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Job Matching And Unemployment Dynamics

John Robert Kennes

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JOB MATCHING AND UNEMPLOYMENT DYNAMICS

by

John Kennes

Department of Economics

Submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy

Faculty of Graduate Studies
The University of Western Ontario
London, Ontario
August 1996

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ABSTRACT

The first chapter of the thesis considers a job search model with entrepreneurial decisions by unemployed workers. A conventional job search model predicts a log-linear relationship between the vacancy: unemployment ratio and the job finding rates of unemployed workers. However, empirical evidence suggests that the relationship is concave. The model presented in this chapter exhibits concavity as a result of entrepreneurial specialization in the pool of unemployed workers.

The second chapter of the thesis develops a general equilibrium model of vacancies and unemployment with competition between on and off-the-job searchers. Underemployed workers at low productivity jobs make up the pool of on-the-job searchers while unemployed workers make up the pool of off-the-job searchers. The model predicts that the job finding rate of unemployed workers is a function of the vacancy: unemployment ratio and the underemployment: unemployment ratio. The model also uses the underemployment: unemployment ratio to predict that labor market tightness can either overshoot or undershoot future values in response to a change in the aggregate productivity of job matches. Overshooting occurs if the underemployment: unemployment ratio follows a downward adjustment path during a recovery while undershooting occurs if the underemployment:unemployment ratio follows an upward adjustment path.
The final chapter of the thesis considers insider decisions in a search equilibrium. The model predicts that insiders have strict preferences for a two-tier wage structure over a single-tier wage structure, if there are complementarities between experienced insiders and inexperienced outsiders. Given complementarities, the model also predicts that a temporary aggregate productivity shock leads to persistent adjustments to unemployment, labor market tightness, and insider wages.
DEDICATION

To Tomoko
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>CERTIFICATE OF EXAMINATION</td>
<td>ii</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>iii</td>
</tr>
<tr>
<td>DEDICATION</td>
<td>v</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>vi</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>ix</td>
</tr>
<tr>
<td>OVERVIEW</td>
<td>1</td>
</tr>
<tr>
<td>CHAPTER 1 - ENTREPRENEURS IN SEARCH EQUILIBRIUM</td>
<td></td>
</tr>
<tr>
<td>Introduction</td>
<td>10</td>
</tr>
<tr>
<td>The model</td>
<td>14</td>
</tr>
<tr>
<td>Equilibrium</td>
<td>20</td>
</tr>
<tr>
<td>A Cobb-Douglas matching technology</td>
<td>25</td>
</tr>
<tr>
<td>Conclusions</td>
<td>30</td>
</tr>
<tr>
<td>Appendix I</td>
<td>32</td>
</tr>
<tr>
<td>CHAPTER 2 - UNDEREMPLOYMENT IN SEARCH EQUILIBRIUM</td>
<td></td>
</tr>
<tr>
<td>Introduction</td>
<td>36</td>
</tr>
<tr>
<td>The model</td>
<td>40</td>
</tr>
<tr>
<td>The Beveridge curve</td>
<td>53</td>
</tr>
<tr>
<td>Conclusions</td>
<td>60</td>
</tr>
<tr>
<td>Appendix II</td>
<td>61</td>
</tr>
</tbody>
</table>
CHAPTER 3 - INSIDERS IN SEARCH EQUILIBRIUM

Introduction 65
The model 68
Equilibrium 79
Dynamics 82
Conclusions 86
Appendix III 87
BIBLIOGRAPHY 90
VITA 96
LIST OF FIGURES

1.1 Labor market tightness and the unemployment rate 23
1.2 The linear model versus the concave model 29
2.1 Phase diagram for the total surpuses of good and bad jobs 51
2.2 Dynamic adjustment of unemployment and vacancies 56
3.1 Equilibrium with complementarities 84
Overview

This thesis presents three essays on the subject of job matching and unemployment dynamics. Three separate models are developed to explain the level of unemployment as a function of the flows into and out of unemployment. In each model, the flow out of unemployment is given by a matching technology while the flow into unemployment is exogenous - the outcome of stochastic structural change and new entry into the labor force (ref: Pissarides 1990). Each of the chapters is motivated by a set of empirical issues.

In the first chapter of this thesis, an entrepreneurial model is presented in which unemployed workers choose between self-employment and paid work. The total supply of entrepreneurs - the self-employed - are a significant fraction of the labor force. As of 1990, business owners comprise approximately 13% of the non-agricultural employees in the United States, a fraction comparable to the percentage of the labor force that is unionized (ref: Blau 1987 and Devine 1991). Likewise, in 1986, one out of every seven workers in Canada was self-employed (Cohen 1988).

The supply of self-employed workers is often linked to job creation. In Canada
the vast majority of recent employment gains has been attributed to small businesses and these firms, although not entirely synonymous with self-employment, are in fact generally operated by working owners (ref: Cohen 1988, Baldwin and Picot 1994). The most recent studies of the Canadian labor market find that across all sectors a disproportionately large number of job creations and job destructions are concentrated within small firms (re: Picot, Baldwin and Dupuy 1995). Interestingly, Picot et al. acknowledge that similar results are not reported in the manufacturing sector of the United States (ref: Davis, Haltiwanger and Schuh 1994, Lane, Stevens and Burgess 1996). The question as to whether self-employed workers are the major driving force behind job creation remains a subject of much debate.

How does an entrepreneurial model help explain the flow out of unemployment? One prediction of a conventional job search model without entrepreneurs such as Pissarides (1987) under the assumption of a constant returns to scale matching technology is that a log-linear relationship exists between the job finding rate and the vacancy: unemployment ratio (ref: Pissarides 1986). However, Davidson (1990) suggests that non-linearities characterize the process of job matching in the US labor market. Recently, Storer (1994) presents Canadian data which indicates that the relationship between the job finding rate of unemployed workers and the vacancy: unemployment ratio is roughly concave. Storer uses a spline estimation procedure which captures a large amount of high frequency variation in the hazard series. He notes that a concave model may also be relevant in the US, because Blanchard and Di-
amond (1990), who estimate a linear model, attribute the large amount of frequency variation of their hazard series to measurement error.

I show that a model in which agents choose between self-employment and paid work can generate either a concave or a linear relationship between the log of the job finding rate and the log of the vacancy: unemployment ratio. The model generates a linear relationship if the cost to entrepreneurs of posting a job vacancy entails only a flow of physical resources but no time cost. A concave relationship arises if there is a time cost. The time cost exists if unmatched agents pursuing self-employment are less effective job searchers than other unmatched agents, or, in other words, if potential employers devote less time to job search than other unemployed workers.

The assumption that the self-employed workers are relatively ineffective job searchers finds empirical support in the work of Holmes and Schmitz (1990). Their main finding is that some workers specialize in self-employment and rarely accept paid work. Holmes and Schmitz argue that choices between self-employment and paid work are crucial to the ongoing process of technological change. My model shows that choices between self-employment and paid work may also be relevant to the observed process of job matching.

The second chapter of the thesis presents a model of on and off-the-job search. The model is motivated by the observation that job to job transitions are nearly as common as unemployment to employment transitions (ref: Pissarides 1994). In my model, on-the-job searchers 'crowd out' off-the-job searchers (ref: Burgess 1993).
OVERVIEW

Therefore, the probability of successful off-the-job search is inversely related to the supply of on-the-job searchers, other things equal.

In an early critique of job search theory, Tobin (1972) presents evidence from 1946-47 that 25% of employed workers have another job in hand before leaving their current job. Mattila (1974) considers further evidence from the late 1960's and he concludes that the figure is 50 percent or higher. More recently, Blanchard and Diamond (1989) have estimates which indicate that new hires from other jobs are about 50% of new hires from unemployment. In Britain, Jackman, Layard and Pissarides (1989) observe that 40% of new hires were directly from other jobs, while smaller figures are reported in other countries. Burda and Wyplosz (1993) estimate that in Germany in 1987 the fraction of new hires from other jobs was only 17%, and that in France the number was only 7%. In Canada, evidence on search methods reveals that about two thirds of job searchers are unemployed and one third are employed (ref: Hasen and Gera 1982).

On-the-job search has been the subject of numerous microeconomic studies. A key finding is that on-the-job search falls rapidly with tenure (ref: Jovanovic 1979). In the US, Hall (1982) finds that in the 40-44 age group, the probability that a worker in a new job completes a tenure of 20 years is 4.6%, after two years employment the probability increases to 15.7% and after five to ten years employment to 35%. In the UK, Pissarides and Wadsworth (1994) conclude that workers in new jobs are more than twice as likely to engage in on-the-job search than workers with job tenures of
OVERVIEW

ten years. The present paper develops a model consistent with these facts as well as
cyclical changes in the rate of on-the-job search.

Recent work indicates that the volume of off-the-job search is largely pro-cyclical.
In particular, Blanchard and Diamond (1989) assume that job to job transitions are a
constant fraction of total quits, which is highly procyclical (ref: Davis and Haltiwanger
1992). In the UK, Burgess (1991) derives estimates of job to job transitions which are
pro-cyclical while Burda and Wyplosz (1994) show that total quits are pro-cyclical
for several European countries.

What issues does a general equilibrium model of on and off-the-job search address?
In a cross country study of 14 OECD countries, Jackman, Pissarides and Savouri
(1990) conclude that when economic activity recovered in most of these countries in
the second half of the 1980's, unemployment, especially in Europe, was slow to fall,
despite a rise in job vacancies. In other words, aggregate labor market tightness - the
vacancy: unemployment ratio - was very responsive to changes in business conditions
in these countries with little effect on the job finding rate of unemployed workers.
Similarly, Andolfatto (1996) finds that, in the United States, vacancies are much
more responsive to changes in aggregate conditions than is suggested by the simple
model of off-the-job search which he calibrates.

My model identifies a mechanism which can either dampen or amplify cyclical
changes in aggregate labor market tightness without substantially changing the cycli-
cal behavior of the job finding rates. A key prediction is that the job finding rate of
unemployed workers is a positive function of the vacancy: unemployment ratio and a negative function of the underemployment: unemployment ratio. I assume that underemployed workers (workers at ‘bad’ low productivity jobs) make up the pool of on-the-job searchers and that unemployed workers make up the pool of off-the-job searchers. In my model, the underemployment: unemployment ratio can increase or decrease during a recovery depending on whether bad jobs ‘crowd out’ good jobs. The behavior of the underemployment: unemployment ratio determines the responsiveness of the vacancy: unemployment ratio to changes in business conditions. On the one hand, fluctuations in the vacancy: unemployment ratio are amplified if the underemployment: unemployment ratio tends to decrease during upturns and increase during downturns. On the other hand, fluctuations in the vacancy: unemployment ratio are dampened if the underemployment: unemployment ratio tends to decrease during downturns and increase during upturns. Therefore, according to my model, cyclical changes in the relative supply of on and off-the-job searchers is a crucial determinant of the magnitude of cyclical fluctuations to both the job finding rate of unemployed workers and the vacancy: unemployment ratio.

The final chapter of the thesis examines how insider decisions affect the seniority profile of wages. There is much debate as to whether wage settlements within insider controlled firms are best characterized by single-tier or two-tier wage structures. In my model, a single-tier wage structure gives a non-discriminatory wage in which all workers are paid equally while a two-tier wage structure gives a discriminatory wage in
which worker are distinguished by seniority. The best examples of non-discriminatory wage structures are found in union wage profiles. In particular, Freeman and Medoff (1984) and Pierce (1990) find that union wages rise more slowly with tenure than non-union wages. The estimates of union wage mark-ups fall from around 30% for newly hired workers to 15% or less for more senior workers. A growing consensus is that unions generally offer ‘standard’ non-discriminatory wages (ref: p. 90, Layard, Nickell and Jackman 1990). Interestingly, Gollier (1991) argues that a flat wage profile enforces the market power of insiders by bidding up the entrants wage.

Recently, the prevailing view of a non-discriminatory union wage structure has been subject to empirical scrutiny by Kuhn and Sweetman (1994). They find that the wages of union workers are in fact strongly increasing with tenure relative to their actual labor market opportunities. Therefore, it is possible that unions are offering discriminatory wages even though union wage profiles are relatively flat. The possibility of discriminatory wages is also motivated by the existence of two-tier wage structures in various industries. For example, Walsh (1988) and Card (1986) document the 1983-86 wave of two-tier wage settlements in the US airline industry while Martin (1990) examines tiered compensation structures in the retail food industry.

Is the distinction between two-tier and single-tier wage structures essential to understanding seniority wage profiles? Booth and Frank (1994) argue that there is no clear pattern of how union wage differentials vary with seniority if such wage structures are not taken into account. Their results suggest that this is because of
the heterogeneity of unions. They find that workers covered by union agreements with formal incremental scales display steep wage profiles. Workers in other types of establishments (union agreements without incremental scales, and non-union establishments whether or not they have scales) have indistinguishable flatter seniority wage profiles. Consequently, the distinction between single-tier and two-tier wage structures leads to a much better understanding of observed seniority wage profiles.

What factors lead insiders to adopt a two-tier wage structure over a single-tier wage structure? I find that insiders strictly prefer a two-tier wage structure over a single-tier wage structure, if there are complementarities in production between experienced and inexperienced workers. In this case, the insider wage is an increasing function of the number of new hires, because experienced insiders demand a wage equal to their marginal product which is an increasing function of their own scarcity. If experienced and inexperienced workers are substitutes, insiders are indifferent over the choice of the wage structure. The choice of wage structure has important implications for the aggregate level of unemployment and vacancies. I find unemployment is lower and vacancies are higher under a two-tier wage structure than under a single-tier wage structure. Firms post more vacancies, because a two-tier wage structure permits lower wages for new hires.

Begg (1988) uses complementarities between experienced and inexperienced workers to explain unemployment dynamics. He finds that complementarities leads to unemployment persistence in response to a temporary productivity shock. My model
OVERVIEW

extends Begg's analysis to a labor market with search and recruiting. The model predicts long-run changes to labor market tightness in response to short-run productivity shocks. The adjustments are the consequence of temporary shortages or surpluses of experienced insiders. An implication of my model is that we cannot look to the behavior of labor market tightness for evidence with which to distinguish between alternative mechanisms of unemployment persistence. For example, Pissarides (1992) generates such movements in a model with loss of skills by the long-term unemployed.

