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Three Essays on Managing Customer-Based Strategies: A Pricing and Revenue Management Approach

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A thesis submitted in partial fulfillment of the requirements for the Doctor of Philosophy degree in Business

THREE ESSAYS ON MANAGING CUSTOMER-BASED STRATEGIES: A PRICING AND REVENUE MANAGEMENT APPROACH

(Thesis format: Integrated Article)

by

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Abstract

Many firms and organizations with already-optimized business functions are under market pressure to protect their narrow profit margins. Their need for supplemental and reliable revenues calls for performance optimization beyond the core business functions. Motivated by applications from online social media and the airline industry, in my dissertation, I focus on the revenue management and pricing decisions of customer-based plans and programs. More formally, the research question addressed in this study is: How can firms effectively use customer-based pricing strategies to boost revenues?

My dissertation consists of three essays. In the first essay, I analyze the ongoing competition among online social media (OSMs) to attract users. Concentrating on the importance of community retention and expansion to OSMs in preserving financially sustainable business models, I investigate whether OSMs should develop revenue sharing programs and reward their contributing users from their limited revenue streams. I present a duopoly OSM game (with a less favourable and a more favourable OSM) in which heterogeneous users choose their levels of contribution with respect to each OSM based on their preferences. In this chapter, I explore how online users' actions and perspectives impact the outcome of the competition among OSMs. Furthermore, I investigate how small social media firms can compete with a dominant firm in the market.

In the second essay, I study the role of ancillary revenue and its significance for industries such as airlines. These firms can barely survive without ancillary fees, even when their capacities are almost fully utilized. I consider the case in which customers-changing rates between flights are stochastic but decreasing with reference to the change fees. In this essay, I examine how firms should design change fees to manage customers' switching behaviour. Specifically, I incorporate change fee revenues as a portion of total revenue structure and investigate how firms should update their markdown pricing strategies when they face price-tracking customers.

In the third essay, I focus on the dynamics between a firm and customers who are uncertain about their future travel plans. While the firm maximizes its revenue by imposing optimal change fees, customers consider their travel plan uncertainties and maximize their utilities by

responding strategically to these fares. In this study, I seek to answer two important policy questions: Although imposing a change fee could increase total revenue, does it burden the firm with a lower customer demand? How should the optimal monopolistic price be set with the presence of a change fee? Without imposing any distributional assumptions, I analytically derive each market player's best reaction to the other to prescribe the characteristics of the firm/customer interaction equilibrium.

Keywords

Ancillary Revenue, Airline Industry, Change Fee, Customer Relationship, Revenue Sharing, Online Social Media, Game Theory, Revenue Management, Marketing

Co-Authorship Statement

I declare that this thesis incorporates some material that is a result of joint research. Essay 1 was in the International Journal of Management Science (OMEGA) co-authored with Dr. Fredrik Odegaard and Dr. Xinghao (Shaun) Yan. As the first author, I was in charge of all aspects of the project including formulating research questions, literature review, research design, analyzing the secondary data, and preparing the first and the following complete drafts of the manuscript. With the above exception, I certify that this dissertation and the research to which it refers, is fully a product of my own work. Overall, this dissertation includes 3 original papers, with the first essay already published in an academic journal.

Essay 1 – Status: Published.

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Chapter 1

1 Introduction

For most categories of products and services, global market competition is continuously on the rise, which leads to lower prices. Profit margins are being squeezed even when the prices are high because the costs of raw material and energy often peak at the same time. The dynamic costs of resources make it difficult to consistently meet profit targets. For example, according to Brian Pearce, the Chief Economist at International Air Transport Association (IATA), airlines' revenue exceeded their costs by just 1% in 2012.[1] This trend drives companies to devise new strategies to generate additional and reliable revenues.

Seeking supplementary revenues is critical for another reason. For many firms, the core business process is not the main driver of the financial stream. For example, according to Reid and Sanders (2013), most of General Motors' monetary returns apparently do not come from their core business functions –manufacturing and selling cars – but rather from post sales parts and services. For movie theatres, while the concession revenue rose 8.8% in 2013, theatres are allegedly not making any more money on movies.[2] The generation of higher revenues from ancillary products and services explains why, from a movie theatre owner's perspective, customers are not at the movies to be entertained. Instead, they are there to buy soda and snacks at marked-up prices.[3]

Firms develop customer-based pricing strategies to ensure a financially healthy and sustainable business model. In this thesis, I focus on two types of customer-based pricing strategies. First, firms reward customers for acting in ways beneficial to the firms. For

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[1] URL: http://www.iata.org/whatwedo/Documents/economics/Aviation-Advocacy-Economics-2013-July.pdf; accessed: 2015–8–8.

[2] URL: http://business.time.com/2009/12/07/movie-theaters-make-85-profit-at-concession-stands; accessed: 2014–7–10.

[3] URL: http://business.time.com/2009/12/07/movie-theaters-make-85-profit-at-concession-stands; accessed: 2014–7–10.

example, *YouTube Partner Program* incentivizes online users to post videos and provides monetary rewards if their videos generate millions of views. Rewarding video creators ultimately makes YouTube a more popular online social media platform and so it is more appealing for advertisers. Although paying active customers is costly for firms, the benefits it produces may exceed the costs.

Second, firms follow an opposite pricing strategy and charge customers when they need a complementary service. For instance, according to the U.S. Bureau of Labor Statistics, in 2011, *ancillary revenue* collected from change and cancellation fees accounted for more than 3% of U.S. airlines' revenue. Higher ancillary fees can generate more revenues for firms, but they may significantly reduce customer demand. In this thesis, I will investigate how firms should optimize the design of these fees. Despite the emerging importance of ancillary revenue, its significance for firms' dynamic pricing has not yet been fully addressed in operations management and management science literature.

Many revenue management models are addressing the performance optimality by solely focusing on the core business function of a firm. Chase and Prentis (1987) call the problem of *sub-optimization* one of the major issues in the management science field, where the performance measure for a part of a system is optimized, but at the expense of total system performance. When a firm designs its core business function without considering the customer-based pricing strategies discussed above, it faces the same challenge. For instance, the introduction of a change fee affects how a firm should update its optimal service price. Therefore, considering customer-based pricing strategies along with the main business process pricing strategies is essential for the firm to prevent the risk of sub-optimization. Unless we optimize the design of all of these strategies together, our prescriptions based on sophisticated revenue management systems may not be beneficial for firms, and may even be misleading and harmful.

The purpose of this thesis is to focus on the revenue management and pricing decisions of customer-based strategies. The results generated from this thesis lead to optimizing the performance beyond the core business functions. Moreover, I provide managerial insights for firms on how to generate supplemental revenues. Higher revenues provide opportunities for firms to be competitive under the market pressure and to access more financial resources for future business investments.

My dissertation consists of three essays examining how organizations can effectively use customer-based pricing strategies to boost revenues. Essay 1 is motivated by applications in online social media and focuses on the first type of customer-based strategies. In this essay, I investigate how a firm should share its advertising revenue with active users. The results from this essay present new insights on how optimal customer reward programs shape the market dynamics in online social media. Essays 2 and 3 are motivated by applications of ancillary fees in the airline industry and focus on the second type of customer-based pricing strategies. In Essay 2, I study customers' monetary incentives to switch between resources and assess how firm can benefit from these incentives by offering lower service prices. In Essay 3, I study customers' uncertainty with respect to their future travel plans and investigate how firms should consider this uncertainty in their pricing policy. Furthermore, I present that how the optimal service prices should be modified jointly with the change fees.

In conclusion, I would like to highlight that I have selected online social media and the airline industry as the focus of this study because both industries have narrow profit margins. Furthermore, both industries are highly dependent on customer-based strategies to generate extra revenues. This thesis extends application to other industries. For example, in Chapter 3, I will discuss how banks and cellphone providers experience an identical problem when dealing with their clients, and in Chapter 4 I show that travel industries, such as auto rental, can benefit from our results.

1.1 Overview of Thesis and Specific Essays

Essay 1 analyzes the ongoing competition among online social media (OSMs) to attract users. Given the importance of community retention and expansion to OSMs in preserving financially sustainable business models, I investigate how OSMs should develop revenue sharing programs and reward their contributing users from their slender revenue streams. In this study, firms share a portion of their advertising revenue with active online contributors and motivate them to generate more content. I present a duopoly OSM game where heterogeneous users choose their levels of contribution with respect to each OSM. I investigate the uniqueness of the Nash equilibrium and derive structural properties of the equilibrium reward solution. Moreover, I investigate the behaviour of online users and their interaction with firms at equilibrium.

In the second essay, I study the role of ancillary revenue and its significance for firms, such as airlines. More specifically, I consider an uncertain changing rate between two resources offered by a firm. The customers' changing rate between two resources is decreasing in terms of change fees. With reference to the change fees, I examine how firms should update its markdown pricing strategies and manage switching behaviour between two resources. Moreover, I investigate whether it is optimal to prevent customers from changing between two resources in any case, and determine how this decision depends on the customers' population structure in the market.

The third essay focuses on the dynamics between a firm and its customers. The firm maximizes revenue by setting the optimal change fee. In contrast to the situation in the second essay, I isolate time uncertainty as the different motivation behind switching fee. Customers are uncertain about their future travel plans and maximize their utilities by purchasing their tickets either earlier or later (In Essay 2 the model captures those customers who change regardless of switching fee, but in Essay 3, customers only change due to plan change). Furthermore, they are heterogeneous with respect to their willingness-to-pay for the service. I analytically derive each player's best reaction to the other to prescribe the characteristics of the firm/customer interaction equilibrium. In this study, I seek to answer the following important questions: while charging customers for changing their travel plans might generate higher revenue per changing customer, does it ultimately burden the firm with lower revenue? How do customers with time uncertainty react to change fees? And finally, how should the firm update its pricing policies with change fees to maximize its revenue?

In the final chapter, I present an overview of the main results from the analysis of the customer-based pricing strategies discussed in the thesis and highlight the managerial implications of their implementations.

Chapter 2

2 User Reward Programs in Online Social Media

Online social media (OSMs) have become a popular and growing Internet phenomenon, as exemplified by the millions of followers of websites like YouTube, Twitter, and Facebook. Given the Internet's ease of access and the high degree of competition to attract users to these sites, a question arises as to whether OSMs should develop revenue-sharing programs as a way to reward their contributing users. I present an ex ante asymmetric duopoly OSM game, where heterogeneous users are either active or passive with respect to each OSM. The game includes two steps: First, the OSMs simultaneously announce their rewards for active users; and second, based on their preference, users choose their level of contribution with respect to each OSM. I show that this game has a unique Nash equilibrium in pure strategies, and identify the conditions under which a symmetric equilibrium exists, despite the asymmetry between the OSMs. Moreover, at equilibrium, no user chooses to contribute content exclusively to the less favourable OSM, even when the more favourable firm shares a lower reward than the less favourable firm. Furthermore, in some circumstances, a higher asymmetry can diminish the net revenue of the more favourable firm and vice versa.

2.1 Introduction

During the last decade, online social media (OSMs) have become a popular and growing Internet phenomenon. Hundreds of millions of users from all over the world visit and contribute to these websites on a daily basis. As of October 10, 2014, LinkedIn, Twitter, YouTube, and Facebook had around 300 million, 950 million, 1 billion, and 1.28 billion unique users, respectively.[4] In this paper, I define an online social medium as any website providing a platform that allows online users to join and post user-created content (UCC). This definition includes social news websites, such as Digg and Reddit; video- and photo-

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[4] URLs: press.linkedin.com/News-Releases/333/LinkedIn-reaches-300-million-members-worldwide; businessinsid-er.com/number-of-users-who-abandon-twitter-2014-2#!JxoTo; youtube.com/yt/press/statistics.html; news-room.fb.com/company-info/; accessed 2014-10-10

sharing websites like YouTube, Flickr, and Metacafe; social network websites, such as MySpace, Facebook, Google+, Tencent QQ, and Twitter; and portal sites such as Yahoo Groups. The key benefit for users comes from the pre-dominantly free service that allows them to stay connected with their communities and friends, sharing knowledge and user-created content like photos, videos, files, software, bookmarks, and blogs. These features contribute to OSMs' appeal for advertisers.

Although some OSMs generate revenue through membership fees, affiliate programs, donations, and merchandise sales, the most common source of revenue is through advertising. Advertisements are displayed to users, and revenue is generated (to an OSM) based on the amount of time the advertisement is displayed and/or the number of times it has been clicked. Based on Deane and Agarwal [8], in the United States alone, annual revenues from ads on social media sites were estimated at over $26 billion in 2010. Hu (2015) argues that advertisers have started to realize that the Internet is a much more accountable and measurable medium compared to other forms of traditional media like television. Therefore, the larger the online community, the more lucrative the site is for advertisers to post their ads, and the greater the opportunity for higher revenue. Consequently, community retention and expansion stand out as key issues for OSMs.

Broadly speaking, there are two types of OSM users: active and passive. Active users post content and observe other active users' contributions, while passive users simply review content generated by active users. Within each category, there is, of course, a continuum. For example, the active category is likely to contain very active users who contribute on a daily or even hourly basis, as well as minimally active users who infrequently post a contribution. Likewise for passive users: some may be "active passive" in that they visit the site frequently, while others visit the site so infrequently that they may not even be considered a user. For the purpose of this pa-per, I restrict the discussion to consider only two categories. More details are provided in Section 3.

Since users visit OSMs because of UCCs created by active users, some OSMs provide active users with monetary incentives through revenue-sharing programs. Under these relatively new practices, OSMs share a portion of their revenue as rewards to their active users. The main purpose of these programs is to increase the traffic and popularity of OSMs, which ultimately drive higher revenue. Table 1 categorizes OSMs with user reward programs. For example, YouTube, although not explicitly declared, pays video developers of original content approximately $2 to $5 per 1,000 views of their posted video. In addition, popular users often may receive additional payment from their own advertisement sales, sponsorships and product placements. For instance, Michael Buckley, famous for his YouTube video blog “What the Buck?!”, reportedly earns $20,000 a month. [5]

I consider two online social media sites that compete simultaneously by rewarding active users (e.g., YouTube and Metacafe). The firms are assumed to be asymmetric in that online users have a general preference for one of the sites. I label one as “more favourable” and the other as “less favourable” and investigate ways for the less favourable firm to compete with the more favourable firm. Both firms decide on the amount of reward they will offer as their strategy. I demonstrate how to derive the optimal reward levels and show that the game has a unique Nash equilibrium in pure strategies. At equilibrium, some users may be indifferent with respect to making a choice between two firms, but I show that the possibility of this case is zero and corresponding firms’ market size based on users’ choices are unique.

[5] URL: http://www.nytimes.com/2008/12/11/business/media/11youtube.html; accessed: 2014–5–10.

Table 1: Categories of Revenue sharing in OSMs (accessed October 10, 2014)

Category	Examples	Structure of Payment
Written Content	*Helium*	Earnings range from $10 to $200 per article plus a $3 bonus.
	Oondi	Writer gets 100% of the advertisement profits.
Video Sharing, Podcasting, Audio and Music	*YouTube*	YouTube pays video developers between $2 and $5 per 1,000 plays.
	About	Video producers are paid a flat fee of $250 per video.
	Blip.tv	On a 50/50 basis, Blip.tv shares its advertising revenue with producers.
Photography	*Shutterfly*	A Shutterfly Affiliate earns $9 per new customer.
	PhotoWorks	Users earn money from photos, illustrations, vectors, and footage.
Professional and Reviewing Community	*RateItAll*	It shares 50% of the Google Adsense revenue generated by the pages users create.
	Newsvine	Users receive 90% of revenue from advertisements.
Answer Services	*Ether*	Users earn money over e-mail.
	Mahalo	The most helpful users receive payment from those who submit the question.
Miscellaneous	*RedBubble*	An online art community that allows users to sell their works.
	Kongregate	A large community of independent game developers with weekly prizes.

This paper makes the following contributions. First, I find that, for any level of asymmetry, there exists a revenue structure such that the game has a symmetric equilibrium [i.e., the two OSMs provide the same level of reward to their active users]. Consequently, although a symmetric allocation of users signals a symmetric game, a symmetric equilibrium does not necessarily imply a symmetric game. Second, when users have economy of scale for contribution to both firms, at equilibrium, no one chooses to contribute content exclusively to the less favourable firm, even when this firm shares a higher reward. This result may seem counterintuitive, but it is anecdotally consistent with what is observed, both with regard to OSMs that provide reward programs (e.g., YouTube versus Metacafe), as well as those that are based merely on users' perceptions (e.g., Facebook versus Google+). Third, although one may expect that, as the asymmetry between the OSMs increases, the more favourable firm would decrease its reward amount, I show that both websites end up sharing higher rewards. Finally, I demonstrate the conditions under which the probability of users being active in the less favourable firm increases when the asymmetry increases between the OSMs (and vice versa). Interestingly, although the less favourable firm never sees an advantage in a higher asymmetry, under certain conditions, a higher level of asymmetry can also diminish the more favourable firm's net revenue.

2.2 Literature Review

Different perspectives of OSMs have been studied in literature. For a general review of topics related to OSMs, see Bainbridge (2007), Clemons (2009), Messinger et al. (2009), and Kim, Jeong, and Lee (2010). These authors agree that OSMs face unique business challenges, such as selection of a financially sustainable business model and managing the customer relationship, and more research needs to be done to address these challenges. The need for further studies about OSMs also has been identified in the operations management and management science literature, although the primary focus of the studies in these fields has been mainly on online retailing, advertising, and network science. (See Ahmed and Kwon (2014), Alderson (2008), Basua, Chakrabortyb, and Sharmac (2015), Fridgeirsdottir and Najafi-asadolahi (2014) and Perdikaki, Kesavan, and Swaminathan (2012) and the papers referenced therein.)

A stream of research that relates – though somewhat orthogonally – to this study is online consumer behaviour, which has been empirically studied by social psychology and marketing science; see Schau and Gilly (2003), Benabou and Tirole (2006), Johnson et al. (2004) and Teo and Yu (2005). This field highlights the crucial role and characteristics of content contributors on the success of any user-generated content website (See Zhang et al. (2012) and Grewal, Lilien, and Mallapragada (2012)). Moreover, some studies, such as Tirunillai and Tellis (2012) and Luo, Zhang, and Duan (2013), investigate the relationship between social media and the market performance of the firm. This study focuses on the effects of monetary incentives and revenue-sharing programs on the dynamics that exist between users and the OSMs on the Internet. This paper contributes to the call for further research in this field, such as OSM profitability and revenue generation.

The subject of competition between media sites has been studied in the literature (see Gal-Or and Dukes (2003), Godes, Ofek, and Sarvary (2009), Zhang and Sarvary (2014), and Zhu and Dukes (2015)), with a focus on different dynamics based on price and other factors of service quality, such as product variety and advertising intensities. Shin and Sudhir (2010) present a related paper that focuses on rewarding customers within symmetric competition. These authors adopt a Hotelling model with two retailers geographically located on the two ends of a unit line. Similar to my setup, they assume there are two types of consumers in the market with value heterogeneity and unstable preference. They present the conditions in which it is optimal for a firm to offer a lower price as reward only to its own customers rather than to the competitor's customers and vice versa. In this paper, however, customers select one retail service.

A recent study, and one that is more closely related, is Zhang and Zubcsek (2010). These authors consider a monopoly setting wherein an OSM provides online users both monetary and non-monetary incentives for their online contributions. Similar to the setup in this paper and in Shin and Sudhir (2010), they also consider two types of customers. They find that when the OSM offers monetary incentives, it is more effective to offer payments exclusively to top contributors instead of to all users. Moreover, their results indicate that one equilibrium strategy involves building a contributor community based solely on monetary incentives rather than on non-monetary incentives, such as intrinsic fun. This study complements their results by providing insights for the duopoly setting. Moreover, while Zhang and Zubcsek (2010) and Shin and Sudhir (2010) consider two types of contributors – that is, a high and low type contributor – I capture customers' heterogeneity through two independent effort levels.

2.3 Online Social Media Duopoly Competition

I assume there are two OSM firms offering the same service and competing simultaneously for online users through their revenue-sharing programs. I index the OSM firms by i, i = 1, 2 and define users to be either active (A) or passive (P) with respect to each firm. Therefore, there are four different states per user. For notational convenience, I use the first and second letters to denote the state of a user in firm 1 and firm 2, respectively: AP, AA, PA, and PP. For example, AP denotes that the user is active in firm 1 and passive in firm 2. Although I consider two types of customers, the ensuing model setting and analysis can be extended to include additional customer contribution levels; however, to keep the model parsimonious, I choose not to include more states. Furthermore, in many cases, OSMs, such as YouTube, impose a threshold on the level of contribution and deal with a continuous spectrum of online users' contributions in a discrete fashion. In such a case, online users whose contributions are below (above) the threshold are considered passive (active).

I consider a market that consists of a large online user population, a place where the decision of one user does not affect how other users behave, and there is no peer pressure in this market (consistent with the setup in, for example, Nasiry and Popescu (2012), Gallego and Sahin (2010), and references in Shen and Su (2007)). Both OSMs have the same pool of online users, since reaching all websites is guaranteed on the Internet. The income per user comes from advertising, and the size of that income depends on the shopping behaviour of the users since revenue is a function of clicks or purchases per advertisement. Urstadt (2008) and Kim, and Jeong and Lee (2010) mention that a fixed portion of online users regularly click on ads (e.g., 0.04 percent on Facebook). As a result, OSMs generate revenue constantly, per user, from advertising. Facebook charges only 14 cents per thousand times that an advertisement is served.

I assume firms receive different benefits from active and passive users' contributions. I define w_A and w_P as the fixed exogenous parameters that represent firms' revenue per active and passive user, respectively. These parameters signify the impact of many factors, which monetize the contribution of different classes of users as revenue for the firms, such as the different levels of online dedication, influence, or rates of purchase for active and passive users. (Venkatesh and Agarwal (2006), Kim, Jeong, and Lee (2010) and Grewal, Lilien, and Mallapragada (2012)) According to the information presented in Table 1, in most cases, OSMs, such as About and Shutterfly, reward active users based on a fixed rate, or they share a fixed percentage of revenue, as, for example, in Oondi, Blip.tv, and Newsvine.

Although advertising revenue may be different for the same type of contribution, in this paper, I consider w_A and w_P as the "average revenues" from active and passive users. For a large market of users, it would seem reasonable to define an average contribution level for active users and maximise the expected firm revenue over an average reward. I assume that, for firms, active users generate higher revenue than do passive users (i.e., $w_A > w_P$). If $w_A \leq w_P$, then the problem becomes pathological since there is no incentive for firms to pay active users a reward. Consequently, all users decide to be passive and no content is generated. Note that, although each firm sets its own reward, its net revenue depends on its competitor's reward too.

The sequence of events is as follows: In the first stage, firms simultaneously announce the amount of their rewards r_i, i = 1, 2. I denote the vector of rewards by $\boldsymbol{r} = [r_1, r_2]$. The second stage accounts for the users' reactions to the reward programs as follows: After observing $\boldsymbol{r}$, each online user selects among AP, AA, PA, and PP. Each website's expected net revenue per user is, for i = 1, 2:

$$\pi_i(\boldsymbol{r}) = \varphi_i(\boldsymbol{r})(w_A - r_i) + \left(1 - \varphi_i(\boldsymbol{r})\right) w_P$$

where φ_i is the probability of being active for an online user in firm i, i = 1, 2. Increasing this probability is an indication of *operational* success of the firm's service and the users' contribution willingness. These probabilities depend on the vector of rewards. Note that I could have $\varphi_1 + \varphi_2 \leq 1$ or even $\varphi_1 + \varphi_2 > 1$, since a user may decide to be active or passive in both firms or in neither firm. To simplify the analysis, I define $w = w_A - w_P$, the *incremental revenue* generated by an active user. Firm *i* aims to find optimal r_i to maximize its expected net revenue; that is:

$$\max_{r_i} \pi_i(\boldsymbol{r}) = \max_{r_i} \ \varphi_i(\boldsymbol{r})(w - r_i) + w_P \tag{1}$$

Next, I use backward induction to solve this game. Given the firms' reward decisions (r_1,r_2), I consider the users' best responses that impact each firm's demand, $\varphi_i{}^*(\boldsymbol{r})$. I discuss how online users choose to be active or passive with respect to an OSM. I assume that being active at an OSM requires effort or is "costly" for online users, and this cost reflects the time or effort spent on participating in a firm's activities (i.e., generating content for an OSM). Also, being passive is effortless and therefore incurs no cost for the users. Although an online user pays a *cost* for an online contribution, he or she could have a specific *motivation* for being active. Studies, including Koufaris (2002) and Hsu, Lu, and Hsu (2007), indicate that the *ease of use*, the *level of enjoyment*, and the *usefulness of the online service* constitute the users' main motivations with respect to Internet contribution. I capture the impact of these incentives as the *general perception* of the OSM of online users. In the duopoly setting, I assume that one firm enjoys better recognition from online users compared to the other firm (i.e., firms are *ex ante asymmetric* in this regard), and this asymmetry could reflect differences in the overall preference, technologies, instructions, fees, or designs that are implemented by each firm. The social media industry is characterised by a trend towards winner-take-all markets and is increasingly dominated by websites such as Facebook or YouTube. However, a large number of niche websites now co-exist with these dominant firms, and I thus model asymmetric competition to investigate how smaller players can compete with major players.

I label the firm with the better perception as "the more favourable firm" and the other firm as "the less favourable firm." For active users, the better the general perception, the lower the cost of contribution; that is, contribution to the more favourable firm is less costly than contribution to the less favourable firm. Therefore, in general, online users are more willing to contribute to the more favourable firm. I denote the *asymmetry* (distinction) between the firms by the fixed (finite) discount parameter a, where $a \geq 1$. Note that in the special case of $a = 1$, the firms are symmetric. Later, in Chapter 6, I study the case where online users have different perspectives with respect to the asymmetry between two firms (i.e., a is a random variable). The taste preferences of the customers are *heterogeneous*, and I capture this heterogeneity through two independent effort levels.

First, the consumers' effort, or "cost," of being active in a firm is captured by the random variable C, where C is uniformly distributed between 0 and 1; $Pr\{C \leq c\} = F_C(c) = c$, for $0 \leq c \leq 1$. From now on, I refer to C as the *consumers' cost*. Without loss of generality, I label firm 2 as the "less favourable" firm and, therefore, the consumers' cost of being active with respect to firm 1 and firm 2 is C and aC, respectively. Second, the consumers' cost of being active in both firms simultaneously is represented by $C(a+K)$, where K is uniformly distributed between 0 and 1 and independent of C; $Pr\{K \leq k\} = F_K(k) = k$, for $0 \leq k \leq 1$. A small K represents economy of scale in being active in both firms, reflecting a case in which a user has better skills, knowledge, or tools in handling two distinct activities at the same time. In Chapter 6, I extend my analysis to the case where $1 \leq k$, in which users have diseconomy of scale for joint participation in both firms, a situation that is common in many manufacturing and service industries.

Online users with higher cost are less willing to be active. The rewards, however, have the opposite effect on the behaviour of online users and compensate for the costs of these contributions. Online users consider *both* rewards *and* costs in their decision-making, and I define a utility function for online users accordingly. After firms have announced their rewards and users have observed those rewards, the users compare their utility of being in any of the four defined states and choose the state with the highest utility:

$$u(c,k) = max\{r_1 - c,\ r_2 - ac,\ r_1 + r_2 - c(a+k),\ 0\} \tag{2}$$

where $r_1 - c$, $r_2 - ac$, and $r_1 + r_2 - c(a+k)$ are the utilities of a user with realized cost variables c and k, of being in states AP, PA and AA, respectively, and 0 is the utility of being passive in both firms.

2.4 Analysis of Firms' and Consumers' Interaction

In this section, I analyze the outcome of interactions between firms and consumers. First, I characterize the best response function for the users. According to the rational reaction of online users, firms maximize their net revenue. Next, I derive the corresponding Nash equilibrium(s) for the simultaneous game between the two firms.

2.4.1 Users' Reaction to Firms' Rewards

A *rational* user is active with the following probabilities:

$$\varphi_1(\boldsymbol{r}) = Pr\{Active\ in\ firm\ 1\} = Pr\{max(r_1 - C, r_1 + r_2 - C(a+K)) \geq max(r_2 - aC, 0)\}$$

$$\varphi_2(\boldsymbol{r}) = Pr\{Active\ in\ firm\ 2\} = Pr\{max(r_2 - aC, r_1 + r_2 - C(a+K)) \geq max(r_1 - C, 0)\}$$

Then, I have the following results:

$$\varphi_i^*(\boldsymbol{r}) = \int_0^1 \int_0^1 g_i(\boldsymbol{r}, c, k)\, f_C(c)\, f_K(k)\, dk\, dc = \int_0^1 \int_0^1 g_i(\boldsymbol{r}, c, k)\, dk\, dc$$

where

$$g_1(\boldsymbol{r}, c, k) = \begin{cases} 1 & if\ \ max(r_1 - c,\ r_1 + r_2 - c(a+k) \geq max(r_2 - a.c, 0) \\ 0 & otherwise \end{cases}$$

$$g_2(\boldsymbol{r}, c, k) = \begin{cases} 1 & if\ \ max(r_2 - ac,\ r_1 + r_2 - c(a+k) \geq max(r_1 - c, 0) \\ 0 & otherwise \end{cases}$$

and $f_C(c)$ and $f_K(k)$ are the pdf of $F_C(c)$ and $F_K(k)$, respectively.

The probability functions φ_i are continuous but not differentiable in r_i. Depending on the particular rewards r_1 and r_2, there are, in total, nine different cases for φ_1 and φ_2. Note that the probabilities also depend on a, but a is a parameter of the model and not a decision variable for the firms. For the ensuing firms' duopoly analysis, only three cases are of interest, and these are summarized as follows. (For completeness, the remaining six cases are summarized in Appendix.)

Lemma 1 *The probability of being active in firms 1 and 2 is as follows:*

- *Case I: if* $r_1 \leq 1$ *and* $r_2 < (a-1)r_1$*, then* $\varphi_1{}^*(\boldsymbol{r}) = r_1$ *and* $\varphi_2{}^*(\boldsymbol{r}) = r_2 log(\frac{a}{a-1})$.
- *Case II: if* $(a-1)\, r_1 \leq r_2 \leq ar_1$*, and* $r_1 + r_2 \leq a$*, then* $\varphi_1{}^*(\boldsymbol{r}) = (r_1 + r_2) log\left(\frac{r_1+r_2}{ar_1}\right) + ar_1 - r_2$ *and*

  $\varphi_2{}^*(\boldsymbol{r}) = (r_1 + r_2)\ log\left(\frac{r_1+r_2}{ar_1}\right) + r_2 log(ar_1) - r_2 log(r_2)$.
- *Case III: if* $(a-1)r_1 \leq r_2 \leq ar_1$*,* $r_1 + r_2 > a$*, and* $r_1 \leq 1$*, then*

  $\varphi_1{}^*(\boldsymbol{r}) = r_1 + a(r_1 - 1) - (r_1 + r_2) log(r_1)$ *and*

$\varphi_2{}^*(\boldsymbol{r}) = r_1 + r_2 - a - (r_1 + r_2) log(r_1) + r_2 log(ar_1) - r_2 log(r_2)$.

Lemma 1 shows how a market of rational online users responds to firms' reward decisions $\boldsymbol{r}$. In Case I, φ_i depends only on r_i and not on r_j, for $i, j = 1, 2$, and $i \neq j$. In Cases II and III, φ_1 and φ_2 depend on r_1 and r_2. For all of the above cases, firms pay less than the maximum of their active users' cost (i.e., $r_1 \leq 1$ *and* $r_2 \leq a$). Depending on which case the user chooses, different dynamics occur. For instance, in Case I, the probabilities of being active for two firms increase *linearly* with corresponding rewards; firms can increase their market shares linearly by increasing their rewards. In contrast, for Cases II and III, the probabilities are nonlinear with respect to the rewards. Notice that $r_1 + r_2 \leq a$ for Cases I and II, and $r_1 + r_2 > a$ for Case III.

I might expect that the probabilities of being in state AP and PA are decreasing in r_2 and r_1, respectively. In other words, by increasing its reward, a firm can diminish the likelihood of users being active exclusively with a competitor. However, I see from (2) that, if the reward of an OSM compensates the cost of a user, then the competing OSM's reward *cannot* diminish this state. Instead, adding that reward can convince the user to choose to be active in both. Proposition 1 shows that φ_i are monotonic functions with respect to r_1, r_2 and a.

Proposition 1 *The probability of being active in firm i,* φ_i*, is non-decreasing in* r_1 *and* r_2*, and non-increasing in* a*, for* $i = 1,2$.

Notice that φ_1 and φ_2 are aggregative probabilities; that is, φ_1 is the sum of the probability of being in states AA and AP, and φ_2 is the sum of the probability of being in states AA and PA. Proposition 1 indicates that these aggregative probabilities are a non-decreasing function of r_1 and r_2, and a non-increasing function of a. However, their components – the probability of being active in states AA, AP, and PA – do not necessarily follow the results in Proposition 1. For example, it can be shown that the probability of being in state AP is decreasing in r_2, and the probability of being in state PA is decreasing in r_1. Counterintuitively, the probability of being active in firm 1, φ_1, can be increasing in r_2 as one of its two components (i.e., the probability of being in state AA) is increasing in r_2 at a greater rate in absolute value than the decreasing rate of its other component (i.e., the probability of being in state AP). For a similar reason, φ_1 is decreasing in a.

Now, knowing the users' best responses about which firm to contribute to, and hence knowing each firm's demand $\varphi_i^*(\boldsymbol{r})$, I solve for the two firms' reward decisions at equilibrium (r_1*, r_2*). Next, I study the firms' competition problem.

2.4.2 Firms' Equilibrium Reward Decisions Under the General Case of *a*

Firms could anticipate the reactions of online users by calculating $\varphi_i^*(\boldsymbol{r})$ and then maximizing their net revenue accordingly. To analyze the simultaneous game between two firms, I start by examining the firms' net revenue functions given in equation (1). As mentioned in 4.1, firm 1 chooses $r_1 \leq 1$ and firm 2 chooses $r_2 \leq a$; that is, the firms never reward beyond their users' maximum cost of being active. This is because, by paying $r_1 > 1$, $\varphi_1 = 1$ and firm 1's payoff is $w - r_1$, which is dominated by paying $r_1 = 1$. By the same logic, firm 2 never pays $r_2 > a$. Note that website i also never chooses r_i higher than w because any reward higher than this level generates a negative revenue in equation (1).

Next, I analyze the equilibrium of the simultaneous game between two firms, denoted by (r_1*, r_2*). Theorem 1 demonstrates that the game has a unique Nash equilibrium. This equilibrium exists in only one of the three cases discussed in Lemma 1.

Theorem 1 *There exists a unique Nash equilibrium in pure strategies for firms' reward structures such that:*

- *If $a \leq 2$, then $r_1^* \leq 1$ and $(a-1)r_1^* \leq r_2^* \leq ar_1^*$, i.e., the equilibrium exists in Cases II and III only.*
- *If $a > 2$, then $r_1^* \leq 1$ and $r_2^* < (a-1)r_1^*$, or $(a-1)r_1^* \leq r_2^* \leq ar_1^*$ and $r_1^* + r_2^* > a$, i.e., the equilibrium exists in Cases I and III only.*

Theorem 1 states that firms' equilibrium is in one of Cases I, II, or III in Figure 1. The structure of the equilibrium depends on a and results in two possibilities: (1) when $a \leq 2$, the equilibrium is in a convex set that includes only Case II and Case III (Panel A, Figure 1); (2) when $a > 2$, the equilibrium is in a *non-convex* set, which is composed of Case I and Case III (Panel A, Figure 1).

Theorem 1 also states that the firms' game has a unique, pure-strategy Nash equilibrium of reward decisions. This implies that the users' (equilibrium) probability of being active with regard to each firm, φ_1 and φ_2, is also unique. In other words, Theorem 1 extends the equilibrium to include the firms' interaction with the users. At equilibrium, however, some users with specific cost levels (c, k) may be indifferent about their status within two firms (see equation 2). Nevertheless, since users are spread over continuous C and K spaces, the possibility of this case is zero. Hence, these "boundary" users do not affect the uniqueness of firms' market sizes and reward decisions at equilibrium.

Next, I consider how to specifically derive the equilibrium solution (r_1^*, r_2^*). First, for a given a, I define a unique threshold, $\hat{w}_a$, which is used to determine which feasible case contains the equilibrium. Specifically, if $w > \hat{w}_a$, the equilibrium is in Case III. If $w \leq \hat{w}_a$, the equilibrium depends on a. That is, if $a \leq 2$, the equilibrium is in Case II; otherwise, the equilibrium is in Case I. The threshold $\hat{w}_a$ can be calculated as follows.

