	• create basic representations of simple mathematical ideas (e.g., using concrete materials; physical actions, such as hopping or clapping; pictures; numbers; diagrams; invented symbols), make connections among them, and apply them to solve problems;
COMMUNICATING	• communicate mathematical thinking orally, visually, and in writing, using everyday language, a developing mathematical vocabulary, and a variety of representations.

This is repeated in each grade section in addition to the Skills Expectations which largely remain unchanged from the 1997 document. The connection of these mathematical processes is very tightly connected to the newer Assessment strategy developed. This is amply seen in the "Achievement Chart – Mathematics, Grades 1–8" (given in Appendix E), (Ministry of Education, 2005, pp. 22-23). In contrast with the 1997 Achievement chart, there are Categories (as compared to "Knowledge/Skills") are "Knowledge and Understanding", "Thinking", "Communication" and "Application" in which the student levels are assessed. As in the 1997 document, these are termed "Level 1", "Level 2", "Level 3" and "Level 4", given here in ascending order of achievement. The "Knowledge and Understanding" category states itself as "Subject-specific acquired in each grade (knowledge), and the comprehension of its meaning and significance (understanding)" (p. 22) in the chart. "Thinking" consists of "The use of critical and creative thinking skills and/or **processes**" (p. 22). The category "Communication" is the "…conveying of meaning through various forms." (p. 23) and "Application" is described as "The use of knowledge and skills to make connections within and between various contexts." (p. 23). The study of the application of this new strategy is left as a topic for future study.

Chapter 5 Comparison of Mathematics Education

Countries emerging from a colonial rule face a double problem with their school systems in terms of identity. Much of the remaining structures and forms of knowledge which have been subjected to the indigenous cultures still represent aspects of power both at the local and the international levels. In a number of such countries, for example, the European colonial language, typically French or English, is the single non-regional language. Additionally, with international communication in science, technology and in mathematics, certain standard nomenclature has become broadly established. It can be argued that it would be counterproductive for these countries to go up against these realities. Opposing this sentiment is an imperative to use schools and knowledge structures to help forge a national identity, which is particularly true in cases in which vast migrations occurred at the time of the formation of such nations.

This is indeed the case in Pakistan, where it is possible to see the interplay of recent colonial and post-colonial influences with the much more ancient traditions of culture and religion.

In contrast, Canada is currently adapting itself to a more economically favorable position in the globe, has undergone a massive demographic change as a consequence of its immigration policies and has separated religion from the state in the traditions of democracies forged in the North American continent. Hence, the cultural aspect and aim of education is kept secular, egalitarian and market economy based. Keeping up with the business demands and more specifically that of its biggest economic partner, USA, it is therefore influenced heavily by the mathematics curricular reform conducted by NCTM. The cultural dimension of mathematics and mathematics education has a long and important history (see, for example, Stigler & Baranes, 1990), with cross cultural studies directed toward understandings of the human mind, both "primitive" and "civilized". There is a certain irony in the preoccupation with constructivist models for the acquisition of mathematical knowledge – the psychological models fail to take into account broader aspects of the world views in which the individuals are immersed, although it must be noted that some who are influential in the reform movement are aware of this problem (see, for example, Romberg, 1997). This is of particular importance when the nature of the current international trends in mathematics education- at least; in Europe and the Anglo-Saxon "western world" is taken into account. It has been argued elsewhere (Pitman, 1989) that the mathematics education reforms of the past twenty years in countries like Australia, the United States and the Netherlands constitute a fundamental redefinition of school mathematics as being based on an empiricist model, requiring independent thinking and the willingness to attempt creative, individualist approaches to the solution of unknown mathematics problems. In this, the "new new mathematics" is closer, both epistemologically and ontologically, to a view of western science.

This chapter looks at the differences as well as similarities of the Ontarian (Canadian) and Punjabi (Pakistani) Mathematics Education in such contexts, with historical sociology laying out the basis for seeking explanation for current reforms in both countries. While it is worthwhile to look into both these systems separately, as was done in earlier chapters, the comparisons draw us into the otherwise subtle features of both the nature of the curricula as well as their reforms.

Despite such diverse backgrounds and needs, the similarities within the curricula are indeed quite striking. For example, I find that the textbooks are very much tied to the assessment methods employed in the two systems, albeit that there are marked differences in their assessment methods.

In this regard, Pakistan faces greater challenges from cultural and religious sources, both as a developing nation as well as being thrust into the global limelight as one of the leading countries in the "war against terror". This led to a lot of pressure on the current government to remove questionable material from textbooks as well as revision of the educational system. The educational policy has aims (much like western models) to make changes so that critical thinking, creativity and problem solving are part and parcel of the newly forged educational policies. Here it is noted in Chapter 3, that the assessment methods have still remained unchanged, which calls into question the entire meaning of this so-called "curricular reform".

Contrasts with Historical Sociology as a background

Multiculturalism versus Homogenization

The objectives in the national policy of Pakistan and Canada differ in their outlook and hence the goals are appreciably different as a result. With an appreciably longer history, as well as shifts in population in most aspects, the Canadian curriculum and textbooks reflect this in the form of multiculturalism, celebration of diversity and a broader cultural outlook. The increased influx of immigrants in North America, as well as democratic values within politics has fostered and strengthened this tradition.

On the other hand, Pakistan, within its fifty years of independence has kept to narrow and focused educational policy of promotion of Islam and patriotism throughout. While there has been an influx of Afghan refugees as well as from Iran and India, there has been little shift in the population. The military has kept a stranglehold on political power and any attempts towards democratic values have proved to be largely unsuccessful. This could also explain the overall failures in educational policies as well as attempts at reforms, which have been abrupt for a relatively narrow framework. On the other hand, the Canadian policies allow for a gradual shift, so that all concerned adapt easily towards the change. This can be seen in the shift from the "mixed" mathematics Ontario curriculum (1997) towards a more constructivist-based document in 2005.

The Nature of Knowledge

The first distinctive difference that one notices between these two curricula is the sets of assumptions on the nature of knowledge and the motivation for studying mathematics.

