Mechanistic Failure Criterion for Unidirectional and Random Fibre Polymer Composites

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Supervisor: Dr. Jeff Wood, The University of Western Ontario
A thesis submitted in partial fulfillment of the requirements for the Doctor of Philosophy degree in Mechanical and Materials Engineering
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Mechanistic Failure Criterion for Unidirectional and Random Fibre Polymer Composites

(Thesis format: Monograph)

by

Jamaloddin Jamali

Graduate Program in Mechanical and Materials Engineering

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy

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Abstract

Polymer composite design in energy absorbing components requires a failure criterion that can predict the energy involved in its fracture under different modes of loading. Present mixed mode criteria are mainly empirical or semi-empirical, and are only suitable for a small range of composite types.

The purpose of this study was to develop a mechanistic failure criterion that is applicable to a wide range of polymer composites. An energy based mechanistic failure criterion is proposed to characterize the toughness of unidirectional (UD) and randomly oriented short fibre composites (random fibre composites).

In UD and random composites, the criterion predicts the energy absorbed in the material during fracture based only on the constituents and interfacial properties. In UD composites, the criterion accounts for the resin fracture energy, hackle formation, interfacial debonding and effects of the plastic zone size. In random fibre composite the criterion also includes the effect of fibre orientation and fibre pull-out energy. The pull-out energy was calculated with the help of a finite element model.

Several experiments were performed to determine the failure mechanisms that influence the energy absorbed in the fracture of the polymer composite materials. Mixed mode loading was applied to the composite specimens using a compact tension shear (CTS) fixture. The comparison of the criterion predictions and experimental data shows a very good match. The criterion is able to predict the critical strain energy release rate (CSERR) of the epoxy and UD composites within 7% error margin. In random fibre composites, the criterion is able to predict an upper and a lower bound for the value of CSERR that fits well with the experiment.

Keywords

Dedication

To my beloved wife, Atieh, for her endless love and support.

To my parents, Ahmad and Sedigheh, whose encouragements supported me throughout my graduate journey.
Acknowledgments

This thesis was written with the assistance of many people, and of course, naming them all, here, seems really impossible. However, some people can not be left unmentioned because of their significant role.

First of all, my honest regards go to my advisor professor Jeff Wood for his patience, guidance, continuous support and generosity over the tenure of this research.

I would like to thank Dr. Tom Bruce for his suggestions in performing mechanical testing and preparation of the thesis.

I would like to thank Mr. Alex Jones, Dr. John Weiler, Mr. Javad Mehrmashhadi, Mr. Chris Rowles, and Mr. Xin Chen for their assistance in simulation using LSDYNA, and testing of epoxy and UD composite specimens.

I would like to thank Mr. Ross Davidson and Mr. Brad Kobe form the Surface Science Western for their assistance in taking SEM images.

I would like to thank Mr. Clayton Cook and his coworkers from the university machine shop for making the test fixtures.

I would like to thank Dr. Ying Fan, Ms. Jillian Quinn, and Mrs. Nasrin Ramezanali for the proofreading of the thesis.

Finally, I would like to thank my wonderful family and friends. Their support was more than crucial in keeping this project moving forward.

A heartfelt thankyou goes to my wife, Atieh, for her endless support, my parents, Ahmad and Sedigheh, my sisters Neda and Najmeh and my brother Babak.
**Table of Contents**

Abstract ......................................................................................................................... ii

Dedication ...................................................................................................................... iii

Acknowledgments ......................................................................................................... iv

Table of Contents ......................................................................................................... v

List of Tables .................................................................................................................. x

List of Figures ............................................................................................................... xi

List of Symbols ............................................................................................................. xxvi

Chapter 1 ....................................................................................................................... 1

1 Introduction .................................................................................................................. 1

Chapter 2 ....................................................................................................................... 5

2 Literature review ......................................................................................................... 5

2.1 Composite Microstructures ...................................................................................... 5

2.1.1 Composite Constituents ..................................................................................... 5

2.2 Deformation of composite materials ....................................................................... 10

2.2.1 Elastic deformation ............................................................................................ 10

2.2.2 Inelastic behavior and fracture ......................................................................... 33

2.3 Fracture ..................................................................................................................... 40

2.3.1 Linear Elastic Fracture Mechanics ................................................................... 42

2.3.2 Stress Based Fracture ....................................................................................... 42

2.3.3 Elastic-Plastic Fracture Mechanics (EPFM) ....................................................... 51

2.3.4 The Thickness Effect ....................................................................................... 51

2.3.5 Energy Based Methods ..................................................................................... 53

2.4 Fracture in Composites .......................................................................................... 55

2.5 Contributions to the Work of Fracture ................................................................... 57
3.3.1 Methods ................................................................................................................. 93
3.3.2 CTS Experiment ................................................................................................. 97
3.3.3 Single-Edge-Notch Bending ........................................................................... 99
3.4 Results (Thickness Study, Data Analysis, and Propagation Angle) ............ 104
  3.4.1 Thickness Study ............................................................................................... 104
  3.4.2 Data Analysis ..................................................................................................... 107
  3.4.3 Crack Growth Angle ........................................................................................ 113
3.5 Discussion ............................................................................................................. 122
  3.5.1 Effects on Fracture Surface Morphology ....................................................... 122
  3.5.2 Mode of Loading Effect .................................................................................... 126
  3.5.3 Existing Failure Criteria ................................................................................... 130
3.6 Summary ................................................................................................................ 132

Chapter 4 ...................................................................................................................... 133
4 Studying UD Composite ............................................................................................. 133
  4.1 Tensile Testing Procedure .................................................................................. 134
  4.1.1 Specimen Preparation ..................................................................................... 134
  4.1.2 Testing ............................................................................................................. 135
  4.1.3 Results (Density, Longitudinal and Transverse Modulus) ......................... 138
  4.2 Fracture Testing Of UD Composite ................................................................. 143
  4.2.1 Methods and Specimen Preparation ............................................................... 144
  4.2.2 CTS Testing .................................................................................................... 148
  4.2.3 Notch Sensitivity .............................................................................................. 149
  4.2.4 Bolt Hole Failure at Specimens ...................................................................... 151
  4.2.5 Modified CTS Specimen ................................................................................. 154
  4.2.6 Results and Data Analysis ............................................................................. 156
  4.3 Fractography (Discussion) ................................................................................ 165
6.3.2 Material Model Validation ................................................................. 226
6.3.3 Finite Element Model Development .................................................... 228
6.3.4 Debond Length ................................................................................... 232
6.4 Fractography .......................................................................................... 235
   6.4.1 Fibre Pull-Out Fractography .............................................................. 235
6.4.2 Transition angle (θtransition) ............................................................... 236
6.5 Failure Criterion vs Experimental Data .................................................... 238
   6.5.1 Loading Mode Function, g(α) .............................................................. 238
   6.5.2 Quantifying the Transition Angle ...................................................... 241
   6.5.3 Comparison with Experimental Data and Discussion .......................... 243
   6.5.4 Mixed Mode Fracture Toughness Criteria ........................................ 247
6.6 Summary ................................................................................................. 248

Chapter 7 ..................................................................................................... 249
7 Conclusions ............................................................................................... 249
  7.1 Research Contribution .......................................................................... 249
     7.1.1 Understanding of failure mechanisms involved in the fracture of polymer composites ............................................................................................... 249
     7.1.2 Developing a Mechanistic failure criterion for UD and random composites ........................................................................................................ 249
     7.1.3 Assisting in the design of optimized energy absorbing polymer composites ........................................................................................................ 251
  7.2 Future Work ............................................................................................ 252
References ...................................................................................................... 254
Curriculum Vitae .......................................................................................... 268
List of Tables

Table 2.2.1 Elastic Young’s modulus prediction for glass fibre epoxy having 40% fibre volume fraction under axial and transverse load using ROM, Reuss and Halpin-Tsai models compared with that of matrix and fibre................................................................. 16

Table 2.2 E-glass fibre, aramid, and epoxy properties ................................................................. 29

Table 3.1.1 Mechanical properties of CLR1180/CLH6560 epoxy cured at room temperature for 48 hours ......................................................................................................................... 91

Table 3.4.1. Measured properties of neat epoxy samples using CTS test. ......................... 112

Table 3.4.2 Values of predicted and measured crack growth angle. ............................ 115

Table 4.2.1: Mechanical properties of epoxy and UD composite used in calculations. .... 161

Table 6.3.1 Mechanical properties of epoxy and glass fibre used in the material model. .... 227
List of Figures

Figure 2.1.1 CES plot, comparison of metals and composite fracture toughness vs. density [1]........................................................................................................................................................................... 2

Figure 2.1.1 a) Cross section SEM of Ti–6Al–4V as MMC (back) and fracture surface (front) [2] b) Fracture surface of fibre reinforced ceramics with SiC-fibres and SiC-matrix [3] c) fracture surface of unidirectional E-glass fibre epoxy (PMC)................................................................. 6

Figure 2.1.2 Composite classification based on reinforcement shape, size, and arrangement. 7

Figure 2.1.3 a) Unidirectional fibre. b) Plain weave fibre. C) Cross-ply composite............ 7

Figure 2.1.4. a) Plain weave fabric b) Twill weave [6] c) and d) 4 and 8 harness satin weave [7]........................................................................................................................................................................................................ 9

Figure 2.1.5 a) Discontinuous unidirectional long fibre. b) Schematic of discontinuous random fibre. c) Roll of textile glass mat [8]............................................................................................................. 9

Figure 2.2.1 stress-strain curve for glass fibre epoxy pre-preg specimen. ....................... 11

Figure 2.2.2a) Schematic of a UD composite b) Slab model having a fibre volume fraction of \( f \) c) The model under longitudinal stress \( \sigma_1 \), fibre and matrix experience equal axial strain \( \varepsilon_1 \)[5]........................................................................................................................................................................................................ 12

Figure 2.2.3 Slab model under transverse loading with equal transverse stress \( \sigma_2 \)[5] ....... 13

Figure 2.2.4 Schematic of strain field of a) unstrained and b) transversely strained composite. ........................................................................................................................................................................................................ 15

Figure 2.2.5 Comparison of experimental data and predictions by the ROM (equal strain), Reuss (equal stress) and Halpin-Tsai (with \( \xi = 1 \)) models [5], [9]......................................................... 16

Figure 2.2.6 Schematics of actual and slab model used to determine shear stiffness of UD composite [5]. ........................................................................................................................................................................................................ 17
Figure 2.2.7 Schematic of off-axis loading on one ply where force is applied in x - y coordinate system and fibres are aligned in 1-2 coordinate (material coordinate) system.... 20

Figure 2.2.8 Off-axis stress-strain curves of glass fibre epoxy specimens with different fibre orientation [6].................................................................................................................................................................. 22

Figure 2.2.9 a) Uniform state of stress for off-axis composite b) effect of clamped ends [13]. ......................................................................................................................................................................................................................... 23

Figure 2.2.10 a) Fibre element under interfacial and axial stress. b) Stress variation along the length of a fibre. b) Hyperbolic change of stress at the end of fibre c) simplified linear change in the fibre end stress. The load transfer from matrix to fiber for fibres shorter, equal and longer than the critical length l1 [15] ........................................................................................................................................................................................................................................... 27

Figure 2.2.11 Comparison of $l_c$ prediction using Equation (2.2.38) in which the interfacial shear stress is assumed to be constant and (2.2.48), in which $\tau(x)$, is dependent on x (place from center of the fibre)....................................................................................................................................................................................... 30

Figure 2.2.12 Schematic illustration of long fibre UD composite inelastic behaviour a)When the fibre has higher strain to failure than matrix b) When the fibre has lower strain to failure than matrix [5].................................................................................................................................................................................................................................................. 36

Figure 2.2.13 Comparison between the transverse stress-strain curve for UD E-glass fibre polyester (top) and three unreinforced polyester (bottom) [5]................................................................................................................................. 37

Figure 2.2.14 a) Schematic illustration of cross-ply laminates having plies with fibres in transverse and longitudinal directions b)Schematic plot for stress-strain curve of cross ply composites........................................................................................................................................................................................................................................ 39

Figure 2.2.15 Schematic picture showing how axial tension of the angle-ply composite introduces interlaminar rotation and raises the interlaminar shear stress [5]......................... 40

Figure 2.3.1 Energy and force of ionic bonds of two atoms vs. interatomic distance [26] .... 41

Figure 2.3.2. The stress distribution around the edge of a circular hole in an infinite pre stressed material [31] ........................................................................................................................................................................................................................................ 43
Figure 2.3.3 The solution of a pre-stressed wedge by Williams uses theory of elasticity (left) can be developed into a model with a crack under an arbitrary state of stress (right) by the limiting case of $\alpha = \pi$ [31].

Figure 2.3.4 Different modes of fracture loading.

Figure 2.3.5 The generalized Westergaard approach. A crack body in the complex domain [31].

Figure 2.3.6 a and b) The numerical photo-elastic pattern calculated from Westergaard equation [35] b) Actual photo-elastic pattern from epoxy sample showing the forward tilt of isochromatic lines.

Figure 2.3.7 Comparison of Various methods defining shape factor $Y$ in terms of $2a/w$ [31].

Figure 2.3.8 Schematic of transition of the plastic zone at the tip of the crack from plane stress region to plane strain region [19].

Figure 2.3.9 Mohr’s circle comparison for plain stress (top) and plane strain (bottom) conditions, suggests higher chance of brittle fracture for plain strain and higher chance of plastic fracture for plane stress regions [31].

Figure 2.4.1 Schematic interaction between a fibre and surrounding matrix in a longitudinal UD composite. Series of microscopic events happen from elastic deformation (a) to final fracture of material (d) [43].

Figure 2.4.2 a) Cook-Gordon model. Stresses acting close to the crack tip. b) The crack is approaching fiber bundle. C) Fibre/matrix debond due to $\sigma_2$ stress, crack blunting is happening [18].

Figure 2.5.1 A unidirectional composite, effect of fibre stiffness on matrix deformation ....

Figure 2.5.2. Schematic of crack passing through short fibre composite, showing interfacial debonding and fibre pull-out [5].
Figure 2.5.3. The effect of fibre orientation on the fracture energy absorbed by UD composite laminate obtained from notched Charpy impact test [57]................................................................. 61

Figure 2.6.1. Double Cantilever Beam Specimen(a) with piano hinges (b) with loading blocks .................................................................................................................................................. 62

Figure 2.7.1 a. Loading in CTS sample [68] b. The CTS specimen fixed in the tensile testing machine in the lab setting. ......................................................................................................................... 64

Figure 2.7.2 a. Proposed fracture specimen by Richard [71] b. The compound CTS specimen proposed by Rikards for composite interlaminar fracture [68]................................................................. 65

Figure 2.8.1 Linear criteria for mixed mode fracture toughness [59]........................................... 69

Figure 2.8.2 Power law criteria for the mixed mode toughness of composites [59] .......... 70

Figure 2.8.3 Polynomial criterion for the mixed mode toughness of composites [59]......... 71

Figure 2.8.4 $K_1$ critical criterion for the mixed mode toughness of composites [59]......... 72

Figure 2.8.5 Hackle criterion models the mixed mode toughness of composites [59]........ 73

Figure 2.8.6 The bi-linear criterion predictions for the mixed mode toughness of composites [59].......................................................................................................................................................... 74

Figure 2.8.7 Comparison of experimental data for UD carbon/bismaleimide resin (T800/5245), and UD carbon/epoxy resin (T800/924) with mixed mode failure criteria [85]. ............................................................................................................................................................................ 75

Figure 2.8.8 Comparison of experimental data for AS4/3501-6 with different mixed mode failure criteria [59]......................................................................................................................................................... 76

Figure 2.9.1 Comparison of the mixed mode criterion and experimental results for nonlayered UD composite [7].................................................................................................................................................. 79

Figure 2.9.2 Comparison of the mixed mode criterion and experimental results for layered UD composite [7].................................................................................................................................................. 79
Figure 3.1.1 a. Finished epoxy specimen ready for the test. b. Epoxy CTS specimen geometry according to ASTM D638 (dimensions are in mm)................................................................. 84

Figure 3.1.2. Servohydraulic loadframe (Instron 8804). The machine grips with specimen. Extensometer with 50 mm gage length is attached to the specimen to properly measure the specimen’s strain........................................................................................................... 85

Figure 3.1.3 Comparison of density between epoxy cured with 15inHg and no Vacuum (both at room temperature)........................................................................................................................................... 86

Figure 3.1.4 The comparison of stress- strain curve for 1180/6560 crosslink epoxy specimens....................................................................................................................................... 87

Figure 3.1.5 Comparison of two different systems of epoxy......................................................... 88

Figure 3.1.6 Comparison of tensile strength of 1180/6560 epoxy specimen using engineering and true stress strain curve............................................................................................................. 89

Figure 3.1.7 Yield stress (0.1% offset yield strength) of CLR1180/CLH6560 epoxy specimen. ........................................................................................................................................ 90

Figure 3.1.8 Failure strain of different CLR1180/CLH6560 epoxy specimens. ....................... 91

Figure 3.3.1 a) The Crosslink resin and hardener kept in room temperature b) The tray is lined with vacuum Teflon bag and leveled. c. the tray is filled with mixed epoxy and is kept in room temperature............................................................................................................. 94

Figure 3.3.2 Mixing tools. Plastic cups for small and large amounts, wood stir stick for mixing, paint brush for spreading the resin on fibre layers ................................................................. 95

Figure 3.3.3 CTS sample geometry (left) Neat epoxy specimen after machining (right) ...... 95

Figure 3.3.4 a. Teflon mold cut and sealed for making thicker SENB and CTS samples b. molds supported by steel bar to withstand epoxy before curing c. cured samples having a minimum thickness of 11 mm. d. final thick CTS specimen before testing. ................................. 96

Figure 3.3.5 CTS fixture. left) Mode I right) mode II ............................................................................ 97
Figure 3.3.6. a. Steel bolt holes and washer plate to hold the sample and pin hole in the CTS specimen. b. The CTS specimen is pinned to a clevis which is fixed into the machine grip. 98

Figure 3.3.7 Schematic of single edge bending test [96] (Top). b. Single edge notch beam lab setting (bottom) .................................................................................................................................................................................. 100

Figure 3.3.8 a and b The pre-crack was made using razor blade. c SEM image shows the crack tip area made by the razor blade is shown beside the fracture area of epoxy subjected to the mode I loading .................................................................................................................................................................................. 103

Figure 3.4.1 Thickness effect on fracture toughness of typical isotropic materials [19] ..... 106

Figure 3.4.2 Thickness effect on determination of mode I fracture toughness of epoxy. .... 106

Figure 3.4.3 Thickness effect on mode I toughness (CSERR) of neat epoxy. ................. 107

Figure 3.4.4 Load-displacement curve for epoxy samples (thickness between 3 mm-4 mm) under pure mode I ($\alpha=0^\circ$). The load is divided by the thickness to ease the comparison of the results of samples with different thickness. This is because by increasing the sample thickness the maximum fracture load is also increased .......................................................................................................................................................... 108

Figure 3.4.5 Load-displacement curve for epoxy sample (thickness between 3-4 mm) under pure mode II ($\alpha=90^\circ$) b) Critical values of critical load for mode II ............................................. 109

Figure 3.4.6 Neat epoxy mode I, mode II (a) and mixed mode (b) fracture toughness versus loading angle. .................................................................................................................................................................................. 111

Figure 3.4.7 Epoxy critical strain energy release rate (toughness) vs. loading angle........ 113

Figure 3.4.8 Comparison of measured and predicted values of crack growth angle. ........ 114

Figure 3.4.9 The crack growth angle in different mixed mode ratio (loading angle). From top left:$0^\circ, 15^\circ, 30^\circ, 60^\circ, 75^\circ, 90^\circ$ .................................................................................................................................................................................. 116

Figure 3.4.10 Effect of the resin deformation and the specimen rotation on the value of the crack growth angle between start of crack initiation and end of fracture test. ............... 117
Figure 3.4.11 a. Photo elastic images from epoxy under pure mode I (α=0°) [103] b. 15° loading angle c. 30° loading angle d. 45° loading angle e. 60° loading angle f. 75° loading angle.......................... 120

Figure 3.4.12 Macro hackles in some of neat epoxy specimen under mode II loading, sample with macro hackle (top) sample without macro hackling (bottom)........................... 121

Figure 3.5.1 Comparison of epoxy specimen’s fracture surface morphologies. Specimens were under mode I (loading angle of 0°) with thickness of 3 mm (a) and 12 mm(b) ....... 123

Figure 3.5.2 Comparison of epoxy specimen’s fracture surface morphologies. Specimens were under loading angle of 15° with thickness of 7 mm (top) and 12 mm (bottom). ...... 124

Figure 3.5.3 III. Specimens with equal thickness (left) kept at room temperature gave a flat fracture surface and toughness of $2.82 \frac{kJ}{m^2}$ (right) kept at an approximate temperature of 40°C gave a complex fracture surface resulting in toughness of $4 \frac{kJ}{m^2}$.......................... 125

Figure 3.5.4 Steromicroscopy (top) and SEM (bottom) images of epoxy fracture surface under pure mode I testing. The flat surface requires small amount of energy to be created. 127

Figure 3.5.5 SEM image of the fracture surface of epoxy under pure mode II testing (a) and 75° of loading angle (b). Closer view of hackles (cusps) in epoxy under mode II loading. c) Extensive plastic deformation on fracture surface during crack propagation releases higher value of energy compared to lower mode II fracture surfaces.......................... 129

Figure 3.5.6 Hackles form by grow and coalescence of cracks due to shear in mode II loading [105].............................................................. 129

Figure 3.5.7 Effect of fracture morphology on the energy released during mixed mode fracture. .......................................................... 130

Figure 3.5.8 Approximation of the mixed mode experimental data using Equation (3.5.1) with different values of $m$ and $n$ exponents. a) Thin specimen with thickness between 3 mm-4 mm. b)Thick specimen having a thickness above 6 mm. .......................................................... 131
Figure 4.1.1 E-glass fibres have diameter of 16-17 micron (top) Periodic threads keeping UD fibres together (bottom) ................................................................. 135

Figure 4.1.2 a UD composite with longitudinal fibres (90° fibre orientation) b Transverse fibres (0° fibre) orientation. Emery cloth is used as an end tab for tensile specimen........... 137

Figure 4.1.3 5kN load cell measured the load applied by tensile machine to transverse UD composite (glass fibre/ CLR1180-CLH6560 epoxy. Extensometer with 50 mm gage length measures tensile extension................................................................. 138

Figure 4.1.4 Lab setting to measure composite density. Measuring specimen coupons weight in water (left) and comparing with its weight in air (right). .................................................. 139

Figure 4.1.5 a. Schematic of longitudinal cracks along the fibre direction in the UD composite due to shear stress between broken part and the rest of the composite. b. Longitudinal cracks in UD composite subjected to longitudinal tension c. Crack in transverse composite extends in the matrix between fibres................................................................. 140

Figure 4.1.6 Comparison of composites’ longitudinal stiffness of different specimens (Test carried out at room temperature)........................................................................... 142

Figure 4.1.7 Comparison of experimental data for longitudinal Young’s modulus (E₁) and prediction by rule of mixture ................................................................. 142

Figure 4.1.8 Comparison of experimental data for transverse Young’s modulus (E₂) and prediction by Reuss and Halpin-Tsai models ................................................................. 143

Figure 4.2.1 Aluminum rollers are utilized to spread the resin and remove air bubbles during hand lay-up. Periodic threads are clear in this photo........................................... 145

Figure 4.2.2 The change in the angle between fibres and loading can make a significant decrease in the composite stiffness for angles close to 0° [5]. .................................................. 146

Figure 4.2.3 Composite specimens are cured by a hot press. The pressure on the cylinder determines the final thickness of the specimen................................................................. 147
Figure 4.2.4 The CTS sample geometry (left), UD composite specimen after machining (right) ................................................................. 148

Figure 4.2.5 The CTS specimen with U-notch and V-notch ................................................. 150

Figure 4.2.6 The comparison of load-displacement curve between specimens with V-notch and U-notch [110] ..................................................................................................................... 151

Figure 4.2.7 Examples of bolt hole failure in 90˚ (top) and 75˚ (bottom) UD composite specimen under mode II loading. No notch opening is observed in the sample and the results are not good for the calculation of fracture properties. The arrows show failure points. ..... 152

Figure 4.2.8 Steps taken to remove the problem of failure at the bolt holes. A. Use of sandpaper between the fixture and the specimen to increase friction in the gripping area. b and c) Use of a combination of UD and plain weave fiber composite to reinforce the gripping area .............................................................................................................................................. 153

Figure 4.2.9 a) Schematic of the modified compact tension shear fixture for high mode II loading. b) Final fixtures after curing, CTS specimen will be glued to each fixture for fracture testing ......................................................................................................................................... 155

Figure 4.2.10 The CTS specimen is glued to the modified CTS fixture. The fixture is pinned to a clevis that is gripped by machine grip. Loading angle a) 75° b)90° ......................... 156

Figure 4.2.11 Mode II load-displacement curve for unidirectional composite. Maximum load have 16% relative error ................................................................. 157

Figure 4.2.12 Thickness effect on mode I fracture toughness of UD composite .......... 158

Figure 4.2.13 Thickness effect on mode II fracture toughness of UD composite ........... 158

Figure 4.2.14 Thickness effect on mode I CSERR (G_I) of UD composite. The results give an average value of 1.02 kJ/m² ........................................................................................................ 162

Figure 4.2.15 Thickness effect on mode II CSERR (G_II) of UD composite. The results give an average value of 2.83 kJ/m² ........................................................................................................ 162
Figure 4.2.16 Maximum values of force vs load angle in CTS specimen shows a gradual increase in value of force from mode I to mode II loading.  

Figure 4.2.17 Mode I and mode II components of fracture toughness of the UD composite under different modes of loading.  

Figure 4.2.18 Mixed mode Effective fracture toughness of UD composite.  

Figure 4.2.19 Experimental results of UD composite mixed mode toughness versus loading angle.  

Figure 4.3.1 Paper pocket was used to cover the fracture surface and protect it from cutting debris.  

Figure 4.3.2 Macroscopically SEM image of mode I fracture surface of UD composite. The consolidated image shows different mechanisms represents on the fracture surface. Crack propagation in this image is from right to left.  

Figure 4.3.3 Fracture surface of UD composite under mode I loading showing interfacial debonding. A. Fibre debonding leaves a smooth channel. B. debris 

Figure 4.3.4 SEM of UD composite subjected to mode I testing, shows cleavage fracture at epoxy. A river patterns caused by the meeting of two fracture surface, moving in the same direction. 

Figure 4.3.5 Fracture surface of UD composite indicates A. flat fracture epoxy surface B river patterns C. fibre debonding D. matrix hackles. 

Figure 4.3.6 Schematic of river line formation, crack plane propagating at different elevations meet each other and form river lines. The arrow shows the local direction of crack propagation [105]. 

Figure 4.3.7 Fracture surface of UD composite under mode II loading. Hackle formation and fibre debonding can be found on the surface. 

Figure 4.3.8 schematic formation of hackles under mode II (a) and mixed mode loading (b) in CTS specimen by the resultant principal stress.
Figure 4.3.9 Hackles in different sizes (A, E and G) and interfacial debonding (B and F) are dominant features in mixed mode and mode II fracture. Other fracture morphologies such as void (H) and debris (I) can be observed on the surface. Fibre layers spacing in the bottom image is approximately 50 micron.

Figure 4.3.10 Fibres broken in specimen under mixed mode loading with $\alpha=30^\circ$, fibre breaking angle is $45^\circ$ mainly due to shear. Arrow shows chop marks on the fibre which can be caused by the compression of the other side of specimen added to the shear force during sliding.

Figure 4.5.1 Crack propagation in UD composite with various fibre angles a) left to right $15^\circ$, $30^\circ$, $75^\circ$, $90^\circ$. b) $0^\circ$, $45^\circ$ and $60^\circ$ [110].

Figure 4.5.2 a) Load-displacement curve for various fibre angles. b) Effective fracture toughness vs. fibre angle.

Figure 5.1.1 CTS specimen under mixed mode load in the lab setting (left) General state of stress at the elements shown in the CTS sample for a mixed mode I and II and crack propagation along the interface under mixed mode loading (right).

Figure 5.1.2 The mixed mode toughness pattern predicted for different modes of loading using failure criteria (Equation (5.1.15)).

Figure 5.2.1 Schematic (left) and real (right) fracture surface morphology and corresponding values of the matrix and interface fracture area. SEM image is taken from a specimen subjected to mode II loading.

Figure 5.2.2 Fibre spacing (m) in square and hexagonal array of fibres [5].

Figure 5.2.3 Glass fibre diameter is measured to be $17$ mm which is used to calculate fibre spacing. The fibre spacing from fibre volume fraction is close to the value shown in the image. Radial pattern in fibre surface is due to brittle fracture.
Figure 5.3.1 Comparison of CSERR predicted for epoxy by failure criterion (Equation (5.1.15) and experimental data from CTS epoxy specimen......................................................... 191

Figure 5.3.2 Comparison of effective fracture toughness results when a) fibre angle is fixed and loading angle changing with b) specimens having fixed loading angle and changing fibre angle. The circles show similar condition and results from both formats of testing.......... 193

Figure 5.3.3 a) specimen with \( \alpha = 0^\circ \) and \( \theta = 0^\circ \) (case 1) and b) a specimen with \( \alpha = 90^\circ \) and \( \theta = 90^\circ \) (case 2).......................................................................................................................... 195

Figure 5.3.4 Comparison of the model prediction and the experiment for 90° UD composite under different modes of loading............................................................................................................. 196

Figure 5.3.5 Periodic polymer threads make changes in the value of fracture work measured from UD composite. These threads have an average diameter of 24\( \mu \)m compared to fibre glass with 17\( \mu \)m diameter................................................................................................................................. 197

Figure 5.3.6 The influence of the interfacial toughness on the total toughness of UD glass epoxy composite. ................................................................................................................................. 198

Figure 6.1.1 a) Fibre pull-out is the major fracture mechanism on the fracture surface. b) Failure mechanisms on the fracture surface A:fibre imprints on epoxy due to fibre/matrix debonding, leaving smooth channel that shows weak bond between epoxy and glass fibre. B: matrix fracture, C: fibre pull-out.................................................................................................................. 201

Figure 6.1.2 A: T-oriented fibres resulting jagged crack, the debonding happens around the full surface area of the fibres. B: L-oriented fibres resulting interfacial debonding and straight propagation of crack. .................................................................................................................. 202

Figure 6.1.3 Schematic of a crack advancing and passing through short fibre composite, the fibre debonds from the matrix and then pulls out from its socket. ............................................. 204

Figure 6.1.4 Schematic of a fibre pulling out from its surrounding matrix due to mode I and mode II loading. The comparison of fibre under mode I(left) and mode II (right), suggests higher possibility of fibre pull-out under mode I loading compared to mode II loading. Fibres have more chance of shearing without pull-out under mode II (right picture). ...................... 208
Figure 6.1.5 Schematic of interfacial debonding due to the crack advancing in a) UD composite and b) Random fibre composite. The interfacial debonding in random short fibre composite is a longer path compared to UD composite. \( \gamma i \) is chosen between 0° and 90°. 210

Figure 6.1.6 a) Delamination debonding b) Cook-Gordon debonding. 215

Figure 6.2.1 Straight random composite specimens before and after performing tensile testing according to ASTM D3039. 216

Figure 6.2.2 Comparison of stress-strain curve (top) and Young’s modulus (bottom) for random glass fibre epoxy. 217

Figure 6.2.3 CTS specimen made from random fibre epoxy under different modes of loading in the beginning and after end of testing. a) 0° pure mode I, b) 15°, c) 45°, d) 60°, e) pure mode II loading 90°. 219

Figure 6.2.4 Optical microscopy image of a chopped-strand mat. Strand mat image used for image analysis to determine fibre angle distribution. 220

Figure 6.2.5 Fibre orientation distribution found from the chopped-strand mat. 221

Figure 6.2.6 Top: Plot of Fracture toughness \((Kl, Kl, Keff)\) vs loading angle, Bottom: CSERR vs loading angle of random composite using CTS fracture testing. 222

Figure 6.3.1 Geometric model for finite element simulations with 5120 elements. 223

Figure 6.3.2 The effective stress (Von-Mises stress) as a function of the number of elements for uni-axial loading condition determined from FE simulations. 224

Figure 6.3.3 Geometric model for finite element simulations of mixed mode loading with 3168 elements. 225

Figure 6.3.4 Effective stress (von-Mises stress) as a function of the number of elements for mode I loading condition determined from FE simulations. The stresses were measured in nodes away from the crack tip. 226
Figure 6.3.5 Comparison of FE tensile curve using piecewise linear plastic model compared with the experiment. ............................................................. 228

Figure 6.3.6 FE model for the CTS specimen with a fibre (shown by a white arrow) parallel to loading direction (θ=90°). The specimen is subjected to pure mode I loading with the lower bolt holes fixed in y direction. ............................................................. 229

Figure 6.3.7 The boundary condition for the nodes at upper bolt holes (A) and lower bolt holes (B) .................................................................................................................. 231

Figure 6.3.8 FE simulations showing von-Mises stress at the tip of the crack close to the fibre under mode I loading. Glass fibre has angle of a) 0° b) 30°c) 45°, and d) 90°. ................. 232

Figure 6.3.9 Sequence of steps taken to determine the fibre pull-out length a) a single fibre passing close to the tip of the crack in CTS specimen with an angle of θ. b) stress components at the interface of fibre was calculated from FE model. C) The calculated stress values were transformed to give stress values normal and parallel to the interface. These values are used to determine whether the desired point on interface detaches from fibre or not. ........................................................................................................ 233

Figure 6.3.10 Fibre debond length in different orientation using FE simulation. .................. 234

Figure 6.4.1 Example of the fibre length on the fracture surface of random composite. The values of pull-out length for some of the fibres are used to calculate the average pull-out length on composite. ............................................................................ 236

Figure 6.4.2 Stereomicroscopy images of the crack path showing range of fibre angles experiencing delamination or Cook-Gordon debonding mechanisms.................................. 237

Figure 6.5.1 Comparison of the value of $g(\alpha)$ for pure mode I ($\alpha = 0^\circ$) and pure mode II ($\alpha = \pi/2$). The transition is used to derive an expression for $g(\alpha)$ in terms of $\alpha$, $G_{II}$, and $G_I$. .......................................................................................................................... 239

Figure 6.5.2 Fibre debond length in different orientation using FE simulation for CTS subjected to mode II loading. .......................................................................................... 240
Figure 6.5.3 Comparison of the model prediction and the experiment for random glass-fibre/epoxy composite under different modes of loading. ......................................................... 244

Figure 6.5.4 Effect of loading angle function, g(α) on the criterion predictions compared with experiment................................................................. 245

Figure 6.5.5 Fibre volume fraction effect on the total value of CSERR ............................................. 247

Figure 6.5.6 Mixed-Mode fracture toughness values at fracture in random glass fibre/epoxy Composite. ................................................................. 247
# List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
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<tr>
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<td>$A_t$</td>
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Mode mixture at which transition from interyarn to interply occurs

Applied load

Critical load

Plastic region radius

Fibre radius, distance from the tip of the crack

Compliance matrix

Fibre aspect ratio

Transformed compliance tensor

Complex variable

Loading angle

Surface energy

Longitudinal strain

Transverse strain

Fibre failure strain

Matrix failure strain

Reinforcement reduction factor

Poisson’s ratio

Radius of curvature of the hole tip

Fibre angle with regard to the specimen geometry

Transition angle
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<td>Fibre strength</td>
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<td>Interfacial normal strength</td>
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<td>$\sigma_m^*$</td>
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<td>Standard deviation of the mean</td>
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Chapter 1

1 Introduction

Materials are properly chosen for engineering applications based on their mechanical or physical properties. These properties include mass, strength, stiffness and toughness. In the transportation sector, light-weighting contributes to the reduction of fuel consumption. Polymer Matrix Composites (PMCs) are generally lighter than metals and have almost equal strength, so they can replace metals in many applications; however, they lack the toughness of metals.

