High resolution tropospheric studies with an MST type radar

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HIGH RESOLUTION TROPOSPHERIC STUDIES WITH AN MST TYPE RADAR
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by

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Abstract

Applications and results of a variety of studies carried out with the Costa Rican VHF profiler radar are included. This radar is unique because of the wide frequency bandwidth available (5 MHz). During the developing stages of the wind profiler radar new techniques of processing and radar engineering were used.

A thorough analysis of a data-dependent method (Capon’s) commonly used in radar was carried out. This method is analyzed based on a reference established by Fourier theory. Disadvantages of using data-dependent methods with small filters were found. A small filter size will cause Capon’s method to underestimate the number of spectral peaks when compared to using a larger filter size. The underestimation will occur even when the filter size is larger than the number of peaks. When used to estimate a known Gaussian spectrum, Capon’s method underestimated the spectral width independently of the filter size.

The backscattered signal measured in radars is the convolution of the transmitted signal with the atmospheric profile of scatterers. The convolution integral model was used to calculate radar backscatter. The implementations were created to simulate radar backscatter using the transmitted signal and the electric permittivity profile of the atmosphere. Tests with realistic physical conditions were carried out to validate the model and verify the implementation. Appropriate range location and Doppler velocities were obtained from simple simulations in one, two and three dimensions. The convolution engine was later used along with a mathematical representation of atmospheric scatterers to study the simulated radar echoes. Correct radial velocities were obtained along with realistic radar effect like the beam broadening effect.

A Large eddy simulation (LES) model was used to simulate a full-physics atmosphere. The LES code follows the fluid dynamics equations. The simulated LES atmosphere was used to calculate the electric permittivity from atmospheric variables. A radar simulation involving the convolution of this electric permittivity profile and the radar pulse allowed us to simulate a radar inside the simulated atmosphere. The initial conditions of the simulation created a clear planetary boundary layer as well as a region of shear instabilities in the upper heights. Both regions generated turbulence during the simulation and allowed the radar simulation to measure it satisfactorily. Anisotropy was observed in the results when comparing vertical beam data to tilted beam data as usually observed in real measurements.

A long term experiment was carried out in Costa Rica to gather information about the tropical atmosphere. This was the first time this type of experiment was carried out in Costa Rica. The information provided a clear perspective of the phenomena found in the lower troposphere. Among others, the planetary boundary layer (PBL), thin layers, isolated patches of turbulence, oscillations and convective events were detected. The presence of layers over Costa Rica is well defined; during the dry season months (winter-spring time of the northern hemisphere) at least one layer can be observed up to 30% of the time, while it decreases below 10% of the time during the rainy season (summer-fall time of the northern hemisphere). The PBL shows great variability depending on the general conditions of the atmosphere; the average PBL top was located near 2 km during the dry season months and increased to almost 4 km during the rainy season. More examples are provided.

**Keywords:** Wind profiler, wide bandwidth, deconvolution, radar simulation, large eddy simulation, spectral tools
To Diana, Luna and Neo.
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Chapter 1

Introduction

1.1 Introduction

The Earth’s atmosphere is full of diverse phenomena. Some of these phenomena are readily detectable by the human eye such as the cloud formations, which have entertained children (of all ages!) for generations. These same formations carry along associated phenomena (e.g. rain and tornadoes) that intrigued and inspired curious minds for as long as humans have walked the surface of the planet. Many other phenomena in the atmosphere are invisible to our senses, yet these events, not observable by eye, have become lucid by the use of instruments.

A wide variety of instruments can be used to probe the atmosphere. Balloon sondes have been used to measure the content of the atmosphere. Rockets with the ability to reach the upper layers of the atmosphere have contributed information about composition and dynamics. The recent advent of satellites allowed more insight into the atmosphere by using high-technology sensors to probe and detect. A common device used for remote sensing is the radar.

Radars can be very diverse as they can be used to identify or detect specific targets. Radars with large power-aperture (Balsley and Gage, 1980) are used to monitor the ionosphere while low power radars with small antennas can be used in marine transportation to locate storms. Miniature radars are installed in cars to monitor the distance to nearby objects. Weather radars can look at the horizon to detect moving clouds or storms. Other radars can look up vertically in order to detect air motion, which is the type of radar used in this research.

Vertically pointing radars can have many different shapes and sizes. Balsley and Gage (1980) included a list of radars commonly used in clear air studies along with their physical characteristics and geographical location. The properties of a radar are determined by the frequency of operation, antenna area, and application. Depending on the combination of properties, radars can detect different structures in the atmosphere. In the upper atmosphere, backscattering will be caused by inhomogeneities in the free electron density. In the lower atmosphere the scattering is caused by bound electrons, hence variations in the index of refraction, driven by the evolution of atmospheric conditions, will cause the backscattering.

Atmospheric properties (i.e. temperature, humidity, pressure) are not fixed, but change dynamically with the evolution of the atmospheric fluids. The index of refraction (defined by the atmospheric properties) will change, creating non-homogenous regions that can cause backscatter of electromagnetic waves. These inhomogeneities can be different in nature de-
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Depending on the phenomena that generated them. Mirror like reflectors, created from sharp changes in the index of refraction, can generate strong radar returns in the atmosphere. Another atmospheric event detectable by radar is turbulence. Turbulence can be generated by different mechanisms and observed under different atmospheric conditions. Radars can even detect non-atmospheric events inadvertently. Aircraft, satellites, birds can generate signals large enough to be detected by the radar. The signals generated by the centre of the galaxy can be detected (as noise) during regular radar operation (will be shown later).

The atmospheric backscatter (also known as echoes) is used by radar to gather information. The information obtained from the echoes include the power received, the structure of the atmosphere, and the velocity of the scatterers. The power received can be calculated based on the detected signal and properties of the radar. The structure of the atmosphere can be obtained by range-gates (old radars) or deconvolution methods (new radars). The velocity of the scatterers can be measured by the Doppler effect on the backscattered signal.

The estimated information however can be contaminated, so caution is imperative when relating the observations to real phenomena. Metallic objects could generate reflections with larger power content than atmospheric events. Range aliasing can generate spurious scatterers in the atmospheric profiles. The width of the radar pulse (and other atmospheric motions) can alter the measurements of scatterer velocity.

The following sections and chapters present an analysis of the these and other topics. The atmosphere, radar and scattering basics, radar simulations and the experimental radar project in Costa Rica, and its results, will be covered. A theoretical treatment of the spectral analysis tools used in radar will be presented, along with basics of interferometry. Furthermore, modelling and computer simulation will be employed as an additional tool used in physics research, to better understand the atmospheric backscattering. The use of the convolution theorem in the simulations will approximate the results to the observations of a radar in Costa Rica.

1.2 Lower Atmosphere

The atmosphere is a mixture of solid, liquid and gaseous matter. The fact that the solid particles are small doesn’t mean that they are not relevant. Small particles of sand and residues from wildfires, are responsible for the formation of rain droplets as shown by laboratory experiments (Lohmann et al., 2004, and references therein) and simulation (Reutter et al., 2009, and references therein). The liquid components of the atmosphere contribute to the dynamics of the hydrological cycle, and are crucial in the regulation of the energy flux in and out of the planet. The gaseous part of the atmosphere is the main engine that, using the solid and liquid constituents, along with the energy provided by the earth and sun, drives the entire mechanism allowing the existence of life.

The extent of the atmosphere is another topic of relevance. Most of the weather occurs in the first few kilometres of altitude. To think of the atmosphere’s dimensions in terms of weather related phenomena is not appropriate. The immediate region of the atmosphere (relative to the surface) behaves the way it does, only because the most external regions and constituents behave the way they do. Understanding that behaviour and structure is paramount to have a comprehensive knowledge of the conditions that created, evolved and sustained the planet’s atmosphere.
The vertical separation between the human habitat and the upper regions is impossible to monitor efficiently using balloons and air-borne instruments. Many of the instruments utilized to measure the atmosphere on the surface will malfunction above the ground. One example is humidity sensors. At tropical latitudes the content of water at the surface can be as large as 20 grams of water per kilogram of dry air (Garbanzo-Salas, 2011), with variations of 5g/kg in just a few hundred meters. The humidity concentration decreases drastically with altitude, and in addition, the low atmospheric pressure and sub-zero temperature makes it possible for water to be solid. These changes from the surface conditions make the measuring devices inadequate to measure water content just 10 km higher, therefore the need of better instruments (Vömel et al., 2007).

Many areas of the geosciences have monitored the atmosphere in order to generate a picture of the mean constituents and temporal variability. This has allowed the creation of different schemes in order to subdivide the atmosphere. The most common variable used to catalogue the atmosphere describes it in layers, distinguishing between regions of positive/negative vertical temperature gradient. The atmosphere can then be divided into the Troposphere, Stratosphere, Mesosphere, Thermosphere and Exosphere as shown in figure 1.1.

For this research the only relevant section is the Troposphere, which is the layer of the atmosphere closest to the surface. The troposphere is characterized by a constant decrease of average temperature with height. This layer extends from the surface to the region where temperature no longer decreases locally with height (tropopause). The troposphere represents more
than 95% of the atmosphere’s mass, and contains most of the weather generating phenomena (e.g. hurricanes, tornadoes, storms, clouds, precipitation).

The maximum height of the troposphere is variable; even if the same geographic location is used to monitor the temperature profile, the values will change in time (Takashima and Shiotani, 2007; Selkirk et al., 2010). This natural variability depends on several factors including time of day, weather conditions, air mass movements, and large scale events, among others. It is important to clarify that the criteria used to define these layers is calculated in a mean atmosphere. In an instantaneous temperature state of the atmosphere, considerable variation in temperature will be found as a function of height at any one time inside the troposphere (Selkirk et al., 2010; Garbanzo-Salas, 2011).

In tropical latitudes the Tropopause (boundary between Troposphere and Stratosphere, and location of a change from negative to positive vertical gradient of temperature) can be found at approximately 17 km altitude (Garbanzo-Salas, 2011). In higher latitudes (e.g. 40 N) it can be located close to 10 km, and 8 km near Earth’s poles. It is important to remember that this is not a barrier or a solid boundary, it is a transition height established according to the average characteristics of the atmosphere and can change dramatically (He et al., 2011).

The relative low power used by the radar in Costa Rica is not enough to measure up to the tropopause. The maximum height used was 6 km. The low troposphere was successfully monitored with this configuration during the experiments (will be shown later).

1.3 Radars

Radar is an acronym for Radio Detection and Ranging. The basic concept of a radar is an implementation of remote sensing. Using the laws of physics, information can be retrieved at a distance from a target of interest. Radars send information in the direction of the target to provide a reference or means of generating a detectable response.

In order to better understand radars, it may be of use to consider a simple example where energy is sent and utilized to gather information about a target. Energy can travel from a source towards the target. After interfering with the target, part of the original energy will be redirected and it will later reach the destination. After the interaction, the received energy is collected and analyzed to obtain information about the target. All the above mentioned elements (energy, source, target, destination) can greatly differ depending on the type of application of the radar.

Another example is the use of kinetic energy as the energy used in a remote sensing application. Suppose a person (source) wants to have an idea of the distance to a wall (target). Perhaps this person picks up a rock and throws it (kinetic energy) at the wall and after some time hears (at the destination) the echo (energy-target interaction) of the hit. The calculation is done in the brain, where the knowledge about sound speed and time produces an estimate of the distance.

A third example is air traffic radar. These types of radar usually have the source and destination co-located, and in many cases the same device (antenna) is both used as transmitter and receiver. The antennas will send energy in the form of electromagnetic waves and if a target is in the way it will interact with it, generating backscattered signal. This signal travels back towards the direction of the antenna, and if it is strong enough for the sensitivity of the equip-
ment, the event can be employed to obtain the range of the target and additional information (e.g., radial velocity after multiple measurements).

The examples described above do not differ much from each other. In the first case, the information about the distance was obtained by using the sound speed in the air and the time. But the speed of sound actually depends on the medium variables. While it will not make much of a difference to the distance to the wall, the fact that it changes in the air can bring extra information about the medium itself. With the air traffic radar, the medium is still the air but now the information carrier is an electromagnetic wave. This wave will travel at the speed of light in air, which is also dependent on the properties of the medium. In this document, the main source of information is the medium itself and this will be thoroughly covered in a later section.

The type of radar used in the experiments described herein is known as a pulsed radar. Continuous wave radars send a constant flow of energy into the atmosphere. Pulsed radars are based on rather short bursts of energy at a certain frequency. A basic diagram of a pulse radar is presented in figure 1.2. This diagram shows a basic approach to vertically pointing pulsed radars. The antennas regularly used in these radars are constituted of a large number of interconnected elements, but individual elements can also be employed. The large arrays are usually interconnected in a way that a phase delay can be introduced to each individual element. This type of arrangement is called as a phased array antenna.

The radar beam (shown in figure 1.2) is defined by the antenna shape. The radar beam width is calculated from the interference pattern. The interference pattern is created by the signals emitted at the antenna array; they are Fourier pairs. A large antenna can generate a narrow beam, and vice versa. A polar diagram (as is represented by the interference pattern) is regularly used to represent the radar’s beam. The beam’s width is usually defined in terms of the relative power to the maximum found at one specific height. A power level of -3 dB relative to the maximum (indicating a drop of 50%) usually defines the width of the radar beam.

The radar equation is used to calculate the power received based on radar properties and characteristics of the scatterers. This equations is shown in appendix A, along with some important topics regarding its implementation. An important part of the radar equation is the Gain functions. When different antenna arrays are used for transmission and reception (as is the case of the radar used in this research, as shown in figure 1.2), there are different Gain functions for each array. The region with the strongest transmission Gain that also coincides with the strongest reception Gain will generate the largest echoes for the radar. Some radars can change this region by means of changing the phase on the transmitting elements or by physically steering the antennas.

The steering is usually accomplished by rotating the antenna as in many weather, military or commercial applications. Another way of steering is changing the orientation using constructive interference from different sources, as it is the case of phased array antennas. Many other techniques exist (e.g., synthetic aperture, aperture synthesis) that allow different antenna configurations. In this document a phased array VHF radar, pointing vertically was used to collect the data.

Important factors in radar operation are the maximum unambiguous range and the radar resolution. The maximum range without ambiguities depends on how often the radar pulse is transmitted. The length of the pulse is related to how much energy one specific event can return; it also sets a limit to the maximum resolution of the radar. More on these topics is
Figure 1.2: Pulsed radar basic diagram. The antennas (can be individual elements but phased arrays are common) are used to transmit the original pulse, and receive the atmospheric echoes. The scatterers inside the radar beam generate echoes of the propagating radar pulse.
cluded in appendix A.

**Frequencies of operation**

Regarding the atmospheric radars that look vertically, several types operate in the troposphere, stratosphere and mesosphere. These radars are a subset of what is known as MST-type radars and there is a scientific community behind these devices and their applications. The term MST comes from the maximum height that the radar can probe; the most powerful radars can measure up to the Mesosphere, as well as the Stratosphere and Troposphere. These types of radars are defined by their antenna dimensions, power, and more importantly, the frequency of operation. Depending on these properties a radar can be considered to be a full MST-, ST- or just T-type radar depending on how deep into the atmosphere it can operate.

The frequency used is determined by the targets of interest, but it is also related to the polar diagram and other radar properties. If scattering from water droplets is desired, a small wavelength (centimetre scale or less) should be used. This range of frequencies is known in radio communication as ultra high frequency (UHF). If larger targets are of interest to the radar, larger wavelengths (meters) known as the very high frequency (VHF) range should be used. There are multiple frequency bands to categorize the spectrum. Table 1.1 shows a short summary of them. This is the subdivision according to the International Telecommunication Union (ITU).

<table>
<thead>
<tr>
<th>Band Name</th>
<th>Symbols</th>
<th>Frequency Range</th>
<th>Wavelength Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Frequency</td>
<td>HF</td>
<td>3 to 30 MHz</td>
<td>10 to 100 m</td>
</tr>
<tr>
<td>Very High Frequency</td>
<td>VHF</td>
<td>30 to 300 MHz</td>
<td>1 to 10 m</td>
</tr>
<tr>
<td>Ultra High Frequency</td>
<td>UHF</td>
<td>300 MHz to 3 GHz</td>
<td>10 to 100 cm</td>
</tr>
</tbody>
</table>

Table 1.1: Frequency Bands regularly used in MST radars

These bands of frequencies are not physical boundaries but a mere convention. Choosing a band in which to operate depends on many factors. By choosing a particular band to transmit, advantages and disadvantages are immediately defined. The type and size of the antenna are defined by the frequency used and the desired polar diagram. The physical mechanisms that will cause backscatter at the chosen frequency will vary according to the chosen band. If the frequency is too high, Mie scattering from water droplets can be obtained in the atmosphere. If the frequency is too low, Rayleigh scattering will be observed from inhomogeneities in the index of refraction. In any case, unexpected sources of noise and scattering can occur, causing the implementation of any radar to be a detailed and careful operation.

**1.4 Costa Rica’s radar**

During the past few years an initiative to create a radar research centre was carried out in Costa Rica. The location chosen for the radar site is a satellite campus of the University of Costa Rica located in Santa Cruz. This small town is located in the North-West of the country and holds a population no larger than seventy thousand inhabitants. Santa Cruz provides the project with a radio-frequency quiet environment and land to grow if the need arises.
The wind profiler radar used during this research was the main focus of the initiative. The wind profiler started as a one receiver radar but has been upgraded into a three receiver system. What makes this radar unique is the unusually large bandwidth available for transmission. In North-America bandwidth allocations of 0.25 MHz can be obtained by radar operators. In Costa Rica a bandwidth allocation of 5 MHz was assigned to the project. An immediate advantage of the large bandwidth is that the spectral content of the pulse sent into the atmosphere can contain multiple frequencies. In the case of the radar in Santa Cruz, this bandwidth was used by creating a several megahertz wide chirp-frequency inside the pulse.

The use of a wide range of frequencies inside the pulse allowed deconvolution to greatly improve the resolution. A radar pulse with a length of one kilometre should have a resolution of 500 metres (see appendix A, equation A.6). By introducing the deconvolution the resolution improved to less than 100 metres. This improvement in resolution generated strong echoes from the atmosphere even with low power transmitters. Transmitters with powers from 40 kW and up to 4 MW can be found in MST radars. The radar used in Costa Rica used a 1 kW transmitter to gather information satisfactorily up to 6 km.

In appendix B an overview of the history of the radar is included. This short view of the project’s history is intended as a complement of this text. Collaborators could find the information contained valuable.

1.5 Wind profilers

Wind profilers often consist of a flat phased array antenna. The phased array antenna is made of many individual radiating elements organized in an orderly manner in order to create a desired radiation pattern. By joining individual elements, flexibility is incorporated in the radar. The radar flexibility allows different techniques to be implemented with minimum changes.

The radiating elements can be as simple as a dipole or as complicated as a Yagi-Uda antenna with multiple elements. The Yagi-Uda concept is based on a single driven element along with multiple additional elements (reflectors and/or directors that are not driven); these extra elements (usually metal rods) are used to enhance transmission/reception in a specific direction. The extra elements in a Yagi-Uda design define specific characteristics like antenna bandwidth (not to be confused with pulse modulation bandwidth), front/back ratio, impedance and other characteristics. It is important to know that not all these variables can be optimized at the same time; usually the ones that are more important for the purpose of the radar are calculated first and then others are adjusted to minimize the negative impact on performance. This process often requires numerical simulations and later experimental calibration. One detailed example of the making of these antennas and radars is found in Fukao et al. (1985).

The polar diagram is dependent on the antenna formation and is of great relevance in any radar. The more energy concentrated per solid angle the more backscatter will be generated, making the echo detectable. This can be accomplished by carefully calculating the spatial distribution and separation of the antennas. The focused energy region generated by the transmitted signal is due to constructive interference and if this region is narrow (small angular separation from zenith) then it is called the radar beam or main lobe.

This main beam can not be infinitely narrow (which would be ideal) but that would require an infinite antenna (the aperture function and radiated pattern are Fourier pairs). Hence the
design of the spatial domain distribution of the radiators is important to optimize this main lobe. As a consequence of the limited antenna, other lobes (side lobes) are usually present in the radiation pattern. As long as the intensity of these secondary maxima is much less than the main lobe, the radar is considered to radiate the majority of its energy in one direction. This direction, along with the pulse length and beam width, defines the scattering volume.

The transmission/reception lines to feed the antennas and the recording of the received signal, require careful design. Different antenna design and distribution will require appropriate matching mechanisms, and signal distribution, among other factors. Wave guides and cables are usually employed but care about different aspects need to be considered. If a large amount of power bounces back from the antennas, or is picked up by other antennas, it can damage the reception system. This can be avoided by disabling the receiver during the peaks of undesired power return. It can also be mitigated by using proper impedance matching in every step of the construction. For example, in the case of coaxial cable, the impedance of the transmitter needs to be matched with the cable that also needs to be matched with the antenna. It is impossible to create a perfect match with the antenna because the individual element design forces optimization of certain properties over others, meaning that getting the maximum gain at one specific frequency could reduce/increase the impedance at other frequencies.

In radar studies two types of transmission/reception configurations are often found. The systems where the transmission and reception are done using the same antenna are said to be monostatic radars. Bistatic or multistatic radars consist of spatially separated equipment for transmission and reception. This can be particularly useful to study anisotropic scatterers and specular or partial reflectors, depending on the configuration and analysis process (Cho et al., 1996; Hocking and Hocking, 2007).

1.6 Radar measurements in the atmosphere

Previous sections introduced the concept of the lower atmosphere and radars. The variables in the troposphere change rapidly, even when the mean value looks steady and uniform. It is apparent that the air is constantly mixing by physical processes (convection, turbulence and diffusion). Those processes cause the variables to change in a non uniform way, generating inhomogeneities. Abrupt changes in variables like temperature, pressure and water vapour affect the value of the refractive index of the atmosphere. Any change in refractive index in the path of a transmitted signal is an interface. Any interface will generate backscatter of electromagnetic waves when illuminated by a transmitted radar signal.

Inside a specific region of the atmosphere, the refractive index change between consecutive heights is small (as will be shown in following chapters). With a small variation in index of refraction with range, the amount of transmitted power needs to be large enough in order to make the backscattered power detectable. An introduction to this topic is included in Chapter 2. A detailed theoretical treatment can be found in Tatarski (1961).

The refractive index in the lower atmosphere depends on temperature, pressure and water content. An empirical equation for the troposphere has been developed over the years to obtain the radio refractivity ($\mathcal{N}$). This variable is related to the actual refractive index by the expression $\mathcal{N} = (n - 1) \times 10^6$, so that $\mathcal{N}$ is just a rescaled deviation from the vacuum index of refraction (Smith and Weintraub, 1953) defined as
where the pressure \((P)\) must be in hPa, \(T\) is the absolute temperature and \(e\) is the vapour pressure (also hPa units). In this equation the first term represents the dry contribution. The second term, the wet contribution, dominates the lower heights where the atmospheric water content can be large. If the water content is small enough, the dry term will dominate the magnitude of \(N\), and consequently its vertical gradient. The humidity in the air depends on different conditions (e.g. surface heating, turbulent transport and convective activity) and hence it is hard to put a maximum height for this term to be relevant, but it should lose relevance past the tropopause.

### 1.6.1 Backscatter

In general terms, scattering is the physical process that consists of energy being sent in different directions due to properties of the medium through which it moves. Backscatter occurs when the energy is scattered in the direction of origin; in the atmosphere it is a consequence of the induced oscillations to electrons which are either free (common in ionospheric heights) or bound (lower atmosphere) encountered in the path of a travelling electromagnetic wave. The travelling wave will generate the most backscatter if the index of refraction perturbations are organized in such a way that a Fourier component of \(\lambda/2\) is present along the path of wave. The value of \(\lambda/2\) is known as the so-called Bragg-scale, and is a consequence of the two-way trip of the radar signal.

The constructive interference of the backscattered waves is greatest at the \(\lambda_{\text{radar}}/2\) scale (Bragg-scale). In the case of the stratosphere and troposphere (ST) these fluctuations will be minimized in the regions where a homogenous profile exist with thermodynamic stability. Different types of phenomena can induce index of refraction fluctuations and create instabilities allowing structures with \(\lambda/2\) size to appear and contribute to the backscatter power. In general, any type of echoes will be due to atmospheric profiles containing Bragg-scale sized perturbations.

This type of backscattering is often referred to in publications as isotropic turbulence backscattering (Hocking and Mu, 1997). Other types of scatterers found in the atmosphere are anisotropic turbulence and specular or partial reflectors. It is worth mentioning that non-natural scatterers (e.g. aircraft) can also be detected by the radars along with various sources of noise. Anisotropic turbulence and specular or partial reflectors are aspect sensitive (the returning power is dependent on the relative location of the scatterer) and different studies discuss how to classify their presence (Hocking and Hocking (2007) and references therein).

The amount of power sent back in the direction of incidence is related to the specific characteristics of the scatterers and the number of those scatterers inside the scattering volume (Tatarski, 1961). From turbulence theory, the amount of those scatterers is related to a structure constant denoted as \(C_n^2\), and this variable is related to the vertical gradient of radio refractivity \((dN/dz)\). These equations can take different forms and different variables are regularly used to represent the same parameter. The ones presented here are those used by Röttger et al. (1978) but others are available (Woodman and Guillen, 1974; Hocking, 1985). The equation for the radar reflectivity \((\eta\) Ottersten, 1969) is
\[ \eta = 0.38 C_n^2 \lambda^{-1/3}, \]  
(1.2)

where \( \lambda \) is the radar’s wavelength; the structure constant \( C_n^2 \) can be thought of as

\[ C_n^2 \propto \left( \frac{dN}{dz} \right)^2. \]  
(1.3)

However \( dN/dz \) needs special calculations. In these equations the contributions generated by turbulent volumetric scattering are accounted for inside the \( C_n^2 \) term. The second equation presents the \( C_n^2 \) dependence on the vertical gradient of radio-refractive index. As will be shown later on \( C_n^2 \) depends on a variety of terms including the turbulent energy dissipation rate. Several of these variables can be determined by radar, as will be introduced in Chapter 2.

### 1.7 Spectral Analysis

One of the objectives of this research was to study optimum procedures to convert the signal and determine the atmospheric properties. One method commonly used is Capon’s method. This method (also known as the minimum variance method or MVM) was first introduced into seismic analysis but later implemented in atmospheric radars processing. Previous tests showed some concerns regarding the capabilities of Capon’s method in resolving continuous spectra (Jian Li and Stoica, 1996). Capon’s method ability to resolve closely spaced peaks was tested in order to better understand its relationship with the filter size. The process is described in Chapter 3 along with some other topics on spectral analysis tools.

Spectral analysis is used during radar processing in different stages. Spectral analysis tools convert the time series into a spectrum to be analyzed. This analysis generates many of the radar products, such as spectral width, power, and Doppler velocity. These tools are used during the deconvolution process to obtain the appropriate spectrum for the received signal. New spectral analysis tools were recently introduced in areas of interferometry (Hocking, 2011). Spectral methods need to be carefully evaluated before being introduced into production environments or drawing conclusions from their results.

In radar observations an atmospheric layer is identified as a region (in the radial coordinate) where the backscattered energy is greater than that of its surrounding environment. These layers are often observed in radar measurements as will be shown in Chapter 6. The capability of a radar to resolve atmospheric layers depends on the radar resolution, as an improvement in resolution will allow the radar to detect narrower layers. In a study by Palmer et al. (2001), Capon’s method was used to enhance an image and obtain layer information. The results obtained are presented in figure 1.3. In this figure a large number of layers are observed. The number of layers and their distribution seems unrealistic at times. Are all these layers really there? Is the method generating spurious peaks which may be later identified as layers?

In order to investigate this matter further, a reference system must be set before proceeding. As will be shown in Chapter 3, when comparing Capon’s method to itself under different circumstances, the products can be quite different. The number of peaks identified by Capon’s method can be changed drastically by changing its filter size. These results (Garbanzo-Salas and Hocking, 2015) showed that under certain conditions, the products generated by Capon’s
Figure 1.3: Figure taken from Palmer et al. (2001). In the upper pane the standard echo power was presented. The lower pane presents the enhanced image obtained with Capon’s method RIM technique.

method are not optimum. Care must be taken when trusting layer information, as the peak’s location could change when introducing extra components in the filter.

Many other uses for spectral tools are available in radar applications. The imaging Doppler interferometry (IDI) method uses frequency domain interferometry to obtain its products. A comparison between products of the FCA technique (commonly used to calculate wind velocity) and those of the IDI method is available (Holdsworth and Reid, 2004). The process of interferometry can be complicated. Some examples regarding these tools are included in Chapter 3. Another use of spectral tools is the addition of different receiver signals to form a single unified receiver. The phase of each receiver can be modified and the optimum phase difference between receivers can be obtained. If averaged over long periods of time, the values of the receiver phases will tend towards the actual receiver phase. This method of software calibration was used in Costa Rica’s radar and the results are presented in Chapter 3.
1.8 Modelling and Simulations

Another important objective of this research was to incorporate the convolution function into radar simulations and retrieve data by deconvolution procedures. As will be shown shortly the radar simulations found in the literature do not use the convolution function to calculate its results. The deconvolution function is the natural way to obtain radar backscatter, as it correctly untangles what occurs in nature when an electromagnetic wave is backscattered by an specific atmospheric profile. Different implementations of the convolution were written and carefully tested in simulations with realistic physical conditions. Chapter 4 contains the details about these simulations and their results.

There are three different areas of contemporary physics involved in research activities. Theoretical and experimental physics contribute towards each others developments with a two way feedback. Theoretical physics creates mathematical descriptions of reality in order to better understand it. Experimental physics complements the theoretical by testing its finding, and often creating knowledge through unexpected results. Theoretical physics leads the experimental into new and unmeasured areas of knowledge. With the advent of modern computers, a new area in research that does not quite fit the previous two was created.

The ability to simulate an event in a computer can be of great benefit. Testing multiple scenarios simultaneously and even measuring the impossible is at hand with simulations. When stepping into simulations, care must be taken to differentiate between a model, a simulation and the implementation. A model is the theoretical representation used to describe an event. The laws of physics used for a simulation are part of the model. The simulations are the conditions given to the model. The simulation thus becomes the actual scenario created to evaluate the model. A fluid simulation can be carried out in a small lake or the entire Pacific ocean, with exactly the same model. The implementation of the simulation can be defined by the algorithms and computation tools (software and hardware) available for the task. The type of operating system, computer language, architecture, and programming paradigm are all aspects associated with the implementation.

In the radar field, simulations have been used on multiple occasions (Nickisch and Franke, 1996; Muschinski et al., 1999; Nickisch and Franke, 2001; Garbanzo-Salas and Hocking, 2015). Simulations can test theories and create hypotheses about relevant aspects to radars. Even when the convolution is the natural way to model the atmosphere, no simulations were found using these method. Instead, Huygens’ principle and its approximations are usually modelled in the time domain. When simulating in the time domain, thousands (or more) steps are needed to obtain a good approximation (Franke et al., 2011). The case of deconvolution-based models was used during the research described herein.

Multiple simulations with three different implementations will be presented. The model used in these simulations is the basic concept of deconvolution. The earlier model presented for the scatterers in these simulations are not necessarily representative of naturally occurring structures (like those generated in a fluid). Even with this conditions these simulations provided a great insight into radar backscatter simulations as the effects of real radar applications are observed in the simulation products. The scatterers used in the simulations are defined and moved according to mathematical/physical equations that do not necessarily comply with the fluid dynamics equations of motion or evolution of scatterers in earth’s atmosphere. An improvement on these simulations can be obtained by replacing the scatterers with atmospheric
structures.

Atmospheric models based on the fluid dynamic equations can be used to simulate the atmosphere. This corresponds to an objective of this thesis where the created implementation of the convolution function was used along a simulated realistic atmosphere to calculate radar backscatter. The importance of this research is due to the two realistic simulations, (i) index of refraction obtained directly from the atmospheric variables in a realistic fluid simulation and (ii) backscatter calculated from this index of refraction convolved with the radar’s pulse. These simulations are described in detail in Chapter 5.

The implementation of atmospheric models usually requires the small scale to be parametrized. This may be useful when the small scale effects are not so relevant to the overall behaviour (e.g. mesoscale motions). In radar models, the small scale needs to be fully resolved in order to account for turbulent motions. By using large-eddy-simulation (LES) models, the fine resolution of the atmosphere is not parametrized; instead the small scale is resolved numerically.

LES models have been used in radar simulations (Muschinski et al., 1999; Franke et al., 2011; Fritts et al., 2011, 2012). The models have used different approaches to simulating backscatter and radial velocities. None of these approaches include a full-convolution involving the transmitted pulse and the atmospheric profile. During this research, a LES generated the atmospheric variables needed to obtain the index of refraction. The simulated index of refraction was convolved with the radar’s pulse in order to obtain the best approximation to the received signal. No references are available where this treatment of the radar model is implemented. In Chapter 4 examples of the simulation and its results are shown.

1.9 Radar experiments in Costa Rica

An experimental radar was built in Costa Rica. The initial stage was completed during 2011, but the full three receiver system was operational at the end of 2012. During 2013 different tests were carried out in order to test the capabilities for research. The results of one test along with the technical aspects have been documented (Hocking et al., 2014).

No atmospheric information (captured with radar) was previously available in Costa Rica. Clearly, no climatology or case-studies had been carried out in Costa Rica using radar data. With an operational radar the main objective was to gather as much information as possible about the Costa Rican atmosphere and create the first view from a radar’s perspective. This information will contribute greatly towards the knowledge of the tropical atmosphere over Costa Rica. The results of this research are explained in detail in Chapter 6.

To gather information a long term experiment was planned in order to record data during a year. For the first few months a single receiver configuration was used. Afterwards, the radar wiring was changed into three receiver mode; several months of atmospheric echoes were logged. During the analysis different aspects of the Costa Rica atmosphere were tracked, they included:

- Atmospheric layer
- Planetary boundary layer (PBL)
- Isolated patches of turbulence (IPoT)
Kelvin-Helmholtz events (common foci in radar publications)

A relevant aspect of the low troposphere is the PBL. The relevance of the PBL comes from the energy that forms it. This energy can be transformed into turbulence and become a hazard for aircraft. In the case of Costa Rica, most of the time during landing/take-off, strong turbulence can be felt by users of commercial flights. The available energy in the PBL can also contribute greatly to thunderstorm generation as was observed in the data (shown in Chapter 6). With the year-long data set, a detailed study of atmospheric echoes related to PBL was carried out. Different types of PBLs were observed and a classification scheme was created. Statistics about the maximum, minimum, and average heights of the PBL top are presented in Chapter 6.

In the measurements of the low troposphere it was common to detect isolated patches of turbulence. The time these IPoTs were observed was quantified and an inverse relationship with the number of atmospheric layers seems to exist. The results about IPoTs and its monthly distribution are presented in Chapter 6.

1.10 Outline

The small scale non-linear dynamics of the atmosphere is covered in Chapter 2. In this Chapter the terms in the fluid dynamics equations related to turbulence are displayed. A theoretical treatment of turbulence is included to better understand the energy dissipation rate and its relation to turbulence intensity. Radar approaches to estimation of the energy dissipation rate are also included in Chapter 2.

Chapter 3 covers some of the spectral analysis tools used in radar applications. A characterization of a data-dependent method is included. Complementary tools (deconvolution, interferometry, receiver phase, and wind estimation) are included towards the end of Chapter 3.

Topics related to computer simulations, numerical techniques and physics modelling are covered in Chapter 4. A first approximation to the polar diagram of an antenna array is described. The convolution was used extensively to simulate radar backscatter. Three different computational engines for radar simulation were created and their description is included in Chapter 4.

In order to complement the measurements, a more advanced simulation was carried out. A large eddy simulation (LES) model was used to simulate a small atmospheric section. The results of the simulation resemble the observations; both results are included in Chapter 5.

In Chapter 6 the experiments carried out in Costa Rica’s radar are described. Two methods for spectral estimation are mentioned; their respective products are also shown. A year-long data set was analyzed and the results were synthesized in Chapter 6.
Chapter 2

Nonlinear small scale dynamics

The atmosphere’s main energy source is the sun. The energy budget is made up of the incident energy, energy transport, radiative balance, greenhouse effect and other phenomena. This energy is not static, but moves and transforms constantly inside the system. The consequence of these movements and transformations shape our atmosphere’s composition, structure, dynamics and thermodynamics.

Humans have named these motions and transformation mechanisms according to their perception (e.g. day-night, winter-summer) and use them regularly with popular meteorological conventions (e.g. windy, rainy, sunny).

Inside the engine that runs the entire Earth system, there is a small range of scales where the atmospheric physics allows the motions to be detectable by atmospheric radars. Nevertheless, by properly processing these data, the radars may be used to determine atmospheric behaviour out to scales of days and even years. That specific radar-sensitive region will be the main focus of this chapter. From the moment the energy starts to interact with the different structures of the Earth, it will begin a journey that will eventually reach microscopic scale where it is converted into sensible heat.

Regarding radar operation, there are two fields of study that are very different but intimately related, which should be discussed in order to understand backscattering.

1. The atmospheric motions and how they transform is directly related to properties like temperature, pressure and humidity. These atmospheric properties define the refractive index of the atmosphere (at tropospheric heights and for VHF wavelengths). Understanding the distribution of the refractive index (and its gradients) is crucial to understanding the backscattering process.

There are different ways to attempt to obtain that information. Complete knowledge of atmospheric properties at all heights and times is not achievable. In-situ measurements provide a good one dimensional sampling but are not capable of accounting for spatial variability. A statistical approach can provide a more general idea of spatial distribution of properties.

2. Propagation of electromagnetic waves is defined by Maxwell’s equations. These equations can provide the necessary tools to understand how the radar energy interacts with the medium.
2.1 Introduction to turbulence

Fukao and Hamazu (2014) defined turbulence as “a chaotic fluid regime producing random and rapid fluctuations of pressure, velocity and other quantities which is not a repeatable process”. A high degree of complication exists in turbulence, but the idea of random motions caused by chaos may not be the most appropriate. The simple definition of turbulence can be challenging. The challenge emerges from different aspects inherent to turbulence. An example is the different scales where turbulence can be found, as it can occur in a glass of water, in planetary atmospheres, and galactic disks. A more precise way to present turbulence would be as fluid motions where no wave-like structures or organized motions are observed. This way the notion of randomness is removed; certainly the initial conditions affect greatly the results making them chaotic, but the motions are not random but clearly defined by the laws of fluid dynamics. Another aspect that makes it hard to define turbulence is the diversity of mechanisms that generate it. Wind shear, motion near boundaries, uneven heating, fluid deformation, convergence/divergence, and wave breaking are among different types of phenomena that could end up generating turbulence. Turbulence provides the pathway for energy to be transported from the larger scales down to the molecular scales.

In order to better understand the role of turbulence and the different scales in which turbulence occurs, it is good to think of the planetary scale and how the energy balance is maintained in the environment. Only a small fraction of the incoming energy from the sun is absorbed by the atmosphere. This energy is large enough to drive the atmospheric (along with the oceanic) engine that runs the entire planet. Among others, the system’s energy is used to create continental air mass movement, mesoscale phenomena like hurricanes and planetary phenomena like the El Niño-Southern Oscillation (ENSO). The length (in time and space) of those events, can be related to the mechanism driving the energy flow.

Let’s start with a simple case to better understand the concept. When considering continental air mass movement, the length scale under consideration is usually in the order of thousands of kilometres (\( \sim 10^6 \) m). With the energy available for that movement it will take several days (\( \sim 10^5 \) s) to complete the displacement based on the pressure difference between different areas of the planet. The velocity of this event can be approximated by

\[
v = \frac{d}{t} \approx \frac{10^6 \text{m}}{10^5 \text{s}} = 10 \text{m}\text{s}^{-1}.
\]

That velocity agrees with the usual horizontal air speed. We can use this to compare the time scales of different mechanisms. If the continental air mass equilibrium was based on molecular diffusion instead of large scale pressure gradients, the time scale would be much larger.

Considering that molecular diffusion moves particles typically only a few metres in an hour, the time required to move a distance of the scale considered before would be in the hundreds of years. Clearly the required time to move air masses is smaller than that because advection
dominates over diffusion. It is appropriate to assume that diffusion is not the major mechanism of transport at planetary length scale.

This simple example can be used to understand that at different scales (time and space) different physical phenomena will be in charge of receiving, transporting, transforming and ultimately dumping the energy. Turbulent transport is somewhere in between these two examples, as will be shown shortly.

The effect of turbulence in the surrounding environment is mainly to produce diffusion of properties (e.g. heat, momentum, particles). Turbulent diffusion of such properties can greatly modify the general circulation or variable state, as redistribution happens rapidly while turbulence is present. The term “rapidly” is always a relative term respect to molecular diffusion.

Two different types of turbulence are regularly observed; two dimensional (large scale), and three dimensional (smaller scale) turbulence. The two dimensional turbulence is usually observed in large scales (greater than tens of kilometres). The turbulent motions under this category grow in scale. Two dimensional turbulence start in small scale (large wavenumber) and grows with time towards larger scales. An example of these two dimensional turbulence can be generated by von Kármán vortices (Karman, 1937). Figure 2.1 contains a satellite image of Cape Verde Islands off the coast of northwestern Africa. The cloud vortex streets in figure 2.1 are shown as a true-color Terra MODIS image from January 5, 2005.

In figure 2.1 the turbulence can be observed as the process allowing cloud formations to develop. These formations can be used to locate where turbulence is occurring. This is only possible because of the height and environmental atmospheric conditions.

Two dimensional turbulence is not the major focus of this chapter and we move now to
three dimensional turbulence. Three dimensional turbulence is usually invisible to the eye and hard to depict in a two dimensional figure. When three dimensional turbulence is invisible it is regularly identified as clear-air-turbulence (CAT).

Clear-air-turbulence can be measured in different ways. Photography is not one of those ways as it is not traced by visible structures. The most common way to measure it is with wind velocity sensors located in aircraft. Also radiosondes and radar can be used to measure the presence of turbulence. Radars offer the great advantage that the probing time of the atmosphere can be as low as a few seconds. More about the measurements of turbulence will be presented towards the end of the chapter. First, let’s understand a little bit more about the mechanisms of formation.

In the previous century three dimensional turbulence and CAT events attracted the attention of atmospheric scientists. The risk implied between CAT and aircraft safety made it particularly important. Measurements were taken under different atmospheric conditions as shown in Pinus et al. (1967) and references therein. These researches found that under different conditions the turbulence measurements showed different characteristics. One interesting finding from Pinus et al. (1967) is that energy input for turbulence seems to be generated in different scales. They categorized different sources of energy for turbulent motions according to the their size:

- $\sim 1000$ km. Planetary inertial waves in the general circulation.
- $> 100$ km. Jet streams and gravity-inertial wave modes.
- $< 60$ km. Meso-scale phenomena of long gravity or lee waves.
- $\sim 1$ km. Short gravity-shearing waves in stable environments, and to the convective “bubbles” in unstable layers.

Those scales reflect the fact that turbulence can be generated by different mechanisms, and successfully develop in very different conditions. This indicates that the fluid should be capable of generating such motions as long as certain criteria is met. More about the required fluid properties necessary to develop turbulence will be introduced shortly. It is worth mentioning that just because turbulence can be developed does not mean that it will. Also, the fact that turbulence develops does not relates to the magnitude or intensity of the turbulence. The intensity of turbulence will be proportional to the energy available to the growth and development; this is not necessarily associated with the fluid conditions.

Three dimensional turbulence affects its surroundings by mixing and heating. The heating comes from the transformation of energy from large scales to smaller scales. The energy from larger scales will be subdivided by instabilities (e.g. convective, shear), non-linear breaking, and critical level interactions into smaller motions. These mechanisms are created by the nature of the fluid (e.g. discrete, compressible). The smaller motions will naturally evolve and subdivide further by the same mechanisms. At the molecular level, convective and shear instabilities are no longer applied and molecular diffusion takes over. The path to equilibrium takes molecules into spreading the macroscopic properties of the fluid based on microscopic motions. The motion of the smaller eddies will be ultimately transformed into molecular velocities, represented macroscopically as sensible heat. Pressure, temperature and water content can be altered by turbulence.
Other mechanisms apart from turbulence can cause the atmospheric properties to change. Convection and molecular diffusion can change atmospheric variables. How the index of refraction depends on atmospheric variables will be treated in the following section.

Radar measurements are possible because of the changes in atmospheric properties. The electromagnetic energy emitted by the radar will interact with the medium. If a change in the index of refraction (defined by the atmospheric properties) is present in the energy’s path, such interaction will result in a fraction of the energy being scattered towards the direction of origin (backscatter). The relationship between turbulence, backscattered energy and how it is detected by the radar will be introduced after the following section.

2.2 Potential refractive index gradient and atmospheric variables

An equation that is regularly used in radar studies relates the refractive index of the neutral (non-ionized) atmospheric air with the atmospheric variables (e.g. density, water content). Even when it should be instinctive for a physicist to think of the existence of this relationship, it is usually not clear how to go from microscopic properties to macroscopic.

The reason why this is not clear is because it is not a simple task. Going from Maxwell’s equations applied to atomic or molecular properties, to index of refraction of a macroscopic amount of those particles is very elaborate. This work has been shown by Hoenders (2008) and described in detail by references there-in.

The physical process can be described as follows. The neutral particle (e.g. molecule) is polarized by the effect of an incident electric field. This electric field will force the charged constituents to move, displacing the electronic cloud and nucleus (in opposite directions) from equilibrium. As a consequence, an induced electric field will be generated and the external field generated by the particle will change. Far away from the particle the field can be approximated as a dipole. This is usually one of the several assumptions taken in the derivation of the equations (Hoenders, 2008).

Another important assumption is that a dipole inside a medium is influenced by other dipoles surrounding it in a sphere (the so-called Lorentz sphere). If this shape is changed in the derivation, the numerical factor in the final equation changes. After considering the net effect of the Lorentz sphere, the final equation relating the relative dielectric permeability \( \varepsilon_r \) to the properties of the medium is known as the Lorentz-Lorenz equation (Böttcher et al., 1978) and is given by

\[
\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{1}{3} \rho a
\]  

\[(2.2)\]

where \( \varepsilon_0 \) is the vacuum permittivity, \( \rho \) represents the density of the dipoles (i.e. atmospheric density of dry air\(^1\) in our case, relating it to temperature and pressure), and \( a \) represents the “dipole strength” generated by an incident field \( E_I \).

Another way to represent this equation, considering a mixture of gases can be found in Owens (1967). In that investigation the atmosphere was considered a mixture of three gases

\(^1\)humidity will be introduced later considering a mixture of gasses.
2.2. Potential refractive index gradient and atmospheric variables

(dry CO\textsubscript{2}-free air, water vapour and carbon dioxide), and an expression for the index of refraction was found. As was shown in that reference, the total index of refraction can be expressed by

\[
\frac{n^2 - 1}{n^2 + 2} = \sum_i R_i \rho_i
\]  

(2.3)

where \( R_i = \frac{4}{3} \pi \frac{N_A}{M_i} \alpha_i \), \( M_i \) is the molecular weight, \( \alpha_i \) the polarizability of the \( i^{th} \) constituent, and \( N_A \) is Avogadro’s number.

An important assumption about this equation is that the factor \( R_i \) must be known in order to calculate the value of \( n \). In Owens (1967), \( R_i \) was obtained for each gas directly from optical wavelengths, and care is needed in extending such values to radio wavelengths.

Owens (1967) obtained expressions for the density of a gas as a function of different variables. The gasses used were dry-air, water vapour and CO\textsubscript{2}. The perturbation of the density variable was assigned different functional forms dependent on \( P \) and \( T \). Depending on the gas under examination some mathematical functions approximated better the experimental results than others. The expression that best described the experimental data was used to calculate the density. The relationships used for the values of \( \rho \) for a mixture of dry air, water vapour and CO\textsubscript{2} are

\[
\rho_{\text{dry air}} \propto A_1 \frac{P}{T} \left[ 1 + P \left( A_2 + \frac{A_3}{T} + \frac{A_4}{T^2} \right) \right],
\]

\[
\rho_{\text{water vapour}} \propto B_1 \frac{P}{T} \left[ 1 + P (1 + B_2 P) \left( B_3 + \frac{B_4}{T} + \frac{B_5}{T^2} + \frac{B_6}{T^3} \right) \right]
\]

and,

\[
\rho_{\text{CO}_2} \propto C_1 \frac{P}{T},
\]

(2.4)

where \( A_j, B_j, \) and \( C_j \) are constants defined by the fitting process of the equations to the measurements (Owens, 1967). After combining all these equations into one, a complicated equation result which relates pressure (\( P \)), temperature (\( T \)) and partial water vapour pressure (\( P_W \)) to index of refraction (\( n \)).

\[
(n - 1) \times 10^8 = \left[ c_1 \frac{P}{c_2} \left( \frac{1 + P (c_3 - c_4 T) \times 10^{-6}}{1 + c_5 T} \right) \right] - c_6 P_W
\]  

(2.5)

The constants \( c_j \) are available in the literature (Owens, 1967). The pressure and partial water vapour pressure units are Torr and the temperature unit is Celsius.

Researchers in MST radar field rarely represent the index of refraction using this equation. Instead the perturbation of electric permittivity (to be covered in Chapters 4 and 5), the radio refractivity and specifically the gradient of radio refractivity are used. One of the main references for the index of refraction is the one presented by Tatarski (1961):

\[
n = 1 + 10^{-6} \times \frac{79}{T} \left( P + \frac{4800 e}{T^2} \right)
\]  

(2.6)
where $T$ is the absolute temperature in Kelvin, $P$ is pressure and $e$ is the water vapour pressure, both in hectopascals. The refractive index is not a passive tracer, but rather this equation is usually converted into other forms that are conserved with vertical displacements. When moving vertically the variables potential temperature and specific humidity can be used as passive tracers. If $n$ is a function of the pressure, potential temperature and specific humidity, it would be expected that if it is displaced vertically, the difference between the $n$ of the parcel and the $n$ of the environment would be

$$ \Delta n = \left[ \frac{\partial n}{\partial P} \frac{\partial P}{\partial z} + \frac{\partial n}{\partial \theta} \frac{\partial \theta}{\partial z} + \frac{\partial n}{\partial q} \frac{\partial q}{\partial z} \right] \Delta z. \quad (2.7) $$

This perspective is overly simplistic as the important factor is not the change in the parcel itself, but the difference between the parcel and its environment. The pressure will adjust automatically as the parcel raises (being the same for both the parcel and the environment), and then the vertical variation becomes

$$ \frac{\partial n}{\partial z} = \left[ \frac{\partial n}{\partial \theta} \frac{\partial \theta}{\partial z} + \frac{\partial n}{\partial q} \frac{\partial q}{\partial z} \right]. \quad (2.8) $$

This term ($\partial n/\partial z$) is usually named $M$ and is called the potential refractive index gradient or simply the vertical gradient of the refractive index. The equation that describes $M$ is

$$ M = \frac{-77.6 \times 10^{-6}}{T^2} P \left( 1 + \frac{15500}{T} q \right) \left( \frac{dT}{dz} + \Gamma_a - \frac{7800}{1 + \frac{15500}{T} q} \left( 1 - \frac{1}{2} \frac{\partial \ln q}{\partial z} \right) \frac{d\ln \theta}{dz} \right). \quad (2.9) $$

The units of temperature and pressure are kept in Kelvin and hectopascals. The two gradients that appear in $M$ are taken across a turbulent layer of the atmosphere. $\Gamma_a$ is the adiabatic lapse rate.

A revised version of this equation is presented by VanZandt et al. (1978) and is also included here for completeness.

$$ M = -77.6 \times 10^{-6} \frac{P}{T} \left( \frac{\partial \ln \theta}{\partial z} \right) \left[ 1 + \frac{15500 q}{T} \left( 1 - \frac{1}{2} \frac{\partial \ln q}{\partial z} \right) \right] \quad (2.10) $$

where $\theta$ represents the potential temperature, $q$ is the specific humidity, $P$ is the atmospheric pressure, $z$ is the vertical coordinate, and $T$ is the temperature. This last equation can be used to get the vertical gradient of the refractive index when a vertical profile of the atmospheric variables is available. One interesting application is the numerical simulation of radar backscatter. In a following chapter this concept will be introduced and results using similar equations will be presented.

It is important to keep in mind that these equations are only approximations of reality, and care must be taken when interpreting the results. Many approximations were used when deriving these physical variables, from the details of the Lorentz sphere to the values of $R_i$ obtained from optical wavelengths.
2.3 Nonlinear dynamics in atmospheric fluids

Fluids are an important part of science. Humans are immersed in the atmosphere of the planet Earth, and as a fluid its properties, processes and their evolution are a main concern in day to day activities. Since the advent of modern computers numerical models of fluids have been implemented to simulate the Earth’s atmosphere. Numerical models can forecast accurately the synoptic scale detail for up to several days, but the meteorological microscale detail still remains unpredictable.

The small detail of numerical models can be affected by several factors (for more information see Chapter 4) including computer round-off error and the non-resolved terms in the fluid equations. Some terms of the equations in the fluid dynamic equations cannot be resolved accurately due to limitations in numerical approximations and large grid spacing. The grid resolution and the number of terms used in the discretization can negatively impact the implementation of the models. These factors joined together cause the infinite forecasting limitation. The small errors caused by unresolved terms will accumulate, spread and add-up to adversely affect the final observed atmospheric conditions.

Here is where the discrete nature of the medium comes in. In order to describe fluid motion perfectly, information regarding each of the particles forming the fluid is needed; along with a computer capable of handling such amount of information. None of those exist yet. Those small scale nonlinear factors are not unknown, they are just too complicated to determine. The equation of Navier-Stokes along with the other equations of fluid dynamics can be of great help when looking for these terms. One representation of the equations is given.

The Navier-Stokes equations (there are three coordinates embedded in this equation) can be presented as:

$$\frac{D\vec{u}}{Dt} = -2\vec{\Omega} \times \vec{u} + \vec{g} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{u} \tag{2.11}$$

where $D/Dt$ is the total derivative, $\vec{u}$ is the total velocity field, $\rho$ is the density, $\vec{\Omega}$ is Earth’s angular velocity, $\vec{g}$ is gravity’s acceleration, $p$ is pressure, $\nu$ represents the kinematic viscosity coefficient, and $\nabla$ is the gradient differential operator. The velocity field $\vec{u}$ contains the three Cartesian components of the wind vector; $u$ is the component along the x-axis known as zonal wind, $v$ is the component along the y-axis known as the meridional wind, and $w$ is the vertical wind measured perpendicular to the x-y plane. The total derivative is necessary as the field’s changes need to take into account not only the local changes but those carried along by the wind. The total derivative is a non-linear operator defined by the equation

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{u} \cdot \nabla$$

where $\nabla$ is the gradient differential operator. Equation 2.11 describes the motion of a fluid under the effects of rotation, gravity, pressure gradients and viscosity. The results of simulating a fluid with this equations will depend on how well many of the terms are manipulated, approximated, discretized, or ignored.

In turbulence studies many of the terms in equation 2.11 are not relevant and can be safely ignored. For example, $\vec{g}$ will act on all the fluid elements simultaneously and the dimensions of
the turbulent regions are small compared to the planet’s radius. For small scale motions with short lifespans the Coriolis effect is unimportant. These assumptions leave equation 2.11 as

$$\frac{D\vec{u}}{Dt} = -\frac{1}{\rho} \vec{\nabla}p + \nu \vec{\nabla}^2 \vec{u}. \quad (2.12)$$

The turbulent motions can be considered deviations (perturbations) from the mean value of the atmospheric variables. The perturbations attributed to turbulence have a time average of zero. The mean wind $\vec{u}$ will have an extra component due to the turbulent perturbation $\vec{u}'$, leaving the total wind as

$$\vec{u} = \vec{u} + \vec{u}'. \quad (2.13)$$

This perturbation treatment can be applied to all the related variables in equation 2.12. In many atmospheric applications the second order terms are ignored. Scale analysis is regularly used to discriminate the small terms. None of those techniques should be applied too early as the main interest is the non linear small factors. The term $\nu \vec{\nabla}^2 \vec{u}$ will be ignored in the following equation as the viscosity factor should be much smaller than other factors involving turbulence. Substituting the perturbed variables into 2.12, ignoring viscosity and taking the time average we obtain

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}, \vec{v}, \vec{w}) \cdot \vec{\nabla} \vec{u} = -\frac{1}{\rho} \frac{\partial p}{\partial x} - \left[ \frac{\partial (u')^2}{\partial x} + \frac{\partial u'v'}{\partial y} + \frac{\partial u'w'}{\partial x} \right]$$

as described by Bradshaw (1971). Similar equations for the $y$ and $z$ components can be derived. Terms of the form $a'b$ will disappear as the time average of a perturbation is zero. Equation 2.14 is very similar to 2.12 as many terms exist on both equations. Factors like $-\frac{\partial (u'w')}{\partial z}$ are similar in form to the viscous term $\nu \vec{\nabla}^2 \vec{u}$. The terms involving the $u'w'$ factor (and other similar terms) are known as Reynold’s stresses, represented often as $\tau_{Reynolds}$. These terms can be measured with radars and used to measure momentum flux (Hocking, 2005).

The physical interpretation of the $\rho u'w'$ terms is relevant as they are regularly associated with turbulent diffusion of momentum. The units of $\rho u'w'$ are

$$\frac{M}{LT^2} = \frac{ML}{T} \frac{1}{L^2T} = \text{Momentum Flux} \quad (2.15)$$

and the term itself represents the vertical flux of horizontal momentum (and simultaneously the other way around). This can be easily be shown from basic principles. Consider a fluid moving horizontally at $u$ and vertically at $w$. The wind speed is constant inside the volume under consideration as shown in figure 2.2. Let’s proceed to calculate the momentum flux on the upper plane of the volume element. The momentum inside the box defined by $A$ and $w * \delta t$ is

$$p_x = \rho [Aw\delta t] u \quad (2.16)$$

where $p$ represents momentum and should not be confused with pressure ($P$). To calculate the momentum flux, $p_x$ is divided by the area $A$ and $\delta t$, resulting in
2.3. Nonlinear dynamics in atmospheric fluids

Figure 2.2: Air parcel defined inside atmospheric fluid. The area $A$ is used along with the short time $\delta t$ to define flux of momentum. For the procedure see text.

Flux = $f = \rho uw$. \hspace{1cm} (2.17)

Perturbations can be introduced into the flux equations similarly as it was done in the fluid dynamic equations. With $u = \bar{u} + u'$ and $w = \bar{w} + w'$ in the flux equation and calculating the time average the mean flux becomes

$$
\bar{f} = \rho (\bar{u} + u')(\bar{w} + w')
$$

\hspace{1cm} (2.18)

$$
= \rho \left[ \bar{uw} + u'w' + \bar{uw}' + u'w' \right]
$$

\hspace{1cm} (2.19)

$$
= \rho \left[ \bar{uw} + u'w' \right] \hspace{1cm} \text{as} \hspace{1cm} u'w' = 0
$$

\hspace{1cm} (2.20)

It can be seen that the two terms represent the vertical flux due to horizontal momentum. The $\bar{uw}$ term corresponds to the contribution of the mean wind and the $u'w'$ term to the fluctuating part.

