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High Resolution Spectroscopy of the Hyades Giants

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Graduate Program in Astronomy

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The four K0 III Hyades red clump giants, $\gamma$, $\delta$, $\epsilon$, and $\theta^1$ Tauri, are ideal candidates for cool star research. They are easily observable at a distance of only 47 pc and one star, $\epsilon$ Tauri, is a suspected Maunder minimum star. These stars also provide a useful opportunity to investigate the nature of the photospheric velocity field, and assess the effectiveness of a range of spectroscopic tools including the merits of the microturbulence-macroturbulence approach to the study of cool star spectra.

High resolution ($\lambda/\Delta\lambda \sim 100,000$) exposures, taken from January 2001 to October 2008 in the 6250 Å region, are used to study photospheric properties of the four Hyades giants, focusing on the three signatures of stellar granulation (Gray 2009): line broadening, line asymmetry, and the variation of line core velocities with depth. Values of projected rotational velocity and macroturbulence are obtained. Microturbulence shows a clear tendency to increase with mean photospheric height of formation.

Also included are estimates of temperature differences between granular and intergranular regions using flux deficit, and temperature differences between the program stars using line-depth ratios.

Measures of convective overshoot velocities, such as the velocity scale are used together with the rotational velocity to estimate the Rossby number for each program star. The results for $\delta$ Tauri are inconclusive, however, $\epsilon$ Tauri shows a larger Ro than the more active $\gamma$ and $\theta^1$ Tauri, a value which is similar to that of the Sun, and consistent with its previously reported low level of magnetic activity.

$\epsilon$ Tauri appears to be more massive the other Hyades giants and may have evolved beyond the coronal boundary into an inactive dynamo stage. The contrast of this scenario with conditions in the Sun is discussed.

Keywords: $\gamma$ Tauri; $\delta$ Tauri; $\epsilon$ Tauri; $\theta^1$ Tauri; Convection; Granulation; Hyades giants; Hyades open cluster; Photospheres; Photospheric velocity fields; Red giants; Clump giants; Rossby number; Radial velocities; Stellar rotation; Stellar atmospheres; Stellar
cycles; Stellar spectroscopy; Line broadening; Macroturbulence; Microturbulence; Line-depth ratios; Stellar temperatures
In loving memory of Pedro Martínez Pérez
1928-2013
ACKNOWLEDGEMENTS

Thanks to all:

This has been the greatest challenge of my life. I am thankful for the opportunities for sharing, teaching and learning, and the many lessons and insights I have received. To the many people who have helped me, adequate thanks would require another volume.

I do hope that my journey, and the results that I am reporting, may prove to be of interest and benefit to others.

Thanks for the guidance, and the unfailing encouragement,

Tony
St Marys, Ontario
March 2018
’Alá’ 174 BE
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ACRONYMS

3SP  Third Signature Plot
ADU  Analog to Digital Unit
CCD  Charge Coupled Device
EST  Eastern Standard Time
EW   Equivalent Width
GD   ‘Goodness’ measure
IUE  International Ultraviolet Explorer
LDR  Line-depth ratio
LTE  Local thermodynamic equilibrium
NIST National Institute of Standards and Technology
Ro   Rossby number
S/N  Signal to noise ratio
VALD Vienna Atomic Line Database
INTRODUCTION

1.1 OUTLINE

1.1.1 Objectives

Red giants are cool, bright, post-main sequence stars. They share many characteristics with cool main-sequence stars with similar temperatures of 5000 to 6000 K. In particular, late G and early K-type giants share several characteristics with the Sun. These stars have convective envelopes, granulation, show evidence of stellar cycles, starspots, solar-like pulsations, and most have similar slow rotation rates of 2-5 km s$^{-1}$. After a period of shell hydrogen burning, these stars settle into a period of core helium burning. The clustering of these stars with similar luminosity and temperature, as shown on the Hertzsprung-Russell (HR) diagram, leads them to be referred to as ‘clump’ giants. Their brightness and relative stability make them ideal targets for high resolution spectroscopy, and they are increasingly being used as dynamical, chemical and distance markers in Galactic and extragalactic studies (Mishenina et al. 2006). A great deal of astronomical knowledge is related to our understanding of the Sun. Red giants, and in particular clump giants, share some solar characteristics and form an important stepping stone in the development of our understanding of other stellar objects. Great insights have been obtained by comparing disk integrated studies of the Sun-as-a-star with stellar observations (Gray 1992b; Takeda 1995; Dumusque et al. 2015; Gray and Oostra 2018). Conversely, studies of stellar evolution and solar-type stellar behaviour, such as the study of stellar activity cycles, which was initiated by Olin Wilson in the 1960s, help in the interpretation of past, current and probable future solar activity (Schröder et al. 2013).
There remain many unanswered questions in the field of red giant and cool star research, many of which apply to the Sun itself. How do rotation and convection combine to drive the dynamo process, which is thought to produce stellar magnetic cycles? Why is the timing and intensity of these cycles irregular? What causes the extended magnetic dormant periods, such as the 17th century Maunder minimum, that coincided with the Little Ice Age? How do we determine when this is likely to reoccur in the Sun? In terms of high resolution spectroscopy, what do the spectral line profiles tell us about photospheric velocity fields, broadening processes and other physical stellar characteristics? How do we distinguish between the effects caused by stellar variability and those caused by orbits of planets or stellar companions? Can the parameters we observe help determine the state of stellar activity? How do these parameters relate to current theoretical models of stellar activity?

In this study, high resolution archival spectra of four K0 III giants from the well-studied Hyades open cluster, taken over a 10 year period, are analysed using a broad range of spectral tools. Photospheric properties are determined and the effectiveness of these tools in determining stellar properties and the presence of stellar activity is evaluated. The relationship between observed photospheric parameters and stellar physical properties is also investigated.

As members of the same cluster, The Hyades giants are expected to have similar ages, metallicities, and parallaxes. They are close to each other on the HR diagram, implying similar masses and a similar stage of evolution. It is of particular interest that $\varepsilon$ Tauri is less active than the others, and does not have significant cyclic activity in chromospheric emission (Baliunas, Hartmann et al. 1983). Other proxies for magnetic activity such as coronal UV and X-ray fluxes are also significantly lower in $\varepsilon$ Tauri (Collura et al. 1993). A supplementary objective of this study is to search for spectroscopic evidence for possible causes of $\varepsilon$ Tauri’s relative lack of activity.

The spectroscopic procedures used here follow closely those developed by David Gray as explained in his books ‘Lectures’ (Gray 1988) and ‘Pho-
tospheres’ (Gray 2005b) and in many other papers. This study of the Hyades giants follows a series on the photospheric properties of red giants (Gray 2013; Gray 2014b; Gray 2015; Gray 2016; Gray 2017).

1.1.2 Program stars

The K0 III giants studied here are γ Tauri (HD 27371), δ Tauri (HD 27697), ε Tauri (HD 28305), and θ1 Tauri (HD 28307). Other designations for the Hyades giants are listed in Appendix A. These stars are bright, of 4th magnitude, about 2.7 times more massive and with radii a little more than ten times larger than the Sun. Their brightness allows spectra with good signal-to-noise (S/N) ratios of ~300 to be obtained from 2 hr exposures using the Coudé spectrograph at the 1.2 m Elginfield telescope. These stars are at a distance of around 47 pc (about 150 light-years) and are part of the well-known ‘V’ asterism in Taurus, SE of the Pleiades.

1.1.3 Procedure

The analysis of spectral lines can give three basic kinds of information. First, the variation of the central wavelength provides the relative radial velocity between the observer and the object due to the Doppler effect; second, the line area or depth determines the line intensity, which is related to the abundance of the absorbing species; and third, the shape of the line gives information both on broadening mechanisms, such as turbulence and rotation, and asymmetries caused by the combined effects of brightness and radial velocity variations in the photosphere.

Chapter 2 discusses the spectra and reduction procedures. Chapters 3 to 5 discuss the broadening, asymmetry and velocity field analyses that characterise important aspects of stellar granulation. These have been called the three signatures of stellar granulation by Gray (2009). In Chapter 6 two approaches are used to determine temperature differences. In the first approach, the difference in temperature between the plasma rising in bright granules and the plasma sinking in the cooler, darker in-
tergranular lanes, is determined by obtaining the flux deficit. The second section of Chapter 6 examines the temperature differences between the program stars from line-depth ratios. Each spectroscopic tool is described in its corresponding chapter. Chapter 7 summarises the results, discusses their significance and proposes areas for future research.

1.2 PREVIOUS STUDIES

1.2.1 The Hyades Open Cluster

The Hyades is the closest well populated open cluster to the Sun. At a distance of ~47 pc, it is the only cluster to be near enough for a full three dimensional distribution of the members to be determined. Cluster membership has been established by combining proper motion and parallax data from the astrometric satellite Hipparcos, with spectroscopic radial velocities (van Leeuwen 2009). Figure 1.1 shows a well defined Main Sequence and the position of the Hyades giants from Perryman et al. (1998), who conducted a detailed study of the Hyades using 180 confirmed members within a 20 pc radius, obtained a distance modulus for the Hyades of $m - M = 3.33 \pm 0.01$ mag, and fitted an age of $625 \pm 50$ Myr based on the Geneva evolutionary codes. Turner et al. (1994) discuss several widely held misconceptions regarding the Hyades and the causes of the continuing controversies regarding its distance modulus.

There is a general consensus that the Hyades has an age in the 600-650 Myr range (Torres et al. 1997; Cayrel de Strobel 1990; de Bruijne et al. 2001), though recent work by T. D. Brandt and Huang (2015) suggests an age of $750 \pm 100$ Myr if allowance is made for rotational effects. Figure 1.2 from de Bruijne et al. (2001), shows a close fit for the Hyades giants using a 631 Myr Padova isochrone (Girardi et al. 2000).

Many studies have found the Hyades to have a metallicity slightly above solar. Taylor (1998) obtained a value of $[\text{Fe/H}] = 0.10 \pm 0.01$ with no observable difference between giants and dwarfs, and Carrera and Pancino (2011) obtained a value of $[\text{Fe/H}] = 0.11 \pm 0.01$ from a study of
1.2 Previous Studies

Figure 1.1: HR Diagram for Hyades Cluster
This figure, adapted from Perryman et al. (1998), shows 131 stars within 10 pc of the cluster centre selected as high probability Hyades members from the following catalogues: Hipparcos (van Leeuwen 2007), Tycho (ESA 1997) and ‘Base des Ames’ (Mermilliod 1995). Grey circles indicate that the data represents known multiple systems and bars indicate standard errors. The four Hyades giants are labelled. Reproduced with permission ©ESO.

The Hyades giants. Liu et al. (2016), using 16 solar-type stars, obtained an abundance of $0.16 \pm 0.01$. Dutra-Ferreira et al. (2016) favour a cluster abundance of $[\text{Fe/H}] = 0.18 \pm 0.01$ and $0.14 \pm 0.01$ for giants. In a comparative study of open clusters, Heiter et al. (2014) found $0.12 \pm 0.04$ from 16 measurements of giants and $0.13 \pm 0.06$ for 92 measurements of 61 dwarfs. Perryman et al. (1998) calculated a cluster value of $0.14 \pm 0.05$ from high resolution spectra. With these (as for many published values) the reported errors do not include systematic errors arising from models or measurement processes. Table 1.1 shows means, medians and standard deviations for stellar parameters taken from the Pastel catalogue (Soubiran...
et al. 2016). The values adopted for each program star are shown in Table 3.1.

Figure 1.2: Isochrone for Hyades Giants
This Figure adapted from (de Bruijne et al. 2001) shows a 631 Myr solar metallicity Padova isochrone (Girardi et al. 2000). The positions of the four Hyades giants are indicated, as are the directions of increasing mass (advanced evolutionary stage). Note that as spectroscopic binaries, the luminosity values of δ and θ¹ Tauri are likely to be less reliable than indicated. Reproduced with permission © ESO

1.2.2 Stellar Evolution

Figure 1.1 shows the Hyades stars on a colour-magnitude diagram. The main sequence where stars are still burning hydrogen in their cores is clearly seen. The main sequence turnoff where stars exhaust their core hydrogen and begin their post-main sequence evolution occurs at around B-V=0.1. This corresponds to early A-type stars of around 2.2 M☉ (Gray 2005b). One can then expect a mass of around 2.3 M☉ for the Hyades giants.

Figure 1.2 shows a solar metallicity Padova isochrone for 631 Myr for 2.32 M☉ stars. The Hyades giants are likely to have passed the helium
flash and are burning helium in the core. This is a relatively long and stable stage of post-main sequence evolution which also increases the probability of finding stars at this stage. Such stars are a significant feature of galaxies and are known as red giant clump stars.

After leaving the main-sequence, red giants pass various boundaries during which their properties change. These boundaries are shown in Figure 1.3 from Gray (1991).

As a typical fast rotating mid A-type main sequence star starts to cool and brighten, it grows in radius, slows slightly in angular velocity, due to conservation of angular momentum, and gradually develops a convective envelope. As discussed in Chapter 4, the transition of the granulation boundary is observed in spectral lines that change from the hot star inverted-C bisectors to the solar-type C shaped bisectors (Gray and Nagel 1989). (Bisectors are discussed in Chapter 4 and Figure 4.8 shows the change of shape across the granulation boundary.) A red giant is still a fast rotator at this stage but as it continues to cool, the developing convection favours the operation of a dynamo, and around spectral type G2 or G3, the star’s rotation rate rapidly slows to a projected equatorial velocity of around 5 km s\(^{-1}\) (Gray 1989). (Stellar dynamos are discussed in Section 1.2.7.) Eventually, a giant passes the coronal boundary, no longer produces hot coronal UV and X-ray emissions, develops a cool wind, and continues to become larger and more luminous along the Asymptotic Giant Branch (Linsky and Haisch 1979; Haisch et al. 1990). The position of the Hyades giants has been added and shown with a circle on Figure 1.3. Comparing Figure 1.3 with Figure 1.2, it is of interest to note that \(\varepsilon\) Tauri, as the brightest and coolest Hyades giant, is also the most evolved, and would be the first to cross the coronal boundary.

1.2.3 The Hyades Giants

The Vogt-Russell theorem states that if a star’s mass \(M_\ast\), age \(t\), and chemical composition [Fe/H] are known, in almost all cases the luminosity and temperature are uniquely determined (Vogt 1926; Russell et
Previous studies have shown that the boundaries on the HR Diagram are crucial for understanding stellar evolution. A circle marks the position of the Hyades giants on an HR diagram from Gray (2014a). The granulation boundary (red line) separates hot and cool stars by the nature of their photospheric velocity fields as shown by their line bisectors. To the right of the rotation boundary (blue line), stars are subject to magnetic braking and are mostly slow rotators. To the right of the coronal boundary (dashed line), stars show weaker magnetic activity, less coronal emission and develop a cool wind. Reproduced with permission © David F. Gray.

Luminosity L or absolute magnitude $M_V$, and effective temperature $T_{\text{eff}}$ or colour temperature (B-V), are used to determine the stage of stellar evolution. In practice, other conditions such as the existence of planetary or stellar companions and the rotation rate are also significant. The distance helps establish the luminosity L, and evolutionary codes help establish the mass. Accurate distances are de-
1.2 Previous Studies

Determined through astrometric satellites and the angular diameter $\theta_D$ can be obtained using lunar occultations and interferometry. The angular diameter in turn establishes the radius $R_*$ for a known distance, which together with the mass, determine the surface acceleration due to gravity $\log g$. The gravitational acceleration, together with the temperature and chemical composition, are used to model spectral lines. There are several ways to determine each of these parameters. The use of multiple approaches helps to ensure a consistent understanding of the star and validate the approaches used.

The recent update of the Pastel catalogue by Soubiran et al. (2016) includes data from 27 studies on the Hyades giants. The average published values are shown in Table 1.1. The $\log g$ values of three of these stars have essentially the same surface gravity, except for $\theta^1$ Tauri that has a higher average value of $2.92 \pm 0.20$ in c.g.s. units. $\epsilon$ Tauri has a higher metallicity than the other three stars.

<table>
<thead>
<tr>
<th>Star</th>
<th>$M_V$ (mag)</th>
<th>$T_{\text{eff}}$ (K)</th>
<th>$\log g$</th>
<th>[Fe/H]</th>
<th>Num</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$ Tau</td>
<td>0.22</td>
<td>4975</td>
<td>2.72</td>
<td>0.09</td>
<td>0.06</td>
</tr>
<tr>
<td>$\delta$ Tau</td>
<td>0.35</td>
<td>4947</td>
<td>2.72</td>
<td>0.09</td>
<td>0.07</td>
</tr>
<tr>
<td>$\epsilon$ Tau</td>
<td>0.09</td>
<td>4915</td>
<td>2.73</td>
<td>0.16</td>
<td>0.06</td>
</tr>
<tr>
<td>$\theta^1$ Tau</td>
<td>0.34</td>
<td>5012</td>
<td>2.92</td>
<td>0.20</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 1.1: Average Parameters for the Hyades Giants

Average values of effective temperature $T_{\text{eff}}$, surface gravity $\log g$, and metallicity [Fe/H] from the PASTEL catalogue (Soubiran et al. 2016). 'e' indicates the uncertainty in the preceding column. 'Num' indicates the number of studies included in the averages. Absolute visual magnitude $M_V$ are taken from the Hipparcos catalogue (van Leeuwen 2007).

Boyajian et al. (2009) conducted an extensive long-baseline interferometric study of the Hyades giants to obtain the limb darkened angular diameters which are shown in Table 1.2. Table 1.2 also shows calculations for the radii of the program stars using Hipparcos parallax distances (van Leeuwen 2007) and distances found by Madsen et al. (2002) using a mov-
The derived radii are consistent with those expected for K0 III giants (Gray 2005b). Similar radii are found for the program stars using the two approaches, with the exception of γ Tauri.

<table>
<thead>
<tr>
<th>Symbol (Units)</th>
<th>γ Tauri</th>
<th>δ Tauri</th>
<th>ε Tauri</th>
<th>θ¹ Tauri</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hipparcos Distance</td>
<td>$d \text{ (pc)}^{(a)}$</td>
<td>49.5 ± 0.7</td>
<td>47.7 ± 1.3</td>
<td>44.9 ± 0.6</td>
</tr>
<tr>
<td>MLD Distance</td>
<td>$d \text{ (pc)}^{(b)}$</td>
<td>45.3 ± 0.4</td>
<td>47.6 ± 0.5</td>
<td>46.2 ± 0.5</td>
</tr>
<tr>
<td>Angular Diameter</td>
<td>$\theta_{\text{LD (mas)}}^{(c)}$</td>
<td>2.52 ± 0.03</td>
<td>2.41 ± 0.04</td>
<td>2.73 ± 0.03</td>
</tr>
<tr>
<td>Hipparcos Radius</td>
<td>$R_*(R_\odot)$</td>
<td>13.3 ± 0.3</td>
<td>12.4 ± 0.4</td>
<td>13.2 ± 0.2</td>
</tr>
<tr>
<td>MLD Radius</td>
<td>$R(R_\odot)$</td>
<td>12.3 ± 0.2</td>
<td>12.3 ± 0.2</td>
<td>13.6 ± 0.2</td>
</tr>
<tr>
<td>Distance Modulus</td>
<td>$m-M \text{ (mag)}$</td>
<td>3.28±0.02</td>
<td>3.38±0.03</td>
<td>3.32±0.03</td>
</tr>
</tbody>
</table>

Table 1.2: Distance and Radii for the Hyades Giants

Astrometric distances from the Hipparcos Catalogue (van Leeuwen 2007)⁵⁵ are compared with those from Madsen et al. (2002) (MLD)⁶, who used the moving cluster method. The MLD values were used in subsequent calculations. The derived radii based on both distance values are shown, using the interferometric angular diameters of Boyajian et al. (2009)⁷. Distance moduli are based on the MLD distances.

The projected rotation velocities $v \sin i$ are determined spectroscopically by studying the Doppler line broadening effect. Rotational periods $P$, are obtained by rotational brightness modulation, which combined with stellar radii, provide the equatorial velocity $v_{eq}$. If all these measurements are available then the aspect angle $i$, between the line of sight and the stellar axis of rotation can be determined, but this is rarely achieved. For a randomly aligned population it was pointed out by van Dien (1948) and Chandrasekhar and Münch (1950) that,

$$ < v_{eq} >= \frac{4}{\pi} < v \sin i > .$$  (1.1)
A sample of four stars is very small but the average of the projected rotational velocities from Gray and Endal (1982) suggest possible equatorial velocities $<v_{\text{eq}}> \sim 3.4 \text{ km s}^{-1}$. Low values such as these, are expected for red giants on the cool side of the rotational boundary. The sun has an equatorial velocity of around 2 km s$^{-1}$.

### 1.2.4 Velocity Fields

Photospheric velocity fields vary with depth, in both radial and tangential directions over a wide range of length scales, and with time due to pulsations and magnetic activity. Microturbulence dispersion $\xi$, was adopted to describe a Doppler enhancement to thermal broadening on scales smaller than the photon mean free path. Microturbulence is small and difficult to measure and usually taken to be isotropic, depth independent, and described by a Gaussian distribution. Microturbulence increases the natural broadening of individual atoms and so increases the equivalent width of the line. Though in many studies $\xi$ is taken to be constant for a star, it has been known for several decades that it increases with height in the photosphere (Unsöld and Struve 1949; Gray 1981; Takeda 1992).

In length scales greater than the photon mean free path, motions have been adequately described by macroturbulence dispersion $\zeta$. Macroturbulence does not affect the absorption profile of an individual atom. The Doppler shifts of many atoms broaden the observed line while maintaining the equivalent width. Originally macroturbulence was taken to be isotropic and Gaussian. It has been found in practice that a radial-tangential macroturbulence dispersion $\zeta_{RT}$, with equal Gaussian dispersions in radial and tangential directions can, after combination with rotational broadening and integration over the observed stellar disk, describe line broadening effectively (Gray 1978).

In a recent high resolution study of the line of sight velocity dispersion from centre to limb in the Sun, Takeda and Ueno (2017) found that the use of $\zeta_{RT}$ caused calculations of the observed turbulence to be over-
estimated and that a Gaussian macroturbulence was more appropriate for solar-type stars, although they use a multi-parameter fitting method rather than the full Fourier analysis. Despite the limitations of the $\xi$ – $\zeta_{RT}$ (microturbulence-macroturbulence) approach to line broadening, it is still widely used. See for example, the recent study on red giants in open clusters by Casamiquela et al. (2017). Combining microturbulence and radial-tangential macroturbulence provides a approach which has been shown to give accurate values for projected rotational velocity of the Sun as a star (Gray 1977), while the large quantity of previous calculations allow useful comparisons to be made. The same approach has been used here in the investigation of line broadening in Chapter 3.

1.2.5 Rotation

Cool stars typically have projected rotational velocities of 5 km s$^{-1}$ or less. With macroturbulence dispersions around 3 km s$^{-1}$, the most effective way of distinguishing between these broadening mechanisms has been by taking a Fourier transform of the line profile. Fourier transforms allow subtle differences in the low frequency components of the line profile to be detected. Microturbulent dispersions are typically around 1 km s$^{-1}$, affect the equivalent width of the line, and can be measured in individual strong lines which begin to show saturation.

