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Informal Hiring Patterns with Endogenous Job Contacts

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Abstract

I present a one-channel model of informal hiring in Chapter 2, a two-channel model with additional formal hiring in Chapter 3, and incorporate a more explicit networking scenario into the informal channel in Chapter 4. I consider a CRS industry in which a single firm weighs the relative gain from a high-quality worker against the odds of a given applicant being low-quality. In the informal channel, the firm is able to condition on two available sources of information: noisy signalling through the endogenous arrival probability of applications, and an exogenous signal of quality through the report conveyed by the application itself. I find that the informal channel may be used in equilibrium to signal high-quality, improve connection between potential workers and the firm, or, either due to insufficient benefit, social norms, or incompatible worker and firm incentives, may not be used at all. I also find complementarity between the report screening power and the composition of the pool of applicants. When used alone, I show that the informal channel is able to endogenously generate a more favourable pool of applicants, with or without homophily effects present in referrals. When used in combination with formal hiring, I show that an inferior pool of informal applicants is also possible, as is sometimes noted empirically. The addition of the formal channel generally affects the informal channel adversely, causing it to shut down in some cases. I find that existence of equilibria and comparative static results are sensitive to the specification of informal arrival costs. Under the more explicit networking scenario, I provide conditions on networking costs for the use of informal hiring in equilibrium. When networking costs increase, any bias in the composition of the informal pool of applicants is intensified. I also find typically non-monotonic effects of parameters on the informal pool composition and profits. My results highlight how informal hiring patterns and equilibrium outcomes depend on the costliness and informativeness of job contacts, and are affected by the firm's need for quality and its ability to discern quality through formal versus informal sources.

Keywords: signalling, networking, selection, formal hiring strategies, informal hiring strategies, job application, asymmetric information

Christine, voilà!

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Contents

List of Tables

List of Figures

List of Appendices

Chapter 1

Introduction

There are many different ways in which workers approach firms to apply for jobs and by which firms advertise job openings and recruit new employees. Informal sources such as referrals have accounted for as much as 80% of hires in blue-collar occupations and 50% in white-collar occupations (Rees, 1966), although empirical estimates of the use and success of different search and recruiting methods vary.[1] Ioannides and Loury (2004) observe that the use of personal contacts in job search has been increasing over time, and also that differences in method use do not fully account for all demographic differences in job search productivity. However, it is difficult to understand the response of job search and hiring strategies to changes in the environment such as reduced communication cost and through the internet and other technology. Although variation in the use and success of different methods has been studied by demographic, by market conditions, and across industries and countries, there is relatively little theoretical grounding to give understanding to the underlying causes.

Of particular interest in many studies are the advantages or disadvantages which may be associated with the use of different search methods, such as the question of whether or not

[1] For comparisons of the use and productivity of different search methods according to gender and race, career stage and employment status, aggregate employment conditions, and country, see for example Corcoran et al. (1980), Reid (1972), Blau and Robbins (1990), Osberg (1992), Weber and Mahringer (2008), Addison and Portugal (2002), and Pellizzari (2010).

informal methods lead to better-quality applicants and hires. Most theoretical models assume or predict advantages of network-based informal search methods, particularly in the form of generating a more favourable pool of applicants. This is usually explained as a result of "homophily" effects in networks, by which people are more likely to share connections with others whose attributes are similar to their own.[2] Thus a referred candidate, whose qualities may be unobserved, is expected to be similar to the person making the referral, whose qualities may be observed. Although the prevalence of homophily in networks has been widely observed and is well-supported by theoretical models of network formation, including within the job-market setting, it is not empirically clear whether or not referrals provide firms with either more favourable candidates or better matches in terms of quality or tenure, or whether there is a wage premium or penalty associated with referral.[3]

This is not entirely surprising given that homophily effects should lead to higher quality in referrals only when the attributes associated positively within the social network are also attributes which are relevant to the desired job skills. Furthermore homophily in networks is not the only determinant of the pool of applicants generated by referrals, neither does the quality distribution of candidates comprise the only source of value in referrals. In particular, information passed through personal contacts can be directly useful.[4] [5]

In general, direct information provided by informal search methods need not be the same

[2]For an overview of homophily in social networks, see McPherson et al. (2001).

[3]Referrals are found by Fernandez and Weinberg (1997) to generate a more appropriate pool of applicants in retail banks, and by Burks et al. (2015) to provide better matches in terms of lower quit rates and higher match-specific skills. Dustmann et al. (2016) and others report similar results reflected in a wage premium for referral use. In contrast, Bentolila et al. (2010) find referrals generate poor matches and reduced wages. Antoninis (2006) finds a wage premium in manufacturing only when referrals shared direct experience, and a wage penalty for unskilled workers. Pellizzari (2010) finds that personal contacts can lead to a wage premium or penalty, both occurring with similar frequency in Europe and correlated with the efficiency of formal search channels.

[4]Informed contacts can reduce uncertainty for firms (Simon and Warner, 1992) and increase opportunities for workers (Calvo-Armengol and Zenou, 2005). Workers may use referrals either to find better jobs or as a last resort (Loury, 2006). Related benefits include decreased length and/or cost of search, see Ullman (1968), DeVaro (2005), and a stream of benefits from future referrals as in Montgomery (1991).

[5]Another potential benefit is decreased cost of monitoring employees. Kugler (2003) points to a reduction in firms' efficiency pay when using referrals, but referrals may also encourage low-quality workers to shirk (Duran and Morales, 2014). Fafchamps and Moradi (2015) found referred recruits more likely to desert or be dismissed from the British colonial army in Ghana.

as information provided by other methods. The attractiveness of a search method to firms or workers should depend on the nature of the information it provides. Rees (1966) points to the importance of qualitative dimensions of information such as whether the information provides more detail about a prospective firm/worker ("intensive" information) or whether it increases the scope of known opportunities ("extensive" information). Search methods which effectively yield intensive rather than extensive information should be more valuable as more heterogeneity is present in the pool of search. Of course, the ability of formal and informal sources to convey different types of information need not be the same across all job settings.[6] In their survey of personnel economics, Oyer and Schaefer (2011) point to a lack of understanding of the firm's optimal hiring strategies. Marsden and Gorman (2001) also note a lack of research addressing firm-level heterogeneity, and reason that job contacts should have greater importance for firms and industries where relevant job skills are more difficult to observe through formal or impersonal credentials, and/or where hiring stakes are high.

If the use of a particular search or application method differs by worker type, the use itself can also become a new source of information to the firm about the suitability of the applicant. If for a particular search method the directly transmitted information is very informative, high-quality workers should have greater incentive to use this method, and it should provide the firm with better candidates. Meanwhile equilibrium effects may be important because the benefits to firms and workers of a given search method are influenced by the others' use of it.

Therefore, in order to better understand the use of different job application methods, I analyse a two-sided job-application and hiring equilibrium which takes into account the informational environment and skill sensitivity of the industry. My model, while compatible with the presence of homophily effects in informal search, shows for example that informal hiring channels may generate a better or worse pool of applicants, depending on the relative importance to the firm for hiring a skilled versus unskilled worker and depending on the informational

[6]For example, referral by friends and family may be preferred by non-profit and religious organizations because valued attributes such as motivation and shared ideology are more difficult to assess through formal methods (Mosca and Pastore, 2009).

differences between search methods.

1.1 One-Channel Hiring in Chapter 2

In Chapter 2 I study a model with a single application channel. I introduce a constant returns to scale industry in which a single firm chooses a hiring policy by weighing the relative gain from a high-quality worker against the odds of a given applicant being low-quality.[7] I study Nash equilibrium in a simultaneous move game. At a cost, workers endogenously determine the probability that the firm sees their application. When calculating the odds of an applicant being low-quality, the firm is able to condition on the "report" which is an exogenous noisy informative signal conveyed by a worker's application. The firm is also able to condition on the event of the application's arrival. The event of arrival can be viewed as an endogenous noisy signal of quality, because the relative strategies chosen by low and high types adjust the quality composition of the pool of applicants relative to that of the general population. This hiring channel will correspond to the "informal" hiring channel in Chapter 3. In this setting, the increase in arrival probability can be interpreted as arising from activities such as asking friends and relatives for help, attending networking events, or building stronger relationships with influential people. When connecting a worker to the firm, it may not always be the case that social contacts provide noisy information to the firm. However as found by Pallais and Sands (2016), referred workers may be of higher than average productivity. Therefore the "report" received by the firm may be the strength of homophily as discussed in Section 3.1.1, or any other effect (apart from chosen effort) through which applicants reaching the firm through the informal channel have a systematically higher quality.

Daley and Green (2014) also study a model in which a costly action providing information about the sender's type is accompanied by a second source of exogenous information. In

[7] I will take the wage as given. A fixed wage may be due to limitations such as might be imposed by a union, internal pay structures, legislation, or competition with other industries. I consider the effects of adjusting the wage in Section 2.3.2.

particular, they consider a variation of the unproductive education model of Spence (1973) such that a public but imperfect “grade” is also available to potential employers. Related adaptations include Feltovich et al. (2002) and Weiss (1983). The endogenous and exogenous sources of information considered in my model differ from these because the worker is able only to choose a probability of arrival, rather than a fixed observable quantity of education, and the noisy report of a worker’s quality is observed only if his application successfully arrives, rather than a grade being publicly available regardless of a worker’s education choice.[8]

Although I do not assume any difference in arrival costs for high- and low-quality workers, I find that endogenous arrival leads to positive selection in general. That is, whenever arrival is non-zero, the quality composition of the applications received by the firm either coincides with the quality composition of the general population, or is more favourable. For all situations of “absolute” hiring, in which the firm will indiscriminately accept all applications it receives, both worker types have identical marginal benefit from arrival. For all situations of “selective” hiring, in which the firm will accept only some applications which arrive and reject others, the probability of being accepted with a high report must be strictly higher than the probability of being accepted with a low report, because reports are informative. Therefore the marginal benefit of arrival must be higher for high types, and they will be willing to put more effort into arrival than low types. Thus informative screening with reports can induce positive selection into the pool of applicants. Similarly, Michelacci and Suarez (2006) show that an improved pool of applicants can also arise as a result of wage bargaining when the firm observes the quality of workers. As in my model, this gives high-productivity workers a higher expected return conditional on arrival.

Unlike standard separating equilibria in which one type may engage in the costly action while the other type does not, I find that the arrival probabilities chosen by worker types are either both zero or both non-zero.[9] This is because unlike wasteful education, arrival has

[8] Noisy signalling models have been considered in Matthews and Mirman (1983) and Carlsson and Dasgupta (1997). Jeitschko and Normann (2012) study the relationship between deterministic and noisy signalling models.

[9] Jeitschko and Normann (2012) find a similar property in their noisy signalling model, that both types choose

intrinsic value for any worker whenever the firm follows a non-zero hiring policy, so that any worker type will find it worthwhile when the marginal cost is not too high, and it is because worker incentives will differ only when the firm rejects some applications. This means that low types can not be discouraged from arrival if the firm believes applications only arrive from high-quality types, because this would lead the firm to accept all applications received regardless of report.

I describe the hiring environment in terms of the profitability of the industry, which I determine by comparing the firm's relative gain from a high- as opposed to low-quality worker against the quality composition of the general population. This is an indication of the firm's sensitivity to quality, which comprises an important dimension of firm heterogeneity next to screening ability because a firm's ability to discern quality is only important to the extent that it has a need to correctly identify quality. Even when outliers are rare, correct identification may be crucial, depending on the nature of the job and skill in question. Firms may be more sensitive to worker quality for example in innovative industries where product payoff has high variance (Andersson et al., 2009) or when filling positions which are strategically critical to the business (Huselid and Becker, 2006). Consistent with Rees' prediction, in a perfectly mixed population with equal arrival probabilities, report accuracy ("intensive" information) has a greater impact on the expected value per hire the greater is the variation between high and low productivities. Additionally, the firm will tend to ignore reports in its hiring decision when there is a strong enough prevalence of one worker type in the general population, which signifies an effectively homogenous skill level in the population.

I provide conditions for the existence and uniqueness of Nash equilibria according to the profitability of the industry and screening power of reports. Brenčič (2012) also identifies sensitivity to quality and ability to discern quality as important factors in the firm's choice of wage determination method. She observes that wage-posting is more likely for jobs with low-level or easy-to-measure skills, high firm search costs (and therefore less selective standards),

a strictly positive level of action.

or when applicants will be screened by agencies. She interprets these factors as evidence that the firm is not very concerned about an adverse pool of applicants, which wage-posting is likely to generate according to Michelacci and Suarez. I find that pure strategy equilibria may not always exist, but mixed equilibria generally exist except when the profitability of the industry is very low. I provide a characterization of equilibria and show that the quality composition of the pool of applicants is generally less favourable the more permissive is the firm's hiring policy.

Within equilibria with selective hiring, I find complementarity between the screening power of reports and the quality composition of the pool of applicants. That is, I find that when the firm's screening technology improves, the positive selection effect of applications is stronger. This is due to the fact that the informativeness of reports is the source of the divergent marginal benefits of arrival for high- and low-quality workers. In contrast, Daley and Green show that education is used by high types to substitute for grades when grades are insufficiently informative. A change in report error influences the firm's hiring policy, both directly and through its effect on the pool of applicants. Due to its complementarity with the pool of applicants, improved screening power can encourage the firm to hire high-report applicants. This may lead to more or less permissive hiring overall; in an industry with very low profitability the firm may become willing to accept more applicants based on their high report, whereas in an industry with a sufficiently high general profitability, improved screening will lead the firm to reject more low-report applicants.[10]

I relate the use of applications in equilibrium to the types of value they provide, as a means of connection (a "door"), as a source of direct information (a "report"), or as a source of indirect information through endogenous selection (a "signal"). I find that for industries with high profitability applications have primary value as a door; all applications are accepted by the firm regardless of any direct information transmitted, and no indirect information is conveyed by

[10]Dineen and Williamson (2012) find evidence that worker awareness of firm screening may induce a better pool of job applicants through self-selection.

applications because they are used equally by all workers. For industries with intermediate and low levels of profitability, I find that the firm does makes use of direct information to hire selectively and that workers do not use applications equally. Although both industry scenarios exhibit similar patterns in the use of applications, they are qualitatively different in their functions. For a low profitability firm applications have primary value as a signal, because in the absence of any endogenous selection bias, the firm would not be willing to accept any applicants. For a firm with intermediate profitability, selective hiring could persist on the basis of direct information conveyed by reports alone.

Finally I present comparative static and welfare results for parameter changes. In particular I find that increased wages affect the firm's profit directly due to the increased cost of labour, and also indirectly due to changes in worker incentives. This indirect effect comprises not only an effect on the volume of applicants (and the subsequent volume of hires) but also an effect on the relative volume of high- and low-quality applicants. I explore how the latter is affected by the cost of accessing the firm, and additional results on this are provided in Chapter 4. Finally I show that alternative timing of the model leads to qualitatively similar equilibrium patterns in the use of applications according to the profitability of the industry and screening technology.

1.2 Two-Channel Hiring in Chapter 3

In Chapter 3 I extend the model to two hiring channels; one, the "informal" channel, has costly endogenous arrival of applications with an exogenous noisy signal as in Chapter 2. The other, the "formal" channel, is equally available to all with an exogenous connection probability, and conveys an exogenous noisy signal of worker quality which is distinct from that of the informal channel.

Although an endogenous or exogenous arrival probability may be descriptive of either formal or informal search methods depending on the specific situation, I will study the worker's choice of informal arrival and assume exogenous formal arrival. Access to the formal channel may be equally available to all by law (for example in the public sector), or it could be that dimensions in which effort can be applied do not affect arrival rates (such as polishing a resume). I discuss the issue of interpretation of these channels in Section 3.1.1. Equal availability is also consistent with the common interpretation of the formal channel as search through "hiring intermediaries," for example public or private agencies such as the Public Labor Exchange (Plesca, 2010) or online outsourcing agencies (Stanton and Thomas, 2016) and job boards (Faberman and Kudlyak, 2016). The exogenous noisy signal in the formal channel can be interpreted for example as the assessment of the candidate by a human resource professional based on the interview and presented credentials. I assume that formal and informal hiring decisions are made separately from each other, with no comparison of information. Such separation does occur in some contexts, for example in hiring public school teachers (Naper, 2010) and workers for retail chains (Deller and Sandino, 2016). Addressing a joint hiring decision results in a significantly more complicated model and is discussed in Section 3.4.3.

Similar to the complementarity exhibited in the single-channel model, the quality composition of informal applicants improves when the screening power of informal reports improves. However, there is an adverse effect on the quality composition of informal applicants when formal reports increase in screening power. This is because the screening power of formal reports gives a relative advantage to high-types in the chance of being hired formally, relatively reducing their reliance on costly informal arrival. Thus formal information can substitute for signalling through the noisy informal channel, as grades can substitute for signalling in Daley and Green (2014).

Given this conflicting influence of formal and informal information on the quality composition of the informal channel, I find that the informal pool is not necessarily favourable (or even neutral).[11] To illustrate, suppose the firm hires selectively in both formal and informal channels. If the formal application arrives with certainty, the informal pool will be unfavourable

[11]These results could explain the existence of diverse findings regarding quality of referred applicants mentioned previously (Fafchamps and Moradi, 2015; Duran and Morales, 2014; Fernandez and Weinberg, 1997; Bentolila et al., 2010; Burks et al., 2015).

when the informal report has weaker screening power than the formal report. If the formal application is not certain to arrive, the informal pool will be unfavourable when the screening power of the formal report is sufficiently stronger than that of the informal report.

Given how the informal pool responds to different combinations of formal and informal hiring patterns, I characterize the existence of pure strategy Nash equilibria according to differing use of the informal channel. I find that when the formal report screening power is high, hiring equilibria with an unfavourable informal pool can be sustained provided that the incentive for high types to use the informal channel is not too weak relative to low types. I also find that not all formal and informal hiring patterns are compatible with each other. Whenever hiring can not be supported in the formal channel, the informal channel can not sustain absolute hiring. This is because an industry in which formal hiring is not used must have very low profitability. Yet absolute informal hiring necessarily induces a weakly unfavourable pool composition, which can not be tolerated for an industry with low profitability. Also, selective hiring in one channel can occur alongside absolute hiring in the other channel only when the other channel has inferior reports.

In a related model, Casella and Hanaki (2008) study the effect of adding a second channel with potential for signalling to the referral hiring model introduced by Montgomery (1991). In their expanded model, endogenous signalling occurs through the formal channel rather than the informal channel, and the two channels also operate with sequential timing; if a worker does not receive an offer through referrals in the informal channel, at a cost he has the option to attempt certification before entering the open market. Success is more likely for high types so obtaining certification is a noisy indication of quality. In contrast to my model, wages are endogenous and all workers will be given a job offer eventually. In this setting endogenous signalling has informational value only, and makes no difference in achieving a connection with the firm.[12]

[12] I am not aware of any other papers with asymmetric information and heterogeneous workers with enodgenous use of the informal channel. Other related literature will be discussed in Section 1.4.

The addition of formal certification has no effect on the pool of informal applicants in their model, because connection with the firm through referrals in the informal channel occurs deterministically according to homophily effects. However, if certification is sufficiently informative, Casella and Hanaki find that the addition of formal certification can shut down the use of referrals when the cost of attempting is within a restricted range. In my model, informal hiring can similarly disappear in equilibrium in the presence of selective formal hiring when formal reports are sufficiently informative relative to informal reports. These similar findings have different underlying causes. In Casella and Hanaki, productive types expect higher wages on the formal market than what they can be offered through referrals, whereas in my model the quality of the pool deteriorates too much as productive types save on search costs.

Although they find that certification can eliminate use of referrals, Casella and Hanaki argue that the use of referrals is resilient to the presence and informativeness of formal certification, and that use of certification can increase referral hiring in equilibrium. They also show that in many cases firms strictly prefer to hire through referrals when homophily effects are strong, even when certification is a perfectly informative signal. In my model, because the addition of the formal channel lowers the volume and quality composition of informal applicants, use of the formal channel generally leads to decreased use of the informal channel and does not clearly improve profit for the firm. Similarly, since improved formal screening has an adverse effect on the pool of informal applicants, it tends to reduce the firm's hiring in the informal channel. Restrictions on informal hiring have also been predicted to have adverse welfare effects by Igarashi (2016) in the context of random search, since the resultant increase in formal job postings can be outweighed by a greater number of unemployed workers in the job queue and increased search frictions.

The assumption that the firm delegates hiring to independent hiring departments leads to the issue that the objective of the decision-makers is not always perfectly aligned with the objective of the firm.[13] I discuss this assumption and also the welfare and comparative static

[13] Hoffman et al. (2016) provide empirical evidence of this issue with human resource managers.

implications of the availability of a second hiring channel. I find that non-monotonicity in the parameter effects of this model is common, and the nature of the specific relationship between cost and arrival probability is a significant factor in the technical conditions for comparative static and welfare predictions.

1.3 Applying Job Contacts in Chapter 4

In order to understand the relationship between cost and informal arrival probability and gain intuition for its effect on equilibrium outcomes in the context of job contacts, Chapter 4 models this relationship explicitly as the outcome of a networking process and applies it in the two-channel model. The worker invests in costly networking, the intensity of which determines his application's probability of arrival. I study in particular the case where each contact in a worker's network gives access to the firm with equal independent probability. This arrival probability is similar to the form of the arrival probability arising from models of information transmission with endogenous job contact network formation based on graph theory such as in Calvo-Armengol (2004) and in Galeotti and Merlino (2014).

I find that networking costs have an intensifying effect on the pool of informal applicants; in situations where the informal pool is favourable, increased networking costs improve the informal pool composition further, whereas in situations where low-quality workers network more, increased networking costs exacerbate this imbalance. This amplification of the pool composition occurs because although increased costs reduce the incentive to network for both worker types, the reduced incentive has a relatively greater effect at lower levels of networking. This means that as technological advances or social innovations decrease networking costs, the informal channel loses value as a signal, and for low profitability industries which rely primarily on the value of networking as a signal, the informal channel may cease to function entirely.[14]

[14]Emergence of online social networks, such as LinkedIn, have likely reduced the costs of networking, but also

Although I focus on a particular class of arrival function, I find that this amplification effect of the cost of networking holds additionally for any networking scenario which has a logarithmically concave marginal probability of arrival, which means that the marginal returns to networking do not diminish too rapidly relative to the arrival probability. Although I show that an amplification effect of networking costs is not universal, I find that it does appear to be satisfied even more generally for some distributions where this sufficient condition of logarithmic concavity does not hold.

I apply the cost and arrival structure arising from this networking scenario to the existence conditions and comparative static and welfare results of the two-channel model. Due to the amplification effect of the cost of networking, in order to support equilibria in which the informal channel's primary value is based on the report, the cost of networking must not be too high. In order to support equilibria in which the informal channel has primary value as a signal, the cost of networking must be sufficiently high as well as not too high. My results suggest the importance of separating different types of job contacts and referrals in empirical studies which differ substantially in the cost of their development and use, such as friends and family versus professional connections, which are typically grouped together.

In the case where contacts provide access to the firm with equal independent probability, I find that reduced networking costs improve profits in high-profitability industries and reduce profits in low-profitability industries. This is because lowered costs increase the volume of informal applicants but reduce the ability of informal arrival to signal quality, which is only an acceptable trade-off when the firm does not need to watch quality very closely.

In contrast to the effect of networking costs, I find that wages have a moderating effect on the informal pool composition; when the informal pool is favourable, increased wages worsen the quality of informal applicants. In particular, the effect on the pool composition of an increase in wages is opposite and proportional to the effect of an increase in networking cost.

improved the firms' screening abilities, implying an overall ambiguous effect on viability and effectiveness of informal hiring (Garg and Telang, 2016).

Since wages affect profits both directly through the increased cost of labour and indirectly through the volume and composition of the informal pool of applicants, the effect of wages on profits is ambiguous. However, in the case where contacts provide access to the firm with equal independent probability, higher wages lead to reduced profit for the firm overall for low-profitability industries due to the adverse effect through the pool of applicants and the negative direct effect.

1.4 Additional Related Literature

There is a growing amount of research incorporating the use of informal methods into search and matching models of the labour market in order to better understand the effects of referrals and social networks. In the context of equally productive workers, Mortensen and Vishwanath (1994) introduce the possibility of receiving indirect offers through employed workers in addition to direct offers in a search market. Their model generates higher wages and longer tenure for jobs obtained through contacts. Although connections are not modeled explicitly, higher wages are also predicted for those workers who are "better connected" when referral arrival rates are taken to be heterogeneous. The recent model by Arbex et al. (2016) is similar and accounts for the complexity of explicit network structure, although each worker's network size is determined exogenously.

Addressing direct versus indirect job offers in a matching model, Galenianos (2014) relates differences in referral use to variations in aggregate matching efficiency across industries. Within a similar framework Calvo-Armengol and Zenou (2005) consider a more involved process of job information transmission through referrals in order to relate job matches to network size, as additional contacts both increase opportunities and introduce rivalry.[15] In both cases, workers are also homogenous in productivity and networks. Galeotti and Merlino (2014) and Galenianos (2017) allow for endogenous networks. The former studies the effect of labour

[15] This is consistent with the networking dynamic introduced in Calvo-Armengol (2004).

conditions on the use and effectiveness of symmetric networks. In the latter, workers also have different productivities, but types are known to the firm.[16]

Following the graph-theoretic network formation literature, Calvo-Armengol (2004) relates network structure to information flow and aggregate unemployment and analyzes equilibrium with non-cooperative network formation. Calvo-Armengol and Jackson (2007) also apply endogenous networks in the context of the labour market. Allowing for formation and dissolution of links over time, they find that differences in initial network states can lead to wage inequalities. While these models account for endogeneity in referral use, the firm's perspective is not modelled in the labour market, as wage offers are exogenous and there is no heterogeneity in the productivity of workers.

Lester and Wolthoff (2012) study hiring strategies for firms with different screening abilities when worker skill is heterogeneous and DeVaro (2005) proposes a wage-posting game in which the firm trades off hiring speed and match quality in its decision to use formal versus informal recruitment methods.[17] However, the effect of informal channels and endogenous networks on the firm's hiring decision has received less focus in general.

[16]See also Stupnytska and Zaharieva (2015).
[17]See also Board et al. (2017).

Chapter 2

One-Channel Model of Hiring

In Section 2.1 I introduce a constant-returns-to-scale industry in which a single firm decides whether or not to accept workers whose applications arrive. I focus on a simultaneous move game between this firm and workers. To maximize profits the firm compares the relative gain from a high-quality worker against the odds of a given applicant being low-quality when making its hiring decision. When calculating the odds of an applicant being low-quality, the firm may use its prior belief (based on the quality composition of the general population) or also condition this prior on other information if it is available. I consider two different sources of information by which the firm can update this prior. First, I suppose the firm is able to condition the odds of an applicant being low-quality on the event of the application's arrival. This is useful if for any reason applications from high- and low-quality workers do not arrive to the firm in the same proportions relative as the proportion of high- and low-quality workers in the population (as may occur in equilibrium). I describe the composition of the pool of applicants as being "favourable" or "unfavourable" in comparison to the composition of the general population. Second, I suppose the firm is able to condition on the "report" which is an exogenous noisy informative signal conveyed by a worker's application. I describe the hiring environment in terms of the "general profitability" of the industry, determined by the relative gain from high workers and quality composition of the general population, and the "decisiveness" of an application's report information, determined by the screening power of reports in relation to the industry's general profitability.

In Section 2.2 I introduce the application decision of workers. At a cost, workers choose the probability that the firm sees their application. Therefore arrival can be viewed as an endogenous noisy signal of quality, because the differences in the choices by low and high types lead to adjustment in the quality composition of the pool of applicants relative to that of the general population. Thus workers choose the arrival probability of their applications in response to the firm's hiring strategy, while the firm chooses its hiring strategy in response to the application arrival probabilities of workers. Allowing for the firm to simultaneously incorporate information about both the endogenous noisy signal determined by worker strategies and the exogenous noisy signal determined by application reports, I show how arrival costs together with the hiring environment determine hiring patterns and application arrivals in equilibrium.

I look at mixed strategy Nash equilibria with non-zero arrival probabilities. I show that for industries with high general profitability, applications function as a simple "doorway" to the firm, with the firm accepting all applications which arrive, and applications arriving from high and low-quality workers in proportion identical to the composition of the population. I find that for all situations of selective hiring, in which the firm accepts only some applicants, the quality composition of the pool of applicants is favourable relative to that of the general population. I show that for intermediate levels of general profitability selective hiring patterns can be supported on the basis of the screening power of reports alone. I show that selective hiring patterns can also be supported for low levels of general profitability, provided that there is a sufficiently favourable adjustment of the composition of the pool of applicants.

Although in both situations of selective hiring the direct information conveyed by reports is useful to the firm, and the arrival of an application is itself a signal of quality, I interpret the function of applications differently. When reports are decisive, applications are inherently a useful "source of information." This means that although the firm certainly benefits from an improved pool of applicants, the report itself would enable the firm to hire even if the pool quality were neutral. In contrast, when general profitability is very low the firm is able to use selective hiring to promote high-quality workers to "signal" through arrival, owing to the advantage given to high-quality workers by the firm's use of reports.

In Section 2.3 I investigate the effects of parameter changes in this model on equilibrium outcomes and welfare. I find complementarity between the screening power of reports and the quality composition of the pool of applicants within equilibria with selective hiring. Due to conflicting influences on the cost of labour, the volume of hires, and sometimes also the quality composition of hires, I find that higher wages may increase or decrease firm profits. Finally, I discuss alternative wage-setting and timing with commitment model variations.

2.1 Basic Hiring Framework

I consider a constant-returns-to-scale industry with a population of workers seeking employment with one representative firm. The firm's profit from a worker depends on that worker's quality type $q \in \{h, \ell\}$. This reflects his skill within that industry, high or low, and is private information. However, the proportion of high-quality workers in the general population, $s \in (0, 1)$ is exogenous and known to all.

The value to the firm of a worker of quality q is v_q, with $v_h > v_\ell$ and $v_\ell > 0$. I suppose that the firm pays a fixed wage $w > 0$, so the profit from a high-quality worker is $v_q - w$ and the profit from a low-quality worker is $v_\ell - w$. The fixed wage may be due to limitations such as might be imposed by a union, internal pay structures, legislation, or competition with other industries not modelled here. I will take $w \in (v_\ell, v_h)$, in which case hiring a high-quality worker is a gain to the firm and hiring a low-quality worker is a loss.[1] The firm's expected profit from a worker which it believes to be high-quality with probability $\mu \in [0, 1]$ is

[1] If $w \geq v_h$ hiring can never be profitable for the firm whereas if $w \leq v_\ell$, the firm maximizes profit by accepting every application.

$$E_q[v_q - w] = \mu(v_h - w) - (1-\mu)(w - v_\ell). \tag{2.1}$$

Let $d \in \{0, 1\}$ indicate a hiring decision for the firm when it considers an individual application, so that $d = 1$ represents a decision to accept the applicant, and $d = 0$ represents a decision to reject the applicant. I assume constant returns to scale technology for the firm, thus there is no competition between individual workers and the firm is willing to accept a given applicant as long as the expected profit from hiring that applicant is not negative. Thus $d = 1$ is (weakly) optimal for the firm if and only if

$$\mu(v_h - w) - (1-\mu)(w - v_\ell) \geq 0 \tag{2.2}$$

And $d = 0$ is (weakly) optimal for the firm if and only if

$$\mu(v_h - w) - (1-\mu)(w - v_\ell) \leq 0. \tag{2.3}$$

For any strictly positive belief $\mu > 0$ we may instead write

$$d = 1 \quad \text{if and only if} \quad \frac{v_h - w}{w - v_\ell} \geq O(\ell{:}h) \tag{2.4}$$

and

$$d = 0 \quad \text{if and only if} \quad \frac{v_h - w}{w - v_\ell} \leq O(\ell{:}h), \tag{2.5}$$

where $O(\ell{:}h)$ denotes the odds that the applicant is low-quality (versus high-quality), which is $O(\ell{:}h) \equiv \frac{Pr(\ell)}{Pr(h)} = \frac{1-\mu}{\mu}$.

Thus the firm's decision follows a cutoff rule. We see that the firm can have positive expected profits from hiring if and only if the relative gain to the firm from hiring a high-quality worker exceeds the odds that a given applicant is low-quality. If the firm's belief μ is updated by conditioning on some new information, then the prior odds in inequalities (2.4) and (2.5) are also updated according to Bayes' Rule. It is then the case that the firm's conditioned expected profits from hiring are positive if and only if the relative gain from a high-quality worker exceeds the conditional (posterior) odds that the applicant is low-quality. I will now consider three situations: first when the firm has no additional information, second when information can be deduced from the relative arrival of applications from high- and low-type workers, and third, when applications themselves bear explicit reports of information.

2.1.1 Hiring from the General Population

In the absence of any additional information and assuming that all workers are equally likely to have their application reach the firm, the firm's belief that a given applicant is high-quality will match the probability that a random worker drawn from the population is high-quality, $\mu = s$. In this case the odds that an applicant is low-quality, $O(\ell{:}h)$, correspond to the quality composition of the general population, $\frac{1-s}{s}$. Thus $d = 1$ is optimal for the firm if and only if

$$\frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s} \tag{2.6}$$

and $d = 0$ is optimal if and only if

$$\frac{v_h - w}{w - v_\ell} \leq \frac{1-s}{s}. \tag{2.7}$$

Given v_h, v_ℓ, w and s for a particular industry setting, the resulting relationship between the firm's relative gain from a high-quality worker and the quality composition of the population defines an important industry characteristic in my model, because it gives an indication of how profitable the industry would be in general if the firm were to employ all workers.

Definition 1. *The industry is "generally profitable" if* $\frac{v_h - w}{w - v_\ell} > \frac{1-s}{s}$.

The industry is "generally unprofitable" if $\frac{v_h - w}{w - v_\ell} < \frac{1-s}{s}$.

When the industry is "generally profitable," the firm would profit in expectation by hiring workers drawn randomly from the population. When the industry is "generally unprofitable," the firm would make negative profit in expectation by hiring workers drawn randomly from the population. Therefore, in the absence of any additional information about applicants, it is strictly optimal for the firm to choose $d = 1$ when the industry is generally profitable, and strictly optimal for the firm to choose $d = 0$ when the industry is generally unprofitable.

Two factors influence the general profitability of the industry. First, the industry may be generally profitable because the industry wage is close to the productivity of the low-quality workers. This reduces the loss of profit from employing a low-quality worker, $w - v_\ell$, while increasing the gain from a high-quality hire, $v_h - w$. Second, the industry may be generally profitable the more abundant are high-quality workers in the population, so that s is higher and $1 - s$ is lower. High-quality workers may be more prevalent in the population when the relevant job skills are common, for example, or when training is easily accessible.

2.1.2 Information About the Pool of Applicants

Suppose that the firm is not automatically aware of individual workers and makes a hiring decision only upon receiving an application. Suppose in addition that worker's application reaches the firm with some probability which may vary according to type, p_q. I call this the "arrival probability." Arrival probabilities may differ by type for a variety of reasons, including differences on either the transmitting or receiving end, and these differences may or may not be endogenous. One worker type may be more likely to know how to apply, or be willing to devote more effort to apply, or one type's application may be less likely to become lost or go unnoticed (efforts more likely to result in the firm becoming aware of interest/availability). I will develop my model with endogenous arrival probabilities, and later link them to an underlying network structure. Whatever the underlying reasons, whenever arrival probabilities differ by type, the firm's hiring decision will be influenced not only by the proportion of high- and low-quality workers in the population, but also by p_h, and p_ℓ.

Provided that the arrival probability for high types is not zero, $p_h > 0$, the firm can update the odds that a given application comes from a low-quality worker according to Bayes' Rule, by conditioning the odds on the event that the application has arrived. Let A denote the event of the application's arrival. The updated conditional (posterior) odds are equal to the prior odds times Bayes' factor, $O(\ell{:}h|A) = O(\ell{:}h) \cdot \Lambda(\ell{:}h|A)$. The prior odds will correspond to the population-based odds, $O(l{:}h) = \dfrac{1-s}{s}$, while Bayes' factor will be determined by the relative arrival probabilities of low- and high-quality workers, $\Lambda(\ell{:}h|A) \equiv \dfrac{Pr(A|\ell)}{Pr(A|h)} = \dfrac{p_\ell}{p_h}$. In this setting, $d = 1$ is optimal for the firm if and only if

$$\frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s}\frac{p_\ell}{p_h} \tag{2.8}$$

and $d = 0$ is optimal if and only if

$$\frac{v_h - w}{w - v_\ell} \leq \frac{1-s}{s}\frac{p_\ell}{p_h}. \tag{2.9}$$

I will refer to Bayes' factor in this case, $\Lambda(\ell{:}h|A) = \dfrac{p_\ell}{p_h}$, as the "*pool adjustment factor*" because it determines the quality composition bias of the pool of applicants relative to the general population.

Definition 2. *The pool of applicants is "neutral" if* $\dfrac{p_\ell}{p_h} = 1$.

When the pool of applicants is "neutral," the quality composition of the pool of applicants exactly matches the quality composition of the general population because high- and low-quality applicants arrive with equal probabilities. When the pool is neutral, the firm can deduce no new information based on the event of an application's arrival and the firm's optimal strategy is determined entirely by the general profitability of the industry.

Definition 3. *The pool of applicants is "favourable" if* $\frac{p_\ell}{p_h} < 1$.

When the pool of applicants is "favourable," its quality composition is superior to the quality composition of the general population, and the posterior odds of an applicant being low-quality are lower than the prior population-based odds. The improved pool composition allows hiring to be optimal for a greater range of industry settings, in the sense that inequality (2.8) can be satisfied for lower relative gain ratio $\frac{v_h-w}{w-v_\ell}$ than inequality (2.6). In particular, a firm may hire despite the general unprofitability of the industry if applications from low-types arrive with sufficiently less probability than applications from high-types. In a generally profitable industry, a favourable pool adjustment factor makes no difference to the firm's hiring decision because the relative gain from a high-quality worker is already high enough that the firm profits from hiring even if the pool of applicants were no better than the general population. This is seen from the fact that inequality (2.8) implies inequality (2.6) when $\frac{p_\ell}{p_h} < 1$. Although the firm's hiring decision itself is not affected in this case, the firm's expected profits are of course higher when low-quality workers are less likely to send applications than when $p_h = p_\ell$.

Definition 4. *The pool of applicants is "unfavourable" if* $\frac{p_\ell}{p_h} > 1$.

When the pool of applicants is "unfavourable," its quality composition is inferior to the quality composition of the general population. The posterior odds of an applicant being low-quality are higher than the prior population-based odds. This deters the firm from hiring in as wide a range of industry settings, in the sense that inequality (2.8) cannot be satisfied for quite as low relative gain ratio $\frac{v_h-w}{w-v_\ell}$ as inequality (2.6). In particular, if $\frac{p_\ell}{p_h}$ is sufficiently high the firm may not hire even when the industry is generally profitable. Given an unfavourable pool bias in a generally unprofitable industry the firm will, of course, remain unwilling to hire.

2.1.3 Noisy Reports of Quality

Now suppose that the application arrival probabilities are the same for each worker type (and non-zero) such that $\frac{p_\ell}{p_h} = 1$, but that the firm gains information from the content of the application. Suppose that applications carry a report which is a noisy signal of worker quality, $R \in \{H, L\}$. Rather than altering the composition of the pool of applicants, this allows the firm to make separate hiring decisions for each report realization, choosing $d_H \in \{0, 1\}$ given $R = H$ and $d_L \in \{0, 1\}$ given $R = L$. In each case, the posterior odds that the worker is low-quality are obtained by conditioning the prior odds on the realization of the report received, $O(\ell{:}h|R) = O(\ell{:}h) \cdot \Lambda(\ell{:}h|R)$.

Bayes' factor in this situation, $\Lambda(\ell{:}h|R) = \frac{Pr(R|\ell)}{Pr(R|h)}$, is determined by the relative probability that the realized report came from a low-quality worker. For $R = H$ it is the relative probability that the report is an error, while for $R = L$ it is the relative probability that the report is true. Assuming that type I and type II errors occur with the same probability $\varepsilon \in (0, \frac{1}{2})$, we will have $\Lambda(\ell{:}h|H) = \frac{\varepsilon}{1-\varepsilon}$ and $\Lambda(\ell{:}h|L) = \frac{1-\varepsilon}{\varepsilon}$. If the firm's prior odds are based on the composition of the general population, $O(\ell{:}h) = \frac{1-s}{s}$, then it is optimal for the firm to accept high-report applications, that is $d_H = 1$ is optimal, if and only if

$$\frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon} \tag{2.10}$$

and $d_H = 0$ is optimal if and only if

$$\frac{v_h - w}{w - v_\ell} \leq \frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}, \tag{2.11}$$

It is optimal for the firm to accept low-report applications, that is $d_L = 1$ is optimal, if and only if

$$\frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon} \tag{2.12}$$

and $d_L = 0$ is optimal if and only if

$$\frac{v_h - w}{w - v_\ell} \leq \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}. \tag{2.13}$$

Since reports are positively correlated with type, due to $\varepsilon < \frac{1}{2}$, a high report lowers the odds that the applicant is low-quality, while a low report increases those odds. Therefore a high report may decrease the posterior odds of low-quality sufficiently for the firm to hire high-report applicants in a generally unprofitable industry, as long as the probability that the report is true, $1-\varepsilon$, is high enough. Similarly a low report may increase the posterior odds of low-quality sufficiently for the firm to reject low-report applicants in a profitable industry, as long as the probability that the report is true, ε, is high enough.

Note also that because the odds of an applicant being low-quality are always higher given a low report than given a high report, inequality (2.10) is always satisfied if inequality (2.12) is satisfied, so it can not be optimal for the firm to hire low-report applicants if it is not optimal to hire high-report applicants. Therefore the firm's optimal hiring decision follows one of three patterns:

(i) "*Absolute Hiring:*" the firm hires all applicants regardless of report, $d_H = d_L = 1$,

(ii) "*Selective Hiring:*" the firm hires only applicants with high reports, $d_H = 1$, $d_L = 0$,

(iii) "*No Hiring:*" the firm rejects all applicants regardless of report, $d_H = d_L = 0$.

For an industry with a given relative gain ratio $\frac{v_h - w}{w - v_\ell}$ and neutral composition of applicants $\frac{1-s}{s}$, the report error ε determines when selective hiring is optimal for the firm (that is, when the firm will hire according to the indication of the report) and when the firm will ignore the report and hire according to the pattern it would adopt in the absence of any report.

Definition 5. *The report is "decisive" if* $\frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon} < \frac{v_h - w}{w - v_\ell} < \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}$.

The report is "not decisive" if $\frac{v_h - w}{w - v_\ell} < \frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}$ *or* $\frac{v_h - w}{w - v_\ell} > \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}$.

When the report is "*decisive*," it is strictly optimal for the firm to hire high-report applicants and rejects low-report applicants. The realization of R sways the firm's decision relative to the decision the firm would make when hiring from the general population; either the firm hires high-report applicants when it would otherwise hire no applicants (such as in a generally unprofiatble industry), or the firm rejects low-report applicants when it would otherwise accept all applicants (such as in a generally profitable industry).

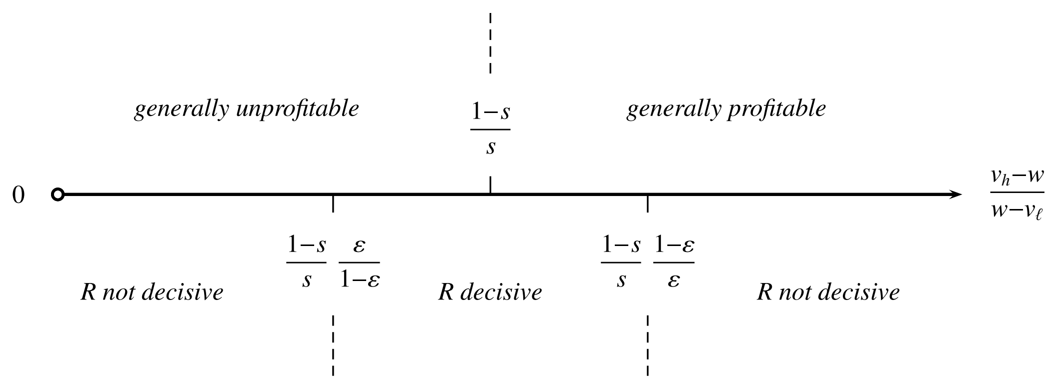

Figure 2.1: General Profitability and Report Decisiveness

The industry setting is described according to the firm's relative gain ratio, report error, and general population composition.

When the report is "*not decisive*," its realization does not affect the firm's decision. The firm will hire all applicants or none based solely on whether or not the industry is generally profitable. When the report is not decisive and the industry is generally unprofitable, $\frac{v_h-w}{w-v_\ell} < \frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}$, the firm will reject both applicants with $R = H$ and applicants with $R = L$. When the report is not decisive and the industry is generally profitable, $\frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon} < \frac{v_h-w}{w-v_\ell}$, the firm will accept both applications with $R = H$ and applications with $R = L$.

Assuming arrival probabilities are non-zero and do not differ by type, $\frac{p_\ell}{p_h} = 1$, the firm's hiring strategy is chosen according to the region in Figure 2.1 in which the firm's relative gain ratio, $\frac{v_h-w}{w-v_\ell}$, falls. When reports are not decisive and the industry is generally unprofitable, no applicants are hired; $d_H = d_L = 0$ ("no hiring.") When reports are decisive, the firm hires only high-report applicants; $d_H = 1$ and $d_L = 0$ ("selective hiring.") When reports are not decisive and the industry is generally profitable, all applicants are hired; $d_H = d_L = 1$ ("absolute hiring.") Since a larger report error reduces the extent to which the firm can rely on the report, an increase in ε decreases the range for which the report is decisive, and therefore decreases the range in which selective hiring is optimal.

2.2 Model Analysis

I now endogenize the application arrival probabilities of workers and examine the possible equilibrium outcomes of the simultaneous move game when firms gain information about applicants from both endogenous differences in high- and low-type workers' arrival probabilities and from noisy reports of quality. I will first develop the firm's best response given worker choices of p_h, p_ℓ, and then develop high- and low-type workers' choices of p_h, p_ℓ as a best response to the firm's hiring strategy. I will then discuss the existence and interpretation of equilibria with hiring. I focus primarily on the simultaneous move game here because it is often plausible to think that neither the firm nor the workers can commit in advance to their strategies. However an alternative timing in which the firm chooses its strategy first is considered in Section 2.3.4.

2.2.1 Best Response for Firm

The firm's strategy specifies a hiring decision for both high-report applications and low report applications given the workers' chosen arrival probabilities p_h and p_ℓ. Since applications carry a report of quality in addition to application arrival probabilities potentially differing by type, the firm can update its beliefs and $O(\ell{:}h)$ to account for both the event of the application's arrival and the observation of the report (given that the application was received). Applying Bayes' rule twice gives the posterior odds $O(\ell{:}h|A\cap R) = O(\ell{:}h) \cdot \Lambda(\ell{:}h|A) \cdot \Lambda(\ell{:}h|A\cap R)$.

The pool adjustment factor is $\Lambda(\ell{:}h|A) = \frac{Pr(A|\ell)}{Pr(A|h)} = \frac{p_\ell}{p_h}$ and the report adjustment factor is $\Lambda(\ell{:}h|A\cap R) = \frac{Pr(R|A\cap\ell)}{Pr(R|A\cap h)}$, which as before will be the relative probability that the report is false when $R = H$ and the relative probability that the report is true when $R = L$. The firm's best response, allowing for mixing when the firm is indifferent between hiring and rejecting, is characterized in the following Lemma.

Lemma 1. *Given $p_h \neq 0$ and $R = H$,*

$$d_H(p_h, p_\ell) = 1 \quad \text{if} \quad \frac{v_h - w}{w - v_\ell} > \frac{1-s}{s}\frac{p_\ell}{p_h}\frac{\varepsilon}{1-\varepsilon} \tag{2.14}$$

$$d_H(p_h, p_\ell) = \{\text{all } \alpha \in [0,1]\} \quad \text{iff} \quad \frac{v_h - w}{w - v_\ell} = \frac{1-s}{s}\frac{p_\ell}{p_h}\frac{\varepsilon}{1-\varepsilon} \tag{2.15}$$

$$d_H(p_h, p_\ell) = 0 \quad \text{if} \quad \frac{v_h - w}{w - v_\ell} < \frac{1-s}{s}\frac{p_\ell}{p_h}\frac{\varepsilon}{1-\varepsilon} \tag{2.16}$$

Given $p_h \neq 0$ and $R = L$,

$$d_L(p_h, p_\ell) = 1 \quad \text{if} \quad \frac{v_h - w}{w - v_\ell} > \frac{1-s}{s}\frac{p_\ell}{p_h}\frac{1-\varepsilon}{\varepsilon} \tag{2.17}$$

$$d_L(p_h, p_\ell) = \{\text{all } \beta \in [0,1]\} \quad \text{iff} \quad \frac{v_h - w}{w - v_\ell} = \frac{1-s}{s}\frac{p_\ell}{p_h}\frac{1-\varepsilon}{\varepsilon} \tag{2.18}$$

$$d_L(p_h, p_\ell) = 0 \quad \text{if} \quad \frac{v_h - w}{w - v_\ell} < \frac{1-s}{s}\frac{p_\ell}{p_h}\frac{1-\varepsilon}{\varepsilon}. \tag{2.19}$$

For $p_h = 0$ and $p_\ell > 0$, the firm's best response is $d_H(p_h, p_\ell) = d_L(p_h, p_\ell) = 0$. For $p_h = p_\ell = 0$, any $d_H(p_h, p_\ell) \in [0,1]$ with any $d_L(p_h, p_\ell) \in [0,1]$ is optimal.

When $p_h \neq 0$ so that the firm does receive applications from high-quality workers, the firm's best response given either report is unique except when $\frac{v_h - w}{w - v_\ell} = O(\ell{:}h|A \cap R)$ and the expected profit of an applicant with report R is exactly zero so that the firm is indifferent between hiring and not hiring.

It is always the case that $d_H(p_h, p_\ell) \geq d_L(p_h, p_\ell)$ for $p_h \neq 0$, so in any equilibrium where $p_h > 0$, the firm's hiring strategy can be summarized by $d \in [0, 2]$ where $d = d_H(p_h, p_\ell) + d_L(p_h, p_\ell)$. Higher values of d correspond to more hiring, and Figure 2.2 shows how for a given pool adjustment factor $\frac{p_\ell}{p_h}$, the firm becomes more willing to hire the greater the relative gain from a high quality worker. For very high $\frac{v_h - w}{w - v_\ell}$ the firm will hire all applicants, $d_H = d_L = 1$, so $d = 2$. I will refer to this hiring pattern as "absolute hiring." For moderate values of $\frac{v_h - w}{w - v_\ell}$, the firm will hire high-report applicants only, $d_H = 1$ and $d_L = 0$, so $d = 1$. I will refer to this hiring pattern as "selective hiring." For very low $\frac{v_h - w}{w - v_\ell}$, the firm will adopt a pattern of "no hiring", $d_H = d_L = 0$, so $d = 0$. In the borderline cases with $\frac{v_h - w}{w - v_\ell} = \frac{1-s}{s} \frac{p_\ell}{p_h} \frac{1-\varepsilon}{\varepsilon}$ where the firm is indifferent concerning low-report applicants, and with $\frac{v_h - w}{w - v_\ell} = \frac{1-s}{s} \frac{p_\ell}{p_h} \frac{\varepsilon}{1-\varepsilon}$ where the firm is indifferent concerning high-report applicants, the firm will mix accordingly between absolute and selective hiring, $d = 1 + \beta$, or between selective hiring and no hiring, $d = \alpha$.

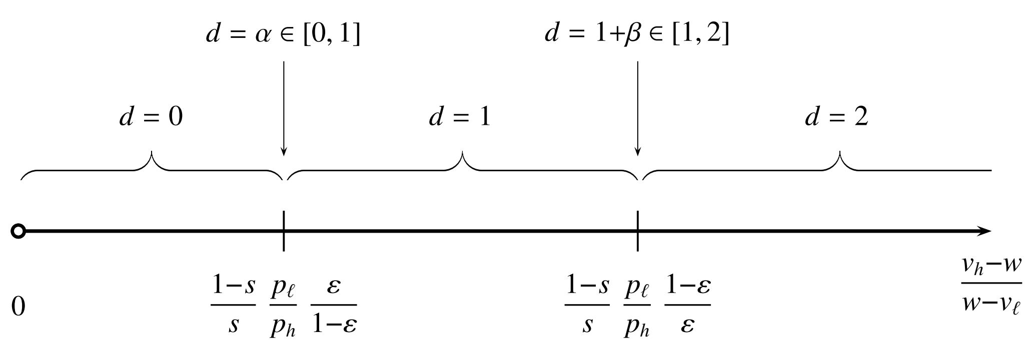

Figure 2.2: Firm Hiring Patterns

The firm's hiring strategy, summarized by $d = d_H + d_L$ for a given pool adjustment, increases from "no hiring" $d = 0$, to "selective hiring" $d = 1$, to "absolute hiring" $d = 2$, as the relative gain from a high worker increases, through mixed hiring patterns in each transition.

