Investigation of Fracture Toughness Measurement for Pipeline Steels Based on SE(B) and SE(T) Specimens

Zijian Yan, The University of Western Ontario

Supervisor: Dr. Wenxing Zhou, The University of Western Ontario

A thesis submitted in partial fulfillment of the requirements for the Master of Engineering Science degree in Civil and Environmental Engineering

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INVESTIGATION OF FRACTURE TOUGHNESS MEASUREMENT FOR PIPELINE STEELS BASED ON SE(B) AND SE(T) SPECIMENS

(Thesis format: Integrated Article)

by

Zijian Yan

Graduate Program in Engineering Science
Department of Civil and Environmental Engineering

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Engineering Science

The School of Graduate and Postdoctoral Studies
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ABSTRACT

This thesis deals with issues related to the experimental determination of the fracture toughness resistance curves, i.e. the J-integral \((J)\)-R curve and crack tip opening displacement \((CTOD)\)-R curves, using the single-edge bend (SE(B)) and single-edge tension (SE(T)) specimens. First, the impact of the crack front curvature on the \(J\)-R curve measured from the SE(B) specimen is investigated through systematic linear-elastic and elastic-plastic three-dimensional (3D) finite element analyses (FEA) of SE(B) specimens containing both straight and curved crack fronts. Three average relative crack lengths are considered, namely 0.3, 0.5 and 0.7, and three specimen width-to-thickness ratios are considered: 0.25, 0.5 and 1. The curved crack fronts are characterized by a power-law expression. The analysis results suggest that the crack length evaluated from the \(CMOD\) compliance of the SE(B) specimen is insensitive to the crack front curvature and that the impact of the crack front curvature on the experimentally-evaluated \(J\) values varies with the specimen configurations. For a given specimen configuration, as the crack front curvature increases, the value of \(J\) evaluated based on the test standard ASTM E1820-11 without considering the crack front curvature becomes less conservative and tends to overestimate the actual \(J\). New crack front straightness criteria that are in most cases less stringent than ASTM E1820-11, are recommended. The accuracy of the double clip-on gauge method for experimentally determining \(CTOD\) is examined through systematic 3D elastic-plastic large-strain FEA of clamped SE(T) specimens. The relative crack lengths of the specimens range from 0.3 to 0.7, and the thickness-to-width ratios are 0.5, 1 and 2. It is observed that the \(CTOD\) values determined from the double clip-on gauge method may involve significant errors. This error primarily depends on the crack length, the material property and the loading level. Based on the analysis results, a modified \(CTOD\) evaluation equation is developed to improve the accuracy of \(CTOD\) evaluated using the double-clip on gauge method.

Keywords

Fracture toughness; J-integral; Curved crack front; FEA; Crack tip opening displacement \((CTOD)\); Double clip-on gauge; SE(B) specimen; SE(T) specimen
CO-AUTHORSHIP STATEMENT

The material presented in Chapter 2, 3 and 4 of this thesis have been published or submitted for potential publication in international conference proceedings or peer-reviewed journals. The permission to reproduce the published work in this thesis has been granted by the copyright holders and included at the end of this thesis.

A version of Chapter 2 has been published and presented as a technical paper co-authored by Zijian Yan and Wenxing Zhou in the CSME International Congress, Toronto, Ontario, Canada, June 1-4, 2014.

Chapter 3 contains materials of a manuscript co-authored by Zijian Yan and Wenxing Zhou that has been accepted for publication in the Journal of Testing and Evaluation, ASTM International. Reproduced in this chapter, with permission, from the Journal of Testing and Evaluation, copyright ASTM International, 100 Barr Harbor Drive, West Conshohocken PA 19428-2959.

A version of Chapter 4, co-authored by Zijian Yan, Yifan Huang and Wenxing Zhou, will be presented as an accepted technical paper and one of seven finalists in the 7th student paper competition in the 10th International Pipeline Conference, Calgary, Alberta, Canada, September 29- October 3, 2014. Paper No. IPC2014-33219.
DEDICATION

To my parents and grandparents
ACKNOWLEDGMENTS

This thesis was carried out in the Department of Civil and Environmental Engineering at Western. Foremost, I would like to express my most sincere gratitude to my supervisor Dr. Wenxing Zhou for his continuously patient guidance, support and encouragement throughout the completion of this program. His vast knowledge, enthusiasm and dedication to research were guiding me on a daily basis during the last two years. He was serious, sharp and precise to every discussion we had, which I believe are the crucial qualities for a researcher. I was fortunate enough to have the opportunity of being a member of his research group.

I would also like to thank my thesis committee members and examiners, Dr. Ashraf A. El Damatty, Dr. Timothy A. Newson and Dr. Liying Jiang for reviewing this thesis. I am certain that the quality of my thesis is improved by their minds and hands. The guidance from Dr. Hanping Hong as the supervisory committee member is also acknowledged.

Sincerely, I wish to extend my appreciation to my colleagues in the research group and all my friends in Canada, for their help, support and most of all the friendship they provided during the past two years. Their presence colored my personal life.

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# TABLE OF CONTENTS

Abstract ......................................................................................................................................................... II
Co-authorship Statement ............................................................................................................................... III
Dedication ......................................................................................................................................................... IV
Acknowledgments ............................................................................................................................................ V
Table of Contents .......................................................................................................................................... VI
List of Tables ................................................................................................................................................ IX
List of Figures ................................................................................................................................................ X
List of Abbreviations and Symbols ............................................................................................................... XIV

Chapter 1 Introduction ............................................................................................................................... 1

1.1 Background ............................................................................................................................................. 1

1.2 Fundamentals of Fracture Mechanics ................................................................................................. 2
  1.2.1 Linear Elastic Fracture Mechanics ................................................................................................. 2
  1.2.2 Elastic Plastic Fracture Mechanics ............................................................................................... 4

1.3 Objective and Research Significance .................................................................................................. 8
  1.3.1 Investigation of the Effect of Crack Front Curvature on the Measurement of J-R Curve of Single-Edge Bend Specimens .............................................................................................................. 8
  1.3.2 Investigation of the Accuracy of the Double-Clip on Gauge Method for Evaluating CTOD of Single-Edge Tension Specimens ........................................................................................................ 8

1.4 Thesis Format and Outline .................................................................................................................. 9

References ....................................................................................................................................................... 9

Chapter 2 Effect of Crack Front Curvature on CMOD Compliance and Crack Length

Evaluation for Single-edge Bend Specimens ............................................................................................. 15

2.1 Introduction ............................................................................................................................................ 15
  2.1.1 Fracture Toughness Resistance Curve Tests on Small-scale Specimens.. 15
2.1.2 Literature Review and Objective .............................................................. 16
2.1.3 Approach ................................................................................................. 17
2.2 Characteristics of Curved Crack Front ....................................................... 18
2.3 Finite Element Analyses ............................................................................ 21
2.4 Analysis and Discussions .......................................................................... 22
   2.4.1 Effect of the Crack Front Curvature on CMOD compliance ............... 22
   2.4.2 Crack Length - COMD Compliance Equations ................................. 23
   2.4.3 Effect of the Crack Front Curvature on the Evaluated Crack Length.... 24
2.5 Summary and Concluding Remarks ......................................................... 25
References ........................................................................................................ 26

Chapter 3 Effect of Crack Front Curvature on Experimental Evaluation of J-integral
for Single-edge Bend Specimens ..................................................................... 45
3.1 Introduction .................................................................................................. 45
   3.1.1 Experimental Evaluation of J............................................................... 45
   3.1.2 Literature Review and Objective ...................................................... 48
   3.1.3 Approach ............................................................................................ 49
3.2 Finite Element Analyses ........................................................................... 50
   3.2.1 Finite Element Model .......................................................................... 50
   3.2.2 Material Model .................................................................................... 51
   3.2.3 Computational Procedure .................................................................. 52
3.3 Results and Discussions ............................................................................ 54
3.4 Summary and Concluding Remarks .......................................................... 56
Reference .......................................................................................................... 57

Chapter 4 Accuracy of the Double Clip-on Gauge Method for Evaluating CTOD
from Single-edge Tension Specimens ............................................................... 81
4.1 Introduction ................................................................................................................. 81
  4.1.1 Experimental Determination of CTOD ................................................................. 81
  4.1.2 Single-edge Tension Specimen ............................................................................ 83
  4.1.3 Literature Review ................................................................................................. 84
  4.1.4 Objective and Approach ..................................................................................... 84
4.2 CTOD Measured from Double Clip-on Gauge Method ........................................... 85
4.3 Finite Element Analyses .......................................................................................... 86
  4.3.1 Finite Element Model ........................................................................................ 86
  4.3.2 Material Model .................................................................................................. 87
  4.3.3 Computational Procedure ................................................................................ 88
4.4 Results and Discussions .......................................................................................... 89
4.5 Summary and Concluding Remarks ........................................................................ 92
Reference ......................................................................................................................... 93
Chapter 5 Summary and Conclusions ........................................................................... 116
  5.1 Effect of the Crack Front Curvature on the CMOD Compliance and J for SE(B) Specimens ........................................................................................................ 116
  5.2 Accuracy of the Double Clip-on Gauge Method for Evaluating CTOD of SE(T) Specimens ............................................................................................................. 119
  5.3 Recommendations for Future Work ....................................................................... 120
Appendix A Derivation of the Relationship between β and λ for Symmetric Bowed Crack Fronts ........................................................................................................... 121
Appendix B Computation of J-integral using Virtual Crack Extension Method .......... 123
  Reference ....................................................................................................................... 124
Appendix C Copyright Permission ................................................................................. 128
Curriculum Vitae ............................................................................................................... 135
LIST OF TABLES

Table 2.1: Summary of the curved crack fronts of the specimens collected in this study 28
Table 2.2: The coefficients used in Eq. (2.9).............................................................. 29
Table 3.1: Recommended crack front straightness criteria for SE(B) specimens ........ 60
LIST OF FIGURES

Figure 1.1. Three typical loading modes in fracture mechanics ........................................... 12

Figure 1.2. Stress field solutions surrounding the crack tip .................................................. 12

Figure 1.3. Arbitrary contour around the crack tip ................................................................. 13

Figure 1.4. Schematically illustration of CTOD definitions .................................................. 14

Figure 2.1. Schematic and experiment set-up of small-scale specimens .................................. 32

Figure 2.2. P-CMOD curve using elastic unloading compliance method (Wang et al. 2012) ................................................................................................................................. 33

Figure 2.3. Schematic illustration of symmetric bowed crack fronts ...................................... 34

Figure 2.4. Typical crack fronts of SE(B) and SE(T) specimens ............................................. 34

Figure 2.5. Comparison of actual and fitted crack fronts for one SE(B) specimen and one SE(T) specimen .................................................................................................................. 35

Figure 2.6. Schematic illustration of symmetric bowed crack fronts with a fixed \( \beta \) and varying \( p \) ............................................................................................................................... 36

Figure 2.7. Geometric and mesh configuration of the finite element model ......................... 37

Figure 2.8. Variation of \( C_\alpha/C_s \) against \( \lambda \) .................................................................... 39

Figure 2.9. Variation of the error \( e_\alpha \) against \( \lambda \) .......................................................... 44

Figure 3.1. Determination of the potential energy ................................................................. 61

Figure 3.2. Plastic area under the load-displacement curve .................................................... 62
Figure 3.3. Variation of the error \( e_J = (J^n - J^{FEA})/J^{FEA} \) against \( P/P_y \) for specimens with different values of \( a_{ave}/W, B/W \) and crack fronts curvature \( \lambda \) (CMOD-based analysis, \( p = 3 \) and \( n = 10 \)) ........................................................................................................... 66

Figure 3.4. Variation of the error \( e_J = (J^n - J^{FEA})/J^{FEA} \) against \( P/P_y \) for specimens with different values of \( a_{ave}/W, B/W \) and crack fronts curvature \( \lambda \) (LLD-based analysis, \( p = 3 \) and \( n = 10 \)) ........................................................................................................... 70

Figure 3.5. Variation of the error \( e_J = (J^n - J^{FEA})/J^{FEA} \) against \( P/P_y \) for specimens with different values of \( a_{ave}/W, B/W \) and crack fronts curvature \( \lambda \) (CMOD-based analysis, \( p = 3 \) and \( n = 20 \)) ........................................................................................................... 74

Figure 3.6. Variation of the error \( e_J = (J^n - J^{FEA})/J^{FEA} \) against \( P/P_y \) for specimens with different values of \( a_{ave}/W, B/W \) and crack fronts curvature \( \lambda \) (LLD-based analysis, \( p = 3 \) and \( n = 20 \)) ........................................................................................................... 78

Figure 3.7. Impact of the shape parameter \( p \) (\( p = 3 \) and \( p = 2.5 \)) on \( e_J \) ......................... 80

Figure 4.1. Schematically illustration of the geometric relationship for the evaluation of CTOD using single clip-on gauge plastic hinge model ......................................................... 98

Figure 4.2. Illustration of the installation of knife edges for the double clip-on gauge method ................................................................................................................................. 99

Figure 4.3. Typical toughness resistance curves for various types of small-scale specimens ................................................................................................................................. 100

Figure 4.4. Schematically illustration of CTOD definitions ................................................................................. 101

Figure 4.5. Schematically illustration of the geometric relationship for the evaluation of CTOD ................................................................................................................................. 102

Figure 4.6. Geometric and mesh configuration of the finite element model with a blunt crack tip ................................................................................................................................. 102

Figure 4.7. A typical side-grooved finite element model for clamped SE(T) specimen 103
Figure 4.8. Schematically illustration of the determination of CTOD in FEA .......... 104
Figure 4.9. Schematically illustration of the double clip-on gauge method in FEA ..... 104
Figure 4.10. Variation of $e_1$ against $P/Py$ for plane-sided specimens with $n = 10$ and the same $B/W$ ...................................................................................................................... 105
Figure 4.11. Variation of $e_1$ against $P/Py$ for plane-sided specimens with $n = 10$ and the same $a/W$ ...................................................................................................................... 106
Figure 4.12. Variation of $e_1$ against $P/Py$ for plane-sided specimens with $B/W = 1$ and the same $a/W$ ...................................................................................................................... 107
Figure 4.13. Geometric relationship surrounding the blunt crack tip .................... 108
Figure 4.14. Variation of the proposed correction factor $\mu$ against $P/Py$ for plane-sided specimens with $n = 10$ ...................................................................................................................... 109
Figure 4.15. Variation of the proposed correction factor $\mu$ against $P/Py$ for specimens with $n = 10$ ...................................................................................................................... 110
Figure 4.16. Variation of the proposed correction factor $\mu$ against $P/Py$ for plane-sided specimens with $B/W = 1$, $a/W = 0.5$, $n = 10$ and different initial radius ................. 111
Figure 4.17. Variation of the proposed correction factor $\mu$ against $P/Py$ for plane-sided specimens with $B/W = 1$ and different $n$ values .................................................. 111
Figure 4.18. Variation of $e_c$ against $P/Py$ for plane-sided specimens with $n = 10$ and the same $B/W$ ...................................................................................................................... 112
Figure 4.19. Variation of $e_c$ against $P/Py$ for plane-sided specimens with $n = 10$ and the same $a/W$ ...................................................................................................................... 113
Figure 4.20. Variation of $e_c$ against $P/Py$ for specimens with $n = 10$, $B/W = 1$ and the same $a/W$ ...................................................................................................................... 114
Figure 4.21. Variation of $e_c$ against $P/Py$ for plane-sided specimens with $B/W = 1$ and the same $a/W$ ................................................................. 115

Figure B.1. The virtual crack extension method in two-dimensional analysis ............ 126

Figure B.2. The virtual crack extension method in three-dimensional analysis .......... 127
LIST OF ABBREVIATIONS AND SYMBOLS

Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASTM</td>
<td>American Society for Testing and Materials</td>
</tr>
<tr>
<td>CMOD</td>
<td>Crack mouth opening displacement</td>
</tr>
<tr>
<td>C(T)</td>
<td>Compact tension</td>
</tr>
<tr>
<td>CTOD</td>
<td>Crack tip opening displacement</td>
</tr>
<tr>
<td>DCG</td>
<td>Double clip-on gauge</td>
</tr>
<tr>
<td>DIC</td>
<td>Digital image correlation</td>
</tr>
<tr>
<td>EPFM</td>
<td>Elastic-plastic fracture mechanics</td>
</tr>
<tr>
<td>FEA</td>
<td>Finite element analyses</td>
</tr>
<tr>
<td>HRR</td>
<td>Hutchinson-Rice-Rosengren</td>
</tr>
<tr>
<td>LEFM</td>
<td>Linear-elastic fracture mechanics</td>
</tr>
<tr>
<td>LLD</td>
<td>Load line displacement</td>
</tr>
<tr>
<td>LSY</td>
<td>Large scale yielding</td>
</tr>
<tr>
<td>PS</td>
<td>Plane-sided</td>
</tr>
<tr>
<td>SE(B)</td>
<td>Single-edge bend</td>
</tr>
<tr>
<td>SE(T)</td>
<td>Single-edge tension</td>
</tr>
<tr>
<td>SSY</td>
<td>Small scale yielding</td>
</tr>
<tr>
<td>SG</td>
<td>Side-grooved</td>
</tr>
<tr>
<td>2D</td>
<td>Two dimensional</td>
</tr>
<tr>
<td>3D</td>
<td>Three dimensional</td>
</tr>
</tbody>
</table>

Symbols

CHAPTER 2

\( a \)  
Crack length

\( a(0) \)  
Crack length at the mid-plane of the specimen

\( a(\pm B/2) \)  
Crack length at the surface of the specimen
Average crack length

Crack length at the nine measurement points \((i = 1, 2, \ldots, 9)\)

Maximum value of the nine physical measurements, \(a_{\text{max}9} = \max (a_i)\)
where \(i = 1, 2, \ldots, 9\)

Maximum crack length over the entire crack front

Minimum value of the nine physical measurements, \(a_{\text{min}9} = \min (a_i)\)
where \(i = 1, 2, \ldots, 9\)

Minimum crack length over the entire crack front

Predicted crack length

Coefficients, \(i = 0, 1, \ldots, 4\)

Specimen thickness

Net thickness for side-grooved specimens

Specimen compliance

Compliance for specimens with curved crack fronts

Measured specimen compliance at load step \(i\)

Compliance for specimens with straight fronts

Error, \(e_a = (a_p - a_{\text{ave}})/a_{\text{ave}}\)

Young’s modulus

Effective modulus

Elastic modulus corresponding to the plane strain condition, i.e. \(E' = E/(1 - \nu^2)\)

Daylight length for clamped SE(T) specimen

J-integral

A shape parameter

Applied load

Specimen span

Specimen width

Coordinate in the specimen thickness direction

Difference of relative crack lengths at the mid-plane and free surface,
\(\beta = a(0)/W - a(B/2)/W\)

\(\Delta = 0.005W\) for ASTM E1820-11
Δa: Crack growth

ν: Poisson's ratio

λ: Characterizing the fatigue crack front curvature level in this study based on ASTM E1820-11, \( \lambda = \text{max} \left( a_{\text{max}} - a_{\text{ave}}, a_{\text{ave}} - a_{\text{min}} \right) / B \)

