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Essays On R&d And Economic Growth

Jinli Zeng

Essays on R&D and Economic Growth

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Jinli Zeng

Department of Economics

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ABSTRACT

This thesis consists of three essays on R&D and economic growth. The objective is to study the relationship between human capital accumulation, human capital allocation and R&D, on the one hand, and economic growth and welfare on the other.

The first essay, entitled "Capital Accumulation, R&D and Economic Growth", integrates two distinct categories of endogenous growth models (the capital-based models and the ideas-based models). A dynamic general equilibrium model is developed, in which both physical and human capital accumulation and investment in R&D are endogenously determined, and successful innovations not only discover new goods and destroy the old counterparts, but also create new knowledge and render part of human capital obsolete. The model shows that both the laissez faire equilibrium and the optimal growth rates depend positively upon the efficiency of human capital accumulation, the size of the economy, the productivity of R&D and the size of innovation and negatively upon the risk aversion coefficient and the rate of time preference; but the monopoly power does not affect the optimal growth rate while it tends to increase the laissez faire growth rate. It also shows that under laissez faire the growth rate may be more or less than optimal, and there always exists a tax/subsidy system which can be used to achieve the optimal growth.

The second essay, "Innovative vs. Imitative R&D and Economic Growth", focuses on the allocative aspect of human capital. The essay presents a model, in which innovations and imitations can occur in the same sector at the same time. We discuss two types of imitations: rent-seeking imitations and productive imitations. We identify the channels through which innovation and imitation interact with each other. In the case where imitations are of the rent-seeking type, we show that subsidizing innovation is not necessarily equivalent to taxing imitation: while taxing imitative R&D

always induces more investment in innovative R&D and less investment in imitative R&D, subsidizing innovative R&D always encourages innovation but it discourages imitation only if the effective employment in innovative R&D is high enough relative to the effective employment in imitative R&D; if the effective employment in innovative R&D is relatively low, then subsidizing innovation also attracts imitation. In the case where imitations are productive, we show that, in addition to the "nonequivalence" result, taxing imitative R&D may induce more imitations. In both cases, we show that a subsidy to innovative R&D always speeds up economic growth while a subsidy to imitative R&D always does the opposite, but the effects on welfare of both subsidies are ambiguous.

The third essay, "R&D and Economic Growth in Open Economies", extends the framework in the second essay to the context of an open economy. A dynamic general equilibrium model is constructed to analyze the impact of patent protection on economic growth in an open economy context. We consider three patent protection scenarios: No patent protection (NPP), Asymmetric patent protection (APP) and Symmetric patent protection (SPP). Given the assumption that imitation is a rent-seeking activity, we show that stronger patent protection induces a higher world growth rate. A calibration exercise also shows that under the APP assumption, any public policy that encourages imitation (discourages innovation) hurts the world economic growth.

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Chapter 1

Capital Accumulation, R&D and Economic Growth

1.1 Introduction

The fast-growing literature on the new growth theory can be broadly divided into two categories according to the underlying sources of growth, as discussed in Romer (1992). One (e.g. Romer 1987 and King and Rebelo 1990) bases growth on endogenous accumulation of both physical and, especially, human capital and emphasizes investment in both types of capital. The other (e.g. Aghion and Howitt 1992, Grossman and Helpman 1992 and Segerstrom et al 1990) takes endogenous technological changes resulting from R&D as the source of growth by treating the product of the R&D as a commodity. The former focuses on the externalities of capital accumulation leaving aside the intentional R&D activity, which is the focus of the latter, while the latter assumes fixed factor endowments. Both categories capture one important aspect of economic growth and are able to generate sustained growth without relying on any exogenous factor growth.

However, factor accumulation , especially human capital accumulation, and technological changes driven by innovative R&D are two integrated elements in driving economic growth in a real world economy. On the one hand, human capital is the most important factor in R&D activities and in applying the new technologies resulting from successful R&D to production. One the other hand, the new technologies open up new economic opportunities for investment in human capital (and physical capital as well) to take place. If these two can be integrated into one single framework,[1] then we will be able to see the interaction between these two types of forces in pushing economic growth and therefore bring the theory a step closer to the reality.

The objective of this paper is to develop a synthesized endogenous growth model, in which both factor accumulation and technology change are endogenously determined and growth is driven by the interaction between these two types of economic forces, by integrating the two distinct literatures mentioned above. Our model is a vertical product differentiation model. In the model economy, there are four types of activities – final good production, intermediate good production, human capital accumulation and innovative R&D. Quality improvement of intermediate goods through innovative R&D is the source of growth. Successful innovations have two types of "creative destruction" effects. On the one hand, they discovers new intermediate goods but make the old counterparts obsolete. On the other hand, they create new knowledge but destroy part of human capital stock. We assume that innovative R&D is the most human capital intensive activity. In the model specification, we make an extreme assumption that innovative R&D uses only human capital (skilled labor) while intermediate good production requires only unskilled labor. Human capital accumulation is necessary because each successful innovation destroys part of the current human capital stock. So is physical capital investment because we suppose that final good production uses capital and intermediate goods as inputs, the quality

[1]Romer (1990) incorporates physical capital into a horizontal product differentiation model, in which, continuous capital investment is required for the production of new intermediate goods discovered through R&D.

improvement of intermediate goods raises the productivity of final good production, which provides new opportunities for capital investment.

We find that the monopolist's market power (measured inversely by α) plays a critically important role. It is the market power that determines whether the laissez faire equilibrium growth is too fast or too slow compared with the socially optimal growth.[2] It is also the market power that determines whether a tax or subsidy scheme is needed to support the optimal growth. Propositions 1.1-1.4 give the finding of this paper.

The rest of this paper is organized as follows: The next section describes the environment and sets up the model. Section 1.3 describes the laissez faire equilibrium conditions and the condition of equilibrium existence. We also perform comparative-static analysis in this section. The welfare property of the laissez faire equilibrium and policy implications are discussed in section 1.4. Finally, some concluding remarks are given in the last section.

1.2 The Model

The basic framework is due to Aghion and Howitt (1992). We consider a closed economy populated with a continuum of identical infinitely lived households with measure 1. Each household is endowed with N unit flow of time which is inelastically supplied to the production sectors and devoted to human capital accumulation activities.

[2] Aghion and Howitt (1992) has a similar result. But as will be seen below, the optimal growth does not depend on the market power. This is different from Aghion and Howitt (1992).

1.2.1 Preferences

We assume that the household's preferences are given by

$$\int_0^\infty e^{-\rho\tau}\frac{C^{1-\sigma}-1}{1-\sigma}d\tau, \tag{1}$$

where C is consumption; ρ the constant rate of time preference; σ the relative risk aversion coefficient and τ represents time. For simplicity, the time subscripts are omitted whenever no confusion can arise and the final good will be used as the numeraire. Given the household's total discounted lifetime income M and the interest rate r (which will be endogenously determined and will be constant in equilibrium), the household's lifetime budget constraint is

$$\int_0^\infty e^{-r\tau}C d\tau \leq M. \tag{2}$$

Maximizing the household's utility (1) subject to its budget constraint (2) gives the optimal time path of consumption, i.e.

$$\frac{\dot{C}}{C}=\frac{r-\rho}{\sigma}, \tag{3}$$

where $\dot{C}$ is the time change rate of consumption C.

1.2.2 Technologies

There are four types of production activities in this economy: final good production, intermediate good production (a continuum of sectors located on $[0, 1]$), human capital accumulation and R&D. It is assumed that perfect competition prevails in all sectors except the intermediate good sectors where there exists temporary monopoly power. The following describes each type of activities.

Final Good Production

The final good production uses the intermediate goods and physical capital as its inputs subject to a constant-returns-to- scale (CRS) technology with the Cobb-Douglas form

$$Y = K^{1-\alpha} \int_0^1 [A(i)x(i)]^\alpha di, \tag{4}$$

where Y is the output of final good production; K and $x(i)$ are respectively the physical capital and the flow of intermediate good i used in the final good production; α is a parameter which measures the contribution of an intermediate good to the final good production and inversely measures the intermediate monopolist's market power; $A(i)$ is the productivity coefficient of intermediate good i. Assume $A(i) = \gamma^{i-1}A, i \in [0, 1]$, where $\gamma > 1$ is the size of each innovation and A is the productivity of the most advanced intermediate good sector. In this specification, we assume that each unit of consumption good foregone can produce one unit of capital and there is no capital depreciation. Profit maximization of the final good sector gives the demand for the capital and the intermediate good i, i.e.

$$(1-\alpha)K^{-\alpha} \int_0^1 [A(i)x(i)]^\alpha di = r, \tag{5}$$

$$\alpha K^{1-\alpha} A(i)^\alpha x(i)^{\alpha-1} = p(i), \text{for all } i \in [0, 1], \tag{6}$$

where $p(i)$ is the price of intermediate good i in terms of the final good.

Intermediate Good Production

Each intermediate good, i, is produced using only labour, $l(i)$, with each unit of labor producing one unit of intermediate good i, i.e. $x(i) = l(i), i \in [0, 1]$. Given the wage rate W, each intermediate monopolist maximizes its profit, i.e. $\alpha K^{1-\alpha}[A(i)x(i)]^\alpha - Wx(i)$, subject to the final good sector's demand for its output, which gives the first-order condition for this maximization problem

$$W = \alpha^2 K^{1-\alpha} A(i)^\alpha x(i)^{\alpha-1}, \text{for all } i \in [0, 1]. \tag{7}$$

Solving the above equation gives intermediate sector i's optimal output

$$x(i) = k\left(\frac{\gamma^{(1-i)\alpha}\omega}{\alpha^2}\right)^{\frac{1}{\alpha-1}}, \text{for all } i \in [0,1], \tag{8}$$

where $\omega = W/A$ and $k = K/A$ are the productivity-adjusted wage of labor and the productivity-adjusted capital stock, respectively. Let $\Pi(i)$ denote the corresponding maximum profit, then

$$\Pi(i) = \alpha(1-\alpha)Ak\gamma^{(1-i)(1-\alpha)}\left(\frac{\gamma^{(1-i)\alpha}\omega}{\alpha^2}\right)^{\frac{\alpha}{\alpha-1}}. \tag{9}$$

Innovative R&D

Attracted by the market incentive , i.e. the temporary monopoly profit obtained by monopolizing the intermediate good sectors once an innovation succeeds, firms invest in R&D. Success of innovation in any intermediate good sector leads to a new intermediate good in that sector which can be used to replace the old one in the final good production and increase the productivity of the final good sector by a factor γ^{α}. As in Aghion and Howitt (1992) (Section 8), we assume different sectors experience innovations in a deterministic order and the innovations always occur in the least advanced sectors.[3] We suppose that innovation follows the Poisson process with the arrival rate Λ, where Λ depends positively on the human capital devoted to the R&D activities and negatively on the sophistication of the current technology, which can be measured by the technology coefficient of the final good production A:

$$\Lambda = \lambda\frac{H}{A}, \tag{10}$$

where H is the human capital used in the R&D activities and $\lambda > 0$ is a parameter. As mentioned above, we assume that innovative R&D is the most human capital intensive activity. Here, we take the extreme case where innovative R&D requires only human

[3]This assumption is initially taken from Shleifer (1986).

capital as its input. It seems reasonable to make this specification by observing that the more the human capital devoted to the R&D, the stochastically faster the innovations come and the more sophisticated the current technology the more difficult to improve it. Here, the Schumpeterian terminology "creative destruction" has double meanings: On the one hand, successful innovations create new intermediate goods which make the final good production more productive but destroy the old ones; on the other, they create new knowledge which helps human capital accumulation (see (13) below) but also destroy some human capital. An R&D firm maximizes $\lambda \frac{H}{A} V - W_H H$. The first-order condition for this maximization problem is

$$\lambda \frac{1}{A} V = W_H, \tag{11}$$

where W_H is the wage rate for skilled labor (human capital) and V is the value of innovation, which is given by[4]

$$\begin{aligned} V &= A \int_0^{\frac{1}{\Lambda}} e^{-r\tau} \tilde{\pi}(\gamma^{(\frac{2\alpha-1}{\alpha})\Lambda\tau}\omega) d\tau \\ &= \alpha(1-\alpha)Ak(\frac{\omega}{\alpha^2})^{\frac{\alpha}{\alpha-1}} \int_0^{\frac{1}{\Lambda}} e^{-r\tau} \gamma^{\frac{2\alpha-1}{\alpha-1}\Lambda\tau} d\tau \\ &= \alpha(1-\alpha)Ak(\frac{\omega}{\alpha^2})^{\frac{\alpha}{\alpha-1}} [\frac{1-\gamma^{\frac{2\alpha-1}{\alpha-1}} e^{-\frac{r}{\Lambda}}}{r - \frac{2\alpha-1}{\alpha-1}\Lambda \ln \gamma}], \end{aligned} \tag{12}$$

because the innovator's productivity-adjusted flow of profits at time τ is[5]

$$\tilde{\pi}(\gamma^{(\frac{2\alpha-1}{\alpha})\Lambda\tau}\omega) = \alpha(1-\alpha)k(\frac{\omega\gamma^{(\frac{2\alpha-1}{\alpha})\Lambda\tau}}{\alpha^2})^{\frac{\alpha}{\alpha-1}}.$$

In Appendix 1.1, we show that the Poisson process with a deterministic innovation order for a continuum of intermediate sectors gives rise to a deterministic result: the length of each interval is $\frac{1}{\Lambda}$ and thus the relative rank of each intermediate good's productivity decreases exponentially. Therefore, the direct formulation of the value of innovation is equivalent to the limiting case ($m \to \infty$) of the m intermediate good

[4]See Appendix 1.1 for proof.

[5]We assume that the innovation occurs at time 0.

model in Aghion and Howitt (1992).

Human Capital Accumulation

Finally, we describe the human capital accumulation process. The formulation of human capital accumulation in the human capital literature has reached a high degree of sophistication. For the problem at hand, we assume that the growth of human capital depends on the time devoted to human capital accumulation activities and the current stock of knowledge which is measured by A:

$$\dot{H} = BSA, \tag{13}$$

where $\dot{H}$ is the time change rate of human capital H; $B > 0$ is a technology coefficient; and S is the time spent on the human capital accumulation. As mentioned above, successful innovations increase the stock of knowledge and therefore speed up the human capital accumulation.

At each point in time, individuals have two choices:[6] supply labor to intermediate good production or accumulate human capital which will be used in future R&D. To choose one of the two activities, each individual compares the earnings of these two activities. If the individual supplies his labor to the intermediate good production, then he earns a wage W per unit of time. If he chooses to accumulate human capital, then each unit of time devoted to this activity will increase his human capital stock by an amount BA, which will bring him an earning W_H per unit of human capital forever. Since earnings in the future have to be discounted at the rate of interest r, and in equilibrium, the earnings of these two activities have to be equal, then we have

$$BA \int_0^{\infty} e^{-r\tau} W_H d\tau = W. \tag{14}$$

[6]Individuals also choose how much to save in the form of physical capital.

Labor Market

Assume full employment, then we have the labor market clearing condition (time constraint)

$$X + S = N, \tag{15}$$

where X is the total employment in the intermediate good production, i.e.

$$X = \int_0^1 x(i)di. \tag{16}$$

Capital Market

We assume that there exists a perfect capital market on which capital for production and R&D is raised.

We have completed the description of the model economy's environment and basic economic activities. Now we turn to the equilibria of this model economy.

1.3 Equilibria

Here we consider only balanced growth stationary equilibria (BGSE). A BGSE is described by the following optimization and market clearing conditions and balanced growth requirement:

$$(1-\alpha)k^{-\alpha}\int_0^1 \gamma^{(i-1)\alpha}x(i)^\alpha di = r, \quad \text{(optimization of final good production)} \tag{17}$$

$$\alpha K^{1-\alpha}A(i)^\alpha x(i)^{\alpha-1} = p(i), \text{for all } i \in [0,1],$$

$$\text{(optimization of final good production)} \tag{18}$$

$$\lambda \frac{1}{A} V = W_H, \qquad \text{(optimization of R\&D)} \qquad (19)$$

$$\frac{\dot{C}}{C} = \frac{r - \rho}{\sigma}, \qquad \text{(optimal consumption path)} \qquad (20)$$

$$BA \int_0^\infty e^{-r\tau} W_H d\tau = W, \qquad \text{(efficiency of time allocation)} \qquad (21)$$

$$\dot{K} + C = K^{1-\alpha} \int_0^1 [A(i)x(i)]^\alpha di, \qquad \text{(final good market clearing)} \qquad (22)$$

$$X + S = N \qquad \text{(time constraint)} \qquad (23)$$

$$\frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{H}}{H} = \frac{\dot{A}}{A} = g_c. \qquad \text{(balanced growth)} \qquad (24)$$

Then a BGSE is a collection of $\{C, K, H, x(i), A, W, W_H, r, p(i)\}$ satisfying equations (17)-(24). Here g_c is the stationary equilibrium growth rate.[7] Since we are interested only in the equilibrium growth rate, we derive the equation that determines the equilibrium growth rate. First of all, we calculate the total employment X in the intermediate good production. From equation (8), we have

$$X = \int_0^1 x(i)di = \frac{(1-\alpha)(1-\gamma^{\alpha/(\alpha-1)})k}{\alpha \ln \gamma} (\frac{\omega}{\alpha^2})^{1/(\alpha-1)}. \qquad (25)$$

Rewriting equation (25), we get the productivity-adjusted wage rate for labor

$$\omega = \alpha^2 [\frac{(\alpha \ln \gamma) X}{(1-\alpha)(1-\gamma^{\alpha/(\alpha-1)})k}]^{\alpha-1}. \qquad (26)$$

Equations (3), (10), (13) and (24) implies

$$h = \frac{g_c}{\lambda \ln \gamma}, \qquad (27)$$

$$S = \frac{g_c^2}{B\lambda \ln \gamma}, \qquad (28)$$

$$r = \sigma g_c + \rho, \qquad (29)$$

where $h = H/A$ is the productivity-adjusted human capital stock. By substituting the relevant variables into equation (19), we get the following equation:

$$\frac{(1-\alpha)(BN\lambda \ln \gamma - g_c^2)}{(\sigma g_c + \rho)[(1-\alpha)(\sigma g_c + \rho) + (2\alpha - 1)g_c]} \frac{[1 - \gamma^{(2\alpha-1)/(\alpha-1)} e^{-(\sigma g_c + \rho)\ln \gamma / g_c}]}{1 - \gamma^{\alpha/(\alpha-1)}} = 1, (30)$$

[7] The growth rate g_c is assumed to satisfy the condition $g_c < \frac{\rho}{1-\sigma}$ to guarantee that the household's utility is finite.

which determines a constant growth rate g_c. Now let's look at the condition under which an equilibrium exists. Appendix 1.2 proves the following proposition.

Proposition 1.1: If $\frac{BN\lambda\ln\gamma}{\rho^2(1-\gamma^{\alpha/(\alpha-1)})} \leq 1$, there is no growth ($g_c = 0$); if $\frac{BN\lambda\ln\gamma}{\rho^2(1-\gamma^{\alpha/(\alpha-1)})} > 1$, there is a unique equilibrium growth rate $g_c \in (0, (BN\lambda\ln\gamma)^{1/2})$.

The intuition behind this proposition is very straightforward. It simply states that if one or more of the following situations occur: (a) the size of the economy (measured by the total time endowment N) is too small; (b) the accumulation of human capital is too inefficient (B is too small); (c) innovation is too difficult (λ is too small); (d) the size of innovation γ is too small; (e) the degree of the monopolist's market power is too low (α is too large); (f) the economy is too impatient (ρ is too large), then there will be no investment in innovative R&D and thus there is no growth. The reason for this is that these situations will either reduce the expected benefit of innovative R&D or increase the cost of innovative R&D or both to such an extent that no firms invest in this activity. As a result, there is no growth. Otherwise, there always are firms investing in innovative R&D, hence the economy experiences a positive growth.

The comparative-static analysis of the equilibrium (see Appendix 1.3) shows the following results.

Proposition 1.2: The laissez faire equilibrium growth rate depends positively on the efficiency of human capital accumulation B, the size of the economy N, the arrival rate parameter λ, the size of innovation γ and the monopolist's market power ($1/\alpha$) and negatively on the risk aversion coefficient σ and the rate of time preference ρ.

These results are intuitive. Each of these parameters $(N, B, \lambda, \gamma, \alpha, \rho, \sigma)$ directly and/or indirectly affects the investment in innovative R&D and growth by changing either the marginal benefit or the marginal cost of innovative R&D or both. For example, an increase in the market power (a decrease in α) increase the marginal benefit of R&D; an increase in the size of the economy N both increases the marginal benefit of R&D and reduces the marginal cost of R&D.

1.4 Social Planner's Problem

In order to examine the welfare property of the laissez faire equilibrium, in this section, we solve the social planner's problem. The social planner maximizes

$$\int_0^\infty e^{-\rho\tau}[\frac{C^{1-\sigma}-1}{1-\sigma}]d\tau, \tag{31}$$

subject to

$$\dot{A} = \lambda H \ln \gamma, \tag{32}$$

$$\dot{H} = BA(N-X), \tag{33}$$

$$\dot{K} = K^{1-\alpha}A^\alpha \int_0^1 (\gamma^{i-1}x(i))^\alpha di - C, \tag{34}$$

given $\quad A_0, H_0$ and K_0.

The Hamiltonian for this maximization problem is

$$\mathcal{H} = e^{-\rho\tau}[\frac{C^{1-\sigma}-1}{1-\sigma}] + \xi\lambda H \ln\gamma + \mu BA(N-X)$$

$$+\nu[K^{1-\alpha}A^\alpha \int_0^1 (\gamma^{i-1}x(i))^\alpha - C], \tag{35}$$

where ξ, μ, and ν are the co-state variables. Then the necessary conditions for a maximum are

$$\frac{\partial \mathcal{H}}{\partial C} = e^{-\rho\tau}C^{-\sigma} - \nu = 0, \tag{36}$$

$$\frac{\partial \mathcal{H}}{\partial x(i)} = -\mu BA + \nu\alpha K^{1-\alpha} A^{\alpha}\gamma^{(i-1)\alpha} x(i))^{\alpha-1} = 0, \tag{37}$$

$$\frac{\partial \mathcal{H}}{\partial A} = \mu B(N - X) + \nu\alpha K^{1-\alpha} A^{\alpha-1} \int_0^1 (\gamma^{i-1} x(i))^{\alpha} di = -\dot{\xi}, \tag{38}$$

$$\frac{\partial \mathcal{H}}{\partial H} = \xi\lambda \ln\gamma = -\dot{\mu}, \tag{39}$$

$$\frac{\partial \mathcal{H}}{\partial K} = \nu(1-\alpha) K^{-\alpha} A^{\alpha} \int_0^1 (\gamma^{i-1} x(i))^{\alpha} di = -\dot{\nu}, \tag{40}$$

and the transversality conditions (TVCs): $\lim_{\tau\to\infty} \xi A = \lim_{\tau\to\infty} \mu H = \lim_{\tau\to\infty} \nu K = 0$. We consider only balanced growth, i.e. $\dot{C}/C = \dot{K}/K = \dot{H}/H = \dot{A}/A = g_p$, then $\dot{\xi}/\xi = \dot{\mu}/\mu = \dot{\nu}/\nu$ implies

$$g_p = \frac{(BN\lambda \ln\gamma)^{1/2} - \rho}{\sigma}, \tag{41}$$

where g_p is the socially optimal balanced growth rate. A positive growth rate requires $(BN\lambda\ln\gamma)^{1/2} > \rho$ and the TVCs imply $(BN\lambda\ln\gamma)^{1/2} < \frac{\rho}{1-\sigma}$. So the optimal growth rate must satisfy $0 < g_p < \frac{\rho}{1-\sigma}$. Obviously, we have

Proposition 1.3: The optimal growth rate increases with an increase in the efficiency of human capital accumulation B, the size of the economy N, the arrival rate parameter λ and the size γ of innovation and decreases with an increase in the risk aversion coefficient σ and the rate of time preference ρ.

These results are also easy to understand. Each of these parameters $(N, B, \lambda, \gamma, \alpha, \rho, \sigma)$ affects the optimal growth rate by changing either the marginal social benefit or the marginal social cost of R&D or both. For example, an increase in the size of innovation increases the marginal social benefit of R&D, therefore, the society should allocate more resource to R&D and increase the growth rate.

Notice that, unlike the laissez faire growth rate, the optimal growth rate does not depend on the monopolist's market power $(1/\alpha)$.

Comparing equations (30) and (41), we know that the laissez faire equilibrium growth rate can be less than, equal to or greater than the optimal growth rate depending on the degree of monopoly power. However, the optimal growth can be supported by a tax/subsidy policy. Let t^* be optimal the tax/subsidy rate on the wage of unskilled workers, then we have

Proposition 1.4: The optimal growth can be supported by a tax/subsidy policy with a tax/subsidy rate on the wage of unskilled workers

$$t^* = 1 - \psi(g_p)\phi(g_p). \tag{42}$$

We show in Appendix 1.4 that $\frac{\partial g_c}{\partial \alpha} < 0$, $\psi(g_p)\phi(g_p)|_{\alpha=0} = +\infty$ and $\psi(g_p)\phi(g_p)|_{\alpha=1} = 0$, there exists $\alpha = \alpha^*$ (the critical point) such that $g_c = g_p$. So if the degree of monopoly power is lower than the critical point $(\alpha > \alpha^*)$, then a tax is required $(t^* > 0)$; if the degree of monopoly power is higher than the critical point $(\alpha < \alpha^*)$, then the optimal growth can be achieved through a subsidy $(t^* < 0)$.

As explained above, the degree of the monopolist's market power has a positive effect on the marginal benefit of R&D. If the degree of the monopolist's market power is too low, then R&D firms do not have enough incentive to invest in R&D. A tax on unskilled labor will increase the supply of human capital (skilled labor), which will reduce the wage rate for skilled labor (the cost of R&D) and thus induce R&D firms to invest more in R&D. If If the degree of the monopolist's market power is too high, then a subsidy is required. The subsidy will do exactly the opposite.

1.5 Conclusions

This paper incorporates capital (both physical and human) accumulation into a vertical product differentiation endogenous growth model (Aghion and Howitt (1992)) with innovative R&D as the source of growth. In the model, both physical and human capital accumulation and investment in R&D are endogenously determined, and successful innovations not only discover new goods and destroy the old counterparts, but also create new knowledge and render part of human capital obsolete.

The model shows that both the laissez faire equilibrium and the optimal growth rates depend positively upon the efficiency of human capital accumulation, the size of the economy, the productivity of R&D and the size of innovation and negatively upon the risk aversion coefficient and the rate of time preference; but the monopoly power does not affect the optimal growth rate while it tends to increase the laissez faire growth rate. It also shows that under laissez faire the growth rate may be more or less than optimal, and there always exists a tax/subsidy system which can be used to achieve the optimal growth.