<table>
<thead>
<tr>
<th>$v \sin i$ (km s$^{-1}$)</th>
<th>$\gamma$ Tauri</th>
<th>$\delta$ Tauri</th>
<th>$\epsilon$ Tauri</th>
<th>$\theta$ Tauri</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gray and Endal 1982</td>
<td>3.4</td>
<td>1.9</td>
<td>3.4</td>
<td>2.4</td>
</tr>
<tr>
<td>Gray 1982c</td>
<td>2.4</td>
<td>2.5</td>
<td>2.5</td>
<td>3.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\zeta_{RT}$ (km s$^{-1}$)</th>
<th>$\gamma$ Tauri</th>
<th>$\delta$ Tauri</th>
<th>$\epsilon$ Tauri</th>
<th>$\theta$ Tauri</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gray and Endal 1982</td>
<td>4.6</td>
<td>5.9</td>
<td>4.8</td>
<td>4.8</td>
</tr>
<tr>
<td>Gray 1982c (Strong/Weak lines)$^1$</td>
<td>5.0/5.9</td>
<td>5.4/6.2</td>
<td>5.5/6.2</td>
<td>4.2/4.9</td>
</tr>
</tbody>
</table>

| $\xi$ (km s$^{-1}$) | Gray 1982c (Strong lines) | 1.6 | 1.2 | 1.3 | 1.8 |

Table 1.3: Previous Line Broadening Analyses of the Hyades Giants

$^1$The results shown in Table 1.3, obtained by Gray and Endal (1982), were updated by Gray (1982c) to include separate $\zeta_{RT}$ values for strong and weak lines.
The rotational velocities of all four Hyades giants published in the revised catalogue of Uesugi and Fukuda (1982) showed only upper bound values of 10 km s$^{-1}$ available at that time. Table 1.3 shows values for $v \sin i$, $\zeta_{RT}$ and $\xi$ obtained using the same microturbulence–macroturbulence Fourier transform approach as used here. The reported increase in $\zeta_{RT}$ with weaker lines formed deeper in the photosphere reported in Gray (1982c) was no longer detected with subsequent studies using the temperature profile for the K0 III star $\beta$ Gemini modelled by Ruland et al. (1980).

Table 1.4: Rotation velocities for the Hyades Giants

<table>
<thead>
<tr>
<th></th>
<th>Units</th>
<th>$\gamma$ Tauri</th>
<th>$\delta$ Tauri</th>
<th>$\epsilon$ Tauri</th>
<th>$\theta^1$ Tauri</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotation Period</td>
<td>d</td>
<td>~160$^{(e)}$</td>
<td>-</td>
<td>-</td>
<td>165±3$^{(f)}$</td>
</tr>
<tr>
<td>$v_{eq}(\text{Hipparcos})$</td>
<td>km s$^{-1}$</td>
<td>4.2</td>
<td>-</td>
<td>-</td>
<td>3.6</td>
</tr>
<tr>
<td>$v_{eq}(\text{MLD})$</td>
<td>km s$^{-1}$</td>
<td>3.9</td>
<td>-</td>
<td>-</td>
<td>3.5</td>
</tr>
<tr>
<td>Projected rotation$^{(d)}$</td>
<td>km s$^{-1}$</td>
<td>2.4</td>
<td>2.5</td>
<td>2.5</td>
<td>3.4</td>
</tr>
</tbody>
</table>

Preston (1984) reported the detection of rotational modulation of Ca II H & K flux for $\gamma$ Tauri with an approximate period of 160 d. A small variability in Ca II H & K emission was also found for $\theta^1$ Tauri by Choi et al. (1995), with a periodicity of approximately 140 days, similar to the periods they found in other clump giants and which they interpret as evidence of rotational modulation due to the existence of magnetically active longitudes. Table 1.4 shows values of equatorial velocities $v_{eq}$ derived from the rotational periods, the angular diameters from Boyajian et al. (2009) and distances from van Leeuwen (2007) and Madsen et al. (2002). The $v \sin i$ values obtained by (Gray and Endal 1982) from line broadening analysis are also shown. It is clear from this table that these
The orbital elements for binary systems are shown for δ and θ₁ Tauri. The orbital data for a planet around ε Tauri is also shown. P is the orbital period in days, K the amplitude of the radial orbital velocity in km s⁻¹, e the eccentricity, ω the argument of periapsis in degrees, T defines the epoch in Modified Julian Days (MDJ = JD-2400000.5 d), a₁ sin i is the apparent semi-major axis in either Gm or AU as marked, and f(m) is the mass function of the companion in M☉. References: Griffin and Stroe (2012)⁹ and Sato et al. (2007)⁹. For further explanation of orbital elements see, for example Urban and Seidelmann (2014).

approaches give consistent results, and the data suggest that θ₁ Tauri has an a rotational aspect angle of close to 90°, while for γ Tauri i ≃ 40°.

1.2.6 Stellar and Planetary Companions

Griffin and Gunn (1977) found δ Tauri and θ₁ Tauri to be spectroscopic binaries, which are stars with periodic radial velocity variations interpreted as being due to orbital motion around a stellar companion. Their measurements were updated and compared with other studies by Griffin and Stroe (2012), who give an engaging description of how our knowledge of these stellar systems gradually developed. Their orbital data for the two stars are collected in Table 1.5, together with the planetary orbital elements found for ε Tauri by Sato et al. (2007).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>δ Tauri⁹</th>
<th>ε Tauri⁹</th>
<th>θ₁ Tauri⁹</th>
</tr>
</thead>
<tbody>
<tr>
<td>P (d)</td>
<td>530.12±0.09</td>
<td>595±5</td>
<td>5921±5</td>
</tr>
<tr>
<td>K (km s⁻¹)</td>
<td>2.74±0.03</td>
<td>0.096±2</td>
<td>7.01±0.04</td>
</tr>
<tr>
<td>e</td>
<td>0.448±0.008</td>
<td>0.15±0.02</td>
<td>0.572±0.004</td>
</tr>
<tr>
<td>γ(km s⁻¹)</td>
<td>39.58±0.02</td>
<td>–</td>
<td>40.21±0.02</td>
</tr>
<tr>
<td>ω(°)</td>
<td>352±1</td>
<td>94±7</td>
<td>69.5±0.5</td>
</tr>
<tr>
<td>T (MJD)</td>
<td>51329±2</td>
<td>52880±10</td>
<td>50993±5</td>
</tr>
<tr>
<td>a₁ sin i</td>
<td>17.9±0.2 Gm</td>
<td>1.93±0.03 AU</td>
<td>468±3 Gm</td>
</tr>
<tr>
<td>f(m) (M☉)</td>
<td>8.1e-4 ± 3e-5</td>
<td>5.3e-8 ± 3e-9</td>
<td>0.117±0.002</td>
</tr>
</tbody>
</table>

Table 1.5: Orbital Elements
1.2 Previous Studies

Percy (1993) detected a ~ 0.2 mag dip in the visual magnitude of θ¹ Tauri in mid October 1992 lasting around 20 days. Griffin and Stroe (2012) point out that if that was a real detection caused by an eclipse from the companion star, it would be observable again in late March 2025. The companion to θ¹ Tauri has been observed several times during lunar occultations and with speckle interferometry and is believed to be a late F-type main-sequence star, offering a partial explanation as to why θ¹ Tauri has a slightly bluer colour than the other Hyades giants. The observations are reviewed by Torres et al. (1997), who resolved the orbit and estimated a mass of $2.9 \pm 0.9 \, M_\odot$ for θ¹ Tauri, with an improved mass estimate for the companion of $1.3 \pm 0.1 \, M_\odot$ and an orbital plane almost aligned with the line of sight. Such an alignment was also implied in the speckle interferometry study by Evans (1984) and the rotational data from the previous section, increasing the likelihood that the dip in magnitude reported by Percy (1993) may have been a transit.

The only reported observation of the companion to δ Tauri is an unrepeated speckle observation by Mason et al. (1993). δ Tauri is a single-lined spectroscopic binary, which could have a companion as small as an M dwarf (Griffin and Stroe 2012).

A planetary companion has been reported for ε Tauri with an orbital period of 595 days (Sato et al. 2007). The orbital elements for ε Tauri are also shown in Table 1.5. The planet would have a minimum mass of $7.6 \pm 0.2 \, M_J$, where $M_J$ is a Jupiter mass, and an orbital eccentricity of 0.15. This planet is of special interest as it is the first exoplanet found in an open cluster.

A single speckle detection of a companion to γ Tauri was made by Morgan et al. (1982), but it is unconfirmed. γ Tauri is the one Hyades giant for which no variation in orbital radial velocity is expected.

1.2.7 Stellar Dynamos and The Rossby Number

Magnetic fields in slowly rotating cool giants have usually been detected through the use of various proxies such as Ca II H & K emission.
(Korhonen 2014), however direct measurements of remarkably small Zeeman parameters have also been made with spectropolarimeters. Aurière, Wade et al. (2009) detected a longitudinal magnetic field of $0.46\pm0.04$ G in $\beta$ Geminorum, a slowly rotating K0 III giant. Aurière, Konstantinova-Antova et al. (2015) found magnetic fields in 29 of the 48 red giants studied. Among these were \( \varepsilon \) and \( \theta^1 \) Tauri for which they obtained mean latitudinal magnetic fields of $1.3\pm0.3$ G and $3.0\pm0.5$ G respectively, with considerable variation and change of polarity over the four year observation period. The longitudinal field and the radial velocity variations for \( \varepsilon \) Tauri showed a certain similarity, but a correlation could not be demonstrated.

Stellar activity is thought to be driven by a self-excited dynamo. The main concept of magnetic flux oscillating between the poloidal field and alternating toroidal fields during successive activity cycles was proposed by Parker (1955) and developed by Babcock (1961). The magnetic flux is carried by the motion of the stellar plasma due to its high conductivity. In the standard dynamo model, there is an \( \omega \) (angular velocity) effect, where differential rotation winds a poloidal magnetic field into an azimuthal field, which is antisymmetric about the equator. Then an \( \alpha \) (flux twisting) effect, through the combined effects of magnetic buoyancy and Coriolis induced vortices, transforms the azimuthal field back into a poloidal magnetic field with an inverted polarity. This is known as the \( \alpha \omega \) dynamo (Weiss 2011).

In order for the dynamo to work efficiently, vortices must be able to form in the plasma. Vortices can form if the rotational velocity is greater than the convective velocity, or equivalently, if the convective turnover time is greater than the rotational period. This condition is known as the dynamo criterion (Gray 2005b; Weiss 2011).

The dynamo criterion can be written in terms of the dynamo number \( N_D \), or the inverse Rossby number \( Ro^{-1} \). \( N_D \) is derived from dynamo models. When the condition \( N_D > 1 \) is met, the dynamo operates and the level of magnetic activity of the star is proportional to \( N_D \) up to a saturation value where the level of activity remains constant with further
increases of $N_D$ (Gray 1982b; Noyes et al. 1984; Wright et al. 2011). The relation between the dynamo number and the Rossby number can be written as,

$$N_D = R^\beta,$$

(1.2)

where $\beta$ is typically taken to be -2, but a value of $-2.7 \pm 0.1$ was found by Wright et al. (2011) in a study of X-ray luminosities of late-type stars. The inverse Rossby number is defined as a measure of the ratio of the rotational Coriolis forces to the inertial convective forces and can be written

$$R_o^{-1} = \frac{\tau_c}{P_{rot}} \simeq \frac{L_c/v_c}{P_{rot}},$$

(1.3)

where $\tau_c$ is the convective turnover time, $P_{rot}$ is the stellar rotational period, $L_c$ is a characteristic convective length scale, which has been taken to be the convective scale height or the depth of the convective zone, and $v_c$ is a characteristic convective velocity (Hartmann and Noyes 1987; Gray 2005b; Donati and Landstreet 2009). It is convenient to use the convective overshoot velocity at the base of the photosphere obtained from the Third Signature Plot (3SP) discussed in Chapter 5 as a characteristic convective velocity $v_c$.

The dynamo criterion can be thought of as $R_o^{-1} > 1$, which for the rotational velocity $v_{rot}$ can be written,

$$v_{rot} \gtrsim \frac{v_c}{L_c/R_*},$$

(1.4)

where $L_c/R_*$ is the fractional depth of the convective zone, as long as theoretical values for the fractional depth of the convective zone are available. For the Sun $L_c/R_* \simeq 0.3$ and if the characteristic convective velocity is taken to be the overshoot velocity from the Standard Curve in Figure 5.1, $v_c \simeq 700 \text{ m s}^{-1}$. The values lead to a dynamo criterion $v_{rot} \gtrsim 2.3 \text{ km s}^{-1}$, slightly higher than the actual equatorial velocity of $v_{eq} \simeq 2 \text{ km s}^{-1}$. 
The Rossby number ($Ro$) has been found to be a good predictor of dynamo activity in both dwarfs and giants (Gondoin 2005). Choi et al. (1995) obtained $Ro = 0.89$ for θ^1 Tauri, by combining observed rotational periods with theoretical convective turnover times from Gilliland (1985) adapted to giants by Hall (1991). A similar value of $Ro = 0.8$ was obtained for θ^1 Tauri by Aurière, Konstantinova-Antova et al. (2015).

1.2.8 Stellar Activity Cycles

The level of solar magnetic activity varies with an approximately 11 year cycle discovered by Schwabe (1844). The maxima and period of the cycle are irregular and the polarity of the magnetic field reverses every cycle (Hale et al. 1919). Increasing levels of solar activity are associated with more sunspots, stronger magnetic fields, a slight increase in temperature, and an increase in the intensity of flares, coronal mass ejections, and coronal emissions in UV and X-ray wavelengths (Hathaway 2010).

Chromospheric bright patches on the Sun called flocculi also increase in area and intensity during solar maxima, a fact which had been known since the 1930s (Abetti 1936). This led to the suggestion that disk integrated Ca II H&K emission lines, which dominate flocculi, could be used as a proxy for magnetic activity in stars. (See for example Sheeley (1967)). In the chromosphere temperatures pass though a minimum before entering a region where the temperature increases by several orders of magnitude in the transition to the corona. The chromosphere is not in equilibrium, is continuously changing, and is intricately connected to photospheric magnetic features. A schematic diagram of the Ca II K line is shown in Figure 1.4 from Duncan et al. (1991). The features that create the double inversion over magnetic active regions are produced from a great range of heights and physical conditions in the photosphere and chromosphere. These lines cannot be modelled using the simplifying assumptions discussed in Section 1.3. For an engaging review of the importance and challenges of studying stellar chromospheres see Judge (2010).
Figure 1.4: Schematic showing Ca II K line for the quiet and active Sun. Typical Ca II K line profiles for quiet and active solar regions are shown. The strength of Ca II H&K emission in dwarfs is measured using the S index, which uses a triangular 1 Å band pass filter as shown. A 1 Å filter is also used to measure the strength of the Ca II H line, which has similar characteristics (Duncan et al. 1991). © AAS. Reproduced with permission.

In the 1960s, Olin Wilson initiated a long-term study at Mount Wilson observatory of chromospheric Ca II H & K emission lines in order to investigate stellar cycles in solar-type stars. This project is referred to as the Mount Wilson HK Project (O. C. Wilson 1968). Wilson and his collaborators found that activity cycles were common in about 80% of solar-type stars. The study was later extended to red giants, and similar results were obtained (Baliunas, Hartmann et al. 1983). The initial study of the Hyades giants led to the interesting conclusion that, though they are of similar mass, age and metallicity, two stars, δ and ε Tauri, are much less active, while one of the stars, ε Tauri, does not appear to display cyclic activity, as shown in Figure 1.5 (Baliunas, Donahue et al. 1998). Possible scenarios for lower activity include either that ε Tauri is in a slightly different evolutionary stage, or that it is possibly going through an extended transitory phase of low activity known as a grand minimum. A grand minimum is also called a Maunder minimum after the period of
70 years or so in the late 17th and early 18th centuries when no sunspots were observed and the Earth experienced the Little Ice Age (Eddy 1976).

The S chromospheric activity index (O. C. Wilson 1968; Duncan et al. 1991) is a good proxy for magnetic activity. The S index measures the ratio of the mean fluxes in 1 Å bands centred on the Ca II H and K lines, to the flux from two 20 Å bands, R and V, taken from the nearby continuum. (See Figure 1.4.) N indicates the integrated flux, and the relation for the S-index is

$$S = \alpha (N_H + N_K)/(N_R + N_V),$$  \hspace{1cm} (1.5)

where \(\alpha\) is a calibration factor determined by O. C. Wilson (1968).

Figure 1.5 from Baliunas, Donahue et al. (1998) contains the Mount Wilson HK data for the Hyades giants and shows a gradual trend of increasing variability from \(\epsilon, \delta, \theta^1\), to \(\gamma\) Tauri, which shows the greatest range of activity. The \(S_{II}\) index plotted in Figure 1.5, is an S index adapted for giants by increasing the width of the band pass filters for the H and K line cores to 2 Å.

Collura et al. (1993) discuss chromospheric, transitional and coronal emission from the Hyades giants observed in UV and X-rays. There have been suggestions (e.g. Baliunas, Hartmann et al. 1983), that the UV excess detected by International Ultraviolet Explorer (IUE) could be from cool main-sequence companions, especially in the case of \(\delta\) and \(\theta^1\) Tauri that are known binaries. At that time, the possible detection of a speckle binary for \(\gamma\) Tauri by Morgan et al. (1982) strengthened the idea that \(\epsilon\) Tauri was not detected in UV or X-rays because it did not have a companion. Stern et al. (1981) obtained EINSTEIN X-ray detections of \(\delta, \theta^1\), and \(\gamma\) Tauri, and Collura et al. (1993) detected X-ray flux from \(\epsilon\) Tauri with ROSAT, but 40 times weaker than that of the other Hyades giants.
Figure 1.5: CaII H&K Emission for the Hyades Giants
Variation in the relative flux of the Ca II H&K emission lines is shown for the Hyades giants over a 12 year period (Baliunas, Donahue et al. 1998). The triangles at the bottom show little variation for ε Tauri. The crosses just above show a distinct though weak cycle for δ Tauri. The larger black dots in the centre of the graph show an active cyclic activity for γ Tauri. The stars at the top look like slightly smaller dots and also show an active cycle for θ¹ Tauri (which is marked θ Tauri on the graph.). Reproduced with permission.

1.3 RADIATIVE TRANSFER

Line profiles show the combined effects of natural broadening, thermal broadening, turbulent dispersion, rotational and collisional effects. Since it is not possible to deconvolve these components in an effective manner, the process of analysis requires modelling and fitting a line profile or its Fourier transform to the observed lines. The initial step is to model a natural line profile based on the characteristics of the atomic species and the star. The natural profile is then broadened using the approach described in Chapter 3.
1.3.1 Formation of the Continuum

In the convective zones of cool stars, increased gas opacity makes radiation transfer less efficient. The gas heats up so the radiative temperature gradient becomes larger than the adiabatic gradient, and since the conditions described by the Schwarzschild criterion are met, energy transfer becomes primarily convective. Rising through the convective zone, the reduction in gas density, pressure and temperature continues until at the base of the photosphere, opacity has decreased sufficiently to allow energy transport to become primarily radiative again. Hence the granular motions in the photosphere are due to convective overshoot. This section gives an overview of radiative transfer in stellar photospheres following the relevant sections of Unsöld (1976), Rybicki and Lightman (1986), Landstreet (2004), Gray (2005b) and Watson (2010).

It is useful to define a monochromatic specific intensity or brightness $I_\nu$, which gives the energy in the frequency interval $(\nu, \nu + d\nu)$, transported by radiation through an area $dA$ at a position $r$ perpendicular to the direction $\hat{n}$, into a solid angle $d\Omega$ centred on $\hat{n}$, in the time interval $(t, t + dt)$. This can be written

$$dE = (I_\nu(r, \hat{n}, \nu, t) \cdot d\bar{A}) \, d\Omega \, d\nu \, dt. \quad (1.6)$$

The radiative loss by absorption in transversing a distance $ds$ is proportional to $I_\nu$. The total, or integrated intensity is

$$I = \int_0^\infty I_\nu \, d\nu. \quad (1.7)$$

Specific intensity is conserved along a ray unless radiation interacts with the medium. This interaction is described using an extinction coefficient $\chi_\nu$ giving the radiation loss from the ray over a distance $ds$, proportional to $I_\nu$, and defined by

$$dI_\nu = -\chi_\nu I_\nu \, ds, \quad (1.8)$$
and an emissivity, or emission coefficient $j_\nu$ per unit length, giving the radiation added by the material into the same ray over a distance $ds$, defined by

$$dI_\nu = j_\nu ds. \tag{1.9}$$

Equation 1.8 is sometimes written using the mass absorption coefficient $\kappa_\nu$, where $\chi_\nu = \kappa_\nu \rho$, and $\rho$ is the gas density. The extinction coefficient $\chi_\nu$ has units of cm$^{-1}$ and is the inverse of the mean free path $l$, that is $\chi = l^{-1}$. The units of the emissivity $j_\nu$ are ergs s$^{-1}$ Hz$^{-1}$ cm$^{-3}$ sr$^{-1}$.

Combining both positive and negative changes to $I_\nu$ from Equations 1.8 and 1.9, leads to the differential form of the equation of radiative transfer

$$\frac{dI_\nu}{ds} = -\chi_\nu I_\nu + j_\nu. \tag{1.10}$$

If Equation 1.10 is divided by $\chi_\nu$, a more common form of the equation is obtained. The source function, $S_\nu = \frac{j_\nu}{\chi_\nu}$, can be thought of as the specific intensity due to the medium. It is also convenient to use the optical depth $\tau_\nu$, which is a dimensionless quantity representing the absorption over a given distance, given by

$$d\tau_\nu = \chi_\nu ds. \tag{1.11}$$

Dividing Equation 1.10 by $\chi_\nu$ and using the foregoing expressions for $S_\nu$ and $d\tau_\nu$, gives

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu. \tag{1.12}$$
If the specific intensity at $\tau = 0$ is $I_\nu(0)$, then integration of Equation 1.12 gives the intensity of the emerging radiation

$$I_\nu(\tau) = I_\nu(0)e^{-\tau\nu} + \int_0^{\tau} e^{-(\tau_\nu - \tau'_\nu)}S_\nu(\tau'_\nu)\,d\tau'_\nu.$$  \hfill (1.13)

If the properties of the medium are constant, then $S_\nu$ is a constant, and

$$I_\nu(\tau) = I_\nu(0)e^{-\tau\nu} + S_\nu(1 - e^{-\tau\nu}).$$  \hfill (1.14)

Equation 1.13 provides the solution to the transfer equation as long as $S_\nu(\tau_\nu)$ is known, however, this is not usually the case. If the atmosphere is in Local thermodynamic equilibrium (LTE) then $S_\nu(T) = B_\nu(T)$, which is the Planck function for black-body radiation,

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1},$$  \hfill (1.15)

where Planck’s constant $h = 6.63 \times 10^{-34}$ Js, and Boltzmann’s constant $k = 1.38 \times 10^{-23}$ J K$^{-1}$.

1.3.2 Line formation in Model Stellar Atmospheres

The energy from a flux $F$ per unit area, per unit time, through an area $dA$, for a time $dt$, is

$$dE = (F \cdot d\vec{A}) \, dt.$$  \hfill (1.16)

For stellar atmospheres, it is natural to use spherical coordinates, where $I_\nu = I_\nu(r, \theta, \phi, \hat{n}, t)$, and $(r, \theta, \phi)$ are the radial distance, the polar angle and the azimuthal angle respectively. If the star is axisymmetric there are no azimuthal variations so that $d\phi/dr = 0$, and if the stellar properties are constant in concentric shells, there is no variation with polar angle, $d\theta/dr = 0$, so the specific intensity $I_\nu = I_\nu(r, \hat{n}, t)$. 
Another simplifying assumption is that since stellar atmospheres typically occupy a tiny fraction of the stellar radius, they can be considered to be plane-parallel. When these coordinates are considered from the point of view of an observer from an arbitrary direction \( \hat{n} \), at a polar angle \( \theta \), the differential from the observer’s point of view, becomes \( ds = dr / \cos \theta = dr / \mu \), where it is customary to write \( \cos \theta = \mu \).

If a star radiates as a black body \( I_\nu = B_\nu \) for \( \mu \geq 0 \), and \( I_\nu = 0 \) for \( \mu < 0 \). The flux \( F_\nu \) is given by

\[
F_\nu = \int B_\nu \mu \, d\Omega, \quad (1.17)
\]

and the total flux \( F \) is

\[
F = \pi B \equiv \sigma T_{\text{eff}}^4, \quad (1.18)
\]

from Stefan-Boltzmann’s law, where the Stefan-Boltzmann constant \( \sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{K}^{-4} \), and \( T_{\text{eff}} \) is the effective temperature of the star.