Embedded in the traditions of the Islamic faith is the reverence of the power to recall and recite the Quran, in particular to be able to identify and recite accurately the appropriate sections for a particular question. The extent to which this view of gaining knowledge in an Islamic country like Pakistan, can be inferred from an Education examination paper, where in Question 1, in part, asks the candidate whether it is right that "Learning is confirmed by recapitulation." (Lahore and Gujranwala, 1994)

Regarding the application of rote memory to learning subjects, Hoodbhoy (1991, p. 38) quotes a 1959 incident recounted by the Indian-born chemist, J.B.S. Haldane:

> I was walking near my house one Sunday afternoon when I heard a male voice raised in a monotonous chant. I supposed that I was listening to some mantras, and asked my companion if he could identify them. The practice of repeating religious formulae is, of course, about as common in Europe as in Pakistan. But my companion stated that the language of the chant was English and the subject organic chemistry. We returned, and I found that he was right. The subject of the chant was the preparation of aliphatic amines, with special reference to various precautions.

Hoodbhoy further comments on this particular mode of learning,

> ... to a significant degree, the rote nature of contemporary education can be traced to attitudes inherited from traditional education, wherein knowledge is something to be acquired rather than discovered, and in which the attitude of the mind is passive and receptive rather than creative and inquisitive. The social conditioning of an authoritarian traditional environment means, as an inescapable consequence, that all knowledge comes to be viewed as unchangeable and all books tend to be

> memorized or venerated to some degree. The concept of secular knowledge as a problem-solving tool which evolves over time is alien to traditional thought (Hoodbhoy, 1991; p. 39).

Based on these premises he draws the comparison between what he terms as "Traditional education" and "Modern Education" given in Chapter 1. This thought process is explicitly seen in the Pakistani 1996 Education Federal Board examination paper I (Grade XI), where a multiple question asks,

> iii. Best time for memorization is: (Early morning-Afternoon-Night) (Federal Board, 1996, p.71)

I wonder at what the correct answer is!

If there is in fact "traditional thought" as a process, interwoven with religious belief, then it is not surprising that it reaches far beyond the teaching of religion. The 1964 thesis by Zaki draws a negative correlational connection between attitudes to science and to religion among science teachers (cited in Hoodbhoy, 1991; p. 39). Nayyar's (1998) chapter dealing with the history of madrasa education provides a useful summary of the evolution of this "Traditional Education", tracing it to as far back as eleventh century Baghdad. The pressure to Islamise the school system has been accompanied by the growth of the madrasa schools, which, although diverse in their nature, are generally characterized by a fundamentalist curriculum which purposefully eschews characteristics of modern education. As Nayyar (1998) points out, these schools have in recent years been receiving financial support through the government's Zakat fund. It is in these schools that I can see, albeit in an extreme form, the elements of Islamic traditionalism that are brought to bear on mathematics education in the system as a whole. The true thought is the sectarian: secular thinking stands in opposition to it and is therefore to be treated with caution. Mathematical thinking is of value only insofar as it can contribute to utilitarian application. This issue of traditional cultural attitudes, authority and mathematics is not just confined to the Pakistani case.

In the context of another Muslim nation, Fasheh (1982, p. 2) discusses the general state of students in what he calls Third World countries, “Why are most students who major in mathematics in these countries usually 'conservative' in their social outlook and their behaviour and 'timid' in their thinking and their analysis?” Fasheh's conclusion from his own experience in Palestine, is that mathematics teaching is a political act which, flowing from the way in which students experience their mathematics, leads to the creation of “... attitudes and intellectual models that will in their turn help students grow, develop, be critical, more aware and more involved ... or ... students who are passive, rigid, timid and alienated. There seems to be no neutral point in between.” (p. 7) In a society which discourages critical, original, and free thinking and statement, especially when that touches on “important” issues in the society. He continues, ‘Students who ask relevant questions about important events in the immediate community and see new alternatives and seek new interpretations of what exists are usually considered 'dangerous'. Teaching people to question, to doubt, to argue, to experiment, and to be critical, and teaching that increases the awareness of students, constitute, in my opinion, the real threat to existing and established institutions, beliefs and authorities everywhere and of every kind.’ (Fasheh, 1982, p. 7)

The brief popularization of mathematics and science clubs in the region is seen by Fasheh to have withered under “... the constant attacks, harassments, and hostile attitudes that began to mount from two directions. Both the Israeli authorities and fanatical conservatives among the local Arab population fought the existence of these clubs-each for its own reasons and in its own ways." (p. 7)

History, however, paints a more positive picture. The Islamic contribution to science and mathematics underwent a "Golden Age" between the 8th and 15th century period, where notable scholars like Robert Briffault, Will Durant, Fielding H. Garrison, Alexander von Humboldt, George Sarton, Muhammad Iqbal, Abdus Salam, and Hossein Nasr, consider the modern scientific method, which is based on empiricism and physical observation, to have primarily originated from Muslim thinkers. In this period, non-Muslims and Muslims working together in a social framework that allowed these activities to be fostered achieved original and pioneering contributions to all major areas of historiography, natural sciences, mathematics and social sciences. While detailing these achievements are beyond the scope of this thesis, it is notable that it is after the 12th century a steady decline in Islamic contributions started to occur. A possible reason for this trend is the eventual domination of the orthodox Islamic school of theology, known as the 'Ashari' school over the rationality-based theology followed by the 'Muzali' school of thought, which triggered the decline.

The recent conflict between "traditional" Islamic thinking (of "fanatical conservatives", for example) within its own framework, as well as that of the Greco-European tradition can be thus traced as far back as the 11th century Islamic theologian Al-Ghazzali, who recorded some of the most forceful criticisms of the nature of scientific and mathematical thinking. Historical events are essential to understand that the concern with mathematical and scientific knowledge is not particularly one of content, but rather one of the ways of thinking.

Al-Ghazzali states that,

> There are two drawbacks which arise from mathematics. The first is that every student of mathematics admires its precision and the clarity of its demonstrations. This leads him to believe in the philosophers and to think that all the sciences resemble this one in clarity and demonstrative power. Further, he has already heard accounts on everyone's lips of their unbelief, their denial of God's attributes, and their contempt for revealed truth; he becomes an unbeliever merely by accepting them as authorities (from Hoodbhoy, 1991; p. 105).