CHAPTER 3

a: Crack length

\( a_{\text{ave}} \): Average crack length

A: Total area under the load versus displacement curve

\( A_{el} \): Elastic area under the load versus displacement curve

\( A_{pl} \): Plastic area under the load versus displacement curve

\( A_{pl}^{CMOD} \): Plastic area under the P-CMOD curve

\( A_{pl}^{LLD} \): Plastic area under the P-LLD curve

B: Specimen thickness

da: Increment of crack length

\( dP \): Increment of applied load

\( dU \): Increment of strain energy

\( d\Delta \): Increment of displacement

e: Base of the natural logarithm, \( e = 2.71828 \)

\( e_J \): Error, \( e_J = \left( J^n - J^{FEA} \right) / J^{FEA} \)

E: Young’s modulus

H: Daylight length for clamped SE(T) specimen

J: J-integral

\( J^{FEA} \): J-integral value from FEA considering crack front curvature

\( J_{el} \): Elastic component of J-integral

\( J_{pl} \): Plastic component of J-integral

\( J^n \): J-integral value evaluated based on plastic geometry factor \( \eta_{pl} \) as specified in ASTM E1820-11

K: Linear elastic stress intensity factor

n: Strain hardening exponent

p: A shape parameter
Applied load

$P_y$ Reference load, $P_y = B(W - a_{ave})^2\sigma_y/S$

Specimen span

Strain energy

Specimen width

A dimensionless parameter

Difference of relative crack lengths at the mid-plane and free surface,

$$\beta = a(0)/W - a(B/2)/W$$

Specimen displacement

Elastic component of the specimen displacement

Plastic component of the specimen displacement

Strain

Reference strain, satisfying $\varepsilon_0 = \sigma_0/E$

A dimensionless geometry factor

Plastic geometry factor used to calculate J-integral

$CMOD$-based plastic geometry factor used to calculate J-integral

$LLD$-based plastic geometry factor used to calculate J-integral

Characterizing the fatigue crack front curvature level in this study based on ASTM E1820-11, $\lambda = \max(a_{max} - a_{ave}, a_{ave} - a_{min})/B$

Poisson's ratio

Stress

Reference stress, equal to the yield strength $\sigma_{YS}$ in this study

Ultimate tensile strength

Effective yield strength, $\sigma_y = (\sigma_{YS} + \sigma_{TS})/2$ as specified in ASTM E1820-11

Yield strength

Crack length

Original Crack length

Specimen thickness

CHAPTER 4
\( e \) Base of the natural logarithm, \( e = 2.71828 \)
\( e_1 \) Error, \( e_1 = (\delta_{DC90} - \delta_{FE90})/\delta_{FE90} \)
\( e_c \) Error of the DCG method by employing the modified equations, i.e. Eqs. (4.8), (4.9) and (4.10)
\( E \) Young’s modulus
\( J \) J-integral
\( K \) Linear elastic stress intensity factor
\( m \) A dimensionless constraint factor
\( n \) Strain hardening exponent
\( P \) Applied load
\( P_y \) Reference load, \( P_y = B(W - a)\sigma_Y \)
\( q_0, q_1 \) and \( q_2 \) Fitting coefficients
\( r_p \) A dimensionless rotation factor
\( V_{pl} \) Plastic component of the measured crack mouth opening displacement
\( V_{pl1}, V_{pl2} \) Plastic component of the two measured crack mouth opening displacements corresponding to the two different knife edge heights in double clip-on gauge method
\( V_1, V_2 \) Two measured crack mouth opening displacements corresponding to the two different knife edge heights in double clip-on gauge method
\( W \) Specimen width
\( z \) The height of the clip-on gauge for single clip-on gauge method
\( z_1, z_2 \) The heights of the clip-on gauge for double clip-on gauge method
\( \delta \) Crack tip opening displacement (CTOD)
\( \delta_{BS} \) Crack tip opening displacement (CTOD) obtained based on BS 7448
\( \delta_{DC90} \) Crack tip opening displacement (CTOD) evaluated from double clip-on gauge method based on the 90 degree definition
\( \delta_{FE90} \) Crack tip opening displacement (CTOD) evaluated from FEA based on the 90 degree definition
\( \delta_{el} \) Elastic component of crack tip opening displacement (CTOD)
\( \delta_{pl} \) Plastic component of crack tip opening displacement (CTOD)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_{90}$</td>
<td>Crack tip opening displacement (CTOD) based on the 90 degree definition</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Strain</td>
</tr>
<tr>
<td>$\varepsilon_0$</td>
<td>Reference strain, satisfying $\varepsilon_0 = \sigma_0/E$</td>
</tr>
<tr>
<td>$\eta_{pl}$</td>
<td>Plastic geometry factor used to calculate J-integral</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Rotational angle</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Proposed correction factor</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson's ratio</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>The initial radius of the blunt crack tip in FEA model</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stress</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>Reference stress</td>
</tr>
<tr>
<td>$\sigma_{TS}$</td>
<td>Ultimate tensile strength</td>
</tr>
<tr>
<td>$\sigma_Y$</td>
<td>Effective yield strength, $\sigma_Y = (\sigma_{YS} + \sigma_{TS})/2$</td>
</tr>
<tr>
<td>$\sigma_{YS}$</td>
<td>Yield strength</td>
</tr>
</tbody>
</table>
Chapter 1 Introduction

1.1 Background

Pipelines are the safest and most effective means to transport large quantity of hydrocarbons (e.g. crude oil and natural gas) over a long distance and are vital to the economic well-being and security of a society. Canada has the largest crude oil pipeline network in the world. According to the Canadian Energy Pipeline Association, there are approximately 98 pipeline companies, which operate approximately 73,000 km of pipelines and approximately 1,400 km of international power lines across Canada (NEB 2012). In 2012, these pipelines shipped approximately $106.3 billion worth of crude oil, petroleum products, natural gas liquids and natural gas to Canadians and export customers at an estimated transportation cost of $6.9 billion (NEB 2012).

Energy pipelines may contain cracks or flaws due to various causes, such as the third party interference, fatigue, corrosion, stress-corrosion cracking (SCC) and welding process. The fracture toughness of the pipeline steel is a key input in the integrity assessment of pipelines containing cracks (i.e. planar defects) and the strain-based design of pipelines. For instance, the fracture toughness is related to the tolerable flaw sizes in the pipeline and governs the tensile strain capacity of the pipeline, e.g. the allowable longitudinal tensile strain in the pipeline girth weld containing circumferential cracks (Fairchild et al., 2012).

For ductile materials such as the modern pipeline steels, the fracture failure process is usually accompanied with significant plastic deformation. The ductile fracture behavior can lead to a slow and stable crack extension, which is a continuous process of ductile tearing. The material can carry more applied load with the growth of the crack length. In such circumstances, a relationship between the crack extension and the fracture toughness value (e.g. J-integral or crack tip opening displacement, see Section 1.2) corresponding to the crack extension, called fracture toughness resistance curve or $R$-curve, can be used to characterize the fracture toughness of ductile materials. The toughness resistance curves are commonly measured on small-scale specimens, e.g. three-point single-edge bend
(SE(B)), compact tension (C(T)) and single-edge tension (SE(T)) specimens. The objective of the study reported in this thesis is to address several issues related to the experimental determination of the fracture toughness resistance curve based on the SE(B) and SE(T) specimens. Some basic concepts of fracture mechanics are briefly reviewed first in Section 1.2.

1.2 Fundamentals of Fracture Mechanics

1.2.1 Linear Elastic Fracture Mechanics

Fracture mechanics deals with the behavior of bodies containing cracks and sharp notches, and is one of the most important development in the theory of mechanics. It can generally be separated into two domains: namely linear-elastic fracture mechanics (LEFM) and elastic-plastic fracture mechanics (EPFM). The former is valid if nonlinear deformation is confined to a small region in the vicinity of the crack tip, which is known as the small scale yielding (SSY) condition (Anderson, 2005); whereas the latter applies to the large scale yielding (LSY), which refers to conditions where fracture is accompanied by considerable plastic deformation (Anderson, 2005).

There are typically three modes of loading defined in fracture mechanics (Anderson, 2005), namely the opening mode or Mode I, the (in-plane) shear mode or Mode II and the tearing (out-of-plane shear) mode or Mode III, as illustrated in Fig. 1.1. The current study focuses on the Mode I loading because it is the most critical and representative fracture mode for pipelines.

For a given cracked isotropic linear elastic body subjected to external forces, if we define a polar coordinate system with the origin at the crack tip, as shown in Fig. 1.2, early studies (Westergaard, 1939; Irwin, 1957; Williams, 1957) showed that the stress field can be expressed as:

\[
\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) + \sum_{m=0}^{\infty} A_m r^{m/2} g_{ij}^{(m)}(\theta) \tag{1.1}
\]

where \(\sigma_{ij}\) \((i, j = 1, 2, 3)\) is the stress tensor; \(r\) and \(\theta\) are coordinates defined in Fig. 1.2; \(f_{ij}\) is a dimensionless function of \(\theta\), and \(K\) is the so-called stress intensity factor in the unit
of force/area×(length)^0.5. For the higher-order terms, \(A_m\) is the amplitude and \(g_{ij}^{(m)}\) is a dimensionless function of \(\theta\) for the \(m\)th term. As \(r\to0\), the leading term approaches infinity, but the higher-order terms remain finite or approach zero. Thus the stress field ahead of the crack tip can be written as:

\[
\lim_{r \to 0} \sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta)
\]

Equation (1.2) describes a stress singularity at the crack tip. The stress intensity factor, \(K\), completely defines the amplitude of the stress singularity, which means that for a given \(K\), the stresses, strains and displacements near the crack tip can be completely determined, and they are independent of the geometry of the cracked body and details of the loading (Hutchinson, 1983; Anderson, 2005).

The single-parameter characterization of the stress field near the crack tip relies on the satisfaction of the SSY condition, which requires the zone of plastic deformation to be sufficiently small compared with the crack length and other relevant geometric length quantities (Hutchinson, 1983). The size of the plastic zone ahead of the crack tip, \(r_p\), can be approximately determined by (Hutchinson, 1983):

\[
r_p = \frac{1}{\alpha \pi} \left( \frac{K}{\sigma_{YS}} \right)^2
\]

where \(\alpha = 1\) for the plane stress condition and \(\alpha = 3\) for the plane strain condition; \(\sigma_{YS}\) is the yield strength of the material. Because the stress field corresponding to the plane strain condition is more severe than that corresponding to the plane stress condition, for a given material at a given temperature, fracture can more easily occur for material under the plane strain condition. Therefore, for a material behaving in a linear elastic manner prior to unstable fracture, a critical point value of stress intensity factor under the Mode I loading and plane strain condition, denoted by \(K_c\), is expected to be an appropriate fracture parameter of the material (Anderson, 2005; ASTM, 2012). To ensure the validity of the evaluated toughness value (i.e. the specimen being under SSY and the plane strain condition), the ASTM standard for experimentally determining the linear elastic plane strain fracture toughness of metallic materials, i.e. ASTM E399 (ASTM,
2012), requires the crack length, the length of the uncracked remaining ligament and the thickness of the test specimen to be no less than \(2.5\left(K_c/\sigma_{YS}\right)^2\) at the point of fracture.

Under the SSY condition, the energy release rate \(G\), defined as the rate of decrease in the potential energy with a unit increase in the crack area (Irwin, 1957), can be related to the stress intensity factor \(K\) through the following relationship:

\[
G = \frac{K^2}{E'}
\]  

(1.4)

For the plane stress condition, \(E' = E\); for the plane strain condition, \(E' = E/(1 - \nu^2)\), where \(E\) denotes the elastic modulus and \(\nu\) is Poisson’s ratio. Although the energy release rate describes global behavior and the stress intensity factor is a local parameter, Eq. (1.4) is proved to be adequate for all cracked configurations under the SSY condition (Anderson, 2005).

### 1.2.2 Elastic Plastic Fracture Mechanics

Linear elastic fracture mechanics (LEFM) is valid as long as nonlinear material deformation is confined to a relatively small region surrounding the crack tip (the SSY condition) (Anderson, 2005). For many ductile materials with medium-to-high toughness, it is virtually impossible to characterize the fracture behavior within LEFM, therefore an alternative approach, i.e. elastic-plastic fracture mechanics (EPFM), is required. Two commonly-used and most important EPFM parameters, i.e. \(J\)-integral and crack tip opening displacement (CTOD), developed primarily in the US and UK, respectively, are briefly introduced in this section.

The \(J\)-integral \((J)\) concept was named by its inventor Dr. James R. Rice (1968) as a path-independent contour integral, which equals the decrease in the potential energy per unit increase in the crack area for nonlinear elastic cracked body, and equals \(G\) for a linear elastic cracked body. For a two-dimensional cracked body with an arbitrary counterclockwise contour \((\Gamma)\) surrounding the crack tip as shown in Fig. 1.3, \(J\) is defined as the following contour integral with the unit of energy/area or equivalently force/length:
\[ J = \int_{\Gamma} \left( wdy - T_i \frac{\partial u_i}{\partial x} ds \right) \] (1.5)

where \( w \) denotes the strain energy density; \( T_i \) and \( u_i \) are the components of the traction and displacement vectors, respectively; \( ds \) is the length increment along the assigned contour \( \Gamma \), as shown in Fig. 1.3. The mathematical proof of the path-independence of J-integral can be found in many fracture mechanics textbooks (e.g., Anderson, 2005). The strain energy density can be written as (Anderson, 2005):

\[ w = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij} \] (1.6)

where \( \varepsilon_{ij} \) is the strain tensor.

Hutchinson (1968) and Rice and Rosengren (1968) independently showed that due to its path-independence, \( J \) can also be used to characterize the crack tip condition in a non-linear elastic cracked body, similar to the stress intensity factor, \( K \), in a linear elastic body. They adopted the following Ramberg-Osgood stress-strain relationship:

\[ \frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \alpha \left( \frac{\sigma}{\sigma_0} \right)^n \] (1.7)

where \( \sigma_0 \) is the reference stress; \( \varepsilon_0 \) is the reference strain, \( \varepsilon_0 = \sigma_0/E \); \( \alpha \) is a dimensionless parameter, and \( n \) is commonly known as the strain hardening exponent of the material. For the region well within the plastic zone, close to the crack tip, the elastic strains are relatively small compared with the total strain. Therefore, Eq. (1.7) reduces to a simple power-law relationship, and the stresses and strains ahead the crack tip can be completely characterized by \( J \) through the following expressions, known as the HRR solutions (singularity):

\[ \sigma_{ij} = \sigma_0 \left( \frac{J}{\alpha \sigma_0 \varepsilon_0 n} \right)^{\frac{1}{n+1}} \bar{\varepsilon}_{ij}(\theta, n) \] (1.8)

\[ \varepsilon_{ij} = \frac{\alpha \sigma_0}{E} \left( \frac{J}{\alpha \sigma_0 \varepsilon_0 n} \right)^{\frac{n}{n+1}} \bar{\varepsilon}_{ij}(\theta, n) \] (1.9)
where $J_n$ is an dimensionless constant depending on $n$ and the stress state (plane stress or plane strain), and $\bar{\sigma}_{ij}$ and $\bar{\varepsilon}_{ij}$ are dimensionless functions of $n$, $\theta$, and stress state. For linear elastic material, i.e. $n = 1$, Eq. (1.8) predicts $1/\sqrt{r}$ singularity, which is consistent with the LEFM theory. Therefore, $J$ provides a single-parameter characterization of the crack-tip fields in EPFM, just as $K$ in LEFM (see Eq. (1.2)), and the experimentally determined critical value of $J$ at the onset of crack growth, known as $J_{ic}$, can be used as the fracture toughness value of the material. For more ductile materials, $J$ always increases with the small amount of crack advance; therefore, the $J$-$R$ curve is commonly used as a characterization of the material toughness, as the $K$-based resistance curve for material under the SSY condition. The $J$-$R$ curve has significant practical implications for engineering structures that are made of ductile materials and can tolerate certain amount of crack growth, because significant additional load carrying capacity can be achieved with the application of the $J$-$R$ curve. The issues related to the experimental determination of the $J$-$R$ curve for pipeline steels will be the main focus of the following chapters of this thesis.

The crack tip opening displacement ($CTOD$) is first proposed by Wells (1961) at the British Welding Institute as a measure of fracture toughness for high toughness materials, because it is found that for such kinds of materials, the sharp crack faces can move apart and becomes blunt due to the plastic deformation prior to fracture as illustrated in Fig. 1.4, and the degree of crack blunting increases in proportion to the toughness of the material (Anderson, 2005). Therefore, $CTOD$ is found to be an appropriate parameter to characterize the crack tip states when $K_{ic}$ is not applicable and LEFM is no longer valid. Based on the strip-yield model analyzed by Burdekin and Stone (1966), the value of $CTOD$ ($\delta$) can be related to $K$ or $G$ in the limit of the SSY condition:

$$\delta = \frac{K^2}{m\sigma_{YS}E'} = \frac{G}{m\sigma_{YS}}$$

(1.10)

where $m$ is a dimensionless constraint parameter that is approximately unity for the plane stress condition and 2 for the plane strain condition.
At present, there are two widely used definitions of CTOD, namely the displacement at the original crack tip and the 90 degree intercept definition, as illustrated in Figs. 1.4 (a) and (b), respectively. The first one was originally proposed by Well (1961), and the second definition was suggested by Rice (1968) and Shih (1981) and commonly used for the CTOD evaluation in the finite element analysis. If a semicircle (blunt) crack tip is assumed, these two definitions are essentially equivalent.

For linear elastic conditions, $J$ is equivalent to the energy release rate $G$, and Eq. (1.10) can be written as follows in the limit of small scale yielding:

$$
\delta = \frac{J}{m\sigma_{YS}}
$$

(1.11)

where $m = 1$ for the plane stress condition. Under the stress state that is neither plane stress nor plane strain, the constraint parameter $m$ usually has a value between 1 and 2 (Hollstein and Blauel, 1977). Equations (1.10) and (1.11), together with Eq. (1.4), demonstrate the equivalence of the fracture parameters ($K$, $G$, $J$ and $\delta$) under the linear elastic conditions. By comparing the displacements at the crack tip obtained from the HRR solutions and the elastic-plastic finite element analyses, Shih (1981) further showed that the linear relationship between CTOD and $J$, i.e. Eq. (1.11), applies well beyond the validity limits of LEFM, so that CTOD and $J$ can be considered as equivalent fracture toughness parameters in EPFM. To consider the stain hardening effect, the yield stress, $\sigma_{YS}$, in Eq. (1.11) is commonly replaced by a so-called effective yield stress $\sigma_Y$ defined as the average of $\sigma_{YS}$ and the ultimate tensile stress $\sigma_{TS}$. Various expressions of the constraint constant $m$ are developed for different specimen configurations through finite element analyses (Kirk and Wang, 1995; Wang et al., 1997; Panontin et al., 2000; Shen and Tyson, 2009; Ruggieri, 2012).
1.3 Objective and Research Significance

1.3.1 Investigation of the Effect of Crack Front Curvature on the Measurement of J-R Curve of Single-Edge Bend Specimens

The single-edge notched three-point bend (SE(B)) specimen is one of the commonly used standard small-scale specimens to experimentally determine the J-R curve. As specified in the test standards, e.g. ASTM 1820-11(ASTM, 2011), BS 7448-97 (BSI, 1997), all machine notched specimens need to be fatigue pre-cracked to simulate natural cracks before the J-R curve testing. The fatigue pre-cracking often introduces curved as opposed to straight crack fronts. The objective of the first study reported in this thesis (Chapters 2 and 3) was to investigate the impact of the specimen crack front curvature on the evaluated crack length and the J-integral values. Based on the analysis results, new crack front straightness criteria that are in most cases less stringent than the existing criteria specified in the relevant test standard, ASTM E1820-11, are recommended. The suggested criteria could potentially lead to reduced specimen rejection rates and cost savings.