In stars, it is also customary to measure optical depth towards the centre of the star. In terms of these coordinates, since distance is measured outwards, Equation 1.11 becomes

\[
d\tau_\nu = -\chi_\nu ds = -\chi_\nu dr / \mu, \quad (1.19)
\]

and substituting in Equation 1.12, the differential form of the transfer equation becomes

\[
\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu, \quad (1.20)
\]

which has a formal solution

\[
I_\nu(\tau_\nu, \mu) = e^{(\tau_\nu - \tau_{\nu_0})/\mu} I_\nu(\tau_{\nu_0}, \mu) + \int_{\tau_\nu}^{\tau_{\nu_0}} e^{-(\tau_\nu - \tau_{\nu'})/\mu} S_\nu(\tau_{\nu'}) d\tau_{\nu'}/\mu. \quad (1.21)
\]
Stellar atmospheres are very thin compared to stellar radii and so the plane parallel approximation is often used. Outside the stellar atmosphere $\tau_\nu \to 0$ as $z \to \infty$ and $\tau_\nu \to \infty$ as $z \to -\infty$. If there is no inward radiation from above the star, then $I_{\nu}(0, \mu) = 0$ for $\mu \leq 0$. The first term in Equation 1.21 goes to zero as $\tau_\nu \to \infty$. For $\mu > 0$, the intensity emerging from the surface is

$$I_{\nu}(\tau_\nu, \mu) = \int_0^{\infty} e^{-\tau'_\nu/\mu} S_\nu(\tau'_\nu) \frac{d\tau'_\nu}{\mu}. \quad (1.22)$$

The integrand of Equation 1.22 is a contribution function which gives the contribution at the surface of radiation from different optical depths. The exponential attenuation increases with $\tau_\nu/\mu$. As the limb is approached and $\mu$ approaches zero, regions of constant brightness will come from layers of lower optical depth nearer the surface, as long as $T(\tau_\nu)$ and $S_\nu(\tau_\nu)$ increase with optical depth. This explains the limb darkening effect observed in the Sun.

The source function can be modelled by using the first Milne-Eddington approximation, which assumes the source function varies linearly with optical depth $\tau_c$ measured away from any line,

$$S_\nu \simeq a_\nu + b_\nu \tau_c, \quad (1.23)$$

where $a_\nu$ and $b_\nu$ are frequency dependent constants. Substituting Equation 1.23 into 1.22 leads to the Eddington-Barbier approximations for the emergent specific intensity

$$I_{\nu}(0, \mu) = a_\nu + b_\nu \mu \simeq S_\nu(\tau_c = \mu), \quad (1.24)$$

and the emergent flux

$$F_{\nu}(0) = \pi S_\nu(\tau = 2/3). \quad (1.25)$$
In other words, the surface specific intensity is equal to the source function at an optical depth where $\tau_\nu = \mu$. Thus, if this approximation holds, the depth dependence of the source function can be obtained by scanning values of $I_\nu$ across the disk. This approach can be used for modelling the solar atmosphere using the resolved disk of the Sun.

If we also have LTE, then

$$I(0, \mu) = B_\nu(T(\tau = \mu)).$$  \hspace{1cm} (1.26)

To consider how lines are formed it is useful to consider an extinction coefficient $\chi$, which is the sum of a continuum component $\chi_c$ and a line extinction coefficient $l_\nu$, so that

$$\chi_\nu = \chi_c + l_\nu.$$  \hspace{1cm} (1.27)

If we also assume that the ratio of line to continuum absorption $\eta_\nu = l_\nu/\chi_c$ does not vary with optical depth (the second Milne-Eddington approximation), then

$$\chi_\nu = (1 + \eta_\nu)\chi_c,$$  \hspace{1cm} (1.28)

and so, the total optical depth $\tau_\nu$ becomes

$$\tau_\nu = (1 + \eta_\nu)\tau_c.$$  \hspace{1cm} (1.29)

Another way of considering Eddington’s approximation, Equation 1.23, is writing $S_\nu(\tau_c = 0) = S_\nu(0)$ and rearranging to give

$$S_\nu \simeq S_\nu(0) \{1 + \beta \tau_c\},$$  \hspace{1cm} (1.30)
where $\beta = \frac{b_\nu}{a_\nu}$ from the original linear relation Equation 1.23. Rearranging Equation 1.30, and substituting for the value of $\tau_c$ in Equation 1.23, gives

$$S_\nu \simeq S_0 (1 + \frac{\beta \tau_\nu}{1 + \eta_\nu}).$$

(1.31)

Substituting Equations 1.29 and 1.31 into Equation 1.22 leads to a very approximate but useful analytical expression for the total emergent specific intensity

$$I_\nu(0) \simeq S_0 (1 + \frac{\beta \mu}{1 + \eta_\nu}).$$

(1.32)

At frequencies outside a line $\eta_\nu = 0$ and the specific intensity assumes the continuum values given by Equation 1.24. At the line centre $\eta$ reaches a maximum value, indicating maximum line absorption and the specific intensity reaches a local minimum. As $\mu \to 0$ toward the limb, the specific intensity weakens giving the observed limb-darkening effect, and line absorption also becomes weaker.

In practice, the natural profile is broadened by the thermal velocity dispersion of the gas atoms to form the ‘thermal profile’. In this study the modelled lines are further broadened according to the procedures described in Chapter 3.
DATA ANALYSIS

The spectra used for this study were taken from a database of observations taken using the high resolution Coudé spectrograph at Western University’s Elginfield Observatory, near London, Ontario. A Charge Coupled Device (CCD) detector with 4096 pixels along the dispersion direction was used from late 1999. From early 2002, the wavelength calibration of the spectra was incorporated by alternating the detection of water vapour lines, using lamp exposures within the Coudé room, with stellar observations (Gray and Brown 2006). Data were obtained until 2010, which made available 9 seasons of wavelength-calibrated CCD data. By the time this project started, the Elginfield observatory was no longer in operation, no further observations were made, and the project was limited to the analysis of archival material.

2.1 OBSERVATIONS

A brief description of the observations follows, though they were not undertaken as part of this project.

2.1.1 Observatory

Elginfield Observatory has a 1.22 m Richey-Chretien telescope made by Boller and Chivens, and is located 25 km north of the University. Elginfield is located on the Mitchel Moraine, and the telescope is mounted on a pier anchored about 10 m into the moraine. The pier is vibrationally isolated from the surrounding building. The secondary mirror provides light through mirrors #3, #4 and #5 to a f/30.5 Coudé focus in the Coudé room below. Figure 2.1 shows the schematic of the telescope including the
first four mirrors. From mirror #5 the light passes through a filter wheel

and a Richardson slicer (Richardson 1966), which slices the stellar image so that all the light passes through the slit of the spectrograph into the Coudé room. In the Coudé room the light is reflected by the collimator mirror, the diffraction grating, and the camera mirror, and then enters the spectrograph detector where a final pick-off mirror directs the light onto the CCD detector (Brown 2006).

2.1.2 Coudé Spectrograph

The Coudé spectrograph at Elginfield uses a blazed grating with 316 lines mm⁻¹ (equivalent to a grating separation distance $d=3.27 \, \mu m$ and a blaze angle
\[ \beta = 63.43^\circ \text{ at the } f/30.5 \text{ focus. The signal passes through the Richardson image slicer into a slit with a width of } W' = 200 \mu\text{m} \text{ (Gray 1986). The collimator focal length is } f_{\text{coll}} = 6960 \text{ mm, in the 9th grating order, } n = 9, \text{ which, with an incidence angle of } \alpha = 60^\circ, \text{ gives a theoretical spectral resolution of} \]
\[ \triangle \lambda = -\cos \alpha \frac{W'}{f_{\text{coll}} n} = 0.051 \text{ Å.} \] (2.1)

In practice, small imperfections in the instruments reduce the resolution. The camera uses a 61 mm \(\times\) 3 mm CCD detector containing 4096 \(\times\) 200 pixels, for a pixel size of 15\(\mu\text{m} \times 15\mu\text{m}. The camera has a focal length \(f_{\text{cam}} = 2080 \text{ mm leading to a linear resolution of} \]
\[ \frac{d\lambda}{dx} = \frac{1}{f_{\text{cam}}} \frac{d\cos \beta}{n} = 0.76 \text{ Å mm}^{-1}, \] (2.2)

which with 15\(\mu\text{m} pixels, provides a typical dispersion of 0.0132 Å pixel\(^{-1}\). The theoretical resolving power implied by Equation 2.1 in the 6250 Å range is \(\lambda/\Delta \lambda \simeq 120,000\). However in practice a resolution of \(\sim 3 \text{ km s}^{-1}\) is obtained, equivalent to a resolving power of \(10^5\) (Gray 2005b).

### 2.2 Measurement Uncertainties

The discussion of the uncertainties in the observations in this section follows Gray and Brown (2006). The capacity to measure radial velocities was added in the early 2000s. Measurement errors are introduced by photon noise, wavelength errors, drift during exposure and the adjustment of the spectroscope optics. Radial velocities are reduced to the centre of gravity of the solar system, known as the barycentre, and care must be taken in calculating the barycentric correction for the midpoint of each exposure. A concerted effort was made to analyse and reduce these errors, which is reported in Appendix A of Brown (2006).

Photon noise is discussed in Section 2.3. The wavelength errors are due to the uncertainty in the laboratory wavelengths added to errors in-
roduced through the polynomial that assigns wavelength values to the **CCD** (Charged-Coupled Device) columns. The reference wavelengths are obtained by detecting telluric (water vapour) absorption lines within the spectrograph by taking two exposures of a flat-field lamp both before and after each stellar exposure. The wavelengths obtained for each lamp exposure are combined using a weighted mean to assign wavelengths to each pixel.

The wavelength shifts during the night caused by the change of refractive index with the density of the air in the Coudé room, are described by the Dale-Gladstone law. This is potentially a large effect with annual ranges of temperature and pressure leading to shifts of the order of \( \text{km s}^{-1} \). Gray and Brown (2006) explain that in order to keep velocity uncertainties within 10 m s\(^{-1}\), pressure variations must be kept within 0.2 mm Hg and temperature fluctuations to within 0.1°C. Humidity also has an effect, though it is much smaller, reaching a maximum potential uncertainty of ±2 m s\(^{-1}\).

Great care was taken to maintain stable conditions in the Coudé room, keeping the temperature variation to about 1° per day. Temperature and pressure measurements were also taken at the beginning and end of each stellar exposure when the telluric lines were measured, so that wavelength corrections could be applied.

Wavelength shifts are also caused by small mechanical flexes. As the liquid nitrogen coolant in the **CCD** dewar evaporates, the stresses on the instrument are reduced, causing it to flex slightly. Temperature changes in the **CCD**, diffraction grating, and other parts of the spectrograph can also cause wavelength shifts.

The motion of the observer around the solar barycentre can give radial velocity offsets of up to 90 m s\(^{-1}\)hr\(^{-1}\) for observations taken at Elginfield. The time at the start of an exposure Eastern Standard Time (EST) and the duration (usually 2 hrs) are recorded in the observer’s logbook. The mid-exposure time is determined to obtain barycentric corrections for radial velocity measurements. Uncertainties in the effective observation time caused by possible uneven photon capture rates can also cause
small wavelength shifts. Uncertainties in the barycentric correction are estimated to be $\sim 10 \text{ m s}^{-1}$.

The gradual deterioration of the mirror surfaces was minimised by aluminising the primary every two years and silvering mirrors 2 through 5 every 5 years. Aluminising the primary was carried out at the observatory, while the silvering required transporting the mirrors to a facility in New Jersey.

The instrument was aligned and focused at least once in the observing season. Occasionally exposures show a reduction of high frequency features, when exposures were made before the adjustments had been made.

The combined effect of the uncertainties is that radial velocity measurements taken with this instrument have an uncertainty of $\sim 25 \text{ m s}^{-1}$ (Gray and Brown 2006).

### 2.3 Photon Noise

The scatter in the detected signal originates from noise in the incoming starlight, noise from the background sky, and noise from the equipment (Gray 2005b). The photon count $N$, is measured using the Analog to Digital Unit (ADU) where in this case $1 \text{ ADU} \simeq 100$ photons (Gray 2005b). The Signal to noise ratio ($S/N$), is determined from Bose-Einstein statistics for incoming photons, but since $h\nu \gg kT$, Poisson statistics can be used.

For a Gaussian distribution, $S/N$ can be calculated approximately by

$$S/N = \frac{N}{\sqrt{N + r}}, \quad (2.3)$$

where $r$ is the the electronic noise of the system Gray (1986). The value for the CCD at Elginfield is $r \sim 2 \text{ ADU}$ (Brown 2006). If, as in these observations, the signal is much larger than the electronic noise, $r \ll N$ then $S/N = \sqrt{N}$.

For each exposure, the total count per CCD column gives a quality ‘Goodness’ measure (GD), which is obtained in the reduction process. The
count gives the number of ADU. The GD is then multiplied by the gain, which for the CCD used here is 6.67 e⁻ ADU⁻¹. That yields

\[ S/N = \sqrt{6.67 \times GD}. \]

(2.4)

The average GD values across the spectrum for each exposure are tabulated in Appendix B. The S/N range from 148 to 478. The mean S/N with their standard deviations are shown in Table 2.1, which summarises the number of exposures used for each star as well as the number that also have lamp exposures for wavelength calibration.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>γ Tauri</th>
<th>δ Tauri</th>
<th>ε Tauri</th>
<th>θ¹ Tauri</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean S/N</td>
<td>294</td>
<td>376</td>
<td>330</td>
<td>418</td>
</tr>
<tr>
<td>σ_{S/N}</td>
<td>74</td>
<td>61</td>
<td>74</td>
<td>75</td>
</tr>
<tr>
<td>Number of Exposures</td>
<td>14</td>
<td>5</td>
<td>14</td>
<td>4</td>
</tr>
<tr>
<td>Exposures with Lamps</td>
<td>14</td>
<td>1</td>
<td>14</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2.1: Data on Exposures

The signal to noise ratios (S/N) for all exposures are listed in Appendix B. Here the data are summarised showing the mean S/N over all the accepted spectra, the standard deviation σ_{S/N}, the number of CCD exposures and the number of exposures that have lamp exposures taken immediately before and after, that allowed wavelength calibration using telluric (water vapour) lines within the spectroscope.

2.4 Spectrum Reduction Procedure

The exposures were taken using a CCD detector with 4096 x 200 pixels. Photons are absorbed and create electron-hole pairs which are separated by an electric field. The electrons accumulated in each pixel are then read out. The CCD used here was cooled with liquid nitrogen to reduce the dark current.

The CCD exposures which have lamp exposures of telluric lines, allowing wavelength calibration, were chosen. δ Tauri only has one of five exposures with associated telluric lamp exposures, so all were used, and
all four CCD exposures were used for θ¹ Tauri, though none could be calibrated for absolute wavelength measurements.

The CCD exposures without absolute wavelengths were used in the broadening and bisector analysis.

2.4.1 Exposure Data files

The data files received for this research include a header identifying the star, the EST at the start of the exposure, the exposure duration, as well as the GD number. The data consist of a single row of 4096 values. The data had already been corrected for the dark current, and ‘flat fielded’ to allow for variations in the sensitivity in the CCD pixels. Spikes arising from cosmic ray hits were also removed. Further details for each exposure were obtained from the observatory logbook.

The reduction procedure was carried out using Fortran routines developed by my supervisor, David Gray. I did not have access to the code and worked with the executable files. However, the programs permit a great deal of flexibility in the manner they are used. Each program has an adjustable Key file, which adjusts several parameters, determining how the program will operate. Input files and physical data inputs are all in separate files. Outputs are written to ‘scratch’ files, which were then renamed and stored, or renamed as input files for other programs. A period of familiarisation and experimentation was necessary with each program to understand its mode of operation and to ensure that it was functioning as expected. Occasional requests for changes to the code were made to David Gray and some programs were updated, but usually adjustments to the Key files were sufficient to ensure the programs were operating as expected.

Since 16 Fortran routines were used, sometimes iteratively, I initially wrote Python programs to call the Fortran programs as subroutines in order to create a streamlined procedure for handling files, allowing automatic variation of parameters for individual spectral lines and stars. The procedure was fast, but it was difficult to evaluate and fine tune the data
handling process at critical stages, so this approach was later put aside. Many of the processes are labour intensive, but promote a deeper understanding of the analytical results. It was necessary to understand the data reduction process in great detail before certain processes could be streamlined.

In order to automate some of the multistep procedures such as modelling the macrobroadening transform (see Section 3.3.6), bash shell routines were written to call several Fortran programs as subroutines, set input parameters, change Key files, and rename input and output files automatically. Eventually, an efficient procedure involving several semi-automated steps was found to provide better and more reliable results than the direct ‘pipeline’ approach initially attempted. Due to the large number of different programs used, each with their own unique key, supplementary data, input and output files, a notebook was kept containing details of the usage, auxiliary files, and parameter information for each program. Appendix E contains a list of the programs with a brief description of each.

Before beginning the reduction process, data were obtained for each exposure from the observer’s logbook, which includes more detailed information than the header of the data file. The logbook contains mid-exposure time, duration, object, temperature, and pressure in the Coudé room before and after the exposure, temperature of the CCD, GD, and the filenames of the telluric lamp exposures that were taken before and after observing the star. Those values were added to a file together with SIMBAD (Wenger et al. 2000) data for the right ascension and declination of the object, for calculating the barocentric correction.

2.4.2 Setting the Continuum

All fluxes are measured as relative fluxes compared to the continuum, either as $F/F_c$ or as a line-depth $1 - F/F_c$. In cool stars there are many metal lines and care must be taken in setting the continuum level since
choosing too many points can remove some of the broader features of the spectrum and reduce the measured strength of some of the broader lines.

Large cosmic ray spikes are found in individual pixels. The automatic removal involves replacing pixel values with the mean of the adjacent values. The process sometimes leaves a slight bump or spike if the ray was strong enough to affect adjacent pixels. Noise and tiny spikes remaining from the removal of cosmic ray hits can also complicate the setting of the continuum and the calculation of line core wavelength, and some data files have an initial value which needed to be removed in order for the continuum to be adjusted correctly.

Continuum portions of the spectrum were identified and the exposures adjusted so the flux at the continuum level was set as closely as possible to unit intensity. Generally, for the better \( S/N \) exposures, the continuum was determined with uncertainties of only a few tenths of a percent. The number of pixels that were averaged to determine the continuum intensity level was usually set to 3, but could be increased if the signal was noisy. There were two possible procedures, either wavelength values for the continuum points were predetermined, or the exposure was divided into a number of sections, usually 8 to 12, and the highest value in each section was chosen to define the continuum level at unit intensity. A cubic spline was then computed to fit the observations and set the continuum. The procedure is very sensitive to the choice of parameters. Figure 2.2 shows a calibrated exposure for \( \gamma \) Tauri.

2.4.3 Radial Velocities

Radial velocities for an individual stellar spectral line require absolute wavelengths obtained from laboratory measurements. The highest precision of \( \sim 0.2 \) mÅ is only available for some atomic iron Fe I lines (Nave, Johansson et al. 1994). Wavelengths are available for most lines, though the accuracy is typically 5 times worse. At 6250 Å, a difference of 0.2 mÅ is equivalent to \( \sim 10 \) m s\(^{-1}\). With precise laboratory wavelengths
The figure shows a typical spectrum with the wavelengths detected in the observatory. The spectrum shown is for \( \gamma \) Tauri. The abscissa shows wavelength marked in Angstroms (Å), while the ordinate gives a normalised flux with respect to the continuum \( F/F_c \). The continuum level has been set and the wavelength scale established by calibrating and averaging four telluric lamp exposures. The triangles indicate the most commonly used lines. The ID indicates the atomic species and three digits give the number of tenths of an Å above 6200 Å. (e.g. Fe I 193 means Fe I 6219.3 Å)
\( \lambda_{\text{lab}} \), the average Doppler velocities \( \Delta v \) of the absorbing atoms can be obtained from the observed wavelength \( \lambda_{\text{obs}} \), where \( \Delta v = c \Delta \lambda / \lambda_{\text{lab}} \), and \( c = 2.998 \times 10^8 \text{ m s}^{-1} \) is the velocity of light, and \( \Delta \lambda = \lambda_{\text{obs}} - \lambda_{\text{lab}} \).

The original Nave wave numbers for Fe I lines have been increased by a factor of \( (1 + 6.7 \times 10^{-8}) \) (Nave and Sansonetti 2004) because of a recalibration of the reference Ar II lines by Whaling et al. (1995). The effect on those wavelengths is a reduction of 0.4 mA or 20 m s\(^{-1} \). To date, the Nave, Johansson et al. (1994) Fe I laboratory wavelengths are the most accurate available. Wavelengths for other elements have been published and assembled in the Vienna Atomic Line Database (VALD) (Ryabchikova et al. 2015) and National Institute of Standards and Technology (NIST) (Kramida et al. 2014) databases however uncertainties are typically of 50 to 100 m s\(^{-1} \).

Radial velocities were corrected to the barycentre of the solar system using the mid exposure time and the coordinates of the observed star based on a procedure developed by Stumpff (1979). Changing line-of-sight of the observers’ relative motion can require corrections of over 100 m s\(^{-1} \) caused by the Earth’s rotational and orbital motions. Exposures were typically two hours long and the time for the barycentric correction was taken as that of mid-exposure. Since the photon capture rate may vary during the exposure, the weighted, effective mid-exposure time can also change introducing errors into the barycentric correction. The weighted mid-exposure time was recorded in the logbook for each exposure. In practice the use of the weighted mid-exposure time changed the barycentric correction by, at the most, a few m s\(^{-1} \), and so it was not used.

2.4.4 Wavelength Calibration

An absolute scale for wavelength calibration was obtained using telluric lines from air within the Coudé room. Though the spectrograph was not initially designed to obtain calibrated spectra, it was found that telluric water vapour lines in the 6th order of diffraction could be used to calibrate the wavelengths of stellar lines observed in the 9th order (Gray and
Brown 2006). The precise wavelengths for the water lines were obtained from Chevillard et al. (1989). Two short lamp exposures were taken before and after each stellar observation. The detection of telluric water vapour absorption lines within the spectrograph was then used to calibrate the wavelength for each pixel across the spectrum. Occasionally, one of the lamps had a low S/N and was discarded. To ensure an accurate determination of the mid-exposure correction, the lamps before and after the observation are averaged with an equal weight. The wavelength is corrected for standard temperature and pressure by measuring the Coudé room temperature and pressure when the lamp exposures were taken.

The barycentric correction for the time of mid-exposure, together with the telluric line information, was used to calibrate the radial velocity of the star during the exposure. A quadratic polynomial was used to fit the variable dispersion in wavelength across the spectrum. The lack of good quality telluric lines at the red end of the spectrum caused the wavelength accuracy to worsen gradually towards the red end of the exposure from around 6260 Å. Tiny noise spikes in the core of the weaker telluric lines sometimes affected the accuracy of the procedure. With care, an average wavelength accuracy of around 30 m s⁻¹ can be obtained.

It is important to note that the logbook uses standard astronomical days that run from noon to noon, so that dates do not change overnight. That must be allowed for, since the program that calculates the barycentric correction uses conventional dates, which change at midnight. A typical reduced exposure is shown in Figure 2.2.

For those exposures where an absolute wavelength scale was not available a relative wavelength scale was obtained from a previously calibrated stellar exposure rather than from a telluric line exposure.

### 2.5 Selection of Spectral Lines

Ideal spectral lines are not blended and have an accurately determined laboratory wavelength. Iron lines are particularly useful, since Fe is abundant in stars, does not suffer from hyperfine splitting, has several
lines in our observed spectral range, and has been studied extensively in the past. Gray (2009) used the Fe I values measured by Nave, Johansson et al. (1994), which have wavelength uncertainties as small as 9 m s$^{-1}$ with a few up to 40 m s$^{-1}$. These are the most accurate laboratory wavelengths available and 14 of these Fe I lines are used here.

Gray and Pugh (2012) used the standard line depth-core velocity relation known as the $3\text{SP}$, to obtain precise wavelengths for non-iron lines. The $3\text{SP}$ is discussed in Chapter 5. Gray and Pugh (2012) studied 19 evolved stars and obtained wavelengths for 21 more non-iron metallic lines based on the Nave, Johansson et al. (1994) wavelengths for Fe I lines.

I made extensive attempts to extend the line list using other published data which are mostly included in the VALD database (Ryabchikova et al. 2015). The NIST database (Kramida et al. 2014) was also consulted. The resulting large dispersion in radial velocities indicated that most laboratory wavelengths are not yet at the accuracy necessary for the present work. A final working list of 35 lines was chosen. Appendix C contains a list of the lines used, together with the spectral analyses that each line was used for.

