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COMMUNICATION AND MATHEMATICS: A CASE FOR ONTARIO SECONDARY TEACHING

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COMMUNICATION AND MATHEMATICS:

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Dedication

I wish to dedicate this work to my family Brett and Sarah without your support and patience I would not have succeeded. Also to my sister Judy for being a supportive critical friend and to my parents Jane and Desmond who stepped in whenever and wherever we needed them.

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List of Abbreviations

EQAO – Education Quality and Accountability Office

TIPS4RM – Targeted Implementation and Planning Strategies for Revised Mathematics

OME – Ontario Ministry of Education

WEFT – a qualitative analysis software application

OSSLT – Ontario Secondary School Literacy Test

ZPD – Zone of Proximinal Development

Chapter 1

Introduction

In 2000 the Government of Ontario introduced a new secondary school curriculum. This secondary curriculum included, for the first time, extensive detail with regards to how teachers should assess student achievement. Under this new assessment system, student work was to be assessed under four categories, one of which was communication. Communication was defined in the 2005 revision of the curriculum to be "The conveying of meaning through various oral, written and visual forms" (*The Ontario Curriculum: Mathematics (Revised)*, p. 25), for the purposes of this research this definition has been adopted. An example of a potential communication question is shown in Figure 1. This question can be considered a communication because it requires students to express their mathematical thinking and in a group discussion situation students can build upon one another's understandings to create a deeper collective understanding of the mathematical ideas of equations and expressions reflected in the question.

Communication was also identified as one of the seven process expectations of the revised curriculum. But what was teachers' understanding of communication in mathematics? How does a classroom teacher know how to teach and assess communication in their classroom? In this introductory chapter I present a brief background about communication as presented in the Ontario Secondary School curriculum and provide context for my own motivations for researching the topic of communication in mathematics.

Small group and whole group discussion: Draw an "X" through the example that does not belong. Justify your answer.

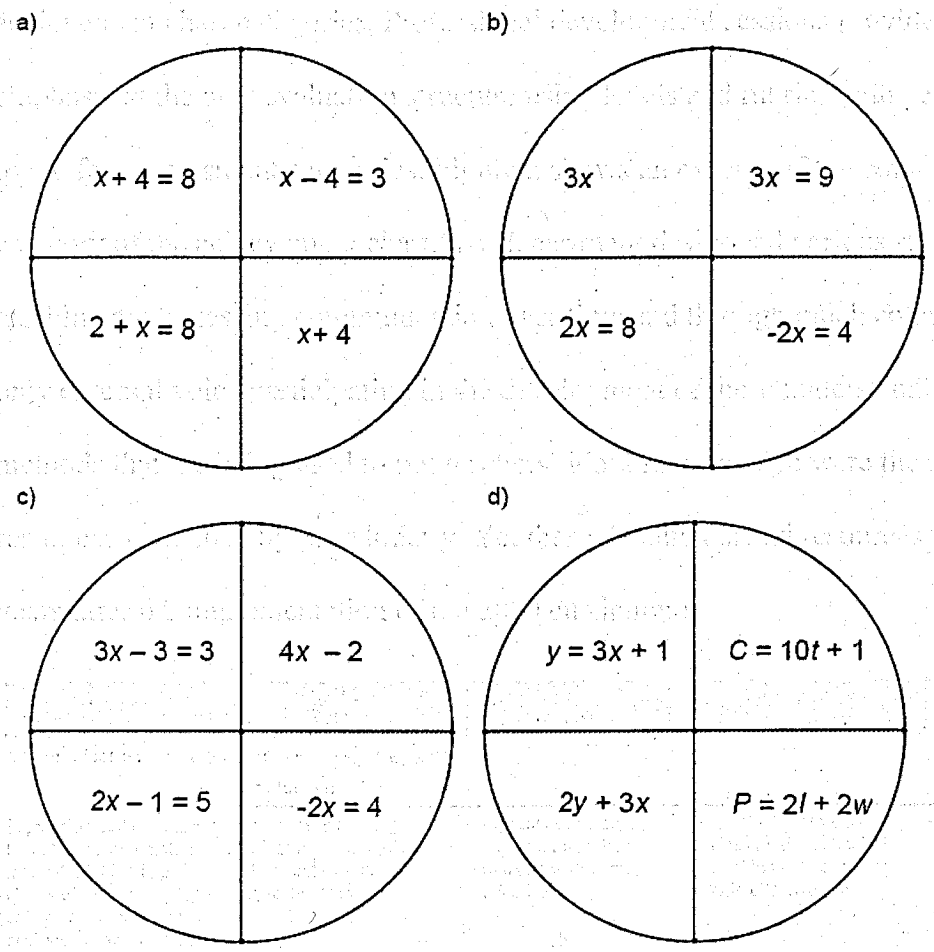

Figure 1. Sample communication question, adapted from TIPS4RM (Targeted Implementation and Planning Strategies for Revised Mathematics) (2005): Grade 9 Applied

Well after the implementation of *The Ontario Curriculum: Mathematics* (2000) and *The Ontario Curriculum: Mathematics (Revised)* (2005), members of the mathematics department at the school where I teach were still having conversations exploring the nature of communication in mathematics (i.e., mathematical form versus explanations written in prose) and the scope of communication (i.e. identifying

communication questions). The uncertainty was caused by the emphasis on communication in this new curriculum, highlighted by its inclusion as one of the achievement chart categories. Professional development sessions provided by our Board emphasized the new evaluation structure using levels and rubrics with general references to the four assessment categories. Figure 2 shows an excerpt of the communication category of the achievement chart. My department developed various strategies for teaching and assessing communication over time and through much collaboration. The only external voice participating in the development of these understandings or the methods that are being used to put teachers' ideas into practice were the documents and resources published by the Ministry. Yet these documents and resources were published years after the implementation of the 2000 curriculum.

Categories	50–59% (Level 1)	60–69% (Level 2)	70–79% (Level 3)	80–100% (Level 4)
Communication *The conveying of meaning through various forms*				
	The student:			
Expression and organization of ideas and mathematical thinking (e.g., clarity of expression, logical organization), using oral, visual, and written forms (e.g., pictorial, graphic, dynamic, numeric, algebraic forms; concrete materials)	– expresses and organizes mathematical thinking with limited effectiveness	– expresses and organizes mathematical thinking with some effectiveness	– expresses and organizes mathematical thinking with considerable effectiveness	– expresses and organizes mathematical thinking with a high degree of effectiveness
Communication for different audiences (e.g., peers, teachers) and purposes (e.g., to present data, justify a solution, express a mathematical argument) in oral, visual, and written forms	– communicates for different audiences and purposes with limited effectiveness	– communicates for different audiences and purposes with some effectiveness	– communicates for different audiences and purposes with considerable effectiveness	– communicates for different audiences and purposes with a high degree of effectiveness
Use of conventions, vocabulary, and terminology of the discipline (e.g., terms, symbols) in oral, visual, and written forms	– uses conventions, vocabulary, and terminology of the discipline with limited effectiveness	– uses conventions, vocabulary, and terminology of the discipline with some effectiveness	– uses conventions, vocabulary, and terminology of the discipline with considerable effectiveness	– uses conventions, vocabulary, and terminology of the discipline with a high degree of effectiveness

Figure 2. The Ontario Ministry of Education (OME) 2005 Achievement Chart for Mathematics – Communication Section

In my roles as teacher, mathematics department head in an Ontario secondary school, and occasional facilitator in-service sessions within my school board. I have experienced both the evolution of the curriculum and teachers' reaction to it. This research investigates the nature of communication in mathematics from the teacher's point of view. The research involved a case study of 12 secondary mathematics teachers working at a board which I refer to as Board P in Ontario. I collected interview data to build a picture of how the teachers in this group view communication in mathematics. My goal was to explore how these teachers understand communication in mathematics and how that understanding has developed.

Even (2003) points out that "mathematics education research is often accused of having only a minor impact on practice" (p. 38). By examining the texts of the teacher interviews in the context of current research on communication in mathematics I wish to determine how theory, research and policy intersects with classroom practice specifically how much theory, research and policy on communication is in evidence in the classrooms within Board P.

1.1 Research Questions

The purpose of this study is to investigate what teachers understand communication in mathematics to be. Specifically I wish to explore: How do secondary mathematics teachers understand communication in mathematics? By this I mean – what do teachers consider to be communication? And how has teachers' understanding of communication developed? To explore these research questions I asked teachers about their practices and opinions surrounding communication in mathematics. I also

investigated the resources available to support teachers in their task of implementing communication in mathematics in their classroom practice.

Chapter 2

Literature Review

Numerous researchers have argued that in order to implement a new direction in curriculum it is essential to bring about change in the beliefs and value systems of the teachers (Powell & Anderson, 2002; Sowell & Zambo, 1997). According to Anderson and Piazza (1996), teachers are the key agents of change in implementing a new curriculum and it is important that individual teachers understand and support the new system sufficiently so they can adapt it for use in their classrooms (Case, 1994). Although teachers can be powerful agents for change, they also can be a source of resistance to change. There could be many reasons for this resistance. Firstly, teachers are being asked to implement a pedagogy different from one they experienced as students (Anderson & Piazza, 1996). Secondly, some teachers hold the opinion that mathematics is rigid and algorithmic – one that conflicts with many recent curriculum reforms (Gregg, 1995). Thirdly, teachers may be concerned that the new curriculum direction will make it harder to cover the curriculum in the time available (Keiser & Lambdin, 1996). Finally, teachers have to recognize a need for change and be willing to let go of old practices (Sowell & Zambo, 1997).

To situate the context of research on curriculum change that centers on communication in mathematics, I have reviewed literature on the role of communication in learning as well as its various modes and benefits. But before the literature review to provide appropriate history on teachers' understandings of mathematics communication and the tools available to them, I examine the curriculum documents that have been used in Ontario secondary schools from 1985 up to the current curriculum introduced in 2000 and its revisions introduced in 2005. Also included is an examination of a variety of

support documents available to classroom teachers. I also examine the research on standardized testing in Ontario as provided by the Education Quality and Accountability Office (EQAO) and the influence such testing has had on teachers' practice.

2.1 Curriculum Documents in Ontario

Key resources teachers use when implementing curriculum are the documents provided by the Ministry of Education. The *Ontario Curriculum: Mathematics (Revised)* (2005) provides a framework for accomplishing the Ministry's goals to "equip them [the teachers] with essential mathematical knowledge and skills; with skills of reasoning, problem solving, and communication; and, most importantly, with the ability and the incentive to continue learning on their own" (p. 3). To situate my research I chose to examine three sets of curriculum documents: *Curriculum Guidelines: Mathematics, Intermediate and Senior Divisions* (1985), *The Ontario Curriculum: Mathematics* (2000) and *The Ontario Curriculum: Mathematics (Revised)* (2005). These documents represent the curriculum that introduced the current emphasis on communication in mathematics, in both its original and revised form as well as the precursor to the 2000 curriculum.

The first complete set of curriculum documents to refer to the idea of communication with respect to mathematics is the *Curriculum Guidelines: Mathematics, Intermediate and Senior Divisions* (1985). In the Process Components section under Language and Mathematics a distinction is made between "Language used to develop an understanding of mathematics"– having students explain their understanding of mathematical concepts – and "Mathematics as a language" – learning the specific verbal and symbolic vocabulary that is unique to mathematics (p. 17). In all courses in the intermediate division, under the aims of each course (7/8, 9/10 basic, general and

advanced), communication is mentioned: "courses should be designed to develop facility in communication skills involving the use of the language and notation of Mathematics" (1985, p. 40). In the senior division, however, communication skills are only an aim in the highest of the three levels of courses: basic, general and advanced.

The Ontario Curriculum: Mathematics (2000) states that "the importance of communication in mathematics is a highlight of the secondary school curriculum" (p. 4). This importance is reaffirmed by the new inclusion of communication as a category in the Achievement Chart to be used to guide assessment practices in Mathematics. Students are expected to "communicate reasoning orally, in writing and graphically" and "using mathematical language, symbols, visuals, and conventions" (p. 77). The documents state that "students will learn to write [using prose] about their use of mathematics, effectively incorporating mathematical forms" (p. 4). The curriculum documents, especially *The Ontario Curriculum: Mathematics* (2000), also attempt to reflect the rich variety embedded in the idea of communication. These documents indicate that communication should take many modes: oral, visual, and written (both using algebraic representation and prose), and that it should include explanations of mathematical reasoning, understanding and purposes. An example of mathematical writing using prose might be when a student explains why he or she knows which mathematical tool to use to solve a problem. An example of writing using algebraic representation might be the algebraic steps to solving the equation $4x^2 + x = 3$.

In the 2005 revision of these documents, *The Ontario Curriculum: Mathematics (Revised)*, the role of communication is expanded even further. Communication is one of the mathematical processes that support effective learning in mathematics. The documents state, "Communication is an essential process in learning mathematics" (p.

16). Students are intended to use communication to "reflect upon and to clarify ideas, relationships, and mathematical arguments" (p. 16). In the Achievement Chart, the role of communication in instruction and assessment is expanded as: "expression and organization of ideas and mathematical thinking, communication for different audiences and purposes and use of conventions, vocabulary and terminology" (p. 21). In all cases communication is expected to be assessed in oral, visual and written modes.

These documents reflect the Ministry of Education's view of communication in mathematics. The review of the curriculum documents, 1985 through 2005, showed clearly that the role of communication in mathematics grew in importance in both instruction and assessment over this time period. What began as a need for students to develop facility with communication has evolved to become an essential process to be assessed and reported on; this new emphasis is shown by inclusion of communication as an explicit assessment category. Although the importance of communication is clear, the underlying benefits are not emphasized – this may make it difficult for teachers to accept this new positioning of, for instance, writing in prose in a subject where writing in prose is very non-traditional.

However clear it may be that the Ministry of Education intends teachers to incorporate communication about mathematics in its many modes into their teaching practices, it appears that an important piece of information is not well articulated: what does communication look like in the classroom? Since the idea of writing about and explaining mathematical concepts using prose rather than just providing a solution using symbolic algebraic representation, for instance, is relatively new, how does a classroom teacher know how to teach and assess communication in their classroom? It is important to remember that most teachers in Ontario were schooled under the 1972 or 1985

curriculum documents where mathematics mostly emphasized computational skills, symbolic procedures, written calculations, and rarely, if ever, were students expected to write about or discuss their ideas in mathematics using prose. To determine how a teacher might then translate the ideas about curriculum reflected in these documents I looked at the tools and resources available to the classroom teacher that are intended to help them practise what is intended in the curriculum.

