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Partial Material Strength Reduction Factors for ACI 318

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Abstract

The strength reduction factors, ϕ, defined in ACI 318-14 for different structural actions and elements lead to inconsistent results. This study proposes partial material strength reduction factors for concrete, ϕ_c, and reinforcing steel, ϕ_s, that yield similar design strengths and more consistent reliability indices. Three structural actions are investigated: moment; shear; and, combined moment and axial force. The first-order, second-moment method is used to compute reliability indices for moment and shear, and Monte Carlo simulation is used for combined moment and axial force. The statistical parameters assumed for the professional factor for shear strength significantly impact the reliability indices. Although no single combination of ϕ_s and ϕ_c is the best for these three actions, the recommended partial material strength reduction factors are ϕ_s of 0.90 and ϕ_c of 0.60, or for spirally reinforced columns, 0.70. Alternatively, for shear, the combination with ϕ_s of 0.80 and ϕ_c of 0.65 is recommended.

Keywords

reinforced concrete; partial material strength reduction factors; moment; shear; slabs; beams; columns; design strengths; reliability.

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Notation

a	depth of equivalent rectangular stress block in ACI 318-14
a_r	depth of equivalent rectangular stress block for partial material strength reduction factors format
a_v	shear span, equal to distance from center of concentrated load to either: (a) face of support for continuous or cantilevered members, or (b) center of support for simply supported members
A	area of compression segment of circular section
A_g	gross area of section
A_i	structural load
A_s	area of nonprestressed longitudinal tension reinforcement
A_{si}	area of the *i*th layer of reinforcement
A_{st}	total area of nonprestressed longitudinal reinforcement
A_v	area of shear reinforcement within spacing s
$A\overline{y}$	moment of compression segment about center of circular section
b	width of compression face of member
b_w	web width
B_i	modeling parameter
c	distance from extreme compression fiber to neutral axis
c_i	influence coefficient
C_c	nominal compressive force in concrete
C_{ij}	name of strength reduction factor combination
C_{rc}	factored compressive force in concrete for partial material strength reduction factors format
d	effective depth, equal to distance from extreme compression fiber to centroid of longitudinal tension reinforcement
d_i	distance from extreme compression fiber to the *i*th layer of reinforcement, where $i = 1$ refers to the reinforcement located furthest
d_t	distance from extreme compression fiber to extreme layer of tension steel
D	effect of service dead load
D_i	simulated value of dead load

e eccentricity

e_i simulated value of eccentricity

e_r design eccentricity for partial material strength reduction factors format, equal to M_r/P_r

e_{rbal} eccentricity corresponding to balanced failure for partial material strength reduction factors format

$(e_{rbal})_{max}$ maximum eccentricity corresponding to balanced failure for partial material strength reduction factors format

$(e_{rbal})_{min}$ minimum eccentricity corresponding to balanced failure for partial material strength reduction factors format

e_u design eccentricity for ACI 318-14, equal to $\phi M_n/\phi P_n$

e_{ubal} eccentricity corresponding to balanced failure for ACI 318-14

$(e_{ubal})_{max}$ maximum eccentricity corresponding to balanced failure for ACI 318-14

$(e_{ubal})_{min}$ minimum eccentricity corresponding to balanced failure for ACI 318-14

E_s modulus of elasticity of reinforcement

$f(\bullet)$ function of resistance or load effect in limit state function

f_c' specified compressive strength of concrete

f_c^* reduced compressive strength of concrete

$f_{c,i\text{-}p}$ in-place compressive strength of concrete

f_s stress in reinforcement at service loads

f_s^* reduced stress in reinforcement

f_{si} stress in the ith layer of reinforcement

f_y specified yield strength for nonprestressed reinforcement

f_y^* reduced yield strength for nonprestressed reinforcement

f_{yt} specified yield strength of transverse reinforcement

$F_{i\text{-}p}$ factor to account for variation of in-place strength

F_r factor to account for rate-of-loading effects

F_{rsi} factored force in the ith layer of reinforcement for partial material strength reduction factors format

F_{si} nominal force in the ith layer of reinforcement

F_1 ratio of mean 28-day control cylinder strength to specified 28-day strength

F_2	ratio of mean in-place strength at 28 days to mean 28-day cylinder strength
$g(\bullet)$	limit state function
G	geometric property
h	overall thickness, height, or depth of member
l	span length of member
L	effect of service live load
L_i	simulated value of live load
M	material strength property
M_i	simulated value of flexural strength
M_n	nominal flexural strength at section
M_r	design flexural strength for partial material strength reduction factors format
M_u	factored moment at section
n	number of samples
P	professional factor
P_f	probability of failure
P_i	simulated value of axial strength
P_n	nominal axial strength of member
$P_{n,max}$	maximum nominal axial compressive strength of member
P_{nt}	nominal axial tensile strength of member
P_r	design axial strength for partial material strength reduction factors format
$P_{r,max}$	maximum design axial compressive strength for partial material strength reduction factors format
P_{rt}	design axial tensile strength for partial material strength reduction factors format
P_u	factored axial force
Q	load effect in limit state function, mean = $\bar{Q}$, standard deviation = σ_Q, coefficient of variation = V_Q
Q_i	load effect for the *i*th type load
Q_M	load effect in limit state function for flexural member
Q_V	load effect in limit state function for member resisting shear force
R	resistance in limit state function, mean = $\bar{R}$, standard deviation = σ_R,

coefficient of variation = V_R

R_M resistance in limit state function for flexural member

R_V resistance in limit state function for member resisting shear force

s center-to-center spacing of transverse reinforcement

S_n nominal strength

T_D factor to account for transformation from dead load to dead load effect

T_{Di} simulated value of transformation from dead load to dead load effect

T_L factor to account for transformation from live load to live load effect

T_{Li} simulated value of transformation from live load to live load effect

U required strength computed using factored load combinations

V coefficient of variation

V_c nominal shear strength provided by concrete

V_n nominal shear strength

V_r design shear strength for partial material strength reduction factors format

V_{rc} design shear strength provided by concrete for partial material strength reduction factors format

V_{rs} design shear strength provided by shear reinforcement for partial material strength reduction factors format

V_s nominal shear strength provided by shear reinforcement

V_u factored shear force at section

w_D specified dead load per unit length

w_L specified live load per unit length

X_i resistance or load variable, mean = $\bar{X}_i$, standard deviation = σ_{X_i}

X_i^* resistance or load variable at linearizing point

Z ratio of strain in extreme tension layer of reinforcement to yield strain

Z limit state function, mean = $\bar{Z}$, standard deviation = σ_Z

α_M design flexural strength ratio, equal to design flexural strength obtained using strength reduction factors in ACI 318-14 to that obtained using partial material strength reduction factors

α_{PM} design combined flexural and axial strength ratio, equal to design combined flexural and axial strength obtained using strength reduction factors in ACI

318-14 to that obtained using partial material strength reduction factors

α_V design shear strength ratio, equal to design shear strength obtained using strength reduction factors in ACI 318-14 to that obtained using partial material strength reduction factors

β reliability index

β_M reliability index for moment

β_{Mr} reliability index for moment obtained using partial material strength reduction factors

β_{Mu} reliability index for moment obtained using strength reduction factors in ACI 318-14

β_{PM} reliability index for combined moment and axial force

β_{PMr} reliability index for combined moment and axial force obtained using partial material strength reduction factors

β_{PMu} reliability index for combined moment and axial force obtained using strength reduction factors in ACI 318-14

β_V reliability index for shear

β_{Vr} reliability index for shear obtained using partial material strength reduction factors

β_{Vu} reliability index for shear obtained using strength reduction factors in ACI 318-14

β_1 factor relating depth of equivalent rectangular compressive stress block to depth of neutral axis

γ ratio of distance between outer layers of reinforcement in column to overall column depth

δ bias coefficient

ε_{cu} maximum usable strain at extreme concrete compression fiber

ε_{si} strain in the ith layer of reinforcement, where i = 1 refers to the reinforcement located furthest from extreme compression fiber

ε_t net tensile strain in extreme layer of longitudinal tension reinforcement at nominal strength

ε_{ty} yield strain in extreme layer of longitudinal tension reinforcement

ε_y	yield strain of reinforcement, equal to f_y/E_s
θ	angle used to calculate compression segment of circular column
λ	modification factor to account for reduced shear strength of lightweight concrete
ρ	longitudinal reinforcement ratio, equal to ratio of A_s to bd
ρ_g	total reinforcement ratio, equal to ratio of total longitudinal reinforcement area to cross-sectional area of column
ρ_t	transverse reinforcement ratio, equal to ratio of area of distributed transverse reinforcement to gross concrete area perpendicular to that reinforcement
σ	standard deviation
ϕ	strength reduction factor in ACI 318-14
ϕ_c	partial material strength reduction factor for concrete
ϕ_s	partial material strength reduction factor for reinforcing steel
ϕM_n	design flexural strength in ACI 318-14
ϕP_n	design axial strength in ACI 318-14
$\phi P_{n,max}$	maximum design axial compressive strength in ACI 318-14
ϕP_{nt}	design axial tensile strength in ACI 318-14
ϕV_n	design shear strength in ACI 318-14
$\Phi(\bullet)$	cumulative distribution function of standard normal distribution

Chapter 1

1 Introduction

1.1 Introduction

In the current Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14) (ACI Committee 318 2014), the basic requirement for strength design is

$$\phi S_n \geq U \qquad [1.1]$$

where: ϕ is the strength reduction factor; S_n is the nominal strength; and, U is the required strength computed using the factored load combinations. The overall strength reduction factor, ϕ, accounts for "the probability of understrength due to variations of in-place material strengths and dimensions, the effect of simplifying assumptions in the design equations, the degree of ductility, potential failure mode of the member, the required reliability, and significance of failure and existence of alternative load paths for the member in the structure" (ACI Committee 318 2014).

For members resisting moment, axial force, combined moment and axial force, or shear, Table 1.1 shows strength reduction factors, ϕ, defined in Chapter 21 of ACI 318-14. The strength reduction factor for shear equals 0.75, while for moment, axial force, or combined moment and axial force, the strength reduction factor ranges from 0.65 to 0.90. The additional requirements shown in Table 1.2, which is identical to Table 21.2.2 in ACI 318-14, need to be applied to determine the exact value for a specific combination of moment and axial force for spirally reinforced or tied columns. The strength reduction factor in these cases depends on the net tensile strain in the extreme layer of longitudinal reinforcement, ε_t, which is determined assuming a linear strain distribution and a strain in the extreme compression fiber of 0.003 at nominal strength. This is shown in Figure 1.1, which is identical to Fig. R21.2.2a in ACI 318R-14. If ε_t is greater than or equal to 0.005, the section is defined as tension-controlled, or if it is less than or equal to ε_{ty}, the section is compression-controlled, where ε_{ty} is the yield strain in the extreme tension layer of reinforcement, equal to f_y/E_s for deformed reinforcement, f_y is the specified yield strength for nonprestressed reinforcement, and E_s is the modulus of elasticity of reinforcement. Between the two limits of 0.005 and ε_{ty}, a transition occurs between the strength reduction factor for moment for lightly reinforced sections, 0.90, and that for axial force combined with moment, 0.65 or 0.75 for tied or spirally reinforced columns, respectively. This is shown in Figure 1.2, which is identical to Fig. R21.2.2b in ACI 318R-14.

The overall strength reduction factor presented in ACI 318-14, ϕ, has some shortcomings that have been identified by others, as follows:

1. For a member subjected to combined moment and axial force, an odd variation happens within the transition region (e.g., Gamble 1998, 2015). Figure 1.3, which is similar to figures generated by Gamble (2015) shows the interaction diagrams for a 325 mm square column with eight bars distributed equally in four faces. The ratio of the distance between the outer layers of reinforcement in a column to the overall column depth, γ, is 0.6 and the ratio of total reinforcement area to the cross-sectional area of column, ρ_g, is 0.01. The specified compressive strength of concrete, f_c', and f_y are 25 MPa and 420 MPa, respectively. The interaction diagram derived using the strength reduction factors in ACI 318-14 shows an inconsistency in the transition region compared to the nominal strength interaction diagram. If partial material strength reduction factors are applied, e.g. $f_c^* = 0.6f_c'$ and $f_y^* = 0.9f_y$ ($f_s^* = 0.9f_s$) recommended by Gamble (2015), the inconsistency disappears, where f_c^* is the reduced compressive strength of concrete, f_y^* is the reduced yield strength for nonprestressed reinforcement, f_s is the stress in reinforcement at service loads, and f_s^* is the reduced stress in reinforcement.

   Similarly, Figure 1.4 is generated for a 325 mm diameter circular column with eight evenly distributed bars and ties. Similar to the square column, the current ACI 318-14 strength reduction factors create an awkward transition that is eliminated when the partial reduced material strengths are used (Gamble 2015).

2. Figure 1.5, originally created by Lequesne and Pincheira (2014), shows the design interaction diagram obtained using ACI 318-11 strength reduction factors which are identical to those for ACI 318-14, for an L-shaped wall section. In this case and for other sections with wide flanges, the results are again unreasonable: on the right side of point B, the flexural and axial strengths increase simultaneously with the increasing eccentricity when the compression zone stress block extends into the web and $\varepsilon_{ty} \leq \varepsilon_t \leq 0.005$. The reason for the increasing design axial strength, ϕP_n, with the increasing eccentricity is that ϕ increases at a proportionally higher rate than the nominal axial strength, P_n, decreases. This results in non-unique moment capacities for one axial strength level between points A and B as shown (Lequesne and Pincheira 2014).

3. For members subjected to shear, the statistical parameters for professional factor have significant changed. The professional factor is defined as a value observed experimentally divided by the value predicted using the actual geometric and material properties and so quantities the accuracy of an equation for resistance. For example, the bias coefficient and coefficient of variation of the professional factor reported by Somo and Hong (2006) are equal to 1.47 and 0.36, respectively, for beams with stirrups and shear span-to-depth ratio larger than or equal to 2. In the original calibration of the ACI strength reduction factors, values of 1.09 and 0.12 were adopted by Israel et al. (1987). Similar values of 1.075 and 0.10 were recommended by Nowak and Szerszen (2003). These changes may significantly affect the reliability, so the strength reduction factor needs to be reevaluated. Specifically the higher bias coefficient reported by Somo and Hong (2006) will increase the reliability and so permit use of a greater strength reduction factor. The higher coefficient of variation, however, has the opposite effect.

4. A single overall value of ϕ cannot clarify the contributions of concrete and reinforcing steel, and variabilities of their strengths, so partial material strength reduction factors may yield advantages for reinforced concrete (Israel et al. 1987).

It is noteworthy that Canadian Standard CSA-A23.3 “Design of Concrete Structures” (CSA 2014) has used partial resistance factors for the concrete and steel material strengths since 1984.

1.2 Objective

The objective of this study is to select partial material strength reduction factors for concrete and reinforcing steel that yield similar design strengths to those obtained using the current ACI 318-14 provisions. Similarly, the reliability indices corresponding to the proposed strength reduction factors should be similar to or more appropriate than those corresponding to the current provisions.

Three structural actions acting on nonprestressed members shall be investigated: moment; one-way shear; and, combined moment and axial force. For members subjected to moment, the full range of flexural reinforcement ratios, corresponding to those in two-way slabs, one-way slabs and beams shall be investigated. Similarly, for members subjected to one-way shear, a realistic range of shear reinforcement ratios shall be investigated. For members subjected to combined moment and axial force, realistic total reinforcement ratios and reinforcement arrangements in the cross sections shall be investigated. Design strengths will be compared based on the current and partial material strength reduction factors. Statistical parameters related to the reliability index calculation will be collected from the literature. The first-order, second-moment (FOSM) reliability analysis method will be applied for members subjected to moment or shear. Monte Carlo simulation will be used to determine the reliability of members subjected to combined moment and axial force because of the complexity of the equations necessary to generate interaction diagrams.

1.3 Outline

In Chapter 2, potential partial material strength reduction factors that yield similar design strengths as the current ACI 318-14 provisions are investigated. For members subjected to moment, the design strengths of singly reinforced two-way slabs, one-way slabs and beams corresponding to ACI 318-14 and the partial material strength reduction factors are calculated for different concrete compressive strengths, f_c', and ratios of nonprestressed longitudinal tension reinforcement, ρ. For members subjected to one-way shear, the design strengths of beams with different transverse reinforcement ratios, ρ_t, and f_c' are studied. For members subjected to combined moment and axial force, five column sections are investigated: square section with three bars in each face; square section with three bars in two end faces only; square section with three bars in two side faces only; circular section with eight evenly distributed bars and ties; and, circular section with eight evenly distributed bars and spiral reinforcement. For each column section, different γ, f_c', and ρ_g are studied. For each structural action, appropriate partial material strength reduction factors are proposed.

Chapter 3 presents the reliability model and the first-order, second-moment (FOSM) method. It then summarizes statistical parameters for geometric properties, material strengths, professional factors and load effects collected from the literature. The calculated reliability indices for members subjected to moment or one-way shear are presented for the different geometric and material properties, and two live-to-dead load ratios. Partial material strength reduction factors are then proposed based on the reliability analyses and the results obtained in Chapter 2.

Chapter 4 presents the reliability analyses for columns conducted by Monte Carlo simulation. Different geometric and material properties, and two live-to-dead load ratios are investigated. The applied moment and axial force are assumed perfectly correlated and reliability indices are computed for a range of specific eccentricities. Again, appropriate partial material strength reduction factors are proposed based on the reliability analyses and the results obtained in Chapter 2.

Chapter 5 presents the summary, conclusions, and suggestions for future work.

Appendices A, B and C present supplementary tables, figures and Matlab (Version R2016b; The Mathworks, Inc. 2016) codes that complement the material presented in Chapters 2, 3 and 4, respectively.

Table 1.1: Strength reduction factors, ϕ, in ACI 318-14

Action or structural element	ϕ
Moment, axial force, or combined moment and axial force	0.65 to 0.90
Shear	0.75

Table 1.2: Strength reduction factors, ϕ, for moment, axial force, or combined moment and axial force, in ACI 318-14

		ϕ	
		Type of transverse reinforcement	
Net tensile strain ε_t	Classification	Spiral	Other
$\varepsilon_t \leq \varepsilon_{ty}$	Compression-controlled	0.75	0.65
$\varepsilon_{ty} < \varepsilon_t < 0.005$	Transition	$0.75+0.15\dfrac{(\varepsilon_t-\varepsilon_{ty})}{(0.005-\varepsilon_{ty})}$	$0.65+0.25\dfrac{(\varepsilon_t-\varepsilon_{ty})}{(0.005-\varepsilon_{ty})}$
$\varepsilon_t \geq 0.005$	Tension-controlled	0.90	0.90

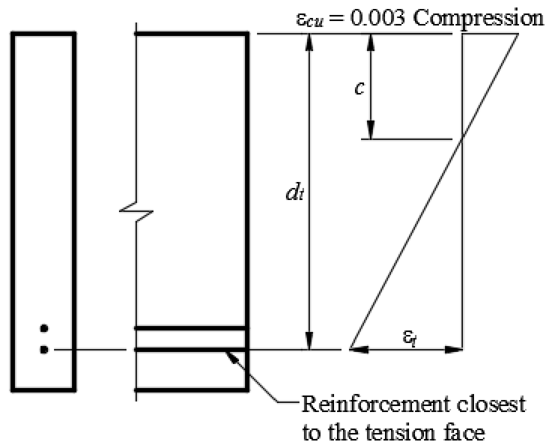


Figure 1.1: Strain distribution and net tensile strain in a nonprestressed member (ACI Committee 318 2014)

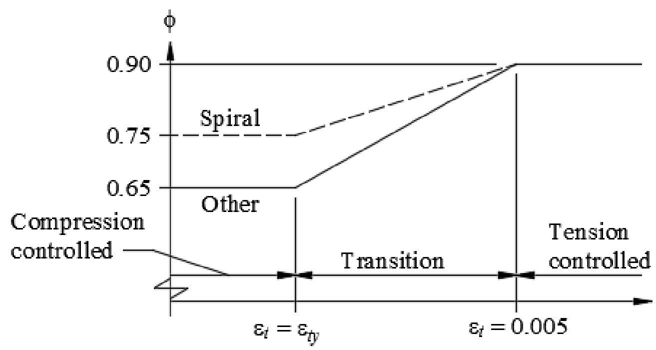


Figure 1.2: Variation of ϕ with net tensile strain in extreme tension reinforcement, ε_t (ACI Committee 318 2014)

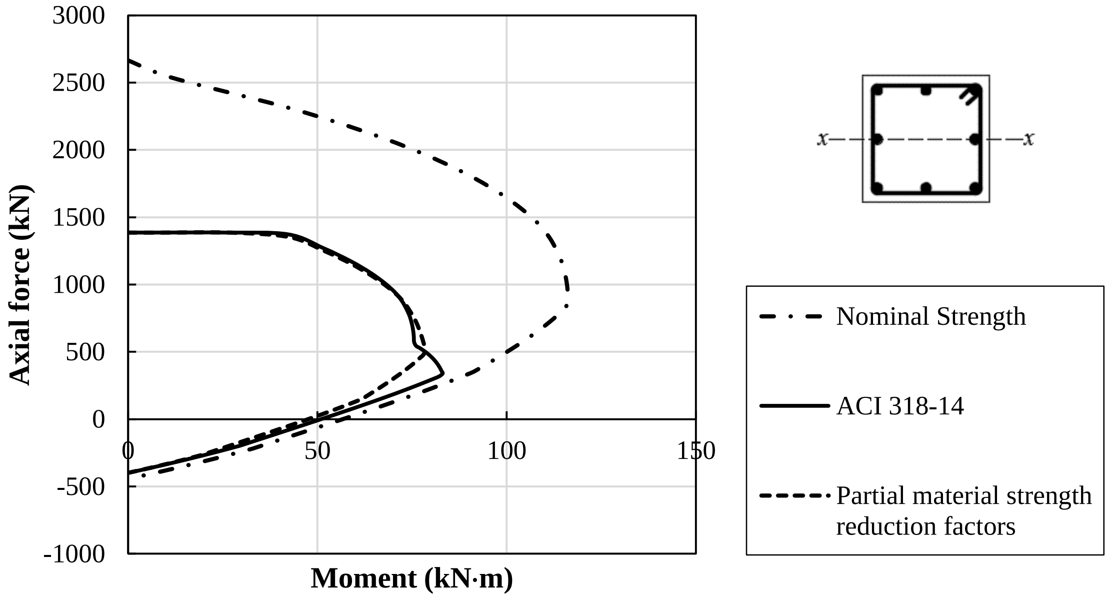


Figure 1.3: Interaction diagrams for a square column

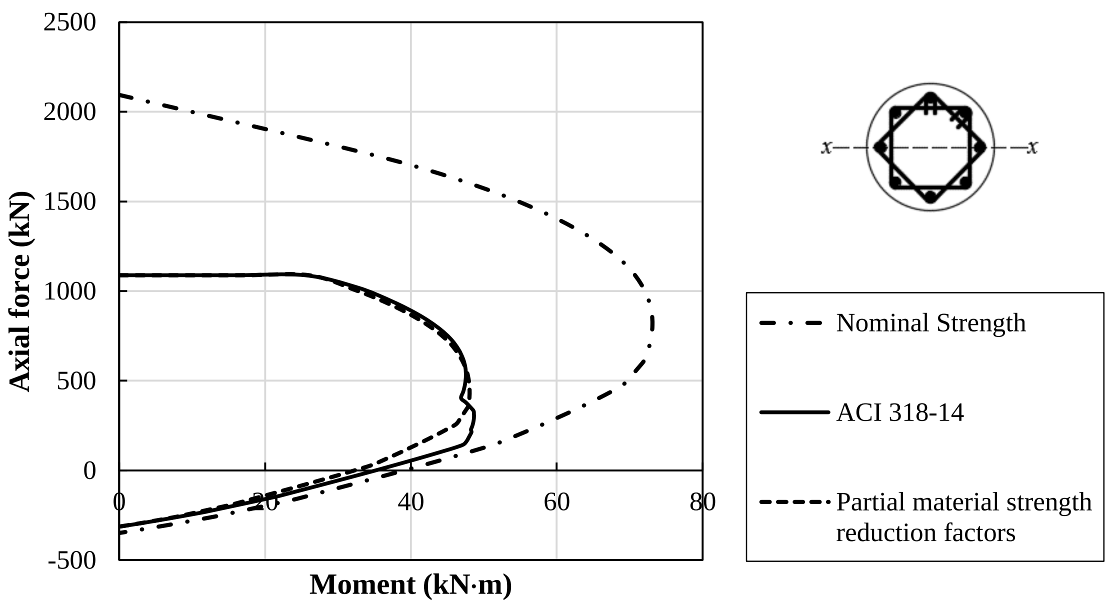


Figure 1.4: Interaction diagrams for a circular tied column

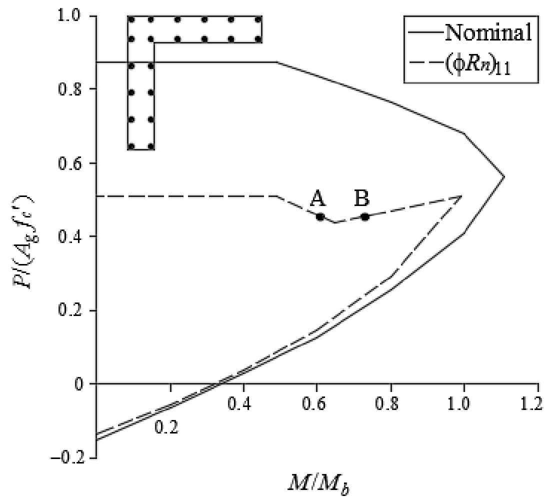


Figure 1.5: Interaction diagrams for an L-shape wall (Lequesne and Pincheira 2014)

Chapter 2

2 Derivation of Partial Material Strength Reduction Factors Based on Design Strengths

2.1 Introduction

Chapter 21 of Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14) (ACI Committee 318 2014) specifies an overall strength reduction factor, ϕ, for reinforced concrete elements, based on the structural action being resisted. The overall objective of this thesis is to propose partial material strength reduction factors for concrete and reinforcing steel that are independent of the structural action.

The objective of this chapter is to identify suitable partial material strength reduction factors that best duplicate the design strengths obtained using the current ACI 318-14 provisions. The preliminary results obtained in this chapter indicate the potential ranges of the best partial material strength reduction factors, and the final decision will be made after conducting reliability analyses presented in Chapters 3 and 4.

2.2 Methodology

This chapter develops partial material strength reduction factors for cross sections resisting three structural actions: moment; one-way shear; and, combined moment and axial force. For each action, design strengths are computed using ACI 318-14 and various partial material strength reduction factors, ϕ_c for concrete and ϕ_s for reinforcing steel. Table 2.1 shows the sixteen partial material strength reduction factor combinations considered in this study. The C_{ij} notation shown represents a particular combination, where i is the i-th value of ϕ_s and j is the j-th value of ϕ_c.

The calculations are conducted using Microsoft Excel (Version 2013; Microsoft 2013) and Matlab (Version R2016b; The Mathworks, Inc. 2016) to compute the design strength ratio, which is defined as the design strength obtained using the strength reduction factor in ACI 318-14 to that obtained using a particular pair of partial material strength reduction factors. Design strength ratios greater than 1 represent cases where the ACI 318-14 design strengths exceed those computed using the proposed values, and so indicate that the proposed values are more conservative. For this investigation, the best combination of partial material strength reduction factors will give design strengths that most closely approximate those obtained using the current ACI 318-14 provisions. This corresponds to the mean design strength ratio approaching 1 with the least standard deviation. Reliability analyses based on these preliminary results will be presented in Chapters 3 and 4.

2.3 Moment

This section presents proposed partial material strength reduction factors that most closely approximate the design flexural strengths obtained using the ACI 318-14 criteria. The ranges of geometric and material parameters are quantified and the design flexural strength equations corresponding to the current ACI 318-14 and the partial material strength reduction factors formats are presented. Typical design flexural strength ratios, α_M, for each combination of partial material strength reduction factors are presented, and the means and standard deviations for each combination are quantified. The sensitivities of the design flexural strength ratios to the partial material strength reduction factors, for various geometric and material properties are investigated. The best factor combinations are recommended.

2.3.1 Geometric and Material Properties

The investigation of moment is limited to rectangular singly reinforced cross sections designed based on ACI 318-14 with a specified reinforcement yield strength, f_y, of 420 MPa and specified concrete compressive strengths, f_c', of 25 and 45 MPa. These material strengths represent the range of strengths commonly used in flexural members. Three ranges of reinforcement ratio are investigated: 0.003 to 0.005, which is representative of two-way slabs; 0.006 to 0.010, which is representative of one-way slabs; and, 0.011 to 0.018, which is representative of beams. The reasons for selecting these three ranges are: they reflect typical reinforcement ratio ranges for slabs and beams; minimum and maximum reinforcement ratio limits are satisfactory in all cases; and, the upper limit of the studied range of beams is defined by the maximum reinforcement ratio for a beam with f_c' of 25 MPa. The same maximum reinforcement ratio is used for beams with f_c' of 45 MPa. One layer of reinforcing steel is assumed.

2.3.2 Design Strength Ratios

Design flexural strength equations corresponding to the current ACI 318-14 and partial material strength reduction factors formats are defined in this section. The equations and definitions below refer to ACI 318-14, MacGregor and Bartlett (2000), and Wight (2016).

In ACI 318-14, the depth of the equivalent rectangular stress block, a, is computed as (Wight 2016)

$$a = \frac{A_s f_y}{0.85 f_c' b} \quad [2.1]$$

where: A_s is the area of the nonprestressed longitudinal tension reinforcement; f_y is the specified yield strength for nonprestressed reinforcement; f_c' is the specified compressive strength of concrete; and, b is the width of the compression face of the member. The design flexural strength, ϕM_n, is (Wight 2016)

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2} \right) \quad [2.2]$$

where: ϕ is the strength reduction factor in ACI 318-14; M_n is the nominal flexural strength at a section; and, d is the distance from the extreme compression fiber to the centroid of the longitudinal tension reinforcement.

For the proposed method, the depth of the equivalent rectangular stress block, a_r, based on the partial material strength reduction factors format presented in MacGregor and Bartlett (2000), is assumed to be

$$a_r = \frac{\phi_s A_s f_y}{0.85 \phi_c f_c' b} \quad [2.3]$$

where: ϕ_s is the strength reduction factor for reinforcing steel; and, ϕ_c is the strength reduction factor for concrete. The design flexural strength for partial material strength reduction factors method, M_r, is (MacGregor and Bartlett 2000)

$$M_r = \phi_s A_s f_y \left(d - \frac{a_r}{2} \right) \tag{2.4}$$

For the longitudinal reinforcement ratio, ρ, defined as (ACI Committee 318 2014)

$$\rho = \frac{A_s}{bd} \tag{2.5}$$

the design flexural strength ratio, α_M, is

$$\alpha_M = \frac{\phi M_n}{M_r} = \frac{\phi \left(1 - \frac{\rho}{1.7} \cdot \frac{f_y}{f_c'} \right)}{\phi_s \left(1 - \frac{\phi_s \rho}{1.7 \phi_c} \cdot \frac{f_y}{f_c'} \right)} \tag{2.6}$$

For each strength reduction factor combination, design flexural strength ratios, α_M, were calculated with respect to longitudinal reinforcement ratios, ρ. Also, each calculation was done twice because two specified compressive strengths of concrete, f_c', were studied. In particular, for f_c' of 25 MPa, the strength reduction factors in ACI 318-14, ϕ, are not always equal to 0.90 in the range of reinforcement ratios studied, e.g., ϕ is equal to 0.86 for ρ of 0.017 and ϕ is equal to 0.83 for ρ of 0.018. As mentioned in Chapter 1, when the section is tension-controlled, i.e. the net tensile strain in the extreme layer of longitudinal tension reinforcement at nominal strength, ε_t, is larger than or equal to 0.005, ϕ is equal to 0.90. Moreover, ϕ less than 0.90 implies that ε_t is less than 0.005, but at least 0.004 (minimum requirement, used to control the upper limit of longitudinal reinforcement ratio) and the section is in the transition region between the tension-controlled and compression-controlled regions (ACI Committee 318 2014). The equation to compute the net tensile strain in the extreme layer of longitudinal tension reinforcement at nominal strength, ε_t, is (Wight 2016)

$$\varepsilon_t = \left(\frac{d_t - c}{c}\right)\varepsilon_{cu} \tag{2.7}$$

where: d_t is the distance from the extreme compression fiber to the extreme layer of tension steel; ε_{cu} is the maximum usable strain at the extreme concrete compression fiber, assumed equal to 0.003 in ACI 318-14; and, c is the distance from the extreme compression fiber to the neutral axis given by (ACI Committee 318 2014)

$$c = \frac{a}{\beta_1} \tag{2.8}$$

where β_1 is the factor relating the depth of the equivalent rectangular compressive stress block to the depth of the neutral axis. It is computed as (ACI Committee 318 2014)

For 17.5 MPa $\leq f_c' \leq$ 28 MPa,

$$\beta_1 = 0.85 \tag{2.9}$$

For 28 MPa $< f_c' <$ 56 MPa,

$$\beta_1 = 0.85 - \frac{0.05(f_c' - 28)}{7} \tag{2.10}$$

For $f_c' \geq$ 56 MPa,

$$\beta_1 = 0.65 \tag{2.11}$$

In this study, one layer of reinforcing steel is assumed, so d_t equals d. From Equations [2.1], [2.7] and [2.8], ε_t can be computed as

$$\varepsilon_t = 0.003 \times \frac{0.85\beta_1 f_c'}{\rho f_y} - 0.003 \tag{2.12}$$

According to Equation [2.12], increasing the longitudinal reinforcement ratio, ρ, reduces ε_t, and when ε_t is less than 0.005, ϕ also reduces. Similarly, when f_c' increases, ε_t

increases, so all of the ϕ values equal 0.90 for f_c' of 45 MPa for the range of reinforcement ratios studied.

The relationships between the design flexural strength ratios, α_M, and the longitudinal reinforcement ratios, ρ, for all ranges of reinforcement ratio are summarized in Figure 2.1 for f_c' of 25 MPa and Figure 2.2 for f_c' of 45 MPa. For each partial material strength reduction factor combination, when ϕ equals 0.90, α_M increases as ρ increases. This occurs because the impact of ϕ_c on the lever arm between the resultant tension and compression forces increases as the reinforcement ratio and associated stress block depth increase. α_M decreases when ϕ is less than 0.90, because the gradual decrease of ϕ leads to the reduction of the design flexural strength in ACI 318-14, ϕM_n. In both figures, four families of trend lines correspond to the four ϕ_s values, while the differences within each family are defined by the four ϕ_c values. Therefore, the design flexural strength ratio, α_M, is more sensitive to ϕ_s than to ϕ_c. The influence of the reinforcement ratios is small when sections are in the tension-controlled region (ϕ of 0.90), but relatively large changes of α_M occur in the transition region. Comparing Figures 2.1 and 2.2, the influence of f_c' is small, but f_c' affects the dispersion of each family: the data are more concentrated for a given ϕ_s value when f_c' is 45 MPa.

2.3.3 Recommended Partial Material Strength Reduction Factors

For f_c' of 25 MPa, the means and standard deviations of α_M for the three ranges of longitudinal reinforcement ratio are summarized in Tables 2.2–2.4. Regardless of the reinforcement ratios, as ϕ_c increases from 0.60 to 0.75, the mean of α_M decreases and as ϕ_s increases from 0.80 to 0.95, the mean of α_M also reduces, but more markedly. This demonstrates α_M is more sensitive to ϕ_s, because the design flexural strength is affected more by ϕ_s than by ϕ_c. This is also evident from the results shown in Figure 2.1.

The standard deviation of α_M, reduces for increased ϕ_c values for reinforcement ratios ranging from 0.003 to 0.005 and from 0.006 to 0.010, respectively. An opposite trend occurs for reinforcement ratios ranging from 0.011 to 0.018 because the strength reduction factor in ACI 318-14, ϕ, is not always equal to 0.90, e.g., ϕ equals 0.86 for ρ of

0.017 and ϕ equals 0.83 for ρ of 0.018. This is evident in Figure 2.1: the slopes become flatter with the increase of ϕ_c when ϕ equals 0.90, so the variations of α_M become smaller. When ϕ_s increases, the standard deviation of α_M increases for the first two reinforcement ratio ranges, but an opposite trend still occurs for reinforcement ratios ranging from 0.011 to 0.018.

The principle for selecting the best partial material strength reduction factors is to find the combinations where the mean design strength ratios approach 1 (or perhaps a value slightly larger than 1 to make the proposed design strengths slightly conservative) and the standard deviations are the least. When reinforcement ratios range from 0.003 to 0.005, the best combination is ϕ_s of 0.90 and ϕ_c of 0.75. When they range from 0.006 to 0.010, the best combination is again ϕ_s of 0.90 and ϕ_c of 0.75. And for beams with reinforcement ratios from 0.011 to 0.018, the best combination is ϕ_s of 0.95 and ϕ_c of 0.65, but the combination with ϕ_s of 0.90 and ϕ_c of 0.75 is nearly optimal. These also can be realized by inspection of Equation [2.6]: when ϕ equals 0.90, ϕ_s and ϕ_c should be located on opposite sides of 0.90 to achieve the similar design strengths. Moreover, the design strength is more sensitive to ϕ_s in the tension-controlled region, so if ϕ_s equals 0.90, ϕ_c should approach 0.90 and the closest value, 0.75, is the best. When the section is in the transition region, ϕ reduces and the influence of the concrete strength increases, so ϕ_c tends to reduce.

For f_c' of 45 MPa, the means and standard deviations of α_M are summarized in Tables 2.5–2.7. The mean has the similar trend to that observed previously for f_c' of 25 MPa. The variation of the standard deviations, for all reinforcement ratio ranges is similar to that observed for f_c' of 25 MPa and reinforcement ratios ranging from 0.003 to 0.005 and from 0.006 to 0.010. The reason is that, for f_c' of 45MPa, the strength reduction factor in ACI 318-14, ϕ, is always equal to 0.90 in the range of reinforcement ratios studied. The best combination is therefore again ϕ_s of 0.90 and ϕ_c of 0.75.

In summary, if the section is tension-controlled with ϕ of 0.90, the best partial material strength reduction factor combination is ϕ_s of 0.90 and ϕ_c of 0.75. If the section is in the transition region between the tension-controlled and compression-controlled regions, the combination with ϕ_s of 0.95 and ϕ_c of 0.65 is the best. Actually, all of the combinations in the family with ϕ_s of 0.90 are potentially suitable. For lower ϕ_c values, although the standard deviations of α_M are larger, they also yield higher means.

2.4 One-way Shear

This section presents partial material strength reduction factors that most closely approximate the design shear strengths obtained using the ACI 318-14 criteria. The analysis process is similar to that previously presented for moment.

2.4.1 Geometric and Material Properties

The investigation of one-way shear is limited to rectangular beams designed in accordance with ACI 318-14 with stirrups perpendicular to the longitudinal axes of the beams. The specified yield strength of transverse reinforcement, f_{yt}, equals 420 MPa and the two grades of normalweight concrete represent the range of commonly used strengths, f_c', of 25 and 45 MPa. The ranges of stirrup ratio, ρ_t, investigated are from 0.001 to 0.007 for f_c' of 25 MPa and from 0.001 to 0.010 for f_c' of 45 MPa. The selected ranges represent the ranges of minimum to maximum transverse reinforcement ratios permitted by ACI 318-14. Maximum stirrup spacing criteria are not always satisfied for some of the transverse reinforcement ratios, because the bar size is assumed fixed and the change of transverse reinforcement ratios is controlled by the spacing.

2.4.2 Design Strength Ratios

Design shear strength equations corresponding to the current ACI 318-14 and partial material strength reduction factors formats are defined in this section. The equations and definitions below refer to ACI 318-14, MacGregor and Bartlett (2000), and Wight (2016).

In ACI 318-14, for nonprestressed members without axial force, the nominal shear strength provided by concrete, V_c, is

$$V_c = 0.17\lambda\sqrt{f_c'}b_w d \qquad [2.13]$$

where: λ is the modification factor to account for the reduced shear strength of lightweight concrete, and equals 1.0 for normalweight concrete; and, b_w is the web width. The nominal shear strength provided by shear reinforcement, V_s, is (ACI Committee 318 2014)

$$V_s = \frac{A_v f_{yt} d}{s} \qquad [2.14]$$

where s, A_v and f_{yt} are the center-to-center spacing, the area within spacing s, and the specified yield strength, respectively, of the transverse reinforcement. The design one-way shear strength at a cross section, ϕV_n, is (ACI Committee 318 2014)

$$\phi V_n = \phi\left(V_c + V_s\right) \qquad [2.15]$$

where V_n is the nominal one-way shear strength, and ϕ equals 0.75 for members resisting shear.

For the proposed method, the design shear strength provided by concrete, V_{rc}, based on the partial material strength reduction factors format presented in MacGregor and Bartlett (2000), is assumed to be

$$V_{rc} = 0.17\lambda\phi_c\sqrt{f_c'}b_w d \qquad [2.16]$$

The design shear strength provided by shear reinforcement, V_{rs}, is (MacGregor and Bartlett 2000)

$$V_{rs} = \frac{\phi_s A_v f_{yt} d}{s} \qquad [2.17]$$

The design one-way shear strength, V_r, is therefore (MacGregor and Bartlett 2000)

$$V_r = V_{rc} + V_{rs} \qquad [2.18]$$

For the transverse reinforcement ratio, ρ_t, defined as (ACI Committee 318 2014)

$$\rho_t = \frac{A_v}{b_w s} \quad [2.19]$$

the design shear strength ratio, α_V, is

$$\alpha_V = \frac{\phi V_n}{V_r} = \frac{\phi\left(0.17\lambda\sqrt{f_c'} + \rho_t f_{yt}\right)}{0.17\lambda\phi_c\sqrt{f_c'} + \phi_s \rho_t f_{yt}} \quad [2.20]$$

For each partial material strength reduction factor combination, the design shear strength ratios, α_V, were calculated with respect to transverse reinforcement ratios, ρ_t. Again, each calculation was conducted twice because two f_c' values were investigated.

The relationships between the design shear strength ratios, α_V, and the transverse reinforcement ratios, ρ_t, are summarized in Figure 2.3 for f_c' of 25 MPa and Figure 2.4 for f_c' of 45 MPa. For each partial material strength reduction factor combination, the design shear strength ratio, α_V, declines as the transverse reinforcement ratio, ρ_t, increases. However, the sensitivity of α_V to ρ_t becomes small for increased ρ_t. As for sections resisting moment, four families of trend lines correspond to the four ϕ_s values, and the differences within each family are defined by the four ϕ_c values. The four families overlap, however, so have been shown in separate figures. The variation of α_V is greatest for low ρ_t, low ϕ_c, and high ϕ_s: as ρ_t or ϕ_c increases or ϕ_s decreases, the sensitivities of α_V to changes of these parameters reduce. Comparing Figures 2.3 and 2.4, the influence of f_c' on α_V is small.

2.4.3 Recommended Partial Material Strength Reduction Factors

For f_c' of 25 MPa and ρ_t of 0.001–0.007, the means and standard deviations of α_V are summarized in Table 2.8. As ϕ_c increases from 0.60 to 0.75, the mean of α_V decreases and as ϕ_s increases from 0.80 to 0.95, the mean of α_V also reduces. The magnitude of the proposed design shear strength is more sensitive to ϕ_s than to ϕ_c, but the difference is less than that for flexural members. This is why the four families of trend lines defined for each ϕ_s in Figure 2.3 tend to overlap. The standard deviation of α_V reduces for increased ϕ_c values, and increases for increased ϕ_s values. This is consistent with a previous observation concerning Figure 2.3: the variation of α_V is greatest for low ϕ_c and high ϕ_s, because as ϕ_c increases or ϕ_s decreases, the sensitivities of α_V to these parameters reduce.

The best strength reduction factor combination has a mean design strength ratio approaching 1 with the least standard deviation. The best partial material strength reduction factor combination is ϕ_s of 0.80 and ϕ_c of 0.65. Moreover, combinations with ϕ_s of 0.80 and ϕ_c of 0.70, and ϕ_s of 0.85 and ϕ_c of 0.60 are close to optimal. These also can be realized by inspection of Equation [2.20]: when ϕ equals 0.75, ϕ_s and ϕ_c should be located on opposite sides of 0.75 to achieve the similar design strengths.

For f_c' of 45 MPa, the means and standard deviations are summarized in Table 2.9. The values have the similar trends to those discussed previously for f_c' of 25 MPa. The best partial material strength reduction factor combination remains ϕ_s of 0.80 and ϕ_c of 0.65. Again, combinations with ϕ_s of 0.80 and ϕ_c of 0.70, and ϕ_s of 0.85 and ϕ_c of 0.60 are close to optimal.

2.5 Combined Moment and Axial Force

This section proposes partial material strength reduction factors that most closely approximate the combined flexural and axial strengths obtained using the ACI 318-14 design criteria. The analysis process is similar to those previously presented for moment and shear. Five column cross sections and eight property combinations for each cross section are investigated. The ratios of the design strengths corresponding to ACI 318-14 to those corresponding to various combinations of partial material strength reduction factors are calculated with respect to specific e/h (eccentricity/overall column depth) values.

2.5.1 Geometric and Material Properties

The investigation of combined moment and axial force is limited to the following five column cross sections, as shown in Figure 2.5:

1. Square section with three bars in each face.
2. Square section with three bars in two end faces only.
3. Square section with three bars in two side faces only.
4. Tied circular section with eight bars evenly distributed around the perimeter.
5. Spirally reinforced circular section with eight bars evenly distributed around the perimeter.

In all cases, the bending is assumed applied about a horizontal axis, *x*–*x*.

For each cross section, eight property combinations designed based on ACI 318-14 shown in Table 2.10 are investigated to account for varying steel location, steel area, and concrete strength. The values of γ, the ratios of the distance between the outer layers of reinforcement to the overall column depth, of 0.6 and 0.9 are considered as they bound commonly used values. The widths and overall depths corresponding to these two γ values are 325 and 1300 mm, respectively, to achieve a 65 mm distance from the outer reinforcement layer to the adjacent column face. Total reinforcement ratios, ρ_g, of 0.01 and 0.04 are investigated because these are the lower and upper limits, respectively, in columns containing lap splices (ACI Committee 318 2014). Specified concrete compressive strengths, f_c', of 25 and 45 MPa are investigated. In all cases, the reinforcement yield strength, f_y, equals 420 MPa. These material strengths are identical to those investigated previously for moment and shear.

2.5.2 Design Strength Ratios

Design combined flexural and axial strength equations corresponding to the current ACI 318-14 and partial material strength reduction factors formats are defined in this section. The equations and definitions below refer to ACI 318-14, MacGregor and Bartlett (2000), and Wight (2016).

Setting the strain in the extreme tension layer of steel, ε_{s1}, equal to $Z\varepsilon_y$, where ε_y is the yield strain of reinforcement, and Z is the ratio of the strain in the extreme tension layer of steel to the yield strain, the distance from the extreme compression fiber to the neutral axis, c, is (Wight 2016)

$$c = \left(\frac{0.003}{0.003 - Z\varepsilon_y} \right) d_1 \qquad [2.21]$$

where d_1 is the distance from the extreme compression fiber to the reinforcement located furthest from the extreme compression fiber. The depth of the equivalent rectangular stress block, a, is computed as (Wight 2016)

$$a = \beta_1 c \qquad [2.22]$$

and must not be greater than the section depth, h. The strain in the ith layer of reinforcement, ε_{si}, is (Wight 2016)

$$\varepsilon_{si} = \left(\frac{c - d_i}{c} \right) 0.003 \qquad [2.23]$$

where d_i is the distance from the extreme compression fiber to the ith layer of reinforcement. Tensile strains, stresses and forces are taken to be negative quantities. The stress in the ith layer of reinforcement, f_{si}, is (Wight 2016)

$$f_{si} = \varepsilon_{si} E_s \text{ but } -f_y \le f_{si} \le f_y \qquad [2.24]$$

where E_s is the modulus of elasticity of reinforcement.

In ACI 318-14, the nominal compressive force in concrete, C_c, is (Wight 2016)

$$C_c = 0.85 f_c' ab \qquad [2.25]$$

If a is less than d_i, the nominal force in the ith layer of reinforcement, F_{si}, is (Wight 2016)

$$F_{si} = f_{si} A_{si} \qquad [2.26]$$

where A_{si} is the area of the ith layer of reinforcement. If a is greater than d_i, it is necessary to account for the concrete displaced by the steel (Wight 2016):

$$F_{si} = \left(f_{si} - 0.85 f_c' \right) A_{si} \qquad [2.27]$$

The design axial strength, ϕP_n, is (Wight 2016)

$$\phi P_n = \phi\left(C_c + \sum_{i=1}^{n} F_{si}\right) \qquad [2.28]$$

where P_n is the nominal axial strength of a member.

The design flexural strength, ϕM_n, is (Wight 2016)

$$\phi M_n = \phi\left(C_c\left(\frac{h}{2}-\frac{a}{2}\right)+\sum_{i=1}^{n} F_{si}\left(\frac{h}{2}-d_i\right)\right) \qquad [2.29]$$

To account for accidental eccentricities, the maximum design axial compressive strength, $\phi P_{n,max}$, for tied columns is (Wight 2016)

$$\phi P_{n,max} = 0.80\phi\left(0.85 f_c'\left(A_g - A_{st}\right) + f_y A_{st}\right) \qquad [2.30]$$

and for spirally reinforced columns is (Wight 2016)

$$\phi P_{n,max} = 0.85\phi\left(0.85 f_c'\left(A_g - A_{st}\right) + f_y A_{st}\right) \qquad [2.31]$$

where: $P_{n,max}$ is the maximum nominal axial compressive strength of a member; A_g is the gross area of the section; and, A_{st} is the total area of the nonprestressed longitudinal reinforcement. The design axial tensile strength, ϕP_{nt}, is (Wight 2016)

$$\phi P_{nt} = -\phi f_y A_{st} \qquad [2.32]$$

where P_{nt} is the nominal axial tensile strength of a member.

For the proposed method, the factored compressive force in concrete, C_{rc}, based on the partial material strength reduction factors format presented in MacGregor and Bartlett (2000), is assumed to be

$$C_{rc} = 0.85\phi_c f_c' ab \qquad [2.33]$$

If a is less than d_i, the factored force in the ith layer of reinforcement, F_{rsi}, is (MacGregor and Bartlett 2000)

$$F_{rsi} = \phi_s f_{si} A_{si} \qquad [2.34]$$

If a is greater than d_i, it is assumed to be (MacGregor and Bartlett 2000)

$$F_{rsi} = \left(\phi_s f_{si} - 0.85\phi_c f_c'\right) A_{si} \qquad [2.35]$$

The design axial strength, P_r, is (MacGregor and Bartlett 2000)

$$P_r = C_{rc} + \sum_{i=1}^{n} F_{rsi} \qquad [2.36]$$

The design flexural strength, M_r, is (MacGregor and Bartlett 2000)

$$M_r = C_{rc}\left(\frac{h}{2} - \frac{a}{2}\right) + \sum_{i=1}^{n} F_{rsi}\left(\frac{h}{2} - d_i\right) \qquad [2.37]$$

To account for accidental eccentricities, the maximum design axial compressive strength, $P_{r,max}$, for tied columns is assumed to be (MacGregor and Bartlett 2000)

$$P_{r,max} = 0.80\left(0.85\phi_c f_c'\left(A_g - A_{st}\right) + \phi_s f_y A_{st}\right) \qquad [2.38]$$

and for spirally reinforced columns is assumed to be (MacGregor and Bartlett 2000)

$$P_{r,max} = 0.85\left(0.85\phi_c f_c'\left(A_g - A_{st}\right) + \phi_s f_y A_{st}\right) \qquad [2.39]$$

The design axial tensile strength, P_{rt}, is (MacGregor and Bartlett 2000)

$$P_{rt} = -\phi_s f_y A_{st} \qquad [2.40]$$

The design eccentricity for ACI 318-14, e_u, equals $\phi M_n/\phi P_n$, and for the proposed method, e_r, equals M_r/P_r. For a specific value of e/h, ratio α_{PM}, of design combined flexural and axial strengths computed using the strength reduction factor in ACI 318-14 and the partial material strength reduction factors is (Hong and Zhou 1999)

$$\alpha_{PM} = \frac{\sqrt{(\phi P_n)^2 + \left(\frac{\phi M_n}{h}\right)^2}}{\sqrt{P_r^2 + \left(\frac{M_r}{h}\right)^2}} = \frac{|\phi P_n|\sqrt{1+(e_u/h)^2}}{|P_r|\sqrt{1+(e_r/h)^2}} \qquad [2.41]$$

Values of α_{PM} greater than 1 represent cases where the design strengths computed using the current ACI 318-14 criteria exceed, and are therefore unconservative with respect to, those computed using the partial material strength reduction factors criteria.

For circular columns, the strain-compatibility solution described above can also be used. The only differences are the area of the compression segment of the circular section, A, and the moment of this area about the center of the column, $A\bar{y}$, which are (Wight 2016)

$$A = h^2\left(\frac{\theta - \sin\theta\cos\theta}{4}\right) \qquad [2.42]$$

$$A\bar{y} = h^3\left(\frac{\sin^3\theta}{12}\right) \qquad [2.43]$$

where angle θ, defined in Figure 2.6 (which is similar to Fig. 11–20 in Wight (2016)), is expressed in radians. If $a \le h/2$, which corresponds to $\theta \le \pi/2$ (Wight 2016),

$$\theta = \cos^{-1}\left(\frac{h/2 - a}{h/2}\right) \qquad [2.44]$$

If $a > h/2$, which corresponds to $\theta > \pi/2$ (Wight 2016),

$$\theta = \pi - \cos^{-1}\left(\frac{a - h/2}{h/2}\right) \qquad [2.45]$$

To compare design strengths for ACI 318-14 and the proposed method, the calculation of α_{PM} is based on identical e_u/h and e_r/h values of 0 (compression only), 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0, −10.0, −5.0, −1.0, −0.5, −0.1 and 0 (tension only). Interpolation in Matlab (Version R2016b; The Mathworks, Inc. 2016) was used to obtain ϕP_n, P_r and α_{PM} values corresponding to these specific e/h values. The Matlab (Version R2016b; The Mathworks, Inc. 2016) codes for Column Section 1 (square section with three bars in each face) and Column Section 5 (spirally reinforced circular section with eight bars evenly distributed around the perimeter), two representative cross sections, are presented in Appendix A.

For Column Section 1 (square section with three bars in each face), the relationships between α_{PM}, and the specific e/h for Property Combination 1 (γ of 0.6, f_c' of 25 MPa, and ρ_g of 0.01) are shown in Figure 2.7a for $e/h > 0$. The dotted vertical line on the figure shows e_{ubal}/h of 0.42, where e corresponds to the balanced failure for ACI 318-14. The solid vertical lines show the range of e values corresponding to the balanced failures for the sixteen partial material strength reduction factor combinations, from $(e_{rbal}/h)_{min}$ of 0.43 to $(e_{rbal}/h)_{max}$ of 0.50. The horizontal axis has a log scale to separate the data corresponding to the lower e/h values, which are the most common cases in short columns. When $e/h > 0$, for each partial material strength reduction factor combination, α_{PM} reduces as e/h increases until the balance point is reached. The large increase of α_{PM} between $e/h \approx 0.4$ and $e/h \approx 0.8$ is due to the increase of ϕ in ACI 318-14 from 0.65 to 0.90. As the eccentricity increases, columns tend to be tension-controlled and α_{PM} declines slightly to a stable level. In the compression-controlled region, four distinct families of the lines correspond to the four ϕ_c values, and the slight differences within each family are defined by the four ϕ_s values, which indicates the significance of the concrete strength. As the eccentricity increases, the steel strength becomes more influential so the four families of the lines are defined by the four ϕ_s values, and the differences within each family are defined by the four ϕ_c values.

When $e/h < 0$, Figure 2.7b shows that α_{PM} increases slightly as the absolute value of e/h increases. For the cross sections subjected to tension, the trend lines are also much more dependent on ϕ_s, because the tensile strength of concrete is ignored. Figures corresponding to the other property combinations are shown in Figures A.1–A.7 of Appendix A.

Figures 2.8a and 2.8b show α_{PM} values for the eight property combinations for Column Section 1 (square section with three bars in each face) for $e/h > 0$ and $e/h < 0$, respectively. The partial material strength reduction factors correspond to ϕ_s of 0.90 and ϕ_c of 0.60. For e/h approximately ranging from 0.1 to 0.38, all of the eight property combinations for both of the ACI 318-14 and partial material strength reduction factors criteria do not reach the minimum value of e_{ubal}/h or e_{rbal}/h, so they are compression-controlled. In this region, the influence of γ is very small, because of the small applied eccentricity. The influence of f_c' is moderate, but ρ_g makes a big difference. For e/h approximately ranging from 0.38 to 1.3, some reach the balance point, but others do not. For e/h approximately ranging from 1.3 to 10, all of the eight property combinations for both of the ACI 318-14 and partial material strength reduction factors criteria equal or exceed the maximum value of e_{ubal}/h or e_{rbal}/h, so they are in the transition or tension-controlled regions. The two lines corresponding to Property Combination 2 (γ of 0.6, f_c' of 25 MPa, and ρ_g of 0.04) and Property Combination 4 (γ of 0.6, f_c' of 45 MPa, and ρ_g of 0.04), differ from the others shown because they remain in the transition region until e/h equals 10: in other words, ε_t equal to 0.005 occurs when $e/h \geq 10$. For e/h less than 0, the sections are subjected to tension. In the tension-controlled region, with the increase of e/h, γ tends to become more significant, and followed by ρ_g and f_c'.

For Column Sections 2, 3 and 4, the α_{PM} values are similar to those for Column Section 1. The results are shown in Figures A.8–A.31 of Appendix A. For Column Section 5 (spirally reinforced circular section with eight bars evenly distributed around the perimeter), the relationships between α_{PM}, and the specific e/h for Property Combination 1 (γ of 0.6, f_c' of 25 MPa, and ρ_g of 0.01) are shown in Figures 2.9a and 2.9b, for $e/h > 0$ and $e/h < 0$, respectively. In the compression-controlled region, α_{PM} increases markedly compared to the values for the tied columns, because ϕ in ACI 318-14 for spirally reinforced columns equals 0.75 instead of 0.65. Figures corresponding to the other property combinations are shown in Figures A.32–A.38.

2.5.3 Recommended Partial Material Strength Reduction Factors

For Column Section 1 (square section with three bars in each face), the minimum and maximum values of e/h for all eight property combinations corresponding to the ACI 318-14 and partial material strength reduction factors criteria at the balance point are 0.383 and 1.361, respectively. It is therefore decided to categorize the data in four e/h ranges: (1) $0 \leq e/h \leq 0.3$, corresponding to compression-controlled failures; (2) $0.3 < e/h \leq 1.0$, an intermediate range; (3) $1.0 < e/h \leq 10.0$, corresponding to data below the balanced point on interaction diagrams, in the tension-controlled region or the transition region; and, (4) $e/h \leq 0$, corresponding to axial tension. For the eight property combinations, the means and standard deviations of α_{PM} are summarized in Tables 2.11–2.14. As ϕ_c increases from 0.60 to 0.75, the mean of α_{PM} decreases. Similarly as ϕ_s increases from 0.80 to 0.95, the mean of α_{PM} also decreases. The magnitude of the design strength ratio is more sensitive to ϕ_c than to ϕ_s in Range (1), the compression-controlled region, as shown in Table 2.11, while the reverse happens in Ranges (3) and (4), as shown in Tables 2.13 and 2.14. All the means are less than 1 in the compression-controlled region, indicating any of the partial material strength reduction factor combination considered yields higher design strengths and so is less conservative compared with ACI 318-14. The standard deviation of α_{PM}, reduces with increased ϕ_c values and increases with increased ϕ_s values in Ranges (1) and (4). In Ranges (2) and (3), the standard deviation of α_{PM} decreases with increased ϕ_s and ϕ_c.

The best partial material strength reduction factor combination corresponds to that having a mean design strength ratio approaching 1 with the least standard deviation. In the compression-controlled region, the family of partial material strength reduction factors with ϕ_c of 0.60 is the best and, at greater eccentricities, the family of partial material strength reduction factors with ϕ_s of 0.90 is the best. To further investigate which combination of partial material strength reduction factors is appropriate, reliability analyses will be presented in Chapter 4.

For Column Sections 2, 3 and 4, the results are similar to Column Section 1 and the results are shown in Tables A.1–A.12 of Appendix A. And the boundary for e/h between Ranges (1) and (2) is 0.4 for Column Section 2.

For Column Section 5 (spirally reinforced circular section with eight bars evenly distributed around the perimeter), the results are shown in Tables 2.15–2.18: In the compression-controlled region, the family of partial material strength reduction factors with ϕ_c of 0.70 is the best and, at greater eccentricities, both the families with ϕ_s of 0.90 and 0.95 are the best. Alternatively an overall factor could be applied to the factored resistance of spirally reinforced columns to account for the advantage of confinement. The magnitude of this factor should be between (0.75/0.65 =) 1.15 and (0.90/0.90 =) 1. However, if a single overall factor is adopted, the associated best partial material strength reduction factors for the compression-controlled and tension-controlled regions will be different. As the different ϕ factors for tied and spirally reinforced columns in ACI 318-14 impact mostly the region of compression-controlled failure, it may be better to apply a unique ϕ_c value to spirally reinforced columns. To further investigate which combination of partial material strength reduction factors is appropriate, reliability analyses will be presented in Chapter 4.

2.6 Summary and Conclusions

This chapter has presented the calculation of design strength ratios for cross sections subjected to moment, one-way shear, and combined moment and axial force. The best partial material strength reduction factors that best duplicate the design strengths obtained using the current ACI 318-14 provisions are proposed. The geometric and material properties of the designed sections represent commonly used values and meet most of the requirements in ACI 318-14. The preliminary results obtained in this chapter will be referred for reliability analyses presented in Chapters 3 and 4.

For members subjected to moment, the best partial material strength reduction factor combination is ϕ_s of 0.90 and ϕ_c of 0.75 if the section is tension-controlled. If the section is in the transition region between the tension-controlled and compression-controlled regions, the combination with ϕ_s of 0.95 and ϕ_c of 0.65 is the best, but the ϕ_s of 0.95 is an extreme value, so maybe not satisfactory for reliability analyses. Actually, all of the combinations in the family with ϕ_s of 0.90 are potentially suitable. For lower ϕ_c values, although the standard deviations of α_M are larger, they also yield higher means.

For members resisting one-way shear, the best strength reduction factor combination is ϕ_s of 0.80 and ϕ_c of 0.65. Combinations with ϕ_s of 0.80 and ϕ_c of 0.70, and ϕ_s of 0.85 and ϕ_c of 0.60 are close to optimal. These results are only based on the design strength calculations, so they just duplicate the design strengths corresponding to ACI 318-14. As mentioned in Chapter 1, however, for members subjected to shear, the statistical parameters for the professional factor have changed significantly, so the reliability indices may be unsuitable for both ACI 318-14 and the partial material strength reduction factors, and lead to different preferred partial material strength reduction factors.

For members resisting combined moment and axial force, ϕ_s of 0.90 is the best for the tension-controlled region and ϕ_c of 0.60 is the best for the compression-controlled region for tied columns. For spirally reinforced columns, ϕ_c of 0.70 is preferred and both the families with ϕ_s of 0.90 and 0.95 seem reasonable.

The results obtained in this chapter are only based on the design strengths. However, if the reliability indices for the current ACI 318-14 criteria are not suitable, the reliability indices for the partial material strength reduction factors just duplicating the design strengths of the current criteria may be also unsatisfactory. In that case, the results obtained in this chapter will be less useful and the best partial material strength reduction factors will be determined only based on the reliability analyses.

Therefore, the final selection of the best partial material strength reduction factors will be made based on the reliability analyses presented in Chapters 3 and 4.

Table 2.1: Partial material strength reduction factor combinations

	ϕ_c			
ϕ_s	0.60	0.65	0.70	0.75
0.80	C_{11}	C_{12}	C_{13}	C_{14}
0.85	C_{21}	C_{22}	C_{23}	C_{24}
0.90	C_{31}	C_{32}	C_{33}	C_{34}
0.95	C_{41}	C_{42}	C_{43}	C_{44}

Table 2.2: Means and standard deviations of design flexural strength ratios, α_M, for $f_c' =$ 25 MPa and $\rho = 0.003$–0.005

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	1.141	0.004	1.136	0.003	1.132	0.002	1.128	0.001
0.85	1.077	0.005	1.072	0.004	1.068	0.002	1.065	0.002
0.90	1.021	0.006	1.016	0.004	1.012	0.003	1.008	0.002
0.95	0.971	0.006	0.966	0.005	0.962	0.004	0.958	0.003

Note: σ, standard deviation.

Table 2.3: Means and standard deviations of design flexural strength ratios, α_M, for $f_c' =$ 25 MPa and $\rho = 0.006$–0.010

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	1.158	0.007	1.148	0.005	1.139	0.003	1.131	0.001
0.85	1.098	0.009	1.088	0.006	1.079	0.004	1.071	0.003
0.90	1.045	0.010	1.034	0.008	1.025	0.006	1.018	0.004
0.95	0.998	0.011	0.987	0.009	0.977	0.007	0.970	0.005

Table 2.4: Means and standard deviations of design flexural strength ratios, α_M, for f_c' = 25 MPa and ρ = 0.011–0.018

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	1.174	0.027	1.153	0.029	1.135	0.030	1.121	0.032
0.85	1.121	0.024	1.100	0.026	1.082	0.027	1.067	0.029
0.90	1.075	0.022	1.053	0.023	1.035	0.025	1.019	0.026
0.95	1.035	0.021	1.012	0.021	0.993	0.022	0.977	0.024

Table 2.5: Means and standard deviations of design flexural strength ratios, α_M, for f_c' = 45 MPa and ρ = 0.003–0.005

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	1.133	0.002	1.131	0.002	1.129	0.001	1.127	0.000
0.85	1.069	0.003	1.066	0.002	1.064	0.001	1.062	0.001
0.90	1.011	0.003	1.009	0.002	1.006	0.002	1.005	0.001
0.95	0.960	0.003	0.957	0.003	0.955	0.002	0.953	0.001

Table 2.6: Means and standard deviations of design flexural strength ratios, α_M, for f_c' = 45 MPa and ρ = 0.006–0.010

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	1.143	0.004	1.137	0.003	1.132	0.002	1.128	0.001
0.85	1.080	0.004	1.074	0.003	1.069	0.002	1.065	0.001
0.90	1.024	0.005	1.018	0.004	1.013	0.003	1.009	0.002
0.95	0.974	0.006	0.968	0.004	0.963	0.003	0.959	0.002

Table 2.7: Means and standard deviations of design flexural strength ratios, α_M, for f_c' = 45 MPa and ρ = 0.011–0.018

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	1.159	0.006	1.148	0.004	1.139	0.003	1.132	0.001
0.85	1.099	0.008	1.088	0.005	1.079	0.004	1.071	0.002
0.90	1.045	0.009	1.035	0.007	1.025	0.005	1.018	0.003
0.95	0.998	0.010	0.987	0.008	0.978	0.006	0.970	0.004

Table 2.8: Means and standard deviations of design shear strength ratios, α_V, for f_c' = 25 MPa and ρ_t = 0.001–0.007

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	1.038	0.047	1.011	0.033	0.985	0.021	0.961	0.010
0.85	0.997	0.055	0.971	0.041	0.947	0.029	0.924	0.018
0.90	0.958	0.061	0.934	0.048	0.912	0.036	0.891	0.026
0.95	0.923	0.066	0.900	0.054	0.879	0.042	0.860	0.032

Table 2.9: Means and standard deviations of design shear strength ratios, α_V, for f_c' = 45 MPa and ρ_t = 0.001–0.010

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	1.039	0.050	1.011	0.035	0.985	0.022	0.961	0.010
0.85	0.997	0.058	0.971	0.044	0.947	0.031	0.924	0.019
0.90	0.959	0.065	0.935	0.051	0.912	0.039	0.891	0.027
0.95	0.924	0.071	0.901	0.058	0.880	0.045	0.860	0.034

Table 2.10: Section properties for columns

Category	γ	b (mm)	h (mm)	f_c' (MPa)	f_y (MPa)	ρ_g
1	0.6	325	325	25	420	0.01
2	0.6	325	325	25	420	0.04
3	0.6	325	325	45	420	0.01
4	0.6	325	325	45	420	0.04
5	0.9	1300	1300	25	420	0.01
6	0.9	1300	1300	25	420	0.04
7	0.9	1300	1300	45	420	0.01
8	0.9	1300	1300	45	420	0.04

Table 2.11: Means and standard deviations of design combined flexural and axial strength ratios, α_{PM}, for Column Section 1 and $0 \le e/h \le 0.3$

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	0.997	0.041	0.943	0.027	0.895	0.017	0.852	0.008
0.85	0.979	0.049	0.926	0.035	0.879	0.024	0.837	0.014
0.90	0.961	0.057	0.910	0.042	0.865	0.031	0.824	0.021
0.95	0.943	0.063	0.894	0.049	0.850	0.037	0.811	0.027

Table 2.12: Means and standard deviations of design combined flexural and axial strength ratios, α_{PM}, for Column Section 1 and $0.3 < e/h \le 1.0$

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	1.073	0.136	1.036	0.134	1.004	0.134	0.975	0.135
0.85	1.037	0.130	1.002	0.128	0.970	0.126	0.941	0.126
0.90	1.005	0.126	0.970	0.122	0.939	0.120	0.911	0.119
0.95	0.975	0.122	0.941	0.118	0.911	0.115	0.884	0.113

Table 2.13: Means and standard deviations of design combined flexural and axial strength ratios, α_{PM}, for Column Section 1 and $1.0 < e/h \leq 10.0$

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	1.157	0.044	1.142	0.041	1.129	0.041	1.117	0.042
0.85	1.099	0.043	1.085	0.040	1.073	0.039	1.061	0.039
0.90	1.047	0.043	1.034	0.040	1.022	0.038	1.011	0.037
0.95	1.000	0.044	0.988	0.040	0.977	0.037	0.966	0.035

Table 2.14: Means and standard deviations of design combined flexural and axial strength ratios, α_{PM}, for Column Section 1 and $e/h \leq 0$

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	1.142	0.022	1.136	0.016	1.131	0.011	1.126	0.010
0.85	1.078	0.025	1.073	0.019	1.068	0.014	1.064	0.010
0.90	1.022	0.028	1.017	0.022	1.012	0.017	1.008	0.012
0.95	0.972	0.031	0.967	0.025	0.962	0.020	0.958	0.015

Table 2.15: Means and standard deviations of design combined flexural and axial strength ratios, α_{PM}, for Column Section 5 and $0 \leq e/h \leq 0.3$

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	1.151	0.046	1.089	0.031	1.033	0.019	0.983	0.009
0.85	1.129	0.055	1.069	0.040	1.015	0.027	0.966	0.016
0.90	1.109	0.064	1.050	0.048	0.998	0.035	0.950	0.024
0.95	1.089	0.072	1.032	0.055	0.981	0.042	0.935	0.030

Table 2.16: Means and standard deviations of design combined flexural and axial strength ratios, α_{PM}, for Column Section 5 and $0.3 < e/h \leq 1.0$

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	1.162	0.075	1.122	0.073	1.086	0.073	1.053	0.074
0.85	1.123	0.073	1.084	0.070	1.049	0.069	1.018	0.068
0.90	1.088	0.073	1.050	0.068	1.016	0.066	0.986	0.065
0.95	1.055	0.072	1.019	0.067	0.986	0.064	0.957	0.062

Table 2.17: Means and standard deviations of design combined flexural and axial strength ratios, α_{PM}, for Column Section 5 and $1.0 < e/h \leq 10.0$

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	1.163	0.036	1.146	0.037	1.129	0.039	1.114	0.042
0.85	1.108	0.033	1.091	0.034	1.075	0.035	1.061	0.037
0.90	1.058	0.031	1.041	0.031	1.027	0.032	1.013	0.034
0.95	1.013	0.029	0.997	0.029	0.983	0.030	0.970	0.031

Table 2.18: Means and standard deviations of design combined flexural and axial strength ratios, α_{PM}, for Column Section 5 and $e/h \leq 0$

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	1.149	0.026	1.141	0.020	1.133	0.018	1.125	0.018
0.85	1.088	0.029	1.080	0.023	1.072	0.019	1.065	0.017
0.90	1.033	0.031	1.025	0.025	1.018	0.020	1.011	0.017
0.95	0.984	0.034	0.976	0.028	0.970	0.023	0.963	0.019

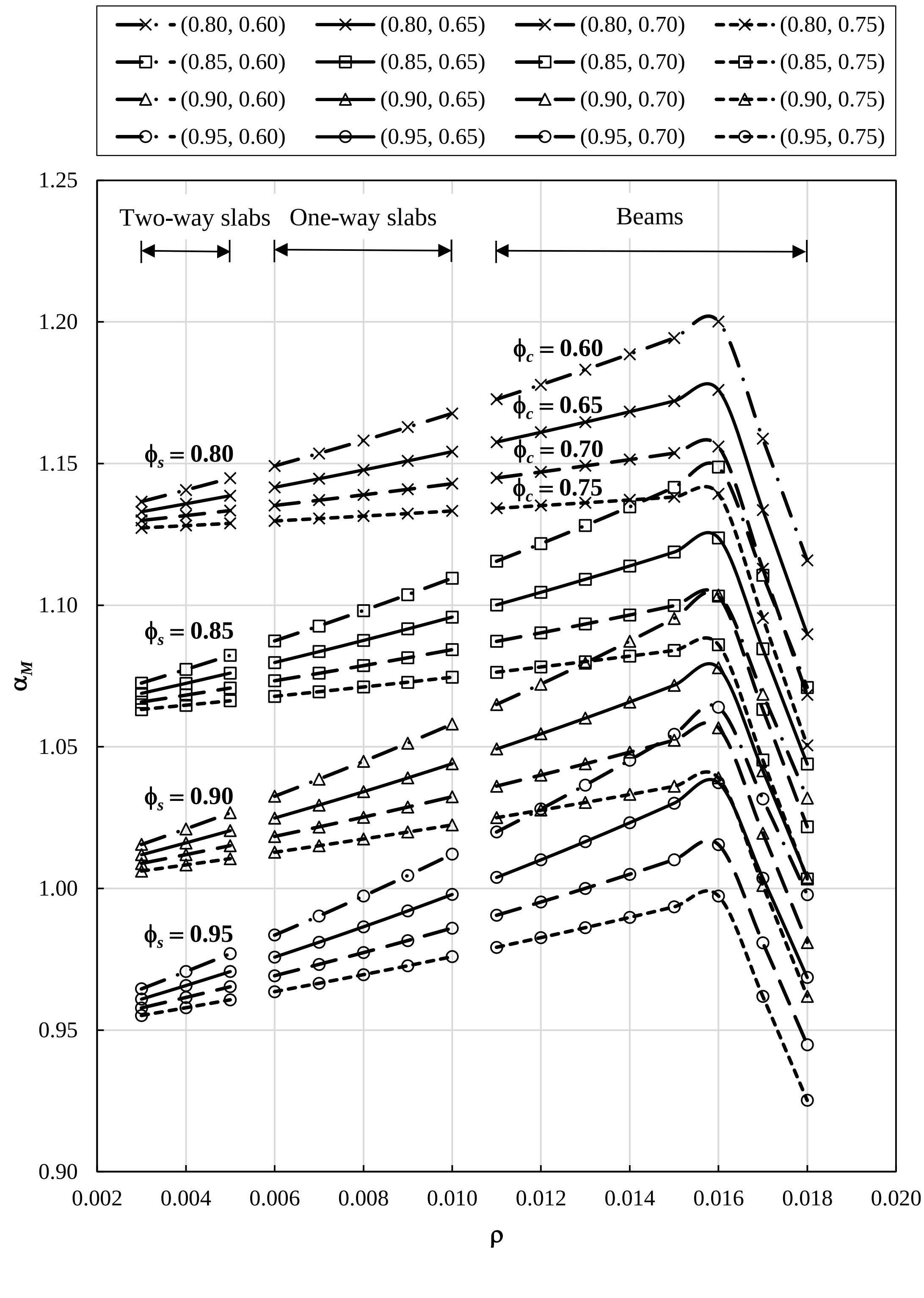


Figure 2.1: Design flexural strength ratios, α_M, for $f_c' = 25$ MPa and $\rho = 0.003$–0.018

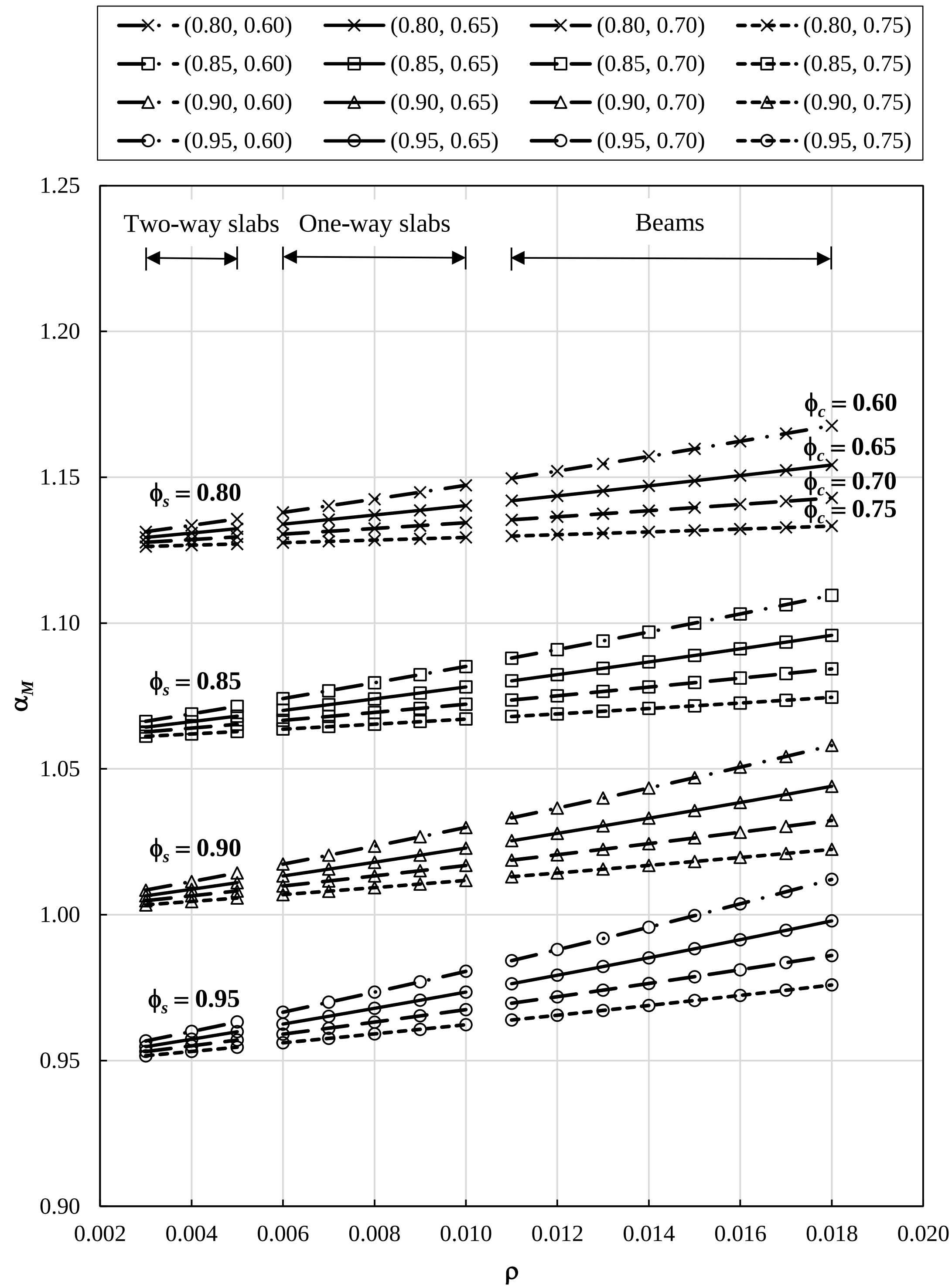
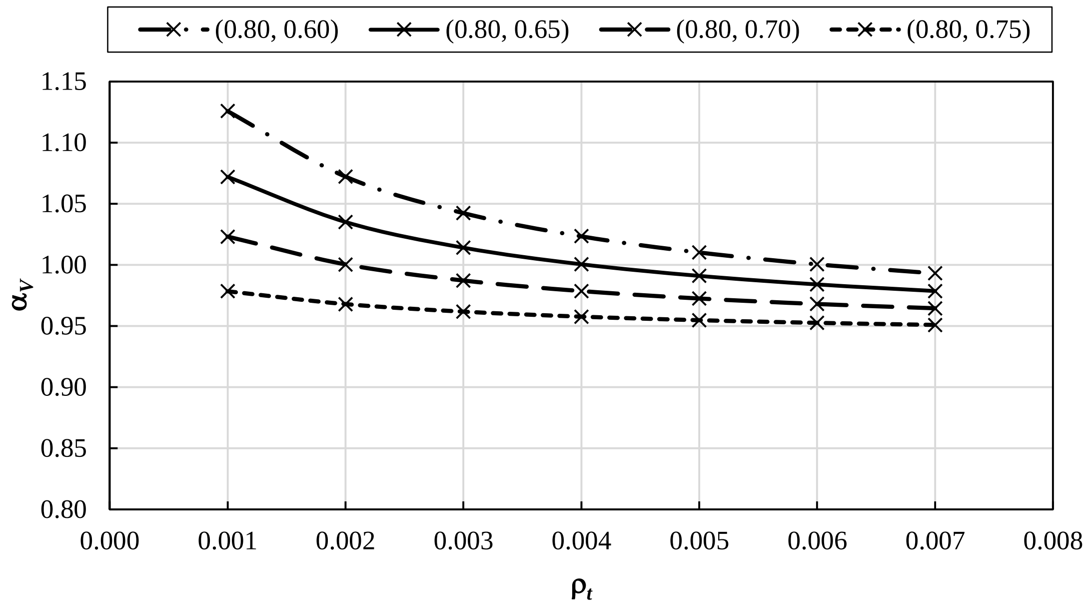


Figure 2.2: Design flexural strength ratios, α_M, for $f_c' = 45$ MPa and $\rho = 0.003$–0.018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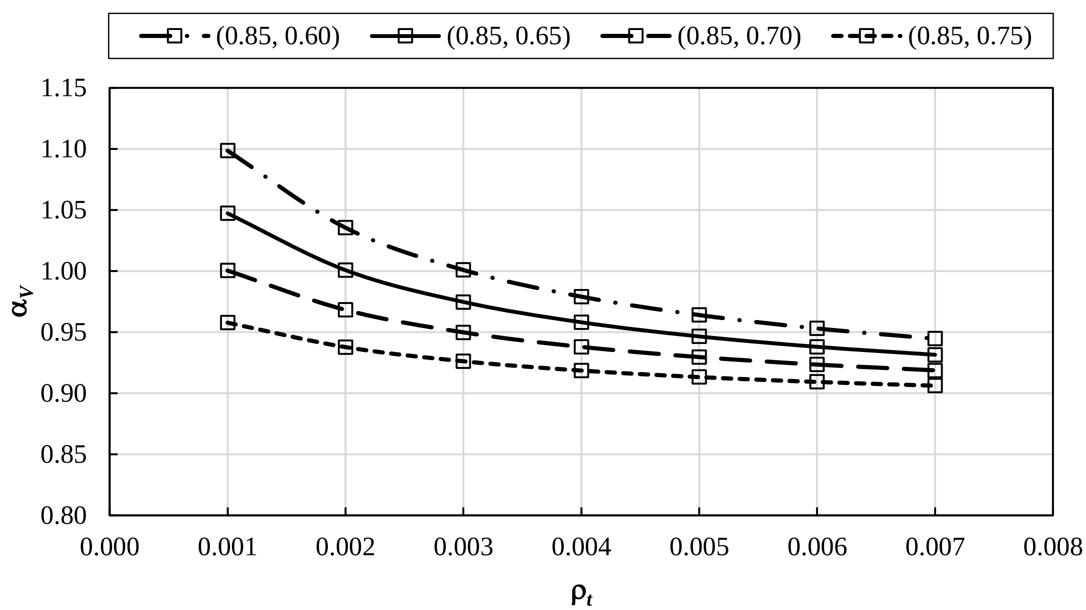
(0.80, 0.65)
(0.80, 0.70)
(0.80, 0.75)
1.15
1.10
1.05
1.00
0.95
0.90
0.85
0.80
α_V
0.000
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
ρ_t

(a)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(0.85, 0.75)
1.15
1.10
1.05
1.00
0.95
0.90
0.85
0.80
α_V
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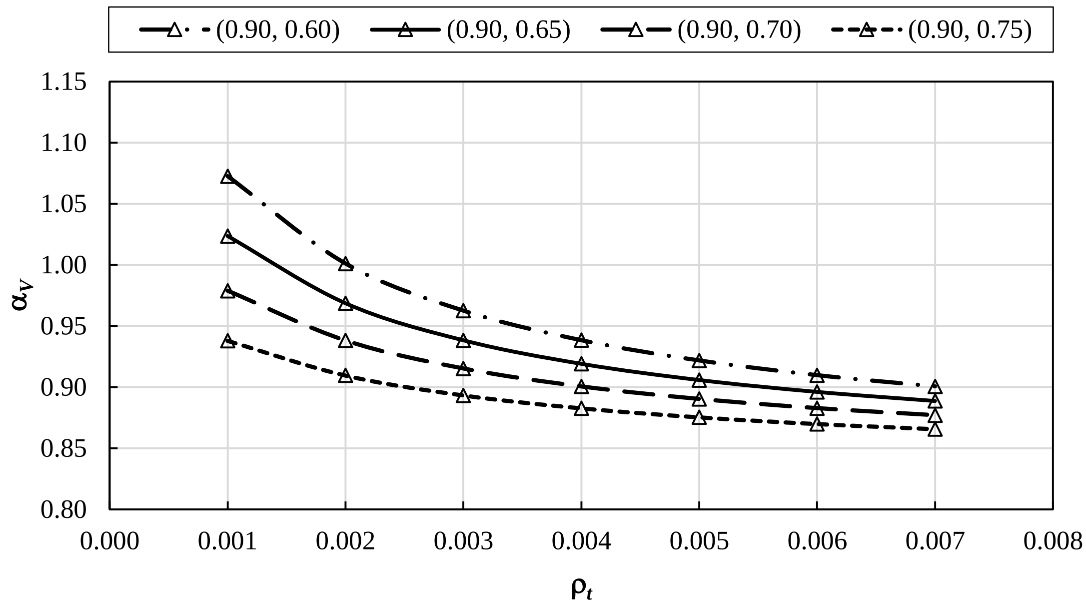
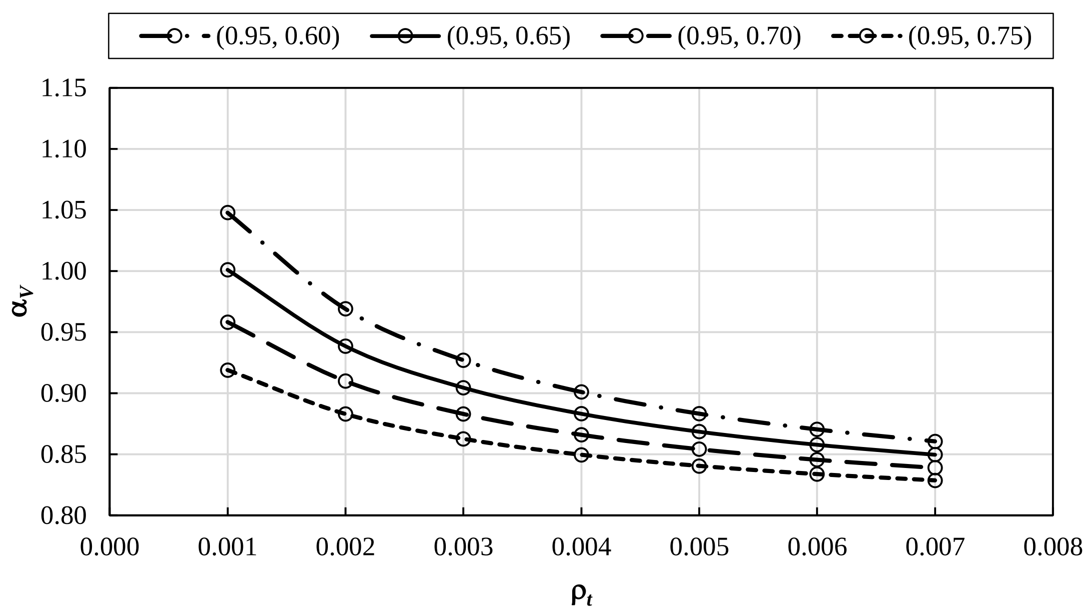
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
ρ_t

(b)