Lines can be used for radial velocity studies if the core is unblended. Lines for the line depth-velocity plot ($3\text{SP}$) need unblended cores and a clear reference continuum near the line. If a line is used for broadening analysis at least one wing must be unblended to allow for blend correction. Weaker lines are important as they form over a smaller vertical range in the photosphere and so are less susceptible to uncertainties in the temperature distribution. Strong lines are also needed as saturated lines have strong sidelobes in their Fourier transforms allowing microturbulence to be determined. Of the lines studied here, the strongest, relatively blend-free line, is Fe I 6252.6 Å, which was used for the bisector analysis.
LINE BROADENING ANALYSIS

3.1 INTRODUCTION

Stellar spectral lines are found to be much broader than predicted by natural broadening. When extra Doppler broadening, due to thermal motions, is added, it is still found to be insufficient to describe the Equivalent Width (EW) of observed lines. (EW is a measure of line strength given by the width in wavelength units of a rectangle with a height from the continuum to the zero flux level, which has an area equal to that of the measured spectral line.) Since the combination of natural and thermal broadening could not explain the large observed EWs, a small scale, isotropic, Gaussian microturbulence was added by Struve and Elvey (1934) to allow for the observed ‘supra-thermal’ motions in stellar photospheric spectral lines. Some broad-lined stars show line widths that imply velocities too large to be motions within the stellar photosphere, but are consistent with rapid stellar rotation. The calculated rotation rates were found to have a lower bound, suggesting another broadening component involving large-scale motions, which like stellar rotation, broadens the line while maintaining the EW. The two turbulent broadening mechanisms are described by two dispersion parameters $\xi$ and $\zeta$, which give the most probable velocity for microturbulence and macroturbulence, respectively (Gray 1988). Though it is not clear what relation those dispersions have to actual physical properties of stellar photospheres or whether these broadening mechanisms represent true hydrodynamic turbulence, they are widely used and allow consistent comparisons to be made between stars.

The effective separation of these broadening mechanisms has been made possible by the use of Fourier transforms. The combined microturbulence-
macroturbulence-rotation approach to line broadening introduced by Gray (1973) originally used an isotropic Gaussian microturbulence and, shortly after, a radial-tangential Gaussian macroturbulence $\zeta_{RT}$ (Gray 1976; Smith and Gray 1976). $\zeta_{RT}$ invokes a Gaussian dispersion only in the radial and the tangential directions, and the combined effect is obtained by disk integration.

There has been much debate on the merits of this approach, often centreing on the lack of a supporting physical model and the difficulty in applying global values to disk resolved observations in the Sun (Worrall and A. M. Wilson 1973; Takeda and Ueno 2017). The $\zeta_{RT}$ approach gives consistent values for a wide range of stars (Gray 1975; Gray 1977), has made possible the separation of rotational velocities from macroturbulent dispersions to within 1 km s$^{-1}$ (Gray 1978), and is still widely used (Steffen et al. 2013; Doyle et al. 2014; Gray 2016). For good discussions on issues relating to turbulence in stellar atmospheres, see Underhill (1960) and Gray (1978).

3D dynamical modelling of stellar atmospheres is an active area of research, and some success has been achieved in reproducing many features, such as line asymmetry, for individual stars (Nordlund and Stein 2000; Allende Prieto, Asplund et al. 2002). Attempts have been made by Steffen et al. (2013) to calculate classical microturbulence and macroturbulence from 3D models.

In this chapter the line broadening of the Hyades giants is modelled with $\xi$, $\zeta_{RT}$ and the projected equatorial rotational velocity $v \sin i$, using the Fourier transform convolution approach. Fourier transforms facilitate the study of slight differences in high resolution line profiles, since they represent their spectral content (Smith and Gray 1976). The slight variations in the slowly changing sides of line profiles are clearly seen in the low frequency part of their Fourier transforms.

A modelled broadened line profile is compared to the observed profile in the Fourier domain, and refined through an iterative process. Once the result is satisfactory, the modelled and observed profiles are compared in the wavelength domain. Representations in both frequency and
wavelength domains are complementary, and emphasise different aspects of the line profile. In the wavelength domain, rapidly changing parts of a line, such as the wings and the core, are easily compared, while the Fourier domain allows the gradually changing low frequency components from the sides of a line to be be examined (Gray 2013).

3.2 LINE BROADENING PROCESSES

CONVOLUTIONS The mathematical process for broadening a line profile is called a convolution. It is the process of taking each wavelength point of the original profile, multiplying it by the broadening profile and adding all the corresponding products.

A convolution is defined as a pointwise product of two functions as one is translated over the other, and can be written as follows: where $f(\sigma) \ast g(\sigma)$ represents convolution of function $f(\sigma)$ with function $g(\sigma)$,

$$f(\sigma) \ast g(\sigma) = \int_{-\infty}^{\infty} f(\sigma)g(\sigma - \sigma_1)d\sigma_1. \quad (3.1)$$

The line profile $D(\lambda)$ which models the relative flux data $F/F_c$ normalised with respect to the continuum, is modelled by convolving the thermal profile $H(\lambda)$ with the macrobroadening profile $M(\lambda)$ and the instrumental profile $I(\lambda)$,

$$D(\lambda) = H(\lambda) \ast M(\lambda) \ast I(\lambda). \quad (3.2)$$

The thermal profile $H(\lambda)$ includes the convolution of natural, thermal and microturbulent dispersions. The macrobroadening profile $M(\lambda)$ represents the combined effects of the non-isotropic radial-tangential macroturbulence $\zeta_{RT}$ and the projected rotation $v \sin i$ through a limb-darkened, weighted disk integration. The instrumental profile $I(\lambda)$ provides the broadening caused by instrumental effects of the telescope and spectro-
meter. Each of the profiles is described here along with other possible broadening mechanisms.

This section follows closely the procedures described in Gray (2005b).

3.2.1 *The Thermal Profile* $H(\lambda)$

Natural broadening is combined with thermal and microturbulent dispersions and forms what is referred to here as the thermal profile $H(\lambda)$.

**Natural broadening** Spectral lines have a natural width arising from the uncertainties in their energy levels, or equivalently, to the typical lifetimes of the levels. Actual lines are usually found to be much broader. Natural broadening, which is also called atomic broadening or radiation damping, gives a profile that has the form of a Lorentzian, also called a dispersion profile, Cauchy, or Agnesi curve (Gray 2005b),

$$L(x) = \frac{1}{\pi} \frac{\gamma}{x^2 + \gamma^2},$$  \hspace{1cm} (3.3)

where $\gamma$ is the half width at half maximum of the spectral line profile, $x$ is the distance in wavelength units from the line centre, and $L(x)$ is normalised to unit area. A Lorentzian has a narrow core and thick wings characteristic of damped oscillators.

The Heisenberg Uncertainty Principle describes how the mean lifetime of an electron at a particular energy level varies inversely with the uncertainty of the energy of that level. It can be written as

$$\Delta E \Delta t \leq \hbar,$$  \hspace{1cm} (3.4)

where $\hbar = \hbar/2\pi$, and $\hbar$ is Planck’s constant. For any energy transition, both upper and lower levels are uncertain, and so have damping constants. The two dispersion coefficients are convolved to give a new damp-
ing constant equal to the sum of those for the upper and lower levels,

\[ \gamma = \gamma_u + \gamma_l. \] (3.5)

Typical atomic energy state lifetimes of \(10^{-8} \text{s}\) give a natural width of \(0.059 \text{ mÅ}\), which in the 6250 Å range is equivalent to a velocity range \(\Delta v \approx 2.8 \text{ m s}^{-1}\). From a classical viewpoint, this is equivalent to a damped oscillator with a width given by the damping constant \(\gamma = 1/\Delta t\) (Zeilik and Gregory 1998). The natural profile determined by the uncertainty in atomic energy levels makes a minimal contribution to the width of stellar spectral lines. The modelled natural profile is important in determining the central wavelength at which other broadening processes operate.

**thermal broadening** Thermal broadening originates in the Maxwellian distribution of velocities for energetic particles, and gives rise to a Gaussian or normal profile,

\[ G(x) = \frac{1}{\sigma\sqrt{\pi}} \exp\left(-\frac{x^2}{\sigma^2}\right), \] (3.6)

where \(\sigma\) is the standard deviation, giving the most probable value or Doppler width of the distribution, and \(x\) is the distance from the line centre. Thermal motions produce wavelength shifts via the Doppler effect. For thermal broadening, \(\sigma\) is the most probable velocity, \(v_{\text{th}} = \sqrt{2kT/m}\), where \(k\) is Boltzmann’s constant, \(T\) is the temperature, and \(m\) is the atomic mass. In a photosphere at 5000 K, a hydrogen atom moves at \(v_{\text{th}}(\text{H}) \approx 9 \text{ km s}^{-1}\), and an iron atom at \(v_{\text{th}}(\text{Fe}) \approx 1.2 \text{ km s}^{-1}\), so an iron line with a wavelength around 6000 Å would have a typical thermal Doppler width of \(\Delta\lambda_D = \lambda v_{\text{th}}/c \approx 25 \text{ mÅ}\). That produces a thermal broadening about 420 times greater than that attributable to natural damping, which can be neglected in such cases.
3.2 LINE BROADENING PROCESSES

microturbulent broadening  Stellar spectroscopic studies measured how the \( \text{EW} \) of spectral lines increased with the number of absorbers as described by curve of growth studies. It was found that modelled lines saturated too soon. By ‘enhancing’ the thermal velocity, the atoms have a broader range of velocities and can absorb a greater amount of flux before saturating. Struwe and Elvey (1934) suggested that such super-thermal motion could be caused by turbulence. The small scale motions are over lengths which are small compared to the photon mean-free-path. The proposed turbulence acts within an optically thick region and so is known as microturbulence. The effect is equivalent to an increase of the thermal width, which increases the line strength as measured by the \( \text{EW} \). Microturbulence is usually considered to be isotropic, Gaussian, and for most dwarfs and giants, has a value of around 1–2 km s\(^{-1}\), increasing with luminosity to around 5 km s\(^{-1}\) in supergiants (Gray 1973; Gray 1975).

In the case of moderately strong lines with \( \text{EW} \gtrsim 0.1 \text{ Å} \), as lines begin to saturate, microturbulence has a greater effect on the curvature of the profile than on the line strength. That is seen clearly in the Fourier transform of the profile as a shift in the position of the first zero. When modelling stronger lines, the microturbulent dispersion \( \xi \) is adjusted together with the \( \text{EW} \) to fit the model to the observed line transform (Gray 2005b).

The cores of strong lines are formed high in the photosphere, and it has been found in this study that the models require larger values of \( \xi \) than for lines of moderate strength formed at lower photospheric depths. The Fourier transform method cannot be used to give precise values of \( \xi \) for weaker lines. Estimated values of \( \xi = 1 \text{ km s}^{-1} \) were used for weak lines \( \text{EW} \lesssim 0.07 \text{ Å} \), but these values had no effect on the model profiles.

collisional broadening  Collisional broadening can also be convolved with the natural profile. It effectively increases the damping constant, making the line a little more “Lorentzian”. See Equation 3.3. The quadratic Stark effect that is sometimes observed in cool giants is due to collisions of absorbers with electrons and ions (Gray 2005b). In the
strongest lines, it is noticeable as a slight rounding of the shoulder of the line, and its inclusion improves the fit to the model in the wavelength domain.

**The Model Atmosphere** Model atmospheres are obtained by scaling the $T(\tau)$ model developed by Ruland et al. (1980) for the K0 III giant β Gem, which defines temperature, and electron, and gas pressures from $\log \tau_{5000} = 1$ to $-5.5$, where $\tau_{5000}$ is the optical depth for radiation with a wavelength of 5000 Å. Each model atmosphere is calculated iteratively for a given wavelength using the values of relative effective temperature $T_{\text{eff}}/T_\odot$, the logarithm of the ratio between the stellar and solar abundances [Fe/H], and the acceleration due to gravity $\log g$. The relative temperature, gas and electron pressures are then obtained as a function of optical depth. The specific intensity is then calculated, which is used to determine the limb darkening effect. The integrated flux is then calculated, and the thermal line profile is obtained for a specific species using atomic data for the species, $\xi$ and $\text{EW}$.

**Constructing the Thermal Profile $H(\lambda)$** When two Gaussian functions are convolved they produce another Gaussian, with a dispersion equal to the root of the sum of the squares of the original dispersions. If the microturbulence is Gaussian and isotropic, it can be added easily to the thermal Doppler width, giving (Gray 1988)

$$\Delta \lambda_D = \frac{\lambda}{c \sqrt{(2kT/m) + \xi^2}}. \quad (3.7)$$

The “enhanced” thermal Gaussian distribution, including the effect of microturbulence, is convolved with the natural line profile, which in some cases includes collisional damping. Since the convolution of a Lorentzian with a Gaussian is a Voigt function, spectral lines typically show Gaussian cores and Lorentzian wings typical of a Voigt profile (Posener 1959).
3.2.2 Macrobroading Distribution $M(\lambda)$

Macroturbulence is caused by large scale motions that act over scales much larger than the photon mean-free-path. The effect of macroturbulence, like rotation, is to increase the line profile width while maintaining the EW constant. A disk integration is carried out to combine the Doppler broadening effects of macroturbulence and rotation weighted by the limb darkening effect (Gray 1988).

![Radial-Tangential Distribution](image)

**Figure 3.1: Radial-Tangential Distribution**

The Gaussian shows the distribution in both $\zeta_R$ and $\zeta_T$ which when integrated over the disk give $\zeta_{RT}$. If, as is usually assumed, $\zeta_R = \zeta_T$, limb darkening has little effect on $\zeta_{RT}$. In this diagram, $M(\lambda)$ refers to the Radial-Tangential Macroturbulence Distribution and not the full disk-integrated Macrobroadening Distribution (Gray 2005b). Reproduced with permission © David F. Gray.

**Radial-Tangential Macroturbulent Distribution** Initially a Gaussian isotropic distribution was assumed for $\zeta$ (Gray 1973), but comparison with centre to limb variations and the nature of granulation cells on the Sun led to a model where the motions are primarily vertical or
horizontal. Since giants also show the characteristic C-shaped bisector found in the Sun and other cool stars, they are likely to have a similar granulation structure. Prescribing a radial-tangential macroturbulent distribution for $\xi_{RT}$, represents observed line profiles well on both the Sun and other cool stars. In practice, both radial and tangential dispersions are given the same Gaussian distribution, which, when integrated over the disk, produces a characteristic cuspy profile, as shown in Figure 3.1 (Gray 1976).

![Figure 3.2: Rotational Distribution](image)

Figure 3.2: Rotational Distribution
This plot shows the rotational profile marked $G(\lambda)$, as a combination of two terms as described by Gray (2005b). The 1st term shows the elliptical profile for a uniformly illumined disk with the limb darkening coefficient $\varepsilon = 0$, while the 2nd term shows the "fully darkened" parabolic profile for $\varepsilon = 1$. Where $\varepsilon$ is defined by Equation 3.8. The sum marked Total shows the combined rotational profile for $\varepsilon = 0.6$, which is typical for a solar type star. The abscissa indicates the variation of velocity or wavelength from a negative limiting value ($-v_L$ or $-\lambda_L$) at one limb through zero at the disk centre to the positive limit at the other limb. (Note: In this graph $G(\lambda)$ refers to the rotational distribution and not the Gaussian distribution.) Reproduced with permission © David F. Gray.
ROTATIONAL DOPPLER-SHIFT DISTRIBUTION  Rotational broadening depends on the aspect angle of the axis of rotation to the line of sight, and on the angular velocity. The extreme values of radial velocity on the observed disk is $v \sin i$ at the limb on the equator. The contribution over the visible disk varies with the linear perpendicular distance from the projection of the axis of rotation on the disk. Integration over the disk gives an elliptical rotational profile which becomes more parabolic with increasing limb darkening as shown in Figure 3.2 (Gray 2005b).

LIMB DARKENING  Limb darkening can be modelled as a linear relation with a single limb darkening coefficient $\varepsilon$,

$$I_\mu = I_{\mu 0}(1 - \varepsilon + \varepsilon \mu), \quad (3.8)$$

where $\theta =$ limb distance and $\mu = \cos \theta$, $I_\mu$ is the specific intensity as a function of $\mu$, and $I_{\mu 0} = I_\mu(\mu = 0)$.

For this study, limb darkening has been modelled using a 4th order polynomial. Limb darkening coefficients were obtained by calculating specific line intensities for the model atmosphere continuum at 6253 Å. This wavelength was chosen to calculate the continuum limb darkening effect since it lies near the centre of the 6250 Å range. Limb darkening depends on wavelength, $T_{\text{eff}}$, and to a lesser extent on surface gravity and metallicity (Gray 2005b). The continuum intensities are calculated for different limb distances and the results are fitted with a fourth order polynomial,

$$I_\mu = I_{\mu 0}(C_0 + C_1 \mu + C_2 \mu^2 + C_3 \mu^3 + C_4 \mu^4), \quad (3.9)$$

where $I_{\mu 0} = I_\mu(\mu = 1)$, that is, at the disk centre, and $C_i$ are the limb darkening coefficients. The calculated limb darkening curve with coefficients for $\gamma$ Tauri are shown in Figure 3.3. The values obtained for the limb darkening coefficients are shown in Table 3.1.
Figure 3.3: Limb Darkening for γ Tauri
The values of specific intensity for specific angular distances from the centre to the limb of the stellar disk were calculated for each stellar model. The values are fitted with a fourth order polynomial (Equation 3.9) and the limb darkening coefficients, as shown in the figure for γ Tauri, are then used in the disk integration of the macrobroadening profile.

**DISK INTEGRATION**  The effects of both rotational and macroturbulent broadening could be combined using a convolution if either the rotational distribution or the macroturbulent distribution were independent of the limb distance \( \theta \). As discussed in Gray (1988), that is clearly not the case for the rotational distribution, and so the macroturbulent distribution would need to be isotropic, which is also clearly not the case for \( \zeta_{RT} \). The combined effects of both rotation and radial-tangential macroturbulence is obtained via a disk integration, which is weighted according to the limb darkening relation to obtain \( M(\lambda) \), the macrobroadening distribution.

To avoid clumpiness in the overall profile, the integration is carried out for about 10,000 disk sectors, and care is taken to maintain resolution.
at the cusp and the region with the maximum flux contribution, which occurs near $\mu = \varepsilon$ (Gray 1976). The result of the disk integration is normalised to unit EW.

![Figure 3.4: Macrobroadening Distribution $M(\lambda)$ for varying $v \sin i/\zeta_{RT}$.
In these disk integrations $\zeta_{RT} = 1\, \text{km s}^{-1}$. As $v \sin i$ is increased, so is the ratio $v \sin i/\zeta_{RT}$ and the cuspiness due to $\zeta_{RT}$ is reduced Gray 1988. Reproduced with permission © David F. Gray.](image)

3.2.3 *Instrumental Profile $I(\lambda)$*

The final broadening mechanism needed to model the line profile is the instrumental profile $I(\lambda)$, which is obtained by measuring through the spectrometer, the profile of the 5461 Å Hg$^{198}$ line, which has a small EW of $\sim 0.05\, \text{Å}$. The broadening resulting from the instrumental profile $I(\lambda)$ is shown in Figure 3.5. The EW of the profile is normalised to 1 Å.

3.2.4 *Other Possible Broadening Mechanisms*

Atomic levels are split in the presence of magnetic fields because of the Zeeman effect. In fields of a few kilogauss, the separation is visible
as separate absorption lines. The separation can be observed in weaker cases with a spectropolarimeter. In the Sun, local fields in active regions can reach 1 or 2 kilogauss, but over most of the surface the field is of a few gauss (Solanki 1998). A field of $10^4$ G gives a line separation of around 0.14 Å. The sensitivity of a line depends on its Landé factor, but there are no magnetically sensitive lines in our spectral range. As a rule of thumb, Zeeman broadening is observable if the effect is larger than the rotational broadening, which for the Hyades giants would require global magnetic fields of around a kilogauss. Recent observations of magnetic fields in slowly rotating giants using high resolution spectropolarimetry indicate the existence of longitudinal fields of a few gauss, including definite de-
tions for $\theta^1$ and $\epsilon$ Tauri (Aurière, Konstantinova-Antova et al. 2015). These magnetic fields would separate lines by a few hundredths of a mÅ, three orders of magnitude too small to be detectable in our instrument with a resolution of around 60 mÅ.

Since solar-like activity cycles are observed in cool stars, including giants, it is likely that, like the Sun, the activity is driven by $\alpha\omega$ dynamos that depend on the interaction of convection and differential rotation. L. L. Kitchatinov and Rudiger (1999) suggested that differential rotation in giants is solar-like but stronger, and preliminary attempts have been made to measure differential rotation in giants using asteroseismology (Deheuvels et al. 2015). Unambiguous detections of differential rotation in giants have yet to be made, and are more likely to be detected in stars with higher $v\sin i$ values than those observed here.

Radial, non-radial and mixed mode pulsations are found in giants, and the time scales for oscillations can be several hours; however, the modes are very difficult to identify and there is uncertainty in the amplitudes involved. Over two hour exposure times, the motions could add to the velocity dispersion observed. It has been suggested that non-radial pulsation may be linked to macroturbulence. Smith (1980) and Asplund et al. (2000) have suggested that the combination of oscillations with convective motions can explain observed macroturbulent dispersions. Beck et al. (2015) found oscillations centring on 90 $\mu$Hz for $\theta^1$ Tauri, which gives periods of around 3 hours, but the amplitudes are in the order of 6 to 9 cm s$^{-1}$ which would not be detectable in our data.

### 3.3 Procedure

The programs used in this procedure were written in Fortran by David Gray for stellar spectroscopic analysis. Though I did not have access to the actual codes, the programs follow closely the procedures outlined in Gray (2005b). Each program has a key file where the behaviour of the program can be determined by varying input parameters. Other parameters are handled as run-time inputs and there is considerable flexibility in the
input and output data files. By varying the values in the key files and adjusting the input parameters, it was possible to study the nature of each code and the reliability of the results.

### 3.3.1 The Fourier Approach

In theory, if a function is formed by the convolution of two other functions, it is possible to deconvolve them. However, the process is computationally complicated, and impractical when dealing with observed rather than precise mathematical profiles.

A powerful alternative is to work with the Fourier transforms of the original profiles. The Convolution Theorem states that a convolution is equivalent to a vector product of the Fourier transforms, and a deconvolution is equivalent to a Fourier vector division.

The Fourier approach is widely applicable and especially effective since broadening effects are spread across the line profile. The procedure allows noise levels to be observed directly in the transform, removing the need for noise filters. The Fourier transform is used to obtain the frequency components of a function, and is defined as

$$f(\sigma) = \int_{-\infty}^{\infty} F(x) e^{2\pi i x \sigma} dx,$$

where $\sigma$ is the Fourier variable which measures frequency when $x$ measures time. In this case, $x$ is in m s$^{-1}$ and $\sigma$ in s m$^{-1}$.

The Fourier transforms are carried out using the Fast Fourier Transform code (Brenner 1968), which is published in Gray (2005b), Appendix C.

We can represent the line profile as a convolution of the thermal, macrobroadening and instrumental profiles as in Equation 3.2. By writing the Fourier transforms of each profile in lower case letters, we have

$$d(\sigma) = h(\sigma) \times m(\sigma) \times i(\sigma),$$
where $d(\sigma)$ is the line profile transform, $h(\sigma)$ the thermal transform, $m(\sigma)$ the macrobroadening transform, and $i(\sigma)$ the instrumental transform. Thus, the Fourier transform of the line profile is obtained by performing a vector multiplication of the transforms of the thermal, the macrobroadening, and the instrumental profiles.

### 3.3.2 Extracting Line Profiles

The lines to be studied are extracted from the relevant exposures into individual files. Close to 30 lines were extracted from each exposure, though in practice about 20 are used. The main limitation is the lack of usable wings because of line blends. The actual lines used vary from star to star and for different parts of the analysis. Once suitable lines are chosen, a mask file is written which can be shifted to allow for changing positions of the lines in different exposures. A data file is created for each selected line for each star. A list of the spectral lines used for broadening analyses is shown in Appendix C. A typical exposure, showing the position of the lines used, is shown in Figure 2.2.

