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Studies of Contingent Capital Bonds

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STUDIES OF CONTINGENT CAPITAL BONDS
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by

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Abstract

A contingent capital bond (CCB) is a subordinated security that converts to common shares when a predetermined trigger is breached. The 2008 financial crisis and the Basel III motivate the issuance of CCBs, aiming to mitigate the too-big-to-fail problem in financial distress and to resolve financial institutions by bailing in with the firm's own capital rather than a bailing out using the taxpayers' money.

Within the structural modelling framework, we consider the pricing of CCBs with an affine geometric Brownian motion by assuming that coupon payments have impact on the asset value dynamics. We extend the capital structure into four tranches including deposits, equity, and senior and subordinated bonds, and calibrate the model to Canadian banking data. Under infinite maturity, we derive a closed-form formula to price CCBs. Regulatory suggestions can be made based on our model in the design of conversion terms in recognition to the creditor-claim seniority and to ensure that equity investors are not rewarded for poor performance. Under the finite-maturity case, the term structures of CCBs are investigated by applying Monte Carlo simulation.

When the conversion price is based on the contemporary market stock price (as it tends to be in practice), CCB investors may have incentives to short the firm's stock to depress the market stock price and earn favourable returns from possible future conversion. Continuing with the structural model, we allow for a deviation between the stock's fundamental value and market value and use it to analyze the CCB investors' incentives to short. We discuss three kinds of market-based conversion prices and find that directly using the contemporary market stock price could tempt manipulations. However, adding a floor to the contemporary market stock price or using the trailing average instead would curb the manipulation incentives.

Among the issuances of CCBs, one noticeable characteristic is that regulators retain the right to force the conversion in view of the issuing firm's solvency prospects and the economic stability. In an intensity-model based approach, we incorporate regulatory discretion into the pricing model and therefore manage to quantify the impact of regulatory uncertainty on the cost of CCBs. Reasonable intervals for conversion terms are also detected under the regulatory trigger. Two categories of intensity functions are considered to distinguish regulators' behaviours towards non-systemically important and too-big-to-fail financial institutions. In general, the CCBs issued by too-big-to-fail financial institutions are more expensive than those issued by non-systemically important financial institutions due to the feature that conversion is sure to happen before liquidation.

Keywords: Contingent capital bond, structural model, manipulation incentives, intensity model, regulatory discretion.

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Chapter 1

Introduction

1.1 Background

The recent financial crisis brings up an important question – what should policymakers do when faced with the potential failure of a large bank? In 2008 officials had to choose between taxpayer bail-outs or systemic financial collapse (Calello and Ervin [2]).

There were a number of taxpayer bail-outs in the 2008 financial crisis. Examples include the Federal Reserve seizing control of Fannie Mae and Freddie Mac by pledging a 200 billion dollars cash injection; the government bailed out American International Group Inc. (AIG) with 85 billion dollars and provided financial institutions with various forms of financing through programs such as the Troubled Asset Relief Program (TARP). Although governments claim that bailouts are essential to provide economic stability and to prevent disruption to the financial system, it has been argued that these bail-out resolutions make taxpayers "foot the bill" and give banks incentives to take undue risks (Dickson [4]). This is especially true for systemically important banks, because of their "too big to fail" status. The implied taxpayer funded protection allows systemically important financial institutions to take on additional risk without having to fully pay for the losses, essentially avoiding market discipline.

The alternative choice, systemic financial collapse, would probably be worse. One typical example is the sudden failure of Lehman Brothers' Holdings, Inc. which is widely viewed as a watershed moment in the 2008 financial crisis. Its insolvency resulted in more than 75 separate and distinct bankruptcy proceedings all over the world (PricewaterhouseCoopers LLP [7]). The aftershocks were swift and far-reaching – the stock market plunged, credit stopped flowing and unemployment surged. One month after its crash, the International Monetary Fund warned that "The world economy is entering a major downturn in the face of the most dangerous financial shock in mature financial markets since the 1930s."

The disadvantages of the two potential results stimulate the initiative of a bail-in financial resolution which can curb the excessive risk-taking activities of too-big-to-fail financial institutions and help reduce the probability of a systemic financial collapse. In addition, a bail-in financial resolution does not make taxpayers in the first place to foot the bill for big financial institutions. One widely discussed security is called contingent capital bond (CCB)[1]. A contingent capital bond is a subordinated security, such as subordinated debt or preferred shares,

[1]CCBs are often referred to as CoCos, where CoCo stands for contingent convertible.

that converts to common shares when a certain predetermined trigger is breached (D'Souza and Gravelle [5]). If the trigger is not breached, the CCB still performs as a traditional bond. This hybrid characteristic enables CCBs to provide an instant capital infusion by converting debt liability into common shares under financial distress, and to shield investors from tax charges under normal financial circumstances. With contingent capital, the resolution for distressed financial institutions starts from within using private capital, not public money (Calello and Ervin [2]). As a result many authors, such as Flannery [6], believe that CCB can help eliminate managers' excessive risk-taking activities (moral hazard) by the potential dilution effect after conversion. In addition, according to Dickson [4], a key aspect of the proposal of contingent capital is that governments would not guarantee any bank or provide emergency capital unless conversion of contingent capital had taken place so that penalties would be carried out appropriately. In other words, the issuance of CCB would have the potential to address market discipline. Contingent capital has been considered as a promising regulatory tool and required in regulations. For example, the Basel Committee on Banking Supervision (2011)[2] requires that the terms and conditions of all non-common Tier 1 and Tier 2 instruments issued by an internationally active bank must have a provision that requires such instruments to either be written off or converted into common equity upon the occurrence of the trigger event. Figure 1.1 presents the revolution of financial resolution after the financial crisis and relevant problems and benefits. In Figure 1.2 we show the working mechanism of contingent capital.

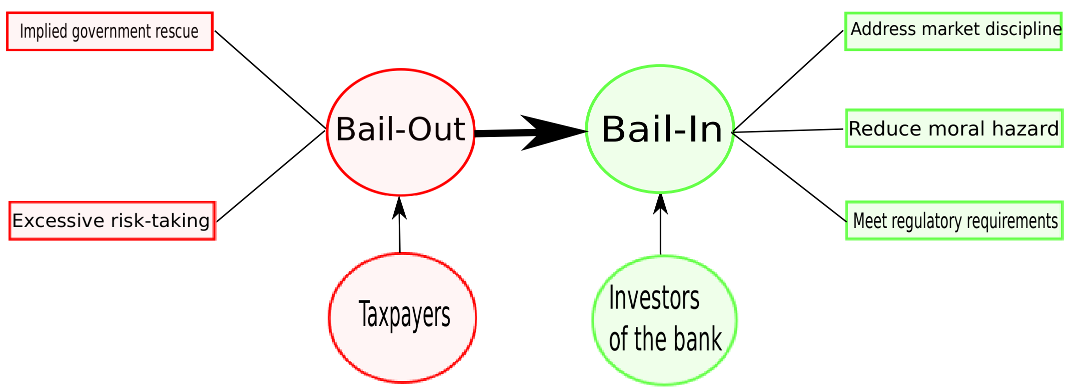

Figure 1.1: Financial resolution: From Bail-out to bail-in.

1.2 Features of Contingent Capital

According to Figure 1.2, there are two main design features of contingent capital bond – the conversion trigger (when to convert) and conversion terms (how to convert). In Figure 1.3, Avdjiev, Kartasheva and Bogdanova [1] categorize the features of contingent capital and our following discussion will be based on the graph.

A wide variety of conversion triggers have been suggested in the literature, and most can be classified as either market-based or accounting-based. One natural choice for the book-value conversion trigger is a capital-ratio trigger because it gives regulators and investors a

[2]http://www.bis.org/press/p110113.htm

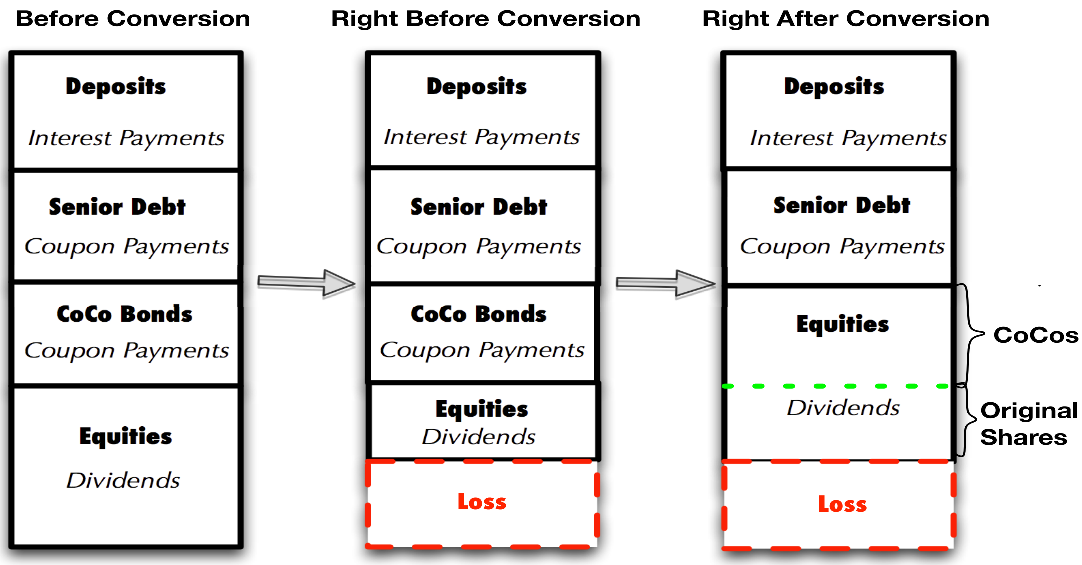

Figure 1.2: Working mechanism of contingent capital.

direct sense of the bank's leverage, profitability, liquidity and solvency and is open to the public in financial statements. Capital ratio triggers are studied and used in the quantitative research of contingent capital, such as Glasserman and Nouri [8], Pennacchi [13] and Metzler and Reesor [12], etc. Book values are only updated periodically and this can lead to a lag between financial statements and a firm's true financial health. To alleviate this issue, as well as potential incentives for managers to manipulate financial statements, several authors have suggested conversion triggers based on market variables. For example, Flannery [6] and Sundaresan and Wang [15] claim that the trigger should be related to the contemporaneous market stock price because triggers based on market values are forward-looking and quickly reflect changes in a firm's condition. In addition to using a single trigger, dual trigger is also discussed, such as in Pennacchi [13], Squam Lake[14] and McDonald [11]. With a dual trigger, conversion occurs when two conditions are met – one based on individual firm and the other based on industry as a whole. The idea behind the dual trigger is to clear deadwood if the industry is not in trouble.

In practice, banks appear to prefer conversion triggers based on book value considering potential manipulation from equity market (Basel Committee on Banking Supervision [9]). The book-value based conversion triggers include Tier I capital ratio, common equity Tier I capital ratio (CET 1), etc. Another trigger suggested in practice is the discretionary trigger, which incorporates the authority's supervision and judgement in the conversion decision in order to mitigate the possible lag-behind of a book-value trigger or avoid an unnecessary conversion under a market-value trigger. Although a discretionary trigger is increasingly used in contingent capital issuances over the past couple of years, such as in Credit Suisse (2011 and 2012), UBS (2012), Royal Bank of Canada (2014), Bank of Montreal (2014) and Canadian Imperial Bank of Commerce (2014), etc., to the best of our knowledge, there has been no rigorous attempt to model discretionary trigger.

Conversion terms (also known as loss absorption mechanisms) determine the value investors receive from conversion. In our thesis, we only discuss the case of conversion to equity. An important variable is the stipulated conversion price (also known as conversion rate) since it determines the number of shares that investors receive for a corresponding notional amount of bonds following conversion (Zahres [16]). One choice is a fixed conversion price. If the conversion price is fixed then CCB investors know exactly the number of shares they will receive at conversion, and equity investors know exactly the dilution effect to their claim. The fixed conversion price is used in typical issuance of contingent capital bonds such as by Lloyds Banking Group in 2009. The other choice is a market-based conversion price. A market-based conversion price relates the number of shares received at conversion to the contemporary or the recent preceding market stock price. For example, in 2011, Credit Suisse Group issued 2 billion dollars of contingent capital bonds, setting conversion price as the volume weighted average stock price for a preceding time period with a floor price. Intuitively, the lower the market-based conversion price, the more shares investors will receive at conversion and the more dilution to original shares. Therefore, the market-based conversion price motivates issuing banks to avoid conversion by taking timely corrective actions, since the potential dilution after conversion cuts the ownership for current equity investors. At the present time most of the academic studies in the pricing of contingent capital take the contemporary market stock price as the conversion price by default, examples include Glasserman and Nouri [8], Pennacchi [13] and Chen et al. [3], etc.

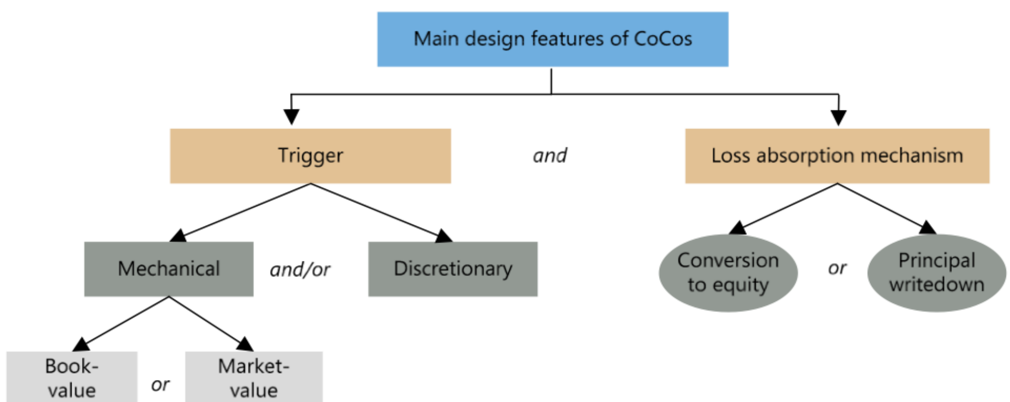

Figure 1.3: Main features of contingent capital. From Avdjiev, Kartasheva and Bogdanova [1].

According to an OSFI advisory [10], there is an explicit principle for the design of contingent capital:

> *Principle No.5*
> *The conversion method should take into account the hierarchy of claims in liquidation and result in the significant dilution of pre-existing common shareholders.*

From the perspective of regulators, the design of conversion term matters because a conversion potentially benefiting contingent capital investors or existing shareholders might violate seniority or tempt manipulations. Taking the fixed conversion price for instance, if the conversion price is too low then the contingent capital investors might take less losses than the senior bondholders at conversion which disobeys the hierarchy of claims in liquidation. However, if the conversion price is too high then equity investors can be effectively rewarded for poor performance, in the sense that they are better off in the presence of contingent capital than its absence. Therefore, there exists a regulatory interval for conversion price within which the principle in OSFI advisory can be followed. Figure 1.4 describes the reasonable interval for conversion price. One of our main contributions in the thesis is the model and estimation of the reasonable regulatory interval for conversion terms.

Figure 1.4: Reasonable region for conversion terms.

It is worth noting that our thesis addresses questions that would be of interest to regulators and issuers of contingent capital, as opposed to traders and/or investors. Most of the discussions are focused on the concerns of regulators and issuers, giving insights in proper and reasonable designs of contingent capital and investigating its fair price and factors influencing its price.

1.3 Structure of the Thesis

This thesis is organized in an integrated article format. We study and discuss three different problems related to contingent capital bonds. In Chapter 2, we focus on the pricing of contingent capital under a structural model extended based on Metzler and Reesor [12] and calibrate the model to Canadian banking data. We investigate the short-selling incentives from investors when the conversion is likely to occur with a market-based conversion price in Chapter 3. A regulatory discretionary trigger is incorporated into the pricing of contingent capital in Chapter 4 using an intensity-model based approach. We summarize the thesis in the last chapter.

Bibliography

[1] S. Avdjiev, A. Kartasheva, and B. Bogdanova. Cocos: a primer. *Working Paper, BIS Quarterly Review*, pages 43–56, September 2013.

[2] P. Calello and W. Ervin. From bail-out to bail-in. *The Economist, Retrieved from http://www.economist.com/node/15392186*, January 28 2010.

[3] N. Chen, P. Glasserman, B. Nouri, and M. Pelger. Cocos, bail-in, and tail risk. *Office of Financial Research Working Paper, Retrieved from SSRN http://dx.doi.org/10.2139/ssrn.2296462*, (4), January 2013.

[4] J. Dickson. Protecting banks is best done by market discipline, April 8 2010.

[5] C. D'Souza and T. Gravelle. Contingent capital and bail-in debt: Tools for bank resolution. *Bank of Canada, Reports for Financial System Review*, December 2011.

[6] M. J. Flannery. Stabilizing large financial institutions with contingent capital certificates. *Working Paper, University of Florida, Retrieved from SSRN http://dx.doi.org/10.2139/ssrn.1485689*, 2009.

[7] PricewaterhouseCoopers' Financial Services Institute (FSI). Lehman brothers' bankruptcy: Lessons learned for the survivors. *Working Paper*, August 2009.

[8] P. Glasserman and B. Nouri. Contingent capital with a capital-ratio trigger. *Management Science*, 58(10):1816–1833, April 2012.

[9] Contingent Capital Working Group. Contingent and bail-in capital: review of market developments. *Working paper, Basel Committee on Banking Supervision*, June 8 2012.

[10] Office of Superintendent of Financial Institutions. Advisory on non-viability contingent capital. *OSFI Reports*, August 2011.

[11] R. L. McDonald. Contingent capital with a dual price trigger. *Journal of Financial Stability*, 9(2):230–241, June 2013.

[12] A. Metzler and R. M. Reesor. Valuation and analysis of zero-coupon contingent capital bonds. *Mathematics and Financial Economics*, 9(2):85–109, March 2015.

[13] G. Pennacchi. A structural model of contingent bank capital. *Working Paper, the Federal Reserve Bank of Cleveland*, (10-04), March 2011.

[14] Squam Lake Working Group on Financial Regulation. An expedited resolution mechanism for distressed financial firms: Regulatory hybrid securities. *Council on Foreign Relations Press*, April 2009.

[15] S. Sundaresan and Z. Wang. On the design of contingent capital with a market trigger. *The Journal of Finance*, 70(2):881–920, April 2015.

[16] M. Zähres. Contingent convertibles, banks bonds take a new look. *Working Series Paper, Deutsche Bank Research*, May 2011.

Chapter 2

Pricing of Contingent Capital Bonds under a Structural Model

2.1 Introduction

The pricing of contingent capital bonds (CCB) is widely studied in terms of structural models. Employing structural models to price CCBs can successfully capture the change of the firm's capital structure after the conversion of contingent capital and the conversion trigger can be modelled based on some firm value related capital ratio. All relevant quantities (equity value, bond value and conversion trigger, etc.) are functions of a single state variable – the value of the firm's assets in the structural model[1].

Various structural models have been used for pricing CCBs. Albul, Jaffee, and Tchitsyi [1] provide a model based on Leland [16] and develop a closed-form solution for the value of CCBs with market-based conversion triggers and fixed imposed losses[2] at conversion. They show that CCBs can reduce the chance of costly bankruptcy or bailout and increase the value of the issuing firm if properly implemented. Pennacchi [25] incorporates possible sudden and discrete asset value declines during financial crises into a structural model by considering an individual bank whose asset value follows a jump-diffusion process. With a practical capital structure composed of short-term deposits, long-term bonds and equity, the author studies the pricing of fixed-coupon contingent capital under various contractual terms (e.g., conversion terms, conversion trigger and maturity) and bank-related risks (e.g. asset jump risks and excessive risk-taking incentives). The featured conclusions include the asset jump risk would obviously increase the credit spreads for contingent capital, a relatively high level of equity value based conversion trigger would protect contingent capital investors from losses of a sudden asset value drop and the issuance of contingent capital is likely to mitigate the moral hazard. Glasserman and Nouri [11] establish a structural model on a broader basis including Merton [20], Black and Cox [6] and Leland [16], and derive an explicit expression for the valuation of CCB. Their research is distinguished by using the capital ratio as conversion trigger and the consideration of gradual and on-going conversion scheme. Chen et al. [7] consider a

[1]From this point of view, using a structural model to price contingent capital bond is a natural choice and would be a good start point for potential further investigations of the pricing problem using other models.

[2]The conversion term of fixed imposed loss will be discussed later.

jump-diffusion model with firm-specific and market-specific jumps of the firm's asset value. They derive closed-form expressions to value the firm and its liabilities with the incorporation of debt rollover and endogenous default. Moreover, they present detailed discussions of potential issues such as debt overhang, the increment of the firm's loss absorbency and the effects of reducing the bankruptcy costs with the issuance of CCBs.

However, because of model and parameter assumptions, not many structural models could work when we incorporate fixed coupon payments, the regulatory requirements in Basel III and real capital structures of financial institutions into calibration at the same time. For example, Metzler and Reesor [21], Madan and Schoutens [18] and Raviv and Hilscher [12] price CCBs as zero-coupon bonds and therefore cannot deal with the coupon issue. For the studies considering CCBs with coupon payments, almost all of them implicitly assume the coupon payments are proportional to the firm's asset value and therefore ignore the impact of debt service on the asset value dynamics. For instance, Glasserman and Nour i[11] and Albul, Jaffee and Tchitsyi [1] assume the firm's value follows the geometric Brownian motion and incorporate the fixed coupon payments into the drift term as a payout ratio proportional to the firm value. This implicitly assumes the issuance of new equity to pay for the coupons when the firm is in financial distress and ignores the fact that it is difficult for a weakened firm to issue new equity[3]. Despite its impressive level of details, the model developed by Pennacchi [25] seems a bit too unwieldy and contains several parameters that appear difficult to calibrate. Most of the other models deal with endogenously determined default, which is not ideal when the goal is pragmatic calibration.

Metzler and Reesor [21] present a novel approach to the valuation of zero-coupon CCBs in the structural framework. They studied the loss absorbency mechanism of contingent capital at conversion and find that conversion terms (specific rules governing the exchange of debt for equity upon conversion) can fundamentally alter the nature of the CCB. Their analysis is mainly from the regulators' perspective and therefore provides preliminary guidance for the design of conversion terms. For example, they find that under the fixed conversion price, there exists a narrow "regulatory interval " within which the fixed conversion price should be located in order to respect the seniority between the senior bond and the CCB in the capital structure, as well as guarantee the existing shareholders are not rewarded at conversion.

In this chapter, we extend the model proposed by Metzler and Reesor [21] to price contingent capital with fixed coupon payments. Once conversion occurs, there will be no coupon payments to investors so the CCB with coupon payments cannot be replicated with a portfolio of zero-coupon CCBs – it means that the zero-coupon CCB model put forward by Metzler and Reesor [21] cannot be directly applied to price the CCB with coupon payments. In the chapter, we use the general asset value dynamics put forward by Merton [20], considering both dividend and coupon payments, with the dividend payments proportional to the asset value but the coupon payments proportional to the notional value of the firm's liabilities. To the best of our knowledge, we are the first to consider the pricing of CCB using this affine geometric Brownian motion by assuming that the coupon payments would have impact on the behaviour of the firm's asset value. We use the model-implied par yields to measure the costs of CCBs and the other liabilities in the firm's capital structure. Additionally, the change of the CCB

[3]The difficulty lies in two incentives against the recapitalization of a distressed firm – debt overhang and adverse selection (Duffie [9])

value is investigated with respect to the change of different firm-specific parameters. Following the suggestions of Financial Stability Board and European Commission [24], we extend our model by allowing part of the senior bond to be converted as well in a trigger event to realize a burden-sharing resolution regime. We manage to calibrate the model using capital structures of representative Canadian financial institutions and the Basel III capital ratio requirements. This calibration is a critical first step in determining realistic estimates of the costs of a particular Canadian institution to issue contingent capital as well as preliminary guidance on setting important design features such as conversion prices and triggers.

For the reader's convenience, we list some of our main findings and conclusions here.

- CCB could either cost more or less than otherwise identical (i.e. non-convertible) junior bond depending on the conversion term and trigger of CCB. Under our calibration, replacing the traditional junior bond with the CCB in the capital structure, the spread of the contingent capital would be greater than that of the otherwise identical junior bond it would replace, whereas the spread on senior bond would be slightly reduced.

- Under fixed conversion price, the costs of contingent capital are sensitive to the trigger location since the equity value received by CCB investors is positively related with the trigger location. However, under fixed imposed loss, the costs of contingent capital are not as sensitive as under fixed conversion price to the trigger location. This is because the redemption to CCB investors upon conversion is fixed as the difference between the notional value and the write-down value upon conversion.

- Under fixed conversion price (fixed imposed loss), the credit spread of contingent capital is positively related to the conversion price (the fixed write-down at conversion). The firm's asset volatility influences the spread significantly. It would be costly for a firm with volatile asset value to issue contingent capital due to the possible early conversion.

- There exist reasonable intervals for conversion terms within which the claim seniority is respected and the existing shareholders will not be rewarded for the firm's poor performance at conversion. The size of the interval or region is remarkably narrow and becomes even smaller as the conversion trigger moves upwards. Therefore, the issuing firm must be careful if it uses a going-concern (relatively high) conversion trigger since there would be a relatively small room from which the firm can choose a proper conversion term.

This paper is organized as follows. Section 2.2 and 2.3 give a brief review and our extension of the structural model put forward by Metzler and Reesor [21]. In Section 2.4, we show the details of our calibration. Section 2.5 demonstrates our numerical results, and the term structure under the finite time horizon. Finally, we summarize and discuss our future work in Section 2.6.

2.2 Non-Contingent Debt Model

Assuming a probability space $(\Omega, \mathcal{F}, \mathbb{Q})$, where $\mathbb{Q}$ is a risk-neutral pricing measure and let $\mathbb{E}$ denote the expectation with respect to $\mathbb{Q}$. Interest rates are taken to be deterministic and independent of both time and tenor, so that the risk-free interest rate term structure is flat at the constant level $r > 0$. We assume that the firm's liabilities have a common maturity of $T > 0$ years.

Metzler and Reesor [21] begin with a firm having a traditional capital structure consisting of a senior bond with notional value L_S, a junior bond with notional value L_J and common equity. In order to more accurately reflect the balance sheets of Canadian banks, one must also include deposits, modelled here as a "super-senior" bond with notional value L_D (interpreted as the total amount on deposit) having a recovery rate of 100% (this reflect the fact that the majority of deposits in Canada are federally insured). We call the capital structure including deposits, the senior bond, the junior bond and common equity as *traditional capital structure*.

We assume that bond i pays a continuous coupon at a constant rate c_i^{trad} per dollar notional value so that the dollar payout rate is $c_i^{trad} L_i$ for $i = D, S, J$. We assume that equity receives dividends continuously at a constant rate $q > 0$ proportional to the firm value V_t so that the dollar payout rate is qV_t at time t. Similar to Merton [20], under the risk-neutral measure, we assume the firm value evolves as affine geometric Brownian motion[4],

$$dV_t^{trad} = \left[(r-q)V_t^{trad} - (c_D^{trad} L_D + c_S^{trad} L_S + c_J^{trad} L_J)\right] dt + \sigma V_t^{trad} dW_t, \tag{2.1}$$

where σ is the volatility of the firm value. Both r and σ are assumed positive constant. It is worth noticing that the coupon payments are not proportional to the asset value in (2.1), implying that the firm cannot necessarily reduce its debt costs as losses mount. In addition we preclude negative dividends, which means that the firm cannot issue new equity to service its debt when it is in distress. Mathematically, the main difference between using a constant coupon payments in the drift term as (2.1) and a proportional coupon payments with respect to the asset value such as in Glasserman and Nouri [11] is the attainability of the origin. When the drift is $(r-q)V_t^{trad}$ as in Glasserman and Nouri [11], the origin is never attainable in finite time, whereas in our case the origin is attainable in finite time for any initial asset value. Moreover, we prove that if $r < q + \sigma^2/2$ then all trajectories are eventually absorbed at the origin in finite time, and if $r \geq q + \sigma^2/2$ then the eventual absorption at the origin occurs with a positive probability. See Theorem B.1.4 in Appendix B for details. Similar firm value dynamics is mentioned in Merton [20] and Leland and Toft [17], etc.

Define the total liability $L = L_D + L_S + L_J$. It is convenient to rewrite the dynamics in a non-dimensional form,

$$\frac{dV_t^{trad}}{L} = \left[(r-q)\bar{v}_t^{trad} - (c_D^{trad} \ell_D + c_S^{trad} \ell_S + c_J^{trad} \ell_J)\right] dt + \sigma \bar{v}_t^{trad} dW_t, \tag{2.2}$$

where $\bar{v}_t^{trad} = V_t^{trad}/L$ is the firm's asset-liability ratio at time t and $\ell_i = L_i/L$ for $i = D, S, J$ is the proportion of each bond in the total liability. It is possible to allow the firm to issue new debt as asset value grows but it would be difficult to model and calibrate such a policy.

As in Glasserman and Nouri [11], we assume that the firm is seized by regulators and therefore ceases operation if the asset-liability ratio falls below the level d^*, where d^* is a predefined level at which regulators deem the bank non-viable. Here we assume the ratio is specified in

[4] We only consider one single state variable driving the evolution of the firm's asset value in the thesis. In practice, the firm's asset value could be influenced by other variables such as the market condition and the change of the firm's liability structure, etc.

regulations such as in Basel III and therefore is known to all investors. Liquidation/seizure therefore occurs at the hitting time,

$$\tau_d^{trad} = \inf\{t \in [0,T] : \bar{v}_t^{trad} \leq d^*\}. \tag{2.3}$$

Observe that $\tau_d^{trad} = \infty$ if asset value never goes below the liquidation threshold. In contrast to Metzler and Reesor [21], we allow for non-zero bankruptcy costs and therefore the violation of strict priority upon seizure. In practice, absolute priority violations do take place (Lando [15]) and there are cases where debt payments are reduced but equity is still "alive" after restructuring and has a positive value (the model of Anderson and Sundaresan [2] captures this phenomenon). Denote constant $\mathcal{R}_D$, $\mathcal{R}_S$ and $\mathcal{R}_J$ as the recovery rate for deposits, the senior, and junior bond respectively. Total bankruptcy cost at liquidation is given by $BC_{\tau_d^{trad}}^{trad} = L - (\mathcal{R}_D L_D + \mathcal{R}_S L_S + \mathcal{R}_J L_J)$. We will calibrate recovery rates in order to match empirically-observed spreads collected in Beyhaghi, D'Souza and Roberts [5] for Canadian banks[5]. The ℓ_i's are obtained from balance sheets. The point of liquidation will be chosen according to the acceptable capital level in Basel III and bankruptcy costs will therefore be implied.

Valuations under Traditional Capital Structure

Bond holders earn their coupons until the random time

$$\bar{\tau}_d^{trad} = \min\{\tau_d^{trad}, T\}, \tag{2.4}$$

at which point they receive either their face value L_i in the event that the firm is not liquidated or $\mathcal{R}_i L_i$ in the event that the firm is liquidated prior to T. Assuming the level of asset value at which the firm is seized by regulators locates above the total notional value of the firm's liability, we can write the principal repayment $\overline{L}_i = L_i[\mathbb{1}_{\{\tau_d^{trad} > T\}} + \mathcal{R}_i \mathbb{1}_{\{\tau_d^{trad} \leq T\}}]$ for $i = D, S, J$.

For simplicity, let $\mathbb{E}[e^{-r(\bar{\tau}_d^{trad} - t)} | V_t^{trad} = v^{trad}] = \mathbb{E}_{t,v^{trad}}[e^{-r(\bar{\tau}_d^{trad} - t)}]$ under the risk-neutral measure. The value of bond i is the expectation of the discounted cash flows to the bond holder

$$\begin{aligned} i_t^{trad} &= \mathbb{E}_{t,v^{trad}}\Big[\int_t^{\bar{\tau}_d^{trad}} c_i^{trad} L_i e^{-r(s-t)} ds + \overline{L}_i e^{-r(\bar{\tau}_d^{trad} - t)}\Big], \\ &= \frac{c_i^{trad} L_i}{r}\Big(1 - \mathbb{E}_{t,v^{trad}}[e^{-r(\bar{\tau}_d^{trad} - t)}]\Big) + L_i \mathbb{E}_{t,v^{trad}}[e^{-r(T-t)} \mathbb{1}_{\{\tau_d^{trad} > T\}}] + \mathcal{R}_i L_i \mathbb{E}_{t,v^{trad}}[e^{-r(\tau_d^{trad} - t)} \mathbb{1}_{\{\tau_d^{trad} \leq T\}}], \end{aligned} \tag{2.5}$$

for $i = D, S, J$, where D_t^{trad}, S_t^{trad} and J_t^{trad} are the value of deposits, the senior bond and junior bond in the traditional capital structure at time t respectively. Given the firm has yet to be liquidated at time t, it follows that the equity value is

$$E_t^{trad} = \mathbb{E}_{t,v^{trad}}\Big[\int_t^{\bar{\tau}_d^{trad}} q V_s^{trad} e^{-r(s-t)} ds + L \max\{\bar{v}_{\bar{\tau}^{trad}}^{trad} - \ell_D - \ell_S - \ell_J, 0\} e^{-r(\bar{\tau}_d^{trad} - t)}\Big]. \tag{2.6}$$

[5]There is no historical data to use since no large Canadian bank has ever defaulted. However, since the recovery rates used in the model is exogenously given and fixed, it is possible to use recovery rates from other countries, such as the United States.

Since $e^{-rt}V_t^{trad}+\int_0^t e^{-rs}(qV_s^{trad}+c_D^{trad}L_D+c_S^{trad}L_S+c_J^{trad}L_J)ds$ is a martingale (see Appendix B.1.2 for details), the equity value follows that

$$E_t^{trad} = V_t^{trad} - D_t^{trad} - S_t^{trad} - J_t^{trad} - BC_t^{trad}, \quad t \leq \bar{\tau}_d^{trad}, \tag{2.7}$$

where $BC_t^{trad} = \mathbb{E}_{t,v^{trad}}[BC_{\tau_d^{trad}}^{trad} e^{-r(\tau_d^{trad}-t)} \mathbb{1}_{\{\tau_d^{trad} \leq T\}}]$ is the expected total bankruptcy cost at time t for the firm.

Par Yields of Traditional Bonds

Par yields are those coupon rates that simultaneously solve the following equations for $i = D, S, J$,

$$\frac{c_i^{trad} L_i}{r}\left(1 - \mathbb{E}_{0,v^{trad}}[e^{-r\bar{\tau}_d^{trad}}]\right) + L_i \mathbb{E}_{0,v^{trad}}[e^{-rT}\mathbb{1}_{\{\tau_d^{trad} > T\}}] + \mathcal{R}_i L_i \mathbb{E}_{0,v^{trad}}[e^{-r\tau_d^{trad}}\mathbb{1}_{\{\tau_d^{trad} \leq T\}}] = L_i, \tag{2.8}$$

or equivalently, after normalization, simultaneously solve the following equations,

$$\frac{c_i^{trad}}{r}\left(1 - \mathbb{E}_{0,v^{trad}}[e^{-r\bar{\tau}_d^{trad}}]\right) + \mathbb{E}_{0,v^{trad}}[e^{-rT}\mathbb{1}_{\{\tau_d^{trad} > T\}}] + \mathcal{R}_i \mathbb{E}_{0,v^{trad}}[e^{-r\tau_d^{trad}}\mathbb{1}_{\{\tau_d^{trad} \leq T\}}] = 1, \quad i = D, S, J. \tag{2.9}$$

The expectations on the left-hand side depend on the dynamics of asset value, and therefore depend on coupon rates of bonds in the capital structure, in general the equation system implicit in (2.8) or (2.9) must be solved numerically. Particularly, in the perpetual case (i.e. $T = \infty$) we have $\bar{\tau}_d^{trad} = \tau_d^{trad}$, and the par yield of the bond i is solved from

$$\frac{c_i^{trad}}{r}\left(1 - \mathbb{E}_{0,v^{trad}}[e^{-r\tau_d^{trad}}]\right) + \mathcal{R}_i \mathbb{E}_{0,v^{trad}}[e^{-r\tau_d^{trad}}] = 1, \quad i = D, S, J. \tag{2.10}$$

By Proposition 2.2.1, the expectations in (2.10) can be calculated with an explicit formula. As a result, we can solve for the par yield and therefore the credit spread of each bond in the capital structure under the perpetual case.

Proposition 2.2.1 *Let $c^{trad} = \ell_D c_D^{trad} + \ell_S c_S^{trad} + \ell_J c_J^{trad}$ be the non-dimensional coupon payment. In the perpetual case, for $\bar{v}_t^{trad} \geq d^*$ and $t < \tau_d^{trad}$,*

$$\mathbb{E}_{t,v^{trad}}[e^{-r(\tau_d^{trad}-t)}] = u(\bar{v}_t^{trad}, d^*, r-q, c^{trad}, \sigma), \tag{2.11}$$

where for positive numbers r, q and c^{trad}, the function u is defined as

$$u(\bar{v}_t^{trad}, d^*, r-q, c^{trad}, \sigma) = \left(\frac{d^*}{\bar{v}_t^{trad}}\right)^{\lambda_+} \frac{M\left(\lambda_+, 2(\lambda_+ + 1) - 2(r-q)/\sigma^2, -2c^{trad}/\sigma^2\bar{v}_t^{trad}\right)}{M\left(\lambda_+, 2(\lambda_+ + 1) - 2(r-q)/\sigma^2, -2c^{trad}/\sigma^2 d^*\right)}, \tag{2.12}$$

where λ_+ is the unique positive solution to $\lambda^2 + \left(1 - 2(r-q)/\sigma^2\right)\lambda - 2r/\sigma^2 = 0$ and the function M is the confluent hypergeometric function.

Proof According to Theorem B.1.6 in Appendix B, this proposition can be proved by taking $x = \bar{v}_t^{trad}$, $y = d^*$, $\alpha = r$ and $\mu = r - q$.

In the case with finite maturity, there is no closed form formula for the expectation of discount factors. However, we can solve for the par yields using Monte Carlo simulation.

2.3 Incorporating Contingent Capital

Now we consider a firm which *replaces the junior bond with a CCB* in the capital structure. The CCB investor initially possesses a subordinated bond paying a coupon rate c_{CCB} on the notional value L_J. If conversion never occurs, the CCB pays its coupon until maturity and the investor receives the principal payment at maturity. If conversion does occur, the CCB investor will return the bond to the firm (who retires it) in exchange for newly issued common shares. To be consistent with the recent issuance of CCBs, we only consider a full conversion of CCB. That is, post conversion the CCB investor only holds common shares (i.e. the bond is fully written off).

In order to incorporate bail-in debt (BID), which is also considered as contingent capital but has a higher seniority than CCB (subordinated debt), we allow for a fraction of the senior bond to be retired in exchange for common equity via conversion. Suppose that the notional of the senior bond is reduced to $(1 - f_S)L_S$ post conversion, where $f_S \in [0, 1]$ is called the conversion proportion, and the coupon rate c_S is applied to the reduced principal. In exchange for the reduction in principal, the senior bond investor receives a fraction of the firm's common equity, so post conversion the senior bond investor will effectively hold two securities - a fraction of common equity and the senior bond with a reduced notional value $(1 - f_S)L_S$. We assume that once the conversion is triggered, CCB and $100f_S\%$ of the senior bond convert at the same time.

In keeping with Metzler and Reesor [21], whose development we follow closely, let N denote the number of common shares in existence today and N^*_{CCB} (N^*_S) the number of shares issued from the conversion of CCB (partial senior bond). Therefore, the CCB (senior bond) investor owns $100\omega_{CCB}\%$ ($100\omega_S\%$) of the firm upon conversion, where $\omega_{CCB} = N^*_{CCB}/(N + N^*_{CCB} + N^*_S)$ ($\omega_S = N^*_S/(N + N^*_{CCB} + N^*_S)$). We call ω_{CCB} (ω_S) the ownership stake distributed to CCB (senior bond) investors and it can be understood as a dilution factor to the existing shareholders.

We use V_t as the firm's asset value at time t with the incorporation of contingent capital and continue using the asset-liability ratio $\bar{v}_t = V_t/L$. We assume that conversion occurs when the firm's asset-liability ratio falls below the predefined critical level b^* for the first time,

$$\tau_c = \inf\{t > 0 : \bar{v}_t \leq b^*\}. \tag{2.13}$$

The choice of the conversion barrier (trigger) will be discussed later. The liquidation occurs when

$$\tau_d = \inf\{t > 0 : \bar{v}_t \leq d^*\}. \tag{2.14}$$

Assuming the critical level $b^* > d^*$, we have conversion always happens before liquidation.

With the incorporation of contingent capital into the capital structure, prior to conversion, the asset value dynamics are

$$dV_t = \left[(r - q)V_t - (c_D L_D + c_S L_S + c_{CCB} L_J)\right] dt + \sigma V_t dW_t, \quad 0 \leq t < \tau_c, \tag{2.15}$$

while post conversion, both CCB and $100f_S\%$ of senior bond convert to common shares, so the asset value dynamics become

$$dV_t = \left[(r - q)V_t - (c_D L_D + c_S(1 - f_S)L_S)\right] dt + \sigma V_t dW_t, \quad t \geq \tau_c. \tag{2.16}$$

We continue to use the simplified assumption, necessary for tractability, that all liabilities have the same maturity T (with $T = \infty$ for the perpetual case). In other words, the firm will cease operation at T if no liquidation happens; however, if the asset value falls below the pre-defined liquidation level prior to maturity, the firm ceases to operate at τ_d. This assumption conflicts with the real world because a firm's bonds usually have different maturities. However, since we are more concerned about the difference contingent capital would make under financial distress (i.e., large default/liquidation risk), identical maturity assumption facilitates us to investigate this, ignoring risk exposure brought by different maturities. Also, we can think of the same maturity realized by the debt rollover before the firm is liquidated. Additionally, the involvement of different maturities would increase the complexity of our model. For example, the capital structure becomes non-stationary and the default level changes as the debt in the bank's capital structure expires.

Valuation of Bonds After Incorporating Contingent Capital

Since we only consider the conversion of CCB and partial senior bond, the valuation of deposits remains the same with that under the traditional capital structure. For the senior bond, its value is the summation of two parts – the part remaining as liability and the part converted to common shares. Prior to conversion and maturity, the present value of the first part is

$$\int_t^{\bar{\tau}_d} c_S(1-f_S)L_S e^{-r(s-t)}ds + (1-f_S)L_S e^{-r(T-t)}\mathbb{1}_{\{\tau_d>T\}} + \mathcal{R}_S(1-f_S)L_S e^{-r(\tau_d-t)}\mathbb{1}_{\{\tau_d\le T\}}, \quad (2.17)$$

and the present value of the second part is

$$\int_t^{\bar{\tau}_c} c_S f_S L_S e^{-r(s-t)}ds + \omega_S E_{\tau_c} e^{-r(\tau_c-t)}\mathbb{1}_{\{\tau_c\le T\}} + f_S L_S e^{-r(T-t)}\mathbb{1}_{\{\tau_c>T\}}, \quad (2.18)$$

where $\bar{\tau}_c = \min\{\tau_c, T\}$ and E_{τ_c} is the equity value of the firm right after conversion. Discounting the cash flows to CCB investors gives the present value of CCB at time $t < \bar{\tau}_c$,

$$\int_t^{\bar{\tau}_c} c_{CCB}L_J e^{-r(s-t)}ds + \omega_{CCB}E_{\tau_c} e^{-r(\tau_c-t)}\mathbb{1}_{\{\tau_c\le T\}} + L_J e^{-r(T-t)}\mathbb{1}_{\{\tau_c>T\}}. \quad (2.19)$$

Taking expectation on (2.17), (2.18) and (2.19), we obtain the value of two parts of the senior bond – the part remaining as straight bond (S_t^B) and the part converted to common shares (S_t^C), and the value of CCB at $t < \bar{\tau}_c$,

$$\begin{aligned}
S_t^B =& \frac{c_S(1-f_S)L_S}{r}\left(1 - \mathbb{E}_{t,v}[e^{-r(\bar{\tau}_d-t)}]\right) + (1-f_S)L_S\,\mathbb{E}_{t,v}[e^{-r(T-t)}\mathbb{1}_{\{\tau_d>T\}}],\\
&+ \mathcal{R}_S(1-f_S)L_S\,\mathbb{E}_{t,v}[e^{-r(\tau_d-t)}\mathbb{1}_{\{\tau_d\le T\}}], \qquad (2.20)\\
S_t^C =& \frac{c_S f_S L_S}{r}\left(1 - \mathbb{E}_{t,v}[e^{-r(\tau_c-t)}\mathbb{1}_{\{\tau_c\le T\}}]\right) + \mathbb{E}_{t,v}[\omega_S E_{\tau_c} e^{-r(\tau_c-t)}\mathbb{1}_{\{\tau_c\le T\}}]\\
&+ \frac{c_S f_S L_S}{r}\left(1 - \mathbb{E}_{t,v}[e^{-r(T-t)}\mathbb{1}_{\{\tau_c>T\}}]\right) + f_S L_S\,\mathbb{E}_{t,v}[e^{-r(T-t)}\mathbb{1}_{\{\tau_c>T\}}], \qquad (2.21)\\
CCB_t =& \frac{c_{CCB}L_J}{r}\left(1 - \mathbb{E}_{t,v}[e^{-r(\tau_c-t)}\mathbb{1}_{\{\tau_c\le T\}}]\right) + \mathbb{E}_{t,v}[\omega_{CCB}E_{\tau_c} e^{-r(\tau_c-t)}\mathbb{1}_{\{\tau_c\le T\}}]\\
&+ \frac{c_{CCB}L_J}{r}\left(1 - \mathbb{E}_{t,v}[e^{-r(T-t)}\mathbb{1}_{\{\tau_c>T\}}]\right) + L_J\,\mathbb{E}_{t,v}[e^{-r(T-t)}\mathbb{1}_{\{\tau_c>T\}}]. \qquad (2.22)
\end{aligned}$$

The value of the senior bond is therefore given by

$$S_t = S_t^B + S_t^C, \quad t < \overline{\tau}_c. \tag{2.23}$$

In the finite-maturity case with $T < \infty$, we must use the Monte Carlo simulation to estimate the expectations of discount factors in the formulas. Unfortunately, Monte Carlo method has relatively high computational costs. In the perpetual case with $T = \infty$, by Proposition 2.2.1, it is clear that

$$\mathbb{E}_{t,v}[e^{-r(\tau_c-t)}] = u(\bar{v}_t, b^*, r-q, c, \sigma), \tag{2.24}$$

where $c = c_D\ell_D + c_S\ell_S + c_{CCB}\ell_J$. If $\tau_c = \infty$ then conversion does not occur and contingent capital investors receive their coupon streams in perpetuity, never becoming part owners of the firm. Being consistent with the discussion under the traditional capital structure, we continue to assume that the firm is liquidated once its asset-liability ratio falls below the threshold d^*. However, it is noteworthy that post conversion, the firm's liabilities become $L_D + (1 - f_S)L_S$. Correspondingly, the bankruptcy cost becomes $BC_{\tau_d} = L_D + (1-f_S)L_S - (\mathcal{R}_D L_D + \mathcal{R}_S(1-f_S)L_S)$. Thus, following the same logic of (2.7), the equity value right after conversion is $E_{\tau_c} = b^*L - D_{\tau_c} - S_{\tau_c}^B - BC_{\tau_c}$, where $BC_{\tau_c} = \mathbb{E}_{\tau_c,b^*L}[BC_{\tau_d}e^{-r(\tau_d-\tau_c)}\mathbb{1}_{\{\tau_d \le T\}}]$.