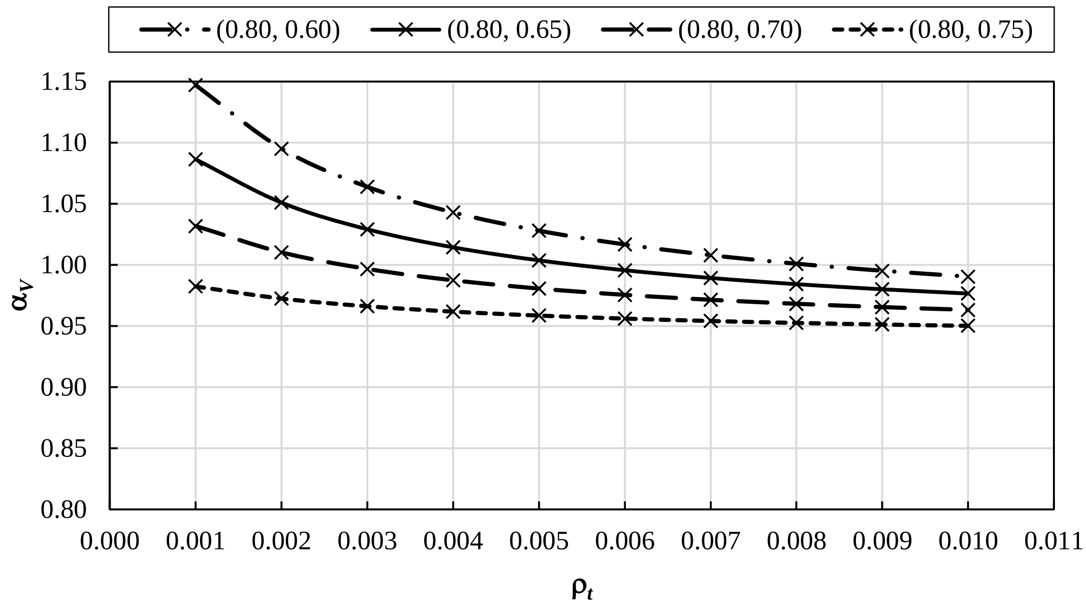


(c)



(d)

Figure 2.3: Design shear strength ratios, α_V, for f_c' = 25 MPa and ρ_t = 0.001–0.007: (a) ϕ_s = 0.80; (b) ϕ_s = 0.85; (c) ϕ_s = 0.90; (d) ϕ_s = 0.95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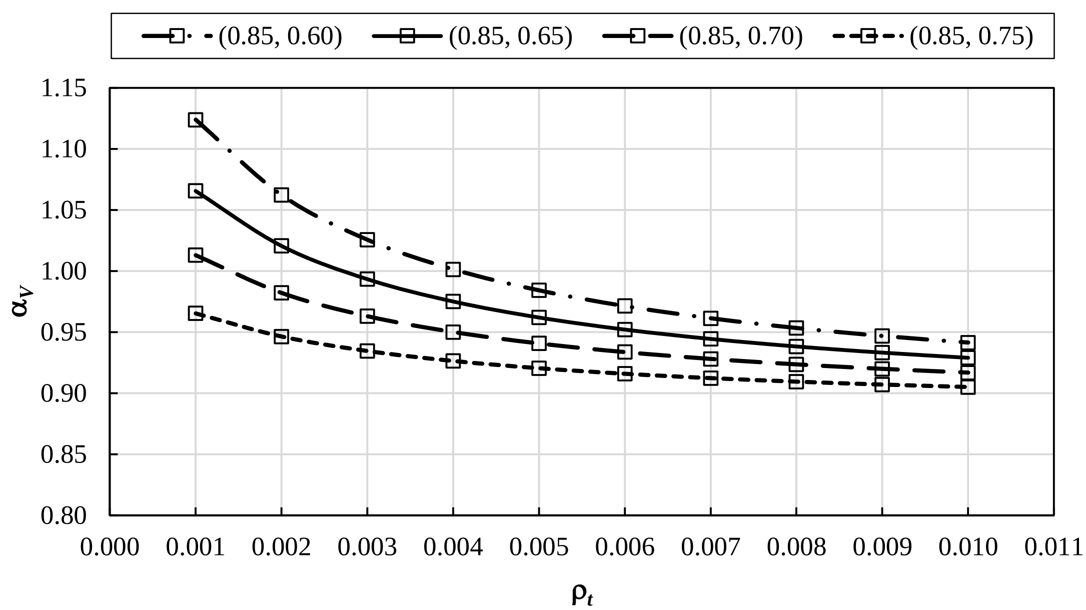
(0.80, 0.65)
(0.80, 0.70)
(0.80, 0.75)
1.15
1.10
1.05
1.00
0.95
0.90
0.85
0.80
α_V
0.000
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.010
0.011
ρ_t

(a)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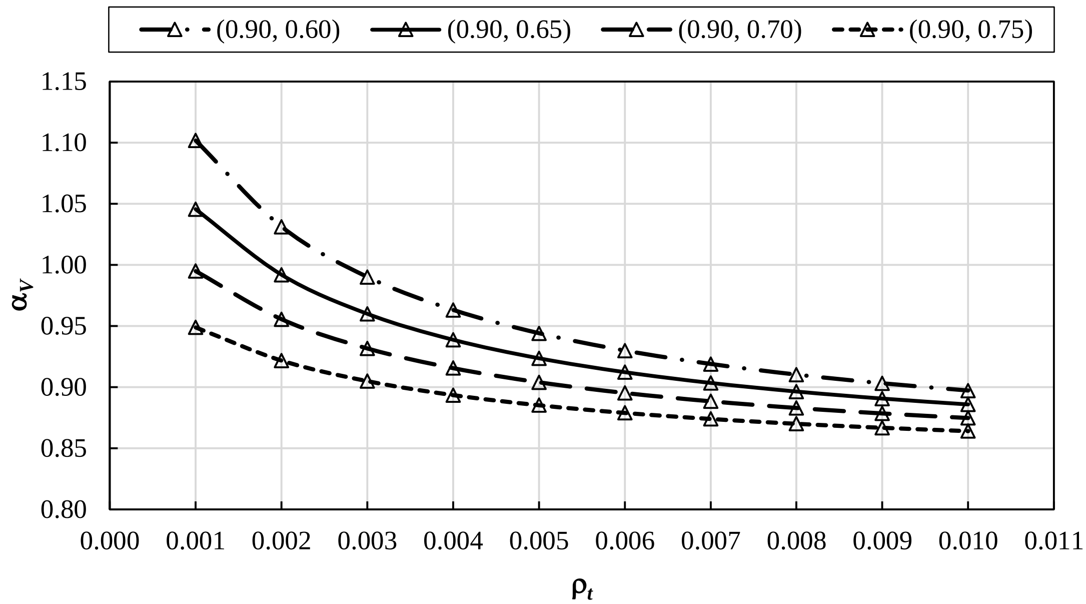
(0.85, 0.75)
1.15
1.10
1.05
1.00
0.95
0.90
0.85
0.80
α_V
0.000
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.010
0.011
ρ_t

(b)



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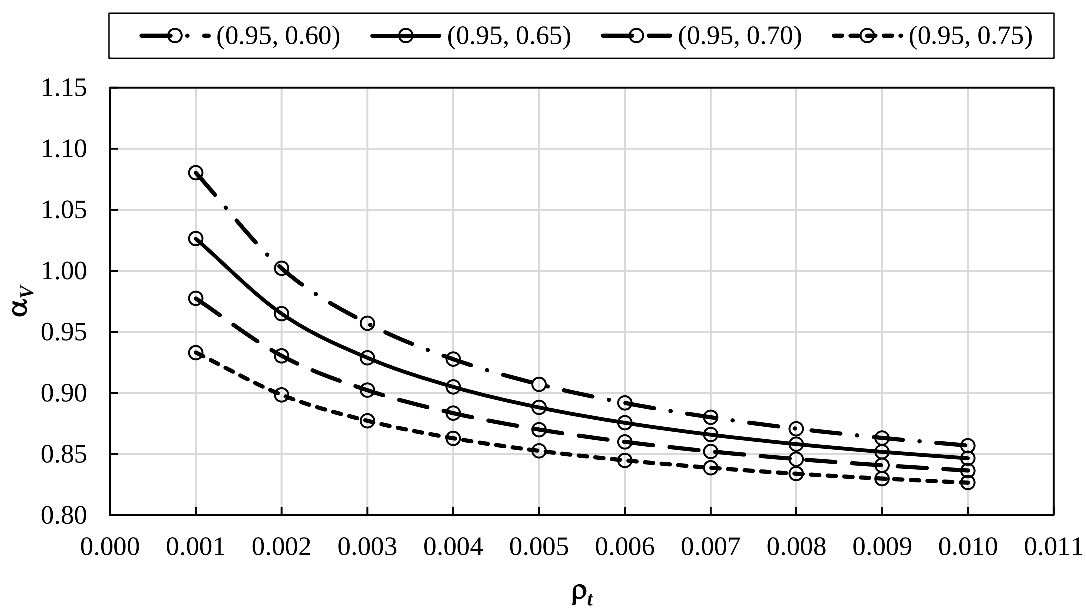


(d)

Figure 2.4: Design shear strength ratios, α_V, for f_c' = 45 MPa and ρ_t = 0.001–0.010: (a) ϕ_s = 0.80; (b) ϕ_s = 0.85; (c) ϕ_s = 0.90; (d) ϕ_s = 0.95

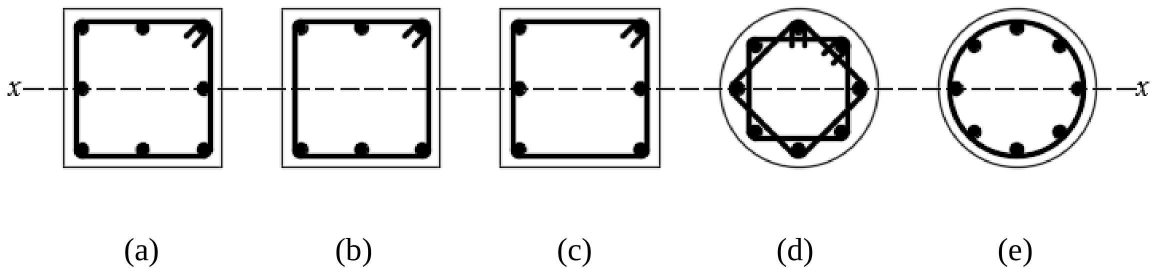


Figure 2.5: Five column cross sections: (a) Column Section 1; (b) Column Section 2; (c) Column Section 3; (d) Column Section 4; (e) Column Section 5

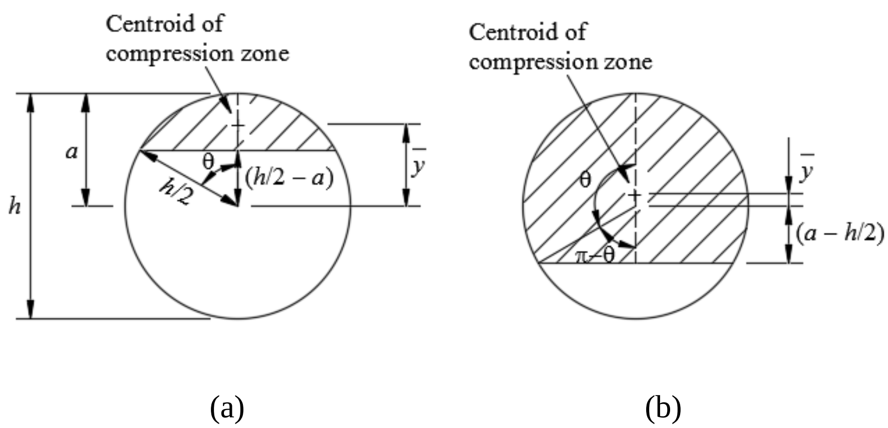
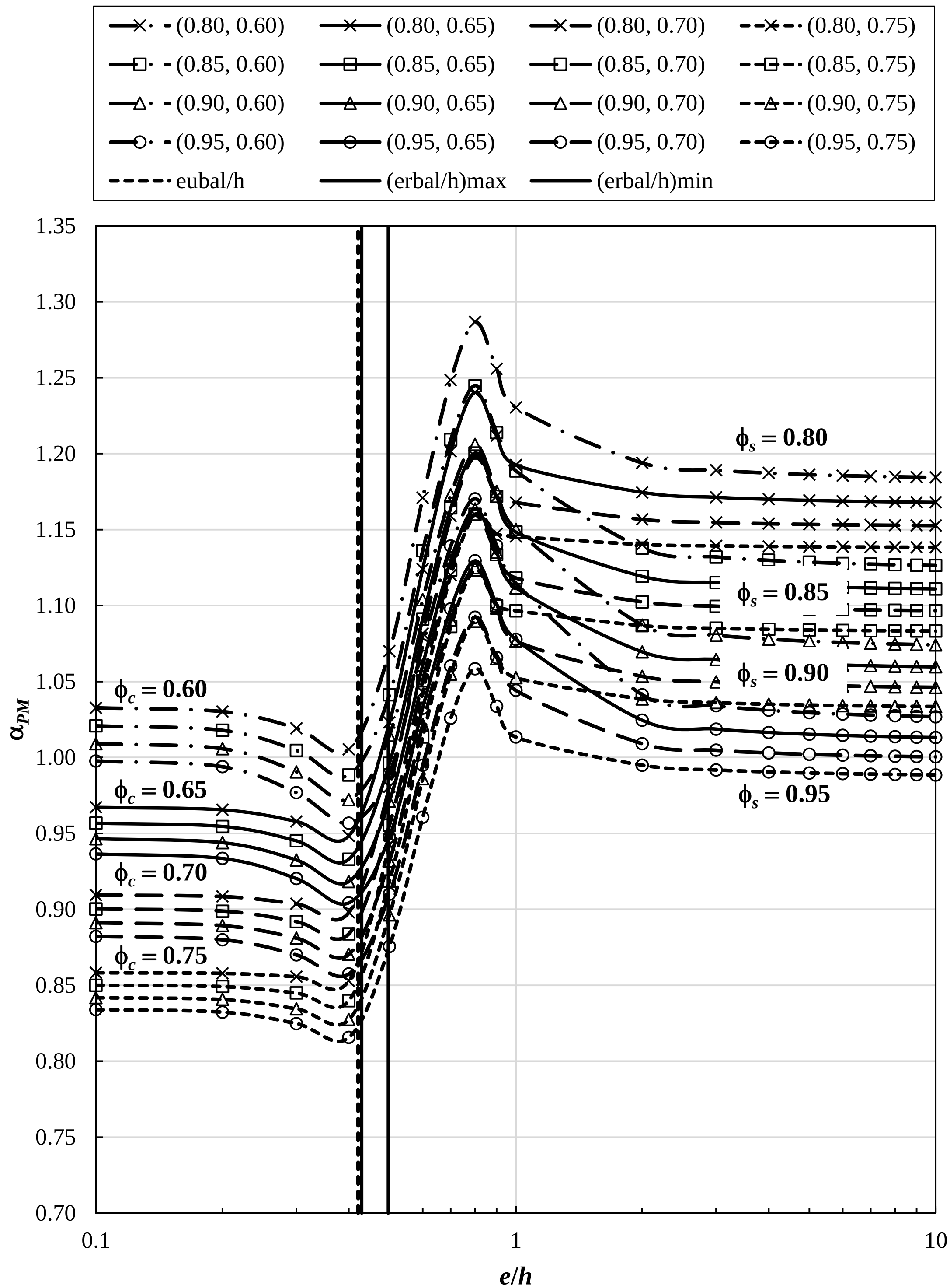


Figure 2.6: Circular segments: (a) $a \leq h/2$, $\theta \leq \pi/2$; (b) $a > h/2$, $\theta > \pi/2$ (Wight 2016)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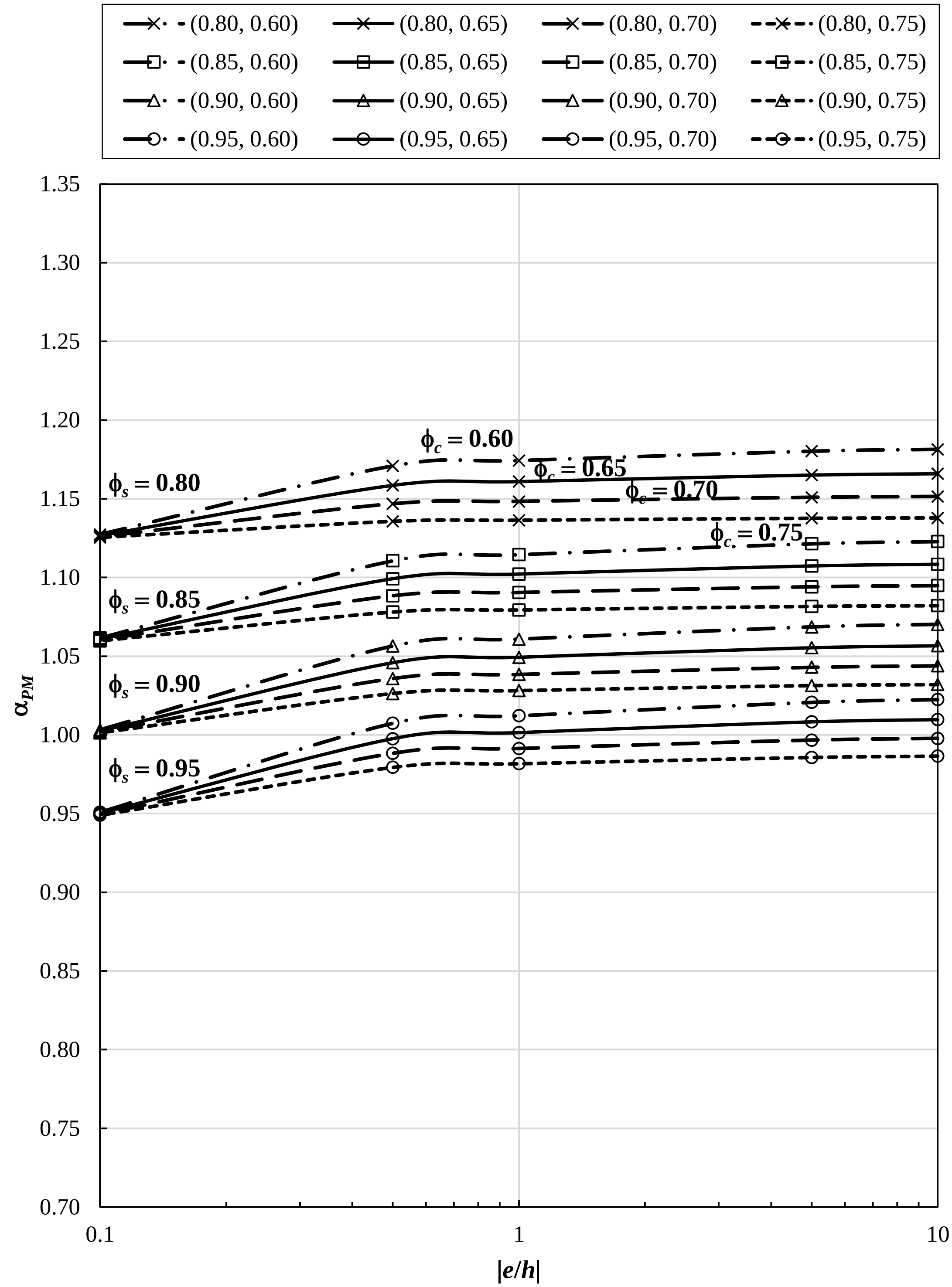
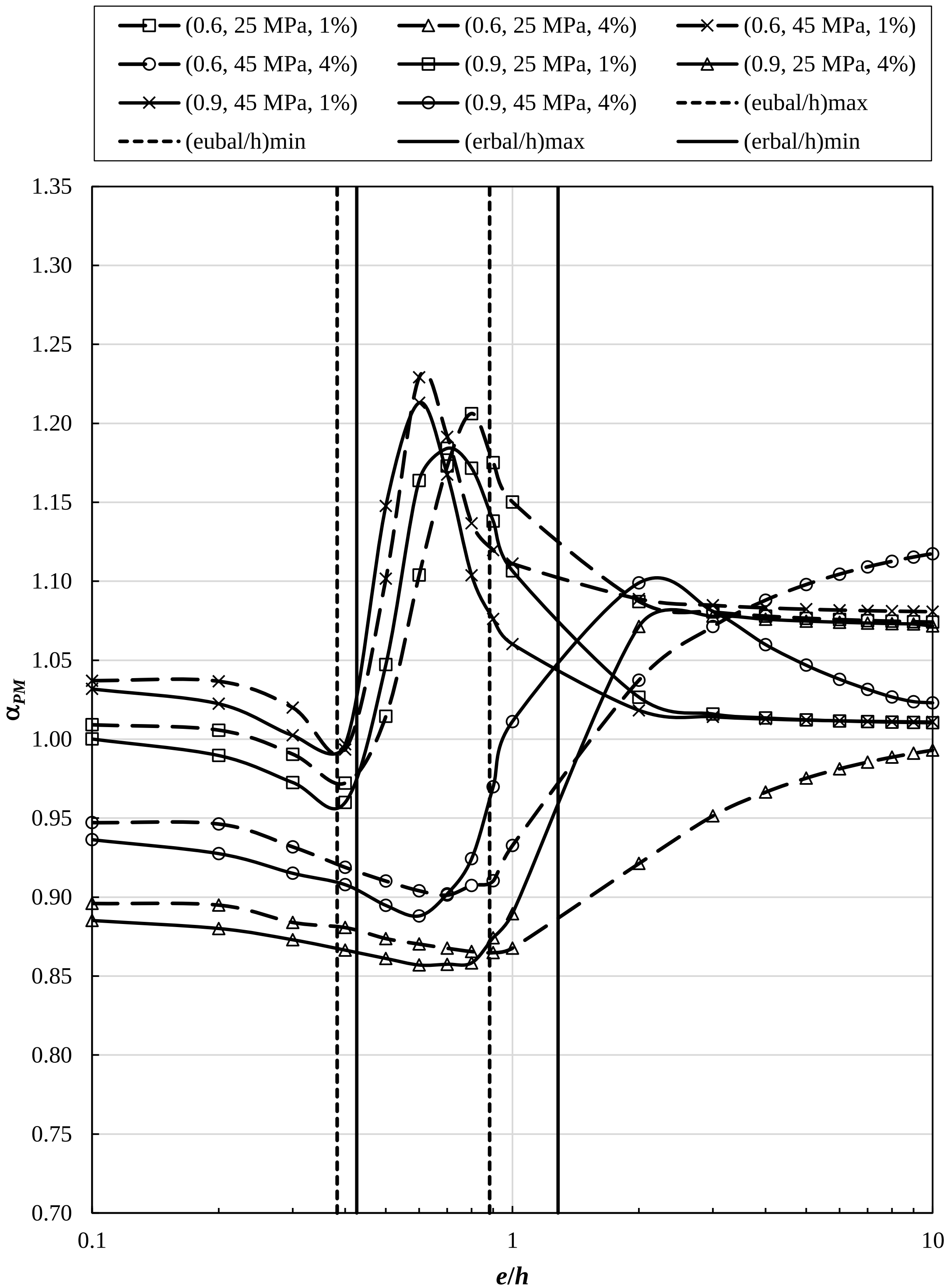
(0.80, 0.65)
(0.80, 0.70)
(0.80, 0.75)
(0.85, 0.60)
(0.85, 0.65)
(0.85, 0.70)
(0.85, 0.75)
(0.90, 0.60)
(0.90, 0.65)
(0.90, 0.70)
(0.90, 0.75)
(0.95, 0.60)
(0.95, 0.65)
(0.95, 0.70)
(0.95, 0.75)
eubal/h
(erbal/h)max
(erbal/h)min
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0.95
0.90
0.85
0.80
0.75
0.70
α_{PM}
ϕ_s = 0.80
ϕ_s = 0.85
ϕ_s = 0.90
ϕ_s = 0.95
ϕ_c = 0.60
ϕ_c = 0.65
ϕ_c = 0.70
ϕ_c = 0.75
0.1
1
10
e/*h*

(a)

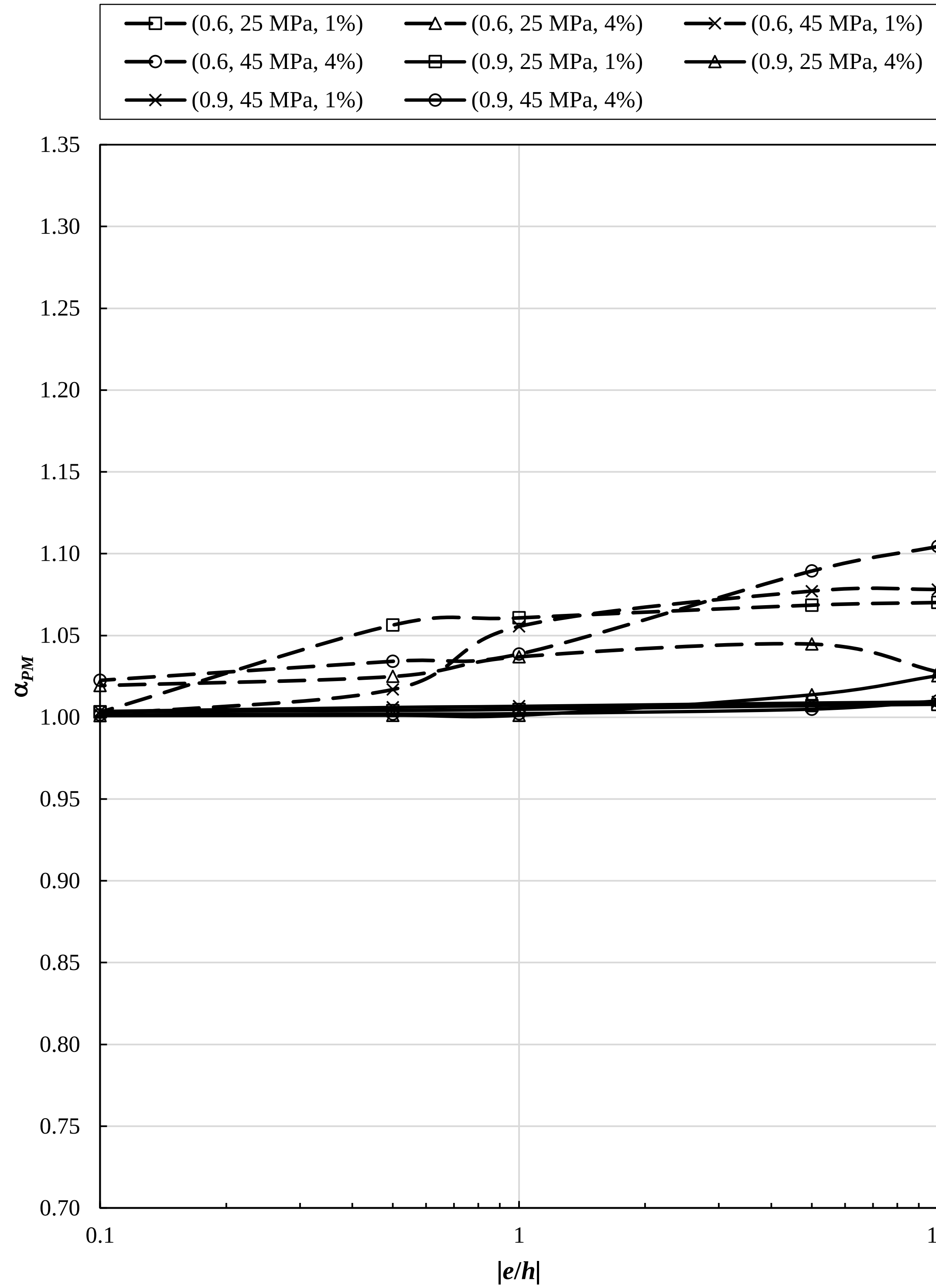


(b)

Figure 2.7: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 1 and Property Combination 1: (a) $e/h > 0$; (b) $e/h < 0$

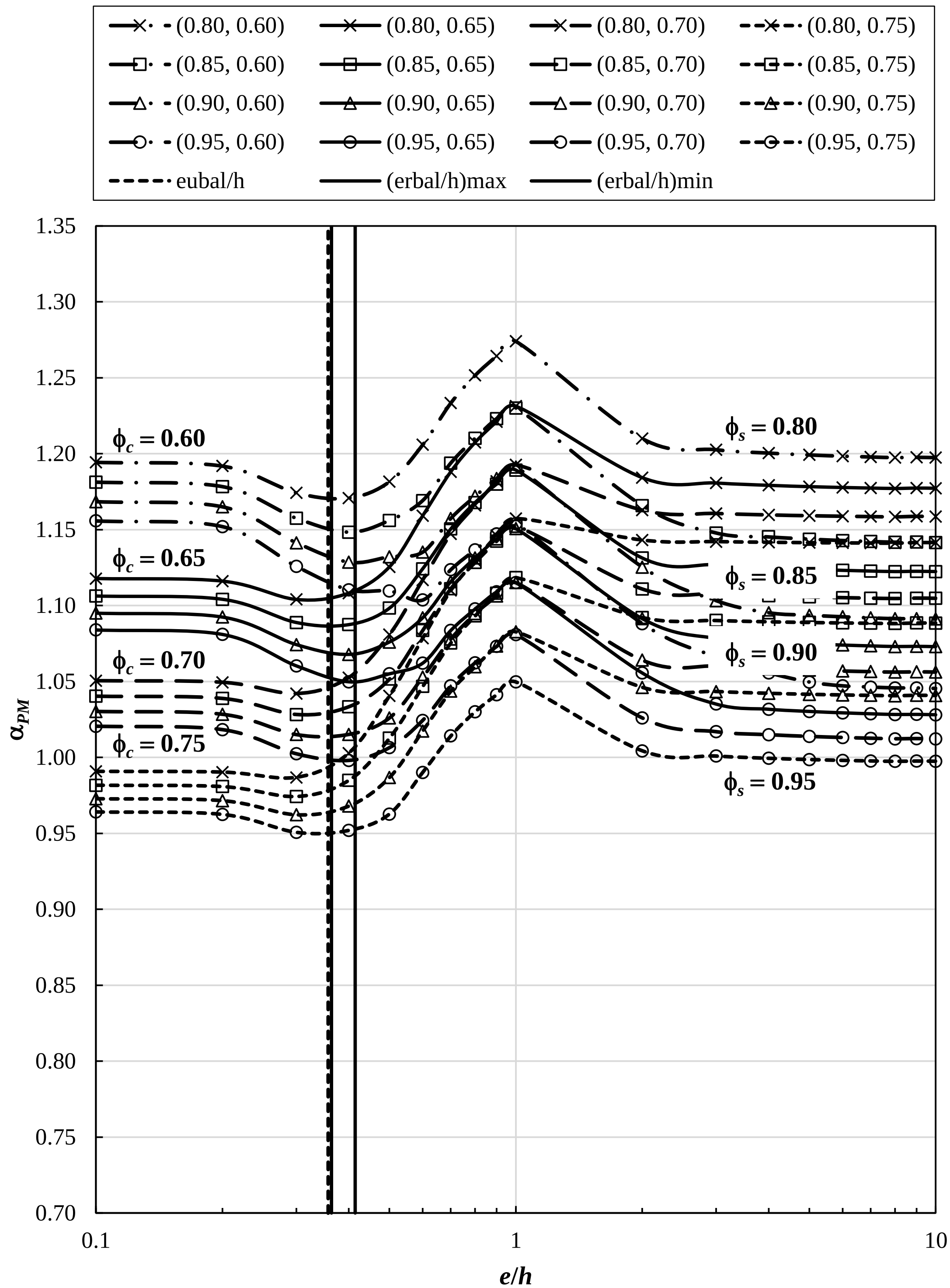

(0.6, 25 MPa, 1%)
(0.6, 25 MPa, 4%)
(0.6, 45 MPa, 1%)
(0.6, 45 MPa, 4%)
(0.9, 25 MPa, 1%)
(0.9, 25 MPa, 4%)
(0.9, 45 MPa, 1%)
(0.9, 45 MPa, 4%)
(eubal/h)max
(eubal/h)min
(erbal/h)max
(erbal/h)min
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0.95
0.90
0.85
0.80
0.75
0.70
αPM
0.1
1
10
e/h

(a)

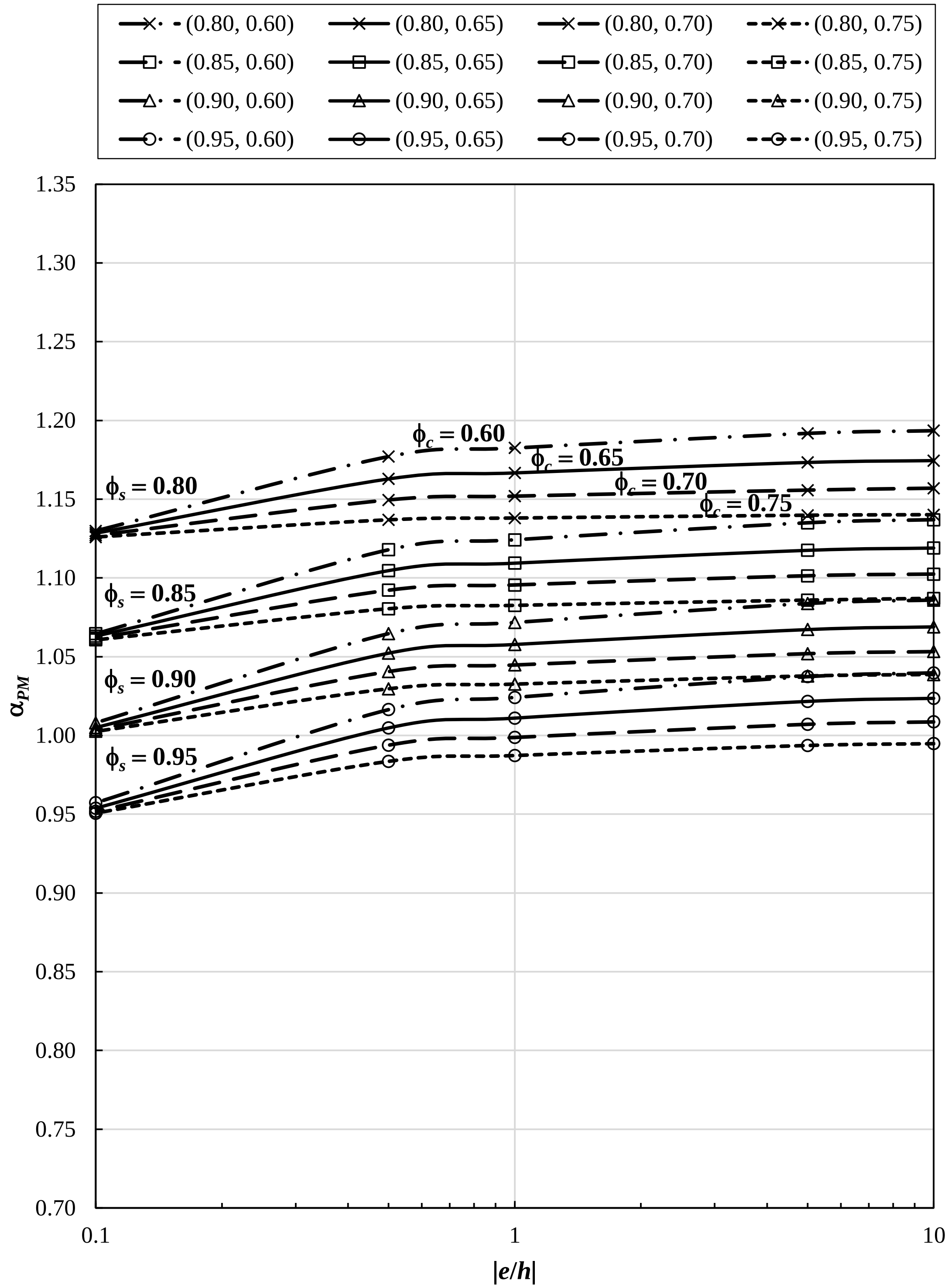


(b)

Figure 2.8: Design combined flexural and axial strength ratios, α_{PM}, corresponding to ACI 318-14, and $\phi_s = 0.90$ and $\phi_c = 0.60$, for Column Section 1: (a) $e/h > 0$; (b) $e/h < 0$


(0.80, 0.60)
(0.80, 0.65)
(0.80, 0.70)
(0.80, 0.75)
(0.85, 0.60)
(0.85, 0.65)
(0.85, 0.70)
(0.85, 0.75)
(0.90, 0.60)
(0.90, 0.65)
(0.90, 0.70)
(0.90, 0.75)
(0.95, 0.60)
(0.95, 0.65)
(0.95, 0.70)
(0.95, 0.75)
eubal/h
(erbal/h)max
(erbal/h)min
ϕ_c = 0.60
ϕ_c = 0.65
ϕ_c = 0.70
ϕ_c = 0.75
ϕ_s = 0.80
ϕ_s = 0.85
ϕ_s = 0.90
ϕ_s = 0.95
α_{PM}
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.1
1
10
e/*h*

(a)



(b)

Figure 2.9: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 5 and Property Combination 1: (a) $e/h > 0$; (b) $e/h < 0$

Chapter 3

3 Derivation of Partial Material Strength Reduction Factors for Moment or One-way Shear Based on Reliability Indices

3.1 Introduction

In Chapter 2, the partial material strength reduction factor combinations were recommended for moment, one-way shear, and combined moment and axial force based on similar design strengths obtained using ACI 318-14 (ACI Committee 318 2014) and the partial material strength reduction factors method. This simple calibration to the ACI 318-14 design strength criteria may not give the best results, however, if the ACI 318-14 criteria yield unsatisfactory reliabilities. This chapter presents the probability-based analyses to obtain reliability indices for moment and one-way shear based on ACI 318-14 and the proposed method. The range of geometric and material properties investigated are identical to those presented in Chapter 2.

The objective of this chapter is to determine appropriate partial material strength reduction factor combinations that approximate reliability indices obtained using the current ACI 318-14 provisions. A second objective is to quantify the ranges of reliability.

3.2 Methodology

Microsoft Excel (Version 2013; Microsoft 2013) is used to compute the reliability indices. The first-order, second-moment (FOSM) method is applied for simply supported members resisting moment or one-way shear. Statistical parameters to quantify the resistances and load effects are obtained from the literature. For the investigation presented in this chapter, the best combination of partial material strength reduction factors will give reliability indices that most closely approximate those obtained using the current ACI 318-14 provisions, if the range of reliability indices corresponding to ACI 318-14 is satisfactory. However, if the range of reliability indices is unsuitable, correction will be applied.

3.2.1 Reliability Model

The limit state function, Z, also denoted as $g(\bullet)$, related to the resistance, R, and the load effect, Q, is defined as (e.g., Ellingwood et al. 1980)

$$Z = g\left(R, Q\right) = \frac{R}{Q} \qquad [3.1]$$

Failure corresponds to $Z < 1$ or $\ln Z < 0$.

The resistance is assumed to be represented by the following product model originally proposed by Galambos and Ravindra (1977):

$$R = GMP \qquad [3.2]$$

where: G is a geometric property; M is a material strength property; and, P is the professional factor.

The load effect for the ith type load, Q_i, quantifies the structural demand and is expressed as (Ellingwood et al. 1980)

$$Q_i = c_i B_i A_i \qquad [3.3]$$

where: c_i is an influence coefficient; B_i is a modelling parameter; and, A_i is the structural load itself. Ellingwood et al. (1980) assumed that "the transformation from load to load effect is linear, and c_i, B_i and A_i are statistically independent." B_i accounts for "the load model which transforms the actual spatially and temporally varying load into a statically equivalent uniformly distributed load", and c_i reflects "the analysis which transforms the equivalent uniformly distributed load to a load effect".

For the limit state function defined by Equation [3.1], the first-order, second-moment reliability index, β, can be computed as (e.g., Ellingwood et al. 1980)

$$\beta = \frac{\ln\left(\bar{R} / \bar{Q}\right)}{\sqrt{V_R^2 + V_Q^2}} \qquad [3.4]$$

where: $\bar{R}$ is the mean resistance; $\bar{Q}$ is the mean load effect; V_R is the coefficient of variation of the resistance; and, V_Q is the coefficient of variation of the load effect. To determine β, these four values must be calculated first before the reliability index can be computed.

3.2.2 Determination of Statistical Parameters for Resistance and Load Effect

The reliability analyses for members resisting moment or shear are based on the Taylor Series expansion to compute the resistance and load effect statistical parameters. This method is described by Ellingwood et al. (1980). The general form of the resistance or the load effect is

$$R\{\text{or } Q\} = f\left(X_1, X_2, \ldots X_n\right) \qquad [3.5]$$

where $f(\bullet)$ is the function of resistance or load effect in the limit state function, and X_i is the resistance or load variable, characterized by its first and second moments. The resistance and the load effect must be linearized at some point for the reliability analysis. The linearization, based on the Taylor Series expansion is (Ellingwood et al. 1980)

$$R\{\text{or } Q\} \approx f\left(X_1^*, X_2^*, \ldots X_n^*\right) + \sum\left(X_i - X_i^*\right)\left(\frac{\partial f}{\partial X_i}\right)_{\underset{\sim}{X}^*} \qquad [3.6]$$

where $\left(X_1^*, X_2^*, \ldots X_n^*\right)$ is the linearizing point, taken as the means of the variables in this study. In other words, $\left(X_1^*, X_2^*, \ldots X_n^*\right) = \left(\bar{X}_1, \bar{X}_2, \ldots \bar{X}_n\right)$. Assuming the variables are statistically independent, the mean and standard deviation of R or Q, $\bar{R}$ or $\bar{Q}$, and σ_R or σ_Q, respectively, are approximated by (Ellingwood et al. 1980)

$$\bar{R}\{\text{or } \bar{Q}\} \approx f\left(\bar{X}_1, \bar{X}_2, \ldots \bar{X}_n\right) \qquad [3.7]$$

$$\sigma_R\{\text{or } \sigma_Q\} \approx \left[\sum\left(\frac{\partial f}{\partial X_i}\right)_{\bar{X}_i}^2 \sigma_{X_i}^2\right]^{1/2} \qquad [3.8]$$

where $\bar{X}_i$ and σ_{X_i} are the mean and standard deviation of the resistance or load variable, respectively.

3.3 Statistical Parameters

This section presents the statistical parameters obtained from the literature for use in the reliability analyses.

3.3.1 Geometric Properties

The geometric properties include the width, b, and the height, h, of the concrete cross section, the effective depth, d, of the flexural reinforcement and the area of the reinforcement, A_s. Table B.1 in Appendix B summarizes the absolute values, means and standard deviations, σ, for concrete geometric properties reported in the literature. They are due to measurement errors in the construction process and are controlled by specified tolerances and so are absolute values in mm. Table 3.1 shows the values used in this study selected from Table B.1. The statistical parameters for b value for beams and b and h values for columns used in this study are derived from Ellingwood et al. (1980) and Mirza and MacGregor (1979). The standard deviation of d is assumed to be 1/2 of the tolerance specified in Table 26.6.2.1a in ACI 318-14: this is consistent with the common approximation that the total tolerance range equals four standard deviations. The bias coefficient, δ, and coefficient of variation, V, for the area of the reinforcement, A_s, is obtained from Nowak and Szerszen (2003).

3.3.2 Material Strengths

The equation used by Bartlett (2007) to characterize the concrete compressive strength is:

$$M = f_{c,i-p} / f_c' = F_1 F_2 F_{i-p} F_r \tag{3.9}$$

where: $f_{c,i-p}$ is the in-place compressive strength of the concrete; f_c' is the 28-day specified strength; F_1 is a parameter representing the ratio of the mean 28-day control cylinder strength to the specified 28-day strength; F_2 is the ratio of the mean in-place strength at 28 days to the mean 28-day cylinder strength; F_{i-p} accounts for the variation of the in-place strength; and, F_r accounts for rate-of-loading effects (Bartlett 2007). It is assumed that F_1, F_2, $F_{i\text{-}p}$ and F_r are statistically independent.

The statistical parameters for F_1, F_2, $F_{i\text{-}p}$ obtained from the literature are shown in Table B.2–B.4 and the corresponding demonstration is also presented in Appendix B. Table B.5 shows statistical parameters for in-situ concrete compressive strength reported by Ellingwood et al. (1980). A summary of the statistical parameters for cast-in-place concrete used in the present study is shown in Table 3.2. Bartlett (2007) computed F_r as 0.88 for dead plus live load combination for f_c' from 20 to 35 MPa with assumption of 1 hour loading duration for live loads. The coefficients of variation for F_1, F_2 and $F_{i\text{-}p}$ are relatively large compared to F_r, so the coefficient of variation for F_r is ignored (Bartlett 2007). The resulting statistical parameters for concrete compressive strength, adopted in the present study are a bias coefficient of 1.15 and a coefficient of variation of 0.211.

Table B.6 shows statistical parameters for reinforcement with yield strength, f_y, of 420 MPa. Nowak and Szerszen (2003) recommended a bias coefficient of 1.145 and a coefficient of variation of 0.05, which implies better control of yield strength than that reported by Ellingwood et al. (1980). The statistical parameters, a bias coefficient of 1.125 and a coefficient of variation of 0.098, reported by Ellingwood et al. (1980) have been adopted for the present study, because they are more conservative.

3.3.3 Professional Factors

Table 3.3 presents statistical parameters for professional factors from several sources. Somo and Hong (2006) explored the professional factor for shear strength based on 1146 beam tests reported in the literature. They categorized the results by the presence of stirrups and shear span-to-depth ratios, a_v/d. Their dataset includes data from Kani et al. (1979) for shallow beams tested at a very young age. Therefore, Somo and Hong (2006) also reanalyzed a reduced dataset that excludes Kani’s beam tests. Collins (2001) studied the professional factor for shear based on two datasets: one containing 776 beam tests and the other 413 beam tests. The larger dataset contains a much higher proportion of beams with depths less than 350 mm, more prestressed beams, and more beams without stirrups. In addition, at least 98% of the data in each dataset are for beams subjected to point loads, whereas in practice, in buildings, it is more common to have beams subjected to uniformly distributed loads (Collins 2001). In a previous calibration of the ACI strength reduction factors, values of 1.09 and 0.12 were adopted by Israel et al. (1987).

Nowak and Szerszen (2003) investigated the bias coefficients and coefficients of variation for different structural members combining results reported by Ellingwood et al. (1980) with “engineering judgement”.

In the present investigation, for beams subjected to shear, beams with stirrups and shear span-to-depth ratios, a_v/d, greater than 2 are of interest, so a bias coefficient of 1.47 and a coefficient of variation of 0.36 reported by Somo and Hong (2006) are used. For members subjected to other structural actions, statistical parameters presented by Nowak and Szerszen (2003) are used.

3.3.4 Load Effects

Based on Table 5.3.1 in ACI 318-14, the load combination investigated is:

$$U = 1.2D + 1.6L \qquad [3.10]$$

where: U is the required strength computed using the factored load combinations; D is the effect of the service dead load; and, L is the effect of the service live load.

The statistical parameters pertaining to the dead load effect and the 50-year maximum live load effect are shown in Table 3.4. For dead load, Ellingwood et al. (1980) assumed all construction materials have the same bias coefficients and coefficients of variation. Szerszen and Nowak (2003) concluded that the statistical parameters for cast-in-place and precast members were similar as shown. A bias coefficient of 1.05 and a coefficient of variation of 0.10 reported by Ellingwood et al. (1980) have been used in this study.

For live load, Table 3.4 shows that Israel et al. (1987) assumed a bias coefficient of 1.00 and a coefficient of variation of 0.25 when A58.1-1982 (ANSI 1982) live load reductions were used and these values are used in this study. Szerszen and Nowak (2003) and Bartlett et al. (2003) selected different parameters based on their literature review and assumptions, as shown. In particular, Bartlett et al. (2003) accounted for the transformation from the load to the load effect separately which is the impact of the influence coefficient, c_i, and the modeling parameter, B_i, shown in Equation [3.3]. Denote the transformations from the dead load and live load to the dead load and live load effect by T_D and T_L, respectively. In contrast, this transformation is already included in the live load effects reported by Israel et al. (1987), and Szerszen and Nowak (2003). In these cases, T_L has a bias coefficient of 1.0 and a coefficient of variation of 0. The parameters reported by Bartlett et al. (2003) are based on the 1995 NBCC (NRCC 1995) live load reduction factors, so they are not suitable for the calibration in this study. The parameters reported by Israel et al. (1987) have been used.

3.4 Moment

This section presents reliability indices for moment derived using the partial material strength reduction factors for comparison with those computed based on the ACI 318-14 criteria. The design of the three cross sections representative of two-way slabs, one-way slabs and beams are quantified and assumptions applied in analyses are presented. The means and standard deviations of the reliability indices are quantified. The sensitivities of the reliability indices to the partial material strength reduction factor combinations for various geometric and material properties are investigated. The best factor combinations are recommended. The factor combinations deemed “best” not only give reliability indices that are similar to those obtained using the ACI 318-14 strength reduction factors but also yield acceptable absolute reliability index values. In other words, if the reliability indices corresponding to the ACI 318-14 strength reduction factors are excessively high or low, partial material strength reduction factor combinations are proposed that yield more satisfactory values.

3.4.1 Assumptions and Design Criteria

The reliability analysis of moment is based on the first-order, second-moment (FOSM) method (Ellingwood et al. 1980). Nominal values of geometric properties, material strengths and live-to-dead load ratios are selected that simulate practical values and are consistent with the requirements in ACI 318-14, as shown in Table 3.5. The design of the rectangular singly reinforced sections is based on Equations [2.1] to [2.5], reinforcement limits, minimum thickness limits, specified concrete cover requirements and material strength requirements specified in ACI 318-14. The ranges of reinforcement ratio investigated are consistent with those in Chapter 2, specifically 0.003 to 0.005 for two-way slabs, 0.006 to 0.010 for one-way slabs and 0.011 to 0.018 for beams. The material strengths also correspond to those investigated in Chapter 2 with f_c' of 25 or 45 MPa and f_y of 420 MPa.

The typical specified live-to-dead load ratio, w_L/w_D, for flexural members ranges from 0.5 to 1.5 (Ellingwood et al. 1980), so the present study investigates ratios within this range. The specified dead and live loads are determined to exactly achieve the design flexural strength, ϕM_n or M_r. Live load reduction factors due to tributary area are neglected. If they are considered, the range of typical w_L/w_D ratios reduces slightly, but the reliability indices computed for a specific w_L/w_D value are correct. Simply supported members are assumed, so the maximum factored moment at mid-span is computed as

$$M_u = \frac{\left(1.2w_D + 1.6w_L\right)l^2}{8} \qquad [3.11]$$

where: w_D is the specified dead load per unit length; w_L is the specified live load per unit length; and, l is the span length of a member.

3.4.2 Reliability Analyses

As mentioned previously, the limit state function is $Z = R/Q$. For flexural members, the flexural resistance, R_M, is

$$R_M = PA_s f_y \left(d - \frac{A_s f_y}{1.7 f_c' b} \right) \qquad [3.12]$$

where P is the professional factor. The load effect for flexural members, Q_M, is

$$Q_M = \frac{\left(w_D T_D + w_L T_L\right)l^2}{8} \qquad [3.13]$$

where: T_D is the factor that accounts for the transformation from the dead load to the dead load effect; and, T_L is the factor that accounts for the transformation from the live load to the live load effect.

The statistical parameters used for reliability analyses for moment summarized from Section 3.3 are shown in Table 3.6. The slab widths (1 m unit length) and member span lengths are assumed deterministic. The statistical parameters for area and line loads are assumed identical. T_D and T_L are already included in the selected parameters for w_D and w_L reported by Ellingwood et al. (1980) and Israel et al. (1987), so bias coefficients of 1.0 and coefficients of variation of 0 are assumed.

The reliability analysis process is as follows:

1. Calculate the design flexural strength ϕM_n using Equations [2.1] and [2.2] or M_r using Equations [2.3] and [2.4].

2. Calculate w_D and w_L by equating the design flexural strength, ϕM_n or M_r, to the factored moment, M_u, with the given load ratio, w_L/w_D, based on Equation [3.11], obtaining

$$w_D = \frac{8\phi M_n \left\{\text{or } M_r\right\}}{\left[1.2 + 1.6\left(w_L / w_D\right)\right] l^2} \qquad [3.14]$$

   and

$$w_L = \frac{8\left(w_L / w_D\right)\phi M_n \left\{\text{or } M_r\right\}}{\left[1.2 + 1.6\left(w_L / w_D\right)\right] l^2} \qquad [3.15]$$

3. Calculate the means and coefficients of variation of the resistances and the load effects using Equations [3.7], [3.8], [3.12] and [3.13].

4. Calculate the reliability index, β_{Mu} or β_{Mr}, using Equation [3.4]. Here, β_{Mu} is the reliability index for moment obtained using the strength reduction factors in ACI 318-

14 and β_{Mr} is the reliability index for moment obtained using the partial material strength reduction factors.

5. Summarize the results.

The variations of reliability indices for moment, β_M, with respect to longitudinal reinforcement ratios, ρ, for w_L/w_D of 0.5 are shown in Figure 3.1 for f_c' of 25 MPa. Very similar results for f_c' of 45 MPa are shown in Figure B.1 of Appendix B. Clearly, an abrupt discontinuity occurs between the reinforcement ratios of 0.010 and 0.011, which represent the upper and lower limits, respectively, of the ranges for one-way slabs and beams. A smaller discontinuity occurs between ρ of 0.005 and 0.006, which represent the upper and lower limits, respectively, of the ranges for two-way slabs and one-way slabs. In reality, the reliability index variation should not show such discontinuities. The discontinuities are due to the different statistical parameters adopted for the geometric properties of two-way slabs, one-way slabs and beams.

To find reasons for these discontinuities, the design conditions and statistical parameters used for the three representative cross sections in Tables 3.5 and 3.6 were critically reviewed. The nominal values of *b*, *h*, *d*, and *l* differ, and the reliability index is sensitive to the statistical parameters for *b* and, particularly, *d*. In reality, any variation of parameters should be gradual instead of abrupt. The analyses were therefore repeated using values of coefficient of variation for *d*, V_d, that vary linearly with the reinforcement ratios, as shown in Figure 3.2. The bias coefficients equal 1 for all ranges, so don't need to be modified. The coefficients of variation for *d* are the standard deviations shown in Table 3.6 divided by the mean values, and are roughly 0.031, 0.028 and 0.015 for the three ranges of reinforcement ratio. The linear transition was therefore assumed to start at ρ of 0.004, an intermediate value for two-way slabs, and to end at ρ of 0.016, which corresponds approximately to the tension-controlled limit of the section with f_c' of 25 MPa. This variation of the coefficients of variation is reasonable because, as *d* increases, the coefficient of variation decreases if the standard deviation of *d* remains constant.

The recalculated results are shown in Figures 3.3 and 3.4 for f_c' of 25 MPa and w_L/w_D of 0.5 and 1.5, respectively. The discontinuities in Figure 3.1 are corrected and the trend lines are continuous. Similar relationships for f_c' of 45 MPa are shown in Figures B.2 and B.3 of Appendix B. Moreover, β_M increases for increased ρ, and this is desirable because increasing ρ causes a flexural failure to be less ductile.

The abrupt increase of slope for reliability indices corresponding to ACI 318-14 strength reduction factors in Figure 3.3 for ρ greater than 0.016 is caused by the reduction of ϕ for sections that are not tension-controlled. The reason is identical to that abrupt decrease occurring in Figure 2.1 of Chapter 2. The four families of trend lines shown correspond to the four ϕ_s values, while the differences within each family are due to the four ϕ_c values. Therefore, the reliability index for moment obtained using the partial material strength reduction factors, β_{Mr}, is more sensitive to ϕ_s than to ϕ_c. The slopes shown for lower ϕ_c values are steeper than those for higher values, but the differences are small. Comparing Figures 3.3 and 3.4, the influence of w_L/w_D is not large, affecting only the dispersion of the trend lines, and the lines with the higher load ratio are more concentrated. As shown in Figures B.2 and B.3, the influence of increasing f_c' to 45 MPa is small, affecting the dispersion in each family: the reliability indices are more concentrated for a typical ϕ_s. In other words, the reliability indices for a typical ϕ_s are less sensitive to ϕ_c for f_c' of 45 MPa.

According to ASCE 7-10 (ASCE 2010), for a 50-year service period, normal buildings with Risk Category II should exhibit a reliability index of: "3.0 if the failure is not sudden and does not lead to widespread progression of damage; 3.5 if the failure is either sudden or leads to widespread progression of damage; and, 4.0 if the failure is sudden and results in widespread progression of damage". Therefore, the desired reliability index ranges are 3.0 for slabs and 3.0 to 3.5 for beams. The reliability indices, β_{Mu}, calculated corresponding to the strength reduction factors from ACI 318-14 for f_c' of 25 MPa and w_L/w_D of 0.5, range from 3.15 for two-way slabs to 3.86 for beams as shown in Figure 3.3. Similarly, the range is 3.05–3.60 for w_L/w_D of 1.5 as shown in Figure 3.4. For f_c' of 45 MPa, the ranges are 3.14–3.28 and 3.04–3.14 as shown in Figures B.2 and B.3 for

w_L/w_D of 0.5 and 1.5, respectively. The lower strength reduction factors, ϕ, less than 0.90, apply for beams with f_c' of 25 MPa with reinforcement ratios higher than 0.016. β_{Mu} is too conservative for these sections. Hence, partial material strength reduction factor combinations should be selected to correspond to reliability indices that satisfy ASCE 7-10 instead of simply duplicating the reliability levels achieved using ACI 318-14.

3.4.3 Recommended Partial Material Strength Reduction Factors

Based on Figures 3.3, 3.4, B.2, and B.3, the best partial material strength reduction factor combinations have ϕ_s of 0.90. The influence of ϕ_c is not as large. The means and standard deviations of the reliability indices for moment, β_{Mr}, for the three ranges of longitudinal reinforcement ratio are summarized in Tables 3.7–3.9 for f_c' of 25 MPa and in Tables B.7–B.9 of Appendix B for f_c' of 45 MPa. The mean reliability index decreases as ϕ_s or ϕ_c increases, and the standard deviation of reliability index increases as ϕ_s increases and decreases as ϕ_c increases. For ϕ_s of 0.90 and ϕ_c of 0.75, the mean reliability indices computed based on the two load ratios and the two f_c' values are approximately 3.14, 3.22, and 3.36 for ρ of 0.003 to 0.005, 0.006 to 0.010, and 0.011 to 0.018, respectively. Thus this combination yields an appropriate range of reliability indices. Again, as noted previously in Chapter 2, the adoption of ϕ_s of 0.90 is desirable, and if a lower ϕ_c is chosen, the reliability index will increase and tend to be conservative, but the standard deviation will also increase.

3.5 One-way Shear

This section compares reliability indices for one-way shear corresponding to the partial material strength reduction factors with those corresponding to the existing ACI 318-14 criteria. The first-order, second-moment (FOSM) analysis procedure is again adopted.

3.5.1 Assumptions and Design Criteria

Similar to moment, the reliability analysis of shear is based on the FOSM method (Ellingwood et al. 1980). Nominal values of geometric properties, material strengths and live-to-dead load ratios are selected that simulate practical values and are consistent with the requirements in ACI 318-14, as shown in Table 3.10. The design of the beams is based on Equations [2.13] to [2.19], shear reinforcement limits, minimum thickness limits, specified concrete cover requirements and material strength requirements specified in ACI 318-14. The stirrup yield strength, f_{yt}, is 420 MPa and the transverse reinforcement ratio, ρ_t, ranges from 0.001 to 0.007 for f_c' of 25 MPa and from 0.001 to 0.010 for f_c' of 45 MPa to represent the ranges permitted by ACI 318-14. However, maximum stirrup spacing criteria are not always satisfied for some of the transverse reinforcement ratios, because the stirrup size is assumed to be a No.3 (9.5 mm diameter) bar and the change of transverse reinforcement ratios is controlled by the spacing.

The typical specified live-to-dead load ratio is identical to that assumed for moment, ranging from 0.5 to 1.5. The specified dead and live loads are determined to exactly achieve the design shear strength, ϕV_n or V_r. Tributary-area-based live load reduction factors are again neglected. Simply supported members are assumed, so the maximum factored shear force can be computed as

$$V_u = \frac{\left(1.2w_D + 1.6w_L\right)l}{2} \qquad [3.16]$$

3.5.2 Reliability Analyses

For members resisting shear force, the resistance, R_V, is

$$R_V = P\left(0.17\lambda\sqrt{f_c'}b_w d + \frac{A_v f_{yt} d}{s}\right) \qquad [3.17]$$

where: λ is the modification factor to account for the reduced shear strength of lightweight concrete, and equals 1.0 for normalweight concrete; b_w is the web width; s is the center-to-center spacing of the transverse reinforcement; and, A_v is the area within spacing s of the transverse reinforcement.

The load effect for members resisting shear force, Q_V, is

$$Q_V = \frac{\left(w_D T_D + w_L T_L\right) l}{2} \qquad [3.18]$$

The statistical parameters for reliability analyses obtained from the literature are shown in Table 3.11. The stirrup spacing, s, length of the beam, l, and modification factor, λ, are assumed deterministic. The professional factor has statistical parameters recommended by Somo and Hong (2006), with a relatively high bias coefficient, 1.47, and a relatively high coefficient of variation, 0.36. The statistical parameters for area and line loads are assumed identical. Similar to moment, T_D and T_L are already included in the selected parameters for w_D and w_L reported by Ellingwood et al. (1980) and Israel et al. (1987), so bias coefficients of 1.0 and coefficients of variation of 0 are assumed.

The reliability analysis process is as follows:

1. Calculate the design shear strength ϕV_n using Equations [2.13], [2.14] and [2.15] or V_r using Equations [2.16], [2.17] and [2.18].

2. Calculate w_D and w_L by equating the design shear strength, ϕV_n or V_r, to the factored shear force, V_u, with the given load ratio, w_L/w_D, based on Equation [3.16], obtaining

$$w_D = \frac{2\phi V_n \ \left\{\text{or } V_r\right\}}{\left[1.2 + 1.6\left(w_L / w_D\right)\right] l} \qquad [3.19]$$

   and

$$w_L = \frac{2\left(w_L / w_D\right)\phi V_n \ \left\{\text{or } V_r\right\}}{\left[1.2 + 1.6\left(w_L / w_D\right)\right] l} \qquad [3.20]$$

3. Calculate the means and coefficients of variation of the resistances and the load effects using Equations [3.7], [3.8], [3.17] and [3.18].

4. Calculate the reliability index, β_{Vu} or β_{Vr}, using Equation [3.4], where β_{Vu} is the reliability index for shear obtained using the strength reduction factors in ACI 318-14

and β_{Vr} is the reliability index for shear obtained using the partial material strength reduction factors.

5. Summarize the results.

The reliability indices for shear, β_V, were calculated with respect to transverse reinforcement ratios, ρ_t, as shown in Figures 3.5 and 3.6 for f_c' of 25 MPa and w_L/w_D of 0.5 and 1.5, respectively. Similar results were obtained for f_c' of 45 MPa as shown in Figures B.4 and B.5 of Appendix B. The reliability index for shear corresponding to ACI 318-14, β_{Vu}, increases as ρ_t increases. As the transverse reinforcement ratio increases, however, the failure of the reinforced member becomes more ductile, so this trend may not be particularly desirable. In contrast, the variation of β_{Vr} can decrease with the increased ρ_t, depending on the various ϕ_s and ϕ_c values. Again, the four ϕ_s values create distinct families of β_{Vr} values that are not as diverse as those for moment, and are shown in separate figures. Differences within each family are due to the different ϕ_c values. Comparing Figures 3.5 and B.4, the influence of f_c' is slight. Comparing Figures 3.5 and 3.6, the higher w_L/w_D value yields slightly higher reliability indices.

According to ASCE 7-10, for a 50-year service period, normal buildings with Risk Category II should exhibit a reliability range of 3.0 to 3.5 for beams, which is achieved by the best strength reduction factor combination for moment. However, the reliability indices shown in Figures 3.5, 3.6, B.4 and B.5 for shear, range from 2.65 to 2.82, and 2.20 to 3.11 computed for the ACI 318-14 criteria and the various partial material strength reduction factor combinations, respectively. In other words, the safety level for moment is markedly greater than that for shear. This is undesirable, because a shear failure is less ductile than a flexural failure and so has a greater target reliability index.

The reliability indices for shear are sensitive to the statistical parameters assumed for the professional factor. For example, if a bias coefficient of 1.075 and a coefficient of variation of 0.10 are assumed for the professional factor, as recommended by Nowak and Szerszen (2003), the reliability index corresponding to the ACI 318-14 criteria ranges from 4.27 to 4.39 for f_c' of 25 MPa and w_L/w_D of 0.5 as shown in Figure 3.7. The reliability indices corresponding to the various partial material strength reduction factor combinations range from 3.21 to 4.99. It is therefore necessary to review the basis for the various statistical parameters for the professional factor reported in the literature.

Nowak and Szerszen (2003) recommended statistical parameters for professional factor by modifying slightly the values recommended in Ellingwood et al. (1980) based on their "engineering judgement". The database used by Ellingwood et al. (1980) contains 62 test beams with stirrups and 96 beams with no stirrups. The database analyzed by Somo and Hong (2006) contains 419 test beams with stirrups and 727 beams with no stirrups, and the total 1146 test beams, includes 878 beams with $h \geq 300$ mm and $f_c' \geq 20$ MPa. Collins (2001) computed professional factors using two databases, one with 413 test results and the other with 776 test results. He observed that the larger database contains much higher number of beams with $h \leq 350$ mm, prestressed beams, and beams without stirrups, and so it is less representative of realistic concrete construction than the smaller database. The three sets of statistical parameters for the professional factor are shown in Table 3.12. The parameters reported by Somo and Hong (2006) are most comprehensively presented, e.g., the parameters are classified by a_v/d and the presence of stirrups and consider prestressed members separately. The parameters recommended by Nowak and Szerszen (2003) have the lowest bias coefficient which is conservative but also the lowest coefficient of variation which is unconservative. Somo and Hong (2006) analyzed the largest number of beams, including the database assembled by Bentz (2000) which is the source of 413 test beams for the study of Collins (2001). Therefore, the parameters for professional factor reported by Somo and Hong (2006) are likely the most appropriate.

3.5.3 Recommended Partial Material Strength Reduction Factors

Although the reliability indices calculated by applying parameters from Somo and Hong (2006) are lower than the desirable values, the influence of the statistical parameters for the professional factor is consistent for both ACI 318-14 and the partial material strength reduction factors. In other words, the reliability indices increase or decrease consistently in both cases when different professional factor parameters are chosen. Therefore, the reliability indices calculated for ACI 318-14 are assumed adequate and the best partial material strength reduction factor combinations are selected as those yielding a mean reliability index ratio, β_{Vu}/β_{Vr}, of 1 with the least standard deviation. The means and standard deviations for β_{Vu}/β_{Vr} are summarized in Table 3.13 for f_c' of 25 MPa and similar results are shown in Table B.10 for f_c' of 45 MPa. The mean increases as ϕ_s or ϕ_c increases. The standard deviation increases as ϕ_s increases and decreases as ϕ_c increases. The best partial material strength reduction factor combination is ϕ_s of 0.80 and ϕ_c of 0.65. Moreover, combinations with ϕ_s of 0.80 and ϕ_c of 0.70, and ϕ_s of 0.85 and ϕ_c of 0.60 are close to optimal. These results are identical to those reported for shear in Chapter 2.

3.6 Summary and Conclusions

This chapter has presented statistical parameters collection and reliability analyses for members subject to moment and one-way shear using the FOSM method.

For members subjected to moment, the best partial material strength reduction factor combination is ϕ_s of 0.90 and ϕ_c of 0.75. Similar to Chapter 2, the family with ϕ_s of 0.90 is most desirable. If a lower ϕ_c value is chosen, the reliability index will increase, and tend to exceed the target values, but the standard deviation will also increase.

For members subjected to one-way shear, the statistical parameters for the professional factor markedly influence the reliability indices. Based on the parameters reported by Somo and Hong (2006), reliability indices corresponding to ACI 318-14 and the partial material strength reduction factors both yield low values, whereas much higher values occur when parameters reported by Nowak and Szerszen (2003) are used. Therefore, because the professional factor has the similar impacts on the reliability indices for ACI 318-14 and the partial material strength reduction factors criteria, and assuming the reliability indices calculated for ACI 318-14 are adequate, partial material strength reduction factors are selected that yield reliability indices that approximate those derived using the ACI 318-14 provisions. The best partial material strength reduction factor combination is ϕ_s of 0.80 and ϕ_c of 0.65. Moreover, combinations with ϕ_s of 0.80 and ϕ_c of 0.70, and ϕ_s of 0.85 and ϕ_c of 0.60 are close to optimal, which are identical to those recommended in Chapter 2.

Table 3.1: Statistical parameters for geometric properties used in this study

Item	Source	Comment		
			Mean	σ
Beams			(mm)	(mm)
b	Ellingwood et al. 1980	Stem width	Nominal+2.54	3.81
			Mean	σ
Columns			(mm)	(mm)
b, h	Ellingwood et al. 1980	Rectangular	Nominal+1.52	6.35
h	Mirza and MacGregor 1979	Circular	Nominal	4.76
			Mean	σ
Slabs, Beams and Columns			(mm)	(mm)
d	ACI Committee 318 2014	$d \leq 203$ mm	Nominal	4.76
		$d > 203$ mm	Nominal	6.35
Reinforcement			δ	V
A_s	Nowak and Szerszen 2003	—	1.0	0.015

Table 3.2: Statistical parameters for concrete compressive strength used in this study

Item	Source	Comment	δ	V
F_1	Bartlett 2007	Cast-in-place concrete	1.27	0.122
F_2	Bartlett 2007	Cast-in-place concrete	1.03	0.113
$F_{i\text{-}p}$	Bartlett and MacGregor 1999	Cast-in-place concrete	1.0	0.130
F_r	Bartlett 2007	1 hour live load loading	0.88	0
Overall			1.15	0.211

Table 3.3: Statistical parameters for professional factors

Source	Comment	δ	*V*
Shear			
Somo and Hong 2006	Beams without stirrups		
	All a_v/d values	2.17	0.75
	$a_v/d \geq 2$	1.74	0.47
	$a_v/d < 2$	4.86	0.53
	Beams with stirrups		
	All a_v/d values	1.51	0.37
	$a_v/d \geq 2$	1.47	0.36
	$a_v/d < 2$	1.79	0.35
	All Beams		
	All a_v/d values	1.92	0.71
	$a_v/d \geq 2$	1.64	0.45
	$a_v/d < 2$	3.96	0.66
	Without Kani's beams		
	All a_v/d values	1.75	0.60
	$a_v/d \geq 2$	1.58	0.46
	$a_v/d < 2$	3.22	0.59
Collins 2001	776 beams dataset	1.30	0.278
	413 beams dataset	1.19	0.339
Israel et al. 1987	Beam, shear	1.09	0.12
Nowak and Szerszen 2003	Beam, shear	1.075	0.10
Others			
Nowak and Szerszen 2003	Beam, flexure	1.02	0.06
	Slab	1.02	0.06
	Column, tied	1.00	0.08
	Column, spiral	1.05	0.06

Table 3.4: Statistical parameters for load effects

Item	Source	Comment	δ	V
Dead load	Ellingwood et al. 1980	All construction materials	1.05	0.10
	Szerszen and Nowak 2003	Cast-in-place concrete	1.05	0.10
		Precast concrete	1.03	0.08
Live load	Israel et al. 1987	A58.1-1982 live load reductions are used	1.00	0.25
	Szerszen and Nowak 2003	50 year maximum load	1.00	0.18
	Bartlett et al. 2003	50 year maximum load, 1995 NBCC live load reductions are used	0.900	0.170
		Transformation to load effect	1.000	0.206

Table 3.5: Design conditions for moment

Section	Item	Nominal value	Unit
	Geometric properties		
Two-way slabs	b	1000 (unit width)	mm
	h	200	mm
	d	155	mm
	l	6	m
One-way slabs	b	1000 (unit width)	mm
	h	200	mm
	d	170	mm
	l	4	m
Beams	b	300	mm
	h	500	mm
	d	435	mm
	l	8	m
	Material strengths		
	f_c'	25 and 45	MPa
	f_y	420	MPa
	Load ratios		
Two-way slabs	w_L/w_D	0.5 and 1.5	
One-way slabs	w_L/w_D	0.5 and 1.5	
Beams	w_L/w_D	0.5 and 1.5	

Table 3.6: Statistical parameters for moment reliability analysis

Item	Source	Comment		
Geometric properties			Mean	σ
Two-way slabs			(mm)	(mm)
b	—	Assumed deterministic	Nominal	0
d	ACI Committee 318 2014	$d \leq 203$ mm	Nominal	4.76
l	—	Assumed deterministic	Nominal	0
			Mean	σ
One-way slabs			(mm)	(mm)
b	—	Assumed deterministic	Nominal	0
d	ACI Committee 318 2014	$d \leq 203$ mm	Nominal	4.76
l	—	Assumed deterministic	Nominal	0
			Mean	σ
Beams			(mm)	(mm)
b	Ellingwood et al. 1980	Stem width	Nominal+2.54	3.81
d	ACI Committee 318 2014	$d > 203$ mm	Nominal	6.35
l	—	Assumed deterministic	Nominal	0
Reinforcement			δ	V
A_s	Nowak and Szerszen 2003	—	1.0	0.015
Material Strengths			δ	V
f_c'	Bartlett 2007 Bartlett and MacGregor 1999	Cast-in-place concrete	1.15	0.211
f_y	Ellingwood et al. 1980	—	1.125	0.098
Professional factor			δ	V
P	Nowak and Szerszen 2003	Beam, flexure, and slab	1.02	0.06
Load effects			δ	V
$w_D T_D$	Ellingwood et al. 1980	All construction materials	1.05	0.10
$w_L T_L$	Israel et al. 1987	A58.1-1982 live load reductions are used	1.00	0.25

Table 3.7: Means and standard deviations of reliability indices for moment using partial material strength reduction factors, β_{Mr}, for f_c' = 25 MPa and ρ = 0.003–0.005

		ϕ_c							
		0.60		0.65		0.70		0.75	
w_L/w_D	ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.5	0.80	4.002	0.039	3.975	0.032	3.951	0.026	3.931	0.021
	0.85	3.639	0.044	3.610	0.036	3.585	0.030	3.564	0.024
	0.90	3.299	0.048	3.268	0.040	3.242	0.033	3.219	0.027
	0.95	2.978	0.053	2.945	0.044	2.917	0.037	2.893	0.031
1.5	0.80	3.743	0.029	3.720	0.023	3.701	0.018	3.685	0.014
	0.85	3.446	0.033	3.422	0.027	3.402	0.022	3.384	0.017
	0.90	3.167	0.038	3.142	0.031	3.120	0.025	3.102	0.020
	0.95	2.904	0.042	2.878	0.035	2.855	0.028	2.835	0.023

Table 3.8: Means and standard deviations of reliability indices for moment using partial material strength reduction factors, β_{Mr}, for f_c' = 25 MPa and ρ = 0.006–0.010

		ϕ_c							
		0.60		0.65		0.70		0.75	
w_L/w_D	ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.5	0.80	4.172	0.069	4.114	0.055	4.064	0.044	4.021	0.035
	0.85	3.829	0.077	3.767	0.063	3.713	0.051	3.668	0.041
	0.90	3.509	0.086	3.442	0.071	3.385	0.058	3.336	0.047
	0.95	3.209	0.095	3.137	0.079	3.077	0.065	3.025	0.053
1.5	0.80	3.869	0.051	3.821	0.040	3.781	0.031	3.746	0.024
	0.85	3.589	0.058	3.538	0.047	3.495	0.037	3.458	0.029
	0.90	3.328	0.066	3.274	0.054	3.228	0.043	3.188	0.035
	0.95	3.084	0.074	3.026	0.061	2.977	0.050	2.935	0.040

Table 3.9: Means and standard deviations of reliability indices for moment using partial material strength reduction factors, β_{Mr}, for f_c' = 25 MPa and ρ = 0.011–0.018

		ϕ_c							
		0.60		0.65		0.70		0.75	
w_L/w_D	ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.5	0.80	4.453	0.098	4.334	0.073	4.233	0.053	4.148	0.036
	0.85	4.153	0.118	4.025	0.090	3.917	0.068	3.825	0.049
	0.90	3.878	0.138	3.740	0.108	3.624	0.084	3.525	0.063
	0.95	3.625	0.159	3.477	0.127	3.353	0.100	3.247	0.078
1.5	0.80	4.081	0.076	3.985	0.056	3.904	0.040	3.834	0.026
	0.85	3.839	0.092	3.735	0.070	3.648	0.052	3.573	0.037
	0.90	3.616	0.109	3.504	0.085	3.411	0.065	3.331	0.049
	0.95	3.411	0.126	3.291	0.100	3.191	0.079	3.106	0.061

Table 3.10: Design conditions for shear

Section	Item	Nominal value	Unit
	Geometric properties		
Beams	b	300	mm
	h	500	mm
	d	435	mm
	l	8	m
	A_v	71 × 2	mm^2
	Material Strengths		
	f_c'	25 and 45	MPa
	f_{yt}	420	MPa
	λ	1.0	
	Load ratios		
	w_L/w_D	0.5 and 1.5	

Table 3.11: Statistical parameters for shear reliability analysis

Item	Source	Comment		
Geometric properties			Mean	σ
Beams			(mm)	(mm)
b_w	Ellingwood et al. 1980	Stem width	Nominal+2.54	3.81
d	ACI Committee 318 2014	$d > 203$ mm	Nominal	6.35
s	—	Assumed deterministic	Nominal	0
l	—	Assumed deterministic	Nominal	0
Reinforcement			δ	V
A_v	Nowak and Szerszen 2003	—	1.0	0.015
Material Strengths			δ	V
f_c'	Bartlett 2007 Bartlett and MacGregor 1999	Cast-in-place concrete	1.15	0.211
f_{yt}	Ellingwood et al. 1980	—	1.125	0.098
λ	—	Assumed deterministic	1	0
Professional factor			δ	V
P	Somo and Hong 2006	Beams with stirrups, $a_v/d \geq 2$	1.47	0.36
Load effects			δ	V
$w_D T_D$	Ellingwood et al. 1980	All construction materials	1.05	0.10
$w_L T_L$	Israel et al. 1987	A58.1-1982 live load reductions are used	1.00	0.25

Table 3.12: Statistical parameters for professional factor for shear

Source	Comment	δ	V
Nowak and Szerszen 2003	Beams, shear	1.075	0.10
Somo and Hong 2006	Beams with stirrups, $a_v/d \geq 2$	1.47	0.36
Collins 2001	413 beams dataset	1.19	0.339

Table 3.13: Means and standard deviations of reliability index ratios for shear, β_{Vu}/β_{Vr}, for f_c' = 25 MPa and ρ_t = 0.001–0.007

		ϕ_c							
		0.60		0.65		0.70		0.75	
w_L/w_D	ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.5	0.80	0.967	0.039	0.991	0.030	1.015	0.021	1.041	0.010
	0.85	1.007	0.051	1.031	0.042	1.057	0.032	1.083	0.022
	0.90	1.048	0.063	1.074	0.054	1.100	0.044	1.128	0.034
	0.95	1.091	0.076	1.118	0.067	1.146	0.057	1.174	0.047
1.5	0.80	0.969	0.037	0.991	0.028	1.014	0.019	1.037	0.010
	0.85	1.006	0.047	1.029	0.038	1.052	0.029	1.076	0.020
	0.90	1.044	0.058	1.067	0.049	1.091	0.040	1.116	0.030
	0.95	1.083	0.069	1.107	0.060	1.132	0.051	1.158	0.042

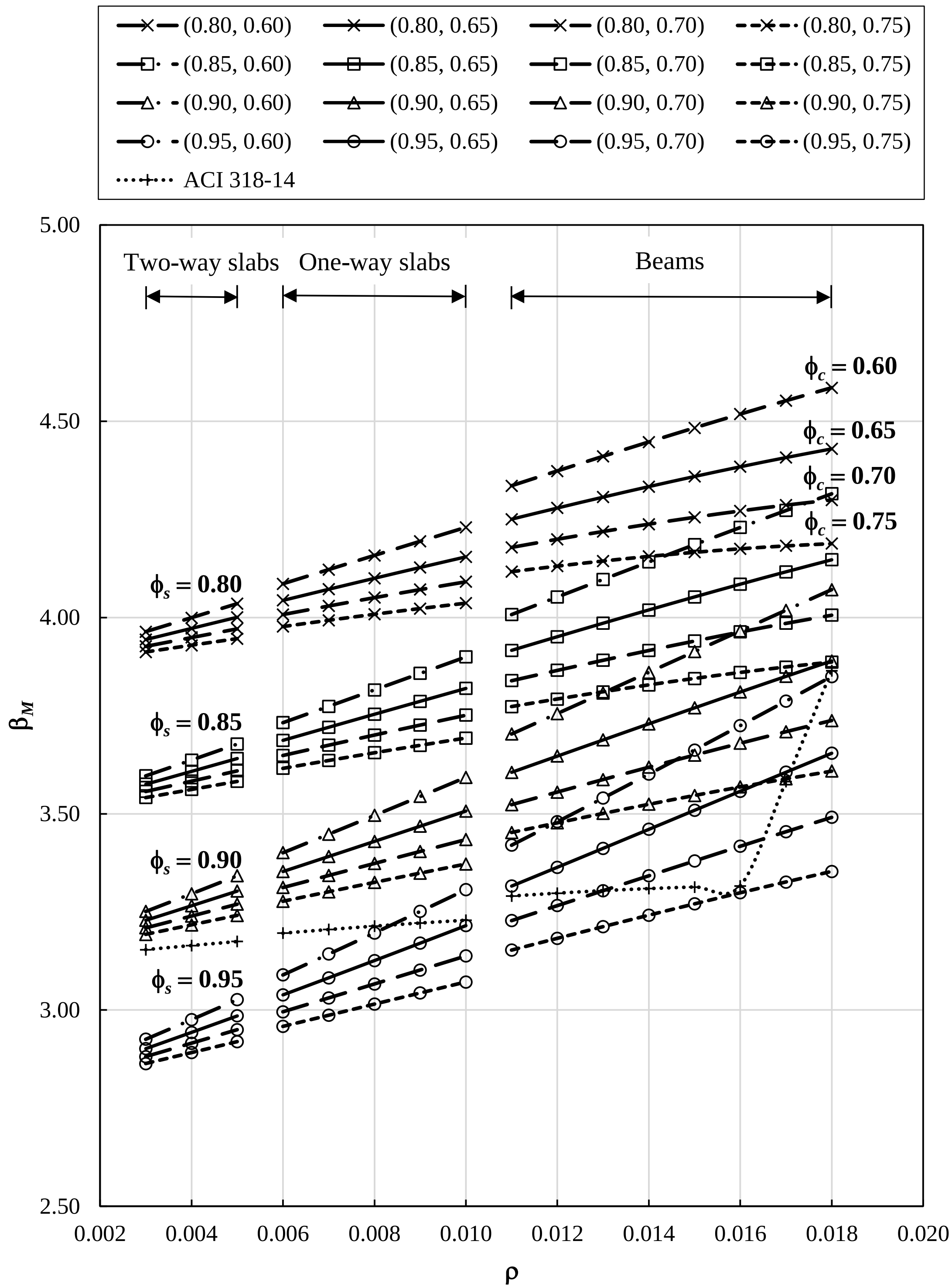


Figure 3.1: Reliability indices for moment, β_M, for $f_c' = 25$ MPa, $w_L/w_D = 0.5$, $\rho = 0.003$–0.018, and constant coefficients of variation for d

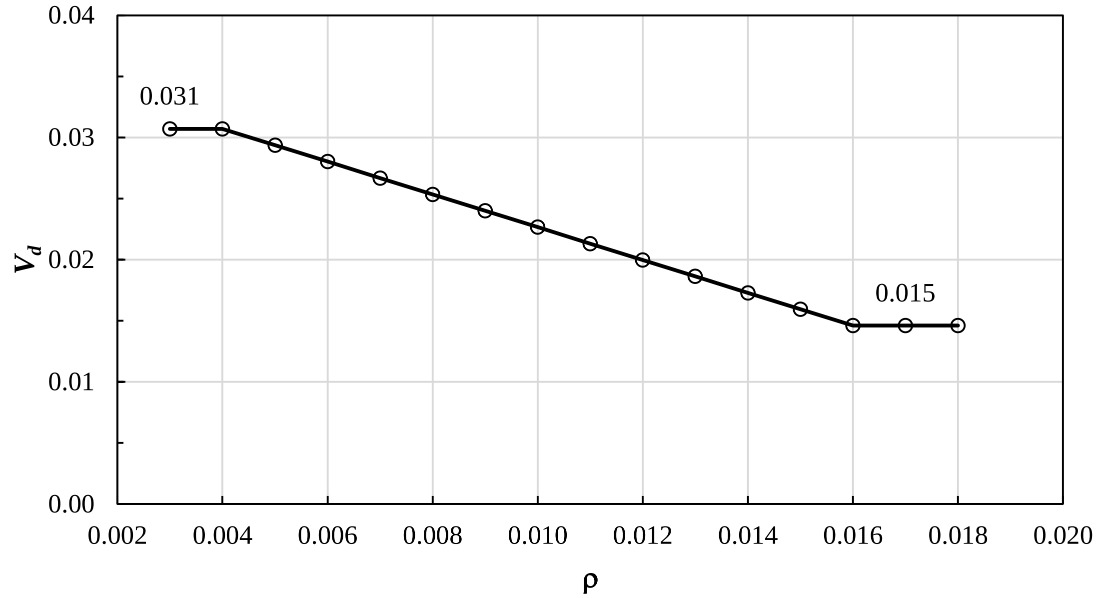


Figure 3.2: Linear variation of coefficients of variation for *d*

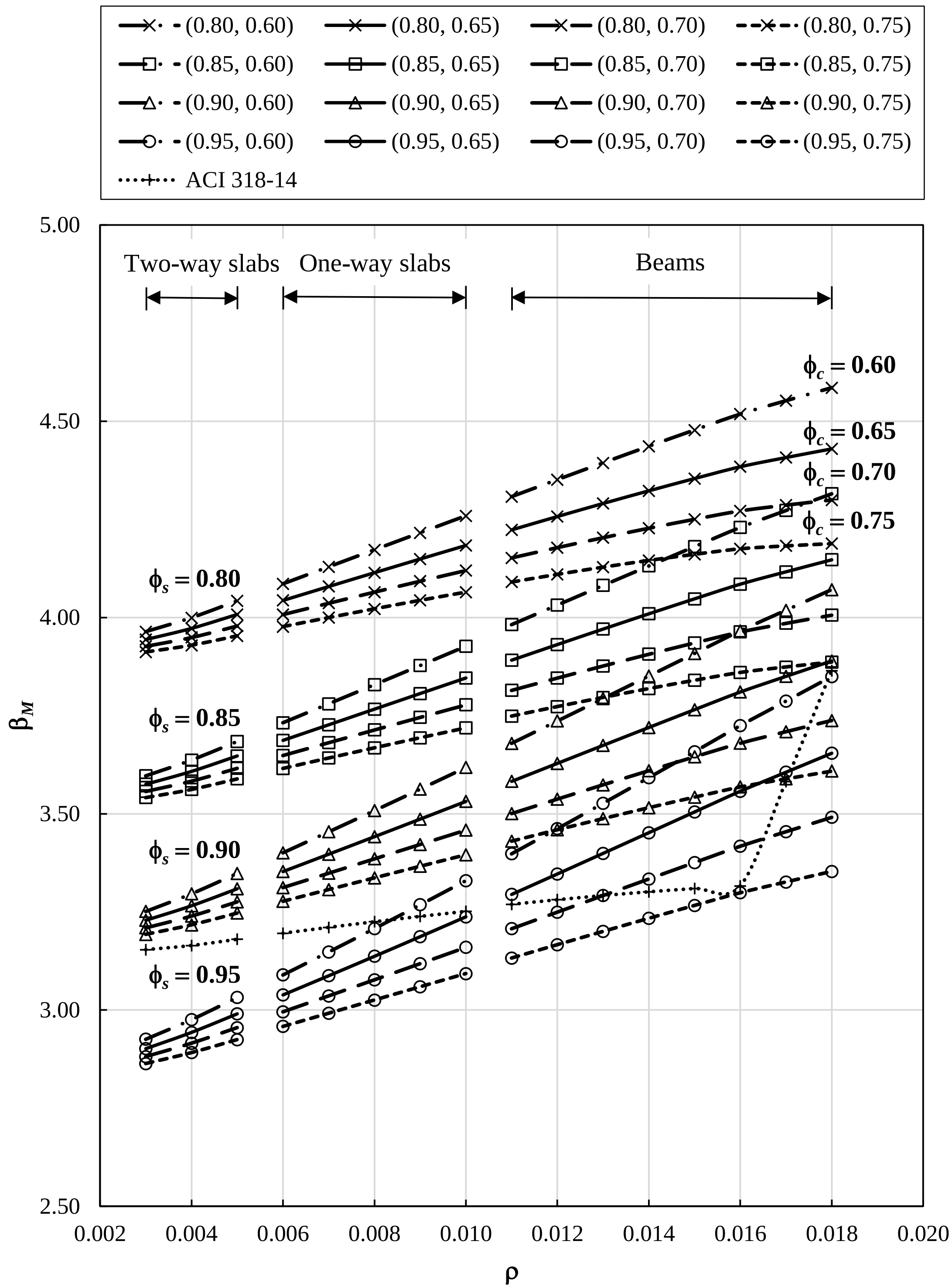


Figure 3.3: Reliability indices for moment, β_M, for f_c' = 25 MPa, w_L/w_D = 0.5, ρ = 0.003–0.018, and linear coefficients of variation for d