These non-linear terms diffuse atmospheric properties due to the fluctuating motion caused by turbulence. By definition the time average of the turbulent perturbation is zero ($\bar{u'} = 0$), but that may not necessarily be true for combined terms like $u'w'$. Terms similar to $u'w'$ can also contribute to the mean flow and provide a forcing which can accelerate the mean flow in a very systematic way; hence, quasi-random motions can drive a non-random forcing.

Now that the turbulent motions have been represented as perturbations in the fluid dynamic equations, two important questions remain. (i) When will these terms be relevant and/or take over the motion? (ii) How intense is the turbulence and how can it be measured?

Turbulent motions have been observed in atmospheric measurements and laboratory experiments. The transition from laminar (non turbulent) flow to turbulent flow has been related to specific properties of the fluid motion. Those properties are quantified by the Reynolds number and the Richardson’s number.
These numbers are calculated from the ratio of properties that are relevant regarding fluid forcing, energy considerations, and/or fluid characteristics. The Reynolds number (from now on represented as $Re$) is a ratio of the inertial forces to viscous forces, defined as

$$Re = \frac{\text{Inertial}}{\text{Viscous}} = \frac{uL}{\nu}. \quad (2.21)$$

Large values of $Re$ are associated with turbulent flows. In contrast, when viscosity increases the values of $Re$ decrease.

When the inertial term dominates instabilities are generated and the fluid tends to be turbulent, usually observed for large $Re$ values (the term “large” is relative as it depends on many factors and changes for different scenarios). On the other hand, when the viscous term dominates the motion the fluid tends to be laminar. Viscous forces stabilize the fluid, creating laminar flow.

Another quantity used regarding turbulence occurrence is the Richardson number ($Ri$). This dimensionless ratio of a buoyant term to a flow-related term is also useful when identifying turbulent motions in fluids. The buoyant term is based on stored potential energy of a fluid element in motion. The flow related term accounts for the extracted kinetic energy by the fluid element in motion.

Many forms of the $Ri$ ratio exist; one of those is

$$Ri = g \frac{\partial \ln \theta}{\partial z} \left( \frac{\partial u}{\partial z} \right)^2 \quad (2.22)$$

where $g$ is the acceleration of gravity, $\theta$ is potential temperature, $u$ is the horizontal wind and $z$ is the vertical coordinate. Theory and experiments suggest that non-turbulent flow will develop turbulence once $Ri$ falls below a certain threshold. The value of $Ri = 0.25$ is regularly accepted as the limiting value between laminar and turbulent flow.

Even when fluid properties can be used to better understand when turbulence can develop, they can not provide an estimate of the turbulence intensity. In order to measure the intensity a theoretical treatment of turbulence can provide a general view of turbulent dissipation of energy. The rate at which turbulence removes energy from certain scales and deposits it in others is a measurement of turbulence intensity. In the following section the theory about turbulent energy dissipation rate is presented. Afterwards the methods for measuring this factor with radar are introduced.

### 2.4 Theoretical treatment of turbulence

The previous section showed how turbulent terms can develop from the fluid dynamic equations. We turn now to the task of creating a physical interpretation and a mathematical representation of turbulence intensity. In fluids, turbulence removes energy from certain scales and deposits it in other scales. When the scale where the energy is deposited is small enough it will be dissipated into heat. The energy dissipation rate, $\varepsilon$, describes the power dissipated per unit mass. This will be introduced to account for the turbulence intensity.
The physical interpretation of turbulence also requires that we discuss the scales in which turbulence occurs. As previously mentioned, if viscosity is too large turbulence will not develop. On the other hand, the motions that generate the required energy to form the turbulence also need to be analyzed. The theory regarding spectral analysis of turbulence will also be considered in this section.

### 2.4.1 Turbulence scales

Before introducing the different turbulent scales a general view of turbulence is needed. Turbulence is one of the most challenging areas of research. The nature of turbulence makes it complicated to comprehend and even explain in simple terms. The mathematical description of turbulence and the physical interpretations available up to date are not perfect. Turbulence is associated with properties of the fluid, and the Richardson number can be used to quantify the probability of turbulent motions. If the fluid is inviscid or too viscous, turbulence will not form. Even then, the turbulence scales and the energy dissipated can be only approximated as will be shown shortly.

Regarding turbulence there are three length scales that are often mentioned:

- Outer scale.
- Inertial range.
- Inner scale.

But these spectral domains are not static. They move and change constantly because of their definition and the variables they depend on. In turbulence theory there are usually those 3 sub-ranges. The outer scale is representative of large motions, including the example mentioned about the continental air masses. Another example could be a thunderstorm, where a large phenomena is in charge of displacing energy from one place to another.

The outer scale is separated from the inertial range by the buoyancy scale, $L_B$. All the scales that are greater than $L_B$ will be considered to occur in the outer scale. This is usually where the kinetic energy enters the different systems.

Once the length scale of the motion is smaller than $L_B$, the scale will be located in the inertial sub-range. Inside this spectral range is where the turbulence takes place.

The lower limit of the inertial sub-range is the inner scale, $\ell$. Past that limit, the viscosity will dominate diffusing the energy into heat. The inner scale $\ell$ is sometimes conceptually confused with Kolmogorov’s microscale $\eta$, and even when they are related (which will be shown shortly) the difference can be even an order of magnitude or more.

**Kolmogorov’s microscale, $\eta$**

We can begin defining the scales of turbulence by using the Kolmogorov’s microscale. This microscale is not included in the previous subdivision because (as will be shown later) it is less than the inner scale. Consider a typical motion with a length $l$ with a time scale $t_f$. The velocity associated with this motion would be just $u_l \approx l/t_f$. 
The energy dissipation rate, representing how fast the energy is transformed by the motion, can be obtained from the variables $\ell$, $t_\ell$ and $u_\ell$. If we consider that after the motion takes place in the inertial range the energy has been redistributed (e.g. dissipated) then:

$$\varepsilon = \frac{\text{Energy}}{\text{Mass} \times \text{Time}} \approx \frac{mu_\ell^2}{mt_\ell} = \frac{u_\ell^3}{\ell} \quad (2.23)$$

where the units of $\varepsilon$ are Watts per kilogram, and the 1/2 constant factor of the kinetic energy is ignored because it is only an approximation. From the equations of fluid dynamics (Navier-Stokes equation) the term $\nu \nabla^2 u$ is the forcing due to viscosity. Because $\varepsilon$ has units of Watts per kilogram ($L^2T^{-3}$) it is representing power per unit mass. It actually represents how much power can be used by the motion. Using the viscous forcing we can also obtain an expression for power dissipation. The force $\nu \nabla^2 u$ multiplied by the length of the motion is the work done by the force. Dividing that by the time scale would represent an average of the power. At a distance $\ell$ in the viscous range the power can be calculated with the equation

$$\text{Power} = \frac{\Delta W}{\Delta t} = \nu \nabla^2 u \frac{\ell}{t_\ell} = \nu \nabla^2 uu_\ell \quad (2.24)$$

and the magnitude of the term $\nu \nabla^2 u$ can be approximated to $\nu u_\ell/\ell^2$ by similitude analysis. The previous two equations are very important as the first one is valid in the inertial range; the second equation represents when the energy exits the system into the viscous range. We are interested in approximating a length scale where all the energy observed in the inertial range is dissipated inside the inner scale (not necessarily in the intersection). By combining the equations a value of $\ell$ that satisfies both equations ($\eta$) is obtained

$$\varepsilon = \frac{u_\ell^3}{\eta^2}. \quad (2.25)$$

In this last equation the value of $\ell$ was changed to $\eta$ to represent the specific length of the transition region. The equation 2.23 applies when there is no energy loss (i.e. inertial range). Equation 2.24 applies when viscous forces dominate (viscous range). Isolating the value of $\eta$ the Kolmogorov’s microscale is

$$\eta = \left[ \frac{\nu^3}{\varepsilon} \right]^{1/4}. \quad (2.26)$$

It is important to keep in mind that $\eta$ represents a specific length. It is different from any other $\ell$ because the kinetic energy is transforming entirely into heat; which is not the case in the inertial or turbulent scale. Relatively large motions will break into smaller size motions, but the energy would still exist as kinetic energy.

**Inner scale, $\ell_\circ$**

In the previous section we derived the Kolmogoroff microscale $\eta$. However, we commented that it is only proportional to the transition between the inertial and viscous ranges.

Now we wish to consider a practical quantity that truly represents the boundary between the regions in a practical sense. We expect it will be proportional to $\eta$, but may differ numerically. We will call this parameter $\ell_\circ$. 
2.4. Theoretical treatment of turbulence

In order to obtain a value for $\ell_o$ it is necessary to develop the concept of turbulent spectra. The wavenumber where the change of regimes from turbulent to viscous occurs is related to $\ell_o$. The $\ell_o$ value is actually the intersection where the equation defining the inertial range equals the equation defining the viscous range if both trends were projected past their limit (Tatarski, 1961).

It has been shown (Hill and Clifford, 1978) that the values $\ell_o$ and $\eta$ are related by a constant, $\ell_o = \gamma \eta$. The value of this constant can vary between 7 and 14, depending of the type of fluctuation under consideration. The relevance comes from the fact that the limit between the inertial range and the inner scale is not the Kolmogorov’s microscale.

An interesting results of the value of $\gamma$, is that the inner scale will be approximately one order of magnitude larger than the Kolmogorov’s microscale. Even when this can seem a big difference, the difference between a millimetre microscale and a centimetre inner scale is not that large, compared to the possible tens or hundreds of metres that the outer scale can achieve, as will be shown shortly.

The development of the turbulent spectra will be shown shortly in the following sections.

**Outer scale, $L_B$**

Regarding the outer scale we can approximate this value from a simple argument, similar to that used for the microscale $\eta$. For relatively large vertical displacements in the atmosphere, a parcel of air will oscillate according to the Brunt-Vaisala frequency $\omega_B$.

This frequency is just the inverse of the time it takes to complete an oscillation. If we name this time $t_L$ then the associated Brunt-Vaisala frequency is

$$\omega_B = 2\pi (t_L)^{-1}$$

(2.27)

The associated amplitude of the oscillation can be named $L$. If all the energy associated to this oscillation is converted entirely into turbulence then we can use again the concept of energy dissipation rate (relevant only in the inertial range) as mentioned before in equation 2.23 to obtain

$$\varepsilon = \frac{u_L^2}{t_L} = \frac{L^2 \omega_B^2}{4\pi^2} \frac{\omega_B}{2\pi} = \frac{L^2 \omega_B^3}{(2\pi)^3}$$

(2.28)

where the expression $u_L = L/t_L = L\omega_B/2\pi$ was used. Isolating $L$ and renaming it as $L_B$ the final expression of the outer scale in term of the Brunt-Vaisala frequency and the energy dissipation rate is obtained.

$$L_B = c\varepsilon^{1/2} \omega_B^{-3/2}$$

(2.29)

Under rigorous calculations (Weinstock, 1978) the value of $c$ has been estimated as $2\pi/0.62$ which is approximately double the one obtained here. A representation of the different scales is included in figure 2.5. In such figure the outer and inner scales are depicted below 90 km of elevation. The narrowing of the inertial range with elevation is observable in figure 2.5 compared to the width in the lower troposphere.
Figure 2.3: A graphical representation of the variation of the inertial range width in the thermosphere and troposphere. Different Reynolds numbers cause the proportionally constant between $L_B$ and $\eta$ (hence $\ell$ too) to get smaller with height.

$L_B$ and $\eta$ ratio

At this point a clear picture of the outer scale and inner scale should be available. But a simpler way to understand the difference of scale would be to obtain a ratio.

One way to approach this is to consider the Reynolds numbers. This number is a ratio of the inertial forces and the viscous forces, and it shows when the viscous forcing take over in the Navier-Stokes equation. If the value of Reynolds number is too large, the fluid is considered inviscid, and for low values viscosity plays a major role.

The equation for Reynolds number is

$$Re = \frac{Lu}{\nu} \quad (2.30)$$

Derivations of the typical values of $L$ and $u$, were used in equation 2.29. The numerator can be written in term of $L_B$ and $\varepsilon$, leaving

$$\text{numerator} = \frac{cL_B^{4/3} \varepsilon^{1/3}}{\pi} \quad (2.31)$$

The denominator of this equation can be described in terms of $\eta$ and $\varepsilon$ using the definition of Kolmogorov’s microscale. Meaning that $\nu = \varepsilon^{1/3} \eta^{4/3}$, and in consequence the ratio of this terms redefine the Reynolds number as follows

$$Re^{3/4} \approx \frac{L_B}{\eta} \quad (2.32)$$

This simple relation between these two different scales can help us emphasize the different lengths. We can use different values of the Reynolds number to estimate the difference. In the troposphere typical values of $Re$ are found in the $10^8$ range. Using equation 2.32 the value of
2.4. Theoretical treatment of turbulence

$L_B$ will be $10^6$ times the value of $\eta$. If the microscale is located in the millimetre range, the outer scale will be located in the kilometre range.

A contrasting case would be to do the same comparison in the thermosphere. The values of $Re$ at those heights is usually found in the $10^5$ order. Thermospheric heights have a very small gap between both scales being $L_B \sim 30\eta$. This is the reason for the existence of the turbopause, above which the region where the inertial range can sustain turbulence is reduced until it disappears. Hocking (1985) included a diagram that describes this height variation of the different scales (reproduced as figure 2.5 for completeness) but a more simple view is presented in figure 2.3 where the narrowing of the inertial range is observed.

2.4.2 Total energy spectrum, $E(K)$

Now that the limits of turbulence have been reviewed we can turn to a more quantitative description. Let’s begin with a conceptual view of what we expect from the outer scale, inertial range and inner scale. Consider a section of the atmosphere that is large enough to contain the outer scale but is static, no motion or energy is being generated, transformed or converted into heat. In that scenario, there is no energy to be represented as a function of its length (or wave number).

If a large motion is suddenly induced in the system, as in Figure 2.4-a, the energy of that large scale motion (small wavenumber) will be the total energy of the motion. The total energy $E$ would be defined as a function of the scale (or wavenumber $\kappa$). Because it is a large oscillation, instabilities respect to the background air will break the large original motion into smaller scale (larger $\kappa$ values) motions. Among the processes that can generate the energy that cascades into smaller motions are orographic effects, gravity waves, tides and planetary waves. These motions and waves can generate turbulence by non-linear breaking, convective instabilities, convective overturning and critical-level interactions. All these turbulence generators will change the energy distribution, and then the energy will move to other wavenumbers (2.4-b,c,d). One important factor to keep in mind is that this energy is being transformed, and will ultimately be deposited as heat in the atmosphere. Hence the total energy will not be constant, but will decrease with time in a proportion equal to the energy dissipated.

In Figure 2.4-c, several different scales can be seen simultaneously. This would mean that some energy is still moving at large scales, even when part of it has been transformed into smaller scale motion. Because we don’t have the spectral shape defined yet, the diagram shows this as a constantly decreasing function that will just eventually represent particles (huge wave numbers). The existence of different regimes will provide us with a more detail view shortly.

Let’s turn now to the units of the total energy spectrum, $E(\kappa)$. The total energy spectrum that is needed will have units of $kg \ m^3 \ s^{-2}$. As it is usual in atmospheric science, a unit mass is considered for the calculation of $E(\kappa)$. With the coarse assumption that turbulence is isotropic and that $E(\kappa)$ is a power law we could obtain the total energy spectrum shape from the energy that is being dissipated ($\varepsilon$) and the wavenumber $\kappa$ at which that energy is being dissipated.

$$E(\kappa) \sim \kappa^p \varepsilon^q$$  \hspace{1cm} (2.33)

Using dimensional analysis, the units need to balance.
Figure 2.4: A simplistic but useful representation of energy cascading through different scales. The energy generated at the kilometre scale and evolves by redistributing to smaller scale motions. As an example in the figure the kilometre (km), hectometre (hm), decametre (dam), and metre (m) scales are included. Note as time evolves the motions the energy moves from low wavenumber (\( \kappa \)) to higher values.

\[
L^3 T^{-2} = (L^{-1})^p (L^2 T^{-3})^q
\]  

(2.34)

The values of \( p \) and \( q \) are -5/3 and 2/3, meaning that from this simple comparison the shape of the total energy spectrum should be

\[
E(\kappa) \sim \kappa^{-5/3} \varepsilon^{2/3}
\]  

(2.35)

It is important that the considerations made here assumed that \( E(\kappa) \) obeys a power law. Also worth mentioning is the fact that the equation just found is valid for regions where all the energy is being transformed (located inside the inertial range). This derivation of \( E(\kappa) \) is a little simplistic (designed for newcomers to the field), and a proper derivation is done through the structure function, which will be discussed later.

### 2.4.3 Statistical description

In the previous section we derived a spectral form for the turbulence spectrum; representative expressions for the boundaries between the different regions were also found. In this section the statistical description of the turbulence and how the structure of the velocity can be understood as a superposition of motions with different wavelengths will be covered. It is also possible to have structure functions for density, temperature, and trace constituents, among others.

A more detailed derivation, based on mathematical functions, can provide a similar expression (Tatarski, 1961). When studying random variables it is useful to describe them in “simple” terms. The simplest term is the mean value. Another simple characteristic of a random event is obtained by calculating the correlation function.
2.4. THEORETICAL TREATMENT OF TURBULENCE

The mean is the average values of the variable under consideration. The mean can be constant or change with time depending on the averaging interval. Similarly, the correlation function can depend on the time difference used to calculate it as well as the spatial displacement of the functions.

A function describing a random or quasi-random process (such as a correlation function) can be defined as stationary if the mean value and standard deviation do not change with time, and the correlation function depends only on the time difference. It was shown by Tatarski (1961), such functions are associated to spectral functions with random complex amplitudes. This means that, instead of observing the energy content in the time domain using the lag of the correlation function, it can be observed in the spectra as a function of wavenumber. Such spectra are often referred as the spectral density of energy distribution. Since correlation functions can be generated in many different ways, depending on which variables are correlated with which other variables (e.g. we can cross-correlate the x-component of the wind with the y or z components), many new types of spectra also emerge. Previously we dealt only with the total energy spectrum - now new types of spectra will be discussed.

Returning to atmospheric variables, the assumptions considered before are not necessarily met for variables such as wind or temperature. The mean value of the wind can increase with time generating larger values of $\bar{|\vec{u}|}$. Similarly, the value of the average temperature could change with time.
In order to better understand the atmosphere, a more general description should be used. In the case of turbulent motion, the variables are described using structure functions instead of correlation functions. They were introduced by Kolmogorov in Kolmogorov (1968) and Kolmogorov (1991). Consider the case of a function $f$ and the one dimensional independent variable $x$. Instead of using the value of $f(x)$ (used in the correlation function) the value of $f(x + r) - f(x)$ is used in structure functions. If the mean value $f(x)$ changes slowly and a small $r$ is used the value of this subtraction would remain unchanged.

Just as a stationary random function can be described using the correlation function, it can also be described by the structure functions (actually they are related as shown in appendix C). Under this consideration, the structure functions can also be associated with spectral densities. The complete description is found in Tatarski (1961).

The structure function $D$ can be obtained from vectorial or scalar fields.

$$D(r) = |f(x + r) - f(x)|^2$$

where $f(x)$ represent an atmospheric variable. In the following sections $f(x)$ will represent any of the total three dimensional wind, wind components, or index of refraction. The latter is very important to radar research because it is related to atmospheric backscatter.

**Homogeneity and isotropy**

The concept of homogeneity and isotropy is very instinctive to humans. If a field of grass has roughly the same height anywhere in one direction it should be homogeneous; and if in addition it has been cut recently, it should be the same in any direction that we move, that would make it isotropic. This is not instinctive when applied to the atmosphere. We perceive the atmosphere as a constantly evolving field, where the different manifestations of energy and motion rarely seem static.

The concept of homogeneity and isotropy in the atmosphere is based on the assumption of strong turbulence present on a parcel of air. This is often called local conditions, meaning that they are only valid in a limited, small section of the atmosphere. The strong turbulence will mix the air efficiently, creating in that specific location the homogeneity and isotropy necessary for the validity of this theoretical treatment.

An example of a random process can be obtained from the atmosphere. The random field $f(r)$ can represent the wind, temperature or index of refraction. Instead of considering these fields stationary, they are assumed homogeneous. The implication of homogeneity relates to the invariability of the correlation function to the chosen origin. The value of the correlation function will be the same inside the local section of the atmosphere independent of the reference point used to calculate it.

A random field that is homogeneous is not required to be isotropic. In order to be isotropic, instead of depending on $\mathbf{r}$ it will only depend on $|\mathbf{r}|$ (from now on simply $r$). Isotropy is the required condition for turbulence to be independent of two coordinates and only be a function of one, as was discussed in the literature (Tatarski, 1961).

These two basic assumptions were used by Tatarski (1961) in order to obtain the structure function for such a random field. It simplifies the calculations of the integrals required to obtain the spectra. After an elaborate derivation involving the corresponding spectra of the
structure function (which will be introduced shortly), and application of the simplifications, an expression for the structure function in terms of a power of $r$ was obtained.

**Vectorial field structure function, Wind.**

The description of the wind field using structure functions requires nine equations.

\[
D_{ij}(\vec{r}) = (u'_i - u_i)(u'_j - u_j)
\]  

(2.37)

where $i,j = x,y,z$. $u_i$ are the wind components and $u'_i$ (the variable $u'$ is no longer used here as a perturbation in the Navier-Stokes equations) are the components at the point $\vec{x} + \vec{r}$. Using the isotropy condition (Tatarski, 1961) two immediate cases $i = j$ and $i \neq j$ emerge.

- **$i = j$**
  This case generates the longitudinal structure function.

\[
D_{\parallel}(r) = \left| \vec{u}_{\parallel}(\vec{x} + \vec{r}) - \vec{u}_{\parallel}(\vec{x}) \right|^2
\]  

(2.38)

where $\vec{u}_{\parallel}$ is the wind component parallel to $\vec{r}$.

- **$i \neq j$**
  The transverse structure function is generate under this conditions.

\[
D_{\perp}(r) = \left| \vec{u}_{\perp}(\vec{x} + \vec{r}) - \vec{u}_{\perp}(\vec{x}) \right|^2
\]  

(2.39)

where $\vec{u}_{\perp}$ is the wind component perpendicular to $\vec{r}$.

These two functions are related and only one is needed to obtain a total description of the wind field (Tatarski (1961), page 31). Differences between the two functions help us understand anisotropic turbulence. Using the total wind field the expression for the structure function is

\[
D_{Total}(r) = \left| \vec{u}(\vec{x} + \vec{r}) - \vec{u}(\vec{x}) \right|^2
\]  

(2.40)

In the literature (Tatarski, 1961; Kolmogorov, 1968), by applying the concept of isotropy, the value of the $D$ functions for turbulence is assumed to be only a function of $r$ and $\varepsilon$. They use the assumption that the only parameter describing the motions in the scale comparable to $r$ is $\varepsilon$. Meaning that

\[
D_{\parallel} = c\varepsilon^p r^q = c\varepsilon^{2/3} r^{2/3}
\]  

(2.41)

where $c$ is a proportionality constant, $\varepsilon$ is the energy dissipation rate and $r$ is the separation used for the function. Dimensional analysis has been used to match the units of $D$ ($m^2 s^{-2}$) on the left to the same units on the right, giving both $p$ and $q$ as $2/3$. This is known as the two thirds law. The results of a similar calculation for $D_{\perp}$ and $D_{Total}$ would result in different values for the constant $c$. For $D_{\perp}$ the value is $4c/3$, and for $D_{Total}$ it is $11c/3$ (due to $4/3+4/3+3/3$).
Chapter 2. Nonlinear small scale dynamics

All these functions have been proven to be relevant and related to different turbulent spectra (Tatarski, 1961; Kolmogorov, 1968). It is usually more useful to express the information in terms of the frequencies contained rather than the related scales. Each structure function can be related to a spectrum. Each spectrum will contain different information, depending on the variables being cross-correlated/compared.

Spectra related to wind structure functions

As it was shown in the literature, each cross-correlation of the wind components is associated to a corresponding spectra. In appendix C it is shown that the structure functions are directly related to the covariance functions by

\[ D_{ij}(r) = 2 \left[ C_{ij}(0) - C_{ij}(r) \right]. \]  (2.42)

It is clear from this equation that the structure function and the autocovariance function are related. For each autocovariance function there is a spectrum associated, and it can be calculated by using Fourier theory as

\[ C(r) = \int_{-\infty}^{\infty} e^{ikr} \phi(k) dk. \]  (2.43)

Consider the one dimensional case

\[ \phi(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-ikr} C(r) dr. \]  (2.44)

This puts in perspective the fact that for every possible \( C(r) \) there is a \( \phi(k) \). In the three dimensional case of the wind field, the tensor \( D_{ij} \) is related to the \( C_{ij}(r) \) and in consequence to the spectrum tensor \( \phi_{ij}(k) \). If the results from equation (2.41) are used, then the resultant spectra is a function of \( e^{2j/3} \) and \( k^{-5/3} \). If we chose to keep the value of \( r \) aligned with the \( \hat{x} \), this results would be valid for the longitudinal structure function \( D_{11} \) for example, but not for the \( D_{22} \) or \( D_{33} \), because they would be transverse structure functions.

In the case of three dimensional structure functions related to \( C_{ij}(\vec{r}) \) it would be associated to \( \phi_{ij}(k) \) by

\[ \phi_{ij}(\vec{k}) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i\vec{k} \cdot \vec{r}} C_{ij}(\vec{r}) d\vec{r} \]  (2.45)

This function is the spectral density in \( k \)-space. In the case of isotropic turbulence this function is spherically symmetric and the integrated value between \( k \) and \( k + dk \) would be more representative.

\[ \Phi_{ij} = \oint \phi_{ij}(\vec{k}) k^2 d\Omega_k \]  (2.46)

This spectra represents the contribution of the \( i-j \) pair. For this to represent the total turbulence energy the values of \( i=j=1,2,3 \) need to be added. The total energy is then

\[ E(k) \propto \Phi_{11} + \Phi_{22} + \Phi_{33} \]  (2.47)

The proportionality factor is \( \frac{1}{2} \). The total energy per unit mass relates to \( \overline{u_{total}^2} \).
Scalar field structure function; Index of refraction.

Finding the structure functions for refractive index is crucial for turbulence studies using radars. This function is directly related to the scale ($k$) and intensity ($\epsilon$) of turbulence. Using the index of refraction provides a tool for measuring it with radar.

These structure functions are taken to have the same shape of the two thirds law. It should be clarified that inside $C_2^2$ is the term $\epsilon$, with the information about the energy dissipation rate. The structure function for a scalar field is

$$D_\theta(r) = C_\theta^2 r^{2/3}$$  (2.48)

The value of the three dimensional turbulence spectral density function can be obtained using the equation

$$\Phi_\theta(\vec{k}) = 0.033 C_\theta^2 |\vec{k}|^{-11/3}$$  (2.49)

Integrating

$$E(k) = \int 0.033 C_\theta^2 k^{-11/3} k^2 d\Omega_k = 0.415 C_\theta^2 k^{-5/3}$$  (2.50)

where $d\Omega_k$ is the differential solid angle in k-space. To measure this spectrum, full knowledge about the atmosphere is necessary.

With an in-situ device the atmosphere would be probed in an approximate straight line. In order to obtain the spectra related to this probing a different derivation is required. The value of $\vec{r}$ would be oriented in one direction $k_1$, and the integration will be in coordinates $k_2$ and $k_3$. The resulting spectrum is

$$S_\theta(k_1) = \int_{-\infty}^{\infty} \Phi_\theta(\vec{k}) dk_2 dk_3.$$  (2.51)

For turbulence, this spectrum is

$$S_\theta(k) = 0.125 C_\theta^2 k^{-5/3}$$  (2.52)

Applying these concepts to the refractive index of the atmosphere ($n$), the spectrum is

$$S_n(k) = 0.125 C_n^2 k^{-5/3}$$  (2.53)

In order to obtain the spectra, the value of $C_n^2$ and hence $\epsilon$ should be determined.

### 2.5 Measuring turbulence intensity using radar

The value of $\epsilon$ measures the intensity of turbulence. Its determination is crucial to understand how much energy moves in turbulent regimes. It is also important in evaluating the precision of the previous statistical treatments. Different instruments can be used to probe the atmosphere and generate valid information for turbulence studies.

In-situ and remote sensing instruments can be used to probe the atmosphere. The high vertical resolution of radiosondes can be used to obtain valuable information such as humidity,
temperature, and more importantly wind. Rockets can reach heights that are out of reach of balloon-carried instruments. Aircraft data also incorporate valuable wind information that can be used for turbulence studies. Remote measurement instruments such as radars and satellites can provide temporal resolutions that are not achievable with in-situ devices. High temporal and spatial resolution makes radar one of the most important tools in gathering information for turbulence studies.

In atmospheric radars there are two different ways of measuring turbulence:

1. The structure function $C_n^2$ can be measured by radars. This value can be used to obtain the value of $\varepsilon$. This first method is based on the relationship between intense turbulence creating greater backscatter than areas of weak turbulence. Intense mixing in adiabatic profiles produces no scatter (Hocking, 2001); in other cases, enhanced mixing in the inertial range scales will create more variation in the index of refraction. With more backscattering structures on the path of the radar’s energy, more energy would be detected by the radar.

2. The spectral width of the radar spectrum can be used to estimate $\varepsilon$. The width of the spectrum obtained from the radar’s time series can be increased by different factors. Beam broadening, vertical velocities and turbulent motions are examples of phenomena associated with variations in the width of the spectrum.

In the following sections these two ways of estimating turbulence intensity are mentioned.

### 2.5.1 Relationship between turbulence theory and backscattered power

It was shown in section 2.2 that the refractive index is related to atmospheric variables. The amount of power returned in the backscattering process is related to the vertical gradient of $n$, called potential refractive index gradient, $M$ (equation 2.10). The value of $M$ is expected to depend on the degree of fluctuations inside the local section of turbulence. All those fluctuations on the Bragg wavenumber ($\lambda/2$) will contribute to the returned power, and a measure of the degree of turbulence is given by the full three-dimensional spectral density function

\[
\Phi_n(k) = 0.033C_n^2 k^{-11/3}
\]

which is different from the one dimensional spectral density function ($k^{-5/3}$ relationship). For tropospheric radars the scattering is due to bound electrons. The scattering crosssection is proportional to $k^4$ (from Maxwell’s equations). In Chapter 1 the radar equation was given, relating the power to technical aspects of the hardware and calibration.

In a more detailed form the radar equation is

\[
P_R \approx 0.015C_n^2 P_T A e^2 \Delta H^{4/3} \frac{A^{1/3}}{H^2}
\]

where the returned power is a function of:

- $k^4$, cross section.
- $0.033C_n^2$, degree of turbulence intensity.
2.5. Measuring turbulence intensity using radar

- $P_T$, transmitted power.
- $G$ or $A_R$, gain or effective area.
- $\Delta H$, pulse length or PL.
- $H$, radar range.
- $e$, radar efficiency.

and the $\lambda^{-1/3}$ term comes from the $k^4 k^{-1/3} = k^{1/3} \propto \lambda^{-1/3}$.

Other forms of this equation are common. Hocking and Mu (1997) included the equation in the form

$$C_n^2 = 66.4 \frac{p_r Z^2 \lambda^{1/3}}{p_t A e_R^2 \alpha_t (0.5 L_t)}$$

(2.56)

where $p$ is power (received and transmitted), $Z$ in the range of the scatterers, $\lambda$ is the pulse’s wavelength, $A$ the effective area, $e_R$ is a loss factor for the radar, $\alpha_t$ a Gain-related factor, and $L_t$ is the pulse length.

Taking a closer look at the $C_n^2$ term we can find out about the factors that it should depend on. When analyzing a turbulent patch is it expected there will be a dependence on the outer scale. Consider a parcel that is displaced adiabatically, the environment (background) has a different profile in temperature and humidity. The scattered signal depends on the difference between refractive index of the parcel and the background. The more the two differ, the bigger the scattered signal. The bigger the potential refractive index gradient, the stronger the returned signal. This dependence on the $M_n^2$ term takes this analysis to

$$C_n^2 \propto L_B^{4/3} M_n^2$$

(2.57)

The value of $q$ that satisfies the unit equality between all these terms is $4/3$. Inside $C_n^2$ there will also be the term relating it to $\varepsilon$. In equation 2.29 this relationship is explicitly given between $L_B$ and $\varepsilon$. Then

$$L_B^{4/3} \propto \varepsilon^{2/3} \omega_B^{-2}$$

(2.58)

Substituting back to the equation the relation is complete

$$C_n^2 = \frac{1}{\gamma} \varepsilon^{2/3} \omega_B^{-2} M_n^2$$

(2.59)

where the value $\gamma^{-1}$ should be determined (Hocking and Mu, 1997). If the radar scattering volume is not filled with turbulence a factor ($F$) can be used to rescale the $C_n^2(turbulence) = C_n^2(radar)/F$. The value of $F$ is the fraction of the scattering volume filled with turbulence.

Even when it looks a bit complicated, the usage of these equation is quite simple. Using the values of $M_n$ and $\omega_B$, $\varepsilon$ can be obtained from $C_n^2$ as measured by the radar (Hocking and Mu, 1997). It is an impressive result because, under the correct atmospheric conditions (assuming to homogeneous and isotropic turbulence in a local region) an estimation of the energy dissipation rate of that atmospheric region can be obtained remotely.
2.5.2 Measuring turbulence intensity with spectral width

The Fourier transform of the time series is used to estimate the spectral content. The radar’s spectral content usually resembles a Gaussian function, at least in the low frequency range. Using the width of the spectra looks like a good approach because it should be related to the wind perturbation of the scatterers with respect to the mean wind, which is a measure of turbulence.

According to the literature (Weinstock, 1981; Hocking, 1999), the standard deviation of the radial velocity \( \sigma_t \) is related to the energy dissipation rate \( \varepsilon \) by

\[
\sigma_t \propto \int_{2\pi/L_B}^{4\pi/4} \varepsilon^{2/3} \kappa^{-5/3} d\kappa \tag{2.60}
\]

where \( \kappa \) is the wavenumber, and the lower and upper limit of the integral are the limits of the inertial subrange measured by the radar. Using this expression the values are directly related by

\[
\varepsilon \approx C\omega_B \sigma_t^2 \tag{2.61}
\]

as shown by Hocking (1999), and where \( \omega_B \) is Brunt-Vaisala’s frequency. The standard deviation of the radial velocity \( \sigma_t \) can be determined from radar’s data by measuring the spectral width. Unfortunately, the spectral content accounts for more than just the turbulence factors. The width of the spectral content can be influenced by other factors related to radar engineering and atmospheric applications. Spectral thickness will depend mainly on four different effects:

- Beam broadening.
- Shear broadening.
- Turbulence.
- Vertical velocities.

These terms can be catalogued as desired and undesired contributions. The turbulence term is the only factor related to turbulence intensity. If any of the other terms are considered, the measurements of turbulence intensity would be biased. The previous effects can be described in more detail to better understand their implications.

Inside the fluid, perturbations in the mean values of the variables will occur. These perturbed motion of scatterers with respect to the mean wind is the desired contribution. The Beam broadening effect occurs even when all the scatterers are moving at the same horizontal velocity. Due to the angular thickness of the beam each will cause a different Doppler shift increasing the width of the spectral signal. This effect usually dominates the spectral width. Horizontal wind shear effect can increase and reduce the width depending on the sign of the shear. Vertical motions under non-convective motions can be small but need to be considered as they can modify the turbulent width of the spectrum. The vertical motions will introduce an extra range of frequencies in the Doppler velocities that could alter the estimation of turbulence intensity.
2.6. Conclusion

In this chapter a short review of the non-linear small scale dynamics related to radar was presented. The process between atomic physics and index of refraction was presented. Atmospheric variables define, at VHF wavelengths, the index of refraction as observed with radars. Warnings regarding the validity of the constants used in the definition of the index of refraction were given, as the original terms were derived from optics.

By applying small perturbations to the fluid dynamic equations the turbulent momentum flux terms were obtained. These terms were observed to represent the physical process of mean momentum transport from basic principles. Their relevance in the fluids depend on the characteristics of the fluid. Various ratios can be used to define the transition between laminar and turbulent flows. No measure of the intensity comes from the equations of fluid dynamics, instead a different treatment was given to obtain those scales.

A physical description of turbulence was given. Different scales generate, transport and finally dissipate atmospheric energy. Using statistics and reasonable assumptions, a mathematical expression for the turbulent energy dissipation rate was obtained. In order to validate those estimates, radar measurements of the dissipation rates need to be carried out by radar.
Two different methods are available to calculate the turbulent energy dissipation rate with radar. The first method described used the backscattered power. This power is related to the magnitude of the inhomogeneities in the index of refraction caused by turbulence. The second method is based on the spectral width of the Fourier transform of the radar time series. The variance of the radial velocities is related to the turbulent energy dissipation rate as shown.

In the following sections theoretical tools used in radar spectral estimation will be analyzed. An analysis of a long dataset of measurements with a Costa Rican radar will be included. Numerical simulations of radar backscatter will be introduced and their results analyzed.
Chapter 3

Spectral analysis

Subatomic particles play little role in the troposphere mainly because of the relative low temperature and short mean free path. Larger particles (atoms, diatomic molecules and larger bodies) are the main constituents of the troposphere. It is interesting to note, that even when they are discrete in nature, the scale difference between humans (and most of our instruments) and particles allow us to perceive them as a continuum.

This illusion of continuity will persist during this thesis because we don’t deal (directly) with the quantum world, but only with the implications of the relatively large scale wavelengths that we use.

Whenever we take a measurement of a variable, we are going from that apparent continuum to a discrete time data series. When a pressure sensor reading is taken, there is a conversion from the continuous pressure function $P(t)$ to a discrete time series $P[n]$ that represents only those values recorded. The rest of the information between the measurements is lost. The same thing will happen when measurements of $T[n]$ and $q[n]$ are taken.

This information loss should be controlled in order to obtain what we need from the data. If a study’s goal is to understand the variation of the atmospheric pressure, will it be enough to measure the pressure once a week or 100 times per second?. The Nyquist-Shannon (Wyner and Shamai, 1998) sampling theorem sets the necessary conditions, stating that a signal $y(t)$ containing frequencies no higher than $F$, will be completely determined by recording samples at intervals $(1/2F)$. The digitizer needs to measure every $t_s = (1/2F)$ to be able to resolve frequencies up to $F$. This can be looked from the other side as well. If you record samples every $t_s$, the maximum frequencies that will be resolved are $(1/2t_s)$.

The Nyquist-Shannon theorem imposes a specific number (and spacing) of measurements in order to properly resolve the spectral content. Modern techniques (e.g. compressive sampling) surpass those limitations (Candès et al., 2006), however the signals under consideration must be sparse. In radar applications, especially in the case of backscattering from turbulent regions, the resulting signal may not be bounded by the sparsity requirements. The following treatment will be based on the Nyquist-Shannon theorem as the Costa Rican radar design is based on it.

When using the Nyquist-Shannon theorem there is an interesting limitation to the maximum frequency measured. There is a phenomenon known as spectral leakage. This is clear from the principle of the convolution occurring in the process of digitization, and will be mentioned later.
In any area where the data are recorded in numerical series there is the need to analyze the content and find its meaning. The method used to study the content of the signal recorded will vary, but most of them focus on finding the major sources of the signal.

Probably the most common method used today is the Fast Fourier Transform (FFT). This method comes from the Discrete Fourier Transform that has a simpler form, with more computational work and produces the same result. Many other methods of spectral analysis exist. In this document the case of Capon’s method will be studied because it is widely (and increasingly) used in our field of research.

Later on, two important applications of spectral analysis are reviewed and studied. The first case is the deconvolution, a property that comes from Fourier Theory. It is related to the correspondence of convolution in the time domain and the multiplication in the frequency domain of two or more functions. The benefits of using this tool in radar applications will be shown. The second case is interferometry. It can be used for different goals, and applied in the time or frequency domain. In this document the three dimensional location of scatterers in the sky is discussed and experimental results are provided.

### 3.1 Data Independent Methods

The data independent methods have the characteristic of assuming no previous knowledge about the signal under consideration. In these type of methods side-lobes near the main peak are usually found in the spectral content. Many of the data dependent methods try to eliminate these side-lobes assuming that no information is contained inside those sections of the spectra or that this information is not relevant. As have been proven before (Jian Li and Stoica, 1996), these methods can have good results if looking for a single peak signal in the spectra but bad results when used for continuous spectral content.

The basis of the data independent methods comes from Fourier Theory. As long as a function \( g(t) \) satisfies the Dirichlet conditions the Fourier transform can be calculated. These conditions are there to ensure that a function cannot contain infinite power in one specific interval, in this way the following integral will exist.

\[
G(f) = \int_{-\infty}^{\infty} g(t) e^{-2\pi i f t} \, dt
\]

(3.1)

There are very important properties and theorems regarding the Fourier transform that should be kept in mind, because they are widely used in radar applications. The double arrow (\( \leftrightarrow \)) is used to identify Fourier transform pairs. To list a few of those:

- **Linearity:** \( c_1 g_1(t) + c_2 g_2(t) \leftrightarrow c_1 G_1(f) + c_2 G_2(f) \)
- **Frequency shifting:** \( g(t) e^{2\pi i f_0 t} \leftrightarrow G(f - f_0) \)
- **Time shifting:** \( g(t - t_0) \leftrightarrow G(f) e^{2\pi i f t_0} \)
- **Convolution:** (covered in a section 3.3)
- **Duality:** if \( g(t) \leftrightarrow G(f) \) exists, then \( G(t) \leftrightarrow 2\pi g(-f) \) also exists
3.2. Data Dependent Methods

- Parseval’s Theorem

In theory, infinite series are feasible. When developing computer models and in applied physics applications, data series are limited to a finite number of entries and length. In these cases the Fourier transform needs to be adjusted to a discrete series of data. Usually the data series will be measured at regular intervals \(T_s\) and for \(N\) consecutive samples. In this case the original function \(g(x)\) will become \(g[nT_s]\) for values of \(n\) from 0 to \(N-1\).

Because the discrete Fourier transform (DFT) is periodic \((G(f + F) = G(f))\) the equation can be limited to calculate in the range \(0 \rightarrow F\) or \(-F/2 \rightarrow +F/2\).

\[
G(k_s) = \sum_{n=0}^{N-1} s[n_s]e^{-2\pi ikn/N}. \tag{3.2}
\]

In the case of radar, the signals are treated as discrete wide-sense stationary random processes. A random process can be defined as an infinite sequence of random variables. A wide-sense stationary process implies that the mean of the random process is independent of time; and also the covariance needs to be dependent only on the temporal separation between samples. Many spectral estimation techniques use these properties to calculate the spectra based on the auto-correlation function of such signals.

By using the power spectral density of these radar signals, the correlogram can be calculated:

\[
G(f) = \sum_{n=-(N-1)}^{N-1} r[k]e^{-2\pi ifk}, \tag{3.3}
\]

where the \(r[k]\) is the term corresponding to lag \(k\) of the covariance matrix. Here is where the data can be used to create a specific filter. Many filters exist that don’t use the data to create the filters. Among those are found Blackman, Barlett, Hamming and Hann.

A weighting function \(w[k]\) is introduced in the calculation of the correlogram, and with appropriate shape more or less importance is given to an specific lag \(k\) of the covariance matrix term. This has the advantage that the terms of this matrix created with only a few points and that generate large uncertainty in the result can be ignored or diminished by adequately setting the values of \(w[k]\).

\[
G(f) = \sum_{n=-(N-1)}^{N-1} w[k]r[k]e^{-2\pi ifk} \tag{3.4}
\]

3.2 Data Dependent Methods

In the previous section the introduction of the \(w[k]\) filters mentioned that by appropriate selection of this filter, the product of the spectral estimator can be improved. Much better results could be obtained if the information contained in the data itself could be used to generate that filter generating a better looking spectral content.

In the case of a signal with only one discrete frequency, the data can generate the needed information while only paying attention to that single frequency. This is a great advantage in
the case that information about a signal is known a priori. In telecommunications this would be of great use, looking for signals and variations that are expected and known to have an specific content, shape and behaviour.

This methods can have the counter effect that when used to study signals with more frequency content, different in nature (continuous instead of discrete) or unknown, the product of the spectral estimation, even when successfully generated by the procedure, could be unrealistic or contain ambiguous information.

### 3.2.1 Minimum variance method

Spectral analysis tools are needed in many areas of industrial processes, commercial production and scientific research. The frequency content and the energy carried within each frequency band are of great importance to the study of various phenomena in physics. The atmosphere is a container where many types of waves and oscillations can occur and spectral analysis is particularly important in identifying and characterizing those events.

Several types of methods exist to estimate the frequency content from discrete signals. If classified by filter design, these methods can be grouped into adaptive and non adaptive methods. The non adaptive methods are often based on Fourier theory, and they are characterized by filters of constant shape and size. Adaptive methods, on the other hand, generate their filters according to the data under investigation. Detailed views of these methods and their application are available (Kay, 1988).

The theory behind the Fourier based methods have been studied for a long time, and has solid basis. These methods have been used in MST radar studies since the very beginning (Woodman and Guillen, 1974). The filters are well known and the implications of the filter shape in effects like spectral leakage are well understood. Such detailed knowledge about the filter’s effect cannot be the case in adaptive filters due to the fact that filter shape will change with the data itself. This is both the greatest strength and also something of a limitation of the process. Understanding the impact of the filter becomes correspondingly more complex.

An example of adaptive processing is Capon’s method (Capon, 1969), also known as Minimum Variance Method (MVM), created originally to study seismic signals and later disseminated into other areas. In MST radars it has been used in beam forming (Palmer et al., 2001) to study spatial and spectral characteristics of the data with the intent to increase the resolution of the imaging products.

In section 3.2.2, the basic principles behind Capon’s method are presented, as well as the main equations used to obtain the filter and power. Because one important application of Capon’s method is optimization of spectral-line thickness, and also optimization of scattering layer thickness, a brief introduction to the optimization process is given in section 3.2.3, including a comparison with Fourier deconvolution. In section 3.2.4 the methodology used in the simulations and radar data is described. In section 3.2.5 the results of the simulations are presented and finally some analysis and conclusions will be addressed in section 3.2.6.

### 3.2.2 Filter Design Process and Spectral Estimators

There are two basic ways in which spectral estimators work. A pre-set filter could be designed to modify the spectral estimation process, giving more weight to those points that hold more
3.2. Data Dependent Methods

relevance (e.g. filter bank processes used in correlograms).

Another way is to adaptively modify the original data series, in order to remove (beforehand) the spectral content of least interest (Capon’s method) and then later estimate the spectra. No matter in what domain or at which stage of processing the filter is created or used, the process will generate a modified data series (and corresponding spectra) with new characteristics.

**Fourier Method**

The spectral characteristics of the filter are dependent exclusively on the type of window chosen to be used as a filter. Different shapes of the window that give the weight to the different lags of the auto correlation function (ACF) will generate the final spectral shape.

**Capon’s Method**

This method is also known as the minimum variance method (MVM). The theory behind Capon’s method is easy to describe in term of general ideas. Based on a data series, an appropriate filter can be constructed to minimize the effect of other frequencies when studying one designated frequency. This procedure measures at just the frequency band of interest, minimizing the effect of the other frequencies. The filter is continually changed as we move to newer frequency bands of interest.

The positive outcome of such filter is clear. Imagine that a filter is created that ignores everything of little interest, and only allows valuable information at pre-selected frequencies to appear.

With that in mind, let’s try to understand the process a bit more by using a complex signal \( y[n] \) in order to estimate its frequency content. The way to proceed is choose a frequency to analyze (call it \( f_c \)), create a frequency domain filter (\( \mathbf{H} \)) with the appropriate size (\( m \)), apply that filter and obtain an estimate of the power which passes out of the filter; move to a new central frequency \( f_c \) and recalculate a new filter with corresponding new power output; and so forth. After sufficient iterations the full estimation of the spectral content will be obtained. This filter will be data dependent and the process is illustrated in figure 3.1. In the case of 5 frequencies of interest, the filter’s frequency \( f_c \) is successively chosen as \( f_c = f_1 \), then \( f_c = f_2 \), \( ..., f_c = f_5 \).

As expected the filter shape is data dependent (changing with every choice of \( f_c \)), and minimizes at all other frequencies that are not \( f_c \). Note that the desired signal is the new time series produced after passing the original signal through the filter, from which the power is calculated. This power is then associated with the relevant frequency \( f_i \).

It can be seen in figure 3.1 that the filter (known as \( \mathbf{H} \))-shape changes for all the different central frequencies used. The Fourier transform of this filter is represented by a sequence of numbers \( h_0, h_1, ..., h_m \). This vector \( \mathbf{h} \) is the temporal equivalent of the filter created.

These functions of the filters (\( \mathbf{H} \) and \( \mathbf{h} \)) are dependent on the frequency (\( f_c \)) that we choose to study. It has been shown (Capon, 1969) that the output power for a selected value of \( f \) is given by the following equation.

\[
\text{Power}_f = \mathbf{h}^\dagger \mathbf{R}_f \mathbf{h}_f
\]  

(3.5)
Figure 3.1: Frequency response filtering process for Capon’s method in the presence of 5 frequencies at $f_1, f_2, ..., f_5$. For each successive relevant frequency $f_i$ the process will generate a filter with frequency response that peaks at the frequency under consideration, while trying to create zero frequency response elsewhere. For example, notice that when the frequency of interest is $f_1$, the filter maximize at $f_1$ and is minima for frequencies $f_2$ to $f_5$.

where the $\dagger$ is the Hermitian operator. $R_y$ is the Toeplitz type autocovariance matrix built from one sequence of data. This matrix is the vehicle by which the filter becomes data dependent and can be defined as

$$
R_y = \begin{bmatrix}
  r_{yy}[0] & r_{yy}[1] & \cdots & r_{yy}[m-2] & r_{yy}[m-1] \\
  r_{yy}[1] & r_{yy}[0] & \cdots & r_{yy}[m-3] & r_{yy}[m-2] \\
  \vdots & \vdots & \ddots & \vdots & \vdots \\
  r_{yy}[m-2] & r_{yy}[m-3] & \cdots & r_{yy}[0] & r_{yy}[1] \\
  r_{yy}[m-1] & r_{yy}[m-2] & \cdots & r^*_y[1] & r_{yy}[0]
\end{bmatrix}
$$

(3.6)

where $m$ is the order or the autocovariance matrix and $r_{yy}[k]$ is the expectation value of the series $y[n]\overline{y}[n-k]$, $y[n]$ being the sequence of data and $\overline{\cdot}$ being the complex conjugate. The calculation of $r_{yy}$ requires the addition of $n \times (n - k)$ products (equation 3.6) and such products must be evaluated $n^2$ times for each matrix.
3.2. Data Dependent Methods

After this consideration, it is only a matter of calculating the values of $h$ that minimizes the output of the power at that frequency and, requiring that $H(f_c)$ has a value of 1. The process is fully described in the literature (Capon, 1969; Kay, 1988; Palmer et al., 1998) and a review of the theory is included in appendix D.

The final estimation from Capon’s method can be obtained by the expression:

$$S = \frac{m + 1}{a_f^*R_y^{-1}a_f}$$

where $S$ is the estimated power and $a_f$ is the optimized filter. Notice that this expression does not explicitly use either $h$ or $H$. These filters are represented through $R_y$, but for physical understanding of the process, knowledge about $h$ or $H$ is critical.

Another form of estimating the magnitude of the spectral height with Capon’s method can be obtained by replacing the $m + 1$ term in this equation with a more elaborate term (Kay, 1988; Stoica and Moses, 2005).

In radar applications Capon’s method is used because it is very effective at resolving peaks that are located close together. Its popularity comes from the higher resolution achieved compared to Fourier transform under an equal filter size. This can be appreciated in figure 3.2-a. Under these circumstances, Capon’s method will resolve two peaks (co-located with the true spectral content) while Fourier method will only resolve one. This result is the main reason why Capon’s method is popular. In the next sections we consider if there are any “down-sides” to Capon’s method, and specially look at the impact on data with many spectral lines and even a continuum of lines.

3.2.3 Deconvolution theory and layer isolation enhancement

While propagating, the radar pulse interacts with the index of refraction inhomogeneities in the atmosphere. This interaction backscatters a small portion of the incident power. This backscattered signal of the atmosphere is known mathematically as a convolution (Champeney, 1973; Bracewell, 1978). The convolution is defined in terms of two different functions: the radar pulse shape, and the atmospheric profile of index of refraction inhomogeneities.

The convolution theorem is a mathematical process relating two functions ($f$ and $g$) to the Fourier transform of these functions ($F$ and $G$). Define $h(t)$ as the convolution of $f(t)$ and $g(t)$ using

$$h(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau$$

it can be shown (appendix E) that this can be converted using the Fourier transform into

$$H(f) = F(f)G(f)$$

where $H(f)$ is the Fourier transform of $h(t)$. Equation (3.9) expresses the Fourier transform of a complicated function (equation (3.8)) as a simple multiplication in the frequency domain. This simplification is used in MST radar applications to better understand the atmospheric scattering process.
Figure 3.2: Comparison of Fourier (solid line) and Capon’s (broken line) results for different combinations of filter size. The locations of the true peaks are shown as diamonds. Fourier’s peaks are presented with a triangle and Capon’s peaks with a circle. a) Both filter sizes were set to 16. b) Fourier filter size is set to a large number and Capon’s filter size is set to \(N/2\). c) Fourier filter size is set to a large width and Capon’s filter size is set to 8.

It is not new to MST radars that the profile obtained in the receivers is a convolution of the transmitted pulse with the atmospheric profile (e.g. Hocking and Röttger (1983) and references therein). Under ideal conditions it should be straightforward to deconvolve the receiver information with the transmitted pulse shape. This has been done before (Röttger and Schmidt, 1979) with some success, but was limited by sampling issues, since optimal sampling was not possible due to the limited computer capabilities at the time.

In radar applications the functions \(f(t)\) and \(g(t)\) represent the radar’s pulse shape \(Tx\) and the atmospheric profile \(Pr\). Suppose that a transmitter pulse \(Tx\) is sent to the atmosphere, and that the atmosphere contains a vertical index gradient profile proportional to \(Pr\), which is a function of height. With regard to pulse delay, \(Pr\) can be considered to be a function of time.
3.2. Data Dependent Methods

The signal received at the digitizer will be the scattered signal \(\text{Rx} + \) sky noise + any receiver noise. The \(\text{Rx}\) is the convolution of the \(\text{Tx}\) and \(\text{Pr}\).

This can be expressed (ignoring the \(1/r^2\) effect) as follows:

\[
\text{Tx} \ast \text{Pr} = \text{Rx} + n
\]

where the convolution is represented by \(\ast\). The noise \(n\) is not known and may be variable with height and time, and of course \(\text{Pr}\) is unknown. In order to recover \(\text{Pr}\) it is necessary to map to the frequency domain and deconvolve (by using the complex division) the obtained product of the atmosphere backscatter with the original pulse sent. In conditions where the noise is small enough to be ignored, the result will reproduce the true atmospheric profile.

The deconvolution process is then carried out by a complex division of \(\text{rX}\) (Fourier transform of \(\text{Rx}\)) by \(\text{tX}\), with appropriate considerations (see below) for complex series and low power in the transmitted signal.

\[
\text{pR} = \frac{\text{rX}}{\text{tX}}
\]

Normally the range of frequencies chosen are selected to ensure that nulls in the Fourier transform of the \(\text{tX}\) are avoided. Once the series of values \(\text{pR}\) is known, it is just a matter of going back to the time (distance) domain and obtaining the atmospheric profile \(\text{Pr}\). More details about this process, and the radars and data used in the deconvolution section of this process is available (Hocking et al., 2014).

The Costa Rican radar, as described in Hocking et al. (2014), generated the data used for the deconvolution. During the digitization process noise is also detected along with the desired signal. Avoiding the noise is crucial in order to obtain a good approximation of the true convolution. In Hocking et al. (2014), the process used to avoid noise outside of the band of interest is explained. By using adequate hardware filtering the noise from other frequency bands is not mapped into the final digitized convolution.

It is well known that deconvolution should be avoided in areas where the Fourier transform of the transmitter pulse approaches zero. The small spectral values in those regions result in amplification of noise at that frequency. We went to considerable lengths to avoid such zeros, generating a wide enough transmitter spectrum and avoiding the areas of the spectrum which result in such amplification.

Capon’s method can be used in the same process as a spectral estimator. Being used in the frequency domain, the method will generate time (distance) domain information. The resulting peaks found in the process will represent scattering layer intensity which is also the desired \(\text{pR}\).

3.2.4 Methodology

All the simulations and products to be presented here were obtained with GNU/Linux type systems\(^1\). The programing language used for coding was python 2 with a combination of Bash Shell Scripting to create the appropriate pre/post processing routines. Extensive use of the python libraries numpy, scipy and matplotlib were required.

\(^1\)Ubuntu LTS in workstations and CentOS in SHARCNET’s cluster Orca and visualization stations.
Four different scenarios were used to study the outputs of Capon’s method for different filter shapes: (i) Evenly spaced frequencies. (ii) Randomly distributed frequencies, (iii) Continuous spectra and (iv) Radar deconvolution data.

Simulated data

The foundation of the simulations is a discrete sequence of complex amplitudes created with known spectral content (called “true spectra”). The spacing (0.125 s) and number of entries (1024) allowed for frequencies up to ±4 Hz. The value of 1024 was chosen because it is similar to the number of sweeps regularly used in Costa Rica’s radar. The effective spectral width that results for each spectral line is set by the inverse of the length of the time-series considered. The total length of the series is 128 seconds.

The Fourier method (using the fast Fourier transform algorithm) was compared to Capon’s method estimation in 4 different tests, as described below. It is important to clarify that, in the first two simulations described, the Fourier method was used with a fixed large filter size in order to set a reference. That reference is used to understand the effects in Capon’s method variation of filter size. The order of the autocovariance matrix was equal to the filter’s size. Examples of the effects of varying the filter size are shown in figure 3.2-b and c. In this figure the Fourier result is shown in both panes in order to compare it with Capon’s results for different filter sizes.

We will give a detailed discussion of figures 3.2-b and 3.2-c shortly: but first we need...
3.2. Data Dependent Methods

to give some more information about how these graphs were prepared. We prepared various
types of data and subjected them to both Capon-transforms and Fourier transforms. There were
4 main tests that were performed, and these are itemized below.