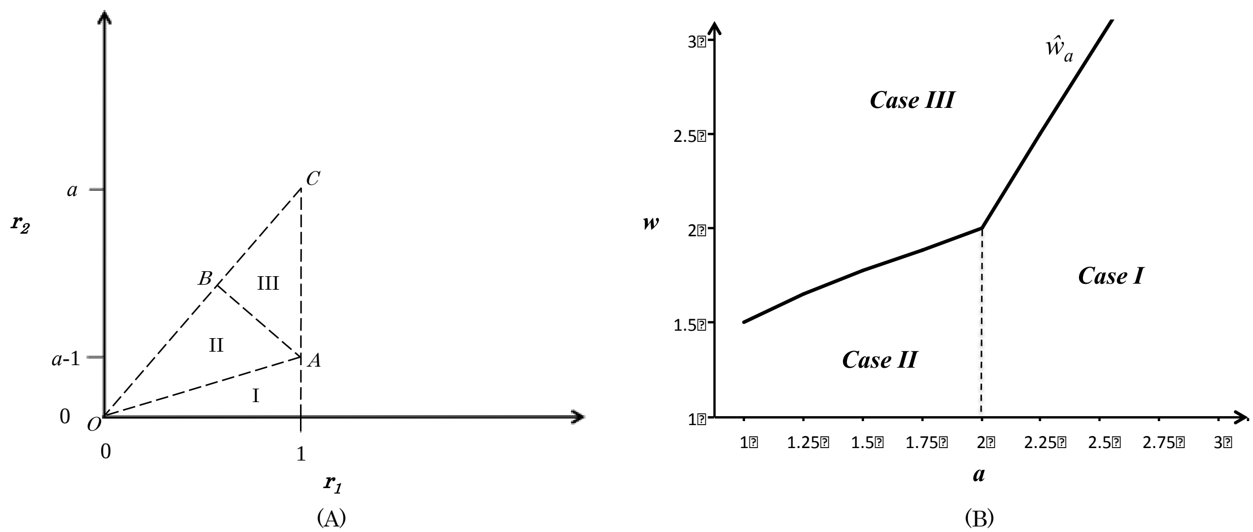

Figure 1: Cases of Lemma 1 (Panel A), and Threshold for w_a based on *a* (Panel B)

Lemma 2 *For a given a, there is a unique threshold $\hat{w}_a$, such that if $w \leq \hat{w}_a$, then $r_1^* + r_2^* \leq a$, and if $w > \hat{w}_a$, then $r_1^* + r_2^* > a$. $\hat{w}_a$ is derived as follows:*

- *For $a \leq 2$, $\hat{w}_a$ is solved from the following system of equations:*

$$\begin{cases} \widehat{w}_a = 2r_2 + \dfrac{r_1 log(r_1)}{log(r_2) - log(a)} \\ \widehat{w}_a = 2r_1 + \dfrac{r_1 r_2 log(r_1)}{ar_1 - r_2 - r_1 log(r_1)} \\ r_1 + r_2 = a \end{cases} \tag{3}$$

- *For $a > 2$,* $\widehat{w}_a = 2(a-1)$ (4)

The solid line in Panel B, Figure 1, illustrates the general shape of $\hat{w}_a$ as a function of a. Note that $\hat{w}_a \geq 1.5$ and is increasing in a. This finding implies that, as the asymmetry increases between two firms, for a fixed w, the equilibrium moves from Case III toward Cases I or II. Moreover, by increasing the w, the sum of rewards increases at equilibrium.

Note that, when $a \leq 2$, no closed-form solution exists for $\hat{w}_a$ from the system of equations (3), and it has to be solved numerically.[6]

The special cases when $w = \hat{w}_a$ are as follows: If $a \leq 2$, then the unique equilibrium solution should satisfy $r_1^* + r_2^* = a$. In this case, the equilibrium is on the boundary of Cases II and III. When $a > 2$, then $r_1^* = 1$ *and* $r_2^* = a - 1$, the equilibrium is a unique boundary point between Cases I and III. Next, I present the conditions to derive the equilibrium for each case by solving the corresponding systems of equations. Lemmas 3 to 5 give the form of equilibrium solution in Cases I to III, respectively.

Lemma 3 *If $a > 2$ and $w \leq \hat{w}_a$ (Case I), then* $r_1^* = min\left(\frac{w}{2}, 1\right)$ *and* $r_2^* = \frac{w}{2}$.

When $a > 2$ and $w \leq \hat{w}_a$, the equilibrium is in Case I, and the firms' net revenues are as follows:

$$\pi_1 = (w - r_1)\, r_1$$

$$\pi_2 = (w - r_2)\, r_2 log(\frac{a}{a-1})$$

In this case, when $w \leq 2$, $r_1^* = r_2^* = \frac{w}{2}$; both rewards increase linearly with respect to w and the equilibrium is symmetric, despite the asymmetry between two firms. In Section 5, I discuss why the less favourable firm pays the same as the more favourable firm, even when it has a smaller market share. When $2 \leq w \leq \hat{w}_a$, the symmetric equilibrium no longer holds, whereas the more favourable firm shares the unit of cost only, the reward for the less favourable firm increases linearly with respect to w, and this firm pays an even higher reward than its competitor.

Now, assume $a \leq 2$ and, $w \leq \hat{w}_a$. The equilibrium can be derived based on Lemma 4.

Lemma 4 *If $a \leq 2$ and $w \leq \hat{w}_a$ (Case II), then r_1^* and r_2^* are the feasible solutions to the following system of equations:*

---

[6] When $a \leq 2$, a close approximation of $\hat{w}_a$ is $\widetilde{w}_a = 1.5\, a^{\sqrt{2}-1}$.

$$\begin{cases} (w-2r_1)\left(r_1\,log\left(\frac{r_1+r_2}{ar_1}\right)-r_2+ar_1\right)-r_1r_2\,log\left(\frac{r_1+r_2}{ar_1}\right)=0 \\ (w-2r_2)\,log\left(\frac{r_1+r_2}{r_2}\right)-r_1\,log\left(\frac{r_1+r_2}{ar_1}\right)=0 \end{cases} \tag{5}$$

where the $\hat{w}_a$ is the unique solution of the system of equations (3) in Lemma 2.

The system of equations (5) cannot be solved in closed form in general. In this case, the equilibrium is in Case II, and the net revenue functions of firms are as follows:

$$\pi_1 = (w-r_1)\left((r_1+r_2)log\left(\frac{r_1+r_2}{ar_1}\right)+ar_1-r_2\right)$$

$$\pi_2 = (w-r_2)\left((r_1+r_2)\,log\left(\frac{r_1+r_2}{ar_1}\right)+r_2log(ar_1)-r_2log(r_2)\right)$$

As I discussed, in Case II, the sum of rewards at equilibrium is less than *a*. In Section 6, I numerically show that the *sum* of rewards at equilibrium increases *linearly* with respect to *w* when *w* is less than $\hat{w}_a$. In other words, for a low *w*, the reimbursement of users in state AA increases linearly with respect to *w* when *w* is low. However, this is not the case when *w* is high. In Case III, the sum of rewards at equilibrium increases *concavely* with respect to *w*, as firms cannot continue increasing their rewards linearly. (See Section 6 for more details.)

Notice that, together, firms never pay beyond $a + 1$, no matter how large the *w* is, since the firms never reward beyond their users' maximum cost. Also, by decreasing *a*, the intensity of the competition increases, and r_1* and r_2* transfer from Case I to Case II. Next, I have the following result, when $w > \hat{w}_a$.

Lemma 5 *If $w > \hat{w}_a$ (Case III), then r_1* and r_2* are found through a two-step process:*

(1) Determine $\hat{r}_1$ and $\hat{r}_2$ as the solution to the following system of equations:

$$\begin{cases} (w-2r_1)(r_1\, log(r_1) + r_2 - ar_1) + r_1(r_1 + r_2 - a - \ r_2\, log(r_1)) = 0 \\ (w-2r_2)log\left(\frac{a}{r_2}\right) - r_1 - r_2 + a + \ r_1\, log(r_1) = 0 \end{cases} \tag{6}$$

(2) Let $r_1^* = \begin{cases} \hat{r}_1 & if\ \ \hat{r}_2 \leq a\hat{r}_1\ and\ a - \hat{r}_2 \leq \hat{r}_1 \leq 1 \\ 1 & otherwise \end{cases}$ (7-1)

and r_2 is found by solving for r_2 in:*

$$(w-2r_2)log\left(\frac{a}{r_2}\right) - r_1^* - r_2 + a + \ r_1^*\, log(r_1^*) = 0. \tag{7-2}$$

Lemma 5 demonstrates that, when $w > \hat{w}_a$, the equilibrium can be calculated through a two-step process. First, I derive the solution of the system of equations (6); I label the solution of this system by $\hat{r}_1$ and $\hat{r}_2$. If this solution is feasible – as it is in Case III – the solution is the equilibrium of the game. However, the solution of system (6) can be outside Case III. I show that this can happen only if the equilibrium of the game is on line AC in Figure 1; that is, when $r_1^* = 1$. In this case, r_1^* is calculated based on Equation (7-1), and r_2^* is derived through solving Equation (7-2) in the second step. In Case III, the net revenue functions of firms are as follows:

$$\pi_1 = \ (w - r_1)\left(r_1 + a(r_1 - 1) - (r_1 + r_2)log(r_1)\right)$$

$$\pi_2 = \ (w - r_2)\left(r_1 + r_2 - a - (r_1 + r_2)log(r_1) + r_2 log(ar_1) - r_2 log(r_2)\right)$$

Analogous to Case II (Lemma 4), I am unable to find the closed-form solution for r_1^* and r_2^*, based on w and a, in Case III (Lemma 5) as well. In general, the closed-form solution for equilibrium can be derived in Case I (Lemma 3) only. However, I illustrate that, in the symmetric game, it is possible to find a closed-form solution for the equilibrium under Case II (Proposition 2). Moreover, in Section 7, I provide numerical results for the asymmetric game and show the condition under which the equilibrium is symmetric despite the asymmetry between the two firms. When $a > 2$ and $w > \hat{w}_a$, Lemma 5 can be simplified as follows:

Corollary 1 *If $a > 2$ and $w > \hat{w}_a$, then $r_1{*} = 1$, and $r_2{*}$ is found by solving for:*

$$(w - 2r_2)log\left(\frac{a}{r_2}\right) - r_2 + a - 1 = 0. \tag{8}$$

Corollary 1 indicates that, when $a > 2$ *and* $w > \hat{w}_a$, firm 1 always shares the amount equal to the unit cost. In this case, the equilibrium is on line AC at Panel (A) in Figure 1. I have already proven the existence of equilibrium in Theorem 1. All equilibrium cases are summarized in Table 2. Next, I study the outcome of equilibrium when either the asymmetry (a) or incremental revenue (w) increases to the highest point.

Corollary 2A *The asymptotic limit of the equilibrium reward solution with respect to a is given by* $\lim_{a\to\infty} r_1^* = min\left(\frac{w}{2}, 1\right)$ *and* $\lim_{a\to\infty} r_2^* = \frac{w}{2}$.

Corollary 2B *The asymptotic limit of the equilibrium reward solution with respect to w is given by,* $\lim_{w\to\infty} r_1^* = 1$ *and* $\lim_{w\to\infty} r_2^* = a$.

Corollaries 2A and 2B simplify the previous results for the extreme cases of a and w. Notice that when a goes to infinity, as I might expect, no user contributes to the less favourable firm, and the duopolistic setting converts to a monopolistic setting. In this case, $\lim_{a\to\infty} \pi_1^* = (w-1)\, min\left(\frac{w}{2}, 1\right)$ *and* $\lim_{a\to\infty} \pi_2^* = 0$. These results also suggest that when a increases significantly, the less favourable firm may completely shut down its rewards program due to a lack of demonstrable profitability. This finding is consistent with what has been observed between YouTube and Metacafe (or even Facebook and MySpace). Also, notice that Corollary 2A and 2B are in conformity with Corollary 1. When w goes to infinity, $\varphi_1{}^*$ and $\varphi_2{}^*$ limit to 1, and both firms share the highest cost incurred by their users. Hence, $\lim_{w\to\infty} \pi_i^* = \infty$, *for i =1,2.*

Regarding the equilibrium, some questions may arise. For example, I have already mentioned that r_1 and r_2 are less than or equal to w. I might wonder whether the firms will be generous in their payments; that is, will they share a higher portion of w with their active users or keep (save) the major amount of it for themselves? Moreover, is it possible that, by sharing a higher payment, the less favourable firm gains a higher market share or revenue than the more favourable firm? I summarize these finding in Corollary 3.

Table 2: Different cases of equilibrium

	$w \leq \hat{w}_a$	$w > \hat{w}_a$
$a \leq 2$	Case II r_1* and r_2* are solved by (5)	Case III r_1* and r_2* are solved by (7.1) and (7.2)
$a > 2$	Case I $r_1^* = min\left(\frac{w}{2}, 1\right)$ and $r_2^* = \frac{w}{2}$	Case II r_1* = 1 and r_2* are solved by (8)

Corollary 3 *At equilibrium:*

- *The proportion of active users in the more favourable firm is higher than or equal to the proportion of active users in the less favourable firm, i.e.,* $\varphi_1^* \geq \varphi_2^*$.
- *The reward payments are less than or equal to half the incremental revenue generated by active users, i.e.,* $r_i^* \leq w/2$, *for* $i = 1, 2$.
- *The set of active users in the less favourable firm is a subset of the set of active users in the more favourable firm.*

First, Corollary 3 asserts that, at equilibrium, the probability of being active is higher for the more favourable firm than for the less favourable firm. This finding holds, even when the more favourable firm shares a lower reward than the less favourable firm.

Second, at equilibrium, firms keep at least half of w for themselves and never pay back more than half of this incremental revenue with their active users. In most cases, firms keep for themselves more than $w/2$. The only case where firms share $w/2$ is when $a \geq 2$ and $w \leq \hat{w}_a$.

Third, Corollary 3 states that the set of active users in the more favourable firm includes the set of active users in the less favourable firm; that is, no users will be passive with respect to firm 1 and active with respect to firm 2. This finding reflects the example of online users, such as Kip Kedersha, who anecdotally decided not to work exclusively for their original community (Metacafe). In doing so, they expanded their activities to an appealing competitor website (YouTube) and found that the competitor's platform was more popular and more convenient to work with.

2.5 Analysis of Asymmetry Between Firms

Due to the complexity of the model, obtaining closed-form equilibrium solutions is challenging. Therefore, to gain further insight regarding the impact of the asymmetry, I analyze certain instances and conduct numerical analyses. More specifically, I first analytically compare two special cases of a. Second, using a thorough numerical analysis, I study the firms' equilibrium reward decisions. Finally, I relax my assumption on the fixed a, and I investigate the case where customers are heterogeneous with respect to the asymmetry between two firms.

2.5.1 Impact of Asymmetry in Cost: Comparison of Special Cases of *a*

In the following section, I study two special cases when $a = 1$ and $a = 2$ in order to achieve some insights on how the asymmetry in firms' costs affects their reward decisions and revenues. When $a = 1$, the game is symmetric. The case with $a = 2$ represents another special situation since, as I see in section 4, this point is a threshold for two different cases of $a < 2$ and $a > 2$, in which the firms' reward decisions at equilibrium are significantly different. Recall that, when $a < 2$ and $w \leq \hat{w}_a$, the equilibrium is in Case II, but when $a > 2$ and $w \leq \hat{w}_a$, the equilibrium is in Case I. Moreover, in the case of $a < 2$, the game is more "intensive" than in the case of $a > 2$; as a result of asymmetry, when $a > 2$, there is lower chance for the more favourable firm to have a better position in the competition.

Symmetric Duopoly Case (a = 1). A question arises regarding how Lemmas 4 and 5 can be simplified when firms are symmetric. Moreover, I would like to examine whether the equilibrium is symmetric (i.e., if $r_1^* = r_2^*$) and study the arrangement of users' states at equilibrium. For example, which portion of users selects states AA, AP, PA or PP? For the symmetric case, $\hat{w}_a = 1.5$. I summarize the result in Proposition 2.

Proposition 2 *The symmetric game (a = 1) has a unique and symmetric Nash equilibrium, which can be calculated as follows:*

(a) If $w \leq 1.5$, then $r_1 = r_2* = w/3$.*

(b) If $w > 1.5$, then $r_1 = r_2* = r*$, where $r*$ is the feasible unique solution to the following equation:*

$$(w - 3r)log(r) + 2r - 1 = 0. \tag{8}$$

Proposition 2 presents a simplified form of Lemma 4 and Lemma 5 for the symmetric game, which has a symmetric equilibrium and fixed threshold $\hat{w}_a$ = 1.5. When $w \leq 1.5$, both firms share one-third of w with their active users. (The equilibrium is on line OB in Panel A, Figure 1.) This result represents a case in which the revenue contribution of an active user is less than 1.5 times of the cost of a user with the highest effort (unit of cost). When $w > 1.5$, the equilibrium is derived based on equation (8), i.e., the equilibrium is on line BC in Panel A, Figure 1.

Note that, for a symmetric game, users decide whether to contribute or not to contribute to *both* firms; that is, users never choose states AP and PA. For example, if w = 1.5, at equilibrium, nearly 70 (30) percent of users decide to be active (passive) in both firms, and the total rewards they receive from firms equal the unit of cost (i.e., $r_1* = r_2* = 0.5$). Each firms share one-third of w and keeps two-thirds. Although the equal probabilities of being in states AP and PA seem natural because of the symmetric setting, selection of neither of these states may seem counterintuitive.

Asymmetric Duopoly Case (*a* = 2). When a = 2, one would presume an asymmetric equilibrium since the users' cost of contribution to the less favourable firm is twice the cost of contribution to the more favourable firm. In this case, $\hat{w}_a$ = 2 and the equilibrium solution is given in Proposition 3:

Proposition 3 *When a = 2, the game has a unique Nash equilibrium, which can be calculated as follows:*

(a) If $w \leq 2$, then $r_1 = r_2* = w/2$.*

(b) If $w > 2$, then $r_1^* = 1$ and r_2^* is the feasible unique solution to this equation:

$$(w - 2r_2)log\left(\frac{2}{r_2}\right) - r_2 + 1 = 0. \tag{9}$$

An interesting result is that, when $w \leq \hat{w}_a = 2$, the equilibrium is symmetric for the asymmetric game. Both firms share half of w with their active users. When $w > 2$, the less favourable firm gives a higher reward than the more favourable firm; however, the less favourable firm always has a lower market share and revenue, even when it pays more than its competitor. For example, when $a = 2$ and $w = 3$, at equilibrium, nearly 83 percent of users decide to be active in both firms, and the remaining users exclusively contribute to the more favourable firm. Note that, when $a = 2$, the equilibrium is on line OA of Panel A in Figure 1 if $w \leq 2$, and it is on line AB if $w > 2$.

Next, I compare the outcomes of the symmetric ($a = 1$) and asymmetric ($a = 2$) games, and I observe that, for both firms, payments in the asymmetric game are higher than those in the symmetric game. Also, the proportion of active users in the asymmetric game is greater (less) than the proportion of active users in the symmetric game for the more (less) favourable firm. Define π_i^{a*} as the revenue of firm i at equilibrium for a specific a. Corollary 4 compares the above cases.

Corollary 4 *The net revenue in the symmetric game is higher than the net revenue in the asymmetric game for both firms, i.e., $\pi_i^{1*} > \pi_i^{2*}$, for $i = 1, 2$.*

Corollary 4 demonstrates that increasing asymmetry results in shrinkage of net revenues for both firms. Revenue shrinkage for the less favourable firm is not surprising due to the higher reward paid out and the loss in customers. However, the diminishing revenue for the more favourable firm is counterintuitive. This result can be explained by the fact that, as a increases, while φ_1 increases, the more favourable firm has to share a higher reward in order to better compete with its rival, which also increases its rewards. The negative impact of the higher reward is greater than the positive impact of the higher market share on the firm's revenue. Therefore, the more favourable firm ultimately loses revenue.

Since the above results come from the comparison of just two special cases for a = 1 and 2, the question arises as to whether these results still hold for other general cases of a. In the next section, I aim to answer this question through a numerical study. I will demonstrate whether increasing asymmetry *always* increases (reduces) the proportion of active users for the more (less) favourable firm and whether it shrinks the revenue for the more favourable firm. I further explore this effect in subsection 5.2.

2.5.2 Numerical Analysis for the General Cases of a

In Lemmas 3, 4, and 5, I presented the Nash equilibrium. In this section, I expand my findings about the revenue-sharing equilibrium through solving the game numerically for specific values of w and a. The following analysis reports on a subset of 64 parameter combinations, spanned by

$$a \in \{1, 1.25, 1.5, 1.75, 2, 3, 5, 20\}, \qquad w \in \{0.5, 1, 1.5, 2, 3, 5, 20, 100\}.$$

I find this selection of parameter combinations to be sufficient for analysing the firms' equilibrium reward decisions. I start this discussion with the symmetric case (a = 1) and then review the results of the asymmetric case (a > 1). In the symmetric case, when a = 1, as prescribed by Proposition 2, $r_1^* = r_2^*$. See the dashed line at a = 1 in Figure 2 for the *first symmetric region*. Furthermore, if $w \leq 1.5$, then $r_1^* = r_2^* = w/3$. When $w > 1.5$, then r_1^* and r_2^* are monotonically increasing in w and converge to 1. I also see that the probability of being active φ_i and net revenues π_i are monotonically increasing in w and converge to 1 and $w - 1$, respectively. In the asymmetric case, when $a > 1$, I observe some interesting and counterintuitive results:

Existence of symmetric equilibrium for the asymmetric game. First, when w is "low" and $1 < a < 2$, r_1^* and r_2^* are very close, since with low w, there is little room to share rewards, and both firms pay approximately the same amount for the asymmetric game. Figure 2 presents the *first asymmetric region*, where the more favourable firm pays slightly more. In this region, the difference between payments of rewards starts increasing when a increases; however, after a certain level of a, the gap starts to diminish, so that, at $a = 2$ for $w \leq 2$, both firms pay exactly the same amount. See Figure 2 for the *second symmetric region*.

It may seem counterintuitive that the symmetric equilibrium is at $a \geq 2$ when w is not very "high." The reason for this result is that, in this case, the market competitiveness of the second firm is weakened to the utmost, and thus, active users prefer to join the more favourable firm. However, to increase their surplus, they also decide whether they may work for the less favourable firm in addition to the more favourable firm or not.

To attract active users, at this point, the less favourable firm should pay them at least the same as the more favourable firm pays in an effort to convince them to join. Note that, when a is smaller, active users can be convinced to join by the less favourable firm with a slightly lower payment compared to the payment of the more favourable firm. I also find that, for small w, $w/3 \leq r_i^* \leq w/2$ where $w/3$ and $w/2$ are the solutions of Propositions 2 and 3, respectively.

Second, when w is high, the more favourable firm shares a higher reward when a is not too high. Nevertheless, when w increases, after a certain level, both firms pay the same amount. This finding indicates an extension of the situation discussed above, in which an asymmetric game leads to a symmetric equilibrium where both firms pay $r_i^* = 1$ (Figure 2); the dashed line represents this case ($1 < a < 2$). When w increases, the less favourable firm pays a higher reward than the more favourable firm.

Figure 2 illustrates the *second asymmetric region*. In this case, firm 1 receives more benefits than firm 2 because firm 1 pays less than its competitor at equilibrium, while it attracts more users (Corollary 3). Conversely, when w is low, firm 1 still receives higher net revenue through paying a higher reward, although the difference between the revenue of the two firms increases since the second firm shares higher rewards without attracting more active users. Figure 3 displays the equilibrium reward level as well as the firm surplus, $w - r_i$, and shows how firms share the marginal revenues between themselves and their active users. As the asymmetry between two firms, a, increases, both firms share higher revenues and keep less revenue for themselves. Notice that they never share beyond $w/2$ (i.e., half of the marginal revenues) with users.

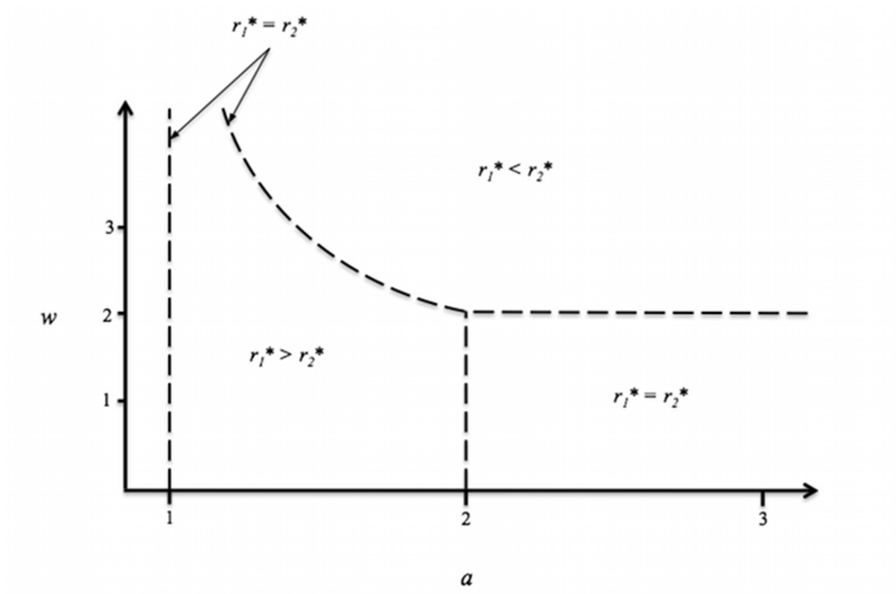

Figure 2: Comparison of equilibrium reward payments

Non-monotonic relationship between φ_i* and *a*. First, when a increases, firms' rewards increase monotonically. The dynamics of the game explain why this happens. As a increases, the less favourable firm pays a higher reward, which provides a trade-off for its higher cost to keep its users. However, this scenario pushes the more favourable firm to increase its payment at the same time in order to avoid becoming non-competitive by failing to share a stimulating reward, as its competitor has done.

Second, for both firms, the probability of being active monotonically increases in w (Figure 4). As I demonstrate in Corollary 3, compared to the less favourable firm, the more favourable firm has a higher probability of gaining active users. While φ_2 does *not* decrease monotonically in a when $w > \hat{w}_a$, φ_1 does not increase monotonically in a when $w \leq \hat{w}_a$; the decreasing φ_1 and increasing φ_2 with respect to a occur when a is small. This exceptional effect is an extension of this finding in Corollary 4 in Section 5.1. Furthermore, an increase in φ_1 with respect to a at equilibrium may seem to conflict with Proposition 1, which suggests decreasing φ_1 with respect to a. Notice that Proposition 1 analyzes the game in a static setting with fixed r_1 and r_2, while, at equilibrium, both firms increase their rewards, and the game ends up with an increasing φ_1 with respect to a.

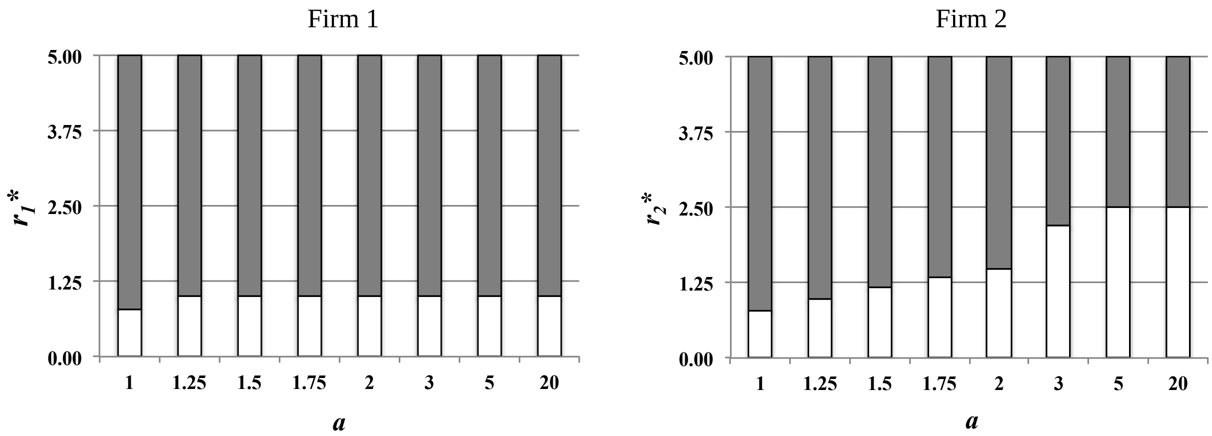

Figure 3: The equilibrium reward level, r_i*, (white region) as well as firms' surplus, $w - r_i$*, (grey region) for case of $w = 5$

Decreasing the net revenue of the more favourable firm in $a < 2$. Increasing w leads to an increase in the net revenue for both firms, while increasing a leads to a decrease in the net revenue for less favourable firms and in the more favourable firm when $a < 2$ (Figure 4). Decreasing the net revenue of the less favourable firm by increasing a is intuitive. Surprisingly, the more favourable firm also loses net revenue, but it is less affected.

As I see from Corollary 4, it may seem counterintuitive to decrease the net revenue for the more favourable firm when $a < 2$. Recall that, as a increases, while φ_1 increases, the more favourable firm must share a higher reward and keeps a lower saving at equilibrium. Since the negative impact of the higher r_1 is greater than the positive impact of φ_1 on revenue of the more favourable firm (equation 1), this firm ultimately loses revenue. When $a > 2$, the net revenue of the more favourable firm is *independent* of a; that is, it is equal to $w^2/4$ when $w \leq 2$ and to $w - 1$ when $w > 2$. Next, I investigate the case where customers are heterogeneous with respect to a (i.e., where a is a random variable).

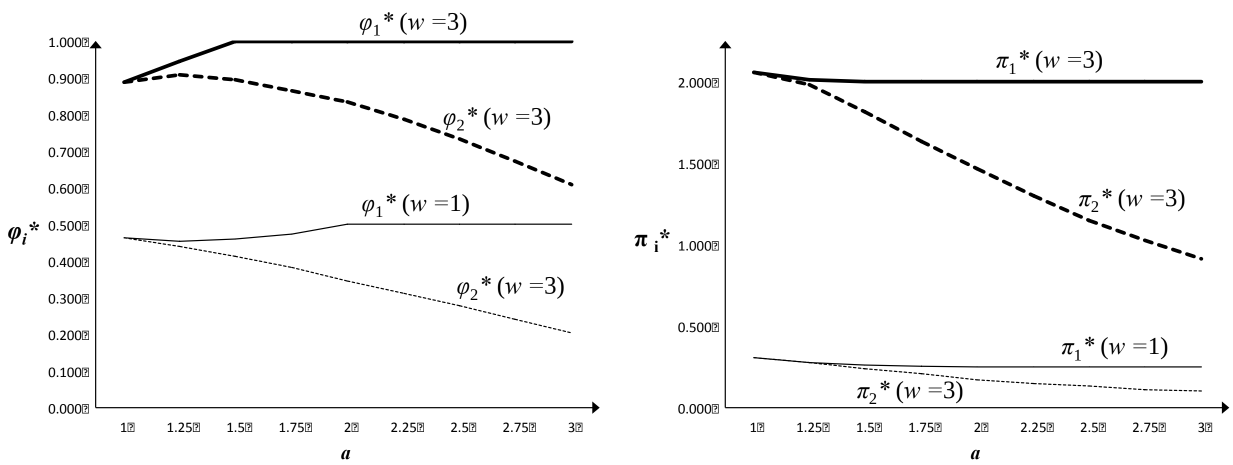

Figure 4: Proportion of active users, φ_i* (left) and net revenues of firms π_i* (right) at equilibrium

2.5.3 Heterogeneous Users with Respect to a

In the previous sections, I have assumed that an asymmetry exists in users' preferences between the two firms. In other words, for all users, one firm is a more favourable firm, and the asymmetry factor between two firms, *a*, is the same for all users. Next, I consider the case where customers are heterogeneous about references of two firms. The heterogeneous case contains two scenarios: First, customers still believe that firm 1 is the more favourable firm, but they may have different level of asymmetry factor, *a*. I refer to this scenario as a *compatible* market. Second, some customers may regard firm 2 as more favourable. I refer to this scenario as a *dissident* market.

To analyze the case where users have different preferences with respect to firms, I solve the game numerically when the asymmetric factor, *a*, follows a three-point distribution with mean a_M. Specifically, *a* takes three values – a_L, a_M, and a_H – with equal probability, where $a_H \geq a_M = a \geq a_L \geq 0$. As an extension of the two-point distribution, a three-point distribution is widely used in literature since it can be used effectively to approximate many well-known and practical distributions, such as normal, lognormal, and beta distribution; see Desai, Koenigsberg, and Purohit (2007), and Yang and Schrage (2009). Notice that, for a dissident market, $a_L < 1$, and for a compatible market, $a_L \geq 1$. I say that *a* becomes more divergent when a_L and a_H are farther from a_M. Notice that $\varphi_i(\boldsymbol{r}, A) =$

$\frac{1}{3}\big(\varphi_i(\boldsymbol{r}, a_L) + \varphi_i(\boldsymbol{r}, a_M) + \varphi_i(\boldsymbol{r}, a_H)\big)$, where $\varphi_i(\boldsymbol{r}, A)$ is the probability of being active in firm *i*, *for i* =1,2, when *a* is a random variable with cumulative distribution function *A(a)*. Also, $\varphi_i(\boldsymbol{r}, A) \leq \varphi_i(\boldsymbol{r})$, since $\varphi_i(\boldsymbol{r})$ is concave in *r* for a compatible market. (See the proof for Proposition 1.) This result implies that, for a compatible market, as *a* diverges, fewer users choose to be active in each firm for the same reward level. This result does not necessarily hold for a dissident market.

I numerically solve the game for the case where the joint contribution factor is fixed, K = 0.7. (I analyze the fixed *K* case in Section 6 and derive the probabilities of being active in Lemma 6A.) I compare the equilibrium solutions between the case where *a* is constant and the cases where *a* follows different three-point distributions. The results are summarized in Table 3. The three sections of Table 3 correspond to cases in which half–width of the distribution are 0, 0.5, 0.75 (for a_M = 1) and 0, 1, 1.5 (for a_M = 2) and 0, 4 and 4.5 (for a_M = 5).

Table 3: Equilibrium reward payments for different three-point distribution of *a*

(r_1*, r_2*) k = 0.7		$[a_L,\ a_M,\ a_H]$								
		[1, 1, 1]	[0.5, 1, 1.5]	[0.25, 1, 1.75]	[2, 2, 2]	[1, 2, 3]	[0.5, 2, 3.5]	[5, 5, 5]	[1, 5, 9]	[0.5, 5, 9.5]
	1	(0.33, 0.33)	(0.44, 0.44)	(0.44, 0.44)	(0.5, 0.5)	(0.45, 0.41)	(0.5, 0.5)	(0.5, 0.5)	No Eq.	(0.5, 0.5)
w	3	(0.85, 0.85)*	(1.1, 1.1)*	(1.22, 1.22)*	(1, 1.5)	(1, 1.31)	(1.5, 1.5)	(1, 1.5)	(1, 1.18)	(1, 1.5)
	5	(0.85, 0.85)*	(1.1, 1.1)*	(1.22, 1.22)*	(1, 1.7)*	(1, 1.33)	(2, 2.55)*	(1, 2.5)	(1, 1.32)	(2, 1.5)

* The equilibrium is not unique.

Table 3 demonstrates the firms' rewards at equilibrium under different marginal revenue (*w*) and different distributions of *a* (with different mean and variance). Comparing the deterministic *a* case with the stochastic *a* case, I observed that at least one equilibrium exists for the deterministic case, but the stochastic case may not have an equilibrium. In spite of this difference between the two cases, for some scenarios of *w* and *a*, the equilibrium is not unique for both cases. When the equilibrium is not unique, in these scenarios, the sum of rewards at equilibrium is constant for both cases. For example, for deterministic cases, when $w = 3$, $k = 0.7$ and $a_L = a_M = a_H = 1$, at equilibrium $r_1* + r_2* = 1.7$, where $0.7 \leq r_1* \leq 1$. In this case, φ_1 and φ_2 are equal and depend on the sum of r_1 and r_2. (I derive the probabilities of being active in both firms in section 6; please refer to Case II in Lemma 6A for this scenario.) Therefore, any combination of r_1 and r_2 with fixed summation generates the same proportion of active and passive users, and each firm's best reaction to a competitor's action is to pay the remaining share of the fixed total rewards.

I also investigate the difference between the deterministic and stochastic *a* cases in terms of the amount of rewards. In the deterministic case, I observe the following result:

Corollary 5 *For the deterministic a case, if* $a \geq 2$, $r_1* \leq r_2*$.

Corollary 5 asserts that, when $a \geq 2$ for the deterministic *a* case, the more favourable firm always pays a lower reward level than the less favourable firm. I find, however, that the less favourable firm may have a lower reward when *a* is stochastic, more specifically, when *w* and a_M are very high and $a_L < 1$. In this special scenario of a stochastic *a* case, the market is in an extremely dissident state, and the more favourable firm should pay a higher reward to accommodate all users with conflicting tastes. Theretofore, the less favourable firm may have a higher reward when *a* is stochastic, while, when *a* is deterministic, the more favourable firm's reward is always lower.

Based on the results presented in Table 3, for a given value of *w*, as *a* diverges, both firms could pay higher or lower rewards, depending on whether the market is compatible or dissident and whether competition is symmetric or asymmetric. Specifically, for an asymmetric competition ($a_M > 1$), when *a* diverges, both firms pay lower rewards in a compatible market, but as *a* diverges further, they both pay higher rewards in a dissident market. For a symmetric completion ($a_M = 1$), when *a* diverges, both firms pay lower rewards.

As previously discussed, for the same reward levels, the probability of being active for the stochastic *a* case is lower than the probability of being active for the deterministic *a* case. In equilibrium, however, firms may receive a higher or lower revenue in the stochastic case since they pay lower or higher rewards in this case. Comparing firms' revenues under deterministic and stochastic cases, I observe that, in most situations, firms' revenues for both more favourable and less favourable firms are lower under the stochastic case. The only exception occurs when the market is in an extremely dissident state and the incremental revenue, *w*, is high. In this scenario, the less favourable firm receives a higher revenue under the stochastic case since it benefits from a higher payment from the more favourable firm.

2.6 Analysis of Users' Cost for Simultaneous Contribution

In the previous sections, I assumed that K (i.e., the parameter that associates with the cost of being active in both firms simultaneously) is uniformly distributed between 0 and 1. This represents a case, for example, in social media and digital markets, where users have economy of scale in being active in both firms since the cost of simultaneous participation in both firms is lower than the sum of separate participation cost in each firm. In this section, I also consider the case where $K > 1$, where the users have diseconomy of scale for joint contribution to both firms. This case is very popular in manufacturing and service industries since it is very costly for a user (employee) of a firm to work for another firm simultaneously. To focus on the comparison of the two cases where $K \leq 1$ and $K > 1$, I assume that K is a constant and I derive the equilibrium rewards.

Similar to Lemma 1, the following lemmas describe only those cases of interest. (The remaining cases are summarized in the Appendix.)