2.2 Curriculum Resources and Tools

Ross, Hogaboam-Gray and McDougall (2002) consider that the most powerful method for implementing curriculum change is teacher in-service training. Sowell and Zambo (1997) and Powell and Anderson (2002) both assert that new textbooks and guidelines are insufficient to motivate change. Price, Ball and Luks (1995) consider that new curriculum implementation in the absence of sustained in-service has little effect. Potvin and Dionne (2007) support this by stating that in-service training is a basic requirement of successful implementation of curriculum changes.

Potvin and Dionne (2007) state that this in-service should be in the form of a coaching model rather than purely presented in the form of theory. Teachers need time to question and reflect on new practices, become convinced of their merits and incorporate them into the teachers' own belief systems; in-service programs should help teachers clarify the vision of the new curriculum for themselves (Sowell & Zambo, 1997). It is important that teachers have time for both private and public reflections on change (Fullan & Connelly, 1990; Grimmett & Erickson, 1988; Kemissis, 1987) and that in-service and the change itself occur in a supportive school and community environment (Powell & Anderson, 2002). Silver and Smith (1996) maintain that the ability of teachers

to cope with the many challenges of implementing curriculum changes may depend on their ability to form supportive, collaborative communities of practice.

Other forms of support for teachers are in the form of supplementary print documents prepared either by the Ministry of Education or by textbook publishers. Publication of texts corresponding to the revised curriculum lagged the implementation dates by at least one year. As a result the first resource available for teachers would have been the curriculum documents discussed above. Other print resources published by the Ministry of Education include *Ontario Curriculum: Exemplars* (samples of student work with notes explaining assessment), Ministry documents published on the *Leading Math Success* website such as *Mathematical Processes* (2006), *Developing mathematical literacy* (2005), *Continuum and connections package (2006)* and textbooks.

The first resource to be considered is the *Ontario Curriculum: Exemplars* published for grades 9 (2000), 11 (2003) and 12 (2005). The purpose of these documents was in part to "show the connections between what students are expected to learn and how their work can be assessed" (Grade 9, p. 4). The Exemplar documents include samples of student responses, an assessment rubric and teacher notes on assessment. Each contains two assignments for different courses within the grade level and each assignment has its own assessment rubric. In the assessment rubrics contained in the Grade 9 Exemplar document, the criteria for effective communication includes "identifies calculations and presents them in a logical, organized way" and "uses mathematical language, symbols and units accurately" (Grade 9 Academic level, p.14) or "prepares an organized and clear report, using appropriate mathematical forms" and "communicates graphically using proper form" (Grade 9 Applied level, p. 46).

In the assessment rubrics contained in the Grade 11 Exemplar document, communication is considered to be that which "effectively integrates prose and mathematical forms in the financial report" and "effectively communicates information to the client in mathematical language" (Functions, Grade 11, University/College Preparation, p. 10) or "prepares budget summary for the trip that is detailed, complete and accurate" (Mathematics for Everyday Life, Grade 11, Workplace Preparation, p. 69). It is clearly a change from the view of communication presented in Grade 9, which was primarily concerned with notation and language, compared with the view of communication presented in the Functions, Grade 11, University/College Preparation course, which adds integration of prose and mathematical modes of writing.

The Exemplar documents for both grade 9 and 10 levels position justifications, explanations and analyses (algebraic or prose) within the Thinking section of the assessment rubric and not in the communication section. Yet as we saw in the section on curriculum documents in Ontario mathematical justifications and explanations are central parts of mathematical communication as conceived in the literature. By not including these topics in the Communication section of the assessment rubric, the Grade 9 exemplars are reflecting a more limited concept of communication: correct mathematical form and logical presentation of solutions rather than the scope of communication presented in *The Ontario Curriculum: Mathematics* (2000) which includes explanation of mathematical reasoning and understandings.

In the Grade 12 Exemplar documents, communication is considered to be that which "clearly communicates information in diagrams or graphs" and "effectively integrates text and mathematical forms" (Advanced Functions and Introductory Calculus, Grade 12, University Preparation, p. 10) or "clearly communicates information in tables

or graphs" (Mathematics for College Technology, Grade 12, College Preparation, p. 85). Also, in the teacher notes it is clear that mathematical form, integration of different mathematical modes, justifications, explanations and analysis are considered when assessing communication. These Grade 12 Exemplar documents were written five years after the implementation of the 2000 curriculum documents and the scope of communication as reflected in the assessment rubrics has expanded to more accurately reflect the combination of written, graphical and other visual modes indicated in the curriculum documents. Since the Exemplar documents only use written assignments, it is difficult for these documents to reflect any expectations of effective oral communication in mathematics.

However, for a teacher looking for guidance at the beginning of the curriculum reforms in 2000, the only Exemplar documents available were the ones for Grade 9, which, as seen above, reflect a limited concept of communication with its emphasis on correct mathematical form and logical presentation of solutions. Exemplar documents for Grade 11 and Grade 12 were published three or five years later respectively. It is unlikely that many teachers are still actively looking for guidance from the Ministry of Education to understand the Achievement Chart (*The Ontario Curriculum: Mathematics (Revised),* 2005, pp. 20-21) categories, especially in communication, five years later but this viewpoint is subject to empirical verification.

The Ministry of Education also publishes other documents on their *Leading Math Success* website, which is a website that provides teaching resources for Grade 7 to 12 mathematics classes. I have closely examined three documents published under the *Targeted Implementation and Planning Strategies for Revised Mathematics (TIPS4RM)* banner that expand on the Ministry's vision of communication in mathematics. While

there are many and varied supports including videos, lesson outlines, instructional supports, and planning guides listed on the website, the documents I chose to examine are those that specifically address communication in mathematics directly.

The first document, *Mathematical Processes* (2006), expands on the seven mathematical process expectations that are included in every course in the 2005 revised mathematics curriculum documents; communication is one of these seven processes. This document summarizes the Ministry of Education's concept of communication expecting that "students will communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions" and "through communication students are able to reflect upon and to clarify ideas, relationships and mathematical arguments" (p. 10).

The second document, *Developing mathematical literacy* (2005), also contains a section which expands on the Ministry's vision "Communication is the process of expressing ideas and mathematical understanding using numbers, pictures, and words, within a variety of audiences including the teacher, a peer, a group or the class" (p. 21). This document underlines how the Ministry's ideas surrounding communication and mathematics are supported by current research in mathematics education.

The third document, *Continuum and connections package (2006),* contains a number of sample questions intended to show connections in the intermediate division on the different strands in the curriculum. The questions in the *Is it always true?* sections are excellent examples of questions a teacher could use to provoke discussion or have students answer using writing in prose to explain their understanding of mathematical concepts. Whereas these documents clarify, expand upon and give concrete examples of the Ministry's expectations surrounding communication in mathematics, my

understanding is that teachers may not know these documents exist or may not have the time to search them out.

The final tool that I consider is the textbooks used in the classroom. One series of texts, Addison-Wesley and Nelson, prepared for the 2000 curriculum, label questions in the exercises and in chapter reviews that are intended to be communication questions. Newer texts, intended for use with the 2005 revised curriculum, include "In your own words" and "Discuss the concepts" questions as part of the lesson structure but have reduced the labelling of communication questions within the student exercises. Whereas the texts give ample examples of questions teachers can use, the understanding of communication reflected is unfortunately narrow – communication is only reflected as writing in prose and is restricted to brief explanations of mathematical ideas. This is in contrast to the research literature which portrays mathematics in communication as a rich blend of oral and written communication including combinations of graphical, written and algebraic representations.

Shortly after the new mathematics curriculum was implemented in 2000, the EQAO standardized test for Grade 9 mathematics was introduced. Several researchers have written about the impact of the standardized EQAO test on Ontario's education system and on teacher practices and teacher attitudes. Nezavdal (2003) states that standardized tests created pressure on schools and teachers to ensure that students perform well on a single test. Shaker (2004) echoes this by asserting that standardized tests are seen as proof of the effectiveness of educational policies and as such standardized tests are assessing school systems, schools and teachers.

Earl (1999) cites evidence that this pressure to perform well on standardized tests can lead to changes in pedagogy, assessment and curricular focus but although these

changes may lead to increased test scores, they do not lead to increased learning. Volante (2004, 2007) agrees with this finding and adds that this pressure has a detrimental effect on the teaching profession and that recent assessment reforms (including standardized testing) have not been embraced by teachers.

Not all of the effects of standardized testing mentioned in the literature are negative. Teachers use data as a basis for action research to improve student learning (Volante, 2007). Pressure from the Ministry of Education has led to increased professional development focus and participation on topics surrounding numeracy and literacy (Hardy, 2009; Giles and Hargreaves, 2006; Volante, 2007). One venue of professional development cited by teachers as positive was marking EQAO tests, and there is evidence of changes to programs and instructional techniques as a result of these marking experiences (Volante, 2007).

In order for tests to be used for improvement they must be viewed as important by teachers and school boards (Earl, 1999). Teachers and other stakeholders are becoming more preoccupied with increasing test scores on standardized tests and moving the educational focus of teachers and boards away from what they want children to learn and instead emphasizing what educators can measure (Volante, 2004, 2007).

This change in focus is reflected by teachers directing their energies toward activities that will lead to an increase in test scores or "teaching to the test". Some ways educators do this is by emphasizing drill activities at the expense of time spent on higher thinking skills, de-emphasizing content that is not assessed and teaching test-taking skills by spending time on practice sessions with cloned test items (Volante, 2007).

2. 3 A Scholarly Review of Communication in Mathematics

Two themes emerged from a review of some of the literature available on mathematics education research and communication: the importance of the role of communication in learning and the different modes of communication considered by researchers. In this section I discuss the role of communication in learning mathematics as a tool to consolidate thinking and to make thinking available to others. I also examine the benefits of having students communicate about mathematics as well as literature on various modes of mathematics including writing about mathematical ideas using prose, writing in the form of mathematical conventions and oral communication about mathematics.

Some writers, Lim and Pugalee (2004) and Whitin and Whitin (2002), take a constructivist stance, maintaining that students use communication to actively construct their knowledge of mathematics. Others such as Sfard (2000a, 2007) subscribe to a socio-cultural view of learning, one that emphasizes the social and cultural aspects of learning to support communication in mathematics (Cobb, 1994). Several authors maintain the connection between communication and learning mathematics: Cai et al. (2005) state that "communication is essential to learning" (p. 238) and "communication is a component that is essential to and necessary in learning, doing and understanding mathematics" (p.245).

Lim and Pugalee (2004), in their research on improving students' journal writing about mathematical ideas, maintain that writing in prose helps students focus and extend their thinking by building mathematical understanding. Whitin and Whitin (2002) studied a Grade 4 class and concluded that communication gives students opportunities to justify thinking, formulate questions and summarize important insights, and furthermore that

writing in prose about mathematical ideas allows students to return to ideas, reflect upon them and refine them.

Another key role of communication in learning is to provide access to student understandings. Kotsopoulos (2007a) states that communication "can be seen as a manifestation of cognition" (p. 2). Sfard (2000a) considers that thinking is just the internal form of communication and that by investigating human communication with others, we can better understand human thinking. Kotsopoulos (2007a) echoes this by asserting that communication is the vehicle whereby internal thought processes become external ones and thus known to others. Sfard extends this internal/external relationship to mathematics by studying how learners solve mathematical problems in groups in order to gain insight into individual learning and problem solving.

Other researchers found a variety of benefits to having students communicate about mathematics. Williams (2003) performed a study on two groups of students, one group who wrote about the process of problem solving and one who did not. The group who wrote about the process of problem solving had improved problem solving performance.

Fernsten (2007) found that having pre-service teachers who communicate through writing in prose about mathematical ideas was useful for dispelling fears about writing. Schoen, Bean and Ziebarth (1996) suggest that having students write in prose about mathematics breaks down the student perception that mathematics is merely symbolic manipulation. Porter and Masingila (2000) echo this by noting that the process of labouring to communicate about mathematics will also show students that there is meaning in the mathematics that they are writing in prose about and that mathematics is not just a series of steps to be memorized. Peressini and Bassett (1996) recommend

having students write in prose about their thinking as a way for teachers to identify mathematical misconceptions, and Goldsby and Cozza (2002) take this idea further by suggesting that teachers use this identification to adjust instruction to improve student learning.

Cai et al. (2005) consider communication in mathematics to encompass "speaking, writing, and depicting visual forms" as well as "reading and interpreting" information presented by others (p. 238). Whitin and Whitin (2002), in their discussion of a Fourth Grade mathematics class, describe communication as talking, drawing and writing, effectively covering the entire range of possible modes of communication. Different modes of communication are also described in the research literature. Sfard (2007) describes graphs, formulas, drawings and diagrams as visual mediators which are objects of one's mathematical discourse, which she considers crucial to communication.

Stonewater (2002) studied a set of calculus essays to determine the criteria for assessing writing by reviewing examples from successful students. He found that successful students were more likely to use sharp, clear, specific mathematical descriptions, use examples to illustrate the points they were trying to make and use algebraic, numeric and graphical representations when writing about mathematics. Lee (2008) also prefers the idea of combining written (using prose) and written (using algebraic modes). He maintains that a page of computations contains no mathematics because it contains no ideas. Lee considers that the mathematical ideas are contained in the written explanation. Masingila and Prus-Wisniowska (1996) also refer to the style of writing that includes a combination of graphical, written in prose, algebraic and diagrammatic representations of mathematics as writing in assessment tasks. They also discuss writing from a prompt, a style which usually requires a shorter explanatory

response to communicate a student's understanding of a specific concept rather than an involved discussion of a series of mathematical processes used to solve an extended problem.

Emig (1977), an early proponent of the writing-to-learn movement, considers that writing is a unique form of learning because it requires students to make connections and establish relationships between concepts as they compose texts. Writing also requires the writer to both analyze and synthesize ideas during composition. She asserts that these higher level cognitive processes can only develop fully when supported by language. Writing allows students to construct their own conceptual understanding of the subject matter under study while learning to communicate in the language of a given discipline (Johanning, 2000). Writing also allows students to reflect upon their learning as they compose and in doing so learn about their own understandings (McIntosh & Draper, 2001). Porter and Masingila (2000), in their study of first year calculus students, concluded that organizing and articulating ideas improved student performance as much as writing about them did.

Esty and Teppo (1996) argue that algebraic representation is just as an important a facet of mathematics communication as language-based modes of communication are. They maintain that algebraic representation is a concise way to express algebraic thought using algebraic notation to conceptualize. Esty and Teppo state that algebraic representations represent communication about some mathematical object or idea and that when algebraic representations of mathematics take on meaning for students it extends their ability to think about mathematical concepts.