**Peak Test 1: Evenly distributed frequencies**

In our first class of tests, a pre-defined number of spectral lines with frequencies of constant
amplitudes were uniformly distributed in the interval between $-3.9$ and $+3.9$ Hz. The number
of frequencies was varied on each new run, being chosen to be anywhere from 1 to 100. The
results were used to compare the number of maxima found by each estimation method.

The shape of the adaptive filter, a map of all filters used and an integrated spectral value was
obtained for each run, where within each run, different values of m were also chosen (Not all
are shown in this document, but sufficient numbers are presented to demonstrate the concepts).

Peak location was performed using a relative maxima search algorithm. The relative max-
ima was found comparing one value in the spectrum to the next consecutive value. In order to
accept a point in the spectra as a peak, at least three points on each side needed to have a smaller
value. The algorithm was tested for many different signals with contents varying from low to
high numbers of frequencies. In each test the algorithm found the true spectra satisfactorily
(while using Fourier method or MVM with a sufficiently large filter).

The time series of length 1024 elements with sampling interval of 0.125 s provided 128 s of
data. With a time series of 128 s a spectral domain with $\approx 7.8 \times 10^{-3}$ Hz resolution is obtained.
The search algorithm just described could resolve more than 150 evenly located peaks in the
specified domain. No more than 100 simultaneous frequencies were used in order to stay away
from the limits of the algorithm.

**Peak Test 2: Randomly distributed frequencies**

In the second class of tests, we used a similar approach to test 1, but this time the locations
of the spectral peaks were randomly chosen. Again, up to 100 peaks were permitted. No
accounting for co-location of frequencies was done. The same algorithm generated the peak
information. One hundred thousand runs with different distributions were used for each filter
size in Capon’s method.

The signal to noise ratio can be used to better understand the spectra. The signal to noise
ratio for low spectral content (10 different peaks) was in the vicinity of $43dB$ for Fourier method
and $68dB$ for MVM. When the number of frequencies was increased the ratio decreased but
was kept large enough for values to be considered appropriate. When including 80 different
frequencies, the SNR for Fourier theory was approximately $19dB$ and $26dB$ for MVM. An
adequate separation between noise level and signals was observed for all the simulated cases.

**Test 3: Continuous spectra fitting**

In order to generate similar spectra to those obtained regularly in radar analysis a perfect com-
plex Gaussian spectra was first generated, and then random phases were given to each of the
elements of this spectra.
This spectrum was inverse Fourier transformed and then 10% of the resultant time series removed from one end. This gave a time series with known spectral characteristics, but the resultant spectrum had realistic variation in spectral density as a function of frequency. This new time-series formed the basis of subsequent tests. It was either Fourier-transformed, or had various types of Capon filters applied to it. The resultant spectrum, whether determined by Fourier or Minimum variance methods, will no longer be a smooth Gaussian, but will show considerable variability in spectral density between adjacent frequencies, thereby simulating a realistic spectrum. This variability in spectral amplitude is not noise, but truly represents the spectrum corresponding to the time-series. Changing the distribution of applied phases alters the time series and the final spectrum. This procedure was similar to the procedure used by Eckermann and Hocking (1989). Examples will be seen shortly.

Again one hundred thousand different sets of phases were used to generate suitable variability for the experiment. The spectral estimation of the frequency content was obtained by both the Fourier Transform and Capon’s Method. The resultant spectra were fitted with a Gaussian function in order to obtain parameters that could be compared and analyzed.

### Test 4: Radar data

We also performed comparison of Fourier transform and Capon’s method using real data. The atmospheric information was obtained from a boundary layer / lower troposphere radar in Costa Rica. The radar is described in appendix B and is described also by Hocking et al. (2014). Using a 4 MHz bandwidth, the recovery of the atmospheric profile was carried out in two different ways. Both ways utilize complex division (deconvolution in frequency domain) of the \( r_X \) by the \( t_X \) spectra.

After this division, the height profile is obtain by inverse Fourier transformation of this data. Because of the duality property this can be done using a Fourier transform in the forward direction. Similarly this time (height) domain content can be estimated using Capon’s method.

The first procedure used the Fourier transform to obtain the height profile. The second consisted of using 4 different filter sizes with Capon’s method to obtain similarly a height profile. In total five different profiles were obtained for this comparison.

### 3.2.5 Results

As previously mentioned, Fourier methods set a reference with which to compare the effect of filter size variation on Capon’s estimations. Figure 3.2(b) shows both methods, and each perfectly resolves the true spectral content. In the lower pane, with a smaller filter size, Capon’s method underestimated the number of peaks and misplaced the maxima. In other words, under conditions where the signal has equal spectral content at discrete frequencies, Capon’s method will improve peak estimation when a sufficiently large filter size is used, but fails if the filter size is too small.

**Discrete spectra**

Figure 3.3 presents the results for the peak resolving capabilities under the random and evenly distributed cases. In the left pane, the region R1 shows one specific spectral analysis where 87
3.2. Data Dependent Methods

Figure 3.4: Frequency response map for 4 (top-left), 10 (top-right), 14 (bottom-left) and 18 (bottom-right) different dominant input frequencies, using a filter size of $m = 16$. Values of the chosen frequency $f_c$ are shown in the ordinate, and the resulting spectrum $H$ is given by the horizontal traverse at that frequency. The integrated power across the bandwidth of every filter is shown next to each map.

true spectral peaks were used. In this case Capon’s method was used with a filter size of 92 (128 produced the same result) and both methods correctly found the 87 peaks. In the same figure, the region R2 contains an example with 60 true frequency maxima but where Capon’s method uses $m = 16$. Application of the Fourier method resolved 60 peaks placed correctly, while Capon’s method, when used with a filter size of 16, only resolved 15 peaks.

Comparing the peaks resolved by FT and Capon’s method, the value of $m$ chosen can be seen to act as an upper limit to the maximum number of peaks that the method can resolve.

When using random frequency location, the peaks resolved are less in number than those found when using regularly spaced frequency lines for Capon’s method. For $m=16$, up to 15 different peaks were found. For $m = 64$ no more than 48 peaks were found. For $m = 128$ no more than 75 peaks could be isolated. The reason for this behaviour can be clarified by looking at the filter maps created by the method to estimate the spectral content.

The complete map of the $H$ filters for $m = 16$, for four different numbers of frequencies, is presented in figure 3.4. In these maps the adaptability of the filters can be seen in the way the
horizontal transverse across the map changes as $f_c$ changes on the ordinate. When 4 frequencies are used, four clear horizontal bands will appear. Those are evident in the integrated filter bandwidth ($\int H(\Omega)d\Omega$, IFB from now on) for every filter in each map.

Capon’s method will generate large peaks (close to 5 times larger than unity) in the $H$ shape when there are a small number of components. When more frequencies are added, it will begin to reduce those large maxima. When the number of frequencies exceeds the filter size it will have a maximum value of 1 in the diagonal, with smaller side-lobes.

Looking at the IFB value (right side of maps in figure 3.4) of each filter, it is clear that after the number of frequencies exceed the filter size ($m$), this method loses its adaptability. The IFB approaches a constant line in the case of a signal containing 18 frequencies, and by looking at the frequency map it is evident that the shape of the filter (not only the IFB) is constant in almost the entire range.

It is important to note that even though the method is clearly not adaptive past this point, it will still resolve the maximum number of frequencies allowed by the degree of freedom. The results of the method could generate peaks that are not necessarily co-located with the true frequencies (e.g. Figure 3.2).

**Gaussian spectra**

The original spectrum, before truncation of the time series, is shown in figure 3.5(a). The standard deviation, sigma, and the mean, are indicated in the figure.

Figure 3.5(b) shows the spectra after the truncation process discussed earlier. Note that the spectral lines change power density significantly between adjacent frequencies. This is not an artifact, nor is it noise. These spectra are the real spectra corresponding to the chosen time series. It will be seen that both the FFT and Capon’s method select basically the same lines as the dominant peaks - the form of the red and black curves are quite similar. However, it can also be seen that Capon’s method makes the larger peaks even larger than the FFT method, and suppresses somewhat the weaker lines (towards the edge of the Gaussian). This is the nature of Capon’s method - to amplify the effect of the stronger spectral lines at the expense of the weaker ones.

In our subsequent tests, we form many such transformations, using both FFT and Capon’s methods, with different phase-sets, perform fits to obtain estimates for $\mu$ and $\sigma$, and then produce histograms of the distribution of these parameters, which are shown in fig 3.5(c) and 3.5(d).

**Centre frequency.**

Histograms of the position of the peak of the best-fit Gaussian are shown in figure 3.5(c) for all analysis methods applied. For a filter size equal to 8 the probability of getting a 0 difference relative to the true value is higher with Capon’s method than Fourier transform. As the value of $m$ increase (16, 32 and 64) this probability will decrease consistently, and the Fourier transform obtains higher probabilities than all those filter sizes.

The probability of finding the centre of the fit away from the true centre increases with increasing filter size of Capon’s method. This probability distribution will have an effect when used to analyze wind data; Capon’s method with a large filter size will have a broader range of velocities where the maximum value can be located than the Fourier method.
3.2. Data Dependent Methods

Figure 3.5: a) Four parameters used to fit the results of the spectral estimation. b) Example of the Fourier (black) and Capon’s method (red) results, along with the fitted Gaussian function. The results from Capon have been rescaled (to satisfy Parseval’s equality) in order for the two functions to have comparable characteristics. This rescaling is just for visualization and was not used when analyzing the Gaussian fitting parameters or the spectrum. Note that the Capon spectrum (red) has been filled with a yellow fill, in order to make it more visible. c) Probability density function for the centre frequency variable determined from one hundred thousand runs. d) Probability density function for the spectral width variable $\sigma$, calculated from one hundred thousand runs.

**Width ($\sigma$).** This value is directly related to the full width at half maximum (FWHM). Histograms of $\sigma$ are shown in figure 3.5(d). The true value for the original function is $\sigma = 1.5\text{Hz}$. The Fourier method fit located the most probable point close to that location. All Capon’s maxima lie at least 0.3 Hz lower and get further away with increasing filter size.

In several radar applications the width of continuous spectra (similar to the Gaussian function) are used to obtain extra information. Capon’s method will always underestimate this width.

The narrowing of the spectral width is not a surprise, in view of the tendency of Capon’s method to amplify the larger peaks (in this case, the more central ones) at the expense of the weaker one (in this case, the ones at the edges).
Deconvolution data

This section contains the results of the deconvolution process, in contrast to spectral analysis of simulated data as shown in previous sections. No spectral estimation was carried out in the spatial domain. The radar signal is obtained in terms of time lag and converted to the equivalent distance only for visualization purposes.

The results obtained from the deconvolution are presented in figure 3.6. The same result of the Fourier transform is present in the four panes. The filters used for Capon’s method vary in size from the same length as the data series (N) to 16. It is clear from the figure that the largest filter (Figure 3.6(a)) generates an output that resembles the Fourier output. However, even in this case, due to the minimization process the Capon profile presents reduced details at some heights (e.g. 1.6 and 2.5 km).

As the filter size is reduced, the number of maxima that can be found will be reduced (as found in previous sections). This is clear in figure 3.6(b). The peaks found using Capon’s method are much clearer, with reduced side lobes. However, this is a deceptive interpretation because Capon’s method only allows a limited number of degrees of freedom, meaning it will sacrifice the smaller peaks in favour of larger ones. What this really means is that the high resolution details found by the deconvolution process are lost by the reduced filter output. The small details are lost in order to maximize the large peaks.

To make it clear, the filter was reduced even more to 32 (c) and 16 (d) in the bottom two panes of figure 3.6. Now the large peaks are the only ones present in the output, and the high resolution details of the deconvolution are lost.

3.2.6 Discussion and conclusions

The accuracy of Capon’s method is defined by the filter size m, and this parameter must be carefully chosen. It must exceed the number of true dominant peaks in the spectrum (often not known in advance). The selected Capon’s method filter size will limit the maximum number of peaks that can be resolved. In the case of evenly spaced frequencies, the number of resolved frequencies will be close to the value of the filter. If these frequencies are distributed randomly it will be much lower (60% for $m = 128$) than the filter size.

The reason for this behaviour is clear in the loss of adaptability observed in the filter maps (Figure 3.4). When more frequencies than degrees of freedom are passed through the filter, Capon’s method will try to generate the maximum number of peaks possible for its filter size. The integrated filter bandwidth will tend to be constant when the number of frequencies exceed the filter size, and the filter shape will be similar for all choices of the central frequency. This would imply that Capon’s method filters lose their adaptability past that point.

For a known continuous spectra, Capon’s method will always underestimate the width and amplitude of the true spectra. The centre of the Gaussian fit will be similar for Fourier and Capon’s method if the filter size is small. As the filter approaches the largest possible value the probability of Capon’s method departing from the true centre value will increase.

Many studies and scientific procedures/tools use Capon’s method spectral estimation as a way to find clear, sharp peaks, claiming that the side-lobes are irrelevant and no positive information can come out of them. In the deconvolution process this is particularly misleading. The “side-lobes” may not always actually be side-lobes, and may contain information about...
the atmosphere, and how the transmitted pulse interacted with a particular volume.

It needs to be noted here that these comments do not negate the application of Capon’s method for its original purpose. Minimum variance methods are powerful for identifying individual spectral lines with high accuracy, and resolving closely spaced peaks. However, the onus is on the user to satisfy themselves that there are indeed only one or two lines. Capon’s method will amplify the larger peaks at the expense of the weaker ones, and if the data truly does contain a mixture of strong and weaker signal -peaks - or if there is significant signal embedded in the sidelobes - then Capon’s method will destroy that information. User beware!

Other concerns exist regarding the use of Capon’s method in radar tools like Range Imaging. Using this spectral estimation method generates images of the atmosphere with a high number of thin layers (Palmer et al., 2001), but the reality of these still needs to be verified independently. As we found, Capon’s method can generate peak information that is misplaced
without warning in the process, especially when used with a small filter size. In the case of RIM, usually the number of frequencies used is four.

In the deconvolution process the high resolution is evident in the details obtained in the vertical profile. With Capon’s method, those details are observable only when an exceedingly large filter (equal to the data series length) is used. This result raises concern about the use of Capon’s method in estimating information for those procedures where the number of peaks is particularly important (e.g. Interferometry).

One limiting factor when using Capon’s method to analyze large data sets is the computing power. When analyzing 24 hours of radar data, and more than \(10^6\) deconvolution results, Capon’s method required more than 13 hours to complete the calculations while Fourier method required less than 2 hours for the same computation. Unless massive computer capacity is available, it would be challenging to incorporate this method in real time analysis and product generation.

Pending tests would include the resolution of multiple Gaussian spectral shapes (e.g. rain + atmospheric signals).

### 3.3 Spectral Applications: Convolution

#### Theory

The theory behind the convolution/deconvolution process comes from the Fourier Theory (will be shown shortly) and can easily be confused with the running mean process (because they are identical for symmetric functions). But the properties of both processes are quite different and care must be taken when using them.

The basic concept of the convolution is that two functions are slid over each other and the integral of the product is found as a function of lag. For each step they are multiplied, integrated and the resultant number located at the point where the sliding function is located. The relationship of this process with radar theory can be understood better if analyzed under the light of an example.

The transmitter antennas emit an electromagnetic signal (function 1) that will travel (slide) through the atmosphere. Every time the signal reaches a point where a sufficiently large change in the index of refraction occurs (function 2), part of the signal will be send back to the receivers. The reflected signal will always be orders of magnitude smaller than the incident signal, which is the reason for the Born approximation (see Chapter 4).

What is more important, at least for atmospheric radar data, than the convolution is the deconvolution process. Instead of going in the forward process of convolving two functions, we are more interested in recovering one of the functions by deconvolving with one known original function.

Before presenting a radar example the proof of the convolution theorem is presented. More information about the convolution theorem and others can be found on Bracewell (1978). If we have two functions \(f(t)\) and \(g(t)\), and calculate the convolution in the time domain, the results of this operation would be the Fourier transform of multiplying the spectra \(F(f)\) by \(G(f)\) in the frequency domain. The mathematical description of the convolution is included in appendix E.

In the case of the atmosphere, as mentioned before, the two functions can be represented
3.3. Spectral Applications: Convolution

by the transmitted pulse $T(t)$ and the atmospheric profile $P(t)$. It is important to clarify that the
time in $Pr(t)$ is the time taken to reach a certain height ($r$) and come back to the radar (time
lag), so time and space are related by $t = 2r/c_o$, where $c_o$ is the speed of light. More about
the Fourier transforms, convolution theorem and their applications can be found in Champeney
(1973) and Bracewell (1978). The true atmospheric profile should be a function of height and
time, if we look in two dimensions simultaneously. This is what the radar does and will be
presented on Chapter 6.

Application

It is not new that the received signal is the convolution of the radar pulse and the atmospheric
profile (Hocking and Röttger, 1983). On the other hand, the successful implementation of the
deconvolution in operational radar applications has been carried out just recently (Hocking
et al., 2014). The usage of the convolution operation in radar simulations can not be found in
the literature as time domain alternatives are commonly used to obtain the received signal. In
this section the basics of the deconvolution procedure are exemplified; similar processing is
used in the operational radar in Costa Rica.

The application of the convolution in the radar process becomes evident when the resultant
of the mathematical operation is assigned to the receiver variable. During the time the radar
signal is travelling upwards, it is being convolved with the atmospheric profile in its path.
The resultant returns of the atmosphere are recorded by the radar at the receiver antennas and
digitized to be processed. Those returns, the convolution, will be represented here as $Rx(t)$
because it is measured at the receivers.

The convolution equation in the time (distance for the profile) domain is

$$Rx(t) = T_x(t) * Pr(t)$$  \hspace{1cm} (3.12)

When moved to the frequency domain ($Rx(t) \rightarrow rX(f), T_x(t) \rightarrow tX(f), Pr(t) \rightarrow pR(f)$), it
simplifies to

$$rX(f) = tX(f)pR(f)$$ \hspace{1cm} (3.13)

In real applications, the receiver antennas will also record noise and there will always be
some noise associated with the receiver hardware as well. If this noise is represented by $\epsilon$, the
spectrum of the atmospheric profile can be found

$$pR(f) = \frac{rX(f) + \epsilon}{tX(f)}$$ \hspace{1cm} (3.14)

This profile measured over several minutes will provide the necessary phase change infor-
mation per height to obtain information about the scatterers in the air. It can also be used to
gather wind information (see Chapter 3) and to apply other techniques like interferometry (see
next section).
Examples

These examples use the convolution/deconvolution process to illustrate the use in regular radar applications. Usually one of the functions (usually \( g(t) \)) will be just a fraction of the length of the other function. In radar it is not rare to have pulse lengths in the order of a few microseconds. On the other hand, the frequency of repetition of the pulse is much larger, allowing us to record the convolution of the signal sent for periods in the order of hundreds of microseconds or more. This will generate a short pulse being convoluted with a longer function.

In this example (Figures 3.7 and 3.8) the large function \( b \) is 50 times the length of the small function. Using a \( g(t) \) function of 5\( \mu s \) and a \( f(t) \) of 250\( \mu s \) the convolution should be expected to measure approximately 255\( \mu s \). In discrete series if \( g[t] \) and \( f[t] \) are \( N \) and \( M \) entries long the convolution will be \( N + M - 1 \) entries long. Shorter convolution results can be used to recover the original function but a distortion towards the edges will occur as noted in figure 3.7.

The perfect deconvolution can be obtained but it is necessary that the full length (\( N + M - 1 \)) time series is known. This case is presented in figure 3.8 where the extra time recorded is indicated by a red horizontal line at the end of \( c \). This case is theoretically possible but unrealistic for implementation in real applications as not only atmospheric events are recorded in the signals. Effects known as ground echoes, ringing of transmitted signal in the cables and mutual coupling effects of the antennas will be recorded.

![Figure 3.7](image)

Figure 3.7: a) \( g(t) \) function with a length of 5\( \mu s \). b) \( f(t) \) function with a length of 250\( \mu s \). c) Convolution result using all possible combinations of \( \tau \). d) Deconvolution result showing amplitude in black, phase in blue and difference between the original function \( f(t) \) and the deconvolution result in red.
3.4 Spectral Applications: Interferometry

Radar back-scatter can not only be generated by point-like objects. In astronomy, meteor physics or military applications many of the echoes observed are generated by point-like scatterers. In atmospheric physics a fraction of the echoes will be generated by volume scatterers. Some other scatterers such as fine layers can also be observed. The volume scatterers become particularly important when long pulses are used, which is not an uncommon practice in experimentation.

In some radar techniques used to process the information, no assumption is made about the nature of the scatterers. When Fourier transforming the radar time series, the spectra contains all the information that the radar detected including layers, volume scattering, meteor echoes, galactic sources, anthropogenic emissions and noise. In other techniques (e.g. Interferometry) the nature of the analysis assumes point-like scatterers. Whether the point-like structure found
by the method is representative of a real structure is still of considerable debate.

Before introducing the method of interferometry as applied in radar, let’s describe the foundations of triangulation and two dimensional location of point-like objects. These methods will prove to be great aid in understanding location and comprehending the great benefit of moving into frequency domain to analyze the data.

3.4.1 Triangulation

The triangulation method has been around since the 1600s. Willebrord Snell derived the method in order to measure the distance to an object based on known geometrical properties.

![Schematic diagram for triangulation of the distance to an object](image)

Figure 3.9: Schematic diagram for triangulation of the distance to an object. This method is purely based on geometrical characteristics of the observers and target.

With known location of the points $\vec{A}$, $\vec{B}$, and $\vec{O}$, the value of $\vec{P}$ can be calculated. It can be observed in figure 3.9, that the following equations are obtained from the diagram

\[
\vec{A} + \vec{C} = \vec{P} \\
\vec{B} + \vec{D} = \vec{P}
\]

(3.15)

The components of both equations should be equal
3.4. Spectral Applications: Interferometry

\[
\begin{aligned}
(A + C) \cdot \hat{i} &= \vec{P} \cdot \hat{i} = 0 \\
(B + D) \cdot \hat{i} &= \vec{P} \cdot \hat{i} = 0 \\
(A + C) \cdot \hat{j} &= \vec{P} \cdot \hat{j} = P \\
(B + D) \cdot \hat{j} &= \vec{P} \cdot \hat{j} = P
\end{aligned}
\]

and with the considerations that $\vec{A} \cdot \hat{j} = 0$, $\vec{B} \cdot \hat{j} = 0$, and $\vec{P} \cdot \hat{i} = 0$ along with the distribution property of the scalar product the equations simplify to

\[
\begin{aligned}
-A + C \cos \alpha &= 0 \\
-A + C \cos \beta &= 0 \\
C &= P/ \sin \alpha = 0 \\
D &= P/ \sin \beta = 0
\end{aligned}
\]

where $A$ is the magnitude of the vector $\vec{A}$ and similarly for $B, C, D,$ and $P$. By combining all these equations a single equation with just $P$ as unknown is obtained

\[
P = \frac{A + B}{\text{Separation}L} \frac{\sin \alpha \sin \beta}{\sin(\alpha + \beta)}
\]  

Solving for $P$ the final equation of triangulation is

\[
P = \frac{A + B}{\text{Separation}L} \frac{\sin \alpha \sin \beta}{\sin(\alpha + \beta)}
\]

The application of this formula is simple. It consists of just measuring the value $L$, $\alpha$, and $\beta$ and then the value of $P$ can be estimated. One of the applications of triangulation is in topographic measurements. When locating visible objects the reflected light in the direction of the observers at $\vec{A}$, $\vec{B}$, and $\vec{O}$ will determine when the object can be identified. The persistence of the incident light will be the main concern in this method.

In ancient warfare, calculating the distance to a ship would entirely depend on the ship reflecting or emitting light, and the observers being able to detect it from different locations. That concept is not far off from the radar back-scatter treatment today. One big difference between the ship and atmospheric back-scatter example is that a ship can’t disappear (unless sunk) and in radar applications the scatterers usually have life spans in the order of seconds. The short life expectancy of the scatterers require fast detection and analysis tools that are better obtained when observing them in a spectrum, as will be shown shortly.

3.4.2 Two dimensional interferometry for general signals

In triangulation the geometry of the parts is used to obtain the range of a target. It was implicit in the method that the speed of light is fast enough that the object looks identical to all the observers and no changes in the amount or type of light can be obtained. In interferometry, the speed of the signal travelling in the medium is key to gathering information. The time difference from the signal coming from the target to the each of the detectors can be used to obtain information about the location.
A diagram of two dimensional interferometry is presented in figure 3.10. In this diagram \( \vec{r} \) represents the direction from where the original signal is approaching the detectors located at \( \vec{A} \) and \( \vec{B} \). The signal is assumed to travel in a straight line parallel to \( \vec{r} \) and the wavefront perpendicular to \( \vec{r} \). The parallel direction of the signal implies that it is not possible to locate the range of the scatterer, only the direction of origin. The value of \( \alpha \) is now our main interest. If multiple systems like the one described here (i.e. another pair of \( \vec{A}' \) and \( \vec{B}' \)) are used simultaneously the range can be accurately determined.

![Diagram of two dimensional interferometry](image)

**Figure 3.10:** Schematic diagram for two dimensional spatial interferometry. This method uses the detectors to observe specific events and then compares the time of arrival to calculate a relative path difference.

For a single pair of receivers \( \vec{A} \) and \( \vec{B} \) the path length can be calculated as the magnitude of the vector

\[
\text{Path difference} = \| (\vec{A} \cdot \hat{r}) \hat{r} - (\vec{B} \cdot \hat{r}) \hat{r} \| = \| \vec{A} - \vec{B} \| \cos \alpha
\]

(3.18)

that is composed of the projection of \( \vec{A} \) and \( \vec{B} \) to the direction of origin. If the path difference equals the distance between both detectors then the source is in the same line that contains them. If the path difference is zero the source of the signal is located perpendicular to a line containing the detectors.

The path difference can also be calculated from the time difference of the signals recorded. This time is usually called the time lag and can be found using the cross-correlation of the two signals. The cross correlation of two functions \( f \) and \( g \) is defined as

\[
h(t) = f \star g = \int_{-\infty}^{\infty} f^*(\tau)g(t + \tau)d\tau
\]

(3.19)

where \( f^* \) denotes the complex conjugate of \( f \) and the lag is represented by \( \tau \). If the signal consisted of only one known function \( f \) but observed at different moments \( t_a, t_b \) in each receiver then \( h(t) \) will be representing the sliding of a signal across the other signal and integrating in
each sliding step. This process can be better understood in term of the Fourier transform of the cross correlation and by using the shift theorem.

The Fourier transform of the cross-correlation is obtained by

\[
\mathcal{F}\{f \ast g\} = \mathcal{F}\{f\}^* \mathcal{F}\{g\}
\]

(3.20)

and the shift theorem can be presented as

\[
\mathcal{F}\{f(t - t_a)\} = e^{-it_a\omega} \mathcal{F}\{f(t)\}
\]

(3.21)

The application of the shift theorem is crucial in the correct interpretation of the product. The displacement \(t_a\) in the time domain, will be represented as a phase change \(t_a\omega\) in the frequency domain. Combining both of the previous properties of the Fourier transform and the property of complex numbers \(\{z_1z_2\}^* = (z_1)^*z_2\) the Fourier transform of the cross-correlation between \(f(t - t_a)\) and \(g(t - t_b)\) is

\[
\mathcal{F}\{f(t - t_a) \ast g(t - t_b)\} = e^{it_a\omega} \mathcal{F}\{f(t)\}^* e^{-it_b\omega} \mathcal{F}\{g(t)\}
\]

(3.22)

\[
= e^{-i(t_b - t_a)\omega} \mathcal{F}\{f(t)\}^* \mathcal{F}\{g(t)\}
\]

(3.23)
Note that if \( f = g \) then the case is reduced to the auto-correlation and the power spectrum \( \| \mathcal{F}\{f\} \|^2 \) is obtained. The term \( (t_b - t_a)\omega \) can be rewritten as \( 2\pi(t_b - t_a)f \). The slope \( m \) of the phase change with frequency around zero frequency can be calculated as

\[
 m = \frac{\Delta \phi}{\Delta f} = -2\pi(t_b - t_a)
\]  

(3.24)

Finally, the time difference between two signals can be obtained from the data by dividing the slope \( m \) by \( 2\pi \). If the known speed of the signal in the medium is \( v \), then the distance travelled in a time \( \Delta t = t_b - t_a \) is just \( d = c\Delta t \). Figure 3.11 shows an example of the procedure. Signals \( A \) and \( B \) are shown in the upper pane. The maximum of signal \( A \) was recorded at time \( t_a \), and \( B \)'s at \( t_b \). When the spectrum of both signals is plotted (centre pane of figure 3.11) the shape of the spectra is equal; this is a consequence of the shift theorem. The difference in the spectra is contained in the phase. The lower pane of figure 3.11 contains the unwrapped phase for Signal \( A \) (black), Signal \( B \) (red) and the Cross-Spectrum of \( A \) and \( B \) (blue). As shown before, the slope of the phase change with frequency is determined by \( -2\pi(t_b - t_a) \). In the example shown in figure 3.11 the value of the slope was calculated to be \( -2.2 \times 10^{-8} \text{radHz}^{-1} \), leaving the value of \( t_b - t_a = -3.5 \times 10^{-9} \text{s} \).

By using the known path difference and the geometry of the receivers the angle of the source can be obtained from

\[
d = (\vec{A} - \vec{B}) \cdot \hat{r} = \left\| (\vec{A} - \vec{B}) \right\| \cos \alpha
\]  

(3.25)

\[
\alpha = \cos^{-1} \left( \frac{d}{\left\| (\vec{A} - \vec{B}) \right\|} \right)
\]  

(3.26)

In the example shown in figure 3.11 the value of \( \alpha \) is 1.928 radians or 110.47 degrees from the \( x^+ \) direction (20.47 degrees CCW of the vertical). Once the value of \( \alpha \) is found the direction of incidence of the signal is known. It can be easily be seen that with two interferometers two different directions can be used. This reduces the case to the triangulation problem and with decreased number of degrees of freedom an estimation in the range can be obtained.

In the next section the generalization of interferometry to three dimensions take place. The range can also be resolved with pulsed radars surpassing the necessity of two systems. Radar data are used in the following section to illustrate examples of interferometry.

### 3.4.3 Radar Interferometry

The triangulation and time domain interferometry cases just presented are useful mostly in simplified scenarios. Multiple emitters can generate (or reflect) complicated signals (usually oscillatory) associated with hard-to-solve time domain problems. Moving to the frequency domain can alleviate many of the complications.

In the frequency domain, different frequencies and phases can be found. This makes it easy to identify multiple objects in the sky by using the spectra of multiple receivers (assuming the scatterers are point sources). In order to determine the precision of the phases to be obtained, it is necessary to clarify the relationship between cross-correlation and cross-spectrum.
Interferometry is a technique that is widely used because it is capable of locating objects in generalized spaces, based only on information that you can gather from the objects and a detailed description of the equipment used to detect it.

Suppose that an object is emitting a signal at the position $\vec{r}$ and that you are able to detect that signal at different positions named $\vec{R}_n$. If the detectors are located in the same plane, you can need as few as 3 receivers. A radar system with three receivers used for interferometry was built in Costa Rica as described in appendix B. The receivers of this radar contains three groups of four antennas as shown in figure 3.13; the value of $\vec{R}_n$ is the centre of coordinates of each group of antennas. Results of this receiver array will be shown in Chapter 6. The equations for the origin of the signal is based on the range $D$ to the object. The variable $d_n$ is the distance from the origin to the centre of coordinates of receiver $n$. These equations for the location of the object and the receivers are

$$\vec{r} = D \left[ \sin(\theta)\cos(\varphi) \hat{i} + \sin(\theta)\sin(\varphi) \hat{j} + \cos(\theta) \hat{k} \right]$$

(3.27)

$$\vec{R}_n = d_n \left[ \sin(\theta_n)\cos(\varphi_n) \hat{i} + \sin(\theta_n)\sin(\varphi_n) \hat{j} + \cos(\theta_n) \hat{k} \right].$$

(3.28)

Using these two equations the phase in the receiver $n$ can be calculated using the scalar
Figure 3.13: Costa Rica’s receiver antennas diagram. Three groups of four antennas are used for detection. The receivers are named 1, 2, and 3. The first group is located towards the south of the centre of coordinates. Receiver number 2 is located towards the north-west. The last receiver can be found towards the north-east of the radar’s detector coordinates.

It is of crucial importance to know the phase difference in the receivers. Using 3 different receivers two equations can be readily obtained.

$$\Delta \phi_{nm} = \frac{2\pi}{\lambda} \left[ (\vec{R}_m - \vec{R}_n) \cdot \hat{r} \right]$$  \hspace{1cm} (3.30)

$$\Delta \phi_{nl} = \frac{2\pi}{\lambda} \left[ (\vec{R}_l - \vec{R}_n) \cdot \hat{r} \right]$$  \hspace{1cm} (3.31)

Rearranging these expressions into a matrix representation can clarify that we have at this point a linear system of 2 equations to be solved.
3.4. Spectral Applications: Interferometry

\[
\begin{pmatrix}
\Delta \phi_{nm} \\
\Delta \phi_{nl}
\end{pmatrix} = \frac{2\pi}{\lambda} \left( \frac{\vec{R}_m - \vec{R}_n}{\vec{R}_l - \vec{R}_n} \right) \hat{r}
\]

The left side is a vector \((\Delta \phi)\) whose entries are the phase difference between different pairs of receivers. In the right side the multiplication of the receiver position difference \((R)\) and the column vector \((r)\). In the case with all the receivers in the plane \(x-y\) the \(z\) component of the vectors in \(R\) is just 0, making this a set of 2 equations to find 2 unknowns \((\theta, \varphi)\).

\[
\hat{r} = (\bar{R})^{-1} (\bar{\Delta \phi})
\]

At this point the values of \(\theta\) and \(\varphi\) can be obtained from the entries of \(r\).

\[
\theta = \arcsin \left( r_x^2 + r_y^2 \right)^{\frac{1}{2}}
\]

\[
\varphi = \arctan \left( \frac{r_y}{r_x} \right)
\]

The value of \(D\) will be obtained from the range gates or the radar of in the case of deconvolution from the respective height.

It may be confusing to the reader why the Interferometry procedure is treated inside the Spectral Analysis chapter. The reason is that in the case of point scatterers, like air planes, it could be simple to get the phase (and consequently the phase difference) directly from the time signal. In the case of MST radars, the signal is not a simple because of the nature of the scatterers. In this case the phase difference can be calculated (look into equation 3.23) from the multiplication of the Fourier transform of signals recorded with different receivers. In the time domain this will be equivalent to find the maximum agreement between the two received signals, with the advantage that after the multiplication of the spectra, the phase difference will be readily available from the in-phase and quadrature components.

**Application**

Several methods exist with atmospheric radars at VHF to find the mean winds. Beam steering (BS) and spaced antenna (SA) are examples of those methods. The space antenna method can be applied in the time domain (full correlation analysis, FCA) or the frequency domain (imaging Doppler interferometry, IDI). In the BS method, the radar energy is focused in two perpendicular tilted beams (e.g. North - East). In the FCA method, the cross correlation of the data sets is used to calculate the maximum between (usually) many receivers and find out how this maximum moves with time in the ground (basically is looking at the projection in the ground of the interference pattern generated by the scatterers and how it moves). The interferometry method will calculate the radial velocity for all the scatterers detected and calculate the mean value. A main concern would be the spatial distribution of those objects, but the method
Figure 3.14: Interferometry products of radial velocity. The results of interferometry in a three dimensional volume of scatterers. In this figure the projection on the three different planes of the coordinates is shown. It is common to see half the projection of scatterers moving radially towards the radar (positive values) and the other half away from the radar (negative values) as a consequence of the mean winds.

generates that distribution, so high level filters can be designed to cover different regions. The interferometry method also assumes that the source of the signals are point scatterers.

What makes interferometry different is that only this method can locate in 3D spaces the scatterers in the sky above the radar. This tool provides a direct look at the structures that are regularly used to detect winds but continue to be a general unknown to the community.

One concise example is the location of the scatterers during a convective storm. The ascendent currents will transport energy from the warm surface and generate different types of motion inside the cloud. That energy will be divided in several ways. Part of it will be released as sensible heat, some other part will be used for motion and start dissipation in turbulent processes. But the ascendent current will also generate, by the continuity law of motion, descendent currents, that will also carry momentum, and it will be dissipated as well, into other forms of energy.

According to the theory of energy cascading down from the inertial range to the viscous range, that energy will generate turbulence at some stage that will be the correct size in order to be visible to the radar. The intensity of the echoes generated by these motions will provide the radar with the necessary information to locate the returns according to the phase in the different receivers.

The introduction of energy into atmospheric motions may seem to indicate that the regions
of greater dissipation will generate the greater return to the radar. Only by implementing methods that allow precise location of the return signals this thesis can be confirmed or denied, because it will provide at the same time information about the speed of the wind that originated the return, as well as the necessary phase information to locate the scatterers.

### Examples

Figures 3.15 and 3.16 show the results of the interferometry process applied to two different times (spaced apart by 2.5 hours). Both figures present in panes a), b) and c) histograms for the X, Y, and Z coordinates from the interferometry procedure. Pane d) in both figures represents the radial velocities obtained from interferometry. In figures 3.15 and 3.16, panes e), f) and g) the spatial distribution of the scatterers is included.

Figure 3.15 corresponds to data captured on May 15th, 2014 at 08:47:33 (UTC). The data observed in figure 3.15 presents more scatterers than those observed in figure 3.16. The data shown in figure 3.16 was recorded on May 15th, 2014 at 11:17:37 (UCT), just two and a half hours after the previous example. For this reason the data measured at 8:47 will be refered to as “active case”, and the 11:17 as “quiet case”.

The probabilities observed for the active case are very similar in the X and Y coordinates. The Z coordinate of the quiet case shows a strong decrease of probability below 2 km.

Both cases showed similar distributions in the radial velocity information. The largest difference observed is in the actual location of the scatterers (panes e to g). The active case contains a large number of scatterers observed in figure 3.15-e,f,g). The vertical slices formed depicted by panes e) and f) reproduce the shape of the radar’s beam. Similarly the top view presented in pane g) follows approximately a diamond shape that represents the centre of the half power full width (for more information see Chapter 4). Figure 3.16-e,f,g) does not show enough information to recreate the beam’s shape, but the vertical view observed in the last pane does contain a localized centre indicating maximum returns from overhead the transmitter.

### 3.5 Spectral Applications: Receiver phase and maximum overhead returns

As shown in section 3.4, in the interferometry equations the terms $\Delta \phi$ determine the relative position of the objects. There is another phase term in interferometric radar work that is crucial in determination of atmospheric information. The receiver phase, and more importantly, the intrinsic phase difference between every pair of receivers.

In this and the following section, two methods to calculate the receiver phase are introduced. Knowledge of the receiver phases is critical in calibrating a radar, and no real position information can be obtained unless they are accurately measured and used in every calculation.

The receiver phase is inherent to the hardware used to build a receiver. It will change with different parts and components, and needs to be measured after any maintenance or repair. These can be tedious and time consuming especially if a large number of receivers is used. Other factors like temperature can alter the phase of the receivers and the following method provides a tool to get a better look at these dependencies. Using only software tools, manip-
Figure 3.15: Interferometry example. Data from 2014-May-15th, 08:47:33 [140515084733].

Panels a, b, and c) Histograms of the X, Y, and Z components. Pane d) shows the radial velocity information distributes in the vertical range. Panes e, f, and g) show the actual location of the scatterers in planes z-x, z-y, and x-y.

ulation of the datasets can provide an approximation of the receiver phase with only a few reasonable assumptions.

The importance of the receiver phase, from now on represented with the letter $\Phi$, resides in its use when calculating the cross-spectrum or unifying the signals from all the receivers. If the relative phase difference of receiver n and m change, all the points in the sky located by the described interferometry procedure will be displaced accordingly to the phase change.

This first method is based on the supposition that the strongest return from the atmosphere will be perfectly overhead the receivers. The radar used to obtain the data used in this document is bi-static, and the separation between the receivers and transmitters is just 100 m. To put this in perspective, at a height of 2 km the results of interferometry showed that scatterers will appear as far as 800 m in the zonal coordinate and 1500 m in the meridional coordinate, with a large concentration towards the centre of the transmitter. The assumption of an overhead maximum is not far off from the true concentration of scatterers.

Let's represent the phases of each receiver with $\Phi_1$, $\Phi_2$, and $\Phi_3$. There is no particular reference phase for the receivers, and what really matters is the phase difference among all of them. Under this circumstances we can set $\Phi_1 = 0$, and later study the effect of the varying the
3.5. Spectral Applications: Receiver phase and maximum overhead returns

Figure 3.16: Interferometry example. Data from 2014-May-15th, 11:17:37 [140515111737].
Panels a, b, and c) Histograms of the X, Y, and Z components. Pane d) shows the radial velocity information distributes in the vertical range. Panes e, f, and g) show the actual location of the scatterers in planes z-x, z-y, and x-y. Two hours and thirty minutes later than the case presented in figure 3.15 the atmosphere echoes hold a very different layout.

remaining two phases. This is equivalent to setting the reference phase at the first receiver.

The signal that we need to obtain is the addition of each of the 3 signals digitized by the radar.

\[ S = e^{i\Phi_1}S_1 + e^{i\Phi_2}S_2 + e^{i\Phi_3}S_3 \]  

where \( S_i \) is the complex time series for receiver \( i \). Using this equation the combined signal can be used to estimate the amount of power returned by the atmosphere. With a phase variation from \(-\pi\) to \(\pi\) both remaining phases can be probed, and a maximum (if there is) should appear for the correct combination of values.

It is important to keep in mind that this maximum should be calculated for long periods of time. If not, strong scatterers located off-centre can alter this picture dramatically.
Application

Using one day of radar data the location of the phases can be accomplished. Each file contains 938 sweeps that together generate the velocity spectrum. One day generates approximately 1000 files, meaning that an average of 1000 different spectrum, each with different phase combinations, can be used.

Once the value of the $S$ is calculated, it can be integrated or fitted with different functions in order to calculate the power. The method used in this work was to integrate $S$ in appropriate boundaries [-1 Hz to -0.1 Hz and 0.1 Hz to 1 Hz]. The reader should keep in mind that what is important is the location of the maxima, not the magnitude of the maxima itself. The integration ($P(h)$) of the spectra at one height can be considerably larger than another value calculated 500 m above, but even then both can be located at the same phase combination.

Each height was later normalized before averaging. The three dimensional results are based on the coordinates $\Phi_2$, $\Phi_3$, and $H$. The resolution of $H$ is fixed by the radar height resolution. The fine structure on the horizontal is controlled by the step size taken in the calculations. In this case the phase step used was 0.01$\pi$ equivalent to 1.8°. When normalizing, only one height should be taken into account at a time. The result will be:

$$P(h) = \frac{P(h) - P(h)_{MIN}}{P(h)_{MAX} - P(h)_{MIN}}$$

(3.37)

This will generate levels where 0 means minimum power and 1 means maximum power. This simplifies the location of the phases that generate the best set.

The last step would be to average over the entire day of files.

$$P(h) = \frac{1}{N} \sum_{i=0}^{N(files)} P(h)$$

(3.38)

The final product, will contain a region of strong maxima around the phases that constructively combined the signals. This region will be fairly constant as a function of height. In regions where the atmospheric echoes are not concentrated in a small volume, this can still be deceiving and generate ambiguities. This is specially true in the dataset used because of the wide beam used for transmitting.

Example

The data set used in this example consists of three receiver data. The days May 17th [959 measurements] and 27th [958 measurements], and June 7th [956 measurements] were used to obtain the averages.

With the aid of a computational cluster it can take several hours to iterate over all the possible phase combinations for one single file, and more than 2800 had to be analyzed. Only 200 consecutive heights were used because of the computational cost. From 1.75 km to 3.15 km (spacing of 7 meters) the different spectra were combined in order to obtain a resultant signal from the three receivers.

A three dimensional matrix with coordinates Phase2, Phase3 and Height were obtained from this process. The resultant average location of the optimum phases is presented in Table
3.1. The products of this tests calculated the ideal phases to be \(-0.268\) rad \((-15.35\) degrees\) and \(-0.645\) rad \((-36.95\) degrees\). The values measured at the radar site were \(-0.146\) rad \((-8.37\) degrees\) and \(-0.730\) rad \((-41.86\) degrees\) which are close considering the error in the measurements to be in the order of 8 degrees (an analog oscilloscope was used).

<table>
<thead>
<tr>
<th>Day</th>
<th>Receiver 2 Phase (radians)</th>
<th>Receiver 3 Phase (radians)</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 17th</td>
<td>-0.265</td>
<td>-0.425</td>
</tr>
<tr>
<td>May 27th</td>
<td>-0.156</td>
<td>-0.716</td>
</tr>
<tr>
<td>June 7th</td>
<td>-0.384</td>
<td>-0.796</td>
</tr>
<tr>
<td>AVERAGE</td>
<td>-0.268</td>
<td>-0.645</td>
</tr>
</tbody>
</table>

Table 3.1: Optimum phase combination relative to receiver 1 for each day and the average.

3.6 Interferometric Winds

The interferometry technique has been used in different areas of radar. In MST type radar is known as imaging Doppler interferometry or IDI. It was first introduced by Adams et al. (1986) using a medium frequency (MF) radar operating at 2.66 MHz.

From the interferometry procedure the location of the scatterers is found. Another product of interferometry is the radial velocity of each scatterer. These two properties can be used to obtain an estimation of the mean winds.

Considering the scatterers are fixed during the recording of the time series and only move due to the mean winds, an estimation of the radial velocity can be obtained. A diagram exemplifying the methodology is presented in figure 3.18. For a single point located at \(\hat{r}\) and with a radial velocity \(v_r\) the following is true

\[
\vec{U} \cdot \hat{r} = v_r
\]  

(3.39)

In radar application the value of \(\vec{U}\) is estimated. The least square method can be used to obtain such estimation. The difference between the measured \(v_r\) and the projected value of the mean wind can be minimized to obtain the best fit. If designated as \(E\) the error in the estimation for is

\[
E = \sum_{i=0}^{N} \left( v_{r_i} - \vec{U} \cdot \hat{r_i} \right)^2
\]  

(3.40)

where \(N\) is the maximum number of scatterers used for the estimation. The value of \(E\) needs to be minimized for the different components of \(\vec{U}\). A system of equations is obtained in the form

\[
\begin{align*}
\frac{\partial E}{\partial u} &= 0 & \text{minimize respect to } u \\
\frac{\partial E}{\partial v} &= 0 & \text{minimize respect to } v \\
\frac{\partial E}{\partial w} &= 0 & \text{minimize respect to } w
\end{align*}
\]
Because in atmospheric motions $u \approx v \gg w$ for non convective conditions the last equation is usually not considered and the $w$ value ignored. Another method can be used to estimate the vertical winds. Consider the region close to the vertical in diagram 3.18. If only points closely located to the vertical are considered, the radial component of those points represent the projection to the vertical $\hat{k}$ of the general wind $\vec{U}$.

The previously described techniques were used to obtain wind values for one specific measurement. The results are presented in figure 3.19. The zonal and meridional components were simultaneously obtained and the vertical component was obtained in a different process.
Figure 3.17: (left pane) Map of the maxima for three different days in the range of $-\pi$ to $\pi$ for the phase angle of receiver 2 (horizontal) and 3 (vertical). (right top pane) Three dimensional visualization of the location of the maximum value. (right lower pane) Horizontal distribution of the maxima for three different days (the blue, red and green dots differentiate the information for each day).
Figure 3.18: Schematic diagram of the interferometric wind procedure. The vertical extent of the data is subdivided in consecutive heights and equation (3.40) is solve in each division. The vectors show in the diagram correspond to examples of $\vec{r} + (\vec{U} \cdot \hat{r})\hat{r}$.
Figure 3.19: Three wind components estimated with interferometry. It is not rare that techniques of smoothing (like removing high frequency components) need to be used to obtain slowly varying values in the vertical. The + represents the raw data as obtained from the solution of equation (3.40) and for the estimation of the vertical component. The continuous line is the final products after the high frequencies are removed and a cubic interpolation is used.
Numerical techniques, physics modelling and radar simulations

For centuries physicists dreamed of devices that could simulate the universe, or at least specific events occurring inside of it. Theoretical computer science deals with the computability and complexity limits of computer simulations (Davis et al., 1994). Charles Babbage’s machine and the interpretation given by Augusta Ada Lovelace [Fuegi and Francis (2003)] are examples of how computers came into existence. These days systems (especially large scale computer clusters) have enough capacity to simulate, in great detail, many of the scenarios needed to better understand the laws of physics and their implications.

Most areas of physics can benefit from simulations. By simulating real or hypothetical scenarios researchers enhance the understanding, and hence, the applications of physics’ laws. The simulated results can lead to theoretical advances, laboratory experiments, and/or more complicated simulations.

Radar related research is an area where simulations are regularly used. Large eddy simulations have been used to calculate index of refraction inhomogeneities and simulate radar backscattering and turbulence (e.g. (Muschinski, 1996; Muschinski et al., 1999)). Spectral simulations have also provided the community with information regarding new methods and their applications (Palmer et al., 1999) as well as theoretical modification (Hocking and Röttger, 1983). One interesting area that is under-investigated is the use of convolution theorem in radar simulations.

The field of radars embraces a wide area of engineering, physics and atmospheric phenomena. Because of the wide variety of scientific areas involved in radar experimentation, there is an equally broad field where simulations can be used. Earth science and meteorology are related to atmospheric dynamics and thermodynamics, which in turn are related to the atmospheric variables responsible for the atmospheric index of refraction and its variability. Numerical weather prediction tools (like Weather Research and Forecast Model, WRF) can be used to obtain realistic atmospheric profiles. Similarly, large eddy simulation programs (e.g. WRF and PALM) can generate high resolution profiles of idealized cases.

The simulation of electromagnetic waves and how the energy travels and interacts with the environment is another example of computer simulation related to radars. The laws of electromagnetism (Maxwell’s equations) mandate how the energy leaves the radar antennas, travels in space and interacts with the disturbances in the index of refraction. Simulations of different
antennas, pulses, signals and configurations allow the researcher to simulate any type of radar. Similarly, regarding the backscattering, simulations can be used to calculate backscattering from the neutral atmosphere (or ionized atmosphere) under specified atmospheric conditions. Even more, different types of theories about the scatterers movement, shape, rotation or consistency can be tested using simulations.

Simulating a radar is not a simple task. Due to the complications entailed, there is no simple computer package or program that can solve all radar related problems. Assuming that the complications could be surpassed with proper coding and computational techniques, the configurations used to initialize the simulation are diverse, as it depends on many other factors. The titanic job of obtaining realistic atmospheric profiles along with the radar backscattering simulation has not yet been packed in a simple computer program.

The different steps of simulating a radar will be carried out in the following sections. Initially, the interference pattern of a radar will be simulated. This simulation will represent the transmitter antenna of the Costa Rica radar. Another hypothetical antenna with more transmitting elements will also be simulated. The receiver of the radar will be simulated along with the transmitter to obtain an adequate polar diagram for the entire radar.

It was mentioned before that the convolution have been underused in radar simulations. A section will be dedicated to study the simulation of atmospheric scatterers by using the convolution theorem. The evolution of the results will be presented in one, two and three dimensional simulations. The convolution simulation will also be used with realistic data obtained from a high resolution balloon-sonde.

In order to achieve a realistic radar simulation, a radar beam will be simulated inside another simulation. An atmospheric model will simulate the atmosphere based on realistic initial conditions. After some time into the simulation, the atmospheric information will be used to initialize the convolution simulation and generate radar backscattering. The use of convolution inside an LES simulation has not been found in the literature.

Before jumping into simulations and results, a short review of simulations in physics is presented. Understanding how a simulation is carried out is as important as knowing when not to simulate and pursue an exact solution. By simulating, errors are introduced in the calculations by the discrete nature of the computers. Even more, a computer program that takes initial conditions can generate an output even when the initial conditions are not an accurate representation of the problem. This is known in computer science as GIGO (Garbage In - Garbage Out). Researchers in general should be aware of the coding techniques, but also of how accurately the initial conditions and the code’s logic reflect the reality of the simulated phenomena.

### 4.1 Introduction to computer simulations

#### Simulations and computer simulations

A simulation can be defined as a process that mimics to a certain degree another process. The mimicking can be achieved by using the same materials but under different conditions (e.g. scale). Wind tunnels are an example of simulation. They are created in engineering/physics laboratories to simulate the physical conditions of building or structures (e.g. cars) submerged in air flows. Wind tunnel simulation can generate valuable information about pressure/stresses
in the test subjects, without the necessity of real scale physical fabrication. The usefulness of application of simulations should be evident, even when for philosophical reasons it is an active matter of debate (Winsberg, 1999; Grune-Yanoff and Weirich, 2010).

In a computer simulation, analogue or digital mechanisms are used to carry out the simulation. Using the wind tunnel example, one further step would be to eliminate the necessity of a physical representation of the test subjects. By using computer digital models, the objects can be submerged into digital airflow and equivalent conditions obtain. Tools to compare these two simulations already exist (Shen et al., 2003).

By using computer simulations a machine can be used to program, initialize, run, store output and analyse a process. Simulations in a computer can be summarized as follows (these topics are intended as a general introductory view for physicists and the topics are not exhaustive or comprehensive; some of the more advances topics will not be defined or covered here)

1. Choose the model. Different types of models can be used to carry out a simulation. A model can be defined as a mathematical representation of an interesting phenomena; in physics it is usually a real world event. This representation attempts to reproduce/predict the behaviour of the phenomena based on a set of parameters and initial conditions. The chosen model will generate an algorithm that will be programmed in order to follow the appropriate steps to simulate the event.

2. Choose the architecture. The hardware and software used to carry out the simulation is essential for the appropriate computer simulation. If not enough memory, storage, or processing capacity is available, compromises on the modelling foundations may lead to incorrect outcomes. Even the stability of the operating systems and its performance can impact a simulation.

3. Implementation. Creating the computer code necessary to represent the event can be challenging. The computer language (e.g. compiled, interpreted), coding methodology (e.g. object oriented, structured, procedural) and the syntax complexity can be allies (or enemies) during implementation. Testing is an important part of implementation to verify (we will return to the issue of verification later) that the code is functioning as intended.

4. Running. Calculating and storing the output during a simulation depends on many factors. Different computer system can have different rules for execution (e.g. first in first out, parallel or serial queue) as well as in network usage and storage. Depending on the type of method-implementation relationship, running a simulation can be further subdivided in tasks of type:

- Memory intensive. Computer memory is used intensively during the calculation but is not saved after the calculation is completed.
- CPU intensive. Large number of CPU operations are carried out during the simulation.
- I/O intensive. Large initialization data and output information are dominant during the simulation.
• Communication intensive. Large amounts of multi-node (different calculating computers) interconnecting messages are needed for the simulation.

• Multi-scenario. Simulations with several or all of the previously mentioned characteristics. One example of is the WRF model that will be covered in a future section.

Different types of queues are used for running depending on the category that best fits the simulation. Variable execution times can be expected depending on the chosen infrastructure. During this research a multi-scenario program was run and on many occasions it took days to process a simulation while using 256 CPUs.

5. Analysis. The final process of computer simulation is the analysis of the model’s output. This analysis can require computers to interpret the results and it is not uncommon to have intermediate programs that interpret the simulated output (e.g. Paraview). One crucial part of the analysis is validating the results. As mentioned before, this can lead to philosophical questions about the how much trust can be given to simulations. Some models, especially those generating predictions, can’t be verified until the forecast time arrives when results can be directly compared with measurements. For example, in cosmological models this time can be impractical and model results cannot be validated in a reasonable time.

Purpose of simulations and testing

Computer simulations are widely used in physics. Running a simulation can have different purposes. In meteorology and atmospheric science one of the most common purposes is for forecasting. Using a model, a prediction of the evolution of a process is desired. This prediction can be just an approximation, an estimate, or exact point observation. One widely known example of this is the forecasting of weather.

Another purpose is to better understand theories or datasets. A simulation can take place to reproduce the physical process that generated some data, or to compare how well a dataset is described by a certain set of equations. In this thesis this was one of the goals. By simulating the atmosphere with similar initial conditions, a comparison can be carried out between simulation and radar measurements. Heuristic purposes are also another reason that simulations take place. A simulation can be used to teach, communicate and better represent information.

One appropriate question regarding simulations and their purpose is, how can the results of the computational approximation be evaluated? This question has been considered (Oreskes et al., 1994) regarding the validation and verification. The authors considered that verification and validation of numerical models of natural systems is impossible. They concluded that the purpose of a model is entirely heuristic, limiting the models to corroborate hypotheses, explore scenarios and direct further experimentation.

Definitions of validation and verification are needed. Validation can be understood as a good enough approximation of the reality. In many areas of research “reality” can be unknown or unmeasurable. The verification can be carried out by comparing the simulation’s output to the exact (analytical) solution. Many simulations exist because of the complexity of the modelled environment and the lack of an exact solution. Another type of verification can be to
verify the code and algorithm. If the implementation of a model is not carried out correctly, the result will undoubtedly not be worth analysis. The garbage-in garbage out (GIGO) phase of computer scientists can be extended into data in - processing garbage - garbage out (DIGPGO). This topic can become so complex than even ethics ends up in question.

On the other hand, validation and verification are an active area of research in many areas of physics. As an earth science example, the WRF model can be validated using measurements. An example of the validation is given by Ruiz et al. (2010). These authors found a great sensitivity in the model to the parameters used and sub-grid models used. The modelled variables of temperature and humidity exhibited different behaviours. The sub-models affect the outcome in great details since agreement of one variable with measurements could imply another variable deviating from reality. And to further complicate matters, tuning these parametrizations affected the entire humidity and heat advection generated by the model over South America (Ruiz et al., 2010).

Another recent example of validation using WRF is available (Gu et al., 2013). Their research shown how the model agrees in some areas of the great lakes and deviates from measurements in others. A claim of correcting the model is made by multiplying the model’s eddy diffusivity by factors ranging from $10^2$ to $10^5$ which is large and probably not realistic suggesting some basic physics is missing from the model. This example shows that, just by slightly modifying certain parameters the model can be “corrected” into approximating the measurements. But another question arises; To what degree should these changes be considered examples of validation?, are they simply modification of the output to decrease the deviation to reality under specific conditions?. In Gu et al. (2013) they even claim: “However, it should be realized that the model is simply being tuned to work for the deeper Great Lakes by lumping all 3-D processes into a 1-D proxy”, and later “... better understanding of the physical processes inside lakes are vital to producing realistic modelling results, and related field work should be strongly emphasized”. This agrees to a certain degree with the point of view presented in Oreskes et al. (1994), mentioned earlier.

Verification of code is complicated and is usually handled internally by the individuals or institutions that carry out the implementation. The terminology is sometimes ambiguous, and verification in modelling can be as simple as "run a process twice and make sure that it is consistent” but this clearly is not related to the definition given here.