It therefore makes sense in the context of official policies in an Islamic country like Pakistan to encourage the Islamisation of the population. This means not only assuring instruction in the beliefs of Islam-religious education-but also instilling the curriculum with the ways of thinking and of dealing with problems which are taken as part of the Islamic tradition as locally interpreted. Thus the historical tension enters the Pakistani curriculum.

In the sense of the *longue durée*, the tensions between the secular and sectarian have a long tradition in Islam, as in other religions. Initially, two forms of knowledge were recognized as coexistent in Islamic schools: the secular and the sectarian. The merging of the two under the legitimizing framework of the sectarian, or religious, started in the twelfth century, and is related to the historical establishment of the madrasa schools. In this shift is embedded the interpretative frames of those holding political power.

Talbani argues that, historically, most Islamic societies never upheld originality, innovation, and change as essential values. The ideal of Islamic culture was not mechanical evolutionary progress but the permanent, final transcendental divinely revealed moral, theological, spiritual values of the Quran and Sunnah (Talbani, pp. 77-78). In his discussion of what he describes as "the crisis of legitimacy of science in Islamic schools", Talbani (1996) identifies the critical and innovative aspects of science as being incommensurable with traditional Islamic schooling.

The links with the power structures need to be understood in the long term; the madrasa schools provide an insight into this. Nayyar (1998) provides a useful account, from their eleventh century foundation to their role in present day Pakistan. From its inception, Pakistani education has been infused with the belief that the " ... system should be inspired by Islamic ideology" (Jalil, 1998, p. 36).

Krugly-Smulska (1994) in her use of Ogannyi's work, shows however, that the differences in traditions of thought need not necessarily lead to the failure of aspects of western thought to intrude into another, indigenous culture. Such notions of duality and commensurability are also taken up by Tambiah (quoted in Krugly-Smulska,1994), who argues that "there appears, for instance, to have been a congruence of cosmological principals of Protestant theology and the new theories of modern science." (p. 16). Krugly-Smulska points to "some consensus that for this social system [that is, a scientific community] to grow, it needed to be separated from the other systems of society, such as religion and politics" (p. 329). Herein lies a possible clarification as to how science and mathematics have been so successful within western cultures in recent times, not least its ability to invite scientific work from the Islamic, Indian and Chinese worlds. It is in the breaking up of this separation between the secular and sectarian divisions in the organization of Pakistan's curriculum that major resistance to reform might be found.

The nature of mathematical knowledge: Canada

In contrast, a modern approach adopted largely by Canada is based on ontological, epistemological and psychological assumptions that are based on diametrically opposing assumptions as compared to Pakistan. Hence, the problem-solving and constructivist learning based Canadian system launches its program in order to make lifelong learners while the Pakistani system expects specific algorithmic skills be mastered and reproduced by the students.

This of course stems from the views of the nature of knowledge and of Mathematics held by the educators and curriculum designers. In the Pakistani system, the prevalence of strict algorithmic skills, combined with a formal and proof-based mathematics curriculum indicates a heavy influence of a pure mathematics view in Mathematics education. In addition, the reliance on formalism is carried to an extreme level of execution when students are expected to reproduce exact proofs as well as to answer questions appearing directly from the textbook on the matriculation examination.

The constructivist thought process that pervades the current Canadian mathematics education curriculum and textbooks contrasts this. There is a great deal of individual attention paid in this system starting from the curriculum to the text and finally to the assessments themselves. This is of course due to the much more open view of mathematical thinking as being constructed within students rather than the other way around. The difference of such fundamental ideas of the nature of mathematics learning percolates into the textbooks as well as the examinations themselves.

Ideological Discourse in Textbooks

Talbani (1996) details the politicized nature of the Islamization of the Pakistani curriculum, in particular drawing attention to the use of Social Studies to rewrite aspects of history. In his paper, he provides a valuable demarcation of the way in which Islamic content enters the curriculum, in the identification of the use of Islam as the legitimating frame for forms of knowledge, and the usage of religion as justification of the state. There has been a significant power shift in Pakistan since independence, from secular visions to a religious state. As Talbani makes clear, even the image of Jinnah, the founder of the country, has been transformed completely in the textbooks and educational system, from a westernized intellectual striving for a secular home for Muslims, to a devout Islamist seeking a separate ideological state.

That this is still the case is evident, at least from the policy point of view in the Matriculation Federal Board's 1994 Education examination paper, in which Question 2 asks "What is the concept of education from Islamic point of view? Explain in the light of Muslim thinkers."

The religious and nationalistic content of the mathematics textbooks is described in Chapter 3 of this thesis, where it is clear that even Mathematics plays a role in fulfilling the need for the establishment of religious and national identity.

In sum, there is indeed a lot more overt political and religious material in the Pakistani mathematics texts than that of the Canadian texts where such aspects of ideological discourse are replaced by secularism and economic benefits of studying mathematics. This can be traced to the strong need for the Pakistani government to inculcate national and religious identity amongst the students. The Canadian texts, on the other hand, highlight secularism, cultural diversity and student individuality in order to fulfill the need for greater economic utility of the students.

Assessment Strategies

The sampling of the textbooks for the Pakistani examination questions, both in the 1990s as well as the most recent ones demonstrates the effect of reinforcing the narrow view of mathematical accomplishment in that it actively discourages students from going beyond the prescribed book. Rather, memorization and reproduction are rewarded. Significant changes are to be seen in the text contents. First, the content and contextual differences in the two language versions observed in those books for the lower grades have disappeared in the Grades 9/10 books. The view of mathematics disseminated throughout the primary and secondary school emerges as one based around a very particular image. The type of mathematics presented is an abstract, logical system based upon definitions and rules of application which are to be followed in order to obtain the one correct result, both in the nature of the method used and in the actual obtained answer/solution. Memorization becomes vital in the effort to reconstruct those aspects of knowledge which are to be acquired, as Hoodbhoy (1991, p. 39) notes.

I have been able to trace the questions on the Matriculation examination to the texts, and in the 1996 texts, as well as the latest ones and equity would demand matching of content as well. This would become a very powerful influence if the emphasis was to be upon recall of actual textual information, as is clearly the case.