Since we assume $b^* > d^*$, by the strong Markov property we know that the times $\tau_d - \tau_c$ and τ_c are independent on the event $\{\tau_c < \infty\}$. Therefore,

$$\mathbb{E}_{\tau_c,b^*L}[e^{-r(\tau_d-\tau_c)}] = u(b^*, d^*, r-q, c', \sigma), \tag{2.25}$$

where $c' = c_D\ell_D + c_S(1-f_S)\ell_S$ is the non-dimensional coupon payment after conversion. We define $\tau_d - \tau_c = 0$ on the event $\tau_c = \infty$.

Proposition 2.3.1 *In the perpetual case, for $\bar{v}_t \in [b^*, +\infty)$ and $b^* > d^*$, we have*

$$\mathbb{E}_{t,v}[e^{-r(\tau_d-t)}] = \mathbb{E}_{t,v}[e^{-r(\tau_c-t)}]\mathbb{E}_{\tau_c,b^*L}[e^{-r(\tau_d-\tau_c)}], \tag{2.26}$$

where $\mathbb{E}_{t,v}[e^{-r(\tau_c-t)}]$ and $\mathbb{E}_{\tau_c,b^*L}[e^{-r(\tau_d-\tau_c)}]$ are given by (2.24) and (2.25) respectively.

Proof According to Lemma B.1.7 in Appendix B, the proposition is proved.

Since conversion always happens prior to liquidation, if $\tau_c < \infty$, then $\tau_c < \tau_d$; if $\tau_c = \infty$, then $\tau_d = \infty$. In the perpetual case and $\bar{v}_t \ge b^* > d^*$, the valuation of deposits, the senior bond and the CCB in the capital structure become

$$D_t = \frac{c_D L_D}{r}\left(1 - \mathbb{E}_{t,v}[e^{-r(\tau_d-t)}]\right) + \mathcal{R}_D L_D \mathbb{E}_{t,v}[e^{-r(\tau_d-t)}], \tag{2.27}$$

$$\begin{aligned} S_t =& \frac{c_S(1-f_S)L_S}{r}\left(1 - \mathbb{E}_{t,v}[e^{-r(\tau_d-t)}]\right) + \mathcal{R}_S(1-f_S)L_S\mathbb{E}_{t,v}[e^{-r(\tau_d-t)}] \\ &+ \frac{c_S f_S L_S}{r}\left(1 - \mathbb{E}_{t,v}[e^{-r(\tau_c-t)}]\right) + \mathbb{E}_{t,v}[\omega_S E_{\tau_c} e^{-r(\tau_c-t)}], \end{aligned} \tag{2.28}$$

$$CCB_t = \frac{c_{CCB}L_J}{r}\left(1 - \mathbb{E}_{t,v}[e^{-r(\tau_c-t)}]\right) + \mathbb{E}_{t,v}[\omega_{CCB}E_{\tau_c}e^{-r(\tau_c-t)}], \tag{2.29}$$

for $t < \tau_c$, respectively.

Par Yields of Bonds After Incorporating Contingent Capital

In the perpetual case, we can solve for par yields by equating the closed-form expressions (2.27), (2.28) and (2.29) with the corresponding notional values. However, in the case of finite maturity ($T < \infty$), there are no closed-form expressions for the expectations of discount factors. One feasible approach to estimate the par yields would be using Monte Carlo simulation.

2.3.1 Conversion Terms

We focus on two kinds of conversion terms in this section – (1) a fixed conversion price and (2) a fixed imposed loss on contingent capital bond at conversion. The former specifies a fixed conversion price and therefore the number of shares received by contingent capital investors. The latter specifies a fixed loss (also known as write-down) to the notional value at conversion while allows the conversion price depending on the stock price.

Before we we detail our discussions on the two conversion terms, we define the effective loss at conversion. In the case that only CCB converts, the value of shares received at conversion is $\omega_{CCB}E_{\tau_c}$, and the face value of bond given up is L_J. Therefore, the effective loss imposed to CCB investors at conversion is

$$\beta_{CCB} = 1 - \frac{\omega_{CCB}E_{\tau_c}}{L_J}. \tag{2.30}$$

Under fixed conversion price, the value of shares received depends on the conversion price and the effective loss is implied by the difference between the fixed conversion price and the stock price at conversion (Metzler and Reesor [21]). In contrast, fixed imposed loss determines the effective loss (left hand side of (2.30)) directly and therefore the value of shares received by CCB investors. The number of shares generated from conversion is determined by conversion price which might depend on the market stock price. Similarly, we define β_S^C as the effective loss imposed to the converted part of the senior bond,

$$\beta_S^C = 1 - \frac{\omega_S E_{\tau_c}}{f_S L_S}, \quad f_S > 0. \tag{2.31}$$

For the special case only considering the conversion of CCBs, we directly assign $\beta_S^C = 0$.

Fixed Conversion Price

As noted by Metzler and Reesor [21], in the mathematical modelling of contingent capital it is most natural to work with ownership stakes (dilution factors) ω_{CCB} and ω_S, whereas the contract usually specifies a rule for determining the conversion price. Define a positive non-dimensional parameter $p_{CCB} = P_{CCB}/P_0$, where P_{CCB} is the fixed conversion price of CCB specified in the contract and P_0 is the market stock price at the issuance assuming all bonds are priced at par. For example, if $p_{CCB} = 0.5$ then the conversion price stated explicitly in the CCB contract is 50% of the firm's market stock price at the issuance of CCB. Similarly, we define a positive non-dimensional parameter p_S as the ratio of the fixed conversion price applied to the (converted part of) senior bond (P_S) over P_0. We suppose that if conversion is triggered, the CCB and $100 f_S\%$ of the senior bond convert to common shares. Using the facts that $N^*_{CCB} = L_J/P_{CCB}$ and $N^*_S = f_S L_S/P_S$, it is straightforward to see that the implied ownership stake afforded to the CCB and senior bond investors at conversion is

$$\omega_{CCB} = \frac{\ell_J}{p_{CCB}(\bar{v}_0 - 1 - BC_0/L) + \ell_J + f_S \ell_S \frac{1}{\bar{p}}}, \tag{2.32}$$

and

$$\omega_S = \frac{f_S \ell_S}{p_S(\bar{v}_0 - 1 - BC_0/L) + \ell_J \bar{p} + f_S \ell_S} = \frac{f_S \ell_S}{\ell_J} \frac{1}{\bar{p}} \omega_{CCB}. \tag{2.33}$$

respectively, where $\bar{p} = p_S/p_{CCB}$ is the ratio of the two conversion prices. If $\bar{p} = 1$ then $\omega_S/f_S\ell_S = \omega_{CCB}/\ell_J$ meaning that the CCB investor and senior bondholder afford the same conversion price and they will receive the same number of shares in the exchange of each unit dollar of notional value upon conversion. However, issuers and regulators might want to impose a higher conversion price to CCB ($\bar{p} < 1$) in order to respect the seniority. Under this circumstance, we have $\omega_S/f_S\ell_S > \omega_{CCB}/\ell_J$, implying that in the exchange of the same amount of notional value, the senior bondholder will receive more shares than the CCB investor. Furthermore, as $p_{CCB} \to 0^+$ ($p_S \to 0^+$), the CCB (senior) investor becomes 100% owners of the firm since $\omega_{CCB} \to 1$ ($\omega_S \to 1$); while as $p_{CCB} \to \infty$ ($p_S \to \infty$), the CCB investor (the converted portion of the senior bond for the senior bondholder) is completely wiped out at conversion in the sense that $\omega_{CCB} \to 0$ ($\omega_S \to 0$) and the bond is worthless at that time. Particularly, if we only consider the conversion of CCB, we assign $f_S = 0$ which leads to $\omega_S = 0$.

Fixed Imposed Loss

Under fixed imposed loss, if the issuer assigns the fixed loss at conversion as $100\beta_{CCB}\%$, the value of common shares received by the CCB investor is $100(1-\beta_{CCB})\% L_J$. The number of shares the investor will receive is equal to the value divided by the prevailing market stock price. Similarly, a fixed loss of $100\beta^C_S\%$ to the converted fraction of the senior bond at the event of conversion would redeem the investor with common shares valuing $100(1-\beta^C_S)\%$ of the converted notional value $f_S L_S$. If we consider the case that both CCB and (partial) senior bond convert to common shares, it is natural to require $\beta_{CCB} > \beta^C_S$ in order to respect seniority of the converted part of the senior bond.

As noted by Metzler and Reesor [21], the fixed-loss mechanism is tantamount to a conversion price that is proportional to the firm's stock price at conversion. For example, for a CCB with \$100 notional value, a 20% write-down to the notional value is equivalent to using the conversion price as 1.25 times the stock price at conversion and zero imposed loss on CCB. As it is reasonable to assume that the firm's stock price decreases as conversion nears and the trailing average stock price will be highly correlated with the stock price at conversion, the 1.25 times the stock price at conversion can be taken as a proxy for a trailing average conversion price. Hence, we can see that if the conversion price is proportional to the stock price, the loss imposed at conversion is non-random. For this reason, we call it the fixed-imposed loss.

The Relation Between Fixed Conversion Price and Fixed Imposed Loss

Although fixed conversion price and fixed imposed loss are defined differently, in the perpetual case, they are connected via the dilution factors. To be more specific,

$$(p_{CCB}, p_S) \xleftrightarrow[(2.32),(2.33)]{one-to-one} (\omega_{CCB}, \omega_S) \xleftrightarrow[(2.30),(2.31)]{one-to-one} (\beta_{CCB}, \beta_S^C). \tag{2.34}$$

Equation (2.34) asserts the existence of a one-to-one correspondence between two conversion terms. For example, given (β_S^C, β_{CCB}), we can "project" the imposed loss to a fixed conversion price by choosing (p_{CCB}, p_S) which gives the same ownership stakes (ω_{CCB}, ω_S), and vice versa. Additionally, from (2.32), (2.33), (2.30) and (2.31), we find that as long as $\bar{p} = 1$ the effective losses imposed to the converted part of the senior bond and the CCB are the same. However, if $\bar{p} < 1$ ($\bar{p} > 1$), for the converted part, the CCB investor would bear a larger (smaller) effective loss than the senior bondholder upon conversion.

2.3.2 Reasonable Terms of Conversion

Following the advisory from OSFI [19], issuers and regulators would like to guarantee that (1) debt seniority is respected upon conversion and (2) equity investors are not rewarded for poor performance at the occurrence of conversion. In order to respect seniority it suffices to ensure that

$$\beta_S < \beta_{CCB}, \tag{2.35}$$

where β_S is the total effective loss imposed on the senior bond at conversion,

$$\beta_S = 1 - \frac{S^B_{\tau_c} + \omega_S E_{\tau_c}}{L_S}. \tag{2.36}$$

If only CCB converts to common shares in a trigger event, we assign $\omega_S = 0$ in (2.36) and β_S measures the loss of a straight senior bond. The following proposition states an equivalent condition to (2.35) in the terms of par yields.

Proposition 2.3.2 *The condition $\beta_S < \beta_{CCB}$ is satisfied if and only if $c_S < c_{CCB}$, where c_S and c_{CCB} are the par yields of the senior bond and CCB, respectively.*

Proof See Appendix B.1.3.

We say that equity investors are rewarded for poor performance if $(1 - \omega_S - \omega_{CCB})E_{\tau_c} > E^{trad}_{\tau_c^{trad}}$ and punished for poor performance otherwise[6]. The term $E^{trad}_{\tau_c^{trad}}$ is the equity value to existing shareholders in an otherwise identical traditional capital structure[7] at τ_c^{trad}, which is the first-passage time of the conversion barrier (the asset-liability ratio b^*) under a traditional capital structure. In other words, $E^{trad}_{\tau_c^{trad}}$ is the value of the otherwise identical firm's equity after an equivalent fall in asset value. The satisfaction of the condition means that the existing shareholders are better off in the presence of contingent capital than they are in its absence. In

[6] See Metzler and Reesor [21] for a more detailed discussion in the case if zero-coupon CCBs.

[7] The capital structure with contingent capital replaced by traditional bond with the same seniority.

non-dimensional terms, it is straightforward to see that equity investors are punished for poor performance if and only if

$$b^* - (1-\beta_D)\ell_D - (1-\beta_S)\ell_S - (1-\beta_{CCB})\ell_J - \frac{BC_{\tau_c}}{L} < b^* - (1-\rho_D)\ell_D - (1-\rho_S)\ell_S - (1-\rho_J)\ell_J - \frac{BC^{trad}_{\tau_c^{trad}}}{L}, \tag{2.37}$$

where ρ_i for $i = D, S, J$ represents the loss imposed on the corresponding straight bond under the traditional capital structure at τ_c^{trad}. After simplification, we have

$$\beta_{CCB} < \frac{\rho_D\ell_D + \rho_S\ell_S + \rho_J\ell_J - \beta_D\ell_D - \beta_S\ell_S}{\ell_J} + \frac{BC^{trad}_{\tau_c^{trad}} - BC_{\tau_c}}{L\ell_J}, \tag{2.38}$$

which provides an upper bound on the imposed loss.

To summarize, given the conversion proportion of the senior bond, the reasonable conversion terms are those that ensure

$$\beta_{CCB} \in (\beta_S, (\rho_D - \beta_D)\frac{\ell_D}{\ell_J} + (\rho_S - \beta_S)\frac{\ell_S}{\ell_J} + \rho_J + \frac{BC^{trad}_{\tau_c^{trad}} - BC_{\tau_c}}{L\ell_J}) \quad and \quad \bar{p} < 1. \tag{2.39}$$

Here we require $\bar{p} < 1$ so that the seniority between the converted part of the senior bond and CCB is guaranteed. As long as the loss imposed to CCB at conversion implied by the conversion term is located within the range given by (2.39), it ensures liability seniority and the existing shareholders are not rewarded at conversion and therefore the contingent capital is properly designed.

We note that conversion prices (in the case of a fixed-price contract) or losses imposed at conversion (in the case of a fixed-loss contract) that ensure (2.39) must in general be determined numerically. Metzler and Reesor [21] prove the existence of the region for reasonable conversion terms under some conditions in the case of zero-coupon contingent capital and only with the conversion of a junior bond. However, in the case of contingent capital with coupon payments, it is more practical to provide numerical proofs rather than rigorous analytical proofs because different conversion terms would influence the coupon rates of contingent capital and therefore the asset value process. In addition, we will see later that the reasonable interval for conversion term might not exist in the case of contingent capital with coupon payments. It is not a contradiction to the discussions made in Metzler and Reesor [21] as in [21] the liability seniority might be violated in order to ensure that the existing shareholders do not benefit from conversion[8].

Similar to Metzler and Reesor [21], under fixed conversion price, we define

$$\hat{f}(f_S, p_S, p_{CCB}) = \beta_{CCB} - \beta_S, \tag{2.40}$$

and

$$\hat{g}(f_S, p_S, p_{CCB}) = E^{trad}_{\tau_c^{trad}} - (1 - \omega_S - \omega_{CCB})E_{\tau_c}. \tag{2.41}$$

[8]In Metzler and Reesor [21], they provide constraints to ensure the liability seniority and the existing shareholders are not rewarded for poor performance at conversion. When the constraints are breached, the liability seniority has to be violated so that the existing shareholders will not benefit from conversion.

Under fixed imposed loss, function $\hat{f}$ and $\hat{g}$ depend on the effective loss β_{CCB} and β_S instead of the conversion price ratio p_{CCB} and p_S. In the case of $f_S = 0$ (i.e. only with the conversion of CCB),

$$\hat{f}(p_{CCB}) = \beta_{CCB} - \beta_S, \tag{2.42}$$

and

$$\hat{g}(p_{CCB}) = E^{trad}_{\tau^{trad}_c} - (1 - \omega_{CCB})E_{\tau_c}, \tag{2.43}$$

By Proposition 2.3.2, (2.40) and (2.42) can be replaced by the difference between the par yield of the CCB and the senior bond. Based on our previous discussions, a reasonable conversion term would make (2.40) and (2.41) (or (2.42) and (2.43)) nonnegative. Rather than giving the proofs of the monotonic features of the functions like in Metzler and Reesor [21], we will directly provide numerical results on the existence of the reasonable intervals in Section 2.5.

2.4 Calibration

In this section, we calibrate our model using data from Canadian banking sector, establishing a base for our investigation of the expenses for Canadian banks if they issue contingent capital bond. We use our calibrated model to generate insights into issues that would be of interest to regulators and issuers of contingent capital, such as the determination of appropriate terms of conversion, the potential cost of contingent capital and the impact its presence would have on other debt costs, etc.

2.4.1 Capital Structure

We assume an endogenous capital structure and calibrate our model to the capital structures of the big-6 Canadian banks according to their shareholders' reports on the second quarter of 2012. Using the approach put forward by Beyhaghi, D'Souza and Roberts [5], we assign each bank's liabilities to four general buckets based on their seniority[9]:

- deposits, and
- senior debt (including senior secured debt and senior unsecured debt), and
- junior debt (including subordinated debt, preferred shares and non-controlling interests)[10], and

[9] Beyhaghi, D'Souza and Roberts [5] put the issues of banks into four general buckets: secured, senior unsecured, subordinated and junior subordinated. In our calibration, we include both the secured and senior unsecured within our senior tranche, subordinated and junior subordinated within our junior tranche. The amounts of deposits are specified in the quarterly reports.

[10] Generally, preferred shares cannot be treated as a liability in capital structures. However, D'Souza and Gravelle [8] point out that preferred shares are also a candidate to be classified as contingent capital, so we include preferred shares in our junior tranche. In fact, the preferred shares occupy only a small fraction in the capital structure, for example, 0.6% in RBC's and 0.5% in CIBC's capital structure respectively, and the final results are not influenced significantly based on this classification.

- common equities Tier 1 capital (including common shares, retained earnings and other components of equity)[11].

It is worth mentioning that in the thesis we “standardize” the dollar values of liabilities and equity into proportions of a unit. The standardization facilitates the par yields searching (root-searching) procedure in our numerical experiments. In Table B.1 of Appendix B.2.1 we present the dollar value of each layer in the capital structure of the big-6 Canadian banks, followed by Table B.2 where we present the proportions of layers in the capital structures. Figure 2.1 illustrates similar capital structures of the big-6 Canadian banks, with deposits, senior debts, junior debts and common shares weighting 61.50%, 32.10%, 1.98% and 4.41% on average (or 61.37%, 31.74%, 1.92% and 4.60% as median) respectively.

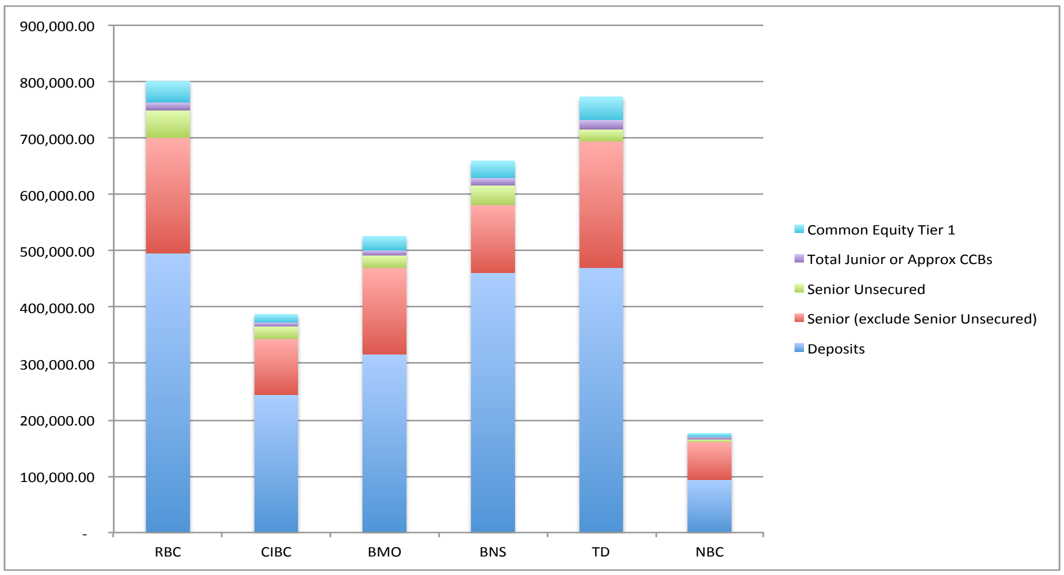

Figure 2.1: Capital structure of the big-6 Canadian banks on the second quarter of 2012.

Without changing the capital structure of each bank, we replace all of its junior debt with CCB. For the part of the senior bond which can be converted at a trigger event, the unsecured senior debt in the senior tranche is used as a proxy. This arrangement is consistent with the regulatory suggestions put forward by Arjani and D’Souza[3], and similar to the way proposed by Flannery[10][12].

The introduction of contingent capital rises the bank’s loss-absorbing capacity. The loss-absorbing capacity refers to the equity value in the bank’s capital structure which absorbs the loss in the very first place without affecting the debt repayment ability of the bank. Converting contingent capital to common shares, the bank’s equity value increases and the liability value decreases. Dividing the contingent capital value with the exiting equity value in the

[11] This approximation is based on the Basel III rules and details can be found in the second quarter shareholders’ report of RBC in 2013 (page 37), http://www.rbc.com/investorrelations/pdf/q213report.pdf .

[12] Flannery[10] assumes that companies can most readily issue contingent capital by replacing their unsecured long-term debt, including the traditional subordinated debt in Tier 1 and Tier 2. In our calibration, we distinguish the unsecured senior debt and subordinated debt, including them in the senior and junior tranche respectively.

capital structure, we get the improvement of the loss-absorbing capacity of the bank by replacing the junior bond (and part of the senior bond) with contingent capital. Perceived from the approximated capital structure (see Table B.4 in Appendix B), for the big-6 Canadian banks, replacing the junior bonds with CCBs, the loss absorption capacity is improved by an average (median) 45.99% (41.31%). Furthermore, including the conversion of partial senior bonds, the loss absorption capacity is raised further by an average (median) 104.62% (102.21%). Therefore, contingent capital would strengthen the firm's stability by boosting up the firm's capital immediately upon conversion.

Considering the similarity among the big-6 Canadian banks in their capital structures, without loss of generality, we show and analyze the results under the capital structure of a representative bank instead of listing all results for the six Canadian banks.

2.4.2 Conversion Trigger

With Basel III coming into effect, one popular choice of conversion trigger is a capital ratio trigger and/or authorities discretion. For example, Lloyds Banking Group used the 5.0% consolidated core Tier 1 ratio, Credit Suisse Group used common equity Tier 1 (CET1) ratio and the authority's supervision, and Royal Bank of Canada directly makes the conversion decided by the authority's judgement of non-viability, etc. In this chapter, we only consider the capital ratio trigger. The regulatory discretionary trigger will be discussed in Chapter 4.

The recent issuances of contingent capital illustrate CET1 ratio as a natural choice[13] for conversion trigger. According to the different locations of the CET1 ratio, we classify it into two types, the going-concern trigger and the gone-concern trigger. A going-concern trigger, or a "high" trigger, corresponds to an early conversion when the capital of the bank is only modestly eroded. However, a gone-concern trigger, or a "low" trigger, is close to the point of non-viability. In addition, since the CET1 ratio is defined as common equity Tier 1 capital divided by the risk-weighted asset (RWA)[14] and the asset-liability ratio used in our model is defined using the total asset, we must identity an empirical relation between the bank's RWAs and total assets. For the representative bank, we plot the ratio of the risk-weighted assets divided by the total assets in Figure 2.2. It illustrates that the ratio is relatively stable over time, so setting it as a constant is reasonable. To this end, we set the ratio as 0.387, which is the mean of the time series[15]. In Table 2.1, we list some correspondence between the CET1 ratio and the asset-liability ratio. We assume the bank will be liquidated when the CET1 ratio falls below 4.0%, or when the asset-liability ratio falls below 1.0157.

As shown in Table B.1, we incorporate CET1 ratio into our model by approximating CET1 capital with the common equity. To be compliant with Basel III criteria and conservative in the choice of conversion triggers, we can consider the 4.5% CET1 ratio, the mandatory requirement in Basel III, as a gone-concern trigger and higher CET1 ratios (for example, 6.0%) as going-concern triggers.

[13] See Appendix A for more details.

[14] Risk-weighted asset is a bank's asset weighted according to the risk.

[15] Based on the discussion, assuming the asset-liability ratio at time t is $\bar{v}_t$, then the CET1 ratio at the moment is $(1 - 1/\bar{v}_t)/0.387$.

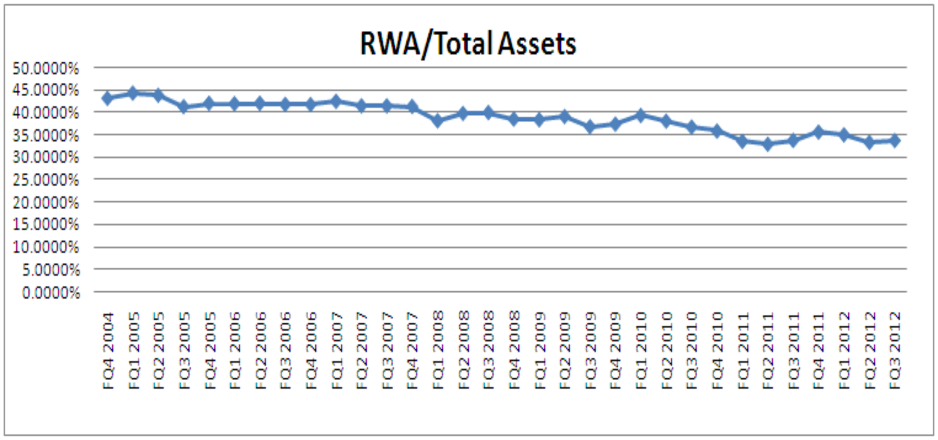

Figure 2.2: The ratio of RWA over the total asset of the representative Canadian bank: the time series has mean 38.71% and standard deviation 3.39%.

Table 2.1: Correspondence Between CET1 Ratios And Asset-Liability Ratios

CET 1 Ratio	4.0%	5.0%	6.0%	8.0%	10.0%	12.0%	14.0%
Asset-Liability Ratio $\bar{v}_t$	1.0157	1.0197	1.0238	1.0320	1.0403	1.0487	1.0573

2.4.3 Other Parameters

Recovery rates. Since there is no large Canadian bank has ever defaulted, the recovery rates used in this paper apparently should be higher than those of the United States[16]. We assume that, upon liquidation, the recovery rate for deposits is 100%, i.e., deposits are guaranteed to be fully repaid. The recovery rate for the senior bond is 98.88%, and for the junior bond is 97.87%. These recovery rates are estimated using our model to match the empirical credit spreads (the median credit spread for senior unsecured debts is 0.21% and for subordinated debts it is 0.40%) reported in Beyhaghi, D'Souza and Roberts [5] for Canadian banks.

Risk-free interest rates. Following the method suggested in Hull [13], the risk-free interest rate is approximated by the yield of treasury bonds. We approximate the risk-free interest rate using the yield of Canadian government one-year treasury bills from July 31, 2011 to July 31, 2012. The yield of one-year treasury bills during this period is relatively stable, allowing us to choose the risk-free interest rate as the average 1.00%[17].

Asset volatilities. Moody's reports estimate asset volatilities from the Merton model. Being

[16] According to Moody's report (2010) [22] which considers comprehensive scenarios (default and no default) in the banking field of United States, the value-weighted recovery rate is 54.7% for senior unsecured debt and 63.8% for senior secured debt. For the senior subordinated debt and subordinated bond, the empirical recovery rate is 39.4% and 32.2%, respectively.

[17] Kim et al. [14] study the Merton model with CIR dynamics for the short rate and find that the stochastic interest rates play a relatively insignificant role in the computed credit spreads. Similar discussions can be found in Lando [15]. Therefore, for simplification, we take the risk-free interest rate as a constant rather than following a diffusion process.

conservative, we assume the asset volatility to be 5.0%, which is the largest reported asset volatility for the big-6 Canadian banks according to Moody's for the period from 2007 to 2012. The largest value 5.0% showed up in the late 2008, during the financial crisis.

Dividend payout ratios. The dividend payout ratio is approximated by the average of the last ten year's dividend payout ratios of common shares (dividends paid to common shares divided by total asset values). As shown in Appendix B.2.2, given the relatively low standard deviation, it appears reasonable to directly use the average dividend payout ratio for each bank.

Conversion fraction of the senior bond. In the event that the senior bond is partially convertible, we assume that only the unsecured portion converts. This assumption follows the suggestion put forward by Arjani and D'Souza[3] and Flannery[10]. For example, according to Table B.3 in Appendix B, the conversion fraction of the senior bond for RBC is 19.47%.

Table 2.2 lists all the calibrated parameters in our base case. The parameters are set to the value shown in the base case by default unless specified otherwise.

Table 2.2: Calibrated Parameters in Base Case

Parameter	**Value**
Recovery rate for deposits $\mathcal{R}_D$	100%
Recovery rate for senior bond $\mathcal{R}_S$	98.88%
Recovery rate for junior bond $\mathcal{R}_J$	97.87%
Default level	4.0% CET1 ratio
Conversion level	5.0% CET1 ratio
Conversion proportion for CCB f_{CCB}	100%
Conversion proportion of senior bond f_S	19.47%
Asset volatility σ	5.0%
Risk-free interest rate r	1.00%
Dividend payout ratio q for RBC	0.3718%

2.5 Numerical Results

In this section we implement our model using the calibrated parameters. In each part, results are shown under fixed conversion price and fixed imposed loss respectively. Moreover, Monte Carlo simulation is employed to study the term structure of CCB yields.

2.5.1 Base Case

Using the calibrated parameters in Table 2.2, the estimated credit spreads under the traditional capital structure are shown in Case 1 of Table 2.3. The spreads are the same with the median of empirical credit spreads summarized in Beyhaghi, D'Souza and Roberts [5], where they find that from 1990 to 2010, for the big-6 Canadian banks, the median credit spread for senior unsecured debts is 0.21% and for subordinated debts it is 0.40%. Since we assume a full recovery of deposits, the credit spread for deposits is always zero and not included in the table.

Only Conversion of CCB Under fixed conversion price, we set $p_{CCB} = 0.5$, meaning the conversion price for CCB is 50% of the market stock price at the issuance of the contingent capital[18]. Under fixed imposed loss, we assume the loss imposed to CCB upon conversion is 5.33% of the notional value. By the arrangement, we match the risk premiums under two conversion terms (Case 2 and Case 4).

Replacing the non-contingent junior bond in the capital structure with CCB, Case 2 and Case 4 in Table 2.3 illustrate a cost increase in the junior tranche but a decline in the senior bond. Therefore, the terms of conversion are sufficiently punitive to CCB investors (i.e. the conversion price is sufficiently high) for the CCB to be more expensive than otherwise identical junior debt[19]. However, the conversion of contingent capital provides a capital cushion and reduces the risk of liquidation, which benefits the senior bondholder leading to a fall in credit spread.

According to the last column of Case 2 and Case 4 in Table 2.3, the issuance of contingent capital would reduce the weighted total costs for the firm. This mainly results from the fall in the cost of the senior bond, which accounts for a much greater proportion of total liabilities than does junior debt, for Canadian banks. Thus, even though it is more expensive than otherwise identical junior debt, issuing contingent capital here would reduce the firm's total cost of debt. From this perspective, issuing contingent capital might benefit the bank regardless of regulatory requirement.

Table 2.3: Credit Spreads in the Base Case

	Case	Conversion Part(s)	Senior	CCB(or Junior)	Weighted Total[b]
Traditional	1[c]	None	21bp	40bp	22bp
Fixed Price	2	CCB	13bp	113bp	18.28bp
	3	CCB & Partial Senior	7bp	152bp	14.65bp
Fixed Loss	4	CCB	13bp	113bp	18.28bp
	5	CCB & Partial Senior	7bp	111bp	12.49bp

[a] Credit spreads here are defined as the difference between the par yields and the risk-free interest rate.
[b] The weighted total cost is calculated with the corresponding notional value as the weight.
[c] The credit spreads from Case I are from Beyhaghi, D'Souza and Roberts [5].

Conversion of CCB and Partial Senior Bond When a fraction ($f_S = 19.47\%$) of the senior bond participating in the conversion, we keep the conversion arrangements for the CCB unchanged. To respect seniority, the fixed conversion price ratio is assumed to be $\bar{p} = 0.95 < 1$ meaning the conversion price for the senior bond is 95% of that for CCB; under fixed imposed loss, we assume the write-down to the converted part of the senior bond is 45.54% of the write-down to CCB at conversion. This arrangement gives the same credit spread for the senior bond (7 basis points) in Case 3 and Case 5.

For these parameters at least, a partially convertible senior bond is less expensive than a traditional senior bond. The possible reasons resulting in the decline of credit spread includes (1) the larger safety cushion reduces the possibility of liquidation and (2) the potential upside

[18]We will see later that this conversion price is located within the reasonable interval for conversion price.
[19]We will soon see this need not always be the case.

from the received equity stake in a reinvigorated firm. In contrast, the risk premium of CCB moves in opposite directions under the two conversion terms. There are two effects for the CCB investor brought by the conversion of senior bond – the ownership stake received at conversion is reduced and the dividend stream after conversion is prolonged. These two effects compete with each other. In our bases case, under fixed conversion price, the first effect dominates leading to a rise in the spread while under fixed imposed loss, the second effect dominates resulting in a slight spread drop[20].

As is shown in Table 2.3, the issuance of contingent capital would reduce the weighted total liability cost for the firm. This mainly results from the fall in the costs of the senior tranche. Comparing Case 2 with Case 3 and Case 4 with Case 5, the larger the fraction of the senior bond converts to common shares in a trigger event, the more borrowing costs will be reduced for the firm.

In the following sections we expand on our base case analysis by considering the impact of the terms of conversion on the firm's debt costs. We start from the conversion term of the fixed conversion price and then turn to the fixed imposed loss.

2.5.2 Fixed Conversion Price

We focus on the fixed conversion price in this section and investigate the credit spread of contingent capital under different conversion prices and its sensitivity to the firm's asset volatility. In addition, the reasonable interval for conversion price is estimated and presented. For each problem, we start from the case $f_S = 0$ (i.e. senior bond is non-convertible) and then the case $f_S > 0$ (i.e. senior bond is partially convertible).

Conversion Prices

Only Conversion of CCB Figure 2.3 plots the change of credit spreads with respect to the change of conversion price. As the conversion price increases, the credit spread of the CCB and senior bond raises. Intuitively, a higher conversion price implies a lower ownership stake to the CCB investor at conversion so the investor would require a higher risk premium as a compensation. For the senior bond, a relatively high conversion price means more expensive contingent debt, which pushes the firm towards liquidation faster and causes the climb of the spread.

Additionally, Figure 2.3 indicates a positive relation between the firm value volatility and the credit spreads. The more volatile the firm value, the higher the conversion probability and the earlier the conversion could be. Therefore, fixing a conversion price, for the firm with a higher volatility, the CCB investor would require a higher (lower) risk premium to compensate for the conversion risk provided that the fixed conversion price implies a positive (negative)

[20] In the case that only CCB converts, matching the credit spreads for CCB under two conversion terms implies $1-\beta_{CCB} = P_{\tau_c}/P_{CCB}$, where P_{τ_c} is the contemporary market stock price at conversion. Adding part of the senior bond into conversion, we turn to match the credit spreads for the senior bond by assigning $1-\beta_S = P_{\tau_c}/P_S$, or equivalently, $1 - x\beta_{CCB} = P_{\tau_c}/(yP_{CCB})$, where in our base case $\beta_{CCB} = 0.0533$, $x = 0.4554$ and $y = 0.95$. We can verify that $1-\beta_{CCB} > y(1 - x\beta_{CCB})$, so with the conversion of the partial senior bond, the effective conversion price for CCB increases, leading to a rise in the credit spread for CCB in Case 3. In addition, we have $1 - 1/x + P_{\tau_c}/(xyP_{CCB}) > P_{\tau_c}/P_{CCB}$, so the effective redemption to CCB increases, leading to a drop in the credit spread in Case 5.

effective write-down at conversion. The intersection point in the right panel of Figure 2.3 represents the point at which the CCB is redeemed at par, in which case the fixed conversion price is the same as the contemporary stock price at conversion. In the base case the bond is redeemed at par provided the conversion price is approximately 46% of the firm's initial stock price. The value appears to be independent of asset volatility as in our model once we fix the CET1 ratio which triggers the conversion, the equity value (share price) at conversion is given. Therefore, under fixed conversion price, the contemporary stock price at conversion implied by the conversion trigger is a constant and not influenced by different asset volatilities. In the left panel, the credit spreads of the senior bond are not as sensitive as the spreads of the CCB to the firm's asset volatility because conversion only influences the length of coupon payment stream to the senior bond without impacting its recovery rate at liquidation.

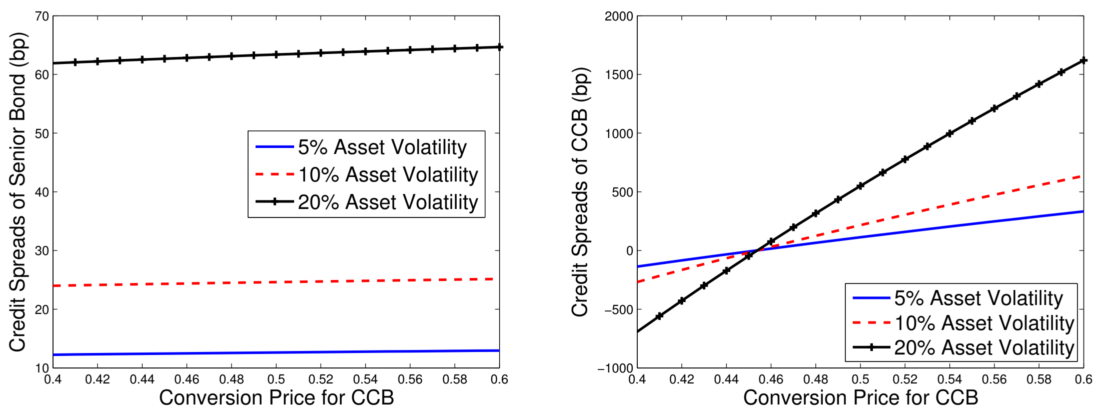

Figure 2.3: Credit spreads as a function of conversion price or the firm's asset volatility. We only consider the conversion of CCB. The x-axis is the conversion price for CCB divided by the market stock price at the issuance of CCB.

Conversion of CCB and Partial Senior Bond It is perceived in Figure 2.4 that the positive relation between the conversion price and the credit spread of CCB and senior bond continues to hold when the senior bond is partially convertible. The credit spread of the senior bond becomes sensitive to the change of conversion price because fixing the conversion price ratio $\bar{p}$, the conversion price for the senior bond increases with the growth of the conversion price for the CCB. For both panels in Figure 2.4, to the left of the intersection point the contingent capital is redeemed above par, in which case early redemption is better than late redemption. To the right of the intersection point, the opposite occurs. The possible early conversion implied by a relatively high asset volatility would increase (decrease) the risk premium for contingent capital provided that at conversion, the effective redemption value implied by the conversion price is below (above) the corresponding notional value. In addition, given an asset volatility, the credit spread of CCB becomes less sensitive to the change of the conversion price than the case with only conversion of CCB (right panel in Figure 2.3). This is because partial conversion of the senior bond provides extra capital infusion to the firm, further delaying the possible liquidation and prolonging the dividend stream for the CCB investor after conversion.

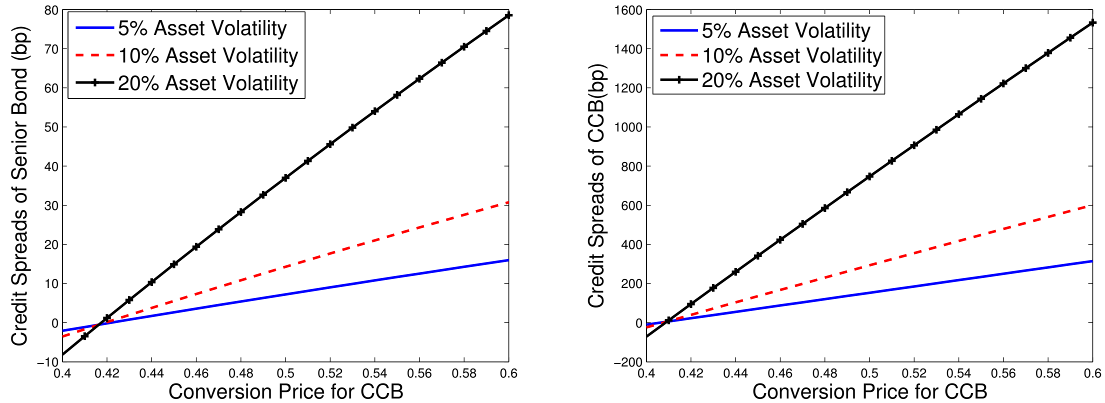

Figure 2.4: Credit spreads as a function of conversion price or firm's asset volatility. We consider the full conversion of CCB and 19.47% of the senior bond. The conversion price ratio is fixed at $\bar{p} = 0.95$. The x-axis is the conversion price for CCB divided by the market stock price at the issuance of CCB.

In Figure 2.5 we vary the conversion price ratio $\bar{p}$ while keeping the conversion price for CCB unchanged. A sufficiently low conversion price ensures that the convertible portion of the senior bond is redeemed above par at conversion, resulting in negative credit spreads in the left panel. In the right panel, as the conversion price ratio declines, the ownership stake distributed to CCB investors declines. As a result, CCB investors bear more losses and the credit spread for the CCB rises dramatically. Hence, when the senior bond is involved in conversion, the issuing firm has to be careful in setting the conversion price ratio in order to ensure that both tranches contribute to the loss absorbency through conversion.

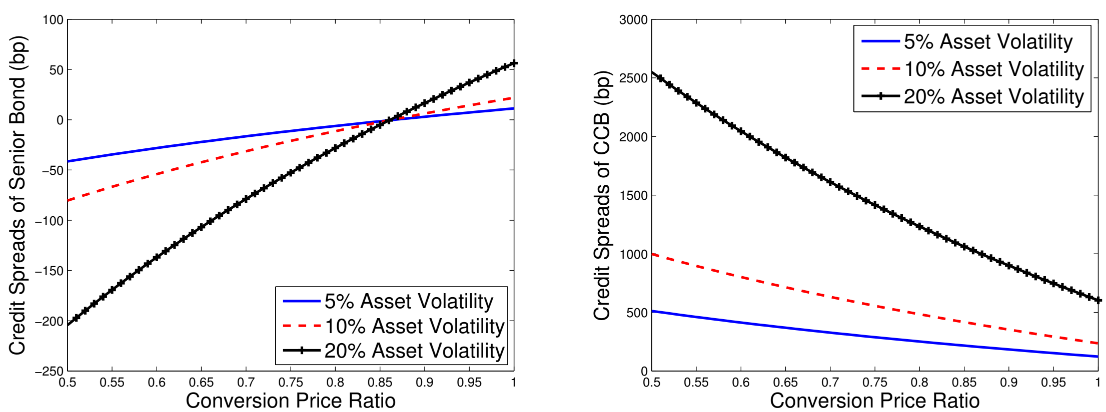

Figure 2.5: Credit spreads as a function of conversion price ratio or firm's asset volatility. We consider the full conversion of CCB and $100f_S\%$ of the senior bond. The conversion price for CCB is fixed at $p_{CCB} = 0.5$. The x-axis is the conversion price ratio $\bar{p}$.