The information gained from adjustments to the pool of applicants affects the firm's hiring decisions differently than information gained from a noisy report of quality. A reduction in the relative arrival of low-quality applications $\frac{p_\ell}{p_h}$ lowers the odds that both high- and low-report applications are truly low-quality, whereas a reduction in report error lowers the odds that a high-report applicant is truly low-quality, while raising the odds that a low-report applicant is truly low-quality.

With a neutral pool adjustment factor, $\frac{p_\ell}{p_h} = 1$, the firm's optimal hiring pattern would be determined by the decisiveness of the report and the general profitability of the industry; the thresholds $\frac{1-s}{s}\frac{p_\ell}{p_h}\frac{\varepsilon}{1-\varepsilon}$ and $\frac{1-s}{s}\frac{p_\ell}{p_h}\frac{1-\varepsilon}{\varepsilon}$ in Figure 2.2 would correspond with $\frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}$ and $\frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}$ Figure 2.1. A favourable pool adjustment factor $\frac{p_\ell}{p_h} < 1$ will shift (and in fact compress) the region of selective hiring to the left relative to the region of selective hiring given $p_h = p_\ell$ (> 0). In contrast, an unfavourable pool adjustment factor $\frac{p_\ell}{p_h} > 1$ will shift (and spread) this region to the right. Similarly, a favourable pool adjustment factor will also expand the region of absolute hiring to the left relative to neutral, while an unfavourable pool will compress it to the right.

Lemma 2. *For the firm, the optimal hiring correspondence in response to any pool factor* $\Lambda = \frac{p_\ell}{p_h}$ *with* $p_h, p_\ell > 0$, *can be characterized by*

$$d(\Lambda) = \begin{cases} 2 & \text{if } \Lambda < \frac{v_h - w}{w - v_\ell}\frac{s}{1-s}\frac{\varepsilon}{1-\varepsilon} \\ [1,2] & \text{if } \Lambda = \frac{v_h - w}{w - v_\ell}\frac{s}{1-s}\frac{\varepsilon}{1-\varepsilon} \\ 1 & \text{if } \frac{v_h - w}{w - v_\ell}\frac{s}{1-s}\frac{\varepsilon}{1-\varepsilon} < \Lambda < \frac{v_h - w}{w - v_\ell}\frac{s}{1-s}\frac{1-\varepsilon}{\varepsilon} \\ [0,1] & \text{if } \Lambda = \frac{v_h - w}{w - v_\ell}\frac{s}{1-s}\frac{1-\varepsilon}{\varepsilon} \\ 0 & \text{if } \Lambda > \frac{v_h - w}{w - v_\ell}\frac{s}{1-s}\frac{1-\varepsilon}{\varepsilon}. \end{cases} \tag{2.20}$$

The firm's optimal hiring correspondence $d(\Lambda)$ is weakly decreasing in Λ.

2.2.2 Best Response for Workers

Now the workers' arrival probabilities p_h and p_ℓ (and therefore the direction of the pool adjustment) will be determined endogenously by workers in best response to the hiring pattern chosen by the firm, according to their own expected utility maximization problem. Suppose that there is some way workers may improve their application arrival probability, but that it is costly. Let $\gamma : [0, 1) \to \mathbb{R}_+$, $\gamma(p)$ denote the cost to a worker whose choice results in a probability p of reaching the firm. I will assume γ is increasing, strictly convex, and twice continuously differentiable, with $\gamma(0) = 0$. If a worker is hired he receives a wage w, and if not he receives an unemployment benefit $b < w$. Therefore the expected utility of a worker of type q is

$$w\Phi_q + b(1-\Phi_q) - \gamma(p_q) \tag{2.21}$$

where Φ_q denotes the probability that a worker of type q is hired, which depends on the worker's arrival probability and the firm's hiring strategy, so that $\Phi_q = \Phi_q(p_q, d_H, d_L)$. Possible alternatives to modelling the common unemployment benefit b and cost function $\gamma(p)$ are that a worker's arrival costs or outside option may be type-dependent, such as $\gamma_q(p)$ and b_q. In Chapter 3, I introduce the existence of a second hiring channel which I show can be considered a special case of a type-dependent outside option for workers.

We obtain the probability of worker q being hired by multiplying the probability that his application arrives to the firm, p_q, by the probability that his application is accepted given that it was received. This conditional acceptance probability, denoted $\phi_q(d_H, d_L)$, is the worker's expectation of the firm's hiring decision. For a high-type worker,

$$\phi_h(d_H, d_L) = d_H(1-\varepsilon) + d_L\varepsilon \tag{2.22}$$

and for a low-type worker,

$$\phi_\ell(d_H, d_L) = d_H\varepsilon + d_L(1-\varepsilon). \tag{2.23}$$

Substituting $\Phi_q(p_q, d_H, d_L) = p_q \cdot \phi_q(d_H, d_L)$ into equation (2.21) for $q \in \{h, \ell\}$ and rearranging, we see that high-quality type chooses p_h to maximize

$$(w-b)p_h(d_H(1-\varepsilon) + d_L\varepsilon) + b - \gamma(p_h) \tag{2.24}$$

and the low-quality type chooses p_ℓ to maximize

$$(w-b)p_\ell(d_H\varepsilon + d_L(1-\varepsilon)) + b - \gamma(p_\ell). \tag{2.25}$$

Therefore since γ is increasing and strictly convex, whenever $\lim_{p\to 1}\gamma'(p)$ is sufficiently high the high-quality worker's optimal choice of p_h must satisfy

$$(w-b)(d_H(1-\varepsilon) + d_L\varepsilon) \leq \gamma'(p_h), \tag{2.26}$$

with equality if $p_h > 0$, and the low-quality worker's optimal choice of p_ℓ must satisfy

$$(w-b)(d_H\varepsilon + d_L(1-\varepsilon)) \leq \gamma'(p_\ell), \tag{2.27}$$

with equality if $p_\ell > 0$. Then, taking $\hat{w} \equiv w-b$, and denoting $\psi \equiv \gamma'^{-1}$, Lemma 3 characterizes the best responses of workers.

Lemma 3. *The high-type worker's best response is characterized by*

$$p_h(d_H, d_L) = \begin{cases} \psi(\hat{w}(d_H(1-\varepsilon) + d_L\varepsilon)) & \textit{if } \hat{w}(d_H(1-\varepsilon) + d_L\varepsilon) > \gamma'(0) \\ 0 & \textit{otherwise.} \end{cases} \tag{2.28}$$

The low-type worker's best response is characterized by

$$p_\ell(d_H, d_L) = \begin{cases} \psi(\hat{w}(d_H\varepsilon + d_L(1-\varepsilon)) & \textit{if } \hat{w}(d_H\varepsilon + d_L(1-\varepsilon) > \gamma'(0) \\ 0 & \textit{otherwise.} \end{cases} \tag{2.29}$$

Therefore when worker best responses are non-zero, we may also express the composition of the pool of applicants as a function of firm strategy d, as follows:

Lemma 4. *Whenever $p_h(d)$, $p_\ell(d) > 0$, the composition of the pool of applicants resulting from worker responses to hiring strategy $d \in (0, 2]$ is characterized by*

$$\Lambda(d) = \frac{p_\ell(d)}{p_h(d)} = \frac{\psi(\hat{w}\phi_\ell(d))}{\psi(\hat{w}\phi_h(d))} \tag{2.30}$$

where

$$\phi_h(d) = \begin{cases} (1-\varepsilon)d & \textit{if } d \in (0, 1] \\ \varepsilon d + (1-2\varepsilon) & \textit{if } d \in [1, 2] \end{cases} \tag{2.31}$$

and

$$\phi_\ell(d) = \begin{cases} \varepsilon d & \textit{if } d \in (0, 1] \\ (1-\varepsilon)d - (1-2\varepsilon) & \textit{if } d \in [1, 2]. \end{cases} \tag{2.32}$$

Since workers' probabilities of acceptance $\phi_h(d)$ and $\phi_\ell(d)$ are increasing in d, both workers have incentive to devote more effort to application for higher d. However, the degree to which effort increases for each type (and the effect this has on the pool composition) is affected by the given initial arrival probability of that type. Therefore depending on the curvature of γ, the resulting pool composition may improve in response to greater hiring by the firm, or it may degrade. This is formalized in the following Lemma and sufficient (but not necessary) conditions are given under which the pool becomes more unfavourable when d increases.

Lemma 5. *For $d \in (0, 2]$ when $p_h(d)$, $p_\ell(d) > 0$, $\Lambda(d)$ may increase or decrease in d.*

(i) Suppose ψ is logarithmically concave. Then for $d \in (1,2)$, we have $\frac{d\Lambda}{dd} > 0$

(ii) Suppose ψ has decreasing elasticity. Then for $d \in (0,1)$, we have $\frac{d\Lambda}{dd} > 0$.

2.2.3 Equilibria

Consider mixed strategy Nash equilibria. A trivial equilibrium certainly always exists with $d_H^* = d_L^* = 0$ and $p_h^* = p_\ell^* = 0$. If no applications arrive to the firm, any firm strategy is a best response; also choosing non-arrival is a best response for workers if the firm accepts no applications.

Non-arrival can also be supported in equilibrium with other firm strategies, provided that it is sufficiently expensive for workers to increase the probability of their application's arrival. An equilibrium with $d_H^* = 1$, $d_L^* = 0$ and $p_h^* = p_\ell^* = 0$ can also be supported whenever $\hat{w}(1-\varepsilon) \leq \gamma'(0)$, because for either worker type, given the selective hiring pattern chosen by the firm, the benefit from an increased chance of having their application arrive is not enough to compensate for the cost. Similarly, if $\hat{w} \leq \gamma'(0)$ then even absolute hiring, $d_H^* = d_L^* = 1$, can be supported in equilibrium with $p_h^* = p_\ell^* = 0$. I will now suppose that p_q is not prohibitively expensive and focus instead on equilibria in which at least some applications are received (p_h^* and p_ℓ^* are not both zero).

Lemma 6. *In any equilibrium in which hiring occurs, $p_h^* \geq p_\ell^* > 0$.*

Proof. There can be no equilibria with hiring in which applications arrive to the firm from only one worker type. If applications only arrive to the firm from low types, then $d_H = d_L = 0$, which is incompatible with $p_\ell > 0$. If applications only arrive to the firm from high types, then $d_H = d_L = 1$, which is incompatible with $p_\ell > 0$. Therefore any such equilibrium has $p_h^* > 0$ and $p_\ell^* > 0$. Since from Lemma 1 we have that $d_H(p_h, p_\ell) \geq d_L(p_h, p_\ell)$ whenever $p_h > 0$, then we know that $d_H^* \geq d_L^*$. By equations (2.28) and (2.29) this implies that $p_h^* \geq p_\ell^*$.

From now on I focus on equilibria in which hiring occurs. Therefore such equilibria must have a favourable or neutral pool adjustment factor, $\frac{p_\ell^*}{p_h^*} \leq 1$ and either absolute or selective hiring ($d_H^* = d_L^* = 1$ or $d_H^* = 1$, $d_L^* = 0$) in pure strategies, or mixed hiring with $d_H = \alpha \in (0,1]$ and $d_L = 0$ or with $d_H = 1$ and $d_L = \beta \in [0,1]$.

Proposition 1. ***Absolute Hiring in Equilibrium.*** *An equilibrium in which p_h^*, $p_\ell^* > 0$ and the firm accepts all applicants, $d_H^* = d_L^* = 1$, exists if and only if $\hat{w} > \gamma'(0)$ and $\frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}$. In this equilibrium $\frac{p_\ell^*}{p_h^*} = \frac{\psi(\hat{w})}{\psi(\hat{w})} = 1$.*

This proposition describes the (non-trivial) equilibrium which arises in sufficiently profitable and sufficiently noisy environments. In this equilibrium both types choose equal arrival probabilities because for $d_H^* = d_L^* = 1$ both types are equally likely to be accepted conditional on the arrival of their application. By equations (2.28) and (2.29), the arrival probability of high and low types in such an equilibrium will be $p_h^* = p_\ell^* = \psi(\hat{w})$. Thus the first condition, $\hat{w} > \gamma'(0)$, ensures that indeed workers are willing to choose non-zero arrival probabilities, p_h^*, $p_\ell^* > 0$. The second condition ensures that the firm is willing to hire low-report applicants (and therefore also high-report applicants) given that $p_h^* = p_\ell^* > 0$. It reflects the requirement that the firm's relative gain from a high-quality worker exceeds $O(\ell{:}h|A \cap L)$, given a neutral pool adjustment factor $\frac{p_\ell^*}{p_h^*} = \frac{\psi(\hat{w})}{\psi(\hat{w})} = 1$.

Proposition 2. ***Selective Hiring in Equilibrium.*** *An equilibrium in which p_h^*, $p_\ell^* > 0$ and only high-report applicants are accepted, $d_H^* = 1$, $d_L^* = 0$, exists if and only if $\hat{w}\varepsilon > \gamma'(0)$ and*

$$\frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\Lambda^1 \leq \frac{v_h - w}{w - v_\ell} \leq \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\Lambda^1, \tag{2.33}$$

where $\Lambda^1 \equiv \frac{p_\ell(d_H^*, d_L^*)}{p_h(d_H^*, d_L^*)} = \frac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))}$.

In any equilibrium with selective hiring and non-zero arrival probabilities, we will have $p_h^* = \psi(\hat{w}(1-\varepsilon))$ and $p_\ell^* = \psi(\hat{w}\varepsilon)$. Since γ is convex, γ' is increasing. Therefore ψ is also increasing and $p_h^* > p_\ell^*$. The condition $\hat{w}\varepsilon > \gamma'(0)$ ensures that both arrival probabilities will indeed be strictly positive. Inequality (2.33) ensures that the firm's selective hiring strategy is compatible with the pool adjustment factor it generates; the relative gain from a high-quality worker is enough to exceed the odds of a high-report applicant being low-quality, but not enough to exceed the odds of a low-report applicant being low-quality.

Corollary 1. *Multiple pure strategy Nash equilibria with non-zero arrival cannot co-exist.*

This follows directly from Propositions 1 and 2. Workers have a unique non-zero best response to any firm strategy, and the levels of relative gain $\frac{v_h - w}{w - v_\ell}$ for which absolute hiring can be supported in equilibrium are disjoint from those for which selective hiring can be supported.

Corollary 2. *A non-zero arrival pure strategy Nash equilibrium may fail to exist.*

This also follows directly from Propositions 1 and 2. Taken together, they show that no pure strategy Nash equilibrium with $p_h, p_\ell > 0$ can be supported for industries with a very low relative gain from high-quality workers, namely $\frac{v_h - w}{w - v_\ell} < \frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\frac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))}$, or for industries with relative gain $\frac{v_h - w}{w - v_\ell} \in \left(\frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\frac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))}, \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\right)$. For such industries the only pure strategy equilibrium is the trivial equilibrium in which no hiring occurs. This is somewhat unsurprising in the former case, that is for industries with very low relative gain, because of their very low general profitability. For some forms of γ it may be possible that this region of non-existence is very small, with $\frac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))} \approx 0$ so that selective hiring induces a pool of applicants that is sufficiently favourable to outweigh the extreme general unprofitability of the industry. But for general γ there will typically be a range of industries for which the relative gain is simply too low for any hiring. Now in the latter case, the inability to sustain selective or absolute hiring in equilibrium comes from the incompatibility of firm and worker strategies. The firm's relative gain is too high to support selective hiring in equilibrium (because given the favourable pool factor resulting from selective hiring, the firm would deviate to hiring all applicants) while at the same time the relative gain is not quite high enough to support absolute hiring in equilibrium (given the neutral pool factor that absolute hiring would generate). Since profit is positive in any equilibrium with non-zero hiring, such a firm would certainly benefit from hiring selectively if it could commit to such a strategy regardless of how attractive the resulting pool would make it to hire absolutely. Some commitment cases are discussed in Section 2.3.4. To some extent, the firm may be able to alleviate these problems and increase the range of relative gain $\frac{v_h - w}{w - v_\ell}$ for which it can hire in equilibrium through the use of mixed strategies.

Proposition 3. ***Mixing Between Selective and Absolute Hiring.*** *An equilibrium in which* p_h^*, $p_\ell^* > 0$ *and* $d_H^* = 1$, $d_L^* = \beta$ *where* $\beta \in (0,1)$, *exists if and only if* $\hat{w}(\varepsilon + (1-\varepsilon)\beta > \gamma'(0)$ *and*

$$\frac{v_h - w}{w - v_\ell} = \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\Lambda^{1+\beta}, \tag{2.34}$$

where $\Lambda^{1+\beta} \equiv \frac{p_\ell(d_H^*, d_L^*)}{p_h(d_H^*, d_L^*)} = \frac{\psi(\hat{w}(\varepsilon + (1-\varepsilon)\beta))}{\psi(\hat{w}((1-\varepsilon) + \varepsilon\beta))}$.

This equilibrium exists for some $\beta \in (0,1)$ *when* $\frac{v_h - w}{w - v_\ell} \in \left(\frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\underline{\underline{\Lambda}}, \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\right)$, *where* $\underline{\underline{\Lambda}} \equiv \inf_{\beta\in(0,1)} \frac{\psi(\hat{w}(\varepsilon + (1-\varepsilon)\beta))}{\psi(\hat{w}((1-\varepsilon) + \varepsilon\beta))}$.

Proposition 4. ***Mixing Between Selective and No Hiring.*** *An equilibrium in which* p_h^*, $p_\ell^* > 0$ *and* $d_H^* = \alpha$, $d_L^* = 0$ *where* $\alpha \in (0,1)$, *exists if and only if* $\hat{w}\varepsilon\alpha > \gamma'(0)$ *and*

$$\frac{v_h - w}{w - v_\ell} = \frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\Lambda^{\alpha}, \tag{2.35}$$

where $\Lambda^{\alpha} \equiv \frac{p_\ell(d_H^*, d_L^*)}{p_h(d_H^*, d_L^*)} = \frac{\psi(\hat{w}\varepsilon\alpha)}{\psi(\hat{w}(1-\varepsilon)\alpha)}$.

This equilibrium exists for some $\alpha \in (0,1)$ *when* $\frac{v_h - w}{w - v_\ell} \in \left(\frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\underline{\Lambda}, \frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\overline{\Lambda}\right)$ *or when* $\frac{v_h - w}{w - v_\ell} = \frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\underline{\Lambda} = \frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\overline{\Lambda}$, *where* $\underline{\Lambda} \equiv \inf_{\alpha\in(0,1)} \frac{\psi(\hat{w}\varepsilon\alpha)}{\psi(\hat{w}(1-\varepsilon)\alpha)}$ *and where* $\overline{\Lambda} \equiv \sup_{\alpha\in(0,1)} \frac{\psi(\hat{w}\varepsilon\alpha)}{\psi(\hat{w}(1-\varepsilon)\alpha)}$.

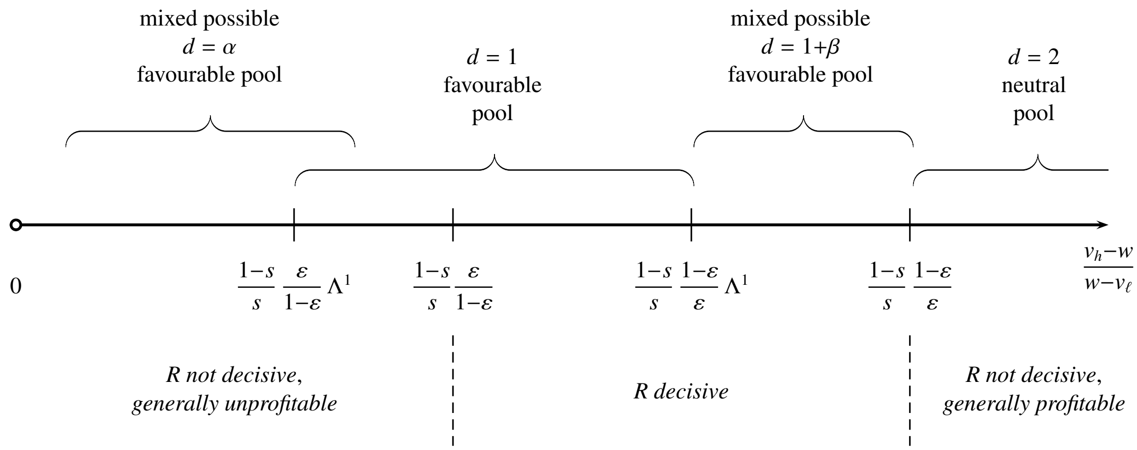

Figure 2.3: Existence of Equilibria

An example of the range of pure strategy Nash equilibria with non-zero hiring is shown. An example range of mixed equilibria existence for logarithmically concave ψ is also given.

Taking the results of Propositions 1-4 together, the following statements can be made regarding the general existence and uniqueness of Nash equilibria with non-zero arrival.

Corollary 3. ***Existence.*** *Suppose $\gamma'(p) > 0$ for all $p > 0$. A Nash equilibrium with non-zero arrival always exists for $\frac{v_h-w}{w-v_\ell} > \frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\underline{\Lambda}$ where $\underline{\Lambda} = \inf_{\alpha\in(0,1)} \frac{\psi(\hat{w}\varepsilon\alpha)}{\psi(\hat{w}(1-\varepsilon)\alpha)}$.*

Corollary 4. ***Uniqueness.*** *Suppose a Nash Equilibrium with non-zero arrival exists. The following are each sufficient conditions for this equilibrium to be the unique Nash equilibrium with non-zero arrival:*

(i) $\frac{v_h-w}{w-v_\ell} \in \left(\frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\overline{\Lambda},\ \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\underline{\underline{\Lambda}}\right)$, *or*

(ii) $\frac{v_h-w}{w-v_\ell} \geq \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}$, *or*

(iii) $\frac{v_h-w}{w-v_\ell} \geq \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\overline{\Lambda}$ *and ψ is logarithmically concave, or*

(iv) ψ has decreasing elasticity,

where $\overline{\Lambda} \equiv \sup_{\alpha\in(0,1)} \frac{\psi(\hat{w}\varepsilon\alpha)}{\psi(\hat{w}(1-\varepsilon)\alpha)}$, *and* $\underline{\underline{\Lambda}} \equiv \inf_{\beta\in(0,1)} \frac{\psi(\hat{w}(\varepsilon + (1-\varepsilon)\beta))}{\psi(\hat{w}((1-\varepsilon) + \varepsilon\beta))}$.

Assuming workers choose non-zero arrival probabilities, that is, assuming that $\psi(\hat{w}\varepsilon) > 0$, the range of existence for equilibria with selective hiring and absolute hiring can look different in different cases. Figure 2.3 depicts the qualitative case where selective hiring can occur when reports are decisive. In contrast, the range of existence for an equilibrium with selective hiring could lie entirely within the industry region that is generally unprofitable with R not decisive. This occurs in the case where the pool of applicants under selective hiring is sufficiently favourable, $\Lambda^1 = \frac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))} < \left(\frac{\varepsilon}{1-\varepsilon}\right)^2$. In this case there can be no pure strategy hiring in equilibrium for any industry with decisive reports. Figure 2.3 also depicts a possible qualitative range of existence for mixed strategy equilibria in cases when ψ is logarithmically concave, assuming non-zero arrival probabilities. Only a possible range is shown as there can be some variation depending on the specification for the cost function γ. To see this, consider the following two examples which each follow directly from Propositions 1-4 for the given functional forms of $\gamma(p)$.

First, consider the case of quadratic arrival costs or for other powers greater than 2, such that $\gamma(p) = cp^x$ (with constant $c > 0$).

Example 2.2.1. *The characterization of non-zero hiring equilibria for $\gamma(p) = cp^x$ for $x \geq 2$ with $c > 0$ is as follows:*

(i) If $\frac{v_h - w}{w - v_\ell} < \frac{1-s}{s}\left(\frac{\varepsilon}{1-\varepsilon}\right)^{\frac{x}{x-1}}$ then there exists no equilibrium with $d_H^*, d_L^* > 0$.

(ii) For any $\alpha \in (0,1)$ there exists an equilibrium with $d_H^* = \alpha$, $d_L^* = 0$ and p_h^*, $p_\ell^* > 0$ if and only if $\frac{v_h - w}{w - v_\ell} = \frac{1-s}{s}\left(\frac{\varepsilon}{1-\varepsilon}\right)^{\frac{x}{x-1}}$.

(iii) There exists an equilibrium with $d_H^* = 1$, $d_L^* = 0$ and p_h^*, $p_\ell^* > 0$ if and only if $\frac{v_h - w}{w - v_\ell} \in \left[\frac{1-s}{s}\left(\frac{\varepsilon}{1-\varepsilon}\right)^{\frac{x}{x-1}}, \frac{1-s}{s}\left(\frac{\varepsilon}{1-\varepsilon}\right)^{\frac{2-x}{x-1}}\right]$

(iv) There exists a $\beta \in (0,1)$ such that $d_H^* = 1$, $d_L^* = \beta$ is an equilibrium with some $p_h^*, p_\ell^* > 0$ if and only if $\frac{v_h - w}{w - v_\ell} \in \left(\frac{1-s}{s}\left(\frac{\varepsilon}{1-\varepsilon}\right)^{\frac{2-x}{x-1}}, \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\right)$.

(v) There exists an equilibrium with $d_H^* = d_L^* = 1$ and $p_h^*, p_\ell^* > 0$ if and only if $\frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}$.

Note that in this case the interval of existence for equilibria with $d^* = \alpha$ is collapsed to a single point, with $\underline{\Lambda} = \overline{\Lambda} = \Lambda^1$. We have $\psi(y) = \left(\frac{y}{xc}\right)^{\frac{1}{x-1}}$ so the workers' best responses are $p_h^* = \left(\frac{\hat{w}\phi_h(d)}{xc}\right)^{\frac{1}{x-1}}$ and $p_\ell^* = \left(\frac{\hat{w}\phi_\ell(d)}{xc}\right)^{\frac{1}{x-1}}$. Also $\Lambda(d) = \left(\frac{\phi_\ell(d)}{\phi_h(d)}\right)^{\frac{1}{x-1}}$ so the pool composition is monotonic in response to d, and in particular for $d \in (0, 1]$ it is constant with $\Lambda(d) = \left(\frac{\varepsilon}{1-\varepsilon}\right)^{\frac{1}{x-1}}$.

Second, consider $\gamma(p) = -c\ln(1-p)$ with constant c chosen such that first order conditions in the worker's optimization problem remain valid.[2]

Example 2.2.2. *The characterization of equilibria for $\gamma(p) = -c\ln(1-p)$ with $c < w\varepsilon$ is as follows:*

(i) For any $\alpha \in (\frac{c}{\hat{w}\varepsilon}, 1)$ there exists an equilibrium with $d_H^* = \alpha$, $d_L^* = 0$ and $p_h^*, p_\ell^* > 0$ if and only if $\frac{v_h-w}{w-v_\ell} = \frac{1-s}{s}\frac{\hat{w}\varepsilon\alpha - c}{\hat{w}(1-\varepsilon)\alpha - c}$.

(ii) There exists an equilibrium with $d_H^* = 1$, $d_L^* = 0$ and $p_h^*, p_\ell^* > 0$ if and only if $\frac{v_h-w}{w-v_\ell} \in \left[\frac{1-s}{s}\frac{\hat{w}\varepsilon - c}{\hat{w}(1-\varepsilon) - c}, \frac{1-s}{s}\left(\frac{1-\varepsilon}{\varepsilon}\right)^2\frac{\hat{w}\varepsilon - c}{\hat{w}(1-\varepsilon) - c}\right]$.

(iii) For any $\beta \in (0, 1)$ there exists an equilibrium with $d_H^* = 1$, $d_L^* = \beta$ and $p_h^*, p_\ell^* > 0$ if and only if $\frac{v_h-w}{w-v_\ell} \in \left(\frac{1-s}{s}\left(\frac{1-\varepsilon}{\varepsilon}\right)^2\frac{\hat{w}\varepsilon - c}{\hat{w}(1-\varepsilon) - c}, \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\right)$.

(iv) There exists an equilibrium with $d_H^* = d_L^* = 1$ and $p_h^*, p_\ell^* > 0$ if and only if $\frac{v_h-w}{w-v_\ell} \geq \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}$.

For this γ it is the case that $\underline{\Lambda} = 0$ and $\overline{\Lambda} = \Lambda^1$, so uniqueness holds on the whole range of $\frac{v_h-w}{w-v_\ell}$. We have $\psi(y) = 1 - \frac{c}{y}$ so the workers' best responses are given by $p_h^* = 1 - \frac{c}{\hat{w}\phi_h(d)}$ and $p_\ell^* = 1 - \frac{c}{\hat{w}\phi_\ell(d)}$. The resulting pool composition is $\Lambda(d) = \frac{\phi_h(d)}{\phi_\ell(d)}\cdot\frac{\hat{w}\phi_\ell(d) - c}{\hat{w}\phi_h(d) - c}$ which is increasing in d.

[2]Note that for both of these functional examples, ψ is logarithmically concave.

2.2.4 Use and Function of Applications in Equilibrium

The previous section established that the equilibrium pool of applicants is always weakly favourable, and showed which hiring patterns can be supported in which industry settings. However there are qualitative differences between equilibria. In this section, I discuss the use and usefulness of applications in equilibrium. First I discuss what constitutes higher use of applications in equilibrium, and then I identify the underlying function which applications serve in different settings.

The extent to which applications are used in equilibrium can be examined from three perspectives; the equilibrium use of applications by the firm, the equilibrium use of applications by workers, and also the overall use of applications in equilibrium. In this model hiring occurs solely through the application process, so we can understand the overall use of applications to be the actual hiring that occurs in equilibrium.

For the firm, "using" applications can be understood as accepting applications when they arrive. The firm's use of applications in equilibrium is directly reflected in its hiring strategy; greater acceptance corresponds to greater d^*. Note that the firm's *use* of applications is distinct from the *usefulness* of applications to the firm, which need not be associated with greater d^*.

For the workers, the use of applications is indicated by the worker's application arriving to the firm; greater equilibrium arrival probabilities p_h^*, or p_ℓ^* correspond to greater use of applications by workers in equilibrium. The relative use of applications by workers, $\frac{p_\ell^*}{p_h^*}$, matters for the overall use of applications in equilibrium because it influences the use of applications by the firm. Meanwhile the absolute levels of use, p_h^* and p_ℓ^*, directly affect the actual level of hiring in equilibrium because only applications which arrive can be accepted.

The actual hiring of workers in equilibrium is the combined effect of the extent to which worker applications arrive to the firm p_h^* and p_ℓ^*, together with the extent to which the firm accepts applications, d^*. Although the absolute levels of use of applications by workers depend on the particular shape of γ, the overall use of applications in equilibrium will increase with d^*

for a given γ because each worker's use of applications is increasing in the firm's use of applications. To the extent that industries with higher relative gains have greater hiring strategies by the firm in equilibrium, we can say that higher profitability industries have higher actual hiring.

However, applications are not necessarily more useful in their function nor lead to greater profits when the general profitability of the industry is high. Next I will discuss the different qualitative functions that applications can be seen to serve in this model, and welfare comparisons will be made in the next section.

Case 1. ***Applications can have primary value as a "Door." It must be the case that*** $\frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}$.

In the equilibria described by Proposition 1 the firm hires absolutely, and this hiring pattern requires high noise in reports and is made possible essentially on the basis of the high general profitability of the industry. The loss from employing low-quality workers or the proportion of low-quality workers in the general population is sufficiently low that the firm may hire blindly without "sifting" through applications by report. Assuming arrival is not prohibitively costly, such absolute hiring leads to all workers choosing the same arrival probability. Because reports are not decisive and the pool adjustment factor is neutral, applications yield no useful information to the firm. Applications have value purely in their primitive function of connecting unemployed workers to the firm, and such connection is desirable for all. Therefore in this setting applications are merely a "door" to the firm.

The equilibria in Proposition 2 demonstrate two other functions applications may serve in addition to merely connecting the firm to workers. Therefore these equilibria can be grouped into two qualitatively different types according to which of these additional functions is dominant. The first additional function is linked to reports. The firm hires selectively in these equilibria, which shows that the information the firm obtains from the application through its report realization is helpful to the firm. Applications therefore offer the firm a useful criterion by which it can sift applicants. The second additional function is linked to the endogenous pool of applicants, and is complementary to the usefulness of reports. When reports enable the firm to hire selectively, high type workers have more incentive to be seen by the firm and therefore they choose a relatively higher arrival probability than low types. This makes applications helpful by allowing high types to signal quality through arrival, providing the firm with a favourably biased pool from which to select its workers.

The equilibria in Proposition 2 can exist both for industries in which reports are decisive and for generally unprofitable industries in which reports are not decisive. Both the report information and the favourable pool are helpful to the firm, but for industries in which reports are decisive, the favourable pool is not critical to the firm's hiring decision. Such a firm would still hire selectively even if the pool of applicants were neutral. However, for generally unprofitable industries with non-decisive reports, the complementarity between the pool and reports is crucial to the firm's hiring decision. Although the firm would be unwilling to hire selectively on the basis of a high report alone, together with the favourable pool which is induced by using report information in the hiring decision, equilibrium hiring can be sustained.

Case 2. ***Applications can have primary value as a "Report." It must be the case that*** $\frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon} < \frac{v_h-w}{w-v_\ell} < \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}$ ***and the pool of applicants must not be too favourable.***

When reports are decisive, the explicit information conveyed by the application hiring gives the application inherent value. Hiring would be worthwhile on the basis of the reported information alone when drawing from a neutral pool of applicants, provided that the hiring decision differs according to the report realization, $d_H^* > d_L^*$. This use of reports helps the firm improve the quality of its hires (relative to hiring randomly form the general population), and is desirable for high-quality workers but not low-quality workers. When sustained in equilibrium, hiring with $d_H^* > d_L^*$ induces a strictly favourable pool, $\frac{p_\ell(d_H^*, d_L^*)}{p_h(d_H^*, d_L^*)} < 1$. However, if it is too favourable the firm will deviate to absolute hiring. Thus applications functioning primarily through their value as a "report" can be observed in equilibrium with a selective or mixed hiring pattern with $d_H^* > d_L^*$ and a favourable pool of applicants, as long as the pool is not too favourable. Thus applications have primary value as a report for moderate levels of industry profitability and moderate noise in reports.

Case 3. ***Applications can have primary value as a "Signal." It must be the case that*** $\frac{v_h - w}{w - v_\ell} < \frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}$ ***and the pool of applicants must be sufficiently favourable but not too favourable.***

A firm in an industry for which the relative gain $\frac{v_h - w}{w - v_\ell}$ is too small to justify hiring even high-report applicants under population parameters may be able to hire on the basis of the endogenously improved pool of applicants, if it is sufficiently favourable. Such hiring is worthwhile by virtue of the favourable difference in high- and low-quality workers' choices of arrival probabilities. The very presence of an application functions as a signal of quality to the firm. However, although applications have value primarily as a signal of quality, the report retains importance even though they are not decisive themselves. In order to support this signalling value in such a setting, the firm must use the report and engage in selective hiring, otherwise high types will not have incentive to maintain a higher arrival probability than low types and applications will not be able to signal quality at all. To prevent the firm from deviating from selective hiring to absolute hiring, it is also necessary that the arrival probability of high types is not too much greater than the arrival probability of low types. Therefore this functional value of applications can be observed in equilibrium with a selective hiring pattern and a favourable pool of applicants, as long as the pool is sufficiently favourable, but also not too favourable.[3] Applications can have value as a signal when the industry has low general profitability but there is low noise in reports.

2.3 Comparative Statics and Welfare

In this section I restrict attention to equilibria with non-zero hiring. I will discuss the effects of parameter changes on equilibrium outcomes (strategies d^*, p_h^*, p_ℓ^*, and the applicant pool $\frac{p_\ell^*}{p_h^*}$),

[3]In contrast to the signalling environment in Daley and Green (2003), "signalling" quality to the firm through applications is not inherently wasteful, since the arrival of applications also affects the volume of hires.

as well as firm profits and payoffs of the workers. Recall that since workers choose $p_h^* = \psi(\hat{w}\phi_h^*)$ and $p_\ell^* = \psi(\hat{w}\phi_\ell^*)$ in response to $d^* = d_H^* + d_L^*$, the quality composition of the pool of applicants in equilibrium is given by $\Lambda^{d^*} = \dfrac{\psi(\hat{w}\phi_\ell^*)}{\psi(\hat{w}\phi_h^*)}$, the firm's profit is given by

$$\pi^* = (v_h - w)s\psi(\hat{w}\phi_h^*)\phi_h^* - (w - v_\ell)(1-s)\psi(\hat{w}\phi_\ell^*)\phi_\ell^* \tag{2.36}$$

and worker utilities are given by

$$u_q = \hat{w}\psi(\hat{w}\phi_q^*)\phi_q^* + b - \gamma(\psi(\hat{w}\phi_q^*)) \tag{2.37}$$

where $\phi_h^* = (1-\varepsilon)d_H^* + \varepsilon d_L^*$, and $\phi_\ell^* = \varepsilon d_H^* + (1-\varepsilon)d_L^*$.

I will focus on changes to the report error and wage. Changes to the unemployment benefit b are directly relevant only to workers and affect p_h^*, p_ℓ^* and the pool of applicants in a manner opposite to that of the wage, so changes to b will be discussed in Section 2.3.2. Changes to v_h, v_ℓ, and s do not affect worker strategies or pool composition in equilibrium, but concerning firm strategy d^* it is straightforward to see that an increase in v_h, v_ℓ, or s increases the firm's general profitability, and d^* has already been discussed in relation to the general profitability of the firm. Concerning welfare, changes to v_h, v_ℓ, and s are directly relevant to firm profits only, although to the extent that these parameters influence the existence of equilibria with different hiring patterns, worker welfare can be affected.

I will address both parameter changes which occur within a given type of equilibrium, such that the firm's hiring strategy remains the same throughout the parameter change, and also parameter changes for which the firm is not able to maintain the same hiring strategy in equilibrium.

2.3.1 Changes in Report Error

Changes in report error have an effect only within equilibria in which $d_H^* > d_L^*$. In any such equilibrium the pool of applicants is strictly favourable, $\Lambda^{d^*} < 1$, because $\phi_h^* > \phi_\ell^*$ when $d_H^* > d_L^*$, and ψ is increasing in its argument. The degree to which the pool is favourable will also be greater the lower the report error is. If reports become more accurate (ε becomes smaller) the pool composition improves, $\frac{d\Lambda^{d^*}}{d\varepsilon} < 0$. This is because high-quality workers become more likely to be accepted and low-quality workers become less likely to be accepted when reports become more accurate. This gives high-quality workers incentive to put greater effort into being seen by the firm, $\frac{dp_h}{d\varepsilon} < 0$, while low-quality workers have incentive to reduce their effort, $\frac{dp_\ell}{d\varepsilon} > 0$. This complementarity between the report error and pool composition is stated formally as follows.

Lemma 7. ***Report and Pool Complementarity.*** *Restrict attention to non-zero arrival in equilibrium and suppose ψ is logarithmically concave and that either*

(i) $\frac{v_h - w}{w - v_\ell} \in \left(\frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\frac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))}, \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\frac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))}\right),$

(ii) $\frac{v_h - w}{w - v_\ell} \in \left(\frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\frac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))}, \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\right)$, *or*

(iii) $\frac{v_h - w}{w - v_\ell} > \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}.$

Then $\frac{d\Lambda^{d^*}}{d\varepsilon} > 0$, *where Λ^{d^*} is the equilibrium composition of the pool of applicants,* $\Lambda^{d^*} = \frac{p_\ell^*}{p_h^*} = \frac{\psi(\hat{w}(\varepsilon d_H^* + (1-\varepsilon)d_L^*))}{\psi(\hat{w}((1-\varepsilon)d_H^* + \varepsilon d_L^*))}$.

Within a given equilibrium with firm strategy $d^* > 0$ and non-zero arrival, an investment which improves the firm's screening technology such that the report error ε decreases is generally beneficial for high-quality workers, $\frac{du_h}{d\varepsilon} \leq 0$ but not for low-quality workers, $\frac{du_\ell}{d\varepsilon} \geq 0$. This result, shown in Lemma 30 (Appendix A) is unsurprising due to the fact that report realizations are correlated with true type, and the inequalities are strict unless $d_H^* = d_L^*$ such that reports are not used in equilibrium. Due to the complementarity between reports and pool composition in Lemma 7, it is therefore unsurprising that firm profits also improve when report error decreases, $\frac{d\pi}{d\varepsilon} \leq 0$. This result is shown in Lemma 31 (Appendix A). Again, the inequality is strict in equilibria where reports are used. In equilibria where the firm ignores reports and treats high- and low-report applications in the same way, $d = 0$ or $d = 2$, there is clearly no effect within the equilibrium of a change in ε.

Now allowing for movement between equilibria with different firm strategies, the effect of a change in ε on welfare is not clear. This is because an improvement in the firm's screening technology can lead to a new equilibrium with either increased or decreased hiring. Since an improvement in report screening technology both lowers the odds that a high-report applicant is low quality and improves the pool composition, d_H^* is weakly increasing in the power of the report $1-\varepsilon$ (thus weakly decreasing in ε). However, since an improvement in report screening technology increases the odds that a low-report applicant is low quality, d_L^* may increase or decrease in ε depending on the strength of the complementarity between reports and the pool composition.

Proposition 1 implies that for industries with high enough general profitability, such that $\frac{v_h-w}{w-v_\ell} > \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}$, a sufficient decrease in ε could make the report become decisive and cause the firm to reduce its hiring to some $d^* \in [1, 2)$. For generally unprofitable industries, if the report error decreases sufficiently to make reports decisive, the decisiveness of the report can have the opposite effect on the firm's decision, and cause the firm to begin accepting high-report applicants, $d^* \in (0, 1]$, when previously it would not accept any.

2.3.2 Changes in the Wage and Unemployment Benefit

Within any equilibrium in which hiring actually occurs, $d^* > 0$, an increase in the wage is strictly beneficial for both worker types, $\frac{du_q}{dw} > 0$, as shown in Lemma 32 (Appendix A). Since the wage is higher, both types have incentive to increase their effort to be seen by the firm. Thus the volume of the pool of applicants increases, although its quality composition may change.

Whether this composition change is favourable or not within an equilibrium with $d^* > 0$ is characterized by the following:

Lemma 8. *Restrict attention to non-zero arrival in equilibrium and suppose ψ is logarithmically concave. Suppose that either*

(i) $\frac{v_h - w}{w - v_\ell} \in \left(\frac{1-s}{s} \frac{\varepsilon}{1-\varepsilon} \frac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))}, \frac{1-s}{s} \frac{1-\varepsilon}{\varepsilon} \frac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))} \right)$, *or*

(ii) $\frac{v_h - w}{w - v_\ell} > \frac{1-s}{s} \frac{1-\varepsilon}{\varepsilon}$.

Then we will have

$$\frac{d\Lambda^{d^*}}{dw} \gtrless 0 \ \textit{iff} \ \frac{\psi'(\hat{w}\phi_\ell^*)\phi_\ell^*}{\psi(\hat{w}\phi_\ell^*)} \gtrless \frac{\psi'(\hat{w}\phi_h^*)\phi_h^*}{\psi(\hat{w}\phi_h^*)}. \tag{2.38}$$

Suppose instead that

(iii) $\frac{v_h - w}{w - v_\ell} \in \left(\frac{1-s}{s} \frac{1-\varepsilon}{\varepsilon} \frac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))}, \frac{1-s}{s} \frac{1-\varepsilon}{\varepsilon} \right)$.

Then $\frac{d\Lambda^{d^*}}{dw} > 0$.

Thus when workers have identical incentives, that is when $d^* = 2$ so that $\phi_h^* = \phi_\ell^*$, the pool composition is unaffected by a change in wage. Meanwhile, within any class of equilibria with $d^* = 1$ it is possible for the pool to become either more favourable or less favourable, depending on the nature of γ. Since $\phi_h^* > \phi_\ell^*$ under selective hiring, the equilibrium pool of applicants Λ^1 will necessarily worsen with an increase in wage, $\frac{d\Lambda^{d^*}}{dw} > 0$, for any γ which has $\frac{\psi'}{\psi}$ increasing. For twice continuously differentiable γ, this sufficient condition is equivalent to the logarithmic convexity of ψ.

On the other hand, when $d^* = 1$, in order for the pool to be improving with an increase in wage it is a necessary but not sufficient condition that $\frac{\psi'}{\psi}$ be decreasing. For twice continuously differentiable γ, this necessary condition is equivalent to the logarithmic concavity of ψ. Again although not exclusive, all concave functions satisfy logarithmic concavity and we can have ψ concave as long as γ is sufficiently convex.[4] With this condition satisfied, it is also possible that

[4]Note that it is possible for a function to be neither logarithmically convex nor logarithmically concave, just as $\frac{\psi'}{\psi}$ need not be monotonic.

the composition of the pool of applicants is constant with respect to the wage. For example, when arrival costs increase according to a power function, such as $\gamma(p) = cp^x$ for $x > 1$, it will be the case that $\dfrac{\psi'(\hat{w}\phi_q^*)}{\psi(\hat{w}\phi_q^*)} = \dfrac{1}{(x-1)\hat{w}\phi_q^*}$ and therefore by condition (2.38) we have $\dfrac{d\Lambda^{d^*}}{dw} = 0$.

How a wage increase affects the profit of the firm is partly determined by whether the wage has a positive or negative effect on the quality composition of the pool of applicants. However, the effect of a wage increase on the profits of the firm is unclear due to the presence of multiple effects which are conflicting. Consider first an equilibrium in which the firm hires absolutely; in this case the quality composition of the pool of applicants is unchanged relative to that of the general population. The effect of a change in wage is given by

$$\frac{d}{dw}\pi\Big|_{d^*=2} = -\psi(\hat{w}) + [(v_h - w)s - (w - v_\ell)(1-s)]\psi'(\hat{w}). \tag{2.39}$$

The first effect is negative and in proportion to the arrival probability of workers, $-\psi(\hat{w})$. This is due to the fact that an increase in wage increases the cost of each unit of labour for the firm. The second effect is due to the increased wage's effect on the volume of hires; the term $(v_h - w)s - (w - v_\ell)(1-s)$ corresponds to the expected profit per hire and the term $\psi'(\hat{w})$ corresponds to the change in volume. This effect is positive because ψ is increasing and because this equilibrium exists only when the industry is generally profitable, which implies that additional volume of hires from a neutral pool is desirable to the firm. Whether or not an increase in wage is beneficial to the firm therefore depends on which of these effects is stronger for a given ψ.

For an example with quadratic arrival costs, take $\gamma(p) = cp^2$ (with constant $c > 0$). In this case the expression for $\pi\big|_{d^*=2}$ is maximized at $w_2 = \frac{1}{2}[(v_h + b)s + (v_\ell + b)(1-s)]$. For $w < w_2$, the volume effect outweighs the effect of labour costs and $\dfrac{d}{dw}\pi\Big|_{d^*=2} > 0$. For $w > w_2$ the increased labour costs of a higher wage outweigh the value of a greater volume of hires. A similar result holds with $\gamma(p) = cp^x$ for other powers $x > 2$, with different values of w_2.

For equilibria in which the firm does not hire absolutely, an increase in the wage has a further complicated influence on profits because the composition of the pool of applicants will change, rather than merely the volume. For example, with selective hiring in equilibrium, the effect of a wage change is given by

$$\begin{aligned}\frac{d}{dw}\pi\Big|_{d^*=1} = &- [s \cdot \psi(\hat{w}(1-\varepsilon))(1-\varepsilon) + (1-s) \cdot \psi(\hat{w}\varepsilon)\varepsilon] \\ &+ (v_h - w)s \cdot \psi'(\hat{w}(1-\varepsilon))(1-\varepsilon)^2 - (w - v_\ell)(1-s) \cdot \psi'(\hat{w}\varepsilon)\varepsilon^2.\end{aligned} \tag{2.40}$$

In this case, whether or not a wage increase is beneficial to the firm depends on whether the effect of labour costs, as adjusted according to the effect of the wage on the composition of applicants, is stronger than the effect of the increased volume of hires, also as adjusted according to the effect of the wage on the composition of applicants, for a particular ψ. In the example of quadratic arrival costs, $\pi\big|_{d^*=1}$ is increasing for wages up to

$$w_1 = \frac{1}{2}\frac{(v_h + b)s(1-\varepsilon)^2 + (v_\ell + b)(1-s)\varepsilon^2}{s(1-\varepsilon)^2 + (1-s)\varepsilon^2}, \tag{2.41}$$

and decreasing for all wages $w > w_1$.

As previously noted, an increase in the unemployment benefit b has the opposite effect on worker strategies and the pool of applicants as an increase in the wage. Thus an increase in b causes both worker types to reduce their arrival probabilities, as shown in Lemma 33 (Appendix A). Thus in contrast with Lemma 2.38, for ψ logarithmically concave we will have

$$\frac{d\Lambda^{d^*}}{db} \gtrless 0 \textit{ iff } \frac{\psi'(\hat{w}\phi_\ell^*)\phi_\ell^*}{\psi(\hat{w}\phi_\ell^*)} \lessgtr \frac{\psi'(\hat{w}\phi_h^*)\phi_h^*}{\psi(\hat{w}\phi_h^*)} \tag{2.42}$$

within any class of equilibria with $d^* = 1$, and $\frac{d\Lambda^{d^*}}{db} = 0$ within any class of equilibria with $d^* = 2$, while we also have $\frac{d\Lambda^{d^*}}{db} = 0$ within any class of equilibria with $d^* = 1{+}\beta$ with $\beta \in (0, 1)$ (see Lemma 35, Appendix A). As with the wage, an increase in the unemployment benefit is beneficial to workers, as shown in Lemma 34 (Appendix A). However, unlike the wage, the unemployment benefit affects firm profits only indirectly through the altered choices of workers. By taking the derivative of π^* by b in equation (2.3) where $\hat{w} = w-b$, we can see that

$$\frac{d\pi}{db} \gtrless 0 \ \textit{iff} \ \frac{v_h-w}{w-v_\ell} \lessgtr \frac{1-s}{s}\frac{\psi'(\hat{w}\phi_\ell^*)\phi_\ell^{*2}}{\psi'(\hat{w}\phi_h^*)\phi_h^{*2}}. \tag{2.43}$$

Thus when $d_H^* = d_L^* = 1$ in equilibrium so that workers have equal incentives $\phi_h^* = \phi_\ell^* = 1$, an increase in unemployment benefit decreases firm profits because such an equilibrium can only occur in a generally profitable industry, $\frac{v_h-w}{w-v_\ell} > \frac{1-s}{s}$. In this case the pool composition is neutral because $\frac{d\Lambda^{d^*}}{db} = 0$ when $\phi_h^* = \phi_\ell^* = 1$, and so the equal reduction in the arrival probabilities of both high- and low-type applications results in lower profit for the firm. However, for equilibria in which the firm does not treat all applications the same, such that $d_H^* > d_L^*$, it is possible for an increase in unemployment to increase firm profits provided that the relative gain from a high-quality worker is sufficiently low.

2.3.3 Optimal and Long Run Wage Determination

Now in this model firms and workers move simultaneously and best-respond to each other for a wage given exogenously. Since for every w there is an equilibrium $(\hat{d}(w), \hat{p}_h(w), \hat{p}_\ell(w))$ of the simultaneous move game, one alternative is for the firm to set the wage optimally, by calculating

$$w^* = \arg\max_w \hat{\pi}(w) \tag{2.44}$$

where $\hat{\pi}(w)$ is the firm's profit given $\hat{d}(w)$, $\hat{p}_h(w)$, and $\hat{p}_\ell(w)$. That is, each wage results in a particular relative gain to the firm from high-quality workers, $\frac{v_h-w}{w-v_\ell}$, which dictates the hiring equilibrium (if there exist any with non-zero hiring) which can be sustained in the simultaneous-move game. According to the balance discussed above between the tradeoffs the firm faces due to a change in wage, for each hiring strategy there is a maximum profit attainable through setting the wage within the range of wages for which this equilibrium can exist. By comparing these profits, the firm will wish to select a wage which will lead to the highest profits in a subsequent simultaneous move equilibrium.

Since the firm makes profit in any equilibrium with hiring, the firm will not choose a wage such that $\hat{d}(w) = 0$. Neither will it choose a wage such that $\hat{d}(w) = \alpha$ for any $\alpha \in (0, 1)$ since the firm would have to be indifferent about every applicant hired, making profit in any such equilibrium also equal to zero. Thus the optimal wage can never be greater than $\overline{w}$ where $\overline{w}$ satisfies $\frac{v_h - \overline{w}}{\overline{w} - v_\ell} = \frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\Lambda^1$, and the resulting optimal hiring strategy will be $\hat{d}(w^*) \in [1, 2]$. However, beyond this nothing can be said in general about which wage and resulting hiring pattern will be chosen optimally in equilibrium.