When using computer simulations it is important to remember that as with any other research tool, care needs to be taken when formulating the design, executing the process, and interpreting the results. In order to close this section a phrase that reflects the great utility of modelling in science is reproduced. The phrase is from the book by Kaufmann and Smarr (1995):

"A simulation that accurately mimics a complex phenomenon contains a wealth of information about that phenomenon. Variables such as temperature, pressure, humidity, and wind velocity are evaluated at thousands of points by the supercomputer as it simulates the development of a storm, for example. Such data, which far exceed anything that could be gained from launching a fleet of weather balloons, reveals intimate details of what is going on in the storm cloud.”
Computer simulations in physics

Simulations in physics are used to simulate reality, ideal cases, events or phenomena. These simulations cover a broad range of areas and it is not rare for them to include multiple areas of physics at one time. The simple pendulum, movement of objects, electromagnetic radiation, and quantum mechanics are just some examples where simulations can be used in physics. The area of physics under simulation is not necessarily related to one specific implementation of computer simulation.

In addition to diversity in areas of physics, the implementation techniques and methods followed to create the algorithm used to simulate interactions are also widely scattered. The algorithm can be as unique as the methods used by the coding individual. The implementation on the other hand, can be understood regarding the type of mathematical approach used to introduce the algorithm into the computer space.

The simple harmonic motion (SHM) (e.g. mass on a spring) is one of the simplest equations in physics and found in many areas. It can be used to understand the implementation (software coding). Because of the simplicity, the position of the mass can be represented exactly with a mathematical expression. The equation

\[ m\ddot{x} = -kx \]  (4.1)

has the general solution

\[ x(t) = Ae^{-\sqrt{\frac{k}{m}}t} + Be^{\sqrt{\frac{k}{m}}t} \]  (4.2)

where the A and B depend on the initial conditions of the system. Because there is an exact solution there is no need for an algorithm to look for the solution. This is what makes it an heuristic example to simulate. Solutions can be obtained to compare the different approaches.

One way to approximate this solution can be obtained by using Euler’s method (Press et al., 1993). This method uses direct integration of first order equations \( (dy/dt = f(t, y)) \) to find an approximate solution. This method uses the formula \( y_{n+1} = y_n + dt \cdot f(t_n, y_n) \) in order to obtain the value at \( n+1 \) from previous value at \( n \). The error obtained increases rapidly because of terms of higher order in \( h \). Under certain circumstances this method is acceptable and appealing by its simplicity (Courant et al., 1967). The Courant-Friedrichs-Lewyt condition (as it is known) is that a time step bigger than some computable quantity should not be taken. In some conditions this time is ridiculously small, and better techniques exist to avoid this method.

A better way to solve the SHM problem would be to use the Runge-Kutta methods (Press et al., 1993). These methods use extra parameters to better approximate the next value of the function (it will be explained appropriately in the next section). The error is much smaller \( (dt^9 \text{ order}) \) that the one generated by Euler’s method \( (dt^2 \text{ order}) \). Carrying out these simulations (along with the exact solutions) provide the individual with better understanding and/or internalization of the concept and implications.

The two mentioned methods (Euler and Runge-Kutta) are examples of numerical techniques used and a basic description is included in appendix I. Different techniques and mathematical tools are available to discretize algorithms and implement methods. A diagram containing some of these mathematical tools is shown in figure 4.1. It is important at this point to
remark about a difference between a mathematical representation and the computational representation of a number. Just because a number can be defined mathematically does not means that the computer can represent it or handle it (see Goldberg (1991) for a review of the floating point numbers and operations in computers). Infinity, \( \pi \) and even complex numbers are examples. Some limitations are due to the discrete nature of computers or the limited capacity of the hardware.

Another common way to solve problems in physics is by integrating equations. Integration can be observed in physics when calculating the area under a velocity function. It can be used to obtain the total power in a time series, integrating under the frequency spectrum. The integration process is usually known as a quadrature problem in computer science. The quadrature problem can be approached in different manners, but because it is not of major use in this thesis it is not described here.

In brief, the exact way to observe a solution to a physical problem is to use the analytical solution. These types of solutions are exact, but usually limited to oversimplified scenarios. When more realistic cases are studied, it is common to obtain solutions in terms of differential equations. These equations can be either ordinary differential equations or, in the case of more complicated problems, partial differential equations. Examples of problems in physics that are represented by ODEs include pendulum motion and motion of particles. Examples of PDEs can be observed in fluid dynamic equations (the continuity equation for incompressible fluids being a simple case) and electromagnetic equations.

Figure 4.1: Schematic diagram of the different paths that can be taken in order to solve differential equations. The up-pointing arrow indicated the exact solutions. The left-pointing arrow follows the path of ordinary differential equations, usually related to physically more advanced simulations representing reality. The right-pointing arrow indicates the partial differential equation branch, usually left for more elaborate problems.
4.1. Introduction to computer simulations

Error in numerical solutions and computer simulations

When performing simulations in physics, many aspects need to be taken into account. As described by Ferziger and Peric (2002) (Chapter 2), the properties of a numerical solution (note that it is not the simulation) depend on different aspects. In a general manner, they can be defined as consistency, stability, convergence, conservation, boundedness, realizability and accuracy. The consistency relates to the fact that the results should become more exact as the grid spacing decreases. As shown before, in finite difference the error is proportional to a power of the spacing. If the spacing tends to zero, the results of the approximation should be more exact.

The stability of a numerical solution can be described in term of amplification of errors. If the solution is rapidly dominated by the errors the stability is compromised. The convergence (as well as the consistency) is difficult to define for problems where the initial conditions can alter drastically the result. In any case, the convergence of a system can be analyzed in terms of the result approaching the exact solution as the grid spacing is decreased. In physics, most of the laws are enforced by conservation. If the numerical solution, and consequently the simulation, can break the conservation laws the solution will certainly not reflect the physical phenomena.

Boundary conditions are found in physics from the simplest of cases to the most advanced. The boundary and initial conditions are used to set the environment’s properties and physical variables. If the product of a numerical solution is not bounded to reasonable limits it is of no use for science. The boundedness of a simulation can be observed during the execution of the simulation as well as the results. For example, if fluid motion velocities greater than the speed of light are registered, the numerical method is flawed.

The realizability is related to how real the solution is. This is a concern particularly with topics whose complexity is large and which can not be treated directly. Turbulence can be used as an example of realizability. If the solution is capable of observing turbulence smaller than the atomic scale, is this related to reality or is it just an artifact of the grid spacing? The reality and applicability of the numerical solutions needs to be tested in order to assure realistic results.

Finally, we consider the important aspect of accuracy. As indicated by Ferziger and Peric (2002), three main sources of errors appear regarding the accuracy of a numerical solution:

- Modelling errors. These are created by the lack of understanding of the rules that govern the phenomena. How well is the problem understood? Is the physics used to create the original equation perfect? The results generated by a numerical solution algorithm can differ from the observations. The model could be wrong and need adjustments in order to better approximate reality. The laws of physics may not have been correctly interpreted causing deviations from observations.

- Discretization errors. Created by ignoring high order terms when creating the discrete approach of the problem. This reflects in deviation of the numerical solution from the reality even before calculations begin.

- Iteration errors. Each time an iteration takes place, the small errors introduced by the two previous source are propagated along the domains. Inside a calculation grid this is
unavoidable and the errors will naturally propagate by the mechanisms inherently chosen in the numerical method.

Other sources of error have not been explicitly mentioned. Round-off errors occur inside the computer and are related to the software/hardware used. It is not part of the errors included in the numerical solution or the numerical method. Round-off errors occur when the software or hardware does not assign the exact value of a calculation to a variable but instead assigns a rounded-off value. These round-off errors are much smaller than the values used but accumulate and propagate with the other sources of errors already mentioned. Floating point arithmetic is another source and propagation of error. This error is related to the capabilities of the computer to represent and manipulate real numbers. A comprehensive review of the source of this error and the techniques used to mitigate its effects was presented by Goldberg (1991). In radar applications, these type of errors can be especially bad near zeros, which is a common problem in deconvolution implementations (Hocking et al., 2014).

With all these warnings and errors sources it may seem impossible to perform useful simulations. It is not impossible but is should be consider a serious matter. A computer will run any code given to it. It is up to the programmers to create code that is adequate for the task.

In the following section, the simulation of the interference pattern of a radar transmitter and transmitter-receiver is presented. This simulation uses scalar and vectorial fields to carry out the simulation. There was no need to solve differential equations in this simulations. Afterwards, the simulations of radars will be presented. In the radar simulation the motion of scatterers is determined with Euler’s method in order to solve the kinematic equations.

### 4.2 Interference pattern simulation

When designing a radar, several aspects need to be taken into account. The frequency band to use is of major importance. The desired range and targets delimits the repetition rate and type of signal to be used. One important aspect of a radar is the design of the antennas. Depending on the phenomena under study, the transmitter could be small (meteor-radars) or large (e.g. ionospheric radars, wind profilers).

The size and constitution of the antenna is directly related to the radar’s detection capabilities. In the case of an antenna formed by array of elements, each antenna will contribute its properties towards the net effect of the array. This net effect is known as the interference pattern, usually presented in the literature as a polar diagram. The importance of the interference pattern is that it defines the regions where targets are more visible through the gain function. This gain function is usually a function of different aspects including azimuth, altitude and wavelength.

The spatial coordinate and wavelength dependence of the interference pattern comes from the nature of electromagnetic waves. The limited aperture size of the antennas, along with the electromagnetic waves emanating from them are responsible for an interference pattern that extends from the antenna. Waves reaching a specific point in space with zero phase difference will interfere constructively. On the other hand, waves with a phase difference of \( \pi \) radians will interfere destructively. In the case of a large number of antennas, the net effect will consist of the sum of the waves emanating from each antenna.
The simulation of the interference pattern generated by the antenna array can be obtained in different ways. In order to simulate the antenna emission, propagation of the electromagnetic waves, and interference pattern, Maxwell’s equations need to be solved numerically (Poggio and Miller, 1973). Several programs exist that can be used to accomplish this; one example is the Numerical Electromagnetic Code (NEC). This code is freely available for GNU/Linux systems under the name of necpp.

Using NEC to simulate antennas provides an excellent approximation of the reality of the devices. Among other advantages, the code approximates the near field effect as the antenna elements and electric currents are solved numerically. NEC simulations can also account for coupling effects between antennas. The effect of coupling is important in the short range as antennas will interfere with each other. Absorbing and re-emitting the signal is another close range effect in real radar applications. One big disadvantage of these simulations is that it usually takes a long time to run if a large number of antennas are used, as in the case for a spaced antenna array. It also takes a long time to set-up the simulation before it can be run.

The coupling effect will not be accounted for in the following simulation. One direct consequence of ignoring these effects is that the near field effect is not as accurate as the far field approximation where coupling is less important. In radar applications, it is usually more important to obtain a good first approximation than knowing the exact interference pattern. Some simplifications and approximations can provide a good view of the pattern without losing the physics.

### 4.2.1 Simulating the electric fields

![Diagram](image)

Figure 4.2: a) Schematic diagram of the different transmitting antennas. Different paths with lengths \( d_i \) will change the phase of the arriving electric field at point \( P \). b) Argand diagram for the electric field at point \( P \). The different electric fields \( E_i \), emitted by antenna \( i \) will sum according to the principle of superposition to obtain \( E_P \).
Each antenna \(i\) can be approximated as a simple radiator of a field \(E_i\), with a realistic polar diagram dictated by its own individual gain function. The field emitted will travel at the speed of light and interfere with any other fields. This is represented schematically in figure 4.2-a. If all the antennas that form the array are considered when analyzing one specific point \(P\) in space the magnitude of the field can be found (diagram shown in figure 4.2-b).

The total field at a point \(P\) in space can be calculated as the complex sum of all the emitted fields:

\[
E_p = \sum_{\text{antennas}} E_{i@P} e^{jkd_i}
\]  

(4.3)

where \(d_i\) is the distance between \(P\) and the \(i\)-th element of the array, \(k\) is the wave number, and \(E_{i@P}\) is the magnitude of the transmitted electric field calculated at the point \(P\). Only for this equation the imaginary unit is represented by \(\hat{i}\), to avoid confusion with the index \(i\) of each antenna. In order to calculate this field we need to go through the calculation of the power sent by the antennas. We can treat the individual antennas as isotropic sources with an initial power of \(P_o\). In that case the intensity at a distance \(r\) will be given by

\[
S_r = \frac{P_o}{4\pi r^2}
\]  

(4.4)

The energy transport per unit area in electromagnetic waves can be obtained from the Poynting vector \(\vec{S}\)

\[
\vec{S} = \vec{E} \times \vec{H} = \frac{1}{\mu_o} \vec{E} \times \vec{B}
\]  

(4.5)

\(\vec{E}\) is the electric field, \(\vec{B}\) is the magnetic flux density, and \(\mu_o\) is the magnetic permeability of vacuum. Using the solution of a plane wave the value of \(S\) can be obtained in terms of \(E\).

\[
S = \frac{1}{c\mu_o}E^2 \approx \frac{E^2}{120\pi}
\]  

(4.6)

Using this equation the magnitude of the electric field in a particular location in space can be determined from the power or viceversa. Using the equation for \(S\), we can find the value of \(E_{i@P}\); we can call it just \(E_r\).

\[
\frac{E_r^2}{120\pi} = \frac{P_o}{4\pi r^2}
\]  

(4.7)

\[
|\vec{E}(\vec{r})| = \frac{\sqrt{30P_oG(\theta,\phi)}}{|\vec{r}|}
\]  

(4.8)

In the previous equation the case of a source with a Gain function is represented by using a \(G\) next to the initial power. Using this value of \(E_r\), the complex addition can be carried inside any volume surrounding the simulated antennas. It is worth mentioning that the results in the near field are not exact. No near field effects are resolved by this simple model.
4.2.2 Computer simulation - The transmitter

In order to simulate the Costa Rica radar, a power of 1000 watts was used. This power is the peak power of the transmitter located in Santa Cruz; for more information see appendix B. This power was divided among 9 antennas located in a square of 1.2 wavelengths (≈ 7.7 m) per side. The distribution is 3 per side with one at the centre.

Two different cases were used for the simulation\(^1\), one with equal number of antennas (nine), as for the current configuration in Costa Rica, and a hypothetical large transmitter with 169 elements. The near field was calculated three-dimensionally in a grid of \(8 \times 10^6 \, \text{m}^3\) with a resolution of 1 metre. The far field grid covered a volume of \(8 \times 10^{12} \, \text{m}^3\) with a resolution of 100 metres in all coordinates.

The location of the simulated antennas was at 1.5 m above the ground, with the centre of the square sitting at the origin of the coordinates. This will change later when the receiver pattern is simulated to resemble the Costa Rica radar (see next section).

In order to compare the different interference patterns that different antennas generate, a large array of 169 antennas, with the same spacing as the small transmitter array was also simulated. Usually in radars a narrow beam for transmission is desired, and as will be shown here with the results, increasing the size of the transmitter array is a good way to achieve that goal.

Horizontal contours of the final interference pattern of the simulation are presented in Fig. 4.3. Three different heights are represented in that figure. The lower section is located just 4 meters over the ground level (where the model is known to be inaccurate due to near field effects). With just four meters the interference pattern for the small antenna array does not look like individual elements any more. The large array on the other hand still shows a similar shape to the physical arrangement of the transmitting elements.

It is important to clarify that because the model only calculated the phase difference between different paths it can’t resolve geometrical effects of the antennas in short distances (near field effect). This makes the 4 m height a coarse approximation, nonetheless is presented here as a complement to the other heights.

The centre pane of Fig. 4.3, represents the same type of power contours but now at 100 meters above the ground level. The maximum value of power for each of the contours is represented by the red color. The black line around the maximum represents the region where the power has decreased to half the maximum value. That level of power corresponds to the -3 dB level that is usually presented in the antennas polar diagram. It is shown that for the small transmitter, at 100 meters of elevation, the -3 dB level forms a circle around the maximum. For the large array, the same level shows a “cross” shape. The size of the cross formed by the large array is less than the size of the small array, indicating that the majority of the energy will be more concentrated in the former case. This becomes more clear in the description that follows.

The “far field” is constituted by the interference pattern of an array when observed from a distance that is larger by several orders of magnitude than the size of the aperture (antenna array size). The “far field”, also known as Fraunhofer diffraction, can be defined in terms of the maximum size of the aperture \(a\), the wavelength of the radiation \(\lambda\) and the separation \(R\), as done by Hecht and Zajac (1974) (page 397). Fraunhofer diffraction will occur in cases

\(^1\)In order to obtain this calculation the cluster Orca and the scientific computer iqaluk of Sharcnet were of great use.
where the following relationship is satisfied:

\[ R > \frac{a^2}{\lambda} \]  

(4.9)

For the small array the maximum size of the aperture was \( a = 13.65 \text{m} \), whilst for the large array the value was \( A = 81.88 \text{m} \). With the value of \( \lambda \) approximately \( 6.4 \text{m} \) the resultant ranges for Fraunhofer diffraction are \( R_d = 28.96 \text{m} \), and \( R_A = 115.79 \text{m} \). The interference pattern for the far field is presented in the top pane of figure 4.3. At 10 km height above the simulated antenna array, both patterns have a maximum at the centre with minor side lobes. This indicates that the vast majority of the energy will be concentrated in the main lobe of the interference pattern.

Using the \(-3\text{dB}\) level for both simulations, the area of major concentration of energy can be compared. In order to achieve a precise measurement the values of power from the simulation
Figure 4.4: Diagrams of the power as a function of horizontal distance. The top panes contain the information as a function of meridional distance; the bottom panes as a function of zonal distance. The panes on the left correspond to the small array (actual size of Costa Rica’s transmitter). The panes on the right are the results for the hypothetical simulation of a large array with equal spacing among the elements.

were extracted for two perpendicular lines crossing the maximum of the main lobe. The result is shown in figure 4.4. The average radius for the small array is 2.55 km, and for the large array is 0.55 km. From the radius of the half power circumference a value of the angle covered from the maximum can be calculated. At 10 km height, that is equivalent to an angle of 14.31° for the small array, and 3.15° for the large array.

These values of the angular distance to the half power of the main lobe of the radar are known as the one way half-power half-width. The importance of the half-power width is that the radar beam is usually defined in term of this width. The beam width is ultimately related to the angular resolution of the radar. Those scatterers inside the radar beam (inside the $-3dB$ delimited circle), will tend to dominate the backscatter.

The magnitude of the half-power half-width can not be associated directly with a positive or negative connotation. In certain research areas a larger value of this angle can be beneficial. As mentioned before, the radar design process is (at least in research) not unique; it changes according to the necessities of those who created it. In that regard, creating a narrow beam, like the one generated by the large array, can be beneficial for the operation of a radar using Doppler beam swinging (known in MST radar as the DBS method) to obtain atmospheric echoes (Luce
et al., 2001). On the other hand, when the interferometric technique is used (usually known as IDI) to locate scatterers, a large volume needs to be illuminated by the radar beam and a large value of the half-power width is ideal (Holdsworth and Reid, 2004).

### 4.2.3 Computer simulation - Transmitter-Receiver interference pattern

Costa Rica’s radar was build to be a bi-static radar. A scaled-map of the antennas is shown in figure 4.5-b, with the centre of coordinates located in between the three receivers. The transmitter array is located 80 meters away from the origin, displaced towards the south-west. For more details related to the radar site see appendix B.

In these type of radars (bi-static), knowing the transmitter polar diagram is not enough information to locate the region of maximum likelihood of backscattering signal. Because the transmitter and receiver arrays are different, the polar diagram need to be calculated in a two step process. Figure 4.5-a presents a diagram of this process with the transmitter antennas in one place and the receiver antennas in a different geographical location. First, the transmitter antennas will generate their interference pattern. Later the polar diagram of the receiver will be calculated by summing the effect of those individual points in the sky towards the receiver antennas. The proper gain had to be used in order to achieve accurate results.

Equations 4.6 and 4.8 were used to obtain a new intensity at the location of any \( \vec{r} \) in the simulated three dimensional space grid. In order to obtain a new power for the backscatterer signal a cross section of 1 \( m^2 \) was chosen along with an isotropic scatterer behaviour. This allows the simulation to map the entire sky as if it was full of identical scatterers all visible from any location. The only variation will be in the intensity of the scatterer signal due to the range effect.

The electric field magnitude at point \( \vec{r} \) was defined previously as \( |\vec{E}(\vec{r})| \). In the case of the receiver interference pattern, the point \( \vec{r} \) will represent the location of the scatterers. We can call the net electric field generated by the transmitter \( E_S \), where this corresponds to an electromagnetic wave with energy flux density \( S_S \). Considering scatterers with isotropic cross sections \( \sigma = 1m^2 \), then the received power at the receiver location \( \vec{R} \) is

\[
P_S = S_S \sigma \tag{4.10}
\]

\[
|\vec{E}(\vec{R})| = \frac{\sqrt{30P_S G_R(\theta R, \phi R)}}{|\vec{R}|} \tag{4.11}
\]

where \( G_R \) is the gain of the transmitter array. The final step in the simulation is to calculate the power detected at the receivers. Equation 4.6 provides the intensity (watts per square metre), and the effective area of the antenna (cross section of the antenna array in square meters) is used to obtain the power. A unitary effective area for each of the three receivers (considered isotropic) was used to calculate the received power.

In the case of Costa Rica, each receiver antenna is made of 4 individual elements for a total of 12 antennas. The net polar diagram will be the accumulated effect of the transmitted polar diagram and the sum of each of the receiver’s polar diagrams (assuming equal phases after passing the receiver).
Figure 4.5: a) Interference diagram for the receiver-transmitter simulation. The transmitting antennas \((T_{x_i})\) interfere at point \(P\) in the sky and the receiving antennas \((R_{x_i})\) detect signals from point scatter scattered from \(P\). b) Schematic diagram of the real Costa Rica’s radar configuration including the transmitter array (red points) and the three receiver antennas (blue points). The same configuration was used for the receiver-transmitter simulations.

The results of this simulation are presented in figure 4.6. Similarly to the previous transmitter simulation results, three different heights are used to visualize the interference pattern. The three receivers are presented in the figure as well as the combined interference pattern assuming a zero phase difference between receivers.

All of the contours per height, presented in figure 4.6, are very similar. This is expected for this type of radar, because the spacing between receivers is comparable to the wavelength of the radar signal, meaning that the power sent back by the scatterers will be similar for all the receivers. The phase of the electromagnetic fields will be different at each receiver, and this is what allows the interferometric technique to be applied using this radar setup.

The data corresponding to a height of 20 meters are presented in the lower panes of figure 4.6. At 20 meters of elevation the effects of the antenna geometry will be very small (< 1% in magnitude), but even then, this 20 meters results should be considered a coarse approximation and in order to gain an exact solution at the near field a full electromagnetic equation solver should be used. The results for 20 m elevation showed a large sidelobe towards the transmitter. This large sidelobe is the result of the receiver’s sidelobe joining the transmitter sidelobes. This region of large sideways amplification will disappear with height (look at the 10 km height data on the same figure).

At 100 m of elevation, figure 4.6 shows that there is clearly a main lobe (large back-scatter region), with two small side lobes towards the north-west and south-east. One big side-lobe is present towards the north-east of the transmitter. It is not as large as the half-power level, but it needs to be considered if scatterers are to be found in those heights. This is usually not possible because of limitations in the radar technical aspects, due to ringing in the cables or ground clutter.

When the far field is analyzed, only one (main) lobe can be observed. This mainlobe is similar to the one observed in the transmitter simulation results presented in figure 4.3 except for the rotation and absent side-lobes. In the transmitter simulation the grid was aligned with the coordinates of the transmitter array. In the second simulation (with the receiver) the latitude
and longitude were used to align the simulation grid. That is the reason for the rotation of the mainlobe. Regarding the side-lobes, the fact that they are not observable in the 10 km height contour does not mean that there are no side-lobes. The side-lobes are tilted towards the sides even more than before escaping the horizontal range of the simulation at that height. This effect will be described later.

Figure 4.6: Interference diagram for each of the simulated pairs of transmitter-receiver, including (left) receiver one, (second from the left) receiver 2, (second from the right) receiver 3, and (right) combined receiver. The simulation was carried out three-dimensionally but specific heights at 20 m (lower panes), 100 m (centre panes) and 10 km (top panes) elevation are shown.

Because this type of simulation is independent of technical limitations in radar detection, we can take a closer look at what is happening at lower heights. Usually MST type radars have a limitation at lower heights due to effects like ringing in the cables, ground clutter and deconvolution effects near the edges.

During the analysis of the data generated by the simulation, a region with a similar backscatter efficiency to the transmitter was observed over the receivers. The response of each individual antenna is not included. That large power is due to a side-lobe of the transmitter coinciding with a large gain region of the receivers. The large gain of the receiver in that region makes it possible to detect it with similar efficiency to the scatterers located directly over the transmitter.

This effect can be observed in figure 4.7. The main lobe is clear in the figure and pointing approximately to the zenith. A secondary large lobe is observed north-east (to the right in the figure) of the transmitter. This lobe gets close to the ground level indicating that scatterers located in this region could be detected in the receivers.

If the inhomogeneities in index of refraction at the levels of occurrence of this side-lobe are large enough to be detected (and they should be since they are closer to the ground), this region
could be filled with atmospheric returns from two different locations. This is the equivalent of having two volumes of scattering.

![Three-dimensional visualization of the simulated interference pattern.](image)

**Figure 4.7:** Three-dimensional visualization of the simulated interference pattern. Three different receivers are presented as corresponding to the real Costa Rican radar. The shape of the diagram in the lower heights should be considered an approximation. The model used to simulate doesn’t take into account the shape of the antenna so immediate field effects were not accounted for.

To finish this section a three-dimensional view of the different interference pattern is presented in figure 4.8. The contour level used to generate this contour was similar to the value of the half power half angle presented in figure 4.3 (top pane). In figure 4.8-a, the pattern generated for the small array of nine antennas shows a large main lobe with smaller side-lobes mainly in 4 directions. The pattern generated when studying the large simulated array of 13 by 13 element transmitter is shown in 4.8-b. The same power will be received from a narrower region, generating a smaller scattering region. It is worth mentioning that even when this concentrated main lobe does not appear to have side lobes, it does have them. The side lobes, at the shown heights, are too close to the main lobe to be easily resolved.

Figure 4.8-c corresponds to the combined far field pattern of the transmitter and receiver. The main lobe (4.8-c) looks wider than the transmitter’s main lobe (4.8-a). The side-lobes in this case are enhanced compared to the transmitter’s pattern due to the transmitter-receiver pattern product. In general the interference pattern of case a) and c) are similar but with differences that should be considered in any research conducted using such devices.

In the next section a different simulation is presented. Instead of simulating the antennas and their respective interference pattern the back-scatter from atmospheric perturbations is introduced. The simulation uses Fourier Theory and the convolution theorem to obtain information from the simulated atmospheric disturbances. The perturbation in the index of refraction are simulated as different electric permittivity values at different locations. The perturbations can move in time creating a dynamic simulated atmosphere.
4.3 Radar backscatter simulation

In this section we turn to a more advanced simulation. The previous simulation dealt with the electric field interference due to point static scatterers in the sky. A simulation with space varying index of refraction was developed in this section. By varying the electric permittivity and moving those variations in space a more realistic atmospheric back-scatter was obtained. Radar spectral powers can be obtained from the time series obtained from the simulation.

The general idea behind this simulation is explained as follows. A multi-dimensional grid is created to contain the space where the atmosphere is located. It is loaded with the various indexes of refraction corresponding to different positions in space. Using the convolution theorem from Fourier Theory (see Chapter 3 for a complete explanation) the grid is then analyzed. The two functions used in the convolution are the radar’s pulse and electric permittivity profile. From the simulated radar information the time series of the corresponding atmospheric returns is obtained.

This method, as will be shown, is both fast and reliable. Instead of calculating thousands of multiplications in the time domain (as was done by Franke et al. (2011)), a single multiplication is carried out in the frequency domain. No literature is available where the convolution operation was used to calculate MST-type radar backscatter. Hocking et al. (2014) showed how a new radar in Costa Rica is using the deconvolution procedure to increase the radar resolution, but no simulation has implemented previously the convolution/deconvolution in calculating radar backscatter.

In order to clarify it even more, an explanation regarding the received signal is appropriate. The convolution of the transmitted pulse (considering the 1/r effect) and the atmospheric index of refraction profile is the received signal. Moving the convolution to the frequency domain is an easy task, and has a direct application in radars (Hocking et al., 2014). One important factor to keep in mind while dealing with the convolution is that only the index of refraction in the direction of propagation is relevant to the back-scatter process. Also there are cautions regarding the noise involved in the convolution; these have been mentioned in Chapter 3.
4.3.1 Convolution theory and retarded signal

In order to obtain the back-scattered electric field from the atmosphere, a convolution needs to be carried out involving the radar pulse and the atmospheric profile of index of refraction.

Using the notation of Valley (1975), the transmitted radar pulse can be simplified for a mono-static radar simulated in one dimension as

$$E_{\text{inc}}(r', t) = A\Omega\left(\frac{r'}{c} - t\right)\cos\left(\omega_o\left(\frac{r'}{c} - t\right)\right)$$

where $A$ is the amplitude of the electric field, $r'$ is a range in space, and $t$ is the time of flight. The function $\Omega$ is usually called the envelope of the pulse. For the present simulation a Gaussian pulse will be used. The full width at half maximum (FWHM from now on) will be used to define the pulse length. The value of $\sigma = \text{FWHM}/\left(2\sqrt{2\ln 2}\right)$.

$$\Omega(r, t) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(r-ct)^2}{2\sigma^2}}$$

The oscillatory term in the pulse equation (4.12) corresponds to the frequency of oscillation of the radar. The pulse used in the simulations is shown in figure 4.9. The pulse shape defined and the scatterer profile is enough information to obtain the convolution and in consequence the scattered electric field.

![Simulated radar pulse](image)

Figure 4.9: Simulated radar pulse (using equation 4.12) at 300 m from the radar with a full width at half maximum (FWHM) of 50 m.

In order to obtain the scattered electric field two processes can be used. The correct way to proceed is using the convolution theorem. The convolution is defined as

$$\{f \star g\}(t) = \int f(\tau)g(t - \tau)d\tau$$

and in this simplified form it only involves the two functions $f$ and $g$, as the $1/r$ dependence will be included later on. The symbol $\star$ represents the convolution of these functions. Carrying out this operation in the time domain is expensive (computationally) and better approaches can be taken. The convolution in the frequency domain is just a multiplication of two different spectra (as will be shown shortly).
Another way to obtain the received signal is to calculate the time domain convolution. One example of equivalency between the results generated by Huygen’s principle and the convolution can be found in Merriman and Ruuth (2000). The solution to the problem is to use a retarded time term to calculate the return. This was used by Franke et al. (2011) to obtain radar echoes from a simulation. The equation used contains the mentioned parameters (Radar pulse in terms of electric field and electric permittivity perturbation) and calculates the received electric field at a position $\vec{r}$.

$$E(\vec{r}, t) = -\mu_o \int \int \epsilon'(r') \frac{\partial^2 E_{inc}(r', t)}{\partial t^2} \frac{\delta(t - \frac{|r-r'|}{c})}{|r-r'|} d^3r'$$

(4.15)

The results generated by this equation reproduced correctly the shape and position of the scatterers. One topic regarding these results that was found inappropriate despite the claim by Franke et al. (2011) is the necessity to modify the radar’s frequency in order to obtain the correct Bragg-scale back-scattering maxima. In addition to that, the computational cost of this simulation is significantly larger with thousands of multiplications in order to obtain a time series of the echoes.

A better approach is to use Fourier theory as Hocking and Vincent (1982) showed. A similar approach was followed in these simulations. By using Fourier transforms the functions are moved into the frequency domain and the convolution is calculated there. This way, the correct equations for back-scattering are obtained from the theory and no need arises to modify the radar’s frequency or use Huygen’s principle.

In order to emphasize the importance of moving into the frequency domain, let’s understand that the convolution can be carried out in the time domain. It would require inverting the function and extensive calculations for every value of $\tau$, and the same solution will be obtained.

For this simulation, the spectral multiplication of $\mathcal{F}\{f(t)\}$ and $\mathcal{F}\{g(t)\}$ was used. One big difference with the regular convolution theory is that instead of looking for $h(t)$ we will be looking for $h(2t)$. This was done in order to obtain the correct location of the scatterers in the time domain, considering the radar’s two way trip. This immediately generates the correct Doppler velocity, pulse resolution and time location of the scatterers.

Consider the modified convolution

$$h(2z) = \int f(t)g(2z - t)dt$$

(4.16)

The value of $\mathcal{F}\{h(2z)\} \rightarrow H(f)$. We can obtain the value of $H(f)$ by using the Fourier transform.

$$H(f) = \int h(2z)e^{-2\pi izf}dz$$

(4.17)

We can then substitute the value of $h(2z)$ by the convolution, and rearrange the result. By using the change of variables $y = 2z - t$, the right hand side integrals change into

$$\frac{1}{2} \int f(t)e^{-2\pi izt}dt \int g(y)e^{-2\pi iy\frac{z}{2}}dy.$$  

(4.18)
This means that under the transformation of \( h(t) \) to \( h(2t) \), the spectral result of \( H(f) \) would be equal to \( 1/2F(f/2)G(f/2) \). To simplify the calculations the variable \( \nu = f/2 \) can be used to obtain

\[
H(2\nu) = \frac{1}{2}F(\nu)G(\nu) \quad (4.19)
\]

The factor \( F(\nu)G(\nu) \) is just the Fourier transform of the convolution of \( f(t) \) and \( g(t) \).

It is important to remark that in the previous theoretical treatment of the convolution one term is missing. The missing term is the \( 1/r \) factor. This inverse distance factor is due to the electric field magnitude falling proportional to \( 1/r \). This was already presented in the previous section of interference pattern simulation and hence will not be discussed here. The correct term for the convolution equation would include then a factor of \( 1/r^2 \), due to the two way path the signal must travel.

\[
h(k) = \int \frac{f(r)}{r^2} g(k-r) dr \quad (4.20)
\]

When considering volumetric radar backscatterers, their contribution itself will be proportional to \( r^2 \).

Before presenting the simulation and how to simulate the atmospheric echoes a word on index of refraction and permittivity perturbation should be given. This topic is covered in the next section, showing how these variables were obtained and used. As will be shown, the term \( g(x) \) introduced in this section is the vertical profile of the permittivity perturbation.

### 4.3.2 Index of refraction and permittivity perturbation

Before presenting the results of the simulation, some detail about the value of the permittivity perturbation used is required. In equation 2.6 (Chapter 2), the values of temperature, pressure and humidity were used to obtain the index of refraction. This index of refraction can be particularly sensitive to humidity, especially near the surface. The equation for index of refraction can be obtained also from the relationship between the speed of light in vacuum and the speed in the medium under investigation.

\[
n = \frac{c}{v} = \frac{(\varepsilon_o\mu_o)^{-1/2}}{(\varepsilon_M\mu_M)^{-1/2}} \quad (4.21)
\]

The index of refraction can then be represented in term of permittivities and permeabilities. It is not unusual to also represent it in terms of relative values \( (\varepsilon_r = \varepsilon/\varepsilon_o, \mu_r = \mu/\mu_o) \).

\[
n^2 = \frac{\varepsilon_M\mu_M}{\varepsilon_o\mu_o} = \varepsilon_r \mu_r \quad (4.22)
\]

When the permeabilities are considered to be unity, the index of refraction depends only on the permittivity term. The relative permittivity \( \varepsilon_r \) can be defined as

\[
\varepsilon_r = \frac{\varepsilon_M}{\varepsilon_o} = \frac{\varepsilon_o + \varepsilon'}{\varepsilon_o} = 1 + \frac{\varepsilon'}{\varepsilon_o} \quad (4.23)
\]
The term $n^2$, on the other hand, can be represented as $(1 + n')^2$. The $n'$ term is just the deviation from the vacuum’s index of refraction. Expanding this equation the index of refraction is obtained in terms of the deviation $n'$

$$n^2 = (1 + n')^2 = 1 + 2n' + n'^2. \quad (4.24)$$

The values of $n'$ are smaller than $10^{-3}$ for most atmospheric conditions, meaning that the term $n'^2$ would be on the order of $10^{-6}$. This value is considered small enough to be ignored. Taking only the first couple of terms the value of $1 + 2n' \approx 1 + \epsilon'$. The value of the permittivity perturbation can be approximated as

$$2n'\epsilon_0 = \epsilon'. \quad (4.25)$$

Figure 4.10 contains the calculated values for $n'$ obtained from weather balloon sonde data. The information was gathered during field experiments carried out in Costa Rica by NASA. These experiments were named TC4 and CR-AVE, and more about these experiments will be mentioned in one of the following case studies.

![Index of Refraction calculated from two different radiosonde measurements.](image)

**Figure 4.10**: Index of Refraction calculated from two different radiosonde measurements. The radiosonde measurements were extracted from data sets obtained during two different field campaigns of NASA in Costa Rica. The campaign CR-AVE (Vömel et al., 2007) was carried out during Winter time, whilst TC4 (Halverson et al., 2007) obtained its information during Summer time.

The maximum values that the $n'$ reaches are found during the summer of the northern hemisphere. The North American summer corresponds to the rainy season in Central America. The summer and winter values are clearly different from the surface up to a height of approximately 10 km. The radiosonde information for both cases (available in Figures 4.14 and 4.15) confirms that the main difference among these cases is the water content in the lower-middle
troposphere. In that range of heights, rapid changes in index of refraction are observed (e.g. Figure 4.10 at 2, 4 and 6 km height). Past 10 km the summer and winter values appear similar.

4.3.3 Case studies

In the following sections the results of specific simulations are presented. One example is included to show the Bragg-scale resolving capabilities of the model. One, two and three dimensional examples were simulated in order to test and prove the model’s resolution and products.

**Bragg-Scale**

The simulation of a radar should reflect the physics behind the transmission, interference and reception. According to radar theory, the maximum return should be obtained from the atmosphere when a Fourier component with a wavenumber length of $2\kappa$ is present in the path of a radar’s pulse with a frequency (Hz) of $\kappa c/2\pi$. This is due to the two way path that the signal will experience. The Fourier component of $2\kappa$ is not necessarily introduced by radial motions in the atmosphere. Turbulence occurring in directions not parallel to propagation can contribute Fourier components to space, hence causing the backscattering of radar signal. This topic will be treated towards the end of the chapter where more advanced simulations are presented.

In order to test this feature of the simulation two different approaches were used. In the first method, a sinusoidal oscillation was used to represent the atmospheric profile. The amplitude of the permittivity perturbation of the sinusoidal were assigned to be between 0 and $0.0007\epsilon_o$. The wavelength of the sinusoidal function was varied from $0.05\lambda$ to $2\lambda$.

The value of $7 \times 10^{-4}\epsilon_o$ was chosen because it correspond to the deviation from $\epsilon_o$ at an atmosphere with a pressure of 905.4 hPa, temperature of 22.7 C, and dew point temperature of 20.6 C [ RH = 88% ]. These values were measured in Costa Rica, 100 meters from the surface, at the Juan Santamaría international airport during a NASA campaign with the name of “Tropical Composition, Clouds and Climate Coupling (TC4)”.

The physical parameters used to configure the simulation were:

1. Amplitude of the electric field (at ~ 1 m). $A = 1000$ V/m.
2. Radar frequency. $f_o = 46.6$ MHz.
3. Full width at half maximum (Gaussian envelope). FWHM = 50 m.
4. Maximum distance. $X_f = 1000$ m.
5. Step size (distance). $\delta x = 0.01$ m

Some of these values are somehow unrealistic or unusual, but are a first approximation for testing purposes of the scattering model. More realistic parameters were used once the physics of the simulation was observed to function properly. An example of a lower than usual value is a FWHM of 50 metres, which is a small pulse when compared to real case scenarios. The same goes for the maximum distance simulated (radar range). A range of 1 km is small for an
atmospheric radar. Most verticals atmospheric radars range go from 5 km up to the turbopause. These variables were used for testing the backscattering engine.

The result for the sinusoidal profile is shown in the upper pane of figure 4.11. With a frequency of 46.6MHz the radar’s wavelength is 6.43m. The maximum return (as observed in the figure) is present at half that value, 3.21m.

Another test was done regarding the Bragg-scale. Thin rectangular boxes were used to simulate boxcar functions. These functions were located at equal spacings and calculated the backscatter for each configuration. The values used for the space between the boxcars were the same ones used for the sinusoidal function wavelength, from 0.05λ to 2λ.

Because a series of boxcar functions defined the scatterers, the Bragg-scale wavelength of λ/2 can be present in more than one location, depending on the spectral content of the profile. This is observed in the lower pane of figure 4.11. The largest return will occur at the Bragg-scale of the radar. Subsequent returns occurred at λ, 3λ/2, and 2λ, with decreasing backscatter intensity as the separation increased.

The success of Bragg-scale backscatter by using the unmodified radar’s frequency is a proof that the model (and hence the simulation) correctly resolved the appropriate aspects of radar theory.

In the following examples, a one dimensional case study of the back-scattered signal is presented as well as the spectra generated by moving scatterers.

**One dimensional - static**

In order to further test the simulation engine, a one dimensional case was used. Four different boxcar function-like objects were created as a profile of index of refraction. Each of those objects had a different magnitude in order to generate different back-scatter intensity returns. The width of each boxcar was set to 2m, meaning that a pulse width of 50 meters would not be able to differentiate between the beginning and end of the box (see appendix A). The 2 m width is smaller than the half wavelength for the chosen frequency, but the important aspect for the simulation is the Fourier components at the beginning and end of the boxcar. The small 2m thickness of the box will make it impossible for a long pulse to resolve both edges.

Scatterers were located at 100, 300, 600, and 800 meters from the centre of coordinates (radar location). The separation of the scatterers is enough for the pulse to resolve each individually, obtaining one single return for each box. The same configuration used for the Bragg-scale example presented previously was used.

The result of the simulation is presented in figure 4.12, top pane. The abscissa used is time, instead of distance. The left y-axis values represent the perturbation of the permittivity and the right y-axis the electric field magnitude of the received signal. The offset of the zeros was intended as a better way to visualize the occurrence of the returns.

The time it takes to observe the echoes doubles from the location (in time) of the box. The pulse shape is observed clearly in each return, representing the convolution between the radar’s pulse and the thin rectangular scatterers. The amplitude of the perturbation given to each scatterer is observed in the magnitude of the received electric field (as pointed out in figure 4.12-a).

As a variation to the described example the radius of the scatterers was changed to 30 meters. An effective width of 60 meters in the scatterer’s length should be reflected in the
4.3. Radar backscatter simulation

Figure 4.11: Top pane: Radar return for sinusoidal oscillations in permittivity profile with wavelengths varying from $0.05\lambda_r$ to $2\lambda_r$; where $\lambda_r$ is the radar’s wavelength. Bottom pane: Radar echo power for the simulation with thin rectangular boxes (boxcar function variation in the electric permittivity) with spacing between $0.05\lambda$ to $2\lambda$. 
Figure 4.12: a) Box like scatterers of different magnitudes (electric permittivity perturbation) simulated backscatter. The location of the scatterers in the detected simulated signal is indicated by arrows. 
b) Similar to the previous case, but the boxes were given a large width. The large width enables the radar to detect both edges of the box as indicated by the arrows.
return echoes. The radar’s resolution should make the edges visible, at the distance of 120 meters.

The results for this second part of the experiment is presented in the bottom pane of figure 4.12. The width of the rectangular boxes is now much wider than in the previous example, but the centre is still located at the same position. Because the width of the boxes is now 60 meters the pulse length allows those limits to be resolved.

As indicated in Chapter 1, a radar pulse of length $T$ would have a resolution of $T/2$. The FWHM used was 50 meters, meaning that if taken as the pulse length, objects (or interfaces) separated by more than 25 meters can be resolved. The echoes observed in the received signal are formed now by a double peaked signal, corresponding to the entrance and exit of the pulse in the region of different index of refraction (boxes).

So far, the simulation products are generating satisfactory results in two important areas. (i) The strongest returns should be generated by a Fourier component of $\lambda/2$ (Bragg-scale) in the atmospheric profile. This is particularly important regarding turbulence studies. If an atmospheric region is filled with turbulence creating $\lambda/2$ wavelength perturbations these will be detected by the radar much more strongly than any other frequency. (ii) The radar can resolve clearly isolated changes in the index of refraction, as well as boundaries. In MST radar this is important because many of the scatterers detected are usually not only related to turbulence, but also, specular reflectors and regions of large index of refraction (layers and tilted layers).

In the following example, the case of moving targets is analyzed. Instead of having static scatterers, the scatterers will have an intrinsic velocity that will move them in space.

### One dimensional - moving objects

The use of radar in atmospheric physics goes beyond target location. One of the most important applications of radar is using the Doppler effect to calculate the velocity of the targets. Determination of radar velocity spectra using the Doppler effect is a major goal of the present simulation.

In this simulation similar parameters to the ones presented for the static case were used, but instead of a resolution of 1cm a resolution of 5cm was used. The decrease in resolution was intended to accelerate the calculations (and it was successful) without a considerable loss in spatial detail. A complex exponential function generated the transmitted pulse. By using the real and imaginary components of the pulse one single peak was generated in the spectrum, as both in-phase and quadrature components are available to calculate the Fourier transform.

The way to obtain the necessary information to calculate the velocity is by sending repeated radar pulses. The technique consists of measuring the phase change of the scatterer with time. This phase change in the time domain is observed in the frequency shift. This property of the Fourier transform is known as the shift theorem and it was presented in Chapter 2 but in a different way; now it is being presented as a phase change obtained by using the in-phase and quadrature components of the radar time series. With a function $f(x)$, the Fourier transform is $\mathcal{F}\{f(x)\} = F(k)$. When this function is multiplied by a phase changing factor $e^{-2\pi i x \phi}$ it is observed that the Fourier transform changed into

$$
\mathcal{F}\{f(t)e^{-2\pi i t \phi}\} = \int f(t)e^{-2\pi i t \phi} e^{-2\pi i f \phi} dt = \int f(t)e^{-2\pi i (f-\phi) t} dt
$$

(4.26)
where the \( f - \phi \) term indicates that the result is just a frequency shift to \( \phi \). By changing the variable of the Fourier transform the final equation is obtained.

\[
\mathcal{F}\{f(t)e^{-2\pi i\phi}\} = F(f - \phi) \quad (4.27)
\]

The radar pulse will be sent repeatedly in order to measure the variable \( \phi \). The rate of repetition is known as Pulse Repetition Frequency (PRF). It is also common to represent the spacing between successive pulses in the time domain, named Inter Pulse Period (IPP). In the simulations a PRF equal to 10 Hz was used (IPP equal to 0.1 seconds). This allowed measurements of velocities up to \( \pm32.1 \text{ m/s} \) with an uncertainty of 0.1 m/s. Higher velocities would cause the phase to change more than \( 2\pi \) between measurements resulting in an incorrect value being calculated.

For this simulation, different scatterers were located in a straight line. The location of the scatterers, velocity and permittivity are listed in table 4.3.3.

<table>
<thead>
<tr>
<th>Number of Scatterer</th>
<th>Radial Distance (m)</th>
<th>Radial Velocity (m/s)</th>
<th>Permittivity Perturbation (F/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>10</td>
<td>( 0.0007\epsilon_0 )</td>
</tr>
<tr>
<td>2</td>
<td>300</td>
<td>5</td>
<td>( 0.0007\epsilon_0 )</td>
</tr>
<tr>
<td>3</td>
<td>600</td>
<td>-8</td>
<td>( 0.0007\epsilon_0 )</td>
</tr>
<tr>
<td>4</td>
<td>900</td>
<td>-13</td>
<td>( 0.0007\epsilon_0 )</td>
</tr>
</tbody>
</table>

Table 4.1: Description of the properties of the simulated scatterers used for the study of motion in one dimension. The values of the permittivity perturbation are in agreement with those presented in figure 4.10. Those values were obtained in Costa Rica and for these simulations are considered representative of near surface - low troposphere values.

The power spectral density of the return was normalized and is presented in figure 4.13. In this figure the initial location of the scatterers is represented using a blue circle.

The distance covered by the scatterer in the 20 seconds of the time series agrees with the assigned velocities; it can be confirmed by comparing the information in the table and the radial velocity information in figure 4.13. It is important to remark that the resolution of the velocity spectra shown in figure 4.13 (as well as the ones in the following sections) are sensitive to the resolution of the grid, especially the radial component of the model. Radial resolution values as low as 1 metre can be used with a low density of scatterers, but once the number of scatterers is increased the need for the increased resolution is evident.

Including this section, another aspect important for radar simulation is observed to behave properly. The velocity spectra of the atmospheric profile can be obtained without major problems. Even though only four scatterers were used in this section, the model can be used with many scatterers, as long as the resolution of the model can resolve the location, size and displacement of each individual object.

**One dimensional - real profile**

In the previous sections the model demonstrated its capabilities in simulating Bragg-scale backscatter, index of refraction barriers, and moving objects. We move now to a simulation of backscattering from a profile of index of refraction obtained from radiosonde data. The radiosonde
data does not have even spacing so a cubic spline interpolation program was used to match the resolution of the profile to that of the model. The cubic spline was used in order to avoid discontinuities that can generate intense artificial backscatter.

An important topic on simulations is the final received power and how it compares to the noise level. Many simulations will generate interesting results that are usually not achievable in real applications. This is one of those simulations. It is included anyways as a test of the simulation engine and as a proof that not necessarily because it can be simulated it represents a measurable event. The real noise level will be many orders of magnitude greater than the backscattered power achieved in the simulation.

The noise level at the Costa Rican radar can be calculated, at temperatures of 40°C (not rare inside the radar trailer in Costa Rica) it can be as high as $1.08 \times 10^{-14} \text{W}$ which is small compared to cosmic noise. The power of the signals generated with these simulations will be many orders of magnitude (nine or more) smaller than the noise. The low values in power backscatter are not a surprise as the radiosonde is missing the half-wavelength component that generates most of the radar backscatter. In real radar application techniques like coherent integration can be used to accumulate the received signal to a power level that is greater than the noise level, but these echoes require that the appropriate Fourier scales are present.
The radiosonde data-set used was part of NASA’s TC4 and CR-AVE experiments\(^2\). The equipment used for the both experiments provided atmospheric data and radiosonde position every two seconds. With an ascension rate of \(7 \text{ m/s}\), typical of the first meters after letting go of the balloon, a resolution of 14 meters would be obtained near the surface. The speed of the radiosonde is not constant, and sometimes it can be as low as \(3 \text{ m/s}\), obtaining a resolution (close to 6 meters) that is approaching the radar’s wavelength. A resolution of 14 meters is considered coarse for this type of simulation, but it is used as a first approximation to realistic back-scatter.

The resolution of the profile is directly related to the backscattering. The strongest backscatter will occur at scales of \(\lambda/2\). If this component is missing or not resolved by the measuring device the simulated returns will not be representative of the real atmospheric backscatter. This profile resolution problem will be considered in more detail in a following section.

The experiment TC4 took place during Summer of 2007. CR-AVE occurred during the Winter of 2006. One sounding from each experiment was used to simulate the radar’s received electric field. Using both profiles generates a different perspective of the atmospheric echoes, due mainly to the dramatic difference in water content. The maximum range of the simulation was set to \(30 \text{ km}\). This limit is below the maximum height reached by the radiosonde, so no extrapolation was necessary.

For this simulation the radar pulse length was increased to 300 meters. The change in pulse length reduced the resolution but increased the capacity to capture large scale characteristics. A value of 300 meters in pulse length is comparable in order of magnitude to the ones used in many real radar experiments and applications. The radial resolution was kept at 5cm.

Using data obtained during the Winter of 2006 a one dimensional simulation (similar to the one described in the previous section but with much larger range) was carried out. The atmospheric information (top three vertical panes) and the magnitude of the simulated electric field (bottom pane) are presented in figure 4.14. Another similar simulation but with a different data-set was also carried out. The data used were gathered during the Summer of 2007. These other results are presented in figure 4.15. Note that the scale values in the Relative Humidity plot are larger for Summer than those for Winter.

The main difference between Summer and Winter sounding data observed in these figures is clearly the water content of the troposphere. The water content is present in the equation used to calculate the index of refraction (2.6) through the water vapour pressure term. In the relative humidity graph, the mid-tropospheric region is where more difference is observed between Summer and Winter values. It is in the mid-troposphere that the radar echoes are noticeably more intense during Summer.

Finding information in high detail about specific phenomena in these figures can be hard to achieve because it is a single measurement. Without a high number of measurements per second (not achievable with radiosonde and one of the strengths of radar devices) it is not possible to obtain velocity information. One example of phenomena that are hard to resolve is the boundary layer upper limit. It can not be pointed exactly only by looking at the radar profile. A better approach is to approximate it with the atmospheric information, and later looking for a radar peak in that area. During the Winter, a big drop in humidity can be found

\(^2\)More information available at: acdb-ext.gsfc.nasa.gov/People/Selkirk/TICOSONDE/Ticosonde_index.html
Figure 4.14: Top pane: Atmospheric information obtained with radiosonde during NASA’s CR-AVE experiment. The experiment took place in Costa Rica during the Winter (dry season) months of 2006. The blue line next to the temperature is the dew point temperature. Bottom pane: Simulated radar back-scatter from the electric permittivity obtain from the sounding’s atmospheric variables.
Figure 4.15: Top pane: Atmospheric information obtained with radiosonde during NASA’s TC4 experiment. The experiment took place in Costa Rica during the Summer (rainy season) months of 2007. The blue line next to the temperature is the dew point temperature. Bottom pane: Simulated radar back-scatter from the electric permittivity obtain from the sounding’s atmospheric variables.
close to 3 km over the surface (4 km height; Weather Station is located in San José at \( \approx 1 \) km elevation). At that same height, an inversion in the temperature indicates that the boundary layer could be ending in that region. Looking at the bottom pane of figure 4.14, in the same section that the humidity changes rapidly (marked as A and B in figure 4.14), there is a spike in the echo power. This spike can be used to define large changes in the humidity, especially in the first few kilometres.

During the summer, the content of water vapour in the atmosphere is increased. There are two regions (around 5 km height) where the relative humidity reaches 100% values and then drops 40% in one case and 20% in the other (marked as A and B in figure 4.15). These same regions can be observed as spikes in received power in the bottom pane of 4.15.

As a test of the simulated radar returns, efforts towards finding atmospheric details were carried out. Finding the tropopause height or the top of the boundary layer are regular uses of radar back-scatter information. Calculating the tropopause height from the received electric field was not obvious either. During Winter a rapid decrease in echo power is observed over 19 km, a region that also shows a rapid decrease in relative humidity towards 0 percent and where the temperature is already increasing. For the Summer, the location of the tropopause can be estimated to be at 17.5 km using the atmospheric data, and the profile obtained by the simulation shows considerable change in echo power in the vicinities of that height. The height of the tropopause would be easily detected by real radars due to the presence of specular reflection which are not detected by radiosonde.

The procedure just described should not be taken as a proper way to obtain radar information of the atmosphere. That is due to the missing spatial components in the information and the introduction of components during interpolation. The simulation can be particularly sensitive to the non-smooth or discontinuous sections of the profiles. It was considered here just as a test for the simulated radar echoes.

A better way to compare the simulated received signals of the atmosphere is by using a ratio of the two electric fields received. In figure 4.16, the values of the Summer to Winter ratio (SoW) are presented in red, and the inverse (Winter to Summer ratio, from now on WoS) are presented in blue.

If the echoes had the same power level, the value of 1 should be obtained meaning a logarithmic value of zero. As can be seen, that relationship is rarely observed. Instead, there are regions where the power of WoS is greater than that of SoW, and vice versa. Interestingly, the regions where the SoW (red curve) exceeds the WoS is larger than the other case. This can be attributed to the excess of water content in the low-mid troposphere increasing the power generated by the simulation.

An important aspect that can be inferred by looking at figure 4.16 is that there are peaks (generated by WoS and SoW) that coincide with the described characteristics of boundary layer top and tropopause height. For example, there is a peak in SoW signal located near 19 km, and another peak in WoS near 17.5 km.

There is also a peak where the boundary layer was estimated to be located for the Winter. Many other peaks are observed but can not be accounted for. The reason for the other peaks could be layers of turbulence and other transient phenomena that would entirely depend on the local conditions or advected phenomena. This topic will be covered in Chapter 5 with examples that illustrate it. In the previous example where radiosonde data was used to calculate radar backscattered power the noise was observed to be many orders of magnitude greater than the
Figure 4.16: Ratio of the simulated powers received by the radar. WoS represents the log of powers from Winter divided by the Summer equivalent. SoW is just WoS$^{-1}$, and obtained in order to use only positive values of power. A value of 0 in power represent a ratio of 1 between the Summer and Winter echoes. Larger values of power (either SoW or WoS) represent superiority of the numerator over the denominator factor.

received signal. This would make it impossible in real applications to compare both signals. Noise would dominate any comparison or ratio taken, as the one shown in figure 4.16.

**Two dimensional - moving objects**

The two dimensional case of the simulation is a natural extension of the one dimensional. In order to obtain a two dimensional received signal (and for the next section three dimensional) it is needed to sample other possible routes that the radar pulse can travel through. By using the convolution in every direction the total received signal is just a superposition of all the possible analyzed directions.

The two dimensional simulation and domain are illustrated in figure 4.17. The computational grid shown in this figure has an angular range of 28 degrees wide along with 40 radial divisions. Values of angular separation used in the simulations go from 50 to 200 subdivisions to generate a better spatial sampling.

A word about how the index of refraction is assigned to the grid is appropriate. Objects with limited spatial properties were created to represent the scatterers; each scatterer had a diameter of 1.6 metres. The way this was achieved was to create a mesh free space with the scatterers defined as objects. Each scatterer would contain its position and velocity (with equal dimensions as the space), as well as a radius and index of refraction.

The fact that the scatterer’s radius and index of refraction is assigned individually gives this
model the capability of having different types of scatterers. Perturbations that are small in size but large in index of refraction can be introduced in the model. Also, perturbations large in size but small in index of refraction can be incorporated. No restrictions on location are applied. Several perturbations can be co-located at one time, one inside of another or just superimposed.

In this model the calculation grid necessary to solve the electric field equation is initialized after the mesh-free scatterers are created. All those points of the grid that are inside the radius of the scatterers will accumulate its value of index of refraction. The value of scatterer radius was incorporated in order to generate a region of influence where the scatterer is large enough to be resolved by the grid.

Ten scatterers were used in this simulation. The information about these objects is presented in table 4.2. The objects are located inside the 2 km range, configured for the radar simulation. The velocities are also smaller than the maximum value that the time series can measure without aliasing. All indexes of refraction are kept equal in order to obtain equal intensity returns from the scatterers.