Complementarities can also be used to generate co-movements between insider wages and labor market tightness. My model predicts that insider wages also converge slowly to a steady state after an initial productivity shock. Complementarities lead to insider wage adjustments, because the insider wage is then a function of the relative scarcity of experienced insiders and inexperienced outsiders. Similar co-movements between insider wages and labor market tightness do not occur in Pissarides' model, because he assumes experienced and inexperienced workers are substitutes. The introduction of complementarities between experienced and inexperienced workers can be seen as one possible method by which to enlarge the set of predictions offered by Pissarides' framework.
Chapter 2

Entrepreneurs in Search Equilibrium

Introduction

The role of entrepreneurs in the creation of new job is often neglected in theories of unemployment and vacancies (e.g. Pissarides 1987, 1990). However, despite the scarcity of theoretical results, there continues to be much public debate as to whether entrepreneurial activities are a significant source of employment opportunities. The goal of the present chapter is to develop a simple model of job creation by entrepreneurs. This theoretical exercise extends previous work on the topic of search and recruiting, and the main results of the analysis throw new light on the importance of entrepreneurs in the creation of jobs. In particular, this chapter shows how the entrepreneurial decisions of unemployed workers affect the relationship between the job finding rate of unemployed workers and the vacancy: unemployment ratio.

A cornerstone prediction of conventional job search models such as Pissarides (1987) is that there should be a log-linear relationship between the job finding rate of unemployed workers and the vacancy: unemployment ratio. This prediction follows
INTRODUCTION

from the assumption that the aggregate flow of job matches is given by a constant returns to scale Cobb-Douglas matching technology and the assumption that the number of unemployed workers and the number of job vacancies are its only inputs. The parameters of a Cobb-Douglas matching technology have been estimated under these assumptions by Blanchard and Diamond (1989) and Jackman, Layard and Pissarides (1989). However, recent empirical evidence in Storer (1994) indicates that the relationship between the log of the job finding rate of unemployed workers and the log of the vacancy: unemployment ratio is concave.\textsuperscript{1} In other words, he finds that an increase in the vacancy: unemployment ratio has a big effect on the job finding rates of the unemployed if the vacancy: unemployment ratio is low and very little impact on these rates if the vacancy: unemployment ratio is high. This result casts doubt on the empirical relevance of the Cobb-Douglas matching technologies which are frequently employed in simple general equilibrium models of unemployment such as Andolfatto (1994).

In this chapter, I will show that a general equilibrium job search model of entrepreneurs can generate either a linear or a concave relationship between the log of the job finding rate of unemployed workers and the log of the vacancy: unemployment ratio even though I assume a Cobb-Douglas matching technology. I assume that search intensity determines the probability that an unemployed agent is a job searcher. I also assume (i) new job vacancies are posted by entrepreneurs who are drawn from the pool of unemployed workers and (ii) unemployed agents sacrifice search inten-

\textsuperscript{1}Storer uses Canadian data.
sity when they pursue entrepreneurship.\textsuperscript{2} Assumptions (i) and (ii) allow a departure from Pissarides' (1987) prediction of a linear relationship because the supply of job searchers is no longer always equal to the number of unemployed workers. My model predicts concavity if unemployed agents sacrifice search intensity when they pursue entrepreneurship. The linear model of Pissarides emerges only if unemployed agents do not sacrifice search intensity when they pursue entrepreneurship.

The intuition behind my concavity result is straightforward. Suppose that unmatched entrepreneurs (potential employers) are scarce in the pool of unemployed workers. In this case, increasing the supply of job vacancies by increasing the supply of entrepreneurs has a big positive effect on the flow of job matches even though a relatively abundant supply of job searchers decreases as a result of more unemployed agents choosing to be entrepreneurs. By contrast, if entrepreneurs are abundant, an increase in the supply of job vacancies has a much smaller positive effect on the creation of new matches, because the corresponding decrease to the relatively scarce supply of job searchers has a much bigger negative effect. In fact at higher levels of labor market tightness it is possible to have a fall in the total flow of new matches as the supply of new vacancies increases. The two cases imply that an increase in the log of the vacancy: unemployment ratio has a much bigger positive effect on the log of the job finding rate if the vacancy: unemployment ratio is low than if it is high. Therefore, the model predicts concavity as is observed in Storer.

\textsuperscript{2}My model is based on the theory of choosing sides in matching games which is developed in Baye and Cosimano (1990) and Kennes (1990).
INTRODUCTION

The theory of specialization is consistent with my assumption that potential employers are not effective job searchers. Rosen (1983) argues that maximizing agents will often choose to specialize in order to capture economies of scale given time constraints. Holmes and Schmitz (1990) use this argument to construct a theory of entrepreneurship and they find persuasive evidence to suggest that entrepreneurs are often specialists. I am going to show that added motivation for a model with specialization by entrepreneurs is the fact that such a model can explain the observed relationship between the job finding rate of unemployed workers and the vacancy: unemployment ratio using the assumption of a Cobb-Douglas matching technology. In particular, a concave relationship supports the view that entrepreneurs are specialists.

The chapter is organized as follows. In the next section, I outline a simple entrepreneurial model of the labor market. A central idea behind this model is that trade in the labor market is time-consuming and costly for both employers and employees. The crucial assumption is that unmatched entrepreneurs face a possible trade-off between job search and job creation. This model gives a dynamic equilibrium description of a labor market with entrepreneurs. Further predictions of the model are then derived by specifying a Cobb-Douglas matching technology. The final section of this chapter offers some concluding remarks.
The model

Consider a continuous time, infinite horizon economy with infinitely lived agents. Each agent has linear preferences over lifetime consumption and wishes to maximize the expected present discounted value of their income stream. The population is distributed on the unit interval and it consists of $e$ employed and $u$ unemployed agents. The stock of employed agents satisfies

$$e = 1 - u. \quad (2.1)$$

Total employment consists of $e_m$ employers and $e_w$ employees. Therefore,

$$e = e_w + e_m. \quad (2.2)$$

A simplifying assumption is that all additions to the stocks of employers and employees are determined by pairwise matching. Each match then consists of one employee and one employer. This condition implies

$$e_w = e_m. \quad (2.3)$$

A match is the basic unit of production which I call a firm. I assume that each firm produces a constant flow of $y$ units of output and that the total output of the firm is split between the employee and the employer via Nash Bargaining. The employee receives a flow of $\xi y$ units of output while the employer receives the remaining flow of $(1 - \xi) y$ units of output.
THE MODEL

Trade and production are separate activities. I emphasize this aspect of the labor market by assuming (i) that employed agents specialize in the production of output and (ii) that unemployed agents specialize in the production of new jobs. On the demand side of the labor market, a supply of \( v \) job vacancies is posted by an endogenous fraction of unemployed agents. I assume that each of these potential employers posts a fixed number of job vacancies, \( c \). The total supply of job vacancies is given by

\[
v = c\theta u, \tag{2.4}
\]

where \( \theta \) is the endogenous fraction of unemployed agents who are potential employers.\(^3\)

Unemployed agents can open and close vacancies instantaneously. The cost of posting a vacancy includes a physical cost and a time cost. The physical cost of posting a vacancy is a fixed flow of \( k \) units of output. All unemployed agents receive a fixed flow of \( a \) units of income, therefore, a potential employer receives a flow of \( a - k \) units of income during unemployment. The time cost of a vacancy is the reduced job search effectiveness of a potential employer. This cost is measured in relation to the job search effectiveness of unemployed agents who are not potential employers. The job search effectiveness of the latter group is normalized to one while the job search effectiveness of the former group is given by an exogenous fraction, \( \alpha \), less than or

---

\(^3\)The parameter \( c \) would be normalized to one if job advertisements measured the number of entrepreneurs. However, the supply of job vacancies is typically measured by the number of pages of job advertisements in newspapers, etc. (ref: Storer 1984). Thus \( c \) may be best interpreted as the size of a job vacancy advertisement.
THE MODEL

equal to one. I follow Pissarides (1990) and assume that the job search effectiveness of an agent is equivalent to the probability that an unemployed agent is a job searcher. The total supply of job searchers is as follows:

\[ s = (1 - \theta)u + \alpha \theta u. \]  

(2.5)

According to this expression, the supply of job searchers decreases as the number of potential employers increases. This trade-off exists only if the search effectiveness of potential employers (\( \alpha \)) is strictly less than other unemployed agents. Otherwise, the supply of job vacancies is equal to the unemployment rate.

In this model, the appropriate measure of labor market tightness is the vacancy: job searcher ratio. Let \( \phi \) denote labor market tightness. Define

\[ \phi \equiv v/s. \]

Notice, if potential employers are less effective job searchers than other unemployed agents, this definition of labor market tightness departs from the usual proxy of labor market tightness - the vacancy: unemployment ratio (\( v/u \)). The hiring process is described by a matching technology giving the total flow of \( m \) pairwise matches as a function of the aggregate supply of \( v \) vacancies and \( s \) job searchers. The matching technology has the form

\[ m = m(s, v). \]  

(2.6)
THE MODEL

The matching technology is assumed to be increasing in both of its arguments, concave, and homogeneous of degree one. The matching process is also random. Therefore, each vacancy has an equal chance of being matched with a job searcher and each job searcher has an equal chance of being matched with a vacancy. The probability $p_v$ that a vacancy is matched with a job searcher satisfies

$$p_v = p_v(\phi) \equiv m(1/\phi, 1), \quad (2.7)$$

because $m(s,v)/v = m(1/\phi, 1)$. The assumption that the matching technology is increasing in its first argument implies that $p_v(\phi)$ is a decreasing function of labor market tightness. The probability $p_s$ that a job searcher is matched with an employer is given by

$$p_s = p_s(\phi) \equiv m(1, \phi), \quad (2.8)$$

because $m(s,v)/s = m(1, \phi)$. The assumption that the matching technology is increasing in its second argument implies that $p_s(\phi)$ is an increasing function of labor market tightness. If an unemployed agent chooses to be a potential employer, the rate at which he/she leaves unemployment is given by $p_v + \alpha p_s$. If an unemployed agent is not a potential employer, there is only one flow probability into employment, $p_s$. The average job finding rate is denoted by $p$. The function determining $p$ has the form
\[ p = \theta(p_v(\phi) + \alpha p_s(\phi)) + (1 - \theta)p_s(\phi), \]  

\[ \text{(2.9)} \]

where \( \theta \) denotes the proportion of unemployed agents who are potential employers. Let \( b \) denote the rate of job separations. This job destruction rate is assumed to be exogenous - the result of negative idiosyncratic shocks. The total flow of agents into unemployment is \((1 - u)b\) while the total flow of agents out of unemployment is \(up\). The rate of change to unemployment is the difference between the flows into and out of unemployment. Therefore, changes to unemployment satisfy

\[ \dot{u} = (1 - u)b - up, \]  

\[ \text{(2.10)} \]

where dot notation is used to denote the rate of change.

Agents earn different flows of income depending on their employment and unemployment status. Let \( J \) and \( W \) denote the present-discounted value of the expected income stream of a matched employer and employee, respectively. Likewise, let \( V \) denote the present-discounted value of the expected income stream of a potential employer and let \( U \) denote the present-discounted value of the expected income stream of all other unemployed agents. If an unemployed agent is not a potential entrepreneur, the agent receives a flow of \( a \) units of income. The probability that this agent finds employment is \( p_v(\phi) \). The asset equation of an unemployed agent, who is not a potential employer, has the form
THE MODEL

\[ rU = a + p_e(\phi)(W - U) + \dot{U}, \]

(2.11)

where \( r \) is the discount factor. The asset equation of an employee is as follows:

\[ rW = \xi y + b(U - W) + \dot{W}, \]

(2.12)

where \( \xi y \) is their flow of income and \( b \) is the probability that the match is destroyed. If an unemployed agent chooses to be a potential employer, the agent receives a flow of \( a - k \) units of income. This agent has two probabilities of entering employment. First, there is the probability \( p_e(\phi) \) that this agent finds employment as an employer, and second, there is a probability \( \alpha p_e(\phi) \) that this agent finds employment as an employee. The asset equation of a potential employer is given by

\[ rV = a - k + p_e(\phi)(J - V) + \alpha p_e(\phi)(W - V) + \dot{V}. \]

(2.13)

An employer receives a flow of \( (1 - \xi)y \) units of income and the match is destroyed with a probability, \( b \). The asset equation of an employer has the form

\[ rJ = (1 - \xi)y + b(V - J) + \dot{J}. \]

(2.14)

Finally, if the potential employer finds employment as an employee, their asset equation is the same as equation (1.12).
Equilibrium

The purpose of this section is to characterize the equilibrium level of labor market tightness and the level of unemployment. Labor market tightness, in turn, gives the equilibrium supply of potential employers. The model is closed by two conditions. First, a Nash bargaining solution determines the split of the total economic surplus of a match. Second, a factor mobility condition equates the expected value of a potential employer with the expected value of all other unemployed agents. A match yields a net return of \( J - V \) to the employer and a net return of \( W - U \) to the employee. The total surplus of a match is given by

\[
\Sigma = J - V + W - U
\]  

(2.15)

The rate of change to the total surplus of a match satisfies \( \dot{\Sigma} = J - \dot{V} + \dot{W} - \dot{U} \). A Nash Bargaining Solution splits the total surplus of a match by a fixed sharing rule. Let \( \beta \) and \( 1 - \beta \) denote the shares of total surplus that go to the employee and the employer, respectively. The sharing rule implies

\[
\beta \Sigma = W - U
\]  

(2.16)

and

\[
(1 - \beta) \Sigma = J - V,
\]  

(2.17)
where $\beta \dot{\Sigma} = \dot{W} - \dot{U}$ and $(1 - \beta) \dot{\Sigma} = \dot{J} - \dot{V}$ characterize rates of change. An unemployed agent chooses to open a vacancy only if the expected return is at least as great as the alternative of not opening a vacancy. Therefore, in equilibrium, all unemployed agents receive the same expected return. This factor mobility condition has the form

$$U = V.$$  \hfill (2.18)

where $\dot{U} = \dot{V}$ gives the rate of change. The factor mobility condition and the Nash bargaining solution can be substituted into the asset equations of unmatched agents - equations (1.11) and (1.13). Subtracting equation (1.11) from equation (1.13) yields

$$p_v(\phi)(1 - \beta)\Sigma + \alpha p_e(\phi)\beta\Sigma = k + p_e(\phi)\beta\Sigma.$$  \hfill (2.19)