Lemma 6A *For $K \leq 1$, the probability of being active in firms 1 and 2 is as follows:*

- *Case I: if $r_1 \leq 1$ and $r_2 \leq (a+K-1)r_1$, then $\varphi_1^*(\boldsymbol{r}) = r_1$ and $\varphi_2^*(\boldsymbol{r}) = \frac{r_2}{a+K-1}$.*
- *Case II: if $(a+K-1)\, r_1 < r_2 \leq \left(\frac{a}{K}\right) r_1$, and $r_1 + r_2 \leq a+K$, then $\varphi_1^*(\boldsymbol{r}) = \varphi_2^*(\boldsymbol{r}) = \frac{r_1+r_2}{a+K}$.*

Lemma 6A presents the way a market of online users responds to firms' reward decisions r when all users incur the same cost of joint involvement in both firms. In Case I, φ_i depends only on r_i and not on r_j, for $i, j = 1, 2$, and $i \neq j$. In Case II, φ_1 and φ_2 depend on both r_1 and r_2. Similar to the stochastic K case, firms pay less than the maximum of their active users' cost (i.e., $r_1 \leq 1$ and $r_2 \leq a$). It can be shown that, for a fixed $K \leq 1$, the same properties regarding φ_i hold as in the stochastic K case; that is, the probability of being active in firm i, φ_i, is non-decreasing in r_i and non-decreasing in r_j. In other words, a firm enjoys a higher demand when its 'competitor' offers higher rewards to lure away users. This result stems from the economies of scale that are important in industries such as online social media. Since digital content can be duplicated without a high cost, consumers can generate content for two competitors – for example YouTube and Dailymotion – and receive a reward for their contribution from both firms (multi-home). Therefore, when YouTube increases its reward, online users become more willing to generate content on Dailymotion as well. As a result, more videos will be created on YouTube, but Dailymotion also benefits since those videos can be uploaded onto its website as well. In this way, economies of scale fundamentally change the competitive dynamics between the social media firms. Finally, the probability of being active in firm i, φ_i, is non-increasing in a and non-increasing in K.

Next, I formally present the equilibrium rewards for the case of a fixed $K > 1$.

Lemma 6B *For $K > 1$, the probability of being active in firms 1 and 2 is as follows:*

- *Case III: if $r_1 \leq 1$ and $r_2 \leq \left(\frac{a+k-1}{K}\right) r_1$, then $\varphi_1^*(\boldsymbol{r}) = r_1$ and $\varphi_2^*(\boldsymbol{r}) = \frac{r_2}{a+K-1}$.*
- *Case IV: if $\left(\frac{a+K-1}{K}\right) r_1 < r_2 \leq r_1 + a - 1$, and $1 < r_1 \leq K$, then $\varphi_1^*(\boldsymbol{r}) = 1 + \frac{r_1}{K} - \frac{r_2 - r_1}{a-1}$ and $\varphi_2^*(\boldsymbol{r}) = \frac{r_2 - r_1}{a-1}$.*

Lemma 6B also studies Cases III and IV, in which $K > 1$. In Case IV, φ_i depends only on r_i and not on r_j, for $i, j = 1, 2$, and $i \neq j$. In Case IV, φ_1 and φ_2 depend on both r_1 and r_2. The $K > 1$ case is different than $K \leq 1$ case in two ways. First, in $K > 1$ case, firms may decide to pay more than the maximum of their active users' cost (i.e., $r_1 > 1$ and $r_2 > a$). When there is no economy of scale, $K > 1$, users should pay a high cost for generating content for both firms. As a result, firms should pay higher rewards and compete harder to encourage active users to join them. Second, the probability of users being active for two firms (as before) increases linearly with their own rewards; however, it is non-increasing with the competitor reward for the $K > 1$ case.

When $K > 1$, the consumers face diseconomies of scale when they participate in multiple firms; as firm *i*'s competitor offers a higher reward, fewer users join firm *i*. This is the case for many traditional manufacturing and service industries, in which users cannot participate freely in both firms. When K is sufficiently large, the users may prefer a single home in one community rather than joining multiple communities. When consumers can select only one firm (single-home), both firms have to compete harder to gain their usership. I highlight the fact that the equilibrium solution can be simplified by fixing K under some cases.

I conclude this section with a note on parameter K, the cost of being active simultaneously in both firms. It has been stated that, for online social media, users have economy of scale for joint participation. For example, in most cases, users encounter economies of scale for posting videos to different sites, since the major cost of video-sharing comes from the cost of video creation. Therefore, once a video is created, it is relatively inexpensive to upload it on different websites. However, for certain types of websites with user-generated content, online users may also face a diseconomy of scale for joint participation. This can occur, for example, when active users keep track of all activities and post in online networks and generate many feedbacks to online comments, which can be a very time-consuming and costly task. Other factors that contribute to a high online users' cost are the large size of the network, peer pressure, and copyright and contract regulations.

The high cost of contribution in user-generated content has been addressed in the literature. For example, Godes, Ofek, and Sarvary (2009) find evidence that the declining online trend for already highly rated books is linked to the high cost of rating. Kumar, Sun, and Srinivasan (2010) find that contribution status or positional utility play an important role in level of contribution. These authors found that older consumers are more sensitive to the cost of contribution, but interestingly, consumers with higher centrality are less sensitive to contribution cost. Thus, online users of social networks may face either economy of scale (i.e., $K \leq 1$) or diseconomy of scale (i.e., $K > 1$), depending on whether joint participation indeed reduces users' cost by considering all the factors mentioned above.

2.7 Conclusion

This study investigates revenue-sharing strategies of online social media and shows how these strategies shape the contribution levels of online users. While the two opposite strategies of *saving* and *sharing* revenue have their pros and cons for the firms, the proposed model aims to provide insights for firms to consider the trade-off between these strategies within a state of asymmetric competition. The game consists of two steps: first, the OSMs announce their rewards for active users; and second, users choose their level of contribution with respect to each OSM, based on their preferences. I derived the revenue function for the firms by assuming they seek to maximize net revenue. The existence and uniqueness of equilibrium was shown for this duopoly game.

My results indicate that users always select the more favourable firm whenever they decide to work exclusively for one firm. Anecdotally, these results are consistent with observed online evidence. For example, within the emergence of YouTube, all online celebrities opened a channel with this medium. Some of these results are even observable for OSMs who do not compete based on revenue-sharing programs. An example includes the ongoing challenge of Google+ to compete with Facebook.[7] Many social media connoisseurs were highly optimistic about Google+ when it first emerged (and even used it exclusively at that point), but over time, they reopened their Facebook accounts.[8] When a firm decides to enter an online competition, it should investigate how online users position it relative to a competitor. My results indicate that, even for a huge asymmetry between two firms, a market share still exists for the less favourable firm (or niche player), which explains why a large number of small websites now co-exist with dominant websites such as Facebook and YouTube.

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[7] URL: http://online.wsj.com/article/SB10001424052970204653604577249341403742390.html; accessed: 2014–5–10.

[8] URL: http://www.forbes.com/sites/insertcoin/2012/04/16/six-months-later-google-plus-still-doesnt-add-up/; accessed: 2014–7–10.

A less favourable firm can better position itself in an online competition if it continuously improves the users' perspective of its online community. For example, while the popularity of iTunes makes it a favourable service for artists, downloads from iTunes are easily accessible on an iPod or other device as well, which makes music more readily accessible for fans. These results suggest that small social media firms can compete with a dominant firm in the market by providing users with a service that has a higher utility.

The revenue generated by users' contributions also plays a significant role with respect to the outcome of the competition. If users' contributions easily generate revenue, then both firms share high rewards and earn high net revenues as well. However, the more favourable firm can better exploit its advantage in this case by sharing a lower reward. On the other hand, when the monetary impact of contributions is too small, both firms follow a parsimonious strategy in sharing rewards. These results imply a bilateral relationship between the monetary values of users' contributions and the rewards they receive; if users demand high rewards, they should deliver rewarding contributions. For example, some media-streaming websites pay substantial payments since the type or level of contributions delivered by their users help to promote the website and thus is in the interest of the website as well as the advertising business.

For many types of online social media, the monetary impact of the contributions is too small. As previously discussed, the less favourable firm can compete with a dominant firm in the market by improving users' perspective of it within the market. When the monetary rewards are insignificant, this strategy is even more beneficial for the less favourable firm. My results suggest that, in this case, the less favourable firm can generate higher revenues, and as a result, the firm can share higher rewards with active users. This trend has been observed recently in the ongoing competition between a dominant firm, such as YouTube, and a smaller online music service, such as Spotify. As Spotify's user base continues to grow, the firm pays increasingly higher reward levels to the artists who use its platform.

Some results regarding properties of equilibrium reward are worth our attention. For example, in order for firms to be successful, it is critical for them to find the right balance between saving and sharing their revenues with active users. I observe that, at equilibrium, firms keep at least half of the marginal revenues for themselves and never share more than half of this incremental revenue with their active users. Moreover, when the asymmetry between two firms increases considerably, it may be optimal for the less favourable firm to completely shut down its rewards program. A higher level of asymmetry not only adversely affects the less favourable firm, but it can also generate monetary challenges for the more favourable firm, especially when the difference between two firms is insignificant originally.

Online user behaviour and online competition are complex issues. In order to make this discussion tractable, I made a few simplifying assumptions in this study. For example, I assumed that users are uniformly distributed with respect to their cost structure. A more general setting would be interesting, but also more challenging, as the probabilities and equilibrium are already non-trivial. Another extension would be to consider more than two OSMs. My main motivation for analyzing a duopoly came from observing the competition between YouTube and Metacafe (or between Spotify and We7), and making note of the particular behaviours of the online users. Further studies can also extend the analysis of users' behaviour by considering both the level and quality of the contributions as a continuum and by endogenously modelling the OSMs' revenue as a function of the level and quality of user-created content. Finally, it might be argued that the size of the OSM should be explicitly modelled. In this setting, the initial size (that is, before announcing $\boldsymbol{r}$) can be considered as part of the parameter a, but the subsequent size (that is, after announcing $\boldsymbol{r}$) is reflected by φ_i. For instance, note that, in this setting, the revenue (π_i) is increasing in size (φ_i). I leave all further enrichments as opportunities for future research.

2.8 Appendix to Chapter 1

PROOF OF LEMMA 1.

Case I. I show that if $r_2 \leq (a\text{-}1)r_1$, and $r_1 \leq 1$, then $\varphi_1 = r_1$ and $\varphi_2 = r_2 log(\frac{a}{a-1})$. In this case no one choose to be active in the second firm and passive in the first firm as follows:

When $c \leq r_2 / a$ (i.e. $r_2\text{-}ac \geq 0$), state PA is preferable than being passive at both firms. Also state PA is preferable than being active at both firms when $c \geq r_1/k$ ($r_2\text{-}ac \geq r_1 + r_2 - ac - c$). As $r_2/a \leq r_2/(a\text{-}1) \leq r_1 \leq r_1/k$, there is no c exists which satisfies above conditions. Note that state PA is never chosen whenever $r_2/a \leq r_1$. For the same logic state AP is never chosen whenever $r_2/a \geq r_1$. Then I have the following: $\varphi_1 = Pr\{max(r_1 - C, r_1 + r_2 - C(a+K)) \geq max(r_2 - aC, 0)\}$

$= Pr\{max(r_1 - C, r_1 + r_2 - C(a+K)) \geq 0\}$.

Note for any A, B, C and D, if $max(B,D) > C$, then $Pr\{max(A,B) > max(C,D)\} = Pr\{max(A,B) > D\}$. Therefore, I have: $\varphi_1 = Pr\{(\mathrm{r}_1 - \mathrm{C} \geq 0)\ or\ (\mathrm{r}_1 + \mathrm{r}_2 - \mathrm{C}(a+\mathrm{K}) \geq 0)\} = Pr\left\{(C \leq \mathrm{r}_1)\ or\ \left(C \leq \frac{\mathrm{r}_1 + \mathrm{r}_2}{a+\mathrm{K}}\right)\right\}$. As $r_1(a+k) \geq r_1 a \geq r_1(a\text{-}1) + r_1 \geq r_1 + r_2$, therefore $r_1 \geq (r_1 + r_2)/(a+k)$. Then I have the following: $\varphi_1 = Pr\left\{(C \leq r_1)\ or\ \left(C \leq \frac{r_1 + r_2}{a+K}\right)\right\} = Pr\{C \leq r_1\} = r_1$.

Now take website 2's perspective. $\varphi_2 = Pr\{max(r_2 - aC, r_1 + r_2 - C(a+K)) \geq max(r_1 - C, 0)\} = Pr\{r_1 + r_2 - C(a+K) \geq max(r_1 - C, 0)\} = Pr\{(r_1 + r_2 - C(a+K) \geq r_1 - C)\ and\ (r_1 + r_2 - C(a+K) \geq 0)\} = Pr\left\{\left(C \leq \frac{\mathrm{r}_2}{a+\mathrm{K}-1}\right)\ and\ \left(C \leq \frac{\mathrm{r}_1 + \mathrm{r}_2}{a+\mathrm{K}}\right)\right\}$. Since $ar_1 \geq r_1 + r_2$, therefore $\frac{\mathrm{r}_2}{a+\mathrm{K}-1} \leq \frac{\mathrm{r}_1 + \mathrm{r}_2}{a+\mathrm{K}}$.

Now I have the following: $\varphi_2 = Pr\left\{C \leq \frac{r_2}{a+K-1}\right\} = Pr\left\{\left(C \leq \frac{r_2}{a}\right)\ and\ \left(C \leq \frac{r_2}{a+K-1}\right)\right\} + Pr\left\{\left(\frac{r_2}{a-1} \geq C \geq \frac{r_2}{a}\right)\ and\ \left(C \leq \frac{r_2}{a+K-1}\right)\right\} + Pr\left\{\left(C \geq \frac{r_2}{a-1}\right)\ and\ \left(C \leq \frac{r_2}{a+K-1}\right)\right\}$

$$= Pr\left\{C \le \frac{r_2}{a}\right\} + Pr\left\{\left(\frac{r_2}{a-1} \ge C \ge \frac{r_2}{a}\right) \; and \; \left(K \le \frac{r_2}{C} + 1 - a\right)\right\} + 0$$

$$= \frac{r_2}{a} + \int_{\frac{r_2}{a}}^{\frac{r_2}{a-1}} \int_{0}^{\frac{r_2}{C}+1-a} dk\, dc$$

$$= \frac{r_2}{a} + \int_{\frac{r_2}{a}}^{\frac{r_2}{a-1}} \left(\frac{r_2}{C} + 1 - a\right) dc = \frac{r_2}{a} + [r_2 log(C) + (1-a)C]_{\frac{r_2}{a}}^{\frac{r_2}{a-1}} = r_2 log\left(\frac{a}{a-1}\right).$$

Case II. In previous cases, it was shown that state PA is never chosen whenever $r_2/a \le r_1$. Now I have the following: $\varphi_1 = Pr\{max(r_1 - C, r_1 + r_2 - C(a+K)) \ge max(r_2 - a.C, 0)\}$

$$= Pr\{max(r_1 - C, r_1 + r_2 - C(a+K)) \ge 0\}$$

$$= Pr\{(r_1 - C \ge 0) \; or \; (r_1 + r_2 - C(a+K) \ge 0)\}$$

$$= Pr\left\{(C \le r_1) \; or \; \left(C \le \frac{r_1 + r_2}{a+K}\right)\right\}$$

$$= Pr\left\{\left((C \le r_1) \; or \; \left(C \le \frac{r_1 + r_2}{a+K}\right)\right) and (C \le r_1)\right\}$$

$$+ Pr\left\{\left((C \le r_1) \; or \; \left(C \le \frac{r_1 + r_2}{a+K}\right)\right) and \left(\frac{r_1 + r_2}{a} \ge C \ge r_1\right)\right\}$$

$$+ Pr\left\{\left((C \le r_1) \; or \; \left(C \le \frac{r_1 + r_2}{a+K}\right)\right) and \left(C \ge \frac{r_1 + r_2}{a}\right)\right\}$$

$$= Pr\{C \le r_1\} + Pr\left\{\left(C \le \frac{r_1 + r_2}{a+K}\right) and \left(\frac{r_1 + r_2}{a} \ge C \ge r_1\right)\right\} + 0$$

$$= r_1 + Pr\left\{\left(K \le \frac{r_1 + r_2}{C} - a\right) and \left(\frac{r_1 + r_2}{a} \ge C \ge r_1\right)\right\}$$

$$= r_1 + \int_{r_1}^{\frac{r_1+r_2}{a}} \int_0^{\frac{r_1+r_2}{c}-a} dk\, dc = r_1 + \int_{r_1}^{\frac{r_1+r_2}{a}} \left(\frac{r_1 + r_2}{c} - a\right) dc$$

$$= r_1 + [(r_1 + r_2) log(c) - ac]_{r_1}^{\frac{r_1+r_2}{a}} = (r_1 + r_2) log\left(\frac{r_1 + r_2}{ar_1}\right) + ar_1 - r_2.$$

Now take website 2's perspective. $\varphi_2 = Pr\{r_1 + r_2 - C(a + K) \geq max(r_1 - C, 0)\}$

$$= Pr\{(r_1 + r_2 - C(a + K) \geq r_1 - C)\ and\ (r_1 + r_2 - C(a + K) \geq 0)\}$$

$$= Pr\left\{\left(C \leq \frac{r_2}{a + K - 1}\right)\ and\ \left(C \leq \frac{r_1 + r_2}{a + K}\right)\right\}$$

$$= Pr\left\{\left(C \leq \frac{r_2}{a + K - 1}\right)\ and\ \left(C \leq \frac{r_1 + r_2}{a + K}\right) and\ \left(C \leq \frac{r_2}{a}\right)\right\}$$
$$+ Pr\left\{\left(C \leq \frac{r_2}{a + K - 1}\right)\ and\ \left(C \leq \frac{r_1 + r_2}{a + K}\right)\ and\ \left(\frac{r_2}{a} \leq C \leq r_1\right)\right\}$$
$$+ Pr\left\{\left(C \leq \frac{r_2}{a + K - 1}\right)\ and\ \left(C \leq \frac{r_1 + r_2}{a + K}\right)\ and\ \left(r_1 \leq C \leq \frac{r_1 + r_2}{a}\right)\right\}$$
$$+ Pr\left\{\left(C \leq \frac{r_2}{a + K - 1}\right)\ and\ \left(C \leq \frac{r_1 + r_2}{a + K}\right)\ and\ \left(\frac{r_1 + r_2}{a} \leq C\right)\right\}$$

Note that as $(C \leq r_1)$, then $C \leq \frac{r_1 + r_2}{a+K}$ is an irrelevant condition. Since $\left(r_1 \leq C \leq \frac{r_2}{a+K-1}\right)$, then $\frac{r_1 + r_2}{a+K} \leq \frac{r_2}{a+K-1}$. Therefore I have:

$$\varphi_2 = Pr\left\{C \leq \frac{r_2}{a}\right\} + Pr\left\{\left(C \leq \frac{r_2}{a + K - 1}\right)\ and\ \left(\frac{r_2}{a} \leq C \leq r_1\right)\right\}$$
$$+ Pr\left\{\left(C \leq \frac{r_1 + r_2}{a + K}\right)\ and\ \left(r_1 \leq C \leq \frac{r_1 + r_2}{a}\right)\right\} + 0$$

$$= \frac{r_2}{a} + \ Pr\left\{\left(K \ \le \ \frac{r_2}{C} + 1 - a\right) \ and \ \left(\frac{r_2}{a} \ \le \ C \ \le \ r_1\right)\right\}$$

$$+ Pr\left\{\left(K \le \ \frac{r_1 \ + r_2}{C} - a\right) \ and \ \left(r_1 \le \ C \ \le \ \frac{r_1 \ + r_2}{a}\right)\right\}$$

$$= \frac{r_2}{a} + \int_{\frac{r_2}{a}}^{r_1} \int_0^{\frac{r_2}{c}+1-a} dk \, dc + \int_{r_1}^{\frac{r_1 + r_2}{a}} \int_0^{\frac{r_1 + r_2}{c} - a} dk \, dc$$

$$= (r_1 + r_2) \ log\left(\frac{r_1 + r_2}{ar_1}\right) + r_2 log(ar_1) - r_2 log(r_2).$$

Case III. $\varphi_1 = Pr\{max(r_1 - \ C, r_1 \ + r_2 - \ C(a + K)) \ge 0\}$

$$= Pr\{(r_1 - \ C \ge 0) \ or \ (r_1 \ + r_2 - \ C(a + K) \ \ge 0)\}$$

$$= \ Pr\left\{(C \ \le \ r_1) \ or \ \left(C \ \le \ \frac{r_1 \ + r_2}{a + K}\right)\right\}$$

$$= Pr\left\{\left((C \ \le \ r_1) \ or \ \left(C \ \le \ \frac{r_1 \ + r_2}{a + K}\right)\right) and (C \ \le \ r_1)\right\}$$

$$+ Pr\left\{\left((C \ \le \ r_1) \ or \ \left(C \ \le \ \frac{r_1 \ + r_2}{a + K}\right)\right) and (\ C \ \ge \ r_1)\right\}$$

$$= Pr\{C \ \le \ r_1\} + Pr\left\{\left(C \ \le \ \frac{r_1 \ + r_2}{a + K}\right) and (C \ \ge \ r_1)\right\}$$

$$= r_1 + Pr\left\{\left(K \le \ \frac{r_1 \ + r_2}{C} - a\right) and (C \ \ge \ r_1)\right\}$$

$$= r_1 + \int_{r_1}^{1} \int_0^{\frac{r_1 + r_2}{c} - a} dk \, dc = r_1 + \int_{r_1}^{1} \left(\frac{r_1 \ + r_2}{c} - a\right) dc = r_1 + [(r_1 \ + r_2) \log(c) - ac]_{r_1}^{1}$$

$= r_1 + a(r_1 - 1) - (r_1 + r_2) log(r_1)$. Now take website 2’s perspective.

$\varphi_2 = Pr\{max(r_2 - \ aC, r_1 \ + r_2 - \ C(a + K)) \ge max(r_1 - \ C, 0)\}$

$$= Pr\{r_1 + r_2 - C(a+K) \geq max(r_1 - C, 0)\} =$$

$$= Pr\{(r_1 + r_2 - C(a+K) \geq r_1 - C) \; and \; (r_1 + r_2 - C(a+K) \geq 0)\}$$

$$= Pr\left\{\left(C \leq \frac{r_2}{a+K-1}\right) \; and \; \left(C \leq \frac{r_1 + r_2}{a+K}\right)\right\}$$

$$= Pr\left\{\left(C \leq \frac{r_2}{a+K-1}\right) \; and \; \left(C \leq \frac{r_1 + r_2}{a+K}\right) and \; \left(C \leq \frac{r_2}{a}\right)\right\}$$
$$+ Pr\left\{\left(C \leq \frac{r_2}{a+K-1}\right) \; and \; \left(C \leq \frac{r_1 + r_2}{a+K}\right) \; and \; \left(\frac{r_2}{a} \leq C \leq r_1\right)\right\}$$
$$+ Pr\left\{\left(C \leq \frac{r_2}{a+K-1}\right) \; and \; \left(C \leq \frac{r_1 + r_2}{a+K}\right) \; and \; (r_1 \leq C)\right\}$$

Based on the proof in the previous case I have the following:

$$\varphi_2 = Pr\left\{C \leq \frac{r_2}{a}\right\} + Pr\left\{\left(C \leq \frac{r_2}{a+K-1}\right) \; and \; \left(\frac{r_2}{a} \leq C \leq r_1\right)\right\}$$
$$+ Pr\left\{\left(C \leq \frac{r_1 + r_2}{a+K}\right) \; and \; (r_1 \leq C)\right\}$$

$$= \frac{r_2}{a} + Pr\left\{\left(K \leq \frac{r_2}{C} + 1 - a\right) \; and \; \left(\frac{r_2}{a} \leq C \leq r_1\right)\right\}$$
$$+ Pr\left\{\left(K \leq \frac{r_1 + r_2}{C} - a\right) \; and \; (r_1 \leq C)\right\}$$

$$= \frac{r_2}{a} + \int_{\frac{r_2}{a}}^{r_1} \int_0^{\frac{r_2}{c}+1-a} dk \, dc + \int_{r_1}^{1} \int_0^{\frac{r_1 + r_2}{c}-a} dk \, dc$$

$= r_1 + r_2 - a - (r_1 + r_2)log(r_1) + r_2 log(a.r_1) - r_2 log(r_2)$. The probabilities of being active for the remaining six cases are as follows (the details of prove for these cases are available in a longer version of Appendix):

- *if $r_1 > 1$ and $r_2 > a$, then $\varphi_1 = 1$ and $\varphi_2 = 1$.*
- *if $r_1 > 1$ and $a\text{-}1 < r_2 \leq a$, then $\varphi_1 = 1$ and $\varphi_2 = r_2 + 1 - a - r_2 log(r_2/a)$.*

- *if* $r_1 > 1$ *and* $r_2 \leq a-1$, *then* $\varphi_1 = 1$ *and* $\varphi_2 = r_2 log(\frac{a}{a-1})$.
- *if* $r_1 \leq 1$ *and* $r_2 > a$, *then* $\varphi_1 = r_1 - r_1 log(r_1)$ *and* $\varphi_2 = 1$.
- *if* $ar_1 < r_2$, *and* $r_1 + r_2 \leq a$, *then* $\varphi_1 = (r_1 + r_2)log(r_1 + r_2) - r_1 log(ar_1) - r_2 log(r_2)$ *and*

  $\varphi_2 = (r_1 + r_2)log((r_1 + r_2)/r_2) + r_2/a - r_1$.
- *if* $ar_1 < r_2$, $r_1 + r_2 > a$, *and* $r_2 \leq a$, *then*

  $\varphi_1 = r_1 + r_2 - a - r_1 log(r_1) - r_2 log(r_2) + r_2 log(a)$ *and* $\varphi_2 = r_2 + \frac{r_2}{a} - a - (r_1 + r_2)log(r_2/a)$. □

PROOF OF PROPOSITION 1**.** The first derivative of φ_i in Lemma 1 for all three cases with respect to r_1 and r_2 is non-negative and with respect to a is non-positive. See Table A1 for the derivative of φ_i for all cases. For Case I, as $a \geq 1$, $\frac{\partial \varphi_2}{\partial r_2}$ and $\frac{\partial \varphi_2}{\partial a}$ are negative and other derivatives are 0. For Case II, I have the following: $\frac{\partial \varphi_1}{\partial r_1} = a + log\left(\frac{r_1 + r_2}{ar_1}\right) - \frac{r_2}{r_1} > 0$ because $r_2 \leq ar_1$ and $r_1 + r_2 \geq ar_1$ (I proved this inequality in Lemma 1). Also $\frac{\partial \varphi_1}{\partial r_2} = log\left(\frac{r_1 + r_2}{ar_1}\right) > 0$, $\frac{\partial \varphi_2}{\partial r_2} = log(ar_1) + log\left(\frac{r_1 + r_2}{ar_1}\right) - log(r_2) > 0$, $\frac{\partial \varphi_2}{\partial r_1} = log\left(\frac{r_1 + r_2}{ar_1}\right) > 0$, $\frac{\partial \varphi_1}{\partial a} = r_1 - \frac{r_1 + r_2}{a} < 0$ and $\frac{\partial \varphi_2}{\partial a} = -\frac{r_1}{a} < 0$ for the same reason. In Case III, the same results hold as $r_2 < a$, $r_1 < 1$, and $r_2 \leq ar_1$. The results hold for all remaining six cases as well.

Table 4: The first derivatives of φ_i.

Case	$\frac{\partial \varphi_1}{\partial r_1}$	$\frac{\partial \varphi_1}{\partial r_2}$	$\frac{\partial \varphi_1}{\partial a}$	$\frac{\partial \varphi_2}{\partial r_1}$	$\frac{\partial \varphi_2}{\partial r_2}$	$\frac{\partial \varphi_2}{\partial a}$
I	1	0	0	0	$log(\frac{a}{a-1})$	$-\frac{r_2}{a^2 - a}$

II	$a + log\left(\frac{r_1 + r_2}{ar_1}\right) - \frac{r_2}{r_1}$	$log\left(\frac{r_1 + r_2}{ar_1}\right)$	$r_1 - \frac{r_1 + r_2}{a}$	$log\left(\frac{r_1 + r_2}{ar_1}\right)$	$log\left(\frac{r_1 + r_2}{r_2}\right)$	$-\frac{r_1}{a}$
III	$a - log(r_1) - \frac{r_2}{r_1}$	$-log(r_1)$	$r_1 - 1$	$-log(r_1)$	$log\left(\frac{a}{r_2}\right)$	$\frac{r_2}{a} - 1$

Moreover φ_i for all nine cases with respect to r_1 and r_2 are continuous on the boundaries of these cases, i.e., φ_1 and φ_2 for any neighbour cases, such as I and II or II and III, are the same on their boundary. Therefore, over all cases, φ_i are non-decreasing in r_1 and r_2. □

PROOF OF THEOREM 1. I have already shown that NE has to be such that $r_1^* \leq 1$, $r_2^* \leq a$. The proof of theorem has the following steps:

1) No equilibrium exists in Case I when $1 \leq a \leq 2$
2) No equilibrium exists in Case II when $a > 2$
3) No equilibrium exists when $ar_1 < r_2$, and $r_1 + r_2 \leq a$
4) No equilibrium exists when $ar_1 < r_2$, $r_1 + r_2 > a$, and $r_2 \leq a$
5) At least one equilibrium exists in the feasible cases, i.e., Case II and III when $1 \leq a \leq 2$, and Case I and III when $a > 2$. The details of proof are as follows:

1) Next, I show that no equilibrium exists in Case I when $1 \leq a \leq 2$. Assume there is an equilibrium (r_1^*, r_2^*) with $(a-1)r_1^* > r_2^*$. Based on Lemma 1, the firm 1 and 2 payoff functions are $\pi_1 = (w-r_1^*)r_1^*$ and $\pi_2 = (w-r_2^*)r_2 log(a/(a-1))$, respectively. Note that both payoff functions π_i are concave in r_i because $\frac{\partial^2 \pi_1}{\partial {r_1}^2} = -2$ and $\frac{\partial^2 \pi_2}{\partial {r_2}^2} = -2log\left(\frac{a}{a-1}\right)$, and both are maximized at $r_i = \frac{w}{2}$ (set $\frac{\partial \pi_i}{\partial r_i} = w - 2r_i = 0$). However when $r_i^* = \frac{w}{2}$, the condition of $(a-1)r_1^* > r_2^*$ is violated because $1 \leq a \leq 2$. Therefore (r_1^*, r_2^*) cannot be an equilibrium. As a result, Nash equilibrium cannot be in Case I.

2) First, if the equilibrium is on the boundary of Case I and II, then $a = 2$ (check the F.O.C for the objective function of firm 1 and 2). When a increases beyond 2, the slope

for the objective function of firm 2 is negative on the boundary with respect to r_2, which implies that the equilibrium can not be on the top of the boundary in Figure 1.

3) Next, I illustrate no equilibrium exists when $ar_1 < r_2$, *and* $r_1 + r_2 \leq a$. In doing so, I show that the assumed equilibrium in this case can never be a best response strategy for firm 1.

Assume there is an equilibrium (r_1*,r_2*) when $ar_1^* < r_2^*$. Based on Lemma 1, the firm 2 payoff is $(w - r_2)\big((r_1 + r_2)log((r_1 + r_2)/r_2) + r_2/a - r_1\big)$. Note that $\frac{\partial^3 \pi_2}{\partial {r_2}^3} = -\frac{(3r_1^2 r_2 + 2r_1^3)w + r_1^3 r_2}{r_2^3 (r_1 + r_2)^2} < 0$, and $\frac{\partial^2 \pi_2}{\partial {r_2}^2} = \frac{-2(a+1)\log\left(\frac{a+1}{a}\right)}{a} < 0$ at $r_2 = ar_1$. Therefore the payoff function of firm 2 is concave in in this case. As $r_2^* = \mathrm{BR}$ ($r1$*) is in this case, the F.O.C is $\frac{\partial \pi_2}{\partial r_2} > 0$ at $r_2 = ar_1$. Positive F.O.C at $r_2 = ar_1$ requires that $w > r_1(2a+1)$ (substitute $r_1 = r_2/a$ in $\frac{\partial \pi_i}{\partial r_i}$).

Now take website 1's perspective. Based on Lemma 1, the firm 1 payoff is $\varphi_1 = (w - r_1)\big((r_1 + r_2)log(r_1 + r_2) - r_1 log(ar_1) - r_2 log(r_2)\big)$.

Note that $\frac{\partial^3 \pi_1}{\partial {r_1}^3} = \frac{(r_2^2 + 2r_1 r_2)w + 2r_2^2 r_1 + r_1^2 r_2}{r_1^2 (r_1 + r_2)^2} > 0$ and $\frac{\partial^2 \pi_1}{\partial {r_1}^2} = -\frac{a^2 w + r_2\left(2(a+1)\log\left(\frac{a+1}{a}\right) - a\right)}{(a+1) r_2} < 0$ at $r_1 = r_2/a$. Therefore, the payoff function of firm 1 is concave. Now $\frac{\partial \pi_1}{\partial r_1} = \big(w - r_1(a+2)\big)\, log\left(\frac{a+1}{a}\right) < \big(r_1(a-1)\big)\, log\left(\frac{a+1}{a}\right) > 0$ at $r_1 = r_2/a$ as $w > r_1(2a+1)$. This is a contradiction because $r_1^* = \mathrm{BR}$ (r_2*) in this case, and $\frac{\partial \pi_1}{\partial r_1}$ should be negative for any $r_1 > r_1^*$ such as $r_1 = r_2/a$. Therefore no equilibrium exists when $ar_1 < r_2$, *and* $r_1 + r_2 \leq a$.

4) Next Note that $\frac{\partial \pi_1}{\partial r_1} = -wln(r_1) - r_2 ln(a) + r_2 ln(r_2) + 2r_1 ln(r_1) > 0$ at $r_1 = a - r_2$, and for $r_2 > ar_1$ as $r_1 = a - r_2 < ar_1$. Now I show that no equilibrium exist when $ar_1 < r_2$, $r_1 + r_2 > a$, *and* $r_2 \leq a$. In doing so, I show that the assumed equilibrium in this case can never be a best response strategy for firm 1.

Assume there is an equilibrium (r_1*,r_2*) when $ar_1^* < r_2^*$. Based on Lemma 1, the firm 1 payoff is $(w - r_2)\left(r_2 + \frac{r_2}{a} - a - (r_1 + r_2)log(r_2/a)\right)$.

Note that $\frac{\partial^2 \pi_2}{\partial {r_2}^2} = -\frac{(ar_2 - ar_1)w - 2a{r_2}^2 \log(r_2) + (2a \log(a) - a + 2){r_2}^2 - ar_1 r_2}{a{r_2}^2} < 0$. Therefore, the payoff function of firm 2 is concave in this case. As r_2* = BR (r_1*) is in this case, $\frac{\partial \pi_2}{\partial r_2} >$ 0 at $r_2 = ar_1$. Positive F.O.C at $r_2 = ar_1$ requires that $w > 2a - r_1 + \frac{-r_1(ar_1 - a + r_1)}{a(a - r_1)\log\left(\frac{a}{a - r_1}\right) + a - r_1 - ar_1}$ when $r_1 \leq a/(a+1)$ and $w > (2a + 1)r_1 + \frac{ar_1 + r1 - a}{\log(r1)}$ when $r_1 > a/(a+1)$.

Now take website 1's perspective. Based on Lemma 1, the firm 1 payoff is $\varphi_1 = (w - r_1)\left(r_1 + r_2 - a - r_1 log(r_1) - r_2 log(r_2) + r_2 log(a)\right)$.

As $\frac{\partial^2 \pi_1}{\partial {r_1}^2} = -\frac{w - 2r_1 \log(r_1) - r_1}{r_1} < 0$ in this case, the payoff function of firm 1 is concave. Now $\frac{\partial \pi_1}{\partial r_1} = -wln(r_1) - r_2 ln(a) + r_2 ln(r_2) + 2r_1 ln(r_1) + a - r_1 - ar_1$ at $r_1 = r_2/a$ is positive because I have already shown that $= -wln(r_1) - r_2 ln(a) + r_2 ln(r_2) + 2r_1 ln(r_1) > 0$ and the conditions on w which are mentioned above. This is a contradiction because r_1* = BR (r_2*), and $\frac{\partial \pi_1}{\partial r_1}$ should be negative at any $r_1 > r_1$* such as $r_1 = r_2/a$. Therefore there is no equilibrium when $ar_1 < r_2$, $r_1 + r_2 > a$, *and* $r_2 \leq a$.

5) I have already shown that no equilibrium exists outside Case I, II and III. Next, I show its existence in these cases. I also assume if there is more than one reward level, which generate the same amount of revenue for a firm in equation (1), the firm always prefers the lowest one. Through doing this, firm can keep a higher saving for itself.

My proof for existence of equilibrium is based on applying results from Szidarovszky (2008). I paraphrase Theorem 3 (p. 100) of Szidarovszky (2008) regarding the existence of equilibrium:

Theorem 3: *Suppose that, in game **Γ**, for all players:*

- *The strategy profile is a nonempty, convex, compact subset of a finite dimensional Euclidean space;*
- *The payoff function is continuous on the strategy profile;*
- *The best response function is single valued.*

Then the game **Γ** has at least one (pure strategy) Nash equilibrium.

Theorem 3 in Szidarovszky (2008) indicates three sufficient conditions for the existence of the Nash equilibrium in pure strategies. I show that the game I study here satisfies the above conditions.

First, based on Proposition 2, strategy profiles are a nonempty, convex, and compact set for both firms. Second, as firms' payoff functions are the product of continuous functions, firms' payoff functions are continuous. Finally, the firms' best response functions are singled-valued (see the assumption after equation 1). Consequently, the game satisfies the above conditions. Therefore, it has a pure strategy Nash equilibrium. The proof of uniqueness will be shown later. □

PROOF OF LEMMA 2. The proof of Lemma has the following steps:

1) For all a, there exists a unique $\hat{w}_a$
2) If $w = \hat{w}_a$, then $r_1^* + r_2^* = a$, and if $r_1^* + r_2^* = a$ then $w = \hat{w}_a$
3) If $w < \hat{w}$, then $r_1^* + r_2^* < a$, and if $w > \hat{w}_a$, then $r_1^* + r_2^* > a$

The details of proof are as follows:

1) System of equations (2) denotes the points on Line AB in Figure 1 where r1 + r2 = a and a/(a+1) $\leq r_1 \leq$ 1. For showing the existence of $\hat{w}_a$, I compare the value of the first and second equations of this system at two extreme values on Line AB in Figure 1, i.e., for $r_1 = a/(a+1)$ and $r_1 = 1$. When $r_1 = a/(a+1)$, I have the following:

$$2r_2 + \frac{r_1 log(r_1)}{log(r_2) - log(a)} = \frac{a(2a+1)}{a+1} \geq \frac{a(2+a)}{a+1} = 2r_1 + \frac{r_1 r_2 log(r_1)}{ar_1 - r_2 - r_1 log(r_1)}$$

and when $r_1 = 1$:

$$2r_2 + \frac{r_1 log(r_1)}{log(r_2) - log(a)} = 2(a-1) \leq 2 = 2r_1 + \frac{r_1 r_2 log(r_1)}{ar_1 - r_2 - r_1 log(r_1)}$$

Note that both equations are continuous functions of r_1 and r_2 over the above interval. Therefore, the above inequalities guarantee the existence of $\hat{w}_a$ based on the Fixed-point Theorem.