1.3.2 Investigation of the Accuracy of the Double-Clip on Gauge Method for Evaluating CTOD of Single-Edge Tension Specimens

The objective of the second study reported in this thesis (Chapter 4) was to examine the accuracy of CTOD measured from the double-clip on gauge method based on a systematic 3D FEA of clamped single-edge tension (SE(T)) specimens. The analysis considers both plane-sided and side-grooved SE(T) specimens with ranges of crack lengths, thickness-to-width ratios and strain hardening characteristic of the material. Based on the analysis results, the existing equation for evaluating CTOD based on the double-clip on gauge method was slightly modified to improve the accuracy of the CTOD evaluation. This study will facilitate the application of the fracture toughness determined from the non-standard SE(T) specimen in the strain-based design of pipelines.
1.4 Thesis Format and Outline

This thesis is prepared in an integrated-article format as specified by the School of Graduate and Postdoctoral Studies at the University of Western Ontario and consists of five chapters. Each chapter, except Chapters 1 and 5, is presented in a manuscript format with its own list of notations and references.

Chapter 1 is the introduction of the entire thesis where a review of fundamentals of LEFM and EPFM, including the concepts of stress intensity factor, energy release rate, J-integral, CTOD and toughness resistance curve is presented. Chapter 2 investigates the effect of the specimen crack front curvature on the crack mouth opening displacement (CMOD)-based compliance and the crack length evaluation for the SE(B) specimen. The effect of the crack front curvature on the evaluation of J-integral for the SE(B) specimen is presented in Chapter 3. Chapter 4 examines the accuracy of the double-clip on gauge method for evaluating CTOD using the SE(T) specimen. Finally, Chapter 5 summarizes the concluding remarks of the thesis and provides recommendations for future studies.

References


Figure 1.1. Three typical loading modes in fracture mechanics

Figure 1.2. Stress field solutions surrounding the crack tip
Figure 1.3. Arbitrary contour around the crack tip
Figure 1.4. Schematically illustration of $CTOD$ definitions
Chapter 2  Effect of Crack Front Curvature on CMOD Compliance and Crack Length Evaluation for Single-edge Bend Specimens

2.1  Introduction

2.1.1  Fracture Toughness Resistance Curve Tests on Small-scale Specimens

The fracture toughness resistance curve of ductile materials, such as the J-integral ($J$) or crack tip opening displacement ($CTOD$) resistance curve, is an important input of the integrity assessment and strain-based design of energy pipelines containing planar defects (i.e. cracks). There are two main components of the fracture toughness resistance curves (i.e. $J$-$R$ or $CTOD$-$R$ curves), namely the crack growth, $\Delta a$, and the toughness value ($J$ or $CTOD$) corresponding to the crack growth.

The $J$-$R$ or $CTOD$-$R$ curves are commonly measured on small-scale specimens, e.g. three-point single-edge bend (SE(B)), compact tension (C(T)) and single-edge tension (SE(T)) specimens. The configurations of typical SE(B), C(T) and SE(T) specimens as well as the difference between the plane-sided and side-grooved specimens are schematically shown in Fig. 2.1, where $a$, $B$, $B_N$, $H$, $S$ and $W$ denote the crack length, specimen thickness, net thickness for side-grooved specimens, daylight length, specimen span and specimen width, respectively. SE(B) and C(T) specimens have been standardized in test standards such as ASTM E1820-11 (ASTM, 2011) and BS 7448 (BSI, 1991). During the $J$- and $CTOD$-$R$ curve test, a displacement controlled load ($P$) is applied to the specimen as illustrated in Figs. 2.1(b) to (d), and the corresponding load line displacement ($LLD$) and crack mouth opening displacement ($CMOD$) can be measured by the test machine and clip-on gauge instrumented at the crack mouth, respectively. Figure 2.1 (f) shows the real test set-up for the single-edge bend (SE(B)) and the clamped single-edge tension (SE(T)) specimens. The obtained $P$-$LLD$ or $P$-$CMOD$ curves are the key input to the experimental evaluation of the $J$-$R$ or $CTOD$-$R$ curves based on these specimens. The evaluation of the $J$ value for the SE(B) specimen and the $CTOD$ value for the SE(T) specimen are discussed in Chapters 3 and 4, respectively.
The elastic unloading compliance method (Clarke et al., 1976) is widely used to experimentally determine the fracture toughness resistance curve from a single test specimen. In this method, a series of partial unloading and reloading sequences are conducted on the specimen during the test as illustrated in Fig. 2.2. At load step $i$, the slope of the corresponding unloading line was evaluated to obtain the measured compliance of the specimen, $C_i$ (i.e. the inverse of the specimen stiffness). The instantaneous crack length can then be estimated from the corresponding compliance through the crack length-compliance equation. Although the compliance of the specimen can be determined based on either LLD or CMOD, the latter is usually preferred over the former because CMOD can be more easily and accurately measured than LLD during tests (Zhu et al., 2008).

2.1.2 Literature Review and Objective

As specified in ASTM E1820-11 (ASTM, 2011), all machine notched specimens need to be fatigue pre-cracked to simulate natural cracks before the resistance curve testing. The fatigue pre-cracking often introduces curved as opposed to straight crack fronts, as illustrated in Fig. 2.3. The shape of the curved initial crack front is largely affected by specimen dimensions, notch machining conditions, fatigue pre-cracking conditions and residual stress distributions (Zhou and Soboyejo, 2002). Furthermore, the crack growth during the test is in general non-uniform across the crack front. The crack generally grows faster at the mid-plane as a result of the high local stress triaxiality, and grows slower near the free surfaces due to the near plane stress conditions (Zhou and Soboyejo, 2002).

Steenkamp (1985) investigated the influence of the crack front curvature on the specimen compliance using two-dimensional (2D) plane strain finite element analyses for SE(B) specimens with the same average crack length but different crack front curvatures. He concluded that for the same average crack length by increasing the crack front curvature, the CMOD compliance would decrease, and for the same crack front curvature, the effect of curvature on the compliance became more pronounced as the average crack length increases. Many crack length-compliance equations for various
specimens have been reported in the literature (e.g. Tada et al., 1973; Saxena and Hudak, 1978; Wu, 1984; Joyce, 1992). However, these equations were all developed based on specimens with straight crack fronts. Applying such equations to specimens with curved crack fronts will inevitably lead to errors in the predicted crack length. Therefore, the main objective of the study reported in this chapter was to quantify the crack front curvature-induced error in the crack length predicted from the crack length-compliance equation. The focus of the study was on the $CMOD$ compliance of the plane-sided SE(B) specimen because 1) the SE(B) specimen is widely used in the resistance curve testing; 2) the crack fronts in the plane-sided specimens are typically more curved than those in the side-grooved specimens (Zhou and Soboyejo, 2002; Park et al., 2011), and 3) $CMOD$ is the preferred deformation measurement than $LLD$.

ASTM E1820-11 (ASTM, 2011) specifies the allowable deviation of a curved crack front from a straight front based on the so-called nine-point measurement method (see Section 2.2). It requires that none of the nine physical measurements of the initial (final) crack size differ by more than $0.05B$ from the average initial (final) crack length $a_{ave}$ obtained from the nine measurements. Test specimens that do not meet this criterion are deemed unacceptable and therefore rejected. In this regard, the other objective of the present study was to examine if the crack front straightness criterion in ASTM E1820-11 is adequate to ensure the accuracy of the predicted crack lengths for SE(B) specimens with curved crack fronts.

2.1.3 Approach

Linear elastic three-dimensional (3D) finite element analyses of plane-sided SE(B) specimens with a wide range of thickness-to-width ratios, average crack lengths and crack front curvatures were carried out to evaluate the elastic compliance of the specimens. For a given specimen with either a straight or curved crack front, the crack length predicted from the $CMOD$ compliance was compared with its actual average crack length to quantify the curvature-induced error in the predicted crack length. The impact of the equivalent elastic modulus corresponding to the plane stress or plane strain
condition used in the crack length-compliance equation on the accuracy of the predicted crack length was also investigated.

The rest of this chapter is organized as follows. Section 2.2 describes the characteristics of the curved crack front as reported in the literature as well as reflected in the actual crack front data; the 3D FEA models and analysis procedures are described in Section 2.3; Section 2.4 presents the analysis results and discussions; the summary and concluding remarks of this chapter are included in Section 2.5.

2.2 Characteristics of Curved Crack Front

Previous experimental studies (Towers, 1983; Nikishkov et al., 1999; Zhou and Soboyejo, 2002; Park et al., 2011; Wang et al., 2012) showed that curved crack fronts are typically bowed and symmetric about the mid-plane of the specimen. Figure 2.4 photographically shows the machined notches, fatigue pre-crack fronts and final crack fronts for three plane-sided specimens (one SE(B) and two SE(T) specimens) fabricated and tested in the Department of Civil and Environmental Engineering at Western. The figure shows that the pre-crack and final crack fronts are all bowed and approximately symmetric with respect to the mid-plane, consistent with the observations reported in the literature. Therefore, only symmetric bowed crack fronts were considered in this study.

The following power-law expression was proposed by Nikishkov et al. (1999) to characterize a typical symmetric bowed crack front (see Fig. 2.3):

$$a(x) = a(0) - \beta W \left( \frac{\text{abs}(x)}{B/2} \right)^p$$  \hspace{1cm} (2.1)

where $x$ is the coordinate in the specimen thickness direction varying from $-B/2$ to $B/2$; $a(x)$ is the crack length as a function of $x$; $a(0)$ and $a(\pm B/2)$ denote the crack lengths at the mid-plane and free surfaces of the specimen, respectively; $\beta = a(0)/W - a(B/2)/W$, and $p$ ($p > 1$) is a shape parameter. For a symmetric bowed crack front such as shown in Fig. 2.3, $\beta$ is equal to $d_{\text{max}}/W - d_{\text{min}}/W$. It follows from Eq. (2.1) that both $\beta$ and $p$ influence the crack front curvature, albeit in different ways: $\beta$ is a linear scaling factor to the
curvature over the entire crack front, whereas the effect of $p$ on the curvature depends on the value of $x$.

By examining the fatigue pre-crack fronts of a total of 110 C(T) test specimens with different specimen thicknesses but the same thickness-to-width ratio ($B/W = 0.5$), Nikishkov et al. (1999) pointed out that the shape parameter $p$ in Eq. (2.1) is insensitive to the specimen thickness and can be assigned a fixed value of 3.0. In this study, we examined the crack fronts in eight SE(B) and nine single-edge tension (SE(T)) specimens tested and reported by Park et al. (2011) and Wang et al. (2012). Details of these specimens are summarized in Table 2.1. Although the focus of this study is plane-sided SE(B) specimens, the fatigue pre-crack fronts of both the SE(B) and SE(T) specimens were included because the fatigue pre-cracking procedure for the two types of specimen is the same. The pre-crack fronts of both the plane-sided and side-grooved specimens were considered because side grooves are fabricated after the pre-cracking and therefore have no impact on the shape of the pre-crack front. However, only the final crack fronts of plane-sided SE(B) specimens were included to examine the applicability of Eq. (2.1).

The crack fronts of the collected specimens were digitized and then curve-fitted using Eq. (2.1). It is observed that Eq. (2.1) fits the actual crack fronts fairly well for all the specimens considered. A comparison of the actual and fitted crack fronts for two selected specimens, i.e. SE(B)-1 and SE(T)-1 (the first two in Fig. 2.4), is depicted in Fig. 2.5 for illustration. The values of $\beta$ and $p$ associated with the fitted crack fronts are summarized in Table 2.1. The average value of $p$ for all pre-crack fronts is 3.09, which is consistent with Nikishkov et al.'s (1999) suggestion that $p$ be assigned a fixed value of 3.0. Given the above, Eq. (2.1) was adopted to characterize the curved crack front in the present study. A wide range of values of $\beta$ were assumed in the study, whereas $p$ was set to equal 3.0 for the majority of the analysis cases. Sensitivity analyses were carried out for several cases with $p = 2.5$. The rationale for selecting $p = 2.5$, as opposed to, say, $p = 3.5$, in the sensitivity analysis is that for a given $\beta$ the central portion of the crack front becomes more curved as $p$ decreases (see Fig. 2.6 for an illustration). A relatively small value of $p$ was therefore selected for the sensitivity analysis.
Because we investigated the impact of the crack front curvature on the CMOD compliance and the evaluated crack length based on the same average crack length but different crack front curvatures, Eq. (2.1) was recast in terms of the average crack length, \( a_{\text{ave}} \), instead of the crack length at the mid-plane, \( a(0) \). Furthermore, the value of \( a_{\text{ave}} \) was calculated in accordance with the nine-point measurement method specified in ASTM E1820-11 (ASTM, 2011). The measurements should be made at nine equally spaced points centered about the mid-plane of the specimen (see Fig. 2.3). The two points farthest from the mid-plane are located at 0.005W from the free surfaces. The value of \( a_{\text{ave}} \) is then obtained as follows:

\[
a_{\text{ave}} = \frac{1}{8} \left( a_1 + a_9 + \sum_{i=2}^{8} a_i \right) \tag{2.2}
\]

where \( a_i \ (i = 1, 2, \ldots, 9) \) denote the crack lengths at the nine measurement points, with \( a_1 \) and \( a_9 \) being the measured crack lengths at the two points farthest from the mid-plane (see Fig. 2.3).

If the crack front is characterized by Eq. (2.1), \( a_i \) is then given by:

\[
a_i = a(0) - \beta W \left[ \text{abs}(i-5) \cdot \left( 0.25 - \frac{\Delta}{2B} \right) \right]^p \quad (i = 1, 2, \ldots, 9) \tag{2.3}
\]

where \( \Delta \) is the distance between the outmost measure point and the specimen free surface specified in test standards. For ASTM E1820-11, \( \Delta = 0.005W \).

By substituting Eq. (2.3) into Eq. (2.2) and Eq. (2.1), and then considering \( a_1 = a_9 \) due to symmetry, Eq. (2.1) can be recast into the following format:

\[
a(x) = a_{\text{ave}} + \beta W \left\{ \left( 0.25 - \frac{\Delta}{2B} \right)^p \cdot \frac{1}{8} \sum_{i=1}^{8} \text{abs}(i-5)^p - \left( \frac{\text{abs}(x)}{B/2} \right)^p \right\} \tag{2.4}
\]

Equation (2.4) completely defines a curved crack front given the average crack length \( a_{\text{ave}} \) obtained from the nine-point measurement method, and the two parameters \( \beta \) and \( p \).

To put Eq. (2.4) in the context of the crack front straightness criterion specified in ASTM E1820-11, a parameter \( \lambda, \lambda = \max(a_{\text{max}9} - a_{\text{ave}}, a_{\text{ave}} - a_{\text{min}9})/B \), was introduced, where \( a_{\text{max}9} \) and \( a_{\text{min}9} \) are the maximum and minimum values of the nine physical
measurements, respectively. Note that a straight crack front corresponds to $\lambda = 0$; the crack front curvature increases with $\lambda$, and $\lambda = 0.05$ corresponds to the maximum allowable crack front curvature as specified in ASTM E1820-11. For specimens with symmetric bowed crack fronts, the values of $\beta$ and $\lambda$ are uniquely related as follows (see Appendix A for the derivation):

$$\beta = \frac{\lambda}{\left(\frac{W}{B}\right)^{p} \left(0.25 - \frac{A}{2B}\right)^{p} \left[\sum_{i=1}^{p-1} \{\text{abs}(i-5)\}^{p}\right]} \tag{2.5}$$

Then for given $a_{ave}/W$, $B/W$ and $p$ ($p > 1$), crack fronts with different curvatures can be generated from Eq. (2.4) and Eq. (2.5) by varying $\lambda$.

### 2.3 Finite Element Analyses

The commercial software package ADINA 8.7.4 (ADINA, 2012) was used to carry out 3D linear elastic finite element analyses (FEA) to evaluate the CMOD compliance of SE(B) specimens. All the SE(B) specimens considered in this study are plane-sided, and have the same width and the standard span-to-width ratio ($S/W = 4$), but three different relative average crack lengths $a_{ave}/W$ (i.e. $a_{ave}/W = 0.3$, 0.5 and 0.7) and three different specimen thicknesses (i.e. $B/W = 1$, 0.5 and 0.25) that are consistent with the range of $B/W$ ratios suggested in ASTM E1820-11 (ASTM, 2011). Specimens with straight and curved crack fronts were considered. For specimens with curved crack fronts, the crack front is characterized by Eq. (2.4) and Eq. (2.5) with $p = 3$ and $\lambda = 0.01$ to 0.10 with an increment of 0.01. In addition, specimens with curved crack fronts characterized by $p = 2.5$ and selected geometric configurations (i.e. $B/W = 0.5$, $a_{ave}/W = 0.3$, 0.5 and 0.7) were also considered to investigate the impact of the shape parameter $p$ on the compliance and accuracy of the crack length predicted from the compliance.

Because of symmetry, only a quarter of the specimen was modeled. The geometric and mesh configurations for a typical specimen are shown in Fig. 2.7 together with the fixation and loading conditions. The model was divided into ten layers in the thickness direction with the mesh density increasing from the mid-plane to the free surface to capture the high stress gradients near the free surface. In the vicinity of the crack tip, the
smallest element has dimensions of about $1/3000W$ and $1/75B$ in the width and thickness directions, respectively. The aspect ratio of the elements is less than 7. There are about 11,000 20-node 3D isoparametric brick elements (ADINA, 2012) included in a typical model. Young's modulus and Poisson’s ratio were set to be 200 GPa and 0.3, respectively.

The load was applied based on a displacement-controlled condition. The $CMOD$ of the specimens included in FEA was obtained at the mid-plane. The calculated $CMOD$ compliance, i.e. $C = CMOD/P$, for a representative specimen was observed to be independent of the magnitude of the applied displacement within the range of 0.001 to 0.1 mm. Therefore, the compliance was evaluated corresponding to the applied displacement of 0.1 mm ($LLD = 0.1 \text{mm}$) for all the specimens included.

2.4 Analysis and Discussions

2.4.1 Effect of the Crack Front Curvature on $CMOD$ compliance

For a given specimen with a curved crack front, the $CMOD$ compliance value, $C_c$, was compared with the compliance, $C_s$, of the specimen with a straight crack front and the same $a_{ave}/W$ and $B/W$ ratios as the specimen with the curved crack front. The values of $C_c/C_s$ are plotted against $\lambda$ for specimens with curved or straight crack fronts in Figs. 2.8(a) through 2.8(c). All the specimens with curved crack fronts shown in Fig. 2.8 have the same shape parameter $p = 3$. The figures suggest that given $a_{ave}/W$ and $B/W$, as $\lambda$ increases from 0 to 0.1, the ratio of $C_c/C_s$ first increases slightly reaching a peak point at around $\lambda = 0.03$ to 0.05, and then decreases as $\lambda$ further increases. After that peak point, the effect of the crack front curvature on the compliance becomes more pronounced as the crack length increases. Due to the dependence of the ASTM E1820-11 straightness criteria on the specimen thickness, i.e. $\lambda$ being a function of $B$, for specimens with the same average crack length and $\lambda$ values, the crack front curvature impacts the compliance of thick specimens more than that of thin specimens as shown in Fig. 2.8. For specimens with $B/W = 1$ and all the $\lambda$ values considered in this study, the $C_c/C_s$ ratio ranges from 0.94 to 1.01; for $B/W = 0.5$, $C_c$ only differs from $C_s$ by less than 2%; and for $B/W = 0.25$, $C_c$ differs from $C_s$ by less than 1%. The differences between $C_c$ and $C_s$ corresponding to
\[ \lambda = 0.05 \] (the maximum allowable crack front curvature as specified in ASTM E1820-11) are within 1% for all the specimen configurations considered in this study. These results suggest that the crack front curvature has a negligible impact on the \textit{CMOD} compliance for \( \lambda \leq 0.05 \).