Researchers consider that oral language is an integral part of communicating about mathematics. Pimm (1987) considers that in order for students to communicate

effectively about mathematics they must learn what he refers to as the "mathematics register" (p. 109) or how to use words and expressions correctly in the context of mathematics communication. Abele (1998) and Thompson and Rubenstein (2000) remind us that language is how we communicate and thus how we communicate about mathematics. Thus students build understanding as they process ideas through language. Hiunker and Laughlin (1996) allude to oral communication about mathematics when they say that talking is a natural way for people to communicate. Pimm (1996) goes farther when he says that "mathematics is brought into being through conversation" (p. 12). O'Connell et al. (2005) found that having students communicate orally about mathematics allowed them to catch their own mistakes as they verbalized their ideas, facilitated learning from their peers and allowed students to refine their thoughts and collaboratively build mathematical understandings.

Pimm (1987) states that oral communication externalizes student thinking which allows teachers to evaluate student understanding. Thompson and Rubenstein (2000) suggest that teachers can also use the opportunity to reinforce correct usage of mathematical words and to help students rephrase their mathematical ideas when necessary. Stein (2007) considers that giving students the tools to communicate orally stops those who disagree from remaining silent by giving them the words to express their ideas. Choppin (2007) maintains that oral communication allows students to receive immediate feedback on their way of thinking about a problem and it allows students to compare ideas to those of their peers as well as to conventional mathematical thinking.

However, talking about mathematics can run counter to many peoples' (both students and teachers) perceptions of what mathematics is (Pimm, 1987). Stein (2007) states that communicating in a mathematics community such as a classroom through

mathematical discourse is as much a part of learning mathematics as understanding concepts. Sfard (2007) expands on this idea by maintaining that learning mathematics is equivalent to changing one's discourse. Pimm supports this by saying that it is important that student talk is seen as a key element of student learning.

Several researchers agree on the importance of discussion in the mathematics classroom. Huinker and Laughlin (1996) maintain that exposure to other students' ideas allows students to clarify and modify existing ideas while creating new understandings. Fonzi and Smith (1998) and Choppin (2007) extend these ideas by stating that classroom discussions construct meaning for all participants in the classroom and that the understandings that result from classroom discussion will be better than those constructed by individuals. Kotsopoulos (2007b) maintains that in order for students to become proficient in mathematics they must participate in classroom discussions. Abele (1998) states that independent discussions by students, in contrast to teacher-led classroom discussions, are more likely to lead to deeper conceptual connections.

Classroom discussion is also important for creating a mathematical community within the classroom. Huinker and Lauglin (1996) state that discussion fosters collaboration and builds a mathematical community that values thinking. Choppin (2007) and Duff (2002) consider that if the teacher uses student responses as a basis for a discussion, the teacher is acknowledging students as mathematical thinkers and establishes the classroom as a mathematical community.

It is clear that mathematics education researchers consider communication to be an important facet of learning about mathematics with implications for improved student results and improved instruction. Researchers also consider that communication in

mathematics takes on many modes – oral, diagrams, graphs, short explanatory text and complex interweaving of algebraic text and explanatory writing in prose to name a few.

Conclusions

The curriculum documents do reflect the importance and breadth of communication that can be found in the research literature reviewed, but the supports for the classroom teacher appear to be narrow in scope. This research sets out to examine which of the ideas present in the research literature and in the curriculum policies on communication and mathematics are reflected in current classroom practice, what teachers consider communication in mathematics to be and how teachers have developed their understanding of communication and mathematics. This study will use case-study methodology involving a series of face-to-face interviews. In the next chapter I describe the theoretical framework, sociocultural perspective, for the research, its basic tenets and connections to communication, teaching and mathematics.

Chapter 3

Theoretical Framework

This research fits within a sociocultural research framework. Lerman (1996) and Glassman (2001) both reference Vygotsky's idea that knowledge is created through social interaction. Knowledge involves using culturally developed tools in an effective manner in order to interact with other members of the social community. In this chapter I describe the central ideas of the sociocultural perspective and then relate them to the topics of teaching, communication and mathematics.

3.1 Sociocultural Perspective

The sociocultural perspective in research is based on the theories of Vygotsky (1978, 1986), which postulate that cognitive development takes place through the use of language and social interaction. Human learning takes place in a social context where children grow into the intellectual culture around them. People are social beings who develop as individuals through interactions with others (Glassman, 2001; Vygotsky, 1978). Since learning takes place through social interaction, this interaction can be seen as leading to the development of an individual's knowledge (Lerman, 1996; Vygotsky, 1978).

Vygotsky (1978) made the connection between language and cognitive development. He considered that in general symbolic activity such as speech has a specific organizing function which gives rise to changes in behaviour. Specifically, when speech and practical activity combine, they give birth to human forms of practical and abstract intelligence. Thus language, as a form of symbolic speech, plays a key role in learning. Lerman (1996) advances these ideas and asserts that consciousness is

constructed through discursive practice and thus language plays a central role in the development of consciousness.

Researchers have connected these twin pillars of language and social context by observing that language itself is learned through social interaction: a child uses signs such as words in a social context before they fully understand their meaning. Children then learn the meanings of these signs in a way that is compatible with that of the community around them and as a result are able to think in terms of language. Children cannot learn the meaning of signs independent of the community (Berger, 2004; Glassman, 2001). It is through interaction with more experienced members of society that children learn the culturally appropriate use of signs. Typically it is an adult who mentors a child in activities that bring language and activity together (Glassman, 2001). This guidance from more experienced social partners plays a critical role in refining children's use of signs (Gauvain, 2001; Glassman, 2001).

A key idea to come out of Vygotsky's writings was his idea of a Zone of Proximinal Development (ZPD). This ZPD is described as the distance between problems a child can solve on their own and a problem they can solve with assistance from others. Because of this emphasis on problem solving through social interaction, the concept of a ZPD reflects both of Vygotsky's key elements of cognitive development: learning through interactions with others and using signs to communicate ideas (Glassman, 2001; Marrone, Harkness, D'Ambrosio & Caulfield, 2004; Vygotsky, 1978).

In summary, Vygotsky's (1978) theories, which became the underpinnings of the sociocultural perspective, consider that language is a key element in learning and that learning leads to cognitive processes that are developed through interaction with others.

As a result of this interaction with others, understanding is defined as the child coming to share in the group meaning through discussion (Lerman, 1996).

3.2 Sociocultural Perspective and Communication

Communication is social interaction mediated by language, so communication can be seen as a key element in a sociocultural perspective. Several researchers have made this link between communication and the sociocultural perspective explicit. People receive all knowledge of the world through language and other modes of communication (Lerman, 2001); as a result, language plays an essential role in socially shaping the minds of students (Crawford, 1996). Communication is the principal force behind learning and knowing (Kotsopoulos, 2007a; Sfard, 2000b).

Vygotsky (1986) considers that inner speech or thought is the internalization of the type of speech that a child uses to process ideas and solve problems. Thus thinking can be seen as a mode of communication - communication with the self. We can consider that thinking which takes place through communication with others can be seen as communication embedded within communication (Sfard and Kieran, 2001). This link between external communication and internal thinking is what makes communication in the mathematics classroom vital.

3.3 Sociocultural Perspective and Teaching

Teaching in a school setting can be viewed through a sociocultural lens. The goal of teaching is student learning. Vygotsky (1986) claims that instruction precedes development, and development of higher mental functions is a product of cooperation between the student and the teacher. Teaching is transmitted through social interaction,

either between the student and the teacher or between students. In essence, education can be seen as the process of mentoring children in socially acceptable activities and cultural tools (Glassman, 2001).

The sociocultural concept of the ZPD can also be applied directly in teaching. Effective teaching requires that teachers recognize students' ZPDs by engaging students in work beyond the level at which they can work on their own. The role of the classroom teacher is to provide supports in the form of instructional activities and opportunities for discourse that engage students to move beyond these individual capabilities. The teacher should use knowledge about students' ZPDs to direct students' future learning (Carter, 2005; Crawford, 1996; Nystrand, 2006).

Another central pillar in the sociocultural perspective is language, thus in a sociocultural classroom communication is seen as being central to learning (Steele, 2001). Students are encouraged to work and talk together as a learning community and the emphasis is on collectively creating meaning though discourse (Bruce, 2007; Carter, 2005; Steele, 2001). The teacher's aim is to structure classroom activities and discourse in such a way as to facilitate participation in the learning community (Lerman, 1996). The teacher can assist the student in furthering learning by furthering the student's knowledge base. This can be done by carefully planning activities that take the student from the familiar to the unfamiliar or in other words creating a ZPD so that the student can develop understanding of culturally established concepts (Steele, 2001).

Under a sociocultural model a teacher considers that communication is central to learning. This teacher encourages her students to collectively create meaning through discussion and tries to move children beyond their individual capabilities. In addition, if

this were a mathematics classroom, the teacher would assess students' communication about mathematics as evidence of their ability to think mathematically.

3.4 Sociocultural Perspective and Mathematics

The area of mathematics fits especially well within a sociocultural perspective. Learning about mathematics also involves acquiring the language and concepts of the wider community of mathematicians. Thus concepts learned in a mathematics classroom are socially and culturally determined. Mathematics learning is also socially acquired. Students are not expected to learn about mathematics on their own (Lerman, 1996, 2001; Steele 2001).

Many researchers have made the connection between a sociocultural research perspective and mathematics. Berger (2004) likens a student's learning of the algebraic representation of mathematics to Vygotsky's consideration of how a child learns language. Mathematical signs are both used as objects with which to communicate, in the form of algebraic representation, and as objects on which to focus and organize ideas, and in fact this use of mathematical symbols is a necessary aspect of mathematical meaning making. Sfard and Kieran (2001) add to this concept when they note that mathematical discourse is made special by its exceptional reliance on symbolic artefacts as communication and mediation tools.

The usage of such symbols is socially regulated by the mathematical community and learners' usage evolves to eventually match the socially accepted meaning. Students often use mathematical symbols before they fully understand them, and by using these symbols in a social context such as a mathematics classroom, students come to use

mathematical symbols in a way that is compatible with the greater community of mathematicians (Berger, 2004).

Within a sociocultural perspective, individual learning is influenced by participation in cultural practices (Steele, 2001). Learning mathematics is very much a social endeavour; the mathematics classroom functions as a community where thinking and discussion are encouraged (Bruce, 2007). As students learn to explain and justify their thinking to others, they construct knowledge and develop mathematical understandings and by doing so they create an understanding of culturally established mathematical practices (Steele, 2001).

If learning can be seen as a change in the way a person communicates then learning mathematics is evidenced by a student becoming more proficient in mathematical discourse. Growth of mathematical understanding occurs through the process of connecting ordinary language to mathematical language. Learning to communicate about mathematics is seen as evidence of learning to think mathematically (Lerman 2001; Marrone, Harkness, D'Ambrosio & Caulfield, 2004; Sfard 2000b; Steele, 2001).

Conclusion

A sociocultural perspective is a good fit for this research that involves communication and mathematics. The pairing of language and interaction with others is key to Vygotsky's writings. Education can be seen as moving students through their ZPD through interactions with peers or a teacher as members of a classroom community (Carter, 2005; Crawford, 1996; Lerman, 1996; Nystrand, 2006). Mathematics education is suited to a sociocultural perspective because mathematics is a discipline that is heavily

dependent on formal language including algebraic representation which students are expected to learn by relying on interaction with other members of the mathematics community (Berger, 2004; Lerman 1996, 2001; Steele, 2001). Several other researchers in the field of mathematics education such as Kotsopoulos (2007a), Steele (2001) and Carter (2005) have used a sociocultural perspective to frame their research.

This research is a series of semi-structured interviews where 12 teachers and I discuss their understandings and practices surrounding communication and mathematics. In a sociocultural context understanding can be seen as coming to share in the group meaning. By analyzing these discussions with the teachers in Board P, I attempt to tap into any shared meanings that they have and thus be better able to ascertain how exactly these teachers understand communication.

In this chapter I have detailed the basic tenets of a sociocultural perspective and I have connected them to communication, teaching and mathematics. I have also connected two important concepts, understanding and teacher practice to the sociocultural context. In the next chapter I detail the methodology that was used in this research.

Chapter 4

Methodology

This research is a case study. The study was intended to capture a rich and detailed description of teacher experiences with respect to communication and mathematics. The research focused on a specific group of participants in a specific school board, in a bounded and integrated system, and sought to investigate the participants' perceptions and understandings. The research could be described as an integrated study because I was only interested in learning about the particular cases of teachers who teach mathematics at a school board that I refer to as School Board P and I was not attempting to generalize to all mathematics teachers (Stake, 1995). As a result of my multiple roles as researcher, mathematics teacher, and facilitator of professional development sessions at Board P, I was intertwined with the research: this involvement made me part of the community from which I drew my case-study participants (Cohen, Manion, & Morrison, 2007).

Cresswell (2007) suggests that case studies are descriptive and focus upon context and setting for a specific situation. A case study tries not to disturb the ordinary course of events but rather sets out to observe, reflect, and interpret while trying to understand how the individuals involved in the case see the situation and represent their different viewpoints (Stake, 1995).

In this case study I investigate how teachers in Board P understand communication and how their understanding of communication has developed by asking them how they have incorporated communication and mathematics into their teaching and assessing practice. After asking about their individual practices, I merged their stories and experiences to describe typical viewpoints or behaviours of mathematics teachers in

Board P (Morse, 1994). I was not interested in changing or interfering with their practice; rather I was interested in talking to teachers and examining the existing different experiences of the different teachers.

To review, the questions that formed the basis for this research are: How do secondary mathematics teachers understand communication in mathematics? By this I mean – what do teachers consider to be communication? And how, has teachers' understanding of communication developed?

4.1 Participants

I chose to do research within my board to take advantage of the convenient access to participants and because learning about the understandings of other teachers in my board with respect to communication and mathematics would be useful to me in my professional capacities as both a classroom teacher and a department head. To select participants I requested a list of all teachers who teach mathematics at secondary schools within Board P from the board's Assessment and Evaluation Coordinator. I received a list of 80 teachers spread over 9 secondary schools. I then sent an e-mail, using the board's internal e-mail system, to 69 of the teachers. In order to make sure that there was no possibility of hidden power issues or concerns over coercion, as suggested by Glesne (1991) I did not solicit participants from within my own department, keeping separate my multiples roles as researcher and department head.

The first request netted six teachers. I sent a second e-mail solicitation which netted me a further six teachers for a total of 12. All teachers who responded were included in the research. As participants responded I assigned them pseudonyms A, B, C, and so on. After this point these were the only names used in my data to refer to them.

The participants were not made aware of the pseudonyms to preserve confidentiality. There was only one file in my data which connected the names of the participants with the labels assigned to them. This file was kept separately and only referred to when confirming demographic information.

The 12 participants, 5 male and 7 female, ranged in experience from 2.5 years to 23 years, (mean 9.0 years, standard deviation 5.6 years). These 12 teachers represented 17.4% of the eligible teachers in the board, covering seven out of eight of the available schools (87.5%). Of the 12 participants, five hold mathematics degrees and several of the 12 teach mathematics exclusively. Only 7 of the participants had taken advanced qualification courses, 10 had taken an in-service course or workshop in mathematics teaching and only 3 had in-service training in mathematics communication. The complete set of demographic data is included in Table 1 on the next page.

