Piston Modelling and Gas-Solid Mixing Characterization in JBR

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Abstract

The Jiggle Bed Reactor (JBR), a batch fluidized bed microreactor, is inexpensive and easy to operate with a small amount of solids. The main goal of this thesis was to provide practical tools for the design and operation of JBR systems that will ensure good gas-solids mixing.

A model was developed to predict the piston motion. The model uses four empirical parameters that change depending on the equipment characteristics. Two parameters characterize the gas supply to the piston and two parameters are used to describe the frictional forces on the piston. The model was validated with high speed video.

Experiments were conducted to study the effect of the reactor motion on the gas-solid mixing in the JBR. A good correlation between gas-solid mixing in the reactor and the maximum acceleration of the piston was identified: good gas-solid mixing was achieved with maximum accelerations greater than 55 m/s².

Keywords

Micro-reactor, Jiggle-Bed-Reactors, JBR, Mechanically Fluidized Reactor, Gas-solid Mixing, Pneumatic Piston, Pneumatic Actuator, Piston Modelling, Cold Model, Mixing, MATLAB, image analysis
Co-Authorship Statement (where applicable)

Mr. Pengzhi Mao, an undergraduate exchange student from University of Tianjin, China, has made significant contributions primarily in the program development and troubleshooting work of chapter 3, piston modelling using MATLAB.
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I am also thankful to all my colleagues in the Institution for Chemicals from Alternative Resources (ICFAR). My special thanks go to the on-site technician Thomas Johnson who devoted much of his work and talent to assist me building the solid vessel and post-doc fellow Dr. Francisco Javier Sanchez Careaga who kindly helped me developed the Arduino based control and DAQ program for the apparatus. Former post-doctoral fellow Stefano Tacchino devoted some of his precious time to help me learning how to conduct research when I first started my master program back in the summer of 2016. My good colleague Aaron Joness provided many valuable and creative advice and ideas for compiling the hardware configuration and MATLAB analytic scripts for this project. Mr. Pengzhi Mao, a visiting undergraduate student from the University of Tianjin also contributed greatly to the work on the piston modelling. Many of my other colleagues like Tracy Hu kindly provided me a lift to the institution during countless weekend shifts. Without their help, this thesis would have been much delayed. I wish them all best in their life and future career.

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LIST OF ABBREVIATIONS / NOMENCLATURE

α  Kinetic friction exponent term
β  Kinetic friction coefficient term
γ  The ratio of gas heat capacity terms (C_p/C_v)
φ  Reduction coefficient for mass flow passing through an orifice that has a changing cross sectional area
ζ  The local resistance coefficient which relates the shape and area ratios of the upstream and downstream cross sectional areas
ρ  Density of air, kg/m^3
Ω  The theoretical transition for the fluid flow
a  Acceleration of the piston, m/s^2
A_e  The effective area of the valve port, m^2
A_l  The effective lower piston block area, m^2
A_u  The effective upper piston block area, m^2
A_r  Cross sectional area of the double rods of the piston block, m^2
A_t  Cross sectional area of the tube, m^2
A_leak0  Potential leakage area around the piston top plate and the double-rod, m^2
A_leak1  Potential leakage area around the piston moving block and cylinder wall, m^2
b  The critical upstream to downstream pressure ratio across the valve
c  Sonic speed at ambient lab conditions, m/s
C  The valve conductance coefficient, NL/min*bar
C_f  Mass flow coefficient for air flow in the overall JBR system
C_{fv}  Mass flow coefficient for air passing through the double solenoid valve
C_p  Molar specific heat capacity at constant pressure of an ideal gas, J/K
C_v  Molar specific heat capacity at constant volume of an ideal gas, J/K
D  The diameter of the fluid flow, m
F_t  Coulomb friction forces, N
F_c  Controlling forces acting on the valve spool within the double solenoid valve, N
g  Gravitational acceleration, 9.81 m/s^2
G  Mass flow of air in general, kg/sec
h  Specific enthalpy of air, J/kg
ICFAR  Institute for Chemical and Fuels From Alternative Resources
JBR  Jiggle Bed Reactor
L_t  Length of the soft tube, m
M  Mass of the piston moving block, kg
M_s  Mass of the valve spool
P_a  Atmospheric pressure at lab conditions, 101325 kPa, 1 atm or 14.7 psi in this study
P_d  The downstream pressure, Pa
P_L  Air pressure acting onto the lower part of the piston block, Pa
P_u  The upstream pressure, Pa
P_U  Air pressure acting onto the upper part of the piston block, Pa
q_{in}  Rate of heat transferred into the boundary, J/s
q_{out}  Rate of heat transferred out of the boundary, J/s
R  Ideal gas constant, 8.314 J/mol*K
Re  Reynolds number for air flow
R_h  Radius of the valve port, m
R_t  Tube resistance for calculating the pressure drop across the tube
T_d  The downstream temperature for the air flow, K
T_u  The upstream temperature for the air flow, K
u  Air flow velocity, m/s
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>v</td>
<td>Piston velocity, m/s</td>
</tr>
<tr>
<td>V</td>
<td>Volume of the air flow in a specified system, m$^3$</td>
</tr>
<tr>
<td>x</td>
<td>Spool displacement within the double solenoid valve, m</td>
</tr>
<tr>
<td>$x_e$</td>
<td>Effective spool displacement within the double solenoid valve, m</td>
</tr>
<tr>
<td>z</td>
<td>Piston displacement, m</td>
</tr>
<tr>
<td>$z_0$</td>
<td>Initial gap between piston and piston’s bottom flange, m</td>
</tr>
<tr>
<td>$z_{0\text{Total}}$</td>
<td>Total virtual tube length for the air to fill-up, m</td>
</tr>
</tbody>
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1 Introduction

Conventional reactors such as gas-solid fluidized bed and other bench scale or pilot plant scale reactors do provide useful means to achieve rapid heat and mass transfer. Nevertheless, they are not efficient to test new catalyst formulations as they usually require large quantity of catalysts.

To resolve this issue with reduced testing costs, microreactors are a better option [Latifi et al., 2017]. However, technologies involving micro-scale fluidized bed reactors are still quite limited in current market. Some microreactors like the Short Contact Time Resid Test unit or simply the SCT-RT unit are used to test catalysts over very short residence time [Imhof et al., 2004]. Others try to solve this short residence time problem by implementing a high recycle rate, but they often encounter an axial coke profile [Berty et al., 1979]. One modified approach to solve this issue is to include an internal impeller to provide more intense gas-solid mixing, but the mechanical seals tend to cause gas leakage problems especially when operated at high pressure and high temperature [Kraemer et al., 1988].

As a recent innovation, the Jiggle Bed Reactor (JBR) provides an alternative to achieve mechanical gas-solid fluidization in a lab setting. This new microreactor has been developed by a team of Western scholars and professors based in the Institute of Chemical from Alternative Resources (ICFAR) [Latifi et al., 2009]. The featured mixing pattern for JBR is “shaken but not stirred”. In addition, unlike its pertinent peers, it is small, easy and inexpensive to operate and troubleshoot. It gives rapid and reproducible mixing for gas-solid mixing. Most of all, the little temperature gradient

The JBR is primarily designed to fulfill the gap of micro fluidizers for lab uses. The JBR consists of a sealed container of solid particles that is attached to a piston that is rapidly moved up and down by a pneumatically powered actuator. As a result, substances in the container are simply mixed by this up and down oscillation instead of using mechanical agitators or a fluidizing gas [Latifi et al., 2015].

Since piston needs to act rapidly under different weight loads, large forces with short responsive time are required. In this case, hydraulic actuators are no longer a choice as they have higher
risks of leakages and more requirements on hardware maintenance. For instance, a hydraulic actuator requires more accessories, such as fluid reservoir, motors, pumps, noise-reduction equipment, release valves, and heat exchangers, apart from the actuator itself. All these companion parts make the hydraulic actuators bulky and more difficult to accommodate.

The electromagnetic systems, although provide quiet and high precision-control for the piston motion, are typically much more expensive than hydraulic and pneumatic actuators. The motor of the electromagnetic system also tend to overheat. Due to such deficiencies, electrical actuators are not suitable for this study because of the high temperature and potentially flammable operations in the nearby work space. As the actuators; force, thrust and speed are primary dependent and therefore limited by the selected motor, for a different set of motion parameters, the motor is required to be changed. On the other hand, pneumatic actuators provide sufficient motion with the access to the readily available dry compressed air on site. Since pneumatic actuators are safe, responsive, low cost and simple to operate and maintain, they are chosen to provide the driving force of gas-solid mixing in this study.

Previously, the ICFAR team had successfully trialed biomass pyrolysis, and catalytic steam reforming using the JBR. Previous experiments showed excellent fluidization of solid particles with reproducible results by using time-consuming trial and error to find an appropriate frequency and amplitude of the actuator as well as pressure of the compressed air feed to the actuator [Latifi et al., 2014]. Nevertheless, researchers are striving to understand the relationship between piston parameters such as actuator frequency and amplitude, and the mixing behavior of the substances in the JBR. Therefore, the main goal of this study is to understand the realationship among the JBR size and the mixing characteristics as functions of the operating parameters of the actuator (acceleration and amplitude). The strategy is to develop and test mathematical models of the JBR.

1.1 Motivations to further develop the JBR system

The solids in the JBR are mechanically fluidized. Unlike conventional micro fluidized bed reactors, the JBR does not require injecting a fluidizing gas to the vessel. When compared to other microreactors with mechanical mixers or fluidization gas, this further preserves bed materials throughout the process, making the JBR particularly suitable for testing precious and
often expensive catalysts. Figure 1-1 is adopted from [Latifi et al., 2014]. It gives a general overview on what a JBR looks like; the unit normally consists of three components: the pneumatic actuator along with its controller which facilitates vertical motion, the vessel which contains the reactants, and the heating units, in this figure, conductive coils are selected. For instance, when performing biomass pyrolysis and gasification, the initial feedstock is loaded to the vessel. As the reaction propagates, the products such as bio-oil vapor and syngas at high temperature can be withdraw continuously from a probe with a built-in check-valve. Hence, the resultant process can be viewed as a semi-batch process. The JBR is quite versatile; in addition to the biomass related operations, it may be used to test catalysts, gas absorbents as well as producing activated carbon. Moreover, previous work done in ICFAR had demonstrated the rapid and reproducible results using JBR over a time as short as 10 mins. [Latifi et al., 2009].

![Figure 1-1](image_url)

**Figure 1-1** A schematic illustration of the JBR system developed earlier [Latifi et al., 2014].

Figure 1-2 shows that, as the JBR comes down, the solid particles do not move down as quickly. Because of their inertia, and solid particles appear to move up relative to the JBR wall: the bed expands [Latifi et al., 2014]. Figure 1-3 shows that, as the JBR comes back up, the solid particles...
do not move up as quickly, because of their inertia, and solid particles appear to move down relative to the JBR wall: the bed contracts [Latifi et al., 2014].

![Figure 1-2 Sequences of mixing of solid particles in the JBR with bed expansion during downward actuator retraction [Latifi et al., 2014].](image1)

![Figure 1-3 Sequences of mixing of solid particles in the JBR with bed contraction during upward actuator extension [Latifi et al., 2014].](image2)

With the JBR, one needs to carefully adjust the frequency and amplitude of the actuator oscillations to ensure a well-agitated gas-solid mixture across the vessel throughout the experiments for a given amount of solid.

### 1.2 Assessing solid mixing

One of the key aspects of JBR is its ability to mix solid within a gas filled vessel. Therefore, finding a way to assess the mixing performance of the JBR becomes one of the primary objectives of this study. Previously, some scholars like [Liu et al., 2015] recommended image processing methods as they do not require sampling of the solid bed which may interrupt the
continuous JBR operations. The vessel is then made with transparent material which allows the high speed camera to capture the bed motion and mixing, frame by frame.

In general, for conducting image analysis of the mixing dynamics, there are generally two types of image processing algorithms: variance or contact [Bridgewater et al., 2012 & Van Puyvelde et al., 1999]. In this study, since gas-solid mixing is involved, the variance method is used. To implement the variance method, the vessel region has to be first subdivided into a certain number of cells. The concentration variance of the component, in this case, the white sand versus the black background, is used to quantify the mixing quality using the following expression adopted from [Liu et al., 2015]:

$$\text{variance} = \frac{1}{n-1} \sum_{i=1}^{n} (\text{concentration}_i - \text{concentration}_{avg})^2$$

Here, n is the number of cells; whereas, the concentration at the i-th cell can be calculated as the following:

$$\text{concentration}_i = \frac{\text{number of white pixels}}{\text{total number of pixels}}$$

Where $$\text{concentration}_{avg} = \frac{1}{n} \sum_{i=1}^{n} \text{concentration}_i$$

Since the sand particles are very fine, each pixel within the image matrix can be counted as one cell. Therefore, once the above variance method can be applied to each action frame, the overall change of concentration variance with time can be plotted. Preliminary calibrations showed that roughly every 1.25 pixel captured in the action frame with the setting provided in this study is equivalent to 1 mm length in real life.

Note that the gas-solid mixing study will be done using white sand, not a binary mixture. The variance method will be used for assessing the vertical dispersion of the sand particles along the vessel wall.
1.3 Objectives and scope

The primary objective is to provide practical tools for the design and operation of lab scale JBR systems that will ensure good gas-solids mixing.

Hence, this study proceeds with the following steps:

1) The motion of the JBR reactor is modeled and compared to experimental measurements.
2) Experiments were conducted to monitor the motion of solids within the reactor and determine which characteristics of the JBR reactor motion are essential to ensure good gas-solids contact within the JBR.
2 Experimental Setup

To achieve the objectives of this study, a small JBR cold model has been built in ICFAR. In addition to the vessel, other major equipment used in this thesis project can be broken down into 3 major parts: the piston and valve complex, the controller, and the data acquisition device (DAQ). Figure 2-1 shows the setup of the JBR system used in this study with each major component labelled.
Figure 2-1 Overview of major components in the experimental apparatus: (a) high speed camera, (b) solid vessel, (c) Red LED, (d) Pneumatic Actuator, (e) Pressure Transducers, (f) Double solenoid Valve, (g) Control board with relays.
2.1 The vessel and others

A vessel made of Plexiglas, see figure 2-2 below, which is 26.5 cm long, 15.4 cm wide and 1.0 cm thick, is mounted on the piston plate through bolts and nuts. As for the dimensions the inner space has a diameter of 3.81 cm (or 1.5 inch), a wall thickness of 0.635 cm (or 0.25 inches), and a spool length (length of the inner volume) of 13.5 cm; The vessel is transparent and flanged for containing the phosphorescent particles. The transparent wall enabled image capture during mixing, which depicts how the solid bed was lifted, expanded and collapsed. Whereas, the overall length of the vessel including both flanges and bolts is 15.0 cm. This gives a total inner volume of 154 ml. Also note that as shown in figure 2-2, all surfaces were covered in black; for instance, the vessel wall at the back of the vessel is taped in black to make strong contrast with the white silica sand. The sand has a density of 2.65 g/cm³ with a Sauter-mean diameter close to 190 μm. Moreover, some of the frontal surface of the two vessel flanges facing the camera are covered in paper painted with non-reflective black dye.

The aluminum alloy plate acts as a support to the vessels and other units. This thickness of the platform plate further ensured the stiffness of the plate during rapid piston strokes while keeping all units on the plate at the same level.

In addition, a small red LED (light emitting diode) is placed on the platform. This red LED is used to trace the piston motion recorded in the high speed video clips. A signal generator acts as a power adaptor which converts AC power to DV power for the LED. The signal generator set the voltage limit to be less than or equals to 3.5 volts while the current is less than or equals to 0.020 mA. These settings were chosen to protect LED from damage caused by power surge. As a result, each video frame captured by the high speed camera shall only reveal the red LED and the white solid bed.
Figure 2-2 Transparent JBR vessel; note that its backside (along with the platform) is covered in black tape to make contrast with the white sand during JBR trials with the red LED wrapped in black showing only its tip (see the boxed region).

Figure 2-3 Signal generator used for powering the red LED.
2.2 Piston and valve complex

The piston used in this study is manufactured by BIMBA with model number FT-3112. It is a double acting FLAT-II model as depicted in part (d) of figure 2-1. For this piston is using dual rods connecting to a single rod end block, non-rotation operation is achieved. Hence, no alignment device such as guides or rods is required.

The body of the piston is built with 304-stainless steel. Anodized aluminum alloy makes up the piston heads. Piston rods are made of ground and polished 303-stainless steel. The sealing for the piston rods are Buna N O-rings while the rod bushing has oil impregnated bronze. The piston is capable to operate under a maximum air pressure rating of 200 psig within a temperature range of -25 °C to + 65 °C.

As for its physical dimensions, the piston cylinder bore size is 5.08 cm or (2 inches) with a stroke length of 30.48 cm (or 12 inches). The two air ports, at the top and bottom are designed to accommodate 0.635 cm (or 1/4 inch) NPT fittings with an internal opening of 0.158 cm (or 1/16 inches). The mechanical drawings with exact dimensions provided by the manufacturer can be found in Appendix D.

2.3 Controller

The controller, shown in part (g) of figure 2-1, consists of two parts: Arduino UNO board and Sainsmart time relay. The Arduino board forms the backbone to implement the control algorithm. The detailed code of the Arduino control program can be found in Appendix C.

The basic idea of the control algorithm which can be demonstrated using figure 2-4: as soon as the toggle switch is turned on by the user, current enters the relay circuit. Red line is positive black line is negative. The Arduino starts a loop which according to the pre-set time differences, and generates on/off commands as an electric power output. This output, essentially an electric signal, is sent to the relay. The relay further activates the double solenoid valve and hence regulates the motion of the piston.
2.4 Valve and gas line

Compressed air is used in this thesis to actuate the piston. The air pressure is first set via adjusting the Metal work diaphragm regulator as seen in figure 2-6. Prior to performing any experiment, a global valve was connected directly downstream to the pressure regulator to act as an emergency shut-off valve. Further down the line is the 5-way 2-position bistable double solenoid valve (made by METALWORK with model #703001200U). The 2 positions are connected to air ports of upper and lower piston chambers, respectively. Note that all gas lines in the system are 0.9525 cm (or 3/8 inch) soft tubes while the valve opening ports are mostly 1.27 cm (or half inch) in diameter. The Manufacturer claimed that it takes the valve 30 millisecond to switch direction completely through energizing one side while de-energizing the other. Also note that, this valve although symmetrically designed, has no central neutral position. The valve can be seen in part (f) of figure 2-1. A closer look of its actual dimensions and its internal structures can both be seen in figure 2-7(a). The outlets are on the other side of the feed port; whereas, the exhaust ports are on the same side of the valve as illustrated by the diagram on the right hand side of figure 2-7(b).
Figure 2-5 METALWORK diaphragm air pressure regulator used for maintaining feed to the system.

Figure 2-6 Actual METALWORK 5/2 3/8 inch bistable double solenoid valve.
Initially, the solenoid valve switches direction, air passage on the right gradually increases, allowing air flowing to the lower chamber leading to upward motion of the piston. Once the designated time for maintaining the power signal is reached, solenoid valve switches direction. As a result, the air passage first decreases to zero and then increases for re-directing the air flow to the other piston chamber. The continuous alternation of the solenoid spool direction results in alternating directions of the piston motion.

2.5 Data Acquisition Devices (DAQ) and others

The Casio Exilm high speed camera is the main data acquisition device. This Casio high speed camera captures video clips at a high frame rate of 240 fps. Videos were stored in the SD card and later transferred to a computer for further frame-by-frame image analysis using MATLAB programs. The distance between the camera and the vessel is set in such a way that with 1x focal length, after adjusting the focus and cropping the frames, the resultant images would consistently capture the entire inner volume of the vessel during JBR operations. Hence, the solid bed motion across the entire vessel can be fully captured. Camera was located on a 1/2 inch stand completely detached from the working table which helped the camera to be firmly in position despite the
mechanical shaking that propagated to the working table. The camera stand has its location taped on the ground, as shown in figure 2-8. This allows researchers to check the camera’s position so that each recording can be kept as consistent as possible. In addition, prior to the start of each set of trials for a given repetition, the pixel to meter ratio is re-evaluated via performing a full stoke up and down test.

Another important DAQ is the absolute pressure transducers. The transducers have an NPT head which is inserted between the lines connecting either piston nozzle to the solenoid valve ports. The maximum rating of the sensor is 100 psi in absolute pressure scale. They can be seen as part (e) of figure 2-1. The transducers are highly sensitive to the pressure change, with a response time of about 2 milliseconds. A complete surge voltage protection is also built in the sensor. The output of the transducer is also proportional to the pressure ranging from 0.5 to 5.0 Volts. The desired input for the transducer is 5.0 Volts. Before using these transducers, calibration is required, see section 4.1.1 for details. And the results can be seen in appendix D. Both transducers are connected in parallel to a constant signal generator to minimize the supply power.

Figure 2-8 The bottom structure of the camera stand has its location taped on the ground.
fluctuations and provide an additional surge protection. A closer look at the transducer sensor can be seen in figure 2-9.

![Pressure transducer used in this study.](image)

Nonetheless, the work space is enclosed by setting up black curtains around. These black curtains kept the Plexiglas vessel from direct exposure of the room light, which minimized the reflection of room light onto the vessel wall. As a result, each frame taken by the camera can be assumed to have the same but low light exposure. This may eventually translate to similar gray threshold if each frame is converted from RGB scale to gray scale.
Figure 2-10 Black curtains setup around the workspace for light blocking: (a) is where the JBR was located; (b) is position of the camera stand.
3 Piston motion modelling

3.1 Overview on piston operating principles

Overall, the solids within the vessel are carried upward and downward by the piston motion; thanks to inertia. To ensure effective and consistent mixing performance, the piston is required to have fast, accurate and periodic response. Low mechanical impedance and friction are also preferred. There are 3 types of actuators available for use: electrical, hydraulic and pneumatic. While electrical actuators may be accurate, they tend to be built with more sophistication and come at a very high cost. Hydraulic actuators are generally more complex to operate and maintain. Pneumatic actuators are relatively inexpensive and are more responsive than hydraulic actuators. Since a constant supply of 100 psig dry and clean air is available in our laboratory, double acting pneumatic actuators become a good choice in our case.

Even though the working principles for pneumatic actuators sound quite easy to comprehend at a first glance, the actual model and control of this double acting piston can face numerous challenges [Richer et al., 1999]. The first challenge is the highly non-linear air flow through the pneumatic components. Other issues include the distance between the compressed air source and pneumatic system, and potential thermodynamic and fluid dynamic changes during the piston operations. These factors complicate the flow calculations of the air through the system, especially when piston frequency increases [Gulati et al., 2005].

For this study, the overall JBR operations during a piston up stroke can be simply illustrated in the above figure 3-1. Here, the piston is initially at bottom prior to the experiments. Once the trial starts, air enters the lower piston chamber through lines and valves. The lower piston chamber expands as more air enters that chamber and drives the piston block upwards. Meanwhile, air in the upper chamber is driven out as a result of the net force experienced from the piston block. All the lines in between the equipment downstream of the air pressure regulator are 3/8 inch soft tubes. Some of the major components of the piston model are explained in the following sections, most of the relationship and equations are adapted from [Richer et al., 1999 & Maré et al., 2014].
Figure 3-1 Overall illustration of the air flow involved in piston modelling during upward stroke.

Figure 3-2 Overall illustration of the air flow involved in piston modelling during downward stroke.
Similarly, figure 3-2 shows that when air flow is reversed: air entering the upper piston chamber pushes the piston block downwards, forcing air out of the valve connected to the lower piston chamber. Alternating the air feed at the two ports under a given time difference thus leads to alternating piston up and down motion.

The piston model in this project considers the following terms: friction forces around the piston seals (coulomb and viscous friction forces), pressure drop across the double solenoid valve, piston gas port nozzle/orifice pressure drop, inactive volumes at either end of the strokes, tube pressure drop, and possible leakage between the upper and lower chambers.

3.2 Part I – Pressure feed adjustment

When calculating the mass flow of air that either enters or exits the piston chamber, one important assumption was made: all air flow upstream of the double solenoid valve is completely static with negligible flow rate. This assumption, according to literatures [Richer et al, 1999 & Mare et al, 2000], is reasonable when air source is a cylinder nearby. However, this is not the case here, as seen in figure 3-1 & 3-2. Prior to entering the solenoid valve, the air flow passes through a very long tube, and air flow is controlled by the ball valve and the air pressure regulator. As a result, the flow is not zero immediately upstream neither of the double solenoid valve nor between the valve and the piston when air exits a piston chamber. These relatively significant tube segments have to be taken into considerations since it is not valid to use compressible flow through orifice model to compute the mass flow of air (see section 3.3.2) without significant compromises on accuracies. To account these tube segments which air has to fill up prior to reaching the piston, a model called “virtual tube” is implemented.

The term virtual tube is denoted as $L_v$. This terms accounts for the entire tube segment upstream of the valve. Initially, air flow has to fill up all the lines and spaces upstream to the double solenoid valve. Besides, air needs to fill up the initial gap within the piston chamber. Overall, to take consideration of all the terms subject to air filling at the beginning of a trial, the equation becomes the following:

$$z_{0_{\text{Total}}} = z_0 + \frac{A_t(L_t+L_v)}{\pi\left(\frac{D}{2}\right)^2}$$  \hspace{1cm} (3-1)
Here $z_0$ is the initial gap between the piston’s moving block and its bottom plate. $A_t$ is the cross sectional area of the tube. $L_t$ is the tube connecting a double solenoid valve exit to a piston chamber. The piston chamber would have an effective cross sectional area with diameter of $D$. $L_v$ is the virtual tube with the same cross sectional area as $L_t$; it is used to represent the space and lines upstream of the double solenoid valve. As part of the empirical parameter, $L_v$ will be solved during the model fitting.

Overall, the required computation for part I can be summarized using the table 3-1:

<table>
<thead>
<tr>
<th>Computation Model</th>
<th>Conditions / Assumptions</th>
<th>Input/ Given conditions</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Virtual tube length, $L_v$</td>
<td>Flow is stagnant before entering the double solenoid valve (essentially a given orifice) after taking account the length of the tube between air source and the valve feed port</td>
<td>$z_0$, initial gap between piston’s moving block and the bottom plate</td>
<td>$z_{0\text{Total}}$, total initial length of the tube</td>
</tr>
<tr>
<td></td>
<td>Ideal gas law</td>
<td>$A_t$, the cross sectional area of the tube</td>
<td>$L_t$, the length of the tube</td>
</tr>
</tbody>
</table>

### 3.3 Part II – Air flow through the double solenoid valve

#### 3.3.1 Overview

After adjusting the feed pressure to the double solenoid valve, the focus is then on how the internal structure, especially the displacement of the valve spool, affects the mass flow rate of the air. The key is to calculate the compressible air flow through a changing orifice within the double solenoid valve. For this reason, the compressible flow function is developed first in section 3.3.2 followed by the model for computing flow through the double solenoid valve in section 3.3.3. The overall valve model is used to calculate the mass flow and pressure of air near the valve exits: there are two streams: a stream going into a piston chamber (e.g. lower chamber in figure 3-1) and a stream exiting the other piston chamber (e.g. upper chamber in figure 3-2). For stream exiting the piston chamber, the valve is assumed to exhaust to room pressure. In the left half of figure 3-3, it corresponds to the upward piston motion (also shown in figure 3-1). In the right half of figure 3-3, it shows the downward piston motion (also shown in figure 3-2).
3.3.2 Calculating the compressible air flow through a given orifice

One of the major challenges in this study involve in air flow computation for the pneumatic system. Air flow is compressible. This makes it especially complex to calculate the pressure and mass flow rate of air when it is passing through a given orifice. In this study, air flow will encounter two changing orifices when it is passing through the valve. One of them is the orifice within the valve which has a changing diameter depending on the location of the spool. This orifice is relatively short, less than 8 mm; the other is at the piston nozzle, which has a fixed diameter, only about 3 ~ 5 mm, and a very small length, less than 5 mm.

To simplify the problem, it is assumed that the compressible, yet ideal air flow is not doing work nor absorbing heat when it passes through the orifice [Li et al., 2004]. Once again, the reason is because air flows out of the piston nozzle over a relatively short segment which is only several millimeters in length. Meanwhile, the air flow velocity is very high, normally close to sonic...
speed. As a result, the kinetic energy dominates the total energy for air flow, and the change in potential energy can be treated as negligible. Therefore, the energy balance of the air flow through a given orifice can be explained using the following simplified equation:

\[ h_u + \frac{u_u^2}{2} = h_d + \frac{u_d^2}{2} = h_{\text{initial}} \] (3-2)

In the above energy balance, the subscript “u” and “d” represent the upstream and downstream side of the given orifice as the air flow is passing through. Whereas, “h” represents the enthalpy per unit mass of the air flow carries with it, or simply the specific enthalpy; “u” is the velocity of the air flow; \( i_0 \) in this case would be the total initial specific enthalpy when the flow velocity is zero, or simply put it as the so-called “stagnation enthalpy” [Hougen et al., 1963]. As a result, the complete expression for the one-dimensional flow velocity becomes:

\[ u = \sqrt{2(h_0 - h) = \sqrt{2C_p(T_0 - T)}} = \sqrt{\frac{2\gamma RT_0}{\gamma - 1}(1 - \frac{T}{T_0})} \] (3-3)

Similar to the stagnation enthalpy, in equation 3-3, \( \gamma \) is the heat capacity ratio of air, which is constant in this study under the ambient lab conditions. This ratio is calculated using the following simple equation: \( \gamma = \frac{C_p}{C_v} \) where \( C_p \) is the heat capacity of air under constant pressure, and \( C_v \) is the heat capacity of air under constant volume. Both terms can be obtained from chemical engineering handbooks [Perry et al., 2007]. \( T_0 \) is named as the stagnant temperature or temperature of the flow when it has a flow velocity of zero. Once again, the reasons for having these terms are to account for different flow situations: for instance, when air is flow out of the piston nozzle due to a short segment with very high flow velocity, the situation can be assumed to be one dimensional and have equal-entropy. Therefore, \( \frac{T}{T_0} = \left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}} \), along with the formula of ideal gas law, \( \frac{P}{\rho} = RT \), can be both used to substitute the relevant terms in equation (3-3), the resultant equation for calculating the air flow velocity becomes:

\[ u = \sqrt{\frac{2\gamma}{\gamma - 1}P_0\rho_0[1 - \left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}}]} \] (3-4a)
In this particular case of flow through a given orifice since the virtual tube has been used to correct the upstream pressure, and the upstream flow can be assumed to have zero velocity. For 
\[
\frac{T_d}{T_u} = \left( \frac{P_d}{P_u} \right)^{\gamma - 1},
\]
derived from the formula of ideal gas law, \( \frac{P_u}{\rho_u} = \frac{R}{\gamma} T_u \), through a quick change in subscripts, the following modified equations can be obtained:

\[
u = \sqrt{\frac{2\gamma}{\gamma - 1} P_u \rho_u \left[ 1 - \left( \frac{P_d}{P_u} \right)^{\frac{\gamma - 1}{\gamma}} \right]}^{(3-4b)}
\]

According to the continuity equation for mass flow of air, \( G = \rho u A \), based on the assumption of equal entropy and equation (3-4b) obtained earlier, the one-dimensional mass flow calculations under the equal-entropy condition can be expressed as:

\[
G = A e \sqrt{\frac{2\gamma}{\gamma - 1} P_u \rho_u \left[ \left( \frac{P_d}{P_u} \right)^{\frac{2}{\gamma}} - \left( \frac{P_d}{P_u} \right)^{\frac{\gamma + 1}{\gamma}} \right]}^{(3-5)}
\]

From the above expressions, the mass flow is a function of the ratio between the upstream and downstream pressure across the orifice where air flow passes. This flow rate is also directly proportional to the effective cross sectional area of the passage.

Nevertheless, the actual mass flow calculations require further examination of the flow velocity. According to Ben-Dov and Salcudean [Ben-Dov et al., 1995], the flow calculations for velocity falling in the sonic and subsonic regime have their physical flow patterns differed significantly. To make a sound determination, the term critical pressure ratio, \( \Omega \), must be first obtained. Once this critical ratio is attained, constants for flow in subsonic or sonic regimes, \( C_1 \) and \( C_2 \) are calculated. These terms are independent of the operations and are generally related to the properties of the fluid using the following sets of equations:

\[
C_1 = \sqrt[\gamma - 1]{\frac{\gamma}{R} \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma + 1}{\gamma - 1}}}^{(3-6a)}
\]
\[ C_2 = \frac{\sqrt{2\gamma}}{R(\gamma - 1)} \]  
\[ \Omega = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}} \]  

For air in this case, \( \gamma = 1.4, C_1 = 0.040418, C_2 = 0.156174 \) and \( \Omega = 0.528 \). The above equation, although often used for ideal circular nozzle, is commonly used for all kinds of circular or elliptical valve orifices [Mare et al, 2000];

Note that here, the upstream, terms denoted with u, is always for the source of the air flow, downstream, terms denoted with d, is always where air flows. And during the formula derivations, the upstream is always assumed to be stagnant, and thus having negligible kinetic energy with flow velocity set to be zero. Since the lab is air-conditioned, the temperature, T is constant throughout the study. The completed mass flow formula for air passing a specified nozzle is then established as the following:

\[
\text{if } \frac{P_d}{P_u} \leq \Omega, \quad \frac{dm}{dt} = C_f A_e C_1 \frac{P_u}{\sqrt{T}}
\]
\[
\text{if } \frac{P_d}{P_u} > \Omega, \quad \frac{dm}{dt} = C_f A_e C_2 \frac{P_u}{\sqrt{T}} \left(\frac{P_d}{P_u}\right)^\frac{1}{\gamma} \sqrt{1 - \left(\frac{P_d}{P_u}\right)^{\frac{\gamma - 1}{\gamma}}}
\]

Overall, these piece wise defined equations can be best visualized as circled lines shown in figure 3-5 and 3-6 below:
Figure 3-4 Change of mass flow when flow is leaving an orifice (under same back pressure with changing upstream pressure).
Figure 3-5 Change of mass flow when entering an orifice (under changing back pressure but with constant upstream pressure).

3.3.3 5/2 double solenoid valve model

Similarly to the previous model for air passing through an orifice, its mass flow rate and pressure changes in a similar manner as it passes through the double solenoid valve involved in this study. The upstream pressure and downstream pressure before and after air enters and exits the valve as well as the internal opening of the valve all contribute to the pressure and flow rate changes throughout the experiment. In fact, the solenoid valve ultimately dictates this double acting piston’s motion. From the manufacturer’s spec sheet, this valve requires about 30 milliseconds to complete a direction switch, i.e. from a state of fully opening one side of the valve passage to a full closure of the passage. Yet, this flow direction switch is mainly achieved by the valve spool moving from one side of the valve’s internal compartment to another. Note that as explained in the later experimental technique sections, the highest frequency set for the experiments is nearly 8.00 Hz which translates to about every 60 milliseconds for the relay to alternate the valve direction once. This implies that the mechanical delay of the valve would be nearly half of the time required for the spool to complete a cycle inside the valve if set during trials with high relay
frequency. Consequently, the effects of valve delay cannot be simply neglected. This inspired a need to establish a valve model for calculating the real time effective area change for the air entering or leaving the valve. Nevertheless, the spool movement and its impact on regulating the air flow through the valve has to be understood.

