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Numerical Modelling of Wind Turbine
Foundations subjected to Combined Loading

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Abstract

With the growing wind industry in Canada, it has become important to optimize wind turbine foundation design. Wind turbine foundations are subjected to combinations of vertical, horizontal and moment loads due to vertical self-weight of the structure and soil surcharge. Additionally, there are significant lateral loads and overturning moments attributed to varying wind forces acting at considerable tower heights above the ground level. In this thesis, the undrained bearing capacity response of circular & octagonal foundations subjected to combined loading is calculated using finite element analyses. Previous works have mostly focused on circular foundations. An octagonal foundation of the style typically used in the wind industry forms the focus of this research. Foundations are either surface based or embedded in homogeneous or heterogeneous soils. The results are expressed in terms of a coherent set of bearing capacity factors and failure envelopes in two dimensional planes (VH, VM and HM). This research also presents a parametric study on the effect of a surficial crust on the bearing capacity of a foundation. Finally, working and design loads for a typical wind turbine foundation are plotted in two dimensional failure planes to investigate if there is a potential ‘spare’ capacity. The finite element study indicates that an increase in soil strength heterogeneity and embedment leads to increases in the uniaxial limit capacities and size of the failure envelopes. For octagonal foundations, the average increase in uniaxial vertical, horizontal and moment capacities due to increases in the embedment is 15%, 52% and 32% respectively. The average increase in uniaxial vertical and moment capacities due to increase in the soil strength heterogeneity is 7.1% and 6.7% respectively, for octagonal foundations. When the shape of a foundation changes from a circle to an octagon, the ultimate uniaxial vertical and moment capacities slightly increase (by 7.7% and 7.2% respectively). Under combined loading, conventional methods are found to underestimate the combinations of horizontal and moment loads that a foundation can resist safely. During eccentric loading, the effective area predicted using the method given by DNV (2002) is also under-predicted.

Keywords: Combined loading, complex loading, octagonal foundations, wind turbine foundations, surficial crust, design optimization, failure envelopes
Dedicated to my mother, my guru, Sri Sri and my beloved teachers at Art of Living, Ami and Aditya, who taught me about love, compassion, simplicity and Self
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Special thanks to my Guru, Sri Sri for His unwavering support and love. Every moment he guided me subtly in the life of which this research is, but a small part. His precious words created peace & harmony within me and allowed me to focus more on my research.
Statement of Originality

The work contained in this thesis has not been previously submitted for a degree or diploma at any other higher education institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due references are made.

April 14, 2016
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Symbols

General parameters

\( V_{ult} \) \hspace{1em} \text{Ultimate vertical load (kN)}
\( H_{ult} \) \hspace{1em} \text{Ultimate horizontal load (kN)}
\( M_{ult} \) \hspace{1em} \text{Ultimate moment load (kN m)}
\( E \) \hspace{1em} \text{Young’s modulus (MPa)}
\( G \) \hspace{1em} \text{Shear modulus (MPa)}
\( G_R \) \hspace{1em} \text{Shear modulus at a depth equal to radius } R \text{ of the footing (MPa)}
\( E_u \) \hspace{1em} \text{Undrained Young’s modulus (MPa)}
\( \nu \) \hspace{1em} \text{Poisson’s ratio}
\( \nu_u \) \hspace{1em} \text{Undrained Poisson’s ratio}
\( \sigma \) \hspace{1em} \text{Stress (kN/m}^2\text{)}
\( \sigma' \) \hspace{1em} \text{Effective vertical stress (kN/m}^2\text{)}
\( \sigma_x \) \hspace{1em} \text{Normal stress in } x\text{-direction (kN/m}^2\text{)}
\( \sigma_y \) \hspace{1em} \text{Normal stress in } y\text{-direction (kN/m}^2\text{)}
\( \sigma_z \) \hspace{1em} \text{Normal stress in } z\text{-direction (kN/m}^2\text{)}
\( \sigma_f \) \hspace{1em} \text{Failure normal stress (kN/m}^2\text{)}
\( \sigma_1 \) \hspace{1em} \text{Major principal stress (kN/m}^2\text{)}
\( \sigma_2 \) \hspace{1em} \text{Intermediate principal stress (kN/m}^2\text{)}
\( \sigma_3 \) \hspace{1em} \text{Minor principal stress (kN/m}^2\text{)}
\( \varepsilon \) Strain
\( \varepsilon_x \) Normal strain in x-direction
\( \varepsilon_y \) Normal strain in y-direction
\( \varepsilon_z \) Normal strain in z-direction
\( \tau_f \) Failure shear stress (kN/m²)
\( \tau_{\text{max}} \) Maximum shear stress (kN/m²)
\( c \) Cohesion (kPa)
\( N_c, N_q, N_y \) Bearing capacity factors
\( N_{v_0} \) Bearing capacity factor due to ultimate vertical load
\( N_{h_0} \) Bearing capacity factor due to ultimate horizontal load
\( N_{m_0} \) Bearing capacity factor due to ultimate moment load
\( t_n \) Time at the beginning of \( n^{th} \) step (s)
\( t_{n+1} \) Time at the beginning of \( n + 1^{th} \) step (s)
\( h \) Depth below ground surface (m)
\( g \) Acceleration due to gravity (m/s²)
\( u \) Pore water pressure (N/m²)
\( k_s \) Spring stiffness (N/m)
\( \gamma \) Shear strain
\( \gamma_c \) Cyclic shear strain
\( m \) Modulus reduction factor
\( e \) Eccentricity (m)
\( i_v, i_q, i_\gamma \) Shape factors
\( d_v, d_q, d_\gamma \) Depth factors
\( \sigma'_{cv} \) Effective vertical consolidation stress (kN/m²)

**Foundation parameters**

\( \gamma_c \) Unit weight of foundation (kN/m³)
\( D \) Diameter of circular foundation or diameter of the inscribed circle of the octagonal foundation (m)
Radius of foundation (m)
Width of foundation (m)
Thickness of foundation (m)
Effective width of foundation (m)
Length of foundation (m)
Effective length of foundation (m)
Depth of embedment (m)
Contact area of foundation bottom with soil (m²)
Effective contact area (m²)
Young’s modulus (GPa)
Poisson’s ratio
Foundation stiffness (N/m)
Vertical stiffness (N/m)
Horizontal stiffness (N/m)
Rotational stiffness (N m²)
Torsional stiffness (N m²)
Saturated unit weight (kN/m³)
Effective unit weight (kN/m³)
Density of soil (kg/m³)
Young’s modulus (MPa)
Small-strain shear modulus (MPa)
Poisson’s ratio
Undrained shear strength (kPa)
Undrained shear strength at soil surface (kPa)
Undrained shear strength of crust layer (kPa)
Undrained shear strength of the layer underlying the crust (kPa)
Thickness of crust (m)
S
γ
ρ
Es
G₀
ν
Sₙₐₙ
Sₘₚ₀
Sₙₜ
Sₙₚₚₜ
tₖ
c_u \quad \text{Undrained cohesion (kPa)}

c_d \quad \text{Design cohesion (kPa)}

c_{ud} \quad \text{Undrained design cohesion (kPa)}

\phi \quad \text{Friction angle (°)}

\phi' \quad \text{Effective friction angle (°)}

\psi \quad \text{Dilation angle (°)}

\alpha \quad \text{Adhesion factor}

\mu \quad \text{Coefficient of friction}

K \quad \text{Earth pressure coefficient}

K_0 \quad \text{Earth pressure coefficient at rest}

k \quad \text{Rate of increase of soil strength with depth (kPa/m)}

K' \quad \text{Soil strength heterogeneity ratio}

\zeta \quad \text{Ratio of undrained shear strength to effective vertical stress}

\textbf{Load parameters}

V \quad \text{Vertical load (kPa)}

H \quad \text{Horizontal load (kN)}

M \quad \text{Moment load (kN m)}

T \quad \text{Torsional moment (kN m)}

V_d \quad \text{Design vertical load (kN)}

M_d \quad \text{Design overturning moment (kN m)}

RM2 \quad \text{Moment about y-axis (kN m)}

q_u \quad \text{Ultimate bearing capacity (kPa)}

q_d \quad \text{Design bearing capacity (kPa)}

Q \quad \text{Ultimate load (kPa)}

\textbf{Displacement parameters}

U_1, u \quad \text{Displacement in x-direction (m)}
$U_2, \nu$ Displacement in $y$-direction (m)

$U_3, w$ Displacement in $z$-direction (m)

$\theta$ Rotation about $y$-axis ($^\circ$)

**Abbreviations**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>BCF</td>
<td>Bearing capacity factor</td>
</tr>
<tr>
<td>OCR</td>
<td>Overconsolidation ratio</td>
</tr>
<tr>
<td>VH</td>
<td>Vertical-Horizontal plane</td>
</tr>
<tr>
<td>VM</td>
<td>Vertical-Moment plane</td>
</tr>
<tr>
<td>HM</td>
<td>Horizontal-Moment plane</td>
</tr>
<tr>
<td>SLS</td>
<td>Serviceability limit state</td>
</tr>
<tr>
<td>ULS</td>
<td>Ultimate limit state</td>
</tr>
<tr>
<td>LRP</td>
<td>Load reference point</td>
</tr>
<tr>
<td>DNV</td>
<td>Det Norske Veritas</td>
</tr>
<tr>
<td>ISO</td>
<td>International Organization for Standardization</td>
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<td>API</td>
<td>American Petroleum Institute</td>
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Chapter 1

Introduction

1.1 Wind Energy and Wind Turbine Foundations

Over the past 10 to 15 years, the wind power sector in Canada has grown significantly. Wind energy has increasingly become a very promising alternative source to produce electric power. While the total installed capacity was just over 200 MW in the year 2000 (Weis, 2015), it surpassed 11 000 MW (Canadian Wind Energy Association, 2015) by December, 2015. This accounts for over 7% of Canada’s total generating capacity (Hislop, 2015). Thus, from initially being on the ‘margins’ of the energy portfolio, wind is moving steadily into the ‘mainstream’.

During the past 20 years, the power output and size of wind turbines have increased by an order of magnitude (Bonnett, 2005). Onshore turbine manufacturers have now developed wind turbines with more than 5 MW rated power and 125 m tower heights. Consequently, the loads that the foundations have to resist, have also increased many times. As the first generation of ‘megawatt’ wind farms are reaching their ‘mid-lives’, owners are starting to consider new, larger turbines being installed on the original foundations to save money. Thus, with the continued growth of the wind industry, it is essential to understand the pressing considerations for the associated growth of foundations and the optimization of the foundation design. Moreover, as the density of wind farms in Ontario increases, geotechnical problems
may become more challenging as the majority of the best sites will already have been taken. In such a scenario, the design of foundations will become even more critical as its impact on the overall profitability of the construction of wind farms will increase.

As a foundation problem, the design of wind turbine foundations is unique because of the combination of loads acting upon them. Besides the relatively low centric vertical loading (V) due to the self weight of the structure, they are also subjected to very large overturning moments (M) (resulting in inclined and/or eccentric loading), which is attributed to the varying wind forces (H) acting at a significant tower height above the ground level. The bearing capacities of foundations under inclined or eccentric loading have traditionally been estimated using conventional methods, such as those given by Meyerhof (1953), Hansen (1970) and Vesic (1973). The guidelines for design of wind turbine foundations given by DNV (2002) and other recommended guides such as ISO (2011) and DNV (1992) are predominantly based on these conventional methods. In these codes, the ultimate limit states (ULS) and serviceability limit states (SLS) are calculated using limit equilibrium analysis and empirical foundation analysis approaches. The eccentricity and inclination of loads are taken into account by using the effective foundation area method and modification factors respectively, within the same type of bearing capacity analysis.

Furthermore, when foundations are embedded or non-homogeneity of soil strength comes into play, the validity of traditional methods to provide accurate estimates of bearing capacity has been questioned by many researchers. It has been found that these approaches are valid only for low H/V ratios. Gourvenec and Randolph (2003) & Shen et al. (2016) showed that for circular foundations on clays with non-homogeneous strengths, conventional methods actually provide unconservative results. Salgado et al. (2004) demonstrated the over-prediction and under-prediction of the depth factors proposed by Meyerhof (1953) and Hansen (1970) respectively, with increase in the embedment ratio; as a foundation becomes further embedded, more soil is involved in the failure mechanism. In addition, design guidelines do not explicitly consider 3-dimensional geometries, but rather suggest equivalence to rectangular foundations with the same area and areal moment of inertia (ISO, 2011). Hence these methods overlook the increase in the bearing capacity due to bearing failure of greater volumes of soil. Furthermore,
the design codes do not have any provision for calculating tensile capacity. Especially in the case of a wind turbine foundation, this becomes significant as the foundation loses its contact with the soil due to the uplifting forces of wind (albeit for very short periods of time).

With innovations occurring in the fast growing wind industry, the design codes and the analysis of embedded foundations may also need to be assessed to optimize wind turbine foundation design. Use of the finite element method to predict the ultimate bearing capacities of foundations has emerged as one of the most popular methods among geotechnical researchers. With higher computer speed, memory and data storage capacity in the recent years, this method has increasingly been used to express explicitly the bearing capacity of a footing with realistic 3D geometries under general loading in terms of failure envelopes in Vertical, Horizontal and Moment (VHM) loading space. Bransby and Randolph (1998), Gourvenec and Randolph (2003), Salgado et al. (2004), Taiebat and Carter (2010), Shen et al. (2016) studied the effects of soil strength heterogeneity, shapes and embedments on the bearing capacity of offshore foundations under general loading and expressed the results in the form of dimensionless loads (bearing capacity factors) and failure envelopes. These results have been compared with those of conventional methods or design codes, to obtain an idea of ‘spare’ or ‘overlooked’ capacity. This helps to reduce foundation sizes, minimizing the construction costs and allows reuse of foundations. However, most of these studies were confined to strip or circular foundations. The interface between the foundation and the soil was usually assumed to be fully bonded [except Taiebat and Carter (2010), Shen et al. (2016)]. Hence, given these facts, different footing geometries (like octagonal), contact conditions like a no-tension interface and soil conditions such as a surficial crust, that are specific to wind turbine foundations in Canada need to be investigated, to gain further insight into the foundation design.

1.2 Aims of Study

This thesis aims to investigate the bearing capacity response of wind turbine foundations subjected to combined static loading under undrained conditions. The response of these
foundations under varying embedment and soil strength heterogeneity conditions is also explored.

Specifically, the aims of this thesis are:

1. To investigate current design and analysis for onshore wind turbine foundations and compare this to offshore geotechnical practice.

2. To investigate the stability of wind turbine shallow foundations under combined Vertical Horizontal Moment (VHM) loading.

3. To determine a coherent set of bearing capacity factors for uniaxial ultimate limit state loads for surface based circular and octagonal foundations on clayey soils.

4. To study the effects of embedment, shape and soil strength heterogeneity on the bearing capacities of these foundations.

5. To assess current design practice and make suggestions for possible changes in practice in light of the findings of this study.

1.3 Thesis Objectives

In order to explore this subject thoroughly, but keep the number of analyses to a minimum, a typical foundation for a wind farm in Canada (Port Alma) has been investigated. Much of the analysis is based on models of a working commercial wind turbine, typical of Ontario conditions with a large shallow gravity based octagonal foundation. To meet the aims of aforementioned study, the following objectives have been completed.

1. A critical literature review was performed of the current state of practice for wind turbine foundation design based on the guidelines given by DNV (2002), DNV (1992) and ISO (2011).
2. A review of the literature on finite element studies investigating undrained bearing capacity response of predominantly offshore foundations subjected to combined loading was performed.

3. Three-dimensional models were developed in the finite element program ABAQUS for surface circular and octagonal foundations on clays and subjected to uniaxial and combined loads.

4. The ultimate uniaxial bearing capacities found were compared with solutions obtained from conventional methods and finite element analysis.

5. The failure envelopes in VH, VM and HM load space planes were compared with those derived by conventional methods and finite element analysis performed by previous researchers.

6. The changes in bearing capacity for four embedment depths and three different soil strength heterogeneity ratios were investigated.

7. A parametric study was performed to investigate the effects of clay crust thickness, depth of embedment relative to crust thickness, average crust strength and increase of shear strength of underlying layer relative to that of the crust.

8. Working and design loads for the commercial wind turbine foundation were plotted in 2-dimensional failure envelope load planes to determine any ‘spare’ capacity.

1.4 Organization of the Thesis

This thesis consists of the following 5 chapters.

Chapter one provides a brief introduction of the design approaches for wind turbine foundations in the recommended codes [DNV (2002); ISO (2011)] and conventional methods. It also highlights possible shortcomings of these methods and introduces finite element analysis of foundation-soil systems as an alternative design approach. It also provides the aims and
objectives of this research. The chapter ends with a brief description of the organization of this thesis.

Chapter two reviews the design guidelines for wind turbine foundations given by DNV (2002) and ISO (2011). It also elucidates the conventional methods used to take eccentricity and inclination of loads into account to assess the bearing capacity. The published work pertaining to investigation of the uniaxial and combined loadings response of foundations for different shapes, embedment conditions and soil strength heterogeneity conditions through numerical methods is also reviewed.

In chapter three, the development of a complete 3-dimensional model for the octagonal foundation at Port Alma in the finite element program ABAQUS is described. Initially, a 2-dimensional plane strain model for a strip footing is created and the obtained uniaxial limit capacities and failure envelopes are compared with published data. This helps in validating the soil constitutive parameters, boundary conditions, contact definitions and mesh techniques. Furthermore, development of model in 3-dimensions for a circular foundation is also explained. Various cases of embedment depths and soil strength heterogeneity ratios to be considered in this research are also presented. Details of a parametric study undertaken to investigate the effects of surficial crust are also given.

Chapter four describes all of the major findings in the form of results obtained from finite element analyses of soil-foundation systems subjected to uniaxial and combined loadings. First, the results of bearing capacity response of circular foundations subjected to combined loading are presented in the form of uniaxial bearing capacity factors and failure envelopes (VH, VM and HM). These results are compared with published data. Next, the ultimate uniaxial limit capacities and failure envelopes obtained for octagonal foundations are presented. The effects of embedments and change in soil strength homogeneity on bearing capacity are discussed.

The last chapter draws conclusions from the results presented in Chapter 4 on the bearing capacity responses of circular and octagonal foundations. The increase in bearing capacity due to change in the shape from a circle to octagon is also described. The effects of embedments, soil strength non-homogeneity and surficial crust are critically evaluated. A brief discussion
of optimization in the size of foundations is provided and the potential scope of future work is presented.
Chapter 2

Literature Review

2.1 Introduction

This chapter provides a detailed literature survey on the bearing capacity response of foundations subjected to combined loading. The literature review is focused on investigating bearing capacities of surface or embedded gravity-based foundations in clayey soils with homogeneous or heterogeneous strengths. Initially, design load cases for wind turbine foundations provided by DNV (2002) and other limit state methods for foundation design are presented. Next, the elastic behaviour of foundations subjected to combined loading is discussed. Later, fundamental analytical and empirical approaches for foundation design and the current state-of-the-practice for wind turbine foundations design are reviewed in the chapter. Finally, this chapter reviews the numerical methods for modelling foundations subjected to combined loading. Especially, past works related to finite element analysis of soil-foundation system are elucidated. The effect of surficial crust on the undrained response of foundations is also explained.

A large portion of global designs of wind turbine foundations are based on the guidelines provided by either DNV (1992), DNV (2002) or ISO (2011). Whilst these codes have been updated from time-to-time to include state-of-the-art knowledge, they all have a basis in classical bearing capacity theory, such as that proposed by Terzaghi (1943), to predict the
capacity under centric vertical load. By introducing modification factors, e.g. as proposed by Meyerhof (1963), Hansen (1970) or Green (1954), the ultimate bearing capacity is adjusted to take into account the effect of depth, inclination or eccentricity of load and shape of the foundation and the variation with respect to the benchmark 2d strip footing problem.

However, when foundations are subjected to complex loading these simple, analytical approaches fail to accurately predict bearing capacities. Indeed, many researchers have highlighted the conservative nature of bearing capacity envelopes deduced from conventional methods e.g. (Bransby & Randolph, 1998); (Taiebat & Carter, 2000); (Gourvenec & Randolph, 2003); (Randolph et al., 2005). Most of these researchers used finite element analysis to investigate offshore foundations where the problem of combined loading is a common scenario. Hence, most of the papers reviewed here have a background in offshore geotechnical engineering.

### 2.2 Wind turbine foundation loading

In common with all foundations, those for wind turbines are subjected to dead and live loads. However, the live loads for wind turbine foundations represent a very significant percentage of the overall loading. The live loads are due to the action of wind and other environmental forces acting at the hub height of the turbine. The dead loads are due to the weight of the superstructure, foundation and the soil backfill. DNV (2002) and IEC 61400-1 provide design load cases consisting of combinations of relevant design situations and external conditions. The following combinations represent a minimum number of relevant combinations (DNV, 2002):

- Normal operation and normal external conditions;
- Normal operation and extreme external conditions;
- Fault situations and appropriate external conditions, which may include extreme external conditions;
• Transportation, installation and maintenance situations and appropriate external conditions.

The design load cases must be analyzed for fatigue (F) or for ultimate loads (U). Using the limit state design method, load combinations are checked for every ultimate limit state (ULS) and also for the serviceability limit state (SLS). The governing variables in this partial safety factor method consists of the loads/driving or destabilizing forces acting on the foundation and the resisting forces due to the strength of the materials in the foundation. This method of design is also known as load and resistance factor design (LRFD). The safety level of a foundation is considered to be satisfactory when

\[ S_d \leq R_d \]  

(2.1)

where \( S_d \) is the design load and \( R_d \) is the design resistance.

The design load is increased by multiplying the characteristic load with partial safety factors for loads \( (\gamma_f) \), while the design resistance is decreased by dividing the characteristic resistance with partial safety factors for materials \( (\gamma_m) \). Note that the partial safety factors mentioned in the code are not safety factors in true sense, but reduction or scaling factor for resisting & driving forces respectively and account for the uncertainties & variability in loads and materials. DNV (2002) does not mention partial safety factors specifically for foundations. However, IEC 61400-1 provides the values of partial safety factors broadly for loads and materials. The minimum general value of partial safety factor recommended is 1.1.

The partial safety factors for loads in IEC 61400-1 are given in Table 2.1 below.

<table>
<thead>
<tr>
<th>Unfavourable loads</th>
<th>Favourable loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal (N)</td>
<td>1.35</td>
</tr>
<tr>
<td>Abnormal (A)</td>
<td>1.1</td>
</tr>
<tr>
<td>Transport &amp; erection (T)</td>
<td>1.5</td>
</tr>
<tr>
<td>All design situations</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Table 2.1: Partial safety factors for different load combinations (IEC 61400-1)

For more details on the use and applicability of partial safety factors, it is recommended to
refer Section 7.6 of IEC 61400-1. The offshore standard DNV-OS-J101 for design of offshore wind turbine structures, DNV (2014), also provides partial safety factors for various limit states and load categories. The resistance factor recommended for undrained conditions by DNV (2014) is equal to 1.25.

To simplify the analysis of wind turbine foundations, loads are subdivided into vertical, horizontal and moment load components [(Zaharescu, 1961); (Ticof, 1977)]. All three loads are assumed to act in the same vertical plane. The loads acting on a foundation with diameter $D$ and the resulting deflections and rotations are shown in Fig. 2.1 below. It must be noted that the effect of torsional loads are not considered in this research and therefore, not shown in the table above and the figure below. However, a number of design codes do consider this additional loading.

![Figure 2.1: Nomenclature for foundation loading and geometry](image)

where

- $V$ = Vertical load
- $H$ = Horizontal load
- $M$ = Moment load
- $w$ = Vertical displacement (in z-direction)
- $u$ = Horizontal displacement (in x-direction)
- $\theta$ = Rotation about y-axis
2.3 Elastic Behaviour of Foundations

The present research work concentrates on the study of the ultimate limit state of foundations (i.e. bearing capacity) under uniaxial and combined loadings. However, in the context of foundation design, the elastic behaviour of foundations is also of some importance. The elastic behaviour of foundations can be expressed in two ways: 1. Elastic deformation and 2. Elastic Stiffness. A preliminary foundation design can be obtained by calculating the elastic deformations of foundations under applied loads using elasticity theory, which can facilitate feasibility assessment. In addition, due to the requirement of rocking stability for wind turbines, elastic analysis can become a governing criterion for foundation designs. Elastic stiffness parameters (e.g. rotational stiffness) help in defining monotonic response of foundations and in understanding the dynamic behaviour of the soil-foundation system. Elastic behaviour of foundations in terms of elastic stiffness is discussed below.

Foundation stiffness ($K_f$) is similar to spring stiffness ($k_s$) and is defined as the ratio of load or moment to the deformation in the direction of the load or rotation respectively. Stiffness of a foundation must keep the deformations of soil below threshold values defined by the serviceability limit state (SLS). For a foundation resting on the surface of a homogeneous isotropic elastic halfspace, four types of loading scenario are possible: vertical, horizontal, moment and torsion; resulting in 4 types of elastic stiffness. For the case of a smooth surface footing, horizontal or torsional loads cannot be applied, since no shear stress can be sustained at the soil-foundation interface. In contrast, all four loadings are possible for a rough footing. Shear stresses are developed (up to the undrained shear strength of the soil) due to the foundation being rigidly connected to the soil. Table 2.2 provides the stiffness parameters for a circular surface footing on an elastic halfspace.
Table 2.2: Elastic stiffness for circular footing on an elastic halfspace

<table>
<thead>
<tr>
<th>Type</th>
<th>Vertical</th>
<th>Horizontal</th>
<th>Rocking</th>
<th>Torsional</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4G R/(1 - \nu)$</td>
<td>$8G R/(2 - \nu)$</td>
<td>$8G R^3/3(1 - \nu)$</td>
<td>$16G R^3/3$</td>
<td></td>
</tr>
<tr>
<td>$\nu = 0.5$</td>
<td>$8G R$</td>
<td>$5.33G R$</td>
<td>$5.33G R^3$</td>
<td>$16G R^3/3$</td>
</tr>
</tbody>
</table>

where

$G$ = Shear modulus
$R$ = Radius of the foundation
$\nu$ = Poisson’s ratio

With regards to these four stiffness values, for wind turbines the rocking stiffness is almost always the critical design stiffness parameter. This is because only rotational stiffness controls the location of the centre of gravity with respect to the foundation of the wind turbine system (Lang, 2012). Turbine manufacturers usually specify a minimum value of rotational stiffness as a design requirement. For example, the value of minimum rotational stiffness for a typical 2.5 MW wind turbine is 900 MNm/deg. Fig. 2.2 shows the stiffness parameters and the associated displacements or rotations. All stiffnesses depend on two key elastic parameters: Poisson’s ratio and shear modulus [except for torsional stiffness, which is independent of Poisson’s ratio]. Besides this, these parameters also depend on the dimensions and embedment of the footing. Rocking stiffness varies cubically with radius and is more sensitive to changes in radius, compared to vertical or horizontal stiffness. For saturated undrained clayey soil, Poisson’s ratio $\nu = 0.5$ and values of stiffness for incompressible soil ($\nu = 0.5$) are also shown in Table 2.2.
Figure 2.2: Foundation stiffness parameters and the associated deformation or rotation (Lang, 2012)

\[ K_V = \text{Vertical Stiffness} \]
\[ K_H = \text{Horizontal Stiffness} \]
\[ K_R = \text{Rotational Stiffness} \]
\[ K_\psi = \text{Torsional Stiffness} \]

Bell (1991) proposed the elastic stiffness estimates for systems under coupled vertical, horizontal and moment loading. Torsional loading was not considered, since it is not of great relevance to many foundations. Bell (1991) postulated the existence of cross-coupling between horizontal and moment loads when combined loading acts on a foundation. That is, in a compressible soil, when a foundation is subjected to horizontal load, it not only undergoes translation, but also rotation about an horizontal axis perpendicular to the direction of the load. Similarly, a moment applied produces both rotation and translation. Bell (1991) expressed the stiffness of
a footing using a matrix approach:

\[
\begin{bmatrix}
\frac{V}{(GR^2)} \\
\frac{H}{(GR^2)} \\
\frac{M}{(GR^3)}
\end{bmatrix}
= 
\begin{bmatrix}
K_1 & 0 & 0 \\
0 & K_2 & K_4 \\
0 & K_4 & K_3
\end{bmatrix}
\begin{bmatrix}
\frac{u_V}{R} \\
\frac{u_H}{R} \\
\theta_M
\end{bmatrix}
\]  

(2.2)

where

\[K_1 - K_4 =\text{Non-dimensional coefficients}\]

\[u_V, u_H \text{ & } \theta_M = \text{Displacements in } z \text{ & } x \text{ axis and rotation about } y \text{ axis respectively}\]

\(K_4\) represents the cross-coupling of horizontal and moment degrees of freedom. For the case of undrained soil conditions i.e. \(v = 0.5\), \(K_4\) becomes zero and horizontal & moment loading become independent of each other.

Bell determined elastic solutions for the vertical and moment loading for both smooth and rough footings using analytical and numerical methods. He also evaluated stiffness coefficients \(K_2, K_3\) and \(K_4\) by performing displacement-controlled finite element analysis. The values of these coefficients (calculated numerically when \(v = 0\) and 0.5) are given below in Table 2.3.

<table>
<thead>
<tr>
<th>(v)</th>
<th>(H/GRu_H) ((K_2))</th>
<th>(M/GR^2u_H) ((K_4))</th>
<th>(H/GR^2\theta_M) ((K_4))</th>
<th>(M/GR^3\theta_M) ((K_3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.271</td>
<td>-0.7364</td>
<td>-0.7364</td>
<td>3.227</td>
</tr>
<tr>
<td>0.5</td>
<td>5.474</td>
<td>-0.0144</td>
<td>-0.0144</td>
<td>5.410</td>
</tr>
</tbody>
</table>

Table 2.3: Numerical results for horizontal and moment loading of a surface footing (Bell, 1991)

It is interesting to note that the values of \(K_4\) obtained by applying horizontal displacement and rotation are exactly the same up to 4 significant digits.

A similar analysis was performed by Doherty and Deeks (2003) for embedded foundations. They used a scaled boundary finite element method to express the response of a circular footing
embedded in non-homogeneous soil, in terms of dimensionless elastic stiffness coefficients. The footing was subjected to vertical, horizontal, moment and torsional loads and was fully bonded with the soil. Four different cases as shown in Fig. 2.3 were considered.

![Figure 2.3: Footing geometry for various embedment conditions (Doherty & Deeks, 2003)](image)

Case (a) represents a circular footing at the bottom of an open trench, case (b) represents a fully embedded footing, case (c) represents an embedded footing (or short pile) with full sidewall-soil contact, and case (d) represents a skirted foundation (or caisson), also with full sidewall-soil contact.