Table 1

Participant Demographic Information

Question	Participant Response											
	A	B	C	D	E	F	G	H	I	J	K	L
Years of Experience	10	2.5	14	5	12	5	14	5	5	6	23	6
Sex	F	M	F	F	M	F	M	M	F	F	F	M
Mathematics Degree	Y	N	Y	N	Y	N	Y	Y	N	N	N	N
Teaches Mathematics Only	Y	N	N	N	Y	Y	Y	Y	N	N	Y	Y
Additional Qualifications, AQ	Y	N	Y	Y	N	Y	N	N	Y	Y	Y	N
In-service - Math	Y	Y	N	N	Y	Y	Y	Y	Y	Y	Y	Y
In-service - Communication	N	N	N	N	Y	Y	N	Y	N	N	N	N

Stake (1995) suggests that researchers do not study individual cases in order to understand other cases but rather the primary goal when selecting samples is to maximize what they can learn as researchers. Creswell (2007) prefers to select unusual or diverse cases when he samples for case studies in order to maximize diversity. As a result of the fairly high level of representation of the board in the pool of participants, the result of this research, would likely offer a great deal of insight about the understandings surrounding communication and mathematics of the mathematics teachers of Board P. Also, the diverse population (years of experience, mathematical background) added interesting and useful content to this case study.

4.2 Data Collection and Analysis

As the confirmation e-mails arrived I contacted each volunteer teacher, sought their consent to participate in the research, and set up a face-to-face interview. The interviews varied in length from 20 to 60 minutes. The interviews were semi-structured, which allowed the participants to influence the interviews and add their perspectives (Hutchison & Wilson, 1994) on communication to the discussion.

In order to create a place where the participant felt comfortable sharing their private thoughts (Kvale & Brinkman, 1996), I interviewed most of the participants at their own schools, either in classrooms or department offices. Most of the participants were able to accommodate my request by scheduling the interviews into their preparation periods or lunch hours. Only one teacher requested an interview in an alternate location (a coffee shop nearer to her home in the evening).

Participants were asked a variety of types of questions including demographic questions, factual questions, descriptive questions, and reflective questions (Cohen et al.,

2007). Appendix A includes the interview protocol. Demographic questions related to the number of years of experience, subjects taught, in-service, and details of any additional education or training. Factual questions related to modes of communication that the participants assessed and taught. Descriptive questions covered methods used to teach communication, example questions and sources of communication questions. The purpose of these questions was to determine how teachers defined communication in terms of their practice and to determine which modes were most important to their understanding of communication. Reflective questions asked the participants how their understanding of communication developed, how important they considered communication to be and how they considered communication in mathematics compared to communication in other disciplines. The purpose of these questions was to elicit information on the development of participants' ideas on communication and their opinions on the value of the increased emphasis on communication. Each interview was audio recorded and the recordings were transcribed as text files with the text organized by question response.

Modelling the data findings and analysis on qualitative data analysis as described by Cresswell (2007), the data was coded by themes for analysis. The first set of themes, Modes, Benefits, and Tools, were suggested by the initial literature review. These themes in turn became the basis for the initial analysis. As I reread the data to verify occurrences of the original themes, other emergent themes (Cresswell, 2007), such as Tensions, In-service, EQAO and Opinions, developed from the transcripts. As the analysis progressed, subthemes were needed to further code the themes. Therefore passages were coded for different themes leading to double coding of some data. The need for subthemes arose as a way to organize the data for some of the larger themes, such as Modes, Benefits and

Tools, and a way to represent diversity in others such as Tensions and Opinions. Figure 3 illustrates the structure of the themes and their related subthemes used in analyzing the data.

- Modes of communication
  - Conventions
  - Writing (ideas)
  - Terminology
  - Oral (student)
  - Writing (algebraic)
  - Oral (teacher)
  - Diagrams
- Benefits
  - Understanding
  - Communication with others
  - Solidify/Clarify
  - Non-algorithmic
- Tensions
  - New Perspective
  - Communicating with words
- Tools
  - Colleagues
  - Textbooks
  - Curriculum documents
  - Other
- In-service
- EQAO
- Opinions
  - Value
  - Evolution
  - Pervasiveness

Figure 3. Themes used in analyzing data from interview transcripts.[1]

The data were analyzed using the data analysis program WEFT, a Computer Assisted Qualitative Data Analysis Software program. WEFT uses text-level analysis to assist in locating and collecting excerpts associated with the themes listed above and organizes the themes and subthemes. WEFT also allowed searches for keywords

[1] The figure's tree structure has been designed to preserve the original organization of categories in WEFT

associated with different themes such as "literacy", "non-fiction writing" and "writing across the curriculum" as indicators of the theme EQAO. The program then created files of text for each theme, allowing me to read all of the text for any given theme together. Within the files for each theme the excerpts were organized by participant so information about participants could still be linked to the themes.

The files created by WEFT were then read closely and quotes were selected in order to reflect the voice of the participants (Stake 1995). Quotes were selected on the basis of being characteristic of responses to a given theme, vividness, and clarity. These quotes formed the foundation of the findings section, which is organized by themes and subthemes. Also included in the findings section are some descriptive statistics which aim to describe the data which has been found in a quantitative fashion (Cohen et al., 2007). Table 2 details the frequencies of occurrences of the themes which formed the basis of the data analysis. Table 3 details the responses to the Likert scale questions asked at the end of the Interview protocol (Appendix C) and includes the mean response and the standard deviation for each question. The connections between the teacher's voices and the research literature, along with a detailed description of the case and its setting (Creswell, 2007), form the discussion section.

4.3 Ethics

In this research, I interviewed adults in their professional capacity about their professional practice, which is already open to a certain amount of scrutiny. As a result, there are only a few issues surrounding ethics including informed consent, voluntary participation, confidentiality, and permission from the school board.

Ethics approval for research involving the use of human subjects was granted by the ethics board at the Faculty of Education at the University of Western Ontario. After approval was granted by the Faculty, ethics approval was then sought from the department of Human Resources at Board P. Only after both of these approvals were granted were potential participants contacted.

Participants were made aware of the intent and scope of the research via a letter of information (Appendix B) which was distributed at the initial invitation to participate. Informed consent was further addressed at the beginning of each of the interviews. I expressed both in writing and verbally that participation was voluntary, that teachers were able to drop out of the study at any point, and that I would respect their wishes with respect to inclusion of their responses in the research findings (Cohen et al., 2007). Participants signed a consent form (Appendix C) before any data was collected. As mentioned already, in order to make sure that there was no possibility of hidden power issues or concerns over coercion I avoided soliciting participants from within my own school and department.

Since it is impossible to ensure anonymity in a face-to-face interview situation, I was only able to guarantee the confidentiality of participant information in the preparation of the research reports at the conclusion of the research. I ensured confidentiality by referring to participants by letters, and keeping the list of participants' names in a secure file, separate from my data.

After I collected the data in the form of audio recordings, transcripts and e-mails, I stored them on a password-protected computer. At the completion of the research I will remove all files from my computer and store them in a secure filing cabinet in my home for five years after the research, at which point they will be destroyed.

Since discussing specific samples of student work was not covered in my ethics approval, even though it is becoming an encouraged professional practice at our board I made a point of only asking teachers about representative questions. We did talk generally about how students respond to specific communication questions and issues and participants showed me sample questions from assessment exercises. It was made clear to the respondents that I did not wish to see student work as it would have become part of my research data and I would have required explicit permissions and releases from the school board, students, and parents to incorporate it into my research.

Because the respondents and I are members of the same teachers' union and because the research was a descriptive case study, it was important to clarify that my research in my back-yard, so to speak, was non-evaluative, especially since I was asking them to be open about how they assess and evaluate student work and I was not in a position to judge the rightness or wrongness of their approach and was merely seeking to explore any approaches they may be using.

4.4 Credibility of Results

When judging the credibility of the results of this research there are several issues to consider. Researcher bias as a result of my prior experience surrounding mathematics teaching and communication in mathematics was a factor when analyzing participant responses. Another form of researcher bias that had to be reflected upon relates to selective reporting when analyzing data. A third issue of the differences between teachers' actual practices and their reported practice in the interviews was also considered.

Bias affecting the credibility of conclusions comes into play in the form of researcher bias. All responses and interpretations were filtered through the lens of the researcher's point of view and any preconceived notions about communication or about teaching and assessing communication that the researcher brings to this research. As a teacher in Board P I have experienced many of the same in-service opportunities as the teachers participating in this research. Additionally, I have considerably more teaching experience with the board (20 years) than most of the participants and I have a teacher's understanding of the history of the implementation of the new curriculum that teachers with fewer years of experience are unlikely to have. Also, as a department head, and as someone who has given workshops on topics such as differentiated instruction, TIPS documents, manipulatives in the Grade 9 classroom and parallel and open questioning, I have a professional development facilitator's view of board initiatives and of some of the motivations behind them. I have also, over the course of this Master of Education degree, reviewed research on communication and mathematics and have broadened my own understandings in a way not available to most of the teachers that I interviewed.

In order to counteract this bias it was important to be aware of and record the cultural models (Gee, 1999) that connected me to the participants in our common role of secondary mathematics teachers as well as those cultural models we do not share because of differences in age, experience, or backgrounds. I also had to look for episodes where the participants and I negotiated situated meanings (Gee, 1999) during the interviews. Some such negotiated meanings revolved around terminology used in the interview questions such as the distinction between mathematical form and modes of communication and the use of the term "communication question" to describe questions that elicit communication.

Less easily checked was researcher bias introduced by selective reporting of data (Cohen et al., 2007). It was necessary to ensure that my data analysis was systemic and included both examples and counterexamples to support any conclusions and that any omission of data was transparent and thoroughly justified in order to support any claims I made of credibility of conclusions. Bias introduced by a lack of standardization of interview questions and thus response data was to a larger extent mitigated by using a semi-structured interview format (Cohen et al., 2007).

In order to avoid misunderstandings when discussing practices surrounding communication in mathematics, I asked teachers to discuss samples of communication tasks that they actually use in the classroom (Interview Question 3). This enabled me to compare actual practices and reported practices. The comparison allowed me to determine the level of non-convergence across participants to add credibility to claims based on statements teachers made about their practice (Lindlof & Taylor, 2002).

Conclusion

This case study involved 12 mathematics teachers from seven schools in Board P. The methodology used for collecting data included semi-structured interviews, one interview with each participant. The data sources for the research were the transcripts of the interviews.

Chapter 5

Findings

The research questions are how do secondary mathematics teachers understand communication in mathematics? By this I mean – what do secondary mathematics teachers consider to be communication? And how has teachers' understanding of communication developed? In the research undertaken to examine these questions, I identified seven themes in the interview data. Study participants named several modes of communication and stated that they considered communication to be beneficial to teaching and learning mathematics. During the interviews teachers also expressed tensions surrounding the topic of communication. Teachers discussed the tools they use to support communication and the in-service they had experienced. Teachers discussed EQAO standardized testing and expressed their opinions about communication in general.

The findings chapter is organized primarily by the themes listed above, although modes of communication and benefits of communication have been combined under a single section named communication modes and benefits. It is my intention by intertwining the teachers discussions of the benefits of communication with the modes of communication they are commenting upon to present similar ideas in a more concise manner. I present a summary of the themes beginning with the most frequently referred to by the teachers. Excerpts from the teacher interviews are provided to illustrate the themes. Table 2 displays the codes used in analyzing the data along with the frequency of occurrences of each code over the 12 interviews and number of transcripts (participants) that referred to each code. In the final section in this chapter I include findings related to the five questions from the interview protocol which were rated on a Likert scale of 1 to 5. Table 3 details each question as well as the mean ratings and standard deviations.

Table 2

Coding summary by frequency and number of participants

Themes	Frequency	Number of Participants
Modes		
Conventions	109	12
Writing (ideas)	75	12
Terminology	21	6
Oral (student)	19	7
Writing (algebraic)	11	6
Oral (teacher)	9	7
Benefits		
Understanding	18	8
Communication with others	16	9
Solidify/clarify	14	7
Non-algorithmic	2	2
Tensions		
New perspective	34	11
Communication	22	9
Tools		
Colleagues	18	8
Textbooks	15	9
Other	13	4
Curriculum documents	4	3
In-service	19	12
EQAO	18	7
Opinions		
Value	37	12
Evolution	32	12
Pervasiveness	12	7

5.1 Communication Modes and Benefits

Participants identified many different modes of communication that they assessed, taught, and emphasized in their classroom practice. The modes that they mentioned were use of conventions of the discipline, writing in prose about ideas and mathematical thinking, use of vocabulary and terminology of the discipline, oral communication with peers, symbolic algebraic representation and oral communication with the teacher.

5.1.1 Mathematical conventions. Mathematical conventions (also sometimes referred to in this section as "form") were the most talked about mode of mathematical communication, with every teacher referring to it multiple times yielding 102 references over the course of 12 interviews. The participants in this research considered mathematical conventions to refer to the rules governing equal signs, brackets, mathematical statements (for example *let statements* such as in the phrase "let x represent the length of the rectangle", and *therefore statements*). Interview questions about which modes of communication participants taught, assessed, modelled, and emphasized all elicited references to mathematical conventions. Participant F described mathematical conventions as:

> I also mark written communication in terms of math conventions and if they are using a variable that hasn't been defined, do they define their variable? Do they, if it's a question that needs a let statement or a therefore sentence, do they include those? Do they include diagrams if appropriate? In terms of equations I look for equal signs being lined up, the conventions, basic conventions of math, I look for those. Units, proper rounding, I consider that a communication thing as well. If they read in the

> question "round your answer to the nearest 10^{th}", if they don't do that, it's a communication error in my mind.

In this excerpt Participant F outlines mathematical conventions she evaluates as communication. She specifically equates errors in conventions to errors in mathematical communication.

Most teachers described conventions as the most commonly modelled mode of communication. Participant K said:

> I explain that however I am modelling a solution, that's what I expect to see from them. I ask them to keep their notebook out when they start to practise so it's right in front of them reminding them how it should look or how detailed it should be.

Here Participant K is describing how students would use a modelled solution to verify their own use of mathematical conventions while doing practice questions.

Some teachers, like one described by Participant A, consider communication to comprise solely of mathematical conventions. Participant A said: "Whereas I know I have another colleague who does mathematical form and that's it." One such teacher is Participant E, who only looks at mathematical conventions when assessing communication. He explained how he uses conventions to gauge student understanding: "A significant communication error in a Grade 9 class when they are first learning the structure [of a solution] can be a pretty insignificant communication error, in your Calculus and Vectors class. [In the latter case] it's just a mistake, I think, rather than a misunderstanding." This understanding that communication is solely defined as convention is not in line with the Achievement Chart (*The Ontario Curriculum:*

Mathematics (Revised), 2005, pp. 20-21), which considers conventions to be one out of three distinct aspects of communication in mathematics (Figure 2).