When there is no co-existence of equilibria with $\hat{d}(w) = \beta$ for $\beta \in (0, 1)$ with either $\hat{d}(w) = 1$ or $\hat{d}(w) = 2$, such as the case with quadratic arrival costs or ψ log concave, there will be a unique (non-zero) hiring strategy possible for each wage below $\overline{w}$. Then without loss of generality taking $\hat{d}(w) = 0$ for any wages equal to $\overline{w}$ or higher, it will be the case that

$$\hat{d}(w) = \begin{cases} 2 & \textit{if } w \leq w_a \\ 1 + \beta(w) & \textit{if } w_a < w < w_b \\ 1 & \textit{if } w_b \leq w < \overline{w} \\ 0 & \textit{if } \overline{w} \leq w \end{cases} \tag{2.45}$$

where w_a is the wage at which $\frac{v_h - w}{w - v_\ell} = \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}$ and w_b is the wage at which $\frac{v_h - w}{w - v_\ell} = \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\Lambda^1$, and where $\beta(w)$ must satisfy $\frac{v_h - w}{w - v_\ell} = \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\frac{\psi(\hat{w}(1-\varepsilon+\varepsilon\beta))}{\psi(\hat{w}(\varepsilon+(1-\varepsilon)\beta))}$.

Although it is difficult to present a closed-form solution even for a particular $\gamma(p)$, focusing on pure strategies it is easy to see that the firm will indeed favour different wages for different settings in the case of quadratic arrival costs. For $\gamma = cp^2$ the expression for $\frac{d}{dw}\pi\big|_{d^*=2}$ given in equation (2.39) is increasing up to the wage w_2 and thereafter decreasing, and since the expression for $\frac{d}{dw}\pi\big|_{d^*=1}$ given in equation (2.40) is increasing up to the wage w_1 and thereafter decreasing, we have

$$\arg\max_{w \leq w^a} \hat{\pi}(w) = \min\{w_a, w_2\} \tag{2.46}$$

and

$$\arg\max_{w_b \leq w < \overline{w}} \hat{\pi}(w) = \begin{cases} w_b & \text{if } w_1 < w_b \\ w_1 & \text{if } w_b \leq w_1 < \overline{w} \end{cases} \tag{2.47}$$

Taking $b = 0$, $s = \frac{1}{2}$, and $v_\ell = 0$, the firm would optimally choose $w^* = w_2 = \frac{1}{4}v_h$ and the resulting equilibrium hiring strategy would be $\hat{d}(w^*) = 2$ when the report error is moderate to high, $\varepsilon > \frac{1}{4}$; whereas for very low report errors, $\varepsilon < \frac{1}{10}$, the firm can be shown to prefer $w^* = w_b = \frac{1}{2}v_h$ and selective hiring in equilibrium rather than setting $w = \arg\max_{w \leq w_a} \hat{\pi}(w)$.

Another alternative to modelling a given exogenous wage is for the wage to evolve in the long run depending on the firm's profits. Now in any mixed strategy Nash equilibrium with $\alpha \in (0, 1)$ the firm is indifferent with every hire and makes zero profit. For any other Nash equilibrium with non-zero arrival and $d^* \neq 0$ the firm will make positive profit. Therefore in the long run, the wage may rise to reflect this profit and the general profitability of the industry will decrease. This wage adjustment may be due to either pressure from the workers or a union for higher wages, or from entry into the industry leading to more intense competition for workers. If the wage adjusts fully and rises such that the firm makes zero profit, absolute hiring can not be sustained as an equilibrium over the long run. This is because in any situation in which absolute hiring can be supported the firm makes strictly positive profit. The highest the wage can rise while absolute hiring remains optimal is $\hat{w}$ such that $\frac{v_h - \hat{w}}{\hat{w} - v_\ell} = \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}$, and although at this wage the firm makes zero expected profit from accepting low-report applicants, it makes strictly positive expected profit from the high-report applicants it accepts.[5] Since for any hiring pattern to be sustained in equilibrium with zero profit in the long run wages must be low enough that the firm does not make any expected profit from any of the workers it hires,

[5] Absolute hiring could however be sustained with zero profit for industries with $\frac{v_h - w}{w - v_\ell} = \frac{1-s}{s}$ in the absence of reports (or if reports were completely noise, $\varepsilon = \frac{1}{2}$).

$d^* = 1+\beta$ is also not sustainable.

However, when wages adjust fully and long run profits are zero, non-zero hiring can be supported in equilibrium with $d^* = \alpha$ for some $\alpha \in (0,1)$ for industries with $\frac{v_h - w}{w - v_\ell} \in \left(\frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\underline{\Lambda}, \frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\overline{\Lambda}\right)$ where as defined previously $\underline{\Lambda} = \inf_{\alpha\in(0,1)} \frac{\psi(\hat{w}\varepsilon\alpha)}{\psi(\hat{w}(1-\varepsilon\alpha))}$, and $\overline{\Lambda} = \sup_{\alpha\in(0,1)} \frac{\psi(\hat{w}\varepsilon\alpha)}{\psi(\hat{w}(1-\varepsilon\alpha))}$. Selective hiring $d^* = 1$ can also be sustained in the long run exactly for $\frac{v_h - w}{w - v_\ell} = \frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\frac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))}$.

2.3.4 Commitment to a Hiring Strategy

Section 2.2 analyzed the equilibrium arrival and hiring patterns in a model where the workers and firm simultaneously choose their arrival probabilities and hiring strategies respectively. An alternative timing is for the firm to choose hiring strategies with commitment, and for workers to then make their choices of arrival probabilities. Note that with this alternative timing with commitment, the firm need not resort to mixed strategies to resolve the strategy compatibility issues identified in the discussion of Corollary 2. Although selective hiring will lead to a too-attractive pool of applicants and absolute hiring will eliminate the favourable bias of the pool of applicants, the firm will not be left with zero profits unable to sustain hiring in equilibrium. Instead, the firm need only decide whether selective or absolute hiring will lead to greater profits, and maintain this hiring strategy when workers best-respond. The following two propositions characterize the pure strategy equilibria for two example functional forms of the arrival cost γ.

Proposition 5. *Suppose $\gamma(p) = cp^2$ for $c > 0$. The optimal pure strategy for the firm in equilibrium in the model with commitment is*

$$d^* = \begin{cases} 0 & \textit{if } \dfrac{v_h - w}{w - v_\ell} \le \dfrac{1-s}{s}\Big(\dfrac{\varepsilon}{1-\varepsilon}\Big)^2 \\ 1 & \textit{if } \dfrac{1-s}{s}\Big(\dfrac{\varepsilon}{1-\varepsilon}\Big)^2 \le \dfrac{v_h - w}{w - v_\ell} \le \dfrac{1-s}{s}\dfrac{1-\varepsilon^2}{1-(1-\varepsilon)^2} \\ 2 & \textit{if } \dfrac{1-s}{s}\dfrac{1-\varepsilon^2}{1-(1-\varepsilon)^2} \le \dfrac{v_h - w}{w - v_\ell}. \end{cases} \tag{2.48}$$

The corresponding optimal arrival probabilities for the workers are

$$p_q^* = \begin{cases} 0 & \textit{if } \dfrac{v_h - w}{w - v_\ell} \le \dfrac{1-s}{s}\Big(\dfrac{\varepsilon}{1-\varepsilon}\Big)^2 \\ \dfrac{\hat{w}\phi_q^*}{c} & \textit{if } \dfrac{1-s}{s}\Big(\dfrac{\varepsilon}{1-\varepsilon}\Big)^2 \le \dfrac{v_h - w}{w - v_\ell} \le \dfrac{1-s}{s}\dfrac{1-\varepsilon^2}{1-(1-\varepsilon)^2} \\ \dfrac{\hat{w}}{c} & \textit{if } \dfrac{1-s}{s}\dfrac{1-\varepsilon^2}{1-(1-\varepsilon)^2} \le \dfrac{v_h - w}{w - v_\ell}, \end{cases} \tag{2.49}$$

for $q \in \{h, \ell\}$ *where* $\phi_h^* = 1-\varepsilon$ *and* $\phi_\ell^* = \varepsilon$.

Comparing this result with Example 2.2.1 parts *(i)*, *(iii)* and *(v)* for $x = 2$, we can see how the model with commitment differs from the main model in the case of quadratic arrival costs. The industry values of general profitability for which there can be no hiring in equilibrium coincide, and the lowest relative gain $\frac{v_h - w}{w - v_\ell}$ for which selective hiring with non-zero arrival is possible is also the same. However, the highest relative gain for which selective hiring can be sustained is lower in the case of commitment since $\frac{1-s}{s}\frac{1-\varepsilon^2}{1-(1-\varepsilon)^2} > \frac{1-s}{s}$. Similarly the lowest relative gain for which absolute hiring can be supported in equilibrium is also lower in the case of commitment, since $\frac{1-\varepsilon}{\varepsilon} > \frac{1-\varepsilon^2}{1-(1-\varepsilon)^2}$ for all $\varepsilon < \frac{1}{2}$.

Proposition 6. *Suppose* $\gamma(p) = -c\ln(1-p)$ *for* $c < \hat{w}\varepsilon$. *The optimal pure strategy for the firm in equilibrium in the model with commitment is*

$$d^* = \begin{cases} 0 & \textit{if } \dfrac{v_h - w}{w - v_\ell} \le \dfrac{1-s}{s}\dfrac{\hat{w}\varepsilon - c}{\hat{w}(1-\varepsilon) - c} \\ 1 & \textit{if } \dfrac{1-s}{s}\dfrac{\hat{w}\varepsilon - c}{\hat{w}(1-\varepsilon) - c} \le \dfrac{v_h - w}{w - v_\ell} \le \dfrac{1-s}{s}\dfrac{1-\varepsilon}{\varepsilon} \\ 2 & \textit{if } \dfrac{1-s}{s}\dfrac{1-\varepsilon}{\varepsilon} \le \dfrac{v_h - w}{w - v_\ell}. \end{cases} \tag{2.50}$$

The corresponding optimal arrival probabilities for the workers are

$$p_q^* = \begin{cases} 0 & \textit{if } \dfrac{v_h - w}{w - v_\ell} \leq \dfrac{1-s}{s} \dfrac{\hat{w}\varepsilon - c}{\hat{w}(1-\varepsilon) - c} \\ \dfrac{\hat{w}\phi_q^* - c}{\hat{w}\phi_q^*} & \textit{if } \dfrac{1-s}{s} \dfrac{\hat{w}\varepsilon - c}{\hat{w}(1-\varepsilon) - c} \leq \dfrac{v_h - w}{w - v_\ell} \leq \dfrac{1-s}{s} \dfrac{1-\varepsilon}{\varepsilon} \\ \dfrac{\hat{w} - c}{\hat{w}} & \textit{if } \dfrac{1-s}{s} \dfrac{1-\varepsilon}{\varepsilon} \leq \dfrac{v_h - w}{w - v_\ell}, \end{cases} \tag{2.51}$$

for $q \in \{h, \ell\}$ where $\phi_h^* = 1-\varepsilon$ and $\phi_\ell^* = \varepsilon$.

Comparing this result with Example 2.2.2 parts *(i)*, *(ii)* and *(iv)* we can see similar differences between the model with commitment and the main model under $\gamma(p) = -c \ln(1-p)$. The industry values of general profitability for which there can be no hiring in equilibrium coincide, but the level of general profitability for which selective hiring with non-zero arrival becomes possible is also lower in the case of commitment because $\dfrac{1-\varepsilon}{\varepsilon} \dfrac{\hat{w}\varepsilon - c}{\hat{w}(1-\varepsilon) - c} < 1$. In contrast, for this $\gamma(p)$ the level of general profitability beyond which absolute hiring occurs in equilibrium coincides in the models with and without commitment.

Note that in both of these examples for $\gamma(p)$, the firm can not gain from mixing with $d^* = 1 + \beta$ for $\beta \in (0, 1)$. In the case of $\gamma(p) = cp^2$, for any parameter setting where $\pi(1)$ is greater than $\pi(2)$, it is also the case that $\pi(1 + \beta)$ is lower than $\pi(1)$. Similarly, $\pi(1 + \beta)$ is lower than $\pi(2)$ wherever $\pi(2)$ is greater than $\pi(1)$. In the case of $\gamma(p) = -c \ln(1-p)$, for any parameter setting where $\pi(1 + \beta)$ is positive, $\pi(2) \geq \pi(1 + \beta)$.

Also in both cases, the range for which no hiring can be supported in equilibrium (that is, for which $d^* > 0$ is the unique equilibrium possible in mixed or pure strategies) is the same in both the model with commitment and the model without commitment. This means that although commitment here helps the firm avoid the strategy deviation problems associated with Corollary 2, commitment itself has not improved the pool of applicants in such a way as to allow the firm to begin hiring at lower levels of general profitability than it would have been able to without commitment. Thus we expect for other specifications of γ the results and their properties to be also qualitatively similar to that of the simultaneous move model.

Chapter 3

Two-Channel Model of Hiring

In this chapter I extend the model to two hiring channels; one, the "informal" channel, is similar to the model introduced in Chapter 2 in that it has costly endogenous arrival of applications and an exogenous noisy signal of quality. The other, the "formal" channel, is equally available to all workers at an exogenous rate and conveys a separate exogenous noisy signal of worker quality.

I develop the two-channel model and provide a characterization of best responses for the firm and workers in Section 3.1. In Section 3.2 I compare the informal channel arrival probabilities of high-quality workers relative to low-quality workers and present some basic observations about the hiring outcomes which are possible in equilibrium. I show how the informal pool responds to different combined hiring patterns. I find that despite having complementarity as in the one-channel model between the quality composition of informal applicants and the screening power of informal reports, workers' best responses do not always lead to a favourable (or even neutral) pool of informal applicants. This is because the screening power of formal reports gives a relative advantage to high-types in the chance of being hired formally.

I characterize the pure Nash equilibria of this model in Section 3.3 and relate the hiring outcomes to the hiring environment, and to the screening power of formal applications versus informal applications. I find that when the formal report screening power is high, hiring patterns with an unfavourable informal pool can be sustained in equilibrium provided that the incentive for high types to use the informal channel remains sufficiently close relative to low types. I also determine the compatibility of formal and informal hiring patterns in equilibrium and compare the possible roles of informal applications (as a doorway, source of direct information, or signal) in different hiring environments. For example, informal applications can not function primarily as a signal when general profitability is high enough for formal applications to function as a door, or when formal applications have value as a sufficiently strong source of direct information.

In Section 3.4 I show the welfare and comparative static implications of the availability of a second hiring channel. I also discuss the effects of this two-channel model's assumption that the firm delegates hiring to independent hiring departments who maximize the expected profit from their own hires.

3.1 Model of Formal and Informal Hiring

In this section I develop a benchmark model to study equilibrium hiring patterns when there is a separate "informal" hiring channel through which applications may arrive to the firm, in addition to a standard "formal" application channel. In particular, the application arrival probabilities in the so-called "informal" channel are determined by each worker endogenously. Thus the firm will choose a hiring pattern for both channels in response to the informal arrival probabilities determined by workers, and workers will choose arrival probabilities in response to the hiring pattern chosen by the firm.

3.1.1 Two Channels

Suppose there are two separate channels through which the worker applications may arrive to the firm, denoted $j \in \{F, I\}$, which I will call "*formal*" and "*informal*" channels. In my model formal and informal applications differ in two important ways; first, in the way in which they reach the firm, and second, the informational content of their reports.

I consider the formal channel as an application route equally available to all, such that any frictions which prevent a worker's formal application from being transmitted or received are assumed to affect all workers equally. Therefore the probability of the firm receiving a formal application from a given worker is type-independent. I will denote this common formal channel arrival probability p_F and I will take $p_F \in (0, 1)$. Applications sent through the formal channel also carry a report of worker quality $R_F \in \{H, L\}$ which has some error probability $\varepsilon_F < \frac{1}{2}$, according to the sources of information associated with formal applications and how likely they are to fail to indicate the applicant's true type.

In contrast, I suppose that the informal channel is accessed endogenously, such that the arrival probability of a worker's informal application may potentially differ between types. I denote the informal arrival probabilities for high- and low-quality workers p_{Ih} and $p_{I\ell}$ respectively.[1]

Applications sent through the informal channel also carry a report of worker quality, $R_I \in \{H, L\}$, which has error $\varepsilon_I < \frac{1}{2}$, according to how likely informal sources of information fail to indicate an applicant's true type. I assume that conditional on the worker's type the formal and informal reports are independent from each other, as are their arrivals. One common networking-based explanation for how informal applications can signal quality is based on the principle of homophily. In models such as Montgomery (1991), where the informal channel operates as a referral network, homophily suggests that applicants referred to the firm by high-quality employees are more likely to be high-quality themselves. In such a situation the referring employee's own type is like a report which is indicative to the firm of the referred applicant's type. Even when the informal hiring channel is specifically referral-based, homophily is not the only rationale for how informal applications may carry a report of quality. For ex-

[1] Although in different contexts access to either or both channels may be modelled as endogenous, I make the association with informal hiring opportunities. In the next chapter, I will specifically model the informal channel transmission probabilities as resulting from workers' networking choices.

ample, individuals who give referrals may possess personal knowledge of the applicant which they are able to credibly convey to the firm.

Whatever processes generate the reports of quality in the formal and informal channels, it is reasonable to suppose that the information conveyed to the firm by R_I may be distinct from the information conveyed by R_F, such that the correlation with quality may differ for reports in the two channels, and $\varepsilon_I \neq \varepsilon_F$. While they are generally distinct, it is conceivable that it is either formal or informal reports which are the better indicator of quality. Which one is more accurate will vary according to the skill characteristics of the job in question, due to the fact that formal and informal sources may be differently suited to convey information about different types of skills. For example, a transcript may give an indication of study skills or work discipline, but a letter of reference may be more informative about interpersonal skills and cooperativeness. Given a similar level of accuracy among the skills assessed, a reference letter may not be intrinsically more valuable than a transcript, however it is clear that the skills assessed by the reference letter are more relevant for a customer service position versus a position in research. If reference letters can only be obtained through networking while all workers can submit a transcript, we may assume $\varepsilon_I < \varepsilon_F$ in the context of customer service hiring, but for research hiring the assumption $\varepsilon_F < \varepsilon_I$ may be more appropriate. The relationship between ε_F and ε_I, that is, whether informal or formal reports are more reliable, will also be an important industry characteristic in my analysis.

There exists a range of interpretations for what are considered "formal" and "informal" job search methods. For example, methods classified as "informal" are often methods which involve interpersonal networks, such as family, friends, and professional contacts.[2] Many studies also include methods such as direct application to the firm in their collection of informal methods, while reserving the term "formal" to describe methods which are non-personal and make use of market intermediaries such as applying or posting through employment agencies and job

[2]This may be through referrals exclusively or may include hearing about opportunities or receiving help, whether or not the contact is a part of the organization or makes a recommendation to the employer (DeVaro, 2008; Simon and Warner, 1992; Corcoran et al., 1980).

advertising boards.[3] Alternatively, direct application may be described as neither formal nor informal.[4] In my model, the key distinction between the two application channels is the cost or effort required to reach the firm with an application through this channel. Among all job search methods, there is a wide variety of mechanisms for reaching the firm, and the mechanism's dependence on effort is not necessarily associated with formal or informal methods for different categorizations. However for traditional job search categorizations in which the distinguishing feature of so-called formal methods is the use of intermediaries such as job boards and employment agencies, access to this channel is usually equally available to everyone and there is some standardized centralized way of relaying applications to the firm. For such methods it is typically appropriate to assume that the probability of reaching the firm with an application is not much affected by a job seeker's effort. Meanwhile for many traditionally "informal" methods, including those not based on networking (such as direct application), the probability of having one's application reach the firm is more naturally seen as increasing in effort. For this reason I associate the endogenous channel with informal methods and the exogenous channel with formal methods. However the model remains somewhat flexible in its interpretation. The endogenous hiring channel could alternatively be interpreted as a formal hiring channel while the exogenous channel is interpreted as an informal channel in a particular situation, if in the given context some application method considered to be "formal" is more appropriately associated with a dependence on effort than some method considered "informal." Additionally, two methods considered both formal or both informal, but which differ in dimensions of effort and information, could be compared with each other in this framework.

3.1.2 The Firm's Problem

Now upon receiving an application through channel $j \in \{F, I\}$ with report $R_j \in \{H, L\}$, the firm must decide whether or not to hire the worker. In this model I will suppose that the firm dele-

[3]For example, Rees (1966).
[4]For example, Holzer (1988).

gates this hiring decision to two independent departments, one handling applications received through the formal channel and the other handling applications received through the informal channel. While it is possible for a given worker to have both a formal and an informal application reach the firm, under this delegation assumption such a worker's formal and informal applications will be considered in isolation. I assume that a worker will become employed as long as he is accepted by at least one department, and whether the worker is accepted by one department or the other or both is irrelevant to the worker's payoff.

Although hiring decisions are often made by a single agent, in some organizations they are made by several distinct parties. In many areas educational contracts are between the teacher and the school district so hiring and placement decisions are made by the school district's centralized human resources office, and yet in some districts the principal also has authority to select hires for his or her own school. In such organizations it might be either hard for the parties with hiring authorities to exchange information about applicants, or it may be that they have difficulties in interpreting some types of information (for example a local manager may have a hard time assessing a formal academic transcript). Alternatively, if the firm were able to jointly observe a given worker's available formal and informal information, together with knowledge of the endogenous informal arrival rates, the firm would be able to make better informed hiring decisions and achieve higher profits. This centralized formulation is much harder to work with and will be discussed further in Section 3.4.3. However, taking into account the cost and feasibility of a centralized endeavour, the firm may find it worthwhile to delegate. Also, it is possible that even if the cost of such an endeavour is negligible, the centralized office may have weaker screening ability. Thus it may be of sufficient overall benefit to the firm to contract out the hiring decision to the offices or agents who have the most expertise in discerning quality through formal and informal reports.

Given that the formal and informal hiring offices do not share data, I will also assume the objective of each hiring department is to maximize the expected profit from its own hiring decisions. This assumption is suitable for situations in which the agents delegated to make the hiring decisions value their own reputation in identifying and recruiting suitable applicants. This private objective of the recruitment offices is not without some cost to the firm. Delegating the hiring decision to independent hiring departments which have their own reputational objective does in some cases result in more workers being hired than would be profit-maximizing for the firm. It would be in the firm's best interest for agents to be more cautious when hiring applicants since hiring applicants who are expected to be profitable should only be counted as increasing the firm's profit to the extent that they are likely not to be accepted by the other agent. Section 3.4.2 will examine the impact of delegation on firm profits in this model and Section 3.4.3 will compare alternatives to this delegation model.

The firm's strategy is a hiring decision for applications received in each department for each report realization. The firm's strategy is therefore $d{:}\{F, I\}{\times}\{H, L\}{\to}\{0, 1\}^4$, so that $d = (d_{FH}, d_{FL}, d_{IH}, d_{IL})$, where $d_{ji} = 1$ indicates a decision to accept an application in channel j with report $R_j = i$, and $d_{ji} = 0$ indicates a decision to reject such an application, for $j \in \{F, I\}$ and $i \in \{H, L\}$.

Since the formal and informal departments act independently and do not share information, each department's calculation of expected profit from accepting an application conditions only on the event of the arrival of the application and the observation of its report (given that it was received). Therefore the optimal hiring decisions d_{jH} and d_{jL} chosen by each department $j \in \{F, I\}$ are determined as in the single-channel benchmark model in Section 2.2, for the appropriate arrival probabilities and report errors in that channel. Following the notation of the previous chapter, let A_j denote the event of an application's arrival through channel $j \in \{F, I\}$. For each department $j \in \{F, I\}$, an application which arrives with report $R_j \in \{H, L\}$ will be accepted according to whether or not the relative gain of a high-quality worker exceeds the (posterior) odds that that applicant is low quality, $O(\ell{:}h|A_j{\cap}R_j)$, calculated by updating from the population-based prior, $O(\ell{:}h|A_j{\cap}R_j) = O(\ell{:}h) \cdot \Lambda(\ell{:}h|A_j) \cdot \Lambda(\ell{:}h|A_j{\cap}R_j)$.

Lemma 9. *For the formal department ($j = F$) the best response is characterized by the following:*

For $R_F = H$,

$$d_{FH}(p_{Ih}, p_{Il}) \ni 1 \quad \textit{iff} \quad \frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}, \tag{3.1}$$

$$d_{FH}(p_{Ih}, p_{Il}) \ni 0 \quad \textit{iff} \quad \frac{v_h - w}{w - v_\ell} \leq \frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}, \tag{3.2}$$

For $R_F = L$,

$$d_{FL}(p_{Ih}, p_{Il}) \ni 1 \quad \textit{iff} \quad \frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}, \tag{3.3}$$

$$d_{FL}(p_{Ih}, p_{Il}) \ni 0 \quad \textit{iff} \quad \frac{v_h - w}{w - v_\ell} \leq \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}. \tag{3.4}$$

In the formal channel, the firm's hiring decision is completely independent of workers' networking strategies, so the firm's best response in the formal department is in fact a dominant strategy. The formal arrival probabilities for applications from high and low types do not differ, so the posterior odds of low-quality are not affected by this factor in the formal channel as $\Lambda(\ell{:}h|A_F) = \frac{p_F}{p_F} = 1$. However, the low-quality odds are affected by the realization of the report according to the factor $\Lambda(\ell{:}h|A_F \cap R_F) = \frac{Pr(R_F|A_F \cap \ell)}{Pr(R_F|A_F \cap h)}$, which is $\frac{\varepsilon_F}{1-\varepsilon_F}$ when $R_F = H$ and $\frac{1-\varepsilon_F}{\varepsilon_F}$ when $R_F = L$.

Since the inequality in condition (3.1) must be satisfied any time the inequality in condition (3.3) is satisfied, the firm's best response for the formal department will have $d_{FH}(p_{Ih}, p_{I\ell}) \geq d_{FL}(p_{Ih}, p_{I\ell})$. As in the single-channel benchmark, this means that the formal department will either hire all formal applicants, selectively hire only high-report formal applicants, or hire no formal applicants. In this chapter "generally (un)profitable" industries and "(not) decisive" reports are defined in a similar fashion as in the previous chapter. Since no information about an applicant's quality can be deduced from the event of the application's arrival, (i) the range of $\frac{v_h - w}{w - v_\ell}$ for which no hiring is optimal for the formal department coincides with the range for which formal reports are not decisive while the industry is generally unprofitable (ii) the range of $\frac{v_h - w}{w - v_\ell}$ for which selective hiring is optimal for the formal department coincides with the range for which formal reports are decisive, and (iii) the range of $\frac{v_h - w}{w - v_\ell}$ for which absolute hiring is optimal for the formal department coincides with the range for which formal reports are not decisive while the industry is generally profitable.

Lemma 10. *For the informal department ($j = I$) the best response is characterized by the following:*

For $p_{Ih} > 0$ and $R_I = H$

$$d_{IH}(p_{Ih}, p_{I\ell}) \ni 1 \quad \textit{iff} \quad \frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s}\frac{p_{I\ell}}{p_{Ih}}\frac{\varepsilon_I}{1-\varepsilon_I}, \tag{3.5}$$

$$d_{IH}(p_{Ih}, p_{I\ell}) \ni 0 \quad \textit{iff} \quad \frac{v_h - w}{w - v_\ell} \leq \frac{1-s}{s}\frac{p_{I\ell}}{p_{Ih}}\frac{\varepsilon_I}{1-\varepsilon_I}, \tag{3.6}$$

For $p_{Ih} > 0$ and $R_I = L$

$$d_{IL}(p_{Ih}, p_{I\ell}) \ni 1 \quad \textit{iff} \quad \frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s}\frac{p_{I\ell}}{p_{Ih}}\frac{1-\varepsilon_I}{\varepsilon_I}, \tag{3.7}$$

$$d_{IL}(p_{Ih}, p_{I\ell}) \ni 0 \quad \textit{iff} \quad \frac{v_h - w}{w - v_\ell} \leq \frac{1-s}{s}\frac{p_{I\ell}}{p_{Ih}}\frac{1-\varepsilon_I}{\varepsilon_I}, \tag{3.8}$$

For $p_{Ih} = 0$ and $p_{I\ell} > 0$, $d_{IH}(0, p_{I\ell}) = d_{IL}(p_{Ih}, p_{I\ell}) = 0$.

For $p_{Ih} = p_{I\ell} = 0$, $d_{IH}(p_{Ih}, p_{I\ell}) \in \{0, 1\}$, and $d_{IL}(p_{Ih}, p_{I\ell}) \in \{0, 1\}$.

When the informal arrival probability of high types, p_{Ih}, is not zero, the posterior odds of low-quality in the informal channel are affected by both the pool adjustment factor, $\Lambda(\ell{:}h|A_I) = \frac{p_{Ih}}{p_{I\ell}}$, and the report adjustment factor, $\Lambda(\ell{:}h|A_I \cap R_I) = \frac{Pr(R_I|A_I \cap \ell)}{Pr(R_I|A_I \cap h)}$, which is $\frac{\varepsilon_I}{1-\varepsilon_I}$ when $R_I = H$ and $\frac{1-\varepsilon_I}{\varepsilon_I}$ when $R_I = L$.

When high-quality workers do not make use of the informal channel, $p_{Ih} = 0$ but low-quality workers do, $p_{I\ell} > 0$, the expected value of hiring any applicant through the informal channel is negative and the firm's best response for the informal department must be to reject all informal applicants. However when neither worker type networks, the firm never receives applications through the informal channel so its decision is irrelevant and any informal hiring strategy is a best response for the informal department.

Since the inequality in condition (3.5) must be satisfied any time the inequality in condition (3.7) is satisfied, the firm's best response for the informal department will have $d_{IH}(p_{Ih}, p_{I\ell}) \geq d_{IL}(p_{Ih}, p_{I\ell})$ for any non-zero arrival probabilities $p_{Ih}, p_{I\ell} > 0$, so that the informal department will either hire all informal applicants, selectively hire only high-report informal applicants, or hire no informal applicants. However, when workers do arrive in the informal channel, the relative arrival probability does affect the optimal hiring decision, and therefore the ranges of $\frac{v_h - w}{w - v_\ell}$ for which no hiring, selective hiring, and absolute hiring are optimal for the informal department do not necessarily coincide with the ranges for which reports are and are not decisive and the industry is generally profitable and unprofitable.

Remark 1. *For any firm best response and non-zero informal arrival probabilities, d_{ji} is increasing in v_h, v_ℓ and s, and decreasing in w for $j \in \{F, I\}$, $i \in \{H, L\}$. Also, d_{jH} is decreasing in ε_j, and d_{jL} is increasing in ε_j for $j \in \{F, I\}$.*[5]

In both channels the hiring conditions for both high-and low-report applications, inequalities (3.1)-(3.7), are each more easily satisfied with a higher relative gain ratio $\frac{v_h - w}{w - v_\ell}$ since this increases the margin by which the profit from a high-quality hire outweighs the loss from a low-quality hire (or decreases the margin by which the loss from a low-quality hire outweighs the profit form a high-quality hire) . Also in either channel $j \in \{F, I\}$, a decrease in the firm's screening ability in that channel, ε_j, increases the chance that the report received is false, which makes the firm less willing to hire high-report workers and more willing to hire low-report workers, as in the case of the single channel analysis.

[5] I follow the convention of denoting weakly increasing or decreasing as simply "increasing" or "decreasing."

Remark 2. *For any firm best response and non-zero informal arrival probabilities, $d_{IH}(p_{Ih}, p_{I\ell})$ and $d_{IL}(p_{Ih}, p_{I\ell})$ are increasing in p_{Ih} and decreasing in $p_{I\ell}$ and $\dfrac{p_{I\ell}}{p_{Ih}}$.*

For the informal channel, a decrease in $\dfrac{p_{I\ell}}{p_{Ih}}$ decreases the odds that an applicant is low-quality for both high- and low- report applications and the firm is more willing to accept both high- and low-report applicants in the informal channel.

3.1.3 The Worker's Problem

The probability that a worker of type q is hired in each channel depends on the probability that his application arrives to the firm and the probability that the firm subsequently accepts it. For a worker of type q, the arrival probabilities in the formal and informal channels are p_F and p_{Iq} respectively. Let ϕ_{Fq} denote the probability that a type q worker's formal application is accepted conditional on the firm receiving it, and let ϕ_{Iq} denote the probability that a type q worker's informal application is accepted conditional on the firm receiving it. Then the (unconditional) probability of the worker being accepted by the formal department is $p_F \cdot \phi_{Fq}$ and the (unconditional) probability of the worker being accepted by the informal department is $p_{Iq} \cdot \phi_{Iq}$.

Now the conditional acceptance probabilities ϕ_{Fq} and ϕ_{Iq} depend on the firm's hiring strategy d, so that $\phi_{Fq} = \phi_{Fq}(d)$ and $\phi_{Iq} = \phi_{Iq}(d)$. When $d_{jH} = 1$, an application received in channel $j \in \{F, H\}$ is accepted with probability $1-\varepsilon_j$ if the worker is high-quality, and with probability ε_j if the worker is low quality. When $d_{jL} = 1$, an application received in channel $j \in \{F, I\}$ is accepted with probability ε_j if the worker is high-quality, and with probability $1-\varepsilon_j$ if the worker is low-quality. If $d_{jH} = 0$ or $d_{jL} = 0$, both worker types have zero probability of being accepted in channel $j \in \{F, I\}$ with report $R_j = H$ or $R_j = L$ respectively. Thus for a worker of high quality $q = h$ we have the conditional acceptance probabilities

$$\phi_{Fh} = (1-\varepsilon_F)d_{FH}+\varepsilon_F d_{FL} \tag{3.9}$$

and

$$\phi_{Ih} = (1-\varepsilon_I)d_{IH}+\varepsilon_I d_{IL}, \tag{3.10}$$

while for a worker of low quality $q = l$ we have the conditional acceptance probabilities

$$\phi_{F\ell} = \varepsilon_F d_{FH}+(1-\varepsilon_F)d_{FL} \tag{3.11}$$

and

$$\phi_{I\ell} = \varepsilon_I d_{IH}+(1-\varepsilon_I)d_{IL}. \tag{3.12}$$

Since a worker obtains the job if he is accepted by either department, but may possibly have applications independently reach and be accepted by both departments simultaneously, the probability that a worker of type q is hired, Φ_q, is equal to the probability of being accepted by the formal department, plus the probability that the worker is accepted by the informal department and not by the formal department:

$$\Phi_q(d, p_{Iq}) = p_F{\cdot}\phi_{Fq} + (1-p_F{\cdot}\phi_{Fq})p_{Iq}{\cdot}\phi_{Iq} \tag{3.13}$$

Rearranging equation (2.21) and substituting $\Phi(d, p_{Iq})$ from above, we see that a worker of type q with informal arrival probability p_{Iq} has expected utility

$$(w-b)(p_F{\cdot}\phi_{Fq} + (1-p_F{\cdot}\phi_{Fq})p_{Iq}{\cdot}\phi_{Iq}) + b - \gamma(p_{Iq}) \tag{3.14}$$

where $\phi_{Fq} = \phi_{Fq}(d)$ and $\phi_{Iq} = \phi_{Iq}(d)$ are determined for $q \in \{h, \ell\}$ by equations (3.9)-(3.12) for given firm strategy d. Thus the best response of a worker of type q to firm strategy d, $p_{Iq}(d)$, must satisfy

$$\hat{w}(1 - p_F{\cdot}\phi_{Fq}(d))\phi_{Iq}(d) \leq \gamma'(p_{Iq}), \quad \text{with equality if } p_{Iq} > 0, \tag{3.15}$$

where $\hat{w} = w - b$.

Lemma 11. *Let $\psi \equiv \gamma'^{-1}$. The worker's best response is given by:*

$$p_{Iq}(d) = \begin{cases} \psi(\hat{w}(1 - p_F \cdot \phi_{Fq}(d))\phi_{Iq}(d)) & \text{if } \hat{w}(1 - p_F \cdot \phi_{Fq}(d))\phi_{Iq}(d) > \gamma'(0) \\ 0 & \text{otherwise.} \end{cases} \tag{3.16}$$

Although the networking choices of worker types affect only the firm's informal hiring decision, worker types take into account both the formal and informal hiring strategies of the firm when making their networking choices. For workers of either type, networking is more appealing when that type's informal application has a high probability of being accepted conditional on being received, ϕ_{Iq}, but networking is less appealing when that type has a high probability of being accepted in the formal channel, $p_F \cdot \phi_{Fq}$. Thus p_{Iq} is increasing in ϕ_{Iq} and decreasing in p_F and ϕ_{Fq}, for $q \in \{h, l\}$. Note that for either worker type, if improving his arrival probability is too costly, specifically if d is such that $\hat{w}(1 - p_F \cdot \phi_{Fq}(d))\phi_{Iq}(d) \leq \gamma'(0)$, then the worker will not use the informal channel at all, $p_{Iq}(d) = 0$.

Comparing the worker's problem in this two-channel model with the one-channel model of the previous chapter, we see that given a particular formal hiring strategy of the firm, d_{FH} and d_{FL}, the formal channel here serves as a type-dependent outside option for workers in a one-channel model. For the worker, note that allowing the unemployment benefit b in equations (2.24) and (2.25) to vary by worker type, worker utility in the one channel model would be given by

$$u_q = (w - b_q)p_q\phi_q + b_q - \gamma(p_q). \tag{3.17}$$

Therefore if we take $b_q = b + (w - b)p_F\phi_{Fq}$ for a fixed d_{FH} and d_{FL}, equation (3.17) is equivalent to equation (3.14) with $p_q = p_{Iq}$ and $\phi_q = \phi_{Iq}$.

Also note that $d_{F\ell} \leq d_{Fh}$ implies $\phi_{F\ell} \leq \phi_{Fh}$, which in turn implies $b_\ell \leq b_h$. Hence the presence of the formal channel provides a (weakly) greater outside option to the high type than to the low type.

3.2 Preliminary Results

3.2.1 Use of Informal Channel by Type

Section 3.1.2 showed how the odds of an applicant being low-quality affect the firm's hiring decision, and how in the informal channel these odds are adjusted according to the relative arrival of applications from low-quality workers in the informal channel, $\frac{p_{I\ell}}{p_{Ih}}$, provided p_{Ih} is non-zero. In order to understand under what circumstances this adjustment factor may be favourable, $\frac{p_{I\ell}}{p_{Ih}} < 1$, unfavourable, $\frac{p_{I\ell}}{p_{Ih}} > 1$, or neutral, $\frac{p_{I\ell}}{p_{Ih}} = 1$, this section will investigate whether high- or low-quality workers choose to use the informal channel more (or equally) in best response to each possible firm strategy d.

For the following discussion I will suppose that the acceptance probabilities resulting from the firm's strategy d are such that use of the informal channel is not prohibitively expensive for either worker type and $p_{I\ell}(d)$ and $p_{Ih}(d)$ are both not zero.[6]

As in the single channel benchmark model, we know that whenever the arrival probabilities of worker applications are not zero, the firm's best response will follow a pattern of either absolute hiring ($d_{jH} = d_{jL} = 1$), selective hiring ($d_{jH} = 1, d_{jL} = 0$), or no hiring ($d_{jH} = d_{jL} = 0$) in each department $j \in \{F, I\}$. For notational ease, I will define $d_F = d_{FH}+d_{FL}$ and $d_I = d_{IH}+d_{IL}$. Thus we may refer to the patterns of "no hiring," "selective hiring," and "absolute hiring" in channel j with $d_j = 0$, $d_j = 1$, and $d_j = 2$ respectively. The firm's strategy d can be conveniently summarized by $(d_F, d_I) \in \{0, 1, 2\}\times\{0, 1, 2\}$ and the best response for worker q may be denoted $p_{Iq}(d_F, d_I)$ rather than $p_{Iq}(d)$.

Since there are three different hiring patterns which may be followed in each channel, there

[6]Lemma 37 in Appendix B shows that no equilibria can be supported where one worker type q chooses to use the informal channel, $p^*_{Iq}>0$, while the other type $\tilde{q}$ does not, $p_{I\tilde{q}} = 0$. Thus I present results only for cases where both types actually do.

are nine (3×3) possible firm strategies for which to consider whether high or low types will choose to use the informal channel more. For the three strategy choices in which the firm adopts a pattern of no hiring in the informal channel, it must be that $p_{Ih}(d_F, 0) = p_{I\ell}(d_F, 0) = 0$; neither worker type will network at all in response to a firm strategy with $d_I = 0$, regardless of d_F. This is because arrival through the informal channel is costly, and there is no chance for either type to be accepted in the informal channel upon arrival, $\phi_{Iq} = 0$ for $q \in \{h, \ell\}$, when the firm adopts a pattern of no hiring in the informal department. So this section will compare high- and low-quality types' best responses to the six remaining firm strategies, $(d_F, d_I) \in \{0, 1, 2\}\times\{1, 2\}$.

As we have seen, the incentives for a worker of type q to use the informal channel increase with his conditional acceptance probability in the informal channel, ϕ_{Iq}, and decrease with his probability of being hired in the formal channel, $p_F{\cdot}\phi_{Fq}$. These incentives may or may not differ by type depending on the hiring patterns adopted by the firm. Equations (3.10) and (3.12) show that according to each of the two possible (non-zero) hiring patterns adopted by the firm in the informal department, $d_I \in \{1, 2\}$, a worker's informal application, if received, will either be accepted with his type's probability of generating a high informal report, or accepted with certainty:

$$\phi_{Ih}(d_I) = \begin{cases} 1-\varepsilon_I & \textit{if } d_I = 1 \\ 1 & \textit{if } d_I = 2 \end{cases} \qquad \phi_{I\ell}(d_I) = \begin{cases} \varepsilon_I & \textit{if } d_I = 1 \\ 1 & \textit{if } d_I = 2. \end{cases} \tag{3.18}$$

Similarly, equations (3.9) and (3.11) show that according to each of the three possible hiring patterns adopted by the firm in the formal department, $d_F \in \{0, 1, 2\}$, a worker's formal application, if received, will either be rejected with certainty, accepted with his type's probability of generating a high formal report, or accepted with certainty:

$$\phi_{Fh}(d_F) = \begin{cases} 0 & \textit{if } d_F = 0 \\ 1-\varepsilon_F & \textit{if } d_F = 1 \\ 1 & \textit{if } d_F = 2 \end{cases} \qquad \phi_{F\ell}(d_F) = \begin{cases} 0 & \textit{if } d_F = 0 \\ \varepsilon_F & \textit{if } d_F = 1 \\ 1 & \textit{if } d_F = 2. \end{cases} \tag{3.19}$$

For any fixed firm strategy (d_F, d_I) a high-quality worker has a conditional acceptance probability advantage over a low-quality worker in both the formal and informal channels, $\phi_{jh}(d_j) \geq \phi_{j\ell}(d_j)$ for $j \in \{F, I\}$. This is because in each channel high reports are realized with higher probability for high-quality workers than for low-quality workers, and high reports lower the odds of an applicant being low quality. The acceptance advantage is a strict advantage for the high type in a given channel precisely when the firm hires selectively in that channel, $\phi_{jh}(d_j) > \phi_{j\ell}(d_j) \Leftrightarrow d_j = 1$ for $j \in \{F, I\}$.

Unlike the single-channel benchmark, the high-quality type's acceptance advantages in the formal and informal channels may result in either the high type or the low type choosing a greater arrival probability in the informal channel. Since the incentive to use the informal channel increases with ϕ_{Iq} but decreases with ϕ_{Fq}, a strict acceptance advantage for the high type in the informal channel creates incentive for the high type to choose a greater arrival probability relative to the low type, whereas a strict acceptance advantage for the high type in the formal channel creates incentive for the high type to choose a lower arrival probability, relative to the low type. For arrival probability choices which are not zero, equation (3.15) gives

$$p_{Ih}(d_F, d_I) \gtreqless p_{I\ell}(d_F, d_I) \quad \textit{if and only if} \quad (1 - p_F{\cdot}\phi_{Fh})\phi_{Ih} \gtreqless (1 - p_F{\cdot}\phi_{F\ell})\phi_{I\ell}. \tag{3.20}$$

We see that the high type may potentially use the informal channel more than the low type if the firm's hiring strategy d gives him a sufficient strict advantage in the informal channel, ϕ_{Ih} sufficiently greater than $\phi_{I\ell}$, due to selective informal hiring, or may potentially use the informal channel less than the low type, if the firm's hiring strategy d gives him a sufficient strict advantage in the formal channel, $\phi_{Fh} >> \phi_{F\ell}$, due to selective formal hiring.

I first compare the arrival probabilities chosen by workers in response to strategies for which the firm does not hire selectively in the informal channel, and then examine strategies for which the firm does hire selectively in the informal channel. The resulting findings are

	$d_I = 2$	$d_I = 1$
$d_F = 0$	$p_{Ih} = p_{I\ell}$	$p_{Ih} > p_{I\ell}$
$d_F = 1$	$p_{Ih} < p_{I\ell}$	$p_{Ih} > p_{I\ell},\ p_{Ih} = p_{I\ell},\ p_{Ih} < p_{I\ell}$
$d_F = 2$	$p_{Ih} = p_{I\ell}$	$p_{Ih} > p_{I\ell}$

Table 3.1: Comparison of Worker Arrival Probabilities

When non-zero, the possible ranking of workers' best-response arrival probabilities, $p_{Ih}(d_F, d_I)$ and $p_{I\ell}(d_F, d_I)$, is shown for each combination of formal and informal hiring strategies.

summarized in Table 3.1.

Suppose the firm does not hire selectively in the informal channel. Since I exclude firm strategies with $d_I = 0$, this means the firm hires absolutely in the informal channel, $d_I = 2$. When all applications are accepted regardless of report, high- and low-quality types will have an equal chance of being accepted in the informal channel, given that the firm received their informal applications, that is, $\phi_{Ih}(2) = \phi_{I\ell}(2) = 1$. If the firm also does not hire selectively in the formal channel, following a strategy of either no hiring or absolute hiring $d_F = 0$ or $d_F = 2$, then both worker types have equal conditional acceptance probabilities in the formal channel as well, either $\phi_{Fh}(0) = \phi_{F\ell}(0) = 0$ or $\phi_{Fh}(2) = \phi_{F\ell}(2) = 1$. In this case both worker types have identical incentives to use the informal channel so $p_{Ih}(d_F, 2) = p_{I\ell}(d_F, 2)$ for $d_F \in \{0, 2\}$. On the other hand, if the firm hires selectively in the formal channel, so that $d_F = 1$, then high-report applications are favoured, so high-quality workers have a greater formal conditional acceptance probability, $\phi_{Fh}(1) > \phi_{F\ell}(1)$. In this case a high-quality worker does not benefit as much as a low-quality worker does from the chance of reaching the firm informally, and so even though all informal applications are equally likely to be accepted, over all the high-quality worker has less incentive to use the informal channel than the low-quality worker, $p_{Ih}(1, 2) < p_{I\ell}(1, 2)$. We see that the high worker type never chooses a higher arrival probability than the low type when the firm adopts a strategy of absolute hiring in the informal channel, a result I state formally as follows:

Lemma 12. *Assuming best response arrival probabilities are non-zero, we have* $p_{Ih}(d_F, 2) \leq$

$p_{I\ell}(d_F, 2)$. *The strict inequality holds if and only if* $d_F = 1$.

Now suppose that the firm hires selectively in the informal channel, $d_I = 1$, so that informal applications with high reports are favoured. Then the high-quality type has a greater conditional acceptance probability in the informal channel than the low type, $\phi_{Ih}(1) > \phi_{I\ell}(1)$. If both worker types have equal formal conditional acceptance probabilities $\phi_{Fh}(d_F) = \phi_{F\ell}(d_F)$, such that the firm treats high- and low-report applications the same in the formal channel (either with no formal hiring $d_F = 0$ or absolute formal hiring $d_F = 2$) then the high-quality worker has more incentive to use the informal channel relative to the low-quality worker due to his advantage in the informal channel. Therefore $p_{Ih}(d_F, 1) > p_{I\ell}(d_F, 1)$ for $d_F \in \{0, 2\}$.

However, when the firm hires selectively in the informal channel, $d_I = 1$, if the firm also favours high-report applications in the formal channel, $d_F = 1$, then the high-quality type has an acceptance advantage in both formal and informal channels and it is not obvious whether or not he will have more incentive over all to choose a greater arrival probability relative to the low-quality type. This will depend on whether the high type's added incentive to use the informal channel due to an informal acceptance advantage outweighs his reduced incentive to use the informal channel due to also having a formal acceptance advantage. Which advantage has a stronger effect will be determined by the magnitudes of formal and informal report errors, ε_F and ε_I, and by the application arrival probability in the formal channel p_F, as follows:

Lemma 13. *Assuming best response arrival probabilities are non-zero, we have*

$$p_{Ih}(1,1) > p_{I\ell}(1,1) \text{ if and only if } \varepsilon_I - \varepsilon_F < \frac{1-p_F}{p_F}(1-2\varepsilon_I),$$

$$p_{Ih}(1,1) < p_{I\ell}(1,1) \text{ if and only if } \varepsilon_I - \varepsilon_F > \frac{1-p_F}{p_F}(1-2\varepsilon_I),$$

$$p_{Ih}(1,1) = p_{I\ell}(1,1) \text{ if and only if } \varepsilon_I - \varepsilon_F = \frac{1-p_F}{p_F}(1-2\varepsilon_I).$$

This result follows directly from evaluating the direction of the inequality in condition (3.20) for $\phi_{Fh}(1,1) = 1 - \varepsilon_F$, $\phi_{Ih}(1,1) = 1 - \varepsilon_I$, $\phi_{F\ell}(1,1) = \varepsilon_F$ and $\phi_{I\ell}(1,1) = \varepsilon_F$. Intuitively, given that the high-quality type is more likely to be hired through the formal channel, he will have more incentive to use the informal channel than the low type if and only if the informal report error is not too much greater than the formal report error. Specifically, the informal report error must not exceed the formal report error by more than the relative probability that formal applications fail to reach the firm times the difference between high- and low-quality workers' acceptance probabilities, $\phi_{Ih}(1,1) - \phi_{I\ell}(1,1) = 1 - 2\varepsilon_I$. When the informal error is smaller than the formal error, high-quality workers certainly will choose higher arrival probabilities. Otherwise, if the informal error probability is too much greater than the formal error probability, the high-quality worker's acceptance advantage in the informal channel is not sufficient to induce him to choose a higher informal arrival probability than the low-quality worker. How much the informal error can exceed the formal error by in order for the high type to use the informal channel more depends on the arrival probability of formal applications, p_F. As this probability approaches certainty, $p_F \to 1$, the high-quality worker will have more incentive to use the informal channel according to whether or not informal reports are more precise than formal reports, $\varepsilon_I < \varepsilon_F$. The lower the formal arrival probability p_F is, the lower the error of informal reports must be in order for the high-quality worker type to overall have more incentive to choose a higher informal arrival probability than the low type.

In the one-channel model from the previous chapter we have seen that the informativeness of the exogenous report and the quality of the pool associated with the endogenous signal were complements. The above considerations suggest that this will continue to be the case in this two-channel model with regard to the relation between the informativeness of the exogenous informal report and the quality of the informal pool of applicants. However the relation between the informativeness of the formal report and the quality of the informal pool is likely to be one of substitutes. This will be investigated and confirmed for the equilibrium pool of informal applicants in Section 3.4.1.

3.2.2 Equilibrium Basics

This analysis will be restricted to pure strategy Nash equilibria. In the equilibrium analyses in the following sections, I will further restrict attention to pure strategy equilibria in which $p^*_{Ih} > 0$ and $p_{I\ell} > 0$.

Definition 6. *A pure strategy equilibrium consists of a firm hiring strategy* $d^* = (d^*_{FH}, d^*_{FL}, d^*_{IH}, d^*_{IL}) \in \{0,1\}^4$ *and strategies for high- and low-quality workers* $p^*_I = (p^*_{Ih}, p^*_{I\ell}) \in \mathbb{R}^2_+$ *such that:*

1. Given workers' strategies $p^*_{Ih}, p^*_{I\ell}$*, each department in channel* $j \in \{F, I\}$ *chooses* d^*_{ji} *to maximize*

$$E_q[v_q - w \mid A_j,\ R_j = i]\cdot d_{ji} \quad \textit{for } i \in \{H, L\}.$$

2. Given the firm's strategy d^**, worker* $q \in \{h, l\}$ *chooses* p^*_{Iq} *to maximize*

$$(w-b)\Phi_q(d^*, p_{Iq}) + b - \gamma(p_{Iq}),$$

where $\Phi_q(d, p_{Iq}) = p_F\cdot\phi_{Fq}(d) + (1 - p_F\cdot\phi_{Fq}(d))p_{Iq}\cdot\phi_{Iq}(d)$*.*

Before turning to the equilibrium analysis I will make the following basic observations about equilibria in this model:

Observation 1. An equilibrium with firm strategy equal to $(d^*_{FH}, d^*_{FL}, 0, 0)$ always exists for some $d^*_{FH} \in \{0,1\}$ and $d^*_{FL} \in \{0,1\}$.