Before modelling the spool, it is necessary to first determine the size and the internal structure of the valve and the spool. Via taking the valve apart, some of the key dimensions are measured: the central block of the spool is about 17mm long; each of the two stems connecting the central block to the two side blocks of the spool is 17mm long each as well. Each port on the valve is half inch or 12.7 mm in diameter. The pitch for the two ports facing piston is 34 mm from center to center; and the pitch for the two ports on either side of the central air feed port is about 32 mm.

Figure 3-6 to 3-8 provide a concise presentation of the valve spool with respect to its location within the valve compartment as well as a brief illustration on how its location change affects the air passage. Note that the valve involved in this study is a 5-port 2-way double solenoid bistable valve, meaning it is symmetrically designed and operated without a neutral position at center. In figure 3-6, when the valve spool moves to its central location within the valve compartment, air feed is blocked completely. Neither air passage on either side of the valve is available.

![Figure 3-6 When valve spool is at the center position.](image-url)
In figure 3-7, as the valve spool moves to its left most position, the left passage is open, connecting the piston chamber and the atmosphere. Meanwhile, air enters the valve through the feed port and continues its flow towards the piston on the right port.

Figure 3-8 shows the exact opposite situation of figure 3-7. In this case, the right passage connecting the piston chamber and the atmosphere is open. Air feed enters the system through the left port. Since the magnetic force is applied to the spool during operations, the force balance for the spool can be developed as the following expression:

\[
M_s \frac{d^2x}{dt^2} + F_{fs} = F_c
\]  

(3-8a)
In this equation, $M_s$ stands for the mass of the valve spool; $F_{fs}$ stands for an overall term for sum of the Coulomb and kinetic friction during valve operations. In addition, $x$ stands for the (horizontal) displacement of the spool within the valve. $\frac{d^2x}{dt^2}$ stands for the acceleration of the spool. $F_c$ is the net controlling force acting on the spool. In order not to damage the integrity of the valve structure, the internal structure was not measured. Therefore, the mass of the spool is unavailable. Through the limited external measurement of the hardware, it was found that the maximum displacement of the spool from left to right is 17 mm. If the spool’s left most location is defined as $x=0$, the direction towards right is positive. The given mechanical delay of 30 milliseconds can be viewed as the time it takes for the valve spool to travel from one side to another.

Also note that according to the manufacturer, this valve is designed to have minimum maintenance and friction. To preserve the outstanding performance of the valve, sufficient lubrication has been applied regularly to minimize the friction and flush out deposits left in the internal structure during each trial. The overall time and distance for the spool to travel is relatively short. This means that the force applied by the solenoids onto the spool, $F_c$ is much larger than the friction forces $F_{fs}$. To simplify the expression, $F_{fs}$ is assumed to be zero in this study for the spool. As a first order approximation, the acceleration for the spool when it is moving along the internal structure of the valve is taken as a constant. Since the denominator, $M_s$ is also a constant, the overall quotient is denoted as a constant $a_s$, which represents the acceleration of the spool. Therefore, equation (3-8a) can be simplified as the following:

$$M_s \frac{d^2x}{dt^2} \cong F_c$$

(3-8b)

Subsequently, a system of differential equations with the above boundary conditions is established as the following:

$$\text{when } 0 < t < 0.03, \quad \frac{d^2x}{dt^2} = \frac{F_c}{M_s} \cong a_s, \text{ else } \frac{d^2x}{dt^2} = 0$$

(3-9a)

$$\frac{dx}{dt} = \int_0^t \frac{d^2x}{dt^2} \, dt, \text{ when } t = 0, \quad \frac{dx}{dt} = 0; \text{ when } t = 0.03, \quad \frac{dx}{dt} = 0$$

(3-9b)
\[ x = \int_{0}^{t} \frac{dx}{dt} \, dt \text{, when } t=0, x = 0; \text{ when } t = 0.03s, x = 0.017m \quad (3-9c) \]

The analytical solution for the above system then becomes the following:

\[ x = \frac{a_s}{2} t^2 + \text{Constant}_1 \times t + \text{Constant}_2 \]

Whereas, “Constant\(_1\)” and “Constant\(_2\)” in the above equation are both arbitrary constants generated from the analytical integrations. After applying the given boundary conditions, the kinetic expressions for the valve spool displacement can be simplified as:

\[ x = \frac{37.78}{2} t^2 \quad (3-10) \]

The above equation calculates the displacement of the spool at a given time; however, this is not sufficient, as the spool moves, the effective opening area for air pass through the valve changes. Fortunately, the effective cross sectional area can be integrated with a given range of spool displacement within the valve. The following sketch, figure 3-9, taken from [Richer et al., 1999], serves as an illustration of the pertinent relationship.

![Diagram](image.png)

**Figure 3-9 Illustrates the relationship between the valve spool and valve opening [Richer et al., 1999].**

In the above figure, \( x_e \) is denoted as the effective displacement of the valve spool; \( R_h \) is the radius of the orifice. In this study, \( R_h \) is 6.35 mm or half inch. From the previous formula, the effective displacement can be attained; after that, the section of the valve opening which has not been blocked by the spool, denoted as \( A_e \), or the effective area of the valve opening, can be calculated using integration. Note that for the port whenever air enters from side of the valve and
exits from the other side, its area changes as the spool’s location changes. In case the side where air enters has a cross sectional area different from the side air exits or vice versa, the smaller area of the two will be selected as the effective opening for air flow calculations. In general, the effective cross sectional area is computed using the following formula:

\[ A_e = 2 \int_0^{x_e} \sqrt{R_h^2 - (\xi - R_h)^2} \, d\xi \]  

(3-11)

After conducting implicit integration, the resultant formula becomes:

\[ A_e = 2R_h^2 \cot \left( \frac{x_e}{2R_h - x_e} \right) - (R_h - x_e) \sqrt{x_e(2R_h - x_e)} \]  

(3-12)

However, the spool moves back and forth over time which implies that this effective area is in a rather periodic relationship with the spool’s displacement. Subsequently, the area needs to be calculated over different boundaries. Nevertheless, whenever the calculated \( A_e \) is larger than the cross sectional area of the tube, \( A_t \), the former will be over-written by the value of \( A_t \) since the cross sectional area of the tube connected to the valve is the maximum cap of the effective cross sectional area of the flow through a given orifice.

Using \( x_{eL} \) as the term that describes the effective displacement of the spool that covers the left hand side valve port, connecting to the piston, and \( x_{eU} \) as the term that describes the effective displacement of the spool that covers the right hand side valve port, assuming that the left part of the valve passage is connected to the lower piston chamber, and the upper piston chamber is connected to the right part, the resultant spool movement can be broken down into seven cases as shown in table 3-2.

Recall this figure shown earlier for reference:
Table 3-2 The seven cases of spool position and the resultant air passage situation within the valve.

<table>
<thead>
<tr>
<th>Spool Displacement (x, mm)</th>
<th>Air Flow at Left Passage Condition</th>
<th>Air Flow at Right Passage Condition</th>
<th>Effective Left Displacement ($x_{el}, mm$)</th>
<th>Effective Right Displacement ($x_{er}, mm$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0&lt;x&lt;1</td>
<td>Outflow</td>
<td>Inflow</td>
<td>6.35+x</td>
<td>6.35-x</td>
</tr>
<tr>
<td>1&lt;x&lt;6.35</td>
<td>Outflow</td>
<td>Inflow</td>
<td>8.35-x</td>
<td>6.35-x</td>
</tr>
<tr>
<td>6.35&lt;x&lt;8.35</td>
<td>Outflow</td>
<td>Close</td>
<td>8.35-x</td>
<td>0</td>
</tr>
<tr>
<td>8.35&lt;x&lt;8.65</td>
<td>Close</td>
<td>Close</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8.65&lt;x&lt;10.65</td>
<td>Close</td>
<td>Outflow</td>
<td>0</td>
<td>x-8.65</td>
</tr>
<tr>
<td>10.65&lt;x&lt;16</td>
<td>Inflow</td>
<td>Outflow</td>
<td>x-10.65</td>
<td>x-8.65</td>
</tr>
<tr>
<td>16&lt;x&lt;17</td>
<td>Inflow</td>
<td>Outflow</td>
<td>x-10.65</td>
<td>3*6.35+4.3-x</td>
</tr>
</tbody>
</table>

Taking the left most location of the spool as zero ensures positive displacement readings for the spool throughout this study. Recall the previously presented equations in (3-6) for computing mass flow of a compressible flow through a given orifice shown in section 3.3.2. To find the mass flow of air through the valve, it can be calculated using the obtained $A_e$ and applying the equations in the previous section to this valve condition:

The upstream pressure $P_u$ becomes $P_{in}$, the air pressure fed to the valve; and the downstream pressure becomes $P_{Lv}$ or $P_{Uv}$ depending on which piston chamber air flow is directed to.

Similarly, the mass flow rate becomes either $G_{Lv}$ or $G_{Uv}$. For instance, as illustrated in the left part of figure 3-4, assuming that air enters the valve and flows to the lower chamber, the overall mass flow rate, $G_{LV}$, leaving the valve can be expressed using the following equations:

$$
\text{if } \frac{P_{Lv}}{P_{in}} \leq \Omega, \quad G_{LV} = C_f A_e C_1 \frac{P_{in}}{\sqrt{T}}
$$

(3-13a)
\[
G\text{}_{\text{LV}} = C_f A_e C_2 P_{\text{in}} \left( \frac{P_{\text{LV}}}{P_{\text{in}}} \right)^{\frac{1}{\gamma}} \sqrt{1 - \left( \frac{P_{\text{LV}}}{P_{\text{in}}} \right)^{\gamma - 1}}
\]

(3-13b)

To better visualize the relationship between the change of effective valve orifice area and the mass flow of air exiting the valve, figure 3-10 is attached. Note that the positive mass flow rate indicates air entering a piston chamber through the valve. Whereas, the negative mass flow rate indicates air leaving a piston chamber before entering the valve. The latter is estimated via setting the exhaust pressure as atmospheric pressure and back calculates the pressure before air flow entering the valve. Table 3-3 is provided to give a recap on what has been covered in this part of the model.

![Figure 3-10 Mass flow through the valve as a function of spool displacement.](image)

Table 3-3 Input and output summary for the double solenoid valve model.

<table>
<thead>
<tr>
<th>Required calculations</th>
<th>Assumptions / Conditions</th>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressible flow through a given orifice</td>
<td>Ideal nozzle</td>
<td>$P_u$, upstream pressure</td>
<td>$\frac{dm}{dt}$, air mass flow rate</td>
</tr>
<tr>
<td><strong>Air flow through the double solenoid valve</strong></td>
<td><strong>Ideal gas law</strong></td>
<td><strong>P_d</strong>, downstream pressure</td>
<td></td>
</tr>
<tr>
<td>---------------------------------------------</td>
<td>------------------</td>
<td>----------------------------</td>
<td></td>
</tr>
<tr>
<td>Ignorable friction for the spool</td>
<td>Isentropic air flow, not absorbing heat nor doing work due to short nozzle and close to sonic air flow</td>
<td>A_e, effective area of the orifice</td>
<td></td>
</tr>
<tr>
<td>Constant spool acceleration</td>
<td>P_d, downstream pressure</td>
<td>C_f, coefficient of mass flow through this orifice</td>
<td></td>
</tr>
<tr>
<td>Nearly stagnant flow at the upstream flow location</td>
<td>Pin, air pressure at feed port</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Incorporating the compressible flow model</td>
<td>Pa, 1atm at the exhaust pressure</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1atm pressure at the valve exhaust</td>
<td>x, the spool displacement</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.4 Part III – Tube pressure drop and mass flow changes

3.4.1 Overview

The tubes connecting double solenoid valve to either piston chamber exert significant influences on the piston motion due to its frictional pressure drop and its volume. Essentially, this made the tube segments acting as a resistance-capacitance system for the air flow prior to entering a piston nozzle. Similar effects occur as air flows out of a piston nozzle and travels towards a valve port through the tube segment. All these factors will impact the rate of pressure change within both chambers and thus affecting the piston motion. Consequently, the effects of tube segments connecting the valve and piston chambers on the air flow cannot be ignored. As illustrated in figure 3-11, the cross sectional area for air flow is the same as A_t throughout this part. The relationship between the change of mass flow of air and the distance air travel within a given tube is assumed to be a partial differential equation. To simplify the calculations, the reduction of air flow will be solved via numerical integrations.
3.4.2 Tube pressure drop model

Previously, many argued that the air flow will become laminar along the tube; some increase the step numbers for solving the equations, some use second order linear approximation to estimate this propagation of the air flow [Schuder et al., 1959 & Anderson et al., 1967 & Hougen et al., 1963]. However, the primary focus of this study is on the piston motion. It is not necessary to bring up the complex correlations for obtaining the pressure drop and flow reduction over a relatively short 3/8 inch tube segment. Meanwhile, flow rate in the study often gets close to or even exceeds the laminar turbulent boundary.

For sake of having a simple yet robust approximation, instead of solving the complex continuous system of equations analytically, a numerical method involving in discretizing the tube segment into dozens of sub-sections are applied to solve this problem. In this case, the tube is divided into a dozen of sub-sections with length s. Within each sub-section, the air density is assumed constant. The pressure drop and reduction of flow will be estimated for that sub-section along the
direction of the flow through incrementally adding the distance and time as illustrated in figure 3-12, the notations are the following: P is the absolute pressure; dP is for the pressure drop; G is mass flow; φ is the reduction coefficient of mass flow of air; ρ is the density of air within a given sub-section.

Figure 3-12 Numerical approximation of pressure drop and flow reduction along the tube through discretization.

Subsequently, the following partial differential equations are used to accomplish this task:

\[
\frac{\partial P}{\partial s} = -R_t u - \rho \frac{\partial u}{\partial t} \quad (3-14)
\]

\[
\frac{\partial u}{\partial s} = -\frac{1}{\rho c^2} \frac{\partial P}{\partial t} \quad (3-15)
\]

In addition to the terms appeared in figure 3-13, u is the air flow velocity within the tube; c is the sonic speed at the ambient lab condition; s is the incremental segment of the tube; R_t is the resistance of the tube. If set A_t as the cross sectional area of the tube segment, the mass flow rate through a given tube segment simply becomes \( G_t = \frac{dm_t}{dt} = \rho A_t u \) Using the substitution methods suggested by the literature [Elmadbouly et al., 1994], inserting the previous mass flow rate expressions into equation 14 and 15, the resultant formula for mass flow of air as time and location changes is the following:

\[
\frac{\partial^2 G_t}{\partial t^2} - c^2 \frac{\partial^2 G_t}{\partial s^2} + \frac{R_t}{\rho} \frac{\partial G_t}{\partial t} = 0 \quad (3-16)
\]
The above equation is in fact a wave function with a dissipative term. Introducing the term, \( G_t(s,t) = \varphi(t)v(s,t) \) and substitute it for the term \( G_t \) in the above equation, and solve it according to [Chester et al., 1970]. When the term \( \varphi(t) = e^{-\frac{(R_t)}{2P}(t)} \) is defined, the resultant solution for \( v(s, t) \) is a discrete hyperbolic function.

Since the tube is relatively short, less than 0.5 meters long, the scattering can be ignored and the equation becomes essentially a classical wave equation. Moreover, assume at \( t=0 \), there is no flow; the flow near the starting point of the tube, \( s=0 \), can be denoted as the term \( q(t) \). All these terms can be seen in figure 3-13 for a simplified illustration. Also, assume there is no reflection nor back flow taking place in the tube, using the term \( L_t \) to represent the length of the tube, the overall initial condition becomes the following:

\[
\begin{align*}
\begin{cases}
    v(s,0) = 0 \\
    \frac{\partial v}{\partial t}(s,0) = 0 \\
    v(0,t) = q(t)
\end{cases}
\end{align*}
\]

(3-17)

Since air propagates as a sound wave travelling within the tube, the flow will reach the exit after a certain time \( \tau = \frac{L_t}{c} \). Solving \( v(s, t) \) for \( \varphi(t) \) gives the following:

\[
v(s,t) = \begin{cases} 
0 & \text{if } \left( t < \frac{s}{c} \right) \\
q \left( t - \frac{s}{c} \right) & \text{if } \left( t > \frac{s}{c} \right)
\end{cases}
\]

(3-18)

\[
\varphi = e^{-\frac{(R_t)}{2P}(\frac{L_t}{c})}
\]

(3-19)
For a 50 cm long tube, the physical time delay as a result of sonic wave propagation is about 1.5 milliseconds. This delay is relatively insignificant, only 5% of the valve response time, 30 milliseconds. Therefore, this time delay is ignored in this study. While the piston operates continuously within the given run time, the corrected formula for computing the mass flow rate becomes:

\[ G_t(L,t) = e^{-\left(\frac{R_t RT}{2P}\right)\left(\frac{L}{c}\right)} h(t) \]  \hspace{1cm} (3-20)

Note that the P in the above expression represents the air pressure near the exit end of the tube segment. Overall, this function relates the exit mass flow of the air to the inlet mass flow of the air. According to [Hougen et al., 1963], this function performs reasonably well over a short tube segment with lower air feed frequency. As for the term \( R_t \), it can be attained via calculating the pressure drop along the tube using Darcy–Weisbach equation with Fanning friction factor [Munson et al., 1990] (note that the specific weight of air is multiplied at both sides of the original equation):

\[ \Delta P = f \frac{L}{D} \frac{\rho u^2}{2} = R_t u L_t \]  \hspace{1cm} (3-21)

In the above equation, \( f \) is the Fanning friction factor; \( D \) is the tube diameter. For fully developed laminar flow, \( f = 64/Re \) where \( Re \) is the Reynolds number. Subsequently, for laminar flow the tube resistance becomes [Schuder et al., 1959]:

\[ R_t = \frac{32\mu}{D^2} \]  \hspace{1cm} (3-22)

In the above equation, \( \mu \) is the kinetic viscosity of air. For turbulent flow passing through, the inner wall of the 3/8 inch soft plastic tube is assumed to be smooth; thus, the friction factor can be calculated as \( f = \frac{0.316}{Re^{0.25}} \), according to Poiseuille's law. After substituting into equation (3-21) using the Blasius equation, the tube resistance formula can be simplified as the following [Jukka et al., 2011]:

\[ R_t = 0.158 \frac{\mu}{D^2} Re^{0.75} \]  \hspace{1cm} (3-23)
This $R_t$ is then substituted back in equation (3-21) to calculate the mass flow of air at a given spot along the tube at a given time. Also note that when calculating along the direction of the flow, pressures drop $\Delta P$ is negative, the reduction of mass flow $\varphi$ at a given time is less than 1. Using the terms labelled in figure 3-13, $P_{\text{out}}$ and $G_{\text{out}}$ are calculated with given $P_{\text{in}}$, and initial flow and tube properties in this case.

However, when computing the tube with air flows out of the piston towards valve in the same time, the way to calculate these terms are slightly different. In this case, $P_{\text{out}}$ and $G_{\text{out}}$ are given (as estimated using the pressure drop obtained from the valve model via setting the exhaust pressure close to 1 atmosphere, see details in section 1.3.3). The pressure drop and flow changes are added incrementally and backwardly since the calculation starts near the valve, which is the end point of air flow in the tube. The model then progressively back calculates the pressure and flow rate one sub-section upstream of current point till it reaches the point when air first enters the tube from the piston nozzle.

### 3.4.3 Selecting the number of steps

As mentioned, there are two major tube segments involved in this valve-piston complex. One segment connects the upper piston to the valve and the other connects the lower chamber to the valve. Besides performing numerical approximations rigorously, the choice of number of steps deserves further discussion. Generally speaking, over the given range of 15 to 55 psi, which covers the pressure feed levels used in this study, after some trial and errors, it is being found that when subdividing the longer tube segment into 20 steps or 20 equal parts, the resultant relative errors for calculating pressure drop would be less than 1% without significantly compromising the time efficiency of the program as observed in figure 3-14. Beyond this number of steps, the percent difference would only shrink subtly. Therefore, it is safe to claim that using 20 steps would be sufficient to approximate the pressure drop over a given tube segment.
Similarly, when checking the minimum step number for computing the mass flow rate. It takes only as many as about 5 ~ 7 steps to reach a percent difference less than 1% as illustrated in figure 3-15. However, to be conservative and minimize the errors, using 20 steps for computing along the tube is selected for modelling the tube pressure drop and mass flow calculations in this study. It shall also be mentioned that the number of steps shall be adjusted according to the length of the tube segments connecting the valve and piston, which is relatively short in this study (the longest one is about 40 cm, and the shortest one is only over 20 cm).

Figure 3-14 Percent errors in pressure drop calculations vs. number of steps.
To summarize, equation (3-20) and (3-23) are both used for taking the flow reduction along the tube into considerations using the Blasius formula. Prior to calculations, mass flow is used to find the Reynolds number. Although this model avoided solving the complex partial differential equations in (3-16), obtaining the changing pressure and flow rate of the compressible air flow along the tube require numerical integration whose accuracy depends on the number of steps. A summary is provided in table 3-4 below.

Table 3-4 Input and output summary for tube delay model.

<table>
<thead>
<tr>
<th>Required calculations</th>
<th>Conditions / Assumptions</th>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure drop and flow reduction along the tube</td>
<td>Sonic speed over the relatively short tubes caused no delay of air flow in time</td>
<td>$G_{Uv}$ or $G_{Lv}$, mass flow of air entering the tube, depending on the direction of the flow</td>
<td>$G_{Ua}$ or $G_{La}$, mass flow of air leaving the tube, depending on the direction of the flow</td>
</tr>
<tr>
<td>(always compute from the valve side to piston side)</td>
<td>No flow at the starting point of the tube at beginning</td>
<td>$P_{Uv}$ or $P_{Lv}$, pressure of air</td>
<td>$P_{Ua}$ or $P_{La}$, pressure of air</td>
</tr>
<tr>
<td>Discretized approximation of the pressure drop and flow reduction</td>
<td>flow at the entrance of the tube</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------------------------------------------------------------</td>
<td>---------------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subdividing the tube into 20 parts</td>
<td>Lₜ, length of the tube segment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ideal gas law</td>
<td>Dₜ, diameter of the tube segment</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>air flow at the entrance of the piston nozzle</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3.5 Part IV – Piston chamber pressure change

3.5.1 Overview

The reduced air flow continues its journey towards the piston chamber. Before that, it first passes through the piston nozzle. The air passage undergoes a sudden change of its effective cross sectional area. All these factors will be taken into considerations when calculating the air flow through this nozzle using the “local resistance” function as explained in section 3.5.2. This function also uses the compressible flow during computation (which can be found in section 3.2.2). Eventually, air enters one piston chamber. As it accumulates in that chamber, air expands and pushes the piston block away. This just explained the fundamental driving force of the piston motion: the net force acting on the piston block. Thus, to fully comprehend the piston motion, determining the pressure change taking place within each chamber is the key. This function, “chamber pressure change”, is unfolded in section 3.5.3. Besides, in case of leakage problems, another function is incorporated to the computation at this part; it is called the “piston leakage” as described in detail in section 3.5.4. In general, all the terms required for computation within this part can be seen in figure 3-16. Also note that conservation of three quantities was used for gas modelling: conservation of mass, conservation of total energy and conservation of momentum.
3.5.2 Local resistance model

In this study, air enters or leaves the piston chambers via a nozzle which connected the tube between the piston and the valve. The soft tube is about 9.5 mm in diameter (or 3/8 inch). When air passes from the tube to the nozzle, the NPT port opening is about 5 mm. the length of the nozzle’s orifice is about 7 mm. Consequently, the air flow undergoes a sudden contraction as it enters the nozzle, followed by a sudden expansion as it leaves the nozzle and enters the chamber. The reversed process happens when air leaves the piston chamber through the nozzle and eventually ends up in the soft tube. See figure 3-17 and 3-18 for illustrations. According to fluid dynamics, energy loss occurs when (high speed, turbulent) flow undergoes sudden contraction and expansion [Perry et al., 2007]. As a result, to avoid the ambiguity, this model is named “local resistance model” which differs from the valve model as this model only considers the energy loss near a given piston nozzle.
According to literature, this pressure drop in these conditions can be expressed as the following [Jiang et al., 2003]:

$$\Delta P = \zeta \frac{\rho u^2}{2}$$

(3-24)

Here, $\zeta$ is the local resistance coefficient which relates the shape and area ratios of the upstream and downstream cross sectional areas when the fluid experiences sudden expansion and sudden contraction. Similar to the principles applied in the previous tube flow reduction and pressure drop model, this reduction coefficient can be written in a similar manner:

$$\varphi = e^{-\left(\frac{\zeta \rho u RT}{4Pc}\right)}$$

(3-25)

Note that all sudden expansions and contractions are square and symmetric according to the design of the nozzles and the push-to-connect straight adapters involved. All orifices at both piston chambers are the same and square-edged as well. Whereas, the local resistance coefficient
in this case can be looked up in table 3-5 and 3-6 adopted from Perry’s handbook [Perry et al., 2007]:

<table>
<thead>
<tr>
<th>(A_u/A_d)</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\zeta)</td>
<td>1.0</td>
<td>0.81</td>
<td>0.61</td>
<td>0.49</td>
<td>0.36</td>
<td>0.25</td>
<td>0.16</td>
<td>0.090</td>
<td>0.040</td>
<td>0.010</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(A_d/A_u)</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\zeta)</td>
<td>0.50</td>
<td>0.47</td>
<td>0.45</td>
<td>0.38</td>
<td>0.30</td>
<td>0.25</td>
<td>0.20</td>
<td>0.15</td>
<td>0.090</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3-5 Values of \(\zeta\) when fluid experiences sudden expansion.

Table 3-6 Values of \(\zeta\) when fluid experiences sudden contraction.

Note that in the above tables, \(A_u/A_d\) is the ratio between cross sectional areas of the flow from upstream to downstream. In this study, when air enters the chamber from the soft tube, cross section area undergoes contraction, and \(A_d/A_u= 0.276\). At that instant, the local resistant coefficient is about 0.397 estimated through linear interpolation using table 3-3. After the flow passes the orifice: \(A_u \ll A_d\), which implies \(A_u/A_d \approx 0\); this means the flow undergoes a sudden expansion and makes the resistance coefficient nearly 1 according to table 3-2; therefore, the overall resistance coefficient when flow enters the piston chamber through the nozzle, is the sum of the previous coefficients and thus, \(\zeta = 1.39708\). Similarly, when air leaves the piston chamber through the nozzle, it undergoes a sudden contraction first. As a result, \(A_u \gg A_d\), in this case. This makes the cross sectional area ratio of \(A_d/A_u\) nearly zero, leaving the resistance coefficient close to 0.5. After that the flow enters the tube segment with a sudden expansion; this time, \(A_u/A_d = 0.276\), with the help of linear interpolation, the coefficient at this stage is about 0.527. Subsequently, the overall coefficient becomes 1.027. Subsequently, these two coefficients are then substituted into equation (3-24) and (3-25) when computing the pressure change and flow change for air leaving and entering a given chamber.

3.5.3 Recap on flow calculations prior to entering either piston chamber

Note that throughout the entire air flow path which involves the valve, tubes and piston nozzles, effects of soft tube resistance on the flow are generally represented by the mass flow reduction and pressure drop of the air flow along the tube, explained in section 3.4.2; the energy loss across the valve when the effective cross sectional area changes will be accounted by the compressible flow model adjusted by the \(C_f\) factor for mass flow reduction, see section 3.3.2 and 3.3.2. The
nozzle participates in the overall system model through the local resistance model developed earlier.

Nevertheless, the effective cross sectional area for calculating the mass flow of the air flow within the lines in the overall piston model takes the effective area provided by the valve with its maximum capped by the 3/8 inch tube size. Since there existed resistance at the direction of the flow, when calculating the mass flow of air in the system, the term upstream and downstream pressure can become ambiguous. To clarify this, the source of air flow, where pressure is higher, is denoted as the upstream; the point where air flows to, and hence having lower pressure and possibility of forming back pressure, is called downstream. And all these terms are strongly related to each other while affecting the mass flow of air in general. As a result, the upstream and downstream pressure across the valve has to be accurately determined. The approach and assumptions to solve such problem used in this study is the following:

- Air flow near the nozzle or a given orifice is essentially assumed to be ideal gas with high (sonic) speed through a short section
- Tube caused pressure drop and mass flow reduction is computed through numerical approximations via discretizing a tube segment into 20 equally spaced sub-sections
  - Density of air is constant within each section and it is calculated using ideal gas law.
  - Pressure drop and mass flow reductions are calculated progressively along the tube from one end point of the previous sub-section to the next
- The area change near the piston nozzles will be used for calculating the pressure and flow loss
- The effective area of the air passage depends on how the spool’s displacement within the valve
- The maximum effective cross sectional area for calculating air flow through a changing valve orifice cannot exceed the tube size which is 3/8 inch in diameter.
- In case of air entering the piston chamber, feed pressure is the upstream pressure, the back pressure, or downstream pressure would be the sum of pressure within that chamber, plus the pressure drop near the nozzle plus the pressure drop as flow passes the tube segment.
• In case of air leaving a piston chamber, the upstream pressure then becomes chamber pressure minus the pressure drop resulted from the local resistance near the nozzle and minus the pressure drop along the tube. Subsequently, the downstream pressure is set to be atmospheric pressure as air exits the system.

3.5.4 Piston dynamics and chamber pressure modelling

This is essentially the force balance between the piston and the load which is a critical component for modelling the driving force of the piston motion in this study:

\[ M(a + g) + \beta v + F_f = P_L A_L - P_U A_U - P_a A_r \]  

(3-26)

Here, since the piston is acting in vertical directions (up and down), air initially was fed to the top chamber with piston initial location at bottom instead of in the middle of the stroke as of the literature. In addition, the mass, \( M \), includes the external load mass and the piston moving block mass. The piston acceleration is denoted as \( a \), and its velocity as \( v \). \( \beta \) represents the viscous friction coefficient; whereas, \( F_f \) is the Coulomb friction force. On the right hand side of the equation, \( P_U \) and \( P_L \) represent the absolute pressures in the upper and lower chambers of the piston, respectively. \( A_U \) and \( A_L \) then become the corresponding effective piston areas of the upper and lower chambers. The last term, \( P_a A_r \) represents the force exerted by the atmospheric pressure on the piston rods’ cross-sectional area. All the forces acting on the piston can be viewed in figure 3-19 below:
Eventually, it is the flow of air through the pneumatic system that provides the chamber pressure necessary to drive the piston up or down. When a piston chamber undergoes a charging process, it can be approximated as an adiabatic process [Ben-Dov et al., 1995]. As for a discharging chamber, based on experiments done by Al-Ibrahim et al, the process closely follows an isothermal process [Al-Ibrahim et al., 1992]. As for the behavior of air in the system, ideal gas law is a good assumption. The air pressure and temperature and density within a given chamber can also be assumed homogeneous. Negligible kinetic and potential energy change of the gas further simplifies the conditions [Richer et al., 1999].

A control volume V of air has a mass of m, pressure of P, temperature of T, and a density of ρ, with R as the ideal gas constant, the ideal gas law can be written as the following form:

\[ P = \rho RT \]  

\[(3-27)\]

For the air flow, the continuity equation can be written as the following:
\[
\frac{dm}{dt} = \frac{d}{dt} (\rho V)
\]  
(3-28)

Note that the above equation describes the net mass flow of air into the control volume V. It is this net accumulation of mass flow of air in one chamber provided the driving force for piston to move away from that chamber. In the case of air entering one piston chamber while simultaneously exiting from the other, following the same subscripts used in previous sections (“L” for lower chamber and “U” for upper chamber), the above equation can be re-written as:

\[
\frac{dm_L}{dt} = \frac{d}{dt} (\rho_L V_L)
\]  
(3-29a)

\[
\frac{dm_U}{dt} = \frac{d}{dt} (\rho_U V_U)
\]  
(3-29b)

Whereas, \( V_L + V_U = \text{constant} \); the total volume of both piston chambers is always a constant; this constant bridges the relationship of volume change in each chamber at any given time.

Since a chamber can only undergo either filling up or emptying at a given time, the mass flow derivations for each chamber shall be derived separately. For illustration purpose, using air flowing to the lower chamber as an example, with piston moving up as a result, the energy balance for the lower chamber can be written as the following:

\[
q_{L,\text{in}} - q_{L,\text{out}} + \gamma C_v \left( \frac{dm_L}{dt} T_L \right) - \frac{dW_L}{dt} = \frac{dU_L}{dt}
\]  
(3-30)

In the above equation, \( q_{L,\text{in}} \) and \( q_{L,\text{out}} \) are the heat transfer terms. \( \gamma \) is the ratio of the specific heat capacity of air at constant pressure, \( C_p \), over that at constant volume, \( C_v \) within the lower chamber; whereas, \( T_{in} \) is the temperature of air flows into the piston chamber. \( W_L \) is the work done by the lower chamber. \( U_L \) is the internal energy within that chamber. The overall change of internal energy over time for the selected lower piston chamber can be expressed as the following formula:

\[
\frac{dU_L}{dt} = \frac{d}{dt} \left( C_v m_L T_L \right) = \frac{1}{\gamma - 1} \frac{d}{dt} \left( P_L V_L \right) = \frac{1}{\gamma - 1} \left( V_L \frac{dP}{dt} + P_L \frac{dV}{dt} \right)
\]  
(3-31)
Using the ideal gas law, \( C_v = \frac{R}{\gamma - 1} \) and substituting \( \frac{dW_L}{dt} = P \frac{dV_L}{dt} \) and equation (3-31) to equation (3-30) yields the following:

\[
q_{L,in} - q_{L,out} + \frac{\gamma}{\gamma - 1} \frac{P_L}{\rho_L T_L} \left( \frac{dm_L}{dt} T_L \right) - \frac{\gamma}{\gamma - 1} P \frac{dV_L}{dt} = \frac{1}{\gamma - 1} V \frac{dP_L}{dt} \]  
(3-32)

If the temperature for air flow entering the chamber is the same as the ambient lab temperature, or simply \( T_{in} = T \), then the above equation can be further reduced to:

\[
\frac{\gamma}{\gamma - 1} (q_{L,in} - q_{L,out}) + \frac{1}{\rho} \left( \frac{dm_L}{dt} \right) - \frac{dV_L}{dt} = \frac{V_L}{\gamma P_L} \frac{dP_L}{dt} 
(3-33)
\]

As for the adiabatic process, heat transfer, \( q_{L,in} - q_{L,out} \), is essentially zero, by substituting the ideal gas law for the density, \( \rho_L \), the formula further reduces to:

\[
\frac{\gamma}{\gamma - 1} \left( \frac{RT_L}{V_L} \right) \left( \frac{dm_L}{dt} \right) - \frac{P_L}{V_L} \frac{dV_L}{dt} = \frac{dP_L}{dt} 
(3-34)
\]

Whereas, for the isothermal process, the change of internal energy within a given chamber becomes the following:

\[
\frac{dU_L}{dt} = C_v T_L \frac{dm_L}{dt} 
(3-35)
\]

In the end, the formula for the changing air pressure within the lower piston chamber can be obtained as the following:

\[
\frac{RT_L}{V_L} \left( \frac{dm_L}{dt} \right) - \frac{P_L}{V_L} \frac{dV_L}{dt} = \frac{dP_L}{dt} 
(3-36)
\]

Through comparing the adiabatic and isothermal processes, the expressions for air pressure change within a given piston chamber under either cases differs only by a factor of \( \gamma \), the ratio of specific heat capacity ratio. However, this alone is not enough, when the piston is at its minimum displacement, or the initial bottom location, is denoted as \( z_0 = 0 \); meanwhile, the maximum piston displacement shall not exceed \( L \), also the full stroke of the pneumatic actuator. Any upward motion will make the vectors of either displacement or acceleration positive; similarly, if the chamber undergoes expansion, the rate of volume change is positive; if it contacts, the rate would be negative. In general, when one piston chamber undergoes (adiabatic) expansion, the
other undergoes (isothermal) compression. However, whether a chamber will undergo expansion or compression, or how much it expands or contacts, depends on not only the piston block motion but also the mass flow rate of air entering or exiting the piston. Consequently, the relationship between chamber volume change and piston velocity are summarized as the following:

\[ V_L = (z_{0L} + z)A_L, \quad V_U = (z_{0U} + (L - z))A_U \]

\[ \frac{dV_L}{dt} = \dot{v}, \quad \frac{dV_U}{dt} = -\dot{v} \]

As a result of this assumed adiabatic expansion and isothermal compression as well as the different cross sectional areas of the upper and lower piston chamber, how chamber pressure changes over a given range of air mass flow can be seen in figure 3-20. If the cross sectional areas of the piston chambers are the same, both expansion curves will be parallel to each other and so will the compression curves.