The left hand side of this expression is the benefit of posting a vacancy. This benefit is characterized by two flows into employment - one as an employer and one as an employee. The right hand side of this expression is the opportunity cost of posting a vacancy. This cost includes the physical cost of a vacancy and the flow of matches accruing to an unemployed agent not advertising a vacancy. Notice, if $\alpha = 1$, equation (1.19) becomes $p_v(\phi)(1 - \beta)\Sigma = k$. This condition is equivalent to the corresponding equilibrium condition of Pissarides' (1987) model without entrepreneurs. Equation (1.19) implies that the total economic surplus of a match is given by the following function of labor market tightness.
\[ \Sigma = \Sigma(\phi) \equiv \frac{k}{(1 - \beta)p_s(\phi) - \beta(1 - \alpha)p_s(\phi)} \]  

(2.20)

This function is differentiated with respect to time to get \( \dot{\Sigma} = \Sigma'(\phi) \dot{\phi} \). A second equilibrium condition is derived from the Nash bargaining solution, the factor mobility condition, and all four asset equations. This expression is found by subtracting equation (1.11) from equation (1.12), subtracting equation (1.13) from (1.14), and then adding the remaining terms. This condition has the form

\[ \dot{\Sigma} = H(\phi)\Sigma(\phi) - Y, \]  

(2.21)

where \( Y = y - a - (a - k) \) and \( H(\phi) \equiv (r + b + \beta(1 + \alpha)p_s(\phi) + (1 - \beta)p_v(\phi)) \). Equation (1.21) characterizes the rate at which the total surplus of a match changes as a function of its current level and the current level of labor market tightness. If this rate of change is zero, the total surplus of a match is proportional to the total net flow of income of a match, \( Y \), and is inversely related to the discount factor \( r \) and the job destruction rate \( b \). The total surplus of a match is also smaller, if an agent can find a new match quickly elsewhere. Equations (1.20) and (1.21) lead to a single expression for the evolution of labor market tightness while equation (1.10) relates labor market tightness to the evolution of unemployment. The two differential equations for labor market tightness and unemployment are given by

\[ \dot{\phi} = (H(\phi)\Sigma(\phi) - Y) / \Sigma'(\phi) \]  

(2.22)
\[ \dot{u} = (1 - u)b - up(\phi) \]  \hspace{1cm} (2.23)

The average job finding rate of unemployed workers is a function of labor market tightness because \( \phi \equiv v/s = c\theta/((1 - \theta) + \alpha \theta) \) implies \( \theta = \phi/(v + (1 - \alpha)\phi) \), which can then be substituted into equation (1.19). The demarcation curves corresponding to equations (1.22) and (1.23) characterize the unique steady state equilibrium of this model. The dynamic adjustment possibilities around this equilibrium are presented in the following phase diagram.

![Figure 1.1: Labor Market Tightness and the Unemployment Rate](image-url)
EQUILIBRIUM

In figure 1, labor market tightness reaches a maximum of $c/\alpha$ if all unemployed agents choose to be potential employers ($\theta = 1$). Figure 1 illustrates that the steady state equilibrium at ($\dot{\phi} = 0, \dot{u} = 0$) is a saddlepoint. Interestingly, the demarcation curve for unemployment can be backward bending for higher values of labor market tightness. This possibility characterizes unemployment and vacancies only if the efficiency of job search by potential employers is sufficiently low and the supply of potential employers is sufficiently high. If entrepreneurs are effective job searchers, such that $\alpha = 1$, the demarcation curve for unemployment is never backward bending.

Except for the backward bending demarcation curve for unemployment, the entrepreneurial model behaves like Pissarides’ (1987) basic job search model. Therefore, an increase in the productivity of a filled job, an increase in the share of economic surplus to entrepreneurs, or a decrease in the physical cost of posting a vacancy all lead to an increase in labor market tightness as the demarcation for labor market tightness shifts upwards. Also, in my model, labor market tightness increases, if the job search effectiveness of potential employers ($\alpha$) increases. A change in this variable pushes the demarcation curve for unemployment in and raises the demarcation for labor market tightness. Likewise, an increase in the number of vacancies per potential employer ($c$) causes an increase in labor market tightness and a decrease in unemployment.
A Cobb-Douglas matching technology

In this section, I derive further predictions from the entrepreneurial model by specifying a Cobb-Douglas matching technology. I obtain two results. First, if the job search effectiveness of potential employers is equal to the job search effectiveness of other unemployed agents, the model predicts a log-linear relationship between the job finding rate of unemployed workers and the vacancy: unemployment ratio. Therefore, in this case, the entrepreneurial model shares an identical prediction with the benchmark model of Pissarides (1987). The second result is for the case where the job search effectiveness of potential employers is less than the job search effectiveness of other unemployed agents. In this case, the entrepreneurial model predicts a concave relationship between the log of the job finding rate of unemployed workers and the log of the vacancy: unemployment ratio.

The key variable is the vacancy: unemployment ratio. This variable measures labor market tightness in theoretical work such as Pissarides (1987) and in the data used by Storer (1994). Let \( \hat{\phi} \) denote the vacancy: unemployment ratio as follows:

\[
\hat{\phi} \equiv \frac{v}{u}.
\]  

In the entrepreneurial job search model of this chapter, the vacancy: unemployment ratio does not always measure labor market tightness. More generally, I use the vacancy: job searcher ratio:
\[ \phi \equiv \frac{v}{s}. \]  

(2.25)

Pissarides' definition of labor market tightness - the vacancy: unemployment ratio - is equivalent to my definition when the number of job searchers equals the number of unemployed agents. According to equation (1.5), these two measures coincide only if entrepreneurs are as effective job searchers as other unemployed workers.

A common assumption in the theory of job matching is to suppose that the aggregate matching technology is a constant returns to scale Cobb-Douglas function of job searchers and job vacancies. This assumption implies that the total flow of job matchings is given by

\[ m(v, s) = \frac{A}{2} v^\gamma s^{1-\gamma} \]  

(2.26)

where \( \gamma \in [0, 1] \) and \( A \) is a positive constant. The average job-finding rate of unemployed workers, \( p \), is derived by dividing the total flow of unemployed agents into employment, \( 2m(v, s) \), by the number of unemployed workers, \( u \). Suppose that entrepreneurs are as effective job searchers as other unemployed workers (i.e. \( \alpha = 1 \)). In this case, where my definition of labor market tightness and Pissarides' definition coincide, I obtain

\[ \ln p = \ln A + \gamma \ln \phi. \]  

(2.27)

Equation (1.27) describes the relationship between the vacancy: unemployment ratio
and the average job finding rate of unemployed workers if entrepreneurs are as effective job searchers as other unemployed workers. This is the linear model which has been estimated in empirical work by Jackman, Layard and Pissarides (1989) and Diamond and Blanchard (1989).

Storer rejects the log-linear model in favor of a more general model. According to Storer’s findings, the relationship between the log of the job finding rates and the log of the vacancy: unemployment ratio is concave. It is possible to generate this prediction in the entrepreneurial model by assuming that entrepreneurs are less effective job searchers than other unemployed agents. In the entrepreneurial model, the general expression for the job finding rate of unemployed workers is given by the total flow of matches divided by the number of unemployed workers.

\[
p = \frac{2m(s, v)}{u} = \frac{Av^\gamma s^{1-\gamma}}{u} = \frac{Av^\gamma (\alpha s u + (1 - \theta) u)^{1-\gamma}}{u} \tag{2.28}
\]

The vacancy: unemployment ratio satisfies \( \hat{\phi} = \frac{v}{u} = \frac{\phi u}{u} = \phi \). Therefore, equation (1.28) can be rewritten

\[
p = A\hat{\phi}^\gamma (1 - \frac{(1 - \alpha)}{c} e^{ln \hat{\phi}})^{1-\gamma} \tag{2.29}
\]

The logarithm of equation (1.29) is

\[
\ln p = \ln A + \gamma \ln \hat{\phi} + Z \tag{2.30}
\]

where \( Z = \ln(1 - \beta_0 e^{ln \hat{\phi}} (1-\gamma)) \) and \( \beta_0 \equiv \frac{(1 - \alpha)}{c} \). The function \( Z \) leads to the prediction
of a concave relationship between the vacancy: unemployment ratio and the job finding rate of unemployed workers. In particular, $Z$ satisfies

$$\frac{dZ}{dx} = -\frac{(1 - \gamma)\beta_\xi e^x}{1 - \beta_\xi e^x} \leq 0,$$

and

$$\frac{d^2Z}{dx^2} = \frac{1}{(1 - \beta_\xi e^x)} \frac{dZ}{dx} \leq 0$$

(2.32)

where $x \equiv \frac{\ln \phi}{\ln z}$. The coefficient $\beta_\xi$ of the function $Z$ consists of two terms - the intensity of job search by potential employers ($\alpha$) and the number of vacancies per potential employer ($c$). If $\alpha$ approaches one, potential employers do not sacrifice search intensity when they advertise job vacancies, $\beta_\xi$ approaches zero. Likewise, $\beta_\xi$ is small if $c$ is large, because a small increase in the number of entrepreneurs causes a large increase in the supply of job vacancies. Concavity characterizes the model, if the job search effectiveness of potential employers deviates from one. If the job search effectiveness of potential employers equals one the entrepreneurial model predicts the log-linear relationship between job finding rates and the vacancy: unemployment ratio of the benchmark model of Pissarides. The two predictions of the entrepreneurial model are compared in the following diagram.
The concave relationship of figure 2 occurs only if the job search effectiveness of potential employers is less than other unemployed agents ($0 \leq \alpha < 1$). This relationship is explained as follows. Suppose that entrepreneurs are scarce such that the vacancy: unemployment ratio is low. In this case, increasing the supply of job vacancies by increasing the supply of entrepreneurs has a big positive effect on the flow of job matches even though the supply of job searchers decreases according to equation (1.5). The effect is large because the supply of potential employers is small in relation to the supply of job searchers. Now suppose that entrepreneurs are abundant such that the vacancy: unemployment ratio is large. In this case, increasing the supply of job vacancies by increasing the supply of abundant entrepreneurs has
a small positive effect on the flow of job matches, because the supply of scarce job
searchers decreases according to equation (1.5). The linear relationship of figure 2
characterizes my model only if the job search effectiveness of potential employers is
equal to other unemployed agents. In this case, the supply of job searchers is not a
function of the relative supplies of potential employers and other unemployed agents.
Since the supply of job searchers is always equal to the number of unemployed workers
under these circumstances, the prediction of my model and Pissarides model coincide.

To the best of my knowledge, a model which predicts concavity has not been de-
veloped elsewhere. One obvious candidate is to endogenize the probability in which
an unmatched worker participates in the matching process. However, the key as-
sumptions of such a model with regards to concavity are not easy to anticipate. For
example, suppose that the matching technology is given by \( m(tu, v) \), where \( t \) is the
endogenous participation decision (possibly an endogenous search intensity decision),
and assume that the general equilibrium solution of the model leads to the prediction
that \( t = \theta^2 \) where \( 0 \leq \theta \leq 1 \). This simple generalization of Pissarides (1987) pre-
dicts the same log-linear relationship of equation (1.27) even though participation is
a concave function of labor market tightness.

Conclusions

The central motivation of this chapter is to provide a theory of en-
trepreneurial activity that explains a concave relationship between the log of the job
CONCLUSIONS

finding rate of unemployed workers and the log of the vacancy: unemployment ratio. A theory is derived by incorporating the insights of Rosen (1983) concerning specialization into the benchmark general equilibrium job search model of Pissarides (1987). This chapter provides a theory of entrepreneurial activity that exhibits a concave relationship between the log of the job finding rate of unemployed workers and the log of the vacancy: unemployment ratio. The model predicts concavity, if potential employers are less effective job searchers than other unemployed agents, and it predicts a linear relationship, if potential employers are equally effective job searchers as other unemployed agents.
Appendix 1.1. Backward bending demarcation curve for unemployment

In the phase diagram of figure 1, the demarcation curve for unemployment \( u = 0 \) is a backward bending function of labor market tightness. This property occurs if the job finding rate first increases and then decreases as the supply of entrepreneurs in the pool of unemployed workers increases. In particular, the job finding rate,

\[
p = \frac{2m(u,v)}{u} = 2m(\alpha \theta, 1 - \theta(1 - \alpha)),
\]
satisfies

\[
\frac{dp}{d\theta} = 2(m_1c - m_2(1 - \alpha)) = \begin{cases} 
\geq 0 & \text{if } \theta \text{ approaches zero, or if } \alpha = 1 \\
< 0 & \text{if } \theta \text{ approaches one and } \alpha \text{ approaches zero}
\end{cases}
\]

(2.33)

\[
\frac{d^2p}{d\theta^2} = 2(m_{11}c^2 - m_{12}c(1 - \alpha) + m_{22}(1 - \alpha)^2) \leq 0
\]

(2.34)

assuming \(-m_{11}, -m_{22}, m_{12} \geq 0\) and \(\lim_{\theta \to 0} m_1(s,v) = \lim_{\theta \to 0} m_2(s,v) = \infty\). (subscripts denote partial derivatives). Assuming \(\alpha\) is sufficiently small, \((\alpha \to 0)\), the job finding rate first increases and then decreases as the supply of entrepreneurs increases in the pool of unemployed workers. Labor market tightness satisfies \(\frac{d\phi}{d\theta} = \frac{c(1 - \theta)}{(1 - (1 - \alpha)\theta)^2} \geq 0\) because \(\phi = \frac{u}{v} = \frac{\partial u}{(1 - \theta) + \alpha \sigma u}\), therefore, the job finding rate first increases and then decreases as labor market tightness decreases. Moreover, the steady state level of unemployment satisfies \(\frac{du}{dp} = \frac{-\phi}{(\phi + \phi)} \leq 0\) because \(u = \frac{\phi}{\phi + \phi}\), therefore, the unemployment rate first decreases and then increases as labor market tightness increases. If \(\alpha\) is large,
(α → 1), the job finding rate is always a positive function of labor market tightness, in which case, the demarcation curve for unemployment is never backward bending.