Figure 3.6 shows how lines extracted from the spectra are combined to give an average profile. Of the 14 calibrated exposures for $\gamma$ Tauri, one gave an irregular line profile for this line and was removed from the profile analysis. It is not necessary to do the blend correction for every exposure as was done here, and it is usual to average the profile first and then do the blend correction once. The lines were centred before being averaged. The setting of the continuum level is not optimal here as an approximate 2% variation in the continuum can be seen.

### 3.3.3 Line Profile Averages

The separate line profiles for each star are combined in a weighted mean. The weights are based on a combination of the GD value for the exposure together with the proportion of the profile which is unblended. Weights between 1 to 5 were determined for each line and used in a
3.3 Procedure

Figure 3.6: Average Blend corrected profile of 6252.6 Å for γ Tauri
The lines are extracted from each exposure. In this case they were individually blend corrected, and the weighted average is shown by blue circles. The colours indicate the dates of the individual exposures which are listed in Appendix B.

weighted average. As an example the input profiles together with the average profile of 6252.6 Å for γ Tauri are shown in Figure 3.6. The blend correction can be done on the average profile, but here the average has been obtained from profiles which were individually blend corrected as described in the next section.

3.3.4 Blend Correction

The optimal procedure is to correct for blends once an average line profile have been obtained. Since the process does not consider any line asymmetry, as long as the line has one good wing, the line can be reconstructed and the blend removed by assuming symmetry in the wings.
Fe I 6240.6 Å has a strong blend on the (short wavelength) blue wing. The line profile is inverted, centred and the blended wing is reconstructed giving the blend corrected profile. Note the additional band blending the far red wing. The asymmetry in the line is at least an order of magnitude smaller than the blend corrections.

All the lines that were studied here are blended to a greater or lesser degree, and so need blend correction. Since this procedure is mostly carried out in the wings of the line away from the core, there can be confidence that the blend correction process will provide a realistic reconstruction of the line. Usable lines have one good wing reaching the continuum. The line is reflected through the line centre and compared with the original in order to reconstruct the blended wing, as shown in Figure 3.7.

Even though the blend correction procedure does not consider line asymmetry, since typical line bisector spans are around 100 m s$^{-1}$ in lines with typical widths of about 10 km s$^{-1}$, the approximation does not have an significant impact on the broadening results.
Modelling the Star and Individual Lines

A model of the stellar atmosphere is generated for each star in the wavelength range of the line to be studied. The model parameters, temperature and electron and gas pressures are all given as a function of optical depth. For each star a temperature scale, gravity, and abundance values are required, which were obtained as discussed in Chapter 2. The adopted parameters for the program stars are given in Table 3.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\gamma$ Tauri</th>
<th>$\delta$ Tauri</th>
<th>$\epsilon$ Tauri</th>
<th>$\theta^1$ Tauri</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{\text{eff}}$ (K)</td>
<td>4965</td>
<td>4918</td>
<td>4878</td>
<td>4978</td>
</tr>
<tr>
<td>Scale $\theta = T_{\text{eff}}/T_{\text{eff}}^{\odot}$</td>
<td>0.859</td>
<td>0.851</td>
<td>0.844</td>
<td>0.862</td>
</tr>
<tr>
<td>$\log g$</td>
<td>2.68</td>
<td>2.61</td>
<td>2.69</td>
<td>2.72</td>
</tr>
<tr>
<td>$[\text{Fe}/\text{H}]$</td>
<td>0.085</td>
<td>0.108</td>
<td>0.162</td>
<td>0.078</td>
</tr>
<tr>
<td>$C_0$</td>
<td>0.1859</td>
<td>0.1830</td>
<td>0.1806</td>
<td>0.1872</td>
</tr>
<tr>
<td>$C_1$</td>
<td>1.5684</td>
<td>1.5645</td>
<td>1.5468</td>
<td>1.5743</td>
</tr>
<tr>
<td>$C_2$</td>
<td>-1.9802</td>
<td>-1.9587</td>
<td>-1.8967</td>
<td>-1.9978</td>
</tr>
<tr>
<td>$C_3$</td>
<td>2.0318</td>
<td>2.0114</td>
<td>1.9333</td>
<td>2.0526</td>
</tr>
<tr>
<td>$C_4$</td>
<td>-0.8086</td>
<td>-0.8024</td>
<td>-0.7667</td>
<td>-0.8183</td>
</tr>
</tbody>
</table>

Table 3.1: Stellar Parameters used for Model Atmospheres
The coefficients $C_0$ to $C_4$ are for the 4th order polynomial describing the limb darkening of each star used in Equation 3.9. The parameters used for modelling were taken from SIMBAD (Wenger et al. 2000).

Residual Transforms $d(\sigma)/h(\sigma)$ and Macrobroadening Transforms $m(\sigma)$

Equation 3.11 shows how the line transform is obtained by multiplying the thermal, macrobroadening, and instrumental transforms. By dividing the observed line transform by the thermal and the instrumental transforms, the macrobroadening transform is obtained. Microturbulence is
modelled together with the EW for the stronger lines before the macroturbulent transform is obtained.

The transform of the observed line $d(\sigma)$ is divided by the thermal transform $h(\sigma)$, to give a residual transform $d(\sigma)/h(\sigma)$. The residual transform is then averaged over several lines, to get the mean residual transform $<d(\sigma)/h(\sigma)>$. The mean residual transform for the star is then divided by the instrumental transform $i(\sigma)$. The result is the observed macrobroadening transform

$$m(\sigma) = <d(\sigma)/h(\sigma)> / i(\sigma).$$  \hfill (3.12)

$m(\sigma)$ combines the broadening effects of both radial-tangential macroturbulence and rotation. The process of obtaining the residual transform is shown for the 6253 Å line from $\gamma$ Tauri in Figure 3.8.

The macrobroadening transform $m(\sigma)$, is compared with the modelled macrobroadening transform $m_{\text{mod}}(\sigma)$, which is the Fourier transform of the disk integration including the Doppler effects of both rotation and macroturbulence, weighted by the limb darkening. The values of $\zeta_{RT}$, and $v\sin i$ are adjusted to obtain the best fit between the observed and the modelled macrobroadening transforms. It is found that the ratio $\zeta_{RT}/v\sin i$ primarily affects the curvature of the model transform, and the value of $\zeta_{RT}$ primarily shifts the transform to adjust the low $\sigma$ values of the transforms. Thus once the value of $\zeta_{RT}$ has been determined, $v\sin i$ is obtained by adjusting the ratio $\zeta_{RT}/v\sin i$. Using this method, values of $\zeta_{RT}$ and $v\sin i$ can be obtained independently to a precision of better than 0.1 km s$^{-1}$. Figure 3.9 shows the optimal fit of a model to the observed $m(\sigma)$ for $\mu$ Cancri from Gray (1988). Models which lie on either side of the observed data show corresponding values of $\zeta_{RT}$ and $v\sin i$ and indicate characteristic uncertainties.
Figure 3.8: How to Obtain a Residual Transform: 6252.6 Å for γ Tauri
The blue line shows the transform of the observed line profile $d(\sigma)$. The green line shows the modelled thermal transform $h(\sigma)$. The observed data transform divided by the thermal transform gives the residual transform which is then fitted by a modelled macrobroadening transform. The macrobroadening transform $d(\sigma)/h(\sigma)$, is shown by the red line. The thermal profile $H(\lambda)$ is adjusted in an iterative process until the thermal transform $h(\sigma)$ fits the line transform $d(\sigma)$. The EW of $H(\lambda)$ is adjusted so that the transforms coincide at low $\sigma$ values, while $\xi$ is adjusted to fit the first zero. Adjusting $\xi$ has a slight effect on the EW because stronger lines begin to saturate. The bump or peak on the residual transform above the first zero is due to division by zero. For the better fitted models that feature almost disappears.

3.3.7 Microturbulence Dispersion $\xi$ and Equivalent Width EW

At first, the process of obtaining the residual transforms is carried out for strong lines. For them, the lines have some saturation and the first zero of the line transform is well delineated above the noise level, as shown by the blue line in Figure 3.8, which shows the process for the transform of the average 6253 Å profile taken from γ Tauri. For strong lines $\xi$, is adjusted until the first zero of the observed and modelled re-
residual transforms coincide, while ensuring the modelled line maintains the correct EW. Weaker lines have smaller values of $\xi$. Using values of $0.8 - 1.2 \text{ km s}^{-1}$ gave consistent residual transforms and line profiles.

3.3.8 Radial-Tangential Macroturbulent Dispersion $\zeta_{RT}$ and Rotation $v \sin i$

By modelling strong lines, the value of $\xi$ for the line is obtained along with a first approximation for the stellar $\zeta_{RT}$ and $v \sin i$ values. Strong lines form over a greater range of photospheric depths than weak lines, so are more sensitive to uncertainties in the temperature distribution (Gray 1976). They also have a pronounced zero before the first sidelobe of the transform. The division by zero in strong lines introduces a discontinuity beyond which the residual transform is less reliable. Weak lines are used for the final macrobroadening studies, as they do not suffer from these disadvantages and so can produce a residual transform that is well defined to higher frequencies. There are 5 strong lines and 13 weak lines used in this analysis, as listed in Appendix C.

The residual transforms are weighted based on the quality of the blend corrected profile and the quality of the residual transform, and then averaged. The averaged residual transform is then divided by the instrumental profile to give the macrobroadening profile as in Equation 3.12.

The values of $\zeta_{RT}$, and $v \sin i$ are adjusted to get the best fit for the weak line macrobroadening profile $m(\sigma)$, and the macroturbulent dispersion $\zeta_{RT}$ and projected rotational velocity $v \sin i$ are obtained as shown in Figure 3.9. Ensuring that the results for weak lines are consistent with the residual transforms for strong lines ensures integrity of the modelling process. Such consistency is required since all lines are expected to have equal amounts of broadening from rotation and macroturbulence.

3.3.9 Checking Results in Wavelength Domain

An extra confirmation of the modelling process is achieved by comparing the modelled and observed line profiles in the wavelength domain.
Figure 3.9: Adjusting the modelled $m_{\text{mod}}(\sigma)$ to the observed Macrobroadening Transform $m(\sigma)$

This figure from Gray (1988), shows how changing the ratio of $\zeta_{\text{RT}}$ to $v \sin i$, as well as the individual values, allow the modelled macrobroadening transform to be fitted to the observed residual macrobroadening transform. Reproduced with permission © David F. Gray.

The modelled line profile is obtained by convolving the thermal the macrobroadening and the instrumental profiles

$$D_{\text{mod}}(\lambda) = H(\lambda) \ast M_{\text{mod}}(\lambda) \ast I(\lambda)$$  \hspace{1cm} (3.13)

However, now the modelled Macroturbulent Profile $M_{\text{mod}}(\lambda)$, is based on the values of $\zeta_{\text{RT}}$ and $v \sin i$ that have been obtained in the broadening analysis. $M_{\text{mod}}(\lambda)$ is convolved with the Thermal Profile $H(\lambda)$, which includes the effects of microturbulence and $\text{EW}$, as well as with the Instrumental Profile $I(\lambda)$, as before. The final comparison in the wavelength domain is an extra confirmation of the integrity of the whole process, as can be seen in the results below. Each domain emphasises different aspects of
the line profile. The Fourier domain clearly shows the coincidence in the slowly varying low and medium frequencies, while the wavelength domain shows up the high frequency components at the core and shoulders of the line. The highest frequency components are lost below the noise in Fourier profiles.

![Graph showing line profile and Fourier domain](image)

Figure 3.10: Strong Line, Thermal and Residual Transforms for γ Tauri
The solid arcs at the bottom of the plot are the line transforms, the dotted lines just above them are the thermal transforms, and the lines at the top of the graph show the residual transforms. The inset table gives the wavelength, the modelled values for EW and $\xi$, and the assigned weight for the transform.

3.4 RESULTS

The results of the broadening analysis are summarised in Table 3.2. For γ Tauri, six plots are shown (Figures 3.10 to 3.15). For δ, ε and θ$^1$ Tauri, only the modelling of the macrobroadening transform $m(\sigma)$ is shown in the main text (Figures 3.16 to 3.18), while the other figures are shown in Appendix D. First, the Residual Transforms are shown for strong lines, and then for weak lines. The third graph shows the best fit between the modelled and observed Macrobroadening transforms, with the resulting
values for $\zeta_{RT}$ and $v \sin i$. These values are then used to model line profiles in the wavelength domain. This is done for all the lines used in the procedure. Three modelled lines are shown for each of the Hyades giants to demonstrate the effectiveness of the line modelling process. All six figures are included here for $\gamma$ Tauri, while for the other three stars only the model fitting figure is included in this chapter. The remaining figures are included in Appendix D.

![Graph](image)

**Figure 3.11: Weak Line Residual Transforms for $\gamma$ Tauri**

Only the residual transforms are shown. The table shows the wavelength, the modelled values for EW and $\xi_r$, and the assigned weight for the transform.

The strong lines have strong sidelobes that permit the values of $\xi_r$ to be obtained. However, the strong sidelobes also leave features in the residual transform that impede the modelling process. Once the strong lines have been analysed the weak lines are studied to ensure a good fit with the strong line results. The best lines are weighted, and an average residual transform is obtained and used to model the macrobroadening transform in an iterative process.
3.4 Results

Figure 3.12: γ Tauri - Modelling Macrobroadening Transform
The lower dashed gray line is the observed residual transform, which, when divided by the instrumental dot-dashed profile above, gives the macrobroadening transform shown by the dark red circles. The error bars show the standard deviation for the mean residual transform. The red line shows the modelled macrobroadening transform for γ Tauri with rotation $v \sin i = 3.0 \, \text{km s}^{-1}$ and $\zeta_{RT} = 4.55 \, \text{km s}^{-1}$.

3.4.1 Results for γ Tauri

Figures 3.10 and 3.11 show the results for the Residual Transforms of strong and weak lines respectively. It is useful to compare Figure 3.10 with Figure 3.8. Weights from 1 to 5 were assigned following two criteria: the quality of the profile, that is, the proportion of the line which needed blend correction, and the quality of the residual transform determined by its similarity to other transforms for the same star.

Figure 3.12 shows the macroturbulence modelling, the lower dashed gray line is the observed residual transform, which, when divided by the instrumental dot-dashed transform above, gives the macrobroadening transform shown by the dark red circles. The error bars show the standard deviation for the mean residual transform. The red line shows the modelled macrobroadening transform for γ Tauri with rotation $v \sin i = 3.0 \, \text{km s}^{-1}$ and $\zeta_{RT} = 4.55 \, \text{km s}^{-1}$. These values were then used to model
**Figure 3.13**: $\gamma$ Tauri-Wavelength Domain model check for Fe I 6226.7 Å
Dots show the observed profiles and continuous line represents the model. Inset label indicates the spectral and broadening parameters used.

**Figure 3.14**: $\gamma$ Tauri-Wavelength Domain model check for Fe I 6240.6 Å
Dots show the observed profiles and a continuous line represents the model. Inset label indicates the spectral and broadening parameters used.
Figure 3.15: γ Tauri-Wavelength Domain model check for Fe I 6266.3 Å
Dots show the observed profiles and continuous line represent the model. Inset labels indicates the spectral and broadening parameters used.

individual lines, and three are shown in Figures 3.13, 3.14 and 3.15. The unblended wing and the core of the line fit extremely well. One wing often has some blending. Figure 3.13 has a particularly good fit. Notice how the model line passes through the centre of the data circles for the observed profile.

3.4.2 Results for δ, ε and θ1 Tauri

Figure 3.16 shows the modelling of macrobroadening for δ Tauri. The red line shows the modelled macrobroadening transform for δ Tauri with rotation $v \sin i = 3.15 \text{ km s}^{-1}$ and $\zeta_{\text{RT}} = 5.30 \text{ km s}^{-1}$. The fit is particularly good at high frequencies though the low frequency fit is worse. The three line models show a very strong fit to the observations and are found in Appendix D, together with figures showing the residual profiles for both strong and weak lines.
3.4 RESULTS

Figure 3.16: δ Tauri - Modelling Macrobroadening Transform

The lower dashed gray line is the observed residual transform, which, when divided by the instrumental dot-dashed profile above, gives the macrobroadening transform shown by the dark red circles. The error bars show the standard deviation for the mean residual transform. The red line shows the modelled macrobroadening transform for δ Tauri with rotation $v \sin i = 3.15 \text{ km s}^{-1}$ and $\zeta_{RT} = 5.30 \text{ km s}^{-1}$.

Figure 3.17 shows the broadening model for ε Tauri. Clearly the model is not a good fit to the observed transform, with deviations at both low and high frequencies. It is not clear what might cause this discrepancy with the model for this particular star. The closest fit for ε Tauri gives values of $v \sin i = 3.40 \text{ km s}^{-1}$ and $\zeta_{RT} = 5.05 \text{ km s}^{-1}$. The models also show more deviations in the wavelength domain, with noticeable differences at the shoulder of the profiles, arising from the difficulties in modelling the high frequency components.

The modelled macrobroadening transform for θ¹ Tauri also fits the observed transform particularly well throughout the frequency range as can be seen in Figure 3.18. The quality of the model is reflected in the good fit of the modelled wavelength domain line profiles, which are available in Appendix D. The red line shows the modelled macrobroadening transform for θ¹ Tauri with rotation $v \sin i = 2.70 \text{ km s}^{-1}$ and $\zeta_{RT} = 4.60 \text{ km s}^{-1}$. 
The lower dashed gray line is the average observed residual transform, which, when divided by the instrumental dot-dashed profile above, gives the macrobroadening transform shown by the dark red circles. The error bars show the standard deviation for the mean residual transform. The red line shows the modelled macrobroadening transform for \( \varepsilon \) Tauri with rotation \( v \sin i = 3.40 \text{ km s}^{-1} \) and \( \zeta_{\text{RT}} = 5.05 \text{ km s}^{-1} \).

The similarity between the curves of residual transforms for all lines, both weak and strong, show that they represent global stellar broadening effects, rather than effects for individual lines. The difference between them is greatest for the high \( \sigma \) values, which correspond to the high frequency components of the line cores and shoulders. That is because the high frequency parts of the transforms drop into the noise levels mak-
Figure 3.18: θ¹ Tauri - Modelling Macrobroadening Transform

The lower dashed gray line is the observed residual transform which when divided by the instrumental dot-dashed profile above gives the macrobroadening transform shown by the dark red circles. The error bars show the standard deviation for the mean residual transform. The red line shows the modelled macrobroadening transform for θ¹ Tauri with rotation v sin i = 2.70 km s⁻¹ and ζRT = 4.60 km s⁻¹.

ing modelling those regions progressively more difficult. The strong line cores are formed high in the photosphere. Differences between the strong and weak line transforms are an indication of the difficulties in modelling the temperature distribution in this region. Strong lines include contributions from a greater range of photospheric heights, so are more sensitive to the details of the modelled temperature distribution.

<table>
<thead>
<tr>
<th>Parameter (Units)</th>
<th>γ Tauri</th>
<th>δ Tauri</th>
<th>ε Tauri</th>
<th>θ¹ Tauri</th>
</tr>
</thead>
<tbody>
<tr>
<td>v sin i (km s⁻¹)</td>
<td>3.0±0.1</td>
<td>3.2±0.2</td>
<td>3.4±0.1</td>
<td>2.7±0.2</td>
</tr>
<tr>
<td>ζRT (km s⁻¹)</td>
<td>4.55±0.05</td>
<td>5.3±0.1</td>
<td>5.05±0.05</td>
<td>4.6±0.1</td>
</tr>
</tbody>
</table>

Table 3.2: Summary of Line Broadening Results

The residual transforms show that there is consistency in the way the macroturbulent approach describes the velocity fields of cool stars. As
3.5 Discussion

can be seen by the close fit of the wavelength-domain line profile models for all four stars, the observed line profiles are reproduced very closely. Note that the Hyades giants that are known to be more active, γ and θ¹ Tauri, have lower values of $v \sin i$, of 3.00 and 2.70 km s$^{-1}$ respectively and their values of $\zeta_{RT}$ at 4.55 and 4.60 km s$^{-1}$ are also less than for the quieter stars δ and ε Tauri.

For each of the Hyades giants, the projected rotational velocities $v \sin i$, and the radial-tangential macroturbulent dispersion $\zeta_{RT}$, were obtained. A marked tendency for $\xi$ to increase with line strength was noted in strong lines formed in the upper photosphere.
LINE ASYMMETRY

4.1 BACKGROUND

Asymmetry in solar spectral lines was noticed by Higgs (1960) and then studied by Higgs (1962) and Olson (1962) using line bisectors. Since the solar surface shows a clear granular structure providing direct evidence of convective cells, the line asymmetry has been explained by the different flux intensities of blue and red-shifted Doppler components of the photospheric velocity field (Olson 1962; Balthasar 1984). Originally line asymmetries were investigated in other stars as evidence of stellar pulsation (Underhill 1947; van Hoof and Deurinck 1952). Smith and Parsons (1975) suggested that a depression in the blue wings of an Ap star could arise from mass motions in the photosphere, while Bray and Loughhead (1978b) described how the use of line asymmetries could be used to study the convective velocity field of cool stars, suggesting the use of the imaginary component of the Fourier transform.

Another method for measuring line asymmetries is to use the difference between a line and its reflection in the wavelength domain, giving a measure of the loss of flux in the depressed wing. Gray (1980) used the technique to compare line asymmetries of the Sun and the K2 III giant, Arcturus. The wing difference approach provided a useful single value for the amount of line asymmetry, however, a great deal of information is lost in this way. Another alternative, the use of the imaginary part of the Fourier transform, was found to be too sensitive to blends in the wings. Subsequently, the studies of stellar line asymmetries have focused on the use of line bisectors (Dravins, Lindegren et al. 1981; Dravins 1982; Gray 1982a). A more recent extensive study of line bisectors in cool stars was made by Gray (2005a).
The construction of a line bisector is shown in Figure 4.1. The points on the blue side are joined to interpolated points on the red side and the centres of such lines are joined to form a line bisector, which is then expanded in the velocity scale so the details can be studied. It is usual to plot the relative flux $F/F_c$ against velocity. The velocity can be calibrated as an absolute radial velocity or a relative velocity change from the line centre. There is a detailed discussion on the topic in Gray (1988).

![Figure 4.1: Constructing a Line Bisector](image)

This figure from (Gray 1992a) shows how bisectors are constructed. Note the need for interpolation between the actual datapoints on one side of the profile in order to define the lines that must be bisected. The bisector on the right has had the wavelengths scaled by a factor of 50 which is typically what is needed to observe the profile. Reproduced with permission.

Line bisectors are widely used in the study of stellar atmospheres (Gray 2010b), to distinguish between stellar and planetary causes of radial velocity variations (Hatzes, Cochran et al. 2006; Hebrard et al. 2014), and recently to study pulsation and stratification in supergiants, Cepheids, and RR Lyrae stars (Gray 2010c; Guggenberger et al. 2013).

Full 3D hydrodynamical models have been very successful in modelling line asymmetry and shifts for the Sun. Figure 4.2 shows three iron lines of increasing strength modelled by Asplund et al. (2000). The models show disk integrated lines that are compared to data including
The appearance of small-scale features on the surface of the Sun fascinated early observers, who struggled to describe what they saw. They were described as clouds, corrugations, willow leaves, rice grains, and eventually, granules (Secchi 1853; Dawes 1864; Nasmyth 1865). Granules, the individual features shown in Figure 4.3, are caused by the underlying convective motion, and their effect on the photosphere is known as granulation. Granules are irregular in shape though roughly polygonal, and of varying sizes, but for the Sun are typically a thousand kilometres across,
lasting on the order of 8 minutes. They represent the upper, or overshoot, region of convective columns also known as Bénard cells.