![Figure 3-20 Predicted relationship between pressure change and mass flow using the assumption of adiabatic expansion and isothermal compression.](image-url)
3.5.5 Model for piston leakages

Recall that the piston motion relies directly on the pressure balance of its two chambers. Therefore, in case of significant leakages occurred in the system, the piston motion can be dramatically affected.

To account this potential problem, leakages are taking into consideration when modeling the piston motion. The only two possible leakages that may occur during this study are located around the piston double rods and between the piston’s moving block and the cylinder wall. During the search for best fitting parameters, it is found that when leakage area is zero, the resultant parameters will cause the minimum values in their least squares as the example shown in figure 3-21. Therefore, the leakages can be viewed as negligible in this study. Detailed explanation of the leakage model can be found in Appendix D.

Figure 3-21 Gap estimation when finding the best fitting parameter during tests (the sum of least squares is minimized when the gap is zero, i.e. no leakage).
3.6 Part V - The piston model

In this part, the primary data obtained will be the frames indicating the motion of the platform, represented by the motion of the red LED dot, and the motion of the solid bed within the vessel. Piston displacement data obtained will be used as reference for searching the best fitting parameters using the finite element search. This can be found in section 3.7.

3.6.1 Model built-up overview

From the above sections, there are eight sub-models used to calculate and compile for the general MATLAB model for the pneumatic actuator. The key of this study is to have feed pressure, relay frequency and piston weight load as three inputs to the model. Subsequently, the model shall predict the piston displacement and its amplitude. The piston displacement is then used to derive for the instantaneous velocity as well as the acceleration. Among these outcomes of the model, the piston acceleration will be one of the key factors that exert a direct impact on the solid bed motion.

The assigned variables can be grouped into three categories: pressure related, mass flow related, and piston-displacement related. All pressure terms are denoted with P, G for all mass flow terms; the relevant changes or differential terms are dP for pressure drop and φ for mass flow reduction coefficient. The “x” is used for spool displacement whereas, parameters, z,v,a are reserved for piston displacement, piston velocity and piston acceleration separately. M is the weight loaded onto the piston block. The valve frequency which is essentially the relay frequency is denoted as f. Subscript “U” and “L” are assigned to terms along the line connecting to the upper and lower chamber of the piston. Moreover, subscript “v” is assigned to terms closer to the valve side. Subscript “a” is assigned to terms near the piston nozzles. Subscript “t” is reserved for terms involving the soft tube. Subscript “l” is used to describe terms near the piston orifices. All these terms are located and labelled on figure 3-23. Quick references for sections where relevant calculations and functions involved in each zone can be found near the bottom of the figure.
Figure 3-22 Overall JBR model illustration with all relevant terms labelled (in this case piston moves upward due to air flowing to the lower chamber).

Once all terms are labelled on the system, calculations can be done along the direction of the air flow using principles of numerical integration and differentiations. Since all terms changes as a function of time, numerical integrations of the differential models are calculated via breaking down 1 second into a finite linearly spaced number of steps. Assuming that within each step of time, all the terms changed little while remaining mostly the same from the beginning of that time interval. To perform calculations consistently, in the same iteration, all terms near the solenoid valve are calculated first, followed by calculations for terms near the piston. In the next iteration, the terms near the piston are calculated first, the terms near the valve are calculated next. And the direction of calculations is reserved once again in the next iteration and so on regardless of whether air is entering or exiting the piston chamber. Once again, when performing the above calculations, all terms are generally vectors: positive direction is set to be when air flow enters a piston chamber; on the other hand, when air flow exits a chamber, the vector G
becomes negative. Similarly, when piston moves up, the displacement is positive; when piston moves downward, the displacement is negative.

As for the air flow, pressure drop is positive along the flow direction, reducing flow rate and pressure as air flow propagates across the line. In this case, the coefficient of mass flow, \( \varphi \) is less than 1. On the other hand, negative pressure drop happens when calculations are done backwardly. This means the coefficient of mass flow is larger than 1. In the end, plots of the MATLAB model generated motion curves can be obtained as well as other terms predicted by the model under the given operating conditions. See the following figure 3-23 and 3-24 as sample plots. Table 3-7 gives a more general recap on the terms involved in the four major parts of the piston motion modelling.

Once the pressure within either chamber is determined, the force balance equation will be completed. The resultant net force acting onto the piston moving block is used to find the instantaneous acceleration of the piston. Through numerical integration of the instantaneous acceleration over a given time, the velocity and eventually the displacement of the piston can be obtained.

Table 3-7 Summary table of terms and calculations involved in the four major parts.

<table>
<thead>
<tr>
<th>Part</th>
<th>Major Calculations</th>
<th>Input</th>
<th>Intermediate Terms</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Virtual tube length</td>
<td>( z_0, L_e, D_i )</td>
<td>-</td>
<td>( L_v )</td>
</tr>
<tr>
<td>II</td>
<td>Flow through valve</td>
<td>( P_{in}, P_a, x, )</td>
<td>( A_{Ue}, A_{Le} )</td>
<td>( G_{Uv}, P_{Uv}, G_{Lv}, P_{Lv} )</td>
</tr>
<tr>
<td>III</td>
<td>Tube pressure drop</td>
<td>( G_{Uv}, P_{Uv}, G_{Lv}, P_{Lx}, L_x, A_i )</td>
<td>( dP_L, dP_{Ux}, \varphi_L, \varphi_U )</td>
<td>( G_{Ua}, P_{Ua}, G_{La}, P_{La} )</td>
</tr>
<tr>
<td>IV</td>
<td>Chamber pressure</td>
<td>( G_{Ua}, P_{Ua}, G_{La}, P_{La} )</td>
<td>( dP_{Li}, dP_{Ui}, \varphi_{Li}, \varphi_{Ui} )</td>
<td>( G_u, P_U, G_L, P_L )</td>
</tr>
</tbody>
</table>
Figure 3-23 Sample calculation results in changes of effective valve orifice area as well as the piston velocity curve over time, obtained from the MATLAB piston model (operating under 25psig feed air pressure, 5.3 Hz relay frequency and 3.951 kg weight load).
Figure 3-24 Sample calculation results on changes of pressure within the piston chambers as well as the piston velocity curve over time, obtained from the MATLAB piston model (operating under 25psig feed air pressure, 5.3 Hz relay frequency and 3.951 kg weight load).

To better understand the entire process loops, a flow chart is provided below:

```
Input: feed pressure, frequency, weight load on the piston, run time

Finite steps of time with initialized chamber pressure at t=0, P_{U0} & P_{L0}

Checking for next iterations

While time interval is not exceeded

Check the direction of piston,
Use equations accordingly

Stop!
```
3.6.2 Methods for searching the best fitting empirical parameters

Note that since the model mostly consists of differential equations, finding the best-fitting parameters can be quite challenging. To simplify the problem, the four empirical parameters and their corresponding four experiments are grouped into two based on whether they are related to the piston motion directly or indirectly; as a result, since the mass flow coefficient $C_f$ and the virtual tube length $L_v$ are obtained via tube and piston chamber filling tests, they become one group. Whereas, the kinetic friction coefficient $\beta$, and the Coulomb friction force, $F_f$ are directly involved in piston motion. These two parameters along with the piston trials form the other group.

To search for the best fitting parameters, a nested looping system is developed in MATLAB. To put it in a simple manner: the internal loop finds several pairs of a group of two parameters. The external loop then searches in a boarder range to identify the best pair as the final resultant parameters for the JBR model. The criteria for selecting the best fitting pair is to check its least squares when comparing the model generated steady state amplitude with the experiment generated steady state amplitude. Details can be seen in Appendix F.
This then concludes the necessary literature review and methodology for preparing the mathematical foundations of the piston model used in this study. As explained, the general piston motion model will have 4 important parameters to find: \( \beta \), \( F_f \), \( C_f \), \( L_v \). While other parameters can be obtained via measurement or calculations, these four parameters are relatively independent and thus requires four independent sets of data. Whereas, \( \beta \) and \( F_f \) are related to the piston motion; \( C_f \) and \( L_v \) can be obtained under static conditions. In order to fine tune and adjust the model more efficiently and effectively, four independent experiments are then designed and carried out, they are: the tube pressure fill-up experiments, piston chamber fill-up tests, full stroke single pass piston tests and the continuous piston tests. These experiments will be described in detail in the next chapter.

4 Experimental Design and Techniques

In general, the experiments involved in this JBR study can be divided into two groups: one group for piston modelling, and the other for solid mixing. In the former group, the main focus is on developing a model for the piston motion, which will eventually be used to calculate the solid mixing model in the latter.

4.1 Experiments for modelling the piston motion

To perform piston modelling, the following input parameters are considered: the pressure level, relay frequency, the combined weight of the piston, the reactor and the bed of particles. According to previous discussion, there are three adjustable parameters: \( C_f \), mass flow coefficient of the nozzle, \( F_f \), coulomb friction coefficient, and \( \beta \), kinetic coefficient. For not all the factors are directly involved in the piston. Many of them are a result of piston design. Prior to the piston motion studies, several preliminary tests have been performed to provide a good initial estimate of these piston modelling factors.

4.1.1 Pressure transducer calibration

To have accurate pressure readings for further analyzing the pneumatic system, all pressure transducers involved in the system must be calibrated. To calibrate the pressure transducers the following configurations are adopted as shown in figure 4-1.
The two transducers are about 15 cm apart. With P1 located near the upstream and P2 is located near downstream. The ball valve is used to close the tube segment during experiments. It can also be used to release pressure after performing each pressure trial. More details for obtaining the calibrated test pressure readings can be found in Appendix D

4.1.2 Tests for modelling pressure drop across a soft tube

To obtain an accurate mass flow rate of air across the systems, the pressure drop across the soft tube, shown in figure 3-1, must be carefully examined. Figure 4-2 shows the test configurations. All the straight black lines downstream to the air pressure regulator shown in the figure represent the soft tube connecting the equipment.
Three ball valves were placed in the air lines to direct the air flow to the designated soft tube sections; there, two digital pressure transducers were installed at both ends of the designated soft tube. These two transducers, P1 and P2, mounted inline using two tees, measure the gage pressure of the air within the tube at their locations. The soft tube connecting ball valve 1 to the solenoid valve is 31.8 cm. The tube connecting P1 and P2 is 34.0 cm long. In order to best simulate the real time pressure drop as a result of valve switching and tube of the piston system, the following procedure was implemented:

1. Prior to the start, the air pressure would be set manually by the regulator; this value would be noted as P-air initial, measured in gage.
2. Ball valve 1 is initially closed, double solenoid valve is having its position open for air passing towards ball valve 2; this would simulate a steady flow established through the double solenoid valve prior to valve direction switching.
3. Ball valve 3 is opened for at least 10 seconds to ensure zero gage pressure within the tube.
4. Prior to the beginning of the experiment, the Arduino program has to be re-initialized, ball valve 3 is closed.
5. The solenoid valve controller is programed, by default, to switch direction from ball valve 2 to ball valve 3 prior to the test.

6. Once the toggle switch is on, pressure transducers’ readings will be displayed on Arduino program’s serial monitor right after.

7. At the beginning of a trial, toggle switch was on, Arduino board sends the power signal to switch the solenoid valve. After a small mechanical delay, about 30 ms, the valve is completely switched from Ball Valve 2 to Ball Valve 3; air flow changes its path; transducers start to record real time pressure readings at P1 and P2.

8. After at least 15 seconds, the data collection ends and the trial is completed, ready to be repeated from step 1, if needed.

Once the pressure readings are plotted and compared, using the time it takes to fill up the tube, causing pressure readings equalize at both ends, is estimated via MATLAB plots (see figure 4-9 and table 4-1 in section 4.2.2). Subsequently, the linear velocity of the air flow at different pressure levels would be computed. Researchers may calculate the pressure drop and a virtual tube length which would be a best approximation for the actual air flow at \( t = 0 \). The pressure fed to the testing apparatus ranges from 15 to 45 psig, with an increment of 5 psig starting from 15 psig. Once again, to ensure reproducibility, at each pressure level, this soft tube test was conducted three times, separately.

### 4.1.3 Piston chamber fill up tests

In these tests, the piston remains static. As air flow enters or exits a piston chamber, it is assumed to undergo either adiabatic expansion or isothermal compression as described in the previous model development sections. Since the flow cross sectional area of the flow changed quite significantly from tube to piston chamber; a contraction near the piston nozzle followed up a rapid expansion into the chamber, the nozzle coefficient cannot be simply assigned as a constant value nor ignored. In addition to the impact from piston nozzle, due to the design of the piston, there is some space within the upper and lower chamber unavailable for facilitating further piston motion once the piston moving block reached its maximum and minimum displacement. For instance, the double-rod structure created a thicker upper plate at top which results in a several-millimeter wide gap between the piston block and the upper plate. Similarly, the piston block
cannot reach the bottom of the chamber due to an intentionally reserved space, several millimeters apart from the bottom plate. These subtle but crucial design features shall be taken account to the considerations when modelling the air flow either entering or exiting the piston chambers.

In general, the nozzle caused resistance for air flow through it. This indirectly affects the piston motion. In order to study this important factor, $C_f$ which describes the overall mass flow coefficient for air in the JBR system, and its impact, the time it takes to fill up both chambers of the piston shall be investigated. Since the two nozzles at the two piston chambers are manufactured nearly identical. It is assumed that when air enters or leaves either piston chamber, the nozzle coefficient remains constant, but the coefficient for mass flow entering a chamber is different from that of mass flow leaving a chamber. Recall in section 3.5.2 for the local resistance model, air flow involved is assumed to be ideal. Subsequently, to simplify the estimations, $C_f$ is defined as the average nozzle coefficient with its value in between the actual nozzle coefficient when air is flowing into the piston or out of the piston. Therefore, $C_f$ is used to represent the overall mass flow coefficient as air travels within this piston-valve complex. As a result, the following setup, as described in figure 4-3, involving pressure transducers, is used for obtaining the time it takes for air, at different gage pressure, enters and fills up the piston chamber.
According to the above setup, ball valve 1 is initially closed, and air pressure directly upstream of this ball valve is set via the diaphragm air pressure regulator. The double solenoid valve is used to switch air flow direction from the closed soft segment to the soft tube segment connecting absolute pressure transducer, P1, and the piston upper chamber nozzle. Whereas, the piston lower chamber nozzle is left open to atmospheric pressure and another pressure transducer, P2 is attached near the exit of a short segment of the soft tube. In case of gas leaking through the piston moving block, most likely around the rubber sealing ring, P2 would sense a changing pressure readings apart from the supposedly steady state atmospheric pressure. The actual trial procedure was further divided into the following steps:

1. Close ball valve 1 and set the air pressure via the diaphragm regulator, and leave the regulator for a few seconds to allow the pressure settle down within the tube segment
2. Open ball valve 2 to release the residual pressure built-up during previous trials, leave it that way for a few seconds to allow equilibrium of air pressure established inside and outside of the valve compartment.
3. Close ball valve 2, ensure that solenoid valve’s initial position is programmed towards ball valve 2
4. Open ball valve 1 and air flow shall be directed towards ball valve 2, perform a manual stroke of the piston block which would disperse and re-initialize the air within the piston chamber, let system to settle down for a few seconds.

5. At t=0, turn on the toggle switch which enables the Arduino controller to change the direction of the solenoid valve to feed air to the upper piston chamber, meanwhile the program will start recording the pressure at P1 and P2. To ensure enough data collected for showing the pressure trend, each trial would last for at least 15 seconds upon switching the solenoid valve.

6. Once a trial is finished, switch off the controller, export the pressure readings into txt files for future data processing, and close ball valve 1 to cut off the air pressure supply.

7. Repeat the above steps 3 times for each pressure level, which ranges from 15 to 45 psig with 5 psig increment.

Once the upper chamber fill up tests were done, the collected data can be processed using wavelet filter following the rules for selecting appropriate coefficient of eliminations, under different feed pressure levels.

To conduct the lower chamber fill up test, the lines and positions of the valve are simply swapped. The procedure is mostly similar except that in this case, the piston block is required to be re-positioned to the top after each trial rather than return to bottom as its initial location. A stopper would be placed to hold the piston block at its maximum displacement during each trial. Figure 4-4 is an illustration of the setup for the lower chamber fill up tests. The related results from these tests can be found in table 4-2 in section 4.2.3.
4.1.4 Single pass full stroke piston trials

Besides acquiring good estimations for the virtual tube length and the piston nozzle coefficients, friction forces play an important role on affecting the piston motion. According to physics, friction forces can be divided into two categories: Coulomb friction forces and kinetic friction forces. The former, denoted in $F_t$, is related to the force perpendicular to the contacting area of the moving piston block and cylinder wall; the latter, denoted as $\beta v^\alpha$ where $v$ is the velocity of the pistons moving block and $\alpha$, the exponent, is usually chosen to be 1. To obtain a better estimation and understanding of these two factors, a series of simple piston stroke tests were conducted.

Since these friction factors can be estimated via analyzing piston motion, a simple yet effective way to track piston position was developed: first, a small but bright red LED has been mounted onto the piston plate. Next, a high speed video camera is mounted at a fixed location on a stand. The Arduino controller was programmed to have a long enough time between relay switching so that piston can reach its maximum stroke length, staying there for a few seconds, returning to its minimum stroke length, staying there for a few seconds and repeating such cycles. Since time

![Figure 4-4 Equipment setup for chamber fill-up test (in this case, lower chamber is being filled up).](image)
relay settings are unchanged and constant, frequency and amplitude are also constant; hence, the only two key variables that will affect the piston motion in this set of trials are the pressure of air feed and the weight load of the piston moving part. As the pressure level of air feed to the piston changes, the speed of piston changes as a result. Since the maximum and minimum piston displacement is fixed for each trial, the time it takes for the piston to travel from bottom to top position provides the essential insights on how kinetic friction changes from trial to trial. On the other hand, the heavier the combined piston weight, the more forces it requires to displace it upward and thus, a slower speed for the same piston pressure.

Before conducting trials, in order to register the frames, camera position is fixed, and the high speed camera offers different levels of frame rates. In this study, all trials involving piston motion video recording has the frame rate set to be 240 fps, or frames per second. Given that the maximum piston displacement from bottom to top position is 30.48 cm (or 12 inches). Since a pair of frames showing piston reached its minimum and maximum displacement can be easily obtained, and the red LED is on during each trials, the piston location can simply be represented as changes in the red LED dot’s row coordinates within the image matrix.

Figure 4-5 Illustration of red dot tracking using high speed camera (a) schematic illustration for the red LED tracking principles (b) red LED when piston is at bottom and when piston is at top location.

Nonetheless, pressure readings, near the piston nozzle where air enters and exits the piston, are monitored simultaneously. The resultant pressure readings are smoothed by wavelet filters
separately based on the applied air pressure. To conduct the trials, pressure levels are varied from 15 to 45 psig the following procedures were implemented:

1. Arduino program is first initialized with time given for the relay to allow piston to perform the full stroke cycle with considerable time for piston to stay at maximum and minimum position, and the serial monitor is refreshed for recording new data sets.
2. Double check the settings which would enable piston to stay at its minimum displacement before switching on the Arduino controller.
3. Ball valve is first closed to allow air pressure adjustment through the regulator.
4. Release the built-up pressure within the piston (simply unplug one pressure transducer from its tee structure). This will not only reinitialize the pressure transducers’ readings but also prevent a constantly pressured tube.
5. Once the pressure is stable according to the regulator’s pressure gage, reconnect the pressure transducer and switch on the ball valve to allow air flow into the piston. Make sure power supplies are provided for both pressure transducers as well as the red LED for position indications.
6. Set the camera in position with its frame rate set to be 240 fps, make sure the stand was placed in the designated area and remained so during all trials since a small shift in camera’s position may result in huge discrepancies between the actual and calculated piston positions.
7. Once the above are set and checked, first start the high speed video clips, and then manually switch on the red LED and controller at the same time.
8. Record the clip for at least 15 seconds, then finish the trial, store the clips, export pressure readings for further analysis using the customized MATLAB program.

Using MATLAB, video clips can then be loaded and read on a frame by frame basis. For each frame, the MATLAB program first crops the frame to a designated area where red LED dots are travelling along. Then, the program switches on the red color filter to remove all background pixel values. Via searching the maximum red value, which is 255 in a gray scale version of the frame, the coordinates of the maximum red value, which represents the red LED, can be obtained. In this case, within each frame, the y-coordinates or row number of the red dot are thus obtained, in pixel values. To convert the height units from pixel based to meter based, a ratio of
pixel per meter can be calculated via first calculating the displacement of the red dot from bottom piston position to top in terms of pixel values and then divide it by the actual piston displacement measured in meters. The resultant ratio will have a unit of pixel per meter. This ratio is then used to find the piston displacement in meter, from frame to frame: with the initial position of the red dot, which the piston position starts at the bottom, every frame afterwards will have its position subtract the initial position, in terms of pixel values. Then, these pixel-based displacement values are converted to meter based values. Consequently, putting these calculations and video frames into a loop, plots of the piston displacement in pixel value and meter units can be both generated. With the help of MATLAB’s built-in data cursor, the time when piston starts from bottom location and reaches it maximum can be easily found via navigating the data cursor along the plot.

Figure 4-6 Schematic illustration of the overall piston setup (note that vessel is not mounted here).

4.2 Preliminary piston modelling test results

4.2.1 Valve modelling results

Please refer to the section 3.3.3 for detailed related model development of the double solenoid valve. Here, figure 4-7 illustrated the effects of spool displacement on the effective orifice area for
air flow passing through the double solenoid valve under a given signal. The blue line is the spool movement, from 0 cm to its most left position to 1.7 cm to its most right position. Note that values here are scaled up. This displacement which was denoted as x earlier, dictates the effective cross sectional area for computing the mass flow rate of the compressible air passing through the valve. The red line is the valve status which is scaled to the same range as the spool movement. The valve status has only two values: either it is fully open (at 1.7) or fully close (at 0). The green line is the change in power signals sent by the Arduino controller. Note that this signal line is 30 millisecond shifted to the left of the valve status. This time delay generated from the valve model confirms the manufacturer specified response time. Finally, the yellow line is the effective displacement of the opening as a result of the valve spool. Using the valve’s exit port connecting to the lower piston chamber as an example, the positive effective displacement of opening indicates that air is entering the lower piston chamber through the valve; negative value implies that air is leaving the lower piston chamber through the valve. Due to the design of this bistable double solenoid valve, spool first moves to the centre position which stops air flowing to either valve exit and then proceed to the other side and vice versa when the signal is switched; hence, the displacement of the opening is mostly symmetric. A more detailed illustration of the relationship between the piston pressure and the valve opening generated by the valve model can be seen in figure 4-8.
Figure 4-7 Effects of signal changes on flow through valve.

Figure 4-8 Detailed illustration of the relationship between piston pressure and valve opening.
4.2.2 Soft tube fill-up test results

Please refer to section 3.2.2 and section 4.1.2 for the experimental procedures as this test was designed to estimate the virtual tube space. Recall that this test uses two pressure transducers to monitor the pressure change cross a given segment of the tube. One pressure transducer is located near at the upstream where air enters from the valve to the tube once valve was first switched. Another transducer is located near the end of the tube segment where the flow is eventually stopped by a valve. The tube segment used in this test is 0.48 meter long. Each test for a given feed pressure is repeated three times, with the averaged time it takes for readings to reach its first peak pressure level, taken as the final results.

A typical plot is included as seen in figure 4-9 below. The overall summary of results is shown in table 4-1:

![Smoothed Plot Values](image)

Figure 4-9 Pressure change monitored within a typical tube fill-up test (using 15 psig for the feed); note that Pt is located at the upstream side of the tube; Pb is at the downstream side of the tube.
### Table 4-1 Averaged tube fill-up time

<table>
<thead>
<tr>
<th>Pressure Feed (psig)</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time to Fill-up (s)</td>
<td>0.0999</td>
<td>0.0856</td>
<td>0.0809</td>
<td>0.0823</td>
<td>0.0823</td>
<td>0.0793</td>
<td>0.0803</td>
</tr>
</tbody>
</table>

From the pertinent results, as pressure feed increases, the time it takes to fill-up the tube or for air pressure within the tube to peak decreases. This makes sense because a higher pressure tends to give a higher flow velocity leading to less time required for the flow to reach the end of the tube segment.

#### 4.2.3 Chamber fill-up test results

Please refer to section 3.5.4 for the development on modelling the piston dynamics, and refer to section 4.1.3 for the detailed experimental procedures. Similar to the tube fill-up tests, the chamber fill-up tests examine the time it takes for the pressure within the chamber to reach a balance with the feed pressure and thus finds the estimates for the nozzle coefficient (or mass flow coefficient of the air flow, $C_f$). The tube connecting the valve and the piston chamber nozzle is 0.242 meter long. Prior to filling up the chamber, the piston is moved away from that chamber to create an empty yet atmospheric pressured volume. Trial for each pressure level is repeated three times with the averaged time for filling up summarized in table 4-2:

### Table 4-2 Summary of averaged chamber filling-up time under a given feed pressure.

<table>
<thead>
<tr>
<th>Pressure Feed(psig)</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time to Fill-up the Upper Chamber (s)</td>
<td>0.386</td>
<td>0.381</td>
<td>0.317</td>
<td>0.338</td>
<td>0.369</td>
<td>0.394</td>
<td>0.375</td>
</tr>
<tr>
<td>Time to Fill-up the Lower Chamber (s)</td>
<td>0.332</td>
<td>0.344</td>
<td>0.360</td>
<td>0.341</td>
<td>0.383</td>
<td>0.408</td>
<td>0.458</td>
</tr>
</tbody>
</table>

The above table hinted that as pressure fed to the piston increases, the time it takes for filling up the upper chamber is generally unchanged whereas, the time it takes for filling up the lower chamber somehow increases. One possible reason for these differences of filling time is likely due to the design of the piston chamber. In the upper chamber, there are two piston rods occupying additional volume as the upper chamber expands. On the other hand, the lower chamber is mostly viewed as a changing cylindrical volume with increasing height the volume.
changes linearly but the cross sectional area remains unchanged. Therefore, relatively smaller upper chamber tends to be filled up quicker under higher feed pressure. This is also worth noted since this difference in fill up time further translates to a different time for piston to complete a full stroke upward and a full stroke downward. The piston often finds itself often easier to move downward while filling up the upper chamber than that of the reverse way of action. Therefore, the selection of the relay time for switching the valve towards upper chamber is often a little bit lower than that of towards the lower chamber in order to maintain relatively steady operations without hitting either side of the piston stroke limits.

4.2.4 Single pass full stroke test results

Please refer to section 4.1.4 for detailed experimental design and procedures. To have a good understand of some of the piston’s motion characteristics as they are related to the pressure level and friction forces, a series of single pass full stroke tests were conducted. In this case, piston stroke reached it top and bottom location, or maximum and minimum displacement over a set number of low frequency. This enables the piston to reach acceleration all the way through along its direction of motion until being physically stopped by its limits. i.e. deceleration was no occurred in this case and the friction exerted by either the coulomb forces or the kinetic forces can be attained. The piston motion curve is also tracked using the proven red dot tracking methods. A typical motion plot can be best demonstrated as figure 4-11 shows below:

![Typical piston motion curve during a single pass full stoke test.](image-url)
The line indicates the motion of the piston which, using MATLAB’s data cursor, the time it first reaches its peak from bottom, or its trough from top can be accurately attained. To analyze the piston chambers separately, averaged time for piston to reach top and bottom are presented in separate tables followed by corresponding plots. For the case of piston moving to the top, the summary of time can be seen in table 4-3 with the corresponding plot in figure 4-12.

Table 4-3 Summary of time needed to move to top at different pressure feed.

<table>
<thead>
<tr>
<th>Feed Pressure (psig)</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight load (kg)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.872</td>
<td>0.219</td>
<td>0.353</td>
<td>0.424</td>
<td>0.317</td>
<td>0.466</td>
<td>0.329</td>
<td>0.411</td>
</tr>
<tr>
<td>3.192</td>
<td>0.349</td>
<td>0.307</td>
<td>0.329</td>
<td>0.384</td>
<td>0.274</td>
<td>0.330</td>
<td>0.373</td>
</tr>
<tr>
<td>4.357</td>
<td>0.283</td>
<td>0.529</td>
<td>0.316</td>
<td>0.375</td>
<td>0.369</td>
<td>0.349</td>
<td>0.328</td>
</tr>
</tbody>
</table>

From the above tables and plots, it is safe to say that piston generally reaches top between 0.2 to 0.5 second upon initiating the air filling to the lower chamber, driving the piston up while leaving the upper chamber open toward the atmosphere. At lower weight load, this time scatters from over 0.2 to over 0.45 second as pressure increases. At intermediate weight load, this time scatters from 0.27 to 0.38 second yet closely along a line of 0.32 second. At higher weight load, however, this trend increase from 0.28 to 0.375 then falls back to 0.32 second while ignoring the
possible outlier at 15 psig case. In general, as weight load increases, the time it takes for the piston to reach top decreases as pressure increases. The shortest time recorded happens at low pressure coupled with light weight; the longest time recorded happens at low pressure coupled with heavy weight. The second longest time happens when feed pressure is as high as 35 psig with a light weight.

On the other hand, as seen in table 4-4 and figure 4-13, this range narrows down to 0.3 to 0.45 second upon initiating the air filling to the upper chamber, driving the piston down while the lower chamber is open towards the atmosphere. At lower weight load, this time scatters from over 0.32 to over 0.42 second as pressure increases. At intermediate weight load, this time scatters from 0.31 to 0.44 second then falls back to about 0.33 and moves up to over 0.4 second as pressure continues to rise. At higher weight load, however, this trend increase from 0.28 to 0.43 second, then likely plateaus onward. The longest time happens at high feed pressure, 45 psig, light weight; the shortest time happens at heavy weight with low feed pressure of 15 psig. Once again, the trend seems to be reversed around a feed pressure of 30 psig. Before that pressure level, heavy weight load gives shortest time for the piston to reach bottom; after that level, however, intermediate weight load gives faster downward motion.

<table>
<thead>
<tr>
<th>Feed Pressure (psig)</th>
<th>Weight load (kg)</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.872</td>
<td>0.425</td>
<td>0.325</td>
<td>0.417</td>
<td>0.381</td>
<td>0.342</td>
<td>0.407</td>
<td>0.466</td>
</tr>
<tr>
<td></td>
<td>3.192</td>
<td>0.312</td>
<td>0.425</td>
<td>0.439</td>
<td>0.443</td>
<td>0.326</td>
<td>0.334</td>
<td>0.427</td>
</tr>
<tr>
<td></td>
<td>4.357</td>
<td>0.294</td>
<td>0.296</td>
<td>0.391</td>
<td>0.323</td>
<td>0.384</td>
<td>0.437</td>
<td>0.402</td>
</tr>
</tbody>
</table>
Figure 4-12 Summary of time needed to move to top at different pressure feed.
According to physics, when object undergoes free fall, its acceleration generally equals to the gravitational acceleration $g$ which is $9.81 \text{ kg}\cdot\text{m/s}^2$. In this case however, when piston is moving downward, the platform is not experiencing free fall; in fact, it is velocity is accelerating downward as air flow continuous to flow into the upper chamber driving the piston block faster down than its free falling speed. Consequently, the lower chamber undergoes a much more rapid compression than before. The downward forces acting on the piston block in this case include forces caused by continuous air pressure built-up in the upper chamber plus the combined weight of the moving part which includes the weight load and the platform and the piston’s moving block. In case of a faster expansion in the upper chamber, the lower chamber pressure may be compressed quick enough to “fight back” or resists the downward motion. Note that the volume of the lower chamber is relatively larger than that of the upper chamber due to its design. Therefore, under a given feed pressure, the lower part may take more time to fill up. Fortunately, as explained in the previous section, the resistance coefficient favored the outflow direction; it is easier to expel the air than to receive the air to a given chamber. As a result, the resistance, or “back pressure” exerted by the lower chamber during an downward motion is likely much less severe than that of the upper chamber during an upward motion, which gives a good assumption why the ripple like motion curve is less obvious during the downward stroke than that of an upward stroke.
Since pressure data were also collected during the test, after smoothing the pressure readings using the wavelet filter based on the level of feed pressure, the pressure profile confirms the above assumptions; as seen in figure 4-15, a typical pressure profile for pressure gage readings near the two nozzles of the piston chambers, pressure readings peaked a little bit soon after that pressure at either side of the piston shortly balanced. Later, the readings dropped, with the magnitude of decrease depending on the level of feed pressure. After piston stopped and valve position remained constant, the pressure readings rise back to the feed level matching the chamber pressure with the feed pressure. The pressure readings recorded near the bottom nozzle somehow showed less fluctuations with shorter stable time compared to its counterpart at the upper nozzle. However, this is somewhat different from the reality (as seen in figure 4-16). In reality, the pressure near the upper and lower piston chamber are crossed and then continued in
opposite direction. The difference is caused by the space for air flow to fill-up along its path before having an impact on piston motion, which confirms the need for a virtual tube space as an empiric parameter.

**Figure 4-14** Predicted pressure profile and air velocity under 45 psig without virtual tube length

**Figure 4-15** Actual pressure profile when operating under 45 psig.
4.3 Experiments for JBR mixing trials

As explained in the previous section, the piston was tracked based on the red LED attached to the piston platform. One essential benefit of using this camera approach is the following: this method not only tracks the piston motion, it also captures the vessel region loaded with white silica sand. The procedure for carrying out such piston trials are generally similar to the single pass full stroke tests except for the time relay settings. In this case, relay is acting more frequently which allows more rapid double acting piston motion (upward and downward). This leads to different amplitudes at different frequencies under different pressure levels. Since the Arduino controller implements a feed forward loop, the control algorithm is not very sophisticated. The time which relay sends on and off command to the 5/2 double solenoid valve, will direct air flow to either entering or exiting a given chamber of the piston. Based on some preliminary studies, solenoid valve has its initial position open towards the left part. This part is connecting to the upper chamber, with $P_{\text{top}}$ measuring the real time pressure. As a result, air enters the upper chamber acting on the piston block and continuously driving it downward. Consequently, the piston is initially at its bottom position. On the other hand, $P_{\text{bot}}$ is attached to the air line connecting the lower piston nozzle and to the double solenoid valve.