Appendix 1.2. Saddle-point property of the phase diagram

The stability of the dynamic system in figure 1 is evaluated around the steady state. The equations for the evolution of unemployment and labor market tightness are as follows:

\[
\dot{u} = F \equiv (1 - u)b - p(\phi)u \quad (1.22)
\]

\[
\dot{\phi} = G \equiv (H\Sigma - Y) / \Sigma' \quad (1.23)
\]

where

\[
H = r + b + \beta(1 + \alpha)p_\ast + (1 - \beta)p_v
\]

\[
H' \equiv \frac{dH}{d\phi} = \beta(1 + \alpha)p'_\ast + (1 - \beta)p'_v
\]

\[
Y \equiv y - a - (a - k)
\]

\[
\Sigma = \frac{k}{(1 - \beta)p_v - \beta(1 - \alpha)p_\ast}
\]

\[
\Sigma' \equiv \frac{d\Sigma}{d\phi} = \frac{k((1 - \beta)p'_v - \beta(1 - \alpha)p'_\ast)}{((1 - \beta)p_v - \beta(1 - \alpha)p_\ast)^2}
\]
where \( p'_s \), \( -p'_v \equiv -\frac{\phi v'(\phi)}{\phi} \geq 0 \). The partial derivatives of \( F \) and \( G \) are as follows:

\[
F_u = -b - p(\phi) < 0
\]  
(2.35)

\[
F_\phi = -p'(\phi)u
\]  
(2.36)

\[
G_u = 0
\]  
(2.37)

\[
G_\phi = \frac{H'\Sigma + H\Sigma Y}{\Sigma'} - \frac{(H\Sigma - Y)\Sigma''}{(\Sigma')^2}
\]  
(2.38)

At \( \dot{\phi} = 0 \), \( (H\Sigma - Y)/\Sigma' = 0 \), therefore

\[
G_\phi = \frac{H'\Sigma + H\Sigma}{\Sigma'}
\]  
(2.39)

This expression can be rewritten as follows:

\[
G_\phi = \frac{-1}{(1 - \beta)p'_v - \beta(1 - \alpha)p'_s} \beta(1 - \beta)(p'_s p_v - p_s p'_v)
\]

\[-(r + b)((1 - \beta)p'_v - \beta(1 - \alpha)p'_s) \geq 0
\]  
(2.40)

The positive sign characterizes \( G_\phi \) because \( p_s \) is increasing function of labor market tightness while \( p_v \) is a decreasing function of labor market tightness. The Jacobian of the linearized dynamic system has the following form.
\[ G_\phi \ F_\phi = + \ \pm/\mp \]
\[ G_u \ F_u = 0 \ - \]

The one positive characteristic root and the one negative characteristic root of the Jacobian establishes the saddle-point property of figure 1 (ref: Chiang 1992). Therefore, the equilibrium is stable.///
Chapter 3

Underemployment in Search Equilibrium

Introduction

It has long been observed that in the United States, as in other countries, job to job transitions are as common as unemployment to employment transitions (ref: Tobin 1972, Mattila 1974, and Burdett 1978). However, only recently has research in macroeconomics begun to explore the implications of competition between on and off-the-job searchers. Pioneering empirical work on this subject by Burgess (1993) demonstrates that on-the-job searchers crowd out off-the-job searchers as they compete for new job vacancies. Pissarides (1994) has developed a general equilibrium model of such competition and the predictions of that model are consistent with Burgess's findings and other stylized facts of the labor market. The present chapter introduces an alternative model of on and off-the-job search with two new predictions. The model predicts that the unemployment outflow rate is a function of the vacancy: unemployment ratio and a variable which I call the underemployment: unemployment ratio. The model also predicts that the vacancy: unemployment ratio
can either overshoot or undershoot its future value in response to a change in the aggregate productivity of job matches. Overshooting occurs if the underemployment: unemployment ratio falls during an upturn while undershooting occurs if this ratio increases during a recovery.

In this chapter, I assume that underemployed workers make up the pool of on-the-job searchers and that unemployed workers make up the pool of off-the-job searchers. Underemployed workers are assumed to be at bad (low productivity) jobs looking for good (high productivity) jobs while unemployed workers search for either. What is interesting about these two groups of agents is that their numbers may exhibit very different cyclical properties. In particular, unlike the supply of unemployed workers, the supply of underemployed workers can either increase or decrease during a recovery depending on whether the supply of good jobs crowds out the supply of bad jobs. My model establishes a link between underemployment and the job finding rate of unemployed workers. If underemployed workers are in competition with unemployed workers for new jobs, the job finding rate of unemployed workers is a function of the underemployment: unemployment ratio in addition to the usual vacancy: unemployment ratio.

Pissarides (1994) considers the evidence of Burgess in the context of cross country evidence such as Bean (1994) and suggests that the sensitivity of aggregate labor market tightness - the vacancy: unemployment ratio - to changes in aggregate productivity varies from country to country. The present chapter offers a potential
explanation of such differences using the underemployment: unemployment ratio. In my model, underemployed workers compete for new jobs and new job vacancies are created in response to such competition. This feature of the model implies that changes in the underemployment: unemployment ratio can either dampen or amplify cyclical changes to aggregate labor market tightness. If the underemployment: unemployment ratio falls during an upturn and then rises during an downturn, the model predicts cyclical swings in aggregate labor market tightness are amplified. By contrast, the model predicts cyclical fluctuations in aggregate labor market tightness are dampened if the underemployment: unemployment ratio rises during an upturn and falls during a downturn.

Presently, there exists little empirical work on the subject of competition between on and off-the-job searchers. However, Burgess (1993) has shown, using British data, that the early increases in the supply of vacancies at the start of a recovery have little effect in reducing unemployment. The explanation is that on-the-job searchers actively compete for new jobs during these periods. Burgess constructs his empirical test from a prediction about on and off the job search, which is related to the observed returns to scale of the matching technology. A major disadvantage of this approach, which Burgess acknowledges, is that the results cannot be directly compared to alternative models of unemployment to employment transitions that emphasize only off-the-job search. My model of on and off-the-job search explains the job finding rate of unemployed workers as a function of the vacancy: unemployment ratio and
the underemployment: unemployment ratio. Therefore, my model encompasses and thus can be directly compared to the conventional off-the-job search model of Pissarides (1987)(1990).

The model of the present chapter is similar in many respects to Pissarides (1994). The major short-coming of Pissarides' model is that it does not generate clear equilibrium predictions concerning the dynamic adjustment of labor market tightness, job finding rates, and unemployment in response to changes in the underlying parameters. My model addresses this problem by refining the matching problem to one of coordinated search rather than one of uncoordinated search, and by assuming probabilistic improvements in the productivity of underemployed workers rather than deterministic changes. Coordinated search comes about in my model, because I assume that underemployed workers do not search in the market for bad, low productivity jobs. Pissarides model has uncoordinated search because good and bad job vacancies are only distinguished at the time of job matching. My model places extra structure on the search process and it is for this reason that I am able to derive new predictions.

The chapter is organized as follows. In the next section, I outline a simple dynamic general equilibrium model of on and off-the-job search. The following section introduces the concept of the underemployment: unemployment ratio and uses this concept to derive specific predictions about job finding rates and the dynamic adjustment of labor market tightness. The final section of this chapter offers some concluding remarks.
The model

Consider a continuous time, infinite horizon economy with infinitely lived agents. Each agent has linear preferences over lifetime consumption and wishes to maximize the expected present discounted value of their income stream. The population is distributed on the unit interval and it consists of $e$ employed and $u$ unemployed agents. The stock of unemployed agents is given by

$$u = 1 - e. \quad (3.1)$$

To motivate the concept of on-the-job search, I assume that there are two types of jobs - good and bad. The workforce consists of $e_b$ workers at bad jobs and $e_g$ workers at good jobs. Total employment satisfies

$$e = e_b + e_g. \quad (3.2)$$

A worker at a bad job is underemployed while a worker at a good job is fully-employed. The distinction arises because the flow of $y_g$ units of output for a good job is greater than the flow of $y_b$ units of output for a bad job. In equilibrium, this disparity implies that the flow of $w_g$ wages for a good job is greater than the flow of $w_b$ wages for a bad job.

Unemployed and underemployed workers have an incentive to search for jobs. I suppose that both types of workers write resumes and answer job advertisements. Underemployed workers do not have an incentive to answer an advertisement for a
bad job. Therefore, only unemployed workers search for bad jobs. The stock of $s_b$ job searchers in the market for bad jobs is given by

$$s_b = u$$  \hspace{1cm} (3.3)

The supply of job searchers in the market for good jobs includes unemployed and underemployed workers. The total supply of $s_g$ job searchers looking for good jobs is given by the following expression:

$$s_g = u + \alpha e_b$$  \hspace{1cm} (3.4)

where $\alpha$ is the effectiveness of on-the-job search by underemployed workers relative to unemployed workers. I assume $\alpha$ is given by an increasing, concave function, $\alpha(s)$, of on-the-job search intensity, $s$. Following Pissarides (1990), the choice of on-the-job search intensity is modeled as a two stage process. In the first stage, the worker makes an expenditure of $s$ units of resources in on-the-job search, then in the second stage, the worker enters the matching process, probabilistically.

There are two types of job vacancies in this economy corresponding to good and bad jobs. The total stock of $v$ vacancies includes $v_b$ vacant bad jobs and $v_g$ vacant good jobs. A good job vacancy costs a flow of $k_g$ units of output while a bad job vacancy costs a smaller flow of $k_b$ units of output. I assume that either type of job vacancy can be opened and closed instantaneously.

Labor market tightness in the good and bad job markets is the vacancy: job
searcher ratio in each job market. Let $\phi_b$ and $\phi_g$ denote labor market tightness for bad and good jobs, respectively. Define

\[
\phi_b \equiv \frac{v_b}{s_b}, \quad (3.5)
\]

\[
\phi_g \equiv \frac{v_g}{s_g}. \quad (3.6)
\]

The total flow of $m$ new matches is given by an aggregate matching technology, which is a function of the number of searchers and vacancies in each labor market. The matching technology has the form

\[
m = m_b(s_b, v_b) + m_g(s_g, v_g). \quad (3.7)
\]

Both components of this matching technology are assumed to be homogeneous of degree one, increasing in the respective arguments and concave.

The transition probabilities into the matched state for vacancies and workers are functions of labor market tightness. Let $q_g$ and $q_b$ denote the transition probabilities at which good and bad jobs are filled, respectively, and let $p_g$ and $p_b$ denote the transition probabilities that an unemployed worker finds a good or bad job, respectively.

\[
p_g = m_g(1, \phi_g) = \phi_g q_g, \quad (3.8)
\]

\[
p_b = m_b(1, \phi_b) = \phi_b q_b. \quad (3.9)
\]
The rate, $p$, at which unemployed workers leave unemployment is given by the
two transition probabilities into good and bad jobs. Thus

$$p = p_y + p_b.$$ \hfill (3.10)

Since it is commonly observed that highly tenured workers are less likely to con-
duct on-the-job search (ref: Pissarides and Wadsworth 1994), I also assume that bad
jobs evolve into good jobs with an exogenous flow probability of $h$. On the job train-
ing is one possible motivation for the transition probability, $h$. The flow probability
that an underemployed worker finds a good job is given by $\alpha p_y + h$ where $\alpha p_y$
the flow probability of changing jobs via on-the-job search.

Let $b$ denote the rate of job separations and assume that it is the same for both
types of jobs. This job destruction rate is assumed to be exogenous - the result
of negative idiosyncratic shocks - and the same for both good and bad jobs. The
flow of workers out of unemployment is $up$ while the aggregate flow of workers into
unemployment is $(1 - u)b$. Therefore, the rate at which unemployment changes has
the form

$$\dot{u} = (1 - u)b - up$$ \hfill (3.11)

where dot notation is used to denote the time derivative. The rate at which the
supply of bad jobs changes is given by
$e_b = u_p b - e_b (b + h + \alpha p_g)$ \hfill (3.12)

where $u_p b$ is the flow of workers into bad jobs and $e_b (b + h + \alpha p_g)$ is the flow of workers out of these jobs. Finally, the rate at which the supply of good jobs changes is given by

$e_g = u_p g + e_b (h + \alpha p_g) - e_g b$ \hfill (3.13)

where $u_p g + e_b (h + \alpha p_g)$ and $e_g b$ are the flows of workers into and out of good jobs, respectively.

Asset Equations

Asset equations characterize the expected present values of each agent’s income as a function of (i) a discount factor, (ii) the flows of income in different states of the world, and (iii) the transition probabilities between these states. The asset equations are used in conjunction with a Nash Bargaining Solution and a zero profit condition to characterize the equilibrium of this model.

Let $V_g$ denote the expected value of posting a vacancy for a good job. The flow cost of this type of vacancy is $k_g$ and the flow probability of filling it is $q_g$. The asset equation of a good job vacancy is
\[ rV_g = -k_g + q_g (J_g - V_g) + \dot{V}_g. \]  \hfill (3.14)

where \( r \) is the discount factor and \( J_g \) denotes the expected value of having this job filled by a worker. A good job produces a flow of \( y_g \) units of output, the worker is paid a flow of \( w_g \) wages and the job switches back to the unmatched state with a constant probability, \( b \). Therefore, the asset equation of a filled good job is given by

\[ rJ_g = y_g - w_g + b(V_g - J_g) + \dot{J}_g. \]  \hfill (3.15)

Firms also post vacancies for bad jobs. A vacancy for a bad job is less costly to create than a vacancy for a good job but a bad job produces a smaller expected flow of output once filled. Let \( V_b \) denote the expected value of posting a vacancy for a bad job. The flow cost of this vacancy is \( \nu_b \) and the flow probability of filling this vacancy is \( q_b \). The asset equation of an unfilled bad job has the form

\[ rV_b = -k_b + q_b (J_b - V_b) + \dot{V}_b \]  \hfill (3.16)

where \( J_b \) denotes the expected value of filling a bad vacancy. A firm’s valuation of a bad job is slightly more complicated than a firm’s valuation of a good job, because (i) there is the possibility of a productivity improvement and (ii) there is the probability that the worker leaves the current match via on-the-job search. A bad job turns into a good job within the current match with a flow probability of \( h \) and the worker leaves the current match via on-the-job search with a flow probability of \( \alpha p_g \). The
THE MODEL

The asset equation of a filled bad job has the form

$$ rJ_b = y_b - w_b + (b + \alpha p_g)(V_b - J_b) + h(J_g - J_b) + \dot{J}_b $$

(3.17)

where $y_b$ is the flow of output and $w_b$ is the worker’s flow of wages.

The asset equations of workers are derived in the same way as the asset equations of firms. Let $U$ denote the expected value of an unemployed worker. The asset equation for each unemployed worker is given by

$$ rU = a + p_g(W_g - U) + p_b(W_b - U) + \dot{U} $$

(3.18)

where $a$ is the flow of unemployment income, $W_g$ is the present value of being fully-employed, $W_b$ is the expected value of being underemployed, and $p_g$ and $p_b$ are the probabilities of finding these types of employment. If the worker is fully-employed, his/her asset equation is

$$ rW_g = w_g + b(U - W_g) + \dot{W}_g $$

(3.19)

where $w_g$ is the wage to this worker and $b$ is the probability that this worker moves back to unemployment. If the worker is underemployed, the worker has a choice to conduct some amount of on-the-job search. The probability that the underemployed worker locates a good job by on-the-job search is $\alpha p_g$ and the probability that the current match is, itself, improved upon is $h$. The asset equation of an underemployed worker is as follows:
\[ rW_b = w_b - s + b(U - W_b) + \]
\[ (h + \alpha(s)p_s)(W_g - W_b) + W_b, \quad (3.20) \]

where \( w_b \) is the underemployed worker's wage and \( s \) is the flow cost of on-the-job search.

