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Fuzzy Family Ties: Familial Similarity Between Melodic Contours of Different Cardinalities

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A thesis submitted in partial fulfillment of the requirements for the Doctor of Philosophy degree in Music

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Abstract

All melodies have shape: a pattern of ascents, descents, and plateaus that occur as music moves through time. This shape—or contour—is one of a melody's defining characteristics. Music theorists such as Michael Friedmann (1985), Robert Morris (1987), Elizabeth Marvin (1987), and Ian Quinn (1997) have developed models for analyzing contour, but only a few compare contours with different numbers of notes (cardinalities), and fewer still compare entire families of contours. Since these models do not account for familial relations between different-sized contours, they apply only to a limited musical repertoire, and therefore it seems unlikely that they reflect how listeners perceive melodic shape.

This dissertation introduces a new method for evaluating familial similarities between related contours, even if the contours have different cardinalities. My Familial Contour Membership model extends theories of contour transformation by using fuzzy set theory and probability. I measure a contour's degree of familial membership by examining the contour's transformational pathway and calculating the probability that each move in the pathway is shared by other family members. Through the potential of differing alignments along these pathways, I allow for the possibility that pathways may be omitted or inserted within a contour that exhibits familial resemblance, despite its different cardinality.

Integrating variable cardinality into contour similarity relations more adequately accounts for familial relationships between contours, opening up new possibilities for analytical application to a wide variety of repertoires. I examine familial relationships between variants of medieval plainchant, and demonstrate how the sensitivity to familial variation illuminated by fuzzy theoretical models can contribute to our understanding of

musical ontology. I explain how melodic shape contributes to motivic development and narrative creation in Brahms's "Regenlied" Op. 59, No. 3, and the related Violin Sonata No. 1, Op. 78. Finally, I explore how melodic shape is perceived within the repetitive context of melodic phasing in Steve Reich's *The Desert Music*. Throughout each study, I show that a more flexible attitude toward cardinality can open contour theory to more nuanced judgments of similarity and familial membership, and can provide new and valuable insights into one of music's most fundamental elements.

Keywords

Melodic Contour, Cardinality, Fuzzy Set Theory, Probability, Transformational Theory, Plainchant, Musical Motive, Johannes Brahms, Minimalism, Steve Reich, Musical Ontology, Music Perception, Music Theory.

Dedication

In Memoriam Rob Schultz, 1977-2016

A man who understood the importance of families, both melodic and human.

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List of Abbreviations

CAS: Contour Adjacency Series

CC: Contour Class

C-space: Contour Space

C-pitch: Contour Pitch

C-seg: Contour Segment

C-subseg: Contour Sub-segment

COM-Matrix: Comparison Matrix

CSIM: Contour Similarity Relation

C+SIM: Contour Ascent Similarity Relation

C-PAS: Contour Pitch Adjacency Subset

FCM Model: Familial Contour Membership model

CHAPTER 1: Introduction

All melodies have shape: a pattern of ascents, descents, and plateaus that occur as music moves through time. This shape—or “contour”—is one of a melody’s defining characteristics. It helps listeners to identify and group related melodies into coherent families. Music theorists have developed models for analyzing melodic contour, but only a few compare contours with different numbers of notes (or cardinality), and fewer still compare entire families of contours. Since these models do not account for relations between family members that have different cardinalities, they apply only to a limited musical repertoire, and it seems unlikely that they reflect the way listeners perceive melodic shape. In this dissertation, I will introduce a new method for evaluating similarities in a family of related contours, even if the contours are different cardinalities.

Robert Morris (1987) and Elizabeth Marvin and Paul Laprade (1987) have developed matrices that quantitatively account for similarity between a pair of contours. Ian Quinn (1997) has generalized these matrices by employing fuzzy set theory to determine a contour’s degree of membership within a family of related contours. Though useful, these matrix-based models cannot meaningfully compare contours of different cardinalities, and therefore have limited application. Additionally, Morris (1993), Rob Schultz (2008), and Mustafa Bor (2009) developed algorithms that reduce a contour to its most basic shape, which permits comparisons between contours regardless of their initial cardinalities. As Quinn demonstrated, however, these models measure contour equivalence and are therefore too restrictive to measure familial membership because

they cannot admit partial members into a set of related contours (Quinn 1997, 237–38). These limitations can be avoided through a shift in perspective. Rob Schultz (2009) provided such a shift, moving away from matrices and toward a linear representation of contour: a transformational process that tracks a contour’s pathway as it unfolds in time.

In this dissertation, I introduce a new method for evaluating familial membership of contour segments (hereafter referred to as c-segs) within a family of related c-segs, without requiring that the c-segs be the same length.¹ Just as Quinn used fuzzy sets to extend the theory behind contour matrices, I extend Schultz’s theories of contour transformation by turning to probability. I measure a contour’s degree of familial membership by examining the contour’s transformational pathway and calculating the probability that each move in the pathway is shared by other family members. Through the potential of differing alignments along these pathways, I allow for the possibility that pathways may be omitted or inserted within a contour that exhibits familial resemblance, despite its difference in length.

This dissertation is divided into two parts. Part I consists of Chapter 1 and Chapter 2, and introduces my new model in the context of existing methodologies. This first chapter contains a brief overview of existing models for contour, and outlines the underlying issues regarding the current theories of contour relations. Chapter 2 then introduces my new Familial Contour Membership Model, and explains how its

¹ As described below, a c-seg is a formal, numerically represented expression of a melodic segment’s specific contour. However, music theorists often use the term ‘c-seg’ to refer both to melodies and their formal representations.

transformational perspective allows for contours with different cardinalities to share familial characteristics.

Integrating variable cardinality into similarity relations for contour more adequately accounts for the similarity of contours within a family, and opens up new possibilities in terms of analytical application to a wide variety of repertoires. In Part II of this dissertation, I will demonstrate the analytical usefulness of the Familial Contour Membership model through three case studies. Chapter 3 will examine familial relationships between variants of medieval plainchant, and will show how the sensitivity to familial variation illuminated by fuzzy models can contribute to our understanding of musical ontology. In Chapter 4, I will explain how melodic shape contributes to motivic development and the creation of narrative in Brahms's "Regenlied" Op. 59, No. 3, and the related Violin Sonata No. 1, Op. 78. Chapter 5 will explore how melodic shape is created and perceived within the repetitive context of minimalist music through a case study of melodic phasing in Steve Reich's *The Desert Music*. Finally, Chapter 6 will provide some concluding thoughts and suggest some potential areas for future research. Throughout each chapter, I will show that a more flexible attitude toward cardinality can open contour theory to more nuanced judgments of similarity and familial membership, and can provide new and valuable insights into one of music's most fundamental elements.

Early Models for Contour Representation

Theorists have often qualitatively described the contour of a motive or phrase, characterizing the way it rises or falls, and commenting on its relationship to the construction of the work. For example, Ernst Toch comments that “with the combination of ascending and descending scale segments melody approaches its real nature: the *wave line*” (1948, 78). Beginning in the 1980s, however, music theorists have begun to approach contour with more systematic precision.² Several theorists, including Michael Friedmann, Robert Morris, Elizabeth Marvin, Paul Laprade, Ian Quinn, Rob Schultz, and Mustafa Bor have developed rigorous models with which to quantitatively describe contour, and to study relationships between contours.

Friedmann’s 1985 article, “A Methodology for the Discussion of Contour: Its Application to Schoenberg’s Music” begins to develop this more systematic approach by calling for theorists to look beyond more abstract pitch classes when analyzing post-tonal music.³ He writes that “we are confronted...with a serious gap between the vivid realities of the surface of twentieth-century music and our accounts of the rather remote abstractions that are supposed to be its prime determinants” (1985, 223). He continues, describing the heavy influence that pitch class has on the analysis of twelve-tone music. He writes:

2 Early precursors to this include the ethnomusicological work of Charles Seeger (1960) and Charles Adams (1976), both of whom would influence the more common models music theorists use today.

3 Seeger (1960) and Adams (1976) notated melodies using ascents and descents as well. Seeger’s “+” and “-” refer to “tension” and “detension”. Adams also enumerates a background gestalt for his melody categorization, using the first, last highest and lowest notes to represent the overall melody. These early precursors have influenced the more rigorous models that we see developed in the 1980s in the pc-set theoretic literature (Marvin 1995, 135–171).

The control of pitch class exercised in twelve-tone music... is as much a delimitation of its sovereignty, as it is a tribute to its importance, and this autonomy of pitch class and interval class has a logical corollary, the autonomy of contour. Thus, the independent associative power of each musical parameter is the major consequence of the classical twelve-tone premises of composition. (1985, 224)

The increased autonomy associated with each parameter of music drives Friedmann to develop a rigorous way to define the parameter of contour, borrowing ideas from pitch-class set theory. He outlines two ways to model the structural characteristics of a given contour. The first is the *Contour Adjacency Series* (CAS), which accounts for relationships between adjacent pitches in a contour by providing an ordered series of direction changes (or moves), using the symbols “+” and “-” to account for the direction changes up and down respectively.⁴ Figure 1.1, for example, shows a nine-note contour, which possesses a CAS of $\langle +++++-+ \rangle$, describing the relative direction from each note to the next. This approach is especially useful when accounting for the transformational development of a contour, and it is in this way that the CAS is used throughout this dissertation.

Figure 1.1 Application of Friedmann’s CAS and CC models (Wallentinsen 2013, 3)



⁴ These symbols were derived from the pitch-class set theoretic concept of ordered pitch intervals, simply removing the crisp intervallic distinction. This, combined with the fact that Friedmann is using the model for serial music, is why the model as it is presented in his article lacks a third “=” symbol to represent the static plateau associated with a pitch repetition.

Despite its usefulness, the CAS does not describe the global relationships between the nine notes of the contour. To address this, Friedmann creates the *Contour Class* (CC), which “describes contour relations among all the pitches...and can reflect the occurrences of pitch repetitions. In the CC, 0 is the lowest pitch, and $n-1$ (n =number of different pitches in the unit) is the highest pitch” (1985, 227). As shown in Figure 1.1, the example contour’s CC would be $\langle 671254103 \rangle$, where 0 reflects the lowest pitch (F4), and the highest pitch (F5) would be represented by $n-1$, which would equal 7 in this case, because two pitches are identical (the G4).⁵ Elizabeth Marvin and Paul Laprade (1987) also used this nomenclature, defining a contour segment, or c-seg, as an ordered set of pitches labeled in the same way that Friedmann does with his CC.⁶

Similarly, Robert Morris introduces new models of contour representation in his 1987 book *Composition with Pitch Classes*. He defines a contour space (or c-space) as “the basic and most musically immediate pitch-space” (1987, 26). This c-space is made up of a “pitch-space of n elements, called c-pitches [for contour-pitches]. C-pitches are numbered in order from low to high” (1987, 26) in the same manner as Friedmann’s CC.

5 Friedmann goes on to discuss other models for contour analysis, including a Contour Interval Array, and various contour vectors based on the CAS and CC. These models fall outside the scope of this dissertation and will not be discussed here.

6 Marvin and Laprade (and Morris too, for that matter) use the same kind of enumeration as Friedmann’s CC for their c-seg representations (i.e., numeric designations for each pitch ranging from 0 as the lowest to $n-1$ as the highest). However, it is important to note that it has become traditionally accepted in the theory community to use the term c-seg to refer both to the contour itself (connoting a process of segmentation on the part of an analyst) as well as the contour-segment’s numeric representation, such as $\langle 671254103 \rangle$ (in the same way that we might use an ordered set of pitch-classes to identify and label an ordered pitch-segment. In this dissertation, I shall use the term c-seg in the same manner.

Morris also creates a matrix-based representation, called the COM matrix, that displays the relationship between each ordered pair of c-pitches in the contour. Morris writes that “COM (a,b) is the comparison function of two c-pitches, a and b, in any c-space. If b is higher than a, COM (a,b) = +1; if b is the same as a, COM (a,b) = 0; and if b is lower than a, COM (a,b) = -1... [in the matrix] the plus and minus signs stand for the +1s and -1s of the various comparisons involved” (1987, 28). This matrix provides analysts with a comprehensive account of the relative position of each c-pitch within c-space. Figure 1.2 shows the COM matrix for the contour in Figure 1.1: following the first row, we can see that the first c-pitch is higher than all other c-pitches except for the second, as indicated by the “-” symbols in the majority of the first row. This matrix allows analysts to examine the internal structure of the contour, enabling them to see the relationship between any two c-pitches in the c-seg.

Figure 1.2. COM Matrix for the ⟨671254103⟩ contour (Wallentinsen 2013, 6)

	6	7	1	2	5	4	1	0	3
6	0	+	-	-	-	-	-	-	-
7	-	0	-	-	-	-	-	-	-
1	+	+	0	+	+	+	0	-	+
2	+	+	-	0	+	+	-	-	+
5	+	+	-	-	0	-	-	-	-
4	+	+	-	-	+	0	-	-	-
1	+	+	0	+	+	+	0	-	+
0	+	+	+	+	+	+	+	0	+
3	+	+	-	-	+	+	-	-	0

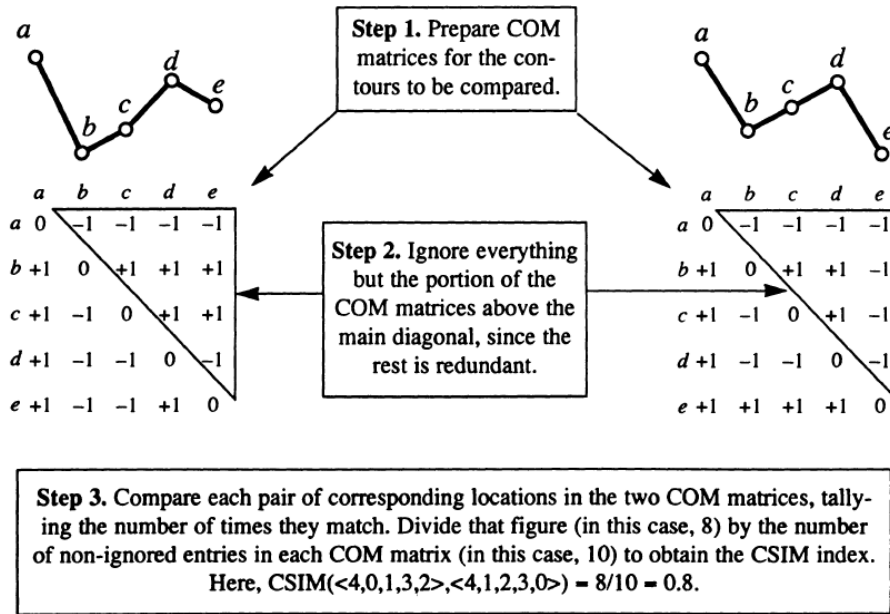
Methods of Contour Comparison

The systematically defined representations of musical contour established by Friedmann, Morris, and Marvin and Laprade paved the way for the development of multiple similarity relations that allowed theorists to compare two contours more thoroughly than previous qualitatively descriptive approaches had been able to do. However, Ian Quinn (1997, 2001) points out that many of them have limitations when it comes to comparing a family of multiple contours at the same time. Many are also limited in that they cannot compare contours of different cardinalities. In this section, I will review pertinent contour comparison methods, and will comment on their limitations with regard to their abilities to compare multiple contours of different sizes.

Marvin and Laprade created a contour similarity measure called CSIM, which measures similarity between two contours by comparing the COM matrices of the contours, counting the number of identical entries in the matrices and dividing by the total number of entries in the matrices, as shown in Figure 1.3. The result is a quantitative gauge of the contours' similarity between 0 (not similar at all) and 1 (identical). In the case of the contours in Figure 1.3, this pair of contours has a CSIM value of 0.8, indicating that, according to this model, they are 80% similar to one another (1987, 234–237). Because CSIM relies on matching positions within the corresponding COM matrices, the contours in question need to be the same cardinality in order for the comparison to work properly. Contours of different lengths, and therefore different-sized

matrices, are not comparable using this method because the models themselves depend on the matrices' identical number of entries.⁷

Figure 1.3. Demonstration of CSIM procedure (Quinn 1997, 241)



Morris (1993), Schultz (2008), and Mustafa Bor (2009) have taken a different approach toward contour comparison, developing algorithms that recursively reduce a contour to its most basic shape, which permits comparisons between contours regardless of their initial cardinalities. Morris's Contour Reduction Algorithm (shown in Figures 1.4a and 1.4b) introduces a systematic process for the reduction of a contour to a prime—a gestalt representing the generalized shape of the contour, outlining the first, last, highest, and lowest c-pitches in the c-seg. Schultz extended the algorithm to account

⁷ Marvin and Laprade also introduced various embedding functions that measure the degree to which one contour is embedded within another. These are useful for the study of contour, but because they are measuring embedding and not similarity (a finely grained distinction, to be sure), they fall outside the scope of this dissertation and will not be discussed further.

more sensitively for primes that were not previously accounted for in Morris's algorithm (2008, 96). Bor introduced a similar reductive approach: a window algorithm that reduces a contour to its prime by pruning c-pitches that are neither the lowest c-pitch nor the highest c-pitch in consecutive strands of three to five c-pitches (Bor 2009, 55–73). In the cases of these reductive approaches, similarity is judged on the basis of primes: two contours are similar if they reduce to the same prime. However, these methods base their similarity judgments on assumptions of equivalence (i.e., “same prime”), and therefore are not a true similarity relation because they miss aspects of similarity that may have been reduced out through the algorithmic process.

These models are incapable of comparing individual contours to an entire family of related contours primarily because these models require pairs of discrete c-segs in order to work. These individual units prohibit the possibility that the relationship between members of a family revolve more around average familial characteristics that do not necessarily manifest the same way in each family member. Quinn argues that the structure of these average familial characteristics is an important way that listeners categorize melodies within a work. He posits that these families, like human families, cohere based on sets of average characteristics, and argues that these earlier models of contour comparison cannot reflect this way of human categorization because they cannot create an average member of the family (Quinn 1997, 240). Quinn then works to develop a new model that can make these comparisons, basing his method on mathematic principles from fuzzy set theory.

Figure 1.4a. Morris's Contour Reduction Algorithm (Morris 1993, 212)

Definition: *Maximum pitch:* Given three adjacent pitches in a contour, if the second is higher than or equal to the others it is a maximum. A set of maximum pitches is called a maxima. The first and last pitches of a contour are maxima by definition.

Definition: *Minimum pitch:* Given three adjacent pitches in a contour, if the second is lower than or equal to the others it is a minimum. A set of minimum pitches is called a minima. The first and last pitches of a contour are minima by definition.

Algorithm: Given a contour C and a variable N:

[STAGE ONE:]

Step 0: Set N to 0.

Step 1: Flag all maxima in C; call the resulting set the *max-list*.

Step 2: Flag all minima in C; call the resulting set the *min-list*.

Step 3: If all pitches in C are flagged, go to step 9.

Step 4: Delete all non-flagged pitches in C.

Step 5: N is incremented by 1 (i.e., N becomes N + 1).

[STAGE TWO:]

Step 6: Flag all maxima in max-list. For any string of equal and adjacent maxima in max-list, either: (1) flag only one of them; or (2) if one pitch in the string is the first or last pitch of C, flag only it; or (3) if both the first and last pitch of C are in the string, flag (only) both the first and last pitch of C.

Step 7: Flag all minima in min-list. For any string of equal and adjacent minima in min-list, either: (1) flag only one of them; or (2) if one pitch in the string is the first or last pitch of C, flag only it; or (3) if both the first and last pitch of C are in the string, flag (only) both the first and last pitch of C.

Step 8: Go to step 3.

Step 9: End. N is the "depth" of the original contour C.

Some “Fuzzy” Preliminaries

In his article “Listening to Similarity Relations” (2001), Quinn challenges common conceptions about similarity relations. He describes similarity relations in terms of fuzzy set theory, a generalization of crisp mathematics developed in 1965 by Lofti Zadeh. In this theory, fuzzy sets, unlike their crisp counterparts, are “class[es] of objects with a continuum of grades of membership” (Zadeh 1965, 338).

George Klir and Bo Yuan give a very clear example of how fuzzy set theory works to arrive at these degrees of membership (called membership functions or membership values) within a fuzzy set. Consider the various descriptions that one might use to describe the weather. They write: “Instead of describing the weather today in terms of the exact percentage of cloud cover, we can just say that it is sunny” (Klir and Yuan 1995, 4). The designation sunny is vague: it does not have a single precise measurement.

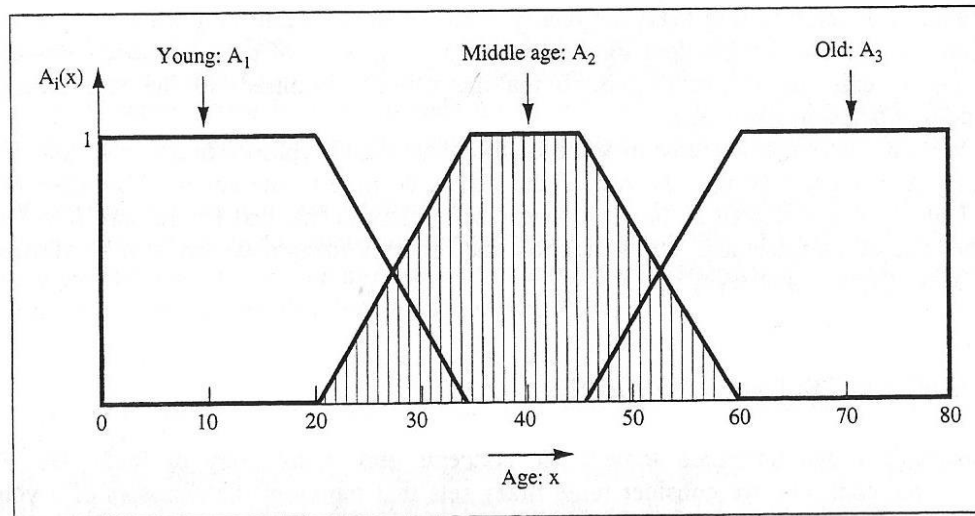
Its meaning is not totally arbitrary, however; a cloud cover of 100% is not sunny, and neither, in fact, is a cloud cover of 80%. We can accept certain intermediate states, such as 10% or 20% of cloud cover, as sunny. But where do we draw the line? If, for instance, any cloud cover of 25% or less is considered sunny, does this mean that a cloud cover of 26% is not? This is clearly unacceptable, since 1% of cloud cover hardly seems like a distinguishing characteristic between sunny and not sunny. We could, therefore, add a qualification that any amount of cloud cover 1% greater than a cloud cover already considered to be sunny (that is, 25% or less) will also be labeled as sunny. We can see, however, that this definition eventually leads us to accept all degrees of cloud cover as sunny. (Klir and Yuan 1995, 4)

The imprecision of the term “sunny” initially looks to be quite problematic from a quantitative point of view, but the introduction of a continuum of graded membership solves this problem. It allows a “gradual transition from degrees of cloud cover that are considered to be sunny and those that are not” (Klir and Yuan 1995, 4). In this sense, the

word “sunny” is understood as a fuzzy set: a container full of values that either fully or partially represent the meaning of the word.

Fuzzy sets can also intersect with each other within a continuum that represents the universal set—the entirety of the space one is considering. Figure 1.5 represents the three intersecting sets of *young*, *middle-aged*, and *old*. Here we can see that as individuals age past 20, they are considered members of both the *young* and *middle-aged* categories to varying degrees: at age 21, for example, they are still considered almost full members of the *young* set, and almost non-members of the *middle-aged* set. As individuals age, their membership function in the *middle-aged* set rises and their membership function in the *young* set falls. An individual’s respective membership functions within both sets meet in the middle around age 27, after which point the person might be considered more middle-aged than young. Finally, after age 34, the person will no longer be considered a member of the fuzzy set *young*. In this way, fuzzy set theory can account not only for partial members within a category, but also for transitions between particular states, and this type of measurement could be quite valuable when examining many different musical parameters.

Figure 1.5. Membership functions representing the fuzzy sets of young, middle-aged, and old (Klir and Yuan 1995, 20)



Just like states of cloud-cover and people of varying ages, objects such as pitches, pitch class sets, c-segs, or motives can be described as partial members within a fuzzy set. This partial membership is based on the degree of membership that these individual objects possess, according to a particular analytical model. Quinn examines musical similarity relations that can produce grades of membership ranging between 0 (not a member) and 1 (a full member). He lists ASIM, IcVSIM, and CSIM just to name a few, to show that methods already exist within the realm of music theory that make judgments similar to those described in fuzzy set theory. Such similarity relations, he states, are extensions and generalizations of equivalence relations, something that was previously considered separate. If similarity indeed maps elements onto a continuum between 0 and 1, then the oppositional binary that creates the crisp equivalence vs. non-equivalence relation lies at either end of that continuum. The area in between is fuzzy—an area where objects bear some resemblance to each other but are not completely equivalent. Similarity

relations, therefore, are fuzzy relations: they are particular types of relations that take pairs of pc-set theoretic elements as their intension, and map them onto a numeric range between 0 and 1 (Quinn 2001).

Quinn's Fuzzy Contour Model

Quinn's exploration of fuzzy relations in contour theory emphasizes certain limitations regarding existing similarity relations. He explains that not all models have the ability to make these kinds of judgments, because they do not base their measurements on the 0 to 1 continuum, but rather measure equivalences in a purely musical context (like the contour reduction algorithms). Furthermore, other existing models that do possess the required membership continuum are limited in that they only map pairs of c-segs, excluding the possibility of adequately studying averaged family structure without further modification. As such, Quinn's search for a way to model fuzzy contour relations leads him to modify CSIM, a similarity relation that compares two c-segs and returns a measurement between 0 and 1, as shown in Figure 1.3.

Quinn bases his theoretical model on the foundational assumption that motive or melody families may be treated as fuzzy sets by allowing motives to be partial members within the family based on their degree of similarity to the overall family. First, Quinn modifies the COM matrix to show ascent vs. non-ascent (a matrix that he calls a C+ matrix). This matrix, such as the one shown in Figure 1.6a, registers a 1 for "higher than" designations and a 0 for all others ("equal to" and "lower than"). The circled "1" value in the figure, for example, tells the analyst that c-pitch *e* is higher than c-pitch *b*. Quinn then

averages the C+ matrices of all existing family members to represent the probability that any given pitch in the family is higher than another, as shown in Figure 1.6b. This allows him to compare potential new members to the entire family by comparing matrices in a procedure which he calls “fuzzy C+SIM”: he compares the matrix of the new member to the probability matrix he created for the family, as shown in Figure 1.6c, arriving at a value between 0 and 1 that represents the new contour’s degree of membership within the family. These membership values give us insight into both how a contour is potentially related to a family of contours and how it is unique within the bounds of the familial resemblance.

Figure 1.6a. Quinn’s C+ matrix (1997, 252)

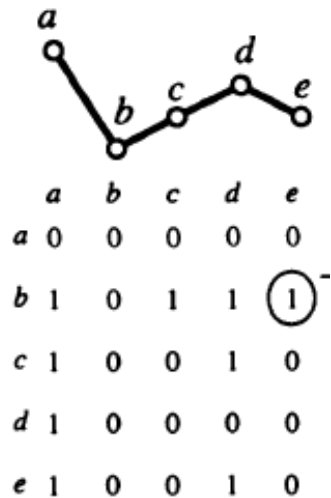


Figure 1.6b. Quinn's method of creating an averaged contour family using C+ matrices (1997, 252)

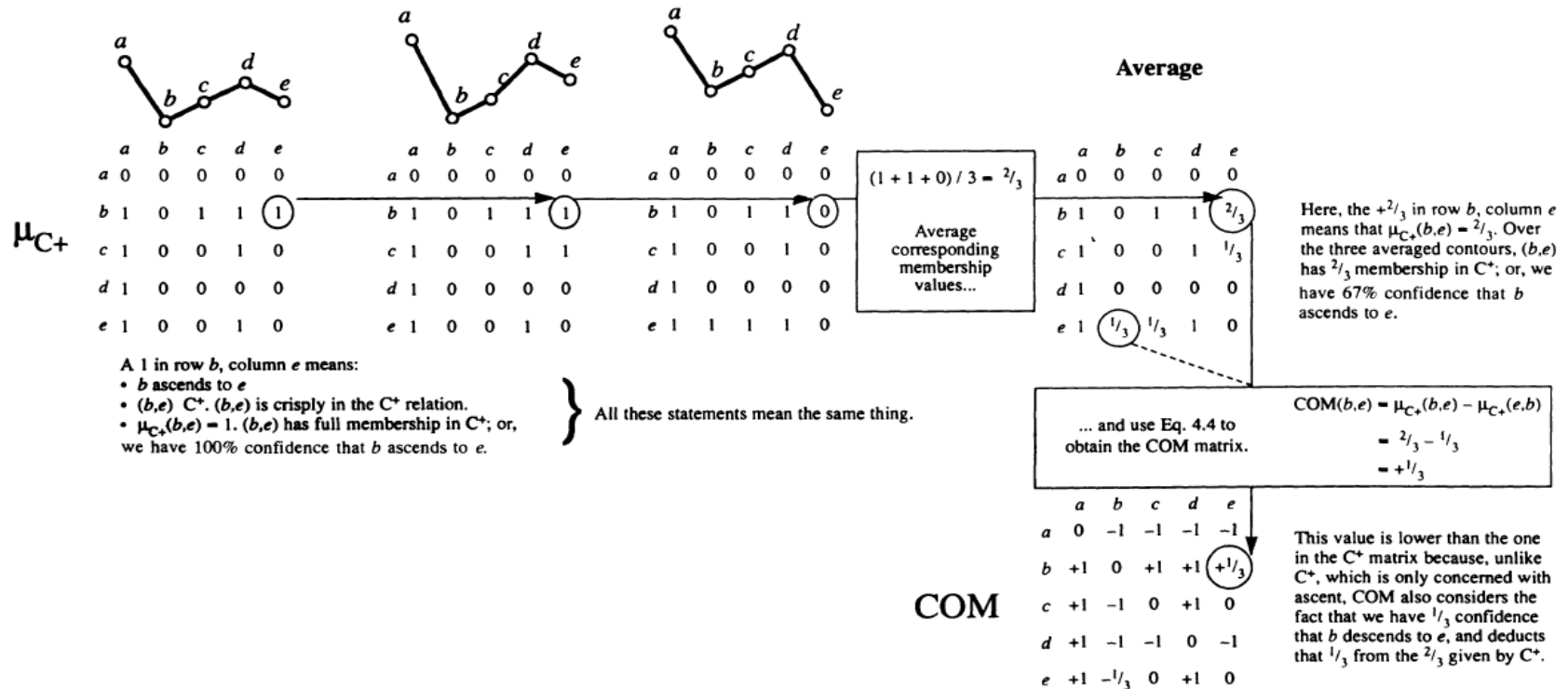


Figure 1.6c. Calculating fuzzy C+SIM using two fuzzy C+matrices (Quinn 1997, 257)

$$\begin{aligned}
 \text{C+SIM} &= \frac{1 - |\mu_{c^+}(a,b) - \mu_{c^+}(a',b')|}{i} + \frac{1 - |\mu_{c^+}(b,a) - \mu_{c^+}(b',a')|}{i} \\
 &= \frac{1 - |0.8 - 0.9|}{2} + \frac{1 - |0 - 0|}{2} \\
 &= \frac{1 - 0.1}{2} + \frac{1}{2} \\
 &= \frac{1.9}{2} = 0.95
 \end{aligned}$$

Explicit and Implicit Categorization

Quinn creates this fuzzy model in order to address a family of contours he identifies from Steve Reich's *The Desert Music* (shown in Figure 1.7), yet the model is based on a series of hidden assumptions that require more careful study.⁸ The fuzzy C+SIM relation maps the comparison between a single crisp new contour and an entire family of contours onto a continuum between 0 and 1, representing the degree of membership of the potential member of the family within the existing family of c-segs. This seems like a very logical construct, and for the purposes of fuzzy C+SIM, it works well.

⁸ Quinn's analysis of the piece itself is also worthy of more careful study. This shall be taken up in Chapter 5, where I will more fully demonstrate the limitations of Quinn's model to provide an analysis that is sensitive to the context in which the melodies occur.

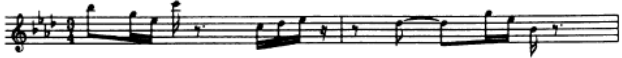

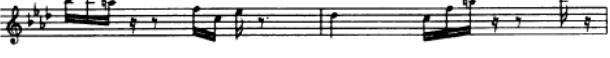
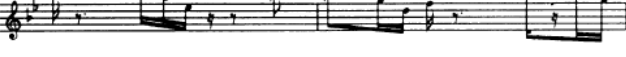

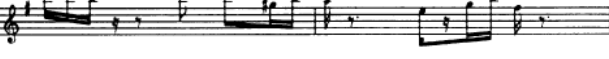
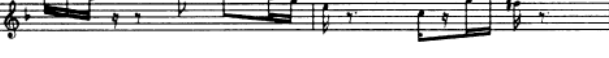
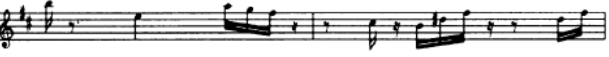


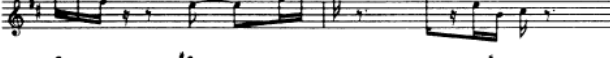
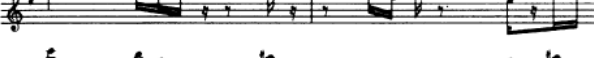

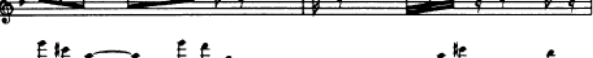
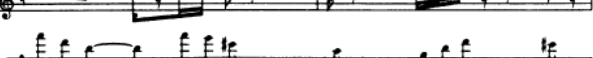
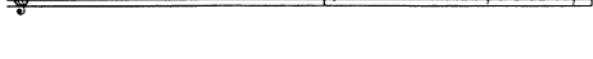
Quinn's model specifically measures membership within the family on the sole basis of contour similarity. By itself, this approach is analytically sound: Quinn's stated intention is to study contour similarity, ignoring other musical categories that contribute to overall similarity. However, the contours that Quinn chooses for his initial family are not admitted to the family under this same sole criterion. Regarding the formation of the initial family, Quinn writes:

No more than a casual hearing of the piece, or a cursory glance at [Figure 1.7], is necessary to inspire in the observer the intuition that all of the first-violin and flute melodies are of a kind. Specifically, they all follow the same rhythmic pattern, and they share a strong family resemblance of contour, despite individual local differences. (1997, 233)

Quinn bases his initial family on the human perception that these passages are "of a kind." He goes on to list several categories to which these similar passages belong; rhythmic pattern and contour most obviously, but the categorization also includes instrumentation (first-violin and flute, which also brings about some tacit categorizations regarding register, tessitura, timbre, etc.), and even location within the piece.

Admission to the initial family on the basis of all of these either explicit or implicit categories (even though it occurs in the guise of a contour similarity judgment) creates a double standard regarding the way that passages are judged within the analysis. New contours must be admitted on the basis of strong contour resemblance alone, while the initial family members were admitted to the family based on an overall perceived similarity across a number of different categorical attributes. This accounts for some of the larger contour variations that occur within the family: some contours within the family have relatively low degrees of membership, such as M_{6b} , which has a membership

Figure 1.7. The sixteen melodies of Quinn’s melody family, labeled “M” (Quinn 1997, 234)

<i>m</i> ₁		x14
<i>m</i> _{2a}		x9
<i>m</i> _{2b}		x3
<i>m</i> _{3a}		x10
<i>m</i> _{3b}		x3
<i>m</i> _{4a}		x10
<i>m</i> _{4b}		x3
<i>m</i> ₅		x14
<i>m</i> _{6a}		x8
<i>m</i> _{6b}		x2
<i>m</i> _{6c}		x2
<i>m</i> _{7a}		x9
<i>m</i> _{7b}		x4
<i>m</i> _{8a}		x9
<i>m</i> _{8b}		x2
<i>m</i> _{8c}		x2

value of 0.545, as shown in Figures 1.8a through 1.8c.⁹ These contours fall below Quinn’s chosen threshold of 0.7, the requirement he chose for membership of new passages within the family. If the goal is to examine contour only, should not the initial family be also governed by contour only? I would posit that all members should be admitted to the family in the same manner, regardless of whether they belong to the initial family or are new potential members of the family, and this requires us to look further into the ways melodies are categorized.

Figure 1.8a. Quinn’s average C+ matrix for his melody family “M” (Quinn 1997, 253)

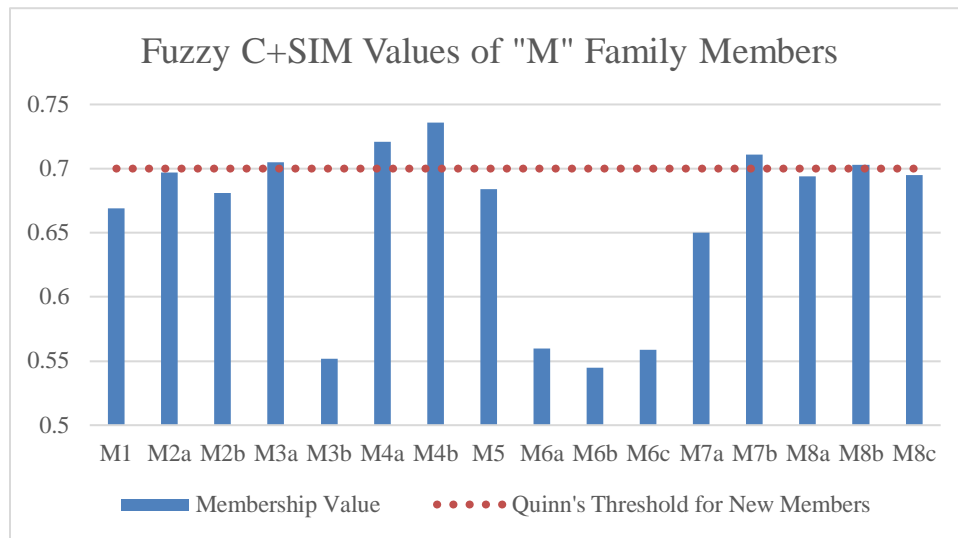
	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>	<i>j</i>	<i>k</i>
<i>a</i>	0	0.14	0.03	0.13	0.00	0.03	0.03	0.00	0.00	0.03	0.06
<i>b</i>	0.86	0	0.13	0.39	0.35	0.00	0.12	0.00	0.13	0.00	0.13
<i>c</i>	0.97	0.87	0	0.38	0.37	0.25	0.12	0.12	0.23	0.41	0.33
<i>d</i>	0.74	0.61	0.62	0	0.34	0.37	0.14	0.30	0.23	0.38	0.46
<i>e</i>	1.00	0.65	0.61	0.66	0	0.49	0.49	0.25	0.35	0.5	0.16
<i>f</i>	0.97	0.95	0.57	0.63	0.51	0	0.25	0.23	0.62	0.67	0.36
<i>g</i>	0.97	0.88	0.72	0.86	0.51	0.75	0	0.36	0.67	0.75	0.63
<i>h</i>	1.00	0.90	0.88	0.66	0.63	0.63	0.64	0	0.77	0.66	0.75
<i>i</i>	1.00	0.73	0.51	0.63	0.52	0.34	0.33	0.23	0	0.62	0.36
<i>j</i>	0.97	0.85	0.24	0.51	0.37	0.13	0.12	0.20	0.38	0	0.36
<i>k</i>	0.94	0.87	0.64	0.54	0.63	0.49	0.35	0.25	0.51	0.64	0

⁹ The calculations presented in Figures 1.8b and 1.8c are derived using Quinn’s fuzzy C+SIM formula presented in Figure 1.6. The calculations are based on the comparison between the crisp C+ matrix of each individual contour and the average C+ matrix shown in Figure 1.8a.

Figure 1.8b. Fuzzy C+SIM values for existing members of M, calculated against Quinn’s average C+ matrix.

Member of Family “M”	Fuzzy C+SIM value against the fuzzy family “M”
M ₁	0.669
M _{2a}	0.697
M _{2b}	0.681
M _{3a}	0.705
M _{3b}	0.552
M _{4a}	0.721
M _{4b}	0.736
M ₅	0.684
M _{6a}	0.560
M _{6b}	0.545
M _{6c}	0.559
M _{7a}	0.650
M _{7b}	0.711
M _{8a}	0.694
M _{8b}	0.703
M _{8c}	0.695

Figure 1.8c. Graph of membership values for each existing member of family M



Categories in Music

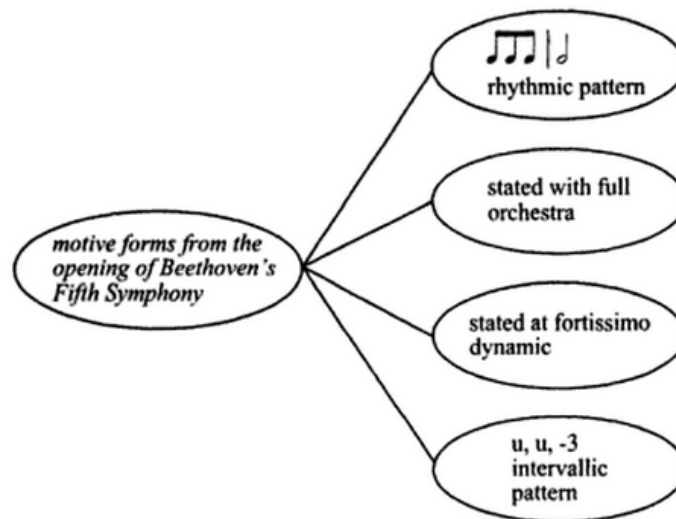
Lawrence Zbikowski (2002) addresses the kind of cognitive processes required to hear Quinn’s family as “of a kind.” He states that understanding music requires the ability to assign motives (or larger passages) to a single cognitive construct, in essence creating a family of passages that fit together on the basis of perceived similarity. Such a family, however, is not solely based on a single category (only rhythm or only contour, for example) and it is the particular categorical structure that provides meaning to an analysis based on motivic similarity or development (Zbikowski 2002, 42–49).

In music, the motive is what is called the *basic-level* category—the category that is the most immediate and easiest to call to mind. These categories lie in the middle of the taxonomic structure of categories within the work. It is made up of smaller parts; pitches

and durations for example, and lies below the more general level, such as exposition, movement, or piece.

This basic-level category is comprised of various subcategories that Zbikowski calls *attributes*. These attributes describe the motive, and can account for the perceived similarity of motives in the category. The hierarchical structure of these attributes in relation to the motive forms Zbikowski's conceptual model. He offers the motive of Beethoven's Fifth Symphony as an example: he breaks down the motivic category into attributes like rhythmic pattern, orchestration, dynamics, and diatonic melodic profile, as shown in Figure 1.9a (Zbikowski 2002, 47). Such attributes then have particular values that individual members of the motivic category possess. Orchestration for example, is divided up into *tutti*, *solo*, and *ensemble*. Motives that are all *tutti* may seem more related to one another than to ones that possess the *solo* attribute.

Figure 1.9a. Conceptual model for the category *motive forms of the opening of Beethoven's Fifth Symphony* (Zbikowski 2002, 47)



These attributes are themselves categories, to which individual motives belong. For example, the value *piano* has ten members: ten individual motives across mm. 1-19 that display the characteristic *piano*. In a way then, these values and attributes are categories that can be described in terms of crisp and/or fuzzy families.¹⁰ If one had the right analytical tools in place, one could create membership functions for each of these families, turning them into fuzzy sets.¹¹ For example, a category *mezzo forte* could be represented as a fuzzy set, where specific motives are placed in the set with regard to their particular decibel level. Such a fuzzy set would look much like the *middle-aged* example discussed earlier, where a range of decibel levels would be categorized as full members of *mezzo forte*, and membership would decrease gradually on the softer and louder sides of that particular decibel range, intersecting with the adjacent categories within the dynamic range.

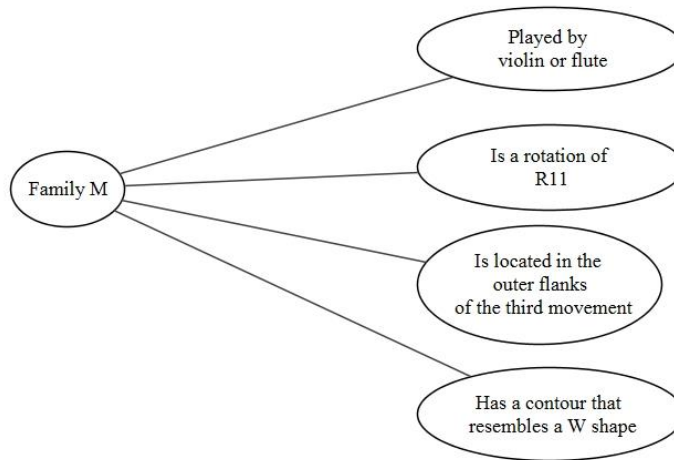
Returning to Quinn's analysis, we can now examine more closely what exactly we are saying when we perceive melodies to be "of a kind." The melodies Quinn uses all belong to the category "similar melodic material from the third movement of Reich's *The Desert Music*," which he calls "Family M." Following Zbikowski's model and diagram, Figure 1.9b shows the attributes Quinn chooses to highlight for this category: rhythmic pattern, contour, instrumentation, and placement within the piece. The values for such

¹⁰ I follow Quinn's usage of the word family, to avoid confusion between the notion of mathematical sets and the musical connotation that the word set has in post-tonal theory. In mathematical parlance, these ten members of the piano category would form a set. Additionally, the term crisp is used in mathematical parlance to refer to sets that are not fuzzy.

¹¹ As stated previously, such a model would need to be able to compare a potential member of the category with a generator of the category, or else in some way compare it to existing members of the category as a whole, and return a value between 0 and 1, representing the degree of membership within that category.

attributes can all be quantified, given the right kinds of similarity relations, and the members' degrees of similarity over all attributes is what contributes to our sense that the melodies are similar and indeed belong to this same category.

Figure 1.9b. A category tree representing Quinn's motivic categorization for his "family M"



Take a hypothetical example of a different piece, where the overarching category is "similar sounding motives from piece x ." Four attributes for this category may include contour, tempo, dynamic, and register.¹² Each member of the family would possess a degree of membership within "family x " for each of these four attributes, as judged by the membership value (the method or equation used to derive the degree of membership) for each of these fuzzy sets. If the family has 10 potential members, their degrees of

¹² These are by no means the only attributes that might categorize motives into groups based on perceived similarity, nor are they attributes that would immediately come to mind when judging similarity. I merely chose these because they are easily quantifiable into a value between 0 and 1 using existing models, and thus are good for explaining the formation of the fuzzy attribute sets, and the subsequent formation of the fuzzy family of motives.

membership are contingent upon the degrees of membership of each member in each attribute category, as shown in Figure 1.10.

Figure 1.10. Hypothetical membership of a motive within a family based on multiple attributes

	Contour	Tempo	Dynamic	Register	Total average membership value
x₁	0.90	0.620	0.280	0.550	0.587
x₂	0.730	0.840	0.640	0.390	0.650
x₃	0.820	0.500	0.370	0.630	0.580
x₄	0.410	0.900	0.590	0.270	0.540
x₅	0.200	0.360	0.820	0.670	0.510
x₆	0.560	0.700	0.670	0.480	0.602
x₇	0.900	0.830	0.720	0.760	0.802
x₈	0.500	0.480	0.640	0.520	0.535
x₉	0.270	0.360	0.290	0.400	0.330
x₁₀	0.870	0.260	0.930	0.750	0.702

Each particular motive has a degree of membership within each fuzzy set, representing its similarity to either the generator of the family, or the average member of the family. One can average the membership values for each of the four attribute categories in order to arrive at an overall degree of membership. This membership value represents the motive's place within the basic-level category of "similar sounding motives from piece *x*." This type of structure models the perception of similarity across all attributes of a category: members that have high values across all attributes (*x*₇, for example) will have a high degree of membership within the overall category. Likewise, members that have low similarity across all attributes (*x*₉, for example) will have a low degree of membership within the overall category. If there were a drastic decrease in the membership value of any one attribute, a motive would need high values in other

categories to offset that low value in order to still possess a high degree of membership. Motive x_{10} for example, has a marked difference in tempo from the average motive in the category (perhaps it is a great deal slower or faster than the other instances of the motive in the piece), yet it still bears a decently high degree of membership within the category: a value of 0.702. Here, the high values of the other three categories (0.87 for contour, 0.93 for dynamic, and 0.75 for register) make up for the marked difference in tempo, and this follows logically with general categorical perception of motives: one may hear the similarity across the other three attributes, and therefore still judge the motive as similar to the motives in the category.

With this understanding of quantifiable category structure, as represented by multiple particular fuzzy sets, we can understand that Quinn's formation of the initial family from *The Desert Music* is not in error. The original intent of Quinn's paper was not to provide a comprehensive listing of potential new members of the family from the music, but rather to illustrate the potential of fuzzy set theory to describe contour relationships, and the relative inadequacy of the current models to do so. However, by ignoring the other attributes upon which his initial family was based, he opened the door for some low-valued members to enter into his initial family, potentially throwing off his subsequent study of contour relations. By understanding that contour is only one attribute in a series of attributes that comprises fuzzy membership within the family, some of these values can be more adequately explained, and the analysis would be more complete.¹³

¹³ A potential solution may be to accept potential contours with a threshold developed by examining the membership values of existing members: in the case of Quinn's family, that would be 0.545. This would

The Hidden Category: Cardinality

In addition to the explicit categories chosen by an analyst such as Quinn, certain hidden categories can arise from the properties of the models one uses to measure the attributes of the chosen categories. As such, it is important to examine the assumptions and limitations of the models used to make these membership judgments, as these judgments may be colored by categories we haven't explicitly accounted for in our analysis. Quinn's C+SIM model (as well as many others) has one very specific such hidden category: cardinality.

The supposition of cardinality equality is built into many of the theoretical models that study contour, yet the notion that a pair of c-segs must have the same number of c-pitches in order to be considered similar is not necessarily reflective of every case of contour similarity. Quinn's analysis works because the c-segs he studies all happen to have the same cardinality, and therefore the use of his model adds an implicit new category to the attributes upon which Quinn bases his initial family: all contours in the family must have eleven c-pitches. Figure 1.11 shows a revised conceptual model of Quinn's family M, reflecting this hidden category. Because he finds only such contours in the course of building his family, the hidden category implicit in the model does not introduce any errors for him. However, this hidden requirement of cardinality equivalence is what limits the wider applicability of Quinn's theory to other analytical

reflect the true variety of contour inherent within this family, and it would be a more analytically sensitive representation of the context in which this model works.

contexts. Consider the following two c-segs, shown in Figure 1.12a: $\langle 012342 \rangle$ and $\langle 0123452 \rangle$. Based on human judgment, these c-segs seem related: the second is simply a slightly larger version of the first. In Quinn’s system, however, these two c-segs are not even comparable, nor are they comparable under Marvin and Laprade’s CSIM upon which Quinn’s model is based.

Figure 1.11. A revised conceptual model for Quinn’s Family M, showing the hidden category of cardinality

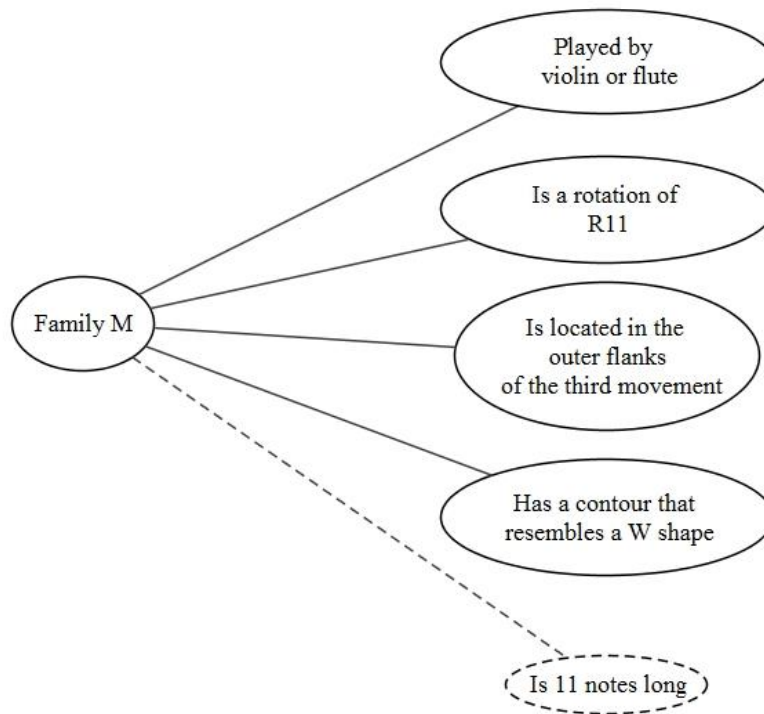


Figure 1.12a. Two c-segs that appear similar despite their difference in cardinality



Consider also Figure 1.12b, which shows slightly altered versions of the melody M_{4a} from Figure 1.7. These alterations occur either through the addition or subtraction of a single pitch, resulting in a ten-note and a twelve-note c-seg respectively.¹⁴ Both c-segs derive from M_{4a} , a member of Quinn’s family M, and feature all of the characteristics that Quinn points to in his family: a background overall descent, and a middle-ground W shape. Unfortunately, however, these two new c-segs would not be admitted to family M using his algorithm because of the cardinality problem, despite the general perception of similarity between them and the c-segs in M.

Figure 1.12b. Cardinality variants of M_{4a} showing their similarity in reduction, using the Contour Reduction Algorithm

	M_{4a}	10-note variant of M_{4a}	12-note variant of M_{4a}
N=0	$\langle 8\ 7\ 5\ 6\ 3\ 5\ 4\ 0\ 2\ 5\ 1 \rangle$	$\langle 7\ 6\ 4\ 5\ 3\ 4\ 0\ 2\ 4\ 1 \rangle$	$\langle 8\ 7\ 5\ 6\ 3\ 5\ 4\ 3\ 0\ 2\ 5\ 1 \rangle$
N=1	$\langle 5\ 3\ 4\ 2\ 3\ 0\ 3\ 1 \rangle$	$\langle 5\ 3\ 4\ 2\ 3\ 0\ 3\ 1 \rangle$	$\langle 5\ 3\ 4\ 2\ 3\ 0\ 3\ 1 \rangle$
N=2	$\langle 3\ 0\ 2\ 1 \rangle$	$\langle 3\ 0\ 2\ 1 \rangle$	$\langle 3\ 0\ 2\ 1 \rangle$
N=3	$\langle 2\ 0\ 1 \rangle$	$\langle 2\ 0\ 1 \rangle$	$\langle 2\ 0\ 1 \rangle$

¹⁴ This is a hypothetical analytical construct displaying the cardinality problem. I am not advocating this in terms of actual musical analysis in reference to Quinn’s selection of melodies, or the relationships of melodies that he shows in Reich’s *The Desert Music*.

Furthermore, the models that we do have to compare c-segs of differing cardinalities appear to corroborate this intuitive sense of similarity: under the Contour Reduction Algorithm (Schultz 2008), the two c-segs reduce to the same prime as their progenitor (M_{4a}) at the same depth level. They also feature two identical intermediary c-subsegs between the prime and the surface. These crisp measurements do not offer a fuzzy analog with which to determine these c-segs' similarity to the other members, or to the average family of M , but these measurements are enough to suggest that perhaps a measurement of membership—fuzzy or otherwise—should not always be dependent on the requirement of equal cardinality.