### 2.4.2 Crack Length - \textit{COMD} Compliance Equations

The relationship between the \textit{CMOD} compliance, \( C \), and the relative crack length \( a/W \) is usually derived from numerical studies (Tada et al., 1973; Wang et al., 2013). The following expression that evaluates \( C \) corresponding to a given \( a/W \) proposed by Tada et al. (1973) is widely used for SE(B) specimens:

\[
\frac{4BE'}{S/W} = f \left( \frac{a}{W} \right) = 24 \left( \frac{a}{W} \right) \left[ 0.76 - 2.28 \left( \frac{a}{W} \right) + 3.87 \left( \frac{a}{W} \right)^2 - 2.04 \left( \frac{a}{W} \right)^3 + \frac{0.66}{\left( 1 - \frac{a}{W} \right)^2} \right] \tag{2.6}
\]

where \( E' \) is the elastic modulus corresponding to the plane strain condition, i.e. \( E' = E/(1 - \nu^2) \). Equation (2.6) was reported to be accurate within 1% for all \( a/W \) values (Joyce, 1992).

By inverting Eq. (2.6), Wu (1984) and Joyce (1992) developed Eqs. (2.7) and (2.8), respectively, to evaluate \( a/W \) from the \textit{CMOD} compliance for SE(B) specimens:

\[
a/W = 0.999748 - 3.950u + 2.9821u^2 - 3.21408u^3 + 51.5156u^4 - 113.031u^5 \tag{2.7}
\]

\[
a/W = 1.01878 - 4.5367u + 9.0101u^2 - 27.333u^3 + 74.400u^4 - 71.489u^5 \tag{2.8}
\]

where \( u = \frac{1}{\left( \frac{4BE'}{S/W} \right)^{1/2} + 1} \).

Equations (2.7) and (2.8) have been adopted in ASTM E1820-11 (with \( E' \) replaced by \( E \)) to evaluate the crack length for deeply-cracked \((0.45 \leq a/W < 1)\) and shallow-cracked \((0.05 \leq a/W < 0.45)\) SE(B) specimens, respectively.

Because the actual stress state in the remaining ligament of a 3D specimen is neither plane stress nor plane strain (Steenkamp, 1985; Shen et al., 2012), the use of either \( E \) or
$E'$ in Eqs. (2.7) and (2.8) will impact the accuracy of the calculated $a/W$ values. Therefore, the following so-called effective modulus of elasticity, $E_e$, was proposed by Wang et al. (2013) to be used in Eqs. (2.7) and (2.8) to improve the accuracy of the calculated $a/W$ values:

$$E_e = (A_0 + A_1 u + A_2 u^2 + A_3 u^3 + A_4 u^4)E$$

(2.9)

where $A_0, A_1, A_2, A_3$ and $A_4$ are empirical coefficients and listed in Table 2.2. It is noted that Eqs. (2.6) to (2.9) are all developed based on specimens with straight crack fronts.

### 2.4.3 Effect of the Crack Front Curvature on the Evaluated Crack Length

In the present study, all three above-mentioned elastic moduli, i.e. $E$, $E'$, and $E_e$, were used in Eqs. (2.7) and (2.8) to predict the average crack length (denoted as $a_p$) for specimens containing curved or straight crack fronts from the CMOD compliance obtained from FEA. Consistent with ASTM E1820-11, Eq. (2.7) was employed for specimens with $a/W = 0.5$ and 0.7, whereas Eq. (2.8) was used for specimens with $a/W = 0.3$. Note that for specimens with curved crack fronts, the predicted crack length $a_p$ can be considered as the equivalent crack length corresponding to a straight crack front, which is generally not the same as the nine point measured average crack length $a_{ave}$ even if Eqs. (2.7) and (2.8) are perfectly accurate. The error in the predicted crack length, $e_a$, was calculated as $e_a = (a_p - a_{ave})/a_{ave}$. The values of $e_a$ are plotted against $\lambda$ in Figs. 2.9(a) to 2.9(i). It is observed that the magnitude of $e_a$ is governed by the choice of the elastic moduli, and insensitive to the crack front curvature regardless of the $a_{ave}/W$ and $B/W$ ratios. Except for specimens with $a_{ave}/W = 0.7$ and $B/W = 1$, the use of $E$ leads to more accurate predictions of $a_{ave}$ than the use of $E'$. The magnitude of $e_a$ corresponding to the use of $E$ is within 1% for specimens with $B/W = 0.5$ and 0.25, and 3% for specimens with $B/W = 1$. The magnitude of $e_a$ corresponding to the use of $E'$ is between 1% and 7%. Furthermore, the use of $E_e$ leads to the most accurate prediction of $a_{ave}$ with the magnitude of $e_a$ being less than 1% for all the specimens considered.

The impact of the shape parameter $p$ involved in Eq. (2.4) on $e_a$ was investigated based on SE(B) specimens with $a_{ave}/W = 0.3, 0.5$ and 0.7, $B/W = 0.5$ and $\lambda = 0$ to 0.10.
The values of $e_a$ corresponding to $p = 3$ and 2.5 for the specimens considered are depicted in Figs. 2.9(d), 2.9(e) and 2.9(f). The results shown in these figures indicate that $p$ has a negligible impact on $e_a$.

2.5 Summary and Concluding Remarks

Three-dimensional linear-elastic FEA was performed on plane-sided SE(B) specimens with straight and curved crack fronts to investigate the impact of the crack front curvature on the CMOD compliance and accuracy of the crack length predicted from the crack length-CMOD compliance equations proposed by Wu (1984) and Joyce (1992). The set of specimens analyzed have three $a_{ave}/W$ values ($a_{ave}/W = 0.3, 0.5$ and $0.7$) and three $B/W$ ratios ($B/W = 1, 0.5$ and $0.25$). Symmetric bowed crack fronts with different curvatures were considered in the analysis. The power-law expression proposed by Nikishkov et al. (1999) was adopted to characterize the curved crack front but was recast by introducing a crack front curvature parameter $\lambda$ that is consistent with the crack front straightness criterion specified in ASTM E1820-11. The CMOD compliance of the specimen with a curved crack front was compared with the compliance of the specimen with a straight crack front and the same average crack length and thickness. For a given specimen with either a straight or curved crack front, the crack length predicted from Wu's or Joyce's equation was compared with its actual average crack length to quantify the error in the predicted crack length.

The numerical results show that for SE(B) specimens with curved crack fronts satisfying the straightness criterion as specified in ASTM E1820-11, i.e. $\lambda \leq 0.05$, the corresponding CMOD compliance differs by less than 1% compared with the compliance of the specimen with the straight crack fronts and the same average crack length. In addition, the crack front curvature has a negligible impact on the accuracy of the crack length predicted from Wu’s or Joyce’s equation for $\lambda$ values ranging from 0.01 to 0.10 regardless of $a_{ave}/W$ and $B/W$ ratios.

The use of three different elastic moduli, i.e. $E$, $E'$ and $E_e$, in the prediction of the average crack length for SE(B) specimens with curved crack fronts was investigated. It is observed that the use of $E_e$ as proposed by Wang et al. (2013) can lead to highly
accurate prediction of the average crack length (with the maximum error of prediction of 1%) for a wide range of $a/W$, $B/W$ and crack front curvatures.

References


Table 2.1: Summary of the curved crack fronts of the specimens collected in this study

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Side surfaces¹</th>
<th>Examined crack front</th>
<th>Fitted parameters</th>
<th>Source</th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
<td>Fatigue pre-crack</td>
<td>α(0)</td>
<td>β</td>
</tr>
<tr>
<td></td>
<td></td>
<td>front</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SE(B)-1</td>
<td>PS</td>
<td></td>
<td>0.50</td>
<td>0.06</td>
</tr>
<tr>
<td>SE(B)-2</td>
<td>SG</td>
<td>Final crack front</td>
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<td>0.08</td>
</tr>
<tr>
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<td>SG</td>
<td></td>
<td>0.32</td>
<td>0.02</td>
</tr>
<tr>
<td>SE(B)-4</td>
<td>SG</td>
<td></td>
<td>0.22</td>
<td>0.04</td>
</tr>
<tr>
<td>SE(B)-5</td>
<td>SG</td>
<td></td>
<td>0.22</td>
<td>0.04</td>
</tr>
<tr>
<td>SE(B)-6</td>
<td>SG</td>
<td></td>
<td>0.23</td>
<td>0.04</td>
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<td>SE(B)-7</td>
<td>SG</td>
<td></td>
<td>0.22</td>
<td>0.04</td>
</tr>
<tr>
<td>SE(B)-8</td>
<td>SG</td>
<td></td>
<td>0.33</td>
<td>0.00</td>
</tr>
<tr>
<td>SE(T)-1</td>
<td>PS</td>
<td>Fatigue pre-crack</td>
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<td>0.05</td>
</tr>
<tr>
<td>SE(T)-2</td>
<td>PS</td>
<td>front</td>
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<td>0.04</td>
</tr>
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<td>SE(T)-8</td>
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<td></td>
<td>0.15</td>
<td>0.01</td>
</tr>
<tr>
<td>SE(T)-9</td>
<td>SG</td>
<td></td>
<td>0.33</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Notes:

1. PS: Plane-sided specimen; SG: Side-grooved specimen. All the specimens collected in this study have the same thickness-to-width ratio ($B/W = 1$).
Table 2.2: The coefficients used in Eq. (2.9)

<table>
<thead>
<tr>
<th>$B/W$</th>
<th>$A_0$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
</tr>
</thead>
<tbody>
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<td>0.2685</td>
<td>-6.9063</td>
<td>28.9474</td>
<td>-32.3795</td>
</tr>
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<td>1.0698</td>
<td>-0.0709</td>
<td>-4.7352</td>
<td>22.4058</td>
<td>-24.9463</td>
</tr>
<tr>
<td>0.25</td>
<td>1.0414</td>
<td>-0.0706</td>
<td>-2.7256</td>
<td>12.2094</td>
<td>-12.147</td>
</tr>
</tbody>
</table>
(a) Schematic of small-scale specimens

(b) Plane-sided three-point single-edge bend (SE(B)) specimen
(c) Plane-sided compact tension (C(T)) specimen

(d) Plane-sided clamped single-edge tension (SE(T)) specimen
Figure 2.1. Schematic and experiment set-up of small-scale specimens

(e) Schematic of side grooves

(f) Experiment set-up for the single-edge bend (SE(B)) and the clamped single-edge tension (SE(T)) specimens
Figure 2.2. *P-CMOD* curve using elastic unloading compliance method (Wang et al. 2012)
Figure 2.3. Schematic illustration of symmetric bowed crack fronts

Figure 2.4. Typical crack fronts of SE(B) and SE(T) specimens
Figure 2.5. Comparison of actual and fitted crack fronts for one SE(B) specimen and one SE(T) specimen.
Figure 2.6. Schematic illustration of symmetric bowed crack fronts with a fixed $\beta$ and varying $p$.
Figure 2.7. Geometric and mesh configuration of the finite element model
(a) $B/W = 1$

\[ \lambda = \max(a_{\text{max}9}, a_{\text{ave}} - a_{\text{ave}} - a_{\text{min}9})/B \]

---

(b) $B/W = 0.5$

\[ \lambda = \max(a_{\text{max}9}, a_{\text{ave}} - a_{\text{ave}} - a_{\text{min}9})/B \]
Figure 2.8. Variation of $C_c/C_s$ against $\lambda$.
(a) $a_{ave}/W = 0.3$ and $B/W = 1$

(b) $a_{ave}/W = 0.5$ and $B/W = 1$
(c) $a_{ave}/W = 0.7$ and $B/W = 1$

(d) $a_{ave}/W = 0.3$ and $B/W = 0.5$
(e) $a_{ave}/W = 0.5$ and $B/W = 0.5$

(f) $a_{ave}/W = 0.7$ and $B/W = 0.5$
(g) $a_{ave}/W = 0.3$ and $B/W = 0.25$

(h) $a_{ave}/W = 0.5$ and $B/W = 0.25$
(i) $a_{ave}/W = 0.7$ and $B/W = 0.25$

Figure 2.9. Variation of the error $e_a$ against $\lambda$
Chapter 3  Effect of Crack Front Curvature on Experimental Evaluation of J-integral for Single-edge Bend Specimens

3.1 Introduction

Single-edge bend (SE(B)) specimens are extensively used to develop the fracture toughness resistance curves such as the J-integral resistance curve (J-R curve), which is an important input of the strain-based design of energy pipelines with respect to planar defects. To simulate natural cracks, all the machine notched specimens need to be fatigue pre-cracked before the toughness testing. As shown in Chapter 2, the fatigue pre-cracking often introduces curved rather than straight crack fronts (see Figs. 2.3 and 2.4). There are two main components of the fracture toughness resistance curve, namely the crack growth, $\Delta a$, and the toughness value corresponding to the crack growth. The impact of the crack front curvature on the accuracy of the crack length predicted from the crack mouth opening displacement (CMOD) compliance has been investigated and reported in Chapter 2. This chapter focuses on the investigation of the effect of the crack front curvature on the experimentally determined J-integral ($J$).

3.1.1 Experimental Evaluation of $J$

Begley and Landes (1972) first recognized that the J-integral ($J$) can be experimentally determined from its interpretation as the energy release rate:

$$J = - \frac{dU}{Bda} \quad (3.1)$$

where $U$, $B$ and $a$ denote the strain energy, specimen thickness and crack length, respectively. A series of testing specimens with the same specimen thickness, initial crack lengths and material are loaded to various values of the load line displacement (LLD) and final crack lengths. From the measured load-displacement data, Begley and Landes (1972) determined the energy absorbed by each specimen and then calculated $J$ using Eq. (3.1). Since multiple specimens need to be tested and analyzed to determine a single experimental result of $J$, this $J$ testing technique is costly and time consuming (Anderson, 2005).
The work by Rice et al. (1973) introduced a more convenient way to evaluate $J$ directly from the load displacement curve of a single test specimen. Either the load controlled (Eq. 3.2a) or displacement controlled (Eq. 3.2b) condition could be used to determine $J$ as follows (see Fig. 3.1):

\[ J = \frac{1}{B} \int_0^P \frac{\partial \Delta}{\partial a} dP \quad (3.2a) \]

or

\[ J = -\frac{1}{B} \int_0^{\Delta} \frac{\partial P}{\partial a} d\Delta \quad (3.2b) \]

where $P$ denotes the applied load; $\Delta$ is the specimen displacement, which could be either the load-line displacement (LLD) or CMOD, and $U$ is defined as the area under the load-displacement curve as shown in Fig. 3.1. Based on the limit load analysis, Sumpter and Turner (1976) proposed an alternative form of Eq. (3.2b):

\[ J = \frac{\eta}{(W-a)B} \int_0^\Delta P d\Delta = \frac{\eta A}{(W-a)B} \quad (3.3) \]

where $W$ is the specimen width; $a$ is the equivalent crack length, which can be determined from the unloading compliance method using a single specimen according to ASTM E1820-11 as discussed in Chapter 2; $\eta$ is a dimensionless geometry factor relating $J$ and the strain energy, and $A$ represents the total area under the load versus displacement curve, which is the total work done by the external force during the test. A typical load-displacement curve is shown in Fig. 3.2. The total area, $A$, under the load-displacement curve can be separated by the elastic unloading path into an elastic component, $A_{el}$, and a plastic component, $A_{pl}$, i.e. $A = A_{el} + A_{pl}$. Similarly, this unloading path separates the total displacement $\Delta$ into an elastic component, $\Delta_{el}$, and a plastic component, $\Delta_{pl}$, i.e. $\Delta = \Delta_{el} + \Delta_{pl}$, and the $J$ value can also be separated into two components accordingly:

\[ J = J_{el} + J_{pl} \quad (3.4) \]
where $J_{el}$ and $J_{pl}$ are the elastic and plastic components of $J$, respectively. The elastic component of $J$ can be directly calculated from the stress intensity factor $K$ as specified in ASTM E1820-11 (ASTM, 2011):

$$J_{el} = \frac{K^2(1-\nu^2)}{E} \quad (3.5)$$

where $E$ and $\nu$ are the Young’s modulus and the Poisson’s ratio, respectively; the evaluation of the stress intensity factor, $K$ is well documented (e.g. Tada et al. 1973) for various structure configurations. The plastic component of $J$ can be determined from Eqs. (3.2a) or (3.2b) and (3.3) as:

$$J_{pl} = \frac{1}{B} \int_0^P \frac{\partial \Delta_{pl}}{\partial a} dP = -\frac{1}{B} \int_0^{\Delta_{pl}} \frac{\partial P}{\partial a} d\Delta_{pl} = \frac{\eta_{pl} A_{pl}}{(W-a)B} \quad (3.6)$$

where $\eta_{pl}$ denotes the dimensionless plastic eta factor that relates $A_{pl}$ to $J_{pl}$. It is noted that both the $P$-$CMOD$ and $P$-$LLD$ curves can be used to evaluate $J$; therefore, $\eta_{pl}$ have $CMOD$-based and $LLD$-based values, denoted as $\eta_{pl}^{CMOD}$ and $\eta_{pl}^{LLD}$, respectively. Therefore, Eqs. (3.4), (3.5) and (3.6) can be rewritten as:

$$J = \frac{K^2(1-\nu^2)}{E} + \frac{\eta_{pl}^{CMOD} A_{pl}^{CMOD}}{(W-a)B} \quad (3.7a)$$

or

$$J = \frac{K^2(1-\nu^2)}{E} + \frac{\eta_{pl}^{LLD} A_{pl}^{LLD}}{(W-a)B} \quad (3.7b)$$

These two plastic eta factor-based equations can be employed to experimentally determine $J$ from the obtained $P$-$CMOD$ or $P$-$LLD$ curves as specified in ASTM E1820-11. The following expression of $\eta_{pl}^{CMOD}$ proposed by Zhu et al. (2008) was adopted in ASTM E1820-11 for both the standard deeply-cracked SE(B) specimens (i.e. the initial crack length greater than or equal to 0.45) and non-standard shallow-cracked SE(B) specimens (i.e. the initial crack length less than 0.45):

$$\eta_{pl}^{CMOD} = 3.667 - 2.199 \left( \frac{a}{W} \right) + 0.437 \left( \frac{a}{W} \right)^2 \quad (3.8)$$
As to $\eta_{pl}^{LLD}$, a constant value of 1.9 is used for standard SE(B) specimens, but there is no recommended value for non-standard specimens in ASTM E1820-11. For the sake of completeness of the current study, the following expression also from Zhu et al. (2008) was employed to evaluate $\eta_{pl}^{LLD}$ for the shallow-cracked specimens included in this study:

$$\eta_{pl}^{LLD} = 1.620 + 0.850 \left( \frac{a}{W} \right) - 0.651 \left( \frac{a}{W} \right)^2$$  \hspace{1cm} (3.9)

### 3.1.2 Literature Review and Objective

Numerical and experimental studies have been performed to investigate the impact of the crack front curvature on the crack driving forces (e.g. stress intensity factor, energy release rate, $J$ and crack tip opening displacement ($CTOD$)) in SE(B) specimens. For example, Crouch (1991) carried out three-dimensional (3D) linear-elastic finite element analyses (FEA) of SE(B) specimens and indicated that if the maximum and minimum crack lengths on a curve crack front differ by 30%, the maximum energy release rate along the curved crack front can be twice the maximum energy release rate on a straight crack front with the same average length. Based on 3D elastic-plastic FEA of the compact tension (CT) and SE(B) specimens containing curved crack fronts, Nikishkov et al. (1999) pointed out that the crack front curvature, as allowed in test standards ASTM E813 (1989), ASTM E1290 (1989) and ESIS P2-92 (1992), has a significant influence on the ratio between the crack tip opening displacements ($CTOD$) obtained at the mid-plane of the crack front and the surfaces of the specimen. Zhou and Soboyejo (2002) developed experimental techniques to fabricate A707 steel SE(B) specimens with controlled crack front curvatures, and reported that the critical $CTOD$ experimentally determined at the mid-plane of the SE(B) specimen increases with the crack front curvature. The critical $CTOD$ corresponding to a crack front curvature of approximately 0.11 mm$^{-1}$ can be 50% higher than that corresponding to a straight crack front.