The participants emphasized the importance of the conventions in mathematical communication. Participant B indicated the need for consistency to aid communication: "And the only way to do that is to follow consistent norms so that anyone looking at someone's work from this school and looking at someone's work from [another school] can make sense of it," thus consistency in conventions will aid communication. Participant C mentioned the role conventions play in the logical layout of a mathematical solution:

> And I have one student right now who argues with me, "Yeah, but I got the right answer." Yeah, but it's flying all over the place. So I use the example of a roof. So, okay, you put vapour barrier down. Then you put shingles, then you put plywood on top, you've got a roof, but it's not right. Well, but, it's a roof. But it's gonna eventually leak, so I want to lay it out properly.

Participant C is emphasizing the conventions which regulate how a solution is laid on a page to aid the communication of the mathematics in a solution.

Three of the participants discussed diagrams as a mode of mathematical conventions, for example a diagram marked with dimensions, vertices labelled, angle measures indicated and so on. Participant C said: "A diagram to me is communication, so what does the diagram look like, did you lay that out nicely for me with unknowns marked on and information marked on?" Here, by referring to the markings on the diagram, Participant C is treating a diagram as an extension of standard mathematical conventions. Even teachers who make a point of including a variety of modes of

communication admitted to emphasizing conventions above the other modes of communication. Participant H said: "I probably emphasize mathematical form the most."

5.1.2 Writing in prose about ideas and mathematical thinking. The second most mentioned mode of communication was writing in prose about mathematical ideas. There were 75 mentions all together, and although all 12 participants mentioned writing in prose about mathematical ideas, for some of them it was in response to the direct question "Do you expect students to write about mathematical ideas?" which was asked if they had not mentioned it during the course of their discussions of different modes of communication. Of the 12 participants three did not expect students to write in prose about mathematical ideas as part of their understanding of communication. One such participant was Participant E, who stated: "No, we wouldn't expect sentences and phrases." Another one of the three, Participant B, was willing to consider writing in prose as a backup in case of difficulty: "If I encourage my students that if they do not know how to mathematically represent the answer, that if they can explain it to me, in a written form. I will give them credit." Interestingly these three teachers (B, E and K) all worked at the same school. These three teachers nonetheless, had varying amounts of experience: B, 2.5 years, E, 10 years and K, 23 years.

Those who included the mode writing in prose about mathematics in their understanding of communication described tasks that they would use to evoke writing in prose. Participant G gave this example: "So 'Describe a difference or describe a similarity' and I would be expecting them to write in sentences. Of course that's the communication part, writing in sentences and talking clearly about their ideas." Participant F described how she elicits writing in prose about mathematical ideas:

> Okay, so in the whole written/oral [task] that would be if I gave a question "explain how" or "describe why" or that sort of thing, where it would be a written out question with words, that would be one part of the communication.

Both of teachers F and G are using writing in prose about mathematical ideas, such as describing or explaining a process, as part of their assessment of students' communication skills.

In order to facilitate writing in prose about mathematical ideas by emphasizing the steps and processes involved in mathematical solutions some teachers described how they intentionally draw attention to the steps of a problem while teaching. Participant D said:

> When going through questions at the front, explaining my thought processes as I actually do a question. For example talking about the specific steps that I would go through, identify what the problem is and also to show them what specific steps I would use to solve the problem.

While others, such as Participant K, drew attention to the way they model the process of deciphering a mathematics question in order to draw attention to the necessary information when writing about the processes used. She said:

> I will read a question sometimes two or three times, explain "This is what I would do." First time I read it I didn't get enough out of it, so I read it again and I look for key words, when it comes to word problems.

In both cases teachers are trying to model to students how to describe mathematical procedures using words to supplement a solution using symbolic algebraic representation.

Teachers discussed the benefits of asking students to write in prose about mathematical ideas. Participant G mentioned that communication allows students to

demonstrate their deeper understanding of mathematical concepts in a way that performing algorithms does not. He said:

> And they can memorize conventions too without actually having the deeper understanding. So these communication type questions are often a little bit of an insight into whether or not the student has that overall understanding perhaps and that never, back in the day if the student could produce beautiful solutions we assumed that they had connected all the dots. And now I know you can sort of dig a little deeper than that.

Participant G is noting that often a student can produce a solution by memorizing an algorithm. Asking a student to add a description of the mathematical ideas behind a solution allows a teacher to determine if a student is demonstrating understanding or memorization.

Participant H connected writing in prose about mathematical ideas as a skill students need to learn for future career paths:

> I use this example with my students: If you can sit in a corner and come up with an answer, that's great, but you need to be able to convince others that that is the correct answer. Having a past career as an actuary I have certainly had the opportunity to come up with numerical answers to questions but unless I could explain the process effectively and convince them that my methods were sound it wouldn't have gone very far. So I expect a similar, I have a similar standard for my students. I expect them to convince me that their numerical work is correct, through written expression.

In this excerpt Participant H draws on his own experience in his previous career as an actuary to explain why it is important for students to develop their ability to communicate about mathematics.

Participant F described how a student might use communication to clarify their ideas about a mathematical concept, allowing them to move forward if they are in difficulty:

> The communication in terms of "Describe..." and "Explain..." are important to those kids who don't always get it right away and have to stop and think about why am I doing this or what am I doing. And those steps, I think it helps those kids to clarify in their head what they are doing

So, if a student has the understanding to explain the steps required to solve a problem they will be better able to set themselves on the right path when they run into difficulties. Participant D stated that she believes communication can be used as a tool for students to solidify their own ideas: "And I think for a lot of kids too, being able to explain what they did solidifies their understanding too." In this case the teacher sees the benefit to having students explain the ideas behind the mathematics they are doing as a way to reinforce their understandings.

5.1.3 Using the terminology of the discipline. Half of the participants mentioned the importance of the language and the vocabulary of mathematics, for example "term, coordinate, coefficient, equation, expression" and so on. Participant A also emphasized the need to teach mathematical vocabulary explicitly:

> It has to be a part of their vocabulary, whether that is using games. [It is important] that they are actually using it [in order to] learn it. It takes a lot of time to do that but at the same time I think there is a benefit to that

> because then when we are doing notes on the board they know what I am talking about, when I'm talking about a variable or they know when I am talking about like terms or unlike terms, they have a conception and that is going to carry them all the way through.

Having access to the terminology of the discipline is seen by Participant A as a key part of having access to the important ideas contained in the mathematical solutions.

Teachers also considered students' use of terminology when assessing student understanding. Participant C said: "That's another thing, how much math vocabulary are they using, how many of the words that we used are they using in their description and that can be a sense of how well they are doing as well." In other words, if they can use the terminology of the discipline correctly it is more likely that they understand the mathematics that they are describing. Participant H described how he modelled the precision of mathematical language and used this modelling to spark discussion:

> When communicating through sentences or even setting up variable definition statements at the beginning, I do take great care in what words I choose and how I word it and make sure they know that I am doing that specifically and intentionally and have them share in the development of what I am writing, so that we can discuss what is appropriate and what is not. So, as opposed to just putting it up there and hoping it sinks in, we do sort of collectively discuss how we should word it. And if they challenge what I have written, they don't understand it, we find another way to phrase it.

In this excerpt the teacher is modelling writing in prose about mathematical ideas and negotiating meanings with the students surrounding mathematical terminology to aid in their understanding of the mathematics being presented.

5.1.4 Communication with peers. Just over half (seven) of the participants in this research discussed students' oral communication as a mode of communication. Despite oral communication being a component of the Achievement Chart (*The Ontario Curriculum: Mathematics (Revised),* 2005, pp. 20-21), some of those teachers explained that they do not consider oral communication when they are assessing. For example, Participant D said: "No I don't think I really have ever assessed oral communication." Some participants indicated that they use oral communication as an alternative to written communication. Participant F said:

> If I think that a kid can know more if they tell me as opposed to write it, I'll get them to do that. I don't often mark oral communication but if I feel that the written communication is lacking enough yet I know the kid actually knows what to do but they struggle with writing then I'll ask them and I will mark that way.

In this case oral communication is seen as a backup if written communication fails but not to be used as a mode of communication in and of itself.

Oral communication appeared to be often used as a method of assessing preliminary understanding. Participant L explained: "Not formally, no. But I do get a sense of how much students understand by asking questions of the class." Communication was seen as a real world skill by Participant B: "I would at a higher level class still focus in on presentation skills because I know they are important in the real

world." Only one participant seemed to see the value of oral communication as a way of building understanding. Participant C explained:

> A good thinking question, you want them talking to somebody about. It is okay if they are talking to somebody. I figure that's involving communication, good. There you go, "Well, I think we should do this, well, no, because, if we did that then this would be wrong, so we should do this instead, oh, okay." Wow, there's good communication

Participant C is unique among the participants in expressing that oral communication is an important mode of communication. She mentioned that students can use it to learn from one another, thus she is using communication to build student understanding.

5.1.5 Writing using symbolic algebraic representations. Six of the teachers pointed out that traditional writing using symbolic algebra representations, for example the steps required to solve an equation, in contrast to writing about mathematical ideas using prose, was still a mode of written communication. Participant K pointed out that writing using symbolic algebra representations is still an explanation of the process used. Participant K said: "Actually on every question unless it is a one word answer or a one-step calculation, you are explaining by showing where the answer came from." Here Participant K is pointing out that writing using symbolic algebraic representation is in effect explaining the series of steps that a student has followed to solve a problem.

Participant E also emphasized this understanding that algebraic solutions are a form of explanation, going so far as to consider algebraic representation a form of non-fiction writing:

> The example that we are presenting to the board, of non-fiction writing is our Grade 9 summary sheets, getting a Grade 9 student to be able to

> summarize a unit, which doesn't necessarily come with many sentences, but it can still be symbolic showing good examples and an understanding of the key types of questions that they have to be prepared to answer.

The non-fiction writing initiative in the participants' school board was a way of getting all departments involved in preparing Grade 10 students for the Ontario Secondary School Literacy Test (OSSLT).

5.2 Tensions

Many teachers expressed tension when discussing communication in mathematics during their interviews. There were a total of 56 excerpts from the transcripts that reflected how teachers struggled while trying to incorporate communication into their teaching practices. I divided the tensions expressed by the participants into two groupings: those that reflected their concerns with the new curriculum direction that was introduced in 2000 and those that reflected their concerns with the idea of communication. There were no questions in the interview protocol that specifically asked about difficulties or struggles the teachers were having; these struggles were volunteered by the teachers as they discussed the various aspects of communication.

Of the 12 teachers that I interviewed, 11 expressed some form of struggle that they were having with the new curriculum. Participant F indicated that there is confusion with respect to creating test items for communication:

> I think a lot of us really want to focus on what, where do you pull the communication mark from, because everyone seems to do it quite differently, and don't always agree on where it should come from or even if we should be using it.

Participant F is expressing her frustration with a lack of consensus in the basis of assigning a communication mark among the teachers in Board P; some assess holistically, others on specific questions, some only assess conventions while others only assess writing in prose about mathematical ideas. Others were still not comfortable with the process of levelling using the 4-level rubric as mandated by the *Ontario Curriculum: Mathematics* (2000). Participant K expressed her discomfort with levels:

> I would have to say that I don't feel confident when I decide on a level, but I have marked something, accidentally, twice and have come up with the same level. And I was happy about that, but obviously I wasn't sure that would happen because the levels also have a large range. One day you might think 62 and the other day 65. Which is a [level] 2 or a 2 minus.

Participant K is a good example of teachers who are not comfortable with what they see as the subjective and hard-to-replicate process of assigning levels to student work; in fact, Participant K seemed pleasantly surprised when the process is consistent. Participant I concisely summed up the situation: "So I think there is definitely some argument as to what assessing communication is supposed to be in math."

Other teachers are concerned with how the new marking system along with this new emphasis on communication is affecting the integrity of the marks that they assign. Participant A said:

> Even tests that I am going to look at this afternoon that I know are level R's [a mark below level one, defined as 'requiring remediation'] and [one of] the students [that] is totally misplaced but if you look at their communication as far as form goes it is definitely not an R and for some

> students that [new emphasis on communication] might mean pushing them along where they shouldn't be pushed along.

Here Participant A is reflecting the understanding that communication does not really correspond to true mathematics understanding and that a student's mark should be based on the other Achievement Chart (*The Ontario Curriculum: Mathematics (Revised),* 2005, pp. 20-21) categories of knowledge and understanding, and application.

Teachers are also concerned, as expressed by Participant C, whether they really understand what communication in mathematics means:

> So would I do a workshop on it? No way, because I don't have enough confidence in the stuff that I have come up with is any great shakes compared to what anyone else comes up with. I always think that I am the weak link there.

Participant C has worked out a definition of communication that works well for her when assessing student work, but she is not confident that it is a correct definition and is definitely not confident enough to share it with others.

Other participants mentioned that they were looking for guidance from either the Ministry of Education or even the school board. Participant L said:

> I would really like to see the Ministry do something in terms of defining what a communication question is, what they would like to see as a communication question. Or even the board, to say a Grade 9 communication question should look like this or should include key words such as this, or something along those lines. Again, because I am so concerned with consistency that I am doing it the right way, that everybody else is teaching it similar to what I do, that I am not putting my

> students behind somebody else's....I would definitely like to see more PD on communication.

Here Participant L is expressing several concerns. He is not sure he really understands communication the way others understand it and he is concerned that his lack of knowledge is disadvantaging his students. Finally he is seeking support either from the board or from his peers with respect to communication.

Most of the teachers struggled with the mix of mathematics and writing in prose about mathematical ideas. Some were concerned with how students' writing and grammar skills would affect their communication in mathematics. Participant H said that he tries to accommodate the differences in skills: "Some students have stronger English skills than others and I try and recognize difference in ability, somewhat, in their written responses." Participant G expressed concern about alienating students with weak writing skills but strong mathematics skills:

> So there is a double-edged sword there, if you don't ask them then you are missing that chance for them to demonstrate further understanding but I also am wary of suddenly making a student feel like they can't do math because they can't write either. It's a balance.

In these interview excerpts both teachers H and G are expressing a concern that students' English writing skills may impact their mathematics mark negatively. This perspective suggests that some teachers may not consider writing in prose about mathematical ideas to be a mathematical task as they would problem solving, computations or algebraic manipulation and so on. Other teachers indicated their own biases in terms of different modes of writing seen in other disciplines. Participant B said: "I want them to be to the

point, concise and not wordy. Whereas in some course like English and history you have to elaborate – I don't care for elaboration [in mathematic]."