Conclusion

As I have demonstrated, the limitations inherent in these existing models highlight a significant gap between our perception of contour similarity and our ability to quantify these perceptions. Quinn rightly suggests that perception of melodic similarity, and subsequent inclusion within a family of related melodies, can be based on multiple parameters, and his supposition that one can create a model capable of reflecting human judgments of inclusion is laudable. However, none of the models explored in this chapter (including Quinn's) has the power to explain fully how contours of differing cardinalities relate to one another within the bounds of familial resemblance. They either have the capacity to relate entire families of equal-length contours, or else have the ability to compare individual contours of different lengths. None has the capacity to do both, and therefore it seems unlikely that they model the nuances of melodic similarity. However, Quinn's use of fuzzy set theory in his fuzzy C+SIM model is useful in two ways: first, it

gives us a way to measure contour membership within an entire family of related contours, confirming that this type of measurement can indeed be useful. Second, it further illuminates the need for sensitivity toward cardinality in these relations.

Removing cardinality from a fuzzy measurement of membership is no simple task. It will not do simply to modify the C+SIM matrix to allow for the possibility of more or less than eleven c-pitches, primarily because the mathematic operations governing these particular matrices prohibit the combination of matrices with different sizes. Developing a model that is sensitive to the requirements of fuzzy membership without invoking cardinality equivalence will require a shift of perspective away from the static representations of contour offered by the COM matrix, toward a more fluid representation of contour structure. It is to this task that I will turn in Chapter 2, examining just such a perspective shift, and developing a new model that does not rely upon strict cardinality equivalence in order to arrive at an averaged contour, nor a membership value of a new contour within an existing contour family.

CHAPTER 2: The Familial Contour Membership Model

Introduction

The first movement of Beethoven's Fifth Symphony contains a family of very familiar motives. The family is structured around several shared characteristics, as shown earlier in Chapter 1 (Figure 1.5a). Contour is one of these defining characteristics, yet the models discussed in Chapter 1 cannot account for the similarity found in the motive family because Beethoven introduces motive-variants that are shorter or longer than the prototypical four-note motive heard at the outset of the piece. To model more adequately the relationships found within families whose members have varying numbers of notes, we must shift our perspective away from the matrix-based representations of contour that limit our comparisons to contours of identical cardinality.

Rob Schultz (2009) provides such a shift, moving toward a linear representation of contour: a diachronic transformational process that tracks the pathway of moves that a contour makes as it unfolds in time. This perspective offers a way of relating contours of differing cardinalities by examining the commonalities of their respective pathways, even if one is longer than the other. In this chapter, I extend Schultz's theories of diachronic contour transformation by turning to probability. My Familial Contour Membership Model (hereafter referred to as the FCM model) measures a contour's degree of familial membership by calculating the probability that each move in the contour's pathway is shared by other family members. In doing so, the model allows for the possibility that pathways may be omitted or inserted within a contour that exhibits familial resemblance, yet is shorter or longer than others in the family. By using this method, I can quantify a

new contour's relationship to an existing family regardless of its cardinality, opening up a new dialogue about familial relationships within motive families across a wide variety of repertoires.

Schultz's Diachronic-Transformational Contour Model

Schultz's dissertation, "A Diachronic-Transformational Theory of Musical Contour Relations" (2009), critiques several existing models, stating that

a crucial phenomenological problem lurks behind each of these approaches, one that has yet to be adequately addressed in the literature. Every analytical observation made above...hinges upon the synchronous, rather than diachronic view of contour—that is, one that treats all c-pitches as fully and simultaneously present within their respective contours. (Schultz 2009, 11)

He explains that contours are, by definition, ordered in time, each unfolding linearly as the music progresses. He writes that "all temporally ordered contours therefore cannot be regarded as autonomous entities in themselves, as is the case in each of the contour measurements explained above [like CSIM], for they are in fact but a single link, so to speak—albeit the crucial culminating link—in this implicit chain of contour transformations" (Schultz 2009, 11–12).

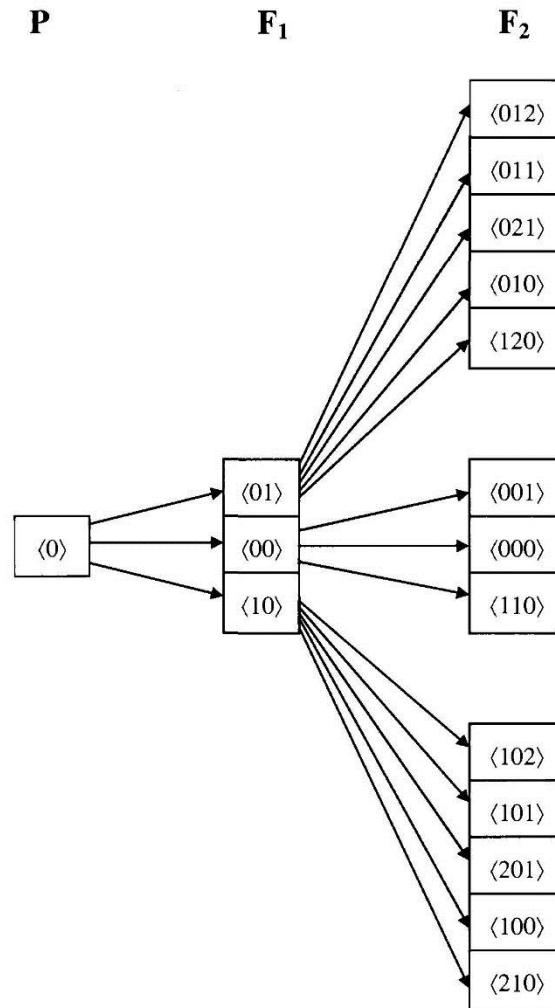
Schultz introduces this diachronic-transformational perspective by observing that every melody begins as a single pitch and then is transformed through the addition of new pitches that "will be either higher than, lower than, or equal to the first note, as dictated by the nature of c-space" (Schultz 2009, 17). Each added pitch follows one of these three paths, as shown in Figure 2.1. The process begins with the parental generation

(labeled P), and each transformation is marked as a new filial generation (labeled F₁, F₂, etc.), borrowing a term from Mendelian genetic analysis (Schultz 2009, 23). As the contour grows, the relations between the individual c-pitches will transform, until the contour reaches its final “culminating link.” For example, a c-seg that begins at ⟨0⟩ and then moves down will require a recontextualization of that first pitch. In the context of the new contour with two c-pitches, the second pitch would become 0, as it is lower than the first, and the c-seg would then be ⟨10⟩. This process continues for each additional added c-pitch. “Every contour presented in [Figure 2.1] (and beyond) thus possesses its own unique transformational path based upon its left-to-right unfolding in time, and families are formed according to these similarities of their respective paths on the graph” (Schultz 2009, 24).¹⁵

This transformational approach offers a more dynamic way to understand contour relations, and Schultz demonstrates its usefulness when relating contours to one another through his generational tree diagrams. However, for the purposes of fuzzy membership values, the lens provided by Schultz’s network is a bit too crisp. He posits a network that theoretically includes all possible c-segs in the entirety of c-space, the beginnings of which are shown in Figure 2.1. Therefore, no two unique c-segs will possess exactly the same transformational pathway. Our lens must therefore be shifted slightly out-of-focus, and this requires some generalization.

¹⁵ It is important to note that Schultz uses the term “family” to refer to contours that share transformational history—his family members are those that travel for a time along the same transformational paths. Therefore, he does not use the term “family” here in the same sense that Quinn uses in his theory, although as we shall see, the two are not as dissimilar as they may initially seem.

Figure 2.1. Schultz's universal contour tree diagram (Schultz 2009, 19)



Fortunately, one can make further observations within the contour tree diagram Schultz introduced, and these observations lead to a fuzzier conception of contour transformation. Under Schultz's system, the family network revolves around the possession of common ancestors. For example, the c-seg $\langle 021 \rangle$ seen in the F₂ generation possesses four other siblings, which all branch from the same parent in the F₁ generation: $\langle 01 \rangle$. Such siblings are derived from the possible ways one can move from $\langle 01 \rangle$: one can

ascend, producing a single F_2 generational member ($\langle 012 \rangle$); one could plateau, again producing a single F_2 generational member ($\langle 011 \rangle$); or one could descend, this time producing three F_2 generational members ($\langle 021 \rangle$, $\langle 010 \rangle$, and $\langle 120 \rangle$). Schultz accounts for this with a measurement that he calls C-pitch adjacency subsets (C-PAS), which highlights contiguous subsets with the cardinality one less than the entire c-seg ($n-1$). In the case of $\langle 021 \rangle$ for example, $C-PAS_A$ would be $\langle 01 \rangle$, the parent of $\langle 021 \rangle$. Its $C-PAS_B$ value on the other hand, would be $\langle 10 \rangle$; the same value one would find for the other two siblings that also moved down from the F_1 generation. This $C-PAS_B$ relationship can be seen in the oval-shaped nodes in Figure 2.2. “In this way, $C-PAS_B$ establishes a hierarchy of relationships amongst the members of these...contour families, and thus the sibling relationships in general” (Schultz 2009, 66). Siblings that share $C-PAS_B$ relationship are more related than those that do not, a relation that Schultz names “sibling0.”

This type of grouping based on which “move” is performed (ascent, descent, plateau), points toward a more generalized model: instead of mapping each individual possible filial generation c-seg ($\langle 021 \rangle$, $\langle 010 \rangle$, and $\langle 120 \rangle$ for example), a fuzzy model might map only the possible moves. One could think of this as mapping the transformation of Michael Friedman’s (1987) Contour Adjacency Series (CAS)—a model that tracks only successive moves between adjacent c-pitches in a c-seg—rather than tracking the specific c-segs that the moves produce. A tree, such as the one in Figure 2.3, provides a more general visualization of how a contour unfolds through time. Our $\langle 021 \rangle$ example would be written as $\langle + - \rangle$, and would follow the transformational path “up, then down,” as highlighted in the figure. This path is then shared by $\langle 021 \rangle$ ’s other two

sibling0-related c-segs $\langle 010 \rangle$ and $\langle 120 \rangle$, by virtue of the fact that this model is now fuzzier than Schultz's original theory.

Figure 2.2. Schultz's universal C-PAS_b-inclusive contour tree diagram (Schultz 2009, 62)

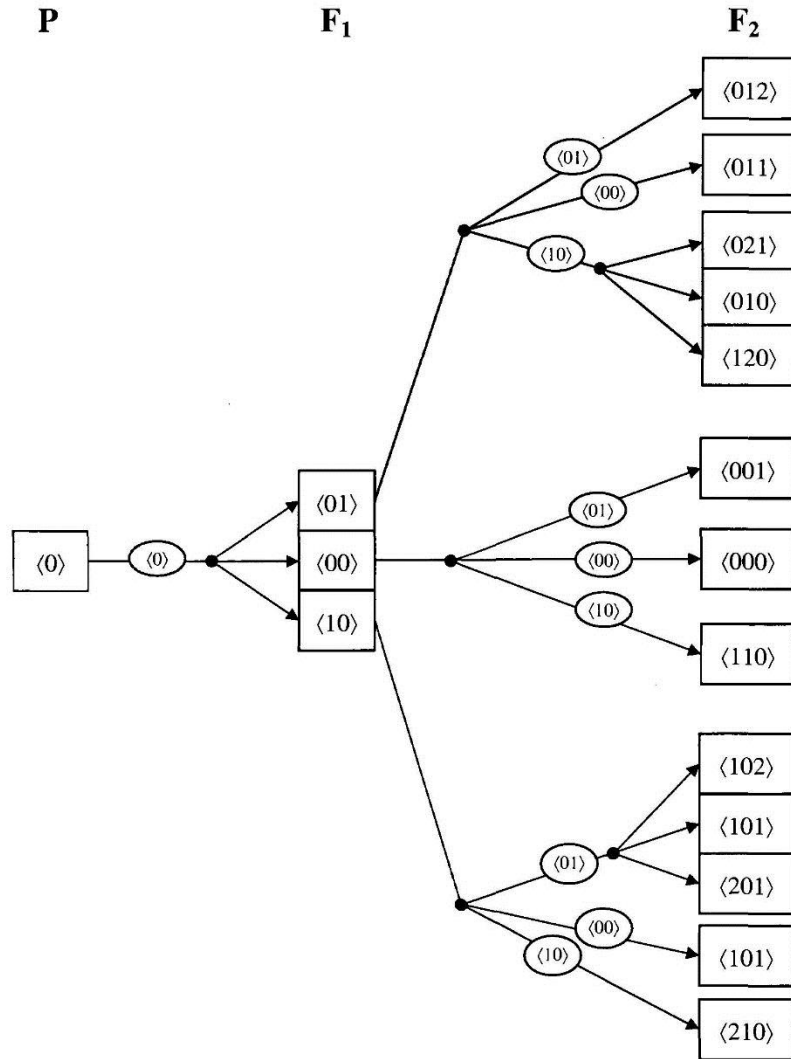
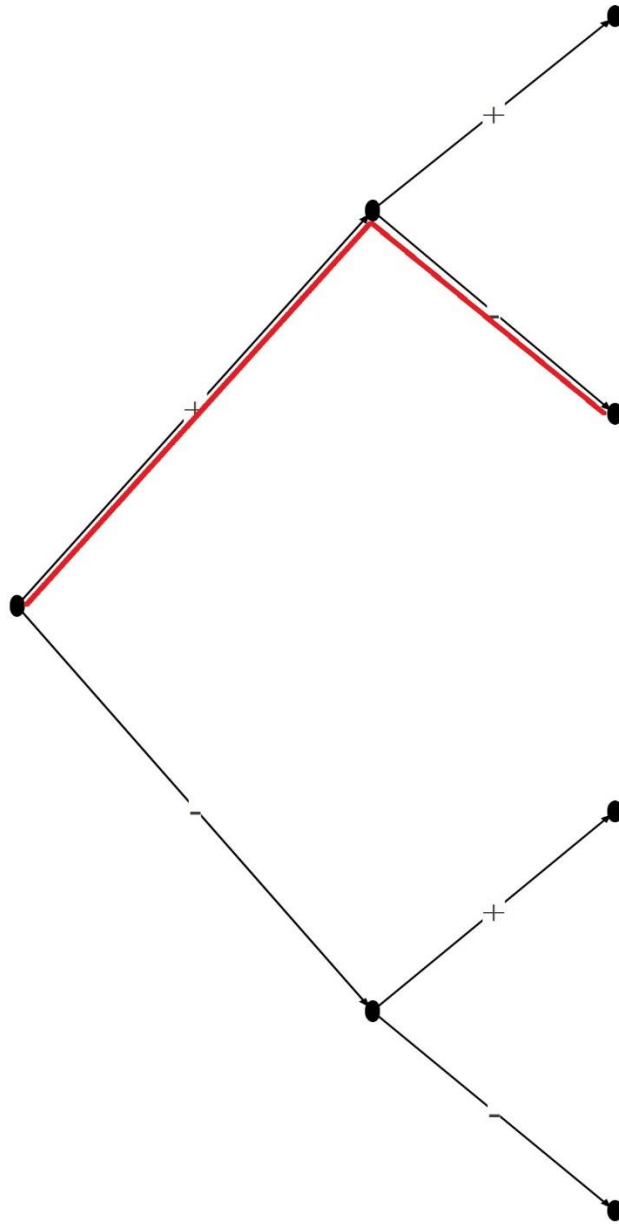


Figure 2.3. Generalized CAS transformational tree diagram



Mapping Familial Probability onto the CAS Tree

The beauty of both Schultz's transformational tree and the CAS tree is that they map the entirety of c-space. Any c-seg can be mapped onto the tree and it will follow one of the paths through c-space. C-segs that map similarly through the space are therefore more related than those that follow substantially different paths, and it is this property of

the CAS tree diagram that allows for the creation of a fuzzy relation in a similar manner to that which Quinn describes.

Imagine, for example, that you are following the path of a c-seg's CAS as it unfolds. This c-seg is related to some degree to a family of similar c-segs, perhaps from an earlier section of a piece from which the c-seg was taken, but the extent to which this new c-seg belongs with the rest of the family is unknown. As you move through the transformational path of the c-seg, you might wonder how many of the other c-segs in the family move as you do. These musings point to our answer: if a c-seg's first move is an ascent, how many others ascended and how many descended? Calculating these numbers from the existing family would tell us how related the new c-seg is to the existing family, based on how it aligned with other members of the family. If the vast majority of the family ascended, a new c-seg that descends might not be as central a member of the family.

When one maps an entire family of c-segs onto the CAS tree, some pathways will be traversed by many members of the family, while other members will traverse pathways that aren't shared by many members. As Quinn does, we need to average these c-segs together to arrive at what Quinn calls the "average member" of the family (Quinn 1997, 240). Figure 2.4a shows as an example a family of related c-segs, each represented by its numeric c-seg designation and their CAS values. Laying each of these on the CAS tree (Figure 2.3) gives us a table of values that carries information about the statistical probability of the average familial shape, shown in Figure 2.4b. We derive these values by tallying the quantity of ascents (+), descents (-), and plateaus (=) for each position in CAS. For example, in CAS position one, three members of the family rise, six members

fall and zero members plateau. Therefore, we can say that three of nine contours in the family ascend, and subsequently that we have a 33% confidence that a contour in the family will rise in position one, as shown in Figure 2.4b.¹⁶

Figure 2.4a. List of six-note contours in a contour family

Member	C-seg	CAS	Membership Value
1	⟨432501⟩	⟨--+-+⟩	0.667
2	⟨453201⟩	⟨+---+⟩	0.667
3	⟨453201⟩	⟨+---+⟩	0.667
4	⟨542031⟩	⟨---+-⟩	0.556
5	⟨243104⟩	⟨+---+⟩	0.667
6	⟨431201⟩	⟨--+-+⟩	0.667
7	⟨431210⟩	⟨--+-+⟩	0.600
8	⟨514320⟩	⟨-+---⟩	0.511
9	⟨542013⟩	⟨---++⟩	0.622
<i>Potential 10</i>	⟨431023⟩	⟨---++⟩	0.622

Figure 2.4b. CAS probability matrix, derived from the six-note contour family

	Pos. 1	2	3	4	5
+	0.333	0.111	0.333	0.222	0.667
=	0	0	0	0	0
-	0.667	0.889	0.667	0.778	0.333

The chart in Figure 2.4b can then be mapped onto the CAS tree diagram shown in Figure 2.4c. Here, it is easy to show the method in which existing members are related to

¹⁶ Quinn discusses the probabilities in his averaged contour as confidence values. Instead of saying that there's a 33% chance of rising, he would say that we can be 33% sure that a contour in the family will rise. The semantic difference is small, yet significant: describing probability in terms of fuzzy set theory can be described as either a fuzzy predicate or a fuzzy confidence level. Fuzzy predicates indicate that content is fuzzy (i.e., not knowing whether a numeric set represents pitch classes, pitches, or contour pitches), while fuzzy confidence levels indicate that the quantity or existence of the content is fuzzy (i.e., not knowing whether a contour is in a family or not).

one another, and the way in which potential new members are judged for membership within the family. If we have a potential new member of the family— $\langle 431023 \rangle$ or $\langle ---++ \rangle$ for instance—we can map its pathway through c-space onto the tree diagram, highlighted in red. On this tree, each arrow connecting the nodes carries a probability that such a move will occur in the family. Examining the sum total of all the arrows that the contour's pathway activates shows us the degree to which it resides in the family. In this case, the pathway sums to 3.11 ($0.667 + 0.889 + 0.667 + 0.222 + 0.667 = 3.11$). Dividing 3.11 by the five moves gives us an average of 0.622, indicating that we have a 62% confidence that this c-seg is in the family. This membership value suggests that the member is partially in the family, but perhaps is not the most centrally located member.

Membership values for existing members are measured in the same way, producing a range of membership values of 0.511 to 0.667, as shown in Figure 2.4a. This range indicates that, while there is some contour resemblance, family members exhibit unique traits that indicate variety within the family as a whole. However, there is a small but significant difference between potential and actualized membership. Members already in a family have a degree of membership within that family *and* help to make up that family. It is, in essence, an actualized member. Potential members however, such as the potential tenth member of the family presented in Figure 2.4a, occur when we compare contours to a family other than the one(s) to which they belong (or to a family in which we are not certain of its membership within the family). In essence, they do not yet play a role in the creation of the average family member. This definition of “potential” helps to distinguish between “membership within” and “similarity to” a family. Judgment of inclusion for potential members can be made on the basis of their similarity to the

family, as calculated by their potential membership value. Given the range of values of the family in Figure 2.4a (0.511 to 0.667), it would stand to reason that the potential tenth c-seg—with its potential membership value of 0.622 falling within the family’s range—would be a good candidate for inclusion within the family.

Once the analyst determines whether a new c-seg should be included in the family, it can be added to the family by adding its CAS values into the probability calculation that creates the table from Figure 2.4b, in essence becoming an actualized member of the family. For example, in position one the probability of descent is no longer calculated as $6/9$, but rather $7/10$ because of the added c-seg. The new table is now shown in Figure 2.4d. In this way, the family can grow and change with the addition of new c-segs. Adding a new contour into an existing family is up to the analyst: like Quinn, one may choose a threshold that is analytically meaningful to the particular context. For example, one may choose to admit only c-segs sharing a high degree of similarity into the family, accepting only c-segs that have a probability above 0.7 (as Quinn suggests). In that case, the potential new member discussed above would not be admitted to the family. One may also choose to judge inclusion based on the range of membership values for existing family members, as was done in our discussion of Figure 2.4. On the other hand, one may choose to include more members within the family—ones that have lower potential membership values—especially if one is not examining the attribute of contour alone. In this case, the unique properties of the c-seg that result in its lower membership value need to be represented in the family, so one should include these unique characteristics in the family. The flexibility of this method thus allows an analyst to use the model in different contexts.

Figure 2.4c. The CAS tree illustrating the probabilities of each transformational pathway in the family, with the red line indicating the pathway of the potential tenth c-seg

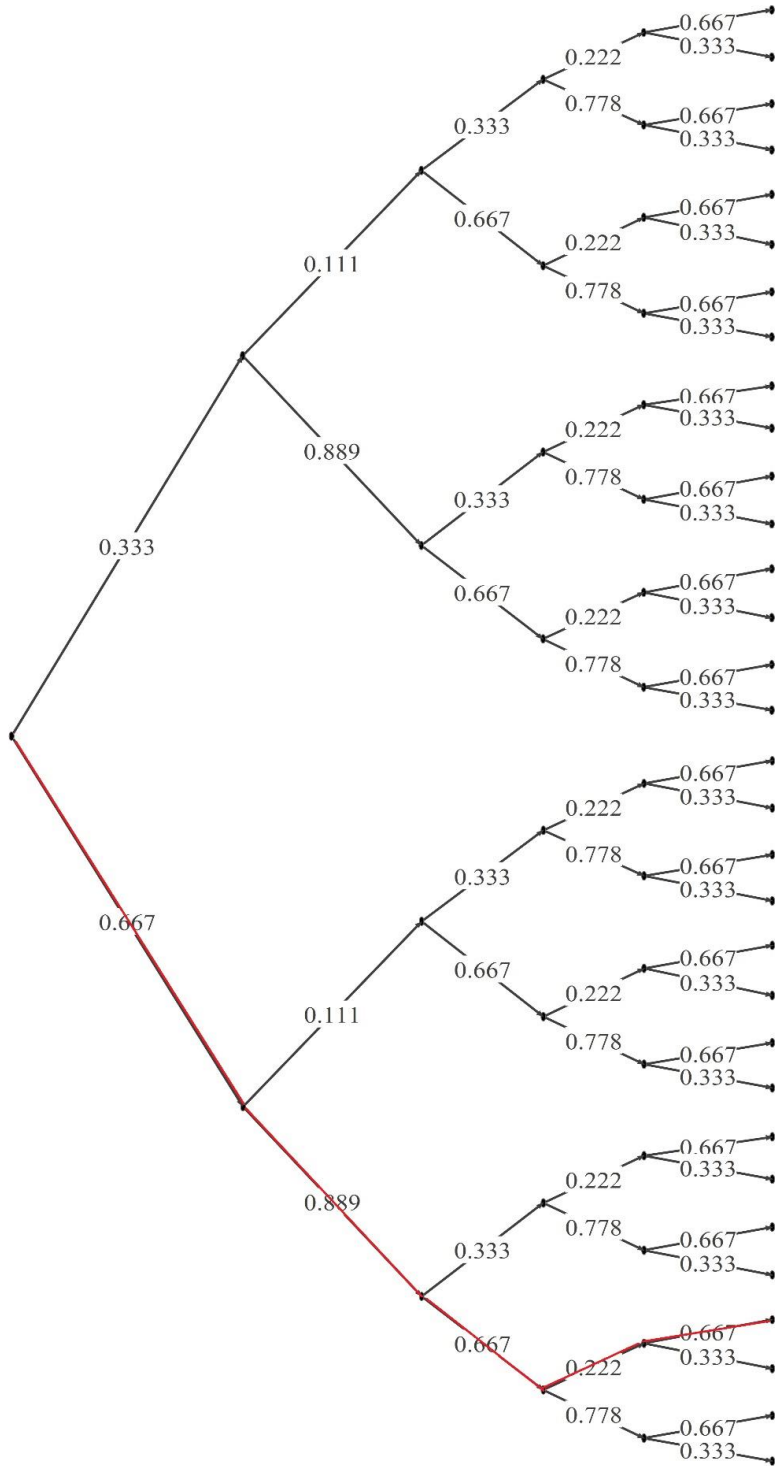


Figure 2.4d. Probabilities derived from the six-note contour family, including the tenth member

	Pos. 1	2	3	4	5
+	0.3	0.1	0.3	0.3	0.7
=	0	0	0	0	0
-	0.7	0.9	0.7	0.7	0.3

Schultz’s recency relation and the notion of best fit

With the logic of the system in place, we may address families with members of differing cardinalities. The family of contours in Figure 2.4 all have the same cardinality. However, a potential new member of the family— $\langle 34201 \rangle$ or $\langle +--+ \rangle$ —appears to be quite similar to members 2, 3, and 5, despite that it is now one c-pitch shorter. Recognizing this similarity, one would logically conclude that it might be a member of the family, but how is that membership quantified?

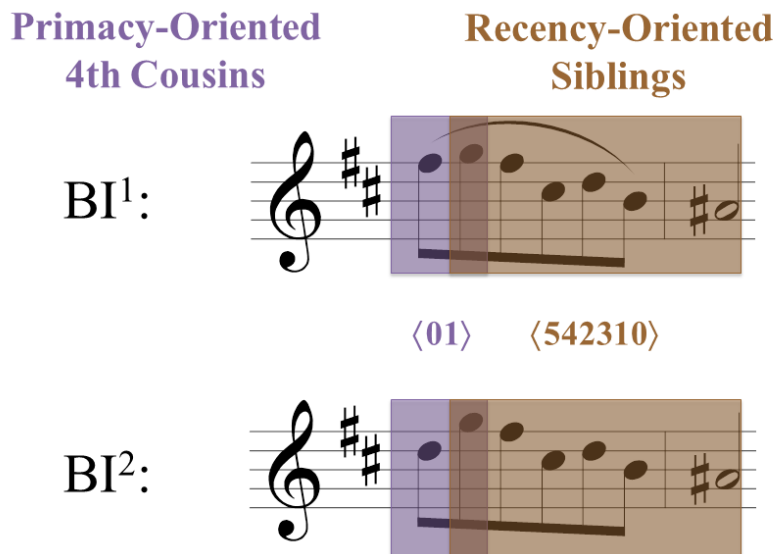
Once again, Schultz’s approach toward contour transformation suggests a way to include these c-segs into a family. His discussion of C-PAS_B is illustrative of the fact that contours can be related by their “right-to-left” orientation as well as their “left-to-right” diachronic unfolding. Figure 2.2 also showed that some of the contours in the F₂ generation are more related than others. An emphasis on C-PAS_B, or the last two c-pitches of the c-segs in the F₂ generation, further refines the relationships between siblings that otherwise share the same generational history.

Schultz gives another example of this phenomenon, shown in Figure 2.5. Here, two basic ideas (labeled in the figure as BI¹ and BI²) from Chopin’s Waltz in B minor Op. 69, No. 2 are compared using Schultz’s diachronic model. In the “left to right”

orientation, what Schultz calls the primacy relation, the two c-segs are only fourth cousins (meaning that they share a very distant common ancestor: in this case a great-great-great-grandparent), a relatively distant relationship. However, in the “right to left” orientation, what Schultz terms the recency relation, the two c-segs are siblings (meaning that their common ancestor, and thus their point of divergence, is only a generation removed from them), a much stronger relationship. He states that:

BI² emerges as distinct from BI¹ relatively early in its process of becoming; the two thus exhibit a rather distant primacy-oriented c-seg relationship. However, following this divergence, BI²'s similarity to BI¹ steadily increases as it continues to unfold due to the extensive number of common recent contour subsets that the two c-segs share. Therefore, their recency-oriented relationship is far closer than their primacy-oriented one. (Schultz 2012, 6)

Figure 2.5. An example of Schultz’s recency relation (Schultz 2012)



Schultz states that the recency-oriented relationship is stronger, and therefore perhaps more representative of the overall relationship between the two passages of

music. The choice between primacy and recency orientations provides contrasting analytical perspectives, and from there, the analyst chooses the best fit: the orientation that provides the strongest relationship, or the one that is most representative of the music and the issues addressed in the analysis. It is this process of best fit that points us to our answer regarding the multiple cardinality issue.

Best-fit procedures for the CAS probability tree

Let us return to the family in Figure 2.4. In order to calculate the membership of the new member $\langle 34201 \rangle$, we must align its positions along the CAS tree, and this requires that somewhere a position on the tree must be left out of the calculation. The question then becomes, which position gets left out?

For example, the potential new member $\langle 34201 \rangle$, with its CAS of $\langle +--+ \rangle$ bears similarity to members of the family, in both its beginning and ending: the $\langle +-- \rangle$ is strongly reminiscent of members 2, 3, and 5; while the ending $\langle -+ \rangle$ bears resemblance to members 1, 2, 3, 5, and 6. However, when one examines the family, no members have the contiguous pattern of $\langle +--+ \rangle$. Aligning this c-seg along the probability tree in that contiguous manner, shown in Figure 2.6a, and making a calculation based on this alignment yields a membership value of 0.55. Such a membership value does not really reflect our judgment that this c-seg is similar to the family. Recalling Schultz's recency relation, aligning the comparison based on a right-to-left approach, shown in Figure 2.6b, also yields a membership value of 0.55. Once again, this does not align with our judgment of the c-seg's similarity to the family.

Figure 2.6b. Contiguous recency relation mapping (in blue) of $\langle + \dots + \rangle$ onto the contour family of figure 2.4d

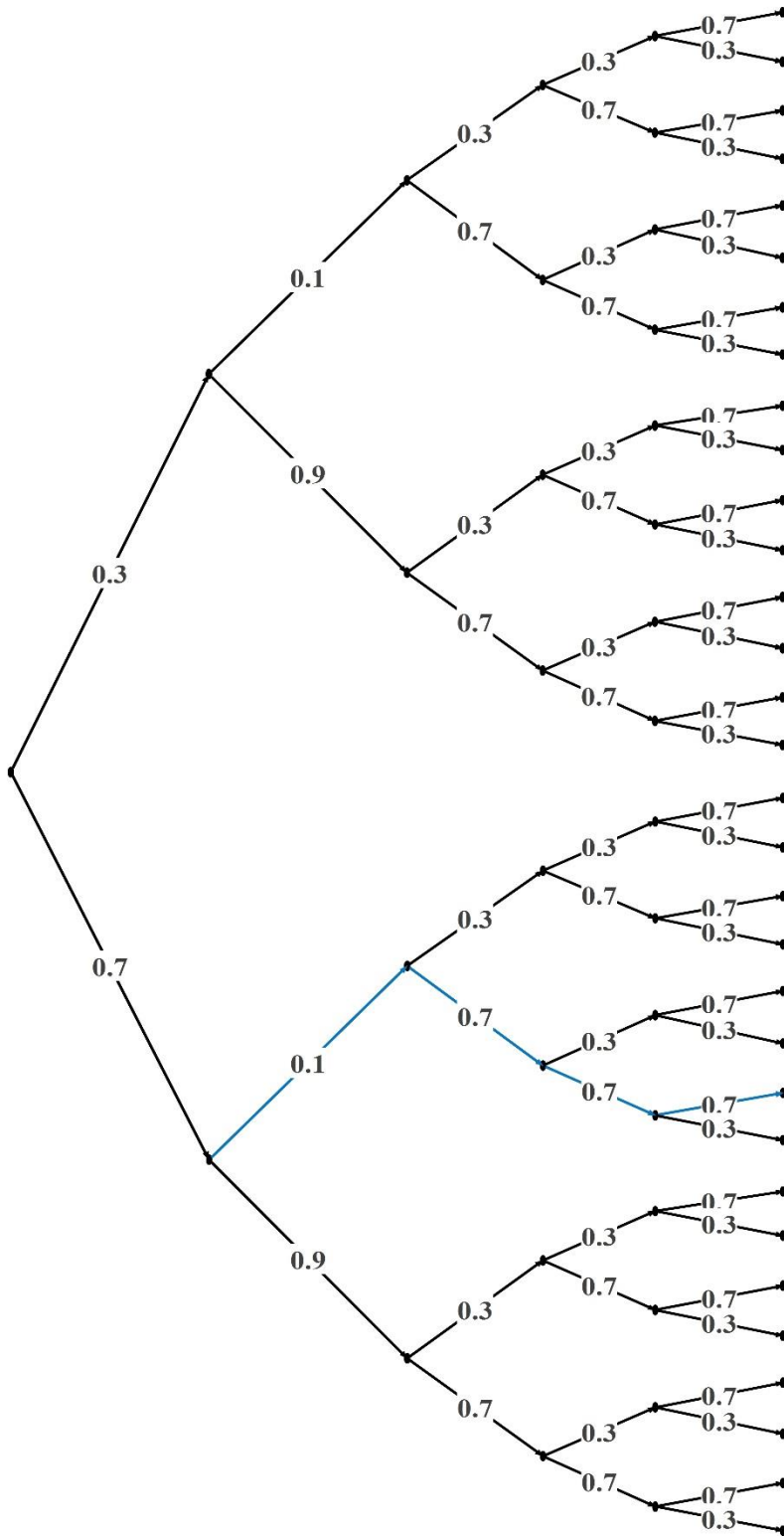
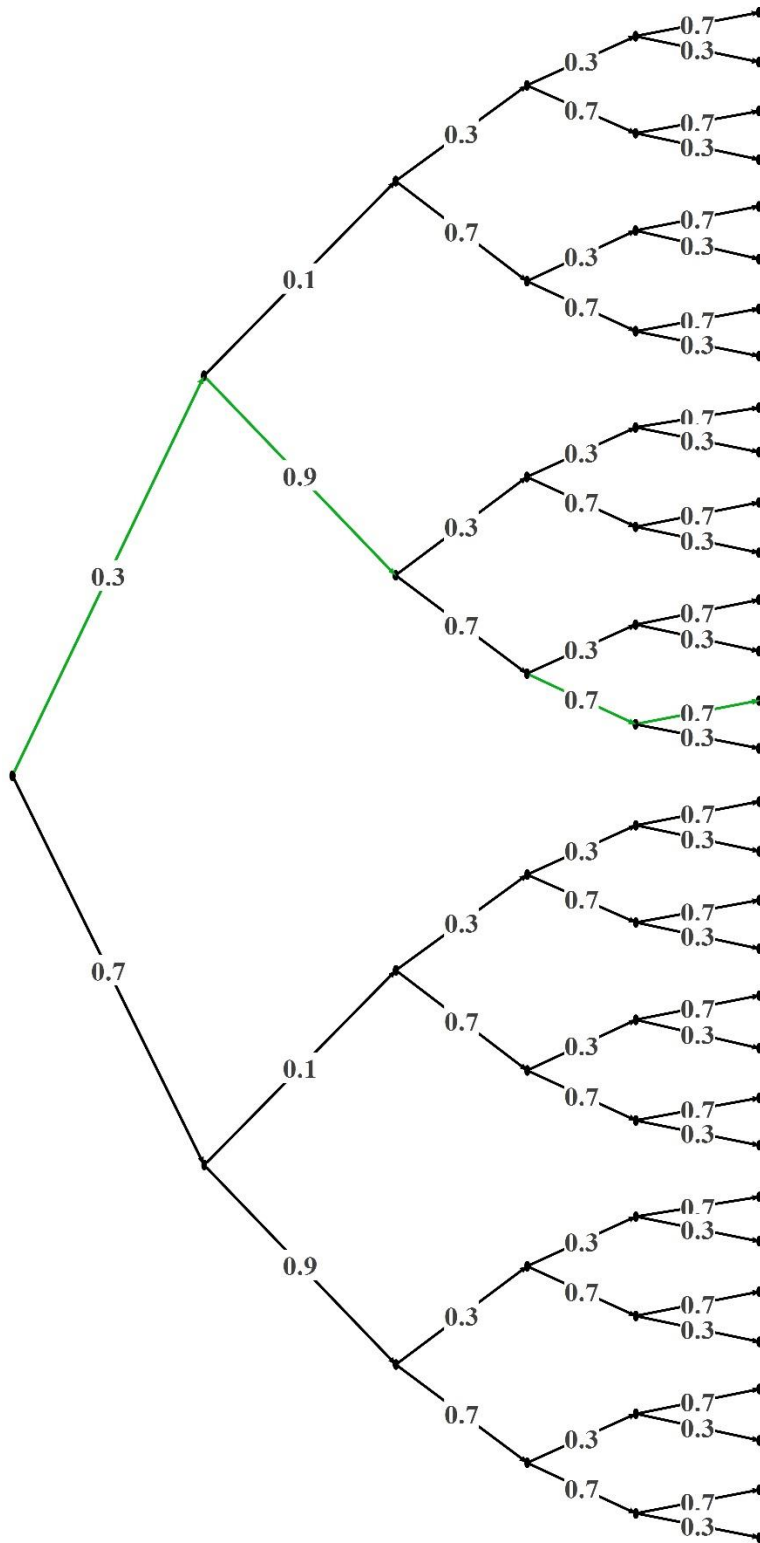


Figure 2.6c. Alignment (in green) of $\langle +\dots + \rangle$ onto the contour family of figure 2.4d, omitting position three



Both of these calculations imply that some sort of contiguous measurement is necessary to make a judgment of similarity: they leave off either the first or last position, and maintain the integrity of the entire smaller c-seg. However, this leaves out the possibility that our judgment of similarity of this c-seg might be based on an alignment that omits one of the middle pitches. Consider Figure 2.6c: this tree shows the new c-seg aligned in a way that omits position three, yielding a membership value of 0.65. While this membership value is also not particularly high, it is the highest membership value out of all the possible alignments. Perhaps this c-seg is not the most average of members within the family, yet this higher membership value does indicate that this alignment may more accurately reflect the similarity perceived between the c-seg and the family.

Put in more general terms: in order to find the membership value of a c-seg with a smaller cardinality than that of the family, one must calculate membership values for all possible alignments, omitting each position in turn. For a family with c-segs containing five positions, five calculations will be made. Figure 2.6d shows this set of calculations for the potential new c-seg discussed above.

Figure 2.6d. Calculations of best fit for each alignment of <+--+> onto the contour family of figure 2.4d

Pos. 1	2	3	4	5	Total	Membership value
Omit	0.1 (+)	0.7 (-)	0.7 (-)	0.7 (+)	2.2	0.55
0.3 (+)	Omit	0.7 (-)	0.7 (-)	0.7 (+)	2.4	0.6
0.3 (+)	0.9 (-)	Omit	0.7 (-)	0.7 (-)	2.6	0.65
0.3 (+)	0.9 (-)	0.7 (-)	Omit	0.7 (+)	2.6	0.65
0.3 (+)	0.9 (-)	0.7 (-)	0.3 (+)	Omit	2.2	0.55

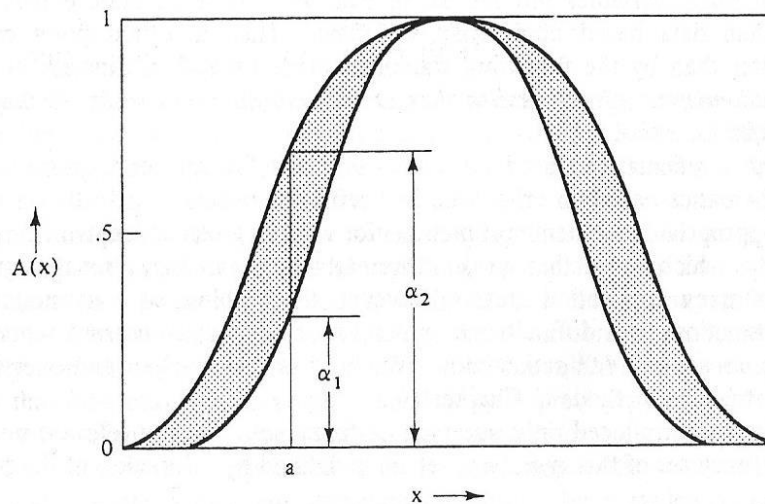
These kinds of membership values reflect a fuzzy degree of confidence that a c-seg will be in the family of related c-segs. However, as with the above illustration, our degrees of confidence can themselves be imprecise, and this type of uncertainty is actually quantifiable using fuzzy set theory. Klir and Yuen discuss this kind of uncertainty, stating that:

membership functions are often overly precise. They require that each element of the universal set [in this case each c-seg within the family] be assigned a particular real number. However, for some concepts and contexts in which they are applied, we may be able to identify appropriate membership functions only approximately. (Klir and Yuen 1995, 16)¹⁷

These approximate membership values accept that there is additional fuzziness in the definition of membership, and instead includes “a closed interval of real numbers between the identified lower and upper bounds.” These sets are called interval-valued fuzzy sets, and an example of this type of set is graphed in Figure 2.7. Here, for each element (x), the membership value of that element in the fuzzy set is represented as the range between the two curves. So the equation belonging to the graph is “ $A(a)=[\alpha_1, \alpha_2]$ ” where the upper and lower bounds are represented by α_1 and α_2 . It is this type of membership that is produced by calculating the “crisp” membership values of each possible alignment. Here, we have a membership value for the c-seg $\langle 34201 \rangle$ that is within the bounds of 0.55 to 0.65.

¹⁷ Klir and Yuen refer to these membership values as membership functions. I use the term value to be more inclusive of potential situations where calculations of membership may not be true mathematical functions.

Figure 2.7. An example of an interval-valued fuzzy set (Klir and Yuan 1995, 16)



Integrating contours of differing cardinality into the CAS probability tree

Two possibilities exist for the integration of this new c-seg into the family's probability tree, and these approaches reflect the possible contexts in which these new potential family members present themselves in musical situations. One may or may not know how a particular c-seg emerged from the context of the other members of a family, and this knowledge will affect how the c-seg is integrated into the larger familial framework.

Beethoven's Fifth Symphony provides an excellent example of a context where the analyst has a good idea of how a particular c-seg emerged from the context of other members of the family. Lawrence Zbikowski provides a comprehensive categorization of the motives of Beethoven's fifth symphony, from mm. 1–37, shown in Figure 2.8a

(Zbikowski 2002, 45). This general motive is a good example because of its familiarity and strong presence in the symphony. He displays a conceptual model of the motive, shown in Figure 2.8b: we typically understand the motive as having the rhythmic pattern of three eighth notes followed by a longer note, and a CAS of $\langle ==- \rangle$. When varied forms of that motive appear, we can understand them in the context of this pattern. Figure 2.9a shows a list of the unique CAS values for the motive family that Zbikowski highlights, and Figure 2.9b shows a CAS probability matrix for the family.

Figure 2.8a. Twenty-seven contours from the motive family in mm. 1–37 of Beethoven’s Fifth Symphony



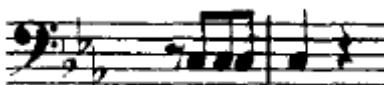




Contour	CAS (Friedmann, 1985)	Number of occurrences	Musical example
$\langle 1110 \rangle$	$\langle ==- \rangle$	15	
$\langle 2210 \rangle$	$\langle =-- \rangle$	3	
$\langle 0000 \rangle$	$\langle === \rangle$	2	
$\langle 0012 \rangle$	$\langle =++ \rangle$	2	
$\langle 0001 \rangle$	$\langle ==+ \rangle$	2	
$\langle 2201 \rangle$	$\langle =-+ \rangle$	2	
$\langle 0111 \rangle$	$\langle += \rangle$	1	

Figure 2.8b. Conceptual model for the category *motive forms of the opening of Beethoven's Fifth Symphony* (Zbikowski 2002, 47)

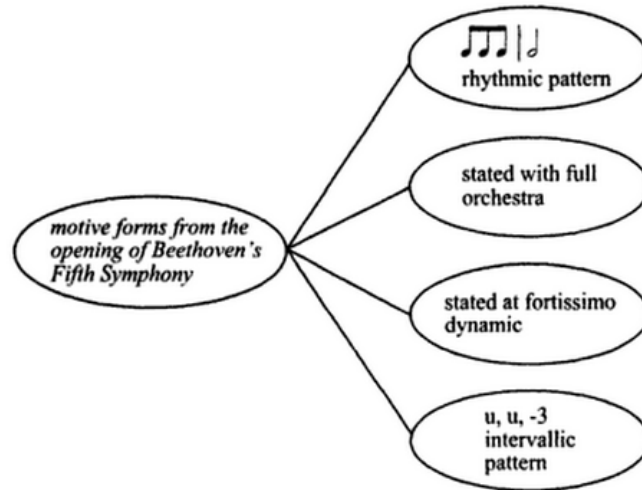


Figure 2.9a. CAS values for the Beethoven's 5th Symphony motive family

CAS	Number of occurrences
<===>	15
<=-->	3
<====>	2
<=++>	2
<===+>	2
<=-+>	2
<=+=>	1
<+==>	1

Figure 2.9b. CAS Probability matrix derived from the Beethoven motive family

	Pos. 1	2	3
+	1/28=0.036	3/28=0.107	6/28=0.214
=	27/28=0.964	20/28=0.714	4/28=0.142
-	0/28=0	5/28=0.178	18/28=0.642

This is the family against which one can compare the contours of motives that occur throughout the rest of the piece. Consider for example, the passage from mm. 171–187, shown in Figure 2.9c. Beethoven has varied the motive, shortening it by removing one of the opening eighth notes characteristic to the motive in the exposition. The motive in the violins in mm. 186–187, for example, features a contour of $\langle 110 \rangle$ and a CAS of $\langle =- \rangle$. The best fit procedure comparing this c-seg to the family is shown in Figure 2.9d.

Figure 2.9c. Beethoven Symphony No. 5 in C Minor, Op. 67, mm. 171–187



Figure 2.9d. Best-fit alignments for $\langle 110 \rangle$ on the Beethoven motive family

Pos. 1	2	3	Total	Membership value
Omit	0.714 (=)	0.642 (-)	1.356	0.68
0.964(=)	Omit	0.642 (-)	1.606	0.80
0.964 (=)	0.178 (-)	Omit	1.142	0.57

The membership value for this motive within the contour family ranges from 0.57 to 0.8. However, the conceptual model posited by Zbikowski may give us some information about what is most “typical,” and we may be able to make a judgment regarding the proper alignment of this particular motive against the family tree of the previous motives in the piece. Using this knowledge, we might be able to rule out the

alignment that omits position three, because the third position is typically the downward motion that is so characteristic to the motive, and which occurs in our potential new member of the family. We may also be able to make a case that the motive in mm. 186–187 comes from the omission of position two, ruling out the omission of position one. Because some motive forms in the family have ascents or descents in position two, this makes position two far more variable than position one. The alignment that omits position two therefore preserves the more stable characteristics of the motive family. These judgments, based on the context from which the motive emerged, point us to a single proper alignment, which we can use to calculate a representative value out of the interval-valued membership that we are examining, and to integrate the respective positions of this c-seg into the family. In this case, we might select the alignment that omits position two as our most representative alignment, yielding a membership value of 0.8.

To integrate this new c-seg into the family, one must follow the chosen alignment and add one to each activated position within the probability matrix. The new matrix is shown in Figure 2.9e. The values for positions one and three have changed to reflect that those positions are present in 29 members of the family, yet the values for position two did not change, reflecting that this position does not exist in the new c-seg we have just integrated into the family. It therefore only possesses 28 members.

Figure 2.9e. Integration of <=> into the Beethoven motive family using the single best-fit method

	Pos. 1	2	3
+	1/29=0.034	3/28=0.107	6/29=0.206
=	28/29=0.965	20/28=0.714	4/29=0.138

-	0/29=0	5/28=0.178	19/29=0.655
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This approach, which I will refer to as the melodic best-fit integration process, is useful when examining the development of a particular motive, because it allows one to pick a best fit based on the musical context; a potential alignment that most accurately reflects the analyst's perception of the relationship between the new member and the existing family. Such a perception can be based on factors outside the realm of contour, and such factors can lead to a choice of alignment that does not necessarily yield the highest possible membership value within the interval-valued fuzzy set. The ability to make such a choice will lead to more analytically rich observations using this model.

The Beethoven example showed that when one has knowledge of musical context, and a desire to make specific claims to similarity based on a relatively stable conceptual model, one can pick a "best fit" from the possible alignments made in the best-fit calculation. However, when knowledge of musical context is lacking or the conceptual model is not stable with regard to contour, it becomes difficult (and perhaps undesirable) to make the same kind of best-fit judgment out of the possible alignments. In these cases the integration of a new member with differing cardinality into an existing family needs to reflect the true interval-valued nature of the fuzzy set: the manner of integration must in some way reflect all possible alignments of the c-seg along the CAS tree. This integration method will be called the mathematic best-fit integration process.

Returning to Figure 2.9d, we can see that each position has two potential motions and one missing motion (the omission). For position one, both possible motions involve

plateaus between the first and second c-pitches. Similarly, in position three, both possibilities involve descent. However, position two is split two ways in terms of its possible motions: one option is to plateau, and the other one is to descend. This means that we know with certainty that position one will be aligned as a plateau, and position three will be aligned as a descent. Because of the two possibilities for position two, we are only 50% certain that it could plateau and 50% certain that it could descend. Such an uncertainty needs to be reflected in how we distribute the value of this new member.

In a similar way to calculating the membership values of the initial family, one can create a matrix that shows how the new c-seg should be distributed within the family, shown in Figure 2.9f. This matrix tallies the number of occurrences of a particular motion within the best-fit calculations, and divides this number by the total number of entries for that position. From here, one needs only to add the value for each position to the CAS matrix of the family, as shown in Figure 2.9g. Using this method, the new c-seg has been distributed throughout all the possible positions, so the total members for all positions is increased to 29, and each probability is then calculated out of 29.

Figure 2.9f. Distribution matrix for integration into the CAS probability matrix

	Pos. 1	2	3
+	0	0	0
=	$2/2=1$	$1/2=0.5$	0
-	0	$1/2=0.5$	$2/2=1$

Figure 2.9g. Integration of ⟨=-⟩ into the Beethoven motive family using the distributive method

	Pos. 1	2	3
+	$1/29=0.0344$	$3/29=0.103$	$6/29=0.206$

=	28/29=0.9655	20.5/29=0.707	4/29=0.138
-	0/29=0	5.5/29=0.189	19/29=0.655

This approach keeps intact the notion that one cannot precisely pinpoint the degree of membership of this new c-seg within the family of existing c-segs. By distributing all the possible best-fit calculations in this way, we have recreated the imprecision reflected by the interval-valued fuzzy membership value within the family structure. All the possible alignments are now reflected in the family, such that new c-segs that align well with one of the possible alignments may have a higher degree of membership than if only one alignment was integrated into the family.

Both approaches toward integration make different assumptions about musical relations, and as such it is important for the analyst to examine exactly what each approach says about the music in question. In this way, the model builds in, and indeed requires, interpretation on the part of the analyst. Such flexibility adds a qualitative component into an otherwise purely quantitative model, and this qualitative subjectivity is what gives the model the sensitivity needed to address a wide variety of musical contexts. However, it is important to note that this sensitivity also requires interpretive responsibility on the part of the analyst to be sensitive to the musical environments in which the contours under examination occur.

C-segs that are larger than the family

Up to now, we have only examined c-segs that are either equal or smaller in cardinality than the family in question, yet the logic works in similar ways for c-segs that are larger than the family as well. Imagine for example that we are trying to determine if

a c-seg <22201>, with a CAS of <==-> has a high degree of membership within the family from Figure 2.9b, reproduced as Figure 2.10a. The procedure is similar, except this time, the larger c-seg is the crisp one, and not the fuzzy family representation as it was in the previous examples. This requires us to reverse the logic we used to calculate the best-fit values.

Figure 2.10a. CAS Probability matrix derived from the Beethoven motive family

	Pos. 1	2	3
+	1/28=0.036	3/28=0.107	6/28=0.214
=	27/28=0.964	20/28=0.714	4/28=0.142
-	0/28=0	5/28=0.178	18/28=0.642

Instead of aligning the new c-seg along the existing family tree, we must instead align the family tree along the new c-seg. Omitted positions then are the positions in the new c-seg that have to be omitted in order for the c-seg to match up with the family. For example, the calculation shown in Figure 2.10b shows the omissions of the four positions respectively: when the matrix states that position one is omitted, the first “=” in the crisp c-seg’s CAS is the omitted value, and the c-seg is treated like a three-position c-seg.

When position one is omitted, the CAS becomes <=->; when position two is omitted, the CAS becomes <=->; when position three is omitted, the CAS becomes <==>; when position four is omitted, the CAS becomes <==->; and these shorter CAS values are then mapped against the CAS family tree yielding an interval valued membership value of 0.45 to 0.77.

Figure 2.10b. Best-fit alignments for $\langle ==-+ \rangle$ on the Beethoven motive family

Pos. 1	2	3	4	Total	Membership Value
Omit	0.964 (=)	0.178 (-)	0.214 (+)	1.356	0.45
0.964 (=)	Omit	0.178 (-)	0.214 (+)	1.356	0.45
0.964 (=)	0.714 (=)	Omit	0.214 (+)	1.892	0.63
0.964 (=)	0.714 (=)	0.642 (-)	Omit	2.32	0.77

From here, the melodic best-fit integration process follows a similar procedure as for smaller c-segs. If one chooses a single best-fit—the alignment that omitted the final position, for example—one would then integrate the entire c-seg into the probability tree, adding a position where the omitted position occurred. Figure 2.10c shows the added position 4 at the end of the c-seg. On its face, this type of integration may seem counterintuitive as a value of 1.00 for the new position seems like an inordinately high probability. However, in this case it is important to remember that not all positions occur in the family, and for each CAS position, the values are stating that if the position does occur in a c-seg, these are the probabilities that it will go in a particular direction. The value of 1.00 for the ascent in position four, especially when written as the fraction 1/1, indicates that this position rarely occurs, but when it does (in the single member of the family that possesses this position) it always goes down.