So far we analyzed the costs of contingent capital under different conversion prices and asset volatilities. Obviously, for a firm with relatively high risk (volatile asset value), it is expensive to issue contingent capital provided that the contingent capital is going to absorb losses (be redeemed under par) upon conversion. Besides, the conversion price cannot be too low and the combination of the conversion price and conversion price ratio cannot be arbitrarily chosen. An extremely low conversion price or unreasonable combination of conversion price and price ratio might benefit investors through conversion and effectively creates a class of investors that benefit from the onset of financial distress. It would make the contingent capital fail to work as a regulatory tool which is supposed to make contributions to loss absorbency through conversion and boost the firm's capital rather than profit from the deteriorating situation of the existing shareholders.

Conversion Fraction of Senior Bond

Figure 2.6 illustrates the impact of f_S on the firm's debt costs. It shows that the spread for the senior bond decreases first and then increases while the cost for the CCB moves in opposite direction by climbing first and then falling. The change of spread for the senior bond can be interpreted by two opposite impacts. On the one hand, the converted part suffers a positive write-down and loses the corresponding coupon payments. On the other hand, the conversion of the senior bond further prolongs the life of the firm and therefore the coupon stream of the unconverted part and the dividend stream for the converted part. When the conversion proportion is relatively low, the second impact dominates, leading to a fall in the credit spread of the senior bond as the conversion proportion increases. However, when the conversion proportion is relatively high and keeps growing, the first impact dominates, leading to a rise in the credit spread of the senior bond. Although the conversion of the senior bond can delay the possible liquidation and extend the dividend stream after conversion, it causes dilution to the ownership stake distributed to the CCB investor upon conversion and the dilution is more severe as a larger fraction of the senior bond converts in a trigger event. The two influences compete with each other resulting in a humped-shaped credit spread of CCB in Figure 2.6. As the senior bond takes a heavy weight in our calibrated capital structure, it appears that the weighted total spread has a similar shape with the spread of the senior bond. The non-monotonic shape indicates that there might exist a conversion fraction for the senior bond with which the weighted total spread is minimized. For example, in Figure 2.6, when the conversion fraction is around 30%, the weighted total spread realizes a minimum value of about 15 basis points.

Reasonable Intervals for Conversion Prices

In this section, we implement our model to investigate the reasonable interval for fixed conversion price. Recall that reasonable terms of conversion imply that seniority is respected at conversion and that equity investors are not rewarded for poor performance. Following the definition of (2.42) and (2.43), we know that the function $\hat{f}(p_{CCB})$ measures the difference between losses imposed at conversion to contingent capital investor and senior bond investor. A positive (negative) value of the function $\hat{f}(p_{CCB})$ means the loss imposed on CCB investors is more (less) than mark-to-market loss sustained by senior investors. In addition, the function $\hat{g}(p_{CCB})$ measures the difference in the equity value at conversion between the case in the absence of contingent capital and the case with contingent capital. A negative (positive) value

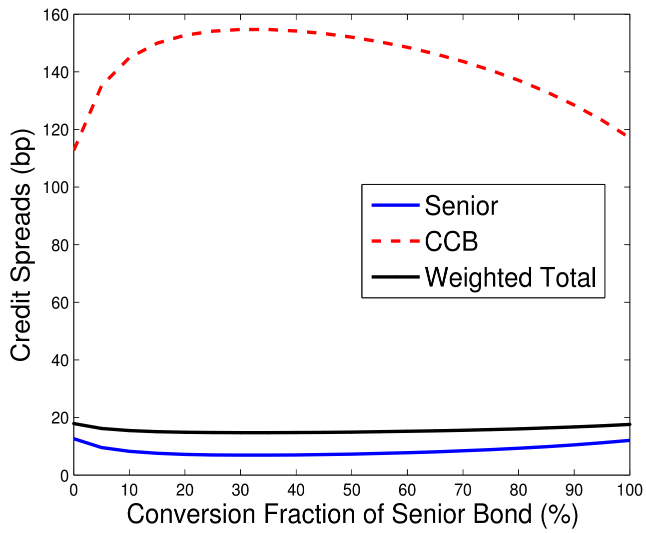

Figure 2.6: Credit spreads as a function of conversion proportion of the senior bond. The conversion price for CCB is $p_{CCB} = 0.5$ and the conversion price ratio $\bar{p} = 0.95$. The weighted total credit spread is estimated with the corresponding notional value as the weight.

of the function $\hat{g}(p_{CCB})$ means the original shareholders are rewarded (not rewarded) for the firm's poor performance.

Only Conversion of CCB Figure 2.7 plots the reasonable interval for the conversion price implied by the function (2.42) and (2.43). The monotonic feature of the function $\hat{f}(p_{CCB})$ and $\hat{g}(p_{CCB})$ is well captured in the graph. The value of the function $\hat{f}(p_{CCB})$ (the green solid line) is positive when the conversion price is located above 45.83% of the stock price at the issuance of CCB. Beyond this point, the CCB investor always bears more losses than the senior bondholder at conversion, guaranteeing the seniority between liabilities. However, the conversion price cannot go beyond 53.32% of the stock price at the issuance of CCB since otherwise it makes the value of $\hat{g}(p_{CCB})$ (the black dot line) go negative rewarding the existing shareholders for the firm's poor performance. Consequently, a reasonable interval for the fixed conversion price is $[45.83\%, 53.32\%]$ of the stock price at the issuance of CCB which is indicated as the red line in the figure.

In Table 2.4, we list reasonable intervals for the fixed conversion price under different trigger locations. A high trigger implies a relatively high market stock price upon conversion, and in order to respect seniority, the conversion price needs to move upwards. Thus, the location of the interval moves towards right. As the trigger location moves upwards, the size of the reasonable interval shrinks and the issuing firm has to be more prudent in choosing the conversion price.

Conversion of CCB and Partial Senior Bond With the senior bond participating in conversion, the functions (2.40) and (2.41) depend on three variables – the conversion proportion (f_S), the conversion price for the senior bond (p_S) and for the CCB (p_{CCB}). In the following, we vary

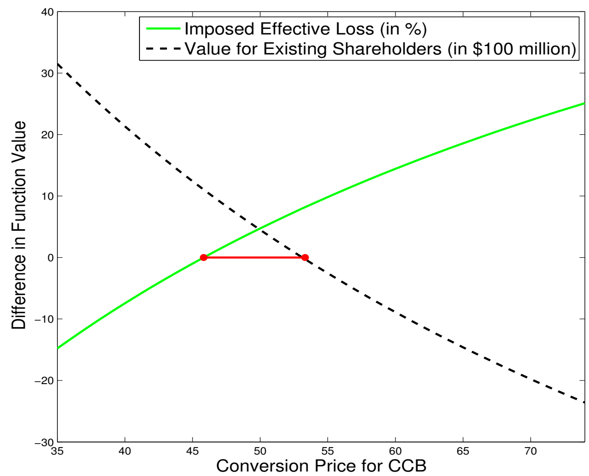

Figure 2.7: Reasonable interval for the conversion price p_{CCB}. We only consider the conversion of CCB. The x-axis is the conversion price as a proportion to the stock price at the issuance of CCB. The green solid line is estimated by function (2.42). The black dot line is estimated by function (2.43). The red line gives the reasonable interval [45.83%, 53.32%] for the location of fixed conversion price.

Table 2.4: Reasonable Intervals for Fixed Conversion Price

Conversion Trigger	Reasonable Interval	Interval Size
4.5% CET1	[41.96%, 49.66%]	7.70%
5.0% CET1	[45.83%, 53.32%]	7.49%
7.0% CET1	[61.43%, 67.64%]	6.21%
9.0% CET1	[77.26%, 81.43%]	4.17%

two parameters each time and investigate the existence and the change of the reasonable region for conversion prices. For convenience, instead of using the conversion price for the senior bond (p_S), we use the conversion price ratio ($\bar{p}$) to control the variation of the conversion price for the senior bond.

Firstly, we fix the conversion proportion of the senior bond (f_S) and vary the conversion price ratio ($\bar{p}$) and the conversion price for CCB (p_{CCB}). We start from the special case $f_S = 0$. The region given in the left upper panel of Figure 2.8 is a bar bounded by two lines vertically located at 45.83% and 53.32% respectively, which are exactly the end points of the reasonable interval in the case with only conversion of CCB. For non-zero conversion proportions, shown in Figure 2.8, the regions satisfying the principle put forward by OSFI are bounded by three lines and are located above the blue dashed-and-point line. The region shrinks as more proportion of the senior bond participates in conversion. As a result, the issuing firm has to be careful in choosing the combinations of the conversion price ratio and the conversion price of CCB if they plan a relatively large fraction of contingent capital in the capital structure.

In Figure 2.9 we fix the conversion price ratio and vary the conversion price for the CCB (p_{CCB}) and the conversion fraction (f_S). The reasonable regions are bounded by three lines and are located below the blue dashed-and-point line. The scale of the region is positively related with the conversion price ratio. If we assign an extremely small conversion price ratio, such as $\bar{p} = 0.001$ in the upper left panel, only a very small fraction (less than 1%) of the senior tranche is allowed to be converted in a trigger event. Otherwise the senior bondholders would be significantly benefited from conversion since the senior bond would be easily redeemed above par.

Since there is a narrow reasonable interval for the conversion price of CCB (p_{CCB}), we only consider two fixed conversion prices ($p_{CCB} = 0.5$ and 0.53) while vary the conversion fraction (f_S) and the conversion price ratio ($\bar{p}$). In Figure 2.10, the reasonable region is located to the left of the red dashed line and black line and the right of the blue dashed-and-point line. Since the black line just appears in the right upper corner, implying that almost all the combinations on the whole panel satisfies the liability seniority between the senior bond and CCB. As the conversion price p_{CCB} increases, the change of the reasonable region is uncertain as is shown in Figure 2.10.

To conclude, we find that the area of the reasonable region depends on the dilution to the CCB investor[21] caused by the conversion of the senior bond. Intuitively, there is a limited region for the fixed conversion price which controls the dilution within a reasonable range. The higher the dilution implied by the parameters, the smaller the range would appear. If we vary the conversion proportion f_S while fixing two other parameters, the dilution to the CCB investor increases as a greater fraction of the senior bond participates in conversion. As a result, we observe a shrinking region in Figure 2.8 as the conversion fraction increases. Similarly, if we vary the conversion price ratio $\bar{p}$ while fixing the the conversion proportion and the conversion price to the CCB, the dilution to the CCB after conversion declines as the conversion ratio climbs so the area of the reasonable region increases. However, if we change the conversion price p_{CCB} while fixing the conversion proportion and the conversion price ratio, the change of the reasonable region is not clear. This is because to keep the conversion price ratio unchanged, the conversion price for the senior bond increases with the growth of that for the CCB making the dilution to the CCB investor ambiguous. As a result, the area change in Figure 2.10 is unclear.

[21]The dilution here refers to the dilution effect to the CCB investor after conversion resulting from the conversion of the senior bond. With the senior bond participating in the conversion, if the ownership stake ω_{CCB} decreases post conversion, we say there is a dilution effect to the CCB investor.

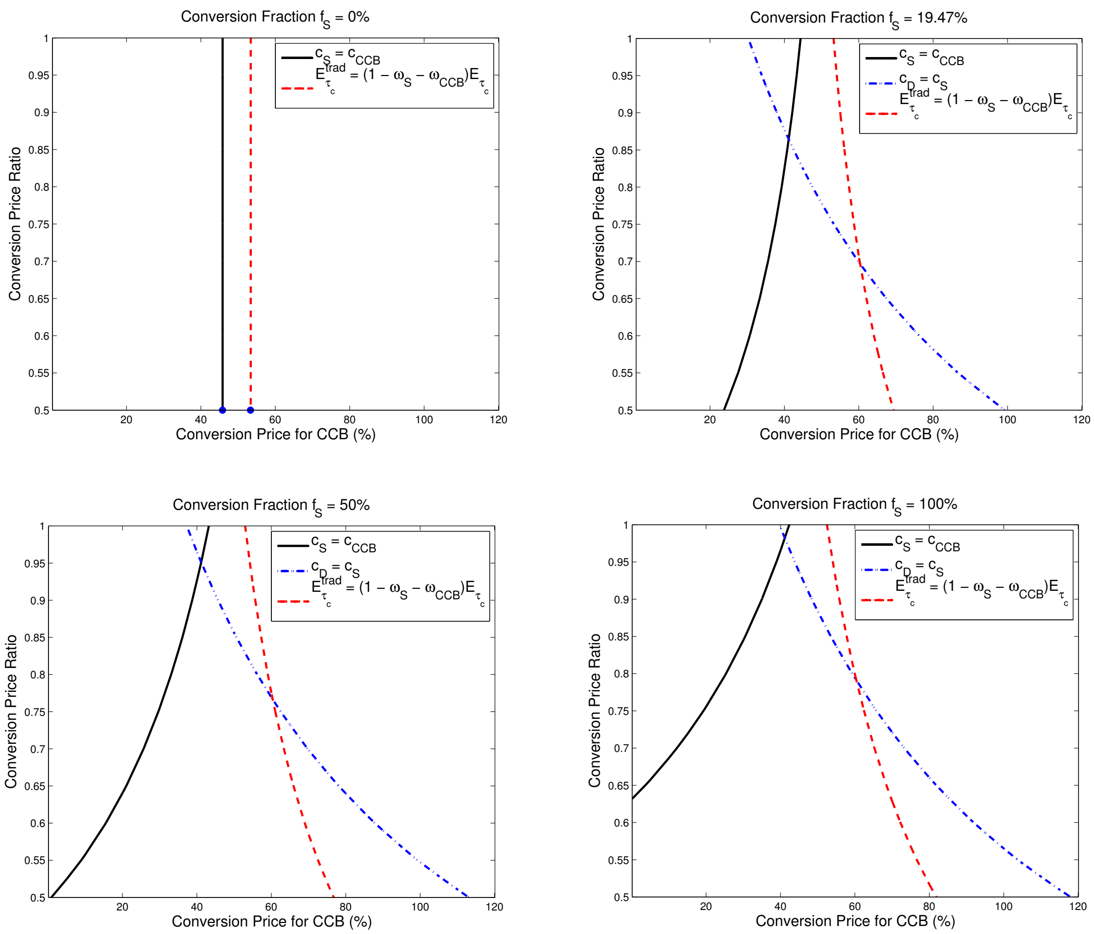

Figure 2.8: Reasonable region for the combination of conversion price and conversion price ratio. The black solid lines represent the function $\hat{f}(f_S, \bar{p}, p_{CCB}) = 0$ and the red dashed lines represent the function $\hat{g}(f_S, \bar{p}, p_{CCB}) = 0$ with given conversion fraction f_S's. The blue dashed-and-point lines represent the equivalence of the par yields for deposits and senior bonds. In the left upper panel, the two blue dots located at 45.83% and 53.32% respectively.

2.5.3 Fixed Imposed Loss

In this section we fix the proportion of imposed loss (or write-down proportion) to the notional value of the contingent capital at conversion, but let the conversion price be the contemporary market stock price. Similar to the section of fixed conversion price, we investigate the credit spread of contingent capital under different fixed losses, its sensitivity to the firm's asset volatility and present the reasonable interval for fixed imposed losses.

Imposed Losses

Only Conversion of CCB According to Table 2.5, the credit spread of the CCB is positively related to the loss imposed at conversion. With a relatively high asset volatility, the credit spread appears to be more susceptible to the change of imposed loss under a higher asset

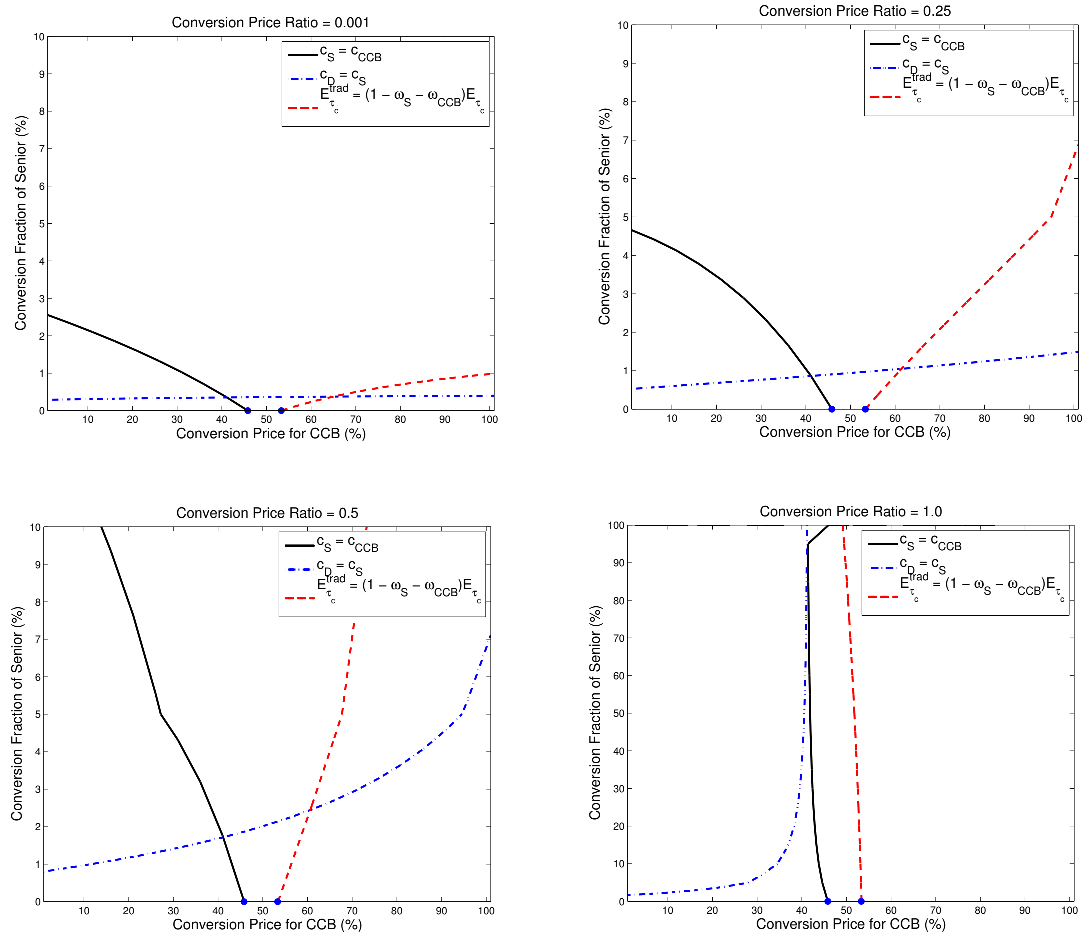

Figure 2.9: Reasonable interval for the combination of conversion price and conversion fraction of the senior bond. The black solid lines represent the function $\hat{f}(f_S, \bar{p}, p_{CCB}) = 0$ and red dashed lines represent the function $\hat{g}(f_S, \bar{p}, p_{CCB}) = 0$ with given conversion price ratio $\bar{p}$'s. The blue dashed-and-point lines represent the equivalence of the par yields for deposits and senior bonds. The two blue dots located at the x-axis have the x-value 45.83% and 53.32%.

volatility because conversion probably occurs earlier and the CCB investor would suffer a write-down earlier than under a low asset volatility. The credit spread of the senior bond is not sensitive to the change of the imposed loss since the conversion of the contingent capital only influences its coupon leg without changing its recovery rate at liquidation.

Conversion of CCB and Partial Senior Bond Table 2.6 lists the credit spreads of the senior bond and CCB under different imposed losses at conversion. The credit spread of the senior bond becomes sensitive to the change of imposed loss and it increases with the the loss imposed at conversion. Comparing Table 2.6 with Table 2.5, the credit spread of CCB does not change significantly as part of the senior bond converts to common shares at the same time with the CCB. This is because the conversion fraction of the senior bond might be significant enough to delay the possible liquidation and prolong the dividend stream for CCB investors.

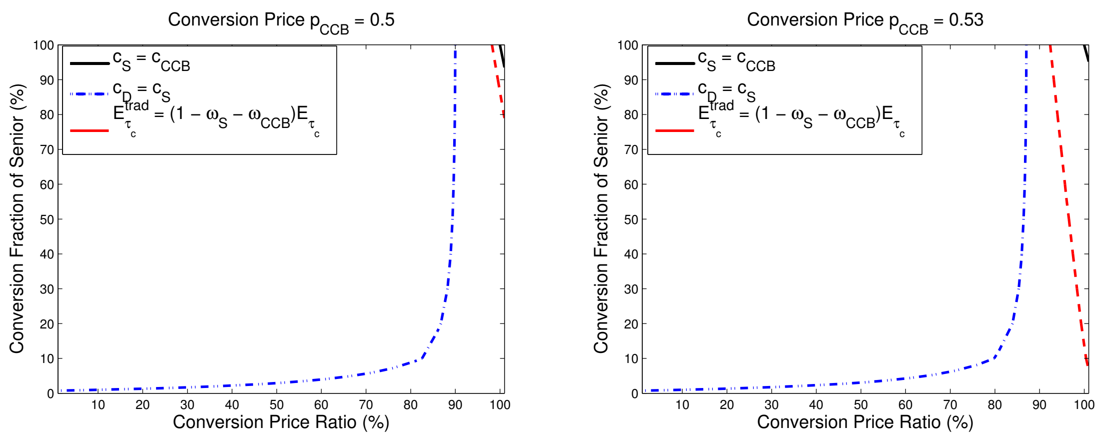

Figure 2.10: Reasonable interval for the combination of conversion price ratio and conversion fraction of the senior bond. The red dashed lines represent the function $\hat{g}(f_S, \bar{p}, p_{CCB}) = 0$ with given conversion price p_{CCB}'s. The blue dashed-and-point line represent the equivalence of the par yields for deposits and senior bonds. The black solid lines represent $\hat{f}(f_S, \bar{p}, p_{CCB}) = 0$.

Table 2.5: Credit Spreads Under Fixed Imposed Loss ($f_S = 0.0\%$)

σ	**Bond**	**Imposed Loss (β_{CCB})**					
		0.0%	5.0%	10%	15%	20%	25%
5.0%	Senior	13 bp	13 bp	13 bp	13 bp	13 bp	13 bp
	CCB	0 bp	106 bp	216 bp	333 bp	455 bp	584 bp
10.0%	Senior	24 bp	25 bp	25 bp	25 bp	25 bp	26 bp
	CCB	0 bp	204 bp	417 bp	638 bp	868 bp	1109 bp
20.0%	Senior	63 bp	63 bp	64 bp	65 bp	65 bp	66 bp
	CCB	0 bp	523 bp	1065 bp	1628 bp	2214 bp	2823 bp

Table 2.6: Credit Spreads Under Fixed Imposed Loss ($f_S = 19.47\%$)

σ	**Tranche**	**Imposed Loss (β_{CCB})**					
		0.0%	5.0%	10%	15%	20%	25%
5.0%	Senior	4 bp	7 bp	10 bp	13 bp	16 bp	18 bp
	CCB	0 bp	103 bp	214 bp	332 bp	459 bp	596 bp
10.0%	Senior	9 bp	14 bp	19 bp	25 bp	30 bp	36 bp
	CCB	0 bp	201 bp	413 bp	637 bp	875 bp	1128 bp
20.0%	Senior	24 bp	37 bp	50 bp	63 bp	77 bp	90 bp
	CCB	0 bp	515 bp	1056 bp	1626 bp	2229 bp	2868 bp

Conversion Fraction of Senior Bond

In Figure 2.11, we plot the credit spreads under different conversion proportions of the senior bond. The credit spread of the senior bond and CCB change non-monotonically with the increase of the conversion proportion. When the conversion proportion is relatively low, the upside (delayed liquidation, prolonged streams of dividends and coupons) brought by the conversion of the senior bond dominates, causing a decline in the credit spreads. However, as the conversion proportion grows, the downside (growing write-down to the notional value and dilution effects) brought by the conversion of the senior bond dominates, causing an increase in the credit spreads. Due to the heavy weight of the senior bond, the weighted total cost moves similarly with the credit spread of the senior bond. The non-monotonic shape makes it possible for the firm to choose a conversion proportion of the senior bond so that the weighted total cost of contingent capital is minimized. In our case, the weighted total cost is minimized when about 30.0% of the senior bond converts to common shares in a trigger event.

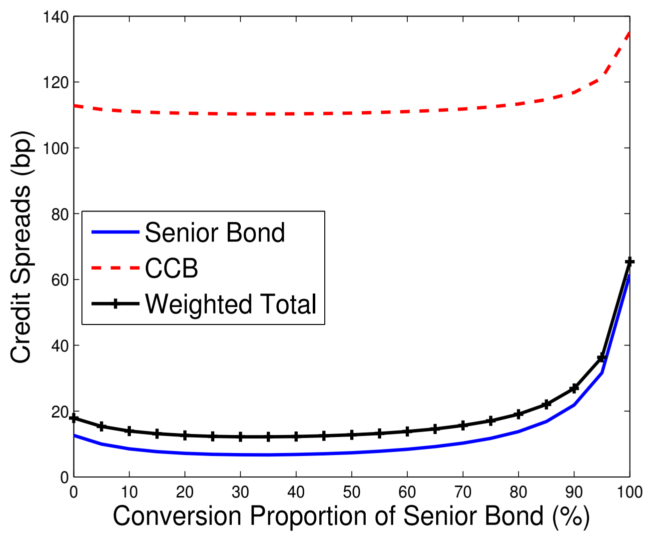

Figure 2.11: Credit spreads as a function of the conversion proportion of the senior bond. The conversion fraction is denoted as the percentage of the senior bond notional value. The weighted total credit spread is estimated with the corresponding notional value as the weight.

Reasonable Intervals for Fixed Imposed Losses

Similar to the analysis under fixed conversion price, now we present the reasonable interval under fixed imposed loss.

Only Conversion of CCB Table 2.7 presents the reasonable intervals under different trigger locations. As the conversion trigger moves up, the interval moves towards the origin. Under a relatively highly-located conversion trigger, conversion might happen when the firm's value is adequate, which implies a low loss proportion imposed to the senior bond and the existing common shares. As a result, two end points of the interval move towards the origin as the trigger moves up. Moreover, the size of the interval declines as the trigger location moves upwards. Therefore, the issuing firm must be careful if it uses a going-concern (highly-located) conversion trigger since there would be a relatively small room from which the firm can choose a fixed imposed loss so that the liability seniority is respected and the existing shareholders are not rewarded upon conversion. Additionally, the higher the trigger, the more concentrated the reasonable interval is around small imposed losses.

Table 2.7: Reasonable Intervals for Fixed Imposed Losses

Conversion Trigger	Reasonable Interval	Interval Size
4.5% CET1	$[0.6465\%, 9.3898\%]$	8.7433%
5.0% CET1	$[0.6020\%, 8.7816\%]$	8.1796%
7.0% CET1	$[0.4247\%, 6.3037\%]$	5.8790%
9.0% CET1	$[0.2481\%, 3.7532\%]$	3.5051%

Conversion of CCB and Partial Senior Bond The reasonable region is determined by the conversion proportion of the senior bond (f_S), the loss imposed on CCB investors at conversion (β_{CCB}) and to the converted part of the senior bond (β_S^C). Similar to our discussions under fixed conversion price, we vary the imposed loss to the converted part of the senior bond through the imposed loss ratio which is defined as β_S^C divided by β_{CCB}.

In Figure 2.12, the reasonable region is bounded by the two lines since under our calibration any combination on the panel would guarantee the seniority between the senior bond and deposits. In the special case of $f_S = 0.0\%$, the region is insensitive to the change of imposed loss ratio and the two lines are horizontally located at 0.6020% and 8.7816% respectively, providing a cross check for the reasonable interval in Table 2.7. As the senior bond participates in the conversion, the red dashed line shows some slope and the black line moves downwards slightly. The black line in the graph is not strictly horizontal but changes slightly under different imposed loss ratios due to the fixed write-down for CCB. As a greater fraction of the senior bond participates in the conversion, the senior bondholder bears more loss at conversion. Under this circumstance, the reasonable combination would move towards the origin as the conversion proportion of the senior bond increases, leading to the red line pressing towards the origin and the black line merges to the zero x-axis.

Fixing the loss imposed on CCB investors, the reasonable region shown in Figure 2.13 is completely determined by the red dashed line which represents the punishment to the existing shareholders at conversion. As the loss imposed on the CCB increases, the reasonable combinations begin to cluster towards the origin because given an imposed loss ratio, with the rise of the loss imposed to CCB, the conversion proportion of the senior bond has to be reduced in order to prevent existing shareholders expropriating wealth from contingent capital investors through conversion.

Assuming the imposed loss ratio between the (converted part of) senior bond and CCB is fixed, the reasonable region is bounded by two lines in Figure 2.14. As the imposed loss ratio goes to one, the loss imposed to the converted part of the senior bond approaches to that of the CCB and the area for the reasonable combinations reduces. Fixing a conversion proportion for the senior bond, if the imposed loss ratio is large or close to one, we have to control the loss imposed to CCB so that the total loss absorbed by contingent capital investors does not exceed the loss sustained by the original shareholders upon conversion.

2.5.4 Conversion Trigger

The second essential design feature of contingent capital is the conversion trigger. In our paper, we only focus on the capital ratio trigger as in Glasserman and Nouri [11]. In the case only CCB converts, Table 2.8 demonstrates that the cost of contingent capital is less sensitive to

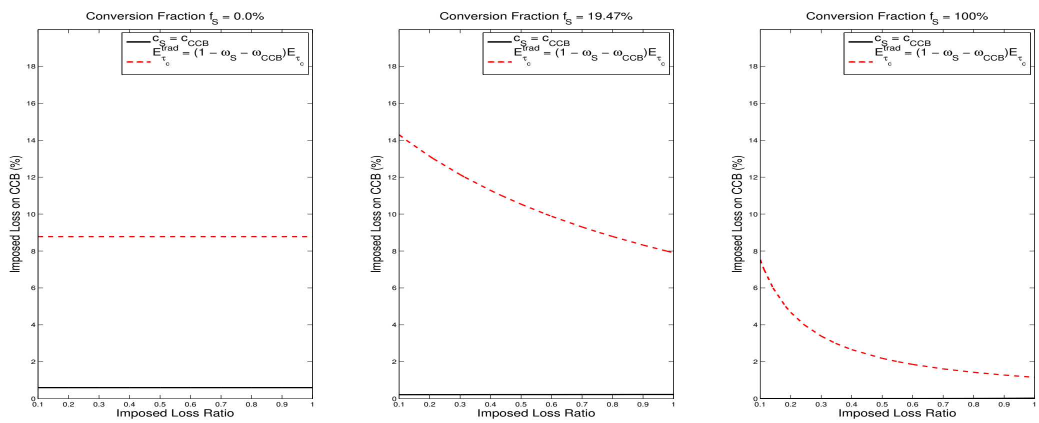

Figure 2.12: Reasonable region for the combination of imposed loss on CCB and the imposed loss ratio. The black solid lines represent the function $\hat{f}(f_S, \beta_S^C, \beta_{CCB}) = 0$ and the red dashed lines represent the function $\hat{g}(f_S, \beta_S^C, \beta_{CCB}) = 0$ with given conversion fraction f_S's.

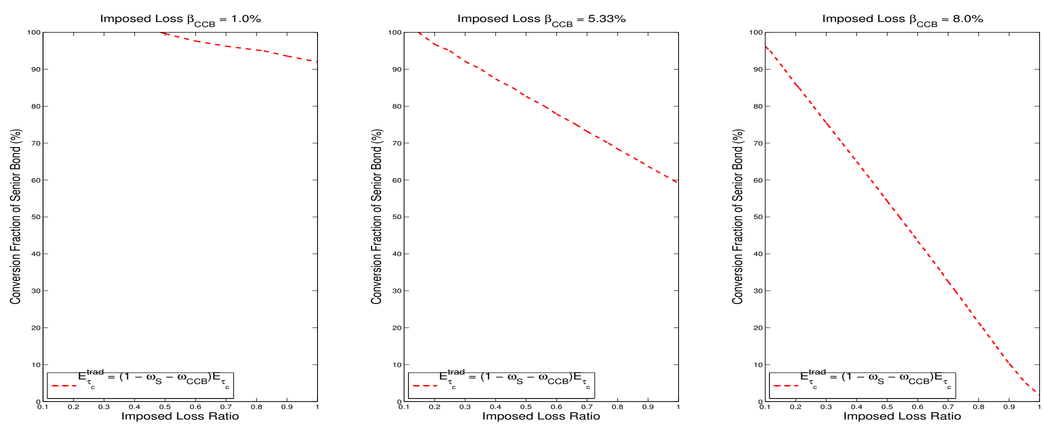

Figure 2.13: Reasonable region for the combination of imposed loss ratio and conversion fraction of the senior bond. The black solid lines represent the function $\hat{f}(f_S, \beta_S^C, \beta_{CCB}) = 0$ and the red dashed lines represent the function $\hat{g}(f_S, \beta_S^C, \beta_{CCB}) = 0$ with given β_{CCB}'s.

the trigger location under fixed imposed loss than under fixed conversion price. This is due to the reason that under a fixed conversion price, CCB investors receive a fixed fraction of the firm's residual value at conversion, and since this residual value increases with the location of the conversion trigger, increasing the trigger level effectively lowers the loss imposed on CCB investors at conversion. However, under fixed imposed loss, the effective redemption to the CCB upon conversion is a constant equal to the difference between the notional value and the fixed write-down. This phenomenon persists even when the senior bond participates in the conversion. As seen in Table 2.9, the credit spread of the CCB is relatively stable under fixed imposed loss but quite volatile under fixed conversion price, with respect to the change

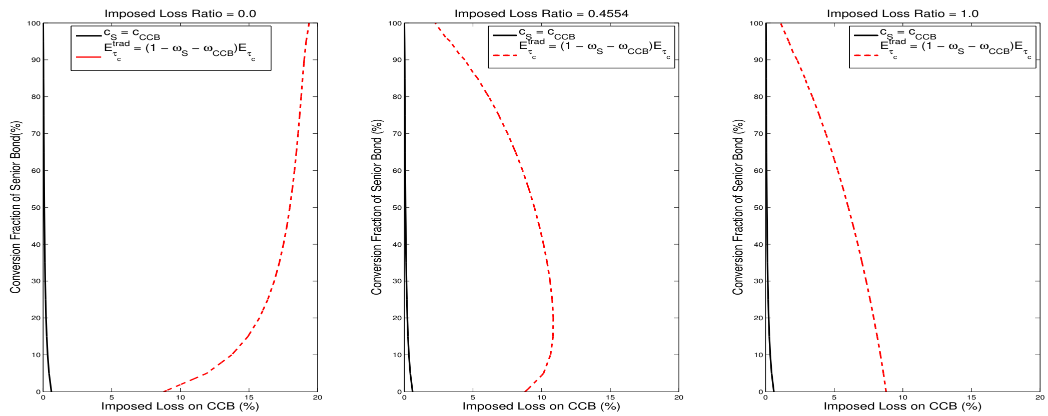

Figure 2.14: Reasonable region for the combination of imposed loss on CCB and the imposed loss ratio. The black solid lines represent the function $\hat{f}(f_S, \beta_S^C, \beta_{CCB}) = 0$ and the red dashed lines represent the function $\hat{g}(f_S, \beta_S^C, \beta_{CCB}) = 0$ with given imposed loss ratios.

of trigger location.

Therefore, the location of the capital-ratio trigger would impact the costs of issuing contingent capital. If the issuing firm wants to incorporate uncertainty into the trigger design, such as saying conversion would be triggered somewhere around 5.0% CET1 ratio rather than explicitly right at 5.0% CET1 ratio, it appears as though a fixed imposed loss would be preferable because the ambiguity of the trigger would not increase the costs of contingent capital significantly.

Table 2.8: Sensitivity to Trigger Location ($f_S = 0$)

	Trigger Location in CET 1 Ratio				
Conversion Terms	4.25%	4.5%	5.0%	5.25%	5.5%
Senior - Fixed Price	13bp	13bp	13bp	13bp	12bp
CCB - Fixed Price	234bp	196bp	113bp	68bp	12bp
Senior - Fixed Loss	13bp	13bp	13bp	13bp	13bp
CCB - Fixed Loss	101bp	105bp	113bp	117bp	122bp

Table 2.9: Sensitivity to Trigger Location ($f_S = 19.47\%$)

	Trigger Location in CET 1 Ratio				
Conversion Terms	4.25%	4.5%	5.0%	5.25%	5.5%
Senior - Fixed Price	13bp	11bp	7bp	5bp	3bp
CCB - Fixed Price	233bp	208bp	152bp	122bp	89bp
Senior - Fixed Loss	7bp	7bp	7bp	7bp	7bp
CCB - Fixed Loss	99bp	103bp	111bp	115bp	119bp

2.5.5 Term Structure under Simulations

So far our work is based on the assumption that all bonds are perpetual. This assumption allows us to derive analytical formulas to price contingent capital and other liabilities in the capital structure. However, since there are few perpetual bonds in the real world, it is meaningful to extend our pricing methodology under a finite time horizon and price contingent capital with finite maturities. I used C++ to implement the simulation.

One possible way is using Monte Carlo simulation. The basic idea is along each path, we discount back all the payments (coupon payments before conversion, dividends post conversion and equity value received upon conversion if conversion happens, or the principal if conversion does not happen). By averaging the discounted values of all simulated paths, we estimate the expectation of the present value for each liability. Equating the expected present value with the corresponding notional value, we may solve for the par yield of each liability in the capital structure. In the procedure, we need to pay attention to the estimation of the equity value received by contingent capital investors upon conversion. Under fixed imposed loss, it is simply as the difference between the notional value and the fixed write-down value (i.e. $(1-\beta_{CCB})L_J$). However, under a fixed conversion price, it depends on the firm's residual value at conversion which must be evaluated through sub-simulations. Figure 2.15 gives a brief introduction of our simulation procedure. Taking one asset value path as an example, it follows the dynamics (2.15) before conversion. When it hits the conversion level around $t = 0.38$ years, we evaluate the residual value at this moment because contingent capital investors receive their "principal payments" in terms of common shares at conversion. Let the conversion (hitting) time be the starting point of our sub-simulation, beyond which we simulate five asset value sub-paths after conversion following the dynamics (2.16). By discounting back the payments along those sub-paths to the conversion time and taking the average, we get the estimated residual value at conversion[22]. After that, we discount all the payments along the main path from the conversion time back to the beginning to calculate the present value of each liability in the capital structure. Repeating this process and equating the average present value of expected payments (including coupon payments, dividends and final principal payments or the value of common shares received from conversion) to the corresponding notional value of each liability in the capital structure, we solve for the par yields. The stopping criteria used in searching for the par yields is

$$\sqrt{\frac{(L_D - D_0)^2}{L_D^2} + \frac{(L_S - S_0)^2}{L_S^2} + \frac{(L_J - CCB_0)^2}{L_J^2}} < 0.1\%, \tag{2.44}$$

where D_0, S_0 and CCB_0 are discounted value of deposits, senior bond and CCB respectively and they are approximated from the simulation. The condition (2.44) implies the solved par yields only allow for a relevant difference between the discounted present values and notional values smaller than 0.1%.

[22] We did not evaluate the residual value at conversion by directly subtracting the value of unconverted liabilities from the total asset value because there are simulation errors which might destroy the strict equivalence of the asset value to the sum of the equity value and the liability value. Therefore, direct subtraction might lead to negative residual values at conversion and cause unnecessary noises in searching for the par yields. However, the simulation errors will vanish when the number of simulation paths is very large, which would be very time-consuming especially when a small time step is used.

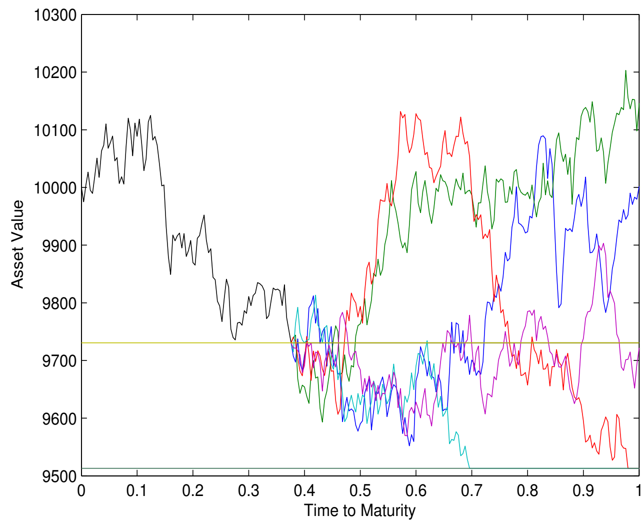

Figure 2.15: Simulation procedure under the fixed conversion price.The upper horizontal line is the conversion level and the lower horizontal line is the liquidation level.

Considering only conversion of the CCB, we use the time step $dt = 0.004$ as an approximation to the asset value's update each business day (252 business days in a year). Under a fixed loss, sub-simulation is not required so we simulate one hundred thousand paths to estimate the par yields. However, under a fixed price, since there are sub-simulations involved for each path hitting the conversion level, in order to keep the computational time reasonable, we use ten thousand paths in the main routine of simulation and one thousand paths in the sub-simulation for each path hitting the conversion level.

The upper panel of Figure 2.16 plots the term structures of the senior bond and CCB under fixed loss. If the firm has a relatively high asset volatility ($\sigma = 5.0\%$ or 3.0%), the term structure for the senior bond is humped-shaped while for the CCB it is monotonically decreasing. For the senior bondholder, it is possible that the firm has difficulties paying back the principals of liabilities within a short term. However, for long-term liabilities, since the repayments of principals are not so imminent, and in the long run, it is possible for the firm to recover after restructuring or corrective actions and be able to repay its liabilities, the risk premium decreases with increasing time to maturity. The humped-shaped term structure is not exclusive to the capital structure model. Analytical and empirical discussions can be found in Merton [20], Sarig and Warga [26] and Kim, Ramaswamy and Sundaresan [14], etc. and this humped-shaped term structure appears in the 2008 financial crisis according to Berg [4][23] and in 2013 debt-ceiling crisis according to Ozdagli and Peek [23][24]. Recalling that the relatively high asset volatility

[23]The author studies the credit default swaps for several maturities to extract the risk premium term structure from market prices and find that short-term risk premium have increased significantly during the financial crisis while the long-term risk premium remained almost unchanged during the financial crisis.

[24]They investigate the effects of the 2013 debt-ceiling crisis on the Treasury bill market and find that the 2013 debt-ceiling crisis reduced the demand for Treasury bills that were scheduled to mature right after the debt-ceiling deadline, but not for longer-term Treasury bills. Accordingly, they see a hump formed at the shorter end of the term structure of Treasury bill yields around the debt-ceiling deadline, with the term structure returning to more

(σ = 5.0%) is calibrated as the maximum value of the asset volatility of the Canadian representative bank during the 2008 financial crisis, it makes sense to perceive a humped-shaped term structure for the senior bond. For the CCB, with a relatively high asset volatility, conversion is likely to occur and the time of conversion is brought forward. In other words, the contingent capital investor bears loss at conversion earlier than under a relatively low asset volatility. Although the conversion could lower the liability commitments and inject capital to the firm immediately, the short maturity might still put the firm into the trouble of paying back all the liability principals due to reasons such as lack of liquidity. However, in the long run, with the capital boost from the conversion of CCB and some operational improvements, it is likely that the firm is able to fully repay its liability commitments. Along with a relatively long coupon or dividend payment stream, we observe a decreasing credit spread for the CCB as the time to maturity increases. Similar term structures are discussed in Merton [20]. If the firm has a relatively low asset volatility (σ = 1.00%), the credit spread of the senior bond and CCB is positively related to the time to maturity. This is a general shape of bond yield term structure, mainly including the concern for the increasing uncertainty in the long run.

In the lower panel of Figure 2.16, we graph the difference between the credit spread of the senior bond and CCB and of the corresponding liability in an otherwise identical traditional capital structure. With the difference staying in the negative panel, replacing the traditional junior bond with CCB would lessen the credit spread of the senior bond. However, the contingent capital bond is more expensive than the otherwise identical traditional junior bond due to the conversion feature and the positive write-down at conversion.

Similar observations can be found in Figure 2.17 under fixed conversion price. For example, we observe a decreasing or humped-shaped term structure under relatively high asset volatilities while an increasing term structure with relatively low asset volatility, which can be explained by the similar analysis as we made under fixed imposed loss.