Participants also struggled with incorporating the newer style of mathematics communication because it was different than the style of mathematics that they had experienced as students. Participant F explained:

> It was like, "Do the math." Don't explain why you are doing it or what you are doing, and just do the math. So that has been a challenge for me to break away from what I know and what I did to getting some kids to communicate in a bit of a different way.

Here Participant F is reflecting upon how difficult it can be for a teacher to incorporate new styles into their teaching practice and to change how they think about mathematics.

5.3 Tools Used by Teachers

Participants were asked "How do you create a good communication question (or one designed to emphasize communication)? What sources do you use?" The sources most commonly mentioned by the participants were colleagues and textbooks. A variety of other resources were mentioned such as the internet, creating questions themselves, or using student questions. Curriculum documents, while mentioned by only four teachers, were not widely considered to be a useful resource.

Of the 12 participants eight of them mentioned talking to colleagues as a source for communication material. Participant A said:

> Sometimes it will be just talking with the people in my math office and I'll say "I'm looking for something in this unit and I want it to do this," and

> sometimes someone else has gathered stuff so there is a certain amount of sharing.

In addition to talking with colleagues to gather communication-type questions, such as those requiring an explanation or justification of mathematical reasoning, some participants communicated with their colleagues as a tool for furthering their own understanding of communication. Participant F explained:

> We debate back and forth about what communication is and where the level should come from. Should we actually put a communication section on our test or just overall communication throughout the knowledge, application and thinking? So we have those discussions. They don't really ever amount to anything that is super conclusive but we have them, which is part of it too. Just being aware of it and thinking about it is a good thing.

Both of these teachers are reflecting that teachers are talking to one another both as a source of material and as a way to develop understandings surrounding communication and mathematics. Others, like Participant B, only discussed communication with their colleagues if an issue arose in the classroom where they needed clarification. In response to the question "do you talk to your colleagues about communication?" Participant B said:

> Very seldom, only if I have a particular issue with how I am trying to do an assessment like, for example, I will tap into my colleagues when there is a major project portion of the data management presentation, so I probably will tap into my colleagues' [understanding] for that. But up until this point, no, very rarely.

In other words, in some schools teachers are working together to understand and implement communication whereas in others it is only being discussed on a need-to-know basis. This underlines the lack of consistency mentioned in previous excerpts.

Textbooks were a popular resource for teachers; nine out of the 12 participants used them regularly. Participant G indicated that the textbook was a "pretty good source." Participants agreed that not all textbooks are created equal; some are better than others. Participant H said: "There are some good textbooks; I find the Marian Small textbooks do contain some good communication questions." Also, not all sections of a textbook are useful as a resource for communication; participants had to pick and choose to find suitable material. As Participant F explained:

> I'll use textbook questions but I find that a lot of them aren't the greatest, in my opinion of what a communication question is there are not a lot of them there so I often will use the "Reflect." In the textbooks there is the lesson and then there is the "Discuss the concepts" and the "Reflection". Those "Discuss the concept" and "Reflection" type questions are usually more about getting them to verbalize what they are thinking and what they know about it.

So, while teachers are using textbooks, they are using them with caution, and as a tool to aid understanding about communication rather than as a source of their understandings.

Participants mentioned several other sources of materials on communication. Several used the internet or made communication questions up themselves. As Participant C said: "My head is usually where they come from." Creating communication questions independently tended to be done by teachers with many years of mathematics teaching experience.

Participant H's indication that he often used student responses to develop questions that would emphasize communication was interesting. Participant H said: "But also I try to pay attention to questions students ask in class that help clarify their understanding and I find that sometimes they make good questions to give the rest of the class." Others modified more traditional questions to emphasize communication. Participant G explained:

> If there is a question that I would like to put on a test or I'd like to do in class, but I just don't have enough time to test it or to do it I'll say let's just list the steps, let's just talk about the steps that we would use to do that.

What Participants C and G are saying here is evidence that teachers are trying to find creative ways to fulfill the explicit requirement that they assess communication in mathematics.

Although it might seem that documents published by the Ministry of Education would be a good source for information surrounding communication, only four of the twelve participants mentioned the documents at all. Those who did mention them were not positive. Participant C reflected: "...all those achievement charts, they're just so vague, they don't say anything in there." Sometimes participants' references to curriculum documents reflected a lack of familiarity, as shown by Participant G: "We've got exemplars that discuss the student's knowledge, application and thinking perhaps, do they talk about the communication or do we get a chance to talk about the communication exclusively?" In fact the Ministry of Education Exemplars do cover assessment examples using all four categories, including communication, from the Achievement Chart (*The Ontario Curriculum: Mathematics (Revised),* 2005, pp. 20-21).

5. 4 In-service

All participants were asked if they had participated in any in-service specifically related to teaching communication in mathematics. More experienced teachers recalled the in-service sessions surrounding the introduction of the Achievement Chart categories in the *Ontario Curriculum: Mathematics* (2000). Participant C said:

> We would have those board meetings, and so they would kind of tell you what they thought they were. I didn't like the general one where we would sit down and they would say "in general the knowledge and understanding are ...." all those achievement charts, they're just so vague, they don't say anything in there. So, okay once we get to math-specific, oh, okay now I see what you mean by knowledge and understanding and once I saw the communication I thought, oh, okay that's no different than what we got, it's just more formal, they just want you to give a mark for it.

In this excerpt Participant C is reflecting on the sessions provided by the board, first to introduce the Achievement Chart (*The Ontario Curriculum: Mathematics (Revised),* 2005, pp. 20-21) categories and then later to discuss how they can be applied to various disciplines including mathematics.

Many teachers were less than positive in their recall of in-service sessions provided by the board. Participant K echoed this sentiment: "It's difficult to remember them, and in my memory it doesn't seem very positive, like did we learn much, I don't know. The board tried." Teachers who had entered the profession after the new curriculum was introduced had no experience with in-service relating to communication and mathematics. Participant B said: "We don't really discuss it, we don't discuss it at PD days." Participant B is in his third year of teaching and the board has moved on from the

implementation of the Achievement Chart (*The Ontario Curriculum: Mathematics (Revised),* 2005, pp. 20-21) to other more current curriculum and pedagogy initiatives. Currently some schools in our board are beginning to focus on communication in their Math Learning Teams. Math Learning Teams are school-based teams that meet several times per semester to investigate teaching problem solving techniques and improving student solutions to open-response questions on the Grade 9 EQAO assessment. Participant F described the focus of one in-service session:

> That actually is the focus of our math workshop thing this year, we are going to look at open response questions but I think a lot of us really want to focus on what, where do you pull the communication mark from, because everyone seems to do it quite differently, and don't always agree on where it should come from or even if we should be using it, kind of thing. I haven't done that yet but that is our focus for this year.

Here, Participant F is explicitly describing the focus of her school's Math Learning Team.

5.5 EQAO

EQAO was a theme that evolved out of the interviews. Even though EQAO was not explicitly mentioned in any of the questions in the interview protocol, seven of the twelve participants made a reference to EQAO in some way, for a total of 18 mentions. Board P has implemented several initiatives targeting improvement in the EQAO Ontario Secondary School Literacy Test. Evidence of the literacy across the curriculum initiative and the non-fiction writing initiatives are evidenced in this excerpt from Participant H:

> Currently we discuss communication at every department meeting because it is one of our department goals: communication and literacy rolled into

> one. We are trying to include a question involving non-fiction writing on every test that we have or every assessment, major assessment anyways.

Participant H's comment reflects that communication in mathematics is benefiting from the board's current emphasis on preparing students for the OSSLT, but it is unclear if communication in mathematics would have been under scrutiny at Participant H's school without the literacy impetus.

Schools are also concerned with results on the Grade 9 EQAO mathematics assessment. Participant I said: "EQAO has probably been a big motivator too. I don't like to teach to the test but at the same time I want to set my kids up to be as successful as possible." EQAO has also had an effect on how teachers incorporate communication into their teaching and assessment practices. Participant L explained:

> I expect a lot more than I did that first year in terms of what they [my students] do and that is basically because of the EQAO testing and the factor that if I can get the kids to really organize their work it seems that their marks on the EQAO are a little bit better than when I started.

Participant L's comment reflects how teachers' practices with respect to communication and mathematics are changing in response to the pressure teachers feel to improve students' scores on EQAO.

5.6 Opinions Expressed by Teachers About Communication

Teachers in the research were asked if they thought communication was important and if they felt that communication was a good addition to the curriculum. They were also asked how their understanding of communication had changed over the time that they had been teaching under the new curriculum. In response to these questions teachers offered

opinions on the value and pervasiveness of communication in mathematics as well as commented on the evolution of their own perspective over time.

All 12 of the teachers in the research expressed a positive opinion about the value of communication in mathematics. Participant F saw the value of communication as a diagnostic tool and as a way of developing understanding. She said:

> I would definitely teach it and I think teaching it in terms of the reflection questions and journal type questions I think it is really important for the development of their knowledge and the development of their skills and everything, but to me I would not mark those questions. They would be useful for me to see them, as I said diagnostics or for the kids to hash out what they are trying to say or even to debate with each other about it.

In this excerpt Participant F is reflecting upon whether or not she would continue to emphasize communication if the restrictions of the Achievement Chart (*The Ontario Curriculum: Mathematics (Revised),* 2005, pp. 20-21) were lifted. Teachers were also positive when asked if they thought this new emphasis on communication was an improvement over past curriculums. Participant A said: "I think it is a good addition." Participant H contrasted the current emphasis to the more traditional approach taken when he was a student:

> Certainly there was more drill work in my education and I think it limited conceptual understanding, which I value. Different teachers value that at different levels. And I think conceptual understanding is better achieved through communication or a blend of communication with the drill.

Participant H said he sees the addition of communication as a positive evolution in the teaching of mathematics, perceiving the more traditional approach to have an emphasis on drill rather than understanding:

Although the teachers were clear about the value that they placed on communication, some expressed doubts about the value the mathematical community within the board placed on communication as expressed through the weightings used for evaluation. Participant D said: "I think there is less of a focus even with the percentage split, communication is less important in math because it is worth less." Participant D was referring to the board's weightings for the Achievement Chart (*The Ontario Curriculum: Mathematics (Revised),* 2005, pp. 20-21) which in mathematics are: Knowledge, 35% for Academic/University level courses or 40% for Applied/College level courses, Application 35% (30%), Communication 15%, and Thinking 15%. In other disciplines, the Achievement Chart category weightings are usually 25% per category, giving the categories equal weighting.

Teachers were asked if their understanding of communication and mathematics had changed or evolved over time. Participant I mentioned how her understanding of communication had evolved to include the connection communication makes between skills and understanding. She explained:

> I probably, as a new math teacher, focused more on the knowledge and I wanted to make sure they know how to rearrange this equation and solve for x. I am not realizing the importance of well, do they understand what the equation means, can they translate it back into words, can they write. I am not saying that I wasn't doing communication but I didn't realize the importance as a link to application skills and thinking skills..

Participant I's comment reflects how she came to understand the importance of the role of communication in learning as she has gained more experience as a teacher. Participant H commented on his increased use of communication as a tool: "I certainly give students more opportunities to communicate in mathematics than I did at the very beginning." This comment reflects on Participant H's increased comfort with communication and mathematics. Participant E indicated how he has come to prefer the new system of assessing communication by assigning levels as compared to the traditional methods of marking. He said:

> I like what I am doing better now. I like how I treat communication more now. I like the kind of assessing communication the way you would assess a painting. I like that better than taking of a mark [say] for leaving off centimetres. I think it is more appropriate.

This comment was probably the most supportive of communication in mathematics from the teachers as they reflected on the new Achievement Chart.

Over half of the participants (seven out of 12) commented on the pervasive nature of communication in their teaching practice. Participant H commented: "Yes, teaching is communication; if I were to ignore communication entirely it would be impossible to teach my subject." Participant C preferred to discuss the pervasiveness of communication in mathematics overall, whether it is written or oral communication. She said:

> You can't do math without communicating. You can't, because you've got to be able to use the language to say it or you have to be able to write down the solution so that some guy can pick it up.

Similarly, Participant E mentioned that communication is a process that is intertwined with all of his teaching: "Yes, I teach it every day. I am constantly talking about structure

and proper form and where it leads to and how we can build on top of strong communication." All of these teachers commented on how communication in its various modes is an essential part of teaching mathematics.

5.7 Responses to Likert Scale Questions

Table 3

Responses to Likert Scale Questions (Scale 1 to 5)

Question	Mean	Std Dev
How prepared do you feel to teach and assess communication?	3.5	1.0
How do you feel supported to teach and assess communication?	2.5	0.9
What is your level of knowledge with respect to teaching and assessing communication?	3.3	1.2
What is your level of experience in teaching and assessing communication?	3.4	0.8
What is your level of confidence in teaching and assessing communication?	3.3	1.0

At the end of each interview teachers were asked five questions where they were to rate their responses on a scale of one to five. The mean response for each question along with the standard deviation is shown in Table 3. Questions relating to their preparedness, knowledge, experience and confidence with respect to teaching and assessing communication had similar responses. All of these questions had a mean rating of close to 3.4 out of five, with a standard deviation near one. This result reflects the comments made in the previous sections. Overall, teachers were comfortable discussing

many modes of communication and how they had incorporated them into their teaching and assessment practices.

The remaining question, referring to the level of support felt by participants stood out from the others with a lower mean of 2.5 out of five. This result is also to be expected given the comments the teachers made during the interviews surrounding in-service, discussing communication with their peers, and consistency in approach. As a result of this uniquely low mean I have chosen to summarize the teachers' responses to the question "How do you feel supported to teach and assess communication?"

Some teachers, such as Participant A, characterized the support they feel as a lack of conflict: "So, support is 'Yeah, I like what you're doing, go ahead. I'm not sure that I agree with you but that's all right.'" Others were more direct in claiming that they feel support from their colleagues but not at the board level. As Participant H said:

> I am a little on the fence on this one because I feel that my colleagues do a good job of supporting but there really aren't, until last year I had not seen a formal attempt at the board level to assist.

Some teachers indicated that the support they are seeking from the board needs to be in the form of more concrete direction as to the nature of communication in mathematics. Participant G explained: "I wouldn't say very supported. We are given an opportunity at times and not very often to discuss communication on a professional level but not often we are given right or wrong answers." Participant D echoed this sentiment: "I don't think there has been much instruction on how to specifically teach communication or how to include it in math." These comments echo comments in earlier sections where teachers emphasized their dissatisfaction with in-service provided by the board. By way of underlining this search for support Participant G, who is a department head at one of the

schools, recognized the importance for teachers to feel supported by those who lead them. He said: "I've put myself in a spot where I am a leader now, and I hope my teachers would feel more support than a 2 [out of 5 score] because I am their support." Participant G is recognizing that teachers are expressing a need for support as well as how he is trying to provide some of this support in his role as department head.