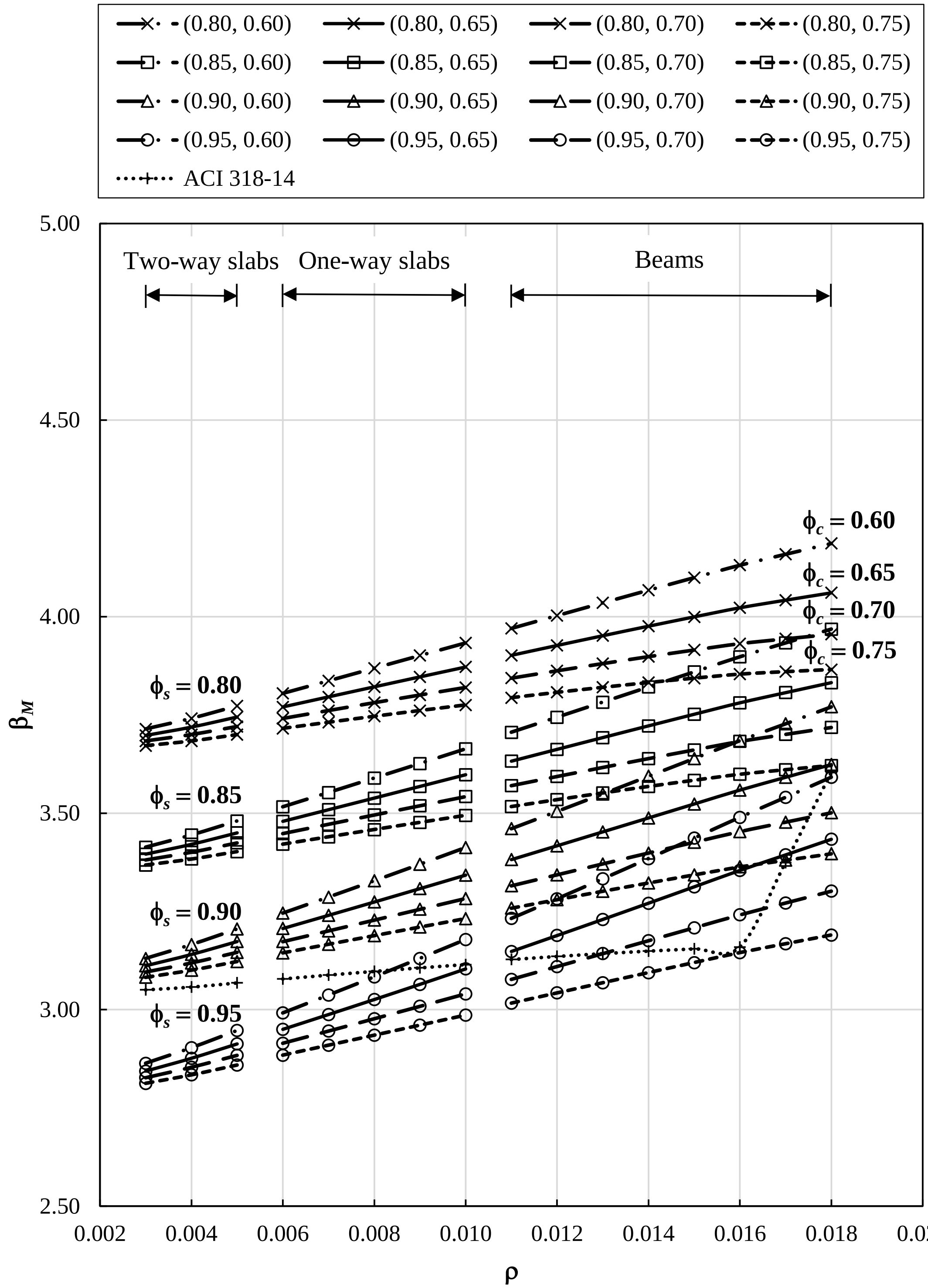
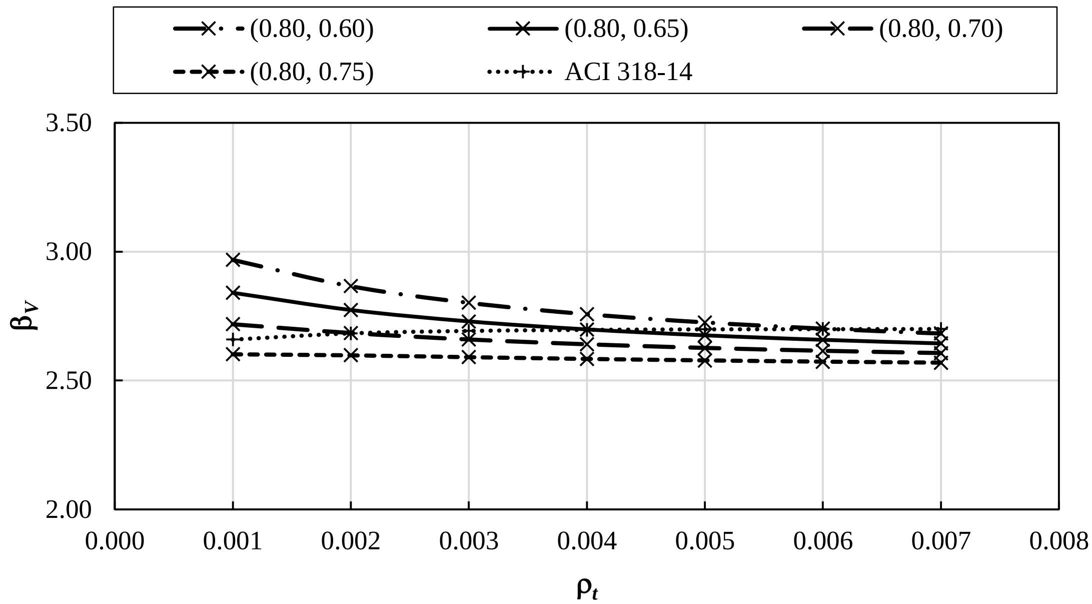


Figure 3.4: Reliability indices for moment, β_M, for $f_c' = 25$ MPa, $w_L/w_D = 1.5$, $\rho = 0.003$–0.018, and linear coefficients of variation for d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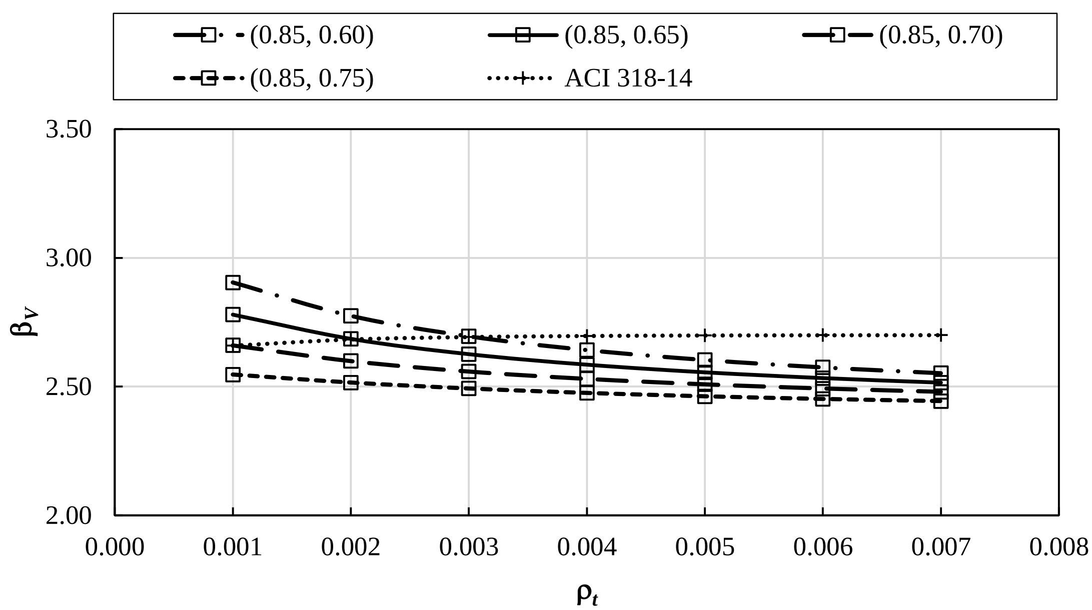
(0.80, 0.65)
(0.80, 0.70)
(0.80, 0.75)
ACI 318-14
3.50
3.00
2.50
2.00
β_V
0.000
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
ρ_t

(a)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ACI 318-14
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2.50
2.00
β_V
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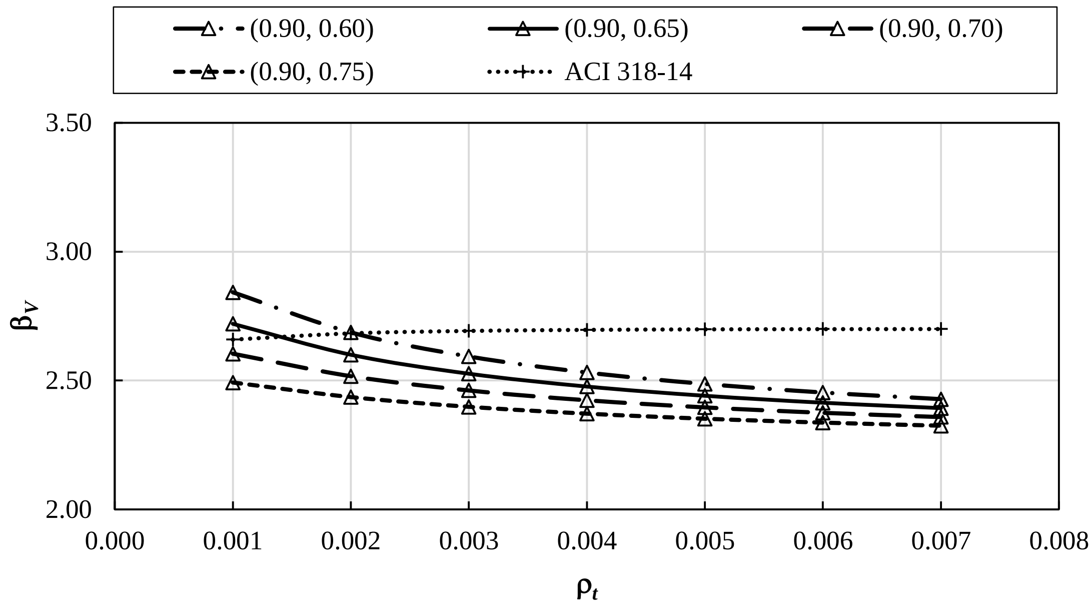
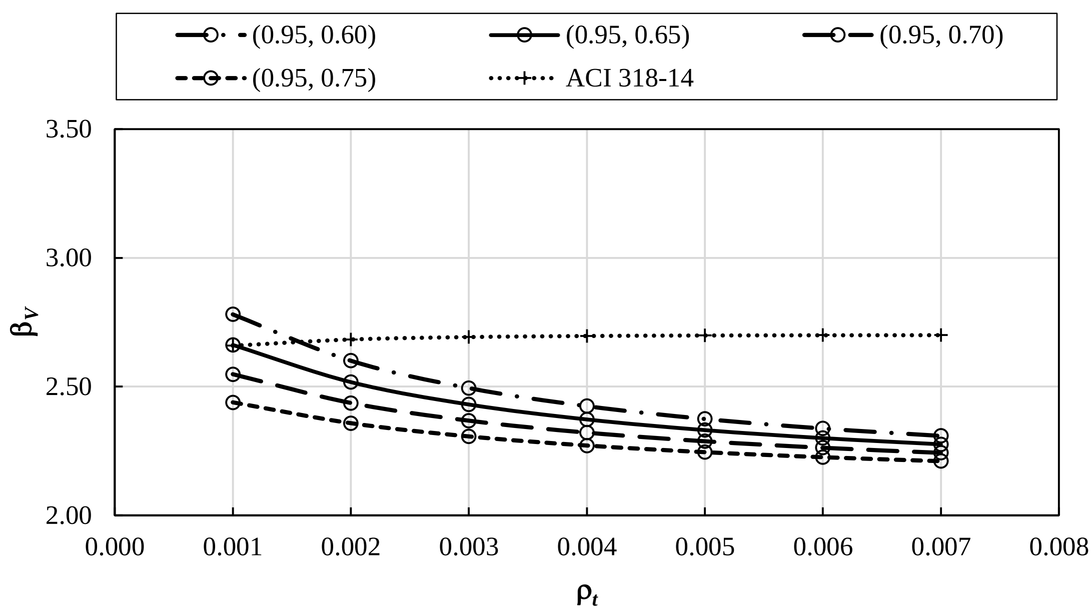
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
ρ_t

(b)

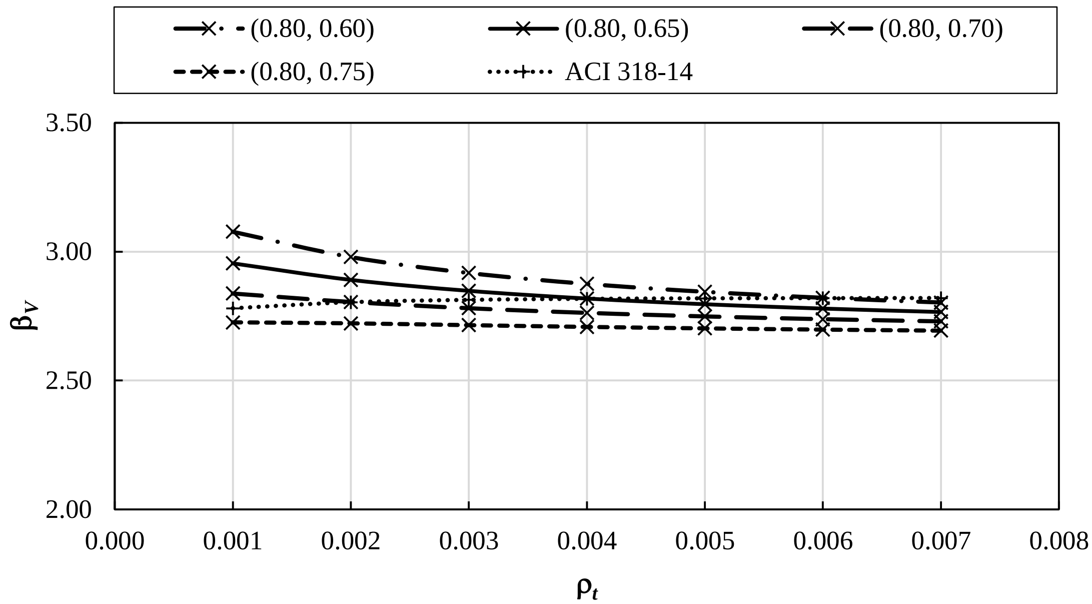


(c)



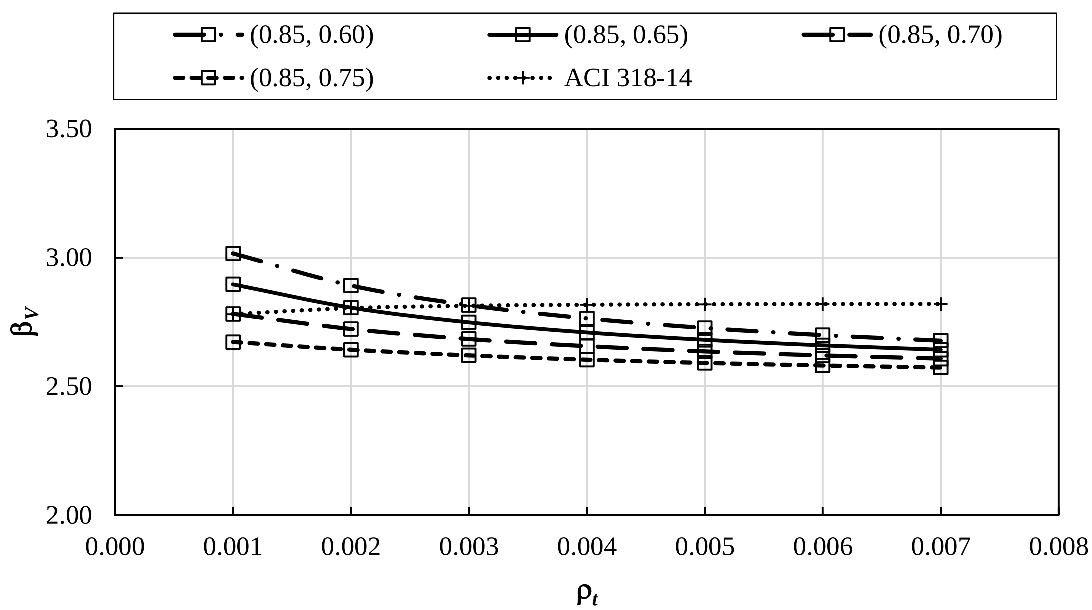
(d)

Figure 3.5: Reliability indices for shear, β_V, for f_c' = 25 MPa, w_L/w_D = 0.5, and ρ_t = 0.001–0.007: (a) ϕ_s = 0.80; (b) ϕ_s = 0.85; (c) ϕ_s = 0.90; (d) ϕ_s = 0.95



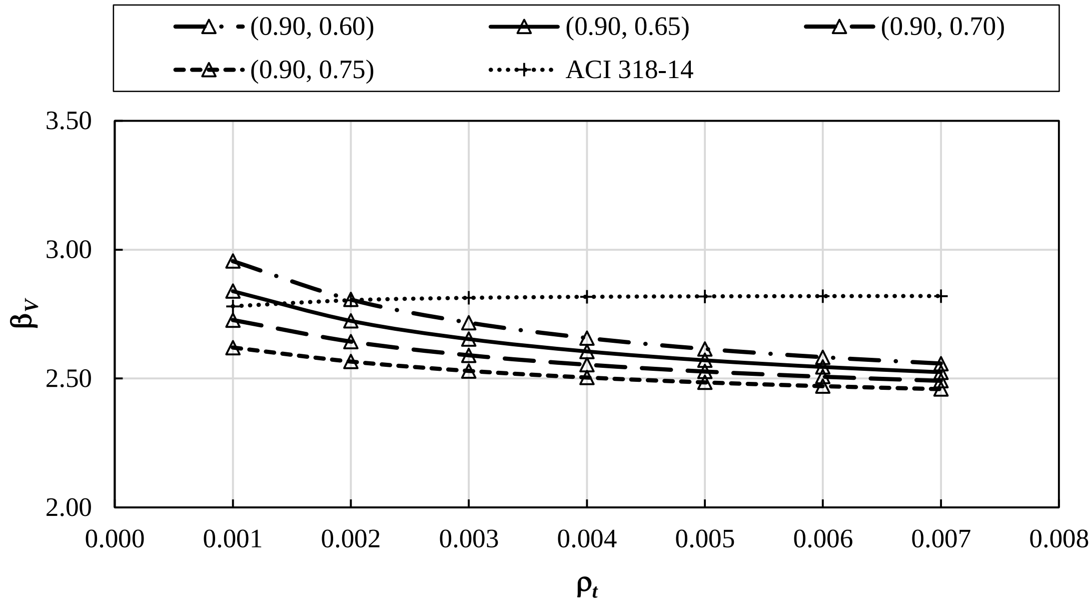
(0.80, 0.60)
(0.80, 0.65)
(0.80, 0.70)
(0.80, 0.75)
ACI 318-14
3.50
3.00
2.50
2.00
βV
0.000
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
ρt

(a)

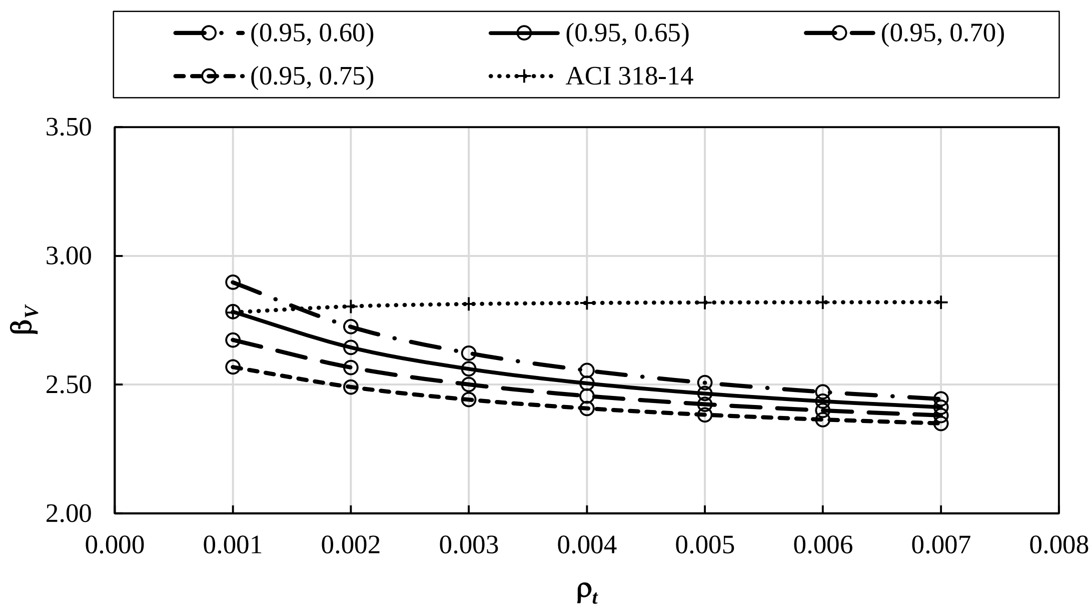


(0.85, 0.60)
(0.85, 0.65)
(0.85, 0.70)
(0.85, 0.75)
ACI 318-14
3.50
3.00
2.50
2.00
βV
0.000
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
ρt

(b)

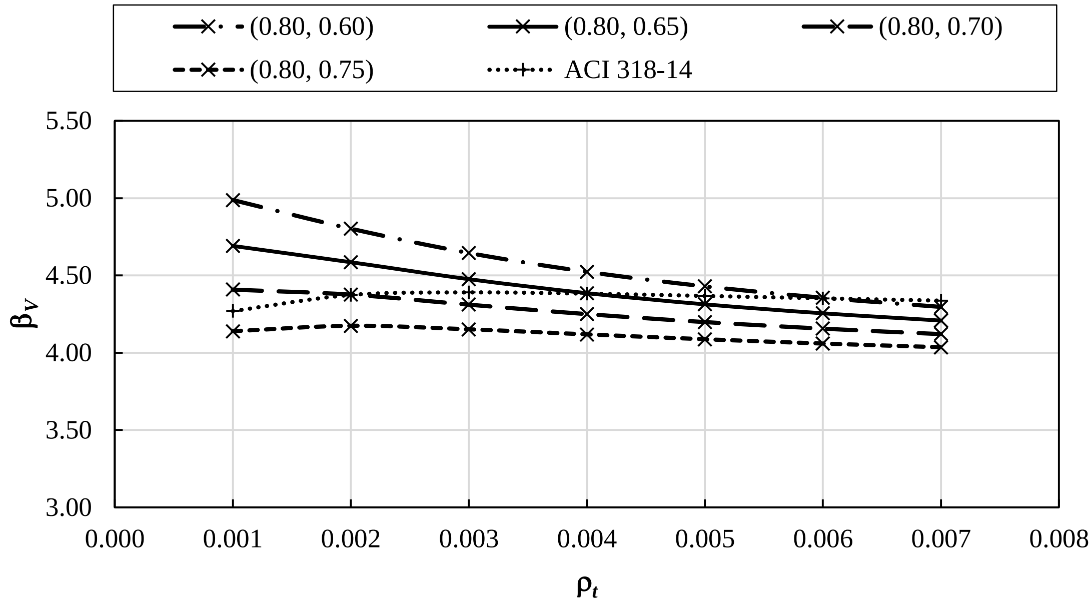


(c)

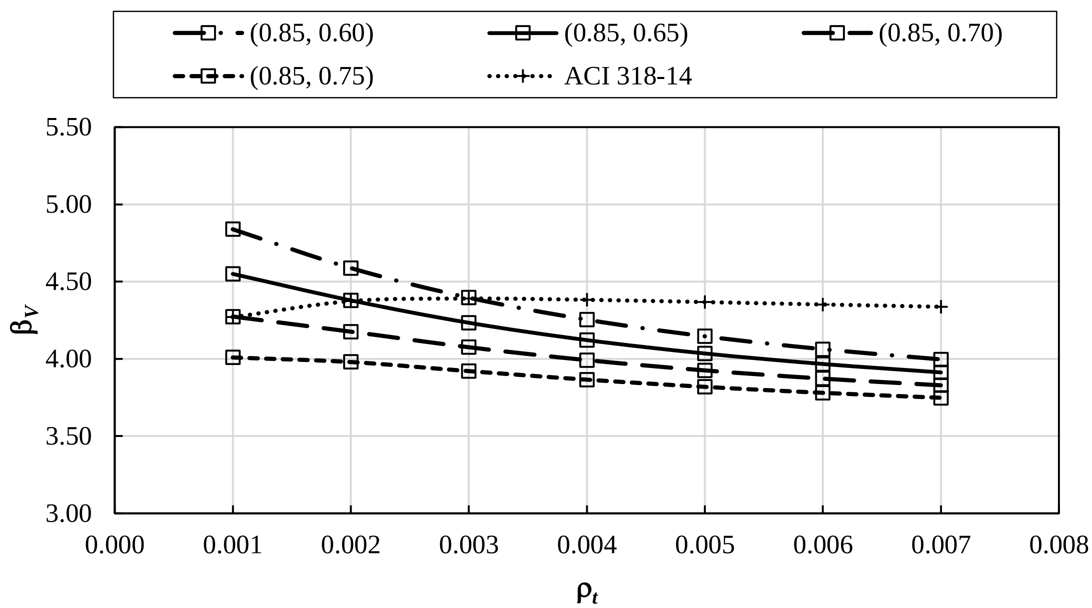


(d)

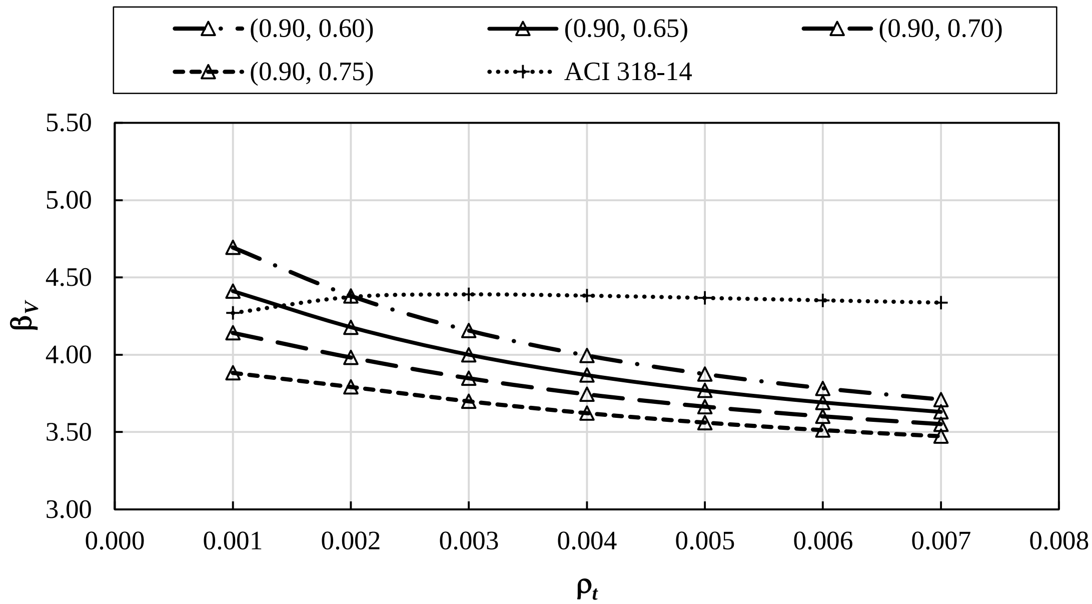
Figure 3.6: Reliability indices for shear, β_V, for f_c' = 25 MPa, w_L/w_D = 1.5, and ρ_t = 0.001–0.007: (a) ϕ_s = 0.80; (b) ϕ_s = 0.85; (c) ϕ_s = 0.90; (d) ϕ_s = 0.95


(0.80, 0.60)
(0.80, 0.65)
(0.80, 0.70)
(0.80, 0.75)
ACI 318-14
5.50
5.00
4.50
4.00
3.50
3.00
β_V
0.000
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
ρ_t

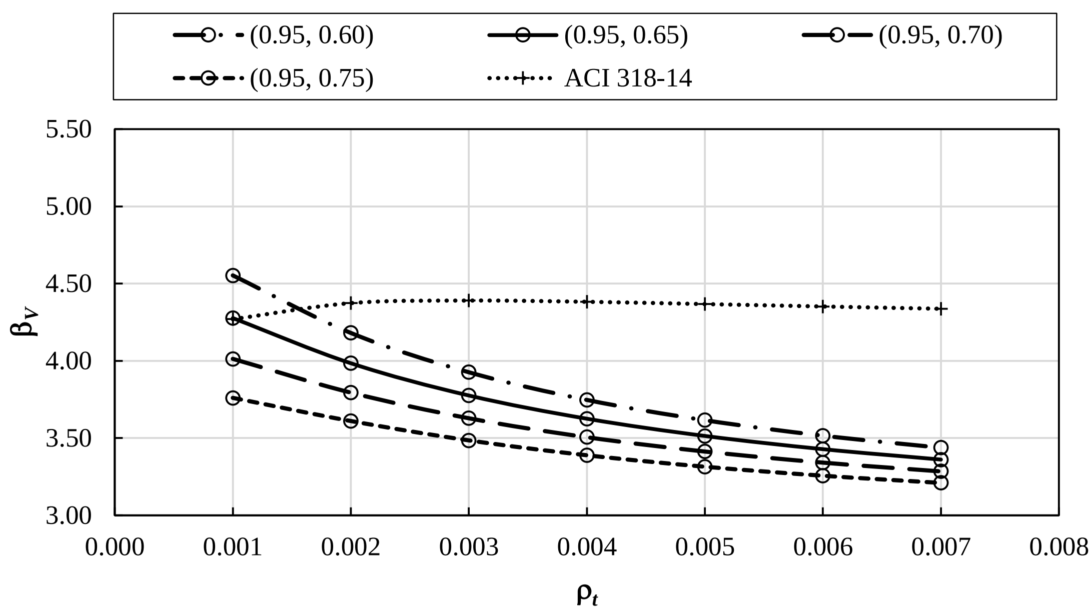
(a)


(0.85, 0.60)
(0.85, 0.65)
(0.85, 0.70)
(0.85, 0.75)
ACI 318-14
5.50
5.00
4.50
4.00
3.50
3.00
β_V
0.000
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
ρ_t

(b)



(c)



(d)

Figure 3.7: Reliability indices for shear, β_V, for $f_c' = 25$ MPa, $w_L/w_D = 0.5$, $\rho_t = 0.001$–0.007, bias coefficient for professional factor = 1.075, and coefficient of variation for professional factor = 0.10: (a) $\phi_s = 0.80$; (b) $\phi_s = 0.85$; (c) $\phi_s = 0.90$; (d) $\phi_s = 0.95$

Chapter 4

4 Derivation of Partial Material Strength Reduction Factors for Combined Moment and Axial Force Based on Reliability Indices

4.1 Introduction

In Chapter 3, the reliability indices presented for moment and one-way shear are based on the first-order, second-moment (FOSM) method. The equations to generate interaction diagrams for combined moment and axial force presented in Chapter 2 are more complicated, however, so a different analysis method, Monte Carlo simulation, is necessary. This chapter presents the reliability analyses for combined moment and axial force to obtain reliability indices based on ACI 318-14 (ACI Committee 318 2014) and the proposed partial material strength reduction factors. The eight geometric and material property combinations of the five column cross sections considered are identical to those presented in Chapter 2.

The objective of this chapter is to select appropriate partial material strength reduction factor combinations that approximately duplicate reliability indices obtained using the current provisions, but may be more uniform for a range of γ, f_c', ρ_g and e/h values, which are the ratio of the distance between the outer layers of reinforcement in a column to the overall column depth, the specified compressive strength of concrete, the total reinforcement ratio, and the eccentricity-to-column depth ratio, respectively.

4.2 Methodology

The simulation is conducted using Monte Carlo techniques (e.g., Hong 2015) which are powerful reliability analysis tools. The basic procedure is to generate *n* sets of random variables and then run the analysis *n* times to simulate the performance (Hong 2015). The transformations from standard uniform random variables or standard normal random variables to normal, lognormal and Gumbel distributed random variables are derived from Hong (2015).

The simulation is run 10^6 times for each case using Matlab (Version R2016b; The Mathworks, Inc. 2016) to compute the reliability indices. By simulating 10^6 times, the reliability indices are not sensitive to a single simulation, so the results tend to be constant.

The means, standard deviations, minimum and maximum values of the reliability indices for ACI 318-14 and each partial material strength reduction factor combination are quantified. The sensitivities of the reliability indices to the partial material strength reduction factor combinations for various geometric and material properties are investigated, and the best partial material strength reduction factor combinations are proposed.

4.3 Assumptions and Design Criteria

Nominal values of geometric and material properties investigated in this chapter are those presented previously in Table 2.10. Again, γ of 0.6 and 0.9, ρ_g of 0.01 and 0.04, f_c' of 25 and 45 MPa, and the specified yield strength of reinforcement, f_y, of 420 MPa are considered. The live-to-dead load ratios, L/D, are assumed to be identical to those adopted previously for moment and shear, of 0.5 and 1.5. The specified dead and live loads are determined to exactly achieve the design strengths. Live load reduction factors due to tributary area are neglected. If they are considered, the range of typical L/D ratios reduces slightly, but the reliability indices computed for a specific L/D value are correct. The applied axial load and moment are assumed perfectly correlated.

4.4 Reliability Analyses

The limit state function is $Z = g(X) = R/Q$, where R is the resistance and Q is the load effect. For short columns, the limit state function at a given eccentricity, $e_i = M_i/P_i$, is (Israel et al. 1987)

$$g(X) = \sqrt{P_i^2 + \left(\frac{P_i e_i}{h}\right)^2} \Bigg/ \sqrt{\left(D_i T_{Di} + L_i T_{Li}\right)^2 + \left(\frac{\left(D_i T_{Di} + L_i T_{Li}\right) e_i}{h}\right)^2} \qquad [4.1]$$

where e_i, P_i, M_i, D_i, L_i, T_{Di}, and T_{Li}, are the simulated values of the eccentricity, axial strength, flexural strength, dead load, live load, transformation from the dead load to the dead load effect, and transformation from the live load to the live load effect, respectively. The professional factor is included in P_i and M_i. In Equation [4.1], h is the nominal column depth (Hong and Zhou 1999). The eccentricity of the applied load effect, e_i, is equal to the nominal value, e, because the axial load and moment are assumed to be perfectly correlated, with identical bias coefficients and coefficients of variation. The e/h values investigated are identical to those in Chapter 2, that is, 0 (compression only), 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0, −10.0, −5.0, −1.0, −0.5, −0.1 and 0 (tension only).

The statistical parameters used in the current reliability analysis were obtained from the literature and are shown in Table 4.1. The statistical parameters for column width, b, depth, h, and effective depth, d, depend on measurement errors in the construction process and the parameters for d are controlled by specified tolerances in ACI 318-14. The standard deviation of d is assumed to be 1/2 of the tolerance specified in Table 26.6.2.1a in ACI 318-14: this is consistent with the common approximation that the total tolerance range equals four standard deviations. The modulus of elasticity of reinforcement, E_s, is assumed deterministic. The statistical parameters for area loads, point loads, and moments are assumed identical. The transformation from the dead load to the dead load effect, T_D, and the transformation from the live load to the live load effect, T_L, are already included in the selected parameters for D and L reported by Ellingwood et al. (1980) and Israel et al. (1987), so bias coefficients of 1.0 and coefficients of variation of 0 are assumed.

The reliability analysis process is as follows:

1. Calculate the design axial strength, ϕP_n or P_r, and the associated design flexural strength, ϕM_n or M_r, for a range of Z values, using Equations [2.21] to [2.45] to generate interaction diagrams. Here, Z is the ratio of the strain in the extreme tension layer of reinforcement to the yield strain. Interpolate for the specific e/h values to

obtain corresponding design axial strength, ϕP_n or P_r. (This step was done in Chapter 2.)

2. Calculate nominal loads, D and L, by equating the design axial strength, ϕP_n or P_r, to the factored axial force from ACI 318-14, $P_u = 1.2D + 1.6L$, for the given load ratio, L/D. The associated equations are:

$$D = \frac{\phi P_n \{\text{or } P_r\}}{\left[1.2 + 1.6\left(L/D\right)\right]} \tag{4.2}$$

and

$$L = \frac{\left(L/D\right)\phi P_n \{\text{or } P_r\}}{\left[1.2 + 1.6\left(L/D\right)\right]} \tag{4.3}$$

3. Calculate the resistance: generate 10^6 sets of random variables using the statistical parameters shown in Table 4.1, and run the simulation to derive 10^6 distinct interaction diagrams by using Equations [2.21] to [2.45] with strength reduction factors equal to 1 and accounting for the professional factor.

4. Calculate the load effects: generate 10^6 sets of random variables for load effects at each specified eccentricity using the statistical parameters shown in Table 4.1.

5. Interpolate on each of the 10^6 interaction diagrams to determine the value of the limit state function, Equation [4.1], at each specified eccentricity.

6. Calculate the number of failures and compute the associated reliability index for combined moment and axial force, β_{PMu} or β_{PMr}, using $\beta = -\Phi^{-1}(P_f)$ (Hong 2015). Here, β_{PMu} is the reliability index for combined moment and axial force obtained using the strength reduction factors in ACI 318-14, β_{PMr} is the reliability index for combined moment and axial force obtained using the partial material strength reduction factors, $\Phi(\bullet)$ is the cumulative distribution function of the standard normal distribution, and P_f is the probability of failure.

7. Summarize the results.

The Matlab (Version R2016b; The Mathworks, Inc. 2016) codes used to conduct the process for Column Section 1 (square section with three bars in each face) and Column Section 5 (spirally reinforced circular section with eight bars evenly distributed around the perimeter), two representative cross sections, are presented in Appendix C.

To save time and avoid unnecessary calculations, only four pairs of partial material strength reduction factors are analyzed for tied columns, combinations with ϕ_s of 0.85 and 0.90, and ϕ_c of 0.60 and 0.65.

For Column Section 1 (square section with three bars in each face), the reliability indices for combined moment and axial force corresponding to ACI 318-14, β_{PMu}, with respect to e/h are shown in Figures 4.1a and 4.1b for the eight property combinations with L/D of 0.5 for $e/h > 0$ and $e/h < 0$, respectively. The vertical dotted lines show the range of e/h at the balance point. For e/h approximately ranging from 0.1 to 0.38, all of the eight property combinations do not reach the minimum value of e_{ubal}/h, where e corresponds to the balanced failure for ACI 318-14, so they are compression-controlled. In this region, similar to the trend of design combined flexural and axial strength ratio, α_{PM}, described in Chapter 2, the most influential property is ρ_g. For ρ_g of 4%, β_{PMu} reaches 4.06, and for ρ_g of 1%, β_{PMu} reaches 2.44, causing inconsistent reliability indices with varying e/h. Varying f_c' causes small changes for lower ρ_g, but large changes for higher ρ_g. And again, the least influential parameter is γ. For e/h approximately ranging from 0.38 to 0.88, some of the cases shown reach the balance point, but others do not. For e/h approximately ranging from 0.88 to 10, all of the eight property combinations equal or exceed the maximum value of e_{ubal}/h, so they are in the transition or tension-controlled regions. The two lines corresponding to Property Combination 2 (γ of 0.6, f_c' of 25 MPa, and ρ_g of 0.04) and Property Combination 4 (γ of 0.6, f_c' of 45 MPa, and ρ_g of 0.04), differ from the others shown because they remain in the transition region until e/h equals 10. For e/h less than 0 shown in Figure 4.1b, the sections are subjected to tension. In the tension-controlled region, the influences of γ, f_c' and ρ_g on β_{PMu} become small. Results for L/D of 1.5 are shown in Figure C.1 of Appendix C. The influence of L/D is small.

For the partial material strength reduction factors ϕ_s of 0.90 and ϕ_c of 0.60, β_{PMr} values for Column Section 1 are shown in Figures 4.2a and 4.2b for the eight property combinations with L/D of 0.5 for $e/h > 0$ and $e/h < 0$, respectively. These β_{PMr} values are relatively uniform, ranging from 2.75 to 3.40 compared to the range of 2.44 to 4.06 for the current ACI 318-14 criteria shown in Figures 4.1a and 4.1b. This indicates an advantage of using partial material strength reduction factors. In the compression-controlled region, the influence of γ is very small and influences of f_c' and ρ are smaller than those shown in Figure 4.1a. In the tension-controlled region, γ has a more significant impact on β_{PMr} for increased e/h values. Results for L/D of 1.5 are shown in Figure C.2 of Appendix C. The influence of L/D is small.

According to ASCE 7-10 (ASCE 2010), for a 50-year service period, normal buildings with Risk Category II should exhibit a reliability index range of 3.0 to 4.0 for columns. Figures 4.3a and 4.3b show the reliability indices for combined moment and axial force, β_{PM}, for Column Section 1, Property Combination 1 (γ of 0.6, f_c' of 25 MPa, and ρ_g of 0.01) and L/D of 0.5 for $e/h > 0$ and $e/h < 0$, respectively. The dotted vertical line in Figure 4.3a shows e_{ubal}/h of 0.42, the e corresponding to the balanced failure for ACI 318-14. The two solid vertical lines represent the range of e values corresponding to the balanced failures for the sixteen partial material strength reduction factor combinations (although there are only four combinations shown in the figure), from $(e_{rbal}/h)_{min}$ of 0.43 to $(e_{rbal}/h)_{max}$ of 0.50. When $e/h > 0$, the reliability index corresponding to ACI 318-14 decreases abruptly for eccentricities greater than that corresponding to the balance point, because the strength reduction factor in ACI 318-14, ϕ, increases from 0.65 to 0.90 for tied columns. In the compression-controlled region, two families of lines are defined by the two ϕ_c values, and the differences within each family are due to the two ϕ_s values. As the eccentricity increases, the influence of steel strength becomes more significant than that of the concrete strength, so the two families are defined by the two ϕ_s values. When $e/h < 0$, β_{PM} increases slightly as the absolute value of e/h increases. In this case, the influence of ϕ_s on β_{PMr} is greater because the tensile strength of concrete is negligible and does not contribute to the strength.

Results for the other seven property combinations and L/D of 0.5 are shown in Figures C.3–C.9 of Appendix C. In the compression-controlled region where the impact of ϕ_c is greatest, β_{PMr} corresponding to the partial material strength reduction factor combinations with ϕ_c of 0.60 approaches β_{PMu} when ρ_g equals 0.01. When β_{PMu} is too conservative, i.e. for ρ_g of 0.04, β_{PMr} values for ϕ_c of 0.60 still fall in an appropriate range. In the tension-controlled region where ϕ_s is more influential, β_{PMr} corresponding to the combinations with ϕ_s of 0.90 approaches β_{PMu}.

Values of β_{PM} for Column Sections 2, 3 and 4 and L/D of 0.5 are shown in Figures C.10–C.33 of Appendix C. The reliability indices, impacts of ϕ_s and ϕ_c, and the best partial material strength reduction factors are similar to those shown for Column Section 1.

For Column Section 5 (spirally reinforced circular section with eight bars evenly distributed around the perimeter), the ϕ values range from 0.75 to 0.90. The four partial material strength reduction factor combinations analyzed for the tied columns are therefore not appropriate. The analysis results for Column Section 5 presented in Chapter 2 indicate that: when the failure is compression-controlled, the partial material strength reduction factor combinations with ϕ_c of 0.70 are the best; and, when the failure is tension-controlled, the combinations with ϕ_s of 0.90 and 0.95 are the best. Therefore, two combinations with ϕ_s of 0.90 and ϕ_c of 0.70, and ϕ_s of 0.95 and ϕ_c of 0.70 are investigated.

Figures 4.4a and 4.4b show β_{PM} for Column Section 5, Property Combination 1 (γ of 0.6, f_c' of 25 MPa, and ρ_g of 0.01) and L/D of 0.5 for $e/h > 0$ and $e/h < 0$, respectively. The reliability index corresponding to ACI 318-14 decreases less abruptly as the eccentricity increases beyond that at the balance point compared with that shown in Figure 4.3a. In this case, the difference between the two ϕ values is (0.90 − 0.75 =) 0.15, which is markedly smaller than (0.90 − 0.65 =) 0.25 for the tied column. For the other seven property combinations and L/D of 0.5, the ranges of reliability index are shown in Figures C.34–C.40 of Appendix C. The influences of geometric and material properties, load ratios, and partial material strength reduction factors are similar to those for Column

Section 1. Inspection of these figures indicates that the reliability indices corresponding to the combination with ϕ_s of 0.90 and ϕ_c of 0.70 are less variable and slightly conservative compared to those corresponding to the ACI 318-14 criteria.

4.5 Recommended Partial Material Strength Reduction Factors

For Column Section 1 (square section with three bars in each face), based on Figures 4.3, and C.3–C.9, the best partial material strength reduction factor combination is ϕ_s of 0.90 and ϕ_c of 0.60. Table 4.2 shows the means, standard deviations, minima and maxima of the reliability indices based on the eight property combinations and the two L/D ratios. The category of the four ranges of e/h is same with that presented in Chapter 2. The combination with ϕ_s of 0.90 and ϕ_c of 0.60 is the best: for any range of e/h, the minimum reliability indices are not smaller than those obtained using ACI 318-14; the means and maxima are not excessively conservative; and, the standard deviations are relatively small. Adopting this combination of partial material strength reduction factors yields reliability indices that are bounded by those obtained using the ACI 318-14 criteria. The proposed partial material strength reduction factors yield reliability indices that are neither unnecessary large to cause the strength to be excessive nor excessively low to make the column unsafe. Similar results corresponding to Column Sections 2, 3 and 4 are shown in Tables 4.3–4.5, respectively, indicating that the longitudinal reinforcement arrangement is not a significant factor.

For Column Section 5 (spirally reinforced circular section with eight bars evenly distributed around the perimeter), the means, standard deviations, minima and maxima of the reliability indices are shown in Table 4.6. The combination with ϕ_s of 0.90 and ϕ_c of 0.70 is the best.

Alternatively, the maximum axial compressive strengths defined in ACI 318-14 are limited to 0.80 and 0.85 of the axial compressive strengths at zero eccentricity for tied and spirally reinforced columns, respectively, and they are approximate axial strengths at e/h of 0.10 and 0.05, respectively (ACI Committee 318 2014). These values can be reviewed and the excessively high reliability indices for columns with the higher reinforcement ratio may reduce.

4.6 Summary and Conclusions

This chapter has presented the reliability indices obtained using Monte Carlo simulation for five column cross sections and eight geometric and material property combinations for each cross section. Two live-to-dead load ratios are considered.

When the section is compression-controlled, the reliability index corresponding to ACI 318-14 is very sensitive to the reinforcement ratio, ρ_g, because the coefficient of variation of f_y is markedly less than that of f_c'. When the reinforcement ratio increases, the reliability index also increases (Israel et al. 1987). When the section is tension-controlled, the influence of γ becomes greater for β_{PMr}, while less for β_{PMu}. The influence of f_c' on the reliability index is larger in the compression-controlled region than in the tension-controlled region. For the partial material strength reduction factors, ϕ_s is more influential in the tension-controlled region, while ϕ_c is more influential in the compression-controlled region. The L/D ratio has negligible effects on the computed reliability indices.

For columns with tied reinforcement, the best partial material strength reduction factor combination is ϕ_s of 0.90 and ϕ_c of 0.60, which is also identical to the values obtained in Chapter 2. The four tied column cross sections investigated yield the similar results, indicating that the longitudinal reinforcement arrangement is not a significant factor.

For columns with spiral reinforcement, the best partial material strength reduction factor combination is ϕ_s of 0.90 and ϕ_c of 0.70.

Therefore, ϕ_s of 0.90 is the best for both tied and spirally reinforced columns. A unique ϕ_c value is appropriate for spirally reinforced columns to account for the advantages of confinement that mainly impact the compression-controlled region.

Table 4.1: Statistical parameters for column reliability analysis

Item	Source	Comment	Distribution		
Geometric properties				Mean	σ
Columns				(mm)	(mm)
b	Ellingwood et al. 1980	Rectangular	Normal	Nominal+1.52	6.35
h	Ellingwood et al. 1980	Rectangular	Normal	Nominal+1.52	6.35
	Mirza and MacGregor 1979	Circular	Normal	Nominal	4.76
d	ACI Committee 318 2014	$d \leq 203$ mm	Normal	Nominal	4.76
		$d > 203$ mm	Normal	Nominal	6.35
Reinforcement				δ	*V*
A_s	Nowak and Szerszen 2003	—	Normal	1.0	0.015
Material strengths				δ	*V*
f_c'	Bartlett 2007 Bartlett and MacGregor 1999	Cast-in-place	Normal	1.15	0.211
f_y	Ellingwood et al. 1980	—	Lognormal	1.125	0.098
E_s	—	—	Deterministic	1	0
Professional factor				δ	*V*
P	Nowak and Szerszen 2003	Tied	Normal	1.00	0.08
		Spiral	Normal	1.05	0.06
Load effects				δ	*V*
DT_D	Ellingwood et al. 1980	All materials	Normal	1.05	0.10
LT_L	Israel et al. 1987	A58.1-1982 live load reductions	Gumbel	1.00	0.25

Table 4.2: Means, standard deviations, minima and maxima of reliability indices for combined moment and axial force, β_{PM}, for Column Section 1

e/h	Statistical parameter	ACI 318-14	$\phi_s = 0.85$, $\phi_c = 0.60$	$\phi_s = 0.85$, $\phi_c = 0.65$	$\phi_s = 0.90$, $\phi_c = 0.60$	$\phi_s = 0.90$, $\phi_c = 0.65$
$0 \leq e/h \leq 0.3$	Mean	3.234	3.140	2.981	3.080	2.920
	σ	0.350	0.188	0.194	0.153	0.158
	Min	2.737	2.836	2.676	2.827	2.660
	Max	3.983	3.507	3.359	3.390	3.220
$0.3 < e/h \leq 1.0$	Mean	3.176	3.291	3.165	3.169	3.040
	σ	0.492	0.130	0.134	0.111	0.108
	Min	2.442	2.959	2.812	2.906	2.757
	Max	4.056	3.562	3.458	3.405	3.261
$1.0 < e/h \leq 10.0$	Mean	2.825	3.263	3.207	3.037	2.982
	σ	0.192	0.187	0.162	0.184	0.159
	Min	2.646	2.943	2.944	2.727	2.719
	Max	3.614	3.614	3.489	3.363	3.254
$e/h \leq 0$	Mean	2.742	3.096	3.075	2.844	2.820
	σ	0.077	0.162	0.141	0.159	0.133
	Min	2.627	2.882	2.887	2.657	2.654
	Max	3.086	3.521	3.395	3.258	3.165

Table 4.3: Means, standard deviations, minima and maxima of reliability indices for combined moment and axial force, β_{PM}, for Column Section 2

e/h	Statistical parameter	ACI 318-14	$\phi_s = 0.85$, $\phi_c = 0.60$	$\phi_s = 0.85$, $\phi_c = 0.65$	$\phi_s = 0.90$, $\phi_c = 0.60$	$\phi_s = 0.90$, $\phi_c = 0.65$
$0 \le e/h \le 0.4$	Mean	3.303	3.177	3.020	3.107	2.951
	σ	0.353	0.189	0.192	0.150	0.159
	Min	2.769	2.861	2.692	2.842	2.661
	Max	4.013	3.548	3.403	3.404	3.262
$0.4 < e/h \le 1.0$	Mean	3.263	3.313	3.202	3.167	3.051
	σ	0.520	0.140	0.135	0.131	0.119
	Min	2.570	2.999	2.861	2.823	2.798
	Max	4.224	3.581	3.445	3.437	3.278
$1.0 < e/h \le 10.0$	Mean	2.772	3.153	3.125	2.906	2.877
	σ	0.108	0.176	0.153	0.171	0.147
	Min	2.666	2.906	2.899	2.677	2.676
	Max	3.492	3.530	3.444	3.294	3.205
$e/h \le 0$	Mean	2.729	3.071	3.061	2.815	2.800
	σ	0.060	0.145	0.134	0.136	0.118
	Min	2.620	2.878	2.879	2.643	2.645
	Max	2.823	3.489	3.414	3.233	3.164

Table 4.4: Means, standard deviations, minima and maxima of reliability indices for combined moment and axial force, β_{PM}, for Column Section 3

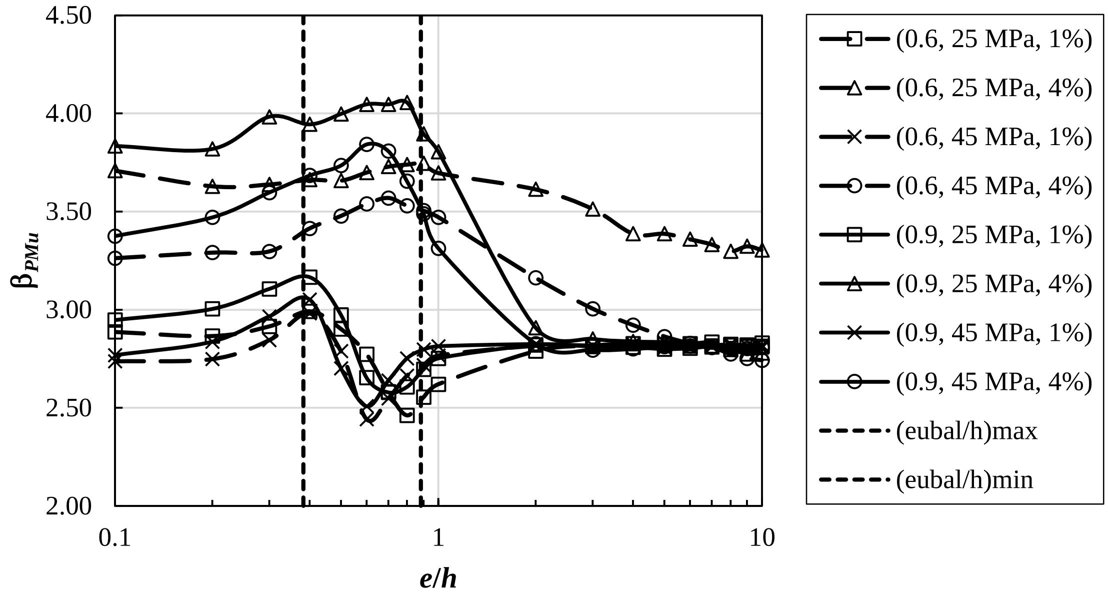
e/h	Statistical parameter	ACI 318-14	$\phi_s = 0.85$, $\phi_c = 0.60$	$\phi_s = 0.85$, $\phi_c = 0.65$	$\phi_s = 0.90$, $\phi_c = 0.60$	$\phi_s = 0.90$, $\phi_c = 0.65$
$0 \le e/h \le 0.3$	Mean	3.218	3.132	2.971	3.069	2.911
	σ	0.349	0.188	0.194	0.152	0.158
	Min	2.729	2.845	2.663	2.813	2.655
	Max	3.976	3.501	3.341	3.374	3.214
$0.3 < e/h \le 1.0$	Mean	3.123	3.281	3.154	3.168	3.036
	σ	0.488	0.125	0.134	0.106	0.107
	Min	2.382	2.956	2.780	2.904	2.748
	Max	4.038	3.600	3.486	3.411	3.285
$1.0 < e/h \le 10.0$	Mean	2.837	3.296	3.229	3.082	3.016
	σ	0.197	0.186	0.159	0.188	0.159
	Min	2.644	2.951	2.939	2.735	2.725
	Max	3.583	3.612	3.539	3.432	3.296
$e/h \le 0$	Mean	2.740	3.115	3.087	2.865	2.836
	σ	0.092	0.174	0.147	0.176	0.144
	Min	2.579	2.884	2.882	2.658	2.655
	Max	3.166	3.551	3.445	3.373	3.224

Table 4.5: Means, standard deviations, minima and maxima of reliability indices for combined moment and axial force, β_{PM}, for Column Section 4

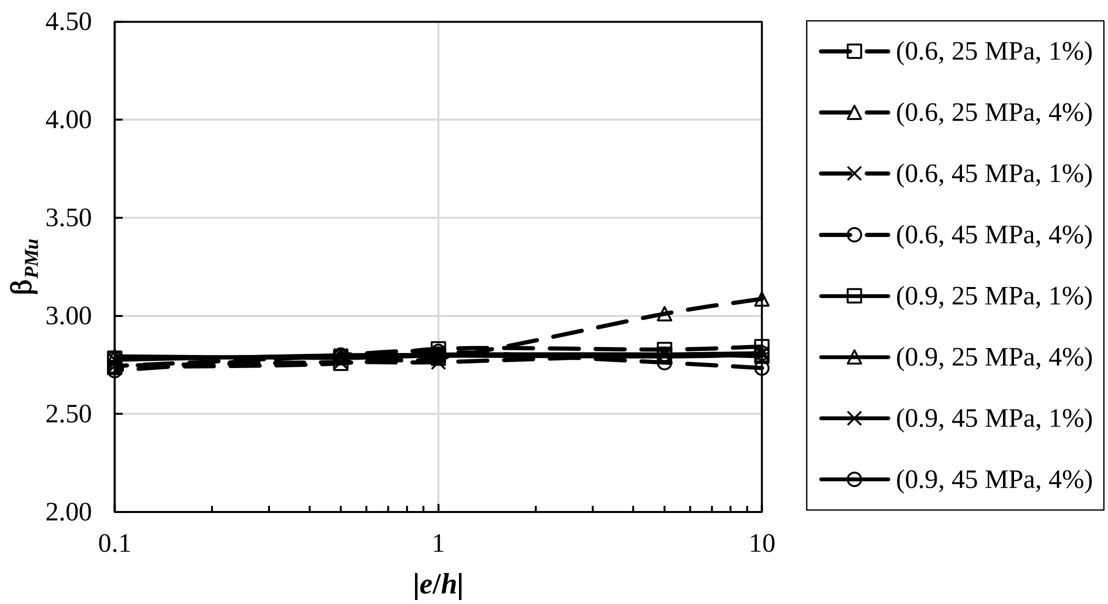
e/h	Statistical parameter	ACI 318-14	$\phi_s = 0.85$, $\phi_c = 0.60$	$\phi_s = 0.85$, $\phi_c = 0.65$	$\phi_s = 0.90$, $\phi_c = 0.60$	$\phi_s = 0.90$, $\phi_c = 0.65$
$0 \leq e/h \leq 0.3$	Mean	3.202	3.110	2.952	3.051	2.889
	σ	0.342	0.184	0.190	0.154	0.158
	Min	2.688	2.775	2.613	2.773	2.591
	Max	3.921	3.494	3.332	3.398	3.203
$0.3 < e/h \leq 1.0$	Mean	3.133	3.289	3.161	3.168	3.037
	σ	0.455	0.128	0.133	0.107	0.105
	Min	2.490	2.941	2.792	2.890	2.737
	Max	3.957	3.556	3.423	3.408	3.249
$1.0 < e/h \leq 10.0$	Mean	2.901	3.297	3.230	3.091	3.020
	σ	0.258	0.149	0.139	0.136	0.122
	Min	2.621	3.051	3.031	2.846	2.822
	Max	3.556	3.559	3.474	3.339	3.244
$e/h \leq 0$	Mean	2.784	3.162	3.127	2.918	2.883
	σ	0.148	0.180	0.157	0.182	0.154
	Min	2.638	2.909	2.899	2.677	2.670
	Max	3.465	3.544	3.446	3.343	3.243

Table 4.6: Means, standard deviations, minima and maxima of reliability indices for combined moment and axial force, β_{PM}, for Column Section 5

e/h	Statistical parameter	ACI 318-14	$\phi_s = 0.90$, $\phi_c = 0.70$	$\phi_s = 0.95$, $\phi_c = 0.70$
$0 \le e/h \le 0.3$	Mean	2.974	2.946	2.889
	σ	0.325	0.211	0.179
	Min	2.464	2.587	2.564
	Max	3.650	3.392	3.254
$0.3 < e/h \le 1.0$	Mean	3.147	3.192	3.074
	σ	0.291	0.158	0.130
	Min	2.730	2.781	2.723
	Max	3.788	3.526	3.335
$1.0 < e/h \le 10.0$	Mean	3.188	3.310	3.107
	σ	0.184	0.176	0.154
	Min	2.888	3.059	2.876
	Max	3.562	3.628	3.395
$e/h \le 0$	Mean	3.133	3.217	2.980
	σ	0.160	0.185	0.174
	Min	2.931	2.957	2.739
	Max	3.589	3.562	3.352

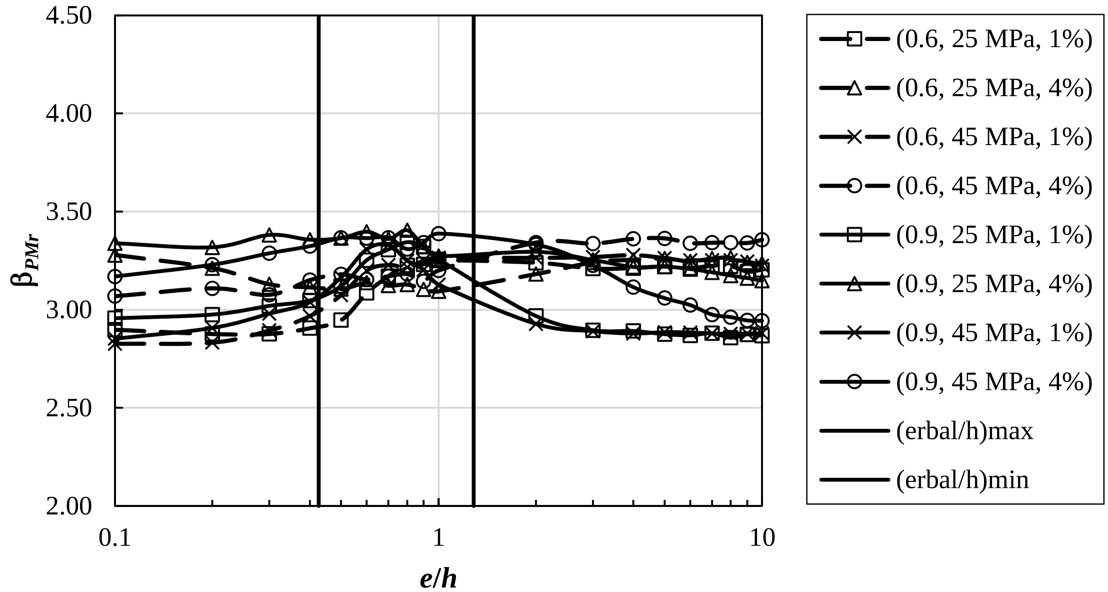


(a)

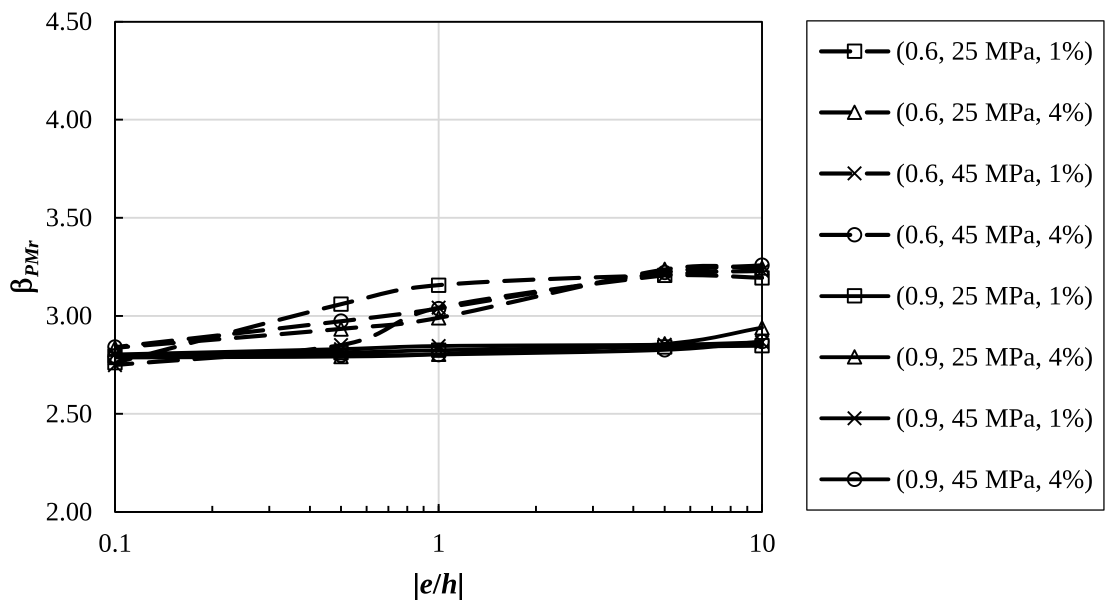


(b)

Figure 4.1: Reliability indices for combined moment and axial force using strength reduction factors in ACI 318-14, β_{PMu}, for Column Section 1 and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

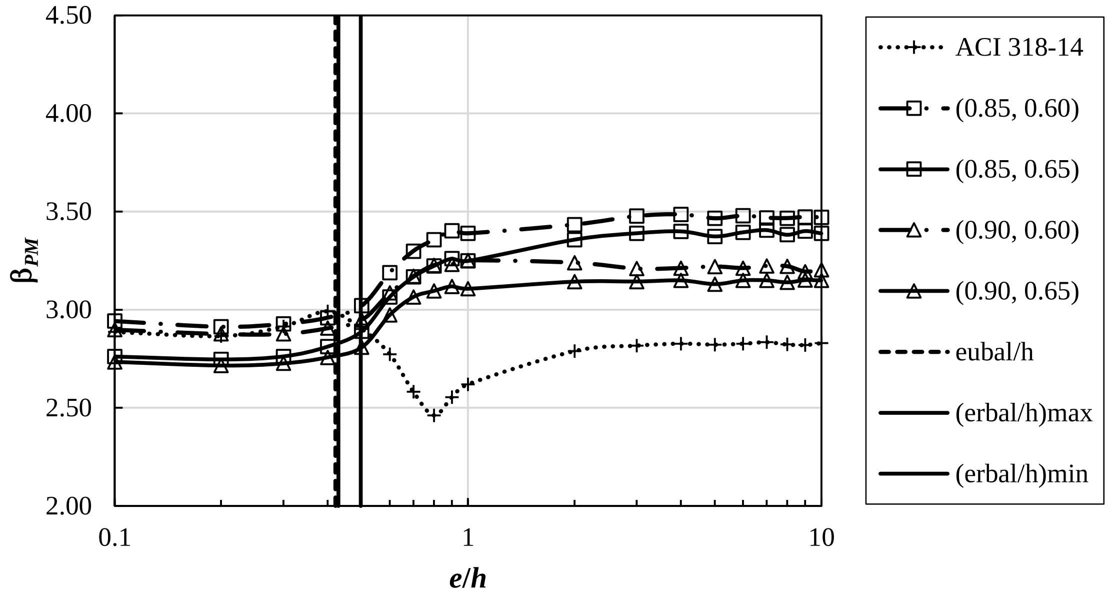


(a)

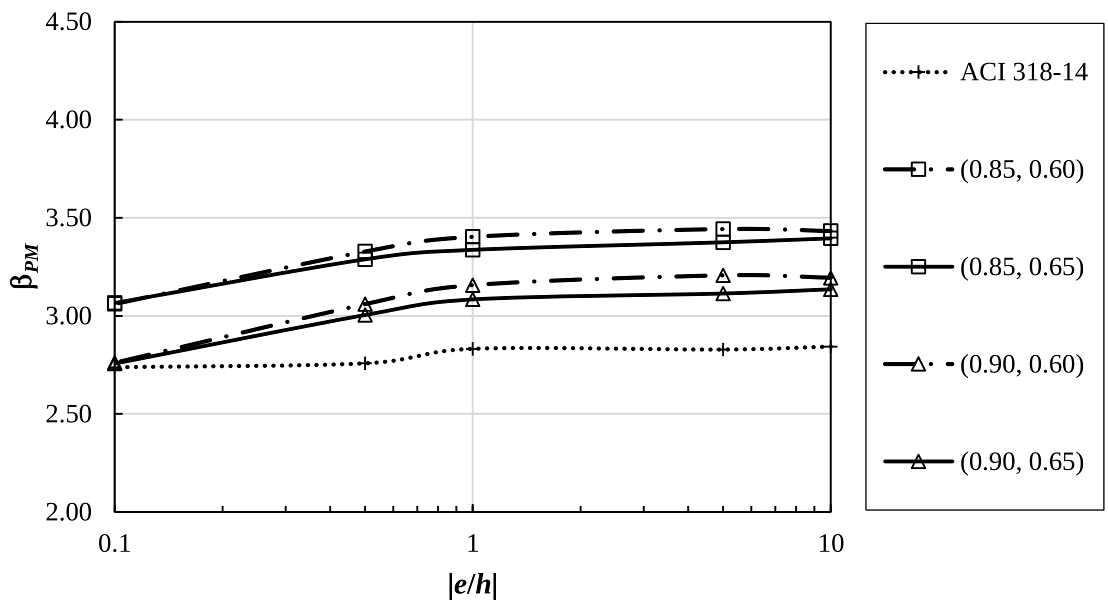


(b)

Figure 4.2: Reliability indices for combined moment and axial force, β_{PMr}, corresponding to $\phi_s = 0.90$ and $\phi_c = 0.60$, for Column Section 1 and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

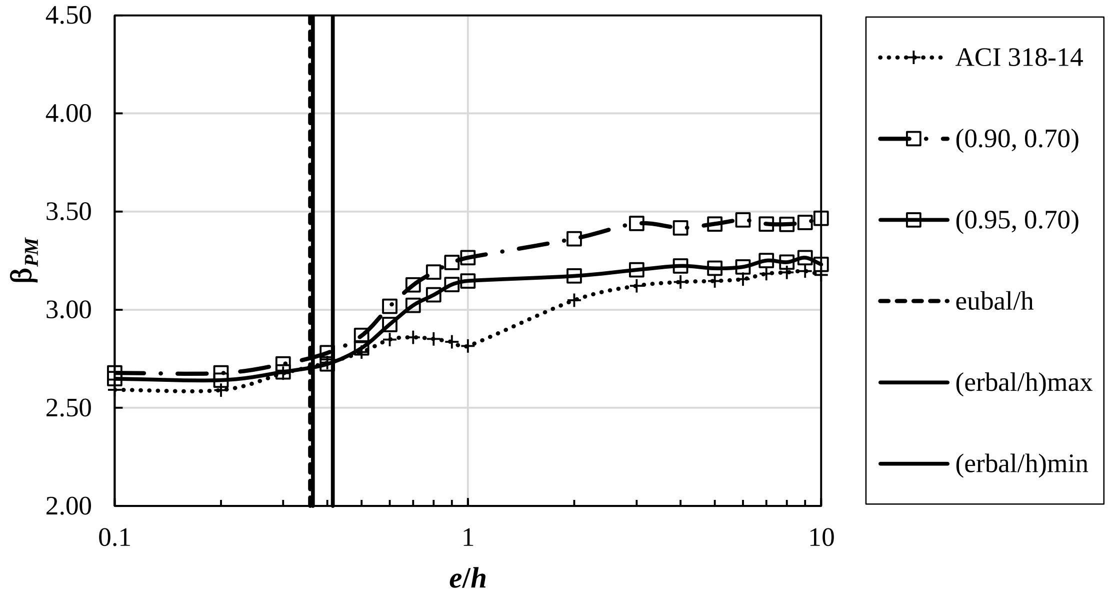


(a)

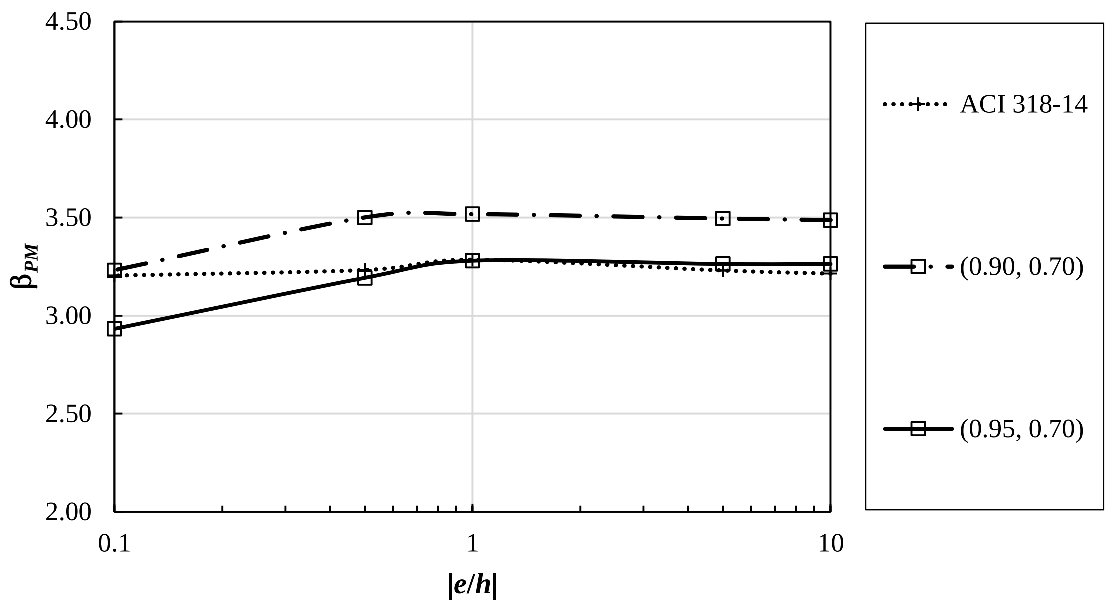


(b)

Figure 4.3: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 1, Property Combination 1, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$



(a)



(b)

Figure 4.4: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 5, Property Combination 1, $L/D = 0.5$, and: (*a*) $e/h > 0$; (*b*) $e/h < 0$

Chapter 5

5 Summary and Conclusions

This chapter presents a summary of the work conducted in this study, lists the conclusions and recommends some suggestions for future work.

5.1 Summary

ACI 318-14 (ACI Committee 318 2014) defines an overall strength reduction factor to account for the probability of understrength. However, it leads to some unsatisfactory results, particularly inconsistencies in the transition region of the interaction diagram, between the compression-controlled and tension controlled regions (e.g., Gamble 1998, 2015), and so non-unique moment capacities for one axial strength level for sections with wide flanges (Lequesne and Pincheira 2014). The statistical parameters that quantify the professional factor for shear since previous calibrations (Israel et al. 1987; Nowak and Szerszen 2003) have also changed markedly (Somo and Hong 2006). Therefore, the present study proposes partial material strength reduction factors for concrete, ϕ_c, and reinforcing steel, ϕ_s, that yield similar design strengths and more consistent reliability indices compared to those based on the strength reduction factors, ϕ, in ACI 318-14. Three structural actions acting on non-prestressed members are investigated: moment; one-way shear; and, combined moment and axial force.

The comparison of design strengths is presented in Chapter 2. For members subjected to moment, singly reinforced sections with concrete compressive strengths, f_c', of 25 and 45 MPa, reinforcement yield strength, f_y, of 420 MPa, and reinforcement ratio, ρ, ranging from 0.003 to 0.018 are investigated. For members subjected to shear, rectangular beam sections with the same material strengths and ranges of transverse reinforcement ratio, ρ_t, from 0.001 to 0.007 for f_c' of 25 MPa and 0.001 to 0.010 for f_c' of 45 MPa are studied. For members subjected to combined moment and axial force, five column cross sections including square section with three bars in each face, square section with three bars in two end faces only, square section with three bars in two side faces only, tied circular section with eight bars evenly distributed around the perimeter, and spirally reinforced circular section with eight bars evenly distributed around the perimeter are investigated. For each column section, eight geometric and material property combinations are investigated, specifically, the ratios of the distance between the outer layers of reinforcement to the overall column depth, γ, of 0.6 and 0.9, f_c' of 25 and 45 MPa, f_y of 420 MPa and total reinforcement ratios, ρ_g, of 0.01 and 0.04. The design strengths of each section are calculated using ACI 318-14 and the partial material strength reduction factors. Then design strength ratios, defined as the design strength obtained using ACI 318-14 to that obtained using a particular pair of partial material strength reduction factors, are calculated. The sensitivities of the design strength ratios to the geometric and material properties and the partial material strength reduction factors are analyzed, and the best partial material strength reduction factor combinations are proposed.

The reliability analyses for members resisting moment and shear are presented in Chapter 3. The reliability model and the first-order, second-moment (FOSM) method are described. Statistical parameters for geometric properties, material strengths, professional factors and load effects collected from the literature are summarized and those used for the subsequent reliability analyses are listed. The reliability indices are calculated for the different geometric and material properties, and two live-to-dead load ratios, w_L/w_D, of 0.5 and 1.5 for varying ρ and ρ_t for moment and shear, respectively, using Microsoft Excel (Version 2013; Microsoft 2013). The means and standard deviations of the reliability indices for each partial material strength reduction factor combination are quantified. The sensitivities of the reliability indices to the geometric and material properties, partial material strength reduction factors, load ratios, and statistical parameters are analyzed, and the best partial material strength reduction factor combinations are proposed.

The reliability analyses for members resisting combined moment and axial force are presented in Chapter 4. The analyses are conducted using Monte Carlo simulation (Hong 2015) because the equations to generate interaction diagrams are relatively complicated. The sections and various geometric and material properties investigated are identical to those for design strength calculations in Chapter 2. Two live-to-dead load ratios, 0.5 and

1.5 are again investigated. The applied moment and axial force are assumed perfectly correlated and reliability indices are computed for a range of specific eccentricities. To save time and avoid unnecessary calculations, only four pairs of partial material strength reduction factors are analyzed for tied columns, combinations with ϕ_s of 0.85 and 0.90, and ϕ_c of 0.60 and 0.65. For spirally reinforced columns, two pairs of partial material strength reduction factors are analyzed, combinations with ϕ_s of 0.90 and 0.95, and ϕ_c of 0.70. The simulation is run 10^6 times for each case using Matlab (Version R2016b; The Mathworks, Inc. 2016). The means, standard deviations, minima and maxima for typical reliability indices for each combination are quantified. The sensitivities of the reliability indices to the partial material strength reduction factor combinations are investigated for various geometric and material properties, and the best combinations are proposed.

5.2 Conclusions

The following conclusions pertain to the design strength analysis results:

1. The design flexural strength ratio, α_M, which is defined as the design flexural strength obtained using the strength reduction factor in ACI 318-14 to that obtained using a particular pair of partial material strength reduction factors, is sensitive to ϕ_s and relatively insensitive to ϕ_c. In the tension-controlled sections, the combination with ϕ_s of 0.90 and ϕ_c of 0.75 is the best. If the section is in the transition region, the combination with ϕ_s of 0.95 and ϕ_c of 0.65 is the best. And any combination with ϕ_s of 0.90 is satisfactory for moment. The results are insensitive to f_c'.

2. The design shear strength ratio, α_V, is also sensitive to ϕ_s and relatively insensitive to ϕ_c, but the influences of these two factors are not as distinct as they are for moment. The best partial material strength reduction factor combination is ϕ_s of 0.80 and ϕ_c of 0.65. Combinations with ϕ_s of 0.80 and ϕ_c of 0.70, and ϕ_s of 0.85 and ϕ_c of 0.60 are close to optimal. The influence of f_c' is again slight.

3. For tied columns, in the compression-controlled region, the design combined flexural and axial strength ratio, α_{PM}, is sensitive to ϕ_c, and ϕ_c of 0.60 is the best. The

influences of γ, f_c', ρ_g are very small, moderate, and relatively large, respectively. In the tension-controlled region, α_{PM} is sensitive to ϕ_s, and ϕ_s of 0.90 is the best. The influence of γ becomes more significant, that of ρ_g is moderate, and that of f_c' is limited. The α_{PM} value varies markedly in the transition region where failure mode changes from compression-induced to tension-initiated, because ϕ in ACI 318-14 increases from 0.65 to 0.90.

4. For spirally reinforced circular columns, α_{PM} increases markedly in the compression-controlled region compared to those for tied columns, which is due to the strength reduction factor in ACI 318-14, ϕ, being 0.75 for spirally reinforced columns instead of 0.65. In the compression-controlled region, ϕ_c of 0.70 is the best and in the tension-controlled region, ϕ_s of both 0.90 and 0.95 are the best.

The following conclusions pertain to the reliability analysis results:

5. The reliability index for members subjected to moment corresponding to partial material strength reduction factors, β_{Mr}, is more sensitive to ϕ_s than to ϕ_c. The best partial material strength reduction factor combination is ϕ_s of 0.90 and ϕ_c of 0.75. As for the design strength comparison, any combination with ϕ_s of 0.90 is satisfactory. The influences of f_c' and w_L/w_D are small.

6. The reliability index for members subjected to one-way shear corresponding to partial material strength reduction factors, β_{Vr}, is more sensitive to ϕ_s than to ϕ_c, but the differences are not as large as they are for moment. The best partial material strength reduction factor combination is ϕ_s of 0.80 and ϕ_c of 0.65. Moreover, combinations with ϕ_s of 0.80 and ϕ_c of 0.70, and ϕ_s of 0.85 and ϕ_c of 0.60 are close to optimal, which are identical to the results based on the design strengths. Again, the influences of f_c' and w_L/w_D are small.

7. The reliability indices for one-way shear range from 2.65 to 2.82, and 2.20 to 3.11 computed for the ACI 318-14 criteria and the various partial material strength reduction factor combinations, respectively. These ranges are markedly lower than

those for moment, which is not desirable because shear failures are less ductile than flexural failures. The reliability indices for shear are very sensitive, however, to the statistical parameters assumed for the professional factor. If the statistical parameters assumed previously by Nowak and Szerszen (2003) are adopted, the reliability indices for shear increase markedly. However, the statistical parameters reported by Somo and Hong (2006) are more appropriate because they are based on larger sample sizes, classification of parameters by a_v/d and the presence of stirrups, and considering prestressed members separately. The influence of the statistical parameters for the professional factor selected is consistent for both ACI 318-14 and the partial material strength reduction factors criteria.

8. For tied columns, in the compression-controlled region, the reliability index for members subjected to combined moment and axial force corresponding to partial material strength reduction factors, β_{PMr}, is sensitive to ϕ_c, and ϕ_c of 0.60 is the best. In the tension-controlled region, β_{PMr} is sensitive to ϕ_s, and ϕ_s of 0.90 is the best. For ACI 318-14, β_{PMu} varies markedly with ρ_g. For the combination with ϕ_s of 0.90 and ϕ_c of 0.60, however, the β_{PMr} values are more consistent. The geometric and material properties and load ratios do not appreciably affect these results. The results are also essentially identical for the four tied column sections investigated, which indicates that the reinforcement arrangement is not a significant factor.

9. For spirally reinforced columns, the influences of material and geometric properties, load ratios, and partial material strength reduction factors are similar to those for tied columns. The best combination is ϕ_s of 0.90 and ϕ_c of 0.70. Therefore, ϕ_s of 0.90 is the best for both tied and spirally reinforced columns, while a unique ϕ_c value is appropriate for spirally reinforced columns to account for the advantages of confinement that mainly impact the compression-controlled region.

10. Although no single combination of ϕ_s and ϕ_c is the best for members resisting moment, shear, or combined moment and axial force, the recommended partial material strength reduction factors are ϕ_s of 0.90 and ϕ_c of 0.60 for slabs and beams subjected to moment, beams subjected to one-way shear, and tied columns, or ϕ_c of

0.70 for spirally reinforced columns. Alternatively, for shear, the combination with ϕ_s of 0.80 and ϕ_c of 0.65 is recommended.

5.3 Suggestions for Future Work

1. The oldest statistical parameters used in this study trace back to 1979. Control of the construction process and material quality may have since improved. Research to determine more current statistical parameters for the geometric properties, material strengths, professional factors, and load effects should be carried out and the recommended partial material strength reduction factors should be reviewed based on these new parameters.

2. The structural actions investigated in this study are moment, one-way shear, and combined moment and axial force. Other actions or structural elements, such as two-way shear, torsion, bearing, brackets and corbels, should be investigated in the future.

3. The reliability index for one-way shear is very sensitive to the statistical parameters used to quantify the professional factor. The basic equations for one-way shear strength in ACI 318 have not changed for more than five decades (Belarbi et al. 2017). Significant changes have occurred in other codes, and the deficiencies of the current provisions include: (1) ignoring the size effect in the calculation of the shear strength resisted by concrete, V_c; (2) ignoring the presence of shear reinforcement in the computation of V_c; (3) assuming the angle of diagonal compression is fixed at 45° irrespective of the amount of reinforcement; and other factors (Belarbi et al. 2017). The provisions for one-way shear should be improved and new statistical parameters for the professional factor should be derived, based on these new criteria.

4. The maximum axial compressive strengths defined in ACI 318-14 are limited to 0.80 and 0.85 of the axial compressive strengths at zero eccentricity for tied and spirally reinforced columns, respectively, and they are approximate axial strengths at e/h of 0.10 and 0.05, respectively (ACI Committee 318 2014). These values should be reviewed and reliability analyses should be conducted.