The products of the simulation are presented in figure 4.18. As before, the initial location of the scatterers is shown on the figure as blue dots. The spectral information obtained is consistent with the set-up parameters of the simulation, as well as the maximum displacement of the scatterers.

Results from this type of simulation are important because they demonstrate the capability of multiple dimension target location and spectral resolution for calculating Doppler velocity. This is crucial in radar applications due to the fact that multiple atmospheric scatterers, each with different radial velocities, will send signals back to the receiver from different locations. A radar simulation that can manage grids with this type of scatterers can be used to obtain more realistic profiles from other sources, as will be shown in future sections.

<table>
<thead>
<tr>
<th>Number of Scatterer</th>
<th>Radial Distance (m)</th>
<th>Radial Velocity (m/s)</th>
<th>Permittivity Perturbation (F/m)</th>
</tr>
</thead>
<tbody>
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<td>-14.0</td>
<td>0.0007ε₀</td>
</tr>
<tr>
<td>2</td>
<td>1.131</td>
<td>25.5</td>
<td>0.0007ε₀</td>
</tr>
<tr>
<td>3</td>
<td>1.779</td>
<td>-6.3</td>
<td>0.0007ε₀</td>
</tr>
<tr>
<td>4</td>
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<td>-26.9</td>
<td>0.0007ε₀</td>
</tr>
<tr>
<td>5</td>
<td>0.893</td>
<td>-3.3</td>
<td>0.0007ε₀</td>
</tr>
<tr>
<td>6</td>
<td>0.342</td>
<td>5.9</td>
<td>0.0007ε₀</td>
</tr>
<tr>
<td>7</td>
<td>1.597</td>
<td>5.9</td>
<td>0.0007ε₀</td>
</tr>
<tr>
<td>8</td>
<td>0.725</td>
<td>14.7</td>
<td>0.0007ε₀</td>
</tr>
<tr>
<td>9</td>
<td>0.810</td>
<td>-16.3</td>
<td>0.0007ε₀</td>
</tr>
<tr>
<td>10</td>
<td>1.485</td>
<td>22.0</td>
<td>0.0007ε₀</td>
</tr>
</tbody>
</table>

Table 4.2: Description of the properties of the simulated scatterers used for the study of motion in two dimensions. Similar values to the one dimension simulation were used for consistency and agreement with Costa Rican atmospheric information.
Figure 4.17: Schematic diagram of the two dimensional simulation. The computation grid is shown as the radial lines. The angular range covered is 28 degrees and the radial resolution is too high to be resolved by the figure. The scatterer location represents the centre of coordinates where the perturbation is defined. The scatterer size (a disk with 1.6 m in diameter; representation in figure is not to scale) represents the region of influence where the grid can accumulate the different perturbations that are created or moved in the simulation. For more detail see the text.

Three dimensional - moving objects

In this section a new coordinate, \( \phi \), was included in the simulation program. The \( \phi \) coordinate allows for a rotation of the vertical axis, giving a realistic three dimensional field of view. With a new coordinate to account for, the computational time increased considerably.

One important difference between the three dimensional simulation and the previous two is that a Gaussian distribution was used to obtain the radial velocities assigned to the scatterers. This is actually a realistic velocity distribution, if scatterers were moving horizontally (not the simulated case), those overhead could be detected more easily by the radar but their radial velocities would be smaller than those scatterers observed towards the sides of the beam. No velocity distributions were used in previous simulations as just a few scatterers were simulated. Because a larger number of scatterers were used in the three dimensional case, the distribution best can be appreciated in the results.

Figure 4.19 contains the diagram for three scatterers located at 600, 400, and 250 meters from the surface level. Each scatterer is located in a different combination of \( \theta \) and \( \phi \) angle. The grid in figure 4.19 is represented by dotted lines. This grid resolution is coarse and was
4.3. Radar backscatter simulation

Figure 4.18: Two dimensional velocity spectra obtained from the simulation. Ten different objects with known location and velocity were used. The original location (before the first time step of the simulation) is shown as blue dots. Correct positioning and displacement is observed. All the objects to the right of the 0 m/s should move upwards and all those on the left should move downwards.

used for illustration purposes only. In this example, only 30 different values of \( r \), 8 values of \( \phi \) and 5 values of \( \theta \) were used. The grids used in actual radar calculations need to be tightly spaced in order to properly resolve the scatterer location and motion. Up to 6000 values of \( r \), 150 values of \( \phi \) and 30 values of \( \theta \) are common.

In this simulation, a realistic view of the radar is obtained. Finally, the cone-like structure generated by the radar volume path in the atmosphere is appropriately visualized by the computational grid in the figure. This grid, representing the region where maximum backscatter return come from, is congruent with the previously presented simulation of the interference pattern of the antennas. This can be observed by comparing figure 4.6 and 4.19. The direction where most of the energy is concentrated in the interference simulation can be approximated by the grid in the three dimensional simulation in this section.

The region covered by the radar pulse can be filled with scatterers at different locations in three dimensions. The values used in the setup of the experiment are shown in Table 4.3. Thirty different scatterers where located randomly in the radial coordinate. The velocity was calculated with random values but distributed in a Gaussian function. The results of this distribution is more objects towards the centre of the simulated velocity spectral density. No accounting for objects leaving the calculation grid was done, as this had no impact on the results of the simulation. It can be seen that object number nine is close to the edge of the maximum range.
Figure 4.19: Schematic diagram of the three dimensional simulation. The computation grid is shown as the lines originating from a single point. A low resolution was used in order to observe the grid points. Five different values of $\theta$, eight different values of $\phi$, and a low radial resolution (30 values) allow the individual computation grid points to be resolved by eye. Three different triangles represent the location of different scatterer locations that are not part of the grid coordinates. For more detail see the text.

and the velocity is positive, meaning that in the first steps of the simulation it was left out of the calculations. Again, this had no impact on the simulation goal.

Another important configuration of this simulation is that even when it can resolve objects near the ground, a height of 1 km was chosen as the minimum for the location of the scatterers. This is just in accordance with a limitation of the real radar applications and configurations.

The results of the simulation are presented in Figure 4.20. The majority of the velocities chosen were located between $\pm 10$ m/s, as most spectra from average tropospheric conditions should reflect. Even with more populated scatterer scenarios the simulation resolved each individual disturbance in agreement with the initial conditions and met the appropriate expectations for the time evolution of the system.

The model that was created is based on Fourier theory’s convolution theorem. It was shown through different case studies that it can generate adequate back-scatter information for static or dynamic simulated atmospheres. No limitation exists on the model regarding pulse length or shape, or atmospheric electric permittivity perturbation profile. A direct implication of this lack of limitations is that the model can be used to probe any type of profile with any type of pulse.
### Table 4.3: Listing of the properties of the simulated scatterers used for the study of motion in three dimensions.

Similar values to the two and one dimension simulation were used for consistency and agreement with Costa Rican atmospheric information. The grid resolution in the radial coordinate allows the position to be precised to 25 cm. The radial velocity uncertainty is also 0.1 m/s.

<table>
<thead>
<tr>
<th>Number of Scatterer</th>
<th>Radial Distance (m)</th>
<th>Radial Velocity (m/s)</th>
<th>Permittivity Perturbation (F/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2596.0</td>
<td>1.2</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>2</td>
<td>2018.0</td>
<td>13.8</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>3</td>
<td>1870.0</td>
<td>-1.6</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>4</td>
<td>2311.0</td>
<td>-1.8</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>5</td>
<td>1633.0</td>
<td>7.8</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>6</td>
<td>1575.0</td>
<td>8.6</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>7</td>
<td>1441.0</td>
<td>2.3</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>8</td>
<td>3039.0</td>
<td>-12.0</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>9</td>
<td>3964.0</td>
<td>14.0</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>10</td>
<td>2166.0</td>
<td>7.3</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>11</td>
<td>2096.0</td>
<td>18.7</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>12</td>
<td>3199.0</td>
<td>-6.9</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>13</td>
<td>1547.0</td>
<td>0.8</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>14</td>
<td>1084.0</td>
<td>-6.0</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>15</td>
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<td>-9.6</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>16</td>
<td>3346.0</td>
<td>-18.9</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>17</td>
<td>2844.0</td>
<td>-6.2</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>18</td>
<td>1174.0</td>
<td>5.6</td>
<td>0.0007 ϵ₀</td>
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<tr>
<td>19</td>
<td>1090.0</td>
<td>-3.3</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>20</td>
<td>1875.0</td>
<td>1.6</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>21</td>
<td>3347.0</td>
<td>0.1</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>22</td>
<td>1056.0</td>
<td>3.8</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>23</td>
<td>2431.0</td>
<td>17.1</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>24</td>
<td>2847.0</td>
<td>7.1</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>25</td>
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<td>-4.6</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
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<td>1.6</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>27</td>
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<td>6.4</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>28</td>
<td>3385.0</td>
<td>-1.1</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>29</td>
<td>2135.0</td>
<td>10.3</td>
<td>0.0007 ϵ₀</td>
</tr>
<tr>
<td>30</td>
<td>1248.0</td>
<td>3.2</td>
<td>0.0007 ϵ₀</td>
</tr>
</tbody>
</table>
Figure 4.20: Three dimensional velocity spectra obtained from the simulation. Thirty different objects with known location and velocity were used. The original location (before the first time step of the simulation) is shown as red dots. The blue path shows the trail observed by the radar due to the scatterer’s motion. The velocity of the objects was obtained using a Gaussian distribution.

4.4 Improved Radar Back-scatter Simulation

The previous simulation showed that the model and the computational engine used for the back-scattering simulation work. By using the convolution, the location, velocity and displacement are correctly obtained. The next logical step is to test the simulation engine under realistic conditions. A random atmosphere distribution of scatterers can be used to obtain such simulation.

Important topics to analyze for this simulation would be the size, shape, density and spatial distribution of the scatterers. Another important aspect to consider is the computational definition of these scatterers. In the previous sections a grid defined the paths that the radar pulse could take. Another grid contained the scatterer information and the product of both grids was used to define the atmospheric profiles. The grid multiplication method proved to be memory inefficient and computationally expensive even for cases where as few as 30 scatterers were used. A better approach would be to implement the scatterer (i.e. shape and location) through algebraic functions.

Modern high level computer languages can carry out algebraic manipulation of mathematical expressions. Commercial and free applications can be found to carry out such manipulation (e.g. Sage). One symbolic expression can be used to represent thousands of scatterers in the
4.4. Improved Radar Back-scatter Simulation

Simulated atmosphere with little memory consumption and without considerable compromises in the computation efficiency.

The Python language created random pairs \((x_o, y_o)\) used to locate the scatterers. The SymPy library created the exact symbolic mathematical representation for the atmosphere. This mathematical representation included the magnitude of the perturbation, the location in space and the spatial dimensions. Once the simulated atmosphere was created the radar simulation takes place by choosing the direction of propagation. The atmospheric profile was directly evaluated from the exact equation for the atmospheric perturbations in the chosen direction of propagation. No need for computational grids or interpolation arose due to the method, only the discretization of the equation was necessary.

4.4.1 Static distribution

In order to obtain a random atmosphere the scatterer’s shape and size must first be considered. As we saw in a previous section the radar backscatter is more intense in a Fourier component of length \(\lambda/2\) (in the radial coordinate). The approximation used created Gaussian perturbations in the index of refraction. Each perturbation represented by the expression (in 2D)

\[
\text{Perturbation} = \epsilon' \exp \left[ -\left( \frac{(x - x_o)^2}{2\sigma_x^2} + \frac{(y - y_o)^2}{2\sigma_y^2} \right) \right]
\]  

(4.28)

where \(x_o, y_o\) are the coordinates of the spatial location, \(\epsilon'\) the magnitude and \(\sigma_{x,y}\) represents the physical extent of the perturbation in both dimensions. For the following cases the value of \(\sigma_x = \sigma_y = \sigma\).

The general mathematical representation of the atmosphere’s perturbation of electric permittivity can be defined as \(P(x, y)\) and defined as

\[
P(x, y) = \sum_{i=1}^{N} \epsilon' \exp \left[ -\left( \frac{(x - x_{oi})^2}{2\sigma_{x,oi}^2} + \frac{(y - y_{oi})^2}{2\sigma_{y,oi}^2} \right) \right]
\]  

(4.29)

where the values of \(\sigma_{x,y}\) where chosen to represent a symmetric function, the location \((x_{oi}, y_{oi})\) of scatterer \(i\) is included, and \(N\) is the number of random scatterers used. Because multiple (usually thousands) of scatterers will be contained inside the radar’s volume an estimation of the best \(\sigma\) is in place. The procedure for the estimation is explain as follows.

A radar pulse full width at half maximum of 200 metres was used. The radial coordinate resolution was set to one centimetre. A two dimensional simulation with an area of 7.2 km\(^2\) (1200 m horizontal and 6 km height) was used. A random distribution of \(x_i\) and \(y_i\) generated the coordinates of three thousand Gaussian scatterers inside the described area. The radar’s beam width was 10 degrees centred at the vertical with one thousand separations. With these parameters the radar’s angular resolution is 0.01 degrees (\(\approx 0.000175\) radians).

To obtain a better understanding of the simulated environment a diagram is presented in figure 4.21. The thin lines mark the edges of the beam and the thick line represents one specific angle within the beam. Note that the scales in figure 4.21 are not 1:1, the actual area is elongated in the vertical. In figure 4.21 a plot of a random distribution of scatterers is presented. No accounting for the separation of the random points took place, and even when this can generate
Figure 4.21: Representation of the simulated area. The radar beam width is marked by red lines at angles of $\pm 5^\circ$. The blue line represents one specific direction of propagation. The size of the dots is not representative of the scatterer’s size.

rapid changes in the function and generate intense back-scatter it can be considered somehow realistic due to turbulent motions.

An example of the profile (divided by $\epsilon'$) obtained from the model is observed in figure 4.22. The $1/r$ factor for the decay of the electric field is not included in figure 4.22 but was included in other simulations. In this figure, because the profile was divided by $\epsilon'$ and the logarithm of the profile was taken before plotting, the value of 0 reflects a maximum value of 1 in the profile. The logarithmic scale makes visible small changes in the values. Most of the scatterer centre coordinates are far from the radar path under consideration. Even then, the scatterers are visible in a logarithmic scale because they are defined by Gaussian functions that will have values different from zero to the point where they can’t be resolved by the computer’s numerical resolution.

The profile is obtained by evaluating the model’s equation (4.29) at points over the trajectory. The trajectory $\vec{R}$ is defined by the angle (measured from the vertical) used to probe with the radar simulation.

$$\vec{R} = N\delta x \hat{r}$$

where the value of $\delta x$ is the resolution of the radial coordinate and was set to 1 cm. The value of $N$ changes from 1 to a value where $N\delta x \leq 6km$. The vector $\vec{r}$ is defined by the angle as
\[ \hat{r} = \sin \theta \hat{i} + \cos \theta \hat{j} \]. Each of the \( x, y \) pairs in \( \vec{R} \) are used to evaluate \( P(x, y) \) and generate the profile \( Pr(\theta) \).

Figure 4.22: Profile \( P(x, y) \) for values of \( x = \vec{R} \cdot \hat{i} \) and \( y = \vec{R} \cdot \hat{j} \). By using direct numerical evaluation of the mathematical representation the need to interpolate (used in some of the previous simulations) was removed. The profile must be multiplied by \( 1/r \) before carrying out the convolution.

The convolution is carried out between the profile \( Pr \) (divided by \( r \)) and the radar’s pulse \( (Tx) \) in the same way it was done for the examples shown previously. The value of the signal received will be a complex superposition of all the convolutions obtained for all directions. The radar’s signal is then

\[ Rx = \sum_{i=\theta_0}^{\theta_f} \frac{Pr_i}{r} \star Tx \] (4.31)

where \( \star \) represents the convolution.

In order to obtain a relationship between the maximum back-scatter and \( \sigma \) the integrated power of the signal \( Rx \) is needed. The value of the integrated power is proportional to the Fourier component of the profile, as shown in previous sections the maximum occurs at \( \lambda/2 \). In order to obtain the relationship the value of \( \sigma \) was changed from 0.05 m to 3.7 m. Higher values were also tested and the tendency (will be shown shortly) to decrease was maintained (not shown). The power of the signal was integrated for each \( \sigma \) in order to observe the variation. The results of the integrated power as a function of \( \sigma \) are shown in figure 4.23. The random distribution was kept unchanged for the different values of \( \sigma \). The maximum value of the integrated power was observed at \( \sigma = 1.4m \) (even for different sets of random locations). Similar values have been obtained by theoretical treatments (Briggs and Vincent, 1973; Hocking, 1987).

The basic necessary tests of the engine were carried out with a random scatterer location and different scatterer size. In order to increase the similarity between the model and reality, the motion of the scatterers needs to be implemented in the simulation. With realistic radar configurations regularly used in operational equipment a more appropriate product will be obtained. This will be covered in the following section.
4.4.2 Uniform motion

The previous section showed how by using static atmospheres the radar simulation was used to calculate the optimum sigma for maximum backscatter. We now turn to a more complicated task of calculating backscatter for a dynamic atmosphere. A purely zonal wind profile, constant in the vertical coordinate, was used to move the scatterers. Euler’s method was used to calculate the position of the scatterers. The number of scatterers was kept constant in the atmosphere as those leaving the volume were reinserted in the opposite end.

In radar applications the spectrum of the time series is the beginning of the information extraction process. The time series is calculated from successive measurements of the radar. The measurements are spaced at the inter-pulse period (IPP), which is usually a small fraction of a second (e.g. IPP \(=0.00033\) s is used regularly in Costa Rica). Once the spectrum is calculated it provides information about the power of the echoes, the radial velocity, and spectral width. Other parameters (e.g. noise level) can also be measured from the spectrum.

The radar configuration used consisted of a 150 m long pulse. The transmitter power was simulated to be 25 kW peak power. The Costa Rican radar operates at 46.6 MHz and the same frequency was used for these simulations. The range resolution of the pulse for the simulation was 10 cm. The same resolution was obtained from the atmospheric profile of electric permittivity.

The simulated atmosphere consisted of an area of 1.2 km by 6 km. Three thousand scatter-
4.4. Improved Radar Back-scatter Simulation

Figure 4.24: Description of three different experiments (A, B, and C). Experiment A consisted of a vertically pointing radar with a wide beam width of 10°. During experiment B the beam was tilted to 5° but the width preserved. During experiment C a narrow beam of only 2° was centred at 5°.

ers occupied simultaneously the space. The shape of each scatterer was determined by equation 4.28 with a width of 1.4 m. The 1.4 metre value is a result of the previous section, it provides the optimum scatterer shape for a $\lambda/2$ Fourier component in the atmospheric profile. A interpulse period 0.01 s was used for the measurements and no coherent integration was used. The total length of the time series used in the calculation was 20 s. The previous settings allowed velocities of more than 150 m/s to be measured unaliased, but the most interesting part of the spectrum was found in the ±20 m/s range.

In this type of simulation the radar configuration is not limited by engineering design. Different types of beams can be programmed into the simulation engine. It is up to the researcher using the simulation code to determine if these settings are representative of real scenarios. Three different simulations were carried out:

1. **Experiment A**: Wide beam pointed vertically (0°). In this configuration the radar beam width was set to 10 degrees. The centre of the beam was pointing to the 0° mark. One thousand different angles from $-5°$ to 5° were used in the simulation for a resolution of 0.01° per scan.

2. **Experiment B**: Wide beam tilted to 5° east from the vertical. This configuration had the radar beam width set to 10 degrees. The centre of the beam was pointing to 5°. One thousand different angles from 0° to 10° were used in the simulation for a resolution of 0.01° per scan.

3. **Experiment C**: Narrow beam tilted to 5° east from the vertical. Two different sub-cases existed for this simulation. (i) Narrow beam with equal resolution then the previous two cases (0.01° per scan). (ii) Narrow beam with the same number of divisions than the
previous two cases (1000 for a resolution of 0.002° per scan). The first case to equivalent to a subsection of experiment B. The second case assumes that all the energy of the radar beam can be concentrated into the narrower beam. Concentrating the energy into a narrower beam would require a bigger transmitter array in order to create a narrower beam with equal power. This case was treated as if a different radar was simulated. This second case will not be shown and was carried out as a test of the simulation engine.

These different experiments are depicted in figure 4.24. The general idea behind the design of these three experiments was to test the simulation of radar backscatter under realistic conditions. The transmitter in Costa Rica is small in size, creating a wide beam. This beam is pointing vertically and could be approximated as experiment A. More advanced radars can tilt their beam. Experiment B was designed with a wide beam of 10° tilted to be centred at 5°. The final experiment (C) as shown in figure 4.24, is still centred at 5° but the width of the beam is just 2°, a subsection of experiment B.

In real applications by steering the beam different types of spectra will be obtained. The width of the beam will also impact the result obtained in the spectrum. It is important to keep in mind that the only velocity present in the simulation is 40 m/s towards the east. If an infinitely narrow beam was pointed vertically it should not be capable of measuring the Doppler velocities of the scatterers as the motion would be perpendicular to the beam. If this hypothetical infinitely narrow beam was tilted to 5° the Doppler velocity measured would be

$$40 \text{ m/s} \times \sin(5^\circ) = 3.486 \text{ m/s}$$

which is the radial component of the wind along the path of the infinitely narrow beam.

In reality an infinitely narrow beam is impossible as it would require a infinitely large antenna. The impact of a wider beam is that the main component of 3.486 m/s will be accompanied by other radial velocities in the spectrum. These extra radial velocities come from the echoes obtained between the centre and edges of the beam (will be shown shortly). The effect of these different experimental configurations can be observed in figure 4.25.

Figure 4.25 shows the results for the three experiments. Different experiment results were sliced at five different heights (shown as rows in figure 4.25). Looking at the results from experiment A the spectra obtained is centred at 0 m/s. The majority of the spectral content is observed to be between ±5 m/s (off-centre vertical lines in figure 4.25). Results of experiment B are similar (in magnitude and width) to those observed for A. One major difference between A and B is that the latter’s spectrum peaks at a non-zero value. This result is expected as a radial velocity component should dominate the Doppler velocities. The results of experiment C are smaller in magnitude and narrower in width. These echoes also generated Doppler velocities with a maximum not centred at zero but displaced towards 5 m/s.

To properly analyze and compare the spectra of the different experiments, a Gaussian fit was applied to certain heights. The Gaussian fit was used to approximate the spectra and obtain parameters that could be readily compared. One example of a Gaussian fit result is shown in figure 4.26.

The results of the Gaussian fit approximate the spectrum with the parameters $A$, $f_0$, $\sigma$, and $D$. For the specific case shown in figure 4.26 the values obtained were $A = 142.05$ power$^7$ per

$^7$arbitrary
4.4. Improved Radar Back-scatter Simulation

Figure 4.25: Spectra obtained from experiments A, B, and C. Five different heights were used to obtain the time series and Fourier transform them to obtain the spectrum. For more information look into text.
Figure 4.26: Gaussian fit of the spectrum generated with a tilted wide beam. The height used for the time series was 2750 m. The equation of the fit is shown with the amplitude $A = 142.1$ power$^5$ per Hertz, Centre radial velocity $= 3.34$ m/s, Width $= 2.77$ m/s, and Floor level $= 0.5$ power$^6$ per Hertz. The standard deviation errors are 1.4, 0.03, 0.03, 0.1 respectively.

$$G(v) = 142.05 e^{-\frac{(v-3.34)^2}{2(2.77)^2}} + 0.52$$

Figure 4.27 holds the results of the four variables of the fitting process. Pane a) of figure 4.27 presents the variation with altitude of the amplitude $A$ of the fit. The value follows approximately the $1/r$ relationship (shown in the figure) that was multiplied by the atmospheric profile before the convolution. The upper heights of all experiments seem to agree with the $1/r$ function. The lower section differs between the wide beams and narrow beams; this difference can be attributed to the decrease in angle coverage. Less beam width with equal angular resolution means less returning power.

The second variable to analyze is presented in figure 4.27-b); this variable shows the radial velocity ($v_o$) where the maximum of the fitted function is found. The vertical lines are located at 0 and 3.486 m/s. The 0 m/s value should be the centre of the vertical beam values. The 3.486 m/s is the expected radial velocity for the tilted beams. The vertical beam is centred at 0 m/s at all heights. Both tilted beams are centred at similar values below 3 km, even the variation is similar. A big difference is observed past 3.5 km of elevation where the wide beam values fall below the 3 m/s mark. Past that height the wide beam deviates from the expected value whilst the narrow beam persist in its vicinity.

Figure 4.27-c) shows the floor of the fitted functions, which can be interpreted as the noise level of the simulated spectrum. The value $D$ decreases with height following approximately the same $1/r$ relationship (shown in the figure) observed for $A$.

The last variable of the analysis is $\sigma$. Its variation with height for the three experiments is presented in figure 4.27-d). The value of $\sigma$ reflects the width of the spectrum and it should be expected that it gets smaller with a narrower beam (except for turbulent contributions to spectral width). This behaviour is only observed above 1.5 km. At lower heights the width of the narrower beam exceeds the wide beam spectrum width. Towards the upper region of

---

8arbitrary
Figure 4.27: Results of the fitting process for the three experiments with a simulated atmosphere in motion. a) Maximum amplitude. b) Radial velocity. c) Floor level. d) Spectral width.
4.27-d) the two wide beams separate and while the vertical increases the value of $\sigma$, the tilted beam’s $\sigma$ decreases.

4.5 Realistic Radar Back-scatter Simulation

In previous sections, different models of the atmosphere were used to simulate radar backscattering. Initially a multi-grid approach was used in section 4.3 to simulate the atmospheric perturbations in electric permittivity. The model was successful in estimating the range and Doppler velocities of a small number of objects. Computational limitations with the implementation of the model were observed, an improved model was created. The improved version of the model was presented in section 4.4, where instead of using grids to define the scatterers, algebraic expressions manipulated by a high level computer language defined the perturbations in electric permittivity in space. This method improved on different aspects the simulation. The number of scatterers included in space can be in the order of thousands without impacting negatively the simulation engine. The time required to simulate was decreased from hours to minutes. As shown in the previous section, this new method can generate realistic looking spectral information.

Even in the improved case, all the previous models of radar backscattering were based on the idea that one dimensional profiles of the atmosphere could be obtained, to later be convolved with a one dimensional pulse. These previous models proved to be useful for radar simulation and results indicated that ranging and Doppler velocities can be successfully obtained. One aspect of the atmosphere was not resolved by such models. Turbulence can generate Fourier components in relevant scales which are not necessarily parallel to the radial vector from the transmitter to the scatterer. Motions not necessarily aligned with the direction of energy propagation can introduce extra Fourier components that will contribute to backscattering. A more realistic model can be generated by using the Fourier transform of all the space.

In order to appropriately account for all the possible Fourier components in space, the entire space would have to be converted into wavenumber domain. This would require to measure the scatterers with enough detail to appropriately capture all the Fourier components. The entire radar’s pulse would be equally transformed into wavenumber domain. Once the two relevant factors of the convolution are moved into the reciprocal domain the multiplication can be carried out. The inverse Fourier transform will generate the final result, the convolution of the radar’s pulse with the atmospheric multi-dimensional profile.

A similar methodology to the previous section was used. Mathematical equations programmed into the computed were used to generate the atmospheric profile. The two dimensional atmospheric profile $P(x, y)$ was obtained with the equation

$$P(x, y) = \sum_{i=1}^{N} \epsilon' \exp \left[ -\left( \frac{(x-x_{oi})^2}{2\sigma^2} + \frac{(y-y_{oi})^2}{2\sigma^2} \right) \right]$$  (4.33)

where $\epsilon' = 7 \times 10^{-4} \epsilon_o$ as introduced in previous sections.

The radar’s pulse was obtained from the multiplication of several functions. The frequency modulation ($F_M(x, y)$), amplitude modulation ($A_M(x, y)$) and the radar’s angular dependence (Gain = $G(x, y)$) were calculated with the following expressions
4.5. **Realistic Radar Back-scatter Simulation**

Figure 4.28: A diagram of the realistic two dimensional method is presented. Fourier transforms of electric permittivity perturbations and radar pulse are obtained. In the wavenumber domain the multiplication is computed and the result is inverse Fourier transformed to obtain the convolution.

\[
F_M(x,y) = \exp\left(-i(k\sqrt{x^2 + y^2} - \omega t)\right)
\]

(4.34)

\[
A_M(x,y) = \exp\left(-\frac{\sqrt{x^2 + y^2} - ct)^2}{2\sigma^2}\right)
\]

(4.35)

\[
G(x,y) = \exp\left(-\frac{-(\theta_o - \arctan(x/y))^2}{2(\delta\theta)^2}\right)
\]

(4.36)

where \(k\) is the radar’s wavenumber, \(\omega\) is the radar’s angular velocity, \(t\) is time, \(c\) is the speed of light in vacuum, \(\sigma\) defines the pulse length, \(\theta_o\) sets the direction of propagation of the radar beam, and \(\delta\theta\) defines the beam width. The equation that accounted for the \(1/r\) dependance of the electric field was multiplied by the space perturbations in electric permittivity.

A diagram showing the method is included in figure 4.28. The pulse length (FWHM) used was \(\approx 60\) m. Three hundred scatterers are located in the space. The radar’s beam width (FWHM) was set to \(13.5^\circ\). The spatial domain of permittivity perturbations is the result of evaluating equation 4.33 in the desired \(x\) and \(y\) locations. It is crucial to the result that the resolution of the space grid is large enough to capture the large wavenumber information. The value of \(\delta x\) and \(\delta y\) was set to \(1\) cm. This same spacing needs to be used in the evaluation of the radar pulse grid. If both separations and length are the same the calculations in wavenumber will be easy to carry out, but different values can be used as long as the appropriate considerations are made.
Figure 4.29: A two dimensional profile of scatterers is presented. Three hundred scatterers were located in the $3.6 \times 10^5 \text{m}^2$ area. The gain of the radar is included in the figure as a multiplication with the profile, hence the angular decay in intensity. The $1/r$ factor was not included in this figure but was included in the calculations of radar backscattering.
Table 4.4: The configuration of the Pulse Length and Beam Width presented in Figure 4.30 are shown. The full width at half maximum of the different Gaussian functions used to setup the simulations are also included.

The method presented in figure 4.28 generated a convolution result shown in the right pane. Due to the small size of the scatterers, large pulse length and wide beam, the echoes observed do not correspond to individual scatterers but to the addition of the returned electric fields. It is interesting to note that even when no individual scatterers are observed in the echoes, point like structures seem to appear in regions of maximum backscattered power. More about this will be mentioned shortly.

The method depicted in figure 4.28 implied a gain in processing time. The total calculation time for a space of 6 by 6 kilometres only takes 8 minutes. A drawback found in the method is the extreme use of temporal memory in the computing node. A small (6 by 6 kilometres) two dimensional simulation used 16 GB of RAM memory to generate the convolution information and plot it. This method is very promising but the memory allocation can be a major obstacle in implementation.

In figure 4.29 one of the profiles used is presented. Three hundred scatterers with equal properties were randomly distributed in the six by six kilometres area. The size of the scatterers was set with a $\sigma = 1.4$ m, which is the optimum value for Gaussian scatterers found in the previous section. No account for co-location or proximity of scatterers was carried out, giving the simulation a true random distribution as could happen with scatterers in a realistic radar measurement.

In order to use this method to create realistic radar backscattering multiple scenarios were created. Three different values of pulse length, and beam width were combined into 9 different simulations. Table 4.4 contains the data for the different experiments. In this table the first column contains the pulse length as specified in the program. A more easily understood pulse length is presented in column two where the full width at half maximum of the Gaussian defining the pulse is presented. Pulse lengths from fifty to one hundred metres were used, which are reasonable lengths used in radars. The beam width as specified in the computer programs is presented in column three of Table 4.4. The equivalent full width at half maximum of the angular displacement is presented in columns 4 and 5. Widths from almost 7 degrees up to 27 degrees were used in the simulations.

Figure 4.30 shows the results for the nine different experiments. The units used for the power are micro Watts. The distance in both axes corresponds to the uncorrected range equivalent. The power of the radar was considered to increase proportionally with the beam width. This was setup this way to observe clearly the dependence on pulse length of the returned signal. With increasing pulse length more power should be received from the scatterers, but less resolution would be obtained. This effect can be observed in any column of figure 4.30 by
Figure 4.30: Nine different simulations with properties defined in table 4.4 are presented. The three upper panes were simulated with a pulse length of $\sigma = 20$ m (FWHM = 47 m). The centre row of panes was calculated for $\sigma = 30$ m (FWHM = 70 m), and the lower three panes with $\sigma = 40$ m (FWHM = 94 m). The left panes represent cases where the beam width was defined by $\delta \theta = 0.05$ rad (FWHM = 6.7°). The centre column of panes with $\delta \theta = 0.1$ rad (FWHM = 13.5°), and the right panes belong to cases with $\delta \theta = 0.2$ rad (FWHM = 27.0°). The distance have not been corrected to compensate for the two way trip. See text for more details.
moving from top to bottom. As an example, let’s compare the top right and bottom right corners. In the upper right pane close to coordinates (-200,800) a clear echo (60 W) is observed. The bottom right corner presents in the same region an elongated echo (90 W) with more than 200 metres in extent while the previous echo measured close to 100 metres. By using pulses of twice the length, the echoes observed are also increased in size.

The data depicted in figure 4.30 shows that by increasing the beam width the angular field of view increases. More scatterer regions are observed as moving from the upper left to the lower right in this figure. This is interesting as the number of scatterers was kept constant for each different width. The increase in power consequence of longer pulses caused the appearance of these extra returns. One important aspect in radar experimentation was mentioned in Chapter 3. Many of the interferometric applications in radar assume that the origin of the scatterers are point-like structures. This simulation shed some light into that consideration.

The origin of the scatterers in the previous simulations were Gaussian like small structures. In the results presented in figure 4.30, large regions with strong echoes can be observed. These regions are not made out of single scatterers but a conglomerate of randomly located scatterers. These regions, none the less, will contribute to the backscattering, and when moving they could be used to track winds by using the Doppler velocities generated by the overall structure. But it is important to contemplate that the centre of the scattering region is not a point-like object, and methods that assume such relationship would be actually locating the origin of the summed echoes, not the individual scatterers.

Due to limitations in hardware the three dimensional simulation could not be carried out. The three dimensional case is now considered a future development project as more computational resources and/or large parallelization would be required. Equivalently the motion of two and three dimensional atmospheres will be developed in future experiments.

In the next chapter a different approach to the simulated atmosphere is taken. Instead of using idealized cases (as the ones presented in this chapter) a numerical model of the atmosphere based on fluid dynamics is in charge of generating the index of refraction inhomogeneities \( n' \) (based on atmospheric properties). The simulation engine developed in this chapter will be used to simulate backscatter with the information of the calculated \( n' \) (or \( \epsilon' \)).
Chapter 5

Atmospheric model and radar simulation

In Chapter 4 a radar simulation engine was created using the convolution theorem. It was thoroughly tested with simple cases and later with more complicated scenarios like volumes filled with Gaussian scatterers. Static and dynamic scatterers served to demonstrate its usefulness in radar simulations.

This chapter introduces a more realistic scenario by including fluid dynamics into the simulation. One goal of this research was to create a realistic atmospheric simulation capable of evolving according to the fluid dynamic equations. With the results of the atmospheric simulation a profile of atmospheric variables would be converted into index of refraction perturbations and these then used to simulate radar back-scattering. The equivalent of these steps together would be to create two complementary computer packages that simulate everything from atmospheric motion to radar back-scatte. This “simulation in a simulation” is represented in figure 5.1.

One of the more advanced cases used in the Chapter 4 used three dimensional simulated atmospheres with varying horizontal winds. No fluid dynamics was applied, hence the equations of motion, continuity and energy are not necessarily satisfied by the simulation. Instead of programming the equations of fluid dynamics from scratch, an atmospheric model created for such simulations, designed for comprehensive and high performance computing, was used.

Before introducing the model a scale analysis of the necessary resolution is needed. An atmospheric model with sufficient resolution can be used to naturally develop turbulence. With the variations of index of refraction naturally represented in the fluid, profiles of the atmospheric variables can be created and the index of refraction derived. The theory for the calculation of the index of refraction based on temperature, pressure and water content was presented in Chapter 2. The simulation of turbulence is a complicated task and an active area of research. In order to properly simulate it, a resolution better than the desired scale is always needed. The resolution needs to be a fraction of the target scale, allowing the small detail of the fluid to be fully resolved and avoiding aliasing issues. No parametrization would be needed at the same order of magnitude. By achieving such resolution the energy cascade of large to small eddies (see Chapter 2) can be modelled.

In order to appreciate the resolution needed from the atmospheric model to obtain appropriate radar back-scatte simulation, let’s consider the following case. A radar with a frequency of 46.6 MHz generates a maximum back-scatte at the scale of $\lambda/2 \approx 3.2m$. It was shown in Chapter 2 that the relationship between the inner scale $\ell_o$ and Kolmogorov’s microscale ($\eta$) is
Figure 5.1: Left: Fluid dynamic model product of realistic atmosphere. The resolution should be large enough that the $\lambda/2$ scale can be fully resolved. Centre: The radar simulation engine that was created for the simulations presented in Chapter 4 can calculate back-scatter from electric permittivity perturbation. The perturbations can be derived from the index of refraction obtain from temperature, pressure and water content information. Right: Combination of both simulations. The fluid dynamic equations generate the atmospheric information needed to calculate radar back-scatter.

\[ \ell_o \approx \gamma \eta \quad \text{with} \quad 7 < \gamma < 14 \quad (5.1) \]

If we consider that the value of $\gamma = 10$ is enough to separate $\ell_o$ and $\eta$, we could use the same factor to separate our desired resolution to the actual resolution used by the model. By using this assumption, a value 10% the size of the required resolution ($\approx 3$ m) the model could run with the order of 30 cm resolution.

Even if the resolution of 30 cm is sufficient for the realistic simulation of turbulence and radar back-scatter, achieving that value is considered in atmospheric science simulations too small to be resolved by a computational grid. To gain some perspective in the area of atmospheric simulation grid resolution let’s observe the planetary circulation model known as Global Forecast System (GFS).

The GFS was created and is maintained by the National Center for Environmental Prediction (NCEP). The model’s resolution is 28 km between horizontal grid point (9.3 $\times$ 10$^4$ times larger than needed at 46.6 MHz radar frequency) and with twelve vertical levels with a separation of 50 hPa. The GFS model serves as a first approximation of earth’s atmosphere. It is regularly used as an initial conditions for mesoscale models and localized forecasting. The resolution of 28 km is used for the first week of forecast; a model product with this resolution is shown in figure 5.2. For the second week of forecast (projecting 2 weeks into the future) the model resolution downscales to 70 km. Much better resolution is achieved by the mesoscale models that look into everyday weather forecasts.

One example of a mesoscale model is the Weather Research and Forecast (WRF) model. This mesoscale model can be used for research and operational forecasting. The WRF model was born from a multiple institution effort to create the next generation of forecasting tools for meteorological centres in the U.S.A. Two different cores exist inside WRF, the ARW (Advanced Research WRF) core and the NMM (Nonhydrostatic Mesoscale Model) core. The
difference between these two cores is inherited from two previous models ETA and MM5; both can be used for real and ideal cases. The resolution of the WRF model can vary from tens of metres to thousands of kilometres. A resolution of tens of metres is not necessarily the needed resolution but is certainly closer than a planetary scale model.

WRF’s cores can be used for real simulations with initial conditions including weather measurements and topography. This model can accurately predict the general circulations (large properties) and even resolve small scale with high accuracy (Done et al., 2004). On the other hand, the small detail is not necessarily precise and great variability enters the model’s small scales. The previous disagreement in the small scale can be due to inappropriate parametrization, inaccuracy of initial conditions and chaos. Other factors like rounding problems in calculations due to discretization of the equations also contribute to the variability.

The WRF model can also be used for simulations of idealized cases. In this type of simulation the parameters, physics and equations can be adjusted to suit a specific scenario. The scenario need not necessarily represent an existing condition on earth. It can be another planet’s atmosphere, two dimensional simulations of wave propagation, or mountain waves generated by an isolated mountain, to mention a few possibilities.

The version installed for the project was the WRF version 3.6.1. Inside this version the single real engine and different ideal engines are available. The “ARW Version 3 Modeling System Users Guide” contains the list of possible ideal engines. This Guide was created by the National Centre for Atmospheric Research of U.S.A and published in January, 2015.

Of particular importance to the present project is the large eddy simulation (LES) mentioned under three dimensional idealized simulations. By using the LES the unnecessary parametrizations can be turned off and full physics can be used to move the fluid, resolving the small scale needed for the radar simulation. A review of LES principles along with the model’s basic configuration, execution and post processing, is presented in the following section.
5.0.1 Large Eddy Simulation (LES)

Global and regional models with coarse resolution (greater than kilometres) can not resolve the fine details of the planetary boundary layer (PBL). In order to include the surface and PBL effects on larger scales, the information needs to be included in the model as a sub-grid parametrization. When the PBL parametrization is active in the model it is assumed that it can not resolve explicitly vertical diffusion. In those cases, vertical and horizontal diffusion are treated separately and independently as mentioned in section 8.5 of Skamarock et al. (2008).

The PBL scheme used is crucial in atmosphere simulations. Surface fluxes (e.g. heat due to solar radiation), moisture and momentum are determined by the chosen value for PBL in the configuration. There are four different schemes of PBL in WRF version 3, as described in section 8.5.(1-4) of Skamarock et al. (2008). Coarse resolution models can not resolve the small motions and eddies associated with small scale. When the grid resolution is increased (less than 100 metres) the eddies start to appear as a result of the motion equations. If the turbulent eddies and small convective events are resolved the physical processes can handle mixing and transport and no need for parametrization exists.

High resolution (enough to resolve specific turbulent eddies) configurations do not need a PBL scheme. Instead of the PBL model, a fully three-dimensional sub-grid turbulence scheme is used. The turbulence scheme for a high resolution model is based on the prognostic equation that governs the evolution of turbulent kinetic energy (TKE). The total derivative ($D/Dt$) of the TKE depends on the shear production, buoyancy and dissipation terms.

In addition to choosing the appropriate PBL scheme, other factors need to be adjusted in order to obtain a realistic simulation of the atmosphere. The WRF model incorporates different important topics related to the simulation and they can be activated when needed and turned off in case they are considered non relevant.

Micro-physics

The micro-physics in the WRF model is dedicated to resolve water related aspect of the atmosphere. Water vapour, clouds and precipitation are handled by the micro-physics option of the model. Nine different ways of handling the micro-physics are available in the model. For the specific case of the LES, due to the small vertical range covered by the domain, it was not necessary to account for deep convection. In addition, no need to calculate precipitable water or rain was found in the simulation.

Radiation

The model’s radiation scheme handles all the energy carried by incoming or outgoing radiation to the gases that make up the atmosphere. The radiative flux divergence and surface heat budget are handled by the radiation scheme. The only source of shortwave radiation considered by the model is the sun and the processes can include absorption, reflection and scattering. The longwave radiation accounts for infrared and thermal wavelengths absorbed or emitted by gases and surfaces.

The surface scheme used is relevant in the radiation handling because it defines the land-use type and the ground temperature. This was not particularly important because of the short period of time for which the simulations were run. The heating of the atmosphere via radiative
process is much smaller than the heat transport from the surface via convection. The energy sources considered relevant in this case are the surface heating (defined separately) and the energy provided by the wind.

**Boundary layer**

The boundary layer scheme is necessary when the model’s resolution can not possibly resolve the evolution of the PBL. With the resolutions that the model achieved the PBL was successfully simulated and evolved appropriately. No boundary layer scheme was used for the simulation, leaving the evolution of the lower tropospheric section entirely to the equations of fluid dynamics.

**Initial conditions**

In order to achieve a realistic simulation, the initial conditions where set to values that were representative of the Costa Rican atmosphere. The more relevant parameters are:

- Direct surface heating.
- Water vapour content profile.
- Temperature profile (unstable-neutral-stable).
- Wind profile.

These initial conditions are defined in the files used to initialize the numerical model and are shown in appendix F.

**Simulation settings**

The WRF simulation holds many variables and settings that can be adjusted to modify the physics and computational aspect of the simulation (see appendix F). The WRF model uses variables defined in the “namelist.input” file to configure each run. The variables control everything from physics, boundary conditions and parametrization to grid resolution, and time steps for the differential equations. The variables inside the “namelist.input” file also control when the grid state is captured to generate output. These variables need to be adjusted to maintain numerical stability in the solutions and to be computed in a reasonable time.

It is in this section that hardware becomes most important. The limitations on calculation time and processing units (CPUs) available for the model will impose an external boundary on the settings used. This is particularly important when stopping and restarting the model is needed, as will be shown shortly. The computer cluster used for this simulation is named *Orca* and is part of Sharcnet’s infrastructure.

Sharcnet is a Canadian consortium of 18 academic institutions. High performance computers are shared among the participants. The cluster Orca uses more than 8000 CPUs with high speed interconnections. As a user of the cluster up to 256 CPUs can be allocated simultaneously for up to one week. No limitation on RAM is imposed past the physical existence. This may seem like a huge amount of resources at first. Using the complete set of CPUs for 7 days
is equivalent (assuming a 1:1 time relationship) to using a single CPU for almost five years. As will be shown shortly, these resources are useful in the desired LES simulation, but not enough to achieve this project’s maximum goal.

In regard to initializing the model a different picture is observed. The WRF model was designed to be executed in parallel over multiple CPUs but the initialization process runs only on a single CPU. The initialization program for idealized runs needs to be carried out in serial mode. Time of initialization and temporal memory usage were huge limitations during this part of the process. Sharcnet’s node Iqaluk was of great use in surpassing the limitations due to the exceedingly large (1 TB) capacity of temporal memory available.

We now return to the namelist.input file used to configure the model. Different sections are used to define the variables used by the code. The time control section describes everything related to the time scale of the simulation, the time step used in the Runge Kutta Integration Scheme, and how often the model should output the atmospheric state. The time control section is also used to define how often the model should output restart information and how many grids per output file should be used.

The section domains defines the computing grids to be used. It can also be used to define nested domains. A nested domain (a computer grid inside another grid) is widely used to localize models. When the time step used in the model is smaller than one second the variable time step needs to be defined as 0. The variables time step fract num and time step fract den are then used to define the fractional time in terms of numerator (first variable) and denominator (second variable). These two variables are very sensitive to changes and the overall impact of the calculation time can differ by orders of magnitude. This variable regularly needs to be adjusted when numerical instabilities occur in the simulation; an error message in the log files presents this evidence every time it occurs.

The physics of the simulation is accounted for in the physics section. The micro-physics (e.g. rain), radiation and other schemes are contained in this section. This section also delimits how often these schemes are applied during a simulation. Surface layer structure and boundary layer schemes are also defined. Important water properties like cumulus (e.g. convection) and water body properties (e.g. sea surface temperature, sea ice) are defined and handled by the variables inside physics.

Of particularly importance to an LES is the dynamics section. Diffusion, damping and advection options are defined inside this portion of the configuration. Without the appropriate settings in the diffusion variables, the model would still parametrize the diffusion instead of leaving it to the fluid dynamic equations. Finally in the bdy_control section the boundary settings are established. In the LES used, the extend of the simulation did not allowed for satisfactory development of turbulence and model evolution. Cyclic coordinates were used in the zonal and meridional coordinates to re-inject the outbound motions into the model.

Appendix F contains the two files used during initialization of one specific case. As mentioned the namelist.input defines the variables and sections just described. The inputsounding file lists the value of the atmospheric variables used to initialize the model.

### 5.0.2 Original goal and approximations

The original goal of the simulation was to generate a simulated volume that could contain the realistic radar beam and maximum range of the Costa Rican radar. This is equivalent to
achieving dimensions of 6 km x 6 km x 6 km. With a resolution of 1 metre (not close to 30 cm but enough for a first approximation) the number of grid points is \( \approx 2 \times 10^{11} \) per variable. And it is a multi-grid simulation.

When the model was ready to run and configured with the adequate variable settings it did not take long to find out that the 1 metre resolution would be out of reach of the computing cluster. The frequency of the simulated radar pulse would have to be changed in order to compensate for the decrease in grid resolution and maintain comparable results. Due to limitations in the initialization program (not running in parallel) the resolution was lowered to a value that was possible to run in a reasonable time (days) in a single CPU.

The configuration approach was changed from trying to obtain the highest resolution to an incremental mode where the grid spacing could be decreased systematically. A first approximation of 16 metres resolution was first configured. This simulation tested the numerical stability of the model, duration of simulation, memory usage and products. In table 5.1 some settings are displayed. Each time step of the model time (0.1 seconds) took approximately 0.7 seconds. The total execution time for the model was 14.1 hours while using 256 CPUs, for a total of 149.5 days of computing time. The memory usage of the simulation was 115.2 GB for the entire job, equivalent to 463.1 MB per computing process.

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<td>history_interval_s</td>
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<td>Restart interval</td>
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<td>Simulation goals</td>
</tr>
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<td>Time Step</td>
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</tr>
<tr>
<td>Grid resolution</td>
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</tr>
</tbody>
</table>

Table 5.1: Settings of the first approximation of the simulation. Coarse resolution (respect to the goal) was used to obtain the first LES results.

The products of the simulation proved to be largely in agreement with the expectations. A cross section of the vertical velocity along the x coordinate is shown in figure 5.3. The data correspond to the atmospheric state after 10 minutes of simulation. The original motion of the simulation was purely horizontal and after 10 minutes the turbulent flux to the vertical created the motion displayed in the figure. Surface heating is contributing to the vertical motion in the lower section of the atmosphere. This region is the developing planetary boundary layer; it will become obvious when the radar simulation inside the digital atmosphere takes place (future section). The vertical motions observed near the 1 km mark are due to instabilities growing from the difference in density and the wind shear in that region (see initial conditions). Because turbulence is allowed to grow naturally inside the simulation it can reach the point where the surface generated perturbations can reach the heights of the wind-density perturbations and consequently interact. This interaction will be shown towards the end of this chapter, where a 25 minute simulation is presented.

Three dimensional grids containing many variables are generated by the model. From those grids the relevant atmospheric variables (among others temperature, pressure, water content, geopotential height and their respective perturbations) can be directly obtained or calculated.
Figure 5.3: Low resolution (16 metre) run of the LES. Vertical wind component slice along X is presented. The parameters used for the simulation were set to \( \delta x = 16 \text{m}, \delta t = 0.1 \text{s}, \text{I/O interval} = 30 \text{s} \). The grid size is 3 km (zonal) x 1.5 km (vertical) x 3 km (meridional). This data corresponds to 10 minutes into the simulation.

The grid can be visualized with software specialized for atmospheric data visualization (e.g. Interactive Data Viewer, IDV) or loaded into computer grids for further processing and analysis. This is exactly what needed to be carried out in order to obtain the index of refraction and perturbation of electric permittivity.

With a resolution of 16x16x8 m the model ran without complications. Systematic increase in resolutions took place. Values of 8x8x8 m resulted in satisfactory runs of the model. Grid spacings of 7,6,5,4,3, and 2 metres were also tested to observe the overall behaviour of the model and the adaptation to the clusters capabilities. Unfortunately the minimum grid spacing that generated an appropriate output without numerical instabilities and that could be completed in the available time of calculation (without restart) was 4 meters. Because the simulation goal of 1 metre resolution was incremented by a factor of 4, the frequency of the radar simulation was decreased by a factor of 4. Instead of simulating a radar in the 46.6 MHz frequency (Costa Rica’s radar settings), a value of 12.5 MHz was chosen to increase the wavelength to \( \approx 24 \text{ m} \). This would leave a factor of 6 between the radar’s wavelength and the model’s resolution.

This reduction in the frequency of the radar has implications on the results of the simulation. By being lower than Costa Rica’s radar frequency the results can not be compared directly to measured data without further consideration. Figure 2.5 taken from Hocking (1985) presents the different scales of turbulence for the first layers of the atmosphere. The range of the Costa Rican radar is below 10 km so only the lower section in figure 2.5 applies.

It can be seen in figure 2.5 that the length of the inertial range is thicker in the troposphere than in any other region. The original frequency of 46.6 MHz generates maximum backscatter at 3.2 m scale. The 12.5 MHz frequency simulation’s backscatter power peaks at 12 m. Both of these scales are well within the inertial range as observed in figure 2.5. Scales close to one hundred meters are near the edge of the buoyancy regime and the value of 12 m used for the
simulation is one order of magnitude smaller. Both scales (3.2 and 12 m) being inside the inertial range make it possible to compare the results as they would be similar in nature.

Before moving to the index of refraction calculation, consider the case of a higher resolution example. The following case will describe the 4 metre resolution case and later we will move on to the calculation of index of refraction.

### 5.0.3 Final configuration

This simulation was successfully carried out in the computing cluster with a resolution of 4 metres. The increase in resolution (smaller grid spacing) caused the model to become numerically unstable on different occasions. There was a need to lower the time step used for the integration of the equations to $1/25$ s in order to achieve stability. The $\approx 3 \times 10^7$ grid point simulation can be run in several days using 192 CPUs. Each time step taken by the model took different values of real time to complete, ranging from 2 to 843 seconds. Figure 5.4 shows a histogram of the real time required to simulate 0.04 s of model time. The most probable value is 3.4 s, for a ratio of 85 in time scales. The ratio of the times make it seem that two days would be necessary for the calculation but it took almost four day to complete.

![Histogram of real time required to compute one time step of the model’s time for a resolution of 4 metres. The most probable value is 3.4 s but values as large as 843 were logged.](image)

Table 5.2 contains some of the parameters used for the 4 metre model run. The total execution time for the model was 82.0 hours while using 192 CPUs, for a total of 654.6 days of computing time. The memory usage of the simulation was 230.4 GB for the entire job, equivalent to 730.4 MB per computing process. The number of CPUs was decreased from 256 to 192 due to several reasons. The most important reason was stability of the cluster run. During the tests of the previous jobs (larger resolution than 4 m), it was noticed that when requesting
the entire capacity of 256 cpus, some of the nodes assigned to calculate for the model crashed during the execution of the thread. When the number is decreased, the number of crashes decreased considerably, resulting in more stable runs and consequently faster product generation.

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</table>

Table 5.2: Settings used in the 4 metre simulation. The resolution of the model was sufficient to simulate radar back-scatter at 12.5 MHz. If a relationship between CPUs and processing time was maintained (not usually the case) it would take a one core computer 1.8 years to complete the simulation and would require one quarter of a terabyte in temporal memory.

In order to summarize the previously mentioned experiments the final output of the cluster queue system is included.

1. Grid spacing set to 16 metres, 60 minutes of simulated time:
   - cpu time: 149.5d / 256.0d (58 %)
   - elapsed time: 14.1h / 24.0h (58 %)
   - virtual memory: 463.1M / 1.0G (45 %)

2. Grid spacing set to 4 metres, 25 minutes of simulated time:
   - cpu time: 344.7d / 576.0d (59 %)
   - elapsed time: 43.2h / 72.0h (59 %)
   - virtual memory: 721.5M / 1.0G (70 %)

3. Grid spacing set to 4 metres, 45 minutes of simulated time:
   - cpu time: 654.5d / 768.0d (85 %)
   - elapsed time: 82.0h / 96.0h (85 %)
   - virtual memory: 730.4M / 1.2G (59 %)

The results of the 25 minutes simulation are exemplified in figures 5.5, 5.6 and 5.7. Figure 5.5 shows the atmospheric state at 9 minutes and 50 seconds into the simulation. In 5.5-a the three components of the wind vector are shown. The maximum in the zonal wind ($u$) used as initial conditions for the simulation is still observed, but perturbations have blurred the maximum originally located at 825 metres with a magnitude of 10 m/s.

Figure 5.6 shows the time evolution for two variables. The zonal and vertical wind were extracted from multiple times into the simulation. The instabilities in the zonal wind are observed as they grow in figure 5.6-a. Figure 5.6-b shows the evolution of the vertical wind for
Figure 5.5: a) Vertical cross-section of vertical ($w$), zonal ($u$), and meridional ($v$) wind. b) Horizontal cross section of the vertical wind ($w$) at 200, 600 and 900 metres of elevation. This data corresponds to 9 minutes 50 seconds into the simulation.
Figure 5.6: a) Time evolution of a vertical cross section of the zonal wind. Four times at 4, 6, 8, and 10 minutes into the simulation are shown. b) Two horizontal cross sections of vertical wind, extracted from 200 and 820 metres of elevation, are monitored as they evolve in time during the simulation. The left panes are found close to the surface, at 200 metres of elevation, and their evolution is dominated by surface heating. The right panes were extracted from a region located close to the maximum in horizontal wind, at 820 metres of elevation. The atmospheric motions at this height are dominated by the growth of instabilities.
two different heights. The turbulence created by the surface heating is observed in the horizontal cross-section of vertical wind at 200 metres; similarly the cross-section for 820 metres shows the evolution of the instabilities created by the wind shear.

A single time is shown in figure 5.7; where the vertical cross-section of the vertical wind at 9:50 into the simulation is shown. The horizontal extent of this cross-section is half of the previously shown in figure 5.3. The detail is considerably increased from the previous (low resolution) case as can be seen by comparing both figures.

Figure 5.7: High resolution (4 metre) run of the LES. Vertical wind component slice along X is presented. The parameters used for the simulation were set to $\delta x = 4\,\text{m}$, $\delta t = 0.04\,\text{s}$, I/O interval = 10s. The grid size is 1.5 km (zonal) x 1.5 km (vertical) x 0.75 km (meridional). This data corresponds to 9 minutes 50 seconds into the simulation.

It was mentioned before that the model had some restart issues. The problems were observed when restarting the model in order to split the total time run in sections. When allocating CPU time in the cluster, the longer the time requested, the longer the queue time is. For example, in order to run the 4 metre resolution model, four days of 192 CPUs were required. After submitting the job for execution it took more than a week for it to start running, a total waiting time of almost two weeks for the product. If shorter jobs could be submitted, the total time would not be as large. This topic is treated in a following section.

5.0.4 Simulated back-scatter and Vertical Beam Data

In section 4.3.3 some simple examples of back-scatter were shown. A more realistic approach was taken in section 4.4 when large numbers of individual scatterers were randomly distributed over the simulated radar. The basic principle of such simulations is that a profile of the atmosphere must first be obtained and later the back-scatter can be calculated as a convolution with the radar pulse (along with the $1/r$ factor). In this section the principles of obtaining the profile are first explained as well as how it was obtained from the LES results.

The results of the LES simulation are contained in a single netCDF type file. Each file is a collection of grids of 1, 2, 3 and 4 coordinates that contain all the information generated by
the model. The fourth coordinate is usually the time step of the simulation. The 25 minute simulation is contained in a 170 GB file while the 45 minute data generated more than 350 GB of information. The most relevant variables for the calculation of the index of refraction are

1. Water vapour mixing ratio. QVAPOR.
2. Perturbation Pressure. P.
3. Base state pressure. PB.
4. Perturbation potential temperature \( (\theta - T_o) \). T. Where \( T_o = 300K \).
5. Perturbation geopotential. PH.
6. Base-state geopotential. PHB.

