Investigation of Discharge Behaviour From a Sharp-Edged Circular Orifice in Both Weir and Orifice Flow Regimes Using an Unsteady Experimental Procedure

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A thesis submitted in partial fulfillment of the requirements for the degree in Master of Engineering Science

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INVESTIGATION OF DISCHARGE BEHAVIOUR FROM A SHARP-EDGED CIRCULAR ORIFICE IN BOTH WEIR AND ORIFICE FLOW REGIMES USING AN UNSTEADY EXPERIMENTAL PROCEDURE

(Monograph)

by

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Graduate Program in Civil and Environmental Engineering

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Engineering Science

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Abstract

An unsteady experimental study was completed with the intention of identifying a transition region between partially full weir flow and fully developed orifice flow for a circular sharp-edged orifice. Three orifices of different diameter were tested. The head-discharge relationships were obtained by pressure recordings and directly compared by using dimensional analysis. The presence of true weir and orifice flow behaviour was evaluated by an original technique where the head exponent in the head-discharge relationship is considered. The study found that true orifice behaviour was achieved in the experiments. Correspondingly, based on the head exponent, no evidence was obtained to support the existence of a different flow behaviour within the transition. Nevertheless, the experimental data have indicated that corrections are required to be applied to the discharge coefficient in the transition domain. A set of steady state experiments verified the unsteady data results in the orifice flow regime. Discharge coefficients were calculated and predicted by a fitted equation for a circular sharp-edged orifice across the entire range of head. Comparisons to the commonly used orifice equation validate its use for design.

Keywords

Unsteady orifice flow, unsteady weir flow, hydraulic head exponent, discharge coefficient, orifice equation, theoretical circular weir flow, theoretical circular orifice flow
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- $H$: Total upstream hydraulic head acting above orifice/weir invert
- $h_1$: Water level above weir/orifice invert
- $H_{avg}$: Hydraulic head level acting above orifice center
- $H^*$: Normalized head level above weir/orifice invert
- $V_1$: Velocity upstream of orifice/weir
- $V_2$: Velocity of freely discharging orifice flow
- $Q_t$: Theoretical discharge through weir/orifice
- $Q_a$: Actual flow through weir/orifice
- $Q^*$: Normalized flow through weir/orifice
- $A_0$: Cross-sectional area of orifice
- $A$: Cross-sectional area of hydraulic tank
- $C_e$: Effective discharge coefficient
- $C_D$: Discharge coefficient
- $C_c$: Contraction coefficient
- $C_v$: Velocity coefficient
- $D$: Circular orifice/weir diameter
- $r$: Circular orifice/weir radius
- $B$: Approach channel width
- $P$: Distance from channel bottom to orifice invert
- $L$: Streamwise length of weir/orifice edge
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Chapter 1

1 Introduction

A popular topic of interest in the recent literature is the control of stormwater runoff. There are two specific reasons for this devotion of attention. The first is the continual increase of urban development across all nations in the world. With urban development comes an increase in the impervious area within a given watershed, and thus an increase in the runoff is also observed. Increased volume of runoff can have negative effects on rivers and stream morphology due to erosion, and increases risks to life and property due to flooding (Ontario Ministry of Environment 2003). An increase in imperviousness results in not only an increase in runoff but also in the sediments and pollution contained within the runoff. This is because water easily picks up any sediment and pollution on pavement or concrete surfaces in urban areas.

It is thus very important to control both the quality and quantity of stormwater runoff resulting from urban areas as accurately as possible. This is typically achieved with the implementation of stormwater detention basins or ponds. These ponds allow large volumes of water to be stored and released slowly at an acceptable rate after a certain amount of settling has occurred. The settling is controlled by the residence time of the water in the pond, and the discharge rate is controlled by the outlet structure. Commonly used outlet structures include culverts, orifices and weirs (Tullis, Olsen and Garadner 2008). An example of a wet pond system utilizing a riser pipe is shown in Fig. 1-1.

The second reason for increased attention in the control of stormwater runoff is the predicted effects of climate change. Although there is still high uncertainty associated with the current modeling and project tools, it is believed that climate change will result in an increase in the severe weather patterns in the southwestern Ontario region (Toronto and Region Conservation Authority 2009). This will effectively result in an increase in the flows corresponding to a similar return period in the future and thus stormwater ponds will become under designed. In order to handle the increased flows expected in the future
effectively, stormwater ponds will need to increase their volume of storage. In most cases this is not possible due to spatial constrictions. In this case attention is drawn to the outlet structure where ongoing research is being conducted to optimize the head-discharge relationship and minimize the required volume of stormwater detention ponds (Tullis, Olsen and Garadner 2008) (Baddour 2008).

Figure 1-1: Plan and profile of example wet pond system (Stormwater Management Fact Sheet: Stormwater Wetland n.d.)
With such emphasis being put on the outlet structures of detention ponds to precisely control the outflow it is crucial to have a well-established, accurate head-discharge relationship for that outlet structure. A quite commonly used outlet device is the circular sharp-edged orifice, whether being used standalone or in a flat plate containing multiple orifices. The equation describing the head-discharge relationship for a circular sharp-edged orifice is well-established and commonly used throughout textbooks and design guidelines.

Some doubt exists as to the performance of the orifice equation under circumstances of low hydraulic head, which is a very common occurrence within detention ponds due to their frequent shallow depths. As will be discussed further in Chapter 2, there is a frequently observed increase in the discharge coefficient at low head in the available literature. This increase is not considered by textbooks and design guidelines which suggest a constant discharge coefficient to be applied across all head levels. The standard orifice equation also includes an approximation which only considers the average hydraulic head acting across the entire orifice. The effects of these two phenomenon are largely unknown and not quantified in the available literature.

Another disconcerting effect that is suggested by Bos (1989) is the formation of air-entraining vortices at low heads for orifice flow. The combination of all these effects at low head led to the formation of a theory that there may be a point below which true orifice flow does not occur. This hypothetical region, which we shall call the transition region, would extend from some level of head in the orifice flow regime to some point within the weir flow regime. This is significant since circular orifices in stormwater detention ponds are frequently flowing under low head or unsubmerged acting as a circular weir. The primary purpose of this study is to determine the extents of and predict the discharge within this transition flow region.

The presence of true orifice and weir flow will be assessed by the power to which the hydraulic head is raised in the head-discharge relationship. The flow behaviour for weirs and orifices of different shapes is characterized by this exponent, which has not been
quantified for a circular weir or orifice due to its difficult geometry. A solution to the theoretical flow for circular weirs/orifices is easily attainable with programs that can execute a simple numerical integration. The exponent can be readily derived from the slope of a logarithmic plot of the head-discharge relationship as will be discussed in Section 5.2. Once the limits of the transition region are established, the discharge within this region will be predicted by evaluating the discharge coefficient.

Chapter 2 of this study provides a review of the available literature on orifice and weir flow and previous studies on such structures using unsteady experimental techniques. Chapter 3 includes an evaluation of the error caused by the average head approximation in the standard orifice equation. Chapter 4 reviews the experimental apparatus and methodology used to collect and analyze the data. Chapter 5 presents the finalized results and includes discussions on the existing trends in the data. Chapter 6 draws appropriate conclusions about the observed orifice and weir flow behaviour and provides recommendations for future research attempts on the topic.
Chapter 2

2 Literature Review

Previous research attempts have not addressed the possible existence of a transition regime between weir and orifice flow for a circular sharp-edged orifice in a vertical plate. There is also no known study in the literature which analyzes the head-discharge relation on logarithmic scale to classify the flow behaviour based on the head exponent.

In order to identify a transitional flow regime, it is required to first review the known solutions for the head-discharge relationship for both circular sharp-edged orifices and sharp-crested weirs. The following sections will first describe the requirements for classification as circular sharp-edged orifice and sharp-crested weir and then summarize research on their head-discharge equations and discharge coefficients. A brief discussion will conclude on previous unsteady orifice discharge experiments.