A comparison of the fracture toughness versus density between PMCs and metals that are largely used in the technological applications show that while PMCs are, on average, 80% lighter than ferrous metals, their average resistance against fracture is less than 20% of that of ferrous metals. This difference increases if PMCs are compared with only steel alloys (see Figure 1).

Replacing any material requires equal performance, and at least one improved property. So it could be of interest if the toughness of polymer composites could be enhanced.

While the elastic properties of PMCs are generally well understood, prediction of their fracture behaviour is largely based on empirical or semi-empirical criteria. Because current fracture models are mainly empirical, a change in the process or constituent requires repetition of the testing procedure to validate their performance. The study of fracture in PMCs is far less developed when compared to metals. There remains a variety of scientific or technological issues associated with predicting the work of fracture in PMCs. These issues include: complexity in the microstructures, anisotropic material behaviour, multiple possible failure modes and toughening mechanisms.
Figure 2.1.1 CES plot, comparison of metals and composite fracture toughness vs. density [1].

The main goal of this research is to develop a fracture criterion for fibre reinforced composites. Ideally this model should be based only on the properties of the constituent materials, the interface between these constituents and their geometric arrangement including their fibre volume fraction, and fibre orientation.

The outcome of this research will be useful to any industry and scientific community that would benefit from lightweight, energy absorbing structural materials, by introducing a mechanistic failure model that can be utilized to predict the behaviour of material without the need for testing the actual material. The criterion will be used to design composite parts with improved energy absorption capability.
Outline of the Thesis

This thesis is written in the manuscript format and is divided into seven chapters.

Chapter Two begins with a general description of the important concepts in fracture mechanics and traces the development of this field to the current state-of-the-art with specific focus on the fracture behaviour of fibre-reinforced composite materials. The existing fracture criteria are introduced and their capabilities are compared and discussed. Then the basis and structure for introducing a mechanistic criterion is presented.

Chapter Three involves the resin study. The resin forms the matrix part of the polymer composites. The study includes the tensile and fracture testing procedures and results of epoxy specimen. The tensile testing of resin is performed to determine mechanical properties of the material. The values of CSERR of epoxy were measured using the compact tension shear testing (CTS) specimens. The fracture surface of resin specimens is studied to investigate the fracture mechanisms. SEM and optical microscopy imaging were used to study these mechanisms representing on the fracture surface.

Chapter Four refers to the tensile and fracture characterization of unidirectional (UD) polymer composites. The tensile testing specimens and procedure is introduced. Tensile testing results provide different mechanical properties of the composite. Mode I, mode II and mixed mode fracture testing of unidirectional composite is presented in this chapter. Fracture testing provides results of energy absorption by the material during the test. It is also performed to study different operative mechanisms during the fracture process. The results and their corresponding interpretation are given in the chapter.

Chapter Five introduces the mechanistic failure criterion for the UD polymer composite. Results and observations from previous chapters are summed here to derive the criterion in form of a mathematical equation to calculate CSERR of UD polymer composites. The criterion prediction is then compared with the experimental results of UD composite and resin specimens.
In Chapter Six, the results from tensile and fracture testing of random fibre composites are presented. The fracture mechanisms operating during the fracture process are studied and discussed. A finite element model is introduced to study the effect of mode of loading and fibre direction on the fibre pull out energy. A mechanistic criterion based on the observations and FE model results is introduced. The criterion predicts the value of mixed mode CSERR of random fibre composite. The criterion predictions and experimental results are compared at the end of the chapter.

Finally, a general conclusion is given in Chapter Seven. Different results and topics are summarized in this chapter. The main contribution of this work is pointed out and future possible works are presented.
Chapter 2

2 Literature review

Introduction

The goal of this research is to develop a fracture criterion for fibre reinforced composites. Ideally the model will be based solely on the properties of the constituent materials, the interface between these constituents and their geometric arrangement.

This Chapter begins with a general description of composite microstructures followed by a description of deformation and fracture of PMCs. This leads to a discussion of the important aspects of fracture mechanics with specific emphasis on the fracture of Fibre Reinforced Polymer (FRP) composites. This discussion will include reference to both analytical and numerical techniques for the prediction of fracture in composite structures subjected to various stress states.

2.1 Composite Microstructures

In this section, composite materials and their constituent elements and microstructures are described. The goal of this section is to present the terminology that is used to characterize composite microstructures.

2.1.1 Composite Constituents

A composite material is a combination of two or more distinct materials or constituents. This usually includes a matrix material and reinforcement. The matrix surrounds the reinforcement and constrains it. The applied load on the composite is transferred through the matrix to the reinforcement.

Composites can be categorized into three groups, based on the type of matrix material; these are Metal Matrix Composite (MMC), Ceramic Matrix Composite (CMC), and Polymer Matrix Composite (PMC). Examples of each one of these are shown in Figure 2.1.1.
Figure 2.1.1 a) Cross section SEM of Ti–6Al–4V as MMC (back) and fracture surface (front) [2] b) Fracture surface of fibre reinforced ceramics with SiC-fibres and SiC-matrix [3] c) Fracture surface of unidirectional E-glass fibre epoxy (PMC)

The reinforcing phase can have a variety of forms ranging from particulates to fibres. Fibres have diameter in the order of micrometers and length up to meters. As an example, E-glass fibre’s diameter ranges between 5 μm to 20 μm [4]. Fibrous reinforcement can be further characterized in terms of length (short vs. long fibres). Long fibres can be Unidirectional (UD), woven and cross ply as illustrated in Figure 2.1.2. There is a differentiation between long and short fibre based on load transfer, which will be discussed in Section 2.2.
Polymer matrices are usually reinforced with reinforcements like carbon and glass. The improved properties of PMC’s have led to their increased utilization in different industries such as sport, automotive and aerospace industries.

2.1.1.1 Continuous Fiber Composites

Continuous fibre composites are in general layered composites comprised of long fibres. Depending on how fibres are aligned (tied together), the composite can be categorized as UD, woven or cross ply composite. Figure 2.1.3 shows unidirectional and woven long fibres compared to cross-ply laminate composite.

![Figure 2.1.3](image)

Figure 2.1.3 a) Unidirectional fibre. b) Plain weave fibre. C) Cross-ply composite.
Unidirectional (UD) composites

In UD or aligned long fibre composites, fibres are continuous and aligned parallel to each other. These composites are manufactured by methods such as hand lay-up, laying prepreg tapes and pultrusion process. In general, the manufacturing process of UD composite compared to other types of composite is expensive and time intensive. UD composites give the highest strength and stiffness in the direction of fibres, therefore they are highly anisotropic. Common applications of UD composites are in the cases where loading direction is well defined. Here fibres are aligned in the direction of the loading.

Laminates

In order to provide improved properties in different directions, laminate composites are formed by stacking a number of UD plies, with each ply in a certain orientation (Figure 2.1.3c). This will reduce the anisotropy observed in UD composite. The anisotropy reduces if the ply direction is evenly distributed. As an example, it was shown that 0/60/120 or 0/90/45 case show isotropic behavior in the plane of laminates [5]. In other word any $n$ layer laminate with layers with $\frac{n}{n}$ fibre orientation shows isotropic behaviour. If the laminate is not symmetric about the thickness center line it will bend or twist under longitudinal loading. Cross-ply (0/90) is a common type of laminate. Laminates can also be manufactured by both wet processing and dry or “prepreg” processing and filament winding method. Filament winding is an automatic method to place fibres impregnated in resin onto a rotating mandrel. Reinforced polymer manufactured by this method is widely used in the form of a pipe. Glass fibre reinforced polymer (GFRP) pipes and storage tanks are widely used in water treatment and sewage industries.

Woven composites

In woven fibres or fabrics, weft yarns weave across a parallel set of yarns called warp yarns using hand or machine. Figure 2.1.4a shows a typical woven (plain weave) fabric. Depending on the pattern of the weave, the woven fabrics can be categorized into different patterns as shown in Figure 2.1.4. In manufacturing composite, woven fabrics form the required shape easier than unidirectional composites.
2.1.1.2 Discontinuous Fibre Composite

Discontinuous fibre composites can be comprised of either short or long fibres. Short and long fibre differentiation depends on the composite system and the aspect ratio of the fibre. The mechanics behind the load transfer of fibre will be discussed in Section 2.2. Typically for PMCs, the fibre aspect ratio required to attain the maximum composite stiffness is about 100, which gives a length of 1.5 mm for E-glass fibres (Section 6.1.4 [5]). Examples of each one of these are shown in Figure 2.1.5.

![Discontinuous Fibre Composite](image1.png)

**Figure 2.1.5** a) Discontinuous unidirectional long fibre. b) Schematic of discontinuous random fibre. c) Roll of textile glass mat [8].
Random or particulate fibre composites are comprised of fibres which are randomly distributed in the matrix. As fibres are randomly dispersed in the composite, they form in-plane isotropic material.

These types of composites are manufactured by injection and compression molding methods. The polymer flow and pressure determines the fibre orientation. These types of fibres are used in automotive parts.

2.2 Deformation of composite materials

In this section, attention is paid to how composites deform under external loading or displacement. The deformation of composite from elastic to inelastic, which in turn leads to fracturing, is discussed here. This section focuses on the types of material that are important to this research. Specifically, this includes UD and fabric material loaded on-axis (transverse and longitudinal) and off-axis.

The goal of this section is to predict the $\sigma - \epsilon$ curve for composites with different microstructures and explain how the $\sigma - \epsilon$ response mirrors the micromechanical behaviour of the material. These properties include fibre volume fraction $v_f$; fibre angle with regard to the composite geometry, $\theta$; matrix Young’s modulus, $E_m$; fibre stiffness, $E_f$; matrix yield and ultimate strength, $\sigma_{yp}, \sigma_{um}$; fibre and matrix failure strain, $\epsilon_f^*, \epsilon_m^*$; and fibre strength, $\sigma_f^*$.

2.2.1 Elastic deformation

When a composite is under tension, it first deforms elastically (Figure 2.2.1). Elastic deformation is linear and reversible. The extent of elastic deformation depends on the
composite microstructures (type, size, alignment and arrangement including fibre volume fraction) that were discussed in the preceding section.

![Figure 2.2.1 stress-strain curve for glass fibre epoxy pre-preg specimen.](image)

2.2.1.a UD Longitudinal and transverse properties

The simplest case to study elastic deformation in composites is to assume a UD composite in which all fibres are continuously aligned in the direction of loading. UD composites show the highest strength and stiffness in one direction.

To determine mechanical properties in the longitudinal direction, the composite (Figure 2.2.2a) can be simplified into a slab model as in Figure 2.2.2b. In this model both fibre and matrix experience equal strain under the applied stress of $\sigma_1$ as shown in Figure 2.2.2c.

As axial strain is equal for both matrix and fibre, we have:

$$
\varepsilon_1 = \varepsilon_{1f} = \frac{\sigma_{1f}}{E_f} = \varepsilon_m = \frac{\sigma_{1m}}{E_m}
$$

(2.2.1)

where the $m$ and $f$ subscripts denote the properties related to the matrix and fibre respectively.
Figure 2.2.2a) Schematic of a UD composite b) Slab model having a fibre volume fraction of $f$ c) The model under longitudinal stress $\sigma_1$, fibre and matrix experience equal axial strain $\varepsilon_1[5]$.

When load is applied to the composite slab model in Direction 1, it is shared between its constituents (As shown in Figure 2.2.2c). The load on each constituent is equal to its stress times its area fraction. Therefore, the applied longitudinal stress $\sigma_1$, can be determined from this load partition in terms of the constituent’s stress as:

$$\sigma_1 = f\sigma_{1f} + (1 - f)\sigma_{1m} \quad (2.2.2)$$

It should be noted that all equations in this chapter are taken from the book written by Hull and Clyne [5] unless otherwise stated.

As in PMCs $E_m < E_f$ from Equation (2.2.1) it can be concluded that the stress value in the fibre is much higher than that of the matrix, so the fibre withstands the major part of the applied load.

To determine the composite stiffness in Direction 1, using Equation (2.2.1) and Equation (2.2.2) we have:
Equations (2.2.1), (2.2.3) lead to the well-known Rule of Mixture (ROM):

\[
E_1 = \frac{\sigma_1}{\varepsilon_1} = \frac{\sigma_1}{\varepsilon_{1f}} = \left[ f \sigma_{1f} + (1 - f) \sigma_{1m} \right] \left( \frac{\sigma_{1f}}{E_f} \right)
\]

(2.2.3)

where \( E_f \) and \( E_m \) are fibre and matrix stiffness and \( f \) is the fibre volume fraction.

The above formula, however, is derived from a simplified model that gives a very good agreement with more advanced treatments and experimental data [9], [10].

To establish the transverse behaviour of the UD composite, the model is simplified into a slab model in Figure 2.2.3.

**Figure 2.2.3** Slab model under transverse loading with equal transverse stress \( \sigma_2 \) [5].

Here we assume in the slab model that composite behaviour is the same in both the 2 and 3 directions, which is not true because Direction 3 is more similar to Direction 1 in the model. It is also assumed that the stress in Direction 2 is the same for both fibre and matrix, which leads to:
\[ \sigma_2 = \sigma_{2f} = \varepsilon_{2f} E_f = \sigma_{2m} = \varepsilon_{2m} E_m \] 

(2.2.5)

The overall strain in Direction 2 can be represented as:

\[ \varepsilon_2 = f \varepsilon_{2f} + (1 - f) \varepsilon_{2m} \] 

2.2.6

By plugging the above relation in

\[ E_2 = \frac{\sigma_2}{\varepsilon_2} \] 

(2.2.7)

the stiffness of the composite can be found using the following formula known as the Reuss formula.

\[ E_2 = \left[ \frac{f}{E_f} + \frac{(1 - f)}{E_m} \right]^{-1} \] 

(2.2.8)

Comparing the results of the Reuss model and the experiment shows that the equal stress assumption is inadequate, since the stress in Direction 2 for the matrix varies from \( \sigma_{2m} = \frac{\sigma_{2f} E_f}{E_m} \) to \( \sigma_{2m} = \sigma_{2f} \). It is shown with the help of the strain field that the matrix which is parallel with fibres in the transverse direction is constrained by the fibres and experiences the same strain as the fibres. This leads to very low stress in the matrix in these regions. The regions in the matrix that are in series with fibres experience equal stress to the fibres and therefore very high strain (Figure 2.2.4). Similar inhomogeneity was observed in the photo elastic images of transparent macromodel composite loaded transverse to the fibres [5].
Figure 2.2.4 Schematic of strain field of a) unstrained and b) transversely strained composite.

The model underestimates the transverse stiffness of the composite. Among other models proposed to overcome this deficiency, the Halpin-Tsai model was the most successful model as its transverse stiffness for the composite was calculated using the following semi-empirical formula [11]:

\[
E_2 = \frac{E_m(1 + \xi \eta f)}{(1 - \eta f)} \tag{2.2.9}
\]

in which \( \eta \) is calculated as:

\[
\eta = \frac{\frac{E_f}{E_m} - 1}{\frac{E_f}{E_m} + \xi} \tag{2.2.10}
\]

where \( \xi \) is close to 1 for this case \( \xi \) is an empirical factor that can be found experimentally. A typical stress-strain curve for a glass fibre epoxy under axial and transverse load is shown in Table 2.2.1. The results are compared with the elastic response of epoxy and glass as epoxy behaves elastically up to a strain of 1.5%. The transverse ultimate stress and failure strain taken from the experimental results are also added to the figure. Axial failure results are not included because they happen at higher values of strain.
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<thead>
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<td>Epoxy</td>
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Table 2.2.1 Elastic Young’s modulus prediction for glass fibre epoxy having 40% fibre volume fraction under axial and transverse load using ROM, Reuss and Halpin-Tsai models compared with that of matrix and fibre.

Figure 2.2.5 Comparison of experimental data and predictions by the ROM (equal strain), Reuss (equal stress) and Halpin-Tsai (with $\xi = 1$) models [5], [9].

Figure 2.2.5 shows a comparison between predictions by ROM, Reuss and Halpin-Tsai and experimental data for the glass fibre/epoxy. The other point to mention is that
transverse testing of the composite is more prone to error. This is mainly due to high stresses in the matrix that may cause the polymer to creep even under lower loads.

**UD shear stiffness**

To determine the shear stiffness of a UD composite, the actual and the slab models can be used as shown in Figure 2.2.6. Shear stiffness can be found using a similar method to the axial and transverse stiffness.

![Figure 2.2.6 Schematics of actual and slab model used to determine shear stiffness of UD composite][5].

Using the slab model and assuming equal shear strain for both fibre and matrix (Figure 2.2.6b), the shear stiffness, $G_{13}$, is calculated as:

$$G_{13} = G_{31} = fG_f + (1 - f)G_m$$ (2.2.11)

Assuming equal stress for the slab model, axial shear modulus is defined by:

$$G_{12} = \left[ \frac{f}{G_f} + \frac{1-f}{G_m} \right]^{-1}$$ (2.2.12)
From the slab model, it can be concluded that the other shear stiffness (i.e. $G_{23}$) can be found using $G_{12}$. Equation (2.2.11) and particularly (2.2.12) (equal strain model) are still not very close to what happens in composites under shearing. Therefore, both equal strain and equal stress condition are not proper to determine shear properties of the composite. Comparing results from these models with experiments shows while Equation (2.2.11) is a significant overestimate for the value of $G_{13}$, Equation (2.2.12) underestimates the values of $G_{12}$ and $G_{23}$ [5].

To determine $G_{12}$, Halpin-Tsai suggests the following relation which gives a better approximation to experimental results compared to the slab model [5], [12]:

$$G_{12} = \frac{G_m(1 + \xi \eta f)}{1 - \eta f} \quad (2.2.13)$$

where

$$\eta = \frac{G_f}{G_m} - 1 + \frac{G_f}{G_m + \xi} \quad (2.2.14)$$

To value of $G_{23}$ can be found using the following relation:

$$G_{23} = \frac{E_2}{2(1 + \nu_{23})} \quad (2.2.15)$$

in which the Poisson ratio $\nu_{23}$ can be found as will be described in the next page. The above relation matches more with the real case compared to the one calculated using the slab model [5].

**UD Poisson effects**

Poisson’s ratio $\nu_{ij}$, which is defined as:

$$\nu_{ij} = \frac{\text{contraction in } j - \text{direction}}{\text{strain in } i - \text{direction}} = -\frac{\varepsilon_j}{\varepsilon_i} \quad (2.2.16)$$
Can be found using the slab model, and \( \nu_{12} \) in a similar way to the equal strain case in stiffness:

\[
\nu_{12} = f \nu_f + (1 - f) \nu_m \quad (2.2.17)
\]

So far, five material constants for the UD composite have been defined and the other two constants, i.e. \( \nu_{21} \) and \( \nu_{23} \), are still to be determined. \( \nu_{21} \) can be found using:

\[
\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2} \quad (2.2.18)
\]

Therefore:

\[
\nu_{21} = \frac{[f \nu_f + (1 - f) \nu_m] E_2}{E_1} \quad (2.2.19)
\]

As \( E_2 < E_1 \), \( \nu_{21} \) will be smaller than \( \nu_{12} \). When the composite is stressed transversely, the fibres resist strongly to the axial contraction, leading to a high contraction in the other transverse direction, so \( \nu_{23} \) is expected to be high compared to the other two Poisson’s ratio values [5].

Clyne determined \( \nu_{23} \) by considering the overall volume change in the material. The relation leads to:

\[
\nu_{23} = 1 - \nu_{21} - \frac{E_2}{K} \quad (2.2.20)
\]

where \( K \) is the bulk modulus of the composite and determined as:

\[
K = \left[ \frac{f}{K_f} + \frac{1 - f}{K_m} \right]^{-1} \quad (2.2.21)
\]

2.2.1.1 Off-Axis Loading of UD Composite

The term off-axis is used to refer to the case when the loading angle and the fibre angle are not in the same direction (Figure 2.2.7). As the thickness of the lamina is usually much smaller than other dimensions, it is therefore; commonly assumed that the lamina is
in the plane stress condition (i.e. $\sigma_{33} = \tau_{23} = \tau_{13}=0$), in this case it is assumed that the fibre and the matrix are elastic and fully bonded. It is also important to recognize that off-axis loading creates shear stress.

![Figure 2.2.7 Schematic of off-axis loading on one ply where force is applied in x - y coordinate system and fibres are aligned in 1-2 coordinate (material coordinate) system.](image)

It is shown in the literature that normal and shear strain in the loading coordinate system can be related to the normal and shear stress using the following relation [5]:

$$
\begin{bmatrix}
\epsilon_x \\
\epsilon_y \\
\gamma_{xy}
\end{bmatrix} = \left[T'\right]^{-1} \left[S\right] \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} = \left[S\right] \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} 
$$

(2.2.22)

where $S$, the compliance matrix is defined as:

$$
S = \begin{bmatrix}
\frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & 0 \\
-\frac{\nu_{21}}{E_2} & \frac{1}{E_2} & 0 \\
0 & 0 & \frac{1}{G_{12}}
\end{bmatrix}
$$

(2.2.23)

and $T$ is defined as:
\[
[T] = \begin{bmatrix}
\cos^2 \theta & \sin^2 \theta & 2 \cos \theta \sin \theta \\
\sin^2 \theta & \cos^2 \theta & -2 \cos \theta \sin \theta \\
-\cos \theta \sin \theta & \cos \theta \sin \theta & \cos^2 \theta - \sin^2 \theta
\end{bmatrix}
\] (2.2.24)

and

\[
[T']^{-1} = \begin{bmatrix}
\cos^2 \theta & \sin^2 \theta & -\cos \theta \sin \theta \\
\sin^2 \theta & \cos^2 \theta & \cos \theta \sin \theta \\
2 \cos \theta \sin \theta & -2 \cos \theta \sin \theta & \cos^2 \theta - \sin^2 \theta
\end{bmatrix}
\] (2.2.25)

in which \( \theta \) is the angle between the fibre and the loading direction as shown in Figure 2.2.7. Therefore, \( \bar{S} \) can be written in the form of:

\[
[\bar{S}] = \begin{bmatrix}
\bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\
\bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\
\bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66}
\end{bmatrix}
\] (2.2.26)

Using the above parameters, the mechanical properties of the ply in the loading direction (i.e. off-axis Young’s modulus, shear modulus and Poisson’s ratio) can be expressed as:

\[
E_x = \frac{1}{\bar{S}_{11}}
\] (2.2.27)

\[
E_y = \frac{1}{\bar{S}_{22}}
\] (2.2.28)

\[
G_{xy} = \frac{1}{\bar{S}_{66}}
\] (2.2.29)

\[
\nu_{xy} = -E_x \bar{S}_{12}
\] (2.2.30)

\[
\nu_{yx} = -E_y \bar{S}_{12}
\] (2.2.31)

Figure 2.2.8 compares the stress-strain response of glass fibre epoxy as fibre angle \( \theta \) changes from 0° to 45°.
Figure 2.2.8 Off-axis stress-strain curves of glass fibre epoxy specimens with different fibre orientation [6].

The figure indicates that the strength of each off-axis ply decreases by changing the fibre angle from 0° to 45°.

As can be found in Equations (2.2.27) to (2.2.31), an important feature when a UD ply is under off-axis loading, is the existence of the non-zero “interaction” terms (i.e. $\bar{S}_{16}$ and $\bar{S}_{26}$). This means that the UD ply normal stress produces shear strain. This introduces errors in the testing of off-axis composites (Figure 2.2.9). The tendency of producing shear leads to a moment in the grips and causes an end constraint effect that should be considered in the testing [13].
Figure 2.2.9 a) Uniform state of stress for off-axis composite b) effect of clamped ends [13].

2.2.1.2 Laminated Composite

As stated in the previous section, off-axis plies may show highly anisotropic properties; however, this can be reduced by stacking plies having different fibre angles. In order to characterize the elastic behaviour of laminates having off-axis and on-axis UD plies, the overall stiffness matrix must be determined first. If the laminate is assumed to be flat and through thickness stress and edge effect is neglected, the stiffness matrix components can be found using the stiffness matrix and the thickness for each ply.

Cross-Ply Laminate

A simple case is a cross-ply laminate in which plies are oriented at right angles to each other. It is assumed in the cross-ply all plies have 0 or 90 degree fibre angles. Consider a cross-ply laminate that has $n$ plies of which $l$ plies have fibres along the loading direction and $m$ plies perpendicular to the loading direction (Figure 2.2.14.a). If the stiffness of
each longitudinal and transverse ply is $E_L$ and $E_T$ respectively, the composite stiffness in the direction of the load using ROM is:

$$E = \frac{l}{n}E_L + \frac{m}{n}E_T$$  \hspace{1cm} (2.2.32)

**General Laminate**

For more general cases where each ply makes an angle of $\phi$ with the direction of laminate loading, the overall stiffness of the laminate can be defined as:

$$\bar{C}_{11g} = \frac{\sum(\bar{C}_{11k}t_k)}{\sum t_k}$$  \hspace{1cm} (2.2.33)

where subscript $k$ shows the parameter belongs to the $k^{th}$ ply.

Other overall (global) stiffness terms can be found using a similar method. The corresponding overall compliances can be found from the overall stiffnesses. These compliances are used to characterize the overall mechanical properties similar to Equations (2.2.27) to (2.2.31). As mentioned in the beginning of this section, anisotropy in the laminate could be lower than in the plies. If the fibre directions are more evenly distributed in the laminate, the interaction term relating shear and tension becomes smaller. Laminates like $0^\circ/45^\circ/90^\circ/135^\circ$ or $0^\circ/60^\circ/120^\circ$ can be assumed to be transversely isotropic materials.

**2.2.1.3 Short fibre composites**

The previous three subsections considered composites having long fibres. The elastic behaviour of short fibre is discussed here. The differentiation between short and long continuous fibres is based on the ability to build that load up to the fracture strength of the fibre. A short fibre does not break under further strain in the composite. A fibre is assumed to be a short fibre when its length is smaller than a critical length.
Fibre Critical Length

To determine the critical length \( (l_c) \) in terms of the short fibre diameter and properties, a small element of the fibre is assumed as shown in Figure 2.2.10.a. If the normal and interfacial shear stresses acting on the element are \( d\sigma \) and \( \tau_i \) respectively, the force balance acting on the element of the fibre is:

\[
2\pi r dx \tau_i = \pi r^2 d\sigma
\]  (2.2.34)

Therefore:

\[
\frac{d\sigma}{dx} = \frac{2\tau_i}{r} = \frac{4\tau_i}{d}
\]  (2.2.35)

where \( x \) and \( r \) are as shown in Figure 2.2.10 and \( d \) is the fibre diameter. The fibre critical length \( (l_c) \) is the minimum length of the fibre that can carry maximum axial stress equal to the fibre strength \( \sigma_f^* \). The tensile stress in the fibre increases from zero at one end to a certain value (depending on the length of the fibre) in the middle and then decreases to zero at the other end.

If the composite is strained beyond the elastic limit where \( (\varepsilon_{overall} = \varepsilon_{m,yield}) \), matrix yielding starts at the end of the fibre-matrix interface. This is because the maximum shear stress happens at these locations. If the composite is strained further, the yielding distributes along the length of the fibre. This will continue until the interfacial shear stress reaches a uniform value of critical shear stress \( (\tau_i^*) \) along the fibre length. At this point the shear stress can build up to the peak normal stress at the fibre center equal to fibre strength \( (\sigma_f^*) \). If the interfacial stress is assumed to be uniform, the stress for a fibre with critical length can be found by integrating the stress increment from zero to fibre strength as:

\[
\int_0^{\sigma_f^*} d\sigma = \frac{4\tau_i^*}{d} \int_0^{\frac{l_c}{2}} dx
\]  (2.2.36)
The fibre critical length is defined by the expression:

\[ \sigma_i^* = \frac{2\tau_i^* l_c}{d} \]  \hspace{1cm} (2.2.37)

So, the fibre critical length is defined by the expression:

\[ l_c = \frac{d \cdot \sigma_i^*}{2\tau_i^*} \]  \hspace{1cm} (2.2.38)

As can be found in the above equation, shear strength between fibre and matrix is proportional to the inverse of critical length, so a shorter critical length indicates good adhesion [14].

**Changing Shear Stress Along the Fibre Length**

The axial stress carried by a fibre under strain is increasing from zero at the ends to its maximum level in the middle of the fibre. For a fibre with \( l < l_c \), the axial stress in the middle of the fibre is less than the maximum normal stress for the case of a fibre with critical length; if \( l > l_c \), the fibre carries maximum normal stress tolerates the same as a fibre with \( l = l_c \) as shown in Figure 2.2.10.b. The maximum normal stress is shown by \( \sigma_{ma} \) in Figure 2.2.10.c. This is however for a longer fibre a longer portion of it carries the maximum normal stress. A point that should be noted is that the interfacial shear stress is positive at one end of the fibre length and it changes into negative value at the other end. This is concluded from the static equilibrium of the fibre.
As can be evaluated by using Equations (2.2.36) to (2.2.38), the value of fibre critical length is identified when the interfacial shear stress is assumed constant along the fibre (i.e. $\tau_i^*=$constant). This is, however the value of shear stress on the interface is not the same along the length of the fibre. The idea here is to investigate the influence of the shear stress affects the value of critical length. Therefore, the hyperbolic change of the stress at both ends of fibre to its maximum value (see Figure 2.2.10b) is considered here. In other words, what happens in reality is that the interfacial shear stress $\tau_i$ is changing along the fibre length and it depends on x (the distance from the fibre center). Cox and Outwater defined a model for short fibres and showed that the interfacial shear stress can be defined in terms of the distance from center of the fibre as [16], [17]:

$$\tau_i = \frac{n\varepsilon_1}{2}E_f \sinh \left(\frac{nx}{r}\right) \text{sech}(ns)$$  \hspace{1cm} (2.2.39)

where $s$ is the fibre aspect ratio ($s=l/d$), $n$ is a dimensionless constant. $n$ can be stated in terms of $E_f$, $E_m$, $\nu_m$ and fibre volume fraction. $\varepsilon_1$ is the overall composite strain. It should be noted that the fibre end surface contribution to the stress that a fibre withstands is not considered in the above relations. To determine the stress for a fibre with critical
length, the fibre stress increment in Equation (2.2.35) is required to be integrated from zero to the fibre strength as:

\[ \int_0^{\sigma_f} d\sigma = \frac{4}{d} \int_0^{\frac{l_c}{2}} \tau_i(x) dx \]  \hspace{1cm} (2.2.40)

Using the above equation and Equation (2.2.39) we obtain:

\[ \int_0^{\sigma_f} d\sigma = 2 \int_0^{\frac{l_c}{2}} \frac{n \epsilon_1}{2} E_f \sinh \left( \frac{nx}{r} \right) \text{sech}(ns) dx \]  \hspace{1cm} (2.2.41)

As \( s \) and \( n \) are independent of \( x \), and we know \( s = \frac{l_c}{2r} \), and \( \int \sinh \left( \frac{nx}{r} \right) = \frac{\cosh \left( \frac{nx}{r} \right)}{\frac{n}{r}} \) so:

\[ \sigma_f = \frac{n \epsilon_1}{r} E_f \text{sech}(ns) \left( \frac{[\cosh(ns) - 1]}{\frac{n}{r}} \right) \]  \hspace{1cm} (2.2.42)

which gives:

\[ \cosh(ns) = \frac{E_f \epsilon_1}{E_f \epsilon_1 - \sigma_f^*} \]  \hspace{1cm} (2.2.43)

We know \( \cosh(ns) \geq 1 \) therefore

\[ E_f \epsilon_1 \geq \sigma_f^* = E_f \epsilon_f^* \]  \hspace{1cm} (2.2.44)

where \( \epsilon_f^* \) is the fracture strain of the fibre (for E-glass fibre epoxy this value is 2.6\%)

So Equation (2.2.43) is meaningful when \( \epsilon_1 \geq \epsilon_f^* = 0.026 \).

Equation (2.2.43) can be written as:

\[ ns = \text{arccosh} \left( \frac{E_f \epsilon_1}{E_f \epsilon_1 - \sigma_f^*} \right) \]  \hspace{1cm} (2.2.45)

We know
\[ \text{arccosh}(X) = \ln \left( X + \sqrt{X^2 - 1} \right); \quad X \geq 1 \quad (2.2.46) \]

so

\[ s = \frac{1}{n} \ln \left[ \frac{E_f \varepsilon_1}{E_f \varepsilon_1 - \sigma_f^*} + \sqrt{\left( \frac{E_f \varepsilon_1}{E_f \varepsilon_1 - \sigma_f^*} \right)^2 - 1} \right] \quad (2.2.47) \]

As \( s = \frac{l_c}{2r} \) then the critical length of the fibre, assuming change in the value of shear stress across fibre’s length can be found as:

\[ l_c = \frac{2r}{n} \ln \left[ \frac{E_f \varepsilon_1}{E_f \varepsilon_1 - \sigma_f^*} + \sqrt{\left( \frac{E_f \varepsilon_1}{E_f \varepsilon_1 - \sigma_f^*} \right)^2 - 1} \right] \quad (2.2.48) \]

The effect of assuming a constant value for the interfacial shear stress on the determination of fibre critical length is studied here. Using the material properties for glass fibre epoxy as given in the Table 2.2, the values of the critical length obtained from Equation (2.2.48) can be compared with the result of Equation (2.2.38). To see including hyperbolic change of stress at the fibre end on the determination of critical length on a different fibre, aramid (Kevlar) fibre is chosen.

<table>
<thead>
<tr>
<th>( \tau^*_i ) (MPa)</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma^*_f ) (MPa)</td>
<td>2000</td>
</tr>
<tr>
<td>( v_m )</td>
<td>0.38</td>
</tr>
<tr>
<td>( f )</td>
<td>0.4</td>
</tr>
<tr>
<td>( E_m ) (MPa)</td>
<td>2900</td>
</tr>
<tr>
<td>( E_{aramid} ) (MPa)</td>
<td>130000</td>
</tr>
<tr>
<td>( r_{glass} ) (( \mu )m)</td>
<td>7.5</td>
</tr>
<tr>
<td>( r_{kevlar} ) (( \mu )m)</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 2.2 E-glass fibre, aramid, and epoxy properties
Figure 2.2.11 compares the effect of using a constant and a hyperbolic $\tau_i$ to determine critical length for glass fibre epoxy at two different fibre volume fractions ($f = 0.4$ and $f = 0.2$) and also aramid (Kevlar) epoxy ($f = 0.4$). As shown in Figure 2.2.11 it can be found that assuming a constant $\tau_i$, results in the underestimation of fibre critical length. As can be seen from the results of composites with two different fibre volume fractions, it can be concluded that for lower values of fibre volume fractions, the simple equation for determining fibre critical length cannot be trusted.

In summary, the idea of critical fibre length and the load transfer along the length of the fibre by shear, using the simplified approach (the constant shear along the fibre length) leads to Equation (2.2.38), while more thorough approach leads to the hyperbolic trigonometric terms (Equation (2.2.48)). This effect is important because if fibres are longer than the critical length it is expected that the shear load can be built up to the point of fracture limit, if they are shorter than the critical length they do not transfer enough load to break the fibres.

![Comparison of $l_c$ prediction using Equation (2.2.38) in which the interfacial shear stress is assumed to be constant and (2.2.48), in which $\tau_i(x)$, is dependent on x (place from center of the fibre).](image)
Elastic response of short fibre aligned composite

To determine the elastic response of a short fibre composite, from Cox model we have:

\[
\sigma_f = E_f \varepsilon_1 \left[ 1 - \cosh \left( \frac{nx}{r} \right) \sech (ns) \right] \tag{2.2.49}
\]

To predict the elastic response of the aligned short fibre composite, if a cross section of the composite normal to the applied load is considered, the applied load can be defined in terms of the average load carried by the constituents as:

\[
\sigma_1 A = f A \bar{\sigma}_f + (1 - f) A \bar{\sigma}_m \tag{2.2.50}
\]

\[
\sigma_1 = f \bar{\sigma}_f + (1 - f) \bar{\sigma}_m \tag{2.2.51}
\]

where \( \bar{\sigma}_f \) and \( \bar{\sigma}_m \) are average stresses carried by fibre and matrix. This is called the ‘Rule of average’.