Observation 2. If there exists an equilibrium with non-zero informal arrival and firm strategy equal to $(d^*_{FH}, d^*_{FL}, d^*_{IH}, d^*_{IL})$, then there does not simultaneously exist any other non-zero informal arrival equilibrium with the same firm strategy.

Observation 3. An equilibrium with firm strategy $(d^*_{FH}, d^*_{FL}, d^*_{IH}, d^*_{IL})$ and an equilibrium with firm strategy $(\hat{d}_{FH}, \hat{d}_{FL}, \hat{d}_{IH}, \hat{d}_{IL})$ where $\hat{d}_{FH} \neq d^*_{FH}$ or where $\hat{d}_{FL} \neq d^*_{FL}$, can only coexist for parameter settings such that the inequalities in conditions (3.1) or (3.3) hold as equality.

The first observation points out that non-use of the informal channel can always be sustained in equilibrium. For any given $v_h, v_\ell, w, s, \varepsilon_F$, there will be some d^*_{FH}, d^*_{FL} which are optimal, so an equilibrium with $(d^*_{FH}, d^*_{FL}, 0, 0)$ can certainly be supported with $p^*_{Ih} = p^*_{I\ell} = 0$. As in the single-channel benchmark, non-use of the informal channel is a best response for both worker types if the firm never hires through the informal channel, and any informal hiring strategy can be included in a best response for the firm if workers never arrive in the informal channel. [7] Therefore non-use of the informal channel in equilibrium may always be explained trivially, occurring perhaps as a social norm. The existence of such equilibria will not be the focus of my analysis in the next section (in fact such a trivial equilibrium must always exist, since non-use of the informal channel by workers and firms can always be paired with a choice of hiring pattern in the formal channel appropriate for the given parameter setting), although we will take interest in circumstances under which non-use of the informal channel is the unique informal hiring pattern possible in equilibrium.

The second observation is that equilibria may be categorized according to the firm strategy used. This observation does not suggest that equilibria are unique. For example, we know from Observation 1 that if there is an equilibrium with firm strategy profile $d^* = (1, 1, 1, 0)$ it must coexist with an equilibrium with firm strategy profile $d^* = (1, 1, 0, 0)$. However, there can never exist two equilibria with the same firm hiring strategy. This is because for any given firm strategy and parameter setting, there is a unique worker best response given by equation (4.3). So equilibria may be categorized by the firm's strategy.

The third observation is that for a given parameter setting, there will typically be only one formal hiring strategy admissible for equilibrium. Again this is because the best response for the firm in the formal channel is determined directly by v_h, v_ℓ, s and ε_F, and is unique except the knife-edge case when either of the inequalities in condition (3.1) or (3.3) holds as equality.

[7] As mentioned before, the optimal decision for the formal channel is independent of the strategy of the informal channel.

3.3 Equilibrium Use of the Informal Channel

In this section I will investigate the existence of equilibria in which the informal channel is actually "used." This requires that the choice of informal arrival probability is strictly positive for at least one worker type, and that the firm accepts at least some applications which it receives through the informal department.

However, there can be no pure strategy Nash equilibria in which the informal channel is used by one worker type and not the other. This is because use of the informal channel by the firm is incompatible with the negative expected profit from informal hiring which is guaranteed if only low types arrive in the informal channel, and also because the absolute hiring pattern which the firm would follow if only high types arrive in the informal channel is incompatible with non-use of the informal channel by low types. So in any equilibrium in which the informal channel is used, we will have non-zero informal arrival probabilities for both worker types, $p^*_{Ih},\ p^*_{I\ell} > 0$ (which I will refer to simply as "non-zero arrival" in the informal channel). In the context of non-zero arrival, recall that we may use the reduced notation $(d_F, d_I) \in \{0,1,2\} \times \{1,2\}$ to express the firm's strategy when convenient rather than $d = (d_{FH}, d_{FL}, d_{IH}, d_{IL}) \in \{0,1\}^4$.

For any equilibrium in which the informal channel is used, the firm's strategy d^* must satisfy the conditions (3.5)-(3.8) evaluated using the pool factor which must arise when workers best-respond to d^*. I will denote the pool adjustment factor which results from the firm strategy $(d^*_F, d^*_I) = (i, j)$ by the term $\Lambda^{ij} \equiv \dfrac{p_{I\ell}(i,j)}{p_{Ih}(i,j)}$ for $i \in \{0,1,2\}$ and $j \in \{1,2\}$.

I will first focus on absolute use of the informal channel in equilibrium, that is, I will examine equilibria in which $d^*_I = 2$, and then I will address equilibria in which the informal channel is used selectively, with $d^*_I = 1$.

In each case, I will also address the qualitative aspects of the use of informal applications. In both the one-channel and two-channel models, applications serve a purpose to the firm and workers in different dimensions; first, by connecting workers to firms so that they can be considered for hire, and second, by providing information to the firm concerning the quality of the applicant. In particular, in addition to the direct information the application may contain in the report, it may also convey indirect information endogenously through the composition of the pool of applicants, as is the case with informal applications. Although an application may have value in all dimensions simultaneously, they may not all be of equal importance depending on the industry setting. For example, when the relative gain from high-quality workers is very high, so that the profit from a high-quality hire easily outweighs the loss in profit from a low-quality hire, informational contributions might be a less important aspect of applications than their connective value. Recall that Section 2.2.4 outlined the circumstances under which applications could be qualitatively interpreted as having primary value as a "door," as a "report" and as a "signal." Here it will be shown in the two-channel model which (primary) functions informal applications can serve when the formal channel has value as a door, as a report, or is not used,[8] and under what circumstances.

3.3.1 Absolute Informal Hiring in Equilibrium

In any equilibrium in which the firm hires absolutely in the formal channel, so that $d_{Ih} + d_{I\ell} = 2$ and $(d_F^*, d_I^*) = (i, 2)$ for some $i \in \{0, 1, 2\}$, it must be the case that the relative gain from high-quality workers exceeds the odds that a low-report informal applicant is truly low-quality when workers are best-responding to $(d_F^*, d_I^*) = (i, 2)$. That is, it is necessary to have

$$\frac{v_h - w}{w - v_l} \geq \frac{1-s}{s} \Lambda^{i2} \frac{1-\varepsilon_I}{\varepsilon_I} \tag{3.21}$$

by Lemma 10, where $\Lambda^{i2} \equiv \dfrac{p_{I\ell}(i,2)}{p_{Ih}(i,2)}$ as noted above. However, by Lemma 12 we know that any equilibrium with non-zero arrival and absolute informal hiring will have $p_{I\ell}(i,2) \geq p_{Ih}(i,2)$, so the informal pool adjustment factor must be at least weakly unfavourable, $\Lambda^{i2} \geq 1$. This

[8]Note that formal applications can never have value as a signal, since the pool of formal applicants is exogenously determined and always neutral. For this reason formal applications also need no restriction on how favourable the pool can be in order to have primary value as a report, in contrast to Chapter 2.

unfavourable informal pool factor implies that the low-quality odds for low-report informal applications are necessarily above $\frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}$. This means that condition (3.21), and therefore absolute informal hiring, can only be satisfied in industries which are generally profitable and for which reports are not decisive:

Lemma 14. *For the existence of any equilibrium with $d_I^* = 2$ and non-zero informal arrival we must have $\frac{v_h-w}{w-v_l} \geq \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}$.*

It follows from Lemma 14 that there can be no equilibrium with non-trivial use of the informal channel which has absolute hiring in the informal channel but no hiring in the formal channel, that is, which has $(d_F^*, d_I^*) = (0, 2)$, because a hiring strategy with $d_F^* = 0$ can never be optimal for the firm in a generally profitable industry.

There is also a further necessary condition for supporting absolute informal hiring together with selective formal hiring in an equilibrium with non-zero informal arrival, namely that formal reports must be more precise than informal reports:

Lemma 15. *For the existence of any equilibrium with $(d_F^*, d_I^*) = (1, 2)$ and non-zero informal arrival we must have $\varepsilon_I \geq \varepsilon_F$.*

Proof. Suppose $(d_F^*, d_I^*) = (1, 2)$ is an equilibrium with $p_h^*, p_\ell^* > 0$. In order for selective formal hiring to be optimal while absolute informal hiring is optimal, the firm must be willing to accept low-report informal applicants, $\frac{v_h-w}{w-v_l} \geq O(\ell{:}h|A_I \cap L)$, while also rejecting low-report formal applicants, $\frac{v_h-w}{w-v_l} \leq O(\ell{:}h|A_F \cap L)$. This is only possible when $O(\ell{:}h|A_I \cap L) < O(\ell{:}h|A_F \cap L)$ Thus by conditions (3.7) and (3.4) it would be necessary that

$$\frac{v_h-w}{w-v_l} \in \left[\frac{1-s}{s}\Lambda^{12}\frac{1-\varepsilon_I}{\varepsilon_I}, \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}\right] \tag{3.22}$$

However, the odds of an applicant being low quality can not be smaller in the informal channel given a low report than in the formal channel given a low report, such that this interval is non-empty, unless the quality composition of the pool of applicants is superior in the informal channel, $\Lambda^{12} < 1$. This is because $\frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I} > \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}$ whenever $\varepsilon_I < \varepsilon_F$. Now by Lemma 12 the composition of the pool of applicants will in fact be inferior in the informal channel, $\Lambda^{12} = \frac{p_{I\ell}(1,2)}{p_{Ih}(1,2)} > 1$, so the interval in condition (3.22) can not exist in equilibrium if $\varepsilon_I < \varepsilon_F$.

The following proposition characterizes the equilibria in which the informal channel is used absolutely with non-zero arrival.

Proposition 7. *Suppose* $\frac{v_h-w}{w-v_l} \geq \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}$. *There are the following two kinds of equilibria with* $d^* = 2$ *and non-trivial use of the informal channel:*

(a) $(d_F^*, d_I^*) = (2,2)$ *and* $p_{Ih}^* = p_{I\ell}^* = \psi(\hat{w}(1-p_F))$.

This equilibrium exists if and only if $\hat{w}(1-p_F) > \gamma'(0)$ *and* $\frac{v_h-w}{w-v_l} \geq \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}$;

(b) $(d_F^*, d_I^*) = (1,2)$ *and* $p_{Ih}^* = \psi(\hat{w}(1-p_F(1-\varepsilon_F)))$, $p_{I\ell}^* = \psi(\hat{w}(1-p_F\varepsilon_F))$.

This equilibrium exists if and only if $\hat{w}(1-p_F(1-\varepsilon_F)) > \gamma'(0)$, $\frac{v_h-w}{w-v_l} \leq \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}$, *and*

$$\Lambda^{12} \equiv \frac{\psi(\hat{w}(1-p_F\varepsilon_F))}{\psi(\hat{w}(1-p_F(1-\varepsilon_F)))} \leq \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}. \tag{3.23}$$

A formal proof is presented in Appendix B. The non-negativity conditions in each case, $\hat{w}(1-p_F) > \gamma'(0)$ and $\hat{w}(1-p_F(1-\varepsilon_F)) > \gamma'(0)$, ensure that workers have strictly positive arrival probabilities in equilibrium, while the conditions on the relative gain ratio $\frac{v_h-w}{w-v_l}$ in relation to the low-quality odds for low-report formal applications, $\frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}$, ensure that the formal hiring strategy in each case is optimal. As seen in Table 3.1, when there is absolute hiring in both channels, the pool of informal applicants will be neutral, $\Lambda^{22} = 1$. Therefore in (a), absolute informal hiring can be sustained together with absolute formal hiring simply because the industry is generally profitable and informal reports are (at least weakly) not decisive, $\frac{v_h-w}{w-v_l} \geq \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}$. However in (b), because selective formal hiring together with absolute informal hiring must lead to an unfavourable informal pool, $\Lambda^{12} > 1$, an additional condition is required in order to sustain this equilibrium. In this setting the firm would be willing to hire absolutely in the formal channel given a neutral informal pool, but an unfavourable pool raises the odds that a given applicant is low-quality. Thus an equilibrium with $(d_F^*, d_I^*) = (1,2)$ requires additionally that the corresponding pool of informal applicants Λ^{12} be not too unfavourable.

Qualitatively, in the equilibria described by Proposition 7 the industry is generally profitable, informal reports are not decisive, and informal applications have primary value as a "door." Use of the informal channel reduces connection frictions between workers and the firm, while the firm accepts any applicant it encounters through this channel. As described in the one-channel model of the previous chapter, applications can have primary value as a door when the industry is generally profitable and the application's reports ε are not decisive, that is when $\frac{v_h-w}{w-v_l} \geq \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}$. In (a) we see that informal applications can have primary value as a door when formal applications also have primary value as a door. This is the case when the industry is generally profitable but neither formal nor informal reports are decisive. When both types of applications function as a door, the pool of informal applicants is neutral.

Recall that when an application's report is decisive, $\frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon} < \frac{v_h-w}{w-v_l} < \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}$, the direct information conveyed to the firm by the application is accurate enough to dictate the hiring decision when drawing from a neutral pool, and the application can have primary value as a "report." From (b) we see that in the two-channel model informal applications can have primary value as a door while the formal channel has value primarily as a report. When informal applications have primary value as a report but formal applications do not, the pool of informal applicants has a worse composition than that of the general population.

3.3.2 Selective Informal Hiring in Equilibrium

Selective use of the informal channel can also be supported in equilibrium with non-zero informal arrival, again with some restrictions. First, it is impossible to sustain an equilibrium with $d_I^* = 1$ in a generally profitable industry when neither formal nor informal reports are decisive. In such a setting we would certainly have $d_F^* = 2$, and as discussed in Section 3.2.1, high-quality workers would have more incentive to use the informal channel than low-quality workers. Thus the informal pool adjustment factor would be favourable and the informal channel odds of low quality, $O(\ell{:}h|A_I{\cap}H)$ and $O(\ell{:}h|A_I{\cap}L)$, would decrease. In particular, we would have $O(\ell{:}h|A_I{\cap}L) < \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}$ and the firm would not wish to reject low-report informal applicants. That is, if the relative gain in the industry is high enough that the firm is willing to hire all informal applicants given a neutral pool, it must be also willing to hire all informal applicants when the pool is adjusted favourably.

Secondly, an equilibrium with selective informal hiring can not be sustained together with absolute formal hiring if $\varepsilon_I \geq \varepsilon_F$. This is again because a hiring strategy $(d_F^*, d_I^*) = (2, 1)$ will result in a favourable informal pool factor, $\frac{p_{I\ell}(2,1)}{p_{Ih}(2,1)} < 1$. If reports have (at least weakly) larger error in the informal channel, and the informal channel additionally receives a favourable pool adjustment, then low-report informal applicants can not be rejected while formal low-report applicants are accepted. These necessary conditions are formalized as follows:

Lemma 16. *For any equilibrium with $d_I^* = 1$ and non-zero informal arrival we must have $\frac{v_h-w}{w-v_l} < \max\left\{\frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}, \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}\right\}$.*

Lemma 17. *For any equilibrium with $(d_F^*, d_I^*) = (2, 1)$ and non-zero informal arrival we must have $\varepsilon_I < \varepsilon_F$.*

The following propositions give necessary and sufficient conditions for the existence of equilibria with non-zero informal arrival in which the informal channel is used selectively. Propositions 8 and 9 examine the existence of equilibria with selective informal hiring when informal reports are decisive, for informal reports which are and are not sufficiently superior to formal reports respectively. Propositions 10 and 11 examine the existence of equilibria with selective informal hiring when informal reports are not decisive, for generally unprofitable and generally profitable industries respectively.

Proposition 8. *Suppose $\frac{v_h-w}{w-v_l} \in \left(\frac{1-s}{s}\frac{\varepsilon_I}{1-\varepsilon_I}, \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}\right)$ and $\varepsilon_I - \varepsilon_F \geq \frac{1-p_F}{p_F}(1-2\varepsilon_I)$. There is only one possible equilibrium with non-trivial use of the informal channel:*

$(d^*_F, d^*_I) = (1,1)$ and $p^*_{Ih} = \psi(\hat{w}(1 - p_F(1 - \varepsilon_F))(1 - \varepsilon_I))$, $p^*_{I\ell} = \psi(\hat{w}(1 - p_F\varepsilon_F)\varepsilon_I)$.

This equilibrium exists if and only if $\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I) > \gamma'(0)$, and

$$\Lambda^{11} \equiv \frac{\psi(\hat{w}(1 - p_F\varepsilon_F)\varepsilon_I)}{\psi(\hat{w}(1-p_F(1-\varepsilon_F)(1 - \varepsilon_I))} \leq \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{1-\varepsilon_I}{\varepsilon_I}. \tag{3.24}$$

Here informal reports are decisive but the informal report error is large relative to the formal report error, such that $\varepsilon_I-\varepsilon_F \geq \frac{1-p_F}{p_F}(1-2\varepsilon_I)$. Because this implies $\varepsilon_I > \varepsilon_F$, and because informal reports are decisive in this setting, $\frac{v_h-w}{w-v_l} \in \left(\frac{1-s}{s}\frac{\varepsilon_I}{1-\varepsilon_I}, \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}\right)$, it must be the case that formal reports are also decisive in this setting, $\frac{v_h-w}{w-v_l} \in \left(\frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}, \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}\right)$. This guarantees that selective hiring is optimal in the formal channel. It also means that selective hiring would be optimal in the informal channel, given a neutral pool factor. By Lemma 13 this will be the case when the condition $\varepsilon_I - \varepsilon_F \geq \frac{1-p_F}{p_F}(1-2\varepsilon_I)$ holds with equality, whereas the informal pool composition will be unfavourable when the inequality is strict. However, condition (3.24) ensures that this pool is not so unfavourable as to prevent the firm from being willing to accept high-report applicants. Again, the non-negativity condition $\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I) > \gamma'(0)$ ensures non-zero arrival in the informal channel, and a full proof is presented in Appendix B.

Proposition 9. *Suppose $\frac{v_h-w}{w-v_l} \in \left(\frac{1-s}{s}\frac{\varepsilon_I}{1-\varepsilon_I}, \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}\right)$ and $\varepsilon_I - \varepsilon_F < \frac{1-p_F}{p_F}(1-2\varepsilon_I)$. There are the following three possible kinds of equilibria with non-trivial use of the informal channel:*

(a) $(d^*_F, d^*_I) = (0, 1)$ and $p^*_{Ih} = \psi(\hat{w}(1-\varepsilon_I))$, $p^*_{I\ell} = \psi(\hat{w}\varepsilon_I)$.

This equilibrium exists if and only if $\hat{w}\varepsilon_I > \gamma'(0)$, $\frac{v_h-w}{w-v_l} \leq \frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}$, and

$$\Lambda^{01} \equiv \frac{\psi(\hat{w}\varepsilon_I)}{\psi(\hat{w}(1-\varepsilon_I))} \geq \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}. \tag{3.25}$$

(b) $(d^*_F, d^*_I) = (1, 1)$ and $p^*_{Ih} = \psi(\hat{w}(1 - p_F(1 - \varepsilon_F))(1 - \varepsilon_I))$, $p^*_{I\ell} = \psi(\hat{w}(1 - p_F\varepsilon_F)\varepsilon_I)$.

This equilibrium exists if and only if $\hat{w}(1 - p_F\varepsilon_F)\varepsilon_I > \gamma'(0)$, $\frac{v_h-w}{w-v_l} \in \left[\frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}, \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}\right]$, *and*

$$\Lambda^{11} \equiv \frac{\psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)}{\psi(\hat{w}(1-p_F(1-\varepsilon_F)(1-\varepsilon_I))} \geq \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}. \tag{3.26}$$

(c) $(d_F^*, d_I^*) = (2, 1)$ *and* $p_{Ih}^* = \psi(\hat{w}(1-p_F)(1-\varepsilon_I))$, $p_{I\ell}^* = \psi(\hat{w}(1-p_F)\varepsilon_I)$.

This equilibrium exists if and only if $\hat{w}(1-p_F)\varepsilon_I > \gamma'(0)$, $\frac{v_h-w}{w-v_l} \geq \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}$, *and*

$$\Lambda^{21} \equiv \frac{\psi(\hat{w}(1-p_F)\varepsilon_I))}{\psi(\hat{w}(1-p_F)(1-\varepsilon_I))} \geq \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}. \tag{3.27}$$

In this setting, informal report errors are either smaller than or not too much greater than formal report errors, and informal reports are decisive. In each part (a), (b) and (c), the conditions on the relative gain ratio $\frac{v_h-w}{w-v_l}$ in relation to the low-quality odds for low-report formal applications, $\frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}$, ensure that the formal hiring strategy in each case is optimal. In (a) the industry is generally unprofitable and formal reports are not decisive, which corresponds to no formal hiring, $d_F^* = 0$; in (b) formal reports are decisive, which corresponds to selective formal hiring, $d_F^* = 1$; and in (c) the industry is generally unprofitable and formal reports are not decisive, which corresponds to absolute formal hiring, $d_F^* = 2$.

Now in each of these cases, the informal pool composition is favourable. We can see from Table 3.1 that $\Lambda^{01} < 1$ and $\Lambda^{21} < 1$, while $\Lambda^{11} < 1$ by Lemma 13 because $\varepsilon_I - \varepsilon_F < \frac{1-p_F}{p_F}(1-2\varepsilon_I)$. Since informal reports are decisive, we know the firm would be willing to hire selectively given a neutral pool, and therefore it is willing to hire selectively given a favourable pool, provided that it is not too favourable (otherwise the firm would deviate to absolute informal hiring). Therefore the requirement in each part of this proposition that the informal pool adjustment factor be not too high ensures that selective informal hiring $d_I^* = 1$ is optimal. In these equilibria, use of the informal channel must also not be prohibitively costly. This is assured for low-type workers by the conditions $\hat{w}\varepsilon_I > \gamma'(0)$, $\hat{w}(1-p_F\varepsilon_F)\varepsilon_I > \gamma'(0)$, and $\hat{w}(1-p_F)\varepsilon_I > \gamma'(0)$ in (a), (b) and (c) respectively. Since $p_\ell(i, 1) < p_h(i, 1)$ for $i \in \{0, 1, 2\}$ these conditions assure non-zero arrival by high-type workers as well.

Note that when $\frac{v_h - w}{w - v_l} \geq \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}$, it is impossible to satisfy the conditions that Λ^{11} or Λ^{21} are "low enough" given that arrival is strictly costly.

Qualitatively, in the equilibria described by Propositions 8 and 9 informal reports are decisive and informal applications have primary value as a "report." Here the informal channel has value to the firm not simply as a method of connecting with unemployed workers, but as a way of helping the firm improve the quality of hires (relative to hiring randomly from the general population). This is possible due to the fact that direct information conveyed by informal reports sufficiently indicates quality to the firm.[9] These propositions show that when informal reports are not too weak, informal applications may have primary value as a door while formal applications have primary value as a door, as a report, or have no value (and are not used) at all, depending on the level of general profitability of the industry. In each of these cases, because the firm makes use of report information, high-quality workers have more incentive to use the informal channel than low-quality workers. This means that although the firm is already willing to hire high-report applicants on the basis of the report information alone, informal applications give additional benefit to the firm by increasing the expected profit per informal hire. As in the one-channel model, this function of the informal channel requires that the informal pool of applicants be not too favourable. On the other hand, when informal reports are sufficiently weak relative to formal reports, informal applications may have primary value as a door only while formal applications have primary value as a report. In this case, due to the strong superiority of formal reports, high-quality workers may have less incentive to use the informal channel than low-quality workers. In such a situation informal applications somewhat reduce the expected profit per informal hire, although the firm sufficiently benefits from the direct information of the reports to remain willing to hire high-report applicants. In contrast with the one-channel model, informal applications can function as a report in this situation only if the pool of informal applicants is not too unfavourable.

[9] In fact, the industry may be generally unprofitable in these propositions. In that case the only way connection to workers is valued is if such connection comes in conjunction with report information.

The following characterizes the existence of equilibria with non-zero informal arrival and selective informal hiring when informal reports are not decisive and the industry is generally unprofitable:

Proposition 10. *Suppose* $\frac{v_h - w}{w - v_l} \leq \frac{1-s}{s}\frac{\varepsilon_I}{1-\varepsilon_I}$. *There are the following two kinds of equilibria with non-trivial use of the informal channel:*

(a) $(d_F^*, d_I^*) = (0, 1)$ *and* $p_{Ih}^* = \psi(\hat{w}(1-\varepsilon_I))$, $p_{I\ell}^* = \psi(\hat{w}\varepsilon_I)$.

This equilibrium exists if and only if $\hat{w}\varepsilon_I > \gamma'(0)$, $\frac{v_h - w}{w - v_l} \leq \frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}$, *and*

$$\Lambda^{01} \equiv \frac{\psi(\hat{w}\varepsilon_I)}{\psi(\hat{w}(1-\varepsilon_I))} \in \left[\frac{v_h - w}{w - v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}, \frac{v_h - w}{w - v_l}\frac{s}{1-s}\frac{1-\varepsilon_I}{\varepsilon_I}\right]. \tag{3.28}$$

(b) $(d_F^*, d_I^*) = (1, 1)$ *and* $p_{Ih}^* = \psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I))$, $p_{I\ell}^* = \psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)$.

This equilibrium exists if and only if $\hat{w}(1-p_F\varepsilon_F)\varepsilon_I > \gamma'(0)$, $\frac{v_h - w}{w - v_l} \in \left[\frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}, \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}\right]$, *and*

$$\Lambda^{11} \equiv \frac{\psi(\hat{w}(1 - p_F\varepsilon_F)\varepsilon_I)}{\psi(\hat{w}(1-p_F(1-\varepsilon_F)(1 - \varepsilon_I))} \in \left[\frac{v_h - w}{w - v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}, \frac{v_h - w}{w - v_l}\frac{s}{1-s}\frac{1-\varepsilon_I}{\varepsilon_I}\right]. \tag{3.29}$$

In this setting, the industry is generally unprofitable and reports are (at least weakly) not decisive. In (a) formal reports are also (at least weakly) not decisive and the industry is generally unprofitable, therefore $d_F^* = 0$ is optimal for the formal channel; whereas in (b) the formal report is (at least weakly) decisive so $d_F^* = 1$ is optimal in the formal channel. Since informal reports are not decisive and the industry is generally unprofitable, the firm would not be willing to hire any applicants given a neutral informal pool of applicants. However, in both cases the pool of informal applicants must be favourable. Therefore the informal pool adjustment factor must be sufficiently favourable to induce the firm to accept high-report applicants, such that $\frac{v_h - w}{w - v_l} \geq O(\ell{:}h|A_I \cap H)$. However in order for the firm to hire selectively in the informal channel, the pool must also not be too favourable, such that low-report applicants are rejected,

$\frac{v_h - w}{w - v_l} \leq O(\ell{:}h|A_I \cap L)$. Therefore in each case the pool adjustment factor Λ^{01} or Λ^{11}, must not be too high, but must also be high enough.

Qualitatively, in the equilibria described by Proposition 10 the industry is generally unprofitable, informal reports are not decisive and informal applications have primary value as a "signal." Here successful functioning of the informal channel relies on the ability of endogenous networking to generate a superior quality composition for the pool of informal applicants. This does not mean that all informal applicants are hired indiscriminately, in fact as shown by Lemma 12, an endogenously superior pool of applicants can not be maintained if all informal applications are accepted. However, by using its informal screening technology to selectively hire only high-report informal applicants, the firm enables high-quality applicants to use the informal channel as a means of further "signalling" their quality through application arrival. As found in the one-channel model, signal use of the informal channel requires a sufficiently favourable pool of applicants, but also the pool of applicants must not be too favourable. We see that informal applications can have primary value as a signal when formal reports have primary value as a report or when formal applications are not used at all. However, informal applications can not have primary value as a signal when formal applications have primary value as a door.

Remark 3. *In a generally unprofitable industry where formal reports are decisive, it is impossible to sustain equilibrium use of the informal channel when informal reports are not decisive and the informal report error is too great.*

Note that in a generally unprofitable industry, having decisive formal reports implies $\frac{v_h - w}{w - v_l} > \frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}$, and having non-decisive informal reports implies $\frac{v_h - w}{w - v_l} \leq \frac{1-s}{s}\frac{\varepsilon_I}{1-\varepsilon_I}$. For a relative gain ratio in this range, only Proposition 10(b) is relevant for existence of equilibria with use of the informal channel, and such equilibria can not be sustained when $\varepsilon_I - \varepsilon_F \geq \frac{1-p_F}{p_F}(1-2\varepsilon_I)$. In the one-channel model, mixed strategy Nash equilibria were considered, and shown in some cases to be able to close the "gaps" in the regions of existence of pure strategy Nash equilibria, for example in regions where selective hiring could not be sustained because the favourable pool would cause the firm to deviate to absolute hiring but absolute hiring could not be sustained because the neutral pool would cause the firm to deviate to selective hiring. Although mixed strategy equilibria have not been presented for the two-channel model, it can be shown that the above remark holds true even when allowing for mixed strategy Nash equilibria.

Finally, the informal channel may be used in equilibrium with selective hiring in the presence of an unfavourable pool.

Proposition 11. *Suppose* $\frac{v_h - w}{w - v_l} \geq \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}$. *There is only one possible equilibrium with* $d_I^* = 1$ *and non-trivial use of the informal channel:*

$$(d_F^*, d_I^*) = (1,1) \text{ and } p_{Ih}^* = \psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I)),\ p_{I\ell}^* = \psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I).$$

This equilibrium exists if and only if $\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I) > \gamma'(0)$, $\varepsilon_I - \varepsilon_F > \frac{1-p_F}{p_F}(1-2\varepsilon_I)$, $\frac{v_h-w}{w-v_l} \leq \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}$, *and*

$$\Lambda^{11} \equiv \frac{\psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)}{\psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I))} \in \left[\frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}, \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{1-\varepsilon_I}{\varepsilon_I}\right]. \tag{3.30}$$

Here informal reports are not decisive and the industry is generally profitable. In order to support selective informal hiring in this setting, the informal pool adjustment factor must be sufficiently unfavourable that $\frac{v_h-w}{w-v_l} \geq O(\ell{:}h|A_I \cap L)$. An unfavourable pool factor, that is $p_{I\ell}^* > p_{Ih}^*$, can only be supported with selective informal hiring for $d_F^* = 1$ (as reviewed in Table 3.1), and requires $\varepsilon_I > \varepsilon_F + \frac{1-p_F}{p_F}(1-2\varepsilon_I)$, while selective formal hiring itself requires $\frac{v_h-w}{w-v_l} < \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}$. The pool adjustment factor Λ^{11} must also not be too unfavourable otherwise the firm will deviate to no hiring in the informal channel, $d_I^* = 0$. A formal proof is given in Appendix B.

Qualitatively this equilibrium shares some similarities to the equilibria with absolute hiring in Proposition 7. The informal channel is used despite the fact that informal reports are not decisive, because the industry is generally profitable and the informal channel offers a way for the firm and workers to connect. As in Proposition 7(b), the informal channel is also used despite its quality composition being worse than that of the general population. However, in this equilibrium the firm also uses the informal report to “sift” applicants, which requires that the unfavourable pool effect is strong enough to make absolute informal hiring unattractive. This may seem to contradict the existence of an equilibrium with $(d_F^*, d_I^*) = (1,2)$ and $p_{I\ell}^* > p_{Ih}^* > 0$ observed under similar conditions in Proposition 7. It is worth noting that equilibria of these two types may in fact coexist. The following Lemma establishes that the region in which these two equilibria can exist is the only region in which two equilibria with non-zero use of the informal channel may coexist within an open set. However, recall that the pool adjustment factor does not exogenously determine whether $d_I^* = 1$ or $d_I^* = 2$ is optimal for the firm, rather, both firm strategies can be compatible in equilibrium with the pool compositions they induce.

Lemma 18. *Suppose there exists an equilibrium with non-trivial use of the informal channel. If* $\frac{v_h - w}{w - v_l} \notin \left(\frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}, \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}\right]$ *and* $\frac{v_h - w}{w - v_l} \neq \frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}$*, then this equilibrium is the unique equilibrium with non-trivial use of the informal channel.*

Proof. Consider first the case where $\frac{v_h - w}{w - v_l} \geq \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}$. Since non-zero use of the informal channel in equilibrium requires $d_I^* \in \{1,2\}$, Propositions 7 and 11 cover all possible non-trivial equilibria in this setting. For $\frac{v_h - w}{w - v_l} \notin \left(\frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}, \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}\right]$, only the conditions of Proposition 7(a) can be satisfied, so the equilibrium must have $(d_F^*, d_I^*) = (2,2)$. Since there can be at most one non-zero best response for workers to this hiring strategy, this equilibrium is unique.

Now consider the case where $\frac{v_h - w}{w - v_l} < \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}$. Then by Lemma 14, any equilibrium with non-trivial use of the informal channel must have $d_I^* = 1$. Given that $\frac{v_h - w}{w - v_l} \neq \frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}$ and $\frac{v_h - w}{w - v_l} \neq \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}$, the firm can not be indifferent in regard to any application received in the formal channel, because no two conditions in Lemma 9 can be satisfied simultaneously. Since there can only be one firm strategy which is optimal in the formal channel, and $d_I^* = 1$ must be optimal in the informal channel, and since there can be at most one non-zero best response for workers to this firm strategy, this equilibrium is unique. □

3.4 Additional Analyses

This section presents comparative statics and welfare results for the two-channel model, and discusses the implications of the assumption that the firm delegates hiring to independent departments.

In Section 3.4.1 I find complementarity as in the one-channel model between the informal screening technology and the informal pool of applicants, while improvements to the formal screening technology have an unfavourable effect on the informal pool. I also find that the two-channel model exhibits generally non-monotonic relationships between equilibrium outcomes and parameters, such that equilibrium predictions and welfare effects are sensitive to parameter settings and the structure of arrival costs, $\gamma(p)$. Accordingly, Chapter 4 will revisit this analysis for particular cases where the structure of the relationship between cost and informal arrival probability is based on networking activities. Section 3.4.2 discusses the effect of the delegation assumption on firm profits, and Section 3.4.3 compares equilibrium outcomes under alternative delegation and centralized hiring assumptions.

3.4.1 Comparative Statics and Welfare

As previously, I will restrict attention to non-trivial use of the informal channel in equilibrium. In accordance with Observation 2 from Section 3.2.2, I will classify equilibria with non-zero arrival according to the firm strategy used. Within a given class of equilibria with firm strategy $(d_F^*, d_I^*) = (i, j) \in \{0, 1, 2\} \times \{1, 2\}$, recall that the pool of informal applicants will be given by $\Lambda^{ij} \equiv \dfrac{\psi(\hat{w}(1-p_F\phi_{F\ell}^*)\phi_{I\ell}^*)}{\psi(\hat{w}(1-p_F\phi_{Fh}^*)\phi_{Ih}^*)}$, where $\phi_{Ah}^* = (1-\varepsilon)d_{AH}^* + \varepsilon d_{AL}^*$ and $\phi_{A\ell}^* = \varepsilon d_{AH}^* + (1-\varepsilon)d_{AL}^*$ for $A \in \{F, I\}$.

As in the one-channel model, the pool of informal applicants will improve if there is an improvement in the firm's ability to screen applicants in the informal channel (at least weakly; if the firm does not make use of informal reports, $d_I^* \neq 1$, a change in screening ability is irrelevant within that class of equilibria). However, a change in formal report error has the opposite effect on the informal pool of applicants. This is because the formal channel is like an outside option for workers which is more attractive to high types than low types the lower its report error. Thus we have the following result in the two-channel model:

Lemma 19. *Within a given class of equilibria with firm strategy* $(d_F^*, d_I^*) = (i, j) \in \{0, 1, 2\} \times \{1, 2\}$*, it is the case that* $\frac{d\Lambda^{ij}}{d\varepsilon_I} \geq 0$ *and* $\frac{d\Lambda^{ij}}{d\varepsilon_F} \leq 0$*.*

As in the one-channel model, for equilibria within a fixed firm strategy, improvements in the firm's ability to screen through informal reports are beneficial to the firm and high-quality workers, but not for low-quality workers. However, the same can not be said for improvements in the firm's formal channel screening ability.

Lemma 20. *Firm profits may be increasing or decreasing in* ε_F*.*

It is intuitive that profit could decrease as a result of weaker formal screening technology, because an increase in ε_F will lower the expected profits from hires made through the formal channel. However, an increase in the formal report error will also improve the pool of applicants in the informal channel, because a reduction in the advantage of high-quality workers in the formal channel and increases their incentive to invest in informal application. Thus the overall effect on profits of an increased formal report error will depend on the size of these two effects. There can also be another effect which arises due to the separate objectives of the two channels, because both channels are aiming to maximize the expected profits from their own acceptances rather than trying to maximize the overall profit of the firm. It is possible that a change in formal report error can exacerbate or reduce the loss of profits from this delegation effect, when it exists. The extent to which this delegation issue affects the firm's profits in general is discussed in Section 3.4.2.

Lemma 21. *The equilibrium pool of informal applicants may worsen or improve with an increase in the wage;* $\frac{d\Lambda^*}{dw} \gtrless 0$ *if and only if*

$$\frac{\psi(\hat{w}(1-p_F\phi_{Fh})\phi_{Ih})}{\psi'(\hat{w}(1-p_F\phi_{Fh})\phi_{Ih})\cdot(1-p_F\phi_{Fh})\phi_{Ih}} \gtrless \frac{\psi(\hat{w}(1-p_F\phi_{F\ell})\phi_{I\ell})}{\psi'(\hat{w}(1-p_F\phi_{F\ell})\phi_{I\ell})\cdot(1-p_F\phi_{F\ell})\phi_{I\ell}}. \tag{3.31}$$

As in the one-channel model, increasing the wage may or may not improve the pool of applicants. Condition (3.31) is very similar to the condition given in Lemma 2.38, except that in this two-channel model it is not always the case that $(1-p_F\phi_{Fh})\phi_{Ih} \geq (1-p_F\phi_{F\ell})\phi_{I\ell}$. Therefore in the two-channel model it may be possible for $\frac{\psi}{\psi'}$ to be decreasing (ψ logarithmically concave) and still have the informal pool improve with an increase in wage, if the equilibrium is such that pool of informal applicants is unfavourable. Thus $\frac{\psi}{\psi'}$ decreasing implies that the informal pool becomes worse with the wage in equilibria with $p^*_{Ih} > p^*_{I\ell}$, such as when informal report errors are lower than formal report errors, and that it improves with the wage in equilibria with $p^*_{Ih} < p^*_{I\ell}$.

The influence of the wage on profits will comprise multiple effects, as in the one-channel model. First, there is a cost of labour effect due to the fact that every hire must be paid the different wage. This effect is negative and will be present when workers are hired through the formal channel, while this and two additional effects can be present when workers are hired through the informal channel. There is a volume effect because the overall number of workers hired is affected by a change in wage, since it changes workers' arrival incentives. This effect will be generally positive, since the firm will generally make positive profit when it is willing to hire, thus making additional volume desirable. There is also a pool composition effect, because the change in wage may not affect high- and low-quality workers' arrival incentives identically. This effect may be positive or negative because as previously shown, an increase in wage can improve or degrade the composition of the pool of applicants. With two channels in operation, a delegation effect may also exist when the wage changes. As will be discussed in Section 3.4.2, the separate objectives of the formal and informal hiring offices can sometimes lead to over-hiring and reduced profit for the firm. A change in wage and its subsequent effect on worker arrival probabilities can affect the degree to which the delegation issue causes actual hiring to be higher than optimal. Now although the effect of wages on firm profit is unclear, for both worker types higher wages are beneficial provided that the equilibrium hiring strategy of the firm remains the same.

An increase in b is always good for workers, but it will cause both types to reduce their effort towards informal applications which may or may not be in such a way as to improve the composition of pool of informal applicants. A change in b will have the exact opposite effect on the pool as a change in the wage, with $\frac{d\Lambda^*}{db} \gtrless 0$ if and only if $\frac{d\Lambda^*}{dw} \lessgtr 0$. The firm is not influenced directly by changes in the outside option of workers, but its profits will be affected to the extent that a change in b alters the volume and quality composition of its hires through the informal channel.

Lemma 22. *Firm profit may be increasing or decreasing in b, with* $\frac{d\pi^*}{db} \gtrless 0$ *if and only if*

$$\frac{v_h - w}{w - v_l} \lessgtr \frac{1-s}{s} \cdot \frac{(1-p_F\phi_{F\ell})^2\phi_{I\ell}^2}{(1-p_F\phi_{Fh})^2\phi_{Ih}^2} \cdot \frac{\psi'(\hat{w}(1-p_F\phi_{F\ell})\phi_{I\ell})}{\psi'(\hat{w}(1-p_F\phi_{Fh})\phi_{Ih})}. \tag{3.32}$$

To illustrate this result, in the case of industries with high general profitability such that $(d_F, d_I) = (2, 2)$, workers have identical incentives to use the informal channel, so that we have $(1-p_F\phi_{Fh})\phi_{Ih} = (1-p_F\phi_{F\ell})\phi_{I\ell}$. Therefore the firm's profit decreases with an increase in b due to the reduced volume of workers, and there is no effect through the composition of the pool of applicants.

Similarly, although the firm is not influenced directly by changes in the rate of accessibility of the formal channel, p_F, an increase in p_F will affect firm profits through the volume of informal applicants and the quality composition of the informal pool.

Lemma 23. *An increase in p_F can worsen or improve the informal pool of applicants, with*

$\dfrac{d\Lambda^*}{dp_F} \gtrless 0$ *if and only if*

$$\frac{\psi(\hat{w}(1-p_F\phi_{Fh})\phi_{Ih})}{\psi'(\hat{w}(1-p_F\phi_{Fh})\phi_{Ih})\cdot\phi_{Fh}\phi_{Ih}} \gtrless \frac{\psi(\hat{w}(1-p_F\phi_{F\ell})\phi_{I\ell})}{\psi'(\hat{w}(1-p_F\phi_{F\ell})\phi_{I\ell})\cdot\phi_{F\ell}\phi_{I\ell}}. \tag{3.33}$$

Since $\phi_{Fh}\phi_{Ih} \geq \phi_{F\ell}\phi_{I\ell}$, improved access to the formal channel will necessarily improve the informal pool of applicants when $\frac{\psi}{\psi'}$ is decreasing. However, if $\frac{\psi}{\psi'}$ is increasing the pool may improve or worsen with an increase in p_F. An increase in the formal arrival probability is beneficial for workers but decreases their incentive to invest effort in informal applications. Depending on the effect of the volume change and to what extent it is accompanied by an improvement or degradation of the informal pool composition, a change in p_F can either increase or decrease firm profits. For industries with very high general profitability, such that $(d_F, d_I) = (2, 2)$ and $\phi_{Fh} = \phi_{Ih} = \phi_{F\ell} = \phi_{I\ell} = 1$, an increase in p_F has no effect on the composition of the informal pool of applicants. In this case only the change in the volume of workers matters for the firm's profit and $\frac{d\pi^*}{dp_F} > 0$.

3.4.2 Delegation and Profits

As previously discussed, the hiring decision for each channel is made with the objective of maximizing the expected profit from hires made through that channel, rather than the objective of maximizing the firm's overall profit. The expected profit from all workers accepted by the formal channel is given by

$$\pi_F(d_F) = (v_h - w)s\cdot p_F\phi_{Fh}(d_F) - (w - v_\ell)(1-s)\cdot p_F\phi_{F\ell}(d_F) \tag{3.34}$$

and for the informal channel this expected profit is given by

$$\pi_I(d_F, d_I) = (v_h - w)s\cdot p_{Ih}(d_F, d_I)\phi_{Ih}(d_F, d_I) - (w - v_\ell)(1-s)\cdot p_{I\ell}(d_F, d_I)\phi_{I\ell}(d_F, d_I). \tag{3.35}$$

Since a worker who happens to be accepted through both channels is hired only once, the actual overall profit of the firm $\pi(d_F, d_I)$ is not equal to $\pi_F(d_F) + \pi_I(d_F, d_I)$ because this would result in double-counting profit in an amount equal to

$$\begin{aligned}\pi_{FI}(d_F, d_I) = (v_h{-}w)s{\cdot}p_F\phi_{Fh}(d_F){\cdot}p_{Ih}(d_F, d_I)\phi_{Ih}(d_I)\\ - (w{-}v_\ell)(1{-}s){\cdot}p_F\phi_{F\ell}(d_F){\cdot}p_{I\ell}(d_F, d_I)\phi_{I\ell}(d_I).\end{aligned} \tag{3.36}$$

This reflects the fact that from the perspective of the firm's profit, an increase in expected profit from one channel's hires is only beneficial if it reflects "new" workers being hired, and not workers which the firm can already gain profit from through the other channel's hires. This means that although the delegation of hiring to channels with these separate objectives may sometimes lead to the same hiring decisions as would be reached with profit-maximization as the common objective, it can sometimes lead to over-hiring. For example, suppose one channel has high arrival rates and is very good at distinguishing quality, so that it is highly likely to accept a great proportion of the high-quality workers in the population. Leaving each channel to hire based on its own separate objective, the firm may not attain profit as high as it would if it could induce the one channel to take into account the other channel's likely hires. In some cases this over-hiring problem may be severe enough that the firm would be better off with only one hiring channel in operation.

Suppose that the firm has an established formal channel hiring process with $d_F \in \{1, 2\}$. It will be beneficial for the firm to also operate the informal channel with $d_I \in \{1, 2\}$ if and only if $\pi_I - \pi_{FI} > 0$. After some algebra, this means it is beneficial to also operate the informal channel with $d_I \in \{1, 2\}$ if and only if

$$\frac{v_h{-}w}{w{-}v_\ell} > \frac{1{-}s}{s} \cdot \frac{1 - p_F\phi_{F\ell}(d_F)}{1 - p_F\phi_{Fh}(d_F)} \cdot \frac{p_{Ih}(d_F, d_I)}{p_{I\ell}(d_F, d_I)} \cdot \frac{\phi_{I\ell}(d_I)}{\phi_{Ih}(d_I)}. \tag{3.37}$$

Lemma 24. *When* $\frac{v_h{-}w}{w{-}v_\ell} > \frac{1{-}s}{s} \cdot \frac{1{-}\varepsilon_I}{\varepsilon_I}$*, the informal channel is beneficial to add, that is,* $\pi_I - \pi_{FI} > 0$.

For $d_F = 2$, the formal channel offers both high- and low-quality workers an equally valuable outside option, so $\dfrac{1 - p_F\phi_{F\ell}(d_F)}{1 - p_F\phi_{Fh}(d_F)} = \dfrac{1 - p_F}{1 - p_F} = 1$. Thus on one hand, whenever the informal channel would find it strictly optimal to hire selectively, $d_I = 1$, that is when $\frac{v_h - w}{w - v_\ell} > \frac{1-s}{s} \cdot \frac{p_{Ih}(d_F, d_I)}{p_{I\ell}(d_F, d_I)} \cdot \frac{\varepsilon_I}{1-\varepsilon_I}$ the inequality (3.37) will be satisfied. On the other hand, the inequality (3.37) will also be satisfied whenever the informal channel would find it strictly optimal to hire absolutely, $d_I = 2$. This is because for $(d_F^*, d_I^*) = (2, 2)$, inequality (3.37) reduces to $\frac{v_h - w}{w - v_\ell} > \frac{1-s}{s}$, which must be satisfied under the assumption $\frac{v_h - w}{w - v_\ell} > \frac{1-s}{s} \cdot \frac{1-\varepsilon_I}{\varepsilon_I}$. Thus in any generally profitable industry where formal reports are not accurate enough to be decisive, so that the formal channel hires absolutely, the addition of the informal channel is beneficial.

However, for industries such that formal reports are decisive, such that $d_F = 1$, it is possible that if the industry is sufficiently generally unprofitable, the over-hiring due to delegation may cause the additional use of the informal channel to result in lower profits for the firm. If used, the informal channel will hire absolutely, $d_I = 2$, when

$$\frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s} \cdot \frac{1-\varepsilon_I}{\varepsilon_I}. \tag{3.38}$$

In this case, inequality (3.37) is equivalent to

$$\frac{v_h - w}{w - v_\ell} > \frac{1-s}{s} \cdot \frac{1 - p_F\varepsilon_F}{1 - p_F(1-\varepsilon_F)} \cdot \frac{\varepsilon_I}{1-\varepsilon_I}. \tag{3.39}$$

Adding the informal channel will therefore be beneficial if the equilibrium is such that high-quality workers network more that low-quality workers, that is, when $\varepsilon_I - \varepsilon_F < \frac{1-p_F}{p_F}(1-2\varepsilon_I)$, because in that case $\dfrac{1 - p_F\varepsilon_F}{1 - p_F(1-\varepsilon_F)} \cdot \dfrac{\varepsilon_I}{1-\varepsilon_I} < \dfrac{1-\varepsilon_I}{\varepsilon_I}$. But when low-quality workers network more, this may not be the case. Alternatively, when

$$\frac{v_h - w}{w - v_\ell} \in \left[\frac{1-s}{s} \cdot \frac{p_{Ih}(d_F, d_I)}{p_{I\ell}(d_F, d_I)} \cdot \frac{\varepsilon_I}{1-\varepsilon_I}, \frac{1-s}{s} \cdot \frac{p_{Ih}(d_F, d_I)}{p_{I\ell}(d_F, d_I)} \cdot \frac{1-\varepsilon_I}{\varepsilon_I}\right] \tag{3.40}$$

the informal channel will hire selectively, $d_I = 1$, if used. In this case, inequality 3.37 is equivalent to

$$\frac{v_h - w}{w - v_\ell} > \frac{1-s}{s} \cdot \frac{1 - p_F \varepsilon_F}{1 - p_F(1-\varepsilon_F)} \cdot \frac{p_{Ih}(d_F, d_I)}{p_{I\ell}(d_F, d_I)} \cdot \frac{\varepsilon_I}{1-\varepsilon_I} \tag{3.41}$$

and since $\frac{1 - p_F \varepsilon_F}{1 - p_F(1-\varepsilon_F)} > 1$ for all $\varepsilon_F < \frac{1}{2}$, it will be the case that $\pi_I - \pi_{IF} < 0$ for industries with sufficient low general profitability (such that the relative gain from a high-quality worker is close enough to the left boundary of the interval in condition (3.40)) and the firm would be better not to use the informal channel due to this delegation issue.

3.4.3 Delegation with Profit Maximization and Centralized Hiring

As mentioned previously, the most profitable hiring structure for the firm (ignoring implementation and organizational costs) would be obtained through complete centralization of the hiring process. For each worker, the firm would receive either no formal report or a formal report which suggests high or low quality, and in addition either no informal report or an informal report which suggests high or low quality. The firm would fully condition the odds of the worker being low-quality on this combined information in best response to the workers' arrival probabilities, taking the resulting informal pool composition into account as well.

Alternatively, if implementing such a centralized hiring process would be costly or difficult, or if the firm finds itself in need of hiring expertise, the firm may choose to delegate either the formal or informal hiring decisions, or both. In the model here, the firm has delegated the decision to two separate hiring offices which aim to maximize the expected profit of their own hiring recommendations. Therefore although from the firm's perspective this results in overhiring, in some cases it may be impossible to align each department's independent objective with the combined true profit of the firm. However, if it is possible, then instead of independent objectives in the delegation model, the department hiring decisions could be profit-maximizing for the firm. In a general model where the firm delegates hiring to two offices with a profit-maximizing objective, there may be more than one combination of hiring strategies (d_F, d_I) which lead to the same level of overall firm profit, and it is also possible that multiple equilibria exist where the firm may achieve higher or lower levels of profit in equilibrium, depending on how the offices coordinate.

Now suppose the firm needed outside expertise in interpreting only one type of report and were able to handle hiring through the other channel itself. Then the firm may be able to make an appropriate adjustment to avoid any profit loss from delegation if the other delegated channel makes its hiring decision first. For example, it may be that the formal channel is outsourced. Then, knowing that d_F will be chosen by the delegate in order to maximize π_F as in equation (3.34), when choosing d_I the firm could maximize $\pi_I - \pi_{FI}$ as in equation (3.36) given d_F, p_{Ih}, and $p_{I\ell}$ rather than maximizing π_I as in equation (3.35), and this would resolve the problem of over-hiring. If the firm were able to align the objectives of both departments, joint profit maximization could also be achieved through coordination of the hiring strategies of the two departments.