Similar to Bell (1991), the stiffness coefficients were expressed in matrix form as given below and can be used to calculate load-displacement response. However, Doherty and Deeks (2003) also derived stiffness coefficients for torsional loads.

\[
\begin{pmatrix}
\frac{V}{G_R R^2} \\
\frac{H}{G_R R^2} \\
\frac{M}{G_R R^2} \\
\frac{T}{G_R R^2}
\end{pmatrix} = 
\begin{bmatrix}
K_V & 0 & 0 & 0 \\
0 & K_H & K_{MH} & 0 \\
0 & K_{MH} & K_M & 0 \\
0 & 0 & 0 & K_T
\end{bmatrix}
\begin{pmatrix}
\frac{w}{R} \\
\frac{u}{R} \\
\theta_M \\
\theta_T
\end{pmatrix}
\]

In their study, the shear modulus was assumed to vary exponentially with depth and was expressed as:

\[
G(z) = G_R \left(\frac{z}{R}\right)^\alpha
\]

where \(G_R\) is the shear modulus at a depth equal to radius of the footing and \(\alpha\) is the non-
homogeneity parameter varying between 0 and 1. The soil was considered linear elastic and hence only two parameters: Poisson’s ratio and Young’s modulus were used to define its properties.

Doherty and Deeks (2003) demonstrated the use of eqn 2.4 by calculating results for four different embedment ratios (0.5, 1, 2 and 4). Poisson’s ratio of 0.2 (to represent sands) and 0.5 (to represent undrained clays) were chosen. For each case, a non-homogeneous soil profile with $a$ ranging between 0 and 1 was considered. With increase in embedment ratio, all of the stiffness coefficients except $K_{MH}$ also increase. The non-homogeneity parameter $\alpha$ has a significant effect on stiffness coefficients. For $d/D = 0.5$ and 1, the stiffness coefficients remains relatively constant with increase in $\alpha$. However, when $d/D = 1$ and 4, all of $K$ values except $K_{MH}$ increase with increase in $\alpha$ indicating that displacement fields get deeper as they encounter material of increasing stiffness. The effect of $\alpha$ is greatest for vertical and moment load cases.

### 2.3.1 Small Strain Stiffness & Degradation

DNV (2002) provides an empirical relationship to determine the small strain shear modulus ($G_0$) for saturated undrained clay.

$$ G_0 = 2600.S_u $$

(2.5)

Shear modulus of soil has been found to be a function of cyclic shear strain ($\gamma_c$) mobilized below the foundation interface. DNV (2002) provides expected range of shear strains ($\gamma$) for the three most important sources of dynamic loading of soils:

- Earthquakes: large strains up to $10^{-2}$ to $10^{-1}$
- Rotating machines: small strains usually less than $10^{-5}$
- Wind and ocean waves: moderate strains up to $10^{-2}$, typically $10^{-3}$ (includes wind turbine foundations)
With increase in cyclic shear strains, the shear modulus is reduced by a modulus reduction factor \((m)\) given by:

\[
m = \frac{G}{G_o}
\]  

(2.6)

Fig. 2.4 shows the variation of the reduction factor with cyclic shear strain found by Vardanega and Bolton (2011) who analyzed a detailed database of stiffness degradation to estimate the behaviour of clays and silts for static, cyclic or dynamic applications. The variation of reduction factor given by DNV (2002) is also shown in the figure. The larger the shear strain, the higher is the reduction in the shear modulus. A small decrease in shear strain can greatly increase the operational shear modulus in the field and in turn, increase the mobilized rotational stiffness of the foundation. DNV (2002) recommends \(\gamma_c = 0.001\) for wind turbines, for which \(m\) can vary anywhere between 0.30 and 0.70 depending on the type and state of soil. Further work is required to determine accurate values of mobilized shear strain below a wind turbine foundation so that foundation size can be optimised.

Figure 2.4: Variation of shear modulus as a function of cyclic shear strain
2.4 Bearing Capacity Analysis for Onshore Wind Turbine Foundations

2.4.1 Conventional Bearing Capacity Methods

In conventional approaches, analysis of complex loading (combined V, H & M) acting on a foundation is based on the simplest case of pure vertical load acting on a strip footing. For the condition of plane strain, Terzaghi (1943) presented the theoretical ultimate vertical bearing capacity of strip footing resting on uniform soil as

\[ q_u = \frac{Q}{BL} = cN_c + \gamma D\eta_q + 0.5\gamma BN_q \]  

(2.7)

where \( q_u \) is the ultimate bearing capacity. For undrained uniform clay soil, eqn. 2.7 reduces to

\[ q_u = \frac{Q}{BL} = c_u N_c \]  

(2.8)

Thus, under undrained conditions, the ultimate vertical bearing capacity for a surface foundation is dependent only on soil undrained shear strength/cohesion and bearing capacity factor. The exact value of \( N_c \) was calculated analytically by Prandtl (1921) to be \( \pi + 2 \). The Prandtl failure mechanism is shown in Fig. 2.5.

![Prandtl mechanism of soil failure](Prandtl_1920.png)

Figure 2.5: Prandtl mechanism of soil failure (Prandtl, 1920)
The Hill mechanism of soil failure is also shown in Fig. 2.6 given above. This is typically associated with soils with strength increasing with depth.

However, foundations of wind turbines are not just subjected to purely vertical loads, but also to inclined and eccentric loads, i.e. there is a component of horizontal force and moment acting simultaneously. Under these influences, the bearing capacity of the foundation usually reduces substantially. To take into account the effect of eccentricity ($e$), Meyerhof (1953) proposed the principle of effective width ($B'$) which is found from:

$$B' = B - 2e$$  \hspace{1cm} (2.9)

Thus, the original width is reduced by an amount equal to twice that of the eccentricity, thereby lowering the contribution of soil shear strength (the 3rd term in eqn 2.7) to the bearing capacity of the foundation. This holds true only if the eccentricity of the load is with respect to a single axis of symmetry. When eccentricity of the load exists with respect to both the axes of symmetry, the length of finite foundations (square, rectangular or circular) also gets modified and is equal to the effective width. Thus, a foundation has an effective contact area
with the soil given by:

\[ A' = B' \times L' \]  \hspace{1cm} (2.10)

The effective breadths, lengths and foundation areas for square, circular and octangular footings are shown in Fig. 2.10, 2.11 and 2.12 respectively in the next section. The geometric centre of the foundation coincides with location of loading (the point on the foundation base where resultant of the vertical and horizontal forces intersect).

For rough foundations, Meyerhof (1963) also proposed analytical solutions in the form of inclination factors to calculate the effect of a load inclined at an angle \( \alpha \) with the vertical (Fig. 2.8). The expressions are:

\[ i_c = i_q = (1 - \theta/90^\circ)^2 \] \hspace{1cm} (2.11)

\[ i_y = (1 - \theta/\phi)^2 \] \hspace{1cm} (2.12)

With inclination factors and the effective width approach, the expression of bearing capacity
Figure 2.8: Inclined load at an angle $\theta$ with centreline of foundation is modified (eqn. 2.13), but still remains relatively simple and straightforward.

$$q = \frac{Q}{B'} L' = i_c N_c + i_q \gamma D N_q + 0.5 i_q \gamma B'$$  
(2.13)

Besides inclination and eccentricity of loads, in most practical geotechnical engineering situations, foundations are usually placed a few meters below ground level. This increases the bearing capacity owing to the higher shearing resistance provided by soil in contact with the foundation sides. Skempton (1951) originally presented depth factors to show this effect and based on many test results, Meyerhof modified them to suit practical design purposes, when the depth of the foundation is less than the width or diameter of the foundation ($d/B, d/D < 1$).

$$d_c = 1 + 0.2 N_\phi^{0.5} D/B$$  
(2.14)

$$d_q = d_\gamma = 1 \quad \text{when} \quad (\phi = 0)$$  
(2.15)

$$d_q = d_\gamma = 1 + 0.1 N_\phi^{0.5} D/B \quad \text{when} \quad (\phi > 10)$$  
(2.16)

With increase in the depth of the foundation, the depth factors increase at a decreasing rate.

Theoretical and semi-empirical factors were introduced into the bearing capacity equation to consider the effects of change of foundation shape from a strip to a circle or square. For a
footing under undrained conditions, the shape factors can be expressed as

\[
sc = 1 + 0.2N_\phi B/L \tag{2.17}
\]
\[
sq = s_y = 1 \tag{2.18}
\]

Thus, Meyerhof’s work extended the bearing capacity theory given by Terzaghi (1943) to take into account the effects of inclination & eccentricity of loads and shape & embedment of foundation. It must be noted here that issues of inclination and eccentricity of loads were addressed separately by Meyerhof. Hansen (1970) also similarly modified the simple empirical formula (eqn. 2.7) to take into account the effects of inclined and eccentric loads. Hansen, however, proposed different depth factors based on the \(D/B\) ratio.

\[
d_c = 1 + 0.4 \frac{D}{B} \quad \text{when} \quad D/B < 1 \tag{2.19}
\]
\[
d_c = 1 + 0.4 \arctan \left( \frac{D}{B} \right) \quad \text{when} \quad D/B \geq 1 \tag{2.20}
\]

Additionally, he proposed base inclination and ground inclination factors for an inclined foundation base and sloping ground respectively. For the case of undrained soil (\(\phi = 0\)), Hansen introduced additive constants, instead of factors, which are theoretically more correct.

\[
q = Q/A = (\pi + 2)Cu(1 + sc + dc - ic) \tag{2.21}
\]

An obvious advantage of introducing additive constants in the equation instead of multiplicative factors is that the effects of shape, depth and inclination can readily be understood by a reader just by looking at the equation. While shape and depth have an increasing effect on bearing capacity, inclination decreases the ultimate capacity of foundation.

### 2.4.2 The DNV Design Guidelines

The DNV (2002) guidelines are widely accepted and used in the wind industry for wind turbine foundation design in conjunction with local/country-specific design codes. Additionally, DNV
(1992) and ISO (2011) provide recommended practices for offshore geotechnical and foundation design.

DNV (2002) considers the stability of foundations by adopting limit equilibrium methods to ensure equilibrium between the driving and resisting forces. Onshore wind turbine foundations usually have relatively smaller footprints and hence the problem of ultimate bearing capacities is solved by assuming idealized conditions. With a combination of vertical (V) and horizontal (H) forces, the bearing capacity is calculated in the same way as suggested by Meyerhof (1963) or Hansen (1970).

![Figure 2.9: Loading under idealised conditions (DNV, 2002)](image)

The forces H and V shown in the Fig. 2.9 above represent design forces, i.e. characteristic forces multiplied by partial safety factors. The eccentricity \((e)\) of the load is calculated using:

\[
e = \frac{M_d}{V_d}
\]  

\((2.22)\)

\(M_d\) is the resulting design overturning moment at the foundation-soil interface due to the combination of vertical and horizontal forces. When \(e \leq 0.3B\), rupture 1 in Fig. 2.9 is considered the most critical failure surface with respect to stability. When \(e > 0.3B\), rupture 2 becomes
the most critical failure surface and bearing capacity is calculated with:

\[ q_d = \gamma' b_{\text{eff}} N_y s_y i_y + c_d N_c s_c i_c (1.05 + \tan^3 \phi) \]  

(2.23)

Under undrained conditions \((\phi = 0)\), this equation reduces to:

\[ q_d = c_d N_c s_c i_c 1.05 \]  

(2.24)

Thus, for undrained soils under extremely eccentric loading, the bearing capacity increases by at least 5% assuming all inclination factors remain the same. However, the values of the inclination factors in both cases are different (Table 2.4).

<table>
<thead>
<tr>
<th>Rupture type</th>
<th>( i_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rupture 1</td>
<td>( \sqrt{0.5 + 0.5[1 - H/(A_{\text{eff}} c_{ud})]} )</td>
</tr>
<tr>
<td>Rupture 2</td>
<td>( \sqrt{[0.5 + 0.5\sqrt{1 + H/(A_{\text{eff}} c_{ud})}]} )</td>
</tr>
</tbody>
</table>

Table 2.4: Inclination factors for different rupture conditions

The value of \( i_c \) for the case of rupture 2 is about 33% higher than that of rupture 1. The overall increase of bearing capacity in the case of extremely eccentrically loaded foundations can be attributed to the failure of additional soil below the unloaded part of the foundation corresponding to the area under the heel.

Under eccentric loading, like Meyerhof (1963), DNV also suggests using an effective foundation area (Eqn 2.10). However, they both differ in the calculation of effective width and length (Table 2.5). When load eccentricity is with respect to only one of the two symmetry axes of the foundation, the effective width is the same as that shown in eqn. 2.9. The effective length is equal to the original width of the foundation. Meyerhof (1963), however, did not explicitly mentioned effective length in this case. For the case of double eccentricity, i.e. when load is eccentric with respect to both the symmetry axes, Meyerhof (1963) suggests using an effective
length $L' = B' = B - 2e$. On the other hand, DNV recommends using $L' = B' = B - e \sqrt{2}$ (Table 2.5).

![Figure 2.10: Load eccentricity with respect to one of two symmetry axes (DNV, 2002)](image)

Figure 2.10: Load eccentricity with respect to one of two symmetry axes (DNV, 2002)

![Figure 2.11: Load Eccentricity with respect to both the symmetry axes (DNV, 2002)](image)

Figure 2.11: Load Eccentricity with respect to both the symmetry axes (DNV, 2002)
Meyerhof’s method underestimates the contact area by approximately 1.2%, as compared to that given by DNV in the case of doubly eccentric load. With a circular foundation, an elliptical effective area is defined.

\[ A_{eff} = 2\left[R^2. \arccos\left(\frac{e}{R}\right) - e\sqrt{\left(R^2 - e^2\right)}\right] \text{ with major axes} \quad (2.25) \]

\[ b_e = 2(R - e) \quad \text{and} \]
\[ l_e = 2R(1 - \sqrt{1 - \frac{b_e^2}{2R^2}}) \]

To simplify this process, a provision to represent the elliptical effective area as a rectangular area is also mentioned:

\[ L_{eff} = \left(\frac{A_{eff}.L_e}{b_e}\right) \quad (2.26) \]
\[ b_{eff} = \left(\frac{l_{eff}}{L_e}\right) b_e \quad (2.27) \]
Figure 2.12: Effective area for circular and octagonal foundations (DNV, 2002)

For octagonal wind turbine foundations, the effective area is the same as that calculated for circular foundations as long as the radius of the inscribed circle of the octagon (Fig. 2.12 is used for the calculations. Analytical methods such as those by Meyerhof (1963) or Hansen (1970) did not address the question of load eccentricity or inclination for the case of polygonal foundations.

Under undrained conditions, the shape factor $s_c$ is given by:

$$s_c = 1 + 0.2 \frac{b_{eff}}{l_{eff}}$$ (2.28)

Thus, the increased bearing capacity of the foundation due to the shape is dependent on effective dimensions of footing contact area.

In addition to the ultimate bearing capacity, DNV also recommends a check of stability of the foundation against horizontal sliding by providing two criteria:

$$H < A_{eff} \cdot c_{ud}$$ (2.29)
These two conditions define the maximum allowable value of destabilizing horizontal force at the interface. While the first condition (eqn. 2.29) limits it to the undrained shear strength of the soil, the latter condition (eqn. 2.30) limits it to 40% of the vertical load. The latter condition also subtly implies that reduction of vertical load beyond a certain value can make sliding failure critical and reduce the reliability of design. Therefore, combinations of horizontal and vertical design loads must be carefully chosen to maintain the $H/V$ ratio.

For embedded foundations, DNV (2002) recommends that the same formula, as used for surface foundations, can be utilized but with a note that it would be conservative. Alternatively, DNV (1992) suggests the following depth factor for foundations.

$$d_c = 0.3 \arctan(D/B')$$  \hspace{1cm} (2.31)

Hansen (1970) also gave a similar depth factor.

$$d_c = 0.4 \arctan(D/B)$$  \hspace{1cm} (2.32)

While the depth factor suggested by Hansen (1970) is dependent on the original width $B$, that given by DNV depends on the effective width of the foundation. For soils with linearly increasing undrained shear strength with depth, DNV (1992) provides a solution for obtaining the ultimate capacity of the foundation.

$$q_u = F.(5.14.s_{uo} + k.B'/4)(1 + s_c + d_c - i_c)$$  \hspace{1cm} (2.33)

The above equation is considered a modified form of the eqn. 2.21 given by Hansen (1970). The terms $kB'/4$ and correction factor $F$ have been added to take into account the effects of linearly increasing strength and are based on the method given by Davis and Booker (1973), see Section 2.5.1 for further details.
2.4.3 The ISO Design Guidelines

The ISO (2011) design guidelines published by the American Petroleum Institute (API) provides geotechnical and foundation design considerations specific to offshore structures. ISO guidelines are also recommended for use with the foundations subjected to less complex loading and soil conditions, such as onshore wind turbine foundations, which are exposed to less severe environmental forces as opposed to their offshore counterparts. Therefore, the methods used to obtain responses for shallow foundations stated by these guidelines are reviewed here. Like DNV, ISO also recommends using limit equilibrium methods to obtain foundation stability under ultimate limit state conditions. The ISO (2011) guidelines take into account the effects of shape and embedment depth of foundations, and inclined & eccentric loads in a very similar manner to that given by DNV (2002). Therefore, the reader is directed to Section 7 of ISO (2011) for more thorough explanations of these effects. Only those aspects where ISO (2011) differs significantly from DNV (2002) are presented here.

- ISO recommends calculating failure envelopes encompassing a range of loads to capture the ultimate capacity of a foundation under eccentric or inclined loading. Safety factors are applied to obtain failure envelopes for allowable loads. Safety factors are applied only to ultimate loads and not to environmental forces & storm loads.

- For embedded foundations, design vertical loads can be modified in a similar fashion as that for skirted foundations. Alternatively, under inclined loading, horizontal loads can be offset by reducing them using safety factors to reflect increase in bearing capacity due to embedment.

- Tensile stresses must be avoided under the foundation. A minimum factor of safety 1.5 and 1.0 is recommended against averaged soil tension and any localized tension under the foundation respectively.

- ISO states that the effective area rule, such as that suggested by Meyerhof can be conservative, especially for the case of large overturning moments and horizontal forces. In such cases, it is recommended to explicitly express combinations of vertical, horizontal and moment
loads in terms of load surfaces or interaction diagrams. Examples of 3-dimensional VHM failure envelopes for no-tension and full adhesion are shown in the guidelines. Key references are cited for further reading on numerical and analytical work on combined loading of shallow foundations and the effects due to embedment, shape & soil strength heterogeneity. ISO recommends seeking specialist geotechnical advice when using the alternative yield surface design approach.

- For sliding failure, unlike DNV (2002), ISO specifies only 1 criteria which is to limit maximum horizontal stress developed at the interface to the undrained soil shear strength.

### 2.5 Foundations Subjected to Combined Loading: the Failure Envelope Approach

In the light of increasing lateral loads and overturning moments on foundations in atypical environments, the validity of traditional design methods to accurately predict bearing capacities under general loading has been questioned by many researchers [e.g. Bransby and Randolph (1999); Houlsby and Puzrin (1999); Gourvenec and Randolph (2003); Randolph et al. (2005)]. The conventional methods given by Meyerhof (1963) & Hansen (1970), and design guidelines from DNV & ISO do not take 3-dimensional geometry of foundations into account explicitly. Due to the shape and depth of embedment, more soil around the foundation edges is mobilized and additional slip surfaces are formed in front of, behind and above the base of the footing, providing higher shear resistance and greater bearing capacity. Conventional methods fail to explain this effect thoroughly. For wind turbine foundations, where horizontal forces and overturning moments play a major role in loading, sliding failure can become critical in many cases and these traditional design methods can give conservative results.

Several numerical studies have been conducted to study the ultimate bearing capacities of foundations subjected to complex loading to better understand their stability. Zaharescu (1961) and Ticof (1977) suggested that eccentric and inclined loads acting on a foundation can be
expressed as a combination of vertical, horizontal and moment loads explicitly. These loads can then be drawn in two or three dimensional load space as failure envelopes or ‘interaction diagrams’. The availability of such diagrams can help designers in making cost-effective design by providing a range of options to choose the appropriate combination of loads safely, rather than relying on a single uniaxial capacity factor (Butterfield et al., 1997). A literature review of works pertaining to the computation of bearing capacities and their expressions in the form of combined load space failure envelopes are covered in the next sections of this chapter. The methods, scope, mesh details, loading method and conclusions are summarized wherever necessary. The papers reviewed here study the response of foundations and the influence of many parameters, viz. shape, depth of embedment and soil strength heterogeneity (e.g. soil with linearly increasing strength with depth) on the bearing capacities of foundations. The research work on the effect of surficial crusts on the bearing capacity of a foundation is also reviewed.

2.5.1 Review of Failure Envelope Papers

The effect of soil strength heterogeneity was studied by Davis and Booker (1973) who used the theory of plasticity to obtain bearing capacity responses of foundations on clays with strength inhomogeneity in the vertical direction. They showed that the rate of increase of cohesion with depth played the same role as that of the density in homogeneous $c - \phi$ soils. A bearing capacity expression for a rigid strip footing when a stiff crust layer is present over soft clayey layer with linearly increasing strength with depth, was calculated (eqn. 2.34). The variation of cohesion with depth is shown in Fig. 2.13.

$$Q/B = F[(2 + \pi)S_{u0} + \rho B/4]$$  \hspace{1cm} (2.34)

where $F$ is a correction factor, $\rho$ is the strength gradient (kN/m)and $B$ is width of the strip footing. As compared to a footing on homogeneous soil, the roughness of the footing has a small but significant effect on the bearing capacity of the footing. In eqn. 2.34, smoothness or roughness of a footing is taken into account by changes in the correction factor $F$. The
variation in the correction factor $F$ with the strength gradient is shown in Fig. 2.14.

The solutions from plasticity theory were also compared with that calculated from the conventional slip circle method. When $\rho B/S_{u0}$ is zero, the slip circle solution is only 8% greater than the exact plasticity solution. However, when $S_{u0}/\rho B$ is zero, the conventional solution is
350% greater. This poor performance was attributed to the conventional method taking the plastic work into account only partially.

Tani and Craig (1995) studied surface strip and circular foundations on soils with linearly increasing strength with depth. The main aim of this study was to understand the behaviour of deep skirted foundations. A linearly increasing strength profile with depth was expressed with a non-dimensional ratio $kB/c_u0$ where $k$ (shown as $\rho$ in eqn. 2.34) represents the rate of increase of undrained strength $c_u0$ with depth $z$ and $c_u0$ is the undrained shear strength at soil surface or mud-line. The value of $k$ can range from 0 to 30. Such a linearly increasing strength profile is suitable for normally consolidated (NC) or lightly over consolidated (OC) clays. Both, smooth and rough contacts were considered. Bearing capacity response of strip and circular foundations resting on soil surface was studied under plane strain and axisymmetric conditions respectively. Centrifuge experiments were also carried out on circular models founded in clay to verify the theoretical results. Models were subjected to uniaxial vertical load only. For clays, the value of $k$ (typically $0.6 \text{kPa m}^{-1}$ to $3 \text{kPa m}^{-1}$) can be estimated as product of effective unit weight ($\gamma' = 4 \text{kN m}^{-2}$ to $10 \text{kN m}^{-2}$) and rate of increase of undrained strength $c_u$ due to effective vertical consolidation stress $\sigma_v'$ under normally consolidated conditions $\Delta c_u/\sigma_v'$ (typically $0.15 \approx 0.30$).

Using the stress characteristic method, bearing capacity of foundation was calculated theoretically in terms of shape and depth factors taking increasing shear strength into account. Since it was essential to accurately characterize the linearly varying strength profile, the cone penetration test technique was used for the centrifuge. The theoretical method proposed overestimated the experimental results by 6 to 17%. Punching shear failure was also observed in the model clay beds confined at shallow depths.
Fig. 2.15 shown above gives bearing capacity factors with variations in $kB/c_0$ or $kD/c_0$ and compares it with the solutions of Davis and Booker (1973) & Houlsby and Wroth (1983) for strip and circular footings. Excellent agreement was found for the results of smooth footings. The results for the rough footing were, however, less consistent as constructing an appropriate stress field is difficult.

The undrained bearing capacity of strip and circular surface foundations on a deposit with varying degrees of soil strength heterogeneity was investigated by Shen et al. (2016) using finite element analyses. A zero-tension interface was used in the analyses by assuming a coefficient of friction, $\mu = 20$. This gives an angle of internal friction, $\phi = 87.1^\circ$ ($\approx 90^\circ$). Displacement controlled probe tests and sideswipe tests were used to identify VH and VM failure envelopes. For the VHM envelope, a constant vertical load was imposed as a proportion of ultimate vertical load and subsequently, displacement probe tests were used to apply horizontal and moment loads.

The traditional approach of superimposing VH and VM solutions to represent combined VHM conditions was shown to be conservative. VHM failure envelopes derived by the conventional
method and Shen et al. (2016) are shown in Fig. 2.16. The asymmetry of envelopes increases with increasing vertical load mobilization and degree of soil strength heterogeneity. An approximate expression was also proposed (shown as Eqn. 9 in the Fig. 2.16).

\[
\left(\frac{h}{h^*}\right)^2 + \left(\frac{m}{m^*}\right)^2 = 1
\]

(2.35)

where

\[ h^* = 1 - 4(v - 0.5)^2, \quad v > 0.5 \]

\[ h^* = 1, \quad v \leq 0.5 \]

and

\[ m^* = 4(v - v^2) \]

\( q = 1.5 \) for strip and circular foundations.

Figure 2.16: Failure envelopes for circular foundations for VHM loading (Shen et al., 2016)
A special case of soil strength heterogeneity occurs with the presence of a surficial crust of high shear strength on a relatively soft clayey soil. The heterogeneity of soil exists in the form of two layers with variable strengths; one being much higher than the other. Feng et al. (2015) investigated the undrained bearing capacity response of rectangular mudmats under fully three-dimensional loading, V-H^2-M^2-T in soils with a surficial crust overlying normally consolidated clay. Horizontal loads were applied along x and y axes (H_x and H_y). Similarly, moment was applied about both the axes (M_x and M_y).

Fig. 2.17 shows the general loading for a rectangular mudmat and the conceptual model the researchers used to define the problem, where

\( S_{ubs} \) = Undrained shear strength of the layer underlying the crust  
\( S_{ut} \) = Average undrained shear strength at top near the soil surface  
\( t_c \) = Thickness of the crust  
\( k \) = Rate of change of undrained shear strength of underlying layer below the crust to crust thickness.

From a parametric study, they found that the pure vertical and moment capacity depends on the shear strength ratio between the underlying soft clay and stronger crustal layer, \( s_{ubs}/s_{ut} \), thickness of the crust relative to the foundation width, \( t_c/B \) and the foundation embedment...
relative to the crust thickness, $d/t_c$. Mathematically, this was expressed as

$$\frac{V_{ult}}{A_{ult}} \text{ or } \frac{M_{ult}}{ABS_{ult}} = a_1 \left( \frac{s_{ult}}{s_{ult}} \right)^{a_2}$$

(2.36)

where the coefficients $a_1$ and $a_2$ are a function of $d/t_c$ and $t_c/B$. Normalized envelopes in VH, VM, VH, HT and HM planes were presented and approximate expressions (Table 2.6) to describe the shape of the envelopes were provided, which can be easily implemented into an automated calculation tool, to provide an optimized foundation design in terms of foundation geometry, soil strength characteristics or load components.

<table>
<thead>
<tr>
<th>2-dimensional plane</th>
<th>Approximate expressions</th>
</tr>
</thead>
<tbody>
<tr>
<td>VH</td>
<td>$h = 1$ for $v \leq 0.4$</td>
</tr>
<tr>
<td></td>
<td>$v = 0.4 + 0.6 \sqrt{1 - h^{2.5 - \cos^2 \theta}}$ for $v &gt; 0.4$</td>
</tr>
<tr>
<td>VM</td>
<td>$v = (1 - m)^p$</td>
</tr>
<tr>
<td>VT</td>
<td>$t = 1$ for $v \leq 0.4$</td>
</tr>
<tr>
<td></td>
<td>$v = 0.4 + 0.6 \sqrt{1 - t^{6(d/t_c - 0.5)^2} + 2}$ for $t &lt; 1$</td>
</tr>
<tr>
<td>HT</td>
<td>$\left( \frac{H_{ult}}{H_{ult}} \right)^m + \left( \frac{M_{ult}}{M_{ult}} \right)^n = 1$</td>
</tr>
<tr>
<td>HM</td>
<td>$\left[ \frac{M^<em>}{M^</em><em>{ult}} - \alpha \frac{H</em>{ult}}{H_{ult}} \frac{M^<em>}{M^</em><em>{ult}} + \beta_0 \left( \frac{H</em>{ult}}{H_{ult}} \right)^2 \right]^3 \times \left( 1 - 0.9 \frac{H_{ult}}{H_{ult}} \frac{M^<em>}{M^</em><em>{ult}} \right) + \left( \frac{H</em>{ult}}{H_{ult}} \right)^2 = 1$</td>
</tr>
</tbody>
</table>

Table 2.6: Approximate expressions for normalized failure envelopes

Bransby and Randolph (1998) investigated the response of skirted foundations subjected to combined loading. They performed 2-dimensional finite element analysis and upper bound analysis on an equivalent surface strip footing in undrained soil with linearly increasing strength. The non-dimensional strength heterogeneity factor $kB/S_{u0}$ was usually kept equal to 6, where $S_{u0}$ is undrained shear strength at the mud-line. This particular value was chosen since it led to mechanistic behaviour typical for a wide range of $kD/S_{u0}$. Footing-soil contact was assumed to be rough. Yield loci in VH, VM and HM planes were obtained by performing ‘side-swipe’ tests originally suggested by Tan (1990) and displacement probes. A side-swipe
test consists of two stages. In the first stage, the foundation is loaded till the ultimate point in one direction (usually vertical) and in the second stage, it is loaded till the ultimate point in second direction (usually horizontal or moment), keeping the load in the first direction constant. Displacement probe tests were used to verify the shape of yield locus predicted by the side-swipe tests. All loads were applied at a single reference point which was the centre of the base of the foundation.

The ultimate vertical load was compared with solutions from plasticity analysis. When $kD/S_{u0} = 6$, the value of $N_c = 10.7$ compares well with that found out by Tani and Craig (1995) [10.49] and Houlsby and Wroth (1983) [10.37]. In the VH and VM planes, the shape of the yield locus was found to be similar to that reported by previous works (Fig. 2.18 & 2.19). Whereas in the HM plane, the yield locus differed significantly (Fig. 2.20). This was attributed to the coupling of horizontal force and moment. A scoop mechanism or wedge-scoop-wedge mechanism was observed post failure due to combined HM loading.

![Figure 2.18: Yield locus under VH loading](image)

Figure 2.18: Yield locus under VH loading
Taiebat and Carter (2000) determined the response of a surface circular foundation resting on a homogeneous clay soil. Three-dimensional finite element analysis was performed to obtain the VH, VM, HM and VHM failure envelopes. The soil-foundation interaction was assumed to be completely rough and fully bonded. While a rough contact allows development of shear stresses, a fully bonded contact sustains unlimited tension at the interface. A rough contact
defines the tangential behaviour while a fully bonded contact defines the normal behaviour at the interface. No separation was assumed to occur between the foundation and soil due to the suctions developed during undrained behaviour. Swipe and displacement probe tests were used to perform displacement controlled analysis. To define the point of uniaxial ultimate vertical and moment load, a very small horizontal loads was applied (e.g. V/H = 60). Calculation of vertical bearing capacity factor and load-displacement response of footing under vertical and horizontal load is illustrated in Fig. 2.21. From the figure, a value of $v_0 = 5.7$ was computed and was considered the ultimate vertical point.