Moreover, $\hat{w}_a$ is unique as the first equation in System (2) is a strictly decreasing function of r_1 and the second equation of in System (2) is a strictly increasing function of r_1. I can check these finding by substitute $r_2 = a - r_1$ and take the first derivative with respect to r_1 as follows:

$$\frac{\partial\left(2(a-r_1) + \frac{r_1 log(r_1)}{log(a-r_1) - log(a)}\right)}{\partial r_1} =$$

$$\frac{\log(r_1)+1}{\log(a-r_1)-\log(a)} - \frac{2r_2[\log(a)-\log(a-r_1)]^2 - r_1\log(r_1)}{r_2[\log(a)-\log(a-r_1)]^2} < 0$$

$$\frac{\partial\left(2r_1 + \frac{r_1(a-r_1) log(r_1)}{ar_1 - (a-r_1) - r_1 log(r_1)}\right)}{\partial r_1} =$$

$$-\frac{(\log(r_1)-1)(a-r_1)^2 + (3ar_1 - 4r_1\log(r_1))r_1 - 2{r_1}^2 log^2(r_1) + 4a{r_1}^2\log(r_1) - 2a^2{r_1}^2}{[a - r_1 + r_1 log(r_1) - ar_1]^2}$$

$$+\frac{{r_1}^2 log^2(r_1) - a{r_1}^2 log^2(r_1)}{[a - r_1 + r_1 log(r_1) - ar_1]^2} > 0$$

Therefore, system (2) has only one unique solution for $a \leq 2$. For $a \leq 2$, it is straightforward that there exists a unique $\hat{w}_a$. Note that in Theorem 1, part 1, I have shown that π_2 is maximized at $r_2 = \frac{w}{2}$. On the boundary of case I and III, $r_2 = a - 1$, which derive the equation 3.

2) Based on Lemmas 2 in Case II, π_1 is concave and continuously differentiable and π_2 is continuously differentiable and $\frac{\partial^3 \pi_1}{\partial {r_1}^3} < 0$. Therefore, the best response functions should satisfy the first order conditions for both firms' payoff functions. If an equilibrium exists such that $r_1^* + r_2^* = a$, it should be the solution of the system of equations (2) as this system is F.O.C of payoff functions for firms.

Now assume $w = \hat{w}_a$. Solving the system of equations (2), I find r'_1 and r'_2, such that $r'_1+r'_2 = a$. Moreover, r'_1 and r'_2 are the best strategy response of each other as both satisfy the F.O.C of firm' payoff functions. Therefore, (r'_1, r'_2) is an equilibrium.

3) Assume there is an equilibrium in Case III when $w < \hat{w}_a$. As $w < \hat{w}_a$, the F.O.C for firm 2 payoff function at $r_2 = a - r_1$ is negative. Note that the payoff function of firm 2 is concave. This implies that the best strategy profile of this firm for any r_1 always satisfying the following condition: $r_2^*(r_1) = a - r_1$. Therefore at equilibrium I have the following: $r_2^* < a - r_1^*$. Based on the same logic, when $w > \hat{w}_a$, I can show that $r_2^* > a - r_1^*$. □

PROOF OF LEMMA 3. The results in Lemma 3 is immediate from Theorem 1, part 1. Note that r_1^* cannot be higher than 1. □

PROOF OF LEMMA 4. Based on Lemma 2, if $w < \hat{w}$, $r_1^* + r_2^* \leq a$. Based on Lemma 1, firms' payoff functions are:

$$\pi_1 = (w - r_1)\big(r_1 + a(r_1 - 1) - (r_1 + r_2)ln(r_1)\big)$$

$$\pi_2 = (w - r_2)\big(r_1 + r_2 - a - (r_1 + r_2)ln(r_1) + r_2 ln(ar_1) - r_2 ln(r_2)\big).$$

Checking the F.O.C for both firms, I have the following for $r_i = r_i^*$, $i = 1, 2$:

$$(w - 2r_1)\left(r_1 \log\left(\frac{r_1+r_2}{ar_1}\right) - r_2 + ar_1\right) - r_1 r_2 \log\left(\frac{r_1+r_2}{ar_1}\right) = 0$$

$$(w - 2r_2)\log\left(\frac{r_1+r_2}{r_2}\right) - r_1 \log\left(\frac{r_1+r_2}{ar_1}\right) = 0 \text{ for } r_2 = r_2^*.$$ □

PROOF OF LEMMA 5. Based on Lemma 2, if $w > \hat{w}_a$, $r_1*+ r_2* > a$. Based on Lemma 1, firms' payoff functions are:

$$\pi_1 = (w - r_1)\left((r_1 + r_2)log\left(\frac{r_1+r_2}{ar_1}\right) + ar_1 - r_2\right) and$$

$$\pi_2 = (w - r_2)\left((r_1 + r_2)\ log\left(\frac{r_1+r_2}{ar_1}\right) + r_2 log(ar_1) - r_2 log(r_2)\right).$$

Checking the F.O.C for both firms, I have the following for $r_i = r_i*$, $i = 1, 2$::

$$(w - 2r_1)(r_1 \log(r_1) + r_2 - ar_1) + r_1(r_1 + r_2 - a - \ r_2 \log(r_1)) = 0$$

$$(w - 2r_2)\left(\log\left(\frac{a}{r_2}\right)\right) - r_1 - r_2 + a + \ r_1 \log(r_1) = 0, \text{ for } r_2 = r_2*.$$

Note that in this case, an equilibrium may exists at $r_1* = 1$, without satisfying the F.O.C for firm 1 because $r_1 = 1$ is a boundary for strategy set of the first firm and firm 1 payoff function is not concave necessary. □

PROOF OF COROLLARY 1. Based on System of equations (6) r_1* and r_2* are non-decreasing function of w. For $a > 2$, when $w = \hat{w}$, $r_1* = 1$ and $r_2* = a - 1$. Therefore, when $w > \hat{w}$, $r_1* = 1$. Then r_2* can be derived from equation (7-2). □

PROOF OF COROLLARY 2. Part *a*. In this case the equilibrium is in Case I. Based on Lemma I and III, the results are immediate. Part *b*. In this case the equilibrium is in Case III. Note that the $r_1* = 1$, $r_2* = a$ is a limiting solution for System (6). As both firms pay at the maximum cost of their active users, the probabilities of being active for both firms are equal 1. The remaining results are immediate. □

PROOF OF UNIQUENESS OF EQUILIBRIUM. Lemma 3 implies the uniqueness of Nash equilibrium in Case I. for Case II and III, I paraphrase the following Theorem in Chenault [6] as proof of the uniqueness of the Nash equilibrium:

Theorem: *If the following assumptions are satisfied, the game has a unique Nash equilibrium:*

- *Assumption 1 (continuity): The response functions are continuous;*
- *Assumption 2 (boundedness): the choice set of an agent is bounded;*
- *Assumption 3 (boundary conditions): if all other agents choose extreme values it is not optimal for an agent to do likewise;*

Assumption 4 (determinant condition): The determinate of the matrix of the derivative of response functions minus strategy sets with respect to strategies is not zero.

This game is a continuous game, i.e., the payoff functions are continuously differentiable over a bounded set in R^2. Notice that the response functions are continuous as $(\partial r_1^*(r_2))/(\partial r_2)$ and $(\partial r_2^*(r_1))/(\partial r_1)$ are continuously differentiable over Cases II and III, respectively (system of equations in Theorem 1 and 2 specify these response functions).

Moreover, as I have shown in Proposition 2, the strategy set of players is bounded (assumption 2). Defining any $r_1 > 1$ and any $r_2 > a$ as extreme cases in assumption 3, I have established in Proposition 2 that choosing these extremes is not an optimal decision for agents (assumption 3).

To check whether assumption 4 holds, define $v(r_1,r_2) = \psi(r_1,r_2) - (r_1,r_2)$ over strategy set, where $\psi(r_1,r_2)$ is the vector of best response functions for given r_1 and r_2. The equilibrium is in Case II or III. Now assume the equilibrium is in Case II, i.e., $w \leq \hat{w}_a$. Then, I have the following:

$$|dv(r_1,r_2)| = \begin{bmatrix} -1 & \dfrac{\partial\psi_1}{\partial r_2} \\ \dfrac{\partial\psi_2}{\partial r_1} & -1 \end{bmatrix} \neq 0$$

because

$$\frac{\partial\psi_1}{\partial r_2}$$

$$= \frac{r_2 w + r_1(r_1+r_2)log\left(\frac{r_1+r_2}{ar_1}\right) - r_1 r_2}{w\left((r_1+r_2)log\left(\frac{r_1+r_2}{ar_1}\right) + (a-1)r_2 + r_1\right) - (r_2+4r_1)(r_1+r_2)log\left(\frac{r_1+r_2}{ar_1}\right) + r_2(3r_2 - 4(a-1)r_1) - 4ar_1^2}$$

$$< 0$$

and

$$\frac{\partial \psi_2}{\partial r_1} = \frac{r_2\left(w - (r_2 + r_1)log\left(\frac{r_1 + r_2}{ar_1}\right) - r_2\right)}{r_1 w + 2r_2(r_1 + r_2)log\left(\frac{r_1 + r_2}{r_2}\right) - r_1 r_2} < 1.$$

as $w < r_2 + 2r_1$. Consequently, the game satisfies all above assumptions and has a unique Nash equilibrium. Now assume the equilibrium is in Case III, i.e., $w > \hat{w}_a$. Then, I have the following:

$$|dv(r_1, r_2)| = \begin{bmatrix} -1 & \frac{\partial \psi_1}{\partial r_2} \\ \frac{\partial \psi_2}{\partial r_1} & -1 \end{bmatrix} \neq 0$$

because

$$\frac{\partial \psi_1}{\partial r_2} = \frac{r_1(r_1 - w)log(r_1)}{w\big(r_2 - ar_1 + r_1 log(r_1)\big) - r_1(2r_1 + r_2)log(r_1) + r_1(2(a-1)r_1 - a - r_2)} < 0$$

and

$$\frac{\partial \psi_2}{\partial r_1} = \frac{(w - r_2)log(r_1)}{log\left(\frac{a}{r_2}\right)(w - 2r_2) + r_1 log(r_1) + a - r_1 - r_2} < 1.$$

Consequently, the game satisfies all above assumptions and has a unique Nash equilibrium. □

PROOF OF COROLLARY 3.

Part *a*. Based on Lemma 1, for $a > 2$, the result is straightforward. For $a \leq 2$, I have the following:

$$\varphi_1 - \varphi_2 = ar_1 - r_2 - r_2 ln\left(\frac{ar_1}{r_2}\right) = \frac{\frac{ar_1}{r_2} - 1 - ln\left(\frac{ar_1}{r_2}\right)}{\frac{1}{r_2}}.$$

Note that $\frac{ar_1}{r_2} - 1 \geq ln\left(\frac{ar_1}{r_2}\right)$ (this is because $x - 1 \geq ln(x)$ as the function $f(x) = x - 1 - ln(x) \geq 0$ for $x \geq 1$ since f(1) = 0 and $\frac{\partial f}{\partial x} = \frac{x-1}{x} \geq 0$). Therefore $\varphi_1 - \varphi_2 \geq 0$.

Part *b*. Based on Lemma 1, for $a \leq 2$, the result is straightforward. There are three cases for this proof: Assume $r_1^* + r_2^* \leq a$. Based on Lemma 1, firms' payoff functions are

$\pi_1 = (w - r_1)\big(r_1 + a(r_1 - 1) - (r_1 + r_2)ln(r_1)\big)$ and

$\pi_2 = (w - r_2)\big(r_1 + r_2 - a - (r_1 + r_2)ln(r_1) + r_2 ln(a.r_1) - r_2 ln(r_2)\big)$.

Now check F.O.C for both firms:

$(w - 2r_1)\left(r_1 \log\left(\frac{r_1+r_2}{ar_1}\right) - r_2 + ar_1\right) - r_1 r_2 \log\left(\frac{r_1+r_2}{ar_1}\right) = 0$ for $r_1 = r_1^*$ and

$(w - 2r_2)\log\left(\frac{r_1+r_2}{r_2}\right) - r_1 \log\left(\frac{r_1+r_2}{ar_1}\right) = 0$ for $r_2 = r_2^*$.

As $r_1 \log\left(\frac{r_1+r_2}{ar_1}\right) - r_2 + ar_1$, $\log\left(\frac{r_1+r_2}{r_2}\right)$ and $\log\left(\frac{r_1+r_2}{ar_1}\right)$ are positive, $(w - 2r_1)$ and $(w - 2r_2)$ should be positive for $r_1 = r_1^*$ and $r_2 = r_2^*$. Therefore r_1^* and r_2^* are less than or equal w/2. Now assume $r_1^* + r_2^* > a$ and $r_1^* < 1$. Based on Lemma 1, firms' payoff functions are

$\pi_1 = (w - r_1)\left((r_1 + r_2)log\left(\frac{r_1+r_2}{ar_1}\right) + ar_1 - r_2\right)$ and

$\pi_2 = (w - r_2)\left((r_1 + r_2)\, log\left(\frac{r_1+r_2}{ar_1}\right) + r_2 log(ar_1) - r_2 log(r_2)\right)$.

Checking F.O.C for both firms:

$(w - 2r_1)(r_1 \log(r_1) + r_2 - ar_1) + r_1(r_1 + r_2 - a - r_2 \log(r_1)) = 0$ for $r_1 = r_1^*$

$(w - 2r_2)\left(\log\left(\frac{a}{r_2}\right)\right) - r_1 - r_2 + a + r_1 \log(r_1) = 0$ for $r_2 = r_2^*$.

As $(r_1 \log(r_1) + r_2 - ar_1)$ and $-r_1 - r_2 + a + r_1 \log(r_1)$ are negative and $(r_1 + r_2 - a - r_2 \log(r_1))$ are positive, $(w - 2r_1)$ and $(w - 2r_2)$ should be positive for $r_1 = r_1^*$ and $r_2 = r_2^*$. Therefore r_1^* and r_2^* are less than or equal w/2. Now assume $r_1^* + r_2^* > a$ and

$r_1^* = 1$. Based on Lemma 2, $w > \hat{w}_a$ and $\hat{w}_a$ is the solution of following system of equations:

$$\begin{cases} \hat{w}_a = 2r_2 \\ \hat{w}_a = 2r_1 \\ r_1 + r_2 = a \end{cases}$$

Therefore $w > \hat{w}_a = 2$. Therefore r_1^* is less than or equal $w/2$. The proof in previous case can be applied for r_2^* in this case.

Part *c*. In Theorem 1, I have shown that the game has Nash equilibrium in Cases I or II or III. Based on Lemma 1, I also have shown that no user choses state PA in these cases. These results conclude that, at equilibrium, no one will be active merely in the less favourable firm. □

PROOF OF PROPOSITION 2. When I substitute $a = 1$ and $r_1 = r_2 = r$ in System of equations (2), it is simplified as:

$$\begin{cases} \hat{w}_a = 2r + \dfrac{r log(r)}{log(r)} = 3r \\ \hat{w}_a = 2r + \dfrac{r^2 log(r)}{-r log(r)} = 3r \\ 2r = 1 \end{cases}$$

Solving this results in $\hat{w}_a = 1.5$. Now, consider $a = 1$ and $r_1 = r_2$ in System of equations 5 as follows:

$$\begin{cases} (w - 2r_1)(r_1\, log(2) - r_1 + r_1) - {r_1}^2\, log(2) = 0 \\ (w - 2r_1)\, log(2) - r_1\, log(2) = 0 \end{cases}$$

This system can be simplified further as:

$$\begin{cases} w - 3r_1 = 0 \\ w - 3r_1 = 0 \end{cases}$$

Which implies that $r_1^* = r_2^* = w/3$. Now, consider $a = 1$ and $r_1 = r_2$ in System of equations 6 as follows:

$$\begin{cases}(w-2r_1)(r_1\,log(r_1)+r_1-r_1)+r_1(r_1+r_1-1-\ r_1\,log(r_1))=0\\ (w-2r_1)\left(log\left(\frac{1}{r_2}\right)\right)-r_1-r_1+1+\ r_1\,log(r_1)=0\end{cases}$$

This system can be simplified further as:

$$\begin{cases}(w-3r_1)\,log(r_1)+2r_1-1=0\\ (w-3r_1)\,log(r_1)+2r_1-1=0\end{cases}$$

which results in equation 8. □

PROOF OF PROPOSITION 3. When I substitute $a = 2$ and $r_1 = r_2 = r$ in System of equations (2), it is simplified as:

$$\begin{cases}\widehat{w}_a = 2r + \dfrac{rlog(r)}{log(r)-log(2)} = 2r\\ \widehat{w}_a = \ 2r + \dfrac{r^2log(r)}{r-rlog(r)} = 2r\\ 2r = 2\end{cases}$$

Solving this results in $\widehat{w}_a = 2$. Now, consider $a = 2$ and $r_1 = r_2$ in System of equations 5 as follows:

$$\begin{cases}(w-2r_1)(-r_1+2r_1)=0\\ 0=0\end{cases}$$

This system can be simplified further as:

$$w-2r_1=0$$

Which implies $r_1* = r_2* = w/2$. Now, consider $a = 2$. Part b is immediate from Corollary 1. □

PROOF OF COROLLARY 4.

Part *a*. Based on Proposition 1 and 2, it is straightforward that r_1* is higher for asymmetric case. r_2* is also higher as it is less than 1 for the symmetric case, however it should be higher than 1 in asymmetric case, since r_2* in asymmetric case is always between *a-1* and *a*.

Part *b*. Based on Proposition 1 and 2, at *a* = 1, $\varphi_1 = 2rlog(2)$ and $2r_1 - 1 - 2r_1log(r_1)$ in case II and III respectively. At *a* = 2, $\varphi_1 = r$ and 1 in case II and III respectively. Note that in both cases, at *a* = 2, φ_1 is higher.

Based on Proposition 1 and 2, at *a* = 1, $\varphi_2 = 2rlog(2)$ and $2r_1 - 1 - 2r_1log(r_1)$ in case II and III respectively. At *a* = 2, $\varphi_1 = r_2log(2)$ and $r_2 - 1 + r_2log(2)$ in case II and III respectively. Note that in both cases, at *a* = 2, φ_2 is smaller.

Part *c*. Decreasing π_2 at *a* = 2 is straightforward. At *a* = 1, $\pi_1 = 4w^2log(2)/9$. At *a* = 2, $\pi_1 = w^2/4$. Therefore π_1 decreases at *a* = 2. □

PROOF OF COROLLARY 5.

When $a \geq 2$, based on Theorem 1, there are two cases as follows: First, based on Lemma 3, if $w \leq \hat{w}_a$ (Case I), then $r_1^* = min\left(\frac{w}{2}, 1\right) \leq \frac{w}{2} = r_2^*$. For the second case, if $w > \hat{w}_a$ (Case III), based on Lemma 2, $r_1^* + r_2^* > a > 2$. Based on Theorem 1, $r_1^* \leq 1$. Therefore, $r_2^* > 1 > r_1^*$. □

PROOF OF LEMMA 6A.

Case I. I show that *if* $r_1 \leq 1$ *and* $r_2 \leq (a+K-1)r_1$, *then* $\varphi_1{}^*(\boldsymbol{r}) = r_1$ *and* $\varphi_2{}^*(\boldsymbol{r}) = \frac{r_2}{a+k-1}$ .In this case no one chooses to be active in firm 2 and passive firm 1 as follows:

When $c \leq r_2 / a$ (i.e. $r_2 - ac \geq 0$), state PA is preferable than being passive at both firms. Also state PA is preferable than being active at both firms when $c \geq r_1/K$ ($r_2 - ac \geq r_1 + r_2 - ac - cK$). As $r_2/a \leq r_1(a+K-1)/a \leq r_1 \leq r_1/K$, there is no *c*, which satisfies above conditions. Then, I have the following: $\varphi_1 = Pr\{max(r_1 - C, r_1 + r_2 - C(a+k)) \geq max(r_2 - aC, 0)\}$

$$= Pr\{max(r_1 - C, r_1 + r_2 - C(a+k)) \geq 0\}.$$

Note for any A, B, C and D, if *max(B,D) > C*, then *Pr{max(A,B) > max(C,D)} = Pr{max(A,B) > D}*. Therefore, I have: $\varphi_1 = Pr\{(r_1 - C \geq 0)\ or\ (r_1 + r_2 - C(a+K) \geq 0)\} = Pr\left\{(C \leq r_1)\ or\ \left(C \leq \frac{r_1 + r_2}{a+K}\right)\right\}$. As $r_2 \leq (a+K-1)r_1$, therefore $r_1 \geq (r_1 + r_2)/(a + K)$. Then I have the following: $\varphi_1 = Pr\left\{(C \leq r_1)\ or\ \left(C \leq \frac{r_1 + r_2}{a+K}\right)\right\} = Pr\{C \leq r_1\} = r_1$. For firm 2, $\varphi_2 = Pr\{max(r_2 - aC, r_1 + r_2 - C(a+K)) \geq max(r_1 - C, 0)\} = Pr\{r_1 + r_2 - C(a+K) \geq max(r_1 - C, 0)\} = Pr\{(r_1 + r_2 - C(a+K) \geq r_1 - C)\ and\ (r_1 + r_2 - C(a+K) \geq 0)\} = Pr\left\{\left(C \leq \frac{r_2}{a+K-1}\right)\ and\ \left(C \leq \frac{r_1 + r_2}{a+K}\right)\right\}$. Since $r_2 \leq (a+K-1)r_1$, $\frac{r_2}{a+K-1} \leq \frac{r_1 + r_2}{a+K}$. Now I have the following: $\varphi_2 = Pr\left\{C \leq \frac{r_2}{a+K-1}\right\} = \frac{r_2}{a+K-1}$.

Case II. In this case no one chooses to be active in one firm and be passive in the another firm as follows: When $c \leq r_2 / a$ (i.e. $r_2 - ac \geq 0$), state PA is preferable than being passive at both firms. Also state PA is preferable than being active at both firms when $c \geq r_1/K$ ($r_2 - ac \geq r_1 + r_2 - ac - cK$). As $r_2/a \leq r_1/K$, there is no c, which satisfies above conditions. Using the same logic, state AP is never chosen. Then I have the following: $\varphi_1 = Pr\{max(r_1 - C, r_1 + r_2 - C(a+k)) \geq max(r_2 - aC, 0)\}$

$= Pr\{max(r_1 + r_2 - C(a+K)) \geq 0\} = \frac{r_1 + r_2}{a+k}$. For firm 2, $\varphi_2 = Pr\{max(r_2 - aC, r_1 + r_2 - C(a+K)) \geq max(r_1 - C, 0)\} = Pr\{max(r_1 + r_2 - C(a+K)) \geq 0\} = \frac{r_1 + r_2}{a+K}$. The probabilities of being active for the remaining four cases are as follows (the details of prove for these cases are available in a longer version of Appendix):

- *if* $\left(\frac{a}{K}\right) r_1 < r_2 \leq a$ *, then* $\varphi_1^*(\boldsymbol{r}) = \frac{r_1}{K}$ *and* $\varphi_2^*(\boldsymbol{r}) = \frac{r_2}{a}$.
- *if* $r_1 \leq K$ *and* $a < r_2$*, then* $\varphi_1^*(\boldsymbol{r}) = \frac{r_1}{K}$ *and* $\varphi_2^*(\boldsymbol{r}) = 1$.
- *if* $r_1 > K$, $r_2 > a + K - 1$ *and* $r_1 + r_2 > a + K$*, then* $\varphi_1^*(\boldsymbol{r}) = \varphi_2^*(\boldsymbol{r}) = 1$.
- *if* $r_1 > 1$ *and* $r_2 \leq a + K - 1$*, then* $\varphi_1^*(\boldsymbol{r}) = 1$ *and* $\varphi_2^*(\boldsymbol{r}) = \frac{r_2}{a+K-1}$. □

PROOF OF LEMMA 6B.

Case III. I show that *if* $r_1 \leq 1$ *and* $r_2 \leq \left(\frac{a+k-1}{K}\right) r_1$, *then* $\varphi_1{}^*(\boldsymbol{r}) = r_1$ *and* $\varphi_2{}^*(\boldsymbol{r}) = \frac{r_2}{a+K-1}$. In this case no one choose to be active in firm 2 and passive in firm 1 as follows:

When $c \leq (r_2 - r_1) / (a-1)$ (i.e. $r_2 - ac \geq r_1 - c$), state PA is preferable than state AP. Since $r_2 \leq \left(\frac{a+k-1}{K}\right) r1$, $c \leq r_1/K$. Also state PA is preferable than being active at both firms when $c \geq r_1/K$. Therefore, there is no c, which satisfies above conditions. Then I have the following: $\varphi_1 = Pr\{max(r_1 - C, r_1 + r_2 - C(a+K)) \geq max(r_2 - aC, 0)\}$

$= Pr\{max(r_1 - C, r_1 + r_2 - C(a+K)) \geq 0\}$.

Therefore, I have: $\varphi_1 = Pr\{(r_1 - C \geq 0) \; or \; (r_1 + r_2 - C(a+K) \geq 0)\} = Pr\left\{(C \leq r_1) \; or \; \left(C \leq \frac{r_1 + r_2}{a+K}\right)\right\}$. As $r_2 \leq \left(\frac{a+k-1}{K}\right) r_1$, therefore $r_1 \geq (r_1 + r_2)/(a+K)$. Then I have the following: $\varphi_1 = Pr\left\{(C \leq r_1) \; or \; \left(C \leq \frac{r_1 + r_2}{a+K}\right)\right\} = Pr\{C \leq r_1\} = r_1$. For firm 2,

$\varphi_2 = Pr\{max(r_2 - aC, r_1 + r_2 - C(a+K)) \geq max(r_1 - C, 0)\} = Pr\{r_1 + r_2 - C(a+K) \geq max(r_1 - C, 0)\} = Pr\{(r_1 + r_2 - C(a+K) \geq r_1 - C) \; and \; (r_1 + r_2 - C(a+K) \geq 0)\} = Pr\left\{\left(C \leq \frac{r_2}{a+K-1}\right) \; and \; \left(C \leq \frac{r_1 + r_2}{a+K}\right)\right\}$. Since $r_2 \leq \left(\frac{a+k-1}{K}\right) r_1$, $\frac{r_2}{a+K-1} \leq \frac{r_1 + r_2}{a+K}$. Now I have the following: $\varphi_2 = Pr\left\{C \leq \frac{r_2}{a+K-1}\right\} = \frac{r_2}{a+K-1}$.

Case IV. Since $r_1 > 1$, no one chooses state PP. Then I have the following: $\varphi_1 = Pr\{max(r_1 - C, r_1 + r_2 - C(a+K)) \geq max(r_2 - aC, 0)\}$

$= Pr\{max(r_1 - C, r_1 + r_2 - C(a+K)) \geq r_2 - aC\}$.

Therefore, I have: $\varphi_1 = Pr\{(r_1 - C \geq r_2 - aC) \; or \; (r_1 + r_2 - C(a+K) \geq r_2 - aC)\} = Pr\left\{(C > r_1/K) \; or \; \left(C \leq \frac{r_2 - r_1}{a-1}\right)\right\}$. As $r_1 / K \leq \frac{r_2 - r_1}{a-1}$, then I have the following: $\varphi_1 = Pr\left\{(C \leq r_1) \; or \; \left(C \leq \frac{r_1 + r_2}{a+K}\right)\right\} = Pr\{C \leq r_1\} + Pr\left\{C \leq \frac{r_1 + r_2}{a+K}\right\} = 1 + \frac{r_1}{K} - \frac{r_2 - r_1}{a-1}$. For firm 2,

$\varphi_2 = Pr\{max(r_2 - aC, r_1 + r_2 - C(a+K)) \geq r_1 - C\} = Pr\{(r_2 - aC \geq r_1 - C) \text{ or } (r_1 + r_2 - C(a+K) \geq r_1 - C)\} = Pr\left\{(C > r_1/K) \text{ or } \left(C > \frac{r_2 - r_1}{a-1}\right)\right\}$. As $r_1/K \leq \frac{r_2 - r_1}{a-1}$, I have the following: $\varphi_2 = \frac{r_2 - r_1}{a-1}$. The probabilities of being active for the remaining five cases are as follows (the details of prove for these cases are available in a longer version of Appendix):

- *if* $\left(\frac{a+K-1}{K}\right) r_1 < r_2 \leq ar_1$, *and* $r_1 \leq 1$, *then* $\varphi_1^*(\boldsymbol{r}) = r_1\left(1 + \frac{1}{K}\right) - \frac{r_2 - r_1}{a-1}$ *and* $\varphi_2^*(\boldsymbol{r}) = \frac{r_2 - r_1}{a-1}$.
- *if* $ar_1 < r_2 \leq a$, *then* $\varphi_1^*(\boldsymbol{r}) = \frac{r_1}{K}$ *and* $\varphi_2^*(\boldsymbol{r}) = \frac{r_2}{a}$.
- *if* $r_1 \leq K$, $r_2 > r_1 + a - 1$ *and* $r_2 > a$, *then* $\varphi_1^*(\boldsymbol{r}) = \frac{r_1}{K}$ *and* $\varphi_2^*(\boldsymbol{r}) = 1$.
- *if* $r_1 > K$, *and* $r_2 > a + K - 1$, *then* $\varphi_1^*(\boldsymbol{r}) = \varphi_2^*(\boldsymbol{r}) = 1$.
- *if* $r_1 > 1$, $r_2 \leq a + K - 1$, *and* $\left(\frac{a+K-1}{K}\right) r_1 > r_2$, *then* $\varphi_1^*(\boldsymbol{r}) = 1$ *and* $\varphi_2^*(\boldsymbol{r}) = \frac{r_2}{a+K-1}$.

□

Chapter 3

3 Airline Switching Revenue with Price-Guarantees

Many airlines permit ticket holders to change the time of their flight by paying a switching fee. For an airline, selecting a switching fee is an important strategic decision for two reasons. Firstly, it is a supplementary but considerable revenue item for firms with a narrow profit margin. Secondly, it significantly impacts their operational planning. Knowing that a low or high fee could cause operational challenges, such as unsold capacity or lost sales, the question that arises is what fee should be set for switching.

I model a single firm, which delivers two comparable services over two sequential periods. The price of the service in the second period is lower and I deal with a theoretical case where switching customers pay a switching fee, but get reimbursed the price difference. This is an extension of the current airlines' switching practice and money-back guarantees, a common practice in many industries (such as consumer electronics). I analyze the firm's revenue function and derive the optimal switching rate (proportion) between two periods.

I demonstrate that the uncertainty in switching behaviour of customers drives the firm's optimal switching policy. When this uncertainty increases, the firm should impose a higher switching fee. Furthermore, I present that the optimal switching fee is increasing in resource prices. Next, I numerically analyze the joint decisions of two service prices and the switching fee. This analysis shows that the optimal switching fee is increasing in the size of the low-price capacity.

3.1 Introduction

In recent years we have seen a growing trend of airline passengers demanding more flexible travel arrangements. This may, in part, be a consequence of dynamic pricing, where customers have come to expect daily or hourly airfare updates, and may also, in part, be due to a general demand for greater consumer flexibility. This trend, however, increases the firms' uncertainty in capacity and resource planning. Given customers' willingness to change the service consumption time, firms aim to control these negative impacts by imposing a *switching* or *change fee*. A second type of fee is the *cancellation fee*, which is applied when a ticket holder decides to completely cancel the service. These ancillary fees have recently provided a better opportunity for firms to collect higher revenues. Switching fees provide more revenue than cancellation fees since customers typically still derive some value from switching tickets.[9] Notice that the current practice is mainly focused on switching from ‘low’ to ‘high’ preference flights, where the customer in addition has to pay the difference in fare.

According to the U.S. Bureau of Labor Statistics, the total ancillary revenue collected from switching and cancellation fees for U.S. airlines has been growing over the past six years (in '000): $915,231 (2007), $1,668,748 (2008), $2,373,019 (2009), $2,297,377 (2010), $2,380,157 (2011), $2,554,658 (2012).[10] Since 2009, these ancillary revenues have accounted for more than 3% of U.S. airlines’ revenue, and it is expected that this percentage will continue to rise in the future.[11] Furthermore, according to an article in the Wall Street Journal, many airlines allegedly can barely survive without these fees, even when they successfully operate at 99% capacity level.[12] Despite the emerging importance of ancillary revenue, its role as a source of revenue and its significance for airlines’ dynamic pricing has not yet been clarified in operations research and revenue management literature. Moreover, as Gallego & Sahin (2010) point out, many firms are not using ancillary fees properly and this is a problem not limited to the aviation industry.

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[9] URL: http://online.wsj.com/article/SB10001424052970204563304574318212311819146.html; accessed: 2015–8–8.

[10] URL: www.rita.dot.gov/bts/sites/rita.dot.gov.bts/files/subject_areas/airline_information/reservation_ cancellation_change_fees/index.html; accessed 2013-05-01.

[11] URL: http://online.wsj.com/article/SB10001424052970204563304574318212311819146.html; accessed: 2015–8–8.

[12] URL: http://www.wsj.com/articles/SB10001424052702303296604577450581396602106; accessed: 2015–8–8.

Airlines have different policies for switching fees. For U.S. domestic flights, some airlines, such as Jet Blue, Alaska and AirTran, do not charge their passengers for switching; while others, such as Virgin America and Hawaiian Airlines, impose a switching fee of around $100, and some others, such as US Airways and Delta, charge even higher fees. This distinction in switching fees policies also exists in the case of international flights. Unless these fares' impacts on optimal dynamic pricing are well understood and the design of these fees is optimized, the prescriptions based on even sophisticated revenue management systems may not be beneficial for firms. As mentioned, under the current practice, the switching is generally from a low to a high price flight where customers must, in addition, pay the fare difference.

Regarding the same flight (for a particular departure date, time and fare class), some airlines such as Air Canada or American Airlines do offer fare guarantees, but mostly on the same day of purchase. These fare guarantees are used by various *price-tracking tools* to refund customers when prices drop. For example, *Yapta.com* and *Tingo.com* track the prices before and after purchasing airline tickets and hotel bookings, respectively. As of October 25th, 2013, Yapta and Tingo have reportedly reimbursed around $250,000,000 and $720,000, respectively.[13] There is a similar price tracking tool in the auto rental industry.[14] If the secondary price drops more than the switching fee, these tools alert the ticket holder and either provide a reimbursement or tell ticket holders exactly who to contact and what to say to obtain the refund. The application of these tools highlights the growing importance of managing switching revenues within different enterprises. Moreover, these tools demonstrate that the customers' abilities to search for better rates have been significantly improved.

I would like to highlight that the focus of this study is on the switching to a different flight or date. In this case, most companies charge a change fee in addition to the fare

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[13] URLs: www.yapta.com and www.tingo.com; accessed 2013–9–9.

[14] URL: http://www.nytimes.com/2012/02/18/your-money/autoslash-a-rate-sleuth-makes-rental-car-companies-squirm-your-money.html; accessed: 2015–8–8.

difference when customers want to switch to a different flight. What I analyze in this paper is the effect of introducing the change fee and *reimbursement* of difference in fare when customers switch to a lower priced flight. The reimbursement of difference in fare is currently not a common airline industry practice. Our approach is, however, similar to Gallego & Sahin (2010) who investigate the benefits of designing partial refundable fares that do not exist currently: I am analyzing this issue to investigate the implications of allowing for fare difference reimbursement. Since customers are price sensitive, I present a hypothetical model that alleviates the burden for customers to change their flight. Furthermore, the practice of *price match guarantee* has been deployed in many industries, such as consumer electronics, which protects customers against price fluctuations over a specific period of time. It is worthwhile to refer to some airlines that offer restrictive versions of this service. For example, Jet Blue and IndiGo (India's largest domestic airline as of September 2015) retains the difference in fare in a credit shell for customers for a couple of months.[15] American Airlines roll-overs ticket holders to a lower fare if this fare was not offered at the time of original booking.[16] In this essay, I study a single-leg flight, not considering a return flight. Also, I do not include fare classes where higher fares classes have right to switch, i.e., analysis is within one fare class.

I analyze the impact of switching fee on firms' operating revenues and investigate how and when imposing these fees is beneficial to firms. When the firm charges a relatively low switching fee, this increases the rate of switching and creates more opportunities for collecting revenue. In addition, the firm can collect more revenue indirectly by replacing the switching ticket holders with waiting list customers during high-price periods. On the other hand, while high switching fees would increase ancillary revenue, they also constitute an opportunity cost to the firms. The higher the switching fee, the less

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[15] URLs: http://www.jetblue.com/travel-agents/lower-fare; http://timesofindia.indiatimes.com/business/india-business/Refund-blues-after-cancellation/articleshow/2199454.cms; http://www.firstpost.com/business/spicejet-has-highest-load-factor-at-92-indigo-retains-a-third-of-market-in-august-2442248.html; accessed 2015-10-10.

[16] URL: www.aa.com/i18n/agency/Booking_Ticketing/Reissue_Rollovers/rollover_lower_fare.jsp; accessed 2013-10-25.

switching behaviour. This may lead to a demand shortage in low-price periods and, ultimately lost sales due to capacity rationing in high-price periods. Therefore, net profits may start to diminish if switching fees are high. This trade-off between ‘the cost of lost demand and unsold capacity’ and ‘direct and indirect revenue from switching’ is the key to understanding the firm's optimal switching fee strategy. Recognizing the growing importance of these supplementary fees for firms, I aim to investigate the optimal switching fees based on airlines' demands and operational factors. To the best of our knowledge, this study is the first to consider revenue management analysis with the presence of switching fee as a source of ancillary revenue for firms.