The aforementioned studies all focused on the effects of the crack front curvature on the local crack driving forces. The present study focused on the effect of the crack front curvature on the average $J$-integral ($J$) over the crack front, which is more relevant to the characterization of the experimentally determined $J$-$R$ curve (Zhou and Soboyejo, 2002).
The impact of the crack front curvature on the average $J$ evaluated using the plastic eta factor-based approach, i.e. Eqs. (3.7a) and (3.7b), for SE(B) specimens was investigated in this study. The values of the plastic eta factors, i.e. Eqs. (3.8), (3.9) or the constant value of 1.9, are obtained from specimens with straight crack fronts (Zhu et al., 2008; ASTM, 2011). Therefore, applying plastic eta factors that are based on straight crack fronts to specimens with curved crack fronts will lead to errors in the calculated $J$ values. This investigation sheds light on the magnitudes of such errors.

ASTM E1820-11 specifies the allowable deviation of a curved crack front from a straight front based on the so-called nine-point measurement method. It requires that none of the nine physical measurements of the initial (final) crack size differ by more than $0.05B$ from the average initial (final) crack length $a_{ave}$ obtained from the nine measurements. Test specimens that do not meet this criterion are deemed unacceptable and therefore rejected. In this regard, the other objective of the present study was to examine the adequacy of the crack front straightness criterion in ASTM E1820-11 as far as the accuracy of the average $J$ evaluation is concerned.

3.1.3 Approach

In this study, systematic 3D finite element analyses of plane-sided SE(B) specimens with a wide range of thickness-to-width ratios, crack lengths and crack front curvatures were carried out. For a given specimen with a curved crack front, the average $J$ values over the crack front evaluated using the virtual crack extension method (see Appendix B (ADINA, 2012)) were compared with the $J$ values evaluated using the plastic eta factor-based approach to quantify the errors in $J$ obtained from the latter approach. The crack front straightness criteria specified in ASTM E1820-11 were examined. Based on the analysis results, crack front straightness criteria that are in most cases less stringent than that specified in ASTM E1820-11 were proposed.

The rest of this chapter is organized as follows. The 3D FEA models and analysis procedures are described in Section 3.2; Section 3.3 presents the comparison of $J$ values evaluated using different methods and the recommendation concerning the crack front
straightness criterion in ASTM E1820-11, and the summary and concluding remarks are presented in Section 3.4.

### 3.2 Finite Element Analyses

#### 3.2.1 Finite Element Model

The 3D finite element analyses were carried out by the commercial software package ADINA 8.7.4 (ADINA, 2012). All the SE(B) specimens considered in this study are plane-sided, and have the same width and the standard span-to-width ratio \(S/W = 4\), but three different relative average crack lengths \(a_{ave}/W\) (i.e. \(a_{ave}/W = 0.3, 0.5\) and 0.7) and three different specimen thicknesses (i.e. \(B/W = 1, 0.5\) and 0.25) covering the range of \(B/W\) ratios suggested in ASTM E1820-11 (ASTM, 2011). Specimens with both straight and curved crack fronts were included. For specimens with curved crack fronts, the so-called nine-point measurement method was employed to evaluate the average crack length as specified in ASTM E1820-11. The bowed and symmetric crack fronts are assumed and characterized by the following two equations, which are the same as Eqs. (2.4) and (2.5), respectively (see Section 2.2 and Fig. 2.6):

\[
a(x) = a_{ave} + \beta W \left\{ \left( 0.25 - \frac{\Delta}{2B} \right)^p \cdot \frac{1}{8} \sum_{i=1}^{8} \left[ \text{abs}(i-5) \right]^p \right\}
\]

\[
\beta = \frac{\lambda}{\left( \frac{W}{B} \right) \left( 0.25 - \frac{\Delta}{2B} \right)^p \left\{ 4^p \cdot \frac{1}{8} \sum_{i=1}^{8} \left[ \text{abs}(i-5) \right]^p \right\}}
\]

where \(p\) is the shape parameter in the assumed power-law function; \(\Delta = 0.005W\) and \(\beta = a(0)/W - a(B/2)/W\). To put the equations in the context of the crack front straightness criterion specified in ASTM E1820-11, a parameter \(\lambda, \lambda = \max(a_{max9} - a_{ave}, a_{ave} - a_{min9})/B\), was introduced, where \(a_{max9}\) and \(a_{min9}\) are the maximum and minimum values of the nine physical measurements, respectively. Note that a straight crack front corresponds to \(\lambda = 0\); the crack front curvature increases with \(\lambda\), and \(\lambda = 0.05\) corresponds to the maximum allowable crack front curvature as specified in ASTM E1820-11.

In this study, \(p = 3\) was assumed for the majority of the specimens with curved crack fronts following the study by Nikishkov et al. (1999) as well as the investigation carried
out at Western (see Section 2.2), whereas \( \lambda \) was assigned values of 0.05 to 0.09 with an increment of 0.01. In addition, specimens with curved crack fronts characterized by \( p = 2.5 \) and selected geometric configurations (i.e. \( B/W = 0.5, a_{ave}/W = 0.3 \) and 0.7) were also considered as sensitivity cases to investigate the impact of the shape parameter \( p \) on the evaluation of \( J \).

The geometric and mesh configurations of the FEA model used in the present study are the same as those presented in Chapter 2 as illustrated in Fig. 2.7. A quarter of the specimen with a sharp crack tip was modeled and divided into ten layers in the thickness direction with the mesh density increasing from the mid-plane to the free surface to capture the high stress gradients near the free surface. To reduce the computation time, 8-node 3D isoparametric brick elements (as opposed to the 20-node brick elements used in Chapter 2) (ADINA, 2012) was employed to carry out this elastic-plastic J-integral analysis. The accuracy of using the 8-node isoparametric brick elements for calculating \( J \) has been shown to be adequate (Kulka and Sherry, 2012). The total number of elements is about 11,000 in a typical specimen model. The mesh surrounding the crack tip consists of 40 concentric semicircles as shown in Fig. 2.7. In the vicinity of the crack tip, the smallest element has dimensions of about \( 1/3000W \) and \( 1/75B \) in the width and thickness directions, respectively. The aspect ratio of these elements is less than 7. Stationary cracks were assumed in all the models.

### 3.2.2 Material Model

The uniaxial stress-strain relationship is described by the Ramberg-Osgood law (Ramberg and Osgood, 1943) as follows:

\[
\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \alpha \left( \frac{\sigma}{\sigma_0} \right)^n
\]

(3.12)

where \( \sigma_0 \) is the reference stress, \( \varepsilon_0 \) is the reference strain, \( \varepsilon_0 = \sigma_0/E \); \( \alpha \) is a dimensionless parameter, and \( n \) denotes the strain hardening exponent. In this study, materials with \( \sigma_0 = \sigma_{YS} = 550 \text{ MPa}, E = 200 \text{ GPa}, \) and \( \alpha = 1 \) were selected to simulate the X80-grade pipeline steel, where \( \sigma_{YS} \) is the yield strength. Two values of \( n \), namely \( n = 10 \) and 20, were
considered in this study to investigate the effect of strain hardening exponent on the $J$ calculation.

### 3.2.3 Computational Procedure

A displacement-controlled line-load was used in all the models. The loading rate varies from about 50 - 100 steps/mm depending on the configuration of the specimen. Using more loading steps was found to have no impact on the load-displacement relationship or $J$ values evaluated from FEA. The incremental theory of plasticity (Lubliner, 2008), von Mises yield criterion and isotropic hardening rule were employed in the analysis. The von Mises yield criterion indicates that yielding starts once the second invariant of the deviatoric stress tensor, $J_2$, reaches a critical value (i.e. $\sigma_{YS}^2/3$). The incremental theory of plasticity combined with the associate flow rule and von Mises yield criterion can be characterized by the following constitutive equation:

$$
d\varepsilon_{ij}^{pl} = d\lambda \cdot s_{ij}
$$

(3.13)

where $\varepsilon_{ij}^{pl}$ and $s_{ij}$ ($i, j = 1, 2$ and $3$) are the plastic strain tensor and the deviatoric stress tensor, respectively, and $d\lambda$ is a scalar factor of proportionality. Fracture toughness tests reported in the literature (e.g. Zhu et al., 2008; Park et al., 2011; Wang et al., 2012) indicate that the maximum applied load ($P$) for the SE(B) specimen is usually about 1.5 to 1.6$P_y$, where $P_y$ is the reference load defined as $B(W - a_{ave})^2\sigma_Y/S$ (Nevalainen and Dodds, 1990). In the current study, the finite element analyses were performed to relatively large plastic deformations corresponding to $P/P_y$ up to 1.8 and 1.6 for $n = 10$ and 20, respectively. Note that $\sigma_Y$ is the effective yield strength, defined as $\sigma_Y = (\sigma_{YS} + \sigma_{TS})/2$ according to ASTM E1820-11(ASTM, 2011), where $\sigma_{TS}$ is the ultimate tensile strength. Ignoring the elastic strain in Eq. (3.12) and applying Considere’s necking criterion (Soboyejo, 2003), one can derive the following equation to evaluate $\sigma_{TS}$:

$$
\sigma_{TS} = \left(\frac{E\sigma_0^{n-1}}{ane}\right)^{1/n}
$$

(3.14)

where $e = 2.71828$ is the base of the natural logarithm.
The sparse matrix solver was employed for its high efficiency in the numerical analysis (ADINA, 2012). The full Newton-Raphson iteration method was selected to find the solution of nonlinear equations with the maximum number of iterations for each step being 15. The displacement convergence criterion was selected, in which the displacement tolerance equal to 0.001 corresponding to a reference displacement of 1 mm (ADINA, 2012). Convergence studies on mesh density and loading rate were conducted and showed good convergence in the elastic-plastic analyses: Dividing the specimen into 20 as opposed to ten layers along the thickness direction, or using 500 as opposed to 50-100 steps/mm, was found to have nearly no impact on the obtained J-integral.

For a given specimen containing either a straight or curved crack, two average $J$ values over the crack front were evaluated in this study, denoted by $J^{FEA}$ and $J^\eta$, respectively. To calculate $J^{FEA}$, the local $J$ values corresponding to the ten layers along the thickness direction in the FEA model were evaluated first using the virtual crack extension method (Anderson, 2005; see Appendix B for a brief illustration of the virtual crack extension method) as implemented in ADINA (ADINA, 2012), whereby virtual shifts were defined by the two outermost semicircular rings surrounding the crack tip, i.e. the 40th ring (see Fig. 2.7). To ensure the path-independence of the calculated $J$ values, the $J$ values obtained from the use of other rings were compared with those obtained from using the 40th ring. The difference was found to be less than 0.5% for all the load steps. $J^{FEA}$ was then evaluated as the weighted average of the local $J$ values, with the weighting factor for each layer equal to the thickness of the layer divided by the half-thickness of the specimen. Note that $J^{FEA}$ was considered the true average $J$ over the crack front. On the other hand, $J^\eta$ was calculated using the plastic eta factor-based approach as specified in ASTM E1820-11, i.e. Eqs. (3.7) through (3.9). To compute $J^\eta$, CMOD, LLD and $P$ corresponding to each loading step were retrieved from the FEA results. It should be noted that for a curved crack front the equivalent crack length $a$ obtained from the unloading compliance method in general does not equal the nine-point measured average crack length $a_{ave}$ as discussed in Chapter 2. However, the results presented in Chapter 2 suggest that the crack front curvature has a negligible impact on the crack length.
evaluated from the elastic compliance of SE(B) specimens (see Section 2.4); therefore, it is assumed in this study that \( a \) equals \( a_{\text{ave}} \) for curved crack fronts.

### 3.3 Results and Discussions

Let \( e_J = (J^\eta - J^{FEA})/J^{FEA} \) denote the error in \( J^\eta \). Positive and negative values of \( e_J \) mean that \( J^\eta \) overestimates and underestimates the actual \( J, J^{FEA} \), respectively. The values of \( e_J \) are plotted against \( P/P_\gamma \) for specimens with curved or straight crack fronts in Figs. 3.3 through 3.6. The specimens shown in Figs. 3.3 and 3.4 have \( n = 10 \), and \( n = 20 \) cases are shown in Figs. 3.5 and 3.6. All the specimens shown in these figures with curved crack fronts have the same shape parameter \( p = 3 \). The errors in the CMOD-based \( J^\eta \) are shown in Figs. 3.3 and 3.5, whereas the errors in the LLD-based \( J^\eta \) are shown in Figs. 3.4 and 3.6.

The five dashed lines in the same subfigure correspond to specimens with the same \( B/W \) and \( a_{\text{ave}}/W \) but different crack front curvatures characterized by \( \lambda \) varying from 0.05 to 0.09, whereas the solid line in the subfigure corresponds to the specimen with a straight crack front and the same \( B/W \) and \( a_{\text{ave}}/W \). The solid line reflects the inherent error associated with the equation (i.e. Eq. (3.8), Eq. (3.9) or the constant value of 1.9) used to evaluate the plastic eta factor. For the CMOD-based analysis, this inherent error ranges from -9\% to -2\% for shallow-cracked specimens (i.e. \( a_{\text{ave}}/W = 0.3 \)), and from -5\% to 3\% for deeply-cracked specimens (i.e. \( a_{\text{ave}}/W = 0.5 \) and 0.7). For the LLD-based analysis, the inherent error is between -4\% and 4\% for specimens with \( a_{\text{ave}}/W = 0.3 \) and \( B/W = 1 \) and 0.5, between -4\% and 15\% for specimens with \( a_{\text{ave}}/W = 0.3 \) and \( B/W = 0.25 \), and between -7\% and 0\% for specimens with \( a_{\text{ave}}/W = 0.5 \) and 0.7. Figures 3.3 to 3.6 suggest that the plastic eta factors for the SE(B) specimen could be computed as a function of \( B/W, n \) and \( P/P_\gamma \), in addition to \( a_{\text{ave}}/W \), such that the accuracy of \( J^\eta \) is improved and more consistent for different loading levels, the stain hardening exponents and \( B/W \) ratios. However, the improvement of the formulas for the plastic eta factors is beyond the scope of the current study.
Figures 3.3 to 3.6 show that variations of $e_J$ with respect to $P/P_y$ for specimens with the same $a_{\text{ave}}/W$, $B/W$ and $n$ but different $\lambda$ are to a large extent similar. Given $a_{\text{ave}}/W$, $B/W$, $n$ and $P/P_y$, $e_J$ consistently increases as $\lambda$ increases from 0 to 0.09, which indicates that $J^\eta$ becomes less conservative and tends to overestimate the actual $J$ as the crack front curvature increases. Figures 3.3 to 3.6 also suggest that $n$ has a negligible impact on $e_J$ for specimens with $B/W = 1$, $a_{\text{ave}}/W = 0.5$ and 0.7, regardless of the crack front curvature. The impact of $n$ on $e_J$ is somewhat more significant for specimens with all the other $a_{\text{ave}}/W$ and $B/W$ ratios, especially for $B/W = 0.25$. At $\lambda = 0.05$, which is the maximum allowable crack front curvature according to ASTM E1820-11, the errors in the $\text{CMOD}$- and $\text{LLD}$-based $J^\eta$ are generally between -7% and 6% for $n = 10$ and 20. However, the $\text{LLD}$-based $J^\eta$ overestimates the actual $J$ by up to 17% for the specimen with $a_{\text{ave}}/W = 0.3$, $B/W = 0.25$, $n = 20$ and $\lambda = 0.05$.

Figures 3.3(a), 3.3(d), 3.5(a) and 3.5(d) indicate that for specimens with $a_{\text{ave}}/W = 0.3$ and $B/W = 1$ and 0.5, $|e_J|$ corresponding to $\lambda = 0.09$ is in general less than that corresponding to $\lambda = 0$ (i.e. straight crack front) for given $P/P_y$ and $n$. These observations suggest that for such specimens, the error in $J^\eta$ attributed to the crack front curvature is opposite to and less than the inherent error in $\eta^\text{CMOD}_{pl}$ calculated from Eq. (3.8). Figures 3.3(a), 3.3(d), 3.5(a) and 3.5(d) further suggest that the crack front straightness criterion as specified in ASTM E1820-11, i.e. $\lambda \leq 0.05$, can be relaxed to $\lambda \leq 0.09$ for plane-sided SE(B) specimens with $a_{\text{ave}}/W = 0.3$ and $B/W = 1$ and 0.5, if $J^\eta$ is evaluated from the $P$-$\text{CMOD}$ curve and no more than 5% overestimation of $J$ is considered acceptable. Consider the $\text{CMOD}$-based $J^\eta$ for the other specimens and let $e_J \leq 5\%$ be satisfied at all $P/P_y$ values. Figures 3.3(b), 3.3(e), 3.5(b) and 3.5(e) suggest that the crack front straightness criterion could be relaxed to $\lambda \leq 0.09$ and 0.07 from $\lambda \leq 0.05$, respectively, for specimens with $a_{\text{ave}}/W = 0.5$, $B/W = 1$ and 0.5; Figures 3.3(c), 3.3(f), 3.5(c) and 3.5(f) suggest that $\lambda \leq 0.06$ is acceptable for specimens with $a_{\text{ave}}/W = 0.7$, $B/W = 1$ and 0.5, regardless of $n$. On the other hand, Figs. 3.3(i) and 3.5(i) indicate that the crack front straightness criterion of $\lambda \leq 0.05$ may remain as is or be slightly tightened for the specimen with $a_{\text{ave}}/W = 0.7$ and $B/W = 0.25$, because $e_J$ corresponding to $\lambda = 0.05$ marginally exceeds 5%. However, Figs. 3.3(g), 3.3(h), 3.5(g) and 3.5(h) suggest that for
specimens with $a_{ave}/W = 0.3$ and 0.5, and $B/W = 0.25$, the crack front straightness criterion is sensitive to the $n$ values and that the criterion corresponding to $n = 20$ should be more stringent than that corresponding to $n = 10$.