Adapting the model for delegation to separate offices with a common profit-maximizing objective is straightforward and tractable in the case where the formal hiring office makes its hiring decision first and the informal channel follows. The formal department will use the same decision rule as in Lemma 9, while the informal department will use the same decision rule as in Lemma 10 whenever $d_F = 0$ or $d_F = 2$. Whenever $d_F = 1$, the informal department will need only a simple correction in the odds used in the cut-off decision rule in Lemma 10. Instead of considering the odds that an informal applicant is low-quality conditional only on arrival in the informal channel and the report received, the informal department must condition the odds on the event that the applicant has not been accepted through the formal channel. Thus in place of $O(\ell{:}h|A_I \cap R_I) = O(\ell{:}h) \cdot \Lambda(\ell{:}h|A_I) \cdot \Lambda(\ell{:}h|A_I \cap R_I)$ in conditions (3.5)-(3.8), the informal department will compare the relative gain from a high-quality worker, $\frac{v_h - w}{w - v_\ell}$, with

$$O(\ell{:}h|\,{A_F}^{\mathsf{c}} \cap A_I \cap R_I) = O(\ell{:}h) \cdot \Lambda(\ell{:}h|\,{A_F}^{\mathsf{c}}) \cdot \Lambda(\ell{:}h|\,{A_F}^{\mathsf{c}} \cap A_I) \cdot \Lambda(\ell{:}h|\,{A_F}^{\mathsf{c}} \cap A_I \cap R_I) \quad (3.42)$$

where ${A_F}^{\mathsf{c}}$ is the event that the worker is not accepted through the formal channel, $O(\ell{:}h)$ is the prior odds of low quality, and the updating likelihood ratios are $\Lambda(\ell{:}h|A_F{}^{\mathrm{c}}) = \dfrac{Pr(A_F{}^{\mathrm{c}}\,|\ell)}{Pr(A_F{}^{\mathrm{c}}|h)}$, $\Lambda(\ell{:}h|A_F{}^{\mathrm{c}} \cap A_I) = \dfrac{Pr(A_I|A_F{}^{\mathrm{c}} \cap \ell)}{Pr(A_I|A_F{}^{\mathrm{c}} \cap h)}$ and $\Lambda(\ell{:}h|A_F{}^{\mathrm{c}} \cap A_I \cap R_I) = \dfrac{Pr(R_I|A_I \cap A_F{}^{\mathrm{c}} \cap \ell)}{Pr(R_I|A_I \cap A_F{}^{\mathrm{c}} \cap h)}$. Since this adjustment is only made when the formal channel is hiring selectively, $d_F^* = 1$, we have $\dfrac{Pr(A_F{}^{\mathrm{c}}\,|\ell)}{Pr(A_F{}^{\mathrm{c}}|h)} = \dfrac{p_F(1-\varepsilon_F) + (1-p_F)}{p_F\varepsilon_F + (1-p_F)}$. Since arrival in the formal channel is independent of informal arrival and informal reports, we have $\dfrac{Pr(A_I|A_F{}^{\mathrm{c}} \cap \ell)}{Pr(A_I|A_F{}^{\mathrm{c}} \cap h)} = \dfrac{Pr(A_I|\ell)}{Pr(A_I|h)}$ and $\dfrac{Pr(R_I|A_I \cap A_F{}^{\mathrm{c}} \cap \ell)}{Pr(R_I|A_I \cap A_F{}^{\mathrm{c}} \cap h)} = \dfrac{Pr(R_I|A_I \cap \ell)}{Pr(R_I|A_I \cap h)}$ as before. Thus with profit maximizing delegation when the formal channel moves first, the only adaptation of the model presented is that when $d_F^* = 1$, the informal department uses the cutoff

$$\frac{1-s}{s} \cdot \frac{p_F(1-\varepsilon_F) + (1-p_F)}{p_F\varepsilon_F + (1-p_F)} \cdot \frac{p_{I\ell}}{p_{Ih}} \frac{\varepsilon_I}{1-\varepsilon_I} \tag{3.43}$$

in place of $\dfrac{1-s}{s}\dfrac{p_{I\ell}}{p_{Ih}}\dfrac{\varepsilon_I}{1-\varepsilon_I}$ in conditions (3.5) and (3.6), and the cutoff

$$\frac{1-s}{s} \cdot \frac{p_F(1-\varepsilon_F) + (1-p_F)}{p_F\varepsilon_F + (1-p_F)} \cdot \frac{p_{I\ell}}{p_{Ih}} \frac{1-\varepsilon_I}{\varepsilon_I} \tag{3.44}$$

in place of $\dfrac{1-s}{s}\dfrac{p_{I\ell}}{p_{Ih}}\dfrac{1-\varepsilon_I}{\varepsilon_I}$ in conditions (3.7) and (3.8). Since $\dfrac{p_F(1-\varepsilon_F) + (1-p_F)}{p_F\varepsilon_F + (1-p_F)} > 1$, correcting for formal channel hires increases the odds that any new applicant arriving to the informal channel is low-quality. So as anticipated, this profit-maximizing adjustment has a conservative influence on the informal department's hiring decisions, but the main qualitative predictions of the adjusted model are expected to be similar to the model analysed here.

Adapting the model for a completely centralized hiring process is more difficult and will not be done here. As an illustration of the difference in profit when the hiring decision is delegated versus centralized, I will compare the centralized outcome and delegation in the special case where formal reports are completely informative, $\varepsilon_F = 0$, and informal reports are completely noisy, $\varepsilon_I = \dfrac{1}{2}$. The firm's overall profit is equal to

$$\pi = (v_h - w)sPr(hire|q{=}h) - (w - v_\ell)(1-s)Pr(hire|q{=}\ell), \tag{3.45}$$

where $Pr(hire|q{=}h)$ and $Pr(hire|q{=}\ell)$ correspond to the probability that the firm actually hires high-quality workers and low-quality workers respectively.

In the centralized decision framework, there are nine possible disjoint events (three possibilities for each channel, namely that the report is high, is low, or does not arrive) which lead to four different outcomes. In any event where the firm receives a high formal report (regardless of what is or is not received in the informal channel), the firm will hire the applicant, and similarly in any event where the firm receives a low formal report it will not hire the applicant. This is because the formal report fully reveals the type of the worker. For high-quality workers high and low reports occur with the probability p_F and 0 respectively, and for low-quality workers these reports occur with the probability 0 and p_F. If the firm does not receive any formal report, but does receive an informal report, the firm may accept the applicant or not, but the decision will not be contingent on the informal report because it is perfectly noisy. A worker of type q will encounter this decision of the firm with probability $(1{-}p_F)p_{Iq}$. Therefore overall, this firm will hire high-quality workers with $Pr(hire|q{=}h) = p_F + (1{-}p_F)p_{Ih}{\cdot}z$ and low-quality workers with $Pr(hire|q{=}\ell) = (1{-}p_F)p_{I\ell}{\cdot}z$ where $z \in [0, 1]$ is the firm's decision of whether to hire applicants who reach the firm through the informal channel but not through the formal channel.

In contrast, suppose that the formal and informal offices are separate. The formal hiring office will accept all applications which arrive to it with a high report. This occurs for high-quality workers with a probability of p_F and for low-quality workers with a probability of 0. The formal hiring office will reject all applications which arrive to it with a low report, which occurs for high-quality workers with a probability of 0 and for low-quality workers with a probability of p_F. Meanwhile regardless of reports, the informal hiring office will either accept or reject all applicants which reach it. A worker of type q will encounter this decision of the firm with probability p_{Iq}. Let $y \in [0, 1]$ denote the informal office's decision of whether to accept informal applicants. Then in the case of delegation, $Pr(hire|q{=}h) = p_F + (1{-}p_F)p_{Ih}{\cdot}y$ and $Pr(hire|q{=}\ell) = p_{I\ell}{\cdot}y$.

When the firm chooses z in the centralized hiring office, it is known that the application did not arrive in the formal channel, whereas when a separate informal hiring office chooses y, it does not know whether or not the applicant has also arrived in the formal channel. If formal arrival probabilities do not differ by type, this will not affect the prior used. However, this does mean that some applicants who did arrive and had a low report in the formal channel may fail to be rejected by the informal channel. Now when the informal office makes its choice of y, if its objective is to maximize the profits from its own hires, it will care only about $d_F = 1$ to the extent that it affects p_{Ih} and $p_{I\ell}$. However, if the informal office's objective is to maximize π given $d_F = 1$, it should generally choose y lower than it would if taking into account only the arrival probabilities and population prior.

Note that choosing $z = y$ will result in identical p_{Ih} under both the centralized and decentralized scenarios, but in a (weakly) lower $p_{i\ell}$ under the centralized scenario. Then $Pr(hire \mid q = h)$ will be the same under the two scenarios but $Pr(hire \mid q = \ell)$ will be lower under the centralized scenario, resulting in higher profits in that case.

In the delegation model with private objectives, complementarity was found between informal screening technology and the pool of informal applicants, while improved formal screening technology had a negative effect on the informal pool of applicants. It is possible for this negative influence to persist under a centralized hiring process. For example, consider a situation in which it is optimal for the firm to accept applications which arrive only through one channel with a high report, while if an applicant reaches the firm through both channels it is optimal for the firm to hire if and only if at least one report is high. In this case we will have

$$Pr(hire|h) = p_F(1-p_{Ih})(1-\varepsilon_F) + p_F p_{Ih}(1-\varepsilon_F\varepsilon_I) + (1-p_F)p_{Ih}(1-\varepsilon_I) \tag{3.46}$$

and

$$Pr(hire|\ell) = p_F(1-p_{I\ell})\varepsilon_F + p_F p_{I\ell}(1-(1-\varepsilon_F)(1-\varepsilon_I)) + (1-p_F)p_{I\ell}\varepsilon_I. \tag{3.47}$$

In this setting workers can increase their chance of being hired by increasing their arrival probability, $\dfrac{\partial Pr(hire|h)}{\partial p_{Ih}} = (1-\varepsilon_I) - (1-\varepsilon_I)(1-\varepsilon_F)p_F > 0$ and $\dfrac{\partial Pr(hire|\ell)}{\partial p_{I\ell}} = \varepsilon_I - \varepsilon_I\varepsilon_F p_F > 0$. In the case of quadratic arrival costs, the optimal arrival for high-quality workers, p^*_{Ih}, will be proportional to $(1-\varepsilon_I) - (1-\varepsilon_I)(1-\varepsilon_F)p_F$ and the optimal arrival for low-quality workers, $p^*_{I\ell}$, will be proportional to $\varepsilon_I - \varepsilon_I\varepsilon_F p_F$. Thus an improvement in the formal screening technology will lead high-quality workers to decrease their arrival probability, $\dfrac{dp^*_{Ih}}{d\varepsilon_F} > 0$, while it will lead low-quality workers to increase their arrival probability, $\dfrac{dp^*_{I\ell}}{d\varepsilon_F} < 0$, leading to an unfavourable change in the relative arrival of high- and low-quality applicants through the informal channel, $\dfrac{d}{d\varepsilon_F}\left(\dfrac{p^*_{I\ell}}{p^*_{Ih}}\right) < 0$.

Chapter 4

Networking in the Informal Channel

In this chapter I model the relationship between cost and arrival probabilities more explicitly as the outcome of a networking process, in order to give structure to the arrival cost function and obtain sharper comparative static results in the one- and two-channel models of the previous chapters. The worker invests in networking, the intensity of which determines the probability of his application's arrival to the firm through the informal channel. I will consider in particular the probability arising from a situation where each contact in a worker's network gives access to the firm with equal independent probability.

I find that networking costs have an intensifying effect on the pool of informal applicants in such a model; when the informal pool is favourable, increased networking costs improve the informal pool composition further. The opposite is true for wages; when the informal pool is favourable, increased wages worsen the quality of informal applicants. These results hold also for any networking scenario with log-concave marginal probability of arrival.

Using this more explicit networking model I revisit the equilibrium results of the two-channel hiring model and provide additional comparative statics. Finally I discuss two examples with an alternative structure for networking-based informal arrival probabilities and the cases of type-dependent costs and type-dependent probabilities of application arrival.

4.1 Model of Network-Based Informal Arrival

4.1.1 Arrival Probabilities and Cost Structure of Networking

In this section I consider how the endogenous arrival of applications in the informal channel of the two-channel model presented in Chapter 3 may be network-based. That is, a type $q \in \{h, \ell\}$ worker's endogenous informal arrival probability p_{Iq}, and resulting incurred cost $\gamma(p_{Iq})$, may be the outcome of that worker's networking decision. First I discuss the networking scenario, then I show how workers' best responses and the pool of informal applicants depend on the cost parameter of the networking scenario.

I abstract from the graph theoretical approach, focusing on the effects of a worker's networking behaviour rather than modelling an explicit network of links. Suppose that the informal arrival probability p_{Iq} depends on a worker's network strength, n_q. I do not restrict n_q to whole values, so $n_q \in \mathbb{R}_+$ and the probability of the firm receiving a given worker's application informally is given by $p_{Iq} = P(n_q) \in [0, 1)$, with $P(0) = 0$. Therefore the arrival probabilities in the informal channel will be $p_{Ih} = P(n_h)$ and $p_{I\ell} = P(n_\ell)$. I will assume that $P(n)$ is thrice continuously differentiable.

I will focus primarily on the functional form $P(n) = 1 - e^{-\lambda n}$ for $\lambda > 0$. This functional form arises in models of endogenous network formation such as Calvo-Armengol (2004) and Galeotti and Merlino (2014). In my reduced-form setting, this specification can be motivated as follows. I will suppose network strength n corresponds to network size, such that n is the number of contacts (measured continuously) in a given network. Suppose that each contact fails to be useful with an independent probability which is identical for all contacts. This probability can be expressed as $(\frac{1}{e})^{\lambda}$, for some $\lambda > 0$. The probability that all of the contacts in a worker's network fail to convey his informal application to the firm is therefore given by $(\frac{1}{e})^{\lambda n}$, or $e^{-\lambda n}$ and the consequent probability that the worker's informal application does reach the firm is equal to $P(n) = 1 - e^{-\lambda n}$.[1]

[1]Alternatively, network strength could be defined as N with change of variables $N = \lambda n$, so that network

For this specification we can interpret the probability of a given worker's informal application reaching the firm as follows. Abstracting from the details of how individual contacts lead to connection with the firm, suppose that the minimum network size required to successfully connect with the firm through the informal channel is a random variable X which takes values on the range $\mathbb{R}_+$. The cumulative distribution function of this random variable represents the probability that the minimum network size has a value less than or equal to some given network size n, $P(X \leq n)$. Therefore the cumulative distribution function of X represents the informal arrival probability of a worker with network size n, so it is equal to the worker's informal arrival probability, $P(X \leq n) = P(n)$. The density of X is given by the marginal informal arrival probability $P'(n)$. The functional form $P(n) = 1 - e^{-\lambda n}$ corresponds to the special case where the random variable X follows an exponential distribution with rate parameter $\lambda > 0$.

This parameter gives the "hazard rate" for the exponential distribution, which is the ratio of the density to the "survival" function $Pr(X > n)$, that is $\dfrac{P'(n)}{1 - P(n)}$. This ratio represents the probability that, conditional on the failure of the worker's informal application to arrive given network size n, the next increment of networking will successfully connect the worker's informal application to the firm. The exponential distribution has the "memoryless" property and therefore this hazard rate is constant, $\dfrac{P'(n)}{1 - P(n)} = \lambda$. In the scenario above, although the marginal benefit of networking is decreasing, due to $P(n)$ strictly concave, each contact provides the same probability of helping a worker to reach the firm, regardless of that worker's existing network size. A worker with a large network who has not yet reached the firm informally has no more or less chance of success from his next (infinitesimal) contact than a worker with a small network who has not yet reached the firm informally.

Alternatively, there are many plausible scenarios where the hazard rate is not constant. An increasing hazard rate for the distribution of X would mean that contacts become more useful toward successfully reaching the firm through the informal channel when the worker's network is larger. This could correspond to a scenario in which the networking process allows the

strength comprises the combination of the size of the network and the effectiveness of contacts, and $P(N) = 1 - e^{N}$.

worker to progressively improve his search. Suppose that some contacts are closer to the firm than others, offering a higher probability of helping a worker submit an informal application to the firm. Their identities may be unknown initially, or perhaps they are not directly accessible. Perhaps starting as an outsider a worker may need to invest substantial time and resources before acquiring well-connected and powerful contacts. As a worker increases the size of his network he may be increasingly able to work his way up into more important inner circles, thus the more contacts he has acquired without connecting to the firm, the greater is the chance that the next contact he reaches will bring success.

In contrast, a decreasing hazard rate for the distribution of X would describe a situation where contacts are most valuable toward reaching the firm through informal channel when the worker's network is small, reflecting decreasing returns to scale in the search for or use of contacts because the more promising opportunities are explored or exploited first. For example this could correspond to a scenario in which the worker is well informed about where best to devote initial networking efforts. It could also occur when there is a high degree of overlap in the services contacts offer or whatever means of access to the informal channel potential contacts provide tends to be very similar, and not cumulatively useful. Perhaps contacts improve a worker's informal arrival probability because they give a worker inside information about how to reach the firm, but there is very little difference in the information the worker will receive from different contacts. This may be plausible when homophily effects are strong in the networking process, so that a worker's networking efforts tend to lead to a network composed of contacts with little diversity, making it more likely that the contribution from each new contact will be redundant. In such cases, the conditional probability that an additional contact will connect the worker to the firm may be smaller the greater is the number of existing contacts which have already not been helpful.

A particularly tractable class of distributions that allow monotonic hazard rates are Weibull distributions. This class of distributions is commonly used in a variety of disciplines due to its flexibility in modelling either increasing or decreasing hazard rates such as may be involved in system failure due to component failure, or ageing and diffusion processes. If for example the minimum required network size X follows a Weibull distribution, such that $P(X \leq n) = 1 - e^{-\lambda n^k}$ for some rate parameter $\lambda > 0$, the shape parameter $k > 0$ determines whether the hazard rate is decreasing ($k < 1$), or increasing ($k > 1$). The Weibull distribution reduces to the exponential distribution, and therefore constant hazard rate, when the shape parameter is precisely $k = 1$.[2]

Although I place some focus on the special case of the functional form $P(n) = 1 - e^{-\lambda n}$, the main results of this chapter hold for specifications of $P(n)$ which in addition to being strictly increasing and strictly concave, also have a logarithmically concave first derivative (corresponding to logarithmic concavity of the probability density function of X). Logarithmic concavity of the density function implies that the distribution's hazard rate is increasing.[3] Logarithmic concavity and the implied increasing hazard rate is a very common property among well-known probability distributions, although logarithmically convex and non-logarithmically-concave probability distributions are also not rare.[4] Although the Weibull density, with cdf $P(n) = 1 - e^{-\lambda n^k}$, is logarithmically convex and has a decreasing hazard rate for $k < 1$, the main results of this chapter hold for this class of informal arrival probability functions also. I will demonstrate this and discuss how the lack of the property of logarithmic concavity of P' can affect equilibrium results for other functional forms in Section 4.2.2, and briefly discuss type-dependent costs in Section 4.2.3.

Since a worker's probability of reaching the informal department depends on that worker's network strength, a worker may achieve a desired informal arrival probability by increasing (or decreasing) his networking activities to an appropriate level. Therefore informal arrival probabilities are endogenously determined by workers through the choice of n, however, developing

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[2]If adopting a Weibull distribution for X with $k > 1$ in this networking model, care must be taken because the assumption that $P(n)$ is strictly concave will hold only for $n > \sqrt[k]{\frac{k-1}{k}}$. However, this is certainly satisfied for all $n \geq 1$.

[3]For a summary of properties, examples, and applications of logarithmic concave functions and distributions, see Bagnoli and Bergstrom (2004).

[4]The density of the Weibull distribution satisfies logarithmic concavity when $k > 1$ and logarithmic convexity when $k < 1$, while the density function of the exponential distribution ($k = 1$) is simultaneously logarithmically convex and logarithmically concave. Note that it is possible for a density or function to be neither logarithmically convex nor logarithmically concave.

a network is costly to workers, according to some function $C_q(n) : \mathbb{R}_+ \to \mathbb{R}_+$ with $C_q(n) = 0$.

In this chapter I will assume a type-independent cost function of the form $C_q(n) = cn$, $c > 0$. Since the informal arrival probability resulting from a network strength of n is $P(n)$, we have $\gamma(P(n_q)) = cn_q$. The specification of $C_q(n)$ and $P(n)$ together can be understood in terms of a specification for the cost function $\gamma(p)$ used in the previous chapters. For example, a linear cost function cn_q together with a specification of arrival probability corresponding to a Weibull distribution function as previously discussed, with $P(n) = 1 - e^{-\lambda n^k}$, could alternatively be modelled with the benchmark informal arrival probability $P(n) = 1 - e^{-\tilde{\lambda} m}$ through change of variables $m = n^k$, together with an adjustment to the cost function, with either appropriate diminishing or increasing returns to the cost of networking.

The primary example throughout this chapter, a linear cost specification $C_q(n) = cn$ together with informal arrival probabilities of the form $P(n) = 1 - e^{-\lambda n}$, corresponds to an informal arrival cost function of the form $\gamma(p) = -\frac{c}{\lambda}\ln(1-p)$. Note that this functional form satisfies assumptions of previous chapters (it is strictly increasing, strictly convex, continuously differentiable, and logarithmically concave) and the characterization of its equilibria was considered in the one-channel model in Example 2.2.2.[5]

This chapter will consider the two-channel model and will focus workers' strategies in terms of their choice of networking, n_q, rather than their arrival probabilities p_{Iq}. It will investigate the equilibrium effects of the cost parameter c as related to the properties of $P(n)$. Note that in this treatment of the cost and benefits of networking, a worker's effort and resource expenditure is only productive toward the transmission of his informal application and report. That is, a greater network strength increases the worker's probability of reaching the firm, but networking does not offer the worker any way to influence the actual report realization. The distribution of the informal report is independent from the distribution of the formal report conditional on the worker's type. Therefore, through the increase in arrival probability, networking does include as a consequence the chance to be considered by the firm on the basis

[5]This comparison requires an adjustment of the constant c to the constant $\frac{c}{\lambda}$.

of a different type of information than in the formal channel. Note also that as networking is modelled here, it is equally difficult for high- and low-quality workers to develop a network. Nevertheless in the presence of reports of quality on informal applications, choice of network strength will still vary by type in general due to differences in the benefits of networking for each type.

4.1.2 The Pool Effect of Network-Based Informal Arrival

In this section I apply the network-based arrival and cost structure introduced above to the informal channel of the model in Chapter 3 in order to show the effect of network-based arrival on the quality composition of the pool of informal applicants. In particular, I show how the cost of networking relates to the intensity of the pool bias, whether favourable or unfavourable.

The worker's expected utility in equation (3.14) can be reformulated in terms of his networking choice as

$$(w-b)\Big(p_F\cdot\phi_{Fq} + (1-p_F\cdot\phi_{Fq})P(n_q)\cdot\phi_{Iq}\Big) + b - cn_q. \tag{4.1}$$

where $\phi_{Fq} = \phi_{Fq}(d_F)$ and $\phi_{Iq} = \phi_{Iq}(d_I)$ for $q \in \{h, \ell\}$ are determined by formulae (3.18) and (3.19) for a given firm hiring strategy $(d_F, d_I) = (d_{FH}+d_{FL}, d_{IH}+d_{IL})$. A worker of type q chooses network strength n_q to maximize this expected utility.

For strictly increasing and strictly concave informal arrival probability $P(n)$, a worker's optimal choice of network strength n_q must satisfy

$$\hat{w}(1-p_F\cdot\phi_{Fq}(d_F))P'(n_q)\cdot\phi_{Iq}(d_I) \leq c, \tag{4.2}$$

with equality if $n_q > 0$, where $\hat{w}=w-b$. When convenient I will formulate the workers' choice of networking as a best response to the conditional acceptance probabilities ϕ_{Fq} and ϕ_{Iq} for

$q \in \{h, \ell\}$ which are determined by the firm's chosen hiring strategy (d_F, d_I), that is, $n_q(\phi_{Fq}, \phi_{Iq})$, rather than as a best response to the firm's hiring strategy directly, $n_q(d_F, d_I)$.

Lemma 25. *Suppose $P(n)$ is strictly increasing and strictly concave. If $\phi_{Iq} > 0$ the worker's best response is given by:*

$$n_q(\kappa, \phi_{Fq}, \phi_{Iq}) = \begin{cases} P'^{-1}\left(\dfrac{c}{\hat{w}(1-p_F \cdot \phi_{Fq})\phi_{Iq}}\right) & \text{if } \hat{w}(1-p_F \cdot \phi_{Fq})\phi_{Iq}P'(0) \geq c \\ 0 & \text{otherwise,} \end{cases} \tag{4.3}$$

where $\kappa = (c, w, b, \varepsilon_I, \varepsilon_F, p_F)$.

For the functional form $P(n)$=$1 - e^{-\lambda n}$ with $\lambda > 0$ this becomes:

$$n_q(\kappa, \phi_{Fq}, \phi_{Iq}) = \begin{cases} \dfrac{1}{\lambda} \ln \dfrac{\hat{w}\lambda(1-p_F \cdot \phi_{Fq})\phi_{Iq}}{c} & \text{if } \hat{w}\lambda(1-p_F \cdot \phi_{Fq})\phi_{Iq} \geq c \\ 0 & \text{otherwise.} \end{cases} \tag{4.4}$$

Note that if networking is too costly, specifically if c is greater than the marginal benefit of networking at zero, $c > \hat{w}(1-p_F \cdot \phi_{Fq})\phi_{Iq}P'(0)$, then worker q will not network at all, $n_q(\kappa, \phi_{Fq}, \phi_{Iq})$=0. In the case of the functional form $P(n) = 1 - e^{-\lambda n}$ we have $P'(0)$=1. Thus the worker does not network if c is above $c_{max}(\phi_{Fq}, \phi_{Iq}) \equiv \hat{w}\lambda(1-p_F \cdot \phi_{Fq})\phi_{Iq}$.

The firm's best response in each department can also be reformulated in terms of networking, denoted as $d_{FH}(n_h, n_\ell)$ and $d_{FL}(n_h, n_\ell)$ instead of $d_{FH}(p_{Fh}, p_{F\ell})$ and $d_{FL}(p_{Fh}, p_{F\ell})$ in Lemma 9 and Lemma 10, with $p_{Ih} = P(n_h)$ and $p_{I\ell} = P(n_\ell)$. Since $P(n)$ is increasing and has $P(0) = 0$, the conditions $p_{Iq} > 0$ and $p_{Iq} = 0$ correspond to the reformulated conditions $n_q > 0$ and $n_q = 0$ for $q \in \{h, \ell\}$. The definition of equilibrium in Section 3.2.2 can also be reformulated to address the worker strategy in terms of networking strengths n_h and n_ℓ rather than arrival probabilities p_{Ih} and $p_{I\ell}$.

Since $P(n)$ is increasing, the conditions that determine which worker type networks more, $n_h \gtrless n_\ell$, will also be the same as the conditions in Section 3.2.1 that determine which worker arrives more in the informal channel, $p_{Ih} \gtrless p_{I\ell}$. Which worker type networks more for each

	$d_I = 2$	$d_I = 1$
$d_F = 0$	$n_h = n_\ell$	$n_h > n_\ell$
$d_F = 1$	$n_h < n_\ell$	$n_h > n_\ell,\ n_h = n_\ell,\ n_h < n_\ell$
$d_F = 2$	$n_h = n_\ell$	$n_h > n_\ell$

Table 4.1: Comparison of Worker Network Strengths

When non-zero, the possible ranking of workers' best-response networking strengths, $n_h(d_F, d_I)$ and $n_\ell(d_F, d_I)$, is shown for each relevant combination of formal and informal hiring strategies. Note these are best-responses only, $(d_F, d_I) = (0, 2)$ can not occur in equilibrium.

hiring pattern $(d_F, d_I) \in \{0, 1, 2\} \times \{1, 2\}$ is shown in Table 4.1 and corresponds to the ranking of informal arrival probabilities which were given in Table 3.1.

Now by Lemma 25, when the informal arrival probability $P(n)$ is increasing and concave, it will be the case that for fixed firm strategy d, an increase in the cost of networking discourages both worker types from networking, that is, $\dfrac{d\,n_q(c,d)}{dc} \leq 0$ for $q \in \{h, \ell\}$. However, an increase in the cost of networking can not typically be expected to reduce high- and low-quality worker incentives in such a way as to maintain a constant composition of the pool of informal applicants.

Let $\Lambda(\kappa, d)$ denote the informal pool adjustment factor resulting from the best responses (assuming they are non-zero) of workers $q \in \{h, \ell\}$ to $\phi_{Fq}(d)$ and $\phi_{Iq}(d)$ for parameter setting $\kappa = (c, w, b, \varepsilon_I, \varepsilon_F, p_F)$, so that $\Lambda(\kappa, d) \equiv \dfrac{P(n_\ell(\kappa, d))}{P(n_h(\kappa, d))}$. The following result describes what effect a change in the cost of networking c will have on this informal pool adjustment factor $\Lambda(\kappa, d)$, for a certain class of network-based arrival probability functions, $P(n)$, given a fixed firm strategy and assuming networking best responses are non-zero. For the functional form $P(n) = 1 - e^{-\lambda n}$, and also for other forms with logarithmically concave first derivative, it turns out that an increase in the cost parameter c will have an intensifying effect on the pool of informal applicants.[6] That is to say, any bias in the informal pool adjustment factor, whether

[6]Note that under the assumptions given for $P(n)$, logarithmic concavity of $P'(n)$ corresponds to logarithmic concavity of $\psi(P(n))$. This is because $P'(n) = c\psi(P(n))$ and $\dfrac{d^2}{dn^2}[\ln \psi(P(n))] < 0$.

favourable or unfavourable, will become more pronounced for higher cost of networking c.

Lemma 26. *Suppose $P(n)$ is strictly increasing and strictly concave, and that $P'(n)$ is logarithmically concave. For fixed firm strategy d, and therefore for fixed ϕ_{Fh}, ϕ_{Ih}, $\phi_{F\ell}$ and $\phi_{I\ell}$ with $n_h(\kappa, \phi_{Fh}, \phi_{Ih})), n_\ell(\kappa, \phi_{F\ell}, \phi_{I\ell})) > 0$, we have*

$$\frac{d\Lambda(\kappa, d)}{dc} \gtrless 0 \quad \textit{iff} \quad n_h(\kappa, \phi_{Fh}, \phi_{Ih}) \lessgtr n_\ell(\kappa, \phi_{F\ell}, \phi_{I\ell}) \tag{4.5}$$

Proof. Under the same assumptions, Lemma 38 in Appendix C shows that when networking is non-zero, whether the informal pool composition will be improved or worsened by a change in some parameter $a \in \{c, w, b, \varepsilon_F, \varepsilon_I, p_F\}$ for fixed firm strategy d will be determined by the condition

$$\frac{d\Lambda(\kappa, d)}{da} \gtrless 0 \quad \textit{if and only if} \quad \frac{P'(n_h(a))x'_h(a)}{P(n_h(a))P''(n_h(a))} \lessgtr \frac{P'(n_\ell(a))x'_\ell(a)}{P(n_\ell(a))P''(n_\ell(a))}, \tag{4.6}$$

where $x_q = \frac{c}{\hat{w}(1 - p_F\phi_{Fq})\phi_{Iq}}$ for $q \in \{h, \ell\}$. In the case of the cost parameter, $a = c$, we will have $\frac{dx_q}{dc} = \frac{1}{\hat{w}(1 - p_F\phi_{Fq})\phi_{Iq}}$ so that

$$x'_h(c) \lessgtr x'_\ell(c) \quad \textit{if and only if} \quad n_h \gtrless n_\ell. \tag{4.7}$$

Since arrival probability is an increasing function of network strength it is also the case that

$$P(n_h(c)) \gtrless P(n_\ell(c)) \quad \textit{if and only if} \quad n_h \gtrless n_\ell. \tag{4.8}$$

Since both $x'_q(a)$ and $P(n_q(c))$ are positive, conditions (4.7) and (4.8) together imply that

$$\frac{x'_h(c)}{P(n_h(c))} \lessgtr \frac{x'_\ell(c)}{P(n_\ell(c))} \quad \textit{if and only if} \quad n_h \gtrless n_\ell. \tag{4.9}$$

Under the assumption that $P'(n)$ is logarithmically concave, it must be the case that $\frac{P''(n)}{P'(n)}$ is a decreasing function of n, so that

$$\frac{P''(n_h(c))}{P'(n_h(c))} \lesseqgtr \frac{P''(n_\ell(c))}{P'(n_\ell(c))} \quad \textit{if and only if} \quad n_h \gtrless n_\ell, \tag{4.10}$$

or equivalently

$$\frac{P'(n_h(c))}{P''(n_h(c))} \gtrless \frac{P'(n_\ell(c))}{P''(n_\ell(c))} \quad \textit{if and only if} \quad n_h \gtrless n_\ell. \tag{4.11}$$

Now note that P is strictly concave so that $P''(n) < 0$ for all n. So multiplication of condition (4.9) by $\frac{P'(n_h(c))}{P''(n_h(c))}$, which is negative, gives

$$\frac{x'_h(c)}{P(n_h(c))}\frac{P'(n_h(c))}{P''(n_h(c))} \gtrless \frac{x'_\ell(c)}{P(n_\ell(c))}\frac{P'(n_h(c))}{P''(n_h(c))} \quad \textit{if and only if} \quad n_h \gtrless n_\ell. \tag{4.12}$$

On the other hand, multiplication of (4.11) by $\frac{x'_\ell(c)}{P(n_\ell(c))}$ gives

$$\frac{x'_\ell(c)}{P(n_\ell(c))}\frac{P'(n_h(c))}{P''(n_h(c))} \gtrless \frac{x'_\ell(c)}{P(n_\ell(c))}\frac{P'(n_\ell(c))}{P''(n_\ell(c))} \quad \textit{if and only if} \quad n_h \gtrless n_\ell. \tag{4.13}$$

Therefore by conditions (4.12) and (4.13) together we have

$$\frac{x'_h(c)}{P(n_h(c))}\frac{P'(n_h(c))}{P''(n_h(c))} \gtrless \frac{x'_\ell(c)}{P(n_\ell(c))}\frac{P'(n_\ell(c))}{P''(n_\ell(c))} \quad \textit{if and only if} \quad n_h \gtrless n_\ell, \tag{4.14}$$

Therefore we have the result that the cost of networking has an intensifying effect on the informal pool composition, as by condition (4.14), the condition (4.6) becomes

$$\frac{d\Lambda(\kappa, d)}{dc} \gtrless 0 \quad \textit{if and only if} \quad n_h \lesseqgtr n_\ell. \tag{4.15}$$

□

Note that this Lemma applies when network-based arrival probability is of the form $P(n) = 1 - e^{-\lambda n}$. In this case $P'(n) = \lambda e^{-\lambda n}$ so the logarithm of $P'(n)$ is $\ln \lambda - \lambda n$. We have $\frac{d^2}{dn^2}(\ln \lambda - \lambda n) = 0$, so the logarithm of $P'(n)$ is (weakly) concave.

Together with the equilibrium analysis in the previous chapter, the pool effect of a change in c shown by Lemma 26 makes it possible to understand how the structure of network-based informal arrival probabilities and costs can affect the existence of equilibria with different hiring patterns and functions of the informal channel, as will be discussed in the next section.

4.1.3 Equilibrium Implications of Network-Based Informal Arrival

In this section I will highlight the implications of logarithmically concave $P'(n)$ on the equilibrium results of informal channel use from the two-channel model. I will maintain the assumptions that $P(n)$ is strictly increasing, strictly concave and thrice continuously differentiable, and consider only pure strategy equilibria with non-trivial use of the informal channel. Recall that there can be no pure strategy Nash equilibrium in which the informal channel is used by one worker type and not the other (see Section 3.3). Therefore only firm strategies such that $\phi_{Iq}(d_F, d_I) > 0$ for both worker types $q \in \{h, \ell\}$, and only worker best responses which are non-zero will be considered here. Qualitative graphical depictions of the regions of existence of non-trivial equilibria for $P(n) = 1 - e^{-\lambda n}$ are shown at the end of this section in the (exhaustive) cases where (i) $\varepsilon_I < \varepsilon_F$, (ii) $\varepsilon_I > \varepsilon_F$ with $\varepsilon_I - \varepsilon_F < \dfrac{1-p_F}{p_F}(1-2\varepsilon_I)$, (iii) $\varepsilon_I > \varepsilon_F$ with $\varepsilon_I - \varepsilon_F = \dfrac{1-p_F}{p_F}(1-2\varepsilon_I)$, and (iv) $\varepsilon_I > \varepsilon_F$ with $\varepsilon_I - \varepsilon_F > \dfrac{1-p_F}{p_F}(1-2\varepsilon_I)$, in Figures 4.1-4.4 respectively.

For workers, the non-zero constraint for their choice of networking in equation (4.3) depends on the firm strategy. For a given firm strategy $(d_F, d_I) = (i, j) \in \{0, 1, 2\} \times \{1, 2\}$ let c^{ij}_{max} denote the cost beyond which networking is prohibitively expensive for either worker type, such that no equilibrium with non-trivial use of the informal channel can be supported for any $c \geq c^{ij}_{max}$. Then for a given firm strategy this upper limit on the cost of networking will be

$$c^{ij}_{max} \equiv \min\{\hat{w}(1-p_F\cdot\phi_{Fh})\phi_{Ih}P'(0)\,,\ \hat{w}(1-p_F\cdot\phi_{F\ell})\phi_{I\ell}P'(0)\}.$$

For convenience I will also sometimes write c^{d}_{max} instead of c^{ij}_{max} when firm strategy d is such that $(d_F, d_I) = (i, j) \in \{0, 1, 2\} \times \{1, 2\}$.

Lemma 27. *Suppose $P(n)$ is strictly increasing, strictly concave, and $P'(n)$ is logarithmically concave, and consider fixed parameters $\kappa_{-c} = (w, b, \varepsilon_I, \varepsilon_F, p_F)$ and fixed firm strategy d such that $d_I \neq 0$.*

(a) Suppose d and κ_{-c} are such that $\Lambda(\kappa, d) < 1$ for all $c \in (0, c^d_{max})$. Then for any value $\hat{\Lambda} \in (0, 1)$, a unique $\hat{c} \in (0, c^d_{max})$ exists such that $\Lambda(\hat{c}, \kappa_{-c}, d) = \hat{\Lambda}$.

(b) Suppose d and κ_{-c} are such that $\Lambda(\kappa, d) > 1$ for all $c \in (0, c^d_{max})$. Then for any value $\hat{\Lambda} > 1$, a unique $\hat{c} \in (0, c^d_{max})$ exists such that $\Lambda(\hat{c}, \kappa_{-c}, d) = \hat{\Lambda}$.

Proof. For any fixed firm strategy d, if networking is not prohibitively costly, $c < c^d_{max}$, then $n_h(\kappa, \phi_{Fh}, \phi_{Ih})$ is not zero and $\Lambda(\kappa, d)$ is a continuous monotonic function of c. In particular, when firm strategy and parameters are such that $\Lambda(\kappa, d)$ is greater than one, then $n_h(\kappa, \phi_{Fh}, \phi_{Ih}) < n_\ell(\kappa, \phi_{F\ell}, \phi_{I\ell})$ and by Lemma 26 we know that $\Lambda(\kappa, d)$ is increasing in c. Now whenever $n_h(\kappa, \phi_{Fh}, \phi_{Ih}) < n_\ell(\kappa, \phi_{F\ell}, \phi_{I\ell})$, it must be the case that $\hat{w}(1-p_F \cdot \phi_{F\ell})\phi_{I\ell} > \hat{w}(1-p_F \cdot \phi_{Fh})\phi_{Ih}$, so the cost at which networking becomes prohibitively expensive for high-quality workers is lower than the cost at which networking becomes prohibitively expensive for low-quality workers. Therefore $c^d_{max} = \hat{w}(1-p_F \cdot \phi_{Fh}(d))\phi_{Ih}(d)P'(0)$ and $\lim_{c \uparrow c^d_{max}} \Lambda(\kappa, d) = \infty$. Now for $q \in \{h, \ell\}$ $P'^{-1}\left(\dfrac{c}{\hat{w}(1 - p_F\phi_{Fq})\phi_{Iq}}\right)$ increases without bound as c goes to zero, thus for each type q we have $\lim_{c \downarrow 0} P\left(P'^{-1}\left(\dfrac{c}{\hat{w}(1 - p_F\phi_{Fq})\phi_{Iq}}\right)\right) = 1$. Therefore we also have $\lim_{c \downarrow 0} \Lambda(\kappa, d) = 1$. Since $\Lambda(\kappa, d)$ is strictly increasing in c, for fixed κ_{-c} and d, we know that $\Lambda(c, \kappa_{-c}, d)$ takes every value in $(1, \infty)$ for a unique c on the interval $(0, c^d_{max})$.

Similarly when firm strategy and parameters are such that $\Lambda(\kappa, d)$ is less than one, then $n_h(\kappa, \phi_{Fh}, \phi_{Ih}) > n_\ell(\kappa, \phi_{F\ell}, \phi_{I\ell})$ and by Lemma 26 we know that $\Lambda(\kappa, d)$ is decreasing in c. Now whenever $n_h(\kappa, \phi_{Fh}, \phi_{Ih}) > n_\ell(\kappa, \phi_{F\ell}, \phi_{I\ell})$, it must be the case that $\hat{w}(1-p_F \cdot \phi_{F\ell})\phi_{I\ell} < \hat{w}(1-p_F \cdot \phi_{Fh})\phi_{Ih}$, so the cost at which networking becomes prohibitively expensive for low-quality workers is lower than the cost at which networking becomes prohibitively expensive for high-quality workers. Therefore $c^d_{max} = \hat{w}(1-p_F \cdot \phi_{F\ell}(d))\phi_{I\ell}(d)P'(0)$ and $\lim_{c \uparrow c^d_{max}} \Lambda(\kappa, d) = 0$.

Again because $P'^{-1}\left(\frac{c}{\hat{w}(1-p_F\phi_{Fq})\phi_{Iq}}\right)$ increases without bound as c goes to zero for $q \in \{h, \ell\}$, we have $\lim_{c\downarrow 0} P\left(P'^{-1}\left(\frac{c}{\hat{w}(1-p_F\phi_{Fq})\phi_{Iq}}\right)\right) = 1$, and therefore $\lim_{c\downarrow 0} \Lambda(\kappa, d) = 1$. Since $\Lambda(\kappa, d)$ is strictly decreasing in c, for fixed κ_{-c} and d we know $\Lambda(c, \kappa_{-c}, d)$ takes every value in $(0, 1)$ for a unique c on the interval $(0, c^d_{max})$.

Now the conditions on the equilibrium pool of informal applicants $\Lambda^{ij} = \frac{p_{I\ell}(i,j)}{p_{Ih}(i,j)}$ for $(d_F, d_I) = (i, j) \in \{0, 1, 2\} \times \{1, 2\}$ in Propositions 7-11 in Chapter 3 can be understood as conditions on the underlying cost of networking c. In the case of network-based arrival probabilities with $p_{Iq}(i, j) = P(n_q(\kappa, d))$ for $q \in \{h, \ell\}$ and firm strategy d such that $(d_F, d_I) = (i, j)$, we have $\Lambda^{ij} = \Lambda(\kappa, d) = \frac{P(n_\ell(\kappa, d))}{P(n_h(\kappa, d))}$. Thus for logarithmically concave $P'(n)$, Lemma 27 together with the pool effect of c given by Lemma 26 allows us to state such conditions on c explicitly. In particular we have the following equilibrium results:

Corollary 5. *Suppose $c < c^d_{max}$ for $(d_F, d_I) = (1, 2)$. A necessary condition for the existence of an equilibrium with $(d^*_F, d^*_I) = (1, 2)$ and non-trivial use of the informal channel is that the cost of networking be not too high, $c \leq \bar{c}^{12}$ for some $\bar{c}^{12} \in (0, c^{12}_{max})$.*

According to condition (3.23) in Proposition 7(b), absolute use of the informal channel while the formal channel is used selectively requires that the informal pool composition in equilibrium, Λ^{12}, not exceed $\frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}$. Since $n_h(d_F, d_I) < n_\ell(d_F, d_I)$ for any parameter setting when $(d_F, d_I) = (1, 2)$, we know that $\Lambda^{12} > 1$. Therefore by Lemma 27(b) there exists $\bar{c}^{12} < c^d_{max}$ such that

$$\frac{P\left(P'^{-1}\left(\frac{\bar{c}^{12}}{\hat{w}(1-p_F\varepsilon_F)}\right)\right)}{P\left(P'^{-1}\left(\frac{\bar{c}^{12}}{\hat{w}(1-p_F(1-\varepsilon_F))}\right)\right)} = \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}.$$

By Lemma 26 we also know that $\Lambda(\kappa, d)$ is increasing in c, such that the informal pool composition worsens with a higher networking cost. This means that for any $c > \bar{c}^{12}$ condition (3.23) will be impossible to satisfy, and the firm can not be willing to hire absolutely in the informal channel. Therefore the cost of networking must not be too high in order for such an equilibrium to exist. For the case where $P(n) = 1 - e^{-\lambda n}$, $\bar{c}^{12}$ as a function of $\frac{v_h - w}{w - v_\ell}$ is represented by the curved boundary of the region 7*b* in Figures 4.2-4.4.

Note that Proposition 7(a) does not place any condition on the pool of informal applicants because in any equilibrium with $(d_F^*, d_I^*) = (2, 2)$, the pool of informal applicants must be neutral, $\Lambda^{22} = 1$. Therefore no condition on the cost of networking c is needed to support absolute formal hiring together with absolute informal hiring in equilibrium except (as always) that it not be prohibitively expensive, which in this case is the condition that $c < c_{max}^{22}$. For the functional form $P(n) = 1 - e^{-\lambda n}$, we have $c_{max}^{22} = \hat{w}\lambda(1-p_F)$, so this is the upper boundary for the region 7*a* in each of the Figures 4.1-4.4.

Now Propositions 8 and 9 cover the existence conditions for equilibria with selective informal hiring in parameter settings for which informal reports are decisive, that is, $\frac{v_h - w}{w - v_l} \in \left(\frac{1-s}{s}\frac{\varepsilon_I}{1-\varepsilon_I}, \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}\right)$. In each case, the existence of an equilibrium with $(d_F^*, d_I^*) = (i, 1)$ for $i \in \{0, 1, 2\}$ requires that the composition of the pool of informal applicants be not too extreme; either Λ is unfavourable and must not exceed a given threshold (must not be too unfavourable), or Λ is favourable and must exceed a given threshold (must not be too favourable). In both cases, the condition that the composition of the informal pool of applicants must not be too extreme is equivalent to a condition that the cost of networking must not be too high, as the following result shows.

Corollary 6. *Suppose informal reports are decisive and* $\varepsilon_I - \varepsilon_F \neq \frac{1-p}{p}(1-2\varepsilon_I)$. *A necessary condition for the existence of an equilibrium with selective informal hiring,* $d_I^* = 1$, $d_F^* = i \in \{0, 1, 2\}$, *and non-zero networking, is that the cost of networking be not too high,* $c \leq \bar{c}^{i1}$ *for some* $\bar{c}^{i1} \in (0, c_{max}^{i1})$.

When as in Proposition 8 we have $\varepsilon_I - \varepsilon_F > \frac{1-p}{p}(1-2\varepsilon_I)$, it must be the case that the firm also hires selectively in the formal channel $d_F^* = 1$, and the pool of informal applicants is unfavourable, $\Lambda^{11} > 1$. Therefore by Lemma 27(b) there exists $\overline{c}^{11} < c^{11}_{max}$ such that

$$\frac{P\left(P'^{-1}\left(\frac{\overline{c}^{11}}{\hat{w}(1-p_F\varepsilon_F)\varepsilon_I}\right)\right)}{P\left(P'^{-1}\left(\frac{\overline{c}^{11}}{\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I)}\right)\right)} = \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{1-\varepsilon_I}{\varepsilon_I},$$

and by Lemma 26 we also know that $\Lambda(\kappa, d)$ is increasing in c, so that the informal pool composition worsens with a higher networking cost. Therefore, for any $c > \overline{c}^{11}$ condition (3.24) will be impossible to satisfy and the firm can not be willing to accept high report informal applicants. Therefore the cost of networking must not be too high in order for such an equilibrium to exist. In the case of $P(n) = 1 - e^{-\lambda n}$, c^{11} as a function of $\frac{v_h-w}{w-v_\ell}$ when $\varepsilon_I - \varepsilon_F > \frac{1-p}{p}(1-2\varepsilon_I)$ is represented by the curved boundary of the region 8 in Figure 4.4. Note that when $\varepsilon_I - \varepsilon_F = \frac{1-p}{p}(1-2\varepsilon_I)$, the pool of informal applicants is exactly neutral $\Lambda^{11} = 1$. Therefore in this case there is no restriction on c for the existence of this equilibrium except for the non-negativity constraint that $c < c^{11}_{max}$. This is because $1 < \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{1-\varepsilon_I}{\varepsilon_I}$ so condition (3.24) will certainly be satisfied for any $c \in (0, c^{11}_{max})$. For the functional form $P(n) = 1 - e^{-\lambda n}$, we have $c^{11}_{max} = \hat{w}\lambda(1-p_F\varepsilon_F)\varepsilon_I$, so this is the upper boundary for the region 8 in Figure 4.3. Note that because $\varepsilon_I - \varepsilon_F = \frac{1-p}{p}(1-2\varepsilon_I)$, $c^{11}_{max} = \hat{w}\lambda(1-p_F(1-\varepsilon_F))(1-\varepsilon_I)$ also.

On the other hand, when as in Proposition 9 we have $\varepsilon_I - \varepsilon_F < \frac{1-p}{p}(1-2\varepsilon_I)$, for any $d_F^* = i \in \{0, 1, 2\}$ the pool of informal applicants is favourable, $\Lambda^{i1} < 1$. Therefore by Lemma 27(b) there exists $\overline{c}^{i1} < c^{i1}_{max}$ such that

$$\frac{P\left(P'^{-1}\left(\frac{\overline{c}^{i1}}{\hat{w}(1-p_F\phi_{F\ell})\varepsilon_I}\right)\right)}{P\left(P'^{-1}\left(\frac{\overline{c}^{i1}}{\hat{w}(1-p_F\phi_{Fh})(1-\varepsilon_I)}\right)\right)} = \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I},$$

for $i \in \{0, 1, 2\}$ and $\phi_{Fq} = \phi_{Fq}(i, 1)$ for $q \in \{h, \ell\}$. Furthermore by Lemma 26 we know that $\Lambda(\kappa, d)$ is decreasing in c, so that in this case the informal pool composition improves with a higher networking cost. Therefore, for any $c > \overline{c}^{01}$ condition (3.25) will be impossible to satisfy, for any $c > \overline{c}^{11}$ condition (3.26) will be impossible to satisfy, and for any $c > \overline{c}^{21}$ condition (3.27) will be impossible to satisfy, so the firm will not be willing to reject low-report applicants and selective hiring can not be supported. Therefore again, the cost of networking must not be too high in order for such an equilibrium to exist. In the case of $P(n) = 1 - e^{-\lambda n}$, $\overline{c}^{01}$, $\overline{c}^{11}$, and $\overline{c}^{21}$ as functions of $\frac{v_h - w}{w - v_\ell}$ when $\varepsilon_I < \varepsilon_F$ are represented by the curved boundary of the regions 9*a*, 9*b*, and 9*c* respectively in Figure 4.1. In Figure 4.2, $\overline{c}^{11}$ as a function of $\frac{v_h - w}{w - v_\ell}$ when $\varepsilon_I > \varepsilon_F$ but $\varepsilon_I - \varepsilon_F < \frac{1-p}{p}(1-2\varepsilon_I)$ can also be seen, represented by the curved boundary of region 9*b*. Note that the equilibria described by Propositions 9 parts (a) and (c) can not exist for $\varepsilon_I > \varepsilon_F$. This is because the equilibrium in (a) requires $\frac{v_h - w}{w - v_l} \leq \frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}$, and the equilibrium in (c) requires $\frac{v_h - w}{w - v_l} \geq \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}$, both of which would be contradicted for $\varepsilon_I > \varepsilon_F$ when informal reports are decisive, $\frac{v_h - w}{w - v_l} \in \left(\frac{1-s}{s}\frac{\varepsilon_I}{1-\varepsilon_I}, \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}\right)$.

Corollary 7. *Suppose informal reports are not decisive. A necessary condition for the existence of an equilibrium with selective informal hiring,* $d_I^* = 1$, $d_F^* = i \in \{0, 1\}$, *and non-zero networking, is that the cost of networking be sufficiently high but not too high,* $\underline{c}^{i1} \leq c \leq \overline{c}^{i1}$ *for some* $\underline{c}^{i1}, \overline{c}^{i1} \in (0, c_{max}^{i1})$.