2.1 Circular Sharp-Edged Orifice

A circular sharp-edged orifice is a hydraulic flow-device used to measure and control the outflow from channels, reservoirs and tanks. A schematic showing the profile of sharp-edged orifice flow is presented in Fig. 2-1. As the upstream flow approaches the orifice the velocity will increase and it will contract from all angles towards the orifice. Due to this contraction the streamlines will also have a perpendicular velocity directed towards the center of the orifice causing the emerging jet to contract. This results in a jet that is smaller in diameter than the orifice through which it passed. The contraction continues to a maximum point known as the vena contracta. This is the point at which the flow is typically calculated since it can be assumed that all streamlines are horizontal at this point.
Specific requirements that must be met for classification of structure as a circular sharp-edged orifice are discussed by Bos (1989). Bounding sides and bottom of the approach channel need to be significantly remote from the edges of the orifice to prevent the interference with the contraction of the jet. For circular orifices this distance should be at least equal to the radius of the orifice. To ensure accurate measurement of discharge the upstream face of the orifice plate must be smooth and perpendicular to the sides and bottom of the approach channel. Increasing roughness on the upstream face will reduce the vertical velocity along the plate and result in a reduced contraction (Bos 1989).

Also, to be classified as sharp-edged the thickness of the orifice edge should be equal to or less than 2mm. If the orifice plate thickness L is larger than 2mm it must be beveled at an angle greater than 45° from the horizontal. A schematic of this requirement for the edge profile is shown in Fig. 2-2 as presented by Bos (1989). This requirement also applies for sharp-crested weirs and ensures a non-adherence to the edge and proper development of the jet.
Furthermore, for true orifice flow to occur the free surface should be significantly above the top of the orifice since at low flow vortices may form causing air entrainment. This is one of the effects that may cause a change to the theoretical orifice discharge at low head values. The development of the theoretical head-discharge relationship for a circular sharp-edged orifice will be discussed in the next section.

2.1.1 Head-Discharge Relationship

The discharge through an orifice can be derived from Bernoulli’s relationship for conservation of energy. If we consider the energy balance at the reference point at the center of the orifice in Fig. 2-1 and ignoring losses, the Bernoulli relationship is

\[ h_1 + \frac{v_1^2}{2g} = \frac{v_2^2}{2g} \]  

[2-1]

If the upstream velocity is negligible \( (V_1 = 0 \text{ and } h_1 = H) \) we can rearrange Eq. [2-1] to obtain a relationship describing flow from an orifice that was first developed by Evangelista Torricelli in 1643, known now as the Torricelli Theorem (Bos 1989). Torricelli discovered experimentally that a fluid exiting a reservoir through a small orifice will attain a velocity \( (V_2) \) equal to that of a particle falling from the height \( (H) \) of fluid surface of that reservoir. Put mathematically, the Torricelli theorem states
\[ V_2 = \sqrt{2gH} \tag{2-2} \]

From this relationship the theoretical discharge from orifices may be readily determined by integrating the velocity over the cross-sectional area of flow for different orifice shapes. The theoretical discharge includes the following assumptions that are accounted for by applying coefficients accounting for various effects and assumptions, including:

- Exit losses through the orifice
- Negligible upstream velocity head \( \left( \frac{V_1^2}{2g} \right) \)
- Flow non-uniformity (velocity profiles)

These effects and others, still to be mentioned, will be accounted for in the effective discharge coefficient. The effective discharge coefficient is applied to the theoretical discharge to achieve the actual discharge as follows

\[ Q_a = C_e Q_t \tag{2-3} \]

where \( Q_a \) is the actual discharge, \( C_e \) is the effective discharge coefficient and \( Q_t \) is the theoretical discharge. To derive \( Q_t \) for a circular sharp-edged orifice we must consider a circular cross-section shown in Fig. 2-3. By applying Eq. [2-2] to an element of the orifice we obtain a relationship for the discharge through that element

\[ dQ = b(m) \times \sqrt{2g(H - m)} \times dm \tag{2-4} \]

where \( b \) is the width of the element and can be calculated by

\[ b(m) = 2 \times \sqrt{r^2 - x^2} \tag{2-5} \]

Since \( x = |r - m| \) we can rewrite Eqs. [2-5] and [2-4] as

\[ b(m) = 2 \times \sqrt{2mr - m^2} \tag{2-6} \]

\[ dQ = 2 \sqrt{2g(2mr - m^2)(H - m)} \times dm \tag{2-7} \]
Integrating across the entire orifice we obtain a relationship for the total discharge

\[ Q_t = 2\sqrt{2g} \int_0^D \sqrt{(2mr - m^2)(H - m)} \, dm \]  

[2-8]

The solution to Eq. [2-8] is very complex and not used in practice for circular orifices. The preferred method is to consider the average head \( H \) measured to the center of the orifice as shown in Fig. 2-1) acting on the orifice as a whole to eliminate the need for integration across the circular cross-section. The theoretical discharge with average head acting across the entire orifice is

\[ Q_t = A_0\sqrt{2gH} \]  

[2-9]
where \( A_0 \) is the area of the orifice and \( H \) is the head level measured from the center of the orifice. This average head assumption will only be accurate at high head. At low head where there is a significant difference between the head acting on the top and bottom of the orifice, a disagreement is expected with the solution to Eq. [2-8]. Chapter 3 includes a numerical solution to Eq. [2-8] and a comparison with Eq. [2-9]. A correction factor is also obtained in Chapter 3 that can be easily applied to resolve the difference occurring at low heads.

To obtain the actual discharge through a circular sharp-edged orifice we can sub Eq. [2-9] into Eq. [2-3] to achieve Eq. [2-10] which is commonly known as the orifice equation.

\[
Q_a = C_e A_0 \sqrt{2gH}
\]  

[2-10]

The effective discharge coefficient \( C_e \) is the product of 3 coefficients accounting for different losses and assumptions. The energy losses across the orifice are taken into account by the velocity coefficient \( C_V \). The fact that the area of flow at the vena contracta is smaller than the area of the orifice due to the contracting jet is taken into account by the contraction coefficient \( C_C \). Finally, effects of flow non-uniformity and neglecting upstream velocity head are combined in the discharge coefficient \( C_D \). The effective discharge coefficient is then

\[
C_e = C_D C_V C_C
\]  

[2-11]

The losses due to flow passing through the orifice are usually small, resulting in a commonly used \( C_V = 0.98 \) in the majority of textbooks (Lienhard V and Lienhard (IV) 1984). However the value of \( C_V \) has been seen to range from 0.951-0.993 for orifices of 0.02-0.06m diameter under 0-30m of head, decreasing slightly at lower head (Smith and Walker 1923). Thus \( C_V \) is usually close to unity and does not lead to a large deviation from theoretical flow. The effects of flow non-uniformity are also very small under most circumstances (as long as the flow is turbulent), and the upstream velocity is generally negligible if the approach channel is constructed properly.
Conversely, the contraction coefficient is significantly less than unity due to the substantial contraction of the jet exiting the orifice. The theoretical contraction coefficient can be calculated based on potential flow theory under the assumption of inviscid and irrotational flow as follows (Smith and Walker 1923)

\[ C_c = \frac{\pi}{\pi + 2} = 0.611 \]  \hspace{1cm} [2-12]

This solution agrees well with the experimental data except at high Reynolds number where lower contraction coefficients are observed (Grose 1985). This value is expected to increase at low head due to lower velocities resulting in a reduced contraction. Presently, there are no methods in the literature to predict the theoretical change of \( C_c \) at low head. However, methods have been presented to determine the contraction coefficient of orifice meters in pipes where the upstream pressure is constant across the orifice. Such studies have been performed by Grose (1985) and Benedict (1970).

A method developed by Grose (1985) uses circular and elliptical imaginary potential surfaces of constant pressure upstream of the orifice. This ‘surface’ bounds a control volume across the orifice and equations for conservation of mass and momentum can then be utilized to determine \( C_c \). This method applied to an orifice discharging freely under gravity could provide an estimation of how the contraction coefficient would change under low head. However it would require that the shape of the potential surface change as a function of head. It would also become quite strenuous due to a complex pressure and velocity profile acting across the control volume. The recent literature has focused more intently on predicting the effective discharge coefficient based on orifice and channel parameters rather than examining each component of \( C_e \) individually.

### 2.1.2 Effective Discharge Coefficient

The focus of current research has been on predicting the effective discharge coefficient (henceforth used interchangably with discharge coefficient) combining all coefficients in an experimental and empirical manner. The common accepted practice by hydraulics textbooks is to use a single constant discharge coefficient of \( C_e = 0.60 \). This technique is
also suggested in the urban drainage design manual provided by the Federal Highway Administration in the United States (Ayres Associates Inc 2009). The local design guidelines provided by the Ministry of Environment use a single constant value of 0.63 (Ontario Ministry of Environment 2003). A range of $C_e$ from 0.60-0.64 is suggested by Bos (1989) depending on the orifice diameter. Many other studies have focused on the effects of viscosity, plate roughness and edge rounding on the discharge coefficient. Very few studies have focused on the change of discharge coefficient under low-head conditions.