Conclusion

The teachers in this research, when looked at as a group, are aware of a variety of modes of communication: use of conventions of the discipline, writing in prose about ideas and mathematical thinking, use of vocabulary and terminology of the discipline, communication with peers, and writing using symbolic algebraic representations. However, their comments reflect a heavy reliance on mathematical conventions when teaching and assessing communication in mathematics. Teachers in this research are seeking knowledge and support with respect to communication and mathematics. They mentioned seeking support from colleagues, the board and also the Ministry of Education in an effort to build their understandings of what was expected of them when it came to assessing and teaching communication. Also notable was the influence that EQAO is having on teacher practices. The influence has led to an increased focus on communicating about mathematical ideas as teachers try to incorporate writing in prose into a discipline not known for an emphasis on prose.

In the next section I draw connections between my findings and the research on communication and mathematics as presented in the Literature Review section. I also draw conclusions from this discussion as well as comment on improvements to this research and directions for further research.

Chapter 6

Discussion

Ten years after the implementation of the *Ontario Secondary Curriculum* (2000), teachers are still struggling with its new direction. The emphasis on communication, highlighted by its inclusion in the assessment structure of the new curriculum, as one of the four achievement chart categories, is perhaps the root cause of this struggle. Teachers are continuing to question the nature of communication in mathematics and the scope of communication in mathematics in the context of their classroom and assessment practices.

The objective of this research was to investigate the nature of teachers' understanding of communication in mathematics from the viewpoint of secondary mathematics teachers. The research questions that informed the investigation were: How do secondary mathematics teachers understand communication in mathematics? By this I mean – what do teachers consider to be communication? And how has teachers' understanding of communication developed?

In this chapter I first discuss how the findings and the literature review come together to respond to each of my research questions. I then summarize the study, present my conclusions, and finally briefly make suggestions for further research.

6.1 How Do Secondary School Mathematics Teachers Understand Communication in Mathematics?

Case (1994) states that in order for teachers to adapt to using new curriculum it is important that they support it. All of the teachers that I interviewed expressed positive opinions about the value of communication in mathematics. Teachers also commented on

the pervasiveness of communication in their classroom practice. One participant said: "You can't do math without communicating...You've got to be able to use the language to say it," echoing Pimm (1996) and Hiunker and Laughlin (1996) when they refer to talking as a natural way for people to communicate about mathematics. This participant also is embracing the principle of the sociocultural classroom, where language is seen as being central to learning (Steele, 2001).

Along with noting the importance and the intrinsic nature of communication in mathematics, the teachers in this research also brought forward several benefits of emphasizing communication within the classroom. The benefits mentioned by the teachers are all echoed in the literature review. However, the teachers took a much narrower understanding of the types of benefits of communicating about mathematics.

Most of the benefits mentioned by the participants were those that benefited the classroom teacher. Teachers are aware that student communication of mathematics gives them insight into student understandings or gives them immediate feedback during lessons, allowing them to identify any misconceptions (Peressini & Bassett, 1996; Pimm 1987). The idea that teachers can determine student understandings through communication is supported by the sociocultural view that understanding can be seen as a student coming to share in the meanings of the larger community through discussion (Lerman, 1996).

The teachers in my research were also aware of the benefits that communicating about mathematics brings to students. Teachers described how students can record their thinking in order to reflect and then refine their ideas as a way to work through difficulties they may have with a problem (Whitin & Whitin, 2002).

Another benefit cited by two of the teachers interviewed for this research was that communication about mathematical ideas allows students to see that mathematics has more dimensions than the purely algebraic, algorithmic aspects (Porter & Masingila, 2000; Schoen, Bean & Ziebarth, 1996). As one Participant said: "And they can memorize conventions too without actually having the deeper understanding; so these communication type questions are often a little bit of an insight into whether or not the student has that overall understanding. One explanation for why more teachers did not mention this as a benefit could be that teachers of mathematics find the algebraic, algorithmic facet of the subject to be second nature.

There are other benefits mentioned in the literature that were not brought up by teachers. Although teachers mentioned using communication to determine student understanding, they did not discuss how they would use that information to improve student learning by either adjusting their own delivery or perhaps correcting student misconceptions along the lines suggested by Goldsby and Cozza, (2002), Thompson and Rubenstein, (2000). Also missing was the idea that students can use communication to learn from one another and collaboratively build understanding (Choppin, 2007; Fonzi & Smith, 1998; Hiunker & Laughlin, 1996; O'Connell, 2005).

Teachers also did not bring up the concept that having students communicate about mathematics builds a discourse community in the classroom and gives the students the tools to become active participants in this discourse community (Choppin, 2007; Duff, 2002; Hiunker & Laughlin, 1996; Stein, 2007). Nor did teachers comment on how communication in mathematics is a key element needed for proficiency (Kotsopoulos, 2007b; Pimm, 1987).

The primary difference between the teacher perspectives and those in the existing research is that teachers took a very pragmatic view and commented on benefits that they saw as directly useful to them or their students; researchers on the other hand, looked at a broader spectrum of benefits. There are two possible explanations for this divergence. First, teachers were not asked specifically to comment on the benefits that they saw in emphasizing communication in mathematics; the benefits listed above came out as participants discussed other aspects of communication. Secondly, teachers are not researchers; for something to be a perceived benefit it may need to impact their practice in a practical, concrete fashion. As well, it might the case that had the teachers been exposed to constructs such as discourse community they might have been able to comment on the constructs.

The teachers in this research referred to a broad range of modes of communication; however, some modes of communication were more universally recognized than others. The modes that came up most often were: conventions, diagrams, writing (both in prose about ideas and using symbolic algebraic representations), oral (both by teacher and by the student), and terminology. The modes mentioned most often were conventions and writing in prose about ideas. This could be because these were the two modes most likely to be assessed during classroom assessments, and conventions are the one mode teachers firmly asserted that they taught and emphasized.

Some teachers almost exclusively used conventions as their method of assessing communication. This is not to say that teachers did not value communication or did not appreciate the new emphasis; conventions were just the mode of communication they taught and assessed. Interestingly, but perhaps not surprisingly, this focus of assessment was shared by teachers from the same school, so assessing communication as conventions

may have been a departmental emphasis. Indeed, this department considered algebraic representation as their "non-fiction writing" example for one of the board initiatives in preparing for the OSSLT. The extent to which other teachers in the board agree with this department's understanding of algebraic representation as a mode of non-fiction writing is questionable.

The modes of communication that the teachers in the research mentioned are supported by the research literature. Several of the teachers in the research referred to students needing to learn the language of mathematics in a manner akin to Pimm, (1987). The findings of Lim and Pugalee (2004), Whitin and Whitin (2002), and Peressini and Bassett (1996), among others, support the activity of students writing in prose about mathematical ideas.

Many of the teachers mentioned oral communication (Pimm, 1987; Porter & Masingila, 2000; Thompson & Rubenstein, 2000) although most used it as a formative assessment or as a substitution for written modes when students were having difficulty. As mentioned above, mathematical conventions and algebraic representation (Esty & Teppo, 1996) were the most mentioned mode of communication by the teachers. In a sociocultural context discussion with a peer or more knowledgeable adult is the catalyst for children's learning. While this inclusion of oral communication is encouraging from a sociocultural viewpoint, it is unfortunate that there was not more emphasis by the participants on facilitating classroom discussion. From a sociocultural perspective students should be encouraged to work and talk together as a classroom community (Bruce, 2007; Carter, 2005; Steele, 2001).

There were some modes of communication mentioned with lesser frequency, such as reading and interpreting as a mode of communication (Cai et al, 2005). Stonewater

(2002) discusses how strong mathematics students combine various modes of mathematical representation: algebraic, graphical and written. Only one of the teachers in my research captured this and she thought it best for her weaker students. Again only one teacher alluded to discussion as a key aspect of communication despite the emphasis on discussion as a mode of communication in the literature (Abele, 1998; Choppin, 2007; Hiunker & Laughlin, 1996).

One element of the research available on communication in mathematics which did not show up in the teachers' conversations was the discussion of how learning and discourse are connected when students are learning and thinking and expressing understandings about mathematics (Cobb et al, 1997; Kotsopoulos, 2007a; Sfard, 2007). There could be two explanations for this. Firstly it could be a gap in the interview technique or the interview protocol that did not elicit connections between learning and discourse. Secondly it could be due to the fact that the teachers that I spoke to are not researchers; they are practitioners and as such tend to think and discuss communication in practical, concrete, experiential terms.

Although teachers have adopted different modes of communication and can cite several benefits of incorporating communication tasks into their teaching practice, they still have reservations about this new emphasis. The fact that all of the teachers in the study expressed reservations indicates that teachers with a wide range of teaching experience are facing tensions with respect to communication and mathematics. Teachers expressed their discomfort with the new emphasis on communication because it is a different style of pedagogy than the one they experienced when they were students. This tension was echoed in the literature on teachers implementing new curricula. This

discomfort may be interpreted to mean that teachers are resisting the curriculum changes (Anderson & Piazza, 1996).

Another discomfort expressed by teachers had to do with the introduction of writing in prose about mathematical ideas. All of the teachers I interviewed had degrees in Math, Science or Engineering, which are not disciplines traditionally noted for their emphasis on the written word. As a result some teachers were also uncomfortable with incorporating what they perceived as English into Mathematics. Even teachers who were positive about this aspect of communication still referred to the non-algebraic portion as English. Clearly much of the teachers' discomfort with written (in prose) communication was linked to being asked to step out of their comfort zone of the algebraic algorithmic nature of mathematics.

Powell and Anderson (2002) and Sowell and Zambo (1997) observe that other struggles expressed by the teachers in this research had to do with aspects of the new curriculum itself. In order for teachers to be agents of change it is important to first bring about change in their own belief and value systems. The teachers that I interviewed expressed conflicted understandings about the direction of the new curriculum, especially surrounding assessment practices. Teachers were concerned about the integrity of the marks assigned to communication tasks, the mechanics of using the 4-level marking system, and how to define communication items on assessments. Teachers were torn between placing specific questions on tests for assessing communication as opposed to assessing communication holistically over the entire body of an assessment.

Perhaps the reason for the tension expressed above is teachers' lack of understanding of the new curriculum. In order for teachers to effectively implement curriculum changes they must understand the new curriculum (Case, 1994). Several of the

teachers interviewed expressed a lack of confidence in their own interpretation of communication.

Even though teachers are beginning to experiment and include different modes of communication in their teaching and assessment practice such as oral communication, blending representation and writing in prose about ideas, they have clearly not embraced this change of direction whole-heartedly. Their comments suggest that this is partly because the new direction is such a departure from the vision of mathematics that they are used to and partly due to lack of understanding of the connections between communication and mathematics. In this research there was no evidence of a stubborn resistance to change.

6.2 How has Teachers' Understanding of Communication Developed?

All of the teachers in this research were able to describe their conception of communication through their discussion of its importance, its benefits and its modes. Most of the teachers also discussed the evolution of their understanding of communication over the years that they have been teaching under the new curriculum. This is encouraging, because in order to effectively implement a new direction in curriculum it is necessary to bring about a change in the beliefs and value systems of teachers (Powell & Anderson, 2002; Sowell & Zambo, 1997). Whereas the change in value systems is by no means complete as teachers continue to struggle and find their way, it would seem it has begun to take hold.

Teachers were asked "How do you create a good communication question? (Or one designed to emphasize communication.) What sources do you use?" The primary responses to this question referred to easily accessed, on-site materials: colleagues and

textbooks. Participants referred to their colleagues for understanding and clarification of issues surrounding communication and as a source for materials. However, they said they were aware that this is an imperfect process. In a sociocultural context, understanding is seen as an individual coming to share in the group meaning (Lerman, 1996). By discussing communication in mathematics with their colleagues, teachers are living this process of building group meaning. Textbooks, while being popular, are regarded critically and cautiously. Additionally, teachers are aware of the inconsistencies among various texts.

Participants referred to the curriculum documents rarely and in the case where curriculum documents were mentioned it was primarily to reference the achievement chart (*The Ontario Curriculum: Mathematics (Revised),* 2005, pp. 20-21). The only other Ministry of Education publication mentioned was the Exemplar documents, but this was only either to ask if they existed or with imperfect recollections.

In my view, the Exemplars themselves are really not helpful and could be improved; they give definitions and a few examples but overall the documents are weak. Unfortunately, teachers seem to be unaware or imperfectly aware of the Ministry of Education resources available to them such as *Mathematical Processes* (2006), *Developing mathematical literacy* (2005), *Continuum and connections package (2006).* Implementing a new curriculum in 2000-2004, and then a revised curriculum in 2005-2008, has in my view left teachers time to do little else than survive the changes. Now may be the perfect time to introduce teachers to other resources since curriculum appears to have stabilized and teachers have an opportunity to fine-tune their understandings and practices. While the Ministry of Education documents may not give teachers the concrete

guidance they are looking for, they would at the very least provide a starting point for discussion with colleagues or a basis for in-service.

For the teachers struggling with a new direction, ideally support should be provided to allow them to explore, clarify and solidify their questions and concerns. Unfortunately the teachers interviewed did not feel supported. In fact the most positive description of the support felt in the research was more like ambivalence. The teachers in the research were also very clear that they would like more support and information. Again, if understanding can be seen as coming to share in the group meaning (Lerman, 1996), these teachers are expressing that they are not really sure what the group meaning for communication and mathematics is.

This feeling of a lack of organized support is more than unfortunate for a curriculum initiative that promotes students' communication. It may also be hindering teachers in their embracing of communication. In-service is a key element in implementing curriculum changes (Ross et al., 2002). Powell and Anderson (2007) add to this idea by saying that it is important that this in-service occurs in a supportive environment. Potvin and Dionne (2007) describe the ideal conditions for such in-service is when it follows a coaching model, rather than a presentation of theory. Silver and Smith (1996) suggest that the ability of teachers to cope with curriculum change may depend on their ability to form supportive communities of practice. The teachers are well aware that learning from each other is the best way to learn. They are echoing the sociocultural perspective that learning takes place through interaction with others (Vygotsky, 1978). So the support for teachers has to mean more than ambivalence; it needs to be active, collaborative, directed support and to take various forms including mentorship and written resources.