Under pure moment, moment capacity factor $m_0 = 0.80$ was found. A very small horizontal load in the ratio $M/H = 100$ was applied to better detect the ultimate moment point. Derived solutions were compared with the conventional methods graphically (Fig. 2.22).

![Figure 2.21: Load-displacement response under VH loading (Taiebat & Carter, 2000)](image)
In common with Bransby and Randolph (1998), Taiebat also reported that with positive moment
and positive horizontal load, maximum moment capacity is achieved (Fig. 2.23), in contrast to positive moment alone. This demonstrated the coupling between horizontal and moment degrees of freedom. A maximum moment capacity factor $m_0 = 0.89$ was found which is 11% higher than that found under pure moment. This coincided with horizontal load of $H = 0.71A_s u$.

Under inclined loading (vertical and horizontal force), critical angle, measured from the vertical direction, was predicted to be $19^\circ$, which is the same as that predicted by the modified expression of Bolton (1979) but a little higher than the $13^\circ$ calculated from Vesic (1975). The modified expression of Bolton (1979) for a circular footing, using a shape factor of $s_c = 1.2$ can be written as:

$$V = 1.2S_u \left[1 + \pi - \arcsin \left(\frac{H}{A_s u}\right) + \sqrt{1 - \left(\frac{H}{A_s u}\right)^2} \right]$$

An important outcome of the research was that conventional methods of calculating bearing capacity does not always yield conservative results. For example, under high horizontal loads, the conventional method overestimates the bearing capacity (Fig. 2.22). However, failure envelopes in the VM planes calculated by conventional methods were found to be conservative compared to those calculated by numerical methods. The extent of the plastic zone and displacement vectors showing movement of soil were also presented for each case.

Taiebat and Carter (2002a) analyzed the response of strip and circular foundations subjected to eccentric loads. Special ‘no-tension’ interface elements were used to model the separation between footing and soil. For strip foundations, 2-dimensional analysis and for circular foundations, 3-dimensional finite element analysis were performed. A new set of shape factors for circular footings with smooth and rough contacts was proposed; 1.11 and 1.18 respectively, which is less than 1.2 proposed by Meyerhof (1953). Failure surfaces deduced from numerical analysis were compared with lower and upper bound solutions. *It was concluded that under eccentric loads alone, conventional methods predict collapse loads with reasonable accuracy.*

Gourvenec and Randolph (2003) explored the influence of foundation shape and soil strength heterogeneity on the shape and size of 2-dimensional failure envelopes (i.e. VH, VM and
HM planes). Two and three dimensional finite element analysis were performed on strip and circular foundations fully bonded with clayey soil. Additionally, upper bound limit analysis was conducted to validate the finite element results. Swipe tests and displacement probe tests were used to obtain uniaxial limit loads and failure envelopes.

In the VH plane, the effects of shape or soil non-homogeneity were found negligible and the Green’s solution (1954) could be scaled as per magnitude of ultimate vertical load to obtain the failure envelope. However, in the VM and HM planes, the degree of heterogeneity significantly influenced the shape of the envelopes. While normalized envelopes in the VM plane could be fitted with a power law relationship, no simple relationship could be derived for the HM plane due to its complex shape. Normalized size of failure envelopes were found to be similar for strip and circular footings (Fig.2.24,2.25).  

*This research work showed that for a footing subjected to combined loading, scaling of the failure envelopes for homogeneous conditions to derive failure envelopes for non-homogeneous conditions would be non-conservative. This can be attributed to changing kinematic failure mechanisms due to changes in loading state and heterogeneity.*

![Normalized failure envelope in VH plane for circular footing (continuous line) and strip footing (dotted line) (Gourvenec & Randolph, 2003)](image)
Salgado et al. (2004) performed rigorous 3-dimensional finite element limit analysis on rough strip, circular, square and rectangular footing in clays under the action of centric vertical loads. Footings were placed at 13 different embedment ratios. Limit base resistances of these footings placed at various depths were deduced to calculate their lower and upper bound limits. Closed
form expressions for shape and depth factors proposed by Salgado and other previous works are given in Table 2.7.

<table>
<thead>
<tr>
<th>Author</th>
<th>Shape factor ( (s_c) )</th>
<th>Depth factor ( (d_c) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meyerhof (1953) &amp; Skempton (1951)</td>
<td>( 1 + 0.2.B/L )</td>
<td>( 1 + 0.2.d/B )</td>
</tr>
<tr>
<td>Hansen (1970)</td>
<td>( 1 + 0.2.B/L )</td>
<td>( 1 + 0.4.d/B ) for ( d/B &lt; 1 ) ( 1 + 0.4. \arctan(d/B) ) for ( d/B \geq 1 )</td>
</tr>
<tr>
<td>Salgado et al. (2004)</td>
<td>( 1 + 0.12.B/L + 0.17\sqrt{d/B} )</td>
<td>( 1 + 0.27.\sqrt{d/B} )</td>
</tr>
<tr>
<td>Gourvenec (2008)</td>
<td>( d_{cVault} )</td>
<td>( 1 + 0.86.d/B - 0.16.(d/B)^2 )</td>
</tr>
<tr>
<td></td>
<td>( d_{cHault} )</td>
<td>( 1 + 4.46.d/B - 1.52.(d/B)^2 )</td>
</tr>
<tr>
<td></td>
<td>( d_{cMult} )</td>
<td>( 1 + 1.27.d/B + 1.27.(d/B)^2 )</td>
</tr>
</tbody>
</table>

Table 2.7: Closed form expressions for shape and depth factors

All of the shape and depth factors proposed by Meyerhof (1953) were found to be conservative. For \( D/B < 0.5 \), the depth factor proposed by Hansen (1970) was found to be conservative, while for \( D/B \geq 0.5 \), it was found non-conservative. A very narrow band of upper and lower bound solutions for strip footings was obtained. In contrast, the two bounding solutions diverged at greater depths for other footing shapes. By proposing definite values of shape and depth factors, Salgado’s work helped to reduce uncertainties in predicting ultimate bearing capacity.

Gourvenec (2004) examined the response of shallow circular foundations for varying soil shear strength conditions. Unlike Gourvenec and Randolph (2003), wherein swipe and probe tests were used, Gourvenec (2004) applied vertical load as force and horizontal & moment loads as displacements at fixed ratios. Since rough contact was considered for the 3-dimensional model, moment resistance is developed even at zero vertical load. Kinematic mechanisms under failure due to maximum moment were shown for different soil conditions and vertical loads. At zero
vertical load and homogeneous conditions, a near-perfect scoop mechanism was observed similar to that reported by Bransby and Randolph (1998). As strength heterogeneity increases, soil deformations are confined towards the upper soil layers having low shear strength and depth of failure decreases (Fig. 2.27). This indicates that shallower layers play a more important role in forming the failure mechanisms as strength heterogeneity increases. Increasing strength heterogeneity also leads to increasing eccentricity of the HM failure envelope.

With increase in vertical load, transformation of the failure mechanism from scoop (homogeneous soil) or wedge-scoop-wedge (heterogeneous soil) mechanisms to wedge-scoop mechanisms were observed. The derived failure envelopes for HM plane at different vertical loads were compared with envelopes obtained from conventional design methods (Fig. 2.28) by scaling and highlighted the extra capacity neglected by conventional design methods confirming the findings of Gourvenec and Randolph (2003).
Figure 2.27: Failure mechanisms under maximum moment (Gourvenec, 2004)
Edwards et al. (2005) inspected the response of strip and circular footings at different embedment ratios ranging from 0 to 4. In finite element analysis, they considered only 2-dimensional
geometry and applied vertical loads in the form of vertical displacements. The soil had uniform undrained strength throughout its depth. The soil-foundation base interface was always rough, whereas the sides were either smooth or rough. They demonstrated that the depth factors for circular footing were significantly larger than those for strip footings. They also showed that these factors are not unique for any foundation shape, but depend on the roughness between the vertical sides of the footing and soil. They suggested that solutions derived by Salgado et al. (2004) for lower bound and that derived by Martin (2001) for circular footings embedded at $d/B > 1$ needed further optimization. For smooth strip footings, solutions given by Skempton (1951) were found to be in good agreement with the finite element results. However, Edwards et al. (2005) did not provide any closed form expressions for the shape and depth factors.

Yun and Bransby (2007) specifically investigated the effects of embedment ratio on shape and size of the HM failure envelopes under combinations of horizontal and moment loads with zero vertical load. Citing similarities of soil deformation mechanisms and associated bearing capacity envelopes for strip and circular foundations, only strip foundations were studied. Both finite element analysis and upper bound plasticity analysis were conducted. Soil-footing contact was fully bonded and this allowed development of horizontal and moment capacity even at zero vertical load. Analysis on uniform soil and normally consolidated undrained soil were performed. The load reference point was taken as the mass centre of the foundation rather than the usually adopted centre of foundation-soil interface at the base. It was found that as the embedment ratio increases, the eccentricity of the failure envelopes also increases near $M_{ult}$ and $H_{ult}$. This was attributed to a scoop-type mechanism near $M_{ult}$ and a reverse-scoop mechanism near $H_{ult}$. The failure conditions for embedded foundations and equivalent surface foundations differed significantly and hence it was emphasized to consider embedments of foundations explicitly.

Research by Gourvenec (2008) addressed the effects of embedment on undrained bearing capacity of surface strip foundations by performing 2-dimensional analysis. Foundations were subjected to uniaxial limit loads as well as combined loadings. Four embedment ratios very similar to that considered by Yun and Bransby (2007) were used. Unlike Yun and Bransby
(2007), Gourvenec (2008) also studied the shape and size of failure envelopes under VH and VM loadings. A completely bonded contact for the bottom as well as the sides was assumed, suitable for offshore scenarios. To achieve accurate results, swipe tests were carried out only for surface foundations to deduce VH and VM failure envelopes. For achieving all other results, fixed-ratio displacement tests were done.

It was concluded that the size and shape of failure envelopes for a strip foundation are dependent on embedment ratio. Uniaxial limit capacities were found to vary with the square of the embedment ratio. Increasing embedment ratios does not significantly add to $V_{ult}$ and $H_{ult}$. On the other hand, additional embedment is increasingly beneficial for pure moment capacity. Similar to the findings of Yun and Bransby (2007), it was concluded that with increase in embedment ratio, the eccentricity of HM failure also increases. It is usually assumed that failure envelopes derived for a foundation fully bonded with soil on surface can be scaled by the respective limit loads to obtain the failure envelope for an embedded foundation. However, this work (Gourvenec, 2008) showed that this assumption is not true and it ignores the extra capacity mobilized due to coupling of horizontal and moment degrees of freedom. This highlighted the conservative nature of depth factor used in conventional design practice.

Taiebat and Carter (2010) investigated failure responses of a surface circular footing resting on undrained soil. Both, contacts with no tension and full bonding (adhesion) were modelled at the interface. ‘Modified’ swipe and displacement probe tests were conducted with a 3-dimensional finite element model to obtain two and three-dimensional failure envelopes. In the modified swipe tests, the displacement was applied in the first direction (usually vertical) till ultimate point in that direction is reached. During the second portion of loading, the incremental displacements in the first direction are reduced gradually from its maximum value using a cosine function, while the incremental displacements in the second direction increases from zero to its maximum value using a sine function. The modified swipe loading improves the accuracy of the failure envelope predicted by swipe loadings. Load controlled tests were also performed to cover the entire failure envelope.

For the case of no-tension interface, the foundation cannot sustain any moment in the absence
of vertical load. For a constant value of moment and horizontal load, the value of vertical load can be increased up to $0.5V_{ult}$ i.e. half of that of the uniaxial ultimate vertical capacity. However, for loads beyond this value, factor of safety decreases and further loading can cause footing failure. A scallop shape VHM failure envelope for a smooth interface was found; this was smaller and slightly non-symmetric (Fig. 2.30) compared to that found for the rough interface (Fig. 2.31). This study revealed that for practical combinations of loads, conventional methods underestimate the bearing capacity, while for large horizontal loads and/or vertical loads, conventional theory overestimates the results. Additionally, it indicated that Meyerhof’s effective area rule provides a good approximation of bearing capacity for a no-tension criterion for circular footings on clays, whilst when full adhesion is assumed, the methods become conservative in cases of large eccentricity.

Figure 2.30: 3-dimensional failure envelope in VHM plane, no tension interface (Taiebat & Carter, 2010)
Gourvenec and Mana (2011) extensively determined the undrained vertical bearing capacity factors for shallowly embedded foundations (strip and circular) with varying soil-foundation roughness and soil strength heterogeneity. Finite element analysis along with finite element limit analysis for 2-d plane strain and axisymmetric models were used to calculate the best estimates of bearing capacity factors. For soil-foundation bottom being rough, the sides were either smooth, of intermediate roughness or rough whereas for soil-foundation bottom smooth interface, the sides were also assumed smooth always. A summary of published studies reporting undrained vertical bearing capacity factors of shallow foundations was given (see Table 2.8).

The relationship between base interface roughness, embedment ratio and soil strength heterogeneity is quite complex and this prevented definition of accurate closed-form expressions for vertical bearing capacity factor. However, an approximate expression was proposed. For a strip foundation,

\[ N_{c0} = N_{c0,a=0} + C \frac{d}{B} \alpha \]  \hspace{1cm} (2.38)
and for a circular foundation

\[ N_{c0} = N_{c0,\alpha=0} + C \frac{d}{D} \alpha \]  

(2.39)

where

\[ N_{c0,\alpha=0} = \text{Bearing capacity factor for a smooth-sided rough-based foundation} \]
\[ C = \text{Constant defined as a function of soil strength heterogeneity} \]
\[ d = \text{Depth of embedment} \]
\[ B = \text{Breadth of strip foundation} \]
\[ D = \text{Diameter of circular foundation} \]

The values of \( N_{c0,\alpha=0} \) can be interpolated from Table 2.9 & 2.10 for strip and circular footing. This has helped in reducing the uncertainties associated with selection of appropriate vertical bearing capacity factors. One important finding of this research was that the vertical bearing capacity factor is influenced the most by change in embedment depths, while change in interface roughness (at the sides) and soil strength heterogeneity does not have much effect. For the cases with both interfaces rough or smooth, the difference between the two solutions is quite small at low embedment ratios, however, it diverges with increasing embedment.
<table>
<thead>
<tr>
<th>Strip</th>
<th>Circular</th>
<th>$d/B$ or</th>
<th>$d/D$</th>
<th>$\alpha = 0$</th>
<th>$0 &lt; \alpha &lt; 1$</th>
<th>$\alpha = 1$</th>
<th>Uniform $s_u$</th>
<th>$kB/\sum$ or $kD/\sum &gt; 0$</th>
<th>Method$^a$</th>
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<tr>
<td>—</td>
<td>Y</td>
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<td>Y</td>
<td>—</td>
<td>—</td>
<td>Y</td>
<td>—</td>
<td>—</td>
<td>SE</td>
<td>Skempton (1951)</td>
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<td>—</td>
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<td>Y</td>
<td>—</td>
<td>—</td>
<td>Y</td>
<td>—</td>
<td>—</td>
<td>SE</td>
<td>Hansen (1970)</td>
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<td>0</td>
<td>Y</td>
<td>—</td>
<td>—</td>
<td>Y</td>
<td>$\leq \infty$</td>
<td>—</td>
<td>MoC</td>
<td>Davis &amp; Booker (1973)</td>
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<td>—</td>
<td>Y</td>
<td>0</td>
<td>Y</td>
<td>—</td>
<td>—</td>
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<td>$\leq 30$</td>
<td>—</td>
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<td>—</td>
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<td>—</td>
<td>—</td>
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<td>—</td>
<td>—</td>
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<td>Y</td>
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<td>Y</td>
<td>$\infty$</td>
<td>—</td>
<td>UB &amp; FEA</td>
<td>Yun &amp;</td>
</tr>
<tr>
<td>Y</td>
<td>—</td>
<td>$\leq 1$</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>—</td>
<td>—</td>
<td>FEA</td>
<td>Bransby (2007)$^d$</td>
</tr>
<tr>
<td>Y</td>
<td>Y</td>
<td>$\leq 1$</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>$\leq 20$</td>
<td>—</td>
<td>FEA</td>
<td>Gourvenec (2008)</td>
</tr>
<tr>
<td>Y</td>
<td>—</td>
<td>$\leq 1$</td>
<td>—</td>
<td>—</td>
<td>Y</td>
<td>Y</td>
<td>$2, 6$</td>
<td>—</td>
<td>FEA</td>
<td>Mana et al. (2011)$^e$</td>
</tr>
</tbody>
</table>

Table 2.8: Summary of published studies reporting undrained vertical bearing capacity factors of shallow foundations (Gourvenec & Mana, 2011)

$^a$ SE, semi-empirical; MoC, method of characteristics; UB, upper bound; FEA, finite-element analysis; FELA, finite-element limit analysis; $^b$ Only base resistance reported; $^c$ Foundation modelled as slot at depth; $^d$ Some analytical results presented for $\alpha = 5$; $^e$ Deformable soil plug modelled
<table>
<thead>
<tr>
<th>kB/s&lt;sub&gt;um&lt;/sub&gt;</th>
<th>d/B</th>
<th>( N_c^0 ) smooth base, smooth sides</th>
<th>( N_c^0 ) rough base, ( \alpha_{base} = 1 ), varying side adhesion factor ( \alpha_{side} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>( \alpha_{base} = 0, \alpha_{side} = 0 )</td>
<td>( \alpha_{side} = 0 )</td>
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<tr>
<td>0</td>
<td>0.0</td>
<td>5.144</td>
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<td></td>
<td>0.1</td>
<td>5.507</td>
<td>5.511</td>
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<td>0.2</td>
<td>5.758</td>
<td>5.762</td>
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<tr>
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<td>0.3</td>
<td>5.952</td>
<td>5.955</td>
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<td>6.222</td>
<td>6.224</td>
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<td>6.616</td>
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<td>9.818</td>
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<td>0.1</td>
<td>8.436</td>
<td>9.499</td>
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<td>0.2</td>
<td>8.204</td>
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<td>0.3</td>
<td>8.045</td>
<td>8.805</td>
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Table 2.9: Bearing capacity factors \( N_c^0 = q_u/s_{u0} \) for strip foundations (Gourvenec & Mana, 2011)
Table 2.10: Bearing capacity factors $N_{c0} = q_u/s_{u0}$ for circular foundations (Gourvenec & Mana, 2011)

<table>
<thead>
<tr>
<th>kB/s&lt;sub&gt;um&lt;/sub&gt;</th>
<th>d/B</th>
<th>$N_{c0}$ smooth base, smooth sides</th>
<th>$N_{c0}$ rough base, $\alpha_{base} = 1$, varying side adhesion factor $\alpha_{side}$</th>
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<td>1.0</td>
<td>8.801</td>
<td>9.498</td>
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</table>
2.6 Summary

This chapter provides an overview of research works undertaken to study the response of shallow foundations subjected to combined/general loading. Key findings from this research review are:

- Zaharescu (1961) and Ticof (1977) suggested expressing inclined-eccentric loads as combinations of coupled vertical, horizontal and moment loads explicitly. This has become popular and it has been used repeatedly by many researchers in recent years.

- To study the size and shape of these failure envelopes, they are usually non-dimensionalized by foundation contact area and undrained shear strength, and normalized by using uniaxial limit loads respectively.

- Under inclined or eccentric loading alone, conventional design approaches give fairly good estimates of bearing capacities of foundations. However when vertical, horizontal and moment loads are applied together, i.e. when inclination and eccentric loads act simultaneously, failure envelopes cannot be accurately predicted even in the simple case of a strip footing. One important reason is that conventional methods ignore the coupling of horizontal and moment degrees of freedom. For the case of fully bonded contact between foundation and soil, a foundation with positively applied lateral load sustains more moment than that without any lateral load.

- With change in embedment ratio and soil strength homogeneity, scaling of envelopes does not hold good for every loading scenario. Traditional methods suggest isotropic expansion or contraction of failure envelopes, which provide conservative results.

- Assumption of equivalent surface foundations to model an embedded foundation does not remain valid as depth of embedment increases. Embedments must be explicitly modelled in such cases. As depth of embedment increases, failure mechanisms become more localized and confined.
• Most of the research work on response of foundations to combined loadings assumes fully bonded contact at the soil-foundation interface owing to its suitability for offshore conditions. The conditions of foundation-soil interface with no tension have to be investigated more.

• For cases of no tension interface, traditional design methods overestimate failure loads when horizontal force is very large. Otherwise, for most practical combinations of loads, the results are on the conservative side.

• The response of circular foundations subjected to combined loadings has been well researched under different conditions of embedment and soil strength heterogeneity. However, octagonal foundations subjected to general loading have been rarely studied specifically and are assumed to behave like circular foundations.

• The presence of a stiff surficial crust over a normally consolidated clay influences the bearing capacity response of a foundation. More research work needs to be done to understand the effects of surficial crust thickness, embedment depth relative to crust thickness, average crust strength and rate of increase of strength in the layer underlying the crust.

A summary of research works done to study response of foundations subjected to combined loading under different conditions of embedment depths, shapes and soil strength heterogeneity is presented in Table 2.11 below. It must be noted that this table is no way exhaustive.
<table>
<thead>
<tr>
<th>Reference</th>
<th>Foundation shape</th>
<th>Embedment ratio $(d/B \text{ or } d/D)$</th>
<th>Heterogeneity ratio $(K' = kD/S_{u0})$</th>
<th>Tangential contact</th>
<th>Normal contact</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tani &amp; Craig (1995)</td>
<td>Strip, Circular</td>
<td>0</td>
<td>0 − 30</td>
<td>Smooth, Rough</td>
<td>Full adhesion</td>
</tr>
<tr>
<td>Bransby &amp; Randolph (1998)</td>
<td>Strip</td>
<td>0</td>
<td>0, 1, 2, 3, 6, 10</td>
<td>Rough</td>
<td>Full adhesion</td>
</tr>
<tr>
<td>Taiebat &amp; Carter (2000)</td>
<td>Circular</td>
<td>0</td>
<td>0</td>
<td>Rough</td>
<td>Full adhesion</td>
</tr>
<tr>
<td>Taiebat &amp; Carter (2002a)</td>
<td>Strip, Circular</td>
<td>0</td>
<td>0</td>
<td>Smooth</td>
<td>No-tension</td>
</tr>
<tr>
<td>Doherty &amp; Deeks (2003)</td>
<td>Circular</td>
<td>0.5, 1, 2, 4</td>
<td>0 − 1*</td>
<td>Rough</td>
<td>Full adhesion</td>
</tr>
<tr>
<td>Gourvenec &amp; Randolph (2003)</td>
<td>Strip, Circular</td>
<td>0</td>
<td>0, 1, 2, 3, 6, 10</td>
<td>Rough</td>
<td>Full adhesion</td>
</tr>
<tr>
<td>Salgado et al. (2004)</td>
<td>Strip, Circular, Square, Rectangular</td>
<td>0, 0.01, 0.05, 0.1, 0.2, 0.4, 0.6, 0.8, 1, 2, 3, 4, 5</td>
<td>0</td>
<td>Rough</td>
<td>Full adhesion</td>
</tr>
<tr>
<td>Gourvenec (2004)</td>
<td>Circular</td>
<td>0</td>
<td>0, 2, 6</td>
<td>Rough</td>
<td>Full adhesion</td>
</tr>
<tr>
<td>Edwards et al. (2005)</td>
<td>Strip, Circular</td>
<td>0, 0.25, 0.5, 0.75, 1, 1.5, 2, 2.5, 3, 3.5, 4</td>
<td>0</td>
<td>Bottom: Rough; Sides: Smooth or Rough</td>
<td>Full adhesion</td>
</tr>
<tr>
<td>Yun &amp; Bransby (2007)</td>
<td>Strip</td>
<td>0, 0.2, 0.5, 1</td>
<td>0</td>
<td>Rough</td>
<td>Full adhesion</td>
</tr>
<tr>
<td>Gourvenec (2008)</td>
<td>Strip</td>
<td>0, 0.25, 0.5, 1</td>
<td>0</td>
<td>Rough</td>
<td>Full adhesion</td>
</tr>
<tr>
<td>Taiebat &amp; Carter (2010)</td>
<td>Circular</td>
<td>0</td>
<td>0</td>
<td>Rough</td>
<td>No-tension, full adhesion</td>
</tr>
<tr>
<td>Gourvenec &amp; Mana (2011)</td>
<td>Strip, Circular</td>
<td>0, 0.1, 0.2, 0.3, 0.5, 1</td>
<td>0, 5, 20, 100, 200</td>
<td>Bottom, $\alpha=0$, Sides, $\alpha=0$, 0.5, 1</td>
<td>Full adhesion</td>
</tr>
<tr>
<td>Shen et. al (2016)</td>
<td>Strip, Circular</td>
<td>0</td>
<td>0, 2, 6, 10</td>
<td>Rough</td>
<td>No-tension</td>
</tr>
</tbody>
</table>

Table 2.11: Summary of published studies for foundations subjected to uniaxial and/or combined loading

* represents non-homogeneous factor defined by eqn. 2.4
Chapter 3

Numerical Model Development and Verification

3.1 Introduction

In this thesis, the bearing capacities and failure mechanisms of circular and octagonal foundations are investigated for a range of embedment ratios and soil strength heterogeneity factors subjected to combined loading under undrained conditions. This chapter describes the methods used to develop 3-dimensional finite element models for the soil-foundation domain of a wind turbine. Small-strain finite element analysis using the commercial software ABAQUS 6.13-4 (Dassault Systèmes, 2013) was used to determine the ultimate uniaxial bearing capacity factors, failure mechanisms and failure envelopes plots shown in Chapter 4.

This chapter describes the soil conditions for a typical southern Ontario case, soil constitutive models, type of 3-dimensional finite elements, boundary conditions, mesh principles, displacement based loading methods and failure criteria for the surface and embedded foundations used in the present research work. A benchmark in the form of a 2-dimensional plane strain strip footing model is also described and is used to study the response of the foundation subjected to ultimate uniaxial and combined loads. The value of bearing capacity factors obtained is...
compared with the theoretical values given in the literature. This helped in validating the soil constitutive parameters and boundary conditions assumed. To further calibrate the model for octagonal foundations, models for 3-dimensional surface circular footings with six degrees of freedom are established. The steps followed for the 2-dimensional model are repeated. Uniaxial limit capacities and 2-dimensional failure envelopes in VH, VM and HM planes are compared with the published data.

Finally, a 3-dimensional model for an octagonal foundation similar to that of a typical commercial wind turbine is developed. As compared to a circular foundation, an octagonal foundation is easy to cast-in-situ and requires less skilled labour. A special case of soil strength heterogeneity, wherein upper soil layers consist of a stiff crust layer is also considered to simulate the common clay till soil conditions. More details are provided in a further section on this analysis.

### 3.2 Site Characteristics

This research has been based on a typical octagonal gravity-based foundation supporting a 2.3 MW horizontal axis wind turbine such as those located in southern Ontario. This type of turbine has a rotor diameter of 93 m and a hub height of 80 m. The foundation can be inscribed with a circle of 19 m diameter and has 3 m depth below grade at its centre. It tapers towards the edges and is 0.4 m thick at the toe (Fig. 3.1).
Figure 3.1: Geometry of a typical octagonal foundation

3.2.1 IEC Classification and Design Wind Speeds & Loads

The 2.3 MW wind turbine and the site (Port Alma) fall under IEC class IIb. The Port Alma Wind Farm is a 101.2 MW project owned by Kruger Energy Inc. located on the north shore of Lake Erie in the Municipality of Chatham-Kent, Ontario, Canada. Table 1 of IEC 61400 – 1 provides design wind speeds and turbulence parameters for each wind turbine class. For class IIb, medium turbulence characteristics exist with an expected value of turbulence intensity at 15 m/s equal to 14% and reference wind speed average over 10 min equal to 42.5 m/s. The design foundation loads are therefore based on annual average wind speeds of 8.5 m/s and 3 sec gust wind speed, with a return period of 50 years of 59.5 m/s; both are calculated at hub height. The design loads for the foundation are shown in Table 3.1.
<table>
<thead>
<tr>
<th>Condition</th>
<th>Normal Force V (kN)</th>
<th>Horizontal Force H (kN)</th>
<th>Overturning moment M (kN m)</th>
<th>Torsional moment T (kN m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal operations</td>
<td>2900</td>
<td>900</td>
<td>60000</td>
<td>7300</td>
</tr>
<tr>
<td>Highest overturning moment</td>
<td>2900</td>
<td>1100</td>
<td>76200</td>
<td>4400</td>
</tr>
</tbody>
</table>

Table 3.1: Magnitude of design loads for the wind turbine

*Torsional loads are shown here for completeness. However, these loads are not modelled in the present research.

### 3.2.2 Site Soil Conditions

A geotechnical investigation was carried out in the vicinity of the wind turbine foundation under study, to explore the subsoil and groundwater conditions (Tyldesley et al., 2013). The borehole encountered a 200 mm thick layer of organic clayey topsoil. Below the surface topsoil layer, a major deposit of firm to hard silty clay with embedded sand and gravel was encountered. This deposit can be subdivided into three zones: 1. Weathered zone, 2. Brown zone (“the crust”) and 3. Grey zone. The uppermost zone of the silty clay is weathered and has a mottled brown-grey appearance, with a stiff to very stiff consistency. The weathered zone has higher moisture content due to infiltration of surface water into fissures of the clay. The weathered zone extends to a maximum depth of around 2 m below ground surface. Underlying the weathered layer lies the crust zone around 2 m thick characterized by a prevalent brown colour with a very stiff consistency and relatively lower natural moisture content. A soil colour change of brown to grey occurs between 3 and 4 m. Below the crust and extending to the maximum depth of the borehole (35.5 m), uniform grey silty clay was encountered. This zone is characterized by a uniform grey appearance, a stiff to very stiff consistency and relatively higher moisture content. Atterberg limit test results indicated that the soil in this layer can be classified as CL-ML to CL (silty clay or low plasticity clay). No groundwater was encountered.
in the borehole. The variation of OCR, moisture content and undrained shear strength with depth is shown in Fig. 3.2. The values of overconsolidation ratio were derived from oedometer testing (Tyldesley et al., 2013). A piezocone penetration test (CPTU) was also conducted near the wind turbine foundation, which provided estimates of the undrained shear strength of the soil (Kiss et al., 2014).
Figure 3.2: Borehole log for the wind turbine foundation location
3.3 Elasto-Plastic Constitutive Model

3.3.1 General

One of the essential ingredients for a successful finite element analysis of a geotechnical problem is an appropriate soil constitutive model (Potts & Zdravković, 1999). Constitutive models can be broadly classified into two categories based on the material behaviour: isotropic and anisotropic. An isotropic model has point symmetry, i.e. every plane in the body is a plane of symmetry for material behaviour. In an anisotropic model, the material behaviour is different in at least two directions. The common constitutive models available for clays are listed below:

1. Isotropic constitutive models:
   
   (a) Elastic
      
      i. Linear elastic (i.e. Hooke’s law)
      
      ii. Non-linear elastic
   
   (b) Elasto-plastic
      
      i. Mohr-Coulomb
      
      ii. Drucker-Prager
      
      iii. Cam-clay forms

2. Anisotropic constitutive models

For a linear elastic model, only 2 material constants, viz. Young’s modulus $E$ and Poisson’s ratio $\nu$ are required. Whereas for an anisotropic material with properties varying in at least two directions, 5 material constants are needed (Potts & Zdravković, 1999). For a fully anisotropic material which can potentially describe real soil behaviour, 21 independent elastic stiffness parameters are required to be defined in ABAQUS (section 22.2.1 of ABAQUS User’s Guide Vol. III, 2013). Thus, use of an anisotropic model requires rigorous & detailed input data obtained
by performing laboratory and field tests. If laboratory and field tests results are not reliable or available, then increases in accuracy due to the use of anisotropic models can easily be lost. However, conducting these tests with the required degree of accuracy and precision also increases the cost of the project for a geotechnical engineer. Due to these reasons anisotropic models are rarely used.