Figure 2.10c. Integration of $\langle ==-+ \rangle$ into the Beethoven motive family using the single best-fit method

	Pos. 1	2	3	4 (the added position)
+	1/29=0.034	3/29=0.103	6/29=0.207	1/1=1.00
=	28/29=0.966	21/29=0.724	4/29=0.137	0/1=0
-	0/29=0	5/29=0.172	19/29=0.655	0/1=0

The alternative method of integration, where all possible alignments are integrated into the family, presents a more challenging calculation when dealing with new c-segs that are larger. In the case of the larger c-segs, the uncertainty involved with the alignment procedure addresses where the missing position in the crisp c-seg should be added into the fuzzy family (since the family is now larger). As a result, the mathematic best-fit method of integration involves the even distribution of the entire fuzzy set across the now larger framework represented by the new crisp c-seg. In essence, it requires the recontextualization of all the other (smaller) crisp c-segs in the family against the crisp c-seg in exactly the method of mathematic best-fit calculation outlined earlier. These calculations are conceptually challenging without the aid of a computerized computational model. Fortunately, as we shall see in the coming chapters, situations where this becomes a problem are relatively rare, and do not come up at all in the analyses in this dissertation.¹⁸

Conclusion

Quinn is certainly correct when he discusses the analytical usefulness of fuzzy set theory, and explains how it can lead one to a more refined understanding of musical

¹⁸ Because of this computational challenge, it is advisable always to begin with the calculation of the CAS family probability matrix with the largest c-seg in the known family, unless it is the goal to show the addition of c-segs into the family in a specific alternate ordering (for example, if the goal of the analysis is to show the diachronic development of the family, in the same manner that is done in Chapter 5). In this way, one is able to include effectively the mathematical imprecision into the family without any added difficulty.

similarity across a variety of parameters. As I have illustrated, accepting the notion of fuzziness and uncertainty into our notion of cardinality can open the door to even more refined judgments of similarity and familial membership. Integrating multiple cardinality capabilities into our similarity relations for contour more adequately accounts for the similarity of c-segs within a family, and offers new possibilities in terms of analytical application.

The FCM model developed in this chapter addresses this cardinality issue by including a number of procedures for the integration of c-segs with variable cardinalities into a single family. Such procedures model the fuzziness that occurs when cardinality differences occur, and integrate that fuzziness into the overall fuzzy family of the c-segs in question. In doing so, the FCM model provides a multifaceted representation of contour similarity, which can illustrate many possible kinds of relationships between c-segs in a melody family. This potential offers new possibilities in terms of analytical application to a wide variety of repertoires, as we shall see in Part II of this dissertation.

CHAPTER 3: Fuzzy Families in Plainchant: What Fuzzy Set Theory Can Say about Musical Ontology

Medieval plainchant “was a kind of traditional practice specific to local liturgical communities but under pressure since the Carolingian era to cleave to the practice of Rome...and thereby to constitute a uniform, universal practice” (Treitler 2003, 131). However, its origin as an oral tradition resulted in differences in practice that have yielded a multiplicity of chant variants. Such variants complicate both the intentions of the Carolingians as well as our modern understanding of plainchant’s ontology. Regarding such chant variants, Leo Treitler writes that “the scores of these melodies themselves challenge one’s sense of what counts as ‘the same melody’” (1993, 491).

This multiplicity has sparked larger discussions regarding musical ontology, which rely on human interpretation to understand the complicated nature of the musical object (Popper 1977, Ingarden 1989, Treitler 1993, Bohlman 1999, Cook 2013). Karl Popper, for example, puts forth a traditional theory that the musical work is “a real ideal object which exists, but exists nowhere, and whose existence is somehow the potentiality of its being reinterpreted by human minds” (Popper 1977, 450). Treitler likens the musical work to “that of a unicorn” (1993, 483): the unicorn’s existence relies on individual interpretations, resulting in many depictions that obscure the “ideal image” of the unicorn. The multiplicity in both the representations of unicorns and of plainchant melodies precludes the possibility of a single ideal form. In other words, the musical idea itself is fuzzy.

This chapter examines how the principles underlying fuzzy set theory (Zadeh 1965), exemplified by the FCM model, can contribute to our understanding of musical ontology, especially with regard to plainchant. Using the model developed in Chapter 2, I quantify a chant passage's familial membership on the basis of contour. In this way, I illuminate the fuzzy relationship between individual representations of a chant, as well as their collective relationship to the chant as an abstract musical idea. Examining chant passages in this way allows us to come closer to an "ideal image" of the chant in question. The convergences highlight structural tendencies common to the family, allowing us to imagine a fuzzy representation of the passage as a whole. It is, in many ways, like Treitler's unicorn: there is no one "ideal" unicorn and no singular representation of a musical idea. Using fuzzy contour membership to quantify convergences and divergences between the notated variants of a chant, one can gain a more thorough understanding of the fuzziness within the musical idea itself.

Plainchant's Ontological Complications

Leo Treitler's seminal article "Homer and Gregory: The Transmission of Epic Poetry and Plainchant" examines several ontological complications with regard to the oral transmission of plainchant by "work[ing] out a detailed account of how such a prolix body of traditional song can have come into existence and have been disseminated in an oral culture" (2003, 131). He draws upon theories of memory and recall, as well as theories of epic poetry performance in order to illustrate possible ways of understanding plainchant's complicated origin, a task fraught with ontological complications when one

considers that our primary body of evidence is the notated chant texts that are the result of hundreds of years of oral dissemination.

The ontological status of a chant (involving both text and melody) in the absence of notation relies on the collective memory and reconstruction of the chant within the community (in this case being the wide spread community of the Roman Catholic church as it existed across Europe before notation).¹⁹ Because of this reliance of communal memory and oral transmission, it is difficult to pinpoint with any certainty an exact melody, as we may be more easily able to do in the era of the Western canon. Theodore Karp writes that “it is difficult to establish why one particular Burgundian, Aquitanian, Beneventan, German, Northern Italian, Northern French, or insular source is more representative of the medieval tradition as a whole than any other source with a different reading” (1989, xiv), suggesting that there is no one prototypical representation, and instead looking for a representation that can speak to average practice.

Historical accounts of this oral transmission point toward a desire for unity, but also illuminate the practical impossibilities of such desire’s exactitude. For example, in the ninth century, John the Deacon wrote “Of the various European peoples it was the Germans and the Gauls who were especially able to learn and repeatedly to relearn the suavity of the schola’s song, but they were by no means able to maintain it without distortion” (quoted in Strunk and Treitler 1998, 179). The effect of constant remembering

¹⁹ Many scholars discuss this in great detail, citing several hypotheses regarding possible modes of oral transmission that contributed to stability in pre-notation times. However, Karp warns that “without a deeper acquaintance of source material than we presently possess, we are able to give free rein to the tendency to seek simple solutions where they do not exist” (Karp 1989, xii).

and reconstruction results in changes to the chant melody that complicate the notion of what defines the melodies. However, the notated evidence we have as the chants were written down across the various regions in Europe point to a certain level of fixity that help us to understand how the process of remembering and recreation may have taken place. Treitler has stated that “efforts to reach behind the notated manuscripts have depended largely on a principle derived essentially from the adaptation to music of the dictum formulated by the liturgist Walter Frere: ‘Fixity means antiquity, that is to say that if the same formulary appears in many sources it must be relatively old’” (Treitler 2003, 144).

Theories regarding the fixity of chant variants fall into two broad categories. Some theories posit a singular origin, wherein fixity observed in the notated variants arises from the existence of a singular archetypical source of notated chant. Kenneth Levy, for example, postulates the existence of “an authoritative neumed recension of the Gregorian propers ca. 800, a century sooner than is presently supposed” (Levy 1998, 82). In essence, this approach toward fixity treats the early descendants of the original notated chant (i.e., our existing body of evidence) as deviations from an “ideal.”²⁰

Other theories propose that fixity arose from the gradual coalescence of multiple traditions into a practice that coheres musically as a single entity. James McKinnon, for example, puts forth that “if one compares the same chants as they appear in a generously

20 It is important to note that Treitler and others are skeptical of this single-origin view, creating significant debate in the musicological community regarding these theories of fixity (Treitler 1988, Levy 1988, Hughes 1988, Zijlstra and Van Der Werf 1997). My aim here is not to contribute to the debate, but rather to show that both sides can potentially benefit from being represented using fuzzy set theory.

representative selection of early notated manuscripts and finds them to be nearly identical, then one must conclude that the chants existed in substantially the same form for a considerable period before the manuscripts were written” (1991, 99). Similarly, David Hughes writes that “the evidence of the manuscripts clearly shows the chant to have acquired fixed form well before the appearance of the earliest surviving notated manuscripts” (1987, 377). Both McKinnon and Hughes point to a desire for fixity before the advent of notation, resulting in the relative uniformity we see in the early notated sources.

These theories all describe the coherence of these variant families qualitatively, but according to Treitler, they “lack clearly formulated criteria for our own judgments about what makes variants ‘slight,’ ‘negligible,’ ‘insignificant,’ ‘minor,’ and ‘trivial,’ and for determining the boundaries of a chant’s ‘identity’” (Treitler 2003, 148). Fuzzy set theory provides a quantitative lens through which to examine these issues. Using this lens, a family of chant variants would be represented as a fuzzy set, wherein each variant is assigned a membership value corresponding to “the degree to which that individual is similar or compatible with the concept represented by the fuzzy set” (Klir and Yuen, 1995, 4). Examining melodic passages from a group of chant variants in this way sheds light on Treitler’s question regarding the criteria for our judgments about a chant’s degree of variance from the imagined “ideal.”

Fuzziness and Familial Grouping

Fuzzy set theory relies on familial resemblance to determine membership values for members of a fuzzy set. In many ways, musicologists are already thinking along these lines in terms of their practice of grouping regional chant variants into families. Theodore Karp, for example, writes that “the variety exhibited in the various readings creates a malaise if one refers to the group in terms of ‘a melody;’ it seems preferable to speak of a melody complex” (Karp 1989, 13). This established tradition of grouping has helped many chant scholars approach their analyses with sensitivity to this ontological problem, yet most rely on qualitative description of their observations of the group in order to theorize regarding the group’s potential origin and dissemination.²¹

Fuzzy set theory offers two models that align in similar ways with the theories of fixity outlined above. The first model provides gradations of membership within a fuzzy set based on a 0-to-1.0 scale, measured against a single crisp source representing the “concept represented by the fuzzy set.” This model is most analogous to the theory of fixity posited by Levy, who theorized a single source from which all variants originally arose. However, there are many issues with this model ontologically when it comes to the “concept” of the melody. Treitler writes that

there is still a missing element: the story needs to persuade its readers that in the [musically] scriptless culture in which it is placed, where music had always been passed from teacher to pupil or from one singer to another; musical notation, immediately upon its invention, would have functioned in the prescriptive mode

21 The notable exception is the work of Kate Helsen, whose big-data approach toward plainchant helps to quantify relations among families in a similar manner to that which I am proposing in this chapter (Helsen 2009, 2014). However, as with other musicological approaches, this methodology does not treat the corpus as fuzzy, and does not examine contour in the same way, and therefore does not make the same claims that I am making.

that we take for granted today and the authority of a written text would at once have been accepted and texts adopted as the preferred medium of transmission. It seems that these ideas have been so inculcated into the habitus of the field of musical practice as it is today that Levy did not see fit to mention them. (Treitler 2003, 147)

Like Levy's theory of fixity, the concept of a single crisp musical idea lacks the flexibility to model the very human manner of oral melody transmission.

The second model proffered by fuzzy set theory treats the "concept represented by the fuzzy set" as fuzzy in and of itself. It bases the gradations of membership of individuals within the family against the average of all members of the family. In other words, because there is no one exemplar, membership is measured against averaged characteristics of members already in the family. In this way, this model is better able to account for dominant family traits, while still allowing individuals to possess more recessive traits—more of a true "family" structure. This model is most in alignment with the theories of McKinnon and Hughes, both of whom posit that melodic stability arose before notation and is based on the human process of remembering and reconstruction. It is this model that also approaches the nuance with which we must think of the musical object in these cases. As Karp mentioned, to think singularly of "the melody" is to miss the depth that the melody idea contains.

Treitler offers one particular melody family that exemplifies exactly this type of fuzziness: his Example II.3, reproduced here as Figure 3.1, shows a family of variants of the trope *Filius ecce patrem*, a genre of plainchant designed to interject between sections of existing plainchants—in this case, introits. Indeed, variation between the passages calls into question the crispness with which we conceive of the term "melody," suggesting that for this repertoire the notion of melody is ontologically fuzzier than we tend to think.

Figure 3.1. Passages from the trope *Filius ecce patrem*, showing regional variants (Treitler 2003, 276)

Ex. II.3. Trope *Filius ecce patrem* from Paris lat. 1871, 887, 1235, and Benevento VI. 34

I
a
1871, fo. 40^v
M N
Fi - li - us ec - ce pa - trem con - pel-

b
887

c
1235

d
Benevento

e
1871, fo. 37^v
[A] - pos - to - li - cis ve - ne - ran -

f
1871, fo. 37^v
[C]ul - men a - pos - to - li - cum An - dre - am su - per as - tra be - a - tus in - sig - nis

a
O
lans ta - li - ter in - quit A-C

b
A-C

c
A-C

d
A-C

e
AB
dis me - los clan - ga - mus ho - no - ri - bus

f
AB
re - ti - net, nos in - de ca - na - mus om - nes

Treitler identifies six trope variants that share similarities in both melody and liturgical use.²² Each variant is labeled *a–f*, and the variants are laid out vertically in the figure in order to align similar melodic segments with one another. Of those that bear the same text (*a–d*), two are Aquitanian, one is from northern France, and one is from southern Italy. Despite the difference in region, the melodies manifest similarly, so much so that a casual observation (such as the kind that Quinn suggests in his article on fuzzy contour) of the melodies would lead the observer to believe that they belong together. Indeed, Treitler writes that “At a quick glance it will strike everyone that in *a–d* we have essentially the same melody. What is difficult is to find ways of describing the similarities and differences that a closer examination will bring into focus” (2003, 266). In essence, Treitler questions how one can understand the slight differences between each variant in the context of this larger family resemblance. The monophonic nature of chant suggests that melodic shape—or contour—plays a critical role in its structure and organization, so analysis of contour could prove to help address Treitler’s quandry. Figure 3.2a–b shows the analysis of these contours using the FCM model: for example, Figure 3.2a shows the contours and membership values of the variants *a–d* for phrase “M” (“Fillius ecce patrem”). For example, the variant *b* possesses a CAS of $\langle =+--++----+++--\rangle$, which is distributed and measured against the CAS grid of the averaged family shown in Figure 3.2b. In this measurement, the averaged values for each motion in the contour (+, =, or -) are represented in the grid of Figure 3.2b, such as the plateau in position one or the ascent in position two that both possess 1.0 membership, or the

22 The last two, designated as “e” and “f”, feature different text, and are used at a different date within the liturgical calendar, yet the function is similar, as will be described below.

plateau in position three that exhibits a 0.250 membership. This calculation follows thus: $(1.0 + 1.0 + 0.250 + 0.333 + 1.0 + 1.0 + 1.0 + 0.333 + 0.667 + 1.0 + 0.333 + 1.0 + 0.750 + 1.0 + 1.0) / 15 = 11.666 / 15 = 0.778$. Examining the membership values of the family in this way, one can see that the variants possess relatively high degrees of membership within the family. Indeed, the average membership of these chants within their family ranges from 0.762 to 0.916 (see Figure 3.2a).²³ Within these phrases, we can see shared structural tendencies that contribute to these high values. Phrase M displays several affinities that each of the melodies in the family share: in Figure 3.2b, we see that positions 1, 2, 5, 6, 7, 10, 12, 14, and 15 all display 1.0 membership/non-membership within the family, indicating that all family members follow the same contour pathway at these points in their respective contour transformations. For example, all of the chants have a strong middle ascent followed by descent in positions 5–7, as seen in Figure 3.2c. Likewise, the majority of the chants finish with a descent followed by a plateau, as shown in positions 14 and 15: all except variant *a* follow this pathway.²⁴

We find similar affinities in the phrases labeled “N” and “O,” as shown in Figure 3.3: the family of phrase N shows considerable uniformity in its averaged family, only varying in the final position. As a result, the members of phrase N have very high membership values, ranging from 0.933 to 1.0. The family of phrase O shows a little

23 Granted, this is a rather small family, so the membership degrees that we see in smaller families have the potential to reside on the more drastic ends of the spectrum. Nonetheless, that these membership values fall toward the high end is significant.

24 Variant *a* does not end in this way because the variant’s ending seems to align better with the middle points of the other three phrases, specifically positions 10-12. It lacks the “ending” of the others, but is still conceptually a member of this family because of these other shared melodic characteristics as well as the shared text.

more variety (only four of the thirteen positions has a 1.0 membership), yet the melodies still maintain a range of membership values between 0.692 to 0.885.

Figure 3.2a. Contours of variants A–D for phrase M of the trope Filius ecce patrem

ID	C-SEG	CAS	Card. of CAS	Membership
A	⟨1012101⟩	⟨-++--+⟩	6	0.916
B	⟨0011034321012311⟩	⟨=+=-++-----+++=⟩	15	0.778
C	⟨120124232102311⟩	⟨+-----+-----+⟩	14	0.762
D	⟨110123122102211⟩	⟨=-+++++=-++=-⟩	14	0.833

Figure 3.2b. Averaged family of phrase M

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
+	0	1.0	0	0.667	1.0	1.0	0	0.667	0	0	0.333	1.0	0.750	0	0
=	1.0	0	0.250	0	0	0	0	0	0.333	0	0	0	0.250	0	1.0
-	0	0	0.750	0.333	0	0	1.0	0.333	0.667	1.0	0.667	0	0	1.0	0

Figure 3.2c. The family of phrase M, showing internal consistencies within the familial membership

The figure shows four musical staves for phrase M. Each staff has a red bracket under the first three notes and a red '+' sign below it. The contours are: a) Fi - li - us ec - ce pa - trem, b) Benevento, c) Benevento, d) Benevento.

Figure 3.3a. Contours of variants A–D for phrase N of the trope Filius ecce patrem

ID	C-SEG	CAS	Card. of CAS	Membership
A	⟨14432121011⟩	⟨+=----+++=⟩	10	0.933
B	⟨01201⟩	⟨++-+⟩	4	1.0
C	⟨121010⟩	⟨+----⟩	5	0.933
D	⟨0121010⟩	⟨+----+⟩	6	0.945

Figure 3.3b. Averaged family of phrase N

	1	2	3	4	5	6	7	8	9	10
+	1.0	0	0	0	0	1.0	0	0	1.0	0
=	0	1.0	0	0	0	0	0	0	0	0.333
-	0	0	1.0	1.0	1.0	0	1.0	1.0	0	0.667

Figure 3.3c. Contours of variants A–D for phrase O of the trope Filius ecce patrem

ID	C-SEG	CAS	Card. of CAS	Membership
A	⟨22101224221321⟩	⟨=---+++=--++-⟩	13	0.885
B	⟨01334532123101⟩	⟨++=+++++++-+⟩	13	0.692
C	⟨0132453203311⟩	⟨++-++---+--=⟩	12	0.750
D	⟨01333213311⟩	⟨++==--++=-=⟩	10	0.700

Figure 3.3d. Averaged family of phrase O

	1	2	3	4	5	6	7	8	9	10	11	12	13
+	0.750	0.750	0	1.0	1.0	0	0.250	0	0.750	0.250	0.500	0	0.250
=	0.250	0	0.500	0	0	0.500	0	0	0.250	0.500	0	0	0.500
-	0	0.250	0.500	0	0	0.500	0.750	1.0	0	0.250	0.500	1.0	0.250

Such analyses suggest that the similarities between variants contribute to a sense among listeners and performers alike that these are “the same melody,” calling into question—as Treitler indicates—the notion that each specific notated variant accounts for the complete notion of “the melody.” What this analysis using the FCM model gives us is the degree to which each specific notated variant resembles the melody, or rather the degree to which

the variant is able to represent the ideal concept of the melody. It also enables us to understand where the shared characteristics occur in the phrases (as they are not always readily apparent), and what those shared characteristics are in terms of relative contour.²⁵

This analysis also allows one to explore the role that these shared characteristics play in the act of remembering and reconstructing a chant in performance. Such considerations are crucial to the understanding of chant's ontology because the manner of oral transmission is mostly responsible for the great degree of multiplicity that we see in chant families like these. Treitler considers various psychological principles at play in the process of memory and recall. He notes that remembering depends on the active perceptual process of organization, and that this organization is dependent upon salient features of chant (such as its beginning and ending) that serve as structural signposts. Furthermore, the process of reconstructing chant from memory is dependent upon patterns of past experience, which leads to formal categorization depending on the chant's salient features (Treitler 2003, 159). It is this process of reconstruction that opens the door for variability, as the recall of past experience is not crisply accurate. In short, the active organization around salient structural signposts gives a singer or listener a framework (or form) from which one can reconstruct the finer details of the melodic passage. This notion of memory for melody is not especially novel, nor does it pertain

25 In using this method, I am not claiming that we can examine ontological coherence on the basis of contour alone—as ontological coherence can occur across multiple musical and extra-musical parameters—but rather am using contour as an example of the way in which studying these parameters using a fuzzy approach can lead us to new questions and new ways to investigate both the ontology of melody and the ontology of chant.

only to the melody of plainchant.²⁶ Nevertheless, plainchant is much more dependent on the perceptual aspects of memory and reconstruction than is the more modern conception of music because of its oral transmission in the absence of written notation.²⁷

Treitler's theories regarding memory in plainchant (especially with regard to what constitutes enough similarity to count as a salient feature) are not always so readily apparent, and the example presented in Figures 3.2 and 3.3 is too small to see some of the complexities that can obscure the judgment of salient features. Figure 3.4 shows a larger family of related chant variants, this time of the opening of the communion *Mirabantur omnes*. Like Figure 3.1, each variant is identified by a letter *a-l*, and arranged such that analogous melodic segments in each variant are aligned vertically. Karp has grouped this family based on mode, genre, and "melody type," suggesting that the family coheres at least in part on melodic shape (1989, 14). Karp refers to this example as a highly variable "melody complex." His examination, however, relies not on a holistic observation as does Treitler's. Instead, he has parsed the phrase by syllable, and then uses the variation within the syllabic structure to critique Treitler's theories of reconstructive memory as a process for oral chant transmission. He writes that reconstructive memory cannot account for several of the differences found within the body of this family. Karp cites mode,

26 Cognitive studies have explored these issues of structure and memory for melody, including Berz 1995; Boltz 1986, 1991; Dowling 1978, 1994; Hasher and Griffin 1978; Oura 1991; Quinn 1997; Schmuckler 1997.

27 In fact, because of the oral nature of chant's transmission, each notated excerpt can be conceived of as a fuzzy representation in itself, a representation of all the possible oral variations that came before it and contributed to the stability of the particular variant that we see written down. In this way, chant families resemble level-2 fuzzy sets, in that each representation included in the family is itself fuzzy. Of course, we cannot know what that lower level-1 fuzzy set entails without the aid of a time machine, but we should remain sensitive to the fact that the fuzziness is there.

starting pitch, cadential preparation, and lack of “melodic personality” as factors that contribute to his impression that this phrase has had an unstable transmission within the oral tradition of chant. What in the music then (aside from text) allows him to group these meaningfully as variants of the same chant and not different chants bearing the same text? In a word, contour: while these melodies may vary, there are similarities in their contours’ structural characteristics that lead one to consider them variants of the same melodic idea rather than separate melodic ideas.

A closer examination of the contour assigned to each syllable reveals an underlying pattern that may account for the logical grouping of these variants together, and suggests that cognitive processes associated with Treitler’s theories of memory may indeed occur in the transmission of this chant. Figures 3.5a–i show a familial analysis of the contours of each syllable of the first word “Mirabantur.” Figure 3.5a, for example, lists the contours of the melody assigned to the syllable “Mi” for each variant *a–l*, and includes each variant’s membership value when measured against the averaged family for this segment shown in Figure 3.5b. Figures 3.5c–h show analogous charts for the syllables “ra” “ban” and “tur.”

Figure 3.4. Karp's example 1: Different regional variants of *Mirabantur omnes* (1989, 14)

The image displays 12 musical staves, labeled 'a' through 'l', each representing a different regional variant of the opening of the chant 'Mirabantur omnes'. The first staff (a) includes the lyrics 'Mi - ra - ban - tur o - mnes' written below the notes. The subsequent staves (b-l) show various melodic and rhythmic interpretations of the same text, with some staves featuring handwritten annotations or specific rhythmic markings above the notes.

EXAMPLE 1. The opening of *Mirabantur omnes*: (a) Chartres, Bibl. mun. MS 47; (b) Laon, Bibl. mun. MS 239; (c) Angers, Bibl. mun. MS 91; (d) Mont Renaud (in private hands); (e) St. Omer, Bibl. mun. MS 252; (f) Montpellier, Fac. de Méd. MS H 159; (g) Darmstadt, Hess. Landesbibl. MS H 1946 (= Milan, Bibl. Ambr. MS D 84 inf.); (h) Ivrea, Bibl. Cap. MS 60; (i) Monza, Bibl. Cap. MS 13/76 (= Vercelli, Bibl. Cap. MS 161); (j) Vatican, Bibl. Apost. MS lat. 4770; (k) Rome, Bibl. Angelica MS 123; (l) Vercelli, Bibl. Cap. MS 186.

Figure 3.5a. Contour family for the syllable “Mi”²⁸

ID	C-SEG	CAS	Card. of CAS	Membership
A	⟨0⟩	NA	0	1.0
B	⟨0⟩	NA	0	1.0
C	⟨012⟩	⟨++⟩	2	1.0
D	⟨0⟩	NA	0	1.0
E	⟨01⟩	⟨+⟩	1	1.0
F	⟨01⟩	⟨+⟩	1	1.0
G	⟨0⟩	NA	0	1.0
H	⟨01⟩	⟨+⟩	1	1.0
I	⟨0⟩	NA	0	1.0
J	⟨0⟩	NA	0	1.0
K	⟨01⟩	⟨+⟩	1	1.0
L	⟨01⟩	⟨+⟩	1	1.0

Figure 3.5b. Averaged family for the syllable “Mi”

	1	2
+	1.0	1.0
=	0	0
-	0	0

28 It is interesting that since all members of this family only involve ascent, they all possess a 1.0 membership value. For example, the third chant, labeled C aligns with the family as follows:

$(1 + 1) / 2 = 2 / 2 = 1.0$. Even the smaller c-segs, bearing only a single ascent, also possess 1.0 membership, no matter which method of calculation is used. The melodic best-fit approach would simply align the ascent with one of the ascents in the fuzzy family represented by Figure 3.5b, resulting in a 1.0 membership.

Under the mathematic best-fit approach, the distribution of the ascent would be uniform across both positions in the CAS grid, again resulting in $(0.5 + 0.5) / 1 = 1 / 1 = 1$.

Figure 3.5c. Contour family for the syllable “ra”²⁹

ID	C-SEG	CAS	Card. of CAS	Membership
A	⟨012⟩	⟨++⟩	2	0.883
B	⟨012⟩	⟨++⟩	2	0.883
C	⟨012⟩	⟨++⟩	2	0.883
D	⟨012323⟩	⟨++++-+⟩	5	0.733
E	⟨012323⟩	⟨++++-+⟩	5	0.733
F	⟨0⟩	NA	0	
G	⟨012323⟩	⟨++++-+⟩	5	0.733
H	⟨0⟩	NA	0	
I	⟨012⟩	⟨++⟩	2	0.883
J	⟨012323⟩	⟨++++-+⟩	5	0.733
K	⟨012323⟩	⟨++++-+⟩	5	0.724
L	⟨0123234⟩	⟨++++-++⟩	6	0.817

Figure 3.5d. Averaged family for the syllable “ra”

	1	2	3	4	5	6
+	1.0	1.0	1.0	0.700	0.600	1.0
=	0	0	0	0	0	0
-	0	0	0	0.300	0.400	0

29 These calculations were made using the mathematic best-fit approach, since the ontology of the relationships between the variants, and thus their respective positions, is fuzzy. For example, the calculation for the ⟨012⟩ c-seg is as follows: $(1 + 1 + 0.850 + 0.800 + 1 + 1 + 0.850 + 0.800 + 1 + 0.850 + 0.800 + 1 + 0.650 + 0.850 + 0.8) / 15 = 13.25 / 15 = 0.883$ This is calculated as the average of the membership values of all possible alignments of the two CAS ascents against the six-position CAS grid, so the first 1 in the equation represents the c-seg aligned against the first two CAS positions, the second 1 in the equation represents the c-segs alignment along positions 1 and 3 in the CAS grid, etc.

Figure 3.5e. Contour family for the syllable “ban”

ID	C-SEG	CAS	Card. of CAS	Membership
A	⟨012⟩	⟨++⟩	2	0.599
B	⟨012⟩	⟨++⟩	2	0.599
C	⟨012⟩	⟨++⟩	2	0.599
D	⟨21210⟩	⟨-+--⟩	4	0.565
E	⟨1210⟩	⟨+--⟩	3	0.610
F	⟨012323321⟩	⟨+++--+==--⟩	8	0.518
G	⟨2321⟩	⟨+--⟩	3	0.610
H	⟨012⟩	⟨++⟩	2	0.599
I	⟨210⟩	⟨--⟩	2	0.667
J	⟨22210⟩	⟨==--⟩	4	0.417
K	⟨22210⟩	⟨==--⟩	4	0.417
L	⟨21210⟩	⟨-+--⟩	4	0.565

Figure 3.5f. Averaged family for the syllable “ban”

	1	2	3	4	5	6	7	8
+	0.583	0.607	0.591	0.467	0.497	0.365	0.333	0.333
=	0.167	0.167	0.143	0.105	0.062	0.107	0	0
-	0.250	0.183	0.266	0.428	0.440	0.528	0.667	0.667

Figure 3.5g. Contour family for the syllable “tur”

ID	C-SEG	CAS	Card. of CAS	Membership
A	⟨10121⟩	⟨-+++⟩	4	0.914
B	⟨10121⟩	⟨-+++⟩	4	0.914
C	⟨10121⟩	⟨-+++⟩	4	0.914
D	⟨10121210⟩	⟨-+++---⟩	7	0.843
E	⟨20210⟩	⟨-+++⟩	4	0.973
F	⟨20121210⟩	⟨-+++---⟩	7	0.843
G	⟨20121210⟩	⟨-+++---⟩	7	0.843
H	⟨10121⟩	⟨-+++⟩	4	0.914
I	⟨101210⟩	⟨-++++⟩	5	0.898
J	⟨20121210⟩	⟨-+++---⟩	7	0.843
K	⟨20121210⟩	⟨-+++---⟩	7	0.843
L	⟨20121210⟩	⟨-+++---⟩	7	0.843

Figure 3.5h. Averaged family for the syllable “tur”

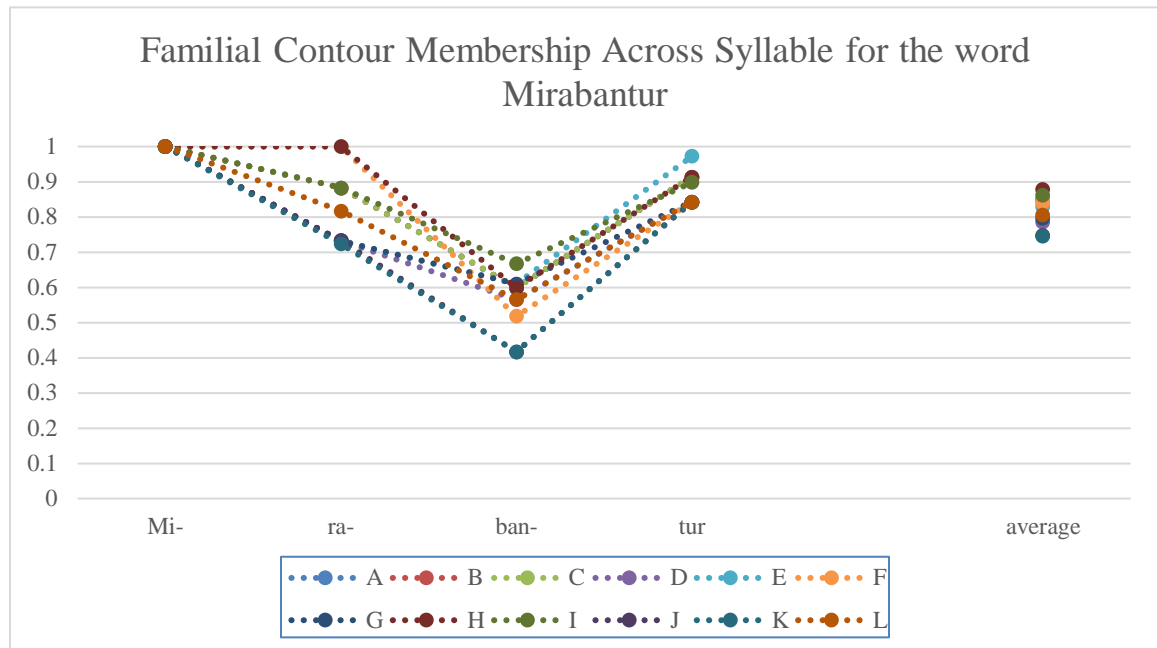
	1	2	3	4	5	6	7
+	0	0.764	0.894	0.404	0.817	0.167	0
=	0	0	0	0	0	0	0
-	1.0	0.236	0.105	0.596	0.183	0.833	1.0

Figure 3.6 shows a graph of membership values for each variant (labeled *a-l*) across the entire word “Mirabantur.” In the graph, we see the membership values for each variant *a-l* plotted across their syllables shown on the x-axis of the graph, revealing the patterns of coherence and variety inherent within this chant.³⁰ Here, we see that many of these membership values are clustered together, such that multiple chants are represented by only one point on the graph. These tight clusters show us that the membership values of these variants are relatively uniform, suggesting that these segments might be varied in similar ways. Additionally, in the familial structures of these first four syllables, for example, we see a pattern of structural stability that is reflective of the psychological principle that beginnings and endings are more memorable (Roberts 1986, 153): the first syllable displays remarkably high contour membership within its own family. Of the variants that move between pitches in this syllable, each contour ascends, resulting in a 1.0 membership value for each of these members. Karp cites the fact that not all members actually move on the first syllable, yet the potential of the first pitch to ascend in the next

30 The dotted lines connecting syllables for each chant variant are meant only as a visual aid to assist the reader in understanding which points belong to specific chant variants.

syllable (whether it is actualized or not) is still suggestive of uniformity on the part of memory.³¹

Figure 3.6. Graph of membership across the word “Mirabantur”



The second syllable, “ra,” displays less similar memberships: between 0.724 and 0.883. These values are still relatively high, suggesting strong connections among the variants in the family, yet they are more varied due to small changes in the middle of the segments. The third syllable, “ban,” shows the lowest degrees of membership: between 0.417 and 0.667, indicating that this syllable is the most widely varied of the four

31 Schultz posits a Husserlian view of anticipation called *protention*, where the indeterminacy of the future motion is captured through its potential possible motions (2009, 14-18). The tendencies of other family members (in this case the family of the first syllable “Mi-”) may influence the expectation for movement in that moment, and it is this heightened potential that maintains the suggestion of ascent in this family in the absence of actualized motion. In other words, since most of the family ascended, those that only possessed one pitch (i.e., no motion) could have the potential to ascend as well.

syllables. This may be where Karp sees the lack of “melodic personality,” as the family displays a less coherent connection in this segment. Finally, the last syllable, “tur,” once again returns to high degrees of membership, as well as a smaller range: 0.843 to 0.973.

These membership values suggest that at the level of the word, structural signposts seem to exist at the beginning and ending syllables, with the potential for greater variety in the inner syllables. Furthermore, if we look at the familial probabilities indicated for each syllable on its own, we also see that in all but the third (most varied) syllable, the families themselves display strong beginning and ending similarities, indicated by the fact that they all begin and end with the same motions. These important similarities contribute to the cohesiveness of the family, despite its other unstable aspects: the average membership across all four syllables, after all, ranges from 0.7 to 0.9, indicating continuity in the face of variation.

If we include the second word “omnes” in our analysis, as shown in Figure 3.7a-d, as Karp suggests, we actually see an even stronger signpost at the cadential point G4. Parsing again by syllable, we see that the first syllable “om” is quite variable, as Karp claims: the family’s variants have membership values ranging from 0.565 to 0.675. However, as the word finishes with its cadence on G4, the family of the final syllable displays a membership of 1.0, indicating exact identity.

Figure 3.7a. Contour family for the syllable “om”

ID	C-SEG	CAS	Card. of CAS	Membership
A	⟨21010⟩	⟨--+-⟩	4	0.565
B	⟨1210⟩	⟨+--⟩	3	0.675
C	⟨21010⟩	⟨--+-⟩	4	0.565
D	⟨012121⟩	⟨+++--⟩	5	0.638
E	⟨0121212⟩	⟨+++++⟩	6	0.629
F	⟨01312⟩	⟨+++⟩	4	0.619
G	⟨012101⟩	⟨++++⟩	5	0.648
H	⟨210101⟩	⟨--+-⟩	5	0.568
I	⟨012101⟩	⟨++++⟩	5	0.648
J	⟨0121212⟩	⟨+++++⟩	6	0.629
K	⟨0121212⟩	⟨+++++⟩	6	0.629
L	⟨0121⟩	⟨++⟩	3	0.672

Figure 3.7b. Averaged family for the syllable “om”

	1	2	3	4	5	6
+	0.667	0.633	0.358	0.533	0.283	0.583
=	0	0	0	0	0	0
-	0.0.333	0.367	0.642	0.467	0.717	0.417

Figure 3.7c. Contour family for the syllable “nes”

ID	C-SEG	CAS	Card. of CAS	Membership
A	⟨10⟩	⟨-⟩	1	1
B	⟨10⟩	⟨-⟩	1	1
C	⟨10⟩	⟨-⟩	1	1
D	⟨10⟩	⟨-⟩	1	1
E	⟨10⟩	⟨-⟩	1	1
F	⟨10⟩	⟨-⟩	1	1
G	⟨10⟩	⟨-⟩	1	1
H	⟨10⟩	⟨-⟩	1	1
I	⟨10⟩	⟨-⟩	1	1
J	⟨10⟩	⟨-⟩	1	1
K	⟨10⟩	⟨-⟩	1	1
L	⟨10⟩	⟨-⟩	1	1

Figure 3.7d. Averaged family for the syllable “nes”

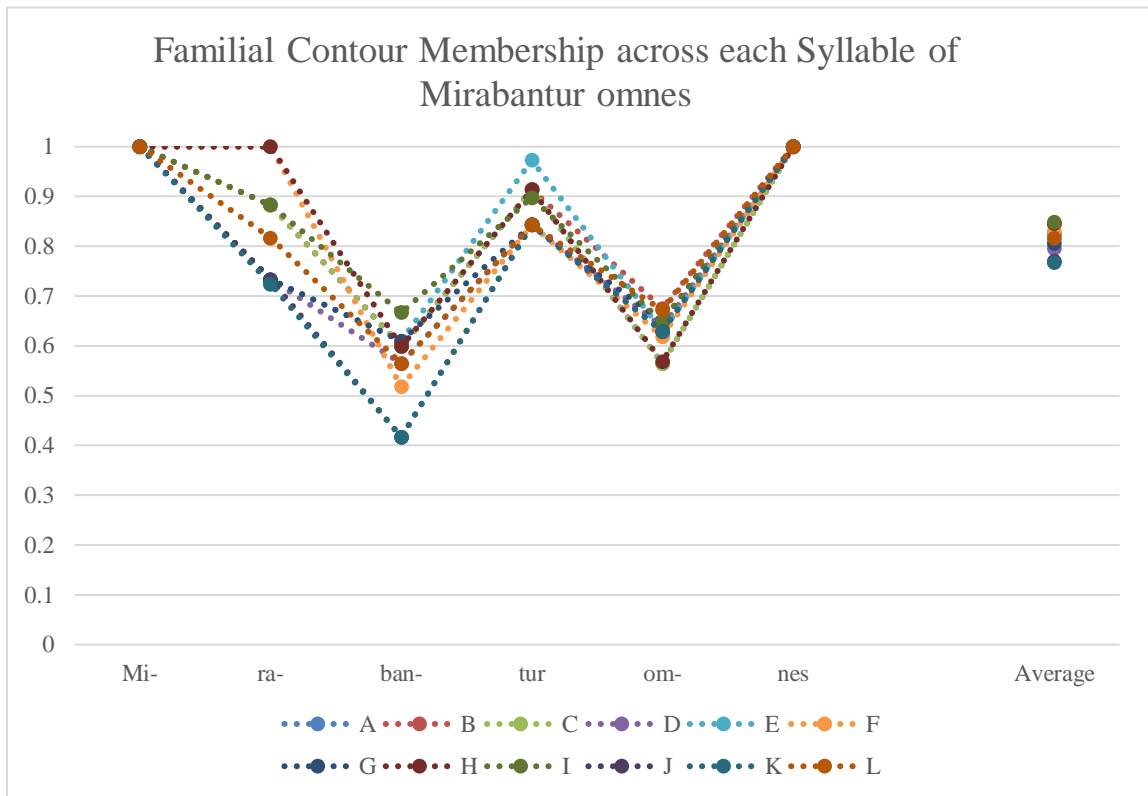
	1
+	1.0
=	0
-	0

Adding these syllables into our graph, shown in Figure 3.8, gives us a strong picture of the shared characteristics that this melody complex possesses. The structural tendencies we see in the contour of this melody family give us a foundation with which to investigate the stability (or lack thereof) of a particular chant. In the case of *Mirabantur omnes*, we see that the primary area of instability occurs in the third syllable, which in all cases centers on C5, which Karp calls “the first contrasting pole to the final” (1989, 14). Likewise, the cadential preparation Karp cited as a feature of the family’s instability, features similarly low membership values, again indicating it as an area of variability within the structure of the phrase. However, other structural elements, such as the beginning ascent, the stability found at the fourth syllable, and the concluding descent provide the contextual evidence for the conceptual grouping of these variants. In doing so, one can see that perhaps Treitler’s theory of reconstructive memory is not so far out of reach as Karp would suggest.

As noted above, we can see indications of Treitler’s theory of reconstruction at work in the body of notated chant variants that we have as our primary body of evidence. The shared characteristics in the above examples show where these salient features may be, pointing us to the structural signposts that facilitate reconstruction of the chant. This

is especially noticeable in the beginnings and endings I mentioned with regard to the *Mirabantur omnes* chant family: syllables at the beginning and ending of words possess higher membership values within their respective families than do syllables in the middle. Within these forms, however, variation is introduced as a result of the reconstruction process: as a singer continually relies on past experiences singing a chant, small variations may be introduced that, while not sacrificing the structural integrity of the chant as a whole, cause the identity of the chant idea to change. This is seen especially in the middle syllables of the *Mirabantur omnes* family (such as the syllable “ban” and the syllable “om”), exemplified by the graph in Figure 3.8.

Figure 3.8. Graph of membership across the phrase “Mirabantur omnes”



In measuring the fuzziness of the multiple notated variants in this example, we can gain a better understanding of what aspects of the melody contribute to the stability of the melody, and conversely which aspects tend to change. In the analysis of both Karp's and Treitler's examples, we see segments that are similar across variants, and areas in which notable differences occur.

Further Ontological Complications

In the analyses of both *Filius ecce patrem* and *Mirabantur omnes*, we see that the notion of a chant's ontology is closely interconnected to the resemblances shared by its melodic variants. However, it was often the case in the practice of plainchant to adapt existing melodies to different texts. In doing so, the ontological status of a melody or melodic segment is severed from its attachment to specific chants, further complicating the ontology of both the melody and the chant or chants to which it belongs.

Recall in Figure 3.1 that Treitler includes two additional tropes within his trope family beyond the four that bear the same text. These two (variants *e* and *f*) deviate from the other members of the family in that they have different texts, but were used in accompaniment with the same introit as the other tropes in the family. Treitler bases the inclusion of these two tropes on melodic similarity, specifically that these tropes contain some of the same range constraints and the same melodic progression through the range of the mode.

Analyzing these phrases against the other four members of the family shows that while melodic underpinnings do exist, these two tropes exhibit greater variety than the

members of the initial family. Figure 3.7, for example, shows this analysis across phrase M. While the original members of the family displayed membership values ranging from 0.762 to 0.916 (as shown in Figure 3.2a, reproduced in contours A-D in Figure 3.9), the values for the tropes labeled E and F are much lower: 0.595 and 0.348 respectively.

Figure 3.9. Analysis of phrase M of variants E and F of *Fillius ecce patrem* (with variants A–D for reference)

ID	C-SEG	CAS	Card. of Cas	Membership
A	⟨1012101⟩	⟨-++++⟩	6	0.916
B	⟨0011034321012311⟩	⟨=+=-++-----+++=⟩	15	0.778
C	⟨120124232102311⟩	⟨+++++-----+=⟩	14	0.762
D	⟨110123122102211⟩	⟨=-+++++=---+=-⟩	14	0.833
E	⟨101123432121011⟩	⟨-+=++++-----+=⟩	14	0.595
F	⟨011123321222110011⟩	⟨+=+=+=-++=-=-+=⟩	17	0.348

Given the remarkably low membership values across the variants of E and F, one may wonder why Treitler is so inclined to include them in this family at all. Treitler explains, however, that his grouping of these in the family is based on shared melodic traits that are deeper than can be seen in the surface-level contour, suggesting that surface level contour is not the only feature we should examine.³² It is the relative depth at which the similarity occurs, I believe, that may be the deciding factor in Treitler’s question of what constitutes minor vs. significant differences. Underlying melodic similarities may

32 In these cases, it would be profitable to examine such variants using a combination of the FCM model and the Contour Reduction Algorithm (Morris 1993, Schultz 2008), which algorithmically reveals deeper structures within the contours in question. In doing so, one could compare prime representations of the variants, and could use the FCM model to arrive at membership values for each prime within the family of the variants. While this kind of analysis falls beyond the scope of this dissertation, it would be an interesting avenue for future research.

indeed be present, and may be structurally significant in the perception and reconstruction of variants E and F, but the lack of surface level contour similarity suggests a much looser relationship of these chants to the family as a whole, calling into question their ontological status within the family. Treitler writes that it may be valuable to reconsider the “variant” relationship in this family, as the variation seen especially in chants E and F stretch the limits of what may be considered as “the same melody.”

Instead, he writes that

the conception that its versions are all actualizations of a matrix defined by the constraints that have been described here is preferable to the conception that the versions are related as variants of one another or some hypothetical archetype... The conception that the versions are all actualizations of an underlying model has four advantages. First, it is of a greater generality. Second, it provides a more adequate account of the observable facts of the transmission (or to put it the other way around, it does not require us to posit masses of irretrievable evidence). Third, it is consonant with a general conception that is proving to be more satisfactory to account for the transmission of other repertoires of medieval melody and polyphony. Fourth, it offers a category for assimilating the facts of variation, other than the stemma that no one has yet been able to reconstruct. (2003, 269–70)

Such a model puts the measure of similarity at a deeper level. To call them melodic variants in the same way that we might call the members of the *Mirabantur omnes* family variants seems illogical, suggesting that there is a distinction to be made between the ontological status of variants that more closely resemble each other, and those that have deeper structural connections, but vary more widely on the surface. Such distinctions are challenged additionally because of differences in text—which in many cases automatically result in minor differences between variants—but the main idea remains the same. In short, these two variants E and F lie at the border between what are minor variations and what are more significant.

Melodic Formula Families with Different Texts

In studying this issue in more depth, I would like to bring in another analysis of chants with different texts—this time a family that coheres more closely. Rebecca Maloy’s book *Inside the Offertory* studies a large body of chants with different texts in both the Old Roman tradition and the Gregorian tradition. In her study, she identifies a range of melodic similarities that inform her analysis of the differences between traditions. One significant point that she makes is that the Old Roman offertories make heavy use of melodic formulae, which Maloy defines as a “relatively” stable melodic phrase that gets used in multiple places within the chant repertoire. Such a formula is different from standard melodic material (which Maloy calls “ideomelic”) in that it always appears in the same or similar form or order (Maloy 2010, 90–92). Ontologically speaking, these formulae would constitute a melody family, despite appearing in different chants with different texts.

Since these formulas are long, Maloy defines smaller shapes within them, which she calls “functions” because each one serves a function within the phrase. She defines eight such functions (although not all formulae must possess all eight functions, and some functions may be repeated) shown in Figure 3.10:

Figure 3.10. Maloy’s formalic functions (2010, 93)

I	Pre-accent	“all segments in columns labeled I serve the role of accent preparation”
II	Accent	“all segments in columns labeled II fall on an accent”
III	Post-accent	“segments under this heading fall after the accent or on the final syllable of the word”
IV	Accent-neutral recitation:	“this neume occurs on both accented and unaccented syllables and is repeated as often as needed to accommodate syllables remaining before the cadential pattern.”
V	Pre-cadential	“may occur on either an accented or unaccented syllable”
VI	Cadential 1	these functions are cadential, “accommodating the final three or four syllables of the clause.”
VII	Cadential 2	
VIII	Cadential 3	

These eight functions (shown with Roman numerals) break up the melodic formula into segments that have specific purposes within a phrase of plainchant, often associated with where they are placed against the specific stresses of the text. Maloy discusses the overall stability of these formulae, but also the interchangeability of functions within certain formulae. In some cases, not all functions are present, and some functions overlap, as in cases where the post-accent function may also serve as the pre-cadential function leading into the end of the phrase. Analyzing these functional segments using the FCM model shows the stability that Maloy discusses. I have chosen a formula specific to mode-2 offertories that Maloy identifies as “formula 2-1,” examples of which are shown in Figure 3.11. In the figure, Maloy identifies four instances of the formula, found in four different offertories. For each, she segments the formula according to the functions identified in Figure 3.10, and aligns the formulae according to functional similarity.

Figure 3.11a. Maloy's function segmentation of formula 2-1 (2010, 97)

Formula 2.1

Function: I II III II III II V VI VII VIII

1. Bonum est v. 1.
Quam ma - gni - fi - ca - ta — sunt o - pe - ra — tu - a — do - - - mi - - - ne —

Benedic anima,
re - tri - bu - ti - o - - - - - nes ce - ius —

2. Benedic anima,
re - tri - bu - ti - o - - - - - nes ce - ius —

Ad te domine,
re - spond de - - - - - us me - - - - - us —

3. Ad te domine,
re - spond de - - - - - us me - - - - - us —

Benedicite gentes,
re - spond be - ne - dic - - - - - tus do - - - - - mi - - - - - nus —

4. Benedicite gentes,
re - spond be - ne - dic - - - - - tus do - - - - - mi - - - - - nus —

As one may notice in the figures above, the length of the text to be fitted to the formula determines the extent to which certain functions are activated within the formulaic segment itself. Yet, as we might see in the figure, the functions themselves have a bit of variability, and this is found in other versions of the formula as well. I have identified seventeen instances of the formula within the Old Roman offertories that Maloy includes in her book, categorized according to Maloy's chant ID number, which is based on placement of the chant within the liturgical calendar. These formulae are shown in Appendix 1. A familial analysis of each of these functions across the seventeen identified members shows the ways in which the melody family of formula 2-1 coheres. Figures 3.12 a–n show the familial analyses of the eight functions across these seventeen variants.³³

³³ Note that Function IV is not present in this formula.

Throughout Figures 3.12a through 3.12n, we see a variety of membership value ranges emerge for each of the eight functions, suggesting that certain functions are more stable than others, and subsequently that certain places within the melody are more stable than others. The pre-accent and post-accent functions (functions I and III) are the most unstable with regard to contour: the pre-accent function (Figures 3.12a and 3.12b) features a range of membership values from 0.352 to 1.0, while the post-accent function (Figures 3.12e and 3.12f) has a range of 0.277 to 0.583. A closer analysis of these functions shows crucial differences between the ways in which the two families vary. The average membership value for the pre-accent function is 0.850, suggesting that there are strong family resemblances with a few drastic outliers. When we examine the table for function I in Figure 3.12a, we see this to be the case. All but four of the thirteen members that possess function I feature 1.0 membership: in essence, they all follow the initial ascent. The four that do not have 1.0 membership are considerably more varied in their opening gestures, although each also ends with an ascent that could be compared analogously to the ascent of the remaining members. In short, these four outliers are elaborations upon the shape presented by the rest of the family. On the other hand, the average membership value for the family of Function III is 0.489, considerably lower than that of Function I. This suggests a different makeup, with much more variety inherently in the family. We see this in the family, as some members plateau (for differing lengths of time), while others feature prominent ascents and descents in different places.³⁴ In other words, there is no real central pattern from which to “deviate”

³⁴ These differences are difficult to compare using a melodic best-fit approach, and so because of this variety, it is necessary to use the mathematic best-fit approach for the analyses of this formula.

as we saw in Function I. Therefore, in these charts we see that not only do the family members of these various functions differ from each other significantly; they also differ in ways that further reinforce the differences between functions.