This brings to view the level of sophistication of assessment methods which is quite different in both systems. For example, the Pakistani texts cater exclusively to the assessment, which takes place in the 9^{th} and 10^{th} grades. The Canadian texts are much more geared towards student development rather than just fulfilling "expectations" which is amply demonstrated in the "hands on" approach of the texts. In the Pakistani system students are merely expected to regurgitate learned material from the texts, which fulfills the assumptions of the subject being formal, proof-driven and a collection of algorithmic skills. This also runs in line with the marking of such exams that are graded on certain marks assigned to each examination question. The Canadian EQAO tests contain sections that put students in unique life-like and practical situations, which require greater conceptual effort on the part of the assessment methods. The questions, mostly multiple choice and written, have a much wider scope in recognizing the "Level" at which each student is currently working. The most recent Ontario document also requires students be assessed according to their mental "processes" engaged in learning Mathematics as detailed in Chapter 4.

The texts in Pakistan are exclusively produced by a small number of publishers that have dominated this area for decades. Hence, there is no variety in mathematical content presented. On the other hand, the more flexible Canadian system allows many publishers to present material for both teachers and the students (although subject to content evaluation), thus more options are available for mathematical content in Canada.

Teacher Education and Support

There is a lot more support for teacher education as well as available resources on tap for Canadian teachers than is the case in Pakistan. While this is to be expected due to the relative difference in budget and resource allocation for education in the two countries, the strikingly different requirements of each mathematical curriculum also drives this difference. The Nawaz Sharif government advocated an unprecedented 4 %

GNP allocation to education. However, before this was put into motion, the Musharraf government took over. The plans to move towards such an allocation are impeded by dependence on external and private funding.

By construction, the Ontario curriculum is individual centered and therefore the teacher is required to treat each student as more of a unique case than in the Pakistani system. In Pakistan, where the ultimate meaning of Mathematics may be passing the matriculation exams to gain access to higher education, the incentive is very low for teachers in the earlier grades to focus on individual cases, especially when the goal is to prepare them for passing the provincial exams. The "Examplars" in the Ontario case provide good examples of what is required by the nature of the mathematics education as for many teachers, adaptation from algorithmic teaching skills to problem-solving required re-education as well. In contrast, there is no need for such a document in the self-contained textbook system in Pakistan.

The similarities of the systems

The textbooks in both systems adhere quite strictly to the curriculum as well as to the assessment methods used by the authorities. This is amply demonstrated in the previous chapters with examples. There some degree of ambiguity of what is stated in the Pakistani mathematics curriculum. However, mathematical topics and content follow faithfully from the curriculum to the texts.

Transition periods: Intermediate curricular reform as reactions

Both systems saw the development of curriculum standards that reacted to the status quo and then reverted to different degrees, to the pre-existing mode of thinking. In the Canadian case, the study of the earlier Ontario standards (1995) reveals that the most evident view of the mathematics is constructivist in nature. This is evident in the emphasis of problem-solving as the basis behind the learning of Mathematics in all the grades of students. This is stated as one of the key components of Mathematics learning, "Problem solving, communication, reasoning, connections, and technology are central and essential components…" "Problem solving should be the central focus of the curriculum" (p.11). This was a close adaptation to the NCTM document produced in the USA. However, with a change in government, 1997 saw a transition period for mathematics and a "best of both worlds" curriculum was devised. This, being a reaction to the newly endorsed ideas of constructivist in mathematical mode of teaching lasted till the latest revision of the document in 2005. This, again, was bought about by a more liberal government and is much clearer about the underlying meaning of mathematical learning as based on problem solving and constructivism.

Similarly, the Pakistani system experienced a flush of new textbooks that minimized political and religious material starting in 2001/2002 when Ms. Zubaida Jalal was the Minister of Education. The texts were remarkably more activities-based as well as much more child-centered. This, of course was a reaction to the inertia of a system that has existed since the inception of the country. In 2003-4, much controversy regarding such textbooks resulted in Ms. Jalal stepping down from her position and the texts returned back to their original form. The reaction lasted for some time but indicated a period, albeit short, for which there was a remarkable and noticeable attempt at change in mathematics education of Pakistan.

Emphasis on Percentages for High School Grades

Despite the declaration of mathematical reforms and most crucially in the assessment sector, both the matriculation exams in Pakistan and high school grades in the Ontario are expressed in percentages. While this is not surprising at the relatively static and one-dimensional educational system in the case of Pakistan (where the matriculation exams are crucial and exacting levels to be passed), the case in Ontario clearly points to the social pressures in reinstating the belief regarding precision and accuracy of schools represented by grades as percentages. The recent 2005 document only addresses Grades 1 through 8 with assessment "Levels" which take into account "mathematical processes" as well other factors stated earlier in Chapter 4. However, *The Ontario Curriculum, Grades 9-10: Mathematics* (1999) states "Levels" alongside percentages attached to each of them.

Economic Motives: Information as a mathematical gold mine

Both countries are driven hard to compete internationally in order to achieve their economic goals. In this regard, there is a notable and uniform emphasis on Information Technology in both Pakistani and Canadian texts and curricula. Although the Pakistani texts are imbued with much more idealistic notions and messages by authorities calling on students to return the nation to past Islamic glories through Mathematics, it is nevertheless emphasized that financially rewarding and emergent sciences like Information Technology must not be missed at any cost.

Hence, in both mathematical policy documents, Information Technology is highlighted as one of the most promising and commercially viable applications of Mathematics learning. This is not only highlighted by greater emphasis on reliance on computers for mathematical learning but also greater mathematical content on data analysis and statistical methods, as indicated in the previous chapters. This is not entirely surprising, as both India and USA, which happen to be geographical neighbors, are world leaders in Information Technology. Hence, there are enough geopolitical and economic motives to emphasize this area of specialization to the students of both countries.

Concluding Remarks

The analysis done in this work points to the multifarious ways in which deeply embedded worldviews, combined with the recent policy decisions of successive governments, have penetrated the mathematics curriculum, textbooks and practices in both Canada and Pakistan. The historical, cultural and social influences emerge not only in the nature of examples and content, but are both included and excluded. They also emerge in implied understandings of the relation between knowledge and the learner.