The effect on the ratio of orifice diameter to riser diameter $D/d$ on the discharge coefficient for circular orifices in riser pipes was investigated by Prohaska II (2008). The increase in $D/d$ was seen to lead to a decrease of the discharge coefficient. This was likely due to the increased contraction angle associated with a higher $D/d$ ratio. The effect of the upstream head above the orifice was also reported. The discharge coefficient increased at low head under all the investigated circumstances, attributed to the increased contraction coefficient at low flow (Prohaska II 2008). A power function was fitted to the data to predict $C_e$ as a function of $D/d$ and $H/D$ for discharge through orifices in a riser pipe.

It has been reported for quite some time that higher discharge coefficients are evident under low-head conditions (Smith and Walker 1923). Despite this knowledge there has been a lack of effort to accurately quantify these effects for use in design applications. No known study has provided a finite relationship between the discharge coefficient and head level for a circular sharp-edged orifice in a flat plate. As previously mentioned this is important for certain applications such as stormwater detention facilities where an orifice is discharging under low-head conditions fairly often.
2.2 Circular Sharp-Crested Weir

A circular sharp-crested weir is a hydraulic device used to measure and control discharge in channels, reservoirs or tanks. It is also indirectly used in situations where sharp-edged orifices are unsubmerged and thus behave as a weir. As the flow passes over the weir it separates due to the sharp crest and forms a nappe. A circular weir has certain advantages that it shares with the circular orifice due to its geometry: the crested can be beveled with extreme precision in a lathe, leveling of the weir crest is not required and the zero-flow point is easy to determine (Stevens 1957).

An overview of the requirements for classification of a sharp-crested weir is provided by Bos (1989). A schematic showing the profile of a sharp-crested weir is shown in Fig. 2-4. To guarantee accurate discharge the upstream face of the weir must be smooth and perpendicular to the sides and bottom of the approach channel. For classification as sharp-crested the head acting on the weir should be at least 15 times the thickness of the weir in the direction of flow ($h_1/L > 15$). This ensures that the length of the weir in the direction of flow does not influence the head-discharge relationship (Bos 1989). The edge profile should also comply with the same criteria mentioned for a sharp-edged orifice shown in Fig. 2-2.

![Figure 2-4: Profile of sharp-crested weir](image)
Another consideration for flow over a sharp-crested weir is the presence of an air pocket beneath the nappe on the downstream side of the weir. Due to continuous removal of air from this pocket by the flow passing over the weir, it is common for pressure to be reduced in this area. This can cause both an increase in the curvature of the flow, and vibration of the flow if the air supply to the pocket is irregular. Both of these effects will affect the discharge and should be abated by supplying air to the air pocket beneath the nappe or ensuring that the tailwater is at least 0.05m below the crest of the weir (Bos 1989).

2.2.1 Theoretical Head-Discharge Relationship

This section will include a description of the development of the theoretical head-discharge relationship for circular sharp-crested weirs. A schematic of a cross-section of a circular weir is provided in Fig. 2-5 below. In order to calculate the discharge it is assumed that a sharp-crested weir behaves as an orifice with a free surface since there is no evident location of critical flow over a sharp-crested weir. In calculating the discharge the following typical assumptions are made:

- Upstream velocity head is negligible \((h_1 = H)\)
- The drawdown is negligible, therefore the height of water over the crest is \(H\)
- Streamlines are horizontal when passing over the weir crest

These effects will all be accounted for in the effective discharge coefficient that is applied to the theoretical discharge in a manner similar to Eq. [2-3] for circular orifice. Since we are considering orifice behaviour we can use Eq. [2-2] to represent the velocity of flow over a sharp-crested weir. Thus the same method of integration can be followed as was done for circular orifices in Eq.’s [2-4] to [2-7]. Integrating across the entire range of head acting on the weir the discharge can be presented as

\[
Q_t = 2\sqrt{2g} \int_0^h \sqrt{(2mr - m^2)(H - m)}\, dm
\]  
[2-13]
Figure 2-5: Circular Weir Cross-Section Schematic

Notice that the only difference between Eq. [2-13] and [2-8] for theoretical orifice flow is the limits of integration. Solving Eq. [2-13] is quite complex and does not lead to a simple function between upstream head and discharge. The functional relationship between head and discharge was obtained by Stevens (1957) who determined the theoretical solution to Eq. [2-13] using complete elliptic integrals of the first and second kind. The solution as presented by Stevens (1957) is

\[
Q_t = \frac{4}{15} \sqrt{2gD^{5/2}} \left[ 2(1-k^2+k^4)E - (2-k^2)(1-k^2)K \right] \quad [2-14]
\]

where \( k = H/D \), and \( K \) and \( E \) are the complete elliptic integrals of the first and second kind respectively. These integrals are expressed as follows

\[
K(k) = \int_0^{\pi/2} \frac{d\varphi}{\sqrt{1-k^2 \sin \varphi}} = \int_0^1 \frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}} \quad [2-15]
\]
\[ E(k) = \int_{0}^{\pi/2} \sqrt{1 - k^2 \sin^2 \varphi} \, d\varphi = \int_{0}^{1} \frac{\sqrt{1 - x^2}}{\sqrt{1 - k^2 x^2}} \, dx \]  

where \( k \) is the elliptic modulus, \( \varphi \) is the amplitude, and \( x = \sin(\varphi) \). Numerical solutions to Eqs. [2-15] and [2-16] are readily available, and thus Eq. [2-14] can successfully provide a tabular theoretical relation between the head and discharge of a circular sharp-crested weir. This theoretical solution is also presented by Bos (1989) in a tabular form, and was reportedly obtained first by Staus and Von Sanden in 1926 (Bos 1989).

### 2.2.2 Discharge Coefficient

The discharge coefficient is utilized to account for the aforementioned assumptions in determining the theoretical discharge. Due to the complex nature of the flow, there is no theoretical development of the discharge coefficient, and all methods of predicting it are experimental in nature. Extensive experimental efforts have been made to accurately predict the discharge coefficient for circular sharp-crested weirs. Numerous experimental studies were summarized in Stevens (1957) who comprehensively analyzed all available experimental data (not including data available in Europe) on the flow through circular weirs. Despite evidence of a non-linear trend in the data, it was indiscernable for the presented data and he calculated a single average discharge coefficient of 0.59 for the entire head range (Stevens 1957).

Discharge coefficients for various weir diameters were experimentally derived by Staus (1931) who first recognized that the discharge coefficient is in fact a function of the filling ratio \((H/D)\). Average values of discharge coefficients resulting from these experiments are summarized and reported in (Bos 1989) and are provided in Appendix A.

More recently a study by Balachandar et al. (1991) presented an accurate relationship between the discharge coefficient and the filling ratio while considering effects of \( H/P \) and \( D/B \). Experiments were carried out for multiple weir diameters with various \( D/B \) ratios and were also compared to data presented by Stevens (1957). The equation presented is
\[ C_e = 0.517 + 0.066 \left( \frac{H}{D} \right) - 0.105 \left( \frac{H}{D} \right)^2 + 0.123 \left( \frac{H}{D} \right)^3 \] \[ [2-17] \]

Equation [2-17] is stated to be valid over the range \( 0 < H/D < 1 \), \( 0 < H/P < 1 \) and \( 0 < H/B < 0.5 \) and predicts discharge within a maximum error of 4% for the experimental data used for calibration (Balachandar, Sorbo and Ramamurthy 1991).

Another study presented by Vatankhah (2010) also determined a relationship between the discharge coefficient and filling ratio by applying a curve fitting method to data obtained by Greve (1932). The relationship was presented as follows and was stated to be valid over a range of \( 0.1 < \frac{H}{D} < 1 \) where 94% of the data used has error less than 2.5%

\[ C_e = \frac{0.728 + 0.240\eta}{1 + 0.668\sqrt{\eta}} \] \[ [2-18] \]

where \( \eta = \frac{H}{D} \). The data presented in Appendix A and Eqs. [2-17] and [2-18] will be used to compare to experiment discharge data obtained in the present study.