Appendix I

Appendix 1.1: Proof of Equation (12)

This appendix is to show, through a Cobb-Douglas example, that the direct formulation is equivalent to the limiting case of the m intermediate good model in Aghion and Howitt (1992). The structure of the Cobb-Douglas case is as follows:

$$Y = K^{1-\alpha}\sum_{i=1}^{m}[A_i x_i]^{\alpha}, \quad 0<\alpha<1, \quad \text{(final good production technology)}$$

$$x_i = l_i, \quad i=1,2,...,m, \quad \text{(intermediate good production technology)}$$

$$m\Lambda = m\lambda(\frac{H}{A}), \quad \text{(R\&D technology)}$$

$$\dot{H} = BSA, \quad \text{(human capital accumulation technology)}$$

$$A_i = \gamma^{(1-i)/m}A. \quad (\text{ } i\text{th most advanced sector, i=1,2,...,m })$$

The final good sector and intermediate good sector i's optimization conditions give intermediate sector i's optimal output

$$x_i = k(\frac{\gamma^{(i-1)/m}\omega}{\alpha^2})^{\frac{1}{\alpha-1}}.$$

Then the average optimal output is

$$\bar{x} = \frac{1}{m}\sum_{i=1}^{m}x_i = k\frac{1}{m}\sum_{i=1}^{m}(\frac{\gamma^{(i-1)\alpha/m}\omega}{\alpha^2})^{\frac{1}{\alpha-1}} = k\frac{1}{m}(\frac{\omega}{\alpha^2})^{\frac{1}{\alpha-1}}\frac{1-\gamma^{\alpha/(\alpha-1)}}{1-\gamma^{\alpha/(\alpha-1)m}}.$$

Solving the above equation for ω, we have

$$\omega = \alpha^2[\frac{(1-\gamma^{\alpha/(\alpha-1)m})m\bar{x}}{(1-\gamma^{\alpha/(\alpha-1)})k}]^{\alpha-1}.$$

Since intermediate sector i's profit is

$$\Pi_i = \alpha(1-\alpha)Ak\gamma^{(i-1)(1-\alpha)/m}[\frac{\gamma^{(i-1)\alpha/m}\omega}{\alpha^2}]^{\frac{\alpha}{\alpha-1}},$$

we have intermediate sector i's productivity-adjusted profit

$$\tilde{\pi}(\gamma^{(i-1)(\frac{2\alpha-1}{\alpha})/m}\omega) = \alpha(1-\alpha)k\gamma^{(i-1)(2\alpha-1)/((\alpha-1)m)}[\frac{(1-\gamma^{\alpha/((\alpha-1)m)})m\bar{x}}{(1-\gamma^{\alpha/(\alpha-1)})k}]^{\alpha}.$$

Then the value of innovation is

$$V = A\sum_{i=1}^{m}(\frac{m\Lambda}{r+m\Lambda})^{i-1}[\frac{\gamma^{\alpha/m}\tilde{\pi}(\gamma^{(i-1)(\frac{2\alpha-1}{\alpha})/m}\omega)}{r+m\Lambda}]$$

$$= \alpha(1-\alpha)Ak[\frac{(1-\gamma^{\alpha/(\alpha-1)m})m\bar{x}}{(1-\gamma^{\alpha/(\alpha-1)})k}]^{\alpha}(\frac{1}{r+m\Lambda})$$

$$*\frac{\gamma^{\alpha/m}[1-(\frac{m\Lambda}{r+m\Lambda})^{m}\gamma^{(2\alpha-1)/(\alpha-1)}]}{1-\frac{m\Lambda}{r+m\Lambda}\gamma^{(2\alpha-1)/(\alpha-1)m}}.$$

Thus we have

$$\lim_{m\to\infty}\omega = \alpha^{2}[\frac{\alpha\bar{x}\ln\gamma}{(1-\alpha)(1-\gamma^{\alpha/(\alpha-1)})k}]^{\alpha-1},$$

and

$$\lim_{m\to\infty}V = \lim_{m\to\infty}\alpha(1-\alpha)Ak[\frac{(1-\gamma^{\alpha/(\alpha-1)m})m\bar{x}}{(1-\gamma^{\alpha/(\alpha-1)})k}]^{\alpha}$$

$$*\gamma^{\alpha/m}[1-(\frac{m\Lambda}{r+m\Lambda})^{m}\gamma^{(2\alpha-1)/(\alpha-1)}]\frac{1/(r+m\Lambda)}{1-\frac{m\Lambda}{r+m\Lambda}\gamma^{(2\alpha-1)/(\alpha-1)m}}$$

$$= \alpha(1-\alpha)^{2}Ak[\frac{\alpha\bar{x}\ln\gamma}{(1-\alpha)(1-\gamma^{\alpha/(\alpha-1)})k}]^{\alpha}\frac{1-\gamma^{(2\alpha-1)/(\alpha-1)}e^{-r/\Lambda}}{(1-\alpha)r+(2\alpha-1)\Lambda\ln\gamma},$$

because

$$\lim_{m\to\infty}(\frac{m\Lambda}{r+m\Lambda})^{m} = \lim_{m\to\infty}(1+\frac{r/\Lambda}{m})^{-m} = e^{-r/\Lambda},$$

and

$$\lim_{m\to\infty}\frac{1/(r+m\Lambda)}{1-\gamma^{(2\alpha-1)/(\alpha-1)m}(\frac{m\Lambda}{r+m\Lambda})}$$

$$= \lim_{m\to\infty}\frac{-\Lambda/(r+m\Lambda)^{2}}{[-\frac{r\Lambda}{(r+m\Lambda)^{2}}+\frac{m\Lambda}{r+m\Lambda}\frac{(2\alpha-1)\ln\gamma}{(\alpha-1)m^{2}}]\gamma^{(2\alpha-1)/(\alpha-1)m}}$$

$$= \lim_{m\to\infty} \frac{-\Lambda/(r+m\Lambda)}{[-\frac{r\Lambda}{r+m\Lambda} + \frac{(2\alpha-1)\Lambda\ln\gamma}{(\alpha-1)m}]\gamma^{(2\alpha-1)/(\alpha-1)m}}$$

$$= \lim_{m\to\infty} \frac{\Lambda^2/(r+m\Lambda)^2}{[\frac{r\Lambda^2}{(r+m\Lambda)^2} + \frac{r\Lambda}{r+m\Lambda}\frac{(2\alpha-1)\ln\gamma}{(\alpha-1)m^2} - (\frac{(2\alpha-1)\ln\gamma}{\alpha-1})^2\frac{\Lambda}{m^3} - \frac{(2\alpha-1)\ln\gamma}{\alpha-1}\frac{\Lambda}{m^2}]\gamma^{(2\alpha-1)/(\alpha-1)m}}$$

(by multiplying the numerator and denominator by m^2)

$$= \frac{1}{r + (2\alpha-1)\Lambda\ln\gamma/(1-\alpha)}.$$

Therefore, the R&D firm's optimization condition (in the limiting case) implies

$$\frac{m\lambda V}{A} = W_H,$$

which, combining with the other equilibrium conditions and the balanced growth restriction, gives the same equilibrium growth rate as in section 1.3.

Appendix 1.2: Existence of Equilibrium

First of all, we show that the LHS of equation (30) is a decreasing function of g_c. Let

$$\psi = \frac{(1-\alpha)(BN\lambda\ln\gamma - g_c^2)}{(\sigma g_c+\rho)[(1-\alpha)(\sigma g_c+\rho)+(2\alpha-1)g_c]},$$

$$\text{and} \quad \phi = \frac{[1-\gamma^{(2\alpha-1)/(\alpha-1)}e^{-(\sigma g_c+\rho)\ln\gamma/g_c}]}{1-\gamma^{\alpha/(\alpha-1)}},$$

then equation (30) becomes $\psi(g_c)\phi(g_c) = 1$. For simplicity, the following notations will be used:

$$N_\psi = (1-\alpha)(BN\lambda\ln\gamma - g_c^2),$$

$$D_\psi = (\sigma g_c+\rho)[(1-\alpha)(\sigma g_c+\rho)+(2\alpha-1)g_c],$$

$$N_\phi = 1-\gamma^{(2\alpha-1)/(\alpha-1)}e^{-(\sigma g_c+\rho)\ln\gamma/g_c},$$

$$D_\phi = 1 - \gamma^{\alpha/(\alpha-1)},$$

$$F = e^{-(\sigma g_c+\rho)\ln\gamma/g_c}.$$

Since

$$\frac{\partial\psi}{\partial g_c} = -\{\frac{2(1-\alpha)g_c}{D_\psi} + \frac{\sigma N_\psi}{(\sigma g_c+\rho)D_\psi} + \frac{[(1-\alpha)\sigma+(2\alpha-1)]N_\psi}{[(1-\alpha)(\sigma g_c+\rho)+(2\alpha-1)g_c]D_\psi}\} < 0,$$

$$\frac{\partial\phi}{\partial g_c} = -\frac{\rho\ln\gamma\gamma^{(2\alpha-1)/(\alpha-1)}F}{g_c^2 D_\phi} < 0,$$

where we assume $\sigma > \frac{(2\alpha-1)}{(\alpha-1)}$, the LHS of equation (30) is decreasing in g_c. Moreover, we have LHS$(g_c = 0) = \frac{BN\lambda\ln\gamma}{\rho^2(1-\gamma^{\alpha/(\alpha-1)})}$ and LHS$(g_c = (BN\lambda\ln\gamma)^{1/2}) = 0$. Then if $\frac{BN\lambda\ln\gamma}{\rho^2(1-\gamma^{\alpha/(\alpha-1)})} \leq 1$, there is no growth; if $\frac{BN\lambda\ln\gamma}{\rho^2(1-\gamma^{\alpha/(\alpha-1)})} > 1$, there is a unique equilibrium where $g_c \in (0, (BN\lambda\ln\gamma)^{1/2})$.

Appendix 1.3: Comparative Statics

From Appendix 1.2, we have the laissez faire equilibrium condition

$$\psi\phi = 1.$$

Differentiate the above equation with respect to ξ, $\xi = N, B, \lambda, \gamma, \alpha, \rho, \sigma$, we have

$$\phi[\frac{\partial\psi}{\partial g_c}\frac{\partial g_c}{\partial\xi} + \frac{\partial\psi}{\partial\xi}] + \psi[\frac{\partial\phi}{\partial g_c}\frac{\partial g_c}{\partial\xi} + \frac{\partial\phi}{\partial\xi}] = 0,$$

which implies

$$\frac{\partial g_c}{\partial\xi} = \frac{\frac{\partial\psi}{\partial\xi}\phi + \frac{\partial\phi}{\partial\xi}\psi}{-(\frac{\partial\psi}{\partial g_c}\phi + \frac{\partial\phi}{\partial g_c}\psi)}.$$

Since $\psi > 0$, $\phi > 0$ and we have shown in Appendix 1.2 that $\frac{\partial\psi}{\partial g_c} < 0$ and $\frac{\partial\phi}{\partial g_c} < 0$, then the denominator of $\frac{\partial g_c}{\partial\xi}$, i.e.

$$-(\frac{\partial\psi}{\partial g_c}\phi + \frac{\partial\phi}{\partial g_c}\psi) > 0.$$

Therefore, the sign of $\frac{\partial g_c}{\partial \xi}$ is determined by the sign of the numerator of $\frac{\partial g_c}{\partial \xi}$, i.e. $(\frac{\partial \psi}{\partial \xi}\phi + \frac{\partial \phi}{\partial \xi}\psi)$. To determine the sign of $(\frac{\partial \psi}{\partial \xi}\phi + \frac{\partial \phi}{\partial \xi}\psi)$, we derive the relevant derivatives:

$$\frac{\partial \psi}{\partial N} = \frac{(1-\alpha)B\lambda \ln\gamma}{D_\psi} > 0,$$

$$\frac{\partial \phi}{\partial N} = 0,$$

$$\frac{\partial \psi}{\partial B} = \frac{(1-\alpha)N\lambda \ln\gamma}{D_\psi} > 0,$$

$$\frac{\partial \phi}{\partial B} = 0,$$

$$\frac{\partial \psi}{\partial \lambda} = \frac{(1-\alpha)BN \ln\gamma}{D_\psi} > 0,$$

$$\frac{\partial \phi}{\partial \lambda} = 0,$$

$$\frac{\partial \psi}{\partial \gamma} = \frac{(1-\alpha)BN\lambda}{\gamma D_\psi} > 0,$$

$$\frac{\partial \phi}{\partial \gamma} = \frac{(\frac{(2\alpha-1)}{1-\alpha} + \frac{\sigma g_c+\rho}{g_c})\gamma^{(2\alpha-1)/(\alpha-1)}F}{\gamma D_\phi} - \frac{(\frac{\alpha}{1-\alpha})\gamma^{\alpha/(\alpha-1)}N_\phi}{\gamma D_\phi^2},$$

$$\frac{\partial \psi}{\partial \alpha} = -\frac{\psi}{1-\alpha} - \frac{(-\sigma g_c - \rho + 2g_c)N_\psi}{[(1-\alpha)(\sigma g_c+\rho) + (2\alpha-1)g_c]D_\psi},$$

$$\frac{\partial \phi}{\partial \alpha} = -\frac{[-\frac{2}{1-\alpha} + \frac{1-2\alpha}{(\alpha-1)^2}]\gamma^{\alpha^2/(\alpha-1)}\ln\gamma F}{D_\phi} - \frac{[\frac{1}{1-\alpha} + \frac{\alpha}{(\alpha-1)^2}]\gamma^{\alpha/(\alpha-1)}\ln\gamma N_\phi}{D_\phi^2} < 0,$$

$$\frac{\partial \psi}{\partial \rho} = -\frac{\psi}{\sigma g_c+\rho} - \frac{(1-\alpha)\psi}{(1-\alpha)(\sigma g_c+\rho) + (2\alpha-1)g_c} < 0,$$

$$\frac{\partial \phi}{\partial \rho} = \frac{\gamma^{(2\alpha-1)/(\alpha-1)}\ln\gamma F}{g_c D_\phi} > 0,$$

$$\frac{\partial \psi}{\partial \sigma} = -\frac{g_c\psi}{\sigma g_c+\rho} - \frac{(1-\alpha)g_c\psi}{(1-\alpha)(\sigma g_c+\rho) + (2\alpha-1)g_c} < 0,$$

$$\frac{\partial \phi}{\partial \sigma} = \frac{\gamma^{(2\alpha-1)/(\alpha-1)}\ln\gamma F}{D_\phi} > 0.$$

Then we determine the sign of $(\frac{\partial \psi}{\partial \xi}\phi + \frac{\partial \phi}{\partial \xi}\psi)$. The results are shown in Table 1.1.

Table 1.1: The Sign of $\frac{\partial g_c}{\partial \xi}$

ξ	N	B	λ	γ	α	ρ	σ
Sign	+	+	+	+	-	-	-

Appendix 1.4: Optimal Tax/Subsidy Scheme

From Appendix 1.3, we have $\frac{\partial g_c}{\partial \alpha} < 0$, Furthermore,

$$\psi(g_p)\phi(g_p)|_{\alpha=0} = +\infty$$

and

$$\psi(g_p)\phi(g_p)|_{\alpha=1} = 0.$$

Therefore, there must exist $\alpha = \alpha^*$ such that $g_c = g_p$.

Consider a tax t levied on unskilled labor, then equation (21) becomes

$$BA\int_0^\infty e^{-r\tau}W_H d\tau = W(1-t).$$

As a result, equation (30) becomes

$$\psi(g_c)\phi(g_c) = 1 - t.$$

Choose $t^* = 1 - \psi(g_p)\phi(g_p)$, then the solution to the above equation gives the desired result: $g_c = g_p$.

Chapter 2

Innovative vs. Imitative R&D and Economic Growth

2.1 Introduction

It has been recognized by more and more economists that technological progress is probably the most important source of economic growth. This can been seen from the literature on endogenous growth. All endogenous growth models base economic growth on technological progress but through different channels. For example, in the first endogenous growth model (Romer (1986)), technological progress was achieved along with physical capital accumulation; Lucas (1988) focused on technological progress through human capital accumulation; Aghion and Howitt (1992) and Grossman and Helpman (1991a) emphasized the importance of industrial innovations; and Dinopoulos (1991), Grossman and Helpman (1991b), Segerstrom (1991) and Davidson and Segerstrom (1993) (hereafter DGSD) considered two channels – innovations and imitations – at the same time. In this paper, we share with the DGSD the same belief that both innovation and imitation are essential to technological progress and thus to economic growth.

Innovation and imitation interact with each other in the process of technological progress. On the one hand, they encourage each other. Successful innovations open up new opportunities for imitations and therefore induce more resources to be spent in imitative R&D, while imitations speed up the spread of the application of innovations. But on the other hand, they also discourage each other. Further innovations render the previous imitations obsolete and thus weaken the incentive for imitators to invest in imitative R&D, and successful imitations increase the product market competition, therefore they discourage the innovators to invest in innovative R&D. While the empirical evidence shows that like innovation, imitation is also an important economic phenomenon, the literature on R&D races pays very little attention to it. In this literature, imitation is either exogenously determined or totally ignored. This paper is an attempt along the line of the DGSD to further understand the interactions of innovation and imitation in the process of promoting technological progress, pushing economic growth and improving welfare, and to see how public policies can influence this process.

We adopt a dynamic general equilibrium framework similar to Segerstrom (1991) and Davidson and Segerstrom (1993), which are due to Grossman and Helpman (1991b). But following Aghion and Howitt (1991) (Appendix 2), we model the processes of innovation and imitation in such a way that innovations and imitations occur randomly and independently across firms, across sectors and over time. In this model, economic growth is driven by the interactions of innovation and imitation, but growth rate is determined by innovation only although both innovation and imitation (in the case where imitations are productive) contribute to welfare. We discuss two types of imitations – rent-seeking imitations and productive imitations. For both types of imitations, we identify the channels through which the investment (i.e. employment)[1] in innovative and imitative R&D affects the values of innovation and imitation. In both cases, the employment in innovative R&D has a "business-stealing" effect on the

[1]Throughout the paper, we use *investment* and *employment* interchangeably.

values of innovation and imitation and a positive competition effect on the value of imitation; the employment in imitative R&D has a negative competition effect on the values of both innovation and imitation.[2] In the case where imitations are of the rent-seeking type, we show that there may exist three stationary equilibria depending on the values of the model's parameters. The first equilibrium involves zero investment in both innovative and imitative R&D. As a result, there is no growth. In the second equilibrium, firms invest only in innovative R&D. Therefore, the model degenerates to an innovation-driven growth model such as Aghion and Howitt (1992) and Grossman and Helpman (1991a). Our focus is on the third equilibrium, in which firms invest in both innovative and imitative R&D. Several interesting results have been obtained. The most striking one may be that subsidizing innovation and taxing imitation are not necessarily equivalent. The effects of these two policies on the investment in innovative R&D are the same: both of them induce more investment in innovative R&D. But their effects on the investment in imitative R&D can be different. While taxing imitation always reduces investment in imitative R&D, subsidizing innovation may or may not reduce investment in imitative R&D depending on the effective employment in innovative R&D relative to that in imitative R&D: if the employment in innovative R&D is relatively high, then, intuitively, subsidization of innovation discourages imitations; but if the effective employment in innovative R&D is relatively low, then the subsidization of innovation not only encourages innovation but also induces imitation.

In the case where imitations are productive, we identify two more channels through which the employment in innovative and imitative R&D affects the values of innovation and imitation. That is, in addition to those effects mentioned above, the employment in both innovative and imitative R&D has a positive and negative dynamic competition effect on the values of both innovation and imitation.[3] We focus

[2] All these effects are explained in section 2.2.

[3] The positive and negative dynamic competition effects are explained in section 2.5.

on the equilibrium with both innovation and imitation. Numerical examples show that an increase in the subsidy to innovative R&D induces more employment in innovative R&D, and it may increase or decrease the employment in imitative R&D; an increase in the subsidy to imitative R&D reduces the employment in innovative R&D, and similar to the subsidy to innovative R&D, it may increase or decrease the employment in imitative R&D.

In both cases, we show through numerical examples that a government subsidy to innovative or imitative R&D may or may not improve welfare.

Closely related to the present paper are the DGSD mentioned above. We adopt the similar basic framework: preferences are similar; production technologies are the same; and as in Dinopoulos (1991), imitations also take the form of producing new varieties. However, the processes of innovation and imitation are modelled in a different way. As in Aghion and Howitt (1991) (Appendix 2), we model innovations and imitations in such a way that innovations and imitations occur randomly and independently across firms, across sectors and over time. The DGSD assumes that both innovative and imitative R&D are targeted to specific sectors. In Grossman and Helpman (1991b), the static Bertrand competition is used to analyze the product markets and imitation is driven by factor price differences across countries, so innovations and imitations can not occur in the same country. In Dinopoulos (1991), Segerstrom (1991) and Davidson and Segerstrom (1993), imitation is assumed to be driven by sharing profit with the innovator (by producing different varieties to compete with the innovator in Dinopoulos (1991) and through collusion in Segerstrom (1991) and Davidson and Segerstrom (1993)), and equilibria are constructed such that at any point in time, some sectors are targeted by innovators and other sectors are targeted by imitators, therefore innovations and imitations can not occur in the same sector at the same time, although they can coexist in the same country. In this model, the way of modelling innovation and imitation leads to a different scenario: at any point in time, each sector has potential innovators and imitators; therefore innovations and imitations can occur in the same sector at the same time. We believe that this is consistent with casual observations.

The rest of this paper is organized as follows. Sections 2.2-2.4 are restricted to the case where imitation is of the rent-seeking type. The next section describes the economic environment and sets up the basic framework. Three stationary equilibria are discussed in section 2.3. Section 2.4 focuses on the equilibrium with both innovation and imitation to analyze the effects of exogenous changes in the model's parameters and of the government policies. In section 2.5, we consider productive imitations. We discuss the welfare properties of the laissez faire equilibrium in section 2.6. Finally, some concluding remarks are given in section 2.7.

2.2 The Model

The basic framework is similar to Segerstrom (1991) and Davidson and Segerstrom (1993) which are due to Grossman and Helpman (1991b). However, following Aghion and Howitt (1991) (Appendix 2), we model the processes of innovation and imitation in such a way that innovations and imitations occur randomly and independently across firms, across sectors and over time.

2.2.1 Preferences

The model economy consists of a continuum of sectors, indexed by i, located on $[0, 1]$ and is populated with identical infinitely-lived individuals with measure N. The representative individual's intertemporal utility function is assumed to be given by

$$U = \int_0^{\infty} e^{-\rho t} u(t) dt, \tag{1}$$

where ρ is the individual's subjective discount rate, and $u(t)$ is the individual's instantaneous utility function which is assumed to take the following form

$$u(t) = \int_0^1 \ln(\sum_{\tau=0}^{q_i} \gamma^{\tau} Z_{i\tau}) di, \tag{2}$$

where τ refers to vintage (quality) τ,[4] q_i is the number of successful innovations in sector i up to time t, $\gamma > 1$ is a measure of the size of innovation which represents the quality improvement of a new product relative to its old counterpart, and $Z_{i\tau}$ is a utility index for consumption of the products of vintage τ in sector i. We assume that this subutility function is

$$Z_{i\tau} = \Omega(M_{i\tau})(M_{i\tau} \prod_{j=1}^{M_{i\tau}} x_{ij\tau}^{\frac{1}{M_{i\tau}}}), \tag{3}$$

where $x_{ij\tau}$ is the consumption of variety j of vintage τ in sector i, $M_{i\tau}$ is the number of varieties of vintage τ in sector i, and $\Omega(M_{i\tau})$ represents the individual's preference for varieties. The function $\Omega(M_{i\tau})$ is assumed to have the following properties: (i) $\Omega'(M_{i\tau}) \geq 0$, that is, the individual at least weakly prefer more varieties (if $\Omega'(M_{i\tau}) = 0$, then imitations are of the rent-seeking type because they do not contribute to welfare; if $\Omega'(M_{i\tau}) > 0$, then imitations are productive because they contribute to welfare.); (ii) $\lim_{M_{i\tau} \to \infty} \Omega(M_{i\tau}) \leq \gamma$, indicating that the individual's preference for varieties is not too strong (or equivalently, the quality improvements are large enough) so that new products can replace old ones; (iii) $\Omega(1) = 1$, this is an assumption without loss of generality. In section 2.3 and section 2.4, we consider the case where $\Omega(M_{i\tau}) = 1$ for all $M_{i\tau}$ (therefore, $\Omega'(M_{i\tau}) = 0$). The case where $\Omega'(M_{i\tau}) > 0$ is left for section 2.5. The functional form (3) is equivalent to assuming that the individuals' preferences over different varieties are highly diversified and the population is large

[4] A product of vintage τ is a product whose quality has been improved τ times since time 0.

so that by the law of large number each variety has the same demand if it has the same price. So (3) represents the "average" individual's preferences over varieties. The representative individual's budget constraint is

$$\int_0^\infty e^{-R(t)} E(t) dt = W_0, \tag{4}$$

where $R(t)$ is the cumulative interest factor, W_0 is the discounted expected life time income at time 0, and $E(t)$ the total expenditure at time t which is given by

$$E(t) = \int_0^1 E_i(t) di = \int_0^1 \sum_{\tau=0}^{q_i} E_{i\tau}(t) di = \int_0^1 (\sum_{\tau=0}^{q_i} \sum_{j=1}^{M_{i\tau}} p_{ij\tau} x_{ij\tau}) di, \tag{5}$$

where $E_i(t)$ is the expenditure on the products of sector i at time t, $E_{i\tau}(t)$ is the expenditure on the varieties of vintage τ in sector i at time t, and $p_{ij\tau}$ is the price (in terms of labor) of variety j of vintage τ in sector i.

Since the individual's preferences defined by (1), (2) and (3) exhibit separability across varieties, across vintages, across sectors and over time, the consumer's problem can be decomposed into four sub-problems: First, given the expenditure on the varieties of vintage τ in sector i, $E_{i\tau}(t)$, the consumer chooses the quantity of each variety, $x_{ij\tau}$, to maximize (3) subject to the constraint, $\sum_{j=1}^{M_{i\tau}} p_{ij\tau} x_{ij\tau} = E_{i\tau}$. The first-order conditions for this maximization problem give the demand functions[5]

$$x_{ij\tau} = \frac{E_{i\tau}}{M_{i\tau} p_{ij\tau}}. \tag{6}$$

Then the indirect utility function associated with vintage τ of sector i is

$$Z_{i\tau} = \Delta_\tau E_{i\tau}, \tag{7}$$

[5] Notice that since the demand functions (6) exhibit a constant price elasticity and unitary expenditure elasticity, they can be aggregated across consumers to obtain aggregate demand functions with exactly the same form with $E_{i\tau}$ being the aggregate expenditure on the varieties of vintage τ in sector i and correspondingly E_i and E being the aggregate expenditure on the products of sector i and the aggregate total aggregate expenditure respectively. So in what follows, we take (6) as aggregate demand functions.

where $\Delta_\tau \equiv \Omega(M_{i\tau})(\prod_{j=1}^{M_{i\tau}} p_{ij\tau}^{-\frac{1}{M_{i\tau}}})$. Second, given the expenditure on the products of sector i, E_i, the consumer allocates it among vintages within sector i. That is, the consumer chooses the expenditure on each vintage, $E_{i\tau}$, to maximize

$$u_i \equiv \ln(\sum_{\tau=0}^{q_i} \gamma^\tau \Delta_\tau E_{i\tau}), \tag{8}$$

subject to $\sum_{\tau=0}^{q_i} E_{i\tau} = E_i$. Since products of different vintages adjusted for quality are perfect substitutes, the consumer chooses the vintage with the highest marginal utility of expenditure. It is easily verified that if the (product replacement) condition:[6] $\gamma\Delta_\tau \geq \Delta_{\tau-1}$, i.e.

$$\gamma\Omega(M_{i\tau})(\prod_{j=1}^{M_{i\tau}} p_{ij\tau}^{-\frac{1}{M_{i\tau}}}) \geq \Omega(M_{i(\tau-1)})(\prod_{j=1}^{M_{i(\tau-1)}} p_{ij(\tau-1)}^{-\frac{1}{M_{i(\tau-1)}}}) \tag{9}$$

holds, then the consumer chooses vintage τ.[7] Assume that once the τth innovation succeeds, the $(\tau-1)$th technology becomes common knowledge, then the prices of the varieties of vintage $(\tau-1)$ are driven to 1 (the marginal cost) when the products of vintage τ become available. We also assume that firms producing different varieties of the same vintage engage in price competition, so they charge the same price $p_{i\tau}$ (i.e. $p_{ij\tau}$=$p_{i\tau}$). Then the condition (9) becomes

$$\frac{\gamma\Omega(M_{i\tau})}{p_{i\tau}} \geq \Omega(M_{i(\tau-1)}) \tag{10}$$

The price $p_{i\tau}$ will be set to satisfy this condition. Then the consumer will always just consume the-state-of-the-art products. Consequently, the indirect utility function associated with sector i is given by

$$u_i = \ln(\gamma^{q_i}\Delta_{q_i}E_i), \tag{11}$$

and, correspondingly, the indirect instantaneous utility function is

$$u(t) = \int_0^1 \ln(\gamma^{q_i}\Delta_{q_i}E_i)di. \tag{12}$$

[6]This product replacement condition is similar to the one in Dinopoulos (1991).