Last but not the least, it is perceived from Figure 2.16 and Figure 2.17 that it requires more than twenty years[25] to maturity before the perpetual case is realized. The review of issuances of contingent capital in Appendix A show that in practice, the contingent capital are mainly long-term debts – all of the issued contingent capital have maturities more than ten years and some are perpetual debts. From this point of view, we can say that the main implications given by our model under the infinite maturity might still hold in practice.

2.6 Conclusions and Future Work

Starting from the structural model developed by Metzler and Reesor [21] to price the zero-coupon contingent capital, we extend the model to price contingent capital with fixed coupon payments. In terms of the perpetual case, the closed-form pricing formula is derived and solved numerically, giving fair prices for contingent capital under different conversion terms and trigger locations. In the finite-maturity case, we implement our model with Monte Carlo simulation and generate term structures having similar shapes with the classical Merton model [20]. We implement the pricing model based on the capital structures calibrated from the balance sheet

normal levels immediately after resolution of the crisis.

[25] Our numerical results demonstrate that when the time to maturity is extended to thirty years, the perpetual case is well replicated.

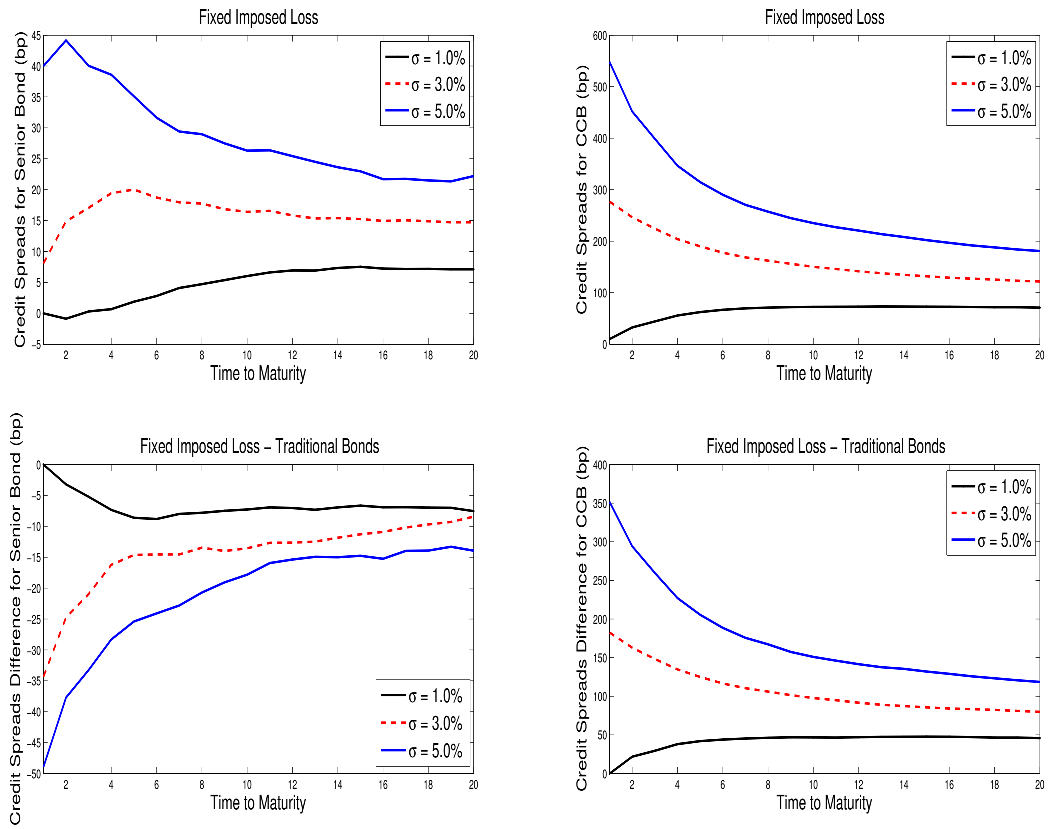

Figure 2.16: Term structures for liabilities under fixed imposed loss. The upper panel plots the credit spread of the senior bond and CCB. In the lower panel, it shows the difference between the credit spread of the senior bond and CCB and that of corresponding liability in an otherwise identical traditional capital structure.

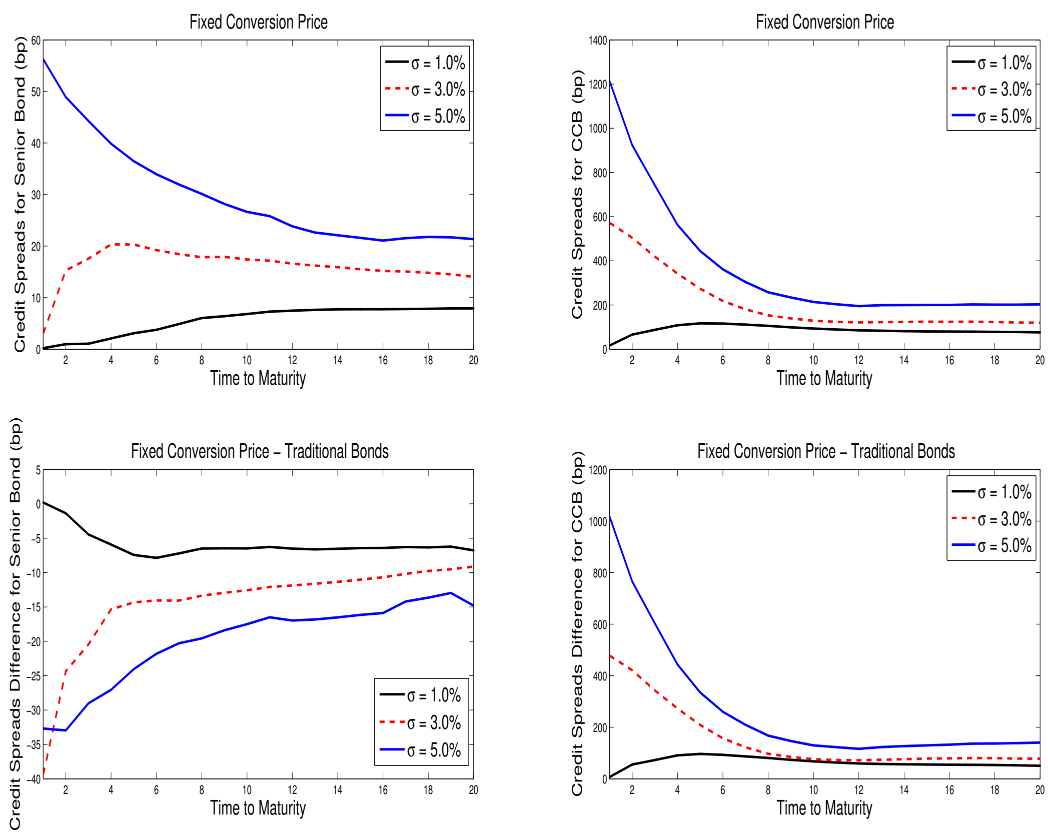

Figure 2.17: Term structures for liabilities under fixed conversion price. The upper panel plots the credit spread for the senior bond and CCB. In the lower panel, it shows the difference between the credit spread of the senior bond and CCB and of the corresponding liability in an otherwise identical traditional capital structure.

of a representative Canadian bank and discuss conversion terms and triggers under a regulatory perspective.

We did not compare the value of a zero-coupon CCB and the value of a CCB with fixed coupon payments on the quantitative level in this chapter but some insights can still be obtained comparing the results in this chapter and the results in Metzler and Reesor [21]. Generally, most of the results for the CCB with coupon payments discussed in this chapter are consistent with the results for the zero-coupon CCB studied in Metzler and Reesor [21]. For example, the remarkably narrow reasonable intervals for conversion terms, the sensitivity of the CCB price to the location of the conversion trigger under different conversion terms and the influence of firm-specific parameters on the value of CCBs.

As new shares are issued through the conversion of contingent capital, there exist dilution and ownership re-distribution after conversion, which might give rise to manipulation incentives to the contingent capital investors and the existing equity holders. For example, if the conversion price depends on the market stock price, contingent capital investors might have incentives to push down the stock price before conversion so that they could obtain a larger ownership stake. If the conversion trigger is based on the market stock price, the existing shareholders might want to trigger the conversion by pushing down the stock price and transfer wealth from contingent capital investors. Hence, an important avenue for future work is investigating the manipulation incentives when the conversion is imminent.

According to OSFI [19], the non-viability contingent capital bond (NVCC) would bring larger net benefits in the Canadian context than a CCB with the trigger well above the point of non-viability. NVCC is a contingent capital bond with conversion trigger at the point of non-viability. It is a special contingent capital because authorities can participate in deciding if the bank is close to or at the so-called point of non-viability and force the NVCC to convert. In other words, instead of using an objective conversion trigger which might be a capital ratio or an observable market variable, the trigger of NVCC depends on the knowledge and judgement of authorities and therefore is subjective. Thus, another division of our future work would be the evaluation of contingent capital with regulatory discretionary trigger.

Bibliography

[1] B. Albul, D. M. Jaffee, and A. Tchistyi. Contingent convertible bonds and capital structure decisions. *University of California, Berkeley, Working Paper*, April 2010.

[2] R.W. Anderson and S. Sundaresan. Design and valuation of debt contracts. *The Review of Financial Studies*, 9(1):37–68, 1996.

[3] N. Arjani and C. D'Souza. Calibrating a higher loss absorbency requirement for canadian banks. *Working Paper, Bank of Canada*, 2011.

[4] T. Berg. The term structure of risk premia new evidence from the financial crisis. *Working Paper Series, European Central Bank*, (1165), March 2010.

[5] M. Beyhaghi, C. D'Souza, and G. S. Roberts. Funding advantage and market discipline in the canadian banking sector. *Journal of Banking & Finance*, 48:396–410, November 2014.

[6] F. Black and John C. Cox. Valuing corporate securities: Some effects of bond indenture provisions. *The Journal of Finance*, 31(2):351–367, May 1976.

[7] N. Chen, P. Glasserman, B. Nouri, and M. Pelger. Cocos, bail-in, and tail risk. *Office of Financial Research Working Paper, Retrieved from SSRN http://dx.doi.org/10.2139/ssrn.2296462*, (4), January 2013.

[8] C. D'Souza and T. Gravelle. Contingent capital and bail-in debt: Tools for bank resolution. *Bank of Canada, Reports for Financial System Review*, December 2011.

[9] D. Duffie. *How Big Banks Fail and What to Do about It*. Princeton University Press, October 2010.

[10] M. J. Flannery. Stabilizing large financial institutions with contingent capital certificates. *Working Paper, University of Florida, Retrieved from SSRN http://dx.doi.org/10.2139/ssrn.1485689*, 2009.

[11] P. Glasserman and B. Nouri. Contingent capital with a capital-ratio trigger. *Management Science*, 58(10):1816–1833, April 2012.

[12] J. Hilscher and A. Raviv. Bank stability and market discipline: The effect of contingent capital on risk taking and default probability. *Journal of Corporate Finance*, 29:542–560, December 2014.

[13] J. C. Hull. *Options, Futures and Other Derivatives, 7th Edition*. Pearson Education International, 2009.

[14] I. J. Kim, K. Ramaswamy, and S. Sundaresan. Does default risk in coupons affect the valuation of corporate bonds?: A contingent claims model. *Financial Management*, 22(3):117–131, 1993.

[15] D. Lando. *Credit Risk Modeling: Theory and Applications*. Princeton University Press, 2004.

[16] H. E. Leland. Corporate debt value, bond convenants, and optimal capital structure. *The Journal of Finance*, 49(4):1213–1252, September 1994.

[17] H. E. Leland and K. B. Toft. Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads. *The Journal of Finance*, 51(3):987–1019, 1996.

[18] D. B. Madan and W. Schoutens. Conic coconuts: the pricing of contingent capital notes using conic finance,. *Mathematics and Financial Economics*, 4(2):87–106, Februray 2011.

[19] Office of Superintendent of Financial Institutions. Advisory on non-viability contingent capital. *OSFI Reports*, August 2011.

[20] R. C. Merton. On the pricing of corporate debt: The risk structure of interest rates. *The Journal of Finance*, 39(2):449–470, May 1974.

[21] A. Metzler and R. M. Reesor. Valuation and analysis of zero-coupon contingent capital bonds. *Mathematics and Financial Economics*, 9(2):85–109, March 2015.

[22] S. Ou, D. Chiu, and A. Metz. Corporate default and recovery rates, 1920-2010. *Working Paper, Moody's Investors Service*, February 2011.

[23] A. Ozdagli and J. Peek. Cliff notes: The effects of the 2013 debt-ceiling crisis. *Public Policy Briefs, Federal Reserve, Bank of Boston*, (13-9), November 2013.

[24] C. Pazarbasioglu, J. Zhou, V. Le Leslé, and M. Moore. Contingent capital: Economic rationale and design features. *Working Paper, IMF Staff Discussion Note*, January 2011.

[25] G. Pennacchi. A structural model of contingent bank capital. *Working Paper, the Federal Reserve Bank of Cleveland*, (10-04), March 2011.

[26] O. Sarig and A. Warga. Some empirical estimates of the risk structure of interest rates. *The Journal of Finance*, 44(5):1351–1360, December 1989.

Chapter 3

Short-Selling Incentives Near Conversion to Equity

In this chapter we address the issue of whether a long position in a CCB creates incentives to short the issuing firm's stock. We consider two types of incentives - "honest" and "dishonest". By an honest short we mean a short position that is taken in an effort to create a natural hedge against a decline in the value of the CCB. By a dishonest short we mean one that is put in place within an effort to push the market value of the stock below its fundamental value.

In order to quantify incentives to take honest short positions, we consider a firm having a traditional capital structure, as in Section 2.2. We compute the fraction of the firm's equity that would need to be shorted in order to completely hedge a long position in the firm's junior debt. Next we replace the junior debt with contingent capital and compute the fraction of the firm's equity that would need to be shorted in order to completely hedge a long position in the firm's contingent capital. We find that for the representative Canadian institution, near the point of conversion CCB investors would need to short nearly three times as much (4.0% versus 1.38%) of the firm's equity as would investors in otherwise identical junior debt. Relatively speaking, then, the introduction of contingent capital would create much stronger incentives to take honest short positions; in absolute terms it is not clear that these incentives would be a significant concern.

Quantifying incentives to take dishonest short positions is much more challenging. The first step is to understand how such incentives might arise. To this end consider an investor that owns a CCB for which the conversion price is the firm's stock price at conversion. Suppose that conversion appears imminent in the sense that the firm's CET1 ratio is 5.2% and the conversion trigger is a CET1 ratio of 5.0%. Further suppose that if the firm's CET1 ratio were to drop to 5.0% then the fair/fundamental value of the firm's stock would be \$5 per share. If, by shorting the firm's stock, the CCB investor is able to push the market price down to 80% of its fundamental value, then the investor will end up paying \$4 per share for something that should cost \$5 per share. Alternatively, the investor will end up owning 25% more of the firm's equity than they would have owned in the absence of manipulation. It is precisely this type of incentive that is a major concern for regulators and potential issuers of contingent capital, and this chapter appears to be the first academic work to rigorously develop a model that can be used to investigate the issue.

3.1 Honest Incentives – Hedge

We continue using the notation from Chapter 2. In the case that the firm's asset value falls, the value of the CCB and equity fall as well. Therefore, the profit from shorting equity in the case that the firm's asset value falls would cover the loss in the long positions of CCB. When the firm's financial situation is deteriorating and conversion is likely, the short-selling behaviour would presumably intensify (Spiegeleer and Schoutens [26] [25]).

Based on (2.29) and by Itô's Lemma, given an asset value at time t, we can derive the diffusion process for CCB (see Appendix C for details) and therefore the diffusion term

$$\sigma_{CCB}(V_t) = (\omega_{CCB} E_{\tau_c} - \frac{c_{CCB} L_J}{r})\sigma V_t U'(V_t), \tag{3.1}$$

where $U(V_t) = u(\bar{v}_t) = \mathbb{E}_{t,v}[e^{-r(\tau_c - t)}]$ with the asset-to-liability ratio $\bar{v}_t = V_t/L$ and $u(\bar{v}_t)$ is given by (2.24). Similarly, we can obtain the diffusion terms for deposits ($\sigma_D(V_t)$) and senior bond ($\sigma_S(V_t)$). To this end, the diffusion term for the equity is

$$\sigma_E(V_t) = \sigma V_t - \sigma_D(V_t) - \sigma_S(V_t) - \sigma_{CCB}(V_t), \quad t \le \tau_c, \tag{3.2}$$

which decomposes the variation of the firm value as the sum of variations from each tranche in the capital structure.

Define

$$\Delta_t = \frac{\sigma_{CCB}(V_t)}{\sigma_E(V_t)}. \tag{3.3}$$

Considering a portfolio consisting of (i) a long position in the CCB and (ii) a short position in $100\Delta_t\%$ of the firm's outstanding equity, is (locally) riskless. Let $\pi_t = CCB_t - \Delta_t E_t$ be the time-t value of the portfolio. Using the fact that in the absence of arbitrage all assets have the same Sharpe ratio[1], we obtain $d\pi_t = (1 - \Delta_t) r dt$. Therefore, by shorting $100\Delta_t\%$ of the firm's equity, CCB investors of the firm offset the gain or loss in their long positions of CCB given the firm value at the moment. From this perspective, Δ_t measures the short-selling pressure from CCB investors given the firm value at time t.

Similarly, under a traditional capital structure, define,

$$\Delta_t^{trad} = \frac{\sigma_J^{trad}(V_t^{trad})}{\sigma_E^{trad}(V_t^{trad})}, \tag{3.4}$$

where $\sigma_J^{trad}(V_t^{trad})$ and $\sigma_E^{trad}(V_t^{trad})$ are the diffusion terms of the junior bond and equity in the traditional capital structure at time t. Intuitively, Δ_t^{trad} can be thought as the sensitivity of a change in the otherwise identical traditional junior bond value for a change in the issuing firm's equity value. Although there is no conversion risk for the otherwise identical traditional junior bondholders, they can offset the possible loss in their long positions of the traditional junior bonds by shorting the firm's shares as well.

Figure 3.1 shows the short sale proportions of the issuing firm's equity given different asset values using the parameters and capital structure given in Table 2.2. At the conversion level

[1] That is, $\frac{\mu_{CCB}(V_t)-r}{\sigma_{CCB}(V_t)} = \frac{\mu_E(V_t)-r}{\sigma_E(V_t)}$, where $\mu_{CCB}(V_t)$ and $\mu_E(V_t)$ are the growth rate of CCB and equity value at time t respectively.

(the starting point of the x-axis), CCB investors would need to short approximately 4.0% of the firm's outstanding equity in order to fully hedge their position; comparatively, the otherwise identical traditional junior bondholders only need to short approximately 1.38% of the firm's equity to fully hedge their position. When the asset value keeps growing, two deltas approach to zero because there is little conversion risk for the CCB and little default risk for the otherwise identical traditional junior bond in which case both of them tend to be risk-free. The potential for conversion is the main difference between the CCB and the otherwise identical traditional junior bond. As long as the conversion level locates above the liquidation level, we always have $\Delta_t > \Delta_t^{trad}$ because conversion always occurs prior to liquidation and it is the extra risk except the default risk confronted by CCB investors.

Remark So far we compare the hedge ratio of CCB with that of the otherwise identical traditional junior bond and confirm the existence of the extra short-selling incentives from CCB investors. In other words, if we replace the traditional junior bond in the capital structure with CCB, the short-selling incentives of CCB investors will be significantly intensified as the firm value approaches to the conversion level.

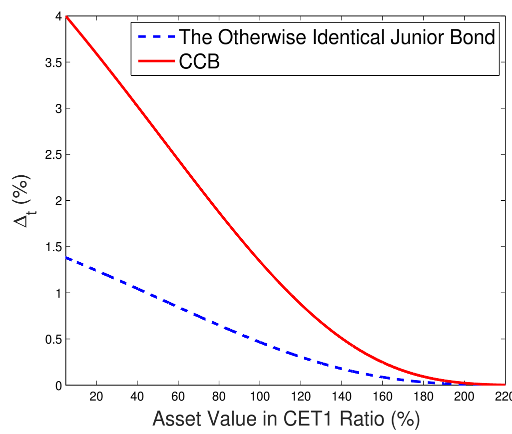

Figure 3.1: Delta of CCB and the otherwise identical traditional junior bond.

3.2 Dishonest Incentives – Convert at A Favourable Price

In the vast majority of existing CCBs the conversion price is related to the firm's stock price at conversion. For example, Credit Suisse Group (2011 and 2012) uses a volume weighted average stock price for a preceding time period with a predefined floor price as the conversion price, and Bank of Cyprus (2011) bases the conversion price on the market price with both a floor and a ceiling price, etc. The issuances made by Canadian banks, such as RBC, CIBC and BMO also relate the conversion price to the market stock price. A review of the issuances and developments of CCBs can refer to Avdjiev, Kartasheva and Bogdanova [2].

If the conversion price is highly correlated with the firm's stock price at or near conversion then, all else being equal, a lower stock price translates into a larger ownership stake post-conversion for CCB investors. With the prospect of conversion, CCB holders might short sell the firm's shares to press down the market stock price in advance. If the conversion occurs eventually, they would receive more shares from the conversion than not doing so. Therefore, CCB investors receive a greater fraction of the equity value than they deserve and the shares they receive are probably under-valued upon conversion. The difference between the conversion price and the rational conversion price based on the firm's fundamentals is seized by CCB investors. Such incentives are a serious concern for regulators. More qualitative discussions about possible manipulation (short-selling) incentives can be found in Squam Lake Working Group on Financial Regulation [27], Basel Committee on Banking Supervision [4][13], Ioannides and Skinner [5], Order and Lai [23], D'Souza and Gravelle [9] and Duffie [10], etc.

Although the aforementioned studies have discussed the CCB investors' possible manipulation incentives qualitatively, the issue has never been rigorously analyzed. In this chapter, we use the structural model developed in Chapter 2 to rigorously investigate incentives for investors that own CCBs to short the issuing firm's stock. Canadian banking data are employed in this chapter in order to provide a closer look of the potential manipulation risk in the real world issuance of CCBs. In order to analyze potential incentives for dishonest short positions we allow temporary deviation between the market stock price and its fundamental value. The effect is to mimic the potential effect of large-scale short selling on the market value of the stock. Our model provides a practical way to analyze investors' possible return from manipulations under different market-based conversion prices, and therefore sheds light on the short-selling incentives from CCB investors.

Three market-based conversion prices are discussed in this paper, the contemporary market stock price, the trailing average stock price and the contemporary market stock price plus a floor. We find that

- the direct use of contemporary market stock price could tempt the manipulation incentives;

- as might be expected, a relatively high floor price can reduce incentives to take dishonest short positions;

- the use of trailing average would cut down the possible return from manipulations to a limited extent, depending on the number of historical data used in calculating the trailing average.

This chapter is organized as follows. Firstly, we model the deviation between the market stock price and its fundamental value. Then we discuss the assumption of the time to close investors' short positions. In Section 3.3.3 we define and investigate two returns from investors' manipulations - the return from conversion and the return from short-selling. We present our analytical and simulation results in Section 3.4 and conclude in Section 3.5.

3.3 Model

Assuming a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where $\mathbb{P}$ is the physical pricing measure. Denote the expectation $\tilde{\mathbb{E}}$ with respect to the measure $\mathbb{P}$. We continue using the notation from Chapter 2. Under the physical measure, before conversion, the asset value dynamics is

$$dV_t = ((\mu - q)V_t - (c_D L_D + c_S L_S + c_{CCB} L_J))\, dt + \sigma V_t dW_t, \quad t < \tau_c \tag{3.5}$$

where the constant $\mu > 0$ is the gross growth rate in the firm's assets. We do not consider the conversion of the senior bond in the capital structure in this chapter so post conversion, the firm ceases to pay the CCB's coupon and the asset value dynamics becomes

$$dV_t = [(\mu - q)V_t - (c_D L_D + c_S L_S)]\, dt + \sigma V_t dW_t, \quad t \geq \tau_c, \tag{3.6}$$

where the conversion time τ_c is defined as the first passage time of the pre-determined asset-liability ratio b^*,

$$\tau_c = \inf\{t \geq 0 : \bar{v}_t \leq b^*\}. \tag{3.7}$$

To analyze the short-selling incentives from CCB investors, we are going to extend the model proposed in Chapter 2, where the fundamental equity value depends on the asset value and cannot be manipulated, by assuming the market equity value could be manipulated by investors and there will be a deviation between the market and fundamental value under manipulations.

3.3.1 Impact of Short-Selling

Conventional wisdom tells that short selling generally drives down the market price of the stock being shorted. Assuming that short-selling only drives down the market share price without influencing fundamentals, short-selling would induce a deviation between the market share price and the fundamental value. One would assume that such a deviation would not persist. In the long run, the market share price would revert back towards its fundamental value. Therefore, in our model we allow the market stock price to first deviate from and then revert back to the fundamental value. This assumption is inspired by the pervasive asset price movement pattern in the market – short-term momentum and long-term reversal after unexpected information and events[2]. Various empirical studies have suggested the existence of the pattern and the interpretation of this pattern has been linked strongly to financial behavioural models[3].

We assume that the investor opens the short position when the CET1 ratio strikes a lower threshold. Define $\tau_{s/o}$ as the time when short-selling happens. Since there is a one-to-one relation between the asset value and the CET1 ratio, the short position is opened upon first passage of asset value to some lower threshold

$$\tau_{s/o} = \inf\{t \geq 0 : V_t = b_{s/o} L\}, \tag{3.8}$$

[2]According to Vayanos and Woolley [29], momentum is a tendency of rising asset price to rise further, and falling asset price to keep falling; reversal describes a long-term phenomenon – assets that perform well in the long run tend to subsequently underperform, or vice versa.

[3]Empirical studies include Jegadeesh and Titman [15], De Bont and Thaler [6], Chan [7], Rouwenhorst [24] and Fama and French [11], etc. For interpretation using financial behaviour models please see Daniel, Hirshleifer and Subrahmanyam [8], Barberis, Shleifer and Vishny [3] and Hong and Stein [14], etc.

where $b_{s/o}$ is the asset-liability ratio corresponding to the critical CET1 trigger where short position is opened. Define the stock price based on the fundamental value as P_t. It is equal to the fundamental equity value divided by the existing number of shares. Based on our assumption, the market stock price $\tilde{P}_t$ and the fundamental value P_t are related by a continuous function $y(t)$,

$$\tilde{P}_{t+\tau_{s/o}} = y(t)P_{t+\tau_{s/o}}, \quad t \geq 0. \tag{3.9}$$

We expect the function $y(t)$ to capture both the deviation and reversion process of the market stock price. In mathematics, it has the property $y(0) = 1$ and $\lim_{t\to\infty} y(t) = 1$. The first property tells that it always takes time to realize the deviation between the market and fundamental value and the second one describes a long-term full reversion process.

In this chapter, we use one of the candidates

$$y(t) = 1 - K\alpha t e^{1-\alpha t}, \quad t \geq 0, \tag{3.10}$$

where the constant K measures the maximum price deviation between $\tilde{P}_{t+\tau_{s/o}}$ and $P_{t+\tau_{s/o}}$ and the constant α controls the speed of deviation and reversion. For simplification, we assume the CCB investors only short once at $\tau_{s/o}$ and there are no transaction costs. Figure 3.2 presents the percentage deviation of the market share price from the fundamental value. Moreover, a piecewise version of the function $y(t)$ in (3.10) would allow us to control the deviation and reversion speed respectively [4].

3.3.2 Closing the Short Position

We assume the investors close their short positions when either (1) the mark-to-market value of the short position falls below some pre-specified stop-loss level or (2) conversion occurs. As we can see the latter is the hitting time of asset value to a fixed lower level corresponding to a pre-determined asset-liability ratio b^*, while the former is the hitting time of asset value to a moving upper barrier.

Let the parameter $h \geq 1.0$ be some stop-loss level chosen by the investors. For example, $h = 1.15$ means that the stop-loss level corresponds to the market stock price increasing by 15%. The stop-loss level is hit at the stopping time

$$\tau_{s/l} = \tau_{s/o} + T_{s/l}, \tag{3.12}$$

where $T_{s/l}$ measures the elapsed time between hitting times $\tau_{s/o}$ and $\tau_{s/l}$,

$$T_{s/l} = \inf\{t \geq 0 : \tilde{P}_{t+\tau_{s/o}} = h\tilde{P}_{\tau_{s/o}}\}, \quad h \geq 1.0. \tag{3.13}$$

In other words, once the short sellers' losses exceed $100(h-1)\%$, they will close their short positions. Equation (3.12) and (3.13) indicate that $\tau_{s/l}$ is decided by the market equity value.

[4] For example, let

$$y(t) = \begin{cases} 1 - K\alpha_d t e^{1-\alpha_d t}, & 0 < t \leq 1/\alpha_d, \\ 1 - K\alpha_r t e^{1-\alpha_r t}, & 1/\alpha_d < t, \end{cases} \tag{3.11}$$

where the parameter α_d and α_r control the deviation and reversion speed respectively. For example, if $\alpha_d = 4$ and $\alpha_r = 7.16$, the deviation time and reversal time is approximately 0.25 years and 1.02 years after $\tau_{s/o}$.

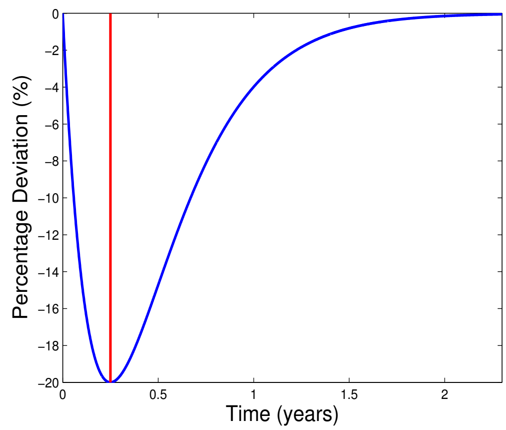

Figure 3.2: The percentage deviation of the market share price from the fundamental value (with $\alpha = 4, K = 0.2$ in (3.10)). The red vertical line stands at the point ($t = 0.25$) when the maximum (20%) difference is realized. The left side of the red line illustrates the deviation process of the market share price from the fundamental value. The right side depicts the reversion process of the market share price towards the fundamental value.

However, the conversion time τ_c is decided by the fundamental asset (equity) value[5]. Therefore, the two hitting times correspond to two different but related processes. Using (3.10), we rewrite

$$\begin{aligned} T_{s/l} &= \inf\{t \geq 0 : \tilde{P}_{t+\tau_{s/o}} = h\tilde{P}_{\tau_{s/o}}\}, \\ &= \inf\left\{t \geq 0 : y(t)P_{t+\tau_{s/o}} = hP_{\tau_{s/o}}\right\}, \qquad (3.14) \\ &= \inf\left\{t \geq 0 : P_{t+\tau_{s/o}} = \frac{hP_{\tau_{s/o}}}{y(t)}\right\}. \qquad (3.15) \end{aligned}$$

With this transformation, the hitting time $\tau_{s/l}$ is determined by the trajectory of the fundamental value as well. Since the fundamental equity value is implied by the fundamental asset value under the structural model, assumption (3.15) can be restated in terms of the asset value,

$$T_{s/l} = \inf\left\{t \geq 0 : E_{t+\tau_{s/o}}(V_{t+\tau_{s/o}}) = \frac{hE_{\tau_{s/o}}(V_{\tau_{s/o}})}{y(t)}\right\}, \qquad (3.16)$$

where $E_t(V_t)$ is the fundamental equity value in term of its corresponding asset value implied by the structural model. Due to the correspondence between the fundamental equity value and asset value, we can understand the assumption to close short positions in term of the asset value instead of the equity value. Once the short sell starts, our framework includes two barriers in term of the asset value – the lower barrier is the conversion level and the upper barrier is the stop-loss requirement level. Hitting either barrier would make CCB investors close their short positions. In the perpetual case, investors' short positions are closed at

$$\tau_{s/c} = \min\{\tau_c, \tau_{s/l}\}. \qquad (3.17)$$

[5]Recall that under the structural model, the firm's equity value depends on the firm's asset value.

If the asset value path hits the conversion level first, we have $\tau_{s/c} = \tau_c$ and investors make a positive profit on the short sale; however, if the asset value path hits the stop-loss requirement level first, we have $\tau_{s/c} = \tau_{s/l}$ then investors lose money on the short sale. It is noteworthy that according to (3.16), our upper barrier is a *moving barrier* rather than a constant barrier as the lower barrier (fixed conversion level) because it varies with the time elapse.

For simplification, we assume that CCB investors will not use shares they receive from conversion to cover their short positions. This assumption might be relaxed if we introduce the parameter named short interest, which is defined as the number of shorted shares. However, it is hard to quantify the short interest corresponding to a delayed price fall after short sales because the price drop might be influenced by subjective factors such as investors' predictions and behaviours in the market. Therefore, we always assume CCB investors would buy shares from the market on the contemporary market stock price to cover their short positions at $\tau_{s/c}$.

3.3.3 Return Analysis

We are going to analyze investors' incentives to enter dishonest short positions by decomposing the potential return into two parts. One is the return from conversion (R_c), which quantifies the difference between the fundamental-based conversion price and the market-based conversion price. It distinguishes CCB investors' short-selling behaviours from the regular short-sellers' because it is a specific return for CCB investors rather than a general return for all regular short sellers. The other is the return from short-selling (R_s), which measures investors' profits or losses by holding their short positions. It is defined the same way as the return for all regular short-sellers and includes short-selling behaviours from both CCB investors and regular short-sellers. The total return from the dishonest incentives is defined as a weighted summation of the two components

$$R_{total} := w(m)R_c + (1 - w(m))R_s, \tag{3.18}$$

where m is the relative size of the short position to the notional value of the CCB and $w(m) = 1/(1 + m)$. For example, if $m = 2$ then investors must short \$2 worth of equity for every \$1 notional of the CCB in order to have the desired effect on the firm's stock price.

Analyzing CCB investors' potential returns under manipulations would shed a light on investors' manipulation incentives. By employing Monte Carlo simulation, we further our studies under different conversion prices and discuss their effects of restraining potential manipulation incentives.

Return from Conversion (R_c)

The return from conversion is defined as the difference between the conversion price (the price CCB investors actually "pay" for the shares) and the rational conversion price in the absence of manipulations (the fair price CCB investors should "pay"). The occurrence of conversion only depends on the behaviour of the fundamental firm value. That is, if the fundamental firm value hits the conversion level, conversion occurs and the conversion price is decided by the market stock price at that moment. Although we define our conversion level using the accounting based CET1 ratio, the conversion time decided by the fundamental value path potentially merges the judgement of regulators about conversion because the fundamental value is not observable from the market. Figure 3.3 describes a situation when the fundamental firm value hits the conversion level and the market stock price is below the fundamental value at conversion. The difference between what the conversion *should be* (i.e. what the price would be in the absence of manipulation) and what the conversion price actually *is*, represents a transfer of value from equity investors to CCB investors.

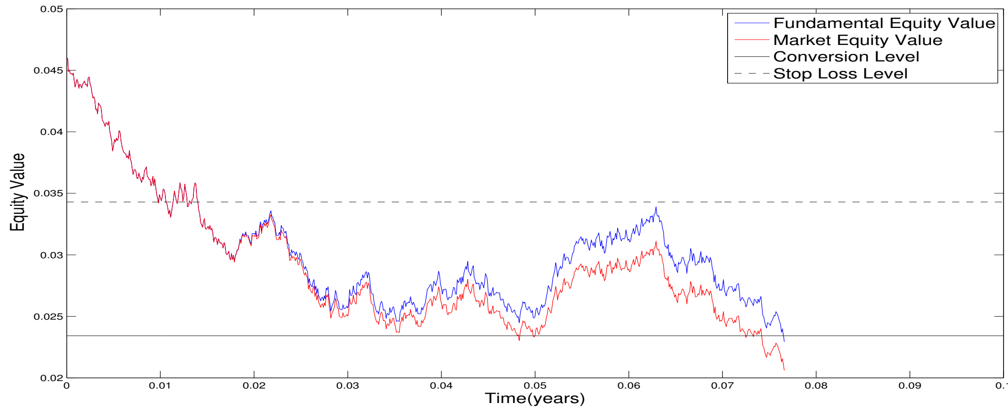

Figure 3.3: Short-selling causes the deviation between the market equity value and the fundamental value. Conversion happens because the fundamental equity value hits the conversion level. Investors close their short positions at conversion.

We define the return from conversion

$$R_c = \frac{P_{conv}}{\tilde{P}_{conv}} - 1, \tag{3.19}$$

where P_{conv} is what the conversion price would have been in the absence of manipulation and $\tilde{P}_{conv}$ is what the conversion price actually is. Since we assume that manipulations would depress the market stock price below the fundamental value which implies $P_{conv} \geq \tilde{P}_{conv}$, the return from conversion will stay non-negative. If conversion never occurs, the return from conversion is defined as zero. The conversion price could be the contemporary market stock price at conversion, the trailing average market stock price in a specified look-back period or a market stock price plus a floor.

Contemporary Market Stock Price We start from considering the simplest case, where the conversion price is the contemporary market stock price at conversion ($\tilde{P}_{conv} = \tilde{P}_{\tau_c}$). Let $T_c = \tau_c - \tau_{s/o}$ measure the amount of time that elapses between the opening of the short position and conversion and the expectation $\tilde{\mathbb{E}}_{\tau_{s/o},v}[R_c] = \tilde{\mathbb{E}}[R_c|V_{\tau_{s/o}} = v]$. By assumption (3.10), we have

$$\tilde{\mathbb{E}}_{\tau_{s/o},v}[R_c] = \tilde{\mathbb{E}}_{\tau_{s/o},v}\left[\frac{1}{y(T_c)} - 1\right]. \tag{3.20}$$

It is clear that $R_c = g_1(T_c)$ and $g_1(t) = (1 - y(t))/y(t)$. Thus the return from conversion is

$$\tilde{\mathbb{E}}_{\tau_{s/o},v}[R_c] = \int_0^\infty g_1(t) f_{\tau_{s/o},v}(t)dt, \tag{3.21}$$

where $f_{\tau_{s/o},v}(t)$ is the density function of T_c given the manipulation occurs at $\tau_{s/o}$ and the firm's asset value is v at that moment. Note that in general the function f can be defective in the sense that it does not integrate to one, and this will occur whenever there is a positive probability that conversion never occurs. Since T_c is the hitting time of an affine GBM beginning at the asset-liability ratio $b_{s/o}$ to the level b^*, the Laplace transform of f is available in closed form (Metzler [22]). In addition, as we only focus on the return from conversion here, the density function is defined in a one-barrier (conversion level) framework. This is because even if investors cover their short positions prior to conversion due to the stop-loss assumption, conversion might happen afterwards when there is still difference between the fundamental value and the market price, in which case CCB investors might still obtain benefits with the conversion price below its fundamental value.

Let $F_{\tau_{s/o},v}(t) = \int_t^\infty f_{\tau_{s/o},v}(s)ds$ and $\hat{F}_{\tau_{s/o},v}(t) = F_{\tau_{s/o},v}(t)/F_{\tau_{s/o},v}(0)$. Then $\hat{F}_{\tau_{s/o},v}(t)$ is the survivor function of a proper density in the sense that $\hat{F}_{\tau_{s/o},v}(0) = 1$, whereas $F_{\tau_{s/o},v}(0)$ could be less than one which will cause problems for the Abate and Whitt [1] we will use later. Using the fact that $F_{\tau_{s/o},v}(0) = \mathcal{L}\left\{f_{\tau_{s/o},v}(t)\right\}(0)$, it is easy to derive

$$\mathcal{L}\left\{F_{\tau_{s/o},v}(t)\right\}(\beta) = \frac{1}{\beta}\mathcal{L}\left\{f_{\tau_{s/o},v}(t)\right\}(0) - \frac{1}{\beta}\mathcal{L}\left\{f_{\tau_{s/o},v}(t)\right\}(\beta). \tag{3.22}$$

As a result, we have

$$\mathcal{L}\left\{\hat{F}_{\tau_{s/o},v}(t)\right\}(\beta) = \frac{1}{\beta} - \frac{1}{\beta}\frac{\mathcal{L}\left\{f_{\tau_{s/o},v}(t)\right\}(\beta)}{\mathcal{L}\left\{f_{\tau_{s/o},v}(t)\right\}(0)}, \tag{3.23}$$

which is in closed form since $\mathcal{L}\left\{f_{\tau_{s/o},v}(t)\right\}(\beta)$ is available in closed form according to Metzler [22]. As the function $\hat{F}$ is the survivor function of a proper density, it follows that it can be recovered numerically from its Laplace transform using the algorithm in Abate and Whitt [1]. Using the techniques of partial integration, we can rewrite (3.21) as

$$\tilde{\mathbb{E}}_{\tau_{s/o},v}[R_c] = \int_0^\infty F_{\tau_{s/o},v}(t)g_1'(t)dt = \mathcal{L}\left\{f_{\tau_{s/o},v}(t)\right\}(0)\int_0^\infty \hat{F}_{\tau_{s/o},v}(t)g_1'(t)dt \tag{3.24}$$

We use numerical techniques to estimate the integral term in (3.24) and start by decomposing it as

$$\int_0^\infty \hat{F}_{\tau_{s/o},v}(t)g_1'(t)dt = \int_0^{T^*} g_1'(t)\hat{F}_{\tau_{s/o},v}(t)dt + \int_{T^*}^{+\infty} g_1'(t)\hat{F}_{\tau_{s/o},v}(t)dt, \tag{3.25}$$

where T^* is a quantity beyond which the integrand value is very small and negligible[6]. As a result, we can approximate the expected return from conversion by

$$\tilde{\mathbb{E}}_{\tau_{s/o},v}[R_c] \approx \mathcal{L}\left\{f_{\tau_{s/o},v}(t)\right\}(0)\int_0^{T^*} g_1'(t)\hat{F}_{\tau_{s/o},v}(t)dt. \tag{3.26}$$

[6]We can prove that $g_1'(t) \to 0$ as $t \to +\infty$.

So far our discussion illustrates that the error of this approximation includes the estimation error of $\hat{F}_{\tau_{s/o},\nu}(t)$ from using the algorithm of Abate and Whitt [1] and the truncation error.

The survivor function $\hat{F}_{\tau_{s/o},\nu}(t)$ approaches to zero as $t \to \infty$. Additionally, we can verify that the derivative of the function $g_1(t)$ is well bounded. Therefore, one way of choosing T^* is to make the value of the survivor function within the interval $(T^*, +\infty)$ approximately zero. By doing so, we can effectively control the truncation error since the integrand is approximately zero in the interval $(T^*, +\infty)$. Since the survivor function can be thought as the complementary cumulative distribution function, we can prove that the condition $\hat{F}_{\tau_{s/o},\nu}(T^*) \approx 0$ is equivalent to $\mathbb{P}_{\tau_{s/o},\nu}(T_c \leq T^*) \approx \mathbb{P}_{\tau_{s/o},\nu}(T_c < +\infty)$. It means that once the short position is opened, if conversion does not happen within T^*, the probability of conversion happening after T^* is almost zero.

Trailing Average Market Stock Price Thus far we have discussed the return from conversion when the conversion price is the contemporary market stock price upon conversion. In practice, however, it is more common for the conversion price to be a n-day trailing average market stock price prior to conversion, that is,

$$P_{conv} = \frac{1}{n}\sum_{i=0}^{n-1} P_{\bar{t}_i}, \tag{3.27}$$

where $\bar{t}_i = \tau_c - i\Delta t$ with $\Delta t = 1/250$ is the i-th day in the look-back period starting from the conversion time τ_c. Unfortunately, here, we cannot simplify the return from conversion to a neat integral as we did when the conversion price is the contemporary market stock price. According to the definition, we have

$$\tilde{\mathbb{E}}_{\tau_{s/o},\nu}[R_c] = \tilde{\mathbb{E}}_{\tau_{s/o},\nu}\left[\frac{\frac{1}{n}\sum_{i=0}^{n-1} NP_{\bar{t}_i}}{\frac{1}{n}\sum_{i=0}^{n-1} N\tilde{P}_{\bar{t}_i}} - 1\right] = \tilde{\mathbb{E}}_{\tau_{s/o},\nu}\left[\frac{\frac{1}{n}\sum_{i=0}^{n-1} E_{\bar{t}_i}}{\frac{1}{n}\sum_{i=0}^{n-1} \tilde{E}_{\bar{t}_i}} - 1\right], \tag{3.28}$$

where as a reminder, N is the number of existing common shares before conversion. Equation (3.28) shows that the return from conversion depends on the historical fundamental and market equity value ($E_{\bar{t}_i}$ and $\tilde{E}_{\bar{t}_i}$) before conversion.