The study of solar bisectors (Dravins, Lindegren et al. 1981; Dravins 1982) led to a two component model describing how the lines are formed. The rising plasma in the granules is hot, bright, and blueshifted, while the intergranular lanes are cooler, darker, and red-shifted. A simplified model displaying strong blueshifted and weaker redshifted components from the granules and lanes respectively, can help to visualise the process as in Figure 4.4, from Dravins, Lindegren et al. (1981). The honeycomb shows hot blueshifted hexagonal granules, with darker redshifted lanes. Both are represented by the flux distributions as shown, where the redshifted fainter line is to the right at positive velocities. The figure on the right shows how the combination of the two lines yields a curved bisector.
Figure 4.4: A Granulation Model
A simple model by Dravins (1982) shows how a regular network of granules and lanes can produce line asymmetry. The central panel shows a line from the brighter blue-shifted (hot and rising) granules and below that the same line from the fainter red-shifted (cool and sinking) lanes. The third panel shows how combining the two profiles broadens the line and increases absorption in the red wing, causing the top of the typical C-shaped bisector found in cool stars. Reproduced with permission.

Figure 4.5: Schematic showing radial velocity effects of changing perspectives on granulation
An observer seeing the disk centre from a perspective 'A' primarily sees the effects of radial velocities, while an observer looking at positions progressively towards the limb have perspectives 'B' and then 'C', which show increasing effects from the transverse velocity component (Bray, Loughhead and Durrant 1984). Reproduced with permission.
The ability to resolve the solar surface permits spectroscopic studies of how line bisectors vary across the observed solar disk. As the limb is approached, a given optical depth samples progressively higher parts of the photosphere, and tangential components of the gas motion dominate because of the changing perspective, as shown schematically in Figure 4.5 from Bray, Loughhead and Durrant (1984). At the limb, inverse C bisectors can be observed. These come from the upper photosphere where wave motion, down flow, and possibly inverse granulation from the chromosphere, make inverse C bisectors more common. (see Uitenbroek 2006).

![Figure 4.6: Comparison of Line Bisectors for Arcturus K2 III and the Sun G2 V (Gray 2005a)](image)

The bisectors are compared with those obtained from the spectral atlases by Hinkle et al. (2000) and Beckers et al. (1976). Note how the left-hand blue bulge is lower (i.e. higher in the photosphere) for the higher luminosity star Arcturus where greater convective velocity reaches higher into the photosphere. © The Astronomical Society of the Pacific. Reproduced with permission. All rights reserved.
4.3 STELLAR LINE BISECTORS

Gray (1982a; 1983) confirmed the C shape for cool star bisectors, and demonstrated a great deal of variability in their shapes. Figure 4.6 compares the average line bisectors for Arcturus (K2 III) and the Sun (G2 V), indicating how the bisectors curve in the same direction but look quite different. In particular, the range of velocities is larger for Arcturus and the blue bulge occurs at greater line depths, which correspond to higher positions in the photosphere. The bisector blueshift increases with the velocity of the convective overshoot into the lower photosphere. The higher luminosity and lower gravity of Arcturus cause the bulge from the blueshifted granules to extend higher into the photosphere, indicating sustained net motion toward the observer. In the Sun, lower luminosity and higher acceleration from gravity cause the velocity of the blueshifted granular component to drop off rapidly.

Figure 4.7: The Effect of Line Depth on Bisectors
Lines of varying strength are shown with arbitrary displacements and superimposed. Strong lines follow the same bisectors as weak lines but extend higher into the photosphere. The mean bisector is also shown (Gray 1992a). Reproduced with permission.

Weak lines form deeper in the photosphere and are subject to the same velocity and temperature distributions as the wings of strong lines. Weak
lines, therefore, show just the upper part of bisectors, while the lower part of the C is lost. Figure 4.7 illustrates how lines of different strengths are combined and averaged to form a mean bisector for a star (Gray 1983).

As the study of granulation was extended across the HR diagram (Gray and Toner 1986; Dravins 1988), an unexpected ‘inverted-C’ bisector was found for stars on the hot side of the HR diagram. Figure 4.8 shows the change of inverted C to C shaped bisectors for decreasing temperature in dwarfs (Gray and Toner 1986). The inverted-C line bisectors show that there is a photospheric velocity field in hot stars, where it appears that the hot bright blueshifted component has a lower filling factor, so the flux is dominated by the cool red-shifted component (Landstreet 2007; Gray 2010a). It is of interest to note that inverted-C bisectors have been observed near the solar limb (Gray 2005b), and have also been detected in the solar chromosphere (Uitenbroek 2006). P. N. Brandt and Solanki (1990) found that increasing magnetic filling factors cause the deep photospheric parts of bisectors (nearer the continuum) to have increased red shifts, while the velocity shifts in the upper photosphere is suppressed, leading to straighter bottom halves.

Gray (2005a) shows how bisectors vary across the cool half of the HR diagram. In higher luminosity classes, the blueshifted hot component reaches higher into the photosphere and the temperature effect causes a gradual change of the bisector slope, especially as the granulation boundary is approached. For further discussion on boundaries on the HR diagram see Section 1.2.2.

Bisector spans have been measured by choosing arbitrary points, such as from 0.4 to 0.8 of the flux, or to the maximum blueshift of the bisector (Gray 1982a; Gray 2005a; Hatzes, Cochran et al. 2006). Gray (2005a) showed how the height of the blue-most point of the bisector span (called the blue bulge) varies systematically with absolute magnitude as in Figure 4.14.

Line bisectors are very sensitive to the formation depth of the line and to stellar variability. For these reasons, line bisectors are often averaged to give an overall picture of atmospheric conditions. Radial and non-
average bisectors for supergiants show how the cool C shape bisector changes to the hot inverted-C shape at a granulation boundary. The inset shows the average bisectors obtained in this study of the Hyades giants. They lie between G5 and K2 as expected. (Gray and Toner 1986) © The Astronomical Society of the Pacific. Reproduced with permission. All rights reserved.

radial pulsation with periods ranging from hours to hundreds of days with velocity dispersions of a few cm s$^{-1}$ to 100 m s$^{-1}$ have been detected in cool giants (Hatzes and Cochran, 1993; Hatzes, 2002). Cavallini et al. (1987) found changes in the bisectors of iron lines in the Sun in phase with the 5 min oscillation, and Hatzes and Cochran (1998) found similar pulsation driven changes in bisectors in Aldebaran, a K giant. Rotational modulation by surface features such as starspots also affect the shape of line bisectors (Toner and Gray 1988; Hatzes 2002). All these effects can contribute to possible variability in line bisectors.

It is generally assumed that a constant bisector shape with a periodic change in radial velocity is indicative of an orbital effect, and that any line bisector variations in phase with the radial velocity changes are evidence of rotational modulation of magnetic effects. However, star-planet interactions (SPIs) are increasingly being reported. Quinn et al. (2015) detected SPIs with a planet around a red giant at 0.5 A.U. Planetary, stellar, and
instrumental effects can prove difficult to disentangle, and care must be shown in interpreting the results. See, for example, Queloz et al. (2001).

4.4 Procedure

Line bisectors can be obtained for any line, but good lines with little or no blending, and strong enough to sample most of the photosphere, are rare in our data. Fe I 6252.6 Å is a good choice in the 6250 Å range used here. As in other analyses carried out in the project, the main programs used are part of a suite of Fortran high resolution spectroscopy routines developed by David Gray. The profiles are extracted from the individual exposures, and then the bisectors were obtained and weighted mean bisectors calculated.

Another route to obtain the average bisector is to first obtain the average profile, and then use that to calculate its bisector. That dual procedure was used to test the internal consistency of the bisector reductions.

4.5 Results

The bisectors for Fe I 6252.6 Å in the four Hyades giants are displayed in Figures 4.9, 4.10, 4.11 and 4.12. Note that the dotted bisector is the average of all the observed bisectors for the star. Details of the exposures are given in Appendix A: there are 14 wavelength calibrated exposures for γ Tauri and also for ε Tauri, there are five CCD exposures for δ Tauri, of which one has a wavelength calibration, and four θ1 Tauri exposures, none of which have absolute wavelengths. The relative wavelength scale for the uncalibrated exposures was obtained from a calibrated exposure of γ Tauri.

Errors in line bisectors are greatest at the top and bottom of the line, both of which correspond to sections of the line profile with the smallest gradient. Ordinate positions are best defined near the middle of the line, and hence provide better values for the bisector. The bisector blue bulge is in this region and serves as a reliable measure of line bisector variations.
The bisector dates of observation are shown. The bisectors show significant variability even for those taken only a few days apart. The blue bulge is found at different line depths, indicating velocity and flux changes at the level where the line is formed. The depth of the blue bulge increases with increasing luminosity (Gray 2005a), as shown in Figure 4.14.

Other measures commonly used to measure bisector variations are the velocity span of the bisector, which is sometimes divided into upper and lower spans, to give a measure of curvature, and the velocity shift.

Only one of the bisectors for δ Tauri in Figure 4.10 includes wavelength information, and so the others were shifted to aid with comparisons. The shape is more constant than for γ Tauri (Figure 4.9), although there is some variability from blends in the wings. δ Tauri is known to be a binary with a period of 532 days, and a radial velocity variation from orbital motion of ± 2.9 km s\(^{-1}\) (Massarotti et al. 2008). The bisector span does not vary as much as the bisectors themselves, staying close to 100 m s\(^{-1}\) from the blue bulge to the line core, whereas the span for γ Tauri can extend to
3 times that value. The height of the blue bulge in these bisectors remains close to the average height of \( F/F_c \approx 0.6 \).

Bisectors for \( \varepsilon \) Tauri (Figure 4.11) show considerable variability. A radial velocity variation due to a planet of almost \( \pm 100 \text{ms}^{-1} \) was found by Sato et al. (2007), as shown in Table 1.5. However, the observed variability exceeds this, and further observations with better sampling rate is needed to investigate possible causes. The lack of evidence for strong magnetic activity suggests that, if these line bisector variations are confirmed they may be caused by other mechanisms such as long period pulsations.

The \( \theta^1 \) Tauri bisectors (Figure 4.12) show little variation in span and bulge height with variations in the extent of the blue bulge, suggestive of effects at mid-photospheric altitudes that reduce the contrast between the granular and intergranular components. The contrast with the results for \( \gamma \) Tauri is interesting, given the similarity in their chromospheric studies.
Figure 4.11: Line Bisectors of 6252.6 Å for ε Tauri
The bisectors of spectral lines from ε Tauri show considerable variability, beyond the uncertainty in the radial velocity measurements of around 30 m s$^{-1}$, and the radial motions attributed to a planetary orbit.

<table>
<thead>
<tr>
<th></th>
<th>γ Tauri</th>
<th>δ Tauri</th>
<th>ε Tauri</th>
<th>θ$^1$ Tauri</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of Blue Bump (F/F$_c$)</td>
<td>0.63</td>
<td>0.66</td>
<td>0.61</td>
<td>0.58</td>
</tr>
<tr>
<td>Span (m s$^{-1}$)</td>
<td>130</td>
<td>97</td>
<td>93</td>
<td>99</td>
</tr>
<tr>
<td>Absolute Magnitude (M$_v$)</td>
<td>0.338</td>
<td>0.538</td>
<td>0.435</td>
<td>0.629</td>
</tr>
</tbody>
</table>

Table 4.1: Line Bisector Data

4.6 Discussion

The height of the blue bulges are plotted against absolute magnitude $M_v$ in Figure 4.14, taken from Gray (2005a). All four stars show results typical of giants.

The changing shape of the bisector in γ Tauri is indicative of changing conditions in the star compatible with magnetic activity and possibly pulsation. Magnetic variability is expected from the variability in
Figure 4.12: Line Bisectors of 6252.6 Å for θ¹ Tauri

θ¹ Tauri had one observation with wavelength calibration and the other line bisectors were shifted to assist with comparisons. The bisector shapes show are more constant that the other Hyades giants. The abscissae show relative velocities.

chromospheric activity found, for example, by Baliunas, Hartmann et al. (1983).

The height of the bulge around F/Fc ≈ 0.6 and the overall shape are similar to those of other K giants, as indicated in the luminosity relation of Figure 4.14. The average bisectors are similar to β Gem in shape, height of the blue bulge, and span (Gray 2005a).

Figure 4.13 indicates the average bisectors calculated in two ways to show the level of internal consistency. The dashed lines show the average of the bisectors for the lines taken from individual exposures, whereas the continuous lines show the bisector for the average profiles. The largest difference seen in the blue bulge for ε Tauri is equivalent to a difference of ∼ 7 m s⁻¹.

The velocity span of the bisector (from blue bulge to line core) is around 100 m s⁻¹ for δ Tauri and θ¹ Tauri, and marginally less for ε Tauri, while γ
Figure 4.13: Average Line Bisectors of 6252.6 Å for Hyades Giants
Comparison of average bisectors for FeI 6252.6 Å. The bisectors for γ and ε Tauri include information on absolute radial velocities corrected for movement around the solar system barycentre. The bisectors for δ and θ¹ Tauri that do not have absolute wavelength information been shifted to fit on the plot. Dashed lines show the average of the calculated bisectors while the continuous lines are bisectors of the average profiles for each star. This plot shows that the internal errors in the wavelength analysis show a consistency of around ±6 m s⁻¹.

Tauri has a span of around 140 m s⁻¹, which is interesting to note since it is the most active of the Hyades giants. Further study would be needed to identify possible causes of the variability in bisector shapes and whether they could be attributed to stellar features such as spots or stellar pulsations.

Line asymmetry in the Hyades giants was studied using the line bisector of Fe I 6252.6 Å, along with the height of the blue ‘bump’ and the bisector span. Both indicators of the strength of the photospheric velocity field, were obtained for each star. The average bisectors are compared with other stellar types in Figure 4.8, where results expected from K0 giants can be seen.
Figure 4.14: The Luminosity Effect on Bisectors
The height of the blue bulge decreases with increasing luminosity. The yellow circles show data from this study. (Gray 2005a) © The Astronomical Society of the Pacific. Reproduced with permission. All rights reserved.
5.1 BACKGROUND

The redshift of solar spectral lines relative to laboratory arc measurements was first reported by Jewell (1896), who noted that the effect was more pronounced for stronger lines. Halm (1907) reported a limb redshift effect, which depends on line strength. St. John (1928) showed that a correction should be made for the gravitational redshift,

\[ v_g = \frac{GM_\odot}{R_\odot c} = 636 \text{ ms}^{-1}, \]  

(5.1)

where \( G \) is the gravitational constant and \( M_\odot \) and \( R_\odot \) are the solar mass and radius. If one does so, there is a region at a height of around 500 km in the solar photosphere with no net radial velocity and the blueshift increases with photospheric depth, corresponding to decreasing line strength. The average blueshift is often thought of as a convective correction to radial velocity. This correction is very sensitive to photospheric conditions and the choice of line lists (Bray and Loughhead 1978a). Dravins, Lindegren et al. (1981) made a detailed study of the effect of granulation on line asymmetries and wavelength shifts, and of dependencies of line strength with velocity and excitation potential, and hence, with formation depth in the solar photosphere. Similar line strength effects on line shifts were shown in stars by Gray and Toner (1985) and Nadeau (1988). The effect of convection on line-shifts and asymmetries were studied by Balthasar (1984) and reviewed by Dravins (1998). A detailed study of the effect in dwarfs was made by Allende Prieto, Lambert et al. (2002) using EWs instead of core line depths. The use of EW makes it harder to
detect the line strength versus radial velocity relation, since it depends on a greater range of photospheric depths.

Gray (2009) undertook a study of photospheric velocity fields, and chose to use line core velocities and depths rather than equivalent widths because of the greater precision in a spectral region where blends are common. The line core-velocity plot or 3SP was found to be a characteristic of the granulation in all cool stars. It was given this name by David Gray Gray (2009), who described this relationship as the third signature of stellar granulation after line broadening and line asymmetry. Once a standard 3SP curve was found for the Sun, the velocity distribution for other cool stars were fitted by scaling the standard curve and shifting it according to radial velocity. The third signature plot provides a disk integrated picture of line formation properties in the velocity field of the granulation zone, and is a powerful tool for detecting stellar variations, differences between stars, and testing the predictive abilities of atmospheric models.

Relative velocity shifts can be obtained with considerable precision, and are currently approaching the m s\(^{-1}\) barrier (e.g. Lemke and Reiners 2016). However, the accuracy required for absolute radial velocities are more difficult to obtain given the limited accuracy of laboratory radial wavelengths added to instrumental effects that typically limit accuracy to \(\sim 100 \text{ m s}^{-1}\). Radial velocity measurements include gravitational redshifts, convective, and oscillatory effects, and include the relative motions between the observer and the source. For an engaging discussion, see Gullberg and Lindegren (2002).

5.2 Description of Third Signature Plot

Figure 5.2 shows the Standard Curve obtained for the Sun and ten other cool dwarfs and giants which have been scaled, shifted, and super-
imposed, as demonstrated by Gray (2009). It is described in the range 
$-700 \text{ m s}^{-1} < v_{\text{core}} < 200 \text{ m s}^{-1}$ by the 3rd order polynomial,

$$F(v) = 0.275 - 0.2521 \times 10^{-3}v + 0.1731 \times 10^{-6}v^2 - 0.1364 \times 10^{-8}v^3,$$ (5.2)

where $F(v)$ models the relative flux $F_c/F_{\text{cont}} = 1 - D_v$, $v$ is the line of sight velocity of the line core in m s$^{-1}$, $F_c$ is the flux in the line core and $D_v$ is the line depth. The equation facilitates comparisons with the data obtained for other stars, allowing the scale and radial velocities to be determined. An updated version of the standard curve was published by Gray and Oostra (2018), after the present analysis had been completed.

The Standard Curve can also be fitted with a logarithmic function. If the standard approach for self-similar turbulent velocity cascades is used

$$\frac{V}{V^*} = C + \frac{\ln((F/F_c) - F_0)}{\kappa},$$ (5.3)

where $V^* = -125 \text{ m s}^{-1}$, $C = 6.0$, and $\kappa = 0.4$, is the von Kármán number (von Kármán 1931), and $F_0$ is the minimum observed relative flux in a saturated line. $V^*$ is the turbulent velocity near the upper boundary, which in this case would be the inversion layer above the photosphere. The good fit of this relationship suggests that the photospheric velocity field may be caused by hydrodynamic turbulence driven by convective overshoot from the base of the photosphere.

Gray and Pugh (2012) showed that the $3\text{SP}$ obtained for the Sun could serve as a standard curve. They showed that if the radial velocities of bright giants and supergiants were scaled and shifted they would fit the standard solar $3\text{SP}$. They also, unexpectedly, found that some supergiants have a reversed $3\text{SP}$. A similar result was found for the F8 I star $\gamma$ Cygni, which is on the hot side of the granulation boundary. For cool dwarfs and giants stars, however, the standard curve appear to represent an empirical signature of the dynamical processes characteristic of stellar granulation. The overshoot velocity from the base of the photosphere shows a characteristic velocity loss with photospheric height as shown by
the 3SP. Blending is a perpetual challenge. Even apparently isolated lines can have hidden blends, which show up in the line bisectors, and shift the line core towards the blend, affecting the quality of the 3SP.

5.3 Procedure

Precise laboratory wavelengths are used to analyse the reduced exposures and obtain absolute velocity shifts and line core depths for each of the studied lines. The velocity and flux for each line core are obtained by fitting a parabola to two points on either side of the line core using a program called Therm. The procedure is repeated for all exposures, and the values for each line are then averaged and a mean and standard deviation are obtained for the radial velocity for each for the measured lines for the star. The dispersion in radial velocity includes instrumental
effects, stellar photospheric variations and shifts caused by bulk motions in the star.

The standard curve is matched to the 3SP, to provide a scale that measures the convective strength and a shift that determines the radial velocity. The scale and the zero point are standardised relative to the Sun. The use of a scaled temperature distribution based on that for β Gem, which is also a K giant, rather than a scaled solar relation, improved the line-broadening analysis and removed inconsistencies in the derived macroturbulent velocity. The main differences between models for dwarfs and giants are precisely in the upper regions of the photosphere, where the core of the strong lines are formed, since giant star photospheres are more diffuse than those of dwarfs. Once red giant 3SPs have been scaled and shifted they fit the solar standard curve well. However, compared to dwarfs, where comparison can be made with the Sun, there is increased uncertainty in the value of the zero-point representing the stellar surface.

As suggested by Gray (2009), once astrometric measurements become more precise, it will be possible to get a more precise zero-point for 3SPs, which, when combined with interferometric measurements of stellar radii, will result in an independent method for determining stellar mass from the gravitational redshift. The gravitational redshift for the Hyades giants is expected to be about $\sim 120 \text{ m s}^{-1}$, which is only a little above the uncertainty in the absolute accuracy in radial velocity measurements. Since atmospheres in giant stars are extended, gravitational redshifts can also depend on the formation height of the individual line (Dravins, Gullberg et al. 1999). The potential of obtaining precise radial velocity information can lead to values for stellar masses from the gravitational Doppler effect. The values obtained are listed in Table 5.1.

5.4 Results

3SPs were obtained for three of the four program stars: γ, ε, and δ Tauri. The CCD exposures with telluric line exposures used for wavelength calibration were used when available. There are 14 wavelength calibrated
exposures for γ Tauri, 14 for ε Tauri, only one for δ Tauri, and no exposures with absolute wavelengths for θ¹ Tauri. For γ and ε Tauri it was possible to average the results to improve the signal-to-noise ratio and calculate a likely radial velocity dispersion value for each line. For δ Tauri there was only one exposure with calibrated velocities.

![Figure 5.2: Third Signature Plot for γ Tauri](image)

The line core line depths and radial velocities shown were all obtained in this study. The information marked in the inset lists the sources of the reference wavelengths used. Red dots show the line using reference wavelengths from Nave, Johansson et al. (1994), the blue circles use wavelengths from Gray and Pugh (2012), and the blue empty circles use the reference wavelengths from Ryabchikova et al. (2015). The blue dotted line gives a scale of 0.65±0.1 and a velocity shift of 38530 ± 40 m s⁻¹, while the red dotted line gives a scale of 0.75±0.1 with a velocity shift of 38560 ± 40 m s⁻¹. The error bars show the standard errors on the mean values.

Figures 5.2, 5.3 and 5.4 show the average 3SPs for γ, δ and ε Tau. The error bars indicate the dispersion in the radial velocity values for each line over all observations. For δ Tauri, where only one exposure was used, the error bars show a typical dispersion of ±100 m s⁻¹. The values
obtained are summarised in Table 5.1. The 3SP for γ Tauri also shows an attempt to extend the line list, and demonstrates the uncertainty in the available wavelengths. For γ Tauri, the scale of the standard curve is 0.7±0.1 with a radial velocity shift of 38.55 ± 0.04 km s⁻¹. The value coincides with the spectroscopic value of +38.6 ± 0.1 km s⁻¹ found by Jofré et al. (2015), and is somewhat less than the astrometric +39.3 ± 0.1 km s⁻¹ and +38.9 ± 0.1 km s⁻¹ obtained by Perryman et al. (1998) and Madsen et al. (2002), respectively. The values show that there are systematic differences between the two approaches. The inclusion of a gravitational redshift implies that the spectroscopic approach is including a 'convective blueshift' of ∼ 700 m s⁻¹. The discrepancies for the radial velocity obtained from the 3SP of ε Tauri, shown in Figure 5.4, is similar. Jofré et al. (2015) also obtains a radial velocity of +38.5 ± 0.1 km s⁻¹ while the same astrometric studies obtain +39.4 ± 0.1 km s⁻¹. With a clearer definition of the zero point of radial velocity on the 3SP for giants, the accuracy of stellar radial velocities obtained using 3SPs could be expected to improve.

δ Tauri has a binary companion with an orbital period of P = 530 d and a radial velocity semi-amplitude of K = 2.8 km s⁻¹ (Griffin and Stroe 2012), which is similar to the 595 day orbital period of the planet around ε Tauri.

A combination of blends and insufficient accuracy in laboratory wavelengths make the dispersion high in the 3SP for ε Tauri, which increases the uncertainty of the best fit for the scale of the standard curve. The velocity dispersion is not constant with line depth, but passes through a minimum at a line depth ∼ 0.6, which approximately coincides with the maximum blueshift of the bisector of Fe I 6252.6 Å.

The Scale values increase with ζRT as expected Gray (2009), though as before the values for δ Tauri are less reliable.

The position of the zero point showing a star’s inertial reference frame has not been calibrated for giants as is it has been for the Sun. The radial velocities in Table 5.1 have been estimated based on the lower end of the standard curve, which is equivalent to a velocity of +180 m s⁻¹ for the Sun.
Figure 5.3: Third Signature Plot for δ Tauri
Since there is only one exposure with wavelengths, the errors estimated are ±100 m s$^{-1}$ and ±0.2 in relative flux. The dotted line is a 3SP with a scale of 0.9±0.1 and a shift of 37950 ± 40 m s$^{-1}$.