Figure 3.12a. Contour family for Function I (pre-accent)

ID	Location	C-SEG	CAS	Card. of CAS	Membership
22	Line 1	⟨01⟩	⟨+⟩	1	1
17	Line 2	⟨01⟩	⟨+⟩	1	1
36	Line 5a	⟨01⟩	⟨+⟩	1	1
6	Line 4 (second line)	⟨101⟩	⟨-+⟩	2	0.602
13	Line 6	⟨01⟩	⟨+⟩	1	1
72	Line 7	⟨01⟩	⟨+⟩	1	1
36	Line 5a	⟨01⟩	⟨+⟩	1	1
36	Line 8	⟨01⟩	⟨+⟩	1	1
14	Line 4	⟨11001⟩	⟨=-=+⟩	4	0.352
23	Line 2 (line 2)	⟨1201⟩	⟨+--+⟩	3	0.658
58	Line 6	⟨1001⟩	⟨-=-+⟩	3	0.444
10	Line 2	⟨01⟩	⟨+⟩	1	1
10	Line 6	⟨01⟩	⟨+⟩	1	1

Figure 3.12b. Averaged family for function I

	1	2	3	4
+	0.769	0.743	0.795	1.0
=	0.077	0.051	0.128	0
-	0.154	0.205	0.077	0

Figure 3.12c. Contour family for Function II (Accent)

ID	Location	C-SEG	CAS	Card. of CAS	Membership
22	Line 1	⟨0121⟩	⟨+++⟩	3	0.918
22	Line 1	⟨010⟩	⟨+-⟩	2	1
22	Line 1 (line 3)	⟨010⟩	⟨+-⟩	2	1
17	Line 2	⟨0121⟩	⟨+++⟩	3	0.918
36	Line 5a	⟨0121⟩	⟨+++⟩	3	0.918
6	Line 4 (line 2)	⟨0121⟩	⟨+++⟩	3	0.918
6	Line 4 (line 3)	⟨010⟩	⟨+-⟩	2	1
13	Line 6	⟨0121⟩	⟨+++⟩	3	0.918
72	Line 1 (line 3)	⟨010⟩	⟨+-⟩	2	1
72	Line 3 (line 2)	⟨021⟩	⟨+-⟩	2	1
72	Line 7	⟨0121⟩	⟨+++⟩	3	0.918
36	Line 5a	⟨0121⟩	⟨+++⟩	3	0.918
36	Line 8	⟨010⟩	⟨+-⟩	2	1
36	Line 8	⟨01231⟩	⟨++++⟩	4	0.797
36	Line 8	⟨010⟩	⟨+-⟩	2	1
14	Line 4	⟨010⟩	⟨+-⟩	2	1
14	Line 4	⟨010⟩	⟨+-⟩	2	1
14	Line 4	⟨010⟩	⟨+-⟩	2	1
14	Line 5	⟨010⟩	⟨+-⟩	2	1
14	Line 5	⟨010⟩	⟨+-⟩	2	1
23	Line 2 (line 2)	⟨0121⟩	⟨+++⟩	3	0.918
23	Line 2 (line 2)	⟨010⟩	⟨+-⟩	2	1
58	Line 3 (line 2-3)	⟨010⟩	⟨+-⟩	2	1
58	Line 6	⟨010⟩	⟨+-⟩	2	1
58	Line 6	⟨010⟩	⟨+-⟩	2	1
10	Line 2	⟨010⟩	⟨+-⟩	2	1
10	Line 2	⟨021⟩	⟨+-⟩	2	1
10	Line 6	⟨010⟩	⟨+-⟩	2	1
10	Line 6	⟨1210⟩	⟨+++⟩	3	0.856
10	Line 6	⟨021⟩	⟨+-⟩	2	1

Figure 3.12d. Averaged family for function II

	1	2	3	4
+	1.0	0.756	0.433	0
=	0	0	0	0
-	0	0.244	0.567	1.0

Figure 3.12e. Contour family for function III (Post-accent)

ID	Location	C-SEG	CAS	Card. of CAS	Membership
22	Line 1	⟨00⟩	⟨=⟩	1	0.583
22	Line 1 (line 3)	⟨00⟩	⟨=⟩	1	0.583
17	Line 2	⟨00⟩	⟨=⟩	1	0.583
13	Line 6	⟨10⟩	⟨-⟩	1	
72	Line 1 (line 3)	⟨00⟩	⟨=⟩	1	0.583
36	Line 8	⟨210⟩	⟨--⟩	2	0.374
36	Line 8	⟨1210⟩	⟨+--⟩	3	0.277
14	Line 4	⟨2101⟩	⟨--+⟩	3	0.277
14	Line 4	⟨000⟩	⟨==⟩	2	0.583
14	Line 5	⟨000⟩	⟨==⟩	2	0.583
23	Line 2 (line 2)	⟨00⟩	⟨=⟩	1	0.583
58	Line 6	⟨0⟩	NA	0	
10	Line 2	⟨0⟩	NA	0	
10	Line 6	⟨210⟩	⟨--⟩	1	0.374
10	Line 6	⟨0⟩	NA	0	

Figure 3.12f. Averaged family for function III

	1	2	3
+	0.083	0	0.083
=	0.583	0.583	0.583
-	0.333	0.416	0.333

Figure 3.12g. Contour family for function V (Pre-cadential)

ID	Location	C-SEG	CAS	Card. of CAS	Membership
22	Line 1 (line 3)	⟨00⟩	⟨=⟩	1	1
36	Line 5a	⟨00⟩	⟨=⟩	1	1
6	Line 4 (line 2)	⟨00⟩	⟨=⟩	1	1
6	Line 4 (line 3)	⟨0⟩	NA		
72	Line 1 (line 3)	⟨0000⟩	⟨===⟩	3	1
72	Line 3 (line 2)	⟨000⟩	⟨==⟩	2	1
72	Line 7	⟨00⟩	⟨=⟩	1	1
36	Line 5a	⟨00⟩	⟨=⟩	1	1
36	Line 8	⟨00⟩	⟨=⟩	1	1
14	Line 4	⟨00⟩	⟨=⟩	1	1
14	Line 5	⟨00⟩	⟨=⟩	1	1
58	Line 3(lines 2-3)	⟨000⟩	⟨==⟩	2	1
58	Line 6	⟨000⟩	⟨==⟩	2	1
10	Line 2	⟨000⟩	⟨==⟩	2	1
10	Line 6	⟨000⟩	⟨==⟩	2	1

Figure 3.12h. Averaged family for function V

	1	2	3
+	0	0	0
=	1.0	1.0	1.0
-	0	0	0

Figure 3.12i. Contour family for function VI (Cadential)

ID	Location	C-SEG	CAS	Card. of CAS	Membership
22	Line 1 (line 3)	⟨13210⟩	⟨+---⟩	4	0.994
36	Line 5a	⟨13210⟩	⟨+---⟩	4	0.994
6	Line 4 (line 2)	⟨1210⟩	⟨+--⟩	3	1
6	Line 4 (line 3)	⟨13210⟩	⟨+---⟩	4	0.994
13	Line 6	⟨023210⟩	⟨++---⟩	5	0.861
72	Line 1 (line 3)	⟨13210⟩	⟨+---⟩	4	0.994
72	Line 3 (line 2)	⟨13210⟩	⟨+---⟩	4	0.994
72	Line 7	⟨1210⟩	⟨+--⟩	4	0.994
36	Line 5a	⟨1210⟩	⟨+--⟩	3	1
36	Line 8	⟨13210⟩	⟨+---⟩	4	0.994
14	Line 4	⟨13210⟩	⟨+---⟩	4	0.994
14	Line 5	⟨13210⟩	⟨+---⟩	4	0.994
58	Line 3 (line 2-3)	⟨13210⟩	⟨+---⟩	4	0.994
58	Line 6	⟨13210⟩	⟨+---⟩	4	0.994
10	Line 2	⟨13210⟩	⟨+---⟩	4	0.994
10	Line 6	⟨13210⟩	⟨+---⟩	4	0.994

Figure 3.12j. Averaged family for function VI

	1	2	3	4	5
+	1.0	0.328	0.021	0	0
=	0	0	0	0	0
-	0	0.672	0.979	1.0	1.0

Figure 3.12k. Contour family for function VII (Cadential)

ID	Location	C-SEG	CAS	Card. of CAS	Membership
22	Line 1	⟨1321210⟩	⟨+++++-- ⟩	6	0.701
22	Line 1 (line 3)	⟨321210⟩	⟨---+--⟩	5	0.7
36	Line 5a	⟨321210⟩	⟨---+--⟩	5	0.7
6	Line 4 (line 2)	⟨32120⟩	⟨---+--⟩	4	0.883
6	Line 4 (line 3)	⟨321210⟩	⟨---+--⟩	5	0.7
13	Line 6	⟨321210⟩	⟨---+--⟩	5	0.7
72	Line 1 (line 3)	⟨321210⟩	⟨---+--⟩	5	0.7
72	Line 3 (line 2)	⟨321210⟩	⟨---+--⟩	5	0.7
72	Line 7	⟨321210⟩	⟨---+--⟩	5	0.7
36	Line 5a	⟨321210⟩	⟨---+--⟩	5	0.7
36	Line 8	⟨321210⟩	⟨---+--⟩	5	0.7
14	Line 4	⟨321210⟩	⟨---+--⟩	5	0.7
14	Line 5	⟨321210⟩	⟨---+--⟩	5	0.7
23	Line 2 (line 2)	⟨1321210⟩	⟨+++++-- ⟩	6	0.701
58	Line 3 (line 2-3)	⟨321210⟩	⟨---+--⟩	5	0.7
58	Line 6	⟨321210⟩	⟨---+--⟩	5	0.7
10	Line 2	⟨321210⟩	⟨---+--⟩	5	0.7
10	Line 6	⟨321210⟩	⟨---+--⟩	5	0.7

Figure 3.12l. Average family for function VII

	1	2	3	4	5	6
+	0.111	0	0.516	0.644	0.033	0
=	0	0	0	0	0	0
-	0.889	1.0	0.483	0.356	0.967	1.0

Figure 3.12m. Contour family for function VIII (Cadential)

ID	Location	C-SEG	CAS	Card. of CAS	Membership
22	Line 1	⟨210⟩	⟨--⟩	2	0.928
22	Line 1 (line 3)	⟨10210210⟩	⟨-+---+--⟩	7	0.772
17	Line 2	⟨210⟩	⟨--⟩	2	0.928
36	Line 5a	⟨1021021⟩	⟨-+---+--⟩	6	0.758
6	Line 4 (line 2)	⟨210⟩	⟨--⟩	2	0.928
6	Line 4 (line 3)	⟨210⟩	⟨--⟩	2	0.928
13	Line 6	⟨10210210⟩	⟨-+---+--⟩	7	0.772
72	Line 1 (line 3)	⟨210⟩	⟨--⟩	2	0.928
72	Line 3 (line 2)	⟨210⟩	⟨--⟩	2	0.928
72	Line 7	⟨1021021⟩	⟨-+---+--⟩	6	0.758
36	Line 5a	⟨1021021⟩	⟨-+---+--⟩	6	0.758
36	Line 8	⟨210⟩	⟨--⟩	2	0.928
14	Line 4	⟨210⟩	⟨--⟩	2	0.928
14	Line 5	⟨1021021⟩	⟨-+---+--⟩	6	0.758
23	Line 2 (line 2)	⟨1021021⟩	⟨-+---+--⟩	6	0.758
58	Line 3 (line 2-3)	⟨10210210⟩	⟨-+---+--⟩	7	0.772
58	Line 6	⟨10210⟩	⟨-+---⟩	4	0.828
10	Line 2	⟨210⟩	⟨--⟩	2	0.928
10	Line 6	⟨10210210⟩	⟨-+---+--⟩	7	0.772

Figure 3.12n. Averaged family for function VIII

	1	2	3	4	5	6	7
+	0	0.456	0.119	0.024	0.309	0.219	0
=	0	0	0	0	0	0	0
-	1.0	0.544	0.881	0.976	0.691	0.781	1.0

The instances of wide variability are typically a product of the differences between texts. Figure 3.13a, for example, shows that in the pre-accent function (Maloy's

function I), not all of the texts are the same length.³⁵ Typically, in the Latin language (both spoken as well as sung) the accent falls on the penultimate syllable: Take the formula from the offertory *Meditabor* (chant ID 22), for example: the text reads “in mandatis tuis.” In this phrase, the accent is placed on the “da” of “mandatis” as it is the penultimate syllable in the first word of substance in the formula. For this phrase, there are only two syllables preceding the accent, “in man,” resulting in the single ascent from C4 to D4, as shown in Figure 3.13b. However, consider the same formulaic phrase from the offertory *Tollite portas* (chant ID 6), possessing the text “et plenitudo eius” (Figure 3.13c). In this phrase, we have three syllables “et ple-ni” before we get to the accent function on the syllable “tu.” As a result of this extra syllable, the pre-accent function has been lengthened by the addition of the initial D4 on “et.” Therefore, the variety with regard to the stability of the pre-accent function is a product of the length of the words selected for the phrase. The fact that these different texts can affect such change is a strong contributing factor to the complexity with which we must regard the ontology of the melodies in this section. As we see from Figures 3.13 a–c, the variety exhibited may not detract from our sense that it is “the same melody,” yet we must remain sensitive to the variation that comes when we put the melody to different texts.

35 For reference, see the notated chants in Appendix 1.

Figure 3.13a. Syllabic breakdown of the pre-accent function (showing the text of the accent function in parenthesis)³⁶

ID	Location	C-SEG	CAS	Membership	Text:	Syllables
22	Line 1	<01>	<+>	1	In man-(datis)	2
17	Line 2	<01>	<+>	1	In-i-(micos)	2
36	Line 5a	<01>	<+>	1	Be-ne-(dictus)	2
6	Line 4 (second line)	<101>	<-+>	0.602	Et ple-ni-(tudo) 2 word...	3
13	Line 6	<01>	<+>	1	Et (exaudivit)	1
72	Line 7	<01>	<+>	1	Be-ne-(dictus)	2
36	Line 5a	<01>	<+>	1	Be-ne-(dictus)	2
36	Line 8	<01>	<+>	1	Iu-bi-(late)	2
14	Line 4	<11001>	<=- =+>	0.352	Quam ma-gni-fi-(ca- ta)	4
23	Line 2 (line 2)	<1201>	<+--+>	0.658	Re-tri-bu-ti-(on-es)	4
58	Line 6	<1001>	<-+=>	0.444	Quo-ni-am (de-us)	3
10	Line 2	<01>	<+>	1	Re-ges	2
10	Line 6	<01>	<+>	1	Et iu-(sti-ti-am)	2

Figure 3.13b. Formulaic phrase from Meditatibor (chant ID 22), showing a two-syllable pre-accent function

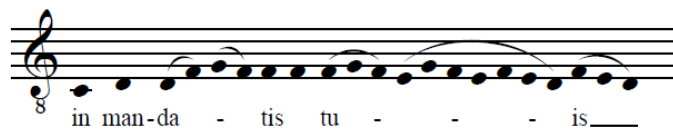


Figure 3.13c. Formulaic phrase from Tollite portas (chant ID 6), showing a three-syllable pre-accent function



³⁶ In this family, we see a significant number of single motion <01> contours, each with membership values of 1.0. These membership values indicate that the ascent occurs in every contour (although other contours have additional motions) and therefore that this ascent is a crucial feature of the family.

With regard to the melody's stability, we see some interesting patterns emerge. The most stable functions are the accent function (function II) and the pre-cadential function (function V).³⁷ The family of function II (Figures 3.12c and 3.12d) ranges in membership from 0.797 to 1.0, with an average membership of 0.967. The family of the pre-cadential function (Figures 3.12g and 3.12h) has a 1.0 membership across all members, suggesting that they possess identical melodic shapes (a plateau motion, differing only in length).³⁸ Such points of structural stability within the melody suggest that these are important melodic signposts within the formula, and thus the uniformity found in both the accent and pre-cadential functions greatly contributes to the stability of the melodic formula as a whole.

The cadential functions also possess a good deal of similarity, although, as is shown in the tables above, there is still room for variability within these three functions. The cadential functions' membership values all seem high, with the lowest values hovering around 0.7, indicating that in addition to being melodically more stable than the pre- and post-accent functions, these functions exhibit variety in a very uniform way. Examining the averaged probability matrix for Function VII (Figure 3.12l), for example, we see that the family possesses 1.0 memberships in the downward motions of position 2 and position 6. Likewise, position 1 and position 5 exhibit strong tendencies toward a

37 To refer back to the definitions of each function, please see Figure 3.10.

38 In this family, each member only features a plateau. What results is a family where there is only one possible motion, resulting in a 1.0 membership/non-membership for all positions in the family. Therefore, each member of the family will also possess 1.0 membership.

downward motion, indicating that only a minority of members ascends in these positions. The greatest variability comes in the middle of the c-segs, in positions 3 and 4, where the values for ascent and descent hover closer toward the middle of the spectrum. This uniformity in variation contributes to the relative stability we see in these cadential sections.

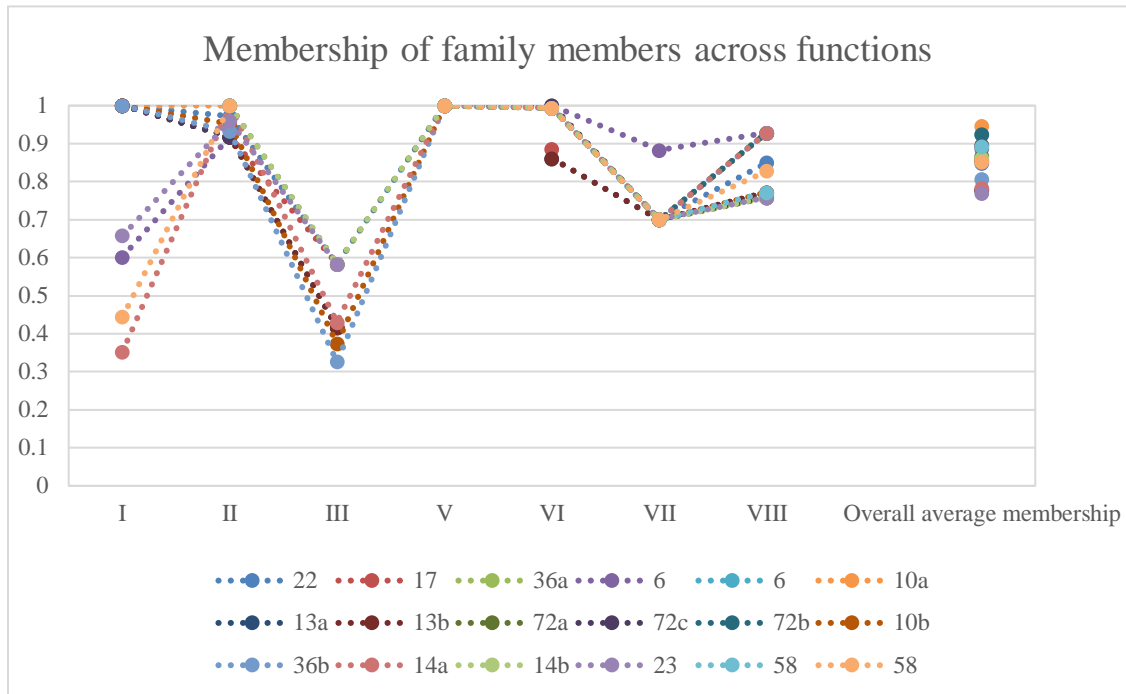
These membership values all seem logical in that the pre- and post-accent functions serve as more flexible connecting material between structural entities like the accent, pre-cadential, and cadential functions. As such, it would stand to reason that this material is more varied according to the specific needs of the text. However, when we examine the overall degree of membership by combining the membership values of each function throughout each formula, we see that these variable sections do not detract from the stability of this melodic formula. Figure 3.14a shows these averages: the average membership value for the family of entire formulae ranges from 0.769 to 0.946. Figure 3.14b graphs the membership values of the functions for each formula, and shows that each formula exhibits a relatively consistent pattern of coherence and variety. Within these membership values we can see how the more variable functions have affected the familial similarity between these formulae. In Figure 3.14a, we see that the formula from chant 23 (shown in the third row from the bottom in Figure 5.14a), for example, features the lowest membership of the entire family, primarily because it makes use of the functions that are less stable such as functions I and III, while at the same time not using functions such as V and VI that have markedly high membership values. In addition, when we look at the chants with the highest degrees of membership, we see that they either possess high-end values in the more variable functions, such as chant 22 (shown in

the top row of Figure 3.14a) which features the highest values for functions I and III, or do not make use of those functions at all, such as chant 72, line 3 (shown in the eighth row from the bottom in Figure 3.14a).

Figure 3.14a. Overall familial membership for the formula 2-1 (X indicates that the function is not present in the given formula)

ID	Location	I	II	III	V	VI	VII	VIII	Overall average membership
22	Line 1	1	0.918, 1, 1	0.583, 0.583	1	0.994	0.701, 0.7	0.928, 0.772	0.848
17	Line 2	1	0.918	0.583	X	0.885		0.928	0.857
36	Line 5a	1	0.918	X	1	0.994	0.7	0.758	0.895
6	Line 4	0.602	0.918	X	1	1	0.883	0.928	0.888
6	Line 4 (partial repeat)	X	1		1	0.994	0.7	0.928	0.924
10	Line 2	1	1, 1	X	1	0.994	0.7	0.928	0.946
13	Line 6	1	0.918	0.416	X	0.861	0.7	0.772	0.778
13	Line 6 (full repeat)	1	0.918	0.416	X	0.861	0.7	0.772	0.778
72	Line 1	X	1	0.583	1	0.994	0.7	0.928	0.867
72	Line 7	1	0.918	X	1	1	0.7	0.758	0.896
72	Line 3	X	1	X	1	0.994	0.7	0.928	0.924
10	Line 6	1	1, 0.856, 1	0.374	1	0.994	0.7	0.772	0.855
36	Line 8	1	1, 0.797, 1	0.374, 0.277	1	0.994	0.7	0.928	0.807
14	Line 4	0.352	1, 1, 1	0.277, 0.583	1	0.994	0.7	0.928	0.783
14	Line 5	X	1,1	0.583	1	0.994	0.7	0.758	0.862
23	Line 2	.658	0.918, 1	0.583	X	X	0.701	0.758	0.769
58	Line 3	X	1	X	1	0.994	0.7	0.772	0.893
58	Line 6	0.444	1,1	X	1	0.994	0.7	0.828	0.852

Figure 3.14b. Graph of membership across functions (also showing the overall average membership values) for each member in the family of formula 2-1



Ontologically speaking, these melodic formulae cohere much more than the variants presented in the Treitler example, and as such, we can group them more soundly according to the notion of “variant,” even though there is still a fair amount of structural actualization happening in the activation of certain functions within the formula. However, the high degrees of membership we see, combined with Maloy’s discussion of the use of melodic formulae as an easily remembered unit inserted in multiple chants across the repertoire, seem to affirm our ontological conceptions of the melodies in this family being “of a kind.” Indeed, Maloy comments that Treitler even distinguishes between formula and what he calls “formulaic system” in that the formulaic system relies on melodic constraints including “the underlying tonal characteristics of a melody, such as its typical range and emphasized pitches... principles of text declamation and the

stylistic or formal traits of a particular genre” (Maloy 2010, 90), just as we saw in the two outlying members of Treitler’s family of tropes. A formula on the other hand, has a more limited connotation. Maloy writes that “a formula is defined simply as a standard phrase that occurs, with some variation, in more than one offertory...a formula consists of a series of small melodic segments that occur in a predictable order” (Maloy 2010, 91). Therefore, while there is indeed a separation between the ontology of chant as a whole and the more specific ontology of the melody as shown by the differences in text, there is indeed a quantifiable difference between the melody families categorized as variants or formulae and those that more loosely cohere around structural underpinnings as described by Treitler’s formulaic system. Those melodies that cohere as a formulaic system have typically lower membership values: as we saw with the two outlying members of Treitler’s trope family, these membership values ranged from 0.348 to 0.595 (as shown in Figure 3.7). Conversely, those melodies that cohere as a formula (or as variants) contain higher membership values such as those we saw in Figure 3.14, ranging from 0.769 to 0.946.

Regional Variants: How Different is too Different?

So far we have examined mostly chants that group coherently into melody families possessing strong ontological connections with each other. We have looked at the ontological coherence of chants possessing the same text and melody, and we have considered the ontological complications of chants with different texts that bear the same melody. However, another ontological complication stems from regional and chronological differences that call into question when melodies may be called variants of

one another, and when they must be considered different melodies, even if they bear the same text and liturgical function.

The set of offertories containing the formula 2-1 come from offertories in the Old Roman Tradition. Rebecca Maloy examines these chants both in that tradition and in the analogous Gregorian tradition.³⁹ She contemplates the melodic relationships between the two sets of sources, and explores the theories behind their potential shared origin. She writes that

There can be little doubt that the offertory formed a part of the core repertory transmitted from Rome to Francia in the eighth century...evidence of the offertory's presence in the core repertory lies in its verbal texts, liturgical assignments, and the division of responds and verses, which are generally the same in Roman and Gregorian traditions. Despite these indices of an early origin, however, many offertories lack the musical resemblance between the two traditions that has been demonstrated in other genres. (2010, 8–9)

What then, if anything, do these pairs of offertories have in common? Maloy explains her judgments of the relationship between what she calls the two differing dialects:

[S]tylistic preferences typically result in differences of surface detail between cognate pairs. Although surface similarities do occur, they are usually brief. Affinity between the two traditions is more often manifest in underlying structural traits: range, tonal structure, melodic contour, and the distribution of neumatic passages and melismas. Despite stylistic differences, then, the two versions can often be seen as different realizations of the same underlying musical structure,

39 Put rather simplistically, Old Roman chant was the tradition practiced in Rome in the eighth century, while Gregorian chant refers to the tradition adapted by the Frankish kingdom by the late ninth century. However, it is important to note that the distinction between the two traditions is much more complicated than that, giving rise to several questions and debates in musicological circles. Maloy writes that “The decades between the Frankish reception of Roman chant and its preservation in notation, however, have left us with scores of unresolved questions. Did the Frankish cantors intend to reproduce the Roman chant verbatim...or did they deliberately modify the melodies to conform to their own sensibilities?...Is the melodic tradition recorded in the late ninth century as ‘Gregorian chant’ the same one the Franks first heard the Romans sing some fourteen decades earlier?” (Maloy 2010, 5–6). While these questions are indeed important, and reflect the complexity of the differences between the chants, these specific questions lie beyond the scope of this dissertation.

with the surface details of the melodies determined by dialectical preference.
(2010, 107)⁴⁰

Maloy uses the word “dialect” in a way that suggests ontological consistency between the Roman and Gregorian variants, as if they are variants of the same chant, therefore belonging to the same family. However, just as Treitler described, Maloy sees the melodic similarities occurring not on the surface, but in the structural underpinnings, suggesting that for the melody at least, the ontological status of each pair of Gregorian and Roman variants do not constitute “versions of the same melody.” Such an ontological split between chant and melody must be explored further in order to truly understand the nature of the relationship between these Gregorian and Roman chant pairs.

Maloy identifies five categories of similarity as shown in Figure 3.15, depending on the degree of similarity between what she calls cognate pairs of Gregorian and Roman chant versions. She continues to claim that certain variants in her collection of Gregorian and Roman offertories are more similar than others along these categorical structures. Figure 3.16 shows a table of membership values for cognate pairs from each category. Those with the identifier “G” belong to the Gregorian tradition, while those with “R” come from the Roman family of formula 2-1 studied above. The chart then shows the membership values for the seven functions (as was noted earlier, Function IV is missing in this set of formulae) functions across each formula, ending with an average of all of each formula’s membership value. For example, Figure 3.17a shows the formula from

40 By “melodic contour,” I believe Maloy is referring more generally to overall shape, in a similar manner to that which I described in chapter 1, and not to the more specific intricacies of the surface-level contours of the phrase.

Meditatibor (chant ID 22), with the Gregorian version on the left and the Roman version on the right. In the Roman version, we have a single pre-accent function (function I, on the syllables “in man”), two accent functions (function II, the first on the syllable “da” and the second on the beginning of the syllable “tu”), a single post accent function (function III, on the syllable “tis”), no pre-cadential function (function V), and only the last two cadential functions (functions VII, and VIII, on “tuis”). The chart in Figure 3.16 shows the distinct membership values for each of these functions (with two values for the accent function, reflecting the two distinct accent functions), as well as the overall average membership value for the entire formula (as calculated by the average of the values across all functions).⁴¹ Through these calculations, we see a steadily decreasing average membership value for the Gregorian variant within each category, just as Maloy describes.

Figure 3.15. Maloy’s categories of similarity (2010, 108–110)

Category of Similarity	Descriptive Features
Category 1	“The two versions correspond in range and contour on a phrase-by-phrase basis and occasionally resemble one another in surface detail.”
Category 2	“exhibit specific points of similarity between cognate pairs, but contain one or more longer passages that are dissimilar in range, contour, or placement of melismas.”
Category 3	“exhibit a general similarity in range and tonal structure but show only traces of specific similarity.”
Category 4	“share a broad tonal similarity but lack evidence of a more specific continuity”
Category 5	“lack even the broadest similarity in tonal structure... exhibit a stark contrast between the two traditions.”

41 This calculation for the Roman version of *Meditatibor* (chant ID 22R) is as follows:

$(1.0 + 0.918 + 1.0 + 0.583 + 0.701 + 0.928) / 6 = 0.855$.

Figure 3.16. Comparison of membership values for Gregorian and Roman cognate pairs against the family of formula 2-1⁴²

ID	Name	Location	Category	I	II	III	V	VI	VII	VIII	Averaged Membership
22G	Meditatibor	Line 1	1	0.448	1, 1	0.583	1	0.994	X	1	0.860
22R		Line 1		1	0.918, 1	0.583	X	X	0.701	0.928	0.855
36G	Benedictenges	Line 5a	2	1	1	0.583	X	0.667	X	0.485	0.747
36R		Line 5a		1	0.918	X	1	0.994	.7	0.758	0.895
6G	Tollite portas	Line 4	3	0.602	0.5, 1	0.249	1	0.326	0.503	0.728	0.614
6R		Line 4		0.602	0.918	X	1	1	0.883	0.928	0.888
36G	Benedictenges	Line 8	4	1	0.5, 0.918, 1, 0.522	0.194, 0.374, 0.374	0	0.745	0.524	1	0.596
36R		Line 8		1	1, 0.797, 1	0.374, 0.277	1	0.994	0.7	0.928	0.807
14G	Bonum est confiteri	Line 5	5	X	0.478, 0.144	0.583, 0.416	0	0	X	0.728	0.474
14R		Line 5		X	1, 1	0.583	1	0.994	0.7	0.758	0.862

⁴² X indicates that this function is not present in the phrase, which should be kept distinct from the notion of 0 membership within the family of the function, which does happen.

Maloy states that the pair shown in Figure 3.17a (the Gregorian and Old Roman versions of *Meditatibor*) has category 1 similarity. The second line of the Roman version, *in mandatis tuis*, is our familiar formula from the formula family 2-1. While the non-formulaic passages have changed, the larger formulaic passage still remains quite close to both the Roman version, and the fuzzy melody family to which the Roman formula belongs. Figure 3.16 shows the Gregorian version's membership values within each function of the formula 2-1 family. We see that across individual functions, the chant displays similar patterns of membership to that of its Roman counterparts, and indeed also possesses a high average membership across the entire formula: 0.860. Indeed, the Gregorian and Old Roman versions differ in overall membership by 0.005, a very low quantitative difference, and this is also seen in the pattern similarity in the graph in Figure 3.17b. What is interesting about this particular example, however, is that a few distinct differences between the two versions give the Gregorian version a slightly higher membership value, indicating it is even closer to the family than its Roman counterpart. The cadential patterns in each phrase differ, making their comparison difficult. The Roman version displays a cadential function initially resembling function VII, while the Gregorian version uses a cadential function that initially resembles function VI. Since function VI is a more stable family from a membership standpoint, the higher value for function VI gives the Gregorian version that higher value in the overall membership average.

Figure 3.17a. Comparison of Gregorian and Roman versions of the formulaic phrase from Meditatibor

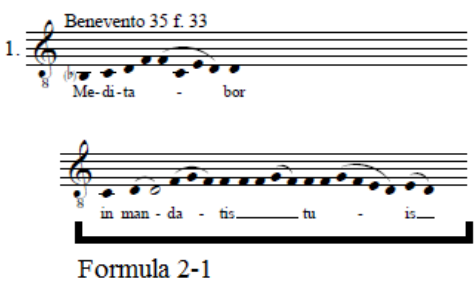
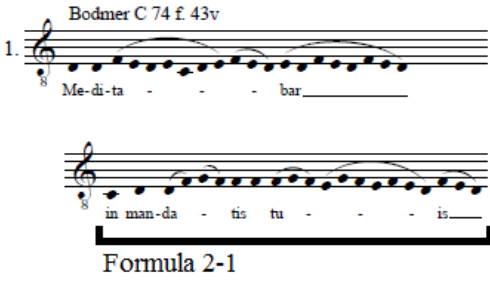
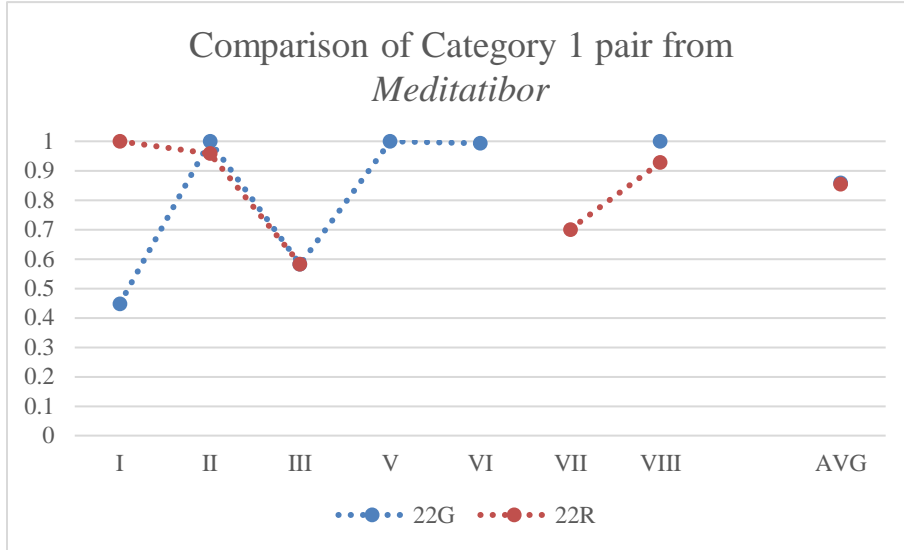
Gregorian Version	22. Meditabor	Old Roman Version
		

Figure 3.17b. Graph of membership across functions (also showing the overall average membership values) for the Gregorian and Roman versions of the phrase from Meditatibor



Despite these differences, it is quite clear that these phrases are related both to each other and to the fuzzy family of formula 2-1. The overall membership value of the Gregorian version falls comfortably within the range of the family, and its similarity

across functions is quite high.⁴³ Consider Figure 3.18a, which shows the Gregorian/Roman comparison of a formula from the chant *Benedicte gentes* (chant ID 36, line 5a), which Maloy has defined as having category 2 similarity. In this comparison, we see that the melodies begin in a similar manner, but differ more dramatically with regard to the ending cadential material. Figure 3.16 and Figure 3.18b show that the memberships across functions corroborate this judgment: we see that the pre-accent and accent functions remain melodically identical, and feature 1.0 membership within the family. Similarly, Function III (the post-accent function) has the same membership as many of function III's existing family members. However, the differences in the cadential section vary more widely: function VI (the first of the cadential functions), which typically has membership values in the 0.9 range, displays a membership of 0.667, and function VIII only has a 0.485 membership, indicating that conceptually speaking, these sections are markedly different from the traditional cadential segments of the Roman formula.⁴⁴ As a result, the overall membership of the phrase within the family of formula 2-1 is 0.747, falling just short of the lower bound of the Roman family's range of familial membership. As Maloy states, the category 2 similarity between these cognate pairs

43 It is interesting to note in this variant, however, that despite its high degree of membership, we can already see in this variant that the strict rules regarding the placement of certain melodic functions against the structure of the text have been loosened. Rearranging certain syllables changes the shape of the individual functions in ways that weaken it, despite its overall similarity in many instances (the accent function and cadential functions especially). From this, one might be able to see how loosening the rules regarding accent placement and structure may lead to increased variation that may result in drastically lower membership values.

44 In the comparison of cadential functions, I have chosen to segment the melodic segment <0100> on the syllable "do" as analogous to function VI, and <0010> on the syllables "mi-nus" as analogous to function VIII. The reason I have decided not to include the conceptual space of function VII is because the segment on "do" more closely resembles the analogous function VI, and separates it from the final syllabic "accent" of the word that would have been on "mi," and would occasionally spill over into the last syllable of the word.

“exhibit[s] specific points of similarity” in their similar openings “but contain one or more longer passages that are dissimilar in range, contour, or placement of melismas” which is seen in the marked difference between the cadential sections (2010, 108).

Figure 3.18a. Comparison of Gregorian and Roman versions of a formulaic phrase from Benedicte gentes

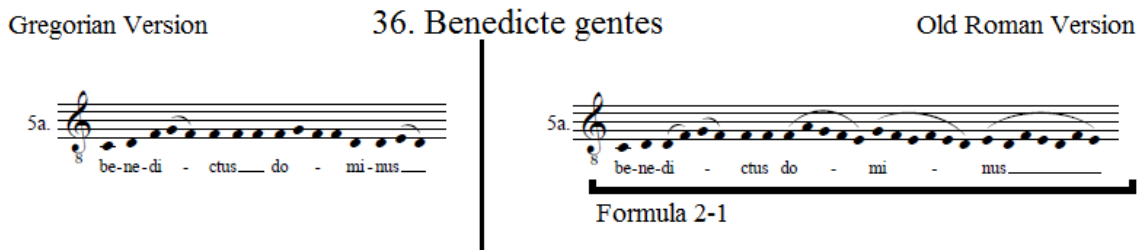
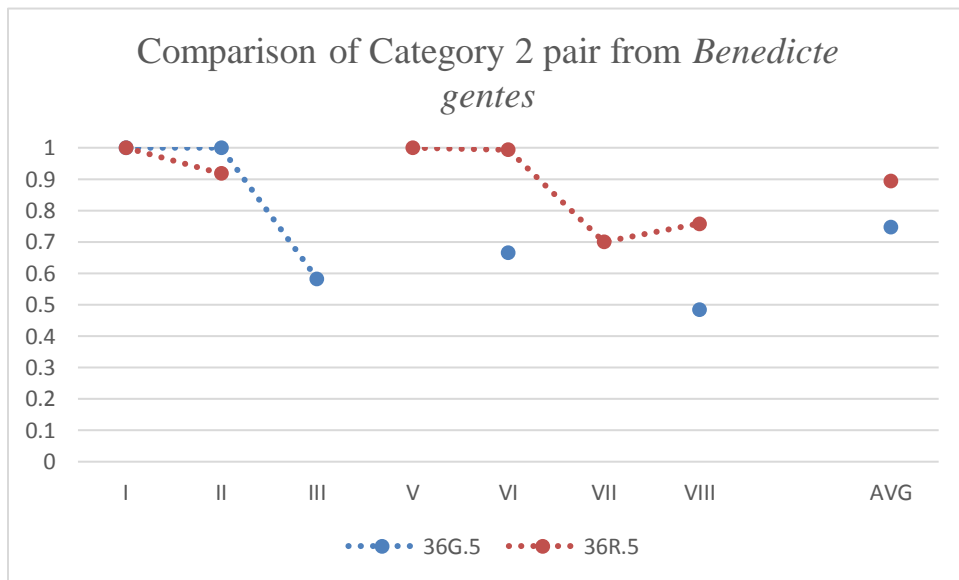


Figure 3.18b. Graph of membership across functions (also showing the overall average membership values) for the Gregorian and Roman versions of the category 2 phrase from Benedicte gentes



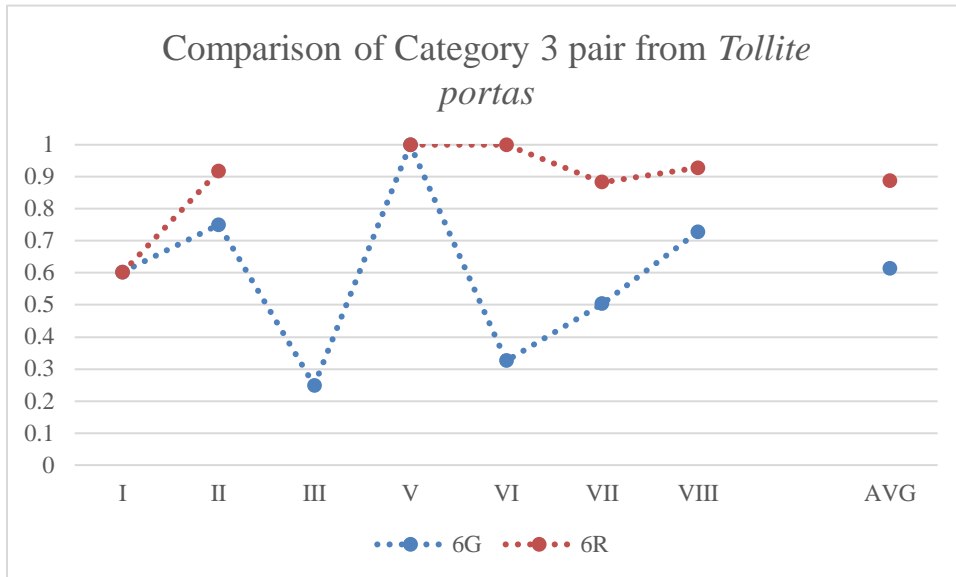
Tollite portas (chant ID 6) is identified as having a category 3 similarity between cognate pairs. Figure 3.19a shows the pair: there are differences in range, as well as in

melodic construction, almost to the point where it becomes difficult to distinguish the functional resemblances on the surface. Nonetheless, we do still see similarities across the phrase that allow us to segment and compare them to functions within the Roman family, as shown in Figure 3.16 and Figure 3.19b. Doing this returns an overall membership value of 0.614, significantly lower than the category 2 similarity. In the comparison of functions, we see that many of the values for the functions in the Gregorian version are significantly lower than those of its Roman counterpart, most notably in the post-accent function (III) and the first two cadential functions (VI and VII). Indeed, we see this in the music itself, the pre-cadential and cadential material looks very different on the last word “eius,” although a small contribution toward similarity between function VII exists in the repeated <+---+> contour beginning in the tenth note before the end. In essence, this motion nods toward the <+---+---+> motion we see in the Roman version for function VII, but has omitted the additional descents, lessening its membership within the family of function VII.

Figure 3.19a. Comparison of Gregorian and Roman versions of the formulaic phrase from Tollite portas

Gregorian version	6. Tollite portas	Roman Version
<p>4.</p> <p>et_ ple-ni-tu - do_____ e - - - ius_____</p> <p>or - bis_____ ter - ra - rum</p>		<p>4.</p> <p>et ple-ni-tu - do e - - - ius</p> <p>Formula 2-1</p> <p>or - bis ter-ra - - - - rum_</p>

Figure 3.19b. Graph of membership across functions (also showing the overall average membership values) for the Gregorian and Roman versions of the phrase from *Tollite portas*



Where our *Tollite portas* example showed a breakdown of similarity in the cadential area, line 8 of *Benedicte gentes* (chant ID 36, Figure 3.20a) begins to wear away the similarities between other structurally important (and traditionally high-valued membership) functions, such as the accent function. For example, the accent on “la” of *Iubilate* in the Roman version features the ⟨+−⟩ shape so prevalent in function II’s family. When we study the Gregorian version, we see ⟨=+⟩ instead, which features a 0.5 membership within function II, as shown in Figure 3.16. The Gregorian version features four accent functions, two of which fall between 0.5 and 0.522, compromising the overall membership of the phrase within the family of formula 2-1. This phrase has a category 4 similarity, and a membership value of 0.596 indicating that it is starting to break down the structurally stable continuities that both held the Roman family together, and that

comprised the structurally similar elements of the previous three similarity categories in their Gregorian counterparts.

Regarding category 5, Maloy writes that the Gregorian versions “lack even the broadest similarity in tonal structure... exhibit a stark contrast between the two traditions.” In some cases, this is very true. Consider Figure 3.21, which shows the beginning of verse 2 (line 6) from *Ascendit deus*. In the Roman version, the formula is very clearly and deliberately laid out, such that there is little question regarding the functional segments Maloy identifies in this phrase. The Gregorian version, however, shows so much variety with regard to this phrase that segmenting into meaningful analogous functional sections is nearly impossible, and certainly not productive. However, not all category 5 chants deviate so drastically, and it is prudent to examine one of them to continue to see where and how the breakdown of similarity is taking place.

Figure 3.22a shows part of Verse 1 (line 5) from *Bonum est confiteri* (chant ID 14). This chant is slightly easier to segment analogously to the Roman chant. In doing so, we see just how much the contour of the phrase has changed. The initial accent function of the Gregorian version on “nimis” features a $\langle +=+ \rangle$ contour rather than the more balanced $\langle +- \rangle$ contour typical of the Roman accent functions.⁴⁵ This $\langle +=+ \rangle$ contour has a membership value of 0.478. We see significant change again with the second accent function on “fun”, where the segment is $\langle +==+ \rangle$ instead of $\langle +- \rangle$, and features a membership value of 0.144. Both of these Gregorian accents, when measured against the

45 Here this pair is missing the pre-accent function, beginning on the accent function instead.

Figure 3.20a. Comparison of Gregorian and Roman versions of a formulaic phrase from *Benedicte gentes*

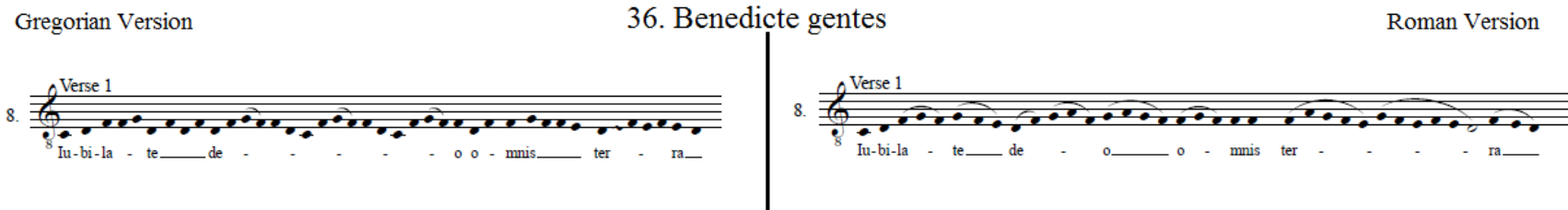


Figure 3.20b. Graph of membership across functions (also showing the overall average membership values) for the Gregorian and Roman versions of the category 4 phrase from *Benedicte gentes*

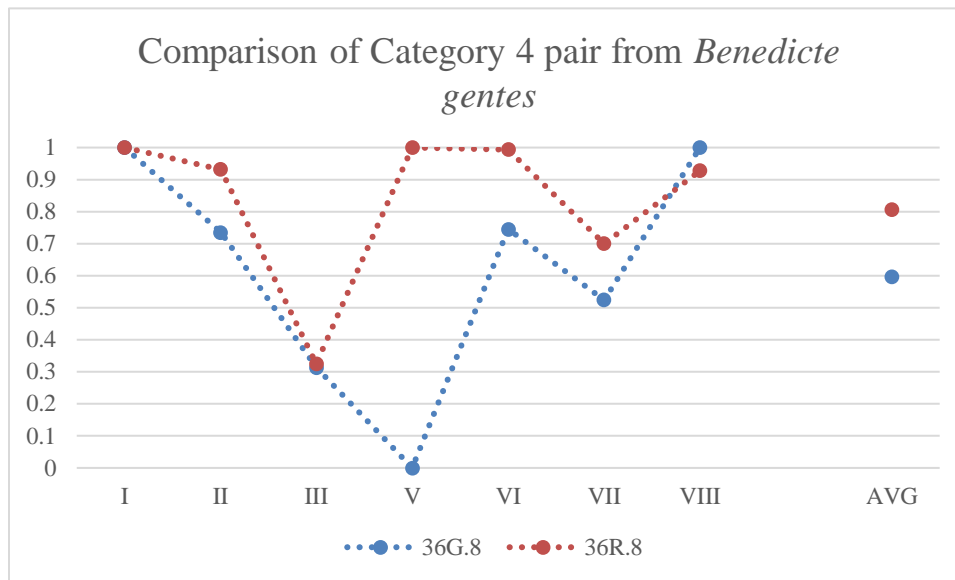


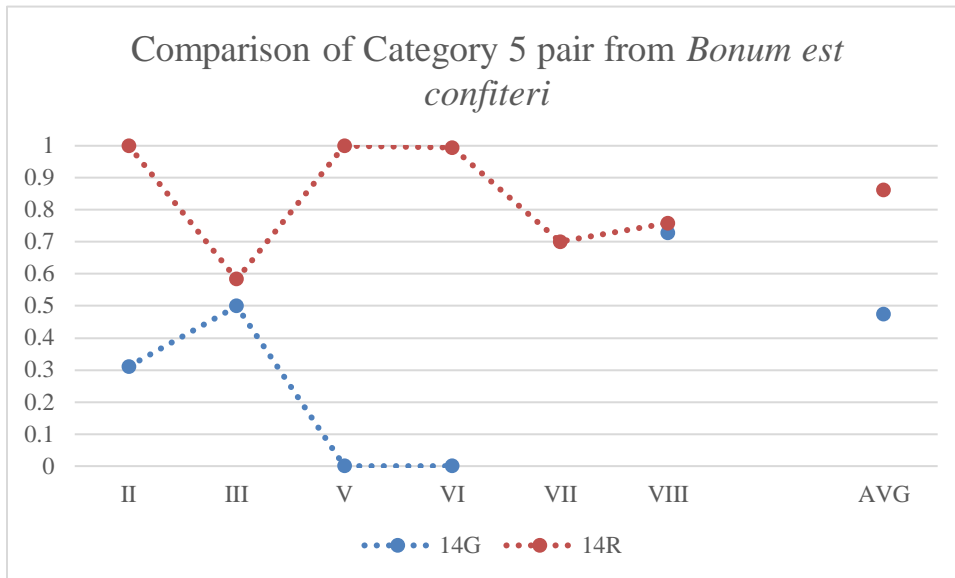
Figure 3.21. Comparison of Gregorian and Roman versions of a formulaic phrase from Ascendit deus

<p>Gregorian Version</p> <p>6. Verse 2</p> <p>Quo - - - - ni - am _____</p> <p>do - - - - mi - nus _____</p> <p>sum - mus ter - ri - bi - lis _____</p>	<p>58. Ascendit deus</p>	<p>Roman Version</p> <p>6.* Verse 2</p> <p>Quo - ni - am</p> <p>de - us</p> <p>sum - mus ter - ri - bi - lis _____</p>
<p>*All three lines together comprise formula 2-1</p>		

Figure 3.22a. Comparison of Gregorian and Roman versions of a formulaic phrase from Bonum est confiteri

<p>Gregorian Version</p> <p>5.</p> <p>ni - mis pro - fun - de - fa - ctae sunt _____</p>	<p>14. Bonum est confiteri</p>	<p>Roman Version</p> <p>5.</p> <p>ni - mis pro - fun - de - fa - cte _____ sunt _____</p> <p>Formula 2-1</p>
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Figure 3.22b. Graph of membership across functions (also showing the overall average membership values) for the Gregorian and Roman versions of the phrase from *Bonum est confiteri*



family of function II, feature remarkably low membership values, signifying that the differences between the versions are significant enough to question whether the melodic fragments are even related. Furthermore, any resemblance of cadential material seems to have disappeared, as evidenced by the complete non-membership of the phrase against functions V and VI. The only cadential motion that gives function VIII a higher value is the descent of the final three notes. The <-+> at the end fits somewhat into the average family of function VIII, mainly because it features the initial descent, and then an ascent that we see in the middle of function VIII (despite the fact that it is the end of the function in the Gregorian version). The overall membership of this chant is 0.474, which

falls drastically outside the familial bounds of formula 2-1, as seen in the graph in Figure 3.22b.

It is interesting to note that these examples are crisp realizations of a category of similarity that is in itself fuzzy. Indeed these categories possess a range of membership values when taking all variants into account, and in some cases, the lines between categories are blurred, suggesting the possibility of potential categorical overlap. One can especially see this in the closeness in membership between the category 3 example in Figures 3.19a and 3.19b (membership value: 0.614) and the category 4 example in Figures 3.20a and 3.20b (membership value: 0.596). Perhaps these two examples fell toward the lower and upper bounds of their category, respectively, and this explains the closeness in membership value of these two pairs. While these categories are important to Maloy's understanding of the differences between the cognate pairs, my approach adds valuable information about the extent to which each version is a member of the same melody family, adding nuance to Maloy's categorical labels.

Certainly there are historical, liturgical, and societal reasons to group these chants together under a single ontological conception, and this throws into stark relief the difference between the ontological status of chant itself and our modern conception of the ontological status of the melody. While some of the Gregorian variants can truly be considered variants of the melody presented in the Roman family, it is clear that as the two traditions separated, they grew to become melodically independent of each other. Maloy posits a number of theories regarding the family's transmission that would explain this separation. She sees a number of patterns in the placement of resemblances, mainly that those with greater degrees of resemblance fall earlier in the liturgical calendar,

growing more divergent after Septuagesima. Furthermore, among those with greater resemblance, the relationship is greatest in the early verses; indeed we see this with the two examples from *Benedicte gentes*. Maloy cites that in many of the “breaches of affinity,” the Gregorian variant no longer relies on the formula that the Roman version possesses, and states that “The lack of resemblance between the two dialects in the later seasons, however, evinces substantial melodic change, in one dialect or both, after their separation in the eighth century” (2010, 122).

Given these theories of separation, is it conceptually valid to consider these variants of the same melody? I do not think so. Instead, as Treitler puts forward, we could conceptualize them as actualizations of more abstract patterns (akin to the real actualization of, for example, a 3-line Schenkerian *Ursatz*), and in that case, it becomes the abstract pattern that possesses the ontological consistency, not the melody itself in any of its realizations.

Fuzziness of Oral Transmission


In this chapter, we have looked at a variety of chant families, and these families each possessed unique traits that called the ontological status of the chant family into question. Through my analyses, I have shown how fuzzy set theory can add meaningful information to this discussion. In the first section, I used my FCM model to examine how stability of melodic variants can be quantified through contour, examining how the ontological idea of melodic identity (and subsequently chant identity) can be represented more appropriately in fuzzy terms than if one were to try to think of melody as a singular

phenomenon. In the second section, I explored how changes in text affect the stability of the melody family, and how thinking of these families using fuzzy set theory can help to differentiate between the ontology of melody and the ontology of the chant itself. Finally, in the third section, I explored how regionally separated development call into question to what extent regional variants can be considered members of the same melody and chant family. My model was able to illustrate quantifiable differences between Gregorian and Roman variants of offertories containing formula 2-1, providing additional nuance that will aid future analysts in grappling with the ontological discussion of developmental differences between regional variants.

To be certain, the ontology of plainchant is complex, with intricacies arising from the oral nature of chant's transmission that the study of later musical ontology may not need to deal with. However, later music is rife with other complications, namely that variety is achieved in many more ways, across multiple parameters such as harmony, rhythm, form, tempo, dynamic contrast, etc. Nonetheless, thinking in fuzzy terms across these parameters is a good way to explain and deal with ontological issues surrounding music in all its forms and representations. To put this back into the context of Treitler's unicorn, the use of fuzzy set theory in the study of ontology can help us to show when a representation of a unicorn ceases to be a unicorn. It helps to answer the question of how different representations of the unicorn can be reconciled in the same ontological family, and can show how different conceptions of the unicorn may cohere more closely than others. Treitler and others have written about musical ontology in ways that come close to this representation, yet each have lacked the analytical rigor that tools rooted in the logic of fuzzy set theory can provide. In other words, the conception that musical

ontology is fuzzy is perhaps not a new idea, though it has yet to be written in just that way, but to study it in a manner that is sensitive to that fuzziness, such as with the FCM model, may bring us closer to fully appreciating its meaning.

CHAPTER 4: Contour's Role in Motivic Development in Brahms's "*Regenlied*" (Op. 59, no. 3) and *Regenliedsonate* (Op. 78)

I have tried the Sonata with Heermann. We were so glad of the opportunity that we went into the work quite thoroughly. The way you have blended all the motifs together strikes me as wonderful. How charming the dreamy accompaniment of the last motif  sounds at the beginning of the first motif. It is as if the spirit of the whole piece were wafted to one's ears at the very opening. The grace and warmth of the melodies, and the masterly way you treat all the motifs, captivate one's heart and soul from the first to the last note. What heavenly passages there are in it, not to mention the beauty of some of the organ points! And then the ascent in the last movement of the first melody where it finally returns and rises and falls full of sadness and yearning! For such feelings sound alone, and not words, are adequate... Many people may be better able to speak about these things than I am, but no one can feel them more deeply than I do. The deepest and most tender chords of my heart vibrate at the sound of such music.

--Clara Schumann (1971, 49)

This beautiful letter from Clara Schumann to Johannes Brahms praises Brahms's first Violin Sonata, Op. 78. As Clara describes, the sonata is a testament to Brahms's melodic development. Indeed, Op. 78 is arguably one of his most lyric pieces. It is popularly titled the *Regenliedsonate* because of its motivic connection to Brahms's earlier song "Regenlied" Op. 59, no. 3 completed in 1873. Walter Frisch lauds the sonata, stating that "Op. 78 seems to assume the thematic or motivic legacy: its harmonic language is not especially remarkable, but the purely horizontal dimension unfolds with a sophistication and flexibility that Brahms himself was never able to surpass" (1984, 120). Because the sonata's familiar song-like melodies seem to take precedence over the formal design and harmonic procedures within the sonata, one can see the mastery that both Clara Schumann and Walter Frisch extol occurring in a crucial domain: melodic contour.

Either directly or indirectly, contour is often considered to be a defining characteristic of a motive. Scholars following Schoenberg's perspective on motivic development—such as Jack Boss (1992), Walter Frisch (1984), and David Epstein (1979)—identify contour directly as an element of motivic identity. For example, Schoenberg writes that a motive is comprised of “intervals and rhythms, combined to produce a memorable shape or contour” (1967, 8). Still others who discuss motivic identity invoke contour more indirectly, as a secondary attribute of directed interval motion. For example, Lora Gingerich (1986), Edward Pearsall (2004), David Lewin (1987, 1992), and others have developed tools that explain changes to the directed interval pattern of a motive as it develops, potentially affecting the contour of the motive as a result.