First suppose that the industry is generally unprofitable, so $\frac{v_h - w}{w - v_l} < \frac{1-s}{s}\frac{\varepsilon_I}{1-\varepsilon_I}$. If as in Proposition 10(a) formal reports are also not decisive and $d_F^* = 0$, then the pool of informal applicants is favourable, $\Lambda^{01} < 1$. When as in Proposition 10(b) formal reports are decisive and $d_F^* = 1$, the pool of informal applicants is also favourable, $\Lambda^{11} < 1$. Therefore by Lemma 27(b) there exists $\overline{c}^{i1} < c_{max}^{i1}$ such that

$$\frac{P\left(P'^{-1}\left(\frac{\overline{c}^{i1}}{\hat{w}(1-p_F \cdot \phi_{F\ell})\varepsilon_I}\right)\right)}{P\left(P'^{-1}\left(\frac{\overline{c}^{i1}}{\hat{w}(1-p_F \cdot \phi_{Fh})(1-\varepsilon_I)}\right)\right)} = \frac{v_h - w}{w - v_l}\frac{s}{1-s}\frac{1-\varepsilon_I}{\varepsilon_I},$$

and there also exists $\underline{c}^{i1} < c^{i1}_{max}$ such that

$$\frac{P\left(P'^{-1}\left(\frac{\underline{c}^{i1}}{\hat{w}(1-p_F\cdot\phi_{F\ell})\varepsilon_I}\right)\right)}{P\left(P'^{-1}\left(\frac{\underline{c}^{i1}}{\hat{w}(1-p_F\cdot\phi_{Fh})(1-\varepsilon_I)}\right)\right)} = \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}$$

for $i \in \{0,1\}$ and $\phi_{Fq} = \phi_{Fq}(i,1)$ for $q \in \{h,\ell\}$. By Lemma 26 we know that $\Lambda(\kappa, d)$ is decreasing in c, so that the informal pool composition improves with a higher networking cost, and it is the case that $\underline{c}^{i1} < \overline{c}^{i1}$. Conditions (3.28) and (3.29) will be impossible to satisfy for any $c > \overline{c}^{01}$ and $c > \overline{c}^{11}$ respectively, because the informal pool composition will be too favourable due to the high networking cost and the firm will not be willing to reject low-report applicants. However conditions (3.28) and (3.29) will also be impossible to satisfy for any $c < \underline{c}^{01}$ and $c < \underline{c}^{11}$ respectively because the informal pool composition will not be sufficiently favourable due to the low networking cost for the firm to accept high-report applicants. Therefore the cost of networking must be high enough and must also be not too high in order for such an equilibrium to exist. In the case of $P(n) = 1 - e^{-\lambda n}$, $\underline{c}^{01}$ and $\overline{c}^{01}$ as functions of $\frac{v_h-w}{w-v_\ell}$ are represented by the curved boundaries of the region 10*a* in Figures 4.1-4.4, while $\underline{c}^{11}$ and $\overline{c}^{11}$, as functions of $\frac{v_h-w}{w-v_\ell}$ when $\frac{v_h-w}{w-v_l} < \frac{1-s}{s}\frac{\varepsilon_I}{1-\varepsilon_I}$, are represented by the curved boundaries of the region 10*b* in Figure 4.2. Note however that the conditions of Proposition 10(b) can not be satisfied when $\varepsilon_I - \varepsilon_F \geq \frac{1-p_F}{p_F}(1-2\varepsilon_I)$ (see Remark 3 in Chapter 3), and therefore no region 10*b* appears in Figure 4.3 or 4.4.

Now suppose the industry is generally profitable, so $\frac{v_h-w}{w-v_l} > \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}$. There can be no equilibrium with $d_F^* = 0$ in this case, and there can be no equilibrium with $d_F^* = 1$ if formal reports are also not decisive. However, if as in Proposition 11 formal reports are decisive and $d_F^* = 1$, then the pool of informal applicants will be unfavourable, $\Lambda^{11} > 1$. Therefore by

Lemma 27(b) there exists $\overline{c}^{11} < c^{11}_{max}$ such that

$$\frac{P\left(P'^{-1}\left(\frac{\overline{c}^{11}}{\hat{w}(1-p_F\cdot\phi_{F\ell})\varepsilon_I}\right)\right)}{P\left(P'^{-1}\left(\frac{\overline{c}^{11}}{\hat{w}(1-p_F\cdot\phi_{Fh})(1-\varepsilon_I)}\right)\right)} = \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{1-\varepsilon_I}{\varepsilon_I},$$

and there also exists $\underline{c}^{11} < c^{11}_{max}$ such that

$$\frac{P\left(P'^{-1}\left(\frac{\underline{c}^{11}}{\hat{w}(1-p_F\cdot\varepsilon_F)\varepsilon_I}\right)\right)}{P\left(P'^{-1}\left(\frac{\underline{c}^{11}}{\hat{w}(1-p_F\cdot(1-\varepsilon_F))(1-\varepsilon_I)}\right)\right)} = \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}.$$

By Lemma 26 we know that $\Lambda(\kappa, d)$ is increasing in c in this case, so that the informal pool composition worsens with a higher networking cost, and it is the case that $\underline{c}^{11} < \overline{c}^{11}$. Condition (3.30) will be impossible to satisfy for any $c > \overline{c}^{11}$ because the informal pool composition will be too unfavourable due to the high networking cost and the firm will not be willing to accept high-report applicants. This condition will also be impossible to satisfy for any $c < \underline{c}^{11}$ because the informal pool composition will not be sufficiently unfavourable due to the low networking cost for the firm to reject low-report applicants. Again the cost of networking must be high enough and must also be not too high in order for such an equilibrium to exist. In the case of $P(n) = 1 - e^{-n}$, $\underline{c}^{11}$ and $\overline{c}^{11}$, as functions of $\frac{v_h-w}{w-v_\ell}$ when $\frac{v_h-w}{w-v_l} > \frac{1-s}{s}\frac{\varepsilon_I}{1-\varepsilon_I}$, are represented by the curved boundaries of the region 11 in Figure 4.4. Note that this equilibrium described by Proposition 11 requires $\varepsilon_I - \varepsilon_F > \frac{1-p}{p}(1-2\varepsilon_I)$.

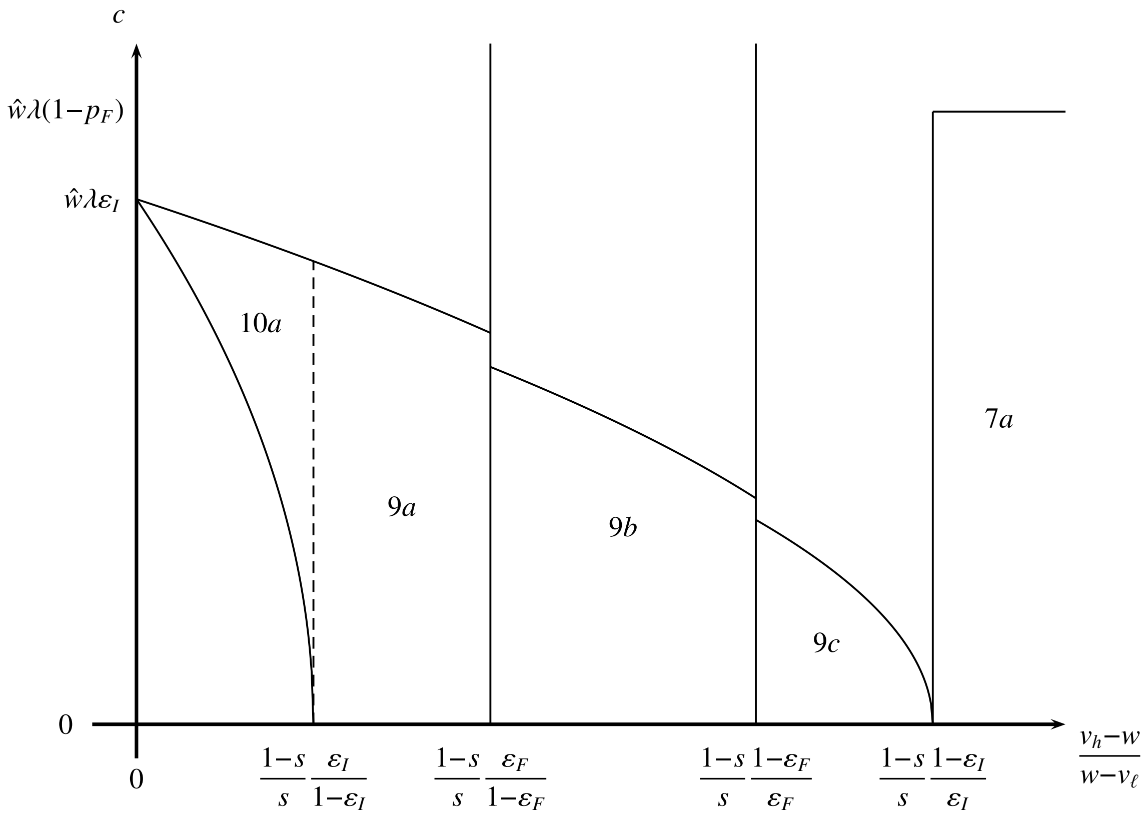

Figure 4.1: Range of Existence of Equilibria when $\varepsilon_I < \varepsilon_F$

A qualitative depiction is shown for $P(n) = 1-e^{-\lambda n}$ and assuming $\varepsilon_I < 1-p_F$. Within each region, the label corresponds to the proposition which describes the non-trivial equilibrium which exists in this region. Within unlabelled regions, no equilibrium exists with non-zero arrival in the informal channel.

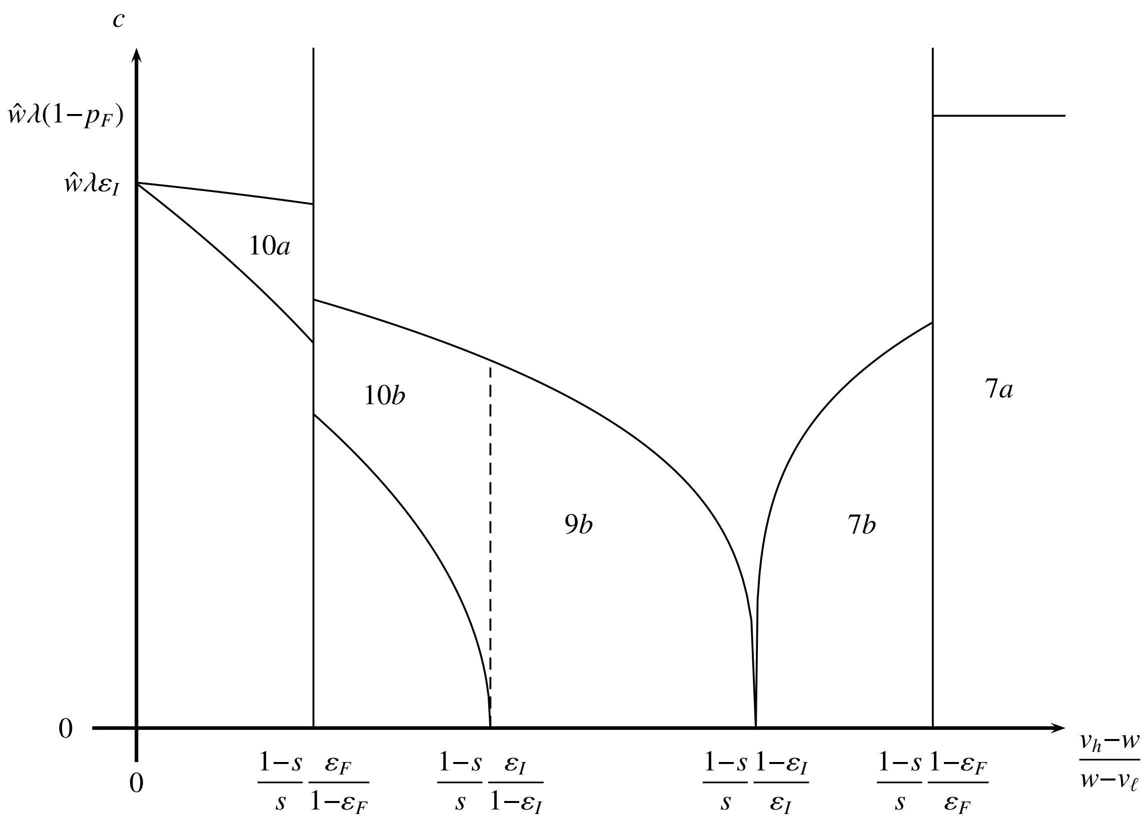

Figure 4.2: Range of Existence of Equilibria when $\varepsilon_I > \varepsilon_F$ and $\varepsilon_I - \varepsilon_F < \frac{1-p_F}{p_F}(1-2\varepsilon_I)$

A qualitative depiction is shown for $P(n) = 1 - e^{-\lambda n}$ and assuming $\varepsilon_I < 1 - p_F$.

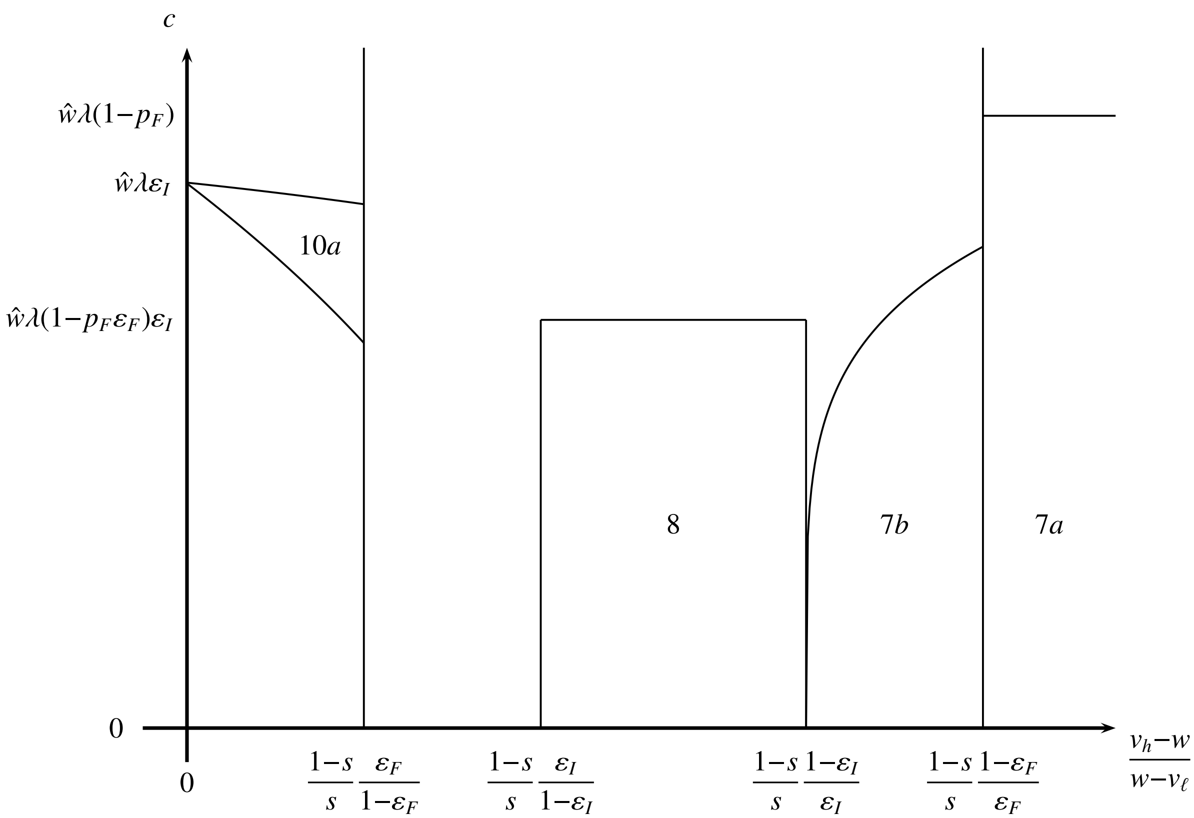

Figure 4.3: Range of Existence of Equilibria when $\varepsilon_I > \varepsilon_F$ and $\varepsilon_I - \varepsilon_F = \frac{1-p_F}{p_F}(1-2\varepsilon_I)$

A qualitative depiction is shown for $P(n) = 1-e^{-\lambda n}$ and assuming $\varepsilon_I < 1-p_F$.

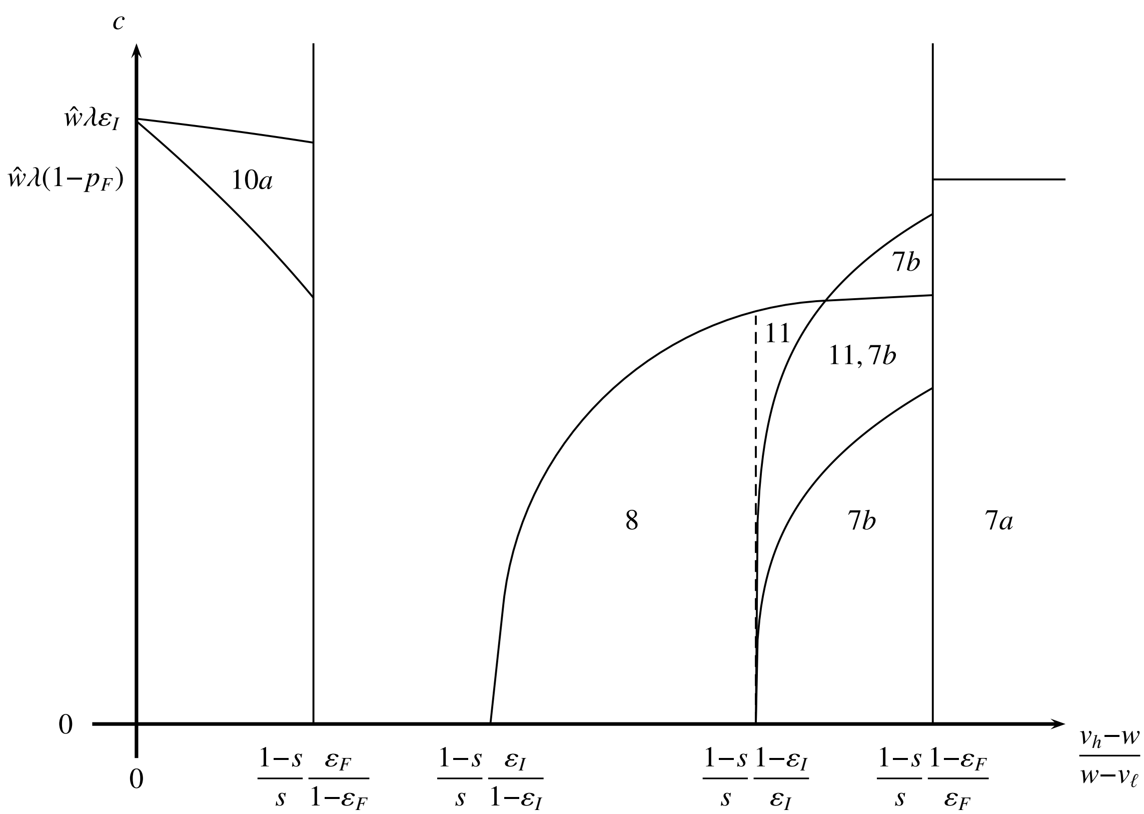

Figure 4.4: Range of Existence of Equilibria when $\varepsilon_I > \varepsilon_F$ and $\varepsilon_I-\varepsilon_F > \frac{1-p_F}{p_F}(1-2\varepsilon_I)$

A qualitative depiction is shown for $P(n) = 1-e^{-\lambda n}$ and assuming $\varepsilon_I > 1-p_F$.

4.2 Additional Analyses

4.2.1 Implications for Comparative Static Results and Welfare

In this section I will compare the pool effect of a change in the cost of networking c with the effect of a change in other parameters on the composition of the informal pool of applicants, and implications for comparative static and welfare results.

Lemma 26 showed that an increase in the cost of networking c has an amplifying effect on the quality composition of the pool of informal applicants when $P'(n)$ is logarithmically concave. Although increased networking costs discourage both worker types from using the informal channel, it turns out that the reduction is stronger at low levels of networking. Therefore if one worker type is relatively prevalent in the informal pool, an increase in c will increase the relative prevalence of that type, while the volume of the informal pool of applicants decreases. Assuming that the increase in networking costs does not change the equilibrium hiring strategy of the firm, an increase in c will lower worker utility for both types because decreasing their arrival probabilities will decrease their acceptance probabilities. For the firm on the other hand, the change in the cost of networking may increase or decrease profits. For equilibria in which the informal pool is favourable, the firm will benefit from an increase in the informal pool bias. For equilibria in which the informal pool is unfavourable, an increase in the informal pool bias is not desirable for the firm. However, in both cases, the volume of informal applicants will be reduced. Therefore a rise in networking costs may increase or decrease firm profit.

Lemma 28. *Within a given class of equilibria with $(d_F^*, d_I^*) = (i, j) \in \{0, 1, 2\} \times \{1, 2\}$ and non-zero networking, it is the case that $\dfrac{du_q}{dc} < 0$ for $q \in \{h, \ell\}$, and profit may increase or decrease in c. For $P(n) = 1-e^{-\lambda n}$ in particular, $\dfrac{d\pi}{dc} \gtrless 0$ if and only if $\dfrac{v_h-w}{w-v_\ell} \lessgtr \dfrac{1-s}{s}$.*

Proof. From the worker utility given in equation (4.1), for fixed firm strategy with $d_I \neq 0$

whenever $c < c^{ij}_{max}$ we have

$$\frac{du_q}{dc} = \left[\hat{w}(1 - p_F\phi_{Fq})\phi_{Iq}P'(n_q) - c\right] n'_q(c) - n_q. \tag{4.16}$$

In any equilibrium with non-zero networking, this is negative because it will be the case that $\hat{w}(1 - p_F\phi_{Fq})\phi_{Iq}P'(n^*_q) = c$.

On the other hand, firm profit is given by $\pi = \pi_F + \hat{\pi}_I$ where as in the previous chapter

$$\pi_F = (v_h - w)sp_F\phi_{Fh} - (w - v_\ell)(1-s)p_F\phi_{F\ell} \tag{4.17}$$

denotes profit from hires through the formal channel and

$$\hat{\pi}_I = (v_h - w)s\phi_{Ih}(1-p_F\phi_{Fh})P(n_h) - (w - v_\ell)(1-s)\phi_{I\ell}(1-p_F\phi_{F\ell})P(n_\ell). \tag{4.18}$$

denotes profit only from “additional” hires through the informal channel.

Since a change in networking cost has no effect on profit from hires in the formal channel, we have $\frac{d\pi}{dc} = \frac{d\hat{\pi}_I}{dc}$ with

$$\frac{d\hat{\pi}_I}{dc} = (v_h - w)s\phi_{Ih}(1-p_F\phi_{Fh})P'(n_h)n'_h(c) - (w - v_\ell)(1-s)\phi_{I\ell}(1-p_F\phi_{F\ell})P'(n_\ell)n'_\ell(c). \tag{4.19}$$

As derived in the proof of Lemma 38 we have

$$n'_q(a) = \frac{1}{P''(n_q)}x'_q(a), \tag{4.20}$$

for any parameter $a \in \{c, w, b, \varepsilon_I, \varepsilon_F, p_F\}$, where $x_q = \frac{c}{\hat{w}(1-p_F\phi_{Fq})\phi_{Iq}}$ for $q \in \{h, \ell\}$. Therefore since $x'_q(c) = \frac{1}{\hat{w}(1-p_F\phi_{Fq})\phi_{Iq}}$, equation (4.19) becomes

$$\frac{d\hat{\pi}_I}{dc} = (v_h - w)s\frac{P'(n_h)}{P''(n_h)}\frac{1}{\hat{w}} - (w - v_\ell)(1-s)\frac{P'(n_\ell)}{P''(n_\ell)}\frac{1}{\hat{w}}. \tag{4.21}$$

Thus within any class of equilibria with non-zero networking, we have

$$\frac{d\hat{\pi}_I}{dc} \gtrless 0 \textit{ if and only if } \frac{v_h - w}{w - v_\ell} \lessgtr \frac{1-s}{s} \frac{\frac{P'(n_\ell)}{P''(n_\ell)}}{\frac{P'(n_h)}{P''(n_h)}}, \tag{4.22}$$

so profit may increase or decrease in c. Note that the direction of the second inequality in (4.22) is due to $P''(n) < 0$.

When $P'(n)$ is logarithmically concave, so that $\frac{P'(n)}{P''(n)}$ is increasing in n, we will have $\frac{P'(n_\ell^*)}{P''(n_\ell^*)} \geq \frac{P'(n_h^*)}{P''(n_h^*)}$ whenever low-quality workers network (weakly) more than high-quality workers in equilibrium, and $\frac{P'(n_\ell^*)}{P''(n_\ell^*)} \leq \frac{P'(n_h^*)}{P''(n_h^*)}$ whenever high-quality workers network (weakly) more than low-quality workers in equilibrium. Thus in industries which are generally profitable, an increase in the cost of networking will lower profit in any equilibrium in which $n_h^* \geq n_\ell^*$, for example equilibria in which the firm hires absolutely in both channels, or, if informal reports are more accurate than formal reports, equilibria in which the firm hires selectively in both channels. In these cases the firm will not benefit from the effect of the amplification of the (favourable) informal pool bias sufficiently to outweigh the loss of value from fewer hires due to a decreased volume of informal applicants.

Now in the case of an arrival probability function of the form $P(n) = 1 - e^{-\lambda n}$, the term $\frac{P'(n)}{P''(n)}$ is constant with respect to n, so condition (4.22) reduces to $\frac{d\hat{\pi}_I}{dc} \gtrless 0$ *if and only if* $\frac{v_h - w}{w - v_\ell} \lessgtr \frac{1-s}{s}$. In this case higher networking costs always improve profit in equilibrium in industries which are generally unprofitable and always decrease profit in equilibrium in industries which are generally profitable.

Now while networking costs have an amplifying effect on the pool of informal applicants, the opposite holds for a change in the wage w when P' is logarithmically concave. In particular, the pool effect of w is negatively proportional to the pool effect of c. Since for fixed firm strategy d we have

$$\frac{d\Lambda(\kappa,d)}{da} = \frac{P(n_h)P'(n_\ell)n'_\ell(a) - P(n_\ell)P'(n_h)n'_h(a)}{[P(n_h)]^2}, \tag{4.23}$$

and since $x'_q(w) = -\frac{c}{\hat{w}^2(1-p_F\phi_{Fq})\phi_{Iq}}$, then by equation (4.20), equation (4.23) for $a = w$ becomes

$$\frac{d\Lambda(\kappa,d)}{dw} = \frac{1}{[P(n_h)]^2}\left[-P(n_h)\frac{P'(n_\ell)}{P''(n_\ell)}\frac{c}{\hat{w}^2(1-p_F\phi_{F\ell})\phi_{I\ell}} + P(n_\ell)\frac{P'(n_h)}{P''(n_h)}\frac{c}{\hat{w}^2(1-p_F\phi_{Fh})\phi_{Ih}}\right]. \tag{4.24}$$

By comparison, since $x'_q(c) = \frac{1}{\hat{w}(1-p_F\phi_{Fq})\phi_{Iq}}$ equation (4.23) for $a = c$ gives

$$\frac{d\Lambda(\kappa,d)}{dc} = \frac{1}{[P(n_h)]^2}\left[P(n_h)\frac{P'(n_\ell)}{P''(n_\ell)}\frac{1}{\hat{w}(1-p_F\phi_{F\ell})\phi_{I\ell}} - P(n_\ell)\frac{P'(n_h)}{P''(n_h)}\frac{1}{\hat{w}(1-p_F\phi_{Fh})\phi_{Ih}}\right]. \tag{4.25}$$

Therefore we can see that $\frac{d\Lambda(\kappa,d)}{dw} = -\frac{c}{\hat{w}} \cdot \frac{d\Lambda(\kappa,d)}{dc}$. From this observation and Lemma 26, Lemma 29 follows immediately.

Lemma 29. *Suppose $P(n)$ is strictly increasing and strictly concave, and that $P'(n)$ is logarithmically concave. For fixed firm strategy d with $n_h(\kappa,d), n_\ell(\kappa,d) > 0$, we have*

$$\frac{d\Lambda(\kappa,d)}{dw} \lesseqgtr 0 \Leftrightarrow n_h \lesseqgtr n_\ell.$$

Thus for logarithmically concave $P'(n)$ the wage has a moderating effect on the informal pool composition, reducing the intensity of any favourable or unfavourable bias.

In contrast, since $x'_q(b) = \frac{c}{\hat{w}^2(1-p_F\phi_{Fq})\phi_{Iq}}$, so that $x'_q(b) = -x'_q(w)$, the informal pool effect of b is exactly the negative of the informal pool effect of w, and within any class of equilibrium with fixed d and non-zero networking we will have

$$\frac{d\Lambda(\kappa,d)}{db} \gtreqless 0 \Leftrightarrow n_h \lesseqgtr n_\ell \tag{4.26}$$

whenever $P'(n)$ is logarithmically concave. So the unemployment benefit has an amplifying effect on the informal pool which is proportional to the pool effect of c, $\dfrac{d\Lambda(\kappa,d)}{db} = \dfrac{c}{\hat{w}} \cdot \dfrac{d\Lambda(\kappa,d)}{dc}$.

On the other hand, within a given class of equilibria with fixed firm strategy and non-zero networking, the effect on the pool of informal applicants of the formal arrival probability p_F can not be generally determined. Of course for equilibria with no formal hiring, $d_F^* = 0$, the formal arrival probability is irrelevant to worker choices of networking, and the informal pool is neutral in any equilibrium with absolute hiring in both channels, $(d_F^*, d_I^*) = (2,2)$, so again the formal arrival probability has no pool effect in such a case. Less trivially, for equilibria with $(d_F^*, d_I^*) = (2,1)$, we have $\dfrac{d\Lambda(\kappa,d)}{dp_F} = -\dfrac{1}{1-p_F} \cdot \dfrac{d\Lambda(\kappa,d)}{dc}$. This can be shown by comparison of equation (4.25) with equation (4.23) for $a = p_F$, because for firm strategy $(d_F^*, d_I^*) = (2,1)$, it is the case that $x_h'(p_F) = \dfrac{-1}{\hat{w}(1-p_F)^2(1-\varepsilon)}$ and $x_\ell'(p_F) = \dfrac{-1}{\hat{w}(1-p_F)^2\varepsilon}$. Thus for generally profitable industries where formal reports are not decisive, a change in the formal arrival probability has a moderating effect on the pool of informal applicants.

Since a change in the unemployment benefit has no effect on profit from hires in the formal channel, we have $\dfrac{d\pi}{db} = \dfrac{d\hat{\pi}_I}{db}$ with

$$\frac{d\hat{\pi}_I}{db} = (v_h - w)s\phi_{Ih}(1-p_F\phi_{Fh})P'(n_h)n_h'(b) - (w-v_\ell)(1-s)\phi_{I\ell}(1-p_F\phi_{F\ell})P'(n_\ell)n_\ell'(b). \tag{4.27}$$

with

$$n_q'(b) = \frac{1}{P''(n_q)} x_q'(b). \tag{4.28}$$

As noted previously in Lemma 38, $x_q'(b) = \dfrac{c}{\hat{w}^2(1-p_F\phi_{Fq})\phi_{Iq}}$ for $q \in \{h,\ell\}$, so equation (4.27) becomes

$$\frac{d\hat{\pi}_I}{db} = (v_h - w)s\frac{P'(n_h)}{P''(n_h)}\frac{c}{\hat{w}}^2 - (w-v_\ell)(1-s)\frac{P'(n_\ell)}{P''(n_\ell)}\frac{c}{\hat{w}}^2 \tag{4.29}$$

and we can see by comparison with equation (4.21) that the effect of a change in the unemployment benefit on profit is proportional to the effect of a change in networking cost, $\dfrac{d\pi}{db} = \dfrac{c}{\hat{w}}\dfrac{d\pi}{dc}$.

A change in w also affects informal profits through the effect on the workers' networking, and similarly this effect is proportional to the effect of a change in network cost. However an increase in w also has a direct negative effect on profit since the firm's costs go up. The effect of a change in w on the firm's profit from formal hires is negative and in proportion to the volume of formal hires. By differentiation of equation (4.17) this is given by

$$\frac{d\pi_F}{dw} = -p_F\left[s\phi_{Fh} + (1-s)\phi_{F\ell}\right]. \tag{4.30}$$

Now by equation (4.18) the effect on profit from "additional" hires through the informal channel is given by

$$\frac{d\hat{\pi}_I}{dw} = (v_h-w)s\phi_{Ih}(1-p_F\phi_{Fh})P'(n_h)n_h'(w) - (w-v_\ell)(1-s)\phi_{I\ell}(1-p_F\phi_{F\ell})P'(n_\ell)n_\ell'(w). \tag{4.31}$$

Since $x_q'(w) = -\frac{c}{w}x_q'(c)$, we have $n_q'(w) = -\frac{c}{w}n_q'(c)$ by equation (4.20). With this substitution in (4.18) we can see that $\frac{d\hat{\pi}_I}{dw} = -\frac{c}{\hat{w}}\frac{d\hat{\pi}_I}{dc}$ by (4.19). Since the overall effect of a change in w on profit is given by

$$\frac{d\pi}{dw} = \frac{d\pi_F}{dw} + \frac{d\hat{\pi}_I}{dw} \tag{4.32}$$

and because $\frac{d\hat{\pi}_I}{dc} = \frac{d\pi}{dc}$, we therefore have

$$\frac{d\pi}{dw} = -p_F\left[s\phi_{Fh} + (1-s)\phi_{F\ell}\right] - \frac{c}{\hat{w}}\frac{d\pi}{dc}. \tag{4.33}$$

4.2.2 Alternative Arrival Structures

As seen in Section 4.1.3 the relationship between use of the informal channel in equilibrium and networking cost depends on how a change in c affects the composition of the pool of informal applicants. Lemma 26 shows that in the case of logarithmically concave P', an intensifying effect holds. However, when P' is not logarithmically concave, this certainly need not be the case.

For example, consider $P(n) = \delta\sqrt{n}$ for an appropriate $\delta > 0$ chosen such that $P(n_q^*) \leq 1$ for both types $q \in \{h, \ell\}$. This corresponds to the arrival cost function $\gamma(p) = \hat{c}p^2$ considered in Example 2.2.1 in Chapter 2 for $x = 2$ and constant $\hat{c} = c/\delta^2$. This functional form for the arrival probability satisfies the assumptions that $P(n)$ is strictly increasing and strictly concave, however $P'(n)$ is not logarithmically concave. This is because $\ln P'(n) = \ln\frac{\delta}{2} - \frac{1}{2}\ln n$ so that $(\ln P'(n))' = -\frac{1}{2n}$, which is increasing in n. For this example, a worker's optimal networking choice (when non-zero) satisfies

$$\hat{w}(1-p_F\phi_{Fq})\phi_{Iq} \cdot \frac{\delta}{2\sqrt{n_q}} = c \tag{4.34}$$

and

$$n_q = \left[\frac{\delta}{2}\frac{\hat{w}(1-p_F\phi_{Fq})\phi_{Iq}}{c}\right]^2 \tag{4.35}$$

by condition (4.2) and Lemma 11. Therefore the pool composition is independent of the cost of networking, with

$$\frac{P(n_\ell)}{P(n_h)} = \frac{\sqrt{n_\ell}}{\sqrt{n_h}} = \frac{(1-p_F\phi_{F\ell})\phi_{I\ell}}{(1-p_F\phi_{Fh})\phi_{Ih}}. \tag{4.36}$$

Alternatively, it is also possible that a change in networking cost may have a moderating effect on the pool of informal applicants when P' is not logarithmically concave. Since concavity of P' will imply logarithmic concavity of P', a necessary condition for the occurrence of a moderating effect is that $P''' > 0$, such that P' is strictly convex. As an example, consider an informal arrival probability of the form $P(n) = \frac{1}{2}(n + \sqrt{n})$ where networking intensity is chosen in the unit interval, $n \in [0, 1]$. I will show in this case that when the informal pool of applicants has a favourable bias, an increase in the cost of networking c causes the composition to become less favourable, whereas when the pool has an unfavourable bias, an increase in the cost of networking c causes the composition to become less unfavourable.

Now as previously shown, for $P(n)$ twice differentiable, strictly increasing and strictly concave, we will have

$$\frac{d\Lambda(\kappa,d)}{dc} \gtrless 0 \quad \textit{if and only if} \quad \frac{P'(n_h(c))x_h'(c)}{P(n_h(c))P''(n_h(c))} \lessgtr \frac{P'(n_\ell(c))x_\ell'(c)}{P(n_\ell(c))P''(n_\ell(c))}, \tag{4.37}$$

with $x_q = \frac{c}{\hat{w}(1-p_F\phi_{Fq})\phi_{Iq}}$ for $q \in \{h, \ell\}$. When workers choose networking optimally, it will be the case that

$$\hat{w}(1-p_F\phi_{Fq})\phi_{Iq}P'(n_q) = c, \tag{4.38}$$

assuming that optimal networking is non-zero, and therefore it is the case that $x'(c) = P'(n_q)/c$. Substituting this into condition (4.37) gives

$$\frac{d\Lambda(\kappa,d)}{dc} \gtrless 0 \quad \textit{if and only if} \quad \frac{[P'(n_h(c))]^2}{P(n_h(c))P''(n_h(c))} \lessgtr \frac{[P'(n_\ell(c))]^2}{P(n_\ell(c))P''(n_\ell(c))}. \tag{4.39}$$

This means that if $\frac{[P'(n)]^2}{P(n)P''(n)}$ is decreasing, a change in the cost of networking will have a moderating effect on the informal pool of applicants, that is, we will have $\frac{d\Lambda(\kappa,d)}{dc} \gtrless 0$ if and only if $n_h \gtrless n_\ell$.

For $P(n) = \frac{1}{2}(n + \sqrt{n})$ with $n \in (0, 1]$, we have $P'(n) = \frac{1}{2} + \frac{1}{4\sqrt{n}} > 0$, $P''(n) = -\frac{1}{8\sqrt{n^3}} < 0$ but $\frac{P''(n)}{P'(n)} = \frac{-\frac{1}{4\sqrt{n^3}}}{1 + \frac{1}{2\sqrt{n}}}$ is not a decreasing function of n so P' is not logarithmically concave. This can be seen because by the quotient rule for derivatives, $\frac{d}{dn}\left[\frac{P''(n)}{P'(n)}\right] \leq 0$ for this functional form if and only if

$$\left(1 + \tfrac{1}{2\sqrt{n}}\right) \cdot \left(\tfrac{3}{8\sqrt{n^5}}\right) - \left(-\tfrac{1}{4\sqrt{n^3}}\right)^2 \leq 0, \tag{4.40}$$

but the expression on the left side simplifies to $\frac{3}{8\sqrt{n^5}} + \frac{3}{16n^3} - \frac{1}{16n^3}$, which is positive.

For this informal arrival probability function, $\frac{[P'(n)]^2}{P(n)P''(n)}$ is decreasing because

$$\frac{[P'(n)]^2}{P(n)P''(n)} = \frac{-\left[\frac{1}{2}(1 + \frac{1}{2\sqrt{n}})\right]^2}{\frac{1}{2}\sqrt{n}\left(\sqrt{n}+1\right) \cdot \left(\frac{1}{8\sqrt{n^3}}\right)} = \frac{-(1 + 4n + 4\sqrt{n})}{\sqrt{n}+1} \tag{4.41}$$

and so by the quotient rule for derivatives, we will have $\frac{d}{dn}\left[\frac{[P'(n)]^2}{P(n)P''(n)}\right] < 0$ if and only if

$$-(\sqrt{n}+1)(4+2\sqrt{n}) + (1+4n+4\sqrt{n})(\tfrac{1}{2\sqrt{n}}) < 0. \tag{4.42}$$

Simplification of the left side to $-4-2\sqrt{n}-\frac{3}{2\sqrt{n}}$ shows that this is indeed negative for $n \in (0, 1]$ and thus $\frac{[P'(n)]^2}{P(n)P''(n)}$ is decreasing and a change in networking cost c will have a moderating effect on the informal pool of applicants in this case.

Finally, note that for $P(n)$ twice differentiable, strictly increasing and strictly concave, logarithmic concavity of P' is a sufficient but not necessary condition for the networking cost to have an intensifying effect on the pool of informal applicants. In particular, consider the arrival probability function corresponding to the cdf of a Weibull distribution, $P(n) = 1 - e^{-\lambda n^k}$, with rate parameter $\lambda > 0$ and shape parameter $k \in (0, 1)$. For this class of Weibull, $P(n)$ is strictly increasing and concave, but P' is not logarithmically concave.

For $P(n) = 1-e^{-\lambda n^k}$ we have $P'(n) = \lambda k e^{-\lambda n^k} \cdot n^{k-1} > 0$ and $P''(n) = P'(n) \cdot \frac{1}{n}[k-1-\lambda k n^k] < 0$. With these substitutions and some cancellation Condition (4.39) reduces to

$$\frac{d\Lambda(\kappa, d)}{dc} \gtrless 0 \quad \textit{if and only if} \quad \frac{\lambda k e^{-\lambda n_h^k} \cdot n_h^k}{(1-e^{-\lambda n_h^k})(k-1-\lambda k n_h^k)} \lessgtr \frac{\lambda k e^{-\lambda n_\ell^k} \cdot n_\ell^k}{(1-e^{-\lambda n_\ell^k})(k-1-\lambda k n_\ell^k)}. \tag{4.43}$$

Lemma 39 in Appendix C shows that for $k \in (0, 1)$ we have

$$\frac{d}{dn}\left[\frac{e^{-\lambda n^k} \cdot n^k}{(1-e^{-\lambda n^k})(k-1-\lambda k n^k)}\right] > 0 \quad \textit{if} \quad 1 - e^{-\lambda n^k} < \lambda n^k \left[1 - \tfrac{k}{k-1}\lambda n^k\right]. \tag{4.44}$$

For $k \in (0, 1)$, it is the case that $\left[1 - \frac{k}{k-1}\lambda n^k\right] > 1$. Since $1 - e^{-x} < x$ for all $x < 0$, we have $1 - e^{-\lambda n^k} < \lambda n^k$ and therefore $1 - e^{-\lambda n^k} < \lambda n^k \left[1 - \frac{k}{k-1}\lambda n^k\right]$. Thus $\frac{e^{-\lambda n^k} \cdot n^k}{(1-e^{-\lambda n^k})(k-1-\lambda k n^k)}$ is increasing in n and we have

$$\frac{e^{-\lambda n_h^k} \cdot n_h^k}{(1-e^{-\lambda n_h^k})(k-1-\lambda k n_h^k)} \gtrless \frac{e^{-\lambda n_\ell^k} \cdot n_\ell^k}{(1-e^{-\lambda n_\ell^k})(k-1-\lambda k n_\ell^k)} \quad \textit{if and only if} \quad n_h \gtrless n_\ell. \tag{4.45}$$

This means that a change in networking cost c has an intensifying effect on the pool of informal applicants when the informal arrival probability is of the form $P(n) = 1 - e^{-\lambda n^k}$ for $k \in (0, 1)$, even though the Weibull density function fails to be logarithmically concave.

4.2.3 Type-dependent Arrival Probablities and Costs

In addition to the assumption that P' is logarithmically concave, the networking model of this chapter has assumed that cost of networking and informal arrival probabilities do not vary by worker type, other than to the extent that workers have incentive to choose different levels of networking. However, in some scenarios it is plausible that this is not the case.

For example, high-type workers may have a higher opportunity cost of networking, or may have a lower networking cost if networking skills are related to job-relevant skills. Suppose that $C_q(n) = c_q n$ with $c_q > 0$ for $q \in \{h, \ell\}$. In this case, Table 4.1 does not describe which hiring patterns result in high- or low-quality worker types networking more. For a given firm strategy we will have

$$n_h(\phi_{Fh}, \phi_{Ih}) \gtreqless n_\ell(\phi_{F\ell}, \phi_{I\ell}) \quad \textit{if and only if} \quad \frac{(1-p_F\phi_{Fh})\phi_{Ih}}{c_h} \gtreqless \frac{(1-p_F\phi_{F\ell})\phi_{I\ell}}{c_\ell}. \tag{4.46}$$

One new implication of different networking costs is that for hiring patterns such that the firm treats both high- and low-report applications the same (that is, with $d_F \in \{0, 2\}$ and $d_I = 2$), worker networking patterns may differ by type. Absolute hiring in both channels will lead to a biased pool, rather than a neutral pool, with a favourable bias if $c_h < c_\ell$ and an unfavourable bias if $c_h > c_\ell$. Note that when high types have an advantage in networking such that $c_h < c_\ell$, then absolute hiring in the informal channel does not necessarily lead to an unfavourable informal pool composition as it does in the model with identical costs (shown in Lemma 12). This means that in equilibrium selective formal hiring can be possible to support with absolute informal hiring with a favourable pool, rather than only with an unfavourable pool. It also means that it can be possible to support a new class of non-trivial equilibria, with absolute hiring in the informal channel even when there is no hiring in the formal channel, $(d_F^*, d_I^*) = (0, 2)$. Finally it opens the possibility for equilibria in which only the high type networks.

On the benefits side, an alternative way to model the informal arrival process is that contacts may be more likely to connect high-quality workers to the firm than low-quality workers. Thus for a given level of networking, high-quality types may have higher informal arrival probability than low-quality types. For example, suppose contacts who make referrals know the applicant and have a personal knowledge or opinion about the applicant, and that the probability that a contact fails to pass a worker's informal application to the firm is $(\frac{1}{e})^{\lambda_q}$ for $\lambda_q > 0$ for $q \in \{h, \ell\}$. In this case, the informal arrival probability function is type-dependent, with $P_q(n) = 1 - e^{-\lambda_q n}$. This means that for $\phi_{Iq} \neq 0$ the optimal non-zero networking for each type q is given by

$$n_q(\phi_{Fq}, \phi_{Iq}) = \frac{1}{\lambda_q} \ln\left(\frac{\hat{w}\lambda_q(1-\phi_{Fq})\phi_{Iq}}{c}\right), \tag{4.47}$$

provided that $c < \hat{w}\lambda_q(1-\phi_{Fq})\phi_{Iq}$. I will consider only non-zero networking here.

For this scenario, which worker networks more for a given firm strategy is given by

$$n_h(\phi_{Fh}, \phi_{Ih}) \gtrless n_\ell(\phi_{F\ell}, \phi_{I\ell}) \quad \textit{if and only if} \quad \left(\frac{\hat{w}\lambda_h(1-p_F\phi_{Fh})\phi_{Ih}}{c}\right)^{\frac{1}{\lambda_h}} \gtrless \left(\frac{\hat{w}\lambda_\ell(1-p_F\phi_{F\ell})\phi_{I\ell}}{c}\right)^{\frac{1}{\lambda_\ell}}. \tag{4.48}$$

Note that $\left(\frac{\hat{w}(1-p_F\phi_{Fq})\phi_{Iq}}{c}\right)^{\frac{1}{\lambda_q}}$ is decreasing in λ_q for $c < \hat{w}(1-p_F\phi_{Fq})\phi_{Iq}$, while $\lambda_q^{\frac{1}{\lambda_q}}$ is increasing in λ_q when $\lambda_q \in (0, e)$ and decreasing when $\lambda_q > e$.

Suppose that contacts are more effective for high-quality types, with $\lambda_h > \lambda_\ell$, such that the probability that a contact fails to be useful is lower for high-quality types than for low-quality types, $(\frac{1}{e})^{\lambda_h} < (\frac{1}{e})^{\lambda_\ell}$. When this probability is sufficiently low for both types, with $\lambda_h, \lambda_\ell > e$, we will have $\lambda_h^{\frac{1}{\lambda_h}} < \lambda_\ell^{\frac{1}{\lambda_\ell}}$. Furthermore because $x^{\frac{1}{\lambda_h}}$ is decreasing in λ_h for all $x > 1$ we have

$$\left(\frac{\hat{w}(1-p_F\phi_{Fh})\phi_{Ih}}{c}\right)^{\frac{1}{\lambda_h}} < \left(\frac{\hat{w}(1-p_F\phi_{Fh})\phi_{Ih}}{c}\right)^{\frac{1}{\lambda_\ell}}. \tag{4.49}$$

Now whenever $(1-p_F\phi_{Fh})\phi_{Ih} \leq (1-p_F\phi_{F\ell})\phi_{I\ell}$, such that conditional on informal arrival, low-quality types have equal or greater chance of being not accepted formally and accepted informally than high-quality types, we will also have

$$\left(\frac{\hat{w}(1-p_F\phi_{Fh})\phi_{Ih}}{c}\right)^{\frac{1}{\lambda_\ell}} < \left(\frac{\hat{w}(1-p_F\phi_{F\ell})\phi_{I\ell}}{c}\right)^{\frac{1}{\lambda_\ell}} \tag{4.50}$$

because $x^{\frac{1}{\lambda_\ell}}$ is increasing in x for all $x > 0$. Since we have $\lambda_h^{\frac{1}{\lambda_h}} < \lambda_\ell^{\frac{1}{\lambda_\ell}}$, by inequalities (4.49) and (4.50), condition (4.48) gives $n_h(\phi_{Fh}, \phi_{Ih}) < n_\ell(\phi_{F\ell}, \phi_{I\ell})$. Thus when contacts are sufficiently effective but more effective for high-quality types, $\lambda_h > \lambda_\ell > e$, the pool of informal applicants will necessarily be unfavourable for $(d_F, d_I) \in \{0, 1, 2\} \times \{2\}$ or for $(d_F, d_I) = (1, 1)$ when $\varepsilon_I - \varepsilon_F \geq \dfrac{1-p_F}{p_F}(1-2\varepsilon_I)$. As with type-dependent networking costs, type-dependent informal arrival probabilities open the possibility for equilibria in which only the high type networks.

Chapter 5

Conclusions

There is a wide variation seen in reports of the use of different job search and recruitment methods by firms and workers. Economic theory tends to suggest that informal methods such as referrals are good, although empirical evidence on this point is mixed. There is currently little understanding of how search strategies respond to changes in the environment, for example as advancing technologies and the internet affect the information and costs associated with social networking. There has been little focus on the optimal hiring strategies of heterogeneous firms in particular. In order to improve understanding in these areas I have studied hiring patterns and workers' endogenous use of informal methods according to the screening abilities and needs in different industries and under different cost structures.

In Chapter 2 have introduced a model of hiring through a single channel in which the noisy signal received by the firm is accompanied by an additional exogenous signal. This model offers an endogenous explanation for the positive selection of informal applicants which is typically assumed in other theories of referral based on homophily in networks. Although compatible, my result does not rely on the presence of homophily effects.

In Chapter 3 have expanded this model to allow for the availability of an additional hiring channel, in order to investigate the simultaneous use of informal and formal methods of search.

This model is able to account for the possibility of negative selection into the pool of informal applicants which is sometimes observed empirically rather than the typically assumed positive selection. To my knowledge, such a model of incomplete information has not been previously studied with heterogeneous worker productivity and endogenous use of the informal channel.

In my treatment of the one- and two-channel hiring models, I found that existence and uniqueness of equilibria as well as many comparative static assessments were sensitive to the specification of the cost to workers of using the informal channel. Therefore in Chapter 4 I also incorporated into the previous models a more explicit scenario of informal networking. Although I maintain a reduced form approach, it is sufficient to provide useful interpretation and give greater structure to the cost function. This allows for sharper comparative static results in the one- and two-channel models.

Although my model has shown the role of endogenous arrival and screening technologies in determining whether informal search methods lead to higher or lower quality job candidates, it is not well-suited to clarify the present questions and ambiguities concerning the wage effect of referrals. However, based on my findings, future empirical researchers studying the effects of formal and informal search methods on wages and other outcomes may find it fruitful to separate quality-sensitive industries from industries where quality is less crucial, and to preserve the distinction between informal methods which rely on costly contacts and those which involve family and friends.

Duration of search has also not been addressed by my model. The equilibrium effects that formal and informal screening technologies have on the applicant pool are likely to remain relevant when search duration is taken into account and should be important in the trade-off firms face between filling a position quickly and filling it well. Therefore further contributions in this area would be valuable also.

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Appendix A

Additional Proof of Results for Chapter 2

Proof of Lemma 5. For $d \in (1,2)$ or $d \in (0,1)$, when p_h, $p_\ell > 0$ we have $\Lambda(d) = \dfrac{\psi(\hat{w}\phi_\ell(d))}{\psi(\hat{w}\phi_h(d))}$ where $\phi_h(d)$ and $\phi_\ell(d)$ are given by equations (2.31) and (2.32). Now differentiation on $d \in (1,2)$ or $d \in (0,1)$ gives $\dfrac{d\Lambda}{dd} \gtreqless 0$ if and only if

$$\psi(\hat{w}\phi_h(d)) \cdot \psi'(\hat{w}\phi_\ell(d))\hat{w}\frac{d\phi_\ell}{dd} - \psi(\hat{w}\phi_\ell(d)) \cdot \psi'(\hat{w}\phi_h(d))\hat{w}\frac{d\phi_h}{dd} \gtreqless 0. \tag{A.1}$$

For $d \in (1,2)$ as in *(i)*, equations (2.31) and (2.32) give $\dfrac{d\phi_h}{dd} = \varepsilon$ and $\dfrac{d\phi_\ell}{dd} = 1-\varepsilon$, so with this substitution and rearranging, inequality (A.1) implies $\dfrac{d\Lambda}{dd} > 0$ if and only if

$$\frac{\psi(\hat{w}\phi_h(d))}{\psi'(\hat{w}\phi_h(d))} > \frac{\psi(\hat{w}\phi_\ell(d))}{\psi'(\hat{w}\phi_\ell(d))} \cdot \frac{\varepsilon}{(1-\varepsilon)}. \tag{A.2}$$

If ψ is logarithmically concave, then $\dfrac{\psi}{\psi'}$ is increasing. Therefore because $\phi_h(d) > \phi_\ell(d)$ for $d \in (1,2)$ and because $\dfrac{\varepsilon}{(1-\varepsilon)} < 1$, inequality (A.2) is satisfied and we have $\dfrac{d\Lambda}{dd} > 0$.