In both countries, I find it instructive to revisit the differentiation made by Hoodbhoy between what he terms "traditional" and "modern" education, where tensions between the two are seen to interplay in different forms and manifestations in both countries. The current or "modern" mathematics curricular reforms in much of the Western countries, including Canada aim at the development of the individual student, with curricula which are developing in response to changes in the subject itself. Further, they are based upon the use of empiricist activities in combination with both inductive and deductive processes, at least partly with the aim of encouraging the student to become an independent solver of non-standard problems.

Teaching styles based upon developmental psychological models assume the internalized development of personal knowledge structures. Hence, assessment strategies have been elevated to the level of addressing of "mathematical processes" in the latest Ontario curriculum. However, this latest stage of "modern education" was not achieved, even in Canada till recently, without reactions impeding its full implementation.

In complete contrast, I have found considerable evidence that the Pakistani mathematics curriculum persistently takes a traditionalist approach to mathematical content, and Islamic principles are incorporated into problems and explanatory text at the primary (elementary) level, despite claims to the contrary in the recent educational policy. The instructive nature of the texts, in accordance with the nature of the matriculation examination point to an externalized view of knowledge, revealed through the authority of the text. Memorization of prescribed procedures therefore encourages conventionality at the expense of innovation in the dealing with standard problems. The educational policy is directed toward a particular form of Islamisation of formal schooling, not least in the aim to bring about a degree of convergence between formal and madrasah (mosque) schooling (Nayyar, 1998, Ministry of Education, 1988), militate against responding in an effective way to the other external pressures to bring the content and purposes of schooling to bear on meeting the educational outcomes requisite to providing the workforce and expertise needed to meet the human capital aspirations of the government.

The study of the mathematics curricula in this thesis finds a basis for the claims made concerning the collapse of the traditional differentiation of the secular and the sectarian forms of knowledge (in Pakistani education). The domination of knowledge which is revealed by authority has important consequences that emerge from the analysis, in particular in the relation between the well arranged text and the examination. It therefore becomes virtually impossible for Pakistan to take up the most basic and defining aspects of the constructivist and problem-solving approach towards the subject, as it stands incompatible with the nature of the international and Western curricular reforms. The incommensurability of the epistemological and ontological assumptions of such new mathematics curricula and those of traditional Islamic educators, as well as the need for reinforcing a national identity aggravates the problem, as it becomes not only one of addressing long held world views, but also the shorter term political concerns of those involved in the struggle for the school curriculum.

On the other hand, I find that the recent Ontario curricular reforms ease the tensions between the "traditionalist" passive-receptive method of teaching mathematics and the constructivist approach to the subject by being totally consistent with the "modern" or new "new Mathematics" approach. This measure is seen as being consistent with the latest reform mathematics curricular reforms set by the United States in 2000. Thus, the differences in findings in both the countries, the primary and high school texts, as well as the curriculum point to the value of an approach to dealing with the evidence which pays attention to the *longue durée* and *mentalité*, as well as the immediate and recent history of events.

REFERENCES

Barlow, M. & Robertson, H.-J. (1994). The right-wing alliance and the politics of education. In *Class warfare: The assault on Canada schools* (pp.112-137). Toronto: Key Porter Books.

Bailin, S., Case, R., Coombs, J., & Daniels, L. (1999). Common misconceptions of critical thinking. *Journal of Curriculum Studies*, 31, 269-283.

Braudel, F.(1980). *On History* (trans. S. Matthews) (pp.1-70). Chicago: University of Chicago Press. Braudel, F. (1980). History and the social sciences. The Longue durée. In *On history* (trans. S. Matthews) (pp. 25-54). Chicago: The University of Chicago Press.

Curriculum and Evaluation Standards for School Mathematics. (Mar 1989), NCTM, 1-12, 238-254.*Curriculum and Evaluation Standards for School Mathematics.* (Mar 1989), 1-12, 238-254.

E.D. Hirsch, Jr., (Fall 1996) Reality's revenge: research and ideology. *American Educator* 158-175.

Engineer, A.A. (1986). Islam and reformation. *Islam and the Modern Age 9* (1), 86-95.

Employability Skills Profile (1999), The Conference Board of Canada.

Fasheh, M. (1982). Mathematics, culture and authority. *For the learning of mathematics*, 3(2), 2-8.

Gardner, R. (1998). *Education. Demographic and health surveys. Comparative studies No. 29.* Calverton, Maryland: Macro International.

Government of Pakistan Ministry of Education (1998). *National Education Policy, 1998-2010.* Islamabad: Author.

Government of Pakistan Ministry of Education (2002). *Mathematics Curriculum 2000. For Classes IX-X.* Islamabad: Author.

Government of Pakistan Ministry of Education (2002). *Education Sector Reforms: Action Plan 2001-2004.* Islamabad: Author.

Grade 9 Assessment of Mathematics, 2000-2001. Education Quality and Accountability Office (2000).

Flewelling, G & Higginson, W (2001). *A handbook on rich learning tasks : realizing a vision of tomorrow's mathematics classroom.* Kingston, Ontario: Centre for Mathematics, Science and Technology Education, Faculty of Education, Queen's University.

Guidelines for Approval of Textbooks, Ontario Ministry of Education, (2006).

Hoodbhoy, P. (1991). *Islam and Science. Religious orthodoxy and the battle for rationality.* Karachi: Zed Books.

Hoodbhoy, P. (Ed.) (1998). *Education and the State. Fifty years of Pakistan.* Karachi: Oxford Press.

Jalil, N. (1998). Pakistan's education - the first decade. In P. Hoodbhoy (Ed.) (1998). *Education and the State. Fifty years of Pakistan* (pp. 23-42). Karachi: Oxford University Press.

Lahore and Gujranwala Board (nd). *Federal un-solved papers. Up-to-date for Matric.* Lahore: Azhar Publishers.

Federal Board (nd). *Federal unsolved question papers: First Year (Grade XI).* Rawalpindi: Sun Rise Publishers.

Krugly-Smolska, E. (1994). An examination of some differences in integrating Western science into societies with an indigenous scientific tradition. *Interchange 25* (4), 325-334.

Le Roy Ladurie, E. (1981). *The mind and the method of the historian* (trans. S & B. Reynolds). Chicago: University of Chicago Press.