New crack front straightness criteria were also suggested by considering the LLD-based $J^n$ and the corresponding $e_J$ shown in Figs. 3.4 and 3.6. Table 3.1 summarizes the suggested crack front straightness criteria based on considerations of CMOD- and LLD-based $J^n$ and specimens with $n = 10$ and 20. The suggested straightness criteria for $n = 20$ are always equally or more stringent than those for $n = 10$. It is noteworthy that $e_J$ corresponding to LLD-based $J^n$ for the SE(B) specimen with $a_{ave}/W = 0.3$, $B/W = 0.25$ and $\lambda = 0.05$ is up to 10% and 17%, respectively, for specimens with $n = 10$ and 20. For this particular specimen, tightening the crack front straightness criterion with the aim of improving the accuracy of $J^n$ is considered unfruitful without first improving the accuracy of the corresponding $\eta_{pl}^{LLD}$.

The impact of the shape parameter $p$ in Eq. (2.4) on $e_J$ was investigated based on SE(B) specimens with $a_{ave}/W = 0.3$ and 0.7, $B/W = 0.5$, $n = 10$ and $\lambda = 0.05$ to 0.09. The values of $e_J$ corresponding to $p = 3$ and 2.5 for the specimens considered are depicted in Fig. 3.7. Only the results corresponding to the straight crack front ($\lambda = 0$) and curved crack front with $\lambda = 0.05$ and 0.09 are plotted, and results corresponding to $\lambda = 0.06$ to 0.08 are not shown to reduce clutter. Figure 3.7 indicates that $p$ has a negligible impact on $e_J$.

### 3.4 Summary and Concluding Remarks

Three-dimensional elastic-plastic FEA of plane-sided X80-steel SE(B) specimens with straight and curved crack fronts were performed to investigate the impact of the crack front curvature on the accuracy of the average $J$ over the crack front, $J^n$, evaluated using the plastic eta factor-based approach as specified in ASTM E1820-11. The models analyzed contain stationary cracks with sharp crack tips, and have three average crack lengths ($a_{ave}/W = 0.3$, 0.5 and 0.7) and three $B/W$ ratios ($B/W = 1$, 0.5 and 0.25). Symmetric bowed crack fronts with different curvatures were considered in the analysis.
The curved crack front was characterized by a power-law expression. For specimens with given \( a_{ave}/W, B/W \) and \( p (p > 1) \), the crack fronts with different levels of curvature can be generated from Eq. (2.4) and Eq. (2.5) by varying \( \lambda \).

The eta factors recommended in ASTM E1820-11 were employed to evaluate the \( CMOD \)- and \( LLD \)-based \( J^n \) for all the specimens considered, except for the \( LLD \)-based \( J^n \) for the specimens with \( a_{ave}/W = 0.3 \), in which case the eta factors proposed by Zhu et al. (2008) were adopted.

The numerical results show that given \( a_{ave}/W, J^n \) becomes less conservative and tends to overestimate the actual average \( J \), which was evaluated using the virtual crack extension method, as the crack front curvature increases. For specimens that have curved crack fronts with the crack front curvature equal to the maximum allowable value (\( \lambda = 0.05 \)) specified in ASTM E1820-11, the errors in \( J^n \) are between -7\% and 6\% for almost all the specimens considered. For the specimen with \( a_{ave}/W = 0.3, B/W = 0.25 \) and \( \lambda = 0.05 \), the \( LLD \)-based \( J^n \) overestimates the actual \( J \) by up to 10\% and 17\%, respectively, for \( n = 10 \) and 20. Results of the sensitivity analysis indicate that the value of the shape parameter of the power-law expression for the curved crack front has a negligible impact on the accuracy of \( J^n \). Based on the analysis results and the consideration that \( J^n \) overestimates the actual \( J \) by no more than 5\%, new crack front straightness criteria for the SE(B) specimen were recommended. The suggested criteria vary with \( a_{ave}/W, B/W \) and \( n \), and are in most cases less stringent than that specified in ASTM E1820-11. This could potentially lead to a decrease in the specimen rejection rate and cost savings.

**Reference**


Table 3.1: Recommended crack front straightness criteria for SE(B) specimens

<table>
<thead>
<tr>
<th></th>
<th>(a_{ave}/W)</th>
<th>(B/W = 1)</th>
<th>(B/W = 0.5)</th>
<th>(B/W = 0.25)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(n = 10)</td>
<td>(n = 20)</td>
<td>(n = 10)</td>
</tr>
<tr>
<td>(CMOD)-based</td>
<td>0.3</td>
<td>0.09^2</td>
<td>0.09^2</td>
<td>0.09^2</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>0.09</td>
<td>0.09</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>(LLD)-based</td>
<td>0.3</td>
<td>0.08</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>0.09^2</td>
<td>0.09^2</td>
<td>0.09^2</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>0.06</td>
<td>0.06</td>
<td>0.09^2</td>
</tr>
</tbody>
</table>

Notes:

1. The criteria are based on the maximum allowable \(\lambda (\lambda = \max(a_{max9} - a_{ave}, a_{ave} - a_{min9})/B)\) consistent with that of ASTM E1820-11, and the overestimation of \(J\) being no more than 5%.

2. Based on the results shown in Figs. 3.3 to 3.6, the value could be further increased with the overestimation of \(J\) still within 5%.

3. \(J^p\) corresponding to \(\lambda = 0.05\) (the maximum allowable crack front curvature in ASTM E1820-11) overestimates the actual \(J\) by slightly higher than 5%.

4. \(J^p\) corresponding to \(\lambda = 0.05\) calculated based on the non-standard \(\eta_{pl}^{LLD}\) from Eq. (3.9) significantly overestimates the actual \(J\).
Figure 3.1. Determination of the potential energy
Figure 3.2. Plastic area under the load-displacement curve
-8%
-6%
-4%
-2%
0%
2%
4%
6%
8%
10%
12%
0.6 0.8 1 1.2 1.4 1.6 1.8
Straight
0.05
0.06
0.07
0.08
0.09
increasing
/W = 0.3, B/W = 1, CMOD-based, n = 10

-8%
-6%
-4%
-2%
0%
2%
4%
6%
8%
10%
12%
0.6 0.8 1 1.2 1.4 1.6 1.8
increasing λ

-8%
-6%
-4%
-2%
0%
2%
4%
6%
8%
10%
12%
0.6 0.8 1 1.2 1.4 1.6 1.8
increasing λ

-8%
-6%
-4%
-2%
0%
2%
4%
6%
8%
10%
12%
0.6 0.8 1 1.2 1.4 1.6 1.8
increasing λ

(a)

(b)

(c)
The diagrams show the behavior of various parameters under different conditions. Each graph represents a specific combination of parameters:

- **(d)**: $a_{av}/W = 0.3, B/W = 0.5, \text{CMOD-based, } n = 10$
- **(e)**: $a_{av}/W = 0.5, B/W = 0.5, \text{CMOD-based, } n = 10$
- **(f)**: $a_{av}/W = 0.7, B/W = 0.5, \text{CMOD-based, } n = 10$

The graphs illustrate the trend of increasing $\lambda$ against the applied force ratio $P/P_F$. The straight line represents the baseline, while different lines denote various values of another parameter, typically indicating a range of variability or conditions. The diagrams help in understanding how these parameters interact under different experimental conditions.
(g) 

\( a_{ave}/W = 0.3, B/W = 0.25, \text{CMOD-based, } n = 10 \)

(h) 

\( a_{ave}/W = 0.5, B/W = 0.25, \text{CMOD-based, } n = 10 \)

increasing \( \lambda \)
Figure 3.3. Variation of the error $e_J = (J^i - J^{FEA})/J^{FEA}$ against $P/P_y$ for specimens with different values of $a_{ave}/W$, $B/W$ and crack fronts curvature $\lambda$ (CMOD-based analysis, $p = 3$ and $n = 10$)
(a) \(a_{ave}/W = 0.3, B/W = 1, LLD-based, n = 10\)

(b) \(a_{ave}/W = 0.5, B/W = 1, LLD-based, n = 10\)

(c) \(a_{ave}/W = 0.7, B/W = 1, LLD-based, n = 10\)
$d_{ave}/W = 0.3, B/W = 0.25, LLD-based, n = 10$

$\alpha_{ave}/W = 0.5, B/W = 0.25, LLD-based, n = 10$
Figure 3.4. Variation of the error $e_J = (J^l - J^{FEA})/J^{FEA}$ against $P/P_y$ for specimens with different values of $a_{ave}/W$, $B/W$ and crack fronts curvature $\lambda$ ($LLD$-based analysis, $p = 3$ and $n = 10$)
(a) $a_{ave}/W = 0.3, B/W = 1, CMOD-based, n = 20$

(b) $a_{ave}/W = 0.5, B/W = 1, CMOD-based, n = 20$

(c) $a_{ave}/W = 0.7, B/W = 1, CMOD-based, n = 20$
(d) $a_{\text{ave}}/W = 0.3, B/W = 0.5, \text{CMOD-based, } n = 20$

(e) $a_{\text{ave}}/W = 0.5, B/W = 0.5, \text{CMOD-based, } n = 20$

(f) $a_{\text{ave}}/W = 0.7, B/W = 0.5, \text{CMOD-based, } n = 20$
(g) \(a_{ave}/W = 0.3, B/W = 0.25, CMOD-based, n = 20\)

(h) \(a_{ave}/W = 0.5, B/W = 0.25, CMOD-based, n = 20\)
Figure 3.5. Variation of the error $e_J = (J^p - J^{FEA})/J^{FEA}$ against $P/P_y$ for specimens with different values of $a_{ave}/W$, $B/W$ and crack fronts curvature $\lambda$ (CMOD-based analysis, $p = 3$ and $n = 20$)
(a) $a_{ave}/W = 0.3, B/W = 1, LLD$-based, $n = 20$

(b) $a_{ave}/W = 0.5, B/W = 1, LLD$-based, $n = 20$

(c) $a_{ave}/W = 0.7, B/W = 1, LLD$-based, $n = 20$
(d) $a_{avg}/W = 0.3, B/W = 0.5, LLD-based, n = 20$

(e) $a_{avg}/W = 0.5, B/W = 0.5, LLD-based, n = 20$

(f) $a_{avg}/W = 0.7, B/W = 0.5, LLD-based, n = 20$
\( \alpha_{ave}/W = 0.3, B/W = 0.25, LLD-based, n = 20 \)

\( \alpha_{ave}/W = 0.5, B/W = 0.25, LLD-based, n = 20 \)

increasing \( \lambda \)
Figure 3.6. Variation of the error $e_J = (J^l - J^{FEA})/J^{FEA}$ against $P/P_y$ for specimens with different values of $a_{ave}/W$, $B/W$ and crack fronts curvature $\lambda$ ($LLD$-based analysis, $p = 3$ and $n = 20$)
(a) $a_{ave}/W = 0.3, B/W = 0.5, CMOD$-based, $n = 10$

(b) $a_{ave}/W = 0.7, B/W = 0.5, CMOD$-based, $n = 10$
Figure 3.7. Impact of the shape parameter $p$ ($p = 3$ and $p = 2.5$) on $e_j$
Chapter 4  Accuracy of the Double Clip-on Gauge Method for Evaluating CTOD from Single-edge Tension Specimens

4.1  Introduction

4.1.1  Experimental Determination of CTOD

The crack tip opening displacement (CTOD) is a widely used parameter for characterizing the fracture toughness of ductile materials and a key input in the integrity assessment of structures containing flaws. For instance, the CTOD concept could be used directly to calculate the tolerable flaw sizes or as a CTOD design curve to determine the allowable crack sizes in welded structures (Burdekin and Dawes, 1971; BSI, 2005). In the strain-based design of energy pipelines, CTOD can be employed to evaluate the tensile strain capacity of the pipeline (Fairchild et al., 2012).

There are currently two main approaches to determine CTOD experimentally from small-scale specimens. The first approach is based on the fact that CTOD (δ) can be uniquely related to the J-integral (J) (Shih, 1981; Anderson, 2005) as \( \delta = J/m\sigma_0 \), where \( m \) is the so-called (dimensionless) constraint parameter (Shih, 1981; Kirk and Dodds, 1993), and \( \sigma_0 \) is a reference stress (e.g. yield strength) of the material. The above J-CTOD is widely used to determine CTOD indirectly from J using both the standard single-edge bend (SE(B) and compact tension (C(T))) specimens (ASTM, 2008; ASTM, 2011) as well as the non-standard single-edge tension (SE(T)) specimens (Shen and Tyson, 2009a; Ruggieri, 2012). The value of J can be evaluated from the experimentally measured load-displacement curve through a plastic geometry factor, \( \eta_{pl} \) (see Section 3.1.1). It follows that by adopting this method the accuracy of the determined CTOD is largely governed by the accuracy of the two dimensionless parameters, i.e. \( m \) and \( \eta_{pl} \), both of which are functions of the specimen geometry and material properties (ASTM, 2008; Shen and Tyson, 2009a; Moreira and Donato, 2010; ASTM, 2011; Ruggieri, 2012; DNV, 2013).

Compared with the indirect method, the plastic component of CTOD can be determined directly from the measured crack mouth opening displacement (CMOD) by
employing a plastic hinge model assuming two halves of the specimen rotate rigidly
about a rotational center (i.e. plastic hinge) during tests (see Fig. 4.1), as specified in BS
7448 (BSI, 1991) and ISO 12135 (ISO, 2002) for bend specimens. The \( CTOD \) values can
then be determined from the following equation:

\[
\delta_{BS} = \delta_{el} + \delta_{pl} = \frac{K^2(1-\nu^2)}{2E\sigma_Y} \frac{r_p(W-a_0)V_{pl}}{r_p(W-a_0)+a_0+z} \quad (4.1)
\]

where \( \delta_{BS} \) is \( CTOD \) determined according to BS 7448; \( \delta_{el} \) is the elastic component of
\( CTOD \), which can be calculated from the stress intensity factor, \( K \); \( \delta_{pl} \) denotes the plastic
component of \( CTOD \) and is evaluated from the plastic component of the measured
\( CMOD \), \( V_{pl} \), by assuming a plastic hinge model as illustrated in Fig. 4.1, and \( W, a_0, \nu, E \)
and \( \sigma_Y \) are the specimen width, original crack length, Poisson’s ratio, Young’s modulus
and the yield strength of the material, respectively. The parameter \( r_p \) in Fig. 4.1 is a
dimensionless rotation factor. Wu et al. (1988a; 1988b) employed the limit load analysis
to derive \( r_p \approx 0.44 \) for deeply-cracked SE(B) specimens and \( r_p \approx 0.46 \) for C(T) specimens.
The values of \( r_p \) adopted by BS 7448 for SE(B) and C(T) specimens with \( a_0/W = 0.45 - 0.55 \)
are 0.4 and 0.46, respectively. However, similar to \( m \) and \( \eta_{pl} \), \( r_p \) is usually load-,
geometry- and material-dependent (Donato and Ruggieri, 2006; Cravero and Ruggieri,
2007).

The double clip-on gauge (DCG) method (Deng et al., 1980; Willoughby and
Garwood, 1983) was first proposed in the 1980s as an alternative method to experimentally determine \( CTOD \). As shown in Fig. 4.2, a pair of specially-designed
knife edges are used to adapt two clip-on gauges at different heights above the specimen
surface, which can simultaneously measure two values of \( CMOD \) at the two heights. Based on certain simplifying assumptions, \( CTOD \) can then be related to the two directly measured \( CMOD \) values through a simple geometric relationship, and the details will be
presented in Section 4.2. The advantage of the DCG method is that it is based on the
physical deformation of the crack tip and simple geometric relationship, and does not
involve the evaluation of \( J \) or assumption of the location of the plastic hinge as required
in the plastic hinge model.
4.1.2 Single-edge Tension Specimen

Previous studies (e.g. Brocks and Schmitt, 1995) have shown that the fracture toughness resistance curve (i.e. J-R and CTOD-R curves) determined from small-scale test specimens are dependent on the specimen geometric configurations and/or the type of loading (e.g. bending or tension) applied to the specimen. This phenomenon is largely due to the so-called crack-tip constraint effect, which is a measure of the resistance to the plastic flow (Brocks and Schmitt, 1995). Generally, a high level of the crack-tip constraint results in a low resistance curve because a high level of constraint restricts the plastic deformation and associated energy dissipation in the vicinity of the crack tip and therefore lowers the resistance to fracture (Brocks and Schmitt, 1995; Kim et al., 2004). The standard fracture toughness test specimens such as those specified in ASTM E1820-11 and BS 7448 are deeply-cracked, predominantly bend-loaded specimens with high crack-tip constraint levels such that conservative measurements of fracture toughness are obtained from such specimens.

Figure 4.3 schematically shows four typical toughness resistance curves obtained from different types of test specimens including C(T), deeply- (i.e. \( a/W \geq 0.45 \)) and shallow-cracked (i.e. \( a/W < 0.45 \)) SE(B) and SE(T) specimens, where \( a \) and \( W \) are the crack length and specimen width, respectively. Due to the similar loading conditions and crack-tip constraint levels between the SE(T) specimen and the full-scale pipeline containing surface cracks under longitudinal tension (Chiesa et al., 2001; Kibey et al., 2009; Shen and Tyson, 2009a; Kibey et al., 2010; Moore and Pisarski, 2012), there are increasing interests in using the non-standard SE(T) specimen to determine the toughness resistance curve in the pipeline industry over the last decade. The growing interests in the SE(T) specimen are the main motivation for the present study. There are two types of SE(T) specimens: pin-ended and clamp-ended specimens. The latter is considered more relevant than the former to the full-scale pipe in terms of the crack-tip stress and strain fields (Shen et al., 2008).
4.1.3 Literature Review

Tang et al. (2010) examined the applicability of the DCG method for SE(T) specimens by carrying out two-dimensional (2D) plane strain finite element analyses (FEA). The crack propagation was simulated in the analysis. They reported that the CTOD values obtained from the DCG method agree well with the corresponding values directly obtained from FEA. Moore and Pisarski (2012) investigated the accuracy of the DCG method experimentally by comparing the CTOD values obtained from DCG with those measured from the specimen notch replicas. They reported that the CTOD values measured using DCG agree well with the physical measurements taken from the notch replicas with errors being less than 10% if a/W is in the range of 0.3 to 0.5. Note that in Moore and Pisarski’s study only the CTOD values corresponding to the last loading step in the experiment were examined. Note further that in the aforementioned two studies, CTOD was defined as the opening length of the original crack tip before blunting, which is different from the commonly used 90º intersect definition of CTOD (Shih, 1981). These two CTOD definitions are schematically illustrated in Fig. 4.4.

4.1.4 Objective and Approach

In the study reported in this chapter, we aimed to investigate the impact of the specimen thickness-to-width ratio, crack length, side-grooving, stain hardening characteristics of the material and loading level on the accuracy of the DCG method for the clamped SE(T) specimen. To this end, systematic three-dimensional (3D) finite element analyses of clamped SE(T) specimens with wide ranges of the crack length and thickness-to-width ratio were carried out. The commonly used 90º intersect definition of CTOD was adopted in this study and is denoted by CTOD$_{90}$ or $\delta_{90}$. The geometric relationship in the vicinity of the deformed crack tip that is key to the DCG method was examined. Based on the analysis results, the existing equation for evaluating CTOD based on the DCG method was slightly modified to improve the accuracy of the CTOD evaluation. This study will facilitate the application of the fracture toughness determined from the SE(T) specimen in the strain-based design of pipelines.
The rest of this chapter is organized as follows. A brief illustration of the DCG method for evaluating \( CTOD_{90} \) is included in Section 4.2; the 3D FEA models and analysis procedures are described in Section 4.3; Section 4.4 shows the analysis results and modification of the existing DCG-based equation for evaluating \( CTOD_{90} \), and the summary and concluding remarks are presented in Section 4.5.