The disconnect between the research literature surrounding in-service as a vehicle for implementing new curriculum and teacher experience among the research participants is disturbing. The participants were asked, "Please indicate any in-service, specifically in communication and mathematics that you have participated in." Half of the teachers responded with a blunt "none." The literature is clear on the crucial positive role in-service can play in curriculum implementation. In-service is considered to be the most powerful method for implementing curriculum change (Ross et al., 2002), and trying to implement new curriculum without supporting professional development will have little effect (Potvin & Dionne, 2007; Price, Ball & Luks, 1995; Ross et al., 1998). Some of the teachers recalled the in-service surrounding the original roll out of the new curriculum, but their recollections were either negative or related to the details of the achievement chart (*The Ontario Curriculum: Mathematics (Revised),* 2005, pp. 20-21) categories. Unfortunately these sessions were often led by consultants who had left the classroom and were not actually experiencing the transition the teachers were; the consultants had never actually tried to put the Ministry of Educations' directives into practice and did not understand many of the complexities. If the purpose of in-service that effectively supports curriculum change is to help teachers clarify the new vision (Sowell & Zambo, 1997), then the board can claim mixed results. Teachers recall trying to understand the new vision; however they perceive the "new" curriculum as a repackaging of the old one.

The positive mentions of professional development provided by the board referred to more recent sessions intended to support EQAO such as spring of 2009 workshop by Marion Small on questioning and a series of small group sessions focusing on open response questions for the Grade 9 EQAO assessment for the 2009-2010 school year. This 2009 workshop does fall into line with the research that cites an increased emphasis

on in-service surrounding literacy and numeracy, in other words targeting EQAO (Giles & Hargreaves, 2006; Hardy, 2009; Volante, 2007).

EQAO was a recurrent theme in the participant interviews as an influence on teachers' understandings about communication. There were no questions directly referencing EQAO in the protocol but both oblique and direct references were made both to the OSSLT and to the Grade 9 Mathematics Test by over half of the teachers interviewed. It would seem that the teachers in this research have internalized standardized testing as a facet of their teaching approach and are active participants in the board's focus on improving the scores on this these tests.

The influence of Board P's emphasis on the OSSLT was apparent in some of the terms the teachers used when discussing communication; the terms "literacy", "writing across the curriculum" and "non-fiction writing" are all phrases that refer to board initiatives aimed at increasing student pass rates on the OSSLT. The fact that these terms are part of the teachers' lexicon reflect how teachers are well aware of the pressures of needing to perform well on these standardized tests (Nezevdal, 2003). This pressure has led to teachers changing their teaching and assessment practice to incorporate skills necessary for EQAO (Earl, 1999).

Teachers do reflect Volante's (2004) concern with teaching to the test but in the same breath reflect that successful EQAO tests are evidence of successful learning (Shaker, 2004). Volante (2007) indicates that teachers have not embraced recent reforms in assessment including standardized testing. I found no negative reaction to EQAO testing and in fact some participants cited EQAO as one of the sources for the evolution of their approach to communication; however since we were not discussing EQAO directly I may not have experienced the full range of teachers' opinions on the subject.

6.3 Concluding Remarks

The purpose of this research was to examine how teachers understand communication and mathematics and to investigate this issue teachers were asked about their teaching assessing practices. My research questions for this research were: how do secondary mathematics teachers understand communication in mathematics? By this I mean – what do teachers consider to be communication? And how, has teachers' understanding of communication developed? There were three key findings: teachers have a rich understanding of communication but only emphasize two modes when teaching and assessing communication, teachers are seeking knowledge and support with respect to communication and mathematics, and EQAO has had an influence on teachers' practices surrounding communication.

Firstly teachers have a much richer understanding of communication than I expected; instead of only focusing on mathematical conventions teachers considered oral communication, writing in prose about mathematical ideas, intertwining modes of communication, reading and interpreting information and the language of mathematics when they discussed communication. Unfortunately, however broadly they discuss communication, teachers are really only emphasizing two modes of communication, mathematical conventions and writing in prose about mathematical ideas, when it comes to teaching and assessment they are leaving out combining multiple representations and mathematical talk in the modes of presentations and classroom discussion.

Teachers are aware that communication is key to teaching and learning about mathematics; the fact that they restrict themselves primarily to these two modes, mathematical conventions and writing in prose about mathematical ideas, for teaching and assessing may come down to what they are comfortable with rather than what they

perceive to represent the full scope of communication. This indicates that although teachers' understandings on communication have evolved since the introduction of the new curriculum, this evolution is by no means complete. To assist the evolution teachers need support to expand their practice surrounding communication in the form of in-service.

A second conclusion is that teachers are seeking knowledge and support with respect to communication and mathematics. The teachers that participated in the study are discussing communication with their colleagues and actively wrestling with what communication is and how to fit it in to their current practices. Teachers are not just finding the easiest way to assess and teach communication and they are not content with their understanding or execution of these new assessment methods. It was not surprising that teachers are using textbooks and discussions with colleagues as their primary sources of information about communication. Teachers have not been trained as researchers to review literature and if they are to expand their resource base they will require guidance and modelling.

As a result, 2010/2011 would be an ideal time for the board to introduce in-service on communication and mathematics. Perhaps by using a combination of peer-to-peer coaching and larger group sessions, teachers could be exposed to existing research and given support and examples in order to encourage them to experiment with different modes of communication. However, this is unlikely to occur because, for this type of in-service to be supported at the board level, communication would have to become a board priority. For this to happen there would have to be a direct, tangible, immediate link to improved student performance, which in today's educational climate means student performance as measured by EQAO.

My third conclusion is only tangentially connected to communication. Although the teachers in this research were not asked about EQAO at all, there were multiple references to EQAO. It is clear that teachers have internalized the assumption that EQAO is the measure of student performance. Teachers cited EQAO as a reason for emphasizing communication, especially in its written (in prose) mode, and as an influence on their own understandings and practices.

I would have expected that when teachers were asked directly about their attitudes about EQAO they would have been fairly critical and somewhat negative about standardized testing. The way teachers have been absorbed into the culture of externalized standardized testing is surprising and not a little disturbing. The fact that they are no longer taking a critical view of EQAO, questioning its value as a standard, but instead have incorporated its language and ideals into their teaching lexicon speaks volumes about how commonplace standardized testing has become.

All of this indicates that the new curriculum, nine years on, is still a work in progress. Teachers, as professional educators are still seeking, thinking and discussing the changes that have occurred. Hopefully, teacher and student learning are being improved by the process and this emphasis on communication will withstand the next "new curriculum" when it arrives.

6.4 Limitations of the Research

Looking back on the design of this research, I can see some areas where adjustments might have strengthened the credibility of the conclusions reached. The first issue has to do with research experience. As a first time interviewer I found my proficiency increased as I conducted more interviews. I became more adept at inserting

clarifying questions and recognizing when they were needed. Also I became aware, as the interviews progressed, of the need to negotiate meanings of some of the terminology that were used.

One example of the need to negotiate meanings can be seen in the term "form", this was both used to indicate different modes of communication as well as being a reference to mathematical form. In the end I had to differentiate between modes of communication and mathematical conventions. I had also assumed that teachers would understand what was meant by "communication question". I learned over the course of the interviews that those teachers who did not use "communication questions" still used questions designed to elicit communication from their students and I had to adjust my questioning techniques accordingly.

My interactions with the teachers did not end once the semi-structured interview was over. Often a teacher participant and I discussed issues that had come up in the course of the interview. One teacher was very terse in her responses to the interview questions but was much more open once the digital recorder was turned off. Had I foreseen this interaction, I could have incorporated summary notes of this unstructured discussion time into my ethics request and then been able to include teachers' candid remarks in my data. Such inclusions would have enriched the data I collected.

6.5 Directions for Further Research

There are several possible directions for future study on the topic of communication and mathematics. One such extension to this research could be a broadening of the investigation to actually explore communication in practice. Teachers could be observed in the classroom and assessment instruments could be examined to

investigate if teachers' actions match their anecdotal descriptions of their teaching and assessment practices. Such an investigation could also be used to indicate priorities when designing in-service to support teachers' quest to improve their knowledge and practice surrounding communication.

Information about teachers' attitudes toward EQAO also became apparent during this investigation. Another future investigation could focus on the impact EQAO has had on classroom practice. It might also be worth investigating how EQAO has affected the culture of teaching and how teachers have absorbed current attitudes toward standardized testing.

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Appendix A: Interview Protocol

A) Collection of consent and explanation of procedure and mention of confidentiality

B) Description of format for the sessions

C) Interview questions:

Demographic questions:

1) Please indicate: your teaching experience; the subjects you teach this year; the subjects you have taught in the past year; what grade levels do you teach?
2) Please indicate: your highest level of education, which grades you are qualified to teach; additional teaching qualifications, graduate courses, professional courses, in-service training in teaching mathematics and specifically in teaching communication in mathematics that you have participated in.

Factual Questions:

1) Do you assess communication?
2) What forms of communication do you assess?
3) How do you assess communication?
4) How often do you assess communication?
5) Do you teach communication?
6) How do you teach communication?
7) What is your perception of whether or not your colleagues are teaching communication?

Descriptive questions

1) Could you share some examples of communication questions that you use in your classroom?

2) How do you create a good communication question? What sources do you use?

3) What forms of communication do you teach; which ones do you emphasize and why? Could you give me an example?
4) How do you model good communication for your students? Could you give me an example?
5) Do your expectations surrounding communication change from grade to grade or from level to level? How?
6) How often do you discuss communication with your colleagues?

Reflective questions

1) How did you develop your understanding of communication in mathematics?
2) Has your understanding of communication and mathematics changed or evolved over the last ___ years? (the number would depend on the amount of time the participant had been teaching.) Why?
3) How important do you think communication in mathematics is? Why?
4) Would you teach communication if it was not dictated?
5) Do you consider the inclusion of Communication to be an improvement over previous curricula?
6) How do you consider communication in mathematics compares to communication in other subjects (e.g., English)?

Likert Scale questions

On a scale of 1 (not at all) to 5 (extremely prepared)

7) How prepared do you feel to teach and assess communication?
8) How do you feel supported to teach and assess communication?
9) What is your level of knowledge with respect to teaching and assessing communication?

10) What is your level of experience in teaching and assessing communication?

11) What is your level of confidence in teaching and assessing communication?

D) Conclusion

Is there anything that we have not discussed that you would like to add?

Thank you for your time

Discuss possible follow up using email

Appendix B: Information and Consent Letter

Communication and Mathematics: A Case for Ontario Secondary Teaching

LETTER OF INFORMATION

Introduction

My name is Lynda Hemming and I am a Masters of Education student at the Faculty of Education at The University of Western Ontario. I am currently conducting research into Communication and Mathematics and would like to invite you to participate in this study.

Purpose of the study

The aims of this study are to explore secondary mathematics teachers' understandings and practices surrounding communication and mathematics.

If you agree to participate

If you agree to participate in this study you will be asked to participate in a face-to-face interview at a location of your choice. Topics covered in the interview include: demographic information, concept of communication in mathematics, practices surrounding communication in mathematics, development of understandings of communication in mathematics and resources used to develop these understandings. These interviews will be audio recorded. A Summary of your interview will provided upon request. You might also be asked to participate in e-mail discussions for follow-up questions and clarifications that come out of the interviews. The face-to-face interview will take about an hour and the e-mail discussions might take not more than another hour in total of your time.

Confidentiality

The information collected will be used for research purposes only, and neither your name nor information which could identify you will be used in any publication or presentation of the study results. All information collected for the study will be kept confidential. Any data collected from your interview and/or email discussions will be identified by a code. The list of codes and names will be kept separate from the data in a locked file. The data will be stored on a password protected computer and after the study will

be transferred to a locked file. The data records will be destroyed five years after the completion of the study or after publication.

Risks & Benefits
There are no known risks to participating in this study. Research participants will benefit from professional development in the form of professional discussions with a colleague on topics such as teaching practice, assessment, developing understandings based on curriculum documents and other resources with respect to communication and mathematics. In my experience participating in research and professional development strengthens teachers' professional identity.

Voluntary Participation
Participation in this study is voluntary. You may refuse to participate, refuse to answer any questions or withdraw from the study at any time with no effect on your employment status.

Questions
If you have any questions about the conduct of this study or your rights as a research participant you may contact the Manager, Office of Research Ethics, The University of Western Ontario at 519-661-3036 or ethics@uwo.ca. If you have any questions about this study, please contact Lynda Hemming (519-571-0969; bwnelson@kw.igs.net or my supervisor, Immaculate Namukasa (519-661-2111x82271; inamukas@uwo.ca)

This letter is yours to keep for future reference.

Lynda Hemming

Appendix C: Consent Form

Communication and Mathematics: A case for Ontario Secondary Teaching

Lynda Hemming Masters of Education University of Western Ontario

CONSENT FORM

I have read the Letter of Information, have had the nature of the study explained to me and I agree to participate. All questions have been answered to my satisfaction.

Name (please print):

Signature: Date:

Name of Person Obtaining Informed Consent:

Signature of Person Obtaining Informed Consent:

Date:

Appendix D: Ethics Approval Form

THE UNIVERSITY OF WESTERN ONTARIO
FACULTY OF EDUCATION

USE OF HUMAN SUBJECTS - ETHICS APPROVAL NOTICE

Review Number: 0905-4
Applicant: Lynda Hemming
Supervisor: Immaculate Namukasa
Title: *Curriculum and Mathematics: A case for Ontario secondary teaching.*
Expiry Date: September 30, 2009
Type: M. Ed. Thesis
Ethics Approval Date: June 11, 2009
Revision #:
Documents Reviewed & Approved: UWO Protocol, Letter of Information & Consent

This is to notify you that the Faculty of Education Sub-Research Ethics Board (REB), which operates under the authority of The University of Western Ontario Research Ethics Board for Non-Medical Research Involving Human Subjects, according to the Tri-Council Policy Statement and the applicable laws and regulations of Ontario has granted approval to the above named research study on the date noted above. The approval shall remain valid until the expiry date noted above assuming timely and acceptable responses to the REB's periodic requests for surveillance and monitoring information.

No deviations from, or changes to, the research project as described in this protocol may be initiated without prior written approval, except for minor administrative aspects. Investigators must promptly report to the Chair of the Faculty Sub-REB any adverse or unexpected experiences or events that are both serious and unexpected, and any new information which may adversely affect the safety of the subjects or the conduct of the study. In the event that any changes require a change in the information and consent documentation, newly revised documents must be submitted to the Sub-REB for approval.

Dr. Jason Brown (Chair)

2008-2009 Faculty of Education Sub-Research Ethics Board

Dr. Jason Brown	**Faculty (Chair)**
Dr. Elizabeth Nowicki	**Faculty**
Dr. Jacqueline Specht	**Faculty**
Dr. John Barnett	**Faculty**
Dr. J. Marshall Mangan	**Faculty**
Dr. Immaculate Namukasa	**Faculty**
Dr. Robert Macmillan	**Assoc Dean, Graduate Programs & Research** (*ex officio*)
Dr. Jerry Paquette	**UWO Non-Medical Research Ethics Board** (*ex officio*)

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