The model variables are converted to the correct dimensions with

\[
e = P \frac{w}{w + 1}
\]

for the water vapour pressure. The equation for the temperature is

\[
T = (300 + \theta) \frac{P}{100000}^{0.286}
\]

and finally the equation for \( n \) introduced in Chapter 2 (equation (2.6)) is used to obtain the index of refraction (reproduced here for completeness)

\[
n = 1 + 10^{-6} \times \frac{79.0}{T} \left( P + \frac{4800.0e}{T} \right)
\]

The value of \( P \) is just the model’s pressure plus the perturbation part of the scalar field. After this step was carried out, grids of the same dimensions as P and T contained the variable \( n \). The equations for calculating the value of \( \epsilon' \) have already been introduced and shown in 4.25 (reproduced here for completeness)

\[
2n' \epsilon_o = \epsilon'
\]

which is valid for small values of \( n' \). The results of the procedure is a three-dimensional matrix of perturbation of electric permittivity that can be used to extract the profile seen by the radar in any direction of propagation. This is a great advantage of the method used. The radar can be pointed in any direction and the profile will be extracted from the grid directly. To illustrate this, let’s turn to figure 5.8.

In the upper pane of figure 5.8 the previously shown plot of the vertical wind is included along with a radar volume diagram and a figurative radar pulse. This is a representation of the simulation in a simulation. The Large Eddy Simulation (LES) is now combined with a Radar Back-scatter Simulation (RBS) to obtain a combined model, henceforth described as (LE-RB)S.

The radar can be located in any coordinates at the lower boundary of the grid. If the grid dimensions goes from 0 to 1536 m (4 × 384 divisions) in the zonal coordinate and from 0 to
768 m in the meridional coordinate, then the radar origin (radar antenna’s centre) can take any values as long as it points in a direction that generates a valid profile (values of permittivity can be obtained). The coordinates do not need to be a multiple of the grid division because it is independent of those coordinates.

The orientation of the radar is also independent of the grid. In order to achieve this the radar direction of propagation is calculated from the desired angle. This directing vector is used to generate the coordinates of the propagation path. Those coordinates are later filled with the corresponding value of electric permittivity perturbation interpolated from the model’s grid. After choosing a value of the desired angle a unitary vector in that direction is created and no radial distances greater than 1.5 km are used to obtain the atmospheric profile. Linear interpolation of three dimensional data sets entails the risk of generating spurious scatterers from discontinuities in the profile.
Figure 5.8 (lower pane) shows the simulated radar back-scatterer of a narrow beam pointed vertically. The radar’s properties used for the simulation were set to

- Frequency: 12.5 MHz
- Pulse Length (FWHM): 50 m
- Maximum range: 1500 m
- Simulation range resolution: 0.1 m

Different properties are observed in the lower pane of figure 5.8. During the initial minutes of the simulation the only back-scatter is generated by the large gradient of humidity present close to 1 km (see initial conditions). This is very characteristic of the boundary layer and regularly observed in Costa Rica’s soundings. With the time evolution of the simulation, two distinct regions start to generate back-scattering. After 10 minutes of simulation, the planetary boundary layer growth is observed. Turbulence from the thermals generated by surface heating create echoes up to 300 metres height. Also at 10 minutes but located at 700 metres over the surface, the evolution of the atmosphere due to the instabilities generated in the upper part of the simulation start to appear. These instabilities cause the non-uniform mixing of humidity, pressure and temperature, generating clear regions of rapid change in radar back-scatter. This rapid change is usually associated with non uniform mixing layers.

The time evolution of the (LE-RB)S in figure 5.8 (lower pane) is interesting. Fifteen minutes into the simulation, turbulence generated by two sources (surface heating and wind shear instabilities) reaches the 500 m level. Once these two sources of turbulence are mixed they are not distinguishable. The scattered power reaches a maximum at 15 minutes at 1 km height. That region is located close to the centre of the wave-like oscillations observed at that height, indicating non uniform mixing creating large gradients.

Towards the end of the simulation a new development in the lower simulated atmosphere is observed. After the mixing of the two different sources of turbulence occurs, low values of power are observed again below 500 metres of elevation. This indicates that the atmospheric variable gradients that generate back-scatter are no longer at the Fourier component most adequate for back-scatter generation. Another way of saying it is that the mixing has been efficient in that region, eliminating the gradients that were initially observed. This is particularly remarkable because the simulation is reproducing a phenomena regularly observed in radar, as will be shown shortly.

5.0.5 Tilted Beam Data

In the previous section the vertical beam of the simulated radar was presented and described. By using a simulated radar inside the LES data the radar beam can be steered simply by choosing a different direction of propagation. In this section the tilted beam simulation is explain and the results presented.

Figure 5.9 (upper pane) shows a diagram in the same way as it was previously presented in figure 5.8. In this figure, a vertical wind profile calculated at 9 minutes and 50 seconds into the simulation is used to represent the LES data. The vertical representation of the radar
in the figure indicates that the radar was tilted towards the x+ direction (from the centre of coordinates in the middle of the surface grid). It is worth mentioning that the diagram is only for illustration purposes, the beam width and pulse shape are not the ones used for the actual simulation.

While the beam was pointed vertically (previous section) the radar direction of propagation was chosen to be parallel to \( \vec{r} = C\hat{k} \), where C is just a constant. For the tilted beam, spherical coordinates were used to locate the chosen propagation path. For the tilted beam \( \theta \) and \( \varphi \) were chosen and the directing vector \( \vec{r} = C [\sin \theta \cos \varphi \hat{i} + \sin \theta \sin \varphi \hat{j} + \cos \theta \hat{k}] \) was used to generate the path. The value of C was kept under 1.5 km for all the cases and the discretization was calculated every one centimetre.

In the lower pane of figure 5.9 the resultant simulated power back-scattered by the LES atmosphere is presented. Similarities exist between this case and the previous one presented in figure 5.8 (lower pane). The radar simulation products for these two cases can be compared side by side in figure 5.10. The major characteristics of the boundary layer development are observed in a similar fashion but the upper limit of the PBL is more clearly defined in the tilted beam data. The top of the PBL also is found at higher elevations for the tilted beam; this can be due to the tilted beam effect (the actual height is just \( R \cos \theta \) where R is the desired range) instead of it actually being found at a different height.

In the upper section of the simulated backscatter power, similarities and differences are also found. The maximum of backscattered power observed from the 1 km height region is still observed in between the oscillatory sections, indicating layer-like structures or very active turbulence causing discontinuities in the index of refraction. The maximum backscatter observed is not located exactly in the same time and height location, it is slightly moved forward in time, and downwards in height.

Towards the end of the simulation the results presented in figure 5.9 (lower pane) show a significant difference to the vertical beam data. The region below 750 metres contains generally less power for the tilted beam than for the vertical beam. The reason for this difference can be due to two different causes. It can be an actual effect found in the simulation, known in radar as anisotropic back-scattering (or aspect sensitivity). This effect causes anisotropic scatter which has a non uniform angular dependance with stronger scatter from overhead and lesser scatter at off-vertical angles. This occurs because of “average” scatter body is stretched elliptically with larger horizontal wave-number than vertical. This can explain the observed difference of calculated backscattered power by the simulation. Another possibility that needs to be consider is that the LES model is not resolving with equal detail the turbulence generated horizontally as it is resolving the vertical. This would generate a weaker Fourier component in the tilted propagation path, resulting in less power being received by the simulated radar. This effect was considered to be mitigated by using the same resolution (4 metres) in all directions. The equations of motion should have the same resolution in all coordinates. This is not usual in atmospheric simulations (even LES) where most of the times the vertical resolution is different from the horizontal grid spacing.

Both vertical and tilted beam backscatter power are presented in figure 5.10-a) and 5.10-b) for comparison. The differences can be clearly observed by contrasting both data sets. A much better way to compare vertical to tilted back-scatter will be introduced shortly.
Figure 5.9: (upper pane) Schematic diagram of the model’s results with the simulated tilted radar inside the grid. The pulse is illustrative and no relationship to the actual pulse used for the back-scatter simulation exists. (lower pane) Simulated radar back-scattered power obtained from the model’s simulated atmosphere. Instead of using a vertical beam as in the previous example, a tilted narrow beam was used. An angle of 10 degrees toward the east was used.
5.0.6 Anisotropy

The difference between the backscattered power by a vertically pointing beam and a tilted beam was mentioned in the previous section. The backscattered power would only be equal in both cases if the scatterers in the path of the radar pulse are not aspect sensitive. Aspect sensitivity is defined as the property of scattering being stronger from vertical directions than from off-vertical angles (Hocking and Hamza, 1997). Different structures in the atmosphere can cause backscattering. Tilted specular reflectors and anisotropic turbulence can be affected by aspect sensitivity, as described in Hocking and Hamza (1997).

A year-long data set was used by Hocking (2001) to study the anisotropy in the troposphere. Higher values of power from the vertical direction were observed. The equation regularly used to calculate the anisotropy of the received powers is related to the variable $\theta_s$. This variable is used to parametrize the angular scattering properties. The equation regularly used when describing a narrow (pencil like) beam, as described in Hocking (2001) and references therein, is

$$P(\theta) = P(0) \exp \left( \frac{-\sin^2 \theta}{\sin^2 \theta_s} \right)$$

where $\theta$ is the angle of propagation from the vertical, and $P$ represents the received power at both vertical (0) and tilted ($\theta$) directions. The assumptions made in equation 5.6 include that the power will fall following an Gaussian function. This assumption is not adequate in cases where complete isotropy is observed nor when the off-vertical exceeds the vertical scattered power (Hocking, 2001).

In order to properly compare the backscattered power shown in the previous sections (lower panes of figures 5.8 and 5.9) a similar factor to the anisotropy term was used. Instead of using the same parametrization, only the powers were compared. The equation used to observe the anisotropy is

$$\text{Anisotropy} = \ln \left[ \frac{P[\theta]}{P[0]} \right]$$

where the angle $\theta$ is the direction of the off-vertical beam measured from the vertical. The value of $\theta_s$ was not calculated as in regular studies of anisotropy (Hocking and Hocking, 2007) where it was used to study its relationship with rainfall. None the less, the results of anisotropy from equation 5.7 can be used to observe the differences between the scattered power obtained from different paths.

The anisotropy result from simulated backscatter is presented in figure 5.10-c). In this figure the value of 0 (presented in white) means that equal power was obtained from both directions. Equal power means isotropic scattering. When $P[\theta]$ exceeds $P[0]$, equation 5.7 generates positive numbers (shades of red in figure 5.10-c)). Positive values are caused by off-vertical backscattered power being larger than the values generated by the vertical path. When $P[0]$ exceeds $P[\theta]$, negative values (shades of blue in figure 5.10-c)) are obtained. Negative values are caused by the vertical backscattered power being larger than the off-vertical. The latter case is the region following the definition on equation 5.6.

The top of the simulated boundary layer in figure 5.10-c) exhibits greater backscattered power while observed with the off-vertical beam than the vertical beam. This feature is ob-
Figure 5.10: a) Tilted beam back-scatter from LES atmosphere. b) Vertical beam back-scatter from LES atmosphere. Aspect sensitivity can be observed at different heights, this can be due to realistic anisotropic back-scatterers or lack of Fourier components in the tilted beam. The latter was mitigated by introducing isotropic grid spacing in the simulation. c) Anisotropy calculated with equation 5.7. The results show aspect sensitivity in all heights. For more detail see text.
served continuously until it mixes with the motions generated by higher-level instabilities. In the region of strong wind ( \( \sim 1 \text{ km} \) ), the simulation generated equal backscattered power during the initial minutes. Instabilities developing at 1 km generated differences in the backscattered power before 10 minutes into the simulation. Alternation between off-vertical generating more backscattering than the vertical, and vice versa, are observed in the vicinity of 1 km for the rest of the simulation.

The results observed in figure 5.10-c) confirm that anisotropy is generated by the simulation. The anisotropy result observed in this figure also shows that tilted beam echoes can exceed in value those generated by vertically pointing beam propagation. The region below 500 m between 15 and 20 minutes generates the strongest backscatter from the vertical. In contrast, below 500 meters between 20 and 25 minutes, the off-vertical is larger than the vertical.

In the following section the results of the simulations will be compared to radar measurements. Even when the time scales are different, similar backscattering properties are observed.

### 5.0.7 Radar vs LES Backscatter

The results obtained with the LES model as described in previous sections were used to simulate radar backscatter. The (LE-RB)S generated results that can be obtained with a radar pointing in any direction inside the volume. The validation of numerical models results is a matter of debate (Oreskes et al., 1994) but the heuristic properties are known the be practical. It can be considered that a simulation in a simulation is even harder to accept as a good approximation because of the explicit twofold abstraction.

![Figure 5.11: Radar measurements taken in Costa Rica’s radar.](image)

Numerical weather prediction (NWP) tools cannot be validated relative to the atmosphere in a perfect sense. NWP models are known to represent the atmospheric state accurately enough to forecast it with a precision of several days. This can be used to understand that
even when a model cannot be validated or even verified entirely, the results that it produces can be representative of reality and consequently of use in research and operational applications. For this to be true the physics of the model and implementation of the scientific tools need to be implemented appropriately.

The model that was described earlier as the (LE-RB)S can be considered to represent specific conditions of measured radar backscatter. One specific case observed in Costa Rica resembles the simulation and is shown in figure 5.11. The underlying radar measurements were taken on April 7th, 2014 with the radar operating in three-receiver mode (for more information about radar data and experiments look at Chapter 6). The horizontal scale of the (LE-RB)S results in figure 5.11 have been increased to better suit the comparison, the vertical scale is in 1:1 in relation to the radar data. It should be remarked that the radar information is not available from the surface to \( \approx 1 \) km due to the long pulse that was used and the engineering aspects of the radar digitization process.

The time scale of the radar measurement in figure 5.11 reflects the local time (UCT-6). It corresponds to the 24 hours of April 7th. Multiple layers are observed from the sunset of the previous day (left edge of the figure) up to the intersection with the top of the planetary boundary layer (look at Chapter 6). Of specific interest for this section is the layer observed close to 3 km of elevation. This layer evolves with time and is located closer to 2.5 km during sunrise (middle of the figure). This layer is observed at that height in the same manner that the layer of the simulated data was initialized in the (LE-RB)S as observed in figure 5.10.

Regarding surface heating the satellite images of early April 7th (shown in figure 5.12) can provide valuable insight. The satellite images presented are captured by the Geostationary Operational Environmental Satellite (GOES) system which is operated by the Department of Information for the United States of America. The GOES-13 satellite or, as it is known, GOES-east, captures different wavelengths that are related to different physical parameters in the images. The infrared channels (IR) are used to estimate temperature with a main focus on cold regions for cloud tops. The IR data can also be used to measure surface temperature in clear days where no clouds are in the way. This is precisely what can be observed in figure 5.12 (upper pane) where the IR channel for 10:15 am (local time) is shown.

As it is mentioned in appendix B, the radar is located in the north-west part of the country. This region is known as the Nicoya Peninsula. The IR channel image contains a temperature scale observed in the lower section of the figure. The surface temperature over the radar can be estimated to be 38 Celsius which is not unusual during a hot dry season day. The lack of cloud cover is evident in the lower pane of figure 5.12 where the visible channel is displayed. The visible image was taken two hours after the infrared image. The sea-breeze caused by the warm land next to the ocean is further evidence of the sustained surface heating.

The initial conditions used to setup the LES parameters agree with the described characteristics. Clear layers observed in radar data, and surface heating over the surrounding land of the radar site were used to configure the LES model. It is important to clarify that the (LE-RB)S is not intended to perfectly simulate this radar data. As mentioned before, the time scale of the simulation is smaller than the equivalent region shown in figure 5.11. The great similarities found between model’s results and radar measurements indicates that the model can satisfactorily simulate radar backscatter from realistic scenarios.

Under ideal circumstances the atmospheric simulation must emulate the real atmosphere and radar’s range. It would also need to account for the large scale winds and gravity waves.
Figure 5.12: (top) IR channel captured by GOES-13 on April 7th. The intense surface heating is reflected as a high surface temperature on the satellite image. Values close to 40 Celsius can be seen in the Nicoya Peninsula. (bottom) Visible channel captured by GOES-13 on April 7th. The sea-breeze caused by discontinuous temperature across the land-sea interface reflects as a coastline shaped cloud formation.
This would require a simulation with a height of 6 km with a resolution of 1 metre. This was the original goal of the LES model, but as described before the conditions, software and infrastructure imposed extra limitations on the goals. None the less, the simulated range and resolution were capable of achieving similarities to measured radar back-scatter and this can be considered an encouraging result. The satisfactory coupling of an LES model with a con-
volution back-scatter model can be also considered an important evolution in the radar field as no reference to such simulations exist.

When restarted, the used LES model discarded intermediate calculation grids. The intermediate grids are used to calculate derivatives in space and time. After a restart, without the previous step’s intermediate grids, extra frequency components are introduced into the numerical grid. In the following section an interesting event related to the simulation is presented.

5.0.8 Error in initialization and chaos

The simulation by using LES is challenging in many ways. Probably the most intense task is the computation of the data. In order to achieve an adequate time for result generation, the simulation needs to be run appropriately. An excessively high resolution will result in unrealistic computational requirements, and if computation times of one hour are desired, poor resolution would be necessary. During the execution of this model the cluster Orca provided the infrastructure necessary to obtain the products. Limitations regarding the use of the cluster exist, and they are closely related to the interesting characteristics of products generated by restarting the LES model.

The queue system of the cluster Orca is not a first come first served. Instead, the scheduler looks for the order of jobs that maximize the usage of resources. There are also some restrictions on certain contributed nodes that alter the regular queue (e.g. services to third party individuals or institutions). With the intention of optimizing the run time of the model, it was split in different sections. Stopping and restarting the model was necessary but (as will be shown shortly) the consequences of splitting the simulation forced the final configuration to be run only in a continuous mode.

The general idea was to avoid a problem of time scale of each run. For example, consider a test that changes the initial conditions of the model. After the initial conditions are set in the input_sounding file (see appendix F), the initialization program that created the necessary files for the model needs to be completed. The “ideal” program requires hundreds of gigabytes of temporal memory (RAM) and takes up to 3 days to run (depending on the usage of the node). Once the initial grids and methods are created, the model is sent to the queue of the cluster where it can wait its turn to run. This time can extend from days up to two weeks. When the call for the model occurs it will need four more days to complete the routines. Under these considerations, one single test with the model can take 2 or 3 weeks. If the model run failed for some reason (e.g. numerically unstable run) the process needs to be restarted again.

Because of these considerations, it was planned to split the run of the model. By splitting the execution, a simulation of 3 minutes or less could be sent to the cluster, executed and analyzed in a matter of days, instead of waiting weeks for a product. If the results of this short simulation were according to the desired case study, then all the rest of the simulation could be sent to the queue in parts.

One major issue arose from the splitting of the time domain. The atmospheric state was
Figure 5.13: Radar backscatter from the LES data. a) The high resolution model with an initial run of 10 minutes and a restarted version for another 10 minutes was used for this calculation. The extra frequency components included by the restart of the model are clearly visible. The strong atmospheric echoes after the 10 minute mark in the simulated radar backscatter power (see text for details) reflect those extra frequencies (discontinuous evolution). b) Simulated radar backscatter power obtained from a continuous (non-restarted) version of the same simulation as the one shown in the top pane. Continuity in the time evolution is observed during the simulation.
saved to the restart files, but not the intermediate grids. As it is mentioned in appendix I, when solving an equation it is usually needed to have not only the grid values, but also values at intermediate points. These intermediate points contain valid information about spatial derivatives of the atmospheric variables. When the model was restarted the values of these secondary grids were initialized but not with the previous state. This resulted in extra Fourier components being incorporated into the simulation as can be seen in figure 5.13-a.

The data presented in figure 5.13 correspond to two different simulations. The data shown in 5.13-a is a 20 minute simulation with a restart at the 10 minute mark. To contrast the previous information a continuous 25 minute run (a vertical beam trajectory was used) without restarts in the LES model is presented in figure 5.13-b. In this figure the first case shows a sharp change in properties at 10 minutes when the restart run occurred. The bottom pane of 5.13 shows a product of the model run with the same simulation but in continuous mode. Without interruptions the grids are maintained in memory, being updated as often as the model needs, to preserve all information in the spatial and temporal domains.

One direct consequence of the extra frequencies incorporated with the restart is that it invalidates the radar back-scatter simulations after the restart. The strong echoes observed after the 10 minute mark in figure 5.13 are not real and are a mere figment of the model introduced by the lack of information. When no restart is used for the modelling of the atmosphere (as in all the other cases shown here), such extra frequencies are not observed in the data of radar back-scatter powers, as can be seen in figure 5.10.
Chapter 6

Radar data and experiments

Several aspects of Costa Rica’s radar design enhance its research capabilities. In Costa Rica, the bandwidth allocation for this radar is large. With up to 5 MHz of bandwidth the spectra is 10 times wider than usually found in north-American countries (Hocking et al., 2014). The large bandwidth contributed to the satisfactory use of the deconvolution process in finding backscatterer profiles.

Another feature of this radar is that the hardware used in the detection has been minimized. Only one digitization channel per receiver is used; normally two are required for in-phase and quadrature signals. Each detector consists of just 4 amplification stages and one filtering circuit. No mixing or beating takes place in hardware, minimizing inputs of extra electronic noise and artificial frequency content in the signals.

The digitization and pre-processing software are provided by the company Mardoc Inc. This software stores the information gathered in binary files for later scientific analysis. Even though the company provides also the necessary software for analysis of echo power, spectral information and ultimately calculating wind, alternative programs were written in order to achieve a higher flexibility. By creating the subroutines, more control on the products and processing was obtained.

All the aforementioned characteristics of a state of the art radar (see appendix A) with custom created analysis tools optimize the products and results obtained. These results can be compared to those obtained with radars with power-apertures of hundreds of times larger. One of such comparisons will be taken independently with a statistical analysis. Fukao et al. (2011), showed that over the MU radar Kelvin Helmholtz oscillations were observed during only 0.6% of the time. An analysis about the presence of this type of oscillations will be presented shortly. A group of data-sets, obtained with Costa Rica’s wind profiler, was used to study how often these oscillations are detected by the radar.

6.1 A quick view of the raw information

In Chapter 3, tools for spectral estimation and methods to use them were introduced. Something that was not covered is how to obtain the simple spectral content of the radar information. From a single radar pulse only one profile (a value per height) is obtained. This height value corresponds to the value of the convolution of the pulse and the reflection profile at the corre-
6.1. A quick view of the raw information

Corresponding time lag. In order to understand how the atmosphere evolves with time, successive pulses are sent and the echoes recorded. This time series of echoes per height is used to obtain the spectral content.

A diagram to illustrate the process is presented in figure 6.1. Real radar information was used in this figure. The deconvolution information is pictured in the vertical direction representing the time lag (height) distribution. After enough values of the deconvolution are obtained (usually around one thousand), a single height transverse in time is used to obtain the time-series signal at that specific range. The foundation of the next step is the shift theorem for Fourier transforms.

![Diagram of the process](image)

Figure 6.1: Schematic diagram to illustrate the process of obtaining the spectrum. The values used in the diagram correspond to real radar information. The deconvolution result, time series generated and Fourier transform are presented.

The velocity information observed in figure 6.1 shows different peaks as a function of height. These peaks can be related to parcels of air in motion, atmospheric layers, meteors detected by the radar and other phenomena. We are interested in atmospheric motions that are usually found towards the centre of the spectrum.

In order to calculate the power of the echoes, these peaks need to be appropriately located and measured. Just by integrating different regions of the spectra, valuable information about the returning power can be obtained. Care must be taken when carrying out this integrations, due to the noise. At different heights the noise can exceed the signal and make it unmeasurable.
An alternative to integration is spectral fitting. By fitting a function to the spectra a more precise value of the result can be obtained.

In the following section, the raw data of several days of measurements is presented. An integration method was used to obtain these results.

### 6.1.1 Direct integration method of power estimation

Before proceeding to individual experiments, larger data sets, and information about the occurrence of events, let’s review some of the raw information of the radar. The raw information contains all the gathered signal. No removal of background noise or even spectral fitting has been applied to the information. This is very useful for diagnostic purposes.

The direct integration method consists of integrating the spectra between certain appropriately chosen values. The integrated value is considered a representation of the back-scattered power by slowly moving objects in the atmosphere. The frequency limit used in the next example (and in some other results) is $\pm 3$ Hz. This limit needs to be converted to velocity with the scaling factor $-\frac{\lambda}{2}$. Velocities less than or equal to $\pm 9.65 \text{ m s}^{-1}$ will be considered inside the integral. No distinction between positive or negative velocities is made. The range chosen for the integration is representative of what is considered the limits ($\pm 10 \text{ m s}^{-1}$) of vertical motion velocity under general circumstances (no strong convection).

$$P[H] = \int_{9.65 \text{ m s}^{-1}}^{9.65 \text{ m s}^{-1}} S_H(v) dv$$

The value of $S_H(v)$ is the spectral content as a function of velocity ($v$) for the desired height ($H$). The value $P[H]$ is the total power carried by all the velocities in the specified range for the specific height $H$.

Just to clarify completely the process. From thousands of consecutive measurements (the Costa Rican radar uses 60000) of atmospheric echoes per height a single number is obtained. In order to gather enough information for a complete day, millions of measurements of the atmosphere need to be taken. In figure 6.2 ten consecutive days are presented. This figure contains data from the 20th to the 30th of December of 2013. The radar configuration was setup to gather echoes using all the receivers acting as one (single receiver mode).

![Figure 6.2: Data gathered using the single receiver mode. The raw data presented has not been subject to any spectral fitting or noise removal procedure.](image)

Several aspects of the raw data are important to note. Only occasional errors in the data set are observed. Between December 24th and 26th, four vertical spikes can be catalogued as
corrupted information. Diurnal variation of the echo power is present during the entire recorded period. Continuity of the events is clearly observed. A large oscillation with 24 hour period (or close to) is observed in figure 6.2.

The oscillation observed is due to the noise generated by the galaxy at the frequency of the radar. Extraterrestrial signals can be digitized along with locally generated noise and atmospheric echo information (Jansky, 1932, 1933a,b). In these three articles, the father of radio-astronomy observed the galactic noise and described it as: “a faint steady hiss”. The radar in Costa Rica is sensitive enough to capture Jansky’s hiss as can be observed in appendix G, and is observed independently of the method of analysis used. These observations allowed us to measure the sidereal time, which is used to define a time relative to the far-away stars instead of the sun (Aoki et al., 1982). Considering the sidereal time being 23 h 56 m 4.1 s, the galactic noise appears approximately 3.93 minutes earlier each day. Using the data measured in Costa Rica, it has been estimated that the source of the galactic noise appears 4.37 minutes earlier every day (11% error). The source of this error can be attributed to a short period of time being used to calculate it.

The technical aspects of the radar allow the galactic noise to enter the digitized signal. The radar’s central frequency is 46.6 MHz, but the bandwidth of the antennas is wide (4 MHz) as shown by Hocking et al. (2014). The hardware filters used in the receivers will allow other frequencies around the centre value to pass into the digitizer. For atmospheric studies this information is not valuable and actually any galactic source is considered noise.

In order to remove this noise different techniques can be used. The galactic noise will be present at any height, even when the convolution of the signals is buried into the noise. By analyzing heights where no atmospheric echoes appear, an estimation of the noise generated is obtained. This noise can then be subtracted from the data obtained for other heights. Another way to remove the noise is by using spectral fitting. A Gaussian function can be used to approach the shape of the signal and use the floor level of the fit to remove the noise. Only the values between the floor and the tip of the Gaussian function can be recognized as power back-scattered by the atmosphere.

### 6.1.2 Gaussian fit method of power and variable estimation

The integration method just presented was based on integrating the spectra. By integrating the spectra between limits an estimation of the power is obtained with fast results but at the cost of losing information. A better way to obtain the spectra would be to fit a function to the data and obtain as much information as possible.

In clear air atmospheric spectra, most of the observed phenomena can be associated with a Gaussian shape. One phenomena that generates an echo similar to a exponential distribution is the meteor trail echo. The meteor trails are usually detected in the time domain and because they are not part of the spectra generated by low-troposphere phenomena will not be considered here. Atmospheric echoes corresponding to rain and air are usually associated with Gaussian distributions (Woodman, 1985; Campos et al., 2007).

Fitting a function to data is carried out by adjusting the necessary parameters and using appropriate initial conditions. In a Gaussian function the values of these parameters come from the equation for such a function
Chapter 6. Radar data and experiments

Figure 6.3: Integration process result. Instead of obtaining multiple parameters (see next method for an example) the integration process can only generate one value of power as a result of the integration per measurement per height. The data pictured was obtained using one receiver mode during an experiment carried out April 3rd, 2014. Thousands of measurements and heights are involved in generating an entire day of information. The power magnitudes were not included as this is just an example of the integration method products; it will be included when data analysis takes place.

\[ f(v) = Ae^{-(v-v_0)^2/\sigma^2} + D \]  

where the values \( A, v_0, \sigma \) and \( D \), are used to calculate the fit. The method used for the fit was Nonlinear Least Squares Fitting. This process can approximate a function to the data and possibly converge to a solution.

From the given variables, valuable information is obtained. The value of \( D \) represents the floor level of the function. In our case, this would be the noise level at the height under consideration. By using \( D \) in the fitting process the noise of galactic sources was immediately removed from the final products, improving considerably the visualization of the results; however, some uncertainty in \( D \) is observed in the results and even structure can be found along with the noise level information.

The value of \( \sigma \) can be associated with the width of the spectral peak found. This width is relevant to turbulence studies because it is related to the velocity distribution of turbulence. The width of the peak is also affected by the interference pattern shape of the radar (Dehghan and Hocking, 2011). In turbulence, the value of sigma can be associated with the turbulent energy dissipation rate. As will be shown shortly, large values of \( \sigma \) can be observed in regions of the atmosphere where strong convective currents are observed as well as in atmospheric layer of turbulence.

The \( v_0 \) fitting parameter represents the centre of the Gaussian function. It is the value (in meters per second) of the location of the maximum. This value determines if the function under consideration represents air moving towards or away from the radar, and how fast it is moving.

Finally the value of \( A \) in the fitting process represents the amplitude of the function at the location of the maximum. The value of \( A \) will be obtained in the Gaussian function only when
\( v \) equals \( v_o \). \( A \) does not represent the power carried out by the spectra but only the amplitude.

In order to approximate the power the integration of the fitted Gaussian function needs to be carried out. The difference between the magnitude of \( A \) and \( D \) needs to be taken into account for the integration.

In order to estimate the power, the fitted equation needs to be integrated. By using

\[
\int_{-\infty}^{\infty} e^{-x^2} \, dx = \sqrt{\pi}
\]

the result of integrating is

\[
P = \int f(v) \, dv = \int_{v_i}^{v_f} \left[ Ae^{-\frac{(v-v_0)^2}{2\sigma^2}} + D \right] \, dv = A\sigma \sqrt{2\pi} + D(v_f - v_i)
\]

where the final term is just the area under the noise level. What is more important is the region that is observable over the \( D \) level. This integration is just the \( A\sigma \sqrt{2\pi} \) value.

One important limitation of the current method is that it is not designed to resolve more than one spectral peak per analysis. If two or more peaks were found the result could not be trusted. This can occur in situations where multiple wind currents are located inside the radar scattering volume or rain is detected by the radar in the vicinities of the peak.

In order to better appreciate the difference between the integration method and the Gaussian fitting method two examples are included. Figure 6.3 contains one example of the Integration method for power estimation. Figure 6.4 presents all the parameters obtained by using the fitting method. The second method generates more information about the atmosphere under study. The first method is fast to implement and can quickly generate a first approximation of a data set.

In the following sections, depending on the detail needed, one of the two methods presented will be used. When the motion or type of the echoes is important the fitting method will be used.

### 6.2 Chronological tests and experiments

In this section a small review of the different experiments carried out during the past few years is included. Some of those experiments were quite successful and some others (like number 5) helped to improve the radar.

Experiments one and two were carried out nearly after the completion of the minimal infrastructure needed to operate the radar. Their success proved that the radar was functional and atmospheric echoes could be obtained satisfactorily. Experiments three and four confirmed that the radar could be used for interferometry and multiple receiver techniques. The usefulness of experiments three and four was provided by using three receivers instead of a unified input.

Experiment five was in place because of a new digitizer card was replaced in the radar hardware. In science (as in many other areas) success does not teach as much as failure. This new card, did not appropriately fit the equipment and it is evident in the results. Measures needed to be taken to surpass this problem and continue with the operation of the radar.

The focus of this document is experiments six and seven. They contain the vast majority of data used for the analysis. Data from those experiments was used in Chapter 3 for the
Figure 6.4: Fitting process result. The parameters from the fitting process are shown. a) Power, b) Amplitude, c) $\sigma$, d) $v_0$, e) $D$, f) Diagnostic graphs. The data correspond to measurements taken on September 14th, 2014. Three receiver mode was used in the radar during this experiment. An example of what the diagnostic graphs can be used for is shown in Figure 6.12. The diagnostic figures are just histograms of the fitted variables and scatter plots of one variable as a function of the other. The magnitudes in the color bars are missing but it is not relevant here as the images are intended as an example of the fitting method products. The magnitudes are included whenever the analysis is presented.

interferometry section as well as the radar overhead calculation. The importance of those experiments is due to the large amount of data they contain and the timespan.

In the following segments a brief view of these experiments and some of the results are presented.

### 6.2.1 Experiments 1 and 2

Information about experiments 1 and 2 is given in table 6.1. The construction of the antennas and cabling was finished in August, 2011. The computational equipment as well as detectors arrived in Costa Rica late in December. The equipment was assembled and tested (after the holidays) and soon proved to be operational.

The first experiment with the radar took place in January 8th, 2012. During this time, it is dry season in the location of the radar and it is normal for temperatures to reach values of 35 degrees around noon. The high temperature at Santa Cruz contributed towards the excitement of running the radar for the first time. Close to mid day the radar was turned on with a pulse
6.2. **Chronological tests and experiments**

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<th>Year</th>
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<tbody>
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<td>Type of experiment</td>
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</table>

Table 6.1: Information about experiments 1 and 2.

![Experiment in Santa Cruz, Costa Rica, January 8-9, 2012.](image)

Figure 6.5: Experiment 1: First measurement with the radar during PhD research. Just 5 months after the beginning of the program.

repetition frequency (PRF) of 3000 Hz and a coherent integration value of 8. This first experiment generated the data presented in figure 6.5. Strong echoes are observed between 1.5 and 2 km, with some other echoes occurring between 2.5 and 3 km. The presence of structures in the power map confirmed that the radar was gathering atmospheric echoes successfully.

After the initial test was completed, tuning of the radar components took place. A new experiment was set up where the PRF was increased slightly to 3072 Hz, and the coherent integration increased to 64. The goal was to improve on the quality of the data as well as the resolution. The results of this second experiment are contained in figure 6.6.

The increase in resolution is evident. More detail is observed in all the vertical extension. By using 16 hours of data, separate structures (and their evolution with time) are identified in the 1 to 3 km range. With this experiment the radar’s capabilities to obtain high resolution data in the lower troposphere was confirmed.

In the following experiments, multiple receivers were used to test the radar’s infrastructure.

### 6.2.2 Experiments 3 and 4

These two experiments took place after some rewiring to connect the three detectors to the three arrays of receiving antennas. Information about these experiments is shown in table 6.2. Proof of the three receiver capability of this radar was imperative to obtain winds. Without multiple receivers there are no multiple signals to compare phase changes.
Figure 6.6: Experiment 2: Radar measurements with the improvements from the previous run.

<table>
<thead>
<tr>
<th>Year</th>
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Table 6.2: Information about experiments 3 and 4.

The results for the first experiment are presented in figure 6.7. The power observed in the figure is obtained by adding (with the correct phase correction) all three receiver data into one signal. The high resolution provided by the high bandwidth is still observed. Different sources of interference were identified in this experiment.

For experiment four, an increase in the gain of the receivers was performed. This increase in gain, along with better noise reduction algorithms provided the required information to obtain the data presented in figure 6.8. This plot was a definitive confirmation that the radar had achieved a good tuning level and could be used as a tool for atmospheric research. In fact, this same plot was published in Hocking et al. (2014) as an indication of the good products generated by the equipment.

Interesting facts about the atmosphere can be readily obtained from figure 6.8. A large number of atmospheric backscatterers can be observed, and on some occasions (e.g. Feb 23rd at 01 : 00 UTC) as many as 8 simultaneous layers occur between 1 to 5 km. There is a quasiconstant region of strong returns centred at 2 km during the entire experiment (observed as shades of red in figure 6.8). This echo region is located approximately 2 km but it varies from 1.5 to 3 km depending on the time of day. This strong echo region can be associated with the boundary layer dynamics as will be presented in a following section.

Another aspect that can be observed in figure 6.8 is the growth of the diurnal (convective) boundary layer. This is present from the 14 : 00 UTC up to the end of the experiment. It can be observed as strong echoes ascending with time. These echoes are observed to ascend at approximately 0.4 km/h.
6.2.3 Experiment 5

In this section a hardware test description is included. The information about experiment five is included in table 6.3. A digitizer card replacement was acquired and this section shows it’s initial results. As every test in scientific equipment it can lead to changes and improvements to better apply the devices to the project. This case is included in this document because of it’s particular impact on the observed information.

The experiment recorded data for 24 hours. During this time the same configuration for all other parameters was unchanged. The results are presented in figure 6.9. It can be observed that there is a large amount of noise present, represented as vertical lines of intense echoes. The atmospheric information can still be resolved in the background. For example, the boundary layer growth observed in Experiment 4 is clearly present from the 14 : 00 UTC to the end of the experiment reaching a height of 3 km towards the end.

All other aspects are smeared out by the interference. This clearly was not a successful test.
Table 6.3: Information about experiment 5.

<table>
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</tr>
<tr>
<td>Type of experiment</td>
<td>Digitizer card testing</td>
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</table>

The digitizer card did not fit appropriately the rest of the equipment.

Figure 6.9: Experiment 5: Testing with a 4 kW transmitter and receiving using the 3 available receivers with a bandwidth of 2.5 MHz.

During a visit to the radar site some changes in the clock signal solved the problem. The different type of digitizer card required a slightly higher level of clock voltage than its predecessor. The changes in the radar’s triggers allowed this problems to be surpassed. Normal operation was observed with these changes in place.

6.2.4 Experiment 6 - DRY SEASON

The experiment described here consisted of several different radar configurations. These configurations were used to test the radar’s performance in atmospheric information gathering. One common factor in the setup of the experiments is the use of only one receiver. Hence the data-set generated during this experiment will usually be referred to as “one receiver data”. This one receiver information covers the dry season in Guanacaste and the beginning of the rainy season; basic information about the experiment is given in table 6.4.

One major change with respect to the previous sections is that for the long term experiments (6 and 7) a low power transmitter (one kilowatt) was used instead of the large one (four kilowatt) used for the previous experiments. Proper measures were taken to compensate for the decrease in signal to noise ratio. The change in the transmitter power did not affect negatively the products of the radar as similar structures were observed with both transmitters.
### 6.2. Chronological tests and experiments

<table>
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<td>2014</td>
<td>5</td>
<td>12</td>
<td>26536</td>
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</table>

Table 6.5: Experiment 6: Dry Season information

An example of one receiver data and its analysis is presented. Figure 6.10 contains the information gathered during December 23rd, 2013. Different layers are observed in the vicinity of 3 km. Three different layers are observed after midnight and the layer located at 3.3 km splits into two new layers separated by 300 metres. The layer splitting is clearly observed in the spectral width of the signals (not shown) and will be explained shortly after Experiment 7. Another important aspect observed in figure 6.10 and present during the vast majority of days recorded is the planetary boundary layer (PBL). It can be observed after 10 am (local time) growing steadily reaching the 2.2 km mark before 1 pm. A second intrusion of the PBL is observed after 2:30 pm when the upper limit is observed to increase again reaching 2.9 km around 5 pm in the afternoon.

Some of the characteristics mentioned about figure 6.10 are commonly observed in the data set for the dry season. A similar analysis was carried out for each of the available complete days and the results of the analysis will be introduced after some of the interesting case studies are covered.

In the following section a different configuration with more receivers is described. Measurements during rainy season will prove to be different in content and evolution (temporal and spatial).
Figure 6.10: Measurements taken with a 1 kW transmitter. Digitization was carried out by using a unified receiver with a bandwidth of 2.5 MHz. Layers can be observed at the same height that the previous day boundary layer had at sunset (6 pm). After sunrise the new boundary layer grows (10 am) and reaches initially 2.2 km, with a second increase in height observed past 2 pm reaching 2.9 km.

<table>
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<th>Year</th>
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Table 6.6: Information about experiment 7.

### 6.2.5 Experiment 7 - RAINY SEASON

Experiment 7 consisted of a single configuration used for measurements during the rainy season of north-western Costa Rica. During this experiment three receivers were used for all the measurements, hence the data set will be usually referred to as “three receiver data”. The experiment can be summarized as shown in table 6.6.

A listing of properties of the gathered information is presented in table 6.7. The information in this table implies that the efficiency of the radar decreased considerably when compared to Experiment 6. This decrease in efficiency can be attributed primarily to two factors. The first external agent is the inadequate cooling of the facilities where the equipment is housed. Without the necessary heat extraction the computer parts are prone to errors causing failures in the operating system. The second factor is the power outages caused by lightning during thunderstorms, common during the rainy season. The radar’s operation was interrupted (on occasions for several days) by these two factors, causing the decrease in efficiency. Countermeasures will be taken in future experiments to obtain improved performance.

An example of the results obtained during the rainy season operation is presented in figure 6.11. The data depicted in this figure correspond to October 12th, 2015. The presence of layers is not common during the rainy season (as will be shown later) but in this image an interesting...
6.2. **Chronological Tests and Experiments**

<table>
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<td>9</td>
<td>11083</td>
<td>64</td>
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<td>19</td>
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<td>2014</td>
<td>11</td>
<td>6</td>
<td>7368</td>
<td>43</td>
</tr>
</tbody>
</table>

Table 6.7: Experiment 7: Rainy Season information

An event is presented. Three different layers are observed at 2.2, 3.1 and 3.8 km during the first half of the day. The layer centred at 3.1 km shows oscillation of 600 metres in amplitude between 4 and 6 am. Afterwards the 3.1 km oscillation seems to disappear after colliding with the intrusions of the PBL observed at 10 am. No further layer activity was observed for the day.

This behaviour of layers terminating after encountering an active PBL was not isolated but repeated on several occasions. During the dry season months this was not the regular interaction between atmospheric layers and PBL. During the dry season many of the layers observed over the PBL top height were bent to follow the PBL shape. When the PBL growth was fast, the layers would interact with the PBL formation and continue to exist in an altered form. One example will be presented shortly in a section dedicated to such case studies.

Figure 6.11: Experiment 7: Measurements taken with a 1 kW transmitter. Digitization was carried out with three different receivers and a bandwidth of 2.5 MHz. Layer presence is not as common during rainy season as it is during dry season. The layer centred at 3.1 km is observed to oscillate with increasing amplitude towards sunrise. Later interaction with the PBL intrusions caused the termination of the layer as they are no longer observed in the following hours.

Experiments 1 to 7 were included to present the variety of experiments carried out in Costa
Rica. These experiments allowed the research team to explore the capabilities of the new radar. Testing and enhancing of the radar was challenging and time consuming but the products proved to be suitable for research. With the results obtained after long periods of operation (Experiment 6 and 7) a thorough analysis of the data took place.

The analysis of long data-sets is demanding. Careful consideration of the information needs to be taken. Local atmospheric conditions and radar design are important aspects affecting the observed products. Knowledge about both of those aspects is crucial in understanding and assimilating the radar’s products into valuable quantitative information.

The methodology used in understanding the information will be presented in the following section. This information can be converted into statistics that later will be used to categorize atmospheric events. The presence of layers in the low troposphere and PBL upper heights are examples of events that will be analyzed for all the available days of data.

### 6.3 Data analysis and methods

The previous sections provided an overview of the different experiments in Santa Cruz’s radar. The long term experiments (6 and 7) recorded atmospheric echoes during months between December 2013 and November 2014. More than 190 complete days were logged into memory and later analyzed. The procedure used to review, categorize and quantize the information is described here.

As mentioned previously, each set of measurements taken by the radar will generate a spectrum. This spectrum was approximated by a Gaussian function and the information plotted as pictured in Figure 6.12. This figure contains the information for one specific day (April 13th, 2014 for this case). By using these images the events can be categorized according to their observed properties. Atmospheric structures and evolution observed in the figures can be associated with different atmospheric processes.

As an example of the analysis carried out for each day, we will describe the data presented in figure 6.12. In figure 6.12, different structures are observed. Clear layers are present during the night time and two of them, located at 2.4 and 3.8 km, persist after sunrise. The lower layer is observed to split into two other layers after 8 am; however, the upper layer is thin and the resolution in the figure does not allows for proper identification. After the separation takes place a clear oscillation is observed in the lower branch with a period of 7.65 minutes encircled in figure 6.12-d). The upper branch seems to hold the same internal oscillation observable at 10 am local time. A shallow planetary boundary layer is observed growing from 7 am to 9:30 am (clear in figure 6.12-c), reaching a local maximum with a depth of 1.8 km. Before noon another growth of the PBL is observed, this intrusion reaches past 2 km, colliding with the layer located at 2.2 km. The results of this interaction is observed from noon to 2:30 pm (all panes of figure 6.12). Strong echoes from 1.8 to 2.9 km are observed during that time. As reflected in figure 6.12-d) within the radial velocity plot there is oscillatory motion inside this region. The period of this oscillation is different from the one observed hours before inside both branches of the layer after the splitting. A larger period of 11.47 minutes is first observed after noon centred at 2.5 km. A few extra cycles are observed towards the end of the oscillatory section but the period is now increased to 12.08 minutes.

Analysis like the one just presented were carried out twice (separately) for each of the
6.3. Data analysis and methods

available daily data-sets. In this manner the information was quantized and statistics were calculated about the presence of layers, characteristics of the PBL and isolated patches of turbulence (IPoTs). The analysis of each image usually consisted of several steps:

1. Observe general aspects of the day (layers, strong echoes, isolated patches of returns, planetary boundary layer and their time evolution).

2. Identify layers and characterize them.

3. Measure layers by their horizontal (time) extent.

4. Matching of the layer observed with phenomena observed in the previous day (e.g. Residual PBL or nocturnal PBL).

5. Identify the boundary layer.

6. Characterize the boundary layer by its maximum depth and growth rate.

7. Identify regions of isolated back-scattering (also mentioned here as isolated patches of turbulence or IPoT) not associated with layers or PBL.

8. Measure the horizontal extent of the IPoT to quantify their presence in the atmosphere.
9. Take notes of interesting events, transitions, or patterns that need to be kept in the analysis log.

Not only the radar information was used to analyze the atmospheric conditions. Satellite images were used to observe general circulation and conditions over the country and in the region located over the radar. Satellite images from the GOES satellites are available every 30 minutes for Costa Rica. Different resolutions and wavelengths allow the information to be used in different ways. Visible channels are useful when looking for cloud formations, cloud bands, and wind shears. Infrared channels can be used to identify different levels of motion as well as convection.

The visible channel is usually available in two resolutions of 1 and 2 km. Among the advantages of this type of image, the precise location of storms, smoke from forest fires or volcanoes and sea breeze fronts can be mentioned. No information is available in the visible channel during the night. The infrared channel detects energy emitted by the planet and works at all times, even during night time. This provides a way to monitor the atmosphere without interruptions. The resolution of the infrared channels is usually less than the visible channels. The available images in the infrared have a resolution of 4 km, enough to track convective storms and observe surface heating.

By using satellite imagery the general conditions of the surroundings can be determined. During April 12th (not shown) and 13th, strong surface heating over Santa Cruz and the Nicoyan peninsula generated a clear sea breeze front. This front can be observed in figure 6.13 at 18:15 pm in the visible image of 1 km resolution. The sea breeze front located near the surface will head inland from the coast line. Over Santa Cruz this will be observed as wind towards the east. The direction of the wind observed over the lower troposphere observed in the 2 km visible image is towards the west. The wind shear generated by this opposing directions can contribute to the strong layer activity observed by the radar. Mountain wave activity can also be related to the observed layers. Another important effect of the wind shear is the inhibition of convection, hence causing a shallow PBL as observed.

The previous description is an example of the methodology followed when analyzing the satellite imagery information. In the following section interesting case studies observed during the analysis are covered. On different occasions, uninterrupted layers were observed during entire days. Also, different shapes of the PBL were observed. These and some other cases of turbulence will be covered in the following section.

### 6.4 Case studies

During the analysis of the year long experiments carried out in Costa Rica, many interesting cases were recorded. These cases can be very different in nature. Some of those cases involve the presence of layers for prolonged periods of time. Other cases are related to planetary boundary layer growth rate, shape, time evolution and structure. The passage of storms over the radar’s beam is also of interest and observed on several occasions.

The following sections will document some case studies with gathered information. It is not intended to be a comprehensive list of Costa Rican atmospheric phenomena, as a limited span of a year was used. In order to achieve higher precision a more detailed study with longer
Figure 6.13: Montage of satellite imagery for April 13th, 2014. Infrared images are presented on the left side. The centre pane contains the visible images at 1 km resolution, and the 2 km visible spectrum can be observed on the right side. Four different hours 00:15 am, 06:15 am, 12:15 pm and 18:15 pm are included as rows.
radar data-sets is recommended but such a database does not exist. The operation of the radar will be continued past the research carried out here with the expectation of achieving such dimensions in the future.

In the following sections the planetary boundary layer will be covered first. Later the occurrence of layers and examples will be shown. Afterwards some oscillations and wave breaking will be mentioned along with some examples.

6.4.1 Planetary Boundary Layer

The planetary boundary layer can be defined in different terms. It can be defined as the atmospheric region that is directly affected by surface related effects. It can also be defined as the region that is altered by diurnal heating in less than a certain amount of time. It is hard to create a fixed definition for something with large variability and with such a fast-paced dynamics, as will be shown shortly.

Stull (1988) depicted the PBL as presented in figure 6.14 where different regions of the PBL can be clearly identified. The diagram of the PBL shown in figure 6.14 is intended for regions with high atmospheric pressure, as described in Stull (1988); an the definition used for the PBL in this reference is: “that part of the troposphere that is directly influenced by the presence of the earth’s surface, and responds to surface forcings with a time scale of about an hour or less”. In figure 6.14 an active mixed layer is observed to begin in the early morning and maximizing before noon. This maximum is located below 1.5 km in height and persists even past sunset. The stable (nocturnal) boundary layer starts to grow before sunset and continues to do so until it is overtaken by the following mixed layer.

The residual layer is defined in figure 6.14 as the space occupied by the residual motions generated by the mixed layer. These divisions of the PBL have a theoretical background to be considered realistic as described in Stull (1988). The observations can be very different from this categorization. Let’s begin by shown one example where a smoothly varying PBL is observed as a first approach to measured PBLs.

The echo power plot shown in figure 6.15 corresponds to February 18\textsuperscript{th}, 2014. This figure contains small gaps in the data caused by the post-processing running in real time and not being able to measure during those brief moments. The slow variation of this image can be beneficial in trying to identify different portions of the PBL. Figure 6.15, as with the others presented in this document, begins on the time-axis at sunset. The centre of figure 6.15 corresponds to the sunrise information with midnight 1/4 of the way to the left and noon 1/4 of the way to the right. The time-axis difference is relevant when comparing to figure 6.14. The reader should keep in mind that the lower level observed in the figure is not the surface but 1.3 km above of the radar’s position, making it impossible to resolve lower level characteristics of the PBL.

By contrasting both images large differences appear immediately, mainly regarding maximum location and structure.

The PBL observed in figure 6.15 does not seem to reach a maximum as fast as depicted in figure 6.14. Instead of reaching the maximum height before noon it gets to 2.8 km at 2 pm. The depth of the PBL is clearly a big difference. The value of 2.8 km would seem large compared to the 1.5 km value of Stull (1988) but this is not rare and actually it is just above the average for the month of February as will be shown shortly. Another major difference is the descent of the residual layer observed in figure 6.15. Instead of keeping the same height, as suggested by
6.4. Case studies

Figure 6.14: Schematic diagram of the different layers and regions that are part of the Planetary Boundary Layer evolution taken from Stull (1988); this diagram is based on atmospheric measurements. The planetary boundary layer starts with the growth of the (Convective) Mixed Layer. After the sunset a residual layer remain at upper heights whilst the lower levels form part of the Stable (Nocturnal) boundary layer. The upper region is limited by a cloud layer, the entrainment zone, and a capping inversion.

Stull (1988), the residual layer of the previous day’s mixed layer drops height considerably as time passes, reaching 1.5 km before sunrise. Considering that the minimum observed height is 1.3 km, the chances of observing a nocturnal boundary layer are considered scarce.

The smooth PBL presented in figure 6.15 is an example of a theory-like PBL case, as it is a slowly varying well shaped structure. This was not a common scenario in the measurements, as most of the PBLs observed do not agree with the depiction shown in figure 6.15.

Figure 6.15 was used because it represents one of the few cases where the PBL is smoothly varying. Most of the observed time the PBL consisted of truncated sections with several discontinuities and inconsistent growth. On occasions the PBL is observed as a Gaussian shaped structure, as just observed in figure 6.15, but in some other cases a linear or discrete growth dictates the pace of evolution. During the rest of this section an attempt to group the different PBLs shapes into categories will be presented.

The PBL can vary in shape, depth and ascension speed according to the general atmospheric conditions. After considering the entire data-set of one receiver and three receiver data the following categories seem to be appropriate in describing the observation. They have been separated into two figures. Figure 6.16 contains the categories that are found commonly and whose shapes can be considered most frequent. Figure 6.17 contains those cases where the shape or evolution of the PBL was considered to be somehow exotic or uncommon.

Categories contained in figure 6.16 are:

- **Smooth growth with full development.**
Chapter 6. Radar data and experiments

Figure 6.15: February 18th, 2014. Slowly varying smooth planetary boundary layer. This case can be considered rare as most PBLs contain large variability.

Figure 6.16-a) contains data for Feb 18th. This case was just presented and is included in this listing for completeness. This PBL top resembles a Gaussian function in shape. At the maximum rate of growth it covers 12 centimetres per second and reaches a maximum height of 2.8 km. Figure 6.16-b) displays the PBL case recorded during December 28th, 2013. A clear PBL shape is observed to grow, but with less intensity than the previous case. It seems to reflect in a better way the diagram of figure 6.14, where after reaching a maximum of 2.9 km it practically keeps the height constant until the sunset’s decay.

In order to consider a full development of the PBL the region under the top needs to be carefully examined. In the present category both cases present a clear top of the PBL. The region between this top and the minimum height available present little to no atmospheric echoes. The extent of the PBL (other than the top) generates no considerable backscatter. This does not mean that there are no inhomogeneities or turbulence inside the PBL. It can be understood as turbulence homogenizing the atmospheric properties causing the index of refraction of this region to be “uniform”. Uniform does not mean that there are no gradients, it means that the $\lambda/2$ Fourier component of index of refraction fluctuation has been removed and no sharp transitions of pressure, temperature and water content are found inside the region. It is worth mentioning that the velocity components generated by turbulence could still be present.

- **Absent**

In some occasions the PBL structures regularly observed were missing. This can be due to weak growth not surpassing the minimum height. Figure 6.16-c) contains data for Feb 8th, 2014. In this example the PBL is not observed in the data. The satellite imagery showed that there was no cloud formation detectable by the satellites over Santa Cruz and the surface temperature reached values of approximately 30 Celsius. A strong wind from the Caribbean Sea is observed in the visible spectrum of the GOES images.

- **Truncated**
The PBL is very dynamic in nature and this case exemplifies it clearly. Figure 6.16-d) taken from data corresponding to April 7th, 2014 shows a clear PBL with linear growth that suddenly ends at 3.75 km of altitude around 1:35 pm. This layer kept a steady growth rate of 0.14 m/s for several hours and was suddenly truncated. The visible satellite imagery for this day confirms that the sea breeze overtook the lower tropospheric motion over Santa Cruz around 1:45 pm. This is an interesting case where the PBL growth is interrupted. No secondary PBL was observed during this day, as was also observed in other cases that will be introduced later.

- **Linear growth**

In the previous example a truncated PBL was shown. The growth rate of the truncated case can be considered a linear function of time. This was not an isolated case, as many other days presented a similar tendency in time evolution. Figure 6.16-e), f), g) and h) show different cases of linear growth corresponding to January 9th, May 5th, March 11th and February 13th (2014 for all the cases) respectively. Four cases are presented to cover the most common cases observed in the data-set. The truncated events were not included in this category as they do not reach the end of the afternoon, but some of them could have a linear growth tendency.

The PBL for the first three cases [ e), f) and g) ] correspond to linear growth with speeds of 9.1, 10.1 and 9.6 cm/s. A large difference between the three cases is observed underneath the top of the PBL. Figure 6.16-e) presents a quiet region under the PBL top, representing a well mixed region (seen as shades of red in the figure). The cases presented in Figure 6.16-f) and -g) do not show a silent PBL below the upper limit. Instead strong echoes are observed in the f) pane before sunset and less intense echoes are observed in the g) pane but with similar implications. Satellite imagery for January 9 (figure 6.16-e) showed a weak sea breeze not reaching Santa Cruz with strong influence by the trade (easterly) winds. The infrared image showed that surface heating was observed during the early morning but rapid cloud formation decreased the amount of radiation reaching the surface. The reduced radiation could be associated with the linear growth rate. The fact that the sea breeze did not reach the radar’s position allowed turbulence to create efficient mixing causing the decrease in echoes at those heights.

The previous hypothesis about the sea breeze can be confirmed by the satellite images for May 5th and March 11th. Similar conditions regarding early surface heating and cloud formation are observed in these two cases. One major difference between these two cases and January 9th is that the sea breeze did reach Santa Cruz in the afternoon, creating deeper convection and wider cloud formations. The enhanced instabilities could be responsible for the echoes during the afternoon inside the PBL.

The last case presented in figure 6.16-h) contains a linear but rather weak growth of the PBL top. An ascension rate of 3.3 cm/s was estimated for this case which represents close to one third of the estimated value for the other three cases just described. In this case, even with a slow growth of the PBL, a region of silent atmosphere below the PBL top is observed in a similar fashion to figure 6.16-e). By observing the evolution in time of these cases it can be concluded that a well mixed layer in the PBL is not necessarily associated with the growth rate. Satellite images for this day showed that diurnal heating
was present during all day with a faint formation of clouds and an equally weak sea breeze. The trade winds seem to dominate the low tropospheric motion.