### 4.2 \( CTOD \) Measured from Double Clip-on Gauge Method

A detailed geometry near the crack tip in the context of the double clip-on gauge method was developed and shown in Fig. 4.5, where point \( O \) is the deformed crack tip and \( OB \) is the 45° interception line used to determine \( CTOD_{90} \). The following equation can be derived from the geometric relationships shown in Fig. 4.5 to evaluate \( CTOD \) considering similar triangles between \( BEF \) and \( FHG \):

\[
\begin{align*}
\delta_{DC90} &= \frac{V_1 - 2(a_0 + z_1) \sin \theta}{1 - \sqrt{2} \sin \theta \cos(45^\circ - \theta)} \\
\sin \theta &= \frac{V_2 - V_1}{2(z_2 - z_1)}
\end{align*}
\]  

(4.2)

where \( V_1 \) and \( V_2 \) are the two measured crack mouth opening displacements corresponding to two different knife edge heights \( z_1 \) and \( z_2 \), respectively (see Figs. 4.2 and 4.5); \( a_0 \) is the initial crack length, and \( \delta_{DC90} \) denotes the \( CTOD \) value obtained from the double clip-on gauge method here. Generally the angle \( \theta \) is small such that Eq. (4.2) can be simplified as:

\[
\delta_{DC90} = V_1 - \frac{(a_0 + z_1)(V_2 - V_1)}{z_2 - z_1}
\]  

(4.3)

Equation (4.3) is the same as that used by Tang et al. (2010), although they adopted a different definition of \( CTOD \).

In a previous (2000) version of DNV-OS-F101 (2000), \( \delta_{DC90} \) is separated into an elastic component and a plastic component as follows:

\[
\delta_{DC90} = \frac{(1-v^2)K^2}{2E\sigma_Y} + V_{pl1} - \frac{(a_0 + z_1)(V_{pl2} - V_{pl1})}{z_2 - z_1}
\]  

(4.4)
where the first term on the right hand side of Eq. (4.4) is the elastic component of $\delta_{DC90}$ and evaluated from the stress intensity factor, $K$; the evaluation of $K$ is well documented for SE(T) specimens (e.g. Tada et al., 1973; Shen and Tyson, 2009b); the second and third terms on the right hand side of Eq. (4.4) are the plastic component of $\delta_{DC90}$, and $V_{pl1}$ and $V_{pl2}$ are the plastic components of the two measured CMOD values. The accuracy of both Eqs. (4.2) and (4.4) was examined in this study.

4.3 Finite Element Analyses

4.3.1 Finite Element Model

The commercial software package ADINA 8.7.4 (ADINA, 2012) was used to carry out the 3D finite element analyses. Both plane-sided and side-grooved clamped SE(T) specimens were modeled in this study. The plane-sided specimens were considered as baseline cases, whereas the side-grooved specimens were employed to investigate the impact of the side grooves on the analysis. All the specimens considered in this study have the same width ($W = 20$ mm) and daylight length ($H = 10W$) (see Fig. 2.1d) (DNV, 2006; Shen et al., 2008; Shen and Tyson, 2009a), but five different relative crack lengths, i.e. $a/W = 0.3$ to 0.7 with an increment of 0.1, and three different thickness-to-width ratios, i.e. $B/W = 0.5, 1$ and 2, where $B$ is the specimen thickness. For side-grooved specimens, the depth of the side groove on each side was selected to be $7.5\%B$ (Shen et al. 2010).

Because of symmetry, only a quarter of the specimen was modeled using 8-node 3D isoparametric brick elements (ADINA, 2012). The model was divided into 17 and 25 layers in the thickness direction for plane-sided and side-grooved models, respectively, with the mesh density increasing from the mid-plane to the free surface to capture the high stress gradients near the free surface. The side groove is modelled as a sharp V-notch with an opening angle of 45°. The geometric and mesh configurations for a typical specimen are shown in Fig. 4.6, and a side-grooved model with $a/W = 0.5$ and $B/W = 1$ is schematically shown in Fig. 4.7.
The $J_2$ incremental theory of plasticity (Lubliner, 2008) and large-displacement large (finite) strain formulation (Anderson, 2005; ADINA, 2012) were employed in FEA. In ADINA, the large-displacement large-strain formulation requires input of the Cauchy (true) stress-logarithmic (true) strain relationship and outputs the Cauchy stress and deformation gradient. A blunt crack tip with initial radii ($\rho_0$) of 2.5, 5 and 10 µm was incorporated in the model to simulate the crack-tip blunting during the loading process and facilitate convergence of the large-displacement large (finite) strain analysis (McMeeking and Parks, 1979; Dodds, 2009; ADINA, 2012). The first value of $\rho_0$ is the baseline case that applies to all the specimens considered, whereas the latter two values were employed for selected geometric configurations only (i.e. $B/W = 1$ and $a/W = 0.5$) to investigate the impact of the initial crack tip radius on the accuracy of the DCG method. Note that for the side-grooved specimens, the blunt crack tip is also prepared through the thickness of the side groove as shown in Fig. 4.7 to mitigate the impact of the singularity caused by the 45° sharp V-notch under tension on the finite strain analysis. The mesh surrounding the crack tip consists of 40 concentric semicircles. In the vicinity of the crack tip, the minimum in-plane size of the elements closest to the crack tip is about 1/10 of the crack-tip radius (Qian and Dodds, 2006; Graba and Galkiewicz, 2007), whereas the in-plane size of the elements in the outermost ring (i.e. 40th ring) is about 2,000 times that of the element closest to the crack tip (Dodds, 2009). The aspect ratio of these elements is less than 7. The total number of elements is approximately 32,000 for a typical plane-sided model and 50,000 for a side-grooved model. Stationary cracks were assumed in the present analysis. Convergence studies on mesh density, e.g. refine the mesh surrounding the crack tip or separate the specimen into more layers, were conducted and showed good convergence in this elastic-plastic analyses.

### 4.3.2 Material Model

The uniaxial stress-strain relationship of the material is described using an elastic-power-law plastic expression as follows:

$$\frac{\varepsilon}{\varepsilon_0} = \begin{cases} \frac{\sigma}{\sigma_0}, & \varepsilon \leq \varepsilon_0 \\ \left(\frac{\sigma}{\sigma_0}\right)^n, & \varepsilon > \varepsilon_0 \end{cases}$$

(4.5)
where $\sigma_0$ is the reference stress; $\varepsilon_0$ is the reference strain, $\varepsilon_0 = \sigma_0/E$; $n$ denotes the strain hardening exponent of the material. In this study $\sigma_0 = \sigma_{YS} = 520$ MPa, $E = 200$ GPa was selected. Three values of $n$, namely $n = 10$, 15 and 20, were considered in this study to investigate the effect of strain hardening exponent on the evaluated $CTOD$. The isotropic hardening rule and associated flow rule were employed in the analysis.

### 4.3.3 Computational Procedure

All the specimens were loaded by a displacement-controlled load up to the level corresponding to large plastic deformations, i.e. $P/P_y = 1.3$ (Shen et al., 2009; Wang et al., 2012), through about 5,000 steps, where $P$ is the applied load, and $P_y$ is the reference load defined as $B(W - a)\sigma_Y$ (Shen and Tyson, 2009a). Note that $\sigma_Y$ is the effective yield strength, defined as $\sigma_Y = (\sigma_{YS} + \sigma_{TS})/2$, where $\sigma_{TS}$ is the ultimate tensile strength. Applying Considere’s necking criterion (Soboyejo, 2003) to Eq. (4.5), one can derive the following equation to evaluate $\sigma_{TS}$:

$$
\sigma_{TS} = \left( \frac{E\sigma_0 n - 1}{ne} \right)^{1/n} \quad (4.6)
$$

where $e = 2.71828$ is the base of the natural logarithm. Equation (4.6) implies that the yield-to-tensile ratio of the material could be expressed as follows assuming $\sigma_0 = \sigma_{YS}$:

$$
\frac{\sigma_{YS}}{\sigma_{TS}} = \left( \frac{ne\sigma_{YS}}{E} \right)^{1/n} \quad (4.7)
$$

For material with $n = 10$, 15 and 20, the corresponding yield-to-tensile ratios are 0.77, 0.86 and 0.91, respectively. The sparse matrix solver was employed for its high efficiency in the numerical analysis (ADINA, 2012). The full Newton-Raphson iteration method was selected to find the solution of nonlinear equations with the maximum number of iterations for each step being 15. The displacement convergence criterion was selected, in which the displacement tolerance equaled 0.001 corresponding to a reference displacement of 1 mm (ADINA, 2012).

As illustrated in Fig. 4.8, due to symmetry, $CTOD_{90}/2$ was evaluated based on the intercept between a straight line at 45° originating from the crack tip in the deformed
position and the deformed crack flank at the mid-plane of the specimen. The interception point was captured using a linear interpolation between two nearest deformed nodes on the deformed flank given the corresponding nodal displacements (Tracey, 1976; Shen and Tyson, 2009a; Ruggieri, 2012). The value of CTOD obtained from FEA based on this approach is denoted by $\delta_{FE90}$ and considered as the true value of CTOD in this study.

As shown in Fig. 4.9, the two measured CMODs, i.e. $V_1$ and $V_2$, are calculated from the nodal displacements of the two outermost nodes on the deformed crack flank at the mid-plane of the specimen in FEA, i.e. points $M$ and $N$, which are corresponding to the two knife edge heights of zero and $-z_2$. Therefore, Eqs. (4.2) and (4.4) can be employed to calculate the double clip-on gauge measured CTOD, $\delta_{DC90}$, according to the FEA results. Several other positions of point $N$ (shown in Fig. 4.9 as $N_i$, $i = 2, 3, 4, 5$) have been analyzed to investigate the impact of the position of point $N$ on the value of $\delta_{DC90}$. The analysis results indicate that $\delta_{DC90}$ is insensitive to the position of point $N$.

### 4.4 Results and Discussions

Let $e_1 = (\delta_{DC90} - \delta_{FE90})/\delta_{FE90}$ denote the error of $\delta_{DC90}$ evaluated using Eq. (4.4). The values of $e_1$ are plotted against $P/Py$ for plane-sided specimens with the same $B/W$ ratio but different $a/W$ ratios in Fig. 4.10 and for plane-sided specimens with the same $a/W$ ratio but different $B/W$ ratios in Fig. 4.11. The figures suggest that once $P/Py$ exceeds 0.3, $\delta_{DC90}$ can markedly overestimate $\delta_{FE90}$, and when the applied load reaches around $0.9Py$, the error reaches a peak value of up to 40%. The specimen $B/W$ ratio has a negligible impact on $e_1$. The values of $e_1$ for specimens with the same $a/W$ and $B/W$ but different $n$ values are shown in Fig. 4.12, which suggests that $n$ has a large impact on $e_1$. Given $a/W$, $B/W$ and $P/Py$, $e_1$ decreases as $n$ increases from 10 to 20. The errors associated with $\delta_{DC90}$ evaluated using Eq. (4.2) (i.e. without separating CTOD into the elastic and plastic components) are also evaluated and it is observed that the error is significantly higher than that associated with $\delta_{DC90}$ evaluated using Eq. (4.4), with the peak value of $e_1$ reaching as high as 100%.

The geometric relationship in the vicinity of the deformed crack tip was examined. The main reason attributing to the error in $\delta_{DC90}$ is that the idealized geometric
relationship as shown in Fig. 4.5 does not always hold in real situations. Figure 4.13(a) schematically illustrates the geometric relationship in the vicinity of the blunt crack tip according to the FEA results, which indicates that the intersection point between the 45° line from the crack tip and deformed crack flank, i.e. point \(D\), is not on the extension of the straight line that connects the two outermost nodes on the deformed crack flank in FEA, i.e. points \(M\) and \(N\). In other words, the assumption that the intersection point \(D\) is collinear with points \(M\) and \(N\) as involved in the DCG method (see Fig. 4.5) does not hold in real conditions. Figure 4.13(a) also clearly shows the relationship between the true \(\text{CTOD}_{90}\), \(\delta_{FE90}\), and the \(\text{CTOD}\) value evaluated using the DCG method. The above observation suggests that although the DCG method is more advantageous than the single clip-on gauge method by avoiding the assumption of the plastic hinge location, the accuracy of the DCG method can be further improved. The relatively large errors associated with the DCG method as reflected in Figs. 4.10 through 4.12 are in contrast to the results reported by Moore and Pisarski (2012), which indicates that the accuracy of the DCG method is within ±10%. Note that the \(\text{CTOD}\) definitions adopted in this study and by Moore and Pisarski (2012) are different; therefore, the difference between the accuracy of the DCG method reported in the two studies suggests that the accuracy is sensitive to the definition of \(\text{CTOD}\).

To improve the accuracy of \(\text{CTOD}\) measured using the DCG method, a correction factor, \(\mu\), as defined in Fig. 4.13(b) by setting \(MC' = \mu a_0\), was introduced to modify the initial crack length \(a_0\) used in Eq. (4.2). Equation (4.2) can then be revised as follows by considering similar triangles between \(D'EM\) and \(NFM\):

\[
\left\{ \begin{align*}
\delta_{\text{DC90}} &= \frac{V_1 - 2(\mu a_0 + z_1) \sin \theta}{1 - \sqrt{2} \sin \theta \cos(45° - \theta)} \\
\sin \theta &= \frac{V_2 - V_1}{2(z_2 - z_1)}
\end{align*} \right. \tag{4.8}
\]

where the correction factor \(\mu\) can be uniquely determined by setting \(\delta_{\text{DC90}} = \delta_{\text{FE90}}\). The values of \(\mu\) are plotted against \(P/P_y\) for clamped plane-sided SE(T) specimens with ranges of \(B/W\) and \(a/W\) ratios in Fig. 4.14. It is observed that \(\mu\) generally decreases towards unity as \(P/P_y\) or \(a/W\) increases, which means that for deeply-cracked specimens, i.e. point \(D\) in Fig. 4.13 being far away from points \(M\) and \(N\), the required correction factor, \(\mu\) is
close to unity. For specimens with \( a/W = 0.5, 0.6 \) and 0.7, the \( B/W \) ratio has a negligible impact on \( \mu \), and for specimens with \( a/W = 0.3 \) and 0.4, the maximum difference between \( \mu \) values corresponding to different \( B/W \) ratios is about 4\%. The values of \( \mu \) for side-grooved SE(T) specimens with ranges of \( B/W \) and \( a/W \) ratios are compared with the results for plane-sided specimens in Fig. 4.15, which indicates that the presence of side grooves has a negligible impact on \( \mu \) since the CTOD values are all measured at the mid-plane.

The impact of the initial blunt crack tip radius in the FEA mesh, \( \rho_0 \), on the proposed correction factor \( \mu \) is also investigated. Based on clamped SE(T) specimens with \( a/W = 0.5 \) and \( B/W = 1 \), the values of \( \mu \) corresponding to three different \( \rho_0 \) are depicted in Fig. 4.16, which shows that \( \mu \) is insensitive to \( \rho_0 \). The impact of the strain hardening exponent \( n \) on the proposed correction factor \( \mu \) is investigated based on the plane-sided specimens with \( a/W = 0.3 \) to 0.7 and \( B/W = 1 \). The values of \( \mu \) corresponding to \( n = 10, 15 \) and 20, \( a/W = 0.3 \) and 0.7 are depicted in Fig. 4.17. The results corresponding to \( a/W = 0.4 \) to 0.6 are not shown to reduce clutter. Figure 4.17 suggests that \( \mu \) depends on \( n \), especially for shallow-cracked specimens.

In summary, the correction factor is a function of the crack length, applied load level and the yield-to-tensile ratios of the material, and insensitive to the specimen thickness-to-width ratio, side-grooving and initial blunt crack tip radius in the FEA mesh. To facilitate the practical application of \( \mu \), the following empirical expression of \( \mu \) was developed based on the results obtained in this study:

\[
\mu = q_0 + q_1 \left( \frac{P}{P_y} \right) + q_2 \left( \frac{P}{P_y} \right)^2 \quad (4.9)
\]

where the fitting coefficients \( q_0, q_1 \) and \( q_2 \) are functions of \( a/W \) and \( \frac{\sigma_{YS}}{\sigma_{TS}} \) given as follows:

\[
\begin{align*}
q_0 &= \left[ 1.8847 - 0.0147 \left( \frac{\sigma_{YS}}{\sigma_{TS}} \right) \right] + \left[ -0.9516 + 0.1054 \left( \frac{\sigma_{YS}}{\sigma_{TS}} \right) \right] \left( \frac{a}{W} \right) \\
q_1 &= \left[ -0.2328 - 0.0388 \left( \frac{\sigma_{YS}}{\sigma_{TS}} \right) \right] + \left[ 0.5702 - 0.7958 \left( \frac{\sigma_{YS}}{\sigma_{TS}} \right) \right] \left( \frac{a}{W} \right) \\
q_2 &= \left[ 1.1733 - 1.5117 \left( \frac{\sigma_{YS}}{\sigma_{TS}} \right) \right] + \left[ -1.7971 + 2.5704 \left( \frac{\sigma_{YS}}{\sigma_{TS}} \right) \right] \left( \frac{a}{W} \right)
\end{align*}
\]  

(4.10)
The fitting error of the equations is generally less than 3%. The error of the DCG method by employing the modified equations, i.e. Eqs. (4.8), (4.9) and (4.10), denoted as $e_c$, is plotted against $P/P_y$ for specimens with various $a/W$, $B/W$ and $n$ in Figs. 4.18 through 4.21. These figures indicate that the modified equations can significantly improve the accuracy of $CTOD$ evaluated from the DCG method, with $e_c$ being generally within ±10%. It should be noted that Equations (4.9) and (4.10) are applicable for SE(T) specimens with $a/W$ values between 0.3 to 0.7 and the yield-to-tensile ratios of the material larger than 0.77.

4.5 Summary and Concluding Remarks

The double clip-on gauge method used to experimentally measure $CTOD$ for clamped SE(T) specimen was reviewed, and the accuracy of this method was systematically investigated by carrying out three-dimensional finite element analyses of clamped SE(T) specimens with a wide range of specimen dimensions ($a/W = 0.3$ to 0.7 with an increment of 0.1, and $B/W = 0.5$, 1 and 2). Side-grooved clamped SE(T) specimens were also modeled in this study as sensitivity cases to investigate the impact of the side grooves on the measured $CTOD$ values. The commonly-used 90 degree intersection definition of $CTOD$ (Shih, 1981) was adopted in this study as opposed to the definition used by Tang et al. (2010) and Moore and Pisarski (2012).

It is observed that the $CTOD$ values evaluated using the existing equations based on the $CMOD$ measurements obtained from the double clip-on gauges can involve significant errors. This error primarily depends on $a/W$, the material straining hardening characteristic (i.e. $n$ or equivalently $\sigma_{YS}/\sigma_{TS}$) and loading level characterized by $P/P_y$, and can be as large as 40 - 100%. The specimen $B/W$ ratio and side-grooving have a negligible impact on the error. Based on the FEA results obtained in this study, the geometric relationship surrounding the blunt crack tip was investigated and the existing DCG-based equations were modified by introducing a correction factor to the original crack length included in the equation. This correction factor was then fitted as a polynomial function of $a/W$, $\sigma_{YS}/\sigma_{TS}$ and $P/P_y$. The modified equation can significantly improve the accuracy of the $CTOD$ evaluated from the double clip-on gauges, with the
error in the estimated CTOD values being generally within ±10%. The results will facilitate the application of the CTOD values determined from the SE(T) specimen in the strain-based design of pipelines.