Let ω_{trail} be the fraction of the firm's equity that CCB investors would receive upon conversion in the absence of manipulation and $\tilde{\omega}_{trail}$ be the ownership investors will receive with manipulations. Following the structural model proposed in Chapter 2, the conversion term determines the ownership stake distributed to contingent capital investors at conversion and therefore the costs (coupon rates) of liabilities in the capital structure, which in turn determines the equity value. As a result,

$$E_t(\omega_{trail}) = V_t - D_t - S_t - CCB_t(\omega_{trail}) - BC_t, \quad t < \tau_c, \tag{3.29}$$

where D_t, S_t and CCB_t are value of deposits, senior bond and CCB in the capital structure and can be estimated by (2.27), (2.28) and (2.29) respectively, BC_t is the discounted bankruptcy cost at time t. Furthermore, by the fact that $\omega_{trail} = N_J^*/(N_J^* + N)$ and $P_{conv} = L_J/N_J^*$, where N_J^* is the number of shares generated from conversion, we have

$$P_{conv} = \frac{1}{N}\frac{1-\omega_{trail}}{\omega_{trail}}L_J. \tag{3.30}$$

Multiplying N on both sides of (3.27) and (3.30) gives

$$\frac{1}{n}\sum_{i=0}^{n-1} E_{\bar{t}_i}(\omega_{trail}) = \frac{1-\omega_{trail}}{\omega_{trail}} L_J, \tag{3.31}$$

from which we can solve the fair ownership stake ω_{trail} when the conversion price is the trailing average stock price. To be more specific, for each path we simulate until the conversion happens (τ_c) or T^*. In the trailing average period we are interested in, we use the asset value $V_{\bar{t}_i}$ to compute the equity value $E_{\bar{t}_i}$ and therefore the left side of the equation (3.31). Then we can solve for ω_{trail} from the equation (3.31).

By our assumption for the relationship between the market and fundamental stock price, we can solve $\tilde{\omega}_{trail}$ from

$$\frac{1}{n}\sum_{i=0}^{n-1} \tilde{E}_{\bar{t}_i}(\tilde{\omega}_{trail}) = \frac{1}{n}\sum_{i=0}^{n-1} E_{\bar{t}_i}(\omega_{trail}) y(T_c - \bar{t}_i) = \frac{1-\tilde{\omega}_{trail}}{\tilde{\omega}_{trail}} L_J. \tag{3.32}$$

Consequently, we can rewrite the return from conversion as

$$\tilde{\mathbb{E}}_{\tau_{s/o},v}[R_c] = \tilde{\mathbb{E}}_{\tau_{s/o},v}\left[\frac{(1-\omega_{trail})/\omega_{trail}}{(1-\tilde{\omega}_{trail})/\tilde{\omega}_{trail}} - 1\right]. \tag{3.33}$$

Thinking of the ratio $(1-\omega_{trail})/\omega_{trail}$ as the residual value distributed to existing shareholders over the CCB investors, then (3.33) shows that the return from conversion is decided by the ratio change under manipulations. By solving ω_{trail} and $\tilde{\omega}_{trail}$ from (3.31) and (3.32), we evaluate the return from conversion with the conversion price as the trailing average by (3.33). Since there is no approach to solve for the ownership stakes analytically, to compute the return from conversion, we use Monte Carlo simulation instead. For each path, we estimate the ownership stake ω_{trail} and $\tilde{\omega}_{trail}$ by (3.31) and (3.32) and then the return from conversion. The final expectation of the return from conversion is approximated by taking the average.

Contemporary Market Stock Price Plus a Floor To guarantee the minimum loss imposed to CCB investors at conversion, regulators might use a floor to make sure that the conversion price is no smaller than the floor price. That is,

$$P_{conv} = \max\left\{P_{\tau_c}, P_{floor}\right\}, \tag{3.34}$$

where P_{floor} represents a constant floor price. With the definition, the expectation of the return from conversion becomes

$$\tilde{\mathbb{E}}_{\tau_{s/o},v}[R_c] = \tilde{\mathbb{E}}_{\tau_{s/o},v}\left[\frac{NP_{\tau_c}}{\max\left\{N\tilde{P}_{\tau_c}, NP_{floor}\right\}} - 1\right]. \tag{3.35}$$

Let ω_{floor} ($\tilde{\omega}_{floor}$) be the fair ownership stake distributed to CCB investors at conversion without (with) manipulations, then

$$\tilde{\mathbb{E}}_{\tau_{s/o},v}[R_c] = \tilde{\mathbb{E}}_{\tau_{s/o},v}\left[\frac{(1-\omega_{floor})E_{\tau_c}}{\max\left\{(1-\tilde{\omega}_{floor})E_{\tau_c} y(T_c), p_{floor}E_0\right\}} - 1\right], \tag{3.36}$$

where $p_{floor} = P_{floor}/P_0$ is the ratio of the floor price divided by the stock price at the date CCB was issued (P_0), and E_0 is the equity value at the same date. Since there is no closed-form expression, we apply Monte Carlo simulation to approximate the value.

Return from Short-Selling (R_s)

Consider an investor with a long position in a CCB that opens his/her short position when the firm's CET1 ratio is 5.5%, believing that conversion is imminent. If the investor is wrong and the firm is able to reverse its fortunes and start performing well, then it is likely that the upper stop-loss limit is hit prior to conversion occurring. In this instance the investor would lose money on the failed attempt to capitalize on more favourable terms of conversion. Because the investor has the potential to lose money on the short position, there is a potential cost associated with the short position that must be weighed against the benefit of more favourable terms of conversion. We measure this cost using what we call the return from short selling, defined as

$$R_s = 1 - \frac{\tilde{P}_{\tau_{s/c}}}{\tilde{P}_{\tau_{s/o}}}, \tag{3.37}$$

where $\tau_{s/c}$ is given by (3.17). If the expected value of R_s is negative then naked short selling is not profitable. It is possible, however, that the expected value of R_c is so large that it outweighs the costs of short selling, in which case owning a CCB can actually induce an investor to short the stock in instances when naked shorting is not profitable.

The return from short-selling is a general concern, not only for CCB investors, but also for all regular short-sellers. If the return from short-selling is significant, normal short-sellers and speculators besides CCB investors might participate in the short-selling and amplify the price drop. The return from short-selling is estimated either at the date conversion happens (Figure 3.3) or the date investors have to close their short positions because of the stop-loss requirement (Figure 3.4). As a result, there are two barriers, the conversion barrier and the stop-loss barrier. According to Karatzas and Shreve [16][7], if the firm value diffusion process starts between the two regular barriers, it will end up with hitting either the lower barrier (the conversion barrier) or the upper barrier (the stop-loss requirement level). Therefore, we always have $\tau_{s/c} < \infty$.

Instead of discussing the return from short-selling under different conversion terms, we generalize our discussion here. By the assumption (3.9),

$$\tilde{\mathbb{E}}_{\tau_{s/o},v}[R_s] = \tilde{\mathbb{E}}_{\tau_{s/o},v}\left[1 - h\mathbb{1}_{\{\tau_c > \tau_{s/l}\}} - y(\tau_c - \tau_{s/o})\frac{P_{\tau_c}}{P_{\tau_{s/o}}}\mathbb{1}_{\{\tau_{s/l} > \tau_c\}}\right], \tag{3.38}$$

Therefore, to compute the return from short-selling, one must be able to compute (i) the probability that the stop-loss barrier is hit prior to the conversion barrier and (ii) the mean value of $y(\tau_c - \tau_{s/o})P_{\tau_c}$ given that the conversion barrier is hit prior to the stop-loss barrier (recall that $P_{\tau_{s/o}}$ is a known constant). In general both of these quantities must be computed using Monte Carlo. The difficulty with (i) is that the stop-loss barrier is moving (i.e. time-dependent) while the conversion barrier is flat[8]. Upper and lower bounds on this probability can be determined as follows. Let B_- and B_+ be the minimum and maximum values of the moving barrier, respectively, and let $\tau^-_{s/l}$ and $\tau^+_{s/l}$ be the hitting times of these asset value to these levels. Then

[7]Theorem 5.29 on page 348.

[8]Although the double-barrier hitting time distribution is studied and applied in the pricing of barrier options, most studies are under the framework of double constant barriers, such as in Lin [19], Lo, Chung and Hui [20], Rogers and Zane [18], Tuckwell and Wan [28], Geman and Yor [12] and Lo, Lee and Hui [21], etc. It seems that there is no similar study to the case of our problem, which includes one fixed barrier and one moving barrier and the hitting time depends on an affine geometric Brownian motion.

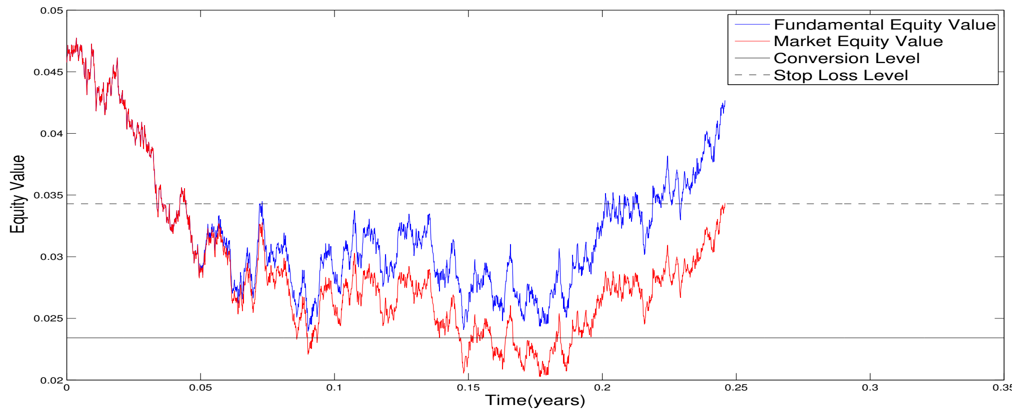

Figure 3.4: Short-selling causes the deviation between the market equity value and the fundamental value. Investors have to cover their short positions before conversion because the stop-loss requirement level is breached by the market equity value.

the probability of interest will lie somewhere between $\mathbb{P}_{\tau_{s/o},v}(\tau^+_{s/l} < \tau_c)$ and $\mathbb{P}_{\tau_{s/o},v}(\tau^-_{s/l} < \tau_c)$. And since B_- and B^+ are regular boundaries for asset value these bounding probabilities can be computed using the following theorem.

Theorem 3.3.1 (Karlin and Taylor [17, Page 192]) *Let X_t be a diffusion on (a, b) where $a < b$ are two regular barriers. Given that $X_0 = x \in (a, b)$, we define $R(x) = \mathbb{P}(\tau_b < \tau_a)$. Then $R(x)$ is the solution of the differential equation*

$$\frac{1}{2}\sigma^2(x)R''(x) + \mu(x)R'(x) = 0, \tag{3.39}$$

with necessary boundary conditions $R(a) = 0$ and $R(b) = 1$.

As for the mean value of $y(\tau_c - \tau_{s/o})P_{\tau_c}$, since P_{τ_c} is a constant when the conversion price is equal to the market price at conversion, computing the second expectation in (3.38) reduces to computing the term $\tilde{\mathbb{E}}_{\tau_{s/o},v}\left[\alpha\left(\tau_c - \tau_{s/o}\right)e^{-\alpha(\tau_c - \tau_{s/o})}|\tau_c < \tau_{s/l}\right]$. This term can be computed using the following theorem. The detailed discussion can be found in Appendix C.

Theorem 3.3.2 *Suppose a diffusion process $dX_t = \mu(X_t)dt + \sigma(X_t)dW_t$, where W_t is the standard Brownian motion. Let $X_0 = x$ and $x \in (a, b)$ where $a < b$ are two regular barriers and define τ_a and τ_b are the hitting times to the barrier a and b respectively. For a fixed parameter α, let*

$$v(x) = \mathbb{E}[\alpha\tau_a e^{-\alpha\tau_a}|\tau_a < \tau_b], \tag{3.40}$$

$$w(x) = \mathbb{E}[e^{-\alpha\tau_a}|\tau_a < \tau_b]. \tag{3.41}$$

Then we have $v(x) = \lim_{\epsilon\to 0} v_\epsilon(x)$ and $w(x) = \lim_{\epsilon\to 0} w_\epsilon(x)$, where $v_\epsilon(x)$ and $w_\epsilon(x)$ are defined on $(a, b - \epsilon)$. Moreover, $v_\epsilon(x)$ is the unique solution to the following ordinary differential equation

$$\frac{\sigma^{*2}(x)}{2}v''_\epsilon(x) + \mu^*(x)v'_\epsilon(x) - \alpha v_\epsilon(x) + \alpha w_\epsilon(x) = 0, \quad a < x < b, \tag{3.42}$$

subject to the boundary conditions $v_\epsilon(a) = 0$ *and* $v_\epsilon(b-\epsilon) = 0$. *And* $w_\epsilon(x)$ *is the unique solution to the following ordinary differential equation*

$$\frac{\sigma^{*2}(x)}{2} w_\epsilon''(x) + \mu^*(x) w_\epsilon'(x) - \alpha w_\epsilon(x) = 0, \quad a < x < b, \tag{3.43}$$

subject to the boundary conditions $w_\epsilon(a) = 1$ *and* $w_\epsilon(b-\epsilon) = 1$.

Additionally, or both (3.42) and (3.43),

$$\sigma^*(x) = \sigma(x), \tag{3.44}$$

and

$$\mu^*(x) = \mu(x) - \frac{s(x)}{S(x)}\sigma^2(x), \tag{3.45}$$

with $s(\eta) = \exp\left\{-\int_\eta^b \frac{2\mu(\xi)}{\sigma^2(\xi)} d\xi\right\}$ *as the scale density function of* x *and* $S(x) = \int_x^b s(\eta) d\eta$.

It is worth noting that Theorem 3.3.2 provides an approach to calculate the expectation term in the case of two fixed barriers. Similar to our discussion for term of probability, we would calculate the expectation term replacing the moving upper barrier with the constant barrier B_- or B_+. By doing so, we obtain a range within which the expectation term would be located.

3.4 Numerical Results

Except the parameters from the base case shown in Table 2.2 in Chpater 2, we assume the growth rate of the representative financial institution as $\mu = 8.0\%$. Without specification, we fix the parameter $\alpha = 4$ implying that market price would start to revert back towards the fundamental value after one quarter. This arrangement lies in the fact that there are quarterly financial statement updates for financial institutions so we assume the market price would be equal to the fundamental value after the quarterly financial announcements of the financial institution. The maximum deviation K between the market and fundamental stock price is assumed as 20% which is the historical maximum daily price drop of the representative Canadian bank of last 15 years. The stop-loss parameter is $h = 1.15$ meaning that the short sellers are going to close their short positions once their losses exceed 15% of their initial investment[9].

3.4.1 The Return from Conversion (R_c)

In this section, first we show the return from conversion when the conversion price is the contemporary market stock price. Applying Monte Carlo simulations, we extend our analysis under different conversion prices, including the trailing average and the contemporary market stock price plus a floor.

[9]The approximation is based on the margin requirement. Under Regulation T, the Federal Reserve Board requires all short sale accounts to have 150% of the value of the short sale at the time the sale is initiated. Many brokerages have maintenance requirements of 30%−40%. In our case, we assume the maintenance margin as 30% so that the the first margin call occurs when the market stock price is 115% of the stock price at the occurrence of short-selling.

In Figure 3.5, both panels show an non-monotonic shape for the expectation of the return from conversion given different values of α and K. This shape is consistent with our assumption that the market stock price would deviate first and then revert back towards the fundamental value under manipulations. In the left panel, we vary the value of parameter K and find a positive relation between the return from conversion and maximum deviation value. The right panel illustrates the change of the return from conversion given difference deviation speeds. For example, $\alpha = 4$ implies the maximum deviation realizes after one quarter since the occurrence of short-selling, $\alpha = 12$ implies one month and $\alpha = 52$ implies one week. The location of the point at which investors could maximize the expectation of return from conversion depends on the speed of deviation. In other words, if the market could quickly reflect the short-selling effects into the market stock price (a large α), the optimal short-selling point would be close to the conversion level.

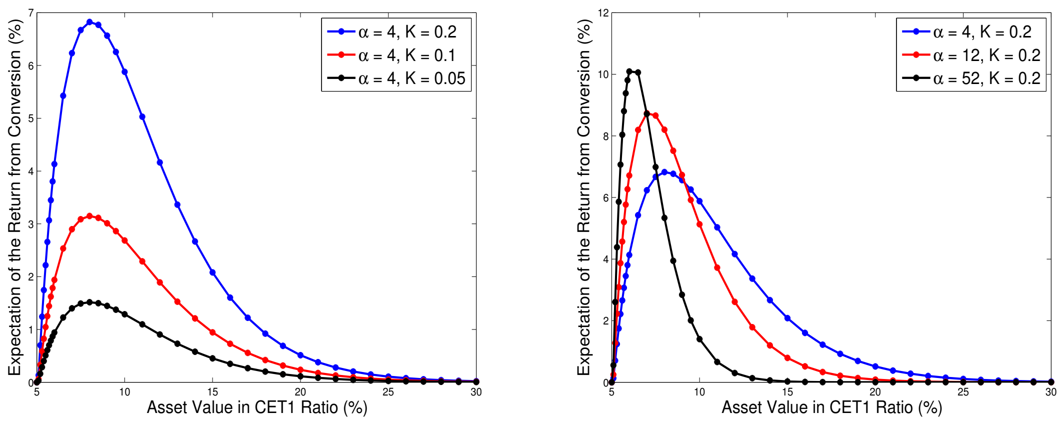

Figure 3.5: Expectations of the return from conversion (R_c) at different short-selling occurrence levels. The conversion price is the contemporary market stock price. The x-axis is the firm value in CET1 ratio at which short-selling occurs. For example, 10% means short position is opened when the firm's CET1 ratio is at 10%.

In addition, we calculate the Sharpe ratio by estimating the standard deviation of the expectation return numerically. The shape of the Sharpe ratio generally follows the non-monotonic shape shown in Figure 3.5, confirming that a rapid and large deviation of the market stock price would raise the return from conversion. We note that when the asset value is relatively large and far away from the conversion level, we have significantly negative Sharpe ratios. Therefore, although the expected return from conversion is nonnegative, it is highly risky to carry out the manipulation when the firm value is high and conversion is unlikely.

Now we extend our discussion to more general conversion prices by applying Monte Carlo simulation. In simulation, we use the parameter T^* as the "maturity". It means that we simulate paths all of which start from the assumed short-selling occurrence level and hit the conversion barrier before T^*. Based on those paths we can obtain a biased average return from conversion because it is conditioning on $T_c \leq T^*$. To correct the bias, we need to multiply the average return with $\mathbb{P}_{\tau_{s/o},v}(T_c \leq T^*)$ (see Appendix C.4 for more details).

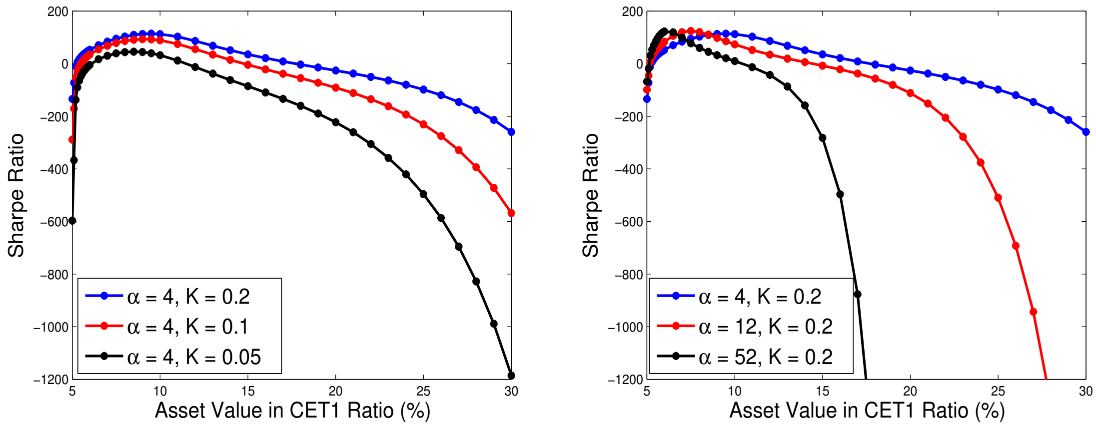

Figure 3.6: Sharpe ratios of the return from conversion (R_c) at different short-selling occurrence levels. The conversion price is the contemporary market stock price at conversion. The x-axis is the firm value in CET1 ratio at which short-selling occurs.

With Monte Carlo simulation, we consider three different conversion prices - the contemporary market stock price, the trailing average market stock price and the contemporary market stock price plus a floor which is 49% of the stock price on the date the bond was issued[10]. Figure 3.7 displays the histograms of the return from conversion under the three different conversion prices when short-selling occurs at different asset levels. When the conversion price is the contemporary market stock price, the return from conversion never goes negative, implying that the worst case for investors is the market stock price fully reverts back to the fundamental value and conversion happens at the fundamental value. However, the manipulation becomes less effective when we add the floor (49% of the stock price at the date CCB was issued) to the conversion price since the return from conversion roughly stays at zero. This results from the high location of the floor which could be easily approached by the fundamental and market stock price. Once the floor is reached, the conversion price is fixed at the floor price and there are no profits from pushing down the market stock price prior to conversion. The 7-business day trailing average conversion price only has a limited effect on restraining the manipulation incentives. When the conversion price is the 7 business day trailing average, although the return from conversion is smaller than when the conversion price is the contemporary market stock price, it is still significantly positive.

Table 3.1 summaries the statistics of Figure 3.7 after the bias correction. The results demonstrate that the contemporary market stock price plus a floor would restrain CCB investors' short-selling incentives by totally wiping out their potential return from conversion under manipulations. If we use the 7-business day trailing average, the manipulation incentives are not significantly held down compared to directly using the contemporary market stock price.

The number of days we use in calculating the trailing average would influence the return

[10]The location of the floor is within the "regulatory window" with conversion trigger as 5.0% CET1 ratio discussed in Chapter 2. With the floor price locating within the window, the original shareholders are not rewarded when conversion occurs.

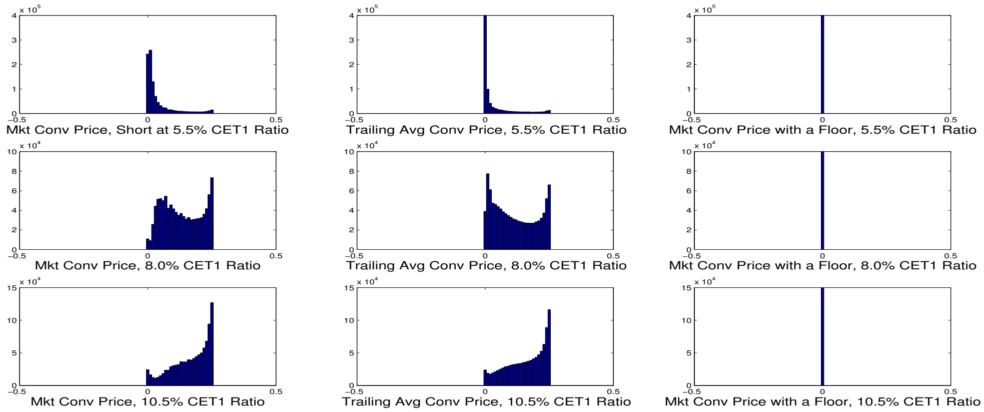

Figure 3.7: The return from conversion (R_c) under different conversion prices and short-selling occurrence levels in CET1 ratios. We use one million paths with $T^* = 8$ years and $dt = 0.001$. Trailing average is calculated based on 7 business days. The floor price is set as 49% of the stock price at the date CCBs were issued.

Short-Selling	Market Price		Trailing Avg Price		Market Price with a Floor	
	Mean	Std	Mean	Std	Mean	Std
5.5%	3.7358%	5.8663%	2.6245%	5.7322%	0.00%	0.00%
8.0%	7.0451%	8.5537%	6.2106%	8.3503%	0.00%	0.00%
10.5%	5.2285%	8.7065%	4.9783%	8.4725%	0.00%	0.00%

Table 3.1: Mean and standard deviation of the return from conversion (R_c) with different conversion prices and different short-selling occurrence levels. We use 7-business-day trailing average and the floor is 49% of the stock price at the date CCBs were issued.

from conversion. Generally, using more historical stock prices in the look back period would increase the trailing average price and therefore decrease the return from conversion. Taking the 30 business day trailing average for instance, the average return from conversion is 1.6046%, 4.3184% and 4.0046% given the short-selling occurs at 5.5%, 8.0% and 10.5% CET1 ratio respectively, and all those returns are smaller than the corresponding ones listed in Table 3.1.

As for the conversion price with a floor, the location of the floor is critical in determining the return from conversion. A low floor would increase the probability of a positive return from conversion by depressing the market stock price in advance. For example, when we use 45% of the stock price at the date CCBs were issued instead, the average return from conversion is 2.4799%, 3.6328% and 2.2697% given that short-selling occurs at 5.5%, 8.0% and 10.5% CET1 ratio respectively, and all those returns are significantly larger than the corresponding ones listed in Table 3.1.

3.4.2 The Return from Short-Selling (R_s)

As the upper barrier is a moving barrier we cannot find a closed-form expression for expected return from short-selling. However, by bounding the moving upper barrier between two flat barriers (B_- and B_+) we can determine analytic bounds on the expected return from short-selling.

We commence with estimating the probability of hitting the stop-loss (upper) barrier prior to the conversion (lower) barrier (i.e. $\mathbb{P}_{\tau_{s/o},v}(\tau_c > \tau_{s/l})$). By Monte Carlo simulation, we can approximate the probabilities under all three kinds of upper barriers – the flat barriers B_-, B_+ and the moving barrier. According to Theorem 3.3.1, we can numerically solve the probability from a second-order ordinary differential equation when the upper barrier is fixed. Therefore, when the upper barrier is B_- or B_+ , the probabilities from solving the ordinary differential equations provide cross checks for our simulation results. As long as the simulation results and the results from solving differential equations are not significantly different, we can assume the simulation approximation for the probability under the moving barrier is accurate. We graph our results in Figure 3.8, including results from simulation and solving ordinary differential equations. Under the two constant upper barriers B_- and B_+, the relative difference of the probabilities given by the simulation and solving ODEs is bounded by 7%. As a result, we could confirm the relative accuracy of the probability estimation under the moving upper barrier.

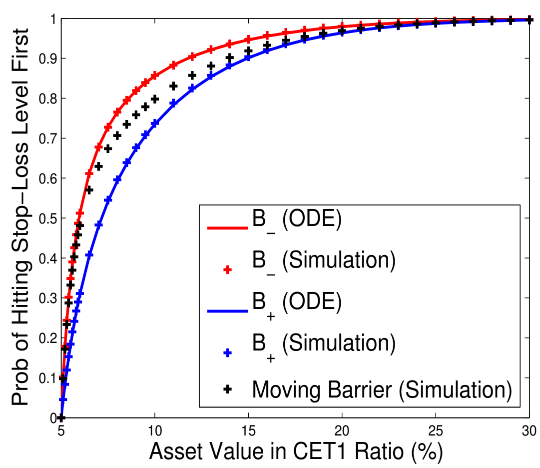

Figure 3.8: The probability of hitting the stop-loss level prior to the conversion level under three kinds of upper barriers – the flat barriers B_-, B_+ and the moving barrier. The dots represent the simulation results. The lines represent the results from solving ordinary differential equations.

Similarly, when the conversion price is the contemporary market stock price at conversion, for the expectation term in (3.38), the estimations under the two constant barriers (B_- and B_+) can provide a range for the value of the expectation term under the moving upper barrier. As an approximation, we replace the moving upper barrier with a weighted average of the constant barriers B_- and B_+ and the weight is decided by the distances between the probabilities of hitting the upper barrier first under the three kinds of upper barriers[11]. Based on this replacement, we estimate the expectation term in (3.38) using Theorem 3.3.2[12].

Figure 3.9 shows the expectation of the return from short-selling when the conversion price is the contemporary market stock price. The green dots represent the simulation results for the return from short-selling under the actual moving upper barrier. All three dots are located within the range bounded by the analytic results under the two alternative constant upper barriers B_- and B_+. Roughly, the pattern of the expectation of the return from short-selling is captured when we approximate the moving upper barrier with the weighted average barrier. As a result, we can think of the estimations under the weighted upper barrier as reasonable approximations to those under the moving upper barrier.

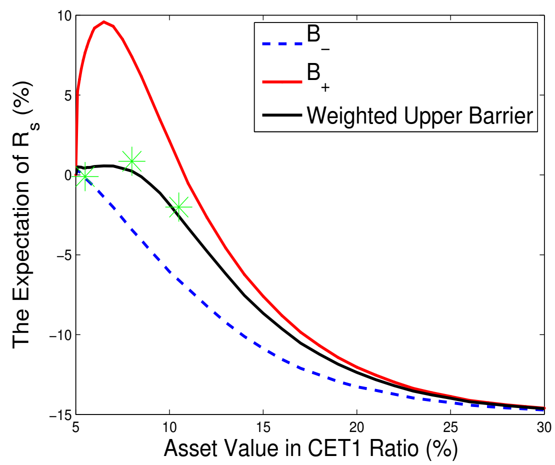

Figure 3.9: The expectation of the return from short-selling (R_s) under different upper barriers given different short-selling occurrence levels. The conversion price is the contemporary market stock price. The green dots are the simulation results.

Using the weighted barrier to approximate the moving upper barrier, we show the change of the expected return from short-selling under different deviation speeds (α) and the maximum deviations (K) in Figure 3.10. We observe a positive relation between the return from short-selling and the value of α and K. Thus, investors would prefer a quick and instantly effective manipulation in order to maximize their return from short-selling.

Figure 3.11 lists the histograms of the return from short-selling. Generally, there are two clusters, one corresponds to the case that the upper barrier is hit first and investors have to close their short positions because of the stop-loss requirement, and the other corresponds to the case that conversion occurs and investors cover their short positions at conversion by buying shares at the contemporary market price. The statistics are listed in Table 3.2. The return from short-selling is not obviously positive and could easily go negative. As a result, if one does not have a

[11]For example, if investors start short common shares at 8.0% CET 1 ratio, according to the simulation results shown in Figure 3.8, the probabilities under B_-, B_+ and the moving barrier are approximately 59.59%, 76.54% and 70.67% respectively. Therefore, the weight given to the maximum and minimum value of the moving upper barrier is about 34.62% and 65.38% respectively.

[12]Please see Appendix C.2 for the rationality of choosing $\epsilon = 10^{-6}$ when we use Theorem 3.3.2.

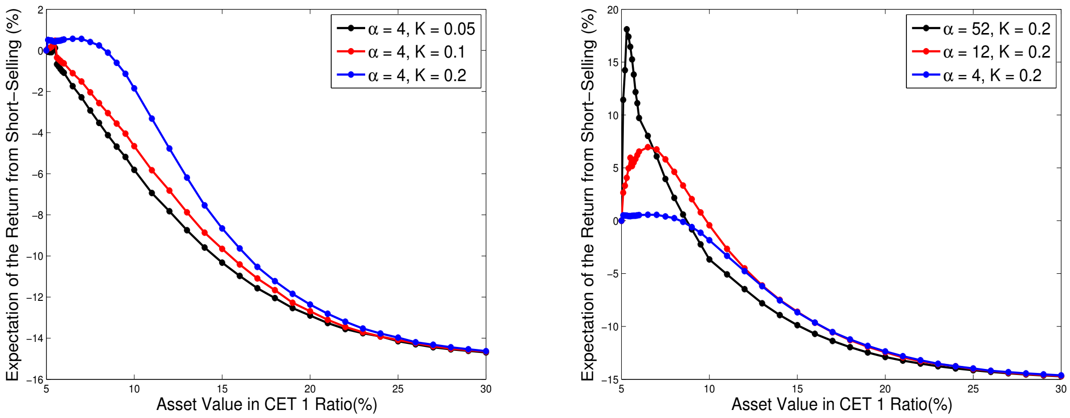

Figure 3.10: Expectations of the return from short-selling (R_s) at different short-selling occurrence levels. The conversion price is the contemporary market stock price. The x-axis is the firm value in CET1 ratio at which short-selling occurs.

long position in a CCB then there are limited incentives to short. However, for CCB investors, although the return from short-selling is relatively small, the possible return from conversion may still tempt them to press down the market stock price so that they could benefit from a lower conversion price.

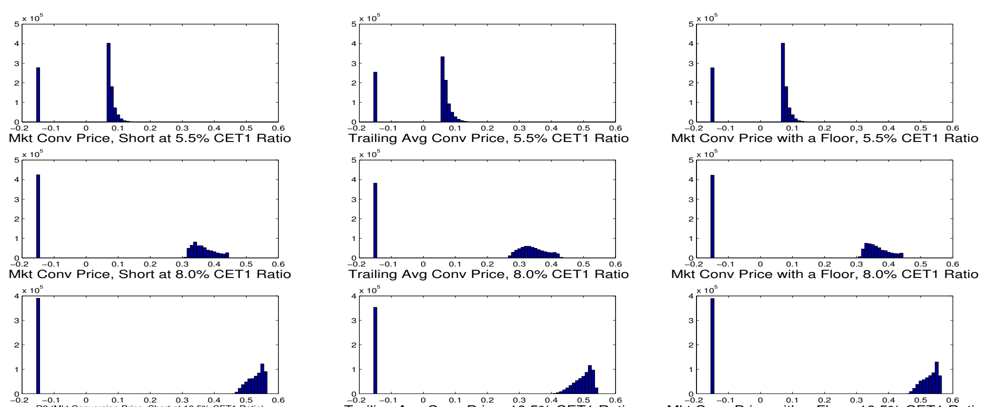

Figure 3.11: The return from short-selling (R_s) under different conversion price designs and short-selling occurrence levels in CET1 ratios. We use 7-business-day trailing average and the floor is set as 49% of the stock price at the date CCBs are issued.

Short-Selling	Market Price		Trailing Avg Price		Market Price with a Floor	
	Mean	Std	Mean	Std	Mean	Std
5.5%	−0.1054%	9.8823%	−0.1368%	9.3438%	−0.1018%	9.8627%
8.0%	0.8523%	21.5707%	1.1933%	20.8229%	0.8630%	21.5466%
10.5%	−2.0107%	23.6657%	−1.8304%	22.9899%	−2.0046%	23.6450%

Table 3.2: Mean and standard deviation of the return from short-selling (R_s) under different conversion prices and short-selling occurrence levels.

3.4.3 The Total Return

According to (3.18), the total return from the short position depends on the dollar value of the shares investors need to short in order to realize the assumed maximum deviation between the market stock price and the fundamental value. If the size of the short position relative to the notional value of the CCB is relatively large then the return from short-selling would take a relatively large weight in the calculation of the total return. However, the return from conversion dominates if the opposite happens.

We plot the weighted total return in Figure 3.12 under the contemporary market stock price. When cost (the size of the short position relative to the notional value of CCB) of realizing the assumed deviation is relatively small, we observe positive total returns which would tempt contingent capital investors to participate into the manipulation. However, the manipulation incentive would diminish as the cost climbs and the total return falls. Corresponding to our assumption of deviation and reversion process between the market stock price and the fundamental value, as the short-selling occurrence level moves away from the conversion level (5% CET1 ratio), the total return increases first (from blue line to green line) and then decreases (from green line to red line). It is not suggested to start short sale when the firm is adequate in capital because investors can only realize positive returns when the size of short position is relatively small. For example, if investors short shares when the firm's CET1 ratio is 10.5%, they can obtain positive returns only when the size of the short position relative to the notional value of the CCB is smaller than about three.

Using the trailing average stock price as the conversion price, we find a general decline of the total return in Figure 3.13 compared to the total return presented in Figure 3.12. This is because the trailing average offsets the market price drop to some extent, making the conversion price not as sensitive to the manipulation as the contemporary market stock price. If more historical data are included in the trailing average, the total return falls further. This can be verified by comparing the two panels in Figure 3.13.

Sometimes the issuing firm applies a floor price in order to curb the short-selling incentives from directly using the contemporary market stock price. For example, several Canadian banks, such as RBC, CIBC and BMO, use the floor price in their issuance of contingent capital bond. In Figure 3.14 we show the total return under two different floor prices. With a relatively low floor (45% price floor), there are some effects on pressing down the potential return from short-selling. However, as the floor moves up to 49%, the return is close to zero if investors short at 5.5% CET1 ratio, in which case the manipulation incentives are effectively curbed. Hence, a relatively high floor has a direct restriction on the incentives to short.

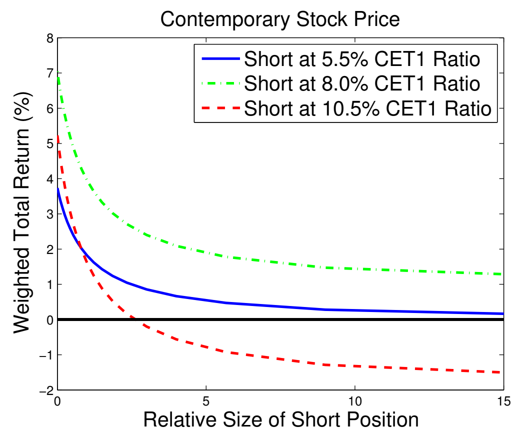

Figure 3.12: Expectations of the total return under different short-selling occurrence levels. The conversion price is the contemporary market stock price. The x-axis is the relative size of short position to CCB notional value (i.e. the parameter m).

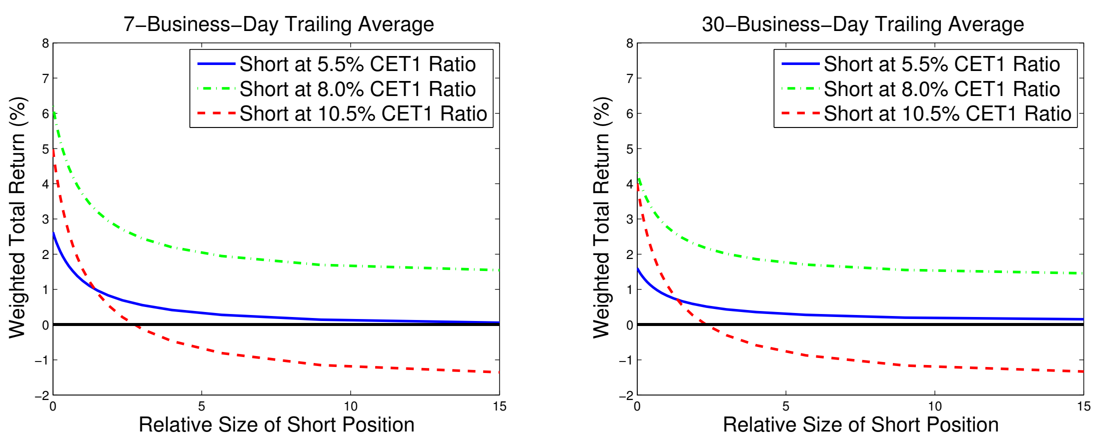

Figure 3.13: Expectations of the total return under different short-selling occurrence levels. The conversion prices are the 7-day and 30-day trailing average for the left and right panels, respectively. The x-axis is the relative size of short position to CCB notional value (i.e. the parameter m).

3.5 Conclusion

By modelling the deviation between the market stock price and the fundamental value, we analyze the extent to which a long position in a CCB might tempt investors to short the issuing institution's stock. Under our calibration to the presented Canadian bank, it is likely to earn profits by depressing the market stock price prior to conversion under a successful and effective manipulation. We observe the return from conversion is the main recourse of the total profits, so CCB investors might be the the ones having manipulation incentives when the conversion is likely. In our calibration, we use parameters (the maximum deviation $K = 20\%$ and the

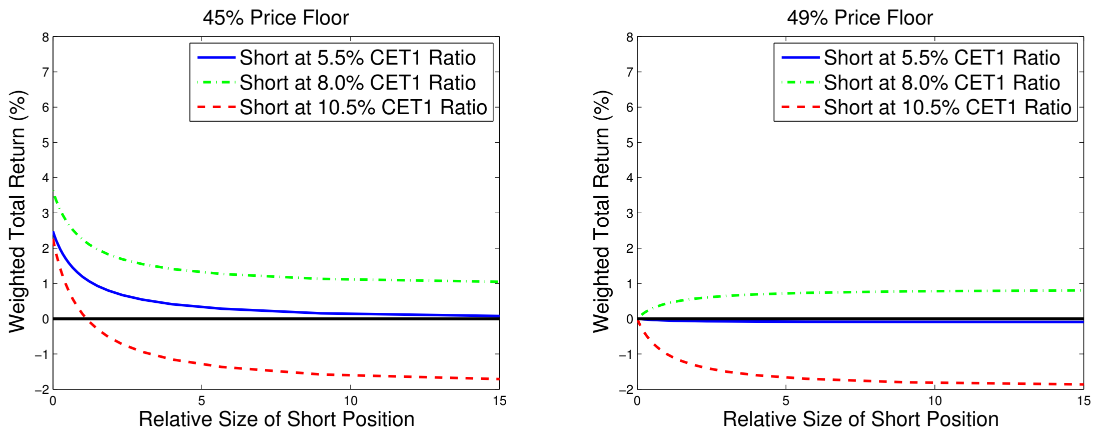

Figure 3.14: Expectations of the total return under different short-selling occurrence levels. The conversion price is the contemporary market stock price plus a floor. The floor is represented as the percentage of the stock price at the issuance of CCB. The x-axis is the relative size of short position to CCB notional value (i.e. the parameter m).

deviation and reversion speed $\alpha = 4$) under some extreme case where the market price drop is huge and the market correction is relatively inefficient so that it might be possible for contingent investors to benefit from their short-selling. The results under this kind of calibration might not be normal, but it indeed provides insights for regulators in the case that the manipulations are successful and exert negative impacts on the market confidence to the financial institution which is the regulators' concern related to CCB and motivates this work.

We discuss three market-based conversion prices in this chapter. When the conversion price is the contemporary market stock price, investors are likely to push down the market stock price below the fundamental value prior to conversion since their return from conversion under manipulation stays non-negative. However, a relatively highly located floor price would restrain this dangerous incentive since the floor guarantees the minimum loss burdened by CCB investors at conversion. The trailing average conversion price offsets the possible dramatic price drop prior to conversion by taking average of the historical stock prices, but its effectiveness of curbing manipulation incentives is medium compared to a relatively high floor price.

Additionally, we prove that the potential return is influenced by the effectiveness of manipulations. If the market price reflects to the manipulation rapidly, the short-selling pressure would mainly appear around the conversion level. Besides, the realized deviation between the market stock price and the fundamental value also plays an important role in calculating the potential return from manipulations.

Bibliography

[1] J. Abate and W. Whitt. Numerical inversion of laplace transforms of probability distributions. *ORSA Journal of Computing*, 7(1):36–43, 1995.

[2] S. Avdjiev, A. Kartasheva, and B. Bogdanova. Cocos: a primer. *Working Paper, BIS Quarterly Review*, pages 43–56, September 2013.

[3] N. Barberis, A. Shleifer, and R. Vishny. A model of investor sentiment. *Journal of Financial Economics*, 49(3):307–343, September 1998.

[4] Basel Committee on Banking Supervision. Globally systemically important banks: Assessment methodology and the additional loss absorbency requirement. *Working Paper*, November 2011.

[5] M. BeyhaghiIoannides and F. S. Skinner. *Contingent Capital Securities: Problems and Solutions*, volume 94. Emerald Group Publishing Limited, 2012.

[6] W. F. M. De Bondt and R. Thaler. Does the stock market overreact? *The Journal of Finance*, 40(3):793–805, 1985.

[7] W. S. Chan. Stock price reaction to news and no-news: Drift and reversal after headlines. *Journal of Financial Economics*, 70(2):223–260, November 2003.

[8] K. D., D. H., and A. Subrahmanyam. Investor psychology and security market under- and overreactions. *The Journal of Finance*, 53(6), December 1998.

[9] C. D'Souza and T. Gravelle. Contingent capital and bail-in debt: Tools for bank resolution. *Bank of Canada, Reports for Financial System Review*, December 2011.

[10] D. Duffie. *How Big Banks Fail and What to Do about It*. Princeton University Press, October 2010.

[11] E. F. Fama and K. R. French. Value versus growth: The international evidence. *The Journal of Finance*, 53(6):1975–1999, December 1998.

[12] H. Geman and M. Yor. Pricing and hedging double-barrier options: A probabilistic approach. *Mathematical Finance*, 6(4):365–378, Octover 1996.

[13] Contingent Capital Working Group. Contingent and bail-in capital: review of market developments. *Working paper, Basel Committee on Banking Supervision*, June 8 2012.

[14] H. Hong and J. C. Stein. A unified theory of underreaction, momentum trading, and overreaction in asset markets. *The Journal of Finance*, 54(6):2143–2184, December 1999.

[15] N. Jegadeesh and S. Titman. Returns to buying winners and selling losers: Implications for stock market efficiency. *The Journal of Finance*, 48(1):65–91, March 1993.

[16] I. Karatzas and S. E. Shreve. *Brownian Motion and Stochastic Calculus*. Springer, 1998.

[17] S. Karlin and H. M.Taylor. *A Second Course in Stochastic Process*. Academic Press Inc., 1981.

[18] L.C.G.Rogers and O.Zane. Valuing moving barrier options. *Journal of Computational Finance*, 1(1):5–11, 1997.

[19] X. S. Lin. Double barrier hitting time distributions with applications to exotic options. *Insurance: Mathematics and Economics*, 23(1):45–58, October 1998.