The Nash Bargaining Solution

A Nash Bargaining Solution (NBS) characterizes (i) the division of total surplus of a match and (ii) the amount of on-the-job search. The total surplus of a match is equal to the total expected value of a match minus the expected values of income that the firm and the worker would get in the unmatched state. Let \( \Sigma_g \) and \( \Sigma_b \) denote the total surplus of a good and bad match, respectively. The total surplus of each match is given by

\[ \Sigma_i = W_i - U + J_i - V_i \quad i \in \{g, b\}. \quad (3.21) \]

where \( \Sigma_i = \dot{J}_i - \dot{V}_i + \dot{W}_i - \dot{U}_i \) gives the rate of change.

One axiom of the NBS is the assumption that the total surplus of a match is split according to a sharing rule. For each type of match let \( \beta \) denote the share of surplus
that goes to the worker and let \( 1 - \beta \) denote the share of surplus that goes to the firm. The sharing rule implies

\[
\beta \Sigma_i = W_i - U, \tag{3.22}
\]

\[
(1 - \beta) \Sigma_i = J_i - V_i. \tag{3.23}
\]

Differentiating with respect to time, I obtain \( \dot{\Sigma} = \dot{W} - \dot{U} \) and \( (1 - \beta) \dot{\Sigma} = \dot{J} - \dot{V} \).

A second axiom of the NBS is the assumption that the participants of a match maximize the total joint surplus of a match. This axiom is relevant to the decision to conduct on-the-job search.\(^1\) The added joint surplus of a worker finding a match through on the job search is given by

\[
\alpha(s) p_g (\beta \Sigma_g - \Sigma_b) - s \tag{3.24}
\]

where \( \alpha(s) p_g \) is the probability \( p_g \) of finding a better match with another firm, \( \beta \Sigma_g - \Sigma_b \) is the added surplus of such a match, and \( s \) is the search investment. A maximum joint surplus is obtained by \( s^* \) such that

\[
\alpha'(s^*) p_g (\beta \Sigma_g - \Sigma_b) = 1. \tag{3.25}
\]

The worker and firm share a common interest in on the job search. This is because

\(^1\)Such 'within the match' decisions are also found in Pissarides (1990) and Andolfatto (1994).
the current match.

The equilibrium supply of vacancies is determined by free entry. A free entry condition implies that the expected value of a vacant job, good or bad, is equal to zero at all times. This condition is given by

$$V_g = V_b = 0.$$  \hspace{1cm} (3.26)

Since this condition characterizes the equilibrium at all times, the time derivatives of $V_g$ and $V_b$ are also equal to zero. The Nash Bargaining Solution and the free entry condition imply that the asset equation of each type of vacancy can be rewritten as follows:

$$k_g = q_g(1 - \beta)\Sigma_g,$$  \hspace{1cm} (3.27)

$$k_b = q_b(1 - \beta)\Sigma_b.$$  \hspace{1cm} (3.28)

The asset equations and the two axioms of the Nash bargaining solution lead to two differential equations which describe changes to the total surplus of each type of match. The differential equations for total surpluses of good and bad matches take the forms

$$\dot{\Sigma}_g = (r + b + \beta p_g + (1 - \beta)q_g)\Sigma_g + \beta p_b \Sigma_b - Y_g$$  \hspace{1cm} (3.29)

and
\[ \dot{\Sigma}_b = (r + b + \beta p_b + (1 - \beta)q_b)\Sigma_b + \beta p_g \Sigma_g \]

\[ -h(\Sigma_g - \Sigma_b) - (\alpha(s^*)p_g(\beta \Sigma_g - \Sigma_b) - s^*) = Y_b \quad (3.30) \]

where \( Y_g \equiv y_g - a - (-k_g) \) and \( Y_b \equiv y_b - a - (-k_b) \) are the net flows of income of each type of match. In a stationary equilibrium, the total surplus of each match is proportional to the net flow of income of a match, inversely related to the discount factor and the job destruction rate, and falls if the probability of matching is high. In a stationary equilibrium, the total surplus of a bad job is also proportional to the rates of productivity improvements, either within the current match, or by on-the-job search. The two dimensional phase diagram in figure 1 illustrates the relationship between the total surpluses of good and bad matches. A derivation of the properties of this phase diagram is given in the appendix.
Figure 2.1: Phase Diagram for the total surpluses of good and bad jobs.

Figure 1 illustrates a unique equilibrium at \((\Sigma^*_g, \Sigma^*_b)\) with all adjustment paths leading away from it. The dynamic adjustment of non-predetermined variables - the total surpluses of good and bad jobs - is instantaneous. The system explodes unless it is always at steady state equilibrium. Therefore, as in Pissarides (1985), the solution to this system is its steady state equilibria. When the state of nature changes, \(\Sigma^*_g\) and \(\Sigma^*_b\) jump immediately from one steady state to the other, and \(\dot{\Sigma}_g = \dot{\Sigma}_b = 0\) all the time.

A permanent aggregate productivity shock which raises \(y_g\) and \(y_b\) causes an immediate adjustment in \(\Sigma_g\) and \(\Sigma_b\) as the two demarcation curves move outward. According to equations (2.27) and (2.28), an increase in the total surplus of a match, \(\Sigma'_i\) increases labor market tightness, \(\phi_i\), increases the job finding rate, \(p_i\), and decreases the rate at which a vacancy is matched, \(q_i\), because \(p_i\) is a positive function of \(\phi_i\) and
\( q_i \) is a negative function of \( \varphi_i \). According to equation (2.25), the other endogenous variable - on-the-job search intensity - is a positive function of the total surplus of a good job and a negative function of the total surplus of a bad job.

**Wages**

The Nash bargain solution splits the total surplus of a match by a sharing rule and this sharing rule determines the wage of each type of worker. The wage of a fully-employed worker is given by

\[
w_g = a + \beta(y_g - a) + \beta_k y_g + \beta_k b.
\]

(3.31)

The fully-employed workers' wage is an increasing function of labor market tightness. The wage of an underemployed worker is similar, but it is also influenced by the amount of on-the-job search. The wage equation of this type of worker has the form

\[
w_b = a + \beta(y_b - a) + \beta_k y_b + \beta_k b + (s^* - \alpha(s^*)p_g)(\beta \Sigma_g - \Sigma_b).
\]

(3.32)

The wage equation of underemployed workers implies that more on-the-job search, other things equal, causes lower wages. This feature arises because the on-the-job
search decision is made within a match and equation (2.28) of the Nash bargaining solution guarantees that the benefits of such decisions are split between the worker and the firm. On-the-job search benefits the worker by allowing him/her to search for a higher paying good job elsewhere and benefits the firm by allowing it to pay this worker a lower flow of current wages (ref: Pissarides 1994).

The Beveridge curve

The Beveridge curve is the steady state relationship between unemployment and vacancies (ref: Pissarides 1987). In this section, I derive the Beveridge curve by isolating values of steady state unemployment and vacancies which are consistent with different levels of aggregate productivity. Assuming that the steady state unemployment rate falls as aggregate labor market tightness increases, the Beveridge curve is downward sloping.\footnote{According to figure 1, the arrival rate of one of the two types of jobs could fall as aggregate productivity increases, because the total surplus of one of these jobs could decrease. Therefore, it is within the realm of possibilities to generate an upward sloping Beveridge curve.} Movements around this Beveridge curve are characterized by changes to aggregate labor market tightness - the vacancy: unemployment ratio. Define

\[
\phi \equiv \frac{v_b + v_s}{u} = \frac{\text{Total vacancies}}{\text{Unemployment}}.
\]

For my purposes it is convenient to define a variable \( \psi \) as the underemployment: unemployment ratio. The underemployment: unemployment ratio is given by

\[
\psi \equiv \frac{c_u}{u} = \frac{\text{Underemployment}}{\text{Unemployment}}.
\]
THE BEVERIDGE CURVE

This ratio gives the relative availability of underemployed and unemployed workers in the work force.

The underemployment: unemployment ratio is a function of job finding rates. In particular, in a steady state, the underemployment: unemployment ratio is given by

$$\psi = \frac{p_b}{b + h + \alpha p_g}.$$  \hfill (3.33)

because equation (2.12) implies $e_b = p_b u / (b + h + \alpha p_g)$. According to this expression, the increases to $\alpha p_g$ and $p_b$, which characterize an economic recovery, can either increase or decrease the underemployment: unemployment ratio. The key issue is how the mix of good and bad jobs changes. Suppose, that an increase in the average productivity of job matches leads to a large increase in the supply of good jobs relative to bad jobs, such that $\alpha p_g$ increases significantly more than $p_b$. In this case, the underemployment: unemployment ratio must decrease during the course of a recovery as the supply of good jobs crowds out the supply of bad jobs. The reverse scenario occurs if an increase in the aggregate productivity of job matches cause $\alpha p_g$ and $p_b$ to increase proportionally, or if $p_b$ increases more than $\alpha p_g$. In this case, the underemployment: unemployment ratio increases during the course of a recovery as the supply of bad jobs crowds out the supply of good jobs. Both scenarios are possible, because two forces are at work. On the one hand, a proportional aggregate productivity shock to job matches causes a larger increase in the absolute productivity of a good job than a bad job without changing the cost of posting either vacancy. Thus
the supply of good jobs in the pool of job vacancies will tend to increase proportionally more than the supply of bad jobs. On the other hand, the Nash bargaining solution guarantees that firms posting bad jobs share some of the total surplus of good jobs via on-the-job search. Therefore, increases in the supply of bad jobs will tend to be larger than is the case without on-the-job search. Consequently, the underemployment: unemployment ratio can follow either an upward or a downward adjustment path during the course of a recovery. The impact of these two possibilities on the other endogenous parameters is the focus of the remaining analysis.

Given the definitions of labor market tightness for good and bad jobs \((\phi_b, \phi_g)\), the intensity of on-the-job search \((\alpha)\), and the underemployment: unemployment ratio \((\psi)\), the aggregate labor market tightness variable is given by the following function:

\[
\phi = \phi_b + \phi_g (1 + \alpha \psi).
\]  

(3.34)

Three of the four variables in this expression are non-predetermined jump variables while the fourth variable is a predetermined variable. The three non-predetermined variables \(\phi_b, \phi_g\) and \(\alpha\) all jump to their steady state levels whenever productivity changes. The only variable which adjusts slowly to changes in economic conditions is the underemployment: unemployment ratio, \(\psi\).\(^3\)

An important feature of the present model is that the aggregate labor market

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\(^3\)The analysis of temporary shocks is similar to the analysis of permanent shocks. If we introduce a stochastic process with transition probabilities between aggregate productivity states, all adjustments of \(\phi_b, \phi_g\), and \(\alpha\) take place instantaneously whenever the state changes. These changes cause an immediate jump in \(\phi\). After that jump, \(\phi\) adjusts up or down depending upon the changes in \(\psi\). The only difference between a permanent shock and a temporary shock is that the original jump in \(\phi\) is generally smaller when the shock is temporary.
tightness variable contains a predetermined component. Therefore, aggregate labor market tightness is itself partly predetermined. According to equation (2.33), if there is a positive productivity shock and the underemployment: unemployment ratio follows an upward adjustment path, aggregate labor market tightness will undershoot the steady state value to which it converges. However, if the underemployment: unemployment ratio adjust downwards then aggregate labor market tightness overshoots its future value. The relationship between changes in aggregate labor market tightness and the underemployment: unemployment ratio are illustrated in figure 2 below.

![Graph showing dynamic adjustment of unemployment and vacancies](image)

Figure 2.2: Dynamic Adjustment of Unemployment and Vacancies

The path from a to b to c is consistent with a positive productivity shock which is then followed by a gradual decrease of the underemployment: unemployment ratio. In
THE BEVERIDGE CURVE

this case, aggregate labor market tightness overshoots its terminal value, $\phi'$. The path from to $a$ to $c$ to $e$ is consistent with another positive productivity shock, however, this time the initial increase in aggregate labor market tightness is then followed by a gradual increase of the underemployment: unemployment ratio. In this case, aggregate labor market tightness undershoots its terminal value, which is also assumed to be $\phi'$. The final path from $a$ to $d$ to $e$ is what happens if there no on-the-job search. In this case the model is essentially identical to the class of search models studied by Pissarides (1990) and others. In these models, aggregate labor market tightness is always non-predetermined, and thus immediately adjusts to a new steady state value whenever the aggregate productivity state changes.

Pissarides (1994) suggests that the sensitivity of aggregate labor market tightness to changes in aggregate productivity may vary from country to country. The present chapter offers a potential explanation of such differences using the underemployment: unemployment ratio. A dampened response of aggregate labor market tightness to changes in aggregate productivity occurs if the underemployment: unemployment ratio rises during an upturn and falls during a downturn. In this case, aggregate labor market tightness increases very little at the start of a recovery because the low proportion of underemployed workers in the pool of job searchers at the start of a recovery attracts a disproportionately small number of additional vacancies. By contrast, an amplified response of aggregate labor market tightness to changes in aggregate productivity occurs if the underemployment: unemployment ratio falls
during an upturn and increases during a downturn. In this case, aggregate labor market tightness increases a lot at the start of a recovery because the high proportion of underemployed workers in the pool of job searchers at the start of a recovery attracts a disproportionately large number of additional vacancies.