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Appendix A-Supplementary Information for Chapter 2

Table A.1: Means and standard deviations of design combined flexural and axial strength ratios, α_{PM}, for Column Section 2 and $0 \leq e/h \leq 0.4$

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	0.989	0.040	0.938	0.027	0.892	0.017	0.850	0.008
0.85	0.969	0.048	0.919	0.035	0.875	0.024	0.834	0.014
0.90	0.950	0.056	0.902	0.042	0.858	0.030	0.819	0.021
0.95	0.931	0.062	0.885	0.048	0.843	0.037	0.805	0.027

Table A.2: Means and standard deviations of design combined flexural and axial strength ratios, α_{PM}, for Column Section 2 and $0.4 < e/h \leq 1.0$

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	1.056	0.133	1.027	0.135	1.001	0.138	0.977	0.140
0.85	1.016	0.123	0.987	0.125	0.962	0.128	0.939	0.130
0.90	0.979	0.116	0.952	0.117	0.927	0.119	0.905	0.121
0.95	0.946	0.109	0.919	0.110	0.895	0.111	0.874	0.113

Table A.3: Means and standard deviations of design combined flexural and axial strength ratios, α_{PM}, for Column Section 2 and $1.0 < e/h \leq 10.0$

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	1.147	0.033	1.139	0.028	1.133	0.024	1.126	0.022
0.85	1.085	0.035	1.078	0.030	1.071	0.026	1.065	0.023
0.90	1.029	0.036	1.022	0.031	1.016	0.027	1.011	0.024
0.95	0.979	0.038	0.973	0.033	0.967	0.029	0.962	0.025

Table A.4: Means and standard deviations of design combined flexural and axial strength ratios, α_{PM}, for Column Section 2 and $e/h \leq 0$

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	1.140	0.020	1.136	0.014	1.132	0.009	1.128	0.004
0.85	1.076	0.022	1.072	0.017	1.068	0.013	1.065	0.008
0.90	1.018	0.025	1.015	0.020	1.012	0.015	1.008	0.011
0.95	0.967	0.026	0.964	0.022	0.961	0.018	0.958	0.014

Table A.5: Means and standard deviations of design combined flexural and axial strength ratios, α_{PM}, for Column Section 3 and $0 \leq e/h \leq 0.3$

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	0.999	0.041	0.944	0.027	0.896	0.016	0.852	0.007
0.85	0.981	0.049	0.928	0.035	0.880	0.024	0.838	0.014
0.90	0.963	0.056	0.912	0.042	0.866	0.031	0.825	0.021
0.95	0.946	0.063	0.896	0.049	0.852	0.037	0.812	0.027

Table A.6: Means and standard deviations of design combined flexural and axial strength ratios, α_{PM}, for Column Section 3 and $0.3 < e/h \leq 1.0$

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	1.086	0.139	1.047	0.135	1.012	0.133	0.982	0.133
0.85	1.051	0.134	1.013	0.130	0.979	0.127	0.949	0.125
0.90	1.019	0.131	0.982	0.126	0.949	0.122	0.920	0.120
0.95	0.990	0.128	0.954	0.122	0.922	0.118	0.893	0.115

Table A.7: Means and standard deviations of design combined flexural and axial strength ratios, α_{PM}, for Column Section 3 and $1.0 < e/h \leq 10.0$

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	1.161	0.045	1.144	0.044	1.128	0.044	1.113	0.046
0.85	1.105	0.044	1.089	0.042	1.073	0.041	1.060	0.042
0.90	1.055	0.044	1.039	0.041	1.025	0.039	1.012	0.039
0.95	1.009	0.045	0.994	0.041	0.981	0.038	0.968	0.037

Table A.8: Means and standard deviations of design combined flexural and axial strength ratios, α_{PM}, for Column Section 3 and $e/h \leq 0$

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	1.145	0.028	1.138	0.021	1.132	0.017	1.126	0.015
0.85	1.083	0.032	1.076	0.025	1.070	0.019	1.065	0.015
0.90	1.027	0.035	1.021	0.028	1.015	0.022	1.010	0.017
0.95	0.977	0.038	0.971	0.031	0.966	0.025	0.961	0.020

Table A.9: Means and standard deviations of design combined flexural and axial strength ratios, α_{PM}, for Column Section 4 and $0 \leq e/h \leq 0.3$

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	0.998	0.040	0.943	0.027	0.895	0.016	0.852	0.007
0.85	0.979	0.048	0.926	0.034	0.880	0.023	0.837	0.014
0.90	0.961	0.055	0.910	0.042	0.865	0.030	0.824	0.020
0.95	0.944	0.062	0.894	0.048	0.850	0.036	0.811	0.026

Table A.10: Means and standard deviations of design combined flexural and axial strength ratios, α_{PM}, for Column Section 4 and $0.3 < e/h \leq 1.0$

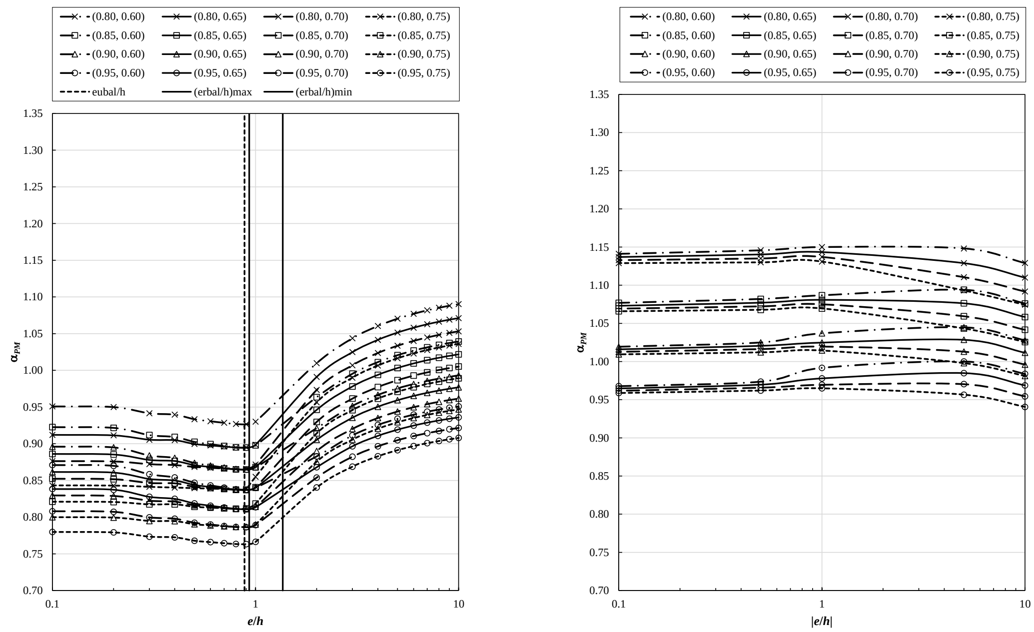
	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	1.084	0.129	1.047	0.127	1.013	0.126	0.983	0.126
0.85	1.048	0.124	1.012	0.121	0.980	0.119	0.950	0.118
0.90	1.015	0.120	0.980	0.116	0.949	0.114	0.920	0.112
0.95	0.984	0.116	0.951	0.112	0.920	0.109	0.893	0.108

Table A.11: Means and standard deviations of design combined flexural and axial strength ratios, α_{PM}, for Column Section 4 and $1.0 < e/h \leq 10.0$

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	1.145	0.065	1.128	0.067	1.112	0.069	1.097	0.071
0.85	1.090	0.060	1.074	0.062	1.059	0.064	1.045	0.065
0.90	1.041	0.055	1.025	0.057	1.011	0.059	0.998	0.060
0.95	0.997	0.051	0.981	0.053	0.968	0.055	0.955	0.056

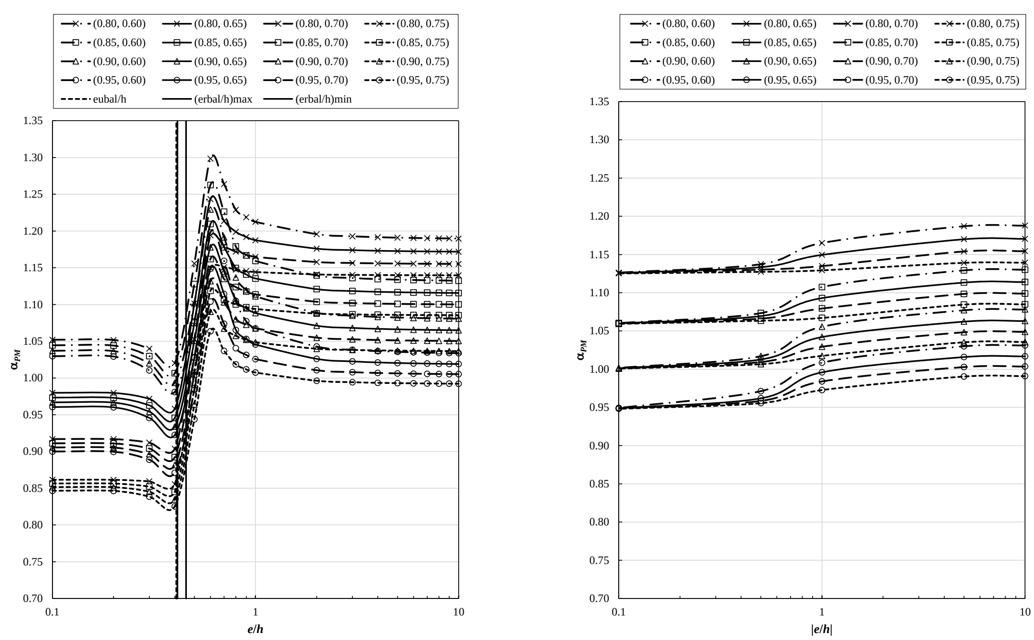
Table A.12: Means and standard deviations of design combined flexural and axial strength ratios, α_{PM}, for Column Section 4 and $e/h \leq 0$

	ϕ_c							
	0.60		0.65		0.70		0.75	
ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.80	1.145	0.032	1.136	0.030	1.128	0.030	1.121	0.032
0.85	1.084	0.033	1.076	0.030	1.068	0.028	1.061	0.028
0.90	1.029	0.034	1.021	0.030	1.014	0.028	1.008	0.027
0.95	0.980	0.036	0.972	0.031	0.966	0.028	0.959	0.026



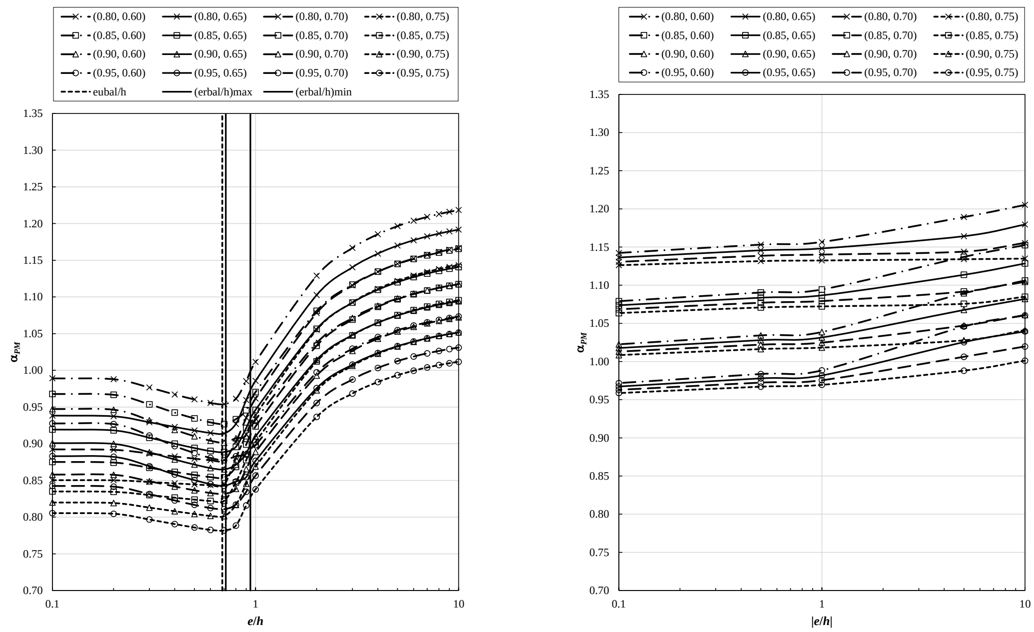
(a) (b)

Figure A.1: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 1 and Property Combination 2: (a) $e/h > 0$; (b) $e/h < 0$



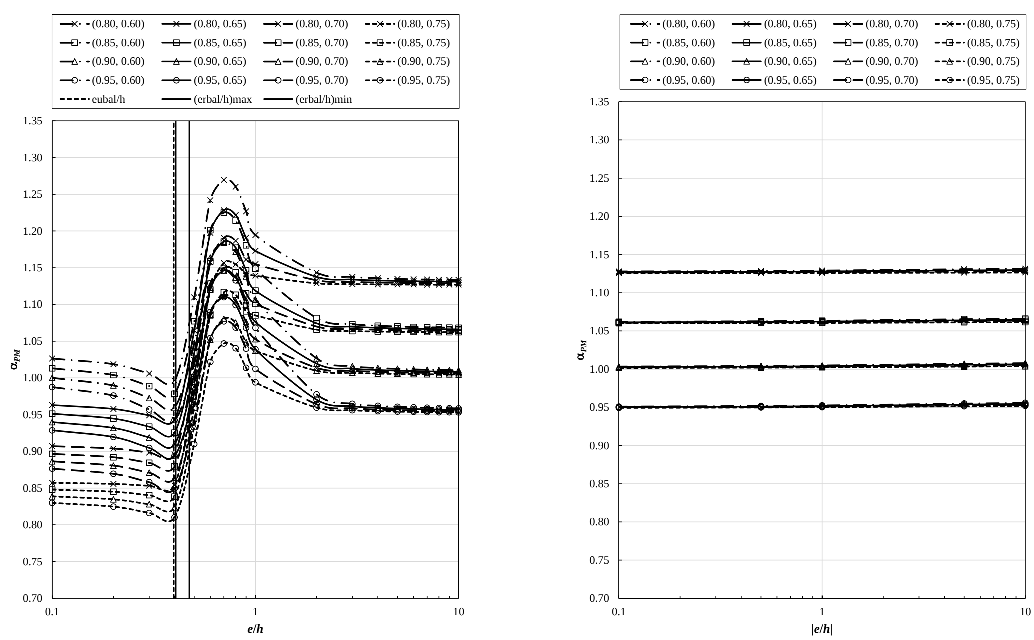
(a) (b)

Figure A.2: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 1 and Property Combination 3: (a) $e/h > 0$; (b) $e/h < 0$



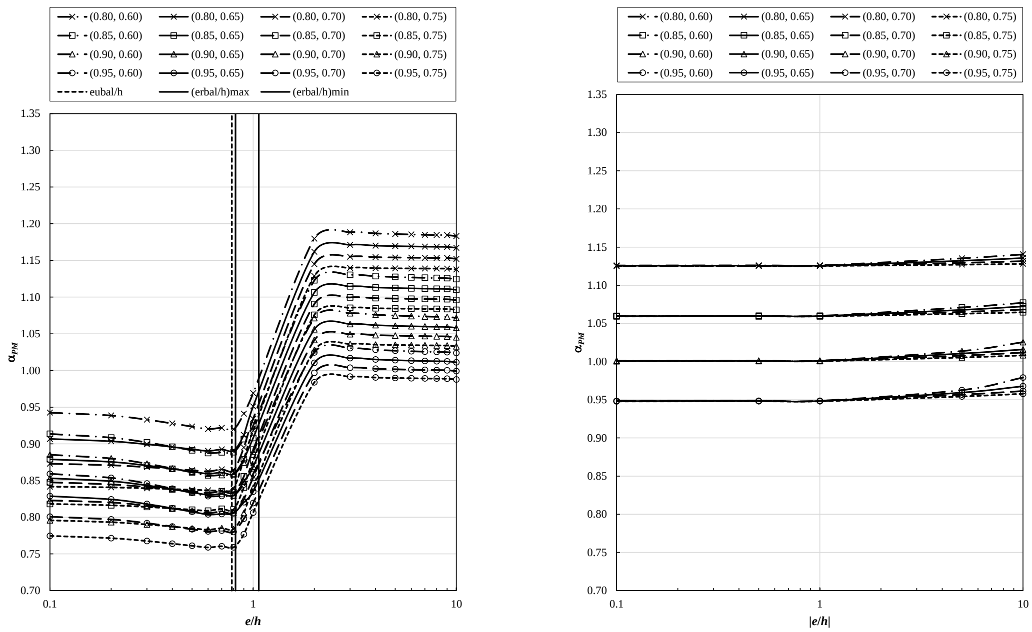
(a) (b)

Figure A.3: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 1 and Property Combination 4: (a) $e/h > 0$; (b) $e/h < 0$



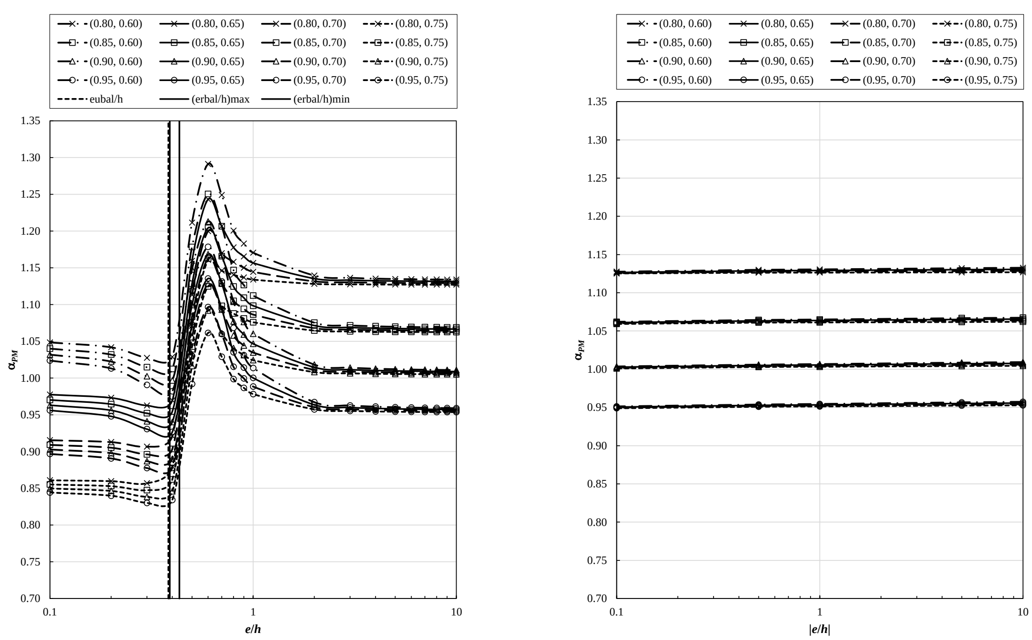
(a) (b)

Figure A.4: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 1 and Property Combination 5: (a) $e/h > 0$; (b) $e/h < 0$



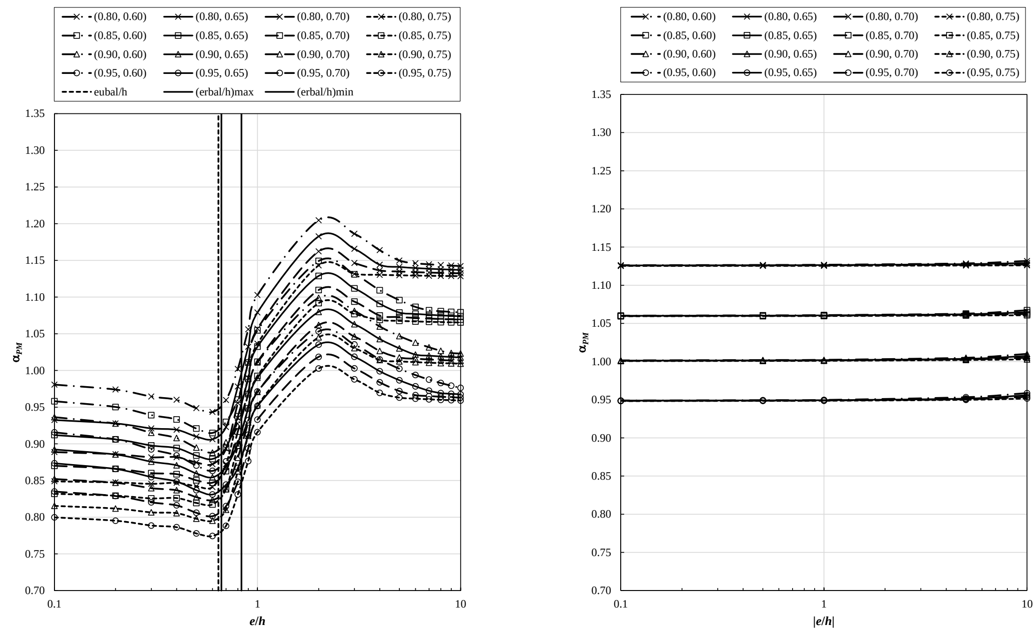
(a) (b)

Figure A.5: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 1 and Property Combination 6: (a) $e/h > 0$; (b) $e/h < 0$



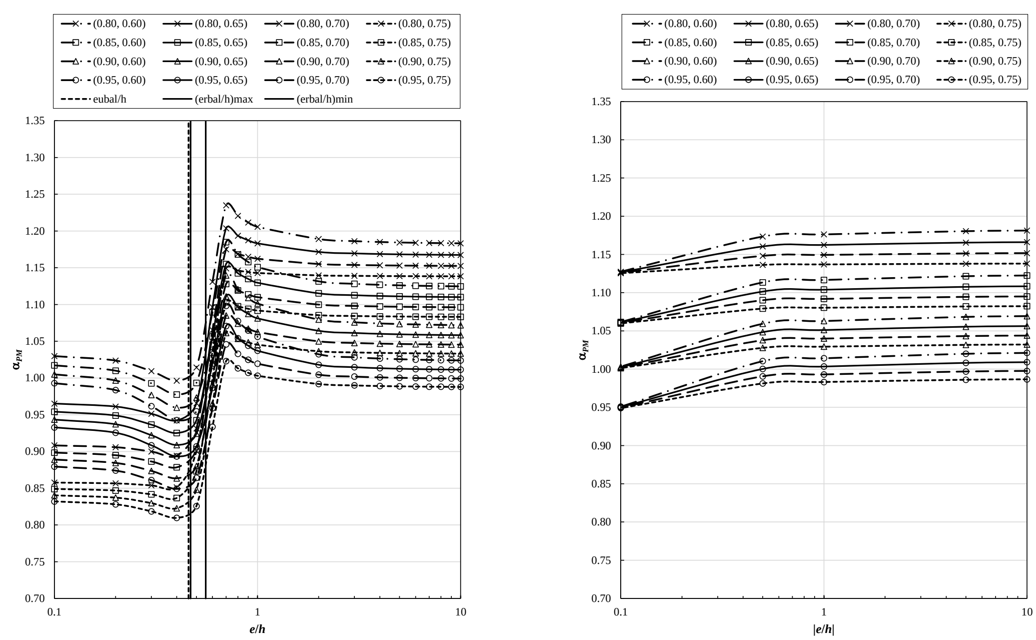
(a) (b)

Figure A.6: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 1 and Property Combination 7: (a) $e/h > 0$; (b) $e/h < 0$



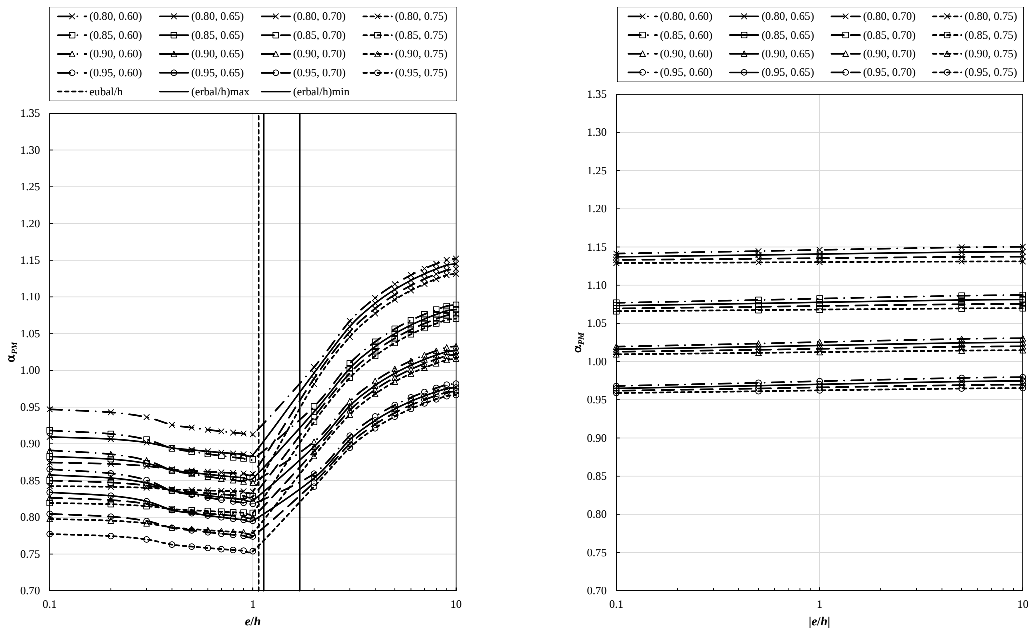
(a) (b)

Figure A.7: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 1 and Property Combination 8: (a) $e/h > 0$; (b) $e/h < 0$



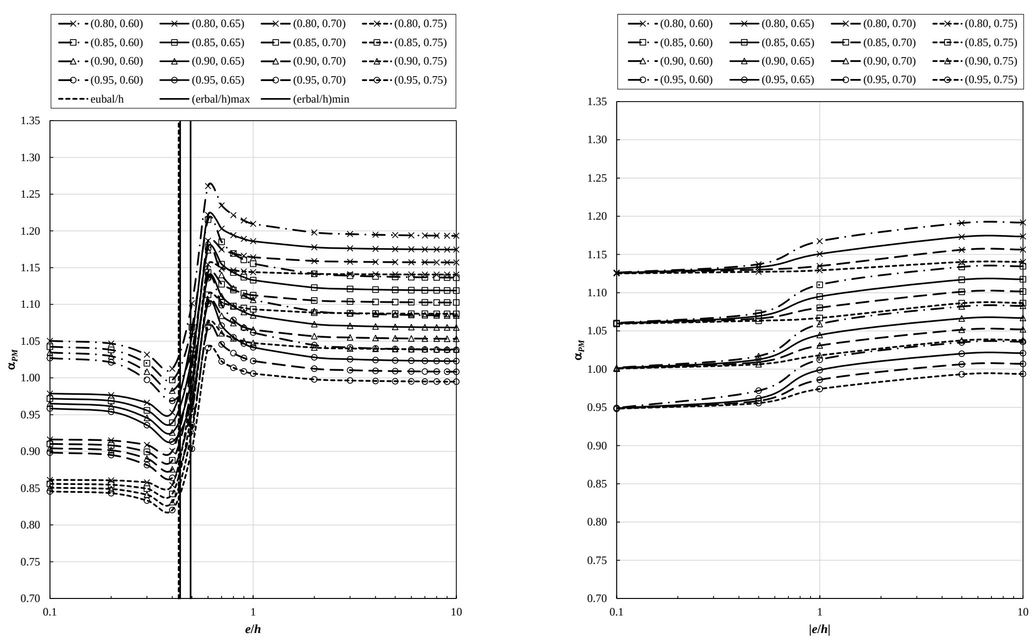
(a) (b)

Figure A.8: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 2 and Property Combination 1: (a) $e/h > 0$; (b) $e/h < 0$



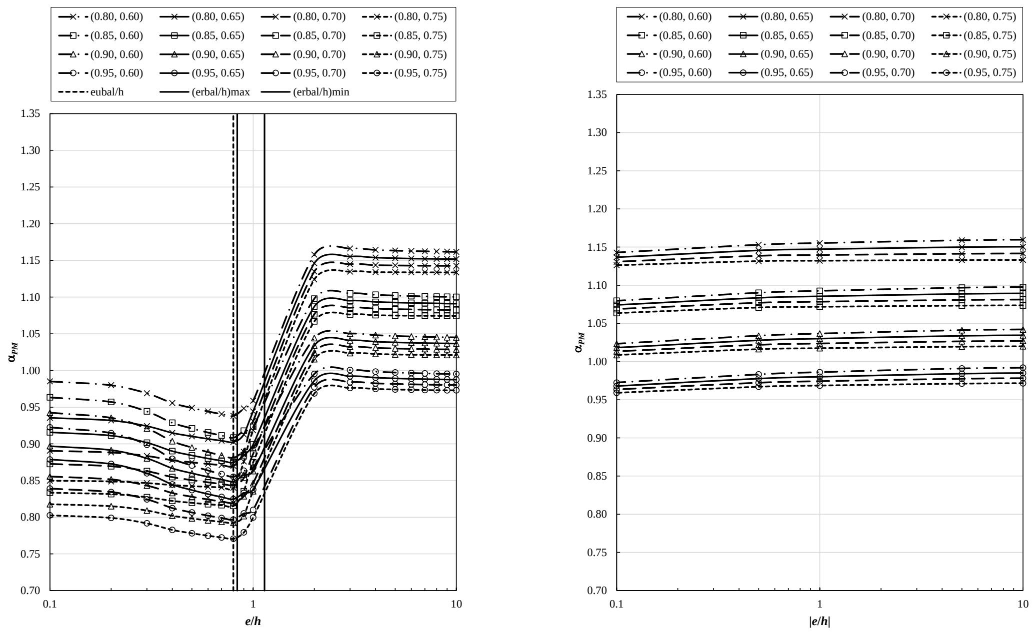
(a) (b)

Figure A.9: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 2 and Property Combination 2: (a) $e/h > 0$; (b) $e/h < 0$



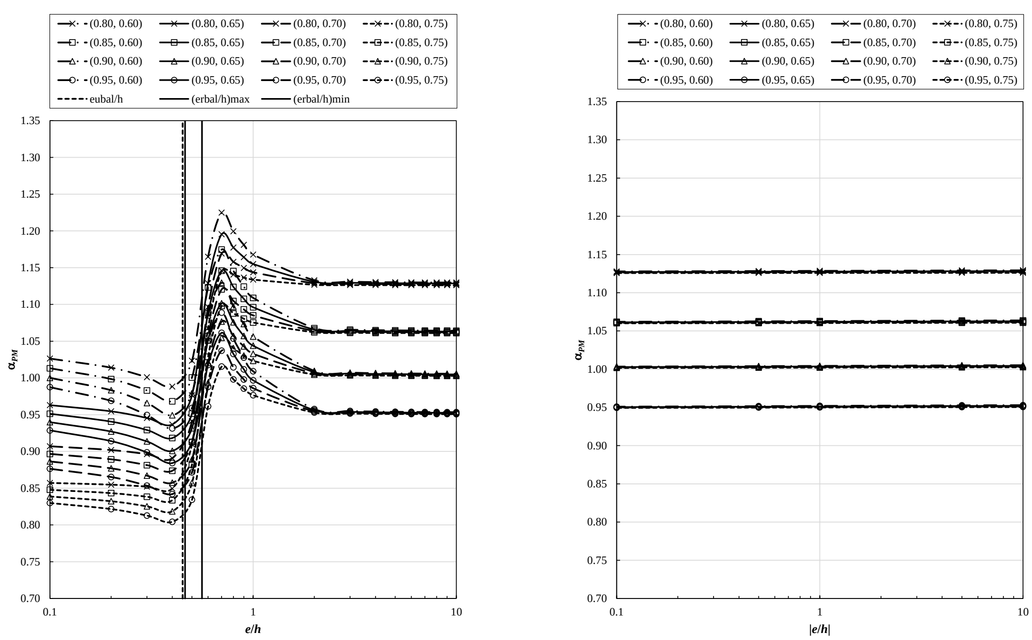
(a) (b)

Figure A.10: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 2 and Property Combination 3: (a) $e/h > 0$; (b) $e/h < 0$



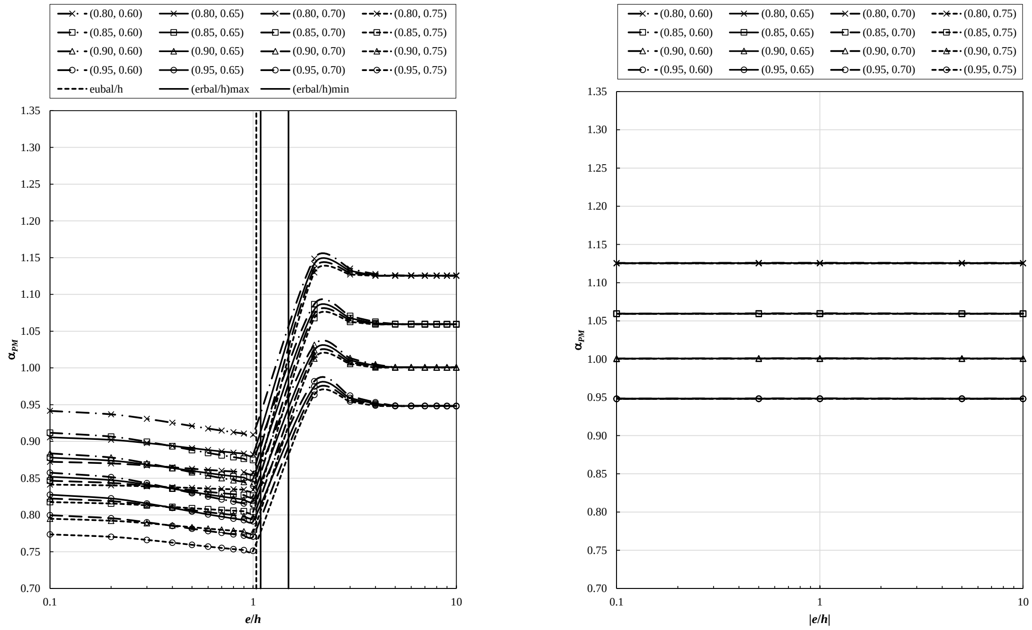
(a) (b)

Figure A.11: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 2 and Property Combination 4: (a) $e/h > 0$; (b) $e/h < 0$



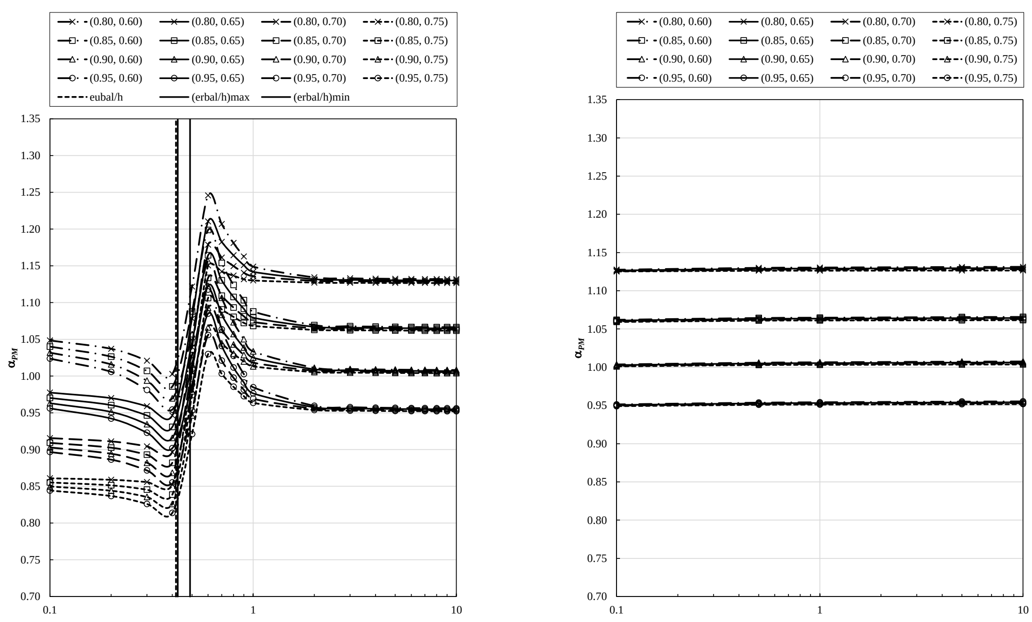
(a) (b)

Figure A.12: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 2 and Property Combination 5: (a) $e/h > 0$; (b) $e/h < 0$



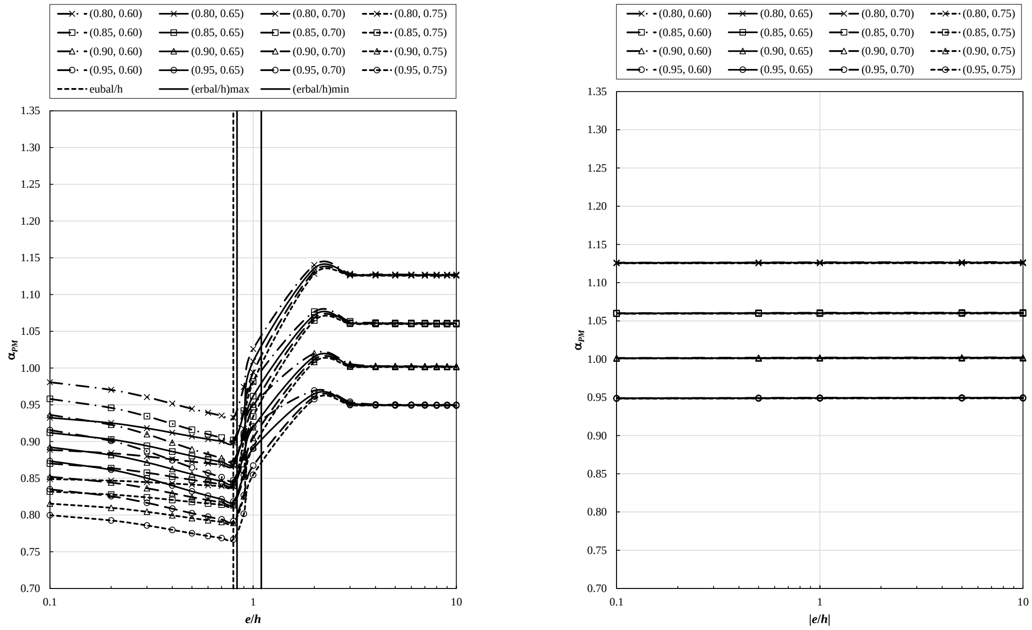
(a) (b)

Figure A.13: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 2 and Property Combination 6: (a) $e/h > 0$; (b) $e/h < 0$



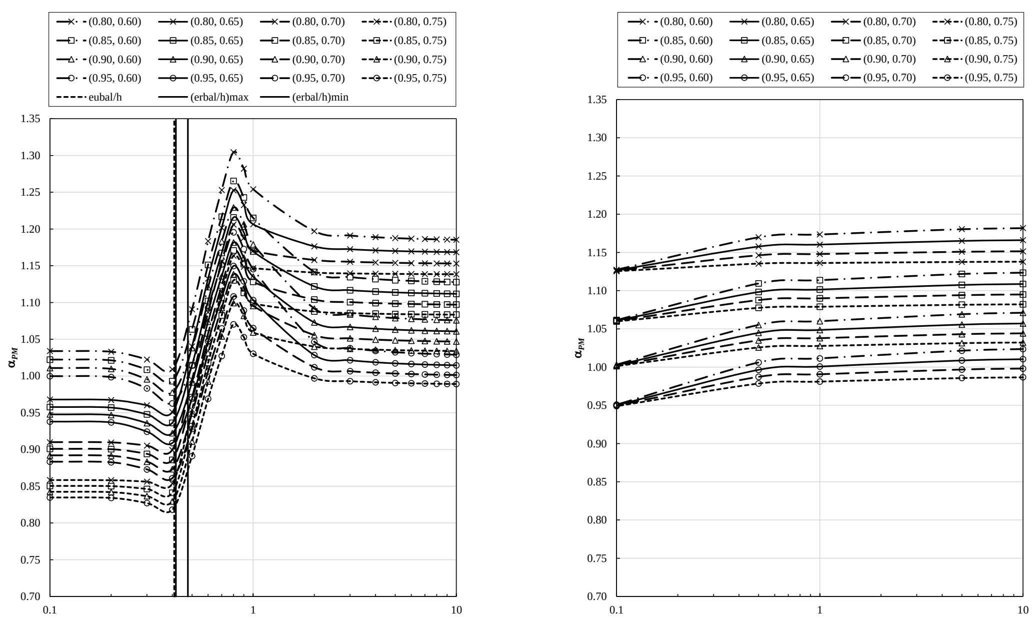
(a) (b)

Figure A.14: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 2 and Property Combination 7: (a) $e/h > 0$; (b) $e/h < 0$



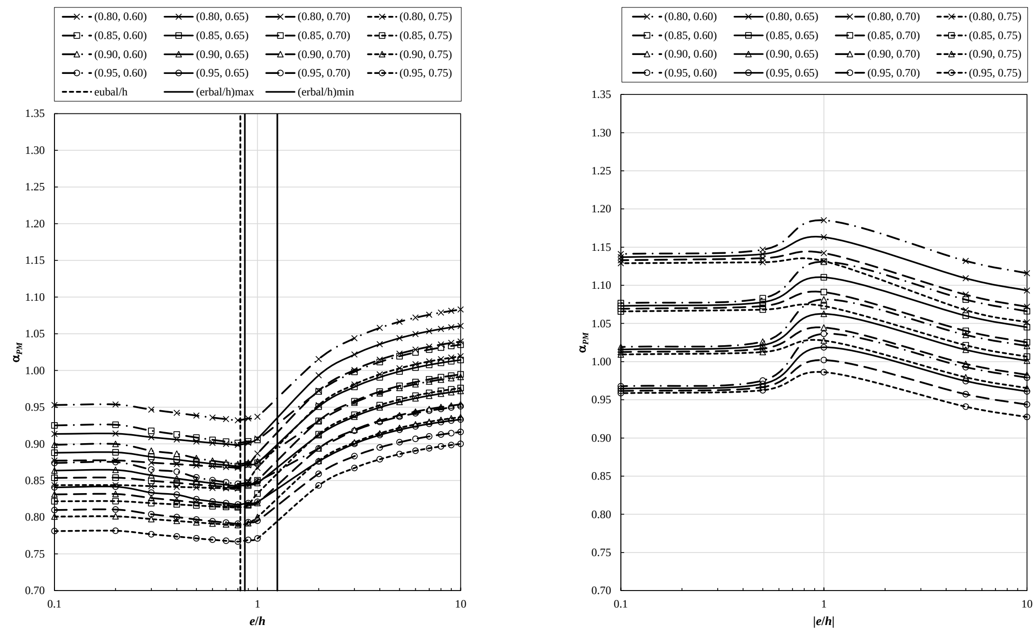
(a) (b)

Figure A.15: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 2 and Property Combination 8: (a) $e/h > 0$; (b) $e/h < 0$



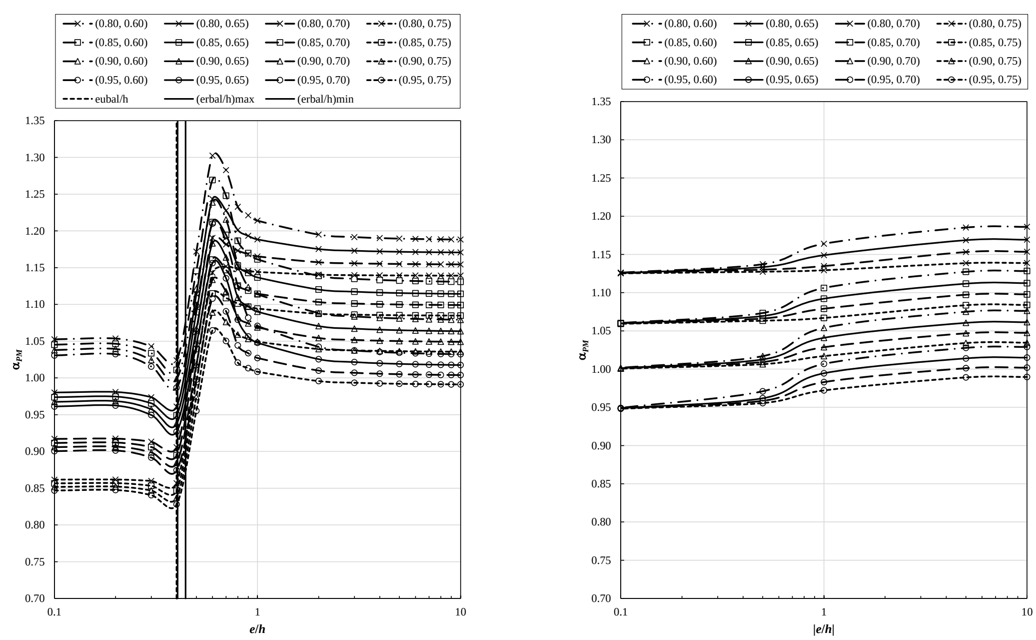
(a) (b)

Figure A.16: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 3 and Property Combination 1: (a) $e/h > 0$; (b) $e/h < 0$



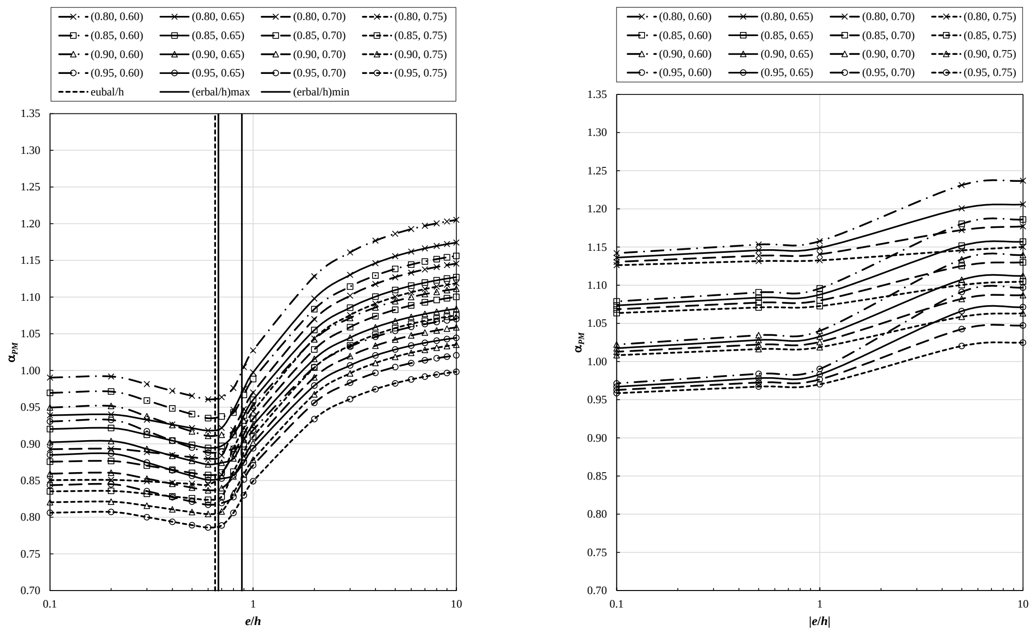
(a) (b)

Figure A.17: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 3 and Property Combination 2: (a) $e/h > 0$; (b) $e/h < 0$



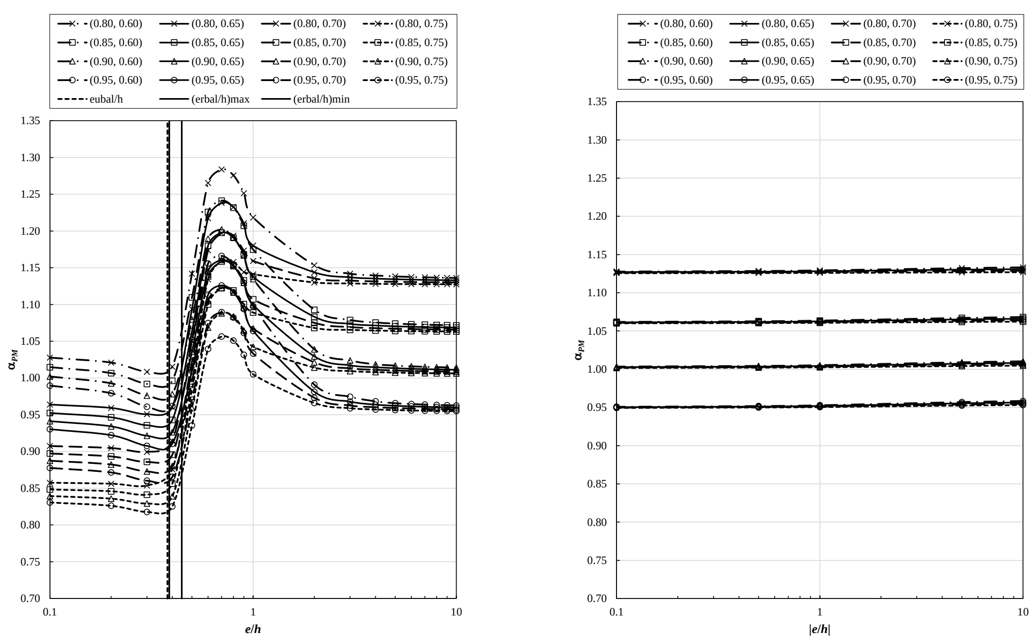
(a) (b)

Figure A.18: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 3 and Property Combination 3: (a) $e/h > 0$; (b) $e/h < 0$



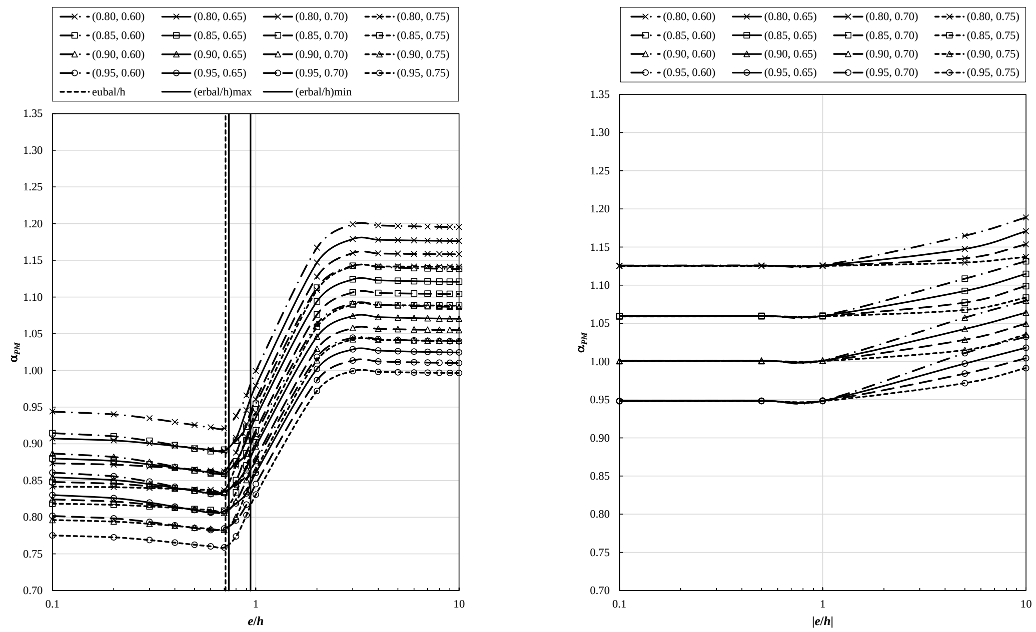
(a) (b)

Figure A.19: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 3 and Property Combination 4: (a) $e/h > 0$; (b) $e/h < 0$



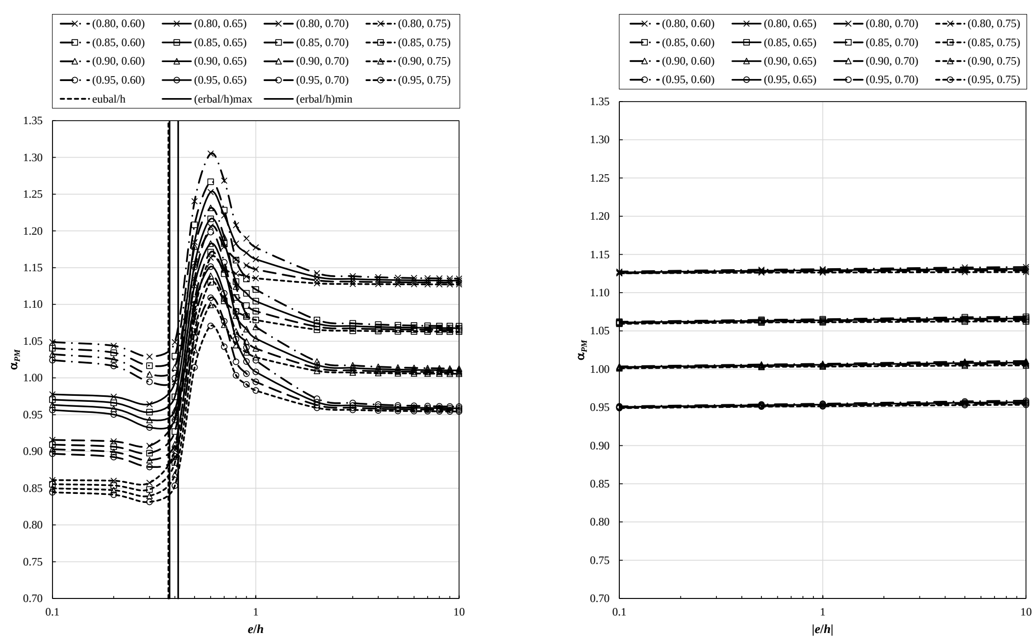
(a) (b)

Figure A.20: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 3 and Property Combination 5: (a) $e/h > 0$; (b) $e/h < 0$



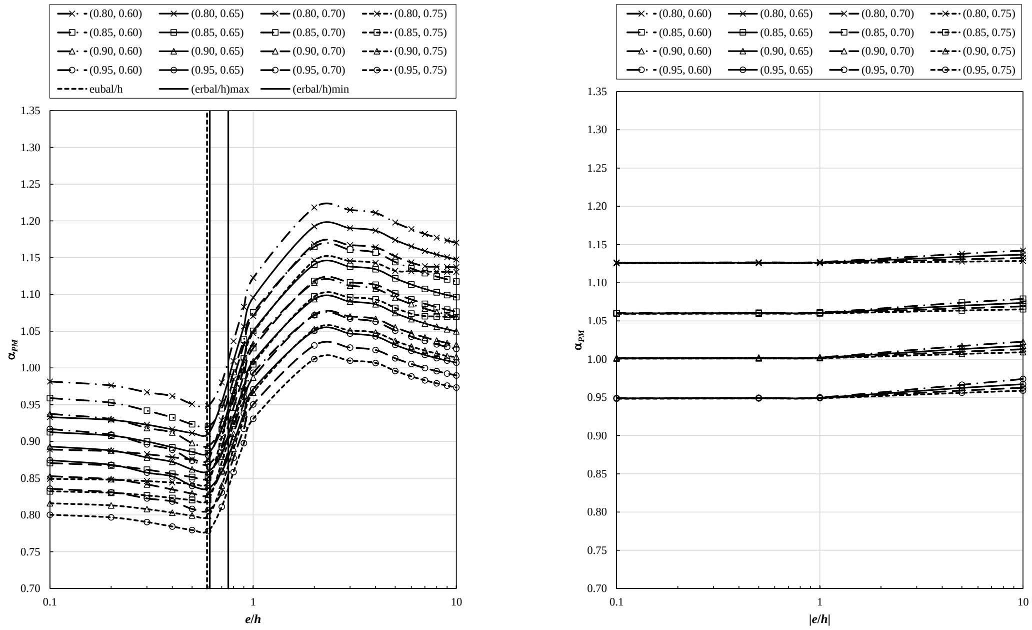
(a) (b)

Figure A.21: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 3 and Property Combination 6: (a) $e/h > 0$; (b) $e/h < 0$



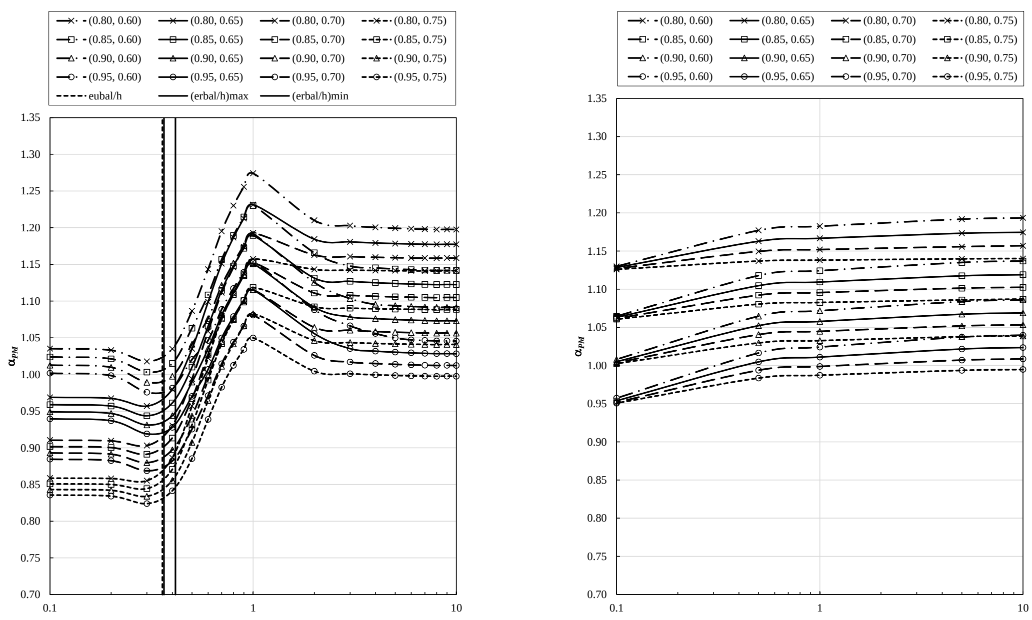
(a) (b)

Figure A.22: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 3 and Property Combination 7: (a) $e/h > 0$; (b) $e/h < 0$



(a) (b)

Figure A.23: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 3 and Property Combination 8: (a) $e/h > 0$; (b) $e/h < 0$



(a) (b)

Figure A.24: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 4 and Property Combination 1: (a) $e/h > 0$; (b) $e/h < 0$

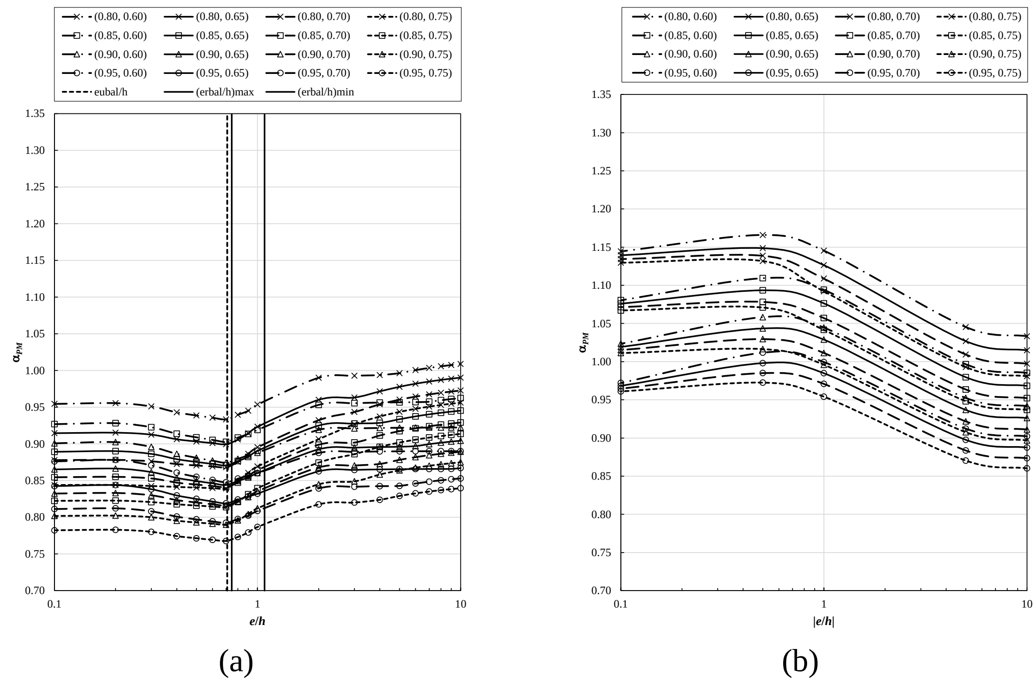


Figure A.25: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 4 and Property Combination 2: (a) $e/h > 0$; (b) $e/h < 0$

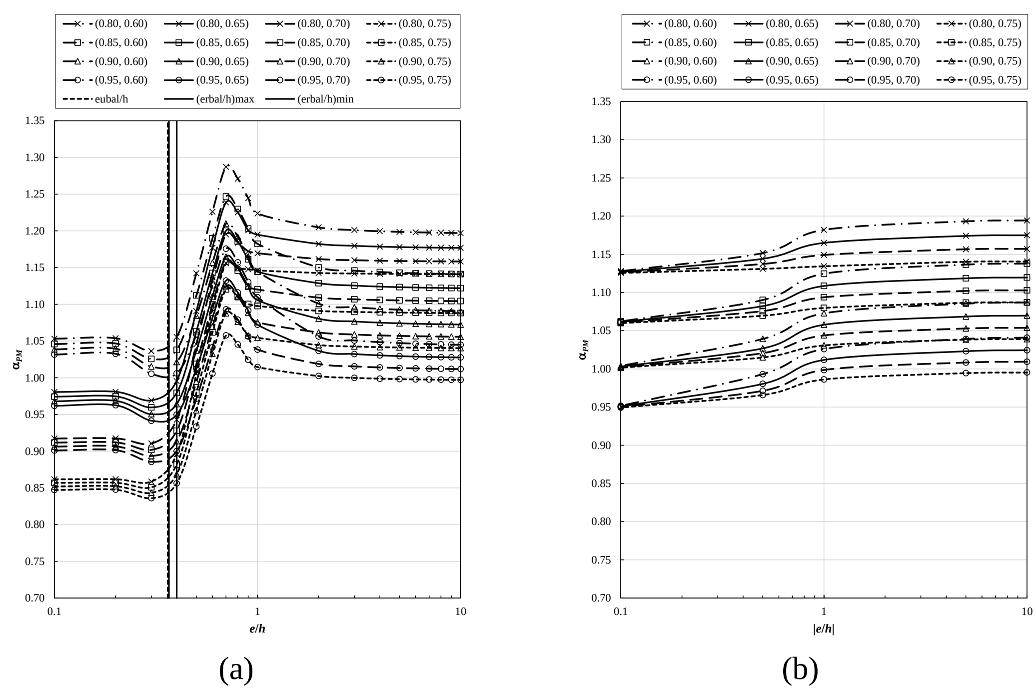
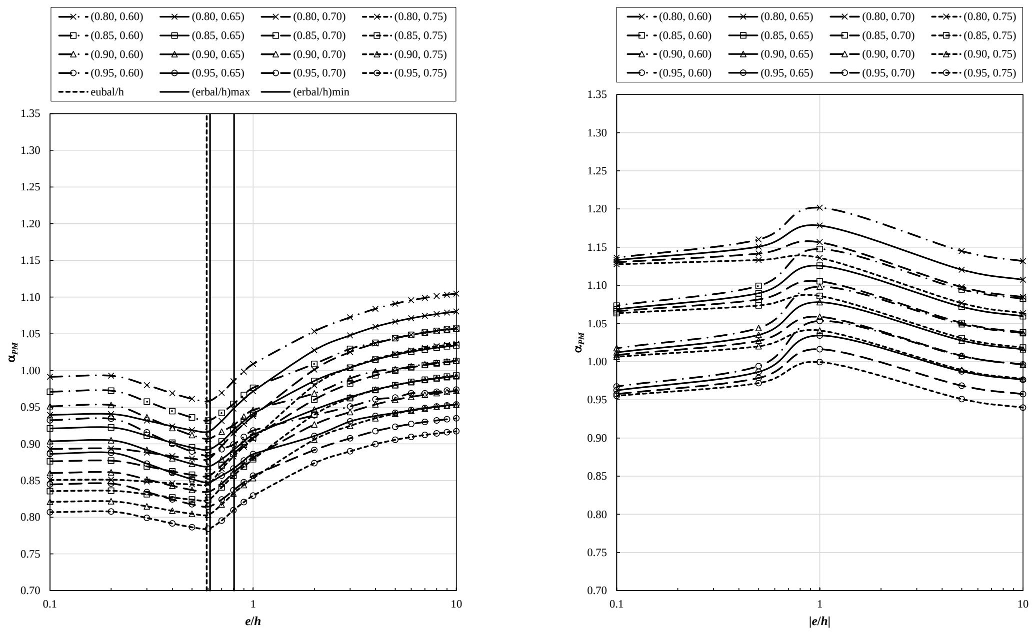
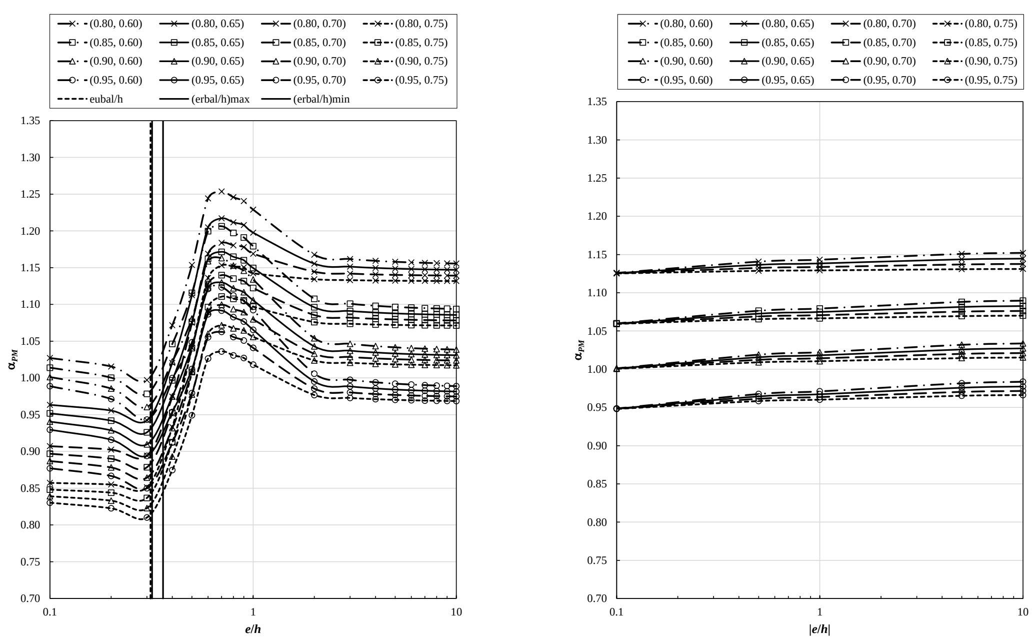


Figure A.26: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 4 and Property Combination 3: (a) $e/h > 0$; (b) $e/h < 0$



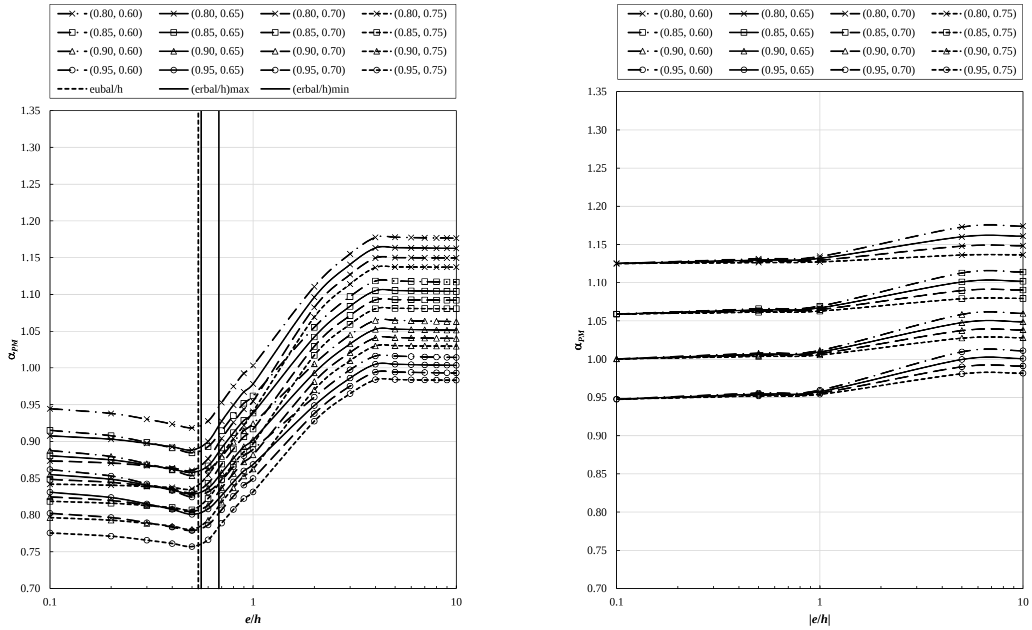
(a) (b)

Figure A.27: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 4 and Property Combination 4: (a) $e/h > 0$; (b) $e/h < 0$



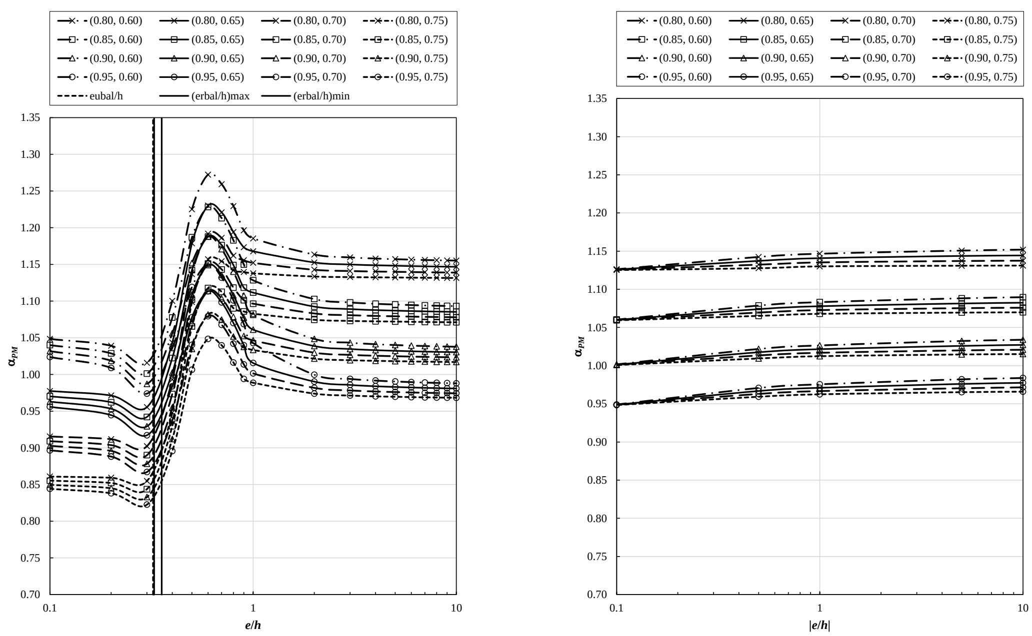
(a) (b)

Figure A.28: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 4 and Property Combination 5: (a) $e/h > 0$; (b) $e/h < 0$



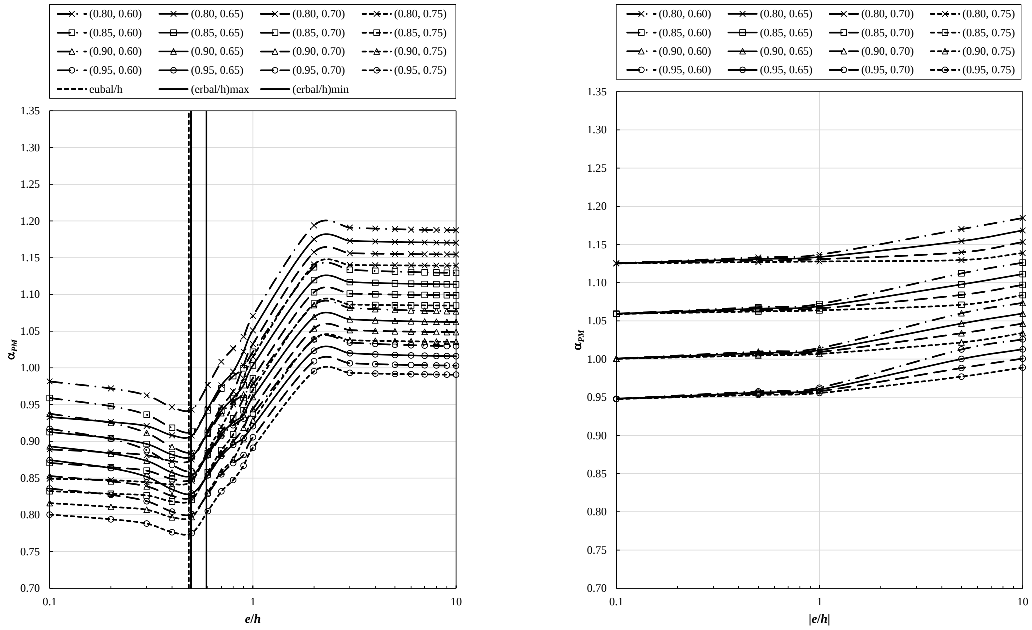
(a) (b)

Figure A.29: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 4 and Property Combination 6: (a) $e/h > 0$; (b) $e/h < 0$



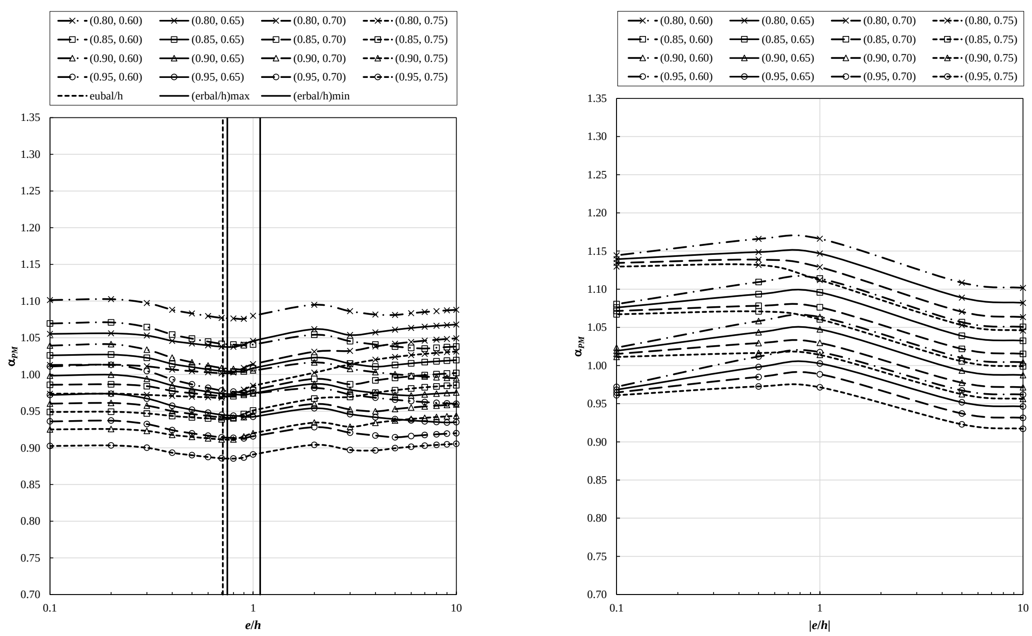
(a) (b)

Figure A.30: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 4 and Property Combination 7: (a) $e/h > 0$; (b) $e/h < 0$



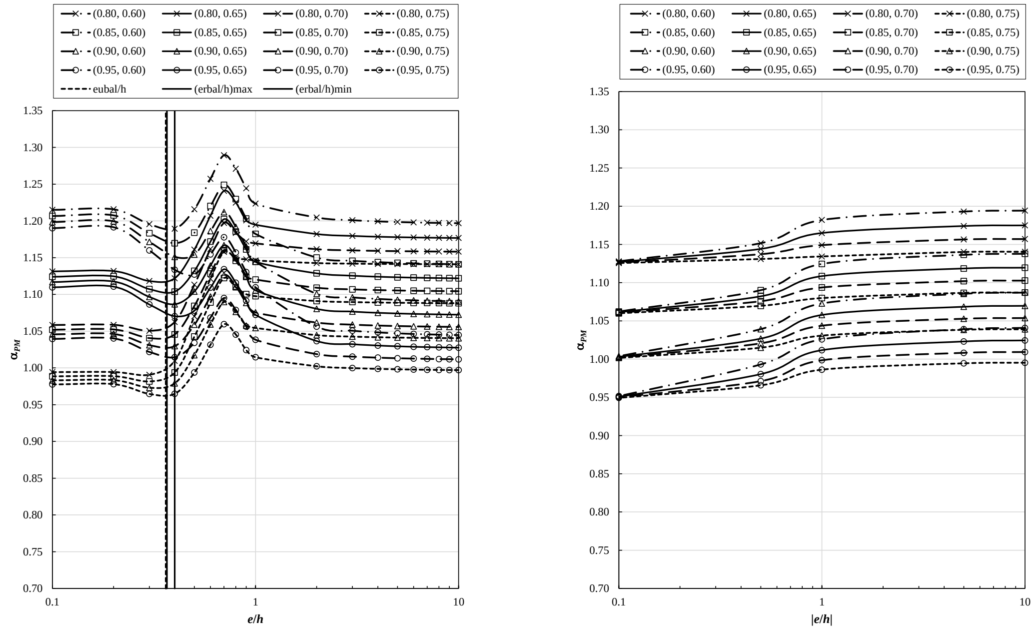
(a) (b)

Figure A.31: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 4 and Property Combination 8: (a) $e/h > 0$; (b) $e/h < 0$



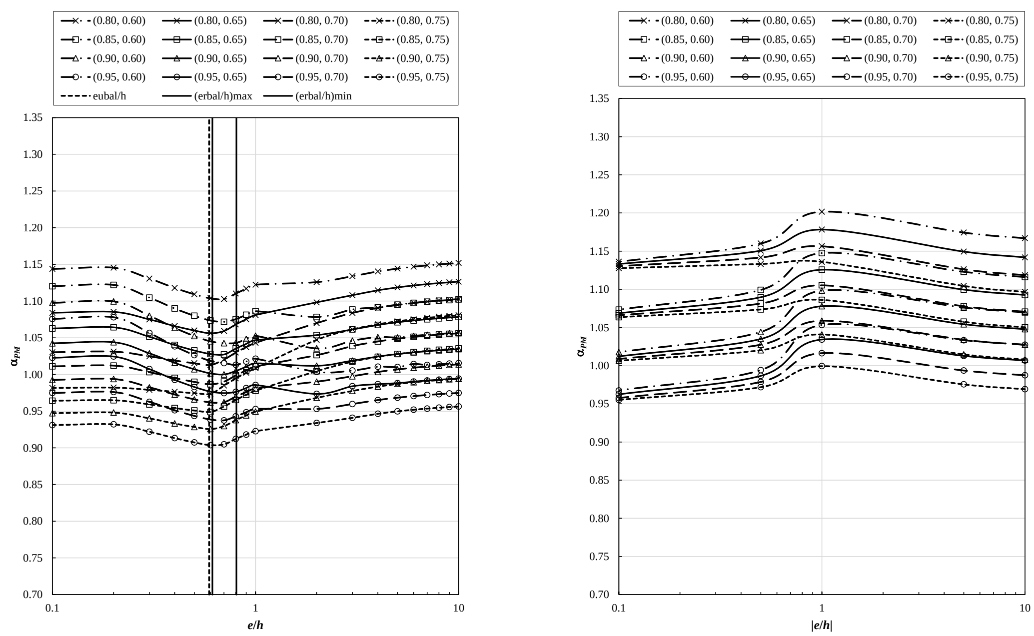
(a) (b)

Figure A.32: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 5 and Property Combination 2: (a) $e/h > 0$; (b) $e/h < 0$



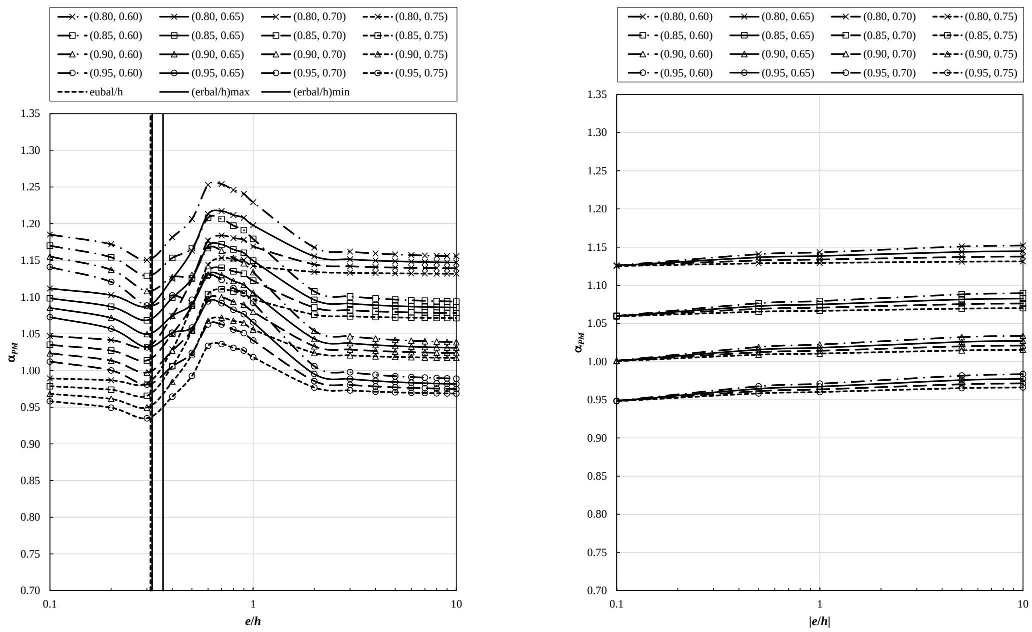
(a) (b)

Figure A.33: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 5 and Property Combination 3: (a) $e/h > 0$; (b) $e/h < 0$



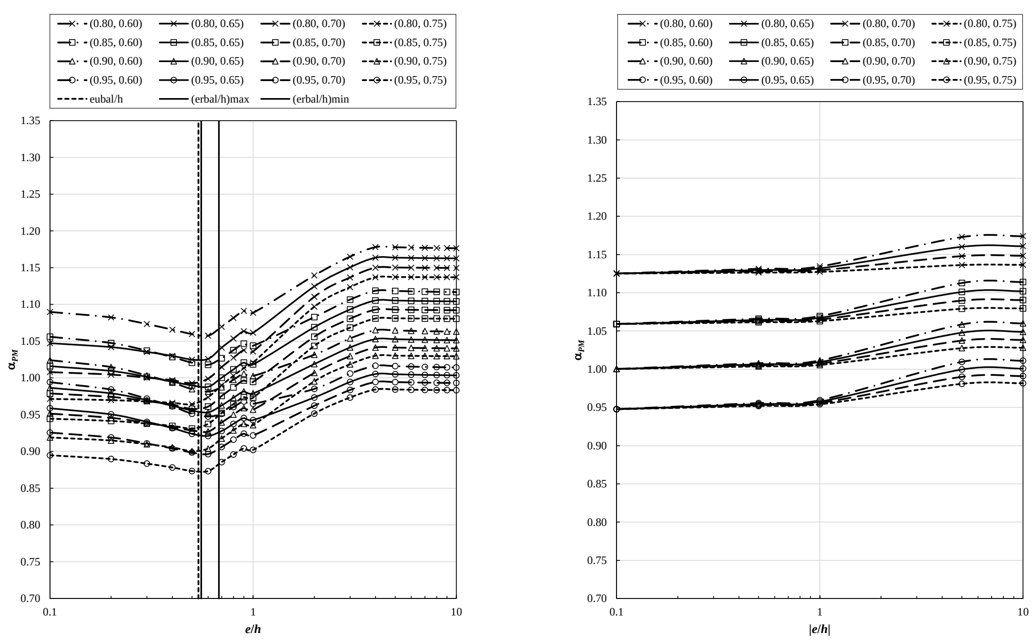
(a) (b)

Figure A.34: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 5 and Property Combination 4: (a) $e/h > 0$; (b) $e/h < 0$



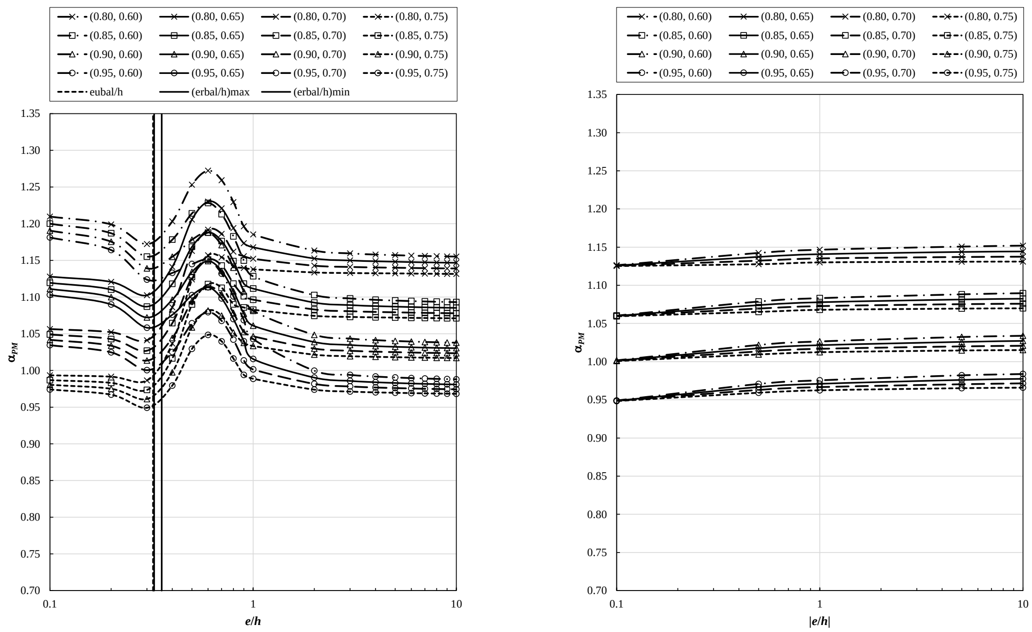
(a) (b)

Figure A.35: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 5 and Property Combination 5: (a) $e/h > 0$; (b) $e/h < 0$



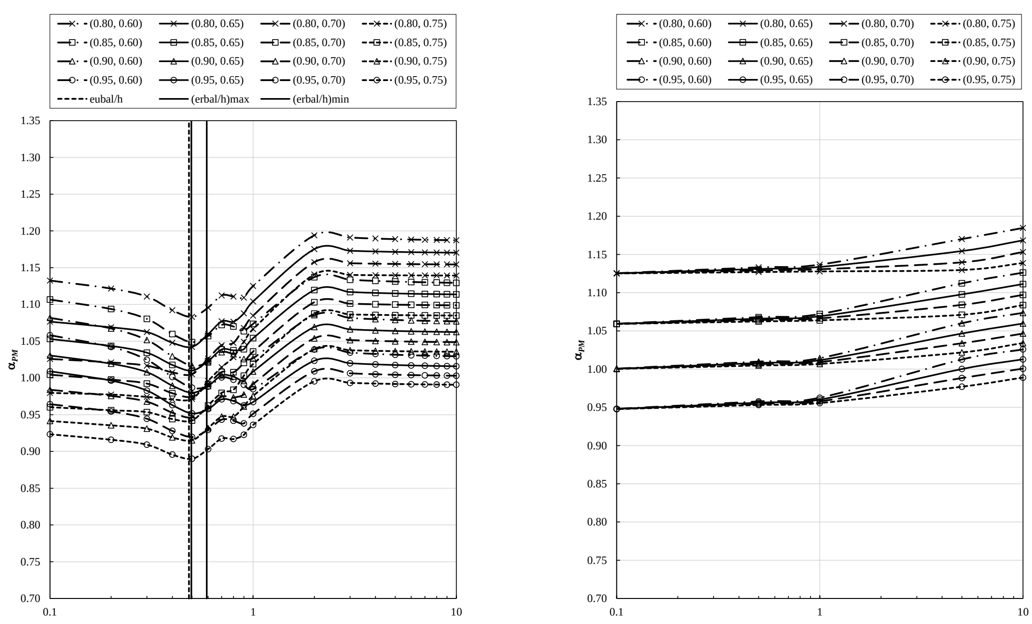
(a) (b)

Figure A.36: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 5 and Property Combination 6: (a) $e/h > 0$; (b) $e/h < 0$



(a) (b)

Figure A.37: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 5 and Property Combination 7: (a) $e/h > 0$; (b) $e/h < 0$



(a) (b)

Figure A.38: Design combined flexural and axial strength ratios, α_{PM}, for Column Section 5 and Property Combination 8: (a) $e/h > 0$; (b) $e/h < 0$

A.1 Codes

The Matlab (Version R2016b; The Mathworks, Inc. 2016) codes used to obtain design combined flexural and axial strength ratios, α_{PM}, corresponding to specific e/h values for Column Section 1 (square section with three bars in each face) and Column Section 5 (spirally reinforced circular section with eight bars evenly distributed around the perimeter), two representative cross sections, are as follows:

A.1.1 Notation

```
% Notation
% a=depth of equivalent rectangular stress block (mm)
% A=area of compression segment of circular section (mm^2)
% A_st=total area of nonprestressed longitudinal reinforcement (mm^2)
% A_s1=area of the 1st layer of reinforcement (mm^2)
% A_s2=area of the 2nd layer of reinforcement (mm^2)
% A_s3=area of the 3rd layer of reinforcement (mm^2)
% A_s4=area of the 4th layer of reinforcement (mm^2)
% A_s5=area of the 5th layer of reinforcement (mm^2)
% b=width of column (mm)
% _b=boundary
% _bal=at balance point
% c=distance from extreme compression fiber to neutral axis (mm)
% _com=combination
% C_c=nominal compressive force in concrete (kN)
% C_rc=factored compressive force in concrete for partial material strength reduction factors format (kN)
% d_1=distance from extreme compression fiber to the 1st layer of reinforcement (mm)
% d_2=distance from extreme compression fiber to the 2nd layer of reinforcement (mm)
% d_3=distance from extreme compression fiber to the 3rd layer of reinforcement (mm)
% d_4=distance from extreme compression fiber to the 4th layer of reinforcement (mm)
% d_5=distance from extreme compression fiber to the 5th layer of reinforcement (mm)
% eoverh=the specific e/h value
% eoverh_2=the specific e/h value, including extreme values
```

```
% e_r=design eccentricity for partial material strength reduction factors format (m)
% e_u=design eccentricity for ACI 318-14 (m)
% E_s=modulus of elasticity of reinforcement (MPa)
% f_c=specified compressive strength of concrete (MPa)
% f_s1=stress in the 1st layer of reinforcement (MPa)
% f_s2=stress in the 2nd layer of reinforcement (MPa)
% f_s3=stress in the 3rd layer of reinforcement (MPa)
% f_s4=stress in the 4th layer of reinforcement (MPa)
% f_s5=stress in the 5th layer of reinforcement (MPa)
% f_y=specified yield strength for nonprestressed reinforcement (MPa)
% F_rs1=factored force in the 1st layer of reinforcement for partial material strength reduction factors
% format (kN)
% F_rs2=factored force in the 2nd layer of reinforcement for partial material strength reduction factors
% format (kN)
% F_rs3=factored force in the 3rd layer of reinforcement for partial material strength reduction factors
% format (kN)
% F_rs4=factored force in the 4th layer of reinforcement for partial material strength reduction factors
% format (kN)
% F_rs5=factored force in the 5th layer of reinforcement for partial material strength reduction factors
% format (kN)
% F_s1=nominal force in the 1st layer of reinforcement (kN)
% F_s2=nominal force in the 2nd layer of reinforcement (kN)
% F_s3=nominal force in the 3rd layer of reinforcement (kN)
% F_s4=nominal force in the 4th layer of reinforcement (kN)
% F_s5=nominal force in the 5th layer of reinforcement (kN)
% h=overall depth of column (mm)
% hovere=the specific h/e value
% hovere_r=h/e_r, where e_r= design eccentricity for partial material strength reduction factors format (m)
% hovere_u=h/e_u, where e_u= design eccentricity for ACI 318-14 (m)
% hovere_2=the specific h/e value, including extreme values
% M_n=nominal flexural strength (kN.m)
% M_r=design flexural strength for partial material strength reduction factors format (kN.m)
% _pri=prime
% pro=property
% P_n=nominal axial strength (kN)
% P_nt=nominal axial tensile strength (kN)
% P_o=nominal axial strength at zero eccentricity (kN)
% P_r=design axial strength for partial material strength reduction factors format (kN)
```

```
% P_rmax=maximum design axial compressive strength for partial material strength reduction factors format
%        (kN)
% P_ro=design axial strength at zero eccentricity for partial material strength reduction factors format
%      (kN)
% P_rt=design axial tensile strength for partial material strength reduction factors format (kN)
% s_=sutscript
% _s=sort
% _sam=samples
% Z=ratio of strain in extreme tension layer of reinforcement to yield strain


% alpha_PM=design combined flexural and axial strength ratio, equal to design combined flexural and axial
%          strength obtained using strength reduction factors in ACI 318-14 to that obtained using partial
%          material strength reduction factors
% beta_1=factor relating depth of equivalent rectangular compressive stress block to depth of neutral axis
% gamma=ratio of distance between outer layers of reinforcement in column to overall column depth
% epsilon_s1=strain in the 1st layer of reinforcement
% epsilon_s2=strain in the 2nd layer of reinforcement
% epsilon_s3=strain in the 3rd layer of reinforcement
% epsilon_s4=strain in the 4th layer of reinforcement
% epsilon_s5=strain in the 5th layer of reinforcement
% epsilon_y=yield strain of reinforcement
% angle_theta= angle theta, angle used to calculate compression segment of circular column
% rho_g=total reinforcement ratio, equal to ratio of total longitudinal reinforcement area to cross-
%       sectional area of column
% phi=strength reduction factor in ACI 318-14
% phi_c=partial material strength reduction factor for concrete
% phi_s=partial material strength reduction factor for reinforcing steel
% phi_sc=a pair of partial material strength reduction factors
% phiM_n=design flexural strength in ACI 318-14 (kN.m)
% phiP_n=design axial strength in ACI 318-14 (kN)
% phiP_nmax=maximum design axial compressive strength in ACI 318-14 (kN)
% phiP_nt=design axial tensile strength in ACI 318-14 (kN)
% phiP_o=design axial strength at zero eccentricity in ACI 318-14 (kN)
```