To determine the average stress carried by the fibre, the term found for the fibre stress in Equation (2.2.49) should be integrated over the length of the fibre as:

\[
\bar{\sigma}_f = \frac{E_f \varepsilon_1}{L} \int_0^L \left[ 1 - \cosh \left( \frac{nx}{r} \right) \right] \frac{\cosh (ns)}{\cosh (ns)} \, dx \tag{2.2.52}
\]

\[
\bar{\sigma}_f = E_f \varepsilon_1 \left( 1 - \frac{\tanh (ns)}{ns} \right) \tag{2.2.53}
\]

Assuming the matrix has equal strain to that of the composite

\[
\bar{\sigma}_m \approx E_m \varepsilon_1 \tag{2.2.54}
\]

Substituting Equations (2.2.53) and (2.2.54) into (2.2.51) gives the following relation for the axial Young’s modulus of the composite:

\[
E_1 = f E_f \left( 1 - \frac{\tanh (ns)}{ns} \right) + (1 - f) E_m \tag{2.2.55}
\]
Taking the effect of fibre end stress into consideration, $E_1$ is expressed as:

$$E_1 = fE_f \left( 1 - \frac{(E_f - E'_m) \tanh(ns)}{E_f n_s} \right) + (1 - f)E_m \quad (2.2.56)$$

where

$$E'_m = \frac{E_f (1 - \text{sech}(ns)) + E_m}{2} \quad (2.2.57)$$

It can be found from Equations (2.2.55) and (2.2.56) that in case $\frac{\tanh(ns)}{n_s} \ll 1$ equations will be similar to the ROM relation for the stiffness (Equation 2.4). In this case, the composite overall Young’s modulus reaches to its maximum value.

The above condition leads to the following term for the fibre aspect ratio:

$$s_{RM} \approx \frac{10}{n} \quad (2.2.58)$$

As $n$ has a value close to 0.1 for PMCs, the above equation gives $s_{RM} \approx 100$, which means for short fibre PMCs to reach the maximum stiffness, fibre length should be around 100 times more than its diameter. This is helpful when the main goal is to maximize the load transfer in PMCs.

The results for the composite Young’s modulus derived from Equations (2.2.55) and (2.2.56) compared with the Eshelby method, which gives more precise results, show that for low values of the aspect ratio the standard shear lag model is inaccurate (this is specifically applicable to MMCs).

The Halpin-Tsai model can also be used to determine the value of longitudinal and transverse Young’s moduli of oriented discontinuous composites as [12]:

$$E_1, E_2 = \frac{E_m (1 + \xi \eta f)}{1 - \eta f} \quad (2.2.59)$$

Where
\[ \xi_{E_1} = \frac{2l}{d} + 40f^{10} \quad (2.2.60) \]

\[ \xi_{E_2} = 2 + 40f^{10} \quad (2.2.61) \]

\[ \eta = \frac{E_f}{E_m} - \frac{1}{\xi} \quad (2.2.62) \]

where \( f^{10} \) is the fibre volume fraction to the power of 10. The term \( f^{10} \) is small and negligible for values of fibre volume fraction under 50%, which is the case here. Shear stiffness for discontinuous fibre composites is determined by:

\[ G_{12}, G_{23} = \frac{G_m(1 + \xi \eta f)}{1 - \eta f} \quad (2.2.63) \]

where

\[ \xi_{G_{12}} = 1 + 40f^{10} \approx 1 \quad (2.2.64) \]

\[ \xi_{G_{23}} = \frac{1}{4 - 3v_m} \quad (2.2.65) \]

then

\[ G_{12} = \frac{G_m(1 + f)}{1 - f} \quad (2.2.66) \]

It should be mentioned that to define the mechanical properties of particulate composites, the value of \( \xi \) is calculated as: \( \xi_G = \xi_{G_{12}} \) as per Equation (2.2.64) and \( \xi_E = \xi_{E_2} \) as defined in Equation (2.2.61)[12].

### 2.2.2 Inelastic behavior and fracture

As the strain is increased in a composite material, eventually inelastic deformation will occur. The events associated with the onset of inelastic deformation of the composite
include plastic deformation of the matrix, fracture of fibres, fibre and matrix interfacial
debonding, sliding in the interface, and crack and cavity formations in the matrix. If any
of these phenomena occur, the overall stress-strain curve of the composite will be
affected. If the applied load increases, the increase in the inelastic deformation and
accumulation of fibre fractures and tiny cracks lead to the final fracture of the composite.

2.2.2.1 UD Longitudinal and Transverse Composites

The initial elastic deformation is adequately modeled by the rule of mixtures and the
Reuss model. We already covered the elastic model for the UD composite in Section
2.2.1a. The overall composite stress in terms of the constituent’s stress is found using
Equation (2.2.67):

\[ \sigma_1 = f \sigma_f + (1 - f) \sigma_m \]  \hspace{0.5cm} (2.2.67)

This is true up to point \( \varepsilon_{mu} \) in Figure 2.2.12.a and point \( \varepsilon_{fu} \) in Figure 2.2.12.b.

The stiffness at this point using ROM is:

\[ E_1 = f E_f + (1 - f) E_m \]  \hspace{0.5cm} (2.2.68)

This can be expressed in the differential format as:

\[ \left( \frac{d\sigma}{d\varepsilon} \right)_1 = f \left( \frac{d\sigma}{d\varepsilon} \right)_f + (1 - f) \left( \frac{d\sigma}{d\varepsilon} \right)_m \]  \hspace{0.5cm} (2.2.69)

The subscript 1 indicates the longitudinal direction, and indices \( m \) and \( f \) show matrix and
fibre respectively. As soon as one constituent starts to deform inelastically (i.e. its stiffness
is reduced) or fails, its \( \left( \frac{d\sigma}{d\varepsilon} \right) \) changes so the left hand side of Equation (2.2.69) changes
and the inelastic behaviour of the composite starts. When a constituent’s \( \sigma - \varepsilon \) response
changes, the composite response, (i.e. overall \( \frac{d\sigma}{d\varepsilon} \)), also changes. Two situations can be
considered after the start of the inelastic deformation.
Situation 1: if the matrix breaks initially at its failure strain $\varepsilon_{mu}$, then only the fibres withstand the rest of the load showing stress $f \sigma_f$ up to the point of fibre fracture at $\varepsilon_{fu}$ (Figure 2.2.12.a).

When a crack in the matrix approaches very close to the fibre, a normal stress parallel to the crack and normal to the far field applied stress in front of the crack detaches the matrix from the fibre; this is the beginning of fibre debonding [18]. Fibre debonding blunts the crack and increases the composite toughness, and this will be discussed in the next section.

If the applied normal stress in the direction of the fibre reaches $\sigma_f^*$, then the fibre breaks and the crack goes through the fibre up to the final fracture of the composite. In practice fibres do not break all at once, so two scenarios might happen here. The first is that, after a random fibre break in the composite, with further straining, a cross section somewhere in the composite fractures if the strength of that section is reached (Cumulative weakening model [19], [20]).

The second case assumes that, by each fibre failure, the stress on the neighbor fibres arises. This local stress concentration can end in the fracture in adjacent fibres (fibre break propagation model [21], [22]).

Situation 2: in case fibres break earlier than the matrix (i.e. $\varepsilon_{fu} < \varepsilon_{mu}$), then upon more straining, the fibres break into even smaller parts until all fibres have lengths smaller than the critical length, where any further stress would have to be carried only by the matrix. This continues up to the point of matrix fracture at $\varepsilon_{mu}$ (Figure 2.2.12.b). In this case, if the fibre volume fraction is low, the composite failure stress may fall to a smaller value than the unreinforced matrix.
Figure 2.2.12 Schematic illustration of long fibre UD composite inelastic behaviour
a) When the fibre has higher strain to failure than matrix b) When the fibre has lower strain to failure than matrix [5]

Transverse Direction

Finding estimation for the ultimate stress of the composite when loaded transversely is not as simple as the longitudinal case. The transverse strength of the composite is affected by parameters such as the interfacial bond, void content, and fibre distribution. In general, the strength and failure strain of the composite is reduced compared to that of the unreinforced matrix.

Figure 2.2.13 shows, however, the value of stiffness is increased in transverse UD composites (see Figure 2.2.13b), but the value of failure stress and failure strain has decreased dramatically compared to the epoxy. While the failure stress of the transverse composite has been reduced to half of that of the epoxy, the failure strain has decreased more than 75%. The difference between the transverse responses of samples A, B and C indicates that transverse testing might have discrepancies partly due to the higher stress in the matrix.
Figure 2.2.13 Comparison between the transverse stress-strain curve for UD E-glass fibre polyester (top) and three unreinforced polyester (bottom) [5].

2.2.2.2 Short Fibre Composite

The inelastic behaviour in short fibre composites is mostly affected by the plastic deformation in the matrix. The reason is that, by further straining of the composite, the fibres’ lengths remain the same and the load carried by the fibres does not change; therefore, any change in the stiffness of the matrix begins inelasticity in the composite. In other words, for a certain value of short fibre volume fraction after the fibres break into shorter lengths up to the critical length, \( \frac{d\sigma}{d\varepsilon} \) remains constant, but changes in \( \frac{d\sigma}{d\varepsilon} \) cause inelasticity in the composite (refer to Equation (2.2.69)). As was explained in 2.2.1.d, the maximum shear stress on the matrix happens at the fibre ends, and matrix plasticity starts at this point.
2.2.2.3 Laminate Composite (Transverse Cracking)

In a laminate consisting of plies in different directions, a fracture of transverse plies causes inelastic behaviour [23]. As mentioned in Section 2.2.1.c, a cross-ply laminate’s stiffness having \( n \) plies of which \( l \) plies have fibres along the loading direction and \( m \) plies perpendicular to the loading direction (Figure 2.2.14.a) can be found using ROM (Equation (2.2.32)) as:

\[
E = \frac{l}{n}E_L + \frac{m}{n}E_T
\]  

(2.2.70)

This equation is valid while plies are intact. If the composite’s strain reaches the fracture strain of the transverse plies \( \varepsilon_{TU} \) (point A in Figure 2.2.14.b), the transverse plies fail. At this point, as the stress carried by these plies is very small, the stress is:

\[
\sigma_A = E\varepsilon_{TU}
\]  

(2.2.71)

Further straining is only carried by the longitudinal plies, so the secondary value for overall stiffness is:

\[
E_s = \frac{l}{n}E_L
\]  

(2.2.72)

The composite fracture happens when the strain is equal to the fracture strain of the longitudinal plies \( \varepsilon_{LU} \). The stress at fracture is expressed by:

\[
\sigma_F = E\varepsilon_{TU} + E_s(\varepsilon_{LU} - \varepsilon_{TU})
\]  

(2.2.73)
Figure 2.2.14 a) Schematic illustration of cross-ply laminates having plies with fibres in transverse and longitudinal directions b) Schematic plot for stress-strain curve of cross ply composites

Delamination

Another mode of fracture in composite laminates is delamination. When a laminate is under stress, the interlaminar shear stresses transfer load between plies and this may cause interlaminar cracking and delamination. The interlaminar shear arises due to the rotation of each ply and it is proportional to the interaction compliance $\tilde{S}_{16}$, which relates overall shear strain to the axial stress in Equation (2.2.22). Figure 2.2.15 shows how interlaminar shear stress arises due to the rotation of plies towards the direction of loading. Pipes and Pagano showed that, for angle-ply laminates of carbon epoxy, the maximum interlaminar shear stress occurs at angles close to 30 degrees [24], [25].
Figure 2.2.15 Schematic picture showing how axial tension of the angle-ply composite introduces interlaminar rotation and raises the interlaminar shear stress [5].

2.3 Fracture

Fracture Mechanism

Fracture is the separation of material into parts when that material reaches its failure strength [19]. The first step in studying fracture is to know what keeps solid materials together. Materials are made of atoms that are attracted together by different bonds, including the ionic bonds. Depending on the interatomic distance, they attract or repel each other. When atoms are in certain interatomic distances (as shown in Figure 2.3.1), atoms are in equilibrium and have minimum potential bond energy (shown as binding energy in Figure 2.3.1).
Atoms on the surface have a higher value of potential bond energy, which is half of the binding energy. The surface energy, $\gamma$, is the summation of this energy for all atoms on the surface. To break the material and generate two new surfaces, the minimum energy of $2\gamma$ is required, i.e.:

$$U = 2\gamma$$

(2.3.1)

It was explained by Griffith that a crack cannot propagate unless the above energy is provided and then released by the crack propagation. Therefore, any mechanism in the material that absorbs a part of this energy would make the material more resistant to further cracking or, in simple words makes it tougher.

Toughness is the amount of energy that a material can absorb either elastically or plastically before it fractures and the work of fracture is the work done on the material to cause fracture. This energy is directly related to the amount of force and deformation of a material before fracture. Therefore, any deformation mechanism that takes additional energy would increase the toughness of the materials.
In polymers, the addition of rubber particles to thermoplastics in polymers absorbs energy and increases the material toughness. When a crack approaches a rubber particle in polymers like PMMA, it acts like a spring and deforms elastically. This process absorbs more energy and toughens the polymer [27]. On the other hand, mechanisms like generation of multiple crazes in polymers accelerate fractures because they increase the number of defects [28]. Apart from thermoplastics, the addition of nanoparticles like nanosilica has been recently used to toughen thermosets like epoxy [29]. PMCs are also good examples of increasing polymer toughness with the help of reinforcement. In glass fibre epoxy for instance, the addition of glass fibre, to epoxy can increase the toughness up to 20 times.

2.3.1 Linear Elastic Fracture Mechanics

Linear elastic fracture mechanics (LEFM) is a branch of solid mechanics that deals with structures containing singularities like a crack. It is based on elastic stress analysis around these singularities. The assumption here is that the material is linear elastic. The following sections explain how LEFM was initiated and developed.

2.3.2 Stress Based Fracture

The study of the fracture behaviour of the material when a crack exists in the material involves a proper understanding of the stress state in the area close to the tip of the crack. The study of crack tip stress was first carried out by introducing stress concentration and developing of the elasticity problems of bodies with a circular hole. This was first done by Inglis in 1913, Inglis showed that the local stresses close to a corner or hole in a part that is under stress could be higher than the average applied stress [30]. He used the theory of elasticity to establish the relation between stress at the edge of a circular hole and the average stress applied to an infinite body as shown in Figure 2.3.2. The approach was developed to establish a solution for the biharmonic equation.
Inglis then evolved the method into the stress state in a material with elliptical hole. He noticed that the amount of rise in the stress depends on the size of the radius of the curvature. In other words the smaller the radius at the elliptical holes the higher the stress concentration. He found the stress concentration factor for the elliptical hole as [30]:

\[
k = 1 + 2 \frac{a}{\sqrt{\rho}}
\]  

(2.3.2)

where a is the radius of the hole and \( \rho \) is the radius of curvature of the hole tip. This relation gives \( k=3 \) for circular hole, the value of \( k \) can be much greater for narrow elliptical holes. Inglis noticed that, however the stress at the tip of the hole might be high, but it decreases rapidly by getting farther from the hole. He also suggested that if there is any crack growth at the tip of the hole, the value of stress concentration would be even higher [31]. However, the Inglis’ work was a great step and can provide useful information about a material with crack like cavity, nevertheless it was not able to address the stress state at the tip of a real crack and to explain the effect of geometry on
this stress. For example, for a real crack the value of $\rho$ gets close to zero, this causes singularity in Equation (2.3.2).

The next step was to advance the theory of elasticity in materials with singular stress field. William was among the first ones to propose a solution to a problem with the singularity. In this problem, he assumed an elastic wedge with an arbitrary angle of $2\alpha$ as shown in Figure 2.3.3. The wedge is pre-stressed at the body and has stress free edges.

![Figure 2.3.3](image)

**Figure 2.3.3** The solution of a pre-stressed wedge by Williams uses theory of elasticity (left) can be developed into a model with a crack under an arbitrary state of stress (right) by the limiting case of $\alpha = \pi$ [31].

The solution to biharmonic equation (i.e. $\nabla^4 F = 0$) results in finding the stress components at different parts of the problem, therefore the effort was made to establish a solution to this equation. Williams assumed an Airy stress function that satisfies the biharmonic equations in polar coordinate $(r, \theta)$ in the following form:

$$F(r, \theta) = r^{\lambda+1} f(\theta) \quad (2.3.3)$$

Where $\lambda$ is determined in the solution. By increasing the angle $\alpha$ close to $\pi$ the problem can be changed into Figure 2.3.3-b which is very similar to a problem with a crack. Using
the problem boundary condition he found a solution for this function that had two symmetric and anti-symmetric stress fields. The symmetric term with respect to the plane of the crack results in the opening of the crack or mode I loading on the crack. Mode I, or “opening mode”, forms when a tensile stress is applied normal to the plane of an existing crack.

Mode I loading is accompanied by rupture displacement, when the crack surfaces move apart in the opposite directions (e.g. in splitting a log).

The anti-symmetric part results in the shearing displacement at the crack tip or mode II loading. Mode II loading or sliding mode, results from a shear stress acting parallel to the crack plane and Perpendicular to the fracture front, as illustrated in Figure 2.3.4.

There is another mode of loading in the crack that is not happening in a 2D elasticity problem. This mode is called antiplane shear mode or mode III. Mode III results from out of plane shear; an example of mode III is cutting paper using scissors. Figure 2.3.4 shows these different modes of fracture.

All these fracture modes can happen separately or occur in any mixture. If the fracture occurs in two or more modes it is called mixed-mode fractures. Thus, a fracture that has both modes I and II can be named as a mixed mode I-mode II fracture.

Figure 2.3.4 Different modes of fracture loading.

In the Williams solution he also noticed that the solution for the symmetric part results in an Airy stress function which is similar to the stress function for a uniform stress in x
direction parallel to the crack plane. This term was later used by Irwin to modify mathematical term for the stress state around the crack tip [32], [33].

Although Williams solution was a significant progress in linear elastic fracture, but it is only good with a single ended part that has stress free crack faces. Therefore, Westergaard used the idea of complex variables in 1939 to introduce a complex airy stress function that satisfies the biharmonic equations. He used his method for an infinite body as shown in Figure 2.3.5 with uniform boundary conditions [24], [31].

![Diagram of a crack body in the complex domain](image)

**Figure 2.3.5** The generalized Westergaard approach. A crack body in the complex domain [31].

Assuming a complex stress airy function and finding the second derivatives of the function results in the following relation for the values of stress components at the tip of the crack:

\[
\sigma_x = Re Z - \gamma (Im Z' + Im Y') + 2Re Y \quad (2.3.4)
\]

\[
\sigma_y = Re Z + \gamma (Im Z' + Im Y') \quad (2.3.5)
\]

\[
\tau_{xy} = -Im Y - \gamma (Re Z' + Re Y') \quad (2.3.6)
\]
Where $Z(z)$ and $Y(z)$ are complex functions, superscript prime is the differentiation with respect to $z$ (the complex variable). The complex variable $z$ is defined as: $z = x + iy$. Westergaard used the same problem as Inglis with a cracked body to determine the stress close to the tip of a central crack. Using problem boundary conditions he found the solution and also the stress components as [19], [24]:

$$\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} = \frac{\sigma \sqrt{\pi a}}{\sqrt{2\pi r}} \begin{bmatrix}
1 - \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right) \\
1 + \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right) \\
\sin\left(\frac{\theta}{2}\right) \cos\left(\frac{3\theta}{2}\right)
\end{bmatrix}$$  \hspace{1cm} (2.3.7)

where $\sigma$ is the far-field stress on body, $r$ and $\theta$ are as depicted in Figure 2.3.5 and $a$ is the crack length. As term $\sigma \sqrt{\pi a}$ has a constant value it can be substituted by $K$ as a constant. The resulting relation for the stress components is very similar to the opening mode solution of Williams problem. Therefore, this invariant is specific mode I and is shown as $K_I$. $K_I$ is also known as stress intensity or mode I fracture toughness. Stress intensity factor, determines the stress components at the tip of the crack in terms of the far-field stress.

As it was discussed in the solution of Williams problem, he noticed a term that was related to a uniform stress parallel to the direction of the crack (i.e. $\sigma_0$). The Westergaard equation assumed an infinite plate under bi-axial load. For the finite plate under uniaxial load, the equations must be modified, so the constant stress of $\sigma_0$ can be superposed to the rest of the terms in the Westergaard. The suitability of the Westergaard equations was also studied using photoelasticity, which led to a modification by Irwin to the Westergaard equation [34]:
\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}
= \frac{K_I}{\sqrt{2\pi r}} \begin{bmatrix}
1 - \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right) \\
1 + \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right) \\
\sin\left(\frac{\theta}{2}\right) \cos\left(\frac{3\theta}{2}\right)
\end{bmatrix}
+ \begin{bmatrix}
\sigma_0 \\
0 \\
0
\end{bmatrix}
\] (2.3.8)

where \( \sigma_0 \) is:

\[
|\sigma_0| = 2\tau_{max}
\] (2.3.9)

\( \tau_{max} \) is the maximum principal shear stress and explained as:

\[
\tau_{max}^2 = \frac{(\sigma_x - \sigma_y)^2}{4} + \tau_{xy}^2
\] (2.3.10)

If photoelasticity fringes are calculated numerically, they give fringes that are symmetrical along and normal to the crack axis. By comparing fringes from Westergaard’s equations and results from photo-elastic images of the crack tip, it can be shown that, unlike the Westergaard model, the fringe pattern at the tip of the crack is tilted towards the front of the crack. The pattern calculated from the Westergaard is compared to the actual photoelastic result from testing on epoxy samples in Figure 2.3.6 [35].

Similar approach can lead to an equation for the values of stress components under shearing mode (mode II) loading. The mode II equations can be found elsewhere [19], [21], [31].
Figure 2.3.6 a and b) The numerical photo-elastic pattern calculated from Westergaard equation [35] b) Actual photo-elastic pattern from epoxy sample showing the forward tilt of isochromatic lines.

To determine the value of stress intensity factor (SIF) for a finite body, it should be noted that SIF is also dependent on the geometry of the body, i.e.:

\[ K = \sigma \sqrt{\pi a} Y \left( \frac{a}{W} \right) \] (2.3.11)

where \( a \) is the crack length, \( \sigma \) is the farfield stress, \( W \) is the specimen width. \( Y \) is a shape factor which is dependent on the crack length over specimen width (W). For example, by comparing Equations (2.3.7) and (2.3.11) we find that the value of shape function for a central crack problem is unit (i.e. \( Y=1 \) and \( K = \sigma \sqrt{\pi a} \)).

To determine the value of \( Y \) for finite bodies with central crack, Irwin noticed that there is a similarity between center cracked panel and a body with periodic arrays of cracks and he concluded the following relation for \( Y \left( \frac{a}{W} \right) \) [34]:

\[ Y = \sqrt{\frac{W}{\pi a}} \tan \frac{\pi a}{W} \] (2.3.12)
Isida and Fedderson later with the help of numerical stress analysis showed that \( Y \) could be stated respectively as \([36], [37] \):

\[
Y \left( \frac{a}{W} \right) = 1 + 0.256 \left( \frac{a}{W} \right) - 1.152 \left( \frac{a}{W} \right)^2 + 12.200 \left( \frac{a}{W} \right)^3 \quad (2.3.13)
\]

And

\[
Y \left( \frac{a}{W} \right) = \sqrt{\sec \left( \frac{\pi a}{W} \right)} \quad (2.3.14)
\]

Various predictions for \( Y \) in the centered crack specimen are compared in Figure 2.3.7.

**Figure 2.3.7 Comparison of Various methods defining shape factor \( Y \) in terms of \( 2a/w \) \([31]\).**

A similar approach can be used to establish a closed form for the geometric shape factor, \( Y \), for different geometries. This method will be used to derive the value for \( Y \) for the fracture specimen in this research \([31]\).
2.3.3 Elastic-Plastic Fracture Mechanics (EPFM)

The previous section discussed the determination of the value of stress intensity factor assuming an elastically deformed crack tip in the material. The theory was dealing with the linear elastic fracture mechanics.

If large zones of plastic deformation at the tip of the crack develop before the crack grows, the elasticity equations are not applicable; in this case elastic plastic fracture mechanics should be used. The formation of larger zones of plastic is common for more ductile materials. This condition usually happens in ductile materials. Several researchers developed analyses to correct for yielding and significant plastic deformation at the crack tip, including Irwin, who discussed the plastic zone near a crack [38], Wells, who introduced crack tip opening displacement (CTOD) [39] and Rice who introduced the J-integral method in 1968 [40]. This approach is not of interest in this research.

2.3.4 The Thickness Effect

Studying the fracture at the crack tip, researchers found that the material thickness affects the state of stress at the tip of the crack. This would in turn affects the value of fracture toughness of the part. This effect is because different thicknesses provide plane stress and plane strain conditions at the tip of the crack. For thin materials or at the surface of thicker materials, the plane stress condition exist, in this condition $\sigma_z = \sigma_3 = 0$ ($z$ is perpendicular to the plate). In thicker materials that have plane strain regions, the value of $\sigma_3$ increases from zero at the surface which is in plane stress condition, to a certain value at the plain strain region, this in turn decreases the plastic zone size at the tip of the crack. Therefore, the size of the plastic region in plane strain is smaller from that of plane stress. This is shown schematically in Figure 2.3.8.
Figure 2.3.8 Schematic of transition of the plastic zone at the tip of the crack from plane stress region to plane strain region [19]

For materials that have plain strain condition in the tip of the crack the chance of failure due to plasticity is inhibited which can be defined by the help of Mohr’s circle. The state of stress for plane strain and plane stress in 3D Mohr’s circle is shown in Figure 2.3.9. In plane stress case by increasing the values of $\sigma_x$ and $\sigma_y$ the value of $\sigma_z$ remains zero, therefore an increase in the other two principal stresses (i.e. $\sigma_x$ and $\sigma_y$) increases the diameter of the circle (Figure 2.3.9 top) and increases the chance of reaching to the limiting shear stress and the plastic flow.
Figure 2.3.9 Mohr’s circle comparison for plain stress (top) and plane strain (bottom) conditions, suggests higher chance of brittle fracture for plain strain and higher chance of plastic fracture for plane stress regions [31].

For plane strain condition the stressed region does not reach to plastic flow, this is due to the change in the value of $\sigma_z$ by increasing the load on the material. In conclusion, in the plane strain condition the radius of the Mohr’s circle does not increase to reach to the maximum shear stress to start plastic deformation, therefore, brittle fracture in this region is more probable.

2.3.5 Energy Based Methods

As it was discussed in the introduction of fracture at the atomic level, Griffith developed a theory of brittle fracture based on the idea of minimum potential energy [41], [42]. He found that the difference in the strain energy of a stressed infinite plate (assuming plane
stress condition with a unit thickness) and the strain energy of the same plate with a centered crack having a length of $2a$ as:

$$U = -\frac{\pi a^2 \sigma^2}{E}$$  \hspace{1cm} (2.3.15)

The negative sign shows the decrease in the amount of energy due to the crack propagation. As there are two surfaces forming due to this strain energy change, the forming surface energy is:

$$W_{\text{surface}} = 2\gamma \cdot 2a \times 1 = 4\alpha\gamma$$  \hspace{1cm} (2.3.16)

At the onset of the crack growth the total change of strain and surface energy is zero. In other words, according to Griffith criteria the strain energy rate provided to the fracture surface is equal to the rate of energy required for making new surfaces. Therefore, he concluded the following relation for the critical stress:

$$\sigma_c = \frac{2E\gamma}{\pi a}$$  \hspace{1cm} (2.3.17)

Orowan and Irwin gathered the origins of resistance to crack extension in critical strain energy release rate ($G_c$) and wrote the Griffith criteria as:

$$\sigma_c = \frac{EG_c}{\pi a}$$  \hspace{1cm} (2.3.18)

Here $G_c$ is a material constant that can be measured for each material using standard fracture testing.

By comparing Equation (2.3.18) with the relation introduced for the critical stress in terms of stress intensity factor (Equation (2.3.11), with $Y=1$) it can be concluded that:

$$K_c^2 = EG_c$$  \hspace{1cm} (2.3.19)
As mentioned the above equations are for plane stress condition. Using a similar procedure for plain strain Equation (2.3.19) is:

\[ K_c^2 = \frac{EG_c}{1 - \nu^2} \]  

(2.3.20)

in which \( \nu \) is the material’s Poisson’s ratio.

The above equations determine the relation between stress based and energy based fracture.

The advantage of energy method is that it can be used for both brittle and ductile materials. Where LEFM is difficult to apply for anisotropic materials (like composites), or for situations where loading or geometry is complicated, energy method can be used.

2.4 Fracture in Composites

A PMC such as glass fibre epoxy has a toughness that is much higher than its constituents. The reason for this high toughness is related to what happens in the interface of fibre and matrix. The following paragraphs explain how a crack advances through a UD composite resulting in an increase in its toughness.

When a unidirectional fibre reinforced composite is subjected to tension up to the elastic point the material behaves linear and both fibre and matrix deform elastically (Figure 2.4.1a). Assuming a higher failure strain in fibres compared to the matrix, an increasing load may cause the matrix to fail and crack microscopically. In a UD composite with fibres aligned in the direction of tensile load, microscopic crack proceeds in the matrix and approaches to some of the fibres (Figure 2.4.1b). Increasing displacement (load) monotonically causes the following microscopic events. The microscopic crack propagation is stopped by the fibre at lower stress, if the stress increases crack may pass around the fibre. Depending on the value of bond strength interfacial debonding might happen due to normal stress acting on the fibres according to Gordon and Cook model [18] (see Figure 2.4.2). Increase in load causes stress concentration in the fibre, leading to local Poisson’s contraction. The local contraction and stretch cause the shear stress
between fibre and matrix to reach the interfacial shear strength. This results in the fibre debonding from the matrix in the crack plane (Figure 2.4.1.c); increase in the load results in debonding propagation in the fibre direction at a distance from the initial crack. The fibre reaches its tensile strength and surpasses it after further debonding this is followed by further cracking extension (Figure 2.4.1 d). When the surrounding matrix cracks the fibres start to pull out from their matrix socket in (Figure 2.4.1 e) [43].

![Figure 2.4.1 Schematic interaction between a fibre and surrounding matrix in a longitudinal UD composite. Series of microscopic events happen from elastic deformation (a) to final fracture of material (d) [43]](image)

![Figure 2.4.2 a) Cook-Gordon model. Stresses acting close to the crack tip. b) The crack is approaching fiber bundle. C) Fibre/matrix debond due to σ₂ stress, crack blunting is happening [18].](image)
In this section, the goal is to introduce the test methods that are used in common to define the fracture properties of the composite. As discussed earlier, delamination (interlaminar fracture) is a common mode of failure in laminated composites. Depending on the mode of loading different methods are introduced.

The through thickness fracture of a composite, where the crack propagates perpendicular to the layers is another type of fracture that has been less explained. High-volume manufacturing processes (e.g. those suitable for automotive applications) based on compression molding techniques typically result in planar random arrays of discontinuous fibres. Thus, an advancing through-thickness crack will meet transverse fibres at a range of angles between 0° and 90°. A study on this type of fracture could help to develop a model for the crack propagation in random fibre composites.

2.5 Contributions to the Work of Fracture

In order to predict the work of fracture in composites, it is necessary to understand the various mechanisms involved in the fracture process. Several works have been done on the fracture properties of different kinds of PMCs [44], [45].

2.5.1 Matrix Deformation

The toughness of composite materials increases with increases in the toughness of the matrix. Increasing the thickness of resin between the composite plies also increases the total toughness of the composite [7], [46]. The contribution of matrix deformation to the total work of fracture in a composite may differ significantly from the case where the same material has no reinforcement. This is mainly due to the movement limitation of the stiffer fibres (fibres usually carry most of the load). Another phenomenon is the set up of triaxial stress that prevents the plastic deformation of the matrix. Using the schematic shape in Figure 2.5.1, this phenomenon can be described; a part of the unidirectional composite under tension is the focus of the second picture. Both fibre and matrix are under tensile stress and the matrix is constrained by the two stiffer adjacent fibres. When the matrix is extended plastically in the x direction (before composite fractures), it introduces a lateral contraction (in the y direction) that is opposed by rigid fibres so it
forms transverse normal tension that reduces deviatoric stress and slows down plastic flow, but this may lead to cavitation and fracture [5]. In PMCs, considerable improvements in toughness can be reached by increasing toughness of the matrix so that it can change micro mechanisms of damage and increase the associated energy absorption [47].

![Diagram of a unidirectional composite, effect of fibre stiffness on matrix deformation](image)

**Figure 2.5.1 A unidirectional composite, effect of fibre stiffness on matrix deformation**

### 2.5.2 Fibre Fracture

Although some reinforcing fibres undergo plastic deformation (e.g. Aramid [48]), the most common reinforcements for PMCs (glass, carbon) are brittle. The contribution of fibre fracture to the fracture energy of the composite material is small for most fibres; this contribution is less in PMCs. For example, the fracture energy of glass, carbon and SiC fibres are a few tens of $\frac{J}{m^2}$ compared to a composite like glass/epoxy having a fracture energy of $40-100 \frac{kJ}{m^2}$ [49]. In general, for most composites, fibre fracture makes little or no contribution to the overall toughness.

### 2.5.3 Interfacial Debonding

“Debonding” is used to describe the detachment of fibres from the surrounding matrix. During composite’s fracture event, interfacial debonding can occur. If the crack is normal to the fibre and the crack is deflected when reaching the interface, debonding would occur [18]. The deflection is because in front of the crack, apart from loading stress ($\sigma_1$),
which is parallel to the loading, a transverse stress \((\sigma_2)\) in the direction of crack propagation exists. Based on the Cook-Gordon mechanism, this may cause debonding in the fibre/matrix interface close to the crack tip, so it blunts the crack propagation and opens up the interface. A shear stress acting on the interface propagates this debonding along the fibre length. If the value of the interfacial fracture energy, \(G_{ic}\), is known, the contribution of debonding to the overall fracture energy can be calculated using the work done when a single fibre is debonded before it is pulled out of its socket in the matrix (Figure 2.5.2). The work done during debonding with embedded length \(x_0\) is:

\[
\Delta U = 2\pi r x_0 \cdot G_{ic}, \quad x_0 \leq L,
\]

where \(2L\) is the fibre total length. To determine the local work of the debonding in the composite \(G_{cd}\) all the fibres intersected by the crack between \(x_0\) and \((x_0 + dx_0)\) should be summed up, using the number of fibres per square meter, we have the following formula:

\[
G_{cd} = f \cdot s \cdot G_{ic}
\]  

(2.5.1)

Figure 2.5.2. Schematic of crack passing through short fibre composite, showing interfacial debonding and fibre pull-out [5]

Where \(G_{cd}\) is the critical energy release rate of debonding (total work done by debonding), \(G_{ic}\) is the interfacial fracture energy, \(f\) is the fibre volume fraction and \(s\) is the aspect ratio \((s = \frac{L}{r})\).
2.5.4 Fibre pull-out

The highest contribution is made when fibre pulls out from its socket in matrix. Similar to interfacial debonding it can be shown that the pull-out work of fracture \( G_{PO} \) is defined using the following formula:

\[
G_{PO} = \frac{fs^2\tau_l}{3}
\]  

(2.5.2)

where \( \tau_l \) is the interfacial shear stress and other terms are the same as in Equation (2.5.1). By substituting values of different parameters in Equation (2.5.2) for a composite, it can be shown that fibre pull-out has a high contribution to toughness. Evans and Marshall in 1989 showed that, a weak interface between fibre and matrix in ceramic composites is necessary to obtain satisfactory toughness, because of this sliding and debonding can occur [50].