[20] C.F. Lo, T.K. Chung, and C.H. Hui. Double barrier hitting times distribution of a mean-reverting lognormal process and its application to pricing exotic options. *World Congress on Engineering, Newswood Limited*, 2:1000–1005, July 2007.

[21] C.F. Lo, H.C. Lee, and C.H.Hui. A simple approach for pricing barrier options with time-dependent parameters. *Quantitative Finance*, 3:98=107, 2003.

[22] A. Metzler. The Laplace transform of hitting times of integrated geometric brownian motion. *Journal of Applied Probability*, 50(1):295–299, 2013.

[23] R. A. Van Order and R. N. Lai. Liquidity, fragility and the credit crunch: A theoretical explanation and the introduction of contingent convertible bonds. *25th Australasian Finance and Banking Conference 2012, Retrieved from SSRN: http://dx.doi.org/10.2139/ssrn.2132867*, August 2012.

[24] K. G. Rouwenhorst. International momentum strategies. *The Journal of Finance*, 53(1):267–284, 1998.

[25] J. D. Spiegeleer and W. Schoutens. Pricing contingent convertibles: A derivative approach. *The Journal of Derivatives*, 20(2):27–36, Winter 2012.

[26] J. D. Spiegeleer and W. Schoutens. Multiple trigger cocos: Contingent debt without death spiral risk. *Financial Markets, Institutions & Instruments*, 22(2):129–141, May 2013.

[27] Squam Lake Working Group on Financial Regulation. An expedited resolution mechanism for distressed financial firms: Regulatory hybrid securities. *Council on Foreign Relations Press*, April 2009.

[28] H. C. Tuckwell and F. Y. M. Wan. First-passage time of markov process to moving barriers. *Journal of Applied Probability*, 21(4):695–709, December 1984.

[29] D. Vayanos and P. Woolley. An institutional theory of momentum and reversal. *The Review of Financial Studies*, 26(5):1087–1145, March 2013.

Chapter 4

Contingent Capital Bond with Regulatory Discretionary Trigger

4.1 Introductions

One of the most important features in the design of a CCB is the conversion trigger, which is the event that causes the CCB convert to common shares. In practice the vast majority of CCBs include regulatory discretion. This means that even if the CET1 ratio were to go below the objective trigger of, say 5.0%, conversion would not be triggered unless approved by the designated regulatory body (for example, OSFI in Canada). In other words, authorities retain the right to force the conversion in view of the issuing firm's solvency prospects and economic stability and sustainability. A trigger activated based on supervisors' judgement is called a regulatory discretionary trigger (Avdjiev, Kartasheva and Bogdanova [1]). With the phase in of Basel III, the share of CCBs with discretionary triggers has increased substantially over the past couple of years. A short list includes Credit Suisse (2011 and 2012), UBS (2012), Royal Bank of Canada (2014), Bank of Montreal (2014) and Canadian Imperial Bank of Commerce (2014), etc. More details can be found in Appendix A.

The use of a regulatory discretionary trigger helps ensure conversion occurs in an efficient way. With a book-value based trigger, conversion might fail to be triggered when it is necessary because a book-value based trigger might not reflect the firm's financial situation promptly or correctly[1]. However, in the case of a market-based trigger, it is possible that conversion is triggered when it is unnecessary. For example, if the issuing firm uses the market stock price as the trigger, the trigger might be breached due to normal market price fluctuations or market manipulations rather than the firm's non-viability implied by the market price (Squam Lake [11]). Therefore, the regulators' participation in making conversion decisions can mitigate the lag of a book-value based conversion trigger to the market and avoid unnecessary conversions under a market-value based trigger. In addition, regulators might prefer a contingent capital claim to forestall bankruptcy only in times of systemic crisis rather than reduce the probability of bankruptcy unconditionally (McDonald [8]). In other words, regulators might permit a poorly-

[1]For example, Citibank, a systemically important financial institution that did receive a significant government bailout during the 2008 financial crisis, had a Tier 1 capital ratio that never fell below 7.0% during the course of the financial crisis (Duffie [3]).

performing individual bank to fail in good times in the case that its failure won't cause systemic damages (as might be appropriate if post-bankruptcy restructuring would be be expected to significantly help the bank). Under this circumstance, regulators' judgement is involved in making conversion decisions in order to realize an optimal resolution for the individual bank.

The role of the regulatory discretionary trigger is most pronounced in some jurisdictions such as Canada. The Advisory from the Office of the Superintendent of Financial Institutions (OSFI) [7] claims that the Superintendent would have the authority to trigger conversion conditioning on the legislative provisions and in association with accompanying interventions by other Financial Institution Supervisory Committee (FISC) agencies. Additionally, the Basel Committee's proposal on gone-concern contingent capital requires that the judgements of regulators must be involved in the conversion decision. In view of a safety-net regulatory framework operated by OSFI, D'Souza and Gravelle [2] argue that it is advisable to issue gone-concern contingent instruments, which focus on resolution, in the Canadian context. Clearly, then, regulatory discretion should be a key component in CCB modelling.

Unfortunately, regulatory discretion is very difficult to model mathematically. If the trigger is solely based on objective capital requirements or observable market prices, investors know exactly if conversion will happen or not given a firm value or market price. On the contrary, under the discretionary trigger, investors only know that conversion might happen given a firm value or market price without knowing exactly if conversion will happen. For example, investors might believe that if conversion is to occur it is likely to be triggered when the firm's CET1 ratio is 5.0%, but they are not certain of this. It is not obvious how to incorporate such perceptions in a mathematical model, and this is the challenge. Despite the fact that the vast majority of actual CCBs incorporate some sort of regulatory discretion in their terms of conversion, almost all the literatures in the field of pricing contingent capital focus on objective or observable triggers such as a capital-ratio trigger (Glasserman and Nouri [4], Metzler and Reesor [10], etc.) or a market trigger (Sundaresan and Wang [12], etc.). To the best of our knowledge, we are the first to incorporate a regulatory discretion into the pricing of contingent capital.

Following the intensity approach (Lando [6]), the conversion time under a regulatory discretionary trigger is modelled as the first jump time of a Cox process. We assume that the intensity of the Cox process is a function of the firm's asset value and this function summarizes investors' beliefs on how regulators are likely to exercise their discretion. In this paper, we consider two types of intensity functions – those corresponding to "too-big-to-fail (TBTF)" institutions and those corresponding to "non-systemically important (NSI)" institutions. The NSI intensity function mimics regulators' possible behaviours in triggering conversion for non-systemically important financial institutions. Regulators might not enforce conversion prior to liquidation (in which case the bond is never converted to equity), i.e. they might simply allow the institution to fail instead of trying to prevent failure via conversion. The TBTF intensity function approximates regulators' considerations for the too-big-to-fail financial institutions. It has the property that conversion is certain to occur prior to liquidation and assumes the likelihood of regulators enforcing the conversion increases as the firm value approaches to the liquidation level.

In this chapter, we focus on the conversion term of fixed imposed loss due to its computational efficiency and the conversion price is equal to the market stock price at conversion. We use the regulatory discretionary trigger rather than the capital ratio trigger which distinguishes this work from our previous ones.

For the reader's convenience, we list our main conclusions here

- The intensity approach is a practical way to incorporate regulatory discretion into the trigger design. The level of conversion uncertainty can be effectively controlled by the parameters in the intensity function.
- For a NSI financial institution, adding conversion ambiguity in a gone-concern context would decrease the cost of CCB. However, adding conversion ambiguity in a going-concern context would increase the cost of CCB. Therefore, the cost change of CCB with discretionary trigger depends on the proposed way of incorporating conversion uncertainty[2].
- For a TBTF financial institution, CCB is more expensive than the one issued by a NSI financial institution. This is because the CCBs issued by TBTF institutions are sure to be converted to common shares as a bail-in financial resolution before other resolutions and liquidation. Hence, there is a chance that conversion occurs "at the last second" in the sense that the institution is on death's door when conversion is finally triggered. In such cases CCB investors are completely wiped out at conversion since the shares they receive are nearly worthless. The presence of such risk increases the cost of CCB substantially.
- It is still possible to set reasonable terms of conversion, in the sense of our discussion in Section 1.2. For a too-big-to-fail financial institution, the reasonable interval is relatively stable which might allow a consistent conversion term even if the level of regulatory discretion changes. However, for a NSI financial institution, the reasonable interval is relatively unstable.

The structure of the paper is as follows: In Section 4.2 we construct the conversion process and introduce the pricing model under the regulatory discretionary trigger. Numerical results are presented in Section 4.3. We conclude in Section 4.4.

4.2 Model

We assume a probability space $(\Omega, \mathcal{F}, \mathbb{Q})$ on which there is a standard Brownian motion W. The σ-algebra $\mathcal{F}_t$ is generated by the Brownian motion up to time t and $\mathbb{Q}$ is a risk-neutral measure which exists in an arbitrage-free economy. The firm's asset value process V_t is defined on the probability space. To be consistent, we continue to use the notation of Chapter 2 and generalize firm's asset value dynamics as

$$dV_t = \mu(V_t, C_i)dt + \sigma(V_t)dW_t, \quad i = 0, 1 \tag{4.1}$$

where $\mu(V_t, C_i) = (r - q)V_t - C_i$ is the drift term before ($i = 0$) and after ($i = 1$) conversion, $\sigma(V_t) = \sigma V_t$ is the diffusion term and W_t is a standard Brownian motion. Before conversion

[2]For NSI financial institutions it is not clear exactly how to quantify "conversion uncertainty". Here, if one quantifies uncertainty via the standard deviation of the CET1 ratio at conversion, we say the conversion ambiguity is added in a gone-concern context; if one quantifies uncertainty via the standard deviation of the intensity function itself, we say the conversion ambiguity is added in a going-concern context. More details will be discussed later.

the dollar value of coupon payments is $C_0 = c_D L_D + c_S L_S + c_{CCB} L_J$. After conversion the dollar value of coupon payments is $C_1 = c_D L_D + c_S L_S$. Since now conversion is decided by regulators rather than the hitting time of the asset value diffusion process, we commence by modelling the conversion time.

4.2.1 Construction of Conversion Process

In this section, we give both an intuitive and a formal description of how the conversion process is modelled. The description here follows Lando [5]. We assume the intensity function $k(\cdot)$ is a non-negative continuous function and depends on the current firm value. It will represent investors' beliefs on how regulators will exercise their discretion given an asset value. Conditional on $\mathcal{F}_t$, N is a Poisson process on $[0, t]$ with intensity $\lambda_s = k(V_s)$,

$$\mathbb{Q}(N_t - N_s = m|\mathcal{F}_t) = \frac{\left(\int_s^t k(V_u)du\right)^m}{m!} \exp\left(-\int_s^t \mathrm{k(V_u)du}\right), \quad \mathrm{m} = 0, 1, ... \tag{4.2}$$

In particular, assuming $N_0 = 0$, we have

$$\mathbb{Q}(N_t = 0|\mathcal{F}_t) = \exp\left(-\int_0^t \mathrm{k(V_u)du}\right). \tag{4.3}$$

With a regulatory discretionary trigger, we define the conversion time as

$$\hat{\xi} = inf\{t \geq 0 : N_t = 1\}, \tag{4.4}$$

Mathematically, the definition (4.4) represents the *first* jump time of the Cox process. Additionally, we define

$$\xi = inf\left\{t : \int_0^t k(V_u)du \geq \tilde{e}_1\right\}, \tag{4.5}$$

where $\tilde{e}_1$ is a unit exponential random variable independent of $\mathcal{F}_t$. It can be shown that $\hat{\xi}$ and ξ have the same probability distribution according to

$$\mathbb{Q}(N_t > 0|\mathcal{F}_t) = 1 - \exp\left(-\int_0^t \mathrm{k(V_u)du}\right) = \mathbb{Q}\left(\int_0^t \mathrm{k(V_u)du} \geq \tilde{\mathrm{e}}_1|\mathcal{F}_\mathrm{t}\right). \tag{4.6}$$

Based on this fact, in our following discussion we will use the definition (4.5) of the conversion time instead of (4.4) under the regulatory discretionary trigger because of the implementation convenience in simulation.

After modelling the conversion time, we are going to give two mathematical results in the following. The probability that conversion does not occur prior to t is

$$\mathbb{Q}(\xi > t|\mathcal{F}_t) = \mathbb{Q}\left(\int_0^t k(V_s)ds \geq \tilde{e}_1\Big|\mathcal{F}_t\right) = \exp\left(-\int_0^t \mathrm{k(V_s)ds}\right). \tag{4.7}$$

Based on this fact, given a firm whose contingent capital bonds have not converted up to time t, the probability of conversion within the next small time interval with length h is

$$\begin{aligned}
\mathbb{Q}(\xi \le t+h|\xi > t, \mathcal{F}_t) &= \frac{\mathbb{Q}(t < \xi \le t+h|\mathcal{F}_t)}{\mathbb{Q}(\xi > t|\mathcal{F}_t)} \\
&= \frac{\mathbb{Q}(\xi > t|\mathcal{F}_t) - \mathbb{Q}(\xi > t+h|\mathcal{F}_t)}{\mathbb{Q}(\xi > t|\mathcal{F}_t)} \\
&= 1 - \frac{\exp\left(-\int_0^{t+h} k(V_s)ds\right)}{\exp\left(-\int_0^{t} k(V_s)ds\right)} \\
&= 1 - \exp\left(-\int_t^{t+h} k(V_s)ds\right) \\
&= k(V_t)h + o(h), \qquad (4.8)
\end{aligned}$$

where (4.8) holds according to the Taylor expansion. As a result, we can think of $k(V_t)$ as a stochastic hazard rate depending on the firm's financial situation at time t. The conditional conversion probability within a small time interval given that conversion has not occurred yet depends on the instantaneous firm's health. This intuition allows us to connect the constructed conversion process with authorities' possible behaviours of triggering the conversion given the current firm value.

4.2.2 Intensity Function

We know that the constructed conversion process depends on the intensity function which approximates the regulators' behaviours of enforcing the conversion. In the following, we are going to make several assumptions for intensity functions in order to discuss regulators' behaviours for different types of financial institutions.

One practical assumption is when the asset value is very large[3], it is unlikely that regulators are going to enforce conversion since the financial institution is very healthy. Mathematically, it implies

$$v \to +\infty, \quad k(v) \to 0. \qquad (4.9)$$

However, when the asset value is relatively low and close to the point of non-viability, it is possible that the firm becomes non-viable due to the failure of keeping its liability commitment, at which point the value of the intensity function should be nonzero. The behaviour near the point of non-viability is less obvious, and to this end we consider two different intensity functions corresponding to two different types of financial institutions. In the following, we discuss intuitions and reasonable assumptions of the intensity function in each category.

NSI Financial Institutions

Regulators are not responsible for saving all financial institutions from failure. For a NSI financial institution, if its financial situation is deteriorated to the point of non-viability, sometimes

[3] Assuming a fixed amount of issued liabilities, a large asset value will be due mostly to a large equity value.

it is wise to let it go bankrupt rather than saving it since the restructured firm that emerges after bankruptcy might bring more benefits than rescuing the financial institution directly. If the institution is truly terrible then simply letting it fail is probably a good idea, since a healthier competitor can take its place. Therefore, regulators might never enforce conversion when the failure of the financial institution will not cause disastrous consequences to financial stability. Under this circumstance, define $DEF_0 = d^*L$ as the liquidation level in the absence of contingent capital (the same idea with (2.3)), and this is what the liquidation level will be if regulators decide not to enforce conversion prior to liquidation. For NSI financial institutions, it is reasonable to assume

$$v \to DEF_0, \quad k(v) \to 0. \tag{4.10}$$

One candidate for the non-monotonic intensity function is the probability density function of the lognormal distribution $\ln N(\mu_k, \sigma_k^2)$ with a parallel shift of distance DEF_0,

$$k(v) = \frac{1}{\sqrt{2\pi}\sigma_k(v - DEF_0)}\exp\left(-\frac{(\ln(\mathrm{v} - \mathrm{DEF}_0) - \mu_\mathrm{k})^2}{2\sigma_\mathrm{k}^2}\right), \quad \mathrm{v} > \mathrm{DEF}_0, \tag{4.11}$$

where μ_k and σ_k are positive constants. One advantage of using (4.11) is that the function $k(v)$ is non-negative for any asset value above the liquidation level. Additionally, by variating μ_k and σ_k, we can control the location of the peak of the intensity function and the mean of possible conversion locations[4].

There are at least two ways to quantify the degree of regulatory uncertainty (i.e. the degree of uncertainty surrounding regulators' decision about whether or not to enforce conversion).

The first approach focus on the expected CET1 ratio at conversion. Since the CET1 ratio is proportional to asset value, this approach is tantamount to fixing $\mathbb{E}[V_\xi]$ and then quantifying uncertainty via the standard deviation of V_ξ. The left panel of Figure 4.1 shows two intensity functions under this approach by fixing the expected CET1 ratio at conversion at 5.0%. Unfortunately there is no closed form expression relating the parameter (μ_k, σ_k) to the mean and variance of V_ξ and the relation between the two sets of parameters must be determined using Monte Carlo simulation. As such this approach is computationally intensive.

The conversion uncertainty also can be incorporated via the spread of the intensity function with the maximum value fixed at a suggested trigger location (such as a CET1 ratio). The fixed suggested trigger can be thought of as a point at which conversion is most likely enforced. As the firm value approaches the suggested objective trigger, it is likely that regulators will enforce the conversion in order to boost the firm's capital above the suggested level. However, if conversion does not happen when the asset value falls below the suggested trigger, the likelihood of conversion decreases with the decline of the firm value and it is more likely regulator is willing to let the firm fail. Given (4.11), the most likely point at which conversion occurs is $DEF_0 + \exp(\mu_\mathrm{k} - \sigma_\mathrm{k}^2)$ (i.e. the location of the maximum value). The parameter σ_k controls the scale of the function and therefore can be interpreted as reflecting uncertainty around the

[4]It is also possible to use other non-monotonic functions instead of the probability density function of the lognormal distribution. For example, the probability density function of the log-logistic and the inverse Gaussian distributions, etc. However, we take the probability density function of the lognormal distribution because fixing the peak value location (mode) and the scale parameter σ_k, the other parameter μ_k can be estimated explicitly. However, the parameter of the log-logistic function and inverse Gaussian distribution can only be estimated numerically rather than explicitly given the fixed mode and the scale parameter.

suggested trigger. A large value of σ_k indicates that investors are not so confident in this belief while a small σ_k indicates that investors are reasonably certain. Therefore, in this approach one first specifies what investors believe is the most likely point at which conversion is enforced, and then varies σ_k in order to quantify the impact of uncertainty on the cost of contingent debt. This situation is demonstrated in the right panel of Figure 4.1.

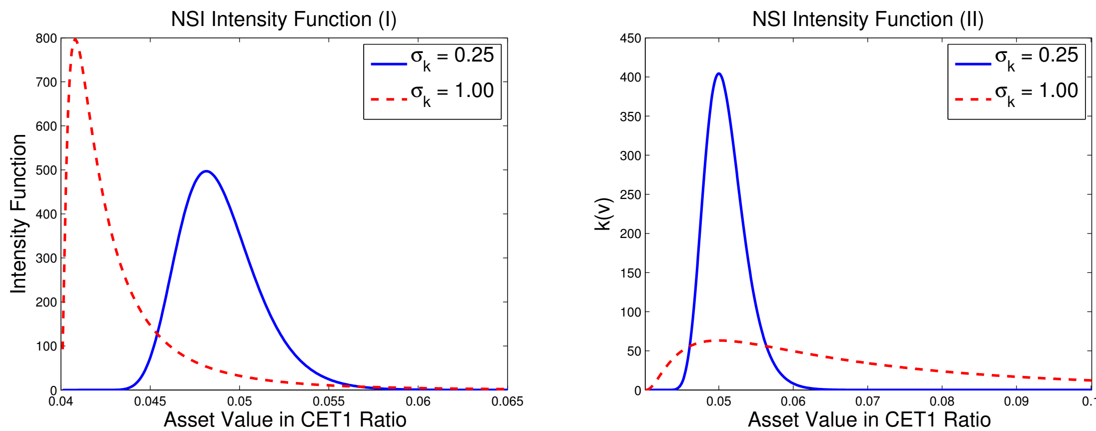

Figure 4.1: Examples of intensity functions for NSI institutions. The intensity functions in the left panel have the same mean of conversion location at 5.0% CET1 ratio while the ones in the right panel have the same peak location at 5.0% CET1 ratio.

TBTF Financial Institution

For the systemically important financial institutions with potential too-big-to-fail problem, the government would support them when they face potential failure in order to avoid severe adverse consequences to the greater economic system. It is reasonable to assume the bail-out financial resolution provided by the government comes after the the bail-in resolution from the conversion of contingent capital. Therefore, conversion will always be enforced by regulator prior to liquidation for the too-big-to-fail financial institutions.

Suppose as the asset value approaches to the level DEF_0 which is the liquidation level for the firm in the absence of contingent capital, it is more and more likely that the regulators are going to trigger the conversion in order to rescue the firm and invoke an orderly resolution. A condition describing this situation would be

$$v \to DEF_0, \quad k(v) \to +\infty, \tag{4.12}$$

which guarantees that conversion always occurs prior to liquidation[5]. To facilitate our comparison with the results under the NSI intensity function, we consider the following intensity function,

$$k(v) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma_k(v-DEF_0)} \exp\left(-\frac{(\ln(\mathrm{v}-\mathrm{DEF}_0)-\mu_\mathrm{k})^2}{2\sigma_\mathrm{k}^2}\right), & v > DEF_0 + \exp(\mu_\mathrm{k} - \sigma_\mathrm{k}^2), \\ \frac{\lambda}{(v-DEF_0)^2}, & DEF_0 < v \le DEF_0 + \exp(\mu_\mathrm{k} - \sigma_\mathrm{k}^2), \end{cases} \tag{4.13}$$

[5]However, the condition (4.12) is necessary but not sufficient for conversion to precede liquidation with probability one.

in which we keep the part of the NSI intensity function at the right side of the peak location unchanged but replace the left part with a monotonic function which approaches to infinity as the asset value moves towards the level DEF_0[6]. The parameter λ is chosen so that the function is continuous at the peak. Two examples are plotted in Figure 4.2.

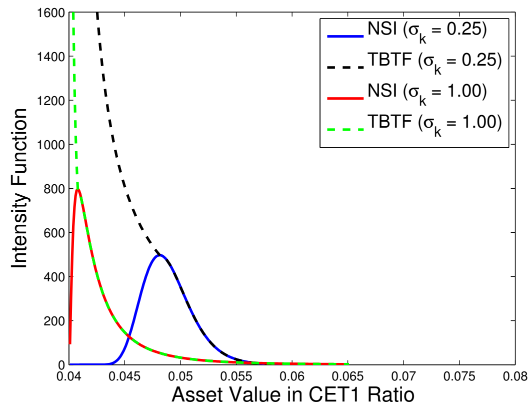

Figure 4.2: Examples of TBTF intensity function.

The two categories of intensity functions describe two possible behaviours of regulators when they consider triggering conversion. The main difference between them is: for the NSI intensity function, regulators might let the institution fail without enforcing the conversion of contingent debt and CCB might end up as a liability; however, in the case of TBTF intensity function, financial resolution always starts with the conversion of CCB.

4.2.3 Pricing of CCBs

Similar to the definition in Chapter 2, the liquidation time τ_d is the first-passage time to a specified asset-liability ratio d^*,

$$\tau_d = inf\{t \geq 0 : \bar{v}_t \leq d^*\}. \tag{4.14}$$

In the case that regulators allow the firm to fail without enforcing conversion ($\tau_d < \xi$), the barrier is the level $DEF_0 = d^* L$. This case is only possible for NSI financial institutions. In the event that conversion is enforced prior to liquidation ($\tau_d > \xi$), the barrier is the level $DEF_1 = d^*(L_D + L_S)$. This case is possible for both NSI and TBTF financial institutions

Being consistent with Chapter 2, we assume the financial institution keeps operating until liquidation happens. Let $\mathbb{E}_x[\cdot]$ be the expectation with the asset value diffusion process starting at $V_0 = x$. Given the fixed imposed loss $100\beta_{CCB}\%$ to the notional value of CCB at conversion,

[6]We don't have a formal mathematical proof that the intensity function in (4.13) is such that conversion to precede liquidation with probability one, but the numerical evidence (in Table 4.10) indicates that this probability is extremely close (if not exactly equal to) one.

the fair coupon rates (par yields) are solved from the following equation system

$$L_D = \frac{c_D L_D}{r}\left(1 - \mathbb{E}_x[e^{-r\tau_d}]\right) + \mathcal{R}_D L_D \mathbb{E}_x[e^{-r\tau_d}], \tag{4.15}$$

$$L_S = \frac{c_S L_S}{r}\left(1 - \mathbb{E}_x[e^{-r\tau_d}]\right) + \mathcal{R}_S L_S \mathbb{E}_x[e^{-r\tau_d}], \tag{4.16}$$

$$L_J = \frac{c_{CCB} L_J}{r}\left(1 - \mathbb{E}_x\left[e^{-r\bar{\tau}_d}\right]\right) + (1-\beta_{CCB})L_J \mathbb{E}_x\left[e^{-r\xi}\mathbb{1}_{\{\xi\leq\tau_d\}}\right] + \mathcal{R}_J L_J \mathbb{E}_x\left[e^{-r\tau_d}\mathbb{1}_{\{\tau_d<\xi\}}\right], \tag{4.17}$$

where $\bar{\tau}_d = \min\{\tau_d, \xi\}$ and the recovery rates at liquidation for deposits, the senior bond and CCB are $\mathcal{R}_D$, $\mathcal{R}_S$ and $\mathcal{R}_J$ respectively. Only when $\tau_d < \xi$ (i.e. regulators never enforce conversion) is there a recovery payment to the CCB investor. To respect seniority, we assume $\mathcal{R}_D > \mathcal{R}_S > \mathcal{R}_J$.

To estimate the credit spreads, we need to calculate the Laplace transform of the liquidation time and conversion time in the equation systems. When the conversion trigger is objective then the Laplace transform of the conversion time can be expressed in terms of Laplace transforms of hitting times of affine geometric Brownian motion to fixed levels, which is available in closed form (Metzler [9]) and happens to involve the confluent hypergeometric function. However, in this chapter, the conversion time ξ is no longer the hitting time of asset value to a fixed level. Thus, the analysis is more complicated and we require the transform of the "killing time" ξ as well as two conditional transforms. Although we are not able to compute these transforms in closed form, we are able to characterize them as solutions to second-order ordinary differential equations. By numerically solving the ordinary differential equations, we obtain the value for the transforms.

In the following, we state the theorems of relevant differential equations and the corresponding boundary conditions. Theoretical proofs are presented in Appendix D. For the reader's convenience, we summarize the results in Table 4.1 at the end of this section.

Theorem 4.2.1 *Let $U(x) = \mathbb{E}_x[e^{-\alpha\xi}\mathbb{1}_{\{\xi\leq\tau_d\}}]$. Then $U(x)$ is the solution of the following second-order ordinary differential equation*

$$\mathcal{G}U(x) - (k(x)+\alpha)U(x) + k(x) = 0, \quad x > DEF_0, \tag{4.18}$$

where the operator $\mathcal{G} = \frac{1}{2}\sigma^2(x)\frac{d^2}{dx^2} + \mu(x, C_0)\frac{d}{dx}$.

Different boundary conditions are applied for different categories of intensity functions. Under the NSI intensity function, the boundary conditions are $U(DEF_0) = 0$ and $\lim_{x\to+\infty} U(x) = 0$. The first condition makes sense because as the asset value approaches to the level DEF_0, it is almost sure that the NSI institution will fail without the conversion of contingent capital (i.e., $\xi > \tau_d$). In other words, as $x \to DEF_0$, the value of the indicator $\mathbb{1}_{\{\xi\leq\tau_d\}}$ approaches to zero. As the asset value approaches infinity, we have the boundary condition $\lim_{x\to+\infty} U(x) = 0$ according to the assumption (4.9). Under the TBTF intensity function, regulators will always enforce conversion prior to liquidation, leading to the boundary condition $U(DEF_0) = 1$. This is because as the asset value approaches to the level DEF_0, contingent debt is more likely to be converted to common shares. As a result, we have $\xi \to 0$ as $x \to DEF_0$. The second boundary condition is $\lim_{x\to+\infty} U(x) = 0$ based on the same reason under the NSI intensity function.

Theorem 4.2.2 *Let $Z(x) = \mathbb{E}_x[e^{-\alpha\tau_d}\mathbb{1}_{\{\tau_d<\xi\}}]$. Then $Z(x)$ is the solution of the following second-order ordinary differential equation*

$$\mathcal{G}Z(x) - k(x)Z(x) - \alpha Z(x) = 0, \quad x > DEF_0, \tag{4.19}$$

where the operator $\mathcal{G} = \frac{1}{2}\sigma^2(x)\frac{d^2}{dx^2} + \mu(x, C_0)\frac{d}{dx}$.

Under the NSI intensity function, we have $\lim_{x\to\infty} Z(x) = 0$ because as the asset value approaches to infinity, the hitting time $\tau_d \to +\infty$. The other boundary condition is $Z(DEF_0) = 1$ due to the reason that as $x \to DEF_0$, we have $\tau_d \to 0$ and the value of the indicator $\mathbb{1}_{\{\tau_d<\xi\}}$ approaches to one. Under the TBTF intensity function, the boundary conditions are $Z(DEF_0) = 0$ and $\lim_{x\to\infty} Z(x) = 0$. As $x \to DEF_0$, it is more likely that regulator will enforce conversion ($\xi \to 0$) before the failure of the institution leading to the indicator $\mathbb{1}_{\{\tau_d<\xi\}}$ approaches to zero.

Theorem 4.2.3 *Let $W(x) = \mathbb{E}_x[e^{-\alpha\tau_d}]$. Then $W(x)$ solves the following second-order ordinary differential equation*

$$\mathcal{G}W(x) - (k(x) + \alpha)W(x) + k(x)W_1(x) = 0, \quad x > DEF_0, \tag{4.20}$$

where the operator $\mathcal{G} = \frac{1}{2}\sigma^2(x)\frac{d^2}{dx^2} + \mu(x, C_0)\frac{d}{dx}$ and $W_1(x) = \mathbb{E}_x[e^{-\alpha\tau_d}]$, which is calculated based on the post-conversion diffusion process, i.e., the fixed coupon payment for the diffusion process is C_1.

Based on the assumption (4.12), $\lim_{x\to+\infty} W(x) = 0$ holds for both TBTF and NSI intensity functions. Under the NSI intensity function, the other boundary condition is $W(DEF_0) = 1$. This is because as $x \to DEF_0$, we have $\tau_d \to 0$. In the TBTF case, as $x \to DEF_0$, τ_d converges to the hitting time of an affine geometric Brownian motion beginning at the level DEF_0 to the level DEF_1. Therefore, $W(DEF_0)$ can be expressed in terms of the known Laplace transform for such hitting times, i.e. $W(DEF_0) = \mathbb{E}_{DEF_0}[e^{-\alpha\tau_d}]$ (see Theorem B.1.6 in Appendix B).

Table 4.1 gives a summary of the relevant differential equations and the corresponding boundary conditions.

Table 4.1: ODEs and Boundary Conditions

Transforms	**ODEs and Boundary Conditions**		
$U(x) = \mathbb{E}_x[e^{-\alpha\xi}\mathbb{1}_{\{\xi\le\tau_d\}}]$	$\mathcal{G}U(x) - (k(x) + \alpha)U(x) + k(x) = 0, \quad x > DEF_0$		
	TBTF	$U(DEF_0) = 1,$	$\lim_{x\to+\infty} U(x) = 0$
	NSI	$U(DEF_0) = 0,$	$\lim_{x\to+\infty} U(x) = 0$
$Z(x) = \mathbb{E}_x[e^{-\alpha\tau_d}\mathbb{1}_{\{\tau_d<\xi\}}]$	$\mathcal{G}Z(x) - k(x)Z(x) - \alpha Z(x) = 0, \quad x > DEF_0$		
	TBTF	$Z(DEF_0) = 0,$	$\lim_{x\to+\infty} Z(x) = 0$
	NSI	$Z(DEF_0) = 1,$	$\lim_{x\to+\infty} Z(x) = 0$
$W(x) = \mathbb{E}_x[e^{-\alpha\tau_d}]$	$\mathcal{G}W(x) - (k(x) + \alpha)W(x) + k(x)W_1(x) = 0, \quad x > DEF_0$		
	TBTF	$W(DEF_0) = \mathbb{E}_{DEF_0}[e^{-\alpha\tau_d}],$	$\lim_{x\to+\infty} W(x) = 0$
	NSI	$W(DEF_0) = 1,$	$\lim_{x\to+\infty} W(x) = 0$

So far we derived the ordinary differential equations and their boundary conditions corresponding to the transforms needed in the equation system. We do not need to derive the differential equation for the term $\mathbb{E}_x[e^{-r\bar{\tau}_d}]$ in (4.17) since it equals to the summation of $U(x)$ and $Z(x)$. By solving the differential equations, we can estimate the par yields under the conversion term of fixed imposed loss.

Similar to our discussions under the objective conversion trigger in Chapter 2, we also investigate the interval for reasonable conversion terms when the conversion trigger depends on regulatory discretion. Recall Figure 1.4 in Chapter 1, a reasonable conversion term must guarantee the liability seniority in the capital structure and the existing shareholders are not rewarded for the poor performance of the firm at the same time.

4.3 Numerical Experiments

To be consistent with our analysis under the objective conversion trigger, we continue to use the parameters from the base case in Chapter 2 if there is no particular specification. Although we only consider the discretionary trigger in this chapter, we use 5.0% CET1 ratio as the conversion trigger suggested in the contract. We start from the NSI intensity function and then move on to the TBTF intensity function, investigating the conversion features, credit spreads of liabilities and the reasonable intervals for conversion terms. We present the numerical results under the 5.33% fixed imposed loss given the efficiency of computation.

We use the finite difference methods to numerically solve (4.18), (4.19) and (4.20). Instead of using a large asset value to approximate the infinite case of our boundary conditions in Table 4.1, we make the substitution $z = 1/x$ and project $x \in [DEF_0, +\infty)$ to $z \in (0, 1/DEF_0]$ in order to transform our boundary value problems to the ones with bounded intervals. When we use the finite difference methods to solve the ordinary differential equations, we keep refining the grids until the solved par yields do not change within the magnitude of 10^{-5} (or 0.1 basis point). Additionally, simulation results have been used as cross checks to the numerical solutions of the ordinary differential equations. When solving for the par yields, we continue using the stopping criteria (2.44) in Chapter 2. That is, the summation of each liability's relative difference between the discounted value and the notional value cannot exceed 0.1%.

4.3.1 Results under NSI Intensity Functions

In the simulation, we take $T = 100$ years as an approximation to the infinite time horizon, the time step $dt = 0.0005$ and use $100,000$ paths. The results are slightly biased since they condition on $\xi < T$. However, this bias can be expected to be very negligible in the sense that $\mathbb{Q}(\xi < \infty) - \mathbb{Q}(\xi \leq T)$ is extremely small. In general we have $\mathbb{Q}(\xi < \infty) < 1$ since there is a positive probability conversion will never be enforced.

We start from the case that the regulatory uncertainty is quantifies via the standard deviation of the conversion location V_ξ. In Table 4.2, we fix the mean of conversion location ($\mathbb{E}[V_\xi]$) around 5.0% CET1 ratio. As the parameter σ_k increases, the standard deviation of conversion location ($SD(V_\xi)$) increases reflecting increased uncertainty about when conversion will occur. In the case that regulatory uncertainty is quantified via the scale parameter (σ_k) of the intensity function, Table 4.3 lists the mean and standard deviation of conversion locations under different σ_k's but with the peak fixed at the 5.0% CET1 ratio. By fixing the peak location of the intensity function, we fix the CET1 ratio at which conversion is most likely to be enforced. As shown in the table, the parameter σ_k is positively related to the conversion uncertainty. However, the mean of conversion location moves upwards with the increase of the parameter σ_k due to the wide spread (fat right tail) of the intensity function.

Table 4.2: Gone-Concern NSI Intensity Function[a]

σ_k	$\mathbb{Q}(\xi \leq T)$	**Conversion Location(CET1)**	
		Mean	**Std**
0.05	82.17%	5.00%	0.05%
0.25	82.70%	5.00%	0.28%
0.5	80.98%	5.01%	0.56%
0.75	77.25%	5.01%	0.88%
1.0	72.18%	5.00%	1.36%
1.5	58.76%	4.96%	1.93%
2.0	40.35%	5.02%	2.61%

[a] Fix $\mathbb{E}[V_\xi]$, vary $SD(V_\xi)$.

Table 4.3: Going-Concern NSI Intensity Functions[b]

σ_k	$\mathbb{Q}(\xi \leq T)$	**Conversion Location(CET 1)**	
		Mean	**Std**
0.05	82.38%	5.01%	0.051%
0.25	85.23%	5.25%	0.34%
0.5	91.58%	6.38%	1.21%
0.75	96.72%	9.37%	3.04%
1.0	98.16%	11.36%	3.94%
1.5	95.47%	11.78%	3.71%
2.0	84.48%	12.60%	4.87%

[b] Fix the peak location, vary σ_k.

As we will see soon, the first approach mainly adds the uncertainty on a gone-concern basis while the second approach adds the uncertainty on a going-concern basis. Therefore, we will refer the first intensity function as gone-concern NSI intensity function and the second one as going-concern NSI intensity function from now on.

In order to have a closer look of the conversion uncertainty, we list the quantiles of conversion location under two types of NSI intensity functions in Table 4.4 and Table 4.5. Under the gone-concern NSI intensity function, we fix the mean of conversion location $\mathbb{E}[V_\xi]$ and vary the standard deviation $SD(V_\xi)$ through the change of σ_k. Table 4.4 shows that more than 85% of conversion locations are located below the 6.0% CET1 ratio for different σ_k's. Moreover, the conversion location moves towards the liquidation level (4.0% CET1 ratio) as the parameter σ_k increases. Therefore, modelling regulatory uncertainty in this way corresponds to the situation where regulators are more likely to enforce conversion on a gone-concern basis. In other words, the conversion uncertainty is incorporated around the firm's point of non-viability. In the case of the going-concern NSI intensity function, we fix the peak location of the intensity function and vary the scale parameter σ_k. It is perceived from Table 4.5 that about 85%

of conversion locations are located above the 5.0% CET1 ratio and the conversion location moves upwards as the parameter σ_k rises. Hence, the conversion is likely to happen when the firm's value is only mildly eroded or at a relatively adequate level. Therefore, in this situation regulators are more likely to enforce conversion on a going-concern basis.

Table 4.4: Quantiles under Gone-Concern NSI Intensity Functions

σ_k	5% **Quantile**	15% **Quantile**	**Median**	85% **Quantile**	95% **Quantile**
0.05	4.92%	4.95%	5.00%	5.06%	5.09%
0.25	4.63%	4.74%	4.97%	5.27%	5.48%
0.5	4.36%	4.51%	4.89%	5.51%	6.06%
0.75	4.19%	4.32%	4.76%	5.71%	6.66%
1.0	4.08%	4.17%	4.59%	5.82%	7.35%
1.5	4.01%	4.04%	4.27%	5.75%	8.34%
2.0	4.01%	4.02%	4.13%	5.72%	9.23%

Table 4.5: Quantiles under Going-Concern NSI Intensity Functions

σ_k	5% **Quantile**	15% **Quantile**	**Median**	85% **Quantile**	95% **Quantile**
0.05	4.93%	4.96%	5.01%	5.06%	5.09%
0.25	4.78%	4.92%	5.21%	5.58%	5.83%
0.5	4.86%	5.23%	6.15%	7.54%	8.66%
0.75	5.45%	6.44%	8.94%	12.12%	14.71%
1.0	6.31%	7.88%	10.98%	14.27%	17.96%
1.5	6.53%	8.37%	11.48%	14.79%	18.25%
2.0	6.37%	8.28%	11.84%	16.66%	21.57%

Credit Spreads

We list the credit spreads of the senior bond and CCB under different parameter σ_k's in Table 4.6 and Table 4.7. Recall that our discussion is under the conversion terms of fixed imposed loss. That is, a fixed amount of loss is imposed to CCB at conversion regardless of the firm's health at conversion. With the gone-concern NSI intensity function, adding conversion uncertainty would make conversion more likely to happen around the liquidation level. It benefits CCB investors because a possibly lower conversion level implies a delayed conversion and therefore a longer coupon stream. Thus, we observe an inverse relation between the parameter σ_k and the credit spread of CCB, as shown in Table 4.6. In addition, the credit spread of the senior bond grows with the parameter σ_k. This is due to the reason that the delayed conversion implied by a relatively large σ_k reduces the possibility of the firm's viability after the recapitalization from conversion. In other words, the senior bondholder is confronted with relatively high liquidation risk under a relatively large σ_k, leading to a positive relation between the spread and the parameter σ_k.

However, conversion uncertainty imposes an opposite impact to credit spreads under the going-concern NSI intensity function as shown in Table 4.7. The going-concern uncertainty makes conversion likely to occur at adequate capital levels, which is not favoured by CCB investors because an early conversion would make them suffer an early write-down, cutting the coupon stream for them. Thus, we observe a positive relation between σ_k and the credit spread of CCB. But for the senior bondholders, an early conversion reduces the liquidation risk leading to a decreasing credit spread.

If we compare the credit spreads shown in Table 4.6 and Table 4.7, we can find that gone-concern CCB is far less sensitive to regulatory uncertainty than going-concern CCB. This can be explained by the less sensitive of conversion location to the uncertainty in the case of adding the uncertainty on a gone-concern basis than on a going-concern basis.

Table 4.6: Credit Spreads under Gone-Concern NSI Intensity Function

σ_k	**Senior (in bp)**	**CCB (in bp)**
0.05	14	95
0.25	14	97
0.5	14	95
0.75	14	90
0.90	15	86
1.0	15	83
1.5	17	69
2.0	18	55

Table 4.7: Credit Spreads under Going-Concern NSI Intensity Function

σ_k	**Senior (in bp)**	**CCB (in bp)**
0.05	14	95
0.25	14	103
0.5	14	130
0.75	13	301
0.90	13	841
1.0	13	1846
1.5	13	4431
2.0	13	2200

Intervals for Reasonable Conversion Terms

Under the fixed imposed loss, Table 4.9 lists the reasonable intervals under different σ_k's with the gone-concern NSI intensity function. The left end point of the interval above which the liability seniority is guaranteed might go negative when this type of conversion uncertainty increases (the increase of the parameter σ_k). In other words, the liability seniority might be kept even when the CCB is redeemed above par in the case of conversion. This is because under the NSI intensity function, conversion might not happen and the CCB eventually ends up as a liability at liquidation. As a result, even if we assign a full redemption (zero write-down) at conversion for contingent capital, the recovery rate (less than the recovery rate for the senior bond) for CCB in the case of liquidation might push the credit spread of CCB above the credit spread of the senior bond. This results in the negative value at the left end point of the interval when conversion is more likely to happen near the liquidation level and liquidation is likely to happen without regulators enforcing the conversion of CCB at the same time. However, the right end point climbs with the increase of the parameter σ_k because the likelihood of conversion occurring below the suggested objective trigger increases according to the change of median in Table 4.4. With the firm's asset value close to the liquidation level, the existing shareholders in the otherwise identical traditional capital structure[7] take a relatively large loss which allows a relatively high imposed loss at the right end point to ensure that the existing shareholders are not rewarded for the firm's poor performance.

As for the going-concern NSI intensity function, the left end point behaves similarly to the ones under the gone-concern NSI intensity function but the right end point of the interval moves oppositely with the increase of the parameter σ_k. Compared to the gone-concern NSI intensity function, the going-concern NSI intensity function makes conversion more likely to occur in a going-concern environment with about 85% probability that conversion happens above the suggested objective trigger 5.0% CET1 ratio. Under this circumstance, the shareholders in the otherwise identical traditional capital structure bear small losses on average so the imposed loss cannot be very large to guarantee the existing shareholders are not rewarded for the poor performance. This trend becomes more obvious and significant with the growth of the parameter σ_k, leading to a decrease in the right end point in Table 4.9.

Table 4.8: Reasonable Intervals under Gone-Concern NSI Intensity Function

σ_k	**Reasonable Interval**	**Size of Interval**
0.05	$[0.27\%, 9.61\%]$	9.34%
0.25	$[0.30\%, 9.53\%]$	9.23%
0.50	$[0.25\%, 9.63\%]$	9.38%
0.75	$[0.11\%, 9.89\%]$	9.78%
1.00	$[-0.14\%, 10.30\%]$	10.44%
1.50	$[-1.09\%, 11.34\%]$	12.43%
2.00	$[-3.39\%, 12.50\%]$	15.89%

Table 4.9: Reasonable Intervals under Going-Concern NSI Intensity Function

σ_k	**Reasonable Interval**	**Size of Interval**
0.05	$[0.25\%, 9.61\%]$	9.36%
0.25	$[0.38\%, 9.09\%]$	8.71%
0.50	$[0.38\%, 7.48\%]$	7.10%
0.75	$[0.17\%, 3.49\%]$	3.32%
1.00	$[0.00\%, 0.71\%]$	0.71%
1.50	$[-0.09\%, 0.22\%]$	0.31%
2.00	$[-0.35\%, 0.22\%]$	0.57%

Due to the change of reasonable intervals under two NSI intensity functions, we find that under the gone-concern NSI intensity function, there is a positive relation between conversion

[7] Please see Section 2.3.2 in Chapter 2 for details.

uncertainty measured by the parameter σ_k and the size of the reasonable interval. However, under the going-concern NSI intensity function, there is an inverse relation. Therefore, as more and more regulatory discretion incorporated into the trigger in a gone-concern framework, the size of the open window for reasonable conversion term grows. However, if the uncertainty is added in a going-concern framework, the interval shrinks to allow for the incorporation of regulatory discretion.