Despite this acknowledged importance of contour to a motive's identity, few have directly explored contour's dynamic potential to influence a motive's development. Schoenberg, when describing the notion of developing variation, calls for composers to alter some features of a motive while at the same time maintaining others, so as not to produce a motive-form that is too foreign to the motive family (Schoenberg 1967, 8–9). Certainly, development across a number of different motivic features could conspire to tip the balance of a motive-form into this “foreign” category, as a motive loses the comprehensibility that identifies it as a meaningful variant within a motive family. Maintaining a balance between development and continuity is therefore crucial to Schoenberg's notion of motivic comprehensibility. Contour, being one feature of a motive, has the potential to influence either side of this continuity-development continuum: when contour remains fairly constant, it contributes to motivic cohesion;

when it varies, it can drive motivic development. Given that contour is also an element that listeners use to make sense of new and recurring melodies, it stands to reason that its contribution to these sides of the continuum will affect how a listener experiences the work.⁴⁶

In this chapter, I use my Familial Contour Membership model to examine how contour contributes to the two sides of the cohesion-development continuum within Brahms's *Regenlied* works. Modeling Brahms's motive families this way illuminates how the motives within each family develop, and how different motive families within the sonata are related. I show how the contours of the "Regenlied" Op. 59 No. 3 follow a developmental trajectory that mirrors the narrative trajectory of the song's text, which sees the present fall away in favor of a past memory. I then explore the related motive families in the Violin Sonata, and draw connections between the two works, showing how contour serves to remind the listener of the narrative presented in the song, while at the same time pushing the narrative further to reflect personal aspects of Brahms's life. In this way, I show how contour exposes aspects of Brahms's developmental tendencies, such as the relationship between contour and narrative in the song, or the developmental relationships that add to the narrative throughout the sonata. Using contour to shed light onto Brahms's motivic development in this way gives us a new way to discuss the melodic aspect of Brahms's developing variation.

⁴⁶ A number of studies have shown that contour can aid in the immediate perception and recall of novel melodies: Dowling and Fujitani 1971, Dowling 1991, Dyson and Watkins 1984, Quinn 1999, and Schmuckler 1999. Exploring the findings in these studies in the context of motivic identification throughout musical works (such as the *Regenlied*) would be an interesting future avenue of study, but lies beyond the scope of this dissertation.

Coherence and Development in *Regenlied*, Op. 59, No. 3:

Brahms wrote “Regenlied” using a poem by Klaus Groth (the text and translation for which is shown in Figure 4.1), wherein the poet experiences “the flood of childhood memories awakened by a summer rain, and the accompanying sadness for the loss of childhood’s capacity for wonder” (Russell 2006, 63).⁴⁷ The song takes an A-B-C-A’ form, as shown in the form diagram in Figure 4.2a. These significant formal markers revolve around differing temporal states within the poet’s life. The poem opens in the present as the poet experiences a rainstorm and longs to be taken away to the memories of his childhood. In the B section beginning in m. 45, however, the poet moves into the past, with a depiction of the physicality of the childhood memory. The C section moves into a more spiritual reflection on that past (when the poet is speaking of his soul opening up and the holy web of creation piercing into his secret life), and the closing A’ section returns to the present.

⁴⁷ Brahms had previously set another of Groth’s poems entitled “Regenlied” to music (WoO 23), yet the poem and musical motives are quite different. Interestingly enough, Brahms revisited this poem when he wrote “Nachklang” of Op. 59, bearing the same motives discussed in this chapter: Brahms’s sole resetting of a same poem (Sams 2000, 129-130, 187-188). One might speculate on the connection of motives of the “Regenlied” Op. 59 no.3 to the earlier poem of WoO 23 in order to add further nuance to the meaning of the music in “Nachklang.” The setting in “Nachklang,” as Russell explains, evokes an “intense grief for something unspecified” (Russell 2006, 64). This connection may add meaning to both the musical motives themselves, and to Brahms’s feelings toward them as he revisits them in the violin sonata. However, a study of the “Nachklang” is beyond the scope of this chapter.

Figure 4.1. Text of the Regenlied (with Fischer-Dieskau translation)

Regenlied (Klaus Groth)	Rain Song (Fischer-Dieskau 1976, 319–20)
Walle, Regen, walle nieder, Wecke mir die Träume wieder, Die ich in der Kindheit träumte, Wenn das Naß im Sande schäumte!	Stream down rain, stream down rain, wake for me those dreams again, which I in my childhood dreamt when water foamed upon the sand!
Wenn die matte Sommerschwüle Lässig stritt mit frischer Kühle, Und die blanken Blätter tauten, Und die Saaten dunkler blauten.	When oppressive summer heat with cool freshness idly strove, and shiny leaves dripped with dew, and crops were of a darker blue.
Welche Wonne, in dem Fließen Dann zu stehn mit nackten Füßen, An dem Grase hin zu streifen Und den Schaum mit Händen greifen.	What bliss then to stand with naked feet in the flow, to brush along against the grass and with my hands to grab the foam,
Oder mit den heißen Wangen Kalte Tropfen aufzufangen, Und den neuerwachten Düften Seine Kinderbrust zu lüften!	or upon my ardent cheeks to catch cold drops, and to the fresh-awakened scents lay bare one's childish breast!
Wie die Kelche, die da troffen, Stand die Seele atmend offen, Wie die Blumen, düftetrunken, In dem Himmelstau versunken.	Like the flower-cups dripping there, open, breathing, stood my soul, like the flowers, fragrance-drunk immersed in heaven's dew.
Schauernd kühlte jeder Tropfen Tief bis an des Herzens Klopfen, Und der Schöpfung heilig Weben Drang bis ins verborgne Leben. Walle, Regen, walle nieder, Wecke meine alten Lieder, Die wir in der Türe sangen, Wenn die Tropfen draußen klangen!	Awesomely each drop struck cold, deep to where the heart was beating and the sacred motion of creation broke through to the hidden life. Stream down rain, stream down rain, waken these old songs of mine, which in the doorway we would sing when, outside, the drops resounded!
Möchte ihnen wieder lauschen, Ihrem süßen, feuchten Rauschen, Meine Seele sanft betauen Mit dem frommen Kindergrauen.	I would like again to listen to their sweet moist rustling noise, like softly to bedew my soul with innocent childish awe.

Figure 4.2a. Formal structure of the song

Formal Section	A	B	C	A'
measures	1–44	45–70	71–94	95–147
Keys	F# minor	→ A major	D major	→ F# minor
Poet's temporal state	present reminiscence	physical past	spiritual past	present
contour membership	Family A	Family B	(Family C)	Family A

This study focuses on the opening A and B sections (mm. 1–70), exploring the motivic relations found between the two contrasting sections. The motives in the A sections—representing the poet in the “present”—resemble the initial germinal A motive, and occur twenty-three times (as shown in Figure 4.2b). These twenty-three motives form family A, and have relatively high membership values within the family (as shown in blue in Figure 4.2b). In many instances, this motive is characterized by an opening series of melodic plateaus that feature a dotted-quarter-eighth-dotted half rhythm, followed by a more varied melodic contour ending the motive.

Section B, beginning in m. 45, bears text that invokes memories from the poet’s childhood of physically feeling the sensations of a rainstorm. Words like “touching,” “smelling,” and “reaching” all convey physical actions that point to the tactile sensation of the memory. Brahms represents this action melodically with a much more active and directed line. Throughout this section, motives resembling the germinal B section motive occur nine times, and these motives form family B (as shown in Figure 4.2b, with membership values shown in red). The motive is characterized by more upward motion, beginning with an ⟨+=+⟩ run in eighth notes, followed by a more varied, yet more pointedly angular shape ending the motive. We are now inside the memory, and this is signified by a shift in motivic content that now forms family B (the motives from mm. 45–70).

Figure 4.2b: Annotated score excerpt for the opening A and B sections, featuring the A and B families (membership values compared to family A are in blue, while membership values compared to family B are in red.)

In mäßiger, ruhiger Bewegung

Singstimme

Pianoforte

0.837 (0.842)

0.776 (0.792)

0.754 (0.866)

0.646 (0.816)

0.837 (0.842)

0.711 (0.463)

0.696 (0.583)

Wal - le

Re - gen, wal - le nie - der, we - cke mir die

Träu - me wie - der, die ich in der Kind - heit

träum - te, wenn das Naß im San - de schäum - tet

0.776 (0.792)

0.754 (0.866)

0.646 (0.816)

0.432 (0.574)

0.432 (0.574)

Wenn die

mat - te Som - mer schwü - le läs - sig - tritt mit

fri - scher Küh - le, und die blan - ken Blät - ter

tau - ten, und die Saa - ten dunk - ler

Figure 4.2b: (cont.)

blau - ten die Saa - ten dunk - ler blau -

etwas lebhafter

cresc.

B

ten. Wel - che Won - ne, in dem Flie - ßen dann zu -

stehn mitnack - ten Fü - ßen an dem Gra - se hin zu strei - fen und den

Schaum mit Hän - den grei - fen, o - der mit den hei - ßen Wan - gen

0.643 (0.362) kal - te Trop - fen auf - zu - fan -

p cresc. *sf*

0.787 (0.256) 0.851 (0.534) - - - gen, und den neu - er wach - ten Düs - ten sei - ne

0.661 (0.404) Kin - derbrust zu lüf - ten, sei - ne Kin - der - brust zu

lüf - ten! Wie die

m. v. *p*

Figures 4.3 and 4.4 show the contour representation of the motive families of these two sections, and it is through each family's strong internal similarities that we see contour's role as a cohesive feature of the motive family. Figure 4.3a lists the contours of family A's motives, including each motive's c-seg and CAS representation.⁴⁸ Columns five and six then quantitatively describe the degree of membership that each motive has within family A, as well as the degree of similarity to family B (as measured by degrees of potential membership within family B, which I will discuss later in this chapter).

These membership values are derived by comparing each individual motive to an averaged representation of the family, called the average family member. Using the Familial Contour Membership model, I created the averaged member of motive family A, which tracks the probability that a c-seg within the family will ascend, descend, or plateau in any given portion of the contour, as labeled using positions within the CAS representation. The columns of Figure 4.3b feature each CAS position within the family, while the rows indicate how many members ascend (+), plateau (=), or descend (-) in that CAS position. For example, 14 of the contours in family A plateau from the first note to the second, so their CAS would feature an “=” in position 1.⁴⁹ Furthermore, there are no

⁴⁸ The contour segment, or c-seg, is a numeric representation of a melody, with 0 as the lowest pitch (Marvin and Laprade 1987). The Contour Adjacency Series tracks the directional motion between adjacent pitches, with “+” for ascent, “=” for plateau, and “-” for descent (Friedmann 1985).

⁴⁹ Because of the cardinality differences, contours can be positioned among the family such that they do not possess position 1 when measuring the contour against the average family member. This is consistent with the notion that relationships can occur without invoking the beginning or ending of a contour segment, as is the case in motives between mm. 4-6 and mm. 7-8. In mm. 7-8, the contour is but the ending of the motive in mm. 4-6, so this contour would not be aligned against position 1 when measuring it against the average family member.

family members that descend or ascend in position 1, making it 100% probable that a new motive will plateau in the family's position 1 (if position 1 is included in the contour). This set of probabilities was derived using a combination of the mathematical best-fit (which takes the potential alignment that produces the highest membership value as the proper alignment within the family) and the musical best-fit models (which uses musical characteristics to determine the desired alignment within the family) I proposed for the fuzzy contour membership tool. I positioned the family in musical accordance with the germinal motive of mm. 4–6, using a more holistic approach that used melodic, harmonic, and rhythmic characteristics in order to determine the proper positioning of each contour against the family. In many cases, musical and mathematical best-fit procedures agreed on the proper positioning of the c-segs within the family.⁵⁰ What results is the averaged family member—a series of probabilities, shown in Figure 4.3b, that describe the directional tendencies of any given position within the CAS of c-segs in the family. From these probabilities one can find the degree of contour membership of each motive within the motive family, the quantities for which are shown in column five of Figure 4.3a. Contours that have a high degree of membership (0.6–1.0 for the purposes of this chapter) share dominant family traits as exhibited by high probabilities within the chart in Figure 4.3b. Conversely, contours that have mid-range or low degrees of membership display less dominant family traits (most common for mid-range values), or are missing family traits altogether (most common for low values).

⁵⁰ Brahms's motivic development here develops motives from the starting point of the germinal motive at the outset of the song, so in this case it is logical to make reference to this germinal motive when forming the averaged member of the family. In other analytical cases however, it may not be as logical to use a germinal member as a reference. In these cases, the mathematical best-fit provides a reliable model for measuring these probabilities in the formation of an average family member.

Despite the occurrence of 23 individual motive variants throughout the song, family A only has nine different contours, suggesting a great degree of coherence. One can already begin to see patterns, such as an opening plateau or the <-++> segment featured in at least 11 individual motives. Furthermore, column five of Figure 4.3a shows that the membership values for each motive within the family also suggest a relatively high degree of cohesion: all but three exhibit a membership value between 0.6 and 0.837.⁵¹ In essence, most of the motives within family A contribute to the cohesion of the motive family by resembling each other, and in turn by resembling the opening germinal motive.

Figure 4.3a. The crisp members within Family A

Example measures	C-SEG	CAS	Membership in family A	Similarity to Family B
mm. 0–2, (2x) (piano)	<333210>	<==--->	0.837	0.842
mm. 4–6 (4x)	<11102432>	<==+--->	0.776	0.792
mm. 7–8 (4x)	<423410>	<-+--->	0.754	0.866
mm. 9–11 (4x)	<432110>	<---=->	0.646	0.816
mm. 12–15 (3x)	<4442432012>	<==+----++>	0.711	0.463
mm. 16–19	<33224102>	<=-+---->	0.696	0.583
mm. 32–35 (2x)	<0033345221>	<=+====-=->	0.432	0.574
mm. 39–44	<1123453210>	<=++++---->	0.53	0.574
mm. 127–131	<44433210>	<==----->	0.679	0.613

⁵¹ It is interesting to note that the lowest members of the family occur toward the end of the first A section, indicating a turn away from the core of family A. This is consistent with the textual narrative, wherein the poet turns toward thoughts of the past. In these sections, from mm. 32 to the end of the A section, the poet begins to reminisce about the past, but we have not arrived at the full-fledged memory yet. These lower values within family A indicate a dropping off of the present.

Figure 4.3b. Family A probabilities

	1	2	3	4	5	6	7	8	9
+	0	0.2	0.043	0.667	0.579	0.087	0	0.5	0.625
=	1.0	0.667	0.174	0.143	0	0.174	0	0.333	0
-	0	0.133	0.783	0.190	0.421	0.739	1.0	0.167	0.375

Figure 4.4a shows an analogous list of the motives' contours for family B, and Figure 4.4b shows its averaged family probability, from which we can find membership values within the family. In these tables, we can see that family B is a highly coherent motive family: all members of the family exhibit relatively high degrees of membership, ranging from 0.643 to 0.898, with no outliers. In addition, we can see this coherence in the averaged table itself: CAS positions 1, 5, 6, 7, 9, and 11 all have a crisp 1.0 membership/non-membership probability, resulting in higher membership values for contours that follow these pathways. For example, all of the family members except two ascend in the family's first position (the motives in mm. 54–55 and 65–67 do not ascend first because they are missing the familial first position, which is akin to a family member exhibiting all the dominant family traits except one). As a result, the members that have this ascent are very similar to the average member of the family. Take the first motive in mm. 45–46 for example: the contour invokes the pathways in CAS positions 1,2,3,4,5,6,7,9, and 11, and therefore follows all of the pathways that exhibit this 1.0 membership/non-membership value, as shown in Figure 4.4c. What results is a very high degree of membership, as $(1 + 0.833 + 0.875 + 0.375 + 1 + 1 + 1 + 1 + 1) / 9 = 0.898$. This potential for high probability suggests that for a potential member that follows these CAS positions, it is guaranteed to follow the path laid out by *all* of the other existing members of the family, or conversely that potential motives that do not follow these paths

will have significantly lower membership values within the family.⁵² In essence, the motions in these positions are dominant family traits that significantly affect the relationship of the c-segs to the family.

Figure 4.4a. The crisp members of Family B

Measures	C-SEG	CAS	Membership in this family	Similarity to family A in song
45–46	⟨0223310231⟩	⟨+=+---++⟩	0.898	0.320
47–49	⟨0224523154⟩	⟨+=++++++⟩	0.870	0.418
49–51 (x2)	⟨0112212121⟩	⟨+=+--++⟩	0.870	0.360
54–55	⟨75316420⟩	⟨---+---⟩	0.761	0.543
56–59	⟨01234454⟩	⟨++++=+-⟩	0.643	0.362
60–62	⟨022453133564⟩	⟨+=+++++==++⟩	0.787	0.256
62–64	⟨13432110⟩	⟨++++=-⟩	0.851	0.534
65–67	⟨654321401⟩	⟨-----++⟩	0.661	0.404

Figure 4.4b. Family B probabilities

	1	2	3	4	5	6	7	8	9	10	11
+	1.0	0	0.875	0.375	0	0	1.0	0	1.0	0.333	0
=	0	0.833	0	0.375	0	0	0	0.25	0	0.333	0
-	0	0.167	0.125	0.25	1.0	1.0	0	0.75	0	0.333	1.0

Figure 4.4c. The contour of mm. 45–46 measured against Family B’s probabilities

	1	2	3	4	5	6	7	8	9	10	11
+	1.0	0	0.875	0.375	0	0	1.0	0	1.0	0.333	0
=	0	0.833	0	0.375	0	0	0	0.25	0	0.333	0
-	0	0.167	0.125	0.25	1.0	1.0	0	0.75	0	0.333	1.0

⁵² Because of the method for measuring membership of contours of lesser cardinality, not every member of a family has all of the family’s CAS positions. For example, the contour described in Figure 4.4c is missing positions 8 and 10, suggesting that the contour does not possess the family traits exhibited in those positions.

In addition to the coherence of each family within itself, family B exhibits a relatively high degree of distinction when compared to family A. When the individual motives of family B are compared to the averaged family A, they exhibit a range of potential membership from 0.256 to 0.543, for an average potential membership of 0.395. This relation says that the motives in family B are relatively unlikely also to be members of family A. The large discrepancy between families in this case is suggestive of a form that is clearly demarcated, and this is consistent with the formal distinctions I have offered in Figure 4.2. Between the A and B sections, we see a dramatic shift of key to the relative major, as well as a shift of the text's narrative context from present to the past. In each case, contour serves as a feature that aids in the coherence of each section, and subsequently contrasts with the other section.

Contour Development and Narrative Context

Despite this convincing sense of coherence, Brahms's motivic treatment throughout mm. 1–70 can be viewed as developmental when we examine the relationship between the two families as a whole. As I showed above, the motives in family B are outlying members of family A, yet when we look at the relationship from the opposite lens—where we examine the individual members of family A against the tendencies of family B—we find a much closer relationship. Column five of Figure 4.3a shows these specific membership values for the individual members of family A. Here we see a somewhat surprisingly high degree of potential membership within the averaged family B, with values ranging from 0.46 to 0.866, for an average of 0.68. Such high degrees of membership indicate that there are still underlying relationships binding these motive

families together, despite their outward appearance as unique and individually cohesive. In essence, while there is enough similarity to make a convincing case for coherence, their outward appearance suggests that development has occurred in order to conceptually separate the two families. For example, Figure 4.5 shows the germinal motives of families A (mm. 4–6) and B (mm. 45–47) (hereafter referred to as A₁ and B₁ respectively). Although they are rhythmically unique and belong to different keys, one can see similarities in contour that suggest deeper relationships between the families in the contour domain. Both feature prominent plateaus in the outset of the motive, and feature a similar <-++> motion toward the end. These prominent similarities between the two motives explain the high potential membership value that motive A₁ has within family B, yet development is still present that suggests emerging distinction of family B away from family A. In B₁ the two plateaus are now separated by two prominent ascents, in CAS positions 1 and 3. Additionally, Brahms adds an extra descent in the middle, while removing one from the end.

Figure 4.5. Similarities in CAS between the two germinal motives of the song

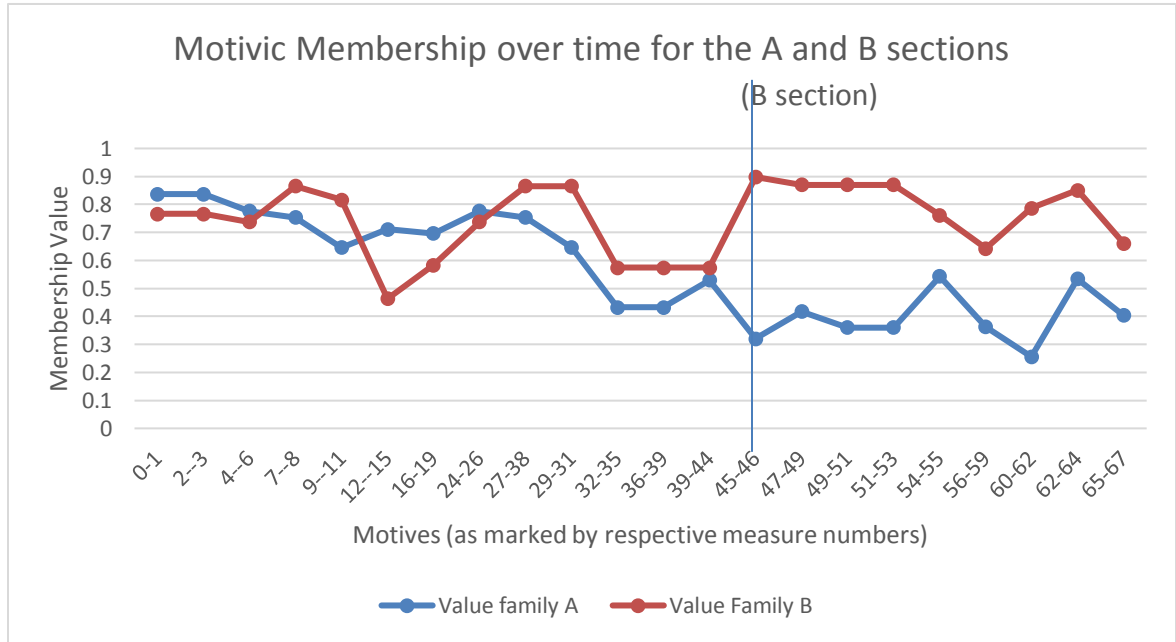
The figure displays two musical excerpts from a song in G major. The first excerpt, measures 4-6, shows the vocal line with the lyrics "Wal - le, Re - - gen, wal - le". Below the notes, a Contour Analysis Sequence (CAS) is provided: "= = - + + - -". The second excerpt, measures 45-47, shows the vocal line with the lyrics "Wel - che Won - ne, in dem Flie - - ßen". Below the notes, a CAS is provided: "+ = + = - - + +". Both excerpts include piano accompaniment and are in 4/4 time.

Consider Figure 4.6, which shows a graph summarizing the developmental paths of motive families A and B in the first two sections of the song. This graph shows the two families' interaction across the whole opening binary section by reinterpreting the data from columns four and five of Figures 4.3a and 4.4a respectively. Each node represents the membership value of the motive occurring in the measures across the x-axis within the two respective families, with blue representing the membership within family A and orange representing the membership within family B. By examining the membership values in context of both families, we see an interesting formal distinction begin to emerge. The lines move closely together throughout the A section, ending in m. 44, and then dramatically split apart as the B section takes over in m. 45. Throughout the A section, the blue "value family A" line measures the crisp motives throughout the A section against the fuzzy family A. The orange "value family B" line measures the crisp motives of the A section against the fuzzy family B. Throughout the B section, the blue "value family A" represents the crisp motives occurring in the B section against the fuzzy family A, while the orange "value family B" line represents the crisp motives of the B section against the fuzzy family B.

Figure 4.6 shows that the motives in the A section, which helped to build fuzzy family A, have the potential to also fall within the norms of family B. However, the opposite cannot be said for the motives in the B section: the graph of the B section suggests that these motives are more outliers within family A. There are a few ways to explain this pattern: one is to suggest that the motives in family B are somehow emergent from those in family A as I had previously suggested in my discussion of Figure 4.5, such that the motives in family A still bear resemblance to family B despite family B's

emerging distinction from family A. There is also a second, more convincing explanation that relies on the text's temporal relationships, and it is to this explanation that I shall now turn.

Figure 4.6. Motivic Membership throughout the A and B sections



The text of the A section (shown in Figure 4.1) describes the present: the rain falling, the state of the poet, and his current desire to relive past memories. The text of the B section, on the other hand, depicts the past: memories of the poet's childhood spent running through the fields, feeling the rain on his feet, and touching the grassy dew with his hand. These convey different specific time periods within the poet's life, and the particular pattern of motivic development reinforces this. In the A section, we see the motives exhibiting high potential membership within both families, suggesting that the state of these motives in the present are colored by memories of the past: the potential of both families coexist in the A motives, just as memories of the past coexist with and

influence experiences in the present. When the B section arrives in m. 45 however, we see family A drop off. This is also consistent with a flashback to the past: high degrees of membership within family B suggest a unified cohesive set of motives that are also not highly related to the present that is represented by family A. In the past, the present of family A has not yet occurred, and therefore cannot have as great an influence on the past as the past does on the present.

If we were to return to the germinal motives of each family, shown in Figure 4.5, we might recontextualize the type of development I suggested between the families. Instead of the germinal contour for family B emerging from family A with the prominent additions of ascents at the beginning, we might flip this development to consider that these prominent ascents might have been smoothed out or forgotten as time spanned from the temporal context of family B to family A, as shown in Figure 4.7. This might again point convincingly to the idea of memory: as time passes, specific details of a memory may fall away or otherwise be remembered differently as we have progressed through time and experience. The memory may have taken on new context and changed, as signified by these significant “missing” contour motions in the germinal motive of family A.

Figure 4.7. The similarities between the germinal motives of the song, with an emphasis on the retroactively discovered relationship of the B motive to family A

45
Wel - che Won - ne, in dem Flie - . - Ben

4
Wal - le, Re - . - gen, wal - le
(+) = (+) = (-) - + + - -

In this depiction of the past, the present represented by family A has not occurred yet, and therefore the membership of the motives of family B within the fuzzy family A are low, indicating that they are outliers in family A: capable of hinting at a potential future, but unable to achieve any value higher. The present, however, possesses reference to both itself and the past, in many cases having a stronger affinity toward family B than family B does to family A (as in the case of the motives that appear in mm. 7–8, 9–11, 27–28, and 29–31). In this sort of retrospective analysis, we can see that these motives are colored by the memory of the past, just as the poet’s present existence is colored heavily by his strong affinity for the memory depicted in the B section. In both cases,

“past” events are strong members within one’s present existence. In the case of the Brahms song, we only discover this as the past is relived through the flashback to the memory: a development that is, in a way, backwards.

Motivic Identity and Development in the Sonata (Sonata No. 1, Op. 78, mvt. III)

These motivic ideas resurface for Brahms five years later in his first violin sonata in G Major, Op. 78. Most notably, the germinal motive-forms of the song appear not in the opening movement as one might guess, but rather in the last movement of the sonata, almost as if Brahms required the two previous movements to build up to their entrance. The movement is a 5-part rondo as shown in Figure 4.8. The refrains in mm. 1–28, 61–83, and 124–164 (including the coda from 140–164) feature material stemming from the two germinal motives of the song, this time in the key of G minor. The work’s two episodes, mm. 29–60 and 84–123, feature what seems to be new and contrasting material, although as we shall see later, the material is not entirely unrelated.

The sonata movement expresses difference between these two motive families in similar ways: both families take as their germ the opening melodies from each family of the song, and develop from there. Although the germinal motives may be highly related, the families built from the contours of the sonata are significantly different, suggesting a more extensive developmental attitude within the contour domain, such that the members of family A and B in the sonata bear lower overall potential membership within their respective averaged families in the song. For example, the germinal member of family A appears in mm. 0–1 of the sonata movement, shown in Figure 4.9a, comes directly from

the germinal motive in mm. 4–6 of the song. In both families, the motive has a high degree of membership: 0.785 in family A of the sonata and 0.776 in family A of the song, as shown in the membership chart in Figure 4.9b. Despite this initially high degree of similarity, the potential membership of the rest of the motives in family A of the sonata within the average family of the song (shown in the fifth column of Figure 4.9b, which derived its values by comparing the individual contours of the sonata against the averaged member of Family A in the song, shown in Figure 4.3b) exhibit a wide continuum, ranging from 0.255 (very dissimilar) to 0.837 (very similar, in fact this member has an identical counterpart in family A of the song). Such a wide range suggests that Brahms takes the motive to a wider variety of variants than were heard in the song.⁵³ Additionally, unlike the song, which only features twenty-three total members and only nine unique members of family A, the family A of the sonata has fifty-five total members and twenty-seven unique contours, suggesting again that the motive is developed much more in this context than it is in the song. This is not unexpected, given the tendency of the instrumental genre to be longer and more developmental, since it is not as constrained by the presence of text or the smaller range of the voice.

⁵³ In fact, of all the twenty-seven distinct contours found in family A of the sonata, only three exact matches are found in the family of the song.

Figure 4.8. Form diagram of the third movement of Brahms's violin sonata No. 1, Op. 78

A				B	A			C	transition	A	Coda	
mm. 1–28				29–60	60–83			84–112	113–123	124–139	140–164	
G minor				D minor	G minor			E-flat Major	→	G minor	G Major	
a	b	a	b		a	b	a			a	B	a
mm. 1–9	10–13	14–22	23–28		60–69	70–73	74–82		124–139	140–158	159–164	
family A	family B	family A	family B						resemblance to family A	family A	family B	family A

Figure 4.9a: Annotated score excerpt for the opening refrain of the sonata. Membership values of each motive compared to family A are shown in blue, while membership values of each motive compared to family B are shown in red.

20 **A** Allegro molto moderato 0.785 (0.533) 0.862 (0.531)

3 0.536 (0.484) 0.770 (0.570)

6 0.770 (0.570) 0.827 (0.597) 0.575 (0.513)

9 **B** 0.881 (0.472) 0.881 (0.472) 0.881 (0.472) 0.881 (0.472)

13 0.783 (0.791) **A** 0.862 (0.531)

The score is in G minor, 3/4 time. It features a piano introduction with a *p dolce* marking. Motive A is marked *Allegro molto moderato*. Motive B is marked *dolce*. The score includes various dynamics such as *p*, *pp*, and *sempre p*.

15 0.862 (0.531) 0.560 (0.586) 21

17 0.720 (0.570) 0.770 (0.570)

20 0.827 (0.597) 0.689 (0.748)

23 **B** 0.881 (0.427) 0.757 (0.284) 0.881 (0.472)

26 0.784 (0.346) 0.62 (0.411) (Episode)

The score continues with measures 15-26. It includes markings for *sempre p*, *pp*, *dolce*, and *poco cresc.*. Measure 26 is labeled as the start of an *(Episode)*.

Figure 4.9b. The members of Family A from the sonata movement

Measures	C-SEG	CAS	Similarity to family A in song	Membership in this family	Similarity to Family B in Sonata
0–1(2x) (germinal member)	<11102432>	<==+--+>	0.776	0.785	0.533
2 (5x)	<323410>	<-+--+>	0.754	0.862	0.531
3–4 (2x)	<201354687>	<-+++++-->	0.255	0.536	0.484
4–5 (8x)	<11211011>	<=+-=-+=>	0.537	0.770	0.570
7 (4x)	<65124301>	<-++--+>	0.647	0.827	0.597
8 (2x)	<43223120>	<-+=+--+>	0.502	0.575	0.513
14 (3x)	<102432>	<-+--+>	0.754	0.862	0.531
16 (2x)	<2013201442>	<-+++++==>	0.183	0.560	0.586
21 (2x)	<3212001>	<-+-=-+>	0.589	0.689	0.748
52–53 (4x)	<111023>	<==++>	0.739	0.794	0.679
54–55	<222110>	<===->	0.725	0.73	0.651
(56)	<20013456>	<-++++++>	0.446	0.586	0.585
57	<1023>	<-++>	0.691	0.93	0.774
58–60	<444332210>	<===-=->	0.570	0.644	0.521
113–115 (2x)	<000001>	<=====+>	0.560	0.544	0.490
115–116 (2x)	<00154321>	<=++++->	0.647	0.591	0.864
123–124	<11203543>	<=++++-->	0.710	0.789	0.615
126–127	<32143215211>	<-+-----=>	0.285	0.414	0.534
127–128 (2x)	<333210>	<==--->	0.837	0.793	0.771
129–130	<5553214210>	<==----->	0.521	0.647	0.605
131	<53214210>	<---+--->	0.543	0.695	0.648
133	<102543>	<-+--+>	0.754	0.862	0.531
134 (2x)	<32103210>	<---+--->	0.543	0.695	0.648
135–136	<5432100>	<-----=>	0.578	0.646	0.773
136–137	<134022>	<++++=>	0.549	0.528	0.766
137	<135402>	<+++++>	0.722	0.78	0.929
160–163	<11101024321>	<==+----->	0.533	0.675	0.519

Figure 4.9c. The averaged member of family A of the sonata

	1	2	3	4	5	6	7	8	9	10
+	0	0.5	0.111	0.136	0	0.979	0.811	0.104	0.051	0.588
=	0.714	0.467	0	0.682	0	0	0	0.270	0.051	0.176
-	0.286	0.033	0.889	0.182	1.0	0.021	0.189	0.625	0.897	0.235

Since family A in the sonata has more developmental tendencies, the family features a more graded internal continuum of membership (as measured against the probabilities of the averaged member of family A shown in Figure 4.9c), with a more substantial minority of motives (thirteen out of the fifty-five) falling below 0.6 within the sonata's family A than that of the song. In these thirteen members falling below 0.6, we see aspects of developing variation at work. For example, mm. 3–4 (shown in Figure 4.10) feature a motive that looks fairly different from the two motive-forms immediately preceding it. Here, Brahms has chosen to elaborate upon a small part of the initial motive that is a fairly dominant trait within the family: the $\langle -++ \rangle$ c-subseg that features prominently in the previous two motives. Brahms has altered the rhythm, the interval size, and has repeated the motion twice in order to create the new motive form. In essence, Brahms took a family trait that was fairly dominant, and repeated it, and it is this repetition that is unique, making the membership value fall. As a result, the contour of the new motive form bears a looser relationship to the rest of the family, a 0.536 membership, which is still a strong enough relationship to be connected (as suggested by the use of the dominant family trait), yet distinct enough to show development (as shown by the repetition). Because of these lower-valued members, we see a family that is wider in range as a result of Brahms's more developmental tendencies. This happens in the song as well, and it is here that we might see Brahms's desire to move away from the

development that occurs in the song. This gesture in mm. 3–4 features a good degree of upward motion, suggesting further potential action in the music, while the analogous gesture in the song, seen in mm. 9–11 of the song features a mostly downward descent ending the phrase.

Figure 4.10. The development of mm. 3–4 from the germinal motive

The image shows a musical score for a piece in 3/4 time, marked "Allegro molto moderato" and "p dolce". The score consists of two systems. The first system shows the vocal line and the piano accompaniment. The vocal line has two red boxes highlighting specific motifs: one in measures 3-4 and another in measures 9-11. The piano accompaniment has a red box highlighting a motif in measures 3-4. Red lines connect the vocal motifs to the piano accompaniment motif, illustrating the development of the germinal motive. The second system shows the continuation of the piano accompaniment, with a red box highlighting a motif in measures 3-4.

The motives' relationships here highlight the mastery with which Brahms approaches thematic development: the family is coherent enough to hold together, yet loose enough to promote development, exemplifying Schoenberg's statement that the motives shall not be developed to the point that they lose comprehensibility. These outliers within family A do not fall below 0.5, keeping them related enough to retain motivic identity within family A.

Family B begins with a truncated version of the motive found in the song: the sonata takes the first $\langle + = ++ \rangle$ motion of the song motive as its germ (shown in Figures

4.11a and 4.11b). This truncated version of the song motive has an extremely high degree of membership within the family of the song, 0.958 (as shown in the membership chart in Figure 4.11c), since it is such a prominent feature in so many of the song's B-section motive-forms. Its frequent repetition throughout the sonata, therefore, connects it very closely to the song. In fact, the majority of family B in the sonata displays the same kind of high degree of coherence that the song's family B showed, with only five members of the family falling below 0.6.

Figure 4.11a. Germinal family B motive of the sonata, truncated from the family B motive of the song

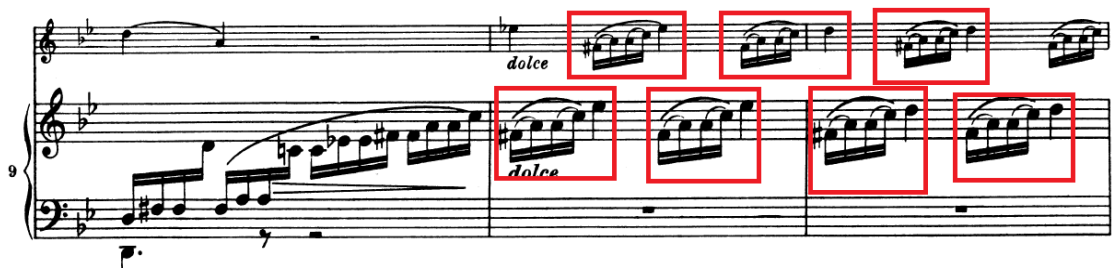


Figure 4.11b. Germinal family B motive of the song



Figure 4.11c. The members of Family B of the sonata

Measure	C-SEG	CAS	Similarity to A in Sonata	Membership in this family	Similarity to B in song
10 (x4) (germinal)	<01123>	<+===>	0.472	0.881	0.958
10 (x7)	<01123>	<+===>	0.472	0.881	0.958
12–13 (2)	<022451336>	<+=====>	0.443	0.783	0.791
23–24	<02245432120>	<+====-+>	0.284	0.757	0.783
25–26	<01134326561>	<+====-+->	0.346	0.784	0.883
27–28	<25406173>	<+---+->	0.411	0.62	0.964
140	<01123>	<+===>	0.472	0.881	0.958
140–141	<12246543201>	<+====-+>	0.403	0.81	0.617
144	<01123>	<+===>	0.472	0.881	0.683
145–146	<011598765423>	<+====-+>	0.403	0.767	0.561
146–47	<43201>	<---+>	0.574	0.722	0.938
147–148	<765430112>	<-----+>	0.528	0.474	0.599
149–150	<32011>	<---=>	0.260	0.444	0.813
152–53	<235544332011>	<+==--=-+>	0.411	0.41	0.303
156–157	<0122344>	<+==++>	0.558	0.501	0.764
157–159	<287655443301>	<+----=-=-+>	0.483	0.519	0.330

Figure 4.11d. The averaged member of family B of the sonata

	1	2	3	4	5	6	7	8	9	10	11
+	1.0	0.087	0.826	0.870	0	0	0.545	0	0.25	0.222	0.778
=	0	0.826	0.087	0	0.25	0	0.182	0	0.416	0	0.222
-	0	0.087	0.087	0.130	0.75	1.0	0.273	1.0	0.333	0.778	0

Furthermore, the relationship between family B in the song and sonata is substantial: although there are no exactly identical members as there are in Family A, most of the motives within the sonata’s family B have a high degree of potential membership within the song’s averaged family B. The seventh column in Figure 4.11c shows that with the exception of two outliers (which also belong to the set of outliers falling below 0.6 in the sonata’s family B), the members all range from 0.6 to 0.958. Yet even within these values, one can see a degree of developmental activity that takes this family further from its relationship to the song’s close-knit family B. Family B’s first

occurrence in the opening refrain shows a high degree of membership and coherence, featuring membership values ranging between 0.958 and 0.783 within the song's family B. The second reprise, beginning in m. 61 features similarly high values, as it features an exact repetition of the opening reprise's B material. The third refrain's B motives, however, feature much lower resemblance to the B material of the song, indicated by a wider range of potential membership values, ranging from 0.958 to 0.303, and include all five of the outliers within the sonata's own family B.

These five outliers deserve a deeper examination. Each falls within the final refrain of the movement, as shown in Figure 4.12. Three of the motives each in turn recall a specific episode from the sonata, bridging the divide between motive families within the work, while at the same time contributing to the variety found within family B. The remaining two motive forms, found in mm. 156–157 and mm. 157–159 respectively work to draw the movement to a close. The first of these motives in mm. 156–157, shown in Figure 4.13, features striking resemblance to the initial $\langle +=+= \rangle$ contour (or its variant contour of $\langle +=++ \rangle$). However, Brahms has drawn out the motive in terms of rhythm by filling in the third jump with a stepwise ascent, elongating the contour of the motive as a result. What we have then is a $\langle ++=++= \rangle$ motion, which still resembles the family, as indicated by its 0.501 membership value, and this relationship can easily be seen at a deeper level.⁵⁴ However, Brahms gives the motive more linearity by adding the ascents in, accounting for its difference from the rest of the motives in family B.

⁵⁴ Using Morris's (1993) contour reduction algorithm, it would be the N=1 level to be specific, which eliminates all local passing c-pitches.

Figure 4.12: Annotated score excerpt for the closing refrain (including the coda). Membership values for motives compared to family A are shown in blue, while membership values for motives compared to family B are shown in red

121 *poco calando*
dim. *poco calando*
dim. *p*
 0.789 (0.615)

124 *in tempo*
p
in tempo
 0.862 (0.531) 0.414 (0.534)

127 *poco cresc.*
poco cresc.
 0.793 (0.771) 0.793 (0.771) 0.647 (0.528)

130 *cresc.*
cresc.
 0.695 (0.648) 0.862 (0.531)

133 0.862 (0.531) 0.695 (0.648) 0.646 (0.773) 28

136 0.528 (0.766) 0.78 (0.929) *poco rit.* 0.695 (0.648)
p *dim.* *poco rit.*

139 *Più moderato* 0.881 (0.472) 0.81 (0.403)
dolcis. *dolcissimo* *p*

142 0.881 (0.472) 0.881 (0.472) 0.767 (0.403)
pp *p cresc.* *p cresc.*

145 0.722 (0.574) 0.474 (0.528)
p

Figure 4.12 (cont.)

30 **0.444 (0.260)**
pp

149 *dim.* *pp ben legato*

0.41 (0.411)
espress. *pp*

152 *pp*

espre s. **0.501 (0.558)** **0.519 poco rit. (0.483)**
poco rit. - *pp*

155

p

158

0.675 (0.519)
dim. *p*

161 *f* *dim.* *p*

Detailed description: This figure shows a page of musical notation for a piano and violin. The piano part is in the lower staves, and the violin part is in the upper staves. The key signature has one sharp (F#). The score is divided into systems. The first system (measures 30-49) features a violin melody with a boxed annotation '0.444 (0.260)' and a piano accompaniment marked 'pp ben legato'. The second system (measures 50-59) has a violin melody with a boxed annotation '0.41 (0.411)' and a piano accompaniment marked 'pp'. The third system (measures 60-69) includes a violin melody with boxed annotations '0.501 (0.558)' and '0.519 poco rit. (0.483)', and a piano accompaniment marked 'pp' with 'poco rit.' markings. The fourth system (measures 70-79) shows a violin melody with a boxed annotation '0.675 (0.519)' and a piano accompaniment marked 'p'. The fifth system (measures 80-89) features a violin melody with a boxed annotation '0.675 (0.519)' and a piano accompaniment marked 'f' and 'dim.'. The sixth system (measures 90-99) shows a violin melody with a boxed annotation '0.675 (0.519)' and a piano accompaniment marked 'p'. The score includes various performance markings such as 'dim.', 'espress.', 'poco rit.', and 'pp'.

Figure 4.13. The motive in mm. 156–157, showing its resemblance to the germinal motive of family B

The image shows a musical score for two staves. The upper staff is in treble clef with a key signature of one sharp (F#) and a common time signature. The lower staff is in bass clef with the same key signature and time signature. The music begins at measure 155. A specific motive in measures 156 and 157 is enclosed in a black rectangular box. Above this box, the text "espre s." is written. Inside the box, the number "0.501" is written in red above the first measure, and "(0.558)" is written in blue above the second measure. Above the box, there are two rows of red symbols: the top row contains "(+ = + =)" and the bottom row contains "+ = + =".

Relationship between Family A and Family B

The “development” that occurred throughout the song dealt with the shift in relationship between the two families, as each motive possessed to a certain degree a membership within each family. Therefore, an examination of the relationship between families A and B throughout the sonata may reveal significant relationships between the song and sonata. Consider Figure 4.14a, which shows a graph of membership over time similar to the graph I introduced for the song (Figure 4.6), this time featuring the motives of the opening refrain of the sonata movement’s rondo form. In this expositional refrain, we see similar tendencies to that of the opening of the song: the A sections in mm. 0–9 and mm. 14–22 (bearing motives belonging to the A family) feature a closer relationship between the two families, as shown in the graph of Figure 4.14b. Overall, the mathematical difference between the two membership values, as exhibited by the columns in the figure, are smaller for the A sections than the B sections, from columns 1–7 and columns 13–19. Similarly, the B sections of the song in mm. 10–14 and mm. 23–29

display that same strong divide between the A and B families that exists in the song, with the membership values of family A dropping off consistently while the membership values of family B are quite high, as indicated in the large differences between values exhibited in Figure 4.14b.

Figure 4.14a. Membership over time for the motives of the first reprise

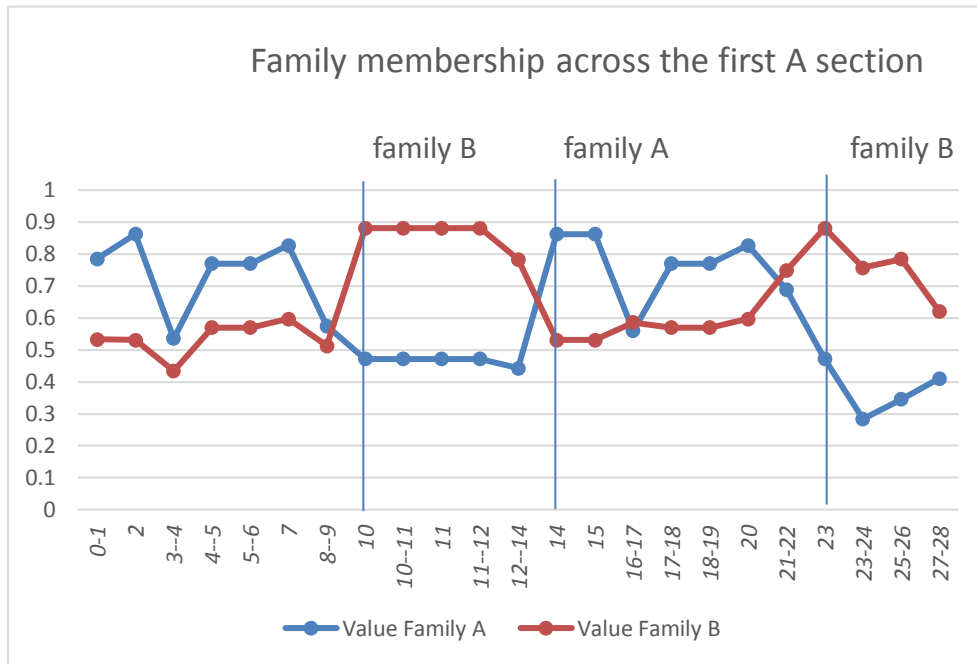
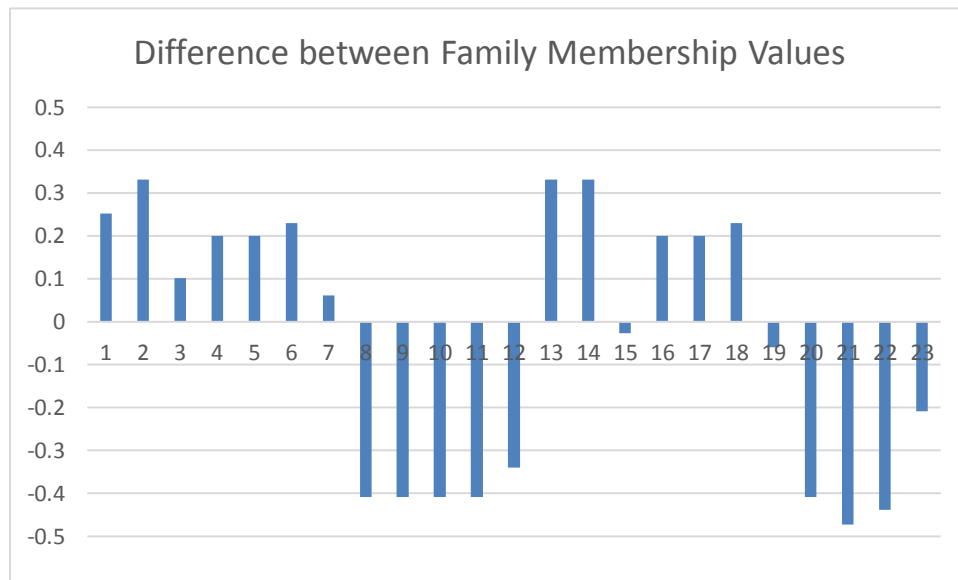


Figure 4.14b. Difference between family A and B membership values (positive bars feature higher values for family A, negative bars feature higher values for family B)

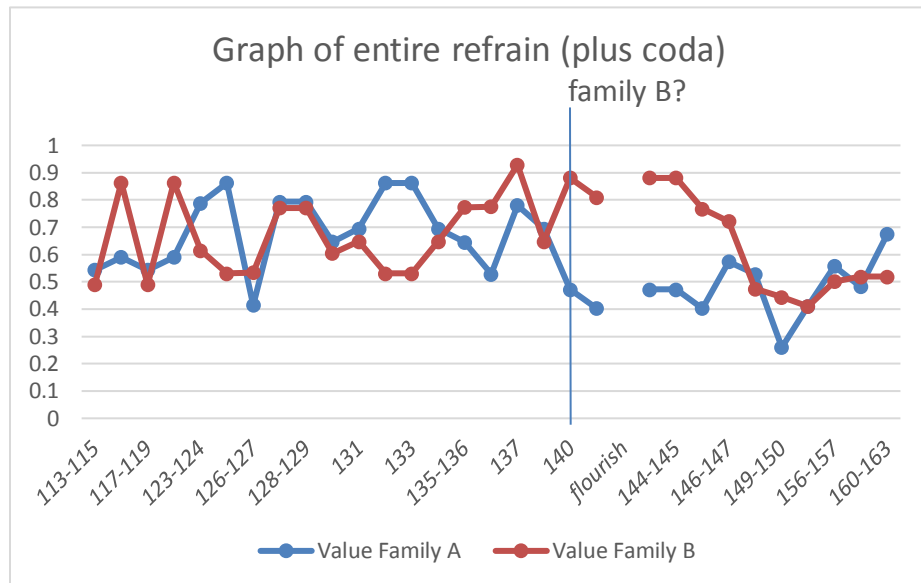


The similarity to the song that these patterns exhibit suggest that Brahms's conceptions of the song and sonata are not so distinctly separated, and therefore one may even suggest that a reminiscence of the narrative associated with these patterns may exist within the sonata as well. Despite this similarity, Brahms has made changes to the families that add a more unique identity within each family in this opening refrain. The two families alternate more rapidly than they did in the song, which divided the respective families among first and second section more or less equally. Unlike the two families in the opening of the song, the two potential families' membership values here only cross twice outside of the transition between A and B sections: the first time in mm. 16–17 (the middle of the second A section), and the second time in mm. 21–22 (at the end of the second A section). This stands in stark contrast to the opening of the song, where values for family B crossed higher than those for family A seven out of thirteen times. This gives family A greater independence from the context of family B in the

sonata, contributing to a higher degree of motivic identity and coherence throughout this opening section. This could be due in part to the truncation of the opening motive of family B, and its subsequent development, emerging as more different from family A in the sonata than family B did in the song. As the opening refrain of an instrumental rondo form, it is not unexpected to see this clearer differentiation between motive families: since it is serving an expository function, it introduces the themes and therefore it is desirable for reasons of comprehensibility for the two families to be more separate. Because the motive is no longer tied to the textual meaning as it was in the song, Brahms can show greater separation here at the beginning.

However, the motive families do not remain as separate as this throughout the entire sonata. The closing refrain and coda show a significantly different relationship between family A and family B than is seen in the outset of the movement. Figure 4.15 shows the graph of the motivic membership within the entire final refrain (including the coda material that features motive family B in the major mode). Here, we see a series of motives that are so intertwined in terms of membership values that it is perhaps difficult to determine from the graph alone where the section markers might occur, whereas the earlier graphs featured prominent shifts in motivic membership among families that serve as clear markers of formal separation. We see that at m. 140, where the family A value drops off for a few measures, yet the values for family B eventually drop to join the levels of family A. In this section, we see how the distinctions that once divided the families more clearly are beginning to fall away and the families are becoming more difficult to distinguish using contour alone.

Figure 4.15. Family membership over time for the final refrain (including the coda)



For example, Figure 4.16a shows the motive in mm. 127–129: it features a 0.793 membership within family A and a 0.771 membership within family B (as shown in columns six and seven of Figure 4.9b). It features the prominent opening $\langle ==- \rangle$ motion that is characteristic of family A, but also shares similarities with the way family B develops. Although it does not resemble the initial germ of family B, it does share a similar descending motion in the second half of the motive with the motive forms from mm. 23–24 and mm. 25–26 (which are also featured in mm. 140–141, and 145–146). We see these developments coming to affect the averaged families of each motive family: Figure 4.16b shows that the characteristic $\langle ==- \rangle$ motion receives relatively high values in the opening three positions of the averaged member of family A, as well as two other prominent descents throughout the averaged family that contribute to the high degree of membership of this motive within family A. Meanwhile, Figure 4.16c shows that the

motive's second half invokes three prominent descents in Family B's averaged member: positions 6, 8, and 10. This, combined with the prominent plateau in the second position contributes to this motive's high degree of relationship with motives of family B, and subsequently the connection between the two motives in general. Because it shares these features from each family, the motivic coherence of each family is subsequently called into question. Overall motivic coherence in these cases must therefore rely on high degrees of membership within the domains of other elements such as rhythm, because contour is no longer serving a unifying role within the motive family, and is now acting as a developmental element.

Figure 4.16a. Relationship between the motives in mm. 127–129 to an earlier development in family B

Figure 4.16b. The contour of mm. 127–129 measured against the averaged member of family A of the sonata

	1	2	3	4	5	6	7	8	9	10
+	0	0.5	0.111	0.136	0	0.979	0.811	0.104	0.051	0.588
=	0.714	0.467	0	0.682	0	0	0	0.270	0.051	0.176
-	0.286	0.033	0.889	0.182	1.0	0.021	0.189	0.625	0.897	0.235

Figure 4.16c. The contour of mm. 127–129 measured against the averaged member of family B of the sonata

	1	2	3	4	5	6	7	8	9	10	11
+	1.0	0.087	0.826	0.870	0	0	0.545	0	0.25	0.222	0.778
=	0	0.826	0.087	0	0.25	0	0.182	0	0.416	0	0.222
-	0	0.087	0.087	0.130	0.75	1.0	0.273	1.0	0.333	0.778	0

Inter-Movement Motivic Connections and Narrative Development

It is here in this final refrain and coda where we see the greatest difference from the structure of the song: the motivic connection between the sonata and the song is clear, enough so that Brahms’s audience would recognize the reference to the song and its narrative when listening to the sonata. However, with Brahms’s motivic development comes a shift in the narrative context originally held in the song, adding new meaning to the story. This shift is not entirely lost on Brahms’s audience: Brahms’s friend Theodor Billroth, for example, writes in a letter to Brahms that

It is a strange thing to be made to hear well-known song motifs in sonata form. In your composition, the song, like the ‘Homeland’ by Klaus Groth, is to me one of the most beautiful of all poetic creations; I can forget both words and tones over the depth and pathos of its feelings, the feeling is transfigured into an almost religious enthusiasm...I find it absolutely impossible to imagine what sort of an impression this sonata will make on people who do not have the song completely and fully within themselves like a creation of their own. To me the entire sonata is like an echo of the song, like a fantasy about it (Billroth, quoted in Floros 2010, 133).