In wood (as a natural composite) for example the fibre pull-out creates new surface area which is responsible for higher fracture toughness. Shearing opens the cracks up and they propagate axially. The crack propagation pulls all cell walls apart without breaking them through [51].

The other parameter that affects the fracture mechanism is fibre critical length \( (l_c) \): fibre pull-out is the prevailing fracture mechanism when fibre length is shorter than the critical fibre length \( (l < l_c) \), whereas fibre breakage occurs when fibre length is larger than the critical one \( (l > l_c) \) [52]. Fibre critical length is defined in the previous section and is shown in Figure 2.2.10.

2.5.5 Effects of Microstructure

One of the microstructural features that influences the fracture energy of composite is the fibre orientation that characterizes the angle between the loading direction and the fibre axis. Fracture energy in uni-directional composite falls off sharply as the loading angle increases (Figure 2.5.3), this is because fibre pull-out becomes inhibited and fracture happens parallel to fibre axis. In short fibre composites in addition to fibre orientation
other effects like fibre surface quality, FVF, fibre content, mean fibre length and fibre length distribution and fibre arrangement affect the composite fracture [53], [54].

Other factors like environmental factors also affect the fracture behaviour of composites. These factors include: moisture that affects interface bonding during manufacturing and service [45], the temperature that can weaken the matrix, acidic environment which can affect crack initiation in unidirectional glass/polymer composite materials [55], [56].

![Graph showing the effect of fibre orientation on the fracture energy absorbed by UD composite laminate obtained from notched Charpy impact test.](image)

**Figure 2.5.3.** The effect of fibre orientation on the fracture energy absorbed by UD composite laminate obtained from notched Charpy impact test [57].

### 2.6 Mode I, Mode II and Mixed Mode Interlaminar Fracture Toughness

For finding the mode I interlaminar strain energy, $G_{IC}$ of polymer composite materials double cantilever beam (DCB) test can be used as described in ASTM 5528 [58]. The test method is used for composites consisting of unidirectional carbon fibre and glass fibre tape laminates with brittle and tough single-phase polymer matrices is as shown in Figure
2.6.1. Load is applied at the hinges and tries to open the crack. The critical load that causes the crack to open is used to determine mode I interlaminar strain energy release rate.

![Diagram of Double Cantilever Beam Specimen](image)

**Figure 2.6.1. Double Cantilever Beam Specimen**

Mode II and mixed mode I/II interlaminar fracture toughness of the composite can also be determined using other methods. Methods like the End Notched Flexure (ENF) and mixed mode testing (MMT) specimen are being used for mode II and mixed mode respectively [59], [60]. These methods are explained thoroughly in ASTM WK22949 and D6671 [61], [62].

### 2.7 Compact Tension Shear Specimen

As discussed in the previous section, to measure the mixed mode I and II fracture properties of composites, beam type specimen are often used [63], [64]. The problem with these methods was that loading different specimens in the beam form were required to measure the whole range of mixed mode loading. Moreover, these methods are not suitable to study different types of composite fracture other than interlaminar fracture. For instance, through thickness fracture of composites cannot be investigated using the above mentioned beam methods.

In order to study mode I, mode II and mixed mode behavior of composite with methods such as ENF and DCB and MMT different specimen were required in the beam form. Also one requires to use different beam type specimens to measure different modes of failure. For example Hwu used four different beam specimens to study the mixed mode
fracture of material [65]. These methods include ENF, DCB, modified ENF and crack lap shear test. Moreover, these methods are not suitable to study a range of composite fracture other than interlaminar fracture. For instance through thickness fracture of composites cannot be investigated using the above mentioned beam methods. However, only pure mode I through-thickness toughness of composites can be studied using compact tension (CT) specimen or single edge notched beam (SENB) specimen [66], [67].

The compact tension shear (CTS) sample can be utilized to obtain results for fracture toughness under all modes of loading applied on the part (see Figure 2.7.1).

The CTS specimen was introduced in the literature by Richard and Benitz to study whole ranges of in-plane loading from mode I to mode II and mixed mode loading on homogenous materials like aluminum, the specimen was later developed by Rikards and coauthors to be used to study interlaminar fracture behaviour of composite materials [68], [69]. The CTS sample that was proposed by Richard is shown in Figure 2.7.2 a. Rikards later modified the CTS specimen and proposed a compound version of CTS to study interlaminar fracture toughness of glass fibre epoxy. The compound version is shown in Figure 2.7.2 b. Advantages regarding this method include its simple shape easily control of the loading mode using a loading device [70].

The CTS sample has an edge crack starting in the middle of one side perpendicular to that side and extending in the cross section of the specimen. The length of the crack is shown by a. The value of crack length is suggested to be between 0.45w and 0.7w [71].
As shown in Figure 2.7.1, the load is applied to the specimen using a fixture made of steel. The steel fixture has lower and upper grips which are fixed at grips of the testing machine. The sample is fixed to the grips using steel bolts. By adjusting the pin location on the fixture, different angles and modes of loading are generated in the specimen. For example, in order to apply pure mode I to the sample the pin should be located at point A (Figure 2.7.1a) and to get pure mode II the pin should be in position B. The pinholes between A and B are used to apply mixed mode loading. Point A in Figure 2.7.1a is equivalent to \( \alpha = 0^\circ \) (mode I) and point B corresponds to \( \alpha = 90^\circ \).
Figure 2.7.2 a. Proposed fracture specimen by Richard [71] b. The compound CTS specimen proposed by Rikards for composite interlaminar fracture [68]

The advantage of CTS method is that the specimen geometry is simple and the test is able to provide all ratios of mode mixity on the sample without changing the specimen geometry.

Apart from what was mentioned above, as the application of CTS specimen in the study of fracture of composite when crack opens by the delamination process, the CTS specimen was used by few researchers to study the fracture of composite material when the crack is advancing through the thickness of the composite. Mixed mode and Mode II through thickness fracture was studied on the wood using CTS specimen by Pitti and Dubois [72]. They made modifications on CTS specimen to be used for fracture response of timber Wood showing viscoelastic behaviour. They report the mixed mode test results and they did not use or developed a failure criterion. Reber and coworkers studied the effect of woven fibre orientation on the mode I fracture toughness of the composite using CTS specimen [73]. They modified the CTS test and used thermography and fractography to study the crack propagation during, and damage surface after, the test. Using an experimental compliance method they found the value of G for knitted composite. They found a very high toughness for these materials compared to
conventional composites. Other fracture studies use CTS specimen to measure interlaminar toughness [68].

The fracture toughness $K$ is a function of crack size, load and geometry. The geometry shape factor for this specific geometry can be determined similar to the approach taken by Irwin, Federson and Isida in the linear elastic fracture section. The geometry shape factor for the CTS specimen in terms of $\left(\frac{a}{w}\right)$ was determined using numerical methods by Richard. The relation for shape factor $Y$ in terms of $\frac{a}{w}$ is used to derive a relation for the mode I and mode II fracture toughness using the following relations [74], [75]:

$$K_I = \frac{P\sqrt{\pi a}}{wt} \cdot \cos \alpha \sqrt{1 \pm \frac{1}{2} \left( \frac{a}{w-a} \right) - 0.05 \left( \frac{a}{w-a} \right)^2}$$

(2.7.1)

$$K_{II} = \frac{P\sqrt{\pi a}}{wt} \cdot \sin \alpha \sqrt{1 - 0.67 \left( \frac{a}{w-a} \right) + 2.08 \left( \frac{a}{w-a} \right)^2}$$

(2.7.2)

Where, $P$ is the applied load, $w$ and $t$ indicate specimen width and thickness and $a$ is the crack length. The loading angle is shown by $\alpha$ in the above equations. If $P_{\text{max}}$ is used in the above equations the critical stress intensity factor or fracture toughness ($K_{lc}$ and $K_{IIc}$) of the material can be calculated. $P_{\text{max}}$ is the critical (maximum) load that opens the crack up.

After the sample is inserted in the fixture the load is applied, The maximum load at which the crack opens up is measured and used for the calculation of the mode I ($K_I$) and mode II ($K_{II}$) fracture toughness of the composite from above equations (Equations (2.7.1) and (2.7.2)).

The values of $K_{lc}$ and $K_{IIc}$ will be used to calculate effective fracture toughness, mode I and mode II toughness of materials.

Effective mixed mode fracture toughness can be found in terms of $K_I$ and $K_{II}$ as:
\[ K_{\text{eff.}} = \sqrt{K_1^2 + K_\text{II}^2} \]  \hspace{1cm} (2.7.3)

To determine the value of critical strain energy release rate (CSERR) for mode I, mode II and total CSERR, \( G_c \) for plane strain material, we have [19]:

\[ G_{\text{Ic}} = \frac{(1 - v^2)K_1^2}{E} \]  \hspace{1cm} (2.7.4)

\[ G_{\text{IIc}} = \frac{(1 - v^2)K_{\text{II}}^2}{E} \]  \hspace{1cm} (2.7.5)

Using (4) and (5) it gives:

\[ G_c = G_I + G_{\text{II}} = \frac{1 - v^2}{E} (K_1^2 + K_{\text{II}}^2) \]  \hspace{1cm} (2.7.6)

Equation (2.7.4) to (2.7.6) are for isotropic materials so they can be used for epoxy specimens, but not for the UD composites that show a high degree of anisotropy. The relations for UD composite will be explained in Chapter 4.

## 2.8 Review on Mixed Mode Failure Criteria

To predict different fracture envelopes by composite materials, several mixed mode failure criteria are proposed in the literature. The existence of several types of failure criteria indicates that these criteria are not able to predict the behaviour of a full range of composite materials properly [57], [76].

Mixed mode failure criteria are mostly energy based criteria and mainly focus on the mixed mode delamination of composite where crack advances between layers of composite. All these failure criteria are empirical and based on the parameters that are found by the curve fitting of the measured CSERR values through experiments. Therefore, they have no accuracy in predicting the behaviour of a new composite system other than the composite they have used for the curve fitting. The mixed mode failure
criteria fall into several categories such as: linear, power law, polynomial and bi-linear criteria [77].

### 2.8.1 Linear Type Criteria

A group of linear type criteria was introduced by Whitcomb, Gillespie and Wu [78]–[80]. The simplest form of these kinds of criteria is when mode I or mode II reaches to the critical value:

\[ \frac{G_I}{G_c} = 1 \implies G_I = G_c \quad \text{[78]} (2.8.1) \]

\[ \frac{G_{II}}{G_c} = 1 \implies G_{II} = G_c \quad \text{[79]} (2.8.2) \]

The above two criteria are conservative, and are not efficient in modeling design parameters. Therefore, the total value of mode I and mode II toughness was proposed by Wu et al [80]:

\[ \frac{G_I}{G_c} = 1 \implies G_I + G_{II} = G_c \quad \text{[80]} (2.8.3) \]

Here, total CSERR remains constant while its mixed mode components are changing. While first and second criteria assume only mode I or mode II are controlling fracture, the third one (Equation [80] (2.8.3)) gives a more reasonable assumption by combining both mode I and II components as controlling parameters in the fracture. Another type of linear criteria has been introduced using normalizing each toughness component in terms of its pure mode toughness [80]:

\[ \frac{G_I}{G_{Ic}} + \frac{G_{II}}{G_{IIc}} = 1 \quad (2.8.4) \]

The normalized equation, which is very common in the literature is a straight line that connects the pure mode II toughness to the pure mode I toughness as shown in Figure 2.8.1 [59].
Figure 2.8.1 Linear criteria for mixed mode fracture toughness \([59]\).

Figure 2.8.1 compares the above mentioned linear criteria. Nomenclatures \(G_I\) and \(G_{II}\) are substituted by \(G^m_I\) and \(G^m_{II}\) in the figure. It is noteworthy that values of \(G_I\) and \(G_{II}\) in all above criteria need to be measured for the composite at the first step.

### 2.8.2 Power Law Criterion

Power law criterion is the generalized format of Equation (2.8.4) as stated below for the Mode I-Mode II CSERR \([78]\):

\[
\left( \frac{G_I}{G_{Ic}} \right)^\alpha + \left( \frac{G_{II}}{G_{IIc}} \right)^\beta = 1 \tag{2.8.5}
\]

By changing the values of parameters \(\alpha\) and \(\beta\), different types of material mixed mode behaviour can be modeled. If the values of the parameters are equal 1 the power law criterion will be changed into the linear criterion of Equation (2.8.4). By changing
\( \alpha \) and \( \beta \) different curves from concave to convex curves are obtained (see Figure 2.8.2). As can be seen in the figure different behaviours are modeled through different values of \( \alpha \) and \( \beta \).

For finding the prediction by power law criterion, the values of Mode I and Mode II CSERR of the composite should be measured. To determine the optimal value for \( \alpha \) and \( \beta \), the mixed mode experimental data required to be fit by the curve.

![Power Law Criterion](image_url)

**Figure 2.8.2 Power law criteria for the mixed mode toughness of composites** [59].

Again here, it is noticed that for every system of composite material, that specific composite is required to be tested and its specific experimental data obtained. However, the criterion covers different ranges of materials response; it does not address how material behaves. As a result, it does not have predictive capability.

### 2.8.3 Polynomial Criterion

This criterion is developed using power law criterion, it assumes that total CSERR is a function of ratios of mode II to mode I CSERR components [81]:
As shown in Figure 2.8.3, the criterion covers a wide range of mixed mode fracture response of material by adjusting the values of $\rho$ and $\tau$.

$$G_{ic} + \rho \left( \frac{G_{II}}{G_I} \right) + \tau \left( \frac{G_{II}}{G_I} \right)^2 = G_c \quad (2.8.6)$$

As shown in Figure 2.8.3, the criterion covers a wide range of mixed mode fracture response of material by adjusting the values of $\rho$ and $\tau$.

The criterion allows the increase in mode I when mode II is introduced. This is however, the criteria is not a suitable option when high mode II values exist in the loading. The criterion is not therefore a good option as a general failure criterion. Apart from this disadvantage, the polynomial criterion like other criteria needs the values of CSERR to be determined through experiment.

### 2.8.4 $K_I$ Critical Criterion

Another criterion that was introduced in the literature assumes that the composite toughness is linearly dependent on the mode I critical stress intensity factor of the composite ($K_{ic}$) [82]:

$$G_{ic}^{m} + G_{ic}^{n} = G_{ic} + \rho \left( \frac{G_{ic}^{m}}{G_{ic}^{n}} \right) + \tau \left( \frac{G_{ic}^{m}}{G_{ic}^{n}} \right)^2$$
\[ G_{IIc} - (G_{IIC} - G_{IC}) \left( \frac{G_I}{G_{IIc}} \right)^{1/2} = G_c \]  

(2.8.7)

The criterion is stated here in terms of mode I and mode II toughness. Depending on the values of \( G_{IIc} \) and \( G_{IIC} \) the criterion can be simplified to linear or power law criterion. The modeling by this criterion is shown in Figure 2.8.4. This criterion is also dependent on the values of the mode I and mode II CSERR of the composite determined by the experiment.

![Graph showing the critical criterion for the mixed mode toughness of composites](image)

**Figure 2.8.4** \( K_1 \) critical criterion for the mixed mode toughness of composites [59].

### 2.8.5 Hackle Criterion

The hackle formulation was proposed by Hahn to model the delamination growth with the help of hackle formation [83]. In this criterion CSERR of the composite is defined in terms of hackle angle, i.e. \( \sqrt{1 + \left( \frac{K_{II}}{K_I} \right)^2} \):

\[ (G_{IC} - X) + X \sqrt{1 + \frac{G_{II}}{G_I} \left( \frac{E_{11}}{E_{22}} \right)} = G_c \]

(2.8.8)
In which \( \frac{k_{II}}{k_I} \) is substituted by the Young’s moduli and pure mode toughness. The parameter X is changed to give different modeling curve. If the value of X is other than zero, the criterion predicts an infinite number for the mode II CSERR \( (G_{II}) \) which is not realistic (see Figure 2.8.5).

![Graph showing the relationship between mode I and mode II toughness](image)

**Figure 2.8.5 Hackle criterion models the mixed mode toughness of composites [59].**

The criterion was later modified through an exponential format (exponential hackle criterion) of the criterion to be able to predict finite values for the mode II CSERR [84].

### 2.8.6 Bi-Linear Interaction Criterion

This criterion was proposed by Reeder. He noticed a change in the failure mechanism at 50% mode II loading. He proposed the bi-linear criterion to be able to model this behaviour. The criterion divides into two equations as below [59]:

\[
G_I = \xi G_{II} + G_{IC}
\]  

(2.8.9)
\[ G_I = \xi G_{II} - \zeta G_{IIc} \]  \hspace{1cm} (2.8.10)

where \( \xi \) and \( \zeta \) are arbitrary parameters that show the slope of each of the line segments in the criterion. Depending on the values of \( \xi \) and \( \zeta \) the criterion may be reduced to the linear criterion of Equation (2.8.4). The bi-linear prediction for different values of arbitrary parameters are shown in Figure 2.8.6.

![Figure 2.8.6 The bi-linear criterion predictions for the mixed mode toughness of composites [59].](image)

The bi-linear criteria models the change in the slope of the toughness from lower mode II values to higher mode II values of CSERR, but it does not address any failure mechanism during the fracture. So, the criterion is not a predictive criterion.

Other mixed mode criteria such as B-K criterion [44] and crack opening displacement were also reviewed in the literature [7], [59].
2.8.7 Comparison of Failure Criteria

An extensive study on the comparison of the existing failure criteria with experimental data were carried by Reeder and Greenhalgh [59], [85].

Figure 2.8.7 Comparison of experimental data for UD carbon/bismaleimide resin (T800/5245), and UD carbon/epoxy resin (T800/924) with mixed mode failure criteria [85].
They studied 6 different systems of UD carbon thermoset and thermoplastic. They measured the values of the mode I, mode II CSERR at different modes of loading. Greenhalgh compared the experimental results against 12 mixed mode criteria [85]. Reeder compared his experimental data against 6 mixed mode criteria [59], [77]. Parts of these comparisons are shown in Figure 2.8.7 and Figure 2.8.8.

They concluded that none of the criteria accurately modeled the mixed mode fracture response of a range of composite materials. Greenhalgh and Singh mentioned that the criteria they studied are empirical and do not address the fracture mechanisms during fracture [86], [87]. It was also noticed that because of the failure mechanism transition when the mixed mode loading changes from pure mode I to pure mode II, multiple criteria may be required for different failure mechanisms involved in the fracture of the composite [59].
2.9 Mechanistic Criterion

According to the preceding section, the lack of criteria that include failure mechanisms in the fracture process and can be applied to a range of composite materials is being noted. Therefore, a mechanistic failure model that incorporates the operative failure mechanisms and is based on the constituent’s properties would be the answer to what is noticed.

2.9.1 Mechanistic Criterion for Delamination

A mechanistic mixed-mode failure criterion for the delamination of continuous fiber-polymer composites was proposed by Bruce [7]. In his criterion he considered the resin fracture strength, the hackle formation, interfacial debonding, and interply vs interyarn delamination. For each of the mentioned parameters he determined a term to find the corresponding energy release rate. Each energy term consisted of the values of CSERR of the constituent and the corresponding area on the fracture surface. To study failure mechanisms during delamination he used mixed mode bending test of UD composite.

Bruce and Wood noticed that during delamination of UD composite at certain range of mixed mode loading the crack changes its path from interyarn (through the fibres) to interply (between the layers of composite) and he assigned a term for each mechanism describing the corresponding amount of energy released [88], [89]. The criterion was stated in the following relations:

\[
G_c = F_{f-1P}G_{ic} + \frac{F_{m-1P}G_{ic}H_f (E_m + E_f)}{2E_f} \tag{2.9.1}
\]

And
\[ G_c = G_{ic} (F_{f-1Y} + \left(1 - \frac{M_t - M}{M_t}\right)(F_{f-1P} - F_{f-1Y})) + F_{m-1Y} G_{ic} H_f \left(\frac{E_m}{2E_f}\right) \]

\[ + G_{ic} H_f (F_{m-1Y} + \left(1 - \frac{M_t - M}{M_t}\right)(F_{m-1P} - F_{m-1Y})) \left(\frac{E_m}{2E_f} + \left(1 - \frac{M_t - M}{M_t}\right)\left(\frac{E_f - E_m}{2E_f}\right)\right) \]  

(2.9.2)

where subscripts i, m, f, IY and IP indicate interface, matrix, fibre, inter-yarn and interply. F is the area fraction, M is the mode mixture, and Mt is the mode mixture at which transition from inter-yarn to interply occurs.

Equation (2.9.1) is used when only inter-yarn failure happens during the delamination of composite and Equation (2.9.2) used when interply failure occurs. Hf is the hackle function or the hackle angle as a function of mixed-mode loading. This function includes the higher value of composite CSERR due to the hackling of resin at higher modes of loading.

The comparison of Bruce Criterion and two systems of UD glass fibre epoxy are shown in Figure 2.9.1 and Figure 2.9.2. He measured experimental results using MMB test of UD composite according to ASTM 6671 [62].
Figure 2.9.1 Comparison of the mixed mode criterion and experimental results for nonlayered UD composite [7].

Figure 2.9.2 Comparison of the mixed mode criterion and experimental results for layered UD composite [7]

As mentioned in the Bruce criterion, delamination of UD composite was considered. In this criterion the LEFM formulation for an isotropic material were used and the
anisotropy of UD composite to derive relation between CSERR and fracture toughness was not considered.

2.9.2 Mechanistic Through-Thickness Fracture Criterion

The mechanistic criteria have the following advantages over the existing failure criteria that were discussed in the previous section.

1. They can be applied for a wider group of composite materials because it considers the mechanisms resulting in the fracture process and these mechanisms would be similar for different system of composite materials.

2. They will be cost and time saving, because they are not based on the curve fitting of the experimental data for each specific system of composite. As an example, when the value of fibre volume fraction in a specific composite material changes, a new series of mechanical testing is required to characterize fracture properties of the new system.

3. They can be used in the design of the composite system by selecting different types of the constituents or by changing the constituent property or their portion in the composite.

In this research, a mechanistic mixed mode criterion based on the fully understanding of the failure mechanisms involved in the through thickness fracture of UD polymer composite will be developed. To develop a mechanistic model, the major failure mechanisms occurring during fracture and their corresponding energy absorbing mechanism and the amount of energy released will be understood and determined. The fractography will be done by the help of scanning electron and optical microscopy images. The major failure mechanisms for the through thickness of UD composite will be resin fracture, and fibre/matrix debonding that will be studied carefully. To derive equations for the values of energy released by the fracture of UD composite, anisotropy of UD composite will be considered and the relations will use LEFM.

The total amount of CSERR will be accumulation of each of the energy terms calculated in the previous step. The result will be CSERR as function of mixed mode loading angle. To validate the criterion, CTS specimen will be made and subjected under different
modes of loading. Their corresponding values of CSERR will be calculated. The criterion predictions will be assessed with the experimental results.

In the next step, the mixed mode criterion for the UD composite will be enhanced to predict the value of CSERR for random layered composite. Similar methodology as the UD composite will be followed to determine total CSERR of the random composite. Major failure mechanism in this case will be fibre pull out. To determine the amount of energy released by the fibre pull out, fractography and finite element model will be used. The enhancement of the criterion from UD composite to random composite is vital because this type of composite is utilized in many mass production applications such as automotive industry.

In this thesis, the concepts and ideas in the process of fracture are discussed in separate chapters. The fracture process of resin is investigated first (Chapter 3) and then completed towards the UD composite in the following chapter (Chapter 4). Each of these chapters includes the experimental studies and related results analysis. In Chapter 5, the accumulations of ideas and concepts are put together in the mechanistic mixed mode criterion. The criterion’s prediction is then compared with the experimental results in Chapter 3 and Chapter 4. The enhancement of the mechanistic criterion for the UD composite to the random composite is explained in a new chapter. The prediction of the new criterion is compared with the experimental results measured from fracture testing of a random composite.
Chapter 3

3 Study of Matrix

In order to develop a predictive model for the fracture properties of composite material based on its constituents, the constituents (i.e. fibre and matrix) should be studied first. The amount of energy released during fracture of composite is the summation of energy released by its constituents. Therefore, matrix as an important source of releasing energy during fracture is studied in this chapter. To measure fracture properties of resin (matrix), its mechanical properties should be characterized at first step. Fracture study of resin is the next step. Mode I, mode II and mixed mode fracture testing of the resin (Crosslink epoxy) that is used to manufacture the UD composite as will be described in Section 3.2. This chapter also includes studying fracture mechanism contribute to the work of fracture in the resin.

Mode I results in a flat fracture surface while for mixed mode loading hackle formation contributes to work of fracture. The goal of this chapter is investigating fracture mechanisms and relating them to the value of energy released during fracture of epoxy. This will help to develop a mechanistic model to predict fracture behaviour of epoxy and composite as will be described in Chapter 5 and Chapter 6.

3.1 Tensile Testing of Epoxy

Resin tensile and fracture behaviour are studied in this chapter. As it was mentioned in the introduction to this chapter, fracture properties of composite constituents cannot be found without knowing mechanical properties of them. In this research mechanical properties are mainly found through tensile testing. In this section epoxy specimen preparation and their tensile behaviour results are given.
3.1.1 Method

3.1.1.1 Specimen Preparation

The neat epoxy specimen in this study is made using CLR1180/CLH6560 epoxy material manufactured by Crosslink Tech. Inc. Resin mixing and curing is discussed in the next Chapter. After epoxy plate is cured at room temperature or in 60 degree temperature, it is cut to desired dimension as described in ASTM standard.

3.1.1.2 Testing

Tensile testing of epoxy specimens was carried out according to standard ASTM D638 [90]. ASTM D638 (equivalent to ISO 527-1) is recommended to measure tensile strength and tensile modulus using dumbbell shape specimens. The advantage of this standard is its ease of use [91]. As there is stress concentration at the radii the specimen, only specimens broke in the middle part of gage length were assumed to show proper tensile behaviour.

The dimension of the dog bone tensile specimen and a finished specimen are shown in Figure 3.1.1. The tensile specimen thickness is between 2.7 mm and 3 mm. The specimen was cut to the final size using milling machine or by cutting the specimen using a fixture and rotating cutting blade. The specimen sides were sanded using low grit sandpaper to make smoother surfaces and reduce surface flaws on the specimen. This ensures repeatable and reliable results.

The specimen was then taken to the tensile machine and gripped by machine grips. The specimen was loaded with a displacement rate between $2 \frac{mm}{min} - 5 \frac{mm}{min}$. Assuming the gage length of about 50 mm, the strain rate is between $6.7 \times 10^{-4} \frac{1}{s}$ and $1.7 \times 10^{-3} \frac{1}{s}$. This range is assumed as quasi static, in which displacement is changing with time, but similar to the static load the effects of inertia can be ignored [92]. As per standard, minimum 5 specimens should be tested in similar conditions. The load is applied by an Instron servohydraulic load frame (Instron 8804) and it is measured by a 250 kN or 5 kN load cell (Figure 3.1.2). The load cell type depends on the approximate value of
maximum load. According to the load cell manufacturer the load cell is accurate for loads greater than 1% of the load cell capacity. An extensometer was used to capture the exact value of strain in the gage length. Depending on the type of the material and its geometry different gage lengths ranging from 12.5 mm to 50 mm can be used. The extensometer is connected to the control unit and it is able to measure the displacement at the specimen gage length automatically. In this case the undesired displacement due to the compliance of the machine is not included in the tensile displacement.

Figure 3.1.1 a. Finished epoxy specimen ready for the test. b. Epoxy CTS specimen geometry according to ASTM D638 (dimensions are in mm)
3.1.2 Results

3.1.2.1 Influence of Voids in the Material Density

As it was mentioned, care should be taken in mixing and making an epoxy plate to reduce the amount of bubbles in the specimen. To make sure that curing in the room temperature results in a low void content specimen, specimens were cured under low vacuum (15inHg) at room temperature. The relative vacuum was provided by a vacuum line inside a desiccator. The values of density of both series of specimens were measured as shown in Figure 3.1.3 (error bars indicate standard deviation). The density was measured
using Archimedes method as described in ASTM D729 [93]. The comparison shows that both series of specimen gave density with 0.4% difference. These value confirm with the value of density claimed by the manufacturer (1.18 $\text{g/cm}^3$).

**Figure 3.1.3 Comparison of density between epoxy cured with 15inHg and no Vacuum (both at room temperature)**

### 3.1.2.2 Stress Strain Curves

Tensile extension of epoxy was utilized to determine stress-strain curve of this material. The testing was carried out at room temperature with strain rate of $0.1 \text{min}^{-1}$. The values of force and displacement data acquisition was set at 100 ms for all data.

The stress-strain curves for different 1180/6560 crosslink epoxy specimen are shown in Figure 3.1.4. The curves show a consistent response from epoxy specimens. Tensile properties that are calculated from these tests are as follows.
3.1.2.3 Modulus of Elasticity

As described in ISO 527 (ASTM D638 equivalent), the value of Modulus of elasticity, or the modulus of elasticity, is the same as the slope of the linear portion of the curve. To be more specified, it "is the slope of a secant line between 0.05% and 0.25% strain on a stress-strain plot"[94].

In other words:

\[ E = \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1} \]  \hspace{1cm} (3.1.1)

where \( \varepsilon_1 \) and \( \varepsilon_2 \) are strains of 0.0005 and 0.0025 respectively parameters \( \sigma_1 \) and \( \sigma_2 \) are the stress at \( \varepsilon_1 \) and \( \varepsilon_2 \).

The results for the epoxy system (1180/6560) are shown in Figure 3.1.5, average values of Young’s modulus of epoxy specimens showing error bars with a standard deviation of
means are also shown in Figure 3.1.5. The values result in average value of 2904 MPa for 1180/6560 epoxy specimens.

![Figure 3.1.5 Young's modulus comparison between different 1180/6560 epoxy specimens (top). Average Young's modulus in the epoxy specimens (bottom).](image)

3.1.2.4 Tensile Strength

Maximum value of tensile stress that specimen showed during the tension test was measured. The results for 1180/6560 specimens calculated from engineering and true
stress-strain curves are brought in Figure 3.1.6. This gives an average value of 63.7 MPa as true tensile strength and 61.7 MPa as engineering tensile strength. The average values for engineering tensile strength and true tensile strength have 8% and 5% difference with the tensile strength reported by the manufacturer (58.7 MPa) respectively [95].

![Comparison of tensile strength of 1180/6560 epoxy specimen using engineering and true stress strain curve](image)

3.1.2.5 Yield Strength

Yield strength is the value of stress at which material shows deviation from linear behaviour. The 0.1% offset yield strength for epoxy specimens are as given in Figure 3.1.7. The discrepancy between values yield strength is more than other parameters. The results give an average value of 31.8 MPa for the yield stress of the specimens.
3.1.2.6 Failure Strain

The values of failure strain will be used as input of material definition in the FE model in Chapter 6. The values are shown in Figure 3.1.8. The average value of failure strain is 3.94%. The value of effective plastic strain at failure, $\varepsilon_{p,\text{eff.}}$, can also be found from failure strain as:

$$\varepsilon_{p,\text{eff.}} = \varepsilon_{\text{eff.}} - \frac{\sigma_f}{E} \quad (3.1.2)$$

Where $\sigma_f$ is the stress at failure point. The above relation gives $\varepsilon_{p,\text{eff.}} = 1.75\%$
Average mechanical properties of epoxy found from tensile testing are summarized in Table 3.1.1.

<table>
<thead>
<tr>
<th>Density ($g/cm^3$)</th>
<th>Young’s modulus (MPa)</th>
<th>Tensile strength (MPa)</th>
<th>Yield strength (MPa)</th>
<th>Failure strain %</th>
</tr>
</thead>
<tbody>
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<td>2904</td>
<td>63.7</td>
<td>31.8</td>
<td>3.94</td>
</tr>
</tbody>
</table>

Table 3.1.1 Mechanical properties of CLR1180/CLH6560 epoxy cured at room temperature for 48 hours

3.2 Fracture Testing Procedure

Background works to study the fracture behaviour of materials are reviewed in Chapter 2. Fracture studies are performed on epoxy, unidirectional glass fibre epoxy and plain weave glass fibre epoxy. Epoxy is studied as a composite’s constituent. Mode I, mode II and mixed mode testing are carried on Crosslink epoxy that is used to make UD composite. The values of fracture toughness and critical strain energy release rate for epoxy is determined and related with the failure mechanism in the fracture of UD
composite. UD composite is also made and tested under mode I, mode II and mixed mode loading is also performed on UD composite to investigate the failure mechanisms that are contributing the toughness of the material. The study also aims to characterize the through thickness fracture behaviour of UD composite and epoxy. Tests have been observed and studied carefully. Failure mechanism involved in the fracture of the composite under mode I, mode II and mixed mode I/II are investigated. Parameters affecting the critical strain energy release rate and fracture toughness in aforementioned materials are investigated.

The main focus of this chapter is to study the fracture behaviour using compact tension shear specimen also known as Richard’s test and single edge notch beam [71].

3.2.1 Compact Tension Shear Specimen (CTS)

As it was mentioned in Chapter 2, the advantage of compact tension shear (CTS) is that using one CTS specimen all modes of loading can be applied to the specimen. The methods and equations to determine the values of the mode I and mode II fracture toughness of the material were explained in details in Chapter 2.

The values of $K_I$ and $K_{II}$ can be found using the following Equations as stated in Chapter 2 [71].

\[
K_I = \frac{P\sqrt{\pi a}}{wt} \cdot \frac{\cos \alpha}{1 - \frac{a}{w}} \sqrt{\frac{0.26 + 2.56 \left( \frac{a}{w-a} \right)}{1 + 0.55 \left( \frac{a}{w-a} \right) - 0.08 \left( \frac{a}{w-a} \right)^2}} \tag{3.2.1}
\]

\[
K_{II} = \frac{P\sqrt{\pi a}}{wt} \cdot \frac{\sin \alpha}{1 - \frac{a}{w}} \sqrt{\frac{-0.23 + 1.4 \left( \frac{a}{w-a} \right)}{1 - 0.67 \left( \frac{a}{w-a} \right) + 2.08 \left( \frac{a}{w-a} \right)^2}} \tag{3.2.2}
\]

The values of $K_{IC}$ and $K_{IIc}$ will be used to calculate effective fracture toughness, mode I and mode II toughness of materials similar to the procedure mentioned for isotropic material. CSERR of the epoxy is the summation of mode I and mode II toughness values.
3.3 Fracture Testing of Resin

In this section, resin sample preparation and fracture testing set up are introduced.

3.3.1 Methods

3.3.1.1 Specimen Preparation

The neat epoxy sample, in this study, is made using CLR1180/CLH6560 epoxy material manufactured by Crosslink Tech. Inc. (Figure 3.3.1.a.). Resin is a two part epoxy, the type of which is the same as epoxy used in UD composite. The epoxy is made by mixing the hardener and resin in a ratio recommended by the manufacturer (here 100 units of resin with 30 units of Hardener).

In order to make sure that the two parts of epoxy are mixed properly, the following mixing procedures should be taken into consideration:

The resin should not be old, it is better to use the resin within a certain period after being manufactured. The appropriate time is indicated by the manufacturer. The resin will be contaminated by dust and debris or moisture in the container over time. These contaminations result in weakening of the resin bonding. The mixing should be at room temperature, changes in temperature may collect moisture in the container and contaminate the resin container [96]. As the mixing ratio is very important, care should be taken in weighing of resin and hardener to the proper ratio. Therefore, hardener and resin parts are measured with digital scale having 0.5 gr error. The resin and hardener are mixed thoroughly using a plastic spoon or stir stick in plastic cups.

After mixing, the resin is poured into trays that were lined with vacuum Teflon bags to avoid resin adhering to the trays. Baking trays are used for this purpose. The trays are leveled to give an even value of thickness in the resin sheet which is then cured at room temperature for 48 hours (See Figure 3.3.1 b and c). After pouring resin wrinkling of the vacuum bag should be avoided wrinkles change the cross sectional even thickness and results higher stress in the area and in error in the test. The weight of the epoxy used to make the sheets is 390 gr to give a thickness of 3.9 mm-4.1 mm and 260 gr of epoxy to
give a thickness of 2.7 mm to 3 mm. After 48 hours the resin is cured, it should be hard and no longer tacky to the touch [96].