<table>
<thead>
<tr>
<th>Star</th>
<th>Scale</th>
<th>$V_{rad}$ m s$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>γ Tauri</td>
<td>0.7±0.1</td>
<td>38660±50</td>
</tr>
<tr>
<td>δ Tauri$^a$</td>
<td>0.9±0.1</td>
<td>38040±50</td>
</tr>
<tr>
<td>ε Tauri</td>
<td>0.8±0.1</td>
<td>38640±50</td>
</tr>
</tbody>
</table>

Table 5.1: Summary of Line Broadening Results
The Scale value shows the amount that the solar Standard Curve is scaled to fit the 3SP of the star. $V_{rad}$ gives the radial velocity of the star based on a zero point equivalent to +180 m s$^{-1}$ on the Sun. $^a$Note that δ Tauri only had one calibrated exposure and its data are less reliable than the values for the other two stars. The errors are estimated based on the uncertainties of fitting the standard curve to the data.

No correction has been made for the gravitational redshift (Equation 5.1), which for the Hyades giants is approximately 130 m s$^{-1}$.
The fit for the Standard 3SP gave a scale of 0.8 ± 0.1 and a velocity shift of 38500 ± 40 m s$^{-1}$. The error bars show the statistical standard error on the mean.

5.5 DISCUSSION

The results from the 3SP study for the Hyades giants are consistent with the results obtained in Chapters 3 and 4. The 3SP is a useful tool for studying granulation zones, providing useful measures of both convective strength and radial velocities. Some possible areas of investigation involving this procedure are considered here.

Increasing excitation potential $\chi$ has been found to increase the blue-shift of weak lines (Dravins, Lindegren et al. 1981). An attempt to determine the effect of $\chi$ on radial velocity was made by Nadeau (1988) for weak lines. However, the stronger effect of line strength suggests that a relation may be found using $\chi$ as a correction to the velocity line strength relation. Balthasar (1984) found that, in the Sun, the dominant parameter affecting
radial velocity was average height of formation of a given line, while the other parameters primarily affected height of formation. The disadvantage of basing the curve on photospheric height is that it would lose the advantages of using a directly observable value such as line depth.

The reduced dispersion in the middle of the 3SPs, at a line-depth of around 0.5, is notable, as the individual plots have a slight tendency to rotate back and forth about that point, increasing the range of values at both ends of the 3SP. Greater errors are expected in the measurement of the weaker lines. Kostik and Khomenko (2012) studied the effect of magnetism on velocity fields in the Sun, and found that magnetic fields have opposite effects on velocities in granular and intergranular regions, and that a crossover region exists (for Ba II 4554 Å), in both velocity and contrast, at a height of around 300 km above the base of the solar photosphere.

Attempts to obtain stellar absolute radial velocity measures to within 1 m s\(^{-1}\) require a consistent approach. Even for dwarfs, where the Sun gives a reliable zero point, the calibration of measurements is not trivial. In the case of giants that are being used as standard candles in distance scales, care must be taken to avoid confusing increasing precision with accuracy. When measuring hundreds of lines, the precision in determining radial velocity can be improved significantly, but the values are dependent on the range of photospheric heights where the lines are formed and the part of the line which is being used. The application of a convective blueshift as a correction to radial velocity is prone to error. Saar and Fischer (2000) showed significant correlations of around 30% between radial velocities and magnetic activity measured using a Ca II emission index, \(S_{IR}\). The use of the 3SP could provide a consistent approach to calibrating radial velocity surveys and correcting existing radial velocity data.

The line-depth velocity curves, also known as 3SPs, were studied for \(\gamma\), \(\delta\) and \(\epsilon\) Tauri. The velocity scale \(S\) was obtained, which compares the range of the stellar photospheric velocities to that of the Sun. The logarithmic nature of these velocity profiles suggest the possibility of a velocity deficit caused by a hydrodynamic turbulent cascade.
TEMPERATURE DIFFERENCES

In this chapter, two different spectroscopic approaches are used to obtain the temperature differences between the granular and intergranular components of each star, as well as the temperature differences between the program stars.

Gray (2010b) described how the study of a line bisector and a star’s 3SP can be combined to obtain the difference in flux between photospheric granular and intergranular components. If a filling factor for the granular component is assumed, a temperature difference between granules and lanes can be obtained. The second approach uses the fact that depths for some spectral lines are more sensitive to temperature variations than others. By choosing suitable pairs of temperature sensitive and temperature insensitive lines, the variation of line depth ratios can be used to measure temperature differences. This method, which was used by Gray and Brown (2001) to measure temperature differences in red giants, is used here to measure the temperature differences between the Hyades giants.

6.1 FLUX DEFICIT

6.1.1 Procedure

The 3SP is derived from the line cores, which originate in the strongest part of the absorption line, corresponding to contributions from the brighter, blue-shifted granular component. The line bisector is obtained from the radial velocity distribution of the total flux. In other words, at a given line depth, a bisector is obtained from a range of photospheric temperatures. The 3SP comes from the hottest, brightest granular components from lines of different line strengths.
In order to separate the two components, one can begin by asking, if the 3SP were a bisector, what would its line profile be? In other words, a simulated line profile is constructed which has the 3SP as its bisector. This process is described as ‘mapping’ the bisector to the 3SP, as shown in Figure 6.1. The difference between the original line produced by flux from the whole disk and the constructed line produced by the granules gives the flux deficit.

The mapping is carried out by adjusting the original line profile to a new profile which has a bisector that coincides with the 3SP. The procedure involves increasing the flux values on the red side of the profile to obtain the required bisector. See, for example, Figure 6.2. There are two profiles: the blue line is the original profile representing the total flux, while the magenta line has been mapped to the 3SP and represents the granular flux. The flux deficit, representing the velocity distribution of the intergranular component of the line (shown in red), is obtained by subtracting the granular line profile from the original profile. That provides an empirical, model-independent measure of the flux deficit between the granular and intergranular streams, and a measure of the mean velocity separation between the two components. Since the errors in relative flux translate into wavelength errors proportional to the gradient of the line profile wavelength, errors are greatest in the wings and core of the line. That leads to effects such as the offset between the two lines at the bottom of Figure 6.1.

The peak flux deficit is normalised with the core depth of the original profile, providing a line depth ratio for the granular to the intergranular components. The area or EW of the flux deficit also needs to be normalised, and it has been found that by comparing with half the EW of the original profile, the flux ratios are in close agreement with values obtained in disk-resolved observations of the Sun.

Gray (2010a) obtained a flux ratio of 0.89 for the Sun using the above procedure. His value coincides very closely with the disk resolved value of $\approx 0.88$ obtained by Sánchez Cuberes et al. (2000) at a limb distance of $\mu = \cos \theta = 0.6$. That ensures consistency of the procedure, since the
linear limb darkening coefficient for solar-type stars is \( \varepsilon \approx 0.6 \), and the strongest contribution to a disk integration is at a limb distance where \( \mu = \varepsilon \).

The mapping process requires attention to detail since there is crosstalk, where the adjustment of a particular point can affect the position of adjacent points. Successive approximations are required until an accurate result is obtained. The procedure could be automated by interpolating points on the red side of the profile to correspond to line depths of points on the blue side.

A measure of the velocity difference between the granular and lane fluxes is obtained by measuring \( v_L \), the velocity at the flux deficit half-maximum, at the low velocity side of the distribution. The half maximum

Figure 6.1: Mapping Procedure for \( \gamma \) Tauri using Fe I 6252.6 Å
The purple line shows the original mean bisector. The cyan line shows the standard 3SP for \( \gamma \) Tau. Once the flux on the red side of the mean profile has been adjusted, the mean bisector is described by the red crosses which coincide with the original 3SP.
Figure 6.2: Bisector Mapping for γ Tauri
The red side of the original profile (blue line) is adjusted until its bisector (originally blue dots) coincides with the star’s 3SP (purple dotted line) giving the granular velocity distribution (purple). The difference between the original and the granular profile is the flux deficit (red).

is taken in order to obtain a more accurate parameter, since the slope is more inclined and the velocity can be determined with greater precision than at the peak value. The blue side of the flux deficit is preferred since the individual line depth values are large and the relative error of the difference is considerably smaller than on the red side (Gray 2015). The values are tabulated in Table 6.1.
6.1.2 Results

The flux deficits for $\gamma$, $\delta$, and $\epsilon$ Tauri are plotted in Figures 6.2, 6.3, and 6.4, and the results are summarised in Table 6.1.

The 3SP for $\delta$ Tauri is based on the only exposure for the star that had calibrated wavelengths, so there is increased uncertainty with its flux deficit values. As in the other studies, $\delta$ and $\epsilon$ Tauri have similar flux deficits and velocity differences $v_L$. The hot stream filling factor is assumed to be 0.5. The estimated temperature differences between the two streams for equal area filling factors are similar for both $\delta$ and $\epsilon$ Tauri, and over 20% larger than the value for $\gamma$ Tauri. If we take the flux ratio to be $f$, then
from the Stefan-Boltzmann law the temperature difference $\Delta T$ is given by,

$$\Delta T = \left(\frac{1}{\sqrt{f}} - 1\right) T. \quad (6.1)$$

<table>
<thead>
<tr>
<th>Star</th>
<th>Flux Deficit</th>
<th>Flux Ratio</th>
<th>$\Delta T$ (K)</th>
<th>$v_L$ (km s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$ Tau</td>
<td>12.0%</td>
<td>0.88</td>
<td>161</td>
<td>2.5 ± 0.2</td>
</tr>
<tr>
<td>$\delta$ Tau</td>
<td>14.8%</td>
<td>0.85</td>
<td>207</td>
<td>2.6 ± 0.4</td>
</tr>
<tr>
<td>$\epsilon$ Tau</td>
<td>14.5%</td>
<td>0.85</td>
<td>203</td>
<td>2.7 ± 0.2</td>
</tr>
</tbody>
</table>

**Table 6.1: Summary of Flux Deficit Results**

The table shows the values of the apparent flux deficit obtained from the bisector mapping. The flux ratio is the ratio of the flux in the hot rising to the cold descending streams. The flux ratio corresponds to a temperature difference given by Equation 6.1.
The flux deficits are compared in Figure 6.5. As with other measures, γ Tauri shows a marked difference, with a peak intensity 20% less than the other two stars.

Figure 6.5: Flux Deficits for three Hyades Giants
Flux deficits are plotted versus the velocity relative to line centre, for ε, δ and γ Tauri.

Figure 6.6 shows the flux velocity measure $v_L$ is plotted against the scale values from Chapter 5. The results are compared with data from other stars taken from (Gray 2010b). The $3SP$ scale factor indicates the range of photospheric velocities compared to the Sun.

The plot has γ and ε Tauri a little higher than in previous published results, while the values for δ Tauri reflect the greater uncertainty caused by the small number of observations which were averaged for the bisector, while the $3SP$ was based on only one exposure.
6.2 LINE-DEPTH RATIOS

6.2.1 Background

Line strength has been used since the pioneering days of stellar spectral classification, based originally on equivalent widths. Over the last few decades, with widespread access to high resolution spectra, precise measurements of line depths can be made. These values rely on the determination of only the local continuum and the line core flux levels, and are less subject to the perennial blends found in cool star spectra.

A well chosen Line-depth ratio (LDR) provides a precise measure of photospheric temperature difference. The strength of lines with low exci-
ation potentials $\chi$, are sensitive to temperature differences. Line pairs are chosen, which are free from blends in the core, to have similar wavelengths to reduce errors from changes in the continuum level, and to have contrasting excitation potentials. The LDRs are calibrated using temperatures of known red giants. Temperature differences can be determined to within a few K, though the absolute values depend on the stellar models, which typically have uncertainties ten times larger.

LDRs were first used by Gray (1989) to determine the precise spectral type of red giants. The procedure was developed together with calibrated LDRs for cool dwarfs by Gray and Johanson (1991). The approach was refined in several articles during the following decade, and then applied to giants by Gray and Brown (2001) and later to the Sun (Gray 2004). The technique is now widely applied to measure stellar activity, study starspots, and to search for rotational modulation in active stars.

6.2.2 Procedure

LDRs were obtained for the four Hyades giants to determine temperature differences. A temperature for each star was also obtained. Several LDR ratios were studied and 7 chosen for study. The primary objective was to determine the temperature differences between the stars and the temperature sensitivity of each LDR. The mean value for each LDR was plotted versus a previously published value for each stellar temperature. The temperatures published in Gray and Brown (2001) derived from R-I photometry were used as those values are less affected by lines than B-V values. The R-I values also gave a temperature for $\delta$ Tauri higher than $\theta^1$ Tauri, which was consistent with the LDR values. The gradient $\frac{dT}{dR}$, (the rate of change of temperature with respect to the LDR), is itself not constant and increases with temperature (Gray and Brown 2001). Over the 90K range found in these stars, it was assumed that the relationship remains linear. Three of the LDRs gave linear regression values of $R^2 > 0.98$. From the linear relations, the temperature sensitivity of each LDR was ob-
Table 6.2: Relative Temperatures of the Hyades Giants

The temperature differences with respect to $\varepsilon$ Tauri are precise to within a few degrees, though the uncertainties in the actual temperature are ten times greater. The differences with respect to Gray and Brown (2001) data are the result of $dT_{\text{eff}}/dR$ being determined for a small sample of similar stars. The data in the fourth column are averages from the Pastel Catalogue (Soubiran et al. 2016).

<table>
<thead>
<tr>
<th>Star</th>
<th>Temperature $T_{\text{eff}}$ (K)</th>
<th>$\Delta T_{\text{eff}}$ (K)</th>
<th>Gray and Brown (2001)</th>
<th>Soubiran et al. (2016)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$ Tauri</td>
<td>4946±25</td>
<td>58±1</td>
<td>4938</td>
<td>4975</td>
</tr>
<tr>
<td>$\delta$ Tauri</td>
<td>4984±25</td>
<td>96±2</td>
<td>4965</td>
<td>4946</td>
</tr>
<tr>
<td>$\varepsilon$ Tauri</td>
<td>4888±25</td>
<td>0</td>
<td>4882</td>
<td>4915</td>
</tr>
<tr>
<td>$\theta^1$ Tauri</td>
<td>4967±25</td>
<td>79±2</td>
<td>4962</td>
<td>5012</td>
</tr>
</tbody>
</table>

The results of the LDR studies are shown in Table 6.2. The temperature sensitivities obtained are compared to those published by Biazzo, Frasca et al. (2007) for stars with $T_{\text{eff}} = 5000$ K. Slight variations caused by differences in metallicity and luminosity are expected between these stars and others of similar spectral types.

Table 6.2 shows the temperature differences obtained for the Hyades Giants compared with results in Gray and Brown (2001) and the mean values of the Pastel Catalogue (Soubiran et al. 2016). It is of interest to note that all the LDRs indicate a higher temperature for $\delta$ Tauri, whereas several studies indicate that $\theta^1$ Tauri is warmer. Beck et al. (2015) suggested increasing the B-V colour magnitude for $\theta^1$ Tauri by a little more than 1% to allow for a F8 V companion. That could explain why most studies find $\theta^1$ Tauri to be the hotter of the two.

It is known that $\theta^1$ Tauri has a binary companion, which may explain the large difference between photometric and spectroscopically derived temperatures. The actual temperatures of these stars are not well determined by this method and can shift up or down by tens of degrees.
The calculated effective temperatures lead to the luminosity and absolute magnitude values shown in Table 6.3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>γ Tauri</th>
<th>δ Tauri</th>
<th>ε Tauri</th>
<th>θ1 Tauri</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective Temperature $T_{\text{eff}}$</td>
<td>K</td>
<td>4946±25</td>
<td>4984±25</td>
<td>4888±25</td>
<td>4967±25</td>
</tr>
<tr>
<td>Luminosity $L$</td>
<td>$L_\odot$</td>
<td>81±2</td>
<td>84±2</td>
<td>94±2</td>
<td>72±2</td>
</tr>
<tr>
<td>Absolute Magnitude $M_V$</td>
<td>mag</td>
<td>- 0.02</td>
<td>- 0.06</td>
<td>- 0.19</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 6.3: Luminosities and Absolute Magnitudes

Effective temperatures are shown together with the luminosity and absolute magnitudes derived using the MLD radii shown in Table 1.2.

Temperature differences were found for the four Hyades giants. The results are consistent with previous studies although here δ Tauri has been found to be 16±3 K hotter than θ1 Tauri.

An effective empirical method for separating granular from intergranular photospheric components was applied to three of the Hyades stars. Peak flux deficit velocities of around 5 km s$^{-1}$ are consistent with values found for other giants that range between 4.5 to 5 km s$^{-1}$ (Gray 2010b). The temperature differences of around 150 to 190 K are somewhat higher than the values of 130-140 K obtained for the Sun.

The method of using LDRs to determine temperature differences is well established. The use of LDRs with similar stars within an open cluster is particularly effective. Biazzo, Pasquini et al. (2007) used the technique to study stellar temperatures of stars in the IC 4651 cluster with positive results. In the present study, the similarity between the program stars constrains the possible range of $dT_{\text{eff}}/dR$ ratios and makes it possible to suggest probable values of $T_{\text{eff}}$.

Temperature differences have been studied using two different spectroscopic approaches. First, a flux deficit for the intergranular flux was obtained for γ, δ, and ε Tauri by comparing line bisectors with 3SPs. The flux deficits were used to derive temperature differences between the upward and downward photospheric flows for each star. The velocity shifts
$v_L$ for the flux deficit were obtained, providing another indicator of the strength of the photospheric velocity field. Second, several LDRs were investigated and six were selected based on freedom from blends and temperature sensitivity, which depends on the difference between the excitation potentials of the individual lines in the pair. The gradient of the effective temperature $T_{\text{eff}}$ to line-depth ratio $R$ ($dT_{\text{eff}}/dR$) was determined for each LDR in an iterative process starting from published effective temperatures. Precise temperature differences $\Delta T_{\text{eff}}$ between the program stars were obtained, together with values for their effective temperatures.
7.1 Discussion of Results

7.1.1 Line Broadening

In Chapter 3, projected rotational velocities $v \sin i$ and macroturbulent dispersions $\zeta_{RT}$ were obtained from the line broadening analysis. The results are summarised in Table 7.1. The $v \sin i$ values are similar to previous results, as shown in Section 1.2.5. The values of $\zeta_{RT}$ also lie within the expected range for K giants (Gray 2005b). The broadening values can be modelled consistently to within $0.1$ km s$^{-1}$. However, for $\delta$, and $\theta^1$ Tauri, both of which have fewer observations, the uncertainty in $v \sin i$ is estimated to be closer to $0.2$ km s$^{-1}$ from the model fitting process explained in Chapter 3.

Microturbulent dispersion $\xi$, values in the range of $0.8 - 1.5 \pm 0.1$ km s$^{-1}$ were found, with a clear tendency for $\xi$ to increase with line strength, corresponding to increasing photospheric height, as has been suggested previously (Gray 1981; Takeda 1992; Gray 2005b).

A number of authors have reported increasing blueshift with increasing excitation potential $\chi$, corresponding to greater photospheric depth (Balthasar 1984; Krempec-Krygier and Turlo 1987; Nadeau 1988; Dravins

<table>
<thead>
<tr>
<th>Parameter (Units)</th>
<th>$\gamma$ Tauri</th>
<th>$\delta$ Tauri</th>
<th>$\epsilon$ Tauri</th>
<th>$\theta^1$ Tauri</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v \sin i$ (km s$^{-1}$)</td>
<td>3.0±0.1</td>
<td>3.2±0.2</td>
<td>3.4±0.1</td>
<td>2.7±0.2</td>
</tr>
<tr>
<td>$\zeta_{RT}$ (km s$^{-1}$)</td>
<td>4.6±0.1</td>
<td>5.3±0.2</td>
<td>5.1±0.1</td>
<td>4.6±0.2</td>
</tr>
<tr>
<td>Luminosity $L$ $L_\odot$</td>
<td>81</td>
<td>84</td>
<td>94</td>
<td>72</td>
</tr>
</tbody>
</table>

Table 7.1: Summary of Broadening Results
and Larsson 1984). The high \( \xi \) values of the V I lines are consistent with the lines being formed high in the photosphere. If confirmed, a correction would need to be applied to lines with differing excitation potential in order to fit on a 3SP.

7.1.2 Line Asymmetry

The velocity range of the bottom half of the averaged C-shaped bisectors varied from 90 m s\(^{-1}\) for \( \varepsilon \) Tauri to 130 m s\(^{-1}\) for \( \gamma \) Tauri, as shown in Table 4.1. Both \( \delta \) and \( \theta^1 \) Tauri have the same velocity range of 100 m s\(^{-1}\). Their values correspond well to the higher magnetic activity levels in \( \gamma \) Tauri and the lower activity in \( \varepsilon \) Tauri, as suggested by UV, X-ray, and radio observations. A campaign using several instruments with a high sampling rate has found solar-like oscillations on several timescales in \( \theta^1 \) Tauri (Beck et al. 2015). The pulsations could partially explain the large variability in line bisectors observed.

<table>
<thead>
<tr>
<th>( \gamma ) Tauri</th>
<th>( \delta ) Tauri</th>
<th>( \varepsilon ) Tauri</th>
<th>( \theta^1 ) Tauri</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of Blue Bump ((F/F_c))</td>
<td>0.63</td>
<td>0.66</td>
<td>0.61</td>
</tr>
<tr>
<td>Span ((m s^{-1}))</td>
<td>130</td>
<td>97</td>
<td>93</td>
</tr>
<tr>
<td>Luminosity ( L ) ((L_\odot))</td>
<td>81</td>
<td>84</td>
<td>94</td>
</tr>
</tbody>
</table>

Table 7.2: Line Bisector Data

7.1.3 Velocity-Core Depth Relation

The 3SP is used to study the photospheric velocity field and determine the velocity scale \( S \), which measures the strength of the convective overshoot in the stellar photosphere. The derived velocity spans and radial velocities are again consistent with expected variations in convective strength. The values are shown in Figure 7.3.

Bisector and 3SP analyses are combined to measure the flux deficit between the granular and intergranular components. The granular flux is
consistently blueshifted by around 5 km s\(^{-1}\) with respect to the flux deficit, which contributes 5.9% of the total flux for \(\gamma\) Tauri, 6.4% for \(\delta\) Tauri, and 7.1% for \(\epsilon\) Tauri, as shown in Table 7.3. The trend of decreasing activity from \(\gamma\) to \(\epsilon\) Tauri is again apparent, though the flux deficit appears to decrease with increasing stellar activity.

The results show that velocity scale \(S\), bisector span, and the flux deficit velocity at half maximum \(v_L\), all show increasing values with increasing magnetic activity from \(\epsilon\), through \(\delta\) and \(\theta^1\), to \(\gamma\) Tauri.

### 7.1.4 Temperature Differences

Two spectroscopic approaches were used to determine temperature variations. The first method used the bisector and 3SP to determine the flux deficit of the photospheric cold intergranular stream. The second method used line-depth ratios to determine temperature differences between the stars.

The analysis of line-depth ratios provided precise temperature differences between the program stars to ±2 K. The actual values could be constrained by about ±25 K, though the probable values are further restricted by \(\epsilon\) Tauri being at the minimum and \(\delta\) Tauri at the maximum of their reported temperature ranges. A comparable result was found by Cayrel de Strobel et al. (1970) and Kovtyukh et al. (2006) based on line-depth ratios, while Lambert and Ries (1981) and Hekker and Meléndez (2007) obtained equal temperatures for \(\delta\) and \(\theta^1\) Tauri using equivalent widths. As shown

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Units</th>
<th>(\gamma) Tauri</th>
<th>(\delta) Tauri</th>
<th>(\epsilon) Tauri</th>
<th>(\theta^1) Tauri</th>
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</thead>
<tbody>
<tr>
<td>Scale (S)</td>
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<td>0.7</td>
<td>0.9</td>
<td>0.8</td>
<td>–</td>
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<tr>
<td>(v_L) km s(^{-1})</td>
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<td>2.75</td>
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<td>–</td>
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<tr>
<td>Flux Deficit</td>
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<td>6.4</td>
<td>7.1</td>
<td></td>
</tr>
<tr>
<td>Luminosity (L) (L_\odot)</td>
<td></td>
<td>81</td>
<td>84</td>
<td>94</td>
<td>72</td>
</tr>
</tbody>
</table>

Table 7.3: Bisector Mapping Results
in Table 1.1, most papers give a higher temperature for θ1 Tauri than for δ Tauri, pointing to the possible influence of a companion on photometric studies.