Now for $d \in (0,1)$, equations (2.31) and (2.32) give $\dfrac{d\phi_h}{dd} = 1-\varepsilon$ and $\dfrac{d\phi_\ell}{dd} = \varepsilon$. So with this substitution and rearranging, inequality (A.1) implies $\frac{d\Lambda}{dd} \gtrless 0$ if and only if

$$\frac{\psi(\hat{w}\phi_h(d))}{\psi'(\hat{w}\phi_h(d))} \gtrless \frac{\psi(\hat{w}\phi_\ell(d))}{\psi'(\hat{w}\phi_\ell(d))} \cdot \frac{(1-\varepsilon)}{\varepsilon}. \tag{A.3}$$

If ψ is logarithmically convex, so $\frac{\psi}{\psi'}$ is decreasing, then because $\phi_h(d) > \phi_\ell(d)$ for all $d \in (0,1)$ and because $\frac{(1-\varepsilon)}{\varepsilon} > 1$, condition (A.3) implies $\frac{d\Lambda}{dd} < 0$. Thus $\Lambda(d)$ may be increasing or decreasing in d.

If however ψ has decreasing elasticity, then we have

$$\frac{\psi'(\hat{w}\phi_h(d))}{\psi(\hat{w}\phi_h(d))} \cdot \hat{w}\phi_h(d) < \frac{\psi'(\hat{w}\phi_\ell(d))}{\psi(\hat{w}\phi_\ell(d))} \cdot \hat{w}\phi_\ell(d). \tag{A.4}$$

because $\hat{w}\phi_h(d) > \hat{w}\phi_\ell(d)$ for all $d \in (0,1)$. Since for any $\alpha \in (0,1)$ we have $\phi_h(\alpha) = \alpha(1-\varepsilon)$ and $\phi_\ell(\alpha) = \alpha\varepsilon$, inequality (A.4) is equivalent to

$$\frac{\psi'(\hat{w}\alpha(1-\varepsilon))}{\psi(\hat{w}\alpha(1-\varepsilon))} \cdot \hat{w}\alpha(1-\varepsilon) < \frac{\psi'(\hat{w}\alpha\varepsilon)}{\psi(\hat{w}\alpha\varepsilon)} \cdot \hat{w}\alpha\varepsilon, \tag{A.5}$$

or alternatively

$$\frac{\psi(\hat{w}\alpha\varepsilon)}{\psi'(\hat{w}\alpha\varepsilon)} \cdot \frac{1-\varepsilon}{\varepsilon} < \frac{\psi(\hat{w}\alpha(1-\varepsilon))}{\psi'(\hat{w}\alpha(1-\varepsilon))}. \tag{A.6}$$

Thus we have

$$\psi(\hat{w}\phi_h(d))\psi'(\hat{w}\phi_h(d)) > \frac{\psi(\hat{w}\phi_\ell(d))}{\psi'(\hat{w}\phi_\ell(d))} \cdot \frac{(1-\varepsilon)}{\varepsilon} \tag{A.7}$$

for all $d = \alpha \in (0,1)$. Therefore we have $\frac{d\Lambda}{dd} > 0$. □

Proof of Proposition 1.

Suppose that $d_H^* = d_L^* = 1$ with some $p_h^*, p_\ell^* > 0$ is an equilibrium. By Lemma 3 it must be the case that $p_h^* = p_\ell^* = \psi(\hat{w}) > 0$, so we have $\hat{w} > \gamma'(0)$. By conditions (2.17) and (2.18), it must also be the case that $\frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\frac{\psi(\hat{w})}{\psi(\hat{w})}$, so $\frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}$.

Now suppose it is the case that $\hat{w} > \gamma'(0)$ and also $\frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}$. Then by condition (2.14), $d_H = 1$ is optimal for the firm in response to $p_h = p_\ell = \psi(\hat{w})$, while by conditions (2.17) and (2.18) $d_L = 1$ is (at least weakly) optimal. On the other hand, by Lemma 3, $p_h = p_\ell = \psi(\hat{w})$ is optimal for the workers in response to $d_H = d_L = 1$. Thus $d_H^* = d_L^* = 1$ with $p_h^* = p_\ell^* = \psi(\hat{w}) > 0$ is an equilibrium. □

Proof of Proposition 2.

Suppose that $d_H^* = 1$, $d_L^* = 0$ with some $p_h^*, p_\ell^* > 0$ is an equilibrium. By Lemma 3 it must be the case that $p_h^* = \psi(\hat{w}(1-\varepsilon)) > 0$ and $p_\ell^* = \psi(\hat{w}\varepsilon) > 0$, so certainly $\hat{w}\varepsilon > \gamma'(0)$. Then by conditions (2.14) and (2.15), it must also be the case that $\frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\frac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))}$ and by conditions (2.18) and (2.19) it must be the case that $\frac{v_h - w}{w - v_\ell} \leq \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\frac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))}$.

Now suppose $\hat{w}\varepsilon > \gamma'(0)$ and $\frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\frac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))} \leq \frac{v_h - w}{w - v_\ell} \leq \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\frac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))}$. Note that $\hat{w}\varepsilon > \gamma'(0)$ implies $\hat{w}(1-\varepsilon) > \gamma'(0)$ so that $\frac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))}$ is well-defined. By conditions (2.18) and (2.19) , $d_L = 0$ is at least weakly optimal for the firm in response to $p_h = \psi(\hat{w}(1-\varepsilon))$ and $p_\ell = \psi(\hat{w}\varepsilon)$, as by conditions (2.14) and (2.15) $d_H = 1$ is also. On the other hand, by Lemma 3, $p_h = \psi(\hat{w}(1-\varepsilon))$ and $p_\ell = \psi(\hat{w}\varepsilon)$ are optimal for the workers in response to $d_H = 1$ and $d_L = 0$. Thus $d_H^* = 1$, $d_L^* = 0$ with $p_h^* = \psi(\hat{w}(1-\varepsilon)) > 0$ and $p_\ell^* = \psi(\hat{w}\varepsilon) > 0$ is an equilibrium. □

Proof of Proposition 3.

Suppose that $d_H^* = 1$, $d_L^* = \beta \in (0,1)$ with some $p_h^*, p_\ell^* > 0$ is an equilibrium. By Lemma 3 it must be the case that $p_h^* = \psi(\hat{w}((1-\varepsilon) + \varepsilon\beta)) > 0$ and $p_\ell^* = \psi(\hat{w}(\varepsilon + (1-\varepsilon)\beta)) > 0$, so certainly $\hat{w}(\varepsilon + (1-\varepsilon)\beta) > \gamma'(0)$. Then by condition (2.18), it must be the case that $\frac{v_h - w}{w - v_\ell} = \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\frac{\psi(\hat{w}(\varepsilon + (1-\varepsilon)\beta))}{\psi(\hat{w}((1-\varepsilon) + \varepsilon\beta))}$.

Now suppose $\hat{w}(\varepsilon + (1-\varepsilon)\beta) > \gamma'(0)$ and also $\frac{v_h - w}{w - v_\ell} = \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\frac{\psi(\hat{w}(\varepsilon + (1-\varepsilon)\beta))}{\psi(\hat{w}((1-\varepsilon) + \varepsilon\beta))}$ for some $\beta \in (0,1)$. Note that $\hat{w}(\varepsilon + (1-\varepsilon)\beta) > \gamma'(0)$ implies $\hat{w}((1-\varepsilon) + \varepsilon\beta) > \gamma'(0)$, so that $\frac{\psi(\hat{w}(\varepsilon + (1-\varepsilon)\beta))}{\psi(\hat{w}((1-\varepsilon) + \varepsilon\beta))}$ is well-defined. By condition (2.14), $d_H^* = 1$ is optimal for the firm in response to $p_h = \psi(\hat{w}(1-\varepsilon)\beta)$ and $p_\ell = \psi(\hat{w}(\varepsilon + (1-\varepsilon)\beta))$, while by condition (2.18), $d_L^* = \beta \in (0,1)$ is optimal also (at least weakly) . On the other hand, by Lemma 3, $p_h = \psi(\hat{w}((1-\varepsilon)+\varepsilon\beta))$ and $p_\ell = \psi(\hat{w}\varepsilon\beta)$ are optimal for the workers in response to $d_H = 1$, and $d_L = \beta$. Thus $d_H^* = 1$, $d_L^* = \beta \in (0,1)$ with $p_h^* = \psi(\hat{w}((1-\varepsilon) + \varepsilon\beta)) > 0$ and $p_\ell^* = \psi(\hat{w}(\varepsilon + (1-\varepsilon)\beta)) > 0$ is an equilibrium.

Since a necessary condition for this equilibrium is that $\beta \in (0,1)$ exists such that $\frac{v_h-w}{w-v_\ell} = \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\frac{\psi(\hat{w}(\varepsilon + (1-\varepsilon)\beta))}{\psi(\hat{w}((1-\varepsilon) + \varepsilon\beta))}$, and since ψ is continuous, this equilibrium can exist for $\frac{v_h-w}{w-v_\ell} > \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\underline{\underline{\Lambda}}$ and $\frac{v_h-w}{w-v_\ell} < \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\overline{\overline{\Lambda}}$, where $\underline{\underline{\Lambda}} \equiv \inf_{\beta\in(0,1)} \frac{\psi(\hat{w}(\varepsilon + (1-\varepsilon)\beta))}{\psi(\hat{w}((1-\varepsilon) + \varepsilon\beta))}$ and where $\overline{\overline{\Lambda}} \equiv \sup_{\beta\in(0,1)} \frac{\psi(\hat{w}(\varepsilon + (1-\varepsilon)\beta))}{\psi(\hat{w}((1-\varepsilon) + \varepsilon\beta))} = 1$. □

Proof of Proposition 4.

Suppose that $d_H^* = \alpha \in (0,1)$, $d_L^* = 0$ with some $p_h^*, p_\ell^* > 0$ is an equilibrium. By Lemma 3 it must be the case that $p_h^* = \psi(\hat{w}(1-\varepsilon)\alpha)$ and $p_\ell^* = \psi(\hat{w}\varepsilon\alpha)$, so $\hat{w}\varepsilon\alpha > \gamma'(0)$. Then by condition (2.15), it must be the case that $\frac{v_h-w}{w-v_\ell} = \frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\frac{\psi(\hat{w}\varepsilon\alpha)}{\psi(\hat{w}(1-\varepsilon)\alpha)}$.

Now suppose $\hat{w}\varepsilon\alpha > \gamma'(0)$ and $\frac{v_h-w}{w-v_\ell} = \frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\frac{\psi(\hat{w}\varepsilon\alpha)}{\psi(\hat{w}(1-\varepsilon)\alpha)}$ for some $\alpha \in (0,1)$. Note that $\hat{w}\varepsilon\alpha > \gamma'(0)$ implies $\hat{w}(1-\varepsilon)\alpha > \gamma'(0)$, so that $\frac{\psi(\hat{w}\varepsilon\alpha)}{\psi(\hat{w}(1-\varepsilon)\alpha)}$ is well-defined. By condition (2.19), $d_L = 0$ is optimal for the firm in response to $p_h = \psi(\hat{w}(1-\varepsilon)\alpha)$ and $p_\ell = \psi(\hat{w}\varepsilon\alpha)$, while by condition (2.15), $d_H = \alpha$ is (at least weakly) optimal. On the other hand, by Lemma (3), $p_h = \psi(\hat{w}(1-\varepsilon)\alpha)$ and $p_\ell = \psi(\hat{w}\varepsilon\alpha)$ are optimal for the workers when $d_H = \alpha$ and $d_L = 0$. So $d_H^* = \alpha \in (0,1)$, $d_L^* = 0$ with $p_h^* = \psi(\hat{w}(1-\varepsilon)\alpha) > 0$ and $p_\ell^* = \psi(\hat{w}\varepsilon\alpha) > 0$ is an equilibrium.

Since a necessary condition for this equilibrium is that $\alpha \in (0,1)$ exists such that $\frac{v_h-w}{w-v_\ell} = \frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\frac{\psi(\hat{w}\varepsilon\alpha)}{\psi(\hat{w}(1-\varepsilon)\alpha)}$, and since ψ is continuous, this equilibrium can exist for $\frac{v_h-w}{w-v_\ell} > \frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\underline{\Lambda}$ and $\frac{v_h-w}{w-v_\ell} < \frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\overline{\Lambda}$. □

Proof of Corollary 3.

Note that $\gamma'(p) > 0$ for all $p > 0$ implies $\psi(x) > 0$ for all $x > 0$. By Proposition 4, an equilibrium with p_h^*, $p_\ell^* > 0$ exists for $\frac{v_h - w}{w - v_\ell} \in \left(\frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\underline{\Lambda}, \frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\overline{\Lambda}\right)$ where $\underline{\Lambda} \equiv \inf_{\alpha \in (0,1)} \frac{\psi(\hat{w}\varepsilon\alpha)}{\psi(\hat{w}(1-\varepsilon)\alpha)}$ and $\overline{\Lambda} \equiv \sup_{\alpha \in (0,1)} \frac{\psi(\hat{w}\varepsilon\alpha)}{\psi(\hat{w}(1-\varepsilon)\alpha)}$. This is because $\psi(\hat{w}\varepsilon\alpha) > 0$ will be satisfied for all $\alpha \in (0,1)$. By Proposition 2 an equilibrium with p_h^*, $p_\ell^* > 0$ exists for all $\frac{v_h - w}{w - v_\ell} \in \left[\frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\frac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))}, \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\frac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))}\right]$ because $\psi(\hat{w}\varepsilon) > 0$ is satisfied. Now ψ is continuous and $\frac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))} \leq \sup_{\alpha \in (0,1)} \frac{\psi(\hat{w}\varepsilon\alpha)}{\psi(\hat{w}(1-\varepsilon)\alpha)}$, so an equilibrium exists for all $\frac{v_h - w}{w - v_\ell} \in \left(\frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\underline{\Lambda}, \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\frac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))}\right]$.

By Proposition 3 an equilibrium with p_h^*, $p_\ell^* > 0$ exists for $\frac{v_h - w}{w - v_\ell} \in \left(\frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\underline{\underline{\Lambda}}, \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\right)$ where $\underline{\underline{\Lambda}} \equiv \inf_{\beta \in (0,1)} \frac{\psi(\hat{w}(\varepsilon + (1-\varepsilon)\beta))}{\psi(\hat{w}((1-\varepsilon) + \varepsilon\beta))}$. This is because for all $\beta \in (0,1)$ we will have $\psi(\hat{w}(\varepsilon + (1-\varepsilon)\beta)) > 0$. Now ψ is continuous and $\frac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))} \geq \inf_{\beta \in (0,1)} \frac{\psi(\hat{w}(\varepsilon + (1-\varepsilon)\beta))}{\psi(\hat{w}((1-\varepsilon) + \varepsilon\beta))}$, so an equilibrium exists for all $\frac{v_h - w}{w - v_\ell} \in \left(\frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\underline{\Lambda}, \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}\right)$.

By Proposition 1 an equilibrium with p_h^*, $p_\ell^* > 0$ exists for all $\frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}$ because $\psi(\hat{w}) > 0$ is satisfied. Therefore an equilibrium with p_h^*, $p_\ell^* > 0$ exists for all $\frac{v_h - w}{w - v_\ell} > \frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\underline{\Lambda}$. □

Proof of Corollary 4.

For any firm strategy $d \in (0,2]$ there is at most one non-zero best response for each worker type, which is given by $p_h = \psi(\hat{w}(d_H(1-\varepsilon) + d_L\varepsilon))$ for high types and $p_\ell = \psi(\hat{w}(d_H\varepsilon + d_L(1-\varepsilon))$ for low types. Therefore no equilibria with non-zero arrival coexist in which d^* is the same. Under condition *(i)* any equilibrium with non-zero arrival must be a pure strategy equilibrium and have $d^* = 1$ by Propositions 1-4. Therefore $d^* = 1$ with $p_h^* = \psi(\hat{w}(1-\varepsilon))$ and $p_\ell^* = \psi(\hat{w}\varepsilon)$ is the unique non-zero arrival equilibrium in this case. Similarly under condition *(ii)*, any equilibrium with non-zero arrival must be a pure strategy equilibrium and have $d^* = 2$ by Propositions 1-4. Therefore $p_h^* = \psi(\hat{w})$ and $p_\ell^* = \psi(\hat{w})$ is the unique non-zero arrival equilibrium in this case.

Note that no equilibrium with non-zero arrival and $d^* < 1$ can exist under condition *(iii)* by Proposition 4. Now if ψ is logarithmically concave, then by Lemma 5 and because γ' is continuous, we know $\Lambda(d)$ is strictly increasing for all $d \in (1,2)$, from $\Lambda(1) = \frac{p_\ell(d_H^*, d_L^*)}{p_h(d_H^*, d_L^*)} = \frac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))}$ to $\Lambda(2) = \frac{\psi(\hat{w})}{\psi(\hat{w})} = 1$. Since by Lemma 2 the firm's optimal hiring strategy is decreasing in Λ, there can at most be one equilibrium with non-zero arrival under condition *(iii)*.

Now when ψ has decreasing elasticity, it must also be logarithmically concave. By Lemma 5(i) $\Lambda(d)$ is strictly increasing for all $d \in (1,2)$ and by Lemma 5(ii) it is strictly increasing for all $d \in (0,1)$. Since ψ is continuous it is strictly increasing for all $d \in (0,2]$. Since by Lemma 2 the firm's optimal hiring strategy is decreasing in Λ, there can at most be one equilibrium with non-zero arrival under condition *(iv)*. □

Proof of Lemma 7.

Restricting attention to non-zero arrival equilibria, by Propositions 2, 4, and 1, condition *(i)* implies $d^* = 1$, condition *(ii)* implies $d^* = 1+\beta$ with $\beta \in (0,1)$, and condition *(iii)* implies $d^* = 2$. In each case $d^* > 0$ and $p_q^* = \psi(\hat{w}\phi_q(d^*)) > 0$ for $q \in \{h, \ell\}$. Under condition *(i)* and *(iii)* there is a unique equilibrium with non-zero arrival because for $d^* = 1$ or $d^* = 2$ there is a unique non-zero best response for workers of each type. Under conditions *(i)* and *(iii)*, a local change in ε will lead to new arrival probabilities in equilibrium, but will not lead to a change in firm strategy in equilibrium. Therefore under *(i)* and *(iii)*, by Lemma 4 we will have $\frac{d\Lambda^{d^*}}{d\varepsilon} = \frac{d}{d\varepsilon}\left[\frac{\psi(\hat{w}\phi_\ell(d^*))}{\psi(\hat{w}\phi_h(d^*))}\right]$ where

$$\phi_h(d^*) = \begin{cases} (1-\varepsilon) & \textit{if } d^* = 1 \\ 1 & \textit{if } d^* = 2 \end{cases} \tag{A.8}$$

and

$$\phi_\ell(d^*) = \begin{cases} \varepsilon & \textit{if } d^* = 1 \\ 1 & \textit{if } d^* = 2. \end{cases} \tag{A.9}$$

Thus by differentiation and rearranging we will have $\frac{d\Lambda^{d^*}}{d\varepsilon} \gtrless 0$ if and only if

$$\frac{\psi'(\hat{w}\phi_\ell^*)}{\psi(\hat{w}\phi_\ell^*)} \cdot \frac{d\phi_\ell^*}{d\varepsilon} \gtrless \frac{\psi'(\hat{w}\phi_h^*)}{\psi(\hat{w}\phi_h^*)} \cdot \frac{d\phi_h^*}{d\varepsilon} \tag{A.10}$$

where

$$\frac{d\phi_\ell^*}{d\varepsilon} = \begin{cases} 1 & \text{if } d^* = 1 \\ 0 & \text{if } d^* = 2 \end{cases} \tag{A.11}$$

and

$$\frac{d\phi_h^*}{d\varepsilon} = \begin{cases} -1 & \text{if } d^* = 1 \\ 0 & \text{if } d^* = 2. \end{cases} \tag{A.12}$$

Note that ψ and ψ' are strictly positive. Now since $\frac{d\phi_\ell^*}{d\varepsilon} > 0$ and $\frac{d\phi_h^*}{d\varepsilon} < 0$ for $d^* = 1$, we will have $\frac{\psi'(\hat{w}\phi_\ell^*)}{\psi(\hat{w}\phi_\ell^*)} \cdot \frac{d\phi_\ell^*}{d\varepsilon} > \frac{\psi'(\hat{w}\phi_h^*)}{\psi(\hat{w}\phi_h^*)} \cdot \frac{d\phi_h^*}{d\varepsilon}$ so that $\frac{d\Lambda^{d^*}}{d\varepsilon} > 0$ in any equilibrium with $d^* = 1$. For $d^* = 2$, we have $\frac{d\phi_\ell^*}{d\varepsilon} = 0$ and $\frac{d\phi_h^*}{d\varepsilon} = 0$, so $\frac{d\Lambda^2}{d\varepsilon} = 0$.

Now under condition *(ii)*, any equilibrium will have $d^* = 1+\beta$ with $\beta \in (0,1)$, and must satisfy

$$\Lambda^{d^*} = \frac{v_h - w}{w - v_\ell} \frac{s}{1-s} \frac{\varepsilon}{1-\varepsilon} \tag{A.13}$$

by Proposition 3. In this setting, logarithmically concave ψ implies that there is a unique equilibrium with non-zero arrival, by Corollary 4. However, any local change in ε will result not only in a change in arrival probabilities, but also a change in the firm's mixing probability, since an equilibrium can only exist if equation (A.13) is satisfied. Thus an increase in ε requires an increase in Λ^{d^*} in order to sustain an equilibrium. □

Lemma 30. *Any equilibrium with $d^* > 0$ and p_h^*, $p_\ell^* > 0$ will have $\frac{du_h^*}{d\varepsilon} \leq 0$ and $\frac{du_\ell^*}{d\varepsilon} \geq 0$.*

Proof. Suppose there is an equilibrium with $d^* > 0$ and p_h^*, $p_\ell^* > 0$. For $q \in \{h, \ell\}$ we have $u_q^*(p_q^*, \varepsilon) = \hat{w} p_q^* \phi_q^* + b - \gamma(p_q^*)$ with ϕ_h^* and ϕ_ℓ^* given by equations (A.8) and (A.9) above, and so by the envelope theorem we have

$$\frac{du_q}{d\varepsilon} = \frac{\partial u_q(p_q^*, \varepsilon)}{\partial \varepsilon}\bigg|_{p_q^*=\psi(\hat{w}\phi_q^*)} = \left(\hat{w} p_q^* \frac{d\phi_q^*}{d\varepsilon}\right)\bigg|_{p_q^*=\psi(\hat{w}\phi_q^*)} = \hat{w} \cdot \psi(\hat{w}\phi_q^*)\frac{d\phi_q^*}{d\varepsilon}. \tag{A.14}$$

Now for $q = h$, by equation (A.11) we will have $\frac{d\phi_h^*}{d\varepsilon} < 0$ for all $d^* \in (0, 2)$ and $\frac{d\phi_h^*}{d\varepsilon} = 0$ for $d^* = 2$. For $q = \ell$, by equation (A.12) we will have $\frac{d\phi_\ell^*}{d\varepsilon} < 0$ for all $d^* \in (0, 2)$ and $\frac{d\phi_\ell^*}{d\varepsilon} = 0$ for $d^* = 2$. Therefore $\frac{du_h^*}{d\varepsilon} \le 0$ and $\frac{du_\ell^*}{d\varepsilon} \ge 0$. □

Lemma 31. *Any equilibrium with $d^* > 0$ and p_h^*, $p_\ell^* > 0$ will have $\frac{d\pi^*}{d\varepsilon} \le 0$.*

Proof. Suppose there is an equilibrium with $d^* > 0$ and p_h^*, $p_\ell^* > 0$. Differentiating equation (2.36) with respect to ε gives

$$\begin{aligned}\frac{d\pi^*}{d\varepsilon} =& (v_h - w)s\Big[\psi'(\hat{w}\phi_h^*)\hat{w} + \psi(\hat{w}\phi_h^*)\Big]\frac{d\phi_h^*}{d\varepsilon} \\ & - (v_\ell - w)(1-s)\Big[\psi'(\hat{w}\phi_\ell^*)\hat{w} + \psi(\hat{w}\phi_\ell^*)\Big]\frac{d\phi_\ell^*}{d\varepsilon}.\end{aligned} \tag{A.15}$$

Since $\frac{d\phi_h^*}{d\varepsilon} < 0$ and $\frac{d\phi_\ell^*}{d\varepsilon} > 0$ for all $d^* \in (0, 2]$, we have $\frac{d\pi^*}{d\varepsilon} \le 0$ □

Lemma 32. *Suppose there is an equilibrium with $d^* > 0$ and p_h^*, $p_\ell^* > 0$. Then we will have $\frac{du_q^*}{dw} \ge 0$ for $q \in \{h, \ell\}$.*

Proof. For $q \in \{h, \ell\}$ we have $u_q^*(p_q^*, w) = \hat{w} p_q^* \phi_q^* + b - \gamma(p_q^*)$ with $\hat{w} = w - b$, so by the envelope theorem we have

$$\frac{du_q}{dw} = \frac{\partial u_q(p_q^*, w)}{\partial w}\bigg|_{p_q^*=\psi(\hat{w}\phi_q^*)} = \left(p_q^*\phi_q^*\right)\bigg|_{p_q^*=\psi(\hat{w}\phi_q^*)} = \psi(\hat{w}\phi_q^*) \cdot \phi_q^* \ge 0. \tag{A.16}$$

□

Proof of Lemma 8.

Restricting attention to non-zero arrival equilibria, by Propositions 2, 4, and 1, condition *(i)* implies $d^* = 1$, condition *(ii)* implies $d^* = 2$, and condition *(iii)* implies $d^* = 1{+}\beta$ with $\beta \in (0, 1)$. In each case $d^* > 0$ and $p^*_q = \psi(\hat{w}\phi_q(d^*)) > 0$ for $q \in \{h, \ell\}$. Under condition *(i)* and *(ii)* there is a unique equilibrium with non-zero arrival because for $d^* = 1$ or $d^* = 2$ there is a unique non-zero best response for workers of each type. Under conditions *(i)* and *(ii)*, a local change in w will lead to new arrival probabilities in equilibrium, but will not lead to a change in firm strategy in equilibrium. Therefore under condition *(i)* or *(ii)*, we will have

$$\frac{d\Lambda^{d^*}}{dw} = \frac{d}{dw}\left[\frac{\psi(\hat{w}\phi_\ell(d^*))}{\psi(\hat{w}\phi_h(d^*))}\right]. \tag{A.17}$$

By differentiation and cross-multiplication we obtain the condition

$$\frac{d\Lambda^{d^*}}{dw} \gtrless 0 \;\; iff \;\; \frac{\psi'(\hat{w}\phi^*_\ell)\phi^*_\ell}{\psi(\hat{w}\phi^*_\ell)} \gtrless \frac{\psi'(\hat{w}\phi^*_h)\phi^*_h}{\psi(\hat{w}\phi^*_h)}. \tag{A.18}$$

Note that ψ and ψ' are strictly positive.

Now under condition *(iii)*, any equilibrium will have $d^* = 1{+}\beta$ with $\beta \in (0, 1)$, and must satisfy

$$\Lambda^{d^*} = \frac{v_h - w}{w - v_\ell}\frac{s}{1-s}\frac{\varepsilon}{1-\varepsilon} \tag{A.19}$$

by Proposition 3. In this setting, logarithmically concave ψ implies that there is a unique equilibrium with non-zero arrival, by Corollary 4. However, any local change in w will result not only in a change in arrival probabilities, but also a change in the firm's mixing probability, since an equilibrium can only exist if equation (A.19) is satisfied. Thus an increase in w requires a decrease in Λ^{d^*} in order to sustain an equilibrium, because $\frac{v_h - w}{w - v_\ell}$ is decreasing in w.

□

Lemma 33. *Suppose there is an equilibrium with $d^* > 0$ and p^*_h, $p^*_\ell > 0$. Then we will have $\frac{dp^*_q}{db} > 0$ for $q \in \{h, \ell\}$.*

Proof. Since it must be the case that $p_q^* = \psi(\hat{w}\phi_q^*)$ in such an equilibrium, we will have $\dfrac{dp_q^*}{db} = -\phi_q^* \cdot \psi'(\hat{w}\phi_q^*) < 0$. □

Lemma 34. *Suppose there is an equilibrium with $d^* > 0$ and $p_h^*, p_\ell^* > 0$. Then we will have $\dfrac{du_q^*}{db} > 0$ for $q \in \{h, \ell\}$.*

Proof. For $q \in \{h, \ell\}$ we have $u_q^*(p_q^*, b) = \hat{w}p_q^*\phi_q^* + b - \gamma(p_q^*)$ with $\hat{w} = w - b$, so by the envelope theorem we have

$$\frac{du_q}{db} = \left.\frac{\partial u_q(p_q^*, b)}{\partial b}\right|_{p_q^*=\psi(\hat{w}\phi_q^*)} = \left.\left(1 - p_q^*\phi_q^*\right)\right|_{p_q^*=\psi(\hat{w}\phi_q^*)} = 1 - \psi(\hat{w}\phi_q^*) \cdot \phi_q^*. \tag{A.20}$$

Now since $p_q^* = \psi(\hat{w}\phi_q^*) < 1$ and $\phi_q^* \leq 1$, we have $\dfrac{du_q^*}{db} > 0$ for $q \in \{h, \ell\}$. □

Lemma 35. *Restrict attention to non-zero arrival in equilibrium and suppose ψ is logarithmically concave. Suppose that either*

(i) $\dfrac{v_h - w}{w - v_\ell} \in \left(\dfrac{1-s}{s}\dfrac{\varepsilon}{1-\varepsilon}\dfrac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))}, \dfrac{1-s}{s}\dfrac{1-\varepsilon}{\varepsilon}\dfrac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))}\right)$, *or*

(ii) $\dfrac{v_h - w}{w - v_\ell} > \dfrac{1-s}{s}\dfrac{1-\varepsilon}{\varepsilon}$.

Then we will have

$$\frac{d\Lambda^{d^*}}{db} \gtrless 0 \ \textit{iff}\ \frac{\psi'(\hat{w}\phi_\ell^*)\phi_\ell^*}{\psi(\hat{w}\phi_\ell^*)} \lessgtr \frac{\psi'(\hat{w}\phi_h^*)\phi_h^*}{\psi(\hat{w}\phi_h^*)}. \tag{A.21}$$

Suppose instead that

(iii) $\dfrac{v_h - w}{w - v_\ell} \in \left(\dfrac{1-s}{s}\dfrac{1-\varepsilon}{\varepsilon}\dfrac{\psi(\hat{w}\varepsilon)}{\psi(\hat{w}(1-\varepsilon))}, \dfrac{1-s}{s}\dfrac{1-\varepsilon}{\varepsilon}\right)$.

Then we will have $\dfrac{d\Lambda^{d^*}}{db} = 0$.

Proof. Restricting attention to non-zero arrival equilibria, by Propositions 2, 4, and 1, condition *(i)* implies $d^* = 1$, and condition *(ii)* implies $d^* = 2$. In each of these cases $d^* > 0$ and $p_q^* = \psi(\hat{w}\phi_q(d^*)) > 0$ for $q \in \{h, \ell\}$. Under both conditions there is a unique equilibrium with non-zero arrival because for $d^* = 1$ or $d^* = 2$ there is a unique non-zero best response for workers of each type. A local change in b will lead to new arrival probabilities in equilibrium, but will not lead to a change in firm strategy in equilibrium. Therefore under condition *(i)* or *(ii)*, we will have

$$\frac{d\Lambda^{d^*}}{db} = \frac{d}{db}\left[\frac{\psi(\hat{w}\phi_\ell(d^*))}{\psi(\hat{w}\phi_h(d^*))}\right]. \tag{A.22}$$

Condition (35) is then obtained directly from differentiation and cross-multiplication. Note that ψ and ψ' are strictly positive.

Now under condition *(iii)*, any equilibrium will have $d^* = 1+\beta$ with $\beta \in (0,1)$, and must satisfy

$$\Lambda^{d^*} = \frac{v_h - w}{w - v_\ell}\frac{s}{1-s}\frac{\varepsilon}{1-\varepsilon} \tag{A.23}$$

by Proposition 3. In this setting, logarithmically concave ψ implies that there is a unique equilibrium with non-zero arrival, by Corollary 4. For each b there is a unique $d^* = 1+\beta(b)$ such that $\Lambda(d^*(b))$ satisfies equation (A.23). Thus, allowing for adjustment of the firm strategy $d^*(b)$ to maintain the existence of equilibrium as b changes, the equilibrium pool of applicants will remain constant, $\dfrac{d\Lambda(d^*(b))}{db} = \dfrac{d}{db}\left[\dfrac{v_h-w}{w-v_\ell}\dfrac{s}{1-s}\dfrac{\varepsilon}{1-\varepsilon}\right] = 0.$ □

Lemma 36. *Let π^d denote the firm's profit given worker best responses $p_h(d)$ and $p_\ell(d)$ to $d \in \{0,1,2\}$. We have*

(i) $\pi^0 = 0$,

(ii) $\pi^1 \geq 0$ *if and only if*

$$\frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\frac{p_\ell(1)}{p_h(1)}, \tag{A.24}$$

(iii) $\pi^2 \geq 0$ *if and only if*

$$\frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s}\frac{p_\ell(2)}{p_h(2)}, \tag{A.25}$$

(iv) $\pi^2 \geq \pi^1$ *if and only if*

$$\frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s}\left[\frac{p_\ell(2) - \varepsilon p_\ell(1)}{p_h(2) - (1-\varepsilon)p_h(1)}\right]. \tag{A.26}$$

Proof. For given worker best responses $p_h(d)$ and $p_\ell(d)$ to $d \in \{0, 1, 2\}$, the firm's profit is given by

$$\pi^d = (v_h - w)sp_h(d)\phi_h(d) - (w - v_\ell)(1-s)p_\ell(d)\phi_\ell(d). \tag{A.27}$$

(i) For $d = 0$, we have $\phi_h(0) = \phi_\ell(0) = 0$ and therefore $\pi^0 = 0$.

(ii) For $d = 1$, we have $\phi_h(1) = 1-\varepsilon$ and $\phi_\ell(1) = \varepsilon$. So equation (A.27) for $d = 1$ becomes

$$\pi^1 = (v_h - w)sp_h(1)(1-\varepsilon) - (w - v_\ell)(1-s)p_\ell(1)\varepsilon. \tag{A.28}$$

Setting $\pi^1 \geq 0$ and rearranging gives $\pi^1 \geq 0$ if and only if

$$\frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s}\frac{\varepsilon}{1-\varepsilon}\frac{p_\ell(1)}{p_h(1)}. \tag{A.29}$$

(iii) For $d = 2$, we have $\phi_h(2) = \phi_\ell(2) = 1$ so equation (A.27) for $d = 2$ becomes

$$\pi^2 = (v_h - w)sp_h(2) - (w - v_\ell)(1-s)p_\ell(2). \tag{A.30}$$

Setting $\pi^2 \geq 0$ and rearranging gives $\pi^2 \geq 0$ if and only if

$$\frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s}\frac{p_\ell(2)}{p_h(2)}. \tag{A.31}$$

(iv) By equations (A.28) and (A.30) we have $\pi^2 \geq \pi^1$ if and only if

$$(v_h - w)s\left[p_h(2) - p_h(1)(1-\varepsilon)\right] - (w - v_\ell)(1-s)[p_\ell(2) - p_\ell(1)\varepsilon] \geq 0 \tag{A.32}$$

or equivalently, since $p_h(2) \geq p_h(1)$, we have $\pi^2 \geq \pi^1$ if and only if

$$\frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s}\left[\frac{p_\ell(2) - \varepsilon p_\ell(1)}{p_h(2) - (1-\varepsilon)p_h(1)}\right]. \tag{A.33}$$

□

Proof of Proposition 5.

In the model with commitment for a given wage, the firm anticipates the following best response of workers to hiring strategy $d \in \{0, 1, 2\}$:

$$p_q(d) = \begin{cases} 0 & \textit{if } d = 0 \\ \dfrac{\hat{w}\phi_q(d)}{c} & \textit{if } d = 1 \\ \dfrac{\hat{w}}{c} & \textit{if } d = 0, \end{cases} \tag{A.34}$$

for $q \in \{h, \ell\}$ where $\phi_h(1) = 1-\varepsilon$ and $\phi_\ell(1) = \varepsilon$.

By Lemma 36, $d = 0$ is optimal for the firm when inequalities (A.24) and (A.25) both fail to hold strictly. Substituting $p_h(1) = \dfrac{\hat{w}(1-\varepsilon)}{c}$ and $p_\ell(1) = \dfrac{\hat{w}\varepsilon}{c}$, we see that inequality (A.24) fails to hold strictly when

$$\frac{v_h-w}{w-v_\ell} \leq \frac{1-s}{s}\Big(\frac{\varepsilon}{1-\varepsilon}\Big)^2, \tag{A.35}$$

and substituting $p_h(2) = p_\ell(2) = \dfrac{\hat{w}}{c}$ we see that inequality (A.25) fails to hold strictly when

$$\frac{v_h-w}{w-v_\ell} \leq \frac{1-s}{s}. \tag{A.36}$$

Since $\dfrac{\varepsilon}{1-\varepsilon} < 1$, this means that $d = 0$ is optimal for the firm when $\dfrac{v_h-w}{w-v_\ell} \leq \dfrac{1-s}{s}\Big(\dfrac{\varepsilon}{1-\varepsilon}\Big)^2$. High-and low-quality workers' best responses to $d = 0$ are $p_h = 0$ and $p_\ell = 0$, so $d^* = 0$ with $p_h^* = p_\ell^* = 0$ is an equilibrium for $\dfrac{v_h-w}{w-v_\ell} \leq \dfrac{1-s}{s}\Big(\dfrac{\varepsilon}{1-\varepsilon}\Big)^2$.

Again by Lemma 36, $d = 1$ is optimal for the firm when inequality (A.24) holds but inequality (A.26) does not hold strictly. Substituting $p_h(1) = \dfrac{\hat{w}(1-\varepsilon)}{c}$ and $p_\ell(1) = \dfrac{\hat{w}\varepsilon}{c}$, we see that inequality (A.24) holds when

$$\frac{v_h-w}{w-v_\ell} \geq \frac{1-s}{s}\Big(\frac{\varepsilon}{1-\varepsilon}\Big)^2. \tag{A.37}$$

Substituting $p_h(2) = p_\ell(2) = \dfrac{\hat{w}}{c}$ and $p_h(1) = \dfrac{\hat{w}(1-\varepsilon)}{c}$ and $p_\ell(1) = \dfrac{\hat{w}\varepsilon}{c}$, we see that inequality (A.26) does not hold strictly when

$$\frac{v_h - w}{w - v_\ell} \leq \frac{1-s}{s}\left[\frac{1-\varepsilon^2}{1-(1-\varepsilon)^2}\right]. \tag{A.38}$$

Now since $\varepsilon < 1/2$, we have $\left(\dfrac{\varepsilon}{1-\varepsilon}\right)^2 < 1$ and $\dfrac{1-\varepsilon^2}{1-(1-\varepsilon)^2} > 1$. Therefore $d = 1$ is optimal for the firm when $\dfrac{1-s}{s}\left(\dfrac{\varepsilon}{1-\varepsilon}\right)^2 \leq \dfrac{v_h-w}{w-v_\ell} \leq \dfrac{1-s}{s}\dfrac{1-\varepsilon^2}{1-(1-\varepsilon)^2}$. High-and low-quality workers' best responses to $d = 1$ are $p_h = \dfrac{\hat{w}(1-\varepsilon)}{c}$ and $p_\ell = \dfrac{\hat{w}\varepsilon}{c}$, so $d^* = 1$ with $p_h^* = \dfrac{\hat{w}(1-\varepsilon)}{c}$, $p_\ell^* = \dfrac{\hat{w}\varepsilon}{c}$ is an equilibrium for $\dfrac{1-s}{s}\left(\dfrac{\varepsilon}{1-\varepsilon}\right)^2 \leq \dfrac{v_h-w}{w-v_\ell} \leq \dfrac{1-s}{s}\dfrac{1-\varepsilon^2}{1-(1-\varepsilon)^2}$.

Again by Lemma 36, $d = 2$ is optimal for the firm when inequalities (A.25) and (A.26) both hold. Substituting $p_h(2) = p_\ell(2) = \dfrac{\hat{w}}{c}$ we see that inequality (A.25) holds when

$$\frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s}, \tag{A.39}$$

and substituting $p_h(2) = p_\ell(2) = \dfrac{\hat{w}}{c}$ and $p_h(1) = \dfrac{\hat{w}(1-\varepsilon)}{c}$ and $p_\ell(1) = \dfrac{\hat{w}\varepsilon}{c}$, we see that inequality (A.26) holds when

$$\frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s}\left[\frac{1-\varepsilon^2}{1-(1-\varepsilon)^2}\right]. \tag{A.40}$$

Since $\dfrac{1-\varepsilon^2}{1-(1-\varepsilon)^2} > 1$, this means $d = 2$ is optimal for the firm when $\dfrac{v_h-w}{w-v_\ell} \geq \dfrac{1-s}{s}\left[\dfrac{1-\varepsilon^2}{1-(1-\varepsilon)^2}\right]$. High-and low-quality workers' best responses to $d = 2$ are $p_h = p_\ell = \dfrac{\hat{w}}{c}$, so $d^* = 2$ with $p_h^* = p_\ell^* = \dfrac{\hat{w}}{c}$ is an equilibrium for $\dfrac{v_h-w}{w-v_\ell} \geq \dfrac{1-s}{s}\left[\dfrac{1-\varepsilon^2}{1-(1-\varepsilon)^2}\right]$. □

Proof of Proposition 6.

In the model with commitment for a given wage, the firm anticipates the following best response of workers to hiring strategy $d \in \{0, 1, 2\}$:

$$p_q(d) = \begin{cases} 0 & \text{if } d = 0 \\ \dfrac{\hat{w}\phi_q(d) - c}{\hat{w}\phi_q^*} & \text{if } d = 1 \\ \dfrac{\hat{w} - c}{\hat{w}} & \text{if } d = 2, \end{cases} \tag{A.41}$$

for $q \in \{h, \ell\}$ where $\phi_h(1) = 1-\varepsilon$ and $\phi_\ell(1) = \varepsilon$.

By Lemma 36, $d = 0$ is optimal for the firm when inequalities (A.24) and (A.25) both fail to hold strictly. Substituting $p_h(1) = \dfrac{\hat{w}(1-\varepsilon) - c}{\hat{w}(1-\varepsilon)}$ and $p_\ell(1) = \dfrac{\hat{w}\varepsilon - c}{\hat{w}\varepsilon}$, we see that inequality (A.24) fails to hold strictly when

$$\frac{v_h - w}{w - v_\ell} \leq \frac{1-s}{s} \frac{\hat{w}\varepsilon - c}{\hat{w}(1-\varepsilon) - c}, \tag{A.42}$$

Substituting $p_h(2) = p_\ell(2) = \dfrac{\hat{w} - c}{\hat{w}}$ we see that inequality (A.25) fails to hold strictly when

$$\frac{v_h - w}{w - v_\ell} \leq \frac{1-s}{s}. \tag{A.43}$$

Since it is the case that $\dfrac{\hat{w}\varepsilon - c}{\hat{w}(1-\varepsilon) - c} < 1$, then $d = 0$ is optimal for the firm when $\dfrac{v_h - w}{w - v_\ell} \leq \dfrac{1-s}{s} \dfrac{\hat{w}\varepsilon - c}{\hat{w}(1-\varepsilon) - c}$. High-and low-quality workers' best responses to $d = 0$ are $p_h = 0$ and $p_\ell = 0$, so $d^* = 0$ with $p_h^* = p_\ell^* = 0$ is an equilibrium for $\dfrac{v_h - w}{w - v_\ell} \leq \dfrac{1-s}{s} \dfrac{\hat{w}\varepsilon - c}{\hat{w}(1-\varepsilon) - c}$.

Again by Lemma 36, $d = 1$ is optimal for the firm when inequality (A.24) holds but inequality (A.26) does not hold strictly. Substituting $p_h(1) = \dfrac{\hat{w}(1-\varepsilon) - c}{\hat{w}(1-\varepsilon)}$ and $p_\ell(1) = \dfrac{\hat{w}\varepsilon - c}{\hat{w}\varepsilon}$, we see that inequality (A.24) holds when

$$\frac{v_h - w}{w - v_\ell} \geq \frac{1-s}{s} \frac{\hat{w}\varepsilon - c}{\hat{w}(1-\varepsilon) - c}. \tag{A.44}$$

Substituting $p_h(2) = p_\ell(2) = \dfrac{\hat{w} - c}{\hat{w}}$ and $p_h(1) = \dfrac{\hat{w}(1-\varepsilon) - c}{\hat{w}(1-\varepsilon)}$ and $p_\ell(1) = \dfrac{\hat{w}\varepsilon - c}{\hat{w}\varepsilon}$, we see that inequality (A.26) does not hold strictly when

$$\frac{v_h - w}{w - v_\ell} \le \frac{1-s}{s}\left[\frac{\hat{w} - c - (\hat{w}\varepsilon - c)}{\hat{w} - c - (\hat{w}(1-\varepsilon) - c)}\right]. \tag{A.45}$$

This inequality reduces to

$$\frac{v_h - w}{w - v_\ell} \le \frac{1-s}{s}\left[\frac{\hat{w} - \hat{w}\varepsilon}{\hat{w} - \hat{w}(1-\varepsilon)}\right] \tag{A.46}$$

and further to

$$\frac{v_h - w}{w - v_\ell} \le \frac{1-s}{s}\left[\frac{1-\varepsilon}{\varepsilon}\right]. \tag{A.47}$$

Now in condition (A.44) we have $\frac{\hat{w}\varepsilon - c}{\hat{w}(1-\varepsilon) - c} < 1$ and in condition (A.47) we have $\frac{1-\varepsilon}{\varepsilon} > 1$. Therefore $d = 1$ is optimal for the firm when $\frac{1-s}{s}\frac{\hat{w}\varepsilon - c}{\hat{w}(1-\varepsilon) - c} \le \frac{v_h - w}{w - v_\ell} \le \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}$. High-and low-quality workers' best responses to $d = 1$ are $p_h = \frac{\hat{w}(1-\varepsilon) - c}{\hat{w}(1-\varepsilon)}$ and $p_\ell = \frac{\hat{w}\varepsilon - c}{\hat{w}\varepsilon}$, so $d^* = 1$ with $p_h^* = \frac{\hat{w}(1-\varepsilon) - c}{\hat{w}(1-\varepsilon)}$, $p_\ell^* = \frac{\hat{w}\varepsilon - c}{\hat{w}\varepsilon}$ is an equilibrium for $\frac{1-s}{s}\frac{\hat{w}\varepsilon - c}{\hat{w}(1-\varepsilon) - c} \le \frac{v_h - w}{w - v_\ell} \le \frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon}$.

Again, by Lemma 36, $d = 2$ is optimal for the firm when inequalities (A.25) and (A.26) both hold. Substituting $p_h(2) = p_\ell(2) = \frac{\hat{w} - c}{\hat{w}}$ we see that inequality (A.25) holds when

$$\frac{v_h - w}{w - v_\ell} \ge \frac{1-s}{s}, \tag{A.48}$$

and substituting $p_h(2) = p_\ell(2) = \frac{\hat{w} - c}{\hat{w}}$ and $p_h(1) = \frac{\hat{w}(1-\varepsilon) - c}{\hat{w}(1-\varepsilon)}$ and $p_\ell(1) = \frac{\hat{w}\varepsilon - c}{\hat{w}\varepsilon}$, we see that inequality (A.26) holds when

$$\frac{v_h - w}{w - v_\ell} \ge \frac{1-s}{s}\left[\frac{\hat{w} - c - (\hat{w}\varepsilon - c)}{\hat{w} - c - (\hat{w}(1-\varepsilon) - c)}\right] \tag{A.49}$$

or equivalently when

$$\frac{v_h - w}{w - v_\ell} \ge \frac{1-s}{s}\left[\frac{1-\varepsilon}{\varepsilon}\right]. \tag{A.50}$$

Since $\frac{1-\varepsilon}{\varepsilon} > 1$, this means $d = 2$ is optimal for the firm when $\frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon} \le \frac{v_h - w}{w - v_\ell}$. High-and low-quality workers' best responses to $d = 2$ are $p_h = p_\ell = \dfrac{\hat{w} - c}{\hat{w}}$, so $d^* = 2$ with $p_h^* = p_\ell^* = \dfrac{\hat{w} - c}{\hat{w}}$ is an equilibrium for $\frac{1-s}{s}\frac{1-\varepsilon}{\varepsilon} \leq \frac{v_h - w}{w - v_\ell}$. □

Appendix B

Additional Proof of Results for Chapter 3

Proof of Proposition 7.

Note that $\frac{v_h - w}{w - v_l} \geq \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}$ implies the industry is generally profitable, $\frac{v_h - w}{w - v_l} > \frac{1-s}{s}$, so any equilibrium must have $d_F \in \{1, 2\}$. This is because $d_F = 0$ would require $\frac{v_h - w}{w - v_l} \leq \frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}$, but it is the case that $\frac{\varepsilon_F}{1-\varepsilon_F} < 1$, so $d_F = 0$ can only be optimal in a generally unprofitable industry. Since for each firm strategy there can be at most one non-zero best response for workers, parts (a) and (b) cover all possible equilibria with $d_I^* = 2$ and non-trivial use of the informal channel.

For part (a) first suppose that $\frac{v_h - w}{w - v_l} \geq \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}$ and $\hat{w}(1-p_F) > \gamma'(0)$. Then high- and low-type workers have identical incentives to use the informal channel given $(d_F^*, d_I^*) = (2, 2)$, and will choose the same arrival probabilities (see Table 3.1) so that $\frac{p_\ell^*}{p_h^*} = 1$. By Lemma 11 $p_{Ih}^* = p_{I\ell}^* = \psi(\hat{w}(1-p_F))$ is an optimal response for high- and low-quality workers to absolute hiring in both channels. Now in this setting the industry is generally profitable but both informal and formal reports are not decisive. Since $\frac{v_h - w}{w - v_l} \geq \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}$ it is optimal for the firm to hire absolutely in the formal channel, $d_F^* = 2$ by Lemma 9. Since $\frac{v_h - w}{w - v_l} \geq \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}$, by Lemma 10 it will certainly be optimal to hire absolutely in the informal channel, $d_I = 2$, if the quality composition of the pool of informal applicants is the same as that of the general population. Thus $(d_F, d_I) = (2,2)$ is also a best response to $p_{Ih} = p_{I\ell} = \psi(\hat{w}(1-p_F))$ due to the neutral pool factor, $\frac{p_\ell}{p_h} = 1$. Thus $(d_F^*, d_I^*) = (2,2)$ and $p_{Ih}^* = p_{I\ell}^* = \psi(\hat{w}(1-p_F))$ is an equilibrium with non-zero use of the informal channel.

On the other hand, suppose that $(d_F^*, d_I^*) = (2,2)$ and $p_{Ih}^* = p_{I\ell}^* = \psi(\hat{w}(1-p_F))$ is an equilibrium with non-zero use of the informal channel. Thus worker arrival probabilities must be strictly positive, so informal arrival must not be prohibitively costly, with $\hat{w}(1-p_F) > \gamma'(0)$, and the informal pool composition must be neutral, $\frac{p_\ell^*}{p_h^*} = 1$. Thus by Lemma 9 it must also be the case that $\frac{v_h-w}{w-v_l} \geq \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}$ in order for the firm to be willing to hire absolutely in the formal channel. Thus the stated conditions of this proposition are necessary for this equilibrium.