I consider a monopoly firm that sequentially delivers a high-price service and a low-price service over two periods. The price gap between two periods triggers demand leakage, which is based on demand sensitivity to price. In this study, I assume customers switch from the high-price period to the low-price period. For example, when prices drop for a flight, ticket holders compare the difference between the prices of the original (first) ticket and the subsequent (secondary) ticket, plus the switching fee. If a ticket holder decides to change the date and the price difference is acceptable based on his or her criteria, he or she pays the fee. The cost of switching is added to the secondary ticket price and increases the total expected cost of the ticket. I assume customers generally decide to switch because they are *price-sensitive*. They track prices to find the *best* rate (Gorin et al. 2012), since they are aware that firms are adjusting prices on everyday items several times a day. In addition to this *monetary* incentive, in many situations, such as illness, rescheduled meetings and family trips, passengers change their travel arrangements without any monetary benefits. I consider all of these personal or carrier obligations as *non-monetary* incentives. In contrast to the situation in this essay, I will isolate time uncertainty as the different motivation behind switching fee in Essay 3, i.e., in Essay 2 the model captures those customers who change regardless of switching fee, but in Essay 3, customers only change due to plan change. Investigating the impact both incentives have on the behaviour of ticket holders as well as waiting list customers, I analyze how a firm should design a switching fee to maximize total revenue over two periods. With reference to switching fees, I later numerically illustrate how firms should update markdown pricing strategies, as well.

The main contributions of this paper are highlighted in the following points. Firstly, I demonstrate that the uncertainty in switching behaviour of customers drives the firm's optimal switching policy. When this uncertainty increases, the firm should impose a higher switching fee. Secondly, I present that the optimal switching fee is increasing in resource prices. Thirdly, my numerical analysis demonstrates that by increasing the capacity size of the low-price service, the firm can impose a higher switching fee and collect more revenue. Finally, I observe that when a firm faces higher demands, it imposes a lower switching rate on average, but on a larger switching ticket holder population. This ultimately generates higher revenue for the firm.

3.2 Literature Review

There are some streams of research that relate to this work. The demand leakage and fencing customers between market segments, by imposing a switching fee, have been studied in literature (see Zhang & Bell (2012) and references therein). The switching fee in this study can be seen as a time-based market segmentation mechanism in which a firm differentiates customers based on their rights to change the time of their consumption.

A more closely related study that considers customers' flexibility on consumption by designing a partial refundable fee is Gallego & Sahin (2010) in which customers pay a smaller up-front payment to have the right to cancel their tickets. The authors verify that partially refundable tickets result in a higher consumer surplus than low-to-high pricing based on non-refundable and fully refundable fares. Partially refundable fares are also socially optimal. My article has the same motivation: paying a small fee to gain the right on one aspect of the service consumption. However, compared with the previous study where the customers pay the fee to have a right to *cancel* the consumption, in my model, customers pay the fee to change the consumption time.

Many assumptions of my framework, such as the firm's sequentially decreasing pricing policy, customers arrivals in the market, and resource capacity rating, follow Liu & Ryzin (2008), and the related literature in capacity rationing, including Xie & Shugan (2001), and Nasiry & Popescu (2012). Alternatively, I consider two extensions: first, I allow the capacity rationing over two resources with a distinct demand for both resources. Second, I assume a stochastic switching behaviour for ticket holders.

In addition to switching and cancellation fees, airlines also collect revenue from add-on services and auxiliary items such as luggage fees or onboard snacks and beverages purchases (for a classification of ancillary services please check Vinod & Moore 2009). The idea of generating supplementary revenue from these services/items has been highlighted in revenue management literature. Two papers are motivated by charging baggage fees: Allon et al. (2011) and Shulman & Gengu (2012). These studies consider how a firm should offer an ancillary service (baggage delivery) along with the main service. Although this study belongs to the ancillary revenue stream of research, it differentiates itself from the above studies by focusing on ancillary fees, such as switching fee and not add-on services or auxiliary items.

3.3 Model

A monopoly firm delivers two comparable services with corresponding capacities (resources) C_i and corresponding prices p_i, i = 1,2, where $p_1 \geq p_2$. For example, consider an airline operating weekly flights from Toronto to New York, where there is a 'sale' period (week) for flights in week 2. I normalize capacity size in the first period to 1, i.e., $C_1 = 1$, and consider demand to be continuous. The market sizes (potential demand) in each period are distinct and both consist of a large customer population. The booking system is sequential and planned over two booking periods such that the firm first accommodates the demand of service 1 (in booking period 1), and then accommodates the demand of service 2 (in booking period 2). This policy triggers demand leakage from period 1 to period 2. Unmet demand leaves the market at the end of each period and the service of resources 1 and 2 are delivered sequentially after the two booking periods. The firm maximizes total revenue over two periods by choosing the switching fee at the beginning of period 1.

The sequence of events is as follows: At the beginning of period 1, the firm announces p_1 and the *switching fee*, p_s. The firm observes the demand for resource 1, D_1, and accommodates it fully when $D_1 \leq 1$, and partially when $D_1 > 1$. At the beginning of period 2, the firm announces p_2 and observes demand for resource 2, D_2, plus a *demand leakage* from market 1 to market 2. To focus mainly on the impact of switching customers on firm's revenue and to keep the analysis tractable, I assume the excess demand (waiting list) of resource 1 and 2 leave the market at the end of each period.[17] Notice that, I am considering a deterministic demand setting in order to isolate the effect from the switching fee, but one can consider it as an average or expected demand. Figure 5 demonstrates various demand streams and Figure 6 represents the sequence of events.

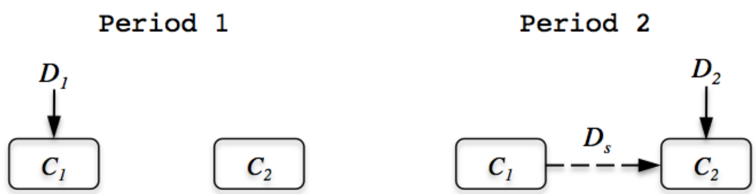

Figure 5: Demand streams in two booking periods

In this study, the demand leakage accounts for *switching* demand, D_s, which represents the switching ticket holders; a portion of period 1 ticket holders who prefer to switch from resource 1 to resource 2. The higher the fee, the lower the switching portion of ticket holders between resource 1 and resource 2 (and vice versa). Also, I assume this switching proportion can be random to capture the uncertainty associated with the behaviour of switching customers. This uncertainty is generated from schedule changes or other personal circumstances of ticket holders, especially business travelers.

Considering the two above assumptions, I denote switching rate (proportion) by Θ, which is defined as follows:

$$\Theta = \theta(p_s)\ \varepsilon$$

[17] In addition to other extensions, in the third essay of this thesis, I will include the waiting list customers.

where $\theta(p_s)$ is a continuously differentiable from 0 to A and decreasing general function and ε is a price independent random variable defined on the range [0,1], with cumulative distribution function $F(.)$ and the probability density function $f(.)$[18]. In addition, I assume, $\frac{d^2\theta}{dp_s^2} \leq 0$, since most of switching customers are business travelers who are less sensitive to switching rate variations at low values. In order to assure that positive switching demand is possible for some range of p_s, I require $\theta(A) = 0$, i.e., A represents a critical threshold of p_s at which the switching rate vanishes, where $0 << A$. A reasonable level for A is p_1 since any switching fee greater than p_1 motivates ticket holders to forfeit their ticket and join D_2 without paying the switching fee.

$\theta(p_s)$ denotes the switching rate between resource 1 and resource 2 without uncertainty. I would like to highlight that the term 'rate' represents the 'proportion' of ticket holders. If a switching customer successfully purchases a unit of capacity in market 2, he or she pays $p_2 + p_s$ and is reimbursed p_1. Otherwise, the ticket holder keeps the original ticket. The switching demand is defined by:

$$D_s = \theta(p_s)\ \varepsilon \min(1, D_1)$$

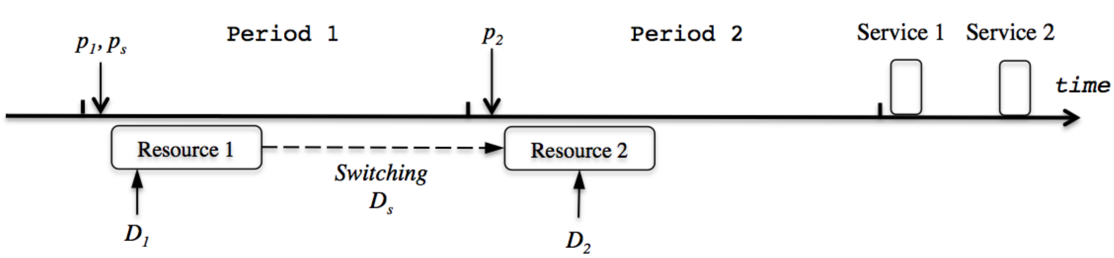

Figure 6: The sequence of events

[18] I define ε to capture the uncertainty in switching behaviour using the setting in Petruzzi and Dada [1999]. The multiplication implies the independence of monetary and non-monetary incentives as well.

In this study, I assume that, when a resource is insufficient to satisfy demand, the limited supply is rationed based on the first-come-first-served practice. Also, all switching customers and original demands are arrived randomly during the booking period. The *accommodation level*[19], denoted by q, can be given by the ratio of available capacity in period 2, C_2, to the total demand in this period, $D_2 + D_s$, namely,

$$q(p_s) = \begin{cases} min\left(\frac{C_2}{D_2+\theta(p_s)\varepsilon D_1}, 1\right), \ D_1 < 1 \\ min\left(\frac{C_2}{D_2+\theta(p_s)\varepsilon}, 1\right), \ otherwise \end{cases} \tag{1}$$

Since C_2 can be lower than the total demand in period 2, in the above equation, a *minimum* function is considered to ensure $0 < q \leq 1$. Notice when $D_1 < 1$, the total demand in period 2 is $D_2 + \theta(p_s)\varepsilon D_1$, and when $D_1 \geq 1$, the total demand in this period is $D_2 + \theta(p_s)\varepsilon$. The above formula is derived from these bases and consistent with the definition of accommodation level in Liu and van Ryzin (2008).

There exists a relationship between the switching rate of period-1 customers, Θ, and the accommodation level at period-2 resource, q. A lower switching rate indicates a higher accommodation level for this resource (and vice versa). There exists a unique threshold on Θ, denoted by $\widehat{\Theta}_q$, such that $D_s + D_2 = C_2$. In other words, when $\Theta \leq \widehat{\Theta}_q$, $q = 1$ and all period 2 customers are accommodated, and when $\Theta > \widehat{\Theta}_q$, $q < 1$ and period 2 customers face capacity rationing, $\widehat{\Theta}_q$ is solved for Θ based on the following equation:

$$\widehat{\Theta}_q = \begin{cases} 0, & if \ C_2 \leq D_2 \\ \dfrac{C_2 - D_2}{D_1}, & if \ D_1 \leq 1 \ and \ D_2 < C_2 \\ C_2 - D_2, & otherwise \end{cases}$$

I assume there is no demand leakage from market 2 to market 1 since customers mostly look forward to transferring their demand, and a service with a higher price would not be

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[19] Note that the term “accommodation level” is different from the common term in airline industry, “load factor”. Load factor for a single flight can be calculated by dividing the number of passengers by the number of seats. Accommodation level evaluates the operational success in terms of demand, and load factor evaluates the operational success in terms of capacity.

attractive, in general, to low-price customers. To focus on switching behaviour and to keep the model parsimonious, I prevent leakage from the low-price period to the high-price period.

The total revenue for the firm is the original revenue for two resources plus the switching revenue:

$$\Pi(p_s) = p_1 min(1, D_1) + \ p_2 D_2 q(p_s) + (p_s + p_2 - p_1) min(1, D_1) \Theta q(p_s) \qquad (2)$$

where $q(p_s)$ is given by equation (1). The first term in (2), $p_1 \min(1, D_1)$, denotes the revenue collected in period 1. The next two following terms in (2) denote revenues collected in period 2, i.e., the second term, $p_2 D_2 q(p_s)$, represents the revenue from the original demand for period 2, and the third term, $(p_s + p_2 - p_1) min(1, D_1) \Theta q(p_s)$, represents the revenue from switching customers. Notice that in (2), the capacity size of resource 1, C_1, is represented by number 1, and the capacity size of resource 2, C_2, is embedded in $q(p_s)$. Also, since demand of resource 1, D_1, is deterministic, the *minimum* function, $min(1, D_1)$, is constant as well. I denote the optimal switching fee by p_s^*.

3.4 Analysis of the Optimal Switching Rate in Symmetrical Case

I first consider the *symmetrical case*, in which the sizes of two resources are constant over time, i.e., $C_1 = C_2 = 1$. Later, I relax this assumption and study the impact of asymmetric capacities in two periods. I also assume $D_2 > D_1$. This assumption implies that, since the price drops in the second period, the demand for the second period is higher than the demand in the first period when the potential markets are independent and equal in size. Next, I consider the firm's optimal decision, p_s^*.

Defining

$$z = \frac{1 - D_2}{min(1, D_1) \theta(p_s)}$$

the accommodation level can be simplified as follows:

$$q(p_s) = \begin{cases} 1, & \varepsilon < z \\ \dfrac{1}{D_2 + min(1, D_1)\theta(p_s)\varepsilon}, & \varepsilon \geq z \end{cases}$$

The expected revenue can be formulated based on (1) for three cases:

• When $D_1 \leq 1$, and $D_2 \leq 1$, it be given by, namely:

$$E[\Pi(p_s)] = \int_0^{\frac{1-D_2}{D_1\theta(p_s)}} (p_s + p_2 - p_1)D_1\,\theta(p_s)uf(u)du$$
$$+ \quad \int_{\frac{1-D_2}{D_1\theta(p_s)}}^{1} \frac{(p_s + p_2 - p_1)D_1\theta(p_s)u + p_2D_2}{D_2 + D_1\theta(p_s)u} f(u)du \qquad (3)$$

The above equation depends on $\theta(p_s)$, as well as $f(.)$. The first integral represents a firm's expected revenue when the accommodation level is one, i.e., the available capacity in period 2 is higher than total demand in this period. Also, the second integral represents a firm's expected revenue when the accommodation level is lower than one, i.e., the available capacity in period 2 is lower than total demand in this period. Notice that the terms inside the first integrals, $(p_s + p_2 - p_1)D_1\theta(p_s)$ and $\frac{((p_s+p_2-p_1)D_1\theta(p_s)u+p_2D_2)}{D_2+D_1\theta(p_s)u}$, are derived from equation (2). These terms represent firm's revenue function in this equation for un-capacitated and capacitated cases, respectively. In this case, the population of switching customers is represented by $D_1\theta$.

• When $D_1 \leq 1$, and $D_2 > 1$, the expected revenue is given by:

$$E[\Pi(p_s)] = \int_0^1 \frac{(p_s + p_2 - p_1)D_1\theta(p_s)u + p_2D_2}{D_2 + D_1\theta(p_s)u} f(u)du$$

Notice that, in this case, the accommodation level is always lower than 1. Also, $\frac{1-D_2}{D_1\theta(p_s)} < 0$, therefore, the first integral in (3) vanishes and the second integral in (3) is simplified as presented above.

• When $D_1 > 1$, and $D_2 > 1$, the expected revenue is given by:

$$E[\Pi(p_s)] = \int_0^1 \frac{(p_s + p_2 - p_1)\theta(p_s)u + p_2 D_2}{D_2 + \theta(p_s)u} f(u)du$$

In this case, the population of switching customers is represented by θ. Next, I study the properties of optimal switching fee under stochastic switching behaviour.

Proposition 1 *The optimal switching fee, p_s^*, exists and is unique.*

I would like to highlight that the existence and uniqueness of optimal switching fee hold for all three cases of D_1 and D_2. Next, I present how to derive the optimal switching fee for two following cases of D_2, $D_2 \leq 1$, and $D_2 > 1$.

Proposition 2A *If $D_2 \leq 1$, the optimal switching fee, p_s^*, is the solution to the following equation:*

$$[(p_s + p_2 - p_1)\theta'(p_s) + \theta(p_s)] \int_0^{\frac{1-D_2}{D_1\theta(p_s)}} uf(u)\,du \\ + \int_{\frac{1-D_2}{D_1\theta(p_s)}}^1 \left(\frac{u}{D_2 + D_1\theta(p_s)u}\right)\left(\theta(p_s) + \frac{(p_s - p_1)\theta'(p_s)D_2}{D_2 + D_1\theta(p_s)u}\right) f(u)du = 0$$

Proposition 2B *If $D_2 > 1$, the optimal switching fee, p_s^*, is the solution to the following equation:*

$$\int_0^1 \left(\frac{\min(1, D_1)u}{D_2 + \min(1, D_1)\theta(p_s)u}\right)\left(\theta(p_s) + \frac{(p_s - p_1)\theta'(p_s)D_2}{D_2 + \min(1, D_1)\theta(p_s)u}\right) f(u)du = 0$$

The optimal switching fee, p_s^*, satisfies above equations, which depends on $\theta(p_s)$, as well as $f(.)$. Next, I study the relationship between demand and the optimal switching fee, p_s^*.

Corollary 1 *The optimal switching fee, p_s^*, is increasing in D_1* and *D_2.*

When D_1 increases, it generates a larger ticket-holder population for flight 1 and a larger population of switching customers accordingly. Therefore, the firm can charge a higher switching fee. When D_2 increases, however, the switching customers have to compete for flight 2 tickets with more flight 2 customers, thus intensifying the competition among customers for this flight. Consequently, the firm can set a higher switching fee to discourage customers to switch in order to decrease intensity of competition. In general, Corollary 1 demonstrates that when facing higher demands, the firm allows a lower switching rate on average, but on a larger switching ticket holder population. This ultimately generates higher revenue for the firm. Next, I examine how the optimal switching fee is affected by the original and markdown price, p_1 and p_2.

Proposition 3 *The optimal switching fee is decreasing in markdown price p_2 and increasing in original price p_1.*

When the price of a resource increases, switching customers should pay a higher fee for switching from this resource to another resource. This can be addressed by a higher cost of losing customers for a more pricy resource. However, as the markdown price goes up, they pay a lower switching fee, since the price gap between the two resources goes down. Notice that one may argue that switching to a more valuable resource should be more costly for customers. Proposition 3, however, does not support this argument. Next, I examine how the optimal switching fee is affected by size of resources.

Proposition 4 *The optimal switching fee is decreasing in C_1 (or C_2).*

Notice that I normalize the capacity C_1 and C_2 to 1. By increasing the capacity size, which is the unit of analysis, D_1 and D_2 will be lower. In the next section, I numerically analyze this result further.

In this study, I analyze how a firm should manage its revenue when facing a *stochastic* switching rate with a *capacity constraint*. One may wonder, under a more simplified and classic scenario of a deterministic switching rate without capacity constraint, which switching fee maximizes a firm's total revenue. For this case, define, p_s^{DU}, the optimal switching fee which maximizes solely deterministic and un-capacitated revenue, given by:

$$(p_s + p_2 - p_1)\theta(p_s)D_1$$

Note that, under a deterministic and un-capacitated case, the first and the second terms of a firm's revenue function, given by (2), are constant and independent of a switching fee. Next, I study the properties of this optimal switching fee and the relationship between optimal switching prices under different scenarios.

Proposition 5A $p_s^* \geq p_s^{DU}$.

Proposition 5B *The asymptotic limits of the optimal switching fee with respect to demand is given by* $\lim_{D_2 \to 0} p_s^* = p_s^{DU}$ *and* $\lim_{D_2 \to \infty} p_s^* = A$.

The limited available resources, as well as the uncertainty in ticket holders' switching behaviours, drive the firm to impose a higher switching fee. Therefore, the optimal switching fee in the stochastic and capacitated case is greater than the deterministic and un-capacitated case. Next, I study the value of the optimal switching fee when the demand in the second period (D_2) increases to the highest point or decreases to the lowest point. Proposition 5 simplifies the previous results for the extreme cases of D_2. Notice that when D_2 approaches zero, there is no limit on capacity allocation. Moreover, the uncertainty in switching behaviour becomes less critical for the firm. Therefore, the stochastic capacitated problem can be simplified to the deterministic un-capacitated case. I would like to highlight that Proposition 5 is in agreement with Corollary 1. When demand goes to infinity, the firm can impose the highest possible fee; however, the optimal switching fee never goes higher than A, since imposing any price higher than A does not generate extra benefit for the firm.

In Proposition 5, I compare the optimal switching fee for a deterministic un-capacitated case with a stochastic capacitated case. This result makes one wonder what happens in the other two remaining cases, i.e., deterministic capacitated case and stochastic un-capacitated case? Under the stochastic un-capacitated case, the firm's revenue function given by (2) can be simplified as:

$$(p_s + p_2 - p_1)\theta(p_s)D_1\varepsilon$$

Notice that the firm's expected revenue is given by:

$$\int_0^1 (p_s + p_2 - p_1)D_1\,\theta(p_s)uf(u)du = (p_s + p_2 - p_1)\theta(p_s)D_1$$

which is equal to the firm's revenue under a deterministic un-capacitated case. Now, consider a deterministic capacitated case. When $q(p_s) = 1$, the firm's revenue function is:

$$(p_s + p_2 - p_1)\theta(p_s)\min(1, D_1)$$

and when $q(p_s) < 1$, the firm's revenue function is:

$$p_2 + \frac{(p_s - p_1)\min(1, D_1)\theta(p_s)}{D_2 + \min(1, D_1)\theta(p_s)}$$

Note that the denominator of the ratio in the above function is decreasing in switching fee, and the numerator of the ratio represents the firm's revenue function under a deterministic un-capacitated case. This implies that the optimal switching fee under a deterministic capacitated case is higher than the optimal switching fee under a deterministic un-capacitated case. Considering all four cases, the optimal switching fee is increasing in uncertainty of switching behaviour only for a capacitated case, while, *ceteris paribus*, this fee is always higher under a capacitated case versus an un-capacitated case.

3.5 Pricing with Switching Fees - Numerical Illustration

In this section, I study two cases as follows. In the first example, I numerically find the optimal switching fee, p_s^*, for a specific problem. Then I extend my findings by analyzing how, for the same problem, the firm should make decisions that jointly treat its markdown pricing policy p_2 (given the original price p_1), and the switching fee, p_s.

3.5.1 Example 1: Optimal Switching Fee

In the first analysis, I assume the ticket prices in the first and the second periods, p_1 and p_2, are exogenous (p_1 and p_2 can also be seen as the optimal prices to Equation (2) without including switching revenues). Also, I assume demand in each period is given by: $D_i = 500 - p_i$, for $i = 1, 2$.

The sequence of analysis is as follows: Given C_1, C_2, p_1, and p_2, the firm selects the optimal switching fee, p_s^*, to maximize its revenue, which is specified by Equation (2). I study two scenarios for ε: (1) a *deterministic* case in which $\varepsilon = 0.5$ and (2) and extreme *stochastic* case in which ε follows a Bernoulli distribution with $p = 0.5$. Note that, in both cases, $E(\varepsilon) = 0.5$, to analyze the impact of uncertainty in switching behaviour of ticket holders.

With respect to the switching rate, I consider the following:

$$\theta(p_s) = 0.8 - 0.0025\, p_s.$$

Notice that in the above setting, when the switching fee is zero, the expected switching rate is $E(\varepsilon)\,\theta(0) = 0.5 \times 0.8 = 0.4$. Also, for a high switching fee (for example above \$300), the expected switching rate is negligible. The choice of the above coefficients depends on these bases. In this analysis, I also study the impact of the size of capacity on joint pricing decisions for specific values of C_1 and C_2. The following analysis reports on 17 cases, spanned by

$$(C_1, C_2) \in \left\{ \begin{array}{c} (50,50), (100,100), (200,200), (300,300), (400,400), (500,500), (50,450), (100,400), \\ (200,300), (300,200), (400,100), (450,50), (250,100), (250,400), (100,250), (400,250), (250,250) \end{array} \right\}.$$

The magnitudes of C_1 and C_2 belong to the range [50, 500], and the ratios of C_1 and C_2 belong to [1/9, 9], which covers a sufficiently wide range of possible values of C_1 and C_2. I find this selection of parameters to be sufficient for analyzing the firm's switching fee pricing decision, p_s^*, based on (2). Notice that the first six cases demonstrate a symmetric case; the second six cases demonstrate a situation in which the sum of capacities equals

500; the next four cases represent a situation in which one capacity is 250 and the second is at a relatively low or a high level; and the last case represents a centre point. The results of the 17 cases are summarized in Table 6.

I observe that the numerical example also demonstrates some of the results discussed previously. For example, consistent with Proposition 1, a unique optimal switching fee exists for all 17 cases. Moreover, as mentioned in Proposition 5, the optimal switching fee never goes lower than the optimal switching fee under deterministic and un-capacitated case. Notice that the firm's revenue under the deterministic and un-capacitated case is:

Table 5: Optimal switching fee for two cases of *ε*: left (*ε* = 0.5) and right (*ε* follows a Bernoulli distribution with *p* = 0.5).

C_1	C_2	p_1	p_2	p_s^*
50	50	450	450	320
100	100	400	400	320
200	200	300	300	310
300	300	250	250	160
400	400	250	250	160
500	500	250	250	160
50	450	450	250	160
100	400	400	250	160
200	300	300	250	160
300	200	250	300	160
400	100	250	400	287
450	50	250	450	288
250	100	250	400	287
250	400	250	250	160
100	250	400	250	320
400	250	250	250	285
250	250	250	250	286

C_1	C_2	p_1	p_2	p_s^*
50	50	450	450	320
100	100	400	400	320
200	200	300	300	310
300	300	250	250	230
400	400	250	250	160
500	500	250	250	160
50	450	450	250	160
100	400	400	250	160
200	300	300	250	160
300	200	250	300	287
400	100	250	400	288
450	50	250	450	291
250	100	250	400	288
250	400	250	250	286
100	250	400	250	320
400	250	250	250	286
250	250	250	250	286

$$\Pi(p_s) = p_s(0.8 - 0.0025\, p_s)D_1$$

which is maximized at $p_s^{*DU} = 160$. Besides, as discussed by Proposition 3, the optimal switching fee is increasing in markdown price p_2 and original price p_1. Finally, the optimal switching fee is decreasing in size of capacity. I also observe the following results:

First, I find that, in most cases, the firm sets the switching fee higher than the price difference $(\Delta p = p_1 - p_2)$ between two periods. The only exception is when the size of the first resource is extremely low and the size of the second resource is extremely high. Such pricing policy prevents customers from switching solely for monetary incentive. When ticket holders switch, however, for non-monetary reasons, monetary incentives still paly a role in the switching behavoiur. These results show that, in most cases, switching cutomers pay significantly higher prices in total (p_2 + p_s) than the original ticket price, p_1.

I also observe that the switching fee increases as the uncertainty in the switching behaviour (standard deviation of ε) increases. The firm sets a higher switching fee to offset the cost of uncertainty in customer behaviour. Note that this result is partially discussed in Proposition 3A. Finally, the firm's expected revenue significantly depends on the switching fee (Figure 7). Changing the switching fee can lead to approximately 10% variation in the firm's expected revenue. This impact is more significant when capacity is adequate.

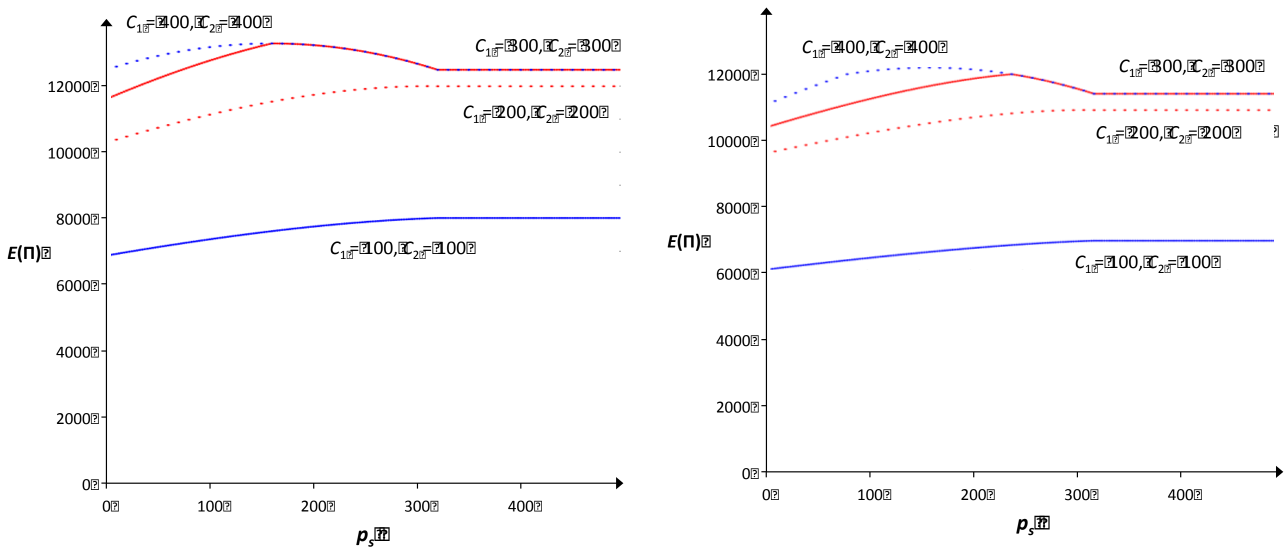

Figure 7: Firm's expected revenue, E(π), left (ε = 0.5) and right (ε follows a Bernoulli distribution with p = 0.5).

3.5.2 Example 2: Joint Optimal Switching Fee and Markdown Price

To focus better on the dynamics between original prices and switching fee, in this numerical example, I consider an endogenous demand setting for the markdown resource. I assume the ticket price in the first period, p_1, is exogenous; p_1 can also be seen as the optimal price to Equation (2) considering the first period exclusively.

The sequence of analysis is as follows: Given C_1, C_2, and p_1, the firm selects the optimal price of the second resource, p_2^*, and switching fee, p_s^*, to maximize its revenue, which is specified by Equation (2). Notice that updating the price of the second period, p_2, also changes the demand in the second period, D_2. I analyze the firm's joint pricing decisions $\left(p_s^*, p_2^*\right)$ based on (2). The results of the 17 cases are summarized in Table 7. I observe the following results:

First, in some case the switching fee increases as the uncertainty in the switching behaviour (standard deviation of ε) increases. This result is different from Corollary 2. To hedge against uncertainty in customer behaviour in these cases, the firm allocates more resources to the original demand of the second resource by setting a lower markdown price, and allocates fewer resources to switching customers by setting a higher switching fee. Note that in this scenario, the firm benefits from a stable demand (D_2) to deal with the unstable demand (θ).

Also, I notice that a higher capacity size results in a lower price p_2, which is intuitive; a more available resource should be priced lower in the market, and a less available resource priced higher. Next, I realize that the switching fee decreases in C_2 and increases in C_2. These findings are also intuitive and consistent with previous findings.

Table 7: Optimal pricing for two cases of ε: left (ε = 0.5) and right (ε follows a Bernoulli distribution with p = 0.5).

C_1	C_2	p_1	p_2^*	p_s^*
50	50	450	450	320
100	100	400	410	320
200	200	300	340	300
300	300	250	310	140
400	400	250	280	145
500	500	250	285	145
50	450	450	250	160
100	400	400	260	155
200	300	300	270	150
300	200	250	370	135
400	100	250	400	287
450	50	250	390	300
250	100	250	390	295
250	400	250	280	145
100	250	400	260	310
400	250	250	310	240
250	250	250	310	240

C_1	C_2	p_1	p_2^*	p_s^*
50	50	450	450	320
100	100	400	410	320
200	200	300	340	300
300	300	250	310	140
400	400	250	280	145
500	500	250	285	145
50	450	450	250	160
100	400	400	260	150
200	300	300	270	150
300	200	250	370	287
400	100	250	400	288
450	50	250	390	300
250	100	250	390	295
250	400	250	280	280
100	250	400	260	310
400	250	250	310	250
250	250	250	310	250

Finally, I observe that the optimal switching fee is lower when it is set jointly with the markdown price. When the firm updates only switching fees, it cannot generate extra revenue from the original demand of the second resource and, therefore, must deal solely with switching customers in order to generate similar revenues. These dynamics push the firm to impose a higher switching fee.

3.6 Discussion

This study focuses on the benefits of managing revenues from switching customers, a phenomenon having recently becomes very significant for many industries such as airlines. As discussed, managing ancillary revenue not only supports a firm with a narrow profit margin, but also could improve customer relationship through providing more options for their consumption. Many industries, such as consumer electronic retail, provide money-back guarantees to customers, even over future dates (for a certain time frame). This practice is used as a marketing tool to generate higher sales.[20] In this study, I consider a similar scenario in which a hypothetical airline might want to use marketing tool as a marketing strategy to attract price-sensitive customers. The proposed case has already been implemented by some airlines. As airlines are exposed to various marketing and pricing strategies, we would not be surprised to see airlines also offering price-guarantees across dates and flights. This study contributes by highlighting some of the operational aspects to take into consideration for these marketing and pricing strategies.

I considered a monopoly firm that sequentially announces prices over two periods. The customers track prices and this behaviour triggers demand leakage between two periods. The firm seeks to find the optimal switching rate, but the existence of original demand complicates the analysis for the firm. It is not clear whether the firm should allow a low or a high accommodation rate for switching customers. I demonstrate that the uncertainty in switching behaviour of its customers drives the firm’s optimal switching policy. When this uncertainty increases, the firm should impose a higher switching fee. Moreover, the optimal switching fee is increasing in the price of resource.

[20] URL: http://www.nytimes.com/allbusiness/AB4353479_primary.html; accessed 2015–10–10.

The managerial insights are that firms could generate higher revenues by segmenting the market and designing ancillary fees for each segment separately. For example, business travelers have a higher switching rate for non-monetary reasons than do other passengers. Airlines can assess their prices according to their estimate of the likelihood that customers from each market segment will change itinerary. The market can also be segmented based on seasons or geographic regions, and so on. Firms predicting the behavior of customers should forecast customer reaction to ancillary fees, as well—even more critical because firms need to take into consideration long-term market competition. A strict pricing policy regarding switching and cancelation might send customers to a competitor. This may partly explain why some airlines, such as Southwest Airlines, have been successful in acquiring higher market shares by designing more suitable ancillary fees for customers.[21]

Finally, I would like to emphasize that firms should not follow pricing strategies that sub-maximize their ancillary revenues. Although some airlines are reportedly more successful in generating higher ancillary revenues, these revenues should be considered as a portion of total revenue structure and be managed for a higher total profit and market performance.

This study on change fees can be extended to answer future research questions. For example, how are the results affected by differences between duopoly and monopoly markets? What if customers partially behave strategically? Many ex-ante asymmetric firms occasionally adjust these fares to position themselves advantageously in market competitions. Combining data sets from the U.S. Bureau of Transportation Statistics, future studies could investigate under which scenarios modifying ancillary fares would be detrimental or beneficial to firms. An extension can also consider multiple fare classes where higher fare classes have no switching fee. This is similar to Gallego and Sahin (2010) where they consider non-refundable, partial refundable and full refundable tickets. Finally, I would like to emphasize that the change fees also generate revenue for the price

[21] URL: http://www.forbes.com/sites/greatspeculations/2014/08/20/southwests-expanding-international-service-will-lift-its-growth-in-the-coming-years; Accessed 201–10–10.

tracking industries, such as Yapta. Future research might explore how airlines should manage their relationship with these parties, and how the revenue that is generated through these services should be shared with these price-tracking parties.

I would like to highlight some of the other industries for which this model and results might apply. For example, banks or cellphone providers experience an identical problem in managing their switching revenues when homeowners decide to change their mortgage plans or when cellphone users change their mobile plans. I would like to highlight that above industries does not have similar capacity contrints discussed here for airline, but the results extend to these industries since I consider the non-capacitatated case as well. In all of these cases, firms should impose an appropriate fee to find the best allocation of their limited resources to different streams of demand, keeping in mind that the related ancillary revenues should be collected to maximize their total profit.

3.7 Appendix

Proof of Proposition 2

The total revenue is given by equation (2) is a function of D_1 and D_2. There are there cases as follows:

- Case I ($D_1 \leq 1$ and $D_2 \leq 1$) In this case, the expected revenue is given by:

$$E[\Pi(p_s)] = \int_0^{\frac{1-D_2}{D_1\theta(p_s)}} (p_s + p_2 - p_1)D_1\,\theta(p_s)uf(u)du + \int_{\frac{1-D_2}{D_1\theta(p_s)}}^{1} \frac{\left((p_s + p_2 - p_1)D_1\theta(p_s)u + p_2D_2\right)}{D_2 + D_1\theta(p_s)u} f(u)du$$

Deriving the first derivative of above function with respect to p_s and using Leibniz integral rule, I have the following results:

$$\frac{\partial E[\Pi(p_s)]}{\partial p_s} = [(p_s + p_2 - p_1)\theta'(p_s) + \theta(p_s)] \int_0^{\frac{1-D_2}{D_1\theta(p_s)}} uf(u)\,du$$
$$+ \int_{\frac{1-D_2}{D_1\theta(p_s)}}^{1} \left(\frac{u}{D_2 + D_1\theta(p_s)u}\right)\left(\theta(p_s) + \frac{(p_s - p_1)\theta'(p_s)D_2}{D_2 + D_1\theta(p_s)u}\right) f(u)du$$
$$= 0$$

where $\theta'(p_s)$ is the derivative of $\theta(p_s)$ with respect to p_s. (Considering $\frac{d}{d\theta}\left(\int_{a(\theta)}^{b(\theta)} f(x,\theta)\,dx\right)$ format for Leibniz integral rule, for the first integral, for example, I have the following terms:

$x = u$, $\theta = p_s$, $a(\theta) = 0$, $b(\theta) = \frac{1-D_2}{D_1\theta(p_s)}$, $f(x,\theta) = (p_s + p_2 - p_1)D_1\theta(p_s)uf(u)$. I use similar analogy for next Leibniz integral rule calculations.

- Case II ($D_1 \leq 1$ and $D_2 > 1$) In this case, the expected revenue is given by:

$$E[\Pi(p_s)] = \int_0^1 \frac{\left((p_s + p_2 - p_1)D_1\theta(p_s)u + p_2D_2\right)}{D_2 + D_1\theta(p_s)u} f(u)du$$

Deriving the first derivative of above function with respect to p_s and using Leibniz integral rule, I have the following results:

$$\frac{\partial E[\Pi(p_s)]}{\partial p_s} = \int_0^1 \left(\frac{D_1u}{D_2+D_1\theta(p_s)u}\right)\left(\theta(p_s) + \frac{(p_s-p_1)\theta'(p_s)D_2}{D_2+D_1\theta(p_s)u}\right) f(u)du = 0.$$

- Case III ($D_1 > 1$ and $D_2 > 1$) In this case, the expected revenue is given by:

$$E[\Pi(p_s)] = \int_0^1 \frac{\left((p_s + p_2 - p_1)\theta(p_s)u + p_2D_2\right)}{D_2 + \theta(p_s)u} f(u)du$$

Deriving the first derivative of above function with respect to p_s and using Leibniz integral rule, I have the following results:

$$\frac{\partial E[\Pi(p_s)]}{\partial p_s} = \int_0^1 \left(\frac{u}{D_2 + \theta(p_s)u}\right)\left(\theta(p_s) + \frac{(p_s - p_1)\theta'(p_s)D_2}{D_2 + \theta(p_s)u}\right) f(u)du = 0$$

The three above cases are summarized in Proposition 2. □

Proof of Proposition 1

I assume if there is more than one switching fee, which generates the same amount of revenue for a firm in equation (2), the firm always prefers the lowest one. Through doing this, the firm can motivates more customers to purchase a ticket.