It is important that the constitutive model chosen at least reproduces the soil behaviour that is predominant in the problem under investigation. The present work generally focuses on investigating the ultimate limit capacities of foundations under undrained conditions for saturated clayey soil and therefore, it is considered that linear elasticity with a perfectly plastic model governed by the Mohr-Coulomb failure criterion can sufficiently represent the requisite facets of soil behaviour. Many researchers have explored constitutive models and found the use of isotropic models, such as linear elastic with Mohr-Coulomb or Drucker-Prager sufficiently accurate [Chen et al. (1983); Hibbitt et al. (2001)]. The assumption of linear elasticity with a Mohr-Coulomb model is further corroborated by the results obtained from 2-dimensional plane strain footing analysis. A brief description of linear elasticity and Mohr-Coulomb models is given below.

### 3.3.2 Linear Elastic Constitutive Model

Although linear elastic soil models do not simulate real soil behaviour, they serve as a useful introduction to more complex constitutive models and are most commonly used in the engineering world to describe simple stress-strain relationships. Hooke's law relates the stresses and strains through two constants, Young's modulus $E$ and Poisson's ratio $\nu$. In an undrained analysis, undrained values of these parameters $E_u$ and $\nu_u$ are used. In 3-dimensional space, the
relationship between $E$ and $\nu$ can be expressed as:

\[
\sigma_x = \left( \frac{E}{(1 + \nu)(1 - 2\nu)} \right) \left[ \varepsilon_x (1 - \nu) + \nu (\varepsilon_y + \varepsilon_z) \right]
\]

(3.1)

\[
\sigma_y = \left( \frac{E}{(1 + \nu)(1 - 2\nu)} \right) \left[ \varepsilon_y (1 - \nu) + \nu (\varepsilon_x + \varepsilon_z) \right]
\]

(3.2)

\[
\sigma_z = \left( \frac{E}{(1 + \nu)(1 - 2\nu)} \right) \left[ \varepsilon_z (1 - \nu) + \nu (\varepsilon_x + \varepsilon_y) \right]
\]

(3.3)

where

$\sigma_x, \sigma_y, \sigma_z =$ normal stress in the $x, y$ and $z$ directions respectively, and

$\varepsilon_x, \varepsilon_y, \varepsilon_z =$ normal strain in the $x, y$ and $z$ directions respectively.

### 3.3.3 Mohr-Coulomb Constitutive Model

At very small magnitudes of load, soil behaves elastically, i.e. if unloading is done at this stage, soil will completely recover and strain will be equal to zero in the absence of any load. When loading stresses exceed the yield stress, soil starts behaving plastically. Soil, when unloaded at this stage, contains both recoverable and non-recoverable strain components. The Mohr-Coulomb failure criterion helps in describing the ultimate states of the soil. The real behaviour of soil is shown in Fig 3.3 below.
The real behaviour of soil can be idealised as a linear elastic-perfectly plastic material (Fig 3.4).

The Mohr-Coulomb failure surface in $\tau$-$\sigma$ space (Fig 3.5) is defined by:

$$\tau = c + \sigma \tan \phi$$  \hspace{1cm} (3.4)
Where
\( \tau = \text{Shear stress} \), \( \sigma = \text{Normal stress} \), and \( c = \text{cohesion} \).

The failure surface is only dependent on the major and minor principle stress \((\sigma_1, \sigma_3)\), and is independent of the intermediate principle stress \(\sigma_2\). The Mohr-Coulomb criterion resolves into an irregular hexagonal pyramid once mapped into 3-dimensional stress space as shown below.
The figure on the left and the right shows the failure surfaces in the principal stress plane and
the deviatoric stress plane respectively. If the stress point lies within the failure envelope, the
soil will behave elastically. When the stress state reaches the failure surface (which is also
used as a yield surface), the material will undergo elastic and plastic deformations. It is not
possible to increase the stress beyond the current failure surface. Further application of stress
beyond the yield limit leads to indefinite plastic straining in the material.

To summarize, the Mohr-Coulomb model requires 5 parameters. Three of these parameters,
c, ψ and φ, control the plastic behaviour and the remaining two, E and ν control the elastic
behaviour. If associated flow conditions are assumed, then ψ = φ and only 4 parameters are
needed.
3.4 Development of the Numerical Model

3.4.1 Introduction

The research in this thesis uses the finite element commercial software package ABAQUS v6.13-4. ABAQUS is a multi-purpose computer package that allows a user to investigate mechanical, structural and geotechnical problems under static and dynamic loadings. It is an ideal package due to its capability in modelling complex interactions between several bodies, and the available constitutive models for both geotechnical and structural materials. The package also allows for initial residual stress fields to be defined. The present work concentrates on obtaining the uniaxial bearing capacity (i.e. vertical, horizontal & moment) factors and create load combinations for interaction diagrams/failure envelopes. To achieve this, small-strain finite element analysis is sufficient and hence is performed for all analysis throughout the research work. As a preliminary exercise, a simple two-dimensional model was developed and the results are used to benchmark the material properties assumed.

3.4.2 Validation Exercise (Two-dimensional Model)

Given the substantial amount of computational effort and time involved in developing a complete 3-dimensional model, a simple 2-dimensional rigid, infinitely long strip footing was modelled first. A strip footing can be modelled as a plane strain body in two dimensions. A solid body is said to be in a state of plane strain if it satisfies all of the assumptions of plane stress theory except that the body’s thickness (length in z direction) is large in comparison to the dimensions in the XY plane. For a strip footing, values of uniaxial ultimate bearing capacity factors have also been well established and therefore, it is easy to make a comparison and verify the suitability of soil constitutive parameters & meshing techniques and boundary conditions assumed. Due to the wealth of literature on this problem, this presented a good choice for validating the various modelling assumptions. Additionally, the insights gained from this analysis can be extended to more general and realistic 3-dimensional models of the
problem.

3.4.2.1 General Model Characteristics

The soil-foundation system for the 2-dimensional model was developed using the following inputs:

- **Geometry**: Vertical boundaries were kept at a distance of $3.5D$ from the foundation edge, while the soil base was placed $4D$ below the footing (Fig 3.7) similar to that assumed by Taiebat and Carter (2002a). The location of the mesh boundaries was chosen to provide sufficient distance from the foundation edges to eliminate possible boundary effects. Table 3.10 in subsection 3.4.4.3 provides a summary of the boundary distances considered by previous researchers for 2- and 3-dimensional models. The location of boundaries affect the stiffness of the foundation during initial (elastic) loading, but the failure loads remain unaffected since most of the deformation happens immediately below the foundation, near the interface.

- **Boundary conditions**: The base of the soil was kept fixed by encastring all six degrees of freedom (Fig 3.7). The vertical sides were given roller supports and horizontal displacement (U1) was restricted. The surface of the soil was kept free.

- **Mesh techniques and Elements**: All of the elements of the foundation and soil were modelled using the structured meshing technique. In ABAQUS, the structured technique gives the programmer the most control over the mesh generated (section 17.3.4 of ABAQUS User’s Guide). Mesh density was varied to achieve optimum balance between computational effort involved and the accuracy of the solution achieved. The element CPE4R was used to mesh the model. It is a plane strain four-node quadrilateral element. Since in the two-dimensional model, the footing is assumed to be plane strain, this element is most suitable for modelling the foundation and the soil. Additionally, reduced integration and hourglass control reduce the running time of the model and distortion of the elements respectively. At the interface of the foundation and soil, where the stress
concentration is very high, small elements were used to capture the behaviour as closely as possible (Fig 3.7). The smallest size of element was kept 0.15 m, which is less than $\frac{1}{100^{th}}$ of the diameter of the foundation ($D = 19$ m). The aspect ratio of elements near the interface was kept around 2.

- **Contact definition**: A contact definition in terms of tangential and normal contact was given to simulate the contact between the foundation and soil. A friction coefficient of 1 was chosen to create rough contact and separation between the two surfaces. This means that in absence of any vertical load, no moment can be sustained. Details of the contact parameters used in ABAQUS are given in table 3.2 below.

<table>
<thead>
<tr>
<th>Tangential Contact</th>
<th>Normal Contact</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friction formulation</td>
<td>Penalty</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td>1</td>
</tr>
<tr>
<td>Shear stress limit</td>
<td>17.760 kPa</td>
</tr>
<tr>
<td></td>
<td>Pressure-overclosure</td>
</tr>
<tr>
<td></td>
<td>‘Hard’ contact</td>
</tr>
<tr>
<td></td>
<td>Constraint enforcement method</td>
</tr>
<tr>
<td></td>
<td>Penalty (standard)</td>
</tr>
<tr>
<td></td>
<td>Allow separation after contact</td>
</tr>
<tr>
<td></td>
<td>Yes</td>
</tr>
</tbody>
</table>

**Table 3.2**: Contact details for 2D model in ABAQUS
• Input data: The foundation was modelled as a very stiff, linear elastic, non-porous material. A summary of the chosen parameters is given in the Table 3.3 below.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Weight, $\gamma_c$</td>
<td>24</td>
<td>kN m$^{-3}$</td>
</tr>
<tr>
<td>Young’s modulus, $E_c$</td>
<td>500</td>
<td>GPa</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu_c$</td>
<td>0.15</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3.3: Properties of concrete used in the analysis

The soil was modelled as a homogeneous layer of saturated clay simulated by a Mohr-Coulomb elastic-perfectly plastic constitutive model. The soil parameters used in the analysis are summarized in Table 3.4 below.
### Table 3.4: Properties of soil used in analysis

<table>
<thead>
<tr>
<th>Properties</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturated unit weight, $\gamma_s$</td>
<td>19</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Young’s modulus, $E_s$</td>
<td>177.60</td>
<td>MPa</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu_s$</td>
<td>0.48</td>
<td>-</td>
</tr>
<tr>
<td>Undrained shear strength, $S_u$</td>
<td>17.760</td>
<td>kPa</td>
</tr>
<tr>
<td>Friction angle, $\phi$</td>
<td>0.1 $\approx$ 0</td>
<td>Degree</td>
</tr>
<tr>
<td>Dilation angle, $\psi$</td>
<td>0.1 $\approx$ 0</td>
<td>Degree</td>
</tr>
<tr>
<td>Absolute plastic strain</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>Material model</td>
<td>Mohr-Coulomb</td>
<td></td>
</tr>
</tbody>
</table>

Note that a low value of $S_u$ and $E_s/S_u = 10000$ was chosen so that smaller displacements were needed to reach failure in the analysis, thereby reducing the computational cost significantly (Zhang et al., 2012). Since the analyses conducted are presented in terms of non-dimensional parameters, the choice of these values does not affect the results when only the ultimate states are investigated.

#### 3.4.2.2 Discussion on the Choice of Modelling Parameters

In many previous research works, the Young’s modulus of the foundation (concrete) was usually kept very high compared to the Young’s modulus of the soil. For example, El-Marassi (2011) assumed Young’s Modulus of the foundation $10^7$ times larger than that of the soil. This was done so that the foundation would behave effectively rigid compared to the soil and relative deformations of the foundation elements with respect to each other were negligible or almost zero. In the present analysis, however, $E_c \approx 2800.E_s$. To achieve rigidity in the foundation, all of the finite elements of the foundation were given a rigid body constraint, which constrained the six degrees of freedom of all of the nodes to a single reference point. Thus, the motion of the foundation was governed by the motion of a single point.
Choice of Poisson’s ratio: For a linear elastic model, the stress-strain relationship can be expressed in matrix form as:

\[
\begin{bmatrix}
1 - \nu & \nu & \nu & 0 & 0 & 0 \\
\nu & 1 - \nu & \nu & 0 & 0 & 0 \\
\nu & \nu & 1 - \nu & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1 - 2\nu}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1 - 2\nu}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1 - 2\nu}{2}
\end{bmatrix}
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{xy} \\
\tau_{xz} \\
\tau_{yz}
\end{bmatrix}
= \frac{E}{(1 + \nu)(1 - 2\nu)}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{xy} \\
\gamma_{xz} \\
\gamma_{yz}
\end{bmatrix}
\]  

(3.5)

It can be clearly seen from Eqn 3.5 that if a value of 0.5 is used in the above equation, the matrix becomes singular and thus convergence problems can occur during numerical analysis. Thus, to represent incompressible behaviour of undrained clay, a high value such as 0.48 (which is very close to 0.5) is chosen. Further explanations of the reasons why 0.49 or 0.495 are not chosen and only 0.48 is used are given in the Section 3.4.4.3 for 3-dimensional modelling. For similar reasons, Shen et al. (2016) & Gourvenec and Randolph (2003) used \(\nu = 0.49\).

An associated flow rule has been assumed. For undrained conditions, this implies that \(\phi = \psi = 0\). If a value of zero is used numerically, then it can cause convergence problems. Therefore, a very small value of 0.1 is used in the numerical analysis.

### 3.4.2.3 Response of Strip Foundation to Combined Loading

The strip foundation was subjected to uniaxial ultimate vertical, horizontal and moment loads separately to obtain its response in terms of bearing capacity factors. Vertical and horizontal loading was applied in the form of displacements, while moment loading was applied as an angular rotation. Values of uniaxial bearing capacity factors are presented in Table 3.5 and compared with solutions given in the literature wherever possible.
Where
\( N_c (v) \) = Vertical bearing capacity factor,
\( N_h (h) \) = Horizontal bearing capacity factor,
\( N_m (m) \) = Moment bearing capacity factor.

The exact value of \( N_c \), 5.14 was obtained that matches with the theoretical solution (Prandtl, 1921). The value of \( N_m \) obtained in the current study is closer to the upper bound solution than that obtained by Gourvenec and Randolph (2003) using finite element analysis. The failure mechanism, which is very similar to that proposed by Prandtl (1920), is obtained under pure vertical load and is shown overlying the displacement vectors at failure (Fig. 3.8).
Swipe tests were also performed in the VH and VM planes. Swipe tests were first used by Tan (1990) to identify the shape of combined loading failure envelopes. A swipe test consists of two steps. Consider the case of VH failure envelope in which vertical direction is the coordinate direction $U_1$ and horizontal is the coordinate direction $U_2$. A displacement is first applied in the vertical direction from zero until the ultimate vertical load is reached. A displacement is subsequently imposed in the horizontal direction during which the increment of displacement in the vertical direction is maintained at zero, until the ultimate load in the horizontal direction is reached. An example of a swipe test is illustrated in Fig 3.9.
The benefit of a swipe test is that a complete failure locus on a certain plane can be determined in a single simulation. Previous studies [e.g. Bransby and Randolph (1998), Gourvenec and Randolph (2003)] have confirmed that the load path tracks very close to the failure envelopes in the VH and VM planes. Non-dimensionalized and normalized failure envelopes are plotted and compared with the results given by Green (1954), Bransby and Randolph (1998), Gourvenec and Randolph (2003) in Fig. 3.10-3.13.
Figure 3.10: Comparison of swipe yield envelopes in VH plane

Figure 3.11: Comparison of normalized failure envelopes in VH plane
The failure envelopes in the VH and VM planes reasonably match the shape and size of those found by other researchers. Thus, from these results, it can be said that the response of the strip foundation model subjected to combined loading is fairly accurate. The analysis of the strip foundation validates the soil constitutive parameters, boundary conditions and mesh principles assumed. This process helped in the development of the 3-dimensional models.
presented in the next section.

3.4.3 Development of Three-dimensional Foundation Models

Soil and foundations exist as 3-dimensional entities in nature. Research on wind turbine foundations can be done by doing full-scale testing under field conditions. However, due to the size and economics involved, this method is rarely (if at all) used. Three-dimensional numerical analysis can simulate field conditions realistically at considerably lower costs. By performing 3-dimensional finite element analysis, researchers can study the response of foundations under various loading scenarios and soil properties. If only 2-dimensional analysis is done as in the case with plane strain and plane stress problems, the third dimension of the soil-foundation system gets neglected and complete and accurate information about stresses, strains and failure mechanisms is not obtained. Thus, for designing cost-effective foundations, 3-dimensional modelling is necessary, especially for the case of a wind farm project, which involves asymmetric environmental loading. Therefore, in the present research, three dimensional models of circular and octagonal foundations are created to explore the response of foundations under various loading combinations.

It is important to verify a numerical model to ensure that its predictions are within an acceptable limit when compared to real life situations. The wind turbine foundation under study is octagonal in shape. Since no research so far explicitly addresses octagonal foundations subjected to combined loading, a circular foundation is modelled initially and its response to combined loading is studied and compared with well-known works, such as Shen et al. (2016), Gourvenec and Randolph (2003), Houlsby and Wroth (1983), Bransby and Randolph (1998) & Taiebat and Carter (2010). This step is important to verify the meshing principles and boundary conditions. As a final step, an octagonal foundation model is created to simulate a wind turbine foundation in the field and subjected to various loading conditions. The following sections show the finite element model and algorithms used to achieve the aims of this work.
3.4.3.1 Numerical Model Algorithms

A solution to numerical problems is usually obtained using two approaches: Implicit and Explicit methods. Both these methods are briefly described below.

- **Implicit Algorithm**: This method assumes a constant average acceleration over each time step, between \( t_n \) and \( t_{n+1} \) where \( t_n \) is the time at the beginning of the \( n^{th} \) step and \( t_{n+1} \) is the time at the end of the \( n^{th} \) step. In each increment, the Newton-Raphson method is used to perform iterations and enforce equilibrium of the internal structural forces with the externally applied load. The resulting accelerations and velocities at \( t_{n+1} \) are calculated and then the unknown displacements at \( t_{n+1} \) are determined. The use of the Newton-Raphson method allows the analysis to continue with larger stable time increments as the analysis progresses.

- **Explicit Algorithm**: The explicit method calculates the state of a system at a later time \( t_{n+1} \) from the state of the system at the current time \( t_n \). It assumes a linear change in displacement over each time step. The governing equation is evaluated and the resulting accelerations and velocities at \( t_n \) are calculated. Finally, the unknown displacement at \( t_{n+1} \) are determined.

The major difference between implicit and explicit schemes lies in the equations that are used to solve the displacement at time step \( t_{n+1} \). While the implicit method inverts the structural stiffness matrix, the explicit method does not. Consequently, the implicit method requires extra computational effort and time than the explicit method. However, the implicit scheme is unconditionally stable and generally follows any non-linearity by making the time increment small enough to capture the non-linearity. Only under extreme non-linearity cases does the implicit scheme halt. On the other hand, the explicit method is conditionally stable and uses time increments smaller than the critical time step to solve the problem.

Given that this research explores the response of foundations subjected to combined loading, it was considered desirable that the system should be in a state of equilibrium. Therefore, the
problem was solved implicitly. This introduced a series of complications where non-convergence could occur. The complications were eliminated by introducing changes in loading steps and mesh densities. For example, to overcome plastic strain non-convergence problems, initial gravity stresses were applied to the model and geostatic equilibrium was achieved so that the stresses in the soil before the loading begins are very close to the in-situ stresses and the deformations are negligible. This particular aspect of modelling is covered further in Section 3.4.3.3.

3.4.3.2  Three-dimensional Finite Elements

To create a 3-dimensional model, a 3-dimensional element type needs to be chosen. ABAQUS has an extensive element library to provide a powerful set of tools for solving many different problems. In the present research, for all the modelling cases, the element C3D8R is chosen to mesh the soil and foundation.

The element C3D8R belongs to the continuum family and has 8 nodes; one at each of the corners and assumes a linear shape function between adjoining nodes (Fig 3.14). It can also be considered as an extension of a rectangular element in two dimensions, to a rectangular parallelepiped element in three dimensions. It is more commonly known as a brick element and also sometimes called a hexahedral element.

Figure 3.14: Continuum element C3D8R
Discussion on the Choice of Element

Hexahedral elements offer many advantages when compared to the other preferred option of using tetrahedral elements belonging to the same family. A good mesh of hexahedral elements provides results with reasonable accuracy at lower numerical costs and exhibits better convergence than tetrahedral elements. When the soil is incompressible and no volume change is allowed to occur, such as in the present case, fully-integrated, first-order tetrahedral elements undergo volumetric locking. Moreover, they are overly stiff and need very fine meshes for achieving accurate results.

The assumption of a linear element can cause convergence problems for materials subjected to high bending stresses. The difference between the true bent shape and deformed shape of a linear element is shown below.

![Figure 3.15: Bending nature of linear elements](image)

To resolve this issue, smaller elements were used at the critical areas such as, the interface of the foundation and soil and the foundation edges, so that linearity was occurring over a smaller span, which allowed the overall mesh to deform more realistically. For example, the thickness of soil elements near the interface was reduced to almost $1/100^{th}$ of the diameter of the foundation ($D = 19$ m), i.e. 0.2 m.

The reduced integration feature of the element, indicated by the suffix ‘R’ in the element name, reduces the running time and also prevents the problem of volumetric locking in incompressible materials (e.g. undrained soil). Reduced integration uses a lower-order integration to form the
element stiffness. The mass matrix and the distributed loadings use full integration.

3.4.3.3 Contact Formulation and Interaction

In geotechnical problems such as soil-structure interaction of foundations, it is necessary to simulate the interaction between two materials. Wind turbines are subjected to horizontal and moment loadings due to the action of wind forces acting at the hub height, high above the foundation level. As a result, the coefficient of friction acting between the two surfaces becomes an important parameter in providing resistance against the imposed sliding forces. As the horizontal load acting on the foundation increases, the shear stress at the foundation-soil interface reaches a critical value. Once the critical shear stress is exceeded, the foundation will move relative to the soil.

The contact relationship used between the foundation and soil in this study has the following features.

- **Surface-based interaction** was adopted for all of the models.

- The foundation was chosen as the master surface and the soil as the slave surface. The nodes on the two contacting surfaces are grouped together to form master and slave surfaces.

- The two surfaces were adjusted to have zero absolute clearance as an initial condition before the application of load. This was done by either adjusting the overclose distance or specifying an “adjustment zone” in ABAQUS.

- The “hard” contact relationship was used to define the normal contact, which minimized the penetration of the slave surface into the master surface at the constraint locations and did not allow the transfer of tensile stress across the interface. An example of the slave and master surfaces and the penetration restrictions are shown in Fig 3.16.

- Most other researchers, such as Bransby and Randolph (1998), Gourvenec and Randolph (2003), Salgado et al. (2004) & Taiebat and Carter (2000) assumed a fully bonded contact
between the soil and foundation. This implies that infinite friction can mobilize between
the two surfaces and no separation occurs once the two surfaces are in complete contact.
This type of interaction is suitable for the case of offshore foundations, where full
bonding can be expected due to suction forces acting at the interface. However, this
does not hold true in the case of wind turbine foundations. Wind turbine foundations
can lose contact with soil and get uplifted, (although for a very short period of time)
due to the wind forces. Therefore, the option to “allow separation after contact” was
toggled on.

• The contact between the foundation and soil is mechanical in nature and was simulated
by using the basic Mohr-Coulomb friction model. The basic concept of the Coulomb
friction model is to relate the maximum allowable frictional (shear) stress across an
interface to the contact pressure between the contacting bodies. The frictional stress
developed is equal to $\alpha S_u$ where $\alpha$ is an adhesion factor or coefficient of friction between
the foundation and soil. The adhesion factor was assumed to be equal to 1 for the bottom
contact surface (Sladen, 1992). When the foundation was embedded, the adhesion factor
for the side surfaces and for the top contact surface was assumed equal to 0.5 (Fig 3.18).

• Finite sliding surface-to-surface discretization was used for the contact formulation.
When the surface-to-surface contact formulation is used, the choice of master and slave
surfaces has less effect on the results.

• A maximum critical shear stress ($\tau_{\text{max}}$) equal to the undrained shear strength $S_u$ of
the soil was defined so that regardless of the magnitude of the contact pressure stress,
sliding will occur if the magnitude of the equivalent shear stress reaches this value (Fig
3.17). This is important when horizontal forces are acting on the foundation. A similar
consideration has been made by previous researchers, such as Bransby and Randolph
Figure 3.16: Penetration restriction between slave and master surfaces (Hibbit et al., 2001)

Figure 3.17: Slip regions for the friction model with a limit on the critical shear stress (ABAQUS User’s Guide Vol. 5, 2013)
For further detailed information of contact formulations and interactions, the reader is recommended to go through Section 36 to 38 of ABAQUS User’s Guide Vol. 5 (2013).

### 3.4.3.4 Initial Stress State-Geostatic Equilibrium

The stress state in a foundation-soil system before static loads are applied can have significant influence on the response of foundations under the loads. The Mohr-Coulomb diagram shown in Fig 3.19 can be used as an illustrative example of how the initial stresses can affect the response of the foundation. The initial stress state causes a shift along the normal stress axis. Hence the Mohr circle shows an increase in radius, and larger loads are required to induce shear failure within the clay soil (Johnson, 2005).
The initial stress state for the foundations modelled in this research study consist of normal gravity field due to the weight of the soil. The initial vertical stress in the foundation-soil model is given by:

\[
\sigma' = \rho \cdot g \cdot h - u
\]  

(3.6)

Where

\( \sigma' \) = effective vertical stress

\( \rho \) = density of soil (1937 kg m\(^{-3}\))

\( g \) = acceleration due to gravity (9.81 m s\(^{-2}\))

\( h \) = depth below the ground surface (i.e. in the z direction)

\( u \) = pore water pressure (the influence of the water table was not investigated as part of this research) = 0 kPa

An example of the initial stress field is shown in Fig 3.20. The stresses are shown in N/m\(^2\). At the surface, \( \sigma' \approx 0 \) and at \( z = h \), \( \sigma' = \rho gh \). The foundation is considered to be weightless.
For the case of embedded foundations, the process of installation of the foundation is not explicitly simulated and is simply “wished in place”. The installation of foundation may cause deformations in the soil and change the initial stress fields in the soil. Given the stiffness of the soils assumed in this study, this aspect is not covered in the present work.

Figure 3.20: Initial stresses after a geostatic step

To achieve the initial stress state in the model, a geostatic step is applied in ABAQUS. This ensures that vertical effective stress increases proportionally with depth, while the deformations in the model are negligible or very small. Fig 3.21 shows that the deformations obtained after a geostatic step are very small and vary in the range of $10^{-6}$ to $10^{-7}$ m.

Figure 3.21: Initial deformations after a geostatic step
It is important that at the start of the loading steps, the foundation and soil are in complete contact with each other with zero clearance to obtain convergence and accurate distribution of stress and strain. To achieve this, two conditions were imposed during the geostatic step.

- All six degrees of freedom of the top soil surface in contact with the foundation were constrained during the geostatic step. This ensured that the deformation of each of the nodes located on the soil contact surface was zero after the geostatic step.

- The contact pair between the foundation and soil which are ‘initially’ in contact was removed during the geostatic step. If the surfaces are in contact when a contact pair is removed, ABAQUS stores the corresponding contact forces for every node on each surface. During the removal step, ABAQUS automatically ramps these forces linearly down to zero magnitude. The contact constraints for the mechanical surface interactions are also removed instantaneously. *This step prevented the occurrence of zero pivot nodes during the geostatic analysis which can cause non-convergence.*

*Imposing these two conditions ensured that the geostatic step converged in a single increment.*

During the next step of loading, the contacts were reactivated and the constraints applied on the contacting surface of soil released.

For the given vertical stress field, there is an effective horizontal stress field. The horizontal stress at any depth can be estimated as being equal to the vertical stress at the same depth times the effective earth pressure coefficient ($K$). For surface or shallowly embedded foundations on lightly overconsolidated soils, the coefficient of earth pressure at rest ($K_0$) for a normally consolidated soil may be used and is given by:

$$
K_0 \approx 1 - \sin \phi' 
\tag{3.7}
$$
3.4.4 Construction of Three-dimensional Finite Element Model in ABAQUS

The following section will discuss the finite element model in ABAQUS constructed specifically for the purpose of this research. Three-dimensional models were created for circular and octagonal foundations. However, the basic philosophy of meshing techniques, failure criteria and boundary conditions for the soil remain the same. Mainly, the various cases considered fall into two categories: 1. Foundations embedded at various depths in homogeneous soil and 2. Foundations resting on the surface of the soil with varying soil strength heterogeneity. The various cases of embedment ratios or soil strength heterogeneity factors assumed are given in Table 3.6 below.

<table>
<thead>
<tr>
<th>Foundation type</th>
<th>Circular</th>
<th>Octagonal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Embedment ratio, $d/D$</td>
<td>0, 0.157, 0.32, 0.5, 1</td>
<td>0, 0.157, 0.32, 0.5, 1</td>
</tr>
<tr>
<td>Strength non-homogeneity ratio, $k$ or $K'D/S_{u0}$</td>
<td>0, 0.5, 1.0, 1.5</td>
<td>0, 0.5, 1.0, 1.5</td>
</tr>
</tbody>
</table>

Table 3.6: Numerical modelling cases considered in this research

In the beginning, circular and octagonal foundations resting on the surface of the soil are subjected to combined loading. DNV (2002) recommends the use of the radius of an inscribed circle of an octagon to calculate the effective area of the foundation. However, an octagon has slightly more area than the inscribed circle. Comparing the results of a circular foundation with that of octagonal foundation analysis will help in finding the increase in bearing capacity due to increased area and checking the degree of conservativeness in the assumption of effective area given by DNV (2002).

Foundations were then simulated at various depths and the procedure to find uniaxial ultimate capacities and failure envelopes was repeated. In an embedded foundation, more soil (soil above the embedded foundation) provides resistance to the loading and so, the bearing capacity is also more than the surface foundation. Embedment was investigated at four different depths.
In particular, the case of the wind turbine foundation in the field case placed around 3 m below the ground surface level corresponding to \( d/D = 0.157 \), was considered. The increase in bearing capacity factors and size of failure envelopes due to embedments is shown in Chapter 4. The changes in mesh density due to change in soil strength heterogeneity and embedment are indicated in Section 3.4.4.3.