A.1.2 Column Section 1

A.1.2.1 Code 1-Design Combined Flexural and Axial Strength Ratios, α_{PM}

```
clc
clear
% Design strength ratios calculation
eoverh=[0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 -10.0 ...
       -5.0 -1.0 -0.5 -0.1]; % The specific e/h values
hovere=1./eoverh;
eoverh_2=[0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 -10.0 ...
         -5.0 -1.0 -0.5 -0.1 -0]; % The specific e/h values, including extreme values
hovere_2=1./eoverh_2;
Z_sam=0.45:-0.001:-60;

% Check whether eoverh locate in the range of e/h corresponding to Z_sam and M>0
% Need to check hovere_usamb and hovere_rsamb after running the code
% Upper boundary
[phiP_nsamb(:,1),phiM_nsamb(:,1),hovere_usamb(:,1),~,~]=feval('DesignStrength_u_S1',Z_sam(1));
% Lower boundary
[phiP_nsamb(:,2),phiM_nsamb(:,2),hovere_usamb(:,2),~,~]=feval('DesignStrength_u_S1',Z_sam(length(Z_sam)));

% Upper boundary
[P_rsamb(:,:,1),M_rsamb(:,:,1),hovere_rsamb(:,:,1),~,~]=feval('DesignStrength_r_S1',Z_sam(1));
% Lower boundary
[P_rsamb(:,:,2),M_rsamb(:,:,2),hovere_rsamb(:,:,2),~,~]=feval('DesignStrength_r_S1',Z_sam(length(Z_sam)));

for i1=1:8
    if or(hovere_usamb(i1,1)<hovere(1),phiM_nsamb(i1,1)<0)
        hovere_usamb(i1,1)=NaN;
    end
    if or(hovere_usamb(i1,2)>hovere(length(hovere)),phiM_nsamb(i1,2)<0)
        hovere_usamb(i1,2)=NaN;
    end

    for i2=1:16
```

```
        if or(hovere_rsamb(i1,i2,1)<hovere(1),M_rsamb(i1,i2,1)<0)
            hovere_rsamb(i1,i2,1)=NaN;
        end
        if or(hovere_rsamb(i1,i2,2)>hovere(length(hovere)),M_rsamb(i1,i2,2)<0)
            hovere_rsamb(i1,i2,2)=NaN;
        end
    end
end
% Check finish

% Interpolation
% Calculate phiP_n
% Calculate sample points (phiP_nsam, phiM_nsam and hovere_usam corresponding to Z_sam, phiP_o and phiP_nt)
% Preallocation
phiP_nsam=zeros(8,length(Z_sam));
phiM_nsam=zeros(8,length(Z_sam));
hovere_usam=zeros(8,length(Z_sam));

for i3=1:length(Z_sam)
    [phiP_nsam(:,i3),phiM_nsam(:,i3),hovere_usam(:,i3),phiP_o,phiP_nt]...
        =feval('DesignStrength_u_S1',Z_sam(i3));
end

% Calculate phiP_nmax
phiP_nmax=0.80*phiP_o;

% Calculate the unknown points (phiP_n corresponding to specific hovere)
% Preallocation
phiP_n=zeros(8,length(hovere));

for i1=1:8
    s_phiM_nsam=find(phiM_nsam(i1,:)>0);
    phiP_nsampri=phiP_nsam(i1,s_phiM_nsam);
    hovere_usampri=hovere_usam(i1,s_phiM_nsam);
    phiP_nsampri=[phiP_o(i1,1) phiP_nsampri phiP_nt(i1,1)];
    hovere_usampri=[1e10 hovere_usampri -1e10];
    [hovere_usampris,I_hovere_usampri]=sort(hovere_usampri,'descend');
```

```
    phiP_n(i1,:)=interp1(hovere_usampris,phiP_nsampri(I_hovere_usampri),hovere,'linear');
    s_phiP_nmax=find(phiP_n(i1,:)>phiP_nmax(i1,1));
    phiP_n(i1,s_phiP_nmax)=phiP_nmax(i1,1);
end

% phiP_n includes phiP_nmax and phiP_nt
phiP_n=cat(2,phiP_nmax,phiP_n,phiP_nt);

% Calculate P_r
% Calculate sample points (P_rsam, M_rsam and hovere_rsam corresponding to Z_sam, P_ro and P_rt)
% Preallocation
P_rsam=zeros(8,16,length(Z_sam));
M_rsam=zeros(8,16,length(Z_sam));
hovere_rsam=zeros(8,16,length(Z_sam));

for i3=1:length(Z_sam)
    [P_rsam(:,:,i3),M_rsam(:,:,i3),hovere_rsam(:,:,i3),P_ro,P_rt]=feval('DesignStrength_r_S1',Z_sam(i3));
end

% Calculate P_rmax
P_rmax=0.80*P_ro;

% Permute the 2nd and 3rd dimensions for P_rsam, M_rsam, hovere_rsam, P_ro, P_rt and P_rmax
P_rsam=permute(P_rsam,[1,3,2]);
M_rsam=permute(M_rsam,[1,3,2]);
hovere_rsam=permute(hovere_rsam,[1,3,2]);
P_ro=permute(P_ro,[1,3,2]);
P_rt=permute(P_rt,[1,3,2]);
P_rmax=permute(P_rmax,[1,3,2]);

% Calculate the unknown points (P_r corresponding to specific hovere)
% Preallocation
P_r=zeros(8,length(hovere),16);

for i1=1:8
    for i2=1:16
```

```
        s_M_rsam=find(M_rsam(i1,:,i2)>0);
        P_rsampri=P_rsam(i1,s_M_rsam,i2);
        hovere_rsampri=hovere_rsam(i1,s_M_rsam,i2);
        P_rsampri=[P_ro(i1,1,i2) P_rsampri P_rt(i1,1,i2)];
        hovere_rsampri=[1e10 hovere_rsampri -1e10];
        [hovere_rsampris,I_hovere_rsampri]=sort(hovere_rsampri,'descend');
        P_r(i1,:,i2)=interp1(hovere_rsampris,P_rsampri(I_hovere_rsampri),hovere,'linear');
        s_P_rmax=find(P_r(i1,:,i2)>P_rmax(i1,1,i2));
        P_r(i1,s_P_rmax,i2)=P_rmax(i1,1,i2);
    end
end

% P_r includes P_rmax and P_rt
P_r=cat(2,P_rmax,P_r,P_rt);

% Check whether the sign of phiP_n (P_r) is identical with the sign of eoverh
% (Need to check the results after calculation)
for i1=1:8
    for i4=1+1:length(eoverh_2)-1
        if sign(eoverh_2(i4))~=sign(phiP_n(i1,i4))
            phiP_n(i1,i4)=NaN;
        end
    end
end

for i1=1:8
    for i2=1:16
        for i4=1+1:length(eoverh_2)-1
            if sign(eoverh_2(i4))~=sign(P_r(i1,i4,i2))
               P_r(i1,i4,i2)=NaN;
            end
        end
    end
end

% Calculate limited balance points
Z_bal=-1;
```

```
[phiP_nbal,phiM_nbal,hovere_ubal,~,~]=feval('DesignStrength_u_S1',Z_bal);
[P_rbal,M_rbal,hovere_rbal,~,~]=feval('DesignStrength_r_S1',Z_bal);
% Permute the 2nd and 3rd dimensions for P_rbal, M_rbal and hovere_rbal
P_rbal=permute(P_rbal,[1,3,2]);
M_rbal=permute(M_rbal,[1,3,2]);
hovere_rbal=permute(hovere_rbal,[1,3,2]);

% Calculate design strength ratios
% Preallocation
alpha_PM=zeros(8,length(eoverh_2),16);

for i1=1:8
    for i2=1:16
        for i4=1:length(eoverh_2)
            alpha_PM(i1,i4,i2)=(abs(phiP_n(i1,i4))*sqrt(1+eoverh_2(i4)^2))/...
                                (abs(P_r(i1,i4,i2))*sqrt(1+eoverh_2(i4)^2));
        end
    end
end

save alpha_PM_S1.mat phiP_n P_r alpha_PM hovere_ubal hovere_rbal
```

A.1.2.2 Code 2-Function of Design Strengths for ACI 318-14

```
% Design strength calculation corresponding to ACI 318-14
function [phiP_n,phiM_n,hovere_u,phiP_o,phiP_nt]=DesignStrength_u_S1(Z)
% Geometric property combinations
b_com=[325 1300];
h_com=[325 1300];
gamma_com=[0.6 0.9];
d_1_com=(1+gamma_com).*h_com/2;
d_2_com=h_com/2;
d_3_com=(1-gamma_com).*h_com/2;
rho_g_com=[0.01 0.04];

% Material property combinations
f_c_com=[25 45];
```

```
beta_1_com=[0.85 0.85-0.05*(f_c_com(2)-28)/7];
f_y=420;
E_s=200000;
epsilon_y=f_y/E_s;

% Summarize property combinations in one matrix
pro_com=[b_com(1) h_com(1) gamma_com(1) d_1_com(1) d_2_com(1) d_3_com(1) f_c_com(1) beta_1_com(1) ...
         rho_g_com(1);
         b_com(1) h_com(1) gamma_com(1) d_1_com(1) d_2_com(1) d_3_com(1) f_c_com(1) beta_1_com(1) ...
         rho_g_com(2);
         b_com(1) h_com(1) gamma_com(1) d_1_com(1) d_2_com(1) d_3_com(1) f_c_com(2) beta_1_com(2) ...
         rho_g_com(1);
         b_com(1) h_com(1) gamma_com(1) d_1_com(1) d_2_com(1) d_3_com(1) f_c_com(2) beta_1_com(2) ...
         rho_g_com(2);
         b_com(2) h_com(2) gamma_com(2) d_1_com(2) d_2_com(2) d_3_com(2) f_c_com(1) beta_1_com(1) ...
         rho_g_com(1);
         b_com(2) h_com(2) gamma_com(2) d_1_com(2) d_2_com(2) d_3_com(2) f_c_com(1) beta_1_com(1) ...
         rho_g_com(2);
         b_com(2) h_com(2) gamma_com(2) d_1_com(2) d_2_com(2) d_3_com(2) f_c_com(2) beta_1_com(2) ...
         rho_g_com(1);
         b_com(2) h_com(2) gamma_com(2) d_1_com(2) d_2_com(2) d_3_com(2) f_c_com(2) beta_1_com(2) ...
         rho_g_com(2)];

% pro_com=[325 325 0.6 260 162.5 65 25 0.85 0.01;325 325 0.6 260 162.5 65 25 0.85 0.04;
         % 325 325 0.6 260 162.5 65 45 0.73 0.01;325 325 0.6 260 162.5 65 45 0.73 0.04;
         % 1300 1300 0.9 1235 650 65 25 0.85 0.01;1300 1300 0.9 1235 650 65 25 0.85 0.04;
         % 1300 1300 0.9 1235 650 65 45 0.73 0.01;1300 1300 0.9 1235 650 65 45 0.73 0.04]

% Preallocation
b=zeros(8,1);
h=zeros(8,1);
gamma=zeros(8,1);
d_1=zeros(8,1);
d_2=zeros(8,1);
d_3=zeros(8,1);
f_c=zeros(8,1);
beta_1=zeros(8,1);
```

```
rho_g=zeros(8,1);
A_st=zeros(8,1);
A_s1=zeros(8,1);
A_s2=zeros(8,1);
A_s3=zeros(8,1);
c=zeros(8,1);
a=zeros(8,1);
epsilon_s1=zeros(8,1);
epsilon_s2=zeros(8,1);
epsilon_s3=zeros(8,1);
f_s1=zeros(8,1);
f_s2=zeros(8,1);
f_s3=zeros(8,1);
C_c=zeros(8,1);
F_s1=zeros(8,1);
F_s2=zeros(8,1);
F_s3=zeros(8,1);
P_n=zeros(8,1);
M_n=zeros(8,1);
phi=zeros(8,1);
phiP_n=zeros(8,1);
phiM_n=zeros(8,1);
phiP_o=zeros(8,1);
phiP_nt=zeros(8,1);
e_u=zeros(8,1);
hovere_u=zeros(8,1);

for i1=1:8
    % Properities
    b(i1,1)=pro_com(i1,1);
    h(i1,1)=pro_com(i1,2);
    gamma(i1,1)=pro_com(i1,3);
    d_1(i1,1)=pro_com(i1,4);
    d_2(i1,1)=pro_com(i1,5);
    d_3(i1,1)=pro_com(i1,6);
    f_c(i1,1)=pro_com(i1,7);
    beta_1(i1,1)=pro_com(i1,8);
```

```
rho_g(i1,1)=pro_com(i1,9);
A_st(i1,1)=rho_g(i1,1)*b(i1,1)*h(i1,1);
A_s1(i1,1)=3*A_st(i1,1)/8;
A_s2(i1,1)=A_st(i1,1)/4;
A_s3(i1,1)=3*A_st(i1,1)/8;

% Calculation process
% Z=? input of function
% Calculate c
c(i1,1)=(0.003/(0.003-Z*epsilon_y))*d_1(i1,1);
% Calculate a
a(i1,1)=beta_1(i1,1)*c(i1,1);
% Compare a with h
if a(i1,1)>h(i1,1)
    a(i1,1)=h(i1,1);
end

% Calculate epsilon_s1, epsilon_s2, epsilon_s3, f_s1, f_s2, and f_s3
epsilon_s1(i1,1)=Z*epsilon_y;
epsilon_s2(i1,1)=0.003*(c(i1,1)-d_2(i1,1))/c(i1,1);
epsilon_s3(i1,1)=0.003*(c(i1,1)-d_3(i1,1))/c(i1,1);
f_s1(i1,1)=epsilon_s1(i1,1)*E_s;
f_s2(i1,1)=epsilon_s2(i1,1)*E_s;
f_s3(i1,1)=epsilon_s3(i1,1)*E_s;

% Compare f_s1, f_s2 and f_s3 with +-f_y
if f_s1(i1,1)>f_y
    f_s1(i1,1)=f_y;
elseif f_s1(i1,1)<-f_y
    f_s1(i1,1)=-f_y;
end

if f_s2(i1,1)>f_y
    f_s2(i1,1)=f_y;
elseif f_s2(i1,1)<-f_y
    f_s2(i1,1)=-f_y;
end
```

```
if f_s3(i1,1)>f_y
    f_s3(i1,1)=f_y;
elseif f_s3(i1,1)<-f_y
    f_s3(i1,1)=-f_y;
end

% Nominal values
% Calculate C_c
C_c(i1,1)=0.85*f_c(i1,1)*a(i1,1)*b(i1,1)/1000;

% Calculate F_s1
if a(i1,1)<d_1(i1,1)
    F_s1(i1,1)=f_s1(i1,1)*A_s1(i1,1)/1000;
else
    F_s1(i1,1)=(f_s1(i1,1)-0.85*f_c(i1,1))*A_s1(i1,1)/1000;
end

% Calculate F_s2
if a(i1,1)<d_2(i1,1)
    F_s2(i1,1)=f_s2(i1,1)*A_s2(i1,1)/1000;
else
    F_s2(i1,1)=(f_s2(i1,1)-0.85*f_c(i1,1))*A_s2(i1,1)/1000;
end

% Calculate F_s3
if a(i1,1)<d_3(i1,1)
    F_s3(i1,1)=f_s3(i1,1)*A_s3(i1,1)/1000;
else
    F_s3(i1,1)=(f_s3(i1,1)-0.85*f_c(i1,1))*A_s3(i1,1)/1000;
end

% Calculate P_n and M_n
P_n(i1,1)=C_c(i1,1)+F_s1(i1,1)+F_s2(i1,1)+F_s3(i1,1);
M_n(i1,1)=(C_c(i1,1)*(h(i1,1)/2-a(i1,1)/2)+F_s1(i1,1)*(h(i1,1)/2-d_1(i1,1))+...
          F_s2(i1,1)*(h(i1,1)/2-d_2(i1,1))+F_s3(i1,1)*(h(i1,1)/2-d_3(i1,1)))/1000;
```

```
    % Calculation corresponding to ACI 318-14
    % Calculate phi
    if -epsilon_s1(i1,1)<=epsilon_y
        phi(i1,1)=0.65;
    elseif -epsilon_s1(i1,1)>=0.005
        phi(i1,1)=0.90;
    else
        phi(i1,1)=0.65+0.25*(-epsilon_s1(i1,1)-epsilon_y)/(0.005-epsilon_y);
    end

    % Calculate phiP_n, phiM_n, phiP_o and phiP_nt
    phiP_n(i1,1)=phi(i1,1)*P_n(i1,1);
    phiM_n(i1,1)=phi(i1,1)*M_n(i1,1);
    phiP_o(i1,1)=0.65*(0.85*f_c(i1,1)*(b(i1,1)*h(i1,1)-A_st(i1,1))+f_y*A_st(i1,1))/1000;
    phiP_nt(i1,1)=-0.90*f_y*A_st(i1,1)/1000;

    % Calculate e_u and hovere_u
    e_u(i1,1)=phiM_n(i1,1)/phiP_n(i1,1); % (m)
    hovere_u(i1,1)=(h(i1,1)/1000)/e_u(i1,1);
end
end
```

A.1.2.3 Code 3-Function of Design Strengths for Partial Material Strength Reduction Factors

```
% Design strength calculation corresponding to partial strength reduction factors
function [P_r,M_r,hovere_r,P_ro,P_rt]=DesignStrength_r_S1(Z)
% Geometric property combinations
b_com=[325 1300];
h_com=[325 1300];
gamma_com=[0.6 0.9];
d_1_com=(1+gamma_com).*h_com/2;
d_2_com=h_com/2;
d_3_com=(1-gamma_com).*h_com/2;
rho_g_com=[0.01 0.04];

% Material property combinations
```

```
f_c_com=[25 45];
beta_1_com=[0.85 0.85-0.05*(f_c_com(2)-28)/7];
f_y=420;
E_s=200000;
epsilon_y=f_y/E_s;

% Summarize property combinations in one matrix
pro_com=[b_com(1) h_com(1) gamma_com(1) d_1_com(1) d_2_com(1) d_3_com(1) f_c_com(1) beta_1_com(1) ...
         rho_g_com(1);
         b_com(1) h_com(1) gamma_com(1) d_1_com(1) d_2_com(1) d_3_com(1) f_c_com(1) beta_1_com(1) ...
         rho_g_com(2);
         b_com(1) h_com(1) gamma_com(1) d_1_com(1) d_2_com(1) d_3_com(1) f_c_com(2) beta_1_com(2) ...
         rho_g_com(1);
         b_com(1) h_com(1) gamma_com(1) d_1_com(1) d_2_com(1) d_3_com(1) f_c_com(2) beta_1_com(2) ...
         rho_g_com(2);
         b_com(2) h_com(2) gamma_com(2) d_1_com(2) d_2_com(2) d_3_com(2) f_c_com(1) beta_1_com(1) ...
         rho_g_com(1);
         b_com(2) h_com(2) gamma_com(2) d_1_com(2) d_2_com(2) d_3_com(2) f_c_com(1) beta_1_com(1) ...
         rho_g_com(2);
         b_com(2) h_com(2) gamma_com(2) d_1_com(2) d_2_com(2) d_3_com(2) f_c_com(2) beta_1_com(2) ...
         rho_g_com(1);
         b_com(2) h_com(2) gamma_com(2) d_1_com(2) d_2_com(2) d_3_com(2) f_c_com(2) beta_1_com(2) ...
         rho_g_com(2)];

% pro_com=[325 325 0.6 260 162.5 65 25 0.85 0.01;325 325 0.6 260 162.5 65 25 0.85 0.04;
         % 325 325 0.6 260 162.5 65 45 0.73 0.01;325 325 0.6 260 162.5 65 45 0.73 0.04;
         % 1300 1300 0.9 1235 650 65 25 0.85 0.01;1300 1300 0.9 1235 650 65 25 0.85 0.04;
         % 1300 1300 0.9 1235 650 65 45 0.73 0.01;1300 1300 0.9 1235 650 65 45 0.73 0.04]

% Preallocation
b=zeros(8,1);
h=zeros(8,1);
gamma=zeros(8,1);
d_1=zeros(8,1);
d_2=zeros(8,1);
d_3=zeros(8,1);
f_c=zeros(8,1);
```

```
beta_1=zeros(8,1);
rho_g=zeros(8,1);
A_st=zeros(8,1);
A_s1=zeros(8,1);
A_s2=zeros(8,1);
A_s3=zeros(8,1);
c=zeros(8,1);
a=zeros(8,1);
epsilon_s1=zeros(8,1);
epsilon_s2=zeros(8,1);
epsilon_s3=zeros(8,1);
f_s1=zeros(8,1);
f_s2=zeros(8,1);
f_s3=zeros(8,1);
C_rc=zeros(8,16);
F_rs1=zeros(8,16);
F_rs2=zeros(8,16);
F_rs3=zeros(8,16);
P_r=zeros(8,16);
M_r=zeros(8,16);
P_ro=zeros(8,16);
P_rt=zeros(8,16);
e_r=zeros(8,16);
hovere_r=zeros(8,16);

for i1=1:8
    % Properities
    b(i1,1)=pro_com(i1,1);
    h(i1,1)=pro_com(i1,2);
    gamma(i1,1)=pro_com(i1,3);
    d_1(i1,1)=pro_com(i1,4);
    d_2(i1,1)=pro_com(i1,5);
    d_3(i1,1)=pro_com(i1,6);
    f_c(i1,1)=pro_com(i1,7);
    beta_1(i1,1)=pro_com(i1,8);
    rho_g(i1,1)=pro_com(i1,9);
    A_st(i1,1)=rho_g(i1,1)*b(i1,1)*h(i1,1);
```

```
A_s1(i1,1)=3*A_st(i1,1)/8;
A_s2(i1,1)=A_st(i1,1)/4;
A_s3(i1,1)=3*A_st(i1,1)/8;

% Calculation process
% Z=? input of function
% Calculate c
c(i1,1)=(0.003/(0.003-Z*epsilon_y))*d_1(i1,1);
% Calculate a
a(i1,1)=beta_1(i1,1)*c(i1,1);
% Compare a with h
if a(i1,1)>h(i1,1)
    a(i1,1)=h(i1,1);
end

% Calculate epsilon_s1, epsilon_s2, epsilon_s3, f_s1, f_s2, and f_s3
epsilon_s1(i1,1)=Z*epsilon_y;
epsilon_s2(i1,1)=0.003*(c(i1,1)-d_2(i1,1))/c(i1,1);
epsilon_s3(i1,1)=0.003*(c(i1,1)-d_3(i1,1))/c(i1,1);
f_s1(i1,1)=epsilon_s1(i1,1)*E_s;
f_s2(i1,1)=epsilon_s2(i1,1)*E_s;
f_s3(i1,1)=epsilon_s3(i1,1)*E_s;

% Compare f_s1, f_s2 and f_s3 with +-f_y
if f_s1(i1,1)>f_y
    f_s1(i1,1)=f_y;
elseif f_s1(i1,1)<-f_y
    f_s1(i1,1)=-f_y;
end

if f_s2(i1,1)>f_y
    f_s2(i1,1)=f_y;
elseif f_s2(i1,1)<-f_y
    f_s2(i1,1)=-f_y;
end

if f_s3(i1,1)>f_y
```

```
    f_s3(i1,1)=f_y;
elseif f_s3(i1,1)<-f_y
    f_s3(i1,1)=-f_y;
end

% Calculation corresponding to partial strength reduction factors
% Partial strength reduction factor combinations
% phi_s=[0.80 0.85 0.90 0.95];
% phi_c=[0.60 0.65 0.70 0.75];
phi_sc=[0.80 0.60;0.80 0.65;0.80 0.70;0.80 0.75;
        0.85 0.60;0.85 0.65;0.85 0.70;0.85 0.75;
        0.90 0.60;0.90 0.65;0.90 0.70;0.90 0.75;
        0.95 0.60;0.95 0.65;0.95 0.70;0.95 0.75];

for i2=1:16
    % Calculate C_rc
    C_rc(i1,i2)=phi_sc(i2,2)*0.85*f_c(i1,1)*a(i1,1)*b(i1,1)/1000;

    % Calculate F_rs1
    if a(i1,1)<d_1(i1,1)
        F_rs1(i1,i2)=phi_sc(i2,1)*f_s1(i1,1)*A_s1(i1,1)/1000;
    else
        F_rs1(i1,i2)=(phi_sc(i2,1)*f_s1(i1,1)-phi_sc(i2,2)*0.85*f_c(i1,1))*A_s1(i1,1)/1000;
    end

    % Calculate F_rs2
    if a(i1,1)<d_2(i1,1)
        F_rs2(i1,i2)=phi_sc(i2,1)*f_s2(i1,1)*A_s2(i1,1)/1000;
    else
        F_rs2(i1,i2)=(phi_sc(i2,1)*f_s2(i1,1)-phi_sc(i2,2)*0.85*f_c(i1,1))*A_s2(i1,1)/1000;
    end

    % Calculate F_rs3
    if a(i1,1)<d_3(i1,1)
        F_rs3(i1,i2)=phi_sc(i2,1)*f_s3(i1,1)*A_s3(i1,1)/1000;
    else
        F_rs3(i1,i2)=(phi_sc(i2,1)*f_s3(i1,1)-phi_sc(i2,2)*0.85*f_c(i1,1))*A_s3(i1,1)/1000;
```

```
        end

        % Calculate P_r, M_r, P_ro and P_rt
        P_r(i1,i2)=C_rc(i1,i2)+F_rs1(i1,i2)+F_rs2(i1,i2)+F_rs3(i1,i2);
        M_r(i1,i2)=(C_rc(i1,i2)*(h(i1,1)/2-a(i1,1)/2)+F_rs1(i1,i2)*(h(i1,1)/2-d_1(i1,1))+...
                   F_rs2(i1,i2)*(h(i1,1)/2-d_2(i1,1))+F_rs3(i1,i2)*(h(i1,1)/2-d_3(i1,1)))/1000;
        P_ro(i1,i2)=(phi_sc(i2,2)*0.85*f_c(i1,1)*(b(i1,1)*h(i1,1)-A_st(i1,1))+...
                    phi_sc(i2,1)*f_y*A_st(i1,1))/1000;
        P_rt(i1,i2)=-phi_sc(i2,1)*f_y*A_st(i1,1)/1000;

        % Calculate e_r and hovere_r
        e_r(i1,i2)=M_r(i1,i2)/P_r(i1,i2); % (m)
        hovere_r(i1,i2)=(h(i1,1)/1000)/e_r(i1,i2);
    end
end
end
```

A.1.3 Column Section 5

A.1.3.1 Code 1-Design Combined Flexural and Axial Strength Ratios, α_{PM}

```
clc
clear
% Design strength ratios calculation
eoverh=[0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 -10.0 ...
       -5.0 -1.0 -0.5 -0.1]; % The specific e/h values
hovere=1./eoverh;
eoverh_2=[0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 -10.0 ...
         -5.0 -1.0 -0.5 -0.1 -0]; % The specific e/h values, including extreme values
hovere_2=1./eoverh_2;
Z_sam=0.30:-0.001:-35;

% Check whether eoverh locate in the range of e/h corresponding to Z_sam and M>0
% Need to check hovere_usamb and hovere_rsamb after running the code
% Upper boundary
[phiP_nsamb(:,1),phiM_nsamb(:,1),hovere_usamb(:,1),~,~]=feval('DesignStrength_u_S5',Z_sam(1));
```

```
% Lower boundary
[phiP_nsamb(:,2),phiM_nsamb(:,2),hovere_usamb(:,2),~,~]=feval('DesignStrength_u_S5',Z_sam(length(Z_sam)));

% Upper boundary
[P_rsamb(:,:,1),M_rsamb(:,:,1),hovere_rsamb(:,:,1),~,~]=feval('DesignStrength_r_S5',Z_sam(1));
% Lower boundary
[P_rsamb(:,:,2),M_rsamb(:,:,2),hovere_rsamb(:,:,2),~,~]=feval('DesignStrength_r_S5',Z_sam(length(Z_sam)));

for i1=1:8
    if or(hovere_usamb(i1,1)<hovere(1),phiM_nsamb(i1,1)<0)
        hovere_usamb(i1,1)=NaN;
    end
    if or(hovere_usamb(i1,2)>hovere(length(hovere)),phiM_nsamb(i1,2)<0)
        hovere_usamb(i1,2)=NaN;
    end

    for i2=1:16
        if or(hovere_rsamb(i1,i2,1)<hovere(1),M_rsamb(i1,i2,1)<0)
            hovere_rsamb(i1,i2,1)=NaN;
        end
        if or(hovere_rsamb(i1,i2,2)>hovere(length(hovere)),M_rsamb(i1,i2,2)<0)
            hovere_rsamb(i1,i2,2)=NaN;
        end
    end
end
% Check finish

% Interpolation
% Calculate phiP_n
% Calculate sample points (phiP_nsam, phiM_nsam and hovere_usam corresponding to Z_sam, phiP_o and phiP_nt)
% Preallocation
phiP_nsam=zeros(8,length(Z_sam));
phiM_nsam=zeros(8,length(Z_sam));
hovere_usam=zeros(8,length(Z_sam));

for i3=1:length(Z_sam)
    [phiP_nsam(:,i3),phiM_nsam(:,i3),hovere_usam(:,i3),phiP_o,phiP_nt]...
```

```
        =feval('DesignStrength_u_S5',Z_sam(i3));
end

% Calculate phiP_nmax
phiP_nmax=0.85*phiP_o;

% Calculate the unknown points (phiP_n corresponding to specific hovere)
% Preallocation
phiP_n=zeros(8,length(hovere));

for i1=1:8
    s_phiM_nsam=find(phiM_nsam(i1,:)>0);
    phiP_nsampri=phiP_nsam(i1,s_phiM_nsam);
    hovere_usampri=hovere_usam(i1,s_phiM_nsam);
    phiP_nsampri=[phiP_o(i1,1) phiP_nsampri phiP_nt(i1,1)];
    hovere_usampri=[1e10 hovere_usampri -1e10];
    [hovere_usampris,I_hovere_usampri]=sort(hovere_usampri,'descend');
    phiP_n(i1,:)=interp1(hovere_usampris,phiP_nsampri(I_hovere_usampri),hovere,'linear');
    s_phiP_nmax=find(phiP_n(i1,:)>phiP_nmax(i1,1));
    phiP_n(i1,s_phiP_nmax)=phiP_nmax(i1,1);
end

% phiP_n includes phiP_nmax and phiP_nt
phiP_n=cat(2,phiP_nmax,phiP_n,phiP_nt);

% Calculate P_r
% Calculate sample points (P_rsam, M_rsam and hovere_rsam corresponding to Z_sam, P_ro and P_rt)
% Preallocation
P_rsam=zeros(8,16,length(Z_sam));
M_rsam=zeros(8,16,length(Z_sam));
hovere_rsam=zeros(8,16,length(Z_sam));

for i3=1:length(Z_sam)
    [P_rsam(:,:,i3),M_rsam(:,:,i3),hovere_rsam(:,:,i3),P_ro,P_rt]=feval('DesignStrength_r_S5',Z_sam(i3));
end
```

```
% Calculate P_rmax
P_rmax=0.85*P_ro;

% Permute the 2nd and 3rd dimensions for P_rsam, M_rsam, hovere_rsam, P_ro, P_rt and P_rmax
P_rsam=permute(P_rsam,[1,3,2]);
M_rsam=permute(M_rsam,[1,3,2]);
hovere_rsam=permute(hovere_rsam,[1,3,2]);
P_ro=permute(P_ro,[1,3,2]);
P_rt=permute(P_rt,[1,3,2]);
P_rmax=permute(P_rmax,[1,3,2]);

% Calculate the unknown points (P_r corresponding to specific hovere)
% Preallocation
P_r=zeros(8,length(hovere),16);

for i1=1:8
    for i2=1:16
        s_M_rsam=find(M_rsam(i1,:,i2)>0);
        P_rsampri=P_rsam(i1,s_M_rsam,i2);
        hovere_rsampri=hovere_rsam(i1,s_M_rsam,i2);
        P_rsampri=[P_ro(i1,1,i2) P_rsampri P_rt(i1,1,i2)];
        hovere_rsampri=[1e10 hovere_rsampri -1e10];
        [hovere_rsampris,I_hovere_rsampri]=sort(hovere_rsampri,'descend');
        P_r(i1,:,i2)=interp1(hovere_rsampris,P_rsampri(I_hovere_rsampri),hovere,'linear');
        s_P_rmax=find(P_r(i1,:,i2)>P_rmax(i1,1,i2));
        P_r(i1,s_P_rmax,i2)=P_rmax(i1,1,i2);
    end
end

% P_r includes P_rmax and P_rt
P_r=cat(2,P_rmax,P_r,P_rt);

% Check whether the sign of phiP_n (P_r) is identical with the sign of eoverh
% (Need to check the results after calculation)
for i1=1:8
    for i4=1+1:length(eoverh_2)-1
        if sign(eoverh_2(i4))~=sign(phiP_n(i1,i4))
```

```
            phiP_n(i1,i4)=NaN;
        end
    end
end

for i1=1:8
    for i2=1:16
        for i4=1+1:length(eoverh_2)-1
            if sign(eoverh_2(i4))~=sign(P_r(i1,i4,i2))
               P_r(i1,i4,i2)=NaN;
            end
        end
    end
end

% Calculate limited balance points
Z_bal=-1;
[phiP_nbal,phiM_nbal,hovere_ubal,~,~]=feval('DesignStrength_u_S5',Z_bal);
[P_rbal,M_rbal,hovere_rbal,~,~]=feval('DesignStrength_r_S5',Z_bal);
% Permute the 2nd and 3rd dimensions for P_rbal, M_rbal and hovere_rbal
P_rbal=permute(P_rbal,[1,3,2]);
M_rbal=permute(M_rbal,[1,3,2]);
hovere_rbal=permute(hovere_rbal,[1,3,2]);

% Calculate design strength ratios
% Preallocation
alpha_PM=zeros(8,length(eoverh_2),16);

for i1=1:8
    for i2=1:16
        for i4=1:length(eoverh_2)
            alpha_PM(i1,i4,i2)=(abs(phiP_n(i1,i4))*sqrt(1+eoverh_2(i4)^2))/...
                                  (abs(P_r(i1,i4,i2))*sqrt(1+eoverh_2(i4)^2));
        end
    end
end
```

```
save alpha_PM_S5.mat phiP_n P_r alpha_PM hovere_ubal hovere_rbal
```

A.1.3.2 Code 2-Function of Design Strengths for ACI 318-14

```
% Design strength calculation corresponding to ACI 318-14
function [phiP_n,phiM_n,hovere_u,phiP_o,phiP_nt]=DesignStrength_u_S5(Z)
% Geometric property combinations
h_com=[325 1300];
gamma_com=[0.6 0.9];
d_1_com=(1+gamma_com).*h_com/2;
d_2_com=(2+2^0.5*gamma_com).*h_com/4;
d_3_com=h_com/2;
d_4_com=(2-2^0.5*gamma_com).*h_com/4;
d_5_com=(1-gamma_com).*h_com/2;
rho_g_com=[0.01 0.04];

% Material property combinations
f_c_com=[25 45];
beta_1_com=[0.85 0.85-0.05*(f_c_com(2)-28)/7];
f_y=420;
E_s=200000;
epsilon_y=f_y/E_s;

% Summarize property combinations in one matrix
pro_com=[h_com(1) gamma_com(1) d_1_com(1) d_2_com(1) d_3_com(1) d_4_com(1) d_5_com(1) ...
         f_c_com(1) beta_1_com(1) rho_g_com(1);
         h_com(1) gamma_com(1) d_1_com(1) d_2_com(1) d_3_com(1) d_4_com(1) d_5_com(1) ...
         f_c_com(1) beta_1_com(1) rho_g_com(2);
         h_com(1) gamma_com(1) d_1_com(1) d_2_com(1) d_3_com(1) d_4_com(1) d_5_com(1) ...
         f_c_com(2) beta_1_com(2) rho_g_com(1);
         h_com(1) gamma_com(1) d_1_com(1) d_2_com(1) d_3_com(1) d_4_com(1) d_5_com(1) ...
         f_c_com(2) beta_1_com(2) rho_g_com(2);
         h_com(2) gamma_com(2) d_1_com(2) d_2_com(2) d_3_com(2) d_4_com(2) d_5_com(2) ...
         f_c_com(1) beta_1_com(1) rho_g_com(1);
         h_com(2) gamma_com(2) d_1_com(2) d_2_com(2) d_3_com(2) d_4_com(2) d_5_com(2) ...
         f_c_com(1) beta_1_com(1) rho_g_com(2);
         h_com(2) gamma_com(2) d_1_com(2) d_2_com(2) d_3_com(2) d_4_com(2) d_5_com(2) ...
```

```
        f_c_com(2) beta_1_com(2) rho_g_com(1);
        h_com(2) gamma_com(2) d_1_com(2) d_2_com(2) d_3_com(2) d_4_com(2) d_5_com(2) ...
        f_c_com(2) beta_1_com(2) rho_g_com(2)];

% Preallocation
h=zeros(8,1);
gamma=zeros(8,1);
d_1=zeros(8,1);
d_2=zeros(8,1);
d_3=zeros(8,1);
d_4=zeros(8,1);
d_5=zeros(8,1);
f_c=zeros(8,1);
beta_1=zeros(8,1);
rho_g=zeros(8,1);
A_st=zeros(8,1);
A_s1=zeros(8,1);
A_s2=zeros(8,1);
A_s3=zeros(8,1);
A_s4=zeros(8,1);
A_s5=zeros(8,1);
c=zeros(8,1);
a=zeros(8,1);
epsilon_s1=zeros(8,1);
epsilon_s2=zeros(8,1);
epsilon_s3=zeros(8,1);
epsilon_s4=zeros(8,1);
epsilon_s5=zeros(8,1);
f_s1=zeros(8,1);
f_s2=zeros(8,1);
f_s3=zeros(8,1);
f_s4=zeros(8,1);
f_s5=zeros(8,1);
angle_theta=zeros(8,1);
A=zeros(8,1);
C_c=zeros(8,1);
F_s1=zeros(8,1);
```

```
F_s2=zeros(8,1);
F_s3=zeros(8,1);
F_s4=zeros(8,1);
F_s5=zeros(8,1);
P_n=zeros(8,1);
M_n=zeros(8,1);
phi=zeros(8,1);
phiP_n=zeros(8,1);
phiM_n=zeros(8,1);
phiP_o=zeros(8,1);
phiP_nt=zeros(8,1);
e_u=zeros(8,1);
hovere_u=zeros(8,1);

for i1=1:8
    % Properities
    h(i1,1)=pro_com(i1,1);
    gamma(i1,1)=pro_com(i1,2);
    d_1(i1,1)=pro_com(i1,3);
    d_2(i1,1)=pro_com(i1,4);
    d_3(i1,1)=pro_com(i1,5);
    d_4(i1,1)=pro_com(i1,6);
    d_5(i1,1)=pro_com(i1,7);
    f_c(i1,1)=pro_com(i1,8);
    beta_1(i1,1)=pro_com(i1,9);
    rho_g(i1,1)=pro_com(i1,10);
    A_st(i1,1)=rho_g(i1,1)*pi*h(i1,1)^2/4;
    A_s1(i1,1)=A_st(i1,1)/8;
    A_s2(i1,1)=A_st(i1,1)/4;
    A_s3(i1,1)=A_st(i1,1)/4;
    A_s4(i1,1)=A_st(i1,1)/4;
    A_s5(i1,1)=A_st(i1,1)/8;

    % Calculation process
    % Z=? input of function
    % Calculate c
    c(i1,1)=(0.003/(0.003-Z*epsilon_y))*d_1(i1,1);
```

```
% Calculate a
a(i1,1)=beta_1(i1,1)*c(i1,1);
% Compare a with h
if a(i1,1)>h(i1,1)
    a(i1,1)=h(i1,1);
end

% Calculate epsilon_s1, epsilon_s2, epsilon_s3, epsilon_s4, epsilon_s5,
% f_s1, f_s2, f_s3, f_s4 and f_s5
epsilon_s1(i1,1)=Z*epsilon_y;
epsilon_s2(i1,1)=0.003*(c(i1,1)-d_2(i1,1))/c(i1,1);
epsilon_s3(i1,1)=0.003*(c(i1,1)-d_3(i1,1))/c(i1,1);
epsilon_s4(i1,1)=0.003*(c(i1,1)-d_4(i1,1))/c(i1,1);
epsilon_s5(i1,1)=0.003*(c(i1,1)-d_5(i1,1))/c(i1,1);
f_s1(i1,1)=epsilon_s1(i1,1)*E_s;
f_s2(i1,1)=epsilon_s2(i1,1)*E_s;
f_s3(i1,1)=epsilon_s3(i1,1)*E_s;
f_s4(i1,1)=epsilon_s4(i1,1)*E_s;
f_s5(i1,1)=epsilon_s5(i1,1)*E_s;

% Compare f_s1, f_s2, f_s3, f_s4 and f_s5 with +-f_y
if f_s1(i1,1)>f_y
    f_s1(i1,1)=f_y;
elseif f_s1(i1,1)<-f_y
    f_s1(i1,1)=-f_y;
end

if f_s2(i1,1)>f_y
    f_s2(i1,1)=f_y;
elseif f_s2(i1,1)<-f_y
    f_s2(i1,1)=-f_y;
end

if f_s3(i1,1)>f_y
    f_s3(i1,1)=f_y;
elseif f_s3(i1,1)<-f_y
    f_s3(i1,1)=-f_y;
```

```
end

if f_s4(i1,1)>f_y
    f_s4(i1,1)=f_y;
elseif f_s4(i1,1)<-f_y
    f_s4(i1,1)=-f_y;
end

if f_s5(i1,1)>f_y
    f_s5(i1,1)=f_y;
elseif f_s5(i1,1)<-f_y
    f_s5(i1,1)=-f_y;
end

% Nominal values
% Calculate C_c
if a(i1,1)<=h(i1,1)/2
    angle_theta(i1,1)=acos((h(i1,1)/2-a(i1,1))/(h(i1,1)/2));
else
    angle_theta(i1,1)=pi-acos((a(i1,1)-h(i1,1)/2)/(h(i1,1)/2));
end
A(i1,1)=h(i1,1)^2*(angle_theta(i1,1)-sin(angle_theta(i1,1))*cos(angle_theta(i1,1)))/4;
C_c(i1,1)=0.85*f_c(i1,1)*A(i1,1)/1000;

% Calculate F_s1
if a(i1,1)<d_1(i1,1)
    F_s1(i1,1)=f_s1(i1,1)*A_s1(i1,1)/1000;
else
    F_s1(i1,1)=(f_s1(i1,1)-0.85*f_c(i1,1))*A_s1(i1,1)/1000;
end

% Calculate F_s2
if a(i1,1)<d_2(i1,1)
    F_s2(i1,1)=f_s2(i1,1)*A_s2(i1,1)/1000;
else
    F_s2(i1,1)=(f_s2(i1,1)-0.85*f_c(i1,1))*A_s2(i1,1)/1000;
end
```

```
% Calculate F_s3
if a(i1,1)<d_3(i1,1)
    F_s3(i1,1)=f_s3(i1,1)*A_s3(i1,1)/1000;
else
    F_s3(i1,1)=(f_s3(i1,1)-0.85*f_c(i1,1))*A_s3(i1,1)/1000;
end

% Calculate F_s4
if a(i1,1)<d_4(i1,1)
    F_s4(i1,1)=f_s4(i1,1)*A_s4(i1,1)/1000;
else
    F_s4(i1,1)=(f_s4(i1,1)-0.85*f_c(i1,1))*A_s4(i1,1)/1000;
end

% Calculate F_s5
if a(i1,1)<d_5(i1,1)
    F_s5(i1,1)=f_s5(i1,1)*A_s5(i1,1)/1000;
else
    F_s5(i1,1)=(f_s5(i1,1)-0.85*f_c(i1,1))*A_s5(i1,1)/1000;
end

% Calculate P_n and M_n
P_n(i1,1)=C_c(i1,1)+F_s1(i1,1)+F_s2(i1,1)+F_s3(i1,1)+F_s4(i1,1)+F_s5(i1,1);
M_n(i1,1)=(0.85*f_c(i1,1)/1000*h(i1,1)^3*sin(angle_theta(i1,1))^3/12+...
          F_s1(i1,1)*(h(i1,1)/2-d_1(i1,1))+F_s2(i1,1)*(h(i1,1)/2-d_2(i1,1))+...
          F_s3(i1,1)*(h(i1,1)/2-d_3(i1,1))+F_s4(i1,1)*(h(i1,1)/2-d_4(i1,1))+...
          F_s5(i1,1)*(h(i1,1)/2-d_5(i1,1)))/1000;

% Calculation corresponding to ACI 318-14
% Calculate phi
if -epsilon_s1(i1,1)<=epsilon_y
    phi(i1,1)=0.75;
elseif -epsilon_s1(i1,1)>=0.005
    phi(i1,1)=0.90;
else
    phi(i1,1)=0.75+0.15*(-epsilon_s1(i1,1)-epsilon_y)/(0.005-epsilon_y);
```

```
    end

    % Calculate phiP_n, phiM_n, phiP_o and phiP_nt
    phiP_n(i1,1)=phi(i1,1)*P_n(i1,1);
    phiM_n(i1,1)=phi(i1,1)*M_n(i1,1);
    phiP_o(i1,1)=0.75*(0.85*f_c(i1,1)*(pi*h(i1,1)^2/4-A_st(i1,1))+f_y*A_st(i1,1))/1000;
    phiP_nt(i1,1)=-0.90*f_y*A_st(i1,1)/1000;

    % Calculate e_u and hovere_u
    e_u(i1,1)=phiM_n(i1,1)/phiP_n(i1,1); % (m)
    hovere_u(i1,1)=(h(i1,1)/1000)/e_u(i1,1);
end
end
```

A.1.3.3 Code 3-Function of Design Strengths for Partial Material Strength Reduction Factors

```
% Design strength calculation corresponding to partial strength reduction factors
function [P_r,M_r,hovere_r,P_ro,P_rt]=DesignStrength_r_S5(Z)
% Geometric property combinations
h_com=[325 1300];
gamma_com=[0.6 0.9];
d_1_com=(1+gamma_com).*h_com/2;
d_2_com=(2+2^0.5*gamma_com).*h_com/4;
d_3_com=h_com/2;
d_4_com=(2-2^0.5*gamma_com).*h_com/4;
d_5_com=(1-gamma_com).*h_com/2;
rho_g_com=[0.01 0.04];

% Material property combinations
f_c_com=[25 45];
beta_1_com=[0.85 0.85-0.05*(f_c_com(2)-28)/7];
f_y=420;
E_s=200000;
epsilon_y=f_y/E_s;

% Summarize property combinations in one matrix
pro_com=[h_com(1) gamma_com(1) d_1_com(1) d_2_com(1) d_3_com(1) d_4_com(1) d_5_com(1) ...
```

```
          f_c_com(1) beta_1_com(1) rho_g_com(1);
          h_com(1) gamma_com(1) d_1_com(1) d_2_com(1) d_3_com(1) d_4_com(1) d_5_com(1) ...
          f_c_com(1) beta_1_com(1) rho_g_com(2);
          h_com(1) gamma_com(1) d_1_com(1) d_2_com(1) d_3_com(1) d_4_com(1) d_5_com(1) ...
          f_c_com(2) beta_1_com(2) rho_g_com(1);
          h_com(1) gamma_com(1) d_1_com(1) d_2_com(1) d_3_com(1) d_4_com(1) d_5_com(1) ...
          f_c_com(2) beta_1_com(2) rho_g_com(2);
          h_com(2) gamma_com(2) d_1_com(2) d_2_com(2) d_3_com(2) d_4_com(2) d_5_com(2) ...
          f_c_com(1) beta_1_com(1) rho_g_com(1);
          h_com(2) gamma_com(2) d_1_com(2) d_2_com(2) d_3_com(2) d_4_com(2) d_5_com(2) ...
          f_c_com(1) beta_1_com(1) rho_g_com(2);
          h_com(2) gamma_com(2) d_1_com(2) d_2_com(2) d_3_com(2) d_4_com(2) d_5_com(2) ...
          f_c_com(2) beta_1_com(2) rho_g_com(1);
          h_com(2) gamma_com(2) d_1_com(2) d_2_com(2) d_3_com(2) d_4_com(2) d_5_com(2) ...
          f_c_com(2) beta_1_com(2) rho_g_com(2)];

% Preallocation
h=zeros(8,1);
gamma=zeros(8,1);
d_1=zeros(8,1);
d_2=zeros(8,1);
d_3=zeros(8,1);
d_4=zeros(8,1);
d_5=zeros(8,1);
f_c=zeros(8,1);
beta_1=zeros(8,1);
rho_g=zeros(8,1);
A_st=zeros(8,1);
A_s1=zeros(8,1);
A_s2=zeros(8,1);
A_s3=zeros(8,1);
A_s4=zeros(8,1);
A_s5=zeros(8,1);
c=zeros(8,1);
a=zeros(8,1);
epsilon_s1=zeros(8,1);
epsilon_s2=zeros(8,1);
```

```
epsilon_s3=zeros(8,1);
epsilon_s4=zeros(8,1);
epsilon_s5=zeros(8,1);
f_s1=zeros(8,1);
f_s2=zeros(8,1);
f_s3=zeros(8,1);
f_s4=zeros(8,1);
f_s5=zeros(8,1);
angle_theta=zeros(8,1);
A=zeros(8,1);
C_rc=zeros(8,16);
F_rs1=zeros(8,16);
F_rs2=zeros(8,16);
F_rs3=zeros(8,16);
F_rs4=zeros(8,16);
F_rs5=zeros(8,16);
P_r=zeros(8,16);
M_r=zeros(8,16);
P_ro=zeros(8,16);
P_rt=zeros(8,16);
e_r=zeros(8,16);
hovere_r=zeros(8,16);

for i1=1:8
    % Properities
    h(i1,1)=pro_com(i1,1);
    gamma(i1,1)=pro_com(i1,2);
    d_1(i1,1)=pro_com(i1,3);
    d_2(i1,1)=pro_com(i1,4);
    d_3(i1,1)=pro_com(i1,5);
    d_4(i1,1)=pro_com(i1,6);
    d_5(i1,1)=pro_com(i1,7);
    f_c(i1,1)=pro_com(i1,8);
    beta_1(i1,1)=pro_com(i1,9);
    rho_g(i1,1)=pro_com(i1,10);
    A_st(i1,1)=rho_g(i1,1)*pi*h(i1,1)^2/4;
    A_s1(i1,1)=A_st(i1,1)/8;
```

```
A_s2(i1,1)=A_st(i1,1)/4;
A_s3(i1,1)=A_st(i1,1)/4;
A_s4(i1,1)=A_st(i1,1)/4;
A_s5(i1,1)=A_st(i1,1)/8;

% Calculation process
% Z=? input of function
% Calculate c
c(i1,1)=(0.003/(0.003-Z*epsilon_y))*d_1(i1,1);
% Calculate a
a(i1,1)=beta_1(i1,1)*c(i1,1);
% Compare a with h
if a(i1,1)>h(i1,1)
    a(i1,1)=h(i1,1);
end

% Calculate epsilon_s1, epsilon_s2, epsilon_s3, epsilon_s4, epsilon_s5,
% f_s1, f_s2, f_s3, f_s4 and f_s5
epsilon_s1(i1,1)=Z*epsilon_y;
epsilon_s2(i1,1)=0.003*(c(i1,1)-d_2(i1,1))/c(i1,1);
epsilon_s3(i1,1)=0.003*(c(i1,1)-d_3(i1,1))/c(i1,1);
epsilon_s4(i1,1)=0.003*(c(i1,1)-d_4(i1,1))/c(i1,1);
epsilon_s5(i1,1)=0.003*(c(i1,1)-d_5(i1,1))/c(i1,1);
f_s1(i1,1)=epsilon_s1(i1,1)*E_s;
f_s2(i1,1)=epsilon_s2(i1,1)*E_s;
f_s3(i1,1)=epsilon_s3(i1,1)*E_s;
f_s4(i1,1)=epsilon_s4(i1,1)*E_s;
f_s5(i1,1)=epsilon_s5(i1,1)*E_s;

% Compare f_s1, f_s2, f_s3, f_s4 and f_s5 with +-f_y
if f_s1(i1,1)>f_y
    f_s1(i1,1)=f_y;
elseif f_s1(i1,1)<-f_y
    f_s1(i1,1)=-f_y;
end

if f_s2(i1,1)>f_y
```

```
    f_s2(i1,1)=f_y;
elseif f_s2(i1,1)<-f_y
    f_s2(i1,1)=-f_y;
end

if f_s3(i1,1)>f_y
    f_s3(i1,1)=f_y;
elseif f_s3(i1,1)<-f_y
    f_s3(i1,1)=-f_y;
end

if f_s4(i1,1)>f_y
    f_s4(i1,1)=f_y;
elseif f_s4(i1,1)<-f_y
    f_s4(i1,1)=-f_y;
end

if f_s5(i1,1)>f_y
    f_s5(i1,1)=f_y;
elseif f_s5(i1,1)<-f_y
    f_s5(i1,1)=-f_y;
end

% Calculation corresponding to partial strength reduction factors
% Partial strength reduction factor combinations
% phi_s=[0.80 0.85 0.90 0.95];
% phi_c=[0.60 0.65 0.70 0.75];
phi_sc=[0.80 0.60;0.80 0.65;0.80 0.70;0.80 0.75;
        0.85 0.60;0.85 0.65;0.85 0.70;0.85 0.75;
        0.90 0.60;0.90 0.65;0.90 0.70;0.90 0.75;
        0.95 0.60;0.95 0.65;0.95 0.70;0.95 0.75];

for i2=1:16
    % Calculate C_rc
    if a(i1,1)<=h(i1,1)/2
        angle_theta(i1,1)=acos((h(i1,1)/2-a(i1,1))/(h(i1,1)/2));
    else
```

```
    angle_theta(i1,1)=pi-acos((a(i1,1)-h(i1,1)/2)/(h(i1,1)/2));
end
A(i1,1)=h(i1,1)^2*(angle_theta(i1,1)-sin(angle_theta(i1,1))*cos(angle_theta(i1,1)))/4;
C_rc(i1,i2)=phi_sc(i2,2)*0.85*f_c(i1,1)*A(i1,1)/1000;

% Calculate F_rs1
if a(i1,1)<d_1(i1,1)
    F_rs1(i1,i2)=phi_sc(i2,1)*f_s1(i1,1)*A_s1(i1,1)/1000;
else
    F_rs1(i1,i2)=(phi_sc(i2,1)*f_s1(i1,1)-phi_sc(i2,2)*0.85*f_c(i1,1))*A_s1(i1,1)/1000;
end

% Calculate F_rs2
if a(i1,1)<d_2(i1,1)
    F_rs2(i1,i2)=phi_sc(i2,1)*f_s2(i1,1)*A_s2(i1,1)/1000;
else
    F_rs2(i1,i2)=(phi_sc(i2,1)*f_s2(i1,1)-phi_sc(i2,2)*0.85*f_c(i1,1))*A_s2(i1,1)/1000;
end

% Calculate F_rs3
if a(i1,1)<d_3(i1,1)
    F_rs3(i1,i2)=phi_sc(i2,1)*f_s3(i1,1)*A_s3(i1,1)/1000;
else
    F_rs3(i1,i2)=(phi_sc(i2,1)*f_s3(i1,1)-phi_sc(i2,2)*0.85*f_c(i1,1))*A_s3(i1,1)/1000;
end

% Calculate F_rs4
if a(i1,1)<d_4(i1,1)
    F_rs4(i1,i2)=phi_sc(i2,1)*f_s4(i1,1)*A_s4(i1,1)/1000;
else
    F_rs4(i1,i2)=(phi_sc(i2,1)*f_s4(i1,1)-phi_sc(i2,2)*0.85*f_c(i1,1))*A_s4(i1,1)/1000;
end

% Calculate F_rs5
if a(i1,1)<d_5(i1,1)
    F_rs5(i1,i2)=phi_sc(i2,1)*f_s5(i1,1)*A_s5(i1,1)/1000;
else
```

```
            F_rs5(i1,i2)=(phi_sc(i2,1)*f_s5(i1,1)-phi_sc(i2,2)*0.85*f_c(i1,1))*A_s5(i1,1)/1000;
        end

        % Calculate P_r, M_r, P_ro and P_rt
        P_r(i1,i2)=C_rc(i1,i2)+F_rs1(i1,i2)+F_rs2(i1,i2)+F_rs3(i1,i2)+F_rs4(i1,i2)+F_rs5(i1,i2);
        M_r(i1,i2)=(phi_sc(i2,2)*0.85*f_c(i1,1)/1000*h(i1,1)^3*sin(angle_theta(i1,1))^3/12+...
                   F_rs1(i1,i2)*(h(i1,1)/2-d_1(i1,1))+F_rs2(i1,i2)*(h(i1,1)/2-d_2(i1,1))+...
                   F_rs3(i1,i2)*(h(i1,1)/2-d_3(i1,1))+F_rs4(i1,i2)*(h(i1,1)/2-d_4(i1,1))+...
                   F_rs5(i1,i2)*(h(i1,1)/2-d_5(i1,1)))/1000;
        P_ro(i1,i2)=(phi_sc(i2,2)*0.85*f_c(i1,1)*(pi*h(i1,1)^2/4-A_st(i1,1))+...
                    phi_sc(i2,1)*f_y*A_st(i1,1))/1000;
        P_rt(i1,i2)=-phi_sc(i2,1)*f_y*A_st(i1,1)/1000;

        % Calculate e_r and hovere_r
        e_r(i1,i2)=M_r(i1,i2)/P_r(i1,i2); % (m)
        hovere_r(i1,i2)=(h(i1,1)/1000)/e_r(i1,i2);
    end
end
end
```

Appendix B-Supplementary Information for Chapter 3

B.1 Supplemtentary Information for Concrete Compressive Strength

Table B.2 shows bias coefficients and coefficients of variation of F_1, for cast-in-place and precast concrete. The weighted average computed based on the data reported by Nowak and Szerszen (2003) has a bias coefficient of 1.238 and a coefficient of variation of 0.127 for cast-in-place concrete and a bias coefficient of 1.217 and a coefficient of variation 0.131 for precast concrete. Bartlett (2007) assumed F_1 for cast-in-place concrete has a bias coefficient of 1.27 and a coefficient of variation of 0.122.

Table B.3 shows the bias coefficients and coefficients of variation of F_2 for cast-in-place and precast concrete (Bartlett 2007). Table B.4 shows the bias coefficients and coefficients of variation of F_{i-p} for cast-in-place and precast concrete (Bartlett and MacGregor 1999).

Table B.5 shows statistical parameters for in-situ concrete compressive strength reported by Ellingwood et al. (1980). The values in this table intend to account for F_1, F_2, F_{i-p} and F_r. There is no distinction between cast-in-place or precast concrete reported.

Table B.1: Statistical parameters for geometric properties

Item	Source	Comment		
			Mean	σ
Slabs			(mm)	(mm)
h	Ellingwood et al. 1980	1696 Swedish slabs	Nominal+0.76	11.94
		99 slabs	Nominal+5.33	6.60
d	Ellingwood et al. 1980	One-way slab, bottom bars	Nominal–3.30	8.89
			Mean	σ
Beams			(mm)	(mm)
b	Ellingwood et al. 1980	Stem width	Nominal+2.54	3.81
h	Ellingwood et al. 1980	108 beams	Nominal–3.05	6.35
		24 beams	Nominal+20.57	13.97
			Mean	σ
Columns			(mm)	(mm)
b, *h*	Ellingwood et al. 1980	Rectangular	Nominal+1.52	6.35
h	Mirza and MacGregor 1979	Circular	Nominal	4.76
			Mean	σ
Slabs, Beams and Columns			(mm)	(mm)
d	ACI Committee 318 2014	$d \leq 203$ mm	Nominal	4.76
		$d > 203$ mm	Nominal	6.35
Reinforcement			δ	*V*
A_s	Nowak and Szerszen 2003	—	1.0	0.015

Table B.2: Statistical parameters for F_1

Source	Comment	n	δ	V
Nowak and Szerszen 2003	Cast-in-place concrete, f_c' = 20.7 MPa	88	1.35	0.102
	Cast-in-place concrete, f_c' = 24.1 MPa	25	1.21	0.079
	Cast-in-place concrete, f_c' = 27.6 MPa	116	1.235	0.145
	Cast-in-place concrete, f_c' = 31.0 MPa	28	1.14	0.042
	Cast-in-place concrete, f_c' = 34.5 MPa	30	1.15	0.058
	Cast-in-place concrete, f_c' = 41.3 MPa	30	1.12	0.042
	Mean		1.238	0.127
	Precast concrete, f_c' = 34.5 MPa	330	1.38	0.120
	Precast concrete, f_c' = 37.9 MPa	26	1.19	0.101
	Precast concrete, f_c' = 41.3 MPa	493	1.16	0.090
	Precast concrete, f_c' = 44.8 MPa	325	1.14	0.081
	Mean		1.217	0.131
Bartlett 2007	Cast-in-place concrete, f_c' = 25–45 MPa	85	1.27	0.122

Note: n, number of samples.

Table B.3: Statistical parameters for F_2

Source	Comment	δ	V
Bartlett 2007	Cast-in-place concrete	1.03	0.113
	Precast concrete	0.95	0.133

Table B.4: Statistical parameters for $F_{i\text{-}p}$

Source	Comment	δ	V
Bartlett and MacGregor 1999	Cast-in-place concrete	1.0	0.130
	Precast concrete	1.0	0.103

Table B.5: Statistical parameters for in-situ concrete compressive strength

Source	Comment	Mean (MPa)	δ	V
Ellingwood et al. 1980	$f_c' = 21$ MPa	19.3	0.92	0.18
	$f_c' = 28$ MPa	23.7	0.85	0.18
	$f_c' = 35$ MPa	28.2	0.81	0.15

Table B.6: Statistical parameters for $f_y = 420$ MPa

Source	Bar size	Mean yield f_y (MPa)	n	δ	V
Nowak and Szerszen 2003	No.3 (9.5mm)	496.1	72	1.20	0.04
	No.4 (12.5mm)	473.3	79	1.145	0.065
	No.5 (15.5mm)	465.1	116	1.125	0.04
	No.6 (19mm)	476.1	38	1.15	0.05
	No.7 (22mm)	481.6	29	1.165	0.05
	No.8 (25mm)	473.7	36	1.145	0.05
	No.9 (28mm)	475.7	28	1.15	0.05
	No.10 (31mm)	470.2	5	1.14	0.04
	No.11 (34.5mm)	473.7	13	1.145	0.035
	Recommended			1.145	0.05
Ellingwood et al. 1980	—	472.5	—	1.125	0.098

Table B.7: Means and standard deviations of reliability indices for moment using partial material strength reduction factors, β_{Mr}, for f_c' = 45 MPa and ρ = 0.003–0.005

		ϕ_c							
		0.60		0.65		0.70		0.75	
w_L/w_D	ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.5	0.80	3.938	0.023	3.924	0.019	3.911	0.016	3.900	0.013
	0.85	3.568	0.025	3.552	0.021	3.539	0.018	3.527	0.015
	0.90	3.219	0.027	3.202	0.023	3.188	0.019	3.176	0.016
	0.95	2.889	0.030	2.872	0.025	2.857	0.021	2.844	0.018
1.5	0.80	3.695	0.017	3.683	0.014	3.672	0.011	3.663	0.009
	0.85	3.391	0.019	3.378	0.016	3.367	0.013	3.357	0.010
	0.90	3.104	0.021	3.091	0.018	3.079	0.015	3.069	0.012
	0.95	2.834	0.023	2.820	0.019	2.808	0.016	2.797	0.013

Table B.8: Means and standard deviations of reliability indices for moment using partial material strength reduction factors, β_{Mr}, for f_c' = 45 MPa and ρ = 0.006–0.010

		ϕ_c							
		0.60		0.65		0.70		0.75	
w_L/w_D	ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.5	0.80	4.042	0.042	4.012	0.035	3.985	0.030	3.963	0.025
	0.85	3.679	0.045	3.647	0.038	3.619	0.032	3.595	0.027
	0.90	3.339	0.049	3.304	0.041	3.275	0.035	3.249	0.030
	0.95	3.018	0.053	2.982	0.045	2.950	0.038	2.923	0.032
1.5	0.80	3.770	0.030	3.745	0.025	3.724	0.020	3.705	0.017
	0.85	3.474	0.034	3.447	0.028	3.424	0.023	3.405	0.019
	0.90	3.196	0.037	3.167	0.031	3.143	0.026	3.122	0.021
	0.95	2.934	0.040	2.904	0.034	2.878	0.029	2.856	0.024

Table B.9: Means and standard deviations of reliability indices for moment using partial material strength reduction factors, β_{Mr}, for f_c' = 45 MPa and ρ = 0.011–0.018

		ϕ_c							
		0.60		0.65		0.70		0.75	
w_L/w_D	ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.5	0.80	4.214	0.061	4.155	0.050	4.104	0.040	4.061	0.032
	0.85	3.868	0.069	3.805	0.056	3.751	0.046	3.704	0.037
	0.90	3.545	0.076	3.478	0.063	3.420	0.052	3.371	0.042
	0.95	3.243	0.084	3.171	0.070	3.110	0.058	3.057	0.048
1.5	0.80	3.896	0.045	3.847	0.036	3.806	0.028	3.771	0.022
	0.85	3.615	0.052	3.563	0.042	3.519	0.033	3.482	0.026
	0.90	3.353	0.058	3.298	0.048	3.251	0.039	3.211	0.031
	0.95	3.107	0.065	3.049	0.054	2.999	0.044	2.956	0.036

Table B.10: Means and standard deviations of reliability index ratios for shear, β_{Vu}/β_{Vr}, for f_c' = 45 MPa and ρ_t = 0.001–0.010

		ϕ_c							
		0.60		0.65		0.70		0.75	
w_L/w_D	ϕ_s	Mean	σ	Mean	σ	Mean	σ	Mean	σ
0.5	0.80	0.967	0.042	0.991	0.032	1.015	0.022	1.041	0.011
	0.85	1.007	0.053	1.031	0.044	1.057	0.034	1.083	0.023
	0.90	1.048	0.066	1.074	0.057	1.100	0.046	1.128	0.036
	0.95	1.092	0.080	1.118	0.070	1.146	0.060	1.174	0.049
1.5	0.80	0.969	0.039	0.991	0.030	1.014	0.020	1.037	0.010
	0.85	1.006	0.049	1.029	0.040	1.052	0.031	1.076	0.021
	0.90	1.044	0.061	1.067	0.052	1.091	0.042	1.116	0.032
	0.95	1.083	0.073	1.107	0.064	1.132	0.054	1.158	0.044

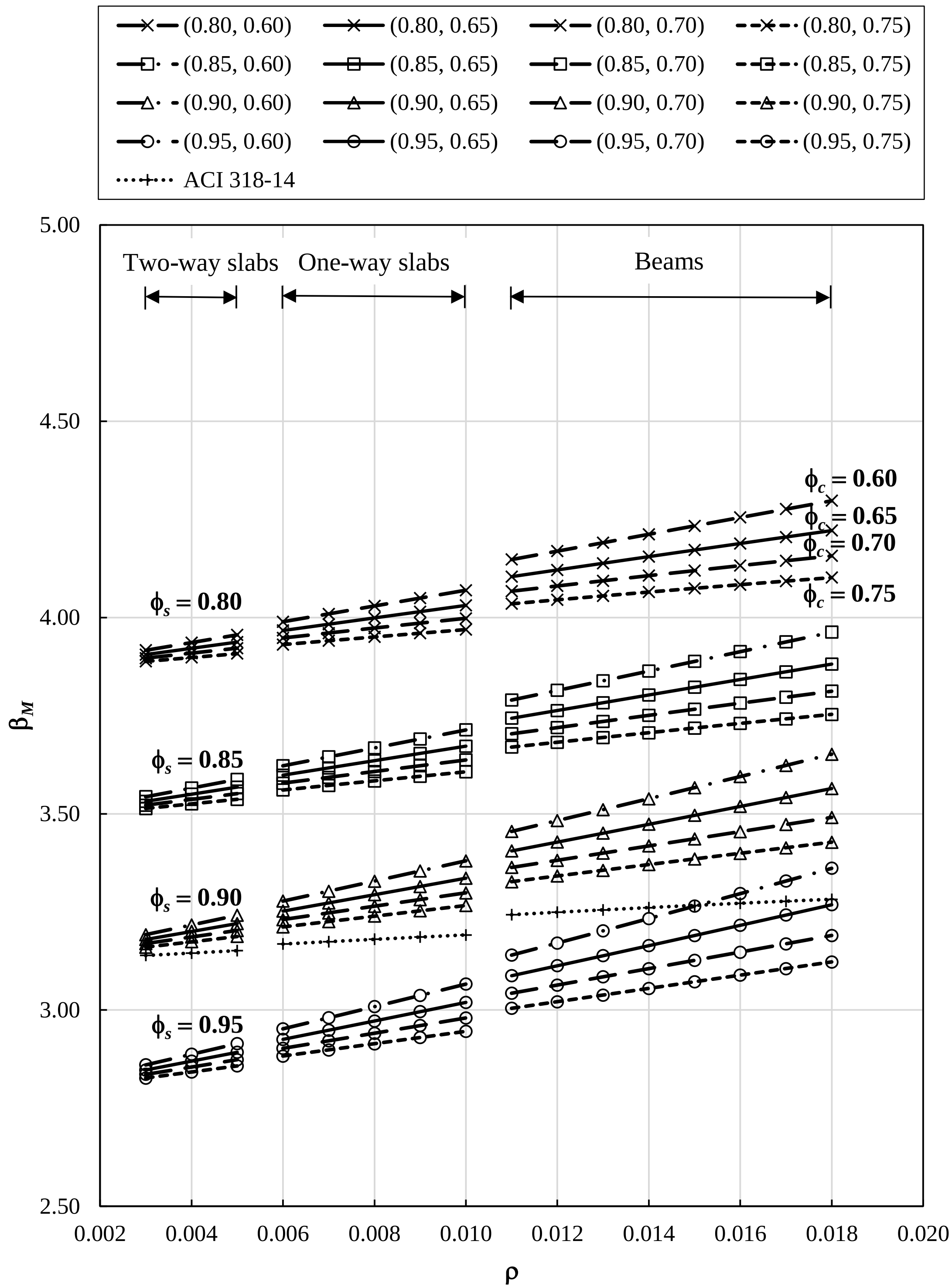


Figure B.1: Reliability indices for moment, β_M, for $f_c' = 45$ MPa, $w_L/w_D = 0.5$, $\rho = 0.003$–0.018, and constant coefficients of variation for d

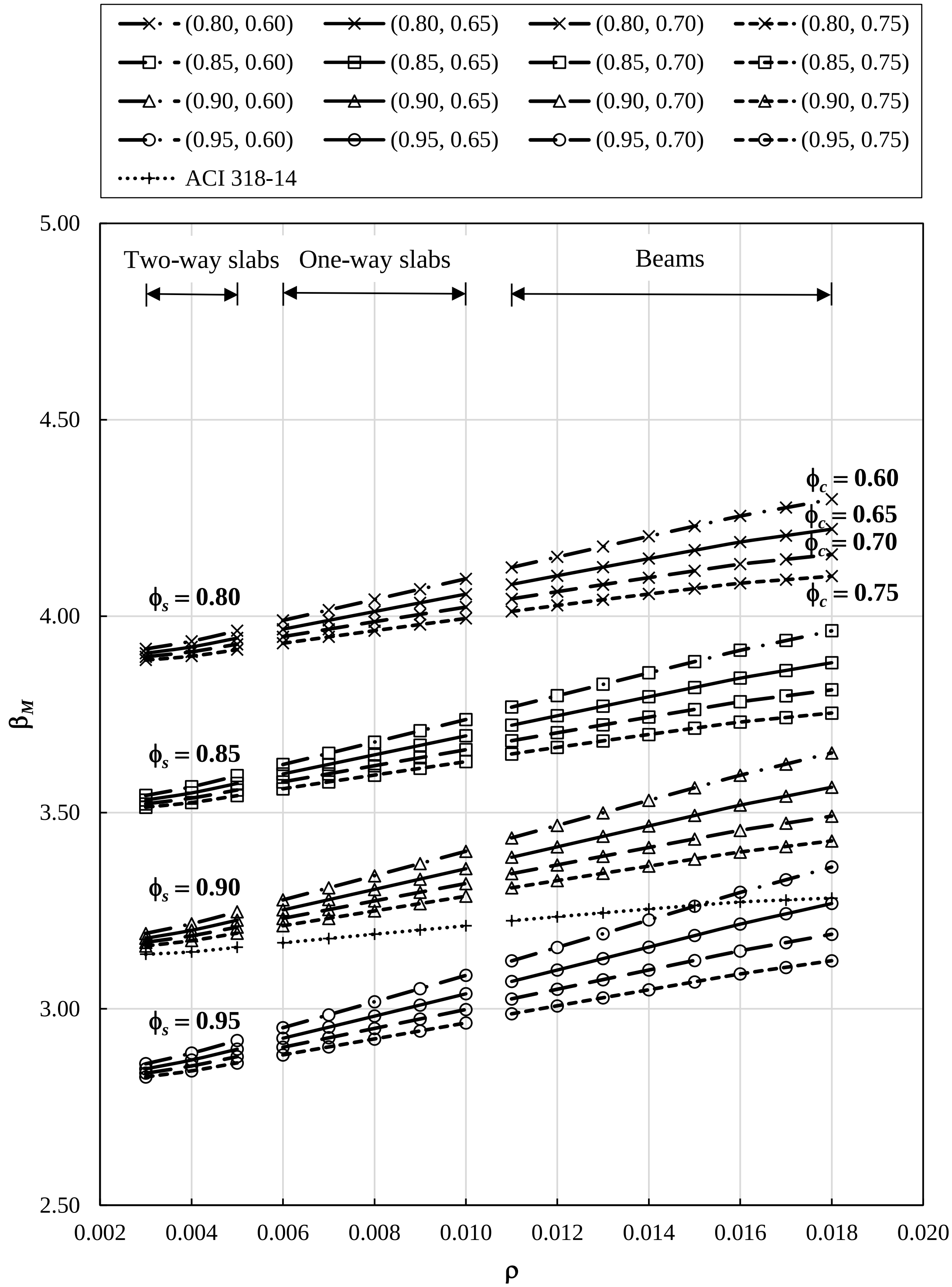


Figure B.2: Reliability indices for moment, β_M, for $f_c' = 45$ MPa, $w_L/w_D = 0.5$, $\rho = 0.003$–0.018, and linear coefficients of variation for d

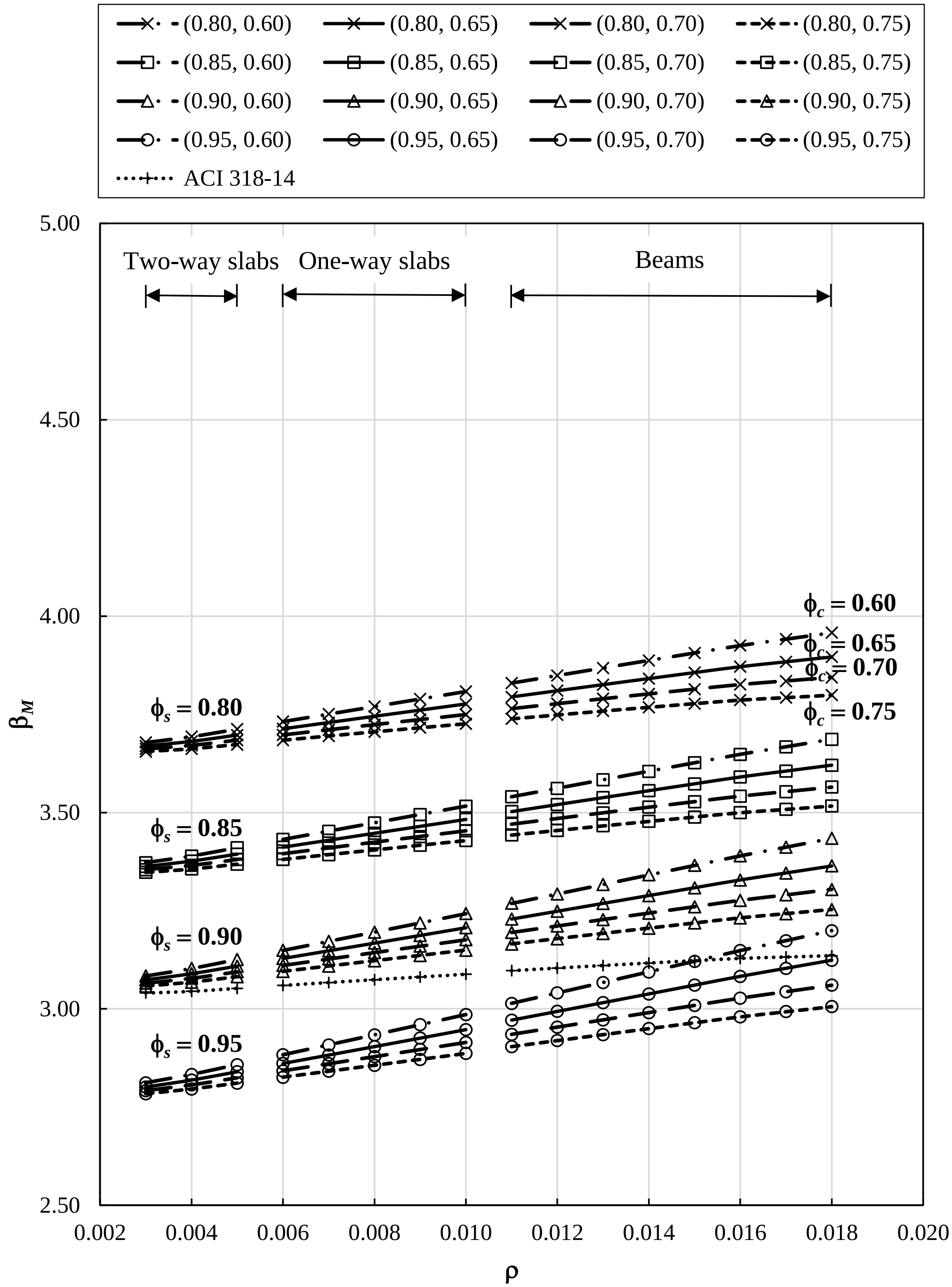
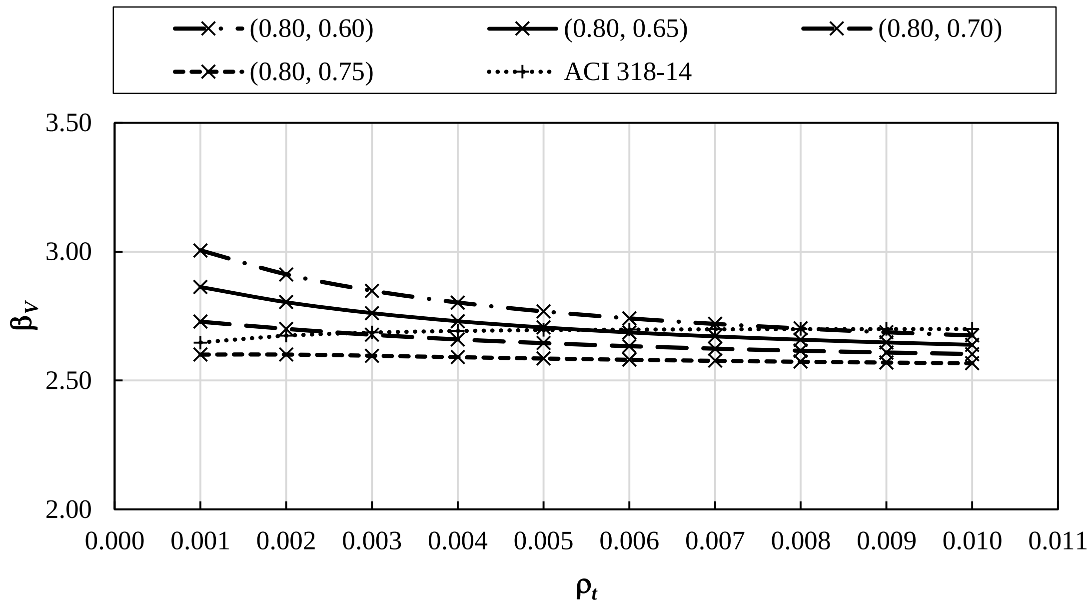
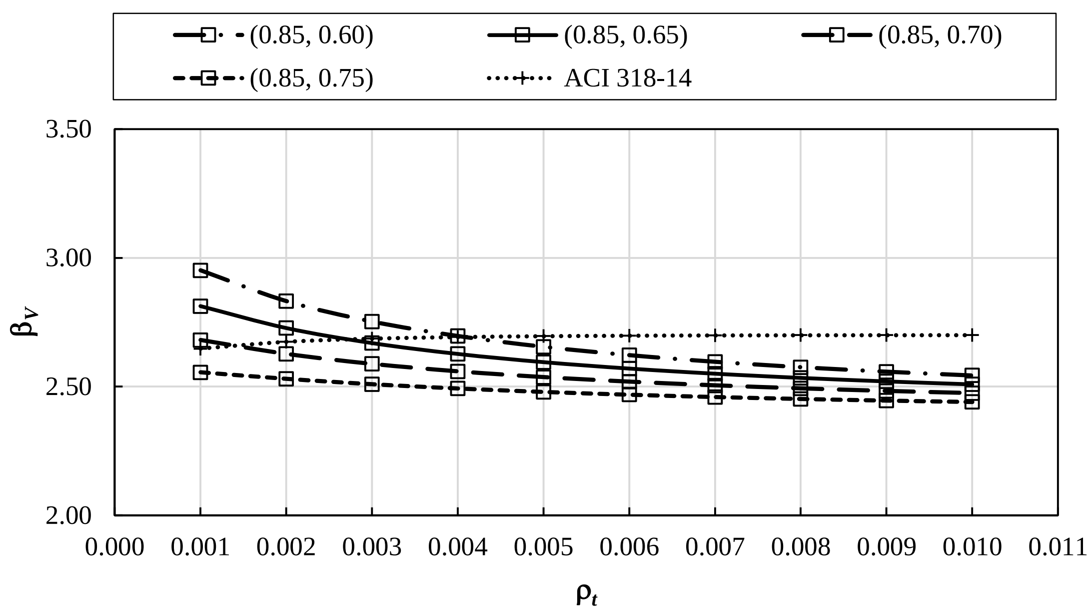


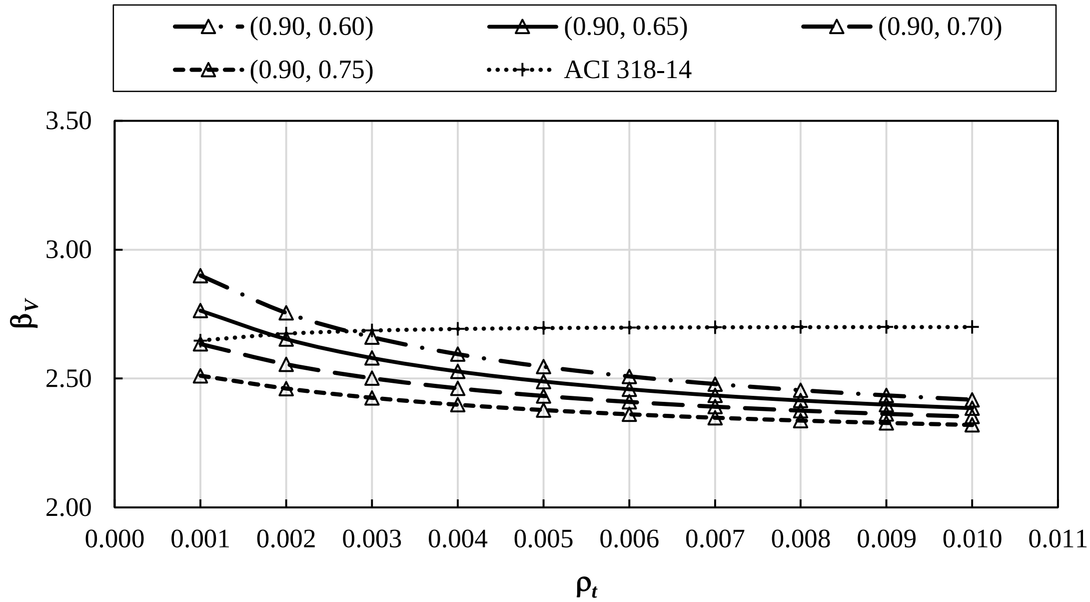
Figure B.3: Reliability indices for moment, β_M, for $f_c' = 45$ MPa, $w_L/w_D = 1.5$, $\rho = 0.003$–0.018, and linear coefficients of variation for d


(0.80, 0.60)
(0.80, 0.65)
(0.80, 0.70)
(0.80, 0.75)
ACI 318-14
3.50
3.00
2.50
2.00
β_V
0.000 0.001 0.002 0.003 0.004 0.005 0.006 0.007 0.008 0.009 0.010 0.011
ρ_t

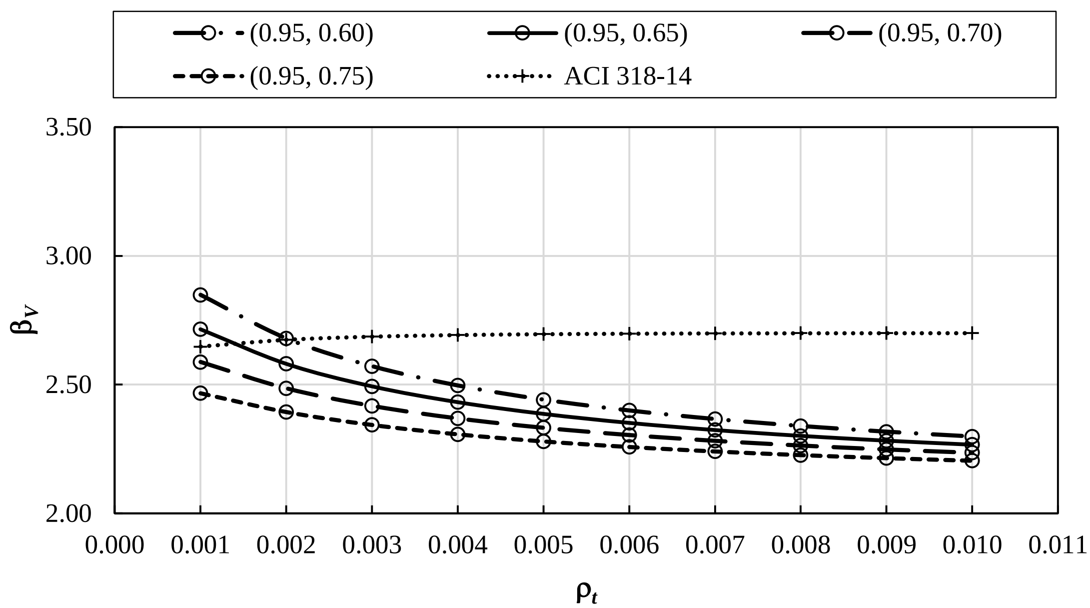
(a)


(0.85, 0.60)
(0.85, 0.65)
(0.85, 0.70)
(0.85, 0.75)
ACI 318-14
3.50
3.00
2.50
2.00
β_V
0.000 0.001 0.002 0.003 0.004 0.005 0.006 0.007 0.008 0.009 0.010 0.011
ρ_t

(b)

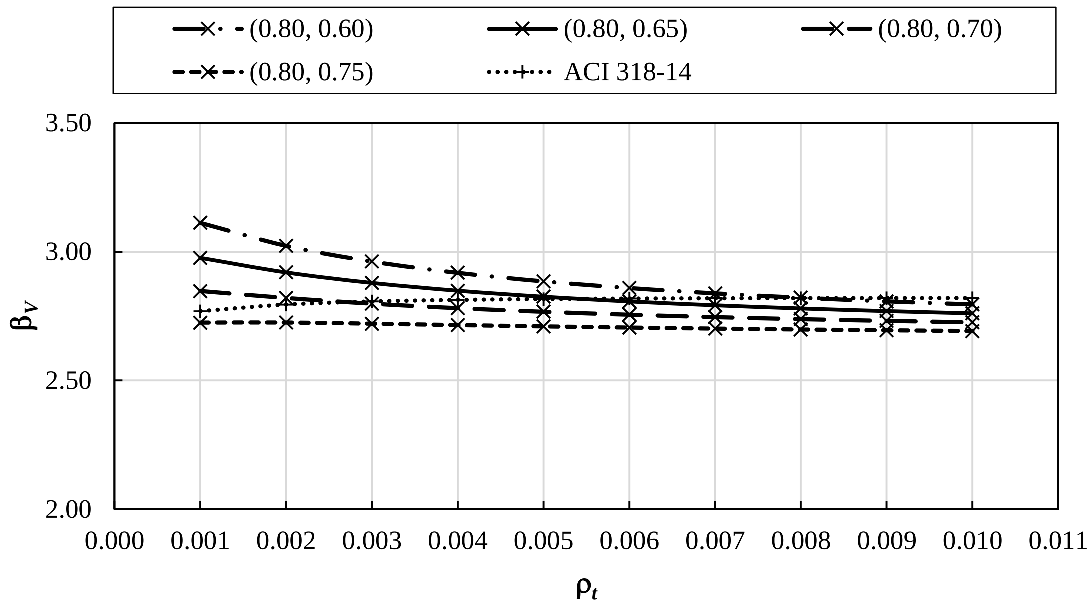


(c)