Now for part (b) first suppose that $\frac{v_h-w}{w-v_l} \leq \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}$, $\hat{w}(1-p_F(1-\varepsilon_F)) > \gamma'(0)$, and $\frac{\psi(\hat{w}(1-p_F\varepsilon_F))}{\psi(\hat{w}(1-p_F(1-\varepsilon_F)))} \leq \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}$. Then by Lemma 11, $p_{Ih} = \psi(\hat{w}(1-p_F(1-\varepsilon_F)))$ and $p_{I\ell} = \psi(\hat{w}(1-p_F\varepsilon_F))$ are non-zero optimal responses for workers given selective formal hiring and absolute informal hiring, $(d_F, d_I) = (1,2)$. In this setting the industry is generally profitable but formal reports are decisive while informal reports are not. Since $\frac{v_h-w}{w-v_l} \leq \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}$ but $\frac{v_h-w}{w-v_l} > \frac{1-s}{s}$, it is optimal for the firm to hire selectively in the formal channel, $d_F = 1$ by Lemma 9. Since $\frac{v_h-w}{w-v_l} \geq \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}$, it will be optimal for the firm to hire absolutely in the informal channel, $d_I = 2$, provided that the quality composition of the pool of informal applicants is not too unfavourable relative to that of the general population, $\frac{p_\ell}{p_h} \leq \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}$, by Lemma 10. Thus $(d_F, d_I) = (1,2)$ is also a best response to $p_{Ih} = \psi(\hat{w}(1-p_F(1-\varepsilon_F)))$, $p_{I\ell} = \psi(\hat{w}(1-p_F\varepsilon_F))$, due to $\frac{p_\ell}{p_h} = \frac{\psi(\hat{w}(1-p_F\varepsilon_F))}{\psi(\hat{w}(1-p_F(1-\varepsilon_F)))}$. So $(d_F^*, d_I^*) = (1,2)$ and $p_{Ih}^* = \psi(\hat{w}(1-p_F(1-\varepsilon_F)))$, $p_{I\ell}^* = \psi(\hat{w}(1-p_F\varepsilon_F))$ is an equilibrium with non-zero use of the informal channel.

On the other hand, suppose that $(d_F^*, d_I^*) = (1,2)$ and $p_{Ih}^* = \psi(\hat{w}(1-p_F(1-\varepsilon_F)))$, $p_{I\ell}^* = \psi(\hat{w}(1-p_F\varepsilon_F))$ is an equilibrium with non-zero use of the informal channel. Then worker arrival probabilities must be strictly positive, so informal arrival must not be prohibitively costly, for high-quality workers in particular, with $\hat{w}(1-p_F(1-\varepsilon_F)) > \gamma'(0)$ (which implies $\hat{w}(1-p_F\varepsilon_F) >$

$\gamma'(0)$ also). By Lemma 9 it must also be the case that $\frac{v_h - w}{w - v_l} \leq \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}$ in order for the firm to be willing to hire selectively in the formal channel. Finally by Lemma 10 it must be the case that $\frac{v_h - w}{w - v_l} \geq \frac{1-s}{s}\frac{p^*_\ell}{p^*_h}\frac{1-\varepsilon_I}{\varepsilon_I}$ in order for the firm to be willing to hire absolutely in the informal channel. Therefore it must be the case that $\frac{\psi(\hat{w}(1-p_F\varepsilon_F))}{\psi(\hat{w}(1-p_F(1-\varepsilon_F)))} \leq \frac{v_h - w}{w - v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}$. Thus the stated conditions of this proposition are necessary for this equilibrium. □

Proof of Proposition 8.

Note that $\varepsilon_I - \varepsilon_F \geq \frac{1-p_F}{p_F}(1-2\varepsilon_I)$ implies that the formal report is strictly more precise than the informal report, $\varepsilon_F < \varepsilon_I$. Therefore since the informal report is decisive in this setting, $\frac{v_h - w}{w - v_l} \in \left(\frac{1-s}{s}\frac{\varepsilon_I}{1-\varepsilon_I}\ ,\ \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}\right)$, then the formal report must also be decisive, $\frac{v_h - w}{w - v_l} \in \left(\frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}\ ,\ \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}\right)$. Thus any equilibrium in this setting must have selective formal hiring, $d^*_F = 1$. By Lemma 14, an equilibrium with non-trivial use of the informal channel with $d^*_I = 2$ can not exist in this setting, so any equilibrium here must have selective hiring in the informal channel also, $d^*_I = 1$. Since for firm strategy $(d_F, d_I) = (1,1)$ there can be at most one non-zero best response for workers, the equilibrium described is the only possible equilibrium with non-trivial use of the informal channel in this setting.

First, suppose that $\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I) > \gamma'(0)$ and also that $\frac{\psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)}{\psi(\hat{w}(1-p_F(1-\varepsilon_F)(1-\varepsilon_I))} \leq \frac{v_h - w}{w - v_l}\frac{s}{1-s}\frac{1-\varepsilon_I}{\varepsilon_I}$. Then $p_{Ih} = \psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I))$, $p_{I\ell} = \psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)$ are non-zero best responses of the workers to a firm strategy of $(d_F, d_I) = (1,1)$ by Lemma 11. Now in this setting formal reports are decisive, therefore $d_F = 1$ is optimal for the firm by Lemma 9. And it is also the case that selective hiring $d_I = 1$ is a best response in the informal channel to $p_{Ih} = \psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I))$, $p_{I\ell} = \psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)$ by Lemma 10. This can be seen by noting that for $\varepsilon_I - \varepsilon_F \geq \frac{1-p_F}{p_F}(1-2\varepsilon_I)$ it must be the case that high types use the informal channel less than low types, $\psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I)) < \psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)$, so the informal pool composition is unfavourable, $\frac{p_\ell}{p_h} > 1$. Since informal reports are decisive, such that $\frac{v_h - w}{w - v_l} \in \left(\frac{1-s}{s}\frac{\varepsilon_I}{1-\varepsilon_I}\ ,\ \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}\right)$, we must also have $\frac{v_h - w}{w - v_l} < \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}\frac{p_\ell}{p_h}$. Since we have $\dfrac{\psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)}{\psi(\hat{w}(1-p_F(1-\varepsilon_F)(1-\varepsilon_I))} \leq \dfrac{v_h-w}{w-v_l}\dfrac{s}{1-s}\dfrac{1-\varepsilon_I}{\varepsilon_I}$, then it is also be the case that $\dfrac{v_h-w}{w-v_l} < \dfrac{1-s}{s}\dfrac{\varepsilon_I}{1-\varepsilon_I}\dfrac{p_\ell}{p_h}$. Thus by conditions (3.5) and (3.8) selective informal hiring, $d_I^* = 1$, is optimal for the firm in response to $p_{Ih} = \psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I))$, $p_{I\ell} = \psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)$. Therefore $(d_F^*, d_I^*) = (1,1)$ and $p_{Ih}^* = \psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I))$, $p_{I\ell}^* = \psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)$ is an equilibrium with non-zero use of the informal channel.

On the other hand, suppose that $(d_F^*, d_I^*) = (1,1)$ and $p_{Ih}^* = \psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I))$, $p_{I\ell}^* = \psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)$ is an equilibrium with non-zero use of the informal channel. Then worker arrival probabilities must be strictly positive, so informal arrival is not prohibitively costly, in particular for high-quality workers $\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I) > \gamma'(0)$ (note that for $\varepsilon_I - \varepsilon_F \geq \dfrac{1-p_F}{p_F}(1-2\varepsilon_I)$ this also implies $\hat{w}(1-p_F\varepsilon_F)\varepsilon_I > \gamma'(0)$). By Lemma 10 it must be the case that $\dfrac{\psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)}{\psi(\hat{w}(1-p_F(1-\varepsilon_F)(1-\varepsilon_I))} \leq \dfrac{v_h-w}{w-v_l}\dfrac{s}{1-s}\dfrac{1-\varepsilon_I}{\varepsilon_I}$ in order for the firm to be willing to hire selectively in the informal channel. Thus the stated conditions of this proposition are necessary for this equilibrium. □

Proof of Proposition 9.

Note that by Lemma 14, an equilibrium with non-trivial use of the informal channel with $d_I^* = 2$ can not exist in this setting, so any equilibrium here must have selective hiring in the informal channel, $d_I^* = 1$. Since for each firm strategy $(d_F, d_I) = (i, 1)$ for $i \in \{0,1,2\}$ there can be at most one non-zero best response for workers, parts (a), (b), and (c) cover all possible equilibria with non-trivial use of the informal channel in this setting.

For part (a), first suppose that $\dfrac{\psi(\hat{w}\varepsilon_I)}{\psi(\hat{w}(1-\varepsilon_I))} \geq \dfrac{v_h-w}{w-v_l}\dfrac{s}{1-s}\dfrac{\varepsilon_I}{1-\varepsilon_I}$, $\hat{w}\varepsilon_I > \gamma'(0)$, and $\dfrac{v_h-w}{w-v_l} \leq \dfrac{1-s}{s}\dfrac{\varepsilon_F}{1-\varepsilon_F}$. Then by Lemma 11 we know that $p_{Ih} = \psi(\hat{w}(1-\varepsilon_I))$ and $p_{I\ell} = \psi(\hat{w}\varepsilon_I)$ are best responses of the workers to a firm strategy of $(d_F, d_I) = (0,1)$. Now given $p_{Ih} = \psi(\hat{w}(1-\varepsilon_I))$ and $p_{I\ell} = \psi(\hat{w}\varepsilon_I)$, we will have a favourable pool of informal applicants, $\dfrac{p_\ell}{p_h} < 1$, because ψ is increasing. Thus $\dfrac{v_h-w}{w-v_l} > \dfrac{s}{1-s}\dfrac{\varepsilon_I}{1-\varepsilon_I}$ implies that $\dfrac{v_h-w}{w-v_l} > \dfrac{s}{1-s}\dfrac{\varepsilon_I}{1-\varepsilon_I}\dfrac{p_\ell}{p_h}$ also, so the condition (3.5) is satisfied. Since we also have $\dfrac{\psi(\hat{w}\varepsilon_I)}{\psi(\hat{w}(1-\varepsilon_I))} \geq \dfrac{v_h-w}{w-v_l}\dfrac{s}{1-s}\dfrac{\varepsilon_I}{1-\varepsilon_I}$, the condition (3.8) is also satisfied. So $d_I = 1$ is a best response in the informal channel to $p_{Ih} = \psi(\hat{w}(1-\varepsilon_I))$ and $p_{I\ell} = \psi(\hat{w}\varepsilon_I)$. Since $\frac{v_h-w}{w-v_l} \leq \frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}$, it is the case that $d_F = 0$ is optimal in the formal channel by Lemma 9. Therefore $(d_F^*, d_I^*) = (0, 1)$ and $p_{Ih}^* = \psi(\hat{w}(1-\varepsilon_I))$, $p_{I\ell}^* = \psi(\hat{w}\varepsilon_I)$ is an equilibrium.

On the other hand, suppose that $(d_F^*, d_I^*) = (0, 1)$ and $p_{Ih}^* = \psi(\hat{w}(1-\varepsilon_I))$, $p_{I\ell}^* = \psi(\hat{w}\varepsilon_I)$ is an equilibrium with non-zero use of the informal channel. Then worker arrival probabilities must be strictly positive, so informal arrival is not prohibitively costly, $\hat{w}\varepsilon_I > \gamma'(0)$ in particular for low-quality workers (note that this implies $\hat{w}(1-\varepsilon_I) > \gamma'(0)$ also). By Lemma 10 it must be the case that $\frac{\psi(\hat{w}\varepsilon_I)}{\psi(\hat{w}(1-\varepsilon_I))} \leq \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{1-\varepsilon_I}{\varepsilon_I}$ in order for the firm to be willing to hire selectively in the informal channel. By Lemma 9 it must be the case that $\frac{v_h-w}{w-v_l} \leq \frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}$ in order for $d_F^* = 0$ to be optimal in the formal channel. Thus the stated conditions of this proposition are necessary for this equilibrium.

For part (b), first suppose that $\hat{w}(1 - p_F\varepsilon_F)\varepsilon_I > \gamma'(0)$, $\frac{v_h-w}{w-v_l} \in \left[\frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}, \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}\right]$, and $\frac{\psi(\hat{w}(1 - p_F\varepsilon_F)\varepsilon_I)}{\psi(\hat{w}(1-p_F(1-\varepsilon_F)(1-\varepsilon_I))} \geq \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}$. Then by Lemma 11 we know that $p_{Ih} = \psi(\hat{w}(1 - p_F(1-\varepsilon_F))(1-\varepsilon_I))$ and $p_{I\ell} = \psi(\hat{w}(1 - p_F\varepsilon_F)\varepsilon_I)$ are best responses of the workers to a firm strategy of $(d_F, d_I) = (1,1)$. Now given $p_{Ih} = \psi(\hat{w}(1 - p_F(1-\varepsilon_F))(1-\varepsilon_I))$ and $p_{I\ell} = \psi(\hat{w}(1 - p_F\varepsilon_F)\varepsilon_I)$, we have a favourable pool of informal applicants, $\frac{p_\ell}{p_h} < 1$, by Lemma 13 because $\varepsilon_I - \varepsilon_F < \frac{1-p_F}{p_F}(1-2\varepsilon_I)$. Thus $\frac{v_h-w}{w-v_l} > \frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}$ implies $\frac{v_h-w}{w-v_l} > \frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}\frac{p_\ell}{p_h}$ also, so condition 3.5 is satisfied. Since we also have $\frac{\psi(\hat{w}(1 - p_F\varepsilon_F)\varepsilon_I)}{\psi(\hat{w}(1-p_F(1-\varepsilon_F)(1-\varepsilon_I))} \geq \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}$, the condition (3.8) is also satisfied. So $d_I = 1$ is a best response in the informal channel to $p_{Ih} = \psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I))$ and $p_{I\ell} = \psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)$. Since $\frac{v_h-w}{w-v_l} \in \left[\frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}, \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}\right]$, it is also the case that selective hiring is also optimal in the formal channel, $d_F = 1$, by Lemma 9. Therefore $(d_F^*, d_I^*) = (1, 1)$ and $p_{Ih}^* = \psi(\hat{w}(1 - p_F(1-\varepsilon_F))(1-\varepsilon_I))$, $p_{I\ell}^* = \psi(\hat{w}(1 - p_F\varepsilon_F)\varepsilon_I)$ is an equilibrium.

On the other hand, suppose that $(d_F^*, d_I^*) = (1, 1)$ and $p_{Ih}^* = \psi(\hat{w}(1 - p_F(1-\varepsilon_F))(1-\varepsilon_I))$, $p_{I\ell}^* = \psi(\hat{w}(1 - p_F\varepsilon_F)\varepsilon_I)$ is an equilibrium with non-zero use of the informal channel. Then worker arrival probabilities must be strictly positive, so informal arrival is not prohibitively costly, $\hat{w}(1-p_F\varepsilon_F)\varepsilon_I > \gamma'(0)$ in particular for low-quality workers (note that when $\varepsilon_I - \varepsilon_F < \frac{1-p_F}{p_F}(1-2\varepsilon_I)$ this implies $\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I) > \gamma'(0)$ also). By Lemma 10 it must be the case that $\frac{\psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)}{\psi(\hat{w}(1-p_F(1-\varepsilon_F)(1-\varepsilon_I))} \le \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{1-\varepsilon_I}{\varepsilon_I}$ in order for the firm to be willing to hire selectively in the informal channel. By Lemma 9 it must be the case that $\frac{v_h-w}{w-v_l} \in \left[\frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}\,,\ \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}\right]$ in order for $d_F^* = 1$ to be optimal in the formal channel. Thus the stated conditions of this proposition are necessary for this equilibrium.

For part (c), first suppose that we have $\hat{w}(1-p_F)\varepsilon_I > \gamma'(0)$, $\frac{v_h-w}{w-v_l} \ge \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}$, and $\frac{\psi(\hat{w}(1-p_F)\varepsilon_I))}{\psi(\hat{w}(1-p_F)(1-\varepsilon_I))} \ge \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}$. Then by Lemma 11 it must be the case that $p_{Ih} = \psi(\hat{w}(1-p_F)(1-\varepsilon_I))$ and $p_{I\ell} = \psi(\hat{w}(1-p_F)\varepsilon_I)$ are best responses of the workers to a firm strategy of $(d_F, d_I) = (2{,}1)$. Now given $p_{Ih} = \psi(\hat{w}(1-p_F)(1-\varepsilon_I))$ and $p_{I\ell} = \psi(\hat{w}(1-p_F)\varepsilon_I)$, we again have a favourable informal pool of applicants, $\frac{p_\ell}{p_h} < 1$, by Lemma 13 because ψ is increasing. Thus $\frac{v_h-w}{w-v_l} > \frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}$ implies that $\frac{v_h-w}{w-v_l} > \frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}\frac{p_\ell}{p_h}$ also, so condition (3.5) is satisfied. Since we also have $\frac{\psi(\hat{w}(1-p_F)\varepsilon_I))}{\psi(\hat{w}(1-p_F)(1-\varepsilon_I))} \ge \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}$, the condition (3.8) is also satisfied. Thus $d_I = 1$ is a best response in the informal channel to worker arrival probabilities $p_{Ih} = \psi(\hat{w}(1-p_F)(1-\varepsilon_I))$ and $p_{I\ell} = \psi(\hat{w}(1-p_F)\varepsilon_I)$. Since $\frac{v_h-w}{w-v_l} \in \left[\frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}\,,\ \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}\right]$, it is also the case that selective hiring is also optimal in the formal channel, $d_F = 1$, by Lemma 9. Therefore $(d_F^*, d_I^*) = (2,1)$ and $p_{Ih}^* = \psi(\hat{w}(1-p_F)(1-\varepsilon_I))$, $p_{I\ell}^* = \psi(\hat{w}(1-p_F)\varepsilon_I)$ is an equilibrium.

On the other hand, suppose that $(d_F^*, d_I^*) = (2,1)$ and $p_{Ih}^* = \psi(\hat{w}(1-p_F)(1-\varepsilon_I))$, $p_{I\ell}^* = \psi(\hat{w}(1-p_F)\varepsilon_I)$ is an equilibrium with non-zero use of the informal channel. Then worker arrival probabilities must be strictly positive, so informal arrival is not prohibitively costly, $\hat{w}(1-p_F)\varepsilon_I > \gamma'(0)$ in particular for low-quality workers (note that this implies $\hat{w}(1-p_F)(1-\varepsilon_I) > \gamma'(0)$ also). By Lemma 10 it must be the case that $\frac{\psi(\hat{w}(1-p_F)\varepsilon_I))}{\psi(\hat{w}(1-p_F)(1-\varepsilon_I))} \le \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{1-\varepsilon_I}{\varepsilon_I}$ in order for the firm to be willing to hire selectively in the informal channel. By Lemma 9 it must be the case that $\frac{v_h-w}{w-v_l} \le \frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}$ in order for absolute hiring to be optimal in the formal channel, $d_F^* = 2$. Thus the stated conditions of this proposition are necessary for this equilibrium. □

Proof of Proposition 10.

Note that by Lemma 14, an equilibrium with non-trivial use of the informal channel with $d_I^* = 2$ can not exist in this setting, so any equilibrium here must have selective hiring in the informal channel, $d_I^* = 1$. Now $\frac{v_h-w}{w-v_l} \leq \frac{1-s}{s}\frac{\varepsilon_I}{1-\varepsilon_I}$ implies that the industry is generally unprofitable, so absolute hiring in the formal channel can not be supported in any equilibrium, $d_F^* \neq 2$. Since for each firm strategy $(d_F, d_I) \in \{(0,1),(1,1)\}$ there can be at most one non-zero best response for workers, parts (a) and (b) cover all possible equilibria with non-trivial use of the informal channel in this setting.

For part (a), first suppose that $\hat{w}\varepsilon_I > \gamma'(0)$, that $\frac{v_h-w}{w-v_l} \leq \frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}$, and that $\frac{\psi(\hat{w}\varepsilon_I)}{\psi(\hat{w}(1-\varepsilon_I))} \in \left[\frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}, \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{1-\varepsilon_I}{\varepsilon_I}\right]$. Then we know that $p_{Ih} = \psi(\hat{w}(1-\varepsilon_I))$ and $p_{I\ell} = \psi(\hat{w}\varepsilon_I)$ are best responses of the workers to a firm strategy of $(d_F, d_I) = (0,1)$ by Lemma 11, and they are strictly positive. Note that $\hat{w}\varepsilon_I\gamma'(0)$ implies $\hat{w}(1-\varepsilon_I) > \gamma'(0)$. Meanwhile, we know that no hiring is optimal for the firm in the formal channel, $d_F = 0$, because $\frac{v_h-w}{w-v_l} \leq \frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}$. Furthermore, we know that $d_I = 1$ is a best response to $p_{Ih} = \psi(\hat{w}(1-\varepsilon_I))$ and $p_{I\ell} = \psi(\hat{w}\varepsilon_I)$ in the informal channel because $\frac{\psi(\hat{w}\varepsilon_I)}{\psi(\hat{w}(1-\varepsilon_I))} \in \left[\frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}, \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{1-\varepsilon_I}{\varepsilon_I}\right]$ implies that conditions (3.5) and (3.8) are satisfied.

On the other hand, suppose that $(d_F^*, d_I^*) = (0,1)$ and $p_{Ih}^* = \psi(\hat{w}(1-\varepsilon_I))$, $p_{I\ell}^* = \psi(\hat{w}\varepsilon_I)$ is an equilibrium with non-trivial use of the informal channel. In order for this equilibrium to have non-trivial use of the informal channel it must be the case that workers have strictly positive arrival probabilities in the informal channel. In particular, this must be true for low-quality workers, which implies that $\hat{w}\varepsilon_I > \gamma'(0)$. In order for this equilibrium to have no hiring in the formal channel, $d_F^* = 0$, it must be the case that conditions (3.2) and (3.4) are satisfied. This implies that $\frac{v_h-w}{w-v_l} \leq \frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}$. Finally, in order for this equilibrium to have selective hiring in the informal channel, $d_I^* = 1$, it must be the case that conditions (3.5) and (3.8) are satisfied for $\frac{p_\ell}{p_h} = \frac{\psi(\hat{w}\varepsilon_I)}{\psi(\hat{w}(1-\varepsilon_I))}$. This implies that we must have $\frac{\psi(\hat{w}\varepsilon_I)}{\psi(\hat{w}(1-\varepsilon_I))} \in \left[\frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}, \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{1-\varepsilon_I}{\varepsilon_I}\right]$. Thus the stated conditions of this proposition are necessary for this equilibrium.

For part (b), first suppose that $\hat{w}(1-p_F\varepsilon_F)\varepsilon_I > \gamma'(0)$, $\frac{v_h-w}{w-v_l} \in \left[\frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}, \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}\right]$, and $\frac{\psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)}{\psi(\hat{w}(1-p_F(1-\varepsilon_F)(1-\varepsilon_I))} \in \left[\frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}, \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{1-\varepsilon_I}{\varepsilon_I}\right]$. Then we know that $p_{Ih} = \psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I))$ and $p_{I\ell} = \psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)$ are best responses of the workers to a firm strategy of $(d_F, d_I) = (1, 1)$ by Lemma 11, and are strictly positive. Note that $\hat{w}(1-p_F\varepsilon_F)\varepsilon_I > \gamma'(0)$ implies $\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I) > \gamma'(0)$. Meanwhile, we know that selective hiring is optimal for the firm in the formal channel, $d_F = 1$, because we have $\frac{v_h-w}{w-v_l} \in \left[\frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}, \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}\right]$. This implies that conditions (3.1) and (3.4) are satisfied. Furthermore, we know that $d_I = 1$ is a best response to $p_{Ih} = \psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I))$ and $p_{I\ell} = \psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)$ in the informal channel because $\frac{\psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)}{\psi(\hat{w}(1-p_F(1-\varepsilon_F)(1-\varepsilon_I))} \in \left[\frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}, \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{1-\varepsilon_I}{\varepsilon_I}\right]$ implies that conditions (3.5) and (3.8) are satisfied. Therefore $(d_F^*, d_I^*) = (1, 1)$ and $p_{Ih}^* = \psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I)) > 0$, $p_{I\ell}^* = \psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I) > 0$ is an equilibrium.

On the other hand, suppose that $(d_F^*, d_I^*) = (1, 1)$ and $p_{Ih}^* = \psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I))$, $p_{I\ell}^* = \psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)$ is an equilibrium with non-trivial use of the informal channel. In order for this equilibrium to have non-trivial use of the informal channel it must be the case that workers have strictly positive arrival probabilities in the informal channel. In particular, this must be true for low-quality workers, which implies that $\hat{w}(1-p_F\varepsilon_F)\varepsilon_I > \gamma'(0)$. In order for this equilibrium to have selective hiring in the formal channel, $d_F^* = 1$, it must be the case that conditions (3.1) and (3.4) are satisfied. This implies that $\frac{v_h-w}{w-v_l} \in \left[\frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}, \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}\right]$. Finally, in order for this equilibrium to have selective hiring in the informal channel, $d_I^* = 1$, it must be the case that conditions (3.5) and (3.8) are satisfied for $\frac{p_\ell}{p_h} = \frac{\psi(\hat{w}\varepsilon_I)}{\psi(\hat{w}(1-\varepsilon_I))}$. This implies that we must have $\frac{\psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)}{\psi(\hat{w}(1-p_F(1-\varepsilon_F)(1-\varepsilon_I))} \in \left[\frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}, \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{1-\varepsilon_I}{\varepsilon_I}\right]$. Thus the stated conditions of this proposition are necessary for this equilibrium. $\square$

Proof of Proposition 11.

Note that $\frac{v_h-w}{w-v_l} \geq \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}$ implies that the industry is generally profitable, so $d_F^* = 0$ is not possible to sustain in any equilibrium. However, if the firm hires absolutely in the formal channel, $d_F^* = 2$, any equilibrium with selective hiring in the informal channel, $d_I^* = 1$, will have a strictly favourable pool of informal applicants (see Table 3.1). Since condition (3.8) can not be satisfied for $\frac{v_h-w}{w-v_l} \geq \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}$ when $\frac{p_\ell}{p_h} < 1$, selective hiring will not be a best response in the informal channel so such an equilibrium is not possible. Thus any equilibrium in this setting with $d_I^* = 1$ and non-trivial use of the informal channel must have $d_F^* = 1$. Since for firm strategy $(d_F, d_I) = (1,1)$ there can be at most one non-zero best response for workers, the equilibrium described in this proposition is the only possible equilibrium with $d_I^* = 1$ and non-trivial use of the informal channel in this setting.

Now first suppose that $\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I) > \gamma'(0)$, that $\varepsilon_I - \varepsilon_F > \frac{1-p_F}{p_F}(1-2\varepsilon_I)$, that $\frac{v_h-w}{w-v_l} \leq \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}$, and that $\frac{\psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)}{\psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I))} \in \left[\frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}, \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{1-\varepsilon_I}{\varepsilon_I}\right]$. Then it will be the case that $p_{Ih} = \psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I))$ and $p_{I\ell} = \psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)$ are best responses for workers to firm strategy $(d_F, d_I) = (1,1)$ by Lemma 11, and they are both positive because $\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I) > \gamma'(0)$ implies $\hat{w}(1-p_F\varepsilon_F)\varepsilon_I > \gamma'(0)$. Meanwhile we know that selective hiring in the formal channel is optimal for the firm. This is due to the fact that $\frac{v_h-w}{w-v_l} \leq \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}$ implies condition (3.4) is satisfied, and also due to the fact that $\frac{v_h-w}{w-v_l} \geq \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}$ implies that the industry is generally profitable so it must be the case that $\frac{v_h-w}{w-v_l} > \frac{1-s}{s}\frac{\varepsilon_F}{1-\varepsilon_F}$ also. Thus formal reports are decisive in this setting and condition (3.1) is satisfied so $d_F = 1$ is indeed optimal. Now since $\frac{\psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)}{\psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I))} \in \left[\frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}, \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{1-\varepsilon_I}{\varepsilon_I}\right]$, it is also the case that conditions (3.5) and (3.8) are satisfied when $p_{Ih} = \psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I))$ and $p_{I\ell} = \psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)$. Therefore selective hiring in the informal channel, $d_I = 1$, is a best response to $p_{Ih} = \psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I))$ and $p_{I\ell} = \psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)$. Therefore $(d_F^*, d_I^*) = (1,1)$ and $p_{Ih}^* = \psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I))$, $p_{I\ell}^* = \psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)$ is an equilibrium.

On the other hand, suppose that $(d_F^*, d_I^*) = (1,1)$ and $p_{Ih}^* = \psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I))$, $p_{I\ell}^* = \psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)$ is an equilibrium with non-trivial use of the informal channel. In order for it to be the case that the informal channel is actually used, we know that workers must have strictly positive arrival probabilities in the informal channel. In particular, this must be true for high-quality workers, which implies that $\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I) > \gamma'(0)$. In order for this equilibrium to have selective hiring in the formal channel, $d_F^* = 1$, it must be the case that conditions (3.1) and (3.4) are satisfied. In particular this implies that $\frac{v_h-w}{w-v_l} \leq \frac{1-s}{s}\frac{1-\varepsilon_F}{\varepsilon_F}$. Finally, in order for this equilibrium to have selective hiring in the informal channel, $d_I^* = 1$, it must be the case that conditions (3.5) and (3.8) are satisfied for $\frac{p_\ell}{p_h} = \frac{\psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)}{\psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I))}$. This implies that we must have

$$\frac{\psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)}{\psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I))} \in \left[\frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{\varepsilon_I}{1-\varepsilon_I}, \frac{v_h-w}{w-v_l}\frac{s}{1-s}\frac{1-\varepsilon_I}{\varepsilon_I}\right]. \tag{B.1}$$

But this condition is equivalent to having $\frac{v_h-w}{w-v_l} \in \left[\frac{1-s}{s}\frac{\varepsilon_I}{1-\varepsilon_I}\Lambda^{11}, \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}\Lambda^{11}\right]$ for $\Lambda^{11} = \frac{\psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I)}{\psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I))}$. Since in the setting of this proposition we have $\frac{v_h-w}{w-v_l} \geq \frac{1-s}{s}\frac{1-\varepsilon_I}{\varepsilon_I}$, this condition can only be satisfied if the informal pool is unfavourable $\Lambda^{11} > 1$, which means that low-quality workers must arrive with a higher probability than do high-quality workers, so that $\psi(\hat{w}(1-p_F\varepsilon_F)\varepsilon_I) > \psi(\hat{w}(1-p_F(1-\varepsilon_F))(1-\varepsilon_I))$. Since ψ is an increasing function, this can only be true when $(1-p_F(1-\varepsilon_F))(1-\varepsilon_I) < (1-p_F\varepsilon_F)\varepsilon_I$. Rearranging algebraically, this condition is equivalent to $\varepsilon_I-\varepsilon_F > \frac{1-p_F}{p_F}(1-2\varepsilon_I)$. Thus the stated conditions of this proposition are necessary for this equilibrium. □

Proof of Lemma 19.

Within a given class of (non-trivial) equilibria with firm strategy $(d_F^*, d_I^*) = (i,j) \in \{0,1,2\}\times\{1,2\}$, we have $\Lambda^{ij} \equiv \frac{\psi(\hat{w}(1-p_F\phi_{F\ell}^*)\phi_{I\ell}^*)}{\psi(\hat{w}(1-p_F\phi_{Fh}^*)\phi_{Ih}^*)}$, where $\phi_{Ah}^* = (1-\varepsilon)d_{AH}^* + \varepsilon d_{AL}^*$ and $\phi_{A\ell}^* = \varepsilon d_{AH}^* + (1-\varepsilon)d_{AL}^*$ for $A \in \{F, I\}$.

For $d_I^* = 2$, Λ^{i2} is constant with respect to ε_I because $\phi_{Ih}^* = \phi_{I\ell}^* = 1$. Now for $d_I^* = 1$

we have $\phi^*_{Ih} = 1-\varepsilon_I$ and $\phi^*_{I\ell} = \varepsilon_I$, therefore differentiation of Λ^{i1} with respect to ε_I gives the condition $\dfrac{d\Lambda^{i1}}{d\varepsilon_I} > 0$ if and only if

$$\begin{aligned}&\psi(\hat{w}(1-p_F\phi^*_{Fh})(1-\varepsilon_I)) \cdot \psi'(\hat{w}(1-p_F\phi^*_{F\ell})\varepsilon_I)\hat{w}(1-p_F\phi^*_{F\ell})\\ &\qquad - \psi(\hat{w}(1-p_F\phi^*_{F\ell})\varepsilon_I) \cdot \psi'(\hat{w}(1-p_F\phi^*_{Fh})(1-\varepsilon))\hat{w}(1-p_F\phi^*_{Fh})(-1) > 0.\end{aligned} \tag{B.2}$$

This inequality is certainly satisfied because both ψ and ψ' are positive and $1-\phi^*_{Fq} > 0$ for $q \in \{h, \ell\}$, so both terms on the left hand side are positive. Thus $\dfrac{d\Lambda^{ij}}{d\varepsilon_I} \geq 0$ within any class of equilibria with $(d^*_F, d^*_I) = (i, j) \in \{0, 1, 2\} \times \{1, 2\}$.

For $d^*_F = 0$, Λ^{0j} is constant with respect to ε_F because $\phi^*_{Fh} = \phi^*_{F\ell} = 0$. Similarly for $d^*_F = 2$, Λ^{2j} is constant with respect to ε_F because $\phi^*_{Fh} = \phi^*_{F\ell} = 1$. Now for $d^*_F = 1$ we have $\phi^*_{Fh} = 1-\varepsilon_F$ and $\phi^*_{F\ell} = \varepsilon_F$, therefore differentiation of Λ^{1j} with respect to ε_F gives the condition $\dfrac{d\Lambda^{1j}}{d\varepsilon_F} < 0$ if and only if

$$\begin{aligned}&\psi(\hat{w}(1-p_F(1-\varepsilon_F))\phi^*_{Ih}) \cdot \psi'(\hat{w}(1-p_F\varepsilon_F)\phi^*_{I\ell})\hat{w}(-p_F\phi^*_{I\ell})\\ &\qquad - \psi(\hat{w}(1-p_F\varepsilon_F)\phi^*_{I\ell}) \cdot \psi'(\hat{w}(1-p_F(1-\varepsilon_F))\phi^*_{Ih})\hat{w}(1-p_F(1-\varepsilon_F))\hat{w}p_F\phi^*_{Ih} < 0.\end{aligned} \tag{B.3}$$

This inequality is certainly satisfied because both terms on the left hand side are negative. Thus $\dfrac{d\Lambda^{ij}}{d\varepsilon_F} \leq 0$ within any class of equilibria with $(d^*_F, d^*_I) = (i, j) \in \{0, 1, 2\} \times \{1, 2\}$. □

Proof of Lemma 20.

Within a given class of equilibria with firm strategy $(d^*_F, d^*_I) = (i, j) \in \{0, 1, 2\} \times \{1, 2\}$, the firm's overall expected profit is given by $\pi^{ij} = \pi_F(d^*_F) + \pi_I(d^*_F, d^*_I) - \pi_{FI}(d^*_F, d^*_I)$ where $\pi_F(d_F)$, $\pi_I(d_F, d_I)$ and $\pi_{FI}(d_F, d_I)$ are given by equations (3.34), (3.35) and (3.36) respectively.

Certainly for any equilibrium with $d^*_F \in \{0, 2\}$, a change in ε_F has no effect on profits.

Instead taking $(d_F^*, d_I^*) = (1, j)$ with $j \in \{1, 2\}$ and grouping like terms, this means that

$$\begin{aligned}\pi^{1j} = (v_h - w)s\Big[p_F(1-\varepsilon_F) + (1-p_F(1-\varepsilon_F))\psi(\hat{w}(1-p_F(1-\varepsilon_F))\phi_{Ih}^*)\phi_{Ih}^* - \Big] \\ - (w-v_\ell)(1-s)\Big[p_F\varepsilon_F + (1-p_F\varepsilon_F)\psi(\hat{w}(1-p_F\varepsilon_F)\phi_{I\ell}^*)\phi_{I\ell}^*\Big].\end{aligned} \tag{B.4}$$

Recall that the conditional acceptance probabilities for high- and low-quality workers are $\phi_{Ah}^* = (1-\varepsilon)d_{AH}^* + \varepsilon d_{AL}^*$ and $\phi_{A\ell}^* = \varepsilon d_{AH}^* + (1-\varepsilon)d_{AL}^*$ for each application channel $A \in \{F, I\}$. Differentiating with respect to ε_F gives

$$\begin{aligned}\frac{d\pi^{1j}}{d\varepsilon_F} &= -(w-v_\ell)(1-s)p_F\Big[1 - \hat{w}(1-p_F\varepsilon_F){\phi_{I\ell}^*}^2\psi'(\hat{w}(1-p_F\varepsilon_F)\phi_{I\ell}^*) - \phi_{I\ell}^*\psi(\hat{w}(1-p_F\varepsilon_F)\phi_{I\ell}^*)\Big] \\ &+ (v_h-w)sp_F\Big[\hat{w}(1-p_F(1-\varepsilon_F)){\phi_{Ih}^*}^2\psi'(\hat{w}(1-p_F(1-\varepsilon_F))\phi_{Ih}^*) + \phi_{Ih}^*\psi(\hat{w}(1-p_F(1-\varepsilon_F))\phi_{Ih}^*) - 1\Big].\end{aligned} \tag{B.5}$$

Taking $\gamma(p) = cp^2$ for $c > 0$ as a simplifying example, we have $\psi(x) = \dfrac{x}{2c}$ and $\psi' = \dfrac{1}{2c}$. In this case equation (B.5) becomes

$$\begin{aligned}\frac{d\pi^{1j}}{d\varepsilon_F} = -(w-v_\ell)(1-s)p_F\Big[1 - \hat{w}(1-p_F\varepsilon_F){\phi_{I\ell}^*}^2 \cdot \frac{1}{2c} - \phi_{I\ell}^* \cdot \frac{\hat{w}(1-p_F\varepsilon_F)\phi_{I\ell}^*}{2c}\Big] \\ + (v_h-w)sp_F\Big[\hat{w}(1-p_F(1-\varepsilon_F)){\phi_{Ih}^*}^2 \cdot \frac{1}{2c} + \phi_{Ih}^* \cdot \frac{\hat{w}(1-p_F(1-\varepsilon_F))\phi_{Ih}^*)}{2c} - 1\Big]\end{aligned} \tag{B.6}$$

which reduces to

$$\frac{d\pi^{1j}}{d\varepsilon_F} = -(w-v_\ell)(1-s)p_F + (v_h-w)sp_F. \tag{B.7}$$

In this case it is easy to see that π^{1j} may be increasing or decreasing in ε_F, namely increasing if the industry is generally profitable and decreasing if the industry is generally unprofitable. Therefore we can have $\dfrac{d\pi^{1j}}{d\varepsilon_F} \gtrless 0$. □

Proof of Lemma 21.

Within a given class of equilibria with firm strategy $(d_F^*, d_I^*) = (i, j) \in \{0, 1, 2\} \times \{1, 2\}$ we have $\Lambda^{ij} \equiv \dfrac{\psi(\hat{w}(1-p_F\phi_{F\ell}^*)\phi_{I\ell}^*)}{\psi(\hat{w}(1-p_F\phi_{Fh}^*)\phi_{Ih}^*)}$, where $\phi_{Ah}^* = (1-\varepsilon)d_{AH}^* + \varepsilon d_{AL}^*$. Therefore, differentiating with respect to w we have

$$a \cdot \frac{d\Lambda^{ij}}{dw} = \psi(\hat{w}(1-p_F\phi^*_{Fh})\phi^*_{Ih}) \cdot \psi'(\hat{w}(1-p_F\phi^*_{F\ell})\phi^*_{I\ell})(1-p_F\phi^*_{F\ell})\phi^*_{I\ell} - \psi(\hat{w}(1-p_F\phi^*_{F\ell})\phi^*_{I\ell}) \cdot \psi'(\hat{w}(1-p_F\phi^*_{Fh})\phi^*_{Ih})(1-p_F\phi^*_{Fh})\phi^*_{Ih} \tag{B.8}$$

where a is a positive constant. Therefore we can have $\frac{d\Lambda^{ij}}{dw} \gtrless 0$ if and only if

$$\frac{\psi(\hat{w}(1-p_F\phi^*_{Fh})\phi^*_{Ih})}{\psi'(\hat{w}(1-p_F\phi^*_{Fh})\phi^*_{Ih})(1-p_F\phi^*_{Fh})\phi^*_{Ih}} \gtrless \frac{\psi(\hat{w}(1-p_F\phi^*_{F\ell})\phi^*_{I\ell})}{\psi'(\hat{w}(1-p_F\phi^*_{F\ell})\phi^*_{I\ell})(1-p_F\phi^*_{F\ell})\phi^*_{I\ell}}. \tag{B.9}$$

□

Proof of Lemma 22.

Recall that within a given class of equilibria with firm strategy $(d^*_F, d^*_I) = (i, j) \in \{0, 1, 2\} \times \{1, 2\}$, the firm's overall expected profit is given by $\pi^{ij} = \pi_F(d^*_F) + \pi_I(d^*_F, d^*_I) - \pi_{FI}(d^*_F, d^*_I)$ where $\pi_F(d_F)$, $\pi_I(d_F, d_I)$ and $\pi_{FI}(d_F, d_I)$ are given by equations (3.34), (3.35) and (3.36) respectively.

Within a given class of equilibria with firm strategy $(d^*_F, d^*_I) = (i, j) \in \{0, 1, 2\} \times \{1, 2\}$ this means that

$$\pi^{ij} = (v_h - w)s\Big[p_F\phi^*_{Fh} + (1-p_F\phi^*_{Fh}) \cdot \psi(\hat{w}(1-p_F\phi^*_{Fh})\phi^*_{Ih}) \cdot \phi^*_{Ih}\Big] - (w - v_\ell)(1-s)\Big[p_F\phi^*_{F\ell} + (1-p_F\phi^*_{F\ell}) \cdot \psi(\hat{w}(1-p_F\phi^*_{F\ell})\phi^*_{I\ell}) \cdot \phi^*_{I\ell}\Big]. \tag{B.10}$$

Recall that the conditional acceptance probabilities for high- and low-quality workers are $\phi^*_{Ah} = (1-\varepsilon)d^*_{AH} + \varepsilon d^*_{AL}$ and $\phi^*_{A\ell} = \varepsilon d^*_{AH} + (1-\varepsilon)d^*_{AL}$ for each application channel $A \in \{F, I\}$. Therefore differentiating with respect to b we have

$$\frac{d\pi^{ij}}{db} = -(v_h - w)s(1-p_F\phi^*_{Fh})^2{\phi^*_{Ih}}^2\psi'(\hat{w}(1-p_F\phi^*_{Fh})\phi^*_{Ih}) + (w - v_\ell)(1-s)(1-p_F\phi^*_{F\ell})^2{\phi^*_{I\ell}}^2\psi'(\hat{w}(1-p_F\phi^*_{F\ell})\phi^*_{I\ell}). \tag{B.11}$$

Thus we can have $\dfrac{d\pi^{ij}}{db} \gtrless 0$ if and only if

$$\frac{(v_h - w)s}{(w - v_\ell)(1-s)} \lessgtr \frac{(1-p_F\phi^*_{F\ell})^2 {\phi^*_{I\ell}}^2 \psi'(\hat{w}(1-p_F\phi^*_{F\ell})\phi^*_{I\ell})}{(1-p_F\phi^*_{Fh})^2 {\phi^*_{Ih}}^2 \psi'(\hat{w}(1-p_F\phi^*_{Fh})\phi^*_{Ih})}. \tag{B.12}$$

□

Proof of Lemma 23.

Within a given class of equilibria with firm strategy $(d^*_F, d^*_I) = (i, j) \in \{0, 1, 2\} \times \{1, 2\}$ we have $\Lambda^{ij} \equiv \dfrac{\psi(\hat{w}(1-p_F\phi^*_{F\ell})\phi^*_{I\ell})}{\psi(\hat{w}(1-p_F\phi^*_{Fh})\phi^*_{Ih})}$ where $\phi^*_{Ah} = (1-\varepsilon)d^*_{AH} + \varepsilon d^*_{AL}$ and $\phi^*_{A\ell} = \varepsilon d^*_{AH} + (1-\varepsilon)d^*_{AL}$ for $A \in \{F, I\}$. Therefore, differentiating with respect to p_F we have

$$\begin{aligned} a \cdot \frac{d\Lambda^{ij}}{dp_F} &= -\psi(\hat{w}(1-p_F\phi^*_{Fh})\phi^*_{Ih}) \cdot \psi'(\hat{w}(1-p_F\phi^*_{F\ell})\phi^*_{I\ell}) \cdot \hat{w}\phi^*_{F\ell}\phi^*_{I\ell} \\ &\quad + \psi(\hat{w}(1-p_F\phi^*_{F\ell})\phi^*_{I\ell}) \cdot \psi'(\hat{w}(1-p_F\phi^*_{Fh})\phi^*_{Ih}) \cdot \hat{w}\phi^*_{Fh}\phi^*_{Ih} \end{aligned} \tag{B.13}$$

where a is a positive constant. Therefore we can have $\dfrac{d\Lambda^{ij}}{dp_F} \gtrless 0$ if and only if

$$\frac{\psi(\hat{w}(1-p_F\phi^*_{Fh})\phi^*_{Ih})}{\psi'(\hat{w}(1-p_F\phi^*_{Fh})\phi^*_{Ih})\hat{w}\phi^*_{Fh}\phi^*_{Ih}} \gtrless \frac{\psi(\hat{w}(1-p_F\phi^*_{F\ell})\phi^*_{I\ell})}{\psi'(\hat{w}(1-p_F\phi^*_{F\ell})\phi^*_{I\ell})\hat{w}\phi^*_{F\ell}\phi^*_{I\ell}}. \tag{B.14}$$

□

Lemma 37. *In any equilibrium with $p^*_{Ih} > 0$, it must also be the case that $p^*_{I\ell} > 0$. In any equilibrium with $p^*_{I\ell} > 0$, it must also be the case that $p^*_{Ih} > 0$.*

Proof. Suppose there is an equilibrium with $p^*_{Ih} > 0$ and $p^*_{I\ell} = 0$. High-quality workers are willing to incur strictly positive arrival costs, so the firm must be willing to hire at least some informal applicants in this equilibrium, $d^*_I \in \{1, 2\}$. Any informal application which arrives is certainly from a high-quality worker, so the firm must hire absolutely in the informal channel, $d^*_I = 2$. Since the firm is hiring absolutely, we must have $p^*_{Ih} \leq p^*_{I\ell}$ by Lemma 12. This contradicts $p^*_{I\ell} = 0$.

Suppose there is an equilibrium with $p^*_{I\ell} > 0$ and $p^*_{Ih} = 0$. Low-quality workers are willing to incur strictly positive arrival costs, so the firm must be willing to hire at least some informal applicants in this equilibrium, $d^*_I \in \{1, 2\}$. Any informal application which arrives is certainly from a low-quality worker, so the firm can not be willing to hire in the informal channel, a contradiction. $\square$

Appendix C

Additional Proof of Results for Chapter 4

Lemma 38. *Let $a \in \{c, w, b, \varepsilon_F, \varepsilon_I, p_F\}$. Suppose $P(n)$ is strictly increasing and strictly concave, and $P'(n)$ is logarithmically concave. For any parameter setting and fixed firm strategy d such that worker best responses are non-zero, $n_h(\kappa, \phi_{Fh}(d), \phi_{Ih}(d))$, $n_\ell(\kappa, \phi_{F\ell}(d), \phi_{I\ell}(d)) > 0$, we have*

$$\frac{d}{da}\left[\frac{P(n_\ell(a))}{P(n_h(a))}\right] \gtrless 0 \quad \Leftrightarrow \quad \frac{P'(n_h(a))x_h'(a)}{P(n_h(a))P''(n_h(a))} \lessgtr \frac{P'(n_\ell(a))x_\ell'(a)}{P(n_\ell(a))P''(n_\ell(a))}, \tag{C.1}$$

where $x_q = \dfrac{c}{\hat{w}(1-p_F\phi_{Fq}(d))\phi_{Iq}(d)}$ *for* $q \in \{h, \ell\}$.

Proof. For fixed d and non-zero networking, the change in the pool composition due to a change in $a \in \{c, w, b, \varepsilon_F, \varepsilon_I, p\}$ is given by

$$\frac{d}{da}\left[\frac{P(n_\ell(a))}{P(n_h(a))}\right] = \frac{P(n_h(a))P'(n_\ell(a))n_\ell'(a) - P(n_\ell(a))P'(n_h(a))n_h'(a)}{[P(n_h(a))]^2}, \tag{C.2}$$

and therefore we have

$$\frac{d}{da}\left[\frac{P(n_\ell(a))}{P(n_h(a))}\right] \gtrless 0 \quad \Leftrightarrow \quad \frac{P'(n_h(a))n_h'(a)}{P(n_h(a))} \lessgtr \frac{P'(n_\ell(a))n_\ell'(a)}{P(n_\ell(a))}. \tag{C.3}$$

Now for non-zero networking, by Lemma 25 we have $n_q = P'^{-1}(x_q)$ for $q \in \{h, \ell\}$ so workers' change in networking in response to a change in a is given by

$$n_q'(a) = P'^{-1\prime}(x_q)x_q'(a) = \frac{1}{P''(n_q)}x_q'(a). \tag{C.4}$$

Therefore by condition (C.3) we have

$$\frac{d}{da}\left[\frac{P(n_\ell(a))}{P(n_\ell(a))}\right] \gtrless 0 \quad \Leftrightarrow \quad \frac{P'(n_h(a))x_h'(a)}{P(n_h(a))P''(n_h(a))} \lessgtr \frac{P'(n_\ell(a))x_\ell'(a)}{P(n_\ell(a))P''(n_\ell(a))}. \tag{C.5}$$

□

Lemma 39. *For $k \in (0,1)$, $\lambda > 0$, and $n > 0$ we have*

$$\frac{d}{dn}\left[\frac{e^{-\lambda n^k}\cdot n^k}{(1-e^{-\lambda n^k})(k-1-\lambda k n^k)}\right] > 0 \quad \textit{if} \quad 1 - e^{-\lambda n^k} < \lambda n^k\left[1 - \tfrac{k}{k-1}\lambda n^k\right]. \tag{C.6}$$

Proof. Focusing on the numerator of the derivative, we can see by differentiation that $\frac{d}{dn}\left[\frac{e^{-\lambda n^k}\cdot n^k}{(1-e^{-\lambda n^k})(k-1-\lambda k n^k)}\right] > 0$ if

$$\begin{aligned}(1-e^{-\lambda n^k})(k-1-\lambda k n^k)[e^{-\lambda n^k}\cdot k n^{k-1} - \lambda k n^{k-1} e^{-\lambda n^k}\cdot n^k] \\ - e^{-\lambda n^k}\cdot n^k[(1-e^{-\lambda n^k})(-\lambda k^2 n^{k-1}) + (k-1-\lambda k n^k)e^{-\lambda n^k}\cdot\lambda k n^{k-1}] > 0.\end{aligned} \tag{C.7}$$

Condition (C.7) is equivalent to each of the following:

$$\begin{aligned}(1-e^{-\lambda n^k})(k-1-\lambda k n^k)e^{-\lambda n^k}\cdot k n^{k-1}[1-\lambda n^k] \\ - e^{-\lambda n^k}\cdot k n^{k-1}[(1-e^{-\lambda n^k})(-\lambda k n^k) + (k-1-\lambda k n^k)e^{-\lambda n^k}\cdot\lambda n^k] > 0,\end{aligned} \tag{C.8}$$

$$\begin{aligned}(1-e^{-\lambda n^k})(k-1-\lambda k n^k)[1-\lambda n^k] \\ - [(1-e^{-\lambda n^k})(-\lambda k n^k) + (k-1-\lambda k n^k)e^{-\lambda n^k}\cdot\lambda n^k] > 0,\end{aligned} \tag{C.9}$$

$$\begin{aligned}(1-e^{-\lambda n^k} - \lambda n^k + \lambda n^k\cdot e^{-\lambda n^k})(k-1-\lambda k n^k) \\ + (1-e^{-\lambda n^k})(\lambda k n^k) - (k-1-\lambda k n^k)e^{-\lambda n^k}\cdot\lambda n^k > 0,\end{aligned} \tag{C.10}$$

$$(1 - e^{-\lambda n^k} - \lambda n^k)(k{-}1 - \lambda k n^k) + (1 - e^{-\lambda n^k})(\lambda k n^k) > 0, \tag{C.11}$$

$$(1 - e^{-\lambda n^k})(k{-}1) - \lambda n^k(k{-}1 - \lambda k n^k) > 0, \tag{C.12}$$

$$(1 - e^{-\lambda n^k})(k{-}1) > \lambda n^k(k{-}1 - \lambda k n^k). \tag{C.13}$$

For $k \in (0, 1)$ this is also equivalent to

$$1 - e^{-\lambda n^k} < \lambda n^k \left[1 - \tfrac{k}{k-1}\lambda n^k\right]. \tag{C.14}$$

Therefore we have

$$\frac{d}{dn}\left[\frac{e^{-\lambda n^k} \cdot n^k}{(1 - e^{-\lambda n^k})(k{-}1 - \lambda k n^k)}\right] > 0 \quad \textit{if} \quad 1 - e^{-\lambda n^k} < \lambda n^k \left[1 - \tfrac{k}{k-1}\lambda n^k\right]. \tag{C.15}$$

□

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