Billroth alludes to the fact that the relationship moves beyond mere motivic borrowing: the sonata movement stands on its own, unique in its differing developmental attitudes—such that it does indeed become “strange” to hear the motivic connections in this new way. As such, the motivic variety throughout this last refrain highlights Brahms’s

narrative development that takes it beyond the story of the song. In order to understand fully this new musical and narrative perspective, we must turn to the first two movements of the sonata, and examine motivic interaction across the movements.

The First Movement

The first movement opens with the familiar dotted plateau opening $\langle = - \dots \rangle$ motive that is so characteristic of Family A of the song: such an opening by itself carries a membership of 0.82 within the song's Family A, and can indeed be seen in many prominent motives of both the song's family A and the third movement's Family A. The motive opening the first movement, bearing a contour of $\langle = - - - \rangle$, is nearly identical to an opening motive found in the piano introduction to the song: Figure 4.17 shows the motive in the piano, and in the violin at the opening of the sonata. Both have high degrees of membership within Family A of the song, 0.837 and 0.768 respectively.

Figure 4.17a. The $\langle = - - \rangle$ motive appearing in the piano introduction of the song

The image shows a musical score for the piano introduction of a song. It consists of two staves: the top staff is for the vocal line (Singstimme) and the bottom staff is for the piano accompaniment (Pianoforte). The key signature is G major (one sharp) and the time signature is 3/4. The tempo is marked 'In mäßiger, ruhiger Bewegung'. The piano part begins with a dotted quarter note followed by a half note, with a red line connecting the notes. Above the piano part, the contour notation $\langle = - - - \rangle$ is shown.

Figure 4.17b. The opening (<=----->) motive of the first movement of the violin sonata

As Brahms builds upon this initial motive in the first movement, one can see clear resemblances to Family A throughout most of the exposition. A sampling of key motives in the sonata's exposition, shown in Figure 4.18a, shows these degrees of membership. Here we see an interesting relationship to family A across the sections of the exposition: each section (the primary theme, the secondary theme, and the closing theme) begins with a motive that has a strong membership within family A, as shown in the graph of Figure 4.18b. As each section progresses, Brahms develops the motives away from that strength, each in a different way. The primary theme section, for example, begins with the germinal motive (a 0.768 membership), and winds up at a motive with membership of 0.427. The second theme section follows very similarly, beginning with a motive featuring a 0.774 membership, and gradually lessening until it winds up with motives featuring 0.524 and 0.570 memberships at the end.

Figure 4.18a. Chart of motivic membership of motives in the first movement's exposition

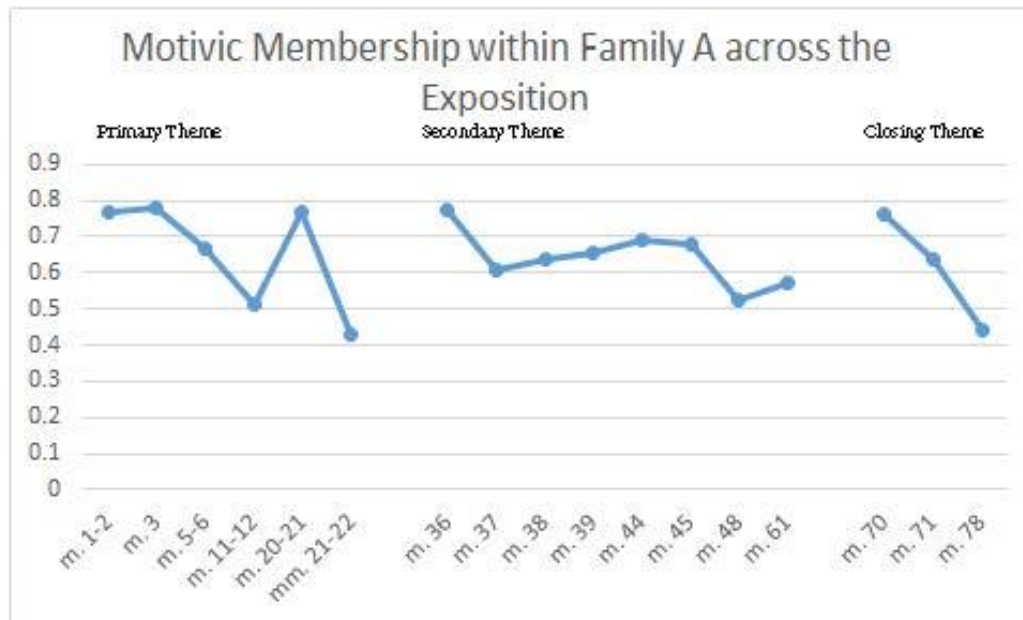
Measures	C-SEG	CAS	Membership within Song A family
mm. 1–2	⟨4443210⟩	⟨==-----⟩	0.768
mm. 3	⟨00011112⟩	⟨==+⟩	0.778
m.5–6 ⁵⁵	⟨412032⟩	⟨-+--+⟩	0.665
mm. 11–12	⟨04765321⟩	⟨++++-----⟩	0.510
mm. 20–21	⟨4443210⟩	⟨==-----⟩	0.768
mm. 21–22 ⁵⁶	⟨10123465⟩	⟨++++++-⟩	0.427
mm. 36	⟨112301⟩	⟨=+++⟩	0.774
m. 37	⟨02331⟩	⟨++=-⟩	0.605
m. 38	⟨012431⟩	⟨++++-⟩	0.637
m. 39	⟨21330⟩	⟨-+=-⟩	0.656
m. 44	⟨2134320⟩	⟨-++-⟩	0.691
m. 45	⟨1012⟩	⟨-++⟩	0.676
m. 48	⟨123440⟩	⟨++++=-⟩	0.524
m. 61 ⁵⁷	⟨23312120⟩	⟨+=-+--+⟩	0.570
m. 70	⟨323101⟩	⟨-+++⟩	0.762
m. 71	⟨343210⟩	⟨+-----⟩	0.640
m. 78	⟨0123456⟩	⟨+++++++⟩	0.443

⁵⁵ This motive is a bit unwieldy because it has such a high quantity of notes: in the original motive there are twelve CAS positions, ten of which make up the global ⟨021⟩ shape that ends the motive in m. 6. An N=1 level reduction (Morris 1993, Schultz 2008) here provided a better sense of this motive's true shape.

⁵⁶ The opening 1 is elided with the 0 of previous c-seg in the chart.

⁵⁷ A three-note connector motive appears in mm. 63-64 that features a CAS of ⟨+-⟩. Since it is so small, it has a high degree of membership, yet this melodic material serves only to connect the end of the second theme to the beginning of the third in a more fluid way, and is not a true motivic member of the second theme's section.

Figure 4.18b. Motivic Membership across the exposition of the first movement



Even the development features prominent contour relationships to Family A. In fact, throughout this development section, Brahms follows and develops the motives heard throughout the opening Primary Theme section. He even begins the development (mm. 82–90) with an exact repetition of the first nine bars of the opening of the exposition, except that the primary motives are now heard in the piano, as shown in Figure 4.19a. Brahms does not introduce developmental material until after that, at the end of m. 90. From there, Brahms continues his development of family A-based motives, as shown in the chart of Figure 4.18b. Measures 90–99 elaborate upon the descending figures heard in mm. 89–90: this development features a series of downward scalar movements in much the same vein as those heard at the end of the exact repetition. Following this, mm. 99–107 cycles through versions of the two germinal motives heard at the opening of the sonata: the $\langle = - - - - \rangle$ and $\langle = = + \rangle$ motives (each featuring 0.768 and 0.778 memberships within family A respectively), as shown in Figure 4.19b. From there,

developmental sections include variations upon the motives in mm. 11–12, m. 70, and m. 78. In short, the development section is built around the opening A section, which as we saw in the exposition, is conceptually built around family A. Brahms is not using contour as a primary developmental agent in this case, and contour is instead contributing to the sense of unity and motivic familiarity throughout this development section. Given that most of the material within the sonata either belongs strongly to Family A or is developed from a motive strongly belonging to family A, it is therefore safe to say that the first movement is conceptually built on the idea of Family A.

Figure 4.19a. The opening bars of the development section, showing the motives of the exposition now in the piano (mm. 73–90)

The image shows a musical score for the opening bars of the development section, measures 73 through 90. The score is written for piano and consists of three systems of staves. The first system (measures 73-78) features a melody in the right hand and a bass line in the left hand. The tempo is marked *un poco calando* and *dim.* (diminuendo). The second system (measures 79-83) continues the melody and bass line, with the tempo marked *in tempo* and *pizz. dolce* (pizzicato dolce). The third system (measures 84-90) features a melody in the right hand and a bass line in the left hand, with the tempo marked *sempre dolce* (sempre dolce) and *arco* (arco).

Figure 4.19b. mm. 99–108 of the development, featuring prominent motives from the exposition

The image displays a musical score for measures 99 through 108, divided into three systems. Each system consists of a vocal line (top staff) and a piano accompaniment (bottom staff). The key signature is one flat (B-flat major/C minor), and the time signature is 4/4. The score includes various musical notations such as notes, rests, slurs, and dynamic markings. The first system (mm. 99-103) begins with a piano (*p*) dynamic. The second system (mm. 104-107) features a *cresc.* marking and a *poco a poco* instruction, leading to a *più sostenuto* section. The third system (mm. 108) concludes with a *sf* (sforzando) dynamic. The piano accompaniment includes complex chordal textures and rhythmic patterns, while the vocal line features melodic phrases with some triplets.

Second Movement: Introducing a Biographical Narrative

If the first movement is conceptualized around Family A, then one might expect Family B to appear in the second movement—if Brahms were indeed keeping to the strict motivic connections with the song. However, the second movement instead presents a family that is aurally distinct from both families of the song. Its germinal motive, shown

in Figure 4.20 features a contour of $\langle +---+= \rangle$, and has a 0.456 membership within the song's family A and a 0.741 membership within the song's family B.

Figure 4.20. The germinal motive of the second movement

Adagio

$\langle + - - + = \rangle$

poco f espress.

Membership family A=0.456

Membership family B=0.741

This high family B value for the germinal motive of this movement suggests that, while audibly differentiated in other motivic parameters, it shares similarities of shape with the family one might have expected following the first movement. As such, this motive family may share a similar narrative connotation: an idea of “past” or nostalgia that was associated with family B of the song. Nostalgia is indeed present within the context of the second movement, but it is a nostalgia more personal to Brahms than that of the protagonist in the song.

Brahms initially wrote the second movement as a stand-alone work in order to comfort Clara Schumann after hearing about her son Felix's diagnosis with terminal tuberculosis (Sandberger 2008, iv; Floros 2010, 129). The meaning present within this compositional act associates this movement not only with the idea of nostalgia, but with a concrete character. Brahms is, in essence, placing himself in the character of the narrator, and his memories of Felix's youth at the center of the nostalgic remembrance.

Brahms was godfather to Felix, often described as the youngest and most talented of Robert and Clara's children. Brahms was very close to his godson: he once said "I don't know how I should contain myself with happiness if I had a son like Felix," (quoted in Swafford 1997, 220), and he set several of Felix's poems to music. The talent and potential young Felix showed was cut short, however, as he was diagnosed with tuberculosis in September of 1873, at the age of nineteen. Constantin Floros makes associations with the time of his initial diagnosis and the Rain Songs of Op. 59, which had been sent to Clara on her birthday, also in September of 1873. On September 17th of 1873, Clara responded to Brahms's gift:

Your dear letter was the first written greeting I found awaiting me on the morning of my birthday. And it pleased me more than I can say. The *Regenlieder* followed yesterday—thank you for everything....Four days before my birthday [Felix] got pleurisy, which, as the doctor said, had been coming on for some time... You can imagine what an anxiety this will be for me. My poor heart seems always to be subjected to fresh trials. How glad I am now that we decided to have Felix with us (Clara Schumann 1971, 271).

Such associations within the timeline do not suggest a compositional narrative for the songs of Op. 59, but rather might posit an associational connection between the feelings both Johannes and Clara had regarding Felix's illness as it progressed through the years of 1873–1879, and the nostalgia for youth presented through the motivic connections in the "Regenlied."

It is, however, not only the circumstances of Felix's illness that contribute to this association, but the circumstances surrounding his birth and early childhood that make this association all the more poignant. Felix was born into a family already fraught with troubles. As the last of the Schumann children, he was born several months after his father was committed to the Sanitorium in Endenich in February of 1854 following his

suicide attempt. During Clara's pregnancy with Felix, she was under a great deal of strain regarding her husband's condition, having to manage the household affairs in her husband's stead. In fact, as time wore on, Clara became quite morose, despairing over her husband's slow deterioration (Worthen 2007, 346–49).

After Robert was taken to Endenich, Brahms went to Düsseldorf to comfort Clara at considerable personal expense, and he remained by her side for the next few years. During this time, he became a routine figure in the lives of the Schumann children, caring for them, and comforting Clara in her grief regarding her husband's condition (Swafford 1997, 109–146). Brahms was present for Felix's birth in 1854, and when Clara began touring again in 1855, Brahms often remained behind in Düsseldorf, and even moved into a room on the first floor of the Schumann household. He worked as a private instructor, and kept Clara apprised on household activities, including those of her children (Swafford 1997, 138). In one such letter to Clara, he wrote: "Eugenie has got a bad cold, she has no appetite, her head is very hot and she is constantly falling asleep. The boys are very well, including Felix. No headway is being made with the alphabet yet in spite of any amount of loaf sugar..." (Brahms 1971, 24). In another, he wrote to her of a storm that wreaked havoc upon their home:

There was a clap of thunder as if the world were splitting in half...they [the children] were crawling about on their knees and screaming, believing the Day of Judgment had come. The two boys were also crawling about in the room. I then took them on my lap and let them fall asleep. Felix was fast asleep and Eugenie was quiet. (1971, 49)

In still another, Brahms wrote of the children: "Ferdinand is too lazy, Ludwig is too self-willed, and Felix is even more so. Genchen [Eugenie] is for the moment just a little bit too passionate. But they are all very good and charming. Yesterday Ferdinand received a

number of smacks because he would not read” (1971, 69). Throughout all of these passages, we see Brahms as an important figure in the day-to-day lives of the younger Schumann children. Brahms therefore had an important role in the initial development of Felix’s young life, and as a result, had powerful memories surrounding Felix’s youth.

Given the role Brahms played in Felix’s developmental years, it is conceivable that the narrative themes of nostalgia presented in the “Regenlied” might resonate with him as he came to terms with Felix’s illness in 1873, and again later as Felix’s condition was pronounced terminal in 1878. Indeed, these feelings toward Felix can especially be seen through a letter, written to Clara on the back of an elaborately written-out leaf containing the first twenty-four measures of the violin sonata’s second movement. He writes:

Dear Clara, if you play what is written overleaf quite slowly, it may tell you more clearly than any other utterance of mine how cordially I am thinking of you and Felix—even of his violin, though it is probably at rest. I thank you from my heart for your letter; while I did not, and do not, like to ask for it, I am always very anxious to hear about Felix... (quoted in Sandberger 2008, iv)

It is a rare occurrence in Brahms’s compositional output to have such a direct tie between a composition and a personal element within his life. Floros writes “This important letter is of exceptional value inasmuch as it is one of the very few documents in which Brahms himself speaks about the personal and semantic background of his music” (Floros 2010, 129). Here in this instance, we see Brahms directly tying the opening twenty-four measures of the second movement (shown in Figure 4.21) to his feelings for Clara and Felix during this difficult time in their lives. The violin—a reference to the primary instrumentation of the sonata—is also mentioned specifically in the letter: a nostalgic longing for his godson, who studied the violin. In making these associations in

Figure 4.21. Score excerpt of the second movement (mm. 1–24)

The musical score excerpt consists of six systems of staves. The first system is marked *Adagio* and includes the instruction *poco f. espress.* Two red boxes highlight specific chordal textures in the piano part. The second system includes a measure number '5' and another red box. The third system is marked with *cresc.* in both staves and includes a measure number '10'. The fourth system is marked with a measure number '14'. The fifth system is marked with *cresc.* and includes a measure number '19'. The sixth system is marked *più andante* and includes the instruction *p m. v.* and a measure number '24'. The score uses various musical notations such as slurs, accents, and dynamic markings.

the second movement—where one might structurally expect the nostalgia themes to appear—Brahms links the narrative of nostalgia to the biographical narrative of his feelings regarding Felix’s illness.

Musically, the motives of the opening twenty-four bars of this second movement (the family for which is shown in Figure 4.22a and 4.22b), which I will term the “Felix family,” warrant closer consideration as they share several familial tendencies with Family B.⁵⁸ Consider the motives highlighted in mm. 0–2, 3–5, and 7–9 of Figure 4.22c. These motives seem unique from the motives of the “Regenlied,” yet the motives here bear at least some resemblance to these earlier motives: the chart in Figure 4.22c shows the membership of these three motives within their own family in the second movement, as well as family A and family B of the song. Here, we see low values corresponding with family A of the song—ranging from 0.355 to 0.531. When we look at their correspondence with family B, we see higher values, ranging from 0.741 to 0.875. Looking musically at these connections, we see that there is some resemblance of these main motives in the Felix family to those of family B in the song, namely the prominent descending figures in the middle of both motive families, as shown in Figure 4.23a. Furthermore, one can see these resemblances in the average members of the families themselves. Similar positions within both families share prominently high/low values, as

⁵⁸ It should be noted that this movement is in ternary construction, and also features a contrasting motive family in the B section of the movement, which I call the “Funeral family” after the funerary quality that Floros associates with this passage (Floros 2010, 127). This “Funeral family” serves to contrast with the Felix family: it is not significantly related to earlier material from a contour perspective and will not be discussed further. It is worth noting, however, that one could observe some rhythmic similarities between the Funeral family and Family A. While analysis of these rhythmic similarities is beyond the scope of this chapter, it would be a fruitful avenue for further study.

highlighted in Figure 4.23b and c: both begin with a strong ascent, feature a prominent ⟨-- +⟩ motion in the middle, and exhibit descents toward the end. The degrees of membership exhibited by these prominent motives in the Felix family, combined with the similarities in the family traits themselves, associates these motives more with family B than with family A, subtly bringing elements of family B into the opening of the second movement.

Figure 4.22a. Motives of the Felix family, with membership values within the family

Measures	C-SEG	CAS	Membership in Felix Family
0–2 (6x)	⟨132011⟩	⟨+--+≡⟩	0.843
2–3 (2x)	⟨012⟩	⟨++⟩	0.875
3–5 (2x)	⟨01243122⟩	⟨++++--+≡⟩	0.770
5–6 (2x)	⟨210⟩	⟨--⟩	0.928
6–7 (2x)	⟨3210⟩	⟨---⟩	0.729
7–9 (2x)	⟨243120⟩	⟨+--+⟩	0.799
9–11 (2x)	⟨1210⟩	⟨+--⟩	0.925
11–13 (2x)	⟨4564323401⟩	⟨+-----++⟩	0.808
13–14 (2x)	⟨1230⟩	⟨++-⟩	0.869
14–15 (2x)	⟨1234⟩	⟨+++⟩	0.631
15–17 (2x)	⟨544322100⟩	⟨-=====⟩	0.527
19–20 (2x)	⟨031243⟩	⟨++++-⟩	0.866
22–24 (2x)	⟨43120⟩	⟨--+⟩	0.770

Figure 4.22b. Membership probabilities for the Felix family

	1	2	3	4	5	6	7	8	9
+	0.917	0.833	0.143	0	0	0.889	0.667	0	0.111
=	0	0.167	0	0	0	0.111	0	0	0.556
-	0.083	0	0.857	1.0	1.0	0	0.333	1.0	0.333

Figure 4.22c. Comparison of motives in mm. 0–2, 3–5, and 7–9 with family A and family B from the song

Measures	C-SEG	CAS	Membership in Felix Family	Similarity to Song A	Similarity to Song B

0-2 (6x)	<132011>	<+---+=>	0.843	0.456	0.741
3-5 (2x)	<01243122>	<++++--+=>	0.770	0.355	0.797
7-9 (2x)	<243120>	<+---+->	0.799	0.531	0.875

Figure 4.23a. A comparison of germinal motives from the Felix family in the second movement and Family B of the song

The image shows two musical excerpts. The top excerpt, labeled 'Germinal Felix motive', is from an 'Adagio' piece in 2/4 time, marked 'poco f' espress.'. It features a sequence of notes: G4, A4, B4, C5, B4, A4, G4. Above this sequence is a binary code: <+ - - + =>. The bottom excerpt, labeled 'Germinal Song B motive', is from a song in 2/4 time with lyrics 'Wel - che Won - ne, in dem Flie - .ßend dann zu_'. It features a sequence of notes: G4, A4, B4, C5, B4, A4, G4. Above this sequence is a binary code: <+ = + = - - + +>. Red boxes highlight the corresponding note sequences in both excerpts.

Figure 4.23b. The probabilities of the Felix family, highlighting common traits with family B of the song

	1	2	3	4	5	6	7	8	9
+	0.917	0.833	0.143	0	0	0.889	0.667	0	0.111
=	0	0.167	0	0	0	0.111	0	0	0.556
-	0.083	0	0.857	1.0	1.0	0	0.333	1.0	0.333

Figure 4.23c. The probabilities of the song's Family B, highlighting common traits with the Felix family

	1	2	3	4	5	6	7	8	9	10	11
+	1.0	0	.875	.375	0	0	1.0	0	1.0	.333	0
=	0	.833	0	.375	0	0	0	.25	0	.333	0
-	0	.167	.125	.25	1.0	1.0	0	.75	0	.333	1.0

While this second movement does not seem to have the same strong audible associations with the motives of the “Regenlied” that the first and third movements have, one can see relationships to family B within it, and these musical relationships help to weave a biographical narrative throughout the entire sonata, tying the circumstances of Felix’s illness and death—as well as reminiscences about his youth—to the story told by the song. The association that the Felix family shares with Family B, both in the contours themselves and in the structural layout of the sonata, ties the very real affinities Brahms had for Felix and his memories of his violin to the theme of nostalgia presented in family B of the song. As a result, the development of the Felix motives in the second movement takes the place of family B both narratively and motivically in the expectation set up in the large-scale layout of the sonata: the first movement introduces and explores motives highly associated with family A, setting up the expectation that a similar exposition and exploration of family B might follow. What we get instead is a new family that has loose associations with family B, but similar narrative connotations. Here, Brahms structurally replaces the abstract idea of the poet’s nostalgia with a very real representation of his own nostalgia for Felix’s youth. The story of nostalgia for one’s youth is no longer a story told by an anonymous protagonist, but now features very important, very personal characters.

This motivic and narrative association between the Felix family and the nostalgia represented by family B is further reinforced in the third movement, and this is where we return to our discussion of the third movement: the iconic member of the Felix family resurfaces unexpectedly during the second episode, and its presence alters the structural

coherence of the contours in both families from this point throughout the remainder of the work. About this resurgence, Walter Frisch writes:

Still more striking is the literal reappearance, in the finale, of the main theme from the Adagio, which enters quite suddenly in its original key (Eb major) at the point where we expect a return of the second theme in D minor (bar 84). The direct reminiscence rings rather hollow here, despite the skill with which Brahms proceeds to weave the Adagio theme seamlessly into its new environment. Brahms uses the technique to much better effect in the finales of the Third Symphony and the Clarinet Quintet, Op. 115, where the opening theme does not reappear suddenly, but emerges gradually to dominate the closing moments. (Frisch 1984, 117)

Once again, the Felix motive family takes the place of an otherwise expected formal event, as Frisch observes. However, I disagree with his assessment of its efficacy in this context: in this episode, the Felix family enters into a dialog with the motives of family A and family B. The Felix family presented in this episode shares strong affinities to both the Felix family of the second movement, as well as B families in both the song and sonata. In short, Brahms uses the guise of an episode as a space to intermingle the motives of the Felix family with those of the third movement's other motive families.

Figure 4.24a shows the opening bars of the "Felix episode" starting in m. 83: as Frisch points out, the Felix motive suddenly appears in its home key of Eb major. It is an exact match to the central motive of the Felix family, first heard in mm. 0–2 of the second movement. As such, it has a high degree of membership in the Felix family: 0.843, as shown in Figure 4.24b. Immediately following it is a dotted <+==++> pattern occurring in the inner-voice of the violin connecting it to the next motive in m. 86. It is this pattern that is most intriguing here, because this is the first time we see a direct instance of Family B material in the space of what is supposed to be the Felix family.

This $\langle + = ++ \rangle$ pattern is the familiar ascending figure seen many times in family B space during the refrains of this final movement. It has a membership of 0.881 within the sonata's family B, while only a 0.66 membership within the Felix family of the second movement.⁵⁹ Furthermore, when taking mm. 85–86 together, one sees a similar pattern emerging to those in sonata family B: In many members of family B (six to be precise), the opening $\langle + = ++ \rangle$ motion serves as an opening to a longer motive-form that introduces variety into the tail-end of family B, mimicking the phrase-structure of the song. These motives state the $\langle + = ++ \rangle$ motive before moving on to new material, and when one examines mm. 85–86 in this fashion, we arrive at a longer motive form that bears a closer resemblance to family B than it does to the Felix family, with a membership of 0.662 within family B and only a 0.561 membership within the Felix family. This process repeats in mm. 107–109: the Felix motive is heard at the outset, before rising again in a characteristically family B fashion (a $\langle + = += \rangle$ motion) before moving into the angular motive in m. 109. Here, the family B motive heard in m. 108 has been raised more audibly to the outer voice, and is indeed more reminiscent of the song's family B (a 0.771 membership within the song's family B, as opposed to a 0.725 membership within the sonata's family B), which features a second plateau in the last of the four positions rather than a second ascent.

⁵⁹ In the context of what I have been designating similar enough to constitute continuity, 0.66 does fall above that line, but I am making a distinction here that is more finely grained than that, because I have already established that the Felix family and the Sonata's family B are loosely related to begin with. As such, finer-grained distinctions allow for further differentiation between two already-related families.

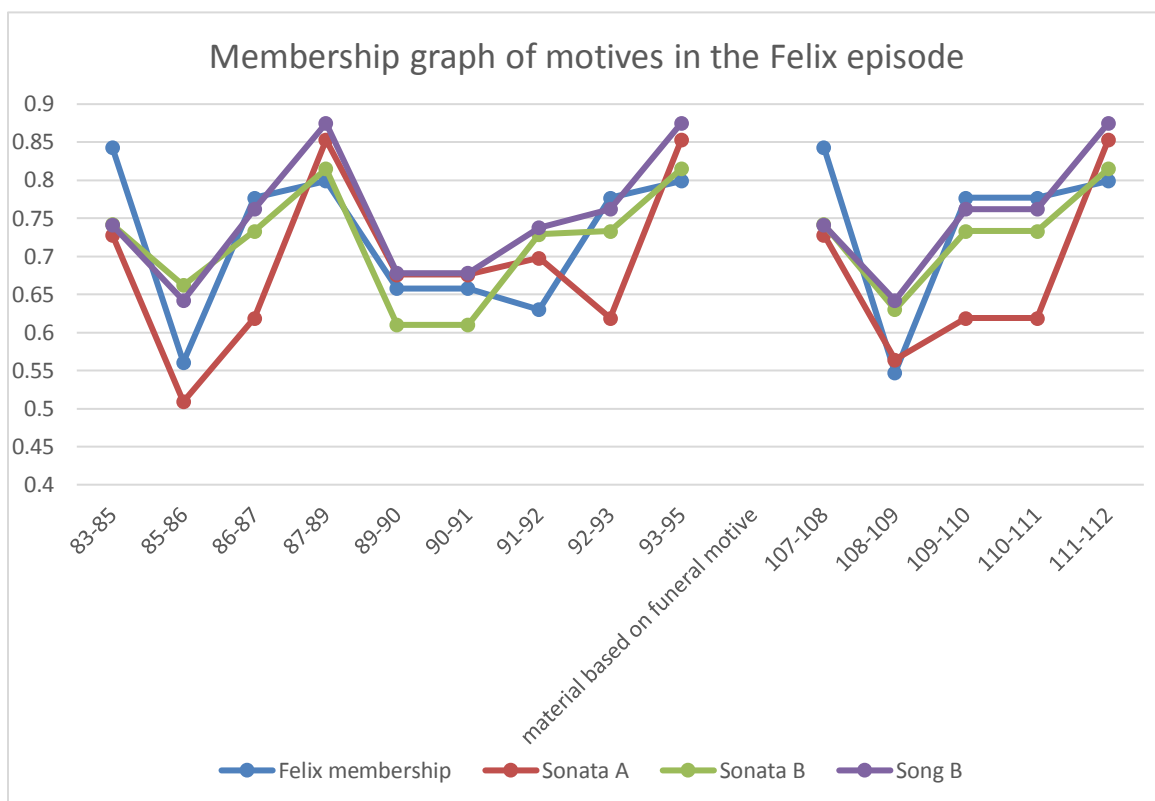
Figure 4.24a. Score excerpt of the opening of the Felix episode, beginning in m. 83

The image shows a musical score excerpt for the opening of the Felix episode, beginning in measure 83. The score is written for piano and consists of two systems. The first system starts at measure 82 and ends at measure 85. The second system starts at measure 86 and ends at measure 89. The key signature is two flats (B-flat and E-flat), and the time signature is 3/4. The piano part features a melodic line in the right hand and a bass line in the left hand. A red box highlights a specific melodic phrase in the piano part starting at measure 83. The score includes dynamic markings such as *p espress.* and performance instructions like $(+)$ and $=$.

Figure 4.24b. Motives in the Felix Episode, showing membership within other families

Example measures	C-SEG	CAS	Membership in Felix 2nd mvt. Family	Similarity to Sonata family A	Similarity to Sonata family B	Similarity to Song family B
83–85 (2x)	⟨132011⟩	⟨+---+=⟩	0.843	0.728	0.742	0.741
85–86	⟨01123⟩	⟨+=++⟩	0.66	0.472	0.881	0.958
85–86 (3x)	⟨23012401⟩	⟨+-----+⟩	0.63	0.697	0.729	0.738
(85–86 whole)	⟨02234123512⟩	⟨+=++-+++++⟩	0.561	0.509	0.662	0.642
86–87 (4x)	⟨36543210⟩	⟨+-----⟩	0.777	0.619	0.733	0.762
87–89 (3x)	⟨243120⟩	⟨+---+⟩	0.799	0.853	0.815	0.875
89–90 (x2)	⟨54320123⟩	⟨-----++⟩	0.658	0.676	0.610	0.678
108–109	⟨01122⟩	⟨+=+=⟩	0.632	0.315	0.725	0.771
109	⟨33012401⟩	⟨=-++++-+⟩	0.527	0.751	0.6	0.595
(108–109 whole)	⟨02244012501⟩	⟨+=+=-+++++⟩	0.547	0.564	0.630	0.642

Figure 4.25. Motivic membership graph of the Felix Episode, showing membership within the Felix family, Sonata family A, Sonata family B, and Song family B⁶⁰



In addition to these very prominent instances of family B motives occurring within Felix family space, a case can be made that the motives present in this episode are actually more coherently related to family B than they are to the Felix family: these motives display consistently high membership values within family B (ranging from 0.6–0.881 in the sonata’s family B, and an even higher range of 0.642–0.958 in the song’s family B), while as a whole, their membership within the Felix family is slightly lower

⁶⁰ I find it particularly important here to compare the motives to both the song’s family B and the sonata’s family B because the Felix material of the second movement compositionally predates the composition of the third movement, and so it is important to compare them to both the song and sonata to account thoroughly for all the similarities found within this episode.

(ranging from 0.52–0.843). Even more striking is that thirteen out of the nineteen motives in this section have higher values for one of the B families (either the song or sonata) than they do in the Felix family. In essence, what Brahms does here with these related families, is to bring out the dominant family B traits within the context of the Felix motive family. We can see this with the comingling of the exact motives from each family, as well as the fact that these motives share dominant traits from both families in the graph of Figure 4.25.

However, this is not the only intermingling occurring in this episode: one also can see prominent instances of family A within the space of the episode as well. Consider Figure 4.26, which shows m. 86 compared to mm. 3–4 and mm. 16–17, both of which exist within family A: the motives are very similar, and related to family A as I have shown earlier in Figure 4.10. In mm. 3–4 and 16–17, this motive had a loose connection to family A: a 0.536 and 0.560 membership value respectively. The instance in m. 86 is higher, with a membership value of 0.697. The repeated instance at m. 109 is even higher, with a membership value of 0.751. By contrast, these motives in m. 86 and m. 109 have lower values within family B and the Felix family. By including these motive-variants into this episode, Brahms brings all three families into dialog with each other: free to intermingle, interact, and change the very nature of each family as they coexist within the space of this episode.⁶¹ Brahms is doing this very subtly however, as he firmly couches this within the framework of the Felix family: the episode begins with the Felix

⁶¹ With this in mind, we actually see the succession of the three motives in mm. 83–86 as “Felix”-B-A: in essence “righting” the temporal reversal of the song.

motive, ends with the Felix motive, is in the key of the second movement, and features an overall resemblance to the Felix family of the second movement, so much so that many prominent scholars, such as Frisch, label this episode based on the second movement and leave it at that. As such, this episode coheres as its own family, despite its strong relationship to the other families within the sonata. In addition, the family—which I will call the “Felix episode” family—possesses its own set of dominant and recessive tendencies, as shown in the table in Figure 4.27: 1.0 values are featured prominently in positions 1, 5, 8, and 10 indicating that the motions in these positions are traits that most of the family members have, while other positions feature more varied membership values indicating that these tendencies would be more recessive.

Figure 4.26. Comparison showing the motivic development of the motive in mm. 3–4 across the movement

The figure displays five excerpts of a musical score in a 3/4 time signature, marked 'Allegro molto moderato' and 'p dolce'. The excerpts are connected by red arrows, illustrating the development of a specific motif. The motif is first introduced in measures 1-4, then appears in measures 3-4, 16-17, 86-87, and finally in measures 109-110. The notation includes treble and bass staves with various musical notations such as notes, rests, and dynamics.

Figure 4.27. Membership probabilities for the Felix episode family

	1	2	3	4	5	6	7	8	9	10
+	1.0	0	0.4	0.6	0	0.733	0.6	1.0	0	1.0
=	0	0.313	0	0.4	0	0	0	0	0.153	0
-	0	0.687	0.6	0	1.0	0.267	0.4	0	0.846	0

In this context, both Family A and Family B are changed as a result of their interaction with the Felix motive within this episode, and this is where the episode's narrative potential comes into play: by prominently associating this section with the second movement, which Brahms linked with Felix Schumann and his illness, one can see the character of Felix come to influence both motive families. The themes of nostalgia for Felix's youth that are present in the second movement come to add nuance to the meaning of the families of this third movement, as the character of the second movement family comes to directly interact with the motives from the song. Here, the associations and interactions between the nostalgia themes of the Felix family and family B are impacting the structure of family A, just as the new associations of the past change how the present is experienced. It is this interaction that contributes to the motivic variety we begin to see as we leave the black box that is this particular episode and enter the final refrain and coda—Brahms's last retelling of the motivic story of the song.

Returning to the discussion of the final refrain, in which the coherence of the family structures begins to break down, we see the effects of the episode on the two families initially presented in the sonata. Going into that final refrain, changes occur in the family A motives: where the motive presented in mm. 3–4 involved the $\langle -++ \rangle$ portion of the opening A motive, Brahms has changed it in this last refrain to $\langle --+ \rangle$, which more closely resembles the middle of the Felix motive $\langle --+ \rangle$ than it does to the middle of the Family A motive, as shown in Figure 4.28a. This change has the power to explain the lower membership value that the motive that mm. 126–127 has within family A: Figure 4.28b shows the pathways the motives take in mm. 3–4, 16–17, and 126–127 (membership values of 0.536, 0.560, and 0.414, respectively). This figure shows the

switch from $\langle + \rangle$ to $\langle - \rangle$ in positions 2 and 6 that cause significant drops in membership values, as the contour of the motive in mm. 126–127 now follows a less dominant pathway, taking the membership value of this contour within family A even lower to 0.414. However, this motive in mm. 126–127 bears a much higher resemblance when compared to the Felix family: a 0.655 membership, as shown in Figure 4.28c.

Figure 4.28a. Comparison of the motive in mm. 126–127 to the motive in mm. 16–17, as well as to the germinal family A motive and the germinal Felix motive

The figure displays three musical excerpts with contour annotations above them:

- Top excerpt:** "Allegro molto moderato" with contour $\langle = = - + + - - \rangle$. The melody is marked *p dolce*.
- Middle excerpt:** "Motive of mm. 16-17 in family A" with contour $\langle + - - - + = \rangle$. The melody is marked *sempre p*.
- Bottom excerpt:** "Motive in mm. 126-127 of the final refrain" with contour $\langle - - + - - - + = \rangle$. The melody is marked *m.f.* and *poco cresc.*.
- Right excerpt:** "Germinal Felix Motive from the second movement Adagio" with contour $\langle + - - + = \rangle$. The melody is marked *poco espress.*.

Red boxes highlight the melodic contours in each excerpt, and red arrows indicate the relationships between these contours, showing how the motive in mm. 126–127 is related to the other motives.

Figure 4.28b. Comparison of motive pathways in mm. 3–4, 16–17, and 126–127 within the sonata’s family A

<i>Mm. 3–4: 0.536 membership</i>										
	1	2	3	4	5	6	7	8	9	10
+	0	0.5	0.111	0.136	0	0.979	0.811	0.104	0.051	0.588
=	0.714	0.467	0	0.682	0	0	0	0.270	0.051	0.176
-	0.286	0.033	0.889	0.182	1.0	0.021	0.189	0.625	0.897	0.235

<i>Mm. 16–17: 0.560 membership</i>										
	1	2	3	4	5	6	7	8	9	10
+	0	0.5	0.111	0.136	0	0.979	0.811	0.104	0.051	0.588
=	0.714	0.467	0	0.682	0	0	0	0.270	0.051	0.176
-	0.286	0.033	0.889	0.182	1.0	0.021	0.189	0.625	0.897	0.235

<i>Mm. 126–127: 0.414 membership</i>										
	1	2	3	4	5	6	7	8	9	10
+	0	0.5	0.111	0.136	0	0.979	0.811	0.104	0.051	0.588
=	0.714	0.467	0	0.682	0	0	0	0.270	0.051	0.176
-	0.286	0.033	0.889	0.182	1.0	0.021	0.189	0.625	0.897	0.235

Figure 4.28c. Motive pathway of mm. 126–127 within the sonata’s Felix family

<i>mm. 126–127: 0.655 membership</i>									
	1	2	3	4	5	6	7	8	9
+	0.917	0.833	0.143	0	0	0.889	0.667	0	0.111
=	0	0.167	0	0	0	0.111	0	0	0.556
-	0.083	0	0.857	1.0	1.0	0	0.333	1.0	0.333

Likewise, mm. 136–137 features a similarly low membership value within family A, despite the fact that these measures share certain structural and formal affinities with family A. Here the low membership value is 0.528, which is again perhaps high enough to still indicate membership, but low enough to indicate that there has been significant

development within the family. Consider Figure 4.29, which shows mm. 136–137 in relation to the germinal motive form in family A: again, Brahms has chosen to elaborate upon the $\langle -++ \rangle$ c-subseg found in the middle of the germinal motive, repeating the idea twice. Just as in mm. 126–127, this motive also shares a high degree of membership within the families representing Felix. It has a membership value of 0.751 within the family of the Felix episode, and an even higher 0.839 membership within the Felix family of the second movement. While the opening $\langle ++ \rangle$ motion may have originated from the middle of the germinal motive of family A (just as we saw in m. 3), Brahms modifies it in the repetition, ending the motive form in mm. 136–137 with the $\langle -+= \rangle$ motion that so characteristically ends the germinal motive of the Felix family, as shown in Figure 4.29.

It is striking that these outlying members of family A within this final refrain should share these higher degrees of membership within the Felix motive families. The outlying members of family A heard previously feature negligibly low values within the Felix motive families as shown in Figure 4.30, such that the presence of Felix is not seen in these earlier developments of the family A motive. Their appearance in this final refrain then indicates that the Felix motive has influenced the turn of events in this final restatement. The Felix motive has come to alter the notion of the “present” that is invoked by family A. Because of the inversely causal narrative relationship seen previously between family A and family B (in that the past represented by family B comes to influence the present of family A), one might then come to expect that the Felix family has altered the structure of family B.

Figure 4.29. Comparison of the motive in mm. 136–137 with the germinal members of Sonata family A and the Felix family

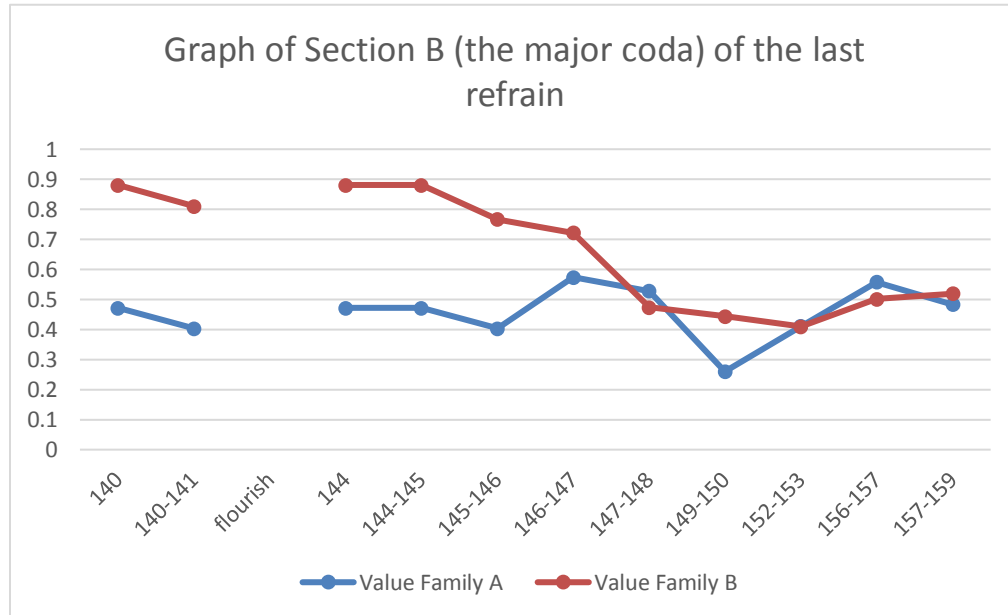
The figure displays three musical excerpts. The top left excerpt is titled "Allegro molto moderato" and features a piano part with a dynamic marking of *p dolce*. A blue bracket highlights a specific melodic motive. The top right excerpt is titled "Adagio" and features a piano part with a dynamic marking of *poco f espress*. A red bracket highlights a similar melodic motive. The bottom center excerpt is a piano part starting at measure 136, with a dynamic marking of *p*. A bracket highlights a motive that is visually connected to the motives in the other two excerpts by blue and red lines.

Figure 4.30. Outlying family A motives before reaching the final refrain

Measures	C-SEG	CAS	Membership in Family A	Membership in Felix 2 nd movement family	Membership in Felix episode family
3–4	<201354687>	<++++++>	0.536	0.439	0.575
16–17	<2013201442>	<++++++>	0.560	0.550	0.523
113–115	<000001>	<=====+>	0.544	0.256	0.312
115–116	<00154321>	<=====+>	0.591	0.615	0.546

In the final restatement of the B motive material, we see that this is the case. Family B resurfaces in m. 140 at the beginning of the coda, and the section exhibits a similar weakening within the coherence of family B, as shown in the graph of this last section in Figure 4.31. The significance of this weakening comes from the fact that it is weakened through the introduction of material introduced in the Felix family of the second movement, and reinforced in the Felix episode. The theme of the second movement is not blatantly reintroduced within family B in this closing coda, but rather works underneath the surface to contribute to the motivic variety we see in this section with regard to contour.

Figure 4.31. Graph of motivic membership values of coda motives within family A and family B



There are several notable instances within this last B section that feature mid-range levels of membership in both families A and B: these are the outliers that I

discussed previously with regard to family B. These mid-range members occur because they seem to include traits from both families, and are also influenced by the Felix family through their association within the Felix episode. As a result, they are not as central within either Family A or Family B. Take, for example, the motive in mm. 157–158, shown in Figure 4.32a: it has 0.483 membership within family A, and 0.519 membership within family B. The motive initially does not appear to have very strong audible connection to either family, and one may initially dismiss it as a closing gesture. However, upon closer examination, one can see resemblances to all three families within it. The opening bears resemblance to the opening germ of family A, with one note changed that alters the contour of the opening significantly, as shown in Figure 4.32b. In the opening, the motive segment in question rises from C5 to Eb5 to G5 before falling stepwise to F5 and Eb5 (<+--+>), eventually leading toward a D5 that begins the next motive form. In mm. 157–158, the motive in question rises from C#5 to A5, and then falls stepwise to G5, F#5, and E5 (<+--->)—a contour change in just a single position. The key has changed, as well as the harmonic context, and the rhythm is not as evenly distributed in eighth notes as it was in the opening, yet the resemblance is present. Additionally, this motivic alteration brings the first half of this motive into alignment with one of the motives from the development of the Felix episode: m. 87 (with an anacrusis in m. 86) features an identical <+---...> beginning, which can be seen in Figure 4.24a. This reproduction of episodic material shows that Brahms is taking that episodic development to highlight similarities between all three families, as seen in this particular motive in mm. 157–158.

Figure 4.32a. The motive in mm. 157–159, exhibiting mid-range membership in both families

This only explains part of the motivic development in these two measures though, and we must turn to Family B to explain the rest, as shown in Figure 4.32b. The second half of the motive, in m. 158, features a chromatic descent from E5 through Eb5 to D5, before falling to F#4 and rising to G4, for a c-seg of <4332201> and a CAS of <-=-=++>. On the surface, this doesn't strongly resemble the core motives of family B, yet the trajectory that the coda takes with regard to family B significantly alters its structure, such that when listening to the coda, I still hear this segment as associated with family B. This segment develops from a previous motive variant in this B section in mm. 152–153. It begins with a composed-out ascent from C5 through D5 to E5 for a CAS of <+=+=>, similar to the development of the B motive I spoke of earlier in m. 156 (shown in Figure 4.13, where the <+=+=> motive was expanded to <+=+=+=> with the use of passing tones), and then ends with a similar chromatic descent to that in m. 158, moving from E5 through Eb5 to D5 and then to C5 before ending with an A#4 rising to B4. This contour is an earlier variant that introduces the idea of the chromatic descent E5-Eb5-D5 as an element of family B, yet it is still an outlier, with a rather low membership value of 0.41 in family B, signifying that this motive variant also has developed away from the core of

family B. This motive variant in mm. 152–153 can trace its development even further back, to the motive in mm. 140–141, where the initial germinal motive of $\langle + = ++ \rangle$ is joined by a descending line that ends with a final chromatic ascent, just as the motives in mm. 152–153 and in mm. 156–158. This ending material initially stems from the Felix episode, which is based on the Felix motive family of the second movement: a $\langle -- += \rangle$ motion that is an iconic feature of the family. Despite its origin in the second movement, this motion also bears a strong resemblance to the ending of multiple core motive variants within earlier refrains, such as mm. 23–24. It is here where we see Brahms once again using the similarities between the families to alter the structure of the B family, subtly underscoring the presence of the Felix family—and the narrative shift that comes with it—within the coda. Specifically, the motive in mm. 23–24 features an ending that varies in only two positions (but only one note has changed) when compared to the motive opening the coda: the motive form in mm. 140–141 features an ending descent from G5 down to A#4 with the chromatic ascent to B4 for a CAS segment of $\langle ----- + \rangle$, whereas the ending of mm. 23–24 features a descent from G5 down to C#5, rising to D5 before falling to the Bb4, for a CAS segment of mm. 23–24 is $\langle ----- +- \rangle$, as shown in Figure 4.17b. This strong relationship between the motive form in the closing refrain and the motive form in the opening refrain suggests a high degree of membership of the motive here in mm. 140–141: indeed, we see that this motive has a membership value of 0.81, which definitely falls within the core of family B. In essence, Brahms is using the relationship between these two motives in order to introduce material from the Felix families into family B in this last refrain, and this introduction changes the way family B develops throughout the coda, growing less and less similar to the germinal motives of the opening

refrain. Returning to the motive in mm. 157–158 then, we can see how it relates to the newly developed family B motive variants, yet also why it has comparatively low membership value within family B. Its low membership across both families indicates a good degree of development within both families, as I have shown in this example.

These developmental instances are significant in that these outlying members of family B often have higher membership values within one or both of the Felix families than they do in family B. The graph in Figure 4.33 shows this in a graph of the coda, indicating membership values of family B, as well as the two Felix families. In the graph we see a gradual rise in membership values of the Felix family within the motives of the coda. These outlying contours therefore seem to have greater resemblance to the Felix families than they do to Family B.⁶² Because of these strong connections between the motives here in the coda and the Felix families, Brahms has colored his earlier theme of nostalgia with the reminiscences he presented in the second movement.

⁶² Granted, a few of these higher values are only marginally higher, yet I believe overall that this result is significant, especially given the larger gaps between the more prominently “Felix-esque” motives.

Figure 4.32b. Motivic Transformations toward the motive form in mm. 157–159

Relationship to Family A

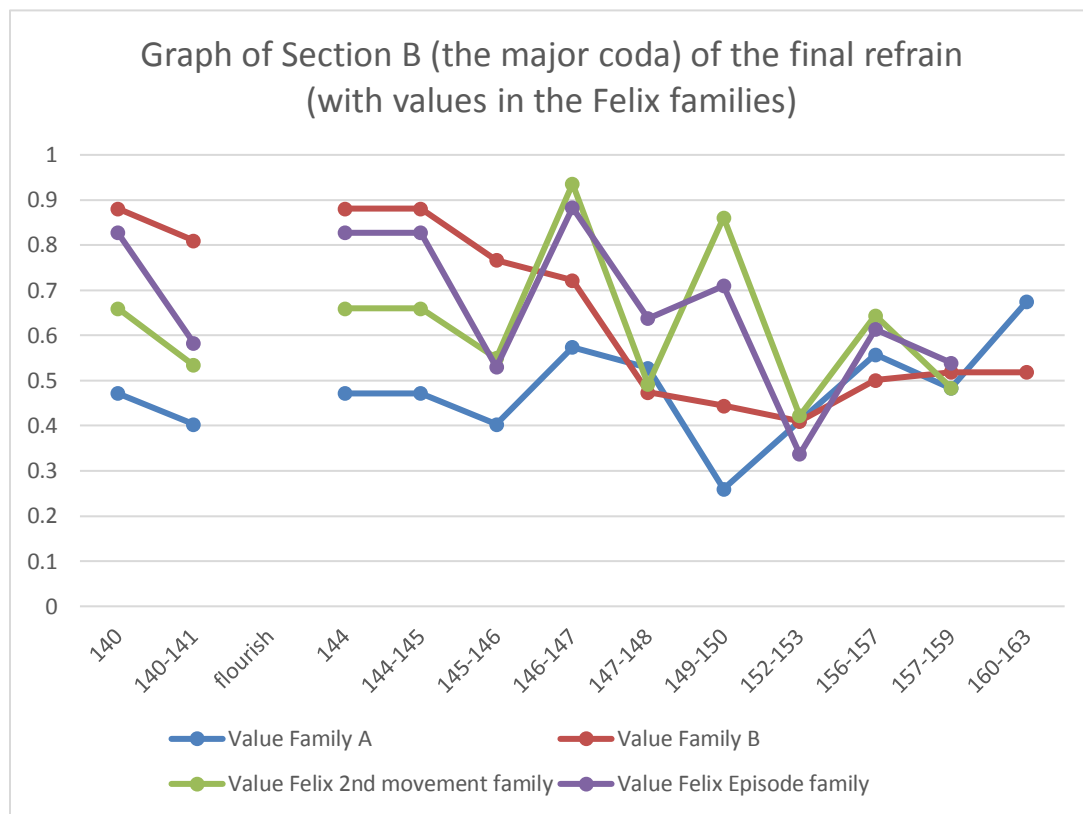
Relationship to Family B

Relationship to Felix Family

The figure illustrates the evolution of a musical motif through three different families. Red boxes and lines connect related motifs across the different sections.

- Relationship to Family B (top):** Shows a piano piece starting at measure 23. The upper staff is marked *dolce*. A red box highlights a specific melodic phrase in the upper staff, with a red circle around the final note. The lower staff is also marked *dolce*.
- Relationship to Family B (middle):** Shows a piano piece starting at measure 140. The tempo is marked *Più moderato*. The upper staff is marked *dolcissimo* and the lower staff *dolcissimo*. A red box highlights a melodic phrase in the upper staff, with a red circle around the final note. This phrase is related to the one in the top section.
- Relationship to Family A (bottom left):** Shows a piano piece starting at measure 1. The upper staff is marked *p dolce* and the lower staff *p dolce*. A red box highlights a melodic phrase in the upper staff.
- Relationship to Family A (bottom middle):** Shows a piano piece starting at measure 152. The upper staff is marked *espress.* and the lower staff *pp*. A red box highlights a melodic phrase in the upper staff.
- Relationship to Family A (bottom right):** Shows a piano piece starting at measure 157. The upper staff is marked *poco rit.* and the lower staff *pp*. A red box highlights a melodic phrase in the upper staff.
- Relationship to Felix Family (right):** Shows a piano piece starting at measure 1. The tempo is marked *Adagio*. The upper staff has a rhythmic pattern $(+ \quad - \quad - \quad + \quad -)$. The lower staff is marked *poco f. espress.*. A red box highlights a melodic phrase in the upper staff, which is related to the motifs in the other families.

Figure 4.33. Motivic membership of coda motives within Family A, Family B, and both Felix families



In this last refrain, then, we see Brahms’s developmental strategies hard at work. The examination of the contours in this section also reveals that the element of contour has become less of a unifying parameter and more of a developmental one, as the degrees of membership become comingled and also tend to drop off in both families A and B as the work draws to a close. Contour no longer serves as an aid to the comprehensibility of motive—such a role might be replaced with a combination of rhythmic factors, formal factors, and others—and now acts as an element that highlights Brahms’s motivic development. Brahms’s intermingling of families and placement of the Felix motive elements within the development of both families A and B transforms family B in order

to attach the biographical nostalgia onto the abstract nostalgia carried over from the song. In doing so, he comes to change the meaning of the nostalgia initially presented in the song, and consequently alters the “present” represented by family A as well. In this way, Brahms changes the very nature of these motivic identities, bringing the character of Felix firmly into both families such that when we reach the final refrain and coda, they are forever changed by Felix’s presence.