A key prediction of Pissarides (1987) is that the job finding rate of unemployed workers is only a function of aggregate labor market tightness. The model of the present chapter predicts that similar unemployment outflow rates may be consistent with different levels of aggregate labor market tightness. The underemployment: unemployment ratio is a key variable behind this phenomenon. This prediction can be derived analytically in relation to Pissarides (1987) model by introducing several simplifying assumptions. The first assumption is that the aggregate matching technology consists of identical constant returns to scale Cobb-Douglas functions (ref: Diamond and Blanchard 1989, Pissarides 1990). The flow of \( m_j \) matches satisfies

\[
m_j(s_j, v_j) = \frac{k}{2} s_j^{1-\gamma} v_j^\gamma \quad j \in \{g, b\}
\]  

(3.35)

where \( s_j \) is the number of searchers, \( v_j \) is the number of vacancies, and \( k \) is an exogenous efficiency parameter. A second assumption is that the two demarcation curves of figure 1 vary symmetrically to changes in aggregate conditions. In this case, labor market tightness in the good and bad labor markets are always equal.

\[
\phi_b = \phi_g.
\]

(3.36)
This assumption guarantees that job vacancies can not be reallocated between the two job markets to get a higher aggregate flow of job matching. This equality implies that there is no labor market mismatch of the kind summarized in Layard, Nickell and Jackman (1991). A final assumption is that the intensity of on-the-job search is exogenous. These three simplifying assumptions imply that the job finding rate of unemployed workers has the form

\[ p = \beta_o \phi (1 + \beta_1 \psi)^{-1} \gamma. \]  

(3.37)

where \(\beta_1 = \alpha/2\), and \(\beta_o = k/2\gamma\). Taking the logarithm of both sides

\[ \ln p = \ln \beta_o + \gamma (\ln \phi - \ln (1 + \beta_1 \psi)). \]  

(3.38)

In this model, an increase in the underemployment: unemployment ratio, other things equal, decreases the job finding rate of unemployed workers. This prediction can be isolated in relation to Pissarides’ (1987) benchmark model by linearizing equation (2.38). A first-order Taylor series expansion of \(\ln (1 + \beta_1 \psi)\) around \(\beta_1 = 0\) (i.e. \(\alpha = 0\)) yields

\[ \ln p = \ln \beta_o + \gamma \ln \phi - \gamma / \beta_1 \psi. \]  

(3.39)

This expression is simplified if on-the-job searchers are not important inputs in the matching process involving unemployed workers and job vacancies. This might happen if the vacancies for good jobs are directed at unemployed workers and on-the-job
searchers find good jobs by other means. In this case, $\alpha = 0$ and the model reduces to

$$\ln p = \ln \beta_0 + \gamma \ln \phi$$  \hspace{1cm} (3.40)

This expression is equivalent to the prediction of Pissarides' (1987) model under the assumption of a Cobb-Douglas constant returns to scale matching technology. It demonstrates how my model reduces to Pissarides model if on-the-job searchers do not enter the matching process.

**Conclusions**

The model predicts that the job finding rate of unemployed workers is related to aggregate labor market tightness and the underemployment: unemployment ratio. The dynamic behavior of the underemployment: unemployment ratio leads to a related prediction that the vacancy: unemployment ratio can either overshoot or undershoot futures values. Consequently, if the underemployment: unemployment ratio increases during a expansion and then decreases during a downturn, the model predicts cyclical changes in aggregate labor market tightness are dampened. By contrast, if the underemployment: unemployment ratio decreases during an expansion and then increases during a recession, the model predicts cyclical changes in aggregate labor market tightness are amplified.
APPENDIX II

Appendix 2.1. Derivation of equations (2.29) and (2.30)

Equation (2.29) and (2.30) are derived from the asset equations, the Nash Bargaining Solution, and the free entry condition for good and bad job vacancies. The derivation involves simple algebraic manipulation of these expressions. Equation (2.15) minus equation (2.14) gives

\[(r + b + q_g)(1 - \beta)\Sigma_g = y_g - w_g - k_g + (1 - \beta) \dot{\Sigma}_g \]  \hspace{1cm} (3.41)

Equation (2.17) minus equation (2.16) gives

\[(r + b + q_b)(1 - \beta)\Sigma_b = y_b - w_b - k_b \]

\[+ h(1 - \beta)(\Sigma_g - \Sigma_b) - \alpha p_g(1 - \beta)\Sigma_g + (1 - \beta) \dot{\Sigma}_b \] \hspace{1cm} (3.42)

Equation (2.19) minus equation (2.18) gives

\[(r + b + p_g)\beta \Sigma_g + p_b \beta \Sigma_b = w_g - a + \beta \dot{\Sigma}_g \]  \hspace{1cm} (3.43)

Equation (2.20) minus equation (2.18) gives

\[(r + b + p_b)\beta \Sigma_b + p_g \beta \Sigma_g = w_b - a - s + (h + \alpha p_g)\beta(\Sigma_g - \Sigma_b) + \beta \dot{\Sigma}_b \] \hspace{1cm} (3.44)
Equation (2.41) plus equation (2.43) gives

\[ \dot{\Sigma}_g = (r + b + \beta p_g + (1 - \beta)q_g)\Sigma_g + \beta p_b \Sigma_b - Y_g \]  

(2.29)

where \( Y_g \equiv y_g - a - (-k_g) \). Equation (2.42) plus equation (2.44) gives

\[ \dot{\Sigma}_b = (r + b + \beta p_b + (1 - \beta)q_b)\Sigma_b + \beta p_g \Sigma_g \]

\[-h(\Sigma_g - \Sigma_b) - (\alpha(s^*)p_g(\beta \Sigma_g - \Sigma_b) - s^*) - Y_b \]  

(2.30)

where \( Y_b \equiv y_b - a - (-k_b) \).

Appendix 2.2. Derivation of the phase diagram in Figure 1.

Two simplifying assumptions:

A1: \( m_i(s_i, v_i) = \Lambda s_i^{\alpha} v_i^{1 - \alpha} \quad i \in \{g, b\} \)

A2: \( \alpha(s) = \frac{1}{1 - \gamma} s^{1 - \gamma} \)

These assumptions simplify the derivation of the phase diagram, because

(i) A1 implies \( p_i = \delta q_i^{-\lambda} \) where \( \delta = A^{1/a}, \lambda = (1 - a)/a \geq 0, \) and

(ii) A2 implies \( \alpha(s^*)p_g(\beta \Sigma_g - \Sigma_b) - s^* = \frac{1}{1 - \gamma}(p_g(\beta \Sigma_g - \Sigma_b))^{1/\gamma}, 0 \leq \gamma \leq 1. \)

Equations (2.27) and (2.28) imply \( q_i = k_i/((1 - \beta)\Sigma_i) \quad i \in \{g, b\} \)

Assumptions A1 and A2 imply that equations (2.29) and (2.30) can be rewritten as follows:

\[ F \equiv \dot{\Sigma}_g = -Y_g + (r + b)\Sigma_g + k_g + \delta \beta \Sigma_g^{1+\frac{\lambda}{k_g}} + \delta \beta \Sigma_b^{1+\frac{\lambda}{k_b}} \left(\frac{1 - \beta}{k_b}\right)^{\frac{\lambda}{k_b}} \]
\[ G \equiv \dot{\Sigma}_b = -Y_b + (r + b)\Sigma_b + k_b + \delta \beta \Sigma_b^{1+\lambda} \left( \frac{1 - \beta}{k_b} \right)^\lambda + \delta / \beta \Sigma_g^{1+\lambda} \left( \frac{1 - \beta}{k_g} \right)^\lambda + h(\Sigma_g - \Sigma_b) - \frac{\gamma}{1 - \gamma} \left( \delta \left( \frac{(1 - \beta)\Sigma_g}{k_g} \right)^\lambda \left( \beta \Sigma_g - \Sigma_b \right) \right)^{\frac{1 - \gamma}{\gamma}} \]

The partial derivatives of \( F \) and \( G \) are as follows:

\[ F_{\Sigma_g} = r + b + (1 + \lambda) \delta \beta \left( \frac{\Sigma_g(1 - \beta)}{k_g} \right)^\lambda \]

\[ F_{\Sigma_b} = (1 + \lambda) \delta \beta \left( \frac{\Sigma_b(1 - \beta)}{k_b} \right)^\lambda \]

\[ G_{\Sigma_g} = (1 + \lambda) \delta \beta \left( \frac{\Sigma_g(1 - \beta)}{k_g} \right)^\lambda - h - \frac{1}{1 - \gamma} \left( \delta \left( \frac{(1 - \beta)\Sigma_g}{k_g} \right)^\lambda \left( \beta \Sigma_g - \Sigma_b \right) \right)^{\frac{1 - \gamma}{\gamma}} \]

\[ \times \left( \delta \lambda \left( \frac{(1 - \beta)\Sigma_g}{k_g} \right)^{\lambda - 1} \left( \beta \Sigma_g - \Sigma_b \right) + \delta \left( \frac{(1 - \beta)\Sigma_g}{k_g} \right)^\lambda \beta \right) \]

\[ G_{\Sigma_b} = r + b + (1 + \lambda) \delta \beta \left( \frac{\Sigma_g(1 - \beta)}{k_g} \right)^\lambda + h + \frac{1}{1 - \gamma} \left( \delta \left( \frac{(1 - \beta)\Sigma_g}{k_g} \right)^\lambda \left( \beta \Sigma_g - \Sigma_b \right) \right)^{\frac{1 - \gamma}{\gamma}} \delta \left( \frac{(1 - \beta)\Sigma_g}{k_g} \right)^\lambda \]
These partial derivatives ensure that

\[
\left| \frac{F_{\Sigma_b}}{F_{\Sigma_g}} \right| < \left| \frac{G_{\Sigma_g}}{G_{\Sigma_b}} \right|
\]

because \( F_{\Sigma_b} < G_{\Sigma_b} \) and \( G_{\Sigma_g} < F_{\Sigma_g} \). Therefore, the slope of the demarcation curve for \( \Sigma_b = 0 \) is greater than the demarcation curve for \( \Sigma_g = 0 \). The partial derivatives of \( F \) and \( G \) also satisfy

\[
\frac{d \Sigma_g}{d \Sigma_g} = F_{\Sigma_g} \geq 0
\]

\[
\frac{d \Sigma_b}{d \Sigma_b} = G_{\Sigma_b} \geq 0
\]

Therefore, \( \Sigma_g \) and \( \Sigma_b \) change according to the arrows of motion in the phase diagram of figure 1.///
Chapter 4

Insiders in Search Equilibrium

Introduction

A debate in insider-outsider theory has developed over whether insiders face single-tier or two-tier wage structures. On one hand, a number of insider-outsider models such as Lindbeck and Snower (1988) and Begg (1988) assume that insiders face two-tier (seniority) wage structures. On the other hand, other insider-outsider models such as Drazen and Gottfries (1992) and Layard, Nickell, and Jackman (1991) assume that the wage structure facing insiders is single-tier (non-discriminatory). More recent research has sought to endogenize the choice of wage structure.\(^1\) For example, Collier (1991) argues that insiders always choose a single-tier wage structure while Andersen and Vetter (1995) derive the opposite result.

The present chapter asks two basic questions. Do complementarities between experienced insiders and inexperienced outsiders lead to a single-tier or a two-tier wage structures? And, how are such complementarities related to the dynamic adjustment of macroeconomic variables such as unemployment, labor market tightness (the vacancy: unemployment ratio), and insider wages? In order to address these questions,

\(^1\)Some preliminary evidence on choices between single-tier and two-tier wage structures for the airline industry in the United States is given by Card (1986).
INTRODUCTION

I develop a simple overlapping generations job search model with complementarities between experienced insiders and inexperienced outsiders. The model has two main components: a labor demand function and an insider objective function. The specification of the labor demand function is quite general. Individual firms hire workers until the expected flow of profits equals the cost of recruiting. The specification of insider objectives is more specific. First, I assume the insider-outsider distinction is between unemployed and incumbent workers. Second, I assume insiders simply maximize their own wages. In this monopoly unionist structure, the key endogenous variables are insider wages, unemployment, and the supply of job vacancies.

I find that insiders strictly prefer a two-tier wage structure over a single-tier wage structure, if there are complementarities between experienced and inexperienced workers. In this case, the insider wage is an increasing function of the number of new hires, because experienced insiders demand a wage equal to their marginal product which is an increasing function of their own relative scarcity. If experienced and inexperienced workers are substitutes, insiders are indifferent over the choice of wage structure. The choice of wage structure has important implications for the aggregate level of unemployment and vacancies. I find unemployment is lower and vacancies are higher under a two-tier wage structure than under a single-tier wage structure. Firms post more vacancies, because a two-tier wage structure permits lower wages for new hires.

Begg (1988) uses complementarities between experienced and inexperienced work-
INTRODUCTION

ers to explain unemployment and wage dynamics. He finds that complementarities leads to unemployment persistence in response to a temporary productivity shock. In this chapter, Begg's analysis is extended to a labor market with search and recruiting. My model predicts long-run changes to labor market tightness in response to short-run productivity shocks. The adjustments are the consequence of temporary shortages or surpluses of experienced insiders. An implication of my model is that we can not look to the behavior of labor market tightness for evidence with which to distinguish between alternative mechanisms of unemployment persistence. For example, Pissarides (1992) generates such movements in a model with loss of skills by the long-term unemployed.

This chapter also shows that complementarities generate co-movements between insider wages and labor market tightness. In particular, the model predicts that insider wages also converge slowly to a steady state after an initial productivity shock. Complementarities lead to insider wage adjustments, because the insider wage is then a function of the relative scarcity of experienced insiders and inexperienced outsiders. Similar co-movements between insider wages and labor market tightness do not characterize Pissarides' model, because he assumes experienced and inexperienced workers are substitutes. The introduction of complementarities between experienced and inexperienced workers can be seen as one possible method in which to enlarge the set of predictions offered by Pissarides' framework.

The rest of the chapter is organized as follows. Section 2 outlines the model.
This section also solves the labor demand function and the insiders’ objective function. Section 3 derives the model’s equilibrium and I evaluate insider preferences for single-tier and two-tier wage structures. Section 4 derives some dynamic implications of the theory. The final section of this chapter offers some concluding remarks.

The model

Consider an infinite horizon model of the labor market with two-period lived overlapping generations of workers. Workers have linear preferences over lifetime consumption and a common discount factor, $\beta$. In any period, $t$, the total population consists of $N_{1t}$ young workers and $N_{2t}$ old workers. I assume zero population growth. Therefore, $N_{1t}$ and $N_{2t}$ are given by a constant, $N$.

Each generation consists of employed and unemployed workers. Employed workers are experienced or inexperienced. The stock of $W_t$ inexperienced workers is given by

$$W_t = W_{1t} + W_{2t} \quad (4.1)$$

where $W_{1t}$ and $W_{2t}$ denote the number of young and old inexperienced workers, respectively. I assume that young inexperienced workers continue their employment in the next period, therefore, the current stock of $M_{2t}$ experienced workers is last period’s stock of young inexperienced workers. In this case,

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2 Other infinite horizon models of the labor market with two-period lived agents include Weiss (1985), Begg (1988), Pissarides (1992), Demougin and Slow (1994), Bean and Pissarides (1993) and Davidson, Martin and Matusz (1994).