4.3.1 Vessel region tracking and calibration

Since the main focus of this study is to analyze and characterize the solid bed during JBR operations, the vessel area depicted in each frame is the most critical raw data. Due to optical shifts and changing viewing angles while the piston is in motion, tracking and locating the boundaries around the vessel in a reliable manner required dedicated calibration experiments.

To obtain the calibration, the vessel covered in white paper was first tracked using high speed camera while putting everything else in a black background. Through MATLAB programming, the white region can be isolated and bounded. Figure 4-17, from left to right, it illustrates a sequence of vessel traveling from bottom to top during a single upward stroke.
After converting the video to frames in MATLAB, a few points on the white paper wrapped vessel region were selected. Their red, green and blue values are stored in a gray scale matrix. A MATLAB program is compiled to seek regions following similar gray scale values from frame to frame. Through comparing the white region from frame to frame, a minimum threshold of the gray scale red, green and blue values can be established. These thresholds are then transferred to the MATLAB program. The MATLAB program then starts to search pixel by pixel across the image matrix. Once the all the pixel values have been searched, whichever has its gray pixel values exceeding the threshold would have its coordinates stored. Later, all the stored locations of these pixels were joined together forming a blob. The location of the blob is largely close to the region of the vessel covered in white. Ultimately, after apply some smoothing and corrections to the image segment, the blob is bounded via the built-in bounding box functions. In the end, the bounded area represents the white region which is the surface area of the vessel region facing the camera. Note that no light shall be directly shed onto the vessel area in order to avoid strong reflections on the Pixel glass recorded during the high speed video clips which may later confuse the MATLAB program. To ensure the solid bed would stand out, any possible shred of reflected lab room light shown on the vessel wall is minimized (also see figure 2-10 in section 2.5).

As for the red LED, when piston is moving, the optical shift is negligible due to its small size in a captured frame. However, the effects of optical shift becomes rather significant for the vessel’s projected surface area. This area changes significantly as piston travels to different locations. Subsequently, the area’s coordinates at the four corners of the bounded vessel region also change. In order to correct this optical shift, a calibration of the vessel coordinates is applied to relate the vessel ‘s coordinates to the piston’s position, represented by the red LED’s coordinates.
in pixel scale. Figure 4-17 showed this problem. To minimize this shift, a correlation between the coordinates of the bounding box and the coordinates of the small red LED dot is established. Once corrections are applied, the shift became minimal as one can see in the bottom right subplot of figure 4-17. Figure 4-18 illustrated how the coordinates of the bounding box of the vessel are related to the coordinates of the bounding box of the red LED. Figure 4-20 gives the relationship between the two key coordinates which are used to provide corrections to counter the effects of optical shifts.

Figure 4-17 Vessel tracking technique in MATLAB (note the shifting of the bounding box at the bottom left and bottom central subplots, if no correction is applied to counter the optical shift during piston motion).
Figure 4-18 Illustration of the bounding box coordinate estimations for the vessel.

Figure 4-19 Vessel coordinate calibrations.

Once the program managed to automatically generate a bounding box for the vessel with respect to the red LED, it can be used to process all other JBR test videos. As a result, the vessel region, drawn by the re-constructed bounding box, is consistently cropped out of the original frame and further analyzed further. The complete illustration for the apparatus setup can be seen in figure 4-20 below:
Once the robust vessel region tracking can be built-into the MATLAB program, further image analysis on solid bed can be achieved. To study how piston motion affects the solid bed motion and gas-solid mixing, a series of experiments are planned and conducted in the steps very similar to the piston full stroke tests. The only differences in this set of experiments are: a vessel loaded with white solid sand was mounted on the piston plate, and the relay frequency was set into three levels higher than the full stroke tests. Overall, the variables are the followings:

1. Compressed air pressure fed to the system: varied from 20 to 40 psig via the regulator with a 5 psig increment
2. Arduino controlled relay frequency, three set values: 5.45 Hz, 6.45 Hz and 8.00 Hz
3. Weight load added to the moving part of the piston block which has three levels: 2.569 kg, 3.951 kg and 5.054 kg
4. Percentage of volume occupied by the solid sand within the vessel, about 5 %, 15 % and 30 % (these levels are achieved via loading 15 g, 45 g and 90 g of solid sand, respectively, with an assumed aerated bed voidage of about 0.4 as a rule of thumb.)

For checking the reproducibility and accuracy of the results, each trial with one of the above conditions is repeated three times. Data collected include video clips and pressure readings at
both piston nozzle as well as controller signal profile. The latter two are merged into a txt file. Nevertheless, the main focus is to analyze the frames recorded in each video clip during each trial.

4.3.2 Solid bed analysis using MATLAB

To analyze the bed situation captured in frames using MATLAB, as shown in figure 4-21 and 4-22, the program first takes out the original frame (first subplot on the top left); it then locates the red LED dot after using color filtering and the built-in find functions; subsequently, it draws a boundary of the vessel region taking the red LED as the reference using the previously obtained correlation. Eventually the program crops the bounded vessel region, enhances it in gray scale (as seen in the bottom central subplot of figure 4-21 and 4-22), and tracks the solid blob by turning the enhanced gray scale picture into a binary, black and white, version. As a result, everything that is less than the white threshold is turned into black, leaving solids bed shown as white blob shown in the cropped vessel region, as shown in the bottom right subplot.

Note that, instead of using a 2-D rectangular column, a 3-D cylindrical vessel is used for gas-solid mixing studies. For lateral mixing occurs in a 2-D column would likely be unchanged during piston motion. Whereas, the circular cross section area of a cylindrical column, having the same volume, allows more space for solid to travel radially along the diameter. Intuitively, the lateral solids mixing in a 2D unit of the same volume would be much worse that the radial solids mixing in a cylinder.

To describe the solid distribution vertically across the vessel, a modified approach of the variance method is used: in this study, 3 mathematical terms are computed for each cycle of piston motion frames since JBR is operated in a cyclic manner. Here, each cycle consists of all the frames taken when piston moves from its previous peak displacement to another, the sum of all the frames during that cycle provided an overall image matrix. This enabled researchers to calculate terms like CV spatial, CV vertical, and CV lateral. Detailed illustrations of the calculation procedures for these CV terms can be found in section 5.2.2.
Figure 4-21 Typical 15g solid bed situation analyzed when vessel is moving up.

Figure 4-22 Typical 15g solid bed situation analyzed when vessel is moving down.
4.4 Brief procedure recap

As mentioned earlier, the preliminary tests include the following: soft tube pressure drop tests, piston chamber filling tests as well as single pass full stroke tests are conducted independently for achieving a good estimation of the 4 modelling parameters. Later JBR mixing trials are conducted with synchronized mixing and piston motion recorded by high speed camera.

After finishing the first 3 preliminary studies, the piston model will be tuned to match up the steady state amplitude obtained from the continuous mixing studies. Meanwhile the frames and video clips will be examined by the appropriate MATLAB programs for image based analysis.
5 Results and Discussions

5.1 Experimental Validation and Tuning of the Model

The piston model described in Chapter 3 is used to match the experimental data by adjusting the 4 empirical parameters: virtual tube length, $L_v$, mass flow coefficient, $C_f$, Coulomb friction force, $F_f$, and coefficient of kinetic friction, $\beta$. Through the rigorous methods of finite element search and other built-in MATLAB techniques, the differences between the model predicted piston motion curves and the actual experimental piston motion curves are minimized. The resulting values for $L_v$, $C_f$, $F_f$, and $\beta$ are 1.26 m, 0.264, 17.6 N, and $49.2$ kg m $^{-2}$ , respectively. Figure 5-1 shows that the best fitting values for $C_f$ and $L_v$ enabled the model to generate very similar results for the time values obtained from the tube fill-up tests and chamber fill-up tests, which were described in section 4.2.2 and 4.2.3. One may then claim that the best-fitting values worked well for approximating the actual piston parameters in these two cases.

![Figure 5-1 Finding the best fitting values for $C_f$ and $L_v$, with error bars applied.](image-url)
However, results obtained from the other two sets of experiments: the single pass full stroke tests and continuous JBR mixing tests, conducted according to section 4.2.4 and section 4.3, showed certain deviations when using those fitting parameters to approximate the piston displacement. According to figure 5-2, the averaged errors between the model and the actual piston curve ranges from 5% to 20%. Note that when comparing to the platform weight load ranges from 2.569kg to 5.054kg, the maximum solid loads, 90 g, is just about 1.78%, less than 2% of the lightest weight load case of 2.569 kg. As mentioned, this combined weight load mentioned includes the vessel, piston block, platform, added weight block and other components part of the moving piston blocks while excluding the solids loaded in the vessel.

Figure 5-2 Comparing the model predicted to the average experimental steady state piston amplitudes resulted from each trial.

As a result, all cases of different solid loads used in this study, are assumed to have no effect on the piston amplitude. A sample of the model predicted motion curve vs. the actual piston curve was shown in figure 5-3 below with a zoomed in version as figure 5-4 below:
Figure 5-3 Example of model predicted motion vs. actual motion (at a weight load of 3.951 kg, 5.48 Hz with 25 psig feed pressure).

Figure 5-4 Enlarged segment of the model predicted motion vs. actual motion (at a weight load of 3.951 kg, 5.48 Hz with 25 psig feed pressure).
After some quick analysis, it is observed that whenever relay frequency is low (around 5.5 Hz) with low feed pressure (less than 30 psig), the model prediction is close to the actual situation, but if the weight load is high, 5.054kg in this case, the prediction deviates significantly. Part of the reason for such a high error percentage is because of the relatively smaller steady state piston amplitude which makes a small deviation significant overall, even though the absolute error is also small in magnitude.

In most cases, the model predicted displacement readings not only matches fairly close to the actual steady state displacement, as shown in figure 5-4, but also properly predicts the time it takes for the piston to reach steady state.

There are many possible reasons that may give rise to these deviations for the model to predict the displacement and amplitude:

- The control algorithm is feedforward which has no control over the actual amplitude during trials. The applied pressure is controlled by a pressure regulator and the controller just controls the switching frequency of the valve connected to the two cylinder chambers. As a result, the amplitude of the reactor motion may vary slightly between cycles. Controlling the amplitude, instead, would be difficult because of inertial effects and both amplitude and frequency would vary from cycle to cycle, which would make modeling much more challenging.

- Model takes a constant frequency for computation, but the actual system undergoes possible drifts which is not strictly following the exact frequency set by the relay since the control method is implemented in a feed-forward manner.

- When building the sub-components of the model, several assumptions are made to reduce the complexity of the calculations; whereas, the finite element methods are used for solving the differential equations numerically, some errors may be generated and propagated.

Overall, this model managed to give reasonable predictions. It served a good foundation for further investigations on relating the piston motion and the solid bed mixing.
5.2 Gas-solid mixing analysis

5.2.1 Overview

In general, there are three parameters input to the JBR system that influence the piston motion: the feed pressure, the relay frequency and the weight load added to the piston’s platform. From section 3.5.4, the force balance suggested that piston experiences upward or downward acceleration as a result of force balance between the two chambers. In order to lift up the solid bed, the force acting on the solid bed, exerted by the piston, has to exceed the bed weight. A typical piston cycle includes two parts: an up stroke and a down stroke. A general illustration of force balance of solid bed during upward piston motion can be seen in figure 5-5. To lift up the solid bed during an upstroke, the piston acceleration has to exceed the gravity acceleration during an upward stroke, so that the resultant net acceleration is positive. Once piston starts to decelerate, inertia carries the solid upward, within the reactor vessel. However, solids near the top of the bed will be lifted up first as they are subject to smallest resistance. Hence, the magnitude of the maximum upward acceleration determines how much solids can be lifted up during an up stroke.

![Figure 5-5 Force balance of the solid bed during upward motion (neglecting the air resistance).](image-url)
On the other hand, during a down stroke, the solid bed can expand as the piston quickly retracts with a downward acceleration much larger than the gravity acceleration. As a result, the bottom of the bed cannot catch up with the bottom of the vessel. The rapid downward stroke leaves a gap between the bottom of the solid bed and the vessel. Subsequently, the solid bed is subjected to free fall motion. The faster the downward acceleration, the more space is created between the solid bed and the bottom of the vessel. The force balance can be seen in figure 5-6 below. To facilitate solid-gas mixing during a piston cycle, the maximum acceleration during up stroke and down stroke is critical.

![Force balance of the solid bed during downward motion (neglecting the air resistance).](image)

**Figure 5-6** Force balance of the solid bed during downward motion (neglecting the air resistance).

### 5.2.2 Image analysis criteria

To analyze how well gas and solid are interacting during a JBR trial, each image depicting the vessel region extracted from each frame is processed using simple matrix operations. Such image consists of about 40 pixels in width and 100 pixels in length. Since each image is consistently cropped using the corrections provided in section 4.2.1, this dimension is constant throughout trials. If each pixel is viewed as one element, then this image can be converted into a matrix of 100 rows with 40 columns. As an example shown in the case of figure 5-3, assuming that this image is converted to a matrix of $N_{\text{row}}$ and $N_{\text{col}}$, using $i$ for row index and $j$ for column index,
each element of the first image matrix, $y_1$ on the left, is added to its corresponding counterpart in the second image matrix, $y_2$ in the middle. The sum of these two images then adds the next image; the process goes on until the last image in this trial is added to this sum. If $k$ represents the sequence number of a given vessel image from a given frame, the resultant matrix containing the time average of image elements can be calculated as the total sum of images divided by the total number of frames, denoted as $N_{frames}$ involved, denoted as $y_s$. This process can be visualized in figure 5-7 below. The relevant calculation can be seen in equation (5-1a).

![Figure 5-7 Example of adding frames using matrix operations.](image)

At this stage, one may use the coefficient of variation, simply referred as CV in future sections, to evaluate the solid distribution across the vessel region. The CV is defined as the standard deviation of the sample data divided by the sample mean. In this study, there are three types of CV calculated: CV spatial which quantifies the overall solid distribution; CV lateral which gives the lateral solid distribution from side to side, and CV vertical which gives the solid distribution from top to bottom. Their values can be obtained using equations and formula listed from (5-1) to (5-3). The lower the CV values, the more evenly distributed the solid throughout the vessel.
region in that given trial. Note that the subscript \( s \) indicates that the quantity is averaged over all the frames of the video. The subscript “avg” indicates that the quantity is averaged over all the pixels. This means that \( y_{s,\text{avg}} \) is the average of the gray value for all the pixels within the reactor, averaged over all the frames.

\[
\begin{align*}
  y_s &= \frac{\sum_{k=1}^{N_{\text{frames}}} \sum_{i=1}^{N_{\text{row}}} \sum_{j=1}^{N_{\text{col}}} \ y_{i,j,k}}{N_{\text{frames}}} \\
  y_{s,\text{avg}} &= \frac{\sum_{i=1}^{N_{\text{row}}} \sum_{j=1}^{N_{\text{col}}} \ y_s(i,j)}{N_{\text{row}} \times N_{\text{col}}} \\
  y_{s,\text{std}} &= \frac{\sqrt{\sum_{i=1}^{N_{\text{row}}} \sum_{j=1}^{N_{\text{col}}} (y_s(i,j) - y_{s,\text{avg}})^2}}{N_{\text{row}} \times N_{\text{col}} - 1} \\
  \text{CV}_{\text{spatial}} &= \frac{y_{s,\text{std}}}{y_{s,\text{avg}}} \\
  y_{s,\text{col},j} &= \frac{\sum_{i=1}^{N_{\text{col}}} y_s(i,j)}{N_{\text{col}}} \quad \text{(for each column } j) \quad \text{(a)} \\
  y_{s,\text{col},avg} &= \frac{\sum_{i=1}^{N_{\text{col}}} y_{s,\text{col},j}}{N_{\text{col}}} = y_{s,\text{avg}} \quad \text{(b)} \\
  y_{s,\text{col},std} &= \frac{\sqrt{\sum_{i=1}^{N_{\text{col}}} (y_{s,\text{col},j} - y_{s,\text{avg}})^2}}{N_{\text{col}} - 1} \quad \text{(c)} \\
  \text{CV}_{\text{lateral}} &= \frac{y_{s,\text{col},std}}{y_{s,\text{col},avg}} \quad \text{(d)} \\
  y_{s,\text{row},i} &= \frac{\sum_{j=1}^{N_{\text{col}}} y_s(i,j)}{N_{\text{col}}} \quad \text{(for each row } i) \quad \text{(a)} \\
  y_{s,\text{row},avg} &= \frac{\sum_{i=1}^{N_{\text{row}}} y_{s,\text{row},i}}{N_{\text{row}}} = y_{s,\text{avg}} \quad \text{(b)} \\
  y_{s,\text{row},std} &= \frac{\sqrt{\sum_{i=1}^{N_{\text{row}}} (y_{s,\text{row},i} - y_{s,\text{avg}})^2}}{N_{\text{row}} - 1} \quad \text{(c)} \\
  \text{CV}_{\text{vertical}} &= \frac{y_{s,\text{row},std}}{y_{s,\text{row},avg}} \quad \text{(d)} 
\end{align*}
\]
Essentially, when calculating the CV lateral, the matrix is first averaged along each column. As a result, it is “collapsed” into a row vector containing all column averages of gray values. This process can be best illustrated in figure 5-8 below. The standard deviation and averages of each element is then computed for CV lateral calculations; whereas, equation (5-2) provided the mathematical illustrations.

![Diagram](image.png)

**Figure 5-8 Illustration of preparing column based averages for CV lateral calculations.**

In this study, CV vertical is more important for gas-solid mixing as the non-rotating piston facilitates the vertical motion, casting solid up and down along the vessel. Similar to the calculations for CV lateral, the time average image matrix is first averaged along each row. The resultant average gray values for each row are then collected in a column vector. This process can be demonstrated with the help of figure 5-9 below. The standard deviation and averages of each element is then computed for CV vertical calculations; whereas, equation (5-3) provided the mathematical illustrations.
Figure 5-9 Illustration of preparing row based averages for CV vertical calculations.

For a relatively poorly-mixed trial, as an example of the time average image shown in figure 5-10 below, the solid presence, indicated with whiter pixels, did not dominate the entire vessel region. On the contrary, in a relatively well-mixed trial, as an example of the time average image shown in figure 5-11, solid presence is observed all over the vessel region. In general, a smaller CV vertical indicates a little variation between rows of image matrix, thus implying a relatively uniform solid distribution and a relatively uniform solid concentration from top to bottom. The closer CV vertical gets to 0 the better, which further implies good gas-solid interactions within the vessel. Therefore, in this study, CV vertical is the primary indicator of how well gas and solid mixed. A summary of CV values for figure 5-10 and 5-11 can be found in table 5-1.
Figure 5-10 CV analysis of time average frame from a relatively poorly-mixed trial.

Figure 5-11 CV analysis of time average frame from a relatively well-mixed trial.
Table 5-1 Cross comparison of piston motion parameters and CV values of a well-mixed case and a poorly-mixed case.

<table>
<thead>
<tr>
<th></th>
<th>Poorly-mixed (Figure 5-10)</th>
<th>Well-mixed (Figure 5-11)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid Loading (g)</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Feed Pressure (psig)</td>
<td>20</td>
<td>40</td>
</tr>
<tr>
<td>Relay Frequency (Hz)</td>
<td>8.0</td>
<td>8.0</td>
</tr>
<tr>
<td>Weight Load (kg)</td>
<td>5.054</td>
<td>2.569</td>
</tr>
<tr>
<td>( CV_{\text{spatial}} )</td>
<td>1.134</td>
<td>0.383</td>
</tr>
<tr>
<td>( CV_{\text{lateral}} )</td>
<td>0.385</td>
<td>0.313</td>
</tr>
<tr>
<td>( CV_{\text{vertical}} )</td>
<td>0.985</td>
<td>0.187</td>
</tr>
</tbody>
</table>

Note that the JBR vessel is a closed system. In most of its applications, gas is not circulated across the vessel and its surroundings. The solid concentration at each pixel can be estimated through the gray level. Subsequently, very low \( CV_{\text{vertical}} \) in JBR can occur in two scenarios: first, solid bed is well expanded throughout the vessel from bottom to top, causing each particle to be as far apart from each other as it can. Then, gas is also uniformly displaced around particles as the solid bed expands. Another case would have solid bulk moving up like a plug, reaching the top of the vessel. Meanwhile, gas is forced go through the solid plug. In either scenario, gas and solid are involved in a relative motion. This means the time averaged solid distribution would have very small deviations from pixel to pixel and thus enabled a good gas-solid contact. Thanks to the good gas-solid contact, at the relatively high vertical piston acceleration, the solid particles are also cast around with a high speed which further favors gas-solid mixing. Therefore, from the above summary table, since the study primary focused on the vertical motion of the solid bed, it can be seen that achieving low \( CV_{\text{vertical}} \) can be seen as an indication of well-performed gas-solid mixing in this study.
5.2.3 Correlations between piston motion and solid mixing

As mentioned, the piston provides the driving force for casting solid into mid-air. This driving force has a primary underlining factor: the maximum piston half cycle acceleration. Here, either an up stroke or a down stroke can be seen as a half cycle. Since solid bed would expand as piston either decelerates during an up stroke, or exceeds gravitational acceleration during a down stroke, the maximum piston half cycle accelerations become an important factor. From experiments, each trial produces a video clip which contains two important pieces of information: frame by frame piston displacement and solid bed distribution. By plotting the CV vertical against the average of maximum piston half cycle accelerations, a threshold for such average can be estimated as seen in figure 5-12 below.

![Figure 5-12](image_url)

**Figure 5-12 CV vertical vs. average of the maximum upward and downward piston accelerations.**

From the above figure, CV vertical decreases sharply before the average of maximum accelerations reach 57 m/s². Beyond this value, CV vertical essentially plateaued around 0.20. For a CV vertical value of less than 0.30, it implies standard deviations is less than 1/3 of the average gray value of among the rows from top to bottom. To further investigate, in a similar manner, CV vertical is plotted against the maximum of the maximum piston half cycle accelerations. The results can be seen in figure 5-13. In this case, this acceleration threshold is
slightly larger than 62 m/s²; beyond this threshold, CV vertical becomes mostly stable below 0.30 and fluctuates around 0.20.

Figure 5-13 CV vertical vs. maximum of the maximum upward and downward piston accelerations.

Another option is to plot the CV vertical against the minimum of the maximum piston half cycle accelerations. The resultant graph can be seen as figure 5-14. Here, the threshold decreases to about 55 m/s². Beyond this threshold, CV vertical becomes mostly stable around 0.20. To be conservative, the threshold for maximum piston half cycle accelerations needs to be larger than 55 m/s².
As shown previously, the maximum half cycle accelerations of the piston are critical for solid mixing. Another important factor worth noting is the average cycle amplitude generated by the piston. This is rather simple to understand: amplitude, the piston displacement, is essentially the acceleration integrated over time. Although the relay set the frequency, or the cycle time, for the double solenoid valve to alternate piston motion, a faster piston half cycle acceleration results in longer distance travelled over the same cycle time.

Through plotting the CV vertical against the all average piston cycle amplitude under all three solid loadings, as demonstrated in figure 5-15 below, it is clear that as amplitude is over 0.06 m or 6 cm, CV vertical decreases below 0.30 and stabilizes around 0.20 as amplitude further increases beyond 0.08 m or 8 cm. Note that the maximum bed height involved in this study is about 5 cm.
Therefore, experiments suggested to operate the piston with at least 55 m/s² for its maximum half cycle acceleration going either way while keeping the amplitude over 5 cm in order to steadily mixing gas and solid with a CV vertical less than 0.30. In general, the more solids loaded to the vessel, the more space required to expand the bed; therefore, more solids load require one to operate at higher piston acceleration in either direction with amplitude larger than the static bed height.

5.2.4 Model predicted piston motion and solid mixing

The model predicts the piston motion under a given input of feed pressure, relay frequency and weight load. To compare predicted and experimental minimum of maximum piston half cycle accelerations, the experimental accelerations are taking the average of the repeated trials under the same condition. As a result, the deviations between the model-predicted accelerations and the experimental ones can be seen in figure 5-16 for matching the higher CV vertical among the repeated trials, and in figure 5-17 for the lower CV vertical among the repeated trials:
In figure 5-16, the threshold for predicted minimum of maximum piston half cycle accelerations is about 52 m/s²; whereas, the threshold for the experimental ones is about 54 m/s². The deviation in acceleration is merely 1/27 of the experimental observations. The difference is rather subtle.

Figure 5-16  Max CV vertical vs. experimental and model predicted minimum of maximum piston half cycle accelerations.
Figure 5-17 Min CV vertical vs. experimental and model predicted minimum of maximum piston half cycle accelerations.

Similarly, for the min CV vertical cases, the threshold resulted from the experiments and the model’s prediction becomes slightly closer to each other, about 50 m/s$^2$ and 52 m/s$^2$ respectively as shown in figure 5-17 above. Subsequently, the deviation is about 1/26 of the experimental observations.
5.3 Recommended operation range

To achieve a CV vertical below 0.30 with the conditions applied to JBR in this study, one needs to operate the piston with a minimum upward and downward acceleration of 55 m/s² while providing a cycle amplitude of over 0.06 m. To be on the safe side, with the help of figure 5-16 and 5-18, a 105% factor is applied to either parameter. The resultant minimum threshold of acceleration is 57.75 m/s² with minimum amplitude of 0.063 m. Through filtering all the cases specified in this study with these two thresholds, the resultant conditions that meet the requirement can be seen in table 5-2.

Overall, it is quite interesting to see that the model suggested none of the cases with high relay frequency settings would result in acceleration and amplitude surpassing the minimum thresholds. In addition, when the piston weight load is light, 2.569 kg, the JBR may be operated under any feed pressure between 20 to 40 psig for a set relay frequency of either 5.48 or 6.45 Hz. As weight load increases to 3.951 kg, all feed pressure would still result in acceptable piston motion with a relay frequency at 5.48 Hz. Under this weight load as relay frequency increases to 6.45 Hz, the desirable feed pressure range narrows down to 30 to 40 psig. Under the heaviest weight load case, 5.054 kg, only high feed pressure, 35 to 40 psig coupled with a relay frequency of 5.48 Hz would set gas-solid mixing in good direction. If the relay frequency was set to be 6.45 Hz, only a feed pressure of 40 psig would provide sufficient piston motion to meet the minimum requirements.
Table 5-2 Model predicted accelerations under given conditions meeting the minimum requirement.

<table>
<thead>
<tr>
<th>Feed Pressure (Psig)</th>
<th>Relay Frequency (Hz)</th>
<th>Amplitude (m)</th>
<th>Weight Load (kg)</th>
<th>Max Upward Acceleration (m/s²)</th>
<th>Max Downward Acceleration (m/s²)</th>
<th>Min of upward &amp; downward acceleration (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>5.48</td>
<td>0.0750</td>
<td>2.569</td>
<td>60.0</td>
<td>54.6</td>
<td>54.6</td>
</tr>
<tr>
<td>25</td>
<td>5.48</td>
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<td>2.569</td>
<td>66.4</td>
<td>60.5</td>
<td>60.5</td>
</tr>
<tr>
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<td>5.48</td>
<td>0.0940</td>
<td>2.569</td>
<td>72.2</td>
<td>66.1</td>
<td>66.1</td>
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<td>71.4</td>
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<td>0.109</td>
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<td>82.7</td>
<td>76.5</td>
<td>76.5</td>
</tr>
<tr>
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<td>2.569</td>
<td>55.6</td>
<td>53.4</td>
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<td>62.8</td>
<td>57.2</td>
<td>57.2</td>
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<tr>
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<td>3.951</td>
<td>66.5</td>
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</tr>
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<td>3.951</td>
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<td>56.1</td>
<td>56.1</td>
</tr>
<tr>
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<td>6.45</td>
<td>0.0720</td>
<td>3.951</td>
<td>63.7</td>
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<td>0.0880</td>
<td>5.054</td>
<td>55.3</td>
<td>49.1</td>
<td>49.1</td>
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<td>5.48</td>
<td>0.0960</td>
<td>5.054</td>
<td>58.7</td>
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<td>52.1</td>
</tr>
<tr>
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<td>6.45</td>
<td>0.0610</td>
<td>5.054</td>
<td>52.2</td>
<td>47.9</td>
<td>47.9</td>
</tr>
</tbody>
</table>
5.4 More insights from the model

One thing needs to be noted; the piston motion depends continuously on the feed pressure and the frequency of the direction switch. Unlike these two independent process variables, the piston’s amplitude is in fact a controlled variable. Nevertheless, in order to meet the minimum requirement for half cycle acceleration under a given weight load and relay frequency, the model suggests that there exists a minimum feed pressure. Future researchers can then use the model to optimize the JBR operations.

![Contour plot of minimum feed pressure (psig) vs. frequency and weight load.](image)

Using the model of Chapter 3 to solve for the required minimum feed pressure which meets the threshold of max half cycle piston accelerations provides the contour plot shown in figure 5-18. While there is a strong impact on the required minimum pressure of the weight load over the entire frequency range, the relay frequency becomes important only for weight loads greater than 6 kg. The model developed in this study can be used to generate similar plots for pneumatic systems with different cylinder, valve and connecting lines.
6 Conclusions

This study provides practical tools for the design and operation of JBR systems that will ensure good gas-solids mixing:

- The model that was developed to predict the piston motion was validated by experimental data. The model uses four empirical parameters that change depending on the equipment characteristics. The first two parameters characterize the gas supply to the piston: a virtual tube length, $L_v$, to model the pressure drop along the gas supply line to the piston, and a mass flow coefficient, $C_f$, which describes the resistance to flow through the double-solenoid valve. The last two parameters are used to describe the frictional forces on the piston, the static Coulomb friction force, $F_f$, and the kinetic friction coefficient term, $\beta$.

- Experiments showed that there was a good correlation between the quality of the gas-solid mixing in the existing reactor and the maximum acceleration of the piston. With the reactor used in this study, very good gas-solid mixing could be achieved with maximum accelerations greater than 55 m/s$^2$.

- To design a new JBR system, the piston model can be used to determine the piston and gas supply lines that will ensure good gas-solid mixing.
7 Future Work and Recommendations

Three types of further studies are recommended:

1. Determine the impact of reactor scale on the maximum acceleration required to achieve good gas-solid mixing.
2. Study the solid mixing in the JBR and determine how it is affected by the piston motion. For example, phosphorescent tracers [Bakhurji et al., 2018 & Harris et al., 2002] could be used to study the dispersion of particles from one section of the reactor to other reactor sections.
3. Investigate the use of non-cylindrical reactor geometries or reactor internals to promote radial mixing of the solids.
References

For JBR and Introduction


For pneumatic piston modelling


For mixing studies


Future work using phosphorescent tracer


Appendices A

MATLAB Piston Model Functions
A.1 General Overview

Detailed MATLAB files are attached in the appendix with extensive comments along the line. The section here tries to provide an outline and summary for illustrating the algorithm of the MATLAB programs for piston modelling and tuning the three parameters for best fitting results.

Piston Model Setup General Overview

From the experimental setup sections, the only input parameters to facilitate the piston motion are the weight loaded onto the piston block, denoted as m, the relay frequency, f, which can be setup by the feedforward Arduino control program and P-inlet, the air pressure feed to the system set by the regulator. All these three input will be used to calculate the piston displacement, z, and the piston’s amplitude, which is derived from the displacement since it is essentially the minimum to maximum piston displacement from cycle to cycle.

To relate the input with the output, force balance of the piston block is used as one of the key principles. For piston motion is a result of net force acting on the piston block, the momentarily acceleration can be obtained. The integration of acceleration over time gives the velocity of the piston block; further integration results in the displacement.

Meanwhile, the force balance onto the piston block is affected by pressure balance in the upper and lower chamber of the piston. The pressure within each chamber is further influenced by the mass flow of air into the chamber. Thus, changes in the mass flow of air entering or exiting the piston are the most critical terms contributing to the piston motion.

Since the tube connecting the piston and valve is less than 50 cm, given that air is propagating as a compressible fluid in sonic speed throughout the tube, the time delay is generally small and subtle, close to 1 millisecond, but the pressure drop across the tube despite small is still significant and non-linear to reduction of air flow across the tube. Nevertheless, the effective orifice area also changes while valve is switching, this orifice area further affects the calculations of the mass flow. Therefore, using just the force balance is far from being sufficient and accurate; the energy and momentum balance of the compressible air flow shall be considered. Since air is compressible, its density is not constant throughout time and locations which results
in changing flow characteristics like Reynold’s number leading to changing mass flow rates. Using the standard equation for the mass flow through an orifice area, the instantaneous mass flow can be obtained at each incremental time slot. From that equation, one can see that the mass flow of air is related to three variables: the upstream and downstream pressure around the valve orifice, effective cross sectional area of the orifice and the non-dimensional, discharge coefficient. The latter two variables can be from the valve’s model and the valve’s specification sheet. In a very similar way, the possible gas leakage is calculated. Since the two possible areas where leakage may happen are at the top plate where the piston double rods were sealed and around the rubber sealing for the piston block within the chamber. In either case of gas leakage, the cross sectional area becomes a ring rather than a circular plane.

As a result the flow path for pressure and mass flow calculations can be completed and cycled as the following:

First of all, the piston has to be initialized, meaning its initial position air pressure feed, weight load, frequency and all other necessary parameters have to be set. Initial mass flow of air is zero as well as the initial acceleration of the piston. Note that the positive directions for air is flowing towards piston, the positive direction for piston motion is upward. In addition, as mentioned in the methodology section, a virtual tube length has to be calculated to provide a better estimation of the initial air flow since the air source is a compressed piped far away upstream of the JBR system. Otherwise, the error would be significant.

As described previously, mass flow of air is a function of not only the cross sectional area of the tube, but also the pressure drop along the tube. The effects of pressure drop induced air flow reduction cannot be ignore and thus, the tube has to be broken down into smaller segments, in our case, a quick sensitivity analysis of the number of cuts for the tube said that for a tube of over 20000 segments, the calculation differences plateaued.

Subsequently, the above formed several smaller and independent functions for calculating those critical terms: local resistance around the piston nozzles, pressure drop within the tube between piston and valve, mass flow of air, chamber pressure, and initial stable pressure feed to the system.
Once all these quantities are set, the mass of air flow can be derived from the pressure and pressure drop calculations across the system. Since the piston and valve complex is the main focus, air pressure six points are calculated.

1. \( P_{UV} \), pressure at the valve port connecting the piston’s nozzle at the upper chamber
2. \( P_{UA} \), pressure right outside of the piston’s nozzle at the upper chamber
3. \( P_{U} \), the air pressure at the upper chamber
4. \( P_{L} \), the air pressure at the lower chamber
5. \( P_{LA} \), the air pressure right outside of the piston’s nozzle at the lower chamber
6. \( P_{LV} \), the pressure at the valve port connecting the piston’s nozzle at the lower chamber

Note that the above pressure values have their corresponding mass flow denoted as \( G_{UV} \), \( G_{UA} \), \( G_{U} \), \( G_{L} \), \( G_{LA} \) and \( G_{LV} \) accordingly. Nonetheless, the pressure drop across the upper tube (between \( P_{UA} \) and \( P_{UV} \)) and the pressure drop across the lower tube (between \( P_{LV} \) and \( P_{LV} \)) are numerically integrated.