7.2 COMPARISON OF HYADES GIANTS

From Ca II H & K and X-ray data, ε Tauri has been shown to be the least active of the Hyades giants (Choi et al. 1995, Collura et al. 1993). It is also the only program star that has a planet detected (Sato et al. 2007).

Direct calculation of $P \sin i$ implied by the values of $v \sin i$ and radius yields periods within the range of 200 – 300 d. The estimated mean equatorial velocity found here of $v \approx 3.9 \text{ km s}^{-1}$, implies rotational periods of $P \approx 150 – 170 \text{ d}$. That is similar to the $P = 140 \text{ d}$ period found by Choi et al. (1995) for θ1 Tauri from Ca II H & K studies, which is found to be consistent with the results obtained here and with $P = 138 \text{ d}$ observed by Beck et al. (2015).

The Rossby number Ro was discussed in Section 1.2.7. It is obtained from the ratio of rotation and convective velocities, but can also be obtained from their corresponding timescales. Ro has been found to be a good predictor of dynamo activity in both dwarfs and giants (Gondoin 2005). Choi et al. (1995) obtained $Ro = 0.89$ for θ1 Tauri, by combining observed rotational periods with theoretical convective turnover times from Gilliland (1985) adapted to giants by Hall (1991). A similar value of $Ro = 0.8$ was obtained by Aurière, Konstantinova-Antova et al. (2015).

It is known that the Sun, though normally magnetically active, will occasionally go into extended inactive periods such as the Maunder minimum of the late 17th century, so it can be surmised that the solar value for the Rossby number is close to the critical value. The $v \sin i$ and rotational periods are compatible with a common equatorial velocity for all the Hyades giants of $v \approx 3.9 \text{ km s}^{-1}$, or about twice the solar value of $v_\odot$. The other scaling factor is the depth of the convective zone. A value $Ro_\odot = 1$, assuming γ and ε Tauri have Rossby numbers just below and just above 1 respectively, implies a relative depth of the convective zone...
### Table 7.4: Calculation of Rossby Number

Ro is calculated, normalising to solar $\text{Ro}_\odot = 1$, and the depth of the convective zone. The depth of the convective zone is taken as $d_{\text{CZ}} = 0.28 R_\star$, for the Hyades giants.

$L_c = 0.28 R_\star$. This depth is a little under the $L_c = 0.36 R_\star$ calculated by Aurière, Konstantinova-Antova et al. (2015) for $\epsilon$ Tauri under the assumption that it is in a core helium burning stage.

The estimated values for Ro are shown in Table 7.4, indicating a magnetically active $\gamma$ Tauri with $\text{Ro} = 0.89$, and a marginally dormant $\epsilon$ Tauri with $\text{Ro} = 1.02$. The value of $\text{Ro} = 1.15$ for $\delta$ Tauri is not consistent with this scenario, as it is slightly more active than $\epsilon$ Tauri. However, the scale $S$ for $\delta$ Tauri was obtained from one observation, and more data would be needed to increase the reliability of these measurements. These values of Ro are consistent with $\gamma$ and $\theta^1$ Tauri being the two most active Hyades giants.

### 7.3 Final Comments

One of the questions driving this study was the wish to investigate whether high resolution spectroscopy could suggest an explanation for the lower activity levels shown by $\epsilon$ Tauri, in the hope that it may provide clues regarding possible causes of solar grand minima events. The results in that respect are intriguing. $\epsilon$ Tauri has a higher mass, luminosity, and enhanced metallicity compared with the other Hyades giants, suggesting that it is slightly more evolved. The star’s position at the edge of the
coronal boundary also suggests that the lower activity level could be an evolutionary effect rather than a transitory grand minimum.

However, recent studies suggest that the Sun itself, the prototype of a grand minimum star, has $R_o \simeq R_{o_{\text{crit}}}$ (Metcalfe et al. 2016). It is possible that main-sequence stars older than the Sun no longer brake according to the Skumanich (1972) $t^{-1/2}$ law because they pass a value of $R_{o_{\text{crit}}}$ (L. Kitchatinov and Nepomnyashchikh 2017). Slight changes in the convective zone characteristics or the distribution of internal angular momentum of a star around the transition stage would turn the dynamo on and off. The scenario appears to be that cool main sequence stars gradually brake through magnetic coupling with a wind until the rotation slows to a stage where the $\alpha \omega$ dynamo turns off. In the case of cool dwarfs it is the gradual braking that reduces the rotation rate until the dynamo criterion is no longer met (L. Kitchatinov and Nepomnyashchikh 2017). With giants it would be the developing strength of convection as the coronal boundary is reached that would stop the dynamo from operating. In both of these cases coronal emission has been observed to shut down.

The similarity between the giant and main sequence situations suggests the following three hypotheses. First, that the coronal boundary is caused by developing convection zones in giants until the dynamo criterion is no longer valid. At this stage toroidal fields no longer develop and an open poloidal field supports a cool wind. Second, that grand minima are found in stars with $R_o \simeq R_{o_{\text{crit}}}$, since at this stage small variations in stellar properties could turn the dynamo on and off. Third, since changes in the convective zone are likely to be monotonic, slight variations in the distribution of angular momentum of a star’s planetary system may be of fundamental importance in producing the grand minima. That may explain why historic proxies for solar activity show common periodicities with angular momentum distribution in the solar system (Wolf 1859; Abreu et al. 2012; Charbonneau 2013).

The position of $\varepsilon$ Tauri on the HR diagram places it at the edge of the coronal boundary. It has a larger radius than the other Hyades giants, and is further along its evolutionary track. At this stage it is not possible
to determine whether or not the processes that make \(\epsilon\) Tauri less active are similar to those that cause grand minima periods in the Sun, which typically appear after hundreds of years (E. H. Lee and D. Y. Lee 2007). There are no observations of an active \(\epsilon\) Tauri in the past, so the evidence currently favours evolutionary factors at the coronal boundary.

O. C. Wilson and Bappu (1957), in describing the relation of the width of the chromospheric emission reversal in Ca II H and K lines with stellar luminosity, suggest that the most probable explanation is chromospheric turbulence. They describe a possible scenario where the energy of the upward convective flow, which depends only on the luminosity, is gradually converted to turbulence, ‘the mean velocity spread of which bears some simple relationship to the original outward flow velocity.’ The dependence of microturbulence on photospheric height and convective overshoot velocity, and whether this physical process involves hydrodynamic turbulence is still to be determined.
Appendices
### Designations for the Hyades Giants

<table>
<thead>
<tr>
<th>Bayer Designation</th>
<th>Symbol (Units)</th>
<th>γ Tauri</th>
<th>δ Tauri</th>
<th>ε Tauri</th>
<th>θ¹ Tauri</th>
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<tr>
<td>Common Name</td>
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<td>Prima Hyadum</td>
<td>Secunda Hyadum</td>
<td>Ain</td>
<td>-</td>
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<tr>
<td>Flamsteed Designation</td>
<td>-</td>
<td>54 Tau</td>
<td>61 Tau</td>
<td>74 Tau</td>
<td>77 Tau</td>
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<td>van Bueren Number</td>
<td>vB</td>
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<td>41</td>
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<td>1373</td>
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<td>Henry Draper Catalogue</td>
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<td>27371</td>
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<td>20455</td>
<td>20889</td>
<td>20885</td>
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<td>Rot</td>
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<td>3851</td>
<td>3871</td>
<td>3872</td>
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<td>04h 22m</td>
<td>04h 28m</td>
<td>04h 28m</td>
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<td>56.093s</td>
<td>36.999s</td>
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<td>+17° 32’</td>
<td>+19° 10’</td>
<td>+15° 57’</td>
<td></td>
</tr>
<tr>
<td></td>
<td>39.52”</td>
<td>33.05”</td>
<td>49.54”</td>
<td>43.85”</td>
<td></td>
</tr>
</tbody>
</table>

Table A.1: Data for the Hyades Giants
Data from the SIMBAD database (Wenger et al. 2000); except items marked (a) Hipparcos Catalogue (van Leeuwen 2007), (b) (Boyajian et al. 2009), (c) Derived values, (d) (Gray and Endal 1982), (e) (Preston 1984), (f) (Beck et al. 2015), #Głębocki and Gnacinski (2003)
LIST OF EXPOSURES

Note: The Exposure File Nomenclature Format: (MMDDYYC.###), where MM are letters for the month, DD and YY are numbers for the day and year, C is the Observer Code and ### is a consecutive number for the observer. The GD number is related to the S/N of the exposure: $S/N = \sqrt{6.67 \times GD}$. The lamps are the exposures taken of the telluric lines before and after each stellar exposure for wavelength calibrations. Some exposures do not have telluric lamps and could not have their wavelengths calibrated.

<table>
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<th>Observer</th>
<th>Code</th>
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<td>David F. Gray</td>
<td>M, Q</td>
</tr>
<tr>
<td>J Postma</td>
<td>P</td>
</tr>
<tr>
<td>P Alexander</td>
<td>E</td>
</tr>
<tr>
<td>–</td>
<td>U</td>
</tr>
<tr>
<td>J Power</td>
<td>N</td>
</tr>
<tr>
<td>Kevin Brown</td>
<td>J, W</td>
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</table>

Table B.1: Observer Codes
<table>
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<th>Exposures</th>
<th>Julian Date</th>
<th>GD</th>
<th>Lamps</th>
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<td>OC1808Q.478</td>
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Table B.2: CCD Exposures for γ Tauri

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Table B.3: CCD Exposures of δ Tauri
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Table B.4: CCD Exposures of ε Tauri

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</table>

Table B.5: CCD Exposures of θ¹ Tauri
In Table C.1, ID# indicates the wavelength in tenths of an Angstrom above 6200 Å. The Ref column is Nave for Nave, Johansson et al. (1994), and G&P for Gray and Pugh (2012).

Excitation potentials χ are from the SIMBAD database (Wenger et al. 2000).

The Application headings indicate LDR for Line-Depth Ratio temperature studies, 3SP for Third Signature Plots and FT for Fourier Transform broadening analyses.

The LDR column indicates numbered pairs for lines used for the line depth ratio temperature analysis. For example, pair number 1 consists of Ni I 6224.0 Å and V I 6224.5 Å. The latter line was also used in pair number 2 with Fe I 6226.7 Å.

The 3SP column has an ‘x’ if that line was used in the line depth-velocity analysis. The FT column indicates “s” for strong lines used for microturbulent dispersion ξ, or “s” for weak lines if they were used in the Fourier analysis to determine rotation v sin i, macroturbulent dispersion ζRT, and microturbulence ξ. Line 526 (Fe I 6252.6 Å) was used for the bisector analysis.

The lines marked Q in the species column have not been identified. However, the properties of these lines is similar to that of Fe I lines.
<table>
<thead>
<tr>
<th>Species</th>
<th>ID#</th>
<th>Ref</th>
<th>Wavelength (Å)</th>
<th>$\chi$ (eV)</th>
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<td>Nave</td>
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<td>x w</td>
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<td>Fe I</td>
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<td>1 x</td>
</tr>
<tr>
<td>V I</td>
<td>245</td>
<td>G&amp;P</td>
<td>6224.5054</td>
<td>0.287</td>
<td>1,2 x w</td>
</tr>
<tr>
<td>Fe I</td>
<td>267</td>
<td>Nave</td>
<td>6226.7359</td>
<td>3.883</td>
<td>2 w</td>
</tr>
<tr>
<td>Q</td>
<td>275</td>
<td>G&amp;P</td>
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<td>-</td>
<td>x</td>
</tr>
<tr>
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<td>Nave</td>
<td>6229.2279</td>
<td>2.845</td>
<td>x w</td>
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<tr>
<td>Ni I</td>
<td>301</td>
<td>G&amp;P</td>
<td>6230.0903</td>
<td>4.105</td>
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<td>Nave</td>
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<tr>
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<td>Nave</td>
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<td>3.654</td>
<td>3 x s</td>
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<td>G&amp;P</td>
<td>6233.1983</td>
<td>0.275</td>
<td>3 x</td>
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<tr>
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<td>373</td>
<td>G&amp;P</td>
<td>6237.3175</td>
<td>5.614</td>
<td>x w</td>
</tr>
<tr>
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<td>384</td>
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<td>6238.3855</td>
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<td>x w</td>
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<tr>
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<td>G&amp;P</td>
<td>6239.3622</td>
<td>2.807</td>
<td>x w</td>
</tr>
<tr>
<td>Fe I</td>
<td>406</td>
<td>Nave</td>
<td>6240.6458</td>
<td>2.223</td>
<td>x w</td>
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<tr>
<td>V I</td>
<td>428</td>
<td>G&amp;P</td>
<td>6242.8270</td>
<td>0.262</td>
<td>x w</td>
</tr>
<tr>
<td>V I</td>
<td>431</td>
<td>G&amp;P</td>
<td>6243.1080</td>
<td>0.301</td>
<td>x</td>
</tr>
<tr>
<td>Si I</td>
<td>438</td>
<td>G&amp;P</td>
<td>6243.8150</td>
<td>5.616</td>
<td>x</td>
</tr>
<tr>
<td>Si I</td>
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<td>G&amp;P</td>
<td>6244.4693</td>
<td>5.616</td>
<td>x</td>
</tr>
<tr>
<td>V I</td>
<td>452</td>
<td>G&amp;P</td>
<td>6245.2190</td>
<td>0.262</td>
<td>x</td>
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<tr>
<td>Sc II</td>
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<td>G&amp;P</td>
<td>6245.6185</td>
<td>1.507</td>
<td>x w</td>
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<tr>
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<td>463</td>
<td>Nave</td>
<td>6246.3184</td>
<td>3.602</td>
<td>x s</td>
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<tr>
<td>Fe II</td>
<td>476</td>
<td>G&amp;P</td>
<td>6247.5592</td>
<td>3.892</td>
<td>x</td>
</tr>
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<td>518</td>
<td>G&amp;P</td>
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<td>4 x w</td>
</tr>
<tr>
<td>Fe I</td>
<td>526</td>
<td>Nave</td>
<td>6252.5550</td>
<td>2.404</td>
<td>4 x s</td>
</tr>
<tr>
<td>Fe I</td>
<td>538b</td>
<td>Nave</td>
<td>6253.8305</td>
<td>4.733</td>
<td>x</td>
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<tr>
<td>Fe I</td>
<td>543</td>
<td>Nave</td>
<td>6254.2581</td>
<td>2.279</td>
<td>x s</td>
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<tr>
<td>Q</td>
<td>560</td>
<td>G&amp;P</td>
<td>6255.9503</td>
<td>-</td>
<td>5 x w</td>
</tr>
<tr>
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<td>564</td>
<td>Nave</td>
<td>6256.3611</td>
<td>2.453</td>
<td>5.6 x</td>
</tr>
<tr>
<td>V I</td>
<td>569</td>
<td>G&amp;P</td>
<td>6256.8995</td>
<td>0.275</td>
<td>6 x</td>
</tr>
<tr>
<td>Ti I</td>
<td>581</td>
<td>G&amp;P</td>
<td>6258.1032</td>
<td>1.443</td>
<td>x</td>
</tr>
<tr>
<td>Ti I</td>
<td>611</td>
<td>G&amp;P</td>
<td>6261.1012</td>
<td>1.439</td>
<td>x</td>
</tr>
<tr>
<td>V I</td>
<td>569</td>
<td>G&amp;P</td>
<td>6256.8995</td>
<td>0.275</td>
<td>x w</td>
</tr>
<tr>
<td>Fe I</td>
<td>651b</td>
<td>Nave</td>
<td>6265.1336</td>
<td>2.176</td>
<td>x s</td>
</tr>
<tr>
<td>Fe I</td>
<td>702</td>
<td>Nave</td>
<td>6270.2246</td>
<td>2.858</td>
<td>x w</td>
</tr>
</tbody>
</table>

Table C.1: Spectral Line Data
Figure D.1: δ Tauri - Strong Line Thermal and Residual Transforms

Figure D.2: δ Tauri - Weak Line Thermal and Residual Transforms
Figure D.3: δ Tauri - Wavelength Domain model check
Figure D.4: $\epsilon$ Tauri - Strong Line Thermal and Residual Transforms

Figure D.5: $\epsilon$ Tauri - Weak Line Residual Transforms
Figure D.6: ε Tauri - Wavelength Domain model
Figure D.7: $\theta^1$ Tauri - Strong Line Residual Transforms

Figure D.8: $\theta^1$ Tauri - Weak Line Residual Transforms
Figure D.9: θ¹ Tauri - Wavelength Domain model
Bash shell routines were written to call several Fortran routines at a time, input data into each program, change Key files, adjust input parameters and rename input and output files. The Fortran programs were written by David Gray and made available to me for the analyses.

Though I did not have access to the actual code, most programs have Keyfile and other datafiles as needed. Input and output files are written to scratch files, which are renamed for storage or for input into other routines. Several of the programs were eventually operated in batch mode, using Unix Bash shell routines.

**FORTRAN PROGRAMS**

**DISPER** - Wavelength scale established using telluric lines and the continuum level is set

**BCC** - Obtains the Barometric Correction and Julian Date for a given observation. David Gray, and research assistant David Holmgren, based on Stumpff (1979)

**SPLT4** - Establishes continuum level for stellar spectra. Includes **FIXER** which removes cosmic ray spikes.

**EXTRACT** - Used to extract the wavelength and flux information of individual lines from an exposure. The position of the lines is determined by a mask which can be shifted to allow for variations in the line position from exposure to exposure.

**LINAVE 4** - Used to find the average profile for a given line.

**SPIMOD** - Models the Specific Intensity. It is also used to determine the limb darkening of a given model
**THERM**4 - Finds the flux values and wavelengths of line cores by fitting a parabola

**FTAVE**

Takes the vector average of Fourier transforms. Used to obtain the average Residual Transform

**MODEL2** - Calculates $T(\tau)$, the temperature distribution with optical depth for a star with a given $T_{\text{eff}}$, $[\text{Fe/H}]$ and $\log g$. Based on model atmosphere by Ruland et al. (1980) for the K0 III star $\beta$ Geminorum.

**DISKI5** - Calculates the disk integrated distribution for a given $v \sin i$, $\zeta_{\text{RT}}$ and fourth order limb darkening coefficients.

**FLUXMOD2** - Determines the line profile for a given absorbing species for a particular stellar model given by MODEL2

**BISM** - Obtains line bisectors for a given line profile

**BIAVE** - Averages line bisectors

**FTP3** - Obtains Fourier transforms and performs vector divisions

It uses the Fast Fourier Transform (FFT) algorithm (see Gray 1976 Appendix C).

**SLINE** - The line is modelled with using atomic data for the individual species and given values of $\xi$ and EW.

**BLUR** - Convolves 2 or 3 profiles
Figure E.1: Broadening Analysis Flowchart

This is a working Flowchart that I prepared to help understand the process, and the various file inputs and outputs and datafiles used in the Line Broadening analysis. Eventually many of these multi-step branches were automated using Bash Unix scripts.
FTPKEYS

0  [0]IA, set to 1 for AVTRAN input (archaic). Normally 0
0  [0]JN, set to 1 for manual input of number of points, N, normally 0
0/ [0]IBP, set to 1 or 2 to bypass computation of 1 or 2 transforms
0  [0]IASK, set to 0 for standard axes ranges; 1 for custom
1  [1]KSTEP, set to 0 for manual input of STEP; 1 for A; 2 for km/s via DLAM
0  [0]Kbell, 0=no bell; 1=do TR1.sc; 2=do TR2.sc; 3=do both
0  [0]Kbase, 0=no base subtraction; otherwise 20 wing-point ave off
0  [0]Krec = 1 to ask for deconvolution back to lambda domain
0  [1]Imat format key; 0=*, 1=single column F12.4
0  [0]Kscale = 1 to scale tr2 to tr1
-6251 [-4550]default wavelength; negative to ask
0  [0]Kbl = 0 for auto find baseline, NOT 0 to ask
1  [1]Kzp = 1 to get zero padding to increase resolution

DIKEYS

0  [0]IA, set to 1 for AVTRAN input (archaic). Normally 0
0  [0]JN, set to 1 for manual input of number of points, N, normally 0
0  [0]IBP, set to 1 or 2 to bypass computation of 1 or 2 transforms
0  [0]IASK, set to 0 for standard axes ranges; 1 for custom
1  [1]KSTEP, set to 0 for manual input of STEP; 1 for A; 2 for km/s via DLAM
0  [0]Kbell, 0=no bell; 1=do TR1.sc; 2=do TR2.sc; 3=do both
0  [0]Kbase, 0=no base subtraction; otherwise 20 wing-point ave off
0  [0]Krec = 1 to ask for deconvolution back to lambda domain
0  [1]Imat format key; 0=*, 1=single column F12.4
0  [0]Kscale = 1 to scale tr2 to tr1
-6251 [-4550]default wavelength; negative to ask
0  [0]Kbl = 0 for auto find baseline, NOT 0 to ask
1  [1]Kzp = 1 to get zero padding to increase resolution

Figure E.2: Examples of Key Files
MSigma.sh

#!/bin/sh

# Shell script to run Obtain m(sigma) - The Tfm of the Macro broadening profile
# Runs DSKi5 for a given vsini and ZetaRS and then runs FTP3 with one input
# Parameters after ./Msigma.sh line vsini ZetaRT

# $1 is Line
# $2 is vsini
# $3 is ZetaRT

# 1. Check dikeys has right wavelength (av of residual profiles)
# 2. Check that ftp3 NOT bypass the FT
# 3. Check that Disper was run for a reasonable value of delta Lambda

Line=$1
vsini=$2
ZetaRT=$3

#
# DISKI5
#
#
Wavelength="62"$Line:0:2":"$Line:2:1"

echo "Running DSKi5 for vsini = "$vsini "and Zeta =" "$ZetaRT"

DSKI5 << EOF
$Wavelength
$vsini
$ZetaRT
$ZetaRT
EOF

echo "* Copying di2.sc to TR1.sc"
cp di2.sc tr1.sc

#
# FTP3
#
#
Inputs=1
cp ftpkeys_standard ftpkeys

echo "* Running FTP3 with $Inputs input for $Wavelength"

FTP3 << EOF
$Wavelength
$Inputs
EOF

EOF

echo "* Import ResTransF from Tr3.sc into AXUM"

Figure E.3: Example of a Bash Script
#!/bin/sh

# Shell script to run convert Residual Profile to m(sigma)obs - # to compare with the Tfm of the
# Macrobroadening profile
# Parameters after ./RT_Msigma.sh line MS

# $1 is Line # $2 is MS
# Copies FTAINLineMS to FTAIN.sc eg FTAIN473WA
# Runs FTAVE
# Copies FAVECTOR to TR1.sc
# Copies IPLine to TR2.sc
# Copies FTPKeys_Bypass to FTPKeys # Runs FTP3

echo " "
echo "*"
echo "* Running RT_Msigma.sh " echo "*

Line=$1 MS=$2
IPFile="IP"$Line FTAINFile="FTAIN"$Line$MS

echo "IPFile is $IPFile"
echo "FTAINFile is $FTAINFile"

#
# FTAVE #
Wavelength="62"${Line:0:2}"."${Line:2:1}
cp $FTAINFile FTAIN.sc
echo "Running FTAVE for Line = $Wavelength"

FTAVE << EOF 1
EOF

#
# FTP3 #

Inputs=2

cp ftpkeys_bypass ftpkeys cp FAVECTOR Tr1.sc
cp $IPLine Tr2.sc

echo "# Running FTP3 in bypass mode with $Inputs inputs for $Wavelength"

FTP3 << EOF $Wavelength $Inputs
0 EOF

echo "Copy Msigma-obs TR3.sc to AXUM"
echo "(Copy Av Res Trans TR6.sc if needed for comparing RT)"

Figure E.4: Bash Script to Obtain Macrobroadening transform
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PUBLICATIONS