Conclusion

When writing the violin sonata in Pörtlach, Brahms wrote that “Here the melodies are flying so thick that one must be careful not to step on one” (quoted in Frisch 1984, 117). Through the lens of contour, we have shed light onto the richly complex tapestry of melodic motives that Brahms has masterfully woven together throughout this sonata. Both the sonata and the song share significant relationships between motive families, and as such, share similar narrative connotations. The song presents a story featuring an anonymous narrator experiencing a rainstorm and longing nostalgically to be whisked away to memories of his youth spent playing in the rain. Groth’s poetic text is set to motives that, when analyzed using the FCM model, mimic the temporal trajectory of the work, attaching narrative meaning to the motive families involved. The sonata then uses these motive families to build upon the narrative of the song: the introduction of the Felix families in the second movement and the third movement episode alter the meaning of the nostalgic motives, giving them a character that is no longer anonymous. The interaction of these families throughout the sonata in effect makes the story presented in

the song about Brahms and his relationship with his young godson—a uniquely personal and elegiac addition to Groth’s poem.⁶³

Using the FCM model to understand the motivic interactions throughout these two pieces illuminates melodic aspects of the work that give additional potential meaning to motivic elements that might otherwise seem to “ring hollow” as Frisch claims. The intermingling of families toward the end of the third movement, therefore, is not a compositional aberration for Brahms, but a deliberate developmental choice that can be illuminated and explained in terms of contour. In this analysis, I have shown how an examination of contour can be used to highlight these features of Brahms’s mastery of motivic development. Using my FCM model for fuzzy contour membership, I have shown how cohesive contour families with high membership values among its members indicates that contour plays a unifying role, serving as an aid to the comprehensibility of the motive, while development occurs in other musical parameters of the work. Alternatively, contour families that feature a wider variety of membership values are not as cohesive and suggest the presence of development in the pitch and contour domains. In this way, the model exposes aspects of development that may have previously been less apparent, such as the relationship between contour and narrative in the song, or the large-scale developmental relationships across the movements of the sonata that point toward a

⁶³ Billroth once commented on the elegiac nature of the sonata, especially at the end of the third movement, where we see the influence of Felix upon family B. In a letter to Hanslick, he wrote “It is a piece of music entirely in elegy...The feelings are too fine, too true and warm, and the inner self is too full of the emotion of one’s heart for publicity” (Billroth 1977, 82-83). Furthermore, Brahms also comments that “My sonata is no more useful for publicity than I am myself” (Brahms 1977, 83). Hans Barkan links this quotation to his deep personal feelings, and the highly personal nature of the sonata.

reconceptualization of the song's original narrative. Using contour as a way to highlight Brahms's developing variation technique therefore gives us new appreciation for the subtle complexities of his motivic mastery.

CHAPTER 5: Implications of Phasing for Contour Perception in Steve Reich's *The Desert Music*

“it is a principle of music
to repeat the theme. Repeat
and repeat again,
as the pace mounts. The
theme is difficult
but no more difficult
than the facts to be
resolved.”

--William Carlos Williams, “The Orchestra”

This excerpt from William Carlos Williams's poem “The Orchestra” (1954) serves as text for the third movement of Steve Reich's *The Desert Music* (1984), and highlights a key aspect of Reich's compositional philosophy. In his processes of melodic phasing, the pattern “repeats and repeats again” until it seems different, due to the placement of a melody against itself at various distances of rhythmic displacement. In *The Desert Music*, Reich uses phasing as an element of melodic process, creating regions of multilinear melodies that challenge the active listener to hear and understand melody in different ways. Contour plays a significant role in the perception of these melodic processes, so in this final study, I would like to return to the article from which this dissertation began: Ian Quinn's article on fuzzy extensions to musical contour, and his analysis of melodies from *The Desert Music*.

Quinn creates a family of contours from the outer flanks of the third movement in order to expound upon his fuzzy model for contour relations. Regarding his melody family, which he labels “M,” Quinn writes that “no more than a casual hearing of the piece, or a cursory glance at Example 1 [reproduced as Figure 5.1a], is necessary to

inspire in the observer the intuition that all of the first-violin and flute melodies are of a kind. Specifically, they all follow the same rhythmic pattern, and they share a strong family resemblance of contour, despite individual differences” (1997, 233). Quinn seems to imply that his analysis of the melody family places all members at about 0.7 in membership, suggesting that he is correct about familial coherence, but this only raises further questions about how the melodies are used in the context of the piece.

Surprisingly, Quinn has little else to say about the work, which leads me to ask: is his familial representation the way listeners really experience the melodies in the context of the work? How is this family built throughout the course of the movement? How does the coherence of this family tie in with the rest of the work? How does it tie in with what Steve Reich says about the work? And what can it say about our experience of minimalist music? Many of these questions can influence one’s perception of familial resemblance, and may therefore affect the structure of the contour family that Quinn has identified. Quinn does not—and I would argue, because of the limitations of his model, cannot—answer these questions. Granted, Quinn is more concerned with using the family as an example for his discourse on the need for fuzziness in contour relations, and in this context, the family works quite well. However, I contend that from the standpoint of an analysis sensitive to the musical contexts found in *The Desert Music*, the family falls short.

In this chapter, I will expand upon Quinn’s analysis, using my model of familial contour membership. I will examine the familial coherence that Quinn addresses, but will also explore how the temporal unfolding and positioning of family members against one another creates a wealth of experiential possibility within the context of these passages.

This possibility suggests further connections with phenomenological theories of multistability (Ihde 2012, Karpinski 2012), in that it is possible to hear multiple contours as the work progresses (few to none of which are actually printed on the page), even though the stimulus remains the same. By examining the relationships between the melodies in terms of these multistable possibilities, the FCM model provides a more sensitive account of the contour relations that a listener may perceive within the music, and this will give us a better understanding of Reich's minimalist processes.

Quinn's Analysis

Ian Quinn claims that his family of melodies, shown in Figure 5.1a, exhibits strong family resemblances. However, as we saw in Chapter 1, the actual membership values of the crisp contours against Quinn's fuzzy family range from 0.544 to 0.736. As shown in Figure 5.1b, twelve of the sixteen melodies fall below the threshold of resemblance of 0.7 that Quinn establishes for potential new members. The FCM model's membership values also suggest a greater degree of variety, as shown in Figure 5.1c and Figure 5.1d. This is not to say that there is no resemblance: ranges between 0.506 and 0.736 still indicate that member melodies are not completely dissimilar (as values below 0.5 may suggest). Indeed, Quinn suggests this by pointing to the middleground "W" shape formed by most of the c-segs in the family (1997, 235).

Figure 5.1a. Quinn's melody family from *The Desert Music* (Quinn 1997, 234)

Example 1. The sixteen melodies (collectively called **M**) played by the first violins and flutes in Reich, *The Desert Music*, iii (outer portions)












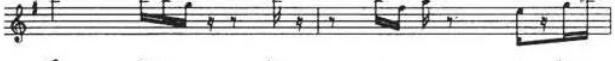




m_1		x14
m_{2a}		x9
m_{2b}		x3
m_{3a}		x10
m_{3b}		x3
m_{4a}		x10
m_{4b}		x3
m_5		x14
m_{6a}		x8
m_{6b}		x2
m_{6c}		x2
m_{7a}		x9
m_{7b}		x4
m_{8a}		x9
m_{8b}		x2
m_{8c}		x2

Figure 5.1b. Membership Values of Quinn’s Family Members Using Quinn’s C+SIM Method

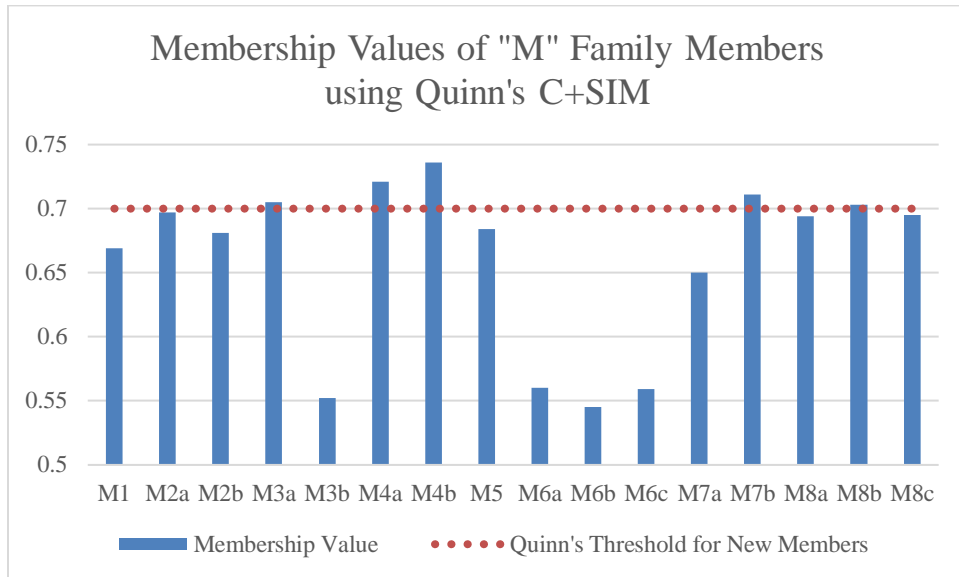


Figure 5.1c. Membership values of contours in Quinn’s family using the FCM model

ID	Occurrences	CSEG	CAS	Card. of CAS	Membership
M1	14	<54361232430>	<--++--->	10	0.592
M2a	9	<67430210345>	<+---+--->	10	0.608
M2b	3	<57430210346>	<+---+--->	10	0.608
M3a	10	<87416302453>	<---+--->	10	0.655
M3b	3	<35412541054>	<+---+--->	10	0.556
M4a	10	<87563540251>	<---+--->	10	0.685
M4b	3	<76454310352>	<--+--->	10	0.687
M5	14	<73654102424>	<+---+--->	10	0.546
M6a	8	<75201365431>	<---+--->	10	0.534
M6b	2	<86301576542>	<---+--->	10	0.534
M6c	2	<86324675201>	<---+--->	10	0.506
M7a	9	<86427413025>	<---+--->	10	0.571
M7b	4	<65314201342>	<---+--->	10	0.654
M8a	9	<75276410253>	<--+--->	10	0.687
M8b	2	<75376210354>	<--+--->	10	0.687
M8c	2	<64265310243>	<--+--->	10	0.687

Figure 5.1d. The Fuzzy Family of Quinn’s contours

	1	2	3	4	5	6	7	8	9	10
+	0.144	0.135	0.385	0.365	0.490	0.25	0.356	0.770	0.712	0.356
-	0.856	0.865	0.615	0.635	0.510	0.846	0.644	0.230	0.384	0.644

But is this how a listener experiences these melodies in the work? I would say no. In the music, each of these melodies identified by Quinn is played against various versions of itself displaced by a certain rhythmic value, as well as a contrapuntal melody (in the second violins and clarinets) that is also played against various displaced versions of itself. Indeed, Steve Reich states that “I wanted to use all the orchestral instruments to play the repeating interlocking melodic patterns found in much of my earlier music” (Reich 2002, 121).

Reich’s Phasing

These interlocking melodic patterns refer to Reich’s earlier phasing music, wherein melodies would be played against themselves at a certain level of rhythmic displacement, termed by Richard Cohn (1992) and John Roeder (2003) as beat-class transposition. Cohn and Roeder have discussed this kind of melodic construction, as well as the emergent listener experience that results from the polyphonic union of phased melodies. Their work on beat-class sets focuses on the properties of the union of the beat-class set with transpositions of itself that displace the metric pattern by a given number of beats. Cohn writes that “As a principal rhythmic pattern is ‘run through’ the phasing process, it enters into different transpositional relations with itself. Each ‘prologational region’ features a new beat-class set that results from a combination of transpositions of the original set...the characteristics of these emergent beat-class sets directly result from

properties of the principal beat-class set” (1992, 153). He continues: “In exploring the pattern of attack-point frequencies through the various prolongational regions, we are essentially inquiring into the cardinality of the union of the principal beat-class set with each of its transpositions” (154). Through his analysis, Cohn’s emphasis on the union of the beat-class set with its transpositions ultimately yields a composite set that represents what is actually heard in terms of attack-points in each region.

In *The Desert Music*, as with other later works of Reich’s, the phasing is not thoroughly exhausted as it is in the earlier works that Cohn describes. Instead, Reich creates the effect of phasing as an element of a larger composition, building it up but not exhaustively running through the entire phasing process. While both Cohn and Roeder are concerned with the beat-class structure of these kinds of patterns, I am interested in the emergent melodic possibilities that arise from the composite patterns that Reich creates. Since Reich’s conception of the melodic unit involves the initial pattern combined simultaneously with its beat-class transpositions, in order to understand the melodic unit, and the subsequent family that Quinn identifies, we need a model capable of encompassing the potential ways that a listener can experience the composite pattern from a contour perspective. The FCM model’s flexibility with regard to cardinality allows us to model this composite in a more nuanced and rhythmically sensitive way.

Consider Figure 5.2a, which shows a two-measure unit beginning at rehearsal 122 in the third movement: an instance of what Quinn labels as the melody M_1 . Here, the flute 1, doubled in the first violins, presents the main melody beginning on the downbeat of the

measure.⁶⁴ Flute 2 presents the same melody, transposed to begin on the second eighth note of the measure—what Cohn and Roeder would call a beat-class transposition of T_1 . Flute 3 presents a similar melody (with the same rhythmic pattern) at a transposition of T_2 , or a quarter note displaced from the downbeat. These transpositions complicate the listener’s awareness of the contour of the passage, as the composite pattern obscures the individuality of each voice’s melody.

Among the three lines, there are 21 attack points in the two-measure unit, and it is against these points that we can begin to model the emergent melodic possibilities inherent within the composite pattern. Each melodic segment can be placed against the 20-position CAS grid (CAS positions occur between the 21 attack points) by placing each motion in its proper metric place in the grid. For example, the descent between the first two attack points in the flute 1 melody (the Bb_5 to the G_5) is placed in position one of the 20-position CAS grid.

Figure 5.2a. The melody M_1 in the flutes (and first violins), showing the T_1 and T_2 phasing

⁶⁴ Note that the three lines in the flutes are always doubled exactly by the first violins (broken into three groups). For the sake of this discourse, I shall only make reference to the flute melodies.

Figure 5.2b shows the placement of the flute 1 melody against the 20-position CAS grid (each position of which occurs between the 21 attack points in the unit).⁶⁵ Figures 5.2c and 5.2d model the flute 2 and flute 3 lines respectively. These placements allow an analyst to create a fuzzy representation of the measure as it is experienced in time, as shown in Figure 5.2e. The fuzziness of the family arises from the fact that multiple lines have differing motions at the same point in time, and this fuzzy multilinear representation captures that by modeling the probability of these differing motions to occur at a particular point throughout the passage. Figure 5.2e shows the fuzzy multilinear representation of the two-measure unit, encompassing all three lines shown in Figures 5.2b, 5.2c, and 5.2d. What we see is that there are certain places where the contour is clear, in the positions that present 1.0 membership. These positions arise either because only one line is sounding at that moment, or that all sounding lines are in agreement with regard to contour at that moment. Other positions become fuzzier, because the lines are moving in different directions due to their transposition.

Furthermore, Roeder points out that in many of his later works, Steve Reich's compositional strategy involves the gradual build-up of the phasing pattern in order to resemble the gradual process of his earlier phasing compositions. Roeder writes that "They are not exhaustively phased; more typically, they appear in two- or three-voice canons at fixed, not varying, temporal intervals. Moreover, the pieces often feature 'build-ups' in which an entering voice, beginning with one attack and adding attacks with each iteration, gradually assembles a complete beat-class transposition of a pattern

⁶⁵ Motions between gaps are marked on their initial side. For example, the gap between position 4 and position 8 indicates that there are rests in between on positions 5-8.

Figure 5.2b. Placement of CAS of M₁T₀ (flute 1) against articulation grid⁶⁶

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	0	0	1.0	0	0	0	0	1.0	1.0	0	0	0	0	1.0	0	0	0	0	0	0
-	1.0	1.0	0	1.0	0	0	0	0	0	1.0	0	0	0	0	0	1.0	1.0	0	0	0

Figure 5.2c. Placement of CAS of M₁T₁ (Flute 2) against the articulation grid

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	0	0	0	0	1.0	0	0	0	0	1.0	1.0	0	0	0	1.0	0	0	0	0	0
-	0	1.0	0	1.0	0	1.0	0	0	0	0	0	1.0	0	0	0	0	0	1.0	1.0	0

Figure 5.2d. Placement of CAS of M₁T₂ (Flute 3) against the articulation grid⁶⁷

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	1.0	0	0	1.0	0	1.0	1.0	0	0	0	0	1.0	1.0	0	0	1.0	0	0	0	0
-	0	0	0	0	0	0	0	1.0	0	0	0	0	0	1.0	0	0	0	0	0	1.0

Figure 5.2e. Fuzzy multilinear representation of M₁ at rehearsal 122

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	0.5	0	1.0	0.333	1.0	0.5	1.0	0.5	1.0	0.5	1.0	0.5	1.0	0.5	1.0	0.5	0	0	0	0
-	0.5	1.0	0	0.667	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	1.0	1.0	1.0	1.0

⁶⁶ Note that these CAS grids do not have a row representing the plateau motion (=) because these melodies do not make use of the plateau motion.

⁶⁷ This contour is slightly different as, in its rotation, begins with an ascent, not a descent.

repeating concurrently in another voice” (2003, 276).⁶⁸ Figure 5.3a shows rehearsals 120–126: the build-up and repetition of the M_1 composite pattern. The first two iterations (rehearsals 120–121) occur in only one voice, such that there is no ambiguity in terms of melodic identity. Beginning at rehearsal 121, however, flutes 2 and 3 begin to build up their variants of the M_1 melody, compromising the clarity with which a listener might hear the original melody presented at rehearsal 120. These variants become fully formed at rehearsal 122, and the three voices continue in their phased relationship until the end of the section at rehearsal 126.

Figures 5.3b–d show an analysis of the diachronic build-up of the passage. In this analysis, membership values for each M_1 variant are continually adjusted as each iteration of the two-measure unit is introduced into the family. These continual adjustments reflect the listener’s experience of increasing complexity as the passage unfolds. The first three iterations of the pattern in the M_1 region possess 1.0 membership, reflecting the clarity of the single line, as shown in the chart in Figure 5.3b and the graph in Figure 5.3d.⁶⁹ After these lines, the membership values of the original M_1 melody begin to drop as the build-up of $M_1(T_1)$ and $M_1(T_2)$ in flutes 2 and 3 begins. As these build-ups in flutes 2 and 3 become incorporated into the family, the crisp positions of the M_1T_0 melody in the CAS grid become compromised, resulting in values lower than 1.0. This reflects the loss of clarity that begins to occur as these additional lines introduce Reich’s phasing. As each iteration is added into the family of the melody, one can see

⁶⁸ The term “build-up” is used by Reich to describe his compositional strategy in these kinds of passages.

⁶⁹ The third iteration is relatively unproblematic, since it is measured against a family that only includes the $M_1(T_0)$ version.

that the clear comprehension of the original M_1 melody becomes compromised. By the time all iterations of M_1 have been heard, the T_0 version's identifiability within the context of the composite pattern has dropped from 1.0 down to 0.757. Meanwhile, the phased versions experience an increase in membership as the repetitions of the full pattern continue.⁷⁰ These combined membership values indicate that the melody itself in this phased context is fuzzy, containing multiple possibilities for hearing the particular contour of the pattern.⁷¹

The patterns we see in the build-up of the M_1 region occur in many other areas of the work as well. The other movements of *The Desert Music* exhibit exactly this kind of phasing, and as such, understanding the tendencies that the phasing process creates is crucial to understanding both the structure and the listening experience of the work. Returning to Quinn's melodies then, the sixteen melodies fall into eight of these phasing regions (reflected in Quinn's labeling of melodies as 1–8, with letter differences differentiating slight modifications that occur within each region). The form diagram in Figure 5.4 shows the breakdown of these regions. Figure 5.5a–h shows the eight fuzzy multilinear families of each of these regions, which incorporate all melodies in each region, as shown in Figure 5.4.

⁷⁰ Again, since the initial fragments occurring in lines 3 and 4 are occurring against the single crisp contour of the $M_1(T_0)$ melody, their initial values are high, but as their full pattern emerges in line 5, the values become much lower.

⁷¹ This potentiality is emphasized more in the timbral and registral uniformity of the instruments involved in the phasing of this melody. Such uniformity limits one's ability to segregate streams on the basis of timbre and register.

Figure 5.3a. Flute lines from rehearsal numbers 120-126, showing the build-up of phasing in the M_1 region

The musical score consists of four systems of three staves each, labeled M₁T₀, M₁T₁, and M₁T₂. The key signature is three flats (B-flat, E-flat, A-flat) and the time signature is 4/4.

- System 1 (Rehearsal 120-121):**
 - Rehearsal 120: M₁T₀ (1) has a melodic line starting with a *mf* dynamic. M₁T₁ (2) and M₁T₂ (3) are silent.
 - Rehearsal 121: M₁T₀ (1) is silent. M₁T₁ (2) and M₁T₂ (3) enter with a *f* dynamic. Red brackets indicate "Build-up of M₁T₁ and M₁T₂" for the first two measures and "Build-up of M₁T₁ and M₁T₂" for the next two measures.
- System 2 (Rehearsal 122):**
 - Rehearsal 122: M₁T₀ (1) has a *mf* dynamic. M₁T₁ (2) has a *f* dynamic. M₁T₂ (3) has a *f* dynamic. A red bracket under the first two measures is labeled "Complete patterns of M₁T₀, M₁T₁, and M₁T₂".
 - Rehearsal 123: M₁T₀ (1), M₁T₁ (2), and M₁T₂ (3) are silent.
- System 3 (Rehearsal 123-124):**
 - Rehearsal 123: M₁T₀ (1), M₁T₁ (2), and M₁T₂ (3) are silent.
 - Rehearsal 124: M₁T₀ (1), M₁T₁ (2), and M₁T₂ (3) are silent.
- System 4 (Rehearsal 125):**
 - Rehearsal 125: M₁T₀ (1), M₁T₁ (2), and M₁T₂ (3) are silent.

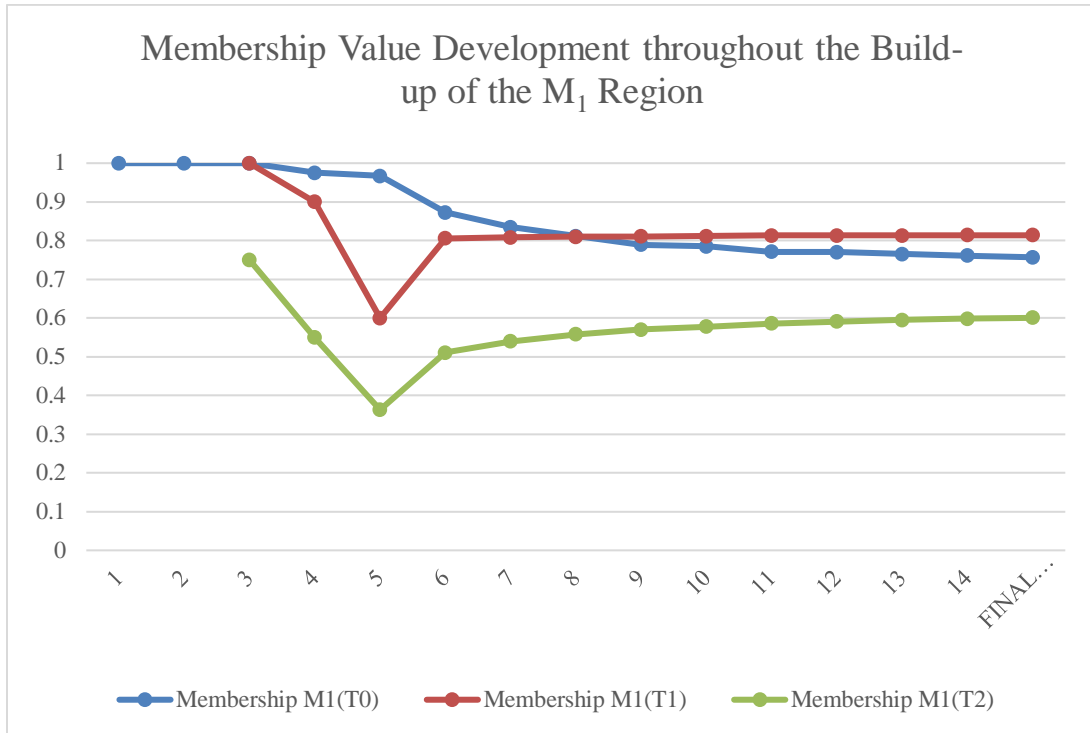
Figure 5.3b. Graduated analysis of membership values through the build-up of the M_1 region

Iteration of M_1 composite	Membership $M_1(T_0)$ (flute 1)	Membership $M_1(T_1)$ (flute 2)	Membership $M_1(T_2)$ (flute 3)
1 st time	1.0	--	--
2	1.0	--	--
3	1.0	1.0	0.75
4	0.975	0.9	0.55
5	0.967	0.6	0.363
6	0.873	0.806	0.511
7	0.835	0.808	0.54
8	0.812	0.810	0.558
9	0.789	0.811	0.570
10	0.785	0.812	0.578
11	0.771	0.813 (0.8126)	0.586
12	0.770	0.813 (0.8129)	0.591
13	0.765	0.813	0.595
14	0.761	0.814 (0.8136)	0.598
FINAL MEMBERSHIP	0.757	0.814 (0.8139)	0.601

Figure 5.3c. The Final Version of the Fuzzy Multilinear Family of M_1

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	0.44	0	1.0	0.278	1.0	0.5	1.0	0.538	1.0	0.417	1.0	0.5	1.0	0.583	1.0	0.417	0	0	0	0
-	0.56	1.0	0	0.722	0	0.5	0	0.461	0	0.583	0	0.5	0	0.417	0	0.583	1.0	1.0	1.0	1.0

Figure 5.3d. Graph of membership value development in the M_1 region



From these families, we can make a few observations about Reich’s melody regions. First, many areas of crispness exist within each family: with the exception of the M_3 family, each family possesses 1.0 crispness in over half of its twenty respective CAS positions. For example, the M_1 fuzzy multilinear family has 1.0 values in twelve CAS positions, and the M_2 fuzzy multilinear family has seventeen 1.0 values in its CAS representation. Such crispness ultimately suggests that listeners can perceive these patterns of clear contour within the passage, especially in cases such as the M_2 region that feature large strings of crisp motions.

Figure 5.4. Form diagram of The Desert Music, movement III, showing where the melodies in Quinn’s family occur

FORM	A									B	A'										
REGION	--	M1	M2		M3		M4		--		--	M5	M6			M7		M8			--
MELODY			a	b	a	b	a	b					a	b	c	a	b	a	b	c	
REHEARSAL NUMBERS	116	120	126	129	130	133	134	137	138	160	211	212	218	223	224	225	230	232	237	238	239–250

Figure 5.5a. M₁ Fuzzy Multilinear Family

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	0.44	0	1.0	0.278	1.0	0.5	1.0	0.538	1.0	0.417	1.0	0.5	1.0	0.583	1.0	0.417	0	0	0	0
-	0.56	1.0	0	0.722	0	0.5	0	0.461	0	0.583	0	0.5	0	0.417	0	0.583	1.0	1.0	1.0	1.0

Figure 5.5b. M₂ Fuzzy Multilinear Family

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	1.0	0	0.645	0	0.5	0	0	1.0	0	1.0	0	1.0	0	0	0.55	1.0	1.0	1.0	1.0	1.0
-	0	1.0	0.355	1.0	0.5	1.0	1.0	0	1.0	0	1.0	0	1.0	1.0	0.45	0	0	0	0	0

Figure 5.5c. M₃ Fuzzy Multilinear Family

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	0.364	0.727	0.15	0.727	0.136	0	0.591	0	0.5	0	0.545	0.333	0.136	0.769	0.455	0.667	0.333	0.667	1.0	0
-	0.636	0.273	0.85	0.273	0.864	1.0	0.409	1.0	0.5	1.0	0.455	0.667	0.864	0.231	0.545	0.333	0.667	0.333	0	1.0

Figure 5.5d. M_4 Fuzzy Multilinear Family

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	0.458	0	0.591	0	0.5	0	0.410	0	0.476	0	0.273	0	0.333	0	0.591	1.0	0	1.0	0	1.0
-	0.542	1.0	0.410	1.0	0.5	1.0	0.591	1.0	0.524	1.0	0.727	1.0	0.667	1.0	0.410	0	1.0	0	1.0	0

Figure 5.5e. M_5 Fuzzy Multilinear Family

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	0.417	1.0	0.5	1.0	0.583	1.0	0.417	0	0	0	0	0	0	0.583	1.0	0.333	1.0	1.0	1.0	1.0
-	0.583	0	0.5	0	0.417	0	0.583	1.0	1.0	1.0	1.0	1.0	1.0	0.417	0	0.667	0	0	0	0

Figure 5.5f. M_6 Fuzzy Multilinear Family

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	0.454	0	0	0	0	0	0.571	1.0	1.0	1.0	0.429	1.0	0.5	1.0	0	0	0.167	0	0.182	0
-	0.545	1.0	1.0	1.0	1.0	1.0	0.429	0	0	0	0.571	0	0.5	0	1.0	1.0	0.833	1.0	0.818	1.0

Figure 5.5g. M_7 Fuzzy Multilinear Family

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	0.414	1.0	0.222	1.0	0	0	0.619	0	0.5	0	0.381	0	0	1.0	0.182	1.0	0.211	1.0	1.0	0.692
-	0.586	0	0.778	0	1.0	1.0	0.381	1.0	0.5	1.0	0.619	1.0	1.0	0	0.818	0	0.789	0	0	0.307

Figure 5.5h. M_8 Fuzzy Multilinear Family

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	0.409	0	0.710	0	0.45	0	0.375	0	0	0	0	0	0	0	0.591	1.0	0.48	1.0	1.0	1.0
-	0.591	1.0	0.290	1.0	0.55	1.0	0.625	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.409	0	0.52	0	0	0

Second, these crisp areas give us an insight into the unique properties of these particular melodies, since they afford the opportunity to create regions with crisp contour, despite the potential of the phasing process to create conflict with regard to contour (as seen in the M_3 region). Consider Figure 5.6, which shows a fully-phased occurrence of M_5 at rehearsal 214. We see multiple forces at work in this passage that contribute to the crispness displayed in M_5 's fuzzy multilinear family. First, strategic rests during attacks 9, 10, 11, and 12 in M_5T_0 open space for the contours of the M_5T_1 and M_5T_2 melodies to emerge, resulting in the long uninterrupted descending string that we see in positions 8-13 in the fuzzy multilinear family.⁷² Similarly, the held note at attack point 7 in M_5T_2 permits these descents to be heard more fully, since they are the moving lines. Finally, repeating motions, such as the double descent from the A5 to the F#5 highlighted with brackets in the figure, allow for overlapping of fragments that results in a long string of non-competing motions. These features of Reich's melodies are important to note because, although they occur frequently throughout this piece, the same cannot be said of just any melody, suggesting a compositional desire on the part of Reich for these emergent patterns to occur within the work.

⁷² This same phenomenon also contributes to the string of ascents heard at the end of the pattern.

Figure 5.6. M₅ in the flutes, showing the T₁ and T₂ phasing in flute 2 and flute 3 respectively

The image shows a musical score for three flutes (Fl. 1, Fl. 2, and Fl. 3) in a key signature of one sharp (F#). The score is annotated with several features:

- Strategic Rests:** Red circles highlight rests in Fl. 1 and Fl. 3.
- Static Note:** A red circle highlights a note in Fl. 3.
- Overlapping Descents:** Red brackets highlight descending lines in Fl. 2 and Fl. 3.
- A box with the number **215** is located in the top left corner.

Third, the placement of these crisp regions can set up expectations with regard to pattern completion and the formal structure of each region. In some families, the crisp regions begin intermittently, with either one or two positions in a row. The M₈ family, for example, features intermittent crisp motions in positions two, four, and six. Such crispness does not continue beyond a single position until position eight. However, two significant areas within the CAS grid possess longer strings of crispness in many of the families. First, a long string of crispness occurs around position eight, which is toward the end of the first measure of the two-measure pattern. For example, a long string of crispness occurs from positions six to fourteen in M₂, and similar strings occur in positions eight through 13 in M₅, in positions eight through ten in M₆, and in positions eight through fourteen in M₈. This string of crispness occurs in about half of the fuzzy families, although others also possess more isolated instances of crispness at position 8, which still contributes to this perception. A second, even stronger tendency for crispness occurs at the end of the pattern itself. In all but M₇, the fuzzy multilinear families possess strings of crispness at least two positions long (but more commonly ranging to four or five positions). Such strong crispness may become an indicator of “ending” that the listener may be able to perceive as the pattern moves through its various repetitions, in essence becoming a convention of these multilinear melody units.

These melodic tendencies come to the forefront as the pattern repeats, making it important to also discuss the build-up of these units, as the gradual process of phasing the melody will result in changes to our perception of the passage's contour. In the M_1 region, we saw that the build-up of the phasing began to challenge the originally clear perception of the M_1T_0 melody. As iterations of the phasing were introduced, lower membership values for M_1T_0 indicated that its primacy within the pattern became less stable, as the two phased melodies gradually became more stable, as shown in Figure 5.3b and Figure 5.3d. Analyzing the build-up of each melody region shows us that all of the melodies follow this pattern (complete analyses of each melody region appear in Appendix 2). In each melody region, the phasing process begins with the crisp statement of the T_0 melody, and then that melody gradually becomes obscured by the phasing process. For example, consider the build-up of the M_7 region, shown in Figures 5.7a and 7b. In this analysis, we see two forces at work. First the phasing process has caused the membership values of the T_0 version to drop from 1.0, while the transposed melodies gradually rise in membership as each iteration is added. Second, the M_7 region features a melodic change after the ninth iteration, as shown in Figure 5.7c. This change alters two of the CAS positions, causing the familial membership values to drop for the T_0 and T_1 versions, as this initial hearing of the altered melody is unexpected. However, the T_2 version rises in membership. Therefore, this addition of altered melody further complicates our ability to perceive these melodic versions. Our perception recovers slightly as new iterations are heard, but these complicating factors challenge one's sense of what "the melody" is, giving way to new possibilities for perception that are reflected in the fuzzy multilinear families of Figure 5.5.

Figure 5.7a. Gradual membership of M₇ melodies within the evolving M₇ multilinear family

Iteration of M ₇ composite	Membership M ₇ (T ₀) (flute 1)	Membership M ₇ (T ₁) (flute 2)	Membership M ₇ (T ₂) (flute 3)
1 st time	1.0	--	--
2	1.0	--	--
3	1.0	0.333	0.333
4	1.0	1.0	1.0
5	1.0	0.400	0.400
6	0.938	0.770	0.720
7	0.910	0.782	0.748
8	0.894	0.789	0.764
9	0.883	0.793	0.773
10	0.691	0.694	0.788
11	0.705	0.706	0.787
12	0.715	0.715	0.787
13	0.724	0.722	0.787
FINAL MEMBERSHIP	0.732	0.727	0.786

Figure 5.7b. Graph of Membership Value across the M₇ Region

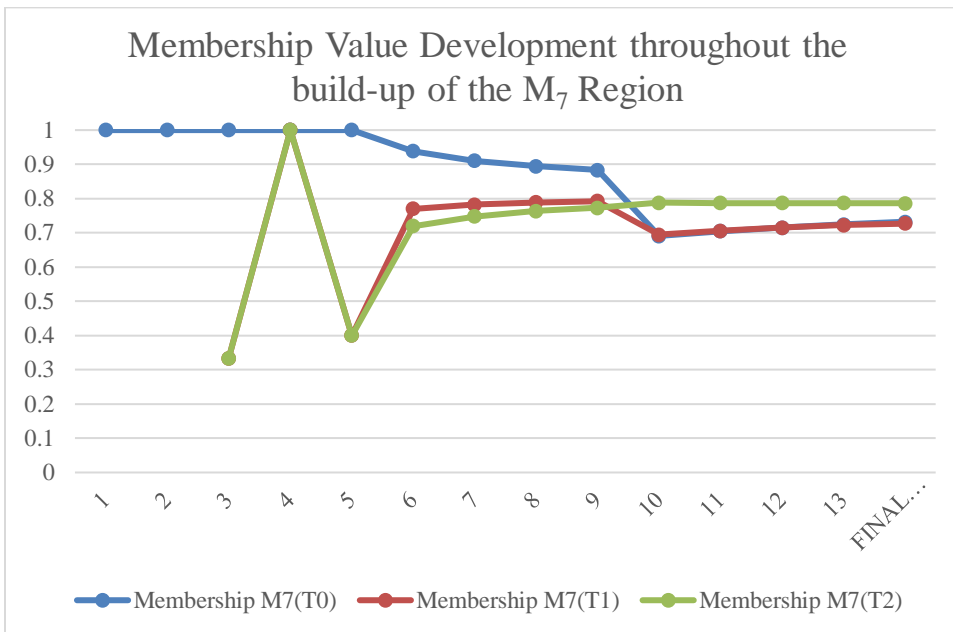


Figure 5.7c. Comparison of phased M_{7a} and M_{7b} melodies, showing where the CAS changes

M7a

Musical score for M7a, measures 227-230. The score is for three flutes (Fl. 1, Fl. 2, Fl. 3) in G major. Red '+' and '-' signs are placed above notes to indicate changes in the Canonical Analysis Set (CAS). In measure 227, Fl. 1 has a '-' sign above the first note, and Fl. 2 and 3 have '+' signs above their first notes. In measure 228, Fl. 1 has a '+' sign above the second note, and Fl. 2 and 3 have '+' signs above their first notes. In measure 229, Fl. 1 has a '+' sign above the second note, and Fl. 2 and 3 have '+' signs above their first notes. In measure 230, Fl. 1 has a '+' sign above the second note, and Fl. 2 and 3 have '+' signs above their first notes.

M7b

Musical score for M7b, measures 230-233. The score is for three flutes (1, 2, 3) in G major. Red '+' and '-' signs are placed above notes to indicate changes in the Canonical Analysis Set (CAS). In measure 230, Fl. 1 has a '-' sign above the first note, and Fl. 2 and 3 have '+' signs above their first notes. In measure 231, Fl. 1 has a '+' sign above the second note, and Fl. 2 and 3 have '+' signs above their first notes. In measure 232, Fl. 1 has a '+' sign above the second note, and Fl. 2 and 3 have '+' signs above their first notes. In measure 233, Fl. 1 has a '+' sign above the second note, and Fl. 2 and 3 have '+' signs above their first notes.

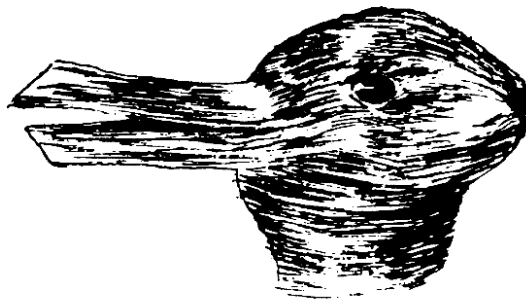
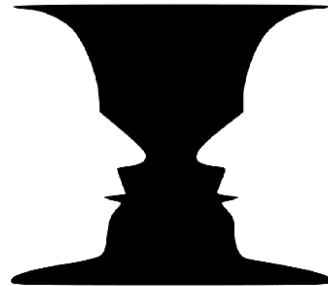
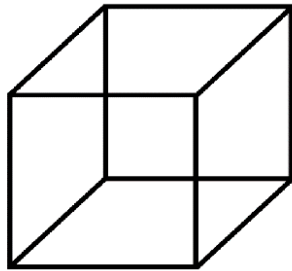
Multistable Perceptions

The possibilities inherent in the fuzzy multilinear families open the door for multiple possible hearings of each pattern, and this multiplicity is an important aesthetic quality that Steve Reich cultivates in his music. About Reich’s music, Paul Hillier writes that his “special brilliance lies in making apparently simple melodic/rhythmic states yield surprising aural ambiguities, so that our sense of a phrase’s identity—its beginning and end, or the precise location of its downbeat or principle accents—may suddenly shift as new light is shed on it from within” (Hillier 2002, 4-5). Similarly, Reich himself writes that “In this way, one’s listening mind can shift back and forth within the musical fabric, because the fabric encourages that... But if you don’t build in that flexibility of perspective then you wind up with something extremely flat-footed and boring” (Reich

2002, 130). The musical design itself is about possibilities, suggesting intentional ambiguity regarding the perception of these kinds of passages.

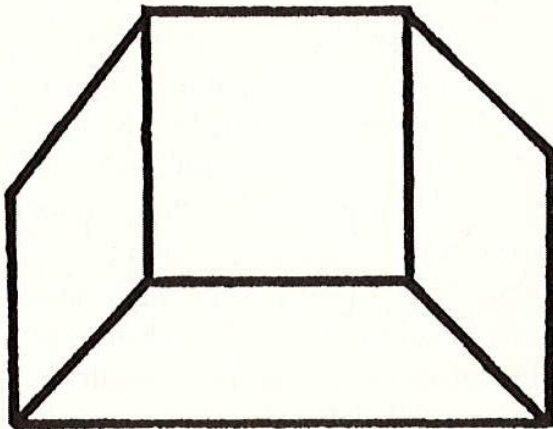
Reich's words call to mind ambiguities inherent in many visual phenomena. Images such as the Necker cube (Necker 1832, Fournier 2010, Karpinski 2012), the duck-rabbit (Ogden 2010, Karpinski 2012), the Rubin vase/face illusion (Rubin 1958, Fournier 2010), and many others (shown in Figure 5.8) possess an inherent ambiguity that causes the viewer of the image to "shift back and forth" between two or more different interpretations (i.e., different faces of the cube, the vase or faces, and the duck or the rabbit). The kind of ambiguity we see in these images is termed multistability.

Figure 5.8. Multistable visual phenomena (Necker 1832, Rubin 1958, Fournier 2010, Ogden 2010, Karpinski 2012)



Consider Figure 5.9a, which shows another visual stimulus that is multistable in its perception. Don Ihde writes that one could view the image either as a hallway, or as a cut-off pyramid (2012, 47). Ihde uses this figure to illustrate a series of hierarchical levels of perceptual awareness that occur when presented with figures such as these. Those experiencing the first level, which he terms “literal-mindedness” only see one of the multiple possibilities in a figure, and cannot see others.⁷³ Those in the second level, which Ihde terms “polymorphic-mindedness” are able to see both the hallway and the cut-off pyramid (albeit not at the same time). The ability of the viewer at this level to switch between perceptions at will becomes permanent at this stage and the viewer will no longer be able to return to the literal-mindedness of the previous level (Ihde 2012, 45-50).

Figure 5.9a. Ihde’s pyramid/hallway diagram (Ihde 2012, 47)



⁷³ In visual phenomena, such as these, Ihde asserts that this “literal-mindedness” level occurs at a level lower than that with which most of us begin, since most will see both the hallway and the pyramid within a few moments. However, I contend that when we discuss multistability in the auditory realm, listeners experience this “literal-mindedness” level much more often, especially when listening passively.

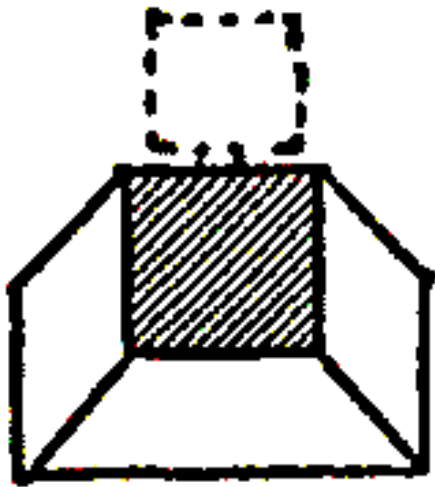
This second level opens a possibility for more active observation of the figure, allowing the viewer not only to see both initial possibilities, but also allows us to ask what further interpretations could exist within the figure. Ihde posits a third interpretation: “Suppose, now, the figure is presented again, only this time there is a third response from a group of people viewing the drawing. This new group, temporarily placed at the literal-minded level, claims the figure is neither a hallway nor a pyramid, but is a *headless robot*” (2012, 51), shown in Figure 5.9b. He continues:

This new modification of polymorphic-mindedness introduces a new variable and a new question. If what was taken to be *the* appearance of the noema has given way to two alternate appearances, and these have now given way to a third, has the range of noematic possibilities been exhausted?...The new element points to the inherent radicalism of variational method. The possibilization of a phenomenon *opens* it to its *topographical* structure. The noema is viewed in terms of an open range of possibilities and these are actively sought noetically. Thus, a special kind of viewing occurs, which looks for what is *not usually* seen. (Ihde 2012, 53)⁷⁴

In this depiction, Ihde illustrates a switch from what Husserl would call a natural attitude to a phenomenological one, wherein the viewer begins to discover the “genuine possibilities and the invariants inhabiting those possibilities” (Ihde 2012, 50) within the figure. This phenomenological attitude seeks a level of richness within the phenomenon that passive observation associated with the natural attitude lacks.

⁷⁴ In Husserlian phenomenology, a noema is an intentional object, an object of thought or perception (Ihde 2012, 25–26)

Figure 5.9b. Ihde's guide picture depicting the multistable image as a headless robot (Ihde 2012, 51)



It is this richness of possibility inherent within multistable visual phenomena that I believe is analogous to Reich's compositional desires, which is further reinforced by yet another quote from Williams's "The Orchestra" that Reich chose to set in the second movement: "I am wide awake. The mind is listening" (Williams 1986, 251). Reich's musical fabric encourages the active phenomenological perspective through the experience of perceptual possibilities, which occur through the context of repetition.

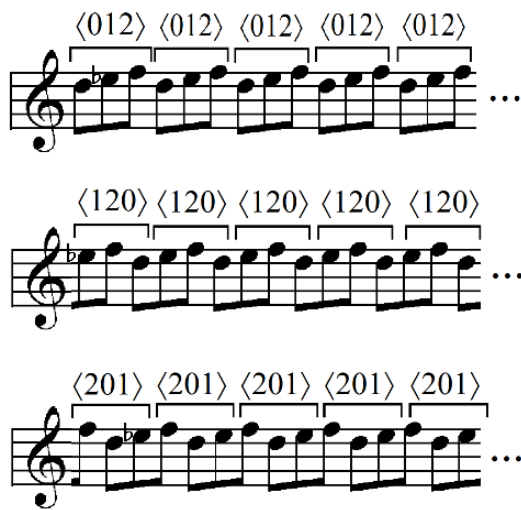
This repetitive structure is crucial to the multistability analogy for minimalist music. Ihde, Karpinski (2012), and others note that in the visual realm, a key characteristic of multistability that distinguishes it from other kinds of ambiguity is that the perceptual shift between possibilities occurs without the figure actually changing at all. Since the nature of music is fleeting, the use of repetition is a way to capture this kind of stasis on the part of the (in this case, musical) stimulus. The repetitions afford the listener the same kind of sense that they are hearing "the same figure" in the repetitions of a musical pattern, just as they would understand that they are looking at "the same

figure” in the visual stimulus examples. In the context of these repetitive figures, it becomes possible for the active listener to hear the repeated passages in different ways. Consider Figure 5.10a, which shows a highly repetitive, yet seemingly simplistic passage from Philip Glass’s *Two Pages*. In this passage, the D-Eb-F passage is repeated so many times (nineteen to be exact), that the listener begins to perceive a kind of stasis in the unchanging repetitions of the figure. The active mind then is able to move beyond the literal-mindedness of the initial contour pattern they hear, in order to perceive two other possible contour patterns within the unchanging passage, as shown in Figure 5.10b. The first possible pattern takes the ascent from D to F to be the structurally organizing framework (i.e., placing the lowest pitch as the perceived downbeat), the second may structure perception around hearing the upper corner of the pattern created by the Eb-F-D figure as important, and finally the third possibility treats the highest notes (F) as structural downbeats, placing them at the beginning of each repeated unit. Additionally, the timespan of the repetitions gives the listener time to explore the perceptual shift that happens when one is aware of each of these patterns. Therefore, actively attending to different structural organizations within the unit opens the listener to possibilities beyond a literal-minded hearing of the passage.

Figure 5.10a. Excerpt from Philip Glass’s *Two Pages*



Figure 5.10b. Multistability of contour perception within the static passage from Glass's Two Pages



Multistability in Reich's Music

Indeed, scholars of Reich's music have commented that this kind of perception takes place in Reich's phasing music as well, given a mind that is attuned to this kind of phenomenological attitude. Regarding Reich's earlier phasing piece *Phase Patterns* (1970), Richard Cohn discusses potential ways one could experience the patterns of the work. He writes that his analysis focused "on the assumption that variations in attack-point frequency are a primary component of the listening experience" (1993, 157).

However, he acknowledges that other possible ways of hearing are possible depending on what kinds of parameters one considers important. He explains that if one were to attend instead to attack density rather than attack frequency, one would "be susceptible to a different phenomenological interpretation: while the total number of attacks intensifies to maximum, the number of doubled attacks deintensifies, at the same rate, to zero. A

listener exclusively attuned to such relations will thus have a different teleological experience than a listener attuned to variations in attack-point density” (157).

Similarly, Roeder’s analysis of accent patterns in Reich’s music illustrates how the repetitiveness that is unique to minimalist music permits listeners to attend to different kinds of accents and subsequent patterns that they create within the repetitive framework of the music. Regarding the build-up of a pattern in *New York Counterpoint* (1985), he writes that “when a pattern is building up, the accent one attributes to its attack varies considerably with the degree of completeness of the pattern. When one attends to accent, one hears hardly any exact repetition in this nominally ‘repetitive’ music” (2003, 287). In this quote, Roeder describes a listener who is attending to a specific parameter, and as a result hears different patterns within the repetitive context of the music. Such analyses describe the potential inherent within Reich’s musical structures that allow for multistable perceptions of the same passage to occur.

Returning then to the quasi-phased passages in *The Desert Music*, we see that Reich’s conception of melody in this passage is inclusive of this multistability, or “flexibility” as he calls it. Such flexibility allows active listeners to attend in different ways to the passage, and the relative stability or ease with which these ways are perceived can be measured by placing the perception against the fuzzy multilinear families shown in Figure 5.5, and calculating the potential membership that this perception may have within the family. When listening to the M_1 region for example, a listener intent on preserving the untransposed M_1 melody that began the section may continue to hear that pattern. Figure 5.11 shows this alignment: the positions highlighted in red indicate M_1T_0 ’s pathway through the M_1 region’s multilinear space that the grid

represents. Aligned against the family in this way, the M_1T_0 pattern produces a membership value of 0.757, suggesting that this hearing is possible (and indeed probable given the relatively high value), but requires some degree of active engagement on the part of the listener (in other words, it may not be a passive hearing) given the interference created by M_1T_1 and M_1T_2 that contribute to M_1T_0 's non-1.0 membership value.⁷⁵ Figures 5.12a and 5.12b present an alternative potential hearing of the M_1 region's multilinear family: a listener attending to the uppermost line of the composite melody presented in Figure 5.12a may hear an oscillation of ascent and descent, as highlighted by the line's CAS $\langle =-+-+--+--+--+--+--+ \rangle$, carrying a membership of 0.688 within the M_1 region's fuzzy multilinear family, shown in Figure 5.12b.⁷⁶ Conversely, a listener attentive to overall contour may perceive the pattern according to the crisp motions within the M_1 region's CAS grid (as shown in Figure 5.13), and following the most dominant path in positions where multiple directional motions are occurring simultaneously. Such a hearing would produce a potential CAS of $\langle --+-----+-----+----- \rangle$, which has a membership value of 0.828 within M_1 's fuzzy multilinear family.⁷⁷ This would reflect the

⁷⁵ The calculation of this membership value is calculated based on the pathways highlighted in Figure 5.11, and is as follows: $(0.56 + 1 + 1 + 0.722 + 0.538 + 1 + 0.583 + 0.583 + 0.583 + 1) / 10 = 0.757$.

⁷⁶ The line of zeros in Figure 5.12b represents the fact that there are no plateaus in any of the individual melodies that make up the fuzzy multilinear family. However, the composite shown in Figure 5.12a highlights a possibility that a listener may hear a plateau in the composite melody. This hearing is accounted for in position one of Figure 5.12b, where the motion between the two B5s is perceived, despite its presence in the family. This lessens this potential hearing's overall membership value. The calculation for this hearing is as follows: $(0 + 1 + 1 + 0.722 + 1 + 0.5 + 1 + 0.461 + 1 + 0.583 + 1 + 0.5 + 1 + 0.417 + 1 + 0.583 + 0 + 1 + 0 + 1) / 20 = 13.766 / 20 = 0.688$.

⁷⁷ This best-fit CAS actually yields four equal possibilities because there are two positions (positions 6 and 12) that are evenly distributed between ascent and descent. This further contributes to the flexibility with which a listener may be able to switch between possibilities upon multiple hearings of the pattern. As with the calculations in Figure 5.11 and Figure 5.12, this membership value is calculated by finding the sum of the values highlighted in red, and dividing by the number of highlighted values (in this case, 20).

Figure 5.11. Alignment of M_1T_0 against the fuzzy multilinear family of the M_1 region

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	0.44	0	1.0	0.278	1.0	0.5	1.0	0.538	1.0	0.417	1.0	0.5	1.0	0.583	1.0	0.417	0	0	0	0
-	0.56	1.0	0	0.722	0	0.5	0	0.461	0	0.583	0	0.5	0	0.417	0	0.583	1.0	1.0	1.0	1.0

Figure 5.12a. Composite melody of the M_1 region

CAS of highest pitches <= - + - + - + - + - + - + - + - + - >

Figure 5.12b. Alignment of composite contour against the fuzzy multilinear family of the M_1 region

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	0.44	0	1.0	0.278	1.0	0.5	1.0	0.538	1.0	0.417	1.0	0.5	1.0	0.583	1.0	0.417	0	0	0	0
=	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-	0.56	1.0	0	0.722	0	0.5	0	0.461	0	0.583	0	0.5	0	0.417	0	0.583	1.0	1.0	1.0	1.0

Figure 5.13. Potential alignment that follows highest-valued motions against the fuzzy multilinear family of the M_1 region

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	0.44	0	1.0	0.278	1.0	0.5	1.0	0.538	1.0	0.417	1.0	0.5	1.0	0.583	1.0	0.417	0	0	0	0
-	0.56	1.0	0	0.722	0	0.5	0	0.461	0	0.583	0	0.5	0	0.417	0	0.583	1.0	1.0	1.0	1.0

least conceptually challenging pathway on the part of the listener, although this may not directly suggest that this is the most probable hearing. More likely, a listener may hear some variant combination of these possibilities, depending on what the active listener is attending to. Furthermore, as the passage repeats nine times, there is potential for listeners to experience the passage slightly differently each time it is heard. The result is a number of possible interpretations of the passage, each possessing a specific relationship (as indicated by the membership value) to the fuzzy multilinear family.

This potential is significant because it is what enables the listener to perceive the potential relatedness between the melodies (and thus corresponding regions) that Quinn highlights. Figure 5.14a, for example, shows the beginning of the next section, containing the melody M_{2a} . The CAS of this melody, <+----+----++>, has a best-fit membership value of 0.771, suggesting that it is a contender among the possibilities inherent within the fuzzy multilinear family of M_1 , as shown in Figure 5.14b.⁷⁸ Indeed, the melody's best-fit membership value falls within the range of membership values exhibited by the various transpositions of M_1 , which range from 0.601 to 0.814 as shown in the chart in Figure 5.15.

This is not to say that the listener will easily hear this, intentionally pick this contour out of the fuzzy multilinear structure, or indeed even perceive this relationship at all in the continuous hearing of the work. This relationship is significant, however, in that M_{2a} 's retrospective relationship to the M_1 region promotes a smoother transition between

⁷⁸ The calculation is as follows: $(0.44 + 1.0 + 0.722 + 0.5 + 1 + 0.461 + 0.583 + 1 + 1 + 1) / 10 = 7.706 / 10 = 0.771$.