In the previous listing several cases of PBL were described along with the corresponding evolution depicted in figure 6.16. Not all the PBL cases observed during the year-long experiments were clearly defined or as simple to define as the ones described. Some other cases seem to fall far from the regular definitions of PBL.

Categories contained in figure 6.17:

- **Multiple growth**
  Figure 6.17-a) correspond to September 14th, 2014 and figure 6.17-b) to January 31st, 2014. Both cases contain boundary layer formations whose growth rate is neither uniform or single. In the case observed during September, the beginning of the PBL is defined by a sharp ascent from the minimum level to 2.8 km. This rapid growth of 66 cm/s is 6 times larger than those observed for linear growth described earlier. This sudden intrusion is isolated, as no other PBL growth is detected past 2 km for 2.43 hours. After that lapse another growth, more intense than the first one, reaches 3.6 km. No satellite imagery was available for this event.

  It should be clarified that because levels below 1.3 km are not observable no assumptions about the motions in those regions are appropriate. The fact that the PBL descended from the first relative maximum indicates weakening in the energy supply, not a total disappearance of the PBL. The January 31st event can be described as a dual PBL growth from the minimum observable level. Figure 6.17-b) shows a first growth that reaches 2.1 km at noon and another intrusion of the PBL reaching 2.5 km at 2:45 pm. Satellite images reveal surface heating along with strong easterly winds that generated cloud bands during the late morning.

  When considering the effects observed at measurable heights it is relevant to keep in mind that the time evolution as well as advection play a major roll in the observations. If the PBL ascension speed is considered constant at 10 cm/s (as in the linear cases described earlier) it would take approximately 3.5 hours to propagate from the surface to levels where it is detectable by the radar. The lag in observation time can be interpreted as retardation in the detection time. The PBL gap observed in figures 6.17-a) and 6.17-b) could be due to a phenomena occurring 3.5 hours before in the same location, or more likely, generated somewhere else in northern Costa Rica and advected over the radar’s beam.

- **Discrete**
  An important case of PBL encountered in the data-set consisted of a non-structured PBL top. In this category the PBL does not hold a consistent top that evolves with time, but rather isolated intrusions are observed. It is very common during the rainy season months to observe this type of discrete intrusions. Both of the cases presented here were measured during the rainy season of 2014. Figure 6.17-c) contains the observations for November 25th and 6.17-d) for September 11th. The November case consisted of more than 9 different intrusions reaching heights between 2.4 km and past 5 km (maximum
Figure 6.16: Eight different cases of planetary boundary layer evolution. Panes a) and b) correspond to smooth growth with full development of the PBL. Pane c) shows a case where no observable PBL was found. Pane d) contains a truncated linearly growing PBL. Panes e), f), g), and h) exemplify linear growth of the PBL top with different ascension rates. For more information see the text.
measurable height). The ascension speed of these events is not close to the values observed in the previously discussed scenarios. The spike-like events usually ascend at rates greater than 1 m/s. The radial velocity for the core of these events was measured at 2 m/s for most of the events. Adjacent to the large positive velocities, regions of large negative values are also present. Convective motions can be associated with these events as strong upwards currents will generate downward motions by mass conservation (continuity), even when not necessarily in the same region. No satellite imagery was available to confirm the presence of convective clouds.

The data depicted in figure 6.17-d) contains less intense intrusions but are equally discrete in nature. The PBL observed in that figure consisted of eight different observable intrusions that reached heights from 1.5 km to 3 km. The second to last intrusion is thicker than most of the others and is made out of two consecutive up and down motions. The radial velocity information is attached to the figure. Three upward motions alternating with three downward motions are observed. These upward-downward combinations are usually associated with thermal currents. Two thermal currents observed at approximately 11 minutes of spacing could be pointed out as responsible for these echoes in the vicinities of 2 km close to 4 pm.

Due to the broad beam used in Costa Rica it should be kept in mind that this values of radial velocities can be artificially generated. Horizontal motions with an order of 10 m/s transporting scatterers in and out of the beam can generate spurious radial velocities of ±2.4 m/s. However this is not considered possible in this case because the time change from positive to negative is in the order of 5 minutes, contrary to the 1 minute period consequence of the beam width expected at the same velocity. A combination of large vertical velocities (2 m/s) and horizontal winds can be responsible for the observed radial oscillations. No satellite imagery was available to confirm the presence of cumulus clouds during these events.

- **Mixed**

The PBL shape described in this section can be considered a mixture of events observed in other cases. The initial PBL observed in figure 6.17-e) at 9 am continues to grow for 1.5 hours. The PBL top decreases for one hour similar to the multiple growth PBL cases mentioned earlier. The PBL regrows steadily past 11:30 am even creating a region of low echo (well mixed by turbulence) under the PBL top between 1.4 km and 2 km at 1:30 pm. The region that follows these events has not been described so far and corresponds to deep convection. The deep convection can be clearly observed in satellite imagery, specially in the infrared channel.

Figures 6.18-a) and b) contain the visible and infrared wavelengths respectively taken during 9:15 am, 12:15 pm and 2:15 pm. The clear atmospheric conditions over the Nicoya peninsula at 9:15 am allowed the initial PBL to grow. The unsettled conditions observed in the region created a convective storm located in the vicinities of the radar and moved exactly through the beam. The echoes of the storm are observed in all the extent of the analysis plots in figure 6.17-e) during two hours. After the convective motion passes, a PBL is still present in the radar echoes towards the end of the afternoon.
Figure 6.17: Seven different cases of planetary boundary layer evolution. Panes a) and b) correspond to multiple growth (double) of the PBL. Panes c) and d) contain discrete growing PBL. Pane e) shows an example of a mixed type growth of the PBL. Panes f) and g) exemplify two cases where the PBL reaches the height of a thin layer and interaction effects are observed. For more information look into text.
Similar cases to the one just described can be found during rainy season and are missing in the dry season data-set. The variability in duration and depth of the convective cases is considerable.

- **Layer-PBL Interaction**

  Last in the PBL section, two cases where the PBL seems to interact with atmospheric thin layers are presented. Figure 6.17-f) contains the information for a case found during April 18\th, 2014. The case presented in 6.17-g) corresponds to October 12\th, 2014.

  The April 18\th case seems odd in the sense that the PBL seems to be present at heights past 1.3 km even before sunrise and the growth of the PBL is delayed momentarily when reaching thin layers. The presence of the PBL immediately after sunrise could be related to a strong nocturnal boundary layer and the growing diurnal boundary layer overtaking it. Multiple layers are observed during the night hours (not shown) and three layers are observed after sunrise (figure 6.17-f). These layers are found at 2.2, 2.7 and 3.6 km. The radial velocity (not shown) describes a clear oscillatory motion in the PBL top and the layer at 2.2 km (this frequency is also observed in the 3.6 km layer). The lower layers fuse into one thick layer at 7 am and continue to oscillate with the same frequency.

  The maximum height of the PBL top seems to be stationary at approximately 2.2 km altitude for three hours. After 1 pm the PBL continues to grow until it reaches 2.7 km where it stops momentarily. The growth continues once more to reach the maximum height at 3.4 km where it begins to descend and around 4 pm splits into two layers (centred at 2.9 and 3.3 km). The two growing stages of the PBL were observed to occur at 29.8 cm/s and 26.6 cm/s, which is large compared to other measurements of PBL growth.

Figure 6.18: Mixed PBL Satellite Imagery. Three different hours are presented for the the visible and infrared wavelengths.
rate but not as fast as strong convective motions. The growth of the PBL is delayed in time when the height of the described layers was reached. Satellite images showed strong wind shear between low tropospheric levels and upper levels. Clear conditions are observed during the morning and the sea breeze did not reach as far inland as Santa Cruz.

The case observed during October 12th is different to all the previous cases. The complete day was already included as an example of the three receiver data in figure 6.11. Up to four layers were measurable during the night. After sunrise at least two layers survive but the interaction with the apparently discrete PBL seems to put an end to the layers. This effect is observable also in the radial velocity and spectral width information (not shown) as a sudden stop at 10:30 am. No layer structures were seen for the rest of the day. No satellite image was available for this date.

In this section some of the planetary boundary layer examples were presented. Along with the PBL structure, thin layers of strong echoes were commonly observed. In the following section these thin layers are observed in more detail along with some of the formation and disappearance dynamics measured by the radar.

### 6.4.2 Layers

The presence of layers is a common topic in MST radar publications because they are often found in measurements (Hélal et al., 2001). A layer is observed in the measurements when a small region (narrow in the vertical coordinate) of the atmosphere generates more backscatter or reflection than its surrounding environment. These regions of the atmosphere usually maintain a similar structure for some time showing in a radar data set as a consistent layer. The physical mechanism begin a layer (as measured by radar) is simply a time-coherent transition in refractive index, causing reflection and/or backscatter of the radar signal. Any atmospheric phenomena that contributes towards non-homogeneous conditions or uneven mixing in the vertical atmospheric coordinate can be responsible for the formation of layers (some examples will be provided shortly). The dynamics of these layers can provide information about diurnal cycles, strong regions of turbulence (along with layer splitting) and other phenomena as was previously shown in the case of the PBL.

Specifics about the appearance of layers during the year-long experiments will be presented in this section. The statistics gathered during the analysis of the information will be presented in a following section. The experiments with one and three receivers generated a long data-set where many layers can be found, especially during the dry season (see section 6.5 on layer statistics). As an initial example of layers observed in the atmosphere, let’s consider an example taken from January 5th, 2014.

In figure 6.19 (top pane) the presence of layers during the 5th day of January (2014) is displayed. A multi-layer system appears during the night generated from two systems of the previous day (not shown). Multiple layer splitting were observed during the night, and at midnight 5 different layers are observed at 2.1, 2.5, 3.0, 3.6, and 4.1 km. Some of these layers contain motions observed in the radial velocity indicating internal structure (e.g. 3.6 km layer after sunset until midnight and 4.1 km layer from 10:30 pm until 4 am, not shown).
At noon the layers presented in figure 6.19 are faint but become more pronounced after the PBL interaction. In this particular case the layer interaction with the PBL does not terminate the layers, contrary to the cases considered in the previous section. The layer located in the vicinities of 3 km is briefly displaced upwards when the PBL reaches its maximum height followed by a drop in height as the PBL height decreases. The lower layer observed at 2.5 km before noon is not measured for a brief moment during the maximum of intensity at PBL deepest reach, but reappears shortly after and intensifies during the following night (not shown). The PBL during January 5 reached a maximum of 2.7 km at 2 pm.

Satellite imagery taken simultaneously to the data in figure 6.19 shows evidence for strong winds from the Caribbean sea. No observable sea breeze in the region of Santa Cruz is evident in the visible spectrum. Fair weather cumulus are observed on most of the Nicoyan peninsula as confirmed by the satellite image at 12:15 pm (figure 6.19-bottom pane). The strong wind from the east could be generating mountain waves that could be observed by the radar. In the surrounding area of the radar a few small mountains could contribute to this effect and should not be ignored. The large mountain chain in the middle of the country could also contribute to layer formation, being approximately 80 km away from the radar site in the north-east direction. To give a reference of the elevation difference between Santa Cruz and the mountain chain in central Costa Rica, the average elevation is 42 metres above sea level for the former while the latter contains the Rincón de la Vieja volcano with 1,916 metres.

The case just described is an example of layers detected by the radar. Satellite imagery was used whenever possible to identify possible sources of the thin layers observed. Atmospheric layers were observed by the radar on a regular basis. Thin layers were so common that they can be observed uninterrupted for days at a time. Most of this layers are very dynamic, transforming constantly by splitting, joining and disappearing. Some of the layers observed will contain oscillations inside the layer that are visible in the radial velocity information. Several of this aspects will be divided into categories of layer events in order to create a classification of observed cases.

- **Layers and the L1, L2, and L3 scheme**

  Figure 6.20 contains two weeks of radar echo information. Information from April 3rd to April 16th is displayed in this figure. The amount of time when no layer is observe is scarce. In occasions as much as five simultaneous layers are observed (e.g. April 14th).

  During the analysis of layers, a scheme was created to quantify when one, two, three or more layers were observed. The number of layers is relevant as more layers in the atmosphere indicates more complicated dynamics; this complications can be observed as multiple regions of change in index of refraction or turbulent dissipation. The scheme was denoted L1, L2, and L3 respectively as shown in table 6.8. This scheme was used to quantify the presence of layers in the atmosphere. The planetary boundary layer was not considered a layer when growing but, as will soon be observed, many layers evolved from the residual motions of the PBL, and they were counted as layers. The layer analysis did not take into account the thickness of the layers, as the main goal was to quantify how often they were observed.

  The methodology used to quantify the layers is based on the number of layers and the fractional amount of the day that they were observed. As figure 6.20 confirms, multiple
layers were observed constantly during a two week period. A case with one layer can be observed during April 12th. Before sunset the layer is displaced upwards, and this case will be categorized as an $L_1$ case with a duration of 0.46 of a day (11.04 hours). During April 7th more than three layers can be observed simultaneously at different heights. These layers persist for $\approx 60\%$ of the day (14.2 hours) and they were categorized as a case of $L_3$ in the scheme. This type of analysis was carried out for all the available days in the radar data-set. Specifics about the duration of all the layer events will be presented later.

This representative data-set of the dry season can be observed in detail for more information. The planetary boundary layers is observed to grow with variable intensity each day. Different cases of PBL are observed in figure 6.20. Strong quasi-linear growth can be observed during April 7th and 15th, whilst not-so-rapid growth can be observed during other days (e.g. April 6th, 10th and 11th). In many days in the April 3-16 period the PBL
Table 6.8: Listing of the names and number of layers included in the category created to quantify the number of layers.

<table>
<thead>
<tr>
<th>Category</th>
<th>Number of layers</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>1</td>
</tr>
<tr>
<td>L2</td>
<td>2</td>
</tr>
<tr>
<td>L3</td>
<td>3 or more</td>
</tr>
</tbody>
</table>

Figure 6.20: Two weeks of consecutive layers. Three days per row are shown and the date specified as the left, centre and right position. The PBL generated and terminated various thin layers during several consecutive days. Layer bending is also observed (e.g. April 12th).

and thin layers seem to be related and interacting.

Examples of this interaction were described in the previous PBL section where PBL-layer interactions were discussed. Another example of this layer termination can be observed during April 14th, when three different layers are suddenly terminated by the presence of the PBL; no clear layers were observed for hours after this event. The relationship of the PBL and the thin layers can be observed during April 3, 4 and 5. During these consecutive days the PBL created a clear residual layer that evolved with time and on occasions split into two or more distinct layers (see night time of those days).

These interactions, and relationships between the PBL and layers, are not relevant in the scheme of quantifying the layers. None the less, the way the layers are formed or destroyed is an important aspect of the atmospheric dynamics. The observed cases of residual PBL layer generation seem to suggest that a slow but constant growth of the PBL top will generate a strong layer for the following day. This layer would be generated over the north-eastern region of Costa Rica and advected over the radar’s beam. On the other hand, the residual layer observed during April 13th seems to suggest that a weak PBL that reaches a thin layer can as a consequence of the interaction create multiple
layers. As this was not the focus of this investigation it can’t be conclusive, but acts as a lead into more possible layer generation/dissipation studies.

• Layer generation

In the previous section certain conditions of the planetary boundary layer were pointed out as a possible cause for thin layer generation. This layers can be generated by other mechanisms. In this section an example observed in the data-set is used to exemplify one alternative mechanism for layer formation.

Figure 6.21 contains information for April 24th. Infrared satellite imagery was used to observe deep convection occurring in the Pacific coast of Costa Rica. The visible channel of the satellite imagery was used to observe advection and mean wind direction. These two types of satellite images are included in figure 6.21 (top and bottom pane respectively). Deep convection was observed at 5:15 UTC in the infrared satellite image over the Guanacaste area. The mean wind was directed to the north causing the air to be advected over the radar’s field of view.

In the centre pane of figure 6.21 the echo power of April 24th is displayed. After midnight echoes are observed reaching 2.8 km and these echoes evolve in time to transform into at least two different layers. These layers were not visible before the convective event. The lower layer generated by the storm is located at 2 km at 7:30 am local time. This layer raises and reaches 3.8 km at 1:30 pm. This section can be easily be confused with the PBL unless it is noticed that no connection exists between the layer and the lower levels at any point in time. The thin layers just described were not visible in satellite imagery as clouds but are clearly detectable by radar.

An important aspect about the layers is presented in figure 6.21-bottom pane. The visible images show a weak sea breeze reach inland but more importantly, the top of the convection over the peninsula is rapidly displaced toward the Pacific ocean. This effect is commonly observed when a large wind shear exists between lower levels of the troposphere. This large wind shear can contribute to the short-lived PBL and layer elevation and intensification.

• Splitting

In the previous sections the common observation of layers was presented. Figure 6.20 showed that apart from layers being present a large percentage of the time these layers are constantly evolving. Part of the evolution of the layers can be observed when these layers split.

Two examples of layer splitting were measured during February 23rd, 2014. The radar configuration was not continuous during these experiment and gaps can be observed in figure 6.22. Different layers were observed during February 23 under 3 km of height. The initial layer (left side top pane figure) descends from the 3 km mark and reaches a local minimum close to 9 pm.

In the following hours this layer created a cat’s eye formation. At 9 pm the layer splits into two parts. The upper section keeps a constant height of 2.9 km. The lower section
Figure 6.21: Information for April 24th, 2014. Infrared satellite imagery taken at 05:15, 6:45, and 14:15 UTC is displayed in the top pane. The bottom pane contains visible imagery taken at 17:45 and 19:45 UTC. The centre pane contains the radar echo power for 24 consecutive hours. The time location is marked with arrows from the image to the time axis of the measurements.

moves at -0.07 m/s and thickens as it descends. The amplitude and radial velocity information (not shown) indicates that instead of a definitive splitting a new larger cat eye was formed. In any case, towards the sunrise a new thin layer is observed at 2.4 km. Another layer at 2.0 km at 6 am will displace upwards and splits into two at 11 am. The data shows in the upper pane of figure 6.22 that the layers kept similar heights after separation. The upper layer remained at 2.7 km and the lower layer at 2.3 km. During these two hours, weak echo power was observed from the approximately 400 metres thick region between the layers.

Figure 6.22-lower pane contains the spectral thickness information from the Gaussian fit. Due to contrast enhancements some of the separations are better observed in this information. All the layers observed during the presented date have considerable spectral width.

- Joining and Breaking
Layer splitting was commonly observed but was not the only evolution pattern of layers. It was common to observe the layers terminate. This termination could simply be observed as dimming of the intensity until no longer measurable by the radar (Figure 6.20 contains several examples). In other cases the termination of layers is not slow but abrupt. One example of this abrupt termination of layers is presented in figure 6.23.

The data measured during April 26th is shown in figure 6.23. On this occasion the layers are visible during the night in the upper region of the plot. At sunset two different layers are located at 3.85 and 4.6 km. This layers evolved with time and seem to join at 1 am. At the same time another layer is observed to appear from the 5 km limit and descend to similarly join the other layer at 3:45 am. After this later joining of layers a different structure with the shape of a hook is observed. It is important to keep in mind that what is being described here as two distinct layers could perfectly be the edges of a wide homogeneous layer. The inside of the wide layer can consist of air with different
properties from its surroundings causing the edges to be measured by radar but the inside
to be silent to the radar.

Whether the layers are independent or part of a wide layer the hook formation at the end
is of interest. The hook is formed at 5 am and extends for several hours. The vertical
extent of the hook shaped formation amplitude is 1.3 km at the maximum spacing. After
this formation crosses the radar’s field of view no layers are observed for several hours.
This is only one case of observed “hook” formations where layers are terminated but
several were documented.

Satellite imagery in the infrared wavelengths showed strong convection in the area. At
the time of the layer termination the dominant air motion from the north-east is overtaken
by north-westerly motions. This can explain the different atmospheric content observed
before and after 7:40 am.

Figure 6.23: Layer join and termination observed during April 26th. Several layers are mixed
during the night into one thick layer located at 2.7 km. During a change in the mean wind this
layers disappear with a hook shaped structure observed before 7:40 am.

In this section different examples of layers and their time evolution were shown. The lay-
ers showed great variability in different aspects during generation, evolution, and termination.
Different meteorological conditions like planetary boundary layer, wind shear, mountain waves
and deep convection can contribute to the presence of layers.

The following section treats the topic of oscillations observed in the radar data. Different
cases were used to exemplify the oscillation’s variability and evolution.

6.4.3 Oscillations and waves

Previous sections remarked on the presence of a planetary boundary layer and thin layers in
most of the observed data. These two phenomena are by far the most common types of ob-
served meteorological events. To a lesser degree, atmospheric oscillations could also be mea-
sured by the radar. Some of the oscillations registered in the data-set are presented here.

Oscillatory motion was observed on different occasions and one way to characterize them
is by the alternation of the radial velocities measured by radar. Most of the observed layers
have in some manner oscillations in height and in radial velocity. Rarely those oscillations are regular. The oscillations observed can be of different nature. Altitude variation of the echo and different radial velocities were used to identify atmospheric oscillations.

Some oscillations have been mentioned already in section 6.4.1 where the PBL intrusions had an alternation between positive and negative radial velocity. Another example of oscillations was presented in figure 6.20 but was not discussed at the time. In such figure the layers are not static with height as occurred during the transition from April 13th to 14th at 3.6 km.

It is now time to specify in more detail cases where these oscillations can be observed clearly. The following section contains the case of oscillations observed in radial velocity data. Afterwards one case of height oscillation will be examined.

- **In layer oscillations**

The in-layer oscillation have been named this way because the radial velocity in the layer indicates the oscillatory motion. In these layers no change with height in the top or bottom make it impossible to observe the oscillation by height variation. The radar's range resolution along with the broad beam used in Costa Rica’s radar could impact negatively these observations.

The case presented in figure 6.24 was measured during February 20th, 2014. Layer splitting at 2.5 km is observed during the night time. The lower branch thickens and presents an oscillatory motion after midnight and lasts 4 hours before splitting itself into two before sunrise. The radial velocity information is presented in figure 6.24-(top left) and has been rescaled by 200%. The period of oscillation for this region is not exactly constant, but in the wave like regions it can be measure to be 5.17 minutes per cycle.

During the same day of measurements observed in figure 6.24, another oscillation occurred inside the PBL. Similar to the case shown in the PBL section, the PBL interacts with the residual layer in the vicinity of 2 km. The region highlighted in figure 6.24-(top right) shows the radial velocity corresponding to the PBL growth at noon. In some regions the oscillatory motion is not regular and a period can not be measured. Around 1 pm the oscillation is more regular and a period of 2.5 minutes per oscillation can be observed. The oscillation is carried out during the PBL growth. Hours later it can still be observed with modified periodicity in the top of the PBL and also in the residual layer inside the mixed region of the PBL.

This previous description is an example where an oscillatory motion is present for most of the day. The frequency of oscillation changed regularly, going from a fast to slow oscillation. During February 21st no oscillatory motion was observed and the planetary boundary layer growth was non-oscillatory in most cases.

Moving to a different case observed during March 15th, 2014 the corresponding data is displayed in figure 6.25. During that day the atmospheric echoes were filled with oscillatory motions most of the time. The oscillations changed in period depending on the region. Multiple layer splitting and joining were observed during this case.

Figure 6.25-A encloses an upper section with two layers. These two layers were generated by layer splitting the day before during afternoon hours. Cat’s eye structures with
Figure 6.24: February 20th (2014) echo power and radial velocity (insets) for oscillations. Top left: A 5 minute oscillatory motion is observed in the lower branch of a split layer. Top right: A period of 2.5 minutes is observed inside the PBL top.

- Height oscillations and Kelvin-Helmholtz events

Large oscillatory motions and Kelvin-Helmholtz instabilities (KHI) are often included in the literature (multiple examples in Fritts et al. (2012) and references therein) as examples of atmospheric phenomena observed by radar; some authors consider them ubiquitous in the atmosphere (Fritts et al., 2012). These KHI and similar oscillatory motions are important because they can redistribute energy and affect human activities, including air-
6.4. Case studies

Figure 6.25: Radar measurements during March 15\textsuperscript{th} (2014). Oscillation in the radial velocity with periods between 6.2 and 12.4 minutes were recorded. Specific events are marked as A (12.4 minutes), B (6.2 minutes), C (6.2 minutes), and D (9.9 minutes). Multiple layer splitting and layer-planetary boundary layer interaction were observed during these 24 hours.

craft operation (Vinnichenko and Dutton, 1969) but other atmospheric phenomena could be associated with similar effects or risks. Fukao et al. (2011) provided a comprehensive review of KHI and radar measurements. As will be shown soon, when considering KHI in the light of a large data set their presence in the atmosphere seems to be far from ubiquitous.

Regarding larger scale oscillatory motion, October 12\textsuperscript{th} provides an interesting example. Figure 6.26 contains the echo power (lower pane) and the spectral width plot (upper pane). The spectral width is usually useful when looking for layer splitting and wave breaking because only events with considerable width are observable. The echo power presents an oscillation that lasts for more than 12 hours (from 8 pm to 10 am) centred at 3.2 km. The displacement amplitude of the oscillation grows from 250 m around midnight to 675 m during sunrise.

The period of the oscillation in figure 6.26 during the first two cycles is 3.15 hours. The period decreases to 2.44 hours before sunrise and to 1.81 hours after 6 am. During the following two hours the motion pattern changes and cat’s eyes are observed. The oscillatory motion has been approximated by a diagram on top of the spectral width information on the top pane of figure 6.26. The cat’s eye structures seem to be skewed
upwards similar to other cases where the layers ascent during the PBL growth.

This case is one of the few where Kelvin-Helmholtz like events were observed in the echoes. The other cases observed are not obvious and care was taken to confirm oscillations, signs of instabilities in the echoes and consequently breaking. The few occurrences were more common during dry season when the easterly winds cause larger wind shears. The enhanced wind shear could contribute significantly to Kelvin-Helmholtz instabilities as well as mountain wave generation.

![Figure 6.26: October 12th atmospheric activity as recorded by Santa Cruz’s radar. Top pane: Spectral width plot obtained from the Gaussian fit of data. Bottom pane: Radar echo power obtained from the Gaussian fit of the spectrum. Oscillatory motion that lasts more than 12 hours increases in amplitude and break towards the end. Changes in the atmospheric circulation prohibited further monitoring of this event.](image)

In total only 11 days out of the 193 contained structures that resemble Kelvin-Helmholtz instabilities, representing a percentage of 5.7\% of the days. When measured in time that the event lasted the percentage drops to 2.5\% of the total logged time. The majority of the events where observed during the dry season, specifically January and February. Only two cases were detected during rainy season. The low number of events during the second part of the year could be due to a decreased number of complete days available for the analysis.

The 2.5\% of the time found in this study is larger than the 0.6\% found by Fukao et al. (2011). The events considered in Fukao et al. (2011) used a short data set of only 600
hour (≈ 25 days). The extended period of time used in this research and the different geographical location could contribute towards the difference. The value of 2.5% is none the less small, reflecting that the Kelvin-Helmholtz like structures are not ubiquitous in the atmosphere and are actually rare to find at this resolution.

Layers in the atmosphere have been linked to KH-like events splitting into layered structures (Fukao et al. (2011) and references therein). During this analysis the presence of KH-like events was shown to be low and the majority of the events during the dry season. It was during dry season that the majority of the layers were also observed. It does not mean that the events are related, but layer splitting, contrary to KH-like events, was observed in a regular basis (e.g. figure 6.20). One example of layer splitting observed during April 13th-14th in such figure was associated with the interaction of layers and the PBL, providing a different mechanism for layer splitting.

### 6.5 Analysis results

The previous case studies were extracted from the year long data-set. This database was a combination of one and three receivers experiments. By analyzing the year long data-set in a quantitative manner statistics about the Costa Rican atmosphere were obtained.

The following aspects were quantified for each daily analysis:

1. Layers (divided into one, two, three or more)
2. PBL depth
3. Isolated patches of turbulence (IPoTs)

**Layers**

As mentioned earlier the L1, L2, and L3 scheme was used to quantify the layers. Each day of the year-long data-set was analyzed and statistics were obtained. Table 6.9 contains the number of complete days per month along with the percentage of the time that layers were observed. The layer information was also plotted in figure 6.27 and it is clear that the presence of layers is greater during the first half of the year. The minimum percentage of time when one layer is found during the dry season months is May with 15.6%. A maximum of 32.14% of the time is observed during February. The two layer category peaks during March with 37.16%. The three layer or more category peaks the following month reaching 24.2% during April.

As a reminder to the reader, June 2014 was missing in the analysis due to data storage issues. July appears in figure 6.27 like a local minimum in layers observed, similar to November. Due to the low number of complete days during those months these values should be consider guidelines at best. Caution must be taken when drawing conclusions out of the poorly populated months. Because the information is presented as a fraction of the total measured time the values are still considered to be representative for each month.

Figure 6.27 clearly demonstrate a difference in layers observed during dry and rainy season. The case studies presented in previous sections can be used to better comprehend some differences in layers during the seasons. Several instances of layer formation were observed as a
Table 6.9: Atmospheric thin layer information for each month of records. The three different categories used in the L1, L2, and L3 scheme (see text) are included along with the total number of complete days of observation. The standard deviation is included.

<table>
<thead>
<tr>
<th>Month</th>
<th>Complete Days</th>
<th>1 Layer (%)</th>
<th>Standard Deviation L1 (%)</th>
<th>2 Layers (%)</th>
<th>Standard Deviation L2 (%)</th>
<th>3 Layers or more (%)</th>
<th>Standard Deviation L3 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>December</td>
<td>18</td>
<td>23.9</td>
<td>0.7</td>
<td>17.5</td>
<td>1.0</td>
<td>4.6</td>
<td>1.1</td>
</tr>
<tr>
<td>January</td>
<td>31</td>
<td>21.5</td>
<td>0.5</td>
<td>24.6</td>
<td>0.5</td>
<td>10.5</td>
<td>0.6</td>
</tr>
<tr>
<td>February</td>
<td>28</td>
<td>32.1</td>
<td>0.6</td>
<td>22.3</td>
<td>0.7</td>
<td>10.3</td>
<td>1.2</td>
</tr>
<tr>
<td>March</td>
<td>22</td>
<td>29.0</td>
<td>1.0</td>
<td>37.2</td>
<td>1.0</td>
<td>4.1</td>
<td>0.4</td>
</tr>
<tr>
<td>April</td>
<td>25</td>
<td>20.7</td>
<td>0.8</td>
<td>33.3</td>
<td>0.9</td>
<td>24.2</td>
<td>1.0</td>
</tr>
<tr>
<td>May</td>
<td>12</td>
<td>15.6</td>
<td>1.1</td>
<td>22.9</td>
<td>1.1</td>
<td>8.3</td>
<td>0.9</td>
</tr>
<tr>
<td>June</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>July</td>
<td>9</td>
<td>1.4</td>
<td>1.4</td>
<td>2.2</td>
<td>2.2</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>August</td>
<td>8</td>
<td>4.1</td>
<td>0.7</td>
<td>10.3</td>
<td>2.5</td>
<td>3.1</td>
<td>3.1</td>
</tr>
<tr>
<td>September</td>
<td>15</td>
<td>3.0</td>
<td>0.3</td>
<td>19.2</td>
<td>0.8</td>
<td>5.5</td>
<td>0.4</td>
</tr>
<tr>
<td>October</td>
<td>19</td>
<td>7.9</td>
<td>1.1</td>
<td>11.1</td>
<td>0.6</td>
<td>7.4</td>
<td>1.1</td>
</tr>
<tr>
<td>November</td>
<td>6</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>8.3</td>
<td>8.3</td>
</tr>
</tbody>
</table>

consequence of PBL growth during the dry season (Figure 6.20). PBL truncation was observed during dry season by the sea breeze in the Nicoyan peninsula. Another mechanism observed that contributes to layer formation was powered by nocturnal convection (Figure 6.22). On the other hand, deep convection during the transition to the rainy season can also contribute to layer formation (Figure 6.21). All these mechanisms for layer generation and suppresion can be used to explain the large presence of layers during the first months of the year.

**Planetary Boundary Layer**

The planetary boundary layer was also examined during the year long experiment. The minimum, maximum and average height of the PBL top was calculated for each month. The tabulated information is presented in table 6.10. In addition to the PBL top information, that table contains the information about the presence of isolated patches of turbulence (IPoTs). The IPoTs were observed commonly during the data analysis and are characterized by a lack of structure as they are not layers or organized events. Even then, an estimation of the percentage of time per day was made and quantified in order to estimate the percentage of the time that IPoTs are observed in the Costa Rican atmosphere. The IPoTs percentage is presented in the last column of table 6.10.

The information regarding PBL height and IPoTs was plotted into figures 6.28 and 6.29 accordingly. The maximum height of the PBL top often reached the 5 km maximum measurable height during the rainy season months. These cases were commonly associated with discrete PBLs or convection as shown previously; continuous PBL growth did not achieve 5 km during the experiment. Figure 6.28 shows that the PBL average height is minimum during dry season and maximum during rainy season. The minimum values and maximum values exhibit the
6.5. **Analysis results**

Figure 6.27: Year long data-set analysis of atmospheric layers. One (L1), two (L2), or more than two (L3) layers where observed during a certain percentage of the time and the total values are presented. The maximum value is reached during the dry season months. The rainy season shows a decrease in the number of measurable layers. The vertical error bars represent the two way standard deviation.

The month of July as observed in figure 6.28 shows a decrease in the average height of the PBL top. As mentioned before, this decrease could be due to the limited data available for this month. Even with that caution, the values for November concur with the overall tendency and have a low density of data. Another possibility is that the July data-set is actually reflecting a physical phenomena known in Central America as “veranillo”. Also known as Midsummer Drought (Magaña et al., 1999) it is a multi-component event whose atmospheric conditions imitate the dry season months. This similitude in atmospheric conditions during July to the summer months could be reflected in PBL growth rate and evolution causing it to be reflected in the data.

Figure 6.28 showed a maximum average height during August of 3.93 km. The minimum was registered during January with 2.34 km. The ≈ 1.6 km difference between rainy and dry season average PBL top is large considering the net increase of 68% penetration into the troposphere by the PBL during the second part of the year. This increase in PBL top depth can be caused by more (and/or stronger) ascending currents and less wind shear generated by
weakened easterly winds from the Caribbean sea.

One important effect of a thicker PBL can be observed by looking at the pollution caused by human activities. Pollutants gases and secondary pollutants (created by chemical reactions of pollutants in the atmosphere) can increase the concentration of certain molecules (e.g. ozone). These secondary pollutants (as well as the primary ones) can have adverse effects on human health and their agricultural activities as described by Jenkin and Clemitshaw (2000). The turbulent nature of the PBL allows pollutants to spread from the source (horizontally and vertically). The vertical dispersion can lower the overall concentration of pollutants near the surface. A thicker PBL as the one observed here during the rainy season could contribute positively towards the health of the population and their activities. The positive effects comes from the reduced concentration of pollutants and the removal of primary and secondary pollutants by the mean winds at upper heights.

The minimum PBL top observed in figure 6.28 seems less conclusive regarding the observed maximum during rainy season. The values of the minimum in such figure equally show a tendency to increase during July-November and maintain lower values during December-April.

**Isolated Patches of Turbulence**

Last in this section the results of the isolated patches of turbulence are analyzed. Because the IPoTs are not regular structures they are much harder to identify and consequently quantify than layers of height values. Care was taken when analyzing each day of the data-set but even then the results should be considered a first approximation. Figure 6.29 contains the plot for the IPoTs measured during the year long experiments in Santa Cruz, Costa Rica. The general tendency is low values during the beginning of the year and high values during the months of

![Table 6.10: Planetary boundary layer and isolated patches of turbulence (IPoTs) information for each month of records. The standard deviation is included.](image-url)
Figure 6.28: PBL top height as a function of month. The average value, maximum and minimum of the planetary boundary layer top is shown. The average PBL top peaks during May and August. A local minimum is observed during July. The minimum value was registered during January. The vertical error bars on the average PBL Top are the two way standard deviation.

the second half of the year.

During the dry season months the IPoTs were not common and values as low as 3.25% were observed during April. November, on the other hand, peaked at 67.5% with the largest percentage of IPoTs measurements during the logged time. The decrease in observations during September can not be associated with any obvious physical mechanism or phenomena. The importance of these patches and layers of turbulence is that they can be associated to aircraft passenger discomfort (Vinnichenko and Dutton, 1969) and aircraft safety.

This chapter was dedicated to analyzing the year long data-set recorded between December 2013 and November 2014. Many interesting case studies were included as evidence of the phenomena described and examples of the large variability observed. The planetary boundary layer seems of major importance in the atmospheric evolution below 3 km during dry-season and up to 5 km during the rainy season. Layers possibly generated by PBL, storms, wind shear and mountain waves were commonly observed in the atmosphere over Santa Cruz. Atmospheric circulation and sea breeze where commonly observe to interrupt the growth of the PBL and terminate atmospheric layers. Layer splitting was observed on a regular basis during the dry season experiments.
Figure 6.29: Isolated patches of turbulence (IPoTs) observed during the year long experiments in Santa Cruz, Costa Rica. Low values were measured during the dry season months and larger values during the rainy season. Possible implications to aircraft safety could be derived from this information. The vertical error bars are the two way standard deviation.
Chapter 7

Conclusions

Windprofiler radars are important tools for weather prediction and archival, and have evolved substantially since their early development in the 1970’s and 1980’s. More efficient and cost effective systems are being developed; mainly driven by faster computers and a better understanding of the analysis techniques/tools and atmospheric phenomena. The objective of this thesis was to make advances in several areas, using a new radar recently developed in Costa Rica.

Specific areas of study include the following. First, the theory of some spectral tools was studied in Chapter 3; special focus was given to Capon’s method and its implementation in radars. Second, computer simulations were used multiple times to simulate radar interference pattern and radar backscatter (among others). Third, experimental information was used to analyze the Costa Rican atmosphere over a year-long period. These interrelated areas of radar science used during this research generated the results shown in previous chapters and the following conclusions.

Topics from Chapter 3

The ability to resolve closely-spaced peaks makes Capon’s method appealing for application in different areas of radar research. The noise reduction obtained by data-dependent filters enhances the peaks making them sharp and easier to identify. A series of tests were carried out to see how well this method behaves under small sized filters. These findings along with other relevant results of their application in the deconvolution process have been published (Garbanzo-Salas and Hocking, 2015).

Results shown in figure 3.2-a (page 50) provide evidence that Capon’s method resolves peaks better than Fourier transform under equal filter size. When comparing Capon to itself with a different filter size, and using Fourier transform with a larger filter as a reference, a different scenario is obtained. The results obtained with Capon’s method with a large filter, shown in figure 3.2-b (page 50), are different from the ones obtained with a small filter (figure 3.2-c, page 50). Under different filter sizes the method can return not only a wrong number of peaks, but misplace the locations.

The previous behaviour was tested for multiple spectral peaks. A degradation of the products generated by Capon’s method is observed in figure 3.3-a,b (page 52). The right pane of
this figure clearly showed that by decreasing the filter size from 512 to 16 the results went from near perfect (defined as a one to one relationship with the Fourier results) to inaccurate. The reason for the degradation in resolution is based on the degrees of freedom imposed by the filter size. This can clearly be seen in figure 3.4 (page 55) as Capon’s frequency response goes from adaptive to static.

Other tests of Capon’s method with Gaussian spectral fitting were carried out. When using Capon’s method to estimate spectra the width will be reduced compared to Fourier’s results. The frequency of maximum power is better estimated by Capon’s only for very small filter sizes (where the peaks resolvability is decreased, as just mentioned).

This results are valuable as Capon’s method can be used in different areas of radar research. The use of Capon’s method usually rely on small filter sizes where the peak resolvability behaves poorly. Care must be taken when considering real or accurate information of peaks under such conditions.

Regarding the process of the convolution and deconvolution, the theory was presented in Chapter 3. The convolution represents accurately how nature handles the backscatter of radar pulses. The deconvolution consists of recovering the original atmospheric profile, and is possible only if the received signal and the transmitted pulse are known. The deconvolution enhances the results and improves the resolution (Hocking et al., 2014; Garbanzo-Salas and Hocking, 2015).

An example of the deconvolution process was included in figures 3.7 and 3.8 (pages 62 and 63). The Python implementation of the deconvolution process is included in appendix H. This result provides the necessary tools to understand and visually comprehend the convolution/deconvolution process.

Three-dimensional location of scatterers is possible when multiple receivers are used. The radar in Santa Cruz can use up to three different receivers simultaneously. As shown in Chapter 3, this technique is complicated, but if implemented correctly the scatterer location and radial velocity can be accurately determined. The product of the interferometry process (with Costa Rican radar measurements) can be projected into three coordinate planes. This result was shown in figure 3.14 (page 72).

In this figure, the radial velocities are included and a clear horizontal wind is observed in the radial components. The projected radial velocities show that half the volume is approximately positive while the other half is negative. This common results of horizontal motion (observed in all panes) is a clear proof that the interferometry procedure generated the correct locations and radial velocities. The variability of the interferometry products was also demonstrated in Chapter 3.

The interferometry procedure can be used for multiple purposes. As shown, a single time series can be used to locate the scatterers; long periods of time can be used to observe for average location of scatterers. If the scatterers are assumed to be random, and move across the entire scattering volume, the average of the scatterer location over a long period of time should be zero. In our case, this assumption proved to be false, as the receiver phase needs to be measured and corrected after the detection.

The receiver phase was changed systematically for the three receivers. The results were used to calculate the location of the scatterers. Under ideal conditions the maximum return should be obtained from overhead (zeros in the x and y coordinates). The results were shown in figure 3.17 (page 79). The products of this tests calculated the ideal phases to be −0.268
rad (−15.35 degrees) and −0.645 rad (−36.95 degrees). These values are close to direct measurements of receiver phase difference. The values measured at the radar site were −0.146 rad (−8.37 degrees) and −0.730 rad (−41.86 degrees) which are close considering the error in the measurements to be in the order of 8 degrees (an analog oscilloscope was used). This result proves that this type of analysis can be used to calibrate the radar. It can be particularly useful in cases where remote recalibration is needed or when a large number of receivers are used.

The final product that was obtained during the spectral tool analysis is the atmospheric wind from interferometric data. The radial velocities and the location of the scatterers were used in a minimization process in order to estimate the mean winds. An example of the generated product was shown in figure 3.19 (page 81). This result showed that the interferometric wind calculation was generating wind information from the three receiver data. No upper air information is available on-site to validate the magnitudes obtained. A radiosonde campaign will be planned in early 2016 to obtain such information and correlate the measurements.

**Topics from Chapter 4**

In Chapter 4 different simulations related to radar were included. The results presented in figure 4.6 (page 98) show evidence that the interference pattern of the radar antenna in Costa Rica was accurately simulated. These results can be compared to those published by Hocking et al. (2014). The results are not perfect as near field effects were not considered in the simulations. Even then, the results are considered a product worthy of attention as they provide a good estimate of the interference pattern.

In figure 4.7 (page 99) the three dimensional visualization products of the interference pattern simulation were included. The side-lobe observed in this figure is caused by the bi-static radar configuration. The detailed structure of the lobe is not accurate due to the near field effect, but it provides a good first approximation.

One important result of this research was presented in figure 4.11 (page 107). This simple figure demonstrated that the radar model used, as well as the simulation parameters and implementation, accurately generated the Bragg-scale backscatter observed by MST type radars. In figure 4.11-a the maximum is clearly observed at the λ/2 scale. The power obtained by the simulation rapidly decreased when deviating from the maximum return region. Figure 4.11-b, showed similar results but for structures where harmonic scales of λ/2 generated backscatter. These results are crucial as a good radar simulation depends on resolving appropriately the Bragg-scale returns.

Better performance and results were obtained by introducing algebraic manipulation into the simulation engine. A large number of scatterers (> 3000) was correctly simulated by the improved version of this radar implementation. The results presented in figure 4.25 (page 4.25) are very similar to real radar measurements. In fact, they are so realistic that the beam broadening effect is observed along with the Doppler velocities. The analysis of the results was included in figure 4.27 (page 131). This figure shows the different effects just mentioned. The Doppler velocities are observed in 4.27-b, and the spectral width (where the beam broadening effect is observed) was presented in 4.27-d.

The previous results opened the door for multiple other simulations. Future simulations can include:
• stretched scatterers to study anisotropy
• wind shear to study shear broadening
• vertical velocity inside the radar’s scattering volume
• spinning scatterers
• accelerating scatterers

where each of these scenarios could bring insight into atmospheric echoes and their relevance/impact on measurements.

The previous simulations were based on the principle of superposition. Multiple one-dimensional paths were taken and the results added in order to obtain two or three dimensions. The way nature handles the atmospheric profiles is not one dimensional. The three dimensional pulse travels in a three dimensional atmosphere. A three-dimensional convolution occurs and the backscattered power is received by the radar.

A better way to imitate nature’s mechanisms would be to consider the entire space. A model was developed where the pulse and space are converted into wavenumber domain and then convolved. This model was later used to implement the code and setup a simulation. The results were shown in figure 4.30 (page 136), where different pulse lengths (vertical axis) and beam widths (horizontal axis) were used. As expected, the returned power increased with a longer pulse. The enhanced beam width allows a wider field of view; as more scatterers are included the returned power also increases.

Computer limitations\(^1\) prohibited the model from achieving the three-dimensional stage. Two-dimensional results were promising and further research regarding its application is encouraged.

**Topics from Chapter 5**

One of the major goals of this research was to obtain a product using computer modelling of the atmosphere and radar backscatter. The atmospheric motions were calculated by a robust atmospheric model (WRF) operating in large-eddy-simulation mode. The backscatter model was created during this research and used in conjunction with the results of the atmospheric model to simulate radar backscatter. The concept is depicted in a diagram shown in figure 5.1 (page 139). This goal was achieved satisfactorily with some limitations on model resolution. High resolution information was generated by the WRF model as shown in 5.7 (page 150). These data were used in conjunction with the previously developed radar backscatter model to simulate radar data. Vertical beam data were simulated satisfactorily and the result was displayed in figure 5.8 (page 152). Because of the versatility of the radar backscatter model developed, a tilted transmission angle was also used to simulate backscatter. The tilted beam data were shown in figure 5.9 (page 155).

Both vertical and tilted beam data can be compared. If the structures found in the atmosphere are isotropic, they would generate the same backscattered power independently of the

\(^1\)Even when using the node Iqaluk with 1 TB of RAM memory
direction of propagation. Under isotropic conditions, the data obtained from both directions of propagation should be identical and a ratio of 1 should be obtained. This was not the case as scatterers generated by atmospheric motions are stretched according to the motion of the fluid. The result of the anisotropy study was shown in figure 5.10 (page 157). In this figure a region equivalent to the PBL top is observed in red from 3 to 15 minutes into the simulation in heights below 500 metres. For this region the tilted beam will generate much stronger echoes than the vertical beam. On the other hand, the upper regions showed great variability between vertical and tilted; this can be attributed to the stretching/tilting of scatterers in the fluid.

An important result was obtained by comparing the vertical beam information to actual radar measurements. It was shown in figure 5.11 (page 158) that the scenario simulated in WRF and later processed for radar backscatter resembles one case measured with the radar. This is an encouraging result and is it highly recommended that further research is carried out in this topic (using the convolution model implemented here). Improvements in the resolution and model boundaries could greatly benefit the results. A higher frequency could be used to simulate the radar backscatter in order to match the radar’s settings.

**Topics from Chapter 6**

Two different approaches were used during the analysis of measurements in Costa Rica; the spectral fitting method and the direct integration method. Both methods can generate valuable information regarding atmospheric echoes. As shown in Chapter 6, the spectral fitting method generates more information that can be used for a thorough analysis.

A year-long data set was analyzed with the Gaussian fitting method. A detailed view about the Costa Rican planetary boundary layer (PBL) was presented in figures 6.16 and 6.17 (pages 187 and 189). Complete cycles of the PBL evolution were observed by the radar. Different scenarios of growth rate and evolution were presented as examples of the great variability found in the tropical PBL. These cases of PBL are not intended to be a comprehensive climatology of PBL, but rather a list of observations and evolution cases of PBL. No previous experiments in Costa Rica have used vertically pointing radar to study the PBL top and evolution; this research provided the first documentation concerning the PBL evolution and statistics on the PBL top as observed by radar.

Satellite imagery (e.g. figure 6.18 on page 190) proved to be of great value while analyzing radar data. General circulation information can be easily observed by cloud motion. Storms, strong convection and even sea-breeze fronts can be located and tracked. This imagery provided the necessary information to described some of the observed echoes and behaviour observed in the radar measurements. One clear example of a change in behaviour was described when the general circulation changed and the nature of the echoes changed drastically in a matter of minutes (e.g. figure 6.17-g, page 189).

The results of the analysis were summarized in three figures: (i) 6.27 (page 205) with layer information, (ii) 6.28 (page 207) with PBL information, and (iii) 6.29 (page 208) with information about the patches of turbulence. The layers observed during the experiments were quantified and the information shown in figure 6.27 (page 205). The behaviour observed in the layer formation seems to be bimodal, following the rainy season pattern and the midsummer drought as shown by Magaña et al. (1999). It should be taken into account that June data...
were missing due to an error in storage and the following months do not have the data density obtained during the first six months of the experiment. In any case, the number of layers observed is not constant. Great variability can be observed between seasons and even during a short period of time, as was shown in figure 6.20 (page 194).

The planetary boundary layer top information was concatenated and displayed in figure 6.28 (page 207). The average PBL top shows a minimum obtained during January (2014) and a maximum during August (2014). The maximum recorded PBL top per month follows a similar trend. The variability of the minimum PBL top is clear in figure 6.28 but the tendency of low values during dry season and high values during rainy season is present. The large standard deviation observed in the PBL top is expected due to the natural variability of the PBL; even then the tendency is maintained.

Isolated patches of turbulence (IPoTs) were observed regularly in the measurements. These patches were also quantified and the result of the monthly average was shown in figure 6.29 (page 208). The minimum observed events happened during April (2014), and the maximum during November (2014). The variability can be due to the low density of data.

The findings of this research (especially the information regarding layers, PBL tops, and IPoTs) are of importance for meteorological applications and national weather services. One possible application of this analysis is related to aircraft. There is an international airport in Liberia; this city is located 50 km away from the radar site. When flying in/out of Liberia it is common to experience strong turbulence. This turbulence can be due to any of the three described events as all of them can be related to turbulence and energy dissipation. A future development of these studies could be to correlate the observed events with turbulence information from aircraft data. This study could relate radar information about the Costa Rican atmosphere with inflight-turbulence, hence having a positive impact on aircraft safety.

Finally, to complete the conclusions, all the previous results have shown that the radar developed in Costa Rica can be used for atmospheric and radar research. This potential for research made possible the measurements in high resolution using multiple receivers. Further research using this state of the art equipment is encouraged. Improvements in the radar site housing, hardware and software can increase the performance of the radar equipment and products.
Bibliography


BIBLIOGRAPHY


Appendix A

Radar equation, unambiguous range and resolution

A.1 The radar equation

The energy coming back from the atmosphere is measured using the received signal. In order to create an expression that estimates the energy coming back from the targets some assumptions are needed. If the antenna emitting the electromagnetic wave (signal) is considered ideal and isotropic, and the transmitting power is $P_S$ (watts, $W$); the intensity of the wave at a distance $R_1$ (meters, $m$) from the source can be represented as:

$$I = \frac{P_S}{4\pi R_1^2}$$  \hspace{1cm} (A.1)

For a non-isotropic (realistic antenna) the intensity $I$ ($Wm^{-2}$) will be multiplied by an angle-dependent gain function $G(\theta, \phi)$, resulting in a directional intensity. The target is described in radar theory as a function of how much of the energy received, it can send back; this is known as the radar cross section $\sigma$ [$m^2$]. Then the power sent back from the target will be:

$$P_T = \frac{P_S}{4\pi R_1^2} G(\theta, \phi) \sigma$$  \hspace{1cm} (A.2)

If the target is not considered an isotropic emitter, another angular dependency known in radars as aspect sensitivity, must be incorporated. Treating it as an isotropic emitter and the separation target-detector being $R_2$, the power returning to the detector would be:

$$P_D = \frac{P_T}{4\pi R_2^2} = \frac{P_S}{(4\pi)^2 R_1^2 R_2^2} G(\theta, \phi) \sigma$$  \hspace{1cm} (A.3)

The detection antenna will have an equivalent to the cross section referred to as the effective antenna aperture $A$ [$m^2$], and is a measure of how much of the energy received is capable of detecting. The ideal scenario would be that all the area covered by the antenna would be the effective antenna aperture; however, limited by the physical characteristics of the antennas, it is usually less. Considering that in many radars $R_1$ is equal to $R_2$, the final power received by the radar can be represented by:
The gain term \( G \) is not only dependent on the direction, it will depend on many other factors, mainly the wavelength \( \lambda \) of operation and the effective antenna aperture \( A \).

The variables \( G \) and \( A \) are determined during the calibration of the radar. If the cross section is known or estimated and the two power terms are known (measured during transmission and reception) the range \( R \) can be readily determined from that equation.

This equation represents the power received from one single scatterer, but care must be taken when studying the returns. There are more mechanisms in the atmosphere to send signals back than single scatterers (e.g. volume scatterers, specular or partial reflectors). A more realistic case is volume scattering, where many structures inside the volume enclosed by the pulse are responsible for the power return. In this case, the equations need to adjust to account for the radar reflectivity (cross section per unit volume, \( \eta = \sigma V \)).

Using dimensional analysis an estimation of the order of magnitude of \( P_D \) can be obtained. For an \( A = 10^4 \text{ m}^2 \) with a \( \sigma = 10^2 \text{ m}^2 \) at a range \( R = 10 \text{ km} \) the ratio of \( P_D/P_S \) could be less than \( 10^{-12} \) (ignoring the \( G \) dependance).

In equation A.2 the Gain factor was introduced as the term that incorporates the angular dependance of an antenna.

### A.2 Range ambiguity

The capability of the radar to resolve two different objects is referred to as resolution. This capability can be separated into range resolution and angular resolution. The angular resolution is closely related to the antenna’s physical characteristics. If the antenna’s polar diagram has a wide central maximum, the radar will not be able to differentiate between two objects at the same range but close together. In the radial coordinate, the radar resolution will depend on the length of the pulse used, usually represented with \( \tau \). This variable (with units of time) is the duration of energy transmission, from the beginning to the end of the pulse. This time span can be converted to distance (length of pulse) just by multiplying it by the speed of light.

Every pulsed radar sends energy at a certain rate. The number of pulses per second is known as, pulse repetition frequency (PRF = \( f_{PR} \)). Another way to represent the same parameter is by its inverse. The inter pulse period (IPP = \( T_{IP} \)) can be calculated as \( 1/PRF \), and represents the time between consecutive pulses. This parameter is directly related to range aliasing, which can generate ambiguities in the separation of a target from the detector. The maximum distance that can be measured without ambiguity is:

\[
R_{amb} > \frac{c \ T_{IP}}{2}
\]
A.3. Resolution

The more effective way to prevent range aliasing is to change the nature of the transmitted pulse. This will not prevent the echoes from reaching the detector but will make them distinguishable, increasing the maximum distance without ambiguity. The IPP variable and an example of range ambiguity are presented in figure A.1.

Figure A.1: Diagram of two consecutive pulses (shown as large red and blue pulses) sent by a radar. The echoes (small red and blue time-inverted pulses) are observed as smaller amplitude inverted copies of the original pulse. Two different atmospheric profiles (1 and 2) are shown at the top of the figure. Two closely spaced scatterers in the profiles are not resolved as they are separated by a distance shorter than \( \frac{c \tau}{2} \). An aliased echo (last pulse observed in red) is included as an example of a common problem in radar detection known as range folding. The time \( t \) shown in the figure can be used to locate the scatterers. The inter-pulse period is shown.

A.3 Resolution

The radial resolution of a radar can be calculated as half the pulse length. For two objects to be fully resolved, the separation \( (R_s) \) between them needs to be:

\[
R_s > \frac{c \tau}{2}. \tag{A.6}
\]

There are many methods to improve the resolution of a radar. An increase in resolution can be achieved by using intra-pulse modulation. This is specially important in applications where the targets of interest are small. The angular resolution can be improved by creating narrower beam widths. Narrow beams require larger antennas than broader beams.

If the target is moving, more information can be extracted from the echoes. The frequency of the scattered signal will be different from the transmitted frequency. The signal will be affected by the Doppler effect. The frequency change for a single frequency pulse (\( \delta f = 2 \nu \lambda^{-1} \)) is a consequence of the radial component of the velocity not the total velocity. It is important to take this into account because a target with a large velocity moving tangentially to the radar
beam, will not cause any frequency change, thereby appearing to have a null speed even when the position inside the scattering volume is changing.
Appendix B

Radar in Costa Rica

Costa Rica is a small country located in Central America. The land extent is 51100 km$^2$ divided into seven provinces. The maximum average temperature is 28°C and the minimum is 17°C. The radar is located in the province of Guanacaste, which is in the North-West side of the country; on the Pacific Coast and close to Nicaragua. Figure B.1 shows the location of the country along with some photographs of Santa Cruz and the region. Santa Cruz was known in the past as “The land of Diriá”. Diri-á can be translated as “small hills” from Chorotega, which is the language of the natives. Santa Cruz was chosen as the host of the radar projects described here.

The theoretical part of a radar operation is quite challenging. Its complexity is only paired by the challenge of actually building a radar. In the next few paragraphs the experience of building a radar will be described. In specific the case of the new wind profiler radar in Santa Cruz, in the province of Guanacaste in Costa Rica. In the theoretical part of the project, the interests of those building the radar play a crucial role. The most important part of an atmospheric radar is the area of the atmosphere that will be studied and what are the related parameters that will be used to study it. This will define among others, the frequency used and the antenna required, that at the same time put a limitation in the amount of land that will be needed to put them in place.

The process of actually building the radar requires knowledge of engineering as well as some handy abilities that most schools of formal education ignore these days. Drilling, hammering and shovelling are common tasks during the early stages of building a radar. Usually contractors will take care of most of the building but in the case of the radar described in this thesis, the entire project was put together with no more than 3 persons at the same time.