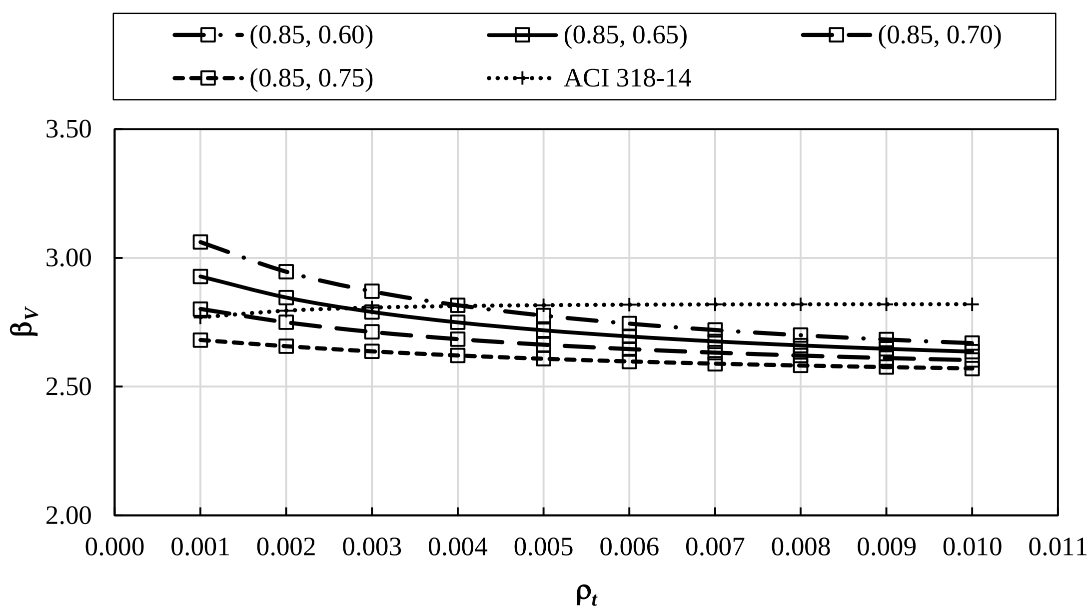


(d)

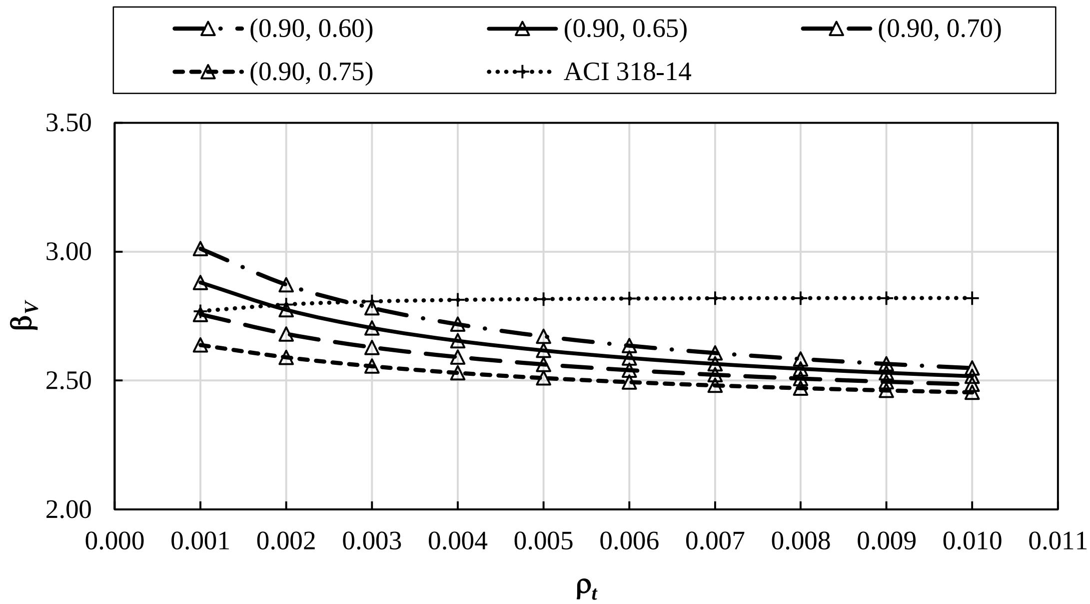
Figure B.4: Reliability indices for shear, β_V, for f_c' = 45 MPa, w_L/w_D = 0.5, and ρ_t = 0.001–0.010: (a) ϕ_s = 0.80; (b) ϕ_s = 0.85; (c) ϕ_s = 0.90; (d) ϕ_s = 0.95


(0.80, 0.60)
(0.80, 0.65)
(0.80, 0.70)
(0.80, 0.75)
ACI 318-14
3.50
3.00
2.50
2.00
β_V
0.000 0.001 0.002 0.003 0.004 0.005 0.006 0.007 0.008 0.009 0.010 0.011
ρ_t

(a)


(0.85, 0.60)
(0.85, 0.65)
(0.85, 0.70)
(0.85, 0.75)
ACI 318-14
3.50
3.00
2.50
2.00
β_V
0.000 0.001 0.002 0.003 0.004 0.005 0.006 0.007 0.008 0.009 0.010 0.011
ρ_t

(b)



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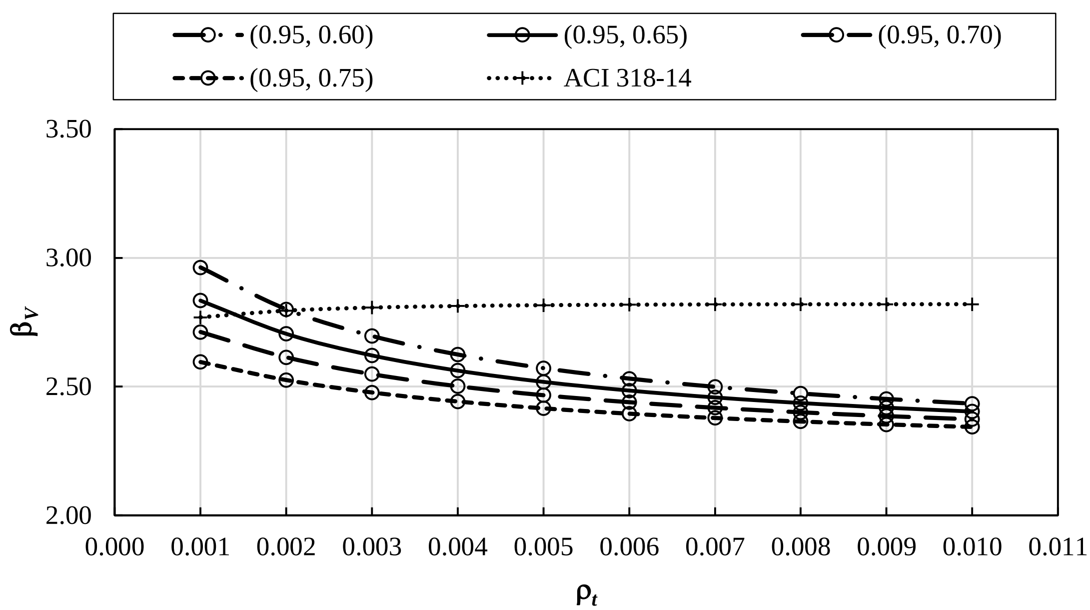
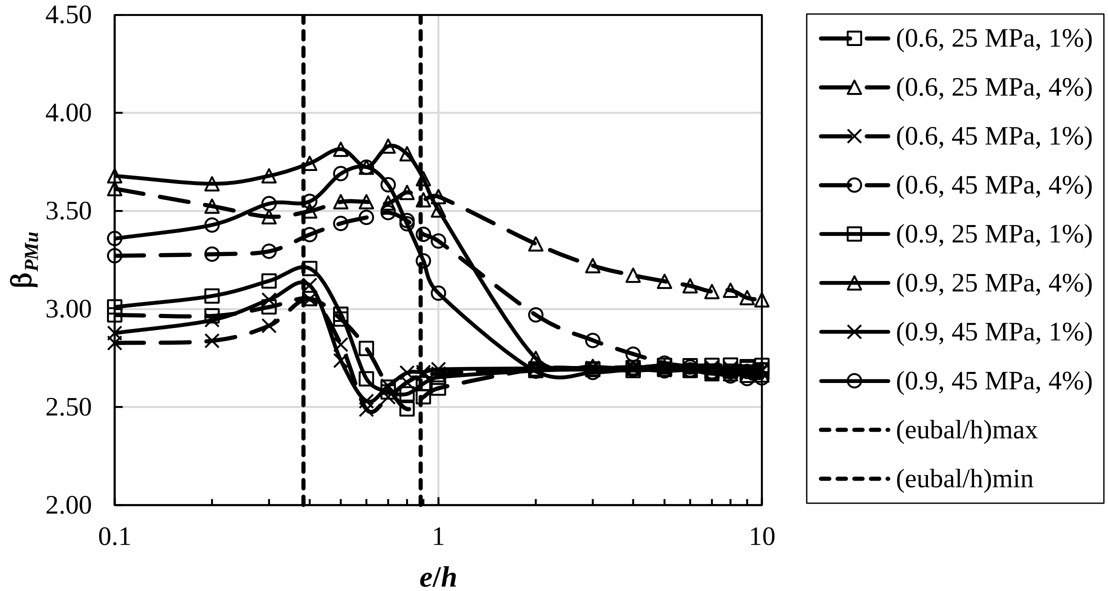


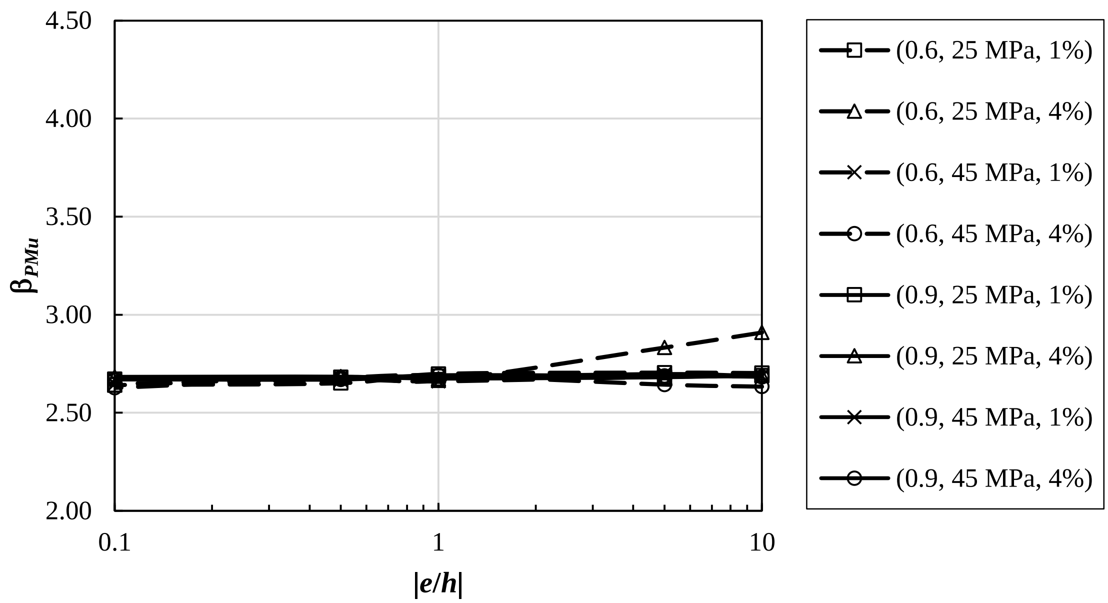
(d)

Figure B.5: Reliability indices for shear, β_V, for $f_c' = 45$ MPa, $w_L/w_D = 1.5$, and $\rho_t =$ 0.001–0.010: (a) $\phi_s = 0.80$; (b) $\phi_s = 0.85$; (c) $\phi_s = 0.90$; (d) $\phi_s = 0.95$

Appendix C-Supplementary Information for Chapter 4

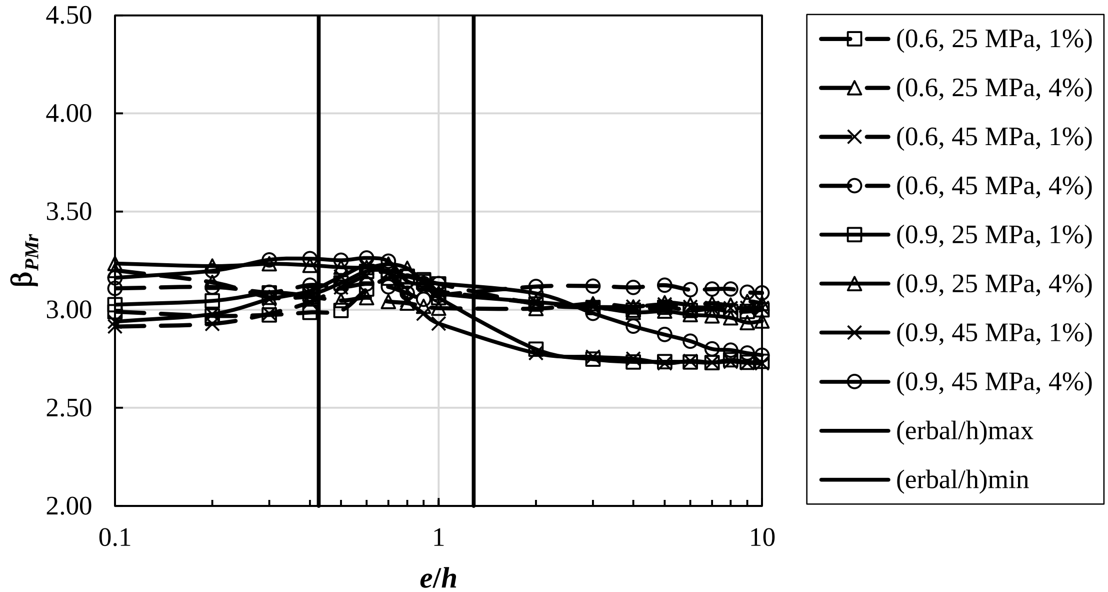


(a)

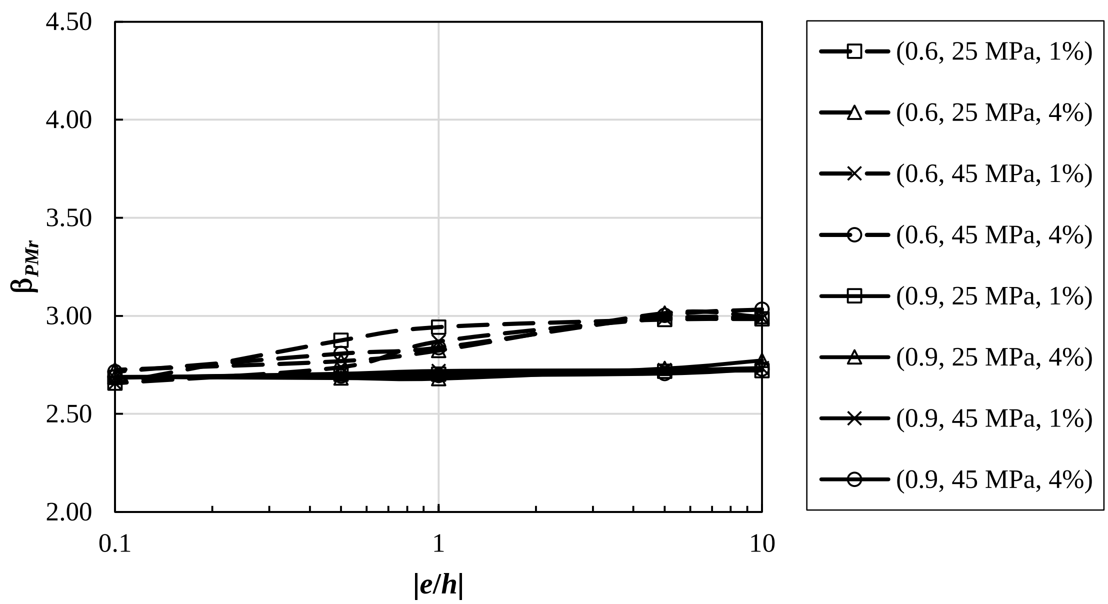


(b)

Figure C.1: Reliability indices for combined moment and axial force using strength reduction factors in ACI 318-14, β_{PMu}, for Column Section 1 and $L/D = 1.5$: (a) $e/h > 0$; (b) $e/h < 0$

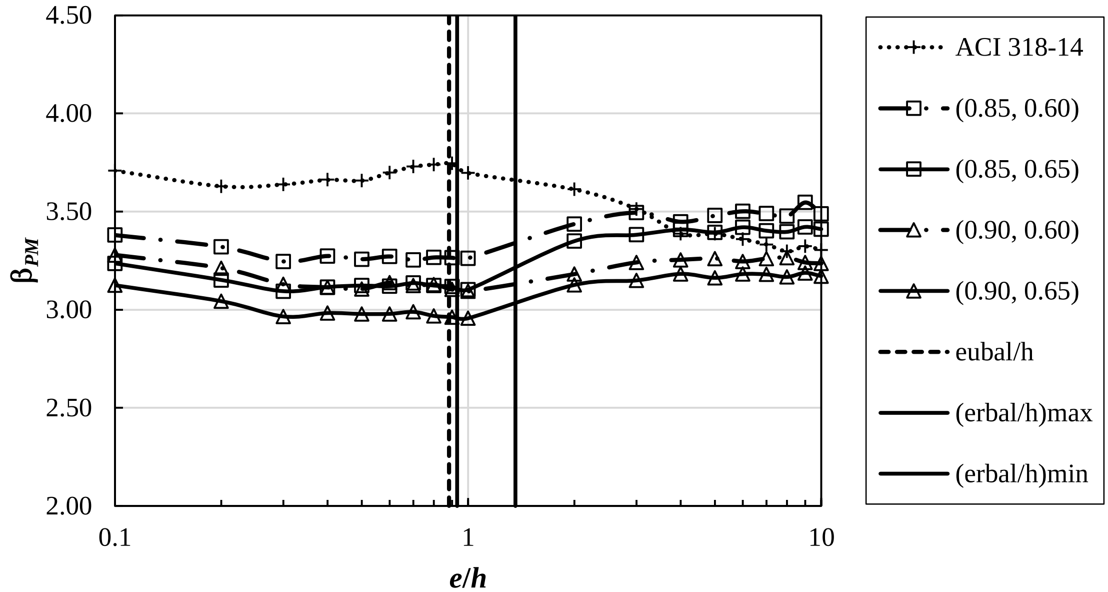


(a)



(b)

Figure C.2: Reliability indices for combined moment and axial force, β_{PMr}, corresponding to $\phi_s = 0.90$ and $\phi_c = 0.60$, for Column Section 1 and $L/D = 1.5$: (a) $e/h > 0$; (b) $e/h < 0$



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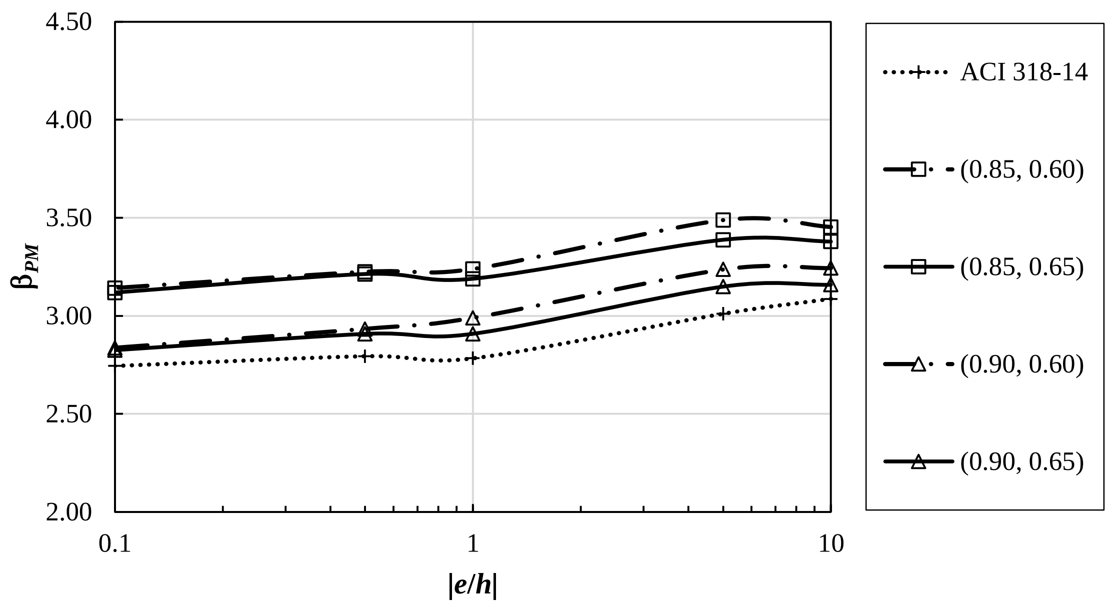
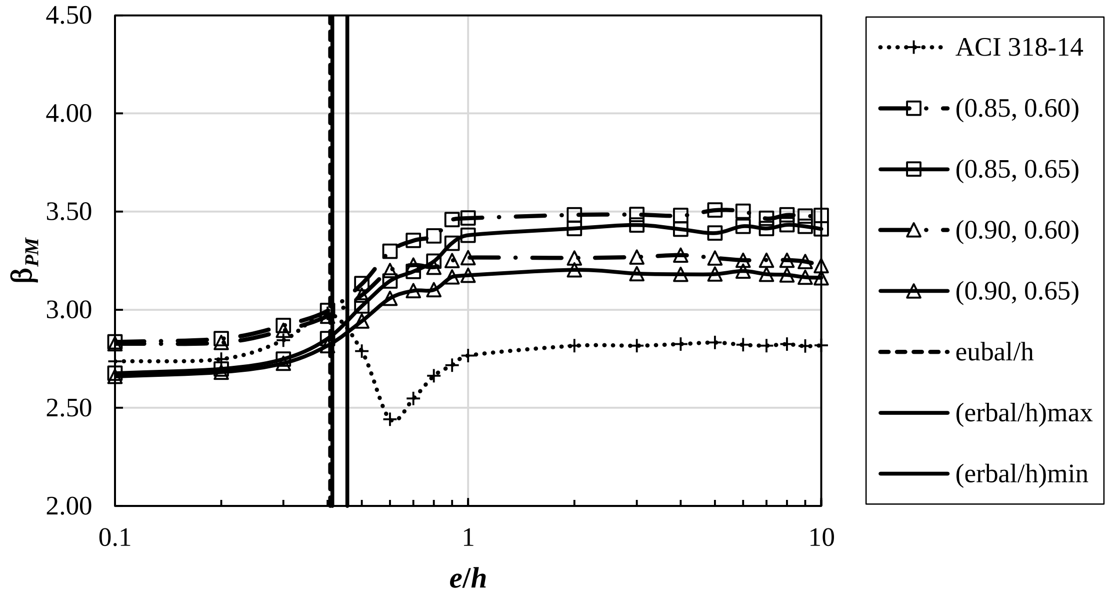


(b)

Figure C.3: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 1, Property Combination 2, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$



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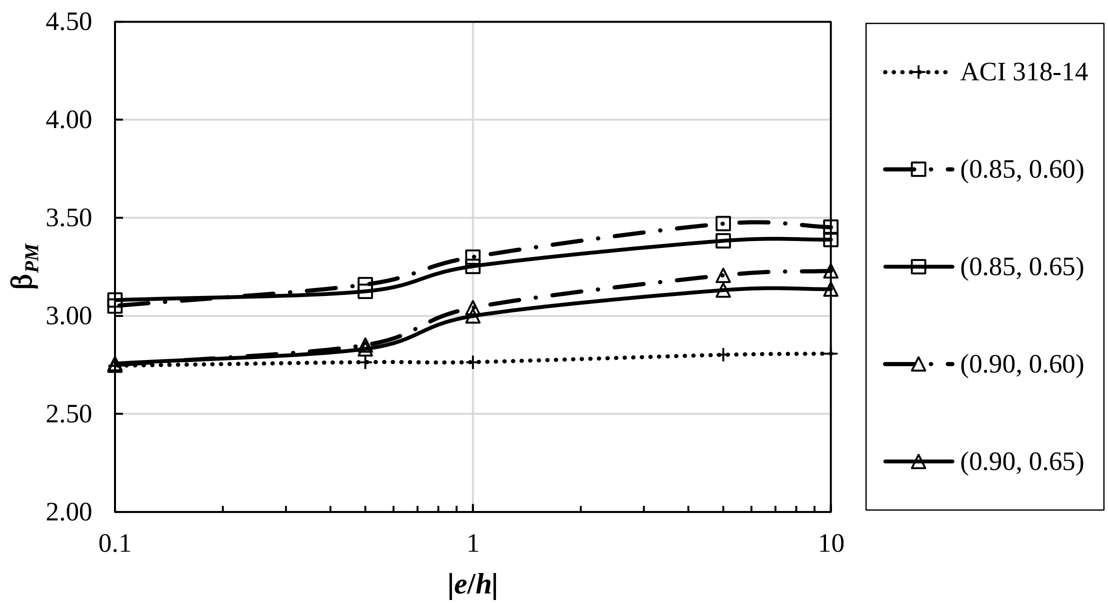
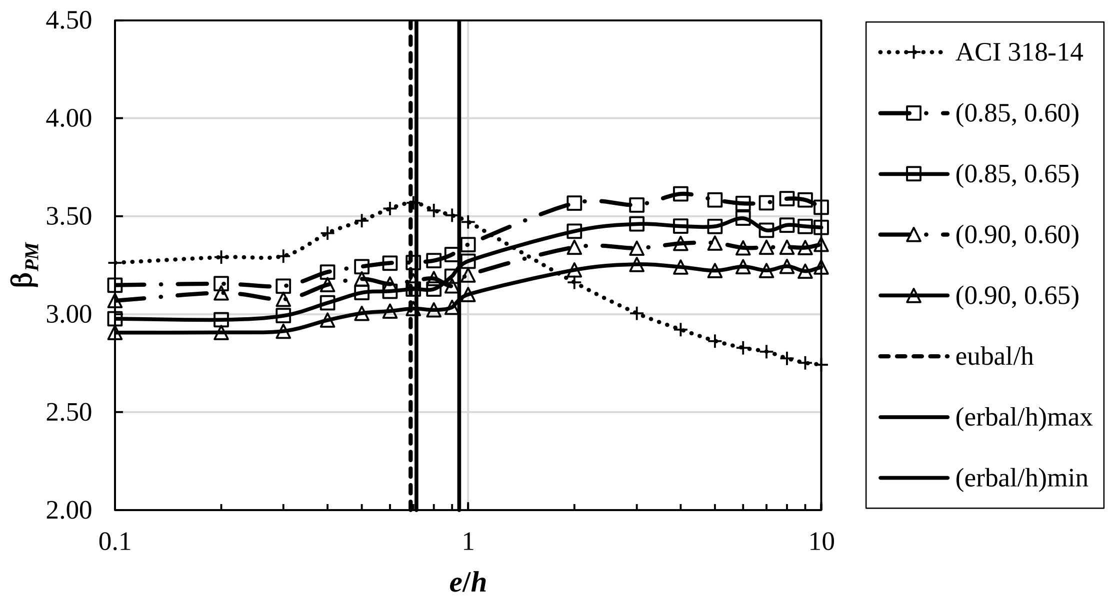


(b)

Figure C.4: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 1, Property Combination 3, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$



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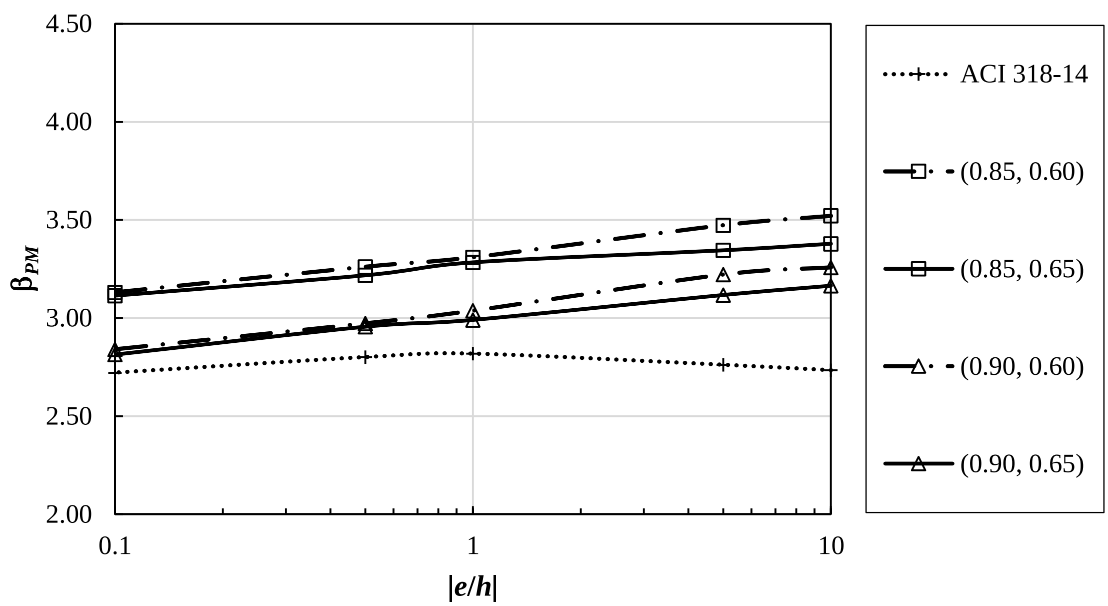
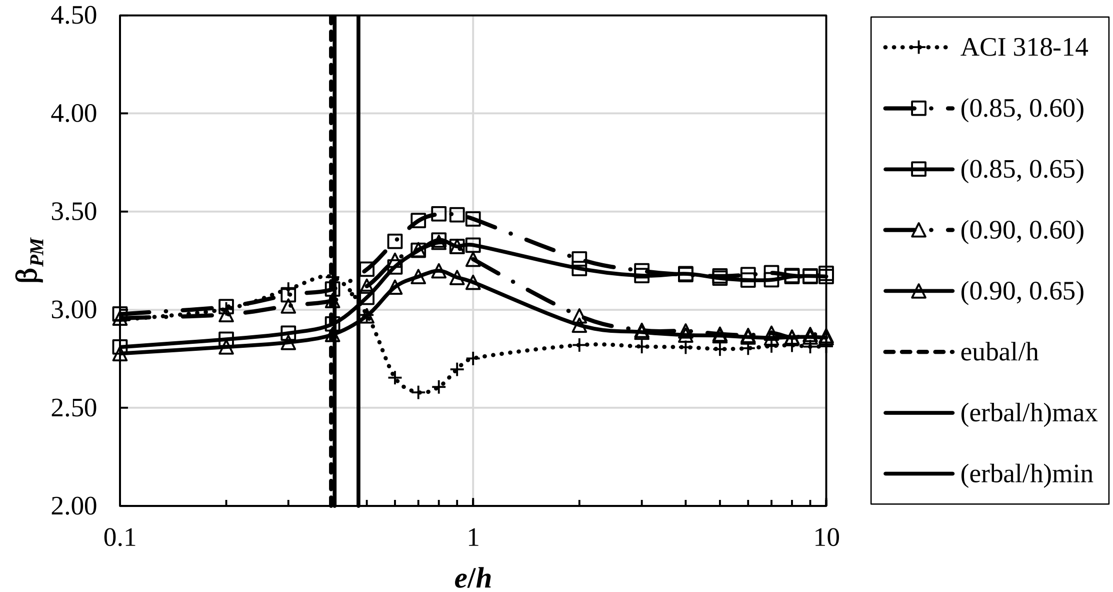
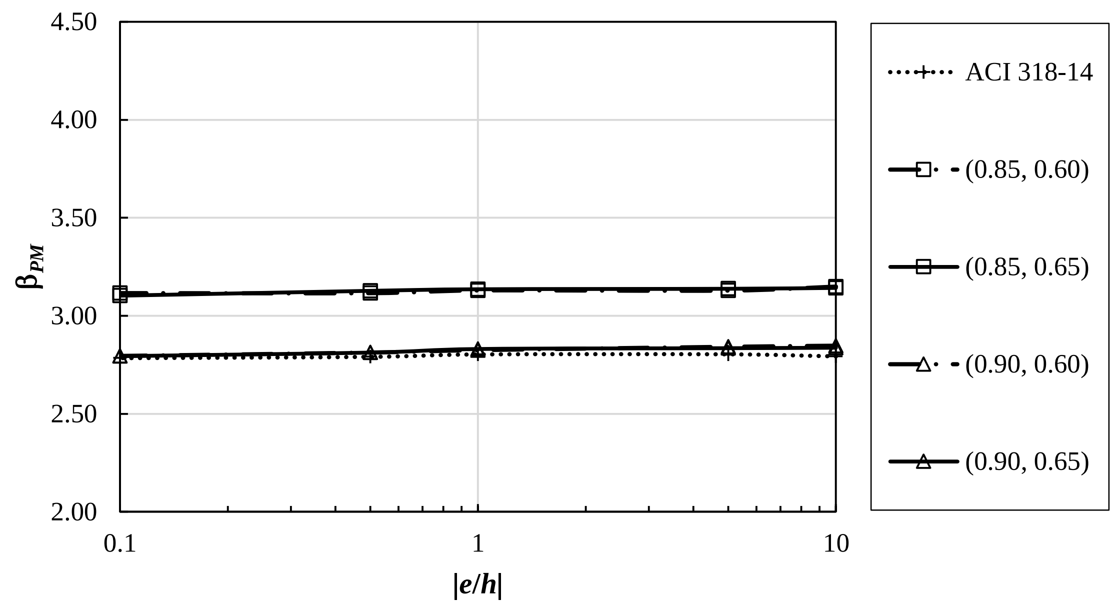


(b)

Figure C.5: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 1, Property Combination 4, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

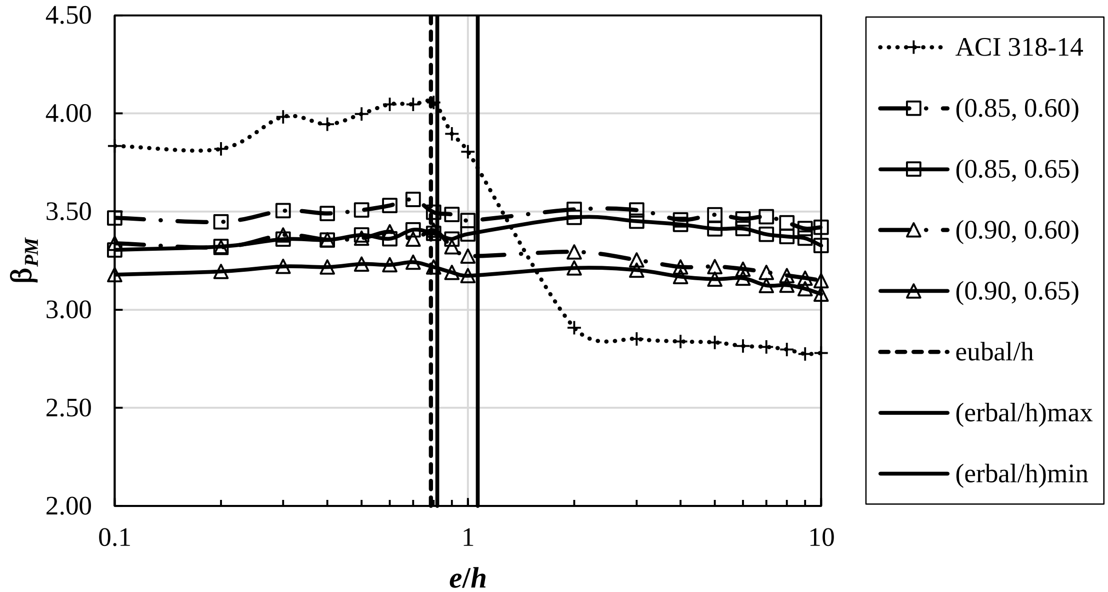


(a)

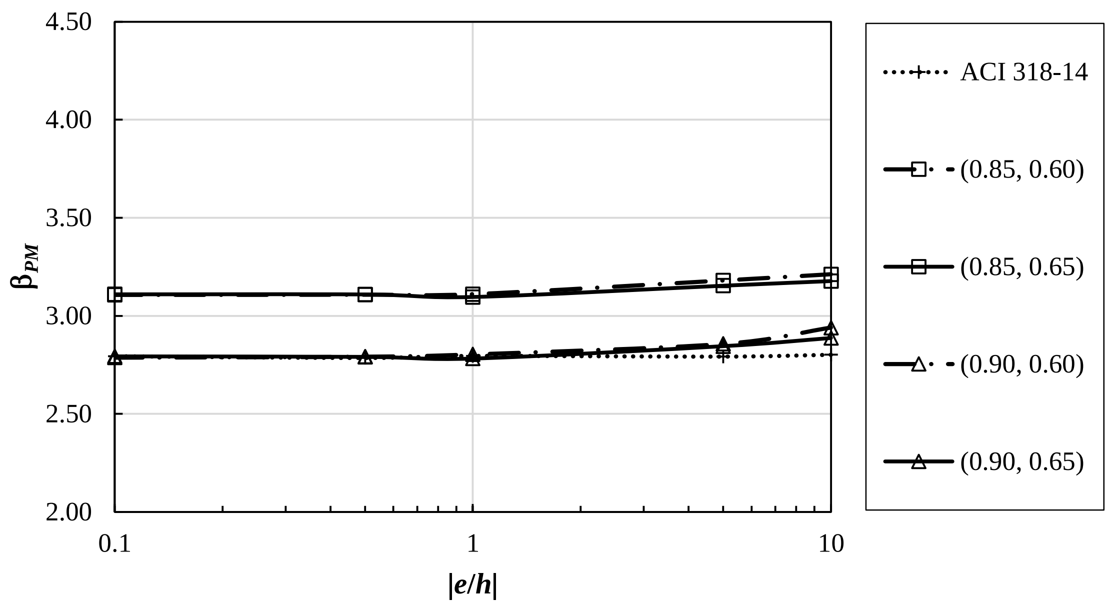


(b)

Figure C.6: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 1, Property Combination 5, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

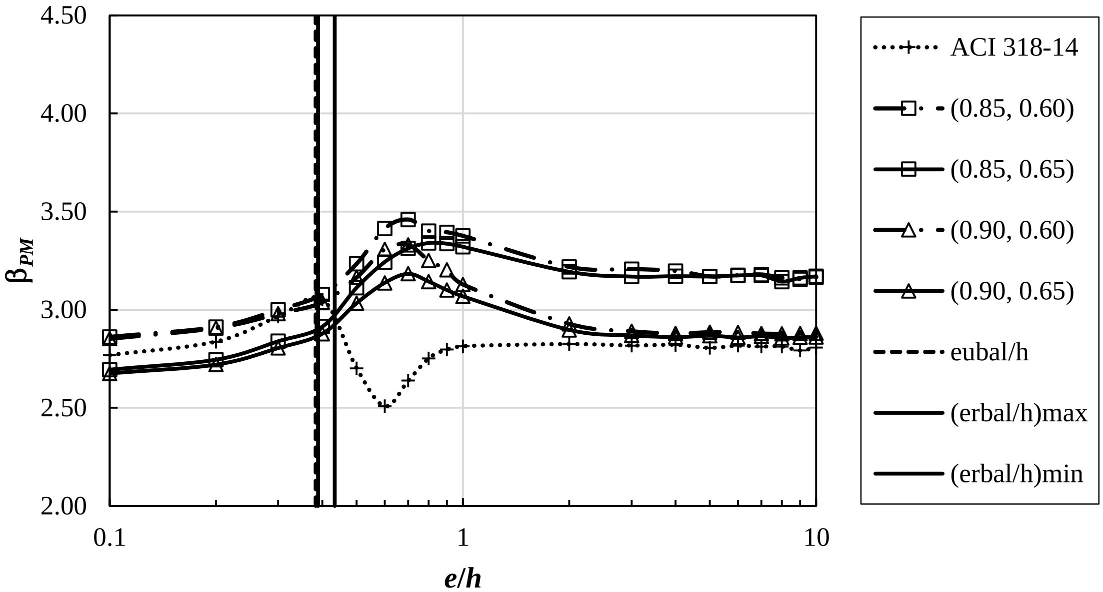


(a)



(b)

Figure C.7: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 1, Property Combination 6, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$



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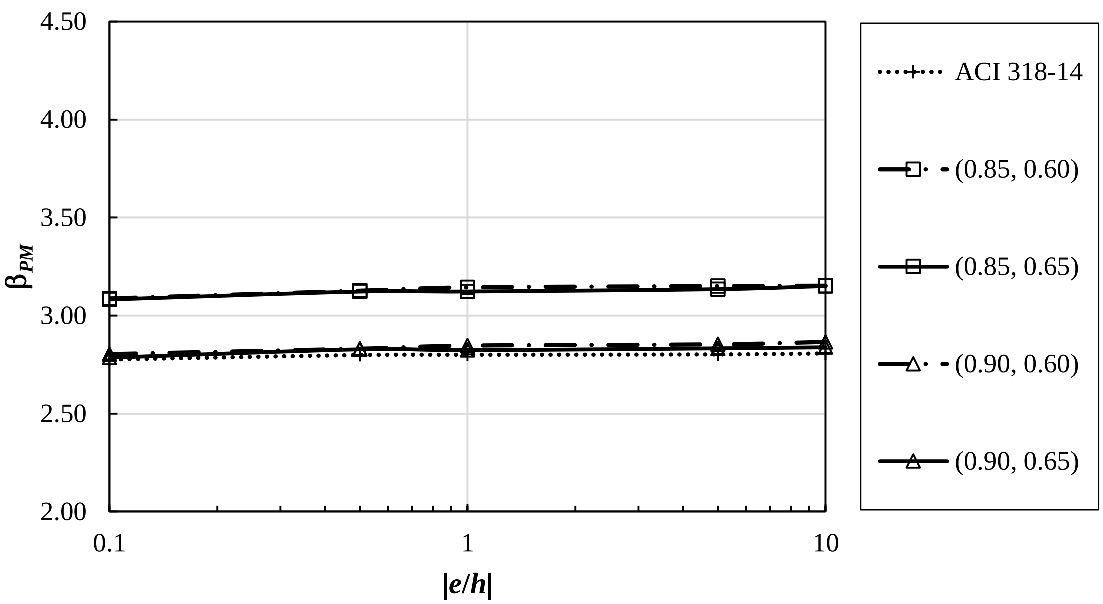
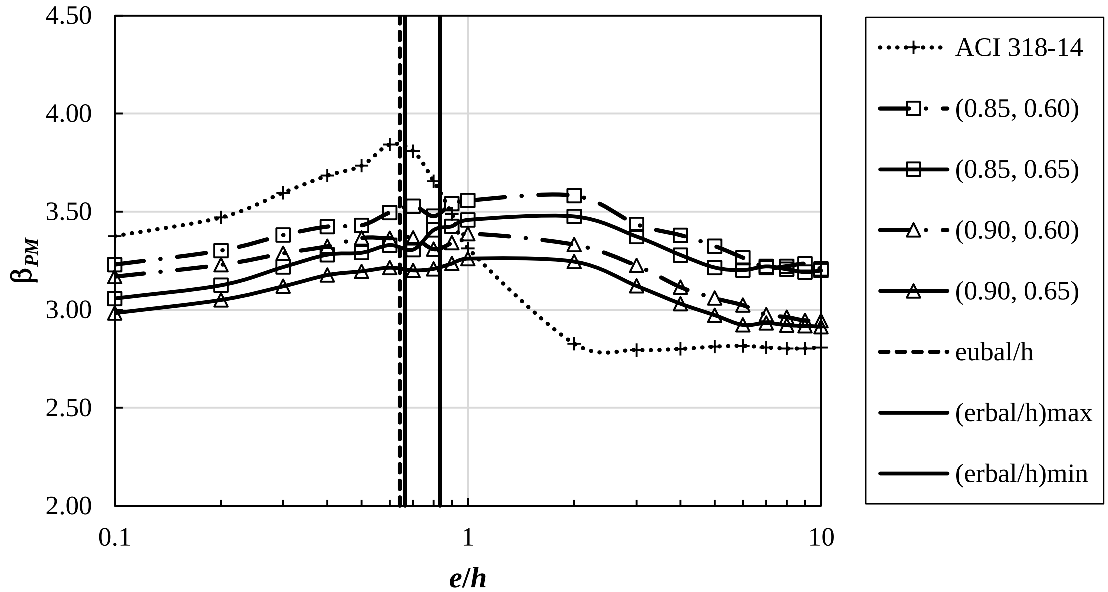
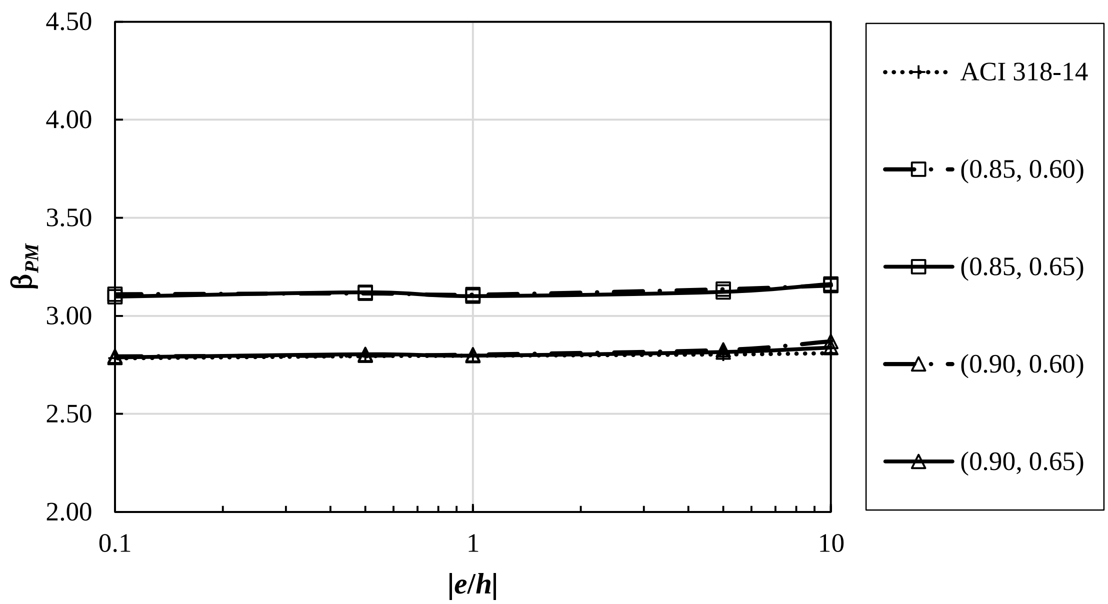


(b)

Figure C.8: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 1, Property Combination 7, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

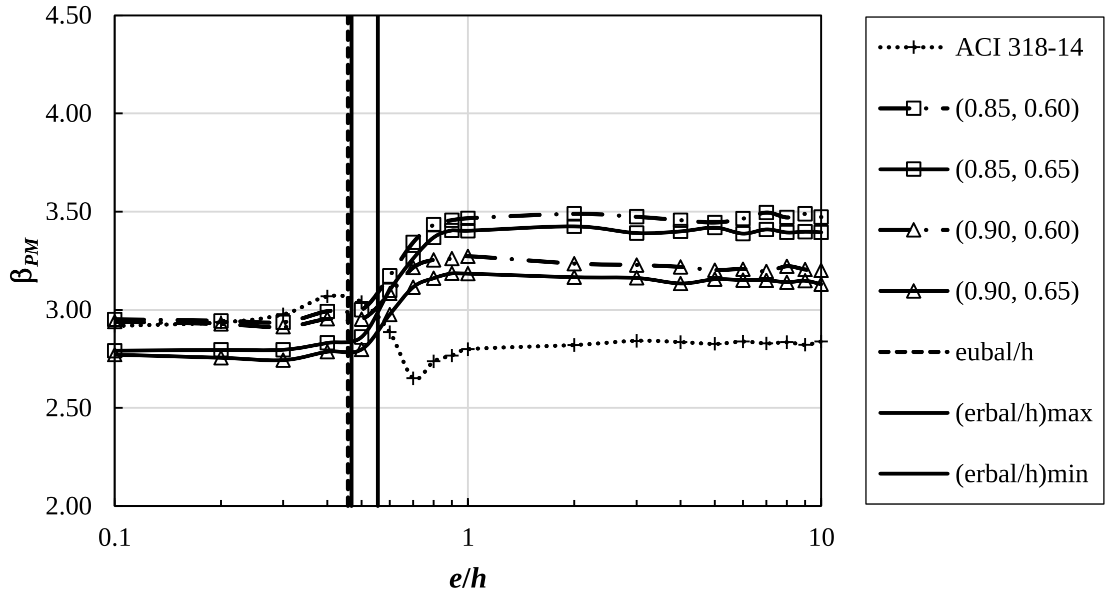


(a)

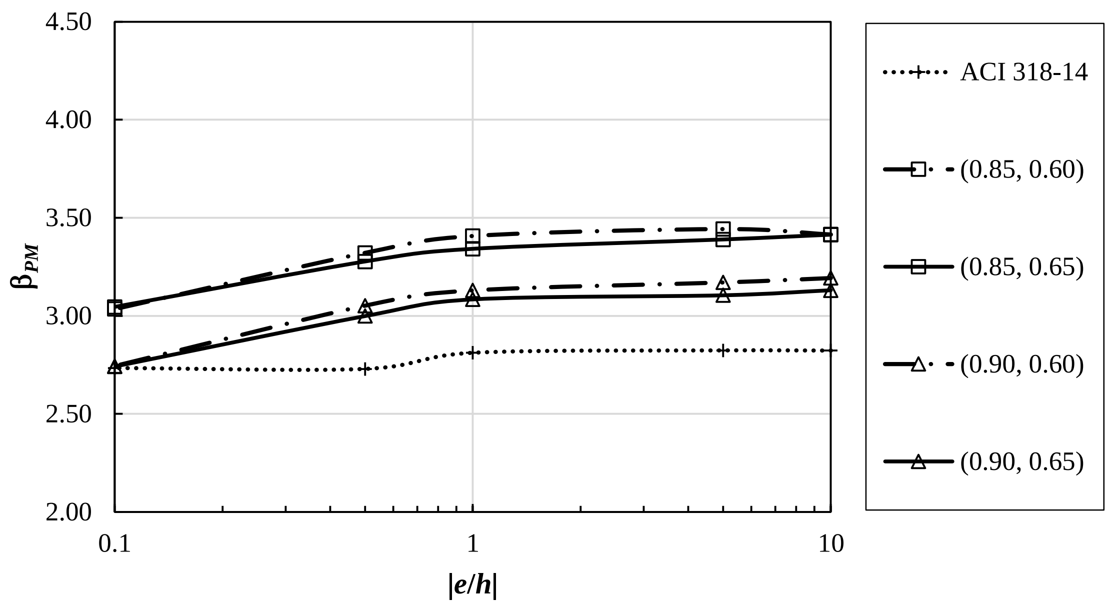


(b)

Figure C.9: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 1, Property Combination 8, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

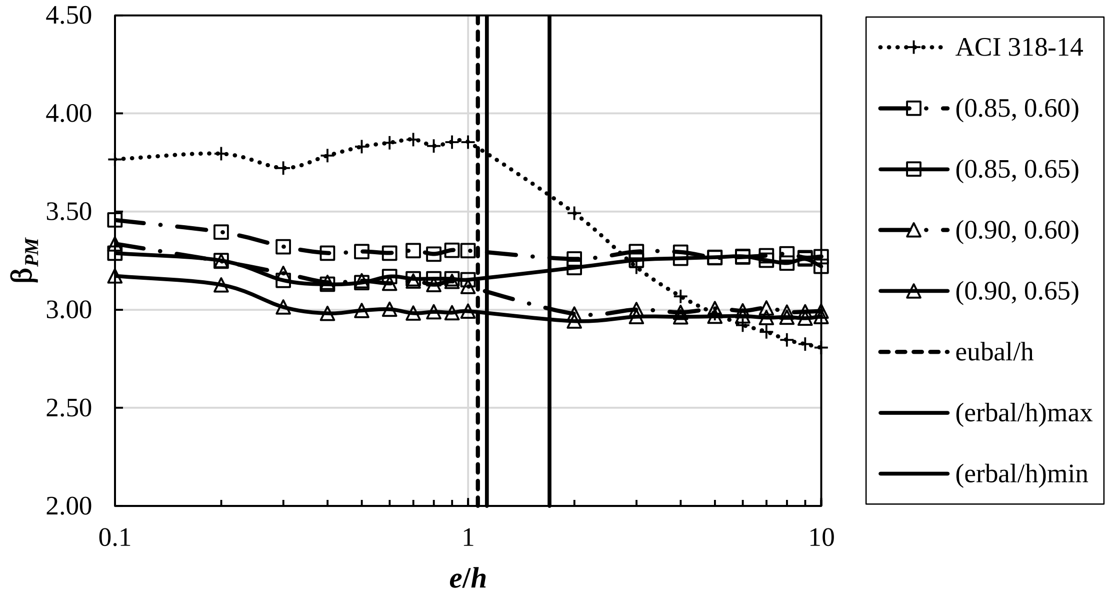


(a)



(b)

Figure C.10: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 2, Property Combination 1, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$



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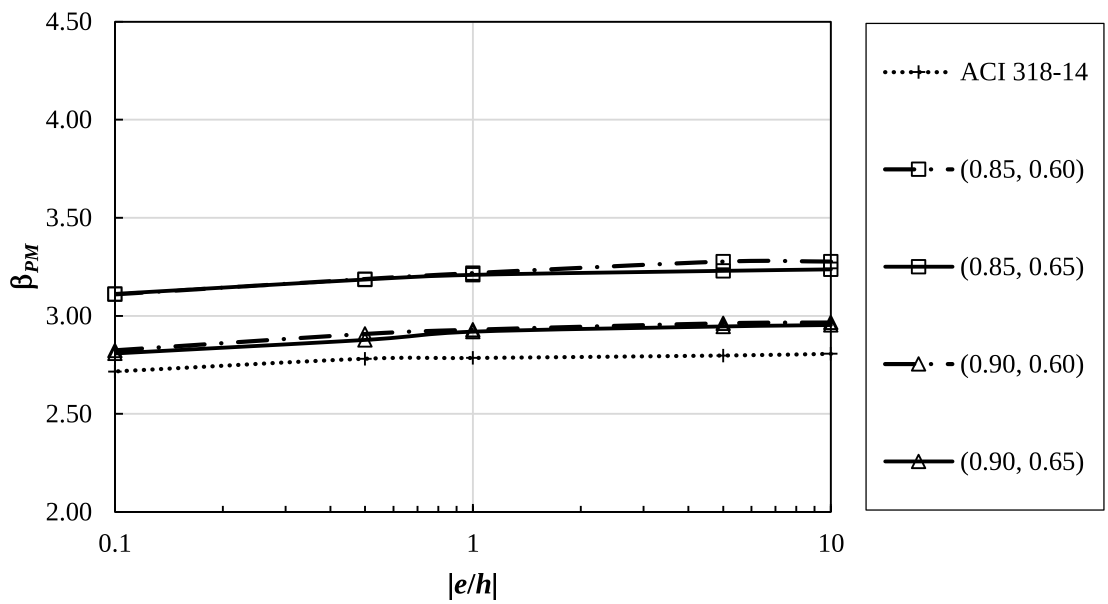
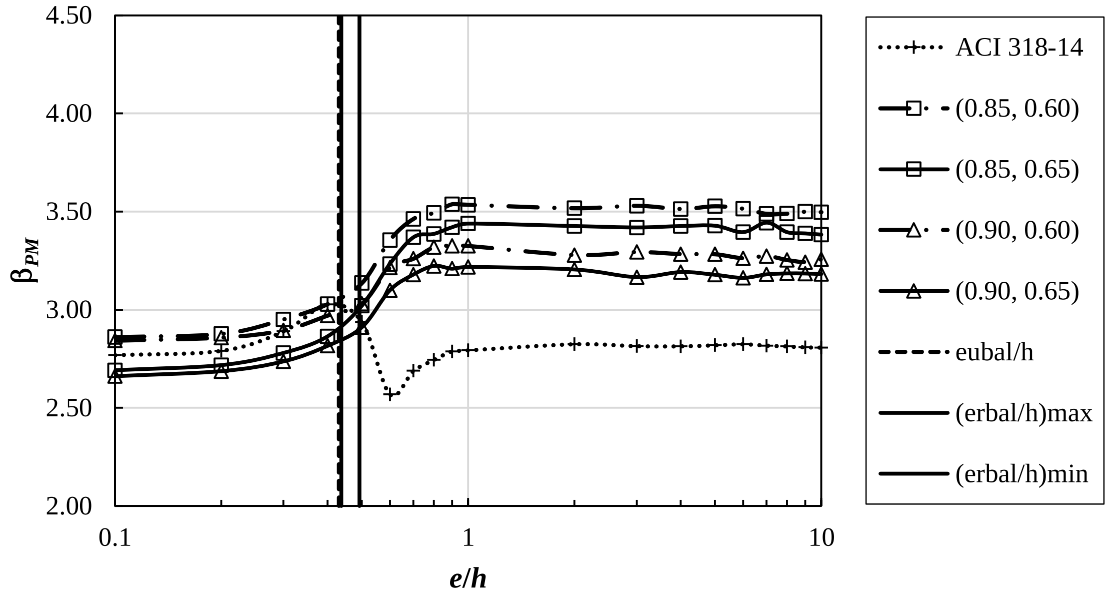
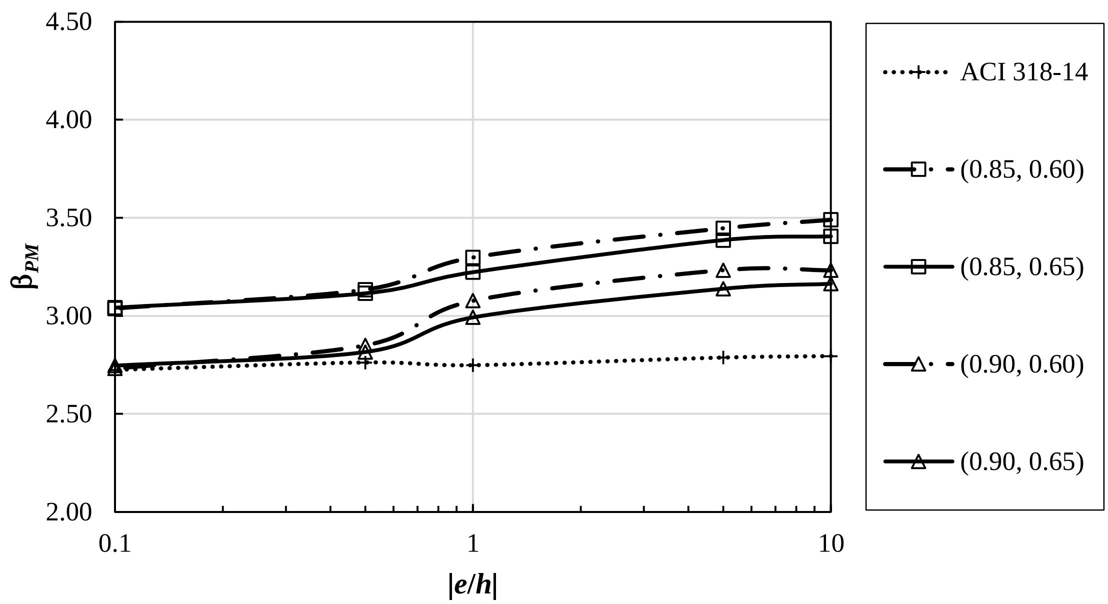


(b)

Figure C.11: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 2, Property Combination 2, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

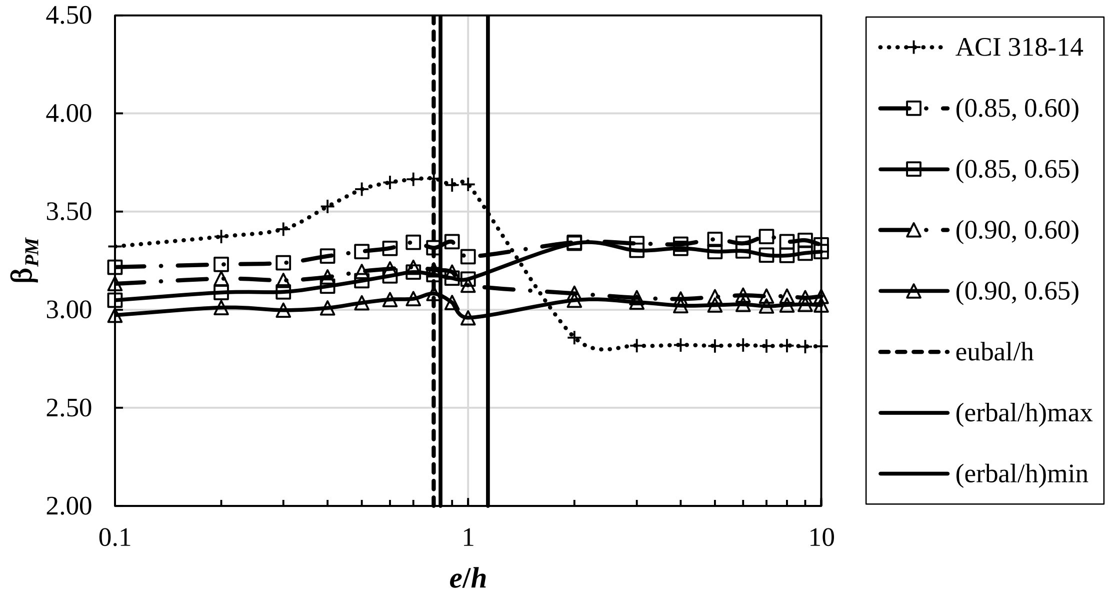


(a)



(b)

Figure C.12: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 2, Property Combination 3, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$



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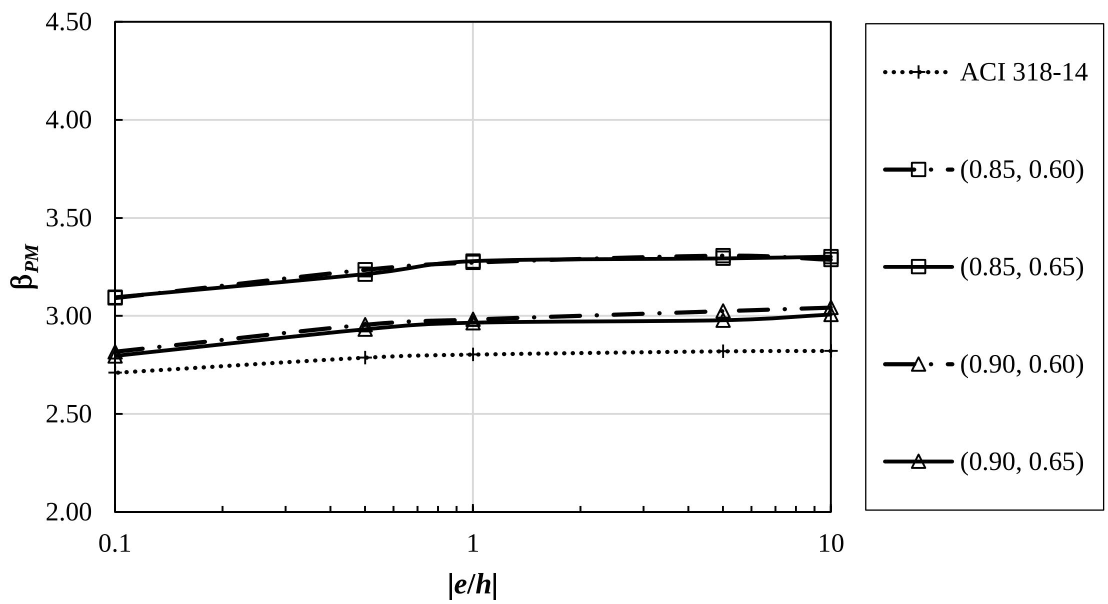
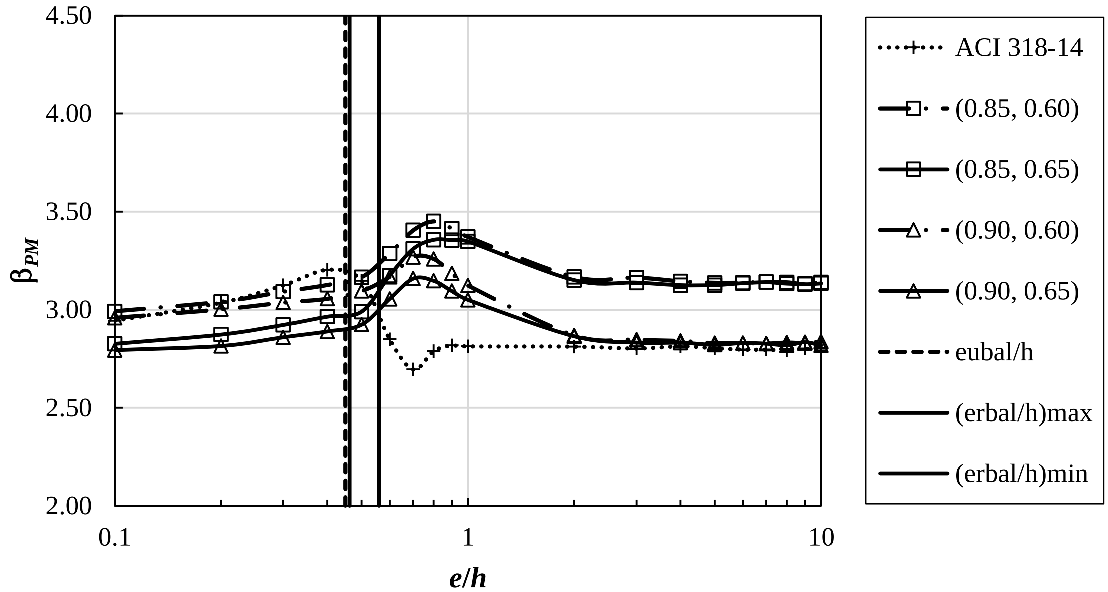


(b)

Figure C.13: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 2, Property Combination 4, and L/D = 0.5: (a) $e/h > 0$; (b) $e/h < 0$



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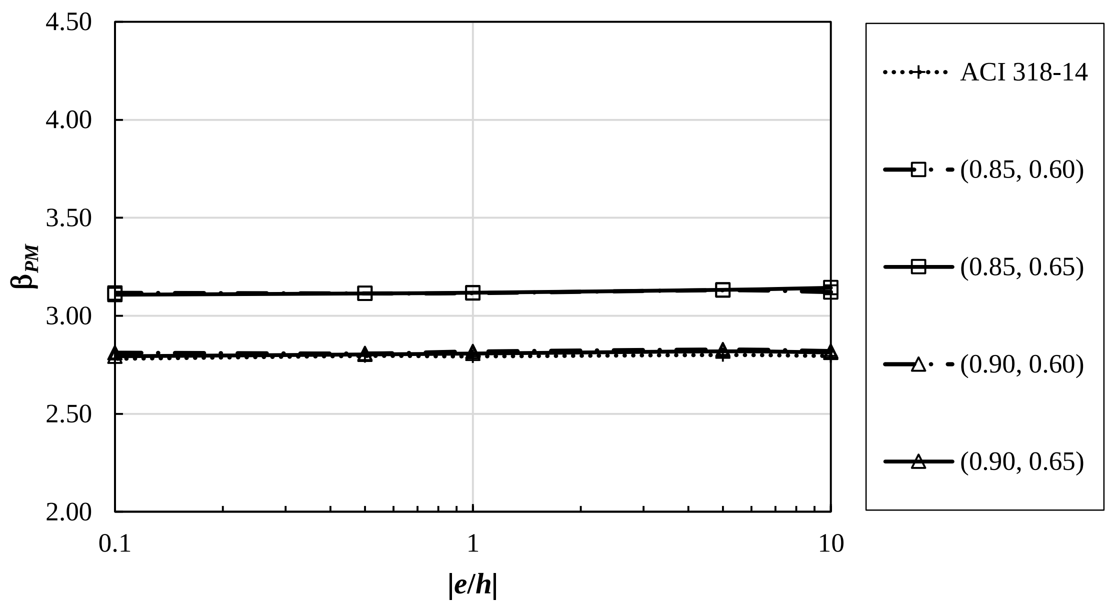
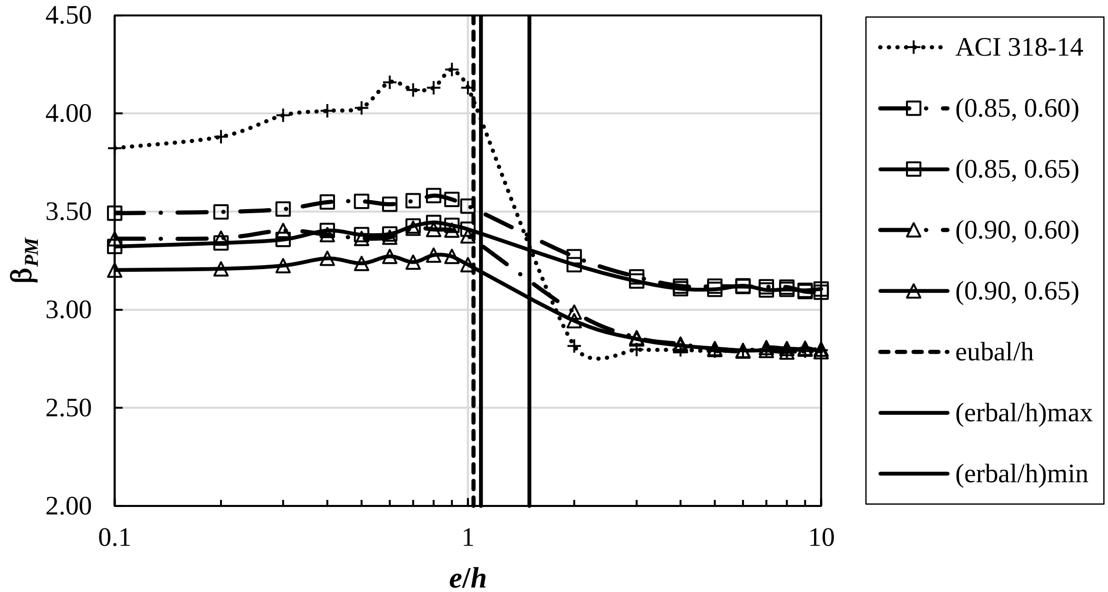


(b)

Figure C.14: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 2, Property Combination 5, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$



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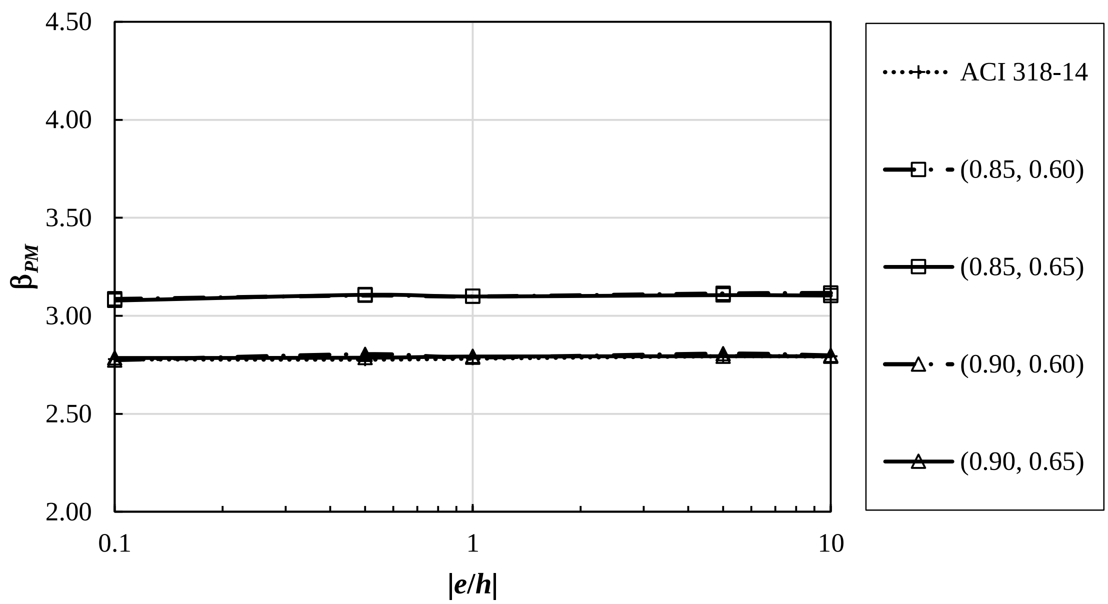
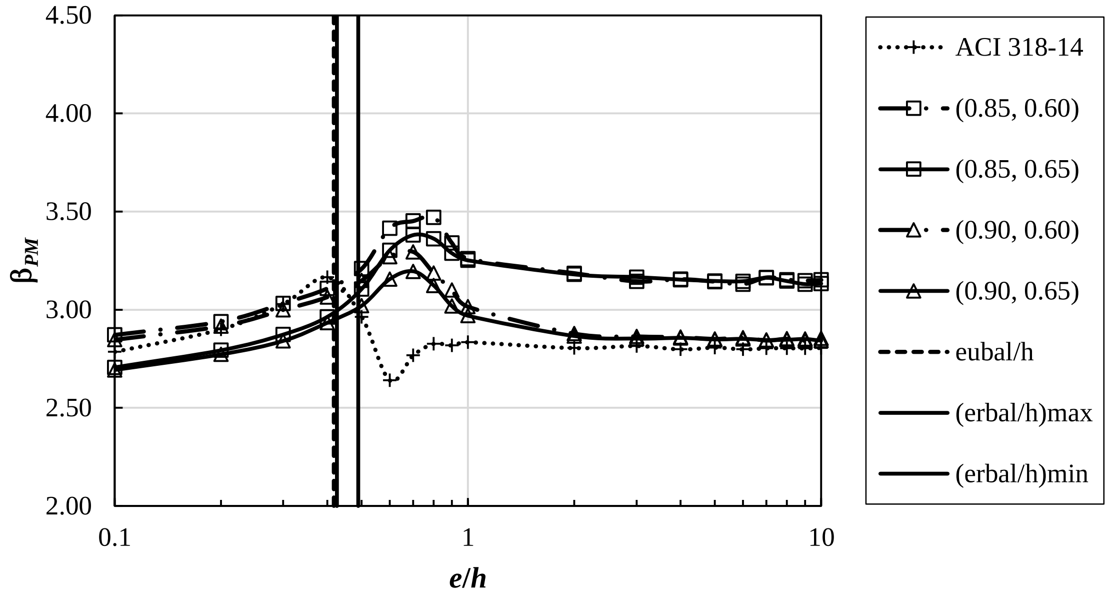
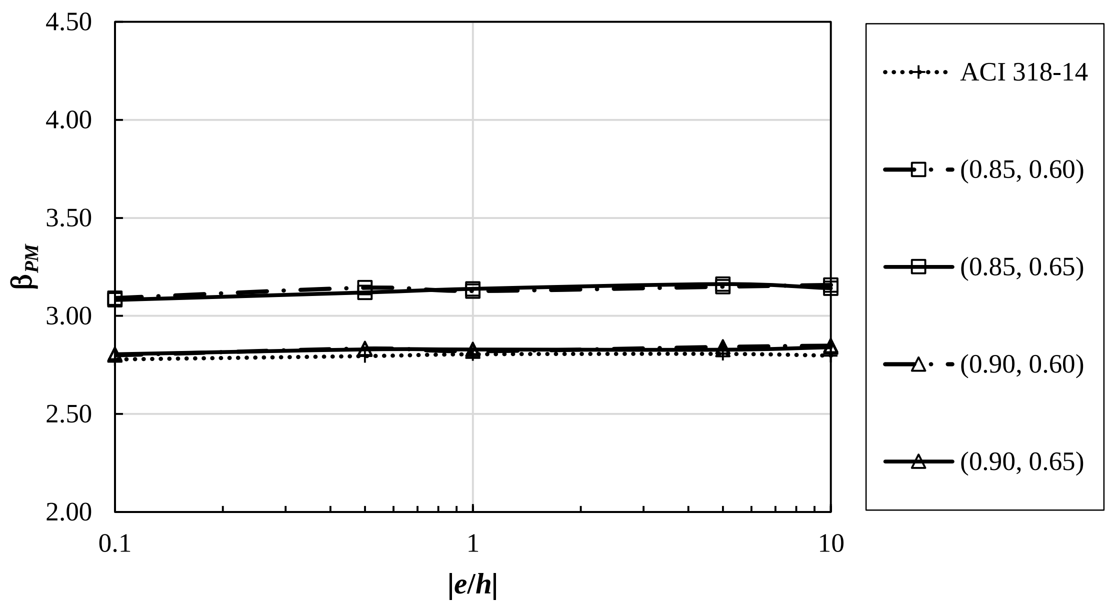


(b)

Figure C.15: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 2, Property Combination 6, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

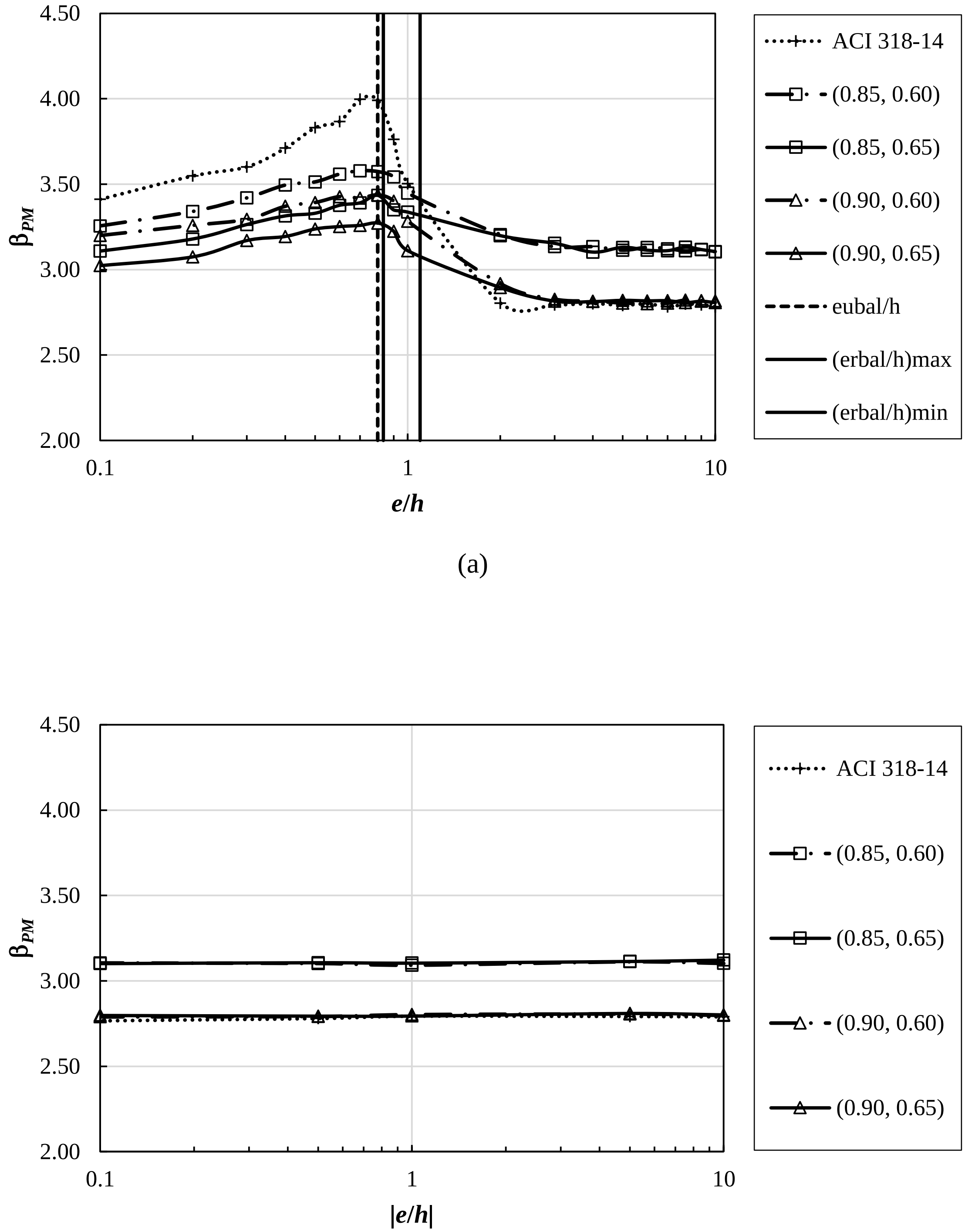


(a)



(b)

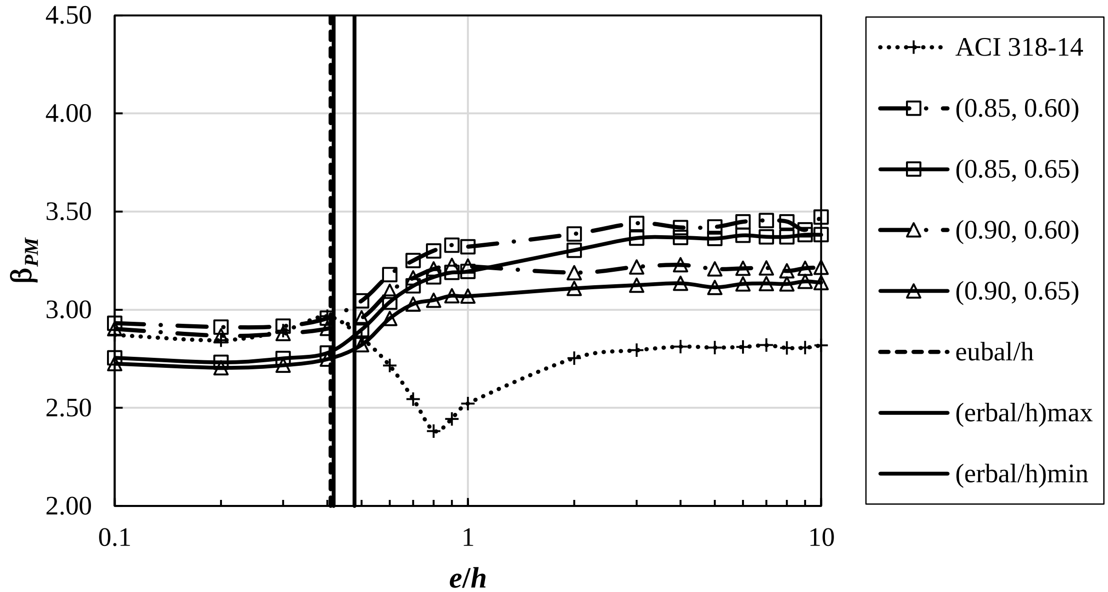
Figure C.16: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 2, Property Combination 7, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$



(a)

(b)

Figure C.17: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 2, Property Combination 8, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$



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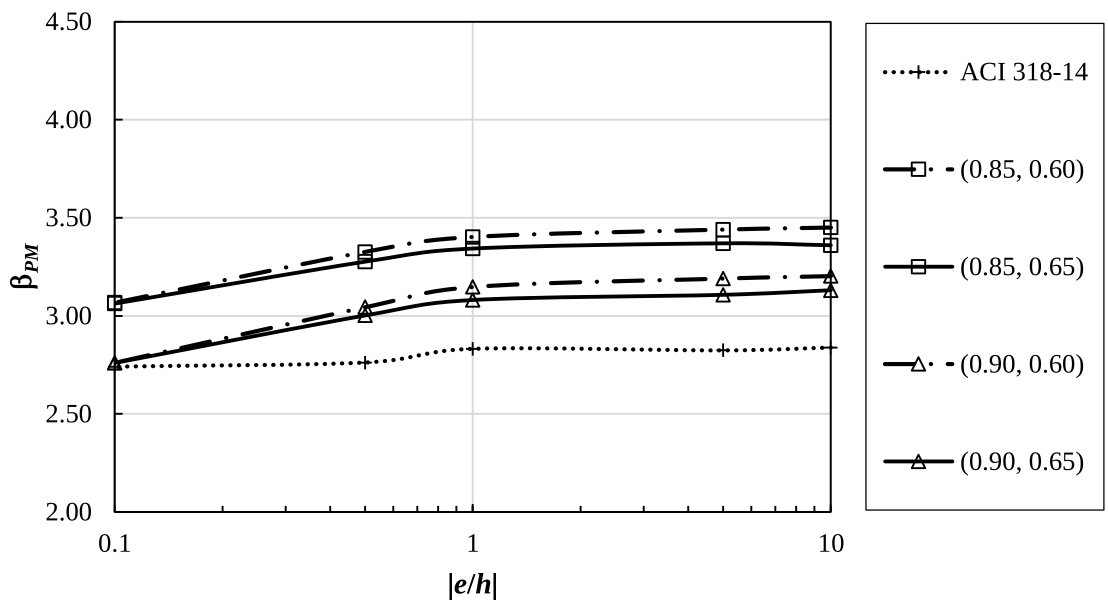
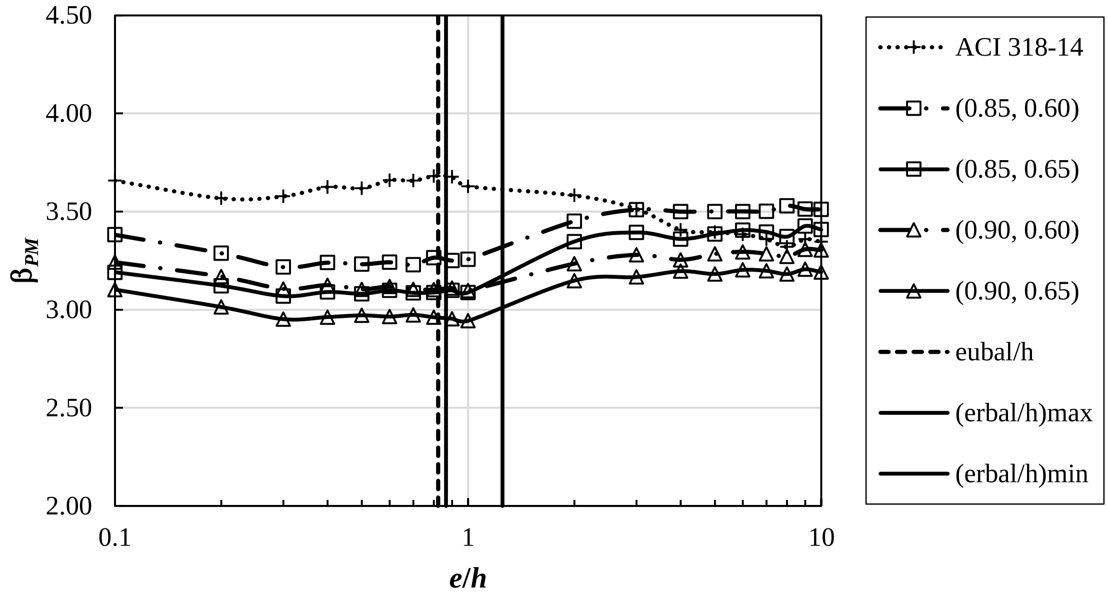
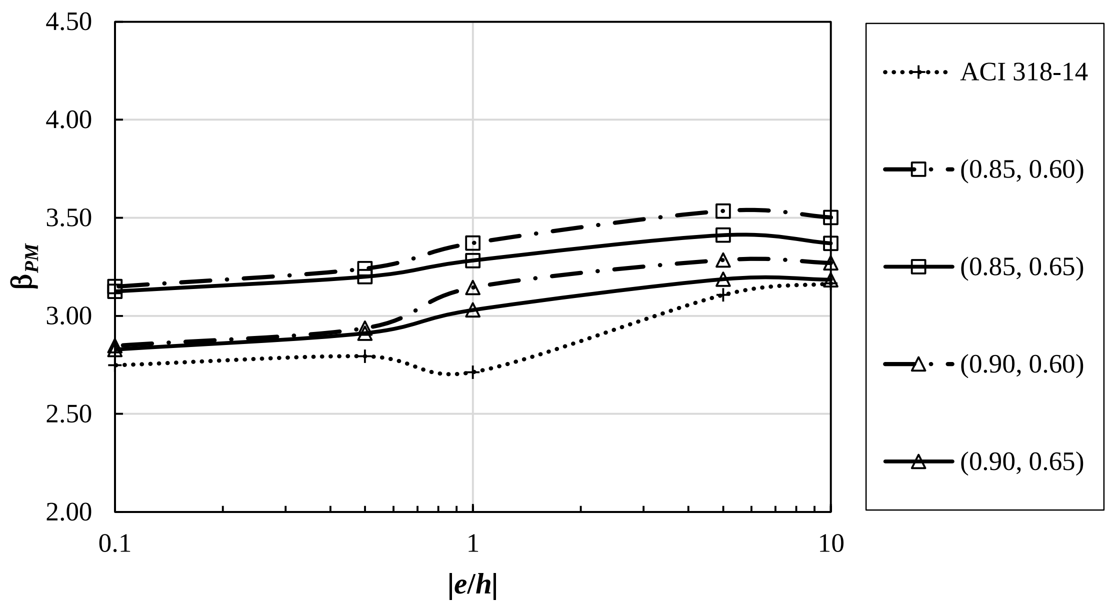


(b)

Figure C.18: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 3, Property Combination 1, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

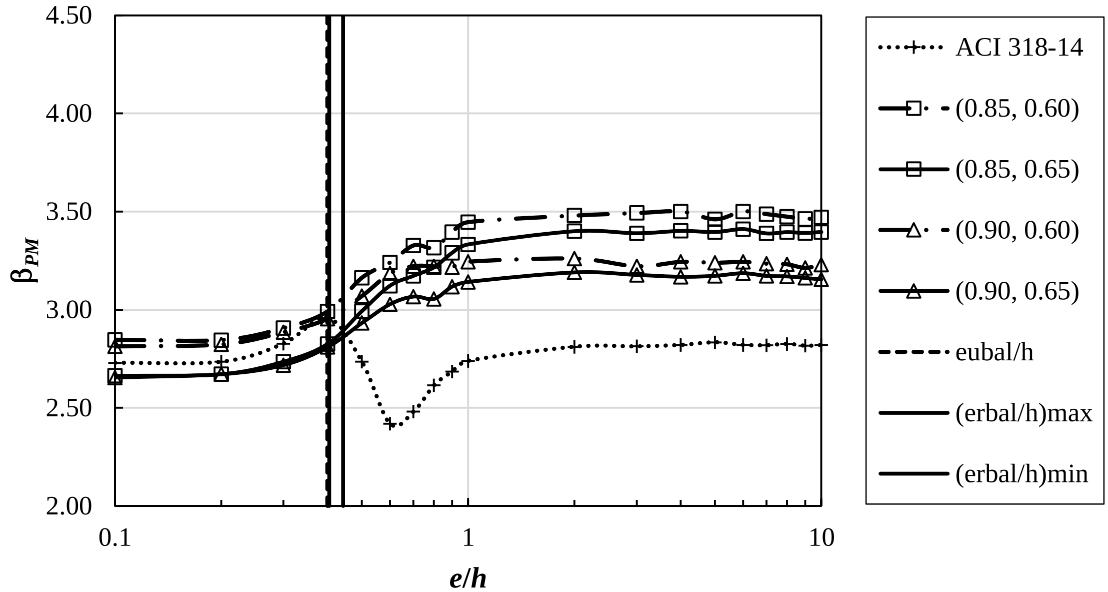


(a)