Another noteworthy conclusion is that under both NSI intensity functions, if a relatively high regulatory discretion is incorporated into the trigger (for example, $\sigma_k = 1.5$ and 2.0), the reasonable intervals demonstrate that even CCB investors are redeemed above par at conversion (with negative write-downs), the liability seniority and the punishment to existing shareholders might be guaranteed. It implies that the adverse impression brought by regulatory discretionary trigger to CCB investors might be offset by the above par redemption.

4.3.2 Results under TBTF Intensity Function

In this section we only consider the TBTF intensity function (4.13) and focus on the comparison between the results under the TBTF and the gone-concern NSI intensity function. By doing so, we investigate the difference between the CCB issued by a NSI financial institution and a TBTF financial institution.

Table 4.10 illustrates a positive relation between the standard deviation of conversion locations and the parameter σ_k, so σ_k still manages to measure the conversion uncertainty as it does with the NSI intensity function. It is worth mentioning that the probability shown in the table measures the likelihood that conversion happens within a given time horizon T. For too-big-to-fail financial institutions, conversion is sure to occur prior to liquidation but it does not mean conversion is certain to occur before the given time horizon T. Therefore, it is reasonable to observe a probability less than one. When we extend the time horizon T, the probability of liquidation increases so does the probability of conversion before T. Perceived from Table 4.11, as σ_k increases, conversion is more likely to occur near (but never below) the liquidation level.

Credit Spreads

We present the credit spreads under the TBTF intensity function and the comparison with the spreads under the gone-concern NSI intensity function in Figure 4.3. As the parameter σ_k measures the conversion uncertainty, for too-big-to-fail financial institutions, the cost of the senior bond is not sensitive to the conversion uncertainty because regulator will always enforce conversion before liquidation. The capital infusion from conversion provides a safety cushion for the senior bondholders reducing the liquidation risk and prolonging their coupon stream. However, for NSI institutions, the conversion uncertainty raises the cost of the senior bond because conversion might never happen resulting in a shorter coupon stream for senior bondholders. The credit spreads shown in the left panel of Figure 4.3 verifies our analysis in Chapter 2 indirectly. That is, the (sure-to-happen) conversion feature of the junior tranche would reduce the cost the senior tranche.

The conversion uncertainty brought by the discretionary trigger reduces the cost of CCBs as shown in the right panel of Figure 4.3. With the increase of the parameter σ_k, conversion is more likely to be delayed to happen around the liquidation level. Hence, the life of CCB as a

Table 4.10: TBTF Intensity Function

σ_k	$\mathbb{Q}_{v_0}(\xi \le T)$[a]	**Conversion Location(CET 1)**	
		Mean	**Std**
0.05	98.35%	4.90%	0.14%
0.25	98.35%	4.84%	0.37%
0.50	98.34%	4.82%	0.65%
0.75	98.32%	4.78%	0.88%
1.00	98.32%	4.72%	1.18%
1.50	98.28%	4.60%	1.61%
2.00	98.24%	4.38%	1.66%

[a] The results shown in the table are under the maturity $T = 100$. When we increase the maturity to $T = 200$, the probabilities are 98.99%, 98.99%, 98.99%, 98.98%, 98.96%, 98.95% and 98.92% respectively from the first row to the last row.

Table 4.11: Quantiles under TBTF Intensity Functions

σ_k	5% **Quantile**	15% **Quantile**	**Median**	85% **Quantile**	95% **Quantile**
0.05	4.64%	4.75%	4.92%	5.03%	5.07%
0.25	4.25%	4.42%	4.86%	5.22%	5.44%
0.50	4.04%	4.17%	4.74%	5.42%	5.95%
0.75	4.00%	4.00%	4.54%	5.51%	6.46%
1.00	4.00%	4.00%	4.32%	5.46%	6.83%
1.50	4.00%	4.00%	4.03%	5.04%	7.16%
2.00	4.00%	4.00%	4.00%	4.28%	6.14%

bond is prolonged and therefore the coupon stream received by investors. This effect reduces the credit spread of CCB leading to generally decreasing lines in the right panel.

As for contingent capital bonds, the conversion uncertainty makes CCBs issued by too-big-to-fail institutions more expensive than those issued by NSI institutions. This is because if conversion happens near the point of non-viability then CCB investors effectively move down the pecking order from bondholders (who has a relatively high recovery rate of their notional value at liquidation) to shareholders (who might recover nothing at liquidation). Under the TBTF intensity function, conversion is sure to occur prior to liquidation, excluding the situation that CCB ends up as a liability with some recovery rate at liquidation. In other words, if liquidation occurs, CCB investors will only hold shares which might be worthless at the moment. Therefore, CCB investors of too-big-to-fail financial institutions would require high coupon payments as compensation. This observation confirms our analysis in Chapter 2 indirectly. That is, the (sure-to-happen) conversion feature makes the junior tranche more expensive.

According to Figure 4.3, the credit spreads under the gone-concern NSI intensity function are more sensitive to the uncertainty than the TBTF intensity function. This is because conversion might not occur under the NSI intensity function, then the recovery rate at liquidation makes the redemption to CCB more volatile than the case that conversion is sure to be enforced by regulators prior to liquidation and the redemption to CCB is fixed as one minus the write-down proportion.

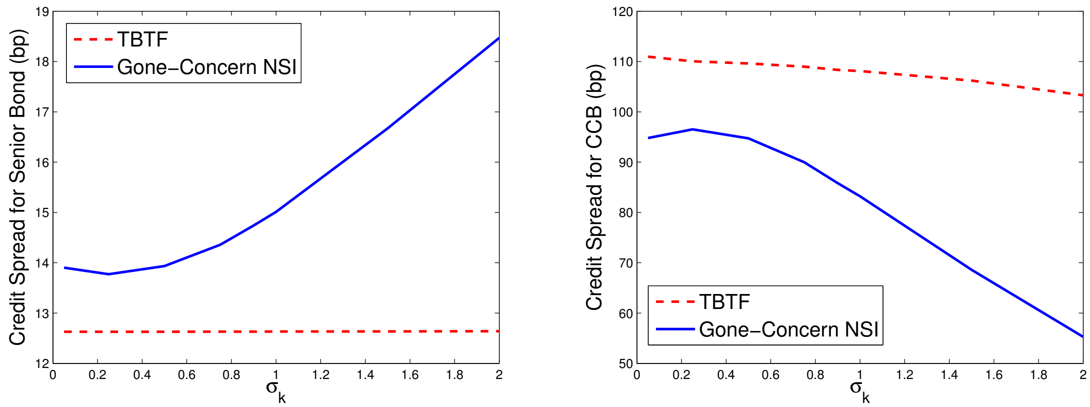

Figure 4.3: Credit spreads as a function of the parameter σ_k.

Reasonable Intervals for Conversion Terms

The reasonable intervals are more stable under the TBTF intensity function (Table 4.12) than that under the gone-concern NSI intensity function (Table 4.8). This observation can be interpreted by the fact that credit spreads are less sensitive to the change of σ_k under the TBTF intensity function than under the gone-concern NSI intensity function.

Table 4.12: Reasonable Intervals under TBTF Intensity Function

σ_k	**Reasonable Interval**	**Size of Interval**
0.05	[0.61%, 8.90%]	8.29%
0.25	[0.62%, 8.98%]	8.36%
0.50	[0.62%, 9.00%]	8.38%
0.75	[0.62%, 9.05%]	8.43%
0.90	[0.63%, 9.09%]	8.46%
1.00	[0.63%, 9.13%]	8.50%
1.50	[0.64%, 9.28%]	8.64%
2.00	[0.66%, 9.53%]	8.87%

4.4 Conclusion

We employ an intensity approach to incorporate the regulatory discretionary trigger into the pricing of contingent capital bonds. Two categories of intensity functions are considered to capture different patterns of regulatory discretion. The NSI intensity functions focus on the NSI financial institutions with the possibility that regulators would not enforce the conversion and let the institution fail, while the TBTF intensity functions focus on the financial institutions having potential too-big-to-fail problem and conversion is sure to happen as a source of financial resolution before liquidation.

As for the NSI intensity function, we discuss two approaches of modelling regulatory discretion. Under the gone-concern NSI intensity function, conversion uncertainty is added on a gone-concern basis and the implied delayed conversion reduces the cost for the contingent capital. However, under the going-concern NSI intensity function, conversion uncertainty is added in a going-concern framework. Due to the implied early write-down, the cost of the CCB increases. Because two types of NSI intensity functions imply opposite conversion uncertainty patterns, the size of the reasonable interval changes oppositely.

Under the TBTF intensity function, the cost of CCB keeps declining as the regulatory discretion increases. However, CCBs issued by too-big-to fail financial institutions would be more expensive than those issued by NSI financial institutions.

Bibliography

[1] S. Avdjiev, A. Kartasheva, and B. Bogdanova. Cocos: a primer. *Working Paper, BIS Quarterly Review*, pages 43–56, September 2013.

[2] C. D'Souza and T. Gravelle. Contingent capital and bail-in debt: Tools for bank resolution. *Bank of Canada, Reports for Financial System Review*, December 2011.

[3] D. Duffie. *How Big Banks Fail and What to Do about It*. Princeton University Press, October 2010.

[4] P. Glasserman and B. Nouri. Contingent capital with a capital-ratio trigger. *Management Science*, 58(10):1816–1833, April 2012.

[5] D. Lando. On cox process and credit risky securities. *Review of Derivatives Research*, (2):99–120, 1998.

[6] D. Lando. *Credit Risk Modeling: Theory and Applications*. Princeton University Press, 2004.

[7] Office of Superintendent of Financial Institutions. Advisory on non-viability contingent capital. *OSFI Reports*, August 2011.

[8] R. L. McDonald. Contingent capital with a dual price trigger. *Journal of Financial Stability*, 9(2):230–241, June 2013.

[9] A. Metzler. The laplace transform of hitting times of integrated geometric brownian motion. *Journal of Applied Probability*, 50(1):295–299, 2013.

[10] A. Metzler and R. M. Reesor. Valuation and analysis of zero-coupon contingent capital bonds. *Mathematics and Financial Economics*, 9(2):85–109, March 2015.

[11] Squam Lake Working Group on Financial Regulation. An expedited resolution mechanism for distressed financial firms: Regulatory hybrid securities. *Council on Foreign Relations Press*, April 2009.

[12] S. Sundaresan and Z. Wang. On the design of contingent capital with a market trigger. *The Journal of Finance*, 70(2):881–920, April 2015.

Chapter 5

Conclusions and Future Work

We extend the model put forward by Metzler and Reesor [2] to model contingent capital bonds with fixed coupon payments. We consider three problems – the pricing of contingent capital with a capital-ratio trigger, the potential manipulation incentives for investors when conversion is likely and the pricing of contingent capital bonds with a regulatory discretionary trigger. Our model adheres to the design advisory from OSFI [1] quantitatively and provides a reasonable range for the choice of conversion terms so that both liability seniority and existing shareholders not being rewarded for the firm's poor performance are guaranteed. We calibrate the model to Canadian banking data to provide a preliminary picture for Canadian banks' consideration for issuing contingent capital bonds.

In general, our work provides an applicable but preliminary approach to model contingent capital bonds. As for some potential extensions, there are several interesting divisions for our future work. Although we incorporate fixed coupon payments into the asset value process, we use constant model factors such as asset volatility (σ), risk-free interest rate (r) and the dividend payout ratio (q) which are important for the behaviour of the firm value and which might vary with the firm value. Therefore, it is worth trying to extend the model by treating factors mentioned above as diffusion processes that change with the firm value. In addition, we assume exogenous recovery rates in our model and calibrate the recovery rates from empirical credit spreads of Canadian banks. However, it would be more realistic to make the recovery rates depend on the firm value and evolve with time which might be another possible future work division of this thesis.

For the investigation of short-selling incentives, the deviation between the market and fundamental stock price may be improved using a more realistic model. For example, we can add random factors to the deviation and reversion process rather than merely making them time-dependent.

In Chapter 4, we assume that regulators are fully in charge of determining conversion and consider two typical intensity functions to mimic regulators' possible behaviours in enforcing conversion. However, some issuances use a combination of capital ratio and regulatory discretion as the trigger. Therefore, one might be interested in considering the combined trigger and investigating the pricing of contingent capital under this type of trigger. In addition, with the increasing complexity of the financial system, it would be hard to recognize if a financial institution is too-big-to-fail. As a result, for each financial institution, we cannot directly use a TBTF or NSI intensity function to approximate regulators' behaviours. One possible way to solve this problem is to combine two categories of intensity functions and assign a weight to each of them. The weight could be related to financial institution's systemically importance or other factors.

Bibliography

[1] Office of Superintendent of Financial Institutions. Advisory on non-viability contingent capital. *OSFI Reports*, August 2011.

[2] A. Metzler and R. M. Reesor. Valuation and analysis of zero-coupon contingent capital bonds. *Mathematics and Financial Economics*, 9(2):85–109, March 2015.

Appendix A

Summary of Issuances of Contingent Capital

Table A.1: Summary of Issuances of Contingent Capital Bonds

Financial Inst.	**Issuance Amt.**	**Maturity**	**Coupon Rates**	**Conversion Trigger**		**Conversion Terms**
				Capital Ratio Trigger	**Authority's Supervision**	
Lloys Banking Group, 2009	7.5 billion GBP	10 - 22 years	6.00% – 16.125%	Published Consolidated Core Tier 1 Ratio is less than 5%.	No.	Convert to common shares. Redemption is 100% of principle amount at par. Fixed conversion price, basically decided by the price at the time of issuance.
Credit Suisse Group, 2011	2.0 billion USD	30 years, with an optional redemption after 5 years.	7.875%	Core Tier 1 ratio (before Basel III Regulations Date) or Common Equity Tier 1 ratio (on or after Basel III Regulations Date) is below 7%.	Yes.	Convert to common shares. Redemption is 100% of principle amount (at par). Conversion price is a volume weighted average stock price for a preceding time period with a floor price.

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Financial Inst.	**Issuance Amt.**	**Maturity**	**Coupon Rates**	**Conversion Trigger**		**Conversion Terms**
				Capital Ratio Trigger	**Authority's Supervision**	
Credit Suisse Group, 2012	0.75 billion CHF	10 years, with an optional redemption after 5 years.	7.125%	Capital Ratio contained in any Interim Capital Report is below 5%.	Yes.	Convert to common shares. Redemption is 100% of principle amount (at par). Conversion price is a volume weighted average stock price for a preceding time period with a floor price.
Credit Suisse Group, 2012	1.725 billion USD	Perpetual with an optional redemption after 6 years.	9.5%	Capital Ratio contained in any Interim Capital Report is below 5%.	Yes.	Convert to common shares. Redemption is 100% of principle amount (at par). Conversion price is a volume weighted average stock price for a preceding time period with a floor price.
Rabobank, 2010	1.25 billion EUR	10 years.	6.875%	Equity Ratio falls below 7%.	No.	Write-down permanently. The amount written-down depends on the amount necessary to relieve relevant trigger event, or the full amount.
Rabobank, 2011	2.0 billion USD	Perpetual with a call after 5.5 years.	8.40%	Equity Capital Ratio falls below 8%.	No.	Write-down permanently. The amount of written-down depends on the amount necessary to relieve relevant trigger event above, or full amount.
UBS, 2012	2.0 billion USD	10 years.	7.625%	Breach of 5% Core Tier 1 ratio prior to the Basel III Implementation Date and 5% CET1 ratio when Basel III rules come into effect.	Yes.	Write-down permanently.

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Financial Inst.	Issuance Amt.	Maturity	Coupon Rates	Conversion Trigger		Conversion Terms
				Capital Ratio Trigger	Authority's Supervision	
Bank of Cyprus, 2011	0.85 billion EUR	Perpetual.	N/A	5% Core Tier 1 Ratio (EU defined) or 4.5% Common Equity Tier 1 ratio (Basel III definition).	Yes.	Convert to common shares. Redemption is 100% of principle amount (at par). Conversion price is based on the market price with both a floor and a ceiling price.
Bank of Macquarie, 2012	0.25 billion USD	N/A.	10.25%	CET1 ratio is equal to or less than 5.125%	Yes.	Convert to common shares. Some formula is used for the calculation of conversion price.
Royal Bank of Canada, 2014	1 billion CAD	10 years.	3.04% for the first 5 years. Then reset to a floating rate.	Point of non-viability.	Yes.	Convert to common shares. Conversion price is the current market price of the common shares plus a floor of $5. The multiplier is set at 1.5 times.
Bank of Montreal, 2014	1 billion CAD	Perpetual.	3.12% for the first 5 years. Then reset to a floating rate.	Point of non-viability.	Yes.	Convert to common shares. Conversion price is the current market price of the common shares plus a floor of $5. The multiplier is set at 1.5 times.

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Financial Inst.	**Issuance Amt.**	**Maturity**	**Coupon Rates**	**Conversion Trigger**		**Conversion Terms**
				Capital Ratio Trigger	**Authority's Supervision**	
CIBC, 2014	1 billion CAD	10 years.	3% for the first 5 years. Then reset to a floating rate.	Point of non-viability.	Yes.	Convert to common shares. Conversion price is the current market price of the common shares plus a floor of $5. The multiplier is set at 1.5 times.

Appendix B

Appendix for Chapter 2

B.1 Mathematical Proofs for Chapter 2

B.1.1 Hitting Time Properties

In this section, we lay out relevant properties of solutions of the diffusion process

$$dX_t = \tilde{\mu}(X_t)dt + \tilde{\sigma}(X_t)dW_t, \tag{B.1}$$

where

$$\tilde{\mu}(x) = \mu x - C$$

and

$$\tilde{\sigma}(x) = \sigma x$$

with constant $\mu, C, \sigma > 0$ and W_t is a standard Brownian motion. We are particularly interested in the hitting time

$$\tau_y = \inf\{t > 0 : X_t \leq y\}, \tag{B.2}$$

where $y \geq 0$ is a constant barrier and located below the initial location of the diffusion process $X_0 = x$.

Here and below, denote the expectation $\mathbb{E}$ with respect to the probability measure $\mathbb{P}$ and we use $\mathbb{P}_x$ and $\mathbb{E}_x$ to represent the probability and expectation conditioning on $X_0 = x$. The Laplace transform of the first-passage time

$$u(x, y, \mu, C, \sigma, \alpha) = \mathbb{E}_x[e^{-\alpha\tau_y}]. \tag{B.3}$$

Rather than working exclusively with processes taking values on the entire real line , we only focus on the interval

$$I = (l, r). \tag{B.4}$$

Similar to Karlin and Taylor [2], denote a and b are arbitrary fixed points inside (l, r), and the particular choice of a and b is of no relevance. The *scale function* is defined as

$$S(x) = \int_a^x s(\eta)d\eta = \int_a^x exp\left\{-\int_b^\eta \frac{2\tilde{\mu}(\xi)}{\tilde{\sigma}^2(\xi)}d\xi\right\}d\eta, \tag{B.5}$$

and the *speed density*,

$$m(x) = \frac{1}{\tilde{\sigma}^2(x)s(x)}. \tag{B.6}$$

According to Karlin and Taylor [2], with the scale function (B.5), the probability of hitting the upper barrier b prior to the lower barrier a is evaluated as $(S(x) - S(a))/(S(b) - S(a))$ for $x \in [a, b]$. From this perspective, the scale function plays a role in measuring the probability with the difference between the value of scale function at x and the lower barrier a.

Karlin and Taylor [2, Definition 6.2] The boundary l is said to be attainable if

$$\Sigma(l) = \int_l^x \left\{ \int_l^\xi s(\eta)d\eta \right\} m(\xi)d\xi < \infty.$$

Proposition B.1.1 (Karlin and Taylor [2, Lemma 6.2]) *If l is attainable, then l is attracting.*

Lemma B.1.2 (Karlin and Taylor [2, Lemma 6.2]) *Let l be an attracting boundary and suppose $l < x < r$. Then the following are equivalent*

1. $\mathbb{P}_x(\tau_l < \infty) > 0$;

2. $\Sigma(l) < \infty$.

Proposition B.1.3 (Karatzas and Shreve [1, Proposition 5.22, case (b)]) *Assume that for any $x \in I$, we have $\tilde{\sigma}^2(x) > 0$ and there exists a $\epsilon > 0$ such that $\int_{x-\epsilon}^{x+\epsilon} \frac{1+|\tilde{\mu}(y)|}{\tilde{\sigma}^2(y)}dy < \infty$. If $S(l+) > -\infty$ and $S(r-) = +\infty$, then $\mathbb{P}_x(\lim_{t\to\tau^*} X_t = l) = 1$, where $\tau^* = \inf\{t \geq 0 : X_t \notin (l, r)\}$.*

This proposition indicates that if the scale function is bounded from below at the lower boundary l while explosive at the upper boundary r, then the probability of hitting the lower boundary l is one.

Theorem B.1.4 *Given the diffusion process (B.1), the origin is attainable in finite time from any initial point $x > 0$. That is*

$$\mathbb{P}_x(\tau_0 < \infty) > 0 \tag{B.7}$$

for all $x > 0$. If $\mu < \sigma^2/2$, this can be strengthened to $\mathbb{P}_x(\tau_0 < \infty) = 1$.

Proof Based on Lemma B.1.2, we only need to prove $\Sigma(l) < \infty$ for $l = 0$. We start from the scale function (B.5),

$$\begin{aligned}
S(\xi) &= \int_l^{\xi} s(\eta)d\eta \\
&= \int_l^{\xi} exp\left\{-\int_b^{\eta} \frac{2(\mu z - C)}{\sigma^2 z^2}dz\right\}d\eta \\
&= \int_l^{\xi} exp\left\{-\frac{2\mu}{\sigma^2}\int_b^{\eta}\frac{1}{z}dz + \frac{2C}{\sigma^2}\int_b^{\eta}\frac{1}{z^2}dz\right\}d\eta \\
&= \int_l^{\xi} exp\left\{-\frac{2\mu}{\sigma^2}ln\left|\frac{\eta}{b}\right| + \frac{2C}{\sigma^2}\left(-\frac{1}{\eta}+\frac{1}{b}\right)\right\}d\eta \\
&= \int_l^{\xi} \left(\frac{\eta}{b}\right)^{-\frac{2\mu}{\sigma^2}} exp\left\{\frac{2C}{\sigma^2}\left(-\frac{1}{\eta}+\frac{1}{b}\right)\right\}d\eta \\
&= b^{\frac{2\mu}{\sigma^2}} exp\left\{\frac{2C}{\sigma^2}\frac{1}{b}\right\}\int_l^{\xi}\eta^{-\frac{2\mu}{\sigma^2}} exp\left\{-\frac{2C}{\sigma^2}\frac{1}{\eta}\right\}d\eta \\
&= B\int_l^{\xi}\eta^{-\frac{2\mu}{\sigma^2}} exp\left\{-\frac{2C}{\sigma^2}\frac{1}{\eta}\right\}d\eta, \qquad \text{(B.8)}
\end{aligned}$$

where we denote the constant $B = b^{\frac{2\mu}{\sigma^2}} exp\left\{\frac{2C}{\sigma^2}\frac{1}{b}\right\}$. Furthermore,

$$\begin{aligned}
\Sigma(l) &= \int_l^x \left(\int_l^{\xi} s(\eta)d\eta\right)m(\xi)d\xi \\
&= B\int_l^x \left(\int_l^{\xi}\eta^{-\frac{2\mu}{\sigma^2}} exp\left\{-\frac{2C}{\sigma^2}\frac{1}{\eta}\right\}d\eta\right)\frac{1}{\sigma^2\xi^2}exp\left\{\int_b^{\xi}\frac{2(\mu z - C)}{\sigma^2 z^2}dz\right\}d\xi \\
&= \frac{B}{\sigma^2}\int_l^x \left(\int_l^{\xi}\eta^{-\frac{2\mu}{\sigma^2}} exp\left\{-\frac{2C}{\sigma^2}\frac{1}{\eta}\right\}d\eta\right)b^{-\frac{2\mu}{\sigma^2}}exp\left\{-\frac{2C}{\sigma^2}\frac{1}{b}\right\}\xi^{\frac{2\mu}{\sigma^2}-2}exp\left\{\frac{2C}{\sigma^2}\frac{1}{\xi}\right\}d\xi \\
&= \frac{B}{\sigma^2}b^{-\frac{2\mu}{\sigma^2}}exp\left\{-\frac{2C}{\sigma^2}\frac{1}{b}\right\}\int_l^x \left(\int_l^{\xi}\eta^{-\frac{2\mu}{\sigma^2}} exp\left\{-\frac{2C}{\sigma^2}\frac{1}{\eta}\right\}d\eta\right)\xi^{\frac{2\mu}{\sigma^2}-2}exp\left\{\frac{2C}{\sigma^2}\frac{1}{\xi}\right\}d\xi \\
&= \frac{1}{\sigma^2}\int_l^x \left(\int_l^{\xi}\eta^{-\frac{2\mu}{\sigma^2}} exp\left\{-\frac{2C}{\sigma^2}\frac{1}{\eta}\right\}d\eta\right)\xi^{\frac{2\mu}{\sigma^2}-2}exp\left\{\frac{2C}{\sigma^2}\frac{1}{\xi}\right\}d\xi, \qquad \text{(B.9)}
\end{aligned}$$

where the last step holds because $Bb^{-\frac{2\mu}{\sigma^2}}exp\left\{-\frac{2C}{\sigma^2}\frac{1}{b}\right\} = 1$. After exchanging the order of integration, we rewrite the integral (B.9) as

$$\Sigma(l) = \frac{1}{\sigma^2}\int_l^x \eta^{-\frac{2\mu}{\sigma^2}} exp\left\{-\frac{2C}{\sigma^2}\frac{1}{\eta}\right\}\left(\int_{\eta}^x \xi^{\frac{2\mu}{\sigma^2}-2}exp\left\{\frac{2C}{\sigma^2}\frac{1}{\xi}\right\}d\xi\right)d\eta. \qquad \text{(B.10)}$$

Let $f(z) = \int_z^x \xi^{\frac{2\mu}{\sigma^2}-2}exp\left\{\frac{2C}{\sigma^2}\frac{1}{\xi}\right\}d\xi$ and $g(z) = z^{\frac{2\mu}{\sigma^2}}exp\left\{\frac{2C}{\sigma^2}\frac{1}{z}\right\}$, then we have $f'(z) = -g(z)/z^2$ and

$$\Sigma(l) = \frac{1}{\sigma^2}\int_l^x \frac{f(\eta)}{g(\eta)}d\eta. \qquad \text{(B.11)}$$

Taking $l = 0$, the integral (B.11) is improper and it is convergent if the integrand $f(\eta)/g(\eta)$ is bounded as $\eta \to 0^+$. By the L'Hopital rule,

$$\begin{aligned}
\lim_{\eta\to 0^+} \frac{f(\eta)}{g(\eta)} &\overset{\frac{\infty}{\infty}}{=} \lim_{\eta\to 0^+} \frac{f'(\eta)}{g'(\eta)} \\
&= \lim_{\eta\to 0^+} \frac{-g(\eta)/\eta^2}{g'(\eta)} \\
&= \lim_{\eta\to 0^+} -\frac{1}{\eta^2} \frac{\eta^{\frac{2\mu}{\sigma^2}} exp\left\{\frac{2C}{\sigma^2}\frac{1}{\eta}\right\}}{\frac{2\mu}{\sigma^2}\eta^{\frac{2\mu}{\sigma^2}-1} exp\left\{\frac{2C}{\sigma^2}\frac{1}{\eta}\right\} - \frac{2C}{\sigma^2}\frac{1}{\eta^2}\eta^{\frac{2\mu}{\sigma^2}} exp\left\{\frac{2C}{\sigma^2}\frac{1}{\eta}\right\}} \\
&= \lim_{\eta\to 0^+} -\frac{1}{\eta^2}\frac{1}{\frac{2\mu}{\sigma^2}\eta^{-1} - \frac{2C}{\sigma^2}\eta^{-2}} \\
&= \lim_{\eta\to 0^+} -\frac{1}{\frac{2\mu}{\sigma^2}\eta - \frac{2C}{\sigma^2}} \\
&= \frac{\sigma^2}{2C} < \infty. && \text{(B.12)}
\end{aligned}$$

So far we proved that $\Sigma(l) < \infty$ for $l = 0$. Therefore, the origin is attainable. By Proposition B.1.1 and Lemma B.1.2, we have $\mathbb{P}_x(\tau_0 < \infty) > 0$.

Now we turn to prove $\mathbb{P}_x(\tau_0 < \infty) = 1$ when $\mu < \sigma^2/2$. According to Proposition B.1.3, we only need to prove $\lim_{x\to 0^+} S(x) > -\infty$ and $\lim_{x\to\infty} S(x) = \infty$ when $\mu < \sigma^2/2$.

$$\begin{aligned}
\lim_{x\to 0^+} S(x) &= \lim_{x\to 0^+} \int_b^x s(\eta)d\eta \\
&= \lim_{x\to 0^+} B\int_b^x \eta^{-\frac{2\mu}{\sigma^2}} exp\left\{-\frac{2C}{\sigma^2}\frac{1}{\eta}\right\} d\eta \\
&\geq \lim_{x\to 0^+} Bexp\left\{-\frac{2C}{\sigma^2}\frac{1}{b}\right\}\int_b^x \eta^{-\frac{2\mu}{\sigma^2}} d\eta \\
&= \lim_{x\to 0^+} b^{\frac{2\mu}{\sigma^2}}\int_b^x \eta^{-\frac{2\mu}{\sigma^2}} d\eta \\
&= \lim_{x\to 0^+} \frac{b^{\frac{2\mu}{\sigma^2}}}{-\frac{2\mu}{\sigma^2}+1}\eta^{-\frac{2\mu}{\sigma^2}+1}\Big|_b^x. && \text{(B.13)}
\end{aligned}$$

It is easy to see that only when $\mu < \sigma^2/2$, the limit is convergent to a negative constant. Similarly, we can prove that only when $\mu < \sigma^2/2$ the scale function explodes as $x \to \infty$. Therefore, we have $\lim_{x\to 0^+} S(x) > -\infty$ and $\lim_{x\to\infty} S(x) = \infty$ when $\mu < \sigma^2/2$.

Based on Theorem B.1.4, for any fixed barrier above the origin, we have the following theorem.

Theorem B.1.5 *Given the diffusion process (B.1), a fixed barrier $y \in [0, x]$ is attainable in finite time from the initial point $x > 0$. That is,*

$$\mathbb{P}_x(\tau_y < \infty) > 0. \tag{B.14}$$

If $\mu < \sigma^2/2$, this can be strengthened to $\mathbb{P}_x(\tau_y < \infty) = 1$.

Theorem B.1.6 *For $\alpha \in \mathbb{R}_+$, and $y \in [0, x]$, the function $u(x, y, \mu, C, \sigma, \alpha)$ defined in (B.3) is equal to*

$$u = \begin{cases} \left(\frac{y}{x}\right)^{\gamma} \dfrac{M\left(\gamma, 2(\gamma+1) - \frac{2\mu}{\sigma^2}, -\frac{2C}{\sigma^2 x}\right)}{M\left(\gamma, 2(\gamma+1) - \frac{2\mu}{\sigma^2}, -\frac{2C}{\sigma^2 y}\right)}, & y > 0, \quad \text{(B.15)} \\ \left(\frac{2C}{\sigma^2 x}\right)^{\gamma} \dfrac{\Gamma\left(\gamma + 2 - \frac{2\mu}{\sigma^2}\right)}{\Gamma\left(2(\gamma+1) - \frac{2\mu}{\sigma^2}\right)} M\left(\gamma, 2(\gamma+1) - \frac{2\mu}{\sigma^2}, -\frac{2C}{\sigma^2 x}\right), & y = 0, \quad \text{(B.16)} \end{cases}$$

where Γ is the gamma function, M is the confluent hypergeometric function and γ is the unique positive root of

$$\xi^2 + (1 - \frac{2\mu}{\sigma^2})\xi - \frac{2\alpha}{\sigma^2} = 0. \tag{B.17}$$

Proof Fix all the parameters rather than the initial value $X_0 = x$. According to Metzler [3] and Theorem B.1.5, it is known that given the diffusion process (B.1), $u(x)$ solves the following second-order ordinary differential equation

$$\frac{\sigma^2 x^2}{2} u''(x) + (\mu x - C)u'(x) - \alpha u(x) = 0, \quad x \in [y, +\infty], \tag{B.18}$$

subject to the boundary condition $u(y) = 1$ and $\lim_{x \to +\infty} u(x) = 0$. Metzler [3] shows that the general solution to (B.18) is

$$u(x) = A\left(-\frac{2C}{\sigma^2 x}\right)^{\gamma} M(\gamma, 2(\gamma+1) - \frac{2\mu}{\sigma^2}, -\frac{2C}{\sigma^2 x}), \tag{B.19}$$

where A is an arbitrary constant. For $y > 0$, we must have $A = \left(-2C/\sigma^2 y\right)^{-\gamma} / M(\gamma, 2(\gamma+1) - 2\mu/\sigma^2, -2C/\sigma^2 x)$ in order to enforce the boundary condition $u(y) = 1$, which leads to (B.15).

Using asymptotic relations for the confluent hypergeometric function M, Metzler [3] shows that we must have $A = (-1)^{\gamma}\Gamma\left(\gamma + 2 - 2\mu/\sigma^2\right)/\Gamma\left(2(\gamma+1) - 2\mu/\sigma^2\right)$ in order to enforce the condition $u(0) = 1$, leading to (B.16).

Lemma B.1.7 (Karlin and Taylor [2, Lemma 7.1]) *Consider two fixed barriers $h_1, h_2 \in (l, r)$ and $h_1 < h_2 < x$. Then $\mathbb{E}_x[e^{-\tau_{h_1}}] = \mathbb{E}_x[e^{-\tau_{h_2}}]\mathbb{E}_{h_2}[e^{-(\tau_{h_1} - \tau_{h_2})}]$.*

B.1.2 Equity Value in the Capital Structure

In the following we are going to prove the relation

$$V_t = L_t + E_t + BC_t, \tag{B.20}$$

where L_t is the total value of liabilities in the capital structure, E_t is the equity value and BC_t is the total bankruptcy cost at time t before conversion (or liquidation under a traditional capital structure). We continue using the diffusion process (B.1) but replacing X_t with V_t and the parameter μ with $r - q$ where r is the risk-free interest rate and q is the dividend payout ratio with respect to the firm value. The parameter C can be though of as the fixed coupon payments to all the liabilities in the capital structure. The following discussion in this subsection is under the risk-neutral measure. By Itô's formula,

$$de^{-rt}V_t = -e^{-rt}(qV_t + C)dt + e^{-rt}\sigma V_t dW_t. \tag{B.21}$$

Write the stochastic differential equation (B.21) in integral form,

$$e^{-rt}V_t = V_0 - \int_0^t e^{-ru}qV_u du + \frac{C}{r}(e^{-rt} - 1) + \int_0^t e^{-ru}\sigma V_u dW_u. \tag{B.22}$$

After some transformation, we have

$$\begin{aligned} V_0 + \int_0^t e^{-ru}\sigma V_u dW_u =& e^{-rt}V_t + \int_0^t e^{-ru}qV_u du - \frac{C}{r}(e^{-rt} - 1), \\ =& e^{-rt}(V_t - L)^+ + \int_0^t e^{-ru}qV_u du \\ &+ e^{-rt}(V_t - (V_t - L)^+) + \frac{C}{r}(1 - e^{-rt}). \end{aligned}$$

Taking expectation on both sides, we have

$$\begin{aligned} V_0 + \mathbb{E}[\int_0^t e^{-ru}\sigma V_u dW_u] =& \mathbb{E}[e^{-rt}(V_t - L)^+ + \int_0^t e^{-ru}qV_u du] + \mathbb{E}[\frac{C}{r}(1 - e^{-rt})] \\ &+ \mathbb{E}[e^{-rt}(V_t - (V_t - L)^+)\mathbb{1}_{\{\tau_d > T\}}] \\ &+ \mathbb{E}[e^{-rt}(V_t - (V_t - L)^+)\mathbb{1}_{\{\tau_d \le T\}} - e^{-rt}BC_t] + \mathbb{E}[e^{-rt}BC_t], \end{aligned} \tag{B.23}$$

where $BC_t = \mathbb{E}_{t,v}[BC_{\tau_d}e^{-r(\tau_d - t)}\mathbb{1}_{\{\tau_d \le T\}}]$. The first expected value term on the right of (B.23) is the present value of equity and the second term is the present value of the coupon leg. The second line shows the expectation of the principal repayment under the circumstance that liquidation does not happen before maturity while the first term on the third line represents the expectation of principal repayment under the circumstance that liquidation happens before maturity. The last term is the expectation of bankruptcy cost at liquidation. Since the expectation of Itô integral is zero, we have $V_0 = L_0 + E_0 + BC_0$. Therefore, we prove $V_t = L_t + E_t + BC_t$ by no arbitrage theory.

B.1.3 Proposition in Reasonable Conversion Terms

In the following we are going to prove the proposition in Section 2.3.2. For convenience we restate the proposition here first and then prove it.

Proposition B.1.8 *The condition $\beta_S < \beta_{CCB}$ is satisfied if and only if $c_S < c_{CCB}$, where c_S and c_{CCB} are the par yields of the senior bond and CCB respectively.*

Proof The definitions of β_S^C, β_S and β_{CCB} are given by (2.31), (2.36) and (2.30), respectively. Let β_S^B be the loss imposed to the unconverted part of the senior bond at conversion,

$$\beta_S^B = 1 - \frac{S_{\tau_c}^B}{(1 - f_S)L_S}, \tag{B.24}$$

where $S^B_{\tau_c}$ is the value of the unconverted part of the senior bond at conversion and is evaluated by

$$S^B_{\tau_c} = \frac{c_S(1-f_S)L_S}{r}\left(1 - \mathbb{E}_{\tau_c,b^*L}[e^{-r(\tau_d-\tau_c)}]\right) + \mathcal{R}_S(1-f_S)L_S\,\mathbb{E}_{\tau_c,b^*L}[e^{-r(\tau_d-\tau_c)}]. \tag{B.25}$$

Moreover, we have

$$\beta_S = 1-(1-f_S)(1-\beta^B_S) - f_S(1-\beta^C_S). \tag{B.26}$$

Denote $u_1 = \mathbb{E}_{0,v}[e^{-r\tau_c}]$ and $u_2 = \mathbb{E}_{\tau_c,b^*L}[e^{-r(\tau_d-\tau_c)}]$, then the par yields can be solved from the following equation system (which omits the deposits)

$$L_S = \frac{c_S(1-f_S)L_S}{r}(1-u_1u_2) + \mathcal{R}_S(1-f_S)L_Su_1u_2 + \frac{c_Sf_SL_S}{r}(1-u_1) + \omega_S E_{\tau_c}u_1, \tag{B.27}$$
$$L_J = \frac{c_{CCB}L_J}{r}(1-u_1) + \omega_{CCB}E_{\tau_c}u_1. \tag{B.28}$$

We take out the ownership stakes (ω_S and ω_{CCB}) out of the expectation because they are determined by the conversion terms. Additionally, under a capital-ratio trigger, the residual value right after conversion (E_{τ_c}) is determined and we can take it out of the expectation as well.

After some transformations to (B.25), we have

$$1-\beta^B_{\tau_c} - \frac{c_S}{r} = -u_2\frac{c_S}{r} + \mathcal{R}_Su_2. \tag{B.29}$$

We normalize (B.27) and (B.28) by dividing them with the corresponding notional value and substitute (B.26) and (B.29),

$$1 = \frac{c_S}{r}(1-u_1) + (1-\beta_S)u_1, \tag{B.30}$$
$$1 = \frac{c_{CCB}}{r}(1-u_1) + (1-\beta_{CCB})u_1. \tag{B.31}$$

Subtracting (B.31) from (B.30) gives

$$(c_S - c_{CCB})\frac{1-u_1}{r} = (\beta_S - \beta_{CCB})u_1, \tag{B.32}$$

which implies that $\beta_S < \beta_{CCB}$ is satisfied if and only if $c_S < c_{CCB}$.

B.2 Calibrations for Chapter 2

B.2.1 Capital Structure

Table B.1: Capital Structure of Big - 6 Canadian Banks (in millions)

Bank	**RBC**	**CIBC**	**BMO**	**BNS**	**TD**	**NBC**
Deposit	495, 875	244, 207	316, 067	460, 907	470, 002	93, 894
Senior (exclude Senior Unsecured)	204, 320	99, 293	153, 343	122, 450	224, 479	66, 825
Senior Unsecured[1]	49, 413	22, 417	22, 426	34, 487	21, 211	5, 095
Senior Tranche[2](Approx BIDs)	253, 733	121, 710	175, 769	155, 220	245, 690	71, 920
Preferred Shares	4, 813	2, 006	2, 465	4, 384	3, 395	762
Subordinated Debt	7, 553	5, 112	5, 276	6, 896	11, 575	2, 461
Non-controlling Interests	1, 773	163	1, 441	1, 717	1, 485	1, 019
Junior Tranche (Approx CCBs)	14, 139	7, 281	9, 182	12, 997	16, 455	4, 242
Common Equity Tier 1[3](CET 1)	36, 624	14, 260	24, 485	30, 566	41, 039	6, 410
Total Assets	800, 371	387, 458	525, 503	659, 690	773, 186	176, 466

[1] Since our capital structure is based on the balance sheet of the second quarter, the value of senior unsecured debts is approximated by the bank's senior unsecured debts which are still outstanding after April 30, 2012. For those issued in foreign currencies, we translate the value back into Canadian dollars using the exchange rate on the day of issuance.

[2] We assume that the senior tranche is constituted of two parts, senior unsecured debt and the remaining part prior to senior unsecured debt.

[3] Since the preferred shares and non-controlling interests are categorized into the junior tranche, equity here does not contain these two parts.

Table B.2: Capital Structure of Big-6 Canadian Banks by Proportion

Bank	**RBC**	**CIBC**	**BMO**	**BNS**	**TD**	**NBC**
Deposit/Total Assets	61.69%	63.03%	60.15%	69.87%	60.79%	53.21%
Total Senior Debt/Total Assets	31.70%	31.41%	33.45%	23.53%	31.78%	40.76%
Total Junior Debt/Total Assets	1.77%	1.88%	1.75%	1.97%	2.13%	2.40%
Equity/Total Assets	4.58%	3.68%	4.66%	4.63%	5.31%	3.63%

Table B.3: Suggested Conversion Proportion of Senior Bond

Bank	**RBC**	**CIBC**	**BMO**	**BNS**	**TD**	**NBC**
Senior Unsecured	49, 413	22, 417	22, 426	34, 487	21, 211	5, 095
Total Senior Debt	279, 764	121, 710	175, 769	156, 937	245, 690	71, 920
Conversion% of Senior	19.47%	18.42%	12.76%	22.22%	8.63%	7.08%

Table B.4: Loss Absorbency Increase from Issuing Contingent Capital

Bank	**RBC**	**CIBC**	**BMO**	**BNS**	**TD**	**NBC**
Total Junior/CET1	38.61%	51.06%	37.50%	42.52%	40.10%	66.18%
Senior Unsecured/CET1	134.92%	157.20%	91.59%	112.83%	51.69%	79.49%

[1] The mean and median for the loss absorbency increase (the first row) from the conversion of CCB is 45.99% and 41.31%, respectively.
[2] The mean and median for the loss absorbency increase (the second row) from the conversion of unsecured senior bond is 104.62% and 102.21%, respectively.

B.2.2 Dividend Payout Ratio

Table B.5: Dividends Payout Ratio

Bank	**RBC**	**CIBC**	**BMO**	**BNS**	**TD**	**NBC**
Average	0.3718%	0.3196%	0.3387%	0.3823%	0.3262%	0.2876%
Median	0.3902%	0.3161%	0.3574%	0.3969%	0.3473%	0.2863%
Std	0.0433%	0.0635%	0.0491%	0.0456%	0.0471%	0.0167%

Bibliography

[1] I. Karatzas and S. E. Shreve. *Brownian Motion and Stochastic Calculus*. Springer, 1998.

[2] S. Karlin and H. M.Taylor. *A Second Course in Stochastic Process*. Academic Press Inc., 1981.

[3] A. Metzler. The Laplace transform of hitting times of integrated geometric brownian motion. *Journal of Applied Probability*, 50(1):295–299, 2013.