3 Bean and Pissarides (1993) show that the introduction of more general concave preferences makes the supply of physical capital interesting as savings decisions would then be related to employment status.
THE MODEL

\[ M_t = W_{1t-1}. \]  \hspace{1cm} (4.2)

Experienced workers are called insiders to denote their bargaining status as incumbent workers. Let \( n_t \) denote the inexperienced: experienced worker ratio. Define

\[ n_t \equiv W_t/M_t. \]

All workers who are not employed are unemployed. Let \( u_{1t} \) and \( u_{2t} \) denote the unemployment rates of young and old workers, respectively. By definition, \( u_{1t} \equiv (N - W_{1t})/N \) and \( u_{2t} \equiv (N - W_{2t} - M_t)/N. \)

Employment is determined by search and recruiting. At the beginning of each period a group of young and old workers search for a job. The total supply of \( S_t \) job searchers is given by

\[ S_t = S_{1t} + S_{2t}, \]  \hspace{1cm} (4.3)

where \( S_{1t} \) and \( S_{2t} \) denote the number of young and old job searchers, respectively.

Let \( \psi_t \) denote the fraction of young agents in the pool of job searchers. Define

\[ \psi_t \equiv S_{1t}/S_t. \]

The stock of \( S_{1t} \) young job searchers, the short-term unemployed, is the total population of young agents. Thus

\[ S_{1t} = N. \]  \hspace{1cm} (4.4)
THE MODEL

The stock of old job searchers is drawn from the pool of long-term unemployed workers. This pool consists of workers who were not employed in the previous period. I assume that long-term unemployed workers engage in job search with a probability less than or equal to one. I write

\[ S_{2t} = \theta(N - M_{2t}), \quad (4.5) \]

where \( \theta \in [0, 1] \). If \( \theta \) is less than one, a long-term unemployed worker is a less effective job searcher than a short-term unemployed worker.

The total level of recruiting is given by a stock of \( V_t \) job vacancies. Firms can open and close job vacancies at any time. The cost of posting a job vacancy is \( k \).

In this model, labor market tightness is measured by the vacancy: job searcher ratio. Let \( \phi_t \) denote labor market tightness. Define

\[ \phi_t \equiv V_t/S_t. \]

A matching technology gives the number of \( W_t \) new hires as a function of the pool of \( V_t \) job vacancies and the pool of \( S_t \) job searchers. This function has the form

\[ m(S_t, V_t). \quad (4.6) \]

The matching technology is homogeneous of degree one and increasing in each argument. The two assumptions imply (i) the absolute size of the population is not a crucial determinant of the unemployment rate and (ii) an increase to the stock of job
THE MODEL

vacancies or the stock of job searchers never lowers the total number of matches. The probability \( q_t \) that a job vacancy is matched is given by

\[
q_t = q(\phi_t) \equiv m(1/\phi_t, 1). \tag{4.7}
\]

The rate at which job vacancies are matched is a decreasing function of labor market tightness, because the matching technology is increasing in its first argument. The probability \( p_t \) that a job searcher finds employment is given by

\[
p_t = p(\phi_t) \equiv m(1, \phi_t). \tag{4.8}
\]

where \( p(\phi_t) \) is an increasing function of labor market tightness, because the matching technology is increasing in its second argument. The job finding rate and the fraction of young agents in the pool of job searchers give \( W_{1t} = p(\phi_t)\psi_t S_t \) and \( W_{2t} = p(\phi_t)(1 - \psi_t) S_t \).

A firm employing \( M_t \) insiders, \( W_{1t} \) inexperienced young workers and \( W_{2t} \) inexperienced older workers produces a total of \( Y_t \) units of output according to a well behaved production function. This function has the general form

\[
Y_t = \lambda_t F(M_t, W_{1t}, W_{2t}), \tag{4.9}
\]

where \( \lambda_t \) is an aggregate productivity parameter. For simplicity, I assume that old and young inexperienced workers are substitutes, but I also allow for the possibility that such workers are not equally productive. Therefore, I rewrite the production
technology as follows:

$$\lambda_t F(M_t, W_{1t}, W_{2t}) = \lambda_t F(M_t, W_t; \psi_t), \quad (4.10)$$

where $\psi_t$ gives the fraction of inexperienced workers who are young. The production technology, $\lambda_t F(M_t, W_t; \psi_t)$, is assumed to be increasing in both arguments, differentiable, concave and homogeneous of degree one in the first arguments.

My attention now turns to the decisions of insiders and firms. The first step in solving this model is to derive the labor demand function. The labor demand function gives wage, employment combinations that are consistent with profit maximization by decentralized firms. Insiders pick points on this function to maximize their objectives. Each decision maker takes as given labor market tightness and the fraction of inexperienced workers who are young. The notation I adopt is as follows: a prime (') indicates a variable at a representative firm, a star (*) indicates a maximizing decision of the firm and a double star (**) indicates a maximizing decision of insiders employed by this firm.

The labor demand function

The firm chooses a sequence $\{W_{1t}', W_{2t}', M_t', V_t'\}$ over periods 1 to $T + 1$ to maximize the expected flow of discounted profits in each period. The firm takes as given the sequence of insider determined wages. Let $w_{1t}'$ and $w_{2t}'$ denote the wages of young and
old inexperienced workers, respectively, and \( y'_t \) denotes the wages of insiders. The firm also takes as given the initial stock of experienced insiders, \( M'_0 \). The maximum level of profits at a representative firm, \( \Pi' \), is given by the following maximization problem

\[
\Pi' = \max_{\{W'_{1t}, W'_{2t}, M'_t, V'_t\}} \sum_{t=1}^{T+1} \beta^t (\lambda_i F(M'_t, W'_{1t}, W'_{2t}) - x'_{1t} W'_{1t} - x'_{2t} W'_{2t} - y'_t M'_t - kV'_t) \quad (4.11)
\]

such that

\[
W'_{1t} = \psi_t V'_t q(\phi_t) \\
W'_{2t} = (1 - \psi_t) V'_t q(\phi_t) \\
M'_{t+1} = W'_{1t}
\]

where \( \lambda_i F(M'_t, W'_{1t}, W'_{2t}) - x'_{1t} W'_{1t} - x'_{2t} W'_{2t} - y'_t M'_t - kV'_t \) is profits in the current period. A firm anticipates that if it posts \( V'_t \) vacancies it will hire \( \psi_t V'_t q(\phi_t) \) young inexperienced workers and \((1 - \psi_t) V'_t q(\phi_t) \) old inexperienced workers. The firm is also aware that only young inexperienced workers are around in the next period. Both \( \psi_t \) and \( \phi_t \) are outside the control of a representative firm. The constraints are binding, therefore, the firm's problem has the form

\[
\Pi' = \max_{\{M'_{t+1}\}} \sum_{t=0}^{T-1} \beta^t (\lambda_i F(M'_t, M'_{t+1}/\psi_t; \psi_t) - x'_{1t} M'_{t+1} - x'_{2t} M'_{t+1} (1 - \psi_t)/\psi_t 
\]
\[-y'_t M'_t - k M_{t+1}'/\psi_t q(\phi_t)\]  

(4.12)

The first order conditions yield a second-order difference equation which represents the labor demand function. The appendix of this chapter shows that the solution to the representative firm's problem is a maximum. The labor demand function is given by

\[k = q(\phi_t)[\lambda_t F_M(M^*_t, M_{t+1}^*/\psi_t; \psi_t) - x'_t\psi_t - x'_M(1 - \psi_t)]\]  

(4.13)

\[+\psi_t\beta(\lambda_{t+1} F_I(M_{t+1}^*, M_{t+2}^*/\psi_{t+1}; \psi_{t+1}) - y'_{t+1})\]

The cost \(k\) of posting a vacancy is on the left hand side of the labor demand function while the benefits of posting a vacancy are on the right hand side. The probability that a vacancy is filled is \(q(\phi_t)\) and the benefits of hiring an inexperienced worker extend into two periods. The benefit in the first period is the marginal product of an inexperienced worker minus the inexperienced workers' wage. The benefits in the second period are discounted by the common discount factor, \(\beta\) and are multiplied by the probability \(\psi_t\) that the inexperienced worker in the first period is young. The benefit of having a young inexperienced worker move into the second period as an experienced worker is the future marginal product of an experienced worker minus the future wage of that experienced worker. What remains to be confirmed is that firms take the wage decisions of insiders as given. It is important to show a firm
THE MODEL

cannot influence future insider wages by its own hiring decision. Otherwise, the
firm’s decision problem is more complicated than as stated in this section.

The objective function of insiders

I assume wages are set by insiders at the beginning of each period. The objective
of insiders at each firm is to choose the maximum wage consistent with their contin-
ued employment. I solve the insiders’ decision problem at a representative firm by
supposing the insiders choose wages to maximize their own wages as if they can also
choose the stock of inexperienced workers and the profits of the firm. I consider two
possible wage structures - a single-tier wage structure and a two-tier wage structure.
A single-tier wage structure pays all workers the same wage regardless of seniority. A
two-tier wage structure allows seniority wages. A single-tier wage structure is defined
as follows:

Definition 1 In a single-tier wage structure, all workers at a representative firm
earn the same wage such that $x'_t = x'_{1t} = x'_{2t} = y'_t$ where $x'_t$ denotes the common wage
of inexperienced workers.

Under a single-tier wage structure, insiders choose a common wage $y'_t$ for themselves
and inexperienced workers. The insider decision problem is as follows:

$$y^{**}_t = \max_{\pi'_t, W'_t, V'_t} \left( \lambda_t F'(M'_t, W'_t; \psi_t) - kV'_t - \pi'_t \right)/(W'_t + M'_t)$$  \hspace{1cm} (4.14)
such that

$$\pi'_i \geq 0$$

$$W'_i = V'_i q(\phi_i).$$

The two constraints in the single-tier wage structure are (a) the firm earns positive profits and (b) the hiring decision obeys the matching process.

An alternative wage structure corresponding to seniority wages is a two-tier wage structure.

**Definition 2** In a two-tier wage structure, experienced and inexperienced workers can earn different wages, but all inexperienced workers at a representative firm receive the same wage such that $$x'_i = x'_{1i} = x'_{2i} \geq \bar{w}$$ where $$\bar{w}$$ denotes a lower bound on the wages of inexperienced workers.

The lower bound on the two-tier wage structure reflects a diverse menu of items including outside options and institutional constraints such as minimum wage regulations. I assume this lower bound is exogenous. In a two-tier wage structure, insiders choose a wage for themselves and a different wage for new hires as follows;

$$y'^* = \max_{\pi'_i, x'_i, W'_i, \psi'_i} (\lambda_i F(M'_i, W'_i; \psi_i) - x'_i W'_i - kV'_i - \pi'_i) / M'_i$$  \hspace{1cm} (4.15)

such that

$$\pi'_i \geq 0,$$
\[ x_i' \geq \bar{w}, \]

\[ W_i' = V_i' q(\phi_i) \]

The three constraints are (a) the firm earns non-negative profits; (b) the wage of inexperienced workers is greater than the lower bound; and (c) the hiring decision obeys the matching process.

The solution of the insiders' decision problem is straightforward under either wage structure. First, insiders choose the following zero profit condition for the firm:

\[ \pi_i^{**} = 0. \quad (4.16) \]

This condition implies that insider decisions are always on the labor demand function. Second, the number of inexperienced workers that the insiders choose to hire, satisfies

\[ q(\phi_i)(\lambda_t F_z(M_i',W_i^{**};\psi_t) - x_i^{**}) = k. \quad (4.17) \]

The left hand side of this expression is the benefit of hiring an additional inexperienced worker and the right hand side is the cost. The maximized insider wage is given by

\[ y_i^{**} = \lambda_t F_1(M_i',W_i^{**};\psi_t). \quad (4.18) \]

Insiders choose a wage equal to their marginal product while the wage for inexperienced workers is dependent on the wage structure. Insiders choose
\[ x_t^{**} = \begin{cases} \bar{w} & \text{two-tier wage structure} \\ y_t^{**} & \text{single-tier wage structure} \end{cases} \] (4.19)

In a two-tier wage structure, the wage of an inexperienced workers is characterized by the exogenous lower bound, \( \bar{w} \). In a single-tier wage structure, the marginal productivity of experienced workers, determines the wage for new hires. Therefore, the wage of new hires is generally lower under a two-tier wage structure than under a single-tier wage structure.

Although the solution of the insiders' problem can be used to characterize all other endogenous variables, it must also be shown that its solution is consistent with the solution of the firm's problem. The first step is to suppose that the insiders' decision problem is satisfied in every period. Therefore, the stock of experienced workers is given by \( M'_{t+1} = \psi_t W^{**}_t \). It is then possible to write equations (3.17) and (3.18) as functions of the stock of experienced workers in the current and future periods.

\[ q(\phi_t)(\lambda_t F_2(M_t, M'_{t+1}/\psi_t; \psi_t) - x_t^{**}) = \kappa, \] (4.20)

\[ y_t^{**} = \lambda_t F_1(M_t, M'_{t+1}/\psi_t; \psi_t). \] (4.21)

These expressions satisfy equation (3.18) - the labor demand function of the firm.

It is straightforward to show that the representative firm cannot influence the wage demands of insiders by varying its hiring decision. First, divide equations (3.20)
and (3.21) by \( M'_i \). Then substitute \( M'_{i+1} = \psi_i W_{i}^{**} \) into these equations to get two expressions for the inexperienced: experienced worker ratio as follows:

\[ q(\phi_i)(\lambda_i F_2(1, n'_i; \psi_i) - x_i^{**}) = k \]  

(4.22)

\[ y_i^{**} = \lambda_i F_1(1, n'_i; \psi_i) \]  

(4.23)

The inexperienced: experience ratio of the firm is independent of \( M'_i \), therefore, the firm can not change the wage demands of experienced workers by changing its hiring decision. Consequently, the firm takes the sequence of insider wage demands as given.

**Equilibrium**

This section characterizes the equilibrium of the model. It is shown that each of the endogenous variables is a function of labor market tightness in the current period and one period past. Therefore, the dynamic analysis of the model is straightforward. One difficulty with the general case is that uniqueness is not generally assured.