Considering the results in Proposition 2, I calculate the limits of the first derivative of the expected revenue with respect to p_s, when it goes to zero and infinity. Based on Proposition 2, when $D_2 > 1$,

$$\frac{\partial E[\Pi(p_s)]}{\partial p_s} = \int_0^1 \left(\frac{u}{D_2 + \min(1, D_1)\theta(p_s)u}\right)\left(\theta(p_s) + \frac{(p_s - p_1)\theta'(p_s)D_2}{D_2 + \min(1, D_1)\theta(p_s)u}\right) f(u)du$$
$$= 0$$

Note that:

$\lim\limits_{p_s \to 0} \frac{\partial E[\Pi(p_s)]}{\partial p_s} > 0$ since $\lim\limits_{p_s \to 0} \theta(p_s) > 0$ and $\lim\limits_{p_s \to 0} \theta'(p_s) < 0$. Also, $\lim\limits_{p_s \to A} \frac{\partial E[\Pi(p_s)]}{\partial p_s} < 0$ since $\lim\limits_{p_s \to A} \theta(p_s) = 0$. As $\frac{\partial E[\Pi(p_s)]}{\partial p_s}$ is a continuous function, this implies that there exist at least a p_s, such that $\frac{\partial E[\Pi(p_s)]}{\partial p_s} = 0$.

$\theta(p_s)$ is non-negative and decreasing in p_s and $\theta'(p_s)$ is non-positive and decreasing in p_s. Therefore, $\left(\frac{u\theta(p_s)}{D_2 + \min(1, D_1)\theta(p_s)u}\right)$ is decreasing in p_s since the first derivative of this ratio with respect to is $\left(\frac{u\theta'(p_s)(D_2+\min(1,D_1)\theta(p_s)u(1-u))}{(D_2+min\,(1,D_1)\theta(p_s)u)^2}\right) \leq 0$. Also, $\left(\frac{u(p_s-p_1)\theta'(p_s)D_2}{(D_2+\min(1,D_1)\theta(p_s)u)^2}\right)$ is decreasing in p_s, since the numerator of the ratio, $u(p_s - p_1)\theta'(p_s)D_2$, is increasing in absolute value in p_s, but the denominator of the ratio, $(D_2 + \min(1, D_1)\theta(p_s)u)^2$, is decreasing in p_s. Therefore, $\int_0^1 \left(\frac{u}{D_2+\min(1,D_1)\theta(p_s)u}\right)\left(\theta(p_s) + \frac{(p_s-p_1)\theta'(p_s)D_2}{D_2+\min(1,D_1)\theta(p_s)u}\right) f(u)du$ is also decreasing in p_s. This implies that there is a unique p_s, such that $\frac{\partial E[\Pi(p_s)]}{\partial p_s} = 0$.

When $D_2 \leq 1$,

$$\begin{aligned}\frac{\partial E[\Pi(p_s)]}{\partial p_s} &= [(p_s + p_2 - p_1)\theta'(p_s) + \theta(p_s)] \int_0^{\frac{1-D_2}{D_1\theta(p_s)}} uf(u)\, du \\ &\quad + \int_{\frac{1-D_2}{D_1\theta(p_s)}}^1 \left(\frac{u}{D_2 + D_1\theta(p_s)u}\right)\left(\theta(p_s) + \frac{(p_s - p_1)\theta'(p_s)D_2}{D_2 + D_1\theta(p_s)u}\right) f(u)du \\ &= 0\end{aligned}$$

Note that:

$\lim_{p_s\to 0} \frac{\partial E[\Pi(p_s)]}{\partial p_s} > 0$ since $\lim_{p_s\to 0} \theta(p_s) > 0$ and $\lim_{p_s\to 0} \theta'(p_s) < 0$. Also, $\lim_{p_s\to A} \frac{\partial E[\Pi(p_s)]}{\partial p_s} < 0$ since $\lim_{p_s\to A} \theta(p_s) = 0$. Notice that the first integral is zero when $p_s \to A$. As the $\frac{\partial E[\Pi(p_s)]}{\partial p_s}$ is a continuous function, this implies that there exist at least a p_s, such that $\frac{\partial E[\Pi(p_s)]}{\partial p_s} = 0$.

In previous case, I have already shown that the second integral in $\frac{\partial E[\Pi(p_s)]}{\partial p_s}$ is decreasing in p_s. Now I will show that the first integral is decreasing in p_s as well.

$\theta(p_s)$ is non-negative and decreasing in p_s and $\theta'(p_s)$ is non-positive and decreasing in p_s. Therefore, $\int_0^{\frac{1-D_2}{D_1\theta(p_s)}} uf(u)\, du$ and $[(p_s + p_2 - p_1)\theta'(p_s) + \theta(p_s)]$ are decreasing in p_s.

As a result, $[(p_s + p_2 - p_1)\theta'(p_s) + \theta(p_s)] \int_0^{\frac{1-D_2}{D_1\theta(p_s)}} uf(u)\, du$ is also decreasing in p_s.

This implies that there is a unique p_s, such that $\frac{\partial E[\Pi(p_s)]}{\partial p_s} = 0$. □

Proof of Corollary 1

Based on Proposition 2, in a general case, p_s^* is given by:

$$[(p_s + p_2 - p_1)\theta'(p_s) + \theta(p_s)] \int_0^{\frac{1-D_2}{D_1\theta(p_s)}} uf(u)\, du + \int_{\frac{1-D_2}{D_1\theta(p_s)}}^{1} \left(\frac{u}{D_2 + D_1\theta(p_s)u}\right)\left(\theta(p_s) + \frac{(p_s - p_1)\theta'(p_s)D_2}{D_2 + D_1\theta(p_s)u}\right) f(u)du = 0$$

In this equation, p_s^* is an implicit function of D_2. Lets call the above integral as R'. Notive that the optimal switching fee is the solution of the implicit function R', i.e., $R' = 0$. Using implicit differentiation, the derivative of this function with respect to D_2 is given by:

$$\frac{dp_s^*}{dD_2} = -\frac{\frac{\partial R'}{\partial D_2}}{\frac{\partial R'}{\partial p_s}}$$

[Proof of above equation. We have $R'(p_s^*, D_2) = 0$. Therefore, $\frac{\partial R'}{\partial D_2}\frac{dD_2}{dD_2} + \frac{\partial R'}{\partial p_s^*}\frac{dp_s^*}{dD_2} = 0$, which simples to: $\frac{\partial R'}{\partial D_2} + \frac{\partial R'}{\partial p_s^*}\frac{dp_s^*}{dD_2} = 0$].

Using Leibniz integral rule, I have the following results:

$$\frac{\partial R'}{\partial D_2} = \left(\frac{1-D_2}{D_1\theta(p_s)}\right)\left(\frac{1-D_2}{D_1\theta(p_s)}\right) f\left(\frac{1-D_2}{D_1\theta(p_s)}\right)[(p_s - p_1)\theta'(p_s)(1-D_2) - p_2\theta'(p_s)]$$

$+ \int_{\frac{1-D_2}{D_1\theta(p_s)}}^{1} \left(\frac{u}{(D_2 + D_1\theta(p_s)u)^3}\right)(\theta(p_s)(D_2 + D_1\theta(p_s)u) + (p_s - p_1)\theta'(p_s)D_2 - (p_s - p_1)\theta'(p_s)\theta(p_s)u) f(u)du > 0.$

Now, Based on Proposition 1, $\frac{\partial R'}{\partial p_s} < 0$. Therefore, $\frac{dp_s^*}{dD_2} > 0$.

For the same logic, I have the following:

$$\frac{\partial R'}{\partial D_1} = \left(\frac{1 - D_2}{D_1\theta(p_s)}\right)\left(\frac{\theta(p_s)(1 - D_2)}{\left(D_1\theta(p_s)\right)^2}\right) f\left(\frac{1 - D_2}{D_1\theta(p_s)}\right)[(p_s - p_1)\theta'(p_s)(1 - D_2) - p_2\theta'(p_s)]$$

$$+ \int_{\frac{1-D_2}{D_1\theta(p_s)}}^{1} \left(\frac{\theta(p_s)u}{(D_2+D_1\theta(p_s)u)^3}\right)\left(\theta(p_s)(D_2 + D_1\theta(p_s)u)(p_s - p_1)\theta'(p_s)(D_2 - \theta(p_s)u)\right) f(u)du > 0.$$

Now, Based on Proposition 1, $\frac{\partial R'}{\partial p_s} < 0$. Therefore, $\frac{dp_s^*}{dD_1} > 0$. □

Proof of Proposition 5

The deterministic and un-capacitated switching revenue is given by:

$$R^{UC} = (p_s + p_2 - p_1)\theta(p_s)D_1$$

Therefore, $\frac{\partial R^{UC}}{\partial p_s} = (p_s + p_2 - p_1)\theta'\left(p_s\right) + \theta(p_s)$ and p_s^{DU} is the solution for this equation:

$$(p_s + p_2 - p_1)\theta'\left(p_s\right) + \theta(p_s) = 0$$

Notice that $(p_s + p_2 - p_1)\theta'\left(p_s\right) + \theta(p_s)$ is decreasing in p_s since the second derivative with respect to p_s is negative i.e.,

$$2\theta'\left(p_s\right) + (p_s + p_2 - p_1)\theta''\left(p_s\right) < 0$$

This implies that p_s^{DU} is the *unique* solution for the deterministic and un-capacitated switching revenue. Now, given that:

$$\frac{\partial E[\Pi(p_s)]}{\partial p_s} = [(p_s + p_2 - p_1)\theta'(p_s) + \theta(p_s)] \int_0^{\frac{1-D_2}{D_1\theta(p_s)}} uf(u)\,du + \int_{\frac{1-D_2}{D_1\theta(p_s)}}^{1} \left(\frac{u}{D_2+D_1\theta(p_s)u}\right)$$

Therefore, $\lim_{D \to 0} \frac{\partial E[\Pi(p_s)]}{\partial p_s} = p_s^{DU}$. Since, $\frac{dp_s^*}{dD} > 0$ based on Corollary 1, i.e., optimal switching fee is increasing in demand, therefore, $p_s^* > p_s^{*DU}$. □

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Proof of Proposition 3

Based on Proposition 2, in the general case, p_s^* is given by:

$$[(p_s + p_2 - p_1)\theta'(p_s) + \theta(p_s)] \int_0^{\frac{1-D_2}{D_1\theta(p_s)}} uf(u)\, du + \int_{\frac{1-D_2}{D_1\theta(p_s)}}^{1} \left(\frac{u}{D_2 + D_1\theta(p_s)u}\right)\left(\theta(p_s) + \frac{(p_s - p_1)\theta'(p_s)D_2}{D_2 + D_1\theta(p_s)u}\right) f(u)du = 0$$

In this equation, p_s^* is an implicit function of p_1. Lets call the above integral as R'. Using implicit differentiation, the derivative of this function with respect to D_2 is given by:

$$\frac{dp_s^*}{dp_1} = -\frac{\frac{\partial R'}{\partial p_1}}{\frac{\partial R'}{\partial p_s}}$$

Using Leibniz integral rule, I have the following results:

$$\frac{\partial R'}{\partial p_1} = -\theta'(p_s) \int_0^{\frac{1-D_2}{D_1\theta(p_s)}} uf(u)\, du + \int_{\frac{1-D_2}{D_1\theta(p_s)}}^{1} \left(\frac{\theta'(p_s)D_2}{D_2 + D_1\theta(p_s)u}\right) f(u)du > 0.$$

Now, Based on Proposition 1, $\frac{\partial R'}{\partial p_s} < 0$. Therefore, $\frac{\partial R'}{\partial p_1} > 0$.

For the same logic, I have the following:

$$\frac{\partial R'}{\partial p_2} = -\theta'(p_s) \int_0^{\frac{1-D_2}{D_1\theta(p_s)}} uf(u)\, du < 0.$$

Now, Based on Proposition 1, $\frac{\partial R'}{\partial p_s} < 0$. Therefore, $\frac{dp_s^*}{dD_1} < 0$. □

Proof of Proposition 4

I normalize the capacity C_1 and C_2 to 1. Therefore, by increasing the capacity, I have a lower demand in equation (2). Based on Corollary 1, the optimal switching fee is decreasing in size of capacities. □

Chapter 4

4 Managing Change Revenue with Presence of Time-Uncertain Customers

4.1 Introduction

According to a recent survey by the travel website Airfarewatchdog of more than 6,100 travelers, 38% of respondents picked the change and cancellation fee when asked to name the fee they most hated.[22] Many travel weblogs and consumer review websites advise passengers on how to avoid paying these fees by carefully planning their travels ahead of time or by purchasing protective travel insurances. Despite all of these efforts by consumers to prevent extra charges, based on the latest data from the U.S. Bureau of Labor Statistics,[23] the airlines' revenue generated in the first quarter of 2015 from cancellation and change fees increased more than 21% over the revenue generated during the same period in 2012.

Firms gain incremental profit from charging change fees. Customers, however, respond strategically to change fees; that is, they are not certain about their future travel plans and they consider the risk and associated costs of having to make changes in their travel plans. Most of the change and cancellation fees are generated from ticket holders affected by external factors, such as personal matters, business meetings, illness, and so on.[24] In contrast to the situation in Essay 2, I isolate time uncertainty as the different motivation behind switching fee in this essay, i.e., in Essay 2 the model captures those customers

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[22] URL: http://www.latimes.com/travel/la-fi-mo-most-hated-passenger-charge-bag-fees-20140124-story.html#axzz2s0EyY6er; accessed 2015-09-09.

[23] URL: www.rita.dot.gov/bts/sites/rita.dot.gov.bts/files/subject_areas/airline_information/reservation_cancellation_change_fees/index.html; accessed 2015-08-08.

[24] URL: http://online.wsj.com/article/SB10001424052970204563304574318212311819146.html; accessed: 2015–8–8.

who change regardless of switching fee, but in Essay 3, customers only change due to plan change. Consider a traveler flying from Atlanta to Paris, who can purchase her ticket early and pay a lower airfare of \$1,000, but she may still be uncertain about her future travel plan if she purchases her ticket early. She speculates that the ticket price will increase to \$1,400 later. Furthermore, she anticipates that there is a 25% chance her travel plan will change in the future. When her uncertainty materializes close to the travel date, she may have to pay a change fee of \$300 to rebook her ticket based on her final travel schedule. Although she receives a discount from purchasing early, her expected cost from purchasing early is $0.75 \times \$1{,}000 + 0.25 \times (\$1{,}000 + \$400 + \$300) = \$1{,}175$.

In this chapter, I study the following research questions: What are the effects of imposing a change fees on travelers' early purchase incentives? Although a higher change fee might discourage customers from purchasing tickets early, how can change fees be beneficial for the airline? How should airlines set the base ticket prices for flights after imposing a change fee? I explore how a firm should manage change fees and service prices to maximize revenue.

I consider a monopolistic firm that offers two *sequential services*, Service 1 and Service 2, over two periods. For example, consider the previous Atlanta to Paris flight, with a Friday departure and a Saturday departure. On Monday, only the Friday flight is available for purchase, but on Wednesday, both Friday and Saturday flights are available for purchase. The firm allows customers to change their initial service choice by paying a change fee; for example, a customer who bought the Friday flight on the Monday might on Wednesday change for the Saturday flight by paying a change fee (in addition to any incremental mark-up premium). In this paper, I investigate how the firm should maximize its revenue by selecting the best change fee. Customers are heterogeneous with respect to their willingness to pay for the service. Moreover, they should decide whether to purchase Service 1 early at a lower price or wait for Service 2 to prevent paying the change fee. However, they have *type uncertainty*; that is, at the beginning, they are not certain whether they will need Service 1 or Service 2. I model type uncertainty as the proportion of customers who will experience a change in demand from Service 1 to Service 2. Customers consider their type uncertainty, service price, premium mark-up price and change fee when deciding to buy early or late. Without imposing any distributional assumptions, I demonstrate how a revenue-maximizing firm should select the change fee and how it should set the service price and the change fee jointly. I would like to highlight that I am looking at a single-leg flight, not considering a case with a return flight in this essay. Also, I do not consider a contingent plan depending on the customer’s eventually realized time preference since airlines do not consider these plans in practice. This might be in contrast with models in economics and organization theory where firms provides different pricing plans based on customers’ realized types.

This paper makes the following contributions. First, I find that when customers are “almost” certain about their types in the future, low-valuations customers might also purchase the service, but if their type changes, they will not pay the change fee and mark-up premium for Service 2. Second, when customers are not “very certain” about their types, I observe that only high-valuation customers purchase Service 1 in the first period. These results suggest that, in this case, the firm should select a relatively low change fee and incentivize all customers to purchase in period 1. This result may seem counterintuitive, but is anecdotally consistent with policies of industries that have comparatively small or negligible change fees, such as car rental (where customers more frequently change their travel plans). Third, although one might expect that by increasing the change fee the demand for Service 1 shrinks, I show that increasing the change fee up to a certain level leads to higher revenues for the firm. Finally, I demonstrate that when the firm sets the service price and the change fee simultaneously, the optimal service price is decreasing in the type uncertainty of customers. However, the optimal change fee is actually increasing in the type uncertainty of customers.

4.2 Literature Review

In this section, I provide a review of relevant literature in strategic behaviour, valuation uncertainty and ancillary revenues in operations management and management science.

This work contributes to both the strategic customer behaviour research and type/valuation uncertainty literature by studying the interplay between customers' strategic waiting decision and their type uncertainty. The literature that considers strategic customers includes Aviv and Pazgal (2008), Elmaghraby et al. (2008), Zhang and Cooper (2008), Su and Zhang (2008), and Cachon and Swinney (2009). These papers consider strategic waiting behaviour, but not valuation uncertainty. Cachon and Swinney (2009) show that retailers should order less when dealing with strategic consumers than with myopic customers. Su and Zhang (2008) study how various supply chain contracts can achieve quantity and price commitments. Considering two types of pricing strategies including a contingent markdown policy and preannounced fixed-discount policy, Aviv and Pazgal (2008) find out that a fixed-discount policy, in general, outperforms contingent pricing under strategic customer behaviour. Contrary to these studies, in this chapter, I consider consumers are not perfectly aware of their type. This uncertainty is an integral part of my model and is generated specifically by external factors, not firms' pricing decisions.

Customers' strategic delay in purchasing due to valuation uncertainty is studied in the revenue management literature. Ozer and Phillips (2012) state that, when a customer plans to buy a ticket (for instance, for an art event that will take place in the next few months), she may not even be certain about her future personal situation, such as health, family issues, or mood. Anderson and Wilson (2003) indicate that use of revenue management without consideration of strategic customer behaviour results in significantly reduced revenues. Xie and Shugan (2001) and Dana (1998) analyze advance purchase discounts to motivate customers purchasing early. Other studies consider selling capacity options; for example, Gallego and Sahin (2010), Yu, Kapuscinski and Ahn (2005). These papers demonstrate that selling capacity options can improve revenues significantly over advance purchase discounts. Moreover, advance selling will be deployed only when capacity is not very scarce. Koenigsberg, Muller, and Vicassim (2008) prove that offering a last-minute discount is beneficial when customers are uncertain about whether such deals will be offered. Moreover, optimal prices should increase over the two periods. Savva and Papanastasiou (2014) develop a model integrating uncertainty in quality, where users wait reviews of others. They demonstrate that this social learning interaction increases the firm's ex ante expected profit. Dilme and Li (2014) consider a profit-maximizing monopolist seller with K identical goods to sell before a deadline. At each point of time, the seller posts a price and the quantity available but cannot commit to future offers. Over time, potential buyers with different reservation values enter the market. They show that prices decline smoothly over the time period between sales and jump up immediately after a transaction occurs. This chapter incorporates the results of some of these studies. However, I explicitly investigate how firms should manage ancillary revenues dealing with time-uncertain customers.

Motivated by applications from apparel retailing, Liu and van Ryzin (2008) study a seller who deliberately under-stocks a product to create a shortage risk for customers. In their model, the seller pre-announces a single-markdown pricing policy over two periods: a premium in period 1 and a discounted price in period 2. As a result, under-stocking motivates high-valuation customers to purchase early at a premium price. Similar to this study, customers are heterogeneous with respect to their willingness to pay and are present at the store from the beginning all at once. Under these assumptions, authors present that with a sufficient large number of high-valuation customers in the market, it is optimal for the firm to have a rationing policy, otherwise, the firm should serve all customers at the discounted price. In this chapter, I consider two resources and an opportunity for customers to change between two resources by paying an ancillary fee. Moreover, in my setting, as presented by Gallego and Sahin (2010), an early discounted price is offered by airlines to motivate customers to purchase early.

Gallego and Sahin (2010) consider the case in which valuation for a service is not fully realized until the end of the selling season. They consider a two-period model in which customers can purchase partially refundable tickets; that is, paying a fixed fee in period 1 provides buyers the right to purchase the service at a lower price than the spot price in period 2. Gallego and Sahin (2010) highlight that partially refundable fares can be viewed as real options. In other words, purchasing a partially refundable fare pays buyers back when the realized valuations in period 2 are exceeding the spot price. Furthermore, Gallego and Sahin (2010) present that partially refundable fares are social optimal and significantly improve firms' expected revenue over current pricing strategies, which are mainly based on refundable and nonrefundable fares. My research differs in that I consider two services, and customers in this study have type uncertainty with respect to these services. I introduce a change fee that provides ticket holders the right to change their service. The change fee also gives an opportunity for firms to increase their revenue over a best pricing strategy, which is solely based on main service optimal pricing.

4.3 Model Description

A revenue-maximizing firm introduces a Service 1 (S1) in period 1, and Service 2 (S2) in period 2. Both services have a base price of p. Only S1 is available for purchase in period 1. In period 2, both S1 and S2 are available for purchase and the firm allows customers who have bought S1 to change to service S2 by paying a *change fee*, p_s. Both services are delivered sequentially after period 2. For example, consider an airline selling tickets on Monday and Wednesday for an Atlanta to Paris flight, with a Friday departure and a Saturday departure. On Monday customers can only buy airfare for a Friday departure, while on Wednesday customers can buy airfare for both Friday and Saturday departures. The firm charges an incremental mark-up premium Δp in period 2, $\Delta p \geq 0$. The premium is applied to both S1 and S2 and represents the "cost" for offering services closer to the delivery date. The base price is exogenous in this chapter and I assume the firm is committed to the above pricing policy.

The market consists of time-uncertain customers who originally seek service S1. However, in period 1, they are uncertain if they ultimately will need S1 or S2. In period 2, they realize their true *types*. Here, I refer to type as to which service (S1 or S2) is needed by customers. Customers' willingness-to-pay (or valuation) **v** for the service is distributed independently and identically with cumulative distribution function $F(v)$, defined over the range $[0, V]$, where $V > p + \Delta p$. In period 1, a customer chooses to either purchase S1 (ticket holders), or wait until period 2 to realize her type (waiting customer). The type of a customer follows a Bernoulli distribution with probability θ, where θ represents the probability that a customer no longer demands S1 but instead demands S2 in period 2. This segment of customers with type change in period 2 represents those who request a change in their travel dates because of personal or business matters. As a result, a customer expects to keep valuing S1 as before with probability $1 - \theta$. I consider a stylized setting in which all consumers have the same θ. Although one may consider that consumers may have different θ, here we can view θ as an average probability across all consumers. In period 2, all customers (ticket holders and waiting customers) realize their true types, and are divided into two groups based on their realized types. I assume θ, $F(v)$, and Δp are common knowledge to both firm and customers from the beginning. I would like to highlight that in period 1, customers are aware that if their type will be changed in period 2, the firm will offer S2 in period 2.

I consider a market that consists of a large customer population. All customers arrive at the beginning of period 1 and they have unit demand. I assume there is no limit on the service capacities for S1 and S2. In the ensuing analysis in this chapter, I normalize the size of market to 1. I would like to note the normalization of total potential demand does not change the conclusion of this chapter and it is not contrary to the above assumption that customers have unit demand; it only brings convenience in model development. Customers take into consideration service prices, premium, change fee, and the uncertainty associated with their type change when deciding to buy S1 in period 1, to wait until period 2 and buy S1 or S2 (depending on their realized type), or not to buy at all. I would like to highlight that in this essay, I consider cases where $\theta \leq 0.5$, this is, the primary focus of this study is S1 customers (not on S2 customers) and switching from S1 to S2 and customers who are primary interested in S2 are not part of the model. A broader model would include a self-selection between two flights. Also, the firm can offer S2 to in period 1, but since $\theta \leq 0.5$, no customer is interested in buying S2 early.

The firm seeks to maximize revenues by choosing the best change fee, while the customers maximize their expected utilities by choosing to buy S1 in period 1, waiting until period 2 to buy either S1 or S2, or not buying at all. The sequence of events is as follows: in period 1, the firm offers S1, and announces the service price p, the change fee p_s, and the premium Δp. After which customers choose to either purchase the S1 or wait until period 2 to realize their type. In period 2, the firm offers both S1 and S2 and changes the service price to $p + \Delta p$ for both services. All customers realize their "true" type and act as follows: a ticket holder with type change pays the change fee and purchases S2 (or possibly simply forfeits her sunk cost *p* for S1 and leave), but a ticket holder without type change keeps her original purchase S1. A waiting customer can purchase S1 or S2 based on her type without paying the change fee (but at the marked-up price $p + \Delta p$). See Figure 8 for the sequence of events.

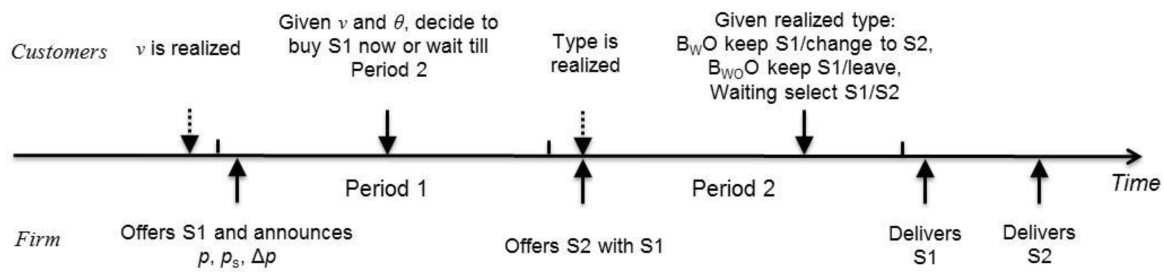

Figure 8: Sequence of events

A customer compares her expected payoff from purchasing S1 in period 1 and following scenarios against her expected utility from waiting for more certainty through purchasing in period 2. Note that when the type of a ticket holder changes in period 2, her utility is $v - p - \Delta p - p_s$ if she changes to Service 2, and it is $-p$ if she doesn't change. In this case, if her valuation for service 2 is not high enough—that is, when $v < \Delta p + p_s$—her not purchasing of Service 2 and leaving the market generate a higher utility because paying extra change and premium fees are too costly for her. The group with low valuation ($v < \Delta p + p_s$) represents *buyers without change option* (B_{WO}O) who might purchase S1 in period 1 in the hope of keeping their original types. If their types, however, change in period 2, they leave the market and lose their tickets. On the other hand, when a customer has a high valuation—that is, when $v \geq \Delta p + p_s$—she ultimately purchase S1 or S2 based on her type and never leaves the market. This group represents *buyers with change option* (B_WO).

A customer purchases S1 in the first period if $v - p \geq 0$ and $(1 - \theta)(v - p) + \theta\, max\,(v - p - \Delta p - p_s, - p) \geq v - p - \Delta p$. The first condition simply indicates that the customer's valuation should be higher than the service price. The second condition represents a strategic customer who purchases the service if she expects a higher utility from purchasing S1 early in period 1 rather than waiting until period 2. See Figure 9 for more details regarding customer's decision tree.

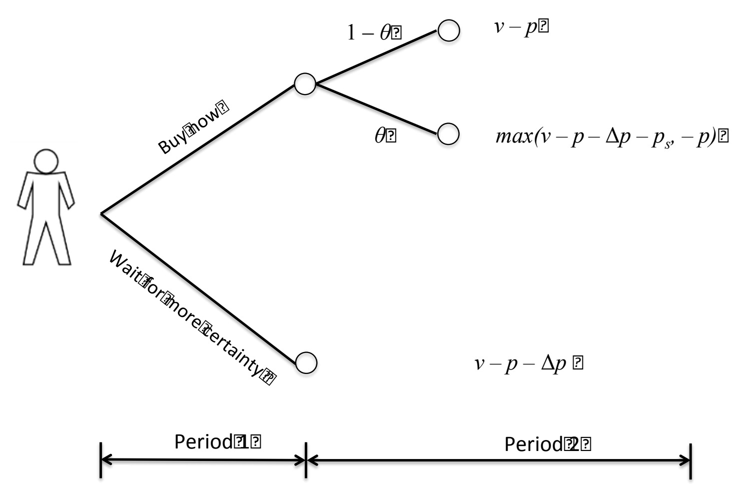

Figure 9: Two-Period decision model

The firm's expected net revenue is given by:

$$\pi(p_s) = \phi_{B_W O}(p_s)(\theta(p + p_s + \Delta p) + (1 - \theta)p) + \phi_{B_{WO} O}(p_s)(\theta p + (1 - \theta)p) + \phi_{Waiting}(p_s)\big(\theta(p + \Delta p) + (1 - \theta)(p + \Delta p)\big)$$

$$= \phi_{B_W O}(p_s)\big(p + \theta(p_s + \Delta p)\big) + \phi_{B_{WO} O}(p_s)(p) + \phi_{Waiting}(p_s)(p + \Delta p) \qquad (1)$$

where $\phi_{B_W O}$ is the probability of being a buyer with change option, $\phi_{B_{WO} O}$ is the probability of being a buyer without change option, and $\phi_{Waiting}(p_s)$ is the probability of being a waiting customer. All these probabilities are dependent on the change fee. Notice that, on average, a buyer with change option pays $p + \theta(p_s + \Delta p)$, a buyer without change option pays p, and a waiting customer pays $p + \Delta p$.

Next, I present how customers with heterogeneous willingness-to-pay in the market react to different levels of change fees set by the firm. In general, I expect that customers with high valuation purchase early and those with lower valuations wait until they realize their valuation. I will show that there are two scenarios based on the proportion of customers with type change: a low θ case, when $\theta \leq \frac{\Delta p}{\Delta p + p}$, and a high θ case, when $\theta > \frac{\Delta p}{\Delta p + p}$. I will present that firm's revenue function has different formats in these two cases and will formally summarize the mathematical details for two cases in Lemma 1 and Lemma 2, respectively.

4.3.1 High Probability of Type Change

First, I study the case where customers have a high chance of type change; that is, when θ is high ($\theta > \frac{\Delta p}{\Delta p + p}$). Given a change fee, I investigate how customers act differently based on their valuations. In general, I expect that, in this case, customers are more willing to wait than the low θ case. I formally summarize customers' reactions to change fees in the following proposition.

Proposition 1 *When* $\theta > \frac{\Delta p}{\Delta p + p}$, *a customer in period* 1 *acts as follows:*

- *Case I: if* $v > p + \theta\ (\Delta p + p_s)$ *and* $p_s \leq \Delta p\ (1 - \theta) / \theta$, *then she* ***buys*** *in period* 1 (purchase in period 1).

- *Case II: if* $v > \Delta p + p$ *and* $p_s > \Delta p\ (1 - \theta) / \theta$, *then she* ***waits*** *until period* 2 (purchase in period 2).

- *Case III: otherwise, she* ***leaves*** *the market* (no purchase).

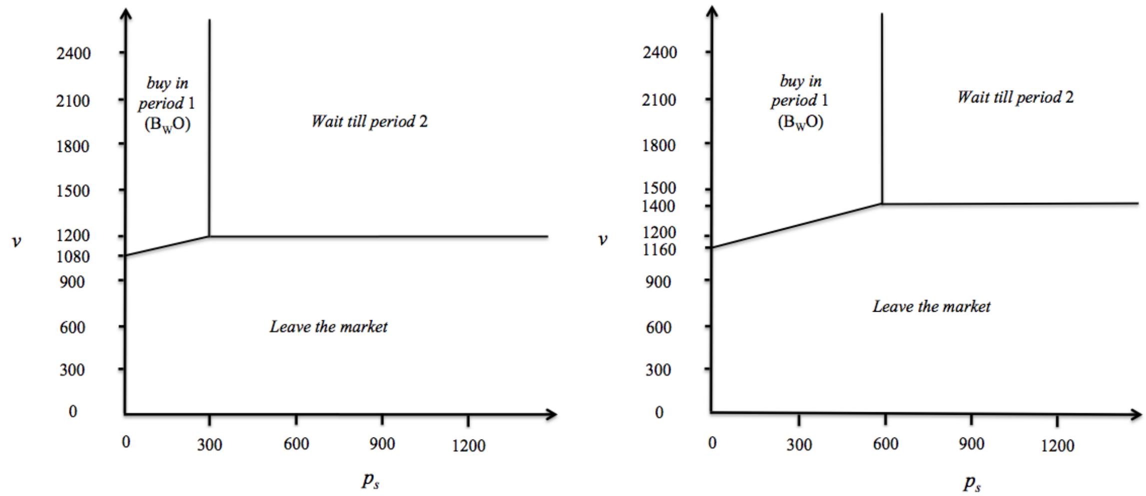

Figure 10: Numerical illustrations of customers' decisions based on different change fees, for θ = 0.4, p = \$1000, Δp = \$200 (left) and Δp = \$400 (right)

Proposition 1 states that when θ is high, there are three types of customers in the market: non-buyers, waiting customers and buyers with option. In this case, since there is a high chance for changing customers' type in period 2, no one buys without change option in period 1. Moreover, high-valuation customers still wait until period 2 if p_s is high as in $p_s > \Delta p (1 - \theta) / \theta$, and they buy in period 1 if p_s is low as in $p_s \leq \Delta p (1 - \theta) / \theta$. Also, all buying customers behave in the same way: either all of them purchase S1 early when the change fee is low or all of them wait when the change fee is high. Therefore, no potential customer purchases S1 without change the fee option. Figure 10 presents the numerical illustrations of customer decisions based on different change fees, for $\theta = 0.4$, p = \$1000, Δp = \$200 (left) and Δp = \$400 (right).

To illustrate Proposition 1, consider our example of a traveler flying from Atlanta to Paris. Assume that she anticipates the chance of her travel plan changing to be greater than $\frac{\Delta p}{\Delta p + p} = \frac{400}{400+1000} \simeq 0.286$. For example, assume there is 40% chance her travel plan change in future, i.e., $\theta = 0.4$. In this case, according to Proposition 1, when $p_s = 0$, she purchases her ticket in period 1 when $v > p + \Delta p\, \theta = 1000 + 400 \times 0.4 = 1160$. Otherwise, she leaves the market. In general, there are two cases over p_s as follows:

First, when $p_s \leq \Delta p\ (1 - \theta) / \theta = 400 \times 0.6 / 0.4 = 600$, she buys early in period 1 if $v > p + \theta\ (\Delta p + p_s) = 1160 + 0.4\ p_s$, and leaves the market otherwise. For example, when $p_s = 300$, if $v > 1160 + 0.4 \times 300 = 1280$, she buys S1 early in period 1. Second, when $p_s > \Delta p\ (1 - \theta) / \theta = 600$, she does not buy S1 early, but rather waits if $v > p + \Delta p = 1400$, and leaves the market otherwise.

The firm anticipates the strategic customers' behaviour with respect to their type uncertainty, and maximizes its total revenue by choosing the best change fee. I defined $\pi(p_s)$ as the firm's revenue in terms of p_s in (1). The firm's revenue can be expressed in terms of p_s for high θ case as follows:

Lemma 1 *When $\theta > \frac{\Delta p}{\Delta p + p}$, the firm's expected revenue is given by,*

$$\pi(p_s) = \begin{cases} \left(p + \theta(p_s + \Delta p)\right)\ \left(1 - F\left(p + \theta(p_s + \Delta p)\right)\right) & if\ p_s \leq \dfrac{\Delta p(1 - \theta)}{\theta} \\ (p + \Delta p)\ \left(1 - F(p + \Delta p)\right) & otherwise \end{cases}$$

For the high θ case, the revenue function is continuous; however, there are two regions over p_s. I would like to highlight that $p_s^* \leq p$. This is because if the firm set the change fee higher than the service price, canceling S1 and purchasing S2 generates higher utilities for ticket holders with type change. For any $\theta < 0.5$, there are low and high type change cases, that is, θ can be lower or higher than $\frac{\Delta p}{\Delta p + p}$. Notice that, at extreme case where $\theta = 0.5$, there should not be a high type change case. This implies that $\frac{\Delta p}{\Delta p + p} \leq 0.5$, which simples to $\Delta p \leq p$. Therefore, with respect to Δp and p, I consider the premium price is not higher that the service price. This result is reasonable since it is very rare that the firm increase the service price by more than 100%. One might wonder, when the proportion of customers with type change is high, can the firm get benefit from this high rate of type change by setting a high change fee. I will analyze this case in Corollary 1.

Corollary 1 *when* $\theta > \frac{\Delta p}{\Delta p + p}$*, firms should set the change fee such that* $p_s \leq \Delta p\ (1 - \theta) / \theta$*, to induce all potential customers purchase early.*

Corollary 1 states that, when a high proportion of customers change from S1 to S2, the firm should never set a high change fee. Notice that in the high θ case, when p_s is high, as in $p_s > \Delta p\ (1 - \theta) / \theta$, the revenue function is independent of p_s. This implies that it is never beneficial for the firm to set a change fee higher than $\Delta p\ (1 - \theta) / \theta$ for the high θ case.