Figure 3.3.1 a) The Crosslink resin and hardener kept in room temperature b) The tray is lined with vacuum Teflon bag and leveled. c. the tray is filled with mixed epoxy and is kept in room temperature
The tools used in mixing epoxy parts are shown in Figure 3.3.2.

Figure 3.3.2 Mixing tools. Plastic cups for small and large amounts, wood stir stick for mixing, paint brush for spreading the resin on fibre layers

Figure 3.3.3 CTS sample geometry (left) Neat epoxy specimen after machining (right)
Figure 3.3.4 a. Teflon mold cut and sealed for making thicker SENB and CTS samples b. molds supported by steel bar to withstand epoxy before curing c. cured samples having a minimum thickness of 11 mm. d. final thick CTS specimen before testing.

After removing the vacuum bag from the epoxy sheet, the sheet is cut to desired dimension as the compact tension shear sample. A finished sample and dimension of the
CTS sample is shown in Figure 3.3.3. For making thicker samples, however in order to reduce material consumption each specimen were made in a separate Teflon mold as shown in Figure 3.3.4, the mixed epoxy is then poured into the mold which its walls are fixed between steel bars and cured in the oven or hot press.

3.3.2 CTS Experiment

Mixed mode loading on CTS is applied using CTS fixture on the epoxy sample. The loading angle $\alpha$, is Figure 3.3.5 shows the sample under mode I and mode II.

Figure 3.3.5 CTS fixture. left) Mode I right) Mode II
Figure 3.3.6. a. Steel bolt holes and washer plate to hold the sample and pin hole in the CTS specimen. b. The CTS specimen is pinned to a clevis which is fixed into the machine grip.

As shown in Figure 3.3.5, the load is applied to the specimen using a fixture made of steel. The steel fixture has lower and upper grips which are fixed at grips of the testing machine. The sample is fixed to the grips using steel M6 bolts. The CTS specimen is pinned to a clevis which is fixed into the machine grip (see Figure 3.3.6). By adjusting the pin location on the fixture, different angles between $\alpha = 0^\circ$ (mode I) and $\alpha = 90^\circ$
(mode II) are generated in the specimen, any angle between them applies mixed mode I and mode II on the samples (Figure 3.3.5).

In order to study fracture properties of epoxy, the load angle altered from 0° to 90° in 15° increments. The load is applied using Instron 8804 tensile testing machine to the specimen and is measured by a 250 kN and 5 kN load cell depending on the size of the load. The load cell is accurate for the loads greater than 1% of load cell capacity. The load is applied at the displacement rate of 2 mm/min. The crack growth and stress pattern was observed using polarized lenses and a light source to see photo-elastic image of the sample.

The displacement is measured at the machine cross head. The load-displacement for each sample is used to determine the material stress intensity factor \( K_{Ic} \) and \( K_{IIc} \)

### 3.3.3 Single-Edge-Notch Bending

Single-edge-notch bending (SENB) test is used to determine mode I fracture toughness of neat epoxy samples according to ASTM 5045 [97]. The test was carried out because preparing beam samples with higher thickness was easier than CTS samples. The method described in ASTM 5045 is appropriate for highly cross linked thermosets like epoxy that behave linear elastic and are among brittle polymers, however the test is not appropriate for most of thermoplastics [98].

#### 3.3.3.1 Test Procedure

In this test, a notched specimen that has been pre-cracked is subjected to three-point bending as shown in Figure 3.3.7.
Figure 3.3.7 Schematic of single edge bending test [97] (Top). b. Single edge notch beam lab setting (bottom).

As shown in Figure 3.3.7.a, the load is applied through an indenter to the specimen, the displacement is measured using a displacement transducer or using the machine cross head displacement sensor. The load is increased until the notch fractures. This load is then used to calculate $K_{IC}$ for the epoxy.
3.3.3.2 Specimen Preparation

For SENB specimen the same type of epoxy as CTS specimen was used, i.e. CLR1180/CLH6560 epoxy material manufactured by Crosslink Tech. Inc. (Figure 3.3.1.a).

In order to minimize the manufacturing time and material usage a plastic mold using Teflon bags are made. To the dimension required for the test is a rectangular with 130 mm long, 30 mm width and 15 mm thick. The Teflon bag used as a mold for making epoxy bars is shown in Figure 3.3.4. The sample is then cured in hot press (no pressure is applied) or oven. The final machining of the sample is done utilizing milling and sanding machine. The notch is made by vertical band saw and the pre-crack is made by a razor blade (Figure 3.3.8.b). The shape and sharpness of the pre-crack affects the value of fracture toughness [97].

To calculate the value of the strain energy release rate for a linear elastic material we have:

\[ U = \frac{1}{2} P \cdot d \]  \hspace{1cm} (3.3.1)

Where \( P \) is a critical load that propagates the crack and \( d \) is the displacement of the tip of the indenter as shown in Figure 3.3.7.a.

\[ G_{ic} = \frac{U}{BW\phi} \]  \hspace{1cm} (3.3.2)

Where \( B \) and \( W \) are specimen thickness and \( \phi \) is an energy calibration factor which considers system compliance and defined in ASTM 5045.

The value of fracture toughness is measured from the applied load, \( P \), and the specimen dimensions [99]:

\[ K = P \cdot \frac{f(x)}{BW^{\frac{1}{2}}} \]  \hspace{1cm} (3.3.3)
where \( x = \frac{a}{w}, \ 0 < x < 1 \):

\[
f(x) = \frac{6x^2[1.99 - x(1-x)(2.15 - 3.93x + 2.7x^2)]}{(1 + 2x)(1 - x)^{3/2}}
\]  \hspace{1cm} (3.3.4)

Where \( P \) is the load in kN, \( B \) and \( W \) are specimen thickness and width in cm, and \( a \) is the crack length in cm.

The value of mode I critical strain energy release rate can also be found directly from mode I fracture toughness using:

\[
G_{IC} = \frac{(1 - \nu^2)K_{IC}^2}{E}
\]  \hspace{1cm} (3.3.5)

The above equation is utilized when the material has enough thickness that results in the plane strain condition. For a plane strain state the value of the thickness should be 2.5 times greater than the square ratio of mode I fracture toughness to the yield stress as described below [19]:

\[
B \geq 2.5 \left( \frac{K_{IC}}{\sigma_{yp}} \right)^2
\]  \hspace{1cm} (3.3.6)

where \( B \) is the thickness of the specimen.
Figure 3.3.8 a and b The pre-crack was made using razor blade. c SEM image shows the crack tip area made by the razor blade is shown beside the fracture area of epoxy subjected to the mode I loading.
3.4 Results (Thickness Study, Data Analysis, and Propagation Angle)

3.4.1 Thickness Study

The first question in finding fracture properties of the material is to ask if the thickness affects the results and if yes, is the thickness of RT sample is suitable for defining $K_c$ value?

It is mentioned in the literature that the critical value for fracture toughness (smallest value) is found when the material is in plane strain condition. Therefore a larger thickness is required to reach to plane strain state in a material with low value of the yield stress and high toughness compared to thinner samples that have high yield stress and low toughness [19].

The effect of sample thickness on the value of stress and resulting stress intensity factor is shown in Figure 3.4.1.

It is experimentally determined that fracture behaviour is typical of plane strain condition.

It has experimentally shown that if $r_p$ is in order of 0.025, where $r_p$ is the size of plastic zone radius at the tip of the crack [19].

$$r_p = \frac{K_I^2}{6\pi\sigma_{ys}^2} = \frac{\sigma^2 a}{3\sigma_{ys}^2} \leq 0.025t \quad (3.4.1)$$

The above relation can be restated as:

$$t \geq 2.12 \frac{K_I^2}{\sigma_{ys}^2} \quad (3.4.2)$$

where $\sigma$ is the far field stress.
By substituting Equation (3.2.1) for pure mode I fracture toughness (i.e. $\alpha=0^\circ$) into Equation (3.4.2) we have:

$$t \geq \frac{2.12}{\sigma_{ys}^2} \times \left( \frac{P\sqrt{\pi a}}{wt} \cdot \frac{1}{\sqrt{1 - \frac{a}{w}}} \cdot \frac{0.26 + 2.56 \left( \frac{a}{w-a} \right)}{1 + 0.55 \left( \frac{a}{w-a} \right) - 0.08 \left( \frac{a}{w-a} \right)^2} \right)^2$$  \hspace{1cm} (3.4.3)

Or

$$t \geq 3 \frac{2.12}{\sigma_{ys}^2} \times \left( \frac{P\sqrt{\pi a}}{w} \times \frac{1}{1 - \frac{a}{w}} \right)^2 \times \frac{0.26 + 2.56 \left( \frac{a}{w-a} \right)}{1 + 0.55 \left( \frac{a}{w-a} \right) - 0.08 \left( \frac{a}{w-a} \right)^2}$$  \hspace{1cm} (3.4.4)

To determine the minimum required thickness of epoxy CTS specimen from Equation (3.4.4), the values of the specimen dimension (specimen width and crack length) and the value of load causing mode I fracture are required. These dimensions are as shown in Figure 3.3.3. Assuming the load causing mode I fracture is 1500 N, Equation (3.4.4) gives the minimum thickness of 9.1 mm.

The above minimum thickness will be approved by mode I testing of samples with different thicknesses and comparing their values of fracture toughness.

The effect of thickness was studied on the value of fracture toughness of epoxy samples. Different samples having a thickness between 2.5 mm and 14 mm were made and their values of mode I fracture toughness were measured by CTS and SNEB test. The results are shown in Figure 3.4.2.a. The figure shows that a minimum thickness of 11 mm is required to ensure state of plane strain to measure the correct value for the mode I fracture toughness.
Figure 3.4.1 Thickness effect on fracture toughness of typical isotropic materials [19].

Figure 3.4.2 Thickness effect on determination of mode I fracture toughness of epoxy.
The effect of thickness on the value of mode I CSERR is also shown in Figure 3.4.3 which results in average value of $2.76 \frac{kJ}{m^2}$ for $G_{Ic}$.

![Figure 3.4.3 Thickness effect on mode I toughness (CSERR) of neat epoxy.](image)

### 3.4.2 Data Analysis

Mode I and mode II fracture toughness, $K_{Ic}$ and $K_{IIc}$, of epoxy are calculated using the maximum load, $P$, at which the crack opens up during the CTS test. Fracture toughness values are calculated using Equations (3.2.1) and (3.2.2). So the first step is to determine the load-displacement curve on the fracture specimen and to find the critical load that causes the fracture.

The CTS test was done on 29 epoxy specimens having a thickness between 3 mm-4 mm. The load displacement comparison of mode I and mode II loading on epoxy samples is shown in Figure 3.4.4 and Figure 3.4.5. The values for mode II show less than 10% difference between the maximum and minimum values of load divided by thickness. Load divided by thickness was used because the thicker sample withstands higher values of load, therefore, to compare the results of different samples with different thickness the value of the load should be normalized with respect to the thickness.
Figure 3.4.5.b shows that an increase in the loading angle or % mode II loading increases the value of maximum load, $P_{\text{max}}$, that cause the crack to open up.

Figure 3.4.4 Load-displacement curve for epoxy samples (thickness between 3 mm-4 mm) under pure mode I ($\alpha=0^\circ$). The load is divided by the thickness to ease the comparison of the results of samples with different thickness. This is because by increasing the sample thickness the maximum fracture load is also increased.
Figure 3.4.5 Load-displacement curve for epoxy sample (thickness between 3-4 mm) under pure mode II (α=90°) b) Critical values of critical load for mode II.

The displacement is the machine cross head displacement. This value is affected by the sample settling in the CTS specimen and also rotation of the sample in the fixture for mode II which can be reasons for different values in displacement in different samples.
The maximum load which is used to determine the value of fracture toughness in each mode of loading depends on the thickness of the sample.

Taking the resin thickness into consideration, the values of mode I and mode II components of the fracture toughness found from Equations (3.2.1) and (3.2.2) are depicted in Figure 3.4.6a. These values are used to determine effective fracture toughness from Equation (2.7.3) as shown in Figure 3.4.6b.
Figure 3.4.6 Neat epoxy mode I, mode II (a) and mixed mode (b) fracture toughness versus loading angle.

The average results for different modes of loading are summarized in Table 3.4.1. The value measured for mode I fracture toughness and mode I CSERR are $3.07 \text{ MPa} \sqrt{m}$ and $2.9 \frac{K_J}{m^2}$ respectively.

Mode II fracture toughness and CSERR using CTS samples were measured as $K_{IIc} = 4.89 \text{ MPa} \sqrt{m}$ and $G_{IIc} = 7.05 \frac{K_J}{m^2}$.

Figure 3.4.7 shows the value of critical strain energy release rate (CSERR) for neat epoxy under different modes of loading. As can be seen the mode II CSERR is almost 2.5 times greater than the value of CSERR for the mode I. The increase in the value of mode II CSERR is related to the size of the plastic radius in front of the crack and the morphology of the fracture surface that will be discussed later.
<table>
<thead>
<tr>
<th>$\alpha^\circ$</th>
<th>Avg. $K_I$ (MPa√m)</th>
<th>Avg. $K_{II}$ (MPa√m)</th>
<th>Avg. $K_{eff}$ (MPa√m)</th>
<th>$G_I$ kJ/m²</th>
<th>$G_{II}$ kJ/m²</th>
<th>$G_c$ kJ/m²</th>
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</tr>
</tbody>
</table>

**Table 3.4.1. Measured properties of neat epoxy samples using CTS test.**

As can be seen in Figure 3.4.6 and Figure 3.4.7, four samples for pure mode I, five samples for pure mode II and the rest of the samples for the mixed mode loading are used. The values of CSERR in Figure 3.4.7 are calculated by substituting the calculated values of $G_{lc}$ and $G_{IIC}$ from Equation 2.7.4 and 2.7.5 into Equation 2.7.6.
Figure 3.4.7 Epoxy critical strain energy release rate (toughness) vs. loading angle

3.4.3 Crack Growth Angle

Crack growth angle is the angle at which the crack opens up under different modes of loading. The prediction of propagation direction was first studied by Erdogan and Sih where they used the maximum principal stress criterion [100] and the minimum strain energy density criterion [101].

The value of crack growth angle in CTS specimen was predicted by Richard and coworkers [102]. Epoxy specimens subjected to the mode I loading begin to fracture at the notch tip, as the crack propagates in a 0° direction (parallel to the notch direction). The propagation angle varies from 0° to 85°. The comparison of the predicted value and measured value show a good agreement as shown in Figure 3.4.8. These values are also given in Table 3.4.2.
Figure 3.4.8 Comparison of measured and predicted values of crack growth angle.

The change in the crack growth angle for mixed mode ratios between mode I and mode II are shown in Figure 3.4.9. It was observed that in order to measure the proper value of the crack growth angle it should be measured during the test and right after the first steps of crack initiation. After the specimen fracture the growth angle might be change compared to the initial crack propagation angle. This is due to the rotation of the sample in the grip and plastic deformation of the stressed area during the crack propagation. Figure 3.4.10 shows how the angle is changed between the start of crack initiation and after the end of the fracture test. This is specifically the case with running fracture test on thinner samples that involves plastic deformation and slow crack propagation.
<table>
<thead>
<tr>
<th>Specimen number</th>
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<td>72.1</td>
<td>79.0</td>
</tr>
<tr>
<td>E14</td>
<td>90.0</td>
<td>72.1</td>
<td>85.0</td>
</tr>
</tbody>
</table>

Table 3.4.2 Values of predicted and measured crack growth angle.
The fact that crack propagates in different angles depends on the mode of loading which can be well studied using the photo-elasticity of the specimen. Figure 3.4.11 shows how isochromatic fringes rotate by rotating angle of loading resulting crack propagation in the direction of principal stress.

For some of thin mode II samples, macro hackles were observed. Multiple macro hackles were observed in the broken samples, starting from a few millimeters away from the tip of the crack as shown in Figure 3.4.12. As thinner specimens have higher toughness they are able to form larger hackles [103].

A model to predict the behaviour of the resin under mixed mode loading will be proposed in Chapter 6 that shows how well the failure model can predict the fracture behaviour under a mixed mode of loading.

Figure 3.4.9 The crack growth angle in different mixed mode ratio (loading angle). From top left: $0^\circ$, $15^\circ$, $30^\circ$, $60^\circ$, $75^\circ$, $90^\circ$
Figure 3.4.10 Effect of the resin deformation and the specimen rotation on the value of the crack growth angle between start of crack initiation and end of fracture test.
Figure 3.4.11 a. Photo elastic images from epoxy under pure mode I ($\alpha=0^\circ$) [104] b. $15^\circ$ loading angle c. $30^\circ$ loading angle d. $45^\circ$ loading angle e. $60^\circ$ loading angle f. $75^\circ$ loading angle
Figure 3.4.12 Macro hackles in some of neat epoxy specimen under mode II loading, sample with macro hackle (top) sample without macro hackling (bottom).
3.5 Discussion

3.5.1 Effects on Fracture Surface Morphology

In this section, it is desired to explore the reason that causes different modes of failure to release different values of energies. Here, we study the reason for lower values of energy released per area of fracture surface for thicker specimen compared with that of thinner specimen. The answer to these questions can be found in the fracture mechanisms represented on the fracture surface. In this section it is aimed to study these failure mechanisms. These mechanisms will help to develop a mechanistic model to predict fracture behaviour of epoxy and composite as described in Chapter 5.

3.5.1.1 Thickness Effect

In the thickness effect study it was noticed that thicker specimen results in a lower value of fracture toughness and toughness (Figure 3.4.2). Comparing the fracture surface of these specimens under the same mode of loading shows a flat fracture area for thicker specimens compared to the fracture surface with plastic deformations, hackles and chevron patterns in thinner specimens. This is shown in Figure 3.5.1 and Figure 3.5.2. Figure 3.5.1 shows the effect of epoxy specimen thickness on the fracture morphology cause by pure mode I loading and Figure 3.5.2 represents the thickness effect for specimens under 15° loading angle. It is obvious that a flat fracture surface required smaller amount of energy to propagate. This results in a smaller value of toughness in thicker specimens.
Figure 3.5.1 Comparison of epoxy specimen’s fracture surface morphologies. Specimens were under mode I (loading angle of 0°) with thickness of 3 mm (a) and 12 mm (b)
3.5.1.2 Temperature Effect

During fracture testing of resin specimens I noticed that specimens that were machined or cut by razor blade right before testing showed higher value of toughness. This was referred to the fact that specimens should be tested at the same temperature. Due to the
heat that is induced to the crack area during cutting by the razor blade, the crack region in
the epoxy specimen is softer and deforms easier under the load. This heat should be
completely dissipated from the epoxy before testing. Therefore, the specimens that were
cut by razor blade had higher fracture toughness than similar specimen that had a 1 day
gap between specimen machining and testing. The latter specimens show more brittle
behaviour and gave correct value of fracture toughness. Figure 3.5.3 compares two
specimen with equal thickness under two different temperature. The specimen having
approximate temperature of 40°C at the crack tip gave mode I toughness of $4 \frac{kJ}{m^2}$. The
specimen temperature was measured using an Infrared Digital Thermometer. The second
specimen that was in room temperature (19°C) gave mode I toughness of 2.82 $\frac{kJ}{m^2}$.

**Figure 3.5.3** Specimens with equal thickness (left) kept at room temperature gave a
flat fracture surface and toughness of 2.82 $\frac{kJ}{m^2}$ (right) kept at an approximate
temperature of 40°C gave a complex fracture surface resulting in toughness of $4 \frac{kJ}{m^2}$. 
3.5.2 Mode of Loading Effect

If attention is paid to the fracture surface of broken resin specimen, it can be found that the fracture surface morphology is changing from pure mode I to pure mode II. To study fracture surface stereomicroscope and scanning electron microscope were utilized. After fracture testing, the fracture surface is carefully cut at a low rate and stored away from humidity to reduce post-failure damage [103]. The fracture surface of the specimen under pure mode I shows a neat cleavage fracture surface, as shown in Figure 3.5.4. A flat fracture surface releases a smaller amount of energy during the fracture compared to the other type of fracture surfaces. By increasing the mode of loading from 0° towards 90°, the resulting fracture morphology is no more flat. SEM and stereomicroscopy images show radial patterns on the fracture surface. These patterns observed in lower angles of loading indicate sharp and fast crack propagation. As can be seen in Figure 3.5.5, for higher degrees chevron and river patterns observed on the surface. These morphologies show extensive resin plastic deformation and extension that require a higher value of energy to propagate. Figure 3.5.7 related fracture surfaces morphologies to their corresponding values of CSERR.

Hackle or cusp as shown in Figure 3.5.5b is the dominant fractographic feature for higher mode II and mode II fracture surfaces [103], [105]. Hackles appear on the fracture surface as inclined broken platelets (as shown in the figure). They are inclined opposing the direction of crack propagation, so they can be used to determine how crack has been locally propagated. The size of the hackles depend on moisture content, temperature and matrix toughness [103]. For example, higher temperature increases the plasticity and gives thicker hackles. Thicker hackle is an important source of increase in the toughness of material. The formation of hackles in polymers during fracture has been discussed by Purslow [106]. According to Purslow if there is shear loading in the area close to the crack tip due to the mode II loading, it can be resolved into principal stress. Principal stress is a tensile stress on the region in front of the crack as shown in Figure 3.5.6. The principal stress acts in 45° angle with regard to the crack direction. This tensile stress develops a crack in the polymer in 45° angle. Hackles are formed by the connection of these cracks as shown in the figure.
The geometry of hackle indicates more area absorbing energy to fracture compared to flat surface in mode I loading. Bruce showed that this change in the surface geometry results in a value of mode II CSERR which is 2.4 times greater than mode I CSERR [7].

Figure 3.5.4 Steromicroscopy (top) and SEM (bottom) images of epoxy fracture surface under pure mode I testing. The flat surface requires small amount of energy to be created.
Figure 3.5.5 SEM image of the fracture surface of epoxy under pure mode II testing (a) and 75° of loading angle (b). Closer view of hackles (cusps) in epoxy under mode II loading. c) Extensive plastic deformation on fracture surface during crack propagation releases higher value of energy compared to lower mode II fracture surfaces.

Figure 3.5.6 Hackles form by grow and coalescence of cracks due to shear in mode II loading [106]
3.5.3 Existing Failure Criteria

To find the appropriate failure criteria in which the results of epoxy samples fit. The plot of mode II CSERR versus mode I CSERR is shown and compared with mixed mode fracture criterion as follows [101], [107]:

\[
\left( \frac{G_I}{G_{ic}} \right)^m + \left( \frac{G_{II}}{G_{IIC}} \right)^n = 1
\]  \hspace{1cm} (3.5.1)

Where \( G_I \) and \( G_{II} \) are the value of mode I and mode II CSERR at each mode of loading, subscript \( c \) indicates the critical value. The values \( m \) and \( n \) are empirical values.
Figure 3.5.8 compares the experimental data and the criterion with different values of m and n. The values of m and n are selected arbitrarily. Thin specimens’ results were used in the figure. As can be seen the value of m=n=0.7 best fits to the experimental data.

![Graph showing energy release rate for different values of m and n.

Figure 3.5.8 Approximation of the mixed mode experimental data using Equation (3.5.1) with different values of m and n exponents. a) Thin specimen with thickness between 3 mm-4 mm. b) Thick specimen having a thickness above 6 mm.
3.6 Summary

Fracture and tensile behaviour of resin as a composite constituent was characterized in this chapter. Fracture morphology and its relation to the value of toughness of the material subjected to different modes of loading was observed and investigated. The increase in the toughness for the mode II loading was related to the hackles formation on the fracture surface of the resin. The results will be utilized in developing a mechanistic model predicting the mixed mode fracture behaviour of UD composites.
Chapter 4

4 Studying UD Composite

Reinforcing polymer with unidirectional fibres increases mechanical and fracture response in the direction of fibres. The material properties however are not improved in transverse direction. UD composite fracture behaviour is studied in this chapter. The specimen fabricated and tested in accordance with CTS test. The crack propagation through the thickness of the composite was investigated, and the energy absorbing processes associated with the through thickness fracture of UD composites was characterized. As it was described previously, experimental study and fracture observations on composite constituents helps to develop a predictive model for the fracture properties of composite material. The model predicts the mixed mode fracture behaviour of UD composite based on its constituents (fibre and matrix) and interfacial properties. In other words, the model is developed based on this idea that the amount of energy released in the composite is the summation of energy released by its constituents and fiber/matrix interface. For the UD composite material studied here, the crack is known to propagate along the fibres, and thus the projected area of fibre on the fracture surface will be zero. Therefore, it is only required to know the fracture properties of resin and interface and relate them to the fracture properties of the UD composite.

Resin fracture is studied in Chapter 3. Chapter 4 focuses on the mechanical and fracture properties of cracked UD composite with fibres aligned in the direction of the material crack. Similar to Chapter 3, tensile properties of UD composite is studied at first, the mechanical response of UD composite is used to calculate material toughness from its fracture toughness. Tensile section is followed with fracture testing and results. Mode I, mode II and mixed mode fracture testing of UD composite is presented and discussed here. Fracture surface morphology is then investigated. Mode I results in interfacial debonding of fibres from the matrix and flat fracture surface of the matrix. For mixed mode loading with high percentage of mode II and for pure mode II, interfacial
debonding of fibres from the matrix and the hackle formation of resin is dominant on the fracture surface.

Observation of fracture mechanisms represented on the fracture surface helps to develop the mechanistic failure criteria that predict mixed mode fracture behaviour of UD composite. The results of this chapter are utilized to validate the failure model predictions. The model will be introduced in Chapter 6.

4.1  Tensile Testing Procedure

The UD composite tensile response is studied here. Tensile testing of composite gives mechanical properties of the material that can be used to determine fracture properties of UD composite through fracture testing of the material. In this section UD composite specimen tensile testing and results are given. Tensile specimen manufactured and tested according to ASTM standard.

4.1.1  Specimen Preparation

In this study, UD composite is made of epoxy reinforced by unidirectional glass fibres. E-glass fibres have a diameter of 16-17 micron (Figure 4.1.1). Fibers hold together using periodic threads in the transverse direction of the fibres as shown in Figure 4.1.1. Epoxy is made by mixing CLR1180 and CLH6560 manufactured by Crosslink Technology Inc. Resin mixing process is discussed in Chapter 3. Unidirectional fibres are then laid over each other and resin is added between them. The manufacturing process, including composite curing and cutting is discussed in the next section. The final specimen for tension test according to ASTM D3039 however, is a rectangular straight shape [108].
Figure 4.1.1 E-glass fibres have diameter of 16-17 micron (top) Periodic threads keeping UD fibres together (bottom)

4.1.2 Testing

Tensile testing of UD composite specimens was carried out according to standard ASTM D3039 [108]. A straight sided specimen is introduced in the standard for tensile testing, the standard is recommended for both discontinuous and continuous-fiber, it is also recommended for highly oriented fiber reinforced polymer composites like UD composite [91]. The dumbbell shape specimen is not recommended for UD composite because crack propagation along the fibres direction may trigger at the radii of the
dumbbell shape specimen. As there is stress concentration at the grip, only specimens broke in the specimen gage length were used for the measurements.

The specimen sides after cutting were sanded using low grit sandpaper (180-grit sandpaper) to reduce surface defects and ensures repeatable results. Straight tensile specimen with fibre orientation of 0 degree has dimension of 15 mm x 250 mm with a thickness of 1mm. For UD composite with fibre orientation of 90 degree dimensions are 25 mm x 175 mm with a thickness of 2 mm. Longitudinal (0° fibre angle) and transverse (90° fibre angle) specimens are shown in Figure 4.1.2. Bonding tabs to the tensile coupon was found to weaken composite in the grip area, therefore as recommended in standard for light serrated grips only emery cloth or coarse sand paper was used between grips and tensile coupon. Use of emery cloth increases the friction between grip and specimen and reduces chance of grip damage [108]. The sand paper used at both ends of the tensile specimen was coarse and medium grade sandpaper (50-80grit).

Tensile testing is performed by Instron machine (as described in Chapter 3), the specimen is fixed to the machine grips. The specimen was loaded with a displacement rate of \( \frac{2 \text{ mm}}{\text{min}} \) as per standard. This range is assumed as quasi static and the effects of inertia can be ignored [92]. The load was measured by 250 kN load cell for 0° UD composite (Figure 3.1.2) and 5 kN load cell for 90° UD composite (Figure 4.1.3). According to standard minimum 5 specimens were tested in similar conditions. To prevent error caused by machine compliance, an extensometer with gage length of 50 mm was used (Figure 4.1.3)
Figure 4.1.2 a UD composite with longitudinal fibres (90° fibre orientation) b Transverse fibres (0° fibre) orientation. Emery cloth is used as an end tab for tensile specimen.
Figure 4.1.3 5kN load cell measured the load applied by tensile machine to transverse UD composite (glass fibre/ CLR1180-CLH6560 epoxy. Extensometer with 50 mm gage length measures tensile extension.

4.1.3 Results (Density, Longitudinal and Transverse Modulus)

*Density*

Mechanical properties of composite are generally dependent on the fibre content (fibre volume fraction). Fibre volume fraction can be adjusted using pressure during the cure of the composite as described in the next section. Density of UD composite was determined for composite materials that have an approximate fibre volume fraction of 43%. The value of density was determined by measuring the weight of the composite coupons in water and air according to ASTM D729 [93] (Figure 4.1.4). The scale was able to weigh
specimen to $10^{-4}$ gr and air free distilled water with temperature of 20° was used. The measurement gives the value of $1.79 \pm 0.03 \frac{gr}{cm^3}$.

Figure 4.1.4 Lab setting to measure composite density. Measuring specimen coupons weight in water (left) and comparing with its weight in air (right).

Longitudinal vs. Transverse Loading

To determine mechanical properties of the UD composite, UD specimens are subjected to longitudinal and transverse loading. The fracture of the composite can be explained as follows.

For a UD composite with longitudinal bundles of fibres subjected to a longitudinal loading, the glass fibres carry most of the load because they are stiffer than their surrounding epoxy. When the composite straining is continued, some fibres break if they are strained to their failure strain. Fracture at each fibre begins at a weak point. Fibre breaking creates micro crack and micro cracks form tiny cracks. When a crack advances to the first group of fibres, it is broken if the stress reaches to the fibre strength. This group of fibres and the surrounding matrix is then contracted due to the removal of axial
load. This is forming a part which is pulled from one side. This part is shown schematically by the broken area in Figure 4.1.5a. The area transfers the axial load to the neighbour part by the shear stress. The shear stress creates longitudinal cracks along the direction of fibres. If the straining is continued, a similar mechanism causes a fracture pattern as shown in Figure 4.1.5a.

Figure 4.1.5 a. Schematic of longitudinal cracks along the fibre direction in the UD composite due to shear stress between broken part and the rest of the composite. b. Longitudinal cracks in UD composite subjected to longitudinal tension c. Crack in transverse composite extends in the matrix between fibres.

In a UD composite under transverse loading, the crack extends in the matrix while some fibres bridge between two crack planes (Figure 4.1.5c). UD composites fractured under
longitudinal and transverse loading are shown in Figure 4.1.5b and Figure 4.1.5c respectively.

*Young’s Modulus (Longitudinal and Transverse)*

Similar to Chapter 3, modulus of elasticity of composite specimens were found from the slope of a secant line between 0.05% and 0.25% strain on a stress-strain plot.

The results for longitudinal modulus of different specimens (glass fibre-1180CLR/6560CLH epoxy) are as shown in Figure 4.1.6. These specimens have unidirectional fibre aligned in tensile direction. It should be noted that specimens that broke at the testing machine grip were not included in the measurement of the average Young’s modulus.

The discrepancy in the values of Young’s modulus is due to their values of fibre volume fraction. This fact was also predicted by the rule of mixture. The dependency of the longitudinal Young’s modulus on the fibre volume fraction is shown in Figure 4.1.7. Rule of mixture (assuming the slab model with equal strain) shows a good agreement with experimental data. As can be seen in Figure 4.1.7, Sample 13 shows a very low Young’s modulus compared to the rest of specimen due to its low fibre content (22% compared to 43% for other specimens).

The results give an average value of longitudinal Young’s modulus of 33800 MPa assuming 43% fibre volume fraction.
Figure 4.1.6 Comparison of composites’ longitudinal stiffness of different specimens
(Test carried out at room temperature)

Figure 4.1.7 Comparison of experimental data for longitudinal Young’s modulus
\((E_1)\) and prediction by rule of mixture

The values of transverse Young’s modulus for different specimens versus their fibre volume fraction is shown in Figure 4.1.8. Experimental data are compared with
predictions from Reuss and Halpin-Tsai models [11] and [12]. As can be seen there is a significant difference between predictions and experiments. Higher values of Young’s modulus in experiments are due to the existence of periodic transverse threads that tie glass fibres together. As shown in Figure 4.1.1, these threads resist against loads in transverse direction and increase transverse Young’s modulus. Another reason for considerable underestimation by Reuss model is due to inadequate assumption of equal stress in the model compared to non-uniform stress in the real specimen [5]. However, Halpin-Tsai that gives a semi empirical relation for transverse stiffness gives closer prediction to the experiment. The value of transverse Yong’s modulus used for calculation in next section for fibre volume fraction of 43% is 9000 MPa.

![Graph](image)

**Figure 4.1.8 Comparison of experimental data for transverse Young’s modulus \( (E_2) \) and prediction by Reuss and Halpin-Tsai models**

### 4.2 Fracture Testing Of UD Composite

In this section unidirectional (UD) composite sample preparation and CTS fracture test set up and their results are introduced.
4.2.1 Methods and Specimen Preparation

For this study, UD E-glass and epoxy material (CLR1180/CLH6560 manufactured by Crosslink Tech. Inc.) were utilized.

Composite panels were manufactured by the hand lay-up of 12 layers of UD glass fibre to give the required thickness for the test. The epoxy is prepared by mixing the hardener and the resin at the ratio specified by the manufacturer, the stirring process should be gentle so that less air voids is induced to the resin. The resin mixing and composite lay-up is done at room temperature. In hand lay-up method the resin is spread on each layer of UD using 2-inch brush (Figure 3.3.2); to relief the air bubbles from the resin and to help the UD glass fibres to soak better into the resin, aluminum rollers are used as shown in Figure 4.2.1. During the lay-up we should make sure that the fibres are aligned in the same direction in all layers, failure to do so results in inconsistency in composite behaviour. This is specifically the case when fibres are in longitudinal direction. Figure 4.2.2 shows a few degrees changes in the fibre angle direction decreases the stiffness of the composite significantly. The whole lay-up process time is limited by the resin gel time that should be noticed during the hand lay-up.
Figure 4.2.1 Aluminum rollers are utilized to spread the resin and remove air bubbles during hand lay-up. Periodic threads are clear in this photo.

The resin is then cured at 60°C for 4 hours under a hydraulic hot press (Figure 4.2.3). The pressure can be altered to give different thicknesses and fibre volume fraction for the composite. In order to keep the value of volume fraction in the sample consistent, the pressure is fixed at 25psi on the UD layers also steel bars with similar thickness were used under the hot press to ensure the consistency of the composite specimen thickness for all samples. After curing the panels are cooled down to the room temperature. The UD composite is then cut to the required dimension for test specimen. The cutting of the sides and notch and drilling of the bolt holes are done by vertical band saw, milling machine and press drill. Applying the appropriate tool and machine speed reduces the amount of the damage that is caused in the composite by machining during the process.
This damage includes delamination. Part of the damage caused to the composite is by the fibres that are bent and not being cut [109].

Figure 4.2.2 The change in the angle between fibres and loading can make a significant decrease in the composite stiffness for angles close to 0° [5].
Figure 4.2.3 Composite specimens are cured by a hot press. The pressure on the cylinder determines the final thickness of the specimen.

A finished unidirectional CTS specimen and its detailed dimension is shown in Figure 4.2.4.
4.2.2 CTS Testing

As discussed in Chapter 2, pure mode I and pure mode II as distinct modes of failure have been studied widely, this is however, many real applications involve mixed mode I/II loading that has been less studied. CTS testing is utilized to study the composite behavior under mode I, mode II and also mixed mode I/II loading [103].

The specimen and test set up is similar to neat epoxy specimens as described in the previous section. Mixed mode in the elastic fracture mechanics can be found by superposition of mode I and II components.

In this study, two sets of tests were performed on UD composite. In order to study the effect of the fibre angle on the value of mode I fracture toughness, the fibre angle, \( \theta \), is altered in specimen from 0° to 90° at an increment of 15° while the loading direction is maintained parallel to the sample geometry (macro-mode I). The second type of the tests was done on 90° UD composite under different modes of loading. 90° UD composite
under different modes of loading. The results were utilized to measure the values of mode I and mode II CSERR which can be used to calculate the value of interface CSERR.

To create a pre-crack at the tip of the notch a sharp cut at the tip of the notch was made. CTS specimens have pre-cracks approximately 0.5 mm long cut into the artificial notch tip using a razor blade. The testing is performed using an Instron servohydraulic loadframe (Instron 8804). The load is measured by 250kN or 5kN load cell depending on the range of the load required for the test. The displacement rate is 2 mm/min and the test is performed at room temperature.

The maximum load at which the crack opens up is measured and used for the calculation of the mode I, $K_{Ic}$, and mode II, $K_{IIc}$, the fracture toughness of the composite.

After fracture testing, the fracture surface is carefully cut at a low rate and stored away from humidity to reduce post-failure damage [103]. The surface is then studied using scanning electron microscopy to observe the constituents on the fracture surface. The fibre volume fractions of the samples were then measured by a burn-off test at 560°C for 1 hour according to ASTM D2584. The fibre volume fraction of the UD samples is between 40% and 50% [110].

4.2.3 Notch Sensitivity

To assess the influence of the notch geometry on the fracture response of the material, an additional study was performed with different shaped notches. A series of tests on UD composites were carried out that had a U-notch (2.5 mm radius), a 45° V-notch, and a V-notch with a slit made by a razor blade (Figure 4.2.5).
Figure 4.2.5 The CTS specimen with U-notch and V-notch

The study of the notch shape on neat epoxy showed that the fracture behaviour of the epoxy is affected by the shape of the notch, while the effect of the notch shape in a UD composite was not observed to be significant on the fracture load. As the crack opening in a UD composite occurs due to interfacial failure, the bonding between the fibre and matrix is the critical parameter, and not the shape of the notch. Therefore, a V-shape notch could be used for all composite specimens. Figure 4.2.6 shows the comparison of load-displacement curve between specimens with V-notch and U-notch.
Figure 4.2.6 The comparison of load-displacement curve between specimens with V-notch and U-notch [111]

4.2.4 Bolt Hole Failure at Specimens

For some combinations of fibre orientation and loading angle, it was found that the specimens would fracture at the bolt holes, rather than at the notch tip as shown in Figure 4.2.7. These failures were attributed to the weakness of the UD composite in the transverse direction, combined with the stress concentration at the bolt holes. This problem was addressed by adding a satin scrim layer to the outer surfaces of the specimens during the manufacturing process, and by incorporating 180 grit sandpaper between the steel fixture and the specimen.
Figure 4.2.7 Examples of bolt hole failure in 90 °(top) and 75°(bottom) UD composite specimen under mode II loading. No notch opening is observed in the sample and the results are not good for the calculation of fracture properties. The arrows show failure points.

To induce adequate roughness and increase the stress applied on the gripping surface between the fixture and CTS specimen, 180 grit sandpaper was used between the fixture and the specimen (Figure 4.2.8a). If the load is carried only by the bolts to the sample, the bolt holes may fail before the notch opens up. To increase friction and roughness between the grip and the sample, satin fabric is also used in the UD layers during the hand lay-up. Use of satin when curing gives a crossing pattern to the smooth surface of the UD composite, so it increases the friction and reduces slip between fixture and the sample.
The second approach to reinforce the gripping area was to use a combination of plain weave fibre composite and UD composite in that area while leaving the central part of specimen with only UD fibre composite as shown in Figure 4.2.8.b.

Figure 4.2.8 Steps taken to remove the problem of failure at the bolt holes. A. Use of sandpaper between the fixture and the specimen to increase friction in the gripping area. b and c) Use of a combination of UD and plain weave fiber composite to reinforce the gripping area.
4.2.5 Modified CTS Specimen

In order to reduce material consumption for testing and ensure proper results for higher modes of loading (modes with high mode II percent), a modified test fixture was introduced. In this fixture as the specimen is glued to the fixture the specimen rotation in the fixture is reduced compared to conventional CTS fixture in which specimen is bolted to the fixture. As can be seen in Figure 4.2.9.a use of the thicker section at the bolt area, removes the bending moment on CTS specimen. A thicker section is provided by gluing another plate of composite to the bolt area of the fixture (see Figure 4.2.9 b).

The test fixtures were made from Aluminum or composite material as shown in Figure 4.2.10. The specimen is glued to the fixture; the glue should be strong enough to ensure the crack propagates in the notch end.

The specimen dimension is 36 mm x50 mm x6 mm. Two different series of specimen with 6 mm and 3 mm thickness were made to study the effect of thickness in the value of fracture toughness. The crack length is almost 18 mm which is half of the length. The fixture is pinned to the tensile machine grips to ensure application of tensile load without moment to the specimen (Figure 4.2.10). Depending on the value of critical load for crack to open up 250 kN or 5 kN were utilized.
Figure 4.2.9 a) Schematic of the modified compact tension shear fixture for high mode II loading. b) Final fixtures after curing, CTS specimen will be glued to each fixture for fracture testing.
Figure 4.2.10 The CTS specimen is glued to the modified CTS fixture. The fixture is pinned to a clevis that is gripped by machine grip. Loading angle a) 75° b) 90°.

4.2.6 Results and Data Analysis

4.2.6.1 Thickness Study

As the plastic region in the matrix inside a reinforced composite is smaller than neat epoxy, the minimum value of thickness, which is required for testing to make sure that the results of the fracture testing is trustworthy is also smaller. As mentioned in section 3.3.3 for effect of thickness on fracture response of neat resin, the specimen should be
thick enough to make sure that material is under plane strain condition. In this case it has experimentally shown that if $\frac{r_p}{\text{thickness}}$ is in order of 0.025, plane strain situation is satisfied and fracture can be assumed as linear fracture. As epoxy in UD composite is constrained between fibres then plastic radius is limited to the fiber spacing. Therefore, it is expected that thinner specimen of UD composite can be utilized for fracture testing. The value of mode I and mode II fracture toughness of the UD composite with different thicknesses are measured using CTS specimen. The load displacement curve from the CTS test is drawn and the value of maximum load is used to determine the value of fracture toughness. The results of mode II test using the modified CTS test are shown in Figure 4.2.11. The value of maximum loads from different specimen show 15% difference. The difference in the value of the displacement is because the x axis in the curve shows the vertical displacement of the tensile machine cross head which is different from specimen crack tip displacement.

![CrossHead Displacement vs Load](image)

Figure 4.2.11 Mode II load-displacement curve for unidirectional composite.

**Maximum load have 16% relative error.**

As can be seen in Figure 4.2.12 and Figure 4.2.13, the value of mode I fracture toughness is not sensitive to thickness in the range of 3 mm to 8 mm. This is while the value of mode II shows a minimum of 5.7 mm thickness specimen is required to gain a minimum value for mode II fracture toughness and CSERR.
Therefore, a specimen with a thickness of minimum 5.7 mm were utilized in the data analysis.

\[ K_{IC} = 3.7MPa\sqrt{m} \]

**Figure 4.2.12** Thickness effect on mode I fracture toughness of UD composite.

\[ K_{IIc} = 3.85MPa\sqrt{m} \]

**Figure 4.2.13** Thickness effect on mode II fracture toughness of UD composite

The value measured for mode I fracture toughness using Equation (3.2.1) is 3.7MPa\sqrt{m}.
Mode II fracture toughness measured by fracture testing CTS samples using Equation (3.2.2) gives 3.85 MPa√m.

The values of the fracture toughness match with the literature [98]. The value of mode I and mode II CSERR can be found using formulation given in the next section.

4.2.6.2 UD Composite Toughness

It was mentioned in the previous section that the value of CSERR for an isotropic material in the plane strain condition can be found using fracture toughness of the material using the following relation:

\[
G_I = \frac{K_I^2(1 - v^2)}{E} \quad (4.2.1)
\]

\[
G_{II} = \frac{K_{II}^2(1 - v^2)}{E} \quad (4.2.2)
\]

Above equations can be used for isotropic materials like epoxy. However, these relations are not valid anymore for unidirectional composite materials that show highly anisotropic behaviour. When the material is anisotropic, the relations that are used to determine the value of the toughness should consider other elastic parameters of the material [112], [113]. Therefore, the relation between CSERR and fracture toughness can be written as:

\[
G_I = K_I^2 \sqrt{\frac{\tilde{S}_{11}\tilde{S}_{22}}{2}} \left[ \sqrt{\frac{\tilde{S}_{11}}{\tilde{S}_{22}}} + \frac{2\tilde{S}_{12} + \tilde{S}_{66}}{2\tilde{S}_{11}} \right]^\frac{1}{2} \quad (4.2.3)
\]

\[
G_{II} = K_{II}^2 \tilde{S}_{11} \left[ \sqrt{\frac{\tilde{S}_{11}}{\tilde{S}_{22}}} + \frac{2\tilde{S}_{12} + \tilde{S}_{66}}{2\tilde{S}_{11}} \right]^\frac{1}{2} \quad (4.2.4)
\]
where $\bar{S}_{ij}$ are elements of transformed compliance tensor, $[\bar{S}]$, which is defined as [5]:

$$[\bar{S}] = [T']^{-1}[S][T]$$  \hspace{1cm} (4.2.5)

and

$$[T']^{-1} = \begin{bmatrix}
\cos^2 \theta & \sin^2 \theta & -\cos \theta \cdot \sin \theta \\
\sin^2 \theta & \cos^2 \theta & \cos \theta \cdot \sin \theta \\
2\cos \theta \cdot \sin \theta & -2\cos \theta \cdot \sin \theta & \cos^2 \theta - \sin^2 \theta
\end{bmatrix}$$  \hspace{1cm} (4.2.6)

and

$$[T] = \begin{bmatrix}
\cos^2 \theta & \sin^2 \theta & -\cos \theta \cdot \sin \theta \\
\sin^2 \theta & \cos^2 \theta & -2\cos \theta \cdot \sin \theta \\
2\cos \theta \cdot \sin \theta & \cos \theta \cdot \sin \theta & \cos^2 \theta - \sin^2 \theta
\end{bmatrix}$$  \hspace{1cm} (4.2.7)

The matrix $T$ relates stresses system ($\sigma_x, \sigma_y, \tau_{xy}$) to stress in fiber axis ($\sigma_1, \sigma_2, \tau_{12}$), where fibres have angle of $\theta$ with loading direction. The angle $\theta$ (Figure 4.2.4) is the fibre angle which is equal to 90° for a UD composite in which the fibre orientation is parallel to the notch direction of the CTS specimen, and matrix $S$ is the compliance tensor of the composite [5].

Using Equation (4.2.3) and (4.2.4) the average value of mode I and mode II CSERR for unidirectional composite is calculated to be $1.02 \frac{K_I}{m^2}$ and $2.83 \frac{K_I}{m^2}$ respectively. These values for mode I and mode II CSERR match with the literature [98]. Values of GI and GII, mode I and mode II CSERR of UD composite is shown in Figure 4.2.14 and Figure 4.2.15.

The values of mechanical properties of the UD composite used in the calculations are given in Table 4.2.1.
### Epoxy (CLR1180/CLH6560 Crosslink)

<table>
<thead>
<tr>
<th>Mechanical Properties</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>2.9  GPa</td>
</tr>
<tr>
<td>σ_y,p.</td>
<td>30.8 MPa</td>
</tr>
<tr>
<td>υ_{12}</td>
<td>0.38 [1]</td>
</tr>
</tbody>
</table>

### UD glass fibre epoxy

<table>
<thead>
<tr>
<th>Mechanical Properties</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>E_1</td>
<td>34.04 GPa</td>
</tr>
<tr>
<td>E_2</td>
<td>5.032 GPa</td>
</tr>
<tr>
<td>E_3</td>
<td>5.032 GPa</td>
</tr>
<tr>
<td>υ_{21}</td>
<td>0.046*</td>
</tr>
<tr>
<td>υ_{31}</td>
<td>0.046*</td>
</tr>
<tr>
<td>υ_{32}</td>
<td>0.35*</td>
</tr>
</tbody>
</table>

**Table 4.2.1: Mechanical properties of epoxy and UD composite used in calculations.**

*values are calculated as described in the Hull and Clyne’s book [5].
Figure 4.2.14 Thickness effect on mode I CSERR ($G_I$) of UD composite. The results give an average value of 1.02 kJ/m$^2$.

$$G_{Ic} = 1.02 \frac{kJ}{m^2}$$

Figure 4.2.15 Thickness effect on mode II CSERR ($G_{II}$) of UD composite. The results give an average value of 2.83 kJ/m$^2$.

$$G_{IIc} = 2.83 \frac{kJ}{m^2}$$
4.2.6.3 Mixed Mode Fracture Toughness

Using CTS test the value of the maximum load that open the specimen crack was measured and the values of mode I and mode II components of the fracture toughness are calculated using Equations (3.2.1) and (3.2.2).

It was observed that crack opens up at an area close to the tip of CTS notch and follows the direction of fibres. The crack path disregards the angle of loading. It was also noticed that for the specimen having equal value of thickness the maximum load for fracture is increasing by increasing the loading angle from 0°(pure mode I) to 90°(pure mode II) as shown in Figure 4.2.16. The reason for lower values of F/t for 75° loading angle compared to 69° loading angle is due to larger values of crack length to width ratio. Higher value of a/W decreases the load to fracture.

![Graph showing maximum values of force vs load angle in CTS specimen](image)

**Figure 4.2.16** Maximum values of force vs load angle in CTS specimen shows a gradual increase in value of force from mode I to mode II loading.

The values of mode I and mode II components fracture toughness under different loading angles obtained from Equations (3.2.1) and (3.2.2) are shown in Figure 4.2.17. These components give effective fracture toughness through Equation (2.7.3). Mixed mode effective fracture toughness of UD composite is given in Figure 4.2.17.
Figure 4.2.17 Mode I and mode II components of fracture toughness of the UD composite under different modes of loading.

Figure 4.2.18 Mixed mode Effective fracture toughness of UD composite.

Substituting values of mode I and mode II fracture toughness give mode I and mode II values for CSERR. Total energy released during mixed mode fracture can be defined as follows:
The values of mixed mode CSERR for different modes of loading are depicted in Figure 4.2.19.

![Graph showing experimental results of UD composite mixed mode toughness versus loading angle.](image)

**Figure 4.2.19** Experimental results of UD composite mixed mode toughness versus loading angle.

### 4.3 Fractography (Discussion)

Fracture behaviour of a composite over the entire range of loading angles, showed that the crack in the sample initiated at the notch tip and then propagated along the fibre/matrix interface. Thus, for loading angles other than 0° and 90°, the cracks propagate under mixed mode loading conditions. When the load is applied to the composite, a white image on the path of the crack is observed before the load reaches maximum. This white image represents the existence of tiny micro-cracks at the fibre matrix interface due to the load. At maximum load, $P_{\text{max}}$, these micro-cracks come together and make a visible crack at the tip of the notch in the fibre direction. The process of crack propagation is faster for thicker samples than thinner ones.
Frac
ture Surface Study

Similar to epoxy specimen the fracture morphology study helps to find out their effect on the value of energy released during fracture of UD composites. After fracture testing, the fracture surface is carefully cut at a low rate. During cutting paper pocket was used to cover the fracture surface and protect it from cutting debris (Figure 4.3.1). Final cut specimen stored away from humidity to reduce post-failure damage [103]. The surface is then studied using scanning electron microscopy to observe the constituents on the fracture surface.

Figure 4.3.1 paper pocket was used to cover the fracture surface and protect it from cutting debris.

4.3.1 Mode I (Opening Mode) Features

A visual image of UD composite mode I fracture surface is shown in Figure 4.3.2. Broken fibres and loose fibres give a rough appearance of the surface. Loose fibres have debonded from matrix and some of them are not attached to the other side of the matrix. In macroscopic view, the pattern of matrix fracture is not very clear, however, some voids (air bubbles) between layers can be seen in the image. To study the fracture mechanisms properly, images with higher magnification are required.
Figure 4.3.2 Macroscopically SEM image of mode I fracture surface of UD composite. The consolidated image shows different mechanisms represents on the fracture surface. Crack propagation in this image is from right to left.

When we look closely at the fracture surface of the composite under mode I loading, fibres debonded from their matrix and matrix fracture area can be observed on the fracture surface. Figure 4.3.3 shows fibres debonded from their socket in the matrix. The smooth surface on the channels indicate a low interfacial bond between the glass fibre and epoxy, a similar debond pattern can be observed elsewhere [114]. The matrix between neighboring fibres is also fractured. Debris on the fracture surface is caused by sliding of surfaces in contact during fracture. When matrix fracture is observed under mode I a flat fracture surface can be found in most of the matrix fracture area (see Figure 4.3.4 and Figure 4.3.5). These flat areas are mostly matrix between UD layers. Apart from cleavage fracture other morphologies like hackles and plastic deformations are also observed which are not dominant in this type of fracture surface. Riverline observed in some parts of the fracture surface is made by the development and convergence of planes of cracks. The crack propagation direction is in the direction of convergence of river lines. The development of river lines has been discussed by Purslow and shown in Figure 4.3.6 [106].
Figure 4.3.3 fracture surface of UD composite under mode I loading showing interfacial debonding. A. Fibre debonding leaves a smooth channel. B. debris

Figure 4.3.4 SEM of UD composite subjected to mode I testing, shows cleavage fracture at epoxy. A river patterns caused by the meeting of two fracture surface, moving in the same direction.
Figure 4.3.5 Fracture surface of UD composite indicates A. flat fracture epoxy surface B river patterns C. fibre debonding D. matrix hackles

Figure 4.3.6 Schematic of river line formation, crack plane propagating at different elevations meet each other and form river lines. The arrow shows the local direction of crack propagation [106].

4.3.2 Mode II and Mixed Mode Loading

Interfacial Debonding and Matrix Hackle
An image with lower magnification of UD composite mode II fracture surface is shown in Figure 4.3.7. When the surface is looked with higher magnification, the hackle formation and fibre debonding is observed as a major failure mechanisms. The formation of hackle during fracture of epoxy subjected to shear loading has been discussed in Chapter 3. The formation of hackles in CTS specimen is given schematically in Figure 4.3.8. The shear and normal stress at the tip of the crack result in principal stress which is tensile and causes the hackle to form as shown in Figure 4.3.8. The angle of principal stress for pure mode II is 45°, this angle changes by the change in the mixed mode loading [103].

Hackles with different size are shown in Figure 4.3.9. The size of the hackles depend on the fibre spacing [105], processing condition and matrix toughness [103]. Debris is observed on surfaces under mode II shear more than mode I, the debris is caused by sliding abrasion between the fracture surfaces during shear loading [103].

**Figure 4.3.7** Fracture surface of UD composite under mode II loading. Hackle formation and fibre debonding can be found on the surface.
Figure 4.3.8 schematic formation of hackles under mode II (a) and mixed mode loading (b) in CTS specimen by the resultant principal stress.
Figure 4.3.9 Hackles in different sizes (A, E and G) and interfacial debonding (B and F) are dominant features in mixed mode and mode II fracture. Other fracture morphologies such as void (H) and debris (I) can be observed on the surface. Fibre layers spacing in the bottom image is approximately 50 micron.
4.3.3  Fibre Fracture

In the study of the fracture surface under pure mode I, pure mode II and mixed mode I/II loading, fibre fracture is not observed very often. However, fibres broken can be found in mixed mode and mode II loading where crack meets a bundle of fibres and fractures fibres by shear load (see Figure 4.3.10).

![Fibre Fracture Image]

**Figure 4.3.10** fibres broken in specimen under mixed mode loading with $\alpha=30^\circ$, fibre breaking angle is $45^\circ$ mainly due to shear. Arrow shows chop marks on the fibre which can be caused by the compression of the other side of specimen added to the shear force during sliding.

4.4  Effect of Fiber Volume Fraction

It was observed that increase in fiber volume fraction can increase the value of mixed mode toughness. Therefore, enough care should be taken in the manufacturing of composite to reach to similar value of fiber volume fraction. This value is measured by burning of composite specimen. The burning is done using a horizontal oven according to
ASTM D2584. The average value of the fiber content of UD composite is between 40% and 45%.

4.5 Effect of Fiber Angle

Fiber angle has an important role in defining composite mechanical and fracture properties. It is observed that in 0° UD composite, few degrees divergence from 0° makes a significant change in mechanical and fracture properties. The material response is less sensitive if the fibre angle is close to transverse direction (90 degree).

To study the effect of the fibre angle, θ, it is altered from 0° to 90° in the specimen at an increment of 15° while the loading direction is maintained parallel to the sample geometry (macro-mode I).

Fracture behaviour of composite over the entire range of fibre angles (i.e. angle θ in Figure 4.2.4) indicated that the crack in the sample initiated at the notch tip and then propagated along the fibre/matrix interface (Figure 4.5.1). Thus, for fibre angles other than 0° and 90°, the cracks propagate under mixed mode loading conditions. When the load is applied on the composite, a white image on the path of crack happens before the load reaches to maximum. This white image represents the existence of tiny micro cracks at the fibre matrix interface due to the load. At maximum load P_max, these micro cracks come together and make a visible crack at the tip of the notch in the fibre direction.
Figure 4.5.1 Crack propagation in UD composite with various fibre angles a) left to right $15^\circ$, $30^\circ$, $75^\circ$, $90^\circ$. b) $0^\circ$, $45^\circ$ and $60^\circ$ [111].
Peak load $P_{\text{max}}$ and subsequent fracture toughness decreased with increasing fibre angle (Figure 4.5.2). As can be seen in Figure 4.5.2.a the displacement at the sample decreases from 90° fibre angle to 0° fibre angle. This decrease in the failure displacement results in lower values of energy absorption and toughness in UD composite with smaller fibre angles. Figure 4.5.2.b shows the value of the mode I fracture toughness for different fibre angles of UD composites. At least three specimens were tested for each fibre angle. As can be seen in Figure 4.5.2 the value of fracture toughness decreases from $21MPa\sqrt{m}$ for pure mode I to approximately $2MPa\sqrt{m}$ for pure mode II. The fibre angle effect will be discussed in Section 5.3.2.1.
4.6 Summary

The fracture and tensile behaviour of UD composite was characterized using tensile and CTS testing. The fracture morphologies of the broken specimens were carefully investigated. It was observed that fibre debonding from the matrix and matrix fracture are two major features representing on the fracture surface. The matrix fracture was observed to change from flat cleavage fracture for mode I loading to hackle form for mode II loading. These observations with the help of the values of fracture properties measured in this Chapter will be used in the next chapter to develop mechanistic model for mixed mode fracture of UD composite.
Chapter 5

5 Mechanistic Model

In the previous chapters it was discussed that many applications involve mixed mode loading that result in mixed mode failure of the material. Therefore, it is important to develop a failure criterion that anticipates different modes of failure. Existing empirical failure criteria in different forms (linear, polynomial or power law) model the mixed mode fracture response of composites. These criteria are mainly based on the values of the mode I and mode II CSERR of the composites and some empirical parameters. The empirical parameters are found through curve fitting of experimental data. These criteria are applicable for one specific system of composite; however, other series of tests are necessary for a new system of composite. Therefore, these existing criteria are labour sensitive and not necessarily accurate for all systems of composites. A detailed study on current failure criteria was done by Bruce [7].

If a crack in composite advances between the composite plies it results in delamination or interlaminar failure of the composite and when the crack propagates perpendicular to the composite plies it results in through-thickness fracture of the composite. As delamination is a common mode of failure in laminated composites, most of the failure criteria have taken delamination into consideration. Compared to an interlaminar crack an advancing through-thickness crack was not studied very often and a failure criterion addressing specifically mixed mode through-thickness failure of the composite has not been introduced. A literature review of the study of through-thickness fracture response using CTS specimen is presented in Chapter 2.

Critical strain energy release rate measured from neat epoxy in Chapter 3 and UD composites in Chapter 4 with the help of fractography are used to develop the mechanistic model. In the first step the model is developed to determine the mixed mode fracture response of the composite based on their pure mode I and pure mode II values. In the next step, the model is then evolved to a mechanistic model that predicts the behaviour of a composite on the basis of its constituent properties.
The failure criterion presented here can be used for a wider range of polymer composites and also as a design method to improve fracture properties of a composite by customizing the percentage and type of its constituents.

5.1 Failure Model

An energy based failure model is proposed here. The model predicts the value of the CSERR of a UD polymer composite based on the fracture properties of its constituents. The properties required to determine CSERR of the composite include resin mode I and mode II CSERR, and fibre/matrix interfacial CSERR. The first step is to develop a model that predicts the mixed mode behaviour of a composite using the value of pure mode I and pure mode II toughness of the composite. The model will then evolve into a mechanistic model.

To begin, we consider a UD composite in which the fibre orientation is parallel to the notch direction of the CTS specimen geometry shown in Figure 5.1.1. The specimen is subjected to a far field stress, $\sigma$, applied at an angle, $\alpha$, to the specimen axis. This applied stress, $\sigma$, can be resolved into the normal and shear components, $\sigma_y$ and $\tau_{xy}$, as shown in Figure 5.1.1.

For the geometry of Figure 5.1.1:

\[
\sigma_y = \sigma \cos \alpha \quad (5.1.1)
\]

\[
\tau_{xy} = \sigma \sin \alpha \quad (5.1.2)
\]
Figure 5.1.1 CTS specimen under mixed mode load in the lab setting (left) General state of stress at the elements shown in the CTS sample for a mixed mode I and II and crack propagation along the interface under mixed mode loading (right)

The mode I and mode II stress intensity factors, $K_I$ and $K_{II}$ at the tip of the crack can be expressed as:

$$K_I = Y_1 \sigma_y \sqrt{\pi a}$$  \hspace{1cm} (5.1.3)

$$K_{II} = Y_2 \tau_{xy} \sqrt{\pi a}$$  \hspace{1cm} (5.1.4)

where, $Y_1$ and $Y_2$ are dimensionless geometry components that depend on crack length, $a$, and the width of the specimen, $w$. The values of $K_I$ and $K_{II}$ for the CTS specimen are found using the following relations as mentioned in Chapter 2 [74] and [75]:

$$K_I = \frac{P \sqrt{\pi a} \cos \alpha}{wt \left(1 - \frac{a}{w}\right)} \sqrt{\frac{0.26 + 2.65 \left(\frac{a}{w-a}\right)}{1 + 0.55 \left(\frac{a}{w-a}\right) - 0.08 \left(\frac{a}{w-a}\right)^2}}$$  \hspace{1cm} (5.1.5)
where \( P_c \) is the critical (maximum) load corresponding to the crack opening, \( t \) is the specimen thickness and \( a \) is the crack length.

Using Equations (5.1.1) and (5.1.2), the stress intensity factors can be restated as:

\[
K_I = Y_1 \sigma \cos \alpha \sqrt{\pi a} \tag{5.1.7}
\]

\[
K_{II} = Y_2 \sigma \sin \alpha \sqrt{\pi a} \tag{5.1.8}
\]

Defining the effective value of stress intensity factor

\[
K_{eff} = \sqrt{K_I^2 + K_{II}^2} \tag{5.1.9}
\]

The CSERR can then be expressed as a function of \( \alpha \) (assuming plane strain):

\[
G_c = K_{eff}^2 \frac{1 - \nu^2}{E} = \frac{(K_I^2 + K_{II}^2)(1 - \nu^2)}{E} \tag{5.1.10}
\]

Substituting Equations (5.1.7) and (5.1.8) into Equation (5.1.10) gives:

\[
G_c = \cos^2 \alpha Y_1^2 \sigma^2 \frac{\pi a(1 - \nu^2)}{E} + \sin^2 \alpha Y_2^2 \sigma^2 \frac{\pi a(1 - \nu^2)}{E} \tag{5.1.11}
\]

or

\[
G_c = \cos^2 \alpha \frac{K_{IIc}^2 (1 - \nu^2)}{E} + \sin^2 \alpha \frac{K_{Ic}^2 (1 - \nu^2)}{E} \tag{5.1.12}
\]

Since [19]:

\[
K_{IIc} = \frac{P_c \sqrt{\pi a}}{wt} \left[ -0.23 + 1.4 \frac{a}{w - a} \right] \left[ 1 - 0.67 \frac{a}{w - a} + 2.08 \left( \frac{a}{w - a} \right)^2 \right] \tag{5.1.6}
\]
Therefore, Equation (5.1.12) can be written as:

\[ G_c = \cos^2 \alpha \ G_{Ic} + \sin^2 \alpha \ G_{IIc} \]  \hspace{1cm} (5.1.15)

Equation (5.1.15) states that the CSERR for any material in which the crack propagates parallel to the original notch, can be determined with the knowledge of the values of mode I and mode II CSERR. Equation (5.1.15) results in the prediction of toughness as a function of a loading angle with the general form shown in Figure 5.1.2.

**Figure 5.1.2** The mixed mode toughness pattern predicted for different modes of loading using failure criteria (Equation (5.1.15))

Equations (5.1.13) and (5.1.14) can be used for an isotropic material like epoxy. When the material is anisotropic, the relations that are used to determine the value of the toughness should consider other elastic parameters of the material, [112] and [113]. Therefore, the relation between CSERR and fracture toughness can be written as:
\[
G_I = K_I^2 \sqrt{\frac{\bar{S}_{11}\bar{S}_{22}}{2}} \left[ \sqrt{\left(\frac{\bar{S}_{11}}{\bar{S}_{22}}\right)^2 + \frac{2\bar{S}_{12} + \bar{S}_{66}}{2\bar{S}_{11}}} \right]^{1/2} \tag{5.16}
\]

\[
G_{II} = K_{II}^2 \frac{\bar{S}_{11}}{\sqrt{2}} \left[ \sqrt{\left(\frac{\bar{S}_{11}}{\bar{S}_{22}}\right)^2 + \frac{2\bar{S}_{12} + \bar{S}_{66}}{2\bar{S}_{11}}} \right]^{1/2} \tag{5.17}
\]

where \( \bar{S}_{ij} \) are elements of the transformed compliance tensor, \( [\bar{S}] \), which is defined as:

\[
[\bar{S}] = [T']^{-1}[S][T] \tag{5.18}
\]

and

\[
[T']^{-1} = \begin{bmatrix}
\cos^2 \theta & \sin^2 \theta & -\cos \theta \cdot \sin \theta \\
\sin^2 \theta & \cos^2 \theta & \cos \theta \cdot \sin \theta \\
2\cos \theta \cdot \sin \theta & -2\cos \theta \cdot \sin \theta & \cos^2 \theta - \sin^2 \theta
\end{bmatrix} \tag{5.19}
\]

Assuming orthotropic symmetry \([S]\) is \([5]\):

\[
[s] = \begin{bmatrix}
\frac{1}{E_1} & -\frac{v_{12}}{E_1} & 0 \\
-\frac{v_{12}}{E_1} & \frac{1}{E_2} & 0 \\
0 & 0 & \frac{1}{G_{12}}
\end{bmatrix} \tag{5.20}
\]

The angle \( \theta \) (Figure 5.1.3) is the fibre angle which is equal to 90° for a UD composite in which the fibre orientation is parallel to the notch direction of the CTS specimen.
5.2 Mechanistic Model

In the previous section a mathematical relation for the total value of mixed mode CSERR in terms of the pure mode I, pure mode II CSERR, and the loading angle was defined. In this section we aim at predicting the whole range of mixed mode behaviour of a UD composite based only on the properties of its constituents. In other words the goal here is to develop the model presented in Equation (5.1.15) into a new model in which the values of mode I and mode II CSERR, i.e. $G_{Ic}$ and $G_{IIc}$ of a composite can be determined in terms of only the properties of its constituents. To do so the mechanisms that are involved during fracture should be identified properly. This is done by looking at the fracture surface. The value of energy released during the creation of each of these mechanisms and their contribution to the work of fracture should be considered in the predictive model.

In order to develop a model to predict the value of energy release rate in terms of the energy released in the constituents, we recognize that mode I and mode II energy released in the composite are the summation of the toughness of each of the constituents represented on the fracture surface. i.e.:

$$G_{Ic} = G_{IC,I} \frac{A_i}{A_t} + G_{IC,m} \frac{A_m}{A_t} + G_{IC,f} \frac{A_f}{A_t} \quad (5.2.1)$$
where $A_m, A_i, A_f,$ and $A_t$ are the matrix, interface, fibre and total area of the fracture surface respectively. Subscripts $i, m,$ and $f$ are the value of toughness which indicates the toughness of interface, matrix and fibre respectively (Figure 5.2.1). Schematic fracture surface morphology used to drive the above relations is shown in Figure 5.2.1. The schematic fracture surface compared with the SEM image of fracture surface of a composite shows two main fracture mechanisms i.e. interfacial debonding and matrix fracture.

\[ G_{IIc} = G_{IIc,m} \frac{A_i}{A_t} + G_{IIc,m} \frac{A_m}{A_t} + G_{IIc,f} \frac{A_f}{A_t} \]  

(5.2.2)

The values of $A_m, A_i, A_f,$ can be estimated from the composite fibre volume fraction, or from the SEM images of the fracture surface. For example, when fibres are assumed to have a square array or hexagonal array inside the matrix (Figure 5.2.2), fibre spacing ($m$), can be found as follows respectively [5]:

\[ m = 2r \left\{ \frac{\pi}{4f} - 1 \right\} \]  

(5.2.3)
The value of $m$ is required to determine the area ratios. In the above relations, $r$ indicates the fibre radius, $m$ is the spacing between fibres, and $f$ is the fibre volume fraction as shown in the square arrayed fibres in Figure 5.2.2.

\[
m = 2r \left[ \left( \frac{\pi}{2f \sqrt{3}} \right)^{\frac{1}{2}} - 1 \right]
\]  

Figure 5.2.2 Fibre spacing ($m$) in square and hexagonal array of fibres [5].

The values of the interface and matrix area for each unit cell on the fracture surface can be found as:

\[
A_t = \pi. r. l \\
A_m = m. l
\]  

where $l$ is the fracture surface length as depicted in Figure 5.2.1 and Figure 5.2.2. It should be noticed that as the interfacial area has curvature, the summation of interfacial area and matrix area will be greater than the total area (i.e., $A_t + A_m \geq A_t$).
As in the UD composite in the material studied here, the crack opens at the notch tip and propagates in the direction of the fibres, so the fiber area fraction, $A_f$, will be zero.

So, Equations (5.2.1) and (5.2.2) simplify to:

$$G_{tc} = G_{tc,i} \frac{A_i}{A_t} + G_{tc,m} \frac{A_m}{A_t}$$  \hspace{1cm} (5.2.6)

$$G_{tii} = G_{tii,i} \frac{A_i}{A_t} + G_{tii,m} \frac{A_m}{A_t}$$  \hspace{1cm} (5.2.7)

where $\eta$ is a reinforcement reduction factor that relates plastic region in unreinforced resin to the reinforced one, and is discussed in more details in the next section.

As we know the total value of toughness in the composite is as follows:

$$G_c = G_{tc} + G_{tii}$$  \hspace{1cm} (5.2.8)

Now $G_{tc}$ and $G_{tii}$ can be substituted from Equations (5.2.6) and (5.2.7) into Equation (5.1.15) and (5.2.8) that give:

$$G_c = \frac{G_{tc,i} A_i}{A_t} \cos^2 \alpha + \frac{G_{tii,i} A_i}{A_t} \sin^2 \alpha + (G_{tc,m} \cos^2 \alpha + G_{tii,m} \sin^2 \alpha) \eta \frac{A_m}{A_t}$$  \hspace{1cm} (5.2.9)

The value of mode II toughness in resin is also shown to have the following relation with mode I toughness [7]:

$$G_{tii,m} = 2.4 G_{tc,m}$$  \hspace{1cm} (5.2.10)

So Equation (5.2.9) is changed into:

$$G_c = \frac{G_{tc,i} A_i}{A_t} \cos^2 \alpha + \frac{G_{tii,i} A_i}{A_t} \sin^2 \alpha + G_{tc,m} (\cos^2 \alpha + 2.4 \sin^2 \alpha) \eta \frac{A_m}{A_t}$$  \hspace{1cm} (5.2.11)

The value of mode I toughness of the matrix can be found directly from mode I testing of the matrix, and mode II interfacial toughness can be defined through experiments like fragmentation, pull-out and push-out test [115] and [116]. To establish the relation
between mode I and mode II interfacial toughness we have Equations (5.1.16) and (5.1.17).

Therefore:

\[
\frac{G_{IIc,i}}{G_{Ic,i}} = \frac{K_{IIc}^2 \bar{S}_{11}}{\sqrt{2}} = \frac{K_{Ic}^2}{K_{IIc}} \left( \frac{\bar{S}_{11}}{\sqrt{2}} \right) = \left( \frac{\tau_i}{\sigma_i} \right)^2 \left( \frac{\bar{S}_{11}}{S_{22}} \right)
\]

\[(5.2.12)\]

where \(\tau_i\) and \(\sigma_i\) are shear and normal interfacial strength of the interface. \(\bar{S}_{ij}\) are the components of transformed compliance matrix as defined in Chapter 2. Substituting Equation (5.2.12) in Equation (5.2.11) gives:

\[
G_c = \frac{G_{IIc,i} \sigma_i^2 A_i}{\tau_i^2} \left( \frac{\bar{S}_{11}}{S_{22}} \right) \cos^2 \alpha + \frac{G_{IIc,i} A_i}{A_t} \sin^2 \alpha + (G_{IC,m} \cos^2 \alpha \eta \frac{A_m}{A_t})
\]

\[(5.2.13)\]

The predictive model presented in Equation (5.2.13), is representing the mixed mode CSERR of the UD composite based on the properties of the matrix, the mode mixity that is defined by the angle of load with respect to the notch and interfacial properties.

Matrix fracture properties are discussed in Chapter 3. Interfacial properties can be found in the literature [115]. It should be noted that Equation (5.2.13) can give the value of interfacial toughness if the value of composite toughness \((G_c)\) is measured through the fracture testing of the UD composite as mentioned in Chapter 4 [117].

### 5.2.1 Reinforcement Reduction Factor

As it was mentioned for Equations (5.2.6) and (5.2.7), the amount of energy released in the matrix was reduced by the reinforcement reduction factor \((\eta)\). When the matrix is
fracturing, a major portion of energy dissipates in the material due to its plastic deformation before fracture. The amount of plastic deformation is however limited in the UD composite by the neighbouring fibres. This difference results in higher value of toughness in a neat resin compared to a reinforced resin. As a result in the case that the CSERR of epoxy is being directly used in the failure criterion (Equation (5.2.13)) that results in a higher value of $G_c$ compared to the experiments. So we scale the CSERR of the composite by the ratio of the material that is being plastically deformed.

The difference between plastic deformations in the above cases is the ratio of the plastic region in an unreinforced material to the average value of fibre spacing in the reinforced material. The reduction factor is therefore defined as: $\eta = \left( \frac{m}{r_p} \right)^2$ where $m$ is the fibre spacing as was shown in Figure 5.2.1, Figure 5.2.2, and Figure 5.2.3.

The plastic region at the crack tip has a plastic radius which is obtained using the following relation [19]:

$$r_p = \frac{1}{6\pi} \left( \frac{K_I}{\sigma_{y.p}} \right)^2 \quad (5.2.14)$$

which gives a radius of 200µm for epoxy.

In reinforced epoxy the plastic zone is limited to the fibre spacing. The value of the fibre spacing, $m$, assuming the fibre volume fraction of 43% (FVF=0.43) from Equation (5.2.3) is calculated to be 5.7 micron. The fibre spacing calculated from FVF is close to the value found from the SEM image (See Figure 5.2.3).
Figure 5.2.3 Glass fibre diameter is measured to be 17 mm which is used to calculate fibre spacing. The fibre spacing from fibre volume fraction is close to the value shown in the image. Radial pattern in fibre surface is due to brittle fracture.

If this value is compared to 200 μm that is calculated in Equation (5.2.14), it is found that the contribution of plastic deformation of epoxy in a reinforced composite is much smaller than that of neat epoxy. Therefore we consider a reduction factor in Equation (5.2.6) and (5.2.7). The value of η assuming 5.7 micron fibre spacing is calculated to be 0.0007.

5.3 The Criterion Predictions vs. Experimental Data

In this section model prediction is compared with the experimental results.
5.3.1 Neat Epoxy

As Equation (5.1.15) is a general equation that applies the free body diagram of the CTS specimen, it can be used for any type of material. First, the mixed mode CSERR for epoxy was predicted from Equation (5.1.15). The behaviour of the epoxy as an isotropic material is studied to make sure the criteria applies to isotropic materials.

The comparison between the results from Equation (5.1.15) and experimental data are shown in Figure 5.3.1 that shows a very good match between theoretical failure criterion and experimental results. In drawing the experimental results in Figure 5.3.1, five epoxy specimen for pure mode I and pure mode II, two epoxy specimen for loading angle of 30° and 15°, and one epoxy specimen for the loading angles of 60° and 45° are used.

![Graph showing comparison between theoretical failure criterion and experimental data for epoxy.](image)

**Figure 5.3.1 Comparison of CSERR predicted for epoxy by failure criterion (Equation (5.1.15) and experimental data from CTS epoxy specimen.**

As can be seen the general S pattern that was introduced in Figure 5.1.2, is also found in the material response. The average value of pure mode I toughness is 2.9 $\frac{kJ}{m^2}$ and the average value of pure mode II toughness is 7.05 $\frac{kJ}{m^2}$, which results in the ratio of 2.43.
mode II to mode I toughness. This ratio of 2.43 is to confirm with was predicted by Equation (5.2.10).

Error bars show the values of standard error or standard deviation of the mean that is determined form the following relation:

$$\sigma_{mean} = \frac{1}{\sqrt{N}} \sigma$$  \hspace{1cm} (5.3.1)

in which $N$ is the number of data and $\sigma$ is standard deviation and is defined as:

$$\sigma = \sqrt{\frac{1}{N} [(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_N - \bar{x})^2]}$$  \hspace{1cm} (5.3.2)

where $\bar{x} = \frac{1}{N} (x_1 + \ldots + x_N)$

The values of $x_i$ are CSERR measured for each angle from CTS specimen.

### 5.3.2 UD Composite (fibre and loading angle effects)

The next step is validating failure criterion in Equation (5.1.15) and mechanistic criterion from Equation (5.2.13) with the results from the UD composite specimen.

In this section the results of two sets of experiments are compared with the criterion experimental data. The first is when the fibre angle ($\theta$) is changing, and the second case is when the fibre angle remains parallel to the specimen notch and the loading angle ($\alpha$) is changing.

#### 5.3.2.1 Fibre Angle Effect

When the fibre angle ($\theta$) is changing from $90^\circ$ to $0^\circ$ (see Figure 4.2.4) and the loading angle remains in $\alpha=0$, different modes of loading from pure mode I to pure mode II is applied at the crack. The mixed mode loading is applied when the fibre angle is changing and the loading angle remains the same. Here, we determine if the criterion can be used to include the effect of the fibre angle. In the previous section of Chapter 4 we presented
the results of testing of such specimens. While the loading direction remained in pure mode I the fibre angle (θ) changed from 0° to 90° with respect to the specimen notch (see Figure 4.5.1 and Figure 4.5.2).

The results from testing of both forms of the UD specimens are shown in Figure 5.3.2. The results of specimens with constant loading direction and changing fibre orientation are shown by diamonds (indicated as $\theta = (0^\circ, 90^\circ), \alpha = 0^\circ$ in the legend). The experimental data for specimens with changing loading direction and constant fibre angle are shown by circles. (indicated as $\alpha = (0^\circ, 90^\circ), \theta = 0^\circ$ in the figure legend). As can be expected there is one common point in both series of tests (i.e. when $\alpha = 0^\circ$ and $\theta = 90^\circ$). In this point similar results are obtained (these values are show inside circles in the figure).

![Graph showing effective fracture toughness results]({attachment:image.png})

**Figure 5.3.2** Comparison of effective fracture toughness results when a) fibre angle is fixed and loading angle changing with b) specimens having fixed loading angle and changing fibre angle. The circles show similar condition and results from both formats of testing.

As can be found from Figure 5.3.2, by decreasing the fibre angle towards 0° (i.e. fibres aligned in axial direction and perpendicular to the notch) the effective fracture toughness
values measured from the test rise significantly. It is evident that by decreasing the fibre angle there is a higher difference between the criterion predictions and the experimental results of the CTS specimen. This significant increase in the value of $k$ (shown as hollow squares in the figure) that is calculated from the fracture load is due to the resistance of the fibres in the direction of the loading. As an example, compare the case of a specimen with $\alpha = 0^\circ$ and $\theta = 0^\circ$ (case 1 as shown in Figure 5.3.3a) and a specimen with $\alpha = 90^\circ$ and $\theta = 90^\circ$ (case 2 as shown in Figure 5.3.3b).

For case 1, we find that as the applied load is in the direction of the fibres it shears the fibres apart from each other (forces at bolts A and B are shearing the fibres against forces at bolts E and F). So the specimen is experiencing mode II loading in the fibres close to the notch and it can be concluded that the mode of loading for both specimen in the figure should be the same. By looking at the results of these tests from Figure 5.3.2 we note that there is a significant difference between the results of both specimens. While $K_{eff} \approx 21 \ MPa\sqrt{m}$ for the first specimen the average $K_{eff}$ for the second specimen (case 2) is close to $4 \ MPa\sqrt{m}$. While the forces at bolts A and B cause shear in the fibres, forces at bolts C and F are pulling fibres along their length. Therefore, the force that is measured during this series of tests is much higher in bolts C and F compared to bolts A and B (or D and F). This is because fibres show highest resistance along their length against external load. The condition in bolts C and F is very similar to the longitudinal tensile testing of a UD composite that results in a very high force. Therefore, these forces should be removed to get to a similar condition to case of Figure 5.3.3b. To reach results similar for case 2, the force should only be applied to the cracked area. So it is suggested that the CTS specimen is only fixed at points A and B to the fixture, and the maximum load is measured to calculate the fracture toughness.

In summary, the criterion is able to predict the fracture response of the UD composite if the appropriate test settings are set up for the composite.
5.3.2.2 Loading Angle Effect

Here we are interested to see if the criterion prediction is comparable with the CTS specimen results when the fibre angle is parallel to the notch and the loading angle is changing from $0^\circ$ (pure mode I loading) to $90^\circ$ (pure mode II loading). The results of the CSERR of testing of the CTS specimen with changing the loading angle are compared with the criterion prediction (Equation (5.2.13) in Figure 5.3.4).

As indicated in Figure 5.3.4, both failure criterion (Equation (5.1.15) and the mechanistic failure criterion accurately predict the CSERR for the UD composite. Both predictive failure models give very close results; with the difference between both models as less than 2%.
Figure 5.3.4 Comparison of the model prediction and the experiment for 90° UD composite under different modes of loading.

The error bars for the experimental data are calculated using Equation (5.3.1). From the resulting comparison it is obvious that the model is able to successfully predict the value of energy release rate in a composite.

The discrepancy of experimental results, especially for loading angles of 45°, 75°, and 90°, can be due to variability in experimental parameters including differences in fibre orientation and fibre volume fraction during the hand lay-up of composites. Another reason for higher value of error bars for the loading angle of 45° was that there were only two specimens that were tested in this direction. To draw the experimental data in Figure 5.3.4, seven specimens for pure mode I, five specimens for mode II, four specimens for loading angle of 75°, and three specimens for the rest of loading angles were used. It was also observed that the continuous polymer threads that hold the UD fibres aligned together can influence the test results (Figure 4.1.1 and Figure 5.3.4).

Apart from the above mentioned parameters the existence of periodic polymer threads that hold the fibre together results in a discrepancy in the fracture response of CTS specimens (See Figure 5.3.5).
Figure 5.3.5 Periodic polymer threads make changes in the value of fracture work measured from UD composite. These threads have an average diameter of 24 µm compared to fibre glass with 17 µm diameter.

Influence of interfacial debonding CSERR

The influence of the interfacial debonding CSERR on the predictions by the mechanistic criterion (Equation (5.2.13), can be determined as shown in Figure 5.3.6. The red line (solid line) shows the predictions of the total toughness using Equation (5.2.13) considering the effect of interfacial toughness and the blue line (dashed line) shows the predictions of the total toughness without considering the effect of interfacial toughness (i.e. $G_{ic} = 0$). The comparison of both predictions shows average decrease of 27% in the prediction of the total toughness.
Figure 5.3.6 The influence of the interfacial toughness on the total toughness of UD glass epoxy composite.
Chapter 6

6 Random Fibre Composite

In the previous chapter, a mechanistic failure model was presented. The model predicts the value of CSERR of UD fibre composites. UD fibre composites, however, comprise only a portion of composites manufactured in industries. As an example, high-volume manufacturing processes (e.g. those suitable for automotive applications) based on compression molding techniques typically result in planar random arrays of discontinuous fibres. Thus, an advancing through-thickness crack will meet transverse fibres at a range of angles between 0° and 90°. In this chapter, the propagation of a through-thickness crack in a random fibre composite was studied as a function of the loading angle with respect to the macroscopic initial crack direction, using the CTS test geometry in order to assess the associated crack propagation energy. The failure criteria introduced in Chapter 5 for a UD composite is developed here to predict the value of energy absorbed in random fibre composites. The random fibre composite assumes an even distribution of fibres in various angles.

6.1 Mechanistic Failure Model

Mass production composites usually use short fibres in random directions. Short fibre composites are cheaper to produce, however, they show lower stiffness and strength compared to UD composites [118].

The energy based failure model proposed in Chapter 5 (Equation 5.2.13) is developed here to be applicable for random composites. The model predicts the value of the CSERR of a random polymer composite based on the fracture properties of its constituents and their interfacial properties. To establish a mechanistic model similar to UD composites, we need to look carefully at the fracture surface of random composites and relate the amount of energy released during fracture to the fracture mechanisms represented on the fracture surface.
As shown in Figure 6.1.1, fibre pull-out is the major failure mechanism that can be easily observed using SEM images, and other mechanisms such as fibre and matrix fracture and fibre/matrix debonding can also be found on the fracture surface.

Fracture can result from different microscopic failure mechanisms related to fibres, matrix, and interface, but the most important mechanism that absorbs energy in random PMCs is related to fibres that are pulled out [5], [43], [119]–[121]. Therefore, it is important to incorporate the value of energy released during fibre pull-out into the mechanistic model proposed in Chapter 5. Other parameters can also affect the amount of energy released during fracture of random fibre composites. As an example, Martson and coworkers reported that the amount of toughness is also dependent on volume fraction, diameter and Young's modulus of fibres [119].

The crack propagation path in random fibres depends on the local orientation of fibres where the crack advances to the fibres. In this case, the fibres (taking fibre direction with regard to the crack direction fibres) are divided into two general categories of the L and T type. In T oriented fibres, where the fibres are transverse or at an angle close to 90° to the crack, the crack forms a jagged pattern where cracks propagate on the fibre interface and the end of the fibres. After deflection of cracks to the fibres’ interfaces, they pull-out from their place, resulting in high toughness (see fibres pull-out on the fracture surface in Figure 6.1.1 and Figure 6.1.2). The mechanism of the fibre/matrix debond and fibre pull-out for fibres in the transverse direction is defined using the Cook-Gordon model which was mentioned in Chapter 4 [18]. In L oriented fibres as shown in Figure 6.1.2, in which fibres are parallel or at an angle close to 0° to the crack plane, the composite fractures in straight crack patterns that propagate mostly between the fibre matrix interface [103], [122].
Figure 6.1.1 a) Fibre pull-out is the major fracture mechanism on the fracture surface. b) Failure mechanisms on the fracture surface A: fibre imprints on epoxy due to fibre/matrix debonding, leaving smooth channel that shows weak bond between epoxy and glass fibre. B: matrix fracture, C: fibre pull-out
In order to derive an analytical equation as a predictive model for a random short fibre composite, the basis of the predictive model in Chapter 5 will be used. Here, the pull-out work will be added to the predictive model used for UD composites. Taking pull-out work into consideration, the total value of the CSERR of a random fibre composite can be written as:

$$G_c = G_{\text{resin}} + G_{\text{interface}} + G_{\text{fibre}} + G_{\text{pull-out}}$$  \hspace{1cm} (6.1.1)$$

Total value of the CSERR is the summation of four different failure mechanisms. First, the amount of energy that is stored by the fracture of resin $G_{\text{resin}}$ is similar to what was discussed in Chapter 3 and 5. Here, a different amount of energy is released by the mode I fracture (cleavage) of epoxy and mode II fracture of epoxy due to the formation of hackles as shown in Figure 3.5.6.
The second term, $G_{\text{interface}}$, is the amount of energy related to the interfacial debonding and is similar to what was mentioned in Chapter 5 with a difference in the determination of the value of the interfacial debond area. The value of the interfacial debond area which is altered due to the change in the debond path and the fibre surface experiencing debonding will be discussed later in this Chapter. The third term, $G_{\text{fibre}}$, is the amount of energy released during the fracture of fibres. Ceramic fibres that are usually used for different PMCs have low values of toughness, so the amount of energy released during their fracture is negligible compared to the other terms in Equation (6.1.1) [5]. However, we leave this term in the equation to study its effect when a different type of fibre is used in the composite. The last term in the above equation is the amount of energy released during fibre pull-out ($G_{\text{pull-out}}$).

6.1.1 Pull-Out Energy Release Rate

To determine the value of the amount of energy released during fibre pull-out, we assume a fibre perpendicular to the advancing crack (see Figure 6.1.3). As the crack advances, the fibre debonds from its surroundings due to the Cook-Gordon effect [18]. The value of the stress at this point should be larger than interfacial shear strength to cause the fibre/matrix debonding. If the fibre length is greater than its critical length, by further straining of the composite it may reach its strength and break into smaller parts. As the stress in the broken fibre does not build up to the fibre strength when it is shorter than critical length, the fibre does not experience any further breaking. The broken fibre then starts to pull-out from its socket (Figure 6.1.3). If the fibre has a remaining length of $x$ embedded in the matrix which is pulled out an increment of distance $dx$, the amount of work during the pull-out of the fibre for $dx$ is stated as:

$$du = 2\pi r x \tau_1^i \, dx$$  \hspace{1cm} (6.1.2)

where $r$ is the average radius of the fibres, $\tau_1^i$ is the interfacial shear stress, which is assumed to be constant along the length of the length of the fibre.
Figure 6.1.3 Schematic of a crack advancing and passing through short fibre composite, the fibre debonds from the matrix and then pulls out from its socket.

If the average value of the pulled-out length is equal to $l_{po}$, the amount of work done during the complete pull-out of this fibre then will be:

$$
\Delta U = \int_0^{l_{po}} 2\pi r x \tau_i^* dx = \pi r \tau_i^* l_{po}^2
$$

(6.1.3)

The total amount of work done on the pull-out of $N$ fibres can be found as:

$$
U = N. \pi r \tau_i^* l_{po}^2
$$

(6.1.4)

Where the number the fibre volume fraction on the fracture surface influences $N$. If the total area of the fracture cross section is assumed to be $A$, then the number $N$ can be stated in terms of fibre volume fraction as [5]:

...
\[ f = \frac{N \pi r^2}{A} \rightarrow N = \frac{4fA}{\pi d^2} \tag{6.1.5} \]

where \( f \) is the fraction of fibres on the fracture surface experiencing pull-out. As there are certain fibre angles that only experience interfacial debonding not fibre pull-out, the proper value of \( f \) is smaller than the average fibre volume fraction in the composite (\( \nu_f \)). For fibres angle close to 0° with the crack direction, the crack passes through their interfacial bonding and does not pull them out of their surrounding matrix.

From Equations (6.1.4) and (6.1.5), the total pull-out work is:

\[ U = \frac{fA \tau_i^* l_{PO}^2}{r} \tag{6.1.6} \]

The amount of energy released in the crack surface area can be found as:

\[ G = \frac{\partial U}{\partial A} = \frac{f \tau_i^* l_{PO}^2}{r} = \frac{2f \tau_i^* l_{PO}^2}{d} \tag{6.1.7} \]

where \( d \) is the fibre diameter, and \( A \) is the fractured area and \( \partial A \) indicates the change in the fractured area due to the increment of crack propagation. Equation (6.1.7) requires knowledge of the average pull-out (\( l_{PO} \)), which is required to be determined from the fracture surface of each type of random fibre composite. A similar relation for the pull-out energy was derived by Cottrell and Kelly [43], [123], [124]. Therefore, most literature introduced the pull-out CSERR that is based on the critical length and not the pull-out length [57], [121], [125]. In this study, it will be discussed in the development of a finite element model that the value of pull-out length can be determined from it, so the above equations will be used in the determination of the CSERR of short fibre composites. The value of the pull-out length can be found by observing the fracture surface using SEM or stereomicroscopy. As the pull-out length in the composite varies between 0 and \( \frac{l_d}{2} \) where \( l_d \) is the average debond length of the fibre, the mean value of \( l_{PO} \) can be determined as:
This assumption is valid for an even distribution of pull-out length in the fractured surface. In random fibre composites manufactured in the industry, such as sheet molding compounds the relation between the pull-out length and debond length can be found by looking at the fracture surface and measuring the pull-out length.

Therefore, with the above assumption, the value of energy released by the pulling out of fibres using Equation (6.1.7) can be rewritten as:

\[
l_{PO} = \frac{l_d}{4}
\]

(6.1.8)

6.1.2 Effect of Mode of Loading on Fibre Pull-Out

Due to the effect of the pull-out energy in the value of total CSERR of a random composite under mixed mode loading it is important to understand whether the pull-out energy is the same for all modes of loading or not. Wang and coworkers showed that there is a significant decrease (approximately 50%) in the value of mode II fracture toughness (measured using Iosipescue specimen) compared to the mode I fracture toughness. They carried out fracture tests on a SMC composite [126]. They addressed this decrease as the change of failure mechanism from “fibre controlled” mode I to mode II that is less controlled by fibres. Similar results were observed by Hoffman and coworkers that measured a 35% decrease in the value of fracture toughness from mode I to mode II in chopped E-glass mat/polyurethane composite [127]. The effect of changing the angle between the loading direction and the fibre orientation was also studied by Fu and coworkers [121]. Therefore, here we are going to introduce a factor that is a function of the mode of loading.

The idea of the effect of the mode of loading on the amount of pull-out work as the main source of energy released during fracture can be explained with the help of Figure 6.1.4.
We assume a group of fibres are perpendicular to the crack plane as shown in Figure 6.1.4. Under mode I loading when the crack advances to a group of fibres, the group of fibres are pulled out perpendicular to the crack direction experiencing the pull-out. Under mode II loading, the fibres might break under shear stress before they completely pull-out from their socket. Therefore, apart from the dependency of the pull-out work on the fibre volume fraction, interfacial shear strength, fibre dimension and debond length, the pull-out work is also a function of the loading angle. So Equation (6.1.9) will be modified as:

\[ G_{pull-out} = \frac{f \tau_1^* l_i^2}{8d} \times g(\alpha) \]  

(6.1.10)

Function \( g(\alpha) \) is the loading mode function, \( d \) is the diameter of the fibre and \( \tau_1^* \) is the fibre/matrix interfacial shear strength.
Figure 6.1.4 Schematic of a fibre pulling out from its surrounding matrix due to mode I and mode II loading. The comparison of fibre under mode I (left) and mode II (right), suggests higher possibility of fibre pull-out under mode I loading compared to mode II loading. Fibres have more chance of shearing without pull-out under mode II (right picture).

6.1.3 Total CSERR of Random Composite

Now at this stage we find a closed form equation for the determination of the total CSERR of the composite. By adding the value of the CSERR calculated from Equation (6.1.10) to Equation 5.2.13 from Chapter 5, we have:
Where $A_i$ is the interfacial debonding area that is different for the random fibre composite from a UD composite. The pull-out term is as introduced in the last section, and the rest of the terms are as discussed in Chapter 5.

Equation (6.1.11) gives the total value of the CSERR of a random composite in terms of fibre, matrix and their interfacial properties. In Equation (6.1.11) the first two terms define the amount of energy released during interfacial debonding, the third term is the amount of energy released due to matrix fracture, and the final term defines the amount of pull-out energy. Assuming an even distribution of fibres in each layer of the random composite, transverse isotropic behaviour of composite can be obtained. Therefore, $\frac{S_{11}}{S_{22}} = 1$ and Equation (6.1.11) can be simplified to:

\[
G_c = \frac{G_{IIc,i}\sigma_i^2 A_i}{\tau_i^2 A_t} \cos^2 \alpha + \frac{G_{IIc,i}A_i}{A_t} \sin^2 \alpha + \left(G_{Ic,m} \cos^2 \alpha + 2.4G_{Ic,m} \sin^2 \alpha\right)\eta \frac{A_m}{A_t} + G_{fibre} + \frac{f_i l_i^2}{8d} \times g(\alpha) \tag{6.1.12}
\]

An expression for the loading mode function $g(\alpha)$, will be determined in Section 6.5. The value of debond length can be determined from experiment or FE simulation that will be introduced in the next section. What remains now is to determine the interfacial debonding area.
6.1.4 Determination of Interfacial Debonding Area, $A_i$

It should be noted that the first two terms in Equation (6.1.12) that define the interfacial debonding energy are very similar to the case of a UD composite, but the determination of interfacial debonding area, i.e. $A_i$, is different from the one determined in Chapter 5 for a UD composite.

Figure 6.1.5 Schematic of interfacial debonding due to the crack advancing in a) UD composite and b) Random fibre composite. The interfacial debonding in random short fibre composite is a longer path compared to UD composite. $\gamma_i$ is chosen between $0^\circ$ and $90^\circ$. 
In the determination of the total interfacial area there are two terms involved, first the average perimeter of the fibre debonded, and the second is the debond length. The following subsections define how each term can be calculated for a random fibre composite.

### 6.1.4.1 Debonding Length

The first step is to calculate the debonding area is finding the debond length. The length can be found if the ratio of the debond length for random composite to the debond length for a UD composite is known. To determine a relation between interfacial debond length in the fracture of a UD CTS specimen and a random short fibre CTS specimen, we assume that the crack is advancing through the CTS specimen as shown schematically in Figure 6.1.5. From the figure it can be found that the interfacial path along the length of a short fibre composite is longer than its corresponding length in a UD composite.

Using Figure 6.1.5.a in a UD composite, the debond length of the fibre is:

\[ l_{d,UD} = L \]  \hspace{1cm} (6.1.13)

where \( L \) is the average length of fibres in the ligament of the CTS specimen.

For random short fibre composite if each fibre makes an angle of \( \gamma_i \) with the direction of the CTS notch as shown in Figure 6.1.5.b the debond length is calculated as:

\[ l_d = \frac{l_1}{\cos \gamma_1} + \frac{l_2}{\cos \gamma_2} + \frac{l_3}{\cos \gamma_3} + \ldots + \frac{l_N}{\cos \gamma_N} \]

Or

\[ l_d = \sum_{i=1}^{N} \frac{l_i}{\cos \gamma_i} \]  \hspace{1cm} (6.1.14)

In which \( l_i \) \((i = 1,2,\ldots,N)\) is the projected length of each fibre in the direction of the notch, for angles of \( \gamma \) close to 90° the crack passes around the fibre, and the amount of
debond length is equal to the fibre diameter which can be neglected compared to the
debond length of fibres at a smaller angle with the crack. For example, the debond length
for a 0° fibre is equal to its length which is almost 40 to 50 times greater than the fibre
diameter. As the cosine value of γ<sub>i</sub> angles is used in the calculation, the Angle γ<sub>i</sub> is
chosen between 0° and 90° to give a positive value for cos γ<sub>i</sub>. It should be noted that this
assumption will be improved to match the actual results after introducing the finite
element model.

For an even distribution of fibre length in the random composite we have:

\[
\bar{l} = l_1 = l_2 = l_3 = l_N \to l_d = \bar{l} \sum_{i=1}^{N} \frac{1}{\cos \gamma_i} \tag{6.1.15}
\]

For an even distribution of fibre orientation we can divide orientations into 4 parts with a
span of 22.5 degrees and choose the medium value of each division as its mean value,
then we have:

\[
\gamma_1 = [(0° - 22.5°)] = 11.25
\]

\[
\gamma_2 = [22.5° - 45°] = 33.75 \tag{6.1.16}
\]

\[
\gamma_3 = [45° - 67.5°] = 56.25
\]

\[
\gamma_4 = [67.5° - 90°] = 78.75
\]

Four parts were chosen to ease the explanation of the model here, a better result can be
found using a larger number of fibre orientation divisions. This will be calculated after
finding a value for the debond length for a short fibre composite using a FE model.

Substituting Equation (6.1.16) and (6.1.15) into (6.1.14) gives:

\[
l_d = \frac{l_1}{\cos 11.25} + \frac{l_1}{\cos 33.75} + \frac{l_1}{\cos 56.25} + \frac{l_1}{\cos 78.75} = 9l_1 \tag{6.1.17}
\]
To compare the average debond length of a random composite estimated in Equation (6.1.17) with the debond length in a UD composite we assume a similar situation in a UD composite with 4 fibres aligned in the same direction with an angle of 0°. In other words, the debond length for a UD composite is $4l_1$.

Therefore,

$$\frac{l_{d,\text{random}}}{l_{d,UD}} \approx \frac{9l_1}{4l_1} = 2.25 \quad (6.1.18)$$

Equation (6.1.18) indicates that the average interfacial area in a random composite is 2.25 times greater than this value for UD composites. Therefore, the amount of energy released during the interfacial debonding in random fibre composite is 2.25 times greater than its corresponding value for UD composites.

As it was mentioned in the above paragraphs, the results of Equations (6.1.14) and (6.1.15) will be more precise if the value of the debond length for each fibre orientation is determined. In Sections 6.3.4 and 6.5.2 more precise predictions of the interfacial debonding area will be presented after the debond length for each fibre orientation is calculated.

### 6.1.4.2 Debonding Perimeter

The second step towards finding the value of interfacial debonding area is to determine whether half or full fibre perimeter is experiencing debonding. Therefore, we are defining two major failure mechanisms regarding the fibre matrix interface and we are going to treat them separately. Using fractography (see Figure 6.1.2), it can be found when the crack approaches to the fibre, depending on the fibre angle ($\theta$), it delaminate the fibre from the surrounding matrix (location B in the figure) or it goes around the fibre (location A). If the fibre-crack angle is close to zero degree the crack separates half of the fibre cylindrical area from the matrix as shown schematically in Figure 6.1.6. We call this type of debonding delamination. The delamination mechanism explained here is similar to the interfacial debonding for the UD composite that was discussed in Chapter 5.
When the crack meets the fibre at steeper angle it debonds the fibre from matrix and goes around the matrix, leaving full surface of the fibre debonded from matrix. Because the debonding process in the second case is similar to the Cook-Gordon model, we call this type of debonding the Cook-Gordon debonding. From Figure 6.1.6 it can be found that the Cook-Gordon interfacial debonding area is twice the area of delaminated fibres.

It is clear that there will be a certain angle that the mechanism switches from the delamination to the Cook-Gordon. We call this the transition angle $\theta_{\text{transition}}$, so if $\theta \geq \theta_{\text{transition}}$, the fibre debonding is mostly due to the delamination mechanism and if $\theta \leq \theta_{\text{transition}}$ then the dominant debonding mechanism is Cook-Gordon mechanism.

In conclusion, for delaminated debonding, the interfacial debonding area is defined similar to a UD composite as:

$$A_i = \pi rl_d \ (\text{for } \theta_{\text{transition}} \leq \theta \leq 90^\circ) \tag{6.1.19}$$

In Equation (6.1.19), $r$ is the radius of fibre and $l_d$ is the debonding length that was introduced in the previous subsection. For the Cook-Gordon debonding mechanism, the interfacial debonding area around the fibre is defined as:

$$A_i = 2\pi rl_d \ (\text{for } 0^\circ \leq \theta \leq \theta_{\text{transition}}) \tag{6.1.20}$$

The other important effect of the transition angle is its effect on the amount of energy released during the pull-out. This is because the angle changes the fraction of the fibres that are experiencing pull-out. If we look at Equation (6.1.10) we notice that $G_{\text{pull-out}}$ is directly related to the fibre fraction ($f$). The range of the transition angle will be discussed in the results section.
6.2 Experimental Methods and Results

After introducing the failure criterion for mixed mode loading of a random composite, experimental data is measured. The experimental results validate the accuracy of the model presented in Equation (6.1.12).

Figure 6.1.6 a) Delamination debonding b) Cook-Gordon debonding.
6.2.1 Tensile Testing and Results

E-glass chopped mat and epoxy (CLR1180/CLH6560 manufactured by Crosslink Tech. Inc.) were utilized for making the composite. Composite manufacturing and testing was described in Chapter 4. Straight specimens were utilized to do tensile testing according to ASTM 3039 [108]. Random composite tensile coupons before and after fracture testing are shown in Figure 6.2.1.

![Figure 6.2.1 Straight random composite specimens before and after performing tensile testing according to ASTM D3039.](image)

The stress-strain curves and values of Young’s modulus resulting from tensile testing of composite specimens are shown in Figure 6.2.2.
6.2.2 Fracture Testing of Random Composite and Results

For this study, a random strand mat made from E-glass fibres and epoxy was utilized. The specimen preparation and testing procedure were explained in Chapter 4. Specimen dimensions are the same as the UD CTS specimens, i.e. 120 mm x 76 mm with thickness between 7 mm-8 mm.

Figure 6.2.2 Comparison of stress-strain curve (top) and Young’s modulus (bottom) for random glass fibre epoxy.
Figure 6.2.3 shows a CTS specimen made of random fibre composite under different modes of loading. The crack growth direction in the random composite is similar to isotropic material. As can be seen in e

Figure 6.2.3, unlike the UD composite the crack propagation angle depends on the mode of loading and is very similar to what was observed in epoxy specimens.
Figure 6.2.3 CTS specimen made from random fibre epoxy under different modes of loading in the beginning and after end of testing. a) 0° pure mode I, b) 15°, c) 45°, d) 60°, e) pure mode II loading 90°
The similarity in the crack growth direction between random composite and isotropic material comes from the fact that the even distribution of fibres in different angles reduces the degree of anisotropy of the material. The even distribution of fibre orientation can also be found by providing statistical analysis on the fibre direction in each mat. To determine fibre orientation, optical microscopy images were used to do image analysis (see Figure 6.2.4).

Figure 6.2.4 Optical microscopy image of a chopped-strand mat. Strand mat image used for image analysis to determine fibre angle distribution.

The results of fibre orientation distribution are shown in Figure 6.2.5. A total of 41 fibre orientations were used in the calculation. The distribution of fibres shows 5% divergence from 100% even distribution in the isotropic material.
Figure 6.2.5 Fibre orientation distribution found from the chopped-strand mat.

The values of the critical load captured during fracture testing of the CTS random composite specimens were used to determine the values of fracture toughness and the CSERR of the composite.

The results of fracture toughness and CSERR for different modes of loading are shown in Figure 6.2.6. The values of mode I and mode II components of fracture toughness under different loading angles are obtained from Equations (3.2.1) and (3.2.2) in Chapter 3.

Unlike the UD composites, the values of the CSERR and fracture toughness for a random composite decreases by changing the mode of loading from pure mode I to pure mode II. This is due to the higher value of pull-out work under mode I compared to mixed mode and mode II loading.
Figure 6.2.6 Top: Plot of Fracture toughness ($K_I, K_{II}, K_{eff}$) vs loading angle, Bottom: CSERR vs loading angle of random composite using CTS fracture testing.

By comparing the values of the CSERR for pure mode I and pure mode II, an average of 33% decrease in the CSERR can be found (i.e. $\frac{g_{II}}{g_I} = 0.67$). This transition can be used for determining an expression for $g(\alpha)$ in Equation (6.1.12). This will be discussed in the failure criteria.
6.3 Finite Element Simulation

The FE model presented here will be used to determine the value of pull-out and the debond length. The pull-out or debond length will be used to calculate the amount of energy released by the fibre pull-out as stated in Equation (6.1.7) and (6.1.9). In the first step, the material model used in the simulation is validated. FE simulations were carried out by LSDYNA software [128]. Then, a CTS epoxy specimen is modeled using LSDYNA. To validate the fracture model, it’s stress-strain result is compared with experimental results. The model is then developed to determine the value of the pull-out length.

6.3.1 Mesh Convergence

A mesh sensitivity study was done on two different geometries. First, a rectangular epoxy block was simulated and the result of von Mises stress versus mesh size was studied. The geometry uses 8-node solid brick elements to simulate an epoxy specimen under the simple tension test (see Figure 6.3.1), with the dimensions as 10 mm x2 mm x0.2 mm. The value of von Mises stress is calculated when the block is stretched 5%. Material type 24 (MAT_PICEWISE-LINEAR-PLASTICITY) is used as the material model for the epoxy, and this material type is chosen because any elasto-plastic material that has arbitrary stress versus strain values can be defined.

Figure 6.3.1 Geometric model for finite element simulations with 5120 elements.
Figure 6.3.2 shows the results of the mesh sensitivity study, in which von-Mises stress for models in terms of the number of elements that are given. The results in Figure 6.3.2 show the convergence at approximately 700 elements for tensile loading. The results consider 2, 80, 640, 2000 and 5120 as the number of elements. The number of elements for the simulation of this geometry is then chosen to be 700 elements.

![Figure 6.3.2 The effective stress (Von-Mises stress) as a function of the number of elements for uni-axial loading condition determined from FE simulations.](image)

The next step was to simulate a CTS specimen and study mesh sensitivity on this geometry. The geometry was modeled by extruding a shell plane to the required thickness in LSDYNA. The geometry dimension is the same as shown in Figure 5.1.3 (120 mm x 76 mm x 4 mm). The notch length is 38 mm with a U-shape end having a diameter of 2.5 mm. The 3D model is shown in Figure 6.3.3.
A fine mesh was used in the crack tip area and a coarser mesh was created for the far-field regions. The bolt-holes had finer mesh compared to the far-field areas. The model shown in Figure 6.3.3 has 3168 elements. The ratio of the notch tip element size to the notch length was fixed at $6 \times 10^{-3}$ to make sure it captures the right value for stress at the tip of the notch. A similar ratio can be found elsewhere [129]–[133]. A comparison of effective stress of the CTS model with different mesh sizes in Figure 6.3.4 suggests that for the range of elements sampled, the effective stress is insensitive to the mesh size. The effective stress was measured in nodes away from the crack tip. A model with 3168 elements was chosen for the simulation from Figure 6.3.4.
Figure 6.3.4 Effective stress (von-Mises stress) as a function of the number of elements for mode I loading condition determined from FE simulations. The stresses were measured in nodes away from the crack tip.

### 6.3.2 Material Model Validation

Material type 24 (MAT_PICEWISE-LINEAR-PLASTICITY) and material type 1 (MAT-ELASTIC) were used to model epoxy and the glass-fibre, respectively. Material type 24 requires the density, yield stress, elastic modulus, Poisson’s ratio, effective plastic strains and their corresponding yield stress as input parameters for the epoxy. Effective plastic strain and yield stress values are found from the simple tension test of the epoxy specimen as previously described in Chapter 3. Material type requires the density, elastic modulus, and Poisson’s ratio that are found from CES software [1]. These values for both materials are given in Table 6.3.1.
Table 6.3.1 Mechanical properties of epoxy and glass fibre used in the material model.

<table>
<thead>
<tr>
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<th>Epoxy</th>
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<th>E-Glass</th>
</tr>
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<tbody>
<tr>
<td>Density $\left(\frac{kg}{m^3}\right)$</td>
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<td>Density $\left(\frac{kg}{m^3}\right)$</td>
<td>2500</td>
</tr>
<tr>
<td>elastic modulus (GPa)</td>
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<td>Elastic modulus (GPa)</td>
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</tr>
<tr>
<td>Poisson’s ratio</td>
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<td>Poisson’s ratio</td>
<td>0.23</td>
</tr>
<tr>
<td>yield stress (MPa)</td>
<td>30.83</td>
<td>Corresponding yield stress (MPa)</td>
<td></td>
</tr>
<tr>
<td>effective plastic strain value</td>
<td></td>
<td>5.7900E-5</td>
<td>40</td>
</tr>
<tr>
<td>0.001180</td>
<td>50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.003694</td>
<td>60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.004716</td>
<td>62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.006561</td>
<td>64</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.010492</td>
<td>64</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 6.3.5 shows the uniaxial tensile curve of the FE simulation compared with the typical experimental tensile curve from the epoxy specimen. The prediction from the linear elastic model for glass is also identical with the experimental data in the literature.
The FE prediction and experimental results in Figure 6.3.5 show a maximum of 4% difference. Therefore, the material type 24 model can precisely predict the behaviour of the epoxy.

### 6.3.3 Finite Element Model Development

Macro-mechanical FE models have been developed to simulate the fracture behaviour of random fibre composites using LS-DYNA. The use of the model is to provide the stress pattern at the tip of the crack and find how the crack advances when it meets a fibre with random orientation. It is also used to determine the value of the debond length to calculate the pull-out energy. Therefore, a single fibre passing near the crack in a neat epoxy CTS specimen is included in the model. The model is under mode I loading. Fibre-glass orientation varies with regard to loading direction from 0° to 90°, in 15° increments. Both 2D and 3D models are used here. The 3D model has approximately 12500 8-node quadratic solid elements with the mesh pattern shown in Figure 6.3.6. The 2D model has 3100 plane strain shell elements within 4 nodal points. The number of elements varies by
changes in the fibre orientation. Due to the stress intensity at the tip of the crack, the areas close to the notch tip and along the direction of notch propagation have finer mesh. The minimum element size is 0.06 mm for the model.

The models are subjected to the mode I loading, which is defined by the nodal displacement at one end of the model. The geometry, boundary condition and application of the displacement are shown in Figure 6.3.6 and is explained in the next section. The Inset boxes A and B in the figure are shown in more details in Figure 6.3.7.

Figure 6.3.6 FE model for the CTS specimen with a fibre (shown by a white arrow) parallel to loading direction ($\theta=90^\circ$). The specimen is subjected to pure mode I loading with the lower bolt holes fixed in y direction.
6.3.3.1 Boundary Condition

The FE model boundary condition was defined to mimic the CTS fracture testing. In the CTS fracture testing (see Figure 6.2.3) the specimen is bolted to the upper and the lower parts of the fixture. The lower bolt holes are fixed while the upper bolt holes are moving upward. The tension results in the upper half of the top bolt holes to be moving upward and the lower half of lower bolt hole cylindrical surfaces to be fixed similar to Figure 6.3.7 A and Figure 6.3.7 B respectively. The fixed nodes are fixed in y direction while rotation around z axis and Poison contraction in z and y direction are allowed. The top bolt hole nodes move along the positive direction of Y axis. The displacement rate of $2 \text{ mm/min}$ can be considered as quasi static displacement, therefore, strain rate effect was not included in the material behaviour. The sharp cut at the tip of the notch that acts as the crack was also included in the model. The cut was made in the specimen using sharp cutting blade.
6.3.3.2 FE Model Results

The results of FE models for four different fibre angles are shown in Figure 6.3.8. As can be seen in the figures, the contour of effective stress rotates by the change in the fibre orientation. These results were used to determine the values of normal and shear stress. When the condition of Equation (6.3.1) is satisfied, the length between failed points is calculated.
Figure 6.3.8 FE simulations showing von-Mises stress at the tip of the crack close to the fibre under mode I loading. Glass fibre has angle of a) 0° b) 30° c) 45°, and d) 90°.

The results of FE simulation are used to determine the debond length as it will be mentioned in the next section.

6.3.4 Debond Length

The value of pull-out energy can be found using the pull-out length or debond length (see Section 6.1.1). In order to determine the pull-out length, two steps are taken. First, using the FE model the value of debond length is calculated and then compared with the statistical study of the pull-out length from fracture surface images. This value will be used to determine the pull-out energy as in Equation (6.1.9).
Figure 6.3.9 Sequence of steps taken to determine the fibre pull-out length a) a single fibre passing close to the tip of the crack in CTS specimen with an angle of $\theta$. b) stress components at the interface of fibre was calculated from FE model. C) The calculated stress values were transformed to give stress values normal and parallel to the interface. These values are used to determine whether the desired point on interface detaches from fibre or not.

To calculate the debond length as shown in Figure 6.3.9, a single fibre passing close to the tip of the crack in the CTS specimen with an angle of $\theta$ was modeled using the FE method, and stress components at the interface of the fibre were calculated from the model. The calculated stress values were transformed to give stress values normal and parallel to the interface. The transformed values of stress are then used to determine whether the interface debonds or not. To determine the failure of the interface, Hashin criteria were used (See Equation (6.3.1)).
\[
\frac{\sigma_{y,\theta}}{\sigma_i} + \frac{\tau_{xy,\theta}}{\tau_i} \geq 1 
\]  \hspace{1cm} (6.3.1)

in which \(\sigma_i\) and \(\tau_i\) are interfacial normal and shear strength that were introduced in Chapter 5.

The points that satisfy Equation (6.3.1) will be debonded from the fibre. The distance between points gives the length that is used in the calculations.

The results of critical length calculated for different fibre angles from FE simulations are shown in Figure 6.3.10. When the fibre has \(0^\circ\) angle, the crack advances along the fibre direction and there is no fibre pull-out. Therefore, it is not included in the average length size. If the average length is found from fibres with \(15^\circ\) to \(90^\circ\), the average value is 507 \(\mu m\) (See Figure 6.3.10).

Now that we have a more precise prediction for the value of the debond length in different fibre angles, the value of the interfacial debonding area can be found more precisely. It should be noted that the interfacial debonding area was discussed in the previous section, which led to Equations (6.1.16), (6.1.17), and (6.1.18).

![Figure 6.3.10 Fibre debond length in different orientation using FE simulation.](image-url)
By doing a similar calculation with the knowledge of debond length in 7 different fibre angles (from $0^\circ$ to $90^\circ$) we conclude:

$$\frac{A_{i,\text{random}}}{A_{i,\text{UD}}} = \frac{l_{d,\text{random}}}{l_{d,\text{UD}}} = \frac{\sum_{i=1}^{7} \frac{d_i}{\cos \gamma_i}}{\sum_{i=1}^{7} d_i} = 1.7$$ \hfill (6.3.2)

This correction will be applied to Equation (6.1.2). From now on $A_i$ indicates the interfacial debond length in random fibre composites.

6.4 Fractography

6.4.1 Fibre Pull-Out Fractography

The values of the debond length calculated from the FE simulation is compared with the experimental results. The comparison helps us to determine if the FE results are within the correct range. The fracture surface of the broken random composite was observed using stereomicroscopy. An extensive number of fibres on the surface are used to determine the pull-out length. This is done with the help of Clemex – Image Analysis Software. An example of random composite fracture surface with its fibre pull-out length is shown in Figure 6.4.1. These images give the average fibre length of 285 microns. To determine the average pull-out length, 120 fibres on the fracture surface were selected randomly. The value of 285 microns is close to the half of the value of 507 microns calculated as an average value of debond length from the FE simulation with fibre angle between 15 and $90^\circ$. 
Figure 6.4.1 Example of the fibre length on the fracture surface of random composite. The values of pull-out length for some of the fibres are used to calculate the average pull-out length on composite.

6.4.2 Transition angle ($\theta_{\text{transition}}$)

As discussed in Section 6.1, the transition angle determines whether the fibre experiences debonding under delamination or Cook-Gordon mechanism. The value of this angle can be determined using fractography. For this purpose, stereomicroscopy images similar to Figure 6.4.1 and Figure 6.4.2 are used. The image shows the range of the fibre angle distribution on the fracture surface. Figure suggests that fibres having an angle of $\theta$ between 57° and 90° experience delamination, and fibres with angle below 57° experience Cook-Gordon debonding. Therefore, it suggests a transition angle close to 57°. As depending on the image, this angle could be different from 57° so we assume that the transition angle ranges between 45° and 75°. The range was selected so that the angle 57° is almost in the middle of this range. The effect of this range will be used to introduce an upper and a lower bound for the prediction of the total CSERR.
Figure 6.4.2 Stereomicroscopy images of the crack path showing range of fibre angles experiencing delamination or Cook-Gordon debonding mechanisms.


6.5 Failure Criterion vs Experimental Data

6.5.1 Loading Mode Function, \( g(\alpha) \)

As discussed in Section 6.1.2 (Effect of Mode of Loading on Fibre Pull-Out) and as shown in Figure 6.2.6, a linear transition is assumed in the decrease of the pull-out energy contribution to the total CSERR. As can be seen in Figure 6.5.1, the transition line that shows the change in the value of \( g(\alpha) \) from pure mode I (i.e. for \( \alpha = 0^\circ \), \( g(\alpha) = 1 \)) to pure mode II (i.e. for \( \alpha = \frac{\pi}{2} \), \( g(\alpha) = \frac{G_{II}}{G_I} \)) can be determined \( g(\alpha) \) for other mixed mode loading (\( 0 \leq \alpha \leq \frac{\pi}{2} \)). Therefore, the linear transition from 1 to \( \frac{G_{II}}{G_I} \) in terms of angle of loading (\( \alpha \)) can be stated as:

\[
g(\alpha) = \frac{2}{\pi} \frac{G_{II} - G_I}{G_I} \alpha + 1 \tag{6.5.1}
\]

In which \( \alpha \) is in Radian.

Therefore, Equation (6.1.12) can be stated as:

\[
G_c = \frac{G_{II} \sigma_i^2 A_i}{\tau_i^2 A_t} \cos^2 \alpha + \frac{G_{II} \sigma_i^2 A_i}{A_t} \sin^2 \alpha + \left( G_{Ic,m} \cos^2 \alpha + 2.4 G_{Ic,m} \sin^2 \alpha \right) \frac{A_m}{A_t} +
\]

\[
G_{\text{fibre}} + \frac{f \tau_i^2 d}{8d} \times \left[ \frac{2 G_{II} - G_I}{\pi} \alpha + 1 \right] \tag{6.5.2}
\]

Because the loading mode function \( g(\alpha) \) was empirically found from fracture testing of glass-fibre epoxy, Equation (6.5.2) will be a semi-empirical failure criterion that is suitable for this system of composite.
To develop a relation for $g(\alpha)$ we need to determine the ratio of the pull-out energy between mode I and mode II. As the ratio is dependent on the ratio of debond length, we need to determine an average value of the debond length under mode I and mode II loading. The average debond length for different fibre angles under mode I was determined in Section 6.3.4. The average debond length under mode II loading remains.

A method similar to Section 6.3.4 can be used to determine the values of the debond length at different fibre angles under mode II using finite element. Here, for different fibre angles in the CTS specimen which is subjected to mode II loading the value of the debond length is determined (see Figure 6.3.9). The values of the debond length calculated from FE simulation of CTS subjected to mode II loading is as shown in Figure 6.5.2. The average values of the pull-out length for the mode II loading for angles between 15° to 90° is 377 microns. As the pull-out length is a fraction of debond length, we have the following relation between mode I and mode II pull-out CSERR:

\[ g(\alpha) = \frac{G_{II}}{G_I} \]

Figure 6.5.1 Comparison of the value of $g(\alpha)$ for pure mode I ($\alpha = 0^\circ$) and pure mode II ($\alpha = \frac{\pi}{2}$). The transition is used to derive an expression for $g(\alpha)$ in terms of $\alpha, G_{II}$, and $G_I$. 

![Diagram showing comparison of $g(\alpha)$ for pure mode I and pure mode II.](image)
\[
\frac{G_{II-pull\ out}}{G_{I-pull\ out}} = \left( \frac{l_{II-pull\ out}}{l_{I-pull\ out}} \right)^2 = \left( \frac{l_{II-debond}}{l_{I-debond}} \right)^2 = \left( \frac{377}{507} \right)^2 = 0.55 \tag{6.5.3}
\]

The value of total CSERR of random composite is found similar to Equation (6.5.2) as:

\[
G_c = \frac{G_{IIc,i} \sigma_i^2 A_i}{\tau_i^* A_t} \cos^2 \alpha + \frac{G_{IIc,i} \pi A_i}{A_t} \sin^2 \alpha \\
+ \left( G_{lc,m} \cos^2 \alpha + 2.4 G_{lc,m} \sin^2 \alpha \right) \frac{A_m}{A_t} + G_{fibre} \tag{6.5.4}
\]

Equation (6.5.4), is a mechanistic model, that unlike Equation (6.5.2) is only dependent on the properties of the composite constituents.

Now remains the definition of the other terms in Equation (6.5.4).

---

**Figure 6.5.2** Fibre debond length in different orientation using FE simulation for CTS subjected to mode II loading.
6.5.2 Quantifying the Transition Angle

The first and second term in Equation (6.5.4) are functions of the debond length and the transition angle. The ratio of the debond length of random composite to that of a UD composite was found in Section 6.3.4. A range for the transition angle was proposed in Section 6.4.1. Therefore, by knowing the value of the debond length, range of the transition angle, and interfacial debonding CSERR for a UD composite, we can find the value of interfacial CSERR for a random composite.

As discussed in Section 6.3.4, the debond length for the random glass/epoxy composite is 1.7 times the debond length for a UD glass/epoxy composite. The second term in defining $A_i$ for a random composite is the transition angle. For all fibre angles below $\theta_{\text{transition}}$ we expect the fibre/matrix debonding under the Cook-Gordon mechanism in which the debonding area is calculated as: $A_i = 2\pi rl_d$. For all fibre angles less than $\theta_{\text{transition}}$ similar to a UD composite we have $A_i = \pi rl_d$. To quantify the effect of the transition angle we use the upper and lower values of the range for $\theta_{\text{transition}}$.

6.5.2.1 Lower Bound

If $\theta_{\text{transition}} = 45^\circ$, then $\frac{1}{2}$ of fibres experience fibre pull-out (Cook-Gordon debonding) and $\frac{1}{2}$ experience delamination debonding. Therefore, using Equation (6.3.2), interfacial debonding CSERR can be found as:

\[
G_{\text{interface-Rnd}} = G_{\text{int.UD}} \times 1.7 \left(\frac{1}{2} \times 2 + \frac{1}{2} \times 1\right)
\]

(6.5.5)

Where $G_{\text{int.UD}}$ is the interfacial debonding CSERR for a UD composite, having similar fibre and matrix to the random composite. In other words $G_{\text{int.UD}}$ can be found as:

\[
G_{\text{int.UD}} = \left(\frac{G_{\text{IC,1}}^2}{\tau_i^2} \cos^2 \alpha + G_{\text{IC,1}} \sin^2 \alpha\right) \left(\frac{A_i}{A_{l,\text{UD}}}\right)
\]

(6.5.6)

Subscript UD indicates a UD composite. $A_i$ is the interfacial debonding area in the UD composite (i.e. $A_i = \pi rl$).
The value of $\theta_{\text{transition}}$ is also influencing the value of the pull-out CSERR. Therefore, if $\theta_{\text{transition}} = 45^\circ$ then the fibres experiencing pull-out are $\frac{1}{2}$ of the total fibre volume fraction:

$$f = \frac{1}{2}v_f \rightarrow G_{\text{pull-out}} = \frac{f\tau_i^* l_d^2}{8d} = \frac{v_f\tau_i^* l_d^2}{16d} \quad (6.5.7)$$

### 6.5.2.2 Upper Bound

If $\theta_{\text{transition}} = 75^\circ$, then $\frac{3}{4}$ of all fibres on the fracture surface experience the pull-out (Cook-Gordon debonding) and $\frac{1}{4}$ of fibres experience the delamination debonding. Therefore, using Equation (6.3.2) we have:

$$G_{\text{interface-Rnd}} = G_{\text{int.UD}} \times 1.7 \left( \frac{3}{4} \times 2 + \frac{1}{4} \times 1 \right) \quad (6.5.8)$$

Where parameters are similar to Equation (6.5.5).

As discussed above, the value of $\theta_{\text{transition}}$ is also influencing the value of the pull-out CSERR. Therefore, if $\theta_{\text{transition}} = 75^\circ$ then the fibres experiencing pull-out are $\frac{3}{4}$ of the total fibre volume fraction:

$$f = \frac{3}{4}v_f \rightarrow G_{\text{pull-out}} = \frac{f\tau_i^* l_d^2}{8d} = \frac{3v_f\tau_i^* l_d^2}{32d} \quad (6.5.9)$$

The values of $f$ and $G_{\text{interface-Rnd}}$ will be used to predict the upper bound for the total value of toughness.

The third term in Equation (6.5.4) is the CSERR of the resin. The determination of resin CSERR is similar to the UD composite. It should be noted that a reduction factor 

$$(\eta = \left( \frac{\text{fibres spacing}}{r_p} \right)^2)$$

is introduced to predict the effect of fibre reinforcement on the reduction of the amount of energy released during plastic deformation of resin compared to when the resin has no reinforcement.
The fourth term in Equation (6.5.4) is the fibre CSERR \( G_{\text{fibre}} \). The value of toughness for ceramic reinforcements such as glass and carbon is 0.01 \( \frac{kJ}{m^2} \) and 0.005 \( \frac{kJ}{m^2} \) respectively [1], [5]. Therefore, if the energy release mechanism is changed within the fibre depending on the loading angle the contribution of fibre fracture to the total CSERR is negligible.

6.5.3 Comparison with Experimental Data and Discussion

Experimental data is utilized to validate the criterion. Therefore, the values of the CSERR of a random glass-fibre/epoxy composite as a function of mixed mode loading angle were predicted. These values were compared to experimental data measured for these materials. The results are measured from only one CTS specimen at 30° loading angle and two specimens from each other angle. The upper and lower error was calculated using the standard deviation of the population (See Chapter 5, Equation 5.3.1).

The results of the mechanistic criterion, Equation (6.5.4) are compared with experimental results in Figure 6.5.3. The upper and lower bounds for the criterion’s predictions are calculated using the approach discussed in Section 6.5.2. The upper bound assumes \( \theta_{\text{transition}} = 75^\circ \) and the lower bound assumes \( \theta_{\text{transition}} = 45^\circ \). As can be seen in Figure 6.5.3 almost all modes of loading fall between the upper and lower bound of the criterion (the only exception is for the loading angle \( \alpha = 45^\circ \)). There were two specimens for each angle except for the result for 30 degree that is measured from one specimen. The size of the error bar for 45 degree and 60 degree experimental data is larger compared to other specimens. Studying larger number of specimens may result in smaller error bar. It should be noted that the mixed mode toughness for the material made and tested is the only available experimental data. The match between the criterion’s prediction and experimental results proves the suitability of the criterion to predict the CSERR of a random fibre composite. Equation (6.5.4) is a mechanistic model that does not require fracture testing of the random composite specimen and is only dependent on the properties of the constituents and interface.
The upper and lower bound suggest that the initial assumption in setting the transition angle between 45° and 75° was correct. Therefore $\theta_{\text{transition}} = 60°$ was chosen to see if it could narrow the prediction bound. As can be seen in Figure 6.5.3 this angle gives a new upper bound for the criterion’s prediction. It can be concluded that the new range for the transition angle falls between 45° and 60°.

The random composite has lower values for mode II toughness and fracture toughness because the matrix dominant fracture in mode II requires smaller energy compared to the fibre dominant fracture in mode I. Similar results were observed in the literature [126], [127]. This effect was predicted in the mechanistic criterion by introducing the loading mode function $g(\alpha)$.

The comparison between results from Equation (6.1.12) with $g(\alpha) = 1$, Equation (6.5.4), and the experimental data are shown in Figure 6.5.4. As shown in the Figure 6.5.4, the blue line that indicates the criterion without considering the loading mode function effect, i.e. $g(\alpha) = 1$ is only good for modes of loading close to mode I. But, Equation (6.5.4)
considers the effect of \( g(\alpha) \) and is able to predict the value of the CSERR for the whole range of loading modes.

The predictions of semi-empirical criterion introduced by Equation (6.5.2) have also been compared with the experimental results. As the loading mode function, \( g(\alpha) \) is smaller in Equation (6.5.2) compared to Equation (6.5.4), Equation (6.5.4)’s predictions for loading angles above 30° is closer to the experimental data.

![](image.png)

**Figure 6.5.4 Effect of loading angle function, \( g(\alpha) \) on the criterion predictions compared with experiment.**

To improve the prediction by the fracture criterion introduced in Equation (6.5.4), further study on the effect of loading direction on the pull-out work, the fracture strength, and Young's modulus of the filaments can be useful [119]. In addition, as the number of experimental data is two at each angle, testing more specimens can improve the experimental results.
6.5.3.1 The Fibre Type Effect

The effect of microstructure on the total CSERR of a random fibre composite can now be predicted using the mechanistic criterion. First, the fibre type in most of PMCs as discussed previously for Equation (6.5.4), is negligible. This is because most of ceramic fibres contribute up to only $0.1 \frac{kJ}{m^2}$ to the total CSERR, this value compared to the total CSERR which is of order of $10 \frac{kJ}{m^2}$ can be neglected. The contribution of ductile fibres such as Kevlar however, is a bit higher with toughness of up to $0.8 \frac{kJ}{m^2}$ [1]. Metal fibres if used in the composite contribute to the total toughness of the composite significantly. As an example, mild steel fibre that has a toughness of $100 \frac{kJ}{m^2}$ can increase the toughness of a random fibre epoxy by 5 times.

6.5.3.2 The Fibre Volume Fraction Effect

The fibre volume fraction has a significant effect on the value of the CSERR due to its effect in the pull-out energy as defined by Equation (6.1.9). On the other hand, the increase in the fibre volume fraction decreases the fibre spacing and decreases the amount of energy released from the resin between the fibres. The decrease in fibre spacing decreases the reduction factor $\eta$. But since the value of $\eta$ is small ($\eta = 0.005$) the change in its value does not affect the total CSERR significantly. In conclusion, the increase in the fibre volume fraction increases the total CSERR. This is shown in Figure 6.5.5, as can be seen by both upper and lower bounds decreasing to less than half by decreasing the fibre volume fraction from 43% to 15%.
6.5.4 Mixed Mode Fracture Toughness Criteria

The results of mixed mode fracture toughness ($K_I$, $K_{II}$) for the random glass-fibre epoxy composite are shown in Figure 6.5.6.

$$\left(\frac{K_I}{21.25}\right)^2 + \left(\frac{K_{II}}{16.4}\right)^2 = 1$$

Figure 6.5.6 Mixed-Mode fracture toughness values at fracture in random glass fibre/epoxy Composite.
As can be seen in the figure based on the experimental data the mixed mode critical values of stress intensity factors conform to a fracture criterion proposed by Wu [80]:

\[
\left(\frac{K_I}{K_{lc}}\right)^2 + \left(\frac{K_{II}}{K_{lIC}}\right)^2 = 1
\]  

(6.25)

Where \(K_{lc}\) and \(K_{lIC}\) are critical values of the stress intensity factor in pure mode I and pure mode II. The average value of \(K_{lc}\) and \(K_{lIC}\) for the random composite introduced here is 20.6 MPa\(\sqrt{\text{m}}\) and 16.4 MPa\(\sqrt{\text{m}}\) respectively.

### 6.6 Summary

A mechanistic failure criterion for a random fibre composite subjected to mixed-mode loading was proposed in this Chapter. This model is the development of a failure criterion for the UD composite introduced in Chapter 5. The failure mechanisms that contribute to the work of fracture in a random composite were studied and included in the failure criterion of the random composites. The criterion predicts the value of mixed mode CSERR of a composite based on the fracture properties of its constituents, interface, and the ratio of mode II to mode I CSERR. The failure criterion predicts mixed mode CSERR that match with experimental results. A range for the fibre transition angle that determines whether the fibre debonds from the matrix due to delamination of or Cook-Gordon mechanism was proposed.

Experimental results with the help of FE simulation were used to calculate the value of mixed mode CSERR of the composite. It was determined that the FE simulation accurately characterized the experimental stress-strain behaviour of the composite subjected to the uniaxial and the fracture loading. Using the FE simulation, an average value of debond length was calculated to determine the fibre pull-out work in the total work of fracture in a random composite.
Chapter 7

7 Conclusions

The work presented in this thesis is part of a larger research project which aims at accurately predicting the mechanical properties of the parts and components made from different types of composites. The objective of this research has been to develop a mechanistic failure criterion that could predict the mixed mode CSERR of a thermoset UD and random fibre composite. The testing has been carried out at different mixed mode loading. Data analysis was used to determine the effect of different parameters such as sample thickness on the value of CSERR.

7.1 Research Contribution

Based on the experimental testing, analytical and numerical modeling and data analysis carried out in this research the contribution of the research to the study of fracture in polymer composites is as follows.

7.1.1 Understanding of failure mechanisms involved in the fracture of polymer composites

Optical microscopy and SEM were used to investigate the fracture mechanisms representing on the fracture surface of the specimens. These mechanisms include from cleavage to hackle formation in Epoxy, fibre matrix delamination in UD and random composite and fibre pull-out in random fibre composite.

7.1.2 Developing a Mechanistic failure criterion for UD and random composites

The investigation on the failure mechanisms represented on the fracture surface with the help of fracture mechanics resulted in the development of a mechanistic failure criterion for UD and random fibre composite.

A failure model was also proposed for isotropic materials (epoxy). The model based on the determination of the value of mode I and mode II toughness for the material and the
loading angle. Experimental values for the CSERR of epoxy were measured using neat epoxy CTS specimens under mixed mode loading. The predictive model for epoxy successfully captured the S-curve behaviour of the material subjected to the mixed mode loading. The predicted values matched very well with the experimental results.

For the UD composites, the proposed mechanistic failure criterion predicts the value of the composite CSERR in terms of the mode of the loading, the composite constituent’s properties, the interfacial properties and fibre content. A reduction factor was introduced to account for the amount of energy released by a reinforced matrix. The reduction factor relates the fibre spacing (the plastic zone area for the reinforced matrix) to the plastic deformation area of the unreinforced matrix.

Experimental values of the toughness were measured with the help of CTS fracture testing of different UD composite specimens. The mechanical properties of materials were measured using uniaxial tension testing of several specimens. The properties were used to calculate the CSERR for the material by measuring the critical load in the CTS specimens. The values of composite toughness (CSERR) predicted from the criterion for the UD composite shows a very good correlation with the experimental results.

For the random fibre composites, the predictive criterion incorporates the amount of energy released in the matrix ($G_{\text{resin}}$), the fibre toughness ($G_{\text{fibre}}$), the interfacial debonding energy release rate ($G_{\text{interface}}$), and the energy released during fibre pull out ($G_{\text{pullout}}$). The fibre fracture energy contribution to the total CSERR of the random composite was found to be negligible. To determine the interfacial debonding energy release rate, the debond length and interfacial debonding area were calculated. A finite element model was developed to estimate the debond length. To determine the interfacial debonding area, two debonding mechanisms were considered. The transition angle, ($\theta_{\text{transition}}$), in which the mechanism switches from the delamination debonding to the Cook-Gordon debonding was introduced. A range for the angle was suggested and an upper and a lower bound for the criterion predictions were calculated.
The pull out energy release rate was observed to be the dominant contributor to the total CSERR of the composite. The debond length calculated from the FE model was used to determine the pull out energy. An isotropic linear elastic and a linear piecewise plastic material models were used to characterize the uniaxial tensile and fracture response of the composite using the finite element simulations. The effect of mode of loading on the pull out energy, as a function of loading angle $g(\alpha)$, was introduced in the model. The function was determined using the calculation of the debond length from the FE model. The effect of transition angle to calculate an upper and lower bound for the pull out energy was considered. Almost all experimental results for the CSERR of random composites fell between the upper and the lower bound of the criterion.

The criteria proposed for the UD and random fibre composite are non-empirical criteria that can successfully predict fracture responses of these materials. Unlike existing criteria, the proposed criteria in this research is capable of predicting the fracture properties of composites without requiring performing mechanical and fracture mechanics experiments for each system of composite.

The fact that these mechanistic criteria are based on the understanding of the effect of operative fracture mechanisms involved in the fracture is the novelty of the criteria over other existing criteria which are empirical or semi-empirical.

### 7.1.3 Assisting in the design of optimized energy absorbing polymer composites

The criteria also provide a capability to design new systems of composite to meet the desired fracture properties.

The effect of different design parameters such as fibre volume fraction, fibre and matrix type, fibre orientation, fibre matrix interfacial properties on the composite structure response is well known. Therefore, the criterion can be used to optimize the design of composite structures.
Examples of the influence of fibre type and fibre volume fraction on the toughness of random fibre composites are discussed in Section 6.5.3.1 and Section 6.5.3.2 respectively. Figure 6.5.5 indicates that 28% increase in the fibre volume fraction increases the upper limit of pure mode II toughness from $8 \frac{kL}{m^2}$ to $17.2 \frac{kL}{m^2}$. The increase in the toughness value is even more for lower modes of loading.

7.2 Future Work

The following suggestions are recommended to enhance the predictive ability of forecasting the mechanical and fracture responses of composites:

1) Experiments should be carried out on other types of thermoset polymer composites to capture the behaviour of other systems of composites, and establish the accuracy of the fracture criterion for the systems other than those studied in this research.

2) Improving the determination and calculation of the loading mode function $g(\alpha)$ to capture a better match between the criterion prediction and experimental results. Further investigations may result in the improvement of the determination of the value of $g(\alpha)$ under pure mode I and pure mode II loading. Further study may also result in a different transition curve for $g(\alpha)$ from the current linear transition between mode I and mode II values. The study may define how the mode of loading may affect the fracture mechanisms from mode I to mode II loading, resulting in better explanation of the fracture mechanisms in the random fibre composite fracture.

3) Establishment of a more effective idea to predict the transition angle $\theta_{transition}$ instead of guessing a range for the value of the transition angle. As the transition angle affects the pull out and interfacial debonding energy in the random composite, predicting the exact value of the angle improves the predictions by the criterion.

4) The mechanistic criterion for the random fibre composite assumed a uniform distribution of fibres’ orientation. This assumption can be modified for random composites manufactured by high volume processes (e.g. those suitable for automotive applications). For instance, in compression molding techniques, the discontinuous fibres
develop a preferred orientation. Methods such as X-ray tomography can help to understand the fibre distribution and its effect on the composite’s CSERR properly. Such methods would also improve the study of the fracture process in different types of composite and the criterion predictions.

5) Further development of the finite element simulation may consider the effect of multiple fibres with random orientation in the value of fibre pull out energy release rate. The FE model might include other parameters such as curing process can result in a more precision in the FE model simulation.

6) Extending the study of the fracture mechanisms in thermoplastic polymer composite. Further study can be done to investigate the influence of failure mechanisms, other than discussed in this thesis, on the response of other types of PMCs.

7) Developing a mechanistic criterion for woven polymer composites. The utilization of woven composites is very common, which shows the importance of extending the current model to enable it to predict the fracture response of woven composites.
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