An essential assumption of the model is that the matching and production technologies exhibit constant returns to scale. Therefore, in equilibrium, the aggregate inexperienced: experienced worker ratio is equal to the inexperienced: experienced worker ratio of the representative firm. The aggregate inexperienced: experienced worker ratio is given by
\[ n_t = \frac{p(\phi_t)}{p(\phi_{t-1})} \frac{1}{\psi_t}. \]  

(4.24)

where \( \psi_t = 1/(1 + \theta(1 - p(\phi_{t-1})) \). Firms earn zero profits in equilibrium and insider wages are maximized subject to this zero profit condition. The wage of experienced workers satisfies

\[ y_t = \lambda_t F_1(1, n_t; \psi_t). \]  

(4.25)

On one hand, insiders are always paid a wage equal to their marginal product in equilibrium. On the other hand, the wage of an inexperienced worker (outsider) is independent of their own productivity and is a function of the wage structure. In equilibrium,

\[ x_t = \begin{cases} 
\tilde{w} & \text{two-tier wage structure} \\
y_t & \text{single-tier wage structure}.
\end{cases} \]  

(4.26)

In a two-tier wage structure, an inexperienced workers wage is given by the lower bound, \( \tilde{w} \). In a single-tier wage structure, inexperienced workers receive the same wage as insiders. The lower bound, \( \tilde{w} \) is equivalent to a minimum wage, and assuming this minimum wage is less than the productivity of experienced workers, the wages of new hires is lower under a two-tier wage structure than under a single-tier wage structure. For either wage structure, the equilibrium number of inexperienced workers is characterized by
\[ G(\phi_t, \phi_{t-1}; \lambda_t) \equiv q(\psi_t)(\lambda_t F_2(1, n_t; \psi_t) - x_t) - k = 0 \] (4.27)

This condition states that inexperienced workers are hired until the probability of finding such a worker times the difference between the worker's marginal product and their wage is equal to the cost of posting a vacancy. According to equation (3.27), labor market tightness is generally higher and unemployment is generally lower under a two-tier wage structure than under a single-tier wage structure, because the wage of inexperienced workers is generally lower under the two-tier wage structure.

Inexperienced workers are always paid a wage less than their marginal product, if recruiting is costly. Therefore, if insiders operate under a single-tier wage structure, costly vacancies are posted if and only if new hires are more productive than experienced workers at the margin. An equilibrium with positive hiring occurs under a single-tier wage structure only if inexperienced workers are more productive than older experienced workers, otherwise, the equilibrium must converge to a zero employment outcome. There are two explanations of employment within insider controlled firms in the face of single-tier wage structures. The first explanation assumes young workers are more productive than experienced workers. The second explanation assumes that long-term unemployment facilitates skill acquisition (Tzanninis 1994).

The absence of a single-tier wage structure within an insider controlled firm can be explained by complementarities between experienced and inexperienced workers. I find that a two-tier wage structure raises insider wages provided complementarities
exist between experienced and inexperienced workers. This result is derived by differentiating the insider wage with respect to the inexperienced: experienced worker ratio.

\[
\frac{dy_t}{dn_t} = \lambda_t F_{12}(1, n_t; \psi_t) \geq 0. \quad (4.28)
\]

The insider wage is a strictly increasing function of the number of new hires if and only if there are complementarities between experienced and inexperienced workers. Assuming \( \hat{w} \) is less than the marginal productivity of experienced workers, complementarities between experienced and inexperienced workers imply that insiders prefer a two-tier wage structure over a single-tier wage structure, because it increases the number of new hires. If all workers are substitutes, as in Fehr and Kirchsteiger (1994), my model predicts that insiders are indifferent between the two wage structures.

Alternative models of the choice of wage structure include Gollier (1991) and Andersen and Vetter (1995). Gollier (1991) argues that insiders prefer single-tier wage structures over multi-tier wage structures to restrict output and raise product prices. However, Andersen and Vetter (1995) are able to support the opposite result in a model, in which, higher product prices are associated with two-tier wage structures.

**Dynamics**

In this section, I look at the dynamic adjustment of labor market tightness and wages. In order to keep the analysis simple, I assume that the long-term un-
employed do not search. This simplification eliminates the possibility of multiple equilibria.\textsuperscript{4} I obtain two main results. First, if there are complementarities between experienced insiders and inexperienced outsiders, the model predicts that a temporary aggregate productivity shock leads to persistent adjustments to unemployment, labor market tightness, and insider wages. Second, if experienced insiders and inexperienced outsiders are substitutes, the model predicts that a temporary aggregate productivity shock does not lead persistent adjustments to unemployment, labor market tightness, and insider wages.

Suppose (a) complementarities between inexperienced and experienced workers and (b) the long-term unemployed workers do not search ($\theta = 0$). In this case, the inexperienced: experienced worker ratio is given by

$$n_t = \frac{p(\phi_t)}{p(\phi_{t-1})}$$  \hspace{1cm} (4.29)

Labor market tightness is characterized by equations (21) and (23) which give

$$G(\phi_t, \phi_{t-1}; \lambda_t) \equiv q(\phi_t)(\lambda_t(F_2(1, p(\phi_t)/p(\phi_{t-1}); 1) - x_t) - k = 0$$  \hspace{1cm} (4.30)

where $x_t$ equals $F_1(1, p(\phi_t)/p(\phi_{t-1}); 1)$ under a single-tier wage structure and $\bar{w}$ under a two-tier wage structure. This expression relates current and past levels of labor market tightness. Implicitly differentiating, I find current labor market tightness is an increasing, concave function of past labor market tightness. The following diagram

\textsuperscript{4}Pissarides (1994) shows that this possibility exists if there is loss of skills by the long-term unemployed. In a previous version of this paper, I show that complementarities can also lead to multiple equilibria with or without loss of skills by the long-term unemployed.
illustrates the unique equilibrium in and out of steady state.

\[ \phi_t \]

\[ G(\phi_t, \lambda_t) = 0 \]

\[ \phi_1 \to \phi \to \phi_{t-1} \]

Figure 3.1: Equilibrium with Complementarities

It is straightforward to show complementarities cause persistent unemployment in the wake of a temporary aggregate productivity shock. In figure 1 there is a steady state equilibrium at a with labor market tightness given by \( \phi \) such that the unemployment rate of young workers is given by \( 1 - p(\phi) \). Suppose a temporary decrease in aggregate productivity happens at \( t = 1 \). In this case, point \( b \) gives equilibrium labor market tightness at \( t = 1 \). Furthermore, labor market tightness then satisfies \( \bar{\phi} > \phi_t > \phi_{t-1} \) for all \( t > 1 \). The unemployment rate of young workers, which is given by \( u_{1t} = 1 - p(\phi_t) \), is a decreasing function of labor market tightness. Therefore, the slow adjustment of \( \phi_t \) to its steady state implies unemployment persistence.

Insider wage dynamics follow changes in labor market tightness. Suppose the temporary productivity shock leaves supply of \( Np(\phi_1) \) experienced workers at \( t = 2 \),
which is less than in steady state. In this period, the return of aggregate productivity to its pre-shock level leaves insider wages above the steady state. In subsequent periods the adjustments of labor market tightness to the steady state imply the inexperience: experienced ratio satisfies $n_{t+1} < n_t < \bar{n}$ for $t > 1$. With complementarities the insider wage is an increasing function of $n_t$ because $dy_t/dn_t = \lambda_t F_{12}(1, n_t; 1) > 0$. Therefore, in periods after $t = 2$, the insider wage gradually converges to the steady state as the initial scarcity of experienced workers at $t = 2$ is reduced.

As in other insider-outsider models, a decrease in hiring in one period leads to higher insider wage demands in the next period. Models with this mechanism include Jackman, Layard and Nickell (1991), Begg (1988), Blanchard and Summers (1986) and D. azen and Gottfries (1992). I find that this ‘insider-outsider’ mechanism of unemployment persistence characterizes my model only if complementarities exist between inexperience and experienced workers.

Now suppose that inexperienced and experienced workers are substitutes rather than complements. In this case, $F_1(1, p(\phi_t)/p(\phi_{t-1}); 1)$ and $F_2(1, p(\phi_t)/p(\phi_{t-1}); 1)$ are given by constants, $\alpha_1$ and $\alpha_2$, respectively. Labor market tightness is then characterized by

$$G(\phi_t, \phi_{t-1}; \lambda_t) \equiv q(\phi_t)(\lambda_t \alpha_1 - z_t) - k = 0$$  \hspace{1cm} (4.31)$$

where $z_t$ equals $\lambda_t \alpha_1$ under a single-tier wage structure and $\bar{w}$ under a two-tier wage structure. If experienced and inexperienced workers are substitutes, the insider wage,
which is given by $\lambda_i\alpha_1$, is not a function of labor market tightness, past or present. Furthermore, the current level of labor market tightness, is not a function of past levels, because productivity and wages are independent of the inexperienced: experienced worker ratio. Thus a temporary change in aggregate productivity leads to temporary adjustments in the endogenous variables. Therefore, if old unemployed workers do not search, long run adjustments to labor market tightness, insider wages and youth unemployment rates occur only if experienced and inexperienced workers are complements.

Conclusions

This chapter develops a simple model of insiders in search equilibrium. The model predicts that insiders have strict preferences for a two-tier wage structure over a single-tier wage structure, if there are complementarities between experienced insiders and inexperienced outsiders. Given complementarities, the model also predicts that a temporary aggregate productivity shock leads to persistent adjustments to unemployment, labor market tightness, and insider wages. Complementarities lead to adjustments in labor market tightness, which are similar to Pissarides' (1992) 'loss of skills' model. However, unlike Pissarides' model, which assumes experienced and inexperienced workers are substitutes, a model with complementarities between experienced and inexperienced workers also predicts that unemployment persistence should be accompanied by continuous adjustments to insider wages.
APPENDIX III

Appendix 3.1. Properties of the representative firms’ maximization problem

The representative firm chooses a vector \( \{M'_t\}_{t=0}^{t=T} \) to maximize profits as follows:

\[
\Pi' = \max_{\{M'_t\}_{t=0}^{t=T}} \sum_{t=0}^{t=T} \beta^t (\lambda_t F(M'_t, M'_{t+1}/\psi_t; \psi_t) - x'_1 M'_{t+1} - x'_2 M'_{t+1} (1 - \psi_t)/\psi_t
\]

\[
- y'_t M'_t - k M'_{t+1}/\psi_t q(\phi_t))
\]  \( (4.32) \)

where \( M'_0 \) is given. The \( T+1 \) first-order conditions of the representative firm's problem over periods 0 through \( T \) are given by

\[
\Pi'_{M_1} = \lambda_0 F_2(M'_0, M'_{1\ast}/\psi_0; \psi_0)/\psi_0 - x'_{10} - x'_{20} (1 - \psi_0)/\psi_0
\]

\[-k/\psi_0 q(\phi_0) + \beta (\lambda_1 F_1(M'_{1\ast}, M'_{2\ast}/\psi_1; \psi_1) - y'_{11}) = 0\]

\[
\Pi'_{M_2} = \beta (\lambda_1 F_2(M'_{1\ast}, M'_{2\ast}/\psi_1; \psi_1) - x'_{21} (1 - \psi_1)/\psi_1
\]

\[-k/\psi_1 q(\phi_1)) + \beta^2 (\lambda_2 F_1(M'_{2\ast}, M'_{3\ast}/\psi_2; \psi_2) - y'_{12}) = 0\]
\[
\Pi'_{M_T} = \beta^{-1}(\lambda_{T-1}F(\psi_{T-1}; \psi_T - x'_{1T-1} - x'_{2T-1}(1 - \psi_{T-1})/\psi_T - k/\psi_T q(\psi_{T-1})) + \beta^T(\lambda_TF_1(M^*_{T-1}, M^*_{T-1} + 1/\psi_T; \psi_T) - y'_{1T}) = 0
\]

\[
\Pi'_{M_{T+1}} = \beta^T(\lambda_TF_2(M^*_{T+1}, M^*_{T+1} + 1/\psi_T; \psi_T - x'_{1T} - x'_{2T}(1 - \psi_T)/\psi_T - k/\psi_T q(\psi_T)) = 0
\]

Notice that in the last period - period \( T \) - new hires work only one period, because the economy ends. Reorganizing terms yields the labor demand function of the text.

I obtain

\[
q(\psi_t)[\lambda_tF_2(M^*_{1t}, M^*_{2t+1}/\psi_t; \psi_t)/\psi_t - \psi_t x'_{1t} - (1 - \psi_t)x'_{2t}]
\]

\[
+ \beta\psi_t(\lambda_{t+1}F_1(M^*_{1t+1}, M^*_{2t+2}/\psi_{t+1}; \psi_{t+1}) - y'_{1,t+1}) = k \quad \forall t = 0, \ldots, T - 1
\]

and

\[
q(\psi_t)[\lambda_tF_2(M^*_{1t}, M^*_{2t+1}/\psi_t; \psi_t)/\psi_t - \psi_t x'_{1t} - (1 - \psi_t)x'_{2t}] = k \quad t = T
\]

The second-order conditions of the firm's problem are given by the following

\[(T + 1) \times (T + 1) \text{ non-bordered Hessian matrix.}\]
This Hessian matrix can be written out explicitly as follows:

\[
H = \begin{pmatrix}
\frac{\lambda_0 F_{22}}{\psi_0} + \beta \lambda_1 F_{11} & \beta \lambda_1 F_{12} / \psi_1 & 0 & 0 \\
\beta \lambda_1 F_{21} / \psi_1 & \beta \lambda_1 F_{22} / \psi_1^2 + \beta^2 \lambda_2 F_{11} & \beta^2 \lambda_2 F_{12} / \psi_2 & 0 \\
0 & \beta^2 \lambda_2 F_{21} / \psi_2 & \beta^2 \lambda_2 F_{22} / \psi_2^2 + \beta^3 \lambda_3 F_{11} & \ldots \\
0 & 0 & \ldots & \ldots \\
& \ldots & \ldots & \ldots \\
& \beta^{T-1} \lambda_{T-1} F_{21} / \psi_{T-1} & \beta^{T-1} \lambda_{T-1} F_{22} / \psi_{T-1}^2 + \beta^T \lambda_T F_{11} & \beta^T \lambda_T F_{12} / \psi_T \\
& 0 & \beta^T \lambda_T F_{21} / \psi_T & \beta^T \lambda_T F_{22} / \psi_T^2
\end{pmatrix}
\]

(4.33)

This Hessian matrix is negative semi-definite provided that $F_{11}, F_{22}, - F_{12} < 0$.

In this case, the solution to the firm's problem is a maximum. This finite period problem and its solution is extended to an infinite horizon simply by allowing $T$ to move to infinity (e.g. Sargent 1987).
Bibliography