Reference


Dodds, R. H. (2009). *WARP3D: Notes on Solution & Convergence Parameters for a Shallow-Notch SE(B) Model*. University of Illinois at Urbana-Champaign, IL.


Figure 4.1. Schematically illustration of the geometric relationship for the evaluation of \( CTOD \) using single clip-on gauge plastic hinge model.
Figure 4.2. Illustration of the installation of knife edges for the double clip-on gauge method.
Figure 4.3. Typical toughness resistance curves for various types of small-scale specimens
(a) Displacement at the intersection of a 90 degree vertex with the crack flanks

CTOD adopted by Tang et al., 2010; Moore and Pisarski, 2012

(a) Opening displacement of the deformed crack at the original crack tip

Figure 4.4. Schematically illustration of CTOD definitions
Figure 4.5. Schematically illustration of the geometric relationship for the evaluation of CTOD

\[ OP = PB = \delta_{DC90}/2 \]

\[ CB = \frac{\sqrt{2}}{2} \delta_{DC90} \cos(45^\circ - \theta) \]

Figure 4.6. Geometric and mesh configuration of the finite element model with a blunt crack tip
Figure 4.7. A typical side-grooved finite element model for clamped SE(T) specimen
Figure 4.8. Schematically illustration of the determination of CTOD in FEA

Figure 4.9. Schematically illustration of the double clip-on gauge method in FEA
Figure 4.10. Variation of $e_1$ against $P/P_y$ for plane-sided specimens with $n = 10$ and the same $B/W$. 

(a) $B/W = 0.5$

(b) $B/W = 1$

(c) $B/W = 2$
Figure 4.11. Variation of $e_1$ against $P/P_y$ for plane-sided specimens with $n = 10$ and the same $a/W$. 

(a) $a/W = 0.3$

(b) $a/W = 0.5$

(c) $a/W = 0.7$
Figure 4.12. Variation of $e_1$ against $P/P_y$ for plane-sided specimens with $B/W = 1$ and the same $a/W$. 

(a) $a/W = 0.3$ 

(b) $a/W = 0.5$ 

(c) $a/W = 0.7$
Figure 4.13. Geometric relationship surrounding the blunt crack tip
Figure 4.14. Variation of the proposed correction factor $\mu$ against $P/P_y$ for plane-sided specimens with $n = 10$
Figure 4.15. Variation of the proposed correction factor $\mu$ against $P/P_y$ for specimens with $n = 10$. 

(a) $B/W = 0.5$

(b) $B/W = 1$

(c) $B/W = 2$
Figure 4.16. Variation of the proposed correction factor $\mu$ against $P/Py$ for plane-sided specimens with $B/W = 1$, $a/W = 0.5$, $n = 10$ and different initial radius

Figure 4.17. Variation of the proposed correction factor $\mu$ against $P/Py$ for plane-sided specimens with $B/W = 1$ and different $n$ values
Figure 4.18. Variation of $\epsilon_c$ against $P/P_y$ for plane-sided specimens with $n = 10$ and the same $B/W$.
Figure 4.19. Variation of $e_c$ against $P/Py$ for plane-sided specimens with $n = 10$ and the same $a/W$
Figure 4.20. Variation of $e_c$ against $P/P_y$ for specimens with $n = 10$, $B/W = 1$ and the same $a/W$. 

(a) $a/W = 0.3$

(b) $a/W = 0.5$

(c) $a/W = 0.7$
Figure 4.21. Variation of $e_c$ against $P/P_y$ for plane-sided specimens with $B/W = 1$ and the same $a/W$. 
Chapter 5  Summary and Conclusions

The fracture toughness is a key input for the structural integrity assessment and strain-based design of steel energy pipelines containing planar defects (i.e. cracks). For modern ductile pipeline steels, the fracture process is usually accompanied with relatively large plastic deformation at the crack tip and considerable crack extension. As the crack grows, a plastic zone at the crack tip increases in size and the driving force must increase accordingly to maintain the crack growth. Therefore, the fracture toughness resistance curve (e.g. J-integral ($J$) or crack tip opening displacement ($CTOD$) resistance curve) is typically used to characterize the fracture property of the pipeline steels. There are two main components of the toughness resistance curves, namely the crack growth, $\Delta a$, and the toughness value ($J$ or $CTOD$) corresponding to this particular crack growth.

The $J$-$R$ and $CTOD$-$R$ curves are commonly measured on small-scale specimens, e.g. the three-point single-edge notched bend (SE(B)), compact tension (C(T)) and single-edge notched tension (SE(T)) specimens. During tests, the applied load ($P$), the corresponding load-line displacement ($LLD$) and the crack mouth opening displacement ($CMOD$) need to be measured simultaneously. The obtained $P$-$LLD$ and $P$-$CMOD$ curves are the main input of the experimental evaluation of $J$-$R$ or $CTOD$-$R$ curves.

5.1 Effect of the Crack Front Curvature on the $CMOD$ Compliance and $J$ for SE(B) Specimens

As specified in test standards such as ASTM E1820-11, all machine notched specimens need to be fatigue pre-cracked to simulate natural sharp cracks before the resistance curve testing. The fatigue pre-cracking often introduces curved as opposed to straight crack fronts. Furthermore, the crack growth during the test is in general non-uniform across the crack front.

Steenkamp investigated the influence of the crack front curvature on the specimen compliance using two-dimensional (2D) plane strain finite element analyses based on seven specimens with the same average crack length but different crack front curvatures.
The previous studies, e.g. Crouch, Nikishkov et al., Zhou and Soboyejo, all focused on the effects of the crack front curvature on the local crack driving forces, and the crack front straightness is generally defined based on the difference between the maximum and the minimum crack length, which is not constant with the crack front straightness criterion specified in ASTM E1820-11. In the present study, the main focus is on the effect of the crack front curvature on the average J-integral over the crack front, which is considered to be more relevant to the characterization of the experimentally determined J-R curve. Symmetric bowed crack fronts with different curvatures were considered in the analysis. The crack front curvature was characterized by a parameter $\lambda$, which is consistent with the crack front straightness criterion specified in ASTM E1820-11. The larger is $\lambda$, the higher is the crack front curvature. The maximum allowable crack front curvature according to ASTM E1820-11 corresponds to $\lambda = 0.05$. A power-law expression as proposed by Nikishikov et al. was adopted to characterize the curved crack front, and it was validated using crack fronts data collected in this study. Two values of shape parameter of the power-law expression for the curved crack front, namely $p = 3$ and 2.5, were considered to investigate its influence on the results. For specimens with given $a_{ave}/W$, $B/W$ and $p$, the crack fronts with different levels of curvature were generated by varying $\lambda$.

The impact of the crack front curvature on the J-R curve experimentally measured from the SE(B) specimen was investigated through systematic linear-elastic and elastic-plastic three-dimensional (3D) finite element analyses (FEA) of plane-sided SE(B) specimens containing both straight and curved crack fronts. A wide range of average crack lengths ($a_{ave}/W = 0.3, 0.5$ and 0.7) and thickness-to-width ratios ($B/W = 1, 0.5$ and 0.25 covering the range specified in ASTM E1820-11) was included.

Linear elastic 3D FEA was employed to investigate the impact of the crack front curvature on the CMOD compliance and the evaluated crack length. The CMOD compliance value for the specimen with a curved crack front was compared with the value obtained from a specimen with a straight crack front and having the same average crack length and thickness. For a given specimen with either a straight or curved crack front, the accuracy of the crack length predicted from the CMOD compliance using the
empirical equations reported in the literature was examined. The use of three different elastic moduli, i.e. $E$, $E'$ and $E_e$, in the prediction of the average crack length from the CMOD compliance was compared.

The numerical results show that the crack front curvature has a negligible impact on the CMOD compliance for $\lambda \leq 0.05$ for the SE(B) specimen, and the crack length predicted from the CMOD compliance is insensitive to the crack front curvature for all the considered $\lambda$ values (up to $\lambda = 0.10$) regardless of $a_{ave}/W$ and $B/W$ ratios. It is observed that the use of $E_e$ as reported by Wang et al. can lead to the most accurate prediction of the average crack length with errors being less than 1% for wide ranges of $a_{ave}/W$, $B/W$ and crack front curvatures for SE(B) specimens.

The Ramberg-Osgood constitutive law was employed as the uniaxial stress-strain relationship for investigating the impact of the crack front curvature on the accuracy of the average $J$ over the crack front, $J^p$, evaluated using the plastic eta factor-based approach as specified in ASTM E1820-11. The plastic geometry factors recommended in ASTM E1820-11 were employed to evaluate the CMOD- and LLD-based $J^p$ for all the specimens considered, except for the LLD-based $J^p$ for the specimens with $a_{ave}/W = 0.3$, in which case the plastic geometry factors proposed by Zhu et al. were adopted. Two values of the strain hardening exponent, namely $n = 10$ and 20, were considered to investigate the influence of $n$ on the $J$ evaluation.

The numerical results show that given $a_{ave}/W$ and $B/W$, as the crack front curvature increases, $J^p$ becomes less conservative and tends to overestimate the actual average $J$. For specimens that have curved crack fronts with the crack front curvature equal to the maximum allowable value ($\lambda = 0.05$) as specified in ASTM E1820-11, the errors in $J^p$ are between -7% and 6% for almost all the specimens considered in this study. Results of the sensitivity analysis indicate that the choice of the value of $p$ has a negligible impact on the accuracy of $J^p$. New crack front straightness criteria for the SE(B) specimen were recommended by considering that $J^p$ overestimates the actual $J$ by no more than 5%. The suggested criteria vary with $a_{ave}/W$, $B/W$ and $n$, and are in most cases less stringent than
those specified in ASTM E1820-11, which could potentially lead to a decrease in the specimen rejection rate and cost savings.

5.2 Accuracy of the Double Clip-on Gauge Method for Evaluating CTOD of SE(T) Specimens

Due to the similar crack-tip constraint levels between the single-edge tension (SE(T)) specimen and full-scale pipes containing surface cracks under longitudinal tension, there is an increasing trend to determine the toughness resistance (e.g. CTOD-R) curve using the SE(T) specimen in the pipeline industry over the last decade. The use of the double clip-on gauge method to experimentally measure CTOD was reviewed, and its accuracy was examined through systematic 3D FEA of clamped SE(T) specimens with a wide range of specimen dimensions ($a/W = 0.3$ to $0.7$ with an increment of $0.1$, and $B/W = 0.5$, $1$ and $2$). The commonly-used 90 degree intersection definition of CTOD was adopted in this study as opposite to the definition used by Tang et al. and Moore and Pisarski. Side-grooved SE(T) specimens were also modeled in this study as sensitivity cases to investigate the impact of the presence of side grooves on the measured CTOD values. A power-law constitutive relationship was assumed, and three values of strain hardening exponent, namely $n = 10$, $15$ and $20$, were considered to investigate the influence of $n$ on the accuracy of the double-clip on gauge method.

It is observed that the CTOD values determined from the double clip-on gauge method may involve errors with as large as $40 - 100\%$. This error primarily depends on the crack length, the material property and the loading level. The primary attributing factor to the error was identified by examining the geometry surrounding the blunt crack tip. Based on the investigation, a modified CTOD evaluation equation was developed to improve the accuracy of CTOD evaluated using the double-clip on gauge method. The error in CTOD evaluated from the modified double-clip on gauge method is generally within $10\%$. The results will facilitate the application of the double clip-on gauge method to experimentally determine CTOD values.
5.3 Recommendations for Future Work

Recommendations for future work are as follows:

1. Further studies can be carried out to investigate the impact of the crack front curvature on the experimentally determined $J-R$ curve for SE(B) specimens based on the crack front straightness criteria specified in the other widely used test standards such as BS 7448 and ISO 12135, and compare the adequacy of these criteria with those specified in ASTM E1820-11.

2. The impact of the crack front curvature on the experimentally determined $J$ or $CTOD$, for the non-standard SE(T) specimen can be investigated, which will facilitate the standardization of the SE(T) specimen-based fracture toughness testing procedure.

3. For the investigation on the accuracy of double clip-on gauge method, experimental studies can be carried out to further validate the numerical results obtained in the present study.
Appendix A  Derivation of the Relationship between $\beta$ and $\lambda$ for Symmetric Bowed Crack Fronts

Consistent with the crack front straightness criterion in ASTM E1820-11, the parameter $\lambda$ is defined as

$$\lambda = \max(a_{\text{max}9} - a_{\text{ave}}, a_{\text{ave}} - a_{\text{min}9})/B$$  \hspace{1cm} (A.1)$$

where $a_{\text{max}9}$ and $a_{\text{min}9}$ are the maximum and minimum values of the nine physical measurements of the crack length. For specimens with symmetric bowed crack fronts characterized by Eq. (2.3), it follows that (see Fig. 2.3)

$$a_{\text{max}9} = a_5 = a(0)$$  \hspace{1cm} (A.2)$$

$$a_{\text{min}9} = a_1 = a(0) - 4^p \cdot \beta W \left(0.25 - \frac{\Delta}{2B}\right)^p$$ \hspace{1cm} (A.3)$$

where $\Delta = 0.005W$.

The average crack length $a_{\text{ave}}$ is then given by

$$a_{\text{ave}} = a(0) - \beta W \cdot \left(0.25 - \frac{\Delta}{2B}\right)^p \cdot \frac{1}{8} \sum_{i=1}^{8} \left|\text{abs}(i - 5)\right|^p$$ \hspace{1cm} (A.4)$$

The following equation can be derived from Eqs. (A.2), (A.3) and (A.4):

$$(a_{\text{max}9} - a_{\text{ave}}) - (a_{\text{ave}} - a_{\text{min}9}) = \frac{\beta W}{2} \left(0.25 - \frac{\Delta}{2B}\right)^p \left(1 + 2^p + 3^p - \frac{3}{2} \cdot 4^p\right)$$ \hspace{1cm} (A.5)$$

Given that the crack front is symmetric and bowed, the value of $p$ is greater than unity as illustrated in Fig. 2.6. Substituting $p = 1$ into Eq. (A.5) leads to $(a_{\text{max}9} - a_{\text{ave}}) - (a_{\text{ave}} - a_{\text{min}9}) = 0$. For $p > 1$, the derivative of the right hand side of Eq. (A.5) with respect to $p$ is always negative. Therefore for $p > 1$, $(a_{\text{max}9} - a_{\text{ave}}) - (a_{\text{ave}} - a_{\text{min}9}) < 0$, which implies that $\lambda = (a_{\text{ave}} - a_{\text{min}9})/B$ for specimens with symmetric bowed crack fronts. Substituting Eqs. (A.3) and (A.4) into $\lambda = (a_{\text{ave}} - a_{\text{min}9})/B$ results in

$$\lambda = \frac{\beta W}{B} \left(0.25 - \frac{\Delta}{2B}\right)^p \left[4^p - \frac{1}{8} \sum_{i=1}^{8} \left|\text{abs}(i - 5)\right|^p\right]$$ \hspace{1cm} (A.6)$$
Equation (A.6) defines a one-to-one relationship between $\beta$ and $\lambda$ for specimens with symmetric bowed crack fronts.
Appendix B  Computation of J-integral using Virtual Crack Extension Method

The virtual crack extension method was first proposed by Parks (1974, 1977) and Hellen (1975) independently during 1970s, which could be employed to calculate the energy release rate in finite element analysis (FEA). Several years later, deLorenzi (1982, 1985) improved this method by considering the energy release rate of a continuum, which is implemented in ADINA (ADINA, 2012) and briefly introduced here.

Figure B.1 schematically illustrates a virtual crack advance in a two-dimensional (2D) continuum. The crack front is surrounded by three zones of material separated by two contours. During the virtual crack extension process, known as virtual shift in ADINA (ADINA, 2012), zone I shifted rigidly by an amount Δx₁ in the x₁ direction, at the meantime material in zone III remain fixed, causing a distortion in the material in zone II. Since the crack front is contained in zone I, the crack length could increase accordingly, denoted as Δa. For the material that obeys deformation plasticity, deLorenzi (1982, 1985) showed that the energy release rate can be expressed as:

\[
J = \frac{1}{\Delta a} \int_{A_C} \left( \sigma_{ij} \frac{\partial u_i}{\partial x_1} - w \delta_{i1} \right) \frac{\partial \Delta x_1}{\partial x_1} dA_C
\]

where \( \sigma_{ij} \) is the stress tensor; \( u_i \) is the components of the displacement \( (i = 1, 2 \) for two dimensional analysis); \( w \) denotes the strain energy density; \( \delta_{ij} \) is the Kronecker delta and \( A_C \) is the area of the cracked body.

A more general expression of Eq. (B.1), i.e. Eq. (B.2), was derived by deLorenzi (1982, 1985) to evaluate \( J \) using virtual crack extension method, which considers a three-dimensional (3D) body and adopted by the commercial software ADINA (ADINA, 2012).

\[
J = \frac{1}{\Delta A_C} \int_{V_C} \left( \sigma_{ij} \frac{\partial u_i}{\partial x_k} - w \delta_{ik} \right) \frac{\partial \Delta x_k}{\partial x_j} dV_C
\]
where \( V_C \) denotes volume of the cracked body; \( \Delta x_k \) is components of the virtual crack extension vector \((k = 1, 2 \text{ or } 3)\), and \( \Delta A_C \) is the increase in crack area corresponding to \( \Delta x_k \). The calculation of \( \Delta A_C \) depends on whether the geometry is 2D or 3D. For a 2D geometry, \( \Delta A_C = b\sqrt{\Delta x_1^2 + \Delta x_2^2} \) where \( b \) is the thickness at the crack tip. For a 3D geometry, the geometric relationship after virtual shift is schematically illustrated in Fig. B.2, and \( \Delta A_C \) could be written as

\[
\Delta A_C = \int \sqrt{\Delta x'_i \Delta x'_j} \, ds,
\]

where \( \Delta x'_i = \Delta x_i - (\sum_{j=1}^{3} t_j \Delta x_j) t_i \); \( t_i \) \((i = 1, 2 \text{ or } 3)\) is the component or directional cosine of the unit tangent vector along the crack front and \( \Delta x'_i \) is always perpendicular to \( t_i \); \( ds \) is the differential length along the crack front. For a real 3D problem, \( J \) typically varies along the crack front. By assuming \( \Delta A_C \) incrementally along the crack front, as shown in Fig. B.2(b), it would result in a local \( J \) value at that particular point along the crack front (Anderson, 2005). In finite element analysis, this approach will provide a local \( J \) value corresponding to each layer along the thickness direction in the FEA model.

By adopting the virtual crack extension method, the contour and surface integrations for two- and three- dimensional problems are converted into an area integration and a volume integration, respectively, which could improve the accuracy of the numerical study significantly. It is noted that Eq. (B.2) is just the basic expression for \( J \) calculation and the impacts of hoop stress, thermal effect, and dynamic effect are not considered (ADINA, 2012). More details about the virtual crack extension method could be found in the literature, e.g. deLorenzi (1982 and 1985), Anderson (2005) and ADINA (2012).

**Reference**


Figure B.1. The virtual crack extension method in two-dimensional analysis

(a) Before virtual shift

(b) After virtual shift

I: zone rigidly shifted by virtual shift
II: zone distorted by virtual shift
III: zone unchanged by virtual shift
Figure B.2. The virtual crack extension method in three-dimensional analysis
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