[7]We assume that even the equality holds (then the consumer is indifferent between the products of two vintages), the consumer still consumes only the higher quality vintage.

Third, the consumer allocates the given total expenditure $E(t)$ among the products of different sectors to maximize the instantaneous utility (12). Obviously, we have $E_i(t) = E(t)$. Then the consumer's life time utility is

$$U = \int_0^\infty e^{-\rho t}\{[\int_0^1 \ln(\gamma^{q_i}\Delta_{q_i})di] + \ln E\}dt. \tag{13}$$

Finally, the consumer chooses the time path of the total expenditure $E(t)$ to maximize his life time utility (13) subject to the budget constraint (4), which gives rise to

$$\frac{\dot{E}}{E} = r(t) - \rho, \tag{14}$$

where $\dot{E}$ denotes the time change rate of the total expenditure $E(t)$ and r(t) is the interest rate at time t. We intend to focus on stationary equilibria,[8] so in what follows, all variables' time subscripts will be dropped. Also, the subscripts for varieties, vintages and sectors can be omitted because of the symmetrical structures of preferences and production technologies.[9] This completes the description of the consumer's preferences. Now we turn to technologies and firms' problems.

2.2.2 Technologies

There are three types of productive activities: consumption good production, innovative R&D and imitative R&D. We assume that all these activities require only one input – labour. It is also assumed that each individual is endowed with one unit of labour which is inelastically supplied to one of the above mentioned activities. So the total labour supply is N. Each type of activities is described as follows.

[8]In a stationary equilibrium, the total expenditure E is constant, so the interest rate $r(t)$ must be constant and equal to the individual's subjective discount rate ρ.

[9]The production technologies will be described in the next subsection.

Consumption Good Production

We assume that consumption good producers in all sectors have access to the same constant-returns-to-scale technology with each unit of labour producing one unit of output regardless of quality and variety. But only successful innovators and imitators are able to produce the state-of-the-art products. Since the consumer buys only those products with the lowest quality-adjusted prices (if lower-quality and higher-quality products have the same quality-adjusted price, then we have assumed that the consumer buys only higher-quality products), for each variety, with the assumptions that $\Omega(1) = 1$ and $\Omega'(M) = 0$, the highest price the producer can charge is $p = \gamma$, where labor is taken as numeraire. From the demand functions (6), we know that the profit flow, $\pi(M)$, for each producer in any sector is

$$\pi(M) = (\frac{\gamma - 1}{\gamma})\frac{E}{M}. \tag{15}$$

Note that, given the size of innovation and the consumer's expenditure, the profit flow depends only on the number of producers in that sector.

Innovative R&D

New higher-quality products have to be discovered through innovative R&D before they can be produced. Innovation is assumed to follow the Poisson process. The arrival rate depends on the productivity of innovative R&D, λ_I, and the amount of labour employed, y_I. The technology for innovative R&D is assumed to be constant returns to scale. So the arrival rate is simply $\lambda_I y_I$. It is also assumed that innovations can not be targeted to specific sectors; they occur randomly and independently across firms, across sectors and over time. Once an innovative R&D firm succeeds in discovering a higher quality product in a certain sector, it becomes the sole producer of that sector and enjoys the monopoly profit until either another firm discovers an even higher quality product in the same sector at which time it is driven out of business or until some other imitative R&D firms copy the state-of-the-art product to produce different varieties at which time it has to share the product market with these imitators. Let W_I be the value of an innovative R&D firm and V_I be the value of a successful innovation, then we have the Bellman equation

$$\rho W_I = \max_{y_I > 0}\{\lambda_I y_I V_I - y_I\}. \tag{16}$$

Assuming free entry into innovative R&D, we have $W_I = 0$. Then

$$0 = \max_{y_I > 0}\{\lambda_I y_I V_I - y_I\}. \tag{17}$$

The first-order condition is

$$\lambda_I V_I \leq 1, y_I \geq 0, \text{with at least one equality.} \tag{18}$$

The value of innovation is given by[10]

$$V_I = [\frac{1}{\lambda_C n_C} \ln(1 + \frac{\lambda_C n_C}{\rho + \lambda_I n_I})](\frac{\gamma - 1}{\gamma})E, \tag{19}$$

where λ_C is the productivity parameter for an imitative R&D firm, n_I and n_C are the aggregate labour employment in innovative R&D and imitative R&D respectively. Here, $\lambda_C > \lambda_I$ is assumed to reflect the fact that imitation is easier than innovation.[11]

Notice that both the employment n_I in innovative R&D and the employment n_C in imitative R&D have a negative effect on the value of innovation (see the signs of $\frac{\partial V_I}{\partial n_I}$ and $\frac{\partial V_I}{\partial n_C}$ in Appendix 2.3). That is, more employment in either type of research makes a successful innovation less valuable. The reasons for this are simple: an increase in the employment in innovative R&D shortens the length of time in which the previous innovator can enjoy its monopoly profit (the *"business-stealing" effect*); and an increase in the employment in imitative R&D reduces the innovator's profit flow by increasing the product market competition (the *negative competition effect*).

[10]See Appendix 2.2 for derivation

[11]Mansfield et al (1981) found that the ratio of imitation time to innovation time was about $\frac{2}{3}$.

Imitative R&D

As mentioned above, a successful innovator can not enjoy the monopoly profit forever. Other R&D firms can engage in copying the state-of-the-art products to produce other varieties. Like innovation, imitation is also assumed to follow the Poisson process and occur randomly and independently across firms, across sectors and over time. The process of imitation has the same structure as that of innovation. By employing y_C units of labour, an imitative R&D firm is successful in imitating the state-of-the-art product to produce a new variety with an instantaneous probability $\lambda_C y_C$. That is, the arrival rate of this Poisson process is $\lambda_C y_C$. In each sector, a successful imitator becomes the sole producer of the new variety but shares the product market of that sector with the innovator and other imitators, if any. The processes of innovation and imitation generate a stationary distribution of the type K of sector across sectors.[12] At any point in time, some sectors have one producer (i.e. the innovator), some sectors have two (the innovator and one imitator), some have three (the innovator and two imitators) and so on. The value of a successful imitation depends crucially on this distribution. Let W_C be the value of an imitative R&D firm and V_C be the value of a successful imitation, and assume free entry into imitative R&D, then we have a Bellman equation and a zero profit condition similar to (16) and (17). The first-order condition is:

$$\lambda_C V_C \leq 1, y_C \geq 0, \text{with at least one equality,} \tag{20}$$

where the value of imitation is given by [13]

$$V_C = \frac{\lambda_I n_I}{\rho \lambda_C n_C}[(1 + \frac{\lambda_I n_I}{\lambda_C n_C})\ln(1 + \frac{\lambda_C n_C}{\lambda_I n_I})$$

[12] The type of sector refers to the number of producers (also the number of varieties) in that sector. The type of sector (more precisely, the variable $K - 1$) is geometrically distributed across sectors. See Appendix 2.1 for derivation.

[13] See Appendix 2.2 for derivation.

$$-(1+\frac{\rho+\lambda_I n_I}{\lambda_C n_C})\ln(1+\frac{\lambda_C n_C}{\rho+\lambda_I n_I})](\frac{\gamma-1}{\gamma})E. \tag{21}$$

As has been shown in Appendix 2.4, higher employment n_C in imitative R&D is associated with a lower value of imitation because the negative competition effect implies that higher employment in imitative R&D leads to stronger competition in the product markets. However, the effect of an increase in the employment in innovative R&D on the value of imitation depends on the effective employment $\lambda_I n_I$ in innovative R&D relative to the effective employment $\lambda_C n_C$ in imitative R&D: if the effective employment in innovative R&D is relatively low, then an increase in the employment in innovative R&D raises the value of imitation; if the effective employment in innovative R&D is relatively high, then a further increase in the employment in innovative R&D lowers the value of imitation. This is because an increase in the employment in innovative R&D has two offsetting effects. On the one hand, the increase in the employment in innovative R&D increases the probability with which an imitator can succeed in sectors with single producers (the *positive competition effect*), and therefore increase the profitability of imitation. But on the other hand, the increase in the employment in innovative R&D in the next period also shortens the length of time in which the imitator can enjoy the profit from producing a new variety (the "business-stealing" effect). So the net effect depends on the relative strength of these two forces. When the effective employment in innovative R&D is relatively low, the positive competition effect dominates the "business-stealing" effect, as a result, the value of imitation rises; when the effective employment in innovative R&D is relatively high, the "business-stealing" effect dominates, therefore the value of imitation decreases.

The effects on the values of innovation and imitation of changes in the employment in innovative and imitative R&D are shown in Figures 2.1 and 2.2 and summarized

Table 2.1: The Effects of Changes in n_I and n_C with $\Omega'(M) = 0$

Employ. Increase	Effect on V_I	Effect on V_C
n_I	"Business-stealing" effect $\Rightarrow V_I$ decreases	"Business-stealing" effect $\Rightarrow V_C$ decreases Positive competition effect $\Rightarrow V_C$ increases
n_C	Negative competition effect $\Rightarrow V_I$ decreases	Negative competition effect $\Rightarrow V_C$ decreases

Note: In addition to these effects, increases in n_I and n_C also decrease V_I and V_C by reducing the employment in consumption good production and thus making each producer's profit flow smaller.

in Table 2 1 for future reference.

Labour Market

Assume full employment, we have the labour market clearing condition

$$n_I + n_C + \frac{E}{\gamma} = N, \tag{22}$$

where E/γ is the total employment in consumption good production.

Capital Market

Finally, we assume there exists a perfect capital market. R&D firms borrow funds from this market to pay their researchers and issue risky securities. The equilibrium interest rate r clears the market at each moment in time. Since there is a continuum of sectors and innovations and imitations occur independently across firms and across sectors, individual investors are able to completely diversify away risk by holding a diversified portfolio of securities.

Figure 2.1: The Value of Innovation

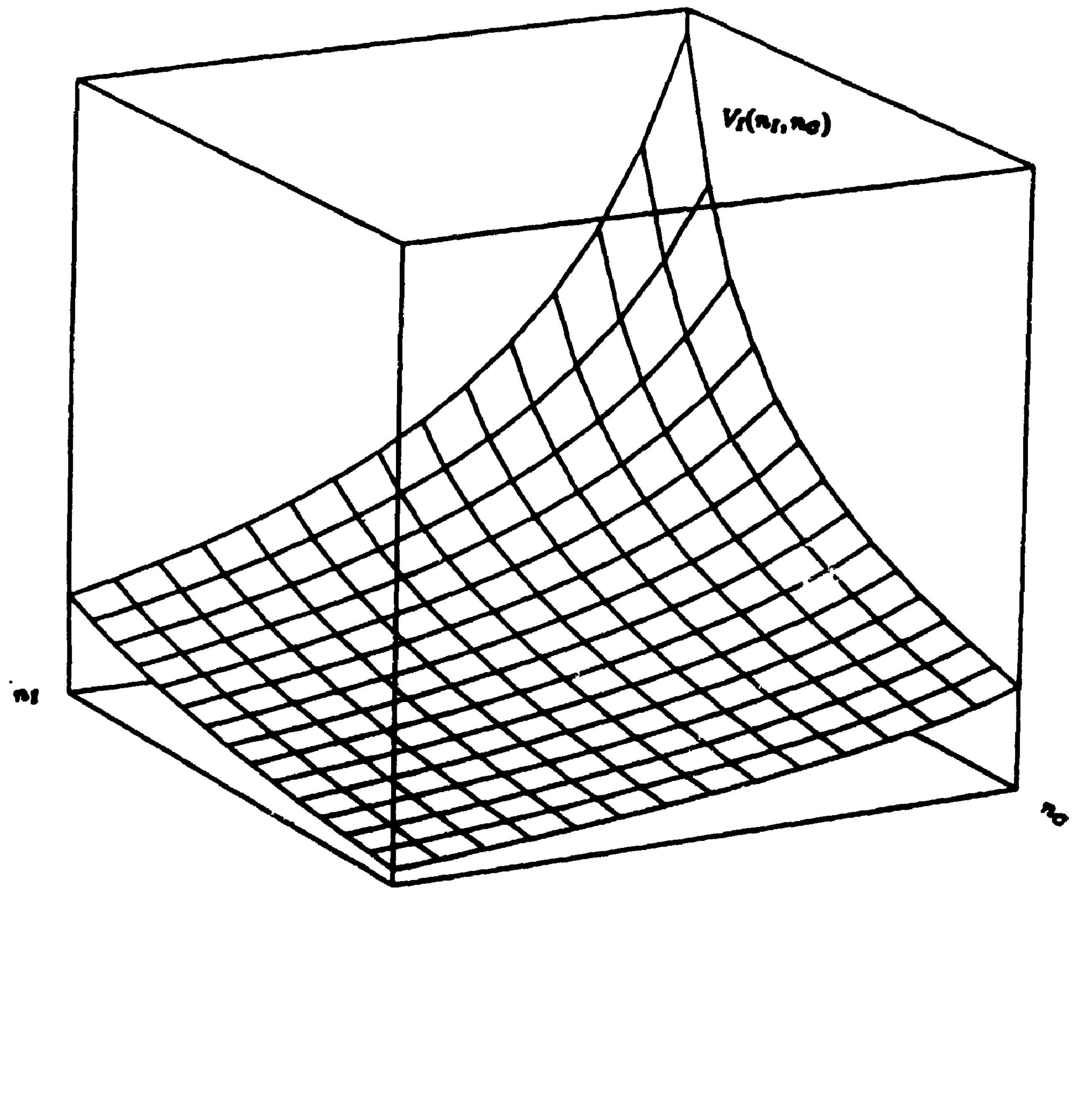

Figure 2.2: The Value of Imitation

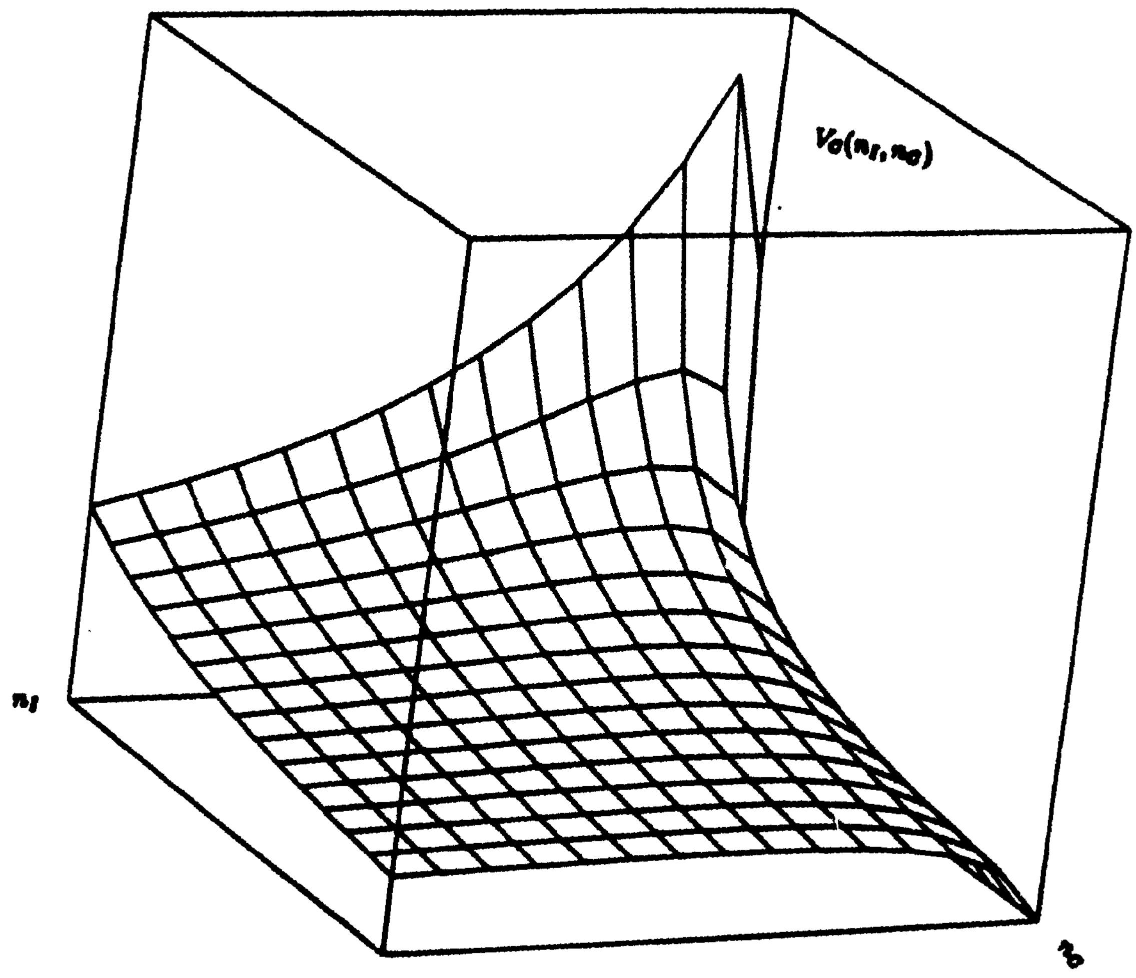

Table 2.2: Three Equilibria ($\Omega'(M) = 0$)

Relative speed λ / Labor endowment N	(0, 1/2)	$[1/2, \infty)$
$(0, \frac{\rho}{\lambda_I(\gamma-1)}]$	$n_I = n_C = 0$	$n_I = n_C = 0$
$(\frac{\rho}{\lambda_I(\gamma-1)}, \infty)$	$n_I > 0, n_C > 0$	$n_I > 0, n_C = 0$

2.3 Stationary Equilibria

We consider only stationary equilibria. In equilibrium, the consumer's total expenditure E, the employment n_I in innovative R&D and the employment n_C in imitative R&D are all constant; the instantaneous interest rate r is also constant and is equal to the individual's subjective discount rate ρ. A stationary equilibrium is described by a constant sequence of $\{n_I, n_C, E\}$ satisfying the following conditions

$$[\frac{\lambda_I}{\lambda_C n_C}\ln(1+\frac{\lambda_C n_C}{\rho+\lambda_I n_I})](\frac{\gamma-1}{\gamma})E \leq 1,\ n_I \geq 0, \text{with at least one equality,} \quad (23)$$

$$\frac{\lambda_I n_I}{\rho n_C}[(1+\frac{\lambda_I n_I}{\lambda_C n_C})\ln(1+\frac{\lambda_C n_C}{\lambda_I n_I}) - (1+\frac{\rho+\lambda_I n_I}{\lambda_C n_C})$$

$$\ln(1+\frac{\lambda_C n_C}{\rho+\lambda_I n_I})](\frac{\gamma-1}{\gamma})E \leq 1,\ n_C \geq 0, \text{with at least one equality,} \quad (24)$$

$$n_I + n_C + \frac{E}{\gamma} = N. \quad (25)$$

There exist three possible equilibria depending on the size of labor endowment and the productivity of innovative R&D relative to that of imitative R&D (see Table 2.2). Now we discuss the conditions for the existence of each equilibrium and the properties of each equilibrium.

2.3.1 Zero R&D Equilibrium

There are two cases in which this equilibrium exists. The first case is that the labor endowment is too small. The second case is a limiting case where imitations are instantaneous. More specifically, letting $\lambda = \lambda_I/\lambda_C$, we have

Proposition 2.1: A zero R&D equilibrium exists if and only if the labor endowment is too small, i.e., $N \leq \frac{\rho}{\lambda_I(\gamma-1)}$, and/ or imitations are instantaneous, i.e., $\lambda = 0$.

The proof is given in Appendix 2.5. The intuitions behind this proposition are as follows.

In the first case, the labor endowment (correspondingly, the total expenditure) is so small that even without imitation, the profit an innovator can make will be too low to cover the cost of R&D. So there is no innovative R&D and therefore there is no imitative R&D.[14] In the second case, imitations are so easy that once an innovation succeeds, an infinitely number of successful imitations follow immediately. Threatened by immediate imitations and thus zero returns to investment, no firms invest in innovative R&D. With no investment in innovative R&D and thus no innovation, firms do not invest in imitative R&D either.

2.3.2 Equilibrium with Innovation only

If the labor endowment is large enough, but imitation is too difficult relative to innovation, then firms invest in innovative R&D but do not invest in imitative R&D. Therefore we have the following proposition.

Proposition 2.2: An equilibrium with innovation only exists if and only if the labor endowment is large enough, i.e., $N > \frac{\rho}{\lambda_I(\gamma-1)}$, but imitation is too difficult, i.e., $\lambda \geq \frac{1}{2}$.

[14]As has been shown ir Appendix 2.5, if $n_I=0$, then $V_C=0$.

We prove this proposition in Appendix 2.6. The intuition is straightforward. If the labor endowment is large enough and innovation is not too difficult relative to imitation, then the profit potential is high enough to attract investment into innovative R&D. But since imitation is not easy enough relative to innovation, the expected benefit will not be enough to cover the cost of investment in imitative R&D. Therefore, in equilibrium, only innovative R&D occurs.

With zero employment in imitative R&D, the value of a successful innovation is $V_I = \frac{(\gamma-1)(N-n_I)}{\rho+\lambda_I n_I}$.[15] The zero profit condition for an R&D firm $\lambda_I V_I = 1$ implies $n_I = (\frac{\gamma-1}{\gamma})N - \frac{\rho}{\lambda_I\gamma}$. This is exactly the result obtained in those endogenous growth models without imitation such as Aghion and Howitt (1992) and Grossman and Helpman (1991a). In equilibrium, the effects of changes in parameters are stated as follows.

Proposition 2.3: In the equilibrium with innovation only, the employment n_I in innovative R&D increases with (i) an increase in the arrival rate parameter λ_I; (ii) an increase in the size γ of innovation; (iii) an increase in labor endowment N and (iv) a decrease in the individual's subjective discount rate ρ. The economic intuitions are explained in Aghion and Howitt (1992) pp. 334-335.

2.3.3 Equilibrium with Both Innovation and Imitation

If we rule out the limiting case (i.e. $\lambda = 0$), then if the labor endowment is large enough, i.e., $N > \frac{\rho}{\lambda_I(\gamma-1)}$, and imitation is not too difficult, i.e., $\lambda < \frac{1}{2}$, then both innovative and imitative R&D occurs and returns to the investment in both types of R&D are equal.

[15] $\lim_{n_C \to 0} V_I = \frac{(\gamma-1)(N-n_I)}{\rho+\lambda_I n_I}$.

In the rest of this section and the next section, we will focus on the equilibrium with both innovation and imitation. Solving (25) for E and substituting it into (23) and (24) (with equalities) gives

$$\frac{1}{\lambda_C}\ln(1+\frac{\lambda_C n_C}{\rho+\lambda_I n_I})$$
$$= \frac{n_I}{\rho}[(1+\frac{\lambda_I n_I}{\lambda_C n_C})\ln(1+\frac{\lambda_C n_C}{\lambda_I n_I})-(1+\frac{\rho+\lambda_I n_I}{\lambda_C n_C})\ln(1+\frac{\lambda_C n_C}{\rho+\lambda_I n_I})], \quad (26)$$

$$[\frac{\lambda_I}{\lambda_C n_C}-\ln(1+\frac{\lambda_C n_C}{\rho+\lambda_I n_I})](\gamma-1)(N-n_I-n_C)=1. \quad (27)$$

The first equation (26) is an equal profitability condition, which says that in equilibrium the expected returns to investment in innovative and imitative R&D are equal. We show in Appendix 2.8 that the employment n_I in innovative R&D and the employment n_C in imitative R&D described by this condition are positively related. That is, more employment in imitative R&D accompanies more employment in innovative R&D in order for this condition to hold.

The intuition behind this can be understood from the effects of changes in the employment in innovative and imitative R&D on the values of innovation and imitation. Let us start from a state in which the equal profitability condition holds (i.e. $\lambda_I V_I = \lambda_C V_C$). Suppose the employment in innovative R&D increases, then, as shown in Table 2.1, the increase in the employment in innovative R&D has two effects: the "business-stealing" effect and the positive competition effect. The "business-stealing" effect tends to lower the values of both innovation and imitation; the positive competition effect does not affect the value of innovation, but it tends to increase the value of imitation. The net result is that the value of innovation decreases more than the value of imitation does. The latter may rise rather than fall if the effective employment in innovative R&D is low relative to the effective employment in imitative R&D!

As a result, the expected return to imitative R&D is higher than that to innovative R&D (i.e. $\lambda_I V_I < \lambda_C V_C$). The higher expected return to imitative R&D creates an incentive for firms to employ more labor. Then the employment in imitative R&D rises and the equality of the expected returns to both type of research is restored. Therefore we have the positive relationship between these two types of employment.

The second equation (27) is a labour market clearing condition. The relationship between these two types of employment, n_I and n_C, are negative as shown in Appendix 2.8. The reason for this negative relationship is obvious. Since the total labour supply N is fixed, then given the employment in the consumption good production, more labour hired by one type of R&D implies less labour available for the other. We show that as long as the total labour endowment N is large enough and $0 < \lambda < \frac{1}{2}$, there always exists an equilibrium with both innovation and imitation. That is,

Proposition 2.4: An equilibrium with both innovations and imitations exists if and only if $N > \frac{\rho}{\lambda_I(\gamma-1)}$ and $0 < \lambda < \frac{1}{2}$. Moreover, the equilibrium is unique if it exists.[16]

The equilibrium properties will be discussed in the next section.

Now we calculate the utility growth rate g. We consider the productive imitation case.[17] Substituting Δ_{q_i} with $p = \gamma\Omega(M)/\Omega(M_{-1})$ and $E_t = E$ into (12) gives

$$u(t) = \ln(\frac{E}{\gamma}) + \int_0^1 q_i \ln \gamma di + \int_0^1 \ln \Omega(M_{-1}) di, \tag{28}$$

where M_{-1} is the number of varieties of vintage $(q_i - 1)$ in sector i. Using the properties

[16]See Appendix 2.8 for proof.

[17]The rent-seeking imitation case can be considered as a special case.

of the Poisson distribution, as in Segerstrom (1991), we have

$$\int_0^1 q_i \ln\gamma di = \sum_{q_i=0}^{\infty} \frac{(\lambda_I n_I t)^{q_i} e^{-\lambda_I n_I t}}{q_i!} q_i \ln\gamma$$

$$= (\lambda_I n_I) t \ln\gamma. \tag{29}$$

And the stationary distribution of the type of sector across sectors implies

$$u_\nu \equiv \int_0^1 \ln\Omega(M_{-1}) di = \sum_{k_{-1}=1}^{\infty} \theta_{k_{-1}} \ln\Omega(k_{-1}), \tag{30}$$

where k_{-1} refers to the type of a sector in the previous generation. Hence, we have

$$u(t) = \ln(\frac{E}{\gamma}) + \lambda_I n_I t \ln\gamma + u_\nu. \tag{31}$$

Then taking the derivative of u(t) with respect to time t gives the growth rate

$$g = (\lambda_I n_I) \ln\gamma. \tag{32}$$

From (32), we know that the growth rate depends positively the productivity λ_I of innovative R&D, the amount of labour n_I employed in innovative R&D and the size γ of innovation. Given the parameters, λ_I and γ, the growth rate depends solely on the equilibrium employment n_I in innovative R&D. Since the growth rate and the employment in innovative R&D change in the same direction, in the next section, the comparative-statics analysis focuses on the effects of exogenous changes in the model's parameters on the equilibrium employment in innovative R&D (and the equilibrium employment in imitative R&D as well).

2.4 Comparative Statics And Public Policies

This section focuses on the equilibrium with both innovation and imitation. We look at the effects of changes in the model's parameters and government policies on the equilibrium employment in innovative and imitative R&D. The effects on welfare will be analyzed through numerical examples in section 2.6. The effects of exogenous changes in the model's parameters ($\lambda_I, \lambda_C, \rho, \gamma$ and N), and of government policies on the employment in innovative and imitative R&D are summarized in the following Propositions.[18]

Proposition 2.5: An increase in the productivity λ_I of innovative R&D increases the equilibrium employment n_I in innovative R&D.

The effect of an increase in the productivity of innovative R&D on the equilibrium employment n_I in innovative R&D is intuitive. The increase in this parameter decreases the marginal cost of innovative R&D by improving the efficiency of employment. It also decreases the marginal benefit to innovative R&D because it increases the rate of creative destruction of the next innovation (reinforcing the "business-stealing" effect). But here the former dominates the latter. Consequently, the equilibrium employment in innovative R&D increases.

Numerical calculations (e.g. Table 2.4 in Appendix 2.9) show that an increase in the productivity λ_I of innovative R&D decreases (increases) the equilibrium employment n_C in imitative R&D if $\theta_1 \geq \theta_1'$ ($\theta_1 < \theta_1'$). [19] The effect on the equilibrium employment in imitative R&D needs some explanations. The intuition may suggest that the increase in the productivity of innovative R&D should always reduce the equilibrium employment in imitative R&D because it reduces the marginal benefit to imitative R&D by increasing the rate of creative destruction. But, as has been explained above, the value of imitation depends not only on the length of time in which the imitator enjoys the profit from being the sole producer of a new variety,

[18]The proofs are given in Appendix 2.9, 2.10 and 2.11.

[19]Here, θ_1' is a critical value of θ_1.

but also on the distribution of the type of sector across sectors. The increase in the productivity of innovative R&D reduces the length of time for the imitator to earn profits, but it also increase the probability of copying the state-of-the-art products from those sectors with single producers (the innovators), and therefore has a larger share of that product market (strengthening the positive competition effect). So when the effective employment $\lambda_I n_I$ in innovative R&D is low relative to the effective employment $\lambda_C n_C$ in imitative R&D, the "business-stealing" effect is more than offset by the positive competition effect. Therefore, the employment in imitative R&D increases rather than decreases. However, when the effective employment $\lambda_I n_I$ in innovative R&D is high relative to the effective employment $\lambda_C n_C$ in imitative R&D, the "business-stealing" effect dominates. Thus, the net effect follows the intuition, that is, the equilibrium employment in imitative R&D decreases as a result of the productivity increase in innovative R&D.

Quite symmetrically, for changes in the productivity of imitative R&D, we have

Proposition 2.6: An increase in the productivity λ_C of imitative R&D decreases the equilibrium employment n_I in innovative R&D.

An increase in the productivity of imitative R&D does not affect the marginal cost of innovative R&D but it decreases the marginal benefit to this activity by reducing the length of time during which the innovator enjoys its monopoly profit (increasing the negative competition effect), so the equilibrium employment in innovative R&D decreases.

It is shown through numerical calculations (e.g. Table 2.5 in Appendix 2.9) that an increase in the productivity λ_C of imitative R&D increases (decreases) the equilibrium employment n_C in imitative R&D if $\theta_1 > \theta_1''$($\theta_1 \leq \theta_1''$). [20] The effect on the equilibrium employment in imitative R&D again depends on two offsetting forces. On the one hand, the increase in the productivity of imitative R&D reduces the marginal cost of imitative R&D by improving the efficiency of employment, but on the other hand, it also decreases the marginal benefit to this activity through the negative competition effect. If the effective employment $\lambda_I n_I$ in innovative R&D is low relative to the effective employment $\lambda_C n_C$ in imitative R&D, the negative competition effect dominates and thus, in equilibrium, the employment in imitative R&D decreases If the effective employment in innovative R&D is relatively high, then the effect of the marginal cost decrease more than offsets the negative competition effect. Therefore, the equilibrium employment in imitative R&D increases.

Proposition 2.7: An increase in the representative individual's subjective discount rate ρ always decreases the equilibrium employment n_C in imitative R&D.

But we show through numerical examples (e.g. Table 2.6 in Appendix 2.9) that an increase in the representative individual's subjective discount rate ρ decreases (increases) the equilibrium employment n_I in innovative R&D if ρ is large (small). The representative individual's subjective discount rate is negatively related to the discounted expected benefits to both innovative and imitative R&D. That is, an increase in the discount rate reduces the discounted expected benefit to each type of research. Since it does not affect the marginal cost of either type, the equilibrium employment in innovative and imitative R&D should decrease responding to lower marginal benefits. This is true if the discount rate is large. However, if the discount rate is small, the increase in the discount rate reduces the equilibrium employment in imitative R&D to such an extent that the negative competition effect outweighs

[20] Like θ_1' , θ_1'' is another critical value of θ_1.

the discount rate effect on the marginal benefit to innovative R&D. As a result, the employment in innovative R&D rises rather than falls.

Proposition 2.8: An increase in the size γ of innovation and the labor endowment N increases the equilibrium employment n_I in innovative R&D and the equilibrium employment n_C in imitative R&D.

This proposition is very intuitive. An increase in the size of innovation or in the labour endowment raises the marginal benefits to both innovative and imitative R&D. The increase in the the size of innovation increases the marginal benefit through charging a higher price, while the increase in the labour endowment increases the marginal benefit by increasing the total expenditure.

Proposition 2.9: An increase in the subsidy s_I to innovative R&D increases the equilibrium employment n_I in innovative R&D.

The effect of an increase in the subsidy to innovative R&D works through the same mechanism discussed in proposition 2.5. It always induces more employment in innovative R&D by reducing the marginal cost of innovative R&D.

Numerical calculations (e.g. Table 2.7 in Appendix 2.9) reveal that an increase in the subsidy s_I to innovative R&D decreases (increases) the equilibrium employment n_C in imitative R&D if $\theta_1 \geq \theta_1'''$ ($\theta_1 < \theta_1'''$). [21] The effect on the equilibrium employment in imitative R&D again depends on the two conflicting effects mentioned above. If the effective employment in innovative R&D is low relative to the effective employment in imitative R&D, the positive competition effect is stronger than

[21] θ_1''' is another critical value of θ_1.

the "business-stealing" effect, thus the equilibrium employment in imitative R&D rises; if the effective employment in innovative R&D is relatively high, the "business-stealing" effect dominates the positive competition effect, therefore the equilibrium employment in imitative R&D decreases.

Proposition 2.10: An increase in the subsidy s_c to imitative R&D decreases the equilibrium employment n_I in innovative R&D and increases the equilibrium employment n_C in imitative R&D.

This is again an intuitive proposition. An increase in the subsidy to imitative R&D does not affect the marginal benefit to imitative R&D, but it decreases the marginal cost, thus it induces more employment in imitative R&D . But the increase in the equilibrium employment in imitative R&D reduces the marginal benefit to innovative R&D through the negative competition effect. Therefore, responding to the decrease in the marginal benefit, the equilibrium employment in innovative R&D declines.

Under most circumstances, taxation and subsidization are two alternatives for public policies. That is, taxing an activity can usually be replaced by subsidizing another activity (or other activities) to achieve the same policy objective. However, this does not apply here. In this model, taxation and subsidization are not always equivalent. If the policy objective is to discourage imitations, then taxing imitative R&D can always satisfy this objective. But subsidizing innovative R&D may do the opposite. So we have the following corollary.

Corollary: Subsidizing innovations and taxing imitations are not necessarily equivalent.

The results regarding the effects of government subsidies will be compared with those in Segerstrom (1991) in the next section.

2.5 Productive Imitation

In this section, we consider the case where individuals prefer more varieties, i.e. $\Omega'(M) > 0$. We restrict our attention to the effects on the levels of innovative and imitative R&D of exogenous changes in the model's parameters and of public policies. To this end, first, we need to calculate the values of innovation and imitation under the new assumption. The comparative statics analysis can be done in the same fashion as in the rent-seeking imitation case.

To derive the value functions of innr ation and imitation under the assumption that $\Omega'(M) > 0$, we need to know the prices innovators and imitators can charge for a new variety or a new product. From the product replacement condition (10), we know that the highest price an innovator or imitator can charge is

$$p = \beta\Omega(M), \tag{33}$$

where the subscripts for sectors and varieties are omitted, $\beta \equiv \gamma/\Omega(M_{-1})$ and M refers to the number of varieties of the-state-of-the-art product while M_{-1} is the number of varieties of the old product (one step down the quality ladder). Notice that with the assumption that $\Omega'(M) > 0$, the price of a new product (or a variety of a new product) depends not only on the size γ of the quality improvement but also on the numbers (M and M_{-1}) of varieties of both the new and old products. As in the case where imitations are of the rent-seeking type, the price of a new product increases with the size of the quality improvement. In addition, the price also increases with the number of varieties of the new product but decreases with the number of varieties of the old one. That is, the larger the number of the old (new) varieties, the lower (higher) the price an innovator or imitator can charge. As a result, the values of both innovation and imitation depend crucially on the distribution of the type of sector across sectors. In this case, the calculation of the values of innovation and imitation turns out to be rather complicated. To illustrate the interactions between innovation and imitation in a manageable way, we assume that the function $\Omega(M)$ takes the following simple form

$$\Omega(M) = \begin{cases} 1, & M = 1, \\ \nu, & M \geq 2, \end{cases} \tag{34}$$

where $1 \leq \nu \leq \gamma$ represents the individual's preference for varieties.[22] As has been shown in Appendix 2.12, the value function of innovation is given by

$$V_I = \{[\frac{1}{\lambda_C n_C} \ln(1+\frac{\lambda_C n_C}{\rho + \lambda_I n_I})][\gamma\nu - \theta_1 - (1-\theta_1)\nu] - \frac{(\nu-1)[\theta_1 + (1-\theta_1)\nu]}{\rho + \lambda_I n_I + \lambda_C n_C}\}\frac{E}{\gamma\nu}, \tag{35}$$

and the value function of imitation is given by

$$\begin{aligned} V_C = & \frac{\lambda_I n_I}{\rho \lambda_C n_C}[(1 + \frac{\lambda_I n_I}{\lambda_C n_C})\ln(1 + \frac{\lambda_C n_C}{\lambda_I n_I}) \\ & -(1 + \frac{\rho + \lambda_I n_I}{\lambda_C n_C})\ln(1 + \frac{\lambda_C n_C}{\rho + \lambda_I n_I})][\gamma\nu - \theta_1 - (1-\theta_1)\nu]\frac{E}{\gamma\nu}, \end{aligned} \tag{36}$$

where $\theta_1 = \frac{\lambda_I n_I}{\lambda_I n_I + \lambda_C n_C}$ is the proportion of type 1 sectors.

Comparing (35) and (36) with (19) and (21) respectively, we can see that the assumption that $\Omega'(M) > 0$ creates two more channels through which the employment in innovative R&D and imitative R&D affects the values of innovation and imitation. In addition to the "business-stealing" effect on the value of innovation V_I and the "business-stealing" effect and the positive competition effect on the value of imitation V_C, identified in section 2.2, an increase in the employment n_I in innovative R&D, on the one hand, increases the profit flows for innovators and imitators by

[22]Note that if $\nu = 1$, then we have the rent-seeking imitation case.

raising the prices they can charge (the *positive dynamic competition effect*[23]) because of (stochastically) fewer old varieties. But on the other hand, the increase in the employment n_I in innovative R&D also affects adversely the profit flows for innovators and imitators by lowering the prices they can charge due to fewer new varieties (the *negative dynamic competition effect*). Correspondingly, besides the negative competition effect on the values of innovation and imitation, an increase in the employment n_C in imitative R&D has another two conflicting effects on the values of innovation and imitation. It decreases (increases) the values of innovation and imitation by increasing (reducing) the competition between the producers of the products of two different generations (qualities) because of more old varieties (more new varieties)(the *negative (positive) dynamic competition effect*) which tends to lower (raise) the prices the producers can charge. For comparisons with Table 2.1 in section 2.2, we list all these effects in Table 2.3.

To close the modified model, we consider the labor market. Since each unit of output of any variety and quality costs one unit of labor to produce, the total employment in consumption good production is simply the total output. Let x denote the output of each variety, Mx is the employment in a single sector. Therefore, the total employment X is[24]

$$X = \int_0^1 x di = \sum_{k_{-1}=1}^{\infty} [\sum_{k=1}^{\infty} (\frac{E}{p})\theta_k]\theta_{k_{-1}}$$

$$= [\theta_1\nu + (1-\theta_1)][\theta_1 + (1-\theta_1)\nu]\frac{E}{\gamma\nu}, \tag{37}$$

[23]We distinguish this effect from the positive competition effect because these two effects work through different channels: the positive dynamic competition effect comes from reducing the competition between the producers of a new product and the producers of the new product's old counterpart (old varieties) while the positive competition effect results from reducing the competition among the producers of different varieties of the same generation (quality). For the same reason, we distinguish the negative dynamic competition effect from the negative competition effect.

[24]See Appendix 2.13 for derivation.

Table 2.3: The Effects of Changes in n_I and n_C with $\Omega'(M) > 0$

Employ.Increase	Effect on V_I	Effect on V_C
n_I	"Business-stealing" effect $\Rightarrow V_I$ decreases Positive dynamic competition effect $\Rightarrow V_I$ increases Negative dynamic competition effect $\Rightarrow V_I$ decreases	"Business-stealing" effect $\Rightarrow V_C$ decreases Positive dynamic competition effect $\Rightarrow V_C$ increases Negative dynamic competition effect $\Rightarrow V_C$ decreases Positive competition effect $\Rightarrow V_C$ increases
n_C	Negative competition effect $\Rightarrow V_I$ decreases Positive dynamic competition effect $\Rightarrow V_I$ increases Negative dynamic competition effect $\Rightarrow V_I$ decreases	Negative competition effect $\Rightarrow V_C$ decreases Positive dynamic competition effect $\Rightarrow V_C$ increases Negative dynamic competition effect $\Rightarrow V_C$ decreases

Note: As in Table 2.1, this table does not list the effects of increases in n_I and n_C on V_I and V_C by reducing the employment in production and thus decreasing the profit flow for each producer.

where θ_k is defined in Appendix 2.1 and $\theta_{k_{-1}}$ is the measure of sectors that are type k in the previous generation. Thus, we have the labor market clearing condition

$$n_I + n_C + [\theta_1\nu + (1-\theta_1)][\theta_1 + (1-\theta_1)\nu]\frac{E}{\gamma\nu} = N. \tag{38}$$

Therefore, the equilibrium conditions are

$$\{[\frac{\lambda_I}{\lambda_C n_C}\ln(1+\frac{\lambda_C n_C}{\rho+\lambda_I n_I})][\gamma\nu - \theta_1 - (1-\theta_1)\nu]$$
$$-\frac{\lambda_I(\nu-1)[\theta_1+(1-\theta_1)\nu]}{\rho+\lambda_I n_I+\lambda_C n_C}\}\frac{E}{\gamma\nu} \leq 1,\ n_I \geq 0,\ \text{with at least one equality,} \tag{39}$$

$$\frac{\lambda_I n_I}{\rho n_C}[(1+\frac{\lambda_I n_I}{\lambda_C n_C})\ln(1+\frac{\lambda_C n_C}{\lambda_I n_I}) - (1+\frac{\rho+\lambda_I n_I}{\lambda_C n_C})\ln(1+\frac{\lambda_C n_C}{\rho+\lambda_I n_I})]$$
$$[\gamma\nu - \theta_1 - (1-\theta_1)\nu]\frac{E}{\gamma\nu} \leq 1,\ n_C \geq 0,\ \text{with at least one equality,} \tag{40}$$

and the above labor market clearing condition (38). In the rest of this section, we will consider equilibria with $n_I > 0$ and $n_C > 0$, which are described by

$$[\frac{1}{\lambda_C n_C}\ln(1+\frac{\lambda_C n_C}{\rho+\lambda_I n_I})][\gamma\nu-\theta_1-(1-\theta_1)\nu]-\frac{(\nu-1)[\theta_1+(1-\theta_1)\nu]}{\rho+\lambda_I n_I+\lambda_C n_C}$$
$$=\frac{n_I}{\rho n_C}[(1+\frac{\lambda_I n_I}{\lambda_C n_C})\ln(1+\frac{\lambda_C n_C}{\lambda_I n_I})-(1+\frac{\rho+\lambda_I n_I}{\lambda_C n_C})\ln(1+\frac{\lambda_C n_C}{\rho+\lambda_I n_I})]$$
$$[\gamma\nu-\theta_1-(1-\theta_1)\nu], \tag{41}$$

and

$$\{[\frac{\lambda_I}{\lambda_C n_C}\ln(1+\frac{\lambda_C n_C}{\rho+\lambda_I n_I})][\gamma\nu-\theta_1-(1-\theta_1)\nu]$$
$$-\frac{(\nu-1)[\theta_1+(1-\theta_1)\nu]}{\rho+\lambda_I n_I+\lambda_C n_C}\}\frac{(N-n_I-n_C)}{[\theta_1\nu+(1-\theta_1)][\theta_1+(1-\theta_1)\nu]}=1. \tag{42}$$

Since innovation and imitation interact with each other in such complicated ways, as shown in Table 2.3, we do not attempt to do comparative-statics and public policy experiments analytically. Instead, numerical examples are used. The numerical examples give us the following results:

For the changes in λ_I, $\lambda_C, \gamma, \rho, N$ and s_I,[25] we have qualitatively the same results as stated in section 2.4 (i.e. the case where imitations are of the rent-seeking type). The effects of changes in ν and s_C on the employment in innovative and imitative R&D are as follows

- An increase in the degree ν of preference for more varieties decreases the employment n_I in innovative R&D and it may increase or decrease the employment n_C in imitative R&D. For example, when $(\lambda_I, \lambda_C, \gamma, \rho, N)$=(0.3, 3, 2, 0.05, 1), an increase in the degree ν of preference for varieties from 1 to 1.1 decreases

[25] With a subsidy s_I to innovative R&D, the equilibrium conditions are given by (41) with its left hand side being multiplied by $\frac{1}{1-s_I}$ and (42) with its right hand side being replaced by $1-s_I$; the equilibrium with a subsidy s_C to imitative R&D is described by (42) and (41) with its right hand side being multiplied by $\frac{1}{1-s_C}$.

the employment n_I in innovative R&D from 0.020,882 to 0.019,221 and also decreases the employment n_C in imitative R&D from 0.190,179 to 0.183,109; when $(\lambda_I, \lambda_C, \gamma, \rho, N)$=(1.3, 3, 2, 0.05, 1), if the degree ν of preference for varieties increases from 1 to 1.1, then the employment n_I in innovative R&D drops from 0.322,868 to 0.263,759, but the employment n_C in imitative R&D rises from 0.156,196 to 0.211,520.

- An increase in the subsidy s_C to imitative R&D decreases the employment n_I in innovative R&D and it may increase or decrease the employment n_C in imitative R&D. For example, when $(\lambda_I, \lambda_C, \gamma, \rho, \nu, N)$=(0.1, 0.3, 2, 0.05, 1.5, 1), an increase in the subsidy s_C to imitative R&D from zero to 0.01 decreases the employment n_I in innovative R&D from 0.011,770 to 0.011,422 and also decreases the employment n_C in imitative R&D from 0.032,141 to 0.031,891. When $(\lambda_I, \lambda_C, \gamma, \rho, \nu, N)$=(0.3, 3, 2, 0.05, 1.5, 1), without a subsidy s_C to imitative R&D, the employment n_I (n_C) in innovative (imitative) R&D is 0.267,863 (0.387,805); with a subsidy $s_C = 0.01$, the employment n_I (n_C) in innovative (imitative) R&D decreases (increases) to 0.262,566 (0.393,147).

The above numerical calculations show that when imitations are productive, the "nonequivalence" result still applies;[26] in addition, taxing imitative R&D may induce more imitations! The reason for this seemingly counterintuitive result is as follows. If imitative R&D is taxed, then the marginal cost of imitative R&D will rise. Meanwhile, the employment in innovative R&D will increase. As listed in Table 2.3, the increase in the employment in innovative R&D has both positive and negative effects on the value of imitation. But with certain sets of parameters, the positive effects dominate the negative effects and increases the marginal benefit to imitative R&D. If the increase in the marginal benefit is greater than that in the marginal cost, then

[26]The "nonequivalence" result refers to the corollary in section 2.4.

the employment in imitative R&D will rise rather than fall.

Let us compare the results regarding government subsidies with those in Segerstrom (1991).[27] Before doing so, it is helpful to calculate the aggregate employment in innovative and imitative R&D and the average growth rate in Segerstrom's model. Using the notations in the present paper, we know that, in his model, $n_I = a_I(\frac{C}{I+C})I$, $n_C = a_C(\frac{I}{I+C})C$ and $g = \frac{1}{a_I}n_I \ln \lambda$, where $a_I(a_C)$ is the amount of labor required for each unit of innovative (imitative) R&D activity, $I(C)$ is the level of innovative (imitative) R&D in a single industry (sector) if this industry is targeted by innovative (imitative) R&D, and λ is equivalent to γ in the present paper. We can easily see that, in Segerstrom's model, the aggregate employment in innovative R&D and that in imitative R&D always change in the same direction. Furthermore, the former is always proportionally greater that the latter because of the assumption that $a_I > a_C$. This property comes directly from the definition of equilibrium. In the present model, the employment levels of innovative and imitative R&D are determined by the costs and benefits of these two activities, therefore, they may or may not change in the same direction.

Now let us see the differences in the effects of government subsidies on the employment in innovative and imitative R&D. While both Segerstrom's and the present model (in both the rent-seeking and productive imitation cases) show that an increase in the subsidy s_I to innovative R&D always increases the employment in innovative R&D and thus the growth rate, these two models have different results concerning the effect on the employment in imitative R&D. Since, in Segerstrom's model, the employment in imitative R&D changes in the same direction as the employment in

[27] As has been introduced at the beginning of the present paper, in Segerstrom's model, a steady state equilibrium is defined in a way such that some industries are targeted by innovative R&D and the others are targeted by imitative R&D, so innovations and imitations can not occur in the same industry at the same time.

innovative R&D does, the employment in imitative increases with the increase in the subsidy s_I. But the present model shows that in both the rent-seeking and productive imitation cases, responding to an increase in the subsidy s_I, the employment in imitative R&D may increase or decrease.

As to the effects of an increase in the subsidy s_C to imitative R&D on the employment in innovative and imitative R&D and the growth rate, the two models also have different results. Segerstrom shows that the effects of an increase in the subsidy to imitative R&D ambiguously affects the employment in innovative and imitative R&D and thus the growth rate; we show that an increase in the subsidy s_C always decreases the employment in innovative R&D and consequently the growth rate in either the rent-seeking imitation or the productive imitation case, and unambiguously increases the employment in imitative R&D when imitations are of the rent-seeking type, although, as in Segerstrom's model, the effect on the employment in imitative R&D can go either way when imitations are productive.

2.6 Welfare

In this section, we analyze the effects of changes in government policies on the consumer's welfare. Before doing so, we need to calculate the consumer's discounted expected life time utility. Since the rent-seeking imitation case can be considered as a limiting case of the productive imitation case, we just need to consider the productive imitation case. From equation (30) in section 2.3 and the assumption in section 2.5 that $\Omega(M) = 1$ if $M = 1$ and $\Omega(M) = \nu$ if $M \geq 2$, we have

$$u_\nu = (1 - \theta_1)\ln \nu \tag{43}$$

Then substituting (43) and (31) into (13) gives

$$U = \frac{1}{\rho}[\ln(\frac{E}{\gamma}) + u_\nu + \frac{\lambda_I n_I \ln\gamma}{\rho}], \tag{44}$$

where θ_1 is defined in Appendix 2.1 and the total expenditure, E, must satisfy the labor market clearing condition (36), or equivalently,

$$\frac{E}{\gamma} = \frac{\nu(N - n_I - n_C)}{[\theta_1\nu + (1-\theta_1)][\theta_1 + (1-\theta_1)\nu]}. \tag{45}$$

Notice that if $\nu = 1$ (i.e. imitations are of the rent-seeking type), then (44) becomes

$$U = \frac{1}{\rho}[\ln(\frac{E}{\gamma}) + \frac{\lambda_I n_I \ln\gamma}{\rho}], \tag{46}$$

where $E/\gamma = (N - n_I - n_C)$.

It is clear from (32) that imitations do not contribute to growth. But we know from (44) that except in the case where $\Omega(M) = 1$ (i.e. $\nu = 1$), they do improve welfare by providing the consumer with more varieties. Since the welfare depends on current consumption E, the number of varieties (indirectly represented by u_ν) and the growth rate g (i.e. $\lambda_I n_I \ln\gamma$), growth and welfare may change in different directions. That is, a higher growth rate does not necessarily implies a higher level of welfare. Therefore, a government policy leading to a higher growth rate may reduce welfare.

As has been shown in sections 2.4 and 2.5, an increase in the subsidy to innovative (imitative) R&D increases (decreases) the employment in innovative R&D, so it speeds up (slows down) economic growth. However, numerical examples show that an increase in the subsidy s_I to innovative R&D has an ambiguous effect on welfare. When $(\lambda_I, \lambda_C, \gamma, \rho, \nu, N)$=(0.1, 0.3, 5, 0.05, 1.5, 1), an increase in the subsidy s_I from zero to 0.01 raises welfare, but when $(\lambda_I, \lambda_C, \gamma, \rho, \nu, N)$=(0.1, 0.3, 2, 0.05, 1.5, 1), the same policy change lowers welfare.

The effect on welfare of an increase in the subsidy s_C to imitative R&D is also ambiguous. For example, when $(\lambda_I, \lambda_C, \gamma, \rho, \nu, N)$=(0.1, 0.21, 2, 0.05, 1.5, 1), an increase in the subsidy s_C from zero to 0.01 increases welfare, but when $(\lambda_I, \lambda_C, \gamma, \rho, \nu, N)$=(0.1, 0.21, 5, 0.05, 1.5, 1), the same policy change does the opposite. These results are quite similar to those obtained by Segerstrom (1991).

2.7 Concluding Remarks

This paper has developed a multi-sector dynamic general equilibrium growth model, in which investments in both innovative and imitative R&D are endogenously determined and economic growth is driven by innovation through its interactions with imitation. Different from the endogenous growth literature, the processes of innovation and imitation are modelled in such a way that innovations and imitations can occur in the same sector at the same time, which we believe is consistent with casual observations.

To understand the relationship between innovation and imitation, we identify the channels through which innovation and imitation interact with each other. We consider both rent-seeking and productive imitations. When imitations are of the rent-seeking type, we show that subsidizing innovative R&D is not necessarily equivalent to taxing imitative R&D; When imitations are productive, we show through numerical examples that, in addition to the "nonequivalence" result, taxing imitative R&D may induce more employment in imitative R&D. We also show that, in both cases, a subsidy to innovative R&D always speeds up economic growth while a subsidy to imitative R&D always does the opposite, but the effects on welfare of both subsidies are ambiguous.

We believe that the model captures many important aspects of a real world economy, especially the complicated interactions between innovation and imitation. It can be extended to study several other issues. For example, patent enforcement can be introduced into the model along the line of Davidson and Segerstrom (1993) to see how patent enforcement affects economic growth through influencing the investment in innovative and imitative R&D. Another promising area is international trade. By extending the model to the context of an open economy, we can analyze the impact of international trade on the investment in innovative and imitative R&D and therefore on a country's growth rate and the roles of government public and commercial policies in stimulating the country's economic growth.

Appendix II

Appendix 2.1: Stationary Distribution of the Type of Sector

This appendix shows that the random variable ($K-1$) follows a geometric distribution. Let θ_k be the proportion of type k sectors, then we have

$$\dot{\theta}_1 = (1-\theta_1)\lambda_I n_I - \theta_1 \lambda_C n_C,$$

$$\dot{\theta}_k = \theta_{k-1}\lambda_C n_C - \theta_k[\lambda_I n_I + \lambda_C n_C], \ \ k = 2, 3, \ldots.$$

Stationary distribution ($\dot{\theta}'s = 0$) implies

$$\theta_1 = \frac{\lambda_I n_I}{\lambda_I n_I + \lambda_C n_C},$$

$$\theta_k = (\frac{\lambda_C n_C}{\lambda_I n_I + \lambda_C n_C})^{k-1}\theta_1, \ \ k = 2, 3, \ldots.$$

Obviously, $(K-1)$ has a geometric distribution.

Appendix 2.2: Derivation of V_I and V_C

We derive the value functions of innovation and imitation in this appendix. Let V_{Ci} be the value of the ith successful imitation, then we have the Bellman equation

$$\rho V_{Ci} = (\frac{\gamma-1}{\gamma})E/(1+i) - \lambda_I n_I V_{Ci} - \lambda_C n_C(V_{Ci} - V_{C(i+1)}),$$

which implies

$$V_{C(i+1)} = \frac{(\rho + \lambda_I n_I + \lambda_C n_C)V_{Ci} - (\frac{\gamma-1}{\gamma})E/(i+1)}{\lambda_C n_C}. \tag{47}$$

Let $\phi_i \equiv \frac{(\frac{\gamma-1}{\gamma})E/(1+i)}{\lambda_C n_C}$ and $\psi \equiv \frac{\rho+\lambda_I n_I + \lambda_C n_C}{\lambda_C n_C} > 1$, then (47) becomes

$$V_{Ci} = \psi V_{C(i-1)} - \phi_{i-1}$$

$$= \psi^{i-1} V_{C1} - \sum_{j=1}^{i-1} \psi^{i-1-j}\phi_j.$$

Or equivalently,

$$\frac{V_{Ci}}{\psi^{i-1}} = V_{C1} - \sum_{j=1}^{i-1} \frac{\phi_j}{\psi^j}. \tag{48}$$

Since the value of a new successful imitation decreases as more imitations succeed and approaches zero as the number of successful imitations goes to infinity, we have

$$\begin{aligned} 0 = \lim_{i\to\infty} \frac{V_{Ci}}{\psi^{i-1}} &= V_{C1} - \lim_{i\to\infty} \sum_{j=1}^{i-1} \frac{\phi_j}{\psi^j} \\ &= V_{C1} - \frac{(\frac{\gamma-1}{\gamma})E}{\lambda_C n_C}[\psi \ln(\frac{\psi}{\psi-1}) - 1], \end{aligned}$$

which gives

$$V_{C1} = [\psi \ln(\frac{\psi}{\psi-1}) - 1]\frac{(\frac{\gamma-1}{\gamma})E}{\lambda_C n_C}. \tag{49}$$

From the Bellman equation for the value of innovation, i.e.

$$\rho V_I = (\frac{\gamma-1}{\gamma})E - \lambda_I n_I V_I - \lambda_C n_C (V_I - V_{C1}),$$

we have

$$V_I = \frac{(\frac{\gamma-1}{\gamma})E + \lambda_C n_C V_{C1}}{\rho + \lambda_I n_I + \lambda_C n_C}. \tag{50}$$

Then substituting V_{C1} with $\psi = \frac{\rho+\lambda_I n_I + \lambda_C n_C}{\lambda_C n_C}$ into (50), we get the value function of innovation

$$V_I = [\frac{1}{\lambda_C n_C} \ln(1 + \frac{\lambda_C n_C}{\rho + \lambda_I n_I})](\frac{\gamma-1}{\gamma})E. \tag{51}$$

Now we derive the value function of imitation. From (48) and (49), we have

$$V_{Ci} = \alpha_i \frac{(\frac{\gamma-1}{\gamma})E}{\lambda_C n_C},$$

where $\alpha_i = \psi^{i-1}[\psi \ln(\frac{\psi}{\psi-1}) - 1] - \sum_{j=1}^{i-1} \psi^{i-1-j} \frac{1}{1+j}$. Let $\delta \equiv \frac{\lambda_C n_C}{\lambda_I n_I + \lambda_C n_C}$, then the assumption that imitations occur randomly and independently across sectors implies

$$V_C = \sum_{i=1}^{\infty} \theta_i V_{Ci}$$

$$= (\sum_{i=1}^{\infty} \delta^{i-1}\alpha_i)\frac{(\frac{\gamma-1}{\gamma})E\theta_1}{\lambda_C n_C}. \tag{52}$$

Using $\ln(1-\frac{1}{\psi}) = -\sum_{i=1}^{\infty}\frac{(1/\psi)^i}{i}$, and $\psi\delta > 1$, we get

$$\sum_{i=1}^{\infty}\delta^{i-1}\alpha_i = \psi(\frac{(1/\psi)^2}{2} + \frac{(1/\psi)^3}{3} + \frac{(1/\psi)^4}{4} + ...)$$
$$+\psi^2\delta(\frac{(1/\psi)^3}{3} + \frac{(1/\psi)^4}{4} + ...)$$
$$+\psi^3\delta^2(\frac{(1/\psi)^4}{4} + \frac{(1/\psi)^5}{5} + ...) + ...$$

$$= \frac{1}{2\psi} + \frac{\delta}{3\psi} + \frac{\delta^2}{4\psi} + ...$$
$$+\frac{1}{3\psi^2} + \frac{\delta}{4\psi^2} + \frac{\delta^2}{5\psi^2} + ...$$
$$+\frac{1}{4\psi^3} + \frac{\delta}{5\psi^3} + \frac{\delta^2}{6\psi^3} + ...$$
$$+...$$

$$= \frac{1}{\psi\delta^2}[-\ln(1-\delta) - \delta]$$
$$+\frac{1}{\psi^2\delta^3}[-\ln(1-\delta) - \delta - \frac{\delta^2}{2}]$$
$$+\frac{1}{\psi^3\delta^4}[-\ln(1-\delta) - \delta - \frac{\delta^2}{2} - \frac{\delta^3}{3}] + ...$$
$$= -[\frac{\ln(1-\delta)}{\delta}\frac{1}{\psi\delta-1} + \frac{1}{\psi\delta-1} + \frac{1}{2\psi}\frac{1}{\psi\delta-1} + \frac{1}{3\psi^2}\frac{1}{\psi\delta-1} + ...]$$
$$= \frac{1}{\psi\delta-1}[\frac{1}{\delta}\ln(\frac{1}{1-\delta}) + \psi\ln(1-\frac{1}{\psi})]. \tag{53}$$

Substituting ψ, δ and (53) into (52) gives

$$V_C = \frac{\lambda_I n_I}{\rho\lambda_C n_C}[(1+\frac{\lambda_I n_I}{\lambda_C n_C})\ln(1+\frac{\lambda_C n_C}{\lambda_I n_I})$$
$$-(1+\frac{\rho+\lambda_I n_I}{\lambda_C n_C})\ln(1+\frac{\lambda_C n_C}{\rho+\lambda_I n_I})](\frac{\gamma-1}{\gamma})E. \tag{54}$$

This is the value function of imitation shown in section 2.2.

Appendix 2.3: Effects of n_I and n_C on V_I

From the value function of innovation (19) (given E), we have

$$\frac{\partial V_I}{\partial n_I} = -\frac{\lambda_I(\frac{\gamma-1}{\gamma})E}{(\rho+\lambda_I n_I)(\rho+\lambda_I n_I+\lambda_C n_C)} < 0,$$

$$\frac{\partial V_I}{\partial n_C} = -\frac{1}{\lambda_C n_C^2}[\ln(1+\frac{\lambda_C n_C}{\rho+\lambda_I n_I}) - \frac{\lambda_C n_C}{\rho+\lambda_I n_I+\lambda_C n_C}](\frac{\gamma-1}{\gamma})E < 0.$$

The second inequality is true because

$$\ln(1+\frac{\lambda_C n_C}{\rho+\lambda_I n_I}) > \frac{\lambda_C n_C}{\rho+\lambda_I n_I+\lambda_C n_C}.$$

Appendix 2.4: Effects of n_I and n_C on V_C

For simplicity, we use the following notations: $x = \lambda_I n_I/\rho$, $s = (\lambda_I n_I + \lambda_C n_C)/\rho$, and $\lambda = \lambda_I/\lambda_C$. Then the value function of imitation (21) can be rewritten as

$$V_C = \frac{x}{\rho(s-x)^2}[s\ln(\frac{s}{x}) - (s+1)\ln(\frac{s+1}{x+1})](\frac{\gamma-1}{\gamma})E. \tag{55}$$

Then given E, we get

$$\frac{\partial V_C}{\partial n_I} = \frac{\lambda_I}{\rho^2(s-x)^2}[(s+x)\ln(\frac{s}{x}) - (x+s+1)\ln(\frac{s+1}{x+1}) - \frac{s-x}{x+1}](\frac{\gamma-1}{\gamma})E.$$

We claim that there exists $0 < x^o < s$ (or equivalently $n_I^0 > 0$) such that

$$\frac{\partial V_I}{\partial n_I} \begin{cases} > 0, & 0 < x < x^o, \\ = 0, & x = x^o, \\ < 0, & s > x > x^o. \end{cases} \tag{56}$$

Note that given the model's parameters, a higher value of x^o is uniquely associated with a higher level of effective employment in innovative R&D.

Proof: Define

$$f(x) \equiv (s+x)\ln\frac{s}{x} - (s+x+1)\ln\frac{s+1}{x+1} - \frac{s-x}{x+1}.$$

Then

$$f'(x) = \ln\frac{s(x+1)}{x(s+1)} - \frac{s-x}{x(x+1)^2},$$

and

$$f''(x) = \frac{1}{x^2(x+1)^3}[(s-x)(3x+1) - x^2(x+1)].$$

Obviously, we have

$$f''(x)\begin{cases} > 0, & 0 < x < x'', \\ < 0, & s > x > x'', \end{cases} \tag{57}$$

where $0 < x'' < 0$ satisfies $f''(x'') = 0$, i.e. $x''[1 + \frac{x''(x''+1)}{3x''+1}] = s$. In addition,

$$\lim_{x\to 0} f'(x) = \lim_{x\to 0}[\ln\frac{s(x+1)}{x(s+1)} - \frac{s-x}{x(x+1)^2}]$$

$$= \lim_{x\to 0}\frac{s-x}{x(x+1)^2}[\frac{\ln\frac{s(x+1)}{x(s+1)}}{\frac{s-x}{x(x+1)^2}} - 1] = -\infty, \tag{58}$$

because

$$\lim_{x\to 0}\frac{\ln\frac{s(x+1)}{x(s+1)}}{\frac{s-x}{x(x+1)^2}} = \lim_{x\to 0}\frac{x(x+1)^2}{x(x+1)+(s-x)(3x+1)} = 0;$$

and

$$\lim_{x\to s} f'(x) = \lim_{x\to s}[\ln\frac{s(x+1)}{x(s+1)} - \frac{s-x}{x(x+1)^2}] = 0. \tag{59}$$

Then (57), (58) and (59) implies

$$f'(x)\begin{cases} < 0, & 0 < x < x', \\ > 0, & s > x > x', \end{cases} \tag{60}$$

where $0 < x' < s$ satisfies $f'(x') = 0$, i.e. $\ln\frac{s(x'+1)}{x'(s+1)} - \frac{s-x'}{x'(x'+1)^2} = 0$. Furthermore,

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}[\ln\frac{s}{x} - (s+x+1)\ln\frac{s+1}{x+1} - \frac{s-x}{x+1}] = \infty, \tag{61}$$

and

$$\lim_{x\to s} f(x) = \lim_{x\to s}[\ln\frac{s}{x} - (s+x+1)\ln\frac{s+1}{x+1} - \frac{s-x}{x+1}] = 0. \tag{62}$$

Combining (61), (62) with (60), we have

$$f(x)\begin{cases} > 0, & 0 < x < x^o, \\ = 0, & x = x^o, \\ < 0, & s > x > x^o, \end{cases} \tag{63}$$

where $0 < x^o < s$. Then (63) is equivalent to (56). **Q.E.D.**

From (55), we also have

$$\frac{\partial V_C}{\partial n_C} = -\frac{\lambda_C x}{\rho^2(s-x)^3}[(x+s)\ln(\frac{s}{x}) - (x+s+2)\ln(\frac{s+1}{x+1})](\frac{\gamma-1}{\gamma})E < 0,$$

because we can show that

$$f(s) \equiv (x+s)\ln(\frac{s}{x}) - (x+s+2)\ln(\frac{s+1}{x+1}) > 0, \forall s > x > 0. \tag{64}$$

Obviously, $f(s = x) = 0$. Moreover, we have

$$f'(s) = \ln[\frac{s(x+1)}{x(s+1)}] - \frac{s-x}{s(s+1)} > 0, \quad \text{because}$$

$$\ln[\frac{s(x+1)}{x(s+1)}] > \frac{s-x}{s(s+1)}.$$

Thus $f(s = x) = 0$ and $f'(s) > 0$ are sufficient for (64) to hold.

Appendix 2.5: Proof of Proposition 2.1

Proof: 1. If $N \leq \frac{\rho}{\lambda_I(\gamma-1)}$ and/or $\lambda = 0$, then $n_I = n_C = 0$.

(i). If $N \leq \frac{\rho}{\lambda_I(\gamma-1)}$, then

$$\max \lambda_I V_I = \lim_{s\to x} \frac{\lambda_I}{\rho(s-x)}(\ln\frac{s+1}{x+1})(\gamma-1)(N-n_I-n_C)$$

$$= \frac{\lambda_I(\gamma-1)(N-n_I)}{\rho+\lambda_I n_I} \leq \frac{\rho}{\rho+\lambda_I n_I} - \frac{\lambda_I(\gamma-1)n_I}{\rho+\lambda_I n_I} < 1, \text{ if } n_I > 0.$$

So, $n_I = 0$. But if $n_I = 0$, then $V_C = 0$ because

$$\lim_{x\to 0} V_C = \lim_{x\to 0} \frac{x}{\rho(s-x)^2}[s\ln(\frac{s}{x})-(s+1)\ln(\frac{s+1}{x+1})](\frac{\gamma-1}{\gamma})(N-n_I-n_C) = 0. \quad (65)$$

Equation (65) is true because

$$\lim_{x\to 0} x\ln\frac{s}{x} = 0 \text{ and } \lim_{x\to 0} x\ln\frac{s+1}{x+1} = 0.$$

Thus $n_C = 0$, therefore, $n_I = n_C = 0$.

(ii). If $\lambda = 0$, then $\lambda_I V_I = 0 < 1$, so $n_I = 0$. From (i), we know that $n_I = 0$ implies $n_C = 0$. Therefore, $n_I = n_C = 0$.

2. If $n_I = n_C = 0$, then $N \le \frac{\rho}{\lambda_I(\gamma-1)}$ and/or $\lambda = 0$.
If $n_C = 0$, then $V_I = \frac{\lambda_I(\gamma-1)(N-n_I)}{\rho+\lambda_I n_I}$. Then if $n_I = 0$, then we have

$$\frac{\lambda_I(\gamma-1)N}{\rho} \le 1, \text{which implies}$$

$N \le \frac{\rho}{\lambda_I(\gamma-1)}$ and/or $\lambda = 0$. **Q.E.D.**

Appendix 2.6: Proof of Proposition 2.2

Proof: 1. If $N > \frac{\rho}{\lambda_I(\gamma-1)}$ and $\lambda \ge 1/2$, then $n_I > 0$ and $n_C = 0$.

(i). Show if $\lambda \ge 1/2$, then $\lambda_I V_I > \lambda_C V_C$ $\forall s > x > 0$. From (19) and (21), we have

$$\lambda_I V_I - \lambda_C V_C = \frac{\lambda_C x s}{\rho(s-x)^2}[(1+\frac{\lambda}{x}+\frac{1-\lambda}{s})\ln\frac{s+1}{x+1} - \ln\frac{s}{x}](\gamma-1)(N-n_I-n_C).$$

Define

$$f(s,x,\lambda) \equiv (1+\frac{\lambda}{x}+\frac{1-\lambda}{s})\ln\frac{s+1}{x+1} - \ln\frac{s}{x}.$$

Since

$$\frac{\partial f}{\partial \lambda} = (\frac{1}{x}-\frac{1}{s})\ln\frac{s+1}{x+1} > 0, \quad \forall s > x > 0,$$

if $f(s,x,1/2) > 0, \forall s > x > 0$, then $f(s,x,\lambda) > 0, \forall \lambda \geq 1/2, s > x > 0$. Now we show that $f(s,x,1/2) > 0, \forall s > x > 0$. Substituting $\lambda = \frac{1}{2}$ gives

$$f(s,x,1/2) = (1 + \frac{1}{2x} + \frac{1}{2s})\ln\frac{s+1}{x+1} - \ln\frac{s}{x}.$$

Because $\lim_{s\to x} f(s,x,1/2) = 0$ and

$$\frac{\partial f(s,x,1/2)}{\partial s} = \frac{1}{2s^2}[\frac{s(s-x)}{x(s+1)} - \ln\frac{s+1}{x+1}]$$

$$> \frac{1}{2s^2}[\frac{s(s-x)}{x(s+1)} - \frac{s-x}{x+1}] = \frac{(s-x)^2}{2xs^2(x+1)(s+1)} > 0,$$

we know that $f(s,x,1/2) > 0, \forall s > x > 0$. Therefore, $f(s,x,\lambda) > 0$, $\forall \lambda \geq 1/2, s > x > 0$.

(ii). From $\lambda_I V_I > \lambda_C V_C$, we have $n_C = 0$. With $n_C = 0$, we know that

$$\lambda_I V_I = \frac{\lambda_I(\gamma-1)(N-n_I)}{\rho + \lambda_I n_I}.$$

Then $\lambda_I V_I = 1$ and $N > \frac{\rho}{\lambda_I(\gamma-1)}$ implies $n_I > 0$.

2. If $n_I > 0$ and $n_C = 0$, then $N > \frac{\rho}{\lambda_I(\gamma-1)}$ and $\lambda \geq 1/2$.

Since $n_I > 0$ and $n_C = 0$, we have

$$\lambda_I V_I = \frac{\lambda_I(\gamma-1)(N-n_I)}{\rho + \lambda_I n_I}.$$

Then $n_I = \frac{\lambda_I(\gamma-1)N-\rho}{\lambda_I\gamma} > 0$ implies $N > \frac{\rho}{\lambda_I(\gamma-1)}$.

Also, $n_C = 0$ implies $\lambda_C V_C \leq 1 = \lambda_I V_I$, $\forall s > x > 0$ (equality holds only if $n_C = 0$). Thus we get

$$\lim_{s\to x}(\lambda V_I - V_C) = [\frac{\lambda}{x+1} - \frac{1}{2(x+1)}](N - n_I) \geq 0, \text{ because}$$

$$\lim_{s\to x}\frac{\lambda}{s-x}\ln\frac{s+1}{x+1} = \frac{\lambda}{x+1},$$

and

$$\lim_{s\to x}\frac{x}{(s-x)^2}[s\ln\frac{s}{x}-(s+1)\ln\frac{s+1}{x+1}]=\frac{1}{2(x+1)}.$$

Therefore, $\lambda \geq 1/2$. **Q.E.D.**

Appendix 2.7: Proof of Three Useful Inequalities

In this appendix, we prove three inequalities which will be used in later proofs. The three inequities are

Inequality 1:

$$(\ln\frac{s+1}{x+1})[s\ln\frac{s}{x}-(s+1)\ln\frac{s+1}{x+1}]>\frac{s-x}{x+1}[(x+1)\ln\frac{s+1}{x+1}-x\ln\frac{s}{x}],\ \forall s>x>0.(66)$$

Inequality 2:

$$(\ln\frac{s+1}{x+1})[x\ln\frac{s}{x}-(x+1)\ln\frac{s+1}{x+1}]>\frac{s-x}{s+1}[(s+1)\ln\frac{s+1}{x+1}-s\ln\frac{s}{x}],\ \forall s>x>0.(67)$$

Inequality 3:

$$(\ln\frac{s+1}{x+1})[(x+s)\ln\frac{s}{x}-(x+s+1)\ln\frac{s+1}{x+1}]>\frac{s-x}{s+1}[(s+1)\ln\frac{s+1}{x+1}-\frac{xs}{x+1}\ln\frac{s}{x}],$$

$$\forall s>x>0. \qquad (68)$$

Proof: We prove Inequality 1 first. Define

$$f(s)\equiv(\ln\frac{s+1}{x+1})[s\ln\frac{s}{x}-(s+1)\ln\frac{s+1}{x+1}]-\frac{s-x}{x+1}[(x+1)\ln\frac{s+1}{x+1}-x\ln\frac{s}{x}].$$

Then we have

$$g(x)\equiv f'(s)$$

$$=\frac{1}{s+1}[s\ln\frac{s}{x}-(s+1)\ln\frac{s+1}{x+1}]+(\ln\frac{s+1}{x+1})[\ln\frac{s}{x}-\ln\frac{s+1}{x+1}]$$

$$-\frac{1}{x+1}[(x+1)\ln\frac{s+1}{x+1} - x\ln\frac{s}{x}] - \frac{s-x}{x+1}[\frac{x+1}{s+1} - \frac{x}{s}],$$

$$h(s) \equiv g'(x)$$

$$= \frac{1}{s+1}[-\frac{s}{x} + \frac{s+1}{x+1}] - \frac{1}{x+1}[\ln\frac{s}{x} - \ln\frac{s+1}{x+1}]$$

$$+(\ln\frac{s+1}{x+1})[-\frac{1}{x} + \frac{1}{x+1}] + \frac{1}{(x+1)^2}[(x+1)\ln\frac{s+1}{x+1} - x\ln\frac{s}{x}]$$

$$+\frac{1+s}{(x+1)^2}[\frac{x+1}{s+1} - \frac{x}{s}] - \frac{s-x}{x+1}[\frac{1}{s+1} - \frac{1}{s}],$$

$$h'(s) = -\frac{1}{(s+1)^2}[-\frac{s}{x} + \frac{s+1}{x+1}] + \frac{2}{s+1}[-\frac{1}{x} + \frac{1}{x+1}] + \frac{2}{(x+1)^2}[\frac{x+1}{s+1} - \frac{x}{s}]$$

$$+\frac{1+s}{(x+1)^2}[-\frac{x+1}{(s+1)^2} + \frac{x}{s^2}] - \frac{s-x}{x+1}[-\frac{1}{(s+1)^2} + \frac{1}{s^2}]$$

$$= -\frac{xs(1+xs+2s) + (s-x)(s^2+x^2+2x^2s+3xs+2x+2s)}{xs^2(x+1)^2(s+1)^2} < 0.$$

Obviously, $f(s=x) = 0$, $g(x=s) = 0$ and $h(s=x) = 0$. Then, $h'(s) < 0$ and $h(s=x) = 0 \Rightarrow h(s) < 0 \Rightarrow g'(x) < 0$ (along with $g(x=s) = 0$) $\Rightarrow g(x) > 0$ $\Rightarrow f'(s) > 0$ (together with $f(s=x) = 0$) $\Rightarrow f(s) > 0$. Therefore, inequality 1 holds.

Inequality 2 is equivalent to

$$(\ln\frac{s+1}{x+1})[s\ln\frac{s}{x} - (s+1)\ln\frac{s+1}{x+1}]$$

$$> [\frac{(x+1)(s+1)(\ln\frac{s+1}{x+1})^2}{(s-x)^2}]\frac{s-x}{x+1}[(x+1)\ln\frac{s+1}{x+1} - x\ln\frac{s}{x}].$$

If we can show that

$$\frac{(x+1)(s+1)(\ln\frac{s+1}{x+1})^2}{(s-x)^2} < 1, \tag{69}$$

then (69) and Inequality 1 ensures that Inequality 2 is also true. But (69) is guaranteed by

$$\ln\frac{s+1}{x+1} < \frac{s-x}{[(x+1)(s+1)]^{1/2}}.$$

We rewrite Inequality 3 as

$$(\ln\frac{s+1}{x+1})[x\ln\frac{s}{x}-(x+1)\ln\frac{s+1}{x+1}]+s(\ln\frac{s+1}{x+1})[\ln\frac{s}{x}-\ln\frac{s+1}{x+1}]$$

$$>\frac{s-x}{s+1}[(s+1)\ln\frac{s+1}{x+1}-s\ln\frac{s}{x}]+\frac{s(s-x)}{(s+1)(x+1)}\ln\frac{s}{x}.$$

Since inequality 2 holds, we just need to show that

$$(\ln\frac{s+1}{x+1})[\ln\frac{s}{x}-\ln\frac{s+1}{x+1}]\geq\frac{s-x}{(s+1)(x+1)}\ln\frac{s}{x}. \tag{70}$$

To show (70), we define

$$f(s)\equiv(\ln\frac{s+1}{x+1})[\ln\frac{s}{x}-\ln\frac{s+1}{x+1}]-\frac{s-x}{(s+1)(x+1)}\ln\frac{s}{x}.$$

Then we have

$$g(x)\equiv f'(s)$$

$$=\frac{1}{s+1}[\ln\frac{s}{x}-\ln\frac{s+1}{x+1}]+(\ln\frac{s+1}{x+1})[\frac{1}{s}-\frac{1}{s+1}]$$

$$-\frac{1}{(s+1)^2}\ln\frac{s}{x}-\frac{s-x}{s(x+1)(s+1)};$$

$$g'(x)=-\frac{(s-x)^2}{xs(x+1)^2(s+1)^2}<0.$$

Moreover, we have $f(s=x)=0$ and $g(x=s)=0$. Then, $g'(x)<0$ and $g(x=s)=0$ $\rightarrow g(x)>0\Rightarrow f'(s)>0$. (along with $f(s=x)=0$) $\Rightarrow f(s)>0$, Hence, inequality 3 also holds. **Q.E.D.**

Appendix 2.8: Proof of Proposition 2.4

This appendix proves the existence of an equilibrium with both innovation and imitation. First, we show that the function n_I defined by (26) is increasing in n_C with $n_I(n_C = 0) = 0$, while the function n_I defined by (27) is decreasing in n_C. Then we prove the necessary and sufficient conditions of Proposition 2.4. Finally, we show that the equilibrium is unique.

Proof: 1. Show that the function n_I defined by (26) has the following properties: $\frac{dn_I}{dn_C} > 0$ and $n_I(n_C = 0) = 0$.

Using the notations introduced in Appendix 2.4, we reformulate (26) as

$$\frac{\lambda}{s-x}\ln\frac{s+1}{x+1} = \frac{x}{(s-x)^2}[s\ln\frac{s}{x} - (s+1)\ln\frac{s+1}{x+1}],$$

which can be simplified to

$$1 + \frac{\lambda}{x} + \frac{1-\lambda}{s} = \frac{\ln(s) - \ln(x)}{\ln(s+1) - \ln(x+1)}. \quad (71)$$

Differentiating (71) with respect to x gives

$$\frac{ds}{dx} = \frac{-\frac{1}{x} + \frac{\lambda}{x^2}\ln(\frac{s+1}{x+1}) + \frac{1}{x+1}(1 + \frac{1}{\lambda} + \frac{1-\lambda}{s})}{-\frac{1}{s} - \frac{1-\lambda}{s^2}\ln(\frac{s+1}{x+1}) + \frac{1}{s+1}(1 + \frac{1}{\lambda} + \frac{1-\lambda}{s})}$$

$$= \frac{\lambda\frac{s}{x}\ln(\frac{s+1}{x+1}) - (1-\lambda)\frac{s-x}{x+1}}{\lambda\frac{s-x}{s+1} - (1-\lambda)\frac{x}{s}\ln(\frac{s+1}{x+1})}. \quad (72)$$

Since $x = \lambda_I n_I/\rho$, $s = (\lambda_I n_I + \lambda_C n_C)/\rho$, $\frac{dn_I}{dn_C} > 0$ is equivalent to $\frac{ds}{dx} > 1$. Three sufficient conditions for $\frac{ds}{dx} > 1$ are

Condition 1: $\lambda\frac{s}{x}\ln(\frac{s+1}{x+1}) - (1-\lambda)\frac{s-x}{x+1} > 0$,

Condition 2: $\lambda\frac{s-x}{s+1} - (1-\lambda)\frac{x}{s}\ln(\frac{s+1}{x+1}) > 0$,

Condition 3: $\lambda\frac{s}{x}\ln(\frac{s+1}{x+1}) - (1-\lambda)\frac{s-x}{x+1} > \lambda\frac{s-x}{s+1} - (1-\lambda)\frac{x}{s}\ln(\frac{s+1}{x+1})$.

From (71), we have

$$\lambda = \frac{x/(s-x)}{\ln(\frac{s+1}{x+1})}[(s)\ln(\frac{s}{x}) - (s+1)\ln(\frac{s+1}{x+1})].$$

Substituting λ into the above three conditions, we can easily verify that the three conditions are equivalent to the three inequalities proven in Appendix 2.7. Therefore, $\frac{dn_I}{dn_C} > 0$. Clearly, $n_I(n_C = 0) = 0$ and $n_C(n_I = 0) = 0$.

2. Show that the function n_I defined by (27) has the property: $\frac{dn_I}{dn_C} < 0$.

Differentiating equation (27), we have

$$\frac{dn_I}{dn_C} = \frac{\ln(\frac{s+1}{x+1}) + \frac{\lambda_I(s-x)(N-n_I-n_C)}{\rho(x+1)(s+1)}}{-\frac{1}{n_C}\ln(\frac{s+1}{x+1})](N-n_I-n_C) - \ln(\frac{s+1}{x+1}) + \frac{\lambda_C(N-n_I-n_C)}{\rho(s+1)}}. \tag{73}$$

Since the numerator is positive, the sign of $\frac{dn_I}{dn_C}$ depends on the denominator. Let $\mathcal{D}$ denote the denominator, then

$$\mathcal{D} = -[\frac{N-n_I}{n_C}\ln(\frac{s+1}{x+1}) - \frac{\lambda_C(N-n_I-n_C)}{\rho(s+1)}] < -[\frac{N-n_I}{n_C}\frac{s-x}{s+1} - \frac{\lambda_C(N-n_I-n_C)}{\rho(s+1)}]$$

$$(\text{because} \ln\frac{s+1}{x+1} > \frac{s-x}{s+1})$$

$$= -\frac{s-x}{s+1} < 0.$$

Hence, $\frac{dn_I}{dn_C} < 0$.

3. Show that $n_I > 0$ and $n_C > 0$ if and only if $N > \frac{\rho}{\lambda_I(\gamma-1)}$ and $0 < \lambda < \frac{1}{2}$.

(i) If $N > \frac{\rho}{\lambda_I(\gamma-1)}$ and $0 < \lambda < \frac{1}{2}$, then $n_I > 0$ and $n_C > 0$.

First, we consider the conditior $0 < \lambda < \frac{1}{2}$. If $0 < \lambda < \frac{1}{2}$, then there always exists a pair of n_I and n_C with $n_I > 0$ and $n_C > 0$ (or equivalently $0 < x < s$) satisfying equation (26). This is because

$$\lim_{x\to 0}\frac{\lambda}{s-x}\ln\frac{s+1}{x+1} = \frac{\lambda}{s}\ln(s+1)$$

$$> \lim_{x\to 0} \frac{x}{(s-x)^2}[s\ln\frac{s}{x} - (s+1)\ln\frac{s+1}{x+1}] = 0$$

and

$$\lim_{x\to s} \frac{\lambda}{s-x}\ln\frac{s+1}{x+1} = \frac{\lambda}{s+1}$$

$$< \lim_{x\to s} \frac{x}{(s-x)^2}[s\ln\frac{s}{x} - (s+1)\ln\frac{s+1}{x+1}] = \frac{1}{2(s+1)}, \text{ if } 0<\lambda<\frac{1}{2}.$$

The condition $N > \frac{\rho}{\lambda_I(\gamma-1)}$ guarantees that the function defined by (27) has the properties: $n_I(n_C = 0) > 0$ and $n_C(n_I = 0) > 0$. This is because if $n_I = 0$, then equation (27) becomes

$$\frac{\lambda_I}{\lambda_C n_C}[\ln(\frac{\rho+\lambda_C n_C}{\rho})](\gamma-1)(N-n_C) = 1. \tag{74}$$

Since $\lim_{n_C\to 0} LHS = \frac{\lambda_I(\gamma-1)N}{\rho}$ and $\frac{\partial LHS}{\partial n_C} < 0$, if $N > \frac{\rho}{\lambda_I(\gamma-1)}$, then $\frac{\lambda_I(\gamma-1)N}{\rho} > 1$. Thus, $n_C > 0$.

Also, if $n_C = 0$, then equation (27) gives

$$n_I = \frac{\lambda_I(\gamma-1)N-\rho}{\lambda_I\gamma}. \tag{75}$$

Then, $N > \frac{\rho}{\lambda_I(\gamma-1)}$ implies that $n_I > 0$.

(ii) If $n_I > 0$ and $n_C > 0$, then $N > \frac{\rho}{\lambda_I(\gamma-1)}$ and $0 < \lambda < \frac{1}{2}$.

From Appendix 2.4, we know that if $n_C > 0$, then $0 < \lambda < \frac{1}{2}$. We also know from (74) and (75) that if $n_I > 0$ and $n_C > 0$, then $N > \frac{\rho}{\lambda_I(\gamma-1)}$.

The uniqueness of the equilibrium is guaranteed by the monotonicity of the functions defined by (26) and (27) **Q.E.D.**

Appendix 2.9: Proof of Propositions 2.5-2.8

We prove the propositions in the comparative steady-state analysis by determining the signs of the derivatives of n_I and n_C with respect to the model's parameters (i.e. $\lambda_I, \lambda_C, \gamma, \rho$ and N). The equilibrium conditions for the model are

$$1 + \frac{\lambda}{x} + \frac{1-\lambda}{s} = \frac{\ln(s) - \ln(x)}{\ln(s+1) - \ln(x+1)}, \tag{76}$$

$$\frac{\lambda}{n_C}[\ln(\frac{s+1}{x+1})](\gamma - 1)(N - n_I - n_C) = 1. \tag{77}$$

Differentiating these two equations with respect to parameter $i (i = \lambda_I, \lambda_C, \gamma, \rho$ and N) gives

$$\begin{bmatrix} a^i_{11} & a^i_{12} \\ a^i_{21} & a^i_{22} \end{bmatrix} \begin{bmatrix} \partial n_I/\partial i \\ \partial n_C/\partial i \end{bmatrix} = \begin{bmatrix} c^i_1 \\ c^i_2 \end{bmatrix},$$

where (the sign of each coefficient is explained at the end of this appendix)

$$a^i_{11} = \lambda_I(a-b)/\rho < 0,$$

$$a^i_{12} = \lambda_C a/\rho > 0,$$

$$a^i_{21} = c < 0,$$

$$a^i_{22} = d < 0,$$

$$c_1^{\lambda_I} = \frac{1}{\lambda_I s}[(\lambda + \frac{(1-\lambda)x}{s})\ln(\frac{s+1}{x+1}) - \frac{(1+\lambda x + (1-\lambda)s)(s-x)}{(s+1)(x+1)}] < 0,$$

$$c_2^{\lambda_I} = \frac{n_I}{\rho}[\frac{s-x}{(s+1)(x+1)} - \frac{1}{x}\ln(\frac{s+1}{x+1})](N - n_I - n_C) < 0,$$

$$c_1^{\lambda_C} = \frac{s-x}{\lambda_C s}[(\frac{\lambda}{x} + \frac{1-\lambda}{s})\ln\frac{s+1}{x+1} - \frac{\lambda(s-x)}{x(s+1)}] > 0,$$

$$c_2^{\lambda_C} = \frac{n_C}{\rho}[\frac{1}{s-x}\ln(\frac{s+1}{x+1}) - \frac{1}{s+1}](N - n_I - n_C) > 0,$$

$$c_1^{\gamma} = 0,$$

$$c_2^{\gamma} = -\frac{1}{\gamma - 1}[\ln(\frac{s+1}{x+1})](N - n_I - n_C) < 0,$$

$$c_1^{\rho} = \frac{1}{\rho}[-(\frac{\lambda}{x} + \frac{1-\lambda}{s})\ln(\frac{s+1}{x+1}) + (1 + \frac{\lambda}{x} + \frac{1-\lambda}{s})\frac{s-x}{(s+1)(x+1)}] < 0,$$

$$c_2^{\rho} = \frac{(s-x)(N - n_I - n_C)}{\rho(s+1)(x+1)} > 0,$$

$$c_1^N = 0,$$

$$c_2^N = -\ln(\frac{s+1}{x+1}) < 0,$$

and where

$$a = -\frac{1}{s} - \frac{1-\lambda}{s^2}\ln(\frac{s+1}{x+1}) + \frac{1}{s+1}(1 + \frac{\lambda}{x} + \frac{1-\lambda}{s}) > 0,$$

$$b = -\frac{1}{x} + \frac{\lambda}{x^2}\ln(\frac{s+1}{x+1}) + \frac{1}{x+1}(1 + \frac{\lambda}{x} + \frac{1-\lambda}{s}) > 0,$$

$$c = -\frac{\lambda_I(s-x)(N - n_I - n_C)}{\rho(s+1)(x+1)} - \ln(\frac{s+1}{x+1}) < 0,$$

$$d = -\frac{1}{n_C}(\ln(\frac{s+1}{x+1}) - \frac{s-x}{s+1})(N - n_I - n_C) - \ln(\frac{s+1}{x+1}) < 0.$$

Then

$$D^{\iota} = \begin{vmatrix} a_{11}^{\iota} & a_{12}^{\iota} \\ a_{21}^{\iota} & a_{22}^{\iota} \end{vmatrix} = a_{11}^{\iota}a_{22}^{\iota} - a_{12}^{\iota}a_{21}^{\iota} > 0,$$

$$D_I^{\lambda_I} = \begin{vmatrix} c_1^{\lambda_I} & a_{12}^{\lambda_I} \\ c_2^{\lambda_I} & a_{22}^{\lambda_I} \end{vmatrix} = c_1^{\lambda_I}a_{22}^{\lambda_I} - a_{12}^{\lambda_I}c_2^{\lambda_I} > 0,$$

$$D_C^{\lambda_I} = \begin{vmatrix} a_{11}^{\lambda_I} & c_1^{\lambda_I} \\ a_{21}^{\lambda_I} & c_2^{\lambda_I} \end{vmatrix} = c_2^{\lambda_I}a_{11}^{\lambda_I} - a_{21}^{\lambda_I}c_1^{\lambda_I},$$

$$D_I^{\lambda_C} = \begin{vmatrix} c_1^{\lambda_C} & a_{12}^{\lambda_C} \\ c_2^{\lambda_C} & a_{22}^{\lambda_C} \end{vmatrix} = c_1^{\lambda_C}a_{22}^{\lambda_C} - a_{12}^{\lambda_C}c_2^{\lambda_C} < 0,$$

$$D_C^{\lambda_C} = \begin{vmatrix} a_{11}^{\lambda_C} & c_1^{\lambda_C} \\ a_{21}^{\lambda_C} & c_2^{\lambda_C} \end{vmatrix} = c_2^{\lambda_C}a_{11}^{\lambda_C} - a_{21}^{\lambda_C}c_1^{\lambda_C},$$

$$D_I^{\gamma} = \begin{vmatrix} c_1^{\gamma} & a_{12}^{\gamma} \\ c_2^{\gamma} & a_{22}^{\gamma} \end{vmatrix} = -c_2^{\gamma}a_{12}^{\gamma} > 0,$$

$$D_C^\gamma = \begin{vmatrix} a_{11}^\gamma & c_1^\gamma \\ a_{21}^\gamma & c_2^\gamma \end{vmatrix} = c_2^\gamma a_{11}^\gamma > 0,$$

$$D_I^\rho = \begin{vmatrix} c_1^\rho & a_{12}^\rho \\ c_2^\rho & a_{22}^\rho \end{vmatrix} = c_1^\rho a_{22}^\rho - c_2^\rho a_{12}^\rho,$$

$$D_C^\rho = \begin{vmatrix} a_{11}^\rho & c_1^\rho \\ a_{21}^\rho & c_2^\rho \end{vmatrix} = c_2^\rho a_{11}^\rho - c_1^\rho a_{21}^\rho < 0,$$

$$D_I^N = \begin{vmatrix} c_1^N & a_{12}^N \\ c_2^N & a_{22}^N \end{vmatrix} = -c_2^N a_{12}^N > 0,$$

$$D_C^N = \begin{vmatrix} a_{11}^N & c_1^N \\ a_{21}^N & c_2^N \end{vmatrix} = c_2^N a_{11}^N > 0.$$

Therefore, we have

$$\frac{\partial n_I}{\partial \lambda_I} = \frac{D_I^{\lambda_I}}{D^{\lambda_I}} > 0, \quad \frac{\partial n_C}{\partial \lambda_I} = \frac{D_C^{\lambda_I}}{D^{\lambda_I}} \text{ (see Table 2.4)};$$

$$\frac{\partial n_I}{\partial \lambda_C} = \frac{D_I^{\lambda_C}}{D^{\lambda_C}} < 0, \quad \frac{\partial n_C}{\partial \lambda_C} = \frac{D_C^{\lambda_C}}{D^{\lambda_C}} \text{ (see Table 2.5)};$$

$$\frac{\partial n_I}{\partial \gamma} = \frac{D_I^{\gamma}}{D^{\gamma}} > 0, \quad \frac{\partial n_C}{\partial \gamma} = \frac{D_C^{\gamma}}{D^{\gamma}} > 0;$$

$$\frac{\partial n_I}{\partial \rho} = \frac{D_I^{\rho}}{D^{\rho}} \text{ (see Table 2.6)}, \quad \frac{\partial n_C}{\partial \rho} = \frac{D_C^{\rho}}{D^{\rho}} < 0;$$

$$\frac{\partial n_I}{\partial N} = \frac{D_I^{N}}{D^{N}} > 0, \quad \frac{\partial n_C}{\partial N} = \frac{D_C^{N}}{D^{N}} > 0.$$

Explanations of the signs of the above coefficients:

- The sign of a_{11}^1 is determined by the size of $(a-b)$. It is easily verified that $a-b<0$ is equivalent to Inequality 3 in Appendix 2.7.
- a_{12}^1 and a has the same sign. It is shown that $a>0$ is equivalent to Inequality 2 in Appendix 2.7.
- The sign of a_{21}^1 (i.e. c) is obvious.

- The sign of a_{22}^{1} (i.e. d) is guaranteed by

$$\ln\frac{s+1}{x+1} > \frac{s-x}{s+1}.$$

- $c_1^{\lambda_I} < 0$ because

$$c_1^{\lambda_I} = \frac{1}{\lambda_I s}[(\lambda + \frac{(1-\lambda)x}{s})\ln(\frac{s+1}{x+1}) - \frac{(1+\lambda x+(1-\lambda)s)(s-x)}{(s+1)(x+1)}]$$

$$< \frac{1}{\lambda_I s}[(\frac{\lambda s+(1-\lambda)x}{s})\frac{s-x}{x+1} - \frac{(1+\lambda x+(1-\lambda)s)(s-x)}{(s+1)(x+1)}]$$

$$(\text{because } \ln\frac{s+1}{x+1} < \frac{s-x}{x+1})$$

$$= -\frac{(s-x)^2}{\lambda_I s^2(x+1)(s+1)}[(1-\lambda)+(1-2\lambda)s] < 0, \text{ if } \lambda < \frac{1}{2}.$$

- $c_2^{\lambda_I} < 0$ because

$$c_2^{\lambda_I} = \frac{n_I}{\rho}[\frac{s-x}{(s+1)(x+1)} - \frac{1}{x}\ln(\frac{s+1}{x+1})](N-n_I-n_C)$$

$$< \frac{n_I}{\rho}[\frac{s-x}{(s+1)(x+1)} - \frac{1}{x}\frac{s-x}{s+1}](N-n_I-n_C)$$

$$= \frac{n_I(s-x)}{\rho(s+1)}[\frac{1}{x+1} - \frac{1}{x}](N-n_I-n_C) < 0.$$

- $c_1^{\lambda_C} > 0$ is proven as follows:

Substituting λ into this inequality, we have

$$\frac{s-x}{\lambda_C s}(\ln\frac{s+1}{x+1})^{-1}[\ln\frac{s+1}{x+1}(\ln\frac{s}{x} - \ln\frac{s+1}{x+1}) - (\frac{s}{s+1}\ln\frac{s}{x} - \ln\frac{s+1}{x+1})] > 0. \quad (78)$$

Define

$$f(x) \equiv \ln\frac{s+1}{x+1}(\ln\frac{s}{x} - \ln\frac{s+1}{x+1}) - (\frac{s}{s+1}\ln\frac{s}{x} - \ln\frac{s+1}{x+1}) > 0, \quad (79)$$

then we need to show that $f(x) > 0$. Two sufficient conditions for $f(x) > 0$ are $f(x = s) = 0$ and $f'(x) < 0$. Let

$$g(s) \equiv f'(x) = -\frac{1}{x+1}[\ln\frac{s}{x} - \ln\frac{s+1}{x+1}]$$

$$+\ln\frac{s+1}{x+1}[-\frac{1}{x} + \frac{1}{x+1}] + \frac{s}{x(s+1)} - \frac{1}{x+1}.$$

Now we need to show $g(s) < 0$, whose sufficient conditions are $g(s = x) = 0$ and $g'(s) < 0$.

$$g'(s) = -[\frac{1}{s(x+1)(s+1)} + \frac{1}{x(x+1)(s+1)} + \frac{1}{x(s+1)^2}] < 0.$$

So $g'(s) < 0$ and $g(s = x) = 0 \Rightarrow g(s) < 0 \Rightarrow f'(x) < 0$ (along with $f(x = s) = 0) \Rightarrow f(x) > 0$.

- $c_2^{\lambda c} > 0$ because

$$c_2^{\lambda c} = \frac{n_C}{\rho}[\frac{1}{s-x}\ln(\frac{s+1}{x+1}) - \frac{1}{s+1}](N - n_I - n_C) > 0$$

$$= \frac{n_C}{\rho(s-x)}[\ln(\frac{s+1}{x+1}) - \frac{s-x}{s+1}](N - n_I - n_C) > 0$$

(again because $\ln\frac{s+1}{x+1} > \frac{s-x}{s+1}$).

- $c_2^{\gamma} < 0$ is obvious.

- $c_1^{\rho} < 0$ because

$$c_1^{\rho} = \frac{1}{\rho}[-(\frac{\lambda}{x} + \frac{1-\lambda}{s})\ln(\frac{s+1}{x+1}) + (1 + \frac{\lambda}{x} + \frac{1-\lambda}{s})\frac{s-x}{(s+1)(x+1)}]$$

$$= -\frac{1}{\rho}[(\ln\frac{s}{x} - \ln\frac{s+1}{x+1}) - \frac{\ln(s/x)}{\ln((s+1)/(x+1))}\frac{s-x}{(x+1)(s+1)}]$$

(because of (76))

Table 2.4: The Effect of λ_I on n_C

λ_I	θ_1	n_I	n_C
0.1	0.004690	0.005263	0.037228
0.2	0.007329	0.013136	0.118613
0.3	0.010861	0.020882	0.190179
0.4	0.015769	0.029701	0.247179
0.5	0.022747	0.040440	0.289560
0.6	0.032864	0.054025	0.317979
0.7	0.047741	0.071538	0.332952
0.8	0.069796	0.094198	0.334777
0.9	0.102510	0.123238	0.323689
1.0	0.150773	0.159777	0.299981
1.1	0.221401	0.204738	0.264000
1.2	0.324015	0.258876	0.216035
1.3	0.472500	0.322808	0.156196
1.4	0.687409	0.397415	0.084336

Note: In Tables 2.4-2.7, the model's parameters which are not shown in the tables are: $(\lambda_I, \lambda_C, \gamma, \rho, N) = (1, 3, 2, 0.05, 1)$.

$$= -\frac{1}{\rho \ln((s+1)/(x+1))}[(\ln\frac{s+1}{x+1})(\ln\frac{s}{x} - \ln\frac{s+1}{x+1}) - \frac{s-x}{(x+1)(s+1)}\ln\frac{s}{x}] < 0$$

(see proof of Inequality 3 in Appendix 2.7).

- $c_2^\rho > 0$ and $c_1^N < 0$ are obvious. **Q.E.D.**

Appendix 2.10: Proof of Proposition 2.9

Proof: With a subsidy s_I to innovative R&D, the equilibrium conditions become

$$1 + \frac{\lambda}{x(1-s_I)} + \frac{1-s_I-\lambda}{s(1-s_I)} = \frac{\ln(s)-\ln(x)}{\ln(s+1)-\ln(x+1)}, \tag{80}$$

$$\frac{\lambda}{n_C}[\ln(\frac{s+1}{x+1})](\gamma-1)(N-n_I-n_C) = 1-s_I. \tag{81}$$

Differentiating (80) and (81) with respect to s_I gives

$$\begin{bmatrix} a_{11}^{s_I} & a_{12}^{s_I} \\ a_{21}^{s_I} & a_{22}^{s_I} \end{bmatrix} \begin{bmatrix} \partial n_I/\partial s_I \\ \partial n_C/\partial s_I \end{bmatrix} = \begin{bmatrix} c_1^{s_I} \\ c_2^{s_I} \end{bmatrix},$$

Table 2.5: The Effect of λ_C on n_C

λ_C	θ_1	n_I	n_C
2.1	0.770232	0.415692	0.059050
2.5	0.329576	0.259223	0.210925
3.0	0.150773	0.159777	0.299981
3.5	0.083149	0.107584	0.338941
4.0	0.051841	0.077494	0.354339
4.5	0.035231	0.058798	0.357810
5.0	0.025498	0.046444	0.355002
5.5	0.019350	0.037858	0.348846
6.0	0.015230	0.031637	0.340934
6.5	0.012338	0.026972	0.332165
7.0	0.010137	0.023373	0.326060
7.5	0.008644	0.020529	0.313922
8.0	0.007420	0.018235	0.304928
8.5	0.006454	0.016353	0.296181
9.0	0.005677	0.014786	0.287739
9.5	0.005043	0.013465	0.279628
10.0	0.004518	0.012338	0.271860

Table 2.6: The Effect of ρ on n_I

ρ	θ_1	n_I	n_C
0.02	0.131826	0.150900	0.331265
0.04	0.144846	0.157289	0.309538
0.06	0.156388	0.161891	0.291100
0.08	0.166838	0.165187	0.274973
0.10	0.176431	0.167469	0.260578
0.12	0.185323	0.168931	0.247539
0.14	0.193626	0.169711	0.235593
0.16	0.201420	0.169911	0.224551
0.18	0.208769	0.169609	0.214272
0.20	0.215723	0.168869	0.204645
0.22	0.222323	0.167740	0.195583
0.24	0.228600	0.166262	0.187014
0.26	0.234587	0.164473	0.178881
0.28	0.240305	0.162399	0.171135
0.30	0.245776	0.160068	0.163736

where

$$a_{11}^{sI} = \lambda_I(a^{sI} - b^{sI})/\rho < 0,$$

$$a_{12}^{sI} = \lambda_C a^{sI}/\rho > 0,$$

$$c_1^{sI} = -\frac{\lambda(s-x)}{xs(1-s_I)^2}\ln(\frac{s+1}{x+1}) < 0,$$

$$a_{21}^{sI} = c < 0,$$

$$a_{22}^{sI} = d < 0,$$

$$c_2^{sI} = -\frac{n_C}{\lambda(\gamma-1)} < 0,$$

and where

$$a^{sI} = -\frac{1}{s} - \frac{1-s_I-\lambda}{s^2(1-s_I)}\ln(\frac{s+1}{x+1}) + \frac{1}{s+1}(1+\frac{\lambda}{x(1-s_I)} + \frac{1-s_I-\lambda}{s(1-s_I)}) > 0,$$

$$b^{sI} = -\frac{1}{x} + \frac{\lambda}{x^2(1-s_I)}\ln(\frac{s+1}{x+1}) + \frac{1}{x+1}(1+\frac{\lambda}{x(1-s_I)} + \frac{1-s_I-\lambda}{s(1-s_I)}) > 0.$$

Note that we consider only small subsidies, so the signs of the above coefficients are guaranteed by Appendix 2.9 and by using continuity arguments. Then

$$D^{sI} = \begin{vmatrix} a_{11}^{sI} & a_{12}^{sI} \\ a_{21}^{sI} & a_{22}^{sI} \end{vmatrix} = a_{11}^{sI}a_{22}^{sI} - a_{12}^{sI}a_{21}^{sI} > 0,$$

$$D_I^{sI} = \begin{vmatrix} c_1^{sI} & a_{12}^{sI} \\ c_2^{sI} & a_{22}^{sI} \end{vmatrix} = c_1^{sI}a_{22}^{sI} - a_{12}^{sI}c_2^{sI} > 0,$$

$$D_C^{sI} = \begin{vmatrix} a_{11}^{sI} & c_1^{sI} \\ a_{21}^{sI} & c_2^{sI} \end{vmatrix} = c_2^{sI}a_{11}^{sI} - a_{21}^{sI}c_1^{sI}.$$

So we have

$$\frac{\partial n_I}{\partial s_I} = \frac{D_I^{sI}}{D^{sI}} > 0, \quad \frac{\partial n_C}{\partial s_I} = \frac{D_C^{sI}}{D^{sI}} \text{ (sec Table 2.7)}. \tag{82}$$

Q.E.D.

Table 2.7: The Effect of s_I on n_C

	$\lambda_C = 2.5$		
s_I	θ_1	n_I	n_C
0.10	0.540869	0.369270	0.125386
0.11	0.571670	0.382967	0.114777
0.12	0.604959	0.397225	0.103756
0.13	0.640994	0.412060	0.092314
0.14	0.680061	0.427489	0.080446
0.15	0.722490	0.443528	0.068144
0.16	0.768656	0.460191	0.055402
0.17	0.818974	0.477492	0.042218
0.18	0.873924	0.495443	0.028590
0.19	0.934059	0.514054	0.014516
	$\lambda_C = 6$		
s_I	θ_1	n_I	n_C
0.10	0.019357	0.042085	0.355341
0.11	0.019890	0.043421	0.356612
0.12	0.020450	0.044824	0.357840
0.13	0.021041	0.046299	0.359020
0.14	0.021664	0.047851	0.360148
0.15	0.022322	0.049484	0.361219
0.16	0.023018	0.051205	0.362229
0.17	0.023754	0.053019	0.363171
0.18	0.024533	0.054934	0.364040
0.19	0.026019	0.056957	0.364830
0.20	0.026945	0.059096	0.365533

Appendix 2.11: Proof of Proposition 2.10

Proof: Let s_C be a subsidy to imitative R&D, then the equilibrium conditions are

$$1+\frac{\lambda(1-s_C)}{x}+\frac{1-\lambda(1-s_C)}{s}=\frac{\ln(s)-\ln(x)}{\ln(s+1)-\ln(x+1)}, \tag{83}$$

$$\frac{\lambda}{n_C}[\ln(\frac{s+1}{x+1})](\gamma-1)(N-n_I-n_C)=1. \tag{84}$$

Differentiating (83) and (84) with respect to s_C, we have

$$\begin{bmatrix} a_{11}^{sc} & a_{12}^{sc} \\ a_{21}^{sc} & a_{22}^{sc} \end{bmatrix} \begin{bmatrix} \partial n_I/\partial s_C \\ \partial n_C/\partial s_C \end{bmatrix} = \begin{bmatrix} c_1^{sc} \\ c_2^{sc} \end{bmatrix},$$

where

$$a_{11}^{sc} = \lambda_I(a^{sc} - b^{sc})/\rho < 0,$$

$$a_{12}^{sc} = \lambda_C a^{sc}/\rho > 0,$$

$$c_1^{sc} = \frac{\lambda(s-x)}{xs}\ln(\frac{s+1}{x+1}) > 0,$$

$$a_{21}^{sc} = c < 0,$$

$$a_{22}^{sc} = d < 0,$$

$$c_2^{sc} = 0,$$

and where

$$a^{sc} = -\frac{1}{s}-\frac{1-\lambda(1-s_C)}{s^2}\ln(\frac{s+1}{x+1})+\frac{1}{s+1}(1+\frac{\lambda(1-s_C)}{x}+\frac{1-\lambda(1-s_C)}{s})$$

$$> 0,$$

$$b^{sc} = -\frac{1}{x}+\frac{\lambda(1-s_C)}{x^2}\ln(\frac{s+1}{x+1})+\frac{1}{x+1}(1+\frac{\lambda(1-s_C)}{x}+\frac{1-\lambda(1-s_C)}{s}) > 0.$$

2 of/de 2

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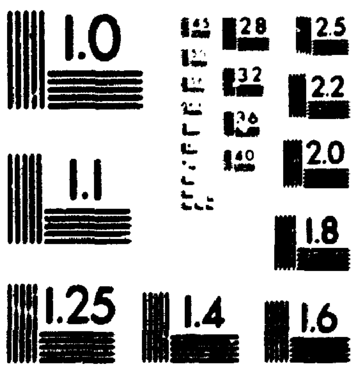

PRECISION℠ RESOLUTION TARGETS

Note that, as in Appendix 2.10, we consider only small subsidies, so by continuity considerations, the signs of the above coefficients are guaranteed by Appendix 2.9. Then

$$D^{sc} = \begin{vmatrix} a_{11}^{sc} & a_{12}^{sc} \\ a_{21}^{sc} & a_{22}^{sc} \end{vmatrix} = a_{11}^{sc} a_{22}^{sc} - a_{12}^{sc} a_{21}^{sc} > 0,$$

$$D_I^{sc} = \begin{vmatrix} c_1^{sc} & a_{12}^{sc} \\ c_2^{sc} & a_{22}^{sc} \end{vmatrix} = c_1^{sc} a_{22}^{sc} < 0,$$

$$D_C^{sc} = \begin{vmatrix} a_{11}^{sc} & c_1^{sc} \\ a_{21}^{sc} & c_2^{sc} \end{vmatrix} = -a_{21}^{sc} c_1^{sc} > 0.$$

So we have

$$\frac{\partial n_I}{\partial s_C} = \frac{D_I^{sc}}{D^{sc}} < 0, \quad \frac{\partial n_C}{\partial s_C} = \frac{D_C^{sc}}{D^{sc}} > 0.$$

Q.E.D.

Appendix 2.12: Derivation of V_I and V_C with $\Omega'(M) > 0$

To derive the value functions of innovation and imitation, we need to know the profit flows for innovators and imitators. Since $p(M_{-1}, M) = \beta\Omega(M)$ and the profit flow for an innovator or imitator is $\pi(M_{-1}, M) = (\frac{p-1}{p})\frac{E}{M}$, with the assumption (34), we have

$$p(M|\text{conditional on } M_{-1}) = \begin{cases} \beta, & M = 1, \text{ with probability } \theta_1, \\ \beta\nu, & M \geq 2, \text{ with probability } (1-\theta_1), \end{cases} \tag{85}$$

and

$$\pi(M|\text{conditional on } M_{-1}) = \begin{cases} (\frac{\beta-1}{\beta})E, & M = 1, \text{ with probability } \theta_1, \\ (\frac{\beta\nu-1}{\beta\nu})\frac{E}{M}, & M \geq 2, \text{ with probability } (1-\theta_1). \end{cases}$$

Then the stationary distribution of the type of sector and $\beta = \gamma/\Omega(M_{-1})$ implies that the price and profit flow are given respectively by

$$p(M_{-1}, M) = \begin{cases} \gamma, & M = 1, M_{-1} = 1, \text{ with probability } \theta_1^2, \\ \frac{\gamma}{\nu}, & M = 1, M_{-1} \geq 2, \text{ with probability } \theta_1(1-\theta_1), \\ \gamma\nu, & M \geq 2, M_{-1} = 1, \text{ with probability } \theta_1(1-\theta_1), \\ \gamma, & M \geq 2, M_{-1} \geq 2, \text{ with probability } (1-\theta_1)^2, \end{cases} \tag{86}$$

and

$$\pi(M_{-1}, M) = \begin{cases} (\frac{\gamma-1}{\gamma})E, & M = 1,\ M_{-1} = 1, \text{ with probability } \theta_1^2, \\ (\frac{\gamma-\nu}{\gamma})E, & M = 1,\ M_{-1} \geq 2, \text{ with probability } \theta_1(1-\theta_1), \\ (\frac{\gamma\nu-1}{\gamma\nu})\frac{E}{M}, & M \geq 2,\ M_{-1} = 1, \text{ with probability } \theta_1(1-\theta_1), \\ (\frac{\gamma-1}{\gamma})\frac{E}{M}, & M \geq 2,\ M_{-1} \geq 2, \text{ with probability } (1-\theta_1)^2. \end{cases}$$

Let $V_{Ci}(k_{-1})$ be the value of ith imitation that occurs in a type k_{-1} sector.[28] Then we have the Bellman equation

$$\rho V_{Ci}(k_{-1}) = (\frac{\beta\nu - 1}{\beta\nu})E/(1+i) - \lambda_I n_I V_{Ci}(k_{-1}) - \lambda_C n_C [V_{Ci}(k_{-1}) - V_{C(i+1)}(k_{-1})].$$

Following the same calculation procedure as in Appendix 2.2, we get the value function of the first imitation that occurs in a type k_{-1} sector

$$V_{C1}(k_{-1}) = [\psi \ln(\frac{\psi}{\psi - 1}) - 1]\frac{(\frac{\beta\nu-1}{\beta\nu})E}{\lambda_C n_C}, \tag{87}$$

where $\psi \equiv \frac{\rho+\lambda_I n_I+\lambda_C n_C}{\lambda_C n_C}$. Let $V_I(k_{-1})$ be the value function of innovation that occurs in a type k_{-1} sector, then we have the Bellman equation

$$\rho V_I(k_{-1}) = (\frac{\beta - 1}{\beta})E - \lambda_I n_I V_I(k_{-1}) - \lambda_C n_C [V_I(k_{-1}) - V_{C1}(k_{-1})],$$

which, along with (87), gives

$$V_I(k_{-1}) = \{[\frac{1}{\lambda_C n_C} \ln(1 + \frac{\lambda_C n_C}{\rho + \lambda_I n_I})](\frac{\beta\nu - 1}{\beta\nu}) - \frac{1}{\rho + \lambda_I n_I + \lambda_C n_C}(\frac{\nu - 1}{\beta\nu})\}E. \tag{88}$$

By assumption, innovations occur randomly and independently across sectors and over time, so the value function of innovation is given by

$$V_I = \sum_{k_{-1}=1}^{\infty} V_I(k_{-1})\theta_{k_{-1}} = \theta_1 V_I(1) + (1-\theta_1)V_I(2). \tag{89}$$

Substituting $V_I(1)$ and $V_I(2)$ given by (88) into (89) and rearranging it gives the value function of innovation (35).

---

[28] A type k_{-1} sector is a sector which is a type k sector in the previous generation.

To derive the value function V_C, let $V_C(k_{-1})$ denote the value of an imitation that occurs in a type k_{-1} sector. Then following exactly the procedure of deriving the value function in the rent-seeking imitation case, we have

$$V_C(k_{-1}) = \frac{\lambda_I n_I}{\rho \lambda_C n_C}[(1 + \frac{\lambda_I n_I}{\lambda_C n_C})\ln(1 + \frac{\lambda_C n_C}{\lambda_I n_I})$$

$$-(1 + \frac{\rho + \lambda_I n_I}{\lambda_C n_C})\ln(1 + \frac{\lambda_C n_C}{\rho + \lambda_I n_I})](\frac{\beta\iota - 1}{\beta\nu})E. \qquad (90)$$

Since it is assumed that imitations also occur randomly and independently across sectors and over time, we get the value function of imitation by taking the expectation of $V_C(k_{-1})$. That is,

$$V_C = \sum_{k_{-1}=1}^{\infty} V_C(k_{-1})\theta_{k_{-1}} = \theta_1 V_C(1) + (1-\theta_1)V_C(2), \qquad (91)$$

which, together with (90), gives (36). Notice that if $\nu = 1$, then (35) and (36) are respectively the same as (19) and (21) in section 2.2.

Appendix 2.13: Derivation of Equation (37)

From (85) and (86) in Appendix 2.12, we have

$$X = \sum_{k_{-1}=1}^{\infty} [\sum_{k=1}^{\infty}(\frac{E}{p})\theta_k]\theta_{k_{-1}}$$

$$= \sum_{k_{-1}=1}^{\infty} [\theta_1 \frac{1}{\beta} + (1-\theta_1)\frac{1}{\beta\nu}]\theta_{k_{-1}} E$$

$$= \{\theta_1[\theta_1 \frac{1}{\gamma} + (1-\theta_1)\frac{\nu}{\gamma}] + (1-\theta_1)[\theta_1 \frac{1}{\gamma\nu} + (1-\theta_1)\frac{1}{\gamma}]\}E \qquad (92)$$

Rearranging (92) gives (37).

Chapter 3

R&D and Growth in Open Economies

3.1 Introduction

Along with the rapid spread of endogenous growth theory, many studies have been devoted to understanding the the relationship between a country's interactions with the rest of the world and its economic growth.[1] These studies have identified various channels through which international interactions influence a country's growth performance. The objective of this paper is to further understand how innovation and imitation interact with each other in an open economy context and how each country's policies influence the world's innovative and imitative R&D which in turn affect the world economic growth. However, we have a different focus. We focus on the impact of patent protection and other government policies on a country's R&D activities and its resulting economic growth rate, which we believe has not yet received enough attention in the new growth literature.[2]

We extend the basic model in the second essay to a two-country world economy.

[1]See, for example, Dinopoulos, Oehmke and Segerstrom (1990), Feenstra (1990), Grossman and Helpman (1989, 1990 and 1991), Lucas (1988), Romer (1990), Segerstrom, Anant and Dinopoulos (1990), Stockey (1988), Young (1990, 1992) and Backus, Kehoe and Kehoe (1991).

[2]Davidson and Segerstrom (1993) analyses the impact of patent protection on economic growth and welfare in a closed economy context.

In the open economy context, innovation and imitation are still treated symmetrically. Both innovation and imitation are intentional and costly. As mentioned above, we will focus on the impact of patent protection and other government policies. But unlike the patent race literature, where the length and width of a patent are considered continuous, we look at discrete cases instead. We compare three scenarios with different patent enforcement considerations. The first one describes a situation where neither country has patent laws (We refer this case to NPP (No Patent Protectio .)). In this case, imitators can imitate either country's innovations. The second scenario involves asymmetric patent protection (\PP). That is, each country protects only its own innovations; imitators are not allowed to copy their own country's innovations, but they imitate foreign country's innovations. In the last scenario, we assume that there is an agreement between the two countries cuncerned, under which each country protects not only its own innovations but also the other country's innovations (symmetric patent protection (SPP)). As a result, there is no imitation.

We use numerical examples to compare the world's investment in innovative R&D and growth rates under the three different patent protection assumptions. We find that stronger patent protection leads to a higher world growth rate. Furthermore, we will show that the equilibrium under the symmetric patent protection assumption is the same as the integrated economy equilibrium. We also look at the effects of government subsidies (under the APP assumption), through a simple calibration exercise. The calibration exercise shows that a subsidy to innovation induces more investment in innovative R&D and thus faster world economic growth while a subsidy to imitation does the opposite.

The rest of the paper is organized as follows. The next section modifies the basic model in the second essay to allow the consideration of two countries. Section 3.3

discusses the equilibrium conditions under the three different patent protection assumptions. We compare the three different scenarios and perform government policy experiments in section 3.4. Finally, section 3.5 provides some concluding remarks.

3.2 The Model

We consider a world consisting of two countries A and B. Each country's economy has the same structure as the closed economy described in the second essay. That is, each country performs three activities - consumption good production, innovative R&D and imitative R&D. In each country, some firms invest in innovative R&D to discover new technologies for producing higher quality products, and some other firms invest in imitative R&D to copy the-state-of-the-art technologies for producing different varieties. Only those successful firms can produce the-state-of-the-art products. Once technologies are obtained, all producers can produce one unit of output by using one unit of labor regardless of quality and variety. The following two subsections describe the representative individual's preferences and technologies available to the three activities.

3.2.1 Preferences

We assume that the world economy consists of a continuum of sectors, indexed by i, located on $[0,1]$. Country j is populated with identical infinitely-lived individuals with measure $\bar{N}^j$, $j = A, B$. Individuals in both countries consume the same set of products and share the same preferences. The representative individual's utility function, budget constraint, demand functions as well as the product replacement condition are described in Section 2.2.1 in the second essay. However, we consider only the case where imitation is of the rent-seeking type (i.e. $\Omega(M) = 1$). For convenience, we list the relevant equations below.

$$U = \int_0^\infty e^{-\rho t} u(t) dt, \quad \text{(intertemporal utility function)} \tag{1}$$

$$u(t) = \int_0^1 \ln(\sum_{\tau=0}^{q_i} \gamma^\tau Z_{i\tau}) di, \quad \text{(instantaneous utility function)} \tag{2}$$

$$Z_{i\tau} = \Omega(M_{i\tau})(M_{i\tau} \prod_{j=1}^{M_{i\tau}} x_{ij\tau}^{\frac{1}{M_{i\tau}}}), \;\; \Omega(M_{i\tau}) = 1,$$

$$\text{(utility index for consumption of good } i\text{, quality } \tau) \tag{3}$$

$$\int_0^\infty e^{-R(t)} E(t) dt = W_0, \quad \text{(life time budget constraint)} \tag{4}$$

$$E(t) = \int_0^1 E_i(t) di = \int_0^1 \sum_{\tau=0}^{q_i} E_{i\tau}(t) di = \int_0^1 (\sum_{\tau=0}^{q_i} \sum_{j=1}^{M_{i\tau}} p_{ij\tau} x_{ij\tau}) di,$$

$$\text{(expenditure composition)} \tag{6}$$

$$x_{ij\tau} = \frac{E_{i\tau}}{M_{i\tau} p_{ij\tau}}. \quad \text{(demand for product } i\text{, variety } j\text{, quality } \tau) \tag{7}$$

$$\gamma(\prod_{j=1}^{M_{i\tau}} p_{ij\tau}^{-\frac{1}{M_{i\tau}}}) \geq (\prod_{j=1}^{M_{i(\tau-1)}} p_{ij(\tau-1)}^{-\frac{1}{M_{i(\tau-1)}}}) \quad \text{(product replacement condition)} \tag{8}$$

$$\frac{\dot{E}}{E} = r(t) - \rho, \quad \text{(time path of expenditure)} \tag{9}$$

where ρ is the individual's subjective discount rate, τ refers to vintage (quality) τ,[3] q_i is the number of successful innovations in sector i up to time t, $\gamma > 1$ is a measure of the size of innovation (the quality improvement of a new product relative to its old counterpart), $x_{ij\tau}$ is the consumption of variety j of vintage τ of sector i, $M_{i\tau}$ is the number of varieties of vintage τ in sector i, $\Omega(M_{i\tau})$ represents the individual's preference for varieties (here, $\Omega(M_{i\tau}) = 1$), $R(t)$ is the cumulative interest factor, W_0 is the discounted expected life time income at time 0, $E(t)$ the total expenditure at time t, $E_i(t)$ is the expenditure on the products of sector i at time t, $E_{i\tau}(t)$ is the expenditure on the varieties of vintage τ in sector i at time t, $p_{ij\tau}$ is the price of variety j

[3] A product of vintage τ is a product whose quality has been improved τ times since time 0.

of vintage τ in sector i, $\dot{E}$ denotes the time change rate of the total expenditure $E(t)$ and r(t) is the interest rate at time t. We intend to focus on stationary equilibria,[4] so in what follows, all variables' time subscripts will be dropped. Also, the subscripts for varieties, vintages and sectors can be omitted because of the symmetrical structures of preferences and production technologies.

Note that since the demand functions in (7) exhibit a constant price elasticity and a unitary expenditure elasticity, they can be aggregated across consumers of the world to obtain aggregate demand functions with exactly the same form with E being the world aggregate expenditure.

3.2.2 Technologies

We assume that all the three activities (consumption good production, innovative R&D and imitative R&D) use labour as their only input. It is also assumed that each individual is endowed with one unit of labour which is inelastically supplied to one of the above mentioned activities. So the total labour supply is N^j. Each type of activities is described as follows.

Consumption Good Production

We assume that all consumption good producers have access to the same constant-return-to-scale technology with each unit of labour producing one unit of output regardless of quality and variety. We also assume that all goods are freely tradable internationally so all producers have access to the world market. But only successful innovators and imitators are able to produce the state-of-the-art products. It is

[4]In a stationary equilibrium, the total expenditure E is constant, so the interest rate $r(t)$ must be constant and equal to the individual's subjective discount rate ρ.

assumed that old technologies are the common knowledge of both countries.[5] Since the consumer buys only those products with the lowest quality-adjusted prices (if lower-quality and higher-quality products have the same quality-adjusted price, then we have assumed that the consumer buys only higher-quality products), for each variety, the highest price the producer can charge is

$$p = \gamma \min_j (w^j, \ j = A, B), \tag{10}$$

where w^j is country j's wage rate. Assume $p > \max_j(w^j, \ j = A, B)$.[6] From the demand functions (7), we know that the profit flow, $\pi(M)$, for each producer in any sector in country j is

$$\pi^j = (\frac{p - w^j}{p})\frac{E}{M}, \ j = A, B. \tag{11}$$

Note that, compared with a closed economy, producers (both innovators and imitators) now enjoy larger markets (larger E) for their products, but at the same time, they face potentially more competitors (larger M). Also, producers in the lower-wage country have higher profit flow because of lower production costs (lower w).

Innovative R&D

Technologies for producing new higher-quality products have to be discovered through innovative R&D. Innovation in country j is assumed to follow the Poisson process with the arrival rate $\lambda_I^j y_I^j$, where λ_I^j and y_I^j are respectively the productivity of innovative R&D and the amount of labour used in an innovative firm in country j. As before, it is also assumed that innovations in either country can not be targeted to specific sectors; they occur randomly and independently across firms, across sectors,

[5] The assumption about information flow has two implications: First, an open economy can benefit from free flows of technologies; Second, the pricing of the-state-of-the-art products is constrained by the lower cost of production (the lower wage rate).

[6] This assumption is needed to guarantee a positive profit flow for innovators in the higher-wage country. Otherwise, innovations occur only in the lower-wage country. This suggests the possibility that innovators in the higher-wage country may not be able to compete with those in the lower-wage country if the wage difference is too large. We will not consider this case in this paper.

across countries and over time. Once an innovative R&D firm succeeds in discovering a higher quality product in a certain sector, it becomes the sole producer of that sector (the monopoly of the world market) and enjoys the monopoly profit until either another firm discovers an even higher quality product in the same sector at which time it is driven out of business or until some other imitative R&D firms copy the state-of-the-art product to produce different varieties at which time it has to share the product market with these imitators. Let W_I^j be the value of an innovative R&D firm in country j and V_I^j be the value of a successful innovation which occurs in country j, then we have the Bellman equation

$$\rho W_I^j = \max_{y_I^j > 0}\{\lambda_I^j y_I^j V_I^j - y_I^j\}. \tag{12}$$

Assuming free entry into innovative R&D, we have $W_I^j = 0$. Then

$$0 = \max_{y_I^j > 0}\{\lambda_I^j y_I^j V_I^j - y_I^j\}. \tag{13}$$

The first-order condition is

$$\lambda_I^j V_I^j \leq w^j,\ y_I^j \geq 0, \text{with at least one equality.} \tag{14}$$

The value function of innovation is derived in Appendix 3.3. As mentioned at the beginning of this paper, we plan to consider three patent protection scenarios. Correspondingly, we have three different value functions of innovation in country j.

Case 1: No Patent Protection

$$V_I^j = [\frac{1}{\sum_j \lambda_C^j n_C^j} \ln(1 + \frac{\sum_j \lambda_C^j n_C^j}{\rho + \sum_j \lambda_I^j n_I^j})](\frac{p - w^j}{p})E, \tag{15}$$

Case 2: Asymmetric Patent Protection

$$V_I^j = [\frac{1}{\lambda_C^i n_C^i} \ln(1 + \frac{\lambda_C^i n_C^i}{\rho + \sum_j \lambda_I^j n_I^j})](\frac{p - w^j}{p})E,\ i \neq j, \tag{16}$$

Case 3: Symmetric Patent Protection

$$V_I^j = \frac{1}{\rho + \sum_j \lambda_I^j n_I^j} (\frac{p - w^j}{p}) E, \tag{17}$$

where the superscript j refers to country j, λ_C^j is the productivity parameter for an imitative R&D firm, n_I^j and n_C^j are the aggregate labour employment in innovative R&D and imitative R&D respectively.

Imitative R&D

The process of imitation is assumed to have the same structure as that of innovation. We assume that imitation in country j follows the Poisson process with the arrival rate $\lambda_C^j y_C^j$, where y_C^j is the amount of labour employed in an imitative R&D firm in country j. Like innovation, we also assume that imitation occurs randomly and independently across firms, across sectors, across countries and over time. In each sector, a successful imitator becomes the sole producer of the new variety but shares the product market of that sector with the innovator and other imitators, if any. The processes of innovation and imitation in both countries generate a stationary distribution of the type K of sector across sectors.[7] At any point in time, some sectors have one producer (i.e. the innovator), some sectors have two (the innovator and one imitator), some have three (the innovator and two imitators) and so on. The value of a successful imitation depends crucially on this distribution. Let W_C^j be the value of an imitative R&D firm in country j and V_C^j be the value of a successful imitation that occurs in country j, and assume free entry into imitative R&D, then we have a Bellman equation and a zero profit condition similar to (12) and (13). The first-order

[7]The type of sector refers to the number of producers (also the number of varieties) in that sector. The type of sector (more precisely, the variable $K - 1$) is geometrically distributed across sectors. See Appendix 3.2 for derivation.

condition is:

$$\lambda_C^j V_C^j \leq w^j,\ y_C^j \geq 0, \text{with at least one equality.} \tag{18}$$

Like the value function of innovation, the value function of imitation varies with the patent protection assumption. [8]

Case 1: No Patent Protection

$$V_C^j = \frac{\sum_j \lambda_I^j n_I^j}{\rho \sum_j \lambda_C^j n_C^j}[(1 + \frac{\sum_j \lambda_I^j n_I^j}{\sum_j \lambda_C^j n_C^j})\ln(1 + \frac{\sum_j \lambda_C^j n_C^j}{\sum_j \lambda_I^j n_I^j})$$

$$-(1 + \frac{\rho + \sum_j \lambda_I^j n_I^j}{\sum_j \lambda_C^j n_C^j})\ln(1 + \frac{\sum_j \lambda_C^j n_C^j}{\rho + \sum_j \lambda_I^j n_I^j})](\frac{p - w^j}{p})E, \tag{19}$$

Case 2: Asymmetric Patent Protection

$$V_C^j = \frac{\xi_I^i \sum_j \lambda_I^j n_I^j}{(\sum_{i,j,i\neq j} \xi_I^i \lambda_C^j n_C^j - \lambda_C^j n_C^j)\sum_j \lambda_I^j n_I^j + \rho \sum_{i,j,i\neq j} \xi_I^i \lambda_C^j n_C^j}$$

$$[(1 + \frac{\sum_j \lambda_I^j n_I^j}{\sum_{i,j,i\neq j} \xi_I^i \lambda_C^j n_C^j})\ln(1 + \frac{\sum_{i,j,i\neq j} \xi_I^i \lambda_C^j n_C^j}{\sum_j \lambda_I^j n_I^j})$$

$$-(1 + \frac{\rho + \sum_j \lambda_I^j n_I^j}{\lambda_C^j n_C^j})\ln(1 + \frac{\lambda_C^j n_C^j}{\rho + \sum_j \lambda_I^j n_I^j})](\frac{p - w^j}{p})E,\ i \neq j, \tag{20}$$

Case 3: Symmetric Patent Protection

$$V_C^j = 0, \tag{21}$$

where ξ_I^i is the fraction of innovations that occur in country i.

[8] See Appendix 3.3 for derivation.

Labour Market

Labor is assumed to be internationally immobile. Then assuming full employment, we have the labour market clearing conditions corresponding to the three patent protection assumptions. [9]

Case 1: No Patent Protection

$$n_I^j + n_C^j + [\xi_I^j \beta + \xi_C^j (1-\beta)]\frac{E}{p} = N^j, \tag{22}$$

where ξ_C^j is the fraction of imitations that occur in country j, E/p is the total world employment in consumption good production, and β is the output share of all innovators, which is given by

$$\beta = \frac{\sum_j \lambda_I^j n_I^j}{\sum_j \lambda_C^j n_C^j} \ln(1 + \frac{\sum_j \lambda_C^j n_C^j}{\sum_j \lambda_I^j n_I^j}).$$

Case 2: Asymmetric Patent Protection

$$n_I^j + n_C^j + [\xi_I^j \beta + (1-\xi_I^j)(1-\beta)]\frac{E}{p} = N^j, \tag{23}$$

where

$$\beta = \frac{\sum_j \lambda_I^j n_I^j}{\sum_{i,j,i \neq j} \xi_I^i \lambda_C^j n_C^j} \ln(1 + \frac{\sum_{i,j,i \neq j} \xi_I^i \lambda_C^j n_C^j}{\sum_j \lambda_I^j n_I^j}).$$

Case 3: Symmetric Patent Protection

$$n_I^j + \xi_I^j \frac{E}{p} = N^j,\ i,j = A,B. \tag{24}$$

[9] See Appendix 3.4 for derivation.

Capital Market

Finally, we assume that capital is internationally mobile and there exists a perfect capital market. R&D firms borrow funds from this market to pay their researchers and issue risky securities. The equilibrium interest rate r clears the market at each moment in time. Since there is a continuum of sectors and innovations and imitations occur independently across firms and across sectors, individual investors are able to completely diversify away risk by holding a diversified portfolio of securities.

3.3 Equilibria

We consider only stationary equilibria. In equilibrium, the world total expenditure E, the employment n_I^j in innovative R&D and the employment n_C^j in imitative R&D are all constant; the instantaneous interest rate r is also constant and equal to the individual's subjective discount rate ρ. With the normalization $E = 1$, a stationary equilibrium is described by a constant sequence of $\{n_I^j, n_C^j, w^j\}_{j=A,B}$ satisfying one set of the following conditions:

Case 1: No Patent Protection

$$\lambda_I^j V_I^j \leq w^j,\ n_I^j \geq 0,\ j = A, B, \text{ with at least one equality,} \tag{25}$$

$$\lambda_C^j V_C^j \leq w^j,\ n_C^j \geq 0,\ j = A, B, \text{ with at least one equality,} \tag{26}$$

and (22), where V_I^j and V_C^j are given by (15) and (19) respectively.

Case 2: Asymmetric Patent Protection

(25) with V_I^j defined by (16), (26) with V_C^j defined by (20), and (23).

Case 3: Symmetric Patent Protection

(25) with V_I^j defined by (17), (26) with V_C^j defined by (21), and (24).

We will consider only the equilibrium with $n_I^j > 0, n_C^j > 0$ (except case 3), $j = A, B$. Similar to the closed economy model, we know that the world growth rate $g = \sum_j \lambda_I^j n_I^j \ln \gamma$.

3.4 Patent Enforcement and Growth

In this section, we use numerical examples and a calibration exercise to see the impact of patent enforcement and other government policies on the world growth rate. To this end, we further simplify the model. The two countries are assumed to be identical (same size of population, equal productivity of innovative R&D and imitati e R&D, i.e. $N^j = N, \lambda_I^j = \lambda_I$, and $\lambda_C^j = \lambda_C, j = A, B$).

3.4.1 The Effect of Patent Protection

We choose a subset of the values of the parameters $(\lambda_I, \lambda_C, \gamma, \rho, N)$ with $(\lambda_I, \lambda_C, \gamma, \rho, N)$ = (1, 4.5, 2, 0.05, 1) as the benchmark values. The equilibrium values of (n_I, n_C) and the resulting growth rates are shown in Tables 3.1-3.5 in Appendix 3.6.

The numerical examples show that SPP always has the highest growth rate, NPP always gives rise to the lowest growth rate and APP is in between. The result is quite intuitive. Since we assume that imitators do not improve the quality of products but produce different varieties of the same quality and share the product markets with innovators, given the total world expenditure E, more imitators implies lower profits flows for the innovators. Stronger patent protection simply reduces imitations, therefore increases the profitability of innovation, which attracts more innovation and hence push economic growth. Furthermore, Appendix 3.5 shows that SPP equilibrium is equivalent to the integrated economy (with patent protection) equilibrium. This is because innovators have the same technology, enjoy the same profit flow and face the same production cost in both a single country economy and the integrated economy. Therefore, in aggregate, they invest the same amount (of labor) in innovative R&D.

3.4.2 The Effect of Public Policy

We also perform public policy experiments under the APP assumption. Unlike the patent protection experiments, we do a simple calibration exercise. We consider the U.S. economy. Suppose it trades its products with a similar economy. We need to determine the values of the model's five parameters $(\lambda_I, \lambda_C, \gamma, \rho, N)$. Of these parameters, the size of population is normalized to 1 and the subjective discount rate is chosen to be 0.05. The other three parameters $(\lambda_I, \lambda_C, \gamma)$ are interdependent. The speed of innovation (or imitation) is meaningful only in terms of the size of innovation. For example, an innovation that takes 1 year and improves the productivity of intermediate goods by 21% is the same as two innovations each of which takes 6 months and improves the productivity by 10%. The speeds of innovation and imitation are also related to each other. So we can also normalize the speed of innovation λ_I to 1. Therefore, only two parameters (λ_C, γ) are left to be determined. To determine the values of these two parameters, we use the results of Mansfield et al (1981) and Lucas (1988). Mansfield et al (1981) found that, on average, the ratio of the imitation time to the innovation time was about 0.70. This ratio is represented by $(\lambda_I n_I)/(\lambda_C n_C)$ in our model, so we expect to have $(\lambda_I n_I)/(\lambda_C n_C)$=0.70 in equilibrium. Furthermore, in Lucas (1988), the employment in research was estimated to be in the range of 18% to 28% of the total employment. We interpret the employment in research as the sum of the employment in innovative and imitative R&D and choose the percentage to be 25%. Therefore, we also expect that in equilibrium we have $(n_I + n_C)$=0.25. The values of (λ_C, γ) that satisfy the above two condition are (4.432972, 1.368150). The equilibrium values of (n_I, n_C), along with the effects of public policies are reported in Table 3.6 in Appendix 3.7.

From Table 3.6, we see that in each country about 19% of labor is employed in innovative R&D and 6% is employed in imitative R&D. A subsidy of 1% to innovation in either country will increase its employment in innovation from 0.189070 to 0.194403 and decrease its employment in imitation from 0.060930 to 0.055343. Moreover, the innovation subsidy will also encourage (discourage) the other country's employment in innovative (imitative) R&D. The employment in innovation (imitation) will increase (decrease) from 0.189070 (0.060930) to 0.194376 (0.55412). As a result, the 1% innovation subsidy raises the world growth rate by about 0.33% (=0.121867-0.118532). A subsidy to imitation in either country will induce (reduce) the employment in imitation (innovation) in both countries. A subsidy of 1% will lower the world growth rate by about 0.32% (=0.118532-0.115289). Hence we have the following conclusion:

Subsidizing one country's innovative R&D not only increases this country's investment in innovative R&D, but also increases the other country's investment in innovative R&D, therefore raises the world growth rate; subsidizing any country's imitative R&D does the opposite.

The reason for this result is as follows. When a country subsidizes its innovative R&D, then within that country, the marginal cost of innovation decreases, as a result, investment in innovative R&D increases. At the same time, the subsidy to innovation indirectly increases the cost of imitation relative to that of innovation, so investment in imitative R&D decreases. Moreover, the subsidy also influences the other country's investment in innovative R&D. Under the APP assumption, imitators can copy only foreign country's innovation. So the other country's investment in innovative R&D increases as a result of the decrease in this country's investment in imitative R&D. Therefore, both countries have more investment in innovative R&D and the world growth rate rises. A subsidy to any country's imitative R&D has the opposite effects.

3.5 Conclusions

This paper has constructed three patent protection scenarios to look at the impact of government intervention (patent protection and public policies) on the world economic growth. Given the assumption that imitation is of the rent seeking type, we provide two intuitive and important messages. The first message is that with the APP assumption stronger patent protection induces a higher world growth rate. The second message, which is also related to the first one, is that any public policy that discourages innovation (encourages imitation) hurts the world economic growth.

However, the results may not be robust to the model specifications. The assumption about the role of imitation implies that imitation is socially useless because it does not contribute to growth. If we assume that imitation also contributes to growth, then the impact of government intervention may be different. Therefore, whether a government should encourage innovation or imitation depends on two factors: the importance of imitation relative to innovation and the extent of market distortions. Thus, empirical evidence is needed in order for decision makers to choose the right policy.

Appendix III

Appendix 3.1: Distributions of Innovations And Imitations Between Countries

Let ξ_I^j and ξ_C^j be the proportion of country j's innovations and imitations respectively, then

$$\xi_I^j = \frac{\lambda_I^j n_I^j}{\sum_j \lambda_I^j n_I^j}, \quad \xi_C^j = \frac{\lambda_C^j n_C^j}{\sum_j \lambda_C^j n_C^j}, \quad j = A, B.$$

Appendix 3.2: Stationary Distribution of the Type of Sector

As in Appendix 2.1 in the second essay, the random variable ($K - 1$) follows a geometric distribution. Let θ_k be the proportion of type k sectors, then we have

$$\dot{\theta}_1 = (1 - \theta_1)I - \theta_1 C,$$

$$\dot{\theta}_k = \theta_{k-1}C - \theta_k(I + C), k = 2, 3, ...,$$

where $I = \sum_j \lambda_I^j n_I^j$ and

$$C = \begin{cases} \sum_j \lambda_C^j n_C^j, & \text{for NPP}, \\ \sum_{i,j,i \neq j} \xi_I^{j i} \lambda_C^j n_C^j, & \text{for APP}, \\ 0, & \text{for SPP}. \end{cases}$$

Let $\dot{\theta}_k = 0$, k=1,2,..., we have

$$\theta_1 = \frac{I}{I + C}, \tag{27}$$

$$\theta_k = (\frac{C}{I + C})^{k-1}\theta_1, \quad k = 2, 3, ... \tag{28}$$

For NPP and APP, $(K - 1)$ follows geometric distributions. For SPP, the distribution is degenerate.

Appendix 3.3: Derivation of V_I^j and V_C^j

Let V_{Ci}^j be the value of the ith successful imitation in country j, $j = A, B$, then we have the Bellman equation

$$\rho V_{Ci}^j = (\frac{p - w^j}{p})E/(1+i) - IV_{Ci}^j - \tilde{C}(V_{Ci}^j - V_{C(i+1)}^j), \tag{29}$$

where

$$\tilde{C} = \begin{cases} \sum_j \lambda_C^j n_C^j, & \text{for NPP,} \\ \lambda_C^j n_C^j, & \text{for APP.} \end{cases}$$

Equation(29) implies

$$V_{C(i+1)}^j = \frac{(\rho + I + \tilde{C})V_{Ci}^j - (\frac{p-w^j}{p})E/(i+1)}{\tilde{C}}. \tag{30}$$

Let $\phi_i^j \equiv \frac{(\frac{p-w^j}{p})E/(1+i)}{\tilde{C}}$ and $\psi^j \equiv \frac{\rho+I+\tilde{C}}{\tilde{C}} > 1$, then (30) becomes

$$\begin{aligned} V_{Ci}^j &= \psi^j V_{C(i-1)}^j - \phi_{i-1}^j \\ &= (\psi^j)^{i-1} V_{C1}^j - \sum_{k=1}^{i-1} (\psi^j)^{i-1-k} \phi_k^j. \end{aligned} \tag{31}$$

We rewrite (31) as

$$\frac{V_{Ci}^j}{(\psi^j)^{i-1}} = V_{C1}^j - \sum_{k=1}^{i-1} \frac{\phi_k^j}{(\psi^j)^k}. \tag{32}$$

Since the value of a new successful imitation decreases as more imitations succeed and approaches zero as the number of successful imitations goes to infinity, we have

$$\begin{aligned} 0 = \lim_{i\to\infty} \frac{V_{Ci}^j}{(\psi^j)^{i-1}} &= V_{C1}^j - \lim_{i\to\infty} \sum_{k=1}^{i-1} \frac{\phi_k^j}{(\psi^j)^k} \\ &= V_{C1}^j - \frac{(\frac{p-w^j}{p})E}{\tilde{C}}[\psi^j \ln(\frac{\psi^j}{\psi^j - 1}) - 1], \end{aligned}$$

which gives

$$V_{C1}^j = [\psi^j \ln(\frac{\psi^j}{\psi^j - 1}) - 1]\frac{(\frac{p-w^j}{p})E}{\tilde{C}}. \tag{33}$$

From the Bellman equation for the value of innovation, V_I^j, i.e.

$$\rho V_I^j = (\frac{p - w^j}{p})E - IV_I^j - \bar{C}(V_I^j - V_{C1}^{ij}),$$

where

$$\bar{C} = \begin{cases} \sum_j \lambda_C^j n_C^j, & \text{for NPP}, \\ \lambda_C^i n_C^i,\ i \neq j, & \text{for APP}, \\ 0, & \text{for SPP}, \end{cases}$$

and

$$V_{C1}^{ij} = V_{C1}^i(\frac{p - w^j}{p - w^i}),\ i \neq j,$$

we have

$$V_I^j = \frac{(\frac{p-w^j}{p})E + \bar{C}V_{C1}^{ij}}{\rho + I + \bar{C}}. \tag{34}$$

Then substituting V_{C1}^{ij} with $\psi^j = \frac{\rho+I+\bar{C}}{\bar{C}}$ into (34), we get the value function of innovation

$$V_I^j = [\frac{1}{\bar{C}}\ln(1 + \frac{\bar{C}}{\rho + I})](\frac{p - w^j}{p})E.$$

Correspondingly, we have

$$V_I^j = \begin{cases} [\frac{1}{\sum_j \lambda_C^j n_C^j}\ln(1 + \frac{\sum_j \lambda_C^j n_C^j}{\rho+\sum_j \lambda_I^j n_I^j})](\frac{p-w^j}{p})E, & \text{for NPP}, \\ (\frac{1}{\lambda_C^i n_C^i}\ln(1 + \frac{\lambda_C^i n_C^i}{\rho+\sum_j \lambda_I^j n_I^j}))(\frac{p-w^j}{p})E,\ i \neq j, & \text{for APP}, \\ \frac{1}{\rho+\sum_j \lambda_I^j n_I^j}(\frac{p-w^j}{p})E, & \text{for SPP}. \end{cases}$$

Now we derive the value function of imitation. From (32) and (33), we have

$$V_{Ci}^j = \alpha_i^j \frac{(\frac{p-w^j}{p})E}{\bar{C}},$$

where $\alpha_i^j = (\psi^j)^{i-1}[\psi^j \ln(\frac{\psi^j}{\psi^j-1}) - 1] - \sum_{k=1}^{i-1}(\psi^j)^{i-1-k}\frac{1}{1+k}$. Let $\delta \equiv \frac{C}{I+C}$, then the assumptions that imitations occur randomly and independently across sectors and only foreign innovations can be imitated to produce different varieties implies

$$V_C^j = \begin{cases} \sum_{i=1}^{\infty} \theta_i V_{Ci}^j, & \text{for NPP}, \\ \xi_I^i \sum_{i=1}^{\infty} \theta_i V_{Ci}^j,\ i \neq j, & \text{for APP}, \\ 0, & \text{for SPP}. \end{cases}$$

The calculation of V_C^j is exactly the same as that of V_C in the second essay, which gives the value function of imitation

$$V_C^j = \begin{cases} \frac{\sum_j \lambda_I^j n_I^j}{\rho \sum_j \lambda_C^j n_C^j}[(1 + \frac{\sum_j \lambda_I^j n_I^j}{\sum_j \lambda_C^j n_C^j})\ln(1 + \frac{\sum_j \lambda_C^j n_C^j}{\sum_j \lambda_I^j n_I^j}) \\ \qquad -(1 + \frac{\rho + \sum_j \lambda_I^j n_I^j}{\sum_j \lambda_C^j n_C^j})\ln(1 + \frac{\sum_j \lambda_C^j n_C^j}{\rho + \sum_j \lambda_I^j n_I^j})](\frac{p-w^j}{p})E, \quad \text{for NPP}, \\ \frac{\xi_I^i \sum_j \lambda_I^j n_I^j}{(\sum_{i,j,i\neq j} \xi_I^i \lambda_C^j n_C^j - \lambda_C^j n_C^j)\sum_j \lambda_I^j n_I^j + \rho \sum_{i,j,i\neq j} \xi_I^i \lambda_C^j n_C^j} \\ \qquad [(1 + \frac{\sum_j \lambda_I^j n_I^j}{\sum_{i,j,i\neq j} \xi_I^i \lambda_C^j n_C^j})\ln(1 + \frac{\sum_{i,j,i\neq j} \xi_I^i \lambda_C^j n_C^j}{\sum_j \lambda_I^j n_I^j}) \\ \qquad -(1 + \frac{\rho + \sum_j \lambda_I^j n_I^j}{\lambda_C^j n_C^j})\ln(1 + \frac{\lambda_C^j n_C^j}{\rho + \sum_j \lambda_I^j n_I^j})](\frac{p-w^j}{p})E, \quad i \neq j, \quad \text{for APP}, \\ 0, \quad \text{for SPP}. \end{cases}$$

Appendix 3.4: Derivation of Shares of Production

Let S^j be country j's share of production, then

$$S^j = \xi_I^j \beta + \xi(1 - \beta),$$

where

$$\xi = \begin{cases} \xi_C^j, & \text{for NPP}, \\ 1 - \xi_I^j,\ i \neq j, & \text{for APP}, \\ 0, & \text{for SPP}, \end{cases}$$

and

$$\beta = \theta_1 + \frac{1}{2}\theta_2 + \frac{1}{3}\theta_3 + \dots + \frac{1}{k}\theta_k + \dots.$$

Since $\theta_k = \delta^{k-1}\theta_1$, and $\delta = \frac{C}{I+C}$, we have

$$\beta = \theta_1(1 + \frac{1}{2}\delta + \frac{1}{3}\delta^2 + \dots)$$

$$= \frac{\theta_1}{\delta}(\delta + \frac{1}{2}\delta^2 + \frac{1}{3}\delta^3 + \dots)$$

$$= \frac{\theta_1}{\delta}\ln(\frac{1}{1-\delta}) = \frac{I}{C}\ln(1 + \frac{C}{I})$$

$$= \begin{cases} \frac{\sum_j \lambda_I^j n_I^j}{\sum_j \lambda_C^j n_C^j} \ln(1 + \frac{\sum_j \lambda_C^j n_C^j}{\sum_j \lambda_I^j n_I^j}), & \text{for NPP,} \\ \frac{\sum_j \lambda_I^j n_I^j}{\sum_{i,j,i \neq j} \xi_I^i \lambda_C^j n_C^j} \ln(1 + \frac{\sum_{i,j,i \neq j} \xi_I^i \lambda_C^j n_C^j}{\sum_j \lambda_I^j n_I^j}), & \text{for APP,} \\ 1, & \text{for SPP,} \end{cases}$$

because $I = \sum_j \lambda_I^j n_I^j$, and

$$C = \begin{cases} \sum_j \lambda_C^j n_C^j, & \text{for NPP,} \\ \sum_{i,j,i \neq j} \xi_I^i \lambda_C^j n_C^j, & \text{for APP,} \\ 0, & \text{for SPP.} \end{cases}$$

Appendix 3.5: Equivalence of SPP and Integrated Economy Equilibrium

The integrated economy of two countries is the same as a one country economy with a population of twice the original size (i.e. $2N$), then we know from the second essay that

$$N_I = (\frac{\gamma - 1}{\gamma})(2N) - \frac{\rho}{\lambda_I \gamma}$$

where N_I is the total employment in innovative R&D in the integrated economy. On the other hand, under the SPP assumption (with the normalization $E = 1$ and symmetric assumption), the equilibrium conditions are

$$\frac{1}{\rho + 2\lambda_I n_I}(\frac{\gamma - 1}{\gamma}) = w, \tag{35}$$

$$n_I + \frac{1}{2}\frac{1}{\gamma w} = N. \tag{36}$$

Solving (35) and (36), we have

$$n_I = (\frac{\gamma - 1}{\gamma})N - \frac{\rho}{2\lambda_I \gamma}$$

Then the world employment in innovative R&D is simply $2n_I$, which is exactly the same as N_I.

Appendix 3.6: The Effect of Patent Protection

Table 3.1: Different Speeds of Innovation

PatentProtection	n_I	n_C	g
Benchmark	λ_I=1.0		
NPP	0.049419	0.396300	0.068509
APP	0.348212	0.138464	0.482724
SPP	0.487500	0.000000	0.675819
λ_I=1.1			
NPP	0.062665	0.396158	0.095559
APP	0.458104	0.030503	0.698574
SPP	0.488636	0.000000	0.745133
λ_I=1.2			
NPP	0.079019	0.389586	0.131452
APP	0.489583	0.000000	0.814447
SPP	0.489583	0.000000	0.814447
λ_I=1.3			
NPP	0.098806	0.377001	0.178066
APP	0.490386	0.000000	0.883765
SPP	0.490566	0.000000	0.884090
λ_I=1.4			
NPP	0.122274	0.358802	0.237311
APP	0.491071	0.000000	0.953077
SPP	0.491071	0.000000	0.953077

Note: λ_C=4.5, γ=2, ρ=0.05, N=1.

Table 3.2: Different Speeds of Imitation

PatentProtection	n_I	n_C	g
λ_C=4.0			
NPP	0.067822	0.390220	0.094021
APP	0.487251	0.000249	0.675473
SPP	0.487500	0.000000	0.675819
λ_C=4.2			
NPP	0.059395	0.393839	0.082339
APP	0.424049	0.063304	0.587857
SPP	0.487500	0.000000	0.675819
λ_C=4.5			
NPP	0.049419	0.396300	0.068509
APP	0.348212	0.138464	0.482724
SPP	0.487500	0.000000	0.675819
λ_C=4.8			
NPP	0.041797	0.396160	0.057943
APP	0.289552	0.196029	0.401404
SPP	0.487500	0.000000	0.675819
λ_C=5.0			
NPP	0.037684	0.395012	0.052241
APP	0.257620	0.227036	0.357137
SPP	0.487500	0.000000	0.675819

Note: λ_I=1, γ=2, ρ=0.05, N=1.

Table 3.3: Different Sizes of Innovati n

Patent Protection	n_I	n_C	g
γ=1.5			
NPP	0.035160	0.236953	0.028512
APP	0.227892	0.087757	0.184805
SPP	0.316667	0.000000	0.256795
γ=2.0			
NPP	0.049419	0.396300	0.068509
APP	0.348212	0.138464	0.482724
SPP	0.487500	0.000000	0.675819
γ=2.5			
NPP	0.057681	0.495845	0.105705
APP	0.420338	0.168986	0.770304
SPP	0.487805	0.000000	0.893942
γ=3.0			
NPP	0.063111	0.563366	0.138669
APP	0.468407	0.189355	1.029195
SPP	0.658333	0.000000	1.446505
γ=3.5			
NPP	0.066961	0.612063	0.167773
APP	0.502738	0.203912	1.259623
SPP	0.707143	0.000000	1.771765

Note: λ_I=1, λ_C=4.5, ρ=0.05, N=1.

Table 3.4: Different Subjective Discount Rates

Patent Protection	n_I	n_C	g
ρ=0.05			
NPP	0.049419	0.396300	0.068509
APP	0.348212	0.138464	0.482724
SPP	0.487500	0.000000	0.675819
ρ=0.06			
NPP	0.051654	0.387313	0.071607
APP	0.347562	0.136478	0.481823
SPP	0.485000	0.000000	0.672353
ρ=0.07			
NPP	0.053677	0.379100	0.074412
APP	0.346869	0.134544	0.480863
SPP	0.482500	0.000000	0.668887
ρ=0.08			
NPP	0.055526	0.371512	0.076975
APP	0.346135	0.132657	0.479845
SPP	0.480000	0.000000	0.665421
ρ=0.09			
NPP	0.057226	0.364442	0.079332
APP	0.345361	0.130818	0.478772
SPP	0.477500	0.000000	0.661956

Note: λ_I=1, λ_C=4.5, γ=2, N=1.

Table 3.5: Different Sizes of Population

PatentProtection	n_I	n_C	g
N=1.0			
NPP	0.049419	0.396300	0.068509
APP	0.348212	0.138464	0.482724
SPP	0.487500	0.000000	0.675819
N=1.2			
NPP	0.056833	0.485438	0.078787
APP	0.418463	0.168192	0.580113
SPP	0.587500	0.000000	0.814448
N=1.4			
NPP	0.064059	0.575297	0.088805
APP	0.488687	0.197954	0.677464
SPP	0.687500	0.000000	0.953077
N=1.6			
NPP	0.071147	0.665705	0.098631
APP	0.558893	0.227736	0.774790
SPP	0.787500	0.000000	1.091707
N=1.8			
NPP	0.078128	0.756542	0.108308
APP	0.629086	0.257533	0.872098
SPP	0.887500	0.000000	1.230336

Note: λ_I=1, λ_C=4.5, γ=2, ρ=0.05.

Appendix 3.7: The Effect of Public Policies

Table 3.6: The Effect of Subsidies

Subsidy	(n_I^j, n_I^i)	(n_C^j, n_C^i)	g
Benchmark	(0.189070, 0.189070)	(0.060930, 0.060930)	0.118532
$s_I^j = 0.01$	(0.194403, 0.194376)	(0.055343, 0.055412)	0.121867
$s_C^j = 0.01$	(0.183873, 0.183921)	(0.066434, 0.066348)	0.115289

Note: s_I^j and s_C^j are respectively country j's subsidies to innovative and imitative R&D; λ_I=1, λ_C=4.432972, γ=1.368150, ρ=0.05, N=1.

Bibliography

[1] Aghion, Phillippe and Peter Howitt, A Model of Growth Through Creative Destruction, Unpublished, the University of Western Ontario, 1991.

[2] ——, A Model of Growth Through Creative Destruction, *Econometrica* 60 (March 1992): 323-351.

[3] ——, Research and Development in the Growth Process, Unpublished, the University of Western Ontario, 1994.

[4] Backus, David, Patrick Kehoe and Timothy Kehoe, In Search of Scale Effects in Trade and Growth, *Journal of Economic Theory* 58 (December 1992): 377-409.

[5] Baumol, William J., Entrepreneurship: Productive, Unproductive, and Destructive, *Journal of Political Economy* XCVIII (1990): 893-921.

[6] Beckenbach, E.F. and R. Bellman, Inequalities, Berlin, Heidelberg, New York, 2nd ed., 1965.

[7] Dasgupta, Partha and Joseph Stiglitz, Uncertainty, Industry Structure and the Speed of R&D, *Bell Journal of Economics* 11 (Spring 1980): 1-28.

[8] Davidson, Carl and Paul Segerstrom, Patent Enforcement and Economic Growth, Michigan State University Working Paper, 1993.

[9] Dinopoulos, Elias, Schumpeterian Product Evolution and Vanishing Growth, University of Florida Working Paper, 1991.

[10] Dinopoulos, Elias, James Oehmke and Paul Segerstrom, High-Technology-Industry Trade and Investment: The Role of Factor Endowments, *Journal of International Economics* 34 (February 1993): 49-71.

[11] Feenstra, Robert, Trade and Uneven Growth, NBER Working Paper No. 3276, March 1990.

[12] Grossman, Gene and Elhanan Helpman, Trade, Innovation and Growth, *American Economic Review* 80 (May 1990a): 86-90.

[13] ——, Comparative Advantage and Long-Run Growth, *American Economic Review* 80 (September 1990b): 796-815.

[14] ——, Quality Ladders in the Theory of Growth, *Review of Economic Studies* 58 (January 1991a): 43-61.

[15] ——, Quality Ladder and Product Cycles,*Quarterly Journal of Economics* 106 (1991b): 557-586.

[16] ——, *Innovation and Growth in the Global Economy*, Cambridge: MIT Press, 1991c.

[17] ——, Imperfect Competition and International Trade, Gene Grossman and Elhanan Helpman ed., *Readings in Economics*, Cambridge: MIT Press, 1992.

[18] ——, Endogenous Innovation in the Theory of Growth, *Journal of Economic Perspectives* 8 (Winter 1994): 23-44.

[19] Jovanovic, Boyan and Rafael Rob, Long Waves and Short Waves: Growth through Intensive and Extensive Search, *Econometrica* 58 (November 1990): 1391-1409.

[20] Jovanovic, Boyan and Glenn M. MacDonald, Competitive Diffusion, *Journal of Political Economy* 102 (February 1994): 24-52.

[21] King, Robert and Sergio Rebelo, Public Policy and Economic Growth: Developing Neoclassical Implications, *Journal of Political Economy* 98 (Part 2) (October 1990): S126-50.

[22] Lee, Tom and Louis Wide, Market Structure and Innovation: A Reformulation, *Quarterly Journal of Economics* 94 (March 1980): 429-436.

[23] Loury, Glen, Market Structure and Innovation, *Quarterly Journal of Economics* 93 (August 1979): 395-410.

[24] Lucas, Robert Jr., On the Mechanics of Economic Development, *Journal of Monetary Economics* 22 (1988):3-42.

[25] Mansfield, Edwin, Mark Schwartz and Samuel Wagner, Imitation Costs and Patents: An Empirical Study, *Economic Journal* 91 (December 1981): 907-18.

[26] Mitrinovic, D.S., Analytic Inequalities, Berlin, New York: Sp[illegible]ger-Verlag, 1970.

[27] Murphy, Kevin M, Andrei Shleifer and Robert W. Vishny, The Allocation of Talent: Implications for Growth, *Quarterly Journal of Economics* (May 1991): 503-530.

[28] Rebelo, Sergio, Long-Run Policy Analysis and Long-Run Growth, *Journal of Political Economy* 99 (1991): 500-21.

[29] Reinganum, Jannifer, The Timing of Innovation: Research, Development and Diffusion, In *Handbook of Industrial Organization*, Vol. I, ed. by R. Schmalensee and R. Willig, Amsterdam: North-Holland, 1989.

[30] Romer, Paul, Increasing Returns and Long-Run Growth, *Journal of Political Economy* 94 (1986): 1002 ¹037.

[31] ——, Growth Based on Increasing Returns Due to Specialization, *American Economic Review* 77 (May 1987): 56-62.

[32] ——, Endogenous Technological Change, *Journal of Political Economy* 98 (1990): S71-S102.

[33] ——, Two Strategies for Economic Development: Using Ideas and Producing Ideas, Working Paper (March 1992).

[34] ——, Idea Gaps and Object Gaps in Economic Development, *Journal of Monetary Economics* 32 (December 1993): 543-73.

[35] ——, The Origins of Endogenous Growth, *Journal of Economic Perspectives* 8 (Winter 1994): 3-22.

[36] Rustichini, Aldo and James A. Schmitz Jr., Research and Imitation in Long-Run Growth, *Journal of Monetary Economics* 27 (April 1991): 271-92.

[37] Schmitz, James A., Jr., Imitation, Entrepreneurship, and Long-Run Growth, *Journal of Political Economy* 97 (1989): 721-739.

[38] Segersytrom, Paul, Innovation, Imitation and Economic Growth, *Journal of Political Economy* 99 (August 1991): 807-827.

[39] Segerstrom, Paul, T.C.A. Anant, and Elias Dinopoulcs, A Schumpeterian Model of the Product Life Cycle, *American Economic Review* 80 (December 1990):1077-91.

[40] Shleifer, Andrei, Implementation Cycles, *Journal of Political Economy* 94 (December 1986): 1163-90.

[41] Stokey, Nancy, Learning by Doing and the Introduction of New Goods, *Journal of Political Economy* 96 (1988): 701-717.

[42] ——, Human Capital, Product Quality, and Growth, *Quarterly Journal of Economics* CVI (May 1991): 587-616.

[43] Taylor, M. Scott, TRIPS, Trade, and Growth, *International Economic Review* 35 (May 1994): 361-81.

[44] Young, Alwyn, Learning by Doing and the Dynamic Effects of International Trade, *Quarterly Journal of Economics* 106 (May 1991): 369-405.

[45] ——, Invention and Bounded Learning by Doing, *Journal of Political Economy* 101 (June 1993): 443-72.

[46] ——, Substitution and Complementarity in Endogenous Innovation, *Quarterly Journal of Economics* 108 (August 1993): 775-807.