In addition, a high change fee will force all buyers to wait, generating no benefit for the firm. Customers react to the firm's policy of setting a high change fee until the second period to avoid paying the change fee. Therefore, it is not beneficial for the firm to set a high change fee and risk provoking customer speculation. Gaining more revenue by reducing the change fee might seem counterintuitive but proves beneficial when a high proportion of customers make a change. This result is anecdotally consistent with the current practice in the auto rental industry, where a significant proportion of customers never show up.[25]

Referring back to the flight from Atlanta to Paris, let us assume two cases where travelers' valuations are uniformly distributed between 0 and 2500 and between 0 and 3000, respectively. Furthermore, for both above cases, consider two values of $\theta = 0.3$ (dashed line) and $\theta = 0.7$ (dotted line), where $p = \$1000$, $\Delta p = \$400$. Figure 11 present the firm's expected revenue for two above cases. Interestingly, in both figures, I observe that a firm can generate high revenues by imposing a low change fee when the proportion of customers with type change is high. Moreover, the firm's expected revenue increases in the proportion of high-value customers. Finally, these results suggest that change fees significantly impact a firm's expected revenue. For example, when travelers' valuations are uniformly distributed between 0 and 2500 and $\theta = 0.3$, selecting a wrong change fee might leads to more than 15% revenue loss.

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[25] URL: http://www.nytimes.com/2012/02/18/your-money/autoslash-a-rate-sleuth-makes-rental-car-companies-squirm-your-money.html; accessed: 2015–8–8.

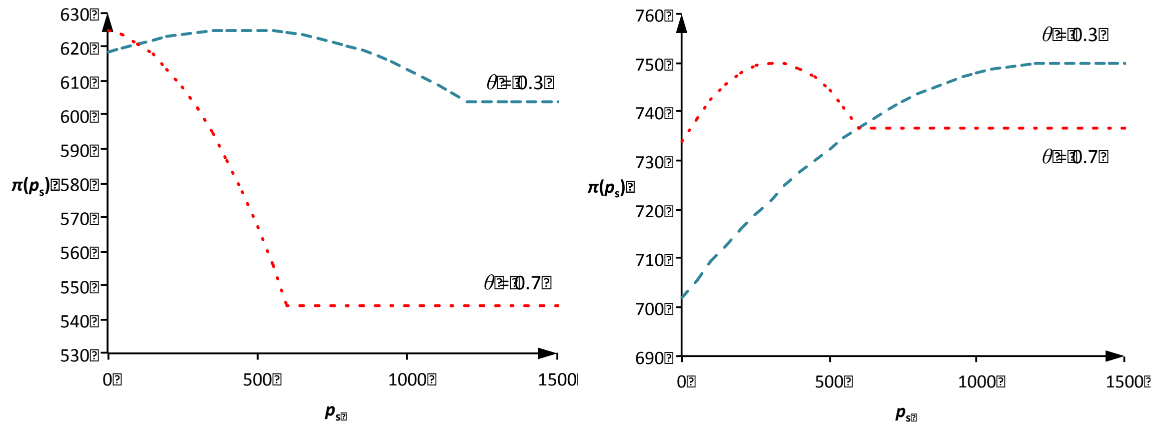

Figure 11: Firm's expected revenue for θ = 0.3 (dashed line) and θ = 0.7 (dotted line), p = \$1000, Δp = \$400, v is uniformly distributed between 0 and 2500 (left) and between 0 and 3000 (right)

A special case is when $\theta = 0$. Based on Lemma 1, the firm's expected revenue is given by:

$$\pi(p_s) = p\left(1 - F(p)\right)$$

which is independent of p_s and represents the revenue of the monopolistic firm posting price p for the service. The revenue functions introduced in this study are extensions to this classical and popular revenue management setting in pricing and supply chain-contracting problems. Let P_M be the optimal monopolistic service price for a single service. In this section, I consider that the service price p is an exogenous parameter. In Section 4.4, however, I consider p as a decision factor and investigate how the firm should set the service price and the change fee together. Next, I present how to derive the optimal change fee for high θ case. First, I define l_s as the solution of p_s to the following equation:

$$\frac{1 - F\left(p + \theta(p_s + \Delta p)\right)}{f\left(p + \theta(p_s + \Delta p)\right)} = p + \theta(p_s + \Delta p) \qquad (2)$$

Note that the optimal change fee cannot be inside the second region of change fee in Lemma 1.

Proposition 2 *If* $\theta > \frac{\Delta p}{\Delta p + p}$*, then the optimal change fee,* p_s^**, is found as follows:*

$$p_s^* = \begin{cases} 0 & if\ \ l_s \leq 0 \\ l_s & if\ 0 < l_s \leq \dfrac{\Delta p(1-\theta)}{\theta} \\ \dfrac{\Delta p(1-\theta)}{\theta} & otherwise \end{cases}$$

Proposition 2 indicates how a revenue-maximizing firm sets the optimal change fee when time-uncertain and heterogeneous customers present in the market for high θ case. Proposition 2 defines a specific change fee, l_s, given by solution of (2). This proposition states that, depending on the proportion of customers with type change, the optimal change fee should be equal to the solution of (2), the specific change fee, l_s, when it is within the lower region of change fee in Proposition 1, i.e., $0 \leq l_s \leq \frac{\Delta p(1-\theta)}{\theta}$. When l_s is outside the lower region of change fee in Proposition 1, the optimal change fee is equal to the boundary of the region which is closer to l_s. This result holds for the general customers willingness-to-pay distribution, $F(v)$.

I would like to highlight that the focus of the foregoing analysis is on the change fee, p_s, and the mark-up premium, Δp, is fixed. Similar to the change fee, firm can also select the mark-up premium endogenously as a decision factor (I will consider the base service price, p, in Section 4.4 as a decision factor as well). I demonstrate that the firm always selects the change fee from the first case of Lemma 1 and we observe the change fee, p_s, is always with the mark-up premium, Δp, in the firm's revenue function. I will show in Lemma 2 that this is also the case for the low probability type change case, i.e., in low probability type change case, I will demonstrate that the firm always selects the change fee from the first and the second cases of Lemma 2 and the change fee is always with the mark-up premium in the firm's revenue function for this case as well. This implies that the *sum* of the change fee and the markup price creates the optimal solution. There, however, there is no unique pair of optimal change fee and the mark-up premium that maximize the firm's revenue; that is, there are more than one pair of optimal change fee and the mark-up premium that maximize firms' revenue, but the optimal sum of the change fee and the mark-up premium, as the solution, is fixed and unique for all of these pairs.

4.3.2 Low Probability of Type Change

Next, I present the case in which the change rate is low ($\theta \leq \frac{\Delta p}{\Delta p + p}$). Given a change fee, I investigate how customers act differently based on their valuations. In general, I expect that, in low θ case, customers are more willing to buy S1 early than the high θ case. I formally summarize customers' reactions to change fees in the following proposition.

Proposition 3 *When* $\theta \leq \frac{\Delta p}{\Delta p + p}$*, a customer in period* 1 *acts as follows:*

- *Case I: if* $v > p + \theta\,(\Delta p + p_s)$*, and* $p_s \leq p\,/\,(1 - \theta) - \Delta p$*, then she* ***buys (with option)*** *in period* 1 (purchase period 1).
- *Case II: if* $v > \Delta p + p_s$*, and* $p\,/\,(1 - \theta) - \Delta p < p_s \leq \Delta p\,(1 - \theta)\,/\,\theta$*, then she* ***buys (with option)*** *in period* 1 (purchase period 1).

- *Case III: if* $p\,/\,(1 - \theta) < v \leq \Delta p + p_s$*, and* $p\,/\,(1 - \theta) - \Delta p < p_s \leq \Delta p\,(1 - \theta)\,/\,\theta$*, then she* ***buys (without option)*** *in period* 1 (purchase period 1).
- *Case IV: if* $p\,/\,(1 - \theta) < v \leq \Delta p\,/\,\theta$*, and* $\Delta p\,(1 - \theta)\,/\,\theta < p_s$*, then she* ***buys (without option)*** *in period* 1 (purchase period 1).

- *Case V: if* $v > \Delta p\,/\,\theta$*, and* $\Delta p\,(1 - \theta)\,/\,\theta < p_s$*, then she* ***waits*** *until period* 2 (purchase period 2).

- *Case VI: otherwise, she* ***leaves*** *the market* (no purchase).

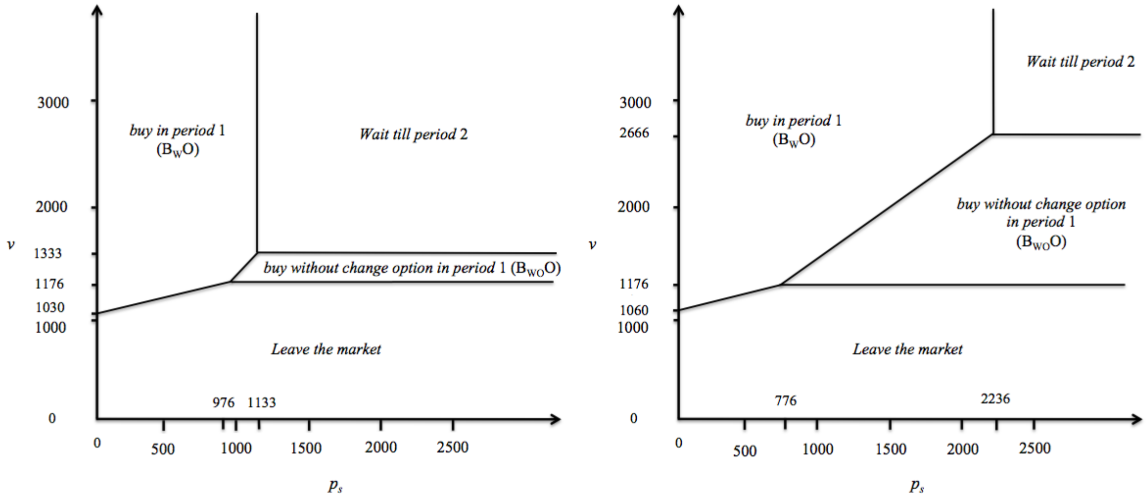

Figure 12: Numerical illustrations of customers' decisions based on different change fees, for θ = 0.15, p = \$1000, Δp = \$200 (left) and Δp = \$400 (right)

When the probability of type change is low, $\theta \leq \frac{\Delta p}{\Delta p+p}$, based on Proposition 3, there are four different cases of customers in the market: waiting customers, buyers without option, buyers with option, and non-buyers. Interestingly, Proposition 3 states that, when the change fee is higher than $\Delta p\,(1 - \theta) / \theta$, then customers with high valuations ($v > \Delta p / \theta$) never buy in period 1 but instead wait until period 2, but customers with lower valuations ($p / (1 - \theta) < v \leq \Delta p / \theta$) buy without option in period 1. Figure 12 presents the numerical illustrations of customer decisions based on different change fees, for $\theta = 0.15$, p = \$1000, Δp = \$200 (left) and Δp = \$400 (right).

Existence of high-valuation waiting customers with low-valuation buyers in period 1 may seem counterintuitive. Notice that, in this case, the change fees are high. Therefore, it is less risky for high-valuation customers to wait and not pay a high change fee, knowing that waiting until period 2 can still generate high utilities for them even after they pay the premium in period 2. In other words, waiting until period 2 and paying the premium fee rather than a change fee generates higher utilities for high-valuation customers.

Low-valuation customers in period 1 do not have the benefit of generating high utilities from waiting, because their valuations are not high enough to cover the premium in period 2. They purchase without option in period 1 as they count on a low chance of changing their type. However, they are aware that, if their type changes in period 2, they should leave the market. Proposition 3 also states that when the change fee is low, no one waits in period 1, high-valuation customers buy with option in period 1, and low-valuation customers leave the market.

To illustrate Proposition 3, consider again our example of a traveler flying from Atlanta to Paris when $\theta < \frac{\Delta p}{\Delta p + p} \simeq 0.29$. In this case, according to Figure 12, when $p_s = 0$, she purchases her ticket in period 1 when $v > p + \Delta p\, \theta = 1000 + 400 \times 0.25 = 1100$. Otherwise, she leaves the market. In general, there are three cases of p_s as follows: First, when $p_s \leq p / (1 - \theta) - \Delta p = 1000 / 0.75 - 400 = 933$, she buys early in period 1 if $v > p + \theta\, (\Delta p + p_s) = 1100 + 0.25\, p_s$, and leaves the market otherwise. For example, when $p_s = 300$, if $v > 1100 + 0.25 \times 300 = 1175$, she buys S1 early in period 1. I would like to highlight that, in my analysis, I consider the possibility of having a change higher than the service price. In practice, firm never charge a change fee higher than the service price since all ticket holders with type change can directly purchase S2 at lower price without paying the change fee.

Second, when $p / (1 - \theta) - \Delta p < p_s \leq \Delta p\, (1 - \theta) / \theta$, (that is when $933 < p_s \leq 1200$), she purchases her ticket in period 1 when $v > p / (1 - \theta) = 1333$. Note that if $v \leq p_s + \Delta p = p_s + 400$, she buys as a $B_{WO}O$ and when $v > p_s + 400$, she buys as a $B_W O$. When $v < p / (1 - \theta) = 1333$, she leaves the market.

Third, when $p_s > 1200$, if $v \leq p / (1 - \theta) = 1333$, then she leaves the market, and if $1333 < v \leq \Delta p / \theta = 1600$, she buys as a $B_{WO}O$, and if $v > 1600$, she waits in period 1.

Notice that in both low θ case and high θ case (discussed in Section 4.3.1), no customers with valuation lower than $p + \theta\, \Delta p$ consider the service, even in the absence of a change fee. Also, in both cases, when the change fee is low, customers only consider the service if their valuation is greater than the expected price in period 2, i.e., $v > p + \theta\, (\Delta p + p_s)$. When the change fee is high, they consider the service only when $v > p / (1 - \theta)$ for the low θ case. However, this threshold on customer valuation is independent of θ for high θ

case. When the change fee is high, they consider the service when $v > \Delta p + p$ for the high θ case.

The firm anticipates the strategic customers' behaviour with respect to their time uncertainty, and maximizes its total revenue (original and ancillary revenue) by choosing the best change fee. I defined $\pi(p_s)$ as the firm's revenue in terms of p_s in (1). The firm's revenue can be expressed in terms of p_s for low θ case as follows:

Lemma 2 *When $\theta \leq \frac{\Delta p}{\Delta p+p}$, the firms expected revenue is given by,*

$$\pi(p_s) = \begin{cases} (p+\theta(p_s+\Delta p))\left(1-F(p+\theta(p_s+\Delta p))\right) & if\ p_s \leq \dfrac{p}{1-\theta}-\Delta p \\ (p+\theta(p_s+\Delta p))\left(1-F(p_s+\Delta p)\right)+p\left(F(p_s+\Delta p)-F\left(\dfrac{p}{1-\theta}\right)\right) & if\ \dfrac{p}{1-\theta}-\Delta p \leq p_s \leq \dfrac{\Delta p(1-\theta)}{\theta} \\ (p+\Delta p)\left(1-F\left(\dfrac{\Delta p}{\theta}\right)\right)+p\left(F\left(\dfrac{\Delta p}{\theta}\right)-F\left(\dfrac{p}{1-\theta}\right)\right) & otherwise \end{cases}$$

For the low θ case, the revenue function is continuous; however, there are three regions over p_s for low θ case. Next, I show how to derive the optimal change fee. First, similar to high θ case, I define l_s as the solution of p_s in equation (2) as follows:

$$\frac{1-F(p+\theta(p_s+\Delta p))}{f(p+\theta(p_s+\Delta p))} = p+\theta(p_s+\Delta p)$$

and u_s as the solution of p_s to the following equation:

$$\frac{1-F(p_s+\Delta p)}{f(p_s+\Delta p)} = p_s+\Delta p \qquad (3)$$

One may consider l_s and u_s as two specific change fees, a lower change fee, and a higher change fee, respectively. Note that the optimal change fee cannot be inside the third region of change fee in Lemma 2. Next, I provide an explicit condition on the optimal change fee solution for the low θ case.

Corollary 2 *For the low θ case, $\theta \leq \frac{\Delta p}{\Delta p+p}$, the optimal change fee, p_s^*, is found as follows:*

– If $p \leq P_M (1 - \theta)$, p_s^*, is given by:

$$p_s^* = \begin{cases} \frac{p}{1-\theta} - \Delta p & if \ \ u_s \leq 0 \\ u_s & if \ 0 < u_s \leq \frac{\Delta p(1-\theta)}{\theta} \\ \frac{\Delta p(1-\theta)}{\theta} & otherwise \end{cases}$$

– If $p > P_M (1 - \theta)$, p_s^*, is given by:

$$p_s^* = \begin{cases} 0 & if \ \ l_s \leq 0 \\ l_s & if \ 0 < l_s \leq \frac{p}{1-\theta} - \Delta p \\ \frac{p}{1-\theta} - \Delta p & otherwise \end{cases}$$

Corollary 2 indicates how a revenue-maximizing firm sets the optimal change fee when time-uncertain and heterogeneous customers present in the market for the low θ case. Corollary 2 defines a threshold on the service price and states that when the service price is lower than this threshold, as in $p \leq P_M (1 - \theta)$, the optimal change fee should be higher than its corresponding threshold, as in $\frac{p}{1-\theta} - \Delta p \leq p_s^*$. We can consider u_s as a specific higher change fee given by solution of (3) and it should be within the second regions of change fee in Lemma 2, that is, $\frac{p}{1-\theta} - \Delta p \leq u_s \leq \frac{\Delta p(1-\theta)}{\theta}$. In this case, when u_s is outside the second region of change fee in Lemma 2, the optimal change fee is equal to the boundary of the region which is closer to u_s. Corollary 2 also states that when the service price is higher than above threshold, as in $p > P_M (1 - \theta)$, the optimal change fee should be lower than its corresponding threshold, as in $\frac{p}{1-\theta} - \Delta p > p_s^*$. We can consider l_s as a specific lower change fee given by solution of (2) and it should be within the first regions of change fee in Lemma 2, that is, $0 \leq l_s \leq \frac{p}{1-\theta} - \Delta p$. In this case, when l_s is outside the lower region of change fee in Lemma 2, the optimal change fee is equal to the boundary of the region which is closer to l_s. This result holds for the general customers willingness-to-pay distribution, $F(v)$. Finally Corollary 2 states that, the firm should balance between service price and change fee, and should not set both at high levels or low levels simultaneously.

I would like to investigate what is the value of the optimal change fee if the service price is set at the optimal monopolistic service price, i.e., $p = P_M$. As I discussed, this represents a case in which no customers demand change ($\theta = 0$) or in which the firm maximizes its main and ancillary revenues separately for technical or operational issues. I will present based on Corollary 2, the optimal change fee lies within the lower cases of Lemma 1 and Lemma 2. I observe that the firm should never set a very high change fee when the service price is equal to the optimal monopolistic service price. The results hold for both high and low θ cases; that is, *If* $p = P_M$, *then* $p_s^* = l_s$. Notice that when $p_s = l_s$, based on Propositions 1 and 3, only true buyers purchase the service in period 1. I will later present whether the introduction of the change fee impacts on the service price and how, in this case, the firm should maximize the total revenue jointly based on service price and change fee.

Corollary 3 *The optimal change fee is decreasing in* p, θ *and* Δp.

Corollary 3 extends the results of Corollary 2 and states that by increasing the service price or the premium, the firm should select a lower change fee. Moreover, the firm should set a lower change fee when the population of customers with type grows. In the next section, I analyze how the firm should select the service price in conjunction with the change fee.

4.4 The Firm's Joint Service Pricing and Change Fee Decision

In this section, I study how the firm should maximize its revenue by simultaneously selecting the change fee and the service price. Define (p^*, p_s^*) as the set of optimal service price and the change fee that maximizes the total revenue for the firm. First, I show how to derive (p^*, p_s^*) given P_M, θ and Δp, and then I analyze the properties of this set.

Lemma 3 *If* $\Delta p \leq P_M$*, the set of optimal service price and change fee,* (p^*, p_s^*)*, is not unique. This set satisfies the following condition:*

$$p^* = P_M + (p_s^* + \Delta p)(1 - \theta).$$

When the premium is lower than the optimal monopolistic service price, and the firm selects both the service price and the change fee, the optimal service price should be set higher than the optimal monopolistic service price; that is, $p^* \geq P_M$. Notice that, in this case, p_s^* is selected at its low value in Corollary 2, i.e., $p_s^* = l_s$. Having both the premium and the optimal change fee at levels lower than their corresponding thresholds motivates the firm to set the service price higher than the optimal monopolistic service price.

Moreover, Lemma 3 indicates that there are multiple optimal change fees as well as the service prices. Although none of these solutions (joint optimal change fees and service prices) are dominated by other solutions, the firm might select a specific solution based on other factors in the market. For example, an extremely high change fee or a service price might encourage customers to stay in the market (even though it does not affect their expected utilities). In this case, the firm may eliminate these extreme rates from the set of possible optimal service prices and change fees. Finally, I would like to highlight that, when the premium is lower than the optimal monopolistic service price, all ticket holders are true buyers and no conditional buyers stay in the market. Next, I study the case in which the premium is higher than the optimal monopolistic service price.

Lemma 4 *If* $\Delta p > P_M$*, the set of optimal service price and change fee,* (p^*, p_s^*)*, is unique and can be found as follows:*

$$p^* = P_M (1 - \theta), \text{ and } p_s^* = 0.$$

When the premium is higher than the optimal monopolistic service price, and the firm selects its service price and change fee together, the optimal service price should be set lower than the optimal monopolistic service price, i.e., $p^* \leq P_M$. Moreover, the firm should allow all ticket holders to freely change between S1 and S2 if they want to do so. This is because the premium is extremely high and the firm cannot impose either high change fees or high service price. Note that, the high premium in period 2, motivates more customers to purchase early. Moreover, contrary to the low premium case, there are buyers without change option in the market, as well.

Corollary 4 *Keeping all other factors the same, the optimal service price increases in* P_M *and decreases in* θ*, and the optimal change fee decreases in* P_M *and increasing in* θ*.*

Corollary 4 outlines how the joint optimal service price and change fee vary based on the optimal monopolistic service price as well as the proportion of customers with type change. Interestingly, Corollary 4 suggests a different result than Corollary 3 with respect to the change fee: When the firm simultaneously selects both rates, a larger population of customers with type change motivates the firm to impose higher change fees.

I numerically analyze the set of optimal service price and change fee, (p^*, p_s^*), for a case in which v is uniformly distributed between 0 and V. A higher V implies a higher proportion of high-valuation customers. In our example of the traveller from Atlanta to Paris, $\Delta p = 400$. The following analysis reports on eight cases, spanned by:

$$(V,\theta) \in \left\{ \begin{matrix} (250, 0.2), (500, 0.2), (1000, 0.2), (1500, 0.2), (2000, 0.2), \\ (250, 0.8), (500, 0.8), (1000, 0.8), (1500, 0.8), (2000, 0.8) \end{matrix} \right\}$$

The numerical results are summarized in Table 6. I highlight the following trends with respect to the set of optimal service price and change fee. First, I observe that when the firm sets service price and change fee together, both fees are increasing in V. This is an intuitive result. Firm can charge higher fees when there are a higher proportion of high-valuation customers in the market.

Table 6: The set of optimal service price/change fee for different *V* and *θ*.

(p^*, p_s^*) $\Delta p = 400$		*V*				
		250	500	1000	1500	2000
θ	0.2	(100, 0)	(200, 0)	(410, 50)*	(870, 250)*	(870, 250)*
	0.8	(25, 0)	(50, 0)	(390, 50)*	(640, 50)*	(640, 50)*

Also, I notice that the optimal service price and the optimal change fee are decreasing in θ. As I discussed, when the risk of type change increases, customers are less willing to buy early to prevent paying the change fee. Furthermore, the service price can be higher or lower than the premium, however, the optimal change fee is always lower than the premium. When the proportion of high-valuation customers increases, the optimal change fee will be close to the premium.

Next, the sum of the service price and the change fee in most cases are less than $V/2$, except when V is very high and θ is very low. When there are many high-valuation customers in the market, the firm can charge high change fees, especially when there is a low chance of the type change. Notice that the optimal service price and the change fee are increasing at a rate higher than V.

Finally, I numerically analyze the impact of the markdown premium on the optimal service price and the change fee. Assume v is uniformly distributed between 0 and 2500. I compare two cases of $\Delta p = 400$ and $\Delta p = 800$ and consider a full range of θ. The results are presented in Figure 13. I observe that the optimal service price is decreasing in Δp. Interestingly, the optimal change fee is decreasing in Δp for low values of θ and increasing in Δp for high values of θ.

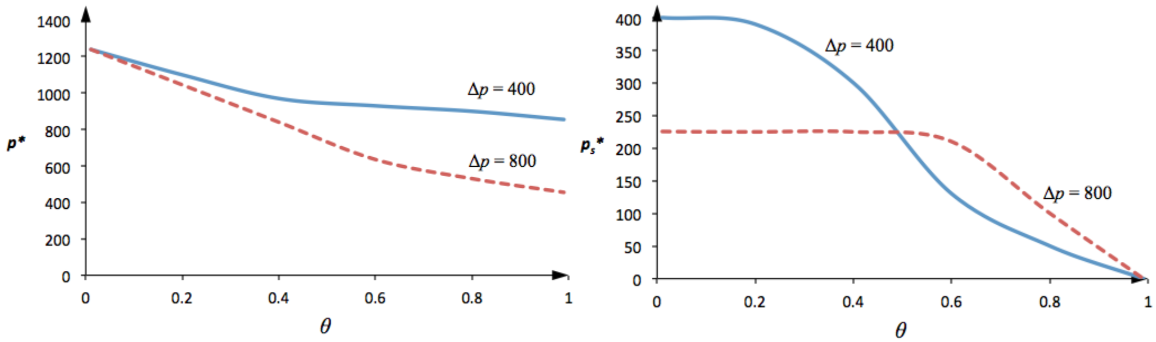

Figure 13: Numerical illustrations of optimal service price (left) and change fees (right) for different θ, V = 2500, Δp = \$400 (solid line) and Δp = \$800 (dashed line).

4.5 Discussion

In this study, I consider a monopolistic firm that generates ancillary revenues by allowing customers with type uncertainty to change between services offered by the firm. Although the firm maximizes revenue by imposing optimal ancillary fares, strategic customers maximize utilities by responding to these fares. Knowing that the introduction of an ancillary fee might distract customers from purchasing the Service 1, I investigate how the firm should introduce such a fee in this study. Also, it is not clear whether the firm should modify its pricing policy after charging its customers a change fee. I derive analytically each market player's best reaction to the other to prescribe the characteristics of firm/customer interaction equilibrium.

My results in Chapter 4 suggest that the firm should consider the change fee in its pricing policy and receive benefits from this fee. Although imposing the change fee might distract customers away from purchasing early, I find that it is beneficial for the firm to increase the change fee up to a threshold. I show that this threshold is a function of the level of customer uncertainty, the optimal monopolistic price and the variability of the price. Increasing the change fee beyond that threshold is, however, harmful for the firm, and the firm should be very thoughtful in this regard.

Ancillary fees do more than simply add to the profit margin; many industries, such as airlines, face massive expansion in demand in many regions of the world, requiring huge investment in different sections of the business. This trend requires access to additional monetary resources for strategic business expansions. However, based on a report by Brian Pearce, IATA's Chief Economist, the aviation industry's average 1990-2012 global net profit margin was 0.0%.[26] Furthermore, as he addresses in the report, airline investors earned almost nothing in the past. This challenge highlights that having an extra source of revenue is critical not only for the short term but also for the healthy financial future of these industries.

The customer uncertainty level plays an important role in how the firm should design a change fee. This result indicates that when this uncertainty is very high, the firm should never set a high change fee. This is anecdotally consistent with the current practice in the auto rental industry, where the change fees are minimal.[27] As I discussed, a high change fee will force all buyers to wait, generating no benefit for the firm.

The optimal monopolistic service, P_M, also has a significant impact on the change fee. When firm jointly selects the service price and the change fee, it is never optimal for the firm to impose a high change fee. The variability of the service price is also critical to the design of the change fee. I find that it is harmful for the firm to set a high change fee when customers face a huge premium. This suggests that, although the firm should exploit the ancillary revenue, it should design change fees wisely, along with other fees. A tradeoff between all fees is necessary when customers face multiple fees. Finally, I demonstrate that when the firm sets the service price and the change fee simultaneously, the optimal service price (with change fee) is increasing in the optimal monopolistic service price and decreasing in the proportion of customers with type change; however,

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[26] URL: https://www.iata.org/whatwedo/Documents/economics/Aviation-Advocacy-Economics-2013-July.pdf; accessed 2015-09-09.

[27] URL: http://www.nytimes.com/2012/02/18/your-money/autoslash-a-rate-sleuth-makes-rental-car-companies-squirm-your-money.html; accessed: 2015–8–8.

the optimal change fee is decreasing in the optimal monopolistic service price and increasing in the proportion of customers with type change.

Notice that, in this chapter, the revenue function has $x\left(1-F(x)\right)$ structure. The $x\left(1-F(x)\right)$ represents the revenue of a service provider selling a product at price *x*. There has been a stream of literature regarding the unimodality of this revenue function configuration. A well-known result is that, if *F* is increasing generalized failure rate (IGFR), the revenue function is unimodal (Lariviere 2006). For an IGFR *F*, the price elasticity of demand is weakly increasing for all *x* such that $F(x) < 1$. Many distributions such as Uniform, Normal and Weibull are IGFR. Please refer to Mihai and Prakash (2013) for a thorough summary of the necessary conditions to ensure the unimodality of the revenue function. As they highlight it, having the IGFR property is tremendously valuable in supply chain and revenue management literature.

This is one of the first studies to explicitly explore the effects of change fee in revenue management. Some firms have started to realize the importance of ancillary revenue as a reliable source—especially as global market competition continuously increases and profit margins tighten. I hope this study addresses some of the calls from practitioners to take into account this new source of revenue in their analyses and contributes to the literature, which has long advocated beneficial revenue management models.

4.6 Appendix

Proof of Proposition 1

This proposition corresponds to high θ case, i.e., $\theta > \frac{\Delta p}{\Delta p+p}$. In part A, part B, and part C of this proof, I analyze the necessary conditions for three cases in which a customer leaves the market in period 1, waits in period 1, and buys in period 1, respectively.

<u>Part A:</u> This case presents two scenarios in which a customer *leaves* the market: First, $v \leq p + \theta\,(\Delta p + p_s)$, *and* $p_s \leq \Delta p\,(1-\theta)/\theta$, and second if $v \leq \Delta p + p$, *and* $p_s > \Delta p\,(1-\theta)/\theta$. I study these two scenarios respectively.

Scenario A1: consider $v \le p + \theta\,(\Delta p + p_s)$, *and* $p_s \le \Delta p\,(1 - \theta) / \theta$: I add a condition to this scenario and analyze all possibilities for this condition.

A11: First, assume $v \le \Delta p + p_s$: A customer purchases in period 1 under this scenario if the expected utility from purchasing in period 1 is non-negative and greater than waiting, i.e., $(1 - \theta)(v - p) + \theta\,(- p) > 0$ and $(1 - \theta)(v - p) + \theta\,(- p) > v - p - \Delta p$, which simplifies to $p / (1 - \theta) < v \le \Delta p / \theta$. Notice that in this case since $v \le p + \theta\,(\Delta p + p_s)$, and $p_s \le p / (1 - \theta) - \Delta p$, thus, $v \le p + \theta\,(\Delta p + p / (1 - \theta) - \Delta p) \le p + \theta\,(p / (1 - \theta)) \le p / (1 - \theta)$ which contradicts $p / (1 - \theta) < v \le \Delta p / \theta$ condition above. Therefore, no one choose to purchase early in this situation.

A customer waits in period 1 under this situation if the expected utility from waiting in period 1 is non-negative and greater than purchasing in period 1, i.e., $v - p - \Delta p > 0$ and $(1 - \theta)(v - p) + \theta\,(- p) < v - p - \Delta p$, which simplifies to $v > \Delta p / \theta > \Delta p / (\Delta p / (\Delta p + p) > \Delta p + p$, which contradicts $v \le \Delta p + p_s$ above. Therefore, no one chooses to wait in this scenario. As a results, all customers in this case leave the market.

A12: Now, assume $v > \Delta p + p_s$. A customer purchases in period 1 under this situation if the expected utility from purchasing in period 1 is non-negative and greater than waiting, i.e., $(1 - \theta)(v - p) + \theta\,(v - p - \Delta p - p_s) > 0$ and $(1 - \theta)(v - p) + \theta\,(v - p - \Delta p - p_s) > v - p - \Delta p$, which simplifies to $p + \theta\,(\Delta p + p_s) < v$, which contradicts $v \le p + \theta\,(\Delta p + p_s)$ condition above. Therefore, no one chooses to purchase early in this situation.

A customer waits in period 1 under this scenario if the expected utility from waiting in period 1 is non-negative and greater than purchasing in period 1, i.e., $v - p - \Delta p > 0$ and $(1 - \theta)(v - p) + \theta\,(v - p - \Delta p - p_s) < v - p - \Delta p$, which says $v > p + \Delta p$. Notice, however, based on the above case, $v \le p + \theta\,(\Delta p + p_s) < p + \theta\,(\Delta p + \Delta p\,(1 - \theta) / \theta) < p + \Delta p$, which contradicts an assumption of Proposition 1, which is $\theta \le \frac{\Delta p}{\Delta p + p}$. Therefore, no one chooses to wait early in this situation. As a results, all customers in this case leave the market.

Scenario A2: Now, consider the second scenario for waiting, i.e., $v \leq \Delta p + p$, and $p_s > \Delta p (1 - \theta) / \theta$. I add a condition to this scenario and analyze all possibilities for this condition.

A21: First, assume $v \leq \Delta p + p_s$: A customer purchases in period 1 under this situation if the expected utility from purchasing in period 1 is non-negative and greater than waiting, i.e., $(1 - \theta)(v - p) + \theta (- p) > 0$ and $(1 - \theta)(v - p) + \theta (- p) > v - p - \Delta p$, which simplifies to $p / (1 - \theta) < v \leq \Delta p / \theta$. The first condition $p / (1 - \theta) < v$ can be written as follows by substituting the lower bound of θ: $p / (1 - \Delta p / (\Delta p + p)) < v$, which is $p + \Delta p < v$. This contradicts $v \leq \Delta p + p_s$ condition above. Therefore, no one chooses to purchase early in this scenario.

A customer waits in period 1 under this situation if the expected utility from waiting in period 1 is non-negative and greater than purchasing in period 1, i.e., $v - p - \Delta p > 0$ and $(1 - \theta)(v - p) + \theta (- p) < v - p - \Delta p$. The first condition contradicts $v \leq \Delta p + p_s$ condition above. Therefore, no one chooses to wait early in this situation. As a results, all customers in this case leave the market.

A22: Now, assume $v > \Delta p + p_s$. In this case, a customer buys early when $(1 - \theta)(v - p) + \theta (v - p - \Delta p - p_s) \geq v - p - \Delta p$, which implies that $\theta \leq \frac{\Delta p}{\Delta p + p}$. This contradicts the condition in the above case. Also, since $v \leq \Delta p + p$, no one waits in period 2, therefore, in this case, all customers leave the market in period 1.

<u>Part B:</u> This case presents a scenario in which a customer *waits* in period 1 in the market: $v > \Delta p / \theta$, *and* $p_s > \Delta p (1 - \theta) / \theta$. I add a condition to this scenario and analyze all possibilities for this condition.

B1: First, assume $v \leq \Delta p + p_s$: A customer purchases in period 1 under this scenario if $(1 - \theta)(v - p) + \theta (- p) > 0$ and $(1 - \theta)(v - p) + \theta (- p) > v - p - \Delta p$, which simplifies to $p /$

$(1 - \theta) < v \leq \Delta p / \theta$. Notice that in this condition contradicts $v > \Delta p / \theta$, and $p_s > \Delta p\, (1 - \theta) / \theta$ condition above. Therefore, no one chooses to purchase early in this situation.

A customer leaves in period 1 under this scenario if the expected utility from leaving in period 1 is greater than purchasing and waiting in period 1, i.e., $v - p - \Delta p < 0$ and $(1 - \theta)(v - p) + \theta\, (- p) < 0$, which simplifies to $v < p / (1 - \theta) < p / (1 - \Delta p / (\Delta p + p)) < \Delta p + p$, which contradicts $v \leq \Delta p + p_s$ above. Therefore, no one chooses to wait early in this situation. As a results, all customers in this case leave the market.

B2: Now, assume $v > \Delta p + p_s$. In this case, a customer buys early when $(1 - \theta)(v - p) + \theta\, (v - p - \Delta p - p_s) \geq v - p - \Delta p$, which implies that $\theta \leq \frac{\Delta p}{\Delta p + p}$. This contradicts the condition in the above case. Also since $v \leq \Delta p + p$, no one waits in period 2; consequently, in this case, no customer leaves the market in period 1. As a results, all customers in this case wait in period 1.

<u>Part C:</u> This case considers when a customer *buys* in period 1, that is, if the expected utility from purchasing in period 1 is non-negative and greater than waiting, i.e., $(1 - \theta)(v - p) + \theta\, (v - p - \Delta p - p_s) > 0$ and $(1 - \theta)(v - p) + \theta\, (v - p - \Delta p - p_s) > v - p - \Delta p$, which simplifies to $p + \theta\, (\Delta p + p_s) < v$ and $p_s \leq \Delta p\, (1 - \theta) / \theta$. Note that theses conditions complete the conditions in previous cases. □

Proof of Lemma 1

The firm's objective function is given by:

$$\pi(p_s) = \phi_{B_W O}(p_s)\big(p + \theta(p_s + \Delta p)\big) + \phi_{B_{WO} O}(p_s)(p) + \phi_{Waiting}(p_s)(p + \Delta p)$$

For $\theta > \frac{\Delta p}{\Delta p+p}$, based on Proposition 1, when $p_s \leq \Delta p\ (1 - \theta) / \theta$, customers are buy the service early if $v > p + \theta\ (\Delta p + p_s)$ (Case I), or leave the market otherwise (Case III). Notice that, there are no buyers without change option in this case.