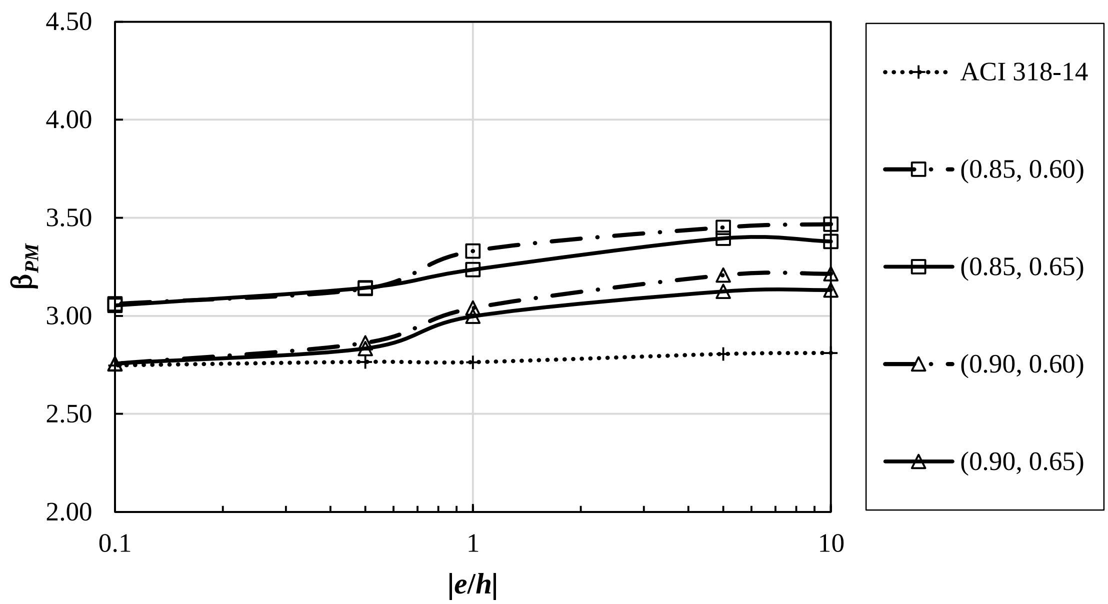


(b)

Figure C.19: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 3, Property Combination 2, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

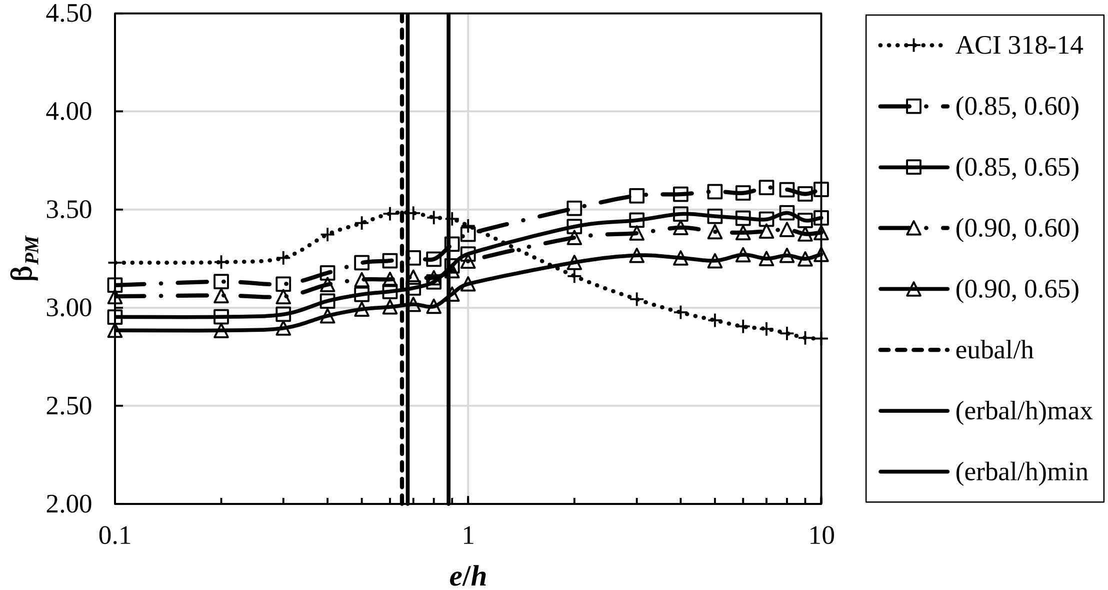


(a)

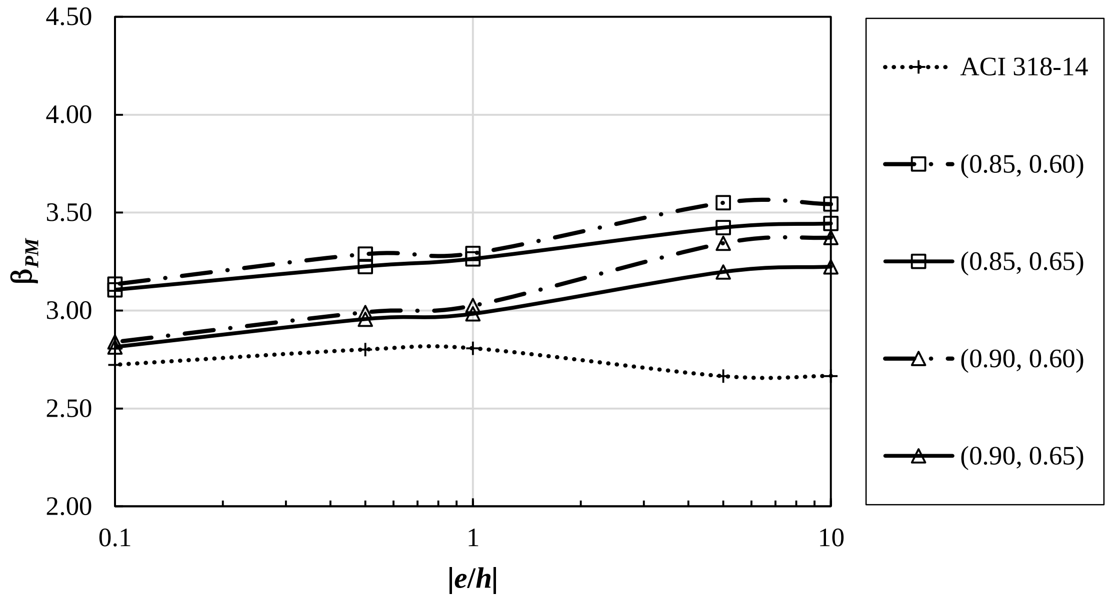


(b)

Figure C.20: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 3, Property Combination 3, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

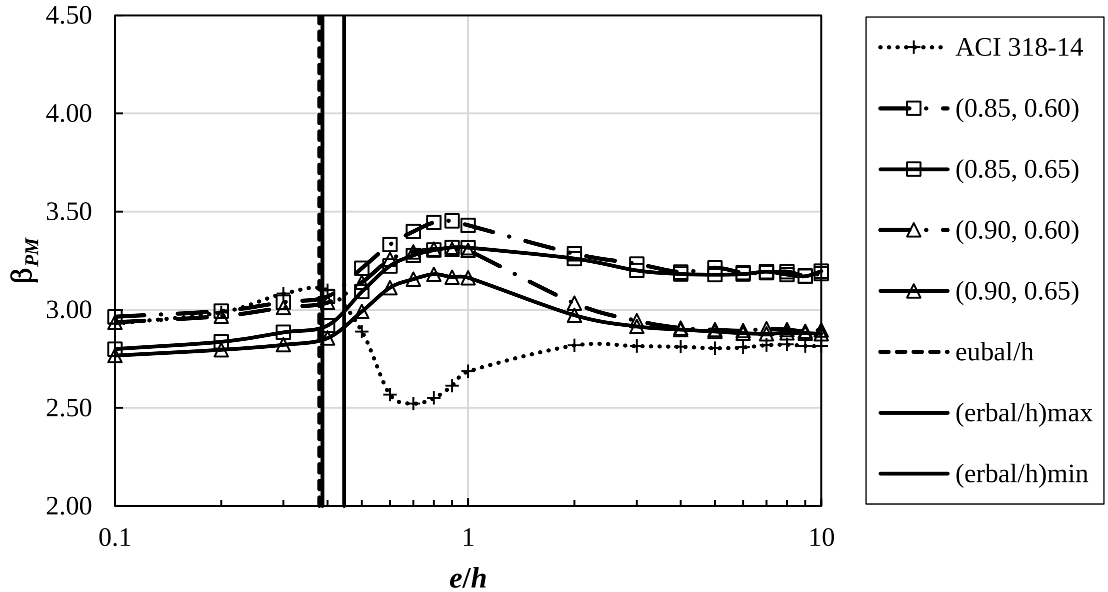


(a)

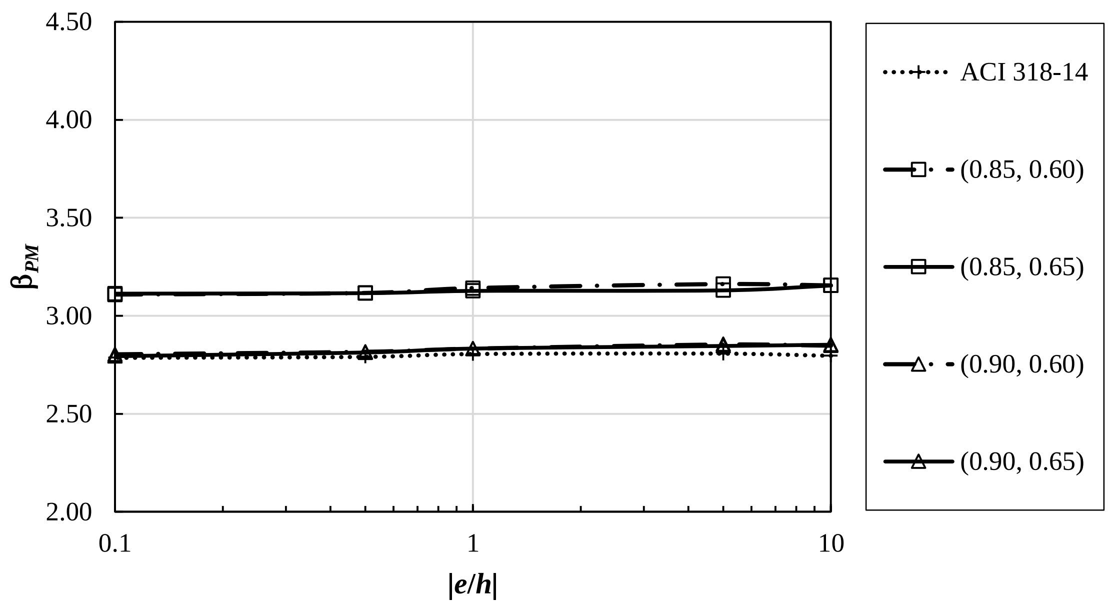


(b)

Figure C.21: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 3, Property Combination 4, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

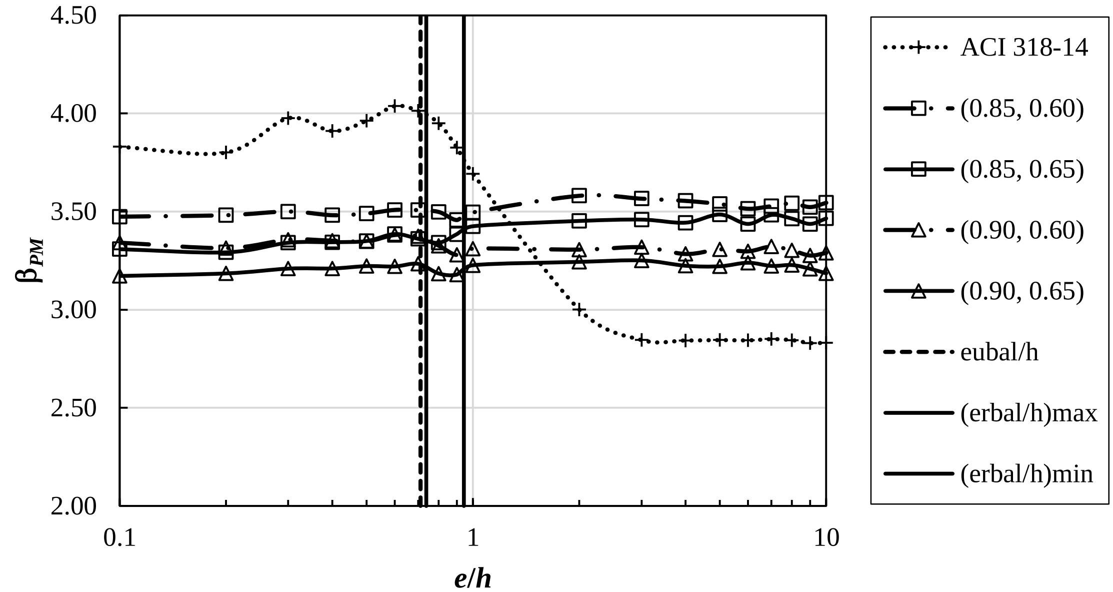


(a)

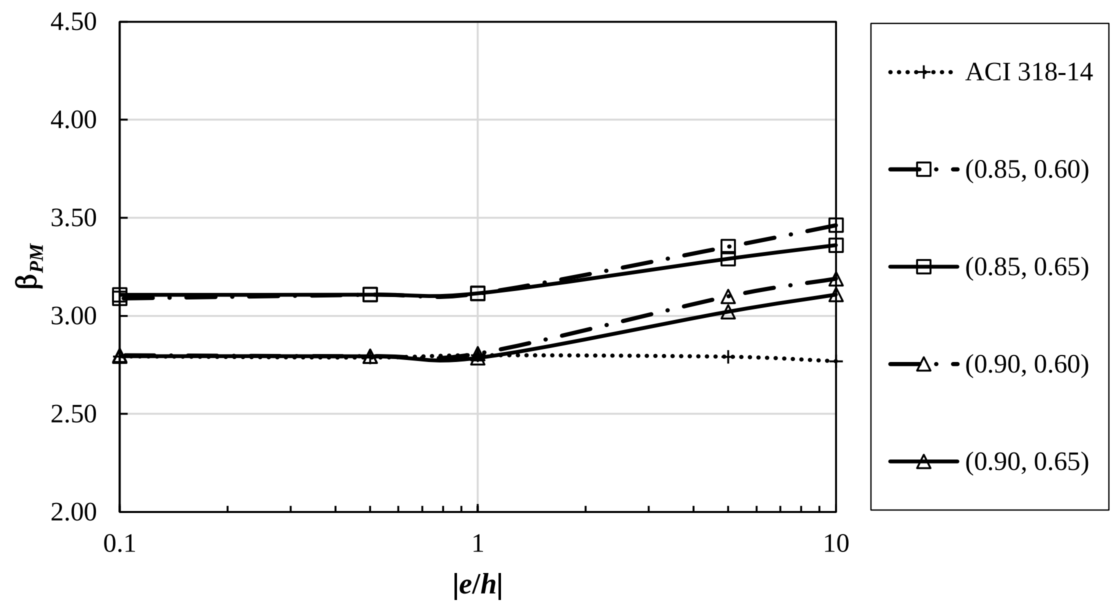


(b)

Figure C.22: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 3, Property Combination 5, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

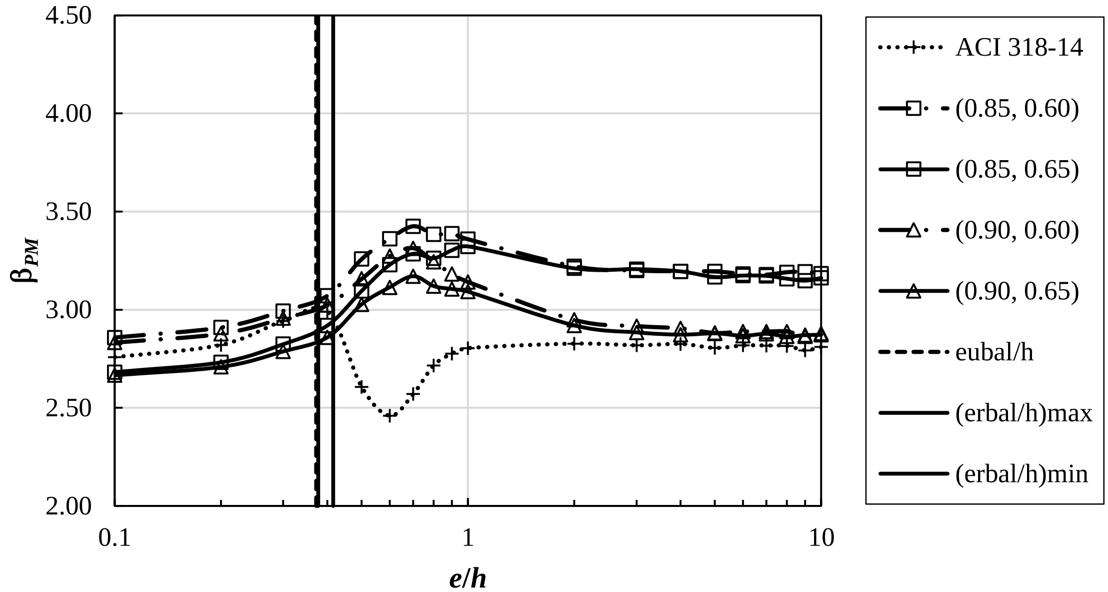


(a)

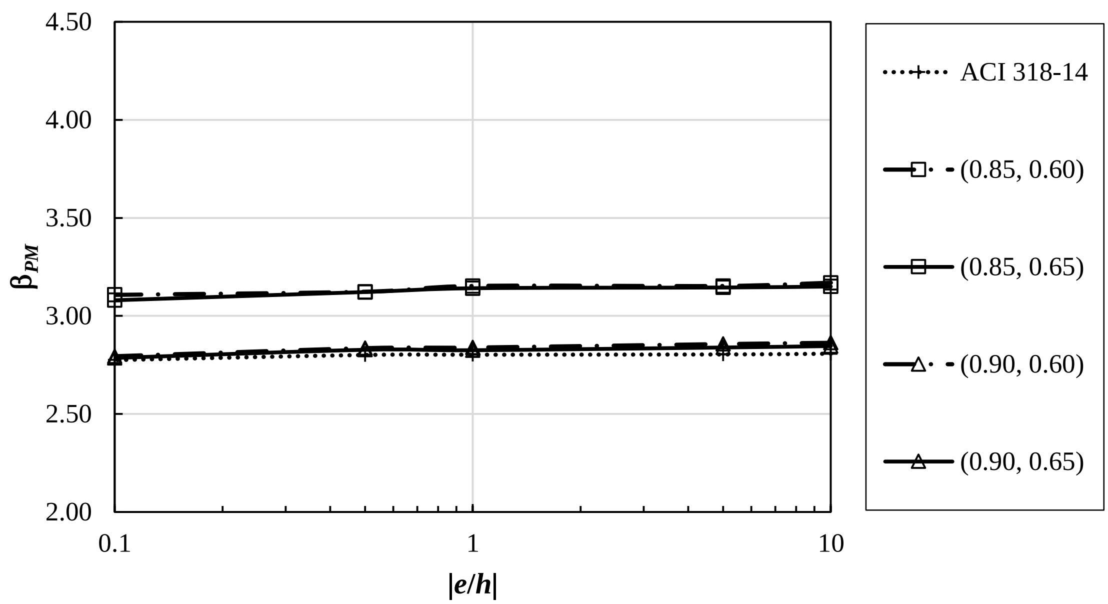


(b)

Figure C.23: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 3, Property Combination 6, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

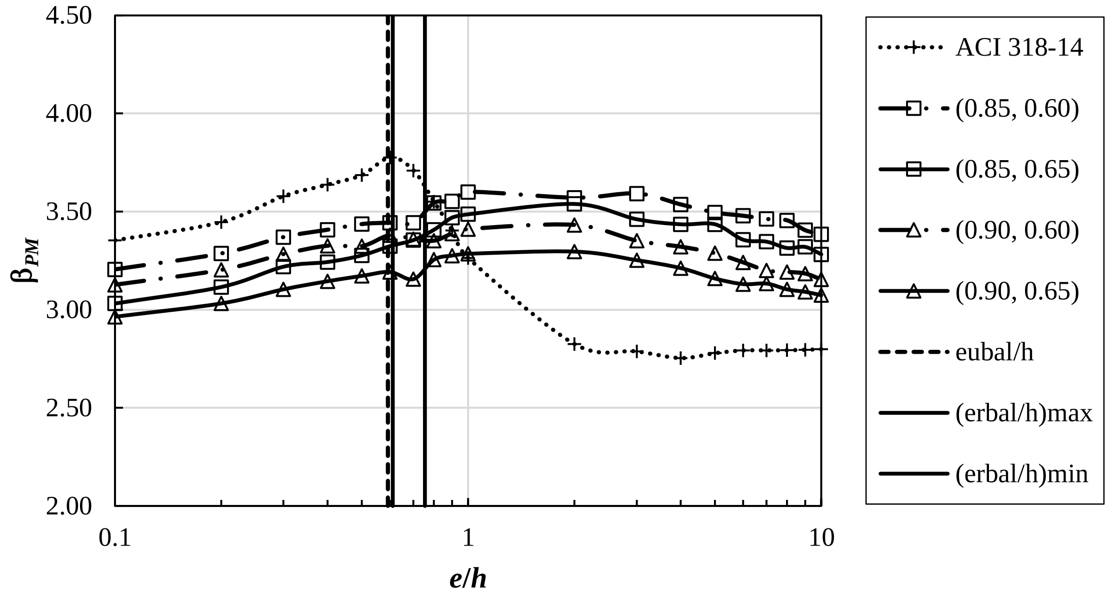


(a)

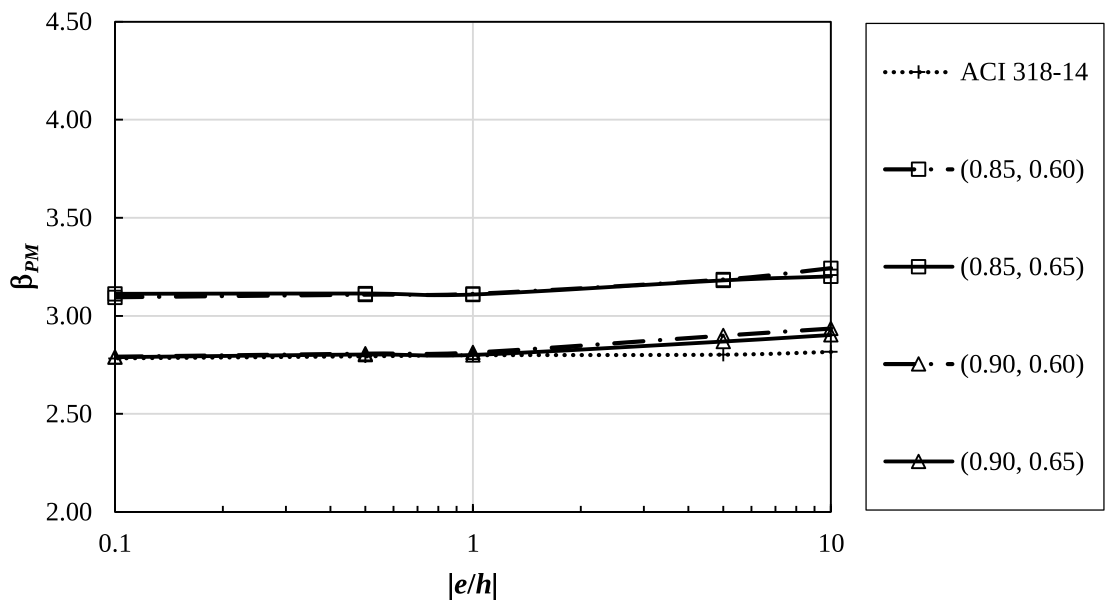


(b)

Figure C.24: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 3, Property Combination 7, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

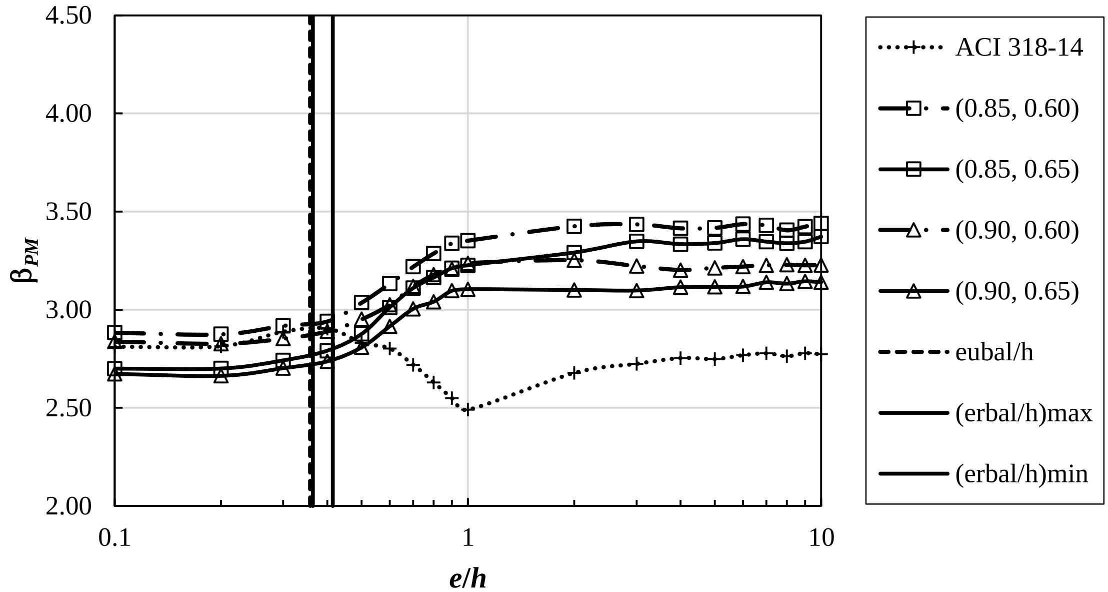


(a)

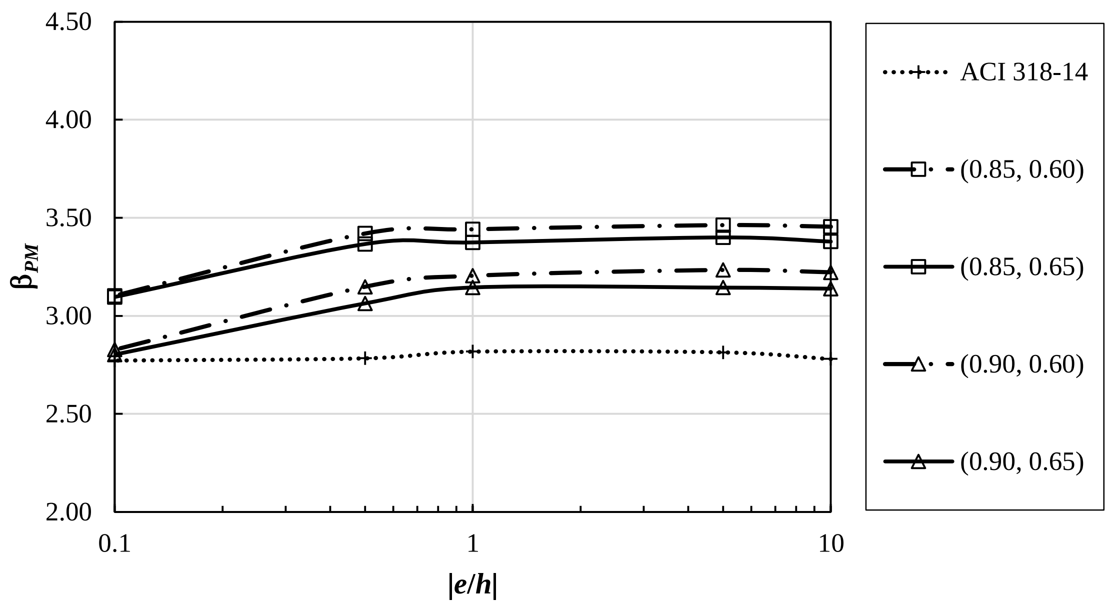


(b)

Figure C.25: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 3, Property Combination 8, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

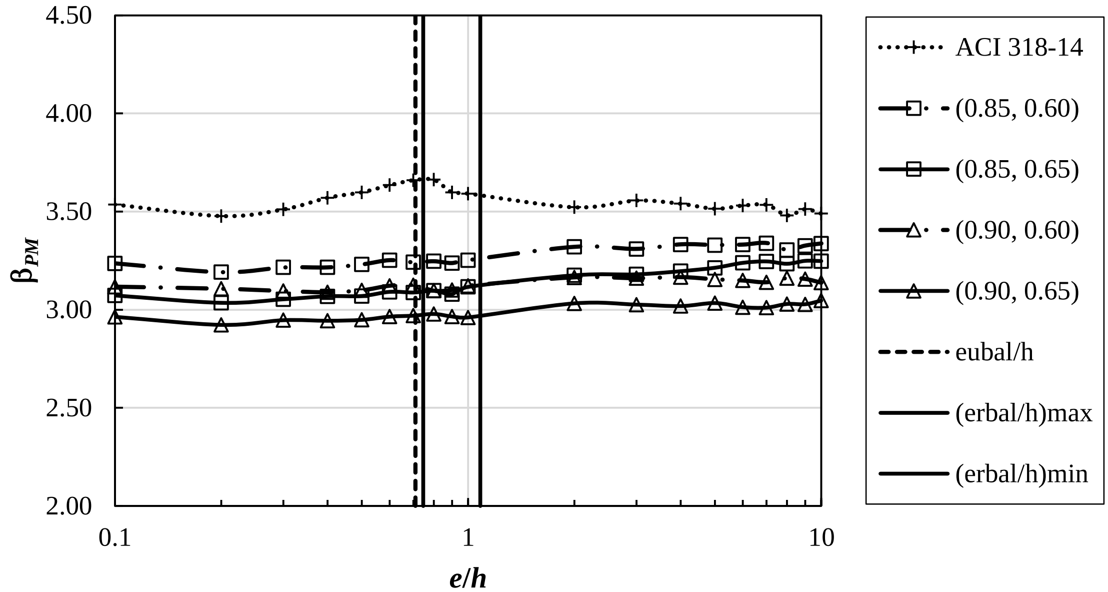


(a)

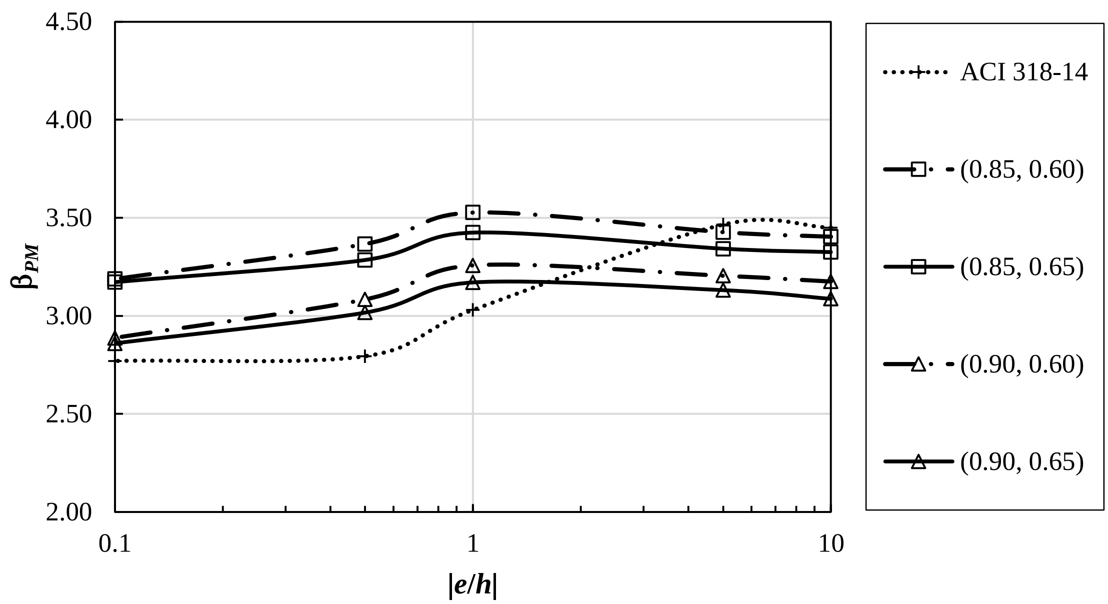


(b)

Figure C.26: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 4, Property Combination 1, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

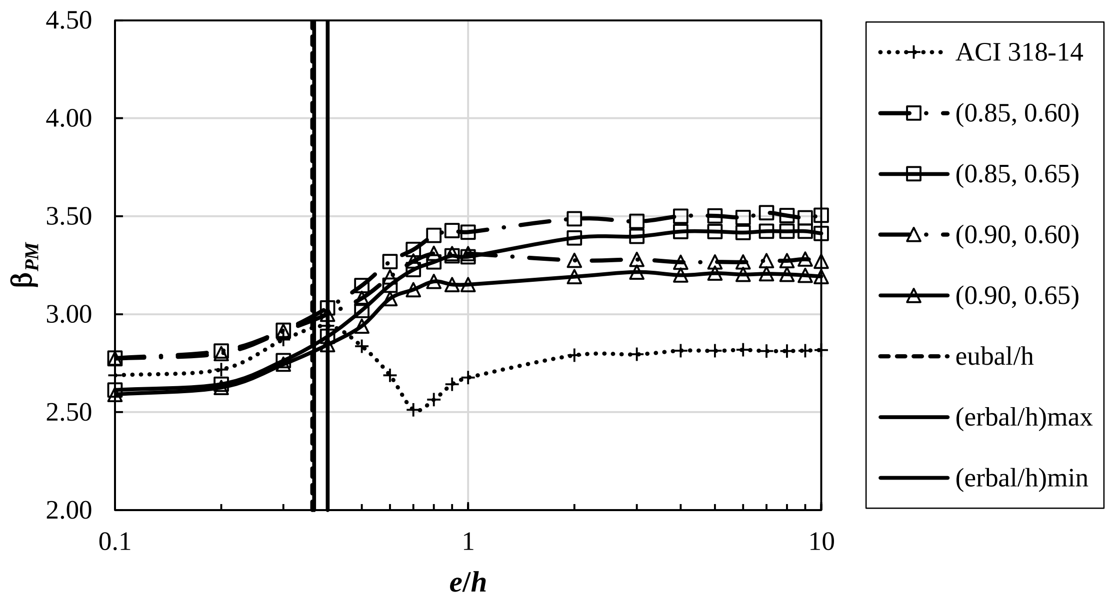


(a)

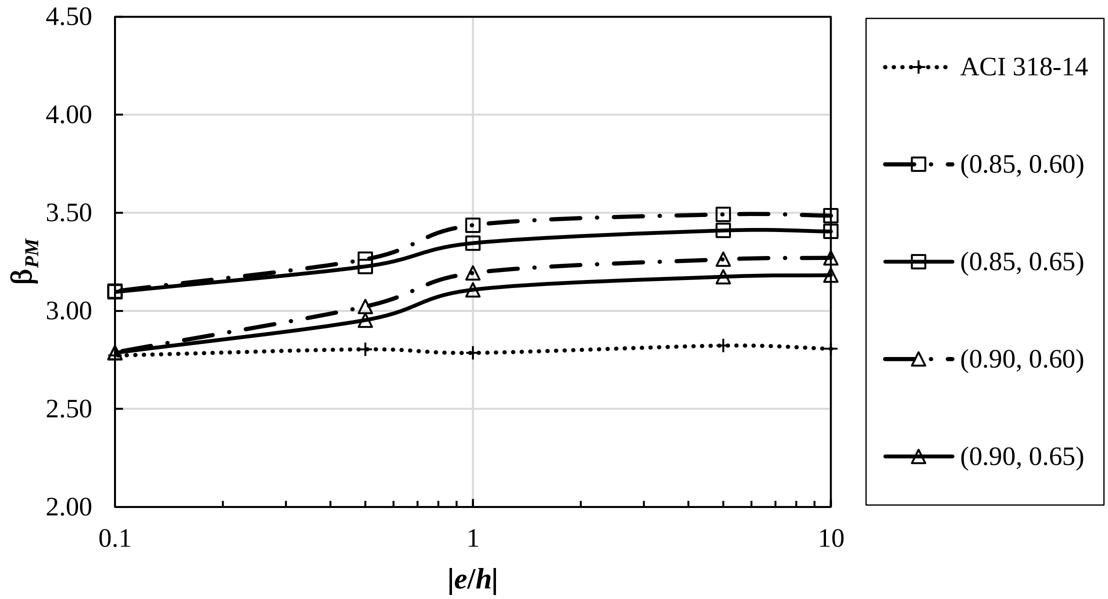


(b)

Figure C.27: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 4, Property Combination 2, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

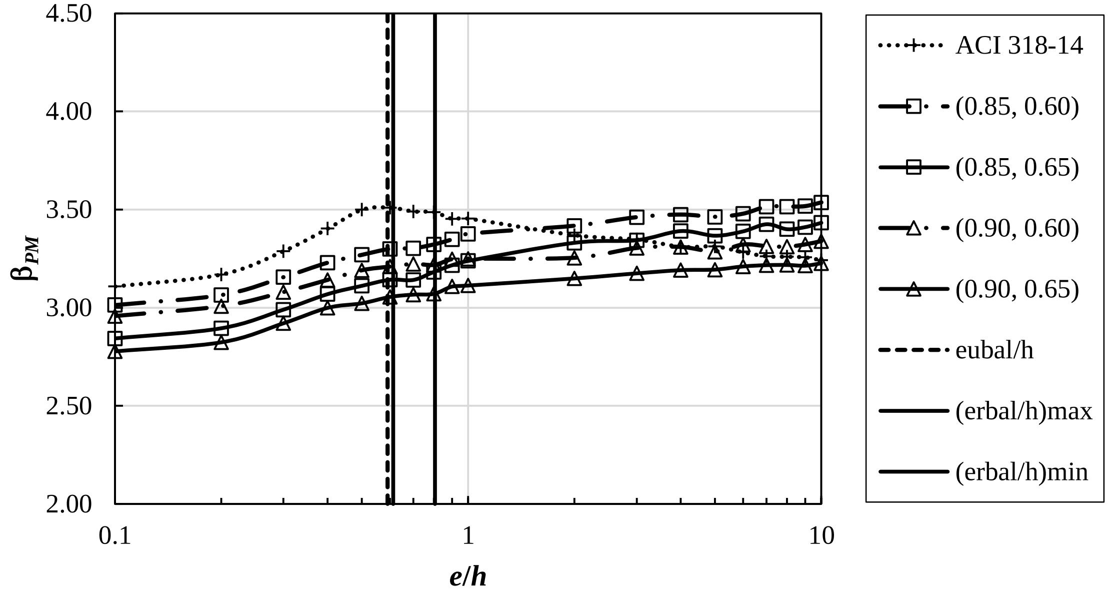


(a)

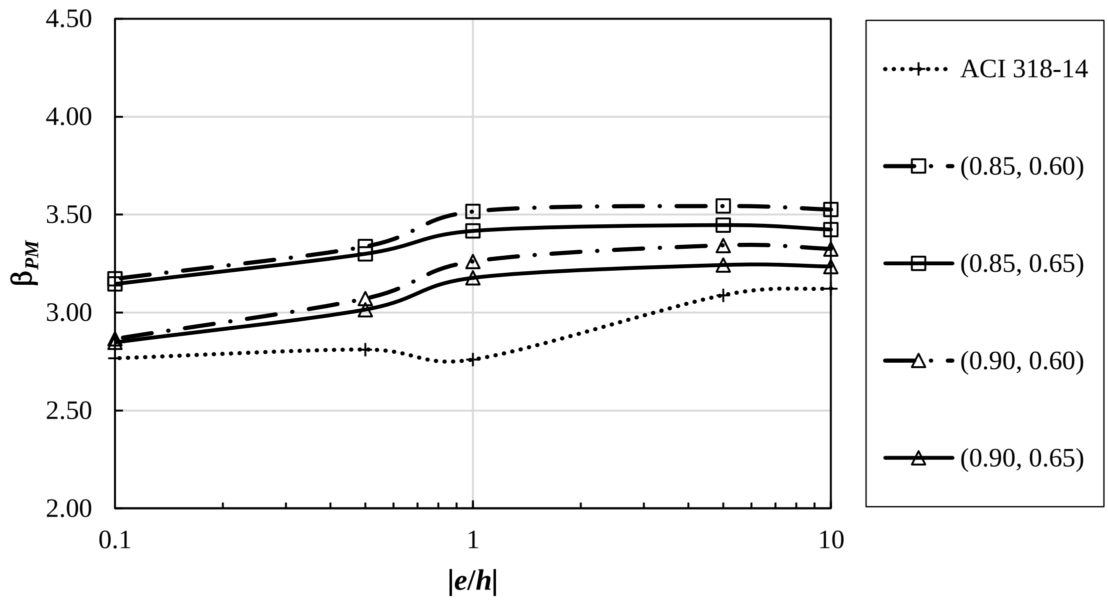


(b)

Figure C.28: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 4, Property Combination 3, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

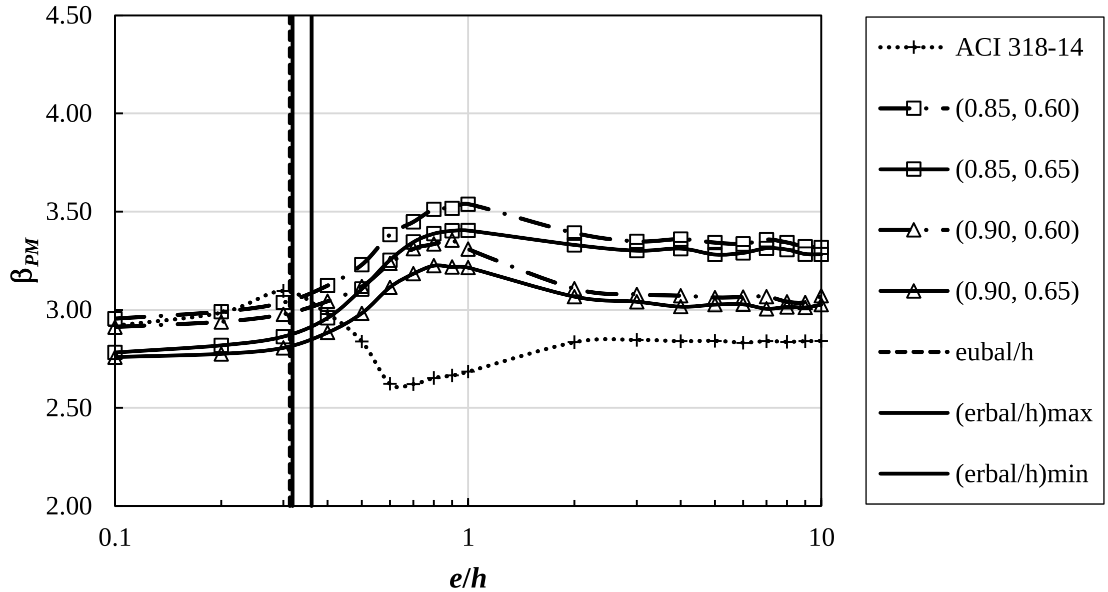


(a)

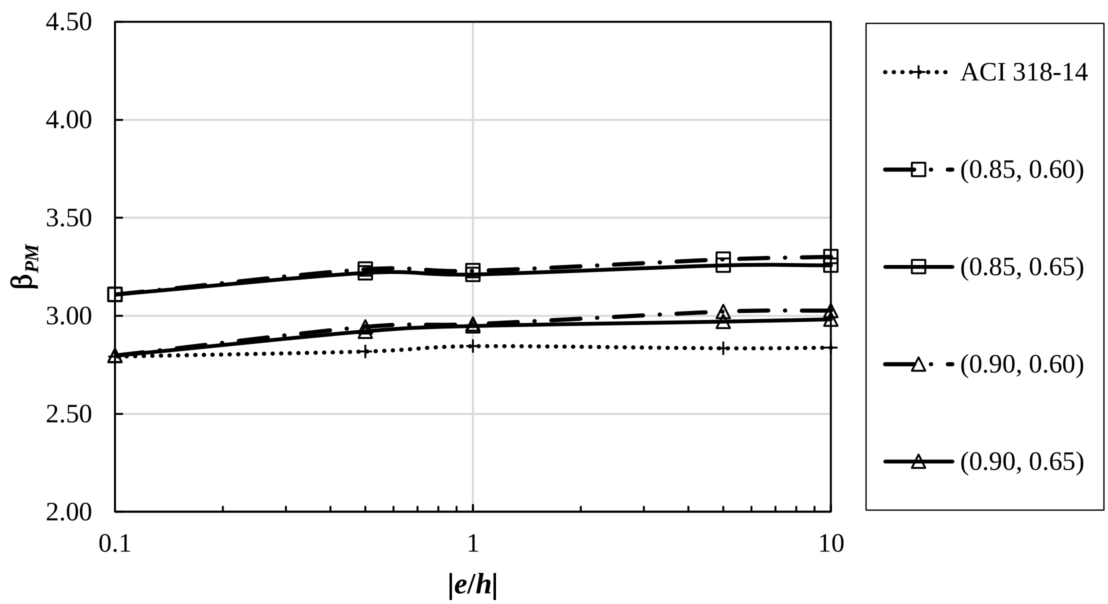


(b)

Figure C.29: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 4, Property Combination 4, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

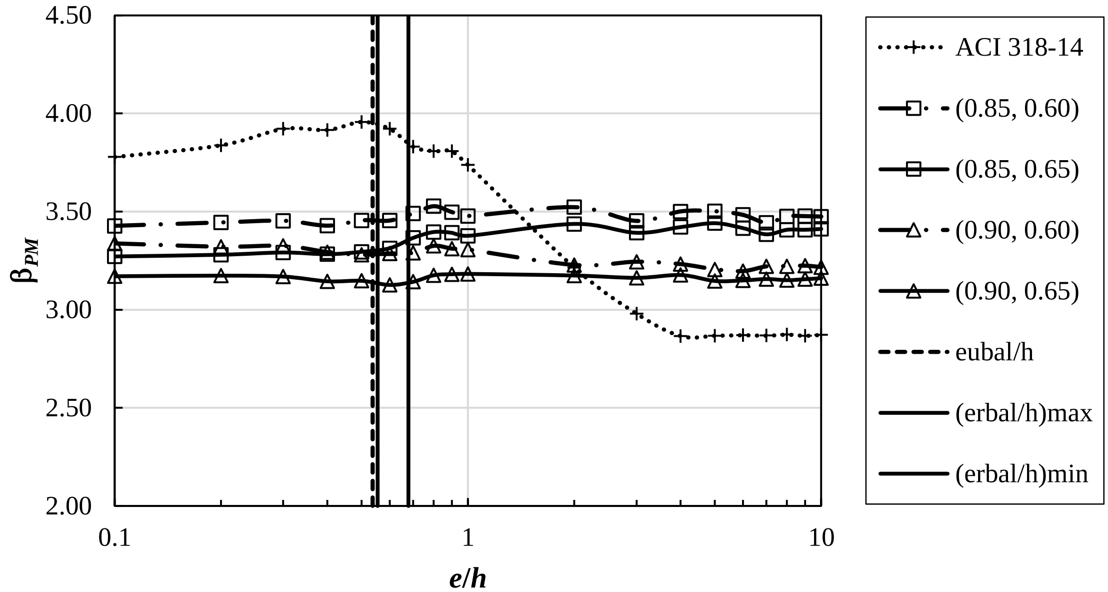


(a)

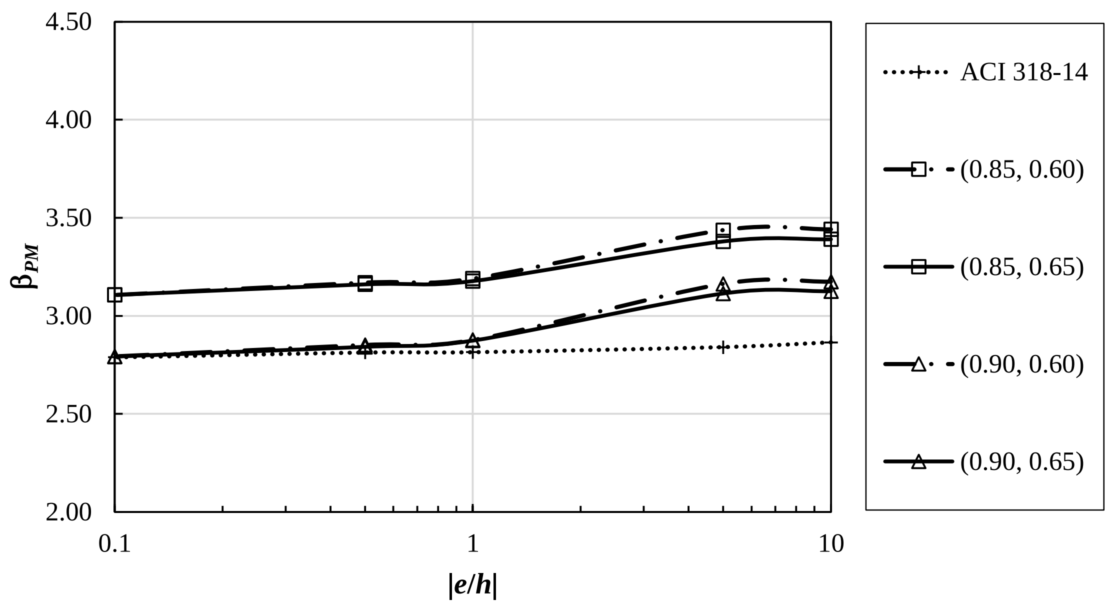


(b)

Figure C.30: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 4, Property Combination 5, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

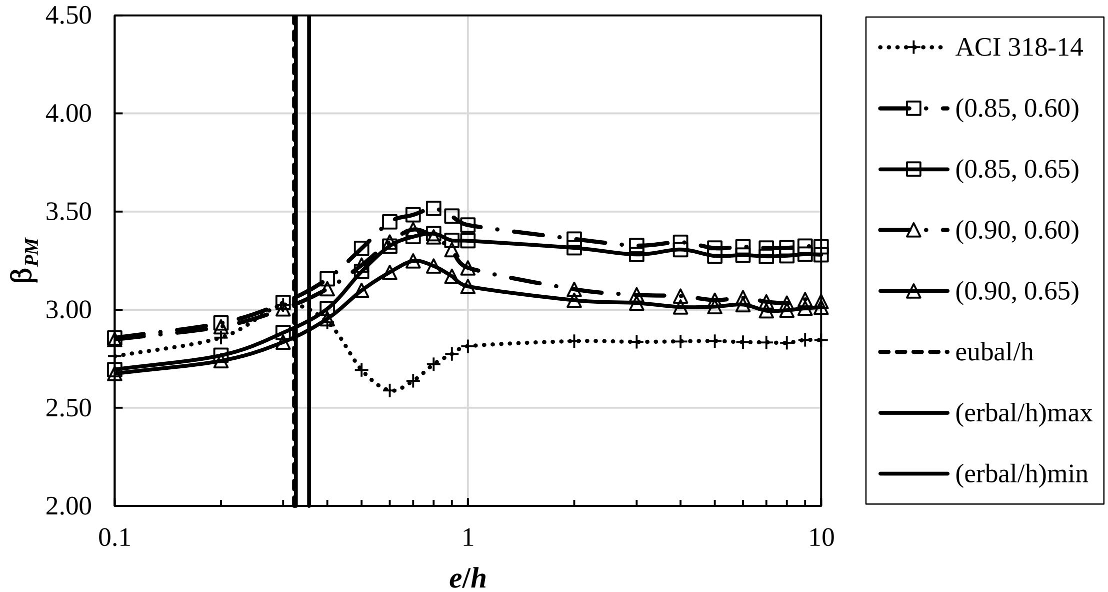


(a)

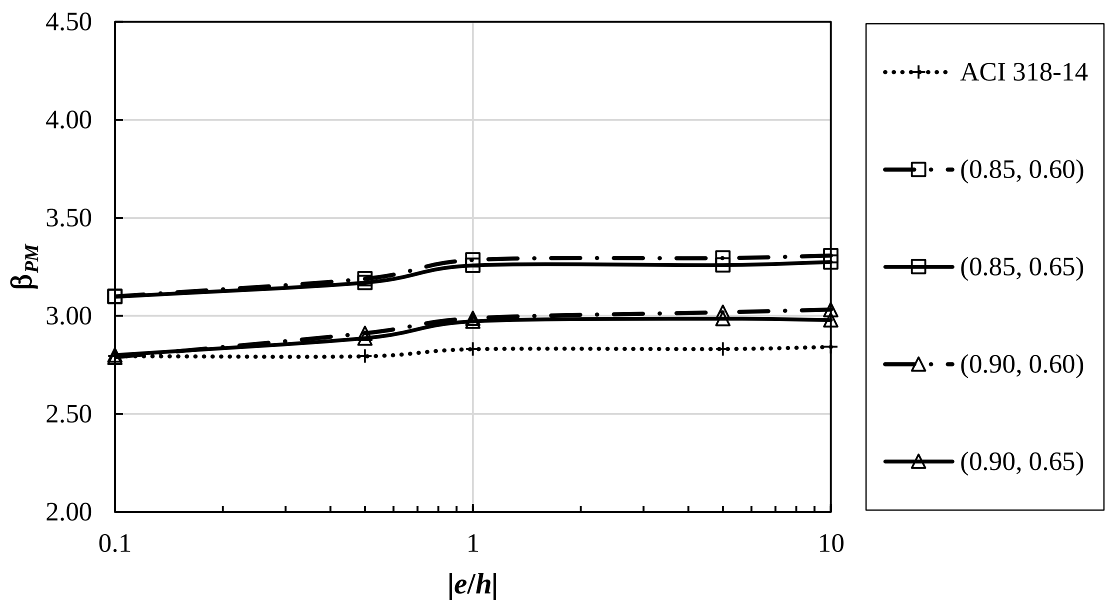


(b)

Figure C.31: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 4, Property Combination 6, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

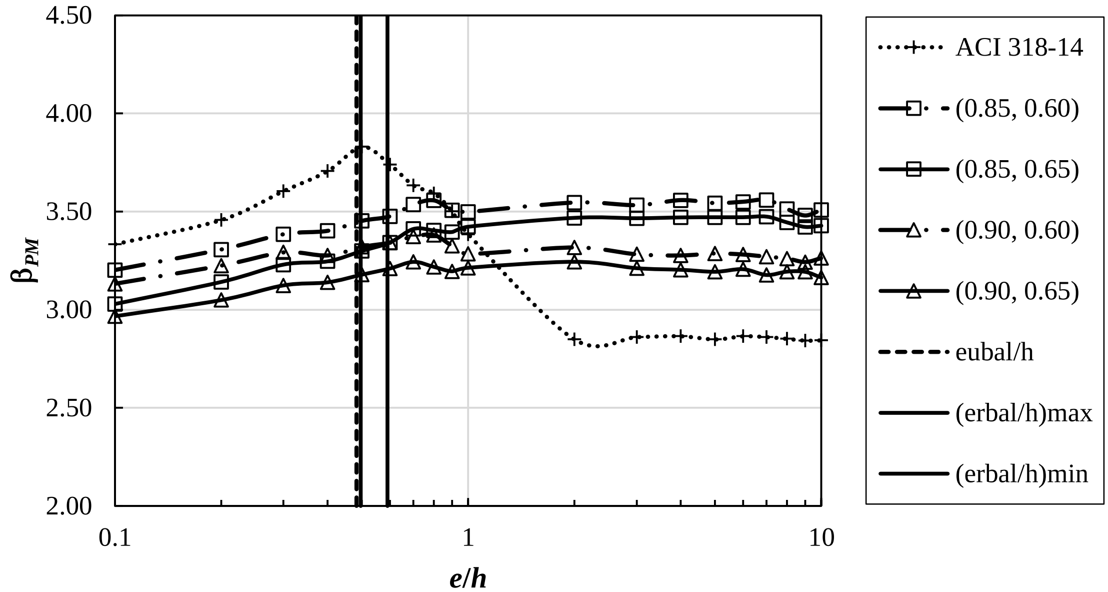


(a)

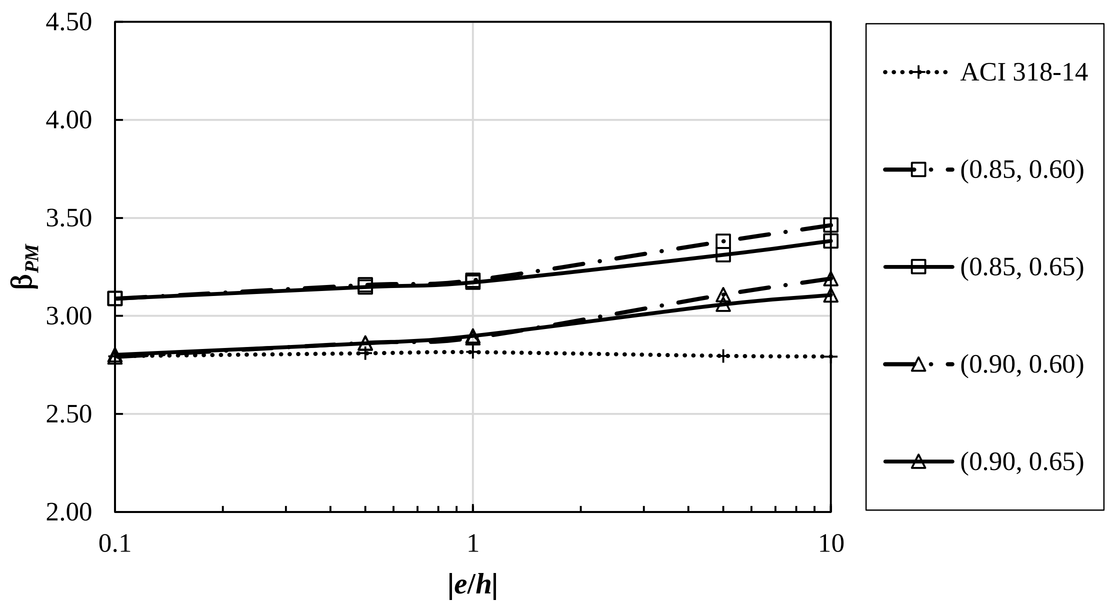


(b)

Figure C.32: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 4, Property Combination 7, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

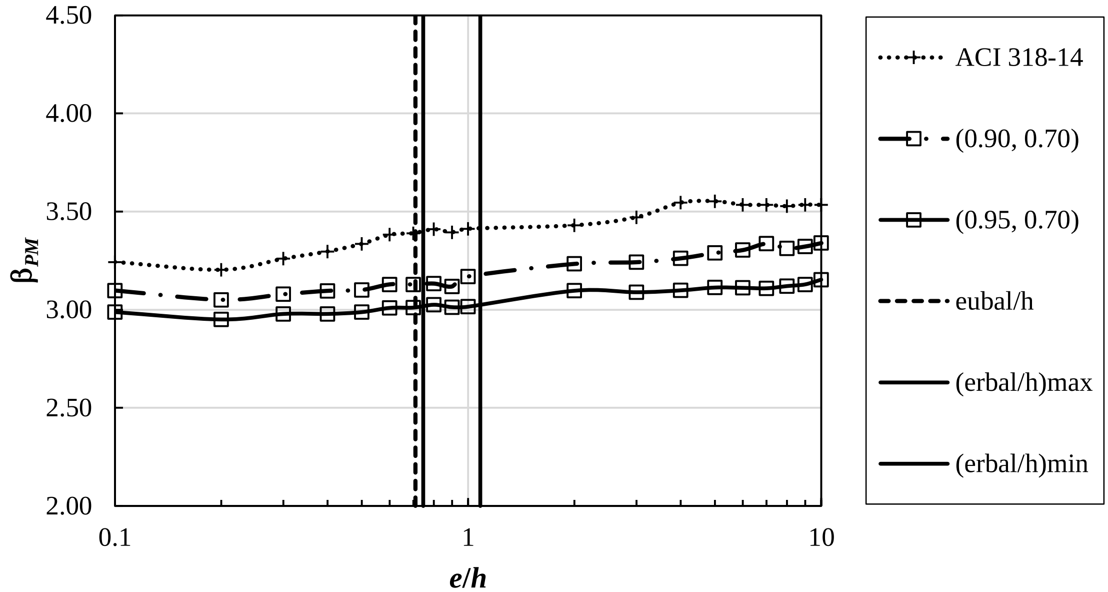


(a)

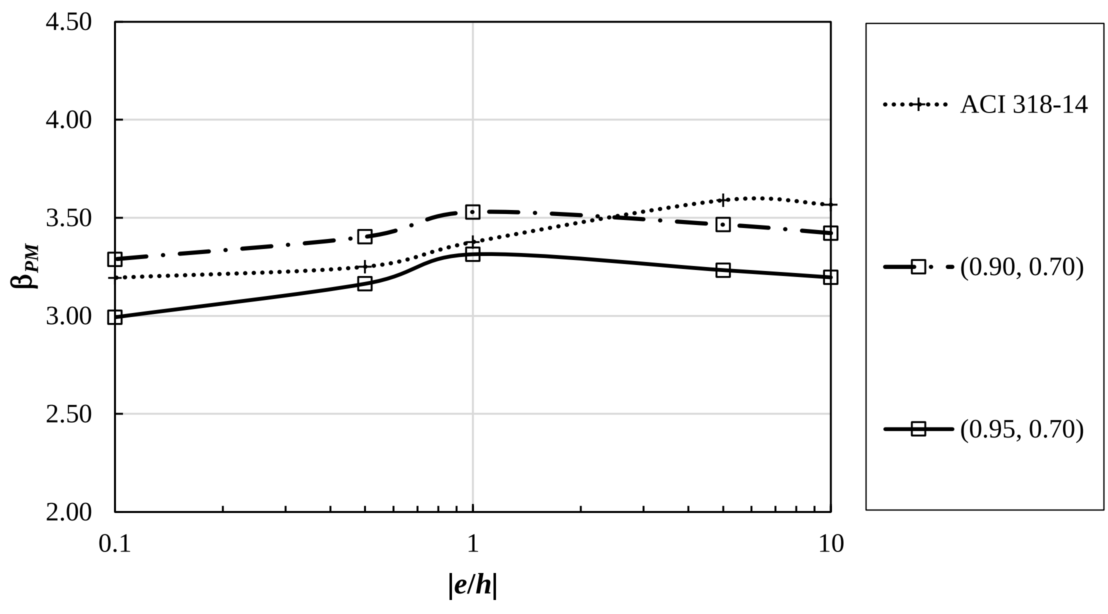


(b)

Figure C.33: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 4, Property Combination 8, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

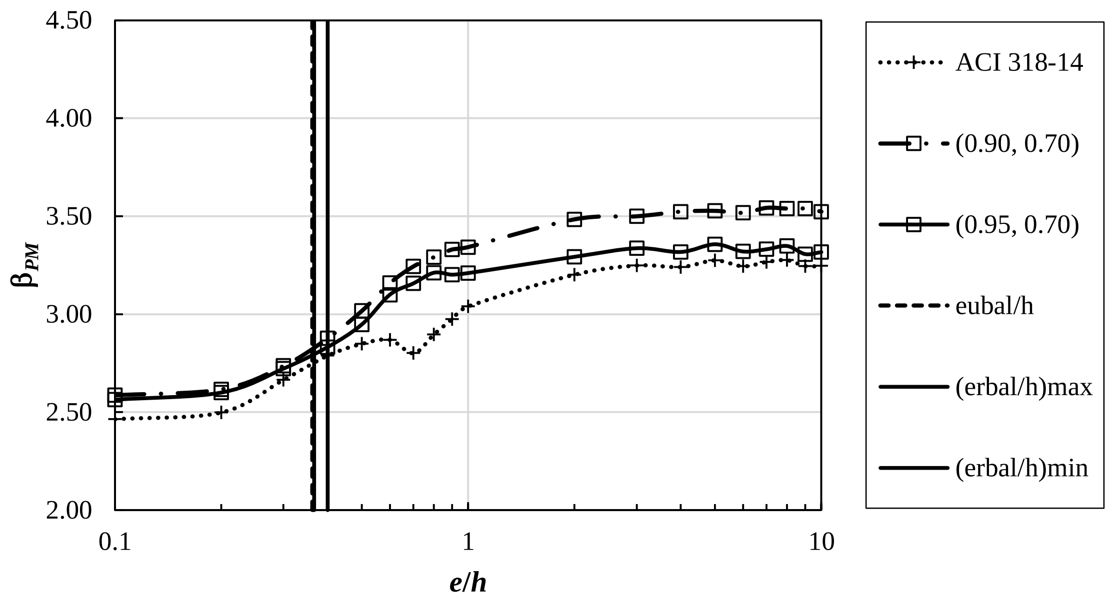


(a)

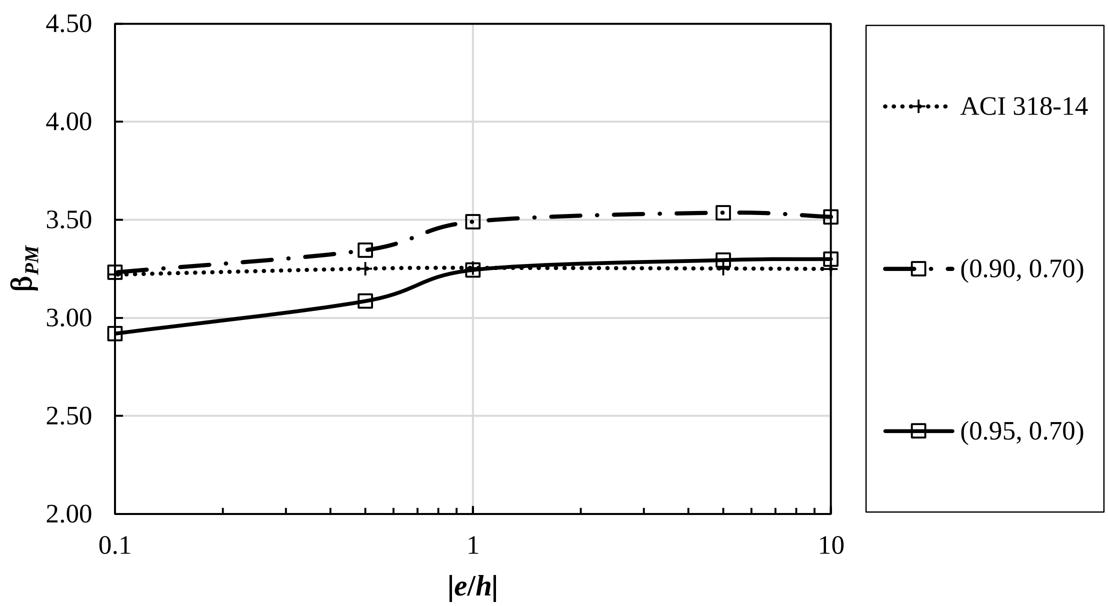


(b)

Figure C.34: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 5, Property Combination 2, and L/D = 0.5: (a) $e/h > 0$; (b) $e/h < 0$

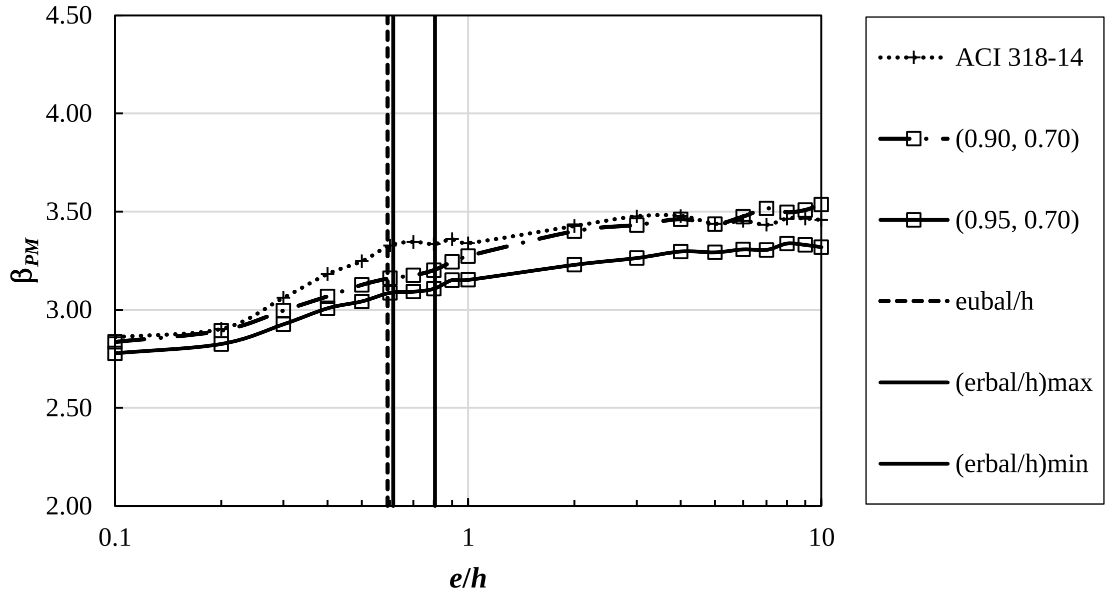


(a)

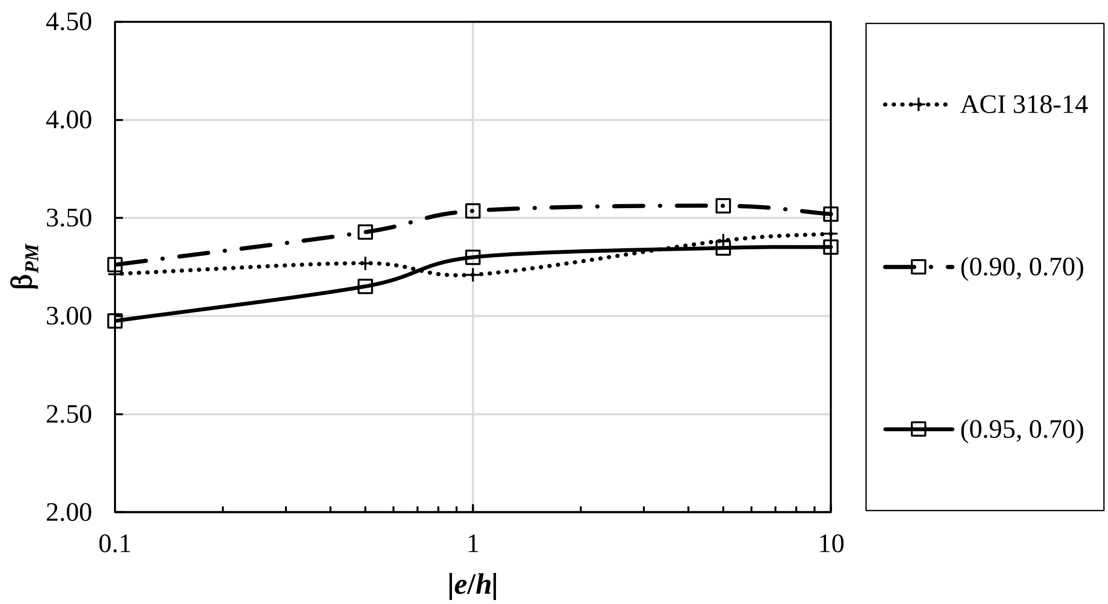


(b)

Figure C.35: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 5, Property Combination 3, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

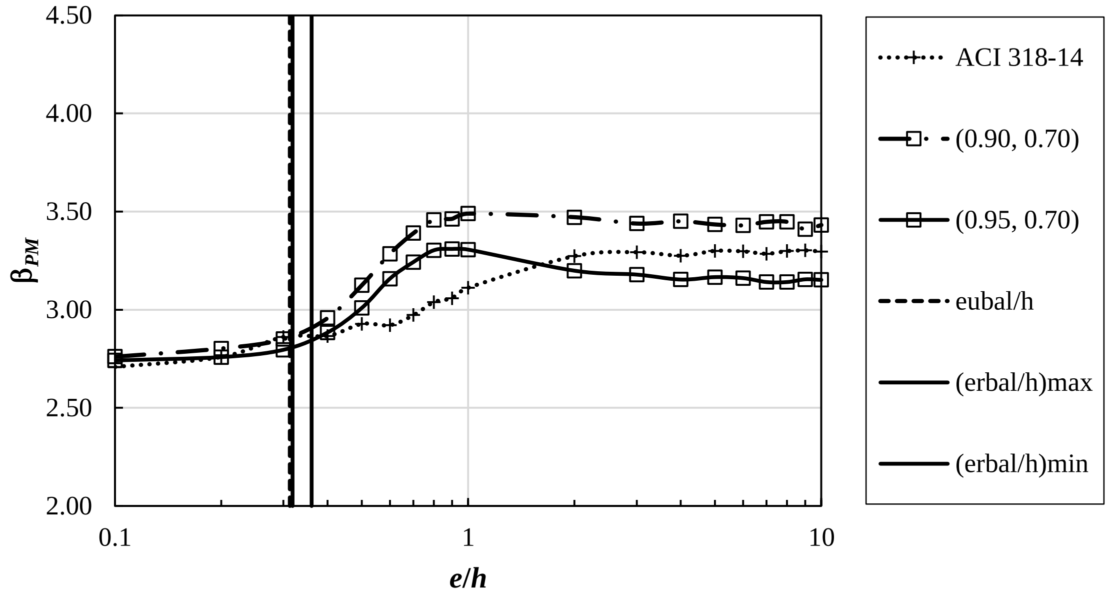


(a)

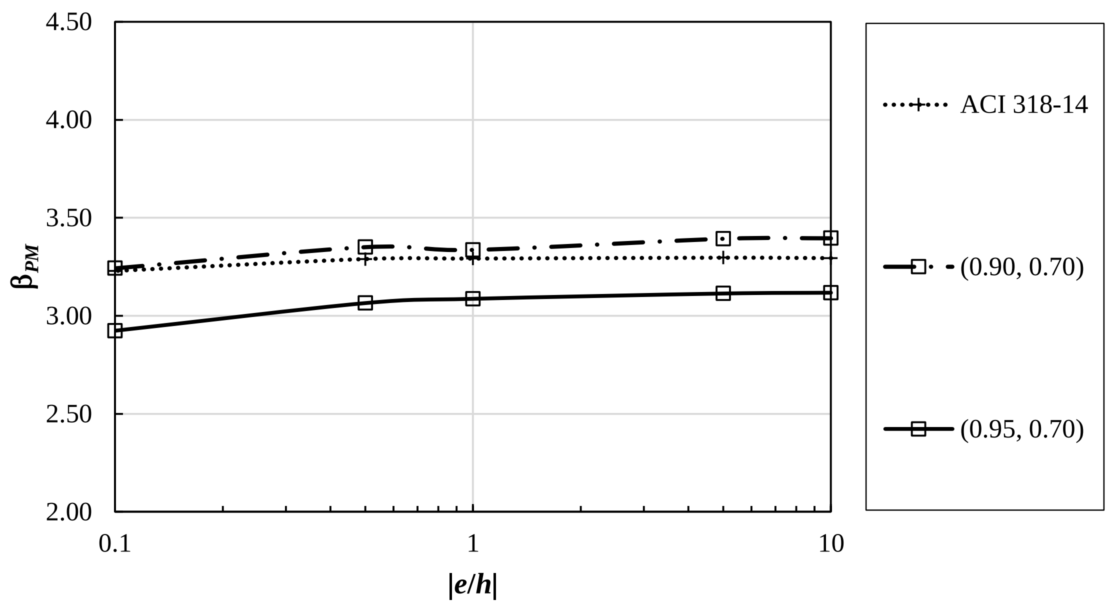


(b)

Figure C.36: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 5, Property Combination 4, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

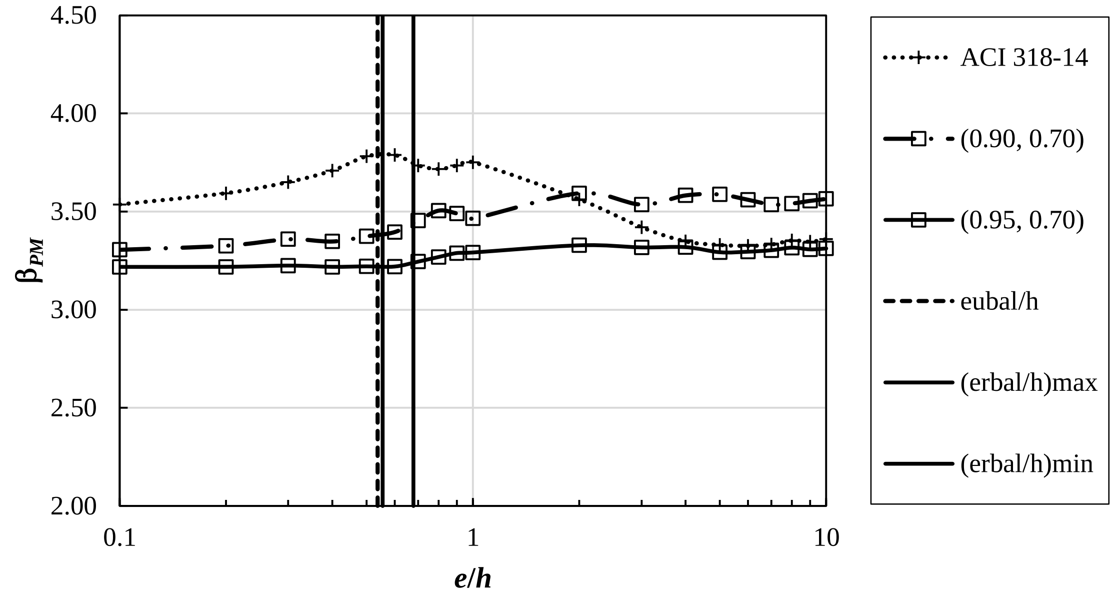


(a)

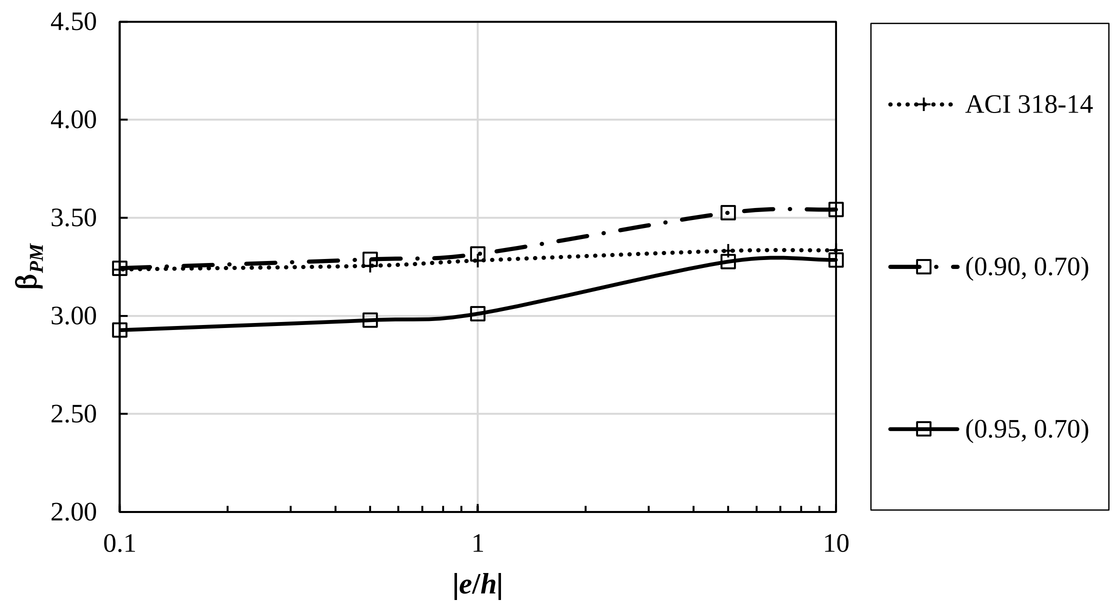


(b)

Figure C.37: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 5, Property Combination 5, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

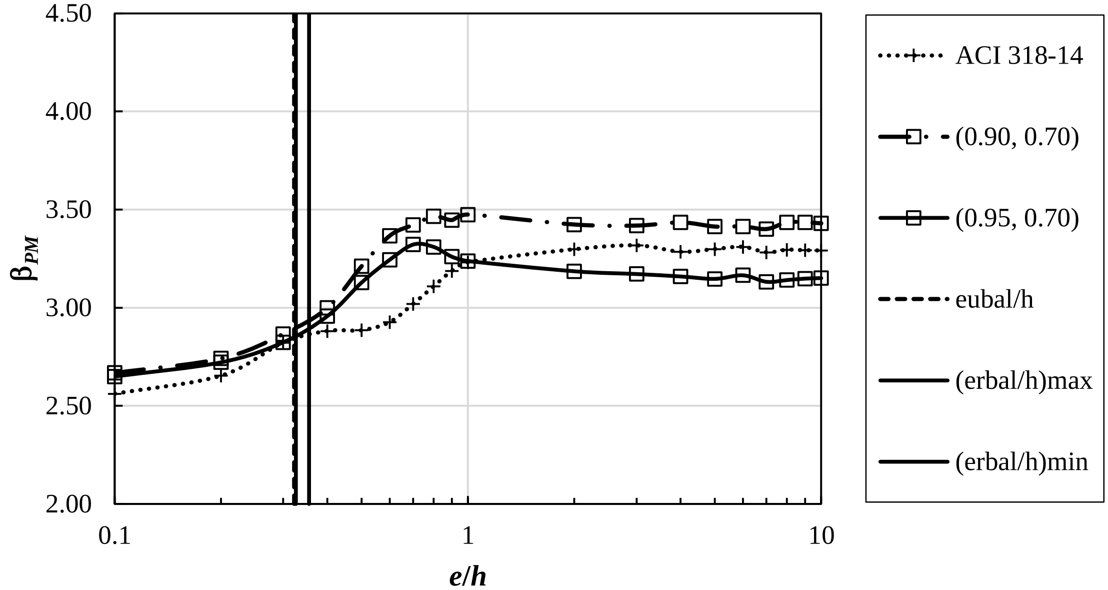


(a)

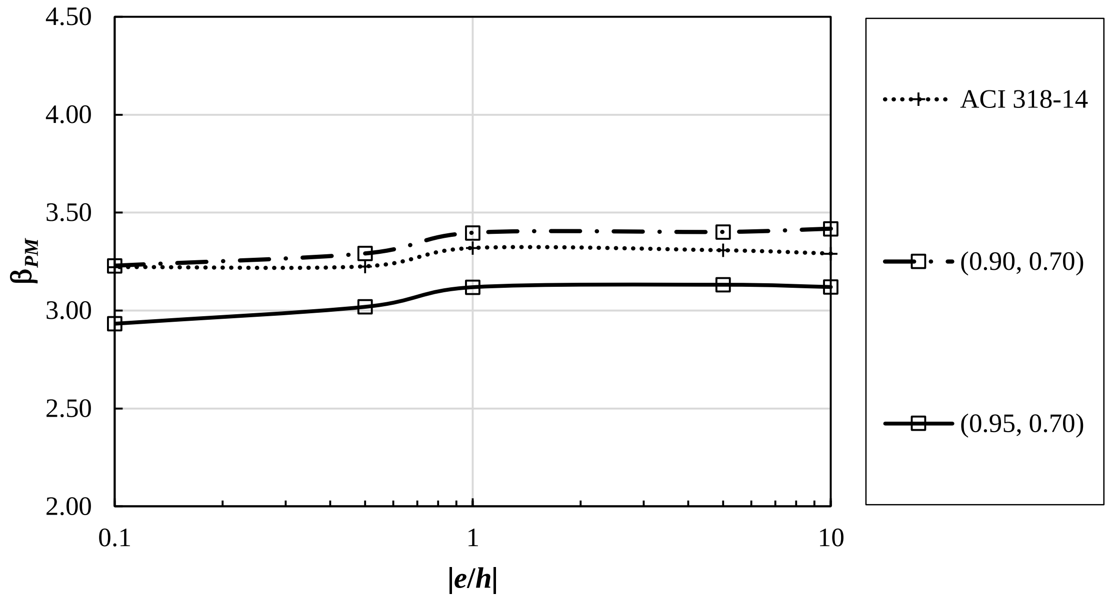


(b)

Figure C.38: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 5, Property Combination 6, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

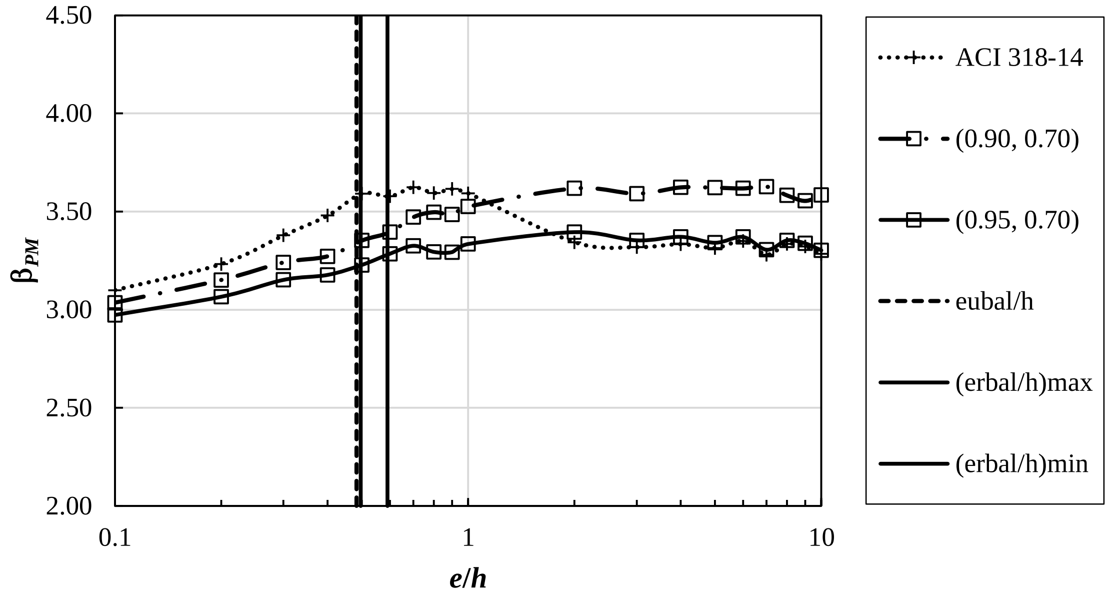


(a)

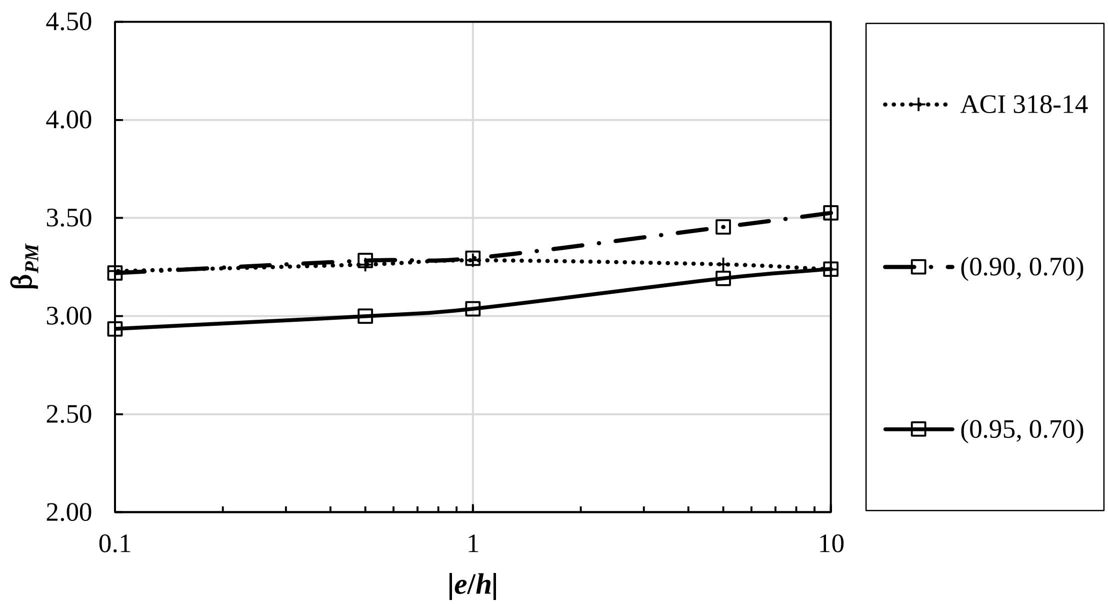


(b)

Figure C.39: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 5, Property Combination 7, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$



(a)



(b)

Figure C.40: Reliability indices for combined moment and axial force, β_{PM}, for Column Section 5, Property Combination 8, and $L/D = 0.5$: (a) $e/h > 0$; (b) $e/h < 0$

C.1 Codes

The Matlab (Version R2016b; The Mathworks, Inc. 2016) codes used to calculate reliability indices for combined moment and axial force, β_{PM}, for Column Section 1 (square section with three bars in each face) and Column Section 5 (spirally reinforced circular section with eight bars evenly distributed around the perimeter), two representative cross sections, are as follows:

The calculation needs to refer the results saved in `alpha_PM_S1.mat` and `alpha_PM_S5.mat`, which are presented in Appendix A.