Figure 5.14a. Melody 2a in the flute



Figure 5.14b. CAS of M_{2a} against the multilinear composite family of M_1 , showing best fit

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	0.44	0	1.0	0.278	1.0	0.5	1.0	0.538	1.0	0.417	1.0	0.5	1.0	0.583	1.0	0.417	0	0	0	0
-	0.56	1.0	0	0.722	0	0.5	0	0.461	0	0.583	0	0.5	0	0.417	0	0.583	1.0	1.0	1.0	1.0

Figure 5.15. Membership values of transposed variants within their own family, as well as best fit of the melody against preceding fuzzy families

ID	Alignment	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8
M_1	T_0 : (1,2,3,4,8,9,10,14,16,17)	0.757							
	T_1 : (2,4,5,6,10,11,12,15,18,19)	0.814							
	T_2 : (1,4,6,7,8,12,13,14,16,20)	0.601							
	Best Fit	0.881							
M_{2a}	T_0 : (1,2,3,7,8,9,13,15,16,17)		0.883						
	T_1 : (1,3,4,5,9,10,11,14,17,18)		0.914						
	T_2 : (3,5,6,7,11,12,13,15,19,20)		0.860						
	Best Fit	0.771							
M_{2b}	T_0 : (1,2,3,7,8,9,13,15,16,17)		0.883						
	T_1 : (1,3,4,5,9,10,11,14,17,18)		0.914						
	T_2 : (3,5,6,7,11,12,13,15,19,20)		0.860						
	Best Fit	0.771							

ID	Alignment	M ₁	M ₂	M ₃	M ₄	M ₅	M ₆	M ₇	M ₈
M _{3a}	T ₀ :(1,5,6,7,11,13,14,15,19,20)			0.763					
	T ₁ :(1,2,3,7,8,9,12,15,16,17)		0.633						
	T ₂ :(1,3,4,5,9,10,11,13,17,18)		0.705						
	Best Fit		0.900						
M _{3b}	T ₀ :(1,5,6,7,11,13,14,15,19,20)			0.700					
	T ₁ :(1,2,3,7,8,9,12,15,16,17)		0.485						
	T ₂ :(1,3,4,5,9,10,11,13,17,18)		0.480						
	Best Fit		0.850						
M _{4a}	T ₀ :(1,2,3,7,9,10,11,15,16,17)			0.752					
	T ₁ :(3,4,5,8,11,12,13,16,18,19)		0.785						
	T ₂ :(1,5,6,7,9,13,14,15,18,20)		0.664						
	Best Fit	0.810							
M _{4b}	T ₀ :(1,2,3,7,9,10,11,15,16,17)			0.757					
	T ₁ :(3,4,5,8,11,12,13,16,18,19)		0.830						
	T ₂ :(1,5,6,7,9,13,14,15,18,20)		0.697						
	Best Fit	0.865							
M ₅	T ₀ :(1,5,7,8,9,13,14,15,16,20)					0.800			
	T ₁ :(1,2,3,6,9,10,11,14,16,17)				0.767				
	T ₂ :(3,4,5,7,11,12,13,16,18,19)				0.800				
	Best Fit			0.959					
M _{6a}	T ₀ :(1,2,3,7,9,10,11,15,16,17)						0.852		
	T ₁ :(3,4,5,8,11,12,13,16,18,19)					0.875			
	T ₂ :(1,5,6,7,9,13,14,15,18,20)					0.838			
	Best Fit			0.767					
M _{6b}	T ₀ :(1,2,3,7,9,10,11,15,16,17)						0.852		
	T ₁ :(3,4,5,8,11,12,13,16,18,19)					0.875			
	T ₂ :(1,5,6,7,9,13,14,15,18,20)					0.838			
	Best Fit			0.767					

ID	Alignment	M ₁	M ₂	M ₃	M ₄	M ₅	M ₆	M ₇	M ₈
M _{6c}	T ₀ :(1,2,3,7,9,10,11,15,16,17)						0.785		
	T ₁ :(3,4,5,8,11,12,13,16,18,19)						0.811		
	T ₂ :(1,5,6,7,9,13,14,15,18,20)						0.838		
	Best Fit					0.883			
M _{7a}	T ₀ :(1,5,6,7,11,13,14,15,19,20)							0.833	
	T ₁ :(1,2,3,7,8,9,12,15,16,17)							0.768	
	T ₂ :(1,3,4,5,9,10,11,13,17,18)							0.731	
	Best Fit						0.699		
M _{7b}	T ₀ :(1,5,6,7,11,13,14,15,19,20)							0.732	
	T ₁ :(1,2,3,7,8,9,12,15,16,17)							0.727	
	T ₂ :(1,3,4,5,9,10,11,13,17,18)							0.786	
	Best Fit						0.699		
M _{8a}	T ₀ :(1,2,3,7,8,9,13,15,16,17)								0.803
	T ₁ :(1,3,4,5,9,10,11,14,17,18)								0.762
	T ₂ :(3,5,6,7,11,12,13,15,19,20)								0.804
	Best Fit							0.915	
M _{8b}	T ₀ :(1,2,3,7,8,9,13,15,16,17)								0.803
	T ₁ :(1,3,4,5,9,10,11,14,17,18)								0.762
	T ₂ :(3,5,6,7,11,12,13,15,19,20)								0.804
	Best Fit							0.915	
M _{8c}	T ₀ :(1,2,3,7,8,9,13,15,16,17)								0.803
	T ₁ :(1,3,4,5,9,10,11,14,17,18)								0.762
	T ₂ :(3,5,6,7,11,12,13,15,19,20)								0.804
	Best Fit							0.915	

regions that are already marked with jarring changes of texture (drastic reduction of forces, sudden lack of phasing, etc.). Additionally, it is the multiplicity of possibilities inherent within the fuzzy multilinear family that has the ability to mask the differences between the crisp M_1 and M_{2a} melodies.

Figure 5.15 shows the analogous best-fit memberships of all of the melodies against the fuzzy multilinear family of the melody immediately preceding it. The chart highlights a pattern of transitional possibility indicated by high potential values of membership for melodies against the preceding region's family. The M_{3a} melody's membership within M_2 for example, is 0.900, a very high membership value that again falls within the range of membership values for the six M_2 variants (ranging from 0.860 to 0.915). The M_{4a} melody's membership within M_3 is 0.810, higher than the six members of M_3 . These values suggest a uniformity of process, as well as a significant degree of shared contour material.

Region M_4 is the last region in the A section of the movement (as shown in the form diagram in Figure 5.4), and the M_5 region picks up the melodic process again in the A' section. Nevertheless, M_5 's relationship to the fuzzy multilinear family of the M_4 region is quite high, 0.959. M_{6a} 's membership against the M_5 family is 0.767, M_{7a} 's membership against the M_6 family is 0.699, and M_{8a} 's membership against the M_7 family is 0.915. This range of transitional values from 0.699 to 0.959 shows that although these melodies may initially seem different when examined by themselves (as indicated by their mid-range membership values calculated in Figure 5.1c), the context of phasing provides a sonic environment that not only facilitates the transition between regions, but offers a heightened perception of familial similarity between members. In other words,

the ability to understand and perceive different possible contours within the context of the fuzzy multilinear family gives us the conceptual apparatus necessary to understand the potential relationships between the melodies themselves.

Level-2 Fuzzy Relations Between Fuzzy Multilinear Families

One may wonder if the above observations are coincidental, but I am not convinced that this is the case. Reich intentionally builds “flexibility” into his musical fabric, and this flexibility opens the door to these kinds of phenomenological possibilities. The fuzzy multilinear families themselves have a good deal in common with one another, such that it is not surprising to see an ability to discern the other melodies from the fabric of one phased pattern. Consider Figure 5.16, which shows a fuzzy family that is comprised of the average of the eight fuzzy multilinear families in the passage.⁷⁹ This kind of fuzziness is termed a level-2 fuzzy family, and this family is the most representative illustration of the melodic regions that Quinn identifies.⁸⁰ It encompasses not only the melodies, but the fuzzy contour possibilities created by Reich’s phasing of the melodies.

⁷⁹ This fuzzy representation is comprised of the mathematical average of each position within the eight fuzzy multilinear families. For example, the ascent value for position one is calculated by averaging all position one ascents from the eight families shown in Figure 5.5:

$$(0.44 + 1 + 0.364 + 0.458 + 0.417 + 0.454 + 0.414 + 0.409) / 8 = 0.495$$

⁸⁰ Klir and Yuen define the level-2 fuzzy set as “fuzzy sets defined within a universal set whose elements are ordinary fuzzy sets” (Klir and Yuen 1995, 18). To put this into terms of melody families, the level-2 fuzzy family comprises elements that are fuzzy, with membership values that relate each fuzzy element to the higher-level fuzzy family, in similar ways that membership values of crisp melodies relate to a level-1 (standard) fuzzy family.

Adapting an equation from Quinn, one can measure the degree of membership of each fuzzy family within this larger level-2 fuzzy family. Quinn's equation, shown in Figure 5.17a, adjusts the calculation of C+SIM according to the mathematical difference between two corresponding positions, thereby accommodating two fuzzy sets. He writes that

it would be illogical to call (a,b) and (a',b'), 0.9 and 0.8 respectively, a mismatch simply because they are not the same, when in fact they are quite close. Let us rather increment C+SIM on that score, but discount the increment in proportion to the slight absolute difference of 0.1 between their membership levels. (Quinn 1997, 225)

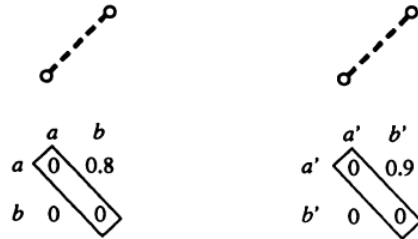
Quinn's previous work with C+SIM allowed him to compare crisp contours with the fuzzy family, just as I have done with the CAS grids in this dissertation. What this equation allows Quinn to do is to compare contour families that are already fuzzy, by accounting for the degree of similarity between two non-identical quantities (in Quinn's case 0.9 and 0.8). Quinn's equation adds 0.9 to the value of C+SIM since the corresponding positions aren't a crisp match, but are close (a difference of only 0.1). Likewise, we can adapt the equation to our CAS grid in order to determine how closely related each fuzzy multilinear family is to the average of all multilinear families. In essence, each position in the CAS grid is treated like one of Quinn's positions in his fuzzy C+matrices (" $\mu_{c+a,b}$ ", for example), and corresponding positions in each fuzzy CAS grid are compared just as Quinn compares corresponding C+matrix values. The overall membership value i is then calculated out of the total number of CAS positions (instead of C+matrices in Quinn's equation). For example, Figures 5.17b and 5.17c show hypothetical pairs of fuzzy contour families, and Figure 5.17d shows the analogous calculation that arrives at the value of similarity between the two families.

Figure 5.16. The Level-2 Fuzzy Family Comprised of the Eight Fuzzy Multilinear Families

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	0.49	0.34	0.47	0.37	0.39	0.18	0.49	0.31	0.43	0.30	0.32	0.35	0.24	0.49	0.54	0.67	0.39	0.70	0.64	0.58
	5	1	7	6	6	8	7	7	5	2	9	4	6	2	6	7	9	8	8	7
-	0.50	0.65	0.52	0.62	0.60	0.81	0.50	0.68	0.56	0.69	0.67	0.64	0.75	0.50	0.45	0.32	0.60	0.29	0.35	0.41
	5	9	3	4	4	3	2	3	6	8	2	6	4	8	4	3	1	2	2	3

Figure 5.17a. Quinn’s Fuzzy C+SIM formula

Figure 12. C+SIM on two tiny fuzzy contours



$$\begin{aligned}
 \text{C+SIM} &= \frac{1 - |\mu_{c^+}(a,b) - \mu_{c^+}(a',b')|}{i} + \frac{1 - |\mu_{c^+}(b,a) - \mu_{c^+}(b',a')|}{i} \\
 &= \frac{1 - |0.8 - 0.9|}{2} + \frac{1 - |0 - 0|}{2} \\
 &= \frac{1 - 0.1}{2} + \frac{1}{2} \\
 &= 1.9/2 = 0.95
 \end{aligned}$$

Figure 5.17b. Hypothetical fuzzy family a

Pos	1	2
+	0.760	0.667
-	0.24	0.333

Figure 5.17c. Hypothetical fuzzy family b

Pos	1	2
+	0.8	0.6
-	0.2	0.4

Figure 5.17d. Adaptation of Quinn's equation that compares fuzzy families a and b

$((1- 0.760-0.8) + (1- 0.667-0.6) + (1- 0.24-0.2) + (1- 0.333-0.4)) / 4 =$
$0.96 + 0.933 + 0.96 + 0.933 / 4 =$
$3.786 / 4 = 0.947$

Each of the four CAS positions are compared using the top part of Quinn's equation, and then the sum of these position comparisons is divided by four, resulting in a membership value of 0.947.⁸¹ Using this method to compare the eight level-1 fuzzy multilinear families presented in Figure 5.5 with the larger level-2 fuzzy family shown in Figure 5.16, Figures 5.18a and 5.18b show that the eight fuzzy multilinear families range in membership from 0.572 to 0.790. This is a more accurate portrayal of the familial relationships between these particular passages: it captures familial resemblances not

⁸¹ Note that this adaptation of Quinn's equation happens to work because the cardinality of the CAS grids is the same in this chapter. I do believe that it will be possible to adjust the equation further in order to accommodate fuzzy families of different cardinalities, yet this falls outside of the scope of this dissertation.

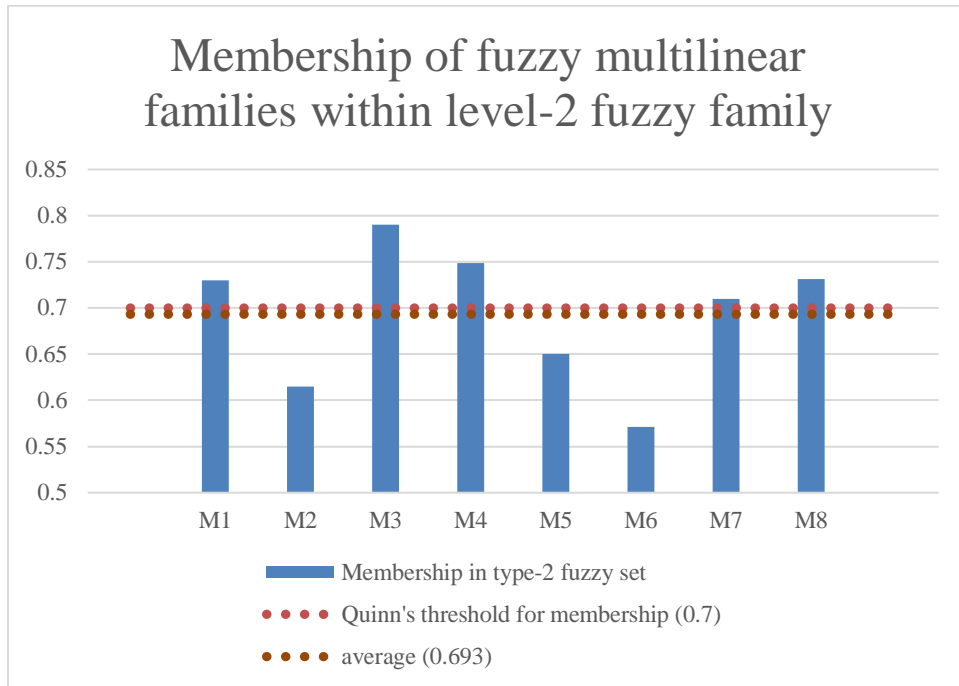
only between crisp melodies, but also the resemblances between the kinds of listener possibilities that each fuzzy multilinear family affords. Since these possibilities are an important compositional outcome for Reich, capturing them within these membership values provides an analysis that is truer to the musical experience of the work.

Furthermore, the membership values for these fuzzy multilinear families fall much more consistently closer to the 0.7 threshold for membership that Quinn proposes; in fact the average for the membership values is 0.693, closer than the average membership values derived by examining the melodies in the absence of phasing (0.660 using Quinn's C+SIM, and 0.612 using the FCM model). This increase highlights not only the fact that attending to the musical context enhances judgments of membership, but also that the method of using the FCM model combined with the type-2 fuzzy set formula yields a more accurate depiction of what Quinn desired to show in the first place.

Figure 5.18a. Membership of the eight fuzzy multilinear families within the level-2 fuzzy family

Fuzzy Multilinear Family	Membership in type-2 fuzzy set
M1	0.730
M2	0.615
M3	0.790
M4	0.749
M5	0.650
M6	0.571
M7	0.710
M8	0.731

Figure 5.18b. Membership within the level-2 fuzzy family, showing average membership and Quinn’s threshold for membership



Conclusion

The nuance of the fuzzy comparisons, both between the crisp melodies and their fuzzy multilinear familial regions and between the fuzzy multilinear families and the level-2 fuzzy family, allow us to understand more fully how these melodies may be “of a kind,” as Quinn claims. The analytical readings above show that the melodic regions identified by Quinn’s melodies share similarities not only in the crisp representation of Reich’s original melodies, but they also share similarities in build-up, in phasing process, and in the affordances inherent within the fuzzy multilinear families that give rise to multistable perception. Indeed, this process does not only occur in the A sections of the

third movement, but rather permeates the entire work. The phased multilinear melody is therefore a crucial aspect of Reich's concept of melody, and has potential to inform both the structure of the work, as well as the musical structure's interaction with Reich's chosen text, and the work's subsequent meaning. These interactions seem deliberate on Reich's part, and are worthy of further research.

As I have shown, the FCM model is uniquely suited to illustrate these phenomena due to its flexibility in terms of cardinality and subsequently its ability to accommodate rhythmically displaced family members. Through this process, one can show how Reich's phasing creates possibilities of perception that align with the composer's intent to build flexibility into his musical fabric for the phenomenologically minded. The FCM model therefore yields deeper insight into the inner workings of the music, Reich's compositional process, and the listener's potential perceptions of the work. Through the application of the FCM model, we become "wide awake" to new possibilities of hearing, and this can have a profound effect on our ability to perceive these families as related. In other words, our minds are listening.

CHAPTER 6: Conclusions

It has been around thirty years since systematic tools for the study of contour made their way into pitch-class set theory. In that time, many analysts have made use of these tools to comment on relations between melodic shapes in various genres including post-tonal music (Marvin and Laprade (1987), Morris (1987, 1993), Quinn (1997), and Schultz (2008, 2009, 2012) just to name a few) and world music (notably in the work of Rob Schultz (2016, 2009), but also in the works of Michael Tenzer (2006), and Aaron Carter-Enyi (2016)). Until recently, however, the limitations of those earlier tools have prevented them from achieving a wider popularity, and subsequently truly having a seat at the table in the discussion of the various musical parameters that contribute to our larger understanding of musical structure. In this dissertation, I hope to have addressed a few of these limitations, namely those that rely on strictly defined cardinality relationships between contours as well as those that only measure relationships between two contours at a time.

The familial contour membership model (FCM model) that I have developed is capable of addressing both of these concerns. The model combines the mathematical power of Ian Quinn's fuzzy set-theoretical approach (1993, 2001) with the analytical power of Rob Schultz's diachronic-transformational model of contour relations (2009, 2012). By creating a generalized transformational tree diagram that expresses all contour motions within a family of related contours in terms of their probability to occur in the family, the model allows an analyst to judge not only specific relationships between

contours, but also relationships between specific contours and expectations created by the perceptual experience of all the other contours in a family of related melodies. Because this model does not rely on internal structural characteristics, but rather transformational possibilities, it has the power to include families of related contours that do not have the same number of notes in ways that are sensitive to both the familial relationship of a set of related contours, as well as the crisp individuality that each contour in the family possesses. It is in this way that the FCM model addresses issues that I believe have limited contour theory's use in wider theoretical and analytical circles in the past.

Through my three analytical case studies in Part II, I have shown that this model of contour relations has the analytical power to apply to a wide variety of repertoire, and to comment on many broader theoretical areas of inquiry, including musical ontology, narrative, compositional strategy, and perception. In Chapter Three, I explored how fuzzy contour relations can contribute to our understanding of the complications surrounding the ontology of plainchant (and of music in general). By quantifying familial relationships between chant variants, we can begin to examine more systematically the distinction that Leo Treitler makes between significant and minor variations between chants that are understood to be "the same" (Treitler 2003, 148). In exploring these issues, the data I collected through the model also allowed me to comment on issues of memory and melodic reconstruction in the performance of the oral tradition of plainchant both before and after notation.

Chapter Four demonstrated the FCM model's ability to address issues of compositional strategy through the study of Brahms's motivic development. By

examining the motivic relationships between Brahms's *Regenlied*, Op. 59 No. 3, and the related Violin Sonata No. 1 in G major, Op. 78 I illustrated how the study of contour families could explain Brahms's use of contour in some places to contribute to motivic cohesion and in other places to drive motivic development. In doing so, I also explored how Brahms's use of contour in these ways contributes to the development of a narrative connection between Klaus Groth's poem and the motivic relationships developed in the music. Furthermore, I explored how Brahms's use of the *Regenlied* motives in the violin sonata shifted the narrative of the song to one of personal grief that reflects Brahms's feelings regarding the death of his godson, Felix Schumann. In this way, I showed how contour exposes aspects of Brahms's compositional tendencies, such as the relationship between contour and narrative in the song, or the developmental relationships across the sonata. Using contour to shed light onto Brahms's motivic development therefore gives us new appreciation for the complexities of his motivic mastery.

In Chapter Five, I returned to the analytical environment in which fuzzy contour theory began: the minimalist compositions of Steve Reich, as illustrated through his 1984 work *The Desert Music*. I showed how the FCM model is uniquely suited to demonstrate the perceptual possibilities inherent within Reich's use of phasing as an aspect of melodic construction. The lack of cardinality constraint enabled me to use the generalized CAS grid as an articulation grid, allowing the beat-class transpositions of each melodic segment to be modeled against one another in their proper rhythmic position as presented in the music. In doing so, I was able to show how the phasing patterns that Reich creates allow listeners a range of multistable possibilities with regard to their hearing of the

melody in each passage; a compositional intention, I might add, that lies at the heart of Reich's compositional philosophy.

Avenues of Further Research

Through these analyses, I have explored contour's ability to speak to many aspects of music-theoretical and musicological areas of inquiry. I hope also to have shown that the FCM model's sensitivity to cardinality differences is crucial to our understanding of melody in each case. The model's particular strength of providing comparative data for large families of related contours paves the way for new analytical avenues when it comes to understanding melody. I believe that the model's adaptability to varying analytical contexts gives it the power to shape the way we understand melody in more nuanced ways, connecting with debates about melodic structure in music cognition, musicology, and ethnomusicology.

The FCM model has particularly unique potential when it comes to corpus study. Bringing a quantitative measure of melodic similarity between members of a large family into corpus studies of many genres has the power to reveal structural characteristics of melody that might otherwise be missed in less quantitative approaches to melodic shape. This kind of analysis could be particularly useful in analyses of current musical traditions that have retained an oral approach toward dissemination and performance. Such corpora could include the historical spread and current practice of folk song traditions, or the improvisational practices found in jazz. These kinds of studies could also have the ability

to comment on modern-day level-2 fuzzy families of the kind that have been lost to us in plainchant, and this will not only yield insights into the ontology of these corpora, but could also yield some insights into the potential structures of plainchant's oral tradition in the era before notation.⁸²

Just as the model can comment on the ontologies and structures of oral corpora, it also has the potential to distinguish between the generic and the specific with regard to melodic shape in many genres of the Western canon of classical music. For example, it could help to define generic characteristics of phrase segments such as basic ideas, contrasting ideas, etc., and the relationships between phrase segments such as basic ideas and contrasting ideas in theme type analysis (Caplin 1998). It could also continue to illuminate compositional tendencies with which certain composers develop their motives throughout their compositional output.

Such analytical capabilities could be expanded even further to include contour in elements other than pitch, as first proposed by Marvin (1991, 1995). Scholars have adapted contour theories in order to study other musical domains such as rhythm, dynamics, and timbre, just to name a few (Marvin 1991, 1995; Boor 2009; Scotto 2016). The FCM model could easily be adapted to analyze these other musical domains: studying them using a fuzzy familial approach could help us to understand how these

⁸² This is in reference to the level-2 type of fuzzy set put forth by Klir and Yuen (1995), that is built upon the notion that a fuzzy set can be comprised of fuzzy sets, in similar ways that a set of notated chants are fuzzy representations of the tradition of oral chant dissemination (which are fuzzy sets in themselves). For more, see Chapter 3.

characteristics influence both the way that music is constructed, and the way that we perceive it.

Finally, it would be very interesting to conduct a related perceptual study to see how listeners categorize gradations of contour similarity, such as those offered by the FCM model. How do listeners hear familial relationships between melodies? At what membership value is their threshold of familial admittance? How do the presence of other musical parameters (such as rhythm, form, tonal progression, dynamics, etc.) influence this threshold? These are all empirical, testable questions that could lead to further refinement of the model, and would give us new insight into how we perceive relatedness in music.

These potential avenues of further research are by no means exhaustive. However, I hope that those I have presented, combined with the analyses I have included, continue to illustrate the point that I have endeavored to make throughout this dissertation: the study of contour should not be relegated to mere description, nor should it be limited by the cardinality constraints of some of the tools that have come before. Instead, more flexible models of melodic contour relations, such as the one I have proposed, can have a profound impact on the way we study melodic shape, providing new and valuable insights into one of music's most fundamental elements.

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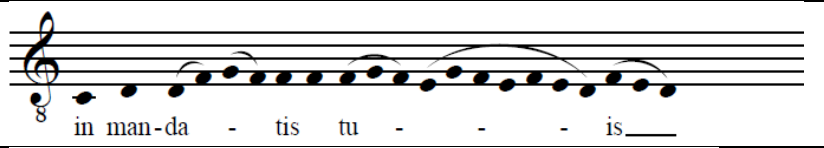
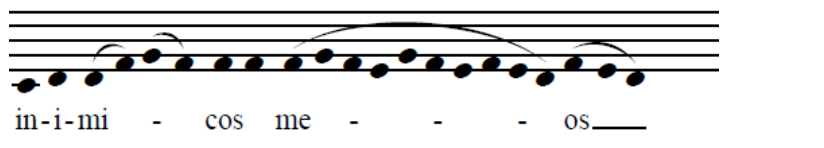

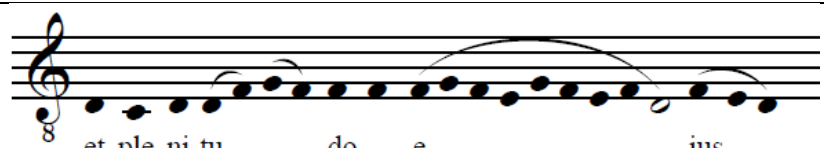
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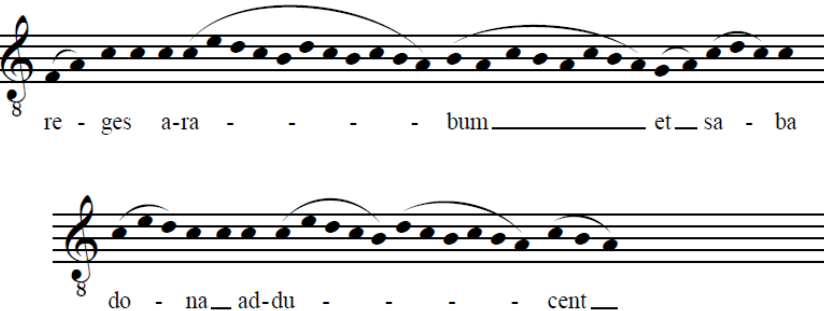


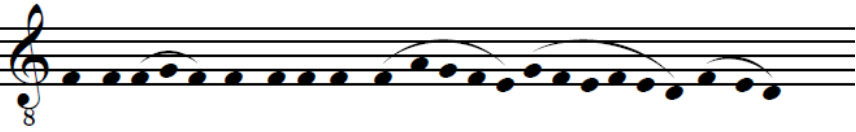
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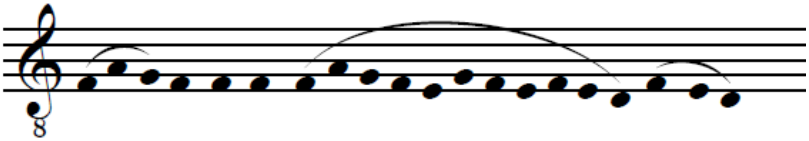


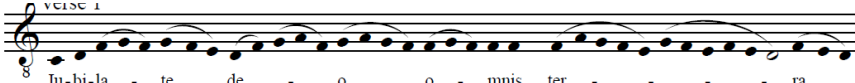
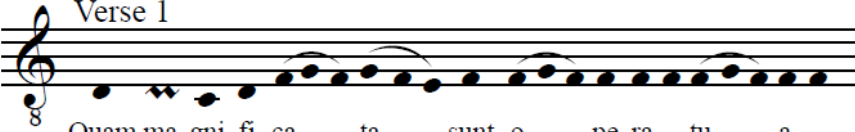

Appendices




APPENDIX 1. Variants of formula 2-1 (Maloy, 2010)

Figure A1.1. Formula 2-1 Variants

ID	Chant	Location	
22	<i>Meditatibor</i>	Line 1 (second line)	 <p>in man-da - tis tu - - - is</p>
17	<i>Exaltabo te</i>	Line 2 (second half)	 <p>in-i-mi - cos me - - - os</p>
36	<i>Benedicte gentes</i>	Line 5a	 <p>be-ne-di - ctus do - mi - nus</p>
6	<i>Tollite portas</i>	Line 4 (second line)	 <p>et ple-ni-tu - do e - - - ius</p>

6	<i>Tollite portas</i>	Line 4 (third line)	 <p>or - bis ter-ra - - - - rum</p>
10	<i>Reges tharsis</i>	Line 2	 <p>re - ges a-ra - - - - bum et sa - ba do - na ad-du - - - - cent</p>
13	<i>Dextera domini</i>	Line 6 (first line)	 <p>et ex - au - di - vit me</p>
13	<i>Dextera domini</i>	Line 6 (second line)	 <p>in la - ti - tu - di - ne</p>
72	<i>Anima nostra</i>	Line 1 (third line)	 <p>de la - que-o ve-nan - ti - um</p>

72	<i>Anima nostra</i>	Line 3 (second line)	 <p>e - rat in no - - - - bis___</p>
72	<i>Anima nostra</i>	Line 7	 <p>be-ne-di - ctus do - mi - nus_____</p>
10	<i>Reges tharsis</i>	Line 6 (second line)	 <p>fi - li-o__re - - - - gis_____</p>
36	<i>Benedicte gentes</i>	Line 8	<p>VERSE 1</p>  <p>lu-bi-la - te___de - o___o - mmis ter - - - - ra___</p>
14	<i>Bonum est confiteri</i>	Line 4	<p>Verse 1</p>  <p>Quam ma-gni-fi-ca - ta___sunt o - pe-ra__tu - a___</p>  <p>do - mi - ne___</p>

14	<i>Bonum est confiteri</i>	Line 5 (partial)	 <p>ni - mis__pro-fun - de__fa - cte_____sunt_____</p>
23	<i>Benedic anima mea</i>	Line 2 (second line)	 <p>re-tri-bu-ti-o - nes e - - - ius_____</p>
58	<i>Ascendit deus</i>	Line 3 (lines 2 and 3)	 <p>plau - di-te__ma - ni - bus_____</p>

58

Ascendit deus

Line 6
(all)

Verse 2

Quo-ni - am

de - - us

sum - mus ter-ri - bi - lis_____

APPENDIX 2: Analysis of diachronic build-up for the eight melodic regions in Steve Reich's *The Desert Music*

Figure A2.1a. Graduated analysis of membership values through the build-up of the M₁ region

Iteration of M ₁ composite	Membership M ₁ (T ₀) (flute 1)	Membership M ₁ (T ₁) (flute 2)	Membership M ₁ (T ₂) (flute 3)
1 st time	1.0	--	--
2	1.0	--	--
3	1.0	1.0	0.75
4	0.975	0.9	0.55
5	0.967	0.6	0.363
6	0.873	0.806	0.511
7	0.835	0.808	0.54
8	0.812	0.810	0.558
9	0.789	0.811	0.570
10	0.785	0.812	0.578
11	0.771	0.813 (.8126)	0.586
12	0.770	0.813 (.8129)	0.591
13	0.765	0.813	0.595
14	0.761	0.814 (.8136)	0.598
FINAL MEMBERSHIP	0.757	0.814 (.8139)	0.601

Figure A2.1b. Graph of membership value development in the M_1 region

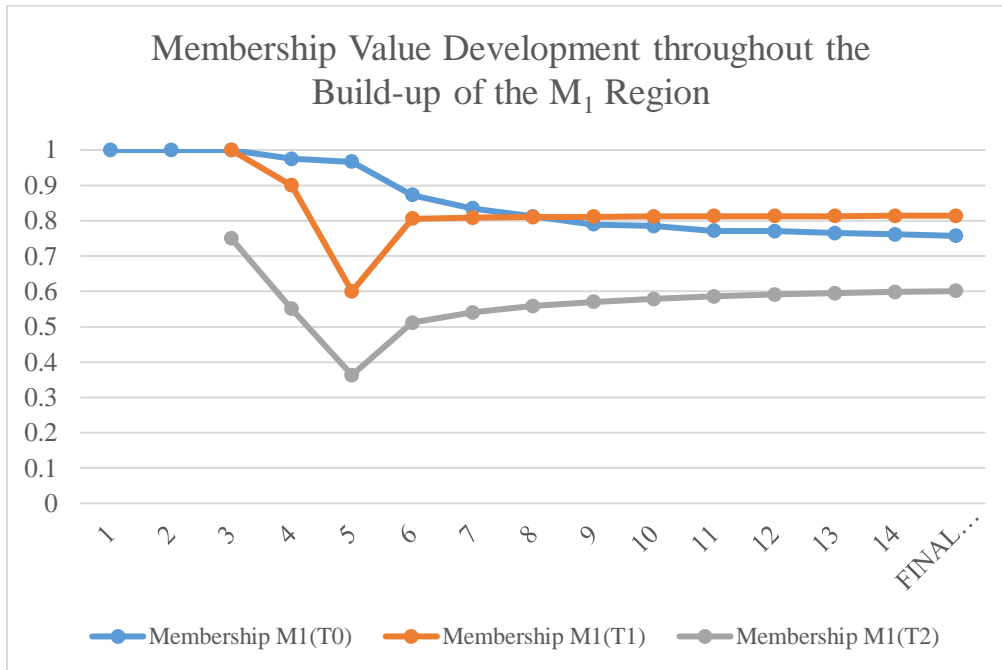


Figure A2.1c. The Final Version of the Fuzzy Multilinear Family of M_1

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	0.44	0	1.0	0.278	1.0	0.5	1.0	0.538	1.0	0.417	1.0	0.5	1.0	0.583	1.0	0.417	0	0	0	0
-	0.56	1.0	0	0.722	0	0.5	0	0.461	0	0.583	0	0.5	0	0.417	0	0.583	1.0	1.0	1.0	1.0

Figure A2.2a. Graduated analysis of membership values through the build-up of the M₂ region

Iteration of M ₂ composite	Membership M ₂ (T ₀) (flute 1)	Membership M ₂ (T ₁) (flute 2)	Membership M ₂ (T ₂) (flute 3)
1 st time	1.0	--	--
2	1.0	0.2	0.2
3	0.967	0.767	0.9
4	0.95	0.6	0.6
5	0.918	0.807	0.832
6	0.907	0.910	0.843
7	0.901	0.912	0.849
8	0.898	0.913	0.853
9	0.895	0.913	0.855
10	0.894	0.914	0.856
11	0.892	0.914	0.858
12	0.891	0.914	0.859
FINAL MEMBERSHIP	0.890	0.915	0.860

Figure A2.2b. Graph of membership value development in the M₂ region

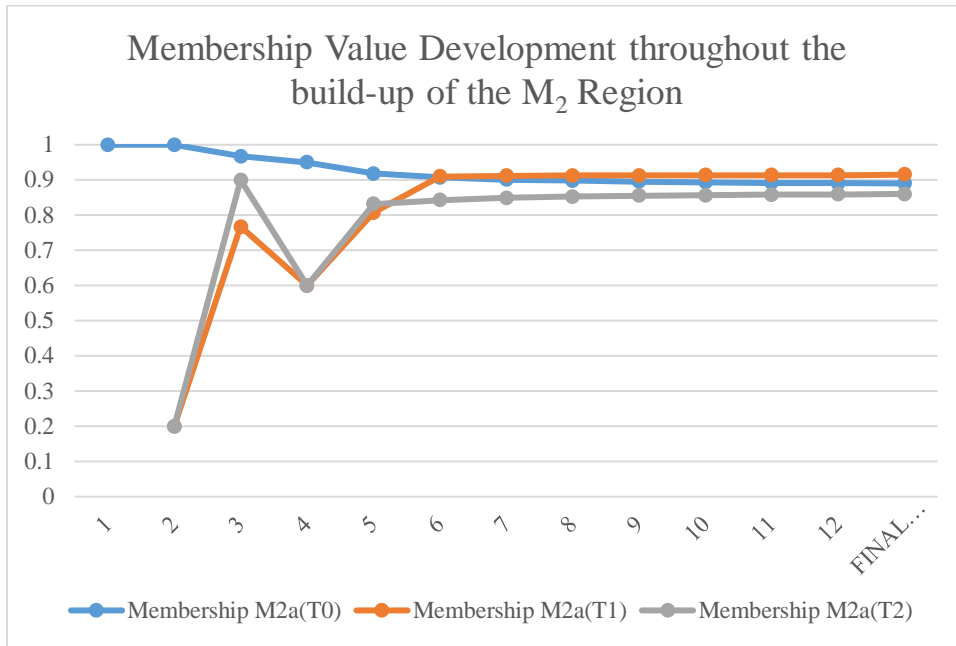


Figure A2.2c. The Final Version of the Fuzzy Multilinear Family of M₂

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	1.0	0	0.645	0	0.5	0	0	1.0	0	1.0	0	1.0	0	0	0.55	1.0	1.0	1.0	1.0	1.0
-	0	1.0	0.355	1.0	0.5	1.0	1.0	0	1.0	0	1.0	0	1.0	1.0	0.45	0	0	0	0	0

Figure A2.3a. Graduated analysis of membership values through the build-up of the M₃ region

Iteration of M ₃ composite	Membership M ₃ (T ₀) (flute 1)	Membership M ₃ (T ₁) (flute 2)	Membership M ₃ (T ₂) (flute 3)
1 st time	1.0	--	--
2	1.0	--	--
3	1.0	0.5	0.0
4	1.0	1.0	1.0
5	1.0	0.3	0.4
6	0.939	0.722	0.728
7	0.908	0.733	0.742
8	0.890	0.740	0.750
9	0.878	0.744	0.756
10	0.869	0.748	0.760
11	0.663	0.350	0.363
12	0.678	0.407	0.411
13	0.690	0.450	0.450
FINAL MEMBERSHIP	0.700	0.485	0.480

Figure A2.3b. Graph of membership value development in the M₃ region

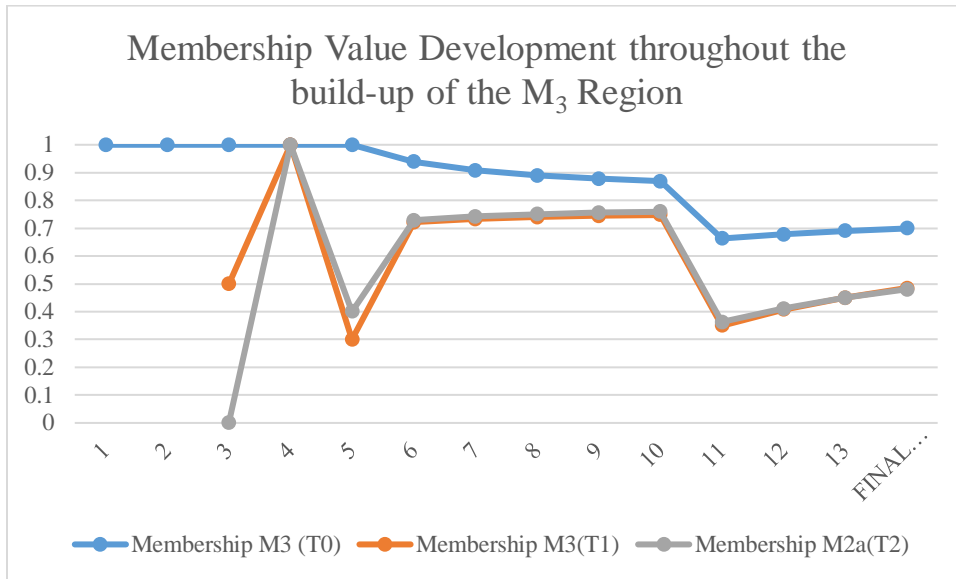


Figure A2.3c. The Final Version of the Fuzzy Multilinear Family of M₃

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	0.364	0.727	0.15	0.727	0.136	0	0.591	0	0.5	0	0.545	0.333	0.136	0.769	0.455	0.667	0.333	0.667	1.0	0
-	0.636	0.273	0.85	0.273	0.864	1.0	0.409	1.0	0.5	1.0	0.455	0.667	0.864	0.231	0.545	0.333	0.667	0.333	0	1.0

Figure A2.4a. Graduated analysis of membership values through the build-up of the M₄ region

Iteration of M ₄ composite	Membership M ₄ (T ₀) (flute 1)	Membership M ₄ (T ₁) (flute 2)	Membership M ₄ (T ₂) (flute 3)
1 st time	1.0	--	--
2	1.0	0.0	0.0
3	1.0	1.0	0.5
4	0.975	1.0	0.625
5	0.967	0.3	0.233
6	0.879	0.740	0.586
7	0.835	0.750	0.615
8	0.808	0.760	0.632
9	0.790	0.767	0.643
10	0.777	0.771	0.651
11	0.743	0.800	0.657
12	0.749	0.813	0.674
13	0.755	0.823	0.688
FINAL MEMBERSHIP	0.757	0.830	0.697

Figure A2.4b. Graph of membership value development in the M₄ region

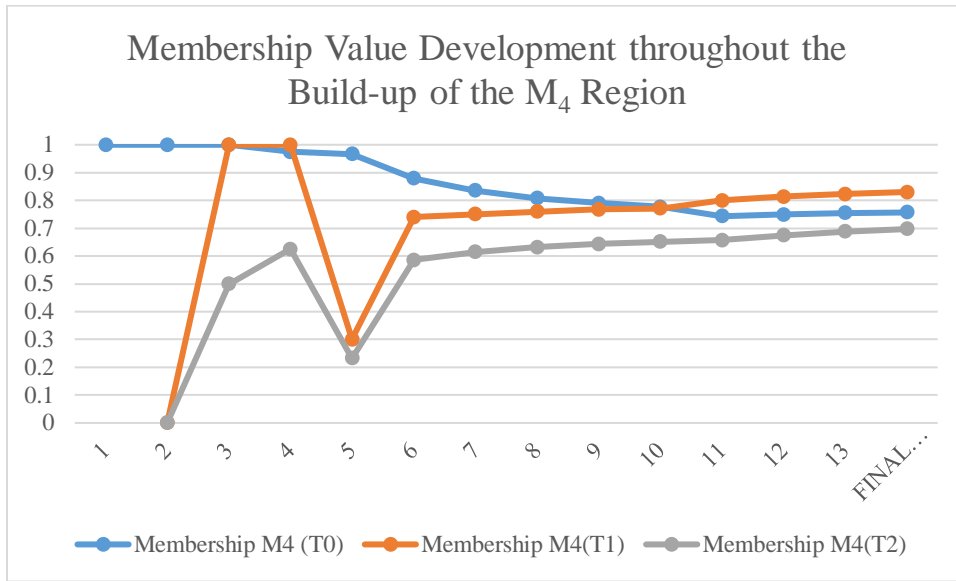


Figure A2.4c. The Final Version of the Fuzzy Multilinear Family of M₄

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	0.458	0	0.591	0	0.5	0	0.410	0	0.476	0	0.273	0	0.333	0	0.591	1.0	0	1.0	0	1.0
-	0.542	1.0	0.410	1.0	0.5	1.0	0.591	1.0	0.524	1.0	0.727	1.0	0.667	1.0	0.410	0	1.0	0	1.0	0

Figure A2.5a. Graduated analysis of membership values through the build-up of the M₅ region

Iteration of M ₅ composite	Membership M ₅ (T ₀) (flute 1)	Membership M ₅ (T ₁) (flute 2)	Membership M ₅ (T ₂) (flute 3)
1 st time	1.0	--	--
2	1.0	--	--
3	1.0	0.0	0.0
4	0.975	0.625	1.0
5	0.967	0.233	0.367
6	0.900	0.717	0.750
7	0.867	0.733	0.767
8	0.846	0.743	0.777
9	0.833	0.75	0.783
10	0.824	0.755	0.788
11	0.817	0.758	0.792
12	0.811	0.761	0.794
13	0.807	0.763	0.797
14	0.803	0.765	0.798
FINAL MEMBERSHIP	0.800	0.767	0.800

Figure A2.5b. Graph of membership value development in the M₅ region

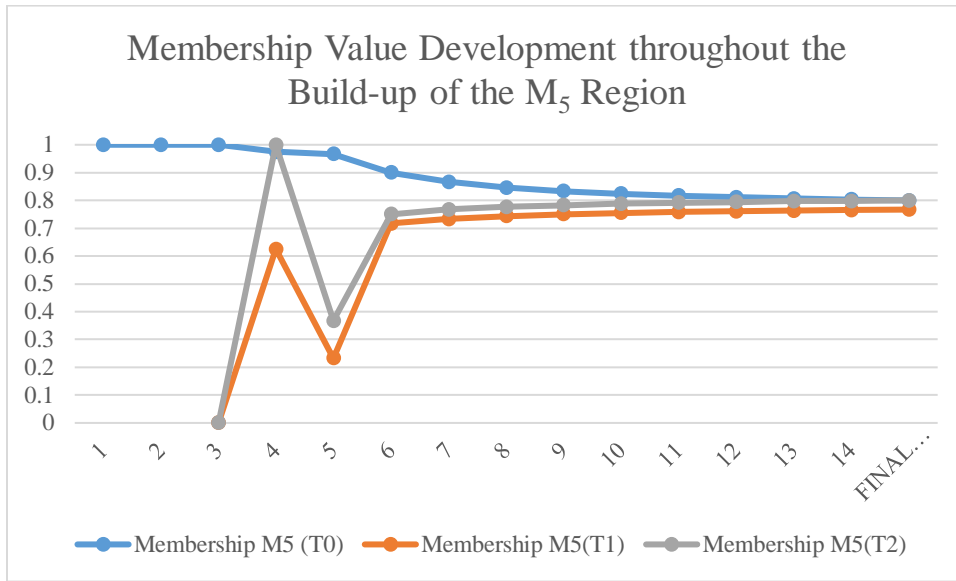


Figure A2.5c. The Final Version of the Fuzzy Multilinear Family of M₅

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	0.417	1.0	0.5	1.0	0.583	1.0	0.417	0	0	0	0	0	0	0.583	1.0	0.333	1.0	1.0	1.0	1.0
-	0.583	0	0.5	0	0.417	0	0.583	1.0	1.0	1.0	1.0	1.0	1.0	0.417	0	0.667	0	0	0	0

Figure A2.6a. Graduated analysis of membership values through the build-up of the M₆ region

Iteration of M ₆ composite	Membership M ₆ (T ₀) (flute 1)	Membership M ₆ (T ₁) (flute 2)	Membership M ₆ (T ₂) (flute 3)
1 st time	1.0	--	--
2	1.0	0.0	0.0
3	1.0	1.0	0.5
4	0.975	0.400	0.425
5	0.927	0.870	0.803
6	0.905	0.879	0.816
7	0.893	0.883	0.823
8	0.887	0.886	0.828
9	0.880	0.888	0.831
10	0.876	0.890	0.834
11	0.773	0.791	0.836
12	0.780	0.802	0.837
FINAL MEMBERSHIP	0.785	0.811	0.838

Figure A2.6b. Graph of membership value development in the M₆ region

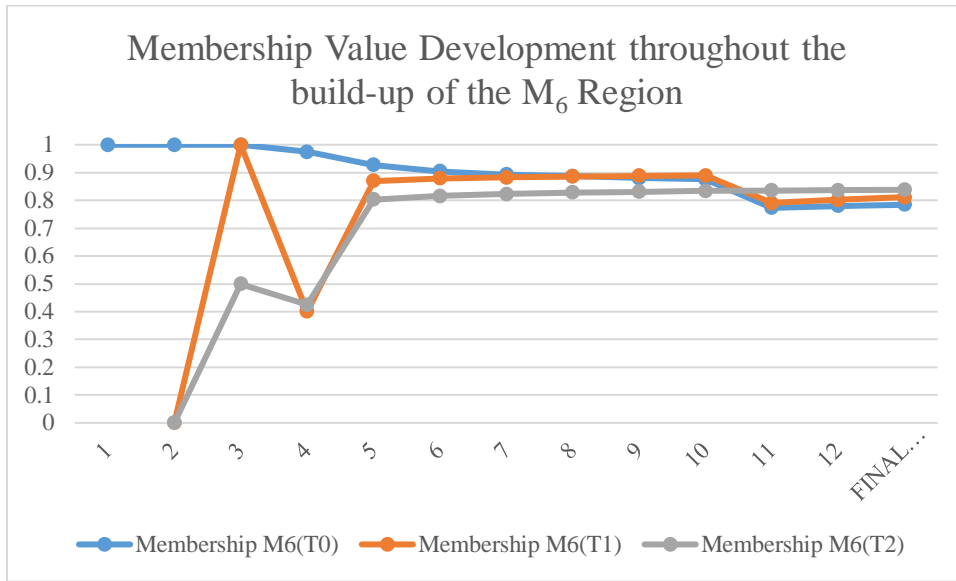


Figure A2.6c. The Final Version of the Fuzzy Multilinear Family of M₆

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	0.454	0	0	0	0	0	0.571	1.0	1.0	1.0	0.429	1.0	0.5	1.0	0	0	0.167	0	0.182	0
-	0.545	1.0	1.0	1.0	1.0	1.0	0.429	0	0	0	0.571	0	0.5	0	1.0	1.0	0.833	1.0	0.818	1.0

Figure A2.7a. Graduated analysis of membership values through the build-up of the M₇ region

Iteration of M ₇ composite	Membership M ₇ (T ₀) (flute 1)	Membership M ₇ (T ₁) (flute 2)	Membership M ₇ (T ₂) (flute 3)
1 st time	1.0	--	--
2	1.0	--	--
3	1.0	0.333	0.333
4	1.0	1.0	1.0
5	1.0	0.400	0.400
6	0.938	0.770	0.720
7	0.910	0.782	0.748
8	0.894	0.789	0.764
9	0.883	0.793	0.773
10	0.691	0.694	0.788
11	0.705	0.706	0.787
12	0.715	0.715	0.787
13	0.724	0.722	0.787
FINAL MEMBERSHIP	0.732	0.727	0.786

Figure A2.7b. Graph of membership value development in the M₇ region

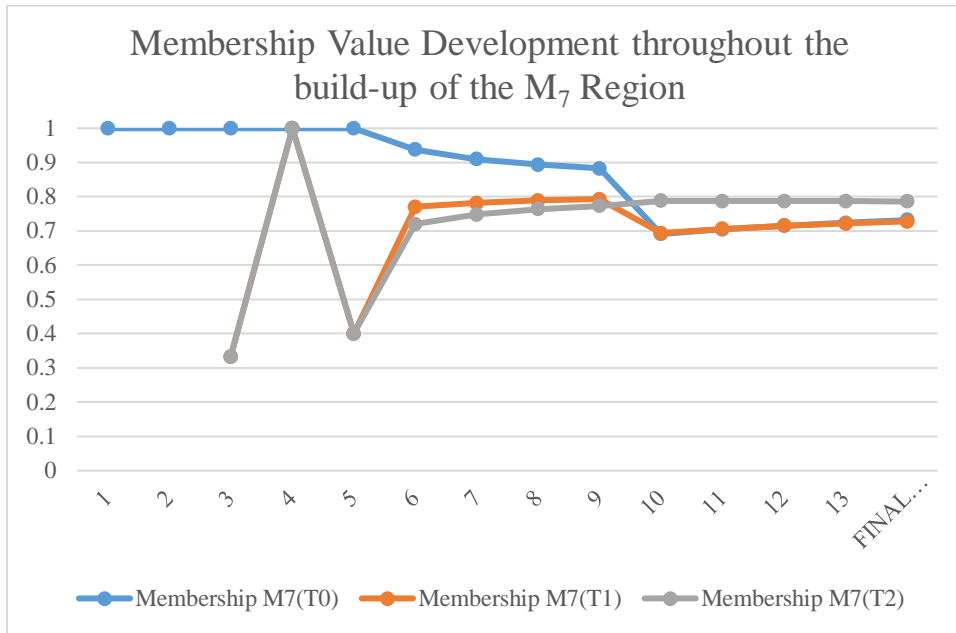


Figure A2.7c. The Final Version of the Fuzzy Multilinear Family of M₇

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	0.414	1.0	0.222	1.0	0	0	0.619	0	0.5	0	0.381	0	0	1.0	0.182	1.0	0.211	1.0	1.0	0.692
-	0.586	0	0.778	0	1.0	1.0	0.381	1.0	0.5	1.0	0.619	1.0	1.0	0	0.818	0	0.789	0	0	0.307

Figure A2.8a. Graduated analysis of membership values through the build-up of the M₈ region

Iteration of M ₈ composite	Membership M ₈ (T ₀) (flute 1)	Membership M ₈ (T ₁) (flute 2)	Membership M ₈ (T ₂) (flute 3)
1 st time	1.0	--	--
2	1.0	0.0	0.0
3	1.0	0.667	1.0
4	0.960	0.800	1.0
5	0.957	0.243	0.500
6	0.895	0.700	0.790
7	0.865	0.724	0.792
8	0.836	0.737	0.794
9	0.833	0.745	0.797
10	0.824	0.751	0.799
11	0.817	0.755	0.801
12	0.812	0.758	0.802
13	0.807	0.761	0.803
FINAL MEMBERSHIP	0.803	0.762	0.804

Figure A2.8b. Graph of membership value development in the M₈ region

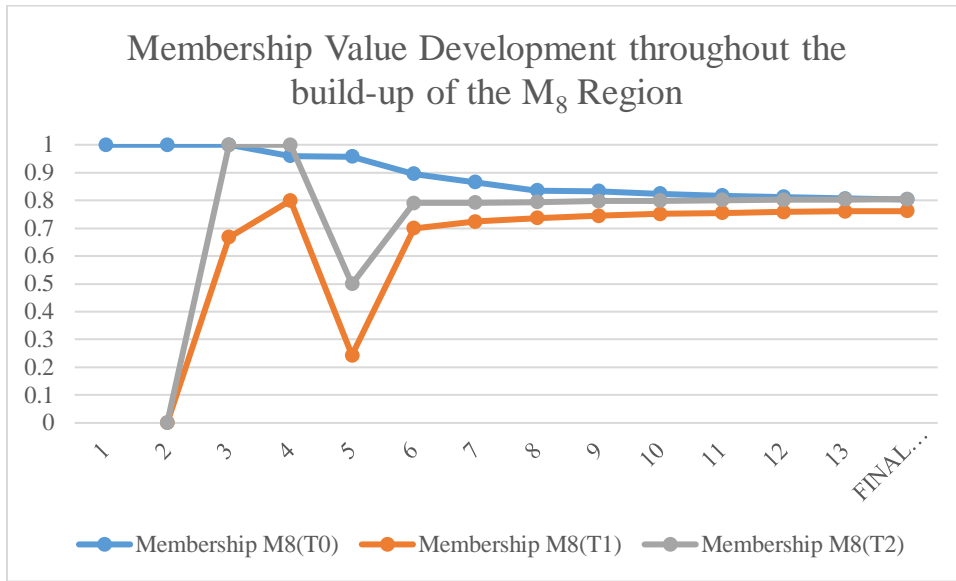


Figure A2.8c. The Final Version of the Fuzzy Multilinear Family of M₈

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
+	0.409	0	0.710	0	0.45	0	0.375	0	0	0	0	0	0	0	0.591	1.0	0.48	1.0	1.0	1.0
-	0.591	1.0	0.290	1.0	0.55	1.0	0.625	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.409	0	0.52	0	0	0

Curriculum Vitae

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2013–2017

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