Before concentrating on the radar and its operation a short view of the University of Costa Rica is presented. The University of Costa Rica (UCR from now on) has a satellite campus in Santa Cruz, Guanacaste, located north-west of the country. This town is very small but very interesting because of the rich culture, beautiful landscapes and easy access to several of the most important touristic beaches in Central-America. The campus at Santa Cruz, used to be an agricultural land for the University, but the UCR, being the bigger institution in Costa Rica in research, education, and community outreach, decided to use a portion of this land for research in areas that are not regularly explored in Guanacaste.
Figure B.1: **Top pane.** Downtown Santa Cruz during the annual celebrations. The locals dress up in their best "vintage" clothes and dance to the rhythm of the marimba. **Top map.** Costa Rica location in Central-America. **Bottom map.** Santa Cruz town location in a map of Costa Rica. **Centre pane.** Costa Rican-style bull riding. The redondel (arena) is filled with people trying to impress their friends with clever jumps and tricks. The bulls are not harmed during the "corridas" (can be interpreted as: very intense run) and the mishaps are the main attractions. **Bottom pane.** Sunset captured at Tamarindo beach. Tamarindo is a coast town facing the Pacific ocean near the Costa Rican dome. It is just 30 minutes away from the radar site (by car). During experiments or students visits it is always a popular place for gathering after a long day of work.
B.1 The University of Costa Rica

The University of Costa Rica (UCR) is the largest and more advanced university in Costa Rica. It holds more than 39,000 students annually. The faculty of UCR consists of thousands of highly trained educators and researchers. Engineering, medicine, economics, social and natural science are examples of the most renown carriers offered by UCR (logo is shown in figure B.2).

![Figure B.2: Logo of the University of Costa Rica.](image)

Different satellite campuses of UCR can be found in Costa Rica. In the north-west of the country, the Guanacaste province, a large campus is located in Liberia. Only 60 km away from Liberia a small research centre in Santa Cruz was used for agricultural research in the past. Nowadays this research centre is expanding into natural science research where physics and astronomy can greatly contribute.

The School of Physics (SoP) is the major entity for physics research inside UCR. In the early 1980s, the faculty of the SoP decided that only scientists holding a PhD degree could become permanent staff. Twenty-eight professors conform the Faculty of Physics; with many others in training elsewhere. This Faculty has incorporated into its department several groups: (a. Astronomy and astrophysics, b. Applied nuclear physics, c. Theoretical physics, d. Condensed matter physics, and d. Oceanic, planetary and atmospheric physics). The activities at the SoP are carried out by different research centres, for example:

- CICANUM. Centre for research in molecular, atomic and nuclear science.
- CICIMA. Centre for research in material science and engineering.
- CINESPA. Centre for space research.
- CIGEFI. Centre for geophysical research.

Inside the School of Physics many laboratories also support the research centres. The Laboratory for Atmospheric and Planetary Research (LIAP, for its initials in Spanish) collaborates with CIGEFI and CINESPA closely. LIAP, CIGEFI, and SoP have supported the radar projects with financial resources, computational equipment, research assistants, and logistics coordinators.

Most of the UCR faculty has carried out their doctoral research in different universities around the world. International relations are usually enhanced by the Foreign Cooperation and International Relations Office (OAICE for its initials in Spanish). Projects involving the UCR with large universities in the Americas, Europe, and Asia are common for this institution.
B.2 Collaboration between UCR and the University of Western Ontario

The University of Western Ontario (UWO) and UCR were involved in a collaborative project several years ago. Dr. Wayne Hocking (UWO) established in Costa Rica the first meteor radar in Central America. The meteor radar project opened the door for further collaborations.

The Santa Cruz’s campus was not considered a physical-sciences research centre before the meteor radar was installed. This campus was usually dedicated to research involving agriculture and zoology. It was chosen as the radar site because it was located in a low RF interference area; optimal for the operation of radars.

An experimental primary school began operations at the university campus a few years after the meteor radar was deployed. It was common to observe kids playing around the antennas, and, occasionally picking up soccer balls thrown by the students inside the fenced areas. The noise detected by the system increased due to extra power lines and voltage transformers being installed around the growing campus. These conditions precipitated the necessity to improve the radar site.

Professor Marcial Garbanzo was at the time a lecturer at the School of Physics of UCR. Professor Garbanzo started to collaborate with Dr. Hocking in technicalities of the radar project and later on became interested in radar data and its applications. Dr. Hocking invited Prof. Garbanzo to attend a radar school and a conference. The MST conference usually takes place every 2 to 4 years; and during May of 2009, it was organized by the University of Western Ontario in London, Canada.

After attending the conference Prof. Garbanzo, having visited several radar sites in Canada, went back to Costa Rica with his knowledge on radar site requirements. A new initiative was created along with the Director of the School of Physics and the Director of the Department of Atmospheric, Oceanic, and Planetary Physics.

B.2.1 Expanding the project

The initiative included two main components:

- Create a new facility where the meteor radar could be relocated.
- Have enough room in this new facility to expand. The extra space would be used in designing and implementing a wind profiler radar.

A request was sent to the authorities of UCR for a larger site in Santa Cruz. The land would be used to move the entire meteor radar with all of its antennas and transmitter/detection equipment. Due to mutual interests, the UCR and UWO requested additional space to design and build a wind profiler radar. This wind profiler radar would be the first equipment to monitor the troposphere in Costa Rica; as no other such type of radar existed in the country or the region.

The approval of the local government and the scientific committee of the regional UCR authorities was required to obtain the land for the new facilities. Figure B.3 shows a photograph taken during the meeting with the scientific committee at Liberia, Guanacaste. Many other meetings took place with politicians and UCR staff. A field of 1.5 Ha was assigned along
B.2. Collaboration between UCR and the University of Western Ontario

Figure B.3: **Left Pane**) Members of the scientific committee of the Guanacaste UCR, including Dr. Raziel Acebedo (Director of Liberia’s campus) and Engineer Edgar Vidal (Director of Santa Cruz’s research campus). **Right Pane**) Dr. Rodrigo Carboni (Director of the School of Physics), Dr. Walter Fernandez (Director of the Department of Atmospheric, Oceanic and Planetary Physics), and M.Sc. Marcial Garbanzo presented the proposal about the new site for the meteor radar and the wind profiler radar.

Figure B.4: A photograph of the land assigned to the project is presented. One and a half acres of grass land were re-purposed into the radar site. No electricity, internet, running water or security were available at the site.
with an initial budget to begin the planning. The area assigned to the project was located approximately 800 metres west of the main office in the research facilities. No electric power, internet, security, or running water were available. A photograph of the land assigned to the initiative is shown in figure B.4. This land was previously used to grow grass to feed the animals on the research centre. The number of animals had decreased in the previous years and it was underused.

B.2.2 Initial stages

Large radar sites (e.g. Jicamarca (Perú) or MU (Japan)) have large buildings hosting the project. Small and medium scale radars can use a trailer to host the components. The transmission, detection and computation equipment are small in size these days. The advent of solid-state amplifiers, reduced hardware in detection, and modern computers makes it simple to fit the equipment in $4m^2$.

A trailer was chosen as the most appropriate unit for housing the equipment. The trailer was purchased and assembled in San José (the capital of Costa Rica). Two photographs of the trailer are shown in figure B.5. In that figure the left was taken during the welding stages of the walls. The right pane of figure B.5 shows the final product. A few months after the beginning of the project, the trailer was ready to be moved into place.

Figure B.6 shows a photograph of the trailer after deployment at the Santa Cruz’s campus of the University of Costa Rica. The land was previously used for agriculture so no surveillance, electricity, running water or internet lines were available at the site. After the installation of the power lines, video surveillance was achieved by a closed-circuit television system. The internet access was temporarily implemented by a long distance microwave link, achieving up to 10 Mbps transfer rates, enough to communicate with the radar equipment.

The fence around the assigned land was a requirement before the meteor radar site could be moved. The construction of the wind profiler radar did not began until the fence was completed. Figure B.5 shows two photographs of the initial stage of the fence. One and a half acres of land were surrounded with a tall fence. This fence proved not to be effective against the intruders in the perimeter. Burglars tried to break into the trailer (even when empty) but the solid structure withstood several attempts.

The subterranean power lines were installed shortly after the fence. Figure B.7 (left pane) shows a photograph of the ditch used to bury the cables. Electronic devices were brought into the trailer after the electric cabling was completed. Air conditioner and a home-surveillance systems were installed immediately. The right pane of B.7 shows a photograph of the air conditioner installation. While fitting an air conditioner to the trailer, some sparks from the power tools used reached the dried grass and caught on fire. A tractor was needed to put out the fire completely. In northern Costa Rica, wild fires are hazards to infrastructures.

A home-monitoring security camera system was installed to provide the local security officer a remote view of the future radar site. This was very important at the time as one security officer in Santa Cruz needs to monitor more than 50 acres of land. Signs that read: “YOU ARE BEING OBSERVED BY UCR SECURITY” were installed around the fence and trailer windows. The combination of signs, surveillance, and fence proved effective and no further attempts to break into the site were registered. A safe and fully functional environment for the
radar site was finally achieved with one exception. Running water was not obtained in the site for another 4 years.

### B.2.3 Meteor radar

The original meteor radar site was located in a modified old classroom. The modifications allowed the site to be functional but it was still located near other operational classrooms and surrounded by possible RF contaminants. A photograph of the equipment room is shown in figure B.9 (left pane). Santa Cruz is a town where the temperature during the day can be extreme. The temperature often reaches more than 30°C during the day. This high temperatures outside can negatively impact closed spaces with equipment. After a few years of operation, the air conditioner unit used in the original radar site, could barely handle the heat generated by the power amplifiers and detection equipment.

Regarding the antennas they were spread around the nearby area of the detection equipment. Small fences were build in order to keep kids and general public from touching the antenna elements. The transmitter antenna (that carries the most power) was located further away with a bigger, more appropriate fence around it. Figure B.9 (right pane) shows a picture of a receiver antenna. During most of the maintenance visits the fences needed to be replaced as the posts were beat down by kids playing soccer. These kids came from a new project at the university involving an experimental primary school. The radar site was no longer appropriate for the project and time to move to the new site had come.

The equipment shown in figure B.9 (left pane) was moved into the new site. Figure B.10 (left pane) shows the same equipment once moved to the trailer. With the change in location several factors improved immediately. The new air conditioner better regulated the temperature. Less RF interference caused the number of meteors detected by the system to increase by a factor of four.

The antennas were located in a open field away from voltage transformers or exposed power lines. A photograph of the far end of the receiving antenna array is shown in figure B.10 (right pane). Since the time of installation, these antennas have been tuned a few times. The advantage of changing sites proved to be of great benefit for the meteor radar project.

### B.2.4 Wind profiler radar - One receiver stage

The original initiative presented to UCR authorities included the implementation of a wind profiler. No previous project on the radar site included a wind profiler. All the different stages of the radar had to start from scratch. A site survey looking for clean frequency bands was realized. A 46.6 MHz frequency was chosen as the centre frequency of the radar. Many aspects of the antennas were immediately set by choosing the frequency. The type of antenna chosen was the Yagi-Uda antenna with one reflector and one director.

The main aluminium elements for the antennas were purchased in Costa Rica. Other parts of the antennas were shipped from Canada in the first stage of development. A photograph of the first crate is shown in figure B.11 (left pane). This crate contained large rolls of coaxial cable to connect the antennas. Other necessary components like a laser level, shovels, long measuring tapes, and boom clamps were inside the crate. The first stage did not include the computer equipment.
Figure B.5: **Left pane**) Initial stages of the trailer to be used as housing for the radar equipment. **Right pane**) Finished inside of the trailer.

Figure B.6: The trailer was transported to Santa Cruz and located in the assigned area. The fence to protect it from burglars was still pending and there is only one security officer in Santa Cruz to cover more than 50 acres of land. The trailer build requirements considered this, and reinforced doors and windows were used. Several attempts to break into the trailer were made (even when it was empty) but all of them were unsuccessful.
B.2. **Collaboration between UCR and the University of Western Ontario**

Figure B.7: During March 20th, 2011, the fence was built. **Left pane:** The fence’s frame was located in a rectangle that covered the 1.5 Ha of land. **Right pane:** Rolls of wired mesh were piled up next to the radar while waiting to be welded into place.

Figure B.8: **Left pane:** The subterranean electrical power lines were installed shortly after the fence was built. Burglars tried to retrieve the wires (common felony in Costa Rica), even though the fence was installed. **Right pane:** The air-conditioning unit was installed after power lines were deployed. During the soldering of the frame support the dried grass caught on fire with the sparks coming from the power tools.
Figure B.9: Left pane) Original installation of the meteor radar. A class room was modified to be used as a radar site. The air conditioner rarely worked due to continuous unexpected power outages. Right pane) The antennas were surrounded by a small fence to protect the kids from an experimental kindergarten.

Figure B.10: Left pane: New meteor radar site inside the trailer. More space was available inside the 3 by 9 metre trailer compared to the small room showed in the previous picture. Right pane: Meteor radar antennas located in the new site. Clear skies without obstacles were an immediate benefit to the project that showed a dramatic increase in the number of meteors detected.
Antenna assembly and trench digging began once the first crate arrived to Santa Cruz. The trenches needed to be deep enough so that the cable would not be accessed easily (except on both ends). A picture of the trenches used for the main cable to the transmitter is shown in picture B.12 (left pane). Unfortunately, even with the trenches, an accident happened two years after when a tractor was used to cut the grass and instead, it severed one of the cables. After long hours of work, all the 21 antennas (12 for receivers, 9 for the transmitter) were built and connected.

The second stage of development started in late 2011. The computational equipment was assembled in the University of Western Ontario. One receiver was also built, calibrated and tested at UWO. All of these components were disassembled and carefully located inside a second crate. This new crate arrived at Santa Cruz’s campus on January 4th, 2012. It took three days to properly unpack and reassemble all of the equipment. The rack that houses the radar components was customized in London, ON. The shelves were located at the exact heights to allow the receivers, computer, KVM switch, pulse shaping equipment, and transmitter to fit appropriately. Figure B.12 (right pane) shows a photograph of the rack along with all the radar components in place. In this figure, a single receiver can be observed in the top shelve.

This configuration was used to test the equipment. The results of the tests are shown in Chapter 5. One experiment took place shortly after all of the equipment was tested. The findings were published by the collaborators of the project [Hocking et al. (2014)]. The same configuration was kept for several months while recording atmospheric information. This will be described in Chapter 5. Multiple receivers need to be used in order to determine winds; a modification to the radar was necessary to add the extra receivers.

### B.2.5 Wind profiler radar - Three receiver stage

The three receiver stage is the final (so far) development of the radar. Three different receivers are used instead of recording a single receiver. This allows the different signals to be analyzed and compared. Wind information can be obtained from this comparison by using interferometry as it will be discussed in Chapter 3.

During the first stage, the twelve receiver antennas were connected to a joint that matched the three feed cables into a single line. This single line was send to the digitizer. For this next stage, twelve antennas were separated into three groups. The groups of antennas can be observed in figure B.13 with four on the foreground of the photograph and the remaining antennas in the background. In order to differentiate the three groups two extra receivers were brought in during a field trip to Santa Cruz.

The extra receivers were co-located with the original receiver. One receiver per each group of four antennas was used to amplify and filter the incoming signal. Three different digitization channels were used one per each receiver. In figure B.14 a photograph of the final state of the three receiver stage is shown. The top shelf of the rack contains one single receiver. Two receivers are located in the second shelf.

Other important changes are observed in figure B.14. Different power backup units are now located in place to mitigate power outages and variations in voltage supply. High speed internet access was also installed in the trailer. The rack holding the IDF (intermediate distribution frame) connected to the MDF (main distribution facility) at UCR’s central office in Santa Cruz.
Figure B.11: Crates with antenna components and computer equipment were shipped from Canada. The first one was received January 9th, 2011. Antenna assembly began shortly after. The second crate was received at Santa Cruz on January 4th. The computer equipment assemble, radar calibration and first operation followed shortly after. Experiments and results are shown in Chapter 5.

Figure B.12: The left pane of the figure shows the transmitter antennas before interconnection. The main trenches were already dug and the cables waiting to be buried. The right pane shows the completed initial stage of the one receiver mode. No broadband internet connection was available at the time.
The three receiver configuration was used for many months to record atmospheric information. In Chapter 3 the interferometric calibration, interferometric scatterer location and wind estimation are described. Different tests were carried out to observe the three receiver capabilities. The products obtained with three receivers will be described in Chapter 5. The data sets recorded with three receivers provided valuable information regarding the rainy season in Costa Rica, as will be shown in Chapter 5.

Figure B.13: Photograph of the receiver antennas. Three different groups of antennas form the receiver. Each group is made of four individual antennas. The set of antennas in the foreground is receiver number 2. The group in the far right is receiver number 1. The group in the background on the left is receiver number 3.

B.3 Community outreach

The governance policy of the University of Costa Rica states the following:

Estimular la formación de una conciencia creativa y crítica, en las personas que integran la comunidad costarricense, que permita a todos los sectores sociales participar eficazmente en los diversos procesos de la actividad nacional.

translates to English as:

To promote vocational training awareness that is creative and critical to the people that form part of the Costa Rican community; so that it will allow for all of society’s sectors efficient participation in the various processes of national engagement.
Figure B.14: Final stage - Three receiver wind profiler radar. The top two shelves hold three receivers. Each receiver is connected to a coaxial cable transporting the signal received by four Yagi-Uda antennas. High speed internet is now available at the radar site with a optic fibre connection. Gigabit Ethernet was recently added to the site.
It is of great importance to the UCR that the knowledge generated by their researchers is disseminated to the general public. It is common practice at UCR for research centres and individual projects to hold community outreach activities regularly. In the case of the project in Santa Cruz, different activities have taken place in the previous years. Among others:

1. Talk and workshop to grade 2 and 3 of the experimental school at Santa Cruz’s campus.
2. Talk and radar visit to grades 2, 5, and 6 of a public school in Santa Cruz.
3. Radar visit to university students from the physics program at UCR.
4. Radar visit by last-year’s students from the Costa Rican Scientific High School.

From the aforementioned activities, the project has been cited on different occasions on national news. Relevant newspaper articles have been written about the project. This media’s portrayal of the science that is conducted at UCR is important to the interest of the general public.

During a workshop carried out in Santa Cruz’s campus, Diana Jimenez (a collaborator), asked kids to make drawings about what they have understood about the project. Some of those drawings are shown in figure B.15. The title of the workshop was ”Getting to know radars”. In the top right drawing, a phrase in Spanish reads: ”the antennas locate the meteors that tell the computers that tell the people” which, in its own way, is quite accurate.

Figure B.16 shows the visit of the Scientific High School students. Their rigorous program in science allows them to better understand the project. It was of great importance to them and their teachers to visit the radar. A small group of students visited the inside of the trailer where images of the radar’s results were discussed. University students also visited the project on different occasions. Students from their first to senior year of the physics program are shown in figure B.17. This photograph was taken during the initial stages of the project, prior to the meteor radar being moved and the wind profiler radar being assembled.

B.4 Radar technicalities

We have described in previous sections the history of the radars located in Santa Cruz’s campus. The technical aspects of the wind profiler radar are now addressed. Generalities of the radar in Santa Cruz have been published along with the initial results [Hocking et al. (2014)].

Many wind profilers exist around the world, even more MST type radars can be found. Hardware mixers are regularly used during the detection stages in MST radars and wind profilers. These mixers are used in a process called heterodyning in order to shift the frequencies to values where the detection process is simpler. The need for the mixers was generated by
Figure B.15: Schools with kids from grade 1 to 6 visited the radar. As part of the community outreach activities kids are often welcomed at the radar site. An activity facilitated by B.Ed. Diana Jimenez on the basics of radar and related activities educated kids from grade 2. The students showed their understanding with these drawings.
B.4. Radar technicalities

Figure B.16: A Costa Rican scientific high school (CCC, for the Spanish initials) from Puntarenas was welcomed at the radar site. The CCC educational system in Costa Rica aims to prepare youth in the science and engineering areas. Many scientists and high profile professionals in Costa Rica have graduated from this successful system.

Figure B.17: Students from first to fourth year of the Physics carrier collaborate with the project in its initial stages. Oscar Alegría, Reymer Vargas, Raquel Hidalgo, André Oliva, Andrés Chavarría, and Esteban Pérez were taking a break from the class rooms to get some field experience.
the limitations in digitization speeds; the frequencies were lowered in order to make them detectable. Other common problem when using multiple receivers is the need for two digitization channels per receiving antenna. Two channels are needed to record separately the in-phase and quadrature components of the signal. Slow variations in the time series of one specific height are regularly regarded as ground echoes. These echoes are a problem when analyzing slowly moving atmospheric phenomena as the low-speed portion of the spectrum is dominated by Fourier components attributed to ground echoes. These (and several other) problems were considered when designing the wind profiler.

The number of digitization channels was reduced to one per receiver by using appropriate envelopes for the radar pulse. The use of the deconvolution in the received signal mitigates the slow drifts in the time series. Digitization rates in the same order of the radar’s central frequency made the use of hardware mixers obsolete in radar detection. Before looking into those topics individually, let’s consider the deconvolution process and the digitization speed.

The deconvolution is one of the foundations of the radar in Santa Cruz, and as a consequence, most of the research described here. The deconvolution is used constantly during radar operation. The topic of deconvolution is covered in Chapter 3. It is used extensively in Chapters 4 and 5 to simulate different radar scenarios. The basic concept of the deconvolution is that a signal with two convoluted functions, can be used to calculate one function if the other function is known a priori.

![Figure B.18: A photograph of the radar pulse generated in Costa Rica, as shown in Hocking et al. (2014). The pulse length of 6μs is shown. The voltage scaling is obtained from an attenuated output of the transmitter used to monitor the amplification stage.](image)

When the radar pulse is transmitted it propagates through the atmosphere. All perturbations in the index of refraction will cause a small fraction to be scattered back. As a result, the received signal is a convolution of the transmitted pulse and the atmospheric profile of index of refraction variation. An example of a radar pulse is shown in figure B.18. For more deconvolution theory and examples see Chapter 3.

The deconvolution process requires that the original pulse is logged as it was transmitted. The process also requires that the received signal contains enough spectral information to carry out the deconvolution. The antennas are related to the spectral information sent to the atmosphere. The type of antenna, the quality of the connections, and antenna tuning needs to be verified carefully in order to guarantee that the majority of the spectral content is radiated.
The pulse used in Costa Rica consists of a down-chirp-frequency pulse. Multiple frequency components need to be radiated into space or the deconvolution process will fail. A measurement of how good a radiator is can be obtained by calculating the voltage standing wave ratio (VSWR). The VSWR measures numerically how well the antenna and signal line are matched. It is regularly used to find “imperfections” in the line, and it is usually presented as a function of frequency.

The VSWR is a function of the reflection coefficient. It describes how much power will be reflected from the antenna. The VSWR can be calculated with the equation

\[ \text{VSWR} = \frac{1 + |R|}{1 - |R|} \]  \hspace{1cm} (B.1)

where \( R \) is the reflection coefficient. When the value of VSWR is close to one the reflection is close to zero and the transmitted power is maximized. When the VSWR gets larger than one, part of the power sent to the antenna will reflect and less power will be radiated. In real applications obtaining perfect transmission is impossible. During the fabrication of the antenna they were tuned at a specific frequency. This frequency in our case is the central frequency of the radar.

This parameter was measured in Santa Cruz’s radar and the results are presented in figure B.19. A region of large efficiency is observed. Almost 4 MHz are available with a VSWR less than 2. A value of 2 is obtained with a reflection coefficient of \( R = 1/3 \), resulting in 11% of the power being reflected and 89% being transmitted.

![Figure B.19: Voltage standing wave ratio (VSWR) measurements carried out in Santa Cruz, Costa Rica. The minimum value is obtained in the vicinity of 46.6 MHz (the radar’s centre frequency). A wide region with VSWR values below 2 shows that the antennas can efficiently radiate other Fourier components along with the main frequency. For more information look into text and Hocking et al. (2014).](image-url)
The values of VSWR showed that the antennas were capable of transmitting the wide spectrum signal. The spectral content was then efficiently sent to the atmosphere. In addition to the antenna bandwidth, the spectral content of the transmitted and received signals will depend on the available bandwidth (usually limited by regulatory laws), proper filtering at the receiver, and it is ultimately related to digitization rate. We will assume for this discussion that proper filtering of the incoming signal was used at the receivers.

Figure B.20: Figure from Hocking et al. (2014). a) Train of pulses. Spectra calculated with high digitization rates b(i) and low digitization speed b(ii) are shown. The value of $f_d$ is the rate of digitization. The spectral regions $A$ and $B$ are the original locations of the spectral content. The spectral regions of $A'$, $A''$, $A'''$, and $B'$, $B''$, $B'''$, are regions that are mapped towards each others regions after digitization; $A''$ and $B''$ are the regions of interest. The high rate limit is determined by the Nyquist frequency. Appropriate filtering is needed in both cases, but is particularly important in the low rate case due to spectral leakage and aliasing. The spectral content needs to be filtered in order to preserve the shape of the backscattered signal and not introduce extra Fourier components.

Figure B.20 was taken from Hocking et al. (2014). In this article the technicalities of the Costa Rican radar and a “twin” radar located in Canada are described. The relevance of this figure is that the digitization rate used in the Costa Rican radar is not as high as the one used
in the Canadian version of the radar. The rate of digitization used in Santa Cruz (12.5 MHz) is slower than the maximum transmitted Fourier component (49.1 MHz = 46.6 MHz + 2.5 MHz), hence the case observed in B.20-b(ii) was obtained. As the digitizer speed increases the ideal case observed in B.20-b(i) is obtained. When careful considerations of this aspects are taken into account while building the radar, slow digitizers along with good hardware filters can be used and equivalent results would be obtained.

The radar equipment as was shown in figure B.14 fits in a single rack. A diagram of the different radar components and their interconnection was published by Hocking et al. (2014). In order to better understand the diagram the different areas have been pointed out in a photograph shown in figure B.21. In this figure the grey area corresponds to the inside of the computer. A red line in figure B.21, connects the grey area with the computer screen. Inside the computer two large PCI cards are found. A waveform generator card creates the triggers and clock pulses used regularly during radar operation. The signals are logged into the computer via a digitizer card connected to the receivers and transmitter monitoring port.

Figure B.21 shows the receivers marked with a letter A in the diagram and the photograph. Each of the receivers is made out of four pre-amplification stages and one hardware filter. Appropriate impedance matching is needed inside the receivers. The receiver’s enclosure is made of thick aluminium as described in Hocking et al. (2014). In the same figure, the letter B indicates the location of the transmitter amplifier. The pulse pre-amplifiers and pulse-shaper circuitry are shown in figure B.21 with a letter C. The flow of the pulse can be described as:

1. The pulse shape, length and other variables are defined in the radar configuration file.
2. Based on the experiment’s configuration the waveform generator creates the clock pulses, triggers, carrier frequency and chirp.
3. The pulse is sent to the pre-amplifier stage (C) where it is amplified.
4. Pre- and post-pulse ringing effects are mitigated in the circuitry at stage C.
5. The pulse enters the transmitter and it is amplified to reach the maximum possible power before being radiated into space.
6. The signal sent to the antennas is sampled at the transmitter (B) and sent to the digitizer to be stored along with all the received signals. This step is crucial for the appropriate results of the deconvolution.

Figure B.22 shows a diagram extracted from Hocking et al. (2014). In this figure, an improvement obtained by the deconvolution process is shown. In figure B.22-a) a time series of 28s in length, and obtained without the use of deconvolution, is observed. The slowly varying signals are approximations of the entire data set. One of the signals observed is a low pass filtered version. The other signal is the result of a sixth degree polynomial fit. Both functions are very similar and are observed in figure B.22-a) and B.22-c) in white color. This slow rate of change in the signals, or drifts, is usually named “ground echoes”. As explained in Hocking et al. (2014), many of this effects are not caused by ground echoes, but are rather due to other causes. Among others, the drifts can be caused by ringing of the radar signal in the antenna array or changes in the components of the power amplifier causing changes in the phases.
Figure B.21: Figure taken from Hocking et al. (2014) modified to incorporate the photograph of the radar components. **A:** Receivers located in the upper shelves of the rack. These receivers are connected to the antennas and digitizer card. **B:** Power amplifier. As indicated in the diagram, the pulse is amplified and routed to the transmitter antenna array. A sample of the pulse is sent back to the digitizer as a reference. **C:** The pulse shape unit and pre-amplification are located underneath the computer. **Red Line:** The computer equipment holds the two main cards (waveform generator and digitizer) along with the software for recording and pre-processing the received and reference signals.
In addition to what is causing the echoes it is important to know what effect they have in the observations. Slow variations in the time series will cause the spectrum in frequencies less than ±1Hz to become contaminated with non-atmospheric information, shown in figure B.22-b) . On the other hand, figure B.22-c) shows a time series of 28s in length, obtained with the use of deconvolution. The reduction in slow drifts of the signal is clear. The resultant spectrum of this time series is shown in figure B.22-d) where most of the powers detected are below the value of 1, much lower than those observed in pane b).

![Figure B.22: An example of the improvements provided by the implementation of the deconvolution extracted from Hocking et al. (2014). By using the deconvolution process in time series analysis the “ground echoes” or slow drifts are mitigated. The time series in a) corresponds to data obtained without the deconvolution. Data presented in c) was obtained as a result of the deconvolution process. Both data sets, depicted in a) and c), were measured at heights where noise dominated the received signals. The spectrum obtained for the regular process (without deconvolution) is shown in b), and with the deconvolution in d). In b) the near zero frequency band was notched out and the location and value is represented by an arrow.](image)

## B.5 Radar operation

In the previous sections the history of the radar and some technicalities were covered. In this section the operation of the radar will be briefly described. The hardware involved in radar operation is minimum as all the variables are specified by software. Variables related to experiment configuration, pre- and post-processing, and analysis are all defined and manipulated by
The radar can be configured with a main program. This procedure is a step by step questionnaire about the parameters used to configure each radar subroutine. The result of the initialization is the required files to run a specific setup (e.g. an experiment).

The radar program is started with the command `runcomrd.bat` for a single measurement. Scripts and more advanced routines can be used to repeat the same experiment or change the configuration.

The result of one radar run is stored in a file named with the format

```
raw.guanacaste.YYMMDDhhmmssV_
```

where YY is the last two digits of the year, MM is the month number, DD is day of month, hh is the hour of the day in UCT, mm is the minutes, ss is the seconds and V indicates that is was transmitted vertically. The Costa Rican radar can not yet steer the beam, this feature will be introduced in early 2016 (according to plan). The data inside this file is organized as a series of binary numbers. At this stage two different paths can be taken: (i) use the software distributed along with the radar, or (ii) read the raw data from the file and carry out all the procedures and analysis independently.

There are advantages in each path to product generation. The software provided with the radar is mature, well developed and ready to generate high quality products. More insight and better understanding of the radar tools is obtained by carrying out all the required processing from raw data to radar products.

The procedure to run the radar’s analysis program is simple. Executing the program `runmast.exe` will look for new files of the raw type and will generate the necessary products. Running the alternative routines requires the execution of a script named `alternativeAnalysis.sh`. These routines are program in a combination of Bash Shell Script and Python. They are only available for the GNU/Linux operating system and can be executed in a virtual machine inside the radar’s operating system. Other way to run it is to move the data into a scientific workstation located at the School of Physics back in the main campus, and then run the scripts.

The subroutines used by the alternative analysis script are

- **readMPI.py**: This routine reads the listing of a directory and uses all the available CPUs in the computer to extract the raw information from the files into netCDF type archives. These new files contain metadata that explains its contents to the user and are fast to read and write. During this step the deconvolution is carried out but methods incorporated into the data structures. The final product contains arrays of data with the in-phase and quadrature components of each receiver and the transmitter.

- **spectralAnalysisMPI.py**: This program reads the netCDF files generated in the previous step and using multiple CPUs extract the spectral information from the files. In this process simple integration or spectral Gaussian fitting can be used to obtain the information. The results are saved into netCDF files.

- **plotPowers.py**: The last step of the alternative analysis is to generate products for visualization. This product can be generated for a single file or a group of files. For a single file the spectrum, fit, and variables can be observed. For a group of files the time variation of received power is plotted.
This last procedure was used to generate the results shown in Chapters 4 and 5.

B.6 Conclusion

This chapter was dedicated to present a short history of the project as well as an introduction to the radar’s details. The history of the radar included some information about the University of Costa Rica, and the School of Physics. These entities host the project and their support have been necessary for the current state of the radar. It will also be necessary for future developments.

The small town of Santa Cruz, was also mentioned as it is the nearest populated area to the research centre where the radar site is located. This town is a great historical relevance in Costa Rica as it was one of the first populated areas near the pacific after the arrival of Christopher Columbus.

Several photographs of different stages are included. These images captured the effort and passion towards a project that have reached many goals and will continue to pursue many others. From the children that attended the radar workshops to the university students that visited the site, the science behind the project have permeated into the Costa Rican society. Television, printed, and radio news have communicated the activities to the people of Costa Rica, achieving the goal of science dissemination that is so important for the project at UCR.

The technical aspects of the radar have been mentioned. Details like the antenna bandwidth and the advantages of using the deconvolution have been included. A physical description of the equipment was carried out along with a diagram (figure B.21) explaining the components of the radar. Basic operation of the radar was also described.

In the next chapters, theory about turbulence, evaluation of spectral analysis tools and many simulations will be covered. The results of a year-long experiment in Costa Rica will also be described.
Appendix C

Relation between Structure functions and Correlation functions

In Chapter 2, during the statistical description of turbulence, the structure and correlation functions were used. In this appendix a simple derivation is used to demonstrate how this two functions are related. The assumptions of homogeneity and isotropy are used to obtain such relationship. It is worth mentioning that the correlation and structure functions can be obtained for vector and scalar variables; all of these different functions can be related to different spectral shapes, as mentioned in Chapter 2.

Starting with the parallel structure function

\[ D_{\parallel}(r) = \left| \overrightarrow{u}(\vec{x} + \vec{r}) - \overrightarrow{u}(\vec{x}) \right|^2 \]  
(C.1)

Let’s consider only the projection of \( \overrightarrow{u} \) over \( \hat{x} \) and limit the \( r \) also to the x axis.

\[ D_{\parallel}(r) = |u(x + r) - u(x)|^2 \]  
(C.2)

Expanding this equation

\[ D_{\parallel}(r) = \bar{u}^2(x + r) - 2u(x)u(x + r) + \bar{u}^2(x) \]  
(C.3)

With homogeneous and isotropic turbulence the average values of \( u^2 \) at \( x \) and \( x + r \) are equal. Then \( \bar{u}^2(x) = \bar{u}^2(x + r) = \bar{u}^2 \). Taking the common numerical factor and \( \bar{u}^2 \) outside of the equation, and recognizing the autocorrelation function

\[ R = \frac{\overline{u(x)u(x + r)}}{\sigma^2} \]  
(C.4)

Then, the value of the structure function is

\[ D_{\parallel}(r) = 2\bar{u}^2 \left[ 1 - R_{\parallel}(r) \right] \]  
(C.5)

Another way to represent it is recognizing \( \bar{u}^2 \) as the covariance at 0 lag, \( C(0) \).

\[ D_{\parallel}(r) = 2 \left[ C_{\parallel}(0) - C_{\parallel}(r) \right] \]  
(C.6)
Appendix D

Capon’s Method

This appendix contains a simplified view of Capon’s method. The complete derivation based on Lagrange multipliers is referenced in the text. It is important to remark that the filter and frequency response variables are not directly observed in the final estimation equation; however understanding their behaviour provides necessary insight into the method. Figures depicting the shape of the frequency response variables were obtained by Garbanzo-Salas and Hocking (2015). The shape of the frequency response is only adaptive when the degrees of freedom are larger than the number of frequency peaks inside the signals.

Consider a filter \( h \) of order \( m \) defined by

\[
h = [h_0^* \ h_1^* \ \cdots \ h_m^*]^T
\]  

(D.1)

where \( T \) represents transpose and the values of \( h_i \) are to be determined for each of the desired frequencies. This filter \( h \) is dependent of the chosen frequency \( \Omega_o \) (indicated by the sub-index). The filter \( h \) acts over the data sequence \( y \) to generate the output of the filter

\[
\bar{y}[n] = h_{\Omega_o}^* \begin{bmatrix} y[n] \\ y[n-1] \\ \vdots \\ y[n-m] \end{bmatrix}
\]  

(D.2)

The output power of the filter for a value of \( f \) is given by

\[
S_{\Omega_o} = E \left( h_{\Omega_o}^* \begin{bmatrix} y[n] \\ y[n-1] \\ \vdots \\ y[n-m] \end{bmatrix} \begin{bmatrix} y^*[n] & y^*[n-1] & \cdots & y^*[n-m] \end{bmatrix} h_{\Omega_o} \right)
\]  

(D.3)

Using the notation of equation (3.6) this expression becomes

\[
S_{\Omega_o} = h_{\Omega_o}^* R_{\Omega_o} h_{\Omega_o}
\]  

(D.4)

Capon’s method consists of minimizing the output power while keeping the frequency response at the chosen frequency equal to unity. The frequency response of the created filter can be obtained with the discrete time Fourier transform of filter \( h \).
where the vector \[ \begin{bmatrix} 1 & e^{-i\Omega} & \cdots & e^{-im\Omega} \end{bmatrix}^T \] will be named \( a_{\Omega_o}(\Omega) \). The process of obtaining the appropriate filter values is now a minimization process with a constraint. It can be solved by using Lagrange multipliers (Palmer et al., 1998, page 1597) to determine the equation for the \( h \) filter coefficients

\[
h_{\Omega_o} = \frac{R_y^{-1} a_{\Omega_o}(\Omega_o)}{a_{\Omega_o}^\dagger(\Omega_o) R_y^{-1} a_{\Omega_o}(\Omega_o)} \tag{D.6}
\]

where the data dependence is obtained through \( R_y^{-1} \). The power is obtained with equation (D.4), and by using the value of \( h_{\Omega_o} \) it becomes

\[
S_{\Omega_o}(\Omega) = \frac{1}{a_{\Omega_o}^\dagger(\Omega) R_y^{-1} a_{\Omega_o}(\Omega)} \tag{D.7}
\]

This equation is presented in the document (including the scaling factor) as equation 3.7.
Appendix E

Deconvolution Theory

Most of the theoretical and experimental results obtained during this research were based on the convolution theorem. The concept of the convolution and deconvolution is surprisingly simple; a multiplication in the frequency domain is the Fourier transform of a complicated mathematical procedure in the time domain. The basic equations of the convolution and their equivalent in the frequency domain are presented in this appendix.

The convolution theorem states that if we have two functions \( f(t) \) and \( g(t) \), and calculate the convolution in the time domain, the results of this operation would be the Fourier transform of the product of \( F(f) \) by \( G(f) \) in the frequency domain, where \( F \leftrightarrow f, G \leftrightarrow g \).

To prove the convolution theorem, we can start with the simple concept of \( H(f) \) being the Fourier transform of \( h(t) \).

\[
H(f) = \int_{-\infty}^{\infty} h(t)e^{-2\pi if}dt \quad (E.1)
\]

Also, consider that \( h(t) \) is the convolution of \( f(t) \) and \( g(t) \):

\[
h(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau \quad (E.2)
\]

The equation E.1 becomes:

\[
H(f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau e^{-2\pi if}dt \quad (E.3)
\]

Replacing the variable \( t-\tau \) by \( \eta \):

\[
H(f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\tau)g(\eta)e^{-2\pi if(\eta+\tau)}d\tau d\eta \quad (E.4)
\]

Finally, because \( \eta \) and \( \tau \) are independent,

\[
H(f) = \int_{-\infty}^{\infty} f(\tau)e^{-2\pi if(\tau)}d\tau \int_{-\infty}^{\infty} g(\eta)e^{-2\pi if(\eta)}d\eta = F(f)G(f) \quad (E.5)
\]

The important point here is that a relatively complicated operation in the time domain (the convolution of two functions), is the inverse Fourier transform of a multiplication of the two
spectra. Computationally this is a large advantage regarding the number of calculations and operations that need to be used to obtain the result.
Appendix F

LES model initialization

The WRF-LES configuration requires two input files; one file to set the atmospheric conditions and the other to setup the simulation parameters (including the physics). The namelist.input file contains the information regarding run time, time resolution, simulation grid size and resolution, physics, parametrization, and boundaries. The input.sounding file contains the state of the atmosphere used to setup the virtual atmosphere. Detailed consideration on both files is required in order to properly simulate in the LES mode. One hundred and ninety two processing units were used to run this configuration; it took several days to obtain the result. The Orca cluster and the workstation Iqaluk (both part of SHARCNET) provided the necessary computational resources.

namelist.input

The file format is variable and value respectively.

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run_days  = 0,
run_hours  = 0,
run_minutes = 45,
5 run_seconds = 00,
start_year  = 0001,
start_month = 01,
start_day   = 01,
start_hour  = 00,
10 start_minute = 00,
 start_second = 00,
 end_year    = 0001,
 end_month   = 01,
 end_day     = 01,
15 end_hour   = 00,
 end_minute  = 45,
 end_second  = 00,
history_interval_m = 00,
history_interval_s = 10,
20 frames_per_outfile = 1000,
 restart = .false.,
 restart_interval_m = 60,
 io_form_history = 2,
 io_form_restart = 2,
25 io_form_input = 2,
 io_form_boundary = 2,
debug_level  = 0,
/
30 &domains
 time_step   = 1,
time_step_fractions = 25,
max_dom = 1,
35 x,w = 1,
e_w = 384,
s,n = 1,
```
Chapter F. LES model initialization

e sn = 192,
ss vert = 1,
s vert = 384,
dx = 4,
dy = 4,
top = 1536,
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parent id = 0,
i parent start = 0,
j parent start = 0,
p parent_grid_ratio = 1,
p parent_time_step_ratio = 1,
feedback = 0,
smooth_option = 0

&physics
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ra lw physics = 0,
ra sw physics = 0,
rndt = 0,
sf sf clay physics = 1,
sf surface physics = 0,
b lbl physics = 0,
bldt = 0,
cu physics = 0,
cudt = 0,

is flux = 2,
num soil layers = 5,
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&ffdd
/
&dynamics
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diff opt = 2,
hm opt = 2,
damp opt = 0,
damp = 15000,
dampcoef = 0.1,
khdf = 1,
kddf = 1,
c = 10,
c k = 0.18,
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smdiv = 0.1,
emdiv = 0.01,
epsm = 0.1,
tke heat flux = 0.24,
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v mom adv order = 3,
b sea adv order = 5,
v sea adv order = 3,
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non hydrostatic = .true..
peri_coriolis = .true..
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symmetric_x = .false..
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symmetric_y = .false..
symmetric_ye = .false..
open_y = .false..
open_ye = .false..
nested = .false..
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&grph2
/
&namelist_quilt
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**input_sounding**

The file content is height in metres, potential temperature in Kelvin, water content in grams per kilogram of dry air, zonal and meridional component of the horizontal wind.

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Appendix G

Long data-sets

Some of the radar measurements are contained in this appendix. The figures are included as evidence of the results obtained during the different experiments and settings. Two major experiments were carried out; the one receiver information is presented first followed by the three receiver results.

For information regarding the results see text in Chapter 5. In Chapter 5 not all the images are used, but a good representation of the results is included. Different color schemes were used in order to properly distinguish the one receiver to the three receiver data; however the depicted variables are the same.

G.1 One receiver data

For this experiments the radar’s three receiver antenna arrays were wired into a single line. One single receiver captured the atmospheric echoes. The spectrum analysis used a Gaussian fitting algorithm to gather information. The data was gathered from Dec 13\textsuperscript{th}, 2013 to May 12\textsuperscript{th}, 2014.
Figure G.1: Maximum power of the Gaussian fit.
Figure G.2: Spectral width of the Gaussian fit.
Figure G.3: Doppler velocity of the Gaussian fit.
Figure G.4: Floor level of the Gaussian fit.


G.2 Three receivers data

For the second set of experiments the radar’s three receiver antenna arrays were used separately. Three receivers captured the atmospheric echoes. The spectrum analysis used a Gaussian fitting algorithm to gather information. The data was gathered from June 1st, 2014 to Dec 8th, 2014; unfortunately storage issues caused loss of data.

Figure G.5: Maximum power of the Gaussian fit.
Figure G.6: Spectral width of the Gaussian fit.

Figure G.7: Doppler velocity of the Gaussian fit.
Figure G.8: Floor level of the Gaussian fit.
Appendix H

Deconvolution code

As part of the research carried out many implementations of simulations were created. The code included here was used to generate the deconvolution result shown in figure 3.8 (page 63). The convolution results show that the error (red line) is null if a full convolution procedure is used. This can be observed in line 29 of the code. The actual deconvolution takes place in line 35 where the complex division of the spectrum is carried out.

The increase in speed obtained by calculating the convolution in the frequency domain is due to the multiplication of spectra. Instead of carrying out the sliding, multiplication and integration for different time lags, only one multiplication is required. The results obtained with this simple program were used to code the final implementation of the radar backscattering simulation program (not included).

Code for the deconvolution.py program:

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy import fftpack, real, imag

# fig = plt.figure()
fig, (ax1, ax2, ax3, ax4) = plt.subplots(nrows=4)

Ts = 1E-8
N = 25000

times = Ts * np.arange(N)
time = times[-1+1/2]
sigma = 1E-6

Norm = 5

# Ts*np.sum(np.exp(-np.square(aTime-Ctime))/2*np.exp(1j)*2*np.random.random(len(aTime)))
gaus = (Norm+np.exp(-np.square(aTime-Ctime))/2*np.exp(1j)*2*np.random.random(len(aTime)))*np.exp(1j)*2*np.random.random(len(aTime))
lnG = len(gaus)

# Just to smooth the function.
b = np.convolve(np.ones(800), b, mode="same")

bSpectra = fftpack.fft(b)
xv = np.fft.fftfreq(len(times), dt=Ts)

T = np.convolve(A,b,mode="full")

print(len(T))

sT = fftpack.fft(T,n=len(T))
sa = fftpack.fft(A,n=len(T))

sh = (sT)/sa
newB = fftpack.ifft(sh)
newB = newB[:499]

# newB = np.append(newB[1:1+1-len(aTime)/2], newB[-1:len(aTime)/2])

ax1.plot(1E5*aTime, np.abs(A,'k-'))
ax1.plot(1E5*aTime, np.angle(A),'b-')
```
ax2.plot(10E2*times, np.abs(b), 'k-')
ax2.plot(10E2*times, np.angle(b), 'b-')

# ax3.plot(xv, np.real(bSpectra), 'k-')
# ax3.plot(xv, np.real(bSpectra), 'k-')
ax3.plot(10E2*Ts*np.arange((N-1+500)), np.abs(T), 'k-')
ax3.set_xlim((0, 0.26))

" print len(newB), len(b), newB[-500:]

ax4.plot(10E2*times, np.abs(newB), 'k-')
ax4.plot(10E2*times, np.abs(newB)-np.abs(b), 'r-')

plt.show()}
Appendix I

Brief review of basic numerical simulation techniques

In this appendix some of the numerical techniques used in physics, and especially some used in this theses, are discussed. The small number of techniques covered is far from being comprehensive and should be considered just as a list of numerical tools. A complete guide to numerical methods can be found elsewhere (Press et al., 2007).

The simplest simulation in physics can be complicated quite rapidly. When simulating simple cases using the kinematic equations, Euler’s method is employed. Euler’s method can be inaccurate under certain circumstances and this can be easily observed in the conservation of energy. Euler’s method is just an implementation of the finite difference method. Different schemes and techniques are based on finite difference (and more complicated ones in finite element). Due to the limited extent of this document only finite difference and two of its more widely spread methods are mentioned in detail.

Finite difference method

Computers are discrete in nature. The circuitry inside a computer’s processor is capable of carrying out standard numerical operations. Calculus is not realizable in a digital computer. Derivation and integration can not be carried out directly by a computer’s processor unit. Higher level techniques are required to obtain products that are dependent on those mathematical tools.

The Finite difference method (FD from now on), transforms a mathematical equation (which can be a Partial Differential Equation, PDE) into an approximated equation that can be solved numerically and implemented in a computer simulation. The FD method will transform an equation with a perfect solution (hard to solve) to another equation with an approximated solution (simple calculation). The equations where FD is used can be obtained from theoretical analysis or experimentation, and converted into approximations that can be easily carried out by discrete summations.

The application of FD is particularly important in PDE. The PDE type equations rarely have an exact solution (Olver, 2014). By converting the PDE to a discrete notation, they can be approximated numerically. Solving these equations provides an immediate insight into the problem that the original PDE represents.
Examples in physics that use PDEs to describe a problem are well known. Two examples are presented:

1. Continuity equation. The continuity equation represents a problem involving two first order terms. It is found in fluid related problems. Conservation of mass is the problem treated by the equation.

\[
\frac{\partial \rho}{\partial t} = \nabla \cdot (\rho u) \quad (I.1)
\]

the term involving the \( \rho u \) will be expanded unless the material is considered incompressible (in which case the equation simplifies considerably). In any case, the derivatives need to be approximated and the equations solved.

2. Simplified Heat Equation. The one dimensional simplified heat equation is a well known problem in physics. It contains two different derivatives. The first derivative is order one respect to time. The second derivative is respect to position and second order.

\[
\frac{\partial T}{\partial t} = C \frac{\partial^2 T}{\partial^2 x} \quad (I.2)
\]

A numerical method that allows this equation to be approximated numerically needs to resolve both approximations.

The problem of obtaining the derivatives in the PDEs presented is covered as follows. In order to obtain a clear perspective of this method, let’s use a simple case starting with the Taylor expansion of a function \( f(x) \) that can be derived infinitely (this approximation is needed for the mathematical formulation but infinite derivatives are not needed in real applications)

\[
f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \cdots \quad (I.3)
\]

we can change the variable \( h = x - a \) to obtain

\[
f(a+h) = f(a) + f'(a)h + \frac{f''(a)}{2!}h^2 + \cdots + \frac{f^{(n)}(a)}{n!}h^n + \cdots \quad (I.4)
\]

rearranging this equation, the first derivative can be isolated

\[
f'(a) = \frac{f(a+h) - f(a)}{h} - \frac{1}{h} \left[ \frac{f''(a)}{2!}h^2 + \cdots + \frac{f^{(n)}(a)}{n!}h^n + \cdots \right] \quad (I.5)
\]

if the part of the equation on top of the brace is ignored, the approximation of first order for the derivative is

\[
f'(a) = \frac{f(a+h) - f(a)}{h} \quad (I.6)
\]

with an error of order \( h \). This error is known as the truncation error and arises because of the mathematical approximation of the first derivative. The error in the approximation is then:
Chapter I. Brief review of basic numerical simulation techniques

\[ E = \frac{1}{h} \left[ \frac{f''(a)}{2!} h^2 + \cdots + \frac{f^{(n)}(a)}{n!} h^n + \cdots \right] \]  

(I.7)

and the order of the largest term is \( h \). This means that the only way to decrease the error is to decrease the value of \( h \).

The method just shown is known as the forward method of finite differences. This method takes the values at \( a \) and increases in intervals of \( h \) to calculate the approximation. The step size \( h \) can be replaced by a negative counterpart. This would result in the backward method. In this method the value of the first derivative would be

\[ f'(a) = \frac{f(a) - f(a-h)}{h} \]  

(I.8)

but the backward FD method is not the equivalent to the first derivative. The signs in the errors will be alternating, meaning that for this case

\[ E = \frac{1}{h} \left[ \frac{f''(a)}{2!} h^2 - \cdots + \frac{f^{(n)}(a)}{n!} (-h)^n + \cdots \right] \]  

(I.9)

The origin of this difference is clear if compared directly:

\[
\begin{align*}
f(a + h) &= f(a) + f'(a)h + \frac{f''(a)}{2!} h^2 + \cdots + \frac{f^{(n)}(a)}{n!} h^n + \cdots \\
f(a - h) &= f(a) - f'(a)h + \frac{f''(a)}{2!} h^2 - \cdots + \frac{f^{(n)}(a)}{n!} (-h)^n + \cdots
\end{align*}
\]  

(I.10)

Multiplying the second one by a minus one and adding it to the first equation yields

\[ f(a + h) - f(a - h) = 2 f'(a)h + \frac{2 f''(a)}{3!} h^3 + \cdots + \frac{2 f^{(n)}(a)}{n!} (h)^n + \cdots \]  

(I.11)

where the values of \( n \) are now limited from 5, 7, 9, \( \cdots \), \( \infty \); the even number terms cancelled out in the addition. Hence the final approximation of the first derivative is

\[ f'(a) = \frac{f(a + h) - f(a - h)}{2h} - \frac{1}{h} \left[ \frac{f''(a)}{3!} h^3 + \cdots + \frac{f^{(n)}(a)}{n!} h^n + \cdots \right] \]  

(I.12)

\[ n=5,7,\ldots,\infty \] Approximation Error

where the error is found to be

\[ E = \frac{1}{h} \left[ \frac{f''(a)}{3!} h^3 + \cdots + \frac{f^{(n)}(a)}{n!} (h)^n + \cdots \right] \]  

(I.13)

which is a remarkable result of finite difference method. By combining two methods with errors in the order of \( h \), the error of the final approximation is reduced to the order of \( h^2 \) by cancelling the second derivative corresponding errors. The method just presented is the central difference to approximate the first derivative.

This method can directly be used to obtain solutions to problems like the continuity equation presented earlier in equation I.1. Physical problems involving the heat equation would require second order solution using the finite difference method.
By using the same scheme just presented for the central finite difference the second derivative approximation can be achieved. By using the equations I.10, but instead of multiplying by a minus one, add the equations directly. The first derivative term will disappear and the result is

$$f(a + h) - 2f(a) + f(a - h) = \frac{2f''(a)}{2!}h^2 + \frac{2f^{(4)}(a)}{4!}h^4 \cdots + \frac{2f^{(n)}(a)}{n!}(h)^n + \cdots$$  (I.14)

Leaving the approximation of the second derivative as

$$f''(a) = \frac{f(a + h) - 2f(a) + f(a - h)}{h^2}$$  (I.15)

and the error being

$$E = \frac{1}{h^2} \left[ \frac{2f^{(4)}(a)}{4!}h^4 \cdots + \frac{2f^{(n)}(a)}{n!}(-h)^n + \cdots \right]$$  (I.16)

for even values of \( n \) greater than 4. The order of the error in this second derivative approximation is \( h^2 \). Equal order was achieved for the first derivative by using the central finite difference method. This can be used to illustrate that not all methods will produce the same order of magnitude errors. Care must be taken in every simulation to understand and correctly analyze the truncation error as well as any other sources of approximation.

Obtaining the truncated equations may be seen as the end of the process. As mentioned earlier, the process of creating a computer simulation is complicated and detailed. Once the discrete equation exits, the need for implementation arises. The implementation would depend on the grid chosen to solve the equation as well as the software/hardware combination. The description of the entire process is far beyond the scope of this document, not to mention the details. The only aspect left to describe is that the approximation equations will be used to create an algebraic system of equations. In that system, an equal number of unknowns and equations will be used to obtain the solution to the problem.

In the next section two common methods for solving linear equations are presented. The simplicity of implementation of these methods is the cause for their widespread usage in simulations. The error can decrease drastically (as was shown in this section) and this will be emphasized in the next section by using the Runge-Kutta method.

**Euler’s and Runge-Kutta method**

Euler’s method is probably the easiest and more widely implemented way to solve ordinary differential equations. Runge-Kutta methods (Euler’s method can be interpreted as the simplest case) are also commonly implemented in solving such equations. The difference, as will be shown shortly, is found in the simplicity of the algorithm and reliability of the result.

In radar research Euler’s method has been implemented in simulations. Palmer et al. (1999) used it to better understand (by using a simulation) the implications of a new spectral analysis tool in finding atmospheric layers. The simulation by Palmer et al. (1999) was based on the algorithm used by Holdsworth and Reid (1995). They simulated the scatterers position using
Euler’s method. This method may not be optimal because the error in Euler’s method is rapidly accumulated for big time steps. In Palmer et al. (1999), the time step used was 0.1 s.

In order to see how Euler’s method is derived, let’s use a simple ODE of the form

$$\frac{dy}{dt} = f(y, t)$$  \hspace{1cm} (I.17)

with the known initial condition that \( y(t_o) = y_o \). This equation can be integrated simply by moving the \( dt \) to the \( f(y, t) \) and carrying out the integral on both sides of the equation.

$$\int_{y_o}^{y} dy = \int_{t_o}^{t} f(y, t) dt$$  \hspace{1cm} (I.18)

if the value of \( f(y, t) \) is considered constant (\( f(y_o, t_o) \)) in the small interval from \( t_o \) to \( t \), it can be taken out of the integral to become

$$y - y_o = f(t_o, y_o)(t - t_o)$$
$$y = y_o + hf(t_o, y_o)$$  \hspace{1cm} (I.19)

where \( h = t - t_o \). This mathematical formulation is often represented in computer science as

$$y_{n+1} = y_n + hf(x_n, y_n)$$  \hspace{1cm} (I.20)

where the independent variable is named \( x \), and the indices \( n, n + 1 \) represent the initial and final value, respectively. The value of \( y_{n+1} \) in a very near future (small \( h \)) can be calculated based on the conditions at \( n \) (\( y_n \)). The algorithm implementation of Euler’s method is simple, direct and accurate if the steps taken are sufficiently small.

Another algorithm used to solve ODEs is known as Runge-Kutta. It was created by the German mathematicians Carl David Tolm Runge and Martin Wilhelm Kutta. This method is similar to Euler’s method. One important difference is that the step size required to obtain precise solutions can be much bigger than the one used by Euler’s method.

In this method the order of the approximation delimits the number of parameters \( k_i \) used. In second order Runge-Kutta, the values of \( k_1 \) and \( k_2 \) are used to calculate the future value of \( y \)

$$k_1 = hf(x_n, y_n)$$
$$k_2 = hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1)$$  \hspace{1cm} (I.21)
$$y_{n+1} = y_n + k_2 + O(h^3)$$

where the \( O(h^3) \) is the Landau symbol indicating the order of the terms left out in the approximation, order 3 in this case. Another commonly found implementation is the fourth-order Runge-Kutta, known as RK4 [Press et al. (1993)]. In this case, four \( k \) indices are used to calculate \( y_{n+1} \).
\[ k_1 = hf(x_n, y_n) \]
\[ k_2 = hf(x_n + \frac{1}{2} h, y_n + \frac{1}{2} k_1) \]
\[ k_3 = hf(x_n + \frac{1}{2} h, y_n + \frac{1}{2} k_2) \]
\[ k_4 = hf(x_n + h, y_n + k_3) \]
\[ y_{n+1} = y_n + \frac{1}{6} k_1 + \frac{1}{3} k_2 + \frac{1}{3} k_3 + \frac{1}{6} k_4 + O(h^5) \]

It can be seen that with RK methods, instead of considering the value of the function as a simple constant, it is approximated several times, in intermediate steps \((1/2h)\), and then a weighted value is used to calculate the final increment. As mentioned earlier, the first term in Runge-Kutta \((k_1)\) is just the term used in Euler’s method.

In the radar simulations presented in section 4.3, Euler’s method was used. The scatterers location for each time step was calculated using Euler’s method. The results showed that for low speed moving scatterers with short time steps, the method satisfies the necessary conditions for the simulations.
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