After analyzing homogeneous soils, the current research focused on heterogeneous soils. Soils are heterogeneous materials with varying strengths. The soil strength is varied linearly along the depth by varying the soil strength heterogeneity factor \( k \). Three values of \( k \) viz. 0.5, 1.0 and 1.5 were chosen here for circular and octagonal foundations and the response of foundations compared in each case.

A surficial ‘crust’ of high shear strength has been reported in the site investigation (Fig 3.2), which is common for Canadian and South Ontario clay soils & tills. This issue can be addressed by considering a crust overlying a soil with linearly increasing undrained shear strength with depth. In the final stage of the research, to simulate the field case (as close as possible), a parametric study was conducted to investigate the bearing capacity response of a circular foundation in terms of the effect of crust thickness, embedment depth relative to crust thickness, crust strength, relative shear strengths of underlying soft clay & crust and rate of \( S_u \) increase with depth for the underlying layer below the crust. Table 3.7 provides the details of the parametric study undertaken. All of the shear strengths and lengths are in kPa and m respectively. The parameter choices for the study are also shown in pictorial form in Fig. 3.22. Refer Section 2.5.1 of Chapter 2 for definitions of the parameters.
Table 3.7: Parametric study table

<table>
<thead>
<tr>
<th>$S_{ut}$</th>
<th>$S_{u0}$</th>
<th>$S_{ubs}$</th>
<th>$S_{ubs}/S_{ut}$</th>
<th>$S_{ut}/S_{ubs}$</th>
<th>$t_c$</th>
<th>$D$</th>
<th>$t_c/D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>175</td>
<td>50</td>
<td>57.98</td>
<td>0.33</td>
<td>3.02</td>
<td>7</td>
<td>19</td>
<td>0.37</td>
</tr>
<tr>
<td>250</td>
<td>50</td>
<td>57.98</td>
<td>0.23</td>
<td>4.31</td>
<td>5</td>
<td>19</td>
<td>0.26</td>
</tr>
<tr>
<td>100</td>
<td>50</td>
<td>57.98</td>
<td>0.58</td>
<td>1.72</td>
<td>2.5</td>
<td>19</td>
<td>0.13</td>
</tr>
</tbody>
</table>

(a) Effect of averaging crust strength

<table>
<thead>
<tr>
<th>$S_{u0}$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1.14</td>
</tr>
<tr>
<td>0</td>
<td>3.20</td>
</tr>
<tr>
<td>30</td>
<td>1.60</td>
</tr>
<tr>
<td>60</td>
<td>0.00</td>
</tr>
</tbody>
</table>

(c) Effect of $S_u$ increase with depth

<table>
<thead>
<tr>
<th>$d$</th>
<th>$t_c$</th>
<th>$d/t_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>7</td>
<td>0.43</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>0.86</td>
</tr>
</tbody>
</table>

(d) Effect of embedment depth relative to crust thickness
Figure 3.22: Parameters chosen for the study

(a) Average crust strength, $S_{ur}$

(b) Crust thickness, $t_c$

(c) Rate of strength increase, $k$

(d) Relative embedment

$S_u = 250, 175, 100$ KPa

$t_c = 2.5, 5, 7$ m

$d = 3.6$ m
3.4.4.1 Failure Criteria for Estimating Foundation Response

One of the many challenges in finite element analysis of a soil-foundation system subjected to combined loading is to define the ultimate loads in the vertical, horizontal and moment directions. In the present research, diverse cases have been modelled, e.g. circular and octagonal, surface and embedded and homogeneous and heterogeneous soil strengths. Due to this variation, following a single failure criteria to define ultimate capacities for all the cases is not possible. Therefore, failure criteria have been defined on a case-by-case basis. Given below are the failure criteria used in the present work.

*Vertical Bearing Capacity of Surface Foundations*

The ultimate vertical load capacity of the foundation $V_{ult}$ was obtained by applying a very small horizontal displacement ($U_1$) along with application of vertical displacement ($U_3$). The ratio $U_3/U_1$ was kept equal to 60 throughout the analysis. The small horizontal component helped to define the ultimate vertical point in a better way. Fig 3.23 shows the load-displacement response of the foundation under vertical and horizontal loading and the method to predict the ultimate vertical capacity.

![Figure 3.23: Load-displacement response for vertical and horizontal loadings](image-url)
This method was adopted by Taiebat and Carter (2000) to define the ultimate vertical point. Further, a personal communication with Hossein (2015) confirmed the validity of this method. The vertical load-displacement response was plotted on the primary axis, while the horizontal load-displacement response was plotted on the secondary axis. The load-displacement curves in the vertical and horizontal load directions are plotted for the same time/load steps and a change of the scale of one without the other is not possible. So at each time step one can know the vertical load corresponding to a horizontal load. Vertical load corresponding to the maximum horizontal load is taken as the ultimate vertical point.

The accuracy of this method was tested on a surface circular foundation and $N_c$ equal to 6.10 was found. This matches fairly well with the solution given by Cox et al. (1961), $N_c = 6.05$. To evaluate any possible effect of the horizontal displacement $U1 = \frac{U3}{60}$ on the vertical bearing capacity, another analysis with a lower value of horizontal displacement, $U1 = \frac{U3}{600}$, was carried out. The same value for the ultimate vertical bearing capacity was obtained, indicating the negligible influence of these relatively small horizontal displacements on the vertical capacity of the footings.

Vertical Bearing Capacity of Embedded Foundations

Bearing capacity of the foundation increases as the embedding depth increases. Additionally, with more embedment, the failure mechanisms tend to become localized compared to that of surface or shallowly embedded foundations. Due to this, in numerical analysis, a very high value of vertical displacement may be required to achieve a plastic “plateau” in the load-displacement curve. A plateau may not be reached within a practical number of increments of the non-linear analysis or it may not be easily identifiable because the load-displacement curve is still rising. The criteria given in the previous subsection holds well only when a single contacting surface (which is the foundation bottom and soil contact) exists between the foundation and the soil. When side and top contacting surfaces also provide resistance against horizontal forces (as is the case with embedded foundation), application of a small value of horizontal displacement does not provide accurate prediction of the ultimate vertical capacity.

To overcome this problem, the ultimate vertical bearing capacity ($v_0$) was defined, as shown
in Fig 3.24. The ultimate bearing capacity can be estimated by extending the straight line through the elastic region of the load-displacement curve, so that it intersects a straight line extending back from the plastic region of the curve (Rowe & Davis, 1982). The corresponding load at the intersection point is defined as the ultimate vertical capacity of the foundation ($v_0$).

![Figure 3.24: Load response under vertical loading for circular foundation $d/D = 0.157$](image)

A value of $v_0 = 7.4$ was obtained for an embedded circular foundation with $d/D = 0.157$. Gourvenec and Mana (2011) provided undrained vertical bearing capacity factors of shallow strip (Table 2.9) and circular foundations (Table 2.10) in tabular form. The value of $v_0$ interpolated from this table for a circular foundation with $d/D = 0.157$ was 7.28. The predicted value is only 2.75% above the value obtained from Gourvenec and Mana (2011). Therefore, this method can be considered to provide relatively accurate estimates of ultimate vertical capacity.

**Horizontal Capacity of Foundations**

The resistance to horizontal loads is mobilized by the shear stress developed at the soil-foundation interface. Therefore, a criteria in terms of maximum shear stress was used to define the ultimate horizontal load. For the case of bottom contact surface, the maximum shear stress that can be mobilized was kept equal to the undrained shear strength of the soil, while for
the side and top contact surfaces, the maximum shear stress was kept equal to half of the undrained shear strength of the soil.

**Moment Capacity of Surface and Embedded Foundations**

Surface and embedded foundations under rotational loads suffer from the same problem of reaching a plastic plateau and defining a true plastic collapse load as explained previously. In the present work, a method called the “modified Southwell plot” is employed to precisely define plastic collapse loads even when the analysis has not approached the full plastic mechanism. This technique was proposed by Doerich and Rotter (2011) who used it to accurately evaluate plastic collapse loads of shell structures even when a fairly complete plastic strain field was not developed. This method was recommended for all numerical studies that seek to evaluate plastic collapse loads.

The application of a modified Southwell (MS) plot is illustrated for a surface circular foundation in Fig 3.25. In this plot, \( \frac{RM2}{UR2} \) is plotted against \( N_m \) where \( RM2 \) is the reaction moment obtained at the reference point about the Y axis, \( UR2 \) is the rotation applied about the Y axis in radians and \( m \) is moment bearing capacity factor. While the factor \( \frac{RM2}{UR2} \) is a measure of the secant stiffness at any given moment \( RM2 \), \( m \) represents the load axis. When a fully plastic region is approached, the MS plot should approach a horizontal line, since the load remains constant but the displacement continues to increase, so \( \frac{RM2}{UR2} \) should steadily decline towards zero at constant load. Here, the fully plastic state has not been quite reached. Therefore, the plastic load is taken as the asymptotic load by extrapolating the plot on the load axis. The value of intercept on the load axis gives the ultimate moment capacity.
3.4.4.2 Mesh Sensitivity Analysis

The foundation-soil model is subjected to uniaxial and combined loading several times for different cases. During the course of this research, several models were created to understand the basic concepts of numerical modeling in ABAQUS and prepare a working model close to realistic conditions. This ranged from creating simple linear elastic 2-dimensional models to more intricate 3-dimensional models with contact surfaces and linear elastic-perfectly plastic behaviour. Considering shape, embedment and soil strength heterogeneity, a total of 168 different cases have been analyzed numerically.

In light of these facts, a mesh sensitivity analysis was vital to minimize the computational effort involved, while achieving the results with reasonable accuracy. Varying mesh densities and boundary distances of the outer edge of the model were used to do sensitivity analysis. For each variation, the mesh was constructed from the beginning in ABAQUS.

Keeping the model geometry, soil constitutive parameters and boundary conditions same throughout, a typical analysis was used to derive three models with coarse, medium and fine
meshes. All of these meshes were created using the structured technique. In the coarse (1) and medium mesh (2), a single layer of elements with size 0.2 m was provided immediately below the foundation to provide horizontal resistance against sliding, while for the fine mesh (3), three layers of elements with size 0.2 m were provided. However, in Mesh 2, the total number of elements were around 35% more than that in Mesh 1 (Table 3.8). The contact in all of the cases is rough with a coefficient of friction or adhesion factor, \( (\alpha) = 1 \). Since the maximum shear stress that can be mobilized at the interface \( \tau_{\text{max}} = \alpha.S_u \), the maximum shear stress cannot exceed the undrained shear strength of the soil. The total number of elements, nodes and mesh verification results in terms of quadrilateral face angle and aspect ratio of elements for each of the threes meshes are given in Table 3.8 below. The fineness of meshes increases from Mesh 1 to Mesh 3.

<table>
<thead>
<tr>
<th></th>
<th>Coarser</th>
<th>Medium</th>
<th>Fine</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mesh 1</td>
<td>Mesh 2</td>
<td>Mesh 3</td>
</tr>
<tr>
<td>No of elements</td>
<td>46326</td>
<td>62204</td>
<td>85400</td>
</tr>
<tr>
<td>No of nodes</td>
<td>49544</td>
<td>65800</td>
<td>90219</td>
</tr>
<tr>
<td>Min angle on quad faces &lt; 10</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Avg min angle</td>
<td>85.09</td>
<td>84.91</td>
<td>85.44</td>
</tr>
<tr>
<td>Worst min angle</td>
<td>54.53</td>
<td>54.92</td>
<td>51.74</td>
</tr>
<tr>
<td>Max angle on quad faces &gt; 160</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Avg max angle</td>
<td>95.59</td>
<td>95.87</td>
<td>95.38</td>
</tr>
<tr>
<td>Worst max angle</td>
<td>144.1</td>
<td>137.35</td>
<td>137.28</td>
</tr>
<tr>
<td>Aspect ratio &gt; 10</td>
<td>1485 (3.20%)</td>
<td>3438(5.53%)</td>
<td>6477(7.58%)</td>
</tr>
<tr>
<td>Avg aspect ratio</td>
<td>3.75</td>
<td>3.87</td>
<td>4.63</td>
</tr>
<tr>
<td>Worst aspect ratio</td>
<td>49.31</td>
<td>49.31</td>
<td>49.31</td>
</tr>
</tbody>
</table>

Table 3.8: Details of meshes used in mesh sensitivity analysis

Mesh 1 is shown in Fig. 3.26 below. Mesh 2 and 3 are given in Appendix A.
The models were subjected to uniaxial and combined loading by applying displacements and/or angular velocities at the reference point. Uniaxial bearing capacity factors are reported in Table 3.9. To examine the performance of the model and benchmark the results, calculated values of $N_c$ were compared with the exact solution ($N_c = 6.05$) given by Cox et al. (1961) for rough circular foundations. The exact value of $N_h$ remains 1, since shear stress developed at the interface cannot exceed the undrained shear strength. Values of $N_m$ are compared with upper bound solution and that reported by Gourvenec and Randolph (2003).
Table 3.9: Comparison of bearing capacity factors with published data

<table>
<thead>
<tr>
<th></th>
<th>$N_c$</th>
<th>$N_h$</th>
<th>$N_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mesh 1</td>
<td>6.08</td>
<td>1.03</td>
<td>0.72</td>
</tr>
<tr>
<td>Mesh 2</td>
<td>6.06</td>
<td>1.02</td>
<td>0.72</td>
</tr>
<tr>
<td>Mesh 3</td>
<td>6.04</td>
<td>1.02</td>
<td>0.71</td>
</tr>
<tr>
<td>Benchmark</td>
<td>6.05</td>
<td>1</td>
<td>0.67</td>
</tr>
<tr>
<td>% difference Mesh 1</td>
<td>0.50</td>
<td>3.00</td>
<td>7.46</td>
</tr>
<tr>
<td>% difference Mesh 2</td>
<td>0.17</td>
<td>2.00</td>
<td>7.46</td>
</tr>
<tr>
<td>% difference Mesh 3</td>
<td>−0.17</td>
<td>2.00</td>
<td>7.46</td>
</tr>
</tbody>
</table>

Swipe tests were also performed on the models to plot non-dimensionalized and normalized failure envelopes in 2-dimensional VH and VM planes. The plotted envelopes are compared with those of Gourvenec and Randolph (2003). To non-dimensionalize the envelopes, vertical and horizontal forces were divided by the factor $A_Su$. While for normalizing the envelopes, they were divided by the respective ultimate capacities. Non-dimensional and normalized load for vertical, horizontal and moment loads are represented by $v$, $v'$, $h$, $h'$, $m$ and $m'$ respectively.

Figure 3.27: Comparison of VH failure envelope
Figure 3.28: Comparison of normalized VH failure envelope

Figure 3.29: Comparison of VM failure envelope
Observations and Discussion

All the three meshes obtained converged solutions. Values of ultimate bearing capacity factors are very close to the exact values or the upper bound solutions. $N_c$ values deviate less than $\pm 0.5\%$ while for $N_h$, the maximum deviation is $3\%$. Values of $N_h$ obtained for Mesh 2 and 3 have the same percentage error of $2\%$. This implies that using a single layer or three layers of 0.2 m thickness does not significantly affect the horizontal resistance developed. The calculated values of $N_m$ have the highest percentage error among the three factors. Mesh 1 and 2 reported the same values of $N_m = 0.72$ and thus, have the same error of $7.46\%$. The $N_m$ value calculated with Mesh 3 is closest to the upper bound solution and deviates only by $5.97\%$. All of the failure envelopes in the VH and VM planes are very close to each other. As compared to the envelopes reported by Gourvenec and Randolph (2003), both, VH and VM envelopes, are found to be conservative. This difference can be explained by the contact conditions assumed in each analysis. Gourvenec and Randolph (2003) considered a fully bonded contact which implied that infinite friction was assumed and no separation was allowed once the two surfaces are in contact. Such an assumption is appropriate for an offshore foundation where a full contact is maintained due to suction. In contrast, the friction coefficient in the present work is only 1 and the two surfaces are allowed to separate. Such a contact condition is appropriate for wind turbine
foundations which are subjected to uplifting forces due to the wind.

With these considerations, it is deemed appropriate to choose Mesh 2. In all of the further analysis, for surface foundations, the contact definitions, boundary conditions and mesh densities are kept the same as that of Mesh 2. When the cases of foundation embeddings or soil strength heterogeneities are considered, the changes in the mesh patterns and fineness are made suitably. For the case of an embedded foundation, thin layers of mesh are used above and below the foundation. For the surface foundation placed on heterogeneous soil, the density of mesh near the interface is increased as the heterogeneity increases. This means that the number of thin layers of soil (of 0.2 m thick) are increased. The number of elements were varied from 62000 for surface foundation to 72500 for foundation with maximum embedment ratio \((d/D = 1)\). For the surface foundation with homogeneous soil, the number of elements were 62000, while for the surface foundation with the highest soil heterogeneity factor \((k = 1.5)\), the number of elements were 85000.

### 3.4.4.3 General Characteristics of the Model

A complete 3-dimensional model was created and analyzed in ABAQUS. If a symmetry exists in geometry as well as the line of action of forces, then a half or quarter or even a sector of a circular or octagonal foundation can be modelled. However, in the present case, the foundation is subjected to vertical, horizontal and moment loading and a symmetry of forces is not possible by reducing the size to half or quarter. Hence a full 3-dimensional model was created. Although such a model has more number of elements and hence takes more time to find the solution, it provides more and better information in terms of stresses and strains occurring in various parts of the model.

The general characteristics of the model are as follows.

*Problem Geometry*

Fig 3.31 and 3.32 shows the geometries for the circular and octagonal foundations.
The octagonal foundation has 19 m diameter of its inscribed circle and 3 m depth. Note that the octagonal foundation is assumed to have a uniform depth/thickness and does not taper towards the edges (to compare with a typical case used in the wind industry, refer Fig. 3.1). Section 8.2.1 of DNV (2002) states that for calculation of the effective area of a polygon (octagon), the radius of the inscribed circle of the polygon must be used. Taking note of this point, the circular foundation having a diameter equal to that of an inscribed circle is modelled in the present research. Thus, the diameter of circular foundation is 19 m and its depth is 3 m.
Boundary Conditions

The vertical boundaries of the soil were placed at a distance $3D$ from each of the foundation edges. A roller support condition was applied on both the vertical sides. This implies that displacements in horizontal directions (X and Y) were restricted. The base of the soil was kept at a distance just over $2D$ from the foundation bottom. The base was fixed and all the six degrees of freedom were constrained. The top surface of the soil was kept free. The boundary distances and conditions used have been used previously by many researchers. A summary of boundary distances from the foundation edge or base for other studies is given in Table 3.10.

Figure 3.33: Boundary distances for the mesh
Published Study | FEA type | Horizontal distance from foundation edges | Vertical boundary from the soil surface
--- | --- | --- | ---
Bransby (1998) | 2D | 2.5D | 2.5D
Taiebat (2000) | 3D | 4.5D | 5D
Taiebat (2002a) | 2D,3D | 3.5D | 4D
Taiebat (2003) | 3D | 3.5D | 4D
Gourvenec (2003) | 2D,3D | 2.5D | 2.5D
Gourvenec (2004) | 3D | 2.5D | 2.5D
Edwards (2005) | 2D | 7B | 5B
Gourvenec (2006) | 3D | 3B | 2.5B
Yun (2007) | 2D | 4.5B | 3B
Taiebat (2010) | 3D | 4D | 4D
Barnett (2011) | 2D | 5B | 5B
Gourvenec (2011) | 2D | 6B | 6B
Cassidy (2013) | 2D | 2.5B | 2.8B

Table 3.10: Summary of boundary distances from published studies

**Finite Element Mesh**

A typical mesh consisted of 65400 nodes and 62200 elements. Three-dimensional eight node continuum element with reduced integration C3D8R was used to discretize foundation and soil into finite elements. The size of the elements were varied depending on their proximity to the load applied. When C3D8R elements are used in ABAQUS, the bulk modulus of soil cannot exceed 25 times the shear modulus. This may cause convergence problems. Hence a Poisson’s ratio of 0.48 which is less than 0.49 was used to solve this issue. The other solution to this problem is to use continuum hybrid elements C3D8H which does not put restrictions on the value of bulk modulus and allows the use of Poisson’s ratio equal to 0.49.

For the case of embedded foundations, the mesh density above and around the foundation was also increased. The figure given below shows the mesh for a circular foundation with
embedment ratio $d/D = 1$. Whilst this is highly unlikely to be used in practice, it was included in the study for completeness.

![Mesh details for embedded foundation $d/D = 1$](image)

Figure 3.34: Mesh details for embedded foundation $d/D = 1$

For the case of the soil with highest strength heterogeneity factor $k = 1.5$, the mesh was made denser near the interface. Three layers of very thin soil elements of 0.2 m thickness were provided. The mesh is shown in the Fig. 3.35 given below.
Contact Conditions

For surface foundations, only the foundation bottom and soil contact occurs. The adhesion factor for this contact surface was assumed equal to 1. For the case of embedded foundations, there is contact at the side surfaces as well as on the foundation top. The adhesion factors in these cases were assumed equal to 0.5. The maximum shear stress mobilized at the side and top surfaces is, thus, half of that developed at the bottom. Further details of the contact definitions used in the 3-dimensional modelling have been mentioned previously in Section 3.4.3.3.

3.4.4.4 Restarting an Analysis in ABAQUS

Since the number of analysis runs are very high, the ‘restart’ analysis feature was used in ABAQUS to save computational effort and time. The restart feature creates additional files on the computer hard disk that are used for the restart and thus, consume extra space. However, it provides a user with many advantages as listed below.
• It allows the user to restart an analysis from the completion of a step thereby providing the ability to troubleshoot efficiently and pinpoint the area where the problem is occurring.

• It saves a lot of time when same procedure has to be followed for different analysis. For example, in the present research, every analysis begins with geostatic equilibrium step. For each single case, uniaxial vertical, horizontal & moment loads and VH, VM & HM failure envelopes need to be found. Thus, by using a single analysis file solved for geostatic step, considerable amount of time can be saved.

• It provides the flexibility to retain only one increment per step by ‘overlaying’ the data from the previous increment thereby minimizing the size of file. In all the analysis here, only the last increment of the step was saved rather than all the increments of the step.

3.5 Summary

In this chapter, a 3-dimensional numerical model for the octagonal foundation in the field was developed in the finite element program ABAQUS, to study its bearing capacity response to combined loading (vertical, horizontal and moment) under undrained conditions. The problem of combined loading was solved implicitly in ABAQUS. For all the analysis, loads were applied in the form of displacements at the load reference point (LRP). The load reference point was chosen as the centre of the base of the foundation. Reaction forces/moments at this point were used to obtain the uniaxial limit capacities. Swipe tests were performed to obtain failure envelopes in VH, VM and HM planes. The real soil was idealised as a linear elastic-perfectly plastic material. To describe the plastic deformations in the soil, the Mohr-Coulomb failure criterion was considered sufficient for the finite element analysis. Choice of other constitutive models, such as those used to describe anisotropic soils, etc. requires more experimental and field data, and was not necessary for this study.

In the initial part of the study, a 2-dimensional plane strain model for strip footing was created and subjected to combined loading. The uniaxial bearing capacities and envelopes in VH &
VM planes when compared, matched reasonably well with published data. This helped in validating the boundary conditions, mesh techniques, geometry and contact definitions. The contact between the foundation and the soil can sustain tension and is, thus, separable.

Subsequently, 3-dimensional models were created for circular foundations to further calibrate the model. As a part of mesh sensitivity analysis, three meshes with different densities were created and subjected to uniaxial & combined loadings. The results from this analysis were compared with those available in the literature. Finally, Mesh 2 was chosen to minimize the computational effort & time while maintaining the accuracy of the solution. In Mesh 2, a single layer of soil elements of thickness 0.2 m (≈ 1/100th of the foundation diameter) was used at the interface to provide sliding resistance against any horizontal forces.

Various cases of foundation embedments and soil strength heterogeneity were considered for both, circular and octagonal foundations. The mesh densities were suitably increased wherever deemed appropriate. Depending on whether the foundation is on the soil surface, shallowly embedded or presence of soil strength heterogeneity, failure criteria to define ultimate vertical, horizontal and moment loads were defined. A stiff crust overlying a soil with linearly increasing strength was considered. A parametric study to study the effects of crust thickness, embedment depth relative to crust thickness, average crust strength and shear strengths of the underlying layer relative to that of the crust was conducted as the final part of the current research.
Chapter 4

Results and Discussions

4.1 Introduction

In this chapter, the results of the finite element analysis of the undrained bearing capacities of footings subjected to uniaxial and combined loadings are presented and discussed. Most published data relates to strip or circular footings subjected to combined loading owing to their simple shapes. Therefore, the first part of this chapter presents findings for circular footings. The results are compared with the solutions given by analytical, conventional and finite element solutions published in the literature. The second part of the chapter covers octagonal foundations, which is the main focus of this research. Here, the results are compared with the finite element solutions obtained for circular foundations in this research.

The final part of this chapter presents the results for the special cases considered and extends the work done in the second part. First, a special case with a surface crust layer of high shear strength is considered. Uniaxial ultimate capacities and failure envelopes are derived for this case and compared with the previously obtained results. Second, the foundation size is reduced whilst keeping the aspect ratio equal to the original embedded foundation. Failure envelopes are plotted to assess if the reduced size of the foundation can sustain the loads under ‘normal’ (or ‘working’) and ‘extreme’ loading conditions. The bearing capacity response
of the original embedded foundation is also obtained under ‘working’ and ‘extreme’ loading conditions, presented in Appendix D. Third, bearing capacities of a circular foundation with radius equal to that of the circumscribed circle of the octagon are determined. Usually, the radius of the inscribed circle of the polygon is considered for the analysis, as mentioned in DNV (2002). This case investigates if using a circumscribed circle has any effects on the bearing capacity of the foundation. For the final special case, horizontal force and moment are applied through or about the diagonal of an octagonal foundation, to compare with the predominant case used in the thesis (horizontal force/moment through the mid-point of a side). Ultimate uniaxial bearing capacities and failure envelopes are derived for this work. The details of the last two cases are presented in Appendix B.

For every case considered, the foundation is first subjected to uniaxial loads using displacement-controlled analysis till the ultimate point is reached. The uniaxial vertical, horizontal and moment limit capacities, henceforth, obtained are presented in the form of bearing capacity factors. Subsequently, swipe tests are done in 2-dimensional planes, viz. VH, VM and HM. For the swipe tests in the VH and VM planes, the foundation is first loaded in the vertical direction till the ultimate vertical capacity is reached. For the second step, it is loaded horizontally or in the moment direction, keeping the increment in the vertical direction equal to zero till the ultimate point is reached in the second applied load direction (i.e. horizontal or moment direction). To find the HM failure envelopes, the foundation is first loaded vertically with a displacement corresponding to half of the ultimate vertical load. In the absence of a fully bonded contact at the interface, the foundation cannot develop any horizontal or moment resistance without vertical load (Taiebat & Carter, 2002a); (Houlsby & Puzrin, 1999). Secondly, the foundation is loaded horizontally till its uniaxial horizontal capacity is reached. Finally, the foundation is swiped in the moment direction, keeping the increment in the horizontal direction equal to zero. Once the ultimate moment is reached, the loading is stopped.

The paths traced by the swipe tests provide failure envelopes in 2-dimensional load planes. Loads which lie inside the envelope are considered safe, while those lying outside define foundation failure. Ultimate limit states and kinematic mechanisms accompanying failure are presented. To study the size of the envelopes, they are non-dimensionalized by the
foundation geometry and soil shear strength, and to investigate the effect of their shapes, they are normalized by ultimate uniaxial loads.

4.2 Undrained Bearing Capacity of Circular Footings

The bearing capacities of circular foundations subjected to uniaxial and combined loadings are investigated under undrained conditions. Case I, II and III represent surface and embedded foundations with homogeneous soil and surface foundation with heterogeneous soils respectively.

4.2.1 Case I: Surface Foundation on Homogeneous Soil

**(3-dimensional)**

_Notations for loads and displacements_

<table>
<thead>
<tr>
<th></th>
<th>Vertical</th>
<th>Horizontal</th>
<th>Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load</td>
<td>V</td>
<td>H</td>
<td>M</td>
</tr>
<tr>
<td>Ultimate load</td>
<td>V₀</td>
<td>H₀</td>
<td>M₀</td>
</tr>
<tr>
<td>Dimensionless load</td>
<td>v = V/(A S_u)</td>
<td>h = H/(A S_u)</td>
<td>m = M/(A D S_u)</td>
</tr>
<tr>
<td>Dimensionless ultimate load (Uniaxial bearing capacity factors)</td>
<td>v₀ = V₀/(A S_u)</td>
<td>h₀ = H₀/(A S_u)</td>
<td>m₀ = M₀/(A D S_u)</td>
</tr>
<tr>
<td>Normalized load</td>
<td>v' = V/V₀</td>
<td>h' = H/H₀</td>
<td>m' = M/M₀</td>
</tr>
<tr>
<td>Displacement</td>
<td>w</td>
<td>u</td>
<td>θ</td>
</tr>
</tbody>
</table>
Uniaxial Bearing Capacity Factors

The plots used to find the values of uniaxial bearing capacity factors are shown below in Fig. 4.1-4.3. Loads are drawn in dimensionless form on the vertical axis, while the fraction of total load is plotted on the horizontal axis. For example, a value of 0.2 on X-axis represents 20% of the total vertical load and a value of 1 represents 100% of the total vertical load. Values of dimensionless load on the Y-axis represent bearing capacity factors. For finding v, a very small horizontal load (V/H or w/u= 60) is applied as described previously in Chapter 3. The value of v corresponding to the maximum value of h is taken as the vertical bearing capacity factor. For example, in Fig. 4.1, \( v_0 = 6.08 \).