To calculate the pressure change within each chamber, \( P \)-inlet is first used to calculate the pressure right upstream of the double solenoid valve using the obtained virtual tube length and the compiled tube pressure drop function over the upstream virtual tube. The following example, when air is entering the lower chamber while exiting the upper chamber resulting in an upward piston motion, is used to illustrate the calculation steps:

Firstly, the air feed pressure right upstream of the solenoid valve is obtained. When the valve is activated, the spool within the valve structure starts to move. The location of the valve spool dictates the amount of effective cross sectional area of the air passing through the valve. Subsequently, the mass flow of air passing through the valve also changes till the valve is fully switched to one direction. Meanwhile, the pressure of air leaving the valve towards piston \( P_{LV} \) is calculated using the valve model which involved valve pressure drop. Since air is travelling from valve to piston, the flow direction is positive; \( P_{LV} \) is larger than \( P_{LA} \) due to tube pressure drop. This pressure drop can be calculated using the previously developed tube pressure drop model. After that, \( P_{LA} \) is obtained as the pressure right outside of the piston nozzle near the bottom chamber. Since the orifice or the effective cross section area of the air flow changes, once again, the energy loss occurs as well as air passes through the piston nozzle entering the lower chamber.
due to this local resistance. Next, the pressure in the lower chamber, $P_l$ is calculated from $P_{La}$ since it is assumed that the pressure inside the chamber is the same as the pressure after air flow just passed the nozzle orifice. Similarly, $P_u$, the pressure inside the upper chamber is also calculated from the valve’s port, but backwardly rather than forwardly as for $P_l$. later, the differences between $P_l$ and $P_u$ cause the piston to move away from the chamber with higher pressure which leads to displacement of the piston platform. In general, when calculating air pressure and mass flow, the tube resistance causes back pressure, a negative pressure drop on the direction of the flow when filling a chamber. And the pressure calculated in current step becomes the initial values for the next step. This calculation cycle continues until the designated piston time is reached. Additional constraints were also setup, for instance, the maximum and minimum displacements of the piston are 0.30 and 0.0 m respectively. These limits will prevent the model to further the piston motion once the piston reached these limits. Frequency input to the system is also translated to a half-cycle time for piston to complete either an upward or a downward motion.

Overall, the program first determines which chamber it is calculating, whether it is upper chamber or lower chamber: if pressure is increasing in one chamber then adiabatic expansion is assumed, and air enters that chamber; if pressure is decreasing isothermal compression is assumed while air exits that chamber. In addition, it also checks the direction of the piston motion since these two are correlated. When it comes to the mass flow calculations, the function also checks whether the air speed is sonic or subsonic, and then uses relevant air flow calculations accordingly.

Notes on MATLAB model’s file naming and their usage:

Main calculations are executed in the file named “cylinderModel” which further calls other sub functions such as “localResistance” for pressure and flow change near a given nozzle or orifice, “valveFlow” for the mass flow of air near the valve ports, “compressibleFlow” for calculating the air flow using numerical integration, and “tubeDelay” for the pressure drop across the tubes. Here are other general guidelines when using the MATLAB programs for different purposes:

1. All file names with prefix “cylinderModel” contains all other calculations necessary to use (where all calculations and computations take place).
2. "run script" computes and analysis singular conditions (not with multiple changes, just single parameter and single case) for detailed one trial analysis: change of pressure, flow, displacement, etc as time progresses in that particular trial with set conditions of air pressure, relay frequency, platform weight load.

3. Solid loads are generally merged into the platform weight load since it is insignificant compared to the much heavier platform load.

4. Files with name involving "test" analyze multiple cases altogether, and only computes the mainly two parameters: steady state amplitude and the time it takes for each case to reach steady state. MATLAB files are grouped in 3 with very similar file names: Ex, some considers the leakage, some gives initial settings (with "In" or "initial" in their file names); some gave more accurate/robust approximations using more refined intervals (with "exact" in the file names)

Model fitting methods and program brief explanation

As for model fitting, MATLAB’s built-in binary search function is used to find the optimum point where the residual of least squares are established. Recall that the model has three empirical factors for tuning: $C_f$, the nozzle coefficient and $L_v$, virtual tube length which accounts for the energy loss of air flow across the tube sections, $F_f$, the Coulomb friction factor and $\beta$, the kinetic friction factor. These four factors are first estimated by three sets of preliminary experiments: tube pressure fill-up tests and piston chamber fill-up tests for $C_f$ and $L_v$, full stroke piston tests for $F_f$ and $\beta$. To improve efficiency, $C_f$ and $L_v$ are grouped together since none of them were affected by piston motion, but to air flow. On the other hand, $F_f$ and $\beta$ are grouped and tuning together via results from full stroke piston trials because they are both influencing the piston when it is moving. In order to optimize these factors, two loops were used in the MATLAB program to search the best possible fit while each factor has a given range for the program to search.

For example, the range for $C_f$ lies between 0.1 and 1.0; when looking for the best fit for $C_f$ and $L_v$, first round will pick out 5 equally spaced points within that range of $C_f$ and run the curve fitting program, find their best matching $L_v$ factors with least squares.
A.2 Key Program Scripts

Individual MATLAB functions are developed for computing independent or dependent single variables during the modeling and results analysis operations.

This appendix not only provides all the code with extensive comments that are used for this study, it also supplies readers with insights and explanations for walking through some of the key algorithms and methods implemented. Without being redundant and tedious, the seven core functions and the finalized piston model are explained with details; they are:

- Function chambersPchange
- Function InitialStableP
- Function InitialStablePExact
- Function localResistance
- Function tubeDelay
- Function valveFlow
- Function compressibleFlow
- Model cylinderModelLeakageInExact

The rest of the scripts are also included with sufficient comments to help readers grasp the complexity of the entire piston model.
Function name: chambersPchange

Purpose: calculating the pressure change within a specified piston chamber

Input:

- G, mass flow of air, measured in kg/s
- A, effective area of the piston chamber, m²
- z, the displacement of the piston, a vector that is positive for upward displacement, negative for downward, 0 when piston is at bottom, m
- v, velocity vector of the piston, positive for upward motion, negative for downward

Output:

- \( \frac{dP}{dt} \), rate of pressure change in that selected chamber, Pa/s

This function also takes other global parameters:

- \( z_{0U} \), initial location of the upper chamber, m
- \( z_{0L} \), initial location of the lower chamber, m
- \( RMA \), molar basis ideal gas constant for air at ambient condition
- \( T \), temperature of the ambient condition of which the piston operates
- \( r \), ratio of the specific heat capacity of air at constant pressure over constant volume
- \( L \), total piston stroke length (maximum piston displacement)

Brief algorithm walkthrough:

The function first determines which chamber was selected for calculations. It then follows the assumption of adiabatic expansion and isothermal compression for air within the selected chamber under a given condition: if air enters a chamber, air expands adiabatically which also drives the piston block apart from where air enter, in the meantime, the other chamber compresses, driving air out of that chamber.

Initially, the piston velocity is zero. In the beginning, adiabatic expansion is applied to the chamber where air starts to fill in. Then, the formula illustrated in the methodology section will be implemented (equation ….), but before calculations, all necessary conditions will be initialized and the selected chamber will lead to a subsequent selection effective area. As a result, the pressure changes within each piston chamber are calculated almost simultaneously. Eventually, this will complete the pressure change ratio for one chamber; the other chamber’s calculations would be mostly coupled to the selected chamber.

**(Script of this function is provided in the following pages.)**
function dPcdt=chambersPchange(G,P,A,z,v,site)
% calculates the pressure change within the upper and lower piston chamber
% notations
% dPcdt, rate of pressure changes, Pa/s
% G, mass flow rate of air, kg/s, a vector, positive for flow entering the
% system, negative for flow exiting the system
% A, effective areas of the piston chamber, m2
% z, piston displacement, a vector, positive for upward motion and negative
% for downward motion, minimum piston displacement when at bottom is set to be
% zero
% v, piston velocity, m/s, a vector that is positive when piston is moving up
% and negative when moving down
% site specifies which chamber is the given calculations focused on, 1 for
% the lower piston chamber and 0 for the upper piston chamber.
% Define global constants.
global z0U z0L RMA T r L

% function
if site==1
% determines which chamber is being calculated, 1 for the lower chamber,
0 for the upper chamber
z0=z0L;
if v>=0
% determines whether the air within the specified chamber undergoes
compress or expansion, if larger than 0, by convention, the lower chamber
undergoes expansion, the upper chamber undergoes compression; if not, vice
versa
% assume expansions are adiabatic while compressions are isothermal,
when velocity is zero, adiabatic conditions are applied
dPcdt=((RMA.*T)/((z0+z).*A)).*(r.*G)-r.*(P.*A./((z0+z).*A)).*v;
else % the lower chamber undergoes isothermal compression
dPcdt=((RMA.*T)/((z0+z).*A)).*(G)-(P.*A./((z0+z).*A)).*v;
end
else
z0=z0U;
if v>0 % and the compression within the upper chamber is isothermal
dPcdt=((RMA.*T)/((z0+(L-z)).*A)).*(r.*G)-r.*(P.*A./((z0+(L-z)).*A)).*(-v);
else % whereas, the upper chamber expands or stops adiabatically
dPcdt=((RMA.*T)/((z0+(L-z)).*A)).*(r.*G)-r.*(P.*A./((z0+(L-z)).*A)).*(-v);
end
end
end

-----------------------------------------------------------------------
Function name: InitialStableP

Purpose: estimates the initial air pressure within either of the piston chambers

Input:

- Pin, pressure feed to the system, Pa
- Pa, atmospheric pressure, Pa, mostly the condition near the valve exits during JBR operations
- initDirect, initial piston direction, at t = 0, this value is 0;
- ALv, the effective cross sectional area for air passing from value to the lower chamber of the piston, m$^2$
- AUv, the effective cross sectional area for air passing from value to the upper chamber of the piston, m$^2$

Output:

- PU, the initial pressure within the upper piston chamber
- PL, the initial pressure within the lower piston chamber

This function also takes other global parameters:

- r, ratio of the specific heat capacity of air at constant pressure over constant volume
- RMA, molar basis ideal gas constant for air at ambient condition
- Psi2Pa, unit conversion factor from psi units to Pascal units, for air pressure
- CfV, orifice coefficient for mass flow of air through the double valve
- Aleak0, the gap between the double-rod of the piston and the top piston plate where possible air leaks may occur, this area essentially consists of two rings, m$^2$
- Aleak1, the gap between the piston block and the cylinder wall, it can be assumed as a ring between the rubber sealing and the cylinder wall, m$^2$
- Cfleak0, mass flow coefficient at the possible leakages as defined in Aleak0
- Cfleak1, mass flow coefficient at the possible leakages as defined in Aleak1

Brief algorithm walkthrough:

The function first initializes all necessary parameters with the given input; for instance, the mass flow coefficients for the upper and lower piston chamber are both set to be equal to the valve’s mass flow coefficient, or CfL = CfU = CfV. Similarly, the effective area near the piston nozzles are equal to the effective orifice areas of their corresponding, connected valve ports. i.e. ALa = ALv, AUa = AUv. Prior to any further calculations, the downstream to upstream pressure ratio is checked against a critical value in order to determine whether the flow has a sonic velocity or a subsonic one.

The rest of the function is implemented to calculate the mass flow through a given orifice or the suspected gap, essentially with shapes of rings, where leakage may have been occurred. However, this function alone has very limited accuracy for the initial pressure calculations. In order to
enhance accuracy of the initial stable pressure estimations, another function is used to provide more precise iterations.

(Script of this function is provided in the following pages.)
function [PU,PL]=InitialStableP(Pin,Pa,initDirect,ALv,AUv)
% this calculates the pressure within the piston chambers when considering
air leakages in the beginning.
% clear all;
% clc;
% Pin=45.*Psi2Pa;
% initDirect=0;

%% notations
% Pin, feed pressure to the system, Pa
% Pa, atmospheric pressure at ambient condition, measured in Pa
% PU, pressure within the upper piston chamber, Pa
% PL, pressure within the lower piston chamber, Pa

%% Define global constants.
global r T  RMA Psi2Pa Cfv Aleak0 Aleak1 Cfleak0 Cfleak1
%
CfL=Cfv;
CfU=Cfv;
ALa=ALv;
AUa=AUv;

%% Function
C1=((r./RMA).*(2./(r+1)).^((r+1)./(r-1))).^0.5;
C2=(2.*r./(RMA.*(r-1))).^0.5;
Pcr=(2./(r+1)).^((r-1)/r);

%syms PU PL
% eq1=(CfL.*AUa.*C2.*Pin./(T.^0.5).*(pp(1)./Pin).^(1./r).*(1-
% (pp(1)./Pin).^(1./(r-1)).)^0.5);
% eq2=(Cfleak0.*Aleak0.*C2.*pp(1)./(T.^0.5).*(Pa./pp(1)).^(1./r).*(1-
% (Pa./pp(1)).^(1./(r-1)).)^0.5);
% eq3=(Cfleak1.*Aleak1.*C2.*pp(1)./(T.^0.5).*(pp(2)./pp(1)).^(1./r).*(1-
% (pp(2)./pp(1))).^(1/(r-1))).^0.5);
% eq4=(CfL.*ALa.*C2.*pp(2)./(T.^0.5).*(Pa./pp(2)).^(1./r).*(1-
% (Pa./pp(2))).^((r-1)/r)).^0.5);
% fun1=@(PU,PL)(eq1-eq2-eq3)
% fun2=@(PU,PL)(eq3-eq4)
% [PU,PL]=fseolve(eq1,eq2,[Pa Pin])
% [PU,PL]=solve(eq1-eq2-eq3==0,eq3-eq4==0,PU,PL)

% pp(1)=PU;
% pp(2)=PL;
if (initDirect==0)
    fun=@(pp)[(CfU.*AUa.*C2.*Pin./(T.^0.5).*(pp(1)./Pin).^(1./r).*(1-
    (pp(1)./Pin).^(1./(r-1)).)^0.5)-
    (Cfleak0.*Aleak0.*C2.*pp(1)./(T.^0.5).*(Pa./pp(1)).^(1./r).*(1-
    (Pa./pp(1)).^(1./(r-1)).)^0.5)-
    (Cfleak1.*Aleak1.*C2.*pp(1)./(T.^0.5).*(pp(2)./pp(1)).^(1./r).*(1-
    (pp(2)./pp(1))).^(1/(r-1))).^0.5)-
    (CfL.*ALa.*C2.*pp(2)./(T.^0.5).*(Pa./pp(2))).^((r-1)/r)).^0.5];
end
options=optimset('tolFun',1e-15);%,'maxIterpositive',30000);%,'display','iter','tolx',1);
% this line above sets the tolerance of the calculation errors
%     options.MaxIterationpositive=30000;
%     options.MaxFunctionEvaluations = 30000; % options.MaxFunctionEvaluations
%     options.MaxFunctionEvaluations = 200 (the default value).
%     [pp,favl,exitflag,output]=fsolve(fun,[Pin,Pa],options)
%     pp=fsolve(fun,[Pin,Pa],options);
else

    fun=@(pp)[(Cfleak1.*Aleak1.*C2.*pp(2)./(T.^0.5)).*(pp(1)./pp(2)).^((r-1)./r)).^0.5-(
        (CfU.*AUa.*C2.*pp(1)./(T.^0.5)).*(1-(Pa./pp(1)).^((r-1)./r)).^0.5)-
        (Cfleak0.*Aleak0.*C2.*pp(1)./(T.^0.5)).*(1-(Pa./pp(1)).^((r-1)./r)).^0.5);]
    options=optimset('tolFun',1e-15);%,'MaxFunctionEvaluations',30000);%,'display','iter','tolx',1);
% the line above sets the tolerance for the function’s calculated results
%     [pp,favl,exitflag,output]=fsolve(fun,[Pa,Pin],options);
%     pp=fsolve(fun,[Pa,Pin],options);
end
pp=real(pp);% taking only the real part of the complex number
% pp./Psi2Pa
PU=pp(1);
PL=pp(2);
end

------------------------------------------------------------------------------------------------------------------
Function name: InitialStablePExact

Purpose: finds a more accurate value for air pressure within the upper and lower chamber while taking the leakage into account

Input:

- Pin, pressure feed to the system, Pa
- Pa, atmospheric pressure, Pa, mostly the condition near the valve exits during JBR operations
- initDirect, initial piston direction, at t = 0, this value is 0;
- rleak0, the effective area ratio of the top gap over the double-rod’s cross sectional area at top
- rleak1, the effective area ratio of the piston-cylinder wall gap over the cross sectional area of the piston cylinder

Output: (6 initial pressure readings at 6 points of the JBR systems)

- PUsta, the initial pressure within the upper piston chamber
- PUasta, initial pressure outside of the piston nozzle near the upper chamber
- PUvsta, initial pressure near the port of the solenoid valve connecting to the upper chamber nozzle
- PLsta, the initial pressure within the lower piston chamber
- PLasta, initial pressure outside of the piston nozzle near the lower chamber
- PLvsta, initial pressure near the port of the solenoid valve connecting to the lower chamber nozzle

This function also takes other global parameters:

- dt, incremental length for calculations across the soft tube segment, m
- r, ratio of the specific heat capacity of air at constant pressure over constant volume
- RMA, molar basis ideal gas constant for air at ambient condition
- pi, π for calculating circular perimeters or area
- Psi2Pa, unit conversion factor from psi units to Pascal units, for air pressure
- CfV, orifice coefficient for mass flow of air through the double valve
- Aleak0, the gap between the double-rod of the piston and the top piston plate where possible air leaks may occur, this area essentially consists of two rings, m²
- Aleak1, the gap between the piston block and the cylinder wall, it can be assumed as a ring between the rubber sealing and the cylinder wall, m²
- AL, effective area of the lower piston chamber, m²
- AU, effective area of the upper piston chamber, m²
- ALe, effective area of the air passage towards the lower piston chamber, m²
- AUe, effective area of the air passage towards the upper piston chamber, m²
- Cfleak0, mass flow coefficient at the possible leakages as defined in Aleak0
- Cfleak1, mass flow coefficient at the possible leakages as defined in Aleak1
Brief algorithm walkthrough:

The function computes the initial pressure in a very similar manner to the InitialStableP function. The difference is that this function further advances the calculation of the results obtained in InitialStableP via considering the following terms and factors along the air flow direction (which enters the piston through valve, then soft tube, through nozzle and eventually to the piston chamber):

1. Pressure drop and mass flow of air changes across the double solenoid valve (essentially the valve pressure drop and flow reduction)
   a. Note that this term is calculated by implementing the “valveFlow” function and assuming PUsta = Pusta=Pvsta as the initial condition.
   b. This means that the pressure levels are initially the same at these three locations: upper chamber, outside of the upper chamber, and valve port connecting to the upper chamber, but with a changing mass flow of air as it passes through these locations after air enters to the system from the valve
   c. As a result, the corresponding mass flow of air through the valve ports towards the piston can be obtained.

2. Tube pressure drop and mass flow reduction to approximate the changes in the mass flow rate and pressure of the air flow (the function “tubeDelay” will take the previously calculated mass flow of air, pressure values and tube length as input while numerically integrate for the accumulated pressure drop and flow reduction across the tube)

3. Local resistance as air passes through the piston nozzles or exits from the piston through those nozzles (this can be computed using the “localResistance” function: taking the previously corrected flow and pressure drop to compute the local resistance near the nozzles)

4. Air leakage is yet another important factor needs attention, but this time with the corrected values and similar assumptions, the accuracy of the results can be improved.

In the end, the entire calculation loop is completed and the adjusted initial air pressure within each chamber are obtained and stored for further modelling related analysis. This initial condition is rather important as it sets the basis for the piston model to initiate its calculation and approximation of the piston displacement and amplitude.

(Script of this function is provided in the following pages.)
function [PUSta,PUvSta,PLvSta,PLvSta]=InitialStablePExact(Pin,Pa,initDirect,rleak0,rleak1)
% calculating the air pressure within the upper and lower piston chamber in
% case of leakages
% clear all;
% clc;
% Pin=45.*Psi2Pa;
% initDirect=0;

% notations
% Pin, air pressure feed to the system, Pa
% Pa, atmospheric pressure, Pa
% PU, air pressure within the upper piston chamber, Pa
% PL, air pressure within the lower piston chamber, Pa

% Define global constants.
global r T RMA pi Psi2Pa Cfv dd D L dt Cfleak0 Cfleak1
global xmin xmax zetaIn zetaOut LtU LtL
global AL AU Aleak0 Aleak1
global th

% initializations
CfL=Cfv;
CfU=Cfv;
if (initDirect==0)
    x=xmin;
    z=0;
    zetaU=zetaIn;
    zetaL=zetaOut;
else
    x=xmax;
    z=L;
    zetaU=zetaOut;
    zetaL=zetaIn;
end
% m2 the effective air leakage area between the double rod and top piston
plate
Aleak0=2.*((dd./2+rleak0).^2-(dd./2).^2).*pi;
% m2 the effective air leakage area between the piston block and wall
Aleak1=((D./2).^2-(D./2-rleak1).^2).*pi;
AL=(D./2-rleak1).^2*pi;% m2, cross sectional area of the lower piston chamber
AU=(D./2-rleak1).^2*pi-2.*(dd./2).^2*pi;% m2, cross section area of the upper
chamber
v=0;

% computations for air pressure within the chamber when leakage reaches a
steady state
% calculating the spool position
valveFlow(Pa,4.*Pa,2.*Pa,2.*Pa,xmin);
% valveFlow calculated the effective air passage area through the valve,
which has nothing to do with air flows
ALv=ALv;
AUv=AUv;
% these are initial rough assumptions and estimates of the pressure inside
% the chambers
% no considerations for tube and nozzle pressure drop was made in the above
cases
[PUSta,PLSta]=InitialStableP(Pin,Pa,initDirect,ALv,AUv);
% assume same pressure level at all three points
PUvSta=PUSta;
PUaSta=PUSta;
PLvSta=PLSta;
PLaSta=PLSta;

% adjust the iterations for more accurate calculations of the pressure at
% the valve ports and near the piston nozzles when flow becomes steady
% at steady state, flow leaks out of the system equals to the flow enters the
% system
% G, the mass flow of air, positive when heading into the system, negative
% when exiting
% initialization
% finds mass flow of air with the initial assumption of same pressure at
% those three locations
[GLvSta,GUvSta]=valveFlow(Pa,Pin,PLvSta,PUvSta,x);
% finds the tube pressure drop when knowing PUv to calculate PUa
[ddPtUSta,PUa_calSta,GUaSta]=tubeDelay(GUvSta,PUvSta,LtU,dt);
[ddPtLSta,PLa_calSta,GLaSta]=tubeDelay(GLvSta,PLvSta,LtL,dt);
% find the local resistance near the piston nozzle with given PUa to find PU
[ddPlUSta,PU_calSta,GUSta]=localResistance(GUaSta,PUaSta,zetaU);
[ddPlLSta,PL_calSta,GLSta]=localResistance(GLaSta,PLaSta,zetaL);

PUaSta=PUSta+ddPlUSta; %Pa, initial pressure near
% the nozzle of the upper
% chamber
PLaSta=PLSta+ddPlLSta; %Pa, initial pressure near the nozzle of the lower
% chamber
PUvSta=PUaSta+ddPtUSta; %Pa, initial pressure near valve port
% connecting to the upper chamber
PLvSta=PLaSta+ddPtLSta; % Pa, initial pressure near valve port connecting to
% the lower chamber
% calculating the air leakages
GleakUSta =
compressibleFlow(PUSta,Pa,Aleak0,Cfleak0)+compressibleFlow(PUSta,PLSta,Aleak1
,Cfleak1);
GleakLSta = compressibleFlow(PLSta,PUSta,Aleak1,Cfleak1);
GtotalSta=GUSta-GleakUSta;
GLtotalSta=GLSta-GleakLSta;
global count
count=0;% number of iterations
while (abs(GUtotalSta)>1E-6 || abs(GLtotalSta)>1E-6)
count=count+1;
[GLvSta,GUvSta]=valveFlow(Pa,Pin,PLvSta,PUvSta,x);% assume same pressure
% level at those three locations with given PUv find the mass flow of air
[ddPtUSta,PUa_calSta,GUaSta]=tubeDelay(GUvSta,PUvSta,LtU,dt);% tube
% pressure drop connecting the upper chamber and valve given PUv find PUa
[ddPtLSta,PLa_calSta,GLaSta]=tubeDelay(GLvSta,PLvSta,LtL,dt);
[ddPlUSta,PU_calSta,GUSta]=localResistance(GUaSta,PUaSta,zetaU); % find
% the pressure near the upper chamber's nozzle given PUa find PU
[ddPlLSta,PL_calSta,GLSta]=localResistance(GLaSta,PLaSta,zetaL);
% calculating the air leakages
GleakUSta = \texttt{compressibleFlow}(PUSta, Pa, Aleak0, Cfleak0) + \texttt{compressibleFlow}(PUSta, PLSta, Aleak1, Cfleak1);
GleakLSta = \texttt{compressibleFlow}(PLSta, PUSTa, Aleak1, Cfleak1);
GUSTotalSta = GUSTa - GleakUSta;
GLTotalSta = GLSta - GleakLSta;
\%
% find the air pressure within the chamber and re-evaluate PU PL
% dP UdSta = \texttt{chambersPchange}(GUSTotalSta, PUSTa, AU, z, v, 0);
% PUSTa = PUSTa + dP UdSta \times \text{th};
% PUSTa = PUSTa + dP UdSta \times \text{th} \times 10;
% dPL dtSta = \texttt{chambersPchange}(GLTotalSta, PLSta, AL, z, v, 1);
% PLSta = PLSta + dPL dtSta \times \text{th} \times 10;
% PLSta = PLSta + dPL dtSta \times \text{th} \times 10;
PUaSta = PUSTa + \text{ddPlUSta} \text{ Pa, initial pressure near the upper chamber’s nozzle}
PLaSta = PLSta + \text{ddPlLSta} \text{ Pa, initial pressure near the lower chamber’s nozzle}
PUvSta = PUASta + \text{ddPtUSta} \text{ Pa, initial pressure near the valve port connecting to the upper chamber}
PLvSta = PLaSta + \text{ddPtLSta} \text{ Pa, initial pressure near the valve port connecting to the lower chamber}
\%
% count % output the number of iterations
% output the adjusted steady pressure at all 6 locations
% PUvSta / Psi2Pa
% PUaSta / Psi2Pa
% PUSTa / Psi2Pa
% PLvSta / Psi2Pa
% PLaSta / Psi2Pa
% PLSta / Psi2Pa
end
Function name: localResistance

Purpose: given the pressure and mass flow of air near the outside of a piston nozzle, find the local resistance and mass flow on the other side, inside, the piston nozzle

Input:

- Go, mass flow of air outside of the piston nozzle, kg/s
- Po, air pressure outside of the piston nozzle, Pa
- zeta, coefficient for local resistance

Output:

- ddPl, local pressure drop, ddPl = Po – Pi, Pa
- Pi, pressure inside the piston nozzle near the chamber, Pa
- Gi, mass flow of air inside the piston nozzle near the chamber, kg/s

This function also takes other global parameters:

- c, sonic speed at the ambient experiment condition, 346.3 m/s
- RMA, molar basis ideal gas constant for air at ambient condition
- T, temperature of the ambient experiment conditions, K
- At, effective area of the soft tube, m²

Brief algorithm walkthrough:

This function is rather simple. It first determines the direction of the flow as one of the following three categories:

1. Positive Po means flow is entering the nozzle from outside, usually the case for air flow into the system for chamber expansion
2. Negative Po, which indicates air flow out of the system during chamber compression
3. Po = 0, which happens when flow direction just switched and there is no flow enters nor exits the system as the spool travels to the central position of the double solenoid valve.

Once the flow situation is determined, the density, linear velocity and pressure drop can all be compared. As a result, the flow reduction and pressure drop are obtained with the resistance factor calculated from local resistance coefficient of the tube nozzle, density and linear velocity of the air flow.

(Script of this function is provided in the following pages.)
function [ddPl,Pi,Gi]=localResistance(Go, Po, zeta)
% Given the
% clc;clear all;close all
% Go=0;
% Po=3*101325;
% Lt=0.4;
% dt=0.01;

% notation
% Go, the mass flow of air near the outside of the piston nozzle, kg/s
% Po, the pressure near the outside of the piston nozzle, Pa
% zeta, coefficient for local resistance, m
% ddPl, local pressure drop around the restriction, ddPl=Po-Pi, Pa
% define that either air flow or pressure drop is positive if towards the system, negative the other way around
% Pi, pressure after passing through the piston nozzle, Pa
% Gi, mass flow of air after passing through the piston nozzle, kg/s
% ul; % local speed of the air flow, m/s
% pgl; % density, kg/m3
% ddPl; % pressure drop, Pa
% Rtl=0; % local resistance within the tube
% feil; % fractional reduction of the mass flow

% Define global constants.
global RMA T c At

%% Function
if Go==0
    Gi = 0;
    Pi = Po;
    ddPl = Po-Pi;
else if Go>0
    pgl = Po./(RMA.*T);
    ul = Go./pgl./At;
    ddPl=zeta.*(pgl.*ul.^2)./2;
    Pi = Po-ddPl;
    Rtl = zeta.*(pgl.*ul)./2;% note that the term Ll was cancelled.
    feil = exp(-(Rtl.*RMA.*T./(2.*Pi)).*(1./c));
    Gi=Go.*feil;
else
    Go=-Go;
    pgl = Po./(RMA.*T);
    ul = Go./pgl./At;
    ddPl=zeta.*(pgl.*ul.^2)./2;
    Pi = Po-ddPl;
    Rtl = zeta.*(pgl.*ul)./2;% note that the term Ll was cancelled.
    feil = exp(-(Rtl.*RMA.*T./(2.*Po)).*(1./c));
    Gi=Go./feil;
    ddPl = Po-Pi;
    Gi=-Gi;
end
end
end
Function name: tubeDelay

Purpose: computes the pressure drop and mass flow reduction as a result of air flow passes through a soft tube.

Input:

- Go, mass flow of air exiting valve port towards the piston chamber, kg/s
- Po, air pressure air exiting valve port towards the piston chamber, Pa
- Lt, length of the tube, m
- dt, diameter of the soft tube, m

Output:

- ddP, local pressure drop, ddP = Po - Pi, Pa
- Pi, pressure entering the piston, but outside of the nozzle, Pa
- Gi, mass flow of air entering the piston, but outside of the nozzle, kg/s

This function also takes other global parameters:

- c, sonic speed at the ambient experiment condition, 346.3 m/s
- RMA, molar basis ideal gas constant for air at ambient condition
- T, temperature of the ambient experiment conditions, K
- pi, π used for circular perimeters and area calculations
- uv, the air viscosity at ambient lab condition

Brief algorithm walkthrough:

This function takes the pressure drop and its impact of mass flow of air into considerations because air flow in the system behaves as a compressible fluid; thus, numerical integration has to be used. The function first initializes all the necessary terms of the air as zeros, where its flow is sourced. Note that the flow’s velocity is generally fast enough to ignore the effects of time delay. Since the longest tube used in this experiment is less than 50 cm, the time it takes for a flow with a linear speed that is nearly sonic would be just around 1 millisecond. One of the key components of the function’s calculated intermediate terms is the term $f_{ei}$ which accounts for the mass flow reduction of air as the flow passes through the tube segment. Once the tube is subdivided into multiple segments, pressure at each spot is calculated numerically with the help of previously obtained local flow velocity, cross sectional area of the flow, friction loss across the tube wall, and Reynold’s number of the flow. Again, the positive mass flow of air indicates air entering the system through the tube; therefore, air flow originates from the valve and the pressure drop across the tube is negative for such flow. On the other hand, a negative mass flow shows that air is leaving the system through the tube. Hence, the source of the air flow is the piston chamber where air leaves the piston chamber through the nearby nozzle and then enters the tube.

(Script of this function is provided in the following pages.)
function [ddP, Pi, Gi] = tubeDelay(Go, Po, Lt, dt)
% given pressure and mass flow of air near the valve port connecting to the
% piston, finds the mass flow about to enter the piston near the nozzle and the
% pressure drop along the tube between the valve and the selected piston
% chamber
% clc; clear all; close all
% Go=0;
% Po=3*101325;
% Lt=0.4;
% dt=0.01;

% notation
% Go mass flow of air at the valve port connection to the piston, kg/s
% Po, pressure at the valve port connection to the piston, Pa
% Lt, total length of the soft tube involved, m
% dt, incremental soft tube length for numerical integration, m
% ddP, pressure drop across the tube, ddP=Po-Pi
% Pi, pressure of air entering the nozzle of the piston, Pa
% Gi, mass flow of air entering the nozzle of the piston, kg/s

% Define global constants.
global pi RMA T uv c

%% Function
stp=20; % step sizes of subdivision for the tube
Ls=Lt./stp; % to further subdivided the tube for enhanced accuracy
% as mass flow changes while air flowing along the soft tube, so does the
% linear speed of air flow as well as the its Re number
us=zeros(stp,1); % air speed, m/s
pgs=zeros(stp,1); % density of air, kg/m^3
Res=zeros(stp,1); % Re number
ddPs=zeros(stp,1); % pressure drop at each sub-section of the tube, Pa
fs=zeros(stp,1); % friction loss caused by the soft tube wall
Ps=zeros(stp,1); % pressure at each subdivided tube segment, Pa
Rts=zeros(stp,1); % tube resistance
fei=zeros(stp,1); % fractional reduction of the mass flow
Gs=zeros(stp,1);

% the following are initialization of all the necessary parameters
us(1)=0; % air velocity within the tube, m/s
pgs(1)=0; % density of air, kg/m^3
Res(1)=0; % Re number
ddPs(1)=0; % pressure drop, Pa
fs(1)=0; % friction loss caused by the soft tube
Ps(1)=Po; % pressure at each subdivided segment, Pa
Rts(1)=0; % tube resistance
fei(1)=1; % fractional reduction of the mass flow

At=pi.*(Lt./2).^2; % cross sectional area of the soft tube, m^2;

% ignore the time delay (since the tube is not long enough to give notable
time delay)
if Go==0
    Gi = 0;
    Pi = Po;
    ddP = Po-Pi;
else if Go>0
    Gs(1)=Go;
    for i=1:1:stp
        pgs(i) = Ps(i)./(RMA.*T);
        us(i) = Gs(i)./pgs(i)./At;
        Res(i)=(pgs(i).*us(i).*dt)./uv;
        fs(i) = 0.316./((Res(i)).^0.25);
        ddPs(i) = fs(i).*Ls.*pgs(i).*us(i).*us(i)./dt./2;
        Ps(i+1) = Ps(i)-ddPs(i);
        Rts(i) = 0.158.*uv.*Res(i).^0.75./dt./dt;
        fei(i) = exp(-(Rts(i).*RMA.*T./(2.*Ps(i+1))).*(Ls./c));
    end
    Gi = Gs(stp+1);
    Pi = Ps(stp+1);
    ddP = Po-Pi;
else
    Gs(1)=-Go;
    for i=1:1:stp
        pgs(i) = Ps(i)./RMA./T;
        us(i) = Gs(i)./pgs(i)./At;
        Res(i)=(pgs(i).*us(i).*dt)./uv;
        fs(i) = 0.316./((Res(i)).^0.25);
        ddPs(i) = fs(i).*Ls.*pgs(i).*us(i).*us(i)./dt./2;
        Ps(i+1) = Ps(i)+ddPs(i);
        Rts(i) = 0.158.*uv.*Res(i).^0.75./dt./dt;
        fei(i) = exp(-(Rts(i).*RMA.*T./(2.*Ps(i))).*(Ls./c));
    end
    Gi = -Gs(stp+1);
    Pi = Ps(stp+1);
    ddP = Po-Pi;
end
end

----------------------------------------------------------------------------------------------------------------------------------
Function name: valveFlow

Purpose: calculates the mass flow rate of air passing through the double solenoid valve

Input:

- \( Pa \), atmospheric pressure, \( 1 \, \text{atm} = 101325 \, \text{Pa} \)
- \( Pin \), air pressure fed to the system, \( \text{Pa} \)
- \( PLi \), air pressure near the lower chamber nozzle, \( \text{Pa} \)
- \( PUi \), air pressure near the upper chamber nozzle, \( \text{Pa} \)
- \( x \), location of the spool, \( \text{m} \)

Output:

- \( GLv \), the mass flow of air exiting the valve port flowing towards the lower piston chamber, \( \text{kg/s} \)
- \( GUv \), the mass flow of air exiting the valve port flowing towards the upper piston chamber, \( \text{kg/s} \)

This function also takes other global parameters:

- \( ALa \), effective area for the piston gas ports at the lower chamber, \( \text{m}^2 \)
- \( AUa \), effective area for the piston gas ports at the upper chamber, \( \text{m}^2 \)
- \( ALe \), effective area of air passage towards the lower piston chamber, \( \text{m}^2 \)
- \( AUe \), effective area of air passage towards the upper piston chamber, \( \text{m}^2 \)
- \( At \), cross section al area of the tube, \( \text{m}^2 \)
- \( CfV \), coefficient of mass flow for the valve
- \( dv \), the size of the valve port (in this study it is always half inch in diameter), \( \text{m} \)
- \( dvl \), length of the valve spool, \( \text{m} \)
- \( dt \), diameter of the soft tube, \( \text{m} \)
- \( flagLv \), indicator of valve status towards the lower chamber
- \( flagUv \), indicator of valve status towards the upper chamber
- \( pi \), \( \pi \) used for circular perimeters and area calculations

Note that an important assumption is made for the spool to satisfy the initial condition:

The spool’s location initially enables the valve directing air flow to the upper chamber while opening the passage for the lower chamber, connecting it to the atmosphere before any piston actions.

The location of the spool refers to its left most possible location within the valve structure. Such left most location is set to have an \( x \) value of zero. In general, positive displacement of the spool is defined as towards right; whereas, negative displacement is towards the left.
Brief algorithm walkthrough:

Prior to implementing any computation, the internal structure of the double solenoid valve and the effects of the valve spool on air flow are thoroughly examined. According to the manufacturer’s specifications and the concept of integration, 8 locations of the valve spool are defined: from $x = 0$ m, the most left location, to $x = 0.017$ m, the most right location, is the total displacement the valve spool travels from one side to another.

These 8 locations enabled more accurate integration of the effective air passage though the valve since some of the flow patterns tend to repeat itself as the spool is travelling back and forth rapidly in this symmetrical and bistable double solenoid valve.

A set of checks will determine the length of the effective opening of the valve covered by the spool. Then, the effective area will be calculated in a numerical manner. Once the effective area was calculated, it was compared with the tube’s cross sectional area. As the soft tube’s diameter is roughly 3/8 inch, smaller than the valve’s half inch ports, the soft tube cross sectional area becomes a maximum cap of the effective area limiting the air flow towards or exits the piston. After obtaining the effective area of the air passage, the mass flow of the air through the valve can be obtained with the help of another function named “compressibleFlow” which will takes the pressure upstream and downstream of the orifice, effective area and orifice coefficient of the valve. The only condition is that flow direction is re-checked; so the upstream or downstream pressure may become the atmospheric pressure depending on the direction of the flow whether it’s entering or leaving the system through the valve.

(Script of this function is provided in the following pages.)
function [GLv,GUv]=valveFlow(Pa,Pin,PLi,PUi,x)
% clc;clear all;close all
% Pin=2*101325;
% Pa=101325;
% PLi = 101325;
% PUi = 1.5*101325;
% x=0.005;

% calculating the mass flow passing through the valve
% input parameters
% GLv, mass flow of air leave valve port connecting to the lower piston chamber, kg/s
% GUv, mass flow of air leave valve port connecting to the upper piston chamber, kg/s
% once again, like many other functions defined, inflow towards the system is positive, outflow is negative.
% Pa, atmospheric pressure at ambient lab condition, Pa
% Pin, pressure fed to the system, Pa
% PLo, pressure near the valve port connecting to the lower piston chamber, Pa
% PLi, pressure near the nozzle leading to the lower chamber, Pa
% PUo, pressure near the valve port connecting to the upper piston chamber, Pa
% PUi, pressure near the nozzle leading to the upper chamber, Pa
% x, location of the valve spool, m
% define the key spool locations as the following: when it is at the left most position, air enters the upper chamber, the lower chamber passage is fully open to the atmosphere. In this case, x = 0 m, when the spool moves to the right, x value increases.
% Define global constants.
global dv dvl dt pi Cfv ALa AUa At
global flagLv flagUv ALe AUe

%% Function
flagLv=1;% air flows toward the lower chamber
flagUv=1;% air flows toward the upper chamber

% reference values of key valve spool locations
xv1=0;
xv2=0.001;
xv3=0.00635;
xv4=0.00835;
xv5=0.00865;
xv6=0.01065;
xv7=0.016;
xv8=dvl;

% effective area for air passages as a result of spool movement
% xe, effective displacement of the spool, m:
% Ae, effective area of valve opening, m2:

if x<xv1
    disp('errors in valve spool displacement, it cannot be less than 0mm');
else if x<=xv2
    xeL=xv3+x;
xeU=xv3-x;
flagLv=0;
flagUv=1;
else if x<=xv3
    xeL=xv4-x;
    xeU=xv3-x;
    flagLv=0;
    flagUv=1;
else if x<=xv4
    xeL=xv4-x;
    xeU=0;
    flagLv=0;
    flagUv=1;
else if x<=xv5
    xeL=0;
    xeU=0;
    flagLv=1;
    flagUv=0;
else if x<=xv6
    xeL=0;
    xeU=x-xv5;
    flagLv=1;
    flagUv=0;
else if x<=xv7
    xeL=x-xv6;
    xeU=x-xv5;
    flagLv=1;
    flagUv=0;
else if x<=xv8
    xeL=x-xv6;
    xeU=3.*0.00635+0.0043-x;
    flagLv=1;
    flagUv=0;
else
    disp('errors in valve spool displacement, it cannot exceed 17mm')
end
end
end
end
end
end
end

% calculate the effective area of air passage through the valve
% Afun=@(xx)(((dv./2).^2-(xx-dv./2).^2).^0.5);
% calculate the openings of the valve
% ALe = 2.*quadgk(Afun,0,xeL);
% AUe = 2.*quadgk(Afun,0,xeU);
% ALe = 2.*(dv./2).^2.*atan(((xeL./2).*(2.*(dv./2)-xeL)).^0.5)-((dv./2)-xeL).*((xeL.*(2.*(dv./2)-xeL)).^0.5);
% AUe = 2.*(dv./2).^2.*atan(((xeU./2).*(2.*(dv./2)-xeU)).^0.5)-((dv./2)-xeU).*((xeU.*(2.*(dv./2)-xeU)).^0.5);

% this sets a cap since the maximum effective air passage area is limited to the soft tube diameter
if ALe>At
    ALe=At;
end
if AUe>At
    AUe=At;
end
% if ALe>ALa
%      ALe=ALa;
% end
% if AUe>AUa
%     AUe=AUa;
% end

% calculates the mass flow through the valve
if flagLv==1
    PLo=Pin;
    GLv=compressibleFlow(PLo,PLi,ALe,Cfv);
else
    PLo=Pa;
    GLv=compressibleFlow(PLo,PLi,ALe,Cfv);
end
if flagUv==1
    PUo=Pin;
    GUv=compressibleFlow(PUo,PUi,AUe,Cfv);
else
    PUo=Pa;
    GUv=compressibleFlow(PUo,PUi,AUe,Cfv);
end
end
Function name: compressibleFlow

Purpose: calculates the mass flow rate of compressible fluid, in this case, air in the ambient condition, through the orifice of a specified piston chamber

Input:

- Po, air pressure outside of the piston orifice of a selected chamber, Pa
- Pi, air pressure after air entering an orifice of a selected chamber, Pa
- Av, effective area of the orifice, m^2
- Cf, coefficient of the orifice for the mass flow of air

Output:

- G, mass flow of air after passing the designated piston orifice, kg/s

This function also takes other global parameters:

- r, ratio of specific heat capacity at constant pressure over constant volume
- RMA, molar based ideal gas constant for air at ambient lab condition
- T, temperature of the air flow at ambient lab condition, K

Brief algorithm walkthrough:

Note that in this case, due to the effects of air passing through an orifice, flow speed has to be checked whether it is within the sonic or subsonic region. To achieve this, the ratio between the upstream and downstream pressure across the orifice has to be calculated and compared to the critical air pressure ratio first. This critical pressure ratio is calculated via a series of computation involving the Cp and Cv values of air under the ambient conditions.

In addition, C1 and C2 as two constants are also calculated according to the equations mentioned in the methodology sections. The rest of the algorithm is simply plug all the given and calculated parameters into that equation set and attain the mass flow of air, G. In the end, the direction of the air flow is checked via either assigning or removing the negative sign of the mass flow since G itself is a vector.

(Script of this function is provided in the following pages.)
function G=compressibleFlow(Po,Pi,Av,Cf)
% calculates the mass flow of air when it passes through an orifice
% % clc;clear all;close all
% % Po=1.5*101325;
% % Pi=101325;
% % Av=10e-5;
% % Cf=1;

% notations
% G, mass flow of air passing through the orifice, kg/s
% Po, air pressure upstream of the given orifice Pa
% Pi, air pressure downstream of the given orifice, Pa
% Av, effective area of the orifice opening, m2
% Cf, coefficient of the mass flow

%Pu, upstream pressure, Pa
%Pd, downstream pressure, Pa

% Define global constants.
global r RMA T

% Function
flag=1;% initialize the air inflow conditions

determine whether air within the selected piston chamber enters or leaves it
if Pi<=Po
    Pd=Pi;% downstream pressure as initial pressure for pressure right
    Pu=Po;% upstream pressure for initialization
    flag=1;% means air enters this chamber
else
    Pd=Po;
    Pu=Pi;
    flag=0;%表 means air leaves this chamber
end

C1=((r./RMA).*(2./(r+1)).^((r+1)./(r-1))).^0.5;
C2=(2.*r./(RMA.*(r-1))).^0.5;
Pcr=(2./(r+1)).^(r./((r-1)/r)).^0.5;

if Pd./Pu<=Pcr % determine whether the flow is in sonic or subsonic region
    G=Cf.*Av.*C1.*Pu./(T.^0.5);
else
    G=Cf.*Av.*C2.*Pu./(T.^0.5).*((Pd./Pu).^(1./r)).*(1-(Pd./Pu).^((r-1)/r)).^0.5;
end

if flag==1;
    G=G;
else
    G=-G;
end
end
MATLAB Piston Model: cylinderModelLeakageInExact

Purpose: puts up all the necessary calculations for modelling the piston motion involved in the study,

Input:

- Pin, air pressure fed to the system, psig
- f, frequency of the time relay, Hz
- m, mass of the weight load onto the piston platform, kg
- tt, total run time for the piston operation, seconds
- initDirect, initial direction of air flow, 1 for entering the lower chamber, 0 for entering the upper chamber

Other global variables and parameters involved in this model: (detailed term notation and explanation can be found in previous sub-function)

Air flow related parameters and constants: Cp, Cv, r, g, pi, pg (density), Pa, MA, RMA, uv, c, Psi2Pa

Piston design and dimension parameters and constants: d, dv, dv1, dvn, dt, z0U, z0L, D, dd, AL, AU, Ar, ALa, AUa, At, L, LtU, LtL

Valve and nozzle related parameters and constants: CfL, CfU, Cfv, xmin, xmax, Ff, beta, alpha

Leakage related parameters and constants: rleak0, rleak1, Aleak0, Aleak1, Cfleak0, Cfleak1

Piston motion and pressure change parameters and constants over the tube: a, v, z, t, xre, falgeLvre, flagUvre, Alere, AUere, ddPlu, ddPIL, ddPtU, ddptL

Mass flow and pressure parameters for air flow within the piston chambers: GL, GU, GLv, GLa, PL, PU, PLa, PUa, PLv, PUv, dPLdt, dPUdt

Indicators for air flow directions and effective parameters: flagLv, flagUv, Ale, AUe

Step size for time: th

Output:

- numerous calculations and plots illustrating the piston displacement over the given total run time
Brief algorithm walkthrough:

The whole model consists of the following stages:

Initialization

All the global variables are defined. These variables range from: air flow air properties, valve orifice coefficients, tube properties, piston dimensions, leakage parameters, direction indicators, etc.

Certain empty matrices are pre-constructed for storing intermediate and final calculation results. Next, experiment conditions are set; unit convertors are applied where necessary since most of the instrument has readings based in British American units.

1st step calculation and onwards

The initial steady pressure has to be obtained to start the loop of iterations within the JBR and valve complex. Then, a “for” loop of piston displacement calculation is formed within the given time range set by the function’s input. The loop first determines the direction of the air flow, which piston chamber undergoes filling up, adiabatic expansion, and the other undergoes isothermal compression. This would set the sign of the vectors since upward motion and entering to the system both are considered to be positive directions for piston displacement and air flow related terms.

Next, the spool valve function takes the initial condition, sets the spool’s location and thus gives the initial mass flow of air through the double solenoid valve towards either chamber. Then, the calculation is moved to the tube segments connecting both upper and lower chambers of the piston to the two ports on the valve which would enable double acting directions upon spool location change. Subsequently, the function “tubeDelay” is used to find the pressure drop which is negative along the air flow is determined. The piston nozzle near the tube and piston connection is the next focus the function “localResistance” is than implemented to find the pressure and mass flow change of the air flow as it either enters or exits a specific piston chamber.

After that, the scope of the calculation is moved into the piston chambers. In this stage, the possible leakage of the piston system is estimated. Here, the leakage is treated essentially as a special case of orifice whereas normal nozzle and orifices are generally a circular open, the leakages involved in this study are generally in the form of rings.

The possible leakage, when the upper chamber undergoes pressure filling, happens most likely around the piston double rods and the top plates as well as near the rubber sealing around the piston block and cylinder wall contact. On the other hand, the possible leakage, for cases when the lower chamber is pressurized, happens primarily around the rubber sealing between the piston block and the cylinder wall. In the end, mass flow of air leakages around both leakages under a given situation of chamber pressurization can be estimated using the “compressibleFlow” function.
After leakages are taking into considerations, the changing pressure within each chamber can be relatively accurately computed, and other pressure check points along the air flow direction can be obtained with the calculations of the relevant pressure drop across the piston orifices nozzles, and soft tubes.

The resultant piston motion is the net force of the pressure exerted or to either side of the piston block. Further constraints for the maximum and minimum piston displacements are provided to prevent unrealistic results of piston motion as piston reaches its full or minimum stroke during the given period.

To better recap the walkthrough see the flow chart next page for illustrations, followed by script of the model in the coming pages.
To better understand the entire process loops, a flow chart is provided below:

Input: feed pressure, frequency, weight load on the piston, run time

Finite steps of time with initialized chamber pressure at t=0, \( P_{U0} \) & \( P_{L0} \)

Checking for next iterations

While time interval is not exceeded

Stop!

Check the direction of piston, Use equations accordingly

Using valve model, determine the spool location and effective orifice area

Using the local resistance model and gas leakage model to find steady state net flow into the piston

Using compressible air flow model and tube pressure drop model to calculate along the flow path before entering the piston nozzle

Find pressure balance within the piston through calculating the pressure in each chamber separately

Find the acceleration and eventually displacement of the piston
function cylinderModelLeakageInExact(Pin,f,m,tt,initDirect)
% piston motion model

%% notations
% Pin, pressure fed to the system, Psi
% f, frequency of the relay, Hz
% m, mass of the weight load added onto the piston platform, kg
% tt, total run time for piston operations, s
% initDirect, initial air flow directions, 1 for air entering the lower chamber, 0 for air entering the upper chamber

%% Define global constants.
global cp cv r g pi R T pg Pa MA RMA uv c Psi2Pa
global d dv dl dvn dt z0U z0L D dd AL AU Ar ALa AUa At L LtU LtL L0 z0

%% Define global constants.
global zetaIn zetaOut %L

global CfL CfU Cfv xmin xmax Ff beta alpha
global rleak0 rleak1 Aleak0 Aleak1 Cfleak0 Cfleak1
global a v z t xre flagLvre flagUvre ALere AUere ddP1U ddP1L ddPtU ddPtL
global GL GU GLv GUv GLa GUa PL PU PLa PUa PLv PUv dPLdt dPUdt
global flagLv flagUv ALe AUe
global th

%% pre-allocation of the storages for calculated or intermediate data
th=1/10000;%s step size break down
PL_cal=zeros(1,tt./th);
PU_cal=zeros(1,tt./th);
PLa_cal=zeros(1,tt./th);
PUa_cal=zeros(1,tt./th);
GleakL=zeros(1,tt./th);
GleakU=zeros(1,tt./th);
GLtotal=zeros(1,tt./th);
GUtotal=zeros(1,tt./th);

%% Initial conditions
% Pin=Pin*Psi2Pa; % feed air pressure conversion
if initDirect==0
    % find the an accurate approximation of the initial pressure within either piston chamber
    [PU(1),PUa(1),PUv(1),PL(1),PLa(1),PLv(1)]=InitialStablePExact(Pin,Pa,initDirect,rleak0,rleak1);
    z(1)=0.00;%m piston displacement, positive for upward motion, negative for downward
    x(1)=xmin;%m valve spool displacement
    zetaU=zetaIn;
    zetaL=zetaOut;
else
    [PU(1),PUa(1),PUv(1),PL(1),PLa(1),PLv(1)]=InitialStablePExact(Pin,Pa,initDirect,rleak0,rleak1);
    z(1)=L;
    x(1)=xmax;
    zetaL=zetaIn;
    zetaU=zetaOut;
end
\[ v(1)=0; \# \text{m/s piston speed initialization} \]
\[ a(1)=0; \# \text{m/s2 piston acceleration initialization} \]
\[ tf=(1/f)/2/th; \# \text{get the number of points for half a cycle, using for calculations} \]
\[ t=0:th:tt; \# \text{scattered time line with the number of points for time direction=initDirect; \# the power signal for flow direction indication, 0 for upper chamber filling up, 1 for lower chamber filling up} \]

global count
count
% initialization of the steady pressure at \( t =0 \)
PUv(1)./Psi2Pa
PUa(1)./Psi2Pa
PU(1)./Psi2Pa
PLv(1)./Psi2Pa
PLa(1)./Psi2Pa
PL(1)./Psi2Pa

%% Calculate
for \( i=1:tt/th \)
    % determines the direction of air flow
    directionp=direction;
    trip=ceil(i/tf); % indicates where the piston motion is with respect to the counting cycles
    direction=mod(trip+initDirect,2);
    % direction is switched upon the implementation of the valve signal
    % ceil takes the integer round down from the values of \( i/tf \)
    % mod is used to indicate whether it is an odd or even number which identifies the trip sequence alternation
    if directionp~=direction
        trip
    end
    tvx=0;
end

\[ x=0.5*37.78*(tvx)^2; \]
if direction==0% air filling up the upper chamber, motion direction will be reversed
    x=dvl-x;
end
if x>xmax
    x=xmax;
end
if x<xmin
    x=xmin;
end
tvx=tvx+th;
xre(i)=x;

% \( PLv(i) = PL(i); \)
% \( PUv(i) = PU(i); \)
[GLv(i), GUv(i)]=valveFlow(Pa,Pin,PLv(i),PUv(i),x);
% \( GLv(i)=compressibleFlow(PLvo,PLv(i),ALA,CfL); \)
% \( GUv(i)=compressibleFlow(PUvo,PUv(i),AUA,CfU); \)
flagLvre(i) = flagLv;
flagUvre(i) = flagUv;
ALere(i) = ALe;
if flagLv==0
    ALere(i) = -ALe;
end
AUere(i) = AUe;
if flagUv==0
    AUere(i) = -AUe;
end

%     GL(i)=GLv(i);
%     GU(i)=GUv(i);
% pressure drop, back pressure along the tube of air flow
[ddPtL(i), PLa_cal(i), GLa(i)] = tubeDelay(GLv(i), PLv(i), LtL, dt);
[ddPtU(i), PUA_cal(i), GUa(i)] = tubeDelay(GUv(i), PUv(i), LtU, dt);

% first calculates the local resistance near the piston nozzle
% first, determines whether air is flowing into the nozzle or from it
if flagUv==1
    zetaU = zetaIn;
else
    zetaU = zetaOut;
end
if flagLv==1
    zetaL = zetaIn;
else
    zetaL = zetaOut;
end
[ddPlU(i), PU_cal(i), GUa(i)] = localResistance(GUa(i), PUa(i), zetaU);
[ddPlL(i), PL_cal(i), GLa(i)] = localResistance(GLa(i), PLA(i), zetaL);

% leakage estimations
GleakU(i) = compressibleFlow(PU(i), Pa, Aleak0, Cfleak0) + compressibleFlow(PU(i), PL(i), Aleak1, Cfleak1);
GleakL(i) = compressibleFlow(PL(i), PU(i), Aleak1, Cfleak1);
GUtotal(i) = GU(i) - GleakU(i);
GLtotal(i) = GL(i) - GleakL(i);

% calculates the chamber pressure changes
dPLdt(i) = chambersPchange(GLtotal(i), PL(i), AL, z(i), v(i), 1);
PL(i+1) = PL(i) + dPLdt(i) .* th;
dPUdt(i) = chambersPchange(GUtotal(i), PU(i), AU, z(i), v(i), 0);
PU(i+1) = PU(i) + dPUdt(i) .* th;

% calculates the pressure after air passing the piston NPT ports
PLa(i+1) = PL(i+1) + ddPlL(i);
PUa(i+1) = PU(i+1) + ddPlU(i);

% calculating the pressure near the valve ports connecting to the piston
PLv(i+1) = PLa(i+1) + ddPtL(i);
PUv(i+1) = PUa(i+1) + ddPtU(i);

% putting all calculated terms together for obtaining the piston acceleration
\[ a(i+1) = (P_L(i) \cdot A_L - P_U(i) \cdot A_U - P_a \cdot A_r + (-\text{sign}(v(i))) \cdot (\beta \cdot |v(i)|^\alpha + F_f)) / m - g; \]
\[ z(i+1) = z(i) + \theta_h \cdot v(i); \]
\[ v(i+1) = v(i) + \theta_h \cdot a(i); \]

% sets the limit for maximum and minimum piston displacement
% to avoid errors or unrealistic results
if \( z(i+1) \leq 0 \)
    \( z(i+1) = 0; \)
    if \( v(i+1) < 0 \)
        \( v(i+1) = 0; \)
    end
end
if \( z(i+1) \geq L \)
    \( z(i+1) = L; \)
    if \( v(i+1) > 0 \)
        \( v(i+1) = 0; \)
    end
end
end
Appendix B

MATLAB Program for Piston Displacement & Solid Bed Analysis
Function Name: pixelToMeter

Purpose: computes the ratio that is used to convert from the image derived pixel displacement to real SI unit based displacement readings

Input:

- image1, the reference frame with red LED on when piston is located at its bottom position
- image2, the reference frame with red LED on when piston is located at its top position

Other global variables and parameters involved in this model:

A: the region, a rectangular box for the program to focus and look for red in a frame

h_piston: the real length of maximum piston stroke, 0.30 m in this case.

Output:

- ratio for converting pixel location based value to SI real world unit based values

Brief algorithm walkthrough:

The whole model first takes the reference frame and crops them by the given coordinates according to the matrix A. Then, the program turns on the red filter to isolate the red LED dot. After that, the maximum value of the red LED is found with its coordinated within that particular reference frame stored. In the end, the difference of the row number between the two dots in the two reference frames are found and divided by the real maximum piston stroke length of 0.30 m. For instance, the difference in row numbers between the two reference images is 360 pixels apart from each other. The resultant ratio is 360 pixels / 0.30 m = 1200 pixels/m.

(See the MATLAB script for this function next page)
function ratio = pixelToMeter (image1, image2)
% this function computes the ratio that converts from pixel values to
% distance which will help the calculation of the piston displacement from
% frame to frame

%% notations
% ratio, the ratio that converts from pixel values, row number location, to
% meters
% image1, the first reference image, preferably when piston is at bottom
% image2, the second reference image, preferably when piston is at top
% redThreshold, the red threshold value for filtering the image to find red
% LED for tracking the piston position
% A, the region for the program to focus and look for red in a frame

%% define global constants
global A h_piston

ref_img1 = imcrop(image1, A);
ref_img_red1 = ref_img1(:, :, 1);
figure, imshow(ref_img_red1);
[red_max1, red_max_idx1] = max(ref_img_red1(:));
[rows_max_red1, col_max_red1] = find(ref_img_red1 == red_max1);
[row1, col1] = ind2sub(size(ref_img_red1), red_max_idx1);

ref_img2 = imcrop(image2, A);
ref_img_red2 = ref_img2(:, :, 1);
figure, imshow(ref_img_red2);

[red_max2, red_max_idx2] = max(ref_img_red2(:));
[rows_max_red2, col_max_red2] = find(ref_img_red2 == red_max2);
[row2, col2] = ind2sub(size(ref_img_red2), red_max_idx2);

h_dot = row1 - row2; % note that the index number is calculated from top to
bottom
fprintf('The overall stroke span of the piston is %6.3f pixel
values.\n\n', h_dot);

ratio = h_dot/h_piston; % pixel per m << watch out this unit, try to
calculate everything in SI base unit
% since the displacement from frames is similar to the actual stroke length
given that the camera position is not changed
fprintf('The overall stroke span of the piston ratio between picture and
reality is %6.3f pixel values per m.\n\n', ratio);
Function Name: solidPixelSum

Purpose: calculates the following summation of pixel entries in an image matrix: a binary pixel value sum, a gray scale pixel value sum and a gray scale pixel difference sum over a given range of frames

Input:

- num, to which number of the cycles the user want the program to perform summation of frames
- T, a matrix that includes all the peaks in terms of frame sequences
- videoObject, the video clips that has been loaded to MATLAB waiting for a frame-by-frame analysis

Other global variables and parameters involved in this model: (detailed term notation and explanation can be found in previous sub-function)

Global variables used to find the red LED dot for tracking the piston displacement: redThres, k, frameRate, rotate_input, ratio, A

Global variables used to indentify video names and name the resultant txt files: baseFileName fpath nCase

Global variables used to locate the vessel region with its specified length and width: vessel_length, vessel_width

Output:

- time, time elapsed upon switching on the red LED and the piston controller, sec
- acceleration, instantaneous acceleration of the piston, m/s^2
- velocity, instantaneous velocity of the piston, calculated based on the instantaneous acceleration, m/s
- d, displacement of the piston according to the red LED dot tracking method, m
- Video_Pixel_Sum, the sum of pixel values in a given image matrix at each of its coordinates in a binary scale
- Video_Pixel.Diff.Sum, the sum of differences in pixel values in a given image matrix at each of its coordinates in a gray scale
- Video_Bin.Diff.Sum, the sum of differences in pixel values in a given image matrix at each of its coordinates in a binary scale
Brief algorithm walkthrough:

The function takes in the number of cycles (periods) specified by the user; it also takes the video clip and a matrix that contains all the frame sequence for the displacement cycles (which is defined as the time it takes to move from one peak to another in the piston displacement term).

First, it computes the piston displacement using the red LED tracking method. Based on the tracked red LED, it crops the given action frame into a small segment which only includes the vessel region. As a result, the solid bed becomes part of this new cropped frame. As the function loops all the frames within the given cycle ranges, it also perform frame to frame addition and subtraction.

In general, each frame is first converted to a gray scale which has its entries ranging from 0, completely dark, to 1, completely white. In this case, 0 indicates little to no presence of the white sand particles. 1 on the other hand, indicates a strong presence of the white sand particles. This gray scale image is then stored as a matrix full of values. Since each video frame is cropped in a similar manner, the matrix dimension becomes the same from crop region to the next. To calculate the sum of pixel values, simply adding all the entries with the entries at the same coordinates in the future frames regions. To calculate the sum of absolute differences between consecutive frames, simple perform the subtraction in a similar manner.

The resultant sum and differences in gray scale would show the overall sand distribution across the vessel region throughout the given cycle time. The differences in BW would give insights on the boundaries of the sand distribution across the vessel over the given time. The above results are then plotted and saved for further analysis.

(See the MATLAB script for this function next page)
function [time, d, velocity, acceleration, Video_Pixel_Diff_Sum, Video_Pixel_Sum, Video_Bin_Diff_Sum] = solidPixelSum(num,T,VideoObject)
% this function gives much simpler analysis of the solid bed through a
% pixel number count of the solid bed occupied area.

% define the global constants
global redThres k frameRate rotate_input
global ratio A
global baseFileName fpath nCase
global initialVolFraction
global vessel_length vessel_width

% before testing the experiment runs, make sure you run a position reference
clip
% while maintaining the camera at the same spot with the same setting:
% Ex: same frame rate), same focal length, etc

row_M = double.empty(0,1);
time = double.empty(0,1);

displacement = double.empty(0,1);
displacement_adjusted = double.empty(0,1);

velocity = double.empty(0,1);
velocity_avg = double.empty(0,1);
acceleration = double.empty(0,1);
acceleration_avg = double.empty(0,1);

d = double.empty(0,1);
d_vessel_top = double.empty(0,1);
d_vessel_bot = double.empty(0,1);

bed_top = double.empty(0,1); % first column for bed top second column for bed bottom
bed_bot = double.empty(0,1);

BW_blob_area = double.empty(0,1);
solidFraction = double.empty(0,1);

% the following groups of arrays are used for calculating the coordinates
% for bounding boxes of vessel and red LED
vessel_box = double.empty(0,4);

delta_h = int16.empty(0,1);
delta_w_r = int16.empty(0,1);
w = int16.empty(0,1);
h = int16.empty(0,1);

Last_Frame = double.empty(0,2);
Last_Bin_Frame = double.empty(0,2);
Video_Pixel_Diff_Sum = double.empty(0,2);
Video_Pixel_Sum = double.empty(0,2);
Video_Bin_Diff_Sum = double.empty(0,2);
counter = 1;
plotSequence = 1;

% figure % prepares a figure to hold the place
for j = 1:nCase
    a = T(num(j),2);
    for i = 1:a %since previous function already found where red LED was on, we
    simply go to the start point this time
    frames = read(videoObject, i);
    thisFrame = imrotate(frames,rotate_input);

    % first, check the section with red LED at bottom
    dot = imcrop(thisFrame,A);
    red_dot = dot(:,:,1);
    red_dot_max = max(max(red_dot(:)));
    if red_dot_max == 255 % && counter <= numFrames
        % use The following codes for finding a specific colored point (in this
        case it is red) and later bound it with a rectangular box
        diffFrame = imsubtract(thisFrame(:,:,1), rgb2gray(thisFrame)); % Get red
        component of the image
        diffFrame = medfilt2(diffFrame, [3 3]); % Filter out the noise by using
        median filter
        binFrame = im2bw(diffFrame, redThres); % Convert the image into binary
        image with the red objects as white
        binFrame = imfill(binFrame,'holes'); %filling up the holes in the
        separated image
        binFrame = bwmorph(binFrame,'close',inf);
        stats_red = regionprops(binFrame,'centroid','BoundingBox','Extrema' );
        thisBB_red_centroid = stats_red.Centroid;
        thisBB_red_extrema = stats_red.Extrema;
        row_M(counter) = thisBB_red_centroid(2);
        if k == 1 %this means when the piston starts from bottom (usually the
        case when you load the entire clip)
            displacement(counter) = row_M(1) - row_M(counter);
            elseif % if piston starts anywhere between the top and bottom location
            (usually the case when you load an editted clip)
            displacement_adjusted(counter) = row1 - row_M(counter);
            displacement(counter) = row_M(1) - row_M(counter) +
            displacement_adjusted(counter);
        end
        if displacement(counter) < 0
            displacement(counter) = 0;
        end
        time(counter) = counter/frameRate; %this calculates the time elapsed
        with the given frame rates, ms
d(counter) = displacement(counter)./ratio;
d_vessel_bot(counter) = d(counter)+0.02;
d_vessel_top(counter) = d(counter)+0.155;

% Compute the displacement, speed and acceleration of the piston
if counter == 1
velocity(counter) = 0;
velocity_avg(counter) = 0;
acceleration(counter) = 0;
acceleration_avg(counter) = 0;
else
velocity(counter) = (d(counter)-d(counter-1))./(1/frameRate);
velocity_avg = medfilt1(velocity,3);
acceleration(counter) = (velocity(counter)-velocity(counter-1))./(1/frameRate);
acceleration_avg = medfilt1(acceleration,3);
end

delta_h(counter) = int16(176.84-0.1745*displacement(counter));
delta_w_r(counter) = int16(26.942-0.0192*displacement(counter));
h(counter) = int16(vessel_length);
w(counter) = int16(vessel_width);

% the following code pass on the values to the array for
% storage
% stores the x y coordinates of the two points of the top red
% vessel bounding box
vessel_box(counter,1) = thisBB_red_extrema(2,1) + delta_w_r(counter) -
w(counter); %x1
vessel_box(counter,2) = thisBB_red_extrema(2,2) - delta_h(counter); %y1
vessel_box(counter,3) = w(counter); %w
vessel_box(counter,4) = h(counter); %h

% export the cropped vessel area in a separate figure
frame_vessel = imcrop(thisFrame, vessel_box(counter,:));
frame_vessel_a = rgb2gray(frame_vessel);
frame_vessel_adj = imadjust(frame_vessel_a);
frame_vessel_avg = medfilt2(frame_vessel_adj); % this performs medium
filter of the image edges
frame_vessel_BW = imbinarize(frame_vessel_avg);

% figure,
% imshow(thisFrame);
% figure,
% imshow(frame_vessel);
% figure,
% imshow(frame_vessel_a);
% figure,
% imshow(frame_vessel_adj);

% hImage=subplot(1, 4, 1);
% imshow(frame_vessel_a);
% caption = sprintf('Original image of the vessel area,at frame
#%d',i);
if counter == 1
% Enlarge figure to full screen.
set(gcf, 'Units', 'Normalized', 'OuterPosition', [0 0 1 1]);
% Give a name to the title bar.
set(gcf, 'Name', baseFileName, 'NumberTitle', 'Off')
end

% Filter out small blobs.
frame_vessel_BW = bwareaopen(frame_vessel_BW,20);

subplot(1,4,2)
imshow(frame_vessel_adj);
caption = sprintf('Enhanced gray scale image of the solid bed,at frame #\d', i);
title(caption);
drawnow;

subplot(1,4,3)
imshow(frame_vessel_avg);
caption = sprintf('Adjusted gray scale image of the solid bed,at frame #\d', i);
title(caption);
drawnow;

subplot(1,4,4)
imshow(frame_vessel_BW);
caption = sprintf('BW image of the solid bed,at frame #\d', i);
title(caption);
drawnow;

% set up an additional guard to eliminate error readings, in case
% blob cannot be detected, simply use the previous one (likely the
% blob was hiding behind the vessel wall and due to an angle of view

if max(max(frame_vessel_BW)) ~= 0

stats_BW =
regionprops(frame_vessel_BW,'Centroid','BoundingBox','Area');
BW_blob_box = stats_BW.BoundingBox;

% assign bed top and bed bottom level separately
% calculate this span in terms of pixel value
bed_top(counter) = BW_blob_box(2);
bed_bot(counter) = BW_blob_box(2)+BW_blob_box(4);
else if counter == 1
    bed_top(counter) = initialVolFraction*d_vessel_top(counter);
    bed_bot(counter) = d_vessel_bot(counter);
else
    bed_top(counter) = bed_top(counter-1);
    bed_bot(counter) = bed_top(counter-1);
end
end
% the following calculates the blob area in the BW image of vessel
% and the ratio between blob and vessel area
BW_blob_area(counter) = bwarea(frame_vessel_BW);

%% finds the accumulated differences in the pixel values of a given
vessel section throughout the entire video
%Pixel_Diff_Avg
Current_Frame =
double(frame_vessel_avg);  %Convert to int16
type integer
Binary_Frame = frame_vessel_BW;

    if counter == 1
        Last_Frame = Current_Frame;
    elseif counter == 2
        Video_Pixel_Diff_Sum = abs(Current_Frame - Last_Frame);  %The absolute difference per pixel of the current and previous frame
        Last_Frame = Current_Frame;
    else
        Diff_Frame_Current = abs(Current_Frame - Last_Frame);
        Video_Pixel_Diff_Sum = Video_Pixel_Diff_Sum + Diff_Frame_Current;  %Creates a matrix of the absolute difference per pixel of the current and previous frame for the length of the spray
        Last_Frame = Current_Frame;
    end

%Pixel_Sum
    if counter == 1
        Video_Pixel_Sum = Current_Frame;
    elseif counter == 2
        Video_Pixel_Sum = Last_Frame + Current_Frame;
    else
        Video_Pixel_Sum = Video_Pixel_Sum + Current_Frame;
    end

%Binary_Diff_Avg
    if counter == 1
        Last_Bin_Frame = Binary_Frame;
    elseif counter == 2
        Video_Bin.Diff_Sum = abs(Binary_Frame - Last_Bin_Frame);  %The absolute difference per pixel of the current and previous frame
        Last_Bin_Frame = Binary_Frame;
    else
        %Saves frame for next loop
Diff_Frame_Bin_Current = abs(Binary_Frame - Last_Bin_Frame);  % The absolute difference per pixel of the current and previous frame
Video_Bin_Diff_Sum = Video_Bin_Diff_Sum + Diff_Frame_Bin_Current;  % Creates a matrix of the absolute difference per pixel of the current and previous frame for the length of the spray
Last_Bin_Frame = Binary_Frame;  % Saves frame for next loop
end
   counter = counter + 1;
else
   continue
end
end
% save the three computed overall files
dlmwrite(['Video Pixel Diff Sum at ',baseFileName,' from beginning to cycle#',num2str(num(j)),'.txt'],Video_Pixel_Diff_Sum,'delimiter','	');
dlmwrite(['Video Pixel Sum at ',baseFileName,' from beginning to cycle#',num2str(num(j)),'.txt'],Video_Pixel_Sum,'delimiter','	');
dlmwrite(['Video Bin Diff Sum at ',baseFileName,' from beginning to cycle#',num2str(num(j)),'.txt'],Video_Bin_Diff_Sum,'delimiter','	');
figure('name',['Solid bed situation at ',baseFileName,' from beginning to cycle# ',num2str(num(j))]);
set(gcf, 'Units', 'Normalized', 'OuterPosition', [0 0 1 1]);
subplot(2,3,1)
I1 = mat2gray(Video_Pixel_Diff_Sum);
imshow(I1)
axis on
colorbar
title(['Pixel difference sum at each given spot of frames from the beginning to cycle# ',num2str(num(j))]);

subplot(2,3,4)
contourf(I1,10);
axis ij
colorbar
title(['Contour fit of Video Pixel Diff Sum from the beginning to cycle# ',num2str(num(j))]);

subplot(2,3,2)
I2 = mat2gray(Video_Pixel_Sum);
imshow(I2)
axis on
colorbar
title(['Pixel sum among all the frames from the beginning to cycle# ',num2str(num(j))]);

subplot(2,3,5)
contourf(I2,10);
axis ij
colorbar
title(['Contour fit of Video Pixel Sum from the beginning to cycle# ',num2str(num(j))]);
subplot(2,3,3)
I3 = mat2gray(Video_Bin_Diff_Sum);
imshow(I3)
axis on
colorbar
title(['Binary pixel sum among all the frames from the beginning to cycle# ',num2str(num(j))]);

subplot(2,3,6)
contourf(I2,10);
axis ij
colorbar
title(['Contour fit of Video Bin Diff Sum from the beginning to cycle# ',num2str(num(j))]);
saveas(gcf,fullfile(fpath,['Comparison plot of pixel difference sum, overall sum at ',baseFileName,' from beginning to cycle# ',num2str(num(j))]),'fig');
saveas(gcf,fullfile(fpath,['Comparison plot of pixel difference sum, overall sum at ',baseFileName,' from beginning to cycle# ',num2str(num(j))]),'jpg');

% Plots some of the overall summary of the pixel sum, binary difference and others
% Give a name to the title bar.
% these are the overall plots
% subplot(nCase,3,plotSequence)
% I1 = mat2gray(Video_Pixel_Diff_Sum);
% plotSequence = plotSequence + 1;
% imshow(I1)
% title(['Pixel difference sum at each given spot of frames from the beginning to cycle# ',num2str(num(j,1))]);
% subplot(nCase,3,plotSequence)
% I2 = mat2gray(Video_Pixel_Sum);
% plotSequence = plotSequence + 1;
% imshow(I2)
% title(['Pixel sum among all the frames from the beginning to cycle# ',num2str(num(j,1))]);
% subplot(nCase,3,plotSequence)
% I3 = mat2gray(Video_Bin_Diff_Sum);
% plotSequence = plotSequence + 1;
% imshow(I3)
% title(['Binary pixel sum among all the frames from the beginning to cycle# ',num2str(num(j,1))]);
% saveas(gcf,fullfile(fpath,['Comparison plot of pixel difference sum, overall sum at ',baseFileName],'fig'));
% saveas(gcf,fullfile(fpath,['Comparison plot of pixel difference sum, overall sum at ',baseFileName],'jpg'));
end

------------------------------------------------------------------------------------------------------------
Function Name: MatrixImageAnalysis

Purpose: give more refined analysis on the vessel region which will give statistical terms such as mean, standard deviations and $CV_{space}$, coefficient of variation in space term, to describe the solid distribution across the vessel region under a given (or a series of) piston cycles.

Input:

- level, the number of gray scale levels specified by the user to subdivide the gray scale range which will help the contour plot, distribution bar charts and pixel counting
- PixelSum, the sum of all pixel values at each coordinate of the given cropped vessel region frame

Other global variables and parameters involved in this model: (detailed term notation and explanation can be found in previous sub-function)

grayThres, the gray threshold value used for removing dark edges of the cropped vessel region

Output:

- solidMean, average gray scale pixel values within the matrix of the vessel region
- solidStd, population based standard deviation gray scale pixel values within the matrix of the vessel region
- $CV_{space}$, a matrix of coefficient of variations of the column pixel values of the given gray scale image matrix
- solidFractionC, a matrix of cumulative distribution of the solid fractions over a specified range of gray scale categories
- solidFractionD, a matrix of discrete distribution of the solid fractions over a specified range of gray scale categories
- weightedSolidFracMean, the weight average of the solid fraction which is calculated via number of pixels counted in a specified category multiply by the fraction of the its weight percentage
- I, the image matrix which represents the latest cropped vessel region converted from the original input image
- I1, the image matrix which represents the processed vessel region which has its dark edges removed
- Cat, the categories for gray scale when performing bar chart plot, essentially the intermediate values between the major gray levels
Brief algorithm walkthrough:

The function first takes the specified levels to create a series of categories that subdivided the range of gray scale from the given gray threshold input by the user to the maximum gray scale of 1.0. Essentially, any matrix entry that is lower than this threshold will be turned into zero, or completely dark. This is applied to the input pixel sum image matrix obtained from the function solidPixelSum and removes its dark edges on both sides. This will further narrow the range of vessel region.

Once the matrix is prepared, it is further thrown into a loop to search for any pixel values that is larger than the gray threshold. The program searches each row and column rigorously in a double for loop. All the non-dark entries are further counted based on the specified gray scale categories. For instance, the loop first searches any pixel that has a value larger than the gray threshold, say 0.05, the number turns out to be 500 which means there are 500 pixels in this image matrix that has pixel value greater than 0.05. The next level is 0.1, and the loop proceeds the counting in a similar manner. In the end, the discrete and cumulative distribution of solid counts can be shown using a bar chart. In addition, the mean and standard deviation of the pixel values based on columns are found to give more insights on the dispersion of the solid across the vessel length, or vertically. With the help of the calculated standard deviation and mean of the pixel values, the CV_{space} can be obtained for the entire image matrix which represents the sum of pixel values of a given cycle (or overall a series of cycles).

The final results are plotted using the MATLAB subplots, the vessel region, contour fit, bar chart of the discrete distribution as well as the change of CV_{space} from cycle to cycle can all be examined visually and quantitatively.

(See the MATLAB script for this function next page)
function [CVspace, solidMean, solidStd, solidFractionC, solidFractionD, weightedSolidFracMean, I, I1, cat] = MatrixImageAnalysis (level, PixelSum)
% the purpose of this function is to convert the pixel sums into matrix and perform further analysis on the solid bed distribution across the vessel region.
% it first trims the input sum via getting rid of the zero columns on the two sides or completely dark edged on both sides
% other functions to an image matrix while performing analysis based on
% gray scale pixel values for the solid distribution in the vessel region

% input parameters
% level, the number of layers to subdivide the image gray scale from 0 to 1
% PixelSum, the txt (or matrix) generated in previous functions for summing
% all the frames in the gray scale, over 1 cycle in most cases

global grayThres

% note that the outputs are essentially column vectors or arrays that will have the same number of rows as the parameter "level" since at each gray scale level, the quantity and deviations are calculated separately
cat = double.empty (0,1);
solidFractionD = double.empty (0,1);
umSolid = double.empty (0,1); % this is a cumulative counter for the solid that being counted within a given level of pixel gray scale

mat = PixelSum; % using a matrix to represent the pixel sum image
category = linspace(grayThres,1,level);
[a,b] = size(category);

for index = 1:b
cat(index) = (category(index)+category(index+1))./2;
end

%% primary focus is to eliminate columns on both sides of the image margin in this case
% Remove zero rows
% mat1( all(~mat1,2), : ) = [];

% Remove zero columns
mat(:,all(~mat,1)) = [];

% converts the matrix to gray scale image once again for further processing
I = mat2gray(mat);
I1 = mat2gray(mat);

% make all entries less than the gray threshold to be zero and then remove
% zero columns again
I1(I1<grayThres) = 0;
I1(:,all(~I1,1)) = [];

% the following counts the total matrix area of the vessel region (in % pixels)
[row1, col1] = size(I1);
area = row1*col1;

for i = 1:row1
    for j = 1:col1
        for k = 1:(level-1)
            if I1(i,j) > category(k)
                numSolid(k) = numSolid(k)+1; % counts the number of solid pixel spots that are larger or equal to the given level of category
            end
        end
    end
end

solidFractionC = numSolid./area; % finds cumulative the solid fraction at each category over the entire vessel area

% finds distributive the solid fraction at each category over the entire vessel area
for index = 1:b
    solidFractionD(index) = solidFractionC(index) - solidFractionC(index+1);
end

weightedSolidFracMean = dot(cat,numSolid'); % this is the weight average by number of pixel spots

solidStd = std2(I1);
solidMean = mean2(I1);
CVspace = solidStd/solidMean;

% M = [cat';numSolid']; % form a resultant matrix by combining the relevant arrays

% the following plot the original and the edited image matrices side by side (optional)
% figure,
% subplot(2,4,1)
% imshow(I)
% axis on
% colorbar
% title(['the original pixel sum image at cycle# ',num2str(CycleNum)]);
% subplot(2,4,4)
% imshow(I1)
% axis on
% colorbar
% title(['the edited pixel sum image at cycle# ',num2str(CycleNum)]);
% subplot(2,4,2)
% contourf(I,level);
% axis ij
% colorbar
% title(['the edited pixel sum image with contour fit at cycle# ',num2str(CycleNum)]);
% subplot(2,4,5)
% contourf(I1,level);
% axis ij
% colorbar
% title(['the editted pixel sum image with contour fit at cycle# 
',num2str(CycleNum))];
end
Main Run Script: pixelCount_test_main_fullyAutomatic

Purpose: generate results and figures from the pixel based gray scale vessel region frame-by-frame analysis in a continuous manner upon running the script

Input:

- folder directories where the high speed trial video clips are located
- certain parameters for the analysis such as the coordinates for the region of focus, red threshold value for finding the red LED dot when determining the location of the piston

Other global variables and parameters involved in this model:

- A, the region, essential coordinates of a rectangular box of which MATLAB searches for the red LED when it tracks the motion of the piston displacement
- grayThres, the minimum gray threshold value, beyond which all were considered to be white or where the solid particles were present since the background of the vessel is completely dark (or black with a gray scale value close to ZERO)
- redThres, red threshold value beyond which all pixel entries in a frame will be considered to be red after applying a red filter
- k, the flag which indicates whether the piston starts its motion from bottom as the LED was turned on or not
- checkerMin, the minimum framerate, the frame rate of the high speed camera, in this case it is 240 fps, this value is used to calculate the time which further synchronizes the piston and bed motion at each given frame
- rotate_input, the degree of rotation applied to each frame, by default this value is -90 for cases when rotation of the frame orientation is necessary and 0 when no rotation shall be applied
- nCase, the number of the cases, or to be more precise the number of cycles you want to study using this script
- MinPeakDistance, a manual set parameter, the minimum peak to peak difference, for the function to find peaks
- baseFileName, the basic file name without the extension, for instance, a video clip is named as ‘15g 40psig 60-65 01.mov’, then its “baseFileName” will be ‘15g 40psig 60-65 01’; this term is mostly used to name figures and txt data files when analysis for a specific video clip is complete
- fpath, the file path or directory or location of the folder to store the resultant figure, txt files and other results of the analysis
- ratio, the pixel to meter conversion ratio calculated
level, the number of “layers” you want the MATLAB program to subdivide the image’s gray scale from its minimum to maximum entries.

caseSolid, this is a flag for indicating which solid case, 15g, 45g ,etc is involved in the current loop of analysis.

vessel_length, the actual length of the vessel in pixel based unit

vessel_width, the actual width of the vessel in pixel based unit

h_piston, the length of the total full piston stroke

weightLoad, this is another flag for indicating which weight load case, 2.569kg, 3.951kg or etc, is involved in the current loop of analysis

Output:

• a series of plots of changing solid bed ratio as the piston complete from cycle to cycle

Brief algorithm walkthrough:

It simply puts the previously displayed functions and models in a continuous manner; the script first tells MATLAB which folder it should look for the trial video clip, it then assigns the folder

(See the MATLAB script for this function starting next page)
clear
close all
clc

% the following continuously and automatically search through a pre-set
% series of folders for videos
% the program loops from video to video while calculating the cycle based
% solid bed situation and generates plots for further analysis between case
% to case
% prepare empty matrices for storing data
t = double.empty(0,1);
CV_space = double.empty(0,1);
solidFrac = double.empty(0,1);
weightedMean = double.empty(0,1);

% define global constants
global A grayThres redThres k checkerMin frameRate startPoint rotate_input
nCase
global MinPeakDistance num1 num2 baseFileName fpath ratio level
global caseSolid vessel_length vessel_width h_piston weightLoad

grayThres = 0.05; % the minimum gray threshold used to trim the frame’s black edges
level = 10; % the number of 'layers' to subdivide the gray scale image

A = [175.5 120.5 48 392]; % this is the area for tracking the red dot and calculating the pixel to meter ratio for the piston displacement

redThres = 0.2; % Threshold for red detection, this value will keep the off LED away from detection (note that you need to give another threshold for other clips as this may vary)
% warning setting the threshold too high may cause red dot blurred away
% from some frames and thus causing troubles for the red dot tracking

k = 1; % a flag value indicating whether the piston starts from bottom or somewhere between, 1 when piston starts from bottom, 0 for all other cases

checkerMin = 0; % a flag value indicating whether the minimum peak distance is applied, 0 is off, meaning the find peaks will be automatic, 1 will allow the global variable to specify the minimum peak to peak distance to the function

frameRate = 240.0; % as the camera setting defined

MinPeakDistance = 25; % this is the minimum number of points between peaks

% the following is the select range for time, measured in seconds
num1 = 0;
num2 = 4;

startPoint = 5*frameRate; % this implies after 5 seconds upon start of piston

% in case the image rotation is unnecessary, if not simply input 0, usually % it is -90
rotate_input = input ('Please enter the number of degree you want to rotate the image counter-clockwise: ');
rotate_input = -90;

vessel_length = 114; % actual vessel length depicted in the cropped region measured in meters
vessel_width = 50; % actual vessel length depicted in the cropped region measured in meters
h_piston = 0.30; % the maximum piston displacement

% part 1 load all the files (manual mode)
all_files = dir('C:\Users\Timothy\Desktop\20180503 piston mixing trials\90g solids\5.054kg\*.mov');

% % input the video files and prepare relevant parameters and empty arrays for data storage
% % Specify input video file name.
% folder = 'C:\Users\Timothy\Desktop\20180503 piston mixing trials\90g solids\5.054kg\';
% % this indicates the current folder
%
% %define constants individually
% % before proceeding with running the code check the following
% caseSolid = input('What is the initial solid volume fraction? 1 for 15g, 2 for 45g, and 3 for 90g: ');
% % caseSolid, this is a switch that distinguish cases when different solid loads were applied
% % caseSolid = 1 when 15g of solids are present in the vessel
% % caseSolid = 2 when 45g of solids are present in the vessel
% % caseSolid = 3 when 90g of solids are present in the vessel
%
% weightLoad = input('What is the weight loaded onto the piston? 1 for 2.569kg, 2 for 3.951kg, and 3 for 5.054kg: ');
% % weightLoad, this is a switch that distinguish cases when different solid loads were applied
% % weightLoad = 1 when 2.569kg of solids are present in the vessel
% % weightLoad = 2 when 3.951kg of solids are present in the vessel
% % weightLoad = 3 when 5.054kg of solids are present in the vessel
%
% part 1 load all the files (automatic mode)

%define constants individually
% before proceeding with running the code check the following
caseSolidMax = 3; % this is the maximum number of cases for solid load
% caseSolid, this is a switch that distinguish cases when different solid loads were applied
% caseSolid = 1 when 15g of solids are present in the vessel
% caseSolid = 2 when 45g of solids are present in the vessel
% caseSolid = 3 when 90g of solids are present in the vessel

weightLoadMax = 3; % this is the maximum number of cases for weight load
% weightLoad, this is a switch that distinguish cases when different solid loads were applied
% weightLoad = 1 when 2.569kg of solids are present in the vessel
% weightLoad = 2 when 3.951kg of solids are present in the vessel
% weightLoad = 3 when 5.054kg of solids are present in the vessel

tic;
for caseSolid = 1:caseSolidMax
    for weightLoad = 1:weightLoadMax
        tic;
        % the following sets up the input folders for the loops
        if caseSolid == 1 && weightLoad == 1
            all_files = dir('C:\Users\Timothy\Desktop\20180331 Piston Trial\15g solid\moving part weight 2.569kg\*mov');
            folder = 'C:\Users\Timothy\Desktop\20180331 Piston Trial\15g solid\moving part weight 2.569kg\';
        elseif caseSolid == 1 && weightLoad == 2
            all_files = dir('C:\Users\Timothy\Desktop\20180331 Piston Trial\15g solid\moving part weight 3.951kg - Corrected order\*mov');
            folder = 'C:\Users\Timothy\Desktop\20180331 Piston Trial\15g solid\moving part weight 3.951kg - Corrected order\';
        elseif caseSolid == 1 && weightLoad == 3
            all_files = dir('C:\Users\Timothy\Desktop\20180331 Piston Trial\15g solid\moving part weight 5.054kg\*mov');
            folder = 'C:\Users\Timothy\Desktop\20180331 Piston Trial\15g solid\moving part weight 5.054kg\';
        elseif caseSolid == 2 && weightLoad == 1
            all_files = dir('C:\Users\Timothy\Desktop\20180502 piston mixing trials\45g solids\2.569kg\*mov');
            folder = 'C:\Users\Timothy\Desktop\20180502 piston mixing trials\45g solids\2.569kg\';
        elseif caseSolid == 2 && weightLoad == 2
            all_files = dir('C:\Users\Timothy\Desktop\20180502 piston mixing trials\45g solids\3.951kg\*mov');
            folder = 'C:\Users\Timothy\Desktop\20180502 piston mixing trials\45g solids\3.951kg\';
        elseif caseSolid == 2 && weightLoad == 3
            all_files = dir('C:\Users\Timothy\Desktop\20180502 piston mixing trials\45g solids\5.054kg\*mov');
            folder = 'C:\Users\Timothy\Desktop\20180502 piston mixing trials\45g solids\5.054kg\';
        elseif caseSolid == 3 && weightLoad == 1
            all_files = dir('C:\Users\Timothy\Desktop\20180503 piston mixing trials\90g solids\2.569kg\*mov');
            folder = 'C:\Users\Timothy\Desktop\20180503 piston mixing trials\90g solids\2.569kg\';
        elseif caseSolid == 3 && weightLoad == 2
            all_files = dir('C:\Users\Timothy\Desktop\20180503 piston mixing trials\90g solids\3.951kg\*mov');
            folder = 'C:\Users\Timothy\Desktop\20180503 piston mixing trials\90g solids\3.951kg\';
        else
            all_files = dir('C:\Users\Timothy\Desktop\20180503 piston mixing trials\90g solids\5.054kg\*.mov');
            folder = 'C:\Users\Timothy\Desktop\20180503 piston mixing trials\90g solids\5.054kg\';
        end

% before testing the experiment runs, make sure you run a position reference clip
% while maintaining the camera at the same spot with the same setting: 
% Ex: same frame rate), same focal length, etc 

% Before analysis make sure you get the ratio correctly 
if caseSolid == 1 && weightLoad == 1 
  % specify the reference frames for calculating the frames 
  image1 = imread('C:\Users\Timothy\Documents\MATLAB\piston related analysis\piston position reference\15g solids\2.569kg\initial 0600.jpg'); 
  image2 = imread('C:\Users\Timothy\Documents\MATLAB\piston related analysis\piston position reference\15g solids\2.569kg\initial 0800.jpg'); 
elseif caseSolid == 1 && weightLoad == 2 
  image1 = imread('C:\Users\Timothy\Documents\MATLAB\piston related analysis\piston position reference\15g solids\3.951kg\initial 0500.jpg'); 
  image2 = imread('C:\Users\Timothy\Documents\MATLAB\piston related analysis\piston position reference\15g solids\3.951kg\initial 0700.jpg'); 
elseif caseSolid == 1 && weightLoad == 3 
  image1 = imread('C:\Users\Timothy\Documents\MATLAB\piston related analysis\piston position reference\15g solids\5.054kg\initial 0650.jpg'); 
  image2 = imread('C:\Users\Timothy\Documents\MATLAB\piston related analysis\piston position reference\15g solids\5.054kg\initial 0815.jpg'); 
elseif caseSolid == 2 && weightLoad == 1 
  image1 = imread('C:\Users\Timothy\Documents\MATLAB\piston related analysis\piston position reference\45g solids\2.569kg\initial 0600.jpg'); 
  image2 = imread('C:\Users\Timothy\Documents\MATLAB\piston related analysis\piston position reference\45g solids\2.569kg\initial 0800.jpg'); 
elseif caseSolid == 2 && weightLoad == 2 
  image1 = imread('C:\Users\Timothy\Documents\MATLAB\piston related analysis\piston position reference\45g solids\3.951kg\initial 02\0550.jpg'); 
  image2 = imread('C:\Users\Timothy\Documents\MATLAB\piston related analysis\piston position reference\45g solids\3.951kg\initial 02\01050.jpg'); 
elseif caseSolid == 2 && weightLoad == 3 
  image1 = imread('C:\Users\Timothy\Documents\MATLAB\piston related analysis\piston position reference\45g solids\5.054kg\initial 02\01100.jpg'); 
  image2 = imread('C:\Users\Timothy\Documents\MATLAB\piston related analysis\piston position reference\45g solids\5.054kg\initial 02\01700.jpg'); 
elseif caseSolid == 3 && weightLoad == 1 
  image1 = imread('C:\Users\Timothy\Documents\MATLAB\piston related analysis\piston position reference\90g solids\2.569kg\initial 02\0340.jpg'); 
  image2 = imread('C:\Users\Timothy\Documents\MATLAB\piston related analysis\piston position reference\90g solids\2.569kg\initial 02\0460.jpg'); 
elseif caseSolid == 3 && weightLoad == 2 
  image1 = imread('C:\Users\Timothy\Documents\MATLAB\piston related analysis\piston position reference\90g solids\2.569kg\initial 02\1000.jpg'); 
  image2 = imread('C:\Users\Timothy\Documents\MATLAB\piston related analysis\piston position reference\90g solids\2.569kg\initial 02\1250.jpg'); 
else 
  image1 = imread('C:\Users\Timothy\Documents\MATLAB\piston related analysis\piston position reference\90g solids\5.054kg\initial 02\1000.jpg'); 
  image2 = imread('C:\Users\Timothy\Documents\MATLAB\piston related analysis\piston position reference\90g solids\5.054kg\initial 02\1430.jpg'); 
end
% the following computes the pixel to meter ratio
ratio = pixelToMeter (image1,image2);
% the following automatically assigns the folder for storing the solid bed
% mixing summary figures
if caseSolid == 1 && weightLoad == 1
% specify the path for saving the data files according to the file names
% and weight load cases
    fpath = 'C:\Users\Timothy\Desktop\20180626 Overall solid bed situation\15g\2.569kg\';
elseif caseSolid == 1 && weightLoad == 2
    fpath = 'C:\Users\Timothy\Desktop\20180626 Overall solid bed situation\15g\3.951kg\';
elseif caseSolid == 1 && weightLoad == 3
    fpath = 'C:\Users\Timothy\Desktop\20180626 Overall solid bed situation\15g\5.054kg\';
elseif caseSolid == 2 && weightLoad == 1
    fpath = 'C:\Users\Timothy\Desktop\20180626 Overall solid bed situation\45g\2.569kg\';
elseif caseSolid == 2 && weightLoad == 2
    fpath = 'C:\Users\Timothy\Desktop\20180626 Overall solid bed situation\45g\3.951kg\';
elseif caseSolid == 2 && weightLoad == 3
    fpath = 'C:\Users\Timothy\Desktop\20180626 Overall solid bed situation\45g\5.054kg\';
elseif caseSolid == 3 && weightLoad == 1
    fpath = 'C:\Users\Timothy\Desktop\20180626 Overall solid bed situation\90g\2.569kg\';
elseif caseSolid == 3 && weightLoad == 2
    fpath = 'C:\Users\Timothy\Desktop\20180626 Overall solid bed situation\90g\3.951kg\';
else
    fpath = 'C:\Users\Timothy\Desktop\20180626 Overall solid bed situation\90g\5.054kg\';
end

% then you start analysis for each file
[numFiles,~] = size(all_files);
for z = 1:numFiles
tic;
    fileName = all_files(z).name;
    baseFileName = fileName;
    baseFileName(end-3:end)=[];

    fullFileName = fullfile(folder, fileName);
    fprintf('The the foler is selected and the program just started to analyze the case of %s.\n',baseFileName);

    % Instantiate a video reader object for this video.
    videoObject = VideoReader(fullfile); display(videoObject);

    % Setup other parameters
    numberOfFrames = videoObject.NumberOfFrame;
\[t_{\text{max}}, T, \text{cycle}, \text{numActionFrames}, \text{statsRedLed}\] = pistonPeakAnalysis(numberOfFrames, videoObject);
    fprintf('The number of frames of which red LED was on and piston is in action is %d.\n', numActionFrames);
    nCase = cycle - 5; % this gives the number of cases for cycle-to-cycle separate analysis

    % the following generates two series of cycle sequence number for % computing cycle to cycle solid bed situations
    if cycle == 0
        z = z - 1;
        fprintf('The file at %s did not produce meaningful cycles with current analysis conditions.\n', baseFileName);
        continue
    elseif cycle < 30
        z = z - 1;
        redThres = redThres - 0.005;
        fprintf('The file at %s did not produce meaningful cycles, and the red threshold is reduced for re-iterations.\n', baseFileName);
        fprintf('The red threshold is reduced to %d for re-iterations.\n', redThres);
        continue
    else
        redThres = 0.20;
        fprintf('The red threshold is brought back to %d for re-iterations.\n', redThres);
    end
    fprintf('Piston analysis iteration is completed for the file at %s.\n', baseFileName);

for numCycle = 1:nCase
    [time, d, velocity, acceleration, Video_Pixel_Sum, Video_Bin_Diff_Sum] = solidPixelSumSimp(T, videoObject, numCycle);
    [CVspace, solidMean, solidStd, solidFractionC, solidFractionD, weightedSolidFracMean, I, I1, cat] = MatrixImageAnalysis(level, PixelSum);
    t(numCycle) = time(T(numCycle));
    CV_space(numCycle) = CVspace;
    solidFrac(numCycle) = solidFractionC(1);
    weightedMean(numCycle) = weightedSolidFracMean;

    if numCycle == 1
        figure('name', ['Solid bed situation at ', baseFileName, ' at cycle#1']);
        set(gcf, 'Units', 'Normalized', 'OuterPosition', [0 0 1 1]);
        subplot(2,2,1)
        imshow(I)
        axis on
        colorbar
        title('Original Video Gray Pixel Sum at cycle#1');
        subplot(2,2,2)
        imshow(I2)
        axis on
        colorbar
title('Editted Video Gray Pixel Sum at cycle#1');
subplot(2,2,3)
contourf(I2,level);
axis ij
colorbar
title('Contour fit of Editted Video Pixel Sum at cycle#1');
subplot(2,2,4)
h = histrogram(solidFractionD,cat);
h.Normalization = 'countdensity';
title('Histogram of Editted Video Pixel Sum Distribution in gray scale at cycle#1');
saveas(gcf,fullfile(fpath,'1st cycle solid bed analysis at ',baseFileName,'fig'));
saveas(gcf,fullfile(fpath,'1st cycle solid bed analysis at ',baseFileName,'jpg'));
elseif numCycle == int8(nCase/2);
figure('name',['Solid bed situation at ',baseFileName,' at cycle#1']);
set(gcf, 'Units', 'Normalized', 'OuterPosition', [0 0 1 1]);
subplot(2,2,1)
imshow(I)
axis on
colorbar
title('Oringal Video Gray Pixel Sum at cycle#1');
subplot(2,2,2)
imshow(I2)
axis on
colorbar
title('Editted Video Gray Pixel Sum at cycle#1');
subplot(2,2,3)
contourf(I2,level);
axis ij
colorbar
title('Contour fit of Editted Video Pixel Sum at cycle#1');
subplot(2,2,4)
h = histrogram(solidFractionD,cat);
h.Normalization = 'countdensity';
title('Histogram of Editted Video Pixel Sum Distribution in gray scale at cycle#1');
saveas(gcf,fullfile(fpath,num2str(numCycle),'th cycle solid bed analysis at ',baseFileName,'fig'));
saveas(gcf,fullfile(fpath,num2str(numCycle),'th cycle solid bed analysis at ',baseFileName,'jpg'));
elseif numCycle == nCase;
figure('name',['Solid bed situation at ',baseFileName,' at cycle#1']);
set(gcf, 'Units', 'Normalized', 'OuterPosition', [0 0 1 1]);
subplot(2,2,1)
imshow(I)
axis on
colorbar
title('Oringal Video Gray Pixel Sum at cycle#1');
subplot(2,2,2)
imshow(I2)
axis on
colorbar
title('Editted Video Gray Pixel Sum at cycle#1');
subplot(2,2,3)
contourf(I2,level);
axis ij
colorbar
title('Contour fit of Edited Video Pixel Sum at cycle#1');
subplot(2,2,4)
h = histrogram(solidFractionD,cat);
h.Normalization = 'countdensity';
title('Histogram of Edited Video Pixel Sum Distribution in gray scale at cycle#1');
saveas(gcf,fullfile(fpath,num2str(numCycle),'th cycle solid bed analysis at ',baseFileName,'fig'));
saveas(gcf,fullfile(fpath,num2str(numCycle),'th cycle solid bed analysis at ',baseFileName,'jpg'));
end
end

figure('name','[Overall solid bed situation at ',baseFileName]);
set(gcf, 'Units', 'Normalized', 'OuterPosition', [0 0 1 1]);
yyaxis left
plot(t,CV_space)
ylabel('CV-space for solid bed change within the vessel from cycle to cycle at %s',baseFileName);
hold on
yyaxis right
plot(t,solidFrac)
ylabel('CV-space for solid fraction change within the vessel from cycle to cycle at %s',baseFileName);
ylim([0 1])
grid on
grid minor
hold off
title('Overall solid bed analysis at %s',baseFileName);
saveas(gcf,fullfile(fpath,'Overall solid bed analysis at ',baseFileName,'fig'));
saveas(gcf,fullfile(fpath,'Overall solid bed analysis at ',baseFileName,'jpg'));

fprintf('Plotting is completed for the trial at %s .
',baseFileName);
close all;
toc;
end
fprintf('The solid case %d with weight load case %d is completed.
',caseSolid, weightLoad);
toc;
end
Appendix C

Arduino Program Code
The Arduino programs are the major controller of the experimental apparatus. The programs have mainly two functions: a function that tests and calibrates the pressure transducers and a function that initiates and controls the piston motion during mixing trials. First, let us take a look at the Arduino program for pressure transducer calibration:

Purpose: record the pressure transducer output in column over the time span of the calibration trials

Input: toggle switch signal and real time

Output: three columns of data; from left to right, time, upstream pressure (measured in mV), downstream pressure (measured in mV)

Algorithm brief explanation:

Once the toggle switch was on, the program initiates a loop to first output the time then reads the pressure transducers’ readings from the Arduino board. As long as the toggle switch remains ON, the loop continues. Note that the sampling rate is set to be about every 2 milliseconds. (See the complete Arduino program starting next page)
// Global Variables
#define sensorP1 A2 //sensor at bottom
#define sensorP2 A3 //sensor at top
int SwitchLimit = 1022;
double Ptop;
double Pbot;
long StartTime, DataLoggerTime;

void setup()
{
    // put your setup code here, to run once:
    Serial.begin(115200);
    StartTime = millis();
}

void loop()
{
    // put your main code here, to run repeatedly:
    int SwitchValue = analogRead(A0);
    if (SwitchValue > SwitchLimit)
    {
        double Ptop=analogRead(sensorP2);
        double Pbot=analogRead(sensorP1);
        DataLoggerTime = millis() - StartTime;
        Serial.print(DataLoggerTime);
        Serial.print("t");
        Serial.print(Ptop);
        Serial.print("t");
        Serial.print(Pbot);
        Serial.println();
    }
}
Function: time based relay control program

Purpose: alternates the double solenoid valve directions via switching the relay ON and OFF over a specified period of time

Input: toggle switch signal, real time, and time differences specified by the user

Output: four columns of data; from left to right, time, signal sent to the relay, (1 means ON, 0 means OFF), upstream pressure (measured in mV), and downstream pressure (measured in mV)

Algorithm brief explanation:

Once the toggle switch was on, the program initiates a loop to first output the time, then checks whether the time difference for the relay to remain ON is reached when the system was first turned on by the user;

If the time difference has not exceeded the allowed value, maintain the current output signal to the relay as it is. If the time difference has been exceeded, the output signal to the relay is changed. This would result in a direction switch of the valve spool which will lead to piston braking and eventually reserving its course of motion. The program reads the pressure transducers’ readings from the Arduino board all the time. As long as the toggle switch remains ON, this loop continues. Note that the sampling rate is set to be about every 2 milliseconds. (See the complete Arduino program starting next page)
// Software used for the Piston Actions
// this program is designed to run the piston within a specified period of time
#include <SD.h>
#include <Wire.h>
#include <RTClib.h>
#include <SPI.h>

// Global Variables
RTC_DS1307 RTC;
#define sensorP1 A2 // pressure at piston bottom
#define sensorP2 A3 // pressure at piston top
#define upTime 1600
#define downTime 1600

int SwitchLimit = 1022;
long StartTime, starttime, endtime, DataLoggerTime;
double Ptop;
double Pbot;

// Main Program Setup
void setup()
{
    Serial.begin(115200);
pinMode(2, OUTPUT);
analogWrite(2, 255);
RTC.begin();
StartTime = millis();
}

// Arduino loop
void loop()
{
    int SwitchValue = analogRead(A0);
    if (SwitchValue > SwitchLimit)
    {
        // Print on Screen
        // Up time (in ms), t1
        starttime = millis();
        endtime = starttime;
        while ((endtime - starttime) <= upTime)
        {
            analogWrite(2, 0);
Ptop = analogRead(sensorP2); // note that these are absolute pressure
Pbot = analogRead(sensorP1);
DataLoggerTime = millis()-StartTime;
Serial.print(DataLoggerTime);
Serial.print("\t");
Serial.print(digitalRead(2));
Serial.print("\t");
Serial.print(Ptop);
Serial.print("\t");
Serial.print(Pbot);
Serial.println();
endtime = millis();
        }
        // Down time (in ms), t2
starttime = millis();
endtime = starttime;
while ((endtime - starttime) <= downTime)
{
    analogWrite(2, 255);
    Ptop = analogRead(sensorP2); // note that these are absolute pressure
    Pbot = analogRead(sensorP1);
    DataLoggerTime = millis() - StartTime;
    Serial.print(DataLoggerTime);
    Serial.print("\t");
    Serial.print(digitalRead(2));
    Serial.print("\t");
    Serial.print(Ptop);
    Serial.print("\t");
    Serial.print(Pbot);
    Serial.println();
    endtime = millis();
}
}
Appendix D

Calibration & Experimental Apparatus Specification Sheets
Pressure transducer calibration

The following table stores all the data collected during the pressure transducer calibration trials, followed by plots which showed that the readings are indeed linearly and directly proportional to the gage pressure that is fed to the transducers as defined by the manufacturer. However, the initial readings of the transducers when placed at ambient lab condition with $P_{\text{atm}} \approx 1\text{ atm}$, tends to drift as time goes on. Thus, in order to avoid these initial drifts, the initial values are subtracted from subsequent transducer readings. The following tables showcase the raw data and the adjusted data after subtracting the initial readings. Later, the adjusted readings are plotted against the gage pressure levels. Both curves for each transducer are almost identical with the same slope and negligible y-intercept.

From the figure D-1, the slope is 0.1262 for converting the transducer readings to gage based pressure readings measured in psig.

<table>
<thead>
<tr>
<th>Absolute pressure (psi)</th>
<th>readings 1</th>
<th>readings 2</th>
<th>gage pressure (psig)</th>
<th>readings 1</th>
<th>readings 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.7</td>
<td>117</td>
<td>117</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>24.7</td>
<td>203</td>
<td>203</td>
<td>5</td>
<td>86</td>
<td>86</td>
</tr>
<tr>
<td>29.7</td>
<td>239</td>
<td>239</td>
<td>10</td>
<td>122</td>
<td>122</td>
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<tr>
<td>34.7</td>
<td>272</td>
<td>272</td>
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<td>156</td>
<td>156</td>
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<tr>
<td>39.7</td>
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<td>20</td>
<td>198</td>
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<tr>
<td>44.7</td>
<td>350</td>
<td>350</td>
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<td>233</td>
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<td>49.7</td>
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<td>30</td>
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<td>54.7</td>
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<td>59.7</td>
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<tr>
<td>64.7</td>
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<td>69.7</td>
<td>548</td>
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<td>50</td>
<td>432</td>
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<td>74.7</td>
<td>591</td>
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<td>475</td>
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<td>79.7</td>
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<td>510</td>
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<tr>
<td>84.7</td>
<td>672</td>
<td>672</td>
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<td>555</td>
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<tr>
<td>89.7</td>
<td>713</td>
<td>713</td>
<td>70</td>
<td>596</td>
<td>596</td>
</tr>
<tr>
<td>94.7</td>
<td>757</td>
<td>757</td>
<td>75</td>
<td>640</td>
<td>640</td>
</tr>
</tbody>
</table>
Figure D-1 Calibration curve for pressure transducer 1 & 2 as both are the same

\[ y = 0.1262x + 0.0201 \]

\[ R^2 = 0.9996 \]
Experimental apparatus specifications

Bimba FLAT-II®

Flat-II® non-rotating, double-acting cylinder provides the answer to applications where rotation cannot be tolerated and space is at a minimum. Non-rotation is achieved with dual piston rods and a rod end block that insures the rods work in tandem. Flat-II® eliminates the need for external alignment devices, such as guides, rods and alignment posts or pins.

- Body — 304 Stainless Steel
- Heads — Anodized Aluminum Alloy
- Piston Rod — Ground and Polished 303 Stainless Steel
- Piston Seals — Buna N (High temperature seals optional)
- Rod Bushing — Oil Impregnated Bronze
- Rod Seals — Buna N O-ring (High temperature seals optional)
- Rod End Block — Anodized Aluminum Alloy
- Pressure Rating — 200 PSI Maximum (Air only)
- Temperature Rating — From -20°F to +150°F (-25°C to +65°C)
- Buna N seals with a temperature range of -20°F to +130°F (-25°C to +65°C) are standard in all Bimba air cylinders. Fluorocelastomer seals rated for higher temperature applications are available. If cylinders are operated below 0°F (-18°C) for extended time periods, special modifications may be required. Special seal materials are available upon request.

How to Order

The Model Number for all Flat-II® cylinders consists of three alphanumeric clusters. These designate type, bore size and stroke length, and mounting and special options. Please refer to the charts below for an example of Model Number FT-040.375-1CE. This is a non-rotating, double-acting, 3/4” bore, 3/8” stroke, pivot mount cylinder with counterbored mounting holes in the rod end block.

<table>
<thead>
<tr>
<th>TYPE</th>
<th>BORE SIZE</th>
<th>STROKE LENGTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>FT - Non-rotating, Double Acting</td>
<td>04 - 3/4&quot;</td>
<td>0.125 - 1/8&quot;</td>
</tr>
<tr>
<td></td>
<td>09 - 1.1/16&quot;</td>
<td>0.25 - 1/4&quot;</td>
</tr>
<tr>
<td></td>
<td>17 - 1.1/8&quot;</td>
<td>0.375 - 3/8&quot;</td>
</tr>
<tr>
<td></td>
<td>31 - 2&quot;</td>
<td>ETC.</td>
</tr>
</tbody>
</table>

**FT-040.375-1 CE**

**MOUNTING OPTIONS**

(Enter in numeric order)

<table>
<thead>
<tr>
<th>No Number - Basic model (Standard counterbored mounting holes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - Pivot mount</td>
</tr>
<tr>
<td>1N - Pivot mount 90° from standard</td>
</tr>
<tr>
<td>2F - Front Flange mount, both ends</td>
</tr>
<tr>
<td>2F - Rear Flange mount, both ends</td>
</tr>
<tr>
<td>3F - Threaded mounting holes, both ends</td>
</tr>
<tr>
<td>3F - Threaded mounting holes, front</td>
</tr>
<tr>
<td>4F - Screw clearance holes, front</td>
</tr>
<tr>
<td>4F - Screw clearance holes, rear</td>
</tr>
</tbody>
</table>

* "Screw clearance" to allow bolt head to pass through; no counter bores (see page 2.21).

**OPTIONS**

(Enter in alphabetical order, except for EE which is last)

<table>
<thead>
<tr>
<th>O - Counter bored rod end block (see page 2.22)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G - Magnitude*3</td>
</tr>
<tr>
<td>K - Endblock rotated 90 degrees</td>
</tr>
<tr>
<td>M, M1, M2, M3 - Magnetic position sensing (see table page 2.9&quot;, 2.10, and Switch Products Section)</td>
</tr>
<tr>
<td>P3 - Front port position (see page 2.21)</td>
</tr>
<tr>
<td>Q - Low temperature option (-40°F to 200°F)</td>
</tr>
<tr>
<td>S - Stainless steel fasteners (125 PSI maximum pressure rating - air only)</td>
</tr>
<tr>
<td>T1, T3, T4 - Additional switch mounting post</td>
</tr>
<tr>
<td>Y - High temperature option (6°F to 400°F)</td>
</tr>
<tr>
<td>M - Moly-coat (MoS, I.D. coating)</td>
</tr>
<tr>
<td>EE237 - 1/8&quot; extra rod extension, etc.</td>
</tr>
<tr>
<td>EE1 - 1&quot; extra rod extension, etc.</td>
</tr>
</tbody>
</table>
* If magnetic position sensing is specified with option V, standard Buna-N based magnet will be provided. Magnetic position sensing is not reliable above 200°F. Overall cylinder length increases with the magnet option.
### Price List

<table>
<thead>
<tr>
<th>Basic Model</th>
<th>Base Price by Bore Size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3/4&quot;</td>
</tr>
<tr>
<td>Base Model</td>
<td>$65.60</td>
</tr>
<tr>
<td>Adder per 1/8&quot; of stroke</td>
<td>0.50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mounting Options</th>
<th>Price Adders by Bore Size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3/4&quot;</td>
</tr>
<tr>
<td>Pivot Mount (Options 1, 1N)</td>
<td>$12.35</td>
</tr>
<tr>
<td>Trunnion Mount (Options 2, 2F, 2R)</td>
<td>10.30 per end</td>
</tr>
<tr>
<td>Threaded Mounting Holes (Options 3, 3F, 3R)</td>
<td>3.35 per end</td>
</tr>
<tr>
<td>Screw Clearance Holes (Options 4, 4F, 4R)</td>
<td>2.75 per end</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Options</th>
<th>Price Adders by Bore Size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3/4&quot;</td>
</tr>
<tr>
<td>EE (each 1/2&quot; each end)</td>
<td>$0.90</td>
</tr>
<tr>
<td>Magnetic Position Sensing (Options M, M1, M3, M4)</td>
<td>7.05</td>
</tr>
<tr>
<td>S</td>
<td>2.30</td>
</tr>
<tr>
<td>Switch Mounting Post (Options T1, T3, T4)</td>
<td>2.50</td>
</tr>
<tr>
<td>High Temperature Seals (Option V)</td>
<td>13.25</td>
</tr>
<tr>
<td>Y (Adder per 1/8&quot; of stroke)</td>
<td>0.55</td>
</tr>
<tr>
<td>Q (low temp seals)</td>
<td>12.10</td>
</tr>
</tbody>
</table>


### Repair Kits

<table>
<thead>
<tr>
<th>Basic Repair Kit (K-B-FT-...)</th>
<th>Part No.</th>
<th>Description</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>PF-29</td>
<td>Rod Seal</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>PF-30</td>
<td>Piston Seal</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>PF-31</td>
<td>Tube Seal</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>PF-31</td>
<td>Bushing</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

*Must specify bore size when ordered. Contact your local BIMBA Distributor for pricing on kits and other repair parts.

### Weights

<table>
<thead>
<tr>
<th>Bore</th>
<th>Approximate Cylinder Weights (oz.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Base</td>
</tr>
<tr>
<td>3/4&quot; (04)</td>
<td>2.7</td>
</tr>
<tr>
<td>1-1/16&quot; (09)</td>
<td>6.4</td>
</tr>
<tr>
<td>1-1/2&quot; (17)</td>
<td>12.2</td>
</tr>
<tr>
<td>2&quot; (31)</td>
<td>18.4</td>
</tr>
</tbody>
</table>
Bimba FLAT-II®

Bimba is a JIT manufacturer and we are able to provide FT model cylinders in ANY 0.001" stroke length increment for all option styles within our standard three-day lead time. Longer stroke lengths are also available upon request at standard lead times. Please consult Technical Assistance at 800-44-BIMBA for help.

Basic Model

Model FT
(Nonrotating, double acting)

The table below represents our standard stroke lengths. Blue stroke lengths are BASIC FT cylinders in stock available for Same Day Shipping.

<table>
<thead>
<tr>
<th>Nominal Bore Diameter</th>
<th>Bore Code</th>
<th>Standard Stroke Length Availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/4&quot;</td>
<td>64</td>
<td>1&quot; 1-1/4&quot; 1-1/2&quot; 1-3/4&quot; 2&quot; 2-1/2&quot; 3&quot; 3-1/2&quot; 4&quot;</td>
</tr>
<tr>
<td>1.1/16&quot;</td>
<td>00</td>
<td>1&quot; 1-1/4&quot; 1-1/2&quot; 1-3/4&quot; 2&quot; 2-1/2&quot; 3&quot; 3-1/2&quot; 4&quot;</td>
</tr>
<tr>
<td>1-1/2&quot;</td>
<td>1&quot;</td>
<td>1&quot; 1-1/4&quot; 1-1/2&quot; 1-3/4&quot; 2&quot; 2-1/2&quot; 3&quot; 3-1/2&quot; 4&quot;</td>
</tr>
<tr>
<td>2&quot;</td>
<td>31</td>
<td>1&quot; 1-1/4&quot; 1-1/2&quot; 1-3/4&quot; 2&quot; 2-1/2&quot; 3&quot; 3-1/2&quot; 4&quot;</td>
</tr>
</tbody>
</table>

Mounting Options

Trunnion Mount
(rear, front or both) (-2R shown)

Pivot Mount
(complete with bronze bushing)
(-1 shown)

Threaded Mounting Holes
(available either or both ends)
(-3R shown)

Screw Clearance Holes
(available either or both ends)
(-4R shown)
**Bimba FLAT-II®**

**Dimensions (in)**

**Counterbored Rod End Block**

![Diagram of counterbored rod end block]

<table>
<thead>
<tr>
<th>Bore</th>
<th>A</th>
<th>B*</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>EC</th>
<th>F</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/4&quot; (04)</td>
<td>1.50</td>
<td>0.94</td>
<td>1.22</td>
<td>#6</td>
<td>#6-32 UNC</td>
<td>#6</td>
<td>0.19</td>
<td>0.332</td>
</tr>
<tr>
<td>1-1/16&quot; (09)</td>
<td>2.00</td>
<td>1.31</td>
<td>1.69</td>
<td>#6</td>
<td>#8-32 UNC</td>
<td>#8</td>
<td>0.25</td>
<td>0.422</td>
</tr>
<tr>
<td>1-1/2&quot; (17)</td>
<td>2.63</td>
<td>1.31</td>
<td>2.19</td>
<td>#10</td>
<td>1/4-20 UNC</td>
<td>1/4</td>
<td>0.38</td>
<td>0.562</td>
</tr>
<tr>
<td>2&quot; (31)</td>
<td>3.13</td>
<td>1.38</td>
<td>2.69</td>
<td>#10</td>
<td>5/16-18 UNC</td>
<td>5/16</td>
<td>0.50</td>
<td>0.750</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bore</th>
<th>J</th>
<th>JJ</th>
<th>K</th>
<th>KK</th>
<th>L</th>
<th>M</th>
<th>N</th>
<th>P</th>
<th>Q</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/4&quot; (04)</td>
<td>0.34</td>
<td>0.47</td>
<td>0.14</td>
<td>0.27</td>
<td>#10-32</td>
<td>0.31</td>
<td>0.13</td>
<td>0.17</td>
<td>0.19</td>
<td>#6-32 UNC</td>
</tr>
<tr>
<td>1-1/16&quot; (09)</td>
<td>0.50</td>
<td>0.69</td>
<td>0.25</td>
<td>0.44</td>
<td>1/8 NPT</td>
<td>0.50</td>
<td>0.25</td>
<td>0.25</td>
<td>0.19</td>
<td>#6-32 UNC</td>
</tr>
<tr>
<td>1-1/2&quot; (17)</td>
<td>0.50</td>
<td>0.69</td>
<td>0.25</td>
<td>0.44</td>
<td>1/8 NPT</td>
<td>0.50</td>
<td>0.25</td>
<td>0.25</td>
<td>0.38</td>
<td>#10-24 UNC</td>
</tr>
<tr>
<td>2&quot; (31)</td>
<td>0.53</td>
<td>0.72</td>
<td>0.25</td>
<td>0.44</td>
<td>1/8 NPT</td>
<td>0.50</td>
<td>0.25</td>
<td>0.25</td>
<td>0.38</td>
<td>#10-24 UNC</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bore</th>
<th>S</th>
<th>T</th>
<th>U</th>
<th>V</th>
<th>VL</th>
<th>VH</th>
<th>W</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/4&quot; (04)</td>
<td>0.38</td>
<td>0.19</td>
<td>0.25</td>
<td>0.38</td>
<td>1.25</td>
<td>0.88</td>
<td>0.75</td>
<td>0.23</td>
<td>0.75</td>
<td>0.19</td>
</tr>
<tr>
<td>1-1/16&quot; (09)</td>
<td>0.38</td>
<td>0.25</td>
<td>0.25</td>
<td>0.38</td>
<td>1.44</td>
<td>1.06</td>
<td>0.81</td>
<td>0.25</td>
<td>0.75</td>
<td>0.19</td>
</tr>
<tr>
<td>1-1/2&quot; (17)</td>
<td>0.75</td>
<td>0.25</td>
<td>0.44</td>
<td>0.50</td>
<td>2.00</td>
<td>1.50</td>
<td>1.19</td>
<td>0.34</td>
<td>1.38</td>
<td>0.38</td>
</tr>
<tr>
<td>2&quot; (31)</td>
<td>0.75</td>
<td>0.31</td>
<td>0.44</td>
<td>0.63</td>
<td>2.50</td>
<td>1.88</td>
<td>1.25</td>
<td>0.34</td>
<td>1.38</td>
<td>0.38</td>
</tr>
</tbody>
</table>

A minimum stroke of 0.36" is required to sense extending end-of-stroke position.
Nonrotation is achieved through the use of dual piston rods incorporated into the body of the Flat-II® cylinder. The rods are securely attached to the piston by our unique spin-riveting process. A rod end block is used to insure the rods work in tandem—as a team. This end block also acts as a useful surface to easily accommodate any mounting attachments required to get the job done. For mounting convenience, the rod end block is provided with threaded mounting holes or optional counterbored holes.

As with any cylinder application, side loading should be avoided. The two smaller rods will have more deflection due to side load than the one standard rod in a comparable Flat-1® model.

The Flat-II® is intended to work satisfactorily against pure torsional loads. The maximum torsional load per bore size is shown in the following table:

<table>
<thead>
<tr>
<th>Bore</th>
<th>3/4&quot; (04)</th>
<th>1-1/16&quot; (09)</th>
<th>1-1/2&quot; (17)</th>
<th>2&quot; (31)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torque (in-lb)</td>
<td>0.3</td>
<td>1</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Κ</td>
<td>5.21</td>
<td>26.61</td>
<td>238.85</td>
<td>1344.63</td>
</tr>
</tbody>
</table>

The amount of angular deflection, in degrees, can be approximated by the following formula:

\[ θ = \frac{TL^2}{K} \]

Where

- \( T \) = Torque (in-lb)
- \( L \) = Length (see sketch below)
- \( K \) = Per chart above
- \( θ \) = Angular deflection

*Note: To prevent rod distortion, the rod end block must be fastened securely.*

### Rotational Tolerance

<table>
<thead>
<tr>
<th>Bore</th>
<th>Maximum Rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/4&quot; (04)</td>
<td>±1°</td>
</tr>
<tr>
<td>1-1/16&quot; (09)</td>
<td>±3/4&quot;</td>
</tr>
<tr>
<td>1-1/2&quot; (17)</td>
<td>±1/2&quot;</td>
</tr>
<tr>
<td>2&quot; (31)</td>
<td>±1/2&quot;</td>
</tr>
</tbody>
</table>

### Deflection L Value

[Diagram of deflection measurement]
The following sections are the specifications of the relay unit part of the Arduino controller for achieving the double acting piston. The relay sets the on-off time for energizing and de-energizing the double solenoid valve.

**SONGLE RELAY**

<table>
<thead>
<tr>
<th>RELAY ISO9002</th>
<th>SRD</th>
</tr>
</thead>
</table>

**1. MAIN FEATURES**
- Switching capacity available by 10A in spite of small size design for high density P.C. board mounting technique.
- UL, CUL, TUV recognized.
- Selection of plastic material for high temperature and better chemical solution performance.
- Sealed types available.
- Simple relay magnetic circuit to meet low cost of mass production.

**2. APPLICATIONS**
- Domestic appliance, office machine, audio, equipment, automobile, etc.
- Remote control TV receiver, monitor display, audio equipment high rushing current use application.

**3. ORDERING INFORMATION**

<table>
<thead>
<tr>
<th>SRD</th>
<th>Model of relay</th>
<th>Nominal coil voltage</th>
<th>Structure</th>
<th>S</th>
<th>L</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>03, 05, 06, 09, 12, 24, 48VDC</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**4. RATING**

<table>
<thead>
<tr>
<th>CCC</th>
<th>FILE NUMBER:CH0052885-2000</th>
<th>7A/240VDC</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCC</td>
<td>FILE NUMBER:CH0033745-99</td>
<td>10A/280VDC</td>
</tr>
<tr>
<td>UL/CUL</td>
<td>FILE NUMBER: E197596</td>
<td>10A/125VAC 28VDC</td>
</tr>
<tr>
<td>TUV</td>
<td>FILE NUMBER: R993379</td>
<td>10A/240VAC 28VDC</td>
</tr>
</tbody>
</table>

**5. DIMENSION (Unit:mm) & DRILLING (Unit:mm) & WIRING DIAGRAM**

![Diagram](image.png)
### 6. COIL DATA CHART (AT20°C)

<table>
<thead>
<tr>
<th>Coil Sensitivity</th>
<th>Coil Voltage Code</th>
<th>Nominal Voltage (VDC)</th>
<th>Nominal Current (mA)</th>
<th>Coil Resistance (Ω) ±10%</th>
<th>Power Consumption (W)</th>
<th>Pull-In Voltage (VDC)</th>
<th>Drop-Out Voltage (VDC)</th>
<th>Max-Allowable Voltage (VDC)</th>
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<tr>
<td>SRD (High Sensitivity)</td>
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<td>03</td>
<td>120</td>
<td>25</td>
<td>abt. 0.36W</td>
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<td>02</td>
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<td>71.4</td>
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<tr>
<td></td>
<td>06</td>
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<tr>
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### 7. CONTACT RATING

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<td>10A 28VDC</td>
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<tr>
<td>Resistive Load (cosφ=1)</td>
<td>10A 125VAC</td>
<td>10A 125VAC</td>
</tr>
<tr>
<td>Inductive Load (cosφ=0.4 L/R=7msec)</td>
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<td>5A 120VAC</td>
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<td>Max. Allowable Voltage</td>
<td>250VAC/110VDC</td>
<td>250VAC/110VDC</td>
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<tr>
<td>Max. Allowable Power Force</td>
<td>800VAC/240W</td>
<td>1200VDC/300W</td>
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<td>Contact Material</td>
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### 8. PERFORMANCE (at initial value)

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<tr>
<td>Release Time</td>
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<td>Dielectric Strength Between coil &amp; contact</td>
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<tr>
<td>Between contacts</td>
<td>1000VAC 50/60HZ (1 minute)</td>
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<tr>
<td>Insulation Resistance</td>
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<td>Max. ON/OFF Switching</td>
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<tr>
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<tr>
<td>Electrically</td>
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<td>Operating Humidity</td>
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<td>Vibration</td>
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<tr>
<td>Endurance</td>
<td>10 to 55Hz Double Amplitude 1.5mm</td>
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</tr>
<tr>
<td>Shock</td>
<td>100G Min.</td>
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<tr>
<td>Endurance</td>
<td>10G Min.</td>
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</tr>
<tr>
<td>Life Expectancy</td>
<td>10⁷ operations. Min. (no load)</td>
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<tr>
<td>Mechanically</td>
<td>10⁶ operations. Min. (at rated coil voltage)</td>
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</tr>
<tr>
<td>Electrically</td>
<td>10⁷ operations. Min. (at rated coil voltage)</td>
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<tr>
<td>Weight</td>
<td>abt. 10grs.</td>
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</table>
Appendix E

Air Leakage Model
This section is then devoted to discuss the possible leakages that may occur in the piston along with a simple model to check for leakages and adjust the calculations if leakages did occur.

According to the literature which uses a very similar pneumatic piston to the one used in this study, there are two possible locations where leakages may have been occurred: either near the piston double-rod and the top plate or between the piston moving block and the cylinder wall [Richer et al, 1999]. In the former case, a pressurized upper chamber causes air leaks to the atmosphere. In the latter case, air leaks from the relatively higher pressured chamber to the lower pressured chamber. While gas leakages including fugitive leakages around the valves and soft tube connections may still occur on a more regular basis, the above two leakages happen primarily when JBR system is in operations. Therefore, the focus of modelling leakages will be on the above two locations.

The model will evaluate leakages generally under two conditions: during the initialization stage when air is filling the upper chamber but not the other, and thus, maintaining piston at its bottom locations; or during JBR operations when piston undergoes continuous up and down motion.

As mentioned, for the initialization stage, theoretically, one piston chamber undergoes filling up, after a few seconds, the pressure within the chamber reaches the same level as the feed pressure if there were no leakage. But, when leakages happened, air pressure within the chamber will reach a steady state as the amount of air leaks out also reaches a steady state.

In this case, both air inflow and leakages can be computed using the compressible air flow through orifices. For instance, if at t = 0, the upper chamber is being filled up; the resultant $P_{L0}$ and $P_{U0}$ can be calculated as:

\[
\begin{align*}
\{ G(P_{in}, P_{U0}, A_U) &= G(P_{U0}, P_a, A_{leak0}) + G(P_{U0}, P_{L0}, A_{leak1}) \\
G(P_{U0}, P_{L0}, A_{leak1}) &= G(P_{L0}, P_a, A_L) \}
\] (E-1)
In the above equation, \( G = f(P_u, P_d, A) \) means that the mass flow of air is a function of upstream and downstream pressure as well as the cross sectional area of the flow passage. Also, \( P_{in} \) is the air pressure of the feed; \( P_{L0} \) is the initial air pressure within the lower piston chamber; \( P_{U0} \) is the initial air pressure within the upper piston chamber. \( P_a \) is the atmospheric pressure at ambient lab condition; \( A_L \) is the cross sectional area of the lower piston chamber. \( A_U \) is the cross sectional area of the upper piston chamber. \( A_{\text{leak0}} \) is the leakage area between the piston rods and the top plate; whereas, \( A_{\text{leak1}} \) is the leakage area between the moving piston block and the cylinder wall. Of the two functions, \( P_{L0} \) and \( P_{U0} \) are the two unknowns to be solved.

Similarly, when air fills up the lower chamber at initial stage, the steady state initial chamber pressure can be found using the following group of functions:

\[
\begin{align*}
G(P_{in}, P_{L0}, A_L) &= G(P_{L0}, P_{U0}, A_{\text{leak1}}) \\
G(P_{L0}, P_{U0}, A_{\text{leak1}}) &= G(P_{U0}, P_a, A_U) + G(P_{U0}, P_a, A_{\text{leak0}})
\end{align*}
\]  
(E-2)

When piston is in motion, the situation becomes more complex as the real time chamber pressure as well as the flow enters and exits a given chamber changes at every instant. Therefore, the mass balance can still be achieved in a similar manner:

\[
\begin{align*}
G_{\text{inTotal}} &= G_{\text{in}} + G_{\text{leak}} \\
G_{\text{outTotal}} &= G_{\text{out}} + G_{\text{leak}}
\end{align*}
\]  
(E-3)
Appendix F

Methods for Searching the Best Fitting Parameters
Preliminary calculation showed that the residual and sum of least squares have a monotonic relationship with the changing parameter estimates. Therefore, whenever the residual terms are minimized, the best fitting parameter pair can be obtained in a very rapid way when the search is done through MATLAB’s built-in bisecting method over the specified range. Overall, this technique is implemented for searching the best fitting parameters of either group. To illustrate this method better, the following example is provided.

Assume that we have two parameters: “A” and “B” grouped together which corresponds to their relevant experiment “a” and “b”. The objective is to find the best fitting parameter A in a very timely manner. This can be achieved in the following way:

1. If A has a specified range, the sub-interval through dividing the range into four parts will have five points, say A1 ~ A5
2. Find the model predicted values using A1~ A5 and compare the residuals between the model predictions and the actual experiment outcome, denoted as Res1~Res5.
3. If Res1 is the least among the five, the next iteration will have the range of A reset between A1 and A2. Subsequently, Res1 = Res1 from the previous iteration and Res5 = Res2 from the previous iteration. In this case, only 3 points and their corresponding residuals are required to be calculated.
4. Else if Res2 is the smallest among the residuals from the first iteration, the next iteration will be performed between A1 and A3. As a result, Res1 = Res1 from the previous iteration, Res3 = Res2 from the previous iteration, and Res5 = Res3 from the previous iteration. In this case, only point# 2 and 4 are required to be found along with their residuals.
5. Else if Res3 is the least, the next iteration will be performed between A2 and A4. Therefore, Res1 = Res2 from the previous iteration, Res3 = Res3 from the previous iteration, Res5 = Res4 from the previous iteration. Once again, only point# 2 and 4 are required to be found along with their residuals.
6. **Else if** Res4 is the smallest, the range for next iteration lies between A3 and A5. In addition, Res1 = Res3 from the previous iteration, Res3 = Res4 from the previous iteration, Res5 = Res5 from the previous iteration. Once again, only point# 2 and 4 are required to be found along with their residuals.

7. **Else if** Res5 is the smallest, the range for next iteration becomes A4 to A5; nonetheless, Res1 = Res4 from the previous iteration, Res5 = Res5 from the previous iteration. In this case, point # 2,3,4 in between A4 and A5 will be calculated along with their residuals.

8. This loop then continues to further shrink the range of iterations for A until a specified number of iterations are reached.

9. In the last iteration, the estimate gives the least residuals will be selected as the best fitting value for the parameter A.

Note that even though parameter A and B are relatively independent according to physics, both participate in the iterations. It is in fact impossible to separate them. To avoid redundancy and save time, this gives the motive to have a nested looping system when finding the best fitting parameters in pairs:

1. The range to find A is equally divided into four parts with end points labelled as A1~ A5.
2. The internal loop first finds the best matches for A1 ~ A5 using the previous method, noted as B1 ~ B5 using experiment set b. Each pair is defined as (A1, B1), (A2, B2), (A3, B3), etc.
3. The external loop, with the help of data from experiment set a, finds the pair among the group of pairs generated in step 2 that produces the least residuals. Moreover, the range of A is shortened and updated around the best pair. As explained in the previous paragraph.
4. Once the designated numbers of iterations are completed, the pair with the least residuals is selected as the final resultant best fitting parameter set.

The efficiency compared to the global searching method is absolutely phenomenal. As one can estimate, for the worst case scenario, to accurately shrink the range of predictions
of a parameter to 1/100 of its initial range, one needs to find about 5 + 6*2.6 ≈ 21 residuals. This is just a little bit more than 1/5 of the number of iterations done using global searching. To shrink the range of predictions of a pair of parameters, it requires the model to calculate 20.6 * 21.6 ≈ 450 residuals. This is merely a quarter of the work required compared to the work performed by using global searching technique. Even though, the drawback from this method is the requirement to perform three additional sets of experiments, which increased the physical work load, the overall time saved is far greater than the amount of time devoted for those experiments. Therefore, this partition method coupled with nested loop algorithm is a desirable tool for performing best fitting of the mode parameters.
Appendix G

CV’s and experimental results table
<table>
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<tr>
<th>Cases</th>
<th>Set Frequency (Hz)</th>
<th>Weight Load (kg)</th>
<th>Max Upward Acceleration (m/s²)</th>
<th>Max Downward Acceleration (m/s²)</th>
<th>Amplitude (m)</th>
<th>CV(_{\text{spatial}})</th>
<th>CV(_{\text{lateral}})</th>
<th>CV(_{\text{vertical}})</th>
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<tbody>
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<td>7.70</td>
<td>2.569</td>
<td>53.7</td>
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<td>0.056</td>
<td>0.417</td>
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In the meantime, to be conservative, if the maximum CV vertical, which implies a less well-mixed scenario, is plotted against predicted average cycle amplitude in a similar manner, this deviation in acceleration seems to be intensified causing predicted amplitude significantly different from the experimental values.

![Graph](image_url)

**Figure G-1 Max CV vertical vs. experimental and predicted average cycle amplitude**

As figure G-1 illustrated, shown above, some CV vertical reached below 0.3 when the predicted amplitude is larger than 0.033 m; however, a considerable number of trial CV vertical did not become less than one despite the predicted amplitude well over 0.033 m.

One may the state that under the same conditions, the model tends to provide slightly smaller accelerations in either direction which leads to smaller amplitude compared to its experimental counterparts. Note that each experimental trial is repeated three times, but model only compute and give results once under the same condition. This difference can be seen in figure G-2. Detailed analysis of model validation mentioned in section 1.1.
Figure G-2 comparing the model-predicted and experimental average cycle amplitude as well as minimum of maximum piston half cycle accelerations
Curriculum Vitae

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University of Western Ontario
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2015-2016

Publications: