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COMMUNICATION IN MATHEMATICS:
A CASE FOR CONCPTUAL QUESTIONING IN ONTARIO MIDDLE SCHOOLS

(Spine Title: Learning Study: Conceptual Questioning in Mathematics)

(Thesis Format: Monograph)

By

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Graduate Program in Education

Submitted in partial fulfillment
Of the requirements for the degree of
Master of Education

School of Graduate and Postdoctoral Studies

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Abstract

In mathematics education, communication is one of the foundational cornerstones. The Ontario Ministry of Education and Training has placed a significant emphasis on communication in the mathematics classroom. In *The Ontario Curriculum, Grades 1-8: Mathematics Revised* (2004), communication is highlighted as one of the seven mathematical processes as well as one of the four categories for assessment on the achievement chart. Communication takes many forms in the mathematics classroom including questioning, written response, and discourse. Despite all of the emphasis on communication, the employment of quality communication in mathematics is somewhat elusive. One method for eliciting quality communication in mathematics is through conceptual questioning. The question then arises, can a generalist intermediate teacher create and implement conceptual questions, and assess student responses in terms of conceptual understanding and communication. Using the action research format known as learning study I investigate two teachers. Learning study provided the opportunity for the teachers to examine the creation and application of conceptual questioning through the development and implementation of the rate and ratio unit. The data collected, some of which contains a narrative format, was viewed through a variation theory and phenomenography lens. The data revealed that within certain specific conditions, the creation and employment of conceptual questions can be accomplished by generalist teachers, albeit to varying degrees of success.

Key Words: Communication, complexity theory, conceptual questioning, learning study, mathematics education, mathematical thinking, phenomenography, problem solving, variation theory

Dedication

To my Dad and Mom, John, Stephen, and Mark in appreciation of their love and support.

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I would like to acknowledge, with gratitude, the encouragement and support given to me by Dr. Immaculate K. Namukasa.

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1.1 Mathematics Education in Ontario

1.1.1 A Brief Overview

The quest to teach mathematics effectively in Ontario has long been a professional goal of educators. All teaching college programs devote a large section of course work time to the training of future educators in the art of teaching mathematics. The Ontario Ministry of Education and Training (OMET) has also provided educators with curriculum guides specifying what content should be taught at specific grades within the five identified strands of mathematics, and combined that with descriptors of the seven mathematical processes through which the content should be taught. To this end there have been several curriculum changes made over the past twenty years through the 1985, 1997, and 2004 curricula. Most recently, there have been the revisions of most Ontario curriculum documents, such as *The Ontario Curriculum, Grades 1-8: Mathematics Revised* (OMET, 2005) which was a revision of the 1997 curriculum. This revised curriculum states that "Students must problem solve, communicate, reason, reflect, and so on [select tools, connect and represent], as they develop the knowledge, the understanding of concepts, and the skills required in all the strands [namely, number sense and numeration, measurement, geometry, patterning and algebra, and probability and data management]" (OMET, p.11). Currently, the Ontario Ministry of Education and Training has designated 300 instructional minutes to mathematics during a six day cycle (The Literacy and Numeracy Secretariat, 2009) and many school boards in Ontario have even specified that there should be 60 minutes of continuous, uninterrupted instructional time each day specifically designated to mathematics instruction and practice (Ophea, 2009). OMET has provided many online resources, although they are mainly at the primary and junior level. OMET has also paired their online resources with funding opportunities geared towards professional development to inform teachers and associated educators, such as subject consultants and specialists at the school boards, about curriculum changes and new resources. For the elementary classroom teacher, shifts in educational emphasis involve curriculum revisions and the pedagogical reforms that follow raise some huge questions: What should mathematical instruction look like? How does one effectively incorporate the seven mathematical processes in the teaching of content? What should be the major learning aims of classroom mathematics instructional time, and how should that learning be assessed and evaluated?

1.1.2 Gaps in Resources at the Intermediate Level

Many resources have been designed to help answer these questions from expert panel reports such as *Leading Math Success: Mathematical Literacy, Grades 7-12 Expert Panel* to practical classroom resources such as *Targeted Implementation and Planning Strategies for the Revised Mathematics* (TIPS4RM) to various guide books and related facilitator handbooks on primary and junior mathematics instruction, and webcasts by such mathematics educators as Dr. Deborah Loewenberg Ball and Dr. Marian Small. Although each of these resources has its own focus on content guidelines, specific strand expectations, practical classroom activities and specific instructional strategies, they also have an overarching, connecting flow. The resources in a manner aligned to the curriculum document emphasize teaching mathematics by encouraging students to solve well designed mathematics problems (what has come to be known as teaching through problem solving). In my experience as a teacher I found the resources lacking on two fronts: One, intermediate mathematics resources, Grades 7 and 8, are significantly under represented. Most Ontario Ministry of Education initiatives and reports of expert panels on mathematics learning have been "rolled out" beginning at the primary level but either do not exist or tend to lose momentum by the intermediate level. Whereas one report is available for Grades 7-12, the *Leading Math Success* (OMET, 2004); two are available for Grades 1-6, the *Early Math Strategy* (OMET, 2003) and *Teaching and Learning Mathematics* (OMET, 2004). It can be argued that the *Targeted Implementation and Planning Supports for Revised Mathematics* (TIPS4RM), (Leading Math Success, 2005) document was designed specifically for Grades 7 to 10, although a significant number of lessons have never been developed for this document including whole units of study. For example, there are major gaps in the Grade 8 number sense and numeration strand, particularly in the integer unit, as the lessons are all listed as not available.

Two, a definitive, practical outline with examples of what communication in mathematics is and how to elicit it, nonetheless is not clearly stated, at least not in one place. Communication as a mathematical process is inadequately addressed on multiple fronts by *The Ontario Curriculum, Grades 1-8: Mathematics Revised* (2005). Yet most elementary teachers in Canada are not specialists at mathematics; as generalists they do not have an extensive mathematical background to draw from. They teach at least three subjects and may not have taken mathematics past the high school level.

1.2 Communication

1.2.1 What is Mathematical Communication?

OMET (2005) defines the process of communication, as:

> the process of *expressing mathematical ideas and understanding* orally, visually, and in writing, using numbers, symbols, pictures, graphs, diagrams, and words.

> Students communicate for various purposes and for different audiences, such as the teacher, a peer, a group of students, or the whole class. Communication is an essential process in learning mathematics. Through communication, *students are able to reflect upon and clarify their ideas, their understandings of* mathematical relationships, and their mathematical arguments. (p. 17, italics added)

Teachers, among other opportunities that exist in the classroom, are encouraged to

> M*odel mathematical reasoning* by thinking aloud, . . . *encourage talk* at each stage of the problem-solving process; *ask clarifying and extending questions* and encourage students to ask themselves similar kinds of questions; ask students open-ended questions relating to specific topics or information (e.g., '*How do you know?' 'Why?' 'What if . . .?' 'What pattern do you see?' 'Is this always true?'*); . . . [and] model ways in which various kinds of questions can be answered. (p. 17)

As a process, communication also involves reasoning, problem solving, and questioning.

Communication is also one of the four key assessment areas in the Ontario curriculum document achievement charts. The achievement charts involve what has come to be referred to as the KUTCA (knowledge and understanding, thinking, communication, and application) model. Communication together with the other three categories – knowledge and understanding, thinking, and application – is identified as an assessment category. Details about subsets of communication as a mathematical process that is to be assessed are offered in the achievement chart. Samples of students' written communication on assessment tasks are compiled in a separate publication, the *Ontario Curriculum Exemplars* (OMET, 2002) to illustrate students' performance at each of the subsets of communication.

> As an achievement category communication in mathematics involves the expression and organization of mathematical thinking (e.g., clarity of expression, logical organization), using oral, visual, and written forms (e.g., pictorial, graphic, dynamic, numeric, algebraic forms, concrete materials) . . . Communication for different audiences (e.g., peers, teachers) and purposes (e.g., to present data, justify a solution, express a mathematical argument) in oral, visual, and written forms . . . Use of conventions, vocabulary, and terminology of the discipline (e.g., terms, symbols) in oral, visual, and written forms. (p. 23, Achievement Chart, OMET, 2005)

Clearly, the learning process and assessment category of communication in mathematics is approached through several forms not limited to the traditional written symbolic forms. Its teaching involves multiple dimensions including mathematical thinking, communication for different audiences, use of conventions, questioning and problem solving. The focus for this thesis is on the process of communication as a phenomenon, intertwined with the process of problem solving.

1.2.2 Communication Through Problem Solving

Communication through problem solving, which is the process of encouraging students to talk, write, draw, listen to mathematics as they work on mathematics tasks, has become a prevailing teaching methodology in mathematics education which is promoted by both the National Council of Teachers of Mathematics (NCTM), and the Ontario Ministry of Education. In Ontario, *The Ontario Curriculum, Grades 1-8: Mathematics Revised* (2005), places a strong emphasis on communication through problem solving as integral parts of the seven mathematical processes. The Ontario

Ministry of Education then, entwines the process of communication and problem solving: "The mathematical processes can be seen as the processes through which students acquire and apply mathematical knowledge and skills. These processes are interconnected. Problem solving and communication have strong links to all other processes" (*The Ontario Curriculum, Grades 1-8: Mathematics Revised*, 2005, p. 11). As evidenced by most curriculum and research developments, communication through problem solving is a broad area of mathematics teaching and learning, therefore a study of the phenomenon of communication needs to narrow its focus. I narrow this focus further to teacher questioning techniques and sensibilities. The general research question is: How can generalist teachers at the intermediate level develop and employ "conceptual questioning techniques and sensibilities"; assess student responses; and promote the communication of student conceptual understanding of the mathematical processes, contained within the five strands of the Ontario Curriculum?

As stated earlier, there are multiple facets of communication. Therefore the refining of the focus of this thesis comprised an examination of teacher questioning as it elicits oral, visual, and written communication in the mathematics classroom. *The Ontario Curriculum, Grades 1-8: Mathematics Revised* (2005) provides some general ideas as to the type of questions that can elicit communication. For instance, it encourages teachers to model mathematical thinking, stresses the correct use of subject specific vocabulary and provides generic prompts for open-ended questioning. However failure to follow this emphasis on eliciting communication is demonstrated by the focus on strands in mathematics for the elementary report card and by stating only one learning expectation for the communication process expectation for each grade as compared to several learning expectations for the strands. In the three reporting cycles, elementary teachers are required to report on each strand in mathematics at least twice. These reports are based on expectation related to the strand rather than being founded on the achievement chart categories that include communication and thinking as is done in high school. Unlike elementary school, high school mathematics teachers in Ontario base their assessment of students on the KUTCA model. Each assessment category has a specific weight for the final grade with a larger emphasis placed on communication and thinking than on knowledge and understanding, and application categories.

1.3 The Research Problem

In my experience, as a teacher and facilitator who provides professional development in mathematics in my school board, elementary teachers lack a clear understanding of the nature of communication, not to mention its relation to problem solving. As a result communication specific questions on provincial and classroom examinations that require a student to explain an answer using words, diagrams and symbols are poorly completed, as little attention is paid to this type of questioning outside of the grades 3, 6, and 9 where provincial testing is conducted. Given the lack of a definitive practical outline, the lack of emphasis on mathematical processes including assessing and reporting on communication on the report cards, and the lack of clear understanding of communication by teachers, there is a need to examine how aspects of communication such as questioning could be developed to promote communication in the mathematics classroom. This study focuses on a specific kind of questioning, identified as conceptual questioning.

From a personal perspective, my interest in communication in the mathematics classroom was originally piqued during course work for my master's degree and also based on reflection on personal teaching practices. From a teaching perspective, there was a natural wonder about communication, and specifically different types of questions and how they should be used in mathematics. From a professional development stance, much of the communication emphasis by the ministry and my school board had been placed on the use of problem solving questions, and student written responses in mathematics. The ideas surrounding the concept of conceptual questioning, however, began during course-related research when I was introduced to Heibert and Stigler (1999) and the 1995/1999 TIMSS video study which is discussed in Chapter 2.

Even in classrooms where generalist elementary teachers are attempting to be aware of helpful teaching practices, such as the nature of questions that elicit mathematical thinking, the *awareness has not been transferred to action* (M.W. Mitchell, personal communication, April 18, 2007). Also the instructional methods in the resources have not been transferred from the resources and the curriculum documents to classroom instruction. Worse still, few resources have been made available for teachers specifically at the intermediate level in terms of how to implement and assess communication. Currently available mathematics textbooks attempt to address some issues surrounding the incorporation of problem solving but are generally lacking in areas of communication and specifically teacher questioning.

In light of Mason and Watson (1998) and Ball and Chazan's (1999) emphasis on the need to possess a deep foundational knowledge of mathematics, it is hard to imagine generalist teachers using conceptual questioning, yet they have to if they are to promote mathematical thinking and communication. Let us imagine a teacher, who has experienced mathematics learning through an instructional questioning or rote method. Due to the nature of rote learning, mimicking the example becomes equated with success. Therefore when it comes to teaching a topic such as the division and multiplication of fractions, many teachers lack conceptual understanding of these operations. Rather than attempting to gain conceptual understanding and explore that understanding with the students, these teachers usually revert back to teaching using instructional questions and rote method even if the easier lessons in a unit have been taught through a more exploratory basis. Ball and Chazan (1999), whose research focuses on pre-service teachers, make the following observations. "Prospective teachers come to their formal preparation to teach mathematics with ideas and ways of thinking that influence what they learn . . . they have developed a web of interconnected ideas about mathematics, about teaching and learning mathematics, and about schools" (p.1). Therefore when teaching, especially a subject area like mathematics that is most likely not a specialty to many teachers, personal experiences and incomplete conceptual understanding of mathematical concepts often lead teachers to teach mathematics the way they were taught mathematics, from a "set of rules' perspective (Ball & Chazan, 1999). Having abandoned the conceptual understanding route, learning becomes more of a rote process, such as following the blackboard example approach with some occasional use of manipulatives and technology. For example, think about the division of fractions, $\frac{1}{2} \div \frac{1}{2}$. It is a difficult concept to teach students without a solid understanding of what division really is. Often teachers will revert back to the rule, or like the little ditty says, "Ours is not to reason why, but to invert and multiply". Therefore students complete $\frac{1}{2} \times \frac{2}{1}$ without the understanding of finding how many $\frac{1}{2}$'s are in a $\frac{1}{2}$. Students, who just follow the "rules" then, become successful imitators of the teacher without conceptual understanding and a lack of ability to communicate about fractional operations. Instructional questioning in mathematics then, is limited to procedural questions concerning the next step in the algorithm or seeking numeric responses.

Therefore the area of mathematics education that this thesis research will explore is how generalist teachers can increase their understanding of communication, particularly how can they create and employ conceptual questions that lead students towards increased communication? The research is guided by the following questions: Since there is a lack of exemplary resources at the intermediate level on communication, (a) How can generalist teachers at the intermediate level develop and employ conceptual questioning techniques and sensibilities? (b) In what ways can generalist teachers assess student responses to conceptual questions; and (c) How can generalist teachers promote the communication of student conceptual understanding of the mathematical processes, contained within the five strands of the Ontario Curriculum?

1.4 The Research Overview

In the literature review chapter, an outline of relevant research related to mathematical thinking, mathematical communication, teacher questioning, and conceptual questioning are examined. The framework chapter explores action research as a means by which research was conducted and synthesized. This is coupled with the methodologies of variation theory and phenomenography through which research data and related questions have been filtered. The method and design chapter provides insight into the action research method and learning study design used to create the unit of study that was implemented and researched. Data analysis and results are outlined in the fifth chapter which includes insights gained from planning, implementing, observing, and reflecting on the unit. The final concluding chapter provides a summary of the findings, contributions of the research, and next steps for the research.

2. Literature Review

2.1 A Very Brief Overview of Trends in Mathematics Education

A large portion of mathematics education research has been focused on student learning of mathematics. Educators and researchers have spent considerable effort in exploring how students of various levels and abilities can best learn mathematics. For instance, researchers have examined and compared schools of varying demographics (Boaler, 1998), rural verses urban (McKenney, Champpell, Berry et al., 2009), various geographic locations across a selected country and even cross-continental comparisons have been made (Heibert & Stigler, 1999; PISA, 2003). With all of the expert panels, research data, and analysis it has come to be understood, at least in the educational community, that students learn mathematics best when they are able to interact with it on a meaningful level (LNS, 2007; Schoenfeld, 1992; Tomlinson, 2001;Van de Walle & Lovin, 2006). In terms of how students learn mathematics then, educators likely agree that students need to develop strong reasoning and proving skills, communication skills, and problem solving skills in order to demonstrate a high standard of understanding. The National Council of Teachers of Mathematics (NCTM) outlines in the *Principles and Standards for School Mathematics* (2000) a focus for educators in terms of how students learn mathematics and thus how they should be taught mathematics. The Ontario Ministry of Education and Training echoes the ideas brought forth by NCTM (2000), emphasizing the need for educators and teachers to use differentiated instructional methods, to provide authentic, real world mathematics problems, to promote solving problems through a variety of methods, and to solve *Fermi type problems* with various solutions (Expert Panel on Students Success in Ontario, OMET, 2004).

2.1.1 Learning Should Influence Teaching Methodologies

Research on teaching methodologies indicates that educators and teachers struggle to change teaching methodologies despite knowing that instruction needs to be student centered (Ball & Chazan, 1999). McKinney, Chappell, Berry, and Hickman, (2008) state that "although significant progress has been made in understanding how children learn mathematics, teaching methodology remains unchanged" (p. 278).

Traditionally changes in teaching methodology have come from outside of the teachers themselves, usually due to a change in curriculum content or in the form of reforms implemented at the school board level in policy. Although these factors can induce some change, it is negligible and only partially assimilated into the teachers' methodology (Pehkonen & Torner, 1999). Most teachers grow and adapt in teaching beliefs and practice over the term of their career to one degree or another. However, to effect change at an increased pace, Pehkonen & Torner (1999) believe that "in order to effect a successful and positive change, teachers first need to be 'perturbed' in their thinking and actions, and secondly they need to commit themselves to doing something about that 'perturbance'" (p. 261). Therefore, change in methodology is a slow process unless teachers encounter an experience which causes them to examine their practice, such as being in the role of a learner rather than the teacher, or becoming a parent of a school-aged child (Pehkonen & Torner, 1999).

2.1.2 Example from USA on Difficulty Changing Methodologies

In their research report, Silver, Morris, Star and Benken (2009) examined intermediate mathematics teacher applications for the National Board for Professional Teaching Standards (NBPTS) certification in the United States. The applications consisted of submitted exemplary lessons in mathematics instruction. These lessons were considered to be the best efforts of competent mathematics teachers and demonstrated the teachers' understanding of problem solving skill development and the related assessment of student work. The conclusions of the researchers were that, while these examples were submitted as "the best of the best", the actual changes desired in the instructional methodologies in mathematics were evident only to a limited degree. The significance of this study then, is the evidence that even designated expert teachers in mathematics struggle with the implementation and consistent use of new teaching methodologies. Reforming pedagogy is not easy. Thus, an examination of specific new teaching methodologies encouraged in Ontario and how teachers and educators can improve upon these methodologies is warranted.

2.2 Constructivism

From a constructivist view point, knowledge is conceptually understood by individuals when they make a connection with that knowledge. Without connections or the activation of prior knowledge, the new information lacks relevance and is not remembered or used by the individual and this holds especially true in mathematics education. As the Literacy and Numeracy Secretariat of Ontario (2007) states, "Mathematics instruction has to start from contexts that children relate to — so that they can 'see themselves' in the context of the question" (p.2). It has been pointed out by Schoenfeld (1992) and reinforced with the NCTM (2000) *Standards and Principles* that one of the main goals of mathematics education should be the development of students as competent problem solvers. In order to clarify what is meant by "competent problem solvers", it is imperative that there is a clear understanding of the terms communication, and mathematical thinking. It is through students' communication in mathematics that teachers are able to discern a student's level of conceptual understanding and ability to demonstrate mathematical thinking.

2.3 Mathematical Thinking

Lampert (1990) and Schoenfeld (1992) stress that mathematical thinking in the classroom needs to be reconnected with what it means to think mathematically in the discipline of mathematics. For the purpose of this thesis the definition of mathematical thinking includes the exploration of patterns, the seeking of solutions, and the formulation of ideas (Schoenfeld, 1992). Thus, there is a strong relationship between mathematical thinking and mathematical communication.

2.3.1 Example of Conceptual Understanding

Mathematical thinking is the actual process of thought through a mathematics problem which varies according to conceptual understandings. For example the addition of 97 + 38 can be approached on two fronts. The first approach would be the addition of the ones column, 7 + 8 is 15, followed by the addition of the tens column. Therefore 9 + 3 is 12, bring over the 1 and the answer is 135. A second approach would be to add the tens column first, 90 + 30 is 120. Then to add the ones column, 7 + 8 is 15, therefore 120 + 15 is 135. Mathematical communication then, is the ability to communicate to others about the process of mathematical thinking. Thus the connections between mathematical thinking and mathematical communication lie in a social constructivist base. From this perspective, learning is based in social interaction where understanding and thinking are developed and enhanced through listening, discussing, and communicating with peers.

2.4 Mathematical Communication

Communication is more than the regurgitation of mere number facts, formulas, and vocabulary. To be meaningful, communication needs to embody ample instructional time for students to participate in problem solving that provides opportunities for them to demonstrate reasoning through mathematical thinking, and for discourse about mathematical ideas, and thus form a deeper understanding of mathematical concepts. Cai and Kenney (2000) define mathematics communication as the ability to not only "use its [mathematics'] vocabulary, notation, and structure to express ideas and relationships but also to think and reason mathematically. In fact, communication is considered the means by which teachers and students can share the process of learning, doing, and understanding mathematics" (p. 534). The Ontario Ministry of Education provides a similar definition that mathematical communication is a central process of expressing and organizing mathematical thinking (The Ontario Curriculum: Grades 1-8, Mathematics, Revised, OMET, 2005).

NCTM outlined in their document *Standards and Principles for School Mathematics* (2000) the importance of communication. "Students should be encouraged to increase their ability to express themselves clearly and coherently. As they become older, their styles of argument and dialogue should more closely adhere to established norms" (Communication section, ¶ 2). The Literacy and Numeracy Secretariat of Ontario (2007) echoes these sentiments proclaiming:

> Classroom instruction needs to provoke students to further develop their informal mathematical knowledge by representing their mathematical thinking in different ways and by adapting their understandings after listening to others. As

> they examine the work of other students and consider the teacher's comments and questions, they begin to: recognize patterns; identify similarities and differences between and among the solutions; and appreciate more formal methods of representing their thinking. Through rich mathematical discourse and argument, students and the teacher come to see the mathematical concepts expressed from many points of view. (p. 2-3)

From this statement it is clear that both the classroom mathematics teacher and the students are expected to grow and develop their mathematical thinking through classroom communication.

2.4.1 Mathematical Literacy

It can be argued that a strong component of communication and mathematical thinking is ***mathematical literacy***. Mathematical literacy is a term that encompasses many aspects of mathematics education and is widely used in professional education circles in Ontario. Several researchers, especially those in Australia, include elements of number sense as part of the definition. For the purposes of this thesis, however, the term mathematical literacy should be taken in a more literal sense. Just as literacy encompasses a strict code of grammar, syntax, and rich vocabulary, so mathematical literacy is comprised of facts, formulas, procedures, and subject specific terminology. Programme for International Student Assessment (PISA) (2003) provides a broader definition of mathematical literacy stating the following:

> [Mathematical literacy is] an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the need of an individual's

life as a constructive, concerned, and reflective citizen. (p. 24)

Several of the mathematical processes outlined in the *Revised Ontario Curriculum: Mathematics Grades 1-8*, based on the above definitions, are also components of mathematical literacy. Examples of this would be problem solving, reasoning and proving, representing, and reflecting.

2.4.2 Balanced Mathematics and Differentiated Instruction (DI)

Another term often associated with mathematical processes in the professional education community in Ontario is that of a ***balanced mathematics*** program. The term borrows from its English subject area counterpart, balanced literacy. A balanced mathematics program would be one that embraces all of the seven mathematical processes in such a way as to provide explicit teacher modeling followed by shared, guided and eventually independent work. Another aspect of the balanced mathematics classroom is the use of ***differentiated instruction*** (DI) methods to teach students. Differentiated instruction is a pedagogical approach to teaching with the central component being the teacher's ability to vary differing aspects of instruction. The main areas of differentiating instruction are to differentiate or alter the content, the process, or the product (Small, 2008; Tomlinson, 2003). As this thesis examines and focuses on specific aspects of communication, balanced mathematics programming and differentiated instruction techniques will surface as central components.

2.5 Teacher Questioning

Communication in mathematics can be viewed by its various forms of argumentation (Lampert, 1990), discourse (Fisher & Kopenski, 2008; Knuth & Peressini, 2001), listening (Davis, 1997), and/or writing (Whitin & Whitin, 2002). However, the force that fosters students' communication in any of those forms is *teacher questioning*. Therefore teacher questioning becomes a central component to quality communication in mathematics.

2.5.1 Types of Questions

Much literature exists on practical management of questions and questioning that spans subject areas. This study focuses on questioning in mathematics teaching, particularly on types of questions that elicit communication. There are several types of questions that are used in the mathematics classroom: process questions that focus on what the next step should be; instructional questions that are used greatly in Socratic lessons - which means for me direct and rote questions where students answer through mimicking the teacher examples, and include specific 'yes or no' and numeric responses; problem solving questions that focus on the standard "What do I know?" and "What do I need to know?" type of questions; and conceptual questioning (Mason 2000). Conceptual questioning is not easy to delineate. Conceptual questioning has promise to go beyond some of the limits of the other types of questioning, yet it is not necessarily separate from other types of questions. Conceptual questioning appears, rather, to be more of a teaching mind set or methodology used for asking questions. It does not focus on the "guess what I'm thinking" questioning game, instead conceptual questioning encourages questions that prompt mathematical thinking asking questions such as "What if . . . ?" and "What happens when . . . ?" (Mason & Watson, 1998; OMET, 2005; Small, 2008). Detailed planning is required in order to effectively implement conceptual questioning because this type of questioning does not necessarily flow naturally unless one has had tremendous practice using conceptual questioning and possesses a deep foundational understanding of the subject area (Ball & Chazan, 1999; Mason & Watson, 1998).

2.5.2. Mason: Three Points about Traditional Questioning

Mason (2000) provides a critique of the use of general teacher questioning techniques. He asserts that:

1. Students require explicit examples of the mathematical thinking process in order to shape and make sense of their own mathematical thinking.

2. Questioning is a natural part of the process for engaging and assessing students' understanding.

3. Teachers impulsively ask questions as their minds travel from concept to concept.

2.5.3 Heibert & Stigler: Cultural View of Teaching/Questioning

Teacher intent is to assist students by guiding the students to the way the teacher thinks. Often, as Mason (2000) points out, this type of teacher questioning also includes pre-determined or expected answers. Through the 1995/1999 TIMSS video study, which most notably compared Japan with successful mathematics educational systems as measured by the TIMSS test to the less than successful educational systems within the United States, Heibert and Stigler (1999) discovered some key elements to successful mathematics teaching. They concur with Mason's ideas on teacher questioning, especially when comparing Japanese teacher lesson formats and interactions with students to those in the United States. Cultural views play a central role, according to Hebert and Stigler (1998), in the teaching and subsequent learning of mathematics. Teachers in the United States uphold the cultural view that the learning of mathematics is a procedural process which emphasizes skill development. The teacher, through questioning strives to keep students engaged in the lessons, demonstrates the necessary skill for the seat work, and then rushes in to aid students who are confused or experiencing frustration. Individual differences are compensated for with modifications and streaming at the upper levels. In contrast, Japanese teaching culture views mathematics as a series of relationships to be discovered through problem solving activities. The teacher poses problems and monitors student struggles and successes in order to facilitate the follow-up discussion. Individual differences are seen as advantages and everyone is expected to gain something from the lessons (p. 125).

2.6 Good Questioning

2.6.1 Mason: Good Questions, Pre-planning and Assessment

Mason (2000) provides insight into solutions for teachers who recognize the need for offering opportunities to develop communication skills and to use questions that encourage students to practice mathematical thinking. The key for teachers lies in teacher questioning techniques. Good questioning techniques require careful pre-planning before implementation can be effective (Mason & Watson 1998; Small 2008). Too often teachers find themselves asking questions that do not provoke mathematical thinking, but rather contain contrived answers or solutions that lead students down the path of rote learning or mimicking the teacher.

2.6.2 Core Mathematical Themes – Schoenfeld – Enculturation

For questioning techniques in mathematics to be beneficial, according to Mason (2000) they should, through explicit means, promote the development of "core mathematical themes" (p. 100). Small (2008) concurs with Mason on this point as she cites the need to acknowledge and define the big ideas and themes in mathematics and center teacher questions around those ideas and concepts. Students would then be more apt, according to Mason (2000), to "integrate them [core mathematical themes, ideas, and concepts] into their sense of mathematical thinking and into their own thinking than if they remain implicit or beneath the surface" (p. 101). Therefore Mason believes that the type of questioning teachers emulate should be modeled after the questioning style mathematicians use among themselves. Schoenfeld's (1992) ideas on mathematics instruction and questioning parallel Masons' as he highlights the need for *enculturation* in mathematics. Enculturation is the immersion of students into the mathematical community. It relies heavily on the idea of building a community through social interaction. Thus the enculturation aspect is akin to the Ontario Ministry of Education's emphasis on mathematical communication that can only be created through teacher modeling of questioning techniques and an appropriate classroom environment of respect created by the teacher (Inman, 2005). Also, enculturation is in line with Lampert (1990) and Schoenfeld's (1992) connection of mathematical thinking in the classroom to thinking mathematically in the discipline of mathematics.

2.6.3 Appropriate Questioning Engages Mathematical Thinking

Appropriate questioning, as Mason (2000) points out, is based on the teacher's attempt to engage students in mathematical thinking. There is a danger for teachers in the persistent use of one type of question as this leads to a predictable pattern for students who will often opt out of the thinking process and instead guess what is on the teacher's mind or disengage completely (Mason). Manouchehir and Lapp (2003) point out that teachers are unique in their communication style because they consistently use questions as a means for communication rather than a more traditional form of communication: conversation. It is the continual use of conversational questioning that Manouchehir and

Lapp state "control[s] students' learning because they focus students' attention on specific features of the concepts . . . [and] establish and validate students' preconceptions about what is important to know to succeed in mathematics" (p. 563-564). Therefore the key for appropriate questioning is the awareness of the type of questions being asked and the further realization in the teaching moment what mode of questioning is being used (Mason). Once this awareness is established and then actively thought about, the techniques for quality questioning, also termed conceptual questioning, can be honed.

2.7 Conceptual Questioning

2.7.1 Filter for Variation of Types of Questions

Conceptual questioning then becomes a filter for the variation of questioning types in order to promote mathematical thinking. Conceptual questions include rich mathematical problems but are not limited to these (Mason & Watson, 1998). They may extend to sub problems, follow-up questions, prompts, or probes that the teacher poses to students as they solve problems or as the teacher instructs the students. An example of a mathematical problem that is a conceptual question is: if one wanted to mass produce cylindrical water containers out of 8 x 11 plastic sheets with minimal waste, what would be the best dimensions? Student approaches and responses will vary. It can be expected, at the most basic level, students will identify and use the circumference and area formulas for circles in order to create a net for a cylinder that may involve minimal waste. At a higher level of mathematical thinking students would apply the necessary formulas to achieve minimal waste from the plastic sheets. However, at a more advanced level students might identify the dimensions for minimal waste, reason through the application of the formulas, justify their reasoning for the dimensions chosen and then explain clearly explain clearly how they arrived at this consensus. According to Mason (2009), "in order to provoke learners into taking initiative, into engaging fully with mathematical ideas and mathematical thinking, it is necessary to construct pedagogic tasks which call upon learners to make use of their undoubted powers of making sense" (p. 1). Therefore it is the attributes of reasoning, problem solving, and communication evoked by the question which promotes mathematical thinking thus making it into a conceptual question.

2.7.2 Heibert & Stigler – Japanese Pre-plan Explicit Questions

Heibert and Stigler (1999) completed an extensive study on mathematics teaching techniques employed in North America. Heibert and Stigler (1999) point out that one of the keys behind the success of the Japanese teachers is that they have not only pre-planned the explicit questions they ask, but they also promote the idea of multiple solutions to the question or mathematics problem presented. In Japanese lessons, the questions used were more adaptive with the acceptance of multiple avenues to reach the desired conclusion. As students worked through the problems, the Japanese teachers let the students struggle with the concepts, using their struggles to guide the follow-up discourse in the lesson. Inman (2005) builds on the idea of the Japanese lesson structure by creating a list of questioning dos and don'ts. Her list includes key ideas such as being open to different responses, giving students thinking time, and not imposing teacher knowledge of mathematical concepts on students. After all, good questioning by teachers allows students to work through mathematical concepts and "we must not rob them of the opportunity to practice alone" (Inman, 2005, p. 29). Small (2008) concurs with Mason (2000), and Hebert and Stigler's (1999) conclusions stating that teachers continue with the tendency to use questions solely to check for students' understanding. This means that there is a limited use of questions which encourage mathematical thinking. Small (2008) agrees with Mason (2000) that educators and teachers must think about the questions they are using. Teachers should be asking why they are using a question and what they hope the students will gain mathematically from the questions used. Small (2009) expands upon this questioning concept by developing mathematical questions and problems that she terms "open questions" and "parallel tasks". Many of these questions and tasks reflect the concepts identified by Heibert and Stigler (1999) as teaching techniques used by the Japanese teachers in their study. However, Small's questions are what she terms "open" as they allow for students of various levels to access them and actively promote variance in the solutions.

Reinhart (2000) takes Mason and Watson's (1998) and Mason's (2000) ideas about pre-planning the use of explicit questions in lessons one step further. He suggests replacing teacher directed lessons with sets of questions aimed at students' discovery of concepts, highlighting the need to ask a variety of types of questions with the motto "never say anything a kid can say" (p. 484).

The communication of mathematical thinking is a primary element of a balanced mathematics program. If students are to ever truly come to a meaningful understanding of mathematics concepts, then teachers must not only provide the type of questions to prompt such thinking, but they must also allow students the opportunity to respond. Manouchehri and Lapp (2003) point out, that current trends in mathematics education include a strong emphasis on the students' abilities to reason, evaluate, draw conclusions, and provide explanations from a mathematical perspective. As a result teacher questioning "must give learners an opportunity to communicate their reasoning processes" (Manouchehri & Lapp, 2003, p. 564).Therefore the key to eliciting quality communication begins with the teachers' careful attention to the types of questions they ask. Questions then should be monitored for form, content, and purpose in order to promote opportunities for quality communication.

2.8 Conclusion

Although there is some research available on questioning in mathematics and a plethora of research on teaching through problem solving and differing types of communication, there is a distinct lack of research on *conceptual* questioning techniques and sensibilities. This lack of research is compounded by a lack of resources at the intermediate level on communication coupled with a focus on teacher questioning techniques. So the research question is then put forth: How can generalist teachers at the intermediate level develop and employ conceptual questioning techniques, assess student responses, and promote the communication of student conceptual understanding of the mathematical processes, contained within the five strands of the Ontario Curriculum? Therefore, since there is a lack of exemplary resources at the intermediate level on communication, (a) How can generalist teachers at the intermediate level develop and employ conceptual questioning techniques and sensibilities? (b) In what ways can generalist teachers assess student responses to conceptual questions?, and (c) How can generalist teachers promote the communication of student conceptual understanding of mathematical processes, contained within the five strands of the Ontario curriculum?

3. Framework and Methodology

This thesis study examines the mathematical processes of communication intertwined with the process of problem solving which is narrowed through a focus of teacher questioning techniques. Therefore the phenomenon of communication is tempered by the following questions: How can generalist teachers at the intermediate level develop and employ conceptual questioning techniques and sensibilities, assess student responses, and promote the communication of student conceptual understanding of the mathematical processes, contained within the five strands of the Ontario Curriculum? I specifically ask: Since there is a lack of exemplary resources at the intermediate level on communication, how can generalist teachers create and employ effective conceptual questions that lead students toward improved communication? In what ways can generalist teachers increase understanding of communication in the mathematics classroom? An examination of how communication can be elicited through teacher questioning techniques and sensibilities, how these techniques and sensibilities can be developed, and how questioning techniques and sensibilities can be implemented so that they become applicable to mathematic pedagogical practices needs to be done. The design purposely incorporates variation theory and phenomenography, and since the study involved two teachers researching, and implementing conceptual questioning in their own classrooms, its method is action research. The action research is being tempered by the structures of learning study.

In this chapter an outline of action research and why it was chosen is followed by an outline of the background and central principles for learning study. The process of implementation of the learning study is further outlined and the issues surrounding participants and ethics are discussed.

3.1 Action Research

Action research, as a framework, has the capability to reflect a variety of personal fundamental points of view. This is due to the very nature of action research that embraces the opinions, ideas and belief systems of the participants. As stated by Somekh (1995):

> Action research is grounded in the culture and values of the social group whose members are both participants in the research field and researchers. It may be instigated by an individual, but its momentum is towards collaboration, because the emphasis on social interactions and interpersonal relationships has the effect of drawing other participants into the research process. The focus of the research is likely to be on an issue which is of concern to the group. (p. 349)

It is hard to point to a specifically defined definition of action research as the definition may flux according to the participants' beliefs and understandings. Bradbury and Reason (2008) define action research as:

> A participatory, democratic process concerned with developing practical knowing in the pursuit of worthwhile human purposes, grounded in a participatory world view which we believe is emerging at this historical moment. It seeks to bring together action and reflection, theory and practice, in participation with others, in the pursuit of practical solutions to issues of pressing concern to people, and more generally the flourishing of individual persons and their communities. (p. 2)

McNiff (2002) defines action research through related terms as a:

> practical way of looking at your own work to check that it is as you would like it

> to be... it is often referred to as practitioner based research; and because it involves you thinking about and reflecting on your work, it can also be called a form of self-reflective practice. (NcNiff, 2002, What is Action Research Section, ¶. 1)

Therefore action research can be viewed as the investigative inquiry into a personal, professional, and practical question which may be investigated through self-conducted research in collaboration with other practitioners.

Action research is the overarching method for the research in this thesis for several reasons:

• Action research is a grass roots process by which a classroom teacher, as researcher, can affect change in their own environment. It lends itself well to the learning study format which will be discussed further in this section.

• Action research has the ability to close the gap between research and practice (Cohen, Manion, & Morrison, 2007; Somekh, 1995) and since the participants are also the researchers, there arises a powerful dynamic for improving practice.

• The ability to address very relevant and practical issues and questions is supported through action research and its personal relationship to the participants (Somekh, 1995).

• Action research can become a very effective mode of professional development (Somekh, 1995). The attraction of action research is that it creates the opportunity to self-examine one's teaching practice which when shared, can create an avenue to effect change on a larger educational scale.

• Action research is fluid in that it does not prescribe itself to one hypothesis but centers around ideas and the development of them (McNiff, 2002). Therefore, action research provides an appropriate avenue for the development of practical research on teaching and learning.

Often action research contains a social justice element which is based in its critical theory roots. Cohen, Manion and Morrison (2007), state that critical theorists view action research as a practical method to bring about change, promoting a more democratic society where one is aware of the elements of power and control, and becomes empowered to change education and ultimately society. Although action research has its roots in critical theory, it has also been heralded by complexity theorists (Bradbury & Reason, 2008). Rather than emphasize the emancipatory aspect of action research, this study emphasizes the changes, differences and relations in experience of participants and in case of classroom action research by both the individual learners and learners as a group, and teachers in the learning process. This variation of action research is obtained through careful implementation of lesson study or its cousin learning study that are discussed in Chapter 4, Method and Design.

3.2 Phenomenography: A Framework for Analyzing Data

3.2.1 What is Phenomenography?

Phenomenography is an established approach to educational research that has its roots in Sweden at the University of Göteborg (Martin, 1994; Pang, 2003). Phenomenography is concerned with relationships between people and the experienced phenomenon. The premise of phenomenography is to "understand the various ways in which different people experience, perceive, or understand the same phenomenon" (Walsh, Howard, & Bowe, 2007, p.1). When examining the term phenomenography, the Greek roots for that word give an increased meaning and significance to the term. Orgill

(2008) states phenomenography, "is derived from the words phainonmenon (appearance) and graphein (description). Thus, phenomenography is a description of appearances" (p. 1). Since the experience is what is being examined, extensive research time and effort is focused on exploratory interviewing of individuals (Orgill, 2008). However, it is also important to note that although different people can experience the same phenomenon and have various experiences related to that phenomenon, there are a limited number of ways any phenomenon can be experienced (Martin, 1994; Walsh, Howard, & Bowe, 2007).

3.2.2 How Phenomenography is Applied to Research in the Mathematics Classroom

As a qualitative approach in an educational setting, phenomenography examines peoples' experiences of a phenomenon. From a phenomenography perspective, knowledge gained is measured in the ways in which the individual understands something rather than the more traditional quantitative approaches where knowledge acquisition is strictly gauged through quantitative measures. In the mathematics classroom, the process of communication can be viewed as a phenomenon. As such, teacher questioning and the ensuing experiences by both the teachers and the students can be explored for variations, growth, and development as aspects of a phenomenon.

3.3 *Variation Theory: A Framework for Analyzing Data*

3.3.1 What Variation Theory Is – And Its Constructivist Roots

According to the learning theory of constructivism, students do not gain a deep personal understanding of concepts unless they are able to make connections with these concepts in terms of prior knowledge. *Variation theory* leads off of a constructivist approach by examining the different ways that students interact with a concept, also generally termed a phenomenon. This added dimension allows the researcher to gain a deeper understanding of the teaching and learning processes (Runesson, 2005).

3.3.2 How Variation is a Methodology and its Relationship to Phenomenography

Variation theory, as a theoretical framework, has its roots embedded in phenomenography (Runesson, 2005; Vikstrom, 2008). Variation theory involves the examination of differences or variations in learning, and thus seeks to identify how learning comes to be (Runesson, 2005). From this perspective, "learning is defined as a change in the way something is seen, experienced, and understood" (Runesson, 2005, p.70). Vikstrom (2008) further clarifies this by stating that variation theory goes beyond just the experience of the phenomenon such as communication in the mathematics classroom by examining how the "different ways of understanding can arise and develop" (p. 212). Originally, phenomenography research, according to Pang (2003), aimed at answering two fundamental questions: "'what are the different ways of experiencing the phenomenon' and 'how are these related to each other?'" (p. 147). With the more recent focus on variation, however, there has been a shift from "studying different ways of understanding something to studying how those different ways of understanding can arise and develop" (Vikstrom, 2008, p. 212). Variation theory then examines how the different ways of understanding a phenomenon or experience can arise and develop. The experience of the phenomenon and the understandings linked to the phenomenon are viewed as one in the same through variation theory (Vikstrom, 2008). Therefore variation theory becomes a natural outflow of the study of a phenomenon such as conceptual questioning and allows the researcher to examine aspects of this phenomenon or phenomenographic experience.

4. Research Method and Design

The framework of action research provides the opportunity to combine classroom practice and reflection with classroom research. Therefore action research becomes a sound vehicle through which to explore students' communication through conceptual questioning. Action research can be carried out in various forms including lesson and learning study.

4.1 From Lesson Study to Learning Study

Learning study, one specific form of action research, allows for a specific focus on the exploration of the phenomenon of teacher questioning as it aids communication and problem solving. In order to gain a clear understanding of the merits of learning study, one can begin by contrasting it with its cousin lesson study. It has been through research on lesson study that I have evolved towards embracing the ideas and the underlying theoretical basis of learning study.

Lesson study is an approach to teacher professional development that is highly successful in Japan. In North America the merits of lesson study have been highlighted by several mathematics education researchers, most notably Heibert and Stigler (1999), and Fernandez and Yoshida (2004) and in Canada by Bruce and Ladky (2009).

Lesson study, as the term implies, is an intense, detailed study of a particular lesson (Fernandez & Yoshida, 2004). It is common practice for lesson study to be completed by 2-3 teachers and an outside facilitator, who has extensive knowledge of the subject area. It usually commences within one school and at a common grade level or division. The goals of lesson study are not necessarily focused on a particular academic outcome but rather centers on one particular teaching skill or learning method which is developed through a particular subject area, usually mathematics (Fernandez & Yoshida). Lesson study focuses on the development of one, possibly two lessons, that are then taught, observed, revised, and re-taught in a cyclical fashion through the various teacher-participants' classrooms. Lesson study's success is due to its nature as a process. It is comprised of various stages that remain in flux depending on the goals of the teachers and educators who are participating. Fernandez and Yoshida, and Heibert and Stigler (1999), identify six steps in lesson study format; preliminary planning of the lesson, lesson implementation, peer observations, lesson revision, re-teaching of the lesson followed by further revision, and the sharing of the lesson with others. In Canada, Bruce and Ladky (2009) identify with and implement four steps in the lesson study cycle: "goal setting, planning, lesson implementation and reflection" (p. 2).

4.2 A Data Analysis Approach

Learning study is a newly developed approach towards improving both teaching and learning which as is the case with phenomenography has developed most notably in Sweden. As a model of professional development, learning study embraces the classroom teacher as the primary catalyst for change in the educational system. It favors a grass roots approach to implementing change rather than a forced top-down approach. Through learning study, teachers "explore the relationship between teaching and learning with the aim to improve students' learning" (Ling & Runesson, 2007, p.157).

The purpose of learning study is to design and study lessons, usually in a unit, that will help students and educators navigate the path between their prior conceptions of a topic to the deeper understanding. The purpose of this learning study is to provide an opportunity to further explore the development of communication about mathematical concepts through the techniques of conceptual questioning. Learning study adopts the language of phenomenography, which "refers to student's conceptions of phenomena" (Davis & Dunnill, 2008, p.5). The use of a phenomenography approach then allows for various conceptions to be developed around a particular concept or phenomenon (Davis & Dunnill, 2008). A 'learning outcome circle', developed by Davis and Dunnill (2008) and based in variation theory, allows for the teacher to evaluate what conceptions students have developed and the interrelationships among them. The intent is that the learning outcome circle be used to emphasize multiple "critical" conceptions of a concept at a variety of levels. This allows for teachers in the learning study to understand the different constructions their students make and to adjust their teaching strategies accordingly. Davis and Dunnill (2008) clarify this by stating that:

> In learning study it is expected that the *target conceptions* [italics mine] for some students will be more complex than the target conceptions for others, given the different starting points in students' thinking. Teachers are expected to identify the features . . . of the conceptions they wish learners to learn and to provide students with an experience in which all of the features of the target conception are varied *simultaneously* [italics in original]. (p. 7)

In my view, the learning study borrows the best aspects of the Japanese lesson study: teacher collaboration, lesson observation, and peer review (Ling & Runesson, 2007; Davis & Dunhill, 2008). However the driving force behind learning study is what sets it apart. Learning study follows a cyclical framework that revolves around the teaching and learning process of both teachers and students (Ling & Runesson, 2007). Learning study begins by examining the participating teachers' foundational knowledge on the way a particular conception of phenomenon is learned and taught, therefore focusing on a conception of phenomenon that might involve several lessons as a unit of study. Davis and Dunnill (2008), identify this examination of conceptions of phenomena by stating that, "In learning study, lesson preparation is preceded by an attempt to identify variation in ways of understanding phenomenon that is the focus of the lesson. This phenomenographic activity frames the learning objectives for the lesson, and also features prominently in teachers' review of the lesson" (p. 3). Davis and Dunnill (2008) further clarify their interpretation of learning study by identifying three central aspects which they believe give learning study its unique stance. First, akin to lesson study, collaboration is emphasized throughout the whole learning study process. Second, in contrast to lesson study, the focus of the planning stages centers around variation theory. The principle of variation theory is "not simply to stress the existence of the feature, but to highlight its relevance through variation" (Davis & Dunnill, 2008, p. 7). According to Ling and Runesson, (2007), variation theory equates the ability to learn something with the ability to "discern its critical aspects" (p.158). It is the ability to discern most aspects of a phenomenon that determines one's ability to learn. The objects of this study, therefore, center on various ways of understanding communication, teachers' ability to discern critical aspects of conceptual questioning, and the resulting student responses. The efforts made to design and implement conceptual questioning with the aim of increased communication are reflected upon. Successes are examined for traits of conceptual development, student engagement, level of communication and evidence of mathematical reasoning and failures are explored, modified, and retested. In focusing on multiple lessons involving the phenomenon of teaching and learning, learning study affords the opportunity for professional development. As well the pre and post reflection process creates a standard by which the successes of the learning study can be adequately judged (Davis & Dunnill, 2008).

4.2.1 Rationale

Learning study, which is a sub-form of action research, is an appropriate method for this research for several reasons. First, learning study provides an effective foundation for examination of teaching pedagogies by exploring the teachers' philosophical view of teaching a particular subject area. Once this has been established, it provides a base from which to build upon during the learning study process. Second, the very collaborative nature of learning study allows for each teacher to have a personal investment and ownership of the planning and implementation. Third, learning study identifies and works with various understandings of phenomena, thus, creates the opportunity for differentiated instruction within the unit of study. Fourth, the creation of meticulous lesson plans means that all teachers complete questioning techniques in a similar manner with the same questions being asked. Fifth, with the learning study format the questions could be replicated in a variety of classroom situations with different teachers and a variety of students. The richness of this experience will lead to designing, implementing, and studying the effects of conceptual questioning on students' communication by teachers who co-plan, co-teach, and co-reflect on a Grade 7/8 mathematics unit.

4.2.2 Structuring Specifics

The specific plan for implementing the research phase of this project is multifaceted. Learning study usually involved the development of a unit of study. The learning study for this thesis was at the elementary level, and completed during the third term. The learning study involved two intermediate elementary classroom teacher-participants, one was both a teacher-researcher and a teacher-participant and the other was only a teacher-participant.

4.2.3 Researcher's Role.

The teacher-researcher began by conducting a preliminary questionnaire (Appendix A) with the teacher-participant to ascertain the teacher-participant's particular background and ideas about communication in mathematics. The teacher-researcher also completed a questionnaire. From a variation theory perspective, the questionnaire was used to ascertain the foundational knowledge of the teachers allowing for the differences in understanding to become more apparent in the post study analysis. A follow up informal interview was also conducted after the questionnaire to further examine the teacher-participants' variations in understanding about communication, teacher questioning techniques and sensibilities in mathematics.

4.3 Research and Design

4.3.1 Meetings

At the initial learning study planning meeting the two teachers involved in the learning study, the teacher-researcher and teacher-participant, discussed the relevant research on teacher questioning techniques in mathematics. This discussion also incorporated the teacher-participants' knowledge on teacher questioning and on the specific strengths and needs of their students. Since this study was conducted in the latter half of the third term, the teacher-participants were generally aware of the participating students' prior conceptual understanding, communication skills, and general comfort level with mathematics. This prior knowledge was used to help design the learning unit, particularly to craft the conceptual questions used within the unit.

The second learning study planning meeting was conducted with the sharing of detailed lesson plans on a computer program, Smart Ideas. This is a commonly used program in the school board where this study took place and both teacher-participants were familiar and comfortable with its use. Several of the conceptual questions were discussed and modified by both teacher-participants. It was also agreed upon that both teachers would adopt similar instructional methods, and lesson formats from existing resources. Differentiated instruction (DI) methods that were already practiced in other subject areas by the teacher-participants, were implemented through the type of teacher questioning used. One aspect of differentiated instructional methods is to provide questions that include multiple layers, so that all students experience some level of success. This layering of questions will be further discussed in a later section on the specific unit used. The teacher-participants also agreed to use the same mathematics text books and provided students with identical homework assignments. After the first two planning meetings, contact between the teacher-participants was maintained through email and telephone conversations. With the creation and implementation of the unit lesson plans, variations in understandings were noted. Student responses to questions and concepts were generally anticipated.

Once the initial lessons had been taught in each classroom, the teacher-researcher and teacher-participant conferenced via the internet. The teachers were able to review their experiences, and post any revisions for the next lesson plans if need arose.

After the learning study had been completed, the teacher-researcher conducted a follow-up interview with the teacher-participant, see Appendix B, and through self-reflection to ascertain if there have been any fundamental shifts in the teachers' view of questioning techniques and how these relate to students' communication and thinking in mathematics. Post unit observations of students' communication and of unit content knowledge were also completed to ascertain grasp of the concept(s) taught.

4.3.2 Data Collection

This learning study gathered a variety of data that is of a qualitative nature: questionnaire, interview, reflection, and observation data. Part of the data was organized by both the instrument used to collect the data and by individual participant. For example, in the findings chapter the interview data is correlated in one subsection by headings relating to the questions asked and by teacher-participant. Other data sets are organized and explored by issue or topic. Since the data was based on personal interviews, conversations, and reflections, parts of the data are presented in a first person narrative format. Emerging themes, trends, and commonalities in the data are presented in a thematic format with emphasis from individualized data.

The learning study format incorporates interviews, self-examination, peer collaboration, and observations. Informal interviews (Appendix B) and conversations, which are reflective in nature, with the teacher-participants is a major component of the phenomenological approach to research. The post learning study interview questions were as follows:

1. What has been the most significant benefit of participating in this learning study?
2. How has your understanding of communication changed?
3. How might the elements of teaching questioning worked on during the learning study be translated into other units of mathematics?

4. Have any of your teaching practices changed since participating in this learning study? How?
5. Do you believe that your students have improved in terms of their communication skills through this unit?

These interview questions were presented to each teacher-participant in a structured manner; however there was also ample discussion of personal experiences, successes, and struggles with lesson structure and with students. Personal experiences related to the phenomenon are central to the phenomenography approach to data analysis. These experiences of teacher questioning techniques were collected through reflective self-evaluation, conversations, informal interviews, and observations. Conversations relating to the reflection of the teacher-participants in this learning study will be presented in a narrative format in the chapter on analysis. Another part of the learning study process involves the use of pre and post observations of students in order to determine their base understanding in a subject area and what was learned. The observations included informal questioning of students during individual and small group work situations about rate and ratios, and individual teacher-student conferencing with students towards the middle and end of the learning study.

4.3.3 Ethical Considerations

Cohen, Manion, & Morrison (2007) provide an extensive review of literature on the ethics of action research which has been categorized into several interrelated ethical principles or considerations. Permission to carry out the research was obtained from the school board and ethical consideration of both the university and school board were adhered to (see Appendix C – ethics approval form, letters of permission, consent forms).

Several of the ethical principles are generic in that they apply to most areas of research. One such issue is that of privacy and confidentiality. When completing this action research project, it was imperative that anonymity of participants be maintained on several levels.

First, action research allows for the researcher to become a participant in the research which has drawbacks and benefits. However, as Cohen, Manion, & Morrison (2007) point out, although the researcher is a part of the area they are studying and cannot therefore be totally objective:

> Other people's perspectives are equally as valid as our own, and the task of research is to uncover those. Validity, then, attaches to accounts, not to data or methods (Hammersley and Atkinson 1983); it is the meaning that subjects give to data and influences drawn from the data that are important. (p.134)

Second, anonymity is necessary so that teacher-participants will feel free enough to share their personal thoughts and challenges and be open to have a peer evaluate their specific use of questioning during their mathematics teaching. Third, paired with the anonymity of the teacher-participants is that of the schools and students where the study is conducted. In order to maintain this anonymity, pseudonyms have been used, specifics on class size have been altered, and only generic information has been shared about the respective schools involved. It is also important to clarify that names of the schools and the teachers have been presented in a purposefully fictional manner to protect the identity of the respective schools and students involved, in light of the dual teacher-researcher – teacher-participant role.

4.3.4 The Participants

In the spirit of carrying out an ethical study, both schools and teacher-participants involved in this study have been assigned pseudonyms.

Bear River is a large school in the heart of an urban community. The school contains a mixture of students from various nationalities including a moderately sized English language learner (ELL) population. Most students who attend Bear River come from lower socio-economic backgrounds. The study took place in a split Grade 7 and 8 classroom with 25-30 students. The teacher from Bear River is Ms. Rock and she was the main classroom teacher. Ms. Rock has been teaching at the intermediate level for approximately eight years. She has her Bachelor of Education degree with a teachable in History. Ms. Rock has her primary, junior, and intermediate qualifications, has obtained a specialist in reading, a specialist in History and completed her Principals Qualification Program Part 1. Ms. Rock has been active in the areas of literacy, social justice, and differentiated instructional techniques task forces at the district school board level.

Fox Wood is a small school situated in the midst of a small community. The school is comprised of students who come from both urban and rural homes. The resulting student population is largely homogeneous with lower to mid socio-economic backgrounds. The study took place in a split Grade 7 and 8 classroom with 25-30 students. The teacher from Fox Wood is Ms. Hare and she was the main classroom teacher during the study. Ms. Hare has been teaching at the intermediate level for approximately eight years. She has her Bachelor of Education degree with a teachable in Art. Ms. Hare has her junior and intermediate qualifications and is currently completing more course work with an education focus. Ms. Hare has been active in the areas of mathematics and science task forces at the district school board level.

4.4 Unit as Planned

4.4.1 Unit Expectation

The mathematics unit of study created for this thesis was on the topic of rate and ratio at the Grade 7/8 level which falls under the Number Sense and Numeration strand of the Ontario mathematics curriculum. The Number Sense and Numeration strand, I believe, is one of the most comprehensive of the five strands as it focuses on a variety of sense-making skills that seem to flow into the other four strands. At the intermediate level, for example, topics such as factors and multiples flow into Patterning and Algebra in terms of finding the n^{th} term and solving for the variable. Percent and ratio slip into Data Management and Probability in terms of collection and analyzing data as well as exploring the concept of chance. Ratio also appears in the Measurement strand when calculating and comparing various measures, converting between units of measure, and between linear and squared cubic measurements. The core of the unit was designed primarily by the teacher-researcher and modified with input from the teacher-participant. Initially the unit was created this way because the teacher-researcher had the research background on the types of questions one could use for conceptual questioning. As this knowledge was shared with the teacher-participant, she was also able to contribute to the formation of questions. The unit was designed with lesson plans for nine teaching days. Appendix D contains some examples of the lesson plans designed for this unit. Since the unit was completed in the month of June, it took longer as some classes were missed for assemblies, sporting events, year end festivities, and class trips.

Since this rate and ratio unit was taught by both the teacher-researcher and teacher-participant to split grades classrooms, the content covered was a blend between the Grade 7 and Grade 8 expectations in *The Ontario Curriculum: Grades 1-8, Mathematics Revised* (2005). The overarching goal of the expectations for mathematics in the Number Sense and Numeration strand, according to *The Ontario Curriculum: Grades 1-8, Mathematics Revised* (2005), is that students grasp an:

> *Understanding of number and operations* as well as the *ability to apply this understanding* in flexible ways to make mathematical judgments and to develop useful strategies for solving problems. In this strand, students develop their understanding of numbers by *learning about different ways of representing numbers and about the relationships* among numbers . . . Experience suggests that students do not grasp all of these relationships automatically. A broad range of activities and investigations, along with guidance by the teacher, will help students construct an understanding of number that allow them to make sense of mathematics and to know how and when to apply relevant concepts, strategies, and operations as they solve problems (p.8, italics added).

In the Number Sense and Numeration strand, there are four specific expectations under the subtitle proportional reasoning (see Appendix E). In Grade 7, students are to demonstrate an understanding of the relationship among decimals, fractions, and percentages concepts; identify rate as a comparison of ratio; and solve problems involving unit rates. For example, a comparison ratio might involve calculating heart rates in beats per minute and making comparisons with classmates. In Grade 8, almost every specific expectation of the four under proportional reasoning highlights problem solving using, proportions, percentages, and rates. For example, a rate problem might ask students to compare the cost of different sized packages of a product to determine which is the best value. This unit on rate and ratio attempts to provide opportunities for students to interact with rate and ratio concepts on a meaningful level and to connect the use of mathematical judgment, mathematical thinking, and different ways of presenting communication skills with students' newly learned rate and ratio concepts as they solve problems.

4.4.2 Conceptual Questions: Background

In terms of the conceptual question techniques created and used in this unit, their development was largely, but not exclusively, influenced by several key researchers in the field of mathematics. Although the researchers and their research have already been discussed in the second chapter, literature review, the influences of some of these key researchers necessitates some restating as it applies to specifics in the development of conceptual questioning for the unit. The brief overview of these essential researchers will be again referenced as initial connections are made between particular questioning techniques discussed, and those used or developed through the rate and ratio unit by the teacher-researcher and teacher-participant.

Hebert and Stigler (1999) explored and highlighted the big ideas and fundamental differences in approaches to teaching mathematics between several countries, most notably Japan and the United States. The differences noted were striking in that the whole Japanese approach to teaching embodied distinct types of mathematics problems in which the solving of the problem became the basis for their lesson structure. One of the Japanese problem solving questions highlighted in the TIMSS Video Study (National Center for Educational Statistics, Heibert & Stigler, 1995) was related to the area of a

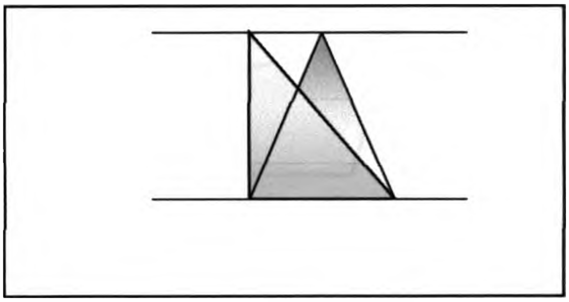

Figure 1: Triangles with the same base and same height have the same area. This is demonstrated with the triangles drawn inside of parallel lines.
Adapted from TIMSS Video 1999

triangle. In this section, I examine the Japanese teachers' approaches to teaching this problem, parallels in their approach to teaching, and elements used in conceptual questioning. In the Japanese classroom, the problem solving question was presented as the main focus for the lesson. In Ontario this approach is referred to as teaching through problem solving. Skills developed in the previous day's lesson were highlighted during the review part of the lesson and students were encouraged to build upon the previous days skills with the current lesson's problem. Previously students had discovered that a triangle between two parallel lines with the same base measure had the same area (Figure 1). The teacher formed links between the problem from the day before and the new problem that he posed. The new problem was to find an equitable way to divide the land with a straight line (see Figure 2) between Eda and Azusa so that the areas were equal. Both Figures 1 and 2 were presented to students on a television screen. The teacher allowed students three minutes to work out a solution individually and then gave them three minutes to work with a partner. After allowing the students to struggle with the solution, the teacher had several students show their solutions on the board. The whole class then discussed the solutions.

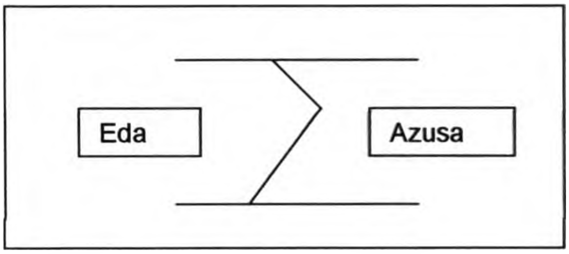

Figure 2: Eda and Azusa's Land
Using the same concept of parallel lines from the triangle question, students need to find an equitable boundary for the land.
Adapted from TIMSS Video 1999

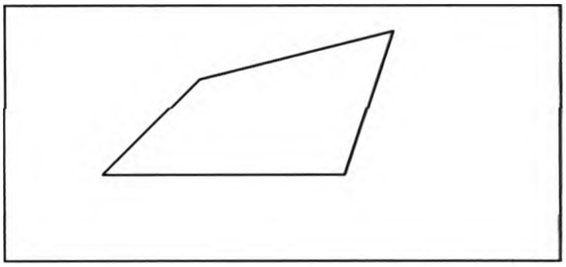

Figure 3: Irregular Quadrilateral
This figure is similar to the quadrilateral presented in the Japanese lesson.
Adapted from TIMMS Video 1999

The teacher then went on to pose the main problem of the day's lesson to the class. He gave them an irregular quadrilateral as shown in Figure 3 and posed the problem: how could one draw a triangle with the same area as the irregular quadrilateral? Again students were given three minutes to solve individually and three minutes with a partner. If the students were not successful after that time period, they were allowed to look at hint cards provided by the teacher and to seek assistance, Figure 4. Those students who solved the irregular quadrilateral problem explained their solutions in a small group setting and were then given extension questions. In one extension question the teacher asked if it was possible to solve a similar problem using different polygons such as a pentagon. Could students get two triangles out of that shape using the same idea of same based triangles inscribed in two parallel lines? What about a hexagon? As students discussed the possibilities, the teacher asked the whole class how they would know if they could solve the problem with two triangles. The conclusion of the lesson was that students would only know the answer if they attempted the problems themselves. The intrinsic value of the Japanese lesson problems is that the problem solving questions are accessible in that they are less wordy and do not *appear* as complex as typical North

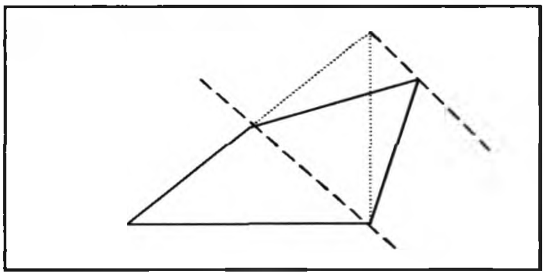

Figure 4: Solution
By using two parallel lines (dashed) the sides of the quadrilateral can be modified into a triangular shape (dotted).
Adapted from TIMSS Video 1999

American textbook and problem solving questions. However, the rich mathematical connections to big ideas, such as fundamental geometric concepts which are demonstrated in the exemplar questions on a variety of conceptual levels, is an essential element to Japanese teaching questions. Students are challenged to increase their conceptual understanding by reasoning through the whole problem solving process. It is important to note that the idea of teaching through problem solving is not new; however it is the manner in which problems are presented and the type of problem solving questions given in the TIMSS Video that made the Japanese teaching methods different from the United States and German lessons shown.

Mason (2000), who was another major source of insight, brings to attention the tremendous role that questions play in classroom communication, especially in the mathematics classroom. Mason and Watson (1998) explain that questions need to go beyond simply "open or closed, [to] promote thought about structure of a concept" (p.5). Mason (2000) highlights the need for teachers to make a conscious effort to think about the questions they ask and specifically to examine the motivation behind the questions asked. In a recent paper on a phenomena approach to teaching and learning mathematics, Mason (2009) states that "mathematical thinking can be initiated through experiencing some phenomena which triggers questions" (p.1). An example, as shown in the question in Figure 5, demonstrates how a central experience of a question or phenomena can help students engage fully with the mathematical concepts and ideas. As with the TIMSS Video study (1995/1999) questions, Mason's (2009) phenomenon of the rolling cup provides an opportunity for students to engage with the mathematics conceptions behind the phenomenon on various levels. At a basic level of conceptual understanding, students develop an understanding of the relationship between the different circumferences of the two ends of the cup and the arc produced during the roll. As students gain conceptual

Rolling Cup	
Phenomena	A plastic cup rolls about on the floor
Questions	What path will the cup follow, and what dimensions of the cup do you need to know in order to predict details of the path? Could two cups of different shapes roll on the same path, in some sense?
Possible Developments	Predicting the path shape is based on experience of the world, though it is not so easy to be precise about why it must have that shape. Predicting details of the curve makes use of properties of circles and the use of the ratios. There are opportunities to work on seeking and expressing relationships and on generalizing [sic] rather than simply dealing with a particular cup.

Figure 5: Sample Phenomenon Question
Presented by Mason (2009, p.2)

understandings, they are challenged by teacher generated "What if . . . ?" and other questions which initiate students into the process of generating their own questions and hypothesizing possible outcomes to their questions. Through the interactions with concepts such as radius, diameter, circumference, arc, and ratios, the application of these concepts, and the related communication both individually and in small groups these questions afforded the opportunity for conceptual development. Not only are students engaged with the concepts but through the problem solving process, their engagement leads to further speculations, questionings, and related hypothesis fostering communication.

Small (2009), another key source of inspiration, provides some practical insight into types of questions, how to modify questions, use of questions, and specific exemplars of questions that one could use in mathematics. These questions provide opportunities to interact with the concepts on a variety of academic levels and also invite communication about concepts. Modifying existing questions, especially those found in textbooks, is one practical method that Small (2009) uses to create deeper mathematical questions. One of her methods for modification includes the rewording and reordering of questions in order to elicit richer response. One such method Small (2009) termed "Making Sense Questions" where the questions are phrased "Why does it make sense that . . .?" see Figure 6. Small's reasoning behind this type of questioning is simple. When one takes away the guessing factor related to the mathematical conceptions, as in the case with the chocolate milk question, that the amounts are related and most likely equivalent, students become free to explain why they know something and are afforded the opportunity to explain their thinking in a less intimidating format. So, when examining the chocolate milk question, students are fairly certain that the amounts are equivalent and what they need to do is state why that might be the case and how they can prove it. Another key phrase used by Small (2009) is "How do you know that . . . ?" which uses the provided answer to the question to allow students to think (e.g. justify, explain), to communicate (e.g. talk, listen, write) mathematically, and to grapple with clarifying their reasoning and conclusions about the question and concepts being investigated.

> *Why does it make sense that chocolate* milk would taste the same if there were 2 scoops of chocolate mix to 3 cups of milk OR 4 scoops of chocolate mix to 6 cups of milk?

Figure 6: Making Sense Question Example
This question was used to illustrate the *making sense question types during a* lecture by Dr. Small.
Small (March 9, 2009)

It is important to note at this time, that the preceding question examples presented by the TIMMS (1999) video study, Mason (2009), and Small (2009) are exemplars created by researchers who have deep knowledge and experience with the development and implementation of conceptual questions. As such, these conceptual question exemplars are the standard to which other conceptual questions are compared, such as those created in the rate and ratio unit.

4.4.3 Conceptual Questions: Explanation

As stated in the introduction to this thesis, there are several types of questions that are used in the mathematics classroom: questions that focus on procedure, formulas, and the process of what the next step should be; instructional questions that focus on numeric responses; problem solving questions that adhere to a standard model of "What do I know?", "What do I need to do?"; and conceptual questioning. Conceptual questioning encompasses multiple aspects of a variety of questions as it focuses on pedagogical practices rather than a type of question and is therefore difficult to define. By its nature, conceptual questioning is more of a skill that requires thought and practice as it is malleable to the mathematics strand, type of question, and desired form of communication sought. Conceptual questioning encourages mathematical thinking asking questions such as "Give me an example of . . . ", "Is it sometimes, always or never true that . . ." (Mason & Watson, 1998, p. 3). Detailed planning is required in order to effectively implement conceptual questioning because this type of questioning does not necessarily flow naturally unless one has had extensive practice using conceptual questioning. As well, this type of questioning appears to require a foundational understanding of the subject area, or at least of the concepts at hand and those that are related. As commented on earlier in this chapter, both Mason (2000) and Heibert & Stigler (1999) discuss the importance of the type of questions used in the mathematics classroom. Their examples are rich in the way that mathematical concepts are developed, which comes from the wealth of mathematical knowledge of the researchers, and a teaching environment, in the case of the Japanese teachers shown in the video, that is conducive to those questioning techniques.

As suggested by Small's (2009) chocolate milk question, one place that a generalist elementary teacher can begin the journey towards the creation of conceptual questions is to glean ideas from existing resources. Although the resources available at the intermediate level are slim, applying some forms of modification, even to textbook questions, can begin to aid teachers in the use of conceptual questioning techniques. In the above three examples forms of modification includes using question starters that encourage thinking and explanations, posing a series of questions that center on big ideas and fundamental concepts, and using presentations of contexts that students can easily think and communicate about. Conceptual questioning involves a huge thinking component on the part of the classroom teacher. An examination as to why a question is being asked and what mathematical concept is being examined becomes fundamental. Therefore, having a resource from which to base conceptual questioning is helpful bearing in mind that to form ordinary textbook questions into conceptual questions involves investigating the reasoning behind asking the question, such as focusing on creating opportunities for communication, mathematical thinking, and concept development. In the rate and ratio unit, the conceptual questions designed were leading towards generating students' communication and developing conceptual understanding based upon the ideas presented by those such as Heibert & Stigler (1999), Mason (2009), and Small (2008). However, developing conceptual questioning is a learning process. In the Findings section I will explain more about the extent to which the questions developed for this rate and ratio unit reached the level of depth that the research exemplars depicted at the start of Section 4.4 portray.

4.4.4 Conceptual Questions: As planned for the Learning Study

4.4.4.1. Introduction of concepts.

The unit on rate and ratio was introduced with a graphic question about popcorn consumption, which was adapted from Addison Wesley 7[1] mainly due to its graphic nature, see Figure 7. The series of ratios related to popcorn consumption was presented to the students in four different formats: 2 out of 3 people surveyed preferred Super popper popcorn (in student language, Ad.1), twice as many people preferred Super Popper popcorn (Ad. 2), 7036 out of 10554 people surveyed preferred Super Popper popcorn (Ad. 3), 3518 more people preferred Super Popper popcorn (Ad. 4).

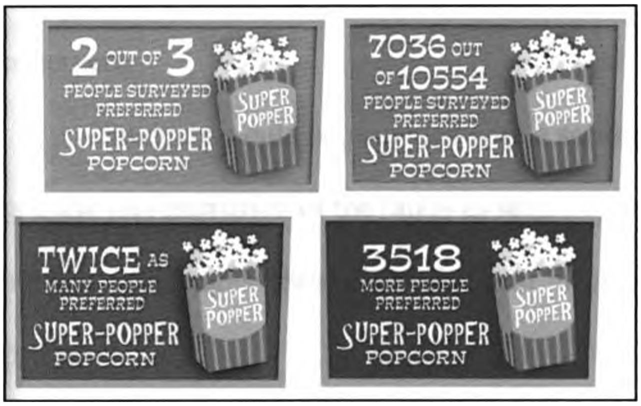

Figure 7: Popcorn Advertising Question
Used with permission from Addison Wesley 7

At the introduction to the rate and ratio unit, the teacher-researcher perceived that the popcorn question would allow students to interact with ratios in a non-threatening manner. From a conceptual development standpoint, the popcorn question was intended to lead to the development of an awareness of both mathematics and linguistic aspects of how ratios may be represented. For example, the statement "twice as many" was

[1] Most teachers refer to textbooks by the publisher's name rather than by author or title. Since that is an educational trend among teachers, and there is a desire to make the following sections flow in a readable and practical manner, the textbooks cited will be referred to by the publisher's name and grade level. Appendix F provides a chart which correlates the textbook publisher and authors for ease of reference.

linguistic while the statement “2 out of 3” was mathematical. The use of what is commonly referred to as a think, pair, share strategy was employed to encourage interaction from all students. Through the accountability built into and opportunity it affords to all students to talk and explain this strategy, it was expected that students would begin to develop communication skills with their peers rather than letting a few students in the class answer all of the questions while the majority were passive observers.

Students would be provided with a series of prompting questions related to the graphic representation of the ratios as seen on the smart board (Appendix G) and on the student work pages. (See Appendix H for black line master version of the question prompts)

1. Which advertisement(s) is/are most effective?
2. Do the advertisements have the same numerical value?
3. How do you know that Ad.1 and Ad. 3 are giving the exact same information?

The first question prompt was designed to encourage the communication of responses which could be entirely opinion based. The second question prompt invites the opportunity for comparison and the development of the idea of equivalency within the ratios. The teacher-researcher and teacher-participant saw the third question prompt presented as a conceptual question for several reasons. It is not asking the student to identify which ratios are the same, but asks for an explanation about how one would know that the statement was true. The “how do you know . . . ?” question starters, are likely to create the opportunity for students to demonstrate a deeper understanding of different representations of ratios while highlighting the development of increased communication skills (Small, 2009).

The teaching component of conceptual questioning, mainly teacher modeling and fostering of student oral communication, begun in the introductory popcorn question continued with the counter question shown in Figure 8. The question was also modified from Addison Wesley 7[2]. Following the counter question were the group of toy vehicles and camel questions. These two questions were significant in that they provided a review and built upon the junior curriculum expectations of fractional comparisons as outlined by the Ontario Ministry of Education curriculum document.

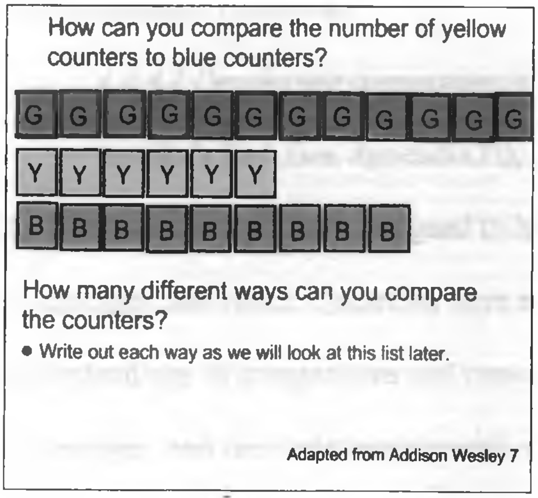

Figure 8: The Counter Question
Adapted from Addison Wesley 7

At the Grade 7 and 8 levels, the fractional comparisons are built upon the junior concept and transferred to fraction, decimal, and percent comparisons and equivalencies. Thus to develop the concept that ratios are comparisons of either part-to-part or part-to-whole, the grouping of toy vehicles was used, also an adaptation form the Addison Wesley 7 text. To practice this connection further, students participated as a class on the camel comparison questions. In the camel questions, two sets of camels were compared; Arabian camels with one hump, and Bactrian camels with two humps. In addition, some camels were sitting while others were standing. Students were asked to create various ratios using the four types of data. The

[2] The question was changed from round coloured dots to coloured tiles largely due to the fact that both teacher participants had coloured tile manipulatives in their classrooms. Thus, the students would be able to physically manipulate the tiles and create various ratios in small groups.

ratios were then classified as part-to-part or part-to-whole. Although the counters, vehicles, and camel questions were not conceptual questions, but instructional questions with mainly numeric responses and some unit terminology, they were included because they reinforce necessary foundational concepts for the unit. Further, it is important to note that more "routine" type questions such as response questions still have a place in the mathematics classroom.

4.4.4.2 Developing connections to other concepts.

Annika's Fish (see Appendix H), a series of questions adapted from a data set in the Nelson 8 textbook, was designed to help develop comparisons between fractions, percentages, and ratios. Questions were modified to bring forth conceptual understandings of comparisons and conversions between fractions, decimals and percentages, and their relationship with ratios. For example, the question: "Explain how you know that Annika's list follows the sales person's recommendations?" elicits an explanation of how and why her list of fish is within the limits stated by the sales person. With the question wording "how do you know" the guess work as to whether or not Annika's list matched the sales person's recommendations is removed. This provides students with the opportunity to explain their reasoning by using the data they had created in the previous questions. Students also had to demonstrate an understanding of how they knew their answers using mathematical concepts such as fractions, ratio and percent.

The question, "How many chocolate chips are in my cookie?" and the ratio table demonstration that followed Annika's Fish questions are focused on the concept of ratio comparisons and foundations in the concept of equivalent ratios. A copy of this activity is included in Appendix H. In the chip and cookie activity, each student was given several cards with picture groups of chocolate chips and cookies. First, students were asked to write a ratio for the chips to cookies. Second, students shared their chip to cookie pictures and ratios with a partner. The partner groups were then instructed to find as many chip to cookies ratios that were the same as they could by examining other groups' ratios. The term equivalence was not yet introduced. The comparing of ratios so that equivalency could be defined required an increase in skill level.

4.4.4.3 Further exploration of ratios and rates.

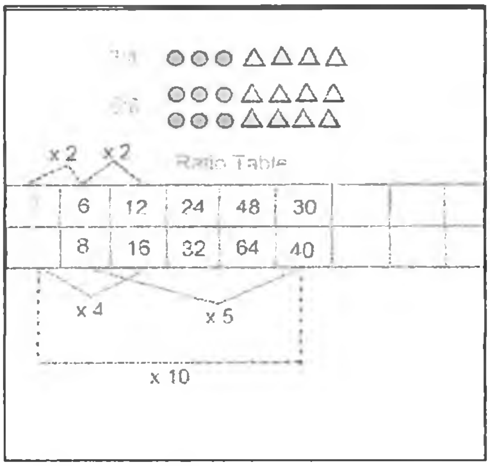

Figure 9: Ratio Table
A ratio table easily demonstrates through direct representation, steps of simple repeated and/or related multiplication to find equivalency.

As equivalent ratios were further explored the idea of the ratio table was introduced. Ratio tables were a method of finding equivalency that was new to the teacher-participants. However, ratio tables are not a new concept. Van de Walle and Lovin (2006) and Small (2008) both describe and demonstrate the principles of the ratio table in their instructional books about teaching mathematics. This concept is also related to T-charts or table of values in the junior curriculum as well as to gradient and differences in the high school curriculum. The tables have the potential to be used to find equivalency based on repeated multiplication (see Figure 9) of the dotted arrows across the top of the ratio chart. Under the bottom row of the chart, the variety of dashed and solid lines represents alternative multiplicative actions. The interesting idea surrounding the ratio table is that it resembles equivalent fractions and mimics the multiplication matrix which students frequently use throughout the year. The use of familiar representations is more helpful for weaker students.

The intent of the next set of tasks and related questions: pattern blocks, hot chocolate, cement ratios, and classroom ratios, was to further develop the concept of equivalent ratios through teacher-directed, as opposed to student-posed, conceptual questioning. The pattern blocks were used to demonstrate equivalent ratios related to area, which was a task adapted from the Nelson 8 textbook. The concept development sought from this group of questions was related to the Ontario Ministry of Education expectations of solving problems involving quantities and proportions. The pattern block question, adapted from Nelson 8, involved the use of manipulatives that many students were already familiar with. Thus, conceptual questioning and teacher scaffolding of the task was done on an individual or small group basis as necessary.

The hot chocolate question, adapted from Addison Wesley 7, was mainly teacher directed as it was used to demonstrate several more ways of finding proportional relationships between two quantities, mostly from a numeric perspective. Comparing two quantities as two different ratios was a key factor in this task and conceptual questioning was related to the comparisons found.

Students' understanding of equivalent ratios was further challenged when a third term was introduced into the ratio such as the cement questions, which were adapted from Nelson 7. In the cement questions, the ratio of bags of cement to sand to gravel was explored. Conceptual questions were introduced following Small's (2009) model of question starters and related to potential problems faced on the job site. For example: "If the ratio of *cement:sand:gravel* was 1:3:4, and Mike arrives to the job site and finds 6 bags of sand, how does he know that he needs to ask his boss for 8 bags of gravel?" The intent of the cement questions then, were to provoke an authentic real life element to what students had been doing thus far with ratios. The classroom ratio questions, adapted from Small (2008) were designed to take a visual representation approach and pair it with the numeric approach for finding equivalency. The cement questions marked the third set of questions related to equivalency. Therefore, students were given time to investigate on their own and share with peers before any instructional hints were given.

4.4.4.4 Problem solving

The concept of problem solving using the knowledge leaned with the calculation of unit rates was introduced through the examination of students' heart rates. Students measured their own heart rates and used the data to complete a graph. This assignment was taken directly from the TIPS4RM document, see Appendix H. Heart rate was then also used to introduce the idea that rate was similar to ratio in that it compared two or more values but that it was different because rate involves two different measures. Another real life component for the exploration of rate was demonstrated with the examination of grocery store item tickets, commonly known as tags (e.g., Figure 10). Students were provided with a series of tickets for related products and examined the various rates on the tags. A discussion was held on how the rates were usually given per 100 grams and that this standard amount helped consumers select the best deal on a product. Conceptual questions such as: "If you usually buy the club packs of yogurt cups,

Figure 10: Grocery Store Tag
Tags were used to discover real life applications of rate.

but the variety packs of 12 are on sale, how can you decide what is the better deal?" were formed around the purchasing of products.

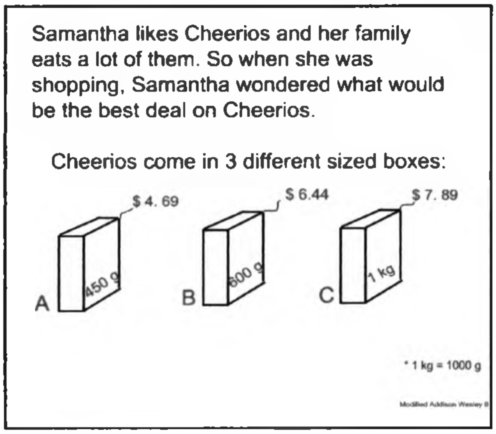

Figure 11: Cereal Questions
This was the starting page for the cereal box questions that initiated rate comparisons.
Adapted from Addison Wesley 8

The culminating task dealt with the examination of cereal box questions which were adapted from the Addison Wesley 8 textbook (Figure 11). Along with asking students to make comparisons as to which box was the best deal, the conceptual questions for the task also prompted students to decide under what circumstances would one choose to buy a certain sized box of cereal even though it may not be the most cost efficient with the greatest amount of cereal for the least amount of money.

The final task in the rate and ratio unit was twofold. One class participated in a percentage problem (see Appendix H) and the other completed a problem solving question on winter sports (also see Appendix H). Students in both classes were challenged to apply their knowledge about rate, ratios, and percentages to their problems. Key to finding a solution was the ability to communicate effectively.

In the percentage question, students were required to explain their thinking about sale prices and what difference, if any, the order of taking percentages off of regular priced merchandise would make. Students justified their responses both in written format and by completing computations as proof that their response was correct.

The winter sport problem was unique in the manner in which it was introduced. Students were divided into groups of four. Each person in the group received a card with a piece of information and a question. The group members could only share their information orally as no one was allowed to look at another person's card. This forced students to communicate not only their question but also the math that was contained on their card. Each group had a problem solving template which had to be completed. Since key pieces of information were necessary, every student had to communicate. The idea of having only one paper limited students so that they also had to communicate the why and how of their mathematics calculations and reasoning.

Table 2 is a summary of the conceptual questions included in the unit. It highlights each questions' unique trait, skill targeted, intent of question, and planned teaching resources. The questions, tasks, activities and prompts planned for the rate and ratio unit were carefully thought out by both the teacher-researcher and teacher-participant. The intent of these questions was to develop a base from which conceptual understanding could be generated and through which quality communication could be elicited. Since the rate and ratio unit was an initial attempt at creating and using conceptual questions, there were many obstacles to overcome. However, the learning study process afforded these educators the opportunity to share ideas about question techniques, how to implement different questions, and discuss what was successful and what further modifications might be beneficial in the future. The highlights of triumphs and frustrations will be discussed in the following sections of this chapter.

Table 2

A Summary of Rate and Ratio Conceptual Questions and Problems

Question	Trait	Skill	Intent	Teaching Resources
Popcorn Advertising Ratios	Graphic, Familiar content, How do you know . . .? prompt	Representing ratios	Introduction to unit, Develop communication skills	Think-pair-share strategy
Counter Questions	Graphic, Manipulatives	Part-to-whole and part-to-part ratios	Self discovery of types of ratio comparisons	Think-pair-share strategy, Manipulatives
Toy Vehicles	Graphic, Familiar content	Part-to-whole and part-to-part ratios	Reinforce use of terminology and application of skills	Teacher Modeling of terminology
Camel Question	Graphic, Virtual manipulatives	Part-to-whole and part-to-part ratios	Reinforce use of terminology and application of skills	Think-pair-share strategy, Virtual manipulatives
Annika's Fish	Chart, Graphic, Real life application	Percentages and ratio comparisons	Application of new skills in ratio and communication/ justification	Think-pair-share strategy
Ratio Table	Chart	Equivalent ratios through linear chart	Introduction of a new method of determining equivalency – based on familiar context of equivalent fractions	Teacher modeling of strategy
Cookies to Chips	Graphic, Manipulative cards with graphics of ratios	Equivalent ratios	Application of equivalency finding skills	Think –pair- share strategy
Pattern Blocks	Graphic, Manipulatives, If . . . then question prompts	Equivalent ratios	Application leading to more abstraction of questioning	Think-pair-share strategy, Manipulatives
Hot Chocolate	Graphic, Virtual manipulative	Equivalent ratios	Introduction to different methods of finding equivalent ratios and making comparisons	Teacher Modeling of two strategies for equivalency
Classroom Ratio	Symbolic representation, If . . . then, Under what condition? Question prompts	Equivalent ratios	Problem solving , Application of skills in more abstract form	Think-pair-share strategy
Cement	Graphic, Real life application, How do you know . . . ? question prompts	Equivalent ratios	Application and Justification of equivalency	Think-pair-share strategy
Heart Rate	Real life application	Rate, and comparisons of rate	Graphing rate comparisons	Think-pair-share strategy

Grocery Tags	Graphic, Real life application	Rate and comparisons of rate	Explanation, Justification, Practice for later questions	Teacher modeling to independent work
Cereal	Real life application, Under what conditions . . . ? question prompts	Rate and comparison of rate	Explanation, Justification, Reasoning and proving	Think-pair-share strategy,
Percentage Problem	Real life application	Percentage relationships and ratio comparison	Problem Solving, Justification	Individual application of skills
Winter Play	Familiar Context	Equivalency, percentage	Problem Solving, Communication, Explanation	Cooperative group effort

4.5 Conclusion

To summarize, action research provides a framework for this thesis through which data is viewed and interpreted. The methodologies of phenomenography and variation theory work simultaneously with one another. Variation theory has developed out of phenomenography as research has shifted focus to attend to the variations in the study of the experience of a phenomenon (Pang, 2003).

Learning study by its nature adopts a personal element by examining one's teaching practice, therefore lending itself well to the framework of action research. Learning study also has foundations steeped in both phenomenography and variation theory. Thus learning study becomes a natural outflow for the research approach used in this thesis. The methodology of variation theory guides the data analysis.

5. Data Analysis and Findings

Action research provides a framework for the learning study on conceptual questioning techniques and sensibilities. The benefit of the framework of action research is that it allows for the pairing of teaching with self-reflection and of theory with practice (McNiff, 2002). The framework of action research is paired with the methodology of variation theory in order to examine and analyze the data and findings of the learning study on conceptual questioning. In this learning study, variation theory focuses research and analysis on the variations of the experience of a phenomenon of teacher questioning techniques and sensibilities, conceptual questioning in particular, and resulting student responses.

Data for the analysis was obtained through: informal interviews and conversations with teacher-participants before, during, and at the end of the learning study; self-analysis; notes based on learning study meetings; pre learning study questionnaire; lesson observations of small group work, class discussions, teacher-student conferencing, and student work. This research project incorporates a variety of data that is qualitative in nature including interview narratives, individual and joint reflection notes and observation of students' activity and participation. In this chapter the main interview, reflection, and observation findings are presented. The purpose of the teacher-participants' narratives in the interview reflection section is to ascertain teacher growth, changes in teacher understanding of communication, conceptual questioning techniques and sensibilities. These aspects of teaching are related back to the thesis question, which examines of the ability of a generalist teacher to elicit student communication through conceptual questions. The personal narratives also include identifying a comfort level with conceptual questioning techniques and sensibilities. The themes of teacher fears, ties to other subjects, and professional benefits add further to the discussion. The observation and reflection section delineates where the students have made progress with communication and their interactions with the new style of conceptual questioning.

5.1 Interview Narratives

The learning study interview questions, which were completed after the rate and ratio learning study, are referenced in this chapter by condensed headings: Benefits of participation in the learning study; Changes in understanding of communication; Transferring teacher questioning techniques; Changes in practice; and student communication skill development. The learning study interview with the teacher-participant was conducted as a conversation between the teacher-researcher and the teacher-participant. In the interview where the teacher-researcher and teacher-participant were one in the same, the interview questions were given by a third party as the teacher-researcher discussed orally and typed responses on the computer. Since the interviews were not tape-recorded, the interview statements are based upon specific, detailed notes from the interview sessions in which the researcher copied both verbatim and in coded short hand the conversation statements. As such, the teacher-researcher wrote the responses to the interview questions from an overview of the questions asked. The data from the interviews is thus represented in a personal narrative format. The narrative format allows aspects of the classroom learning community to be explored following a complexity theory framework with an emphasis on interactions and connections within the learning systems. The narrative form creates a form from which the examination of adaptations by both teachers and students to the complexity of conceptual questioning can be analyzed. Central to this format is an accurate reflection of the teacher-participants' thoughts, ideas, and experiences related to the phenomenon of conceptual questioning as conveyed through their narrative. Analysis of the data involved reading and re-reading the narratives looking for themes and specific instances that exemplify the themes. Three central themes about the learning study emerged from the interview process which are discussed following the interviews: the identification and acknowledgment of general fears teachers have about teaching mathematics; significant ties of communication and teaching pedagogies between teaching mathematics and teaching other subject areas; the professional benefits of teaching mathematics using conceptual questioning. These themes are also presented in a narrative format in the reflection section by the teacher-participant. The format is designed to emphasize parallels and differences among teacher experiences and student responses.

5.1.1. Interview Reflections: Bear River, With Ms. Rock

Benefits of participation in the learning study.

Ms. Rock had this to say about the benefits of participation.

> The question techniques used were one of the most beneficial parts of the study. I was able *to gain more confidence* with a broader range of questions and my questioning became more effective. I also felt that an added benefit was the *pictorial nature of the tasks and questions*. Most of the tasks and questions contained diagrams or picture references which engaged the students and supported students who usually tended to struggle with mathematics concept development. One dramatic example of the benefits of pictorial representations was with a Grade 7 boy who was on an individualized education program. His

work in mathematics had been modified to the Grade 5 programming level throughout the year, but paralleled the topics taught to the other students. In the rate and ratio unit, he became engaged especially with the camel question. He was able *to develop and articulate an understanding of the difference between part-to-part and part-to-whole ratio comparisons* and even completed textbook questions at the Grade 7 level on this concept.

Changes in understandings of communication.

Ms. Rock also commented in depth about changes in her understanding of communication.

> My ideas about communication have changed over the duration of this unit. Although I know what good communication looks like in literacy, I was somewhat unclear about *what good communication may look like in mathematics.* I believe I have become more *effective at using leading questions and eliciting quality communication.* In my mathematics lessons prior to this rate and ratio unit, I would tend to have a teacher-directed lesson in which I elicited student comments and responses intermittently. Generally it was the same three or four students who, even with wait time, always answered the questions. Other students only participated when directly called upon. *Now I actively strive towards incorporating more student talk* through pairs and small group activities. Another change in teaching practice is the use of leading questions. By using the term leading questions, I am trying to differentiate between teaching students as telling and teaching them through guided discovery. *I am able to use the leading questions to help my students think deeper about mathematics and to develop*

understanding rather than trying to give them understanding. A recent example would be in my measurement unit where I have now incorporated activities that help develop area formulas rather than giving students the formula and working strictly on application. I already do this in literacy, since my strengths are in language. *However, I had not done this consistently in math before*. This may be because I had been teaching math the way I was taught, having been taught in the Socratic manner of the British school system. There is a certain comfort level in teaching that way. For example, I have learned to think about and complete mathematics computations and concepts in a certain way. Trying to teach using leading questions, then can become tricky. What if I was asked a question by a student that I was unsure about? I also sometimes wonder if I was asked the wrong question, could I get confused with my own understanding?

Transferring teacher questioning techniques.

The interview also involved a question on whether teachers though what they had learnt during the rate and ratio unit would be transferred to other units. Ms. Rock was of the view that questioning techniques could be transferred.

I don't see why you couldn't transfer the questioning techniques from the rate and ratio unit to another unit of mathematics. If the questioning techniques worked with a unit that our students traditionally struggle with, why would it not transfer to another less difficult unit? An example would be teaching the Number Sense and Numeration unit on integers. Although I have not routinely used leading questions in other units in mathematics, *I have used leading questions, some of which are conceptual questions, and provided discovery tasks in my integers unit.*

Changes in teaching practice.

Ms. Rock also commented about changes in her teaching practice.

> Since participating in the learning study on rate and ratio, *my comfort level with the use of leading questioning techniques has improved.* I also feel that I have *more appreciation of how and when students communicate* their understandings of mathematical concepts. I think the discussion *allowed my students to engage more, especially the small group work and paired activities* where they were able to grapple with the concepts. This was evidenced by one student who had average mathematics capabilities but would never voluntarily answer a question in class. After the small group activity during the popcorn question, this boy volunteered to take on the role of presenting the group's work in whole class discussion. I think this was due in part to the confidence built in the small group setting.

Ms. Rock emphasized the changes in using leading questions and offered an example of a student who benefited from communicating in a small group setting.

Student communication skill development.

> Student *improvement of communication and expression of their conceptual understandings were evident in their written work.* The questions that they completed for the rate and ratio unit fostered a *focused communication on explanations and description* as opposed to the more traditional textbook question that emphasizes knowledge and understanding, or basic application questions and answers. The *assessment for understandings and the assessment of understandings in this unit were both more anecdotal* than I usually have in

> mathematics as I was able to conference with students individually about what they had learned and explore and apply concepts with them.

The benefits that Ms. Rock experienced were seen in both written work and student conferencing.

5.1.2. Interview Reflections: Fox Wood, With Ms. Hare

Benefits of participation in the learning study.

Ms. Hare mentioned collaboration and in-depth reflection as a major benefit of the learning study.

> One of the most beneficial aspects of participating in the learning study was the *opportunity to collaborate with my peers*. Teaching can become a lonely profession and the opportunity to discuss teaching pedagogy with another peer is rare but important. Although I routinely examine what was successful and less than successful in my daily teaching, *a more in depth reflection of my personal practice* was a welcomed opportunity.

Changes in understandings of communication.

As was the case with Ms. Rock, Ms. Hare gave a detailed account of the changes in understanding or communication that resulted from her participation in the learning study.

> I would have to say that my understanding of communication has changed. One of the major challenges for me in this unit was in the creation of defining parameters for what communication is and what it should look like in the mathematics classroom. Originally I thought I knew what communication was, that defining it in terms of mathematics would be as easy as defining it in literacy.

For example I thought that communication was: can the student write using the mathematics terminology and use the formula(s)? However, *I have come to realize that, that is only a small part of communication. There is the whole aspect of reasoning, justification and proving as well as the whole process of talking out ideas in order to develop concepts.*

Communication is a broad topic. Exactly what communication looks like in mathematics is open to some interpretation. As my class and I worked through the questions in the rate and ratio unit, *my expectations for student communication were raised* and I was consistently impressed with the ideas and connections my students were able to make. For example, I was surprised with a pair of students. One student was quick to figure mathematical concepts out, but lacked the necessary communication skills to explain how he arrived at his conclusions. The other student in the pair was one who often remained silent preferring that others explain or the answer questions. However, while working on the popcorn questions, these two boys quickly realized that all four of the ratios were equivalent and then set about demonstrating how they knew that to be true. *The ability to clarify their thinking as a pair was unusual for them* and proved to be an inspiration for me as a teacher.

In addition to detailing aspects in which her understanding had changed, Ms. Hare gave an example of how the positive changes in two students with different communication needs – a student who even when strong in mathematical thinking would rarely participate in oral discussions and another who was simply quite reluctant to participate in activities that required mathematical thinking and communication – challenged her understanding of communication.

Transferring teacher questioning techniques.

Ms. Hare was also positive that teacher questioning techniques were transferable to other strands in mathematics but was rather conservative about the length of the process.

> Questions become conceptual as they are deliberated on by the teacher and molded to lead students to a deeper understanding and more articulate communication. Can this be applied to other mathematics units? *Of course, but it will require a concerted effort on the part of the teacher to create the conceptual questions and anticipate when those questions are best used.* Due to the nature of conceptual questioning, the *application of these techniques would be a long term process rather than something that could be implemented instantaneously.*

Changes in teaching practice.

Ms. Hare considered her level of awareness of forms of questions, and the use of questions to be heightened by teaching the rate and ratio unit.

> My teaching practices have evolved with the creation and implementation of the rate and ratio unit. I have developed a *heightened awareness of the various forms of questions I am asking in class and the type of answers* that I am eliciting. I often find myself pausing before asking a question to *think about the purpose of what I am asking*. Another change in teaching practice has been the use of a technique called think alouds. The idea is for the *teacher to model their thinking by communicating their thought process out loud for students to hear.* In my current mathematics unit, I have also been focusing on creating opportunities for

students to collaborate on mathematical concept development, while actively structuring my mathematics lessons so that *there is less teacher talk and more collaborative sharing and communication of ideas and concepts.*

Student communication skill development.

Ms. Hare shared her pleasant surprises.

In mathematics classes, there tends to be the handful of students who always answer the teacher's questions while the rest of the class remains relatively disengaged. This was not the case with the rate and ratio unit. In my classroom *I was surprised by the number of students who volunteered to answer questions* for the rate and ratio unit. This was *especially noticeable with some of the higher needs students who became actively engaged in both discussions and in their independent work.* One student in particular stands out as she was highly verbal by nature but on an individualized education program which included modifications in mathematics programming. This student could talk circles around any topic in literacy, although not necessarily in a logical manner. However, she would often voice her displeasure with math as she lacked the conceptual development to communicate productively. During the rate and ratio unit, there were several instances [camel question and heart rate] where *this student triumphantly announced that she "got the math" and could share her answers* with the class. Unfortunately this was the last unit in math in June as it would have been interesting to see if the changes sparked by this unit would be transferred by these students to other units in mathematics.

To Ms. Hare, the changes in students who were weak in mathematics were more noticeable.

Data from the interviews after the learning study suggest that the teacher-participants' comfort level and confidence with conceptual questioning increased. Both teachers attribute the changes not only to conceptual questioning but also to the other teaching strategies such as the think-pair-share. The teachers were surprised that both the weak and strong students appeared to have benefited from the use of conceptual questioning. Although the teachers were well aware of the time, effort, and challenges of trying to use conceptual questioning they agreed that the process was worth the effort.

5.2. Reflections: Identified by Theme

As stated earlier in this chapter, there were three central themes which emerged from the analysis of the interview and reflection data: general fears and limitations of generalist elementary teachers; relationships identified between other subject areas and mathematics; and professional benefits identified by both teacher-participants. The narratives from the individual teachers' reflections are presented by theme.

5.2.1 *Fears and Limitations of Generalist Teachers: Ms. Rock*

> There are several challenges that teachers face when trying to implement conceptual questioning. I believe that the primary issue would be one of *confidence*. Experienced teachers, such as myself, have a greater level of confidence which affords them the opportunity to take greater risks or experiment more with their teaching methods. Teachers tend to always teach a subject area in the same manner. This is true especially in mathematics. It is only when teachers are challenged, or presented with a different perspective, that they begin to see

things in other ways. However, *attempting to change the way one teaches is a difficult task and it can shake you at first*. For those *new to teaching*, the difficulty in implementing newly acquired questioning techniques is increased as they are also faced with the challenge of general teaching practice, behaviour management, and teaching methodology.

Beyond the issue of confidence, looms the issue of *time management*. Although I found it profitable to use different types of conceptual questions, there was *a significant amount of time needed by students* to complete these questions. Time constraints come in a variety of forms. There is the rush to cover topics and numerous strands before report cards are due to the office, the intensity of an enormous mathematics curriculum, grappling with what should be taught, and deciding what is not going to be covered. There is also the limit of teaching time; actual minutes in the daily cycle and the number of interruptions for various reasons. Instructional time is further compounded by the type of questions used. Teachers have *a finite amount of time to plan*, and planning time needed far exceeds the number of minutes provided during the typical work day. Planning for the use of conceptual questions takes more time than planning a unit or lesson in a more traditional manner.

To Ms. Rock, lack of confidence, lack of experience and limited time are major challenges to implementing a new and more involved teaching method. Whereas to Ms. Hare the constraint of time is aggravated by the teachers' lower level of comfort with mathematics and by the numerous changes in teaching practice that teachers are encouraged to implement at the same time.

5.2.2 Fears and Limitations of Generalist Teachers: Ms. Hare

Teachers are not immune to the *fear of mathematics*. As elementary teachers we are required to be experts in a multitude of subject areas. This becomes increasingly difficult, especially at the intermediate level where the curriculum content is laden with technical and subject-specific concepts, definitions as well as jargon. Currently *teachers are consumed by trying to implement changes* in teaching practices which are promoted at the district and provincial level. For example, the mandated creation of a balanced literacy and balanced mathematics program coupled with teaching specialized content that is intense such as science, places an enormous strain for teacher planning and implementing such programming. Teachers are also peppered with teaching methodologies such as differentiated instruction. All of this requires re-thinking of classroom activities and pedagogy. Once more, it also requires an *exorbitant amount of time for planning and structuring of the classroom for success*. Therefore, any weakness in a subject area becomes magnified. To compensate, many teachers teach mathematics the way in which they were taught mathematics: teacher example on the board, practice an example as a class, followed by multiple independent rote questions. *Changing the way you teach a subject area like mathematics is a slow process*. It involves lots of trial and error, the ability to accept both failure and success, and learn from them.

> Learning to teach mathematics using conceptual questioning is not an easy process. There is an *element of risk taking on the part of the classroom teacher.* The mathematics teacher needs to have a *certain comfort level* with the mathematical content before conceptual questioning can take place. If the teacher lacks an understanding of the mathematical concept behind the questions, then there is little hope of that teacher being able to provide the necessary foundation for conceptual questioning.

5.2.3 Ties and Comparisons to Other Subject Areas: Ms. Rock

Both Ms. Rock and Ms. Hare bring insightful points of view to the relationship of mathematics and other subject areas based on their academically diverse backgrounds

> My experiences teaching math in this unit from a conceptual questioning standpoint *is like teaching students to write in literacy.* When teaching writing a teacher needs to, even at the intermediate level, go through the pre-writing process with the students. The teacher models how to write, and often lays a foundation of knowledge and ideas with students so that when they are writing independently, they have something to write about and the skills to write it. The *mathematics unit had a large oral focus which acted like the pre-writing stage in literacy.* Since the students and I talked about the conceptual questions, the students knew what to do when it came time to work independently. This lessened the students' anxiety levels about the rate and ratio concepts. Rate and ratio are not easy concepts in mathematics, but the students got it! They grasped the concepts without realizing how hard the rate and ratio principles really were and the students used the mathematical terminology with accuracy and fluency.

5.2.4 Ties and Comparisons to Other Subject Areas: Ms. Hare

> Traditionally mathematics is the one school subject area in which it has been socially acceptable for one to not only admit his or her lack of skill, but to wear that lack of skill as if it were a badge of honor. I have had parents comment that they were never good at mathematics so they wouldn't expect their son or daughter to be good at it. Yet I have never had a parent say the same thing about other subjects such as reading. Thus the fear of mathematics is transferred and even promoted from generation to generation. For some students, taking risks in mathematics becomes next to impossible, especially for the communication of concepts and ideas and even answering some conceptual questions.

Whereas Ms. Rock likened the introduction of conceptual questioning to the pre-writing stage in literacy, Ms. Hare commented on the uniqueness of the aversions towards mathematics that appears to limit students' oral communication in mathematics.

5.2.5 *Professional Benefits: Ms. Rock*

> The mathematics rate and ratio unit provided me with a *different perspective on teaching mathematics* since it was heavily based on conceptual questioning techniques. I welcomed the *opportunity to work collaboratively* on the unit, especially since the rate and ratio mathematics unit was one that I have less of a comfort level teaching. As the unit progressed, both the students and I enjoyed working with the various questions.
>
> In the more literacy-based subject areas such as literacy, history, geography, and even science, teachers tend to ask open-ended questions with ease. This is not

necessarily the case in mathematics. In mathematics there are usually definitive answers, either right or wrong, and of course these answers are numeric values. Traditionally the teacher's role has been that of a leader, the font of all right mathematical answers and keeper of the methods and rules for reaching those answers. With conceptual questioning, what needs to happen is a mind shift in teaching mathematics. Teachers need to let go of the notion that they are all-knowing. They need to *create a mathematical learning community* in which the teacher and the students both partake, on some level, in the learning journey. Despite the challenges of changing teacher questioning techniques there are many benefits to applying different questioning techniques. Changing the types of questions used in the mathematics classroom, *allows more access to mathematical concepts on one level or another for most students*. If teachers, however, do not allow students enough time to work with questions or problems presented, the results would be that the students were short changed in learning how to reason and thus communicate in mathematics. Therefore, teaching mathematics using conceptual questioning has its challenges but it is worth the effort and time.

5.2.6 *Professional Benefits: Ms. Hare*

I enjoyed working on the mathematics rate and ratio unit, although there were also challenges. The use of conceptual questioning requires some content knowledge, flexibility, and practice on the part of the classroom teacher. There is a lot of pre-planning of questions as the teacher needs to approach answers from different perspectives and predict how some students may respond. The questions do not necessarily flow naturally without some practice.

> One thing I found surprising was just how oral the rate and ratio unit was. When teaching, there is a tendency for the teacher to do a lot of the talking during lessons. The emphasis on the think, pair, share strategy and other collaborative groupings provided *multiple opportunities for the students to discuss and communicate ideas and provided some forms of accountability* that do not exist in a more teacher-directed lesson. Thus there was a *huge collaborative effort with students willingly sharing ideas* and examining various solutions. The exciting fact about this unit was not only how actively engaged the students were, but also in the level of success. All my students, even those who struggled academically in mathematics all year, were able to grasp rate and ratio concepts at some level.

When commenting about the professional benefits of the learning study, both the teacher-researcher and the teacher-participant noted the benefits for themselves and for their students, specifically the benefit of collaborative effort and sharing ideas. The teachers noted the learning study was worth the effort and time. They also alluded to several key factors, on the part of the teacher, which were crucial for conceptual questioning to take place. The key factors include teacher's comfort level which is related to teaching experience, the teacher's conceptual understandings of the mathematics being taught paired with an understanding of why conceptual questioning is necessary, and the availability of teacher planning time.

5.3 Observations and Conversations: Unit as it Happened

Learning study has its foundations steeped in both phenomenography and variation theory. As such, learning study fosters an examination of the phenomenon of teacher questioning techniques in teaching mathematics as it provides an effective foundation for examination of views about pedagogy. Thus the examination of key developments in the unit as it took place provide interesting insight into the analysis of teacher questioning techniques as they relate to communication in the mathematics classroom. This section will examine the teacher observations including, teacher–student conferences, small group and whole class discussions, and students' work.

5.3.1 Learning Study: Background Information.

It is important to point out that since the rate and ratio unit was taught in June, neither school in this study was air-conditioned, and most students were well aware that report cards had been submitted to the office before the unit was completed. The fact that this conceptually challenging unit was taught during a less than ideal time period, lends credence to the many positive responses to the attempt to implement conceptual questioning.

5.3.2 The Introduction: Popcorn Question and Counters

The students in both classes were interested in the various popcorn advertisement statements as evidenced by their immediate interaction with the tasks, clearly on-task conversations, and attentiveness to peer comments. The ratio statements related to popcorn consumption, see Chapter 4, Unit as Planned, Figure 7, and the questions that followed the introduction of these ratios, found in Appendix H, asked students to make comparisons between advertisements, and justify which advertisement they thought was most effective. Through the "how do you know . . .?" question focusing on Advertisement 1 and Advertisement 3, and the think, pair, share activity, students were engaged with the mathematics of ratios at various levels according to their abilities. This the student responses both orally and written to various questions. At the most basic level, students identified the first popcorn ratio, Advertisement 1, "2 out of 3 people preferred . . ." as the one they liked the best because they deemed it to be "mathematically based", which in student terms means that the ratio looked like math. Mid-level or average mathematics ability students reported that the first ratio was the best because it looked like a ratio they would see on packaging or in advertising. Higher ability students demonstrated more conceptual mathematical understandings through reasoning and then proving that all of the popcorn information presented was in fact the same, just stated differently. Most students were not intimidated by the ratios, and in fact many did not grasp the fact that they were even working with ratios in the beginning.

Through working in pairs, students continued to establish a comfort level with the mathematics concepts which became foundational to the students' abilities to communicate more effectively about ideas and concepts. Sharing ideas with the whole class provided a foundation for communication by allowing for the consolidation of concepts and the introduction of terms such as ratio. The accompanying black line master, found in Appendix H, provided students with several questions relating to the popcorn ratios. Students appeared to be more confident answering the questions after the class discussion which was evidenced by the limited amount of assistance sought from the teacher. The written student responses and answers to the follow-up textbook assignment reflected the value of the concepts discussed as a whole class. For example, over half of the students in each class were able to recognize that all of the ratios in the popcorn graphic contained the same information, but stated differently. Furthermore, many students also commented that working with the reduced ratios was more meaningful. Responses to questions also indicated that students were split in their reasoning as to which advertisement ratio was the best. The classes' consensus, at both Bear River and Fox Wood centered on the idea that smaller numbers were better. Students were also influenced by the numeric quality of Advertisement 1 which they deemed more appropriate than the literacy-based Advertisement 3. What might have been seen at the movies and in-store advertisements was also a factor. Generally, students were able to back up their ideas with evidence from the ratios coupled with their personal life experiences with advertisements, statistics, and love of popcorn.

Manipulatives are an important part of both teacher-participants' mathematics teaching repertoire. The concept of part-to-part and part-to-whole ratio comparisons was introduced using counting tiles. Counting tiles, also commonly referred to as patterning tiles, are plastic 1 by 1 inch squares. They come in a set of four colours: red, yellow, blue, and green. This lesson included a series of counting tiles displayed as a three-termed ratio. The initial question was posed on the smart board with the counters visible: How many different ways can you compare the counters? Students worked in pairs and triads to physically manipulate the counters to show the various ratios and identified the type of ratio. Most students at this point compared part-to-part ratios but as the teacher-participants circled the room and offered scaffolded hints, students began to also create part-to-whole ratios. This was evidenced in particular by one student who, in previous units, would wait for others to share solutions and complete answers. She would only answer questions with direct teacher assistance. However, after insisting on working alone to physically create and then draw as many ratios as she could find with the square counters, she became quite animated with her discoveries and excitedly shared them first with the teacher and then with the whole class. She had compiled a complete list of not only the part-to-part ratios but also the part-to-whole ratios and labeled them using the correct terminology. This was a defining moment for her in the unit.

Group discussion was furthered by the question posed on the subsequent smart board page: Is the comparison ratio of blue to green the same as the comparison ratio of green to blue? How do you know? This question spawned much whole class discussion, initiated the introduction of key mathematics vocabulary into the conversation, and helped students differentiate between part-to-part and part-to-whole ratio comparisons.

Since both teacher-participants had access to smart board technology, some students were able to also manipulate the counters on the smart board. The benefits of hands on materials became readily apparent during both teacher observations and during follow-up class discussions. All students, regardless of their comfort level with mathematics or their abilities, were active participants in the whole-class discussion. The ability for all students to participate orally, and use mathematical terminology correctly, was further reflected in the presentation of similar questions such as the camel questions and the toy vehicle questions discussed earlier in Chapter 5. Students also commented about the assignments. Sometimes students tended to find that the mathematics work makes sense to them in class, but when they get home they struggle to remember how to apply the concepts to their homework assignments. This was not the case with the rate and ratio homework, assigned out of the students' regular textbook.

5.3.3 Mid-Unit Highlights: Cookies and Cement

The chocolate ship cookie ratio activity (Figures 12 and 13) was one that the student's enjoyed much success with. The think, pair, share strategy allowed for all of the students to gain a level of confidence before sharing their ratios ideas with the class. Initially students were given four chip to cookie cards. Each card had a drawing of chocolate chips and cookies on them. Students were instructed to create a ratio for each card. During the pair time, students shared their ratios with a partner and then tried to see if there were any ratios that might be the same. Since equivalent ratios had not been fully discussed at this point, some students *discovered* equivalency and even went as far as relating them to fractions. Sharing with the class, afforded the opportunity to discuss student discoveries and allowed for students to educate their peers on what they had discovered.

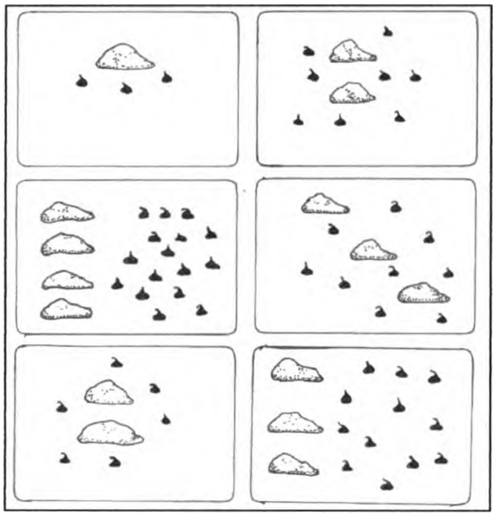

Figure 12: Chop to Cookie Ratio Card 1

The cement question was designed to allow for application of equivalent ratio concepts in a real life situation. The newer element was the introduction of a third term in the ratio. In the cement question, the ratio was 1 bag of cement: 2 bags of sand: 4 bags of gravel. The conceptual questions accompanying the cement ratio implemented Small's (2008) "how do you know . . . ?"

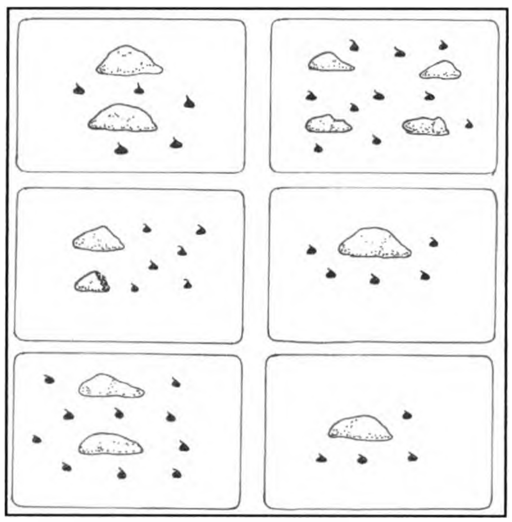

Figure 13: Chip to Cookie Ratio Card 2

question starter and applications of real life context. Thus a large part of the success of the cement ratio questions was due to both the visual nature of the question, and the aspects of real life application. Students appeared to enjoy the real life situations presented and engaged actively, especially with the "how do you know . . . ?" questions. One student on an individualized education plan (IEP), who had started out with work refusal in mathematics, and became totally engaged with the cement questions. Not only did she provide a pictorial representation for one of the questions, she also wanted to share her answers with the whole class (Figure 14). Another student who had previously been uninterested in participating in math class, also shared his interpretation of the answer using a chart format that was demonstrated in the patterning unit several months pervious (Figure 15).

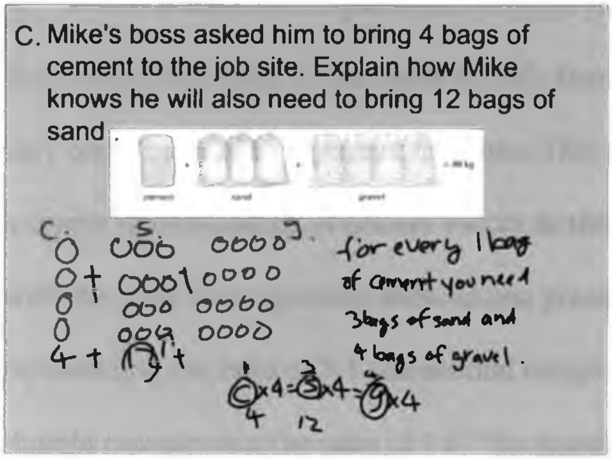

Figure 14: Cement Ratio Solution

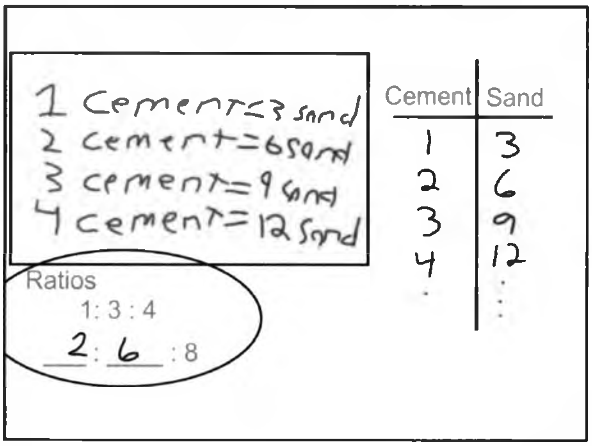

Figure 15: Student Solution from Smart board

Both Figures 14 and 15 were shared with their respective classes on the smart board.

5.3.4 Mid-Unit Struggles: Pattern Blocks and Classroom Ratio

The pattern block questions, adapted from Nelson 8, proved to be challenging. Students in Ms. Rock's class were already familiar with pattern blocks and the idea that they could be used to represent fractions. This prior knowledge allowed students to have a deeper understanding of pattern blocks as they related to ratios. The image associated with the ratio block question showed one green triangle with one red trapezoid representing the ratio of 3:1; the second image was of a yellow hexagon with two blue rhombi representing the ratio of 4:6. The question following the ratio statements was: If the green triangle to red trapezoid is 1:3, how do you know that the ratio of blue rhombi to yellow hexagon is 4:6? Once again, the "how do you know . . .?" question prompt was designed to help students elaborate on the equivalence of the ratios. Students were placed in pairs and triads, based on their mathematics abilities, so that Level 1 students worked with Level 3 and Level 2 students worked with Level 4 students. This type of pairing allowed for the more advanced students to explain their thinking while the Level 1 and 2 students explored the concepts in more depth. This was evidenced through teacher observations of small group activity combined with whole class discussions. The pairing of students according to ability for the questions became a central key in both students' understanding and communication.

Ms. Hare's class, however, struggled significantly with the pattern block question. Most students were unable to discern how the ratio for the first set of blocks was related to the ratio for the second set of patterning blocks, and thus gave up attempting the question. This may be due in part to the fact that they had not used pattern blocks during the fraction unit as Ms. Rock's class had done. Also, there was a distinct tendency in Ms.

Hare's class for students to easily give up attempting anything that they deemed too difficult, preferring the teacher give information and formulas rather than striving for that understanding themselves.

The purpose of the classroom ratio question was to provide the opportunity for further application of the concept of equivalence. The ratios were not equivalent in the initial ratios of boys to girls presented: Class A with a ratio of 3:4 and Class B with a ratio of 4:3. Students were first asked to explain if there were 28 students in Class A and 35 in Class B, how could they prove that the total ratio of boys to girls was 32:31? Students were then asked to discover under what circumstances would or could the ratios become equivalent. The final question asked students to identify under what conditions the total ratio of boy to girl between the two classes could become 1:1. The classroom ratio questions were difficult for both classes. This was evidenced by student refusal to complete that task, verbal analysis of the difficulties by the students, and teacher observations. Students were generally less engaged with the questions, which may be due to a number of factors. The classroom ratio questions were perhaps more abstract than other questions previously explored. During the reflection, Ms. Hare felt that the graphic included in this question was symbolic in nature rather than a visual representation, which may have been a factor for the lack of student success. Ms. Rock hypothesized that if the question had been related to specific classes in the students' school and those ratios, rather than a stranger's class, that students would have experienced more engagement and success. The classroom ratio question is an example of a question that fell short of eliciting oral communication and mathematical thinking.

5.3.5 Final Questions: Cereal Boxes and Winter Playday Problem

The question about cereal boxes was a great success with the students in both classes. Students were intrigued with price tag bar code cards. They seemed fascinated by the amount of information that was available on them with regards to unit pricing, and sizing. Both Ms. Rock and Ms. Hare discussed with their classes the significance of unit pricing by 100 grams. The real life demonstration, by Ms. Rock, of how to compare yogurt containers to find the best deal was significant as it related mathematics to the real world and set the stage for the cereal questions.

As for the cereal questions, most students were able to work out which cereal box was the best price for the most product. However, it was their responses to the application questions that were interesting. In particular, Ms. Rock noted several students' responses to the "Under what circumstances . . ." questions. She felt that due in large part to the generally low to mid range of socio-economic backgrounds of her students, they were able to identify with purchasing a smaller box of cereal at the greater cost per 100 grams. Students cited their reasons being a money issue with the monthly checks having not come out yet and the family being desperate for cereal. It was interesting to note that these students would have gleaned this information from their home life rather than being specifically told by parents that this was the situation. It becomes apparent that mathematics in the real world was the balancing of the bank account and not purchasing the best ratio.

Although it is recognized that the conceptual questions designed for the rate and ratio unit were beginning steps, the teacher-participants were excited about the changes that took place in teaching pedagogy and related results within the classroom. It was noted by one teacher-participant that due to the rich discussion that centered on the terminology and application of rate and ratio students made connections to the new concepts and were then far more confident in their abilities when it came to completing their independent work. This was evidenced by the increased amount of independent work, and the minimal number of "How do I do . . . ?" questions students asked of the teacher. The students' continued successes throughout the unit, at all levels of mathematics competency, in both their understanding of the mathematics concepts and their ability to clearly communicate about those concepts, was a source of inspiration. The teacher-participants credit the pictorial nature of the questions coupled with the large amount of oral discussion, and the types of questions asked as contributing factors to the overall success.

5.3.6 Concluding Discussion

Overall the implementation of the rate and ratio unit was viewed as a success by both Ms. Rock and Ms. Hare. Each teacher-participant commented positively on how oral the unit was. The teacher-participants felt that the quality of mathematics concepts shared by students was unique and surpassed expectations. For instance, both teacher-participants said they had students who traditionally had struggled with mathematics concepts, and yet they were able to articulate their understanding, although basic in nature, about rate and ratio using the correct mathematics terminology. This had previously been a rare occurrence in both classrooms. Although both teacher-participants acknowledged the fact that conceptual questioning techniques did not flow naturally, they both agreed that the techniques became more natural as they were practiced and that they were definitely worth the effort to incorporate. Both teacher-participants mentioned that they are intending to continue developing the use of conceptual questioning in their respective classrooms.

6. Discussion of Findings

As we discuss the findings a review of the fundamental research question is important. The research problem for this thesis examines the process of communication, specifically through conceptual questioning in mathematics as a phenomenon. Therefore the research problem involves multiple dimensions including varied forms, audiences, and purposes of students' communication. How can generalist teachers at the Intermediate level develop and employ conceptual questioning techniques; assess student responses; and promote the communication of student conceptual understanding of the mathematical processes, contained within the five strands of the Ontario Curriculum? Action research provided a framework to explore these questions. The research focused on one unit of mathematics – the Grade 7/8 rate and ratio unit.

The discussion of the findings in the following section has been organized as it relates to three central facets of research: conceptual questioning, communication, and the learning study. Through this analysis there are four key traits or critical aspects of conceptual questioning that emerged: the importance of oral aspects of the unit, discussion, reflection and collaboration; the visual nature of the questions and problem; the importance of real life application and of familiarity with the contexts used; and the significance of teacher modeling. Figure 15, at the end of this chapter provides a conceptual map of the relationship of the four traits to conceptual questioning. These four key traits are further discussed in terms of where they fit into the research, and of how they are related to the thesis question. The analysis by trait will allow for the correlation of related data from interviews, reflections, and observations.

6.1 The Thesis Question: Discussion

6.1.1 An Understanding of Where It Began

The goal of this study has been to develop conceptual questioning techniques and sensibilities that foster student communication in mathematics. The conceptual questioning ideal is demonstrated in part by the Japanese teachers in the TIMSS video (1999), as discussed by Heibert and Stigler (1999), and also supported with rich conceptual question examples from Mason (2000; 2009). In light of the low volume of research on conceptual questioning conducted to date, it is important to point out that this process of conceptual questioning is in its infancy stage especially within the rate and ratio unit.

6.1.2 Conceptual Questioning

The thesis question, as stated earlier in this section, is multi-faceted, and as such the analysis must include a dissection of its parts. The first part of the thesis question examines the idea of conceptual questioning. The journey into conceptual questioning has been a long and twisting road. Initially it was thought by the teacher-researcher that conceptual questioning might be a type of question similar to problem solving questions. It was hypothesized that as a type of question there might exist a type of formula for creating and implementing such questions. However, upon examination of other research, especially Mason and Watson (1998) it soon became apparent that there was not a magic formula for the creation of conceptual questions. Both Mason and Watson (1998) and Small (2008) provide a form of question starter or stem from which a teacher can build conceptual questions. However, cautions against the use of these question starters as a blanket formula leading to conceptual questioning is made by Mason and Watson (1998).

"The questions are certainly not a recipe for teaching mathematics, nor even a menu from which one can choose. Rather they are intended as a source of inspiration and as an aid to change" (p. 3). There is an element of thought that needs to take place on the part of the teacher before a question is asked. For example: What is the purpose of asking that question? What are the big ideas or concepts that I am trying to teach? What are the possible answers students could give and what possible misconceptions might arise? This research confirmed that, conceptual questioning is actually an awareness of questioning style that involves contemplation, content, conceptual understanding, consistency, and dedication on the part of the teacher employing such questioning practices. Conceptual questioning then impacts what aspects of a concept are asked about, when the question might be asked, and finally it provides some pedagogical direction on how to ask the question. As evident from the rate and ratio study, not all conceptual questions work the way they were planned and just because a question worked one time with one class is not a guarantee that it will be effective again. For example, the classroom ratio question, although conceptual in nature and aimed at application of the equivalent ratio concepts, proved to be exceedingly difficult for both classes. With the cereal question, Ms. Rock's class was able to grasp some of the follow-up conceptual questions surrounding the choice of different boxes of cereal due to their generally lower socio-economic backgrounds. Ms. Hare's class, however, needed some prompting and real life examples from the teacher in order to reason through the questions effectively. Therefore, as discussed later in this chapter, conceptual questioning techniques and sensibilities becomes a method of teaching rather than a set of questions that can be dropped into a lesson or unit without thought and careful structured pre-planning.

6.1.3 Critical Aspects of Conceptual Questions and Sensibilities

The driving force behind learning study is that it follows a cyclical framework which revolves around the teaching and learning process of both teachers and students. Thus the rate and ratio learning study began by giving the teacher-participants the opportunity to examine their conceptual understandings and critically examine their current pedagogy regarding the teaching of mathematics.

From Chapter 4, Unit as Planned we can see that the teacher-researcher and teacher-participant adapted some aspects of conceptual questioning. These include but are not limited to: selecting questions that (a) elicit mathematical thinking, (b) ask for explanations rather than for answers only, (c) give the answer but instead ask for the justification, (d) aid conceptual development such as through the use of counters, toy vehicles, and camel questions, (e) engage various students, especially how the questions are presented in familiar math context, visual context, or real life contexts; as well as being prepared to, (f) anticipate student responses and misconceptions, and (g) offer scaffolding hints to those students who are challenged by the question. Further from the unit as implemented, the teacher-researcher and teacher-participant added to this list and refined some traits. For instance, the teachers realized that visual contexts that were symbolic were not as helpful as those that were more concrete. We may therefore say that the teacher-researcher and teacher-participant learned some aspects of conceptual questioning (Ling & Runesson, 2007). Although the teacher-researcher and teacher-participant put forth tremendous effort, time, and diligence into the creation of conceptual questions, what the teachers learned and accomplished regarding conceptual questioning fell shy of the kind of questioning described in the TIMSS (1999) video study or in Mason (2000).

6.1.4 Communication

The second part of the thesis question addresses the issue of communication. Communication is highlighted as a central component in mathematics. It is one of the seven mathematical processes in the Ontario mathematics curriculum, occupies one of the categories on the achievement chart, and is a term that is bantered about in the Ontario mathematics educational community. However, when it comes to defining what communication is in mathematics, the term becomes complex. As discussed in the literature review, communication could be considered what students are able to write, their responses to key questions which are labeled communication, what students can describe orally, or what they are able to share in discussions. This research suggests that under the auspices of conceptual questioning, communication could be defined as a student's interactions with mathematical concepts both orally and written. That is not to be confused with just any generic response. Quality communication must demonstrate a deeper understanding of a concept through reasoning, justification and proving, by students as they not only apply the concept "discovered" but also create connections between concepts. Several aspects of communication are further highlighted in the section regarding the learning study analysis such as the apparent oral and visual nature of conceptual questions within this unit of study and the noticeable increase in oral communication, including the use of correct mathematical vocabulary and concept focused conversation both individually and in group and whole class discussions.

6.1.5 *Application*

The third part of the thesis question examines the implementation and effect of conceptual questioning across the five strands of mathematics. For the purpose of data collection, I selected the number sense and numeration strand out of the five strands.

6.2 Learning Study on Conceptual Questioning Analysis

6.2.1 *Student Understandings and Teacher Perceptions*

The purpose of a learning study is to design and critically evaluate lessons that will help students navigate the paths between their prior conceptions of a topic, and the newly learned concept or phenomenon, thus creating a deeper understanding. For this study the emphasis was on communication through conceptual questioning.

From a lesson evaluation perspective, the student responses during various discussions both as a whole class and in small group settings were revealing. The classes as a whole engaged in the problems and questions presented and generally participated with enthusiasm in discussions. The teacher-researcher and teacher-participant noted that this was a significant shift from previous mathematics units where students were reluctant to engage fully and even with 'think, pair, share' time given, often sat passively in whole group discussions.

Conferencing with individual students also confirmed a strong understanding of rate and ratio concepts covered in the unit. In discussion on rate comparisons a student demonstrated further application of ratio using appropriate terminology. During his student-teacher conferencing he was pleased to point out that the rate comparison was applicable to cans of frozen juice. He said that at the local grocery store the cans of pink lemonade were a much better deal than the cans of juice concentrate because the ratio of lemonade was 1 can to 4 cups of water and the ratio for juice was 1 can to 3 cups of water. Therefore since both cans were of equal amount, there was more lemonade made from one can than the juice so it was a better deal. The mathematical process expectations in *The Ontario Curriculum: Grades 1-8 Mathematics Revised* (2005), specifies that students should "communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions" (p.110). During individual conferencing sessions, most students were able to articulate their thinking on rate and ratio, and all students used mathematical vocabulary directly related to rate and ratio demonstrating solid conceptual understanding.

Meaningful connections were further evidenced by students who typically struggled with mathematics and yet found success with the rate and ratio unit. One such example was a student who responded to the cement ratio question on equivalence, shared in Chapter 5 by drawing a diagram presented in Figure 13 to find and then prove her equivalent ratio. The teacher observed that this same student had previously struggled with equivalence in the fractions unit and was generally reluctant to attempt any mathematics questions, especially those outside of her math comfort zone.

When reflecting on the possible reasons behind the improved levels of participation and successful understanding of rate and ratio concepts, the teacher-researcher and teacher-participant identified several attributes of the learning study. These included the oral emphasis the unit took, the visual aspects of questions used, the importance of teacher modeling and the variety of questions presented within a familiar context. Although these are important aspects to conceptual questioning, it is interesting to note that the oral and visual aspects, teacher modeling and use of familiar contexts are not specific to teacher questioning. These aspects are each a helpful way of teaching mathematics in general.

The most significant attribute was how much of the unit was based around oral discussions. This should not be confused with teacher-centered or teacher-dominated lesson structures. Rather the oral nature of the unit refers to a focus on student responses which incorporated the sharing of ideas and discoveries. The teacher-participants were facilitators of the discussions and activities who provided questions that helped guide students to develop their understanding rather than telling students a rule.

Another key attribute was the visual nature of the problems and related questions. Almost every question presented was related to a visual model; be it a pictorial representation or the use of hands on manipulatives. The teacher-participant and teacher-researcher credit the visual models used in the unit for the success of students who had typically struggled with mathematics concepts during previous mathematics units. This was especially evident in the math counter question where a student who struggled with many mathematics concepts throughout the year, was able to find success with the counter question. In relation to the visual nature of the unit questions, during interviews and planning sessions the teacher-researcher and teacher-participant both commented numerous times on the success level of all of their students. The teachers concluded that this might be due to the visual nature of the questions and problems presented during the unit, which had not been a conscious focus during the creation and adaptation of the questions used.

An important observation was that students actively engaged in the counter questions and the pattern block questions which the teacher-researcher and teacher-participant believed was because the students were accustomed to the use of manipulatives and were given ample time to discuss. One example was in Ms. Rock's class, as her class had previously used pattern blocks to represent fractions and equivalency of fractions. In their small group activity with pattern blocks, students who understood the concept early on were observed helping others. For instance individual work time, coupled with peer to peer assistance allowed for all students to talk about, ask questions, and to explain the mathematics as well as to experience a level of success. The confidence level and communication skills of the stronger students were also built through this activity.

The final significant attribute of the learning study was the level of real life application and the air of familiarity in many of the questions. Students were highly engaged right from the introduction of the unit with the popcorn and advertising ratio questions. The familiarity with popcorn and the elements of advertising allowed students of various mathematical levels to engage actively with the questions. Students commented to both teacher-participants on how easy the mathematics was, especially at the beginning of the unit with the introduction of the popcorn graphic shown in Chapter 4, Figure 7. Even students, who admittedly did not understand the larger numbered ratios, commented on how they could understand the small ratios of *2 out of 3* and *twice as many*. This initial success provided the necessary foundation for students to take risks with other questions in the unit.

Another factor aiding in familiarity of questioning was the amount of teacher modeling that was employed. The teacher-participants worked diligently with their students to establish expectations about the questions used, through the modeling of responses to questions. For example, both teacher-participants modeled responses to various questions presented in the class mathematics textbook. A central component to modeling was the academic level of communication demonstrated with the use of unit vocabulary and detailed explanation of what the teacher-participant was thinking while solving the questions. Teacher modeling of expected behaviors, in this instance types of responses to questions was significant. After all, the idea of conceptual questioning was not only a new technique for the teacher-participants, but also for their students.

6.2.2 *Analysis: The Attributes Contributing to Success and Difficulties with Questions*

Upon reflection, it was surprising to both teacher-participants how pictorial the questions selected and modified for the unit were. Although pictures, diagrams, and symbolic representations often assist student learning and are applied to a degree in textbooks, the fact that almost every question in the rate and ratio unit had a visual representation was noteworthy. The visual representations not only helped the students to gain conceptual understanding but it also gave them a forum from which to discuss their ideas and solutions. In one case, a student in Ms. Hare's class who was quite strong in mathematics but consistently struggled to explain her understandings to her peers was able to articulate her solution to the cement problem using the diagram as an aid. From a teaching perspective, the use of pictorial representations allowed another avenue for discussion as students could be asked to relate their solutions or ideas to the pictorial representations. The visual aspects of the questions used coupled with the teacher modeling and emphasis on discussion as done with questions like the cement ratio, were what seemed to initially determine which questions were successful. Questions which were less visual or pictorial in nature required more review, revisions or as in the case of the classroom ratio questions, just needed to be abandoned altogether.

One problem and related questions that both classes of students were not successful with was the *classroom* ratio problem. The students struggled with the questions even though the concepts of finding equivalent ratios were already familiar to them. The teacher-researcher and teacher-participant shared that the students' challenges were due to two main factors. First, there was a distinct lack of interest in the question as there seemed to be no relevance to student lives. Second, the graphic used was colour coded blue Bs for boys and pink Gs for girls. It was hypothesized that this representation was symbolic rather than pictorial in nature, which may have contributed to the lack of student success.

From a teaching perspective, mathematics questions and the related concepts presented were often taught beginning with the very concrete and moving towards a more abstract application. This is not always an easy transition for students and they will often struggle with questions that are more abstract in nature. An example of this in the rate and ratio unit was the pattern block question. Initially students had difficulty with finding the equivalence in the ratios. However, those that persisted, especially in Ms. Rock's class, were able to complete the question and demonstrate conceptual understanding. Therefore, having students wrestle with a question does not warrant the abandoning of abstract questions. As learned from the Heibert and Stigler (1999) TIMSS video study, allowing students to work for conceptual understanding can strengthen their mathematical abilities, as was evident in the Japanese lessons.

There are, however, exceptions to the benefit of attaining conceptual understanding through some form of a struggle through a question or problem. One such question was the classroom ratio question. Not only did this question lack a strong visual, it was abstract in nature, and the students were reluctant to engage in the question. In both teacher-participants' classrooms, it became apparent that the classroom ratio question was flawed. The classroom ratio question had appeared well-developed. The teacher-researcher and teacher-participant had identified the concepts of equivalency behind the question and defined some ideas as to how students might approach the question. It was also understood that the questions were more abstract. However, as the students of various mathematical abilities worked with the problem, their frustration and lack of engagement quickly became evident. Both teacher-participants, independently of one another, discontinued the lesson with the classroom ratio question and proceeded to the next set of questions.

As professionals, it is important that teachers recognize the difference between a struggle that is worth the enduring understanding in the end, and a question in which the casualties from the battles waged to get the final solution are more detrimental than the concept which may be gained. Although the concepts needed to solve the classroom ratio questions were important, there were other questions which could also accomplish the same outcome and would not deflate the newly developed confidence of the students in the process.

Students from both classes were particularly engaged in the cement and cereal questions. They were animated about the cement questions as the real life applications were directly evident. Many students could identify with the construction worker and his desire to appear competent, with student help, to his boss. Another interesting aspect of the cement questions was the various approaches to solving the questions that students used. One of the questions stated: "Mike's boss asked him to bring 4 bags of cement to the job site. Explain how Mike also knows he will need to bring 12 bags of sand." Students already had the ratio of 1 bag of cement: 3 bags of sand: 4 bags of gravel. When sharing with the whole class how they solved the problem, one student demonstrated equivalency by drawing the increasing ratios under one another until she reached 4 bags of cement and 12 bags of sand. Another student followed in a similar manner by writing the numeric version of the ratio until the necessary conditions were met. This student pointed out the number of gravel bags also needed as he had written the whole ratio. However, it was the use of the patterning T chart to solve the problem that was the most abstract solution. The student was able to apply a concept from patterning to help him solve and explain the equivalency. This was significant on two fronts. First this student was usually able to demonstrate sound mathematical thinking related to mathematics concepts. However, in this situation he demonstrated even more complex mathematical thought by making a direct connection between two concepts, one from patterning and the other from rates. Second, this student often found it difficult to share his thinking process with others, therefore being able to communicate by sharing ideas meaningfully so that others could understand was an interesting development.

The cereal questions were also related to real life for many students. All students were aware that cereal comes in a variety of sized boxes, and they were eager to prove which box was the best price point. Initially, some students stated that the largest box of cereal would automatically be the best buy, without working out the cost per 100 grams. The logic for these students was that being the most expensive, if the large box wasn't the best deal, then no one would buy it. They also likened the purchasing of the large box to what one would find in a warehouse store when buying in bulk. However, many students were also able to identify with the purchase of the smallest box, which was not the best deal but had the smallest price tag.

Thus the elements of oral communication, pictorial references and real life experiences and applications were significant at making the learning study on conceptual questioning successful. It appears these attributes allowed students who had previously struggled tremendously in mathematics to gain some level of confidence and proficiency with ratios. From my teaching experience, rate and ratio concepts are not easy ones to grasp, in fact many adults struggle with them. Therefore it is significant that the students in both classes were actively engaged in most of the questioning tasks in the rate and ratio unit despite the fact that they were learning difficult concepts in late June.

6.3 *The Attributes: A Concluding Discussion*

Initially this thesis questioned how a generalist intermediate elementary teacher can teach mathematics successfully. It has been established through Ontario Ministry of Education documents that at the intermediate level, elementary teachers are, so to speak, required to be experts in a variety of areas of study from science to history and geography. This applies to mathematics too. However, in my experience, most elementary teachers are not mathematics specialists nor have many of them encountered mathematics courses beyond high school. Although a majority would have taken at least a teaching methods course in mathematics during their pre service programs, given the short length of the bachelor of education programs these courses are usually cursory. Since mathematics is to a larger extent a content-based subject that builds from one concept to the next, an understanding of mathematics is a critical factor in the ability to teach it. So, can a generalist intermediate teacher teach mathematics fostering conceptual understanding and communication? The Ontario Ministry of Education (2005) believes this is possible as stated in the revised version of the mathematics curriculum: "fostering student's communication skills is an important part of the teacher's role in the mathematics classroom. Through skillfully led classroom discussions, students build understanding and consolidate learning" (*The Ontario Curriculum: Grades 1-8 Mathematics Revised*, 2005, p. 25). Therefore, the attention given by educators and teachers to communication and conceptual understanding through teacher questioning is of paramount importance regardless of the level of mathematics instruction teachers themselves have received even in pre-service programs. However, based on the reflections in this learning study, the answer to the original pondering of how can a generalist teacher teach with conceptual questioning does not have an easy answer. This study has provided preliminary answers to the question of "how" – which in its most basic form would be – with a great deal of difficulty, effort, time, and risk taking.

Creating and using conceptual questioning at the TIMSS (1999) video study or Mason (2009) level becomes an ideal that is not yet possible for most teachers. However, as demonstrated with the rate and ratio unit, conceptual questioning techniques and sensibilities can be employed by generalist intermediate teachers with various levels of success. This success is contingent on several requirements including comfort level, level of conceptual understanding, and planning time on the part of the teacher.

First, the teaching experience of a teacher trying to implement conceptual questioning is an important factor. Of tremendous importance is the teacher's comfort level with teaching intermediate students in general. Added to the comfort level is the ability of a teacher to focus on lesson preparation and the planning and creation of questions based on understandings of the content. Teachers therefore need to also possess a certain comfort level with the curriculum content and the concepts being taught.

Conceptual questioning techniques require the teacher to think about what they are asking and to analyze the questions before they are posed. Often, as Mason (2000) pointed out, teachers ask questions routinely as part of their communication with students, without stopping to figure out what they are really asking and why. In some instances teachers also keep their questions limited to what they are comfortable with, such as a more Socratic approach. Ms. Rock commented that as an experienced teacher, she was better able to handle conceptual questions with confidence. She also noted the significance of her shift in thinking about her role as a mathematics teacher. Ms. Rock realized that as the teacher she did not have, nor need to have, all of the answers, all of the time for every mathematics question.

Second, having experience coupled with some foundational understandings of mathematics affords teachers time to think through the types of questions they would like to employ and plan for the anticipated student reactions. The anticipation of how students might understand concepts and deciphering possible misconceptions is a second critical requirement for implementing conceptual questioning. Misconceptions will often creep into student's understanding of mathematics and teachers need to anticipate this happening and have a plan of action to counter such misconceptions. For example, many students arrive to the intermediate grades firmly believing that when you multiply two numbers together, the answer will always be larger than the value of the two terms multiplied. This misconception leads to much student frustration when multiplying fractions and decimals, where the answer is not necessarily larger than the terms multiplied.

Since pre-planning for conceptual questioning is necessary, a third critical requirement would be the element of time. Some time was saved for the teacher-participant because the research behind conceptual questioning was presented by the teacher-researcher to the teacher-participant. This allowed for a common foundation from which to form their conceptual questions. Creating that foundation however, took time and effort on the part of the teacher-researcher before conceptual questions used for the unit were even created. It takes time and effort to select the type of questions and the most beneficial methods for introducing those questions. Both Ms. Rock and Ms. Hare commented on the amount of time it took to create and employ conceptual questions especially considering that the unit was short, comprised of only nine teaching days. Ms. Hare found that much of her preparation time at school was used to investigate, modify, and create questions that would aid in the development of mathematics concepts. Ms. Rock echoed these sentiments also commenting on the issue of collaborative time needed to co-plan, which could only be done on the teacher's personal time. Therefore the teachers often sent small thoughts via email rather than being able to sit down together and compare notes.

Throughout the learning study, the main goal of the teacher-researcher and teacher-participant was to employ conceptual questions in the hopes of eliciting quality communication from their students. Through the analysis of the data, the teacher-researcher, teacher-participant, and their respective classes have begun to demonstrate the development of skills which enable them to question, converse, and apply mathematical concepts to the questions presented. This change in teaching practice, although in its infancy, has inspired and motivated the teachers involved to strive for the application of these ideas and related skills in other mathematics units.

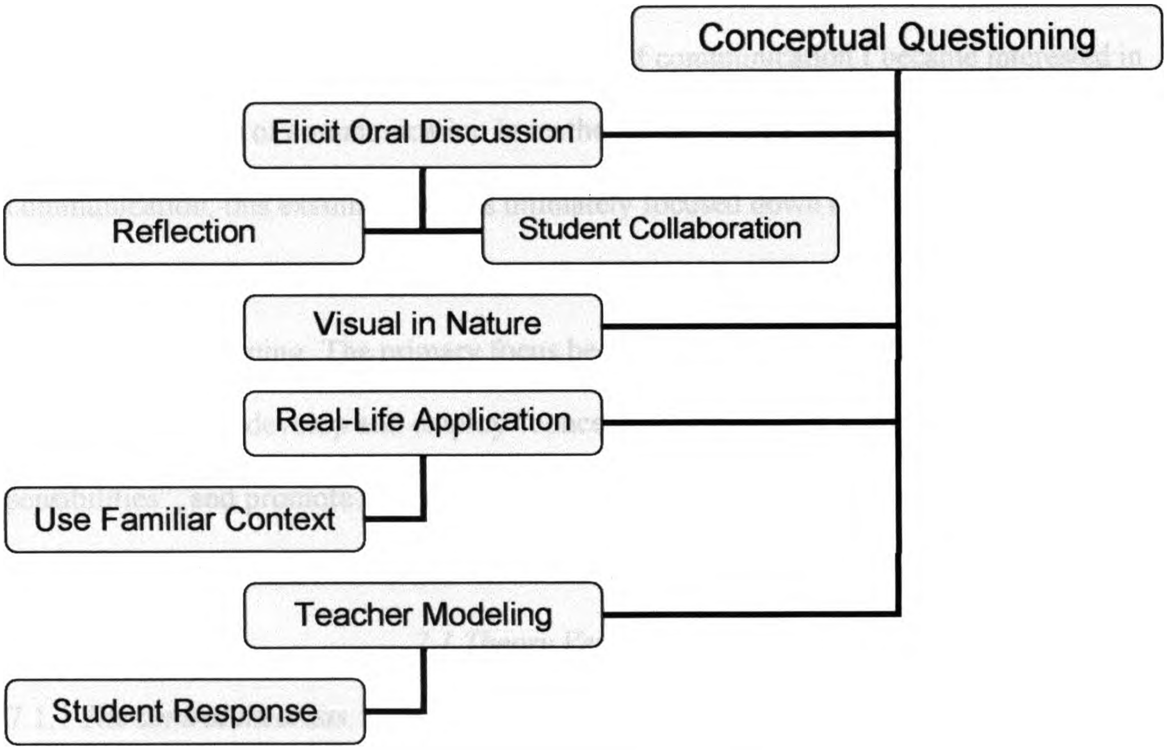

Figure 16 Four Traits of Conceptual Questioning
This diagram demonstrates the connections between conceptual questioning and its four traits.

7. Summary and Conclusion

In the beginning of this journey I was interested in the challenges and triumphs of teaching mathematics at the intermediate level as a generalist teacher. Given the Ontario Ministry of Education emphasis on the process of communication I became interested in examining aspects of communication in mathematics. With a review of the literature on communication, this examination was ultimately focused down onto conceptual questioning and the role that teachers partake in implementing techniques involved in conceptual questioning. The primary focus became: How can generalist teachers at the intermediate level develop and employ "conceptual questioning techniques and sensibilities", and promote the communication of student conceptual understanding of the mathematical processes, contained within the five strands of the Ontario Curriculum?

7.1 Theory Verses Practice

7.1.1 *The central theorists*

As stated in Chapter 5, the rate and ratio unit was the first attempt by the two teachers in this study at the application of research on communication in general and more specifically conceptual questioning in a Grade 7/8 mathematics classroom. Of particular note, Heibert and Stigler (1999), Mason (2000; 2009), and Small (2008) all outline varying aspects of conceptual questioning. Heibert and Stigler (1999), dissect the success of the Japanese teaching structure. They contrast the Japanese structure with the teaching practices in North America. Ultimately there is a cultural difference in the approaches to teaching and learning which influences the success of the Japanese teaching pedagogies, making the application of the Japanese teaching pedagogies difficult to implement in the North American classroom. Mason (2000) provides a variety of question prompts and a solid theory on the implementation of conceptual questioning. Most notably he cautions teachers to develop an awareness of the questions they ask and their subconscious reasons for asking those questions. In order to be successful at thinking through questions prior to asking them, teachers need to possess a conceptual understanding of the mathematics being taught. Small (2008) provides several ways to approach the development of teacher questions such as starting with the answer and then asking process-oriented questions. Small, like Mason, emphasizes the necessity of developing an awareness of the question types that teachers enlist. Implicit in Mason and Small's recommendations on conceptual questioning is the assumption that the teachers posing these questions have conceptual understanding of mathematics and have established a teaching environment which is conducive to particular teaching methods such as group work.

Chapter 4, the section on Unit as Planned drew from the conceptual questions promoted and used by Heibert & Stigler, Mason, and by Small to design a unit that is taught using conceptual questioning. Key elements which were evident in each of the question exemplars used in the literature were: a considerable conceptual understanding of the mathematical concept by the teacher; the obvious forethought about what concepts were being examined and how best to target them; and an awareness of where those concepts connected in the larger picture of mathematical learning.

7.1.2 *The application*

This research focused on the ability of generalist teachers, not experts in the area of mathematics, to elicit communication from their students. In light of the research conducted on communication and the ultimate focus on conceptual questioning, as it relates to the questions created and fielded in the rate and ratio unit, several significant conclusions are reached.

The original thesis question revolved around generalist intermediate teachers' ability to teach mathematics in a manner that encourages mathematical communication from students. In the rate and ratio unit, the teacher-researcher began the learning study by sharing with the teacher-participant the research compiled on communication and conceptual questioning. This allowed for a more equal foundation of knowledge from which the participating teachers could base the creation of conceptual questions. Questions were designed for the rate and ratio unit after examining the research and specific questions such as those viewed in the TIMSS video (1999) and highlighted by Mason (2009) and Small (2008). Many of the questions created, nonetheless, lacked deep conceptual development. This is not to say that the questions created and employed in the rate and ratio unit were ineffective as a whole, rather they were effective in eliciting more communication from the students than had been evident in the teacher-participants' classrooms prior to the research and also in the TIMSS video.

The use of conceptual questions marked a significant change in teaching pedagogies for the participating teachers and the learning experience of the students. The richness of these initial attempts at conceptual questioning, lies in the noteworthy changes in the type, such as discourse on reasoning and proving, explanation, and justification, as well as the amount of communication demonstrated by the students. Better still, through the attempts at employing conceptual questions and the resulting data analysis, several elements necessary for success became evident.

Communication, as discussed, is an expansive topic. There are multiple facets to communication, both oral and written, which include: argumentation, discourse, listening, and/or writing. As evidenced by the oral nature of the rate and ratio unit questions, teacher questioning techniques are the vehicle through which student oral and written communication is elicited and responses evaluated. Therefore it can be concluded that teacher questioning becomes a central component to communication that is a means through which mathematical thinking is evoked and shared and also that encourages problem solving and is a means through which solutions are presented in the mathematics classroom.

7.2 Necessary Prerequisites

7.2.1 The Quandary of a Generalist Teacher: Understanding of Concepts

There are three main conditions under which generalist teachers must operate in order to gain an understanding of conceptual questioning and then begin to change their teaching practices to reflect that understanding. Foremost, generalist teachers need to develop an understanding of the mathematical concepts they are teaching, both within the unit but also within the framework of mathematics as a whole. As the analogy states, a house cannot be built upon the sand and be expected to withstand the tropical storms it will encounter. In the same manner, generalist intermediate teachers cannot formulate or adopt conceptual questions, anticipate students' responses to these questions, assess students' responses without the foundational understanding of the mathematics they are teaching. Since conceptual questioning requires teachers to examine questions used from multiple aspects, including the main concepts and ideas the question is designed to elicit, a foundation in the understanding of the big mathematics ideas in a topic and how these ideas relate to other topics is imperative. Therefore, this research confirms Mason's (2000) finding that teachers need to possess a conceptual understanding of the mathematics being taught if they are to implement conceptual questioning.

7.2.2 The Quandary of a Generalist Teacher: Personal Commitment

The second condition under which generalist teachers can develop an awareness and understanding of communication is directly related to a personal commitment to the necessary changes in teaching pedagogy for a teacher to implement conceptual questioning. This commitment to change an aspect of a teacher's practice is not a small undertaking and cannot be viewed lightly. Change can be an uncertain experience with a level of fear attached to it. This can be especially evidenced in the world of teaching intermediate students. When faced with challenges and uncertainty, teachers generally revert back to the way in which they were taught a subject. This trend has been noted by Ball and Chazan (1999), and commented on by Ms. Rock in her personal narrative reflection in Chapter 5. A second note of significance that was also referenced by both the teacher-researcher and teacher-participant, was that of the amount of time it took to prepare the types of questions that led to quality communication; be it questions posed to the whole class, to small groups, or to individuals. The availability to collaborate with a peer and opportunity to become reflective practitioners, aided both the teacher-researcher and teacher-participant in their quest to not only develop a deeper understanding of the mathematical concepts taught, but also to adjust their pedagogical teaching practices.

7.2.3 The Quandary of a Generalist Teacher: Significance of Communication

Changes in teaching pedagogy are related directly to the third condition, an understanding of the significance of communication in the development of mathematical thinking, and the strong ties between communication and conceptual questioning. Without an appreciation of the necessity of quality communication or an understanding of what quality communication entails, the commitment to implement the changes necessary for communication will fade with the stresses of other teaching commitments such as preparation and planning time, classroom management, and the necessity to cover specified content. A commitment to eliciting communication, at least in part, by using conceptual questioning techniques was reinforced through the experiences of success by both teachers and students, and the evaluations of these successes.

7.3 Revelations and Responses

7.3.1 Employing Questions with a Communication Focus

Questions that evoke a generic "yes/no" or numeric responses, do not allow for the development of mathematical communication, at least not the oral and visual form. Most questions, when examined, are really designed to elicit knowledge and informational responses. In fact, even when asked to explain their responses students will often give the technique or formula used in class so the teacher cannot be really sure if there is understanding of the concept or just mimicking of what the student has seen. Conceptual questions, like the ones employed in the rate and ratio unit in Chapter 5, were different in the type of communication they elicited that went beyond procedural responses. Students became engaged in the mathematics to the point that they were willing to share their thinking such as when they shared a variety of ways of solving the cement question.

When examining student interactions and levels of communication and the elements of the conceptual questions used, several interesting traits emerged. First the questions had a decidedly oral component to them. Students appeared to develop a level of increased confidence in their thinking that meant that they were not shy to share their thinking and to explain solutions to the problems solved. This was evident through the various times students - weak or strong - were willing to share what they had learned despite their previous refusal or lack of ability to do so in other units. A case in point was when the weaker students also shared their solutions such as on the tiles questions, and when stronger students assisted in explaining to weaker students the pattern block question. The increased confidence might be attributed to the process of having students work through the concepts first individually, then collaboratively in pairs, and then sharing in whole class discussion.

Second, teachers need to model quality communication. Students do not know inherently how to communicate, much less how to communicate about mathematics. Therefore teacher modeling is key. That being stated, teachers also need to know what to model and how to model it. Both teacher-participants in this study modeled quality communication and mathematical thought processes while teaching, often using textbook questions to begin and then expanding upon their thought process as a demonstration for students.

Third, was the visual nature of the conceptual questions employed. The questions that were successful in the rate and ratio unit all had a visual component to them. This was particularly evident with the first question surrounding the popcorn advertisements. The visual element in successful conceptual questions was an interesting trend noted by both the teacher-researcher and the teacher-participant.

In tandem to the visual nature of the conceptual questions which were successful was the fourth element of familiarity and relevance of these questions to the students. The successful conceptual questions had relevance to students as evidenced by such questions and those related to the cereal box sizes and contrasted with the less than successful questions related to classroom ratios.

Upon reflection, most questions explored in this unit were designed to help students develop an understanding of a concept or to connect understanding of one concept to another. These questions were usually enthusiastically completed with ensuing talk and writing about concepts and ideas. Examples of these questions include the cement, heart rate, popcorn, and cookie questions. The questions that required further application contained an element of struggle for the students. This was evident with questions like the pattern blocks. However, questions that lacked relevance and visual reference, like the classroom ratio questions, became labor-intensive and consequently students' communication became very limited. Thus, questions that provide opportunity for students to think mathematically, and allow for oral communication of responses, are questions that reflect a students' level of conceptual understanding. Therefore, questions that elicit quality communication embody several components: questions need to allow for oral response before written responses are required; they need to be visual in nature or at least have a visual component; and the context of the question needs to bear some relevance for the students.

7.3.2 Assessing Question Responses for Communication

Part of the foundation of learning study planning stages is variation theory which allows for the teacher to evaluate what conceptions students have developed and their interrelationship throughout the implementation of the unit. The principle of variation theory is to create relevance so that students can discern the crucial concepts through variation (Ling & Runesson, 2007). A learning outcome circle, identified by Davis and Dunnill (2008), helps to focus the development of concepts for planning lessons. The learning outcome circle focuses the big ideas or “target conceptions” in the unit of study (Davis and Dunnill, 2008, p.7). As the students vary in their interactions with the concepts, so the learning outcome circle flexes. This allows for teachers to understand the different constructions their students make and to adjust the teaching strategies accordingly. Since students in the rate and ratio learning study were at differing grade and achievement levels academically, the variations in conceptual understandings were accepted and built upon. In regards to the teachers’ learning outcome circle, the development of variations in designing and implementing conceptual questions included aspects such as visual and pictorial representations, a spectrum of paired to whole class activities and collaboration, and were preceded with authentic teacher modeling.

Communication by most basic definition is the exchanging of thoughts and ideas. Based on this definition it has been my experience that generally, students are better at communicating by talking about their ideas than they are at writing out their thoughts. When talking, students are able to articulate their thoughts and are more willing to explain in detail the processes they use or describe the basis for their ideas. Thus the oral component to this study allowed for the assessment of student communication to take place easily. Student responses were monitored through both group interactions and whole class discussions. Further assessment and the clarification of student thought processes were also completed through individual student conferencing. An interesting aspect of the conferencing is the continuation of communication as a two-way conversation. With individual conferencing, the teacher was able to ask for further clarification when necessary and pose deeper probing questions to continue the dialogue as needed. Thus the assessment of communication was based more upon the oral than the written student responses, which when documented, became a powerful assessment tool for communication in mathematics.

7.4 Conclusion: General Comments and Next Steps

Learning studies view the classroom teacher as the primary source for change. This grassroots, teacher-focused approach affords teachers the opportunity to examine their pedagogies in a practical manner with the goal of improving student learning. The teacher-participants in this research embraced the opportunity of participating in the learning study on rate and ratio. Their engagement became a form of professional development. Traits of conceptual questions including the visual nature and use of familiar contexts, demonstrated a growth in the teacher-participants' understandings of what conceptual questioning entailed. By using the questions they designed, and attempting to implement what they had learned, the teacher-participants developed a willingness to pursue conceptual questioning with confidence.

One lesson learned from implementing conceptual questioning was the realization that not every question created would work, and that just because a question was valuable once did not guarantee it would be significant again. Thus, teachers need to be aware of when to pull back from a question and regroup with their class. However, from the teacher-participants' perspective, the effort needed to create the conceptual questions and the risks taken to implement them in the rate and ratio unit were worthwhile. Having worked to define and then implement conceptual questioning techniques left both teacher participants with the tools to apply their understandings to other units in mathematics, which both teacher-participants stated they were eager to do.

A contributing factor to the success of the learning study was its emphasis on collaboration. Teaching can be a very solitary activity as it is rare for teachers to get the opportunity to collaborate with their peers. Even though time for collaboration was limited in this study the chances to collaborate and chat either by phone or email were extremely beneficial. Through sharing with one another, the teacher-participants were able to further refine their questioning skills. This allowed room for small changes in conceptual question wording and emphasis to be implemented freely and thus the conceptual questions became more effective at eliciting communication, at least oral communication.

Aside from the learning study process, the engagement of students with the rate and ratio concepts is noteworthy. This rate and ratio unit was taught in the month of June. As most elementary teachers know, many students at the intermediate level begin to wind down their learning about May. It is exceedingly difficult to get students engaged in learning when the weather is nice and report cards are completed. There were days, late in June, when the only really academically-oriented activity was the teaching and learning surrounding the rate and ratio unit in both teacher-participants' classrooms. Thus it can be conjectured that the use of conceptual questioning and its related emphasis on communication was a main contributing factor to students' active engagement.

In conclusion, the two teachers in this learning study began by initially exploring conceptual questioning techniques. The designing and implementation of conceptual questions in the classroom over the rate and ratio unit was one step towards fully understanding the conceptual questioning phenomenon. Several notable observations about conceptual questioning were made in the learning study unit which raises a few questions for future investigation. In the data analysis, significant attributes of conceptual questioning were highlighted, specifically the oral nature of the unit, the visual aspects of most of the questions, and the relevant context for the students. This then raises the question, could it be that conceptual questions need to have certain parameters or set of attributes to be successful? For example, how important were the visual aspects of the conceptual questions? And just how significant is the role of oral communication in order for student conceptual understandings to develop? Finally, what are the other attributes of conceptual questioning, attributes that go beyond what has been researched both in this thesis and in the reviewed literature? Thus this initial foray into the process of communication in mathematics and ultimately the application of conceptual questioning has proven to be a rich and rewarding experience that altered the teaching pedagogies of the participating teachers.

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Appendix A

Questionnaire

Learning Study Teacher Questionnaire
on
Communication in the Intermediate Mathematics Classroom

This questionnaire is part of a University of Western Ontario research project on communication in mathematics at the intermediate level. The items in this questionnaire relate to: your understanding of the role of communication and teacher questioning in mathematics; your teaching practices and understandings regarding mathematics instruction; your perceived relevance of currently available resources; your assessment practices.

Please rank your agreement with the statements in this questionnaire. Circle only the number that best relates to your beliefs.

Scale: **1** = never ------ **5** = always/ usually
(adapted from: Kovarik (2008); Lock (2002); Zambo & Hong (1996))

#	Statements: Communication and Questioning	Rank				
1	I have a firm understanding of what communication in mathematics is all about.	1	2	3	4	5
2	I use a variety of questions during mathematics instruction.	1	2	3	4	5
3	I am comfortable with the use of problem solving questions in mathematics.	1	2	3	4	5
4	I use real world examples whenever possible for instructional purposes.	1	2	3	4	5
5	I feel very confident when I am discussing word problems with the class.	1	2	3	4	5
6	I often anticipate student questions and misunderstandings.	1	2	3	4	5
7	I often have strategies for responding to students' questions irrespective of the level of difficulty.	1	2	3	4	5
8	I often have strategies for dealing with student misconceptions.	1	2	3	4	5
9	I often use questions to funnel students to the right answer.	1	2	3	4	5
10	I often know about students' mathematical background and I use this knowledge when formulating questions.	1	2	3	4	5
11	I believe that basic mathematics skills need to be taught first before students can engage in open-ended problem solving activities and in answering questions.	1	2	3	4	5

#	Statements: Communication and Mathematics	Rank				
12	I think that questioning in Mathematics predominately aims at getting students to learn the system of rules and procedures.	1	2	3	4	5
13	I think that questioning in mathematics predominately aims at interpreting and developing mathematical concepts	1	2	3	4	5
14	I think that questioning in mathematics is predominately aimed at getting students to think and communicate mathematically.	1	2	3	4	5
15	I think that questioning in mathematics is predominately aimed at getting students to see the pattern and beauty in mathematics.	1	2	3	4	5
16	I think that getting the correct answer should be the main focus for the use of questioning.	1	2	3	4	5
17	Teachers should use a variety of question types for instructional purposes.	1	2	3	4	5
18	Teachers should use specific types of questions for instructional purposes.	1	2	3	4	5

#	Statements: Communication and Teaching Practice	Rank				
19	I make regular use of journals in mathematics.	1	2	3	4	5
20	I strive to incorporate cooperative learning strategies in mathematics instruction.	1	2	3	4	5
21	I incorporate problem solving strategies routinely in mathematics instruction.	1	2	3	4	5
22	I actively incorporate the seven mathematical processes in my instructional strategies.	1	2	3	4	5
23	I routinely use differentiated instructional methods in mathematics.	1	2	3	4	5
24	I use manipulatives in mathematics instruction regularly.	1	2	3	4	5
25	I implement a balanced mathematics program in my classroom.	1	2	3	4	5
26	My students spend the majority of their seat work time working individually.	1	2	3	4	5
27	I cluster students' desks or use tables so students can work together.	1	2	3	4	5
28	I make it a priority in my classroom to give students time to work together.	1	2	3	4	5

#	Statements: Communication and Resources	Rank				
29	I have been in-serviced on the revised mathematics curriculum.	1	2	3	4	5
30	I am comfortable with the content of mathematics outlined in the curriculum.	1	2	3	4	5
31	I use the TIPS4RM document regularly.	1	2	3	4	5
32	I regularly incorporate the lesson ideas from the Cross Curricular Approaches documents provided by the Ontario Ministry of Education.	1	2	3	4	5
33	I supplement instructional resources with personally obtained materials.	1	2	3	4	5
34	I supplement students' textbook assignments with additional practice materials.	1	2	3	4	5
35	The textbook I use in my classroom supplies all that I need to know about teacher questioning techniques.	1	2	3	4	5
36	The textbooks I use provide good support for implementing a variety of questions.	1	2	3	4	5
37	I believe the curriculum emphasizes the process of communication.	1	2	3	4	5

#	Statements: Communication and Assessment	Rank				
38	I often assess student understanding from class discussion.(P)	1	2	3	4	5
39	I have regular conferences with my students about their progress.	1	2	3	4	5
40	For assessment purposes, I am primarily interested in what students can do individually.	1	2	3	4	5
41	I use math projects as a form of assessment.	1	2	3	4	5
42	I use homework as a main source of assessment.	1	2	3	4	5
43	I incorporate the use of knowledge questions for assessment regularly.	1	2	3	4	5
44	I incorporate the use of application questions for assessment regularly.	1	2	3	4	5
45	I incorporate the use of explanation questions for assessment regularly.	1	2	3	4	5
46	I believe group work is an important part of assessment	1	2	3	4	5
47	I assess students' knowledge primarily with paper and pencil tests.	1	2	3	4	5

Appendix B

Interview Questions

Learning Study Teacher Interview Questions

Communication in the Intermediate Mathematics Classroom

1) What has been the most significant benefit of participating in this learning study?

2) Do you believe your understanding of communication has changed?

3) Do you believe that the elements of teaching questioning worked on during the learning study can be translated into other units of mathematics?

4) Have any of your teaching practices changed since participating in this learning study?

5) Do you believe that your students have improved in terms of their communication skill development through this unit?

Appendix C

Ethics Approval and Letters of Permission

THE UNIVERSITY OF WESTERN ONTARIO
FACULTY OF EDUCATION

Western Education

USE OF HUMAN SUBJECTS - ETHICS APPROVAL NOTICE

Review Number:	**0808-2**
Applicant:	**Stephanie Insley**
Supervisor:	**Immaculate Namukasa**
Title:	***Communication in Mathematics: A Case for Conceptual Questioning in Ontario Middle Schools***
Expiry Date:	**December 31, 2009**
Type:	**M.Ed. Thesis**
Ethics Approval Date:	**February 17, 2009**
Revision #:	
Documents Reviewed & *Approved:*	***UWO Protocol, Letters of Information & Consent***

This is to notify you that the Faculty of Education Sub-Research Ethics Board (REB), which operates under the authority of The University of Western Ontario Research Ethics Board for Non-Medical Research Involving Human Subjects, according to the Tri-Council Policy Statement and the applicable laws and regulations of Ontario has granted approval to the above named research study on the date noted above. The approval shall remain valid until the expiry date noted above assuming timely and acceptable responses to the REB's periodic requests for surveillance and monitoring information.

No deviations from, or changes to, the research project as described in this protocol may be initiated without prior written approval, except for minor administrative aspects. Investigators must promptly report to the Chair of the Faculty Sub-REB any adverse or unexpected experiences or events that are both serious and unexpected, and any new information which may adversely affect the safety of the subjects or the conduct of the study. In the event that any changes require a change in the information and consent documentation, newly revised documents must be submitted to the Sub-REB for approval.

Dr. Jason Brown (Chair)

2008-2009 Faculty of Education Sub-Research Ethics Board

Dr. Jason Brown	**Faculty (Chair)**
Dr. Elizabeth Nowicki	**Faculty**
Dr. Jacqueline Specht	**Faculty**
Dr. John Barnett	**Faculty**
Dr. J. Marshall Mangan	**Faculty**
Dr. Immaculate Namukasa	**Faculty**
Dr. Robert Macmillan	**Assoc Dean, Graduate Programs & Research (*ex officio*)**
Dr. Jerry Paquette	**UWO Non-Medical Research Ethics Board (*ex officio*)**

The Faculty of Education	Karen Kueneman, Research Officer
1137 Western Rd.	Faculty of Education Building
London, ON N6G 1G7	kueneman@uwo.ca
	519-661-2111, ext.88561 FAX 519-661-3029

Copy: Office of Research Ethics

Western

APPROVAL OF M.Ed. THESIS PROPOSAL **FORM A**

If the proposed research does not involve human subjects or the direct use of their written records, video-tapes, recordings, tests, etc., this signature form, along with ONE copy of the research proposal should be delivered directly to the Graduate Programs & Research Office for final approval.	**If the proposed research involves human subjects, this signature form, along with ONE copy of the research proposal and Ethical Review Form must be submitted to the Graduate Programs & Research Office for final approval.**

IT IS THE STUDENT'S RESPONSIBILITY TO PROVIDE A COPY OF THE RESEARCH PROPOSAL (INCLUDING REVISIONS) TO THE THESIS SUPERVISOR AND ALL MEMBERS OF THE ADVISORY COMMITTEE.

Student's Name: Stephanie Insley ID# ____________

Field of Study: Education

TITLE OF THESIS: Communication in Mathematics: A Case for Conceptual Questioning in Ontario Middle Schools

DOES THIS RESEARCH INVOLVE THE USE OF HUMAN SUBJECTS: **YES** ☒ **NO** ☐

Name of Thesis Supervisor: Immaculate k. Namukasa

Name(s) of Members of the Thesis Advisory Committee: George Gadanidis

APPROVAL SIGNATURES:

Graduate Student: ____________

Thesis Supervisor: ____________

Advisory Committee: (at least one) ____________

Ethical Review Clearance: ____________

Review #: 0808-2 Date: Feb 17/09

Associate Dean (GPR): ____________ Date: ____________

A STUDENT MAY PROCEED WITH RESEARCH WHEN A COPY OF THIS FORM CONTAINING ALL APPROVAL SIGNATURES HAS BEEN RECEIVED.

A COPY OF THIS PROPOSAL MAY BE MADE PUBLIC AND KEPT ON A TWO-HOUR RESERVE IN THE FACULTY OF EDUCATION LIBRARY.

Letter of Permission – Teacher

Communication in Mathematics: A Case for Conceptual Questioning in Ontario Middle Schools

LETTER OF INFORMATION

Introduction

My name is Stephanie Insley and I am a graduate student at the Faculty of Education at The University of Western Ontario. I am currently conducting research into forms of communication in mathematics, specifically teacher questioning, and would like to invite you to participate in this study.

Purpose of the study

The aims of this study are to examine elements of communication in the intermediate mathematics classroom with a focus on teacher questioning techniques.

If you agree to participate

If you agree to participate in this study you will be asked to participate in a Learning Study research project which involves the development and implementation of a unit of study, in the subject area of mathematics. As a participant in the Learning Study research project, you will be asked to participate in several focus group meetings; collaborate on the unit of study design; conducting pre and post unit observations of your students in your class who have agreed to participate; complete a pre and post study questionnaire coupled with informal pre and post study interview; and consenting to video-tape your teaching of several lessons in the unit of study. Your time commitment would be approximately 6 hours in total.

Confidentiality

The information collected will be used for research purposes only, and neither your name nor information which could identify you will be used in any publication or presentation of the study results. Video tapes and all information collected for the study will be kept confidential and destroyed after a five year period.

Risks & Benefits

There are no known risks to participating in this study. One benefit of participating in this study may be the opportunity to develop professionally and get a co-planned unit.

Voluntary Participation

Participation in this study is voluntary. You may refuse to participate, refuse to answer any questions or withdraw from the study at any time with no effect on your employment status.

Questions

If you have any questions about the conduct of this study or your rights as a research participant you may contact the Manager, Office of Research Ethics, The University of Western Ontario at 519-661-3036 or ethics@uwo.ca.

If you have any questions about this study, please contact:

Ms. Stephanie Insley
Home:
School: '
Email: or_

My supervisor, Dr. Immaculate K. Namukasa
Tel:
Email:_

This letter is yours to keep for future reference.

Stephanie Insley
Graduate Student
University of Western Ontario

Communication in Mathematics: A Case for Conceptual Questioning in Ontario Middle Schools

Stephanie Insley Graduate Student,
Faculty of Education, University of Western Ontario

CONSENT FORM

I have read the Letter of Information, have had the nature of the study explained to me and I agree to participate. All questions have been answered to my satisfaction.

Name (please print):

____

Signature: ____________________________ Date: ___________________

Stephanie Insley

Signature: ____________________________Date: ___________________

Letter of Permission – Students from Teacher-Participant's Classroom

Communication in Mathematics: A Case for Conceptual Questioning in Ontario Middle Schools

LETTER OF INFORMATION

Introduction
My name is Ms. Stephanie Insley and I am a graduate student at the Faculty of Education at The University of Western Ontario as well as a classroom teacher in the Thames Valley District School Board. I am currently conducting research into how to improve the teaching and learning of Mathematics through the use of teacher questioning. I would like to invite you, as a student, to participate in this study.

Purpose of the study
The aims of this study are to examine elements of communication in the intermediate mathematics classroom with a focus on teacher questioning techniques.

If you agree to participate
If you agree to participate as a student in this study, your involvement in lessons taught during the Mathematics Rate and Ratio unit would be observed and video- taped. All students, regardless of whether they choose to participate in the study or not, will continue to participate in the Mathematics Rate and Ratio unit of study that is going to be researched. If you have not signed the Thames Valley District School Board form indicating that you can be video taped, then I will not video tape you.

Voluntary Participation
Participation in this study is voluntary. You may refuse to participate, refuse to answer any questions, or withdraw from the study at any time with no effect on your academic status. Your decision to participate or not, will not

have any affect on your grade in this unit of mathematics or on your relationship with your classroom teacher or Ms. Insley.

Confidentiality
The information collected will be used for research purposes only, and neither your name nor information which could identify you will be used in any publication or presentation of the study results. All information collected for the study will be kept confidential. All video tapes will be kept in a locked office and not shared with third parties. After a period of five years the video tapes will be destroyed.

Risks & Benefits
There are no known risks to participating in this study.

Questions
If you have any questions about the conduct of this study or your rights as a research participant you may contact the Manager, Office of Research Ethics, The University of Western Ontario at 519-661-3036 or ethics@uwo.ca.

If you have any questions about this study, please contact:

Ms. Stephanie Insley
Tel:
Email: or

Or, my supervisor :

Dr. Immaculate K. Namukasa
Tel:
Email:

This letter is yours to keep for future reference.

Ms. Stephanie Insley
Graduate Student
University of Western Ontario

Letter of Permission – Students from Teacher-Researcher's Classroom

Western
Education

Communication in Mathematics: A Case for Conceptual Questioning in Ontario Middle Schools

LETTER OF INFORMATION

Introduction
My name is Ms. Stephanie Insley and I am a graduate student at the Faculty of Education at The University of Western Ontario as well as a classroom teacher. I am currently conducting research into how to improve the teaching and learning of Mathematics through the use of teacher questioning. I would like to invite you, as a student, to participate in this study.

Purpose of the study
The aims of this study are to examine elements of communication in the intermediate mathematics classroom with a focus on teacher questioning techniques.

If you agree to participate
If you agree to participate as a student in this study, your involvement in lessons taught during the Mathematics Ratio and Proportions unit would be observed and video- taped. All students, regardless of whether they choose to participate in the study or not, will continue to participate in the Mathematics Probability unit of study that I am going to research. If you have not signed the Thames Valley District School Board form indicating that you can be video taped, then I will not video tape you or observe you for study purposes.

Voluntary Participation
Participation in this study is voluntary. You may refuse to participate, refuse to answer any questions, or withdraw from the study at any time with no

effect on your academic status. Your decision to participate or not, will not have any affect on your grade in this unit of mathematics or on your relationship with Ms. Insley. Anonymity regarding your choice to participate or not, will be guaranteed as consent forms will be collected by another teacher, will be kept by the office secretary and will not be released to Ms. Insley until final grades are submitted in June.

Confidentiality
The information collected will be used for research purposes only, and neither your name nor information which could identify you will be used in any publication or presentation of the study results. All information collected for the study will be kept confidential. All video tapes will be kept in a locked office and not shared with third parties. After a period of five years the video tapes will be destroyed.

Risks & Benefits
There are no known risks to participating in this study.

Questions
If you have any questions about the conduct of this study or your rights as a research participant you may contact the Manager, Office of Research Ethics, The University of Western Ontario at 519-661-3036 or ethics@uwo.ca.

If you have any questions about this study, please contact:

Ms. Stephanie Insley
Tel:
Email: or

My supervisor, Dr. Immaculate K. Namukasa
Tel:
Email:

This letter is yours to keep for future reference.

Ms. Stephanie Insley
Graduate Student
University of Western Ontario

Communication in Mathematics: A Case for Conceptual Questioning in Ontario Middle Schools

Ms. Stephanie Insley: Graduate Student
Faculty of Education
University of Western Ontario

CONSENT FORM

Please check one of the boxes below and return to your classroom teacher.

I have read the letter of information. I choose not to participate in the study at this time. I am aware that I am still required complete the unit of study in mathematics.

I have read the letter of information, have had the nature of the study explained to me and I agree to participate. All questions have been answered to my satisfaction.

Name of Student (please print):

____

Signature of Student:

____

Name of Parent/Guardian (please print):

____

Signature of Parent/Guardian:

__________________________________ Date: _______________

Name of person obtaining consent: Ms. Stephanie Insley

Signature of person obtaining consent: ______________________

Date: ___________________________

Appendix D

Sample Lesson Plans

This appendix contains a sample of the lesson plans used in the rate and ratio unit, days one to five. Features include the conceptual question prompts in the resource column and some notes in the lesson plan on what expected student responses may entail when necessary.

Day 1	**What is a Ratio?**	
Materials	**Lesson Plan**	**Resources**
Smart File Calculator Lined paper or notebooks Pencils coloured tiles BLM1	Introduce the notion of ratios by viewing the popcorn advertisements (Smart file - page 2) **1. Think/Pair/Share** Which advertisement(s) is/are the most effective? Do the advertisements have the same numerical values? How do you know that ad #1 and ad #3 are giving the exact same information? Note: Students will naturally gravitate to the 2 advertisements on the left as they are ratios. Both ratios are dealing with the same numbers(and are equivalent), but one is making a part-to-part ratio and the other is making a part-to-whole ratio comparison. You may choose to come back to this point later. **2. Tile Comparisons** This is set up so that you can use coloured tiles for hands on manipulatives along with the smart file. Have students work individually or in pairs to create as many ratios as possible with the data set. You may need to guide them into a comparison of a part-to-whole. Be sure to emphasize the question on part 2 of the smart file "do you think that . . ." as this will allow for communication and give you insight as to how well they understand ratios. Answer: There are 2 different ways to express the ratios concerning the # of green and the # of blue tiles. Green:Blue = 9:12 = 4:3 and Blue:Green 4:3 They are different ratios but they provide us with the same	Addison 7 Nelson 7 & 8 **Conceptual Questions** How do you know that ad # 1 . . .? Do you think that . . .? **Communication Points:** - ads - final ratio comparison - ability to show part-to-part and part-to-whole ratios

Day 2	**Exploring Ratio**	
Materials	**Lesson Plan**	**Resources**
Smart File Notebooks/ paper Pencils Calculators BLM 2	**1. A collection of Hot Wheels** Using the Smart file, compare the collection of Hot Wheels. Begin with **part-to-whole**. In this case you can compare the number of cars to the total number of vehicles. There are 9 cars and 13 vehicles in total. The ratio can be written 9 to 13 or 9:13. **Part-to-part:** One can compare a part of a group to another part of a group. For example there are 9 cars to 4 airplanes. We can write this as 9 to 4 or 9:4. 9 and 4 are called terms of the ratio. 9 is the first term and 4 is the second. You will need to add notes to the Smart file in this section as you see fit. **2. Practice: Camels Question** This is a class practice question. After students have figured out the ratios, have them go back and label their ratios as part-to-whole or part-to-part. **3. Annika's Fish** This has been modified from the Nelson 8 "Getting Started" on page 46 and the visual is in the Smart file. Have students work towards independent practice for this question. It requires some figuring out of percentages from fractions which you may need to review with your students prior to the activity. The follow-up questions for this activity are designed to be more conceptual in nature. Some students may require more structured hints for this part. I included the last question as a slide - to take up after the BLM 2.	Addison 7 Nelson 7 & 8 **Conceptual Questions** **Communication Points:** Camel activity Annika's Fish

Day 3 & 4	**Equivalent Ratio**	
Materials	**Lesson Plan**	**Resources**
Multiplication matrix Pattern Blocks Calculator Math notebook/ paper Pencil Smart File BLM 3 Appendix 1.3 Nelson 8 Textbook	**1 a. Cookies** Equivalent ratios, because they tend to be similar but not the same as fractions, can worry students. As an intro activity, your stronger students should be able to see and make comparisons between ratios based on their equivalent fraction experience while the weaker students will be happy with just making the ratios. They may need a push to make the comparisons. **1 b. Ratio Table** A Ratio table is a neat way of finding equivalent ratios. For your IEP students, they can find equivalent ratios using the multiplication matrix. Your stronger math students can use the ratio table. See Appendix 1.1 for an explanation. **2. Pattern Block Ratios** This lesson is modified from **Nelson 8** section 2.3 on page 56 and there is a pattern block visual in the smart file. Have students look at the green triangle and red trapezoid and identify the relationship. (3 green triangles fit into 1 red trapezoid.) Then have students examine the second grouping of pattern blocks. Pose the question: If the green triangle to the red trapezoid ratio is 1:3, how do you know that the ratio of the blue rhombus to the yellow hexagon is 4:6? Depending on the amount of time and the understanding of students, you may to continue with the 2.3 exploration. "D" also provides an excellent opportunity to demonstrate conceptual understanding. (I may include that one regardless of the time constraints.) **3.Recipe** Begin with the comparing of ratios **smart file** - "hot chocolate". This will allow for the application of the ratio tables to a "real life" problem. This can then be applied to quantities in a recipe for numbers of people served.	Addison 7 Nelson 7 & 8 MathPower 7 Making Math Meaningful **Conceptual Questions** If the green triangle to the red trapezoid ratio is 1:3, how do you know that the ratio of the blue rhombus to the yellow hexagon is 4:6? If Ms. Wilson has 28 students and Ms. Insley has 35 students, how do you know that the total combined ratio of boys to girls is 32:31? **Communication Points:** -class discussion about cookies - ratio table - discussion of pattern blocks

Day 5	Using Proportion to Solve Ratio	
Materials	**Lesson Plan**	**Resources**
BLM 4	**1. Dr. Small's class problem** Pose the class ratio of boys to girls question. Smart file "Small Ratios" . For some students it may help to have them draw it out. Work through both questions - have students do that last one completely independent (except for IEP). 1) If Ms. Wilson has 28 students and Ms. Insley has 35 students, how do you know that the total combined ratio of boys to girls is 32:31? 2) If we combine the 2 classes, what are the necessary conditions for the ratio to be 1:1? **2. Cement** This section is modified from the Nelson 8 text section 2.4 on page 58. Questions have been modified to reflect the curriculum expectations. **The ratio of stone to sand in *hardfast* Concrete is 2 to 3. How much stone is needed if 15 bags of sand are used?** (Ministry Document - example question)	**Conceptual Questions** If we combine the 2 classes, what are the necessary conditions for the ratio to be 1:1? Mike's boss asked him to bring 4 bags of cement to the job site. Explain how Mike knows he will also need to bring 12 bags of sand .

Appendix E

Curriculum Expectations

These curriculum expectations are from *The Ontario Curriculum, Grades 1-8: Mathematics Revised.*(2004) and reflect the specific expectations for both grade 7 and grade 8 under the number sense and numeration strand, proportional relationships. The bolded sections reference the key concepts that were used in the development of the conceptual questions for the rate and ratio unit.

Grade 7 Expectations
Proportional Relationships

– determine, through investigation, the relationships among fractions, decimals, percents, and ratios;

– solve problems that involve determining whole number percents, using a variety of tools (e.g., base ten materials, paper and pencil, calculators) (*Sample problem:* If there are 5 blue marbles in a bag of 20

– demonstrate an **understanding of rate as a comparison, or ratio**, of two measurements with different units (e.g., speed is a rate that compares distance to time and that can be expressed as kilometres per hour);

– solve problems involving the calculation of **unit rates** (*Sample problem:* You go shopping and notice that 25 kg of Ryan's Famous Potatoes cost $12.95, and 10 kg of Gillian's Potatoes cost $5.78. Which is the better deal? Justify your answer.).

Grade 8 Expectations
Proportional Relationships

– identify and describe real-life situations involving two quantities that are directly proportional (e.g., the number of servings and the quantities in a recipe, mass and volume of a substance, circumference and diameter of a circle);

– solve problems involving **proportions**, using concrete materials, drawings, and variables (*Sample problem:* The ratio of stone to sand in HardFast Concrete is 2 to 3. How much stone is needed if 15 bags of sand are used?);

– solve problems involving **percent** that arise from real-life contexts (e.g., discount, sales tax, simple interest) (*Sample problem:* In Ontario, people often pay a provincial sales tax [PST] of 8% and a federal sales tax [GST] of 7% when they make a purchase. Does it matter which tax is calculated first? Explain your reasoning.);

– solve problems **involving rates** (*Sample problem:* A pack of 24 CDs costs $7.99. A pack of 50 CDs costs $10.45. What is the most economical way to purchase 130 CDs?).

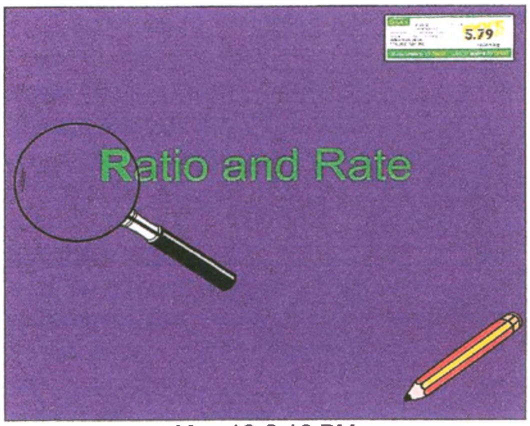
Appendix F

A Correlation of Textbook Authors, Titles, and Publishers

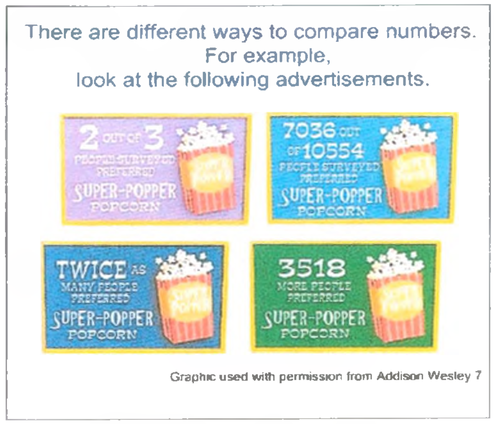
Authors	Title	Grade	Publisher
M. Small, M. Kestall, D. Zimmer, D. Cooper, B. Beales, M. Bodiam, D. Duff, R. Foster, C. Hall, J. Hope, C. Kirkpatrick, B. Kroll Myhill, G. Suderman-Gladwell, J. Toner	Mathematics 7	7	Thomas Nelson
M. Small, M. Kestall, D. Zimmer, D. Cooper, B. Beales, M. Bodiam, D. Duff, R. Foster, C. Hall, J. Hope, C. Kirkpatrick, B. Kroll Myhill, G. Suderman-Gladwell, J. Toner	Mathematics 8	8	Thomas Nelson
J. Johnston, M. Doucette, S. Thomas, T. Brown, D, Jones, J. Paziuk, K. Harper, B. Keyes, A. Lenjosek, M. Sinclair, C. Heideman, M. Davis, S. Jeroski	Math Makes Sense 7	7	Addison Wesley
T. Brown, C. Featherstone, C. Heidaman, S Jeroski, J. Johnston, D. Jones, A.J. Keene, B. Keyes, G. Konis-Chatzis, A. Lenjosek, J. Memmie, E. Milne, M. Sinclair, E. Wood	Math Makes Sense 8	8	Addison Wesley

Appendix G

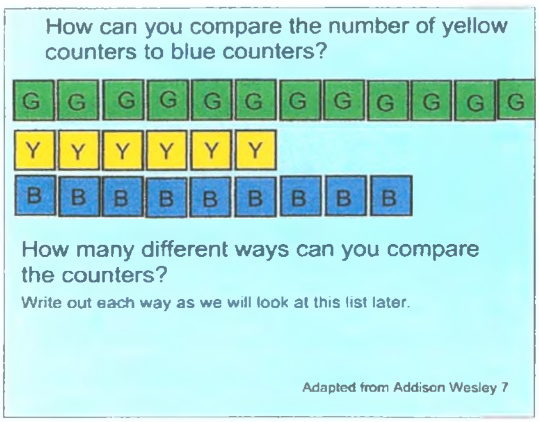
Smart Board File Used for the Lesson



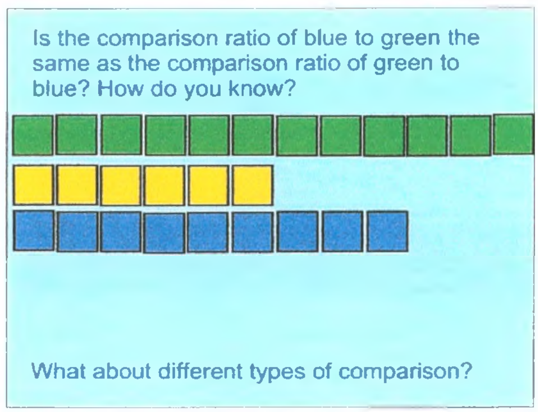
May 16-2:16 PM



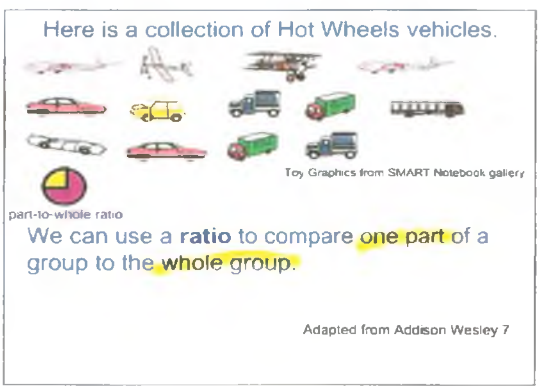
Pop Corn Advertisements



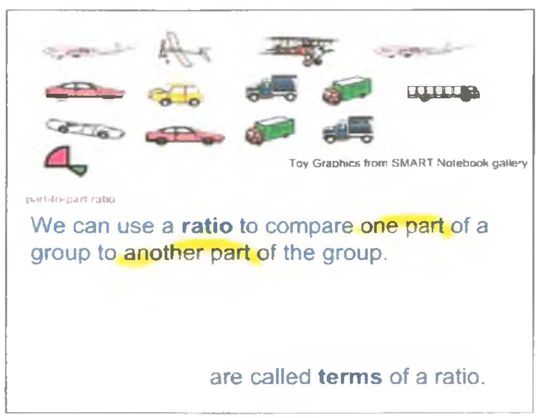
Counters



Counters 2

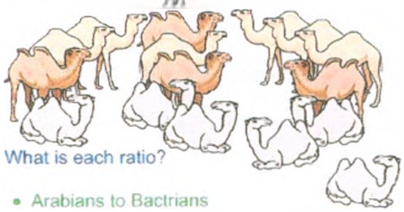


part-to-whole



part-to-part

Arabian camels have 1 hump and
Bactrian camels have 2 humps



What is each ratio?

- Arabians to Bactrians
- Arabians standing to Bactrians sitting
- Bactrians sitting to Bactrians standing
- Arabians to total number of camels

Camel Practice

Annika's Fish

Annika is buying a new fish tank and tropical fish. She wants 20 different kinds and sizes of fish. Based on the size of the tank she wants, the salesperson recommends that no more than 35% of the fish should be 8 cm or longer.

Type of fish	Number of fish Annika would like in her tank	Average length (cm)
neon tetra	8	4
orange swordtail	3	10
black molly	6	7
angel fish	2	11
algae eater	1	15

Wording and chart used with permission from Nelson 8

Annika's Fish

Write the ratio of neon tetra to the total number of fish in the tank.

Neon Tetra: Total Number of Fish

Orange Swordtails: Total Number of Fish

Black Molly : Total Number of Fish

Angel Fish: Total Number of Fish

Wording and chart used with permission from Nelson 8

Annika's Fish Ratios

Annika's Fish

Annika is buying a new fish tank and tropical fish. She wants 20 different kinds and sizes of fish. Based on the size of the tank she wants, the salesperson recommends that no more than 35% of the fish should be 8 cm or longer.

Type of fish	Number of fish Annika would like in her tank	Average length (cm)
neon tetra	8	4
orange swordtail	3	10
black molly	6	7
angel fish	2	11
algae eater	1	15

Wording and chart used with permission from Nelson 8

Does Annika's list follow the salesperson's recommendations? Explain how you know.

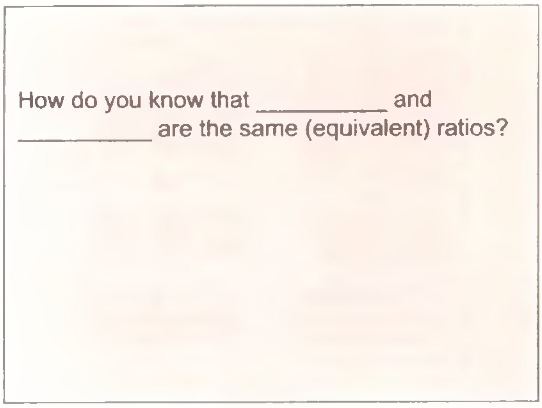
May 24-9:23 PM

Equivalent Ratios

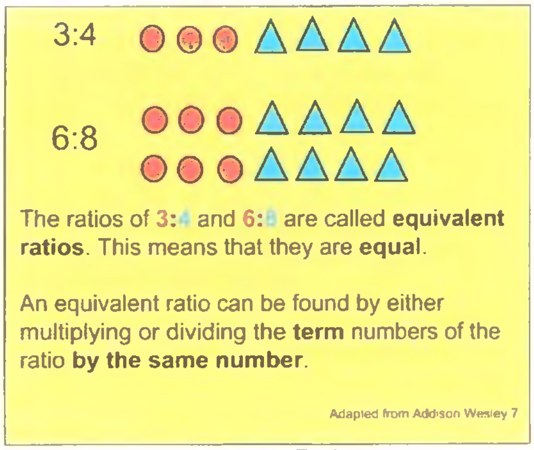
Which cards have the same ratios of chips to cookies?

Cookies

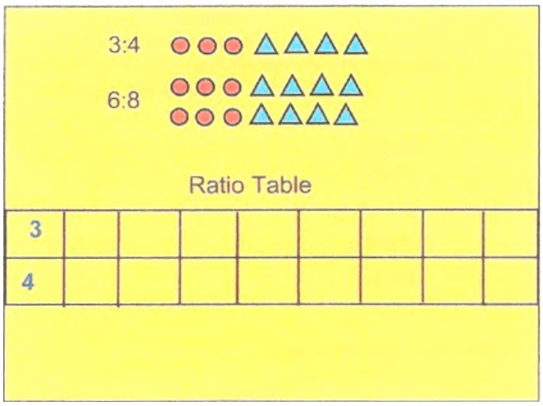
Ratio Cookies



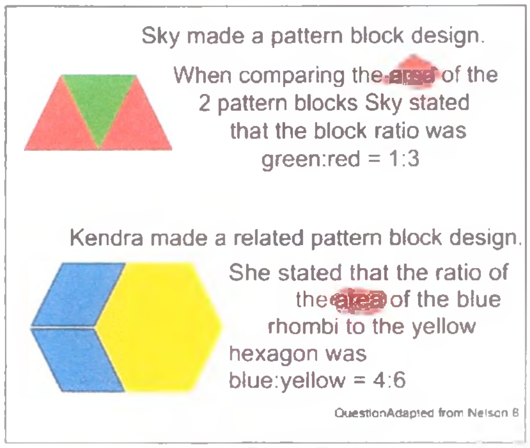
Cookies Continued



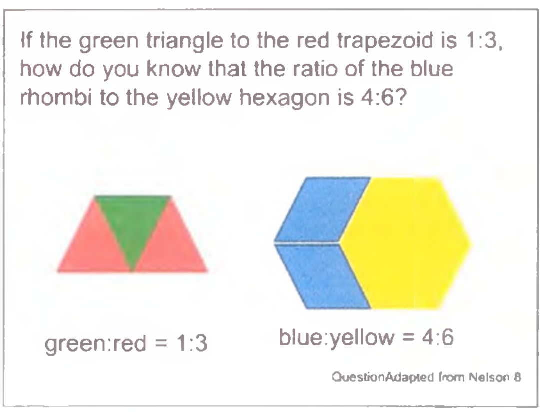
Equivalent Ratios



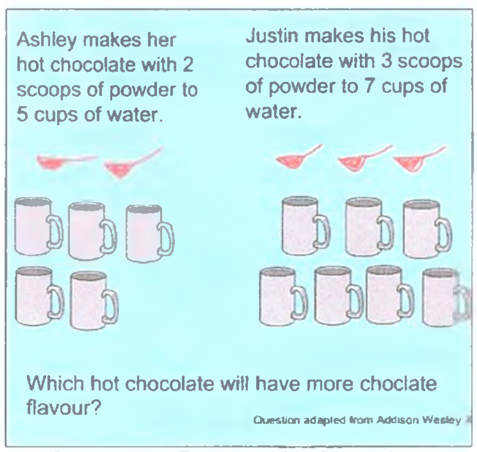
Ratio Table



Pattern Block Ratio



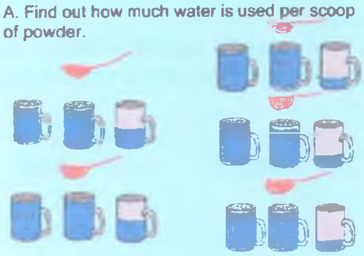
Pattern Block Continued



Comparing Ratios - Hot Chocolate

There are 2 ways to solve this problem:

A. Find out how much water is used per scoop of powder.



1 scoop of hot chocolate powder to 2 1/2 cups of water.

1 scoop of hot chocolate powder to 2 1/3 cups of water.

Question adapted from Addison Wesley 7

Method 1

Ashley uses 1 scoop of hot chocolate powder to 2 1/2 cups of water.

Justin uses 1 scoop of hot chocolate powder to 2 1/3 cups of water.

Ashley uses more water per scoop. Therefore Justin's hot chocolate is stronger than Ashley's hot chocolate.

Question adapted from Addison Wesley 7

Method 1

B. Find out how much powder is used for each cup of water.

- Write the mixture as a ratio
- Write each ratio with the same second term, then compare first terms.

Ashley	Justin
2:5	3:7
4:10	6:14
6:15	9:21
12:30	12:28
14:35	15:35

Question adapted from Addison Wesley 7

Method 2

Since 2:5 = 14:35, Ashley uses 14 scoops of powder to 35 cups of water.

Since 3:7 = 15:35, Justin uses 15 scoops of powder to 35 cups of water.

Justin uses more hot chocolate powder. So Justin's hot choclate is stronger.

Question adapted from Addison Wesley 7

Method 2

In Mr. Watson's class the ratio of boys to girls was 3:4 and in Ms. Smyth's class the ratio of boys to girls was 4:3.

Watson	Smyth
BBB GGGG	BBBB GGG
3 : 4	4 : 3

- If Mr. Watson had 28 students and Ms. Smyth had 35 students, how do you know that the **total** ratio of boys to girls is 32:31?
- If we combine the two classes, what are the necessary conditions for the ratio to become 1:1?

Questions adapted from Small (2008)

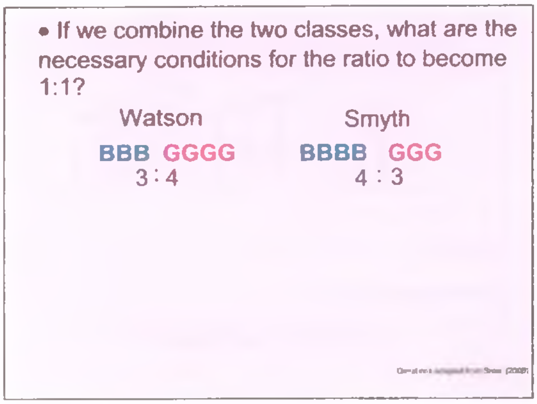
Classroom Ratio

- If Mr. Watson had 28 students and Ms. Smyth had 35 students, how do you know that the **total** ratio of boys to girls is 32:31?

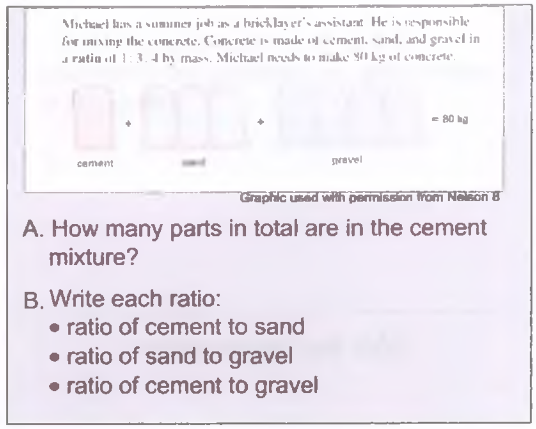
Watson	Smyth
BBB GGGG	BBBB GGG
3 : 4	4 : 3

Questions adapted from Small (2008)

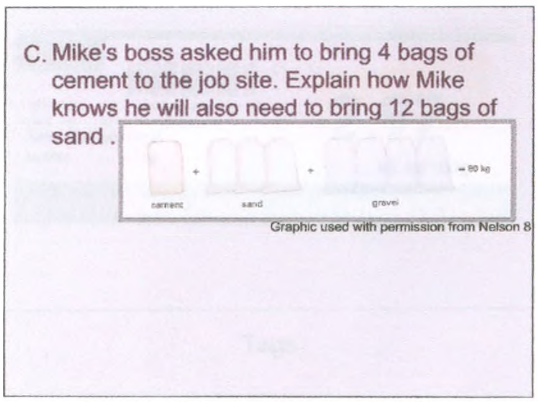
Classroom Ratio



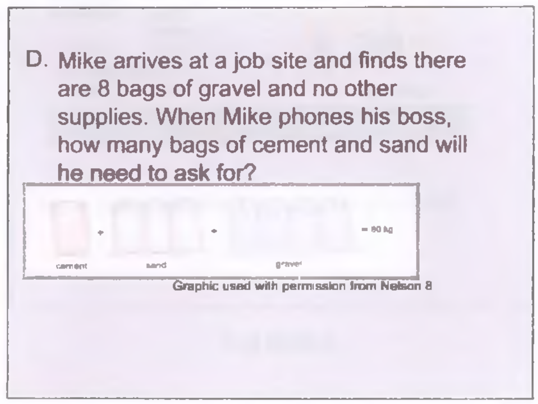
Classroom Ratio



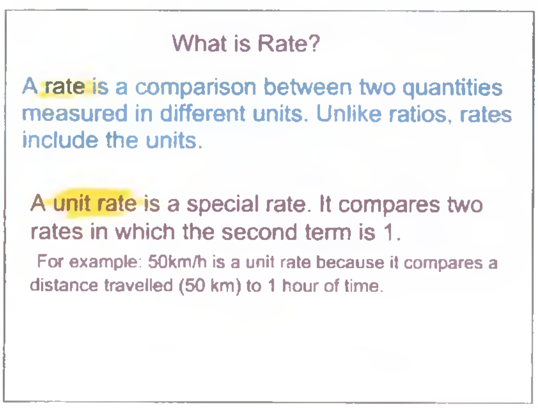
Cement Ratios



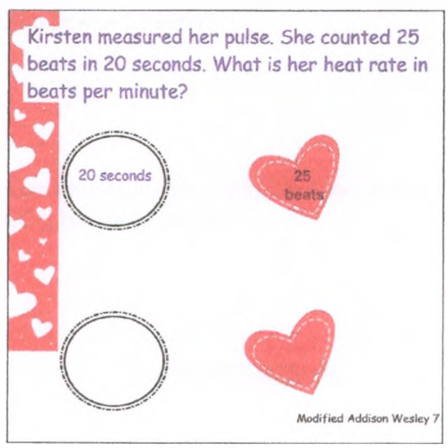
Cement #2



Cement #3



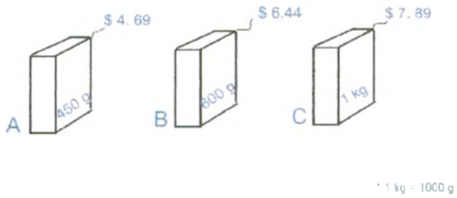
What Is Rate?



Heart Rates

Samantha likes Cheerios and her family eats a lot of them. So when she was shopping, Samantha wondered what would be the best deal on Cheerios.

Cheerios come in 3 different sized boxes:



Adapted from Addison Wesley 8

Cereal Box Questions

A gram is quite a small unit of measure.

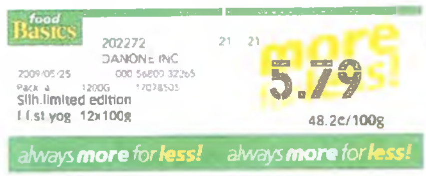
Since a gram is really too small to be practical for our purposes, we often measure in 100 grams.

Measurement unit 100g

When shopping, price tags give us a lot of information

Tags

So, what can we tell about this product?



How many different ratios can you find?

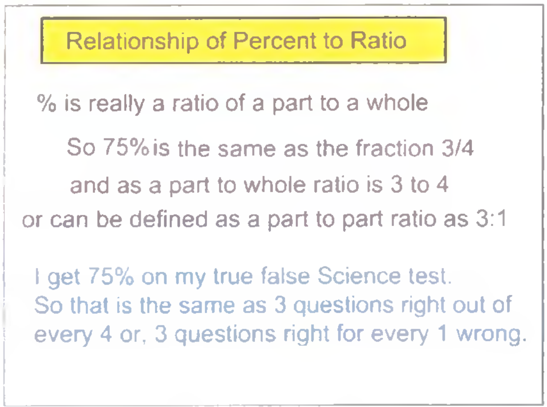
Tag Ratios

Sample Tag Comparisons

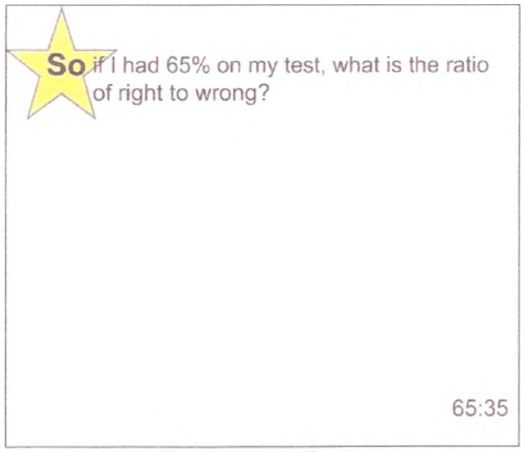
Under what conditions might someone buy the 600 g box of Cheerios?

Under what conditions might someone buy the 1000 g box of Cheerios?

Practice Questions



Relationship of % to Ratio



Percent and Ratio

Appendix H

Black Line Masters 1

popcorn questions.

Math: Rate and Ratio Name: _______________

There are different ways to compare numbers. For example, look at the following advertisements.

Graphic used with permission from Addison 7

How are the numbers in each advertisement compared?

____

____

Which advertisement is most effective? Explain.

____

____

____

How do you know that advertisement #1 and advertisement #3 are giving you the same information?

____

____

Black Line Masters 2

annika's fish questions.

Annika's Fish

Type of Fish	Number of Fish Annika Would Like	Average Length (cm)
neon tetra	8	4
orange swordtail	3	10
black molly	6	7
angel fish	2	11
algae eater	1	15

Chart used with permission form Nelson 8

Annika is buying a new fish tank and tropical fish. She wants 20 different kinds and sizes of fish. Based on the size of the tank she wants, the sales person recommends that no more than 35% of the fish should be longer than 8 cm.

1. Write the ratio of the number of neon tetra to the total number of fish on Annika's list. Write a similar ratio for each type of fish.

   Neon tetra: all fish ____________

2. Calculate the percentage of each type of fish on Annika's list.

3. How many fish on Annika's list are longer than 8 cm?

4. What percentage of the fish are longer than 8 cm?

5. What percentage of the fish are shorter or equal to 8 cm?

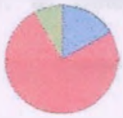
6. Does Annika's list follow the sales person's recommendations? Explain how you know?

Black Line Masters 3a
heart rate questions lesson plan.

(Leading Math Success, OMET, 2005, Chapter5, Day 4 p. 12-13)

Unit 5: Day 4: Modelling Linear Relationships

Grade 7



Math Learning Goals
- Model relationships that have constant rates, where the initial condition is zero.
- Illustrate linear relationships graphically and algebraically.

Materials
- BLM 5.4.1, 5.4.2

Assessment Opportunities

Minds On...

Whole Class → Brainstorm
With the students, brainstorm and compile a list of everyday relationships that involve a constant rate, e.g., a person's resting heart rate, a person's stride length, speed of a car driving at the speed limit, rate of pay at a job that involves no overtime, hours in a day.

Action!

Whole Class → Demonstration
Using the context of stride length, measure one student's stride length, e.g., 25 cm. Complete a table of values for 0–8 strides for this person and calculate the distance walked. Graph the relationship between this person's stride length and the distance walked. (There is no correct answer to the question.) Ask: Should "stride length" or "distance walked" be on the horizontal axis?

Discuss the meaning of:
- constant rate (same value added to each successive term, e.g., 25 cm);
- initial condition (the least value that is possible, e.g., zero);
- linear relationship.

Illustrate how to determine an equation for this relationship ($d = 25s$).

Together, calculate values that are well beyond the values of the table, e.g., what distance would 150 strides cover?

Discuss the advantages and disadvantages of the table of values, the graph, and the algebraic equation.

Representing/Observation/Anecdotal Note: Assess students' ability to represent a linear pattern in a chart and in a graph.

Some students may experience difficulty in determining the algebraic model.

Pairs → Investigation
Students complete question 1 on BLM 5.4.1 and BLM 5.4.2.

Students with the same heart rate should have the same numerical and algebraic representations, but not necessarily the same intervals on their graphs.

Consolidate Debrief

Small Groups → Presentation
By a show of hands, determine which students have the same heart rates. These students form small groups and present their tables, graphs, and algebraic expressions to each other. Groups discuss any results that differ and determine the correct answers.

Reflection
Practice

Home Activity or Further Classroom Consolidation
Complete questions 2 and 3 on worksheets 5.4.1 and 5.4.2.

Black Line Masters 3b
heart rate questions student worksheet.

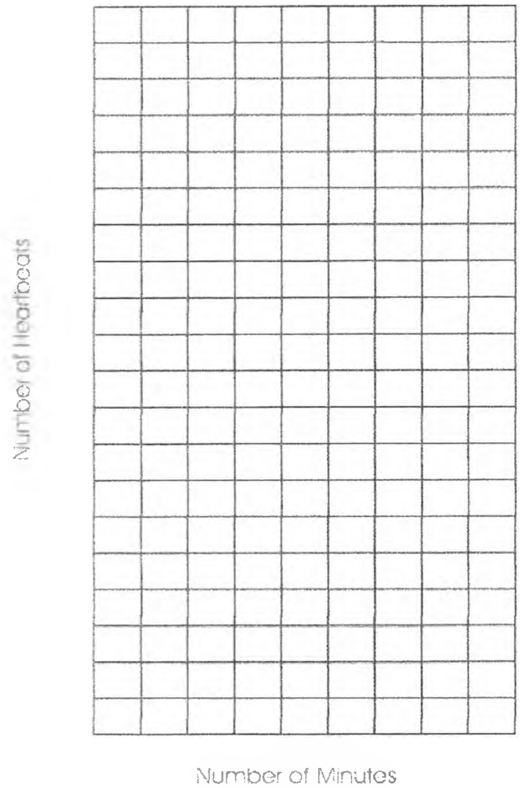
5.4.1: Getting to the Heart of the Math

1. a) Determine your heart rate for 1 minute at rest:

   ______ beats per minute.

   b) Complete a table of values to display the number of heartbeats, *H*, for *t* minutes.

Number of Minutes	Number of Heartbeats
0	
1	
2	
3	
4	
5	
6	
7	
8	



   c) Graph the relationship. Choose suitable intervals for each axis.

   d) Write an algebraic expression for the relationship:

   e) How many times will your heart beat during:
   i. 30 minutes:

   ii. 45 minutes?

   iii. 1 hour?

   iv. 90 minutes?

2. After one minute of vigorous exercise, e.g., running on the spot, take your pulse to determine your heart rate after exercise. Complete a table of values for your increased heart rate, and graph the relationship on the grid.

3. In your journal, compare the two graphs. Include "initial condition" and "constant rate of change."

Black Line Master 4
ratio and percents culminating questions.
(From Metz, 2003)

Making Sense of Percents

Ashley and her Mom are shopping for new jeans. They notice a rack of jeans with this sign on top.

40% off the lowest ticketed price

Ashley is excited because that means that the jeans are an additional 40% off of an already reduced price. She finds a pair of jeans in her size, but part of the price tag has been torn off. The remaining part of the tag looks like this.

Orig. Price

$40.00

Reduced 25% to

Ashley and her Mom both realize that the jeans' ticketed price is missing. However they both think that they can figure out the final cost of the jeans. Ashley's Mom says that since the jeans were already reduced 25% and are being reduced again by 40%, then they are really on sale for 65% off. Ashley disagrees. Ashley thinks that the reduction of 25% followed by a reduction of 40% is the way to calculate the final price.

What would the final sale price be using Ashley's method?

If Ashley's Mom calculated the sale price of the jeans, what would it be?

Is Ashley or her Mom correct in their thinking? How do you know?

Black Line Masters 5a
winter play culminating questions

From Mathematics In-service on Proportional Reasoning

8.3.5: PROBLEM: GROUP 5

Winter Play Day

There are 500 students at Pine Ridge Elementary School.

How many students went snowboarding?

Winter Play Day

75% of the students at Pine Ridge Elementary signed up for Winter Play Day.

How many students participated in each event on Winter Play Day?

Winter Play Day

3/5 of the students who signed up for Winter Play Day actually showed up to participate.

How many students participated in each event on Winter Play Day?

Winter Play Day

60% of the students who participated in Winter Play Day went snowboarding; 8% went skiing and the rest of the students went skating.

How many students went snowboarding?

Black Line Masters 5b
winter play problem solving

United We Solve
Problem Solving Template

Problem Title:
What are you asked to find?
What are "the facts"? List them here.
If necessary, organize "the facts" in a way that you can use them to solve the problem.
Do any calculations here:
Clearly state your answer(s) here:

Black Line Masters 5c
winter play answers.

United We Solve
Problem Solving Template

Problem Title: Winter Play Day	**Team #**

What are you asked to find?
The number of students who participated in the Play Day and the number who went snowboarding.

What are "the facts"? List them here.
-there are 500 students in total
-75% signed up in total
-3/5 of those who signed up, actually showed up
-60% of the students went snowboarding, 8% went skiing and the rest went skating

If necessary, organize "the facts" in a way that you can use them to solve the problem.

-3/5 of the 75% actually showed up
-32% went skating

Do any calculations here:

-Find 75% of 500 to get the number who signed up

0.75 x 500 = 375 (or 3/4 of 500)

-only 3/5 of those actually attended. This is 60%. So 0.60 x 375 = 225

-Therefore 60% of these snowboarded: 0.60 x 225 = 135
-Therefore 8% went skiing: 0.08 x 225 = 18
-Therefore 32% went skating: 0.32 x 225 = 72 (or 225 - 135 - 18)

Clearly state your answer(s) here:

There were 225 who participated. Of these, 135 snowboarded, 18 skiied and 72 skated.

Appendix I

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