Figure 4.1: Case I: Load response under vertical and horizontal loading
Figure 4.2: Case I: Load response under horizontal loading

Figure 4.3: Case I: Load response under moment and horizontal loading
Table 4.1: Case I: Comparison of values of \( v_0 \) for a circular surface footing on homogeneous clays with published data

<table>
<thead>
<tr>
<th></th>
<th>( v_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEM (this study)</td>
<td>6.08</td>
</tr>
<tr>
<td>Gourvenec and Randolph (2003)</td>
<td>5.91</td>
</tr>
<tr>
<td>Cox et al. (1961)</td>
<td>6.05</td>
</tr>
<tr>
<td>Houlsby and Wroth (1983)</td>
<td>6.05</td>
</tr>
<tr>
<td>Taiebat and Carter (2000)</td>
<td>5.70</td>
</tr>
<tr>
<td>Vesic (1973)</td>
<td>6.17</td>
</tr>
<tr>
<td>Shen et al. (2016)</td>
<td>5.87</td>
</tr>
</tbody>
</table>

Table 4.2: Case I: Comparison of values of \( h_0 \) for a circular surface footing on homogeneous clays with published data

<table>
<thead>
<tr>
<th></th>
<th>( h_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEM (this study)</td>
<td>1.03</td>
</tr>
<tr>
<td>Gourvenec and Randolph (2003)</td>
<td>1.02</td>
</tr>
<tr>
<td>Shen et al. (2016)</td>
<td>1.02</td>
</tr>
</tbody>
</table>

Table 4.3: Case I: Comparison of values of \( m_0 \) for a circular surface footing on homogeneous clays with published data

<table>
<thead>
<tr>
<th></th>
<th>( m_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEM (this study)</td>
<td>0.69</td>
</tr>
<tr>
<td>Gourvenec and Randolph (2003)</td>
<td>0.69</td>
</tr>
<tr>
<td>Upper bound (Gourvenec &amp; Randolph, 2003)</td>
<td>0.67</td>
</tr>
<tr>
<td>Taiebat and Carter (2000)</td>
<td>0.80</td>
</tr>
<tr>
<td>Shen et al. (2016)</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Table 4.1 provides the calculated value of dimensionless ultimate vertical load \( (v_0) \) from the
current study along with available published data for vertical bearing capacity under equivalent conditions from various other studies. The value of \( v_0 \), 6.08 is slightly higher than the exact value calculated by Cox et al. (1961) & Houlsby and Wroth (1983) for a rough contact surface, which is 6.05. This slight over-prediction can be attributed to different contact conditions assumed in the present and past works. In the previous works, a rough and fully bonded contact was usually assumed, whereas in the present research, separation between the foundation and soil is allowed although a rough interface has been modelled. Shen et al. (2016), who used a no-tension interface under-predicted the vertical collapse load by 3%. The accuracy of the finite element solution may also be affected by the use of 3-dimensional analysis, wherein more soil is involved in the failure compared to that in a 2-dimensional analysis. However, the value is relatively closer and more accurate than 6.17, found with the conventional solution given by Vesic (1973).

The calculated values of dimensionless ultimate lateral (\( h_0 \)) and ultimate moment load (\( m_0 \)) are shown in Table 4.2 and 4.3 respectively. For a surface foundation, ultimate lateral capacity is estimated based on the assumption that the maximum sliding resistance or shear stress that can be mobilized at the interface is equal to the undrained shear strength \( S_u \). Thus, \( h_0 \) is always theoretically equal to 1.0. In the current finite element analysis, the \( h_0 \) value of 1.03 is only 3% higher than the theoretical solution. This error is 1% higher than Gourvenec and Randolph (2003) and Shen et al. (2016) who reported a value of 1.02. Since the contact between the foundation and soil is separable, no moment can be sustained in the absence of any vertical load. In order to obtain the maximum value of ultimate moment (\( m_0 \)), displacement corresponding to a value of 0.5\( V_0 \) was applied at the reference point as suggested by Taiebat and Carter (2010). The value of \( m_0 = 0.69 \) matches exactly with that obtained by Gourvenec and Randolph (2003), although they considered a fully bonded contact with no vertical load applied to find ultimate moment. However, this value is higher than that calculated by Shen et al. (2016). When compared with the upper bound solution, \( m_0 \) is around 3% higher. However, Taiebat and Carter (2000) calculated a value of 0.80, significantly higher than the current solution.

Note that all of the previous works mentioned above, except Shen et al. (2016), assumed a
fully bonded contact i.e. no separation between foundation and soil was allowed. Shen et al. (2016) used a no-tension interface to model the contact between circular foundation and soil with a coefficient of friction, $\mu = 20$ (i.e. equivalent friction angle of $\phi = \tan^{-1} 20 = 87.1^\circ$ very close to $90^\circ$). In contrast, a no-tension interface behaviour with $\mu = 1$, which corresponds to $\phi = \tan^{-1} 1 = 45^\circ$ is assumed in the present research. Additionally, Shen et al. (2016) found that the value of vertical load $V$ at which the ultimate moment ($m_0$) occurs, is equal to $0.49V_0$ whereas $V=0.50V_0$ at $m_0$ in the present case, is slightly higher.

**Failure Envelopes**

The failure envelopes in VH, VM and HM load space planes are plotted below along with published data. The failure envelopes in the VH and VM planes for the conventional method are obtained from the plots given by Taiebat and Carter (2002b) who used the traditional method given by Vesic (1973). For the HM failure envelope, the data values are derived from the plot given for the traditional method (Meyerhof, 1953) in Shen et al. (2016) for $V= 0.5V_0$ (which gives the maximum possible ultimate moment capacity).

![Figure 4.4: Case I: Failure envelope in the normalized VH plane](image)
Figure 4.5: Case I: Failure envelope in the normalized VM plane

Figure 4.6: Case I: Failure envelope in the normalized HM plane
The predicted failure envelope for VH loading is presented in Fig. 4.4 in normalized form and compared with Gourvenec and Randolph (2003), Taiebat and Carter (2002b), Shen et al. (2016) & the conventional method (Vesic, 1973). It shows that the conventional method neglects some potential additional bearing capacity when horizontal loads are increased. At very high horizontal loads, the bearing capacity decreases indicating that at this stage, the failure is dictated by the horizontal loads rather than the vertical loads. The conventional method neglects this effect and hence, slightly over-predicts the bearing capacity.

The angle of inclination that an inclined/resultant load makes with the vertical direction can be calculated as inverse tangent of the ratio of horizontal to vertical force. For a given vertical force, the maximum value of this angle is known as the critical angle of inclination and it signifies the stage at which horizontal load starts dominating. This value is found to be 15.3°; this is lower when compared to that given by Taiebat and Carter (2000) and Bolton (1979) (19°), but higher than that calculated by Vesic (1973) (13°). If the inclination angle is more than the critical value (i.e. very high horizontal force is acting), the vertical force does not have any influence on the horizontal capacity of the foundation.

The failure locus obtained by combined vertical and moment loading is shown in Fig. 4.5 and compared with those of Gourvenec and Randolph (2003), Taiebat and Carter (2002b), Shen et al. (2016) and the conventional method (Vesic, 1973). The envelope derived from the current study becomes conservative compared to that derived by the conventional method. This shows that the effects of extreme eccentricity are not taken into account by conventional methods and thus, the bearing capacity is overestimated. Indeed, Section 8.2.1 of DNV (2002) modifies the bearing capacity formula for extremely eccentrically loaded foundation ($e > 0.3B$), where more soil under the heel of the foundation is involved in the failure (eqn. 2.23).

Fig. 4.6 presents the HM failure envelope. The failure envelope has been plotted at displacement corresponding to a fixed vertical load of $V = 0.5V_0$. Under eccentric and inclined loading, conventional methods overlook the bearing capacity and become conservative with respect to the envelopes calculated with the numerical analysis. Thus, conventional methods take eccentricity or inclination into account separately, but will breakdown when eccentric and
inclined loads act together. The failure envelope derived in the current work using swipe test is slightly smaller than that given by Shen et al. (2016) who used displacement probe tests to derive the HM failure envelopes. As pointed out by Gourvenec and Randolph (2003), swipe tests must be used with care when deriving envelopes in the HM plane. If the elastic deformation becomes greater than the plastic deformation, the load path moves inside the true failure envelope (Taiebat & Carter, 2010).

**Contact Area**

![Figure 4.7: Case I: Comparison of contact areas calculated by numerical method and DNV (2002) under eccentric loading](image)

Figure 4.7: Case I: Comparison of contact areas calculated by numerical method and DNV (2002) under eccentric loading
The contact areas predicted during eccentric loading (combined vertical and moment loads) by the finite element analysis and DNV (2002) which uses the conventional method of calculating effective area are plotted together in Fig. 4.7. The increasing eccentricity as a fraction of the diameter $D$ is also plotted on the secondary Y axis. The contact area calculated in the current study is always higher than that estimated with the DNV (2002) method. When vertical load is high, the contact areas predicted by both methods are close to each other and the difference is only 3.90%. As the eccentricity increases, the two values diverge more from each other. When the moment load reaches its ultimate value, the difference is 33.91%. The larger area predicted by the finite element (FE) analysis contributes towards the larger prediction of moment capacity. The contact areas are also compared for three values of eccentricity in Fig. 4.8. The area shown in red and green was predicted using finite element analysis, while the elliptical shaped area with black outline and shaded with white stripes is estimated by using the DNV method.
Von Mises Stress Distribution

Absolute values of Von Mises stress are shown from Fig. 4.10 to 4.12 for various load conditions. The stress distribution when no load is acting (i.e. after the geostatic step) is also shown in Fig. 4.9 to benchmark the changes in the stress when uniaxial ultimate loads act on the foundation. The values of Von Mises stress shown in the legend are in kN/m² and the contours are plotted on the undeformed mesh for a section through diametrical plane of the foundation with $y = 0$ (XZ plane). The contours shown are continuous and results are calculated by averaging the output at each node of the extrapolated results of the corresponding element. The averaging is done only if the results from one element are within 75% of that from the other i.e. no more than 25%. Note that for the foundation itself, no stress output was obtained since the degrees of freedom for all of its nodes were constrained to that of a single load reference point (LRP).

Absolute values of Von Mises stress are shown from Fig. 4.10 to 4.12 for various load conditions. The stress distribution when no load is acting (i.e. after the geostatic step) is also shown in Fig. 4.9 to benchmark the changes in the stress when uniaxial ultimate loads act on the foundation. The values of Von Mises stress shown in the legend are in kN/m² and the contours are plotted on the undeformed mesh for a section through diametrical plane of the foundation with $y = 0$ (XZ plane). The contours shown are continuous and results are calculated by averaging the output at each node of the extrapolated results of the corresponding element. The averaging is done only if the results from one element are within 75% of that from the other, i.e. no more than 25%. Note that for the foundation itself, no stress output was obtained since the degrees of freedom for all of its nodes were constrained to that of a single load reference point (LRP).
Figure 4.9: Case I: Von Mises stress distribution at zero load

Figure 4.10: Case I: Von Mises stress distribution at ultimate vertical load
Due to the action of the ultimate vertical load, stresses immediately below and around the foundation increase (by more than $35\%$). When ultimate horizontal or moment loads act, tension is developed at the interface (grey shaded areas shown in Fig. 4.13 and 4.14). Tension is developed in the soil below the foundation edge on the $-X$ side due to the horizontal load acting in the $+X$ direction at load reference point (LRP). When moment is acting in the
clockwise direction about the Y axis at LRP, tension is developed in the semicircular contact area on the −X side. Most of this soil area is not in contact with the foundation at the end of ultimate moment load. Tension is also developed in the area outside the semicircular area on the +X side of the foundation.

Figure 4.13: Case I: Development of tension at ultimate horizontal load

Figure 4.14: Case I: Development of tension at ultimate moment load
### 4.2.2 Case II: Embedded Foundations in Homogeneous Soil

Analyses were conducted for circular foundations embedded at four different depths in homogeneous soil as shown in Table 4.4.

<table>
<thead>
<tr>
<th>Embedment ratio, $d/D$</th>
<th>Embedding depth, $d$(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.157</td>
<td>3</td>
</tr>
<tr>
<td>0.32</td>
<td>6</td>
</tr>
<tr>
<td>0.5</td>
<td>9.5</td>
</tr>
<tr>
<td>1.0</td>
<td>19</td>
</tr>
</tbody>
</table>

Table 4.4: Case II: Details of embedment cases considered

The results from the surface foundations usually provide lower bearing capacity values as compared to those of embedded foundations. An embedment ratio of 0.157 represents the present condition in the field (at Port Alma), where the foundation is placed at 3 m below ground surface level. Data obtained from foundation embedment ratios 0.32 and 0.5 are used to check the trend of change in the bearing capacity factors and the size and shape of the failure envelopes. An embedment ratio of 1.0, i.e. embedding depth equal to the diameter of the foundation (19 m) is not possible practically for wind turbine foundations. However, it serves as an upper bound to the other cases considered.
Uniaxial Bearing Capacity Factors

\[
\begin{array}{cccc}
  d/D & v_0 & h_0 & m_0 \\
 0 & 6.08 & 1.03 & 0.69 \\
0.157 & 7.42 & 1.97 & 1.04 \\
0.32 & 8.48 & 3.10 & 1.36 \\
0.5 & 9.51 & 3.78 & 1.69 \\
1.0 & 11.40 & 5.03 & 2.00 \\
\end{array}
\]

Table 4.5: Case II: Uniaxial bearing capacity factors under varying embedment conditions

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.08</td>
<td>6.168</td>
<td>6.168</td>
<td>6.05</td>
<td>5.757</td>
<td>5.945</td>
</tr>
<tr>
<td>0.157</td>
<td>7.42</td>
<td>6.362</td>
<td>6.555</td>
<td>–</td>
<td>6.756</td>
<td>7.280</td>
</tr>
<tr>
<td>0.32</td>
<td>8.48</td>
<td>6.563</td>
<td>6.958</td>
<td>–</td>
<td>7.206</td>
<td>8.246</td>
</tr>
<tr>
<td>0.50</td>
<td>9.51</td>
<td>6.785</td>
<td>7.402</td>
<td>–</td>
<td>7.592</td>
<td>9.105</td>
</tr>
<tr>
<td>1.0</td>
<td>11.40</td>
<td>7.402</td>
<td>8.106</td>
<td>–</td>
<td>8.421</td>
<td>11.18</td>
</tr>
</tbody>
</table>

Table 4.6: Case II: Comparison of \(v_0\) for embedded foundations in homogeneous clay with published data

<table>
<thead>
<tr>
<th>d/D</th>
<th>FEM (this study)</th>
<th>Gourvenec (2008)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.03</td>
<td>1.00</td>
</tr>
<tr>
<td>0.157</td>
<td>1.97</td>
<td>1.66</td>
</tr>
<tr>
<td>0.32</td>
<td>3.10</td>
<td>2.27</td>
</tr>
<tr>
<td>0.5</td>
<td>3.78</td>
<td>2.85</td>
</tr>
<tr>
<td>1.0</td>
<td>5.03</td>
<td>3.94</td>
</tr>
</tbody>
</table>

Table 4.7: Case II: Comparison of \(h_0\) with published data
Table 4.8: Case II: Comparison of $m_0$ with published data

<table>
<thead>
<tr>
<th>$d/D$</th>
<th>FEM (this study)</th>
<th>Gourvenec (2008)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.69</td>
<td>0.69</td>
</tr>
<tr>
<td>0.157</td>
<td>1.04</td>
<td>1.23</td>
</tr>
<tr>
<td>0.32</td>
<td>1.36</td>
<td>1.54</td>
</tr>
<tr>
<td>0.5</td>
<td>1.69</td>
<td>1.95</td>
</tr>
<tr>
<td>1.0</td>
<td>2.00</td>
<td>3.54</td>
</tr>
</tbody>
</table>

The uniaxial bearing capacity factors ($\nu_0$, $h_0$ & $m_0$) are presented in Table 4.5 for different embedment conditions. Note that some of the reported values in this table from the published data have been obtained by interpolation. With increasing depth of embedment, all of the 3 bearing capacity factors increase. Embedment of the foundation causes more soil around and above the foundation to be involved in the failure, which results in an increase of bearing capacity.
Figure 4.15: Case II: Comparison of $v_0$ with published data

Figure 4.16: Case II: Comparison of $h_0$ with published data
Fig. 4.15 shows a comparison between $v_0$ obtained from the current study and the previous works. Values for Meyerhof (1951) and Hansen (1970) have been derived by multiplying shape and depth factors with $v_0$ for a strip footing. While Salgado et al. (2004) used 3-d finite element limit analysis, Gourvenec and Mana (2011) used both finite element limit analysis and finite element analysis to derive vertical bearing capacity factors for circular footings. It is clearly evident from the figure that with increase in the embedment, $v_0$ always increases. Values of $v_0$ match fairly well with those of Gourvenec and Mana (2011). However, all of the values are slightly higher; the maximum being 4.26% in the case of $d/D = 0.5$. Values given by the conventional methods of Meyerhof and Hansen are lower, since they do not take 3-dimensional effects into account. The results from the finite element analyses from this study suggest a possible quadratic relationship between ultimate uniaxial vertical load and embedment ratio, for $0 \leq d/D \leq 1$, which can be expressed as

$$v_0 = [1 + 1.5(d/D) - 0.65(d/D)^2] \times 6.08$$  \hspace{1cm} (4.1)
v₀ obtained from the expression above and that from the finite element study is compared graphically in Fig. 4.18. Gourvenec (2008) investigated strip footings. A fully bonded contact was assumed between the foundation and soil. Due to unavailability of data for circular footings, the uniaxial horizontal and moment capacity factors are presented graphically with Gourvenec (2008) in Fig. 4.16 and 4.17 respectively. However, the data is not directly comparable and is used only as a reference. Like v₀, h₀ can also be related to embedment ratio using a quadratic expression given by

\[
h₀ = [0.84 + 7.67(d/D) - 3.83(d/D)^2] × 1.03
\]  

(4.2)

Figure 4.18: Case II: Comparison of v₀ from the current finite element study and closed-form expression
The analytical expression values are plotted against those of the finite element study in Fig. 4.19. Similarly, a quadratic relationship exists between $m_0$ and embedment ratio which can be expressed as

$$m_0 = [0.90 + 3.93(d/D) - 1.90(d/D)^2] \times 0.69$$

(4.3)

The plot of $m_0$ values calculated using the above expression and finite element study are shown below (Fig. 4.20).
Figure 4.20: Case II: Comparison of $m_0$ from the current finite element study and closed-form expression
Failure Envelopes

Figure 4.21: Case II: Failure envelope in the non-dimensional VH plane

Figure 4.22: Case II: Failure envelope in the normalized VH plane
Figure 4.23: Case II: Failure envelope in the non-dimensional VM plane

Figure 4.24: Case II: Failure envelope in the normalized VM plane
Figure 4.25: Case II: Failure envelope in the non-dimensional HM plane

Figure 4.26: Case II: Failure envelope in the normalized HM plane
The ultimate limit states at different embedment ratios plotted in the VH, VM and HM planes in terms of non-dimensional and normalized forms are shown from Fig. 4.21 to Fig. 4.26. In all of the three planes, the failure envelopes are expanding indicating the increase in load-carrying capacity with increase in the embedment ratio. In the VH and VM planes, the shape of the failure envelopes are similar but not unique. In the normalized HM plane, considerable variation occurs in the size of the failure envelopes with change in the embedment ratio. The shape of the failure envelopes is also not similar except for the case when \(d/D = 0.32\) and 0.5. As a result, deriving a single curve-fitting expression for the shape of the HM envelope is not practical.

*Failure Mechanisms*

The failure mechanisms in Fig. 4.27 to 4.30 are shown for a section through the diametrical plane of the foundation with \(y = 0\) (XZ plane) and the mechanisms are plotted for this section throughout this research.

*Figure 4.27: Case I: Failure mechanism for a surface foundation under ultimate horizontal load, \(d/D = 0\) (\(V = 0.5V_0\))*
Figure 4.28: Case II: Failure mechanism for an embedded foundation under ultimate horizontal load, \( d/D = 0.5 \) \((V= 0.5V_0)\)

Figure 4.29: Case I: Failure mechanism for a surface foundation under ultimate moment load at \( V=0.5V_0 \), \( d/D = 0 \) \((V= 0.5V_0)\)
**Figure 4.30:** Case II: Failure mechanism for an embedded foundation under ultimate moment load at \( V = 0.5V_0 \), \( d/D = 0.5 \) (\( V = 0.5V_0 \))

*Von Mises Stress Distribution*

The Von Mises stress distribution is shown from Fig. 4.31 to 4.33.

**Figure 4.31:** Case II: Von Mises stress distribution at zero load, \( d/D = 0.5 \)
Figure 4.32: Case II: Von Mises stress distribution at ultimate vertical load, $d/D = 0.5$

Figure 4.33: Case II: Von Mises stress distribution at ultimate horizontal load, $d/D = 0.5$
For an embedded footing, failure under ultimate horizontal load causes rotation of footing (Fig. 4.28). In contrast, for a surface foundation, failure is governed by pure sliding (Fig. 4.27). With increase in the embedment ratio, sliding decreases and rotation increases. Similarly, the mobilization of ultimate moment is accompanied by horizontal displacement of the footing. Thus, for a constant vertical load, coupling of horizontal and moment degrees of freedom occur during HM loading. For a surface foundation subjected to uniaxial ultimate moment, a negligible horizontal displacement is observed. The failure mechanism under pure moment is shown in Fig. 4.29 for a surface foundation. Unlike Gourvenec and Randolph (2003), a scoop mechanism is not observed for the surface foundation. This is because the foundation gets uplifted as the moment load increases and loses contact with the soil partially (Fig. 4.35). However, an embedded footing displays a scoop failure mechanism (Fig. 4.30), with the centre of rotation moving towards the foundation as the embedment increases. No separation between the foundation and soil is observed. Fig. 4.35 shows the change in contact area during combined vertical and moment loading for an embedded foundation with $d/D = 1$, along with that for the surface foundation. While for the surface foundation, the contact area changes drastically as the foundation gets separated, the contact area for the embedded foundation remains almost the same throughout the loading process. Thus, it can inferred from this that
in an embedded foundation, the contact at the interface behaves as if it is fully bonded.

Figure 4.35: Case II: Comparison of contact area for a surface and embedded foundation ($d/D = 1$)

The Von Mises stress distribution (in kN/m$^2$) for a particular case of embedded footing, $d/D = 0.5$ is shown from Fig. 4.31 to 4.34 under no load and at uniaxial limit states.

**Plastic Strain**

The plastic strain output PEEQ in ABAQUS for the ultimate load states for the surface foundation and a foundation with $d/D = 0.5$ is shown from Fig. 4.36 to 4.41. PEEQ is a conjugate strain of Von Mises stress. It is a scalar measure of accumulated plastic strain. It is derived from the deviatoric part of strain tensor and is a measure of distortion. When a foundation is embedded, the plastic strain is develops around and above the foundation, in addition to the area below it. However, the magnitude of strains are less by an order of magnitude. The plastic strain distribution for the ultimate moment load for the embedded foundation confirms a scoop type mechanism as mentioned before. Whereas for a surface foundation, it occurs further to one side, which is under compression compared to the other where the foundation
loses contact with the soil. The plastic strains for embedded foundations are higher than that for surface foundations except for the case of ultimate moment.

Figure 4.36: Case I: Plastic strain at ultimate vertical load for a surface foundation ($d/D = 0$)

Figure 4.37: Case II: Plastic strain at ultimate vertical load for an embedded foundation ($d/D = 0.5$)
Figure 4.38: Case I: Plastic strain at ultimate horizontal load for a surface foundation \((d/D = 0)\)

Figure 4.39: Case II: Plastic strain at ultimate horizontal load for an embedded foundation \((d/D = 0.5)\)
Figure 4.40: Case I: Plastic strain at ultimate moment load for a surface foundation ($d/D = 0$)

Figure 4.41: Case II: Plastic strain at ultimate moment load for an embedded foundation ($d/D = 0.5$)
4.2.3 Case III: Surface Foundations on Heterogeneous Soil

This case investigates the effects of soil strength heterogeneity on the bearing capacity of shallow/surface foundations on clays under undrained conditions subjected to combined loading. The soil strength with depth is varied linearly. Linear increase in strength with depth is typical for normally consolidated soils (Chenari et al., 2014). The linear increase in strength with depth is dictated by a non-dimensional ratio/soil strength heterogeneity factor \( K' = kD/S_{u0} \), where \( k \) is the rate of increase of soil strength with depth and \( S_{u0} \) is the soil strength at the ground surface. For clays, the \( k \) value typically varies between 0.6 kPa/m to 3 kPa/m (Tani & Craig, 1995). In the present analysis, 3 values of \( k \) viz. 0.5, 1 and 1.5 are assumed, which correspond to \( K' \) equal to 0.475, 0.950 and 1.425 respectively.

A value of \( S_{u0} \) equal to 20 kPa is assumed for the present analysis. As stated earlier in Section 3.4.2.1, a low value of \( S_u \) allows using lower displacements to reach failure and saves computational costs significantly without affecting the results. Values of other parameters for each case are shown in Table 4.9.

<table>
<thead>
<tr>
<th>( k )</th>
<th>( K' )</th>
<th>( S_{u0} = 40 \text{ m} )</th>
<th>( E_s \text{(MPa)} )</th>
<th>( E_s/S_{u0} )</th>
<th>( E_c \text{(GPa)} )</th>
<th>( E_c/E_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.475</td>
<td>40</td>
<td>88.8</td>
<td>2220</td>
<td>250</td>
<td>2815.315</td>
</tr>
<tr>
<td>1.0</td>
<td>0.950</td>
<td>60</td>
<td>133.2</td>
<td>2220</td>
<td>375</td>
<td>2815.315</td>
</tr>
<tr>
<td>1.5</td>
<td>1.425</td>
<td>80</td>
<td>177.6</td>
<td>2220</td>
<td>500</td>
<td>2815.315</td>
</tr>
</tbody>
</table>

Table 4.9: Case III: Details of heterogeneity parameters considered for finite element analysis

The modulus ratio \( (E_c/S_u) \) was kept constant at 2220. \( E_c/E_s \) represents the ratio of Young’s modulus of concrete to that of the soil. A very high value such as that chosen here allows the foundation to behave rigidly in the finite element model.
Uniaxial Bearing Capacity Factors

V, H and M are non-dimensionalized by dividing them by $S_{u0}$ (Gourvenec, 2004) and expressed as bearing capacity factors. The values of $v_0$, $h_0$ and $m_0$ are tabulated below in Table 4.10. Note that the values of uniaxial bearing capacity factors for the previous works shown here are derived by interpolating between suitable values.

<table>
<thead>
<tr>
<th>$K'$</th>
<th>$v_0$</th>
<th>$h_0$</th>
<th>$m_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.08</td>
<td>1.03</td>
<td>0.69</td>
</tr>
<tr>
<td>0.475</td>
<td>6.39</td>
<td>0.99</td>
<td>0.76</td>
</tr>
<tr>
<td>0.950</td>
<td>6.71</td>
<td>1.00</td>
<td>0.78</td>
</tr>
<tr>
<td>1.425</td>
<td>7.35</td>
<td>0.99</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Table 4.10: Case III: Uniaxial bearing capacity factors under varying soil strength heterogeneity conditions

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.08</td>
<td>5.91</td>
<td>6.05</td>
<td>5.87</td>
</tr>
<tr>
<td>0.475</td>
<td>6.39</td>
<td>6.32</td>
<td>6.48</td>
<td>6.24</td>
</tr>
<tr>
<td>0.950</td>
<td>6.71</td>
<td>6.73</td>
<td>6.91</td>
<td>6.61</td>
</tr>
<tr>
<td>1.425</td>
<td>7.35</td>
<td>7.06</td>
<td>7.24</td>
<td>6.97</td>
</tr>
</tbody>
</table>

Table 4.11: Case III: Comparison of $v_0$ for surface foundations on heterogeneous soil with published data
<table>
<thead>
<tr>
<th>$K'$</th>
<th>FEM (this study)</th>
<th>Shen et al. (2016)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.03</td>
<td>1.02</td>
</tr>
<tr>
<td>0.475</td>
<td>0.99</td>
<td>1.02</td>
</tr>
<tr>
<td>0.950</td>
<td>1.00</td>
<td>1.02</td>
</tr>
<tr>
<td>1.425</td>
<td>0.99</td>
<td>1.02</td>
</tr>
</tbody>
</table>

Table 4.12: Case III: Comparison of $h_0$ for surface foundations on heterogeneous soil with published data

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.69</td>
<td>0.69</td>
<td>0.67</td>
<td>0.61</td>
</tr>
<tr>
<td>0.475</td>
<td>0.76</td>
<td>0.76</td>
<td>0.72</td>
<td>0.64</td>
</tr>
<tr>
<td>0.950</td>
<td>0.78</td>
<td>0.82</td>
<td>0.77</td>
<td>0.66</td>
</tr>
<tr>
<td>1.425</td>
<td>0.83</td>
<td>0.88</td>
<td>0.82</td>
<td>0.69</td>
</tr>
</tbody>
</table>

Table 4.13: Case III: Comparison of $m_0$ for surface foundations on heterogeneous soil with published data
Figure 4.42: Case III: Comparison of $v_0$ with published data

Figure 4.43: Case III: Comparison of $h_0$ with published data
As the heterogeneity increases, the uniaxial bearing capacity factors also increase (see Table 4.10). Values of \( v_0 \), \( h_0 \) and \( m_0 \) are compared with published data in Table 4.11, 4.12 and 4.13 respectively. The limit factors are compared with those obtained by Shen et al. (2016) who also considered a no tension contact. However, Shen et al. (2016) used a coefficient of friction, \( \mu = 20 \) (i.e. equivalent friction angle of \( \phi = \tan^{-1} 20 = 87.1^\circ \) very close to 90\(^\circ\)). In the present research, a no-tension interface behaviour with \( \mu = 1 \) which corresponds to \( \phi = \tan^{-1} 1 = 45^\circ \) is considered sufficient to provide accurate results (as evident from the tables shown above). Comparisons are also done with Gourvenec and Randolph (2003) & Houlsby and Wroth (1983) who assumed a fully bonded contact. Their data was found to be accurate and close to the conditions assumed. When compared with Shen et al. (2016), estimated \( v_0 \) values are within 3\% when \( K' = 0.475 \) and 0.95 whereas when \( K' = 1.425 \), \( v_0 \) is higher by 5.4\%. The uniaxial lateral capacity, \( h_0 \) is under-predicted by 1\% when \( K' = 0.475 \) and 1.425 compared to the theoretical solution of \( h_0 = 1 \). When \( K' = 0 \) and 0.95, the exact value of \( h_0 = 1 \) is obtained. The uniaxial moment capacity of the foundation has been compared with the values reported by Shen et al. (2016) and finite element & upper bound plasticity solutions found by Gourvenec and
Randolph (2003). For all values of $K'$, $m_0$ exceeds the upper bound solutions. When $K' = 0.475$, $m_0$ is higher by a maximum of 5.56%, while when $K' = 1.425$, $m_0$ is higher by a minimum of 1.22%. However, these values are in general closer to the upper bound solutions than the finite element solutions.

The higher or lower values of limit capacities estimated from the finite element analysis can be attributed to various reasons. To simulate the combined loading analysis, 3-dimensional analysis has been adopted rather than 2-dimensional or axisymmetric. Unlike Gourvenec and Randolph (2003) and Shen et al. (2016) who adopted semicircular 3-d models, a complete 3-d model is used in the present research to derive the results. Consequently, under pure vertical loading, a large amount of soil undergoes out-of-plane failure. This can lead to higher values than the exact collapse loads. Near the interface immediately below and outside the foundation edge where small elements are used, stress concentration is very high and elements undergo large distortion. The accuracy of numerical integration at Gauss points is reduced as the elements become distorted, as the weighting factors relating to the position of integration points become less valid. Although reduced integration elements with hourglass control were used to minimise the error, the values were still higher compared to upper bound solutions for moment capacities.
Failure Envelopes

Figure 4.45: Case III: Failure envelope in non-dimensional VH plane

Figure 4.46: Case III: Failure envelope in normalized VH plane
Figure 4.47: Case III: Failure envelope in non-dimensional VM plane

Note: For Fig. 4.46, the conventional method plots are shown for $K' = 0, 2$ from left to right. For Fig. 4.47, the plots for Shen et al. (2016) and the conventional method are shown $K' = 0, 2$ and that for the current study are shown for $K' = 0, 0.475, 0.95, 1.425$. 
Figure 4.48: Case III: Failure envelope in normalized VM plane

Figure 4.49: Case III: Failure envelope in non-dimensional HM plane
Fig. 4.45 to 4.47 and Fig. 4.48 to 4.50 show the non-dimensional and normalized failure envelopes respectively. In the VH and VM planes, the envelopes are compared with those given by Shen et al. (2016) using the finite element method and the conventional method. For inclined loads, i.e. in the VH plane, the failure envelope for the traditional method was derived using the original solution given by Green (1954) expressed as

\[ v = 0.5 + 0.5 \sqrt{1 - h} \]  

(4.4)

Whereas for eccentric loads, i.e. in the VM plane, it was derived by using the effective area method (Meyerhof, 1953). Values of K' are close to those that Shen et al. (2016) considered (K'=0, 2) and hence it is straightforward to make comparisons. In the non-dimensional VH plane, the envelopes with K' = 0, 0.475 and 0.95 lie within the envelopes found with the finite element analysis. The envelope with K' = 2 tracks a similar path to the envelope with K' = 1.425, while the envelope derived from the traditional method with K' = 2 becomes unconservative. This shows that simple scaling of envelopes as the heterogeneity increases does not yield
accurate results. When normalized by the ultimate values, all of the envelopes fall in a very tight band with the shape following the solution given by Shen et al. (2016). The shape of the normalized envelope can be described by using a power law relationship (eqn. 4.5) plotted in Fig. 4.51. The size of normalized envelopes given by the conventional method are smaller highlighting the additional capacity it neglects.

\[ v' = (1 - h')^{0.12} \]  

(4.5)

Like the VH envelopes, the non-dimensional VM envelopes also expand in size as the heterogeneity increases. For all of the \( K' \) values, the envelopes are slightly bigger. In the normalized plane, as the heterogeneity increases, the size of the envelopes shrink. Thus, the range of loads which can be considered safe for the foundation decrease as the heterogeneity increases. The shape of the normalized envelopes can be described by using a parabolic equation as given below. Note that Shen et al. (2016) also used the same equation to plot the shape of the
normalized VM envelopes.

\[ m' = 4(v' - v'^2) \]  

(4.6)

In the HM plane, the failure envelopes given by the traditional method are smaller than those derived by the finite element analysis. This is because the traditional method overlooks the effects of coupling of the H-M loading as the heterogeneity changes. The envelopes by Shen et al. (2016) are close to those derived in the present research in both non-dimensional and normalized forms. As the heterogeneity increases, the envelopes are found to shrink in size.

Failure mechanisms for surface foundations with heterogeneous soils under ultimate horizontal and moment loading have very similar patterns of displacement vector as shown for the surface foundation with homogeneous soil (Fig. 4.27, 4.29). Hence they are not shown separately. This indicates that change in heterogeneity does not lead to change in failure mechanisms at least for low values of heterogeneity factor (\( K' \)). Unlike Gourvenec (2004), a pure scoop mechanism under ultimate moment is not observed, since separation between the foundation and soil is allowed. However, as the heterogeneity increases, the lateral displacement accompanying
rotational failure under ultimate moment and rotation accompanying horizontal failure under ultimate horizontal load increases (Table 4.14). Since the change in heterogeneity is not high, the increase in rotation or translation is also small.

<table>
<thead>
<tr>
<th>$K'$</th>
<th>Horizontal displacement during ultimate moment load (mm)</th>
<th>Rotation during ultimate horizontal load (radians)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.475</td>
<td>7.7</td>
<td>0.00035</td>
</tr>
<tr>
<td>0.950</td>
<td>15</td>
<td>0.00039</td>
</tr>
<tr>
<td>1.425</td>
<td>20</td>
<td>0.00046</td>
</tr>
</tbody>
</table>

Table 4.14: Horizontal displacements and rotations during ultimate moment and horizontal load respectively

**Von Mises Stress Distribution**

The Von Mises stress distribution at the end of uniaxial loads for a surface foundation on heterogeneous soil with $K' = 0.475$ is shown from Fig. 4.53 to 4.56. With increase in the heterogeneity, the Von Mises stress also increase for all of the uniaxial ultimate limit states. The average increase in stress is 19%, 12% and 16% for uniaxial ultimate vertical, horizontal and moment loads respectively.
Figure 4.53: Case III: Von Mises stress distribution at zero load, $K' = 0.475$

Figure 4.54: Case III: Von Mises stress distribution at ultimate vertical load, $K' = 0.475$
Figure 4.55: Case III: Von Mises stress distribution at ultimate horizontal load, $K' = 0.475$

Figure 4.56: Case III: Von Mises stress distribution at ultimate moment load, $K' = 0.475$
4.3 Undrained Bearing Capacity of Octagonal Foundations

Similar to circular foundations, the bearing capacities of octagonal foundations subjected to uniaxial and combined loadings are also investigated under undrained conditions. Case IV, V and VI represent surface and embedded foundations with homogeneous soil and surface foundation with heterogeneous soils respectively. The results from the finite element analysis on circular footings act as benchmark and are used to compare the results with the octagonal foundations.

4.3.1 Case IV: Surface Foundation on Homogeneous Soil

Uniaxial bearing capacity factors

The plots to find the values of uniaxial bearing capacity factors are shown below.

![Figure 4.57: Case IV: Load response under vertical and horizontal loading](image)

Figure 4.57: Case IV: Load response under vertical and horizontal loading
Figure 4.58: Case IV: Load response under horizontal loading

Figure 4.59: Case IV: Load response under moment loading
The values of $v_0$, $h_0$ and $m_0$ are tabulated in Table 4.15 below and compared with those obtained from the finite element analysis of circular foundations.

<table>
<thead>
<tr>
<th></th>
<th>$v_0$</th>
<th>$h_0$</th>
<th>$m_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEM (Octagon)</td>
<td>6.55</td>
<td>1.00</td>
<td>0.74</td>
</tr>
<tr>
<td>FEM (Circle)</td>
<td>6.08</td>
<td>1.03</td>
<td>0.69</td>
</tr>
</tbody>
</table>

Table 4.15: Summary of uniaxial bearing capacity factors for octagonal and circular foundations

The finite element analysis of octagonal foundations predicts higher vertical and moment limit capacities compared to that for circular foundations. $v_0$ and $m_0$ exceed by 7.7% and 8.8% respectively. This is because of the increased contact area due to the octagonal shape of the foundation which is 5.5% greater than that of the inscribed circle of the octagon. For lateral uniaxial capacity, an exact value of $h_0 = 1.00$ is achieved, which indicates that ultimate lateral capacity is independent of contact area.
Failure Envelopes

Figure 4.60: Case IV: Failure envelope in the non-dimensional VH plane for an octagonal foundation resting on soil surface
Figure 4.61: Case IV: Failure envelope in the normalized VH plane

Figure 4.62: Case IV: Failure envelope in the non-dimensional VM plane
Figure 4.63: Case IV: Failure envelope in the normalized VM plane

Figure 4.64: Case IV: Failure envelope in the non-dimensional HM plane
Figure 4.65: Case IV: Failure envelope in the normalized HM plane

The non-dimensional and normalized failure envelopes in the VH, VM and HM planes are shown from Fig. 4.60 to 4.65. While the non-dimensionalization enables the investigation of the size of the failure envelopes, normalization by the ultimate values allows study of the shape of the envelopes. For octagonal foundations, the VH and VM failure envelopes in non-dimensional planes are slightly larger than their circular counterparts. In the VH plane, due to the higher predicted $v_0$, the gap between the two paths is high initially, but narrows as the horizontal load starts dominating. In the VM plane, the wide gap between the envelopes is consistently maintained while the moment load increases. This implies that under eccentric loading, the assumption of using an inscribed circle instead of an octagon becomes unconservative. The HM failure envelopes found at $0.5V_{ult}$ load are very close to each other. If the horizontal capacity of the circular footing would not have been over-predicted, then the failure envelope would lie entirely inside of that derived for the octagonal footing. The normalized envelopes have similar shapes in all of the 3 planes. The normalized envelopes of the octagonal footing are, however, slightly smaller than those for the circular footing.
Contact Area

Figure 4.66: Case IV: Comparison with contact area between DNV (2002), FEM (Octagon) and FEM (Circle)

The contact area calculated during loading of ultimate moment is shown for octagonal & circular foundations and compared with that predicted by DNV (2002). The area calculated by the finite element analysis for a circular footing is slightly less than that for a octagonal footing. Clearly, DNV (2002) underestimates the effective area throughout the loading process. As the eccentric loading increases, the area given by the DNV method declines much more sharply compared to that given by the finite element method. Thus, under moment loading, traditional methods predict much of the foundation will lift off and lose contact with the soil. This results in under-prediction of the moment capacity. The contact areas are also compared for three values of eccentricity in Fig. 4.67. Area shown in red and green was predicted by using finite element analysis, while the elliptical shaped area with black outline and shaded with white stripes is estimated by using the DNV method.
Failure Mechanisms

The displacement vectors representing failure mechanisms under ultimate horizontal and moment loadings are similar for circular and octagonal footings (Fig. 4.27, 4.68 and 4.29, 4.69). This shows that changes in the shape of the footing do not affect the failure mechanisms under pure horizontal and moment loads.

Figure 4.67: Case IV: Comparison with contact area between DNV (2002) and FEM (Octagon)
The distribution and magnitudes of the Von Mises stress for octagonal footing are very similar to that of the circular footing as shown in Fig. 4.32, 4.33 and 4.34. However, the magnitudes and distribution of plastic strain under ultimate load states is quite different (Fig. 4.70 to 4.72). Under limit vertical load, the plastic strain is distributed over a wider area and is relatively shallow. The magnitudes of plastic strain are slightly higher (from 0.06% to 2.4%) than that for
the circular foundations in all the cases.

Figure 4.70: Case IV: Plastic strain at ultimate vertical load for surface octagonal foundations

Figure 4.71: Case IV: Plastic strain at ultimate horizontal load
Figure 4.72: Case IV: Plastic strain at ultimate moment load
4.3.2 Case V: Embedded Foundations in Homogeneous Soil

As shown in Table 4.4, like circular foundations, the octagonal foundations were also embedded at 4 different depths. The uniaxial limit capacities and failure envelopes in the 2-d planes are presented below.

*Uniaxial Bearing Capacity Factors*

The uniaxial bearing capacity factors expressed as dimensionless loads are given in Table 4.16.

<table>
<thead>
<tr>
<th>$d/D$</th>
<th>$v_0$</th>
<th>$h_0$</th>
<th>$m_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.55</td>
<td>1.00</td>
<td>0.74</td>
</tr>
<tr>
<td>0.157</td>
<td>7.60</td>
<td>1.93</td>
<td>1.10</td>
</tr>
<tr>
<td>0.32</td>
<td>8.60</td>
<td>3.25</td>
<td>1.46</td>
</tr>
<tr>
<td>0.5</td>
<td>9.45</td>
<td>3.75</td>
<td>1.75</td>
</tr>
<tr>
<td>1.0</td>
<td>11.37</td>
<td>4.92</td>
<td>2.20</td>
</tr>
</tbody>
</table>

Table 4.16: Case V: Uniaxial bearing capacity factors for embedded octagonal foundations

Comparison of $v_0$ with that of Gourvenec and Mana (2011) and finite element analysis on circular foundations is given below (Table 4.17). Solutions of $h_0$ are compared with results from the FEM study on circular foundations in this research since no published data is available. For $m_0$, upper bound solutions on strip foundations given by Yun and Bransby (2007) and results from circular foundations in this study are used.
Table 4.17: Case V: Comparison of $v_0$ with published data

<table>
<thead>
<tr>
<th>$d/D$</th>
<th>FEM (Octagon)</th>
<th>Gourvenec (2011)</th>
<th>FEM (Circle)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.55</td>
<td>5.95</td>
<td>6.08</td>
</tr>
<tr>
<td>0.157</td>
<td>7.60</td>
<td>7.28</td>
<td>7.42</td>
</tr>
<tr>
<td>0.32</td>
<td>8.60</td>
<td>8.25</td>
<td>8.48</td>
</tr>
<tr>
<td>0.50</td>
<td>9.45</td>
<td>9.11</td>
<td>9.51</td>
</tr>
<tr>
<td>1.0</td>
<td>11.37</td>
<td>11.18</td>
<td>11.40</td>
</tr>
</tbody>
</table>

Table 4.18: Case V: Comparison of $h_0$ with published data

<table>
<thead>
<tr>
<th>$d/D$</th>
<th>FEM (Octagon)</th>
<th>FEM (Circle)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00</td>
<td>1.03</td>
</tr>
<tr>
<td>0.157</td>
<td>1.93</td>
<td>1.97</td>
</tr>
<tr>
<td>0.32</td>
<td>3.25</td>
<td>3.28</td>
</tr>
<tr>
<td>0.5</td>
<td>3.75</td>
<td>3.78</td>
</tr>
<tr>
<td>1.0</td>
<td>4.92</td>
<td>5.03</td>
</tr>
</tbody>
</table>

Table 4.19: Case V: Comparison of $m_0$ with published data

<table>
<thead>
<tr>
<th>$d/D$</th>
<th>FEM (Octagon)</th>
<th>FEM (Circle)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.74</td>
<td>0.69</td>
</tr>
<tr>
<td>0.157</td>
<td>1.10</td>
<td>0.90</td>
</tr>
<tr>
<td>0.32</td>
<td>1.46</td>
<td>1.36</td>
</tr>
<tr>
<td>0.5</td>
<td>1.75</td>
<td>1.69</td>
</tr>
<tr>
<td>1.0</td>
<td>2.20</td>
<td>2.00</td>
</tr>
</tbody>
</table>
Figure 4.73: Case V: Comparison of $v_0$ with published data

Figure 4.74: Case V: Comparison of $h_0$ with published data
Figure 4.75: Case V: Comparison of $m_0$ with published data

For $d/D = 0.157$ and $0.32$, $v_0$ is about 3% and 2% higher than the FEM (Circle) analysis respectively. When embedment ratio increases to 0.5 and 1, $v_0$ is lower than the FEM (Circle) results by 1% and 0.5% respectively. The results for the octagonal foundations were obtained using the same 3-dimensional mesh density as that for the circular foundations. This can cause some error since more stress concentration occurs at the edges of the octagon resulting in more distortion of elements which decreases the accuracy of the solution. When compared with Gourvenec and Mana (2011), all values of $v_0$ are higher. However, it must be noted that they studied circular foundations. A quadratic relationship between $v_0$ and $d/D$ can be proposed, which can be expressed as

$$v_0 = [1 + 1.2d/D - 0.7(d/D)^2] \times 6.55$$ \hspace{1cm} (4.7)

This expression is plotted and shown in Fig. 4.73. The lateral limit capacities are slightly lower (between 0.8% to 3%) than those for the FEM (Circle) for all of the cases. The $h_0$ values can also
be represented by a quadratic relationship with $d/D$ as given below and plotted in Fig. 4.74.

$$h_0 = 0.96 + 7.63(d/D) - 3.69(d/D)^2 \quad (4.8)$$

The maximum variation in values between octagonal and circular foundations is found in the case of uniaxial limit moment values. At $d/D = 0.5$, $m_0$ is 3.55% higher while at $d/D = 0.157$, it is 22.22% higher than FEM (Circle) value. For the most deeply embedded foundation, i.e. $d/D = 1$, this variation is 9.5%. The average increase in $m_0$ is 10.28%. This indicates that the change in shape of the foundation from a circle to octagon significantly increases its moment bearing capacity. Especially when the foundation is shallowly embedded, this effect is more pronounced than when it is deeply embedded. Thus, it can be said that under eccentric loading the conventional approach of assuming an octagon equivalent to a circle gives unconservative results. The closed form expression, which can be used to relate $m_0$ with $d/D$ is

$$m_0 = 0.73 + 2.601(d/D) - 1.13(d/D)^2 \quad (4.9)$$
Failure Envelopes

Figure 4.76: Case V: Failure envelope in the non-dimensional VH plane

Figure 4.77: Case V: Failure envelope in the normalized VH plane
Figure 4.78: Case V: Failure envelope in the non-dimensional VM plane

Figure 4.79: Case V: Failure envelope in the normalized VM plane
Figure 4.80: Case V: Failure envelope in the non-dimensional HM plane

Figure 4.81: Case V: Failure envelope in the normalized HM plane
Failure Mechanisms

Failure mechanisms in the form of displacement vectors in the XZ plane are shown in Fig. 4.82 and 4.83 for ultimate horizontal and moment capacity for a foundation with $d/D = 0.5$ respectively. The mechanisms have similar patterns of displacement vectors as the circular footing of the same embedment ratio.

Figure 4.82: Case V: Displacement vectors under ultimate horizontal load, $d/D = 0.5$

Figure 4.83: Case V: Displacement vectors under ultimate moment load, $d/D = 0.5$
Von Mises Stress Distribution

The Von Mises stress distribution is shown from Fig. 4.84 to 4.86.

Figure 4.84: Case V: Von Mises stress distribution at ultimate vertical load, $d/D = 0.5$

Figure 4.85: Case V: Von Mises stress distribution at ultimate horizontal load, $d/D = 0.5$
Figure 4.86: Case V: Von Mises stress distribution at ultimate moment load, $d/D = 0.5$

**Plastic Strain**

The plastic strain distribution is shown from Fig. 4.87 to 4.89. The plastic strain magnitudes between circular and octagonal foundations for $d/D = 0.5$ vary within 1% of each other.

Figure 4.87: Case V: Plastic strain at ultimate vertical load, $d/D = 0.5$
Figure 4.88: Case V: Plastic strain at ultimate horizontal load, \(d/D = 0.5\)

Figure 4.89: Case V: Plastic strain at ultimate moment load, \(d/D = 0.5\)

The VH failure envelope in non-dimensional and normalized form is shown in Fig. 4.76 and 4.77 respectively. For each embedment ratio, the envelopes for circular and octagonal footings lie very close to each other with the envelope for an octagonal footing being slightly larger. As the embedment ratio increases, the gap between the two failure envelopes decreases being smallest at \(d/D = 1\). In normalized form, the envelopes lie in a very tight band. Like the VH
envelope, VM envelopes for the octagonal footing are also slightly larger in size than that for the circular footing (Fig. 4.78), signifying the extra capacity available due to the increased area of the octagon. In normalized form (Fig. 4.79), the yield surfaces lie very close to each other. Except the envelope for a surface foundation ($d/D = 0$), the failure envelope size shrinks as the embedment ratio increases for both circular and octagonal foundations. The HM envelopes for octagonal footings lie in a tight band with those of circular footings, both in non-dimensional and normalized form (Fig. 4.80 and 4.81).
4.3.3 Case VI: Surface Foundations on Heterogeneous Soil

Similar to surface circular foundations (Section 4.2.3), this case investigates the effects of soil strength heterogeneity on the bearing capacity of octagonal surface foundations on clays under undrained conditions subjected to combined loading.

A value of $S_{u0}$ equal to 20 kPa is assumed for the present analysis. The values of $k$, $K'$ and $S_u$ (in kPa) are shown in Table 4.20.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$K'$</th>
<th>$S_{u_{z=40m}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.475</td>
<td>40</td>
</tr>
<tr>
<td>1.0</td>
<td>0.950</td>
<td>60</td>
</tr>
<tr>
<td>1.5</td>
<td>1.425</td>
<td>80</td>
</tr>
</tbody>
</table>

Table 4.20: Case VI: Details of heterogeneity parameters considered for finite element analysis

The variation of overconsolidation ratio (OCR), undrained shear strength $S_u$ and the ratio of undrained shear strength to effective vertical stress, $\zeta$ with depth is shown for various values of $k$ in Fig. 4.90, 4.91 and 4.92 below. In Fig. 4.90, values of OCR are compared with oedometer derived data obtained from Tyldesley et al. (2013).
Figure 4.90: Case VI: Variation of OCR with depth

Figure 4.91: Case VI: Variation of $S_u$ with depth
Uniaxial Bearing Capacity Factors

A summary of uniaxial bearing capacity factors is given in Table 4.21. Further, Table 4.21, 4.22 and 4.23 compare values of $v_0$, $h_0$ and $m_0$ with that obtained for circular foundations under the same conditions. While $K'$ represents soil strength heterogeneity ratio, $k$ refers to rate of increase of shear strength expressed in kPa/m.

<table>
<thead>
<tr>
<th>$K'$</th>
<th>$k$</th>
<th>$v_0$</th>
<th>$h_0$</th>
<th>$m_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>6.55</td>
<td>1.00</td>
<td>0.74</td>
</tr>
<tr>
<td>0.475</td>
<td>0.5</td>
<td>7.05</td>
<td>0.99</td>
<td>0.80</td>
</tr>
<tr>
<td>0.950</td>
<td>1.0</td>
<td>7.70</td>
<td>0.99</td>
<td>0.86</td>
</tr>
<tr>
<td>1.425</td>
<td>1.5</td>
<td>8.05</td>
<td>0.99</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Table 4.21: Case VI: Summary of uniaxial bearing capacity factors for surface octagonal foundation on heterogeneous soils
On average, the $v_0$ values for octagonal footings are 10.28% higher. This increase is much higher than that obtained between embedded octagonal and circular foundations on homogeneous soil. This indicates that the change in shape has more effect on $v_0$ for a surface foundation on
heterogeneous soil than that for an embedded foundation in homogeneous soil. The variation of $v_0$ with $K'$ can be defined by a linear expression.

\[ v_0 = 6.565 + 1.08K' \]  \hspace{1cm} (4.10)

When compared with the theoretical solution $h_0 = 1$, the uniaxial lateral capacity is slightly under-predicted for all of the 3 cases by 1%. Whereas for $m_0$, an average increase of 7.80% is observed. $m_0$ values were also compared with those for strip footings published by Gourvenec and Randolph (2003). These two sets of values are very close (Fig. 4.94). The increase of $m_0$ with $K'$ can be represented with a linear relationship which can be expressed as

\[ m_0 = 0.744 + 0.11K' \]  \hspace{1cm} (4.11)
It must be noted here that the linear relationships for $v_0$ and $m_0$ are not tested for $K' > 1.5$ and must be used with caution.
Failure Envelopes

Figure 4.95: Case VI: Failure envelope in the non-dimensional VH plane

Figure 4.96: Case VI: Failure envelope in the normalized VH plane
Figure 4.97: Case VI: Failure envelope in the non-dimensional VM plane

Figure 4.98: Case VI: Failure envelope in the normalized VM plane
Figure 4.99: Case VI: Failure envelope in the non-dimensional HM plane

Figure 4.100: Case VI: Failure envelope in the normalized HM plane
The non-dimensional VH envelopes (Fig. 4.95) of octagonal footings are similar in shape but slightly bigger than their circular counterparts. This signifies the extra capacity available due to the octagonal shape when inclined loads are acting on foundations. Unlike embedded foundations, where the value of \( h_0 \) keeps increasing with increasing embedment, the lateral limit capacity remains at unity irrespective of the soil strength heterogeneity. However, \( v_0 \) values increase with increase in heterogeneity, which leads to VH envelopes expanding laterally (in the +X direction). Indeed the critical angle of inclination with the vertical (when horizontal load starts dominating) decreases as the heterogeneity increases (Table 4.25).

<table>
<thead>
<tr>
<th>( K' )</th>
<th>Critical angle of inclination (°)</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>13.27</td>
</tr>
<tr>
<td>0.475</td>
<td>12.34</td>
</tr>
<tr>
<td>0.950</td>
<td>11.18</td>
</tr>
<tr>
<td>1.425</td>
<td>10.78</td>
</tr>
</tbody>
</table>

Table 4.25: Case VI: Critical angle of inclination for varying soil strength heterogeneity

Note that the critical angle of inclination for an octagonal footing (13.27°) when \( K' = 0 \) is less than that for a circular footing (15.3°).

The normalized VH envelopes (Fig. 4.74) of an octagonal footing are smaller in size than that for a circular foundation and fall in a very tight band. The shape of the normalized envelope can be described by a power law relationship given by

\[
v' = (1 - h')^{0.12}
\]

Like the VH envelopes, the VM envelopes in the non-dimensional plane (Fig. 4.75) are also bigger than that for circular footings and have a similar shape. The increase in heterogeneity leads to bigger sizes of envelopes. However, in normalized form (Fig. 4.76), the envelopes of an octagonal footing are smaller than those for a circular footing and are very close to each other.
As the heterogeneity increases, the size of the envelope keeps shrinking. The normalized vertical and moment forces can be related by a power law expression which can be expressed as

\[ v' = (1 - m')^{0.22} \]  \hspace{1cm} (4.13)

Unlike Gourvenec and Randolph (2003), envelopes in the HM plane have similar shape as the heterogeneity changes (Fig. 4.77 and 4.78). As pointed out in Subsection 4.2.3, this is due to the different contact conditions assumed. While they considered a fully bonded contact, contact between the foundation and soil is separable in the present analysis. Hence the extra capacity beyond ultimate moment value is not mobilized. In normalized form, the envelopes of octagonal and circular footings are generally close to each other and shrink as the heterogeneity increases.

Failure mechanisms for octagonal footings are similar to those for circular footings and hence, are not shown specifically. Thus, the failure mechanisms are independent of the shape of the footing. The contours for Von Mises stress distribution also have the same patterns as that for circular footings.
4.4 Special Cases

4.4.1 Case VII: Circular foundation with Surficial Crust Layer

A circular foundation with a surficial crust overlying a soft clayey soil with linearly increasing strength was also considered. The foundation was embedded 3 m below ground surface (i.e. $d/D = 0.157$). A parametric study was conducted to study the effects of surficial crust on the bearing capacities of the foundation. To achieve this, four parameters related to surficial crust were varied: crust strength ($S_{ut}$), crust thickness ($t_c$), shear strength at the soil surface ($S_{u0}$) and rate of increase of shear strength ($k$). Additionally, the depth of embedment of the foundation ($d$) was also varied. The details of various cases considered are reiterated in Fig. 4.101. The crust strength ($S_{ut}$) is increased in two stages: 75% the first time and around 43% the second time, keeping the strength of underlying layer ($S_{ubs}$) constant. To study the effect of crust thickness, it is increased from 2.5 m to 5 m and then to 7 m. The effect of the strength of the underlying layer is studied by varying the rate of increase of shear strength ($k$) and the shear strength at the top surface ($S_{u0}$) as shown in Fig. 4.101(c). A case for soil with uniform strength throughout i.e. $k = 0$ was also considered. Finally, the foundation is embedded at 3 m and 6 m to study the effect of embedments, during which the crust thickness is kept constant at 7 m. The results of the parametric study are given below in the form of uniaxial limit capacities and failure envelopes for each case.
Figure 4.101: Parameters chosen for the study

(a) Average crust strength, $S_{ul}$

(b) Crust thickness, $t_c$

(c) Rate of strength increase, $k$

(d) Relative embedment
**Uniaxial Bearing Capacity Factors**

<table>
<thead>
<tr>
<th>$S_{ut}$</th>
<th>$S_{ubs}/S_{ut}$</th>
<th>$v_0$</th>
<th>$h_0$</th>
<th>$m_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.58</td>
<td>6.45</td>
<td>1.64</td>
<td>0.72</td>
</tr>
<tr>
<td>175</td>
<td>0.33</td>
<td>5.20</td>
<td>1.61</td>
<td>0.70</td>
</tr>
<tr>
<td>250</td>
<td>0.23</td>
<td>4.15</td>
<td>1.60</td>
<td>0.54</td>
</tr>
</tbody>
</table>

(a) Varying average crust strength

<table>
<thead>
<tr>
<th>$t_c$</th>
<th>$t_c/D$</th>
<th>$v_0$</th>
<th>$h_0$</th>
<th>$m_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>0.13</td>
<td>3.10</td>
<td>0.80</td>
<td>0.34</td>
</tr>
<tr>
<td>5</td>
<td>0.26</td>
<td>4.10</td>
<td>1.53</td>
<td>0.48</td>
</tr>
<tr>
<td>7</td>
<td>0.37</td>
<td>5.20</td>
<td>1.61</td>
<td>0.70</td>
</tr>
</tbody>
</table>

(b) Varying crust thickness

<table>
<thead>
<tr>
<th>$S_{u0}$</th>
<th>$k$</th>
<th>$v_0$</th>
<th>$h_0$</th>
<th>$m_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.20</td>
<td>3.74</td>
<td>1.60</td>
<td>0.51</td>
</tr>
<tr>
<td>30</td>
<td>1.60</td>
<td>4.25</td>
<td>1.60</td>
<td>0.55</td>
</tr>
<tr>
<td>50</td>
<td>1.14</td>
<td>5.20</td>
<td>1.61</td>
<td>0.70</td>
</tr>
<tr>
<td>60</td>
<td>0.00</td>
<td>4.03</td>
<td>1.61</td>
<td>0.59</td>
</tr>
</tbody>
</table>

(c) Varying $S_u$ increase with depth of underlying layer

<table>
<thead>
<tr>
<th>$d$</th>
<th>$d/t_c$</th>
<th>$v_0$</th>
<th>$h_0$</th>
<th>$m_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.43</td>
<td>5.20</td>
<td>1.61</td>
<td>0.70</td>
</tr>
<tr>
<td>6</td>
<td>0.86</td>
<td>4.64</td>
<td>1.85</td>
<td>0.50</td>
</tr>
</tbody>
</table>

(d) Varying relative embedments

Table 4.26: Uniaxial bearing capacity factors for parametric study

Table 4.26 shows the uniaxial bearing capacity factors for the parametric study. With increase in the average crust strength, the uniaxial limit capacities generally tend to decrease (Table 4.26a). The decrease in uniaxial capacity is highest (55.42%) for the case of $v_0$ when $S_{ut}$ changes from 100 kPa to 250 kPa. The value of $h_0$ changes very slightly and remains relatively constant around 1.60. The moment capacity decreases by around 30% when $S_{ut}$ changes from 100 kPa to 175 kPa. However, it decreases very slightly when $S_{ut}$ increases further to 250 kPa. Thus, when a very strong crust is present over a layer with lower shear strength, i.e. the gap between shear strengths of the crust and the underlying layer is high, the uniaxial capacities are usually low. The relationship between $v_0$ and $S_{ut}/S_{ubs}$ can be expressed exponentially by eqn. 4.14 and is shown in Fig. 4.102.

$$v_0 = 8.7e^{-0.171 \frac{S_{ut}}{S_{ubs}}}$$

(4.14)

Feng et al. (2015) related the uniaxial vertical and moment capacities of a rectangular mudmat
with the ratio \( S_{ut}/S_{ubs} \) by a relationship given as

\[
v_0 = a_1 \left( \frac{S_{ut}}{S_{ubs}} \right)^{a_2} \tag{4.15}
\]

\[
m_0 = b_1 \left( \frac{S_{ut}}{S_{ubs}} \right)^{b_2} \tag{4.16}
\]

\( a_1, a_2, b_1, b_2 \) are coefficients which depend on the crust thickness, embedment depth and width of the mudmat.

Through finite element analyses, Merifield et al. (1999) provided rigorous upper and lower bounds for undrained bearing capacity of a strip footing resting on a two-layer clay deposit. The upper and lower bound solutions are also shown in Fig. 4.102. The values of \( v_0 \) for the circular foundation with a crustal layer are much higher than the upper bound solutions for a strip footing. However, the rate of decrease of \( v_0 \) with \( S_{ut}/S_{ubs} \) shows similar patterns in both the cases.

With increase in the crust thickness, all of the 3 factors increase. The \( h_0 \) and \( m_0 \) almost doubles.
when $t_c/D$ changes from 0.13 to 0.37. The increase of $v_0$ can be expressed by an exponential relationship given by:

$$v_0 = 2.3e^{2.18t_c/D}$$

(4.17)

The closed-form expression is plotted in Fig. 4.103 along with values obtained from the current study.

![Figure 4.103](image)

Figure 4.103: Vertical bearing capacity factors for foundations with varying crust thickness

The uniaxial lateral capacity remains relatively unchanged with change in the shear strength of the layer underlying the crust. Whereas, with linear increase in the rate of shear strength with depth, the foundations gain ultimate vertical and moment bearing capacity. Note that a case of uniform $S_{u0} = 60$ kPa throughout the soil depth was also considered. When compared with $S_{u0} = 30$ kPa with $k = 1.60$, $v_0$ are underestimated with uniform $S_{u0} = 60$ kPa. Thus, the assumption of a single layer with uniform shear strength and its value must be chosen carefully for bearing capacity analysis. Merifield et al. (1999) obtained a number of failure mechanisms for a strip footing that are functions of crust thickness and the ratio $S_{ut}/S_{ubs}$. They concluded that existing upper bound and semi-empirical solutions that are based on a single assumed
failure surface cannot model the likely failure mode over a large range of footing geometries.

When the depth of embedment increases to 6 m from 3 m, the uniaxial vertical and moment capacities decrease. This can be attributed to the increased proximity of the foundation to the weaker clayey layer ($t_c = 7$ m). It is interesting to note that the lateral capacity, however, increases by 15%. This is probably due to the extra soil above the foundation surface (when $d = 6$ m) that mobilizes the extra moment capacity.

*Failure Envelopes*

Failure envelopes in non-dimensional planes are shown from Fig. 4.104 to 4.106. As the average crust strength ($S_{ut}$) increases from 100 kPa to 250 kPa, the failure envelopes contracts in all of the 3 planes. Note that the thickness of the crust, $t_c = 7$ m and the value of $S_{ubs} \approx 58$ kPa with $k = 1.14$. As the value of shear strength of the crust diverges from that of the underlying layer, the failure envelopes become smaller. For a constant crust strength ($S_{ut} = 175$ kPa), the failure envelopes decrease in size as the thickness of the crust decreases. The failure envelopes expand as the value of $k$ increases from 1.14 to 3.20. In the VH and VM planes, the shapes of the envelopes are identical. Whereas in the HM plane, due to variable ultimate moment capacities, the envelope shapes are different and they increase in size as the moment load increases. When the foundation gets embedded nearer to the weaker underlying layer (i.e. $d/t_c = 0.86$), the vertical and moment limit capacities decrease. As a result, the failure envelopes in the VH and VM planes are smaller in size compared to the case when $d/t_c = 0.43$. However, in the HM plane, the envelope becomes larger when the foundation gets embedded more deeply. This is due to the increased lateral resistance.
Figure 4.104: VH failure envelopes in the non-dimensional plane
Figure 4.105: VM failure envelopes in the non-dimensional plane
Figure 4.106: HM failure envelopes in the non-dimensional plane
Figure 4.107: VH failure envelopes in normalized plane
Figure 4.108: VM failure envelopes in normalized plane
Figure 4.109: HM failure envelopes in normalized plane
Failure Mechanisms

The failure mechanisms for the different cases are shown from the Fig. 4.110 to 4.113. As the strength of the crust increases from 100 kPa to 250 kPa or the ratio $S_{ut}/S_{ubs}$ increases from 1.72 to 4.31, the failure mechanisms become shallower. When the crust strength is low, more soil in the underlying layer is involved in the failure. However, as the crust strength increases, most of the bearing resistance comes from the stronger crust layer. Thus, the crust layer plays a major role in providing bearing resistance as the difference in strengths between the two layers increases. As a result, when $S_{ut} = 250$ kPa, the failure mechanism during ultimate horizontal load is close to a pure sliding mechanism and that during ultimate moment load is close to a scoop mechanism. Thus, the centre of rotation becomes closer to the foundation base when $S_{ut}$ increases. Similarly, when the thickness of the crust layer decreases from 7 m to 2.5 m, the failure mechanisms for ultimate horizontal and moment loads move closer to the soil surface and become shallower. For the case when rate of increase of shear strength ($k$) changes, failure mechanisms do no differ markedly with each other. When the foundation is embedded deeper at 6 m, it come closer to the weaker underlying layer. Due to this, more soil in the underlying layer takes part in the bearing failure. During the ultimate horizontal failure, the backfilled soil placed above the foundation undergoes rotation. During the ultimate moment failure, the horizontal displacement increases from 25 mm to 36 mm.
Figure 4.110: Failure mechanisms under ultimate horizontal and moment loads for varying crust strengths ($S_{ut}$)

(a) $S_{ut} = 100$ KPa

(b) $S_{ut} = 250$ KPa

Ultimate horizontal load

Ultimate moment load
Figure 4.111: Failure mechanisms under ultimate horizontal and moment loads for varying crust thickness ($t_c$)
Figure 4.112: Failure mechanisms under ultimate horizontal and moment loads for varying rate of increase of shear strength ($k$)
Figure 4.113: Failure mechanisms under ultimate horizontal and moment loads for varying depth of embedment ($d$)
Plastic Strains

Plastic strain PEEQ is shown from Fig. 4.114 to 4.117 for different cases of parametric study. PEEQ is the conjugate strain of Von Mises stress as mentioned before. With increase in the crust strength, the maximum value of plastic strain decreases by 47.4% and 40% for ultimate horizontal and moment loads respectively. A similar observation is noted when thickness of the crust increases from 2.5 m to 7 m. However, the percentage decrease is relatively less; 8% and 27% for ultimate horizontal and moment loads respectively. For the case when $k = 0$ and 3.20kPa/m, the variations in strains are within 10% of each other. The maximum variations in strain is found for the case of varying embedments where it become almost double when the depth of embedement changes from 3 m to 6 m.
Figure 4.114: Plastic strain under ultimate horizontal and moment loads for varying crust strengths ($S_{ut}$)
Figure 4.115: Plastic strain under ultimate horizontal and moment loads for varying crust thickness ($t_c$)

(a) $t_c = 2.5$ m
(b) $t_c = 7$ m
Figure 4.116: Plastic strain under ultimate horizontal and moment loads for varying rate of increase of shear strength ($k$)

(a) $S_{u0} = 0$ KPa, $k = 3.20$ KPa/m

(b) $S_{u0} = 60$ KPa, $k = 0$ KPa/m

Ultimate horizontal load

Ultimate moment load
Figure 4.117: Plastic strain under ultimate horizontal and moment loads for varying depth of embedment ($d$)
4.4.2 Case IX: Foundations with Reduced Size

The final case is an examination of reducing the foundation size. Since the primary design criteria is likely to be the rocking stiffness (typically recommended by the manufacturer), the bearing capacity may exceed typical factors of safety. It is therefore an interesting hypothetical exercise to investigate how the bearing capacity may be optimized. To investigate this, a circular foundation with a diameter $D = 19$ m, equal to that of the inscribed circle of an octagonal foundation is considered. The size of the circular foundation is reduced keeping the aspect ratio the same as that of the original foundation, to check if it can sustain design loads for conditions herein described as normal operating conditions and conditions with maximum overturning moment. The size of foundation has been reduced in two stages (from 100%); first to 75% and second to 50% of the dimensions of the original foundation. Each foundation is subjected to uniaxial limit and combined loads to obtain uniaxial limit capacities and 2-dimensional failure envelopes respectively. To gain more insight into the behaviour of embedded foundations under inclined and eccentric loading, HM envelopes are found at different proportions of ultimate vertical load ($V_{ult}$) viz. 0.1, 0.25 and 0.75 besides 0.5. Additionally, for each of these foundation cases, HM failure envelope is found at a vertical force equal to the sum of the design vertical load and weight of the foundation and the backfilled soil. The weight of the backfilled soil is assumed to be 17.5 kN/m$^3$. This helps to ascertain whether the foundations loaded to the design vertical load can sustain the specified horizontal and moment loads under various conditions.

<table>
<thead>
<tr>
<th>Relative size</th>
<th>$D$ (m)</th>
<th>$d$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original foundation (A)</td>
<td>100%</td>
<td>19.00</td>
</tr>
<tr>
<td>Reduced foundation (B)</td>
<td>75%</td>
<td>14.25</td>
</tr>
<tr>
<td>Reduced foundation (C)</td>
<td>50%</td>
<td>9.50</td>
</tr>
</tbody>
</table>

Table 4.27: Case IX: Dimensions of the original and reduced size foundations

Soil with a uniform $S_u = 75$ kPa (which becomes 60 kPa when reduced by the partial safety factor, $\gamma_m = 1.25$) is assumed. All of the foundations are assumed to have proportional
embedment ratio (i.e. \(d/D = 0.157\)). In addition, foundations resting on the soil surface are also modelled for each case.

<table>
<thead>
<tr>
<th>Foundation loads</th>
<th>(d/D)</th>
<th>(V) (kN)</th>
<th>(H) (kN)</th>
<th>(M) (kN m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal operating conditions</td>
<td>0.157</td>
<td>2900</td>
<td>900</td>
<td>42000</td>
</tr>
<tr>
<td>Highest overturning moment</td>
<td>2900</td>
<td>1100</td>
<td>76200</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.28: Load acting on foundation under various operating conditions

The maximum ‘working’ moment for normal operating conditions has been assumed from moment versus wind speed data for typical operating periods (Fig. 4.118).

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(d/D)</td>
<td>(V_{ult}) (kN)</td>
<td>(H_{ult}) (kN)</td>
<td>(M_{ult}) (kN m)</td>
</tr>
<tr>
<td>Original foundation (A)</td>
<td>0</td>
<td>103523</td>
<td>16916</td>
<td>221838</td>
</tr>
<tr>
<td>Reduced foundation (B)</td>
<td>0</td>
<td>58374</td>
<td>9468</td>
<td>93716</td>
</tr>
<tr>
<td>Reduced foundation (C)</td>
<td>0</td>
<td>25657</td>
<td>4237</td>
<td>27943</td>
</tr>
<tr>
<td>Original foundation (A)</td>
<td>0.157</td>
<td>126445</td>
<td>28565</td>
<td>290320</td>
</tr>
<tr>
<td>Reduced foundation (B)</td>
<td>0.157</td>
<td>71230</td>
<td>15720</td>
<td>120095</td>
</tr>
<tr>
<td>Reduced foundation (C)</td>
<td>0</td>
<td>31751</td>
<td>6890</td>
<td>35433</td>
</tr>
</tbody>
</table>

Table 4.29: Uniaxial ultimate load values for the original and reduced size foundations
Note that the ultimate load values given in Table 4.29 have been increased by an appropriate partial safety factor $\gamma_m = 1.25$ (DNV, 2014). The design and maximum operating loads under normal operating conditions and conditions with maximum overturning moment are shown in the plot for VH and VM failure envelopes in Fig. 4.119 for the case of surface and embedded foundations. Failure envelopes are shown in dimensional form which allows the comparison of absolute values of various combinations of vertical, horizontal and moment loads which the foundations can resist safely. In the plot, normal operating conditions and conditions with maximum overturning moments are referred as “Normal cond.” and “Extreme cond.” respectively.

It can be easily seen from Fig. 4.119 (a) and (c) that the vertical and horizontal design loads are safely resisted by all of the three foundations. However, it is the value of overturning moment that decides the optimized foundation dimensions. HM failure envelopes for different proportions of $V_{ult}$ are shown in non-dimensional and dimensional forms in Fig. 4.120 and 4.121 respectively for the embedded foundations ($d/D = 0.157$). For the foundations resting on the soil surface, HM envelopes are calculated only at $0.5V_{ult}$ and are shown in non-dimensional and dimensional forms in Fig. 4.122 and 4.123 respectively.
Failure envelopes in the HM plane expand as the vertical load increases, up to a maximum value of $V = 0.5V_{ult}$. Beyond this value, the envelopes start shrinking as is evident from the envelope for $V = 0.75V_{ult}$. This implies that for a given value of horizontal force and moment, vertical load acting on a foundation can safely be increased to $0.5V_{ult}$. If vertical load is increased beyond this limit, then it would lead to a decrease in the factor of safety. The total vertical load acting on the embedded foundations which includes the design vertical load and the weight of the foundation and the backfilled soil, is represented by $0.15, 0.27$ and $0.61V_{ult}$ for case A, B and C respectively. Fig. 4.121 indicates that a foundation with reduced dimensions of $D = 14.25$ m and $t = 2.25$ m placed at 3 m can safely resist the design loads under both the conditions.

For the original size of the foundation ($D = 19$ m and $d = 3$ m) considered in this research, the ultimate moment load is 290 MN m which is almost four times of that under "Extreme cond.". Thus, in terms of bearing capacity the same foundation could potentially be used for a wind turbine with a higher rated power. For example, for a 5 MW wind turbine, the maximum overturning moment is around $250 \sim 300$ MN m. In addition, the response of the original foundation under these ‘working’ and ‘extreme’ loading conditions is shown in Appendix D.
Figure 4.119: VH and VM failure envelopes in dimensional form for surface and embedded foundations, $d/D = 0, 0.157$
Figure 4.120: HM failure envelopes for embedded foundations in non-dimensional form, \( d/D = 0.157 \)
Figure 4.121: HM failure envelopes for embedded foundations in dimensional form, $d/D = 0.157$
Figure 4.122: HM failure envelopes for surface based foundations in non-dimensional form

Figure 4.123: HM failure envelopes for surface based foundations in dimensional form
4.5 Summary

This chapter presented and discussed the results of finite element analysis on circular and octagonal foundations under undrained conditions. Foundations were subjected to combined loading by using swipe tests. Broadly, the bearing capacity response of foundations was obtained for three different scenarios: 1. Surface foundations on homogeneous soils 2. Embedded foundations in homogeneous soils, and 3. Surface foundations on heterogeneous soils. The undrained bearing capacities of footings were expressed in terms of dimensionless loads (bearing capacity factors). Failure envelopes were plotted in the VH, VM and HM planes in non-dimensional as well as normalized forms. Findings from circular foundations validated the finite element model created in ABAQUS. The effects of surficial crust on the bearing capacity of a circular foundation were also investigated. Finally, in order to investigate foundation design, three different foundation sizes were chosen and failure envelopes were plotted for various proportions of $V_{ult}$. Comparisons were made under the ‘normal’ and ‘extreme’ loading conditions.
Chapter 5

Conclusions and Future Work

5.1 Introduction

This thesis has investigated the undrained bearing capacity response of shallowly embedded circular & octagonal wind turbine foundations subjected to uniaxial and combined loadings. To achieve this, a finite element model was developed in the program ABAQUS. This research is novel since many other researchers:

- Did not use a no-tension contact to allow the foundation to lift off and lose contact with the soil (which is necessary to model a wind turbine foundation).
- Used a foundation shape that was circular rather than octagonal.
- Did not use a full 3-dimensional model, but rather utilized a semi-circular model or a section of the full foundation.

To achieve the objectives of this research, first, a 2-dimensional plane strain model was created to validate the soil constitutive parameters and mesh principles. Later, a complete 3-dimensional model of a circular footing was created and subjected to combined loading. Comparison with published data on offshore foundations helped in verifying the boundary
conditions and contact definitions. The effects of embedment and heterogeneity on the bearing capacity of circular and octagonal foundations were also studied. Furthermore, finite element analysis of foundations overlying a surficial crust was performed. In this final chapter, the findings and conclusions from the thesis are presented. Additionally, some ideas for future research and developments are proposed.

5.2 Conclusions from the Research

The use of the Mohr-Coulomb criterion for a linear elastic-perfectly plastic model shows promising abilities to obtain undrained response of a surface foundation subjected to uniaxial and combined loadings. The numerical model was able to provide results with reasonable accuracy, even when foundations were embedded or placed on heterogeneous soils.

The finite element analysis of a surface based circular footing subjected to combined loading shows that even for a simple case of a circular footing with no embedment and no soil strength heterogeneity, traditional methods tend to underestimate the combinations of horizontal and moment loads that a foundation can resist safely. Conventional methods neglect the coupling of horizontal and moment degrees of freedom and hence, break down under the superposition of solutions for inclined and eccentric loads. This can potentially explain variations in the size of the foundations used in practice whose design is based on conventional methods.

DNV (2002), which uses conventional methods, also tends to under-predict the effective area for a surface foundation subjected to eccentric loading compared to that derived by finite element analysis. This results in lower moment capacity.

The uniaxial limit capacities of embedded foundations are found to vary quadratically with the embedment ratio. Under embedded conditions, the foundation behaves as if it is fully bonded with the soil. A scoop type mechanism is found for ultimate moment failure. With increase in the embedment ratio, uniaxial bearing capacities also increase and the failure envelopes become larger, signifying the extra capacity available for the same foundation dimensions.
For a surface foundation, failure mechanisms under pure loads do not change with change in the soil strength heterogeneity. However, as the heterogeneity increases, the rotation accompanying horizontal failure during ultimate lateral load increases and vice-versa.

When the shape of the foundation changes from a circle to octagon for a surface foundation, the ultimate uniaxial vertical and moment capacities slightly increase (by 7.7% and 7.2% respectively). Especially when the foundation is shallowly embedded at $d = 3 \text{ m}$ ($d/D = 0.157$), the increase in ultimate moment capacity is significant (22.22%). The ultimate lateral resistance remains unchanged. Note that the top surface of the octagonal foundation is assumed to be flat and does not taper towards the edges. For a foundation with a shallow embedment depth ($d = 3 \text{ m}$), it is assumed that the effect of backfilled soil on the bearing capacity of the foundation is negligible. To add, failure mechanisms for surface or embedded foundations remain the same irrespective of their shapes (circle or octagon).

For a surface foundation, the change in the shape from a circle to an octagon does not have a significant effect on the bearing capacity with various soil strength heterogeneities. The average percentage increase in the ultimate vertical and moment capacities for an octagonal foundation is 7.1% & 6.7% respectively and that for a circular foundation is 6.5% & 6.9% respectively with increase in the soil strength heterogeneity (from $K' = 0.95$ to 1.425).

The presence of a surficial crust can result in a gain or loss of bearing capacity. Its effect cannot be ignored, especially for Canadian soils where this scenario is quite common. The bearing capacity depends on a complex relationship between many factors, such as the average crust strength, crust thickness, embedment of a foundation relative to the crust thickness, and relative shear strengths of the crust and the underlying layer. These factors must be given due consideration during foundation design. If a single layer with an averaged uniform shear strength is chosen to design the foundation, then its values must be chosen carefully.

As the vertical load on a foundation with no-tension interface increases, the HM failure envelopes become larger in size, signifying the extra capacity mobilized. The maximum value of ultimate moment is mobilized around a vertical load $V \approx 0.5V_{ult}$. Beyond this value, the HM envelopes shrink in size. Thus, for a constant horizontal and moment load, the vertical load
can be increased up to $0.5V_{ult}$. If $V > 0.5V_{ult}$ is applied, the factor of safety decreases. Note that the maximum value of ultimate moment is different from the maximum moment which is mobilized in a fully bonded contact.

5.3 Suggestions for Future Research

In this thesis, a coherent set of bearing capacity factors were found for foundations with different shapes, under varying embedment and soil strength heterogeneity conditions. Some simplifications were introduced in the modelling to obtain results, such as that relating to soil constitutive parameters. This opens up avenues for further improvement and development of better models as a part of future research work. The following points can be taken into account for research projects in the near future.

- The present research work essentially focused on static loading. However, wind turbine foundations undergo cyclic/dynamic loading. Numerical modelling of foundation subjected to dynamic loads will, thus, help in investigating its stability under fatigue and cyclic loads.

- Field monitoring of turbine & foundation movements can be set up and the data can be used to validate the dynamic model.

- The soil was simulated as a linear elastic-perfectly plastic material using the Mohr-Coulomb failure criterion as a yield surface and associated flow was assumed. However, real soil is anisotropic in nature. Experimental work involving laboratory tests like triaxial testing, resonant column, and bender element tests coupled with field monitoring data can be used to study cyclic degradation and critical states of soils under dynamic loads and obtain soil properties such as shear strain, friction angle and dilation angle. This data can help in defining more appropriate yield surfaces, flow rules and evolution laws that simulate the hardening or softening behaviour of soils. This will allow use of more complex models in ABAQUS.
• The reduced strength and stiffness of the backfilled soil was not taken into account.

• The installation process which results in the disturbance of surrounding soil was not specifically simulated in the finite element program.

• The octagonal foundation was assumed to be flat on the top. However, it tapers towards the edges in reality and this should be explicitly modelled. The backfilled soil provides resistance against uplifting due to the action of overturning moments. Especially, as the foundation is embedded deeper, the weight of the soil above the foundation increases and this can increase the rocking stiffness of the foundation.

• Only swipe tests were used to track the load paths in 2-dimensional planes. A combination of displacement probe tests and load-controlled analysis in addition to swipe tests can be performed to verify the current envelopes and possibly increase the accuracy.
References


ISO. (2011). ISO 19901–4:2003 (Modified), petroleum and natural gas industries—specific requirements for offshore structures, part 4—geotechnical and foundation design considerations


ics and geotechnical engineering (Vol. 16, pp. 123–176).


Tani, K., & Craig, W. H. (1995). Bearing capacity of circular foundations on soft clay of


Appendix A

Finite Element Meshes in ABAQUS

Mesh 2 (medium) and 3 (fine) which were part of mesh sensitivity analysis (Section 3.4.4.2) are shown below. Meshes were constructed using the structured technique. The details of mesh are given in Table 3.8.

Figure A.1: Mesh 2 (medium)
Figure A.2: Mesh 3 (fine)
Appendix B

Special Cases

B.1 Circumscribed Circle

A finite element analysis was performed on a circular foundation with the radius equal to that of the circumscribed circle of the octagonal foundation. This exercise was performed to compare the results with that found for a circular foundation with radius equal to that of the inscribed circle of the octagon (which is recommended by DNV (2002)). The diameter and area of the two circular foundations are compared in Table B.1 below. The octagonal and the circular foundations are shown in Fig. B.1.

<table>
<thead>
<tr>
<th></th>
<th>Diameter, m</th>
<th>Area, m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumscribed circle</td>
<td>20.56</td>
<td>332</td>
</tr>
<tr>
<td>Inscribed circle</td>
<td>19</td>
<td>283.5</td>
</tr>
</tbody>
</table>

Table B.1: Comparison of geometry between circumscribed and inscribed circle
Figure B.1: Comparison of geometry between circumscribed and inscribed circle

The value of $S_u = 17.760$ kPa and $E_s = 177.60$ MPa are same as that given in Table 3.4.

**Uniaxial Bearing Capacity Factors**

<table>
<thead>
<tr>
<th></th>
<th>$v_0$</th>
<th>$h_0$</th>
<th>$m_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>This study (Circumscribed circle)</td>
<td>6.20</td>
<td>1.00</td>
<td>0.69</td>
</tr>
<tr>
<td>This study (Inscribed circle)</td>
<td>6.08</td>
<td>1.03</td>
<td>0.69</td>
</tr>
</tbody>
</table>

Table B.2: Uniaxial bearing capacity factors

Value of $v_0$ is around 20% higher for the circumscribed circle than that for inscribed circle. The increased bearing capacity is due to the increased area of the circle which is also around 20%. $m_0$ remains the same irrespective of the diameter of the foundation. Values of $h_0$ vary slightly (3%) with each other. Thus, the bearing capacity factors differ slightly with each to other when the area of circle increases. However, it must be noted that bearing capacity factors are non-dimensional values. The absolute values of ultimate vertical, horizontal and moment loads for both the cases are shown below in Table B.3.
Table B.3: Ultimate vertical, horizontal and moment loads

<table>
<thead>
<tr>
<th></th>
<th>( V_{ult} )</th>
<th>( H_{ult} )</th>
<th>( M_{ult} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>This study (Circumscribed circle)</td>
<td>36531</td>
<td>5878</td>
<td>83765</td>
</tr>
<tr>
<td>This study (Inscribed circle)</td>
<td>30512</td>
<td>5200</td>
<td>65745</td>
</tr>
</tbody>
</table>

*Failure Envelopes*

Figure B.2: Failure envelopes in non-dimensional VH plane
Figure B.3: Failure envelopes in normalized VH plane

Figure B.4: Failure envelopes in non-dimensional VM plane
Figure B.5: Failure envelopes in normalized VM plane

Figure B.6: Failure envelopes in non-dimensional HM plane
The failure envelopes in non-dimensional and normalized envelopes are similar to each other in all of the 3 planes. The HM envelope in non-dimensional space (Fig. B.6) for an inscribed circle is slightly larger than that for a circumscribed circle due to slightly higher uniaxial lateral capacity obtained.

### B.2 VHM Loading through the Vertex of the Octagon

For all of the finite element analysis performed on octagonal foundations so far, the forces and moment were applied with a coordinate system as shown in Fig. B.8 (a). In the special case here, horizontal force and moment are applied along or about a diagonal passing through a vertex of the octagon. The coordinate system is shifted by 22.5° as shown in Fig. B.8(b). Case A represents a surface octagonal foundation with normal coordinate system, while Case B refers to a surface octagonal foundation with the new coordinate system. Results are expressed in the form of uniaxial bearing capacity factors and failure envelopes.
Figure B.8: Coordinate systems for the original and special case

Uniaxial Bearing Capacity Factors

<table>
<thead>
<tr>
<th></th>
<th>$v_0$</th>
<th>$h_0$</th>
<th>$m_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case A</td>
<td>6.55</td>
<td>1.00</td>
<td>0.74</td>
</tr>
<tr>
<td>Case B</td>
<td>6.55</td>
<td>1.00</td>
<td>0.74</td>
</tr>
</tbody>
</table>

Table B.4: Uniaxial bearing capacity factors

The uniaxial bearing capacity factors match exactly with each other.
Failure Envelopes

![Figure B.9: Failure envelope in the non-dimensional VH envelope](image1)

![Figure B.10: Failure envelope in the non-dimensional VM envelope](image2)
The failure envelopes match exactly with each other. This implies that for a given plane, horizontal and moment loads are independent of the direction in which they are acting.
Appendix C

Heterogeneous Soils

In the finite element analysis for circular foundations on heterogeneous soils (Case III), a Hill mechanism was not observed under ultimate vertical load since the failure was reached with low values of displacements. However, when a very high value of displacement (0.02D) was applied, a Hill mechanism was obtained as shown in Fig. C.1 and C.2 below.
Figure C.1: Contours of Hill mechanism of soil failure

Figure C.2: Displacement vectors obtained from finite element analyses superimposed by Hill mechanism given by Tani and Craig (1995)
Appendix D

Working and Design Loads Analysis

A circular foundation representing a footing with $D = 19$ m and $d = 3$ m has been subjected to loads under the normal operating and extreme conditions shown described in Section 4.4, shown in Table D.1). All of the loads are applied as a force or moment unlike previous analysis which were displacement-controlled. Initially, a vertical force is applied and then, the horizontal force and moment are applied simultaneously The properties of the soil assumed are shown in Table D.2 and compared with those of the finite element analysis done previously in this research. The stress states and displacements are calculated for four different paths as shown in Fig. D.1. Path 1 is horizontal (lying in XY plane). It has a length equal to $D$ and runs along the centreline of the foundation-soil interface from $(-D/2, 0, 0)$ to $(D/2, 0, 0)$. Path 2, 3 and 4 are vertical (lying in XZ or YZ planes). Each of these three paths have a length of $1.5D$. Path 2 and 3 start at the circumference of the foundation base with coordinates $(D/2, 0, 0)$ and $(0, D/2, 0)$ respectively. Whereas path 4 begins at the centre of the foundation base $(0, 0, 0)$.

<table>
<thead>
<tr>
<th>Foundation loads</th>
<th>$d/D$</th>
<th>$V$(kN)</th>
<th>$H$(kN)</th>
<th>$M$(kN m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal operating conditions</td>
<td>0.157</td>
<td>2900</td>
<td>900</td>
<td>42000</td>
</tr>
<tr>
<td>Highest overturning moment</td>
<td></td>
<td>2900</td>
<td>1100</td>
<td>76200</td>
</tr>
</tbody>
</table>

Table D.1: Load acting on foundation under various operating conditions
<table>
<thead>
<tr>
<th>Properties</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturated unit weight, $\gamma_s$</td>
<td>19</td>
<td>kN m$^{-3}$</td>
</tr>
<tr>
<td>Young’s modulus, $E_s$</td>
<td>133.20</td>
<td>MPa</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu_s$</td>
<td>0.48</td>
<td>-</td>
</tr>
<tr>
<td>Undrained shear strength, $S_u$</td>
<td>60</td>
<td>kPa</td>
</tr>
<tr>
<td>Friction angle, $\phi$</td>
<td>0.1 ≈ 0</td>
<td>Degree</td>
</tr>
<tr>
<td>Dilation angle, $\psi$</td>
<td>0.1 ≈ 0</td>
<td>Degree</td>
</tr>
<tr>
<td>Absolute plastic strain</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>Material model</td>
<td>Mohr-Coulomb</td>
<td></td>
</tr>
</tbody>
</table>

Table D.2: Properties of soil used in the current and previous analysis

Figure D.1: Paths used in the analysis
Figure D.2: Normal vertical stresses for various paths under ‘working’ and ‘extreme’ conditions
Figure D.3: Nodal displacements for various paths under ‘working’ and ‘extreme’ conditions
Figure D.4: Von Mises stresses for various paths under ‘working’ and ‘extreme’ conditions
Fig. D.2 shows the normal vertical stresses along various paths. Under both the conditions, the normal stresses match fairly well with each other for paths 2, 3 and 4. This is because the ultimate uniaxial moment capacity is relatively very high compared to the moments under ‘working’ and ‘extreme’ conditions. Thus, the stresses are almost the same along the depth of the soil. Along the centreline, however, the stresses do vary slightly for the two conditions. Especially near the edge on +X axis ($D/2, 0, 0$), this variation is high (around 41%). Indeed, under ‘extreme’ conditions, the nodal displacement at this point is the highest (3.60 mm). Under ‘working’ conditions, the displacement at the same point (or node) is 2.17 mm (Fig. D.3 (a)). With increasing depth, the displacements decrease. The rotation of foundation due to the overturning action of moment is shown in Fig. D.5 below.

![Figure D.5: Rotation of foundation under ‘working’ and ‘extreme’ conditions](image)
Curriculum Vitae

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National Institute of Technology (NITK), Surathkal, Karnataka
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