C.1.1 Notation

```
% a=depth of equivalent rectangular stress block (mm)
% A=area of compression segment of circular section (mm^2)
% A_st=total area of nonprestressed longitudinal reinforcement (mm^2)
% A_s1=area of the 1st layer of reinforcement (mm^2)
% A_s2=area of the 2nd layer of reinforcement (mm^2)
% A_s3=area of the 3rd layer of reinforcement (mm^2)
% A_s4=area of the 4th layer of reinforcement (mm^2)
% A_s5=area of the 5th layer of reinforcement (mm^2)
% b=width of column (mm)
% bias_=bias coefficient
% c=distance from extreme compression fiber to neutral axis (mm)
% _com=combination
% _cur=calculation corresponding to ACI 318-14
% _cur1=calculation corresponding to ACI 318-14 and L/D=0.5
% _cur2=calculation corresponding to ACI 318-14 and L/D=1.5
% C_c=compressive force in concrete (kN)
% CoV_=coefficient of variation
% d_1=distance from extreme compression fiber to the 1st layer of reinforcement (mm)
% d_2=distance from extreme compression fiber to the 2nd layer of reinforcement (mm)
% d_3=distance from extreme compression fiber to the 3rd layer of reinforcement (mm)
% d_4=distance from extreme compression fiber to the 4th layer of reinforcement (mm)
% d_5=distance from extreme compression fiber to the 5th layer of reinforcement (mm)
```

```
% D=dead load
% e=eccentricity (mm)
% eoverh=the specific e/h value
% eoverh_2=the specific e/h value, including extreme values
% E_s=modulus of elasticity of reinforcement (MPa)
% f_c=specified compressive strength of concrete (MPa)
% f_s1=stress in the 1st layer of reinforcement (MPa)
% f_s2=stress in the 2nd layer of reinforcement (MPa)
% f_s3=stress in the 3rd layer of reinforcement (MPa)
% f_s4=stress in the 4th layer of reinforcement (MPa)
% f_s5=stress in the 5th layer of reinforcement (MPa)
% f_y=specified yield strength for nonprestressed reinforcement (MPa)
% F_s1=force in the 1st layer of reinforcement (kN)
% F_s2=force in the 2nd layer of reinforcement (kN)
% F_s3=force in the 3rd layer of reinforcement (kN)
% F_s4=force in the 4th layer of reinforcement (kN)
% F_s5=force in the 5th layer of reinforcement (kN)
% g=limit state function
% h=overall depth of column (mm)
% hovere=h/e
% _i=simulated value
% k_f=value to count numbers of failure
% L=live load
% LoverD=ratio of live load to dead load, L/D
% mean_=mean
% M=flexural strength (kN.m)
% n=numbers of simulation in one subset
% _neg=negative
% n_f=numbers of failure
% N=total numbers of simulation
% _pri=prime
% pro=property
% _pro=calculation corresponding to partial material strength reduction factors
% _pro1=calculation corresponding to partial material strength reduction factors and L/D=0.5
% _pro2=calculation corresponding to partial material strength reduction factors and L/D=1.5
% P=axial strength (kN)
% P_f=probability of failure
% P_max=maximum axial compressive strength (kN)
```

```
% P_o=axial strength at zero eccentricity (kN)
% P_r=design axial strength for partial material strength reduction factors format (kN)
% Prof=professional factor
% P_t=axial tensile strength (kN)
% rn_n=standard normally distributed random number
% rn_u=standard uniformly distributed random number
% s_=sutscript
% _s=sort
% _sam=samples
% std_=standard deviation
% T_D=factor to account for transformation from dead load to dead load effect
% T_L=factor to account for transformation from live load to live load effect
% Z=ratio of strain in extreme tension layer of reinforcement to yield strain

% alpha_=dispersion parameter for Gumbel distribution
% beta_PMr1=reliability index for combined moment and axial force obtained using partial material strength
%           reduction factors and L/D=0.5
% beta_PMr2=reliability index for combined moment and axial force obtained using partial material strength
%           reduction factors and L/D=1.5
% beta_PMu1=reliability index for combined moment and axial force obtained using strength reduction factors
%           in ACI 318-14 and L/D=0.5
% beta_PMu2=reliability index for combined moment and axial force obtained using strength reduction factors
%           in ACI 318-14 and L/D=1.5
% beta_1=factor relating depth of equivalent rectangular compressive stress block to depth of neutral axis
% gamma=ratio of distance between outer layers of reinforcement in column to overall column depth
% epsilon_s1=strain in the 1st layer of reinforcement
% epsilon_s2=strain in the 2nd layer of reinforcement
% epsilon_s3=strain in the 3rd layer of reinforcement
% epsilon_s4=strain in the 4th layer of reinforcement
% epsilon_s5=strain in the 5th layer of reinforcement
% epsilon_y=yield strain of reinforcement
% angle_theta= angle theta, angle used to calculate compression segment of circular column
% mu_=mean of the associated normal distribution for lognormal distribution
% mu_=location parameter for Gumbel distribution
% rho_g=total reinforcement ratio, equal to ratio of total longitudinal reinforcement area to cross-
%       sectional area of column
% sigma_=standard deviation of the associated normal distribution for lognormal distribution
% phi_sc=a pair of partial material strength reduction factors
```

```
% phiP_n=design axial strength in ACI 318-14 (kN)
```

C.1.2 Column Section 1

C.1.2.1 Code 1-Reliability Indices for ACI 318-14 and *L*/*D* = 0.5

```
clc
clear
tic
% Reliability index calculation corresponding to ACI 318-14 and L/D=0.5
n=1e4;
N=1e6;
Z_sam=[0.5:-0.01:-1 -1.05:-0.05:-10 -10.1:-0.1:-50 -51:-1:-100];
eoverh=[0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 ...
       -10.0 -5.0 -1.0 -0.5 -0.1]; % The specific e/h values
hovere=1./eoverh;
eoverh_2=[0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 -10.0 ...
         -5.0 -1.0 -0.5 -0.1 -0]; % The specific e/h values, including extreme values
LoverD=0.5;

% Preallocation
n_fcur1=zeros(8,length(eoverh_2),(N/n));

for i6=1:(N/n)
    % Calculate sample points for resistance
    % (P_isam, M_isam and hovere_isam corresponding to Z_sam, P_oi and P_ti)
    % Preallocation
    P_isam=zeros(8,n,length(Z_sam));
    M_isam=zeros(8,n,length(Z_sam));
    hovere_isam=zeros(8,n,length(Z_sam));
    P_oi=zeros(8,n,1);
    P_ti=zeros(8,n,1);

    for i1=1:8
        rn_n1=randn(1,n);
        rn_n2=randn(1,n);
```

```
    rn_n3=randn(1,n);
    rn_n4=randn(1,n);
    rn_n5=randn(1,n);
    rn_n6=randn(1,n);
    rn_n7=randn(1,n);
    rn_n8=randn(1,n);
    rn_n9=randn(1,n);
    rn_n10=randn(1,n);
    f_y=420;
    bias_f_y=1.125;
    CoV_f_y=0.098;
    mean_f_y=f_y*bias_f_y;
    std_f_y=mean_f_y*CoV_f_y;
    mu_f_y=log(mean_f_y^2/sqrt(mean_f_y^2+std_f_y^2)); % Lognormal distribution
    sigma_f_y=sqrt(log(std_f_y^2/mean_f_y^2+1));
    f_yi(1,:)=lognrnd(mu_f_y,sigma_f_y,[1,n]);

    for i3=1:length(Z_sam)
        [P_isam(i1,:,i3),M_isam(i1,:,i3),hovere_isam(i1,:,i3),P_oi(i1,:,1),P_ti(i1,:,1)]...
            =feval('ResistanceSim_S1',i1,rn_n1,rn_n2,rn_n3,rn_n4,rn_n5,rn_n6,rn_n7,rn_n8,rn_n9,...
             rn_n10,f_yi,Z_sam(i3));
    end
end

% Calculate P_maxi
P_maxi=0.80*P_oi;

% Permute the 2nd and 3rd dimensions
P_isam=permute(P_isam,[1,3,2]);
M_isam=permute(M_isam,[1,3,2]);
hovere_isam=permute(hovere_isam,[1,3,2]);
P_oi=permute(P_oi,[1,3,2]);
P_ti=permute(P_ti,[1,3,2]);
P_maxi=permute(P_maxi,[1,3,2]);

% Calculate load effect
% LoverD=0.5; % Defined previously
```

```
[D_cur1i,L_cur1i,T_Di,T_Li]=feval('LoadEffectSim_cur_S1',LoverD,n);

% Interpolation
% Calculate the unknown points (P_i)
% Preallocation
P_i=zeros(8,length(hovere),n);

for i1=1:8
    for i5=1:n
        s_M_isam=find(M_isam(i1,:,i5)>0);
        P_isampri=P_isam(i1,s_M_isam,i5);
        hovere_isampri=hovere_isam(i1,s_M_isam,i5);
        P_isampri=[P_oi(i1,1,i5) P_isampri P_ti(i1,1,i5)];
        hovere_isampri=[1e10 hovere_isampri -1e10];
        [hovere_isampris,I_hovere_isampri]=sort(hovere_isampri,'descend');
        P_i(i1,:,i5)=interp1(hovere_isampris,P_isampri(I_hovere_isampri),hovere,'linear');
        s_P_maxi=find(P_i(i1,:,i5)>P_maxi(i1,1,i5));
        P_i(i1,s_P_maxi,i5)=P_maxi(i1,1,i5);
    end
end

% P_i includes P_maxi and P_ti
P_i=cat(2,P_maxi,P_i,P_ti);

% Limit state function and numbers of failure
% Preallocation
g_cur1i=zeros(8,length(eoverh_2),n);
k_fcur1i=zeros(8,length(eoverh_2),n);

for i4=1:length(eoverh_2)
    g_cur1i(:,i4,:)...
        =(abs(P_i(:,i4,:))*sqrt(1+eoverh_2(i4)^2))./...
         (abs(D_cur1i(:,i4,:)*T_Di+L_cur1i(:,i4,:)*T_Li)*sqrt(1+eoverh_2(i4)^2));
    % T_Di, T_Li are numbers, not vectors
end
s_g_cur1i=find(log(g_cur1i)<0);
k_fcur1i(s_g_cur1i)=1;
```

```
    n_fcur1(:,:,i6)=sum(k_fcur1i,3);
end
% Probability of failure
P_fcur1=sum(n_fcur1,3)/N;
% Reliability index
beta_PMu1=-norminv(P_fcur1,0,1);
toc
save beta_PMu1_S1 n_fcur1 P_fcur1 beta_PMu1
```

C.1.2.2 Code 2-Reliability Indices for ACI 318-14 and *L*/*D* = 1.5

```
clc
clear
tic
% Reliability index calculation corresponding to ACI 318-14 and L/D=1.5
n=1e4;
N=1e6;
Z_sam=[0.5:-0.01:-1 -1.05:-0.05:-10 -10.1:-0.1:-50 -51:-1:-100];
eoverh=[0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 ...
       -10.0 -5.0 -1.0 -0.5 -0.1]; % The specific e/h values
hovere=1./eoverh;
eoverh_2=[0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 -10.0 ...
         -5.0 -1.0 -0.5 -0.1 -0]; % The specific e/h values, including extreme values
LoverD=1.5;

% Preallocation
n_fcur2=zeros(8,length(eoverh_2),(N/n));

for i6=1:(N/n)
    % Calculate sample points for resistance
    % (P_isam, M_isam and hovere_isam corresponding to Z_sam, P_oi and P_ti)
    % Preallocation
    P_isam=zeros(8,n,length(Z_sam));
    M_isam=zeros(8,n,length(Z_sam));
    hovere_isam=zeros(8,n,length(Z_sam));
    P_oi=zeros(8,n,1);
    P_ti=zeros(8,n,1);
```

```
for i1=1:8
    rn_n1=randn(1,n);
    rn_n2=randn(1,n);
    rn_n3=randn(1,n);
    rn_n4=randn(1,n);
    rn_n5=randn(1,n);
    rn_n6=randn(1,n);
    rn_n7=randn(1,n);
    rn_n8=randn(1,n);
    rn_n9=randn(1,n);
    rn_n10=randn(1,n);
    f_y=420;
    bias_f_y=1.125;
    CoV_f_y=0.098;
    mean_f_y=f_y*bias_f_y;
    std_f_y=mean_f_y*CoV_f_y;
    mu_f_y=log(mean_f_y^2/sqrt(mean_f_y^2+std_f_y^2)); % Lognormal distribution
    sigma_f_y=sqrt(log(std_f_y^2/mean_f_y^2+1));
    f_yi(1,:)=lognrnd(mu_f_y,sigma_f_y,[1,n]);

    for i3=1:length(Z_sam)
        [P_isam(i1,:,i3),M_isam(i1,:,i3),hovere_isam(i1,:,i3),P_oi(i1,:,1),P_ti(i1,:,1)]...
            =feval('ResistanceSim_S1',i1,rn_n1,rn_n2,rn_n3,rn_n4,rn_n5,rn_n6,rn_n7,rn_n8,rn_n9,...
             rn_n10,f_yi,Z_sam(i3));
    end
end

% Calculate P_maxi
P_maxi=0.80*P_oi;

% Permute the 2nd and 3rd dimensions
P_isam=permute(P_isam,[1,3,2]);
M_isam=permute(M_isam,[1,3,2]);
hovere_isam=permute(hovere_isam,[1,3,2]);
P_oi=permute(P_oi,[1,3,2]);
P_ti=permute(P_ti,[1,3,2]);
```

```
P_maxi=permute(P_maxi,[1,3,2]);

% Calculate load effect
% LoverD=1.5; % Defined previously
[D_cur2i,L_cur2i,T_Di,T_Li]=feval('LoadEffectSim_cur_S1',LoverD,n);

% Interpolation
% Calculate the unknown points (P_i)
% Preallocation
P_i=zeros(8,length(hovere),n);

for i1=1:8
    for i5=1:n
        s_M_isam=find(M_isam(i1,:,i5)>0);
        P_isampri=P_isam(i1,s_M_isam,i5);
        hovere_isampri=hovere_isam(i1,s_M_isam,i5);
        P_isampri=[P_oi(i1,1,i5) P_isampri P_ti(i1,1,i5)];
        hovere_isampri=[1e10 hovere_isampri -1e10];
        [hovere_isampris,I_hovere_isampri]=sort(hovere_isampri,'descend');
        P_i(i1,:,i5)=interp1(hovere_isampris,P_isampri(I_hovere_isampri),hovere,'linear');
        s_P_maxi=find(P_i(i1,:,i5)>P_maxi(i1,1,i5));
        P_i(i1,s_P_maxi,i5)=P_maxi(i1,1,i5);
    end
end

% P_i includes P_maxi and P_ti
P_i=cat(2,P_maxi,P_i,P_ti);

% Limit state function and numbers of failure
% Preallocation
g_cur2i=zeros(8,length(eoverh_2),n);
k_fcur2i=zeros(8,length(eoverh_2),n);

for i4=1:length(eoverh_2)
    g_cur2i(:,i4,:)...
        =(abs(P_i(:,i4,:))*sqrt(1+eoverh_2(i4)^2))./...
```

```
            (abs(D_cur2i(:,i4,:)*T_Di+L_cur2i(:,i4,:)*T_Li)*sqrt(1+eoverh_2(i4)^2));
        % T_Di, T_Li are numbers, not vectors
    end
    s_g_cur2i=find(log(g_cur2i)<0);
    k_fcur2i(s_g_cur2i)=1;
    n_fcur2(:,:,i6)=sum(k_fcur2i,3);
end
% Probability of failure
P_fcur2=sum(n_fcur2,3)/N;
% Reliability index
beta_PMu2=-norminv(P_fcur2,0,1);
toc
save beta_PMu2_S1 n_fcur2 P_fcur2 beta_PMu2
```

C.1.2.3 Code 3-Reliability Indices for Partial Material Strength Reduction Factors and $L/D = 0.5$

```
clc
clear
tic
% Reliability index calculation corresponding to partial strength reduction factors and L/D=0.5
n=1e4;
N=1e6;
Z_sam=[0.5:-0.01:-1 -1.05:-0.05:-10 -10.1:-0.1:-50 -51:-1:-100];
eoverh=[0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 ...
       -10.0 -5.0 -1.0 -0.5 -0.1]; % The specific e/h values
hovere=1./eoverh;
eoverh_2=[0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 -10.0 ...
         -5.0 -1.0 -0.5 -0.1 -0]; % The specific e/h values, including extreme values
LoverD=0.5;
s_phi_sc=10;

% Preallocation
n_fpro1=zeros(8,length(eoverh_2),(N/n));

for i6=1:(N/n)
    % Calculate sample points for resistance
    % (P_isam, M_isam and hovere_isam corresponding to Z_sam, P_oi and P_ti)
```

```
% Preallocation
P_isam=zeros(8,n,length(Z_sam));
M_isam=zeros(8,n,length(Z_sam));
hovere_isam=zeros(8,n,length(Z_sam));
P_oi=zeros(8,n,1);
P_ti=zeros(8,n,1);

for i1=1:8
    rn_n1=randn(1,n);
    rn_n2=randn(1,n);
    rn_n3=randn(1,n);
    rn_n4=randn(1,n);
    rn_n5=randn(1,n);
    rn_n6=randn(1,n);
    rn_n7=randn(1,n);
    rn_n8=randn(1,n);
    rn_n9=randn(1,n);
    rn_n10=randn(1,n);
    f_y=420;
    bias_f_y=1.125;
    CoV_f_y=0.098;
    mean_f_y=f_y*bias_f_y;
    std_f_y=mean_f_y*CoV_f_y;
    mu_f_y=log(mean_f_y^2/sqrt(mean_f_y^2+std_f_y^2)); % Lognormal distribution
    sigma_f_y=sqrt(log(std_f_y^2/mean_f_y^2+1));
    f_yi(1,:)=lognrnd(mu_f_y,sigma_f_y,[1,n]);

    for i3=1:length(Z_sam)
        [P_isam(i1,:,i3),M_isam(i1,:,i3),hovere_isam(i1,:,i3),P_oi(i1,:,1),P_ti(i1,:,1)]...
            =feval('ResistanceSim_S1',i1,rn_n1,rn_n2,rn_n3,rn_n4,rn_n5,rn_n6,rn_n7,rn_n8,rn_n9,...
             rn_n10,f_yi,Z_sam(i3));
    end
end

% Calculate P_maxi
P_maxi=0.80*P_oi;
```

```
% Permute the 2nd and 3rd dimensions
P_isam=permute(P_isam,[1,3,2]);
M_isam=permute(M_isam,[1,3,2]);
hovere_isam=permute(hovere_isam,[1,3,2]);
P_oi=permute(P_oi,[1,3,2]);
P_ti=permute(P_ti,[1,3,2]);
P_maxi=permute(P_maxi,[1,3,2]);

% Calculate load effect
% LoverD=0.5; % Defined previously
% s_phi_sc % Defined previously
[D_pro1i,L_pro1i,T_Di,T_Li]=feval('LoadEffectSim_pro_S1',LoverD,s_phi_sc,n);

% Interpolation
% Calculate the unknown points (P_i)
% Preallocation
P_i=zeros(8,length(hovere),n);

for i1=1:8
    for i5=1:n
        s_M_isam=find(M_isam(i1,:,i5)>0);
        P_isampri=P_isam(i1,s_M_isam,i5);
        hovere_isampri=hovere_isam(i1,s_M_isam,i5);
        P_isampri=[P_oi(i1,1,i5) P_isampri P_ti(i1,1,i5)];
        hovere_isampri=[1e10 hovere_isampri -1e10];
        [hovere_isampris,I_hovere_isampri]=sort(hovere_isampri,'descend');
        P_i(i1,:,i5)=interp1(hovere_isampris,P_isampri(I_hovere_isampri),hovere,'linear');
        s_P_maxi=find(P_i(i1,:,i5)>P_maxi(i1,1,i5));
        P_i(i1,s_P_maxi,i5)=P_maxi(i1,1,i5);
    end
end

% P_i includes P_maxi and P_ti
P_i=cat(2,P_maxi,P_i,P_ti);

% Limit state function and numbers of failure
% Preallocation
```

```
    g_pro1i=zeros(8,length(eoverh_2),n);
    k_fpro1i=zeros(8,length(eoverh_2),n);

    for i4=1:length(eoverh_2)
        g_pro1i(:,i4,:)...
            =(abs(P_i(:,i4,:))*sqrt(1+eoverh_2(i4)^2))./...
             (abs(D_pro1i(:,i4,:)*T_Di+L_pro1i(:,i4,:)*T_Li)*sqrt(1+eoverh_2(i4)^2));
        % T_Di, T_Li are numbers, not vectors
    end
    s_g_pro1i=find(log(g_pro1i)<0);
    k_fpro1i(s_g_pro1i)=1;
    n_fpro1(:,:,i6)=sum(k_fpro1i,3);
end
% Probability of failure
P_fpro1=sum(n_fpro1,3)/N;
% Reliability index
beta_PMr1=-norminv(P_fpro1,0,1);
toc
save beta_PMr1_S1 s_phi_sc n_fpro1 P_fpro1 beta_PMr1
```

C.1.2.4 Code 4-Reliability Indices for Partial Material Strength Reduction Factors and $L/D = 1.5$

```
clc
clear
tic
% Reliability index calculation corresponding to partial strength reduction factors and L/D=1.5
n=1e4;
N=1e6;
Z_sam=[0.5:-0.01:-1 -1.05:-0.05:-10 -10.1:-0.1:-50 -51:-1:-100];
eoverh=[0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 ...
       -10.0 -5.0 -1.0 -0.5 -0.1]; % The specific e/h values
hovere=1./eoverh;
eoverh_2=[0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 -10.0 ...
         -5.0 -1.0 -0.5 -0.1 -0]; % The specific e/h values, including extreme values
LoverD=1.5;
s_phi_sc=10;
```

```
% Preallocation
n_fpro2=zeros(8,length(eoverh_2),(N/n));

for i6=1:(N/n)
    % Calculate sample points for resistance
    % (P_isam, M_isam and hovere_isam corresponding to Z_sam, P_oi and P_ti)
    % Preallocation
    P_isam=zeros(8,n,length(Z_sam));
    M_isam=zeros(8,n,length(Z_sam));
    hovere_isam=zeros(8,n,length(Z_sam));
    P_oi=zeros(8,n,1);
    P_ti=zeros(8,n,1);

    for i1=1:8
        rn_n1=randn(1,n);
        rn_n2=randn(1,n);
        rn_n3=randn(1,n);
        rn_n4=randn(1,n);
        rn_n5=randn(1,n);
        rn_n6=randn(1,n);
        rn_n7=randn(1,n);
        rn_n8=randn(1,n);
        rn_n9=randn(1,n);
        rn_n10=randn(1,n);
        f_y=420;
        bias_f_y=1.125;
        CoV_f_y=0.098;
        mean_f_y=f_y*bias_f_y;
        std_f_y=mean_f_y*CoV_f_y;
        mu_f_y=log(mean_f_y^2/sqrt(mean_f_y^2+std_f_y^2)); % Lognormal distribution
        sigma_f_y=sqrt(log(std_f_y^2/mean_f_y^2+1));
        f_yi(1,:)=lognrnd(mu_f_y,sigma_f_y,[1,n]);

        for i3=1:length(Z_sam)
            [P_isam(i1,:,i3),M_isam(i1,:,i3),hovere_isam(i1,:,i3),P_oi(i1,:,1),P_ti(i1,:,1)]...
                =feval('ResistanceSim_S1',i1,rn_n1,rn_n2,rn_n3,rn_n4,rn_n5,rn_n6,rn_n7,rn_n8,rn_n9,...
                 rn_n10,f_yi,Z_sam(i3));
```

```
    end
end

% Calculate P_maxi
P_maxi=0.80*P_oi;

% Permute the 2nd and 3rd dimensions
P_isam=permute(P_isam,[1,3,2]);
M_isam=permute(M_isam,[1,3,2]);
hovere_isam=permute(hovere_isam,[1,3,2]);
P_oi=permute(P_oi,[1,3,2]);
P_ti=permute(P_ti,[1,3,2]);
P_maxi=permute(P_maxi,[1,3,2]);

% Calculate load effect
% LoverD=1.5; % Defined previously
% s_phi_sc % Defined previously
[D_pro2i,L_pro2i,T_Di,T_Li]=feval('LoadEffectSim_pro_S1',LoverD,s_phi_sc,n);

% Interpolation
% Calculate the unknown points (P_i)
% Preallocation
P_i=zeros(8,length(hovere),n);

for i1=1:8
    for i5=1:n
        s_M_isam=find(M_isam(i1,:,i5)>0);
        P_isampri=P_isam(i1,s_M_isam,i5);
        hovere_isampri=hovere_isam(i1,s_M_isam,i5);
        P_isampri=[P_oi(i1,1,i5) P_isampri P_ti(i1,1,i5)];
        hovere_isampri=[1e10 hovere_isampri -1e10];
        [hovere_isampris,I_hovere_isampri]=sort(hovere_isampri,'descend');
        P_i(i1,:,i5)=interp1(hovere_isampris,P_isampri(I_hovere_isampri),hovere,'linear');
        s_P_maxi=find(P_i(i1,:,i5)>P_maxi(i1,1,i5));
        P_i(i1,s_P_maxi,i5)=P_maxi(i1,1,i5);
    end
end
```

```
    % P_i includes P_maxi and P_ti
    P_i=cat(2,P_maxi,P_i,P_ti);

    % Limit state function and numbers of failure
    % Preallocation
    g_pro2i=zeros(8,length(eoverh_2),n);
    k_fpro2i=zeros(8,length(eoverh_2),n);

    for i4=1:length(eoverh_2)
        g_pro2i(:,i4,:)...
            =(abs(P_i(:,i4,:))*sqrt(1+eoverh_2(i4)^2))./...
             (abs(D_pro2i(:,i4,:)*T_Di+L_pro2i(:,i4,:)*T_Li)*sqrt(1+eoverh_2(i4)^2));
        % T_Di, T_Li are numbers, not vectors
    end
    s_g_pro2i=find(log(g_pro2i)<0);
    k_fpro2i(s_g_pro2i)=1;
    n_fpro2(:,:,i6)=sum(k_fpro2i,3);
end
% Probability of failure
P_fpro2=sum(n_fpro2,3)/N;
% Reliability index
beta_PMr2=-norminv(P_fpro2,0,1);
toc
save beta_PMr2_S1 s_phi_sc n_fpro2 P_fpro2 beta_PMr2
```

C.1.2.5 Code 5-Function of Simulated Resistances

```
% Resistance simulation
function [P_i,M_i,hovere_i,P_oi,P_ti]=ResistanceSim_S1(i1,rn_n1,rn_n2,rn_n3,rn_n4,rn_n5,rn_n6,rn_n7,...
          rn_n8,rn_n9,rn_n10,f_yi,Z)
% Nominal value combinations
% Geometric property combinations
b_com=[325 1300];
h_com=[325 1300];
gamma_com=[0.6 0.9];
d_1_com=(1+gamma_com).*h_com/2;
```

```
d_2_com=h_com/2;
d_3_com=(1-gamma_com).*h_com/2;
rho_g_com=[0.01 0.04];

% Material property combinations
f_c_com=[25 45];
% f_y=420; % Defined in beta
E_s=200000;

% Summarize property combinations in one matrix
pro_com=[b_com(1) h_com(1) gamma_com(1) d_1_com(1) d_2_com(1) d_3_com(1) f_c_com(1) rho_g_com(1);
         b_com(1) h_com(1) gamma_com(1) d_1_com(1) d_2_com(1) d_3_com(1) f_c_com(1) rho_g_com(2);
         b_com(1) h_com(1) gamma_com(1) d_1_com(1) d_2_com(1) d_3_com(1) f_c_com(2) rho_g_com(1);
         b_com(1) h_com(1) gamma_com(1) d_1_com(1) d_2_com(1) d_3_com(1) f_c_com(2) rho_g_com(2);
         b_com(2) h_com(2) gamma_com(2) d_1_com(2) d_2_com(2) d_3_com(2) f_c_com(1) rho_g_com(1);
         b_com(2) h_com(2) gamma_com(2) d_1_com(2) d_2_com(2) d_3_com(2) f_c_com(1) rho_g_com(2);
         b_com(2) h_com(2) gamma_com(2) d_1_com(2) d_2_com(2) d_3_com(2) f_c_com(2) rho_g_com(1);
         b_com(2) h_com(2) gamma_com(2) d_1_com(2) d_2_com(2) d_3_com(2) f_c_com(2) rho_g_com(2)];

% pro_com=[325 325 0.6 260 162.5 65 25 0.01;325 325 0.6 260 162.5 65 25 0.04;
         % 325 325 0.6 260 162.5 65 45 0.01;325 325 0.6 260 162.5 65 45 0.04;
         % 1300 1300 0.9 1235 650 65 25 0.01;1300 1300 0.9 1235 650 65 25 0.04;
         % 1300 1300 0.9 1235 650 65 45 0.01;1300 1300 0.9 1235 650 65 45 0.04]

% Professional factor
Prof=1;

% Nominal values
% Geometric properties
b(1,1)=pro_com(i1,1);
h(1,1)=pro_com(i1,2);
% gamma(1,1)=pro_com(i1,3);
d_1(1,1)=pro_com(i1,4);
d_2(1,1)=pro_com(i1,5);
d_3(1,1)=pro_com(i1,6);
rho_g(1,1)=pro_com(i1,8);
A_st(1,1)=rho_g(1,1)*b(1,1)*h(1,1);
```

```
A_s1(1,1)=3*A_st(1,1)/8;
A_s2(1,1)=A_st(1,1)/4;
A_s3(1,1)=3*A_st(1,1)/8;

% Material properties
f_c(1,1)=pro_com(i1,7);
% f_y=420; % Defined in beta
% E_s=200000; % Defined previously

% Professional factor
% Prof=1; % Defined previously

% Statistical parameters (Bias coefficient and CoV)
% Geometric properties
% bias_b % Use mean directly
% CoV_b % Use standard deviation directly

% bias_h % Use mean directly
% CoV_h % Use standard deviation directly

% bias_gamma=1; % Deterministic
% CoV_gamma=0;

bias_d_1=1;
% CoV_d_1 % Use standard deviation directly

bias_d_2=1;
% CoV_d_2 % Use standard deviation directly

bias_d_3=1;
% CoV_d_3 % Use standard deviation directly

% bias_rho_g=1; % Deterministic
% CoV_rho_g=0;

bias_A_s=1.0;
```

```
CoV_A_s=0.015;

% Material properties
bias_f_c=1.15;
CoV_f_c=0.211;

% bias_f_y=1.125; % Defined in beta
% CoV_f_y=0.098; % Defined in beta

% bias_E_s=1; % Deterministic
% CoV_E_s=0;

% Professional factor
bias_Prof=1.00;
CoV_Prof=0.08;

% Statistical parameters (Mean and Standard deviation)
% Geometric properties
mean_b(1,1)=b(1,1)+1.52;
std_b(1,1)=6.35;

mean_h(1,1)=h(1,1)+1.52;
std_h(1,1)=6.35;

mean_d_1(1,1)=d_1(1,1)*bias_d_1;
std_d_1(1,1)=6.35;

mean_d_2(1,1)=d_2(1,1)*bias_d_2;
if i1<=4
    std_d_2(1,1)=4.76;
else
    std_d_2(1,1)=6.35;
end

mean_d_3(1,1)=d_3(1,1)*bias_d_3;
std_d_3(1,1)=4.76;
```

```
mean_A_s1(1,1)=A_s1(1,1)*bias_A_s;
std_A_s1(1,1)=mean_A_s1(1,1)*CoV_A_s;
mean_A_s2(1,1)=A_s2(1,1)*bias_A_s;
std_A_s2(1,1)=mean_A_s2(1,1)*CoV_A_s;
mean_A_s3(1,1)=A_s3(1,1)*bias_A_s;
std_A_s3(1,1)=mean_A_s3(1,1)*CoV_A_s;

% Material properties
mean_f_c(1,1)=f_c(1,1)*bias_f_c;
std_f_c(1,1)=mean_f_c(1,1)*CoV_f_c;

% mean_f_y=f_y*bias_f_y; % Defined in beta
% std_f_y=mean_f_y*CoV_f_y; % Defined in beta

% Professional factor
mean_Prof=Prof*bias_Prof;
std_Prof=mean_Prof*CoV_Prof;

% Simulation
% Geometric properties
b_i(1,:)=mean_b(1,1)+std_b(1,1)*rn_n1; % Normal distribution
h_i(1,:)=mean_h(1,1)+std_h(1,1)*rn_n2; % Normal distribution
% gamma_i(1,1)=gamma(1,1); % Deterministic
d_1i(1,:)=mean_d_1(1,1)+std_d_1(1,1)*rn_n3; % Normal distribution
d_2i(1,:)=mean_d_2(1,1)+std_d_2(1,1)*rn_n4; % Normal distribution
d_3i(1,:)=mean_d_3(1,1)+std_d_3(1,1)*rn_n5; % Normal distribution
% rho_gi(1,1)=rho_g(1,1); % Deterministic
A_s1i(1,:)=mean_A_s1(1,1)+std_A_s1(1,1)*rn_n6; % Normal distribution
A_s2i(1,:)=mean_A_s2(1,1)+std_A_s2(1,1)*rn_n7; % Normal distribution
A_s3i(1,:)=mean_A_s3(1,1)+std_A_s3(1,1)*rn_n8; % Normal distribution

% Material properties
f_ci(1,:)=mean_f_c(1,1)+std_f_c(1,1)*rn_n9; % Normal distribution
beta_1i(1,:)=0.85-0.05*(f_ci(1,:)-28)/7;
s_beta_1_28=find(f_ci(1,:)<=28);
```

```
beta_1i(1,s_beta_1_28)=0.85;
s_beta_1_56=find(f_ci(1,:)>=56);
beta_1i(1,s_beta_1_56)=0.65;

% mu_f_y=log(mean_f_y^2/sqrt(mean_f_y^2+std_f_y^2)); % Lognormal distribution
% sigma_f_y=sqrt(log(std_f_y^2/mean_f_y^2+1));       % Defined in beta
% f_yi(1,:)=lognrnd(mu_f_y,sigma_f_y,[1,n]);

E_si=E_s; % Deterministic
epsilon_yi(1,:)=f_yi(1,:)/E_si;

% Professional factor
Prof_i(1,:)=mean_Prof+std_Prof*rn_n10; % Normal distribution

% Resistance calculation
% Calculate c_i
c_i(1,:)=(0.003./(0.003-Z*epsilon_yi(1,:))).*d_1i(1,:);
% Calculate a_i
a_i(1,:)=beta_1i(1,:).*c_i(1,:);
% Compare a_i with h_i
s_a_i=find(a_i(1,:)>h_i(1,:));
a_i(1,s_a_i)=h_i(1,s_a_i);

% Calculate epsilon_s1i, epsilon_s2i, epsilon_s3i, f_s1i, f_s2i and f_s3i
epsilon_s1i(1,:)=Z*epsilon_yi(1,:);
epsilon_s2i(1,:)=0.003*(c_i(1,:)-d_2i(1,:))./c_i(1,:);
epsilon_s3i(1,:)=0.003*(c_i(1,:)-d_3i(1,:))./c_i(1,:);
f_s1i(1,:)=epsilon_s1i(1,:)*E_si;
f_s2i(1,:)=epsilon_s2i(1,:)*E_si;
f_s3i(1,:)=epsilon_s3i(1,:)*E_si;

% Compare f_s1i, f_s2i and f_s3i with +-f_yi
s_f_s1iu=find(f_s1i(1,:)>f_yi(1,:)); % Upper boundary, f_yi
f_s1i(1,s_f_s1iu)=f_yi(1,s_f_s1iu);
s_f_s1il=find(f_s1i(1,:)<-f_yi(1,:)); % Lower boundary, -f_yi
f_s1i(1,s_f_s1il)=-f_yi(1,s_f_s1il);
```

```
s_f_s2iu=find(f_s2i(1,:)>f_yi(1,:)); % Upper boundary, f_yi
f_s2i(1,s_f_s2iu)=f_yi(1,s_f_s2iu);
s_f_s2il=find(f_s2i(1,:)<-f_yi(1,:)); % Lower boundary, -f_yi
f_s2i(1,s_f_s2il)=-f_yi(1,s_f_s2il);

s_f_s3iu=find(f_s3i(1,:)>f_yi(1,:)); % Upper boundary, f_yi
f_s3i(1,s_f_s3iu)=f_yi(1,s_f_s3iu);
s_f_s3il=find(f_s3i(1,:)<-f_yi(1,:)); % Lower boundary, -f_yi
f_s3i(1,s_f_s3il)=-f_yi(1,s_f_s3il);

% Calculate C_ci
C_ci(1,:)=0.85*f_ci(1,:).*a_i(1,:).*b_i(1,:)/1000;

% Calculate F_s1i
F_s1i(1,:)=(f_s1i(1,:)-0.85*f_ci(1,:)).*A_s1i(1,:)/1000;
s_F_s1i=find(a_i(1,:)<d_1i(1,:));
F_s1i(1,s_F_s1i)=f_s1i(1,s_F_s1i).*A_s1i(1,s_F_s1i)/1000;

% Calculate F_s2i
F_s2i(1,:)=(f_s2i(1,:)-0.85*f_ci(1,:)).*A_s2i(1,:)/1000;
s_F_s2i=find(a_i(1,:)<d_2i(1,:));
F_s2i(1,s_F_s2i)=f_s2i(1,s_F_s2i).*A_s2i(1,s_F_s2i)/1000;

% Calculate F_s3i
F_s3i(1,:)=(f_s3i(1,:)-0.85*f_ci(1,:)).*A_s3i(1,:)/1000;
s_F_s3i=find(a_i(1,:)<d_3i(1,:));
F_s3i(1,s_F_s3i)=f_s3i(1,s_F_s3i).*A_s3i(1,s_F_s3i)/1000;

% Calculate P_i and M_i, P_oi and P_ti
P_i(1,:)=Prof_i(1,:).*(C_ci(1,:)+F_s1i(1,:)+F_s2i(1,:)+F_s3i(1,:));
M_i(1,:)=Prof_i(1,:).*(C_ci(1,:).*(h_i(1,:)/2-a_i(1,:)/2)+F_s1i(1,:).*(h_i(1,:)/2-d_1i(1,:))+...
         F_s2i(1,:).*(h_i(1,:)/2-d_2i(1,:))+F_s3i(1,:).*(h_i(1,:)/2-d_3i(1,:)))/1000;
P_oi(1,:)=Prof_i(1,:).*(0.85*f_ci(1,:).*(b_i(1,:).*h_i(1,:)-(A_s1i(1,:)+A_s2i(1,:)+A_s3i(1,:)))+...
          f_yi(1,:).*(A_s1i(1,:)+A_s2i(1,:)+A_s3i(1,:)))/1000;
P_ti(1,:)=-Prof_i(1,:).*f_yi(1,:).*(A_s1i(1,:)+A_s2i(1,:)+A_s3i(1,:))/1000;
```

```
% Calculate eccentricities, e_i, and hovere_i
e_i(1,:)=M_i(1,:)./P_i(1,:); % (m)
hovere_i(1,:)=(h(1,1)/1000)./e_i(1,:); % Use nominal h
end
```

C.1.2.6 Code 6-Function of Simulated Load Effects and Nominal Values Based on ACI 318-14

```
% Load effect simulation
% (nominal loads calculation is based on design strengths corresponding to ACI 318-14)
function[D_curi,L_curi,T_Di,T_Li]=LoadEffectSim_cur_S1(LoverD,n)
% Load design strengths
load alpha_PM_S1.mat phiP_n

% Nominal values
% Loads
% LoverD=[0.5 1.5]; % Input of funtion
D_cur=abs(phiP_n)/(1.2+1.6*LoverD); % Use absolute values
L_cur=LoverD*D_cur;

% Transformations from load to load effect
T_D=1;
T_L=1;

% Statistical parameters (Bias coefficient and CoV)
% Loads
bias_D=1.05;
CoV_D=0.10;

bias_L=1.00;
CoV_L=0.25;

% Transformations from load to load effect
% bias_T_D=1; % Effect is accounted in D
% CoV_T_D=0;
```

```
% bias_T_L=1; % Effect is accounted in L
% CoV_T_L=0;

% Statistical parameters (Mean and Standard deviation)
% Loads
mean_D_cur=D_cur*bias_D;
std_D_cur=mean_D_cur*CoV_D;

mean_L_cur=L_cur*bias_L;
std_L_cur=mean_L_cur*CoV_L;

% Transformations from load to load effect
% T_D Effect is accounted in D
% T_L Effect is accounted in L

% Simulation
% Preallocation
D_curi=zeros(8,size(D_cur,2),n);
alpha_L_cur=zeros(8,size(D_cur,2));
mu_L_cur=zeros(8,size(D_cur,2));
L_curi=zeros(8,size(D_cur,2),n);

for i1=1:8
    for i4=1:size(D_cur,2)
        rn_n11=randn(1,1,n); % Standard normally distributed random numbers
        rn_u1=rand(1,1,n); % Standard uniformly distributed random numbers

        % Dead Loads
        D_curi(i1,i4,:)=mean_D_cur(i1,i4)+std_D_cur(i1,i4)*rn_n11; % Normal distribution

        % Live Loads
        alpha_L_cur(i1,i4)=(1/sqrt(6))*(pi/std_L_cur(i1,i4)); % Gumbel distribution
        mu_L_cur(i1,i4)=mean_L_cur(i1,i4)-0.5772/alpha_L_cur(i1,i4);
        L_curi(i1,i4,:)=mu_L_cur(i1,i4)-log(-log(rn_u1))/alpha_L_cur(i1,i4);
    end
end
```

```
s_negcur=find(phiP_n(1,:)<0);
D_curi(:,s_negcur,:)=-D_curi(:,s_negcur,:); % Negative values indicate tension
L_curi(:,s_negcur,:)=-L_curi(:,s_negcur,:);

% Transformations from load to load effect
T_Di=T_D; % Effect is accounted in D
T_Li=T_L; % Effect is accounted in L
end
```

C.1.2.7 Code 7-Function of Simulated Load Effects and Nominal Values Based on Partial Material Strength Reduction Factors

```
% Load effect simulation
% (nominal loads calculation is based on design strengths corresponding to
% partial strength reduction factors)
function[D_proi,L_proi,T_Di,T_Li]=LoadEffectSim_pro_S1(LoverD,s_phi_sc,n)
% Load design strengths
load alpha_PM_S1.mat P_r

% Nominal values
% Loads
% LoverD=[0.5 1.5]; % Input of funtion
% s_phi_sc % Input of funtion
D_pro=abs(P_r(:,:,s_phi_sc))/(1.2+1.6*LoverD); % Use absolute values
L_pro=LoverD*D_pro;

% Transformations from load to load effect
T_D=1;
T_L=1;

% Statistical parameters (Bias coefficient and CoV)
% Loads
bias_D=1.05;
CoV_D=0.10;

bias_L=1.00;
```

```
CoV_L=0.25;

% Transformations from load to load effect
% bias_T_D=1; % Effect is accounted in D
% CoV_T_D=0;

% bias_T_L=1; % Effect is accounted in L
% CoV_T_L=0;

% Statistical parameters (Mean and Standard deviation)
% Loads
mean_D_pro=D_pro*bias_D;
std_D_pro=mean_D_pro*CoV_D;

mean_L_pro=L_pro*bias_L;
std_L_pro=mean_L_pro*CoV_L;

% Transformations from load to load effect
% T_D Effect is accounted in D
% T_L Effect is accounted in L

% Simulation
% Preallocation
D_proi=zeros(8,size(D_pro,2),n);
alpha_L_pro=zeros(8,size(D_pro,2));
mu_L_pro=zeros(8,size(D_pro,2));
L_proi=zeros(8,size(D_pro,2),n);

for i1=1:8
    for i4=1:size(D_pro,2)
        rn_n11=randn(1,1,n); % Standard normally distributed random numbers
        rn_u1=rand(1,1,n); % Standard uniformly distributed random numbers

        % Dead Loads
        D_proi(i1,i4,:)=mean_D_pro(i1,i4)+std_D_pro(i1,i4)*rn_n11; % Normal distribution
```

```
        % Live Loads
        alpha_L_pro(i1,i4)=(1/sqrt(6))*(pi/std_L_pro(i1,i4)); % Gumbel distribution
        mu_L_pro(i1,i4)=mean_L_pro(i1,i4)-0.5772/alpha_L_pro(i1,i4);
        L_proi(i1,i4,:)=mu_L_pro(i1,i4)-log(-log(rn_u1))/alpha_L_pro(i1,i4);
    end
end
s_negpro=find(P_r(1,:,s_phi_sc)<0);
D_proi(:,s_negpro,:)=-D_proi(:,s_negpro,:); % Negative values indicate tension
L_proi(:,s_negpro,:)=-L_proi(:,s_negpro,:);

% Transformations from load to load effect
T_Di=T_D; % Effect is accounted in D
T_Li=T_L; % Effect is accounted in L
end
```

C.1.3 Column Section 5

C.1.3.1 Code 1-Reliability Indices for ACI 318-14 and *L*/*D* = 0.5

```
clc
clear
tic
% Reliability index calculation corresponding to ACI 318-14 and L/D=0.5
n=1e4;
N=1e6;
Z_sam=[0.5:-0.01:-1 -1.05:-0.05:-10 -10.1:-0.1:-50 -51:-1:-100];
eoverh=[0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 ...
       -10.0 -5.0 -1.0 -0.5 -0.1]; % The specific e/h values
hovere=1./eoverh;
eoverh_2=[0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 -10.0 ...
         -5.0 -1.0 -0.5 -0.1 -0]; % The specific e/h values, including extreme values
LoverD=0.5;

% Preallocation
n_fcur1=zeros(8,length(eoverh_2),(N/n));
```

```
for i6=1:(N/n)
    % Calculate sample points for resistance
    % (P_isam, M_isam and hovere_isam corresponding to Z_sam, P_oi and P_ti)
    % Preallocation
    P_isam=zeros(8,n,length(Z_sam));
    M_isam=zeros(8,n,length(Z_sam));
    hovere_isam=zeros(8,n,length(Z_sam));
    P_oi=zeros(8,n,1);
    P_ti=zeros(8,n,1);

    for i1=1:8
        rn_n1=randn(1,n);
        rn_n2=randn(1,n);
        rn_n3=randn(1,n);
        rn_n4=randn(1,n);
        rn_n5=randn(1,n);
        rn_n6=randn(1,n);
        rn_n7=randn(1,n);
        rn_n8=randn(1,n);
        rn_n9=randn(1,n);
        rn_n10=randn(1,n);
        rn_n11=randn(1,n);
        rn_n12=randn(1,n);
        rn_n13=randn(1,n);
        f_y=420;
        bias_f_y=1.125;
        CoV_f_y=0.098;
        mean_f_y=f_y*bias_f_y;
        std_f_y=mean_f_y*CoV_f_y;
        mu_f_y=log(mean_f_y^2/sqrt(mean_f_y^2+std_f_y^2)); % Lognormal distribution
        sigma_f_y=sqrt(log(std_f_y^2/mean_f_y^2+1));
        f_yi(1,:)=lognrnd(mu_f_y,sigma_f_y,[1,n]);

        for i3=1:length(Z_sam)
            [P_isam(i1,:,i3),M_isam(i1,:,i3),hovere_isam(i1,:,i3),P_oi(i1,:,1),P_ti(i1,:,1)]...
                =feval('ResistanceSim_S5',i1,rn_n1,rn_n2,rn_n3,rn_n4,rn_n5,rn_n6,rn_n7,rn_n8,rn_n9,...
                 rn_n10,rn_n11,rn_n12,rn_n13,f_yi,Z_sam(i3));
```

```
    end
end

% Calculate P_maxi
P_maxi=0.85*P_oi;

% Permute the 2nd and 3rd dimensions
P_isam=permute(P_isam,[1,3,2]);
M_isam=permute(M_isam,[1,3,2]);
hovere_isam=permute(hovere_isam,[1,3,2]);
P_oi=permute(P_oi,[1,3,2]);
P_ti=permute(P_ti,[1,3,2]);
P_maxi=permute(P_maxi,[1,3,2]);

% Calculate load effect
% LoverD=0.5; % Defined previously
[D_cur1i,L_cur1i,T_Di,T_Li]=feval('LoadEffectSim_cur_S5',LoverD,n);

% Interpolation
% Calculate the unknown points (P_i)
% Preallocation
P_i=zeros(8,length(hovere),n);

for i1=1:8
    for i5=1:n
        s_M_isam=find(M_isam(i1,:,i5)>0);
        P_isampri=P_isam(i1,s_M_isam,i5);
        hovere_isampri=hovere_isam(i1,s_M_isam,i5);
        P_isampri=[P_oi(i1,1,i5) P_isampri P_ti(i1,1,i5)];
        hovere_isampri=[1e10 hovere_isampri -1e10];
        [hovere_isampris,I_hovere_isampri]=sort(hovere_isampri,'descend');
        P_i(i1,:,i5)=interp1(hovere_isampris,P_isampri(I_hovere_isampri),hovere,'linear');
        s_P_maxi=find(P_i(i1,:,i5)>P_maxi(i1,1,i5));
        P_i(i1,s_P_maxi,i5)=P_maxi(i1,1,i5);
    end
end
```

```
    % P_i includes P_maxi and P_ti
    P_i=cat(2,P_maxi,P_i,P_ti);

    % Limit state function and numbers of failure
    % Preallocation
    g_cur1i=zeros(8,length(eoverh_2),n);
    k_fcur1i=zeros(8,length(eoverh_2),n);

    for i4=1:length(eoverh_2)
        g_cur1i(:,i4,:)...
            =(abs(P_i(:,i4,:))*sqrt(1+eoverh_2(i4)^2))./...
             (abs(D_cur1i(:,i4,:)*T_Di+L_cur1i(:,i4,:)*T_Li)*sqrt(1+eoverh_2(i4)^2));
        % T_Di, T_Li are numbers, not vectors
    end
    s_g_cur1i=find(log(g_cur1i)<0);
    k_fcur1i(s_g_cur1i)=1;
    n_fcur1(:,:,i6)=sum(k_fcur1i,3);
end
% Probability of failure
P_fcur1=sum(n_fcur1,3)/N;
% Reliability index
beta_PMu1=-norminv(P_fcur1,0,1);
toc
save beta_PMu1_S5 n_fcur1 P_fcur1 beta_PMu1
```

C.1.3.2 Code 2-Reliability Indices for ACI 318-14 and $L/D = 1.5$

```
clc
clear
tic
% Reliability index calculation corresponding to ACI 318-14 and L/D=1.5
n=1e4;
N=1e6;
Z_sam=[0.5:-0.01:-1 -1.05:-0.05:-10 -10.1:-0.1:-50 -51:-1:-100];
eoverh=[0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 ...
       -10.0 -5.0 -1.0 -0.5 -0.1]; % The specific e/h values
```

```
hovere=1./eoverh;
eoverh_2=[0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 -10.0 ...
         -5.0 -1.0 -0.5 -0.1 -0]; % The specific e/h values, including extreme values
LoverD=1.5;

% Preallocation
n_fcur2=zeros(8,length(eoverh_2),(N/n));

for i6=1:(N/n)
    % Calculate sample points for resistance
    % (P_isam, M_isam and hovere_isam corresponding to Z_sam, P_oi and P_ti)
    % Preallocation
    P_isam=zeros(8,n,length(Z_sam));
    M_isam=zeros(8,n,length(Z_sam));
    hovere_isam=zeros(8,n,length(Z_sam));
    P_oi=zeros(8,n,1);
    P_ti=zeros(8,n,1);

    for i1=1:8
        rn_n1=randn(1,n);
        rn_n2=randn(1,n);
        rn_n3=randn(1,n);
        rn_n4=randn(1,n);
        rn_n5=randn(1,n);
        rn_n6=randn(1,n);
        rn_n7=randn(1,n);
        rn_n8=randn(1,n);
        rn_n9=randn(1,n);
        rn_n10=randn(1,n);
        rn_n11=randn(1,n);
        rn_n12=randn(1,n);
        rn_n13=randn(1,n);
        f_y=420;
        bias_f_y=1.125;
        CoV_f_y=0.098;
        mean_f_y=f_y*bias_f_y;
        std_f_y=mean_f_y*CoV_f_y;
```

```
    mu_f_y=log(mean_f_y^2/sqrt(mean_f_y^2+std_f_y^2)); % Lognormal distribution
    sigma_f_y=sqrt(log(std_f_y^2/mean_f_y^2+1));
    f_yi(1,:)=lognrnd(mu_f_y,sigma_f_y,[1,n]);

    for i3=1:length(Z_sam)
        [P_isam(i1,:,i3),M_isam(i1,:,i3),hovere_isam(i1,:,i3),P_oi(i1,:,1),P_ti(i1,:,1)]...
            =feval('ResistanceSim_S5',i1,rn_n1,rn_n2,rn_n3,rn_n4,rn_n5,rn_n6,rn_n7,rn_n8,rn_n9,...
             rn_n10,rn_n11,rn_n12,rn_n13,f_yi,Z_sam(i3));
    end
end

% Calculate P_maxi
P_maxi=0.85*P_oi;

% Permute the 2nd and 3rd dimensions
P_isam=permute(P_isam,[1,3,2]);
M_isam=permute(M_isam,[1,3,2]);
hovere_isam=permute(hovere_isam,[1,3,2]);
P_oi=permute(P_oi,[1,3,2]);
P_ti=permute(P_ti,[1,3,2]);
P_maxi=permute(P_maxi,[1,3,2]);

% Calculate load effect
% LoverD=1.5; % Defined previously
[D_cur2i,L_cur2i,T_Di,T_Li]=feval('LoadEffectSim_cur_S5',LoverD,n);

% Interpolation
% Calculate the unknown points (P_i)
% Preallocation
P_i=zeros(8,length(hovere),n);

for i1=1:8
    for i5=1:n
        s_M_isam=find(M_isam(i1,:,i5)>0);
        P_isampri=P_isam(i1,s_M_isam,i5);
        hovere_isampri=hovere_isam(i1,s_M_isam,i5);
```

```
            P_isampri=[P_oi(i1,1,i5) P_isampri P_ti(i1,1,i5)];
            hovere_isampri=[1e10 hovere_isampri -1e10];
            [hovere_isampris,I_hovere_isampri]=sort(hovere_isampri,'descend');
            P_i(i1,:,i5)=interp1(hovere_isampris,P_isampri(I_hovere_isampri),hovere,'linear');
            s_P_maxi=find(P_i(i1,:,i5)>P_maxi(i1,1,i5));
            P_i(i1,s_P_maxi,i5)=P_maxi(i1,1,i5);
        end
    end

    % P_i includes P_maxi and P_ti
    P_i=cat(2,P_maxi,P_i,P_ti);

    % Limit state function and numbers of failure
    % Preallocation
    g_cur2i=zeros(8,length(eoverh_2),n);
    k_fcur2i=zeros(8,length(eoverh_2),n);

    for i4=1:length(eoverh_2)
        g_cur2i(:,i4,:)...
            =(abs(P_i(:,i4,:))*sqrt(1+eoverh_2(i4)^2))./...
             (abs(D_cur2i(:,i4,:)*T_Di+L_cur2i(:,i4,:)*T_Li)*sqrt(1+eoverh_2(i4)^2));
        % T_Di, T_Li are numbers, not vectors
    end
    s_g_cur2i=find(log(g_cur2i)<0);
    k_fcur2i(s_g_cur2i)=1;
    n_fcur2(:,:,i6)=sum(k_fcur2i,3);
end
% Probability of failure
P_fcur2=sum(n_fcur2,3)/N;
% Reliability index
beta_PMu2=-norminv(P_fcur2,0,1);
toc
save beta_PMu2_S5 n_fcur2 P_fcur2 beta_PMu2
```

C.1.3.3 Code 3-Reliability Indices for Partial Material Strength Reduction Factors and $L/D = 0.5$

```
clc
```

```
clear
tic
% Reliability index calculation corresponding to partial strength reduction factors and L/D=0.5
n=1e4;
N=1e6;
Z_sam=[0.5:-0.01:-1 -1.05:-0.05:-10 -10.1:-0.1:-50 -51:-1:-100];
eoverh=[0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 ...
       -10.0 -5.0 -1.0 -0.5 -0.1]; % The specific e/h values
hovere=1./eoverh;
eoverh_2=[0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 -10.0 ...
         -5.0 -1.0 -0.5 -0.1 -0]; % The specific e/h values, including extreme values
LoverD=0.5;
s_phi_sc=15;

% Preallocation
n_fpro1=zeros(8,length(eoverh_2),(N/n));

for i6=1:(N/n)
    % Calculate sample points for resistance
    % (P_isam, M_isam and hovere_isam corresponding to Z_sam, P_oi and P_ti)
    % Preallocation
    P_isam=zeros(8,n,length(Z_sam));
    M_isam=zeros(8,n,length(Z_sam));
    hovere_isam=zeros(8,n,length(Z_sam));
    P_oi=zeros(8,n,1);
    P_ti=zeros(8,n,1);

    for i1=1:8
        rn_n1=randn(1,n);
        rn_n2=randn(1,n);
        rn_n3=randn(1,n);
        rn_n4=randn(1,n);
        rn_n5=randn(1,n);
        rn_n6=randn(1,n);
        rn_n7=randn(1,n);
        rn_n8=randn(1,n);
        rn_n9=randn(1,n);
```

```
    rn_n10=randn(1,n);
    rn_n11=randn(1,n);
    rn_n12=randn(1,n);
    rn_n13=randn(1,n);
    f_y=420;
    bias_f_y=1.125;
    CoV_f_y=0.098;
    mean_f_y=f_y*bias_f_y;
    std_f_y=mean_f_y*CoV_f_y;
    mu_f_y=log(mean_f_y^2/sqrt(mean_f_y^2+std_f_y^2)); % Lognormal distribution
    sigma_f_y=sqrt(log(std_f_y^2/mean_f_y^2+1));
    f_yi(1,:)=lognrnd(mu_f_y,sigma_f_y,[1,n]);

    for i3=1:length(Z_sam)
        [P_isam(i1,:,i3),M_isam(i1,:,i3),hovere_isam(i1,:,i3),P_oi(i1,:,1),P_ti(i1,:,1)]...
            =feval('ResistanceSim_S5',i1,rn_n1,rn_n2,rn_n3,rn_n4,rn_n5,rn_n6,rn_n7,rn_n8,rn_n9,...
             rn_n10,rn_n11,rn_n12,rn_n13,f_yi,Z_sam(i3));
    end
end

% Calculate P_maxi
P_maxi=0.85*P_oi;

% Permute the 2nd and 3rd dimensions
P_isam=permute(P_isam,[1,3,2]);
M_isam=permute(M_isam,[1,3,2]);
hovere_isam=permute(hovere_isam,[1,3,2]);
P_oi=permute(P_oi,[1,3,2]);
P_ti=permute(P_ti,[1,3,2]);
P_maxi=permute(P_maxi,[1,3,2]);

% Calculate load effect
% LoverD=0.5; % Defined previously
% s_phi_sc % Defined previously
[D_pro1i,L_pro1i,T_Di,T_Li]=feval('LoadEffectSim_pro_S5',LoverD,s_phi_sc,n);

% Interpolation
```

```
    % Calculate the unknown points (P_i)
    % Preallocation
    P_i=zeros(8,length(hovere),n);

    for i1=1:8
        for i5=1:n
            s_M_isam=find(M_isam(i1,:,i5)>0);
            P_isampri=P_isam(i1,s_M_isam,i5);
            hovere_isampri=hovere_isam(i1,s_M_isam,i5);
            P_isampri=[P_oi(i1,1,i5) P_isampri P_ti(i1,1,i5)];
            hovere_isampri=[1e10 hovere_isampri -1e10];
            [hovere_isampris,I_hovere_isampri]=sort(hovere_isampri,'descend');
            P_i(i1,:,i5)=interp1(hovere_isampris,P_isampri(I_hovere_isampri),hovere,'linear');
            s_P_maxi=find(P_i(i1,:,i5)>P_maxi(i1,1,i5));
            P_i(i1,s_P_maxi,i5)=P_maxi(i1,1,i5);
        end
    end

    % P_i includes P_maxi and P_ti
    P_i=cat(2,P_maxi,P_i,P_ti);

    % Limit state function and numbers of failure
    % Preallocation
    g_pro1i=zeros(8,length(eoverh_2),n);
    k_fpro1i=zeros(8,length(eoverh_2),n);

    for i4=1:length(eoverh_2)
        g_pro1i(:,i4,:)...
            =(abs(P_i(:,i4,:))*sqrt(1+eoverh_2(i4)^2))./...
             (abs(D_pro1i(:,i4,:)*T_Di+L_pro1i(:,i4,:)*T_Li)*sqrt(1+eoverh_2(i4)^2));
        % T_Di, T_Li are numbers, not vectors
    end
    s_g_pro1i=find(log(g_pro1i)<0);
    k_fpro1i(s_g_pro1i)=1;
    n_fpro1(:,:,i6)=sum(k_fpro1i,3);
end
% Probability of failure
```

```
P_fpro1=sum(n_fpro1,3)/N;
% Reliability index
beta_PMr1=-norminv(P_fpro1,0,1);
toc
save beta_PMr1_S5 s_phi_sc n_fpro1 P_fpro1 beta_PMr1
```

C.1.3.4 Code 4-Reliability Indices for Partial Material Strength Reduction Factors and $L/D = 1.5$

```
clc
clear
tic
% Reliability index calculation corresponding to partial strength reduction factors and L/D=1.5
n=1e4;
N=1e6;
Z_sam=[0.5:-0.01:-1 -1.05:-0.05:-10 -10.1:-0.1:-50 -51:-1:-100];
eoverh=[0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 ...
       -10.0 -5.0 -1.0 -0.5 -0.1]; % The specific e/h values
hovere=1./eoverh;
eoverh_2=[0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 -10.0 ...
         -5.0 -1.0 -0.5 -0.1 -0]; % The specific e/h values, including extreme values
LoverD=1.5;
s_phi_sc=15;

% Preallocation
n_fpro2=zeros(8,length(eoverh_2),(N/n));

for i6=1:(N/n)
    % Calculate sample points for resistance
    % (P_isam, M_isam and hovere_isam corresponding to Z_sam, P_oi and P_ti)
    % Preallocation
    P_isam=zeros(8,n,length(Z_sam));
    M_isam=zeros(8,n,length(Z_sam));
    hovere_isam=zeros(8,n,length(Z_sam));
    P_oi=zeros(8,n,1);
    P_ti=zeros(8,n,1);

    for i1=1:8
```

```
    rn_n1=randn(1,n);
    rn_n2=randn(1,n);
    rn_n3=randn(1,n);
    rn_n4=randn(1,n);
    rn_n5=randn(1,n);
    rn_n6=randn(1,n);
    rn_n7=randn(1,n);
    rn_n8=randn(1,n);
    rn_n9=randn(1,n);
    rn_n10=randn(1,n);
    rn_n11=randn(1,n);
    rn_n12=randn(1,n);
    rn_n13=randn(1,n);
    f_y=420;
    bias_f_y=1.125;
    CoV_f_y=0.098;
    mean_f_y=f_y*bias_f_y;
    std_f_y=mean_f_y*CoV_f_y;
    mu_f_y=log(mean_f_y^2/sqrt(mean_f_y^2+std_f_y^2)); % Lognormal distribution
    sigma_f_y=sqrt(log(std_f_y^2/mean_f_y^2+1));
    f_yi(1,:)=lognrnd(mu_f_y,sigma_f_y,[1,n]);

    for i3=1:length(Z_sam)
        [P_isam(i1,:,i3),M_isam(i1,:,i3),hovere_isam(i1,:,i3),P_oi(i1,:,1),P_ti(i1,:,1)]...
            =feval('ResistanceSim_S5',i1,rn_n1,rn_n2,rn_n3,rn_n4,rn_n5,rn_n6,rn_n7,rn_n8,rn_n9,...
             rn_n10,rn_n11,rn_n12,rn_n13,f_yi,Z_sam(i3));
    end
end

% Calculate P_maxi
P_maxi=0.85*P_oi;

% Permute the 2nd and 3rd dimensions
P_isam=permute(P_isam,[1,3,2]);
M_isam=permute(M_isam,[1,3,2]);
hovere_isam=permute(hovere_isam,[1,3,2]);
P_oi=permute(P_oi,[1,3,2]);
```

```
P_ti=permute(P_ti,[1,3,2]);
P_maxi=permute(P_maxi,[1,3,2]);

% Calculate load effect
% LoverD=1.5; % Defined previously
% s_phi_sc % Defined previously
[D_pro2i,L_pro2i,T_Di,T_Li]=feval('LoadEffectSim_pro_S5',LoverD,s_phi_sc,n);

% Interpolation
% Calculate the unknown points (P_i)
% Preallocation
P_i=zeros(8,length(hovere),n);

for i1=1:8
    for i5=1:n
        s_M_isam=find(M_isam(i1,:,i5)>0);
        P_isampri=P_isam(i1,s_M_isam,i5);
        hovere_isampri=hovere_isam(i1,s_M_isam,i5);
        P_isampri=[P_oi(i1,1,i5) P_isampri P_ti(i1,1,i5)];
        hovere_isampri=[1e10 hovere_isampri -1e10];
        [hovere_isampris,I_hovere_isampri]=sort(hovere_isampri,'descend');
        P_i(i1,:,i5)=interp1(hovere_isampris,P_isampri(I_hovere_isampri),hovere,'linear');
        s_P_maxi=find(P_i(i1,:,i5)>P_maxi(i1,1,i5));
        P_i(i1,s_P_maxi,i5)=P_maxi(i1,1,i5);
    end
end

% P_i includes P_maxi and P_ti
P_i=cat(2,P_maxi,P_i,P_ti);

% Limit state function and numbers of failure
% Preallocation
g_pro2i=zeros(8,length(eoverh_2),n);
k_fpro2i=zeros(8,length(eoverh_2),n);

for i4=1:length(eoverh_2)
```

```
        g_pro2i(:,i4,:)...
            =(abs(P_i(:,i4,:))*sqrt(1+eoverh_2(i4)^2))./...
             (abs(D_pro2i(:,i4,:)*T_Di+L_pro2i(:,i4,:)*T_Li)*sqrt(1+eoverh_2(i4)^2));
        % T_Di, T_Li are numbers, not vectors
    end
    s_g_pro2i=find(log(g_pro2i)<0);
    k_fpro2i(s_g_pro2i)=1;
    n_fpro2(:,:,i6)=sum(k_fpro2i,3);
end
% Probability of failure
P_fpro2=sum(n_fpro2,3)/N;
% Reliability index
beta_PMr2=-norminv(P_fpro2,0,1);
toc
save beta_PMr2_S5 s_phi_sc n_fpro2 P_fpro2 beta_PMr2
```

C.1.3.5 Code 5-Function of Simulated Resistances

```
% Resistance simulation
function [P_i,M_i,hovere_i,P_oi,P_ti]=ResistanceSim_S5(i1,rn_n1,rn_n2,rn_n3,rn_n4,rn_n5,rn_n6,rn_n7,...
          rn_n8,rn_n9,rn_n10,rn_n11,rn_n12,rn_n13,f_yi,Z)
% Nominal value combinations
% Geometric property combinations
h_com=[325 1300];
gamma_com=[0.6 0.9];
d_1_com=(1+gamma_com).*h_com/2;
d_2_com=(2+2^0.5*gamma_com).*h_com/4;
d_3_com=h_com/2;
d_4_com=(2-2^0.5*gamma_com).*h_com/4;
d_5_com=(1-gamma_com).*h_com/2;
rho_g_com=[0.01 0.04];

% Material property combinations
f_c_com=[25 45];
% f_y=420; % Defined in beta
E_s=200000;
```

```
% Summarize property combinations in one matrix
pro_com=[h_com(1) gamma_com(1) d_1_com(1) d_2_com(1) d_3_com(1) d_4_com(1) d_5_com(1) ...
         f_c_com(1) rho_g_com(1);
         h_com(1) gamma_com(1) d_1_com(1) d_2_com(1) d_3_com(1) d_4_com(1) d_5_com(1) ...
         f_c_com(1) rho_g_com(2);
         h_com(1) gamma_com(1) d_1_com(1) d_2_com(1) d_3_com(1) d_4_com(1) d_5_com(1) ...
         f_c_com(2) rho_g_com(1);
         h_com(1) gamma_com(1) d_1_com(1) d_2_com(1) d_3_com(1) d_4_com(1) d_5_com(1) ...
         f_c_com(2) rho_g_com(2);
         h_com(2) gamma_com(2) d_1_com(2) d_2_com(2) d_3_com(2) d_4_com(2) d_5_com(2) ...
         f_c_com(1) rho_g_com(1);
         h_com(2) gamma_com(2) d_1_com(2) d_2_com(2) d_3_com(2) d_4_com(2) d_5_com(2) ...
         f_c_com(1) rho_g_com(2);
         h_com(2) gamma_com(2) d_1_com(2) d_2_com(2) d_3_com(2) d_4_com(2) d_5_com(2) ...
         f_c_com(2) rho_g_com(1);
         h_com(2) gamma_com(2) d_1_com(2) d_2_com(2) d_3_com(2) d_4_com(2) d_5_com(2) ...
         f_c_com(2) rho_g_com(2)];

% Professional factor
Prof=1;

% Nominal values
% Geometric properties
h(1,1)=pro_com(i1,1);
% gamma(1,1)=pro_com(i1,2);
d_1(1,1)=pro_com(i1,3);
d_2(1,1)=pro_com(i1,4);
d_3(1,1)=pro_com(i1,5);
d_4(1,1)=pro_com(i1,6);
d_5(1,1)=pro_com(i1,7);
rho_g(1,1)=pro_com(i1,9);
A_st(1,1)=rho_g(1,1)*pi*h(1,1)^2/4;
A_s1(1,1)=A_st(1,1)/8;
A_s2(1,1)=A_st(1,1)/4;
A_s3(1,1)=A_st(1,1)/4;
A_s4(1,1)=A_st(1,1)/4;
A_s5(1,1)=A_st(1,1)/8;
```

```
% Material properties
f_c(1,1)=pro_com(i1,8);
% f_y=420; % Defined in beta
% E_s=200000; % Defined previously

% Professional factor
% Prof=1; % Defined previously

% Statistical parameters (Bias coefficient and CoV)
% Geometric properties
% bias_h % Use mean directly
% CoV_h % Use standard deviation directly

% bias_gamma=1; % Deterministic
% CoV_gamma=0;

bias_d_1=1;
% CoV_d_1 % Use standard deviation directly

bias_d_2=1;
% CoV_d_2 % Use standard deviation directly

bias_d_3=1;
% CoV_d_3 % Use standard deviation directly

bias_d_4=1;
% CoV_d_4 % Use standard deviation directly

bias_d_5=1;
% CoV_d_5 % Use standard deviation directly

% bias_rho_g=1; % Deterministic
% CoV_rho_g=0;
```

```
bias_A_s=1.0;
CoV_A_s=0.015;

% Material properties
bias_f_c=1.15;
CoV_f_c=0.211;

% bias_f_y=1.125; % Defined in beta
% CoV_f_y=0.098; % Defined in beta

% bias_E_s=1; % Deterministic
% CoV_E_s=0;

% Professional factor
bias_Prof=1.05;
CoV_Prof=0.06;

% Statistical parameters (Mean and Standard deviation)
% Geometric properties
mean_h(1,1)=h(1,1)+0;
std_h(1,1)=4.76;

mean_d_1(1,1)=d_1(1,1)*bias_d_1;
std_d_1(1,1)=6.35;

mean_d_2(1,1)=d_2(1,1)*bias_d_2;
std_d_2(1,1)=6.35;

mean_d_3(1,1)=d_3(1,1)*bias_d_3;
if i1<=4
    std_d_3(1,1)=4.76;
else
    std_d_3(1,1)=6.35;
end

mean_d_4(1,1)=d_4(1,1)*bias_d_4;
```

```
if i1<=4
    std_d_4(1,1)=4.76;
else
    std_d_4(1,1)=6.35;
end

mean_d_5(1,1)=d_5(1,1)*bias_d_5;
std_d_5(1,1)=4.76;

mean_A_s1(1,1)=A_s1(1,1)*bias_A_s;
std_A_s1(1,1)=mean_A_s1(1,1)*CoV_A_s;
mean_A_s2(1,1)=A_s2(1,1)*bias_A_s;
std_A_s2(1,1)=mean_A_s2(1,1)*CoV_A_s;
mean_A_s3(1,1)=A_s3(1,1)*bias_A_s;
std_A_s3(1,1)=mean_A_s3(1,1)*CoV_A_s;
mean_A_s4(1,1)=A_s4(1,1)*bias_A_s;
std_A_s4(1,1)=mean_A_s4(1,1)*CoV_A_s;
mean_A_s5(1,1)=A_s5(1,1)*bias_A_s;
std_A_s5(1,1)=mean_A_s5(1,1)*CoV_A_s;

% Material properties
mean_f_c(1,1)=f_c(1,1)*bias_f_c;
std_f_c(1,1)=mean_f_c(1,1)*CoV_f_c;

% mean_f_y=f_y*bias_f_y; % Defined in beta
% std_f_y=mean_f_y*CoV_f_y; % Defined in beta

% Professional factor
mean_Prof=Prof*bias_Prof;
std_Prof=mean_Prof*CoV_Prof;

% Simulation
% Geometric properties
h_i(1,:)=mean_h(1,1)+std_h(1,1)*rn_n1; % Normal distribution
% gamma_i(1,1)=gamma(1,1); % Deterministic
d_1i(1,:)=mean_d_1(1,1)+std_d_1(1,1)*rn_n2; % Normal distribution
```

```
d_2i(1,:)=mean_d_2(1,1)+std_d_2(1,1)*rn_n3; % Normal distribution
d_3i(1,:)=mean_d_3(1,1)+std_d_3(1,1)*rn_n4; % Normal distribution
d_4i(1,:)=mean_d_4(1,1)+std_d_4(1,1)*rn_n5; % Normal distribution
d_5i(1,:)=mean_d_5(1,1)+std_d_5(1,1)*rn_n6; % Normal distribution
% rho_gi(1,1)=rho_g(1,1); % Deterministic
A_s1i(1,:)=mean_A_s1(1,1)+std_A_s1(1,1)*rn_n7; % Normal distribution
A_s2i(1,:)=mean_A_s2(1,1)+std_A_s2(1,1)*rn_n8; % Normal distribution
A_s3i(1,:)=mean_A_s3(1,1)+std_A_s3(1,1)*rn_n9; % Normal distribution
A_s4i(1,:)=mean_A_s4(1,1)+std_A_s4(1,1)*rn_n10; % Normal distribution
A_s5i(1,:)=mean_A_s5(1,1)+std_A_s5(1,1)*rn_n11; % Normal distribution

% Material properties
f_ci(1,:)=mean_f_c(1,1)+std_f_c(1,1)*rn_n12; % Normal distribution
beta_1i(1,:)=0.85-0.05*(f_ci(1,:)-28)/7;
s_beta_1_28=find(f_ci(1,:)<=28);
beta_1i(1,s_beta_1_28)=0.85;
s_beta_1_56=find(f_ci(1,:)>=56);
beta_1i(1,s_beta_1_56)=0.65;

% mu_f_y=log(mean_f_y^2/sqrt(mean_f_y^2+std_f_y^2)); % Lognormal distribution
% sigma_f_y=sqrt(log(std_f_y^2/mean_f_y^2+1));       % Defined in beta
% f_yi(1,:)=lognrnd(mu_f_y,sigma_f_y,[1,n]);

E_si=E_s; % Deterministic
epsilon_yi(1,:)=f_yi(1,:)/E_si;

% Professional factor
Prof_i(1,:)=mean_Prof+std_Prof*rn_n13; % Normal distribution

% Resistance calculation
% Calculate c_i
c_i(1,:)=(0.003./(0.003-Z*epsilon_yi(1,:))).*d_1i(1,:);
% Calculate a_i
a_i(1,:)=beta_1i(1,:).*c_i(1,:);
% Compare a_i with h_i
s_a_i=find(a_i(1,:)>h_i(1,:));
a_i(1,s_a_i)=h_i(1,s_a_i);
```

```
% Calculate epsilon_s1i, epsilon_s2i, epsilon_s3i, epsilon_s4i, epsilon_s5i,
% f_s1i, f_s2i, f_s3i, f_s4i and f_s5i
epsilon_s1i(1,:)=Z*epsilon_yi(1,:);
epsilon_s2i(1,:)=0.003*(c_i(1,:)-d_2i(1,:))./c_i(1,:);
epsilon_s3i(1,:)=0.003*(c_i(1,:)-d_3i(1,:))./c_i(1,:);
epsilon_s4i(1,:)=0.003*(c_i(1,:)-d_4i(1,:))./c_i(1,:);
epsilon_s5i(1,:)=0.003*(c_i(1,:)-d_5i(1,:))./c_i(1,:);
f_s1i(1,:)=epsilon_s1i(1,:)*E_si;
f_s2i(1,:)=epsilon_s2i(1,:)*E_si;
f_s3i(1,:)=epsilon_s3i(1,:)*E_si;
f_s4i(1,:)=epsilon_s4i(1,:)*E_si;
f_s5i(1,:)=epsilon_s5i(1,:)*E_si;

% Compare f_s1i, f_s2i, f_s3i, f_s4i and f_s5i with +-f_yi
s_f_s1iu=find(f_s1i(1,:)>f_yi(1,:)); % Upper boundary, f_yi
f_s1i(1,s_f_s1iu)=f_yi(1,s_f_s1iu);
s_f_s1il=find(f_s1i(1,:)<-f_yi(1,:)); % Lower boundary, -f_yi
f_s1i(1,s_f_s1il)=-f_yi(1,s_f_s1il);

s_f_s2iu=find(f_s2i(1,:)>f_yi(1,:)); % Upper boundary, f_yi
f_s2i(1,s_f_s2iu)=f_yi(1,s_f_s2iu);
s_f_s2il=find(f_s2i(1,:)<-f_yi(1,:)); % Lower boundary, -f_yi
f_s2i(1,s_f_s2il)=-f_yi(1,s_f_s2il);

s_f_s3iu=find(f_s3i(1,:)>f_yi(1,:)); % Upper boundary, f_yi
f_s3i(1,s_f_s3iu)=f_yi(1,s_f_s3iu);
s_f_s3il=find(f_s3i(1,:)<-f_yi(1,:)); % Lower boundary, -f_yi
f_s3i(1,s_f_s3il)=-f_yi(1,s_f_s3il);

s_f_s4iu=find(f_s4i(1,:)>f_yi(1,:)); % Upper boundary, f_yi
f_s4i(1,s_f_s4iu)=f_yi(1,s_f_s4iu);
s_f_s4il=find(f_s4i(1,:)<-f_yi(1,:)); % Lower boundary, -f_yi
f_s4i(1,s_f_s4il)=-f_yi(1,s_f_s4il);

s_f_s5iu=find(f_s5i(1,:)>f_yi(1,:)); % Upper boundary, f_yi
```

```
f_s5i(1,s_f_s5iu)=f_yi(1,s_f_s5iu);
s_f_s5il=find(f_s5i(1,:)<-f_yi(1,:)); % Lower boundary, -f_yi
f_s5i(1,s_f_s5il)=-f_yi(1,s_f_s5il);

% Calculate C_ci
angle_theta_i(1,:)=acos((h_i(1,:)/2-a_i(1,:))./(h_i(1,:)/2));
s_a_ipri=find(a_i(1,:)>h_i(1,:)/2);
angle_theta_i(1,s_a_ipri)=pi-acos((a_i(1,s_a_ipri)-h_i(1,s_a_ipri)/2)./(h_i(1,s_a_ipri)/2));
A_i(1,:)=h_i(1,:).^2.*(angle_theta_i(1,:)-sin(angle_theta_i(1,:)).*cos(angle_theta_i(1,:)))/4;
C_ci(1,:)=0.85*f_ci(1,:).*A_i(1,:)/1000;

% Calculate F_s1i
F_s1i(1,:)=(f_s1i(1,:)-0.85*f_ci(1,:)).*A_s1i(1,:)/1000;
s_F_s1i=find(a_i(1,:)<d_1i(1,:));
F_s1i(1,s_F_s1i)=f_s1i(1,s_F_s1i).*A_s1i(1,s_F_s1i)/1000;

% Calculate F_s2i
F_s2i(1,:)=(f_s2i(1,:)-0.85*f_ci(1,:)).*A_s2i(1,:)/1000;
s_F_s2i=find(a_i(1,:)<d_2i(1,:));
F_s2i(1,s_F_s2i)=f_s2i(1,s_F_s2i).*A_s2i(1,s_F_s2i)/1000;

% Calculate F_s3i
F_s3i(1,:)=(f_s3i(1,:)-0.85*f_ci(1,:)).*A_s3i(1,:)/1000;
s_F_s3i=find(a_i(1,:)<d_3i(1,:));
F_s3i(1,s_F_s3i)=f_s3i(1,s_F_s3i).*A_s3i(1,s_F_s3i)/1000;

% Calculate F_s4i
F_s4i(1,:)=(f_s4i(1,:)-0.85*f_ci(1,:)).*A_s4i(1,:)/1000;
s_F_s4i=find(a_i(1,:)<d_4i(1,:));
F_s4i(1,s_F_s4i)=f_s4i(1,s_F_s4i).*A_s4i(1,s_F_s4i)/1000;

% Calculate F_s5i
F_s5i(1,:)=(f_s5i(1,:)-0.85*f_ci(1,:)).*A_s5i(1,:)/1000;
s_F_s5i=find(a_i(1,:)<d_5i(1,:));
F_s5i(1,s_F_s5i)=f_s5i(1,s_F_s5i).*A_s5i(1,s_F_s5i)/1000;
```

```
% Calculate P_i and M_i, P_oi and P_ti
P_i(1,:)=Prof_i(1,:).*(C_ci(1,:)+F_s1i(1,:)+F_s2i(1,:)+F_s3i(1,:)+F_s4i(1,:)+F_s5i(1,:));
M_i(1,:)=Prof_i(1,:).*(0.85*f_ci(1,:)/1000.*h_i(1,:).^3.*sin(angle_theta_i(1,:)).^3/12+...
         F_s1i(1,:).*(h_i(1,:)/2-d_1i(1,:))+F_s2i(1,:).*(h_i(1,:)/2-d_2i(1,:))+...
         F_s3i(1,:).*(h_i(1,:)/2-d_3i(1,:))+F_s4i(1,:).*(h_i(1,:)/2-d_4i(1,:))+...
         F_s5i(1,:).*(h_i(1,:)/2-d_5i(1,:)))/1000;
P_oi(1,:)=Prof_i(1,:).*(0.85*f_ci(1,:).*(pi*h_i(1,:).^2/4-(A_s1i(1,:)+A_s2i(1,:)+A_s3i(1,:)+...
          A_s4i(1,:)+A_s5i(1,:)))+...
          f_yi(1,:).*(A_s1i(1,:)+A_s2i(1,:)+A_s3i(1,:)+A_s4i(1,:)+A_s5i(1,:)))/1000;
P_ti(1,:)=-Prof_i(1,:).*f_yi(1,:).*(A_s1i(1,:)+A_s2i(1,:)+A_s3i(1,:)+A_s4i(1,:)+A_s5i(1,:))/1000;

% Calculate eccentricities, e_i, and hovere_i
e_i(1,:)=M_i(1,:)./P_i(1,:); % (m)
hovere_i(1,:)=(h(1,1)/1000)./e_i(1,:); % Use nominal h
end
```

C.1.3.6 Code 6-Function of Simulated Load Effects and Nominal Values Based on ACI 318-14

```
% Load effect simulation
% (nominal loads calculation is based on design strengths corresponding to ACI 318-14)
function[D_curi,L_curi,T_Di,T_Li]=LoadEffectSim_cur_S5(LoverD,n)
% Load design strengths
load alpha_PM_S5.mat phiP_n

% Nominal values
% Loads
% LoverD=[0.5 1.5]; % Input of funtion
D_cur=abs(phiP_n)/(1.2+1.6*LoverD); % Use absolute values
L_cur=LoverD*D_cur;

% Transformations from load to load effect
T_D=1;
T_L=1;

% Statistical parameters (Bias coefficient and CoV)
% Loads
bias_D=1.05;
```

```
CoV_D=0.10;

bias_L=1.00;
CoV_L=0.25;

% Transformations from load to load effect
% bias_T_D=1; % Effect is accounted in D
% CoV_T_D=0;

% bias_T_L=1; % Effect is accounted in L
% CoV_T_L=0;

% Statistical parameters (Mean and Standard deviation)
% Loads
mean_D_cur=D_cur*bias_D;
std_D_cur=mean_D_cur*CoV_D;

mean_L_cur=L_cur*bias_L;
std_L_cur=mean_L_cur*CoV_L;

% Transformations from load to load effect
% T_D Effect is accounted in D
% T_L Effect is accounted in L

% Simulation
% Preallocation
D_curi=zeros(8,size(D_cur,2),n);
alpha_L_cur=zeros(8,size(D_cur,2));
mu_L_cur=zeros(8,size(D_cur,2));
L_curi=zeros(8,size(D_cur,2),n);

for i1=1:8
    for i4=1:size(D_cur,2)
        rn_n14=randn(1,1,n); % Standard normally distributed random numbers
        rn_u1=rand(1,1,n); % Standard uniformly distributed random numbers
```

```
        % Dead Loads
        D_curi(i1,i4,:)=mean_D_cur(i1,i4)+std_D_cur(i1,i4)*rn_n14; % Normal distribution

        % Live Loads
        alpha_L_cur(i1,i4)=(1/sqrt(6))*(pi/std_L_cur(i1,i4)); % Gumbel distribution
        mu_L_cur(i1,i4)=mean_L_cur(i1,i4)-0.5772/alpha_L_cur(i1,i4);
        L_curi(i1,i4,:)=mu_L_cur(i1,i4)-log(-log(rn_u1))/alpha_L_cur(i1,i4);
    end
end
s_negcur=find(phiP_n(1,:)<0);
D_curi(:,s_negcur,:)=-D_curi(:,s_negcur,:); % Negative values indicate tension
L_curi(:,s_negcur,:)=-L_curi(:,s_negcur,:);

% Transformations from load to load effect
T_Di=T_D; % Effect is accounted in D
T_Li=T_L; % Effect is accounted in L
end
```

C.1.3.7 Code 7-Function of Simulated Load Effects and Nominal Values Based on Partial Material Strength Reduction Factors

```
% Load effect simulation
% (nominal loads calculation is based on design strengths corresponding to
% partial strength reduction factors)
function[D_proi,L_proi,T_Di,T_Li]=LoadEffectSim_pro_S5(LoverD,s_phi_sc,n)
% Load design strengths
load alpha_PM_S5.mat P_r

% Nominal values
% Loads
% LoverD=[0.5 1.5]; % Input of funtion
% s_phi_sc % Input of funtion
D_pro=abs(P_r(:,:,s_phi_sc))/(1.2+1.6*LoverD); % Use absolute values
L_pro=LoverD*D_pro;

% Transformations from load to load effect
```

```
T_D=1;
T_L=1;

% Statistical parameters (Bias coefficient and CoV)
% Loads
bias_D=1.05;
CoV_D=0.10;

bias_L=1.00;
CoV_L=0.25;

% Transformations from load to load effect
% bias_T_D=1; % Effect is accounted in D
% CoV_T_D=0;

% bias_T_L=1; % Effect is accounted in L
% CoV_T_L=0;

% Statistical parameters (Mean and Standard deviation)
% Loads
mean_D_pro=D_pro*bias_D;
std_D_pro=mean_D_pro*CoV_D;

mean_L_pro=L_pro*bias_L;
std_L_pro=mean_L_pro*CoV_L;

% Transformations from load to load effect
% T_D Effect is accounted in D
% T_L Effect is accounted in L

% Simulation
% Preallocation
D_proi=zeros(8,size(D_pro,2),n);
alpha_L_pro=zeros(8,size(D_pro,2));
mu_L_pro=zeros(8,size(D_pro,2));
L_proi=zeros(8,size(D_pro,2),n);
```

```
for i1=1:8
    for i4=1:size(D_pro,2)
        rn_n14=randn(1,1,n); % Standard normally distributed random numbers
        rn_u1=rand(1,1,n); % Standard uniformly distributed random numbers

        % Dead Loads
        D_proi(i1,i4,:)=mean_D_pro(i1,i4)+std_D_pro(i1,i4)*rn_n14; % Normal distribution

        % Live Loads
        alpha_L_pro(i1,i4)=(1/sqrt(6))*(pi/std_L_pro(i1,i4)); % Gumbel distribution
        mu_L_pro(i1,i4)=mean_L_pro(i1,i4)-0.5772/alpha_L_pro(i1,i4);
        L_proi(i1,i4,:)=mu_L_pro(i1,i4)-log(-log(rn_u1))/alpha_L_pro(i1,i4);
    end
end
s_negpro=find(P_r(1,:,s_phi_sc)<0);
D_proi(:,s_negpro,:)=-D_proi(:,s_negpro,:); % Negative values indicate tension
L_proi(:,s_negpro,:)=-L_proi(:,s_negpro,:);

% Transformations from load to load effect
T_Di=T_D; % Effect is accounted in D
T_Li=T_L; % Effect is accounted in L
end
```

Curriculum Vitae

Name:	Tong Zhang
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	The University of Western Ontario London, Ontario, Canada 2014-2017 M.E.Sc.
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