McNay, M. (2000). The conservative political agenda in curriculum: Ontario's recent experience with science education. *Journal of Curriculum Studies, 32*(6), 749-756.

Ministry of Education (1988). *Deeni madrasis Pakistan ki Jam'e report 1988.* Islamic Education research Cell. Islamabad: Author.

Nayyar, A.H. (1998). Madrasa education-frozen in time. In P. Hoodbhoy (Ed.), *Education and the State. Fifty years of Pakistan* (pp. 215-250). Karachi: Oxford University Press.

Ontario Education Act (R. S. O. 1990, c.E. 2)

Phillips, D.C. (1995). The Good, the Bad, and the Ugly: The Many Faces of Constructivism. *Educational Researcher, 24(7)*, 5-12.

Pitman, A. (1989). Mathematics education reform in its social, political and economical contexts. In N.F. Ellerton & M.A. Clements (Eds.) *School Mathematics: The challenge to change.* Geelong, Australia: Deakin University Press.

Pitman, A. & Chishtie, F. (2004). Pakistan: The State, religion, and school mathematics. *Canadian and International Education* 33(2), 31-56.

Pitman, A. & Chishtie, F. (2004). Pakistan: A clash of globalising forces? In H. Daun & G. Walford (Eds.), *Educational strategies among Muslims in the context of globalization* (pp. 103-121). Leiden, Netherlands: Brill.

Punjab Textbook Board, Lahore (1996). *Mathematics (Grades1 to 8).* Lahore: Master Book Depot.

Punjab Textbook Board, Lahore (1996). *Mathematics 9th & 10th*. Lahore: Master Book Depot.

Punjab Textbook Board, Lahore (2003). *Mathematics (Grades1 to 8).* Lahore: Punjab Textbook Board.

Punjab Textbook Board, Lahore (2003). *Mathematics 9th & 10th*. Lahore: Punjab Textbook Board.

Romberg, T.A. (1997). The influence of programs from other countries on the school mathematics reform curricula in the United States. *American Journal of Education* 106 (1), 127-147.

Smith, D. (1991). *The rise of historical sociology*. Philadelphia: Temple University Press.

Stigler, J. W. & Baranes, R. (1998). Culture and Mathematics Learning. In E. Z. Rothkopf (Ed.), *Review of research in education 15, 1998-89* (pp. 253-306). Washington, DC: American Educational Research Association.

Talbani, A. (1996). Pedagogy, power and discourse: Transformation of Islamic education. *Comparative Education Review* 40 (1), 66-82.

Taylor, A. (1998). Employability Skills: from corporate 'wish list' to government policy. *Journal of Curriculum Studies, 30,* 143-164.

The Common Curriculum: Provincial Standards Mathematics, Grades 1-9 (1995).

The Ontario Curriculum, Grades 1-8: Mathematics (1997).

The Ontario Curriculum, Grades 1-8: Mathematics (1999).

The Ontario Curriculum – Exemplars, Grades 1-8: Mathematics (2001).

Teacher Package, Grades 1-8: Mathematics (2002).

The Ontario Curriculum, Grades 1-8: Mathematics (Revised) (2005).

von Glaserfeld, E. (1984). An introduction to radical constructivism. In P. Watzlawick (Ed.), *The invented reality,* (pp. 17-40). NewYork: W.W. Norton.

von Glaserfeld, E. (1991). Introduction. In E. von Glaserfeld (Ed.), *Radical constructivism in mathematics education,* (xiii-xx). Dordrecht, The Netherlands: Kluwer.

Windschitl, Mark. (1999). The challenges of sustaining a constructivist classroom culture. *Phi Delta Kappan*_150-157.

Zaki, W. M. (1964). *The attitudes of Pakistani science teachers towards religion and science.* Unpublished thesis, School of Education, Harvard University.

APPENDIX A

The 2005 Ontario Mathematics curriculum states problem solving to be the key in teaching, learning and applying Mathematics. Here are the stated benefits.

Problem solving:

- is the primary focus and goal of mathematics in the real world;
- helps students become more confident in their ability to do mathematics;
- allows students to use the knowledge they bring to school and helps them connect mathematics with situations outside the classroom;
- helps students develop mathematical understanding and gives meaning to skills and concepts in all strands;
- allows students to reason, communicate ideas, make connections, and apply knowledge and skills;
- offers excellent opportunities for assessing students' understanding of concepts, ability to solve problems, ability to apply concepts and procedures, and ability to communicate ideas;
- promotes the collaborative sharing of ideas and strategies, and promotes talking about mathematics;
- helps students find enjoyment in mathematics;
- increases opportunities for the use of critical-thinking skills (estimating, evaluating, classifying, assuming, recognizing relationships, hypothesizing, offering opinions with reasons, and making judgements). (p.11-12).

APPENDIX B

The "Overall Expectations" and "Specific Expectations" for Grade 3 "Number Sense and Numeration" in the *Ontario Curriculum: Mathematics Grades 1-8* (1997) are as follows:

"By the end of Grade 3, students will:

- represent whole numbers using concrete materials, drawings, numerals, and number words;
- compare and order whole numbers using concrete materials, drawings and ordinals;
- represent common fractions and mixed numbers using concrete materials;
- understand and explain basic operations (addition, subtraction, multiplication, division) involving whole numbers by modelling and discussing a variety of problem situations (e.g., show divisions as sharing, show multiplication as repeated addition);
- develop proficiency in adding and subtracting three-digit whole numbers;
- develop proficiency in multiplying and dividing one-digit whole numbers;
- select and perform computation techniques (addition, subtraction, multiplication, division) appropriate to specific problems and determine whether the results are reasonable;
- solve problems and describe and explain the variety of strategies used;
- justify in oral or written expression the method chosen for addition and subtraction, estimation, mental computation, concrete materials, algorithms, calculators;
- use a calculator to solve problems beyond the required pencil-and-paper skills."

The Specific Expectations are stated as follows: "Students will:

Understanding Number

- read and print numerals from 0 to 1000;
- read and print number words to one hundred;
- count by 1's, 2's, 5's, 10's and 100's to 1000 using various starting points and by 25's to 1000 using multiples of 25 as starting points;
- count backwards by 2's, 5's, and 10's from 100 using multiples of 2, 5, and 10 as starting points and by 100's from any number less than 1001;
- locate whole numbers to 100 on a number line and partial number line (e.g., from 79 to 84);
- show counting by 2's, 5's, and 10's to 50 on a number line and extrapolate to tell what goes before or after the given sequence;
- identify and describe numbers to 1000 in real-life situations to develop a sense of number (e.g., tell how high a stack of 1000 pennies would be);
- model numbers grouped in 100's, 10's and 1's and use zero as a place holder;
- use ordinal numbers to hundredth;
- represent and explain common fractions presented in real life situations, as a part of a whole, part of a set, and part of a measure using concrete materials and drawings (e.g., find one-third of a length of ribbon by folding);

Computations

- investigate and demonstrate the properties of whole number procedures (e.g., $7 + 2 = 9$ is related to $9 - 7 = 2$);

- use a calculator to examine number relationships and the effect of repeated operations on numbers (e.g., explore the pattern created in the units column when 9 is repeatedly added to a number);
- interpret multiplication and division sentences in a variety of ways (e.g., using base ten materials, arrays);
- identify numbers that are divisible by 2, 5, or 10;
- recall addition and subtraction facts to 18;
- determine the value of the missing term in an addition sense (e.g., 4 + _ = 13);
- demonstrate and recall multiplication facts to 7 x 7 and division facts to 49 / 7 using concrete materials;
- mentally add and subtract one-digit and two-digit numbers;
- add and subtract three-digit numbers with and without regrouping using concrete materials;
- add and subtract money amounts and represent the answer in decimal notation (e.g., 5 dollars and 75 cents plus 10 cents is 5 dollars and 85 cents, which is $5.85);

Applications

1. pose and solve problems involving more than one operation (e.g. if there are 24 students in our class and five boys and 9 girls wore boots, how many students did not wear boots?);
2. use appropriate strategies (e.g., pencil and paper, calculator, estimation, concrete materials) to solve number problems involving whole numbers;

3. use various estimation strategies (e.g., clustering in tens, rounding to hundreds) to solve problems, then check results for reasonableness."

APPENDIX C

The Trillium "eligibility requirements" for mathematics textbooks are stated as follows:

a) The content must be consistent with that in one or more elementary subjects or secondary courses described in Ontario curriculum policy documents, or with that in one or more Kindergarten learning areas, and must support at least 85 per cent of the expectations for a Kindergarten learning area, an elementary subject in a specific grade, or a secondary course (i.e., a course in a specific grade in a secondary subject/discipline).

b) In the case of a series, at least one title in the series must support at least 85 per cent of the expectations for a Kindergarten learning area, an elementary subject in a specific grade, or a secondary course. Individual titles in a series that do not support 85 per cent of the expectations for a learning area, subject, or course are not eligible for evaluation.

c) If non-print material (for example, a CD-ROM or manipulative material) makes up part of a textbook, and if the whole textbook package of which it is a part supports at least 85 per cent of the expectations for a Kindergarten learning area, an elementary subject in a specific grade, or a secondary course, this material will be eligible for consideration and will be included in the evaluation.

d) A multimedia package that supports at least 85 per cent of the expectations for a Kindergarten learning area, an elementary subject in a specific grade, or a secondary course will also be considered to be a textbook and will be eligible for evaluation.

Provision of a Teacher's Resource Guide

Textbooks must be accompanied by a teacher's resource guide. This guide must be provided in both languages of instruction (English and French) if the textbook is translated.

Canadian Orientation

The content must have a Canadian orientation. It must acknowledge Canadian contributions and achievements and use Canadian examples and references wherever possible. It must use Canadian spelling conventions and SI units (units of measurement of the Système international d'unités, or International System of Units) for measurement references. The vocabulary and examples should be familiar to Canadians.

Canadian Product

Textbooks must be manufactured in Canada and, wherever possible, are to be written, adapted, or translated by a Canadian citizen or citizens or by a permanent resident or residents of Canada. (pp. 7-8)

The full "Evaluation criteria" for the textbooks, is stated as follows:

Content Quality

The content must be of sound scholarship and must have contemporary relevance. The information must be presented in adequate depth and sophistication for the grade or learning area/subject/course and build on students' previous knowledge and skills. Graphics, such as charts, diagrams, and illustrations and photos should be used where appropriate to support students' understanding of the content.

Reference to Use of Technology

The content must reflect uses of technology related to the Kindergarten learning area, elementary subject, or secondary course, where appropriate, and allow students to use and develop these skills.

Health and Safety

Attention to safe practices must be evident through appropriate warnings and information; portrayal of people in learning, working, and playing situations; and the suitability of the learning activities.

Environmental Responsibility

The content must reflect concepts of environmental responsibility, where appropriate, within the context of the Kindergarten learning area, elementary subject, or secondary course.

Language Level

The language used must be appropriate for the reading level of the grade for the elementary subject or secondary course, or for the Kindergarten program. The material must also be written in a style appropriate for the learning area/subject/ discipline. Language, symbols, and technical terms that are subject- or discipline specific must be used in contexts that students would understand.

Instructional and Assessment Strategies

The content must support a broad range of instructional strategies and learning styles. The activities must be appropriate for the skills and knowledge described in the curriculum or learning expectations. The activities must also provide opportunities for students to engage in higher-order thinking and problem solving, to apply concepts and procedures, and to communicate their understanding. There should be a range of tasks – that is, open-ended tasks, teacher-directed tasks, and tasks for students to do independently. The content and activities should be appropriate for students from diverse backgrounds and at different levels of physical ability. It should include, as appropriate, ways of helping students make connections within and between the strands of the subject or course, or within and between learning areas in Kindergarten, and between the subject/course content and the community and workplace. The connections between instructional strategies and assessment should be meaningful and should be consistent with the assessment strategies for the subject or course.

Bias

The content must be free from racial, ethnocultural, religious, regional, genderrelated, or age-related bias; bias based on disability, sexual orientation, socioeconomic background, occupation, political affiliation, or membership in a specific group; and bias by omission. The material should present more than one point of view, and be free from discriminatory, exclusionary, or inappropriately value-laden language, photographs, and illustrations. (p. 8-9)

APPENDIX D

Under the heading "Expectations addressed in the Exemplar Task" the package states that the tasks will involve three expectations from the Ontario curriculum (1997) for Grade 3. These expectations are Measurement, Patterning and Algebra and Data Management and Probability. The stated expectations are (p.53):

Measurement

Students will:

- demonstrate an understanding of and ability to apply measurement terms: centimetre, metre, kilometre; milliliter; gram, kilogram; degree Celsius; week, month, year (3m34);
- identify relationships between and among measurement concept (3m35);
- solve problems related to their day-to-day environment using measurement and estimation (e.g., in finding the height of the school fence) (3m36);
- estimate and measure the passage of time in five-minute intervals, and in days, weeks, months and years (3m44);
- demonstrate the relationship between all coins and bills up to $100 (3m48);
- make purchases and change money amounts up to $10, and estimate, count, and record the value up to $10 of a collection of coins and bills (3m49);
- read and write money amounts using two forms of notation (89 c and $0.89) (3m50).

Patterning and Algebra

- identify, extend and create linear and non-linear geometric patterns, number and measurement patterns and patterns in their environment (3m78).

Data Management and Probability

Students will:

- collect and optimize data (3m91);
- demonstrate an understanding of probability and demonstrate the ability to apply probability in familiar day-to-day situations (3m93);
- organize data in Venn diagrams and charts using several criteria (3m100);
- conduct simple probability experiments (e.g. rolling a number cube, spinning a spinner) and predict the results (3m103);
- apply the concepts of likelihood to events in solving simpler problems (3m104);
- predict the probability will occur (3m105);
- use mathematical language (e.g., possible, impossible) in discussion to describe probability (3m106).

APPENDIX E

The 2005 Ontario Mathematics curriculum document has the following assessment standards for mathematics learning.

Achievement Chart – Mathematics, Grades 1–8

Categories	Level 1	Level 2	Level 3	Level 4
Knowledge and Undestanding	*Subject-subject content acquired in each grade (knowledge), and the comprehension of its meaning and significance (understanding)* **The student:**			
Knowledge of content (e.g., facts, terms, procedural skills, use of tools) Understanding of mathematical concepts	– demonstrates limited knowledge of content – demonstrates limited understanding of concepts	– demonstrates some knowledge of content – demonstrates some understanding of concepts	– demonstrates considerable knowledge of content – demonstrates considerable understanding of concepts	– demonstrates thorough knowledge of content – demonstrates thorough understanding of concepts
Thinking	*The use of critical and creative thinking skills and/or processes**			
Use of planning skills – understanding the problem (e.g., formulating and interpreting the problem, making conjectures) – making a plan for solving the problem	– uses planning skills with limited effectiveness	– uses planning skills with some effectiveness	– uses planning skills with considerable effectiveness	– uses planning skills with a high degree of effectiveness
Use of processing skills* – carrying out a plan (e.g., collecting data, questioning, testing, revising, modelling, solving, inferring, forming conclusions) – looking back at the solution (e.g., evaluating reasonableness, making convincing arguments, reasoning, justifying, proving, reflecting)	– uses processing skills with limited effectiveness	– uses processing skills with some effectiveness	– uses processing skills with considerable effectiveness	– uses processing skills with a high degree of effectiveness
Use of critical/creative thinking processes* (e.g., problem solving, inquiry)	– uses critical/creative thinking processes with limited effectiveness	– uses critical/ creative thinking processes with some effectiveness	– uses critical/creative thinking processes with considerable effectiveness	– uses critical/creative thinking processes with a high degree of effectiveness

Categories	Level 1	Level 2	Level 3	Level 4
Communication	*The conveying of meaning through various forms* **The student:**			
Expression and organization of ideas and mathematical thinking (e.g., clarity of expression, logical organization), using oral, visual, and written forms (e.g., pictorial, graphic, dynamic, numeric, algebraic forms; concrete materials)	– demonstrates limited knowledge of content – demonstrates limited understanding of concepts	– demonstrates some knowledge of content – demonstrates some understanding of concepts	– demonstrates considerable knowledge of content – demonstrates considerable understanding of concepts	– demonstrates thorough knowledge of content – demonstrates thorough understanding of concepts
Communication for different audiences (e.g., peers, teachers) and purposes (e.g., to present data, justify a solution, express a mathematical argument) in oral, visual, and written forms	– communicates for different audiences and purposes with limited effectiveness	– communicates for different audiences and purposes with some effectiveness	communicates for different audiences and purposes with considerable effectiveness	– communicates for different audiences and purposes with a high degree of effectiveness
Use of conventions, vocabulary, and terminology of the discipline (e.g., terms, symbols) in oral, visual, and written forms	– uses conventions, vocabulary, and terminology of the discipline with limited effectiveness	– uses conventions, vocabulary, and terminology of the discipline with some effectiveness	– uses conventions, vocabulary, and terminology of the discipline with considerable effectiveness	– uses conventions, vocabulary, and terminology of the discipline with a high degree of effectiveness
Application	*The use of knowledge and skills to make connections within and between various contexts* **The student:**			
Application of knowledge and skills in familiar contexts	– applies knowledge and skills in familiar contexts with limited effectiveness	– applies knowledge and skills in familiar contexts with some effectiveness	applies knowledge and skills in familiar contexts with considerable effectiveness	applies knowledge and skills in familiar contexts with a high degree of effectiveness
Transfer of knowledge and skills to new contexts	– transfers knowledge and skills to new contexts with limited effectiveness	– transfers knowledge and skills to new contexts with some effectiveness	-transfers knowledge and skills to new contexts with considerable effectiveness	– transfers knowledge and skills to new contexts with a high degree of effectiveness
Making connections within and between various contexts (e.g., connections between concepts, representations, and forms within mathematics; connections involving use of prior knowledge and experience; connections between mathematics, other disciplines, and the real world)	– makes connections within and between various contexts with limited effectiveness	– makes connections within and between various contexts with some effectiveness	– makes connections within and between various contexts with considerable effectiveness	– makes connections within and between various contexts with a high degree of effectiveness

* The processing skills and critical/creative thinking processes in the Thinking category include some but not all aspects of the mathematical processes described on pages 11–17 of this document. Some aspects of the mathematical processes relate to the other categories of the achievement chart.