2.2.3 Simplified Head-Discharge Equation

Due to the fact that the solution to Eq. [2-14] is in tabular form, it is not ideal for practical purposes. Recent efforts have been made to determine a simple and accurate equation for the discharge from a circular sharp crested weir. The simplest equation is of the same form except an equation was developed by Vatankhah (2010) using a curve fitting method to express the function containing elliptic integrals. The solution for the theoretical discharge was presented as follows

\[ Q_t = 2\sqrt{2g} (\eta)^{1/2} H^{3/2} D F(\eta) \] \[ [2-19] \]

\[ F(\eta) = 0.1963 \left( \sqrt{1 - 0.2200\eta} + \sqrt{1 - 0.7730\eta} \right) \] \[ [2-20] \]

where \( \eta = \frac{H}{D} \). Equation [2-19] is very accurate with a maximum error of 0.08% when compared to the numerical solution (Vatankhah 2010).
Another method was employed by Ghobadian and Meratifashi (2012) using the assumption of critical flow existing over the weir crest to determine the theoretical head-discharge relation. The head and discharge equations at the critical flow condition for a circle channel as presented by Ghobadian and Meratifashi (2012) are

\[
H = \frac{D}{2} \left(1 - \frac{\cos \theta_c}{2}\right) + \frac{D}{16} \left(\frac{\theta_c - \sin \theta_c}{\sin \theta_c/2}\right) \tag{2-21a}
\]

\[
Q = \left\{g \left[\frac{D^2}{8} (\theta_c - \sin \theta_c)\right] \right\}^{1/2} \tag{2-21b}
\]

where \(\theta_c\) is the central angle of the channel. Numerous experiments were also conducted as part of the study at various values of weir diameter \(D\) and crest to channel bottom distance \(P\). A discharge coefficient was provided by Ghobadian and Meratifashi (2012) to correct Eq. [2-21] based on \(H/P\) for the conducted experiments

\[
C = \cos \left[\frac{H}{D + 6.46289} \right] + \tan^{-1} \left[\frac{-2.293426}{\frac{H}{P}} + 3 \frac{H}{P}\right] \tag{2-22}
\]

This is a different approach since it does not consider the filling ratio \(H/D\) as could be considered generally accepted practice.

### 2.3 Unsteady Orifice Flow

In almost all of the above-mentioned studies, experiments were carried out in a steady-state manner such that the upstream head remained constant. The effect of a falling head on the discharge for both the circular orifice and weir remains largely unstudied and unknown. It is hypothesized that there may be a slight increase in the discharge coefficient due to upstream velocity head resulting from the falling water surface. This effect would likely be reduced for a vertical tank of smaller orifice to tank diameter ratio.
The study performed by Prohaska II (2008) on discharge of orifices in a riser pipe was done in an unsteady manner using a pressure transmitter to measure the water level with time. Another experimental investigation was conducted by Aoki et al. (2002) on the unsteady flow patterns seen in the flow through a rectangular bottom orifice. Unfortunately, no comparisons were made in these studies between the obtained data and any steady-state data for similar conditions.

The present study includes unsteady discharge experiments such that the upstream head is falling and is not constant. The non-steady experiments will be directly compared to steady-state experiments over the range of $H/D$ investigated. This will allow for the effect of unsteadiness to be quantified if present in the results.
Chapter 3

3 Average Head Approximation for Circular Orifice

A mentioned in section 2.1.1, the head represented in Eq. [2-9] is the average head acting over the entire orifice. This assumption becomes less accurate at lower head where the difference in pressure acting on the bottom and top of the orifice becomes significant. In this section, Eq. [2-8] will be solved numerically and compared to Eq. [2-9] under various conditions of head and diameter to determine the significance of any inaccuracy resulting from the average head approximation. Recall Eq. [2-8] and [2-9] below.

\[
Q_t = 2 \sqrt{2g} \int_0^D \sqrt{(2mr - m^2)(H - m)} dm \quad [2-8]
\]

\[
Q_t = A_0 \sqrt{2gH_{avg}} \quad [2-9]
\]

To provide a direct comparison of the flow resulting from each equation for any orifice size the non-dimensionalised head-discharge relation is compared. The non-dimensionalized discharge and head will be referred to as \(Q^*\) and \(H^*\) respectively

\[
Q^* = \frac{Q}{g^{0.5}D^{2.5}} \quad [3-1]
\]

\[
H^* = \frac{H}{D} \quad [3-2]
\]

where \(H\) is the head measured from the orifice invert. See Appendix B for the development of these equations. Equation [2-8] was solved numerically in MATLAB and plotted against the flow resulting from Eq. [2-9] in Fig. 3-1. It can be seen that the discrepancy between these two solutions becomes larger at lower head values. This result shows that the average head approximation results in an over-approximation of discharge when compared to the theoretical flow. The difference between the two methods is calculated and the error caused by using an average head approximation is presented in Fig. 3-2.
Since the error caused by the approximation is simply a function of $H^*$ a factor $\beta$ can be easily developed to correct for the difference. Regression analysis was completed on the error caused by the average head approximation. A shifted power non-linear regression type provides the best fit with a maximum of only 1% error. The resulting correction factor and new theoretical flow equation are

$$\beta = 1 - 0.006 \left( \frac{H}{D} - 0.654 \right)^{-1.772} \quad [3-3]$$

$$Q_t = A_0 \sqrt{2gH \ast \beta} \quad [3-4]$$
Figure 3-2: Error associated with using the average head approximation for orifice flow

Equation [3-3] successfully corrects for the average head approximation with a maximum error of 0.035\% when compared with the solution to Eq. [2-8]. This solution provides an accurate method of estimating the theoretical orifice discharge without the need for lengthy integration or numerical solutions. It also allows for continuation of use of the standard orifice equation but with a simple correction factor to be applied.
Chapter 4

4 Experimental Setup and Methodology

The purpose of the experiments conducted was to evaluate the accuracy of the circular orifice and weir equations under a range of upstream head conditions. The head was required to range between that of an unsubmerged weir to fully developed orifice flow. A procedure was developed to determine the head-discharge relationship by utilizing a vertical tank with flow exiting through various sized circular orifice plates. This was designed to be a non-steady state experiment whereby a filled tank is allowed to drain naturally while the water level and discharge are measured. A pressure transmitter was procured for the task of measuring the dynamic water level in the tank. The discharge could be obtained from the relationship between the water level and time. In order to verify these methods, a separate set of steady-state experiments were also conducted in a large hydraulic container. Orifice plates were installed in the container and tested under constant flow rates, to compare with the head-discharge data obtained via the non-steady state experiments. Both experiments were conducted in the hydraulics laboratory of Western University.

4.1 Physical Apparatus

4.1.1 Tank with Pressure Transmitter (Non-Steady State Experiments)

The non-steady state experiments were conducted in a 1.5m x 30cm x 30cm vertical tank with a large opening in the side near the bottom. A schematic and a photograph of this apparatus are displayed in Fig 4-1. Three 2mm thick aluminum orifice plates were constructed with diameters of 3cm, 4.5cm and 6cm. The plates fit flush with the inside of the tank. To ensure that the plates would function as sharp-edged orifices, they were constructed with a smooth 1mm thick edge followed by a 45 degree bevel to conform to criteria specified in Fig. 2-2. In order to satisfy the required $h_1/L > 15$ for sharp-crested weir flow a minimum head of 1.5cm is required for all orifices. The distance to the sides and bottom of the tank was sufficiently large to prevent interference to the jet contraction.
This distance was at least twice the diameter of the orifice for all orifice sizes. The tank was also tall enough to provide a meter of water above the orifice, providing a sufficient maximum head to diameter ratio for the purpose of these experiments.

Figure 4-1: Schematic and Photograph of Non-Steady State Experiment Apparatus

Photograph Legend
1 – Hydraulic Tank
2 – Pressure Transmitter
3 – Data Acquisition System
4 – Computer
5 – Tank Aperture
6 – Orifice plate
7 – Door Mechanism
8 – Water Basin
9 – Splash Board
10 – Power Supply
An AMETEK Model 831 electronic pressure transmitter was obtained for the purpose of measuring the water level in the tank throughout the experiments. The pressure transmitter was calibrated to a measurement range of 0-6psi gauge pressure to optimize the accuracy of the device at the low pressures being measured. It was installed near the bottom of the tank opposite the orifice plate, below the elevation of the orifice invert. A simple 10V power supply powered the device. For all orifice sizes the data recording was taken at a frequency of 25Hz, corresponding to a reading taken every 0.04 seconds. The reported response time of the pressure transmitter is a couple of milliseconds as defined by the manufacturer AMETEK. Thus 25Hz is an effective sampling rate that should be significantly lower than the response time of the unit to minimize noise. The pressure transmitter was connected to an Omega OMB-DAQ-56 Personal Data Acquisition system, which read the voltage output from the transmitter. The data was then sent to a computer where it was recorded.

A flexible pipe was attached to a nearby water tap and used to fill the tank. Gradations were marked on the tank above the orifice invert to allow for visual measurement of the water level for calibration purposes. A flat steel plate with a rubber covering was installed inside of the tank to act as a seal on the orifice while being filled for each subsequent test. The door mechanism was connected to a wire rope which passed through a pulley installed on the ceiling. This setup allowed for the door to be pulled free and removed from the tank with haste such that the test could continue without further disturbance.

4.1.2 Hydraulic Container (Steady State Experiments)

The steady-state experiments were conducted in a large 0.57m by 0.88m hydraulic container. A photograph and schematic of the setup are shown in Fig. 4–2. A hole was cut out from one side of the container to allow an orifice plate to be attached. The orifice plate containing the 4.5cm diameter orifice was mounted and tested to verify the data retrieved from the non-steady state experiments. It was installed with a distance of 0.17m to the bottom and significantly large distance to the sides to prevent interference to the jet
contraction. The validation was primarily intended for the data at lower head values due to a limitation in the attainable discharge from the available taps.

Figure 4-2: Schematic and Photograph of Steady-State Apparatus

A flexible pipe attached to a water tap pumping system kept water in a continuous flow cycle through the apparatus. A handmade diffuser was attached to the end of the inflow pipe and placed in rock fill to reduce inflow velocity. Wire mesh was installed to
eliminate turbulence in the flow in the approach channel. Gradations were marked on the side of the flume to visually measure the water level above the orifice during each trial. The flow rate was measured by simple volume over time calculations using a large cylindrical bucket to take volumetric measurements over a specified duration.

4.2 Experimental Procedure

4.2.1 Non-Steady State Experiments

The procedure for the non-steady state experiments was devised to accurately determine the head-discharge relationship of multiple orifices. Five trials were carried out for each orifice plate of different diameter. Due to the slight static variability in the pressure transmitter signals, it was required to calibrate the voltage-head relationship prior to each test. The steps that were taken for each experiment are as follows:

1. The door mechanism was securely fit into place and the flexible pipe was inserted into the vertical tank from the top and began to fill while the computer displayed voltage readings.

2. The inflow of water was stopped periodically at various water levels to measure the water height (via gradations on tank) and voltage (displayed on computer). These readings were recorded and used to develop a calibration equation for each test.

3. At 1m head the inflow was stopped and the flexible pipe removed from the tank. The Personal DaqView program began recording voltage measurements.

4. The door mechanism was pulled free from the orifice and removed from the tank, allowing water to flow freely out the orifice.
5. Recordings were taken until the water had fallen to low levels in the weir range and discharge became negligible. The program created a data file containing the voltage readings from the experiment.

Initial disturbances were caused at the beginning of each test due to removal of the door mechanism and the resulting pressure wave. After the initial disturbance the water surface stayed calm for the remainder of the test.

4.2.2 Steady State Experiments

The procedure for the steady state experiments was relatively straightforward. Twelve different head levels were tested ranging from the weir flow regime to the orifice regime. Once the flow rate was adjusted for each test, several minutes were allowed for development of the steady-state condition. Five volumetric measurements were taken for a specified time interval ranging from 15-60 seconds for each head level. The upstream water level was measured by gradations marked on the side. Flow rates were calculated at the end of each experiment by simple volume over time arithmetic.

4.3 Analytical Procedure

4.3.1 Non-Steady State

Many steps were taken as part of the analysis of the core data that was obtained from experiments. The data files that were created from the Personal DaqView contained only the voltage readings that were taken by the pressure transmitter throughout the test. These readings, the voltage-head calibration equation and knowledge of the frequency at which readings were taken provided enough information to determine the head-discharge relationship for each orifice.

Prior to the conversion and analysis of the raw voltage data, the data was truncated due to the initial disturbances caused by removal of the gate. A sample of raw test data is plotted and shown in Fig. 4-3. The disturbances to the voltage readings can be seen near the beginning of the test and are quite significant. Thus, the raw data was truncated and test
data was considered to start at a head of 80cm above the orifice invert (well into the smooth part of the curve in Fig. 4-3).

The first step was to convert the voltage readings to water level, which was done with the calibration equation. Since the pressure transmitter has a linear relationship between measured pressure and voltage output, the water level was expected to be a linear function of the measured voltage. This was shown by the fact that the calibration curves were in fact all linear. A sample calibration curve plot and all of the calibration data is provided in Appendix C. The variability in readings between subsequent tests was due to the inherent inaccuracy of the pressure transducer. This error and others resulting from the apparatus and methods used are discussed in the error analysis (Section 4.4) at the end of this chapter.

Once the data was converted to water level, the discharge could be found by Eq. [4-1]. The volume $V$ can be calculated by the product of the constant cross-sectional area of the
tank and the change in water level in the tank over a specified period of time $t$ (given by the inverse of the frequency).

$$Q = \frac{V}{t} = \frac{A \cdot dh}{1/f} = \frac{A \cdot dh}{dt} \quad [4-1]$$

However, the signals given by the pressure transmitter was slightly noisy at a small scale. The differential in head over one period would rarely be an accurate indicator of the average fall in head. Thus a moving average technique was utilized to calculate the slope $dh/dt$ for a series of points surrounding the point of interest by a linear fit. An example of this method is shown in Fig. 4-5 for a single data point in the 4.5cm diameter orifice experiment displayed in Fig. 4-3. Due to the increase in flow, fewer points were used as the size of orifice was increased. This yielded the 2.4, 1.6, and 0.8 second average slope for the 3cm, 4.5cm and 6cm diameter orifices respectively. Once $dh/dt$ was obtained, Eq. [4-1] was used to determine the discharge and the head-discharge relationship was then plotted for each trial for analysis.

![Figure 4-4: Calculation of dh/dt for single data point at t=20.8 seconds](image)
4.4 Error Analysis

As part of any experiment, there are possible inaccuracies present. A few errors were encountered as part of the experimental and analytical methodology. These errors were quantitatively assessed and combined to determine the maximum amount of error. All errors encountered in the non-steady experimental procedure were random by nature:

- Electronic pressure transmitter reported accuracy of +/- 0.3% of the operating range (6 psig), corresponding to 1.3cm of water head. This error includes the effects of linearity, hysteresis and repeatability. Thus 0.3% is a drastic overestimation since the pressure transmitter was calibrated prior to each test. The largest difference in readings between subsequent tests was 0.003V, corresponding to roughly 3mm.

- Initial disturbance to the flow caused by opening and removal of the door mechanism from the apparatus. This phenomenon is unquantifiable, but expected to be minimal once the flow settles for the remainder of the experiment.

- Visual measurement of the water level when calibrating the voltage-head curve, estimated to be +/- 1mm.

- Inaccuracy of signal measurement by the data acquisition system of 0.01% of reading plus 0.002% of range (5V). This corresponds to +/- 0.31mm water head for the largest voltage readings taken.

- Errors associated with the analytical procedure such as combining the data of difference orifice diameters by using LOESS smoothing which is discussed in Chapter 5.

The sum of these random errors is equivalent to +/- 4.3mm of head, which is relatively large. However this represents the maximum error possible. The random errors are
expected to be a bit larger than this though due to the effects of the initial pressure disturbance.

The steady-state experiments conducted in the hydraulic container contain the same error from visual measurement of the water level. There was also the presence of small errors associated with the timing of the volumetric measurements done for each trial. This was estimated to be +/- 0.2 seconds corresponding to a maximum of 1.5% error in the discharge measurement. These errors will be discussed further in Chapter 5.
Chapter 5

5 Analysis and Discussion

In this chapter the method of analysis following the calculation of the head-discharge data and a discussion of the findings will be presented. The analytical procedure utilized to obtain the head-discharge data for the non-steady state experiments was discussed in Section 4.3. This included all steps followed to obtain the discharge at various head levels using the voltage data obtained from pressure transducer readings. Figures 5-1 to 5-3 exhibit the head-discharge data points resulting from all five trials for each orifice plate of different diameter. From these figures we can see the high level of agreement between subsequent trials. The maximum deviation in discharge data between subsequent trials for the 3cm diameter orifice is 1-3% in the orifice regime, and increases to as high as 10-30% in the weir flow regime. Similar results are seen in the data for the 4.5cm diameter orifice with deviations of 1-3% at moderate head values and up to 18% at low head. For the 6cm diameter orifice differences of up to 20% were observed at low head with the similar 1-3% error seen at higher head levels. There was also the presence of a few outliers at higher head which can be seen in Fig. 5-3. This is believed to be due to the initial pressure disturbance caused by the door mechanism which were larger in magnitude for the largest orifice.

The errors observed between subsequent trials generally follow what was expected from the random error discussed in Section 4.4. A random error of a few millimeters is fairly small in magnitude when considering a moderate head level. However as the head approaches zero and the low levels in the weir regime this random error becomes quite significant and leads to a large discrepancy between subsequent trials. For this reason it is expected that flow in the weir regime possesses large enough inaccuracies to affect the validity of the results.
Figure 5-1: Discharge data for 5 trials of 3cm diameter orifice

Figure 5-2: Discharge data for 5 trials of 4.5cm diameter orifice
In the interest of obtaining a single head-discharge relation for each orifice diameter a LOESS (local regression) smoothing technique was used to combine the data from all trials for each diameter. This technique uses a least squares regression for small subsets of data in order to predict the local values. For this study the data from all 5 trials was combined and every fifth data point was considered. A 2nd order polynomial was fit to a subset of data points surrounding each point being considered. The discharge corresponding to the head level of the point being considered was taken as the value on the polynomial curve. Thus a singular head-discharge relation was achieved for each orifice diameter as shown in Fig. 5-4. Further detail on the LOESS smoothing technique is discussed in Appendix E.

The root mean square (RMS) of the error compared to the averaged values was determined for each of the three the data sets following the LOESS technique. The RMS of the deviations were calculated as $6.94 \times 10^{-6}$, $9.41 \times 10^{-6}$ and $6.59 \times 10^{-5}$ m$^3$/s for the 3cm, 4.5cm and 6cm diameter orifice respectively. These values are included in Fig. 5-4 as...
error bars. It is evident from the error bars that there is very little deviation between subsequent trials for both the 3cm and 4.5cm diameter orifices.

![Graph showing discharge vs. head for different orifices](image)

**Figure 5-4: Head-discharge relation for all orifices after LOESS smoothing technique**

As mentioned in Chapter 3 the head-discharge relationship can be non-dimensionalized in order to compare the flow between various orifice diameters. Recall Eq.’s [3-1] and [3-2] as the dimensionless discharge and head respectively

\[ Q^* = \frac{Q}{g^{0.5}D^{2.5}} \]  
\[ H^* = \frac{H}{D} \]

The dimensionless head-discharge relation was calculated for all orifice diameters and is presented in Fig. 5-5. The high level of agreement between results from different orifices is evident. The difference between the normalized results is as little as 1-2% above \( H^*=2 \), increasing to a maximum of 25% in the weir regime at \( H^*=0.5 \). However, these
differences are not random in nature. The discharge curve of the 6cm diameter orifice is lower than that of the other orifices over the entire range of head. This phenomenon is exemplified further in Fig 5-6. An explanation for this could be present in the effect of the ratio of orifice to riser diameter D/d as discussed in Section 2.1.2.

The study by Prohaska II (2008) identified a consistent decrease in the discharge coefficient with an increase in the D/d ratio. Even though this was partly due to the curvature of the orifice in a riser pipe, a parallel can be drawn to this study. Also note during a non-steady state experiment the water surface is falling faster as the size of the orifice is increased. Hence, the increase to a 6cm diameter orifice may have been enough to cause a significant momentum in the vertical flow downward through the tank. The vertical flow through the tank would lead to an increased contraction of the jet emanating from the orifice. An increased contraction of the jet causes a reduced cross-sectional area of flow at the vena contracta. This leads to a smaller contraction coefficient and thus reduced discharge. Due to the observed difference and lack of full understanding, further analysis was conducted exclusively on the results from the 3cm and 4.5cm diameter orifices.

LOESS smoothing was utilized again to combine the dimensionless head-discharge data resulting from these two orifices into a single result for the non-steady experiments. The RMS of the error between the mean dimensionless discharge and the independent results from the 3cm and 4.5cm orifice was calculated to be 0.0035. This value is extraordinarily small and shows a very strong agreement between the results from these two orifices of different diameter. The results and conclusions will be drawn from this finalized dimensionless head vs. discharge relationship shown in Fig. 5-7.
Figure 5-5: Normalized head-discharge relation for all orifices

Figure 5-6: Normalized head-discharge relation for all orifices below $H^* = 5$
Figure 5-7: Final Normalized Head vs. Discharge
5.1 Steady vs. Non-Steady State Results

As part of this study, steady-state experiments were carried out in order to verify the non-steady results. As mentioned in Chapter 4, the steady state head-discharge was calculated as a simple volumetric measurement over specified time periods. The measurements taken are provided in Appendix D. The steady-state experiments were conducted exclusively on the 4.5cm diameter orifice. In order to compare these results with those of the unsteady experiments the head and discharge were similarly normalized. The results are plotted in Fig. 5-8 along with the numerical solution for weir and orifice flow. The numerical head-discharge relationship was obtained by the numerical integration method discussed in Chapter 3. The numerical solution is the theoretical flow through the orifice and thus the factor relating this solution with the experimental results will be the effective discharge coefficient.

The results of the steady and unsteady flow compare well with differences of only 0.1-8%. There is no evident patterns in the error within the orifice flow regime and the magnitude of the error is very small (<3%). The only non-random pattern that is evident from the error between the two methods is the slight apparent over-prediction of the flow at low heads in the weir regime (H*<1). Whether this difference is caused by the unsteadiness of the flow or by the inherent larger errors at very low head in the unsteady experimental procedure is somewhat unknown. However, there is an interesting phenomenon that occurs at the transition between weir and orifice flow that is much clearer in an enlargement of the region in Fig. 5-8. In the numerical data there is a marked decrease in the slope behaviour exactly at H*=1. Comparatively, the unsteady data does not exhibit this change in behaviour until somewhere between 0.8 ≤ H* ≤ 0.9. This suggests that the flow device is still behaving as an orifice at the top of the weir flow regime primarily due to the unsteadiness. This hypothesis is supported by the fact that the steady state results are lower in this area. If the slope in the unsteady experimental data were to change at H*=1 and shadow the numerical solution, the results would agree much better with the steady state experimental data.
Figure 5-8: Comparison of steady and unsteady experimental flow results

Figure 5-9: Enlargement of weir-orifice transition in Fig. 5-8
5.2 Analysis of Exponent

The major benefit that is achieved by conducting an unsteady experiment is the vast amounts of data that can be easily collected. Thousands of data points can be achieved from only a few experiments such that a continuous head-discharge relation can be constructed from the data as has been done above. This provides a specific advantage over steady-state results because the presence of true orifice and weir flow can be detected by analyzing the exponent on the head in the flow equation. If we take the logarithm of both sides of the simplified orifice equation as done in Eq. [5-1], we can see that this becomes a linear equation of the form $y = mx + b$. The slope is then expected to be the exponent which the head is raised to in the original orifice discharge equation.

$$Q = C_e A_0 \sqrt{2gH}$$  \hspace{1cm} [5-1a]

$$\log Q = \log(C_e A_0 \sqrt{2g}) + 0.5 \log H$$  \hspace{1cm} [5-1b]

Thus if we were to plot the logarithm of the head-discharge relation, the discharge coefficient becomes irrelevant and the exponent can be observed independently by the slope of the curve. This method applies to the dimensionless head-discharge relationship as well utilizing Eq. [B-7] developed in Appendix B as follows

$$\frac{Q}{D^{2.5} g^{0.5}} = \frac{\pi C_e \sqrt{\frac{2}{4}}}{4} \left( \frac{H}{D} \right)^{0.5}$$  \hspace{1cm} [5-2a]

$$\log \frac{Q}{D^{2.5} g^{0.5}} = \log \left( \frac{\pi C_e \sqrt{\frac{2}{4}}}{4} \right) + 0.5 \log \left( \frac{H}{D} \right)$$  \hspace{1cm} [5-2b]

This method of examining the exponent allows a direct comparison to be made between the experimental results and theoretical flow without considering the effective discharge coefficient. If we consider a plot of the logarithmic head-discharge relation for the orifice flow as shown in Fig. 5-10 it is obvious that the exponent is not constant. A linear curve was fit to the portion of the data at higher head exhibiting linear behaviour. The slope of this line was determined to be 0.51 which is very close to the expected value of 0.5.
However, the linear fit is seen to deviate significantly from the experimental data at low head values. This shows once again that the commonly used orifice equation does not fully exemplify true orifice flow at lower head values. Thus the theoretical exponent for the orifice and weir flow were calculated from the numerical solutions to the head-discharge flow equations.

![Logarithmic head-discharge relationship in experimental orifice flow regime (H*>1)](image)

In order to calculate the exponent from the experimental and numerical results the slope of the data curves must be determined. This was done by again utilizing a local regression technique where a 2nd order polynomial was fit to subsets of data surrounding the point of interest. The slope was then taken as the derivative of the fitted curve at that point. A comparison of the resulting theoretical and experimental exponents are shown in Fig.’s 5-11 and 5-12 for the orifice and weir flow regimes respectively.
It is evident from the comparison of theoretical and experimental exponents in the orifice flow regime that true orifice flow is quite closely followed in the unsteady experiments. This infers that unsteady flow does in fact behave as true orifice flow when ignoring the effects captured in the discharge coefficient. Since true orifice flow is so closely followed there is a definitive lack of evidence to support any theory of a transition region extending into the orifice flow regime when considering a descending unsteady flow. However if the water level were to be ascending there is a possibility that weir behaviour could extend into the orifice flow regime.

Figure 5-11: Comparison of theoretical and experimental head exponent for orifice discharge ($H^*>1$)
Based on the results in Fig. 5-12 the same conclusions cannot be made about the results in the weir flow regime. The experimental exponent in the weir flow regime does not mimic the theoretical exponent and thus true weir flow has not been achieved. As suggested in Section 5.1, it is likely that orifice behaviour has extended a small distance into the weir flow regime. This can be seen in Fig. 5-12 by an under predicted exponent in the upper weir range which coincides with the theoretical value slightly above $H^* = 0.8$. However the two solutions do not continue to agree as the head decreases which makes interpretation difficult. The inherent limitations of the apparatus leading to larger errors at the low head values in the weir regime is the likely cause of this discrepancy. The present study was unable to accurately predict the discharge in the weir regime.

5.3 Discharge Coefficient

As mentioned in Section 5.1, the factor relating the theoretical flow solution to the experimental results in Fig. 5-8 is the effective discharge coefficient. This coefficient takes into account all of the assumptions and effects ignored in the development of the
theoretical orifice discharge. The effective discharge coefficient was calculated for both the weir and orifice flow regimes as the ratio between experimental to theoretical flows. The effective discharge coefficient in the weir regime was calculated and plotted in Fig. 5-13 against the results from other studies mentioned in Section 2.2.2.

It is apparent from the discharge coefficient results that the validity of the data in the weir flow regime is questionable. Although the results from previous studies do not conform to a definitive relationship between the head and discharge coefficient, the results of the unsteady experiments are not close to the vicinity of previously accepted approximations. However it can be seen that the results of the steady state experiments are much closer to the results from previous experiments in the upper weir range. The leading hypothesis follows what has been mentioned previously, proposing that orifice flow extended into the weir flow regime for the unsteady data. The level of intimate accuracy required for predictions in this range of head was not attained in this study. Suggestions are made for progress in this area of research in Chapter 6.

Figure 5-13: Comparison of weir discharge coefficient with results from other studies
The effective discharge coefficient in the orifice flow regime is presented in Fig. 5-14. As expected, there is an increase in the discharge coefficient as the head level decreases. The cause of the increase can be attributed to a decrease in the contraction at lower head. As the approach velocities decrease at lower head, there will be less of a jet contraction and correspondingly a higher contraction coefficient. There is currently no theoretical prediction for this phenomenon in the available literature as mentioned in Section 2.1.1. This change in behaviour begins in the region $5 \leq H^* \leq 10$. Anything below $H^*=5$ can be considered to be low head orifice flow with a substantial change in behaviour when compared to high head orifice flow.

A simple curve-fitting method was applied to the discharge coefficient data in the orifice flow regime to predict the discharge coefficient as a function of the normalized head. A close fit is provided by the Root function as follows

$$C_e = 0.597(1.0614)^{\frac{D}{H}}$$

[5-3]
Equation [5-3] is plotted in Fig. 5-14 alongside the experimental data, and has a maximum error of 0.7% when compared with the experimental data. This provides a very simple and usable prediction of the discharge coefficient for fully contracted circular sharp-edged orifices. The nature of this data fit is very appropriate since the discharge coefficient becomes essentially constant at higher head levels. The shape of the curve is characterized by a factor raised to the power of the inverse of the normalized head. The maximum predicted increase in discharge coefficient is 5% at very low head in the orifice flow regime.

This discharge coefficient relates the theoretical flow to the actual flow, thus if Eq. [5-3] is to be used in the standard orifice equation the factor $\beta$ must be applied as well (see Chapter 3). Thus the increase in flow at low head levels when compared to the standard orifice equation will be less than the 5% predicted by Eq. [5-3] alone. Including the factor $\beta$ and using Eq. [5-3] to calculate the effective discharge coefficient, the maximum error in the discharge prediction by the standard orifice equation with a singular discharge coefficient of 0.6 is only 3%. This error is extraordinarily low because the effect of the increasing discharge coefficient and the average head approximation work to balance each other out to a certain degree. The implications of this are discussed in Chapter 6.
Chapter 6

6 Conclusions and Recommendations

A study was completed to investigate the discharge through a circular sharp-edged orifice in the region between partially full weir flow and orifice flow. The intention of the study was to identify a transition region which exhibits neither true orifice nor weir flow behaviour. This investigation was unique since an unsteady experimental technique was utilized to determine the head-discharge relation. The unsteady technique utilized a falling head process where water in a tank was allowed to drain freely from an orifice whilst the dynamic hydraulic head was recorded by a pressure transducer. Three orifices of 3cm, 4.5cm and 6cm diameter were tested for which the head-discharge relationships were directly compared by using dimensional analysis. A separate set of steady state experiments were conducted on the 4.5cm diameter orifice plate in order to verify the unsteady data that was obtained.

The normalization of the data resulted in a very close comparison of the normalized head-discharge results from the 3cm and 4.5cm diameter orifices, with less than 3% error over the entire range of head. However, the results of the dimensionless flow identified a slightly lower normalized discharge in the largest of the three orifices. The reasoning for this decrease was postulated to be a result of more pronounced unsteady flow behaviour causing an increased contraction of the jet. The results for the 6cm diameter orifice were not considered as part of the study due to this discrepancy.

The full range dimensionless head-discharge relation for the unsteady experiments was constructed exclusively from the 3cm and 4.5cm diameter orifices. The unsteady experiment results were compared to the steady state experimental results. Very minor errors of less than 3% were observed between the two methods in the orifice flow regime. However in the weir flow regime there was a consistent over-prediction of discharge in the case of unsteady flow of up to 8%. The unsteadiness may have resulted in orifice behaviour extending down into the upper portion of the weir flow regime. However this
could be a result of the increased magnitude of error in the experimental procedure at low head.

The presence of true orifice and weir flow behaviour was determined by analyzing the fundamental relationship between the normalized head and discharge. The power to which the hydraulic head is raised in the discharge equation was evaluated for the circular weir and orifice as a function of the normalized head. The study found that the discharge behaviour in the orifice flow regime closely resembled that of a true orifice. This result shows that there is a definitive lack of evidence to support theories of existence of transitional flow behaviour in the orifice regime under falling head. Unfortunately, the results from the weir regime were not so conclusive. The discharge behaviour in the weir flow regime could not be linked to true weir flow. Thus no conclusions could be made regarding any point of deviation from true weir flow.

The discharge coefficient across the entire range of head was also examined as part of the study. When compared to the results published by Bos (1989), Balachandar et al. (1991) and Vatankhah (2010) the discharge coefficient for the unsteady experimental flow in the weir regime was high. This result is in agreement with the comparison of steady and unsteady experimental flow, suggesting that unsteadiness is the probable culprit for the increased discharge. However, there are no visible effects of unsteadiness in the data within the orifice flow regime.

The experimentally derived discharge coefficient in the orifice flow regime was unable to be compared to other studies due to the lack of $C_e$ prediction at low head in the available literature. All the discharge coefficient values attained fell within acceptable levels, and a clear increase in the $C_e$ value was observed at low head. This was expected as the reduced velocity at low head provide a reduced contraction of the jet, resulting in an increase of the contraction coefficient and consequently the discharge. A point segregating low and high head orifice flow is proposed to exist in the range $5 \leq H^* \leq 10$. A regression-based curve fitting technique was implemented and used to predict the discharge coefficient as a function of the normalized head. This fit predicts the discharge coefficient for sharp-
edged circular orifices with significantly remote channel sides and bottom to prevent interference with the jet contraction.

6.1 Use of Standard Orifice Equation in Design

The standard orifice equation (Eq. [2-10]) used in textbooks and design manuals includes the approximation of average head acting across the entire orifice. This is because the theoretical solution of discharge through a circular orifice requires difficult integration and can only provide a tabular solution of the head-discharge relationship. As part of this study a short analysis was completed to evaluate the difference between the theoretical orifice flow and the flow resulting from the standard orifice equation. It was found that the standard orifice equation slightly over-predicts the discharge at low head and that the discrepancy between the two methods is a function of the normalized head. A curve fitting method was employed to model this increase. A simple factor was presented to be applied to the standard orifice equation in order to obtain theoretical orifice flow.

The common practice used in textbooks and most design manuals is to consider a constant discharge coefficient of 0.60. This method results in an under-prediction of the discharge at low head due to the increase in $C_e$. When considering both the effect of using an average head approximation and a constant $C_e$ of 0.6, the resulting error when compared to the experimental results obtained in this study is quite low. Maximum errors of only 3% are observed since the over and under-prediction of flow associated with the two approximation techniques at low head work against each other. Although it is not based upon solid theory, the standard orifice equation is considered acceptable to be used at low head based on the results of this study. However if the highest level of accuracy is desired, the methods presented in this study to predict the discharge coefficient and account for the average head approximation should be utilized.

Limitations of the present study should be noted however, when considering upscale and use of results for designing outlets in storm water detention facilities. Other effects not investigated here may play a large role in the discharge from these outlets. These effects
could include an undulated bottom of the orifice, sediment deposition and build-up behind the outlet, algae build-up on the orifice plate increasing roughness and change in water properties such as temperature and viscosity.

6.2 Recommendations for Further Research

Further investigation of the flow through circular orifices under low head is recommended. The primary issue with the present study is the lack of complete accuracy in the weir flow regime. The increase in discharge and proposed brief continuation of orifice behaviour into the weir flow regime could simply be a result of the high levels of random error in the unsteady experimental procedure at low head. A much clearer understanding of the effects of unsteadiness could be drawn by considering the case of ascending water level. Under these circumstances it may be possible that the true weir flow behaviour will exist across the entire region and extend into the orifice flow regime. In this manner there may be a transitional flow regime that only exists under unsteady conditions and is a function of hysteresis.

It is recommended that unsteady experiments be carried out in a significantly larger tank allowing for much larger diameter orifices to be tested. A larger-scale analysis of the comparison of the normalized head-discharge relationship among various orifice diameters would then be possible. Tests on larger orifices would provide higher quality data in the weir region and hopefully allow for convergence to true weir flow at some point. This would provide great insight to the head-discharge relationship for circular sharp-crested weirs and low head circular sharp-edged orifices since no other study in the available literature has attempted this method of characterizing the flow.
7 References


Bos, M. G. 1989. Discharge measurement structures. Wageningen: ILRI.


Appendices

Appendix A: Circular Weir Discharge Coefficients by Bos (1989)

Table A-1: Circular weir discharge coefficients for various head levels, after Bos (1989)

<table>
<thead>
<tr>
<th>$H/D$</th>
<th>$C_e$</th>
<th>$H/D$</th>
<th>$C_e$</th>
</tr>
</thead>
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<td>0.593</td>
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Appendix B: Dimensional Analysis of Weir/Orifice Flow

The first step in any dimensional analysis is to select the independent variables. The discharge through a circular orifice or weir is a function of the orifice diameter $D$ and the upstream head $H$, driven by the effects of gravity $g$ as follows

$$ Q = f(H, D, g) $$  \[B-1\]

The density and viscosity would also be relevant parameters but no such viscous or compressibility effects are considered as part of this study. Since there are only two dimensions present (length and time) there will only be one ($N = n - m = 3 - 2 = 1$) PI group

$$ \Pi_Q = f(\Pi_1) $$  \[B-2\]

By inspection the first PI group will be

$$ \Pi_1 = \frac{H}{D} $$  \[B-3\]

The discharge PI group will be

$$ \Pi_Q = \frac{Q}{D^{2.5} g^{0.5}} $$  \[B-4\]

Note that the upstream head $H$ could be used interchangeably with the diameter in Eq. [B-4]. Subbing these results back into Eq. [B-2] yields

$$ \frac{Q}{D^{2.5} g^{0.5}} = f\left(\frac{H}{D}\right) $$  \[B-5\]

This relationship is readily developed from the standard orifice equation in the following steps starting with Eq. [2-10]

$$ Q_0 = C_o A_0 \sqrt{2gH} $$  \[2-10\]
Considering the cross-sectional area for a circular orifice and rearranging

\[
\frac{Q_a}{D^2 \sqrt{g}} = \pi C_e \sqrt{2H}
\]  

[B-6]

If we then divide both sides of the equation by \( \sqrt{D} \) we obtain

\[
\frac{Q_a}{D^{2.5} \sqrt{g}} = \pi \sqrt{2} C_e \frac{H}{\sqrt{D}}
\]  

[B-7]

Equation [B-7] is equivalent to Eq. [2-10] in all respects, and can be effectively used to compare the flow between orifices of different diameter. For circular weir flow and theoretical orifice flow the function applied to the normalized head will be different. The power which is applied to the normalized head will not be a constant value as it is with the standard orifice equation. This is discussed further in Section 5.2 where the theoretical and experimental powers are compared to establish the presence of true orifice and weir flow.
Appendix C: Non-Steady Experimental Calibration Data

Table C-1: Calibration data for 3cm diameter orifice experiments

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Figure C-1: Sample plotted calibration data for 3cm diameter orifice experiments
### Table C-2: Calibration data for 4.5cm diameter orifice experiments

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### Table C-3: Calibration data for 6cm diameter orifice experiments

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## Appendix D: Steady-State Experimental Measurements

Table D-1: Head measurements and discharge results for steady state experiments

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<th>Calculated Discharge (m³/s)</th>
<th>Head above invert(cm)</th>
<th>Calculated Discharge (m³/s)</th>
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Appendix E: LOESS Smoothing Technique Details

As discussed in Chapter 5 LOESS smoothing technique was used to combine the data from all trials for each diameter. It was also used to combine the normalized results of the 3cm and 4.5cm diameter orifices. This technique uses a least squares regression for small subsets of data in order to predict the local values. A 2nd order polynomial was fit to a subset of data points surrounding each point being considered. The size of the subset of data was determined in such a manner as to minimize the standard deviation of the error. If either too small or too large a number of data points are included in each subset it will cause increased error, if a 2nd order polynomial fit is maintained. For an example a summary of the values calculated for the 4.5cm diameter orifice is shown in Table E-1.

The total number of data points in the 4.5cm orifice data set from all 5 trials is 4348. There is quite obviously a large range of acceptable number of points to include, this would not always be the case with data that is more irregular with higher error.

Table E-1: RMS of error associated with various data subset sizes

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</table>
Curriculum Vitae

<table>
<thead>
<tr>
<th>Name:</th>
<th>Phil Spencer</th>
</tr>
</thead>
</table>
| Post-secondary Education and Degrees: | Western University  
London, Ontario, Canada  
|              | Western University  
London, Ontario, Canada  
2011-Present M.E.Sc. |
| Honours and Awards: | QEII Graduate Scholarship in Science & Technology, 2012  
Stantec Consulting Ltd. Civil Engineering Achievement Award, 2010  
Vander Laan Undergraduate Scholarship in Engineering, 2009  
Lloyd W. Bracewell-Bracewell Engineering Inc. Award, 2009 |
| Related Work Experience | Teaching Assistant  
Western University  
2011-2013 |
|              | NSERC Undergraduate Student Research Award  
Western University  
2009 |