Appendix C

Appendix for Chapter 3

C.1 Diffusion Terms for Liabilities in the Capital Structure

Considering the asset value process (3.5) before conversion, in the perpetual case, the evaluation for each liability in the capital structure is stated by (2.27), (2.28) and (2.29) with the expectations of the discount factor $\mathbb{E}_{t,v}[e^{-r(\tau_c-t)}]$ given by (2.24). Ignoring all the constant parameters, let $U(v) = u(\bar{v}) = \mathbb{E}_{t,v}[e^{-r(\tau_c-t)}]$ for simplification[1]. According to Metzler [2], it is the solution of

$$\frac{\sigma^2 v^2}{2} U'(v) + ((r-q)v - (c_D L_D + c_S L_S + c_{CCB} L_J))U'(v) - rU(v) = 0, \quad v \in [b^* L, +\infty], \tag{C.1}$$

subject to the boundary condition $U(b^*L) = 1$ and $\lim_{v\to+\infty} U(v) = 0$. The CCB value at time t is

$$CCB_t = \frac{c_{CCB} L_J}{r} + (\omega_{CCB} E_{\tau_c} - \frac{c_{CCB} L_J}{r})U(v), \quad t < \tau_c. \tag{C.2}$$

By Itô's Lemma, we have

$$dCCB_t = (\omega_{CCB} E_{\tau_c} - \frac{c_{CCB} L_J}{r})(((r-q)V_t U' - (c_D L_D + c_S L_S + c_{CCB} L_J)U' + \frac{1}{2}\sigma^2 V_t^2 U'')dt + \sigma V_t U' dW_t). \tag{C.3}$$

By (C.1), we rewrite (C.3) as

$$dCCB_t = (\omega_{CCB} E_{\tau_c} - \frac{c_{CCB} L_J}{r})(rUdt + \sigma V_t U' dW_t). \tag{C.4}$$

As a result, the diffusion term for the CCB is

$$\sigma_{CCB}(V_t) = (\omega_{CCB} E_{\tau_c} - \frac{c_{CCB} L_J}{r})\sigma V_t U'(V_t). \tag{C.5}$$

Similarly, we can derive the diffusion term for deposits ($\sigma_D(V_t)$) and senior bond ($\sigma_S(V_t)$) in the capital structure are

$$\sigma_D(V_t) = \left(\max\{0, \min\{D, \mathcal{R}_D L_D\}\} - \frac{c_D L_D}{r}\right)\sigma V_t \mathbb{E}_{\tau_c, b^* L}[e^{-r(\tau_d - \tau_c)}]U'(V_t), \quad t < \tau_c \tag{C.6}$$

[1] As a remainder, $\bar{v}$ is the asset-liability ratio while v is the asset value at time t.

and

$$\sigma_S(V_t) = \left(\max\{0, \min\{D - \mathcal{R}_D L_D, \mathcal{R}_S L_S\}\} - \frac{c_S L_S}{r}\right)\sigma V_t \mathbb{E}_{\tau_c, b^* L}[e^{-r(\tau_d - \tau_c)}]U'(V_t), \quad t < \tau_c \tag{C.7}$$

respectively.

C.2 Proof of Theorem 3.3.2

We continue using the notations from Section B.1.1 in Appendix B. Instead of considering only one barrier, here we extend our discussion to a framework with two constant barriers $0 < a < b$ where both a and b are entrance boundaries for X_t in the sense of Karlin and Taylor [1]. We assume the starting point of the diffusion process $X_0 = x \in (a, b)$. The hitting times are defined as

$$\tau_a = \inf\{t \geq 0 : X_t \leq a\}, \tag{C.8}$$

$$\tau_b = \inf\{t \geq 0 : X_t \geq b\}. \tag{C.9}$$

Now we consider a process $\{X_t^*, t \geq 0\}$ which is prescribed to be a process confined to the sample paths which hit the lower barrier a prior to the upper barrier b. Accordingly, X_t^* exhibits only a part of the original sample path space since it is conditioned on the restriction of hitting the lower barrier prior to the upper barrier. We shall refer X_t^* as the associated conditioned diffusion process to X_t. Moreover, X_t^* is a Markov process since X_t is, and past history beyond the current state cannot affect where absorption occurs.

According to Karlin and Taylor [1], we can still use the infinitesimal displacement properties and derive relevant differential equations to solve for certain functions. However, due to the conditioning, there are two changes. The first one is in the infinitesimal displacement properties. Here we directly present the infinitesimal displacement properties for the conditional diffusion process and more details can be found in Karlin and Taylor [1]. Conditioning on the diffusion process hitting the lower barrier first,

$$\mu^*(x) = \mu(x) - \frac{s(x)}{S(x)}\sigma^2(x), \tag{C.10}$$

with $s(\eta) = \exp\left\{-\int_\eta^b \frac{2\mu(\xi)}{\sigma^2(\xi)}d\xi\right\}$ and $S(x) = \int_x^b s(\eta)d\eta$. Intuitively, since the process X_t^* hits the lower barrier a first, as $x \to b$, the denominator in the fraction of (C.10) approaches to zero and the drift $\mu^*(x)$ moves towards negative infinity. As a result, the process X_t^* moves down towards the lower barrier a very rapidly to avoid hitting the upper barrier b which is a conflict to the condition $\tau_a < \tau_b$. As for the diffusion term, we have

$$\sigma^*(x) = \sigma(x). \tag{C.11}$$

Since the derivation of the differential equation depends on a Taylor expansion and the infinitesimal displacement properties, in the following, we present the differential equations for the general diffusion process X_t based on which we can then move on to the differential equations for the conditioned diffusion process X_t^* by replacing $\mu(x)$ with $\mu^*(x)$ and $\sigma(x)$ with $\sigma^*(x)$ in the differential equations associated with X_t.

Lemma C.2.1 (Karlin and Taylor [1, Page 204]) *Let*

$$P(x) = \mathbb{E}_{0,x}\left[exp\left\{-\int_0^{\tau_a\wedge\tau_b} g(X_t)dt\right\}\right], \tag{C.12}$$

and

$$Q(x) = \mathbb{E}_{0,x}\left[exp\left\{-\int_0^{\tau_a\wedge\tau_b} g(X_t)dt\right\}\int_0^{\tau_a\wedge\tau_b} f(X_t)dt\right], \tag{C.13}$$

then Q(x) is the solution of the following differential equation

$$\frac{1}{2}\sigma^2(x)Q''(x) + \mu(x)Q'(x) - g(x)Q(x) + f(x)P(x) = 0, \quad a \le x \le b. \tag{C.14}$$

Replacing $\mu(x)$ with $\mu^*(x)$, $\sigma(x)$ with $\sigma^*(x)$ and let $g(x) = \alpha$ and $f(x) = \alpha$ in Lemma C.1.1, we get the differential equations in Theorem 3.3.2 for $x \in (a, b)$. For the process X_t^*, we define the hitting times to a and b as τ_a^* and τ_b^* respectively.

The other change lies in the boundary condition (i.e. $x \to a$ and $x \to b$). Obviously for $v(x) = \mathbb{E}_{0,x}[\alpha\tau_a^* e^{-\alpha\tau_a^*}|\tau_a^* < \tau_b^*]$ and $w(x) = \mathbb{E}_{0,x}[e^{-\alpha\tau_a^*}|\tau_a^* < \tau_b^*]$, we have the boundary condition $v(a) = 0$ and $w(a) = 1$. However, the boundary condition at the upper barrier b does not exist because the upper barrier is unattainable due to the conditioning. One possible resolution to approximate $v(x)$ and $w(x)$ with $x \in (a, b)$ is to solve the differential equations on the interval $(a, b(1-\epsilon))$ where $\epsilon > 0$ is arbitrarily small instead. Since the barrier $b(1-\epsilon)$ is attainable for the conditioned diffusion process, we have $v(b(1-\epsilon)) = 0$ and $w(b(1-\epsilon)) = 1$. The rationality is that according to (C.10), when the diffusion process is very close to the upper barrier b such as at $b(1-\epsilon)$, the drift term $\mu^*(x)$ becomes very negative forcing the diffusion process to move downwards to the lower barrier a very rapidly. Under this circumstance, the time of hitting the lower barrier is so small that we have $v(b(1-\epsilon)) = 0$ and $w(b(1-\epsilon)) = 1$. Let $v_\epsilon(x)$ and $w_\epsilon(x)$ be the solutions of the differential equations on the interval $(a, b(1-\epsilon))$. As ϵ decreases (i.e. the interval $(a, b(1-\epsilon))$ approaches to (a, b)), the solutions $v_\epsilon(x)$ and $w_\epsilon(x)$ converge to $v(x)$ and $w(x)$ respectively except at the points close to the upper barrier b. Therefore, for the solutions at a particular point, saying $\tilde{x}$, as we decrease the value of ϵ, if the solution $v_\epsilon(\tilde{x})$ and $w_\epsilon(\tilde{x})$ do not change significantly, we can use them to approximate the solution $v(\tilde{x})$ and $w(\tilde{x})$ respectively. The following two figures show the solutions $v_\epsilon(x)$ and $w_\epsilon(x)$ under different upper barriers (i.e. different ϵ's) which are close to b.

According to the numerical results, we can see $\epsilon = 10^{-6}$ would provide good approximations to the solutions of the differential equations at the points we are interested in, such as at $x = 5.5\%$, 8.0% and 10.5% CET1 ratio.

C.3 Discussion Related to the Parameter T^*

In this section firstly we present our numerical experiments of choosing T^* to estimate the numerical integral (3.26). Then we present the way to correct the bias in estimating the return from conversion using Monte Carlo simulation.

Recall that we aim to find a T^* such that the value of the survivor function $\hat{F}_{\tau_{s/o},v}(t)$ at T^* approximately zero. Define the function

$$H(T^*) = \frac{\mathbb{P}_{\tau_{s/o},v}(T_c \le T^*)}{\mathbb{P}_{\tau_{s/o},v}(T_c < +\infty)} = 1 - \frac{F_{\tau_{s/o},v}(T^*)}{F_{\tau_{s/o},v}(0)} = 1 - \hat{F}_{\tau_{s/o},v}(T^*). \tag{C.15}$$

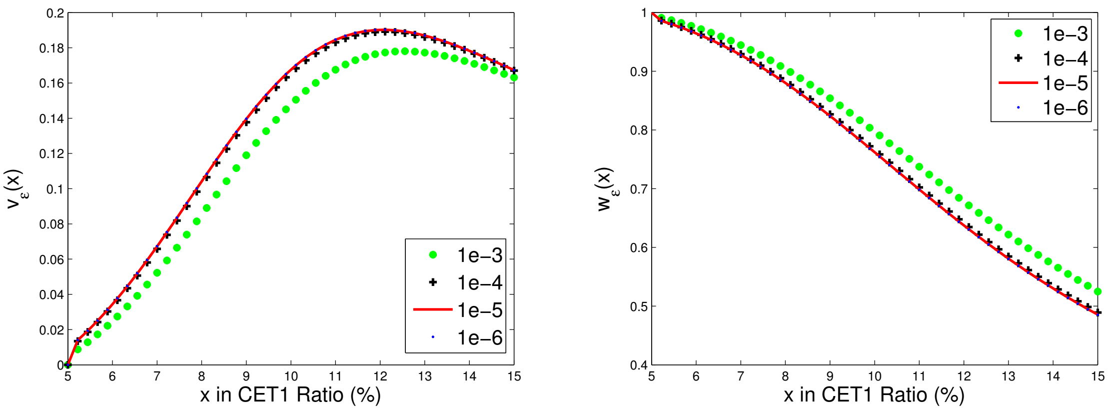

Figure C.1: Solutions $v_\epsilon(x)$ and $w_\epsilon(x)$ on the interval $(a, b(1-\epsilon))$ with different ϵ's. The parameters of the differential equations are from Table 2.2 except the asset value growth rate $\mu = 8.0\%$. The lower barrier a corresponds to the conversion barrier and the upper barrier b corresponds to the minimum value of the upper moving barrier.

Therefore, the condition $\hat{F}_{\tau_{s/o},v}(T^*) \approx 0$ is equivalent to the condition $\mathbb{P}_{\tau_{s/o},v}(T_c \leq T^*) \approx \mathbb{P}_{\tau_{s/o},v}(T_c < +\infty)$ or $H(T^*) \approx 1$. Figure C.2 illustrate the residual value $1 - H(T^*)$ versus different maturity T^*'s. According to our analysis, the closer the residual value $1 - H(T^*)$ is to zero, the better our choice of T^* would be. At $T^* = 8$, the residual value is 6.23×10^{-8}, 5.72×10^{-7} and 1.43×10^{-6} if short-selling starts at 5.5%, 8.0% and 10.5% CET1 ratio respectively. Even if the short-selling starts at 30.0% CET 1 ratio, and the corresponding residual value is approximately 4.13×10^{-5}. Therefore, we consider $T^* = 8$ as a reasonable choice for the maturity in our simulation.

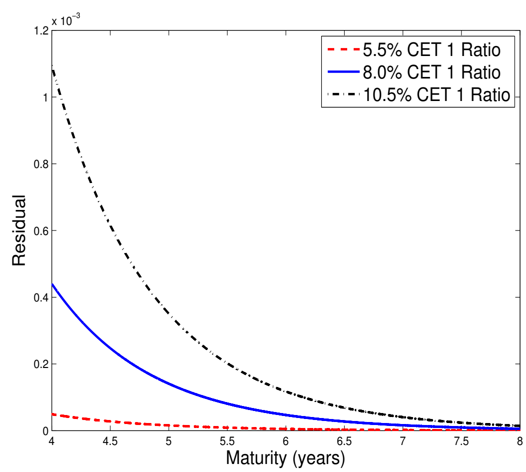

Figure C.2: The choice of the maturity T^* in simulation. The x-axis is the maturity T^*. The y-axis is the value for $1 - H(T^*)$.

C.4 Bias Correction for Simulation

Now we move on to the bias correction to our simulation results. If conversion never happens ($T_c = +\infty$) then R_c is zero. Therefore, we only need to consider two cases in estimating the expectation of the return from conversion: one is $T_c \leq T^*$ and the other is $T^* < T_c < +\infty$. By the law of total expectation,

$$\tilde{\mathbb{E}}_{\tau_{s/o},v}[R_c] = \tilde{\mathbb{E}}_{\tau_{s/o},v}[R_c|T_c \leq T^*]\mathbb{P}_{\tau_{s/o},v}(T_c \leq T^*) + \tilde{\mathbb{E}}_{\tau_{s/o},v}[R_c|T_c > T^*]\mathbb{P}_{\tau_{s/o},v}(T^* < T_c < +\infty). \tag{C.16}$$

According to our previous discussion, we choose a T^* such that $\mathbb{P}_{\tau_{s/o},v}(T_c \leq T^*) \approx \mathbb{P}_{\tau_{s/o},v}(T_c < +\infty)$. Therefore, the second term in (C.16) is very close to zero since the condition $T_c > T^*$ is approximately equivalent to $T_c = +\infty$. Thus,

$$\tilde{\mathbb{E}}_{\tau_{s/o},v}[R_c] \approx \tilde{\mathbb{E}}_{\tau_{s/o},v}[R_c|T_c \leq T^*]\mathbb{P}_{\tau_{s/o},v}(T_c \leq T^*). \tag{C.17}$$

The expectation term in (C.17) is estimated from simulation based on paths all of which hit the conversion level before or at T^*. Therefore, to obtain an unbiased estimation for the expected return from conversion, we have to multiply the expectation term in (C.17) with the probability $\mathbb{P}_{\tau_{s/o},v}(T_c \leq T^*)$. Based on our criteria of choosing T^*, the probability can be replaced with $\mathbb{P}_{\tau_{s/o},v}(T_c < +\infty)$ which can be solved analytically. Let $z(\beta) = \tilde{\mathbb{E}}_{\tau_{s/o},v}[e^{-\beta T_c}]$ and by the law of total expectation,

$$z(\beta) = \tilde{\mathbb{E}}_{\tau_{s/o},v}[e^{-\beta T_c}|T_c < +\infty]\mathbb{P}_{\tau_{s/o},v}(T_c < +\infty) + \tilde{\mathbb{E}}_{\tau_{s/o},v}[e^{-\beta T_c}|T_c = +\infty]\mathbb{P}_{\tau_{s/o},v}(T_c = +\infty), \tag{C.18}$$

$$= \tilde{\mathbb{E}}_{\tau_{s/o},v}[e^{-\beta T_c}|T_c < +\infty]\mathbb{P}_{\tau_{s/o},v}(T_c < +\infty), \tag{C.19}$$

where (C.19) holds because the second term in (C.18) approaches to zero in the infinite case ($T_c \to +\infty$). Taking $\beta = 0$ at both sides of (C.19) gives

$$z(0) = \mathbb{P}_{\tau_{s/o},v}(T_c < +\infty). \tag{C.20}$$

Given a fixed parameter β, $z(\beta)$ can be solved analytically according to Metzler [2]. Hence, we can solve for $z(0) = \mathbb{P}_{\tau_{s/o},v}(T_c < +\infty)$ analytically as well.

Bibliography

[1] S. Karlin and H. M.Taylor. *A Second Course in Stochastic Process*. Academic Press Inc., 1981.

[2] A. Metzler. The Laplace transform of hitting times of integrated geometric brownian motion. *Journal of Applied Probability*, 50(1):295–299, 2013.

Appendix D

Appendix for Chapter 4

In this appendix, we consider the asset value diffusion process (4.1). The liquidation time τ_d is defined as the first-passage time to the liquidation level by (4.14) and the conversion time ξ is defined by (4.5). The expectation term $\mathbb{E}_x[\cdot]$ represents the expectation under the risk-neutral measure with the diffusion process currently located at x. For each theorem, we restate the theorem first and then provide the proof for the reader's convenience.

According to the definition of the liquidation time τ_d, for an arbitrarily small time elapse $h > 0$, we have $\mathbb{Q}_x(\tau_d < h) = o(h)$.

D.1 Theorems and Proofs

Lemma D.1.1 *For a small time elapse $h > 0$,*

$$\mathbb{E}_x[e^{-\alpha\xi}\mathbb{1}_{\{\xi\leq h\}}] = k(x)h + o(h). \tag{D.1}$$

Proof By (4.7), the density function of the conversion time ξ conditioning on the asset value trajectory up to time t is

$$f_x(t) = k(V_t)exp\left\{-\int_o^t k(V_s)ds\right\}. \tag{D.2}$$

For a sufficiently small h, use the rectangle method to approximate the integral,

$$\begin{aligned}
\mathbb{E}_x[e^{-\alpha\xi}\mathbb{1}_{\{\xi\leq h\}}] &= \int_0^{+\infty} e^{-\alpha s}\mathbb{1}_{\{s\leq h\}}f_x(s)ds,\\
&= \int_0^{h} e^{-\alpha s}k(V_s)exp\left\{-\int_o^s k(V_u)du\right\}ds,\\
&\approx k(x)h + o(h). \quad \blacksquare
\end{aligned}$$

According to Karlin and Taylor [1], under the asset value process (4.1), where all the parameters such as the interest rate, dividend payout ratio and the coupon payments in the drift term are constant, the infinitesimal displacement $\Delta_h V_t$ from time t to $t + h$ conditioning on $V_t = x$ have the following properties:

$$\lim_{h\to 0}\frac{\mathbb{E}_x[\Delta_h V_t|\mathcal{F}_t]}{h} = \mu(x, C_i), \quad i = 0, 1, \tag{D.3}$$

and

$$\lim_{h\to 0}\frac{\mathbb{E}_x[(\Delta_h V_t)^2\,|\mathcal{F}_t]}{h}=\sigma^2(x). \tag{D.4}$$

Theorem D.1.2 *Let $U(x)=\mathbb{E}_x[e^{-\alpha\xi}\mathbb{1}_{\{\xi\le\tau_d\}}]$. Then $U(x)$ is the solution of the following second-order ordinary differential equation*

$$\mathcal{G}U(x)-(k(x)+\alpha)U(x)+k(x)=0,\quad x>DEF_0, \tag{D.5}$$

where the operator $\mathcal{G}=\frac{1}{2}\sigma^2(x)\frac{d^2}{dx^2}+\mu(x,C_0)\frac{d}{dx}$.

Proof Firstly, choosing an arbitrarily small time elapse $h>0$, we rewrite $U(x)$ into two parts

$$U(x)=\mathbb{E}_x[e^{-\alpha\xi}\mathbb{1}_{\{\xi\le\tau_d\}}\mathbb{1}_{\{\xi\le h\}}]+\mathbb{E}_x[e^{-\alpha\xi}\mathbb{1}_{\{\xi\le\tau_d\}}\mathbb{1}_{\{\xi>h\}}]. \tag{D.6}$$

For the first term in (D.6),

$$\begin{aligned}\mathbb{E}_x[e^{-\alpha\xi}\mathbb{1}_{\{\xi\le\tau_d\}}\mathbb{1}_{\{\xi\le h\}}]&=\mathbb{E}_x[e^{-\alpha\xi}\mathbb{1}_{\{\xi\le h\}}|\tau_d\ge h]\mathbb{Q}_x(\tau_d\ge h)+\mathbb{E}_x[e^{-\alpha\xi}\mathbb{1}_{\{\xi\le\tau_d\}}|\tau_d<h]\mathbb{Q}_x(\tau_d<h)\\&=\mathbb{E}_x[e^{-\alpha\xi}\mathbb{1}_{\{\xi\le h\}}|\tau_d\ge h]\mathbb{Q}_x(\tau_d\ge h)+o(h)\\&=\mathbb{E}_x[e^{-\alpha\xi}\mathbb{1}_{\{\xi\le h\}}]+o(h).\end{aligned} \tag{D.7}$$

The last equation holds because the time elapse $h>0$ is chosen to be arbitrarily small. As long as the starting point x of the diffusion process is above the liquidation barrier, the condition $\tau_d\ge h$ is satisfied automatically. By Lemma D.1.1 and (D.7), we have

$$\mathbb{E}_x[e^{-\alpha\xi}\mathbb{1}_{\{\xi\le\tau_d\}}\mathbb{1}_{\{\xi\le h\}}]=k(x)h+o(h). \tag{D.8}$$

For the second term in (D.6), define the infinitesimal displacement $V_h=x+\Delta_h V$ and the σ-algebra generated by the σ-algebra $\mathcal{F}_h$ and the indicator $\mathbb{1}_{\{\xi>h\}}$ as $\sigma(\mathcal{F}_h,\mathbb{1}_{\{\xi>h\}})$. By iterated conditioning and (4.7) and using Taylor expansion, we have

$$\begin{aligned}\mathbb{E}_x[e^{-\alpha\xi}\mathbb{1}_{\{\xi\le\tau_d\}}\mathbb{1}_{\{\xi>h\}}]&=\mathbb{E}_x\left[\mathbb{E}_x[e^{-\alpha\xi}\mathbb{1}_{\{\xi\le\tau_d\}}\mathbb{1}_{\{\xi>h\}}|\sigma(\mathcal{F}_h,\mathbb{1}_{\{\xi>h\}})]\right]\\&=\mathbb{E}_x\left[e^{-\alpha h}\mathbb{1}_{\{\xi>h\}}\mathbb{E}_x[e^{-\alpha(\xi-h)}\mathbb{1}_{\{\xi\le\tau_d\}}|\sigma(\mathcal{F}_h,\mathbb{1}_{\{\xi>h\}})]\right]\\&=\mathbb{E}_x\left[e^{-\alpha h}\mathbb{1}_{\{\xi>h\}}\mathbb{E}_{V_h}[e^{-\alpha(\xi-h)}\mathbb{1}_{\{\xi-h\le\tau_d-h\}}|\sigma(\mathcal{F}_h,\mathbb{1}_{\{\xi-h>0\}})]\right]\\&=\mathbb{E}_x\left[e^{-\alpha h}\mathbb{1}_{\{\xi>h\}}U(V_h)\right]\\&=e^{-\alpha h}\mathbb{E}_x\left[\mathbb{E}_x[\mathbb{1}_{\{\xi>h\}}U(V_h)|\mathcal{F}_h]\right]\\&=e^{-\alpha h}\mathbb{E}_x\left[U(V_h)\mathbb{E}_x[\mathbb{1}_{\{\xi>h\}}|\mathcal{F}_h]\right]\\&=e^{-\alpha h}\mathbb{E}_x\left[U(V_h)exp\left\{-\int_0^h k(V_s)ds\right\}\right]\\&=e^{-\alpha h}\mathbb{E}_x\left[U(V_h)(1-k(x)h+o(h))\right]\\&=e^{-\alpha h}(1-k(x)h+o(h))\mathbb{E}_x\left[U(V_h)\right]\\&=e^{-\alpha h}(1-k(x)h+o(h))\mathbb{E}_x[U(x)+U'(x)\Delta_h V+\frac{1}{2}U''(x)(\Delta_h V)^2+o(\Delta_h V)]\\&=e^{-\alpha h}(1-k(x)h+o(h))\left(U(x)+U'(x)\mathbb{E}_x[\Delta_h V]+\frac{1}{2}U''(x)\mathbb{E}_x[(\Delta_h V)^2]\right)\\&=(1-\alpha h+o(h))(1-k(x)h+o(h))(U(x)+h\mathcal{G}U(x)+o(h))\\&=U(x)+h\left(\mathcal{G}U(x)-\alpha U(x)-k(x)U(x)\right)+o(h).\end{aligned} \tag{D.9}$$

We obtain the third line according to the strong Markov property by thinking of the diffusion process starting from the place at time h and the part before the time h becomes irrelevant. Let $\xi_1 = \xi - h$ measure the time elapse between the conversion time and time h and $\tau_{d1} = \tau_d - h$ measure the time elapse between the liquidation time τ_d and time h, then

$$\mathbb{E}_{V_h}[e^{-\alpha(\xi-h)}\mathbb{1}_{\{\xi-h\leq\tau_d-h\}}|\sigma(\mathcal{F}_h, \mathbb{1}_{\{\xi-h>0\}})] = \mathbb{E}_{V_h}[e^{-\alpha\xi_1}\mathbb{1}_{\{\xi_1\leq\tau_{d1}\}}|V_h, \mathbb{1}_{\{\xi_1>0\}}] = \mathbb{E}_{V_h}[e^{-\alpha\xi_1}\mathbb{1}_{\{\xi_1\leq\tau_{d1}\}}] = U(V_h)$$

giving us the equation on the forth line. In the second-last line of (D.9), we use the results of the infinitesimal displacements (D.3) and (D.4). Combining (D.8) and (D.9), we have

$$U(x) = k(x)h + o(h) + U(x) + h\left(\mathcal{G}U(x) - \alpha U(x) - k(x)U(x)\right), \tag{D.10}$$

which verifies that $U(x)$ is the solution of (4.18). ∎

Theorem D.1.3 *Let $Z(x) = \mathbb{E}_x[e^{-\alpha\tau_d}\mathbb{1}_{\{\tau_d<\xi\}}]$. Then $Z(x)$ is the solution of the following second-order ordinary differential equation*

$$\mathcal{G}Z(x) - k(x)Z(x) - \alpha Z(x) = 0, \quad x > DEF_0, \tag{D.11}$$

where the operator $\mathcal{G} = \frac{1}{2}\sigma^2(x)\frac{d^2}{dx^2} + \mu(x, C_0)\frac{d}{dx}$.

Proof Taking a small time elapse $h > 0$, we decompose $Z(x)$ into two parts,

$$Z(x) = \mathbb{E}_x[e^{-\alpha\tau_d}\mathbb{1}_{\{\tau_d<\xi\}}\mathbb{1}_{\{\xi\leq h\}}] + \mathbb{E}_x[e^{-\alpha\tau_d}\mathbb{1}_{\{\tau_d<\xi\}}\mathbb{1}_{\{\xi>h\}}]. \tag{D.12}$$

By the fact that $\mathbb{Q}_x(\tau_d < h) = o(h)$ and the law of total expectation,

$$\mathbb{E}_x[e^{-\alpha\tau_d}\mathbb{1}_{\{\tau_d<\xi\}}\mathbb{1}_{\{\xi\leq h\}}] = \mathbb{E}_x[e^{-\alpha\tau_d}|\tau_d < \xi \leq h]\mathbb{Q}_x(\tau_d < \xi \leq h) = o(h). \tag{D.13}$$

For the second term in (D.12), define $V_h = x + \Delta_h V$ and the σ-algebra generated by the σ-algebra $\mathcal{F}_h$ and the indicator $\mathbb{1}_{\{\xi>h\}}$ as $\sigma(\mathcal{F}_h, \mathbb{1}_{\{\xi>h\}})$. Using iterated conditioning, take out what is known, by (4.7) and a Taylor expansion,

$$\begin{aligned}
\mathbb{E}_x[e^{-\alpha\tau_d}\mathbb{1}_{\{\tau_d<\xi\}}\mathbb{1}_{\{\xi>h\}}] &= \mathbb{E}_x\left[\mathbb{E}_x[e^{-\alpha\tau_d}\mathbb{1}_{\{\tau_d<\xi\}}\mathbb{1}_{\{\xi>h\}}|\sigma(\mathcal{F}_h, \mathbb{1}_{\{\xi>h\}})]\right] \\
&= \mathbb{E}_x\left[e^{-\alpha h}\mathbb{1}_{\{\xi>h\}}\mathbb{E}_{V_h}[e^{-\alpha(\tau_d-h)}\mathbb{1}_{\{\tau_d-h<\xi-h\}}|\sigma(\mathcal{F}_h, \mathbb{1}_{\{\xi-h>0\}})]\right] \\
&= e^{-\alpha h}\mathbb{E}_x\left[\mathbb{1}_{\{\xi>h\}}Z(V_h)\right] \\
&= e^{-\alpha h}\mathbb{E}_x\left[\mathbb{E}_x[\mathbb{1}_{\{\xi>h\}}Z(V_h)|\mathcal{F}_h]\right] \\
&= e^{-\alpha h}\mathbb{E}_x\left[Z(V_h)\mathbb{E}_x[\mathbb{1}_{\{\xi>h\}}|\mathcal{F}_h]\right] \\
&= e^{-\alpha h}\mathbb{E}_x\left[Z(V_h)\mathbb{Q}_x(\xi > h|\mathcal{F}_h)\right] \\
&= e^{-\alpha h}\mathbb{E}_x\left[Z(V_h)exp\left\{-\int_0^h k(V_s)ds\right\}\right] \\
&= e^{-\alpha h}\mathbb{E}_x\left[Z(V_h)\left(1 - k(x)h + o(h)\right)\right] \\
&= e^{-\alpha h}\left(1 - k(x)h + o(h)\right)\mathbb{E}_x\left[Z(V_h)\right] \\
&= e^{-\alpha h}\left(1 - k(x)h + o(h)\right)\mathbb{E}_x[Z(x) + Z'(x)\Delta_h V + \frac{1}{2}Z''(x)(\Delta_h V)^2 + o(\Delta_h V)] \\
&= (1 - \alpha h + o(h))(1 - k(x)h + o(h))(Z(x) + h\mathcal{G}Z(x) + o(h)) \\
&= Z(x) + h\left(\mathcal{G}Z(x) - (\alpha + k(x)Z(x))\right) + o(h),
\end{aligned} \tag{D.14}$$

where the second line holds by the strong Markov property and thinking of a diffusion process starting at the place at time h. Let $\xi_1 = \xi - h$ measure the time elapse between the conversion time and the time h and $\tau_{d1} = \tau_d - h$ measure the time elapse between the liquidation time τ_d and the time h, then

$$\mathbb{E}_{V_h}[e^{-\alpha(\tau_d-h)}\mathbb{1}_{\{\tau_d-h<\xi-h\}}|\sigma(\mathcal{F}_h, \mathbb{1}_{\{\xi-h>0\}})] = \mathbb{E}_{V_h}[e^{-\alpha\tau_{d1}}\mathbb{1}_{\{\tau_{d1}<\xi_1\}}|V_h, \mathbb{1}_{\{\xi_1>0\}}] = \mathbb{E}_{V_h}[e^{-\alpha\tau_{d1}}\mathbb{1}_{\{\tau_{d1}<\xi_1\}}] = Z(V_h)$$

giving us the equation on the third line. The last but second line holds according to the results of the infinitesimal displacements (D.3) and (D.4). Combing (D.13) and (D.14),

$$Z(x) = o(h) + Z(x) + h\left(\mathcal{G}Z(x) - (\alpha + k(x)Z(x))\right), \tag{D.15}$$

which proves that $Z(x)$ is the solution of (D.11). ∎

Theorem D.1.4 *Let $W(x) = \mathbb{E}_x[e^{-\alpha\tau_d}]$. Then $W(x)$ solves the following second-order ordinary differential equation*

$$\mathcal{G}W(x) - (k(x) + \alpha)W(x) + k(x)W_1(x) = 0, \quad x > DEF_0, \tag{D.16}$$

where the operator $\mathcal{G} = \frac{1}{2}\sigma^2(x)\frac{d^2}{dx^2} + \mu(x, C_0)\frac{d}{dx}$ and $W_1(x) = \mathbb{E}_x[e^{-\alpha\tau_d}]$, which is calculated based on the post-conversion diffusion process, i.e., the fixed coupon payment for the diffusion process is C_1.

Proof Taking a small time elapse $h > 0$, we start from separating $W(x)$ into two parts,

$$W(x) = \mathbb{E}_x[e^{-\alpha\tau_d}\mathbb{1}_{\{\xi\le h\}}] + \mathbb{E}_x[e^{-\alpha\tau_d}\mathbb{1}_{\{\xi>h\}}]. \tag{D.17}$$

For the first term in (D.17), define $V_h = x + \Delta_h V$ and the σ-algebra generated by the σ-algebra $\mathcal{F}_h$ and the indicator $\mathbb{1}_{\{\xi\le h\}}$ as $\sigma(\mathcal{F}_h, \mathbb{1}_{\{\xi\le h\}})$. Let $W_1(x)$ be the Laplace transform of the post-conversion liquidation time τ_d with the current firm value located at x. Using iterated conditioning, take out what is known, by (4.7) and a Taylor expansion,

$$\begin{aligned}
\mathbb{E}_x[e^{-\alpha\tau_d}\mathbb{1}_{\{\xi\le h\}}] &= \mathbb{E}_x\left[\mathbb{E}_x[e^{-\alpha\tau_d}\mathbb{1}_{\{\xi\le h\}}|\sigma(\mathcal{F}_h, \mathbb{1}_{\{\xi\le h\}})]\right] \\
&= \mathbb{E}_x\left[e^{-\alpha h}\mathbb{1}_{\{\xi\le h\}}\mathbb{E}_{V_h}[e^{-\alpha(\tau_d-h)}|\sigma(\mathcal{F}_h, \mathbb{1}_{\{\xi\le h\}})]\right] \\
&= \mathbb{E}_x\left[e^{-\alpha h}\mathbb{1}_{\{\xi\le h\}}W_1(V_h)\right] \\
&= \mathbb{E}_x\left[\mathbb{E}_x[e^{-\alpha h}\mathbb{1}_{\{\xi\le h\}}W_1(V_h)|\mathcal{F}_h]\right] \\
&= \mathbb{E}_x\left[e^{-\alpha h}W_1(V_h)\mathbb{E}_x[\mathbb{1}_{\{\xi\le h\}}|\mathcal{F}_h]\right] \\
&= \mathbb{E}_x\left[e^{-\alpha h}W_1(V_h)\mathbb{Q}_x(\xi\le h|\mathcal{F}_h)\right] \\
&= \mathbb{E}_x\left[e^{-\alpha h}W_1(V_h)(1 - exp\left\{-\int_o^h k(V_s)ds\right\})\right] \\
&= \mathbb{E}_x\left[e^{-\alpha h}W_1(V_h)(k(x)h + o(h))\right] \\
&= (1 - \alpha h + o(h))(k(x)h + o(h))\mathbb{E}_x\left[W_1(V_h)\right] \\
&= (k(x)h + o(h))\mathbb{E}_x[W_1(x) + W_1'(x)\Delta_h V + \frac{1}{2}W_1''(x)(\Delta_h V)^2 + o(\Delta_h V)] \\
&= (k(x)h + o(h))\left(W_1(x) + h\mathcal{G}W_1(x) + o(h)\right) \\
&= k(x)W_1(x)h + o(h).
\end{aligned} \tag{D.18}$$

In the second line of (D.18), the conditional expectation implies conversion happens before time h, leading to the third line with the function $W_1(V_h)$ representing the Laplace transform of the time elapse $\tau_d - h$. By the strong Markov property, the expectation only depends on the asset value at the time h so the function W_1 only depends on the asset value V_h. The last but second line in (D.18) holds due to the infinitesimal displacement results (D.3) and (D.4). Applying similar techniques to the second term in (D.17)

$$\begin{aligned}
\mathbb{E}_x[e^{-\alpha\tau_d}\mathbb{1}_{\{\xi>h\}}] &= \mathbb{E}_x\left[\mathbb{E}_x[e^{-\alpha(\tau_d-h)}e^{-\alpha h}\mathbb{1}_{\{\xi>h\}}|\sigma(\mathcal{F}_h, \mathbb{1}_{\{\xi>h\}})]\right] \\
&= \mathbb{E}_x\left[e^{-\alpha h}\mathbb{1}_{\{\xi>h\}}\mathbb{E}_{V_h}[e^{-\alpha(\tau_d-h)}|\sigma(\mathcal{F}_h, \mathbb{1}_{\{\xi-h>0\}})]\right] \\
&= \mathbb{E}_x\left[e^{-\alpha h}\mathbb{1}_{\{\xi>h\}}W(V_h)\right] \\
&= \mathbb{E}_x\left[\mathbb{E}_x[e^{-\alpha h}\mathbb{1}_{\{\xi>h\}}W(V_h)|\mathcal{F}_h]\right] \\
&= \mathbb{E}_x\left[e^{-\alpha h}W(V_h)\mathbb{E}_x[\mathbb{1}_{\{\xi>h\}}|\mathcal{F}_h]\right] \\
&= \mathbb{E}_x\left[e^{-\alpha h}W(V_h)\mathbb{Q}_x(\xi>h|\mathcal{F}_h)\right] \\
&= e^{-\alpha h}\mathbb{E}_x\left[W(V_h)(1-k(x)h+o(h))\right] \\
&= e^{-\alpha h}\left(1-k(x)h+o(h)\right)\mathbb{E}_x\left[W(x)+W'(x)\Delta_h V+W''(x)(\Delta_h V)^2\right] \\
&= e^{-\alpha h}(1-k(x)h+o(h))(W(x)+\mathbb{E}_x[\Delta_h V]W'(x)+\mathbb{E}_x[(\Delta_h V)^2]W''(x)). \qquad \text{(D.19)}
\end{aligned}$$

We need to deal with the term $\mathbb{E}_x[\Delta_h V]$ in the last line of (D.19) carefully because it depends on the coupon payment pattern[1]. Rewrite

$$\mathbb{E}_x[\Delta_h V] = \mathbb{E}_x[\Delta_h V\mathbb{1}_{\{\xi\le h\}}] + \mathbb{E}_x[\Delta_h V\mathbb{1}_{\{\xi>h\}}]. \tag{D.20}$$

For the first term in (D.20), by the law of total expectation and (4.8)

$$\begin{aligned}
\mathbb{E}_x[\Delta_h V\mathbb{1}_{\{\xi\le h\}}] &= \mathbb{E}_x[\Delta_h V\mathbb{1}_{\{\xi\le h\}}|\xi\le h]\mathbb{Q}_x(\xi\le h) + \mathbb{E}_x[\Delta_h V\mathbb{1}_{\{\xi\le h\}}|\xi>h]\mathbb{Q}_x(\xi>h) \\
&= \mathbb{E}_x[\Delta_h V\mathbb{1}_{\{\xi\le h\}}|\xi\le h]\mathbb{Q}_x(\xi\le h) \\
&= \mathbb{E}_x[\Delta_h V|\xi\le h](k(x)h+o(h)) \\
&= h\mu(x, C_1)(k(x)h+o(h)), \\
&= o(h). \qquad \text{(D.21)}
\end{aligned}$$

The second-last line in (D.21) holds because we assume that h is sufficiently small so if conversion happens before h, it is equivalent to happening at the very beginning, then the coupon payment pattern is C_1. Similarly, for the second term

$$\begin{aligned}
\mathbb{E}_x[\Delta_h V\mathbb{1}_{\{\xi>h\}}] &= \mathbb{E}_x[\Delta_h V\mathbb{1}_{\{\xi>h\}}|\xi\le h]\mathbb{Q}_x(\xi\le h) + \mathbb{E}_x[\Delta_h V\mathbb{1}_{\{\xi>h\}}|\xi>h]\mathbb{Q}_x(\xi>h) \\
&= \mathbb{E}_x[\Delta_h V\mathbb{1}_{\{\xi>h\}}|\xi>h]\mathbb{Q}_x(\xi>h) \\
&= \mathbb{E}_x[\Delta_h V|\xi>h]\mathbb{Q}_x(\xi>h) \\
&= h\mu(x, C_0)(1-k(x)h+o(h)), \\
&= h\mu(x, C_0)+o(h). \qquad \text{(D.22)}
\end{aligned}$$

[1]In Theorem D.1.2 and Theorem D.1.3, we only consider the diffusion process prior to conversion, so we take the coupon payment pattern C_0 by default. Here, if conversion happens before liquidation is unknown, so we need to discuss two possibilities.

We use the coupon payment pattern C_0 in (D.22) since the expectation conditions on $\xi > h$ implying that conversion happens after h. By (D.22) and (D.21),

$$\mathbb{E}_x[\Delta_h V] = \mu(x, C_0)h + o(h). \tag{D.23}$$

Now we go back to continue with (D.19),

$$\begin{aligned}\mathbb{E}_x[e^{-\alpha T_{def}} \mathbb{1}_{\{\xi>h\}}] &= (1 - (\alpha + k(x))h + o(h)) \left(W(x) + \mathbb{E}_x[\Delta_h V]W'(x) + \mathbb{E}_x[(\Delta_h V)^2]W''(x)\right),\\ &= (1 - (\alpha + k(x))h + o(h)) (W(x) + h\mathcal{G}W(x) + o(h)),\\ &= W(x) + h(\mathcal{G}W(x) - (\alpha + k(x))W(x)) + o(h). \end{aligned} \tag{D.24}$$

Finally, combining (D.24) and (D.18), we prove that $W(x)$ is the solution of the ordinary differential equation (D.16). ∎

Bibliography

[1] S. Karlin and H. M.Taylor. *A Second Course in Stochastic Process*. Academic Press Inc., 1981.

Appendix E

Glossary

Bail-In A bail-in forces the financial institution's creditors to bear some of the burden by having part of the debt they are owed written off[1].

Bail-Out A bail-out is when outside investors (such as taxpayers) rescue a financial institution by injecting money to help service a debt.

Bankruptcy Bankruptcy is a legal status of a person or other entity that cannot repay the debts it owes to creditors.

Basel III Basel III is a comprehensive set of reform measures, developed by the Basel Committee on Banking Supervision, to strengthen the regulation, supervision and risk management of the banking sector[2].

Bond A bond is a debt investment in which an investor loans money to an entity (typically corporate or governmental) which borrows the funds for a defined period of time at a variable or fixed interest rate.

Capital Financial assets or the financial value of assets, such as cash.

Capital Structure In finance, capital structure is the way a corporation finances its assets through some combination of equity, debt, or hybrid securities.

Contingent Capital Bond A contingent capital bond (CCB) is a subordinated security, such as preferred share or subordinated debenture, that converts to common equity in financial distress.

Convertible Bond A convertible bond can be converted into a certain amount of the company's equity at certain times during its life, at the discretion of the bondholder.

Coupon A coupon payment on a bond is a periodic interest payment that the bondholder receives during the time between when the bond is issued and when it matures.

[1] http://www.economist.com/blogs/economist-explains/2013/04/economist-explains-2

[2] http://www.bis.org/bcbs/basel3.htm

Default Risk The event in which companies or individuals will be unable to make the required payments on their debt obligations.

Dividend A dividend is a distribution of a portion of a company's earnings, decided by the board of directors, to a class of its shareholders. Dividends can be issued as cash payments, as shares of stock, or other property.

Equity Equity is the value of an asset less the value of all liabilities on that asset.

Market Discipline Market discipline is a market-based promotion of the transparency and disclosure if the risks associated with a business or entity. It works in concert with regulatory systems to increase the safety and soundness of the market.

Moral Hazard It is a situation in which one party gets involved in a risky event knowing that it is protected against the risk. For example, a manager of a big financial institution might take excessive risk knowing that the government will support the financial institution if it is in trouble.

Notional Value The notional value on a financial instrument is the nominal or face amount that is used to calculate payments made on that instrument.

Par Yield Par yield is the coupon rate for which the price of a bond is equal to its notional value (or par value).

Risk-Free Rate The rate of interest that can be earned without assuming any risk.

Risk Neutral Valuation The valuation of a derivative assuming the world is risk neutral. Risk neutral valuation gives the correct price for a derivative in all worlds, not just in a risk-neutral world.

Risk-Weighted Asset Risk-weighted asset (RWA) is a financial institution's assets weighted according to risk. Each asset is assigned a risk weight reflecting its credit risk.

Short-Selling Short-selling is the sale of a security that is not owned by the seller, or that seller has borrowed. This behaviour is motivated by the belief that a security's price will decline in the future, enabling it to be bought back at a lower price to make a profit.

Too-Big-To-Fail It refers to the situation that certain financial institutions are so large and so comprehensive that their failure would be disastrous to the greater economies system, and that they therefore must be supported by government when they face potential failure.

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