Therefore, we have the following:

$$\phi_{Waiting}(p_s) = \phi_{BWoO}(p_s) = 0,\ \phi_{BWO}(p_s) = 1 - F\big(p + \theta(p_s + \Delta p)\big)$$

and the firm's expected revenue is simplified to:

$$\pi(p_s) = \Big(1 - F\big(p + \theta(p_s + \Delta p)\big)\Big)\big(p + \theta(p_s + \Delta p)\big) + 0 + 0$$
$$= \big(p + \theta(p_s + \Delta p)\big)\ \Big(1 - F\big(p + \theta(p_s + \Delta p)\big)\Big)$$

For $\theta > \frac{\Delta p}{\Delta p+p}$, based on Proposition 1, when $\Delta p\ (1 - \theta) / \theta < p_s$, customers are either leave the market (Case III) or wait in period 1 (Case II) if $v > \Delta p + p$. Again, there are no buyers without change option in this case.

Therefore, we have the following:

$$\phi_{BWO}(p_s) = \phi_{BWoO}(p_s) = 0,\ \ \phi_{Waiting}(p_s) = 1 - F(p + \Delta p)$$

and the firm's expected revenue is simplified to:

$$\pi(p_s) = 0 + 0 + \big(1 - F(p + \Delta p)\big)(p + \Delta p)$$

$$= (p + \Delta p)\ \big(1 - F(p + \Delta p)\big).\square$$

Proof of Corollary 1

Based on Lemma 1, for $\theta > \frac{\Delta p}{\Delta p+p}$, when $\Delta p\ (1 - \theta) / \theta < p_s$,

$$\pi(p_s) = (p + \Delta p)\ \big(1 - F(p + \Delta p)\big)$$

Notice that, in this case, the firm's revenue function is independent of change fee. This implies that any change fee higher than $\Delta p\ (1-\theta)/\theta$ is dominated by $p_s = \Delta p\ (1-\theta)/\theta$. Therefore, the change fee should never be higher than $\Delta p\ (1-\theta)/\theta$. □

Proof of Proposition 2

First, based on Corollary 1, a firm never sets a change fee higher than $\Delta p\ (1-\theta)/\theta$. Therefore, based on Lemma 1, the firm's expected revenue is given by:

$$\pi(p_s) = \big(p + \theta(p_s + \Delta p)\big)\ \Big(1 - F\big(p + \theta(p_s + \Delta p)\big)\Big)$$

I assume F is IGFR, therefore, the firm's expected revenue is unimodal. Checking the F.O.C. with respect to p_s, I have the following:

$$\theta\left(1 - F\left(p + \theta\big(p_s + \Delta p\big)\right)\right) + \theta\left(p + \theta\big(p_s + \Delta p\big)\right)\left(-f\left(p + \theta\big(p_s + \Delta p\big)\right)\right) = 0$$

which simplifies to equation (2) as follows:

$$\frac{1 - F\big(p + \theta(p_s + \Delta p)\big)}{f\big(p + \theta(p_s + \Delta p)\big)} = p + \theta(p_s + \Delta p)$$

I labeled the solution of (2) with respect to change fee as l_s. Therefore, l_s is the optimal change fee since it satisfies the F.O.C. for the firm's expected revenue. Note that when l_s is negative, since the firm's expected revenue is unimodal, the optimal change fee is zero. Furthermore, when l_s is greater than $\Delta p\ (1-\theta)/\theta$, the optimal change fee is $\Delta p\ (1-\theta)/\theta$, because the firm's expected revenue is unimodal. □

Proof of Proposition 3

This proposition corresponds to the low θ case, i.e., $\theta \leq \frac{\Delta p}{\Delta p + p}$. In part A, part B, and part C of this proof, I analyze the necessary conditions for three cases in which a customer leaves the market in period 1, waits in period 1, and buys in period 1, respectively.

Part A: This case presents two situations in which a customer *leaves* the market: First, *if* $v \leq p + \theta\,(\Delta p + p_s)$, and $p_s \leq p\,/\,(1-\theta) - \Delta p$ *, or if* $v \leq p\,/\,(1-\theta)$, and $p_s > p\,/\,(1-\theta) - \Delta p$.

A1: I first consider $v \leq p + \theta\,(\Delta p + p_s)$, and $p_s \leq p\,/\,(1-\theta) - \Delta p$: I add a condition to this scenario and analyze this situation under all possibilities.

A11: First, assume $v \leq \Delta p + p_s$. A customer purchases in period 1 under this situation if the expected utility from purchasing in period 1 is non-negative and greater than waiting, i.e., $(1-\theta)(v-p) + \theta\,(-p) > 0$ and $(1-\theta)(v-p) + \theta\,(-p) > v - p - \Delta p$, which simplifies to $p\,/\,(1-\theta) < v \leq \Delta p\,/\,\theta$. Notice that in this case, since $v \leq p + \theta\,(\Delta p + p_s)$, and $p_s \leq p\,/\,(1-\theta) - \Delta p$, therefore, $v \leq p + \theta\,(\Delta p + p\,/\,(1-\theta) - \Delta p) \leq p + \theta\,(p\,/\,(1-\theta)) \leq p\,/\,(1-\theta)$ which contradicts $p\,/\,(1-\theta) < v \leq \Delta p\,/\,\theta$ condition above. Therefore, no one chooses to purchase early in this scenario.

A customer waits in period 1 under this situation if the expected utility from waiting in period 1 is non-negative and greater than purchasing in period 1, i.e., $v - p - \Delta p > 0$ and $(1-\theta)(v-p) + \theta\,(-p) < v - p - \Delta p$, which simplifies to $v > \Delta p\,/\,\theta > \Delta p\,/\,(\Delta p\,/\,(\Delta p + p)) > \Delta p + p$, which contradicts $v \leq \Delta p + p_s$ above. Therefore, no one chooses to wait early in this situation. As a results, all customers in this case leave the market.

A12: Now, assume $v > \Delta p + p_s$. A customer purchases in period 1 under this scenario if the expected utility from purchasing in period 1 is non-negative and greater than waiting, i.e., $(1-\theta)(v-p) + \theta\,(v - p - \Delta p - p_s) > 0$ and $(1-\theta)(v-p) + \theta\,(v - p - \Delta p - p_s) > v - p - \Delta p$, which simplifies to $p + \theta\,(\Delta p + p_s) < v$, which contradicts $v \leq p + \theta\,(\Delta p + p_s)$ condition above. Therefore, no one chooses to purchase early in this situation.

A customer waits in period 1 under this situation if the expected utility from waiting in period 1 is non-negative and greater than purchasing in period 1, i.e., $v - p - \Delta p > 0$ and $(1-\theta)(v-p) + \theta\,(v - p - \Delta p - p_s) < v - p - \Delta p$, which sets a condition on θ, $\theta > \frac{\Delta p}{\Delta p + p}$. Notice, however, this contradicts an assumption of Proposition 1, which is $\theta \leq \frac{\Delta p}{\Delta p + p}$.

Therefore, no one chooses to wait early in this scenario. As a results, all customers in this case leave the market.

A2: Now, consider the second situation, i.e., $v \leq p / (1 - \theta)$, *and* $p_s > p / (1 - \theta) - \Delta p$:

I add a condition to this situation and analyze this scenario under all possibilities for this condition.

A21: First, assume $v \leq \Delta p + p_s$. A customer purchases in period 1 under this situation if the expected utility from purchasing in period 1 is non-negative and greater than waiting, i.e., $(1 - \theta)(v - p) + \theta (- p) > 0$ and $(1 - \theta)(v - p) + \theta (- p) > v - p - \Delta p$, which simplifies to $p / (1 - \theta) < v \leq \Delta p / \theta$. This contradicts $v \leq p / (1 - \theta)$ condition above. Therefore, no one chooses to purchase early in this situation.

A customer wait in period 1 under this scenario if the expected utility from waiting in period 1 is non-negative and greater than purchasing in period 1, i.e., $v - p - \Delta p > 0$ and $(1 - \theta)(v - p) + \theta (- p) < v - p - \Delta p$, which simplifies to $v > \Delta p / \theta > \Delta p / (\Delta p / (\Delta p + p) > \Delta p + p$, which contradicts $v \leq \Delta p + p_s$. Therefore, no one chooses to wait early in this situation. As a results, all customers in this case leave the market.

A22: Now, assume $v > \Delta p + p_s$. In this case, $v > \Delta p + p_s > \Delta p + p / (1 - \theta) - \Delta p > p / (1 - \theta)$ which contradicts $v \leq p / (1 - \theta)$ condition in case 1B. Therefore, v cannot be greater than $\Delta p + p_s$ and the second condition ($v > \Delta p + p_s$) does not exist.

Summarizing all different scenarios, in above cases all customers leave the market in period 1.

<u>Part B:</u> Next consider a situation in which a customer *waits* in period 1 in the market: $v > \Delta p / \theta$, *and* $p_s > \Delta p (1 - \theta) / \theta$. I add a condition to this scenario and analyze this situation under all possibilities.

B1: First, assume $v \leq \Delta p + p_s$. A customer purchases in period 1 under this situation if $(1 - \theta)(v - p) + \theta(-p) > 0$ and $(1 - \theta)(v - p) + \theta(-p) > v - p - \Delta p$, which simplifies to $p / (1 - \theta) < v \leq \Delta p / \theta$. This contradicts $v > \Delta p / \theta$, and $p_s > \Delta p (1 - \theta) / \theta$ condition above. Therefore, no one chooses to purchase early in this scenario.

A customer leaves in period 1 under this situation if the expected utility from leaving in period 1 is greater than purchasing and waiting in period 1, i.e., $v - p - \Delta p < 0$ and $(1 - \theta)(v - p) + \theta(-p) < 0$, which simplifies to $v < p / (1 - \theta) < p / (1 - \Delta p / (\Delta p + p)) < \Delta p + p$, which contradicts $v \leq \Delta p + p_s$ above. Therefore, no one chooses to wait early in this situation. As a results, all customers in this case leave the market.

B2: Now, assume $v > \Delta p + p_s$. A customer purchases in period 1 under this scenario if $(1 - \theta)(v - p) + \theta(v - p - \Delta p - p_s) > 0$ and $(1 - \theta)(v - p) + \theta(v - p - \Delta p - p_s) > v - p - \Delta p$, which simplifies to $\theta > \frac{\Delta p}{\Delta p + p}$. Notice, however, this contradicts an assumption of Proposition 1, which is $\theta \leq \frac{\Delta p}{\Delta p + p}$. Therefore, no one chooses to purchase early in this situation.

A customer leaves in period 1 under this situation if the expected utility from leaving in period 1 is greater than purchasing and waiting in period 1, i.e., $v - p - \Delta p < 0$ and $(1 - \theta)(v - p) + \theta(-p) < 0$, which contradicts $v > \Delta p + p_s$ above. Therefore, no one chooses to leave in this scenario. As a results, all customers in this case wait in period 1.

<u>Part C:</u> Next, consider a situation in which a customer *buys* early in period 1. There are two cases here: First, I have the following: A customer purchases in period 1 under this situation if the expected utility from purchasing in period 1 is non-negative and greater than waiting, i.e., $(1 - \theta)(v - p) + \theta(-p) > 0$ and $(1 - \theta)(v - p) + \theta(-p) > v - p - \Delta p$, which simplifies to $p / (1 - \theta) < v \leq \Delta p / \theta$.

Second, a customer purchases in period 1 under this scenario if the expected utility from purchasing in period 1 is non-negative and greater than waiting, i.e., $(1 - \theta)(v - p) + \theta(v - p - \Delta p - p_s) > 0$ and $(1 - \theta)(v - p) + \theta(v - p - \Delta p - p_s) > v - p - \Delta p$, which simplifies to $p + \theta(\Delta p + p_s) < v$ and $p_s \leq \Delta p(1 - \theta) / \theta$. □

Proof of Lemma 2

The firm's objective function is given by:

$$\pi(p_s) = \phi_{B_W O}(p_s)\big(p + \theta(p_s + \Delta p)\big) + \phi_{B_{WO} O}(p_s)(p) + \phi_{Waiting}(p_s)(p + \Delta p)$$

For $\theta \leq \frac{\Delta p}{\Delta p + p}$, based on Proposition 3, when $p_s \leq p/(1 - \theta) - \Delta p$, customers are buy the service early with change option if $v > p + \theta(\Delta p + p_s)$ (Case I), or leave the market otherwise (Case VI). Notice that, there are no waiting customers and buyers without change option in this case.

Therefore, we have the following:

$$\phi_{Waiting}(p_s) = \phi_{B_{WO} O}(p_s) = 0, \phi_{B_W O}(p_s) = 1 - F\big(p + \theta(p_s + \Delta p)\big)$$

and the firm's expected revenue is simplified to:

$$\pi(p_s) = \Big(1 - F\big(p + \theta(p_s + \Delta p)\big)\Big)\big(p + \theta(p_s + \Delta p)\big) + 0 + 0$$
$$= \big(p + \theta(p_s + \Delta p)\big)\Big(1 - F\big(p + \theta(p_s + \Delta p)\big)\Big)$$

Now, for $\theta \leq \frac{\Delta p}{\Delta p + p}$, based on Proposition 3, when $-\Delta p + p/(1 - \theta) < p_s \leq \Delta p / \theta - \Delta p$, customers are either leave the market when $v \leq p/(1 - \theta)$ (Case VI) or buy with change option when $\Delta p + p_s < v$ (Case II) or without change option $p/(1 - \theta) < v \leq \Delta p + p_s$ (Case III) in period 1. Notice that, there are no waiting customers in this case.

Therefore, we have the following:

$$\phi_{Waiting}(p_s) = 0, \phi_{B_W O}(p_s) = 1 - F(p_s + \Delta p), \phi_{B_{WO} O}(p_s) = F(p_s + \Delta p) - F\left(\frac{p}{1 - \theta}\right)$$

and the firm's expected revenue is simplified to:

$$\pi(p_s) = \left(p + \theta(p_s + \Delta p)\right)\left(1 - F(p_s + \Delta p)\right) + p\left(F(p_s + \Delta p) - F\left(\frac{p}{1-\theta}\right)\right) + 0$$
$$= \left(p + \theta(p_s + \Delta p)\right)\left(1 - F(p_s + \Delta p)\right) + p\left(F(p_s + \Delta p) - F\left(\frac{p}{1-\theta}\right)\right)$$

Now, for $\theta \leq \frac{\Delta p}{\Delta p + p}$, based on Proposition 3, when $-\Delta p + p / (1 - \theta) \leq p_s$, customers are either leave the market when $v \leq p/ (1 - \theta)$ (Case VI) or wait when $\Delta p / \theta < v$ (Case V) or buy without change option $p/ (1 - \theta) < v \leq \Delta p / \theta$ (Case IV) in period 1. Notice that, there are no buyers with change option in this case.

Therefore, we have the following:

$$\phi_{Waiting}(p_s) = 1 - F\left(\frac{\Delta p}{\theta}\right), \phi_{B_W O}(p_s) = 0, \phi_{B_{WO} O}(p_s) = F\left(\frac{\Delta p}{\theta}\right) - F\left(\frac{p}{1-\theta}\right)$$

and the firm's expected revenue is simplified to:

$$\pi(p_s) = (p + \Delta p)\left(1 - F\left(\frac{\Delta p}{\theta}\right)\right) + p\left(F\left(\frac{\Delta p}{\theta}\right) - F\left(\frac{p}{1-\theta}\right)\right) + 0$$
$$= (p + \Delta p)\left(1 - F\left(\frac{\Delta p}{\theta}\right)\right) + p\left(F\left(\frac{\Delta p}{\theta}\right) - F\left(\frac{p}{1-\theta}\right)\right). \square$$

Proof of Corollary 2

First, the optimal monopolistic service price, P_M, maximizes the classical expected revenue for a seller with a single resource: $p\left(1 - F(p)\right)$. Since F is IGFR, $p\left(1 - F(p)\right)$ is unimodal and P_M satisfies the F.O.C. with respect to service price as follows:

$$\left(1 - F(p)\right) - pf(p) = 0$$

which can be rearranged to:

$$\frac{1 - F(p)}{f(p)} = p$$

Second, I focus on my study of a firm with two resources, which maximizes its expected revenue based on the change fee. I would like to highlight that based on Lemma 2, a firm never sets a change fee higher than $\Delta p\ (1 - \theta) / \theta$. This is because the firm's expected revenue is independent of change fee when $\Delta p\ (1 - \theta) / \theta < p_s$. Therefore, the optimal change fee should be within the first region of change fee when $p_s \leq p/ (1 - \theta) - \Delta p$ (low change fee case), or the second region of change fee when $-\Delta p + p / (1 - \theta) < p_s \leq \Delta p / \theta - \Delta p$ (high change fee case).

In low change fee case, based on Lemma 2, the expected revenue is given by:

$$\pi(p_s) = \Big(1 - F\big(p + \theta(p_s + \Delta p)\big)\Big)\big(p + \theta(p_s + \Delta p)\big)$$

I assume F is IGFR, therefore, the firm's expected revenue is unimodal. Checking the F.O.C. with respect to p_s, I have the following:

$$\theta\left(1 - F\left(p + \theta\left(p_s + \Delta p\right)\right)\right) + \theta\left(p + \theta\left(p_s + \Delta p\right)\right)\left(-f\left(p + \theta\left(p_s + \Delta p\right)\right)\right) = 0$$

which simplifies to equation (2) as follows:

$$\frac{1 - F\big(p + \theta(p_s + \Delta p)\big)}{f\big(p + \theta(p_s + \Delta p)\big)} = p + \theta(p_s + \Delta p)$$

I labeled the solution of (2) with respect to change fee as l_s. Therefore, l_s is the optimal change fee when it is within the low change fee case since it satisfies the F.O.C. for the firm's expected revenue in this case.

In high change fee case, based on Lemma 2, the expected revenue is given by:

$$\pi(p_s) = \left(p + \theta(p_s + \Delta p)\right)\left(1 - F(p_s + \Delta p)\right) + p\left(F(p_s + \Delta p) - F\left(\frac{p}{1-\theta}\right)\right)$$
$$= (1-\theta)\left(\frac{p}{1-\theta}\right)\left(1 - F\left(\frac{p}{1-\theta}\right)\right) + \theta(p_s + \Delta p)\left(1 - F(p_s + \Delta p)\right)$$

Notice that the first part of objective function is independent of change fee. I assume F is IGFR, therefore, the firm's expected revenue is unimodal in this case, as well. Checking the F.O.C. with respect to p_s, I have the following:

$$\theta\left(1 - F\left(p_s + \Delta p\right)\right) + \theta\left(p_s + \Delta p\right)\left(-f\left(p_s + \Delta p\right)\right) = 0$$

which simplifies to equation (3) as follows:

$$\frac{1 - F\left(p + \theta(p_s + \Delta p)\right)}{f\left(p + \theta(p_s + \Delta p)\right)} = p + \theta(p_s + \Delta p)$$

I labeled the solution of (3) with respect to change fee as u_s. Therefore, u_s is the optimal change fee when it is within the high change fee case since it satisfies the F.O.C. for the firm's expected revenue in this case.

Now, I would like to investigate under which scenarios the optimal change fee lies within the low change fee case and within the high change fee case. Notice that the boundary of two cases are at $-\Delta p + p/(1-\theta) = p_s$. Also, the slope of the firm's objective function is the same for both change fee cases and is given by:

$$\theta\left(1 - F\left(\frac{p}{1-\theta}\right)\right) + \theta\left(\frac{p}{1-\theta}\right)\left(-f\left(\frac{p}{1-\theta}\right)\right)$$

Therefore, whenever this slope is negative, since objective functions in both change fee cases are IGFR, the optimal change fee should be within the low change fee case and whenever this slope is positive, the optimal change fee should be within the high change fee case. Notice that, however, if we substitute $x = p/(1-\theta)$ the slope of the firm's objective function has the following structure:

$$\theta\left(1 - F(x)\right) + \theta x\left(-f(x)\right)$$

and for $x = P_M$, we have

$$\theta\big(1 - F(P_M)\big) + \theta P_M\,\big(-f(P_M)\big) = 0$$

since, as we discussed above, P_M satisfies the following F.O.C. condition.

$$\big(1 - F(p)\big) - pf(p) = 0$$

This implies that whenever $p/(1-\theta)$ is less that or equal to P_M, the slope of the firm's objective function at $-\Delta p + p/(1-\theta) = p_s$ is negative, and the optimal change fee lies within the low change fee case. On the other hand, whenever $p/(1-\theta)$ is higher P_M, the slope of the firm's objective function at $-\Delta p + p/(1-\theta) = p_s$ is positive, and the optimal change fee lies within the high change fee case.

When l_s (or u_s) are outside their region of change fee in Propositions 1 (or 3), the optimal change fee is equal to the boundary of the region that is closer to l_s (or u_s) since I assume customers' valuation follows an IGFR distribution. □

Proof of Corollary 3

The optimal monopolistic service price, P_M, satisfies the following condition:

$$\frac{1 - F(p)}{f(p)} = p$$

Now consider the low and high change fee regions for low θ case, which are discussed in Corollary 2. The optimal change fees in both cases satisfy their corresponding optimality conditions:

For low change fee case:

$$\frac{1 - F\big(p + \theta(p_s + \Delta p)\big)}{f\big(p + \theta(p_s + \Delta p)\big)} = p + \theta(p_s + \Delta p)$$

and for the high change fee case:

$$\frac{1 - F(p_s + \Delta p)}{f(p_s + \Delta p)} = p_s + \Delta p$$

This implies that based on Corollary 2, either we have: $p + \theta(p_s + \Delta p) = P_M$, or $p_s + \Delta p = P_M$. Since P_M is constant, it is immediate that the optimal change fee is decreasing in p, Δp and θ. □

Proof of Lemma 3 and 4

Part A: First, I show that *if* $\theta \leq \frac{\Delta p}{\Delta p + p}$, then the optimal service price should be one of two following service prices: $\underline{p}$ and $\overline{p}$ and *if* $\theta > \frac{\Delta p}{\Delta p + p}$, then the optimal service price is $\underline{p}$.

Consider two cases for analyzing service price as follows: $\Delta p / \theta - \Delta p < p_s$ (high case) and $p_s \leq \Delta p / \theta - \Delta p$ (low case).

For the low case $p_s \leq \Delta p / \theta - \Delta p$, when $(p_s + \Delta p)(1 - \theta) > p$, Based on Lemma 2, I have:

$$\pi(p) = \big(p + \theta(p_s + \Delta p)\big)\,\bar{F}(p_s + \Delta p) + p\left(F(p_s + \Delta p) - F\left(\frac{p}{1 - \theta}\right)\right) =$$

$$= (1 - \theta)\left(\frac{p}{1 - \theta}\right)\left(1 - F\left(\frac{p}{1 - \theta}\right)\right) + \theta\big(p_s + \Delta p\big)\left(1 - F\big(p_s + \Delta p\big)\right)$$

Taking the F.O.C. with respect to p, I have:

$$\left(1 - F\left(\frac{p}{1 - \theta}\right)\right) - p\left(f\left(\frac{p}{1 - \theta}\right)\right) = 0$$

which is rearranged to:

$$\frac{1-F\left(\frac{p}{1-\theta}\right)}{f\left(\frac{p}{1-\theta}\right)}=\frac{p}{1-\theta}$$

I label the solution for the optimal price in above formula as $\underline{p}$.

Now, for the low case $p_s \leq \Delta p / \theta - \Delta p$, where $(p_s + \Delta p)(1 - \theta) < p$, based on Lemma 1 and Lemma 2, I have:

$$\pi(p) = \big(p + \theta(p_s + \Delta p)\big)\,\bar{F}\big(p + \theta(p_s + \Delta p)\big)$$

Based on the condition of unimodality discussed in Lariviere (2006), the optimal service price should satisfy the following conditions:

$$\frac{1-F\big(p+\theta(p_s+\Delta p)\big)}{f\big(p+\theta(p_s+\Delta p)\big)}=p+\theta(p_s+\Delta p)$$

We label the solution for the optimal price in above formula as $\underline{p}$, which should be feasible in above region.

Now, consider the high service price case $\Delta p / \theta - \Delta p < p_s$. When $(p_s + \Delta p)(1 - \theta) > p$, based on Lemma 2, I have:

$$\pi(p) = \big(p + \theta(p_s + \Delta p)\big)\,\bar{F}(p_s + \Delta p) + p\left(F(p_s + \Delta p) - F\left(\frac{p}{1-\theta}\right)\right)$$

which is the same revenue function discussed in previous case. Therefore, the service fee is $\underline{p}$. Now, when $(p_s + \Delta p)(1 - \theta) < p$,

$$\pi(p) = (p + \Delta p)\,\bar{F}(p + \Delta p)$$

However, since p_s never satisfy this condition based on Corollary 1, we can ignore this case.

<u>Part B:</u> I present that if $p_s \leq P_M - \Delta p$, *then* $p^* = u$, and if $p_s > P_M - \Delta p$, then $p^* = l$.

The optimal monopolistic service price, P_M, satisfies the following condition:

$$\frac{1-F(p)}{f(p)} = p$$

and any customer with valuation lower than P_M leaves the market and any customer with valuation higher than P_M purchases the service.

Consider two cases of discussed in Part A as follows: $\Delta p / \theta - \Delta p < p_s$ and $p_s \leq \Delta p / \theta - \Delta p$. The optimal service price in both cases, satisfy their optimality conditions:

For the low case:

$$\frac{1-F\left(\frac{p}{1-\theta}\right)}{f\left(\frac{p}{1-\theta}\right)} = \frac{p}{1-\theta}$$

and for the high case:

$$\frac{1-F\big(p+\theta(p_s+\Delta p)\big)}{f\big(p+\theta(p_s+\Delta p)\big)} = p+\theta(p_s+\Delta p)$$

Notice that at the boundary of two cases, i.e., $(p_s + \Delta p)(1 - \theta) < p$, I have $p_s + \Delta p = v$. This implies that whenever $p_s + \Delta p < P_M$, the optimal change fee lies within the lower case since P_M demonstrates the same optimality structure, otherwise it lies within the higher case.

Part C: Based on part A and Part B, I have the following:

if $p \leq P_M (1 - \theta)$, $p_s^* = u_s$, *otherwise*, $p_s^* = l_s$.

If $p_s \leq P_M - \Delta p$, $p^* = \overline{p}$, *otherwise* $p^* = \underline{p}$.

Let's assume, the joint optimal decision is inside the second region of low θ case. Therefore, based on Part B:

$$-f\left(\frac{p}{1-\theta}\right)\frac{\theta p}{1-\theta}+\left(1-F\left(\frac{p}{1-\theta}\right)\right)\theta=0$$

However, I have shown that in Corollary 2 the slope at the boundary for two regions of change fee satisfies the same condition and should be equal to zero as follows:

$$-f\left(\frac{p}{1-\theta}\right)\frac{\theta p}{1-\theta}+\left(1-F\left(\frac{p}{1-\theta}\right)\right)\theta=0$$

This implies that the joint optimal decision is not inside the second region of the low θ case. Therefore, the joint optimal decision only can be inside the first region of the low θ case or the first region of the high θ case. This summarized to:

$p^* = \overline{p}$ *and* $p_s^* = \underline{p_s}$. Based on conditions of optimality in Part B, Lemma 1 and Lemma 2, I have two necessary conditions corresponding the first region of the low θ case and the first region of the high θ case as follows:

$$P_M = p^* + (p_s^* + \Delta p)(1 - \theta).$$

$$P_M = p^*/(1 - \theta).$$

Now, I will specify when the joint optimal decision should be inside the first region of the low θ case or when it should be inside the first region of the high θ case. The boundary of two cases can be specified as:

$p = P_M (1 - \theta)$ and $p_s = P_M - \Delta p$.

Therefore, when $P_M > \Delta p$, $p_s = 0$. Knowing that $p = (0 + \Delta p)(1 - \theta)$, and $p = P_M (1 - \theta)$, therefore, on the boundary, $\Delta p = P_M$. The results discussed in Lemma 3 and 4 are immediate from above conditions. □

Proof of Corollary 4

Based on Lemma 3 and Lemma 4, the optimal service price is increasing in P_M since p^* is either

$$p^* = P_M + (p_s^* + \Delta p)(1 - \theta).$$

or

$$p^* = P_M\,(1 - \theta), \textit{ and } p_s^* = 0.$$

It is also immediate that p^* decreasing in θ. Furthermore, based on the results in Lemma 3 and Lemma 4, the optimal change fee is decreasing in P_M and increasing in θ. □

Chapter 5

5 Conclusion

Many firms and organizations have allegedly optimized their core business function. However, they are constantly under extreme pressure, driven by external factors, to protect their narrow profit margins. For instance, while many airlines faces government taxes, fee and safety costs required by regulation, they cannot financially survive even when they successfully operate at 99% capacity level. Their need for supplemental and reliable revenues calls for performance optimization beyond the core business functions.

The primary purpose of my dissertation has been to investigate the revenue management and pricing decisions of customer-based strategies that generate extra revenue for firms. Motivated by applications from online social media and the airline industry, over three different studies, I focused on the following research questions in this dissertation: How can firms effectively use customer-based pricing strategies to boost revenues? The results generated from this thesis lead to optimizing the performance beyond the core business functions.

Three different model frameworks were developed in three chapters. In Chapter 2, I investigated the revenue sharing strategies of online social media and showed how these strategies shape the contribution levels of online users. I proposed a stylized model in which two online social media firms compete in the market by offering rewards to online users. Each firm could choose to be either generous in revenue sharing or save more for itself. The game consists of two steps: first, the OSMs announce their rewards for active users and, second, users choose their level of contribution with respect to each OSM and based on their preferences. I derived the revenue function for the firms and presented the existence and uniqueness of equilibrium.

In Chapter 3, I studied the switching behaviour of ticket holders in the aviation industry. I modelled a monopoly firm that sequentially announces prices over two periods for two identical resources. The customers track prices, and this behaviour triggers demand leakage between two resources when the price drops. The firm seeks to manage the uncertain switching behaviour by setting the best change fee. In general, low change fees attract more customers to a service, but high change fees generate more revenue per switching ticketholder. It was not clear whether the firm should accommodate a low or high population of switching customers, as both resources have their own source of demand. I demonstrated that the uncertainty in switching behaviour of customers drove the firm's optimal switching policy.

In Chapter 4, I extended the model I introduced in Chapter 3 to include strategic customers with type uncertainty. I considered a monopolistic firm that generates ancillary revenues by allowing customers with valuation uncertainty to change between services offered by the firm. While the firm maximizes revenue by imposing optimal change fees, strategic customers maximize their utilities by responding to these fares. Knowing that the introduction of a change fee by firms can discourage customers from purchasing the service early, I investigated how the firm should impose such a fee. I analytically derived each market player's best reaction in order to prescribe the characteristics of firm/customer interaction equilibrium.

My results suggest that firms can benefit from imposing a change fee. Although imposing the change fee might discourage customers from purchasing early, I found that it is beneficial for the firm to increase the change fee up to a threshold. I proved that this threshold is a function of the level of customer uncertainty, the optimal monopolistic price, and the variability of the price. Increasing the change fee beyond that threshold is, however, harmful for the firm, and the firm should be very thoughtful in this regard.

5.1 Managerial Insights

The results in this dissertation provide many useful insights for industries interested in developing new customer-based initiatives to boost their revenues. Specifically, I studied two types of customer-based pricing strategies: revenue sharing programs in online social networks and change fees in airlines. First, I provide some insight into the revenue-sharing programs.

5.1.1 Revenue Sharing programs

I investigated revenue-sharing programs of online social media and showed how these programs shape the contribution levels of online users. Firms have two opposing strategies in dealing with active online users; they can either save advertising revenue or share it. I studied the trade-off between these two strategies. My results indicated that, in any asymmetric completion, users always select the more favourable firm whenever they decide to work exclusively for one firm. As I discussed, these results are consistent with observed online evidence. For example, all online celebrities opened their accounts with YouTube in the early stages of the medium's introduction. Moreover, some of my results are even observable for OSMs who do not compete based on revenue sharing programs. An example includes the ongoing challenge of Google+ to compete with Facebook. Many social media connoisseurs were highly optimistic about Google+ when it first emerged (and even used it exclusively) but over time realized that they should reopen their Facebook accounts

These results indicate that a firm entering an online competition should investigate how online users position its online community relative to a competitor one. Even when there is a significant asymmetry between two firms, both firms should be aware of the fact that a market share still exists for the less favourable firm (or niche player). As I discussed, this explains why a large number of small websites now co-exist with dominant websites such as Facebook and YouTube.

These results also make significant suggestions for small players in the market. A firm considered to be less favourable in the market could better position itself in online competition if it continuously improved the users' perspective of its online community. For example, although the popularity of iTunes makes it a favourable service for artists, downloads from iTunes are readily available on an iPod (or other device), allowing fans easier access to music. These results suggest that small social media firms can compete with a dominant firm in the market by providing users with a service that has a higher utility.

In addition to market perception, the revenue generated by users' contributions also has a significant impact on the outcome of online competition. Firms should have a clear assessment of true user contribution. As I discussed, if users' contributions easily generate revenue, then both less favourable and dominant firms share high rewards and earn higher net revenues, as well. However, the more favourable firm can better exploit its advantage in this case by sharing a lower reward. On the other hand, when the monetary impact of contributions is too small, both firms follow a parsimonious strategy in sharing rewards. These results imply a bilateral relationship between the monetary values of users' contributions and the rewards they receive; if users demand high rewards, they should deliver rewarding contributions. For example, some media-streaming websites make substantial payments because the type or level of contributions delivered by their users helps to promote the website, which is in the best interest of the website as well as the advertising business.

I have also found that, in order for firms to be successful, it is critical for them to strike the right balance between saving and sharing their revenues with active users. I observe that, at equilibrium, firms keep at least half of the marginal revenues for themselves and never share more than half of this incremental revenue with their active users. Moreover, when the asymmetry between two firms increases considerably, it might be optimal for the less favourable firm to completely shut down its rewards program. A higher level of asymmetry not only adversely affects the less favourable firm, but it can also generate monetary challenges for the more favourable firm, especially when the difference between the two firms is insignificant originally.

In the digital world, the monetary impact of the contributions is too small. As previously discussed, the less favourable firms can compete with a dominant firm in the market by improving user perspective of its online community. When the monetary rewards are insignificant, this strategy is even more beneficial for the less favourable firm. My findings suggest that, in this case, the less favourable firm can generate higher revenues and, as a result, the firm can share higher rewards with active users. This trend has been observed recently in the ongoing competition between a dominant firm, YouTube, and a smaller online music service, Spotify. As Spotify's user base continues to grow, the firm pays increasingly higher reward levels to the artists who use its platform. Next, I summarize my findings regarding ancillary fees.

5.1.2 Ancillary Revenue

In this thesis, I focused on the benefits of managing revenues from switching customers. As I discussed, ancillary fees have recently become very significant for many industries, such as airlines. Managing ancillary revenue supports a firm with a narrow profit margin to boost its revenue. For example, Airlines will allegedly need to raise funds to invest in 34,000 new aircraft for expansion ahead for emerging markets until 2030.[28] Collecting ancillary fees can financially support airlines for this market expansion which requires $4.5 trillion investment. Furthermore, ancillary revenue improves customer relationship through providing more options. Many industries, such as consumer electronics retail, provide money-back guarantees to customers far into the future (for a limited time). This practice is used as a marketing tool in generating higher sales. In Chapter 3, I considered that a hypothetical airline which might want to use this as a marketing strategy to attract price-sensitive customers. The proposed case has already been implemented by some airlines. As airlines are exposed to various marketing and pricing strategies, I would not be surprised to see airlines also offering price guarantees across dates and flights. This paper has contributed by highlighting some of the operational aspects of these marketing and pricing strategies to take into consideration.

This thesis provides the managerial insights for firms to generate higher revenues by segmenting the market and designing ancillary fees for each segment separately. For example, for non-monetary reasons business travelers have a higher switching rate than other passengers. Airlines, for instance, should have a good estimate of the likelihood of a customer from each market segment changing her itinerary and should assess their prices accordingly. The market can also be segmented based on seasons, geographic regions, and so on. In addition to predicting the behaviour of customers, firms should

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[28] URL: https://www.iata.org/whatwedo/Documents/economics/Aviation-Advocacy-Economics-2013-July.pdf; accessed: 2015–8–8.

forecast their reactions to ancillary fees. This is even more critical because firms need to take into consideration long-term market competition, as well. A strict pricing policy regarding switching and cancelation might send customers away to a competitor. This could partly explain why some carriers, such as Southwest Airlines, have been successful in acquiring higher market shares by designing more suitable ancillary fees for customers. Finally, I would like to emphasize that firms should not follow pricing strategies that sub-maximize their ancillary revenues. Although some airlines are reportedly more successful in generating higher ancillary revenues, these revenues should be considered as a portion of the total revenue structure and be managed for a higher total profit and market performance.

I would like to highlight some other industries to which my model and results might apply. For example, banks or cellphone providers experience an identical problem in managing their switching revenues when homeowners decide to change their mortgage plans or when cellphone users change their mobile plans. In all of these cases, firms should impose an appropriate fee to find the best allocation of their limited resources to different streams of demand, knowing that the related ancillary revenues should be collected to maximize their total profit.

In Chapter 5, I considered customer uncertainty in dealing with a future booking. As discussed, this uncertainty plays an important role in how the firm should design a change fee. This finding indicates that when this uncertainty is very high, firms should never set a high change fee. This is anecdotally consistent with what is the current practice in the auto rental industry, in which the change fees are minimal.[29] As I discussed, a high change fee forces all potential buyers to wait, and this generates no benefit from change fees for the firm.

I made a connection between my results and models studying a single seller in the market. As discussed, the original price of a service, which is captured in my model

[29] URL: http://www.nytimes.com/2012/02/18/your-money/autoslash-a-rate-sleuth-makes-rental-car-companies-squirm-your-money.html; accessed: 2015–8–8.

through the optimal monopolistic service, P_M, also has significant impact on the change fee. A higher service price represents a more premium market. My results suggest that when the service price and the change fee are set simultaneously, it is never optimal for the firm to impose a high change fee. Finally, the variability of the service price is also critical to the design of the change fee. I find that it is harmful for the firm to set a high change fee when customers face a huge premium. This suggests that, although firms should exploit ancillary revenue, they should design change fees wisely and along with other fees. A tradeoff between fees is necessary when customers should pay multiple fees. Finally, I demonstrated that when the firm selects the service price and the change fee simultaneously, the optimal service price (with change fee) increases in the optimal monopolistic service price and decreases in the proportion of switching customers; however, the optimal change fee decreases in the optimal monopolistic service price and increases